

# Precise vehicle localization using fusion of multiple sensors for self-driving

Dr. Peshala, Dr. Ranga, Chanaka, Himasha, Pasan, Ramesh

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## 1 State vector

The state vector to be estimated is

$$\mathbf{x} = \begin{bmatrix} \mathbf{p} \\ \mathbf{v} \\ \mathbf{q} \\ \mathbf{g} \\ \mathbf{n} \\ \mathbf{a}_b \\ \boldsymbol{\omega}_b \end{bmatrix}_{22 \times 1} \quad (1)$$

where

$$\begin{aligned} \mathbf{p} &= (p_x, p_y, p_z) \text{ position relative to the inertial frame} \\ \mathbf{v} &= (v_x, v_y, v_z) \text{ velocity relative to the inertial frame} \\ \mathbf{q} &= (q_w, q_x, q_y, q_z) \text{ quaternion relative to the inertial frame} \\ \mathbf{g} &= (g_x, g_y, g_z) \text{ gravitational vector relative to the inertial frame} \\ \mathbf{n} &= (n_x, n_y, n_z) \text{ magnetic north vector relative to the inertial frame.} \\ \mathbf{a}_b &= (a_{bx}, a_{by}, a_{bz}) \text{ acceleration biases of the IMU} \\ \boldsymbol{\omega}_b &= (\omega_{bx}, \omega_{by}, \omega_{bz}) \text{ angular velocity biases of the IMU} \end{aligned} \quad (2)$$

## 2 Coordinate frames

Following four types of coordinate frames are used.

$$\begin{aligned} \mathcal{F}_{inert} &= \text{the coordinate frame fixed relative to the earth surface} \\ \mathcal{F}_{gnss} &= \text{the coordinate frame in which gnss readings will be provided} \\ \mathcal{F}_{body} &= \text{a coordinate frame fixed relative to the body of the vehicle} \\ \mathcal{F}_{<sensor>} &= \text{coordinate frames attached to sensors in which their readings are provided} \\ (< sensor > = \text{IMU, LiDAR etc.}) \end{aligned} \quad (3)$$

Rotation matrix from  $\mathcal{F}_a$  to  $\mathcal{F}_b$  is denoted as  $R_{b,a}$ . Example:

$$\mathbf{r}_b = R_{b,a} \mathbf{r}_a \quad (4)$$

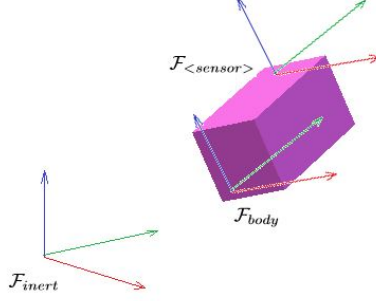


Figure 1: Coordinate frames

### 3 Quaternion operations

Let the unit quaternion  $\mathbf{q}$  be defined as

$$\mathbf{q} = \begin{bmatrix} q_w \\ q_x \\ q_y \\ q_z \end{bmatrix} \quad (5)$$

$$\text{and } \mathbf{q}_v = \begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix} \quad (6)$$

Then some operations on  $\mathbf{q}$  can be defined as given in Figure 2.

### 4 ENU frame as the inertial frame, state vector excluding $\mathbf{g}$ and $\mathbf{n}$

In this approach we use,

$$\begin{aligned} \mathcal{F}_{inert} &= \text{East-North-Up coordinate frame with the origin at a point with precisely known GNSS} \\ &\quad \text{coordinates (considered as an inertial frame)} \\ \mathcal{F}_{gnss} &= \text{WGS84 coordinate system (in latitudes, longitudes and altitude)} \\ \mathcal{F}_{body} &= \text{coordinate frame of the IMU .} \end{aligned} \quad (7)$$

$$\mathbf{p} \otimes \mathbf{q} = \begin{bmatrix} p_w q_w - p_x q_x - p_y q_y - p_z q_z \\ p_w q_x + p_x q_w + p_y q_z - p_z q_y \\ p_w q_y - p_x q_z + p_y q_w + p_z q_x \\ p_w q_z + p_x q_y - p_y q_x + p_z q_w \end{bmatrix}$$

$$[\mathbf{a}]_{\times} \triangleq \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

$$\begin{aligned} \mathbf{R}\{\mathbf{q}\} &= (q_w^2 - \mathbf{q}_v^\top \mathbf{q}_v) \mathbf{I} + 2 \mathbf{q}_v \mathbf{q}_v^\top + 2 q_w [\mathbf{q}_v]_{\times} \\ \mathbf{R}\{-\mathbf{q}\} &= \mathbf{R}\{\mathbf{q}\} && \text{double cover} \\ \mathbf{R}\{1\} &= \mathbf{I} && \text{identity} \\ \mathbf{R}\{\mathbf{q}^*\} &= \mathbf{R}\{\mathbf{q}\}^\top && \text{inverse} \\ \mathbf{R}\{\mathbf{q}_1 \otimes \mathbf{q}_2\} &= \mathbf{R}\{\mathbf{q}_1\} \mathbf{R}\{\mathbf{q}_2\} && \text{composition} \\ \mathbf{R}\{\mathbf{q}^t\} &= \mathbf{R}\{\mathbf{q}\}^t && \text{interpolation} \end{aligned}$$

Let  $\phi = \phi \mathbf{u}$  be a rotation vector representing a rotation of  $\phi$  rad around the axis  $\mathbf{u}$ . Then, the exponential map can be developed using an extension of the *Euler formula* (see (37–42) for a complete development),

$$\boxed{\mathbf{q} \triangleq \text{Exp}(\phi \mathbf{u}) = e^{\phi \mathbf{u}/2} = \cos \frac{\phi}{2} + \mathbf{u} \sin \frac{\phi}{2} = \begin{bmatrix} \cos(\phi/2) \\ \mathbf{u} \sin(\phi/2) \end{bmatrix}}. \quad (101)$$

We call this the *rotation vector to quaternion* conversion formula, and will be denoted in this document by  $\mathbf{q} = \mathbf{q}\{\phi\} \triangleq \text{Exp}(\phi)$ .

Figure 2: Operations related to quaternions

Hence,

$$\mathcal{F}_{IMU} = \mathcal{F}_{body} . \quad (8)$$

Also the state vector is a reduced version of the original one.

$$\mathbf{x} = \begin{bmatrix} \mathbf{p} \\ \mathbf{v} \\ \mathbf{q} \\ \mathbf{a}_b \\ \boldsymbol{\omega}_b \end{bmatrix} \quad (9)$$

$\mathbf{p}$ ,  $\mathbf{v}$  and  $\mathbf{q}$  are expressed in  $\mathcal{F}_{inert}$  frame. However the IMU biases,  $\mathbf{a}_b$  and  $\boldsymbol{\omega}_b$  are expressed in  $\mathcal{F}_{body}$  frame.

#### 4.1 Prediction

State updates can be given as

$$\check{p}_k = \hat{p}_{k-1} + \hat{v}_{k-1} \Delta t + \frac{1}{2} (R_{inert,body} (a_{m_{k-1}} - \hat{a}_{b_{k-1}}) + g) \Delta t^2 \quad (10)$$

$$\check{v}_k = \hat{v}_{k-1} + (R_{inert,body} (a_{m_{k-1}} - \hat{a}_{b_{k-1}}) + g) \Delta t \quad (11)$$

$$\check{q}_k = \hat{q}_{k-1} \otimes q \{ (\omega_{m_{k-1}} - \hat{\omega}_{b_{k-1}}) \Delta t \} \quad (12)$$

$$\check{a}_{b_k} = \hat{a}_{b_{k-1}} \quad (13)$$

$$\check{\omega}_{b_k} = \hat{\omega}_{b_{k-1}} \quad (14)$$

with

$$R_{inert,body} = R \{ \hat{q}_{k-1} \} . \quad (15)$$

Covariance matrix update is

$$\check{\mathbf{P}}_k = F_x \hat{\mathbf{P}}_{k-1} F_x^T + F_i \mathbf{Q}_i F_i^T \text{ with} \quad (16)$$

$$F_x = \left. \frac{\partial f}{\partial \delta x} \right|_{x_{k-1}, u_{k-1}} \quad (17)$$

$$= \begin{bmatrix} \mathbf{I} & \mathbf{I}\Delta t & 0 & 0 & 0 \\ 0 & \mathbf{I} & -[R_{inert,body}(a_{m_{k-1}} - \hat{a}_{b_{k-1}})]_{\times} \Delta t & -R_{inert,body} \Delta t & 0 \\ 0 & 0 & \mathbf{I} & 0 & -R_{inert,body} \Delta t \\ 0 & 0 & 0 & \mathbf{I} & 0 \\ 0 & 0 & 0 & 0 & \mathbf{I} \end{bmatrix}_{15 \times 15} \quad (18)$$

$$F_i = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \mathbf{I} & 0 & 0 & 0 \\ 0 & \mathbf{I} & 0 & 0 \\ 0 & 0 & \mathbf{I} & 0 \\ 0 & 0 & 0 & \mathbf{I} \end{bmatrix}_{15 \times 12} \quad (19)$$

$$\mathbf{Q}_i = \begin{bmatrix} \sigma_{a_n}^2 \Delta t^2 \mathbf{I} & 0 & 0 & 0 \\ 0 & \sigma_{\omega_n}^2 \Delta t^2 \mathbf{I} & 0 & 0 \\ 0 & 0 & \sigma_{a_\omega}^2 \Delta t \mathbf{I} & 0 \\ 0 & 0 & 0 & \sigma_{\omega_\omega}^2 \Delta t \mathbf{I} \end{bmatrix}_{12 \times 12} \quad (20)$$

$$\sigma_{a_n}^2 = \text{white noise variance of IMU acceleration measurement} \quad (21)$$

$$\sigma_{\omega_n}^2 = \text{white noise variance of IMU angular velocity measurement} \quad (22)$$

$$\sigma_{a_\omega}^2 = \text{velocity random walk variance of IMU} \quad (23)$$

$$\sigma_{\omega_\omega}^2 = \text{angular random walk variance of IMU} \quad (24)$$

## 4.2 Correction

Steps for correction upon receiving a measurement can be given as follows.

$$K_k = \check{\mathbf{P}}_k H^T (H \check{\mathbf{P}}_k H^T + V)^{-1} \quad (25)$$

$$\hat{\mathbf{P}}_k = (\mathbf{I} - K_k H) \check{\mathbf{P}}_k \quad (26)$$

Here,

$$H = H_x X_{\delta x} \text{ and} \quad (27)$$

$$X_{\delta x} = \begin{bmatrix} \mathbf{I} & 0 & 0 & 0 & 0 \\ 0 & \mathbf{I} & 0 & 0 & 0 \\ 0 & 0 & Q_{\delta\theta} & 0 & 0 \\ 0 & 0 & 0 & \mathbf{I} & 0 \\ 0 & 0 & 0 & 0 & \mathbf{I} \end{bmatrix}_{16 \times 15} \quad (28)$$

$$Q_{\delta\theta} = \frac{1}{2} \begin{bmatrix} -q_x & -q_y & -q_z \\ q_w & q_z & -q_y \\ -q_z & q_w & q_x \\ q_y & -q_x & q_w \end{bmatrix}_{4 \times 3} \quad (29)$$

For a GNSS measurement where position is measured relative to  $\mathcal{F}_{inert}$  in all three directions,

$$H_x = [\mathbf{I} \ 0 \ 0 \ 0 \ 0]_{3 \times 16} \text{ and} \quad (30)$$

$$\hat{\delta x}_k = K_k (y - \check{p}_k) \quad (31)$$

Steps for injecting the error to the state are as follows.

$$\hat{p}_k = \check{p}_k + \delta \hat{p}_k \quad (32)$$

$$\hat{v}_k = \check{v}_k + \delta \hat{v}_k \quad (33)$$

$$\hat{q}_k = q \left\{ \delta \hat{\theta}_k \right\} \otimes \check{q}_k \quad (34)$$

$$\hat{a}_{bk} = \check{a}_{bk} + \delta \hat{a}_{bk} \quad (35)$$

$$\hat{\omega}_{bk} = \check{\omega}_{bk} + \delta \hat{\omega}_{bk} \quad (36)$$

$$(37)$$

Finally the error state is reset to zero ( $\delta \mathbf{x} \leftarrow 0$ ).

### 4.3 Initialization

$$\delta \mathbf{x} = \mathbf{0} \quad (38)$$

$$p_0 = \text{GNSS position of the starting point expressed in } \mathcal{F}_{inert} \text{ frame.} \quad (39)$$

$$v_0 = 0 \quad (40)$$

$$q_0 = \text{Orientation determined by the gravity and magnetic North vectors.} \quad (41)$$

$$a_{b_0}, \omega_{b_0} = \text{given in datasheet of the IMU/ previous estimate.} \quad (42)$$

$$g = (0, 0, -9.8) \quad (43)$$

#### 4.4 Orientation from magnetic field vector

Let the normalized magnetic field and gravitational vectors in  $\mathcal{F}_{inert}$  frame be  $m_i$  and  $g_i$ . Then

$$m_i = [0 \quad -1 \quad 0]^T \text{ and} \quad (44)$$

$$g_i = [0 \quad 0 \quad -1]^T . \quad (45)$$

Let the unit quaternion corresponding to the orientation of the vehicle be  $q = [w \quad x \quad y \quad z]^T$  and  $R_{inert,body} = R_{inert,body} \{q\}$  (Rotation matrix corresponding to  $q$ ). If the measured, normalized magnetic field and acceleration vectors in  $\mathcal{F}_{IMU} = \mathcal{F}_{body}$  are given by  $m_s = [m_x \quad m_y \quad m_z]^T$  and  $f_s = [f_x \quad f_y \quad f_z]^T$ , when the vehicle is at rest we can have the following set of equations.

$$R_{inert,body} \cdot m_s + m_i = 0 \quad (46)$$

$$R_{inert,body} \cdot f_s + g_i = 0 \quad (47)$$

$$w^2 + x^2 + y^2 + z^2 - 1 = 0 \quad (48)$$

The quaternion components are approximated by fitting a least-square-error line for the above set of over-determined non-linear equations. Furthermore, since the accelerometer reading is much reliable than the magnetometer reading, the orthogonality of the gravity and magnetic field vectors can be utilized apply a correction to the magnetic field vector. In the following approach, we expect to remove any magnetic field components in the direction parallel to that of gravity. Let  $m_s^{raw}$  and  $f_s^{raw}$  be the raw magnetic field and acceleration vectors measured by the sensor at rest and  $m_s$  be the corrected, normalized magnetic field vector.

$$m'_s = m_s^{raw} - (m_s^{raw} \cdot f_s^{raw}) \frac{f_s^{raw}}{\|f_s^{raw}\|^2} \quad (49)$$

$$m_s = \frac{m'_s}{\|m'_s\|} \quad (50)$$

### 5 Initial body frame as inertial frame, full state vector

In this approach we use,

$\mathcal{F}_{inert}$  = a coordinate frame with axes parallel to those of the IMU frame at the initialization, and the origin at a point with precisely known GNSS coordinates (considered as an inertial frame)

$\mathcal{F}_{gnss}$  = WGS84 coordinate system

$\mathcal{F}_{body}$  = coordinate frame of the IMU . (51)

Hence,

$$\mathcal{F}_{IMU} = \mathcal{F}_{body} . \quad (52)$$

$\mathbf{p}$ ,  $\mathbf{v}$ ,  $\mathbf{q}$ ,  $\mathbf{g}$  and  $\mathbf{n}$  are expressed in  $\mathcal{F}_{inert}$  frame. However the IMU biases,  $\mathbf{a}_b$  and  $\boldsymbol{\omega}_b$  are expressed in  $\mathcal{F}_{body}$  frame.

State vectors are initialized in the following manner.

$$\mathbf{p} = \text{initial GNSS reading converted to } \mathcal{F}_{inert} = C_{inert,gnss}(\mathbf{y} \ominus \mathbf{y}_0) \quad (53)$$

$$\mathbf{v} = (0, 0, 0) \quad (54)$$

$$\mathbf{q} = (0, 0, 0) \quad (55)$$

$$\mathbf{g} = \text{initial true acceleration due to gravity measured by the IMU} \quad (56)$$

$$\mathbf{n} = \text{North vector calculated using the initial reading of the magnetometer} \quad (57)$$

$$\mathbf{a}_b, \omega_b = \text{from data given by the manufacturer/ previous estimate, if available} \quad (58)$$

Here,  $\mathbf{y}$  is the GNSS reading at the starting point and  $\mathbf{y}_0$  is the GNSS coordinate of the origin of  $\mathcal{F}_{inert}$ . Note that  $C_{inert,gnss}$  (and therefore  $C_{gnss,inert}$ ) depends on  $\mathbf{g}$  and  $\mathbf{n}$ .

## 6 GNSS measurement auto-regressive model

As the GNSS measurement error (with respect to the ground truth) shows non-zero higher order correlations, it is modeled as a first order auto-regressive process [Zui Tao. Autonomous road vehicles localization using satellites, lane markings and vision, 2016]. Let the GNSS measurement, ground truth and the gnss error at time  $k$  be  $x_k, \hat{x}_k$  and  $e_k$  respectively. Then,

$$e_{k+1} = a_1 e_k + \omega_{k+1} \quad (59)$$

$$(x_{k+1} - \hat{x}_{k+1}) = a_1 (x_k - \hat{x}_k) + \omega_{k+1} \quad (60)$$

$$\hat{x}_{k+1} = x_{k+1} - a_1 (x_k - \hat{x}_k) - \omega_{k+1} \quad (61)$$

Here,  $\omega_k$  is a gaussian white noise term with variance equal to that of the receiver. Although, in the above mentioned paper, they are using a separate states for estimating the GNSS error terms, we intend to simply calculate the parameters ( $a_1$ ) using statistical methods and use the above result to generate a corrected version (with zero correlation) of the measurement.

## 7 NCLT dataset

### 7.1 Sensors

#### IMU: Microstrain 3DM-GX3-45

Accelerometer:

$$\begin{array}{ll} \text{range :} & \pm 50 \text{ } ms^{-2} \\ \text{data rate :} & 50 \text{ } Hz \end{array}$$

### 7.2 Stationary times periods

#### Sequence 2013-01-10

from 1357847238.335046s to 1357847253.355280s, duration: 15.020234s

from 1357847328.254951s to 1357847363.314949s, duration: 35.059998s

from 1357847766.775407s to 1357847771.775615s, duration: 5s

from 1357847983.373600s to 1357847988.375297s, duration: 5s



from 1357847999.505274s to 1357848004.505470s, duration: 5s  
from 1357848259.975057s to 1357848260.025456s, duration: 5s