

# MACRO III

## *Exercise 6*

- Structural Unemployment -

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## Exercise 6.1

*In the figure below, downloaded from the U.S. Bureau of Labor Statistics, <https://www.bls.gov/charts/employment-situation/civilian-unemployment-rate.htm>, you can observe the evolution of the monthly total unemployment rate over the past 20 years.*

*a) How is the rate of unemployment defined and measured by the U.S. Bureau of Labor Statistics? In particular, how is the number of unemployed counted? How is the labor force measured?*

The US Bureau of Labor Statistics (BLS) releases a new unemployment rate each month. It would be way too expensive and take too long to ask each citizen whether or not they are unemployed. Thus, the BLS conducts the so-called Current Population Survey (CPS) where they question 60,000 eligible households that are representative of the US population.

The way they identify the employed and unemployed is straight forward:

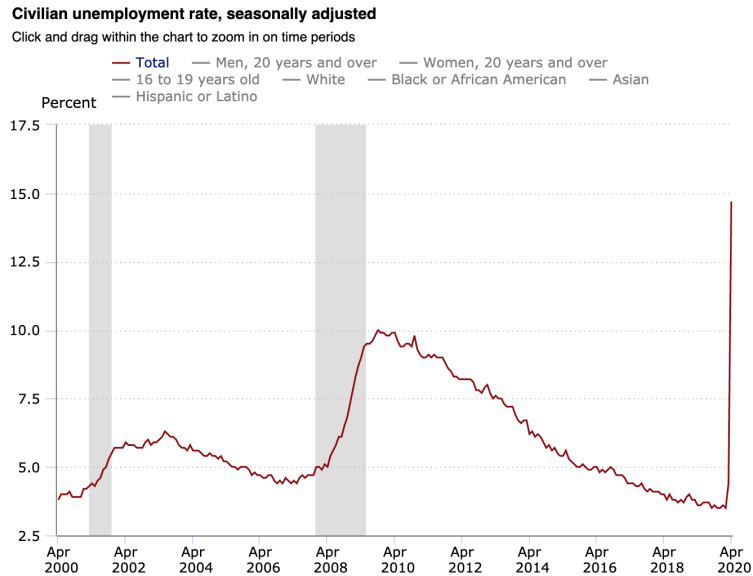
- People with jobs are employed
- People who are jobless, looking for a job, and for work are unemployed
- People who are neither employed nor unemployed are not in the labor force

The sum of the employed and unemployed makes up the labor force. They exclude people living in institutions and active members of the armed forces. People who are ill, on vacation or on maternity/paternity leave are still counted as employed. People are classified as unemployed if they do not have a job, have actively looked for work in the prior 4 weeks, and are currently available for work. Actively looking for work may consist of any of the following activities:

- Contacting
- Submitting resumes or filling out applications
- Placing or answering job advertisements
- Checking union or professional registers
- Some other means of active job search

b) Explain in one paragraph the evolution of the unemployment rate over this period.

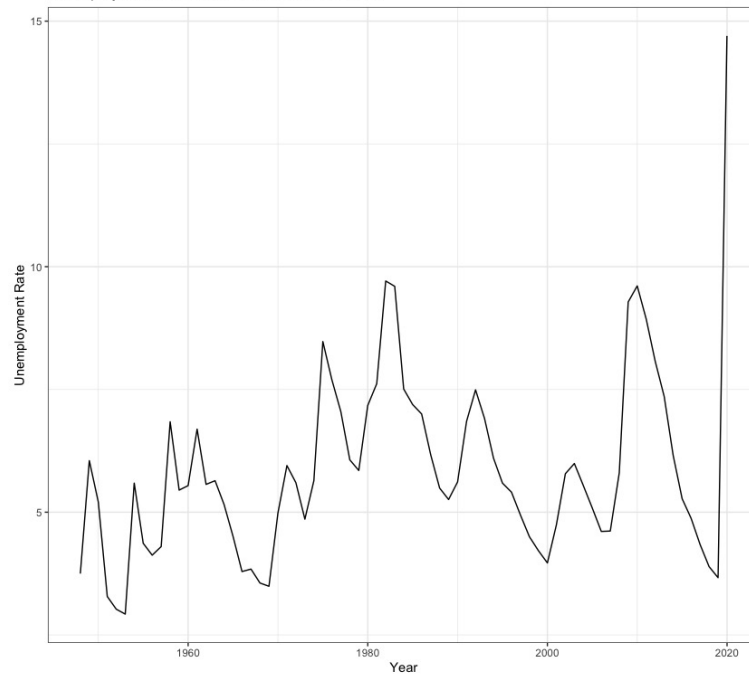
Figure 1: Unemployment Rate in the US from 2000-2020



In the graph above, one can observe the fluctuations of the unemployment rate in the US from 2000-2020. We can see a moderate increase in unemployment after the Dot-Com Bubble in the beginning of the 2000s. Later the rate decreased again until the Financial Crisis started in 2008, where it went all the way up to 10%. After that the unemployment rate decreased steadily and reached their lowest rate during this period in the beginning of 2020. However, as COVID-19 led to a immense shut-down of the economy, many workers were laid off. This is visible as the unemployment rate spikes and is higher than ever before during this period. It is currently at around 15% of the labor force.

c) Download the unemployment rate series from <https://data.bls.gov/> for the sample period 1948-2020. Plot the series. Compute the average unemployment rate over this period and briefly discuss the value.

Figure 2: Unemployment Rate in the US from 1948-2020  
Unemployment Rate in the US from 1948-2020



The average unemployment rate during this time period is **5.74**. This means that the cyclical unemployment rate fluctuates around 5.74, which is the structural or long-run unemployment rate.

## Exercise 6.2

Consider an efficiency wage model with one firm, as discussed in chapter 11.2 of the textbook. The firm's profits are  $zR(a(w')L) - w'L$ , where  $w'$  denotes the real wage paid by this firm. Furthermore, assume that the efficiency function is

$$a = (w' - v)^\eta, \quad 0 \leq \eta < 1 \quad (1)$$

where  $v$  is the employee's outside option.

a) Find the first order conditions for the firm's profit maximization problem with respect to  $w'$  and  $L$ . Derive the Solow condition by combining the two conditions. Find the expression for the optimal wage as a function of  $v$  and  $\eta$ .

We first define the firm's profit as  $\pi(w', L) = zR(a(w')L) - w'L$ . The first order conditions are given by

$$\frac{\partial \pi}{\partial w'} = zR'(a(w')L) a'(w')L - L = 0 \quad (2)$$

$$\frac{\partial \pi}{\partial L} = zR'(a(w')L) a(w') - w' = 0 \quad (3)$$

Rearranging the expressions we get

$$zR'(a(w')L) a'(w') = 1 \quad (4)$$

$$zR'(a(w')L) a(w') = w' \quad (5)$$

Dividing (4) by (5) yields:

$$\begin{aligned}\frac{zR'(a(w')L)a'(w')}{zR'(a(w')L)a(w')} &= \frac{1}{w'} \\ \iff \frac{a'(w')}{a(w')} &= \frac{1}{w'} \\ \iff \frac{a'(w')}{a(w')}w' &= 1\end{aligned}\tag{6}$$

Equation (6) is the Solow condition. To find the optimal wage, it is necessary to insert the efficiency function (1) in the Solow condition (6) and solve for  $w'$ . First we note that:

$$a(w') = (w' - v)^\eta, \quad a'(w') = \eta(w' - v)^{\eta-1}$$

Inserting the efficiency function and its derivation in (6) yields:

$$\begin{aligned}\frac{\eta(w' - v)^{\eta-1}}{(w' - v)^\eta}w' &= 1 \\ \iff \frac{\eta}{(w' - v)}w' &= 1\end{aligned}$$

Solving for  $w'$  gives us the optimal wage:

$$\begin{aligned}\eta w' &= w' - v \\ \iff \eta w' - w' &= -v \\ \iff w' - \eta w' &= v\end{aligned}$$

$$\Longleftrightarrow w'(1 - \eta) = v$$

$$\Longleftrightarrow w' = \frac{v}{(1 - \eta)} \tag{7}$$

Equation (7) is the optimal wage as a function of  $v$  and  $\eta$ .

b) *How does the optimal wage depend on  $v$  and  $\eta$ ? Explain the economic mechanisms.*

Observing Equation (7) it is clear that the optimal wage level depends positively on  $v$  and  $\eta$ . The intuition behind it lies in the assumption of efficiency wages, which presumes that productivity depends positively on (real) wage. Thus,  $v$  can be interpreted as the starting wage, where labour starts to get productive (as a proof, one can take the efficiency equation (1), set to zero, solve for  $w'$  and see it is exactly equal to  $v$ ), so if  $v$  gets bigger, representative firm has to adjust the optimal wage accordingly.

On the other hand,  $\eta$  represents the wage elasticity of productivity. One can interpret it as a psychological positive effect on workers productivity, such as "enjoying working". For  $\eta$  close to one, workers have a lot of fun working, so a small change in wage imply a big gain in productivity. If instead  $\eta$  is close to zero, large wage changes are necessary to have a modest increase in productivity.

One should also keep in mind that under efficiency wage assumption representative firms are not willing to pay the lowest wage possible. They rather try to minimise the price per efficient unit  $\frac{w}{a(w)}$  (one can also interpret it as a maximization of  $\frac{a(w)}{w}$ ).

*From now on, assume that the outside option is*

$$v = (1 - u)w + ucw, \quad 0 < c < 1 \quad (8)$$

*where  $u$  is the unemployment rate,  $c$  is the replacement ratio, and  $w$  is the general real wage level.*



c) Using the outside option, express the optimal wage rate of the representative firm as a function of  $\eta$ ,  $u$ ,  $w$  and  $c$ . Then use the equilibrium condition  $w = w'$  to derive the equilibrium rate of unemployment  $u^*$ .

In order to express the optimal wage rate of the representative firm as a function of  $\eta$ ,  $u$ ,  $w$ , and  $c$ , we insert (8) in the Solow condition solved for  $w'$  (7):

$$\begin{aligned}
 w' &= \frac{v}{(1-\eta)} = \frac{(1-u)w + ucw}{(1-\eta)} \\
 \Leftrightarrow w' &= \frac{w - u(w - cw)}{(1-\eta)} \\
 \Leftrightarrow w' &= \frac{w - u(w(1-c))}{(1-\eta)} \tag{9}
 \end{aligned}$$

Equation (9) demonstrates the optimal wage rate of the representative firm. Then setting  $w = w'$  and solving for  $u$  gives the equilibrium rate of unemployment,  $u^*$ :

$$\begin{aligned}
 w' &= \frac{w' - u(w'(1-c))}{(1-\eta)} \\
 \Leftrightarrow w'(1-\eta) &= w' - u(w'(1-c)) \\
 \Leftrightarrow w' - w'\eta &= w' - u(w'(1-c)) \\
 \Leftrightarrow -w'\eta &= -u(w'(1-c)) \\
 \Leftrightarrow \frac{w'\eta}{w'(1-c)} &= u \\
 \Leftrightarrow u^* &= \frac{\eta}{1-c} \tag{10}
 \end{aligned}$$

Thus, the equilibrium rate of unemployment,  $u^*$ , is given by Equation (10).

*d) How do the parameters  $\eta$  and  $c$  affect  $u^*$ ? Explain the economic mechanisms.*

Looking at the formula,  $u^* = \frac{\eta}{1-c}$ , we can see that both  $\eta$  and  $c$  are positively correlated with  $u^*$ . The  $\eta$  parameter is the wage elasticity of effort. Assuming  $0 < \eta < 1$ , the bigger  $\eta$  is, the more a worker's effort responds to an increase in wage, implying that his wage has been minimized close to a minimal level  $v$ . Knowing this, he has an incentive to become unemployed and receive a compensation close to the minimum wage not doing any effort, thus increasing the equilibrium unemployment rate. The  $c$  parameter represents the replacement ratio, that is the income of an unemployed worker as a proportion of the income when in work. Assuming  $0 < c < 1$ , the closer  $c$  gets to 1, the bigger the equilibrium unemployment rate becomes. This makes sense as a  $c$  with a value close to one means that someone would receive almost the same income being unemployed than he would being employed, which would give him an incentive not to work anymore.

*e) Insert the expression you found for  $u^*$  into the outside option  $v$ ; this gives you  $v^*$ . The plug this  $v^*$  into the efficiency function  $a$ . Assuming  $\eta = 0.01$ , what is the minimum the firm has to pay if it wants its workers to exert any effort?*

Plugging our equilibrium unemployment rate  $u^*$  into the outside option  $v$  yields:

$$v^* = w(1 - \eta)$$

Plugging our equilibrium outside option  $v^*$  into the efficient function  $a$  yields:

$$a = (w'\eta)^\eta$$

Assuming  $\eta = 0.01$  and that showing effort means  $a=1$ , then the minimum wage the firm has to pay if it wants its workers to exert any effort is  $w'=100$ . This was obtained by replacing  $\eta$  and  $a$  into the efficiency function and solving for  $w$ .