Precise vehicle localization using fusion of multiple sensors for self-driving

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1 State vector

The state vector to be estimated is

$$\mathbf{x} = \begin{bmatrix} \mathbf{p} \\ \mathbf{v} \\ \mathbf{q} \\ \mathbf{g} \\ \mathbf{n} \\ \boldsymbol{a_b} \\ \boldsymbol{\omega_b} \end{bmatrix}_{22\times 1} \tag{1}$$

where

$$\begin{aligned}
\mathbf{p} &= (p_x, p_y, p_z) \text{ position relative to the inertial frame} \\
\mathbf{v} &= (v_x, v_y, v_z) \text{ velocity relative to the inertial frame} \\
\mathbf{q} &= (q_w, q_x, q_y, q_z) \text{ quaternion relative to the inertial frame} \\
\mathbf{g} &= (g_x, g_y, g_z) \text{ gravitational vector relative to the inertial frame} \\
\mathbf{n} &= (n_x, n_y, n_z) \text{ magnetic north vector relative to the inertial frame.} \\
\boldsymbol{a_b} &= (a_{bx}, a_{by}, a_{bz}) \text{ acceleration biases of the IMU} \\
\boldsymbol{\omega_b} &= (\omega_{bx}, \omega_{by}, \omega_{bz}) \text{ angular velocity biases of the IMU}
\end{aligned} \tag{2}$$

2 Coordinate frames

Following four types of coordinate frames are used.

$$\begin{aligned}
&\mathcal{F}_{inert} = \text{ the coordinate frame fixed relative to the earth surface} \\
&\mathcal{F}_{gnss} = \text{ the coordinate frame in which gnss readings will be provided} \\
&\mathcal{F}_{body} = \text{ a coordinate frame fixed relative to the body of the vehicle} \\
&\mathcal{F}_{<sensor>} = \text{ coordinate frames attached to sensors in which their readings are provided} \\
&(< sensor > = \text{IMU, LiDAR etc.})
\end{aligned} \tag{3}$$

Rotation matrix from $\mathcal{F}_a$ to $\mathcal{F}_b$ is denoted as $R_{b,a}$. Example:

$$\boldsymbol{r_b} = R_{b,a}\boldsymbol{r_a} \tag{4}$$

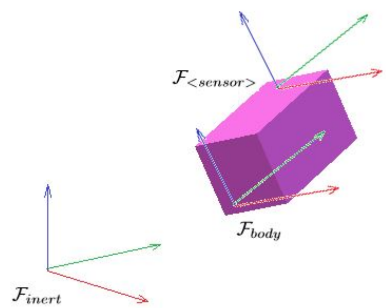

Figure 1: Coordinate frames

3 Quaternion operations

Let the unit quaternion $\mathbf{q}$ be defined as

$$\mathbf{q} = \begin{bmatrix} q_w \\ q_x \\ q_y \\ q_z \end{bmatrix} \tag{5}$$

$$\text{and } \mathbf{q}_v = \begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix} \tag{6}$$

Then some operations on $\mathbf{q}$ can be defined as given in Figure 2.

4 ENU frame as the inertial frame, state vector excluding g and n

In this approach we use,

$$\begin{aligned} \mathcal{F}_{inert} &= \text{East-North-Up coordinate frame with the origin at a point with precisely known GNSS} \\ &\quad\ \text{coordinates (considered as an inertial frame)} \\ \mathcal{F}_{gnss} &= \text{ WGS84 coordinate system (in lattitudes, longitudes and altitude)} \\ \mathcal{F}_{body} &= \text{ coordinate frame of the IMU .} \end{aligned} \tag{7}$$

$$\mathbf{p} \otimes \mathbf{q} = \begin{bmatrix} p_w q_w - p_x q_x - p_y q_y - p_z q_z \\ p_w q_x + p_x q_w + p_y q_z - p_z q_y \\ p_w q_y - p_x q_z + p_y q_w + p_z q_x \\ p_w q_z + p_x q_y - p_y q_x + p_z q_w \end{bmatrix}$$

$$[\mathbf{a}]_\times \triangleq \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

$$\begin{aligned}
\mathbf{R}\{\mathbf{q}\} &= (q_w^2 - \mathbf{q}_v^\top \mathbf{q}_v)\,\mathbf{I} + 2\,\mathbf{q}_v \mathbf{q}_v^\top + 2\,q_w\,[\mathbf{q}_v]_\times \\
\mathbf{R}\{-\mathbf{q}\} &= \mathbf{R}\{\mathbf{q}\} && \text{double cover} \\
\mathbf{R}\{1\} &= \mathbf{I} && \text{identity} \\
\mathbf{R}\{\mathbf{q}^*\} &= \mathbf{R}\{\mathbf{q}\}^\top && \text{inverse} \\
\mathbf{R}\{\mathbf{q}_1 \otimes \mathbf{q}_2\} &= \mathbf{R}\{\mathbf{q}_1\}\,\mathbf{R}\{\mathbf{q}_2\} && \text{composition} \\
\mathbf{R}\{\mathbf{q}^t\} &= \mathbf{R}\{\mathbf{q}\}^t && \text{interpolation}
\end{aligned}$$

Let $\boldsymbol{\phi} = \phi\mathbf{u}$ be a rotation vector representing a rotation of ϕ rad around the axis $\mathbf{u}$. Then, the exponential map can be developed using an extension of the *Euler formula* (see (37–42) for a complete development),

$$\boxed{\mathbf{q} \triangleq \mathrm{Exp}(\phi\mathbf{u}) = e^{\phi\mathbf{u}/2} = \cos\frac{\phi}{2} + \mathbf{u}\sin\frac{\phi}{2} = \begin{bmatrix} \cos(\phi/2) \\ \mathbf{u}\sin(\phi/2) \end{bmatrix}}\,. \tag{101}$$

We call this the *rotation vector to quaternion* conversion formula, and will be denoted in this document by $\mathbf{q} = \mathbf{q}\{\boldsymbol{\phi}\} \triangleq \mathrm{Exp}(\boldsymbol{\phi})$.

Figure 2: Operations related to quaternions

Hence,

$$\mathcal{F}_{IMU} = \mathcal{F}_{body} \ . \tag{8}$$

Also the state vector is a reduced version of the original one.

$$\mathbf{x} = \begin{bmatrix} \mathbf{p} \\ \mathbf{v} \\ \mathbf{q} \\ \boldsymbol{a_b} \\ \boldsymbol{\omega_b} \end{bmatrix} \tag{9}$$

$\mathbf{p}$, $\mathbf{v}$ and $\mathbf{q}$ are expressed in $\mathcal{F}_{inert}$ frame. However the IMU biases, $\boldsymbol{a_b}$ and $\boldsymbol{\omega_b}$ are expressed in $\mathcal{F}_{body}$ frame.

4.1 Prediction

State updates can be given as

$$\check{p}_k = \hat{p}_{k-1} + \hat{v}_{k-1}\Delta t + \frac{1}{2}\left(R_{inert,body}\left(a_{m_{k-1}} - \hat{a}_{b_{k-1}}\right) + g\right)\Delta t^2 \tag{10}$$

$$\check{v}_k = \hat{v}_{k-1} + \left(R_{inert,body}\left(a_{m_{k-1}} - \hat{a}_{b_{k-1}}\right) + g\right)\Delta t \tag{11}$$

$$\check{q}_k = \hat{q}_{k-1} \otimes q\left\{\left(\omega_{m_{k-1}} - \hat{\omega}_{b_{k-1}}\right)\Delta t\right\} \tag{12}$$

$$\check{a}_{b_k} = \hat{a}_{b_{k-1}} \tag{13}$$

$$\check{\omega}_{b_k} = \hat{\omega}_{b_{k-1}} \tag{14}$$

with

$$R_{inert,body} = R\left\{\hat{q}_{k-1}\right\} \ . \tag{15}$$

Covariance matrix update is

$$\check{\mathbf{P}}_k = F_x \hat{\mathbf{P}}_{k-1} F_x^T + F_i \mathbf{Q}_i F_i^T \text{ with} \tag{16}$$

$$F_x = \left.\frac{\partial f}{\partial \delta x}\right|_{x_{k-1}, u_{k-1}} \tag{17}$$

$$= \begin{bmatrix} \mathbf{I} & \mathbf{I}\Delta t & 0 & 0 & 0 \\ 0 & \mathbf{I} & -\left[R_{inert,body}\left(a_{m_{k-1}} - \hat{a}_{b_{k-1}}\right)\right]_\times \Delta t & -R_{inert,body}\Delta t & 0 \\ 0 & 0 & \mathbf{I} & 0 & -R_{inert,body}\Delta t \\ 0 & 0 & 0 & \mathbf{I} & 0 \\ 0 & 0 & 0 & 0 & \mathbf{I} \end{bmatrix}_{15\times 15} \tag{18}$$

$$F_i = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \mathbf{I} & 0 & 0 & 0 \\ 0 & \mathbf{I} & 0 & 0 \\ 0 & 0 & \mathbf{I} & 0 \\ 0 & 0 & 0 & \mathbf{I} \end{bmatrix}_{15\times 12} \tag{19}$$

$$\mathbf{Q}_i = \begin{bmatrix} \sigma_{a_n}^2 \Delta t^2 \mathbf{I} & 0 & 0 & 0 \\ 0 & \sigma_{\omega_n}^2 \Delta t^2 \mathbf{I} & 0 & 0 \\ 0 & 0 & \sigma_{a_\omega}^2 \Delta t \mathbf{I} & 0 \\ 0 & 0 & 0 & \sigma_{\omega_\omega}^2 \Delta t \mathbf{I} \end{bmatrix}_{12\times 12} \tag{20}$$

$$\sigma_{a_n}^2 = \text{white noise variance of IMU acceleration measurement} \tag{21}$$

$$\sigma_{\omega_n}^2 = \text{white noise variance of IMU angular velocity measurement} \tag{22}$$

$$\sigma_{a_\omega}^2 = \text{velocity random walk variance of IMU} \tag{23}$$

$$\sigma_{\omega_\omega}^2 = \text{angular random walk variance of IMU} \tag{24}$$

4.2 Correction

Steps for correction upon receiving a measurement can be given as follows.

$$K_k = \check{\mathbf{P}}_k H^T \left(H \check{\mathbf{P}}_k H^T + V\right)^{-1} \tag{25}$$

$$\hat{\mathbf{P}}_k = \left(\mathbf{I} - K_k H\right) \check{\mathbf{P}}_k \tag{26}$$

Here,

$$H = H_x X_{\delta x} \text{ and} \tag{27}$$

$$X_{\delta x} = \begin{bmatrix} \mathbf{I} & 0 & 0 & 0 & 0 \\ 0 & \mathbf{I} & 0 & 0 & 0 \\ 0 & 0 & Q_{\delta\theta} & 0 & 0 \\ 0 & 0 & 0 & \mathbf{I} & 0 \\ 0 & 0 & 0 & 0 & \mathbf{I} \end{bmatrix}_{16\times 15} \tag{28}$$

$$Q_{\delta\theta} = \frac{1}{2} \begin{bmatrix} -q_x & -q_y & -q_z \\ q_w & q_z & -q_y \\ -q_z & q_w & q_x \\ q_y & -q_x & q_w \end{bmatrix}_{4\times 3} \tag{29}$$

For a GNSS measurement where position is measured relative to $\mathcal{F}_{inert}$ in all three directions,

$$H_x = \begin{bmatrix} \mathbf{I} & 0 & 0 & 0 & 0 \end{bmatrix}_{3\times 16} \text{ and} \tag{30}$$

$$\hat{\delta x}_k = K_k \left(y - \check{p}_k \right) \tag{31}$$

Steps for injecting the error to the state are as follows.

$$\hat{p}_k = \check{p}_k + \hat{\delta p}_k \tag{32}$$

$$\hat{v}_k = \check{v}_k + \hat{\delta v}_k \tag{33}$$

$$\hat{q}_k = q \left\{ \hat{\delta\theta}_k \right\} \otimes \check{q}_k \tag{34}$$

$$\hat{a}_{bk} = \check{a}_{bk} + \hat{\delta a}_{bk} \tag{35}$$

$$\hat{\omega}_{bk} = \check{\omega}_{bk} + \hat{\delta\omega}_{bk} \tag{36}$$

$$\tag{37}$$

Finally the error state is reset to zero ($\boldsymbol{\delta x} \leftarrow 0$).

4.3 Initialization

$$\boldsymbol{\delta x} = \mathbf{0} \tag{38}$$

$$p_0 = \text{GNSS position of the starting point expressed in } \mathcal{F}_{inert} \text{ frame.} \tag{39}$$

$$v_0 = 0 \tag{40}$$

$$q_0 = \text{Orientation determined by the gravity and magnetic North vectors.} \tag{41}$$

$$a_{b_0}, \omega_{b_0} = \text{given in datasheet of the IMU/ previous estimate.} \tag{42}$$

$$g = (0, 0, -9.8) \tag{43}$$

4.4 Orientation from magnetic field vector

Let the normalized magnetic field and gravitational vectors in $\mathcal{F}_{inert}$ frame be m_i and g_i. Then

$$m_i = \begin{bmatrix}0 & -1 & 0\end{bmatrix}^T \text{ and} \tag{44}$$

$$g_i = \begin{bmatrix}0 & 0 & -1\end{bmatrix}^T \ . \tag{45}$$

Let the unit quaternion corresponding to the orientation of the vehicle be $q = \begin{bmatrix}w & x & y & z\end{bmatrix}^T$ and $R_{inert,body} = R_{inert,body}\{q\}$ (Rotation matrix corresponding to q). If the measured, normalized magnetic field and acceleration vectors in $\mathcal{F}_{IMU} = \mathcal{F}_{body}$ are given by $m_s = \begin{bmatrix}m_x & m_y & m_z\end{bmatrix}^T$ and $f_s = \begin{bmatrix}f_x & f_y & f_z\end{bmatrix}^T$, when the vehicle is at rest we can have the following set of equations.

$$R_{inert,body} \cdot m_s + m_i = 0 \tag{46}$$

$$R_{inert,body} \cdot f_s + g_i = 0 \tag{47}$$

$$w^2 + x^2 + y^2 + z^2 - 1 = 0 \tag{48}$$

The quaternion components are approximated by fitting a least-square-error line for the above set of over-determined non-linear equations. Furthermore, since the accelerometer reading is much reliable than the magnetometer reading, the orthogonality of the gravity and magnetic filed vectors can be utilized apply a correction to the magnetic field vector. In the following approach, we expect to remove any magnetic field components in the direction parallel to that of gravity. Let m_s^{raw} and f_s^{raw} be the raw magnetic field and acceleration vectors measured by the sensor at rest and m_s be the corrected, normalized magnetic field vector.

$$m_s' = m_s^{raw} - (m_s^{raw} \cdot f_s^{raw}) \frac{f_s^{raw}}{||f_s^{raw}||^2} \tag{49}$$

$$m_s = \frac{m_s'}{||m_s'||} \tag{50}$$

5 Initial body frame as inertial frame, full state vector

In this approach we use,

$\mathcal{F}_{inert}$ = a coordinate frame with axes parallel to those of the IMU frame at the initialization, and the origin at a point with precisely known GNSS coordinates (considered as an inertial frame)

$\mathcal{F}_{gnss}$ = WGS84 coordinate system

$\mathcal{F}_{body}$ = coordinate frame of the IMU . (51)

Hence,

$$\mathcal{F}_{IMU} = \mathcal{F}_{body} \ . \tag{52}$$

$\mathbf{p}$, $\mathbf{v}$, $\mathbf{q}$, $\mathbf{g}$ and $\mathbf{n}$ are expressed in $\mathcal{F}_{inert}$ frame. However the IMU biases, $\boldsymbol{a_b}$ and $\boldsymbol{\omega_b}$ are expressed in $\mathcal{F}_{body}$ frame.

State vectors are initialized in the following manner.

$$\mathbf{p} = \text{initial GNSS reading converted to } \mathcal{F}_{inert} = C_{inert,gnss}\,(\boldsymbol{y} \ominus \boldsymbol{y_0}) \tag{53}$$
$$\mathbf{v} = (0,0,0) \tag{54}$$
$$\mathbf{q} = (0,0,0) \tag{55}$$
$$\mathbf{g} = \text{initial true acceleration due to gravity measured by the IMU} \tag{56}$$
$$\mathbf{n} = \text{North vector calculated using the initial reading of the magnetometer} \tag{57}$$
$$\boldsymbol{a_b}, \boldsymbol{\omega_b} = \text{from data given by the manufacturer/ previous estimate, if available} \tag{58}$$

Here, $\boldsymbol{y}$ is the GNSS reading at the starting point and $\boldsymbol{y_0}$ is the GNSS coordinate of the origin of $\mathcal{F}_{inert}$. Note that $C_{inert,gnss}$ (and therefore $C_{gnss,inert}$) depends on $\mathbf{g}$ and $\mathbf{n}$.

6 GNSS measurement auto-regressive model

As the GNSS measurement error (with respect to the ground truth) shows non-zero higher order correlations, it is modeled as a first order auto-regressive process [Zui Tao. Autonomous road vehicles localization using satellites, lane markings and vision, 2016]. Let the GNSS measurement, ground truth and the gnss error at time k be $x_k, \hat{x}_k$ and e_k respectively. Then,

$$e_{k+1} = a_1 e_k + \omega_{k+1} \tag{59}$$
$$(x_{k+1} - \hat{x}_{k+1}) = a_1(x_k - \hat{x}_k) + \omega_{k+1} \tag{60}$$
$$\hat{x}_{k+1} = x_{k+1} - a_1(x_k - \hat{x}_k) - \omega_{k+1} \tag{61}$$

Here, ω_k is a gaussian white noise term with variance equal to that of the receiver. Although, in the above mentioned paper, they are using a separate states for estimating the GNSS error terms, we intend to simply calculate the parameters (a_1) using statistical methods and use the above result to generate a corrected version (with zero correlation) of the measurement.

7 KAIST Urban dataset