Precise vehicle localization using fusion of multiple sensors for self-driving

Dr. Peshala, Dr. Ranga, Chanaka, Himasha, Pasan, Ramesh

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1 State vector

The state vector to be estimated is

$$\mathbf{x} = \begin{bmatrix} \mathbf{p} \\ \mathbf{v} \\ \mathbf{q} \\ \mathbf{g} \\ \mathbf{n} \\ \boldsymbol{a_b} \\ \boldsymbol{\omega_b} \end{bmatrix}_{22\times 1} \tag{1}$$

where

$$\begin{aligned} \mathbf{p} &= (p_x, p_y, p_z) \text{ position relative to the inertial frame} \\ \mathbf{v} &= (v_x, v_y, v_z) \text{ velocity relative to the inertial frame} \\ \mathbf{q} &= (q_w, q_x, q_y, q_z) \text{ quaternion relative to the inertial frame} \\ \mathbf{g} &= (g_x, g_y, g_z) \text{ gravitational vector relative to the inertial frame} \\ \mathbf{n} &= (n_x, n_y, n_z) \text{ magnetic north vector relative to the inertial frame.} \\ \boldsymbol{a_b} &= (a_{bx}, a_{by}, a_{bz}) \text{ acceleration biases of the IMU} \\ \boldsymbol{\omega_b} &= (\omega_{bx}, \omega_{by}, \omega_{bz}) \text{ angular velocity biases of the IMU} \end{aligned} \tag{2}$$

2 Coordinate frames

Following four types of coordinate frames are used.

$$\begin{aligned} &\mathcal{F}_{inert} = \text{ the coordinate frame fixed relative to the earth surface} \\ &\mathcal{F}_{gnss} = \text{ the coordinate frame in which gnss readings will be provided} \\ &\mathcal{F}_{body} = \text{ a coordinate frame fixed relative to the body of the vehicle} \\ &\mathcal{F}_{<sensor>} = \text{ coordinate frames attached to sensors in which their readings are provided} \\ &(< sensor > = \text{IMU, LiDAR etc.}) \end{aligned} \tag{3}$$

Rotation matrix from $\mathcal{F}_a$ to $\mathcal{F}_b$ is denoted as $R_{b,a}$. Example:

$$\boldsymbol{r_b} = R_{b,a}\boldsymbol{r_a} \tag{4}$$

`coordinate-frames.JPG`

Figure 1: Coordinate frames

3 Quaternion operations

Let the unit quaternion $\mathbf{q}$ be defined as

$$\mathbf{q} = \begin{bmatrix} q_w \\ q_x \\ q_y \\ q_z \end{bmatrix} \tag{5}$$

$$\text{and } \mathbf{q}_v = \begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix} \tag{6}$$

Then some operations on $\mathbf{q}$ can be defined as given in Figure **??**.

quaternion-operations.png

Figure 2: Operations related to quaternions

4 ENU frame as the inertial frame, state vector excluding g and n

In this approach we use,

$$
\begin{aligned}
\mathcal{F}_{inert} &= \text{East-North-Up coordinate frame with the origin at a point with precisely known GNSS} \\
&\quad\ \text{coordinates (considered as an inertial frame)} \\
\mathcal{F}_{gnss} &= \text{ WGS84 coordinate system (in lattitudes, longitudes and altitude)} \\
\mathcal{F}_{body} &= \text{ coordinate frame of the IMU .}
\end{aligned} \tag{7}
$$

Hence,

$$\mathcal{F}_{IMU} = \mathcal{F}_{body} \ . \tag{8}$$

Also the state vector is a reduced version of the original one.

$$\mathbf{x} = \begin{bmatrix} \mathbf{p} \\ \mathbf{v} \\ \mathbf{q} \\ \boldsymbol{a_b} \\ \boldsymbol{\omega_b} \end{bmatrix} \tag{9}$$

$\mathbf{p}$, $\mathbf{v}$ and $\mathbf{q}$ are expressed in $\mathcal{F}_{inert}$ frame. However the IMU biases, $\boldsymbol{a_b}$ and $\boldsymbol{\omega_b}$ are expressed in $\mathcal{F}_{body}$ frame.

4.1 Prediction

State updates can be given as

$$\check{p}_k = \hat{p}_{k-1} + \hat{v}_{k-1}\Delta t + \frac{1}{2}\left(R_{inert,body}\left(a_{m_{k-1}} - \hat{a}_{b_{k-1}}\right) + g\right)\Delta t^2 \tag{10}$$

$$\check{v}_k = \hat{v}_{k-1} + \left(R_{inert,body}\left(a_{m_{k-1}} - \hat{a}_{b_{k-1}}\right) + g\right)\Delta t \tag{11}$$

$$\check{q}_k = \hat{q}_{k-1} \otimes q\left\{\left(\omega_{m_{k-1}} - \hat{\omega}_{b_{k-1}}\right)\Delta t\right\} \tag{12}$$

$$\check{a}_{b_k} = \hat{a}_{b_{k-1}} \tag{13}$$

$$\check{\omega}_{b_k} = \hat{\omega}_{b_{k-1}} \tag{14}$$

with

$$R_{inert,body} = R\left\{\hat{q}_{k-1}\right\} \ . \tag{15}$$

Covariance matrix update is

$$
\begin{aligned}
\check{\mathbf{P}}_k &= F_x \hat{\mathbf{P}}_{k-1} F_x^T + F_i \mathbf{Q}_i F_i^T \text{ with} && (16)\\
F_x &= \frac{\partial f}{\partial \delta x} \Big|_{x_{k-1}, u_{k-1}} && (17)\\
&= \begin{bmatrix} \mathbf{I} & \mathbf{I}\Delta t & 0 & 0 & 0 \\ 0 & \mathbf{I} & -\left[R_{inert,body}\left(a_{m_{k-1}} - \hat{a}_{b_{k-1}}\right)\right]_\times \Delta t & -R_{inert,body}\Delta t & 0 \\ 0 & 0 & \mathbf{I} & 0 & -R_{inert,body}\Delta t \\ 0 & 0 & 0 & \mathbf{I} & 0 \\ 0 & 0 & 0 & 0 & \mathbf{I} \end{bmatrix}_{15\times 15} && (18)\\
F_i &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ \mathbf{I} & 0 & 0 & 0 \\ 0 & \mathbf{I} & 0 & 0 \\ 0 & 0 & \mathbf{I} & 0 \\ 0 & 0 & 0 & \mathbf{I} \end{bmatrix}_{15\times 12} && (19)\\
\mathbf{Q}_i &= \begin{bmatrix} \sigma_{a_n}^2 \Delta t^2 \mathbf{I} & 0 & 0 & 0 \\ 0 & \sigma_{\omega_n}^2 \Delta t^2 \mathbf{I} & 0 & 0 \\ 0 & 0 & \sigma_{a_\omega}^2 \Delta t \mathbf{I} & 0 \\ 0 & 0 & 0 & \sigma_{\omega_\omega}^2 \Delta t \mathbf{I} \end{bmatrix}_{12\times 12} && (20)\\
\sigma_{a_n}^2 &= \text{white noise variance of IMU acceleration measurement} && (21)\\
\sigma_{\omega_n}^2 &= \text{white noise variance of IMU angular velocity measurement} && (22)\\
\sigma_{a_\omega}^2 &= \text{velocity random walk variance of IMU} && (23)\\
\sigma_{\omega_\omega}^2 &= \text{angular random walk variance of IMU} && (24)
\end{aligned}
$$

4.2 Correction

Steps for correction upon receiving a measurement can be given as follows.

$$
\begin{aligned}
K_k &= \check{\mathbf{P}}_k H^T \left(H \check{\mathbf{P}}_k H^T + V\right)^{-1} && (25)\\
\hat{\mathbf{P}}_k &= \left(\mathbf{I} - K_k H\right) \check{\mathbf{P}}_k && (26)
\end{aligned}
$$

Here,

$$H = H_x X_{\delta x} \text{ and} \tag{27}$$

$$X_{\delta x} = \begin{bmatrix} \mathbf{I} & 0 & 0 & 0 & 0 \\ 0 & \mathbf{I} & 0 & 0 & 0 \\ 0 & 0 & Q_{\delta\theta} & 0 & 0 \\ 0 & 0 & 0 & \mathbf{I} & 0 \\ 0 & 0 & 0 & 0 & \mathbf{I} \end{bmatrix}_{16\times 15} \tag{28}$$

$$Q_{\delta\theta} = \frac{1}{2} \begin{bmatrix} -q_x & -q_y & -q_z \\ q_w & q_z & -q_y \\ -q_z & q_w & q_x \\ q_y & -q_x & q_w \end{bmatrix}_{4\times 3} \tag{29}$$

For a GNSS measurement where position is measured relative to $\mathcal{F}_{inert}$ in all three directions,

$$H_x = \begin{bmatrix} \mathbf{I} & 0 & 0 & 0 & 0 \end{bmatrix}_{3\times 16} \text{ and} \tag{30}$$

$$\hat{\delta x}_k = K_k \left(y - \check{p}_k \right) \tag{31}$$

Steps for injecting the error to the state are as follows.

$$\hat{p}_k = \check{p}_k + \hat{\delta p}_k \tag{32}$$

$$\hat{v}_k = \check{v}_k + \hat{\delta v}_k \tag{33}$$

$$\hat{q}_k = q \left\{ \hat{\delta\theta}_k \right\} \otimes \check{q}_k \tag{34}$$

$$\hat{a}_{bk} = \check{a}_{bk} + \hat{\delta a}_{bk} \tag{35}$$

$$\hat{\omega}_{bk} = \check{\omega}_{bk} + \hat{\delta\omega}_{bk} \tag{36}$$

$$\tag{37}$$

Finally the error state is reset to zero ($\boldsymbol{\delta x} \leftarrow 0$).

4.3 Initialization

$$\boldsymbol{\delta x} = \mathbf{0} \tag{38}$$

$$p_0 = \text{GNSS position of the starting point expressed in } \mathcal{F}_{inert} \text{ frame.} \tag{39}$$

$$v_0 = 0 \tag{40}$$

$$q_0 = \text{Orientation determined by the gravity and magnetic North vectors.} \tag{41}$$

$$a_{b_0}, \omega_{b_0} = \text{given in datasheet of the IMU/ previous estimate.} \tag{42}$$

$$g = (0, 0, -9.8) \tag{43}$$

4.4 Orientation from magnetic field vector

Let the normalized magnetic field and gravitational vectors in $\mathcal{F}_{inert}$ frame be m_i and g_i. Then

$$m_i = \begin{bmatrix} 0 & -1 & 0 \end{bmatrix}^T \text{ and} \tag{44}$$

$$g_i = \begin{bmatrix} 0 & 0 & -1 \end{bmatrix}^T . \tag{45}$$

Let the unit quaternion corresponding to the orientation of the vehicle be $q = \begin{bmatrix} w & x & y & z \end{bmatrix}^T$ and $R_{inert,body} = R_{inert,body}\{q\}$ (Rotation matrix corresponding to q). If the measured, normalized magnetic field and acceleration vectors in $\mathcal{F}_{IMU} = \mathcal{F}_{body}$ are given by $m_s = \begin{bmatrix} m_x & m_y & m_z \end{bmatrix}^T$ and $f_s = \begin{bmatrix} f_x & f_y & f_z \end{bmatrix}^T$, when the vehicle is at rest we can have the following set of equations.

$$R_{inert,body} \cdot m_s + m_i = 0 \tag{46}$$

$$R_{inert,body} \cdot f_s + g_i = 0 \tag{47}$$

$$w^2 + x^2 + y^2 + z^2 - 1 = 0 \tag{48}$$

The quaternion components are approximated by fitting a least-square-error line for the above set of over-determined non-linear equations. Furthermore, since the accelerometer reading is much reliable than the magnetometer reading, the orthogonality of the gravity and magnetic filed vectors can be utilized apply a correction to the magnetic field vector. In the following approach, we expect to remove any magnetic field components in the direction parallel to that of gravity. Let m_s^{raw} and f_s^{raw} be the raw magnetic field and acceleration vectors measured by the sensor at rest and m_s be the corrected, normalized magnetic field vector.

$$m_s' = m_s^{raw} - (m_s^{raw} \cdot f_s^{raw}) \frac{f_s^{raw}}{||f_s^{raw}||^2} \tag{49}$$

$$m_s = \frac{m_s'}{||m_s'||} \tag{50}$$

5 Initial body frame as inertial frame, full state vector

In this approach we use,

$$\begin{aligned}
\mathcal{F}_{inert} &= \text{a coordinate frame with axes parallel to those of the IMU frame at the initialization, and} \\
&\quad \text{the origin at a point with precisely known GNSS coordinates (considered as an inertial frame)} \\
\mathcal{F}_{gnss} &= \text{ WGS84 coordinate system} \\
\mathcal{F}_{body} &= \text{ coordinate frame of the IMU .}
\end{aligned} \tag{51}$$

Hence,

$$\mathcal{F}_{IMU} = \mathcal{F}_{body} \ . \tag{52}$$

$\mathbf{p}$, $\mathbf{v}$, $\mathbf{q}$, $\mathbf{g}$ and $\mathbf{n}$ are expressed in $\mathcal{F}_{inert}$ frame. However the IMU biases, $\boldsymbol{a_b}$ and $\boldsymbol{\omega_b}$ are expressed in $\mathcal{F}_{body}$ frame.

State vectors are initialized in the following manner.

$$\mathbf{p} = \text{initial GNSS reading converted to } \mathcal{F}_{inert} = C_{inert,gnss}\,(\boldsymbol{y} \ominus \boldsymbol{y_0}) \tag{53}$$

$$\mathbf{v} = (0,0,0) \tag{54}$$

$$\mathbf{q} = (0,0,0) \tag{55}$$

$$\mathbf{g} = \text{initial true acceleration due to gravity measured by the IMU} \tag{56}$$

$$\mathbf{n} = \text{North vector calculated using the initial reading of the magnetometer} \tag{57}$$

$$\boldsymbol{a_b}, \boldsymbol{\omega_b} = \text{from data given by the manufacturer/ previous estimate, if available} \tag{58}$$

Here, $\boldsymbol{y}$ is the GNSS reading at the starting point and $\boldsymbol{y_0}$ is the GNSS coordinate of the origin of $\mathcal{F}_{inert}$. Note that $C_{inert,gnss}$ (and therefore $C_{gnss,inert}$) depends on $\mathbf{g}$ and $\mathbf{n}$.