The Transconductance Amplifier

Neuromorphic Engineering I

Giacomo Indiveri

Institute of Neuroinformatics
University of Zurich and ETH Zurich

Zurich, October 20, 2019

Outline

Outline

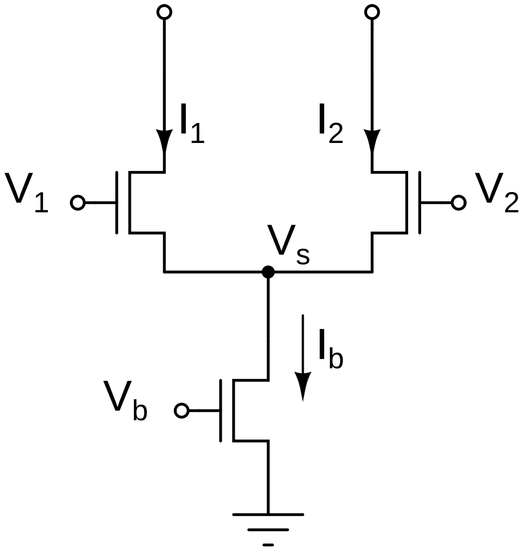
Outline

Outline

Outline

1. Sigmoids
2. The Transconductance Amplifier
3. Assumptions
4. Voltage Amplifier

The diff-pair



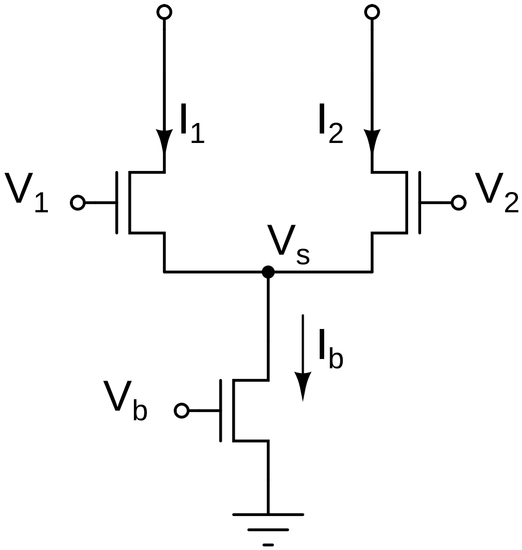
$$I_1 = I_0 e^{\frac{\kappa V_1 - V_s}{U_T}}$$

$$I_2 = I_0 e^{\frac{\kappa V_2 - V_s}{U_T}}$$

$$I_b = I_1 + I_2 = I_0 e^{\frac{\kappa V_b}{U_T}}$$

$$\boxed{I_b = I_0 e^{-\frac{V_s}{U_T}} \left(e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}} \right)}$$

The diff-pair



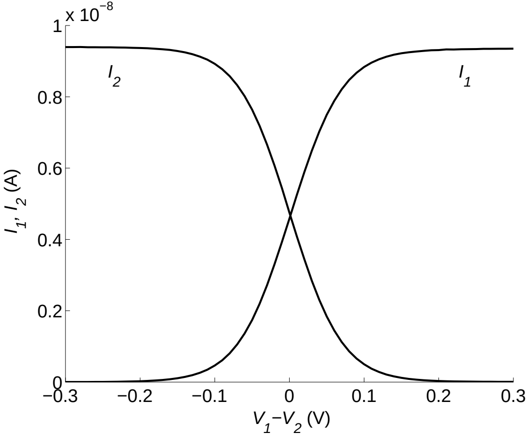
$$e^{-\frac{V_s}{U_T}} = \frac{I_b}{I_0} \frac{1}{e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}}$$

$$I_1 = I_b \frac{e^{\frac{\kappa V_1}{U_T}}}{e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}}$$

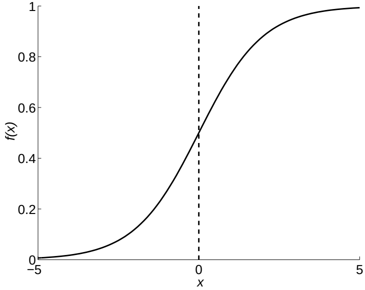
$$I_2 = I_b \frac{e^{\frac{\kappa V_2}{U_T}}}{e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}}$$

Fermi Functions

The diff-pair (contd.)



Sigmoids



The term sigmoid means "S-shaped". Sigmoid functions are typically used in the (conventional) neural network research community. They are smooth, saturating, monotonic activation functions, that map the interval $(-\infty, \infty)$ onto $(0, 1)$. The canonical *logistic* sigmoid is defined as

$$f(x) = \frac{1}{1 + exp(-\alpha x)}$$

Sigmoids (contd)

Diff-pair output currents

The output currents of the diff-pair can be rewritten in the canonical sigmoid form:

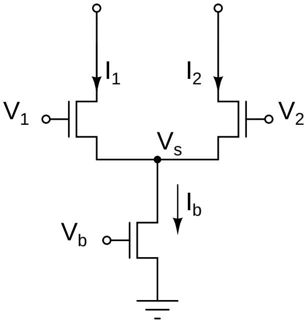
$$I_1 = I_b \ \frac{1}{1 + e^{\frac{\kappa}{U_T}(V_2 - V_1)}} \qquad I_2 = I_b \ \frac{1}{1 + e^{\frac{\kappa}{U_T}(V_1 - V_2)}}$$

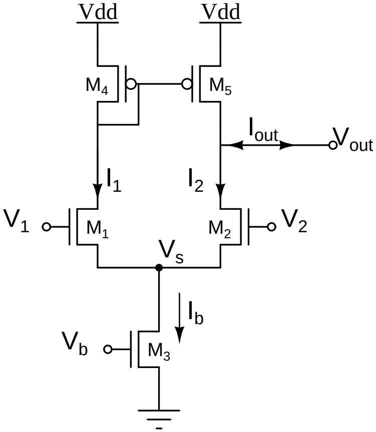
Difference of diff-pair currents

$$I_1 - I_2 = I_b \ \frac{e^{\frac{\kappa V_1}{U_T}} - e^{\frac{\kappa V_2}{U_T}}}{e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}} \quad = I_b \tanh\left(\frac{\kappa}{2U_T}(V_1 - V_2)\right)$$

Difference of currents

To implement the difference of currents $\left(I_1 - I_2\right)$ we can use …

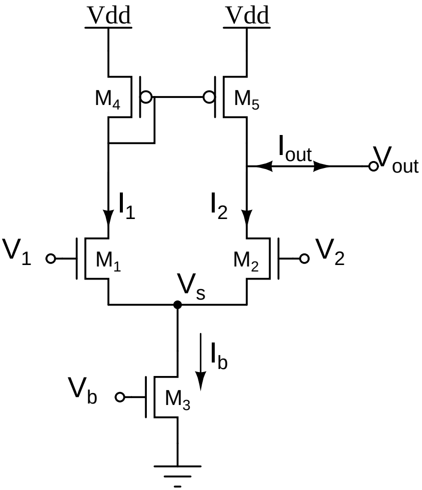


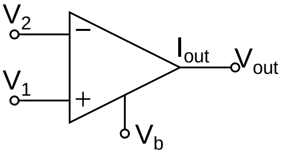


… a current-mirror

Outline

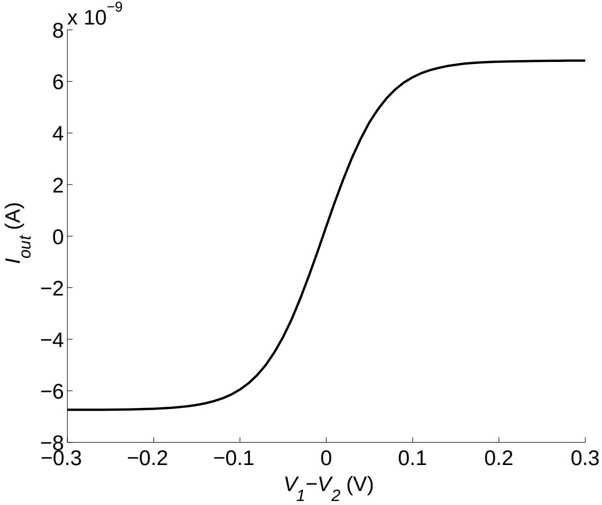
The Transconductance Amplifier





$$\boxed{I_{out} = I_b \tanh\left(\frac{\kappa}{2U_T}(V_1 - V_2)\right)}$$

The Transconductance Amplifier



The Transconductance Amplifier

For small differential voltages (*e.g.* $|V_1 - V_2| < 200mV$), the $\mathtt{tanh}(\cdot)$ relationship is approximately linear and the equation

$$I_{out} = I_b \tanh\left(\frac{\kappa}{2U_T}(V_1 - V_2)\right)$$

can be reduce to:

$$I_{out} \approx g_m(V_1 - V_2)$$

Ohms Law => basically a resistor

where

$$\boxed{g_m = \frac{I_b \kappa}{2U_T}}$$

The Transconductance Amplifier

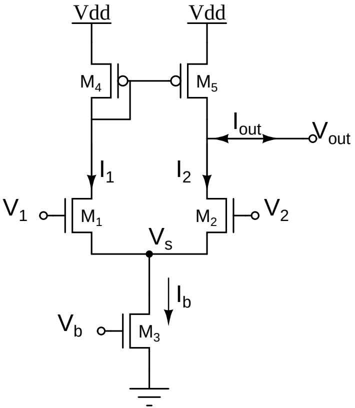
- The term $g_m = \frac{I_b \kappa}{2U_T}$ is the *transconductance* of the amplifier. It has the dimensions of a conductance, but the output current is meaured at a terminal that is different from the pair across which the input voltage difference is applied.
- The output conductance is of the amplifier is

$$g_d = -\frac{\partial I_{out}}{\partial V_{out}} \approx \frac{I_b}{V_E}$$

where V_E is the Early voltage of M_2 and M_5

Outline

Assumptions



In order to obtain the nice $\mathtt{tanh}(\cdot)$ equation, we implicitly made a few assumptions...

$$I_1 = I_0 e^{\frac{\kappa V_1 - V_s}{U_T}}$$

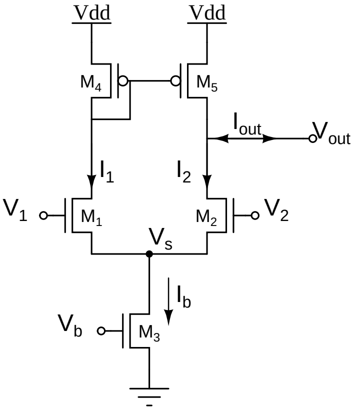
$$I_2 = I_0 e^{\frac{\kappa V_2 - V_s}{U_T}}$$

$$I_b = I_0 e^{\frac{\kappa V_b}{U_T}}$$

$$I_{out} = I_1 - I_2$$

implies that all M_1 thorugh M_5 are in *saturation*.

Bias transistor (M3)



The equation for bias current flowing through M3 is:

$$I_b = I_0 e^{\frac{\kappa V_b}{U_T}} \left(1 - e^{-\frac{V_S}{U_T}}\right)$$

From $I_b = I_1 + I_2$, and assuming that M_1 and M_2 are in saturation:

$$e^{\frac{\kappa V_b}{U_T}} \left(1 - e^{-\frac{V_S}{U_T}}\right) = e^{\frac{\kappa V_1 - V_S}{U_T}} + e^{\frac{\kappa V_2 - V_S}{U_T}}$$

And solving for $e^{-\frac{V_S}{U_T}}$:

$$e^{-\frac{V_S}{U_T}} = \frac{e^{\frac{\kappa V_b}{U_T}}}{e^{\frac{\kappa V_b}{U_T}} + e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}}$$

Bias transistor (contd.)

The saturation condition $V_s > 4U_T$ imposes $e^{-\frac{V_s}{U_T}} \ll 1$.

$$e^{-\frac{V_s}{U_T}} = \frac{e^{\frac{\kappa V_b}{U_T}}}{e^{\frac{\kappa V_b}{U_T}} + e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}} \ll 1$$

$$e^{\frac{\kappa V_b}{U_T}} \ll e^{\frac{\kappa V_b}{U_T}} + e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}$$

$$1 + \frac{e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}}{e^{\frac{\kappa V_b}{U_T}}} \gg 1$$

$$\frac{e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}}{e^{\frac{\kappa V_b}{U_T}}} \gg 1$$

Bias transistor (contd.)

The saturation condition $V_s > 4U_T$ imposes $e^{-\frac{V_s}{U_T}} \ll 1$.

$$e^{-\frac{V_s}{U_T}} = \frac{e^{\frac{\kappa V_b}{U_T}}}{e^{\frac{\kappa V_b}{U_T}} + e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}} \ll 1$$

$$e^{\frac{\kappa V_b}{U_T}} \ll e^{\frac{\kappa V_b}{U_T}} + e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}$$

$$1 + \frac{e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}}{e^{\frac{\kappa V_b}{U_T}}} \gg 1$$

$$\frac{e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}}{e^{\frac{\kappa V_b}{U_T}}} \gg 1$$

Bias transistor (contd.)

The saturation condition $V_s > 4U_T$ imposes $e^{-\frac{V_s}{U_T}} \ll 1$.

$$e^{-\frac{V_s}{U_T}} = \frac{e^{\frac{\kappa V_b}{U_T}}}{e^{\frac{\kappa V_b}{U_T}} + e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}} \ll 1$$

$$e^{\frac{\kappa V_b}{U_T}} \ll e^{\frac{\kappa V_b}{U_T}} + e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}$$

$$1 + \frac{e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}}{e^{\frac{\kappa V_b}{U_T}}} \gg 1$$

$$\frac{e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}}{e^{\frac{\kappa V_b}{U_T}}} \gg 1$$

Bias transistor (contd.)

The saturation condition $V_s > 4U_T$ imposes $e^{-\frac{V_s}{U_T}} \ll 1$.

$$e^{-\frac{V_s}{U_T}} = \frac{e^{\frac{\kappa V_b}{U_T}}}{e^{\frac{\kappa V_b}{U_T}} + e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}} \ll 1$$

$$e^{\frac{\kappa V_b}{U_T}} \ll e^{\frac{\kappa V_b}{U_T}} + e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}$$

$$1 + \frac{e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}}{e^{\frac{\kappa V_b}{U_T}}} \gg 1$$

$$\frac{e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}}{e^{\frac{\kappa V_b}{U_T}}} \gg 1$$

Bias transistor (contd.)

The saturation condition $V_s > 4U_T$ imposes $e^{-\frac{V_s}{U_T}} \ll 1$.

$$e^{-\frac{V_s}{U_T}} = \frac{e^{\frac{\kappa V_b}{U_T}}}{e^{\frac{\kappa V_b}{U_T}} + e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}} \ll 1$$

$$e^{\frac{\kappa V_b}{U_T}} \ll e^{\frac{\kappa V_b}{U_T}} + e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}$$

$$1 + \frac{e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}}{e^{\frac{\kappa V_b}{U_T}}} \gg 1$$

$$\frac{e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}}{e^{\frac{\kappa V_b}{U_T}}} \gg 1$$

Bias transistor (contd.)

Now, from

$$e^{-\frac{V_s}{U_T}} = \frac{e^{\frac{\kappa V_b}{U_T}}}{e^{\frac{\kappa V_b}{U_T}} + e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}}$$

we can derive

$$\boxed{V_s = -\kappa V_b + U_T \ln\left(e^{\frac{\kappa V_b}{U_T}} + e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}}\right)}$$

From the last slide we know that $e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_2}{U_T}} \gg e^{\frac{\kappa V_b}{U_T}}$. If we also assume $|V_1 - V_2| > 4U_T$, we can simplify the $\ln(\cdot)$.

Bias transistor (contd.)

The common node voltage V_s of the transconductance amplifier is

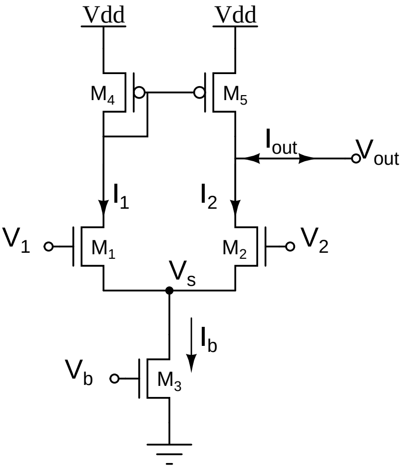
$$\boxed{V_s \approx \kappa(\max(V_1, V_2) - V_b)}$$

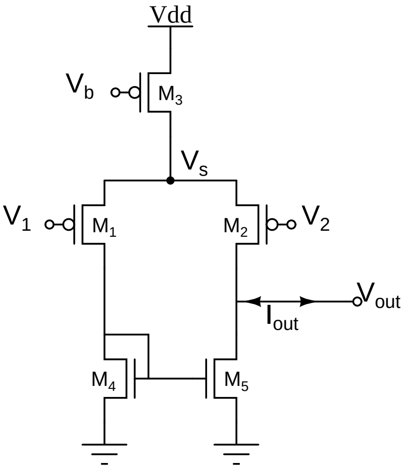
I follows max V

and the saturation condition for M_3, $V_s > 4U_T$ is:

$$\boxed{\max(V_1, V_2) > V_b + \frac{4U_T}{\kappa}}$$

Complementary circuit





Output transistors

The output transistors M_2 and M_5 are the only other transistors that we have to worry about, because we can safely assume that M_1 and M_4 are always in saturation.
The saturations conditions for M_2 and M_5 restrict the output voltage range:

- To keep M_5 in saturation $V_{dd} - V_{out} > 4U_T$.
- To keep M_2 in saturation $V_{out} - V_s > 4U_T$, which implies that

$$\boxed{V_{out} > \kappa(\max(V_1, V_2) - V_b) + 4U_T}$$

(the famous V_{min} problem).

$$\boxed{\kappa(\max(V_1, V_2) - V_b) + 4U_T < V_{out} < V_{dd} - 4U_T}$$

Output transistors

The output transistors M_2 and M_5 are the only other transistors that we have to worry about, because we can safely assume that M_1 and M_4 are always in saturation.
The saturations conditions for M_2 and M_5 restrict the output voltage range:

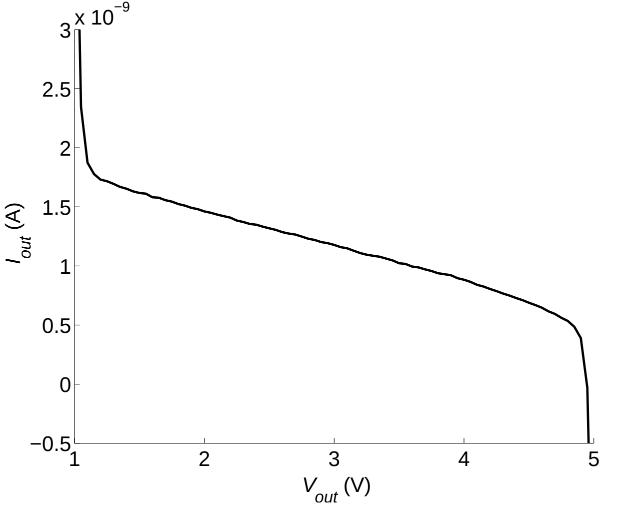
- To keep M_5 in saturation $V_{dd} - V_{out} > 4U_T$.
- To keep M_2 in saturation $V_{out} - V_s > 4U_T$, which implies that

$$\boxed{V_{out} > \kappa(\max(V_1, V_2) - V_b) + 4U_T}$$

(the famous V_{min} problem).

$$\boxed{\kappa(\max(V_1, V_2) - V_b) + 4U_T < V_{out} < V_{dd} - 4U_T}$$

Output current vs output voltage



Above Threshold

If the differential pair is operated above threshold, it can be shown that

$$I_{out} = \frac{\beta}{2}(V_1 - V_2)\sqrt{\frac{4I_b}{\beta} - (V_1 - V_2)^2}$$

where $\beta = \mu C_{ox}\frac{W}{L}$

For $|V_1 - V_2| < \sqrt{2I_b/\beta}$ the transconductance is given by

$$g_m = \sqrt{\beta I_b}$$

Outline

Voltage Amplifier

The transconductance amplifier circuit can also be used as a differential-votage amplifier:

$$V_{out} = A(V_1 - V_2)$$

where A is the *open-circuit voltage gain*.

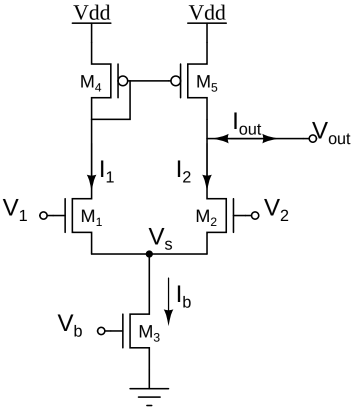
$$A = \frac{dV_{out}}{d(V_1 - V_2)} = \frac{dI_{out}}{d(V_1 - V_2)} \frac{dV_{out}}{dI_{out}}$$

In subthreshold $\boxed{A \approx \frac{\kappa V_E}{2U_T}}$ and above threshold $A \approx \sqrt{\frac{\beta}{I_b}} V_E$.

Voltage amplifier uses

- The open-circuit voltage gain A increases with the Early voltage, and therefore with the length of the output transistors.
- Typical subthreshold values are between 100 and 1000.
- Due to the large gain and *transistor mismatch* effects this circuit is **not** normally used as an open-circuit voltage amplifier.
- In voltage mode, its mainly used as a *comparator*: V_{out} is “high” if $V_1 > V_2$ and “low” if $V_2 > V_1$.

Output voltage limits



We will now compute the limits of the output voltage swing.

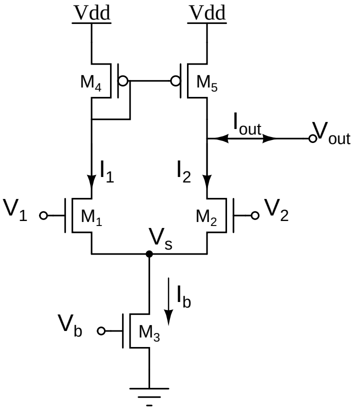
$V_1 > V_2$

For $V_1 > V_2 + 4U_T$ the current through M_2 is much smaller than the one through M_1, hence V_{out} goes almost all the way to V_{dd} to shut M_5 off. M_5 goes out of saturation.

$V_2 > V_1$

The forward current of M2 is $\gg$ than that of M1. But the current mirror imposes $I_1 \approx I_2 \approx I_b/2$. If V_2 is significantly larger than V_1, the voltage drop across M_2 is close to zero and $V_{out} \approx V_s$. M_2 goes out of saturation.

Output voltage limits



We will now compute the limits of the output voltage swing.

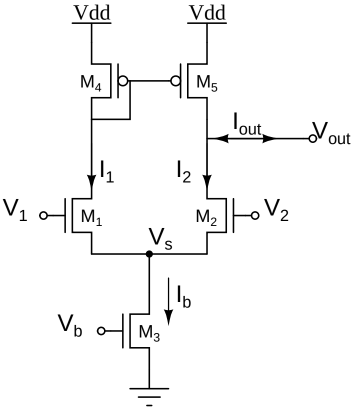
$V_1 > V_2$

For $V_1 > V_2 + 4U_T$ the current through M_2 is much smaller than the one through M_1, hence V_{out} goes almost all the way to V_{dd} to shut M_5 off. M_5 goes out of saturation.

$V_2 > V_1$

The forward current of M2 is $\gg$ than that of M1. But the current mirror imposes $I_1 \approx I_2 \approx I_b/2$. If V_2 is significantly larger than V_1, the voltage drop across M_2 is close to zero and $V_{out} \approx V_s$. M_2 goes out of saturation.

Output voltage limits



We will now compute the limits of the output voltage swing.

$V_1 > V_2$

For $V_1 > V_2 + 4U_T$ the current through M_2 is much smaller than the one through M_1, hence V_{out} goes almost all the way to V_{dd} to shut M_5 off. M_5 goes out of saturation.

$V_2 > V_1$

The forward current of M2 is $\gg$ than that of M1. But the current mirror imposes $I_1 \approx I_2 \approx I_b/2$. If V_2 is significantly larger than V_1, the voltage drop across M_2 is close to zero and $V_{out} \approx V_s$. M_2 goes out of saturation.

Output voltage limits (contd.)

If $V_2 > V_1$, $V_{out} \approx V_s$. But we don't know the value of V_s. We can't assume that M_3 is in saturation, so we have to write:

$$I_b = I_0 e^{\frac{\kappa V_b}{U_T}} \left(1 - e^{-\frac{V_s}{U_T}}\right) = I_1 + I_2$$

with $I_1 = I_0 e^{\frac{\kappa V_1 - V_s}{U_T}}$ (M_1 **is** in saturation), and $I_2 \approx I_1$. So

$$e^{-\frac{V_s}{U_T}} = \frac{e^{\frac{\kappa V_b}{U_T}}}{e^{\frac{\kappa V_b}{U_T}} + e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_1}{U_T}}}$$

$$e^{-\frac{V_s}{U_T}} = \frac{e^{\frac{\kappa V_b}{U_T}}}{e^{\frac{\kappa V_b}{U_T}} + 2e^{\frac{\kappa V_1}{U_T}}}$$

$$e^{-\frac{V_s}{U_T}} = \frac{\frac{1}{2} e^{\frac{\kappa V_b}{U_T}}}{\frac{1}{2} e^{\frac{\kappa V_b}{U_T}} + e^{\frac{\kappa V_1}{U_T}}}$$

Output voltage limits (contd.)

If $V_2 > V_1$, $V_{out} \approx V_s$. But we don't know the value of V_s. We can't assume that M_3 is in saturation, so we have to write:

$$I_b = I_0 e^{\frac{\kappa V_b}{U_T}} (1 - e^{-\frac{V_s}{U_T}}) = I_1 + I_2$$

with $I_1 = I_0 e^{\frac{\kappa V_1 - V_s}{U_T}}$ (M_1 **is** in saturation), and $I_2 \approx I_1$. So

$$e^{-\frac{V_s}{U_T}} = \frac{e^{\frac{\kappa V_b}{U_T}}}{e^{\frac{\kappa V_b}{U_T}} + e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_1}{U_T}}}$$

$$e^{-\frac{V_s}{U_T}} = \frac{e^{\frac{\kappa V_b}{U_T}}}{e^{\frac{\kappa V_b}{U_T}} + 2e^{\frac{\kappa V_1}{U_T}}}$$

$$e^{-\frac{V_s}{U_T}} = \frac{\frac{1}{2} e^{\frac{\kappa V_b}{U_T}}}{\frac{1}{2} e^{\frac{\kappa V_b}{U_T}} + e^{\frac{\kappa V_1}{U_T}}}$$

Output voltage limits (contd.)

If $V_2 > V_1$, $V_{out} \approx V_s$. But we don't know the value of V_s. We can't assume that M_3 is in saturation, so we have to write:

$$I_b = I_0 e^{\frac{\kappa V_b}{U_T}} \left(1 - e^{-\frac{V_s}{U_T}}\right) = I_1 + I_2$$

with $I_1 = I_0 e^{\frac{\kappa V_1 - V_s}{U_T}}$ (M_1 **is** in saturation), and $I_2 \approx I_1$. So

$$e^{-\frac{V_s}{U_T}} = \frac{e^{\frac{\kappa V_b}{U_T}}}{e^{\frac{\kappa V_b}{U_T}} + e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_1}{U_T}}}$$

$$e^{-\frac{V_s}{U_T}} = \frac{e^{\frac{\kappa V_b}{U_T}}}{e^{\frac{\kappa V_b}{U_T}} + 2e^{\frac{\kappa V_1}{U_T}}}$$

$$e^{-\frac{V_s}{U_T}} = \frac{\frac{1}{2} e^{\frac{\kappa V_b}{U_T}}}{\frac{1}{2} e^{\frac{\kappa V_b}{U_T}} + e^{\frac{\kappa V_1}{U_T}}}$$

Output voltage limits (contd.)

If $V_2 > V_1$, $V_{out} \approx V_s$. But we don't know the value of V_s. We can't assume that M_3 is in saturation, so we have to write:

$$I_b = I_0 e^{\frac{\kappa V_b}{U_T}} (1 - e^{-\frac{V_s}{U_T}}) = I_1 + I_2$$

with $I_1 = I_0 e^{\frac{\kappa V_1 - V_s}{U_T}}$ (M_1 **is** in saturation), and $I_2 \approx I_1$. So

$$e^{-\frac{V_s}{U_T}} = \frac{e^{\frac{\kappa V_b}{U_T}}}{e^{\frac{\kappa V_b}{U_T}} + e^{\frac{\kappa V_1}{U_T}} + e^{\frac{\kappa V_1}{U_T}}}$$

$$e^{-\frac{V_s}{U_T}} = \frac{e^{\frac{\kappa V_b}{U_T}}}{e^{\frac{\kappa V_b}{U_T}} + 2e^{\frac{\kappa V_1}{U_T}}}$$

$$e^{-\frac{V_s}{U_T}} = \frac{\frac{1}{2}e^{\frac{\kappa V_b}{U_T}}}{\frac{1}{2}e^{\frac{\kappa V_b}{U_T}} + e^{\frac{\kappa V_1}{U_T}}}$$

Output voltage limits (contd.)

So, if

- $V_1 < V_b - (4 + \ln(2))U_T/\kappa$,

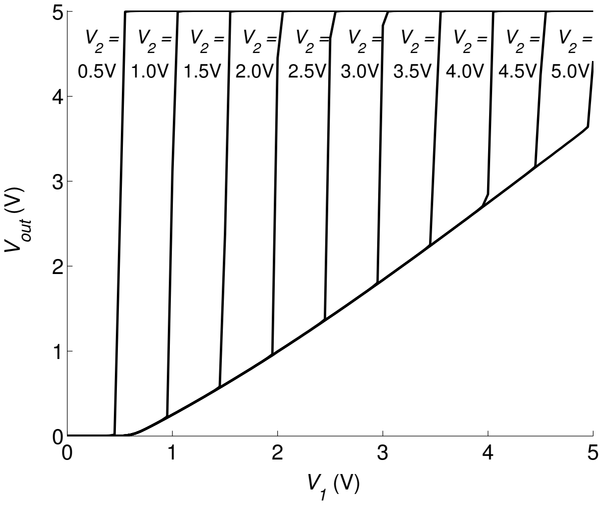
$$V_{out} \approx 0$$

- $V_1 > V_b - (4 + \ln(2))U_T/\kappa$,

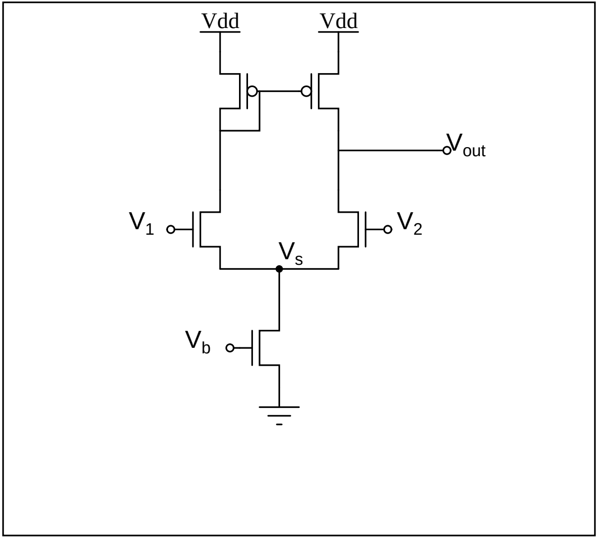
$$\boxed{V_{out} \approx \kappa V_1 - \kappa V_b + U_T \ln(2)}$$

V_{out} increases linearly with V_1, with a slope of κ.

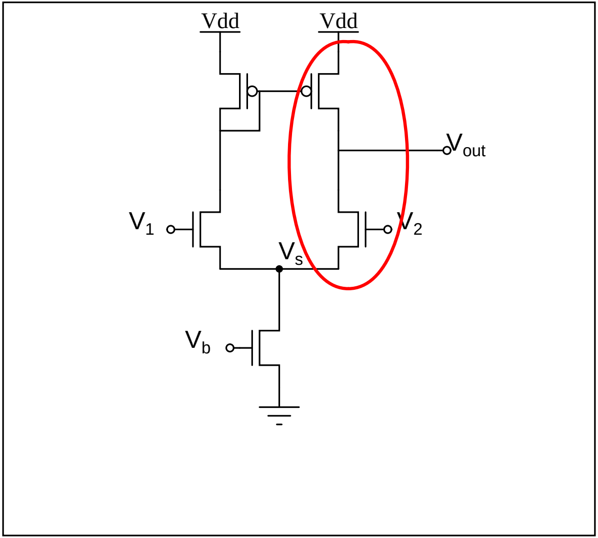
Output-circuit output voltage



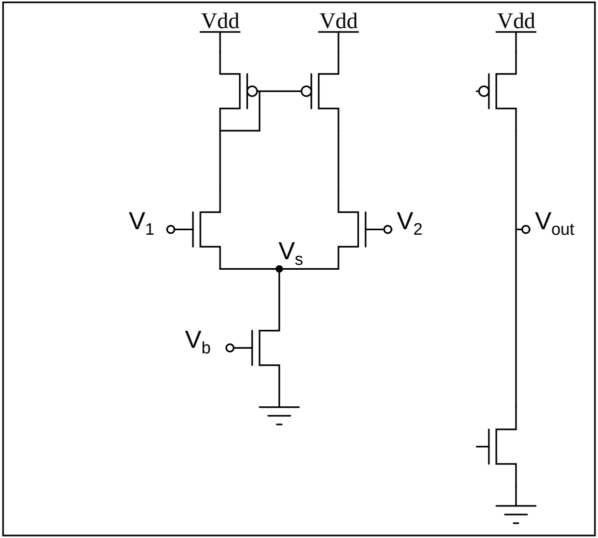
Wide-output-range circuit



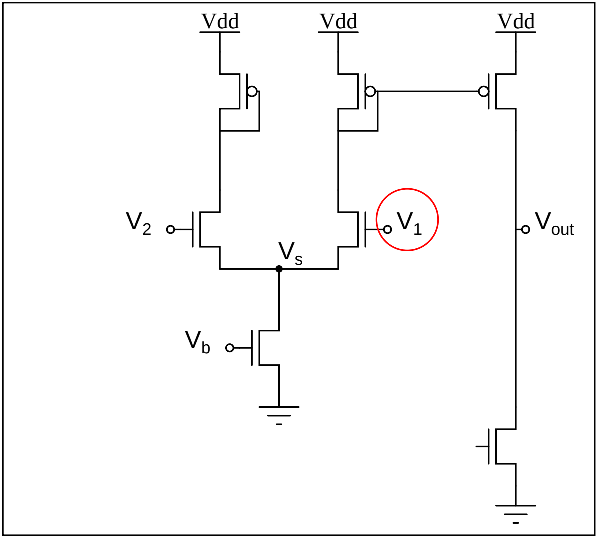
Wide-output-range circuit



Wide-output-range circuit



Wide-output-range circuit



Wide-output-range circuit