Berkshire-Hathaway Holdings PLC – Analysis of Volatility Models

By

Pasan M. de Silva

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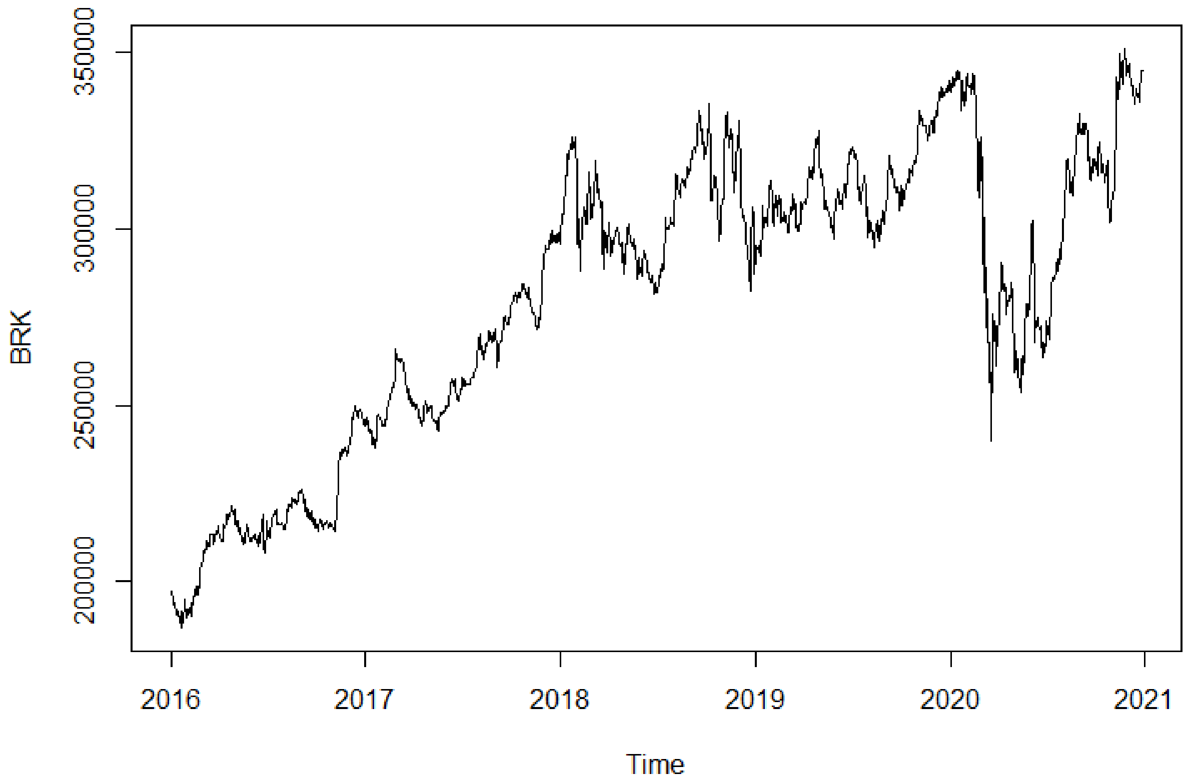
1.0 Introduction to the Analysis

The following report would consider the daily price data of Berkshire-Hathaway PLC (BRK-A), where the shares are traded in New York Stock Exchange, and continue with the construction of univariate ARIMA model into recognize the existence of volatility in the prices, and apply several statistical volatility models, and to recognize which model would be best suited to explore the volatility of the prices (if there is evidence of volatility). The initial task of the analysis would be to load the data into the financial model, where Adjusted close price data of BRK-A from 1st January 2016 – 31st December 2020 will be considered.

1.1 Loading the data to the Model

```
> #install.packages("fGarch")
> #install.packages("rugarch")
> #install.packages("aTSA")
> library("fGarch")
> library("rugarch")
> library("aTSA")
> library(readr)
>
> #Loading Data to a Frame
> data <- read_csv("C:/Users/User/Desktop/BRK-A.csv")

-- Column specification --------------------------------------------------------
cols(
  Date = col_date(format = ""),
  Open = col_double(),
  High = col_double(),
  Low = col_double(),
  Close = col_double(),
  `Adj Close` = col_double(),
  Volume = col_double()
)

> #Assigning Adjusted Close Prices, for the frequency of annual trading days
> BRK <-ts(data$`Adj Close`, frequency = 252,  start = c(2016,01,4))
```

The Adjusted Close price variation of BRK-A for the considered period is illustrated as follows:



With a glimpse of the price variation, it can be seen that over the period of 2016 – 2020 a continuous growth existed, while at the beginning of 2020 a sudden decrease had occurred and again recovered in the beginning of 2021; which is presumably the consequences of COVID-19 pandemic outbreak. However, the stationarity of the model is unclear at this point hence, the Dickey-Fuller test is deployed.

1.2 Dickey-Fuller Test(s) for Stationarity

The results for the Dickey-Fuller test are as follows:

```
> #Dickey-Fuller Test for BRK-A
> adf.test(BRK)
```

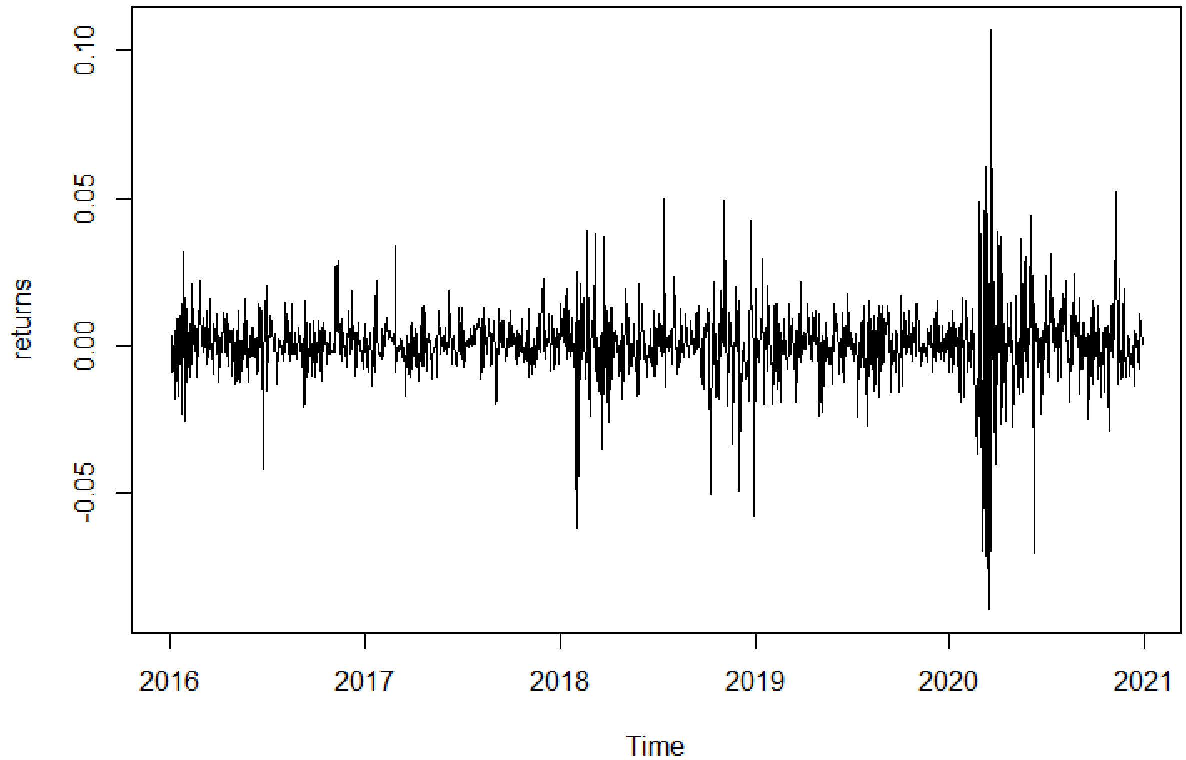
```
Augmented Dickey-Fuller Test
alternative: stationary

Type 1: no drift no trend
     lag   ADF p.value
[1,]   0 0.849   0.888
[2,]   1 1.017   0.916
[3,]   2 0.889   0.900
[4,]   3 0.870   0.894
[5,]   4 0.970   0.910
[6,]   5 0.973   0.911
[7,]   6 1.090   0.926
[8,]   7 0.871   0.895
Type 2: with drift no trend
     lag   ADF p.value
[1,]   0 -1.88   0.376
[2,]   1 -1.72   0.439
[3,]   2 -1.82   0.399
[4,]   3 -1.88   0.375
[5,]   4 -1.84   0.392
[6,]   5 -1.86   0.385
[7,]   6 -1.75   0.427
[8,]   7 -2.01   0.324
Type 3: with drift and trend
     lag   ADF p.value
[1,]   0 -2.95   0.177
[2,]   1 -2.62   0.313
[3,]   2 -2.85   0.216
[4,]   3 -2.94   0.181
[5,]   4 -2.78   0.248
[6,]   5 -2.79   0.241
[7,]   6 -2.57   0.335
[8,]   7 -3.07   0.127
----
Note: in fact, p.value = 0.01 means p.value <= 0.01
```

H_0: The model is not associated with stationary properties

H_1: The model is associated with stationary properties

Based on the above result; in neither the cases of no drift-no trend, drift-no trend nor drift-trend p-values are significant, hence the null hypothesis of the model is not associated with stationary properties cannot be rejected; which suggests that the model is *Non-Stationary*, where σ^2 is not constant. Hence, in order to continue with the analysis, the first differences of the Adjusted price are calculated, and the results are as follows:

```
> #Plotting the First Differences of the Adjusted Prices
> returns <- diff(log(BRK), differences = 1)
> plot.ts(returns)
```



The first differences of the data shows that there are some fluctuations of the adjusted close prices, where in the beginning of 2020, substantial fluctuations could be witnessed; suggesting the presence of high volatility; which signifies that there is at least one error term in the model, which is serially correlated. With this identification, the augmented Dickey-Fuller will be utilized for the first difference returns of Adjusted Closing prices of BRK-A.

```
> #Augmented - Dickey-Fuller Test for BRK-A
> adf.test(returns)
Augmented Dickey-Fuller Test
alternative: stationary

Type 1: no drift no trend
     lag   ADF p.value
[1,]   0 -41.1    0.01
[2,]   1 -24.5    0.01
[3,]   2 -19.4    0.01
[4,]   3 -18.4    0.01
[5,]   4 -16.2    0.01
[6,]   5 -16.2    0.01
[7,]   6 -12.2    0.01
[8,]   7 -12.2    0.01
Type 2: with drift no trend
     lag   ADF p.value
[1,]   0 -41.2    0.01
[2,]   1 -24.5    0.01
[3,]   2 -19.4    0.01
[4,]   3 -18.5    0.01
[5,]   4 -16.3    0.01
[6,]   5 -16.3    0.01
[7,]   6 -12.2    0.01
[8,]   7 -12.3    0.01
Type 3: with drift and trend
     lag   ADF p.value
[1,]   0 -41.2    0.01
[2,]   1 -24.5    0.01
[3,]   2 -19.4    0.01
[4,]   3 -18.5    0.01
[5,]   4 -16.3    0.01
[6,]   5 -16.3    0.01
[7,]   6 -12.2    0.01
[8,]   7 -12.3    0.01
----
Note: in fact, p.value = 0.01 means p.value <= 0.01
```

Here, the Null and Alternative Hypotheses are;

H_0: The model is not associated with stationary properties

H_1: The model is associated with stationary properties

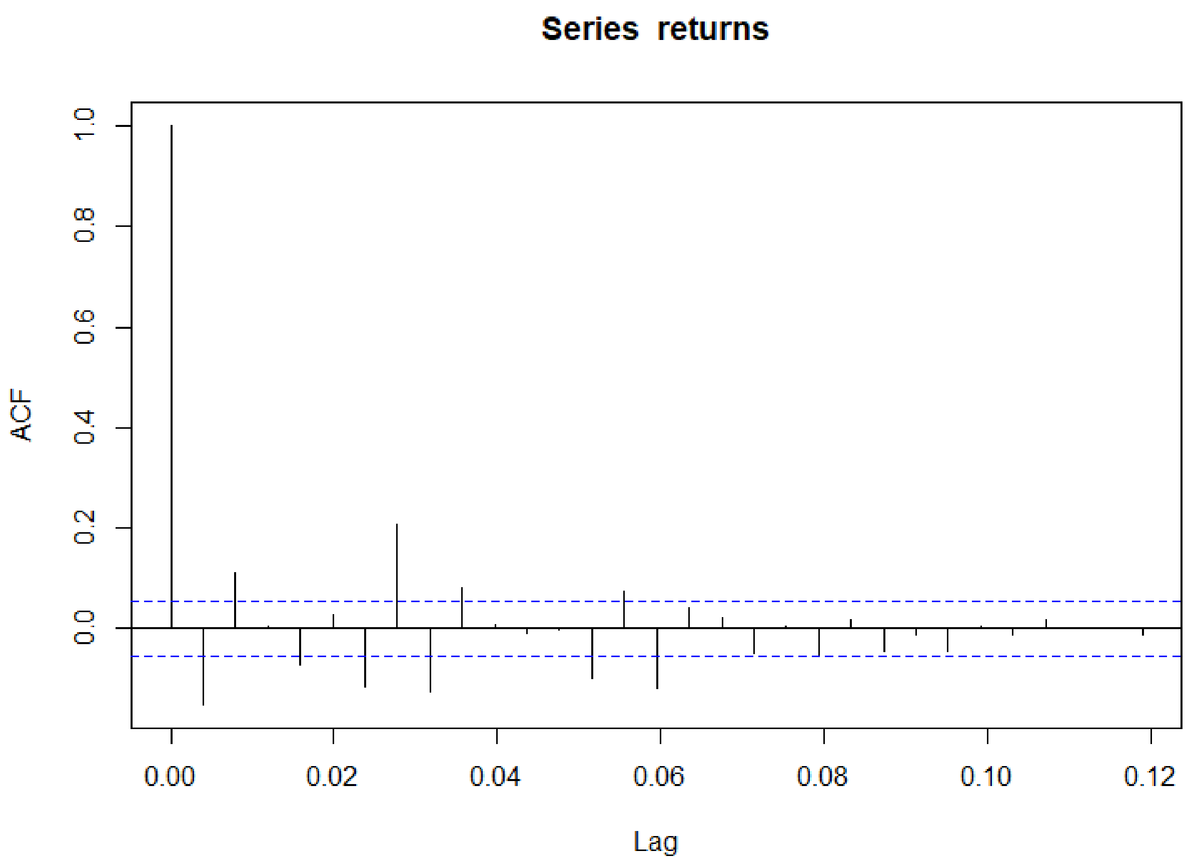
Based on the p-value(s) of each of the cases it can be identified that the first differenced price returns of BRK-A are stationary; as the null hypothesis is rejected at the significance level of 1%.

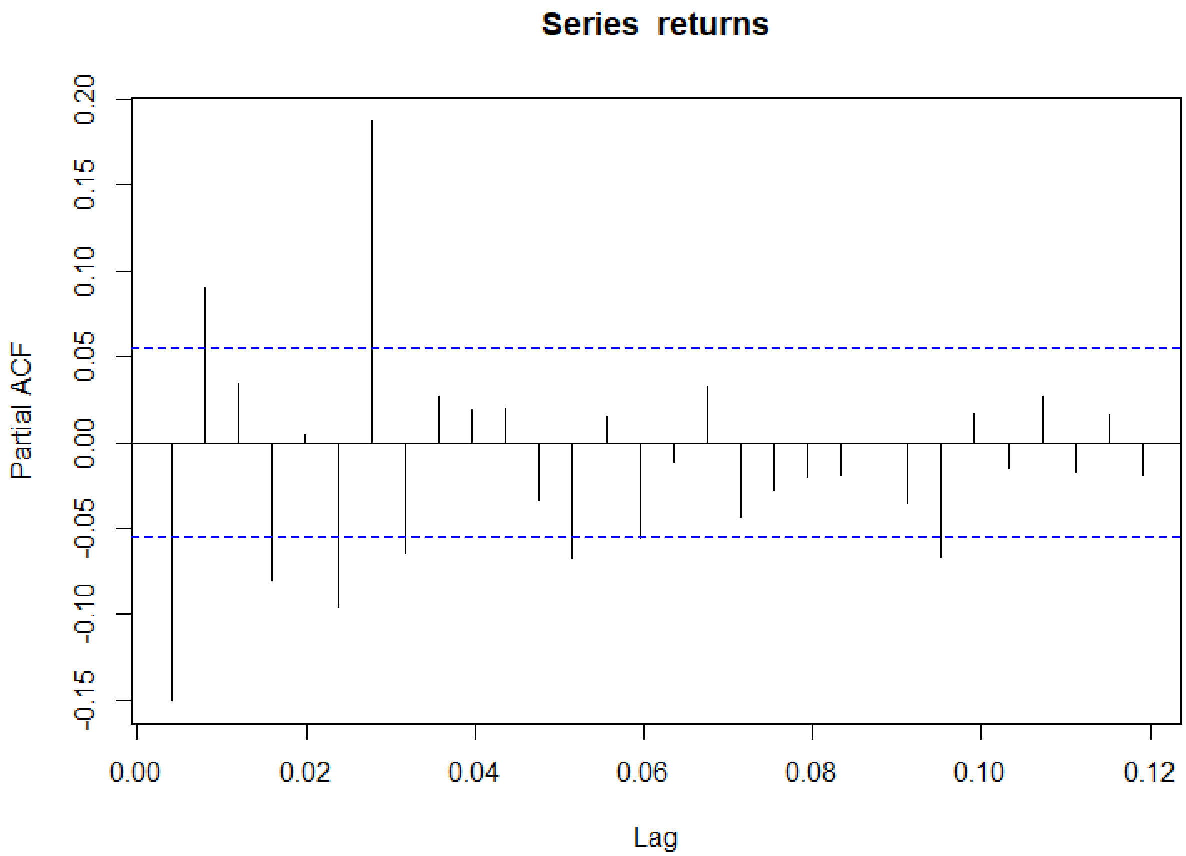
1.3 Selecting P,Q and D values for the ARIMA Model

Prior to analyze the volatility in the model, the ARIMA model needs to be constructed for BRK-A; where initially P, Q, and D values for the model must be identified; which are, the order of the

AR term, order of the MA term and the number of differencing needed for the model to become stationary, respectively. The Partial Autocorrelation Function (PACF) will be utilized to identify the P-Component, while The Autocorrelation Function (ACF) will be utilized to identify the Q-Component; and the results have been illustrated through Correlograms:

```
> #Autocorrelation Formula
> acf(returns)
> #Partial Autocorrelation Formula
> pacf(returns)
>
> #p=2 and q=2 based on ACF and PACF Returns
```





The P-component recognizes the purest form of correlation between the lags and the series, while the Q-component recognize how many MA terms are required to remove any correlation in the stationary series; and based on the lags which are above the 95% confidence level;

- P – 2
- Q – 2

Hence, for BRK-A, for the considered period, the ARIMA application would be (2,0,2); and with the base of this, the next section of the study will be conducted.

2.0 Model Checking for ARCH Effects : Portmanteau-Q and Lagrange Multiplier Tests

From this section onwards, the process of checking for Volatility and selecting the best volatility model to explore the behavior of BRK-A. Initially, the model is to be tested; based on the previously constructed ARIMA model values for the presence of ARCH (Autoregressive Conditional Heteroscedastic) effects; which suggests where Squared residuals/error terms of the constructed time series model exhibit any evidence for the presence of autocorrelation. For this purpose, Portmanteau-Q and Lagrange Multiplier Tests are used, where the Hypotheses are as follows:

H_0: There is no serial correlation between error terms.

H_1: There is serial correlation between error terms; presence of ARCH Effects

```
> #ARCH heteroscedasticity test for residuals
> m1 <- arima(returns, order=c(2,0,2), include.mean=FALSE)
> arch.test(m1, output=TRUE)
ARCH heteroscedasticity test for residuals
alternative: heteroscedastic

Portmanteau-Q test:
     order   PQ p.value
[1,]     4  531       0
[2,]     8  847       0
[3,]    12 1017       0
[4,]    16 1078       0
[5,]    20 1112       0
[6,]    24 1116       0
Lagrange-Multiplier test:
     order    LM  p.value
[1,]     4 907.3 0.00e+00
[2,]     8 297.9 0.00e+00
[3,]    12 194.5 0.00e+00
[4,]    16 141.4 0.00e+00
[5,]    20 110.7 5.88e-15
[6,]    24  90.4 6.09e-10
```

Both in Portmanteau-Q and Lagrange Multiplier Tests are associated with extremely small values which are very close to zero, the Null hypothesis is rejected, which means there is the presence of ARCH effects in the time series model.

2.1 Analysis for ARCH effects

Here, as it was confirmed that the time series data for BRK-A are associated with ARCH effects, the next procedure would be to fit the time series model to explore the ARCH effects, and identify which ARCH model would be able to recognize the time series without the presence of such ARCH effects; or in other words, considered with the conditional mean and variance of the ARCH models, the task would be to recognize the level of ARCH which would be presented with white noise residuals and squared residual terms. The first step in this process would be to consider ARCH(1)/GARCH (1,0); and the results are as follows.

```
> #Auto Regressive Conditional Heteroskedastic Models (ARCH)
> #ARCH(1)
> model1 <- garchFit(~arma(2,2)+garch(1,0), returns, include.mean=FALSE, trace=FALSE)
Warning message:
Using formula(x) is deprecated when x is a character vector of length > 1.
  Consider formula(paste(x, collapse = " ")) instead.
> model1


 Title:
  GARCH Modelling

 Call:
  garchFit(formula = ~arma(2, 2) + garch(1, 0), data = returns,
     include.mean = FALSE, trace = FALSE)

 Mean and Variance Equation:
  data ~ arma(2, 2) + garch(1, 0)
 <environment: 0x000001d5c4839e68>
  [data = returns]

 Conditional Distribution:
  norm

 Coefficient(s):
         ar1          ar2          ma1          ma2        omega       alpha1
  2.0345e-01  -7.3963e-01  -2.8482e-01   7.7475e-01   8.9187e-05   4.7644e-01

 Std. Errors:
  based on Hessian

 Error Analysis:
          Estimate  Std. Error  t value Pr(>|t|)
 ar1      2.034e-01   1.718e-01    1.184   0.2363
 ar2     -7.396e-01   1.436e-01   -5.152 2.58e-07 ***
 ma1     -2.848e-01   1.499e-01   -1.900   0.0574 .
 ma2      7.747e-01   1.431e-01    5.412 6.23e-08 ***
 omega    8.919e-05   5.058e-06   17.632  < 2e-16 ***
 alpha1   4.764e-01   6.556e-02    7.267 3.67e-13 ***
 ---
 Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

 Log Likelihood:
  3849.321    normalized:  3.062308
```

Based on the significance level of the relevant Alpha and Omega values of the ARCH(1) model, they are significant at 1% α, and hence can be recognized that the model is still associated with ARCH effects. The continuation of the model will be associated with Jarque-Bera Test and Shapiro-Wilk Test, where it is tested the model to be associated with skewness and kurtosis matching a normal distribution and tested for the normality of the model, respectively. This is then followed by the more familiar Ljung-Box (LB) Test is utilized; to check the residuals and squared residuals under the ARCH(1) are autocorrelated. The hypotheses in this scenario would be;

H_0: Residuals are associated with White Noise

H_1: There is at least one autocorrelation term in the residuals which in Non-Zero

```
Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  1101.183  0
 Shapiro-Wilk Test  R    W      0.9368463 0
 Ljung-Box Test     R    Q(10)  23.80576  0.008132995
 Ljung-Box Test     R    Q(15)  32.73348  0.005108804
 Ljung-Box Test     R    Q(20)  34.8539   0.02089635
 Ljung-Box Test     R^2  Q(10)  72.33717  1.565403e-11
 Ljung-Box Test     R^2  Q(15)  108.8929  2.220446e-16
 Ljung-Box Test     R^2  Q(20)  159.5104  0
 LM Arch Test       R    TR^2   70.66508  2.404578e-10

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-6.115069 -6.090552 -6.115115 -6.105855
```

At both residuals and Squared Residual levels, the p-values are significant at 5% α, hence can be identified that in ARCH(1) there is at least once autocorrelation residual term in the time series model that is nonzero, rejecting the null hypothesis. Thus, the next order of ARCH model will be considered; ARCH(2).

```
> #ARCH(2)
> model1a <- garchFit(~arma(2,2)+garch(2,0), returns, include.mean=FALSE, trace=FALSE)
Warning message:
Using formula(x) is deprecated when x is a character vector of length > 1.
  Consider formula(paste(x, collapse = " ")) instead.
> summary(model1a)

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~arma(2, 2) + garch(2, 0), data = returns,
    include.mean = FALSE, trace = FALSE)

Mean and Variance Equation:
 data ~ arma(2, 2) + garch(2, 0)
<environment: 0x000001d5c454d388>
 [data = returns]

Conditional Distribution:
 norm

Coefficient(s):
        ar1          ar2          ma1          ma2        omega       alpha1       alpha2
-2.1897e-01  -9.5973e-01   2.0361e-01   9.7639e-01   7.1392e-05   2.6023e-01   2.9107e-01

Std. Errors:
 based on Hessian

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
ar1    -2.190e-01   1.440e-02  -15.201  < 2e-16 ***
ar2    -9.597e-01   1.607e-02  -59.717  < 2e-16 ***
ma1     2.036e-01   9.568e-03   21.280  < 2e-16 ***
ma2     9.764e-01   1.226e-02   79.656  < 2e-16 ***
omega   7.139e-05   4.528e-06   15.766  < 2e-16 ***
alpha1  2.602e-01   4.920e-02    5.289 1.23e-07 ***
alpha2  2.911e-01   4.814e-02    6.047 1.48e-09 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:
 3893.413    normalized:  3.097385


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  1185.865  0
 Shapiro-Wilk Test  R    W      0.9436281 0
 Ljung-Box Test     R    Q(10)  13.59219  0.1924185
 Ljung-Box Test     R    Q(15)  17.15289  0.309807
 Ljung-Box Test     R    Q(20)  20.00005  0.4579265
 Ljung-Box Test     R^2  Q(10)  20.86259  0.02207235
 Ljung-Box Test     R^2  Q(15)  37.75916  0.0009792347
 Ljung-Box Test     R^2  Q(20)  81.65643  2.051117e-09
 LM Arch Test       R    TR^2   30.43908  0.002397154

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-6.183632 -6.155028 -6.183693 -6.172882
```

In this case of ARCH(2), as in ARCH(1), the model is associated with significant Alpha1, Alpha2 and Omega Values; suggesting the presence of ARCH effects; and although the p–values of residuals are non-significant, still the Squared residuals are significant at 5% α level, hence the next order of ARCH; ARCH(3) will be considered.

```
> #ARCH(3)
> model1b <- garchFit(~arma(2,2)+garch(3,0), returns, include.mean=FALSE, trace=FALSE)
Warning message:
Using formula(x) is deprecated when x is a character vector of length > 1.
  Consider formula(paste(x, collapse = " ")) instead.
> summary(model1b)

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~arma(2, 2) + garch(3, 0), data = returns,
    include.mean = FALSE, trace = FALSE)

Mean and Variance Equation:
 data ~ arma(2, 2) + garch(3, 0)
<environment: 0x000001d5c947e980>
 [data = returns]

Conditional Distribution:
 norm

Coefficient(s):
        ar1          ar2          ma1          ma2        omega       alpha1       alpha2       alpha3
-2.2801e-01  -9.5182e-01   2.0912e-01   9.7180e-01   6.2706e-05   2.2994e-01   1.9350e-01   1.7016e-01

Std. Errors:
 based on Hessian

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
ar1    -2.280e-01   1.746e-02  -13.058  < 2e-16 ***
ar2    -9.518e-01   1.841e-02  -51.700  < 2e-16 ***
ma1     2.091e-01   1.054e-02   19.849  < 2e-16 ***
ma2     9.718e-01   1.574e-02   61.726  < 2e-16 ***
omega   6.271e-05   4.916e-06   12.757  < 2e-16 ***
alpha1  2.299e-01   4.511e-02    5.097 3.45e-07 ***
alpha2  1.935e-01   4.321e-02    4.478 7.53e-06 ***
alpha3  1.702e-01   4.858e-02    3.503 0.000461 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:
 3911.438    normalized:  3.111725


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  1138.458  0
 Shapiro-Wilk Test  R    W      0.9483488 0
 Ljung-Box Test     R    Q(10)  11.71122  0.3048485
 Ljung-Box Test     R    Q(15)  16.99728  0.3190268
 Ljung-Box Test     R    Q(20)  19.92132  0.4628608
 Ljung-Box Test     R^2  Q(10)  11.49348  0.320384
 Ljung-Box Test     R^2  Q(15)  25.26573  0.04648654
 Ljung-Box Test     R^2  Q(20)  57.64668  1.636815e-05
 LM Arch Test       R    TR^2   22.09254  0.03649562

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-6.210720 -6.178030 -6.210801 -6.198434
```

In ARCH(3) also, the Alpha1, Alpha2 and Alpha3 values and Omega are significant; presenting evidence for the existence of ARCH effects in the time series model, and here also; though the significance of the residuals is non-significant, Squared residuals are still significant at 5% α level, hence the next order of ARCH model; ARCH(4) is considered.

```
> #ARCH(4)
> model1c <- garchFit(~arma(2,2)+garch(4,0), returns, include.mean=FALSE, trace=FALSE)
Warning message:
Using formula(x) is deprecated when x is a character vector of length > 1.
  Consider formula(paste(x, collapse = " ")) instead.
> summary(model1c)#At ARCH(4) we get squared residuals that are white noise

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~arma(2, 2) + garch(4, 0), data = returns,
    include.mean = FALSE, trace = FALSE)

Mean and Variance Equation:
 data ~ arma(2, 2) + garch(4, 0)
<environment: 0x000001d5c933b5e8>
 [data = returns]

Conditional Distribution:
 norm

Coefficient(s):
        ar1          ar2          ma1          ma2        omega       alpha1       alpha2       alpha3       alpha4
-0.22797644  -0.93407956   0.20153705   0.95592106   0.00005276   0.16766013   0.13058287   0.13654722   0.24121396

Std. Errors:
 based on Hessian

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
ar1    -2.280e-01   2.192e-02  -10.400  < 2e-16 ***
ar2    -9.341e-01   3.377e-02  -27.656  < 2e-16 ***
ma1     2.015e-01   1.555e-02   12.957  < 2e-16 ***
ma2     9.559e-01   2.984e-02   32.035  < 2e-16 ***
omega   5.276e-05   4.788e-06   11.018  < 2e-16 ***
alpha1  1.677e-01   3.962e-02    4.232 2.31e-05 ***
alpha2  1.306e-01   3.778e-02    3.457 0.000547 ***
alpha3  1.365e-01   4.696e-02    2.908 0.003638 **
alpha4  2.412e-01   5.387e-02    4.478 7.55e-06 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:
 3927.892    normalized:  3.124815


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  857.682   0
 Shapiro-Wilk Test  R    W      0.9551304 0
 Ljung-Box Test     R    Q(10)  11.18828  0.3430384
 Ljung-Box Test     R    Q(15)  16.39759  0.356131
 Ljung-Box Test     R    Q(20)  18.6695   0.5433927
 Ljung-Box Test     R^2  Q(10)  9.555923  0.4802777
 Ljung-Box Test     R^2  Q(15)  20.86022  0.1413551
 Ljung-Box Test     R^2  Q(20)  28.88447  0.09005928
 LM Arch Test       R    TR^2   14.71988  0.2571143

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-6.235310 -6.198533 -6.235411 -6.221488
```

At the end of the model ARCH(4), it can be clearly identified that based on the LB test results for the residuals and squared residuals of the model, both are non-significant at 5% α level, hence can be clearly recognized that the ARCH effects are no longer acknowledged in the time series model.

Thus, it is clear that at ARCH(4); i.e., the fourth order of ARCH with zero lags is identified as the best model of recognizing volatility in the time series data of BRK-A.

Until this point of the analysis, the conditional variance of the time series model is specified as a linear function of the past simple variances of the pricing data, and it is to be expanded to allow lagged conditional variances into the model, with the application of GARCH (Generalized Autoregressive conditional heteroscedasticity) model(s).

3.0 Application of GARCH model(s) to the Time Series Analysis

The GARCH model is a better fitted modelling mechanism for the time series data, for the existence of heteroscedasticity and volatility clustering; which also accommodates the price spikes of the model (i.e., Kurtosis) and is better suited to analyze financial time series data. There are number of expansions into the standard GARCH(1,1) model, developed by economists and researchers with the intention of augment the explanation power of the model in different aspects.

The study begins with the standard GARCH(1,1) model; which addresses as a measure for the current term variance of the model as well as a function of the variance of period term and the squared value of the previous model, along with the long term mean variance; where the model tracks mean variance for the entire considered period with a decreasing weight going backwards from the most recent observation of the model which would never reach zero; and this recency of the model makes it much suited to analyze clustered volatility (if presented) of the model. In her also the lags used in the model are denoted with p and q values; which signify:

- P – Number of AR lags imposed
- Q – Number of MA lags specified

In this case, the study recognizes the GARCH(1,1) application; and the results are as follows:

```
> #Generalized ARCH (GARCH) Modeling
> #GARCH (1,1)
> model2 <- garchFit(~arma(2,2)+garch(1,1), returns, include.mean=FALSE, trace=FALSE)
Warning message:
Using formula(x) is deprecated when x is a character vector of length > 1.
  Consider formula(paste(x, collapse = " ")) instead.
> model2 #Both alpha1 and beta1 are statistically significant

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~arma(2, 2) + garch(1, 1), data = returns,
    include.mean = FALSE, trace = FALSE)

Mean and Variance Equation:
 data ~ arma(2, 2) + garch(1, 1)
<environment: 0x000001d5c9444350>
 [data = returns]

Conditional Distribution:
 norm

Coefficient(s):
        ar1          ar2          ma1          ma2        omega       alpha1        beta1
-0.21414687  -0.93874357   0.19541366   0.96503042   0.00000628   0.12837077   0.82718777

Std. Errors:
 based on Hessian

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
ar1    -2.141e-01   1.847e-02  -11.596  < 2e-16 ***
ar2    -9.387e-01   2.272e-02  -41.316  < 2e-16 ***
ma1     1.954e-01   1.370e-02   14.266  < 2e-16 ***
ma2     9.650e-01   1.607e-02   60.048  < 2e-16 ***
omega   6.280e-06   1.524e-06    4.121 3.77e-05 ***
alpha1  1.284e-01   2.183e-02    5.881 4.09e-09 ***
beta1   8.272e-01   2.703e-02   30.607  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:
 3946.954    normalized:  3.139979


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  805.8918  0
 Shapiro-Wilk Test  R    W      0.9603126 0
 Ljung-Box Test     R    Q(10)  11.27273  0.3366687
 Ljung-Box Test     R    Q(15)  15.74924  0.3989102
 Ljung-Box Test     R    Q(20)  17.39215  0.6273823
 Ljung-Box Test     R^2  Q(10)  4.222698  0.9367419
 Ljung-Box Test     R^2  Q(15)  6.91683   0.9599166
 Ljung-Box Test     R^2  Q(20)  13.12827  0.871813
 LM Arch Test       R    TR^2   4.280891  0.9778178

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-6.268821 -6.240217 -6.268882 -6.258070
```

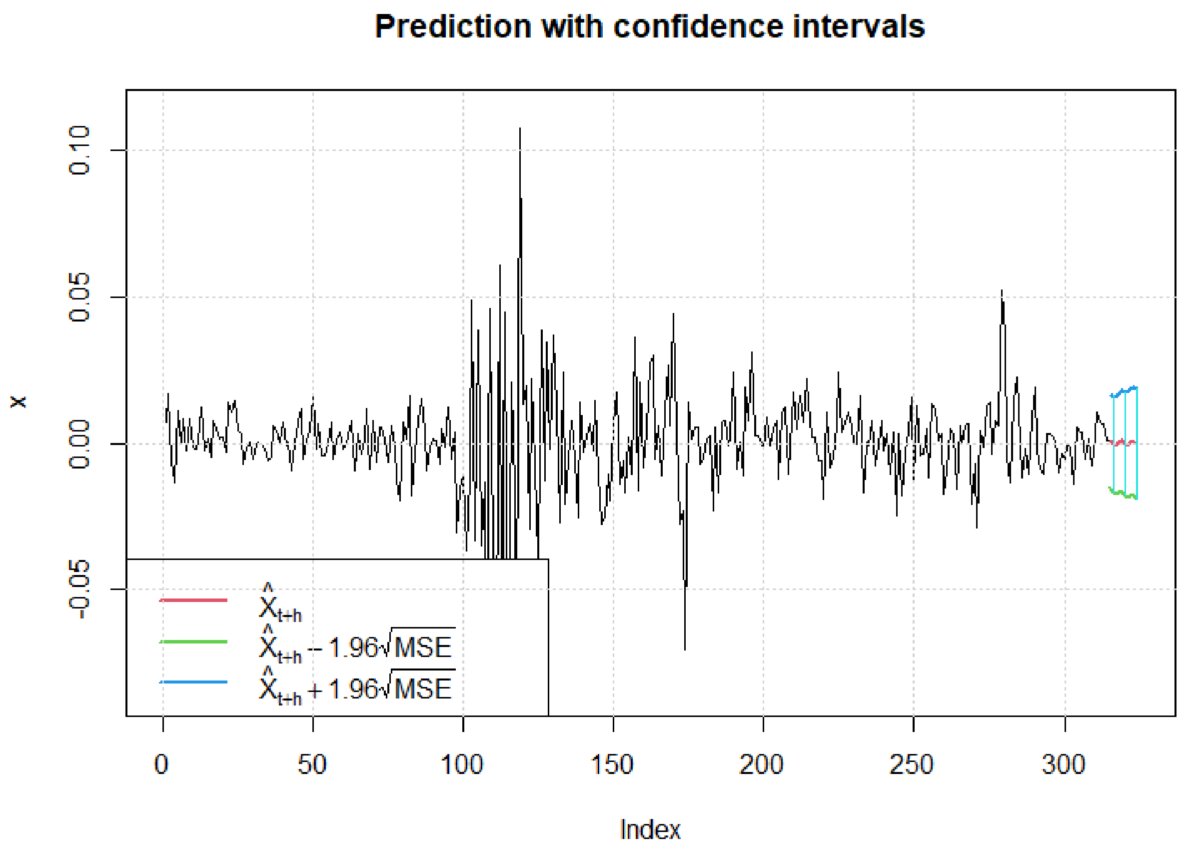
In GARCH(1,1) it can be clearly witnessed that the LB test results for both Residuals and Squared residuals are statistically non-significant at 5% significance level; hence the null hypothesis of the residuals and squared residuals of the model are associated with white noise process is not rejected. At the same time, the efficiency of the application of GARCH is recognized to be significantly higher compared with ARCH modelling; where it has taken fourth order of ARCH to provide residuals (and squared residuals) which are non-significant, while GARCH has done it at its first order; suggesting GARCH(1,1) is better suited to explore the volatility of the time series model. The mere importance in this matter, is that having able to explore the white noise of the residuals and squared residuals of the model minimizes the risk of having negative coefficients, which would ensure that the conditional variance of the model stays positive; making GARCH(1,1) more parsimonious.

As stated in the beginning of the model, there are several expansions into the standard GARCH model, but prior to that the prediction power of the model will be analyzed.

4.0 Prediction with Confidence Intervals

In predicting conditional mean and variance of the time series model; the following prediction function is used; and the results are as follows:

```
> #Prediction with Confidence Intervals (10 periods)
> prediction <- predict(model2, n.ahead=10, plot=TRUE)
```



Here, for 10 time period ahead of the time period, the conditional mean and variances of the time series model for BRK-A are predicted.

5.0 Application of different expansions of GARCH Model

Analysis of the volatility of financial time series models using GARCH family models have been in the practice from the beginning of 1970s, mainly due to the associated uncertainties with the financial data; which are often out of the control scope of the entities. The application of GARCH family models is usually considered to be prominent in these cases of analysis procedures, which are categorized into Symmetric and Asymmetric GARCH models. (GARCH model is symmetric and does not recognize any presence of asymmetry in financial returns, while the concept of asymmetry signifies that unexpected conditions with the financial returns of a specified asset increase the conditional volatility of similar magnitude; which is explored by augmentations to the standard GARCH model, and this section will be analyzing the application of such models to explore the volatility in the time series data of BRK-A.

5.1 Glosten-Jagannathan and Runkle (GJR) GARCH Model

In their study of 1993 *"On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks"* the GJR GARCH Model is presented with not assuming that when a shock would occur in making an effect to the prices; where the sign of that shock would consider to be independent to the response variable; while only becoming a function of the size of the shock; or in other words recognizing the leverage effect into the model. With this prospect; the residual results for GJR-GARCH model of BRK-A's time series data are as follows:

```
> #Glosten-Jaganathan-Runkle GARCH (GJR-GARCH) Model
> gjrgarch1 <- garchFit(formula = ~arma(2,2)+garch(1,1), leverage=T,returns,trace=F,include.mean=F)
Warning message:
Using formula(x) is deprecated when x is a character vector of length > 1.
  Consider formula(paste(x, collapse = " ")) instead.
> summary(gjrgarch1)

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~arma(2, 2) + garch(1, 1), data = returns,
    include.mean = F, leverage = T, trace = F)

Mean and Variance Equation:
 data ~ arma(2, 2) + garch(1, 1)
<environment: 0x000001d5c47fcc58>
 [data = returns]

Conditional Distribution:
 norm

Coefficient(s):
        ar1          ar2          ma1          ma2        omega       alpha1       gamma1        beta1
 1.1350e-01  -4.9858e-01  -1.4230e-01   5.6763e-01   5.5885e-06   9.6262e-02   2.1127e-01   8.5925e-01

Std. Errors:
 based on Hessian

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
ar1     1.135e-01   3.340e-01    0.340  0.73396
ar2    -4.986e-01   2.181e-01   -2.286  0.02226 *
ma1    -1.423e-01   3.182e-01   -0.447  0.65473
ma2     5.676e-01   2.052e-01    2.766  0.00567 **
omega   5.589e-06   1.264e-06    4.421 9.84e-06 ***
alpha1  9.626e-02   1.909e-02    5.042 4.60e-07 ***
gamma1  2.113e-01   8.315e-02    2.541  0.01106 *
beta1   8.593e-01   2.314e-02   37.135  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:
 3946.959    normalized:  3.139983


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  1001.723  0
 Shapiro-Wilk Test  R    W      0.955492  0
 Ljung-Box Test     R    Q(10)  5.242129  0.8744277
 Ljung-Box Test     R    Q(15)  9.996821  0.8199399
 Ljung-Box Test     R    Q(20)  11.51337  0.9318083
 Ljung-Box Test     R^2  Q(10)  5.424734  0.861062
 Ljung-Box Test     R^2  Q(15)  7.619242  0.9381395
 Ljung-Box Test     R^2  Q(20)  12.63443  0.8925125
 LM Arch Test       R    TR^2   5.445772  0.9414113

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-6.267238 -6.234548 -6.267318 -6.254952
```

In the application of GJR-GARCH, it can be seen that alpha1, beta1 and gamma1 are to be statistically significant with their respective p-values, and since gamma is lower than 0 it can recognize that there is presence of leverage effect in the time series model. In the meantime, the LM test results for the residuals and squared residuals of the model are presented with non-significant p-values, where providing evidence that the model is associated with white noise processes.

5.2 Exponential (EGARCH) GARCH Model

Proposed by Nelson in 1991 in "*Conditional Heteroskedasticity in Asset Returns: A New Approach''* the model assumes with the natural log value of the dependent variable to be resulted with a positive value, while capturing the asymmetric effect created on the variance of the model, by positive/negative market shocks. (This includes various market news and fluctuations). In the application of the model to the time series model of EGARCH, the following results can be witnessed.

```
> #Exponential GARCH (EGARCH) model
> egarch1 <- ugarchfit(ugarchspec(mean.model=list(armaOrder=c(2,2),include.mean=F),
+                                variance.model=list(model="eGARCH",garchOrder=c(1,1))),returns)
> egarch1

*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : eGARCH(1,1)
Mean Model      : ARFIMA(2,0,2)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
ar1     0.124728    0.026495   4.7075 0.000003
ar2    -0.534235    0.058617  -9.1140 0.000000
ma1    -0.153107    0.025814  -5.9311 0.000000
ma2     0.599405    0.054122  11.0750 0.000000
omega  -0.353813    0.061938  -5.7123 0.000000
alpha1 -0.042665    0.018386  -2.3205 0.020314
beta1   0.959062    0.006802 141.0057 0.000000
gamma1  0.222264    0.025405   8.7489 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
ar1     0.124728    0.010806  11.5421 0.000000
ar2    -0.534235    0.018556 -28.7904 0.000000
ma1    -0.153107    0.011594 -13.2063 0.000000
ma2     0.599405    0.022732  26.3689 0.000000
omega  -0.353813    0.071805  -4.9274 0.000001
alpha1 -0.042665    0.027297  -1.5630 0.118054
beta1   0.959062    0.007927 120.9902 0.000000
gamma1  0.222264    0.047804   4.6495 0.000003

LogLikelihood : 3945.482

Information Criteria
------------------------------------

Akaike       -6.2649
Bayes        -6.2322
Shibata      -6.2650
Hannan-Quinn -6.2526
```

```
Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.1544  0.6944
Lag[2*(p+q)+(p+q)-1][11]   3.6838  1.0000
Lag[4*(p+q)+(p+q)-1][19]   6.2486  0.9597
d.o.f=4
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                       statistic p-value
Lag[1]                     4.961 0.02592
Lag[2*(p+q)+(p+q)-1][5]    5.029 0.15076
Lag[4*(p+q)+(p+q)-1][9]    5.992 0.29908
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]   0.03191 0.500 2.000  0.8582
ARCH Lag[5]   0.04676 1.440 1.667  0.9955
ARCH Lag[7]   0.98909 2.315 1.543  0.9153

Nyblom stability test
------------------------------------
Joint Statistic:  1.1834
Individual Statistics:
ar1    0.22276
ar2    0.05280
ma1    0.23111
ma2    0.05543
omega  0.12762
alpha1 0.06104
beta1  0.11384
gamma1 0.07991

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.89 2.11 2.59
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value    prob sig
Sign Bias           1.0217 0.30710
Negative Sign Bias  1.7052 0.08841   *
Positive Sign Bias  0.2541 0.79943
Joint Effect        3.2642 0.35265


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     72.75    3.187e-08
2    30     83.88    3.086e-07
3    40     87.95    1.216e-05
4    50    115.04    3.128e-07


Elapsed time : 0.1823449
```

In the EGARCH model, with the application of the LB test, the null hypothesis of the presence of serial correlation in the residuals and squared residuals in the model is rejected, as the P-values are non-significant, hence behaves in white noise processes. Another interesting observation is that remaining ARCH components are also considered under EGARCH model, their residuals are also identified to be statistically non-significant. At the same time, as it is a specific component of the EGARCH model, it can be witnessed that the negative sign bias of the model is significant at 10%, which suggests that the negative market shocks would create more significant impact to the time series model.

The standard GARCH model, along with GJR-GARCH and EGARCH would be considered as the main model mechanisms to consider the volatility in the time series model, nevertheless, some other applications of the GARCH model will also be considered.

5.3 Threshold (T-GARCH) GARCH Model

In his 1994 paper "*Threshold heteroskedastic models"*, Zakoian presented the T-GARCH model, where the T-GARCH model is another expansion of the GARCH model, which is mostly used in regime analysis, where the state of the world, in which the considered time series model data are operated in (for instance, in this case; the NYSE market where BRK-A is operated in) is determined by an observable threshold variable, while the conditional variance is followed with a GARCH process. The application of the model to the time series data of BRK-A are as follows:

```
> #Application of additional enhancements of GARCH Modeling
> #Threshold GARCH (T-GARCH) Model
> fit.tgarch = garchFit(~arma(2,2)+garch(1,1),delta=1,leverage=T,data=returns,trace=F,include.mean=F)
Warning message:
Using formula(x) is deprecated when x is a character vector of length > 1.
  Consider formula(paste(x, collapse = " ")) instead.
> fit.tgarch

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~arma(2, 2) + garch(1, 1), data = returns,
    delta = 1, include.mean = F, leverage = T, trace = F)

Mean and Variance Equation:
 data ~ arma(2, 2) + garch(1, 1)
<environment: 0x000001d5c9695898>
 [data = returns]

Conditional Distribution:
 norm

Coefficient(s):
        ar1          ar2          ma1          ma2        omega       alpha1       gamma1        beta1
 0.01049214  -0.59726020  -0.04207056   0.65316083   0.00053298   0.13092575   0.15322563   0.85705316

Std. Errors:
 based on Hessian

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
ar1     0.0104921   0.1463691    0.072   0.9429
ar2    -0.5972602   0.1221964   -4.888 1.02e-06 ***
ma1    -0.0420706   0.1444886   -0.291   0.7709
ma2     0.6531608   0.1116414    5.851 4.90e-09 ***
omega   0.0005330   0.0001284    4.152 3.29e-05 ***
alpha1  0.1309258   0.0188796    6.935 4.07e-12 ***
gamma1  0.1532256   0.0907522    1.688   0.0913 .
beta1   0.8570532   0.0222758   38.475  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:
 3937.068    normalized:  3.132114


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  956.0989  0
 Shapiro-Wilk Test  R    W      0.952708  0
 Ljung-Box Test     R    Q(10)  7.938094  0.6348839
 Ljung-Box Test     R    Q(15)  13.59686  0.5562977
 Ljung-Box Test     R    Q(20)  15.94657  0.7199331
 Ljung-Box Test     R^2  Q(10)  9.548544  0.480952
 Ljung-Box Test     R^2  Q(15)  14.62012  0.479111
 Ljung-Box Test     R^2  Q(20)  17.42758  0.6250599
 LM Arch Test       R    TR^2   9.164678  0.6888041

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-6.251500 -6.218809 -6.251580 -6.239214
```

Based on the LB test on the behavior of residuals and squared residuals, it can be identified that the null hypothesis of the residuals and squared residuals are following a white noise process cannot be eliminated in the T-GARCH model, as the p-values are non-significant at 5% α level.

5.4 Taylor-Schwert (TS-GARCH) GARCH Model

Taylor in 1986 and Schwert in 1989 proposed that large shocks in the market would constitute significantly extraordinary events and truncate on the conditional variance of time series models by influencing them. Their proposed model is extended to the scale of accounting for the empirical regularity of the constructed time series analysis procedures, by modelling the time varying standard deviation. The application of the model to the time series data of BRK-A are as follows:

```
> #Taylor-Schwert GARCH (TS-GARCH) Model
> fit.tsgarch = garchFit(~arma(2,2)+garch(1,1),delta=1,data=returns,trace=F,include.mean=F)
Warning message:
Using formula(x) is deprecated when x is a character vector of length > 1.
  Consider formula(paste(x, collapse = " ")) instead.
> fit.tsgarch

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~arma(2, 2) + garch(1, 1), data = returns,
    delta = 1, include.mean = F, trace = F)

Mean and Variance Equation:
 data ~ arma(2, 2) + garch(1, 1)
<environment: 0x000001d5d3c1b918>
 [data = returns]

Conditional Distribution:
 norm

Coefficient(s):
       ar1         ar2         ma1         ma2       omega      alpha1       beta1
-0.2118853  -0.9452339   0.1979636   0.9749818   0.0005205   0.1489877   0.8433769

Std. Errors:
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
ar1    -0.2118853   0.0106094  -19.972  < 2e-16 ***
ar2    -0.9452339   0.0120264  -78.596  < 2e-16 ***
ma1     0.1979636   0.0070423   28.111  < 2e-16 ***
ma2     0.9749818   0.0069188  140.917  < 2e-16 ***
omega   0.0005205   0.0001363    3.819 0.000134 ***
alpha1  0.1489877   0.0203036    7.338 2.17e-13 ***
beta1   0.8433769   0.0238283   35.394  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:
 3944.605    normalized:  3.138111


Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  683.9839  0
 Shapiro-Wilk Test  R    W      0.9610209 0
 Ljung-Box Test     R    Q(10)  13.00669  0.2232983
 Ljung-Box Test     R    Q(15)  17.2125   0.3063208
 Ljung-Box Test     R    Q(20)  19.1994   0.5089011
 Ljung-Box Test     R^2  Q(10)  7.426423  0.6846673
 Ljung-Box Test     R^2  Q(15)  11.85692  0.6898252
 Ljung-Box Test     R^2  Q(20)  15.39025  0.7536589
 LM Arch Test       R    TR^2   5.90439   0.9208247

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-6.265084 -6.236480 -6.265146 -6.254334
```

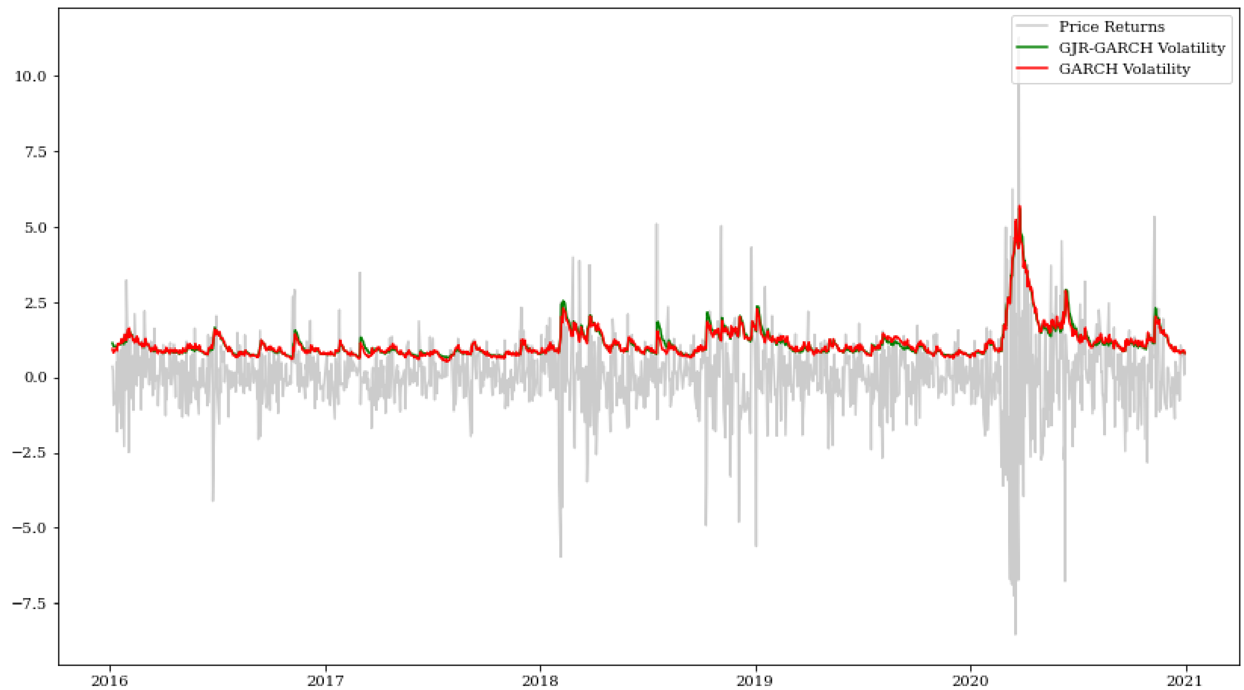
Based on the LB test on the behavior of residuals and squared residuals, it can be identified that the null hypothesis of the residuals and squared residuals are following a white noise process cannot be eliminated in the T-GARCH model, as the p-values are non-significant at 5% α level.

The next chapter of the study will consider a comparison between the constructed models, and attempt to recognize which volatility model is more suited to explore the existing heteroscedasticity in the time series data of BRK-A.

6.0 Comparison between Models

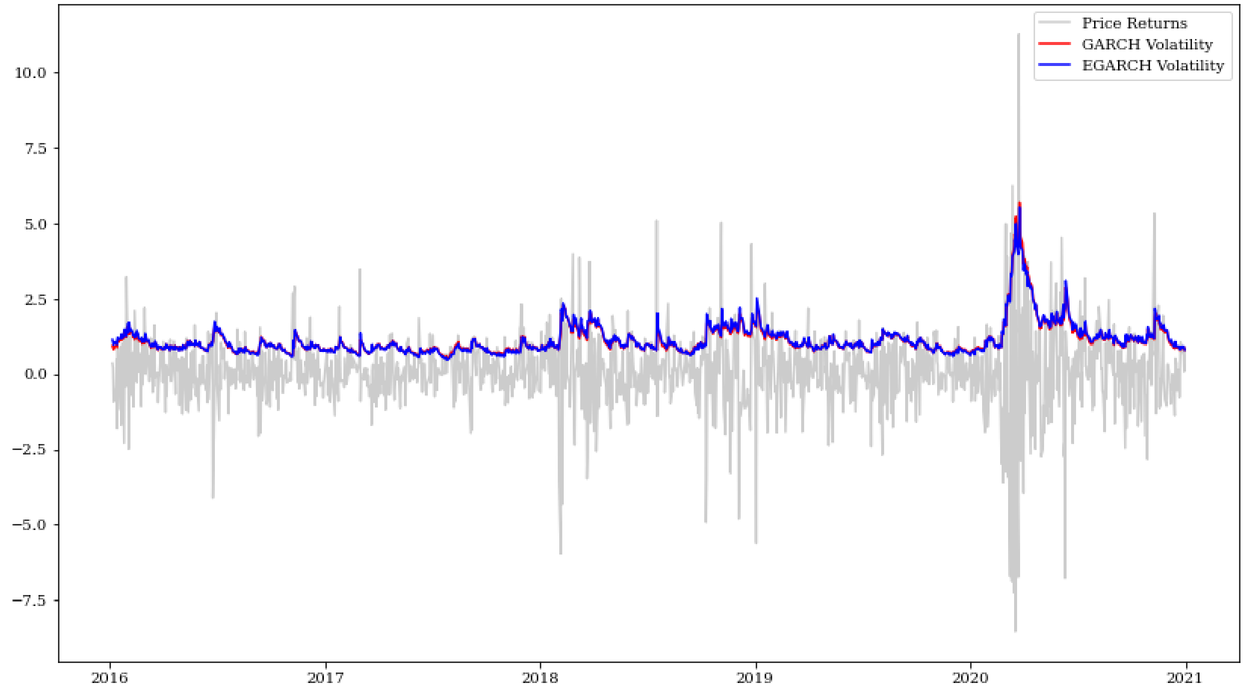
In the comparison procedure of the constructed volatility models, initially it will be attempted to recognize significant variabilities between the ARCH, GARCH(1,1), GJR-GARCH and EGARCH model in a visual representation, and then the more reliable information criterion(s) will be used for all the volatility models to recognize which model is best suited.

6.1 GARCH and GJR-GARCH



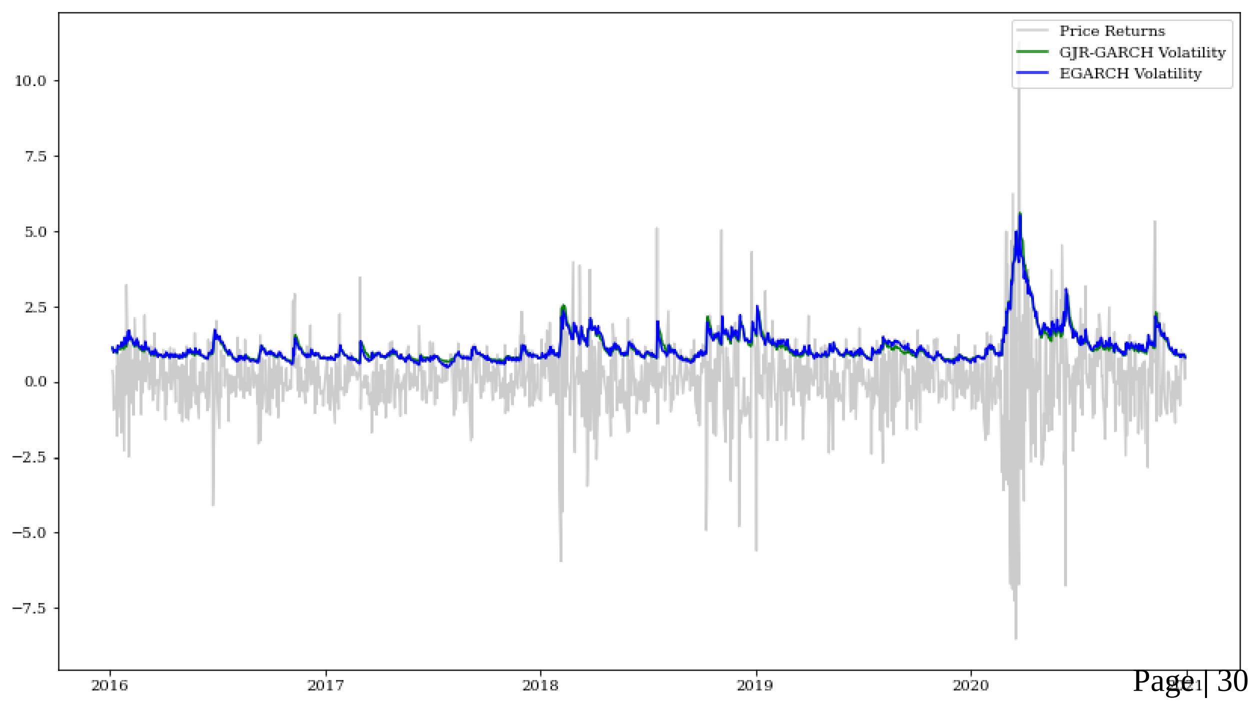
In a visual representation it can be vividly recognized that both the models are mostly follow a same pattern, except for few occurrences of deviation throughout 2018.

6.2 GARCH and EGARCH



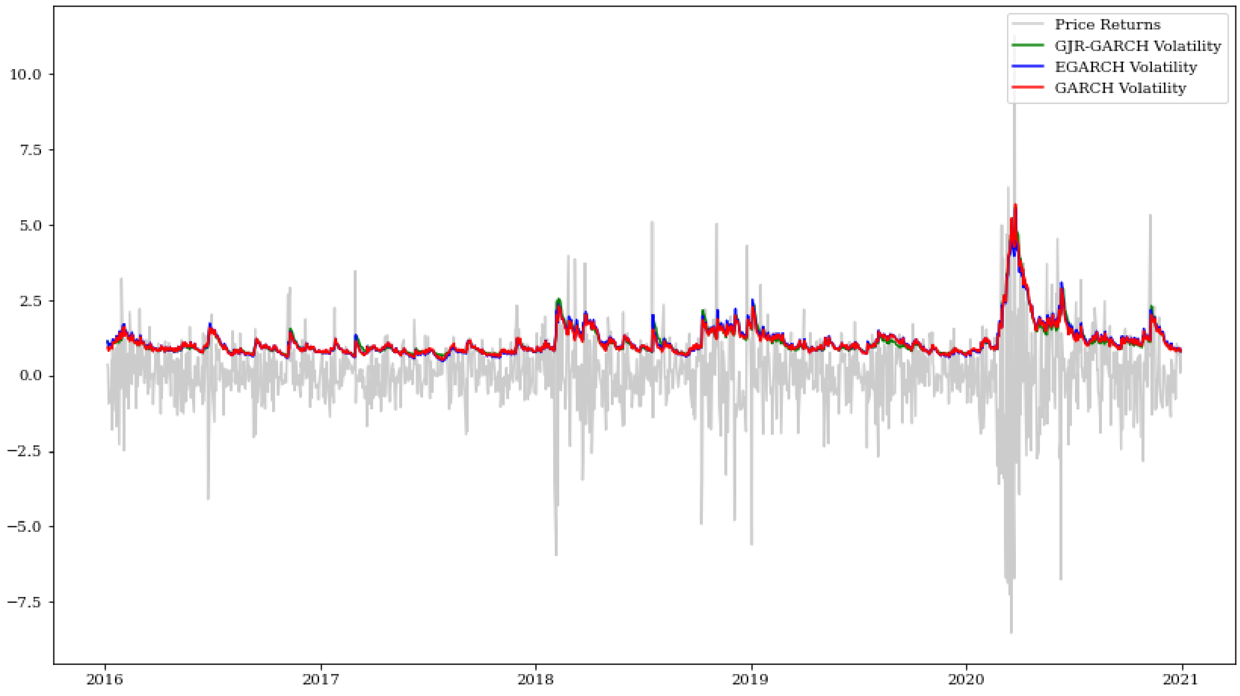
In a visual representation it can be vividly recognized that both the models are mostly follow a same pattern in addressing the volatility of the two models.

6.3 GJR-GARCH and EGARCH



The models GJR-GARCH and EGARCH can recognized to be followed with the closest resemblance of similar pattern in exploring the volatility of the time series data.

6.4 GARCH, GJR-GARCH and EGARCH



While GJR-GARCH and EGARCH are mostly identical (at 99.99% confidence level) GARCH model also closely follow the same pattern, except for few relatively very small fluctuations from the other two models.

The purpose of utilizing the exploring volatility of these models through visual technics was to recognize that both GARCH, GJR-GARCH and EGARCH follows same capability in recognizing the presence of serial autocorrelation in the residuals and squared residuals of the time series model. Thus, the application of Information Criterion is required to recognize the best model to estimate the volatility in the time series model, and the following four statistical Information Criterion will be utilized.

- AIC - Akaike Information Criterion
- BIC – Bayes Information Criterion
- SIC – Shibata Information Criterion

❖ HQIC - Hannan-Quinn Information Criterion

Here all the constructed GARCH models, and the fourth order of ARCH model - ARCH(4) will be compared with each other. From the other constructed ARCH models, ARCH(1), ARCH(2) and ARCH(3) has not been considered for the comparison, as their respective residuals and squared residuals are associated with serial autocorrelation. The comparison results can be summarized as follows:

	ARCH(4)	GARCH(1,1)	GJR-GARCH	EGARCH	T-GARCH	TS-GARCH
AIC	-6.235310	-6.268821	-6.267238	-6.2649	-6.251500	-6.265084
BIC	-6.198533	-6.240217	-6.234548	-6.2322	-6.218809	-6.236480
SIC	-6.235411	-6.268882	-6.267318	-6.2650	-6.251580	-6.265146
HQIC	-6.221488	-6.258070	-6.254952	-6.2526	-6.239214	-6.254334

Based on the results on the Information Criterions, the GARCH(1,1) model is identified to be the best suited to explore the conditional variance of the residuals of the time series data of BRK-A, which are presented with the lowest criterion values. However, in the GJR-GARCH model, the gamma1 was statistically significant, suggesting the presence of leverage factors in the time series data, and hence if it is considered to consider the effects of leverage factors in the data, the second lowest information criterion value presented with the models can be presented, which is interestingly the GJR-GARCH model. Hence, based on the statistical evidence, the most suited model(s) would be the GARCH(1,1) and GJR-GARCH models; based on the consideration of leverage factor to the time series model.

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