mtcars analysis (extension)

Pascal P

24 August 2018

Contents

1 Summary

This document is an (optional investigative) extension of another study `analysis_mtcars.Rmd` (located in: https://github.com/pascal-p/PA_RM).

Having noticed a possible interaction between `wt` and `am` in the `mtcars` dataset, I would like to present this alternative model and compare it with initial model (in `analysis_mtcrs.Rmd` document).

In the following, I am going to present the alternative model, investigate the regression diagnostics and then present a comparison before presenting the conclusions which state what I believe was achieved.

2 Analysis (alternative model)

2.1 Preparation

The dataset (`mtcars`) once prepared, meaning all numerical (discrete) values (for features `cyl`, `vs`, `gear`, `carb` and `am`) converted as factor look as followed (extract):

	mpg	cyl	disp	hp	drat	wt	qsec	vs	am	gear	carb
Mazda RX4	21.0	6	160	110	3.90	2.620	16.46	0	manual	4	4
Mazda RX4 Wag	21.0	6	160	110	3.90	2.875	17.02	0	manual	4	4
Datsun 710	22.8	4	108	93	3.85	2.320	18.61	1	manual	4	1
Hornet 4 Drive	21.4	6	258	110	3.08	3.215	19.44	1	auto	3	1
Hornet Sportabout	18.7	8	360	175	3.15	3.440	17.02	0	auto	3	2
Valiant	18.1	6	225	105	2.76	3.460	20.22	1	auto	3	1

2.2 Model

Let us consider, the interaction between `wt` and `am` (as noted on our first plots for exploratory data analysis in document `analysis_mtcars.Rmd`).

```
library(car)
best.model_with_interaction <- lm(mpg ~ wt + qsec + am + wt:am, data = mtcars)

summary(best.model_with_interaction)$coefficient
```

```
             Estimate Std. Error   t value     Pr(>|t|)
(Intercept)  9.723053  5.8990407  1.648243 0.1108925394
wt          -2.936531  0.6660253 -4.409038 0.0001488947
qsec         1.016974  0.2520152  4.035366 0.0004030165
ammanual    14.079428  3.4352512  4.098515 0.0003408693
wt:ammanual -4.141376  1.1968119 -3.460340 0.0018085763
```

```
summary(best.model_with_interaction)$r.squared
```

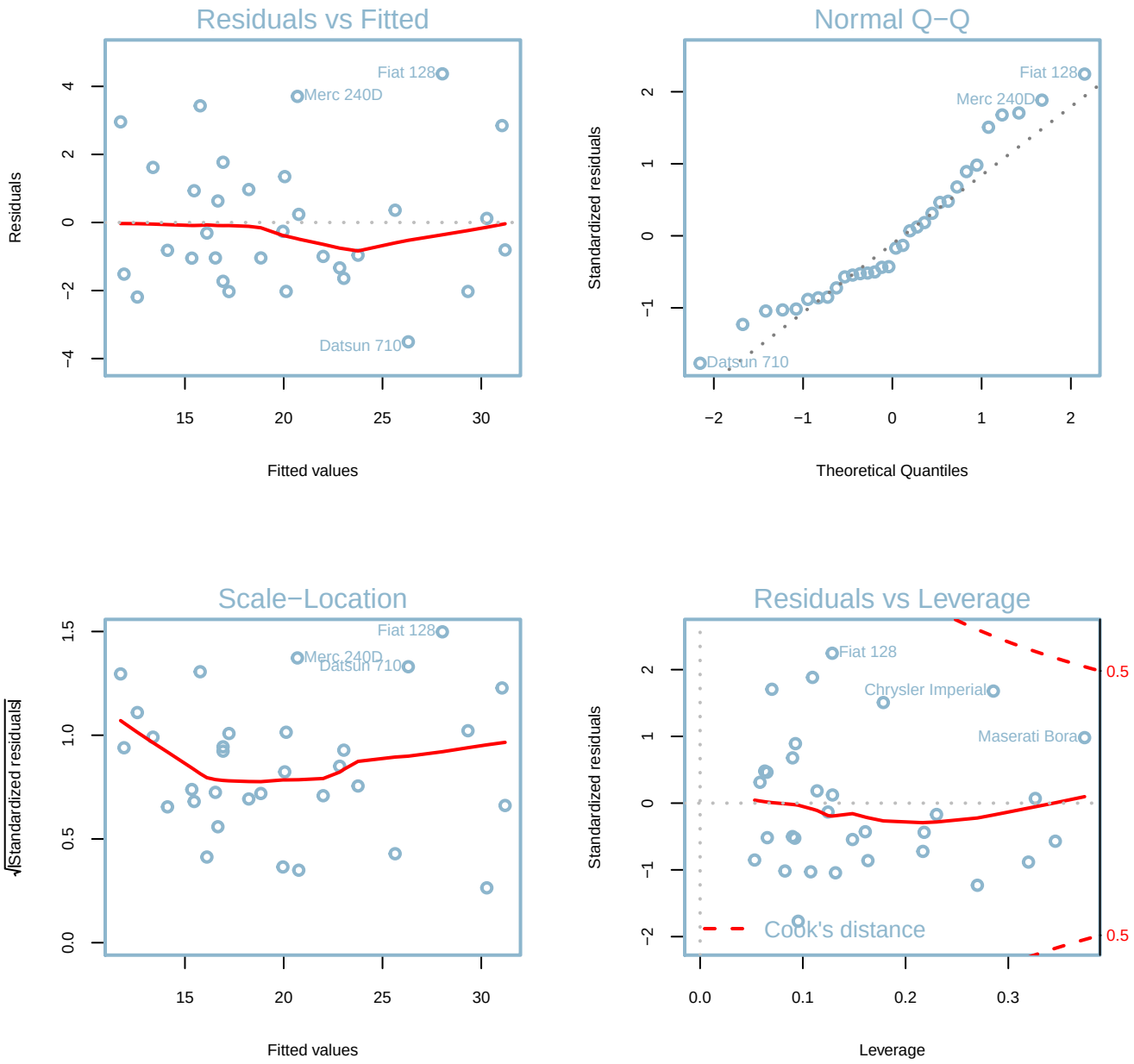
```
[1] 0.8958514
```

All estimates is this model have small p-value and are therefore significantly different from zero. The percentage of variance explained (linked to R^2 statistic) is about 89,6%.
When holding `wt` and `qsec` constant, manual transmission cars have on average $14.079 - 4.141 = 9.938$ more`mpg` than automatic transmission ones.

2.3 Regression diagnostics

2.3.1 Multiplots

The standard plots for residual diagnostics are presented below:



We observe that:

- there is no particular pattern for `Residuals vs fitted` (assumption of linearity), `Scale-Location` (assumption of homoscedasticity) plots,
- we may have in `Residuals vs Leverage` plots (unusual observations), some points to look at (for later).
- Independence of the dependent variable values (`mpg`) can be assumed given the dataset (my assumption, no reason to believe that the characteristics of one car model affects directly another one).
- however it is clear that the assumption of normality (on the residuals) does not hold (`Normal Q-Q` plot), this may suggest:
  1 - that we need to further assess the robustness of that assumption (with `Shapiro-Wilk normality test`) or alternatively
  2 - we need to transform the response variable `mpg` (using `powerTransform` function from `car` package).

2.3.2 Going further with Normality assumption

Let's try the `Shapiro-Wilk normality test` on the residuals:

```
shapiro.test(best.model_with_interaction$res)
```

```
    Shapiro-Wilk normality test

data:  best.model_with_interaction$res
W = 0.94444, p-value = 0.1001
```

And here we would conclude that the normality assumption (on the residuals) is holding (p-value of 0.1).

2.3.3 Transforming the response variable

The alternative suggested was to transform the response variable (denoted Y, to Y^{λ}. Let's check this with the following code:

```
summary(powerTransform(mtcars$mpg))
```

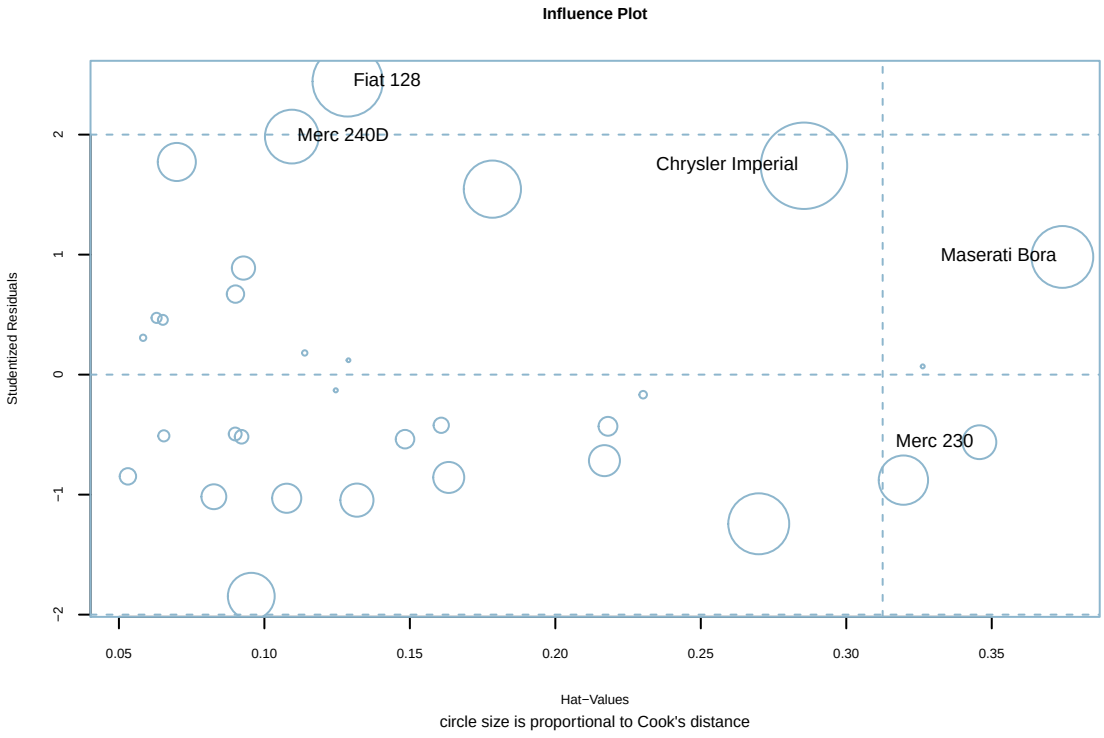
```
bcPower Transformation to Normality
           Est Power Rounded Pwr Wald Lwr Bnd Wald Upr Bnd
mtcars$mpg    0.0296           1      -1.0107       1.0698

Likelihood ratio test that transformation parameter is equal to 0
 (log transformation)
                              LRT df    pval
LR test, lambda = (0) 0.00310595  1 0.95556

Likelihood ratio test that no transformation is needed
                             LRT df    pval
LR test, lambda = (1) 3.212664  1 0.07307
```

A log transformation is suggested although the non-transformation hypothesis ($\lambda = 1$) cannot be rejected (p-value of 0.07), which means that there is not so much evidence that a transformation is required.

2.3.4 Unusual observation



	StudRes	Hat	CookD
Merc 240D	1.9835159	0.1094330	0.0872122
Merc 230	-0.5632192	0.3457671	0.0344001
Chrysler Imperial	1.7404165	0.2854605	0.2251060
Fiat 128	2.4433618	0.1286155	0.1488367
Maserati Bora	0.9806330	0.3742640	0.1151987

We observe:

- `Fiat 128` is an outlier,
- `Merc 230` and `Maserati Bora` have high leverage and
- and observations with relative large circle may have disproportionate influence.

2.4 Comparisons

Let's compare this model with the one presented in original analysis (document `analysis_mtcars.Rmd`), with `anova` and `AIC` (Akaike Information Criterion) functions. First let's show the previous model, as followed:

```
             Estimate Std. Error   t value     Pr(>|t|)
(Intercept)  9.617781  6.9595930  1.381946 1.779152e-01
wt          -3.916504  0.7112016 -5.506882 6.952711e-06
qsec         1.225886  0.2886696  4.246676 2.161737e-04
ammanual     2.935837  1.4109045  2.080819 4.671551e-02
```

```
[1] 0.8496636
```

The following comparison tells us that our model with interaction is indeed better (small p-value) by adding the interaction term.

```
anova(prev_best.model, best.model_with_interaction)
```

Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
28	169.2859	NA	NA	NA	NA
27	117.2763	1	52.00966	11.97396	0.0018086

With the following comparison we look for the model with the smallest `AIC` values, and once again it is suggested that adding the interaction term is relevant.

```
AIC(prev_best.model, best.model_with_interaction)
```

	df	AIC
prev_best.model	5	154.1194
best.model_with_interaction	6	144.3736

3 Conclusion

- Interpreting the results correctly looks more like an art (with science) than a science
- The size of the sample (32 observations) is quite small and even more in comparison with the number of features (11).
  This small sample size may limit the power of our analysis and the accuracy of our conclusion
- The alternative model with interaction term seems better (than original model) and almost compatible with all the related assumptions of a multi-linear regression model: Independence, Linearity, Homoscedasticity (constant variance), and Normality (although not clear cut to me for this one).

- If we accept the previous point, we can state that:

1. the alternative model does not change the main claim of the original one, namely that manual transmission cars have a better `mpg` (lower consumption) than automatic ones (a 2.94 more mpg vs 9.94 more mpg with the alternative model).
2. however the alternative model (with interaction term between `wt` (weight and `am` transmission)) is slightly better.

Would this hold on a larger dataset? would we be able to better select alternative models and use cross-validation to quantify how better models are? It looks like we need, as usual more data (if not more features).