

Hands-On Fluids Challenge

Abstract

Hello

1 Theoretical Calculation

The first objective of the experiment is to model the height of the jet as a function of the speed of the displacement of the piston of the syringe. This is accomplished by defining a relationship between the velocity of the fluid leaving the tip of the needle at ② and the velocity of the fluid at the highest point of the jet stream at ③ through the mass balance equation and Bernoulli's Principle. These locations are illustrated in Figure 1.

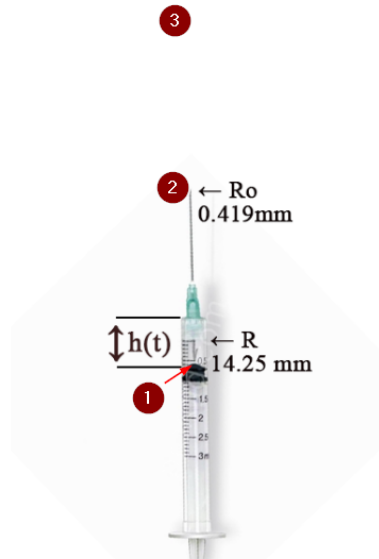


Figure 1: An illustration of the syringe. The label points are used in mass balance and Bernoulli.

From the mass balance equation,

$$0 = \frac{\partial}{\partial t} \int_{cv} \rho dv + \int_{cs} \rho \bar{v} \cdot d\bar{A}$$

The control volume consists of the entire syringe (i.e. all volumes enclosed between ① and ②). As there is fluid existing the control volume, $\frac{\partial}{\partial t} \int_{cv} \rho dv$ is non zero. Integrating the two terms shown in (1) yields the following results:

$$\frac{\partial}{\partial t} \int_{cv} \rho dv = \frac{dm}{dt}$$

$$\int_{cs} \rho \bar{v} \cdot d\bar{A} = \rho \bar{v}_2 \pi R_o^2$$

Substituting the individual terms into (1) results in

$$\frac{dm}{dt} = -\rho \bar{v}_2 \pi R_o^2$$

$$\rho \pi R^2 \frac{dh}{dt} = \rho \pi \bar{v}_2 R_o^2$$

Since the jet stream is a free jet, the velocity profile can be assumed to be uniform and therefore, $\bar{v}_2 = v_2$.

$$\frac{dh}{dt} = \left(\frac{R_o}{R}\right)^2 v_2$$

$$v_2 = \frac{dh}{dt} \left(\frac{R}{R_o}\right)^2 \quad (1)$$

The highest point of the jet stream (i.e. ③) is related to the exist velocity at ② by Bernoulli's Principle.

$$\frac{P_2}{\rho} + gz_2 + \frac{\alpha v_2^2}{2} = \frac{P_3}{\rho} + gz_3 + \frac{\alpha v_3^2}{2}$$

Taking z_2 as the reference point reduces the term to zero and the velocity at the highest point is also zero.

$$\frac{P_2}{\rho} + \cancel{gz_2} + \frac{\alpha v_2^2}{2} = \frac{P_3}{\rho} + gz_3 + \frac{\alpha \cancel{v_3^2}}{2} \quad (2)$$

where

$$P_2 = P_{atm} + P_c$$

$$P_c = \frac{2\gamma}{d} = \frac{2\gamma}{2R_o} = \frac{\gamma}{R_o} \text{ and } \gamma = 72 \frac{mN}{m} \text{ for air-water interface}$$

and

$$\alpha = 2 \text{ for laminar flow as shown by experimental calculation}$$

Substituting back to (2):

$$z_3 = \frac{1}{g} \left(\frac{\gamma}{R_o \rho} + \left(\frac{dh}{dt} \right)^2 \left(\frac{R}{R_o} \right)^4 \right)$$

$$z_3 = \frac{1}{9.81} \left(\frac{72 \times 10^{-3}}{0.419 \times 10^{-3} \times 1000} + \left(\frac{dh}{dt} \right)^2 \left(\frac{14.25 \times 10^{-3}}{0.419 \times 10^{-3}} \right)^4 \right)$$

$$= 0.102 \times (0.172 + \left(\frac{dh}{dt} \right)^2 \times 1.34 \times 10^6) \text{ where } \frac{dh}{dt} \text{ varies in each experiment}$$