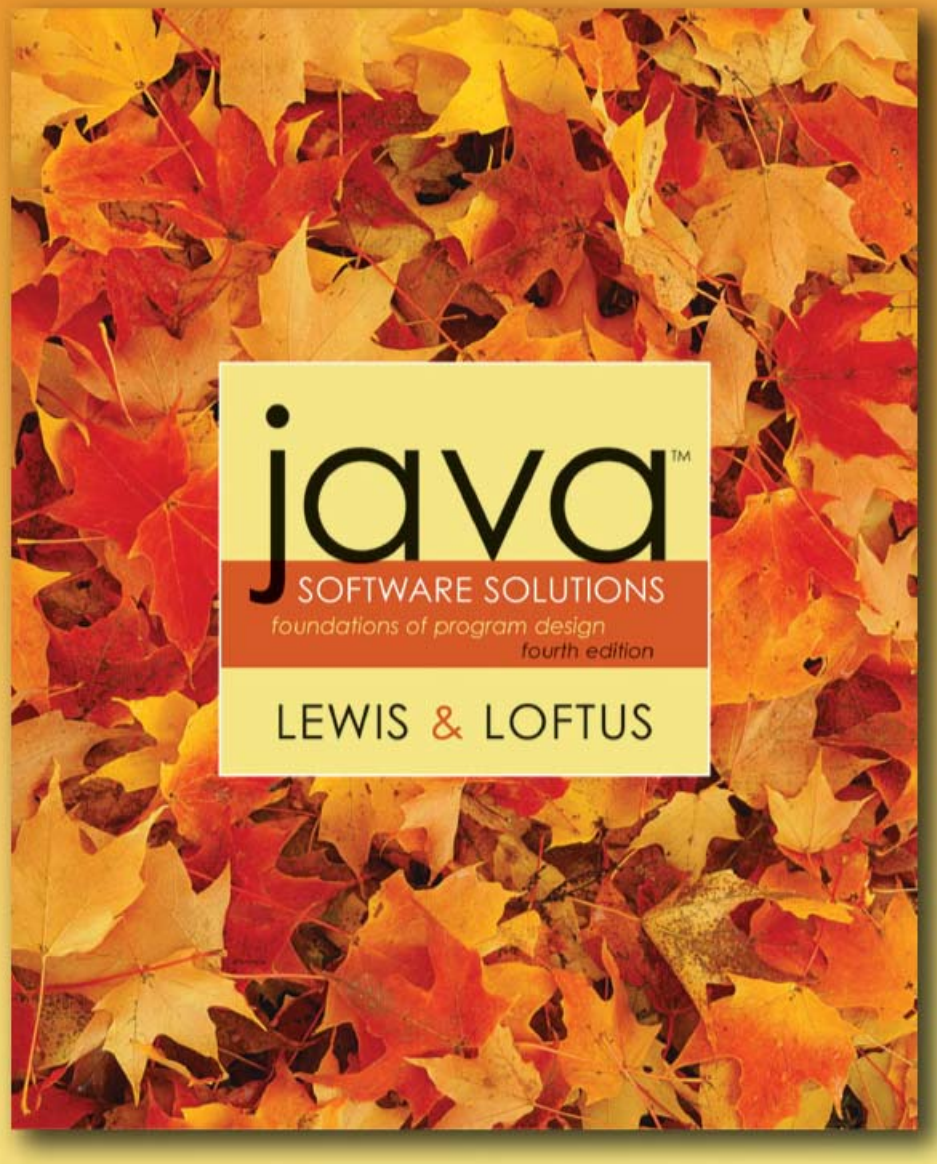
Lecture 11

Recursion

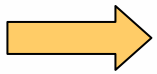


PEARSON Addison Wesley

Recursion

- **Recursion is a fundamental programming technique that can provide an elegant solution certain kinds of problems**
- **Lecture 11 focuses on:**
  - **thinking in a recursive manner**
  - **programming in a recursive manner**
  - **the correct use of recursion**
  - **recursion examples**

Outline



Recursive Thinking

Recursive Programming

Using Recursion

Recursive Thinking

- A *recursive definition* is one which uses the word or concept being defined in the definition itself
- When defining an English word, a recursive definition is often not helpful
- But in other situations, a recursive definition can be an appropriate way to express a concept
- Before applying recursion to programming, it is best to practice thinking recursively

Recursive Definitions

- Consider the following list of numbers:

```
24, 88, 40, 37
```

- Such a list can be defined as follows:

```
A LIST is a:  number
      or a:  number  comma  LIST
```

- That is, a LIST is defined to be a single number, or a number followed by a comma followed by a LIST

- The concept of a LIST is used to define itself

Recursive Definitions

- **The recursive part of the LIST definition is used several times, terminating with the non-recursive part:**

```
number comma LIST
  24     ,   88, 40, 37

             number comma LIST
               88     ,   40, 37

                          number comma LIST
                            40     ,   37

                                       number
                                         37
```

Infinite Recursion

- **All recursive definitions have to have a non-recursive part**
- **If they didn't, there would be no way to terminate the recursive path**
- **Such a definition would cause *infinite recursion***
- **This problem is similar to an infinite loop, but the non-terminating "loop" is part of the definition itself**
- **The non-recursive part is often called the *base case***

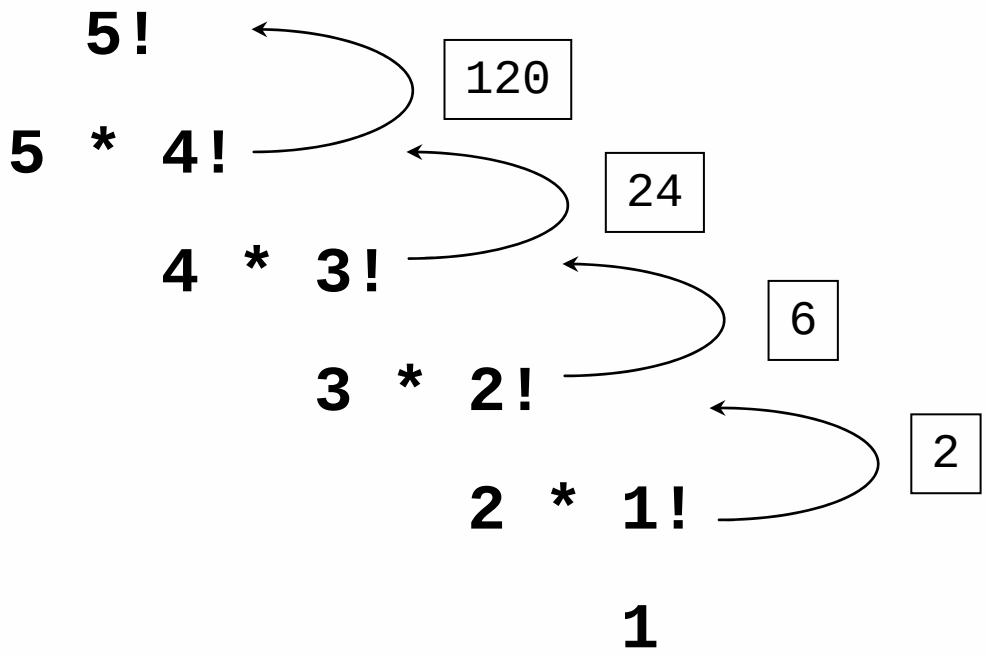
Recursive Definitions

- N!, for any positive integer N, is defined to be the product of all integers between 1 and N inclusive
- This definition can be expressed recursively as:

```
1!  =  1
N!  =  N * (N-1)!
```

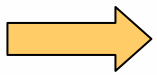
- A factorial is defined in terms of another factorial
- Eventually, the base case of 1! is reached

Recursive Definitions



Outline

Recursive Thinking



Recursive Programming

Using Recursion

Recursive Programming

- A method in Java can invoke itself; if set up that way, it is called a *recursive method*
- The code of a recursive method must be structured to handle both the base case and the recursive case
- Each call to the method sets up a new execution environment, with new parameters and local variables
- As with any method call, when the method completes, control returns to the method that invoked it (which may be an earlier invocation of itself)

Recursive Programming

- **Consider the problem of computing the sum of all the numbers between 1 and any positive integer N**
- **This problem can be recursively defined as:**

$$\sum_{i=1}^{N} i = N + \sum_{i=1}^{N-1} i = N + N-1 + \sum_{i=1}^{N-2} i$$

$$= N + N-1 + N-2 + \sum_{i=1}^{N-3} i$$

$$\vdots$$

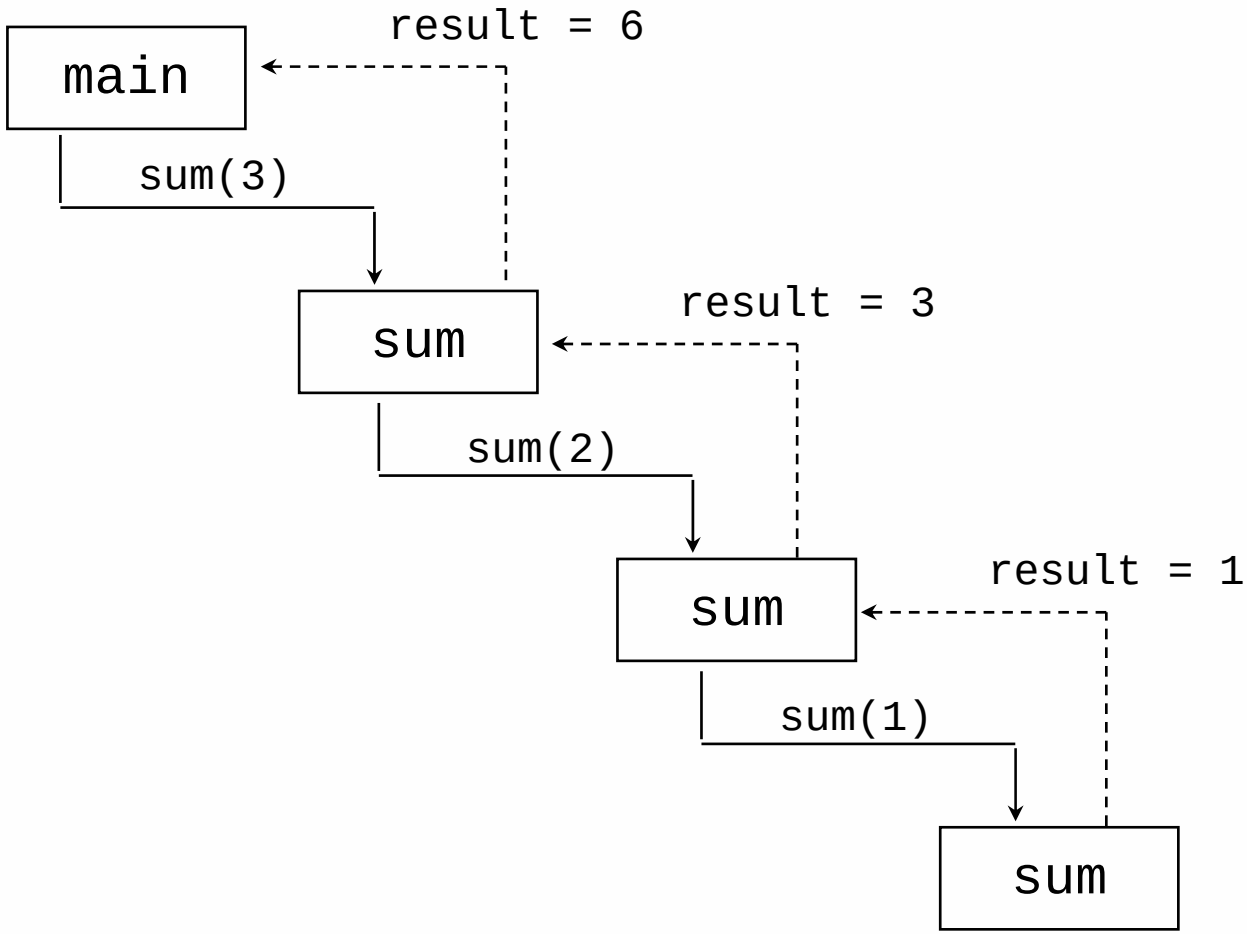
Recursive Programming

```
// This method returns the sum of 1 to num
public int sum (int num)
{
    int result;

    if (num == 1)
        result = 1;
    else
        result = num + sum (n-1);

    return result;
}
```

Recursive Programming



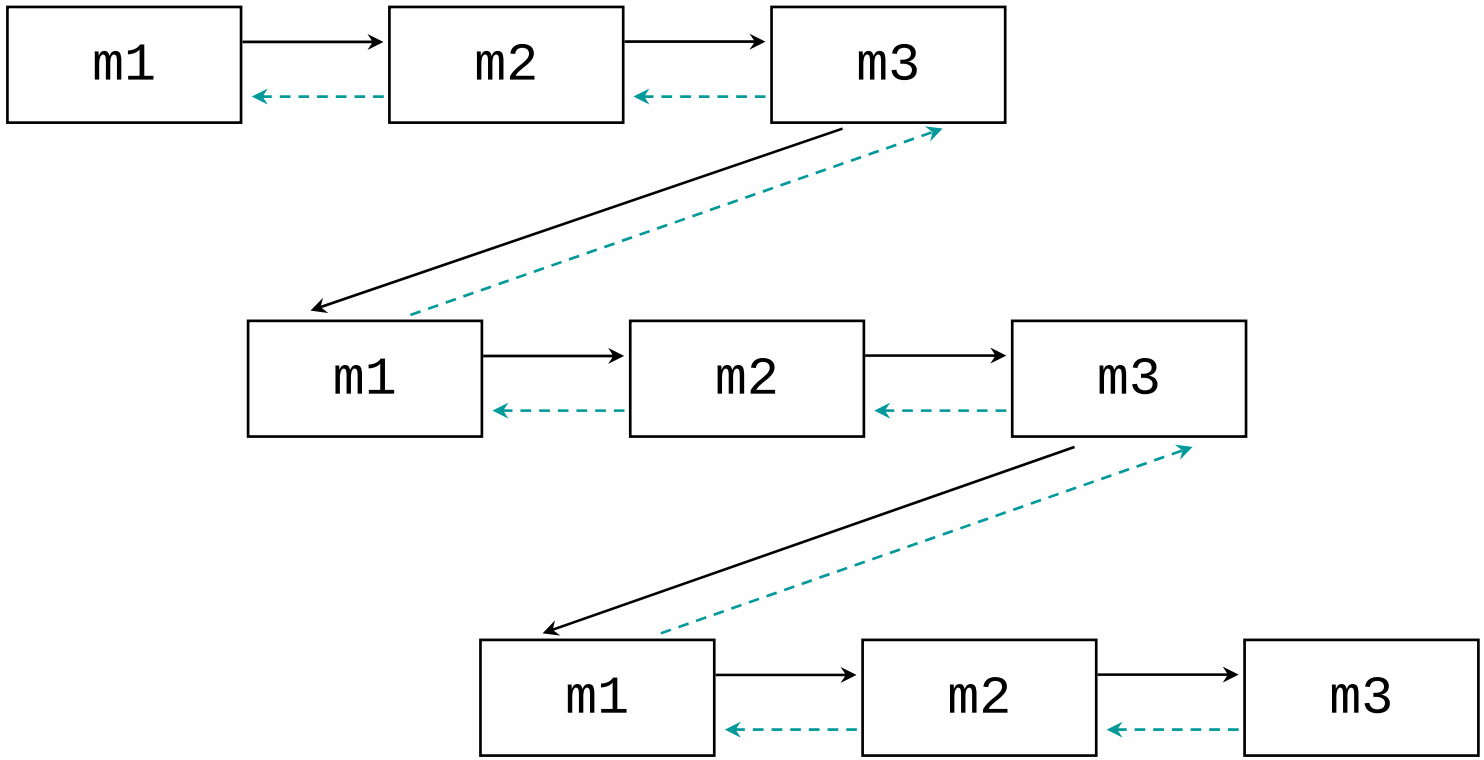
Recursive Programming

- Note that just because we can use recursion to solve a problem, doesn't mean we should
- For instance, we usually would not use recursion to solve the sum of 1 to N problem, because the iterative version is easier to understand
- However, for some problems, recursion provides an elegant solution, often cleaner than an iterative version
- You must carefully decide whether recursion is the correct technique for any problem

Indirect Recursion

- A method invoking itself is considered to be *direct recursion*
- A method could invoke another method, which invokes another, etc., until eventually the original method is invoked again
- For example, method `m1` could invoke `m2`, which invokes `m3`, which in turn invokes `m1` again
- This is called *indirect recursion*, and requires all the same care as direct recursion
- It is often more difficult to trace and debug

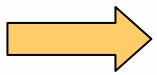
Indirect Recursion



Outline

Recursive Thinking

Recursive Programming



Using Recursion

Maze Traversal

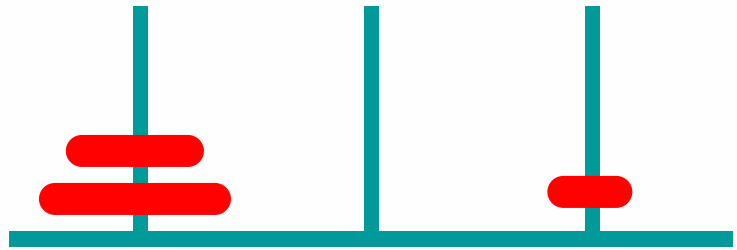
- We can use recursion to find a path through a maze
- From each location, we can search in each direction
- Recursion keeps track of the path through the maze
- The base case is an invalid move or reaching the final destination
- See `MazeSearch.java` (page 583)
- See `Maze.java` (page 584)

Towers of Hanoi

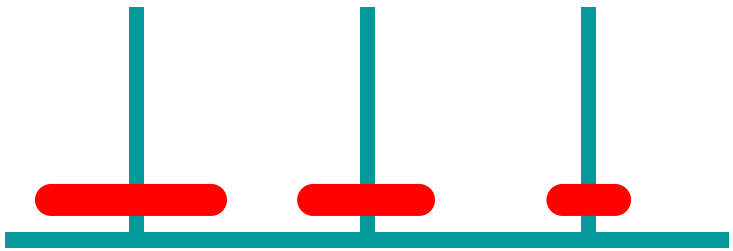
- The *Towers of Hanoi* is a puzzle made up of three vertical pegs and several disks that slide on the pegs
- The disks are of varying size, initially placed on one peg with the largest disk on the bottom with increasingly smaller ones on top
- The goal is to move all of the disks from one peg to another under the following rules:
  - We can move only one disk at a time
  - We cannot move a larger disk on top of a smaller one

Towers of Hanoi

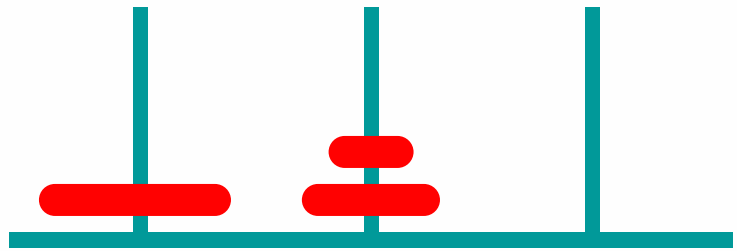
Original Configuration



Move 1

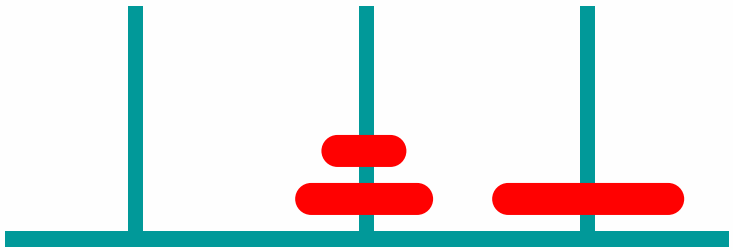


Move 2

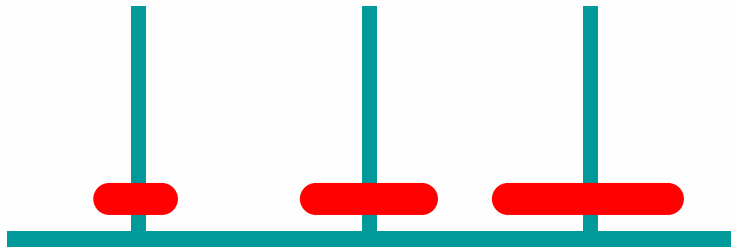


Move 3

Towers of Hanoi



Move 4



Move 5

Move 6

Move 7 (done)

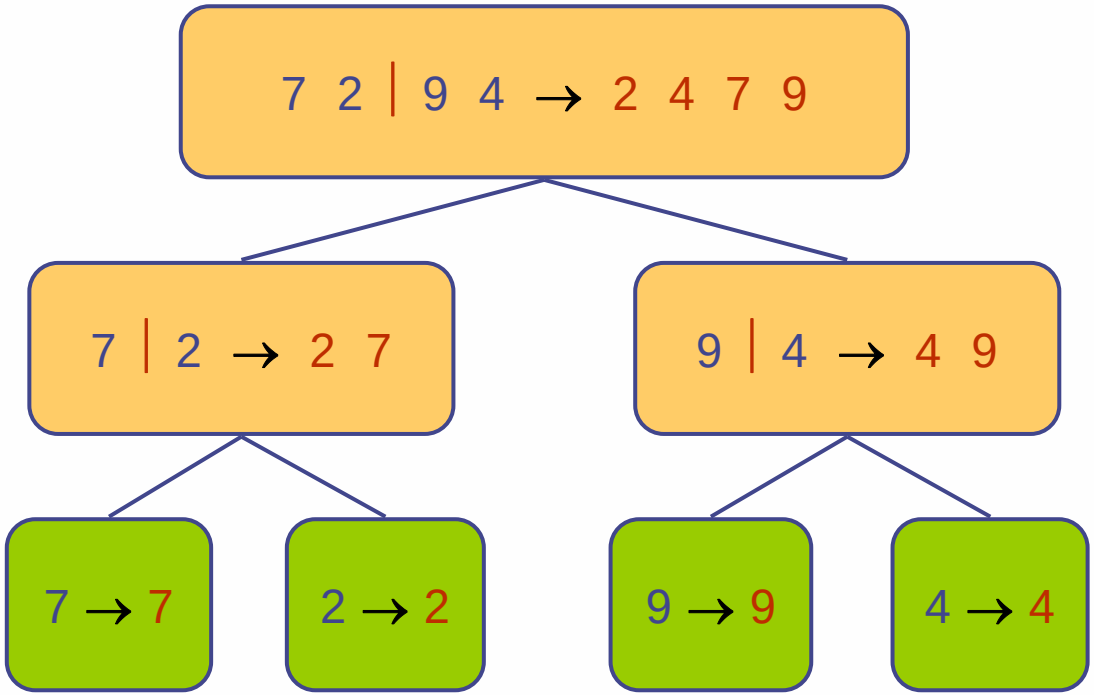
Towers of Hanoi

- An iterative solution to the Towers of Hanoi is quite complex
- A recursive solution is much shorter and more elegant
- See `SolveTowers.java` (page 590)
- See `TowersOfHanoi.java` (page 591)

Merge Sort

- Merge Sort is a recursive algorithm to sort a list (array) of items
- It performs significantly better than Insertion Sort and Selection Sort
- We will outline the key idea of the algorithm in this lecture and sketch it's recursive strategy
- The Java implementation of Merge Sort will be the main task for the next exercise sheet on recursion
- Try to understand the logic of the algorithm now; think about Java details later when you solve the exercises

Merge Sort

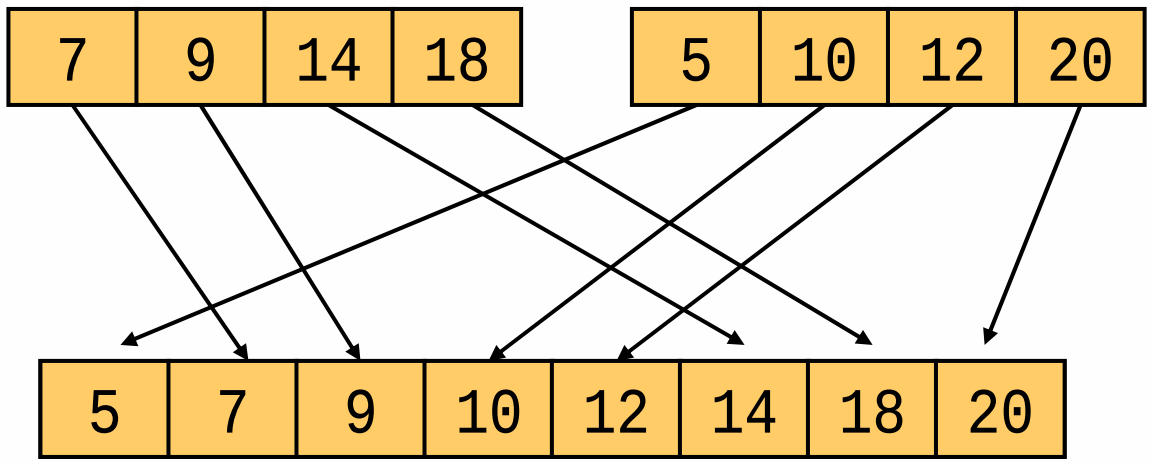


Divide and Conquer

- The Merge Sort algorithm sorts an array by
  - cutting the array in half
  - recursively sorting each half
  - merging the sorted halfs
- The recursion terminates if we are left with subarrays of length one
- Thus, the algorithm keeps dividing the array into smaller and smaller subarrays, sorting each half and merging them back together
- This algorithm carries out dramatically fewer steps than the insertion or selection sort algorithms

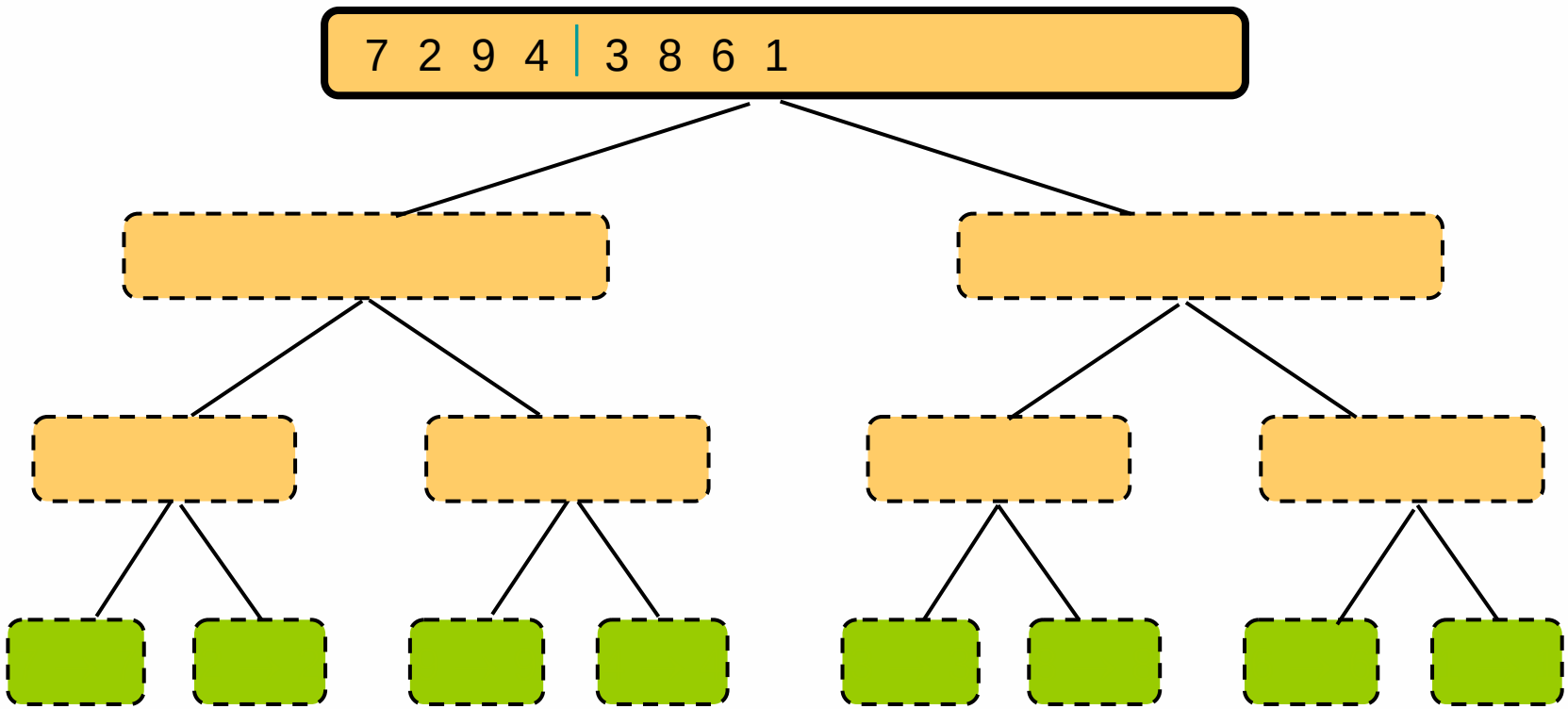
Merging two sorted subarrays

- **merging two sorted subarrays is a simple operation which works in linear time in the size of the two subarrays**



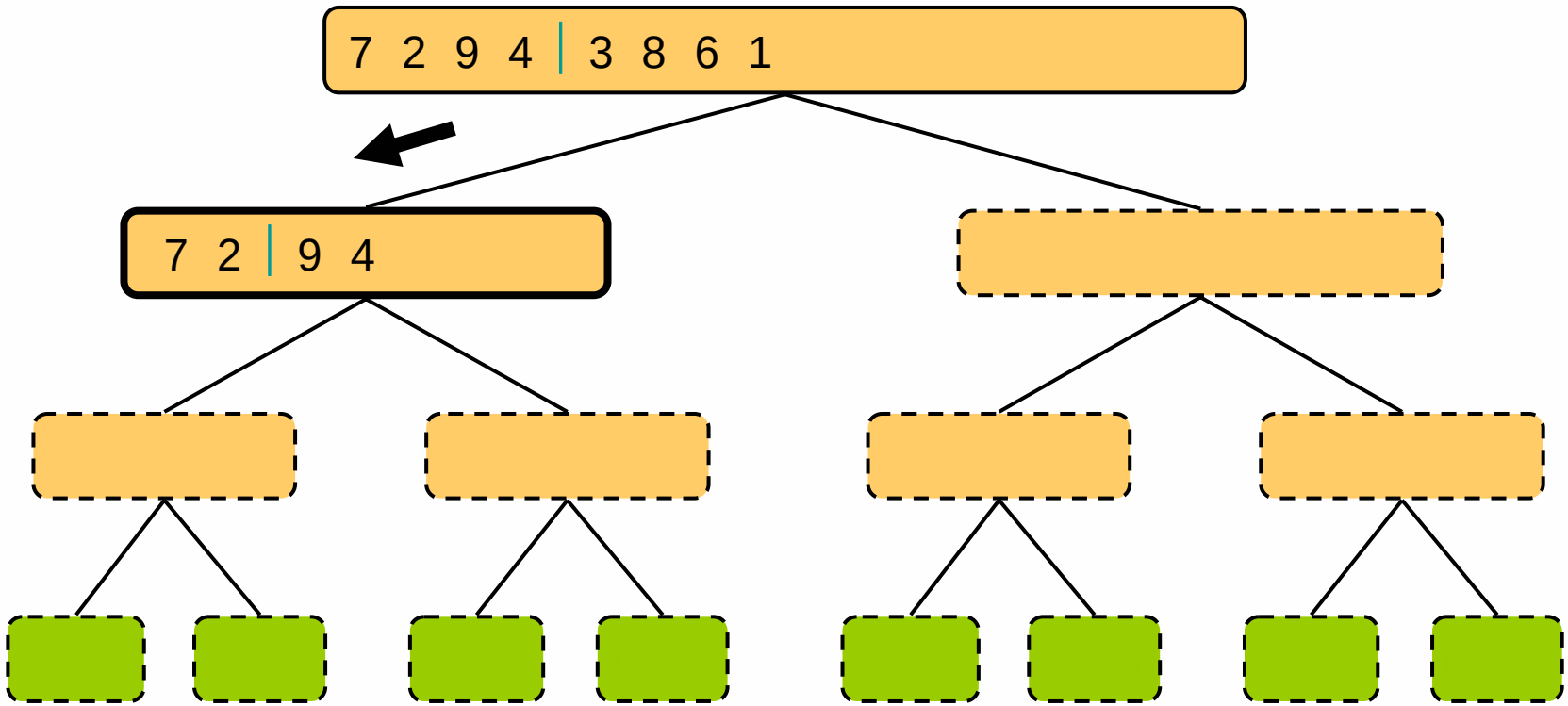
A full example

- **Partitioning**



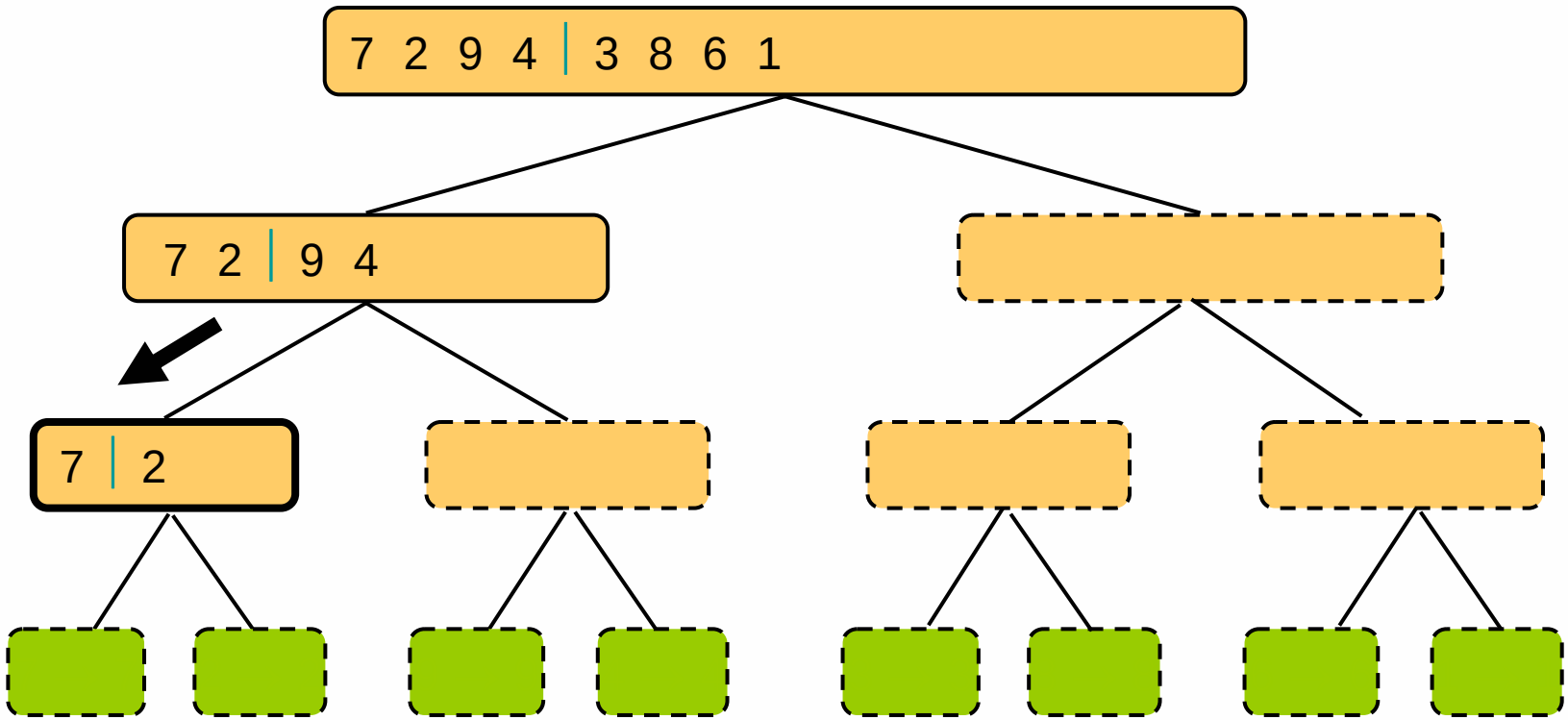
Example (cont.)

- **Recursive call, partitioning**



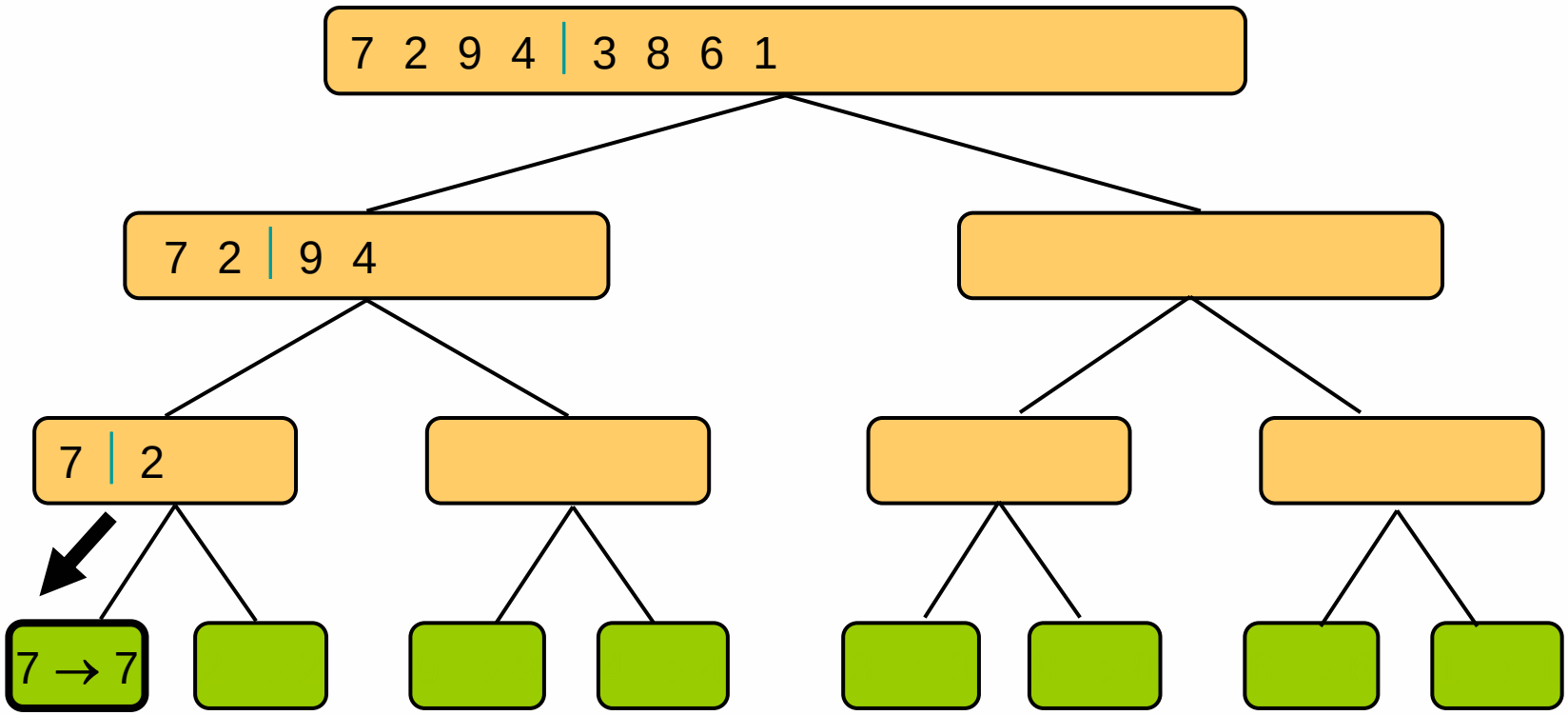
Example (cont.)

- **Recursive call, partitioning**



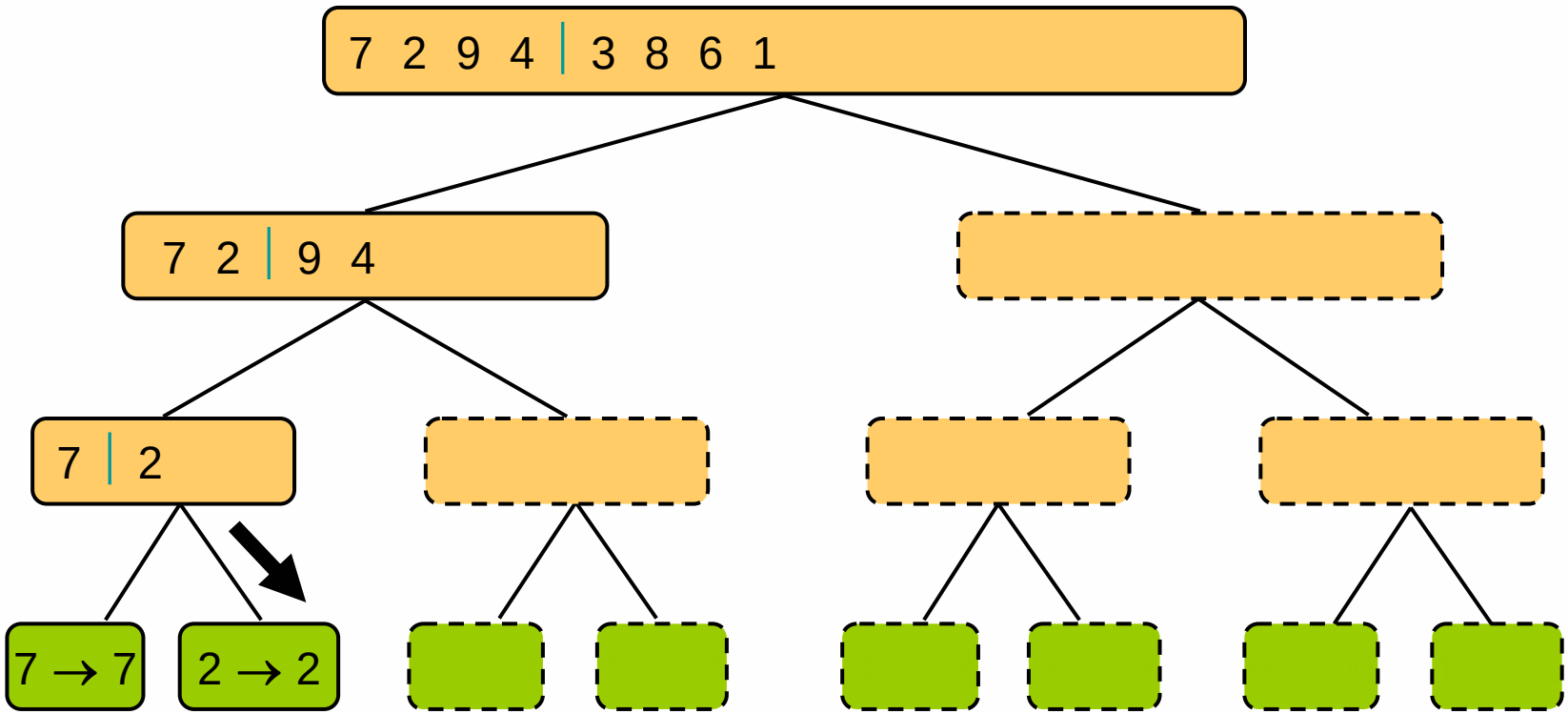
Example (cont.)

- **Recursive call, base case**



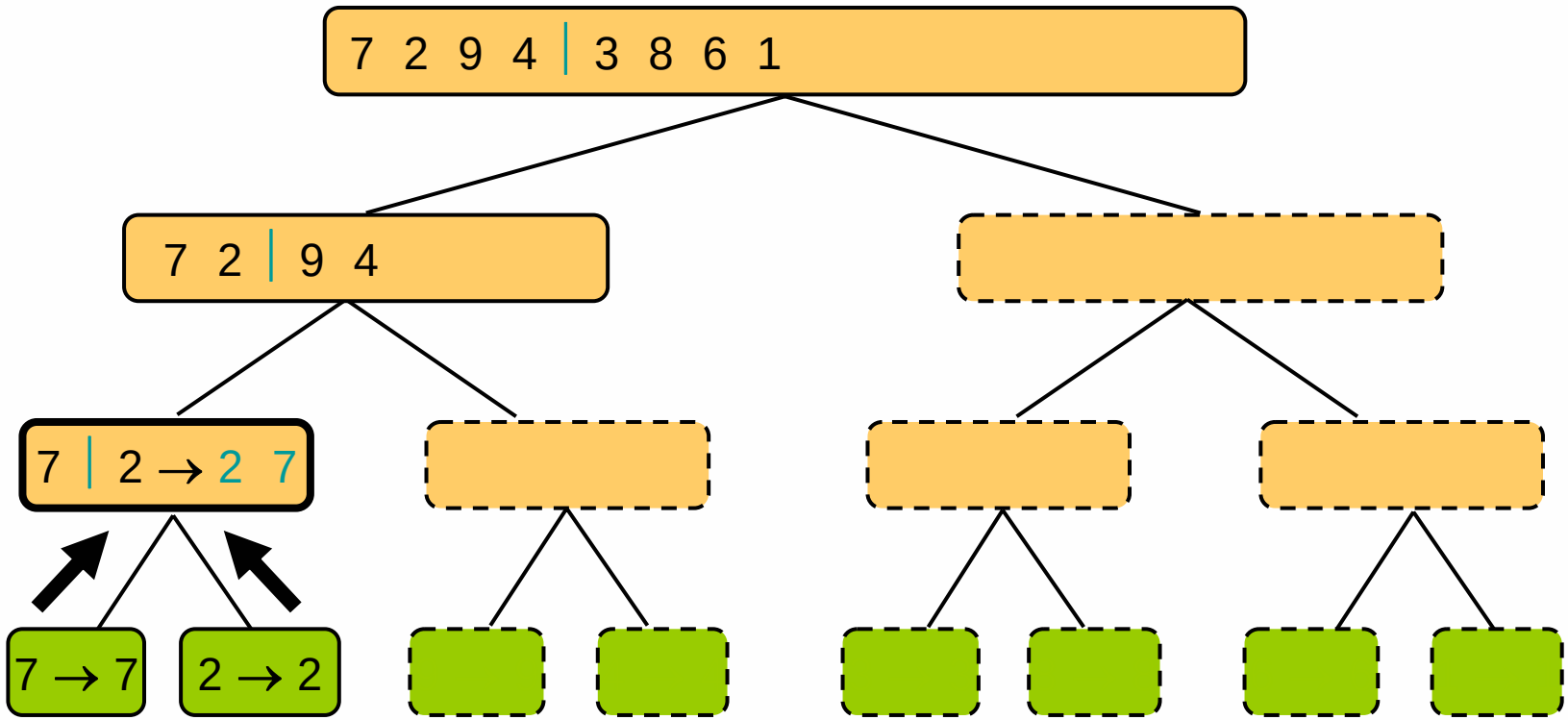
Example (cont.)

- **Recursive call, base case**



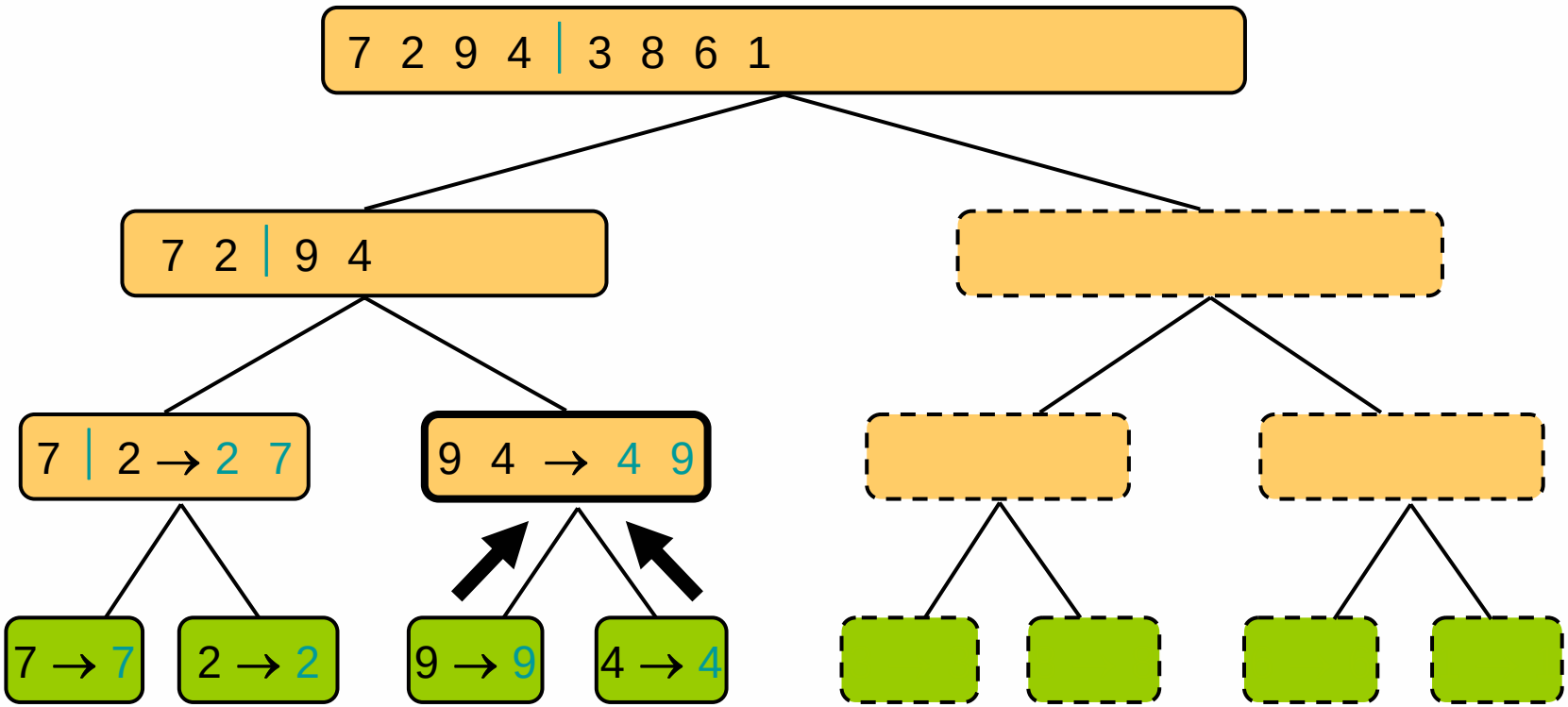
Example (cont.)

- **Merge**



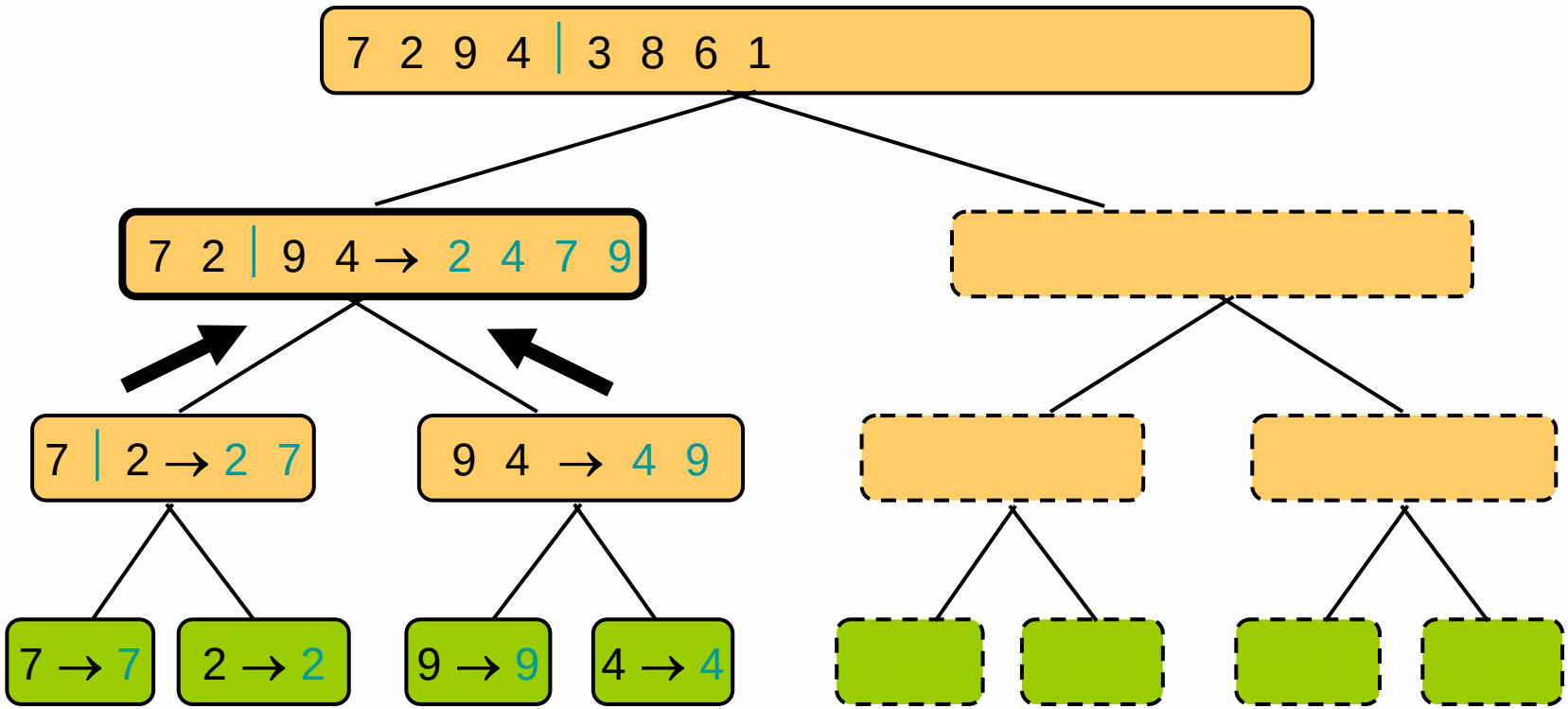
Example (cont.)

- **Recursive call, …, base case, merge**



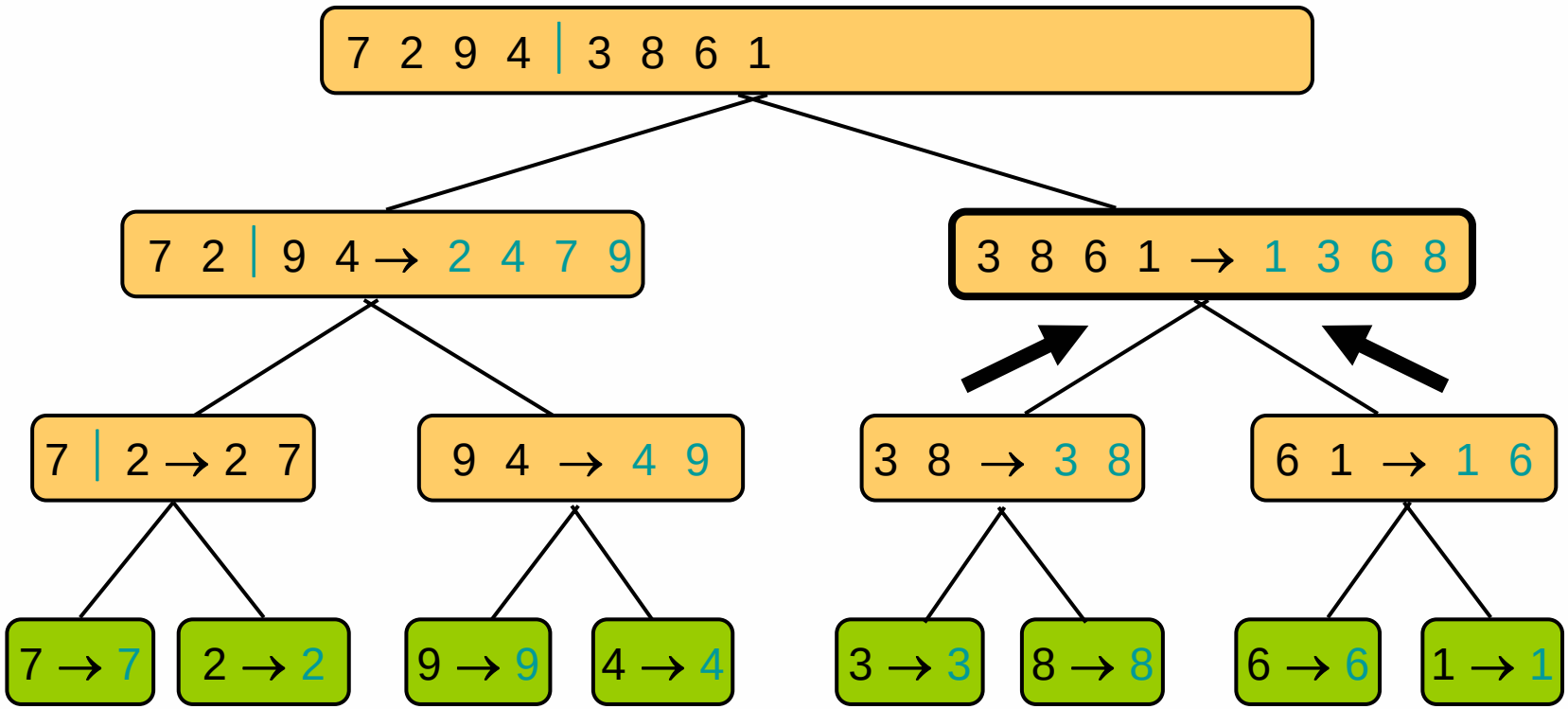
Example (cont.)

- **Merge**



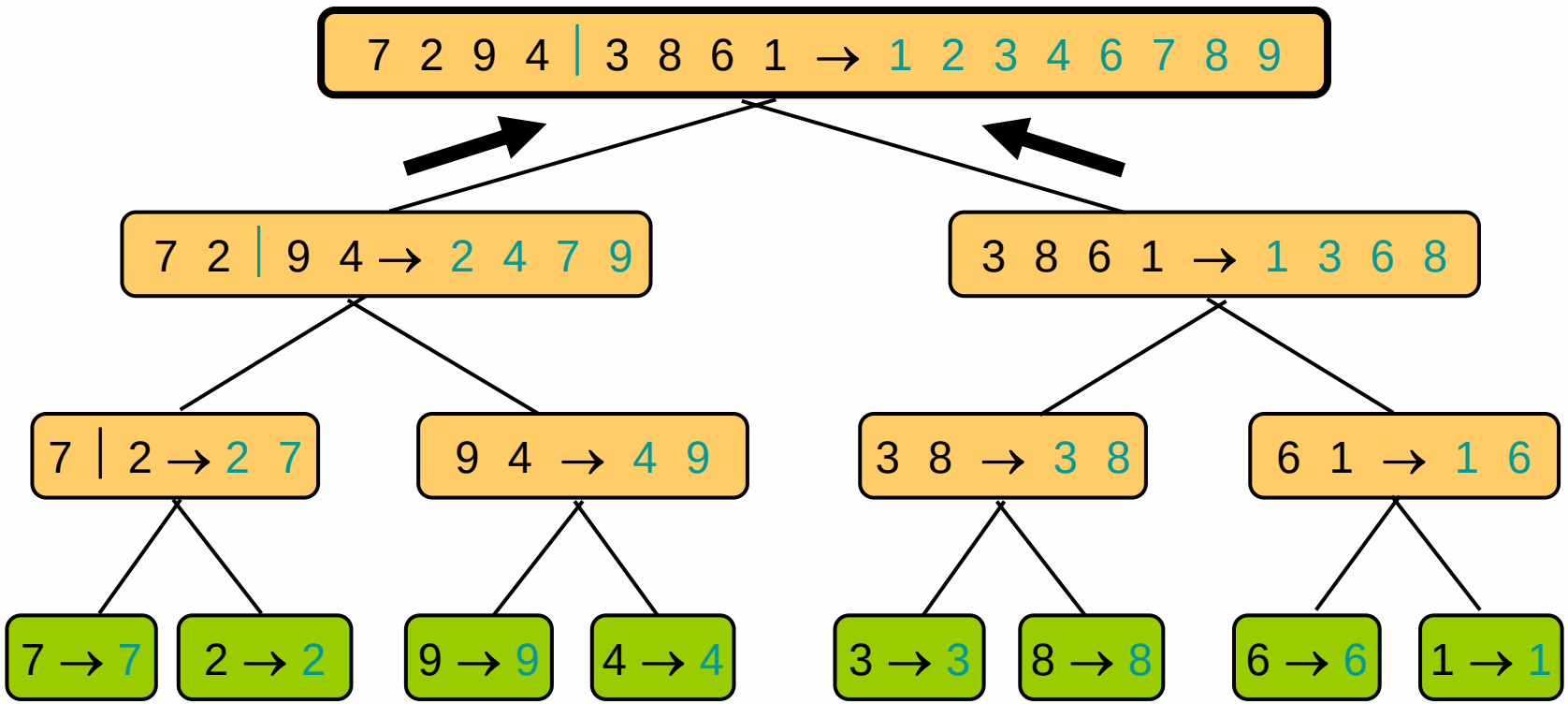
Example (cont.)

- **Recursive call, …, merge, merge**



Example (cont.)

- **Merge**



Complexity of Merge Sort

- The complexity of Merge Sort is O(n log n), where n is the size of the input
- Recall that Selection Sort and Insertion Sort have complexity $O(n^2)$
- There are further efficient, recursively formulated sorting algorithms, e.g.
  - Quick Sort
- You will learn more about searching and sorting in the lecture “Datenstrukturen und Algorithmen”

Summary

- **Lecture 11 has focused on:**
  - **thinking in a recursive manner**
  - **programming in a recursive manner**
  - **the correct use of recursion**
  - **recursion examples**