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Hands-On Fluids Challenge

Abstract

Hello

1 Theoretical Calculation

The first objective of the experiment is to model the height of the jet as a function of the speed of the displacement of the piston of the syringe. This is accomplished by defining a relationship between the velocity of the fluid leaving the tip of the needle at ② and the velocity of the fluid at the highest point of the jet stream at ③ through the mass balance equation and Bernoulli's Principle. These locations are illustrated in Figure 1.

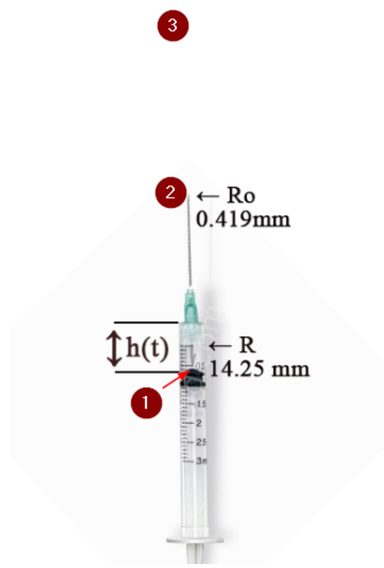

Figure 1: An illustration of the syringe. The label points are used in mass balance and Bernoulli.

From the mass balance equation,

$$0 = \frac{\partial}{\partial t}\int_{cv} \rho dv + \int_{cs} \rho \bar{v} \cdot d\bar{A}$$

The control volume consists of the entire syringe (i.e. all volumes enclosed between ① and ②). As there is fluid existing the control volume, $\frac{\partial}{\partial t}\int_{cv} \rho dv$ is non zero. Integrating the two terms shown in (1) yields the following results:

$$\frac{\partial}{\partial t}\int_{cv} \rho dv = \frac{dm}{dt}$$

$$\int_{cs} \rho \bar{v} \cdot d\bar{A} = \rho \bar{v_2} \pi R_o^2$$

Substituting the individual terms into (1) results in

$$\frac{dm}{dt} = -\rho \bar{v_2} \pi R_o^2$$

$$\rho \pi R^2 \frac{dh}{dt} = \rho \pi \bar{v_2} R_o^2$$

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Since the jet stream is a free jet, the velocity profile can be assumed to be uniform and therefore, $\bar{v}_2 = v_2$.

$$\frac{dh}{dt} = -(\frac{R_o}{R})^2 v_2$$

$$v_2 = -\frac{dh}{dt}(\frac{R}{R_o})^2 \tag{1}$$

The highest point of the jet stream (i.e. 3) is related to the exist velocity at 2 by Bernoulli's Principle.

$$\frac{P_2}{\rho} + gz_2 + \frac{\alpha v_2^2}{2} = \frac{P_3}{\rho} + gz_3 + \frac{\alpha v_3^2}{2}$$

Taking z_2 as the reference point reduces the term to zero and the velocity at the highest point is also zero.

$$\frac{P_2}{\rho} + g\cancelto{0}{z_2} + \frac{\alpha v_2^2}{2} = \frac{P_3}{\rho} + gz_3 + \cancelto{0}{\frac{\alpha v_3^2}{2}} \tag{2}$$

where

$$P_2 = P_atm + P_c$$

$$P_c = \frac{2\gamma}{d} = \frac{2\gamma}{2R_o} = \frac{\gamma}{R_o} \text{ and } \gamma = 72\frac{mN}{m} \text{ for air-water interface}$$

and

$$\alpha = 2 \text{ for laminar flow as shown by experimental calculation}$$

Substituting (1) back to (2):

$$z_3 = \frac{1}{g}(\frac{\gamma}{R_o\rho} + (\frac{dh}{dt})^2(\frac{R}{R_o})^4)$$

where z_3 is the highest point the jet stream can reach in meters.

$$\begin{aligned} z_3 = \frac{1}{9.81}(\frac{72 \times 10^{-3}}{0.419 \times 10^{-3} \times 1000} + (\frac{dh}{dt})^2(\frac{14.25 \times 10^{-3}}{0.419 \times 10^{-3}})^4) \\ = 0.102 \times (0.172 + (\frac{dh}{dt})^2 \times 1.34 \times 10^6) \text{ where } \frac{dh}{dt} \text{ varies in each experiment} \end{aligned}$$

2 Discussion