The Transconductance Amplifier

Neuromorphic Engineering I

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Zurich, October 20, 2019

Outline

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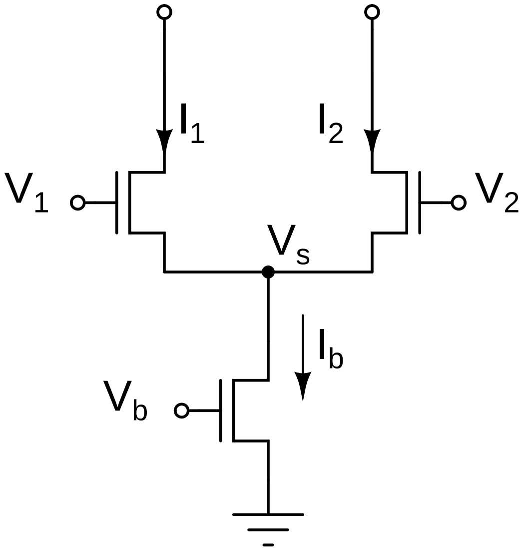
The diff-pair



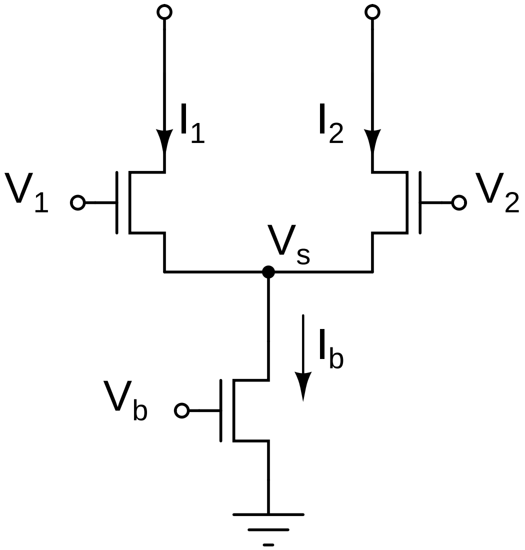
$$I_1 = I_0 e^{\frac{\kappa V_1 - V_s}{U_T}}$$

$$I_2 = I_0 e^{\frac{\kappa V_2 - V_s}{U_T}}$$

$$I_b = I_1 + I_2 = I_0 e^{\frac{\kappa V_b}{U_T}}$$

$$\boxed{I_b = I_0 e^{-\frac{V_s}{U_T}} \left(e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}} \right)}$$

The diff-pair



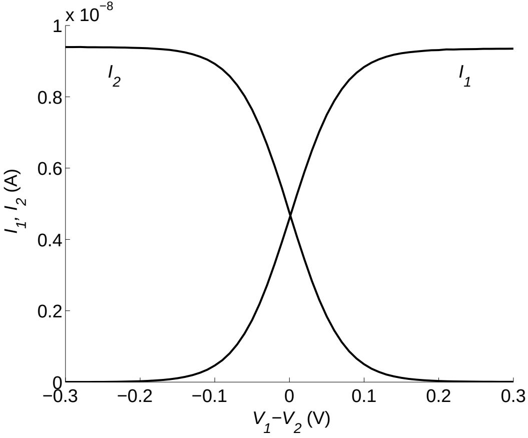
$$e^{-\frac{V_s}{U_T}} = \frac{I_b}{I_0} \frac{1}{e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}}$$

$$I_1 = I_b \frac{e^{\frac{\kappa V_1}{U_T}}}{e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}}$$

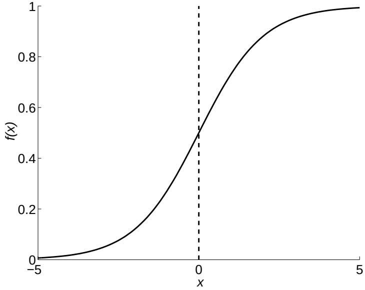
$$I_2 = I_b \frac{e^{\frac{\kappa V_2}{U_T}}}{e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}}$$

Fermi Functions

The diff-pair (contd.)



Sigmoids



The term sigmoid means “S-shaped”. Sigmoid functions are typically used in the (conventional) neural network research community. They are smooth, saturating, monotonic activation functions, that map the interval $(-\infty, \infty)$ onto $(0, 1)$. The canonical *logistic* sigmoid is defined as

$$f(x) = \frac{1}{1 + exp(-\alpha x)}$$

Sigmoids (contd)

Diff-pair output currents

The output currents of the diff-pair can be rewritten in the canonical sigmoid form:

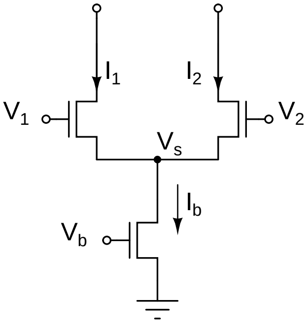
$$I_1 = I_b \frac{1}{1 + e^{\frac{\kappa}{U_T}(V_2 - V_1)}} \qquad I_2 = I_b \frac{1}{1 + e^{\frac{\kappa}{U_T}(V_1 - V_2)}}$$

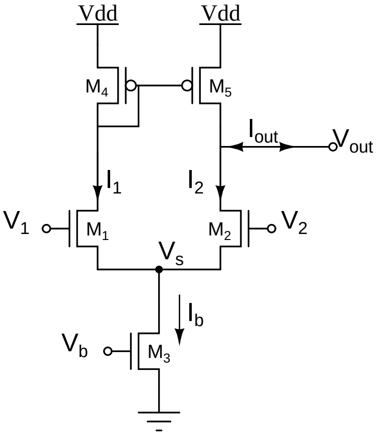
Difference of diff-pair currents

$$I_1 - I_2 = I_b \frac{e^{\frac{\kappa V_1}{U_T}} - e^{\frac{\kappa V_2}{U_T}}}{e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}} = I_b \tanh\left(\frac{\kappa}{2U_T}(V_1 - V_2)\right)$$

Difference of currents

To implement the difference of currents $\left(I_1 - I_2\right)$ we can use …

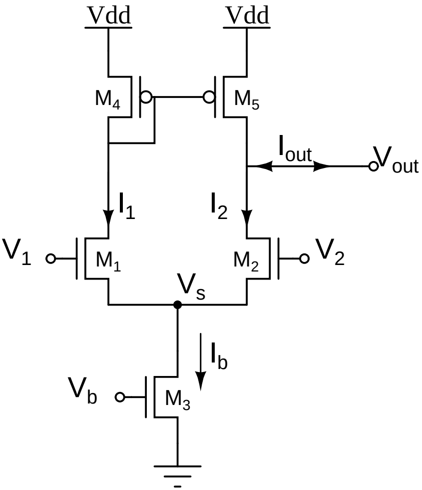


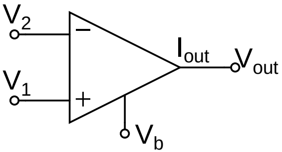


… a current-mirror

Outline

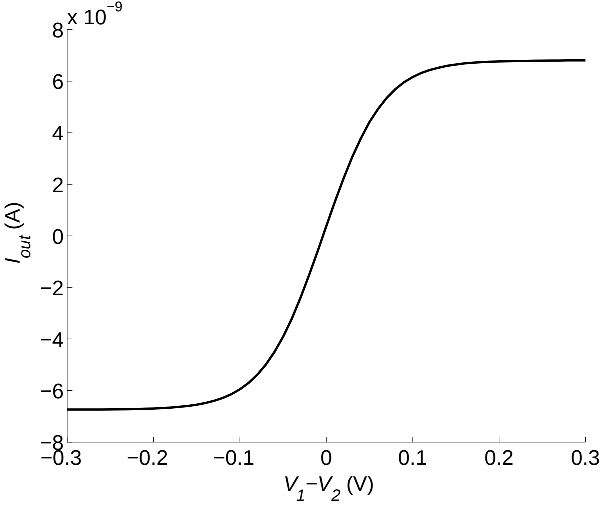
The Transconductance Amplifier





$$\boxed{I_{out} = I_b \tanh\left(\frac{\kappa}{2U_T}(V_1 - V_2)\right)}$$

The Transconductance Amplifier



The Transconductance Amplifier

For small differential voltages (*e.g.* $|V_1 - V_2| < 200mV$), the $\tanh(\cdot)$ relationship is approximately linear and the equation

$$I_{out} = I_b \tanh\left(\frac{\kappa}{2U_T}(V_1 - V_2)\right)$$

can be reduce to:

$$I_{out} \approx g_m(V_1 - V_2)$$

where

$$\boxed{g_m = \frac{I_b \kappa}{2U_T}}$$

The Transconductance Amplifier

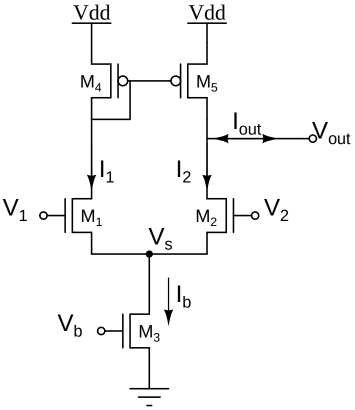
- The term $g_m = \frac{I_b \kappa}{2U_T}$ is the *transconductance* of the amplifier. It has the dimensions of a conductance, but the output current is meaured at a terminal that is different from the pair across which the input voltage difference is applied.
- The output conductance is of the amplifier is

$$g_d = -\frac{\partial I_{out}}{\partial V_{out}} \approx \frac{I_b}{V_E}$$

where V_E is the Early voltage of M_2 and M_5

Outline

Assumptions



In order to obtain the nice $\mathtt{tanh}(\cdot)$ equation, we implicitly made a few assumptions...

$$I_1 = I_0 e^{\frac{\kappa V_1 - V_s}{U_T}}$$

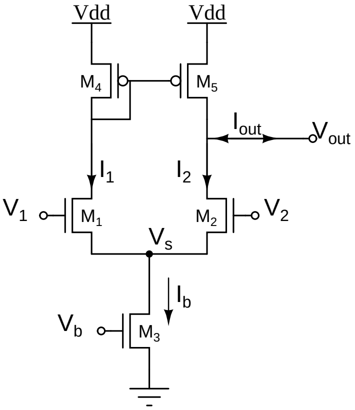
$$I_2 = I_0 e^{\frac{\kappa V_2 - V_s}{U_T}}$$

$$I_b = I_0 e^{\frac{\kappa V_b}{U_T}}$$

$$I_{out} = I_1 - I_2$$

implies that all M_1 thorugh M_5 are in *saturation*.

Bias transistor (M3)



The equation for bias current flowing through M3 is:

$$I_b = I_0 e^{\frac{\kappa V_b}{U_T}} (1 - e^{-\frac{V_S}{U_T}})$$

From $I_b = I_1 + I_2$, and assuming that M_1 and M_2 are in saturation:

$$e^{\frac{\kappa V_b}{U_T}} (1 - e^{-\frac{V_S}{U_T}}) = e^{\frac{\kappa V_1 - V_S}{U_T}} + e^{\frac{\kappa V_2 - V_S}{U_T}}$$

And solving for $e^{-\frac{V_S}{U_T}}$:

$$e^{-\frac{V_S}{U_T}} = \frac{e^{\frac{\kappa V_b}{U_T}}}{e^{\frac{\kappa V_b}{U_T}} + e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}}$$

Bias transistor (contd.)

The saturation condition $V_s > 4U_T$ imposes $e^{-\frac{V_s}{U_T}} \ll 1$.

$$e^{-\frac{V_s}{U_T}} = \frac{e^{\frac{\kappa V_b}{U_T}}}{e^{\frac{\kappa V_b}{U_T}} + e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}} \ll 1$$

$$e^{\frac{\kappa V_b}{U_T}} \ll e^{\frac{\kappa V_b}{U_T}} + e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}$$

$$1 + \frac{e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}}{e^{\frac{\kappa V_b}{U_T}}} \gg 1$$

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Bias transistor (contd.)

Now, from

$$e^{-\frac{V_s}{U_T}} = \frac{e^{\frac{\kappa V_b}{U_T}}}{e^{\frac{\kappa V_b}{U_T}} + e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}}$$

we can derive

$$\boxed{V_s = -\kappa V_b + U_T \ln\left(e^{\frac{\kappa V_b}{U_T}} + e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}\right)}$$

From the last slide we know that $e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}} \gg e^{\frac{\kappa V_b}{U_T}}$. If we also assume $|V_1 - V_2| > 4U_T$, we can simplify the $\ln(\cdot)$.

Bias transistor (contd.)

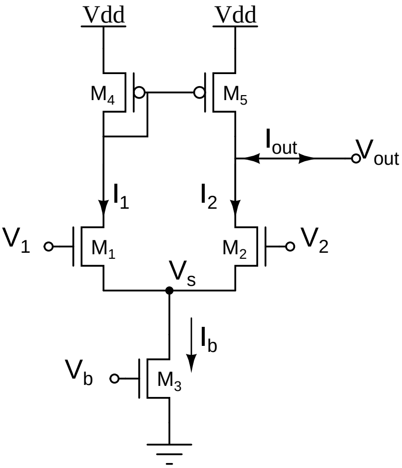
The common node voltage V_s of the transconductance amplifier is

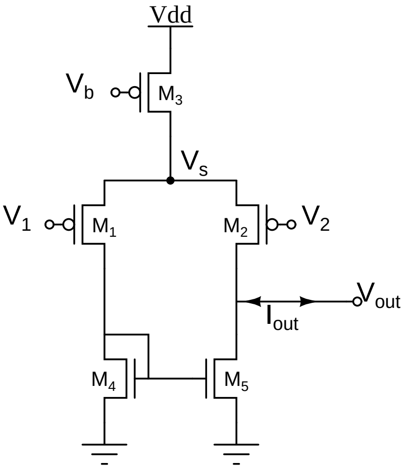
$$\boxed{V_s \approx \kappa(\max(V_1, V_2) - V_b)}$$

and the saturation condition for M_3, $V_s > 4U_T$ is:

$$\boxed{\max(V_1, V_2) > V_b + \frac{4U_T}{\kappa}}$$

Complementary circuit





Output transistors

The output transistors M_2 and M_5 are the only other transistors that we have to worry about, because we can safely assume that M_1 and M_4 are always in saturation.
The saturations conditions for M_2 and M_5 restrict the output voltage range:

- To keep M_5 in saturation $V_{dd} - V_{out} > 4U_T$.
- To keep M_2 in saturation $V_{out} - V_s > 4U_T$, which implies that

$$\boxed{V_{out} > \kappa(\max(V_1, V_2) - V_b) + 4U_T}$$

(the famous V_{min} problem).

$$\boxed{\kappa(\max(V_1, V_2) - V_b) + 4U_T < V_{out} < V_{dd} - 4U_T}$$

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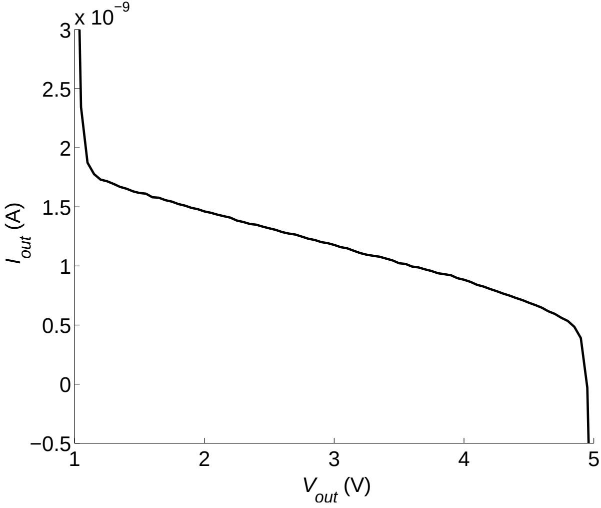
Output current vs output voltage



Above Threshold

If the differential pair is operated above threshold, it can be shown that

$$I_{out} = \frac{\beta}{2}(V_1 - V_2)\sqrt{\frac{4I_b}{\beta} - (V_1 - V_2)^2}$$

where $\beta = \mu C_{ox} \frac{W}{L}$

For $|V_1 - V_2| < \sqrt{2I_b/\beta}$ the transconductance is given by

$$g_m = \sqrt{\beta I_b}$$

Outline

Voltage Amplifier

The transconductance amplifier circuit can also be used as a differential-votage amplifier:

$$V_{out} = A(V_1 - V_2)$$

where A is the *open-circuit voltage gain*.

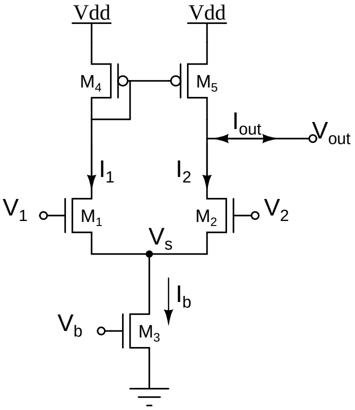
$$A = \frac{dV_{out}}{d(V_1 - V_2)} = \frac{dI_{out}}{d(V_1 - V_2)} \frac{dV_{out}}{dI_{out}}$$

In subthreshold $\boxed{A \approx \frac{\kappa V_E}{2U_T}}$ and above threshold $A \approx \sqrt{\frac{\beta}{I_b}} V_E$.

Voltage amplifier uses

- The open-circuit voltage gain A increases with the Early voltage, and therefore with the length of the output transistors.
- Typical subthreshold values are between 100 and 1000.
- Due to the large gain and *transistor mismatch* effects this circuit is **not** normally used as an open-circuit voltage amplifier.
- In voltage mode, its mainly used as a *comparator*: V_{out} is "high" if $V_1 > V_2$ and "low" if $V_2 > V_1$.

Output voltage limits



We will now compute the limits of the output voltage swing.

$V_1 > V_2$

For $V_1 > V_2 + 4U_T$ the current through M_2 is much smaller than the one through M_1, hence V_{out} goes almost all the way to V_{dd} to shut M_5 off. M_5 goes out of saturation.

$V_2 > V_1$

The forward current of M2 is $\gg$ than that of M1. But the current mirror imposes $I_1 \approx I_2 \approx I_b/2$. If V_2 is significantly larger than V_1, the voltage drop across M_2 is close to zero and $V_{out} \approx V_s$. M_2 goes out of saturation.

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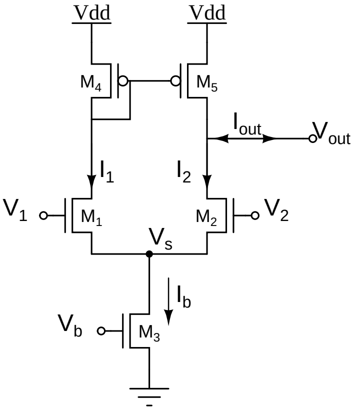

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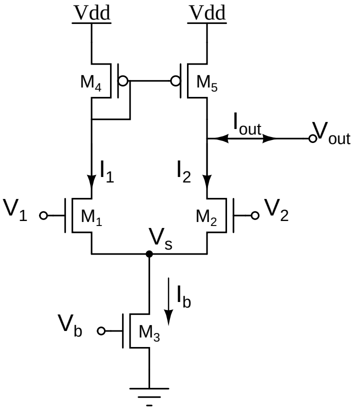

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Output voltage limits (contd.)

If $V_2 > V_1$, $V_{out} \approx V_s$. But we don't know the value of V_s. We can't assume that M_3 is in saturation, so we have to write:

$$I_b = I_0 e^{\frac{\kappa V_b}{U_T}} (1 - e^{-\frac{V_s}{U_T}}) = I_1 + I_2$$

with $I_1 = I_0 e^{\frac{\kappa V_1 - V_s}{U_T}}$ (M_1 **is** in saturation), and $I_2 \approx I_1$. So

$$e^{-\frac{V_s}{U_T}} = \frac{e^{\frac{\kappa V_b}{U_T}}}{e^{\frac{\kappa V_b}{U_T}} + e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_1}{U_T}}}$$

$$e^{-\frac{V_s}{U_T}} = \frac{e^{\frac{\kappa V_b}{U_T}}}{e^{\frac{\kappa V_b}{U_T}} + 2e^{\frac{\kappa V_1}{U_T}}}$$

$$e^{-\frac{V_s}{U_T}} = \frac{\frac{1}{2}e^{\frac{\kappa V_b}{U_T}}}{\frac{1}{2}e^{\frac{\kappa V_b}{U_T}} + e^{\frac{\kappa V_1}{U_T}}}$$

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Output voltage limits (contd.)

So, if

- $V_1 < V_b - (4 + \ln(2))U_T/\kappa$,

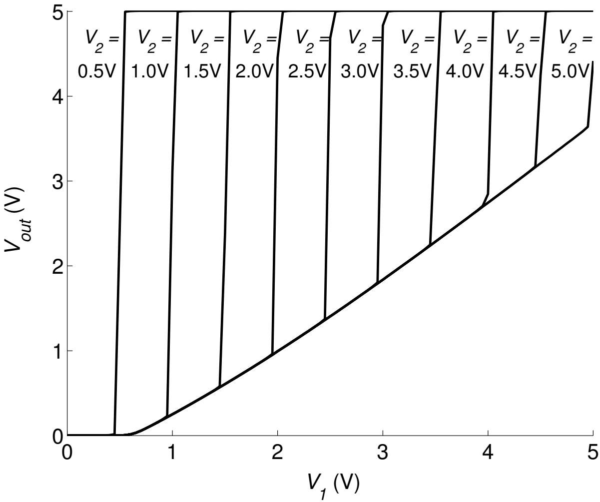
$$V_{out} \approx 0$$

- $V_1 > V_b - (4 + \ln(2))U_T/\kappa$,

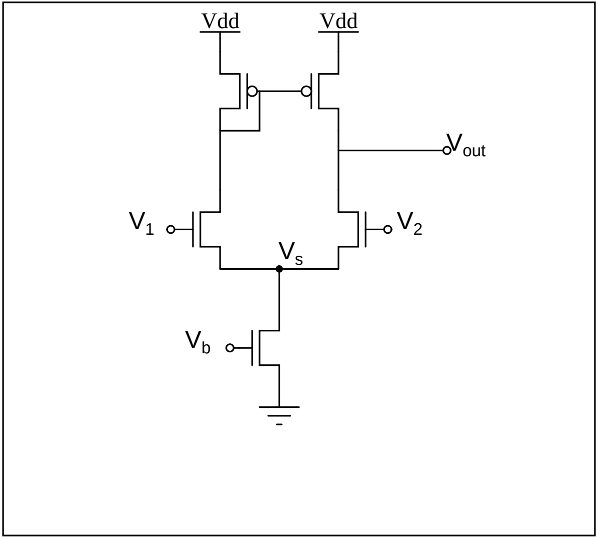
$$\boxed{V_{out} \approx \kappa V_1 - \kappa V_b + U_T \ln(2)}$$

V_{out} increases linearly with V_1, with a slope of κ.

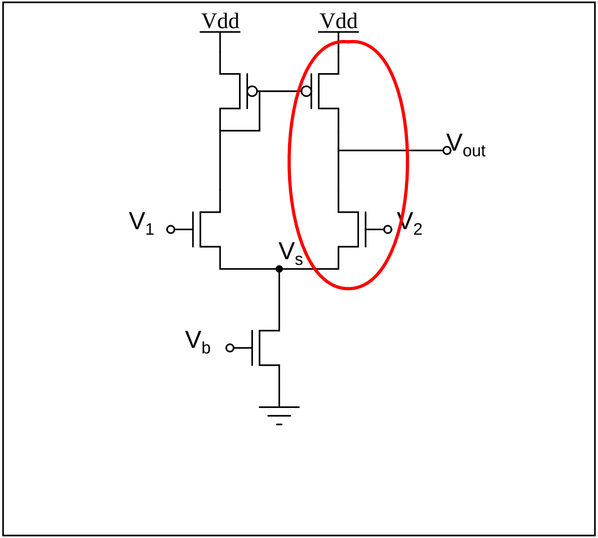
Output-circuit output voltage



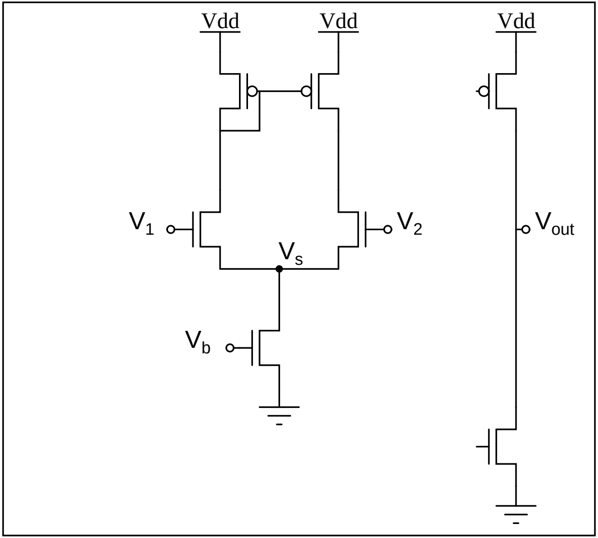
Wide-output-range circuit



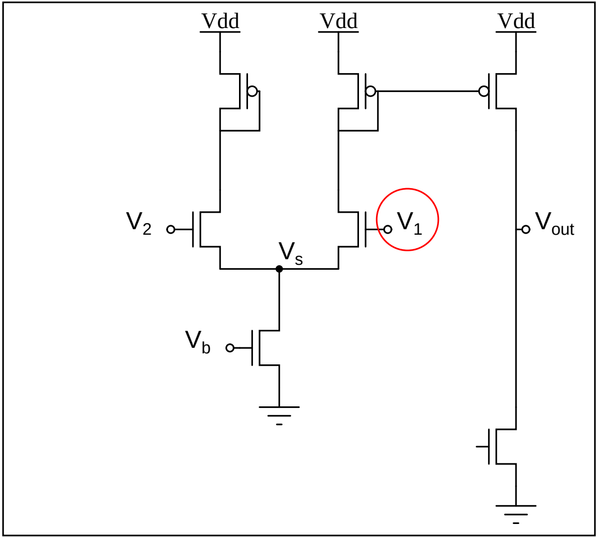
Wide-output-range circuit



Wide-output-range circuit



Wide-output-range circuit



Wide-output-range circuit