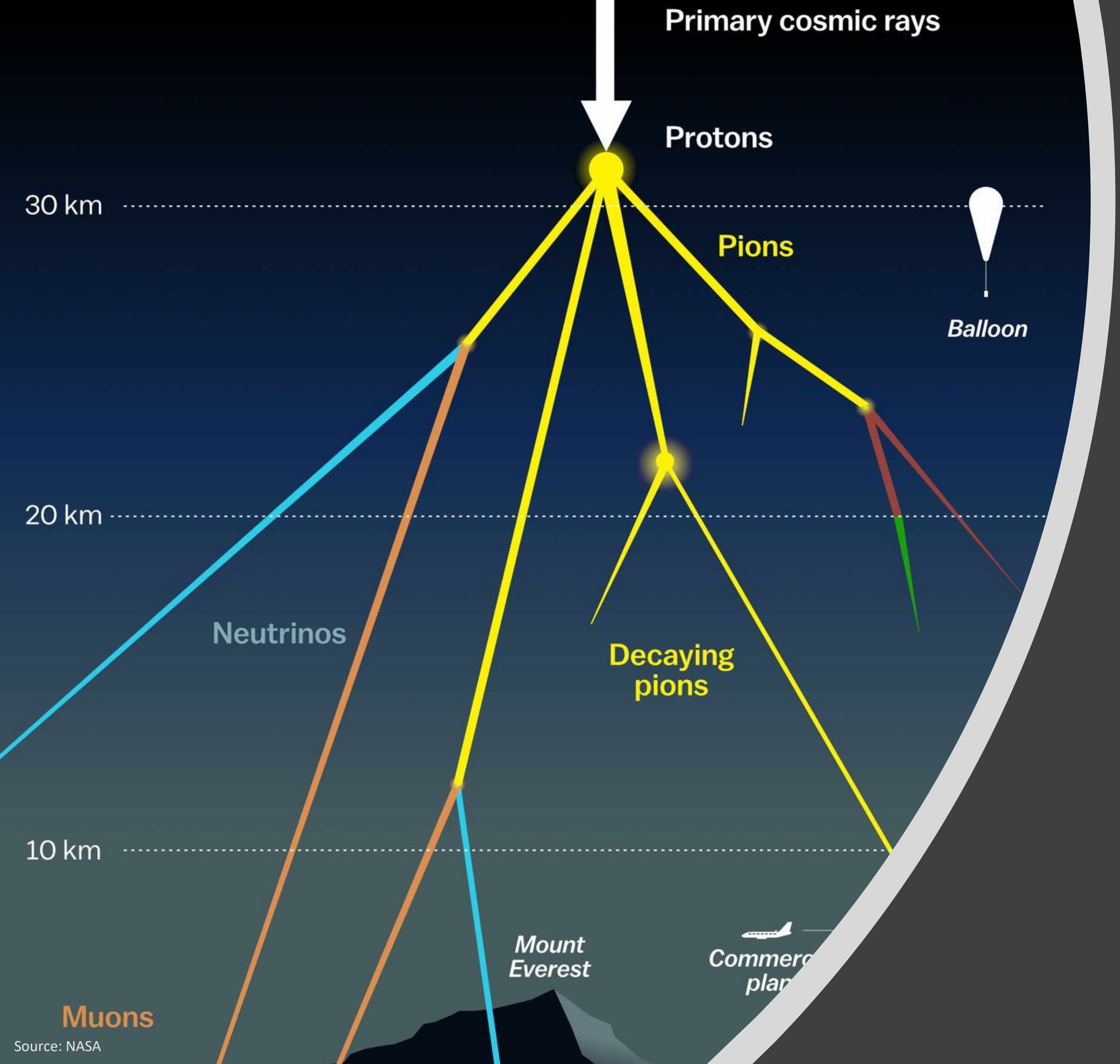


Source: NASA

Measuring the prompt component of the atmospheric muon flux

Pascal Gutjahr and Mirco Hünnefeld

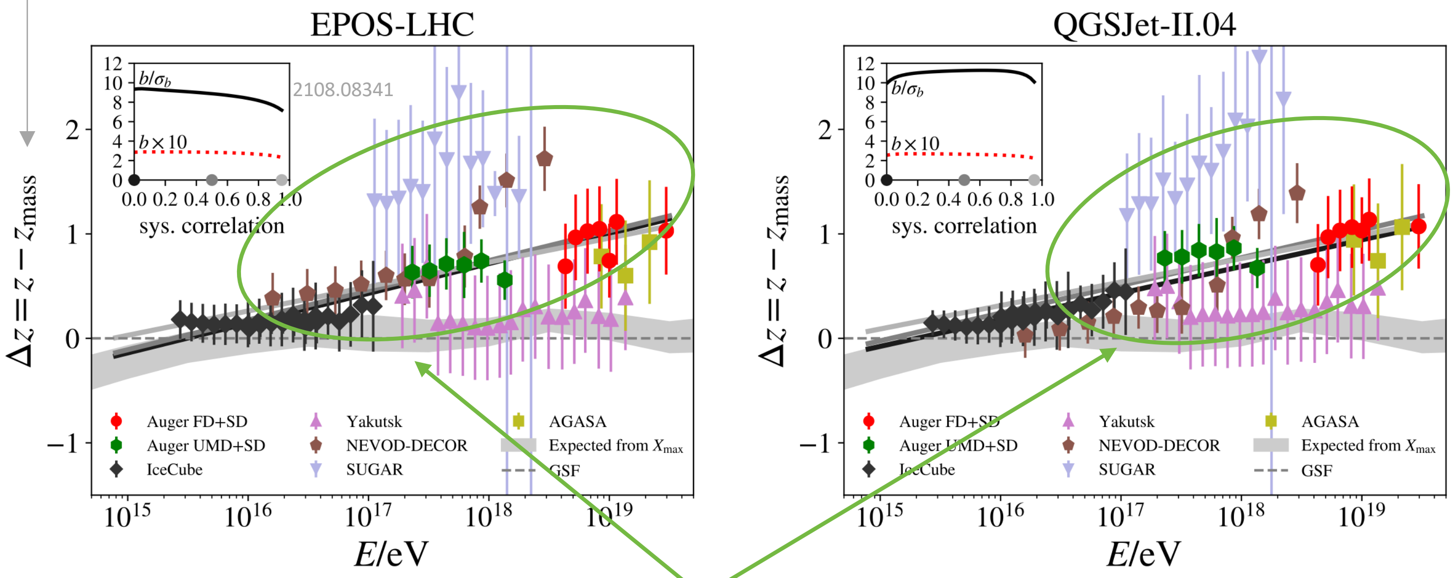
Spring Collaboration Meeting Münster
March 18, 2024

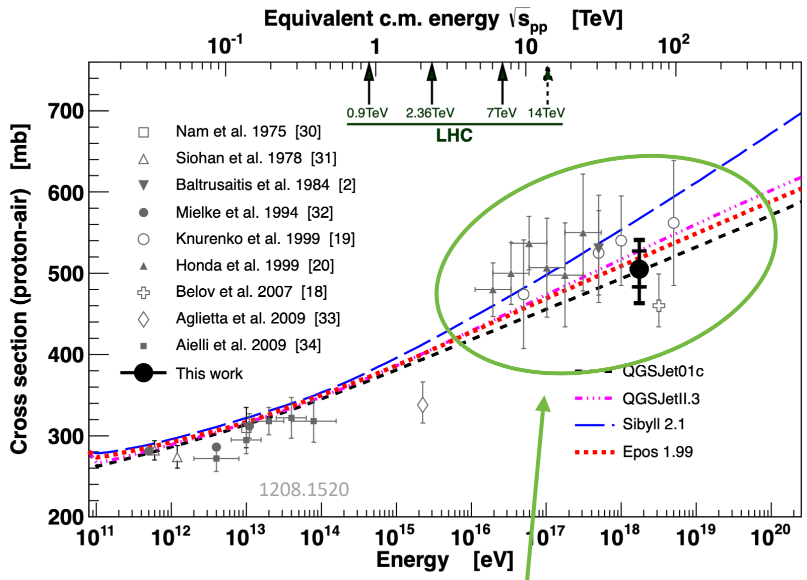
Motivation: muon Puzzle and model uncertainties

"muon number"

Expected z ("muon number")

$$z = \frac{\ln\langle N_\mu \rangle - \ln\langle N_\mu \rangle_p}{\ln\langle N_\mu \rangle_{\mathrm{Fe}} - \ln\langle N_\mu \rangle_p}$$



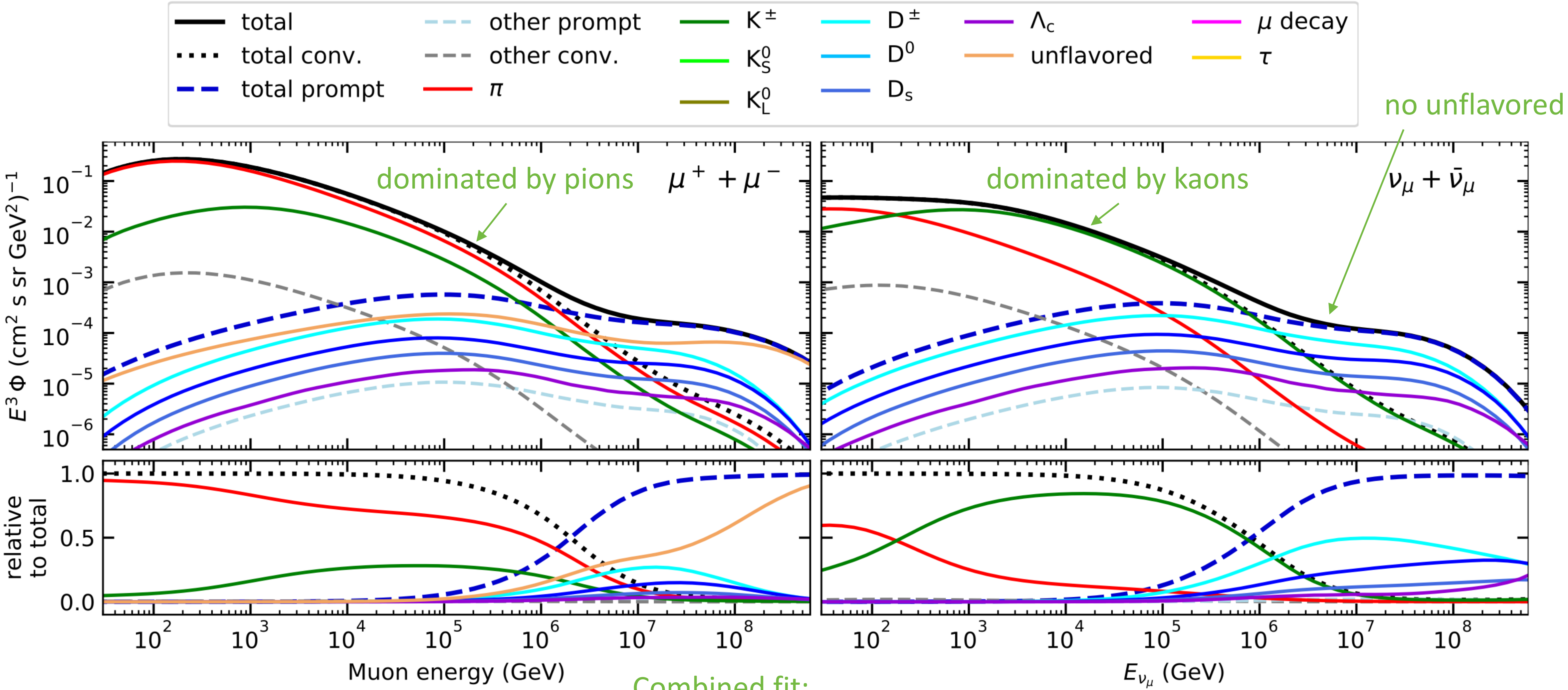


- More muons measured than simulated for E > 40 PeV ~ cms 8 TeV
- Precise pion/kaon ratio measurement needed

- Large uncertainties at E > 10 PeV

The origin of atmospheric muons and neutrinos

10.1103/PhysRevD.100.103018



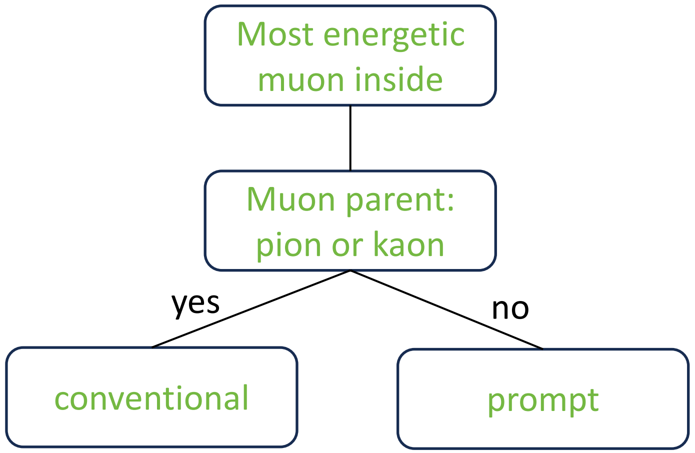
Combined fit:

- handle on pion/kaon ratio
- handle on charmed mesons

Analysis goals

1. Unfolding of an atmospheric muon spectrum
   - No prior / model independent

2. Measuring the normalization of the prompt component of the atmospheric muon flux
   - Forward fit for a specific model



➢ requires new simulation

Future goals

1. Measure normalization of atmospheric prompt neutrino flux
2. Combined muon + neutrino fit

Overview

Simulation

Dataset
- CORSIKA 77420 Ehist
- SIBYLL 2.3d
- 600 GeV – 50 EeV

Validation
- Tag prompt / conv particles
- MCEq comparisons

Reconstruction

DNN reconstructions
- Muon energies
- Muon direction
- Track geometry

Selection

Level 3
- L2 muon filter
- 200 TeV bundle energy cut at surface
- Add labels

Level 4
- Add DNN reconstructions

Level 5
- Data-MC quality cuts

Analysis

Forward fit

Fit prompt normalization
- Poisson LLH fit

Unfolding

Muon spectrum
- Unfold muon flux

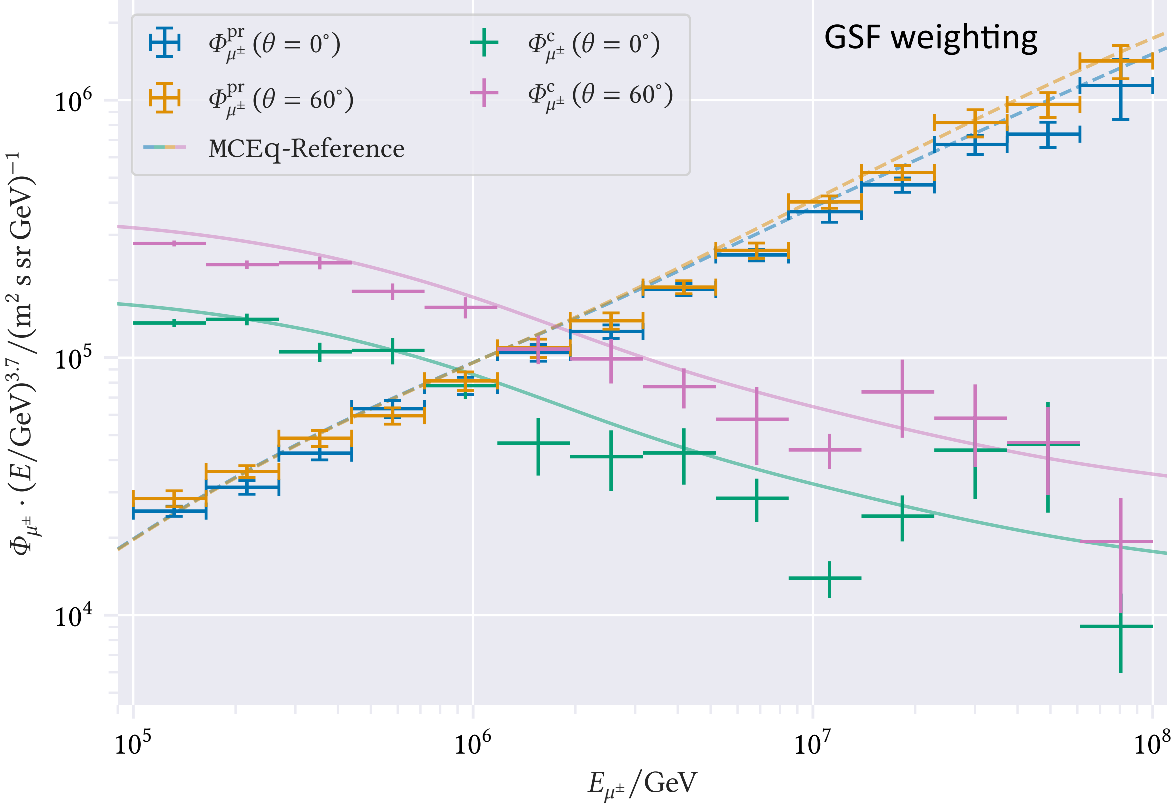
tu dortmund university

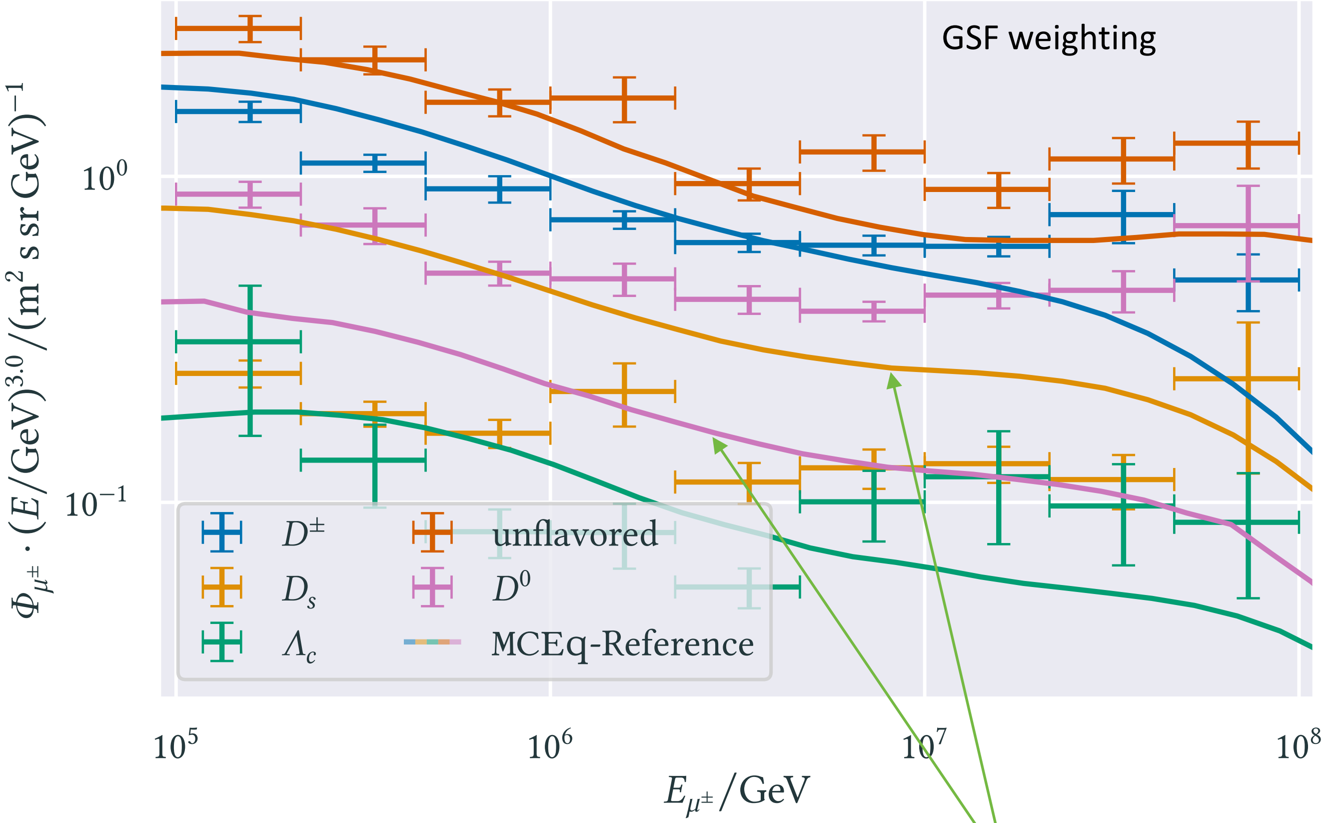
ICECUBE NEUTRINO OBSERVATORY

Simulation

CORSIKA 7 tagging and MCEq comparison

MCEq: tool to numerically solve the cascade equations that describes the evolution of particle densities as they propagate through a gaseous, dense medium
https://github.com/mceq-project/MCEq





- Good agreement in total prompt and conventional muon flux
- D^0 and D_s are swapped here but this is fixed in MCEq

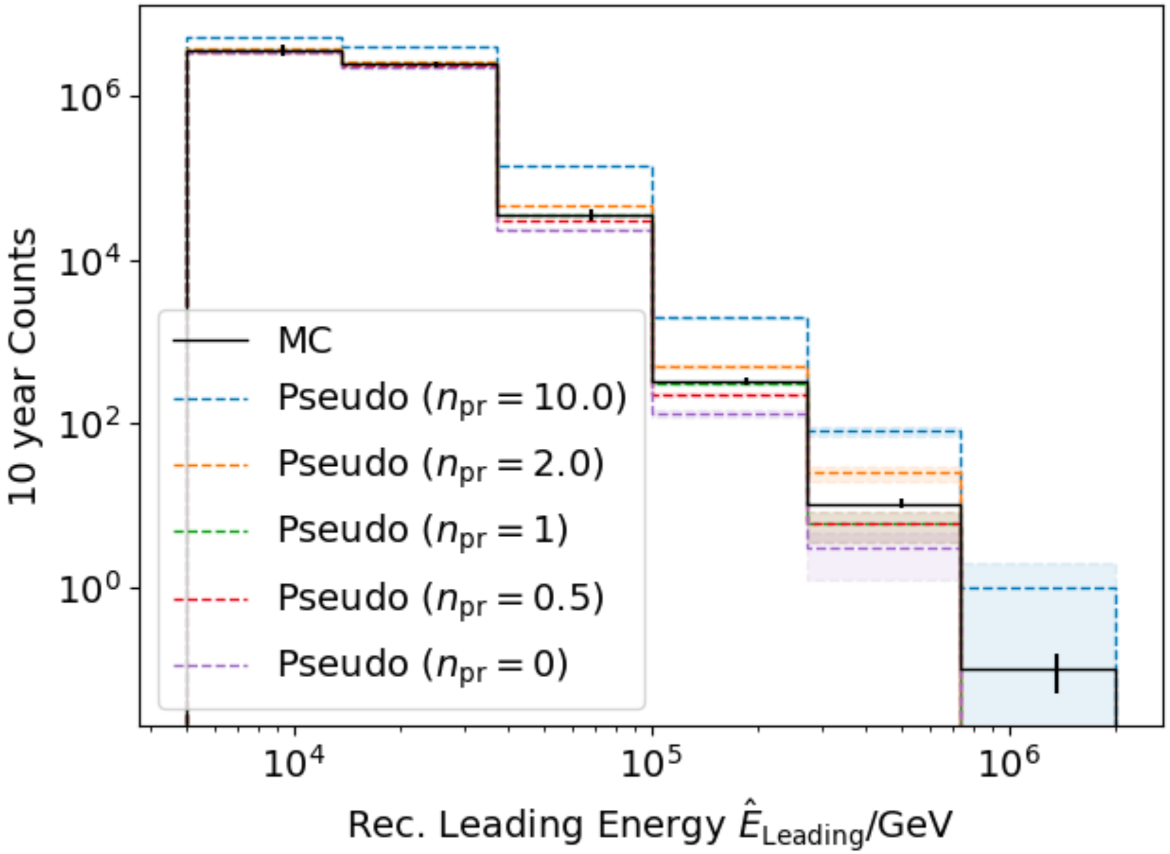
Forward Fit

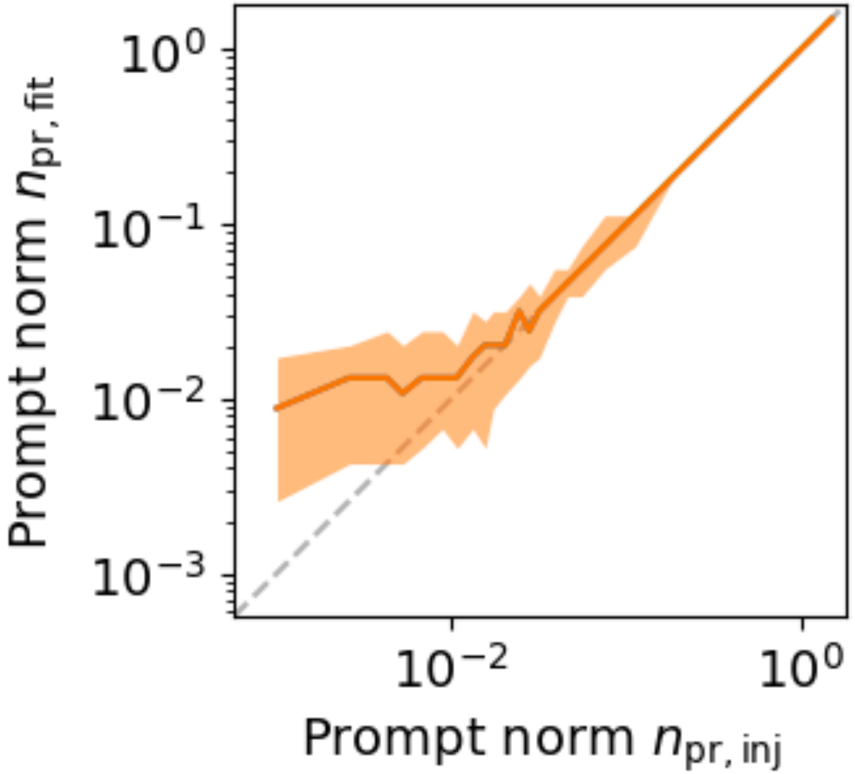
Cuts:
L2 MuonFilter
Bundle energy at entry > 100 TeV

Poisson likelihood forward fit

$$C_1^{\mathrm{MC}} = n_{\mathrm{pr}} C_1^{\mathrm{MC,pr}} + n_{\mathrm{conv}} C_1^{\mathrm{MC,conv}}, \dots, C_M^{\mathrm{MC}} = n_{\mathrm{pr}} C_M^{\mathrm{MC,pr}} + n_{\mathrm{conv}} C_M^{\mathrm{MC,conv}}$$

$$p(C_i) = p_{\mathrm{poisson}}\left(C_i; \lambda(n_{\mathrm{pr}}) = C_i^{\mathrm{MC}}(n_{\mathrm{pr}})\right) = \frac{\lambda(n_{\mathrm{pr}})^{C_i} e^{-\lambda(n_{\mathrm{pr}})}}{C_i!}$$





- Tagging allows scaling of prompt by factor n_{pr}
- Bias starts at a prompt normalization of 0.1

Cuts:
L2 MuonFilter
Bundle energy at entry > 100 TeV

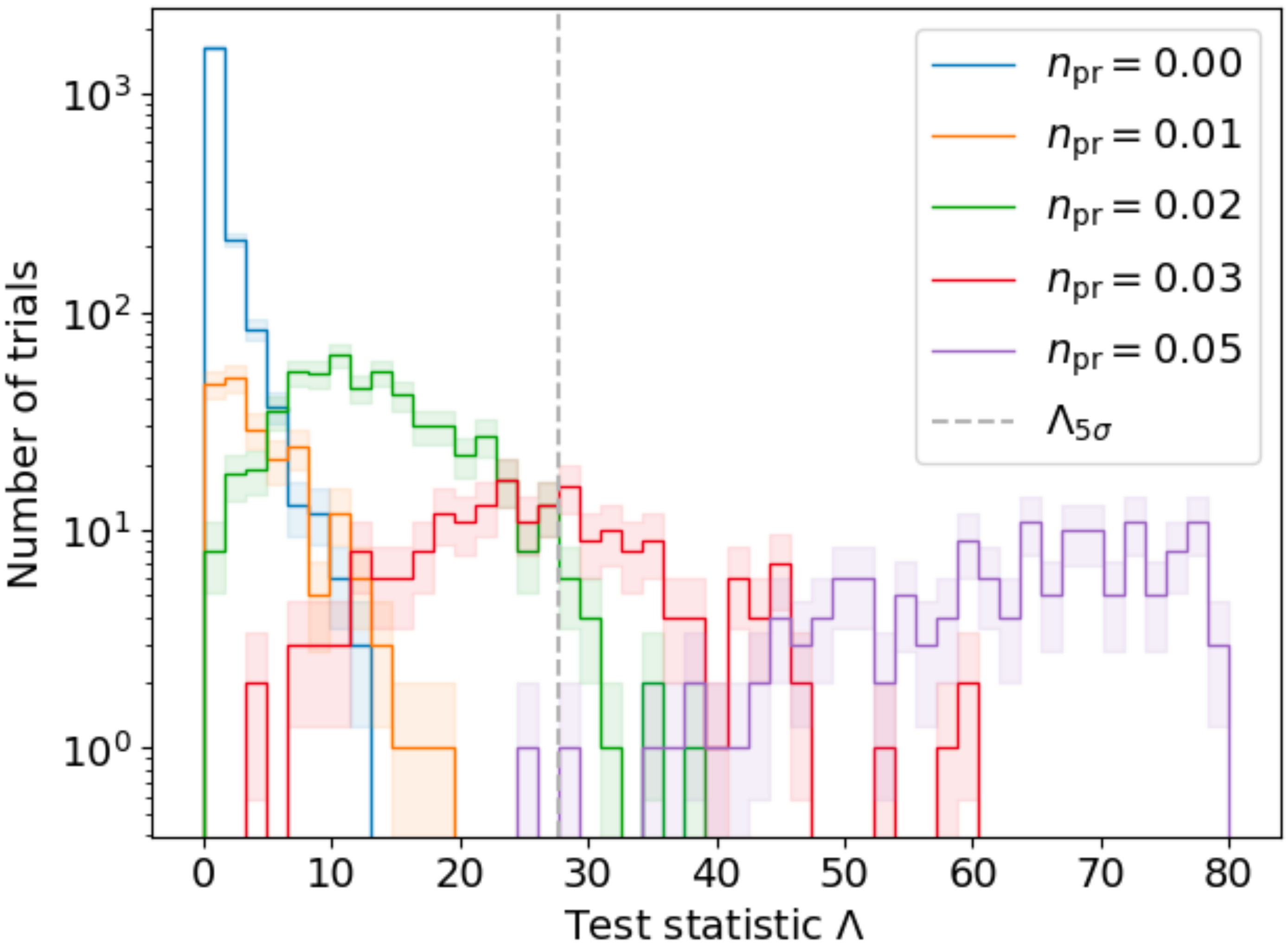
Discovery potential and sensitivity

Expectation for 1 year:
- 5 sigma discovery potential: 0.102 ± 0.005
- Sensitivity: 0.024 ± 0.001

Expectation for 10 years:
- 5 sigma discovery potential: 0.032 ± 0.001
- Sensitivity: 0.007 ± 0.000

Caution:
- Limited MC statistics -> events are oversampled in pseudo dataset
- No systematics

➢ Tested NNMFit



Unfolding

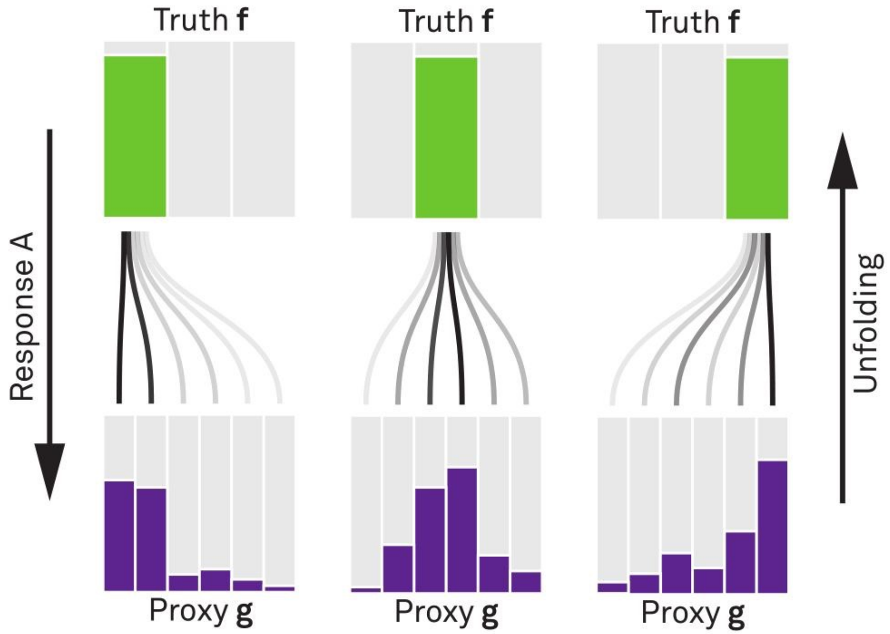
Unfolding in a nutshell

measured proxy — detector response — background

$$\underbrace{g(y)}_{\text{measured proxy}} = \int_{E_0}^{E_1} \underbrace{A(E_\mu, y)}_{\text{detector response}} \underbrace{f(E_\mu)}_{\text{true energy distribution}} \, \mathrm{d}E_\mu + \underbrace{b(y)}_{\text{background}}$$

true energy distribution

1. discretized form: $\overbrace{\vec{g} = A\vec{f}}^{\text{folding}} \leftrightarrow \overbrace{\vec{f} = A^{-1}\vec{g}}^{\text{unfolding}}$



Credit: T. Hoinka

2. maximum likelihod method:

$$\mathcal{L}(\vec{g}|\vec{f}) = \prod_{j=1}^{M} \frac{\lambda_j^{g_j}}{g_j!} \exp(-\lambda_j)$$

$$= \prod_{j=1}^{M} \frac{(A\vec{f})_j^{g_j}}{g_j!} \exp(-(A\vec{f})_j)$$

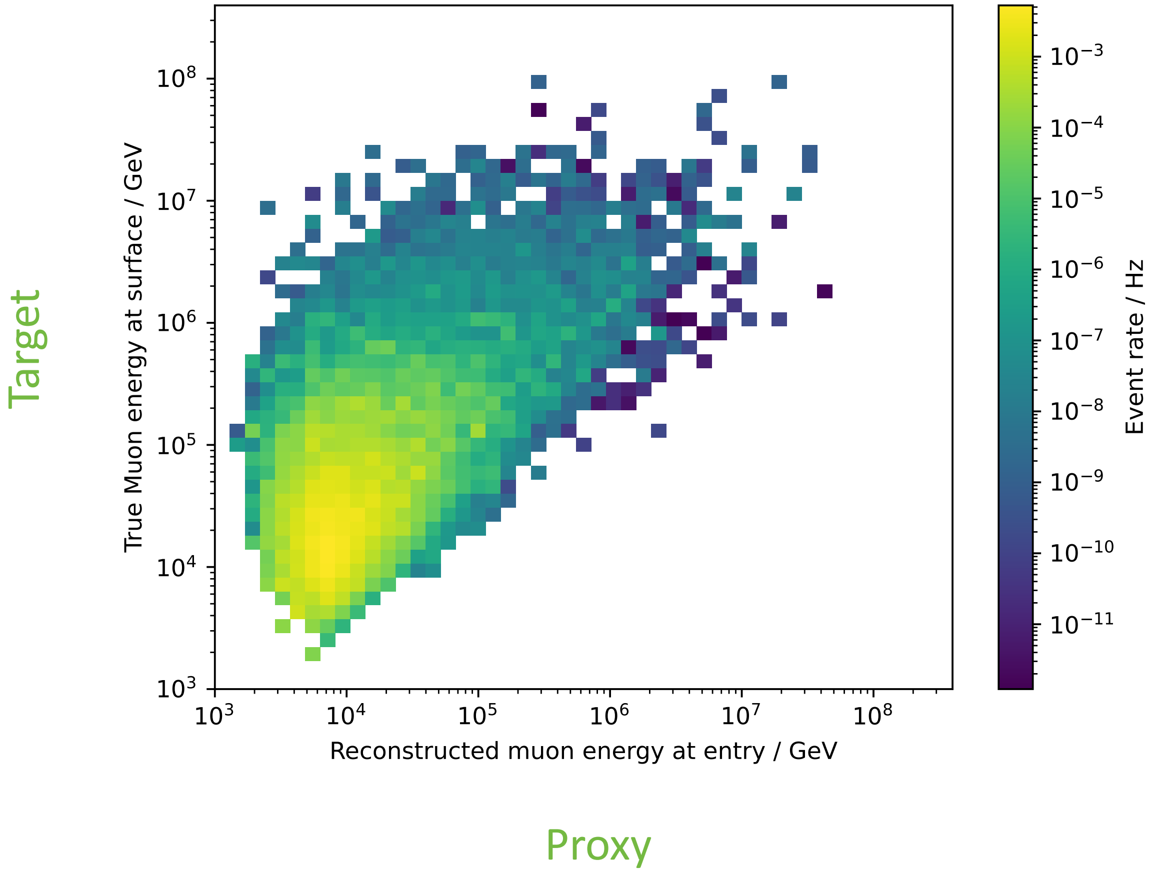
3. Thikonov regularization:

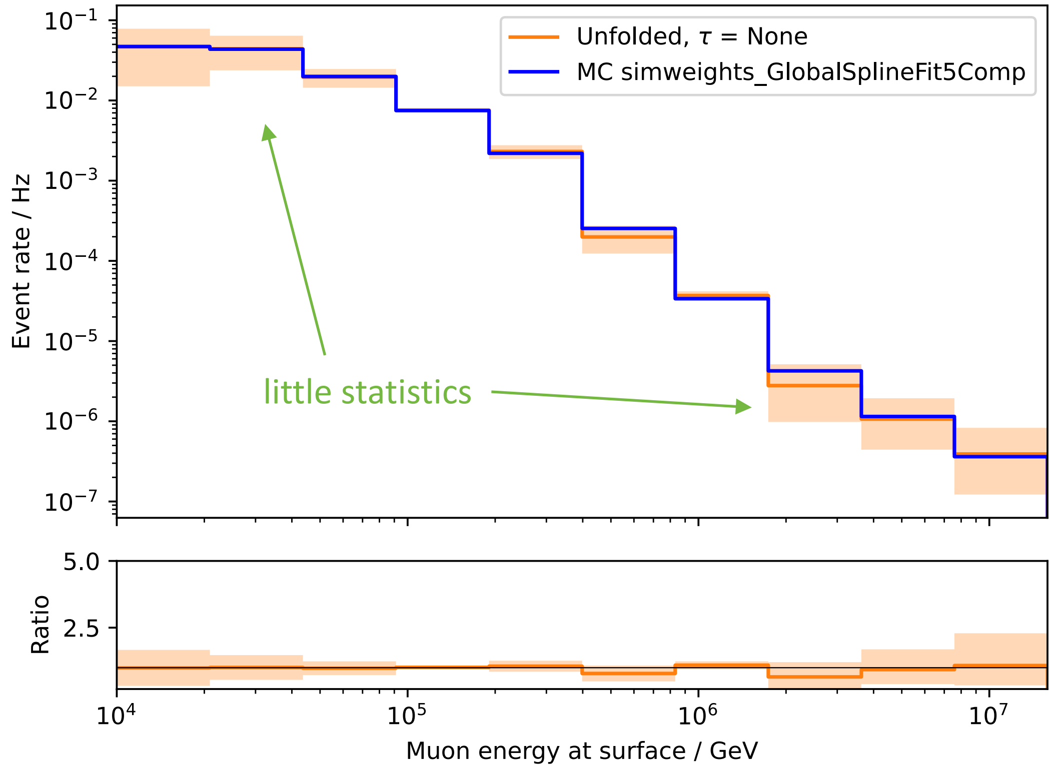
$$t(\vec{f}) = -\frac{1}{2}(C\vec{f})^T (\tau 1)^{-1} (C\vec{f})$$

4. maximize $\log(\mathcal{L}(\vec{g}|\vec{f})) + t(\vec{f})$
with respect to $\vec{f}$ using
Markov Chain Monte Carlo (MCMC)

GitHub **funfolding**
by M. Börner

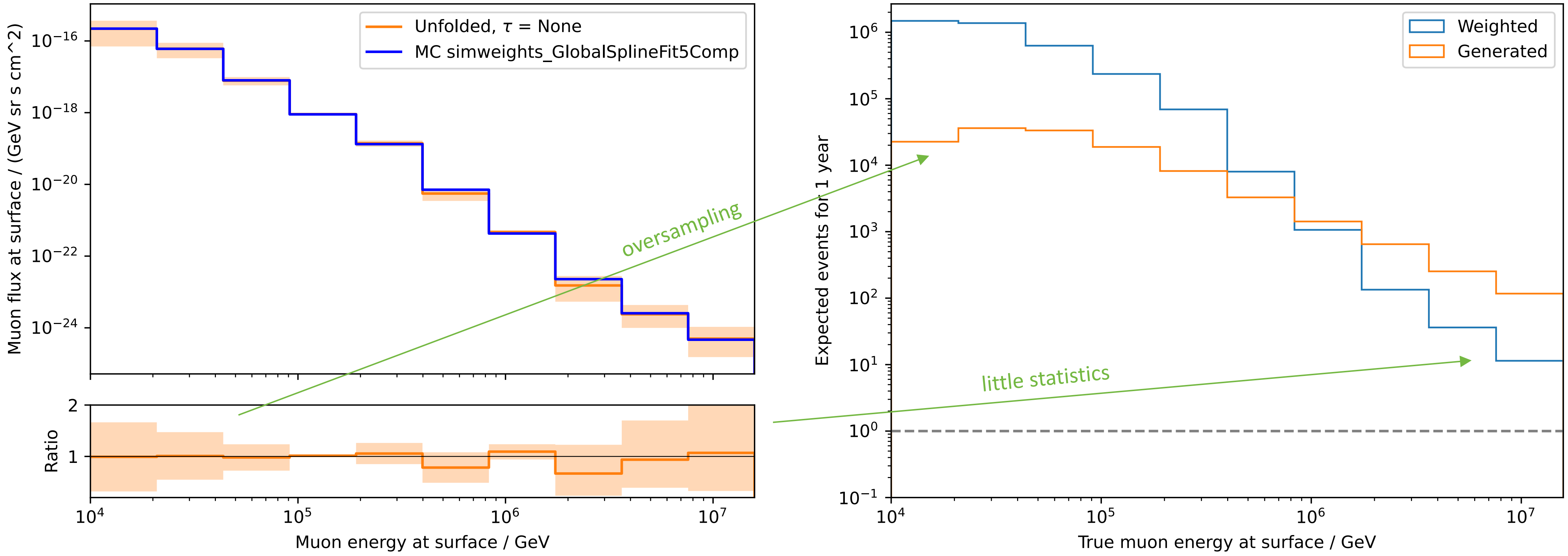
Unfolding of event rate – leading muons





- ➢ Unfolding of event rate works well

Unfolding of flux – leading muons



➢ Successful unfolding of muon flux

Conclusion & outlook

- [x] CORSIKA 7 Ehist test simulation
  - [x] Tag prompt / conv particles
  - [x] Compare with MCEq
- [x] Reconstructions
- [ ] Selection
  - [x] Level 3 (muon filter + 200 TeV cut + labels)
  - [x] Level 4 (add reconstructions)
  - [ ] Level 5 (quality cuts)
- [x] Data-MC
- [ ] Forward fit
  - [x] Performed on pseudo dataset
- [ ] Unfolding
  - [x] Performed on pseudo dataset

ToDos

- [ ] Systematics (too little statistics)
- [ ] Large-scale simulation (IceProd)
  - [x] Latest ice model
  - [x] Latest software release
  - [ ] Storage request

- Prompt muons wiki

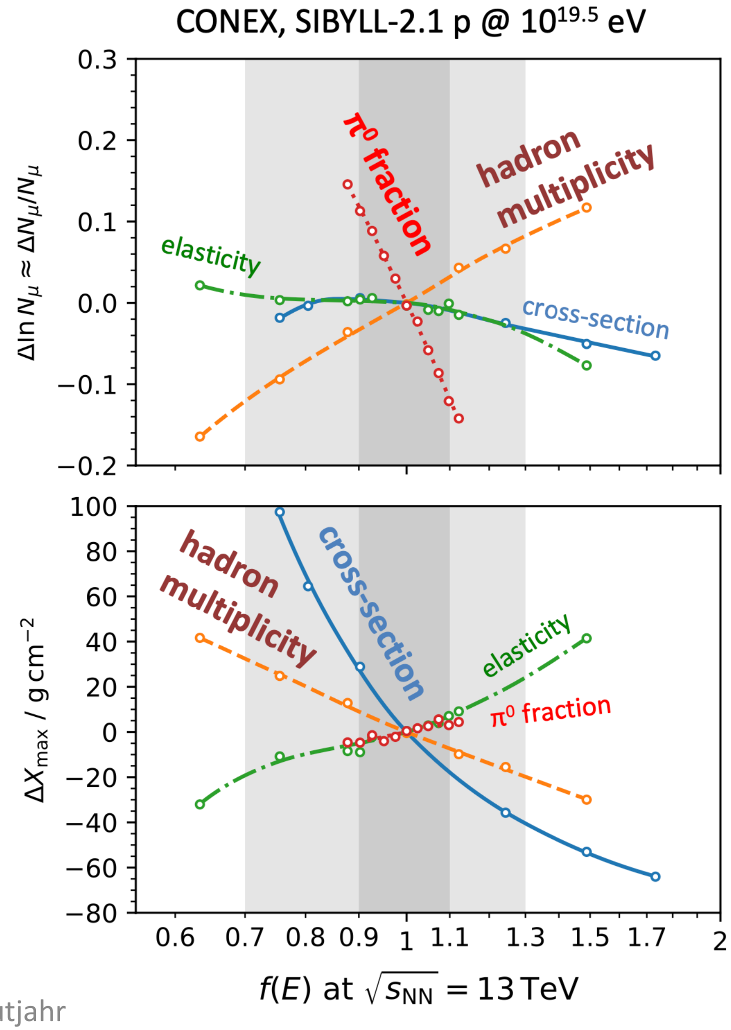
Request for WG reviewer

Thank you for your attention! ☺️

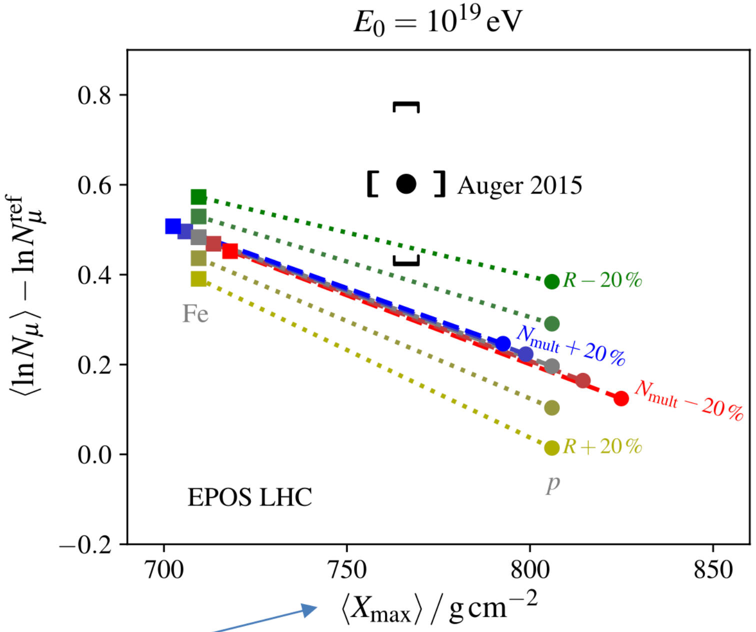
Backup

Possible Solutions

R. Ulrich, R. Engel, M. Unger, PRD 83 (2011) 054026



S. Baur, HD, M. Perlin, T. Pierog, R. Ulrich, K. Werner, arXiv:1902.09265



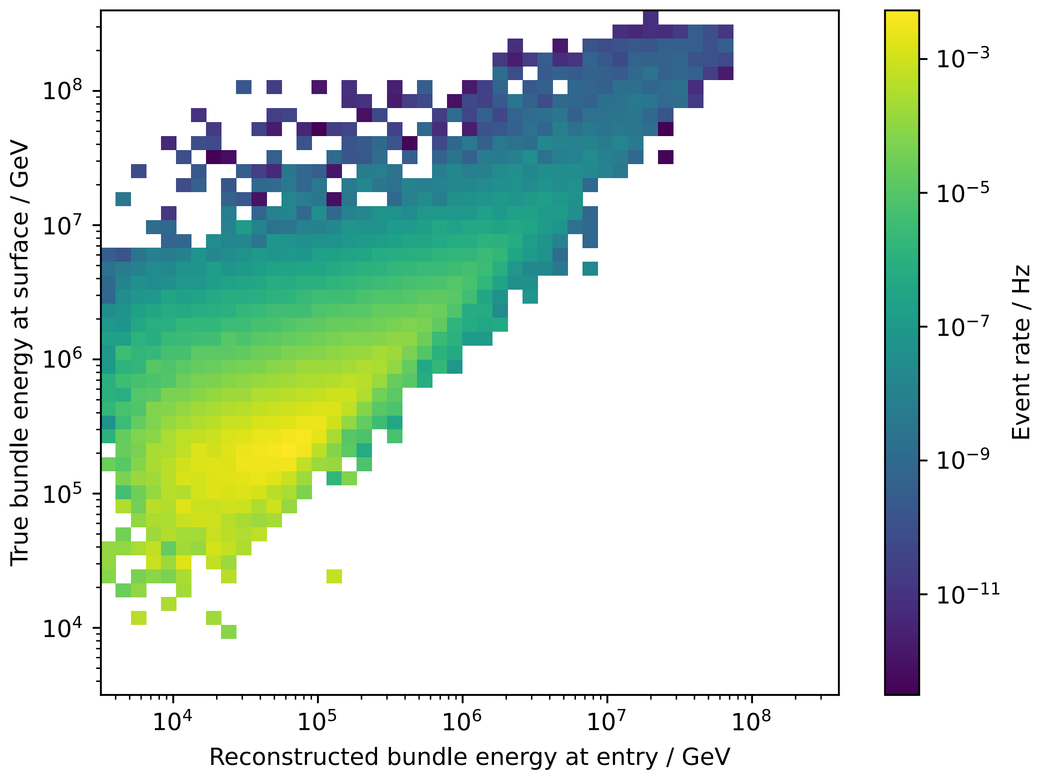
$$R = \frac{E_{\pi^0}}{E_{\text{other hadrons}}}$$

- Only changes to R can solve muon puzzle
- Small changes have large effect,
  R needs to be known to about 5 %

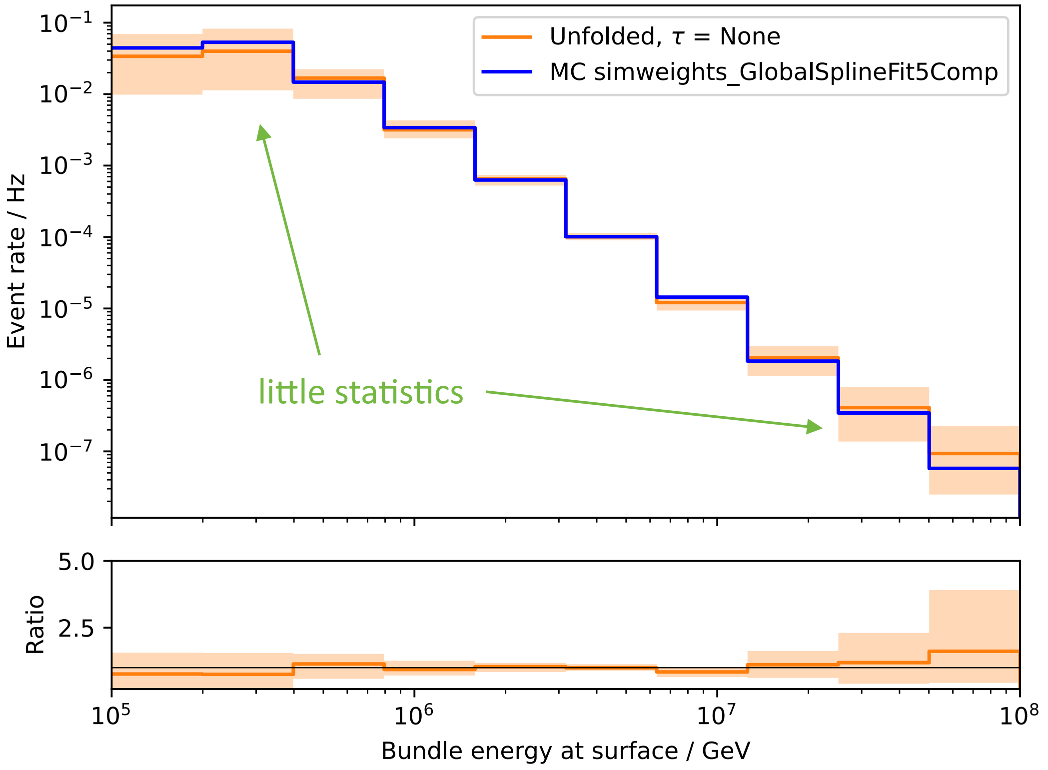
Hans Dembinski

Unfolding of event rate – muon bundles

Target



Proxy



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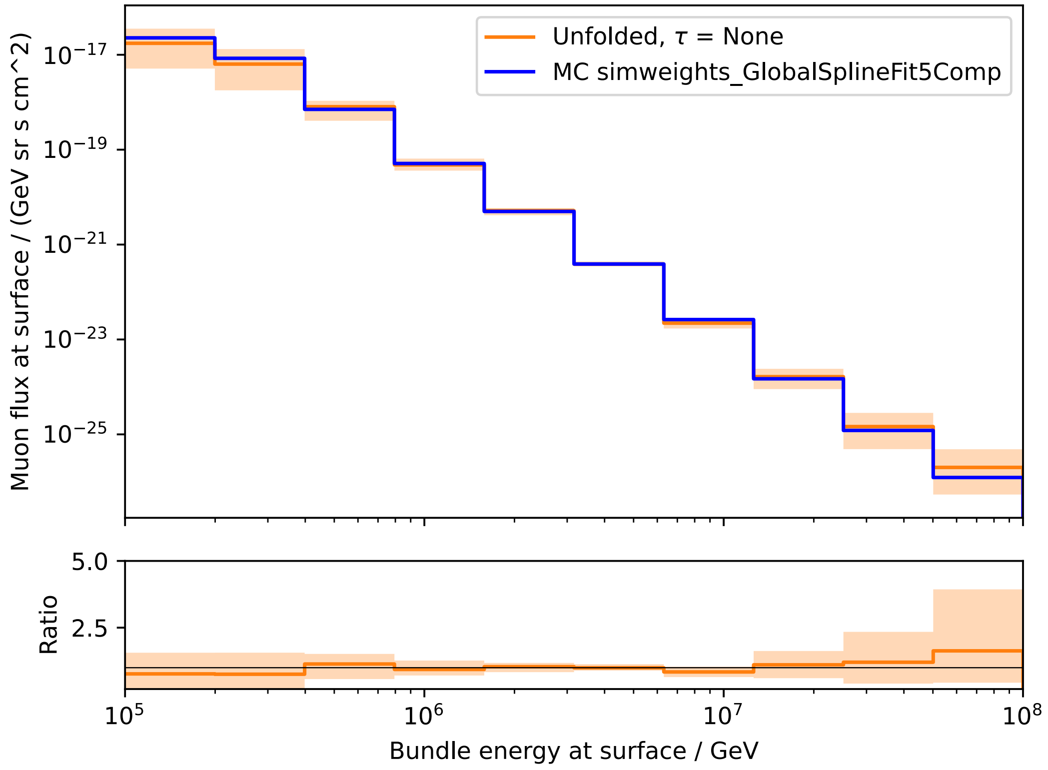
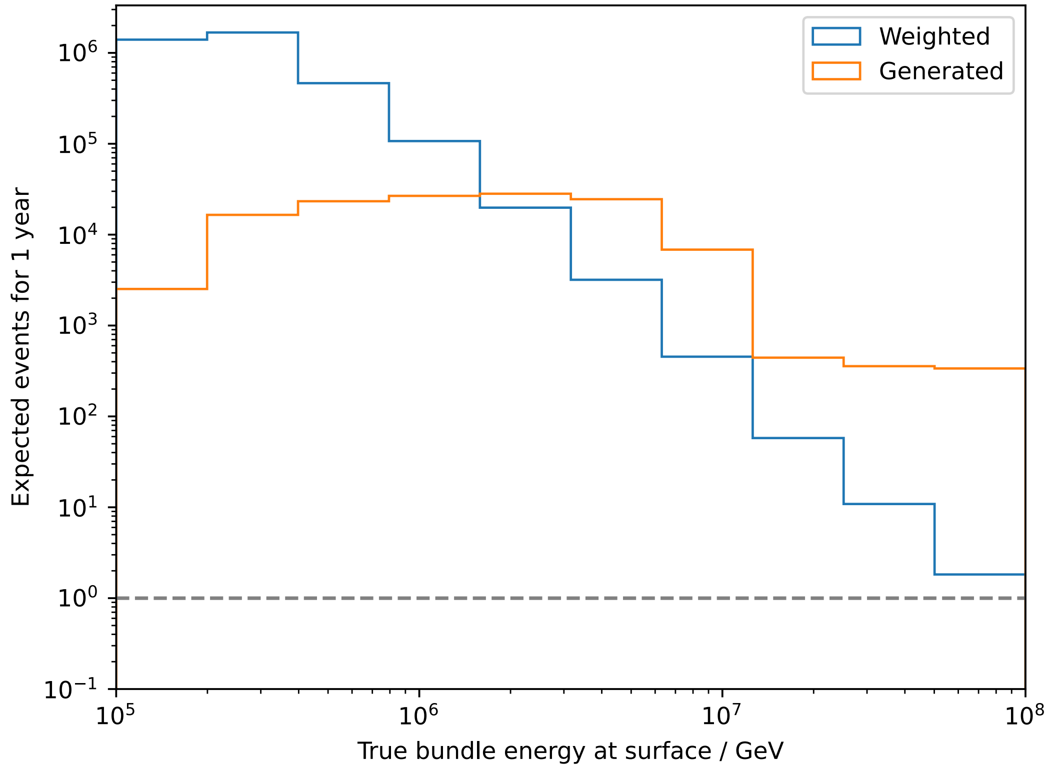
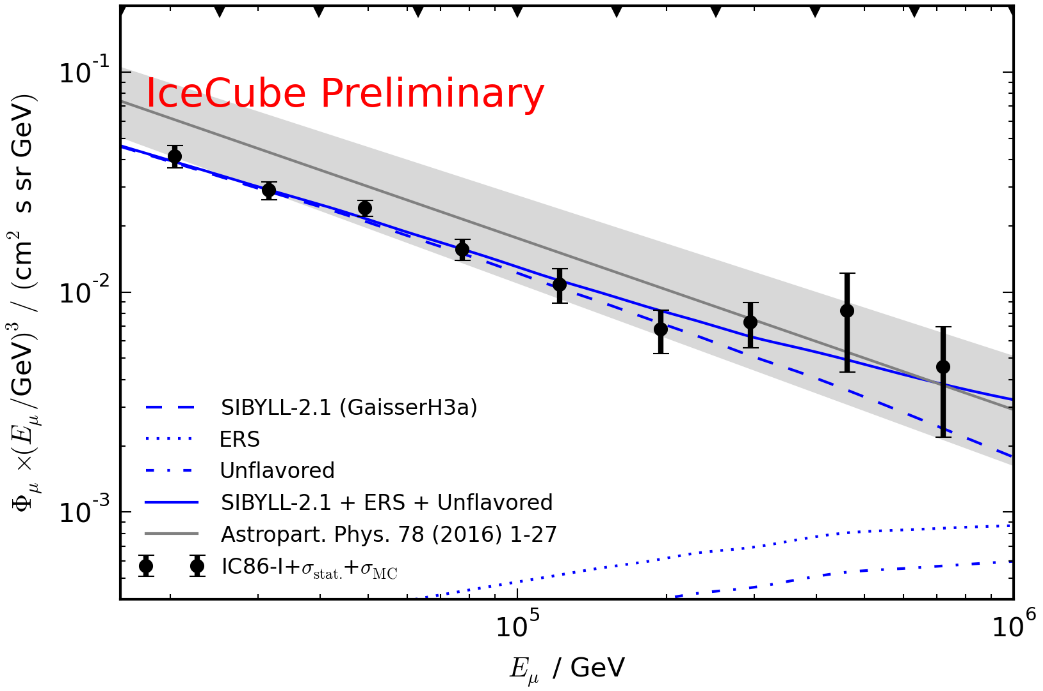
Unfolding of flux – muon bundles

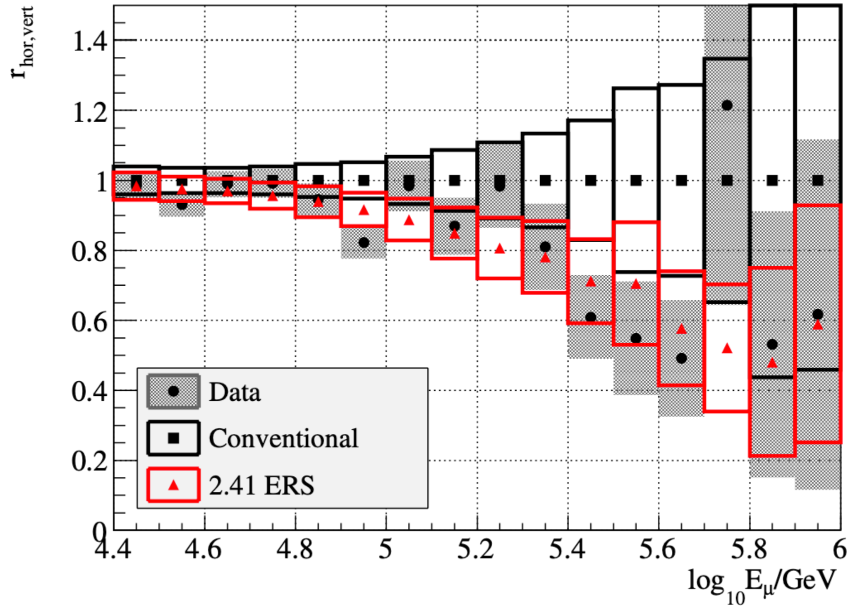




Motivation

- ➢ Detect and measure normalization of prompt component of atmospheric muon flux
- ➢ Constrain uncertainties on hadronic interaction models at very high energies
- Former analyses:
  - Leading muon analysis: limited MC statistics (by Tomasz Fuchs, https://wiki.icecube.wisc.edu/index.php/Analysis_of_Leading_Muons)
  - Characterization of the muon flux: zenith problem (by Patrick Berghaus, https://arxiv.org/abs/1506.07981)

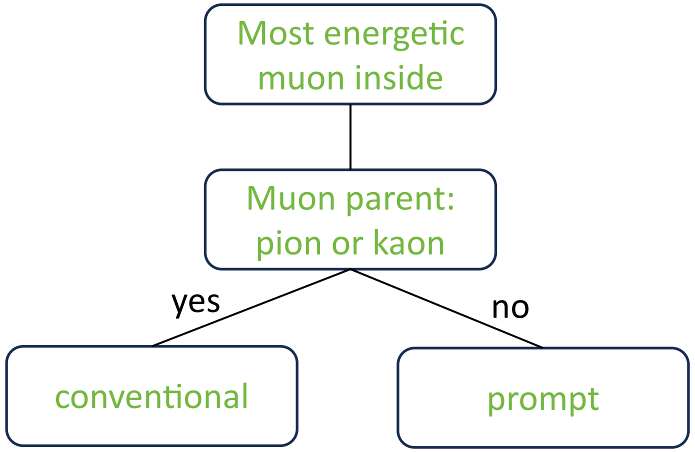




Sample	Best Fit (ERS)	1σ Interval (90% CL)	$\sigma(\Phi_{prompt} > 0)$
Uncorrected	4.93	4.05-5.87 (3.55-6.56)	9.43
Marginalized Ang. Corr.	3.19	1.64-5.48 (0.98-7.26)	3.46

- ➢ Good analyses with some challenges
- ➢ Prompt MC truth not available -> new CORSIKA simulations

Overview



Final goals

- ☐ Measure normalization of the atmospheric prompt muon flux
- ☐ Unfold muon energy spectrum

Steps for normalization measurement

- ☑ Verify CORSIKA extended history simulations
- ☑ Tag prompt muons
- ☑ Comparisons to MCEq
- ☑ Set up preliminary analysis chain
- ★ Reconstruct muon energy and direction
- ★ Data/MC comparisons
- ☐ Include systematics
- ☐ Run full statistics simulation

Additional steps for unfolding

- ☐ Effective area

Terminology

- Muon bundle: all muons in a bundle
- Leading muon: most energetic muon in a muon bundle
- Single muon: no single muons at high energies
- Prompt muon: parent is not pion or kaon

New (preliminary) CORSIKA Ehist simulation

- CORSIKA 77420
- SIBYLL 2.3d
- 600 GeV – 50 EeV
- /data/sim/IceCube/2023/generated/CORSIKA_EHISTORY/

DNN reconstructions

Reconstructed properties

Energy

- entry_energy: Leading muon energy at the detector entry
- bundle_energy_at_entry: Muon bundle energy at the detector entry
- muon_energy_first_mctree: Leading muon energy at surface
- bundle_energy_in_mctree: Muon bundle energy at surface

Network 1

Track geometry

- Length: Propagation length of muon in the ice
- LengthInDetector: Propagation length of muon in the detector
- center_pos_x: Closest x position of muon to center of the detector
- center_pos_y: Closest y position of muon to center of the detector
- center_pos_z: Closest z position of muon to center of the detector
- center_pos_t: Time of closest approach to the center of the detector
- entry_pos_x: x position of muon at the detector entry
- entry_pos_y: y position of muon at the detector entry
- entry_pos_z: z position of muon at the detector entry
- entry_pos_t: Time of muon at the detector entry

Network 2

Direction

- zenith: Zenith angle of muon
- azimuth: Azimuth angle of muon

Network 3

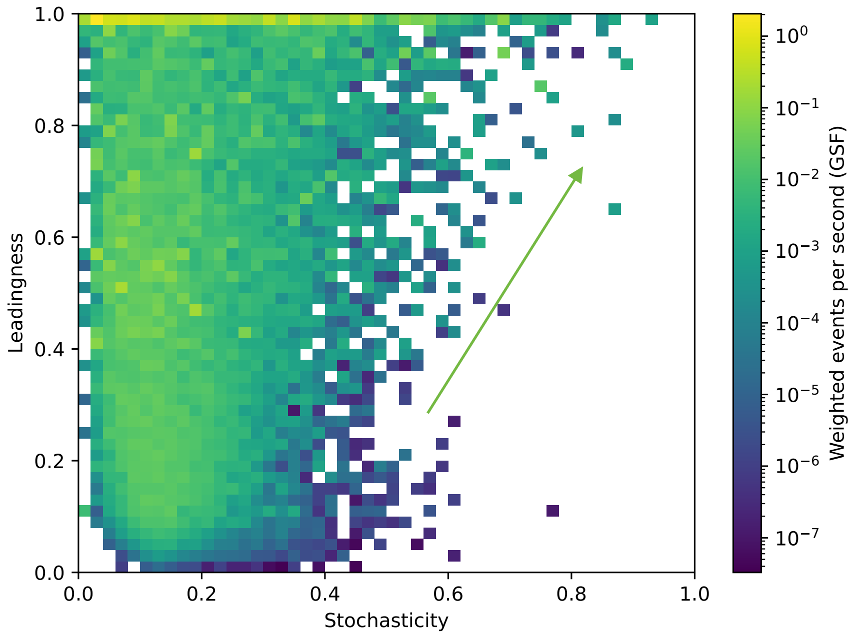
DNN Reconstructions

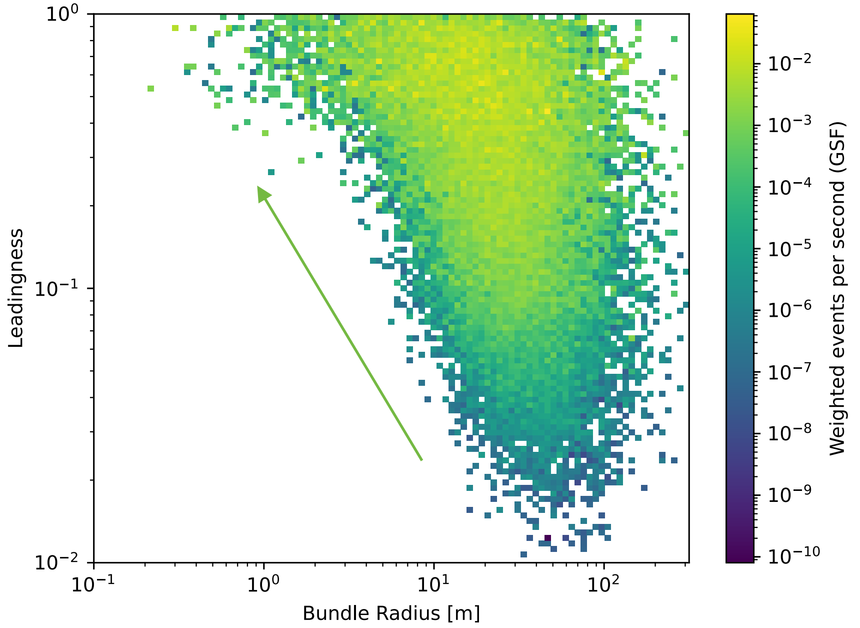
Reconstruct

- ❑ Energy
- ❑ Track geometry
- ❑ Direction

Physics motivation

- Muons lose energy stochastically
  - High leadingness: energy depositions are dominated by large stochastic losses
  - Low leadingness: stochastic losses sum up and appear continuously
- High energies: forward production
  - High leadingness: small bundle radius
  - Low leadingness: larger bundle radius





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Input data per DOM

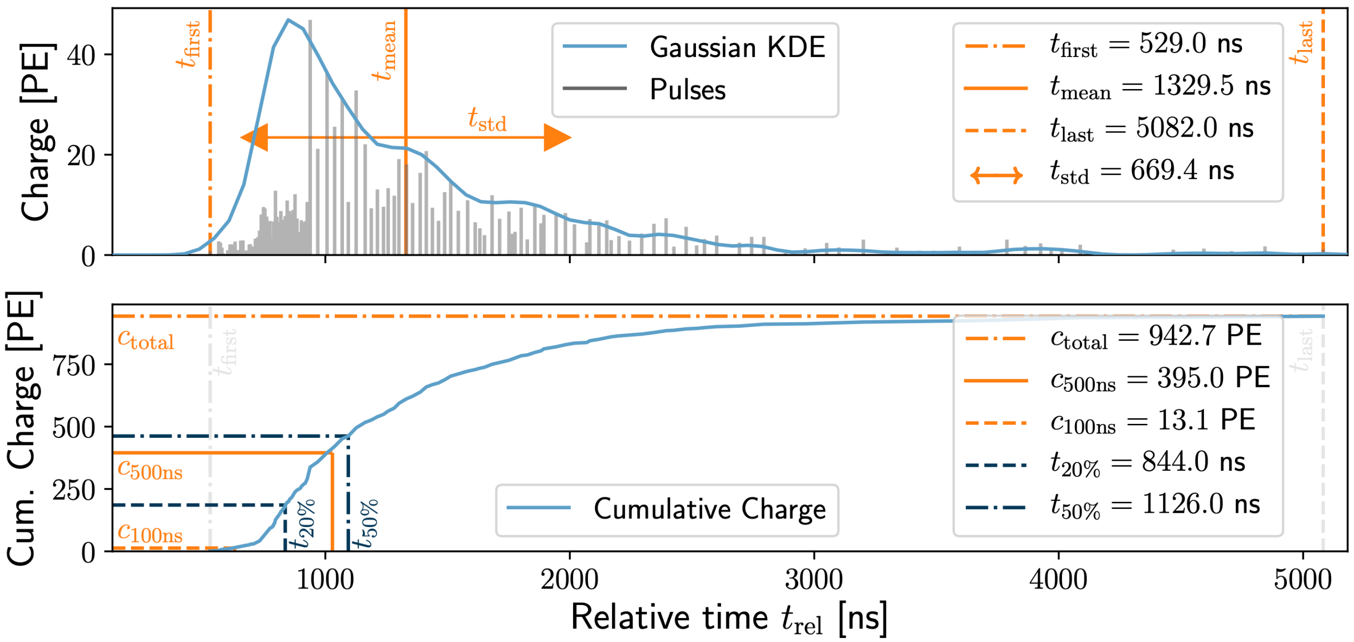
10.1088/1748-0221/16/07/P07041

3 inputs

- c_{total}: Total charge
  - Sum of charge
- t_{first}: Relative time of first pulse
  - Relative to total time offset, calculated as the charge weighted mean time of all pulses
- t_{std}: Standard deviation of first pulse
  - Charge weighted standard deviation of pulse times relative to total time offset

9 inputs

- t_{last}: Relative time of last pulse
  - Relative to total time offset, calculated as the charge weighted mean time of all pulses
- $t_{20\%}$: Relative time of 20% charge
  - Relative to total time offset, calculated as the charge weighted mean time of all pulses
- $t_{50\%}$: Relative time of 50% charge
  - Relative to total time offset, calculated as the charge weighted mean time of all pulses
- t_{mean}: Mean time
  - Charge weighted mean time of all pulses relative to total time offset
- c_{500ns}: Charge at 500ns
  - Sum of charge after 500ns
- c_{100ns}: Charge at 100ns
  - Sum of charge after 100ns



Input pulses

- SplitInIceDSTPulses
- SplitInIceDSTPulsesTWCleaning6000ns
- (DNN framework performs an internal cleaning)

Training datasets

- 20904
- 21962
- 22020
- 22187

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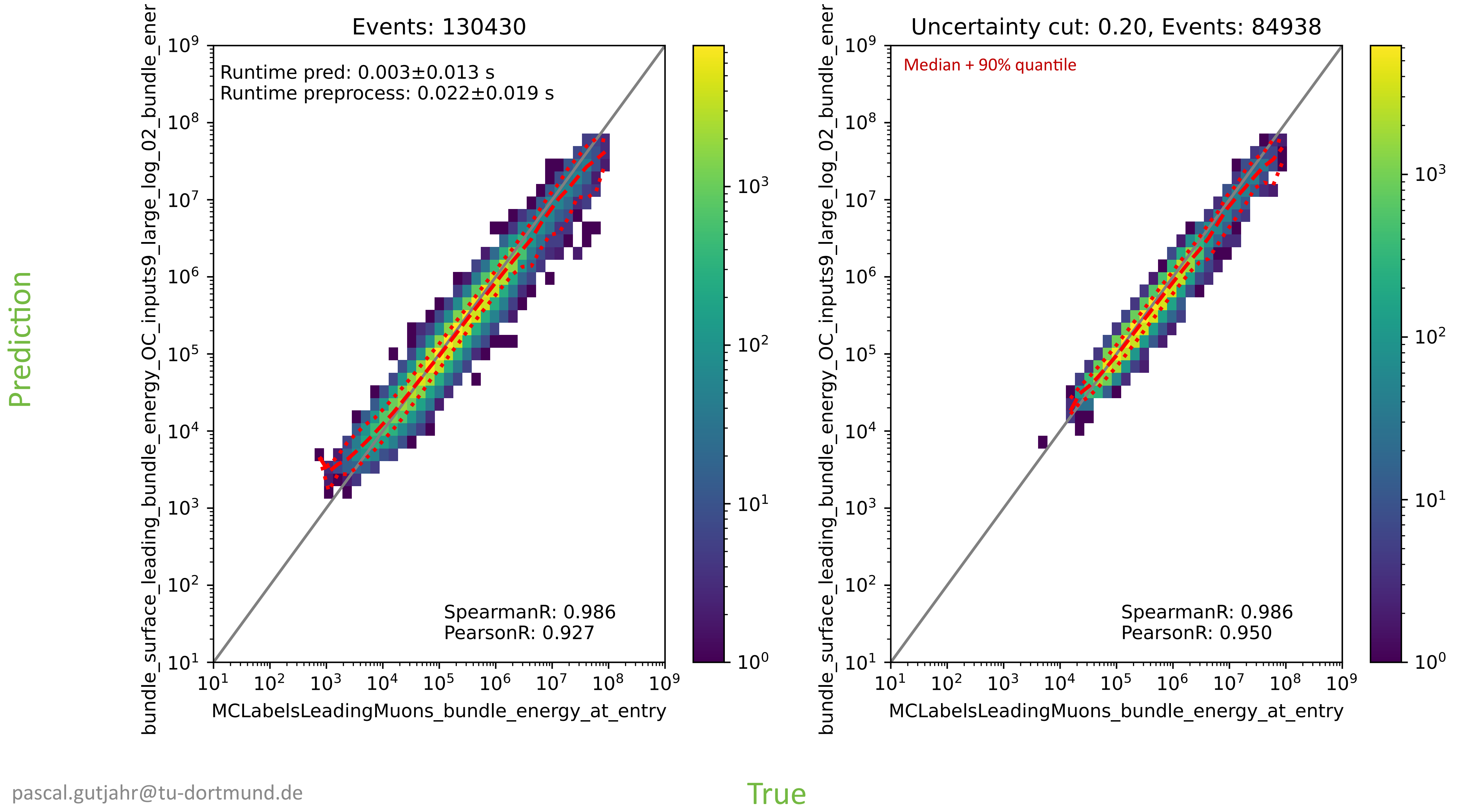
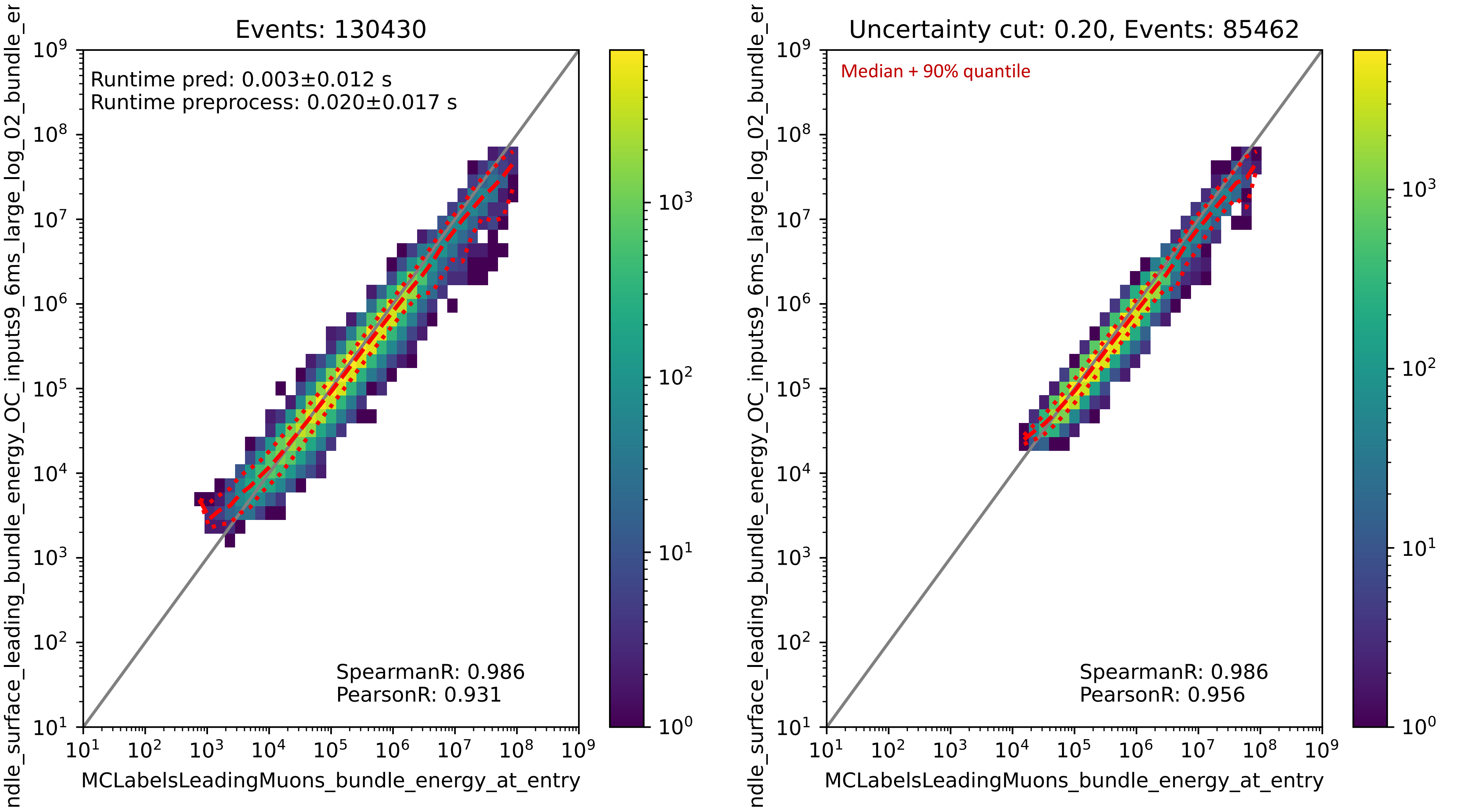
Network evaluation - Energy

Bundle energy at entry – internal cleaned pulses



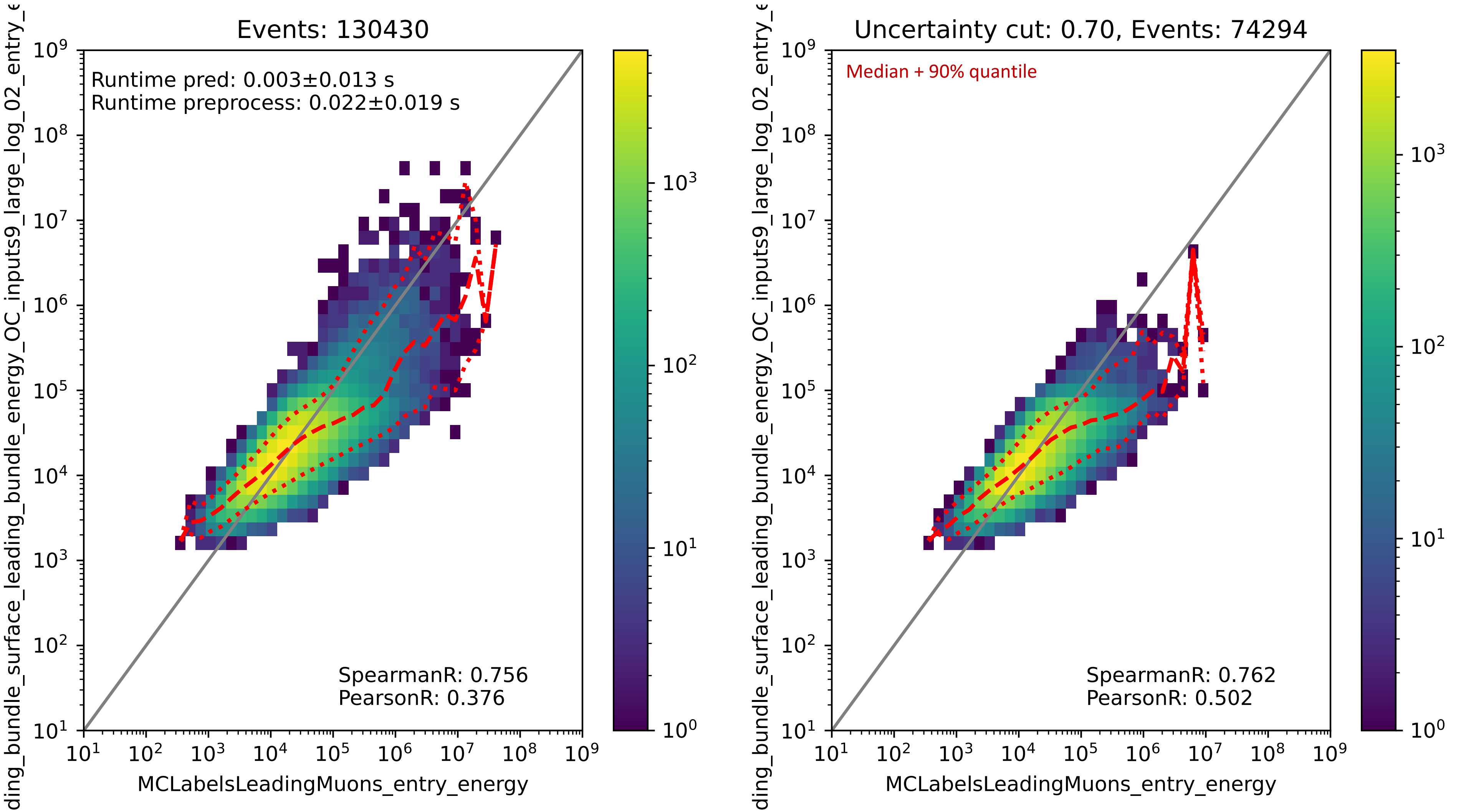
Bundle energy at entry – 6 ms cleaned pulses

Prediction



True

Leading muon energy at entry – internal cleaned pulses

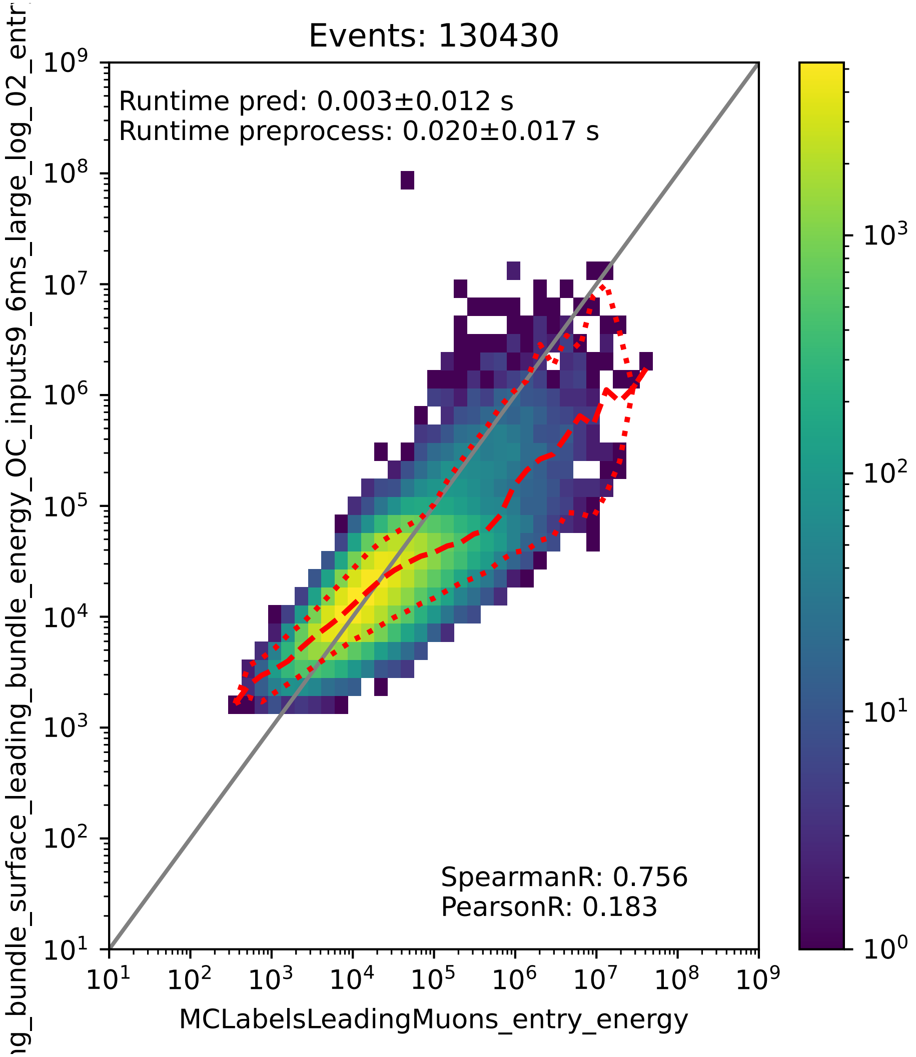


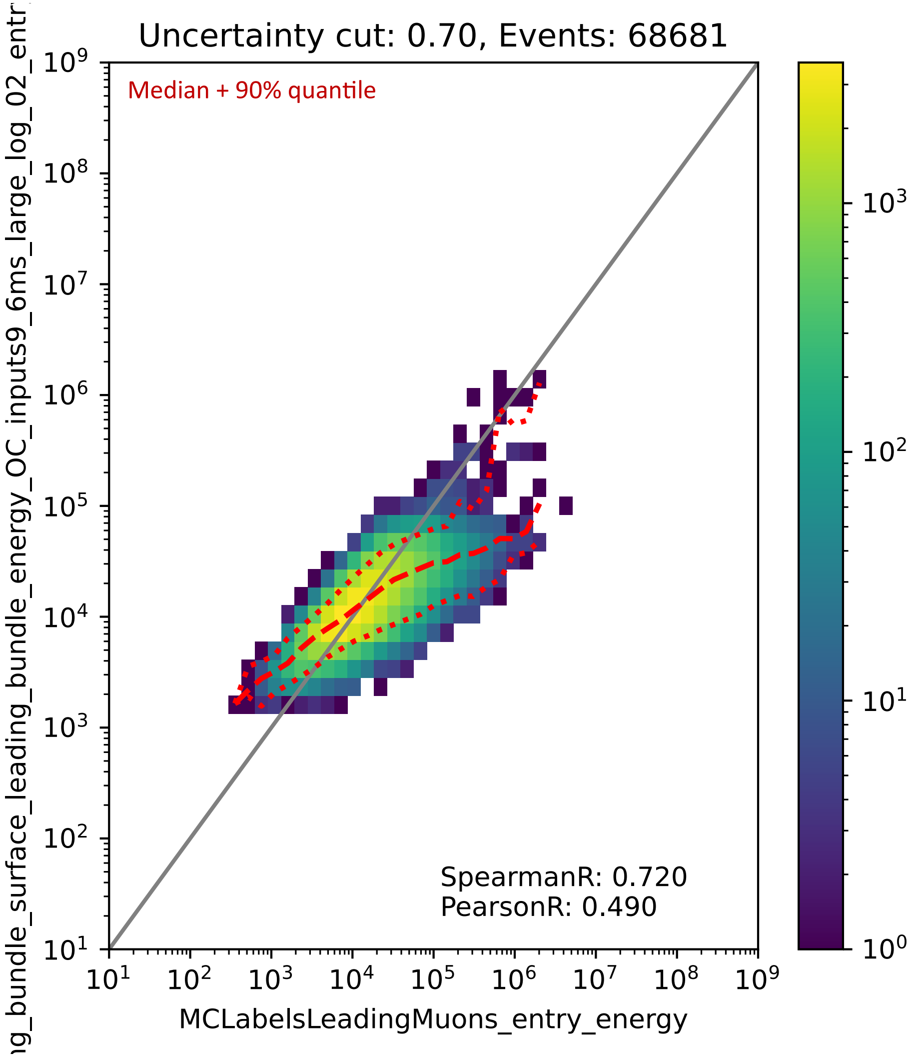
Prediction

True

Leading muon energy at entry – 6 ms cleaned pulses

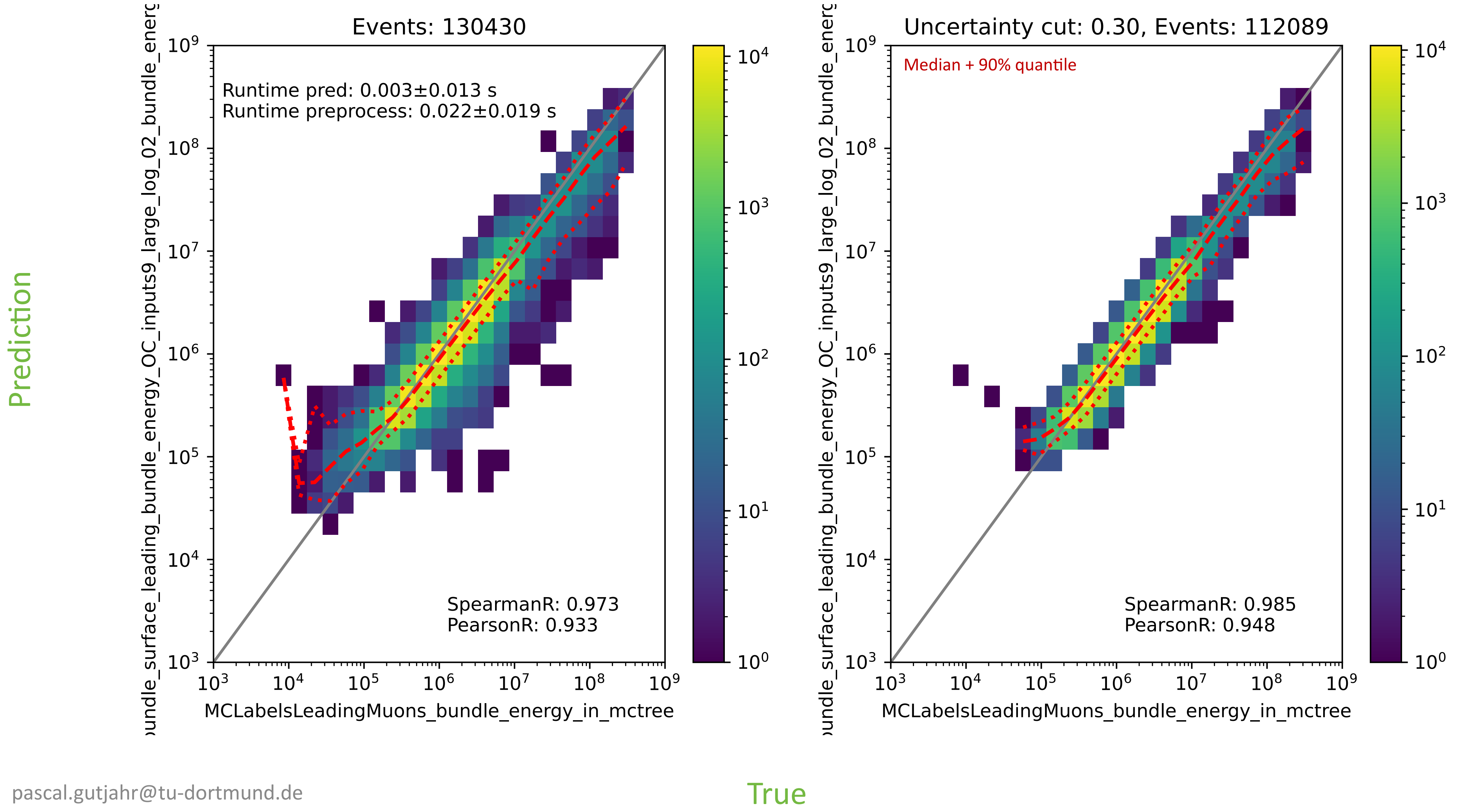
Prediction



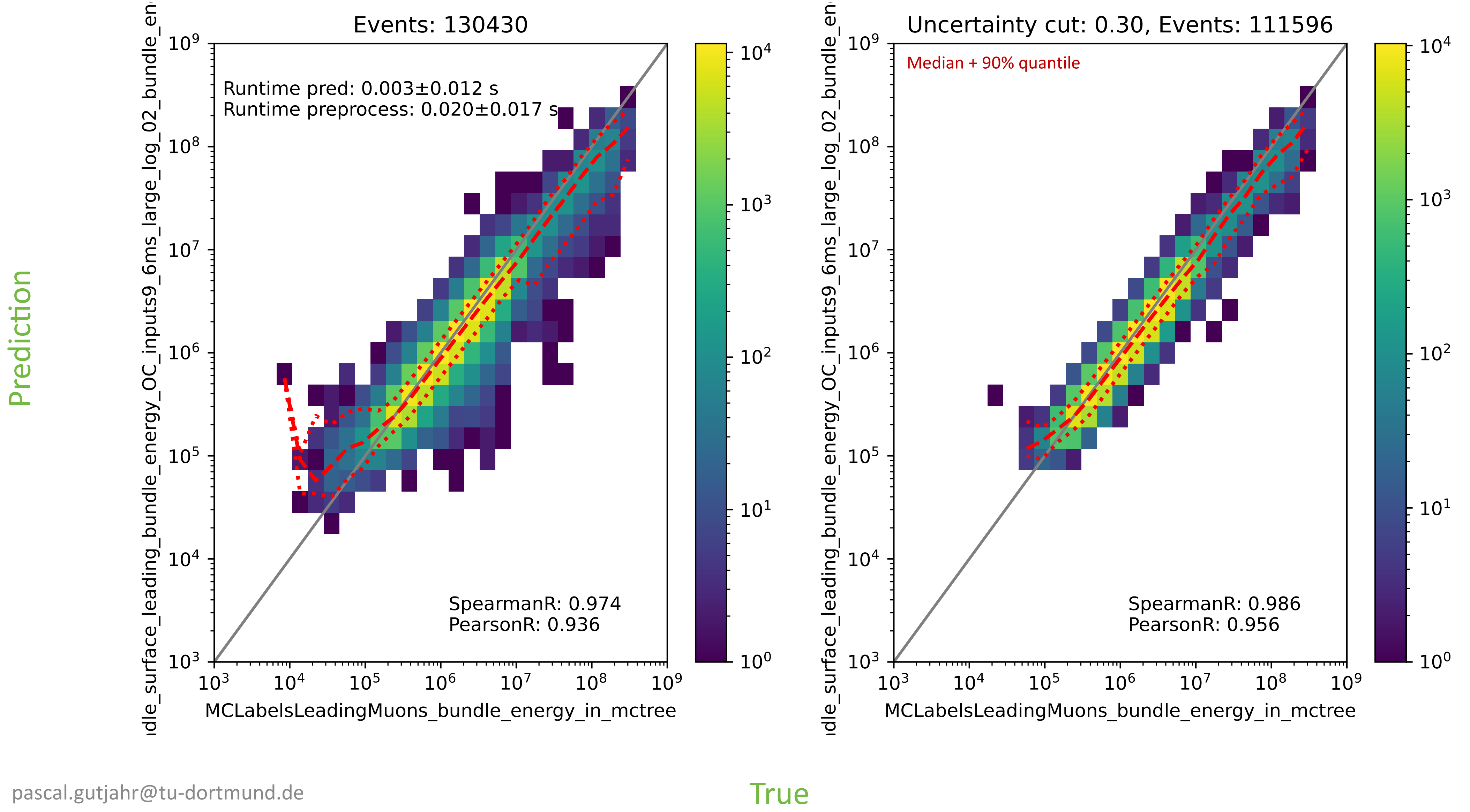


True

Bundle energy at surface – internal cleaned pulses

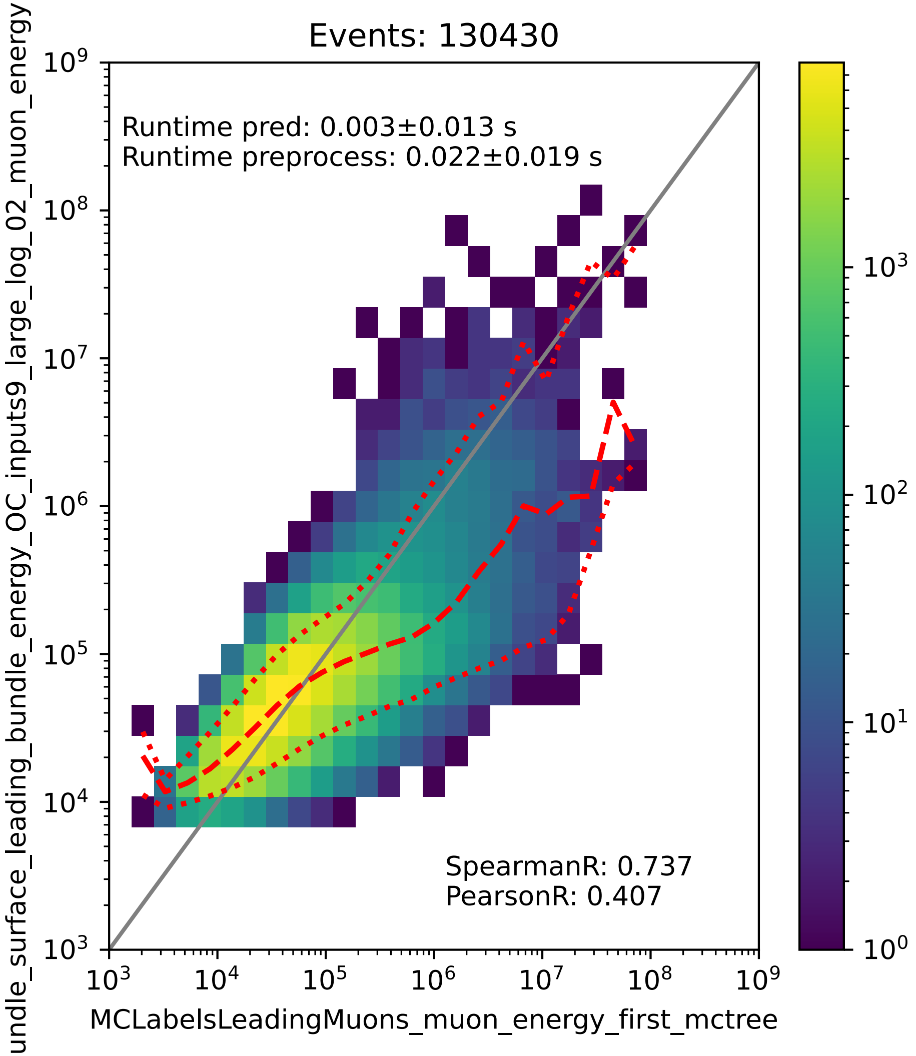


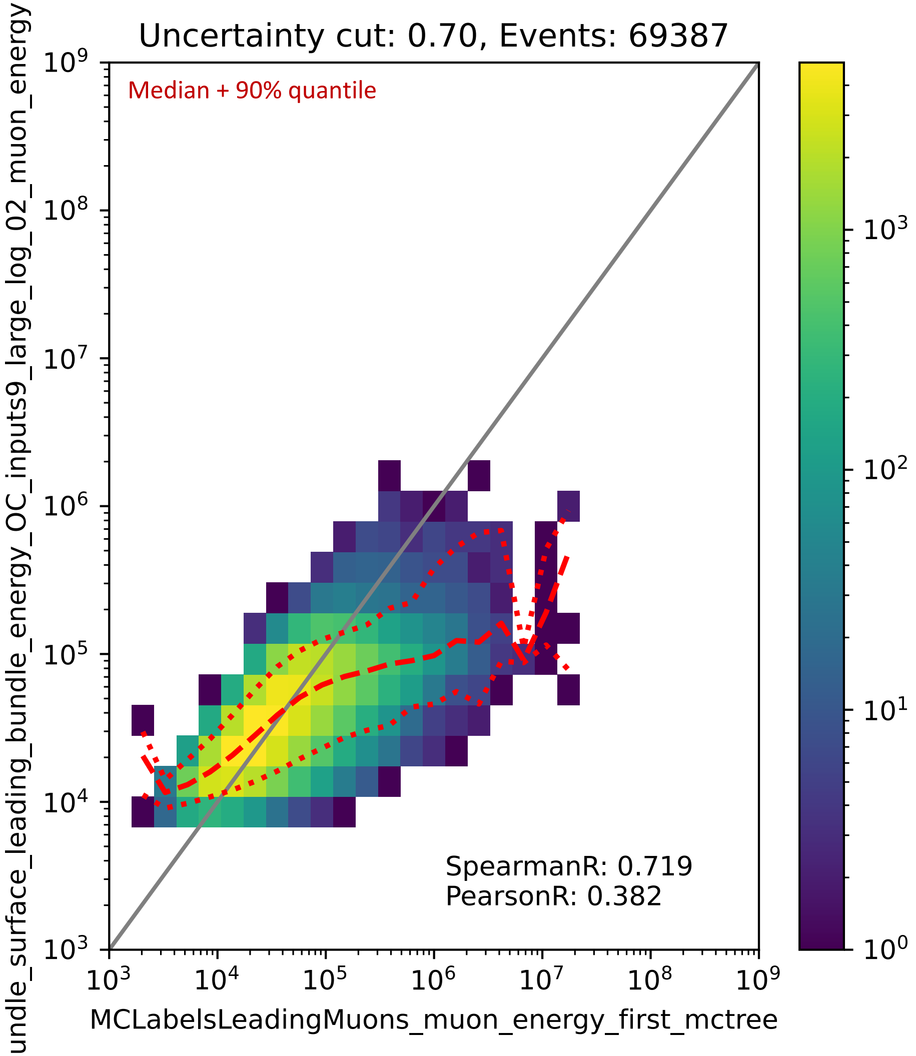
Bundle energy at surface – 6 ms cleaned pulses



Leading muon energy at surface – internal cleaned pulses

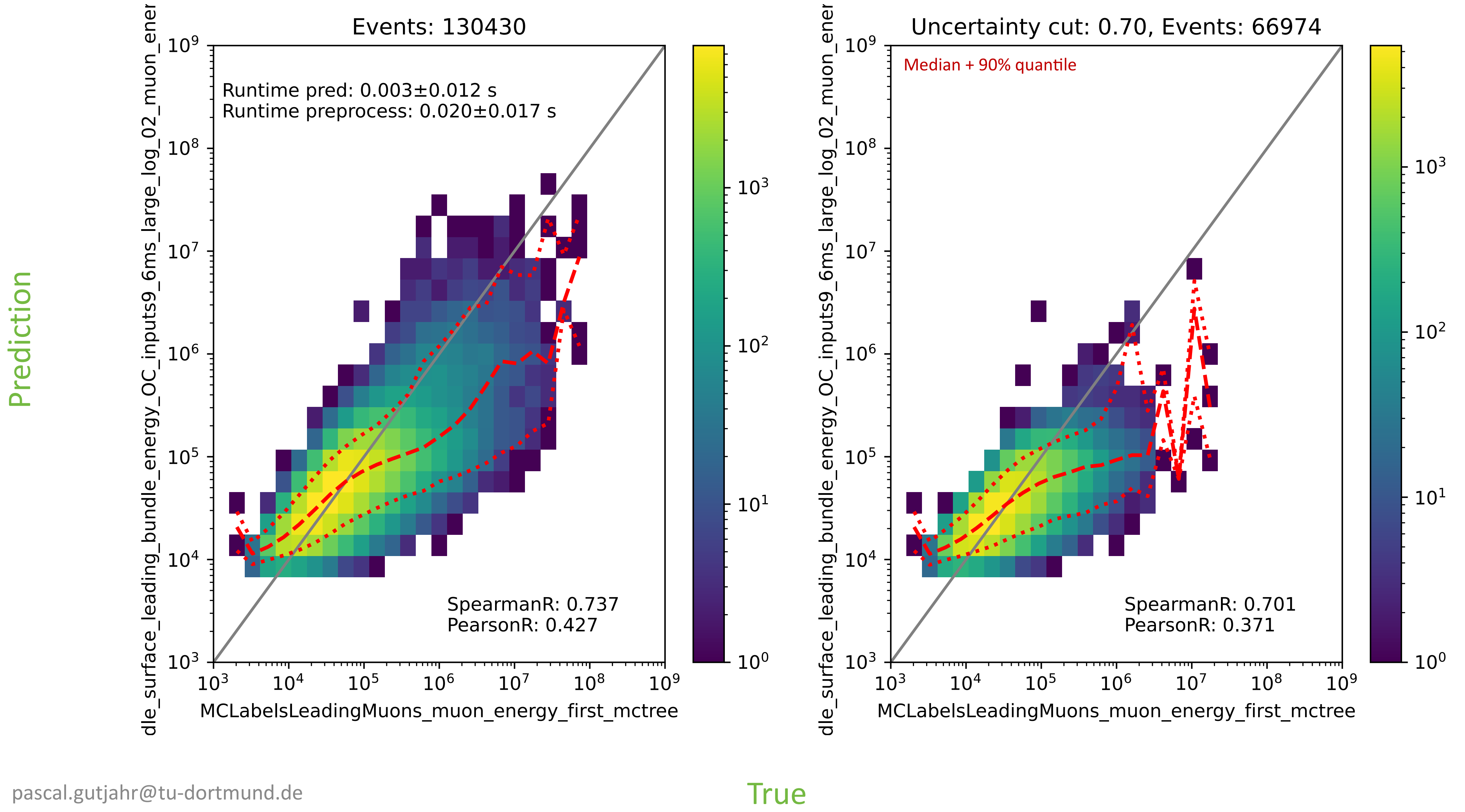
Prediction





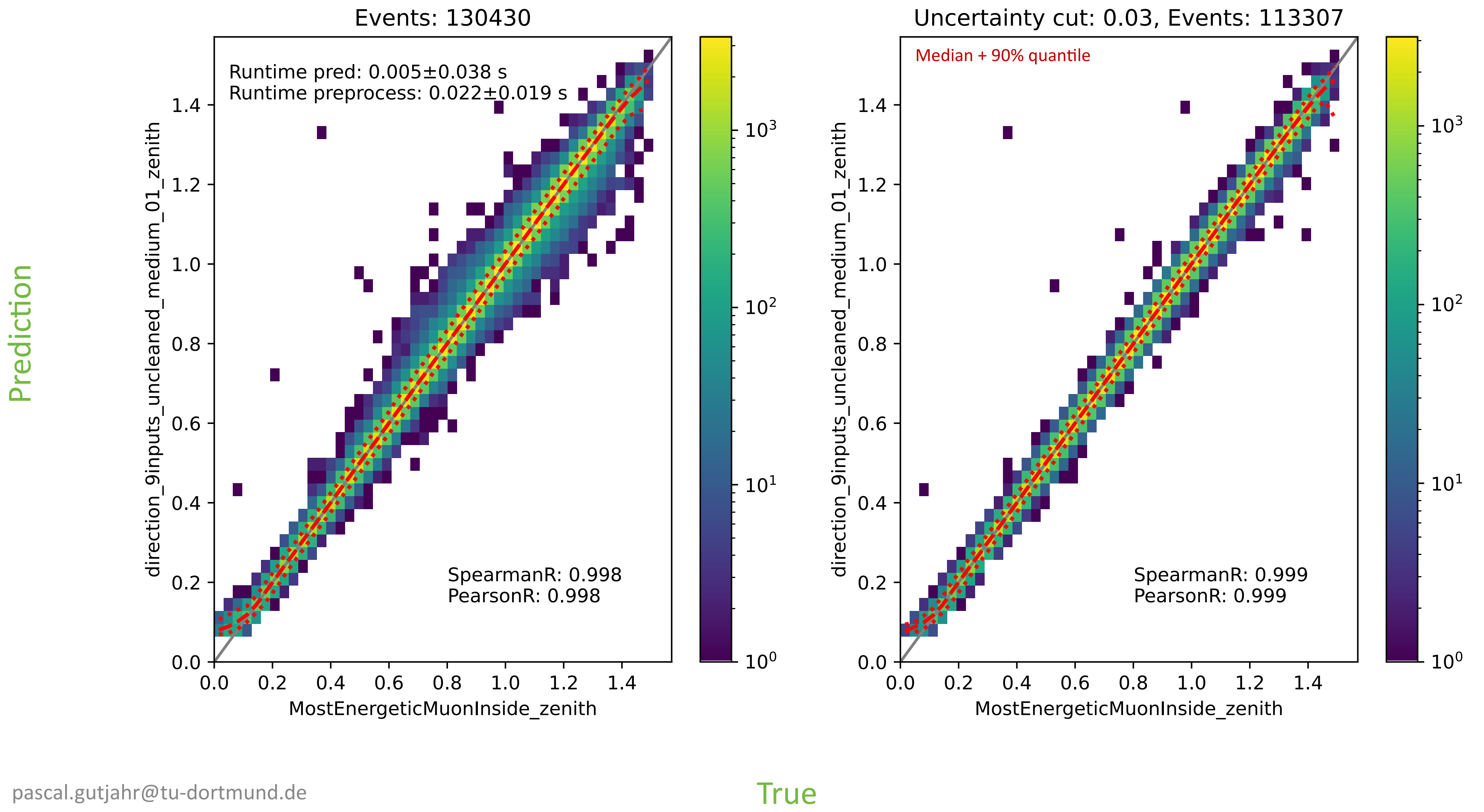
True

Leading muon energy at surface – 6 ms cleaned pulses



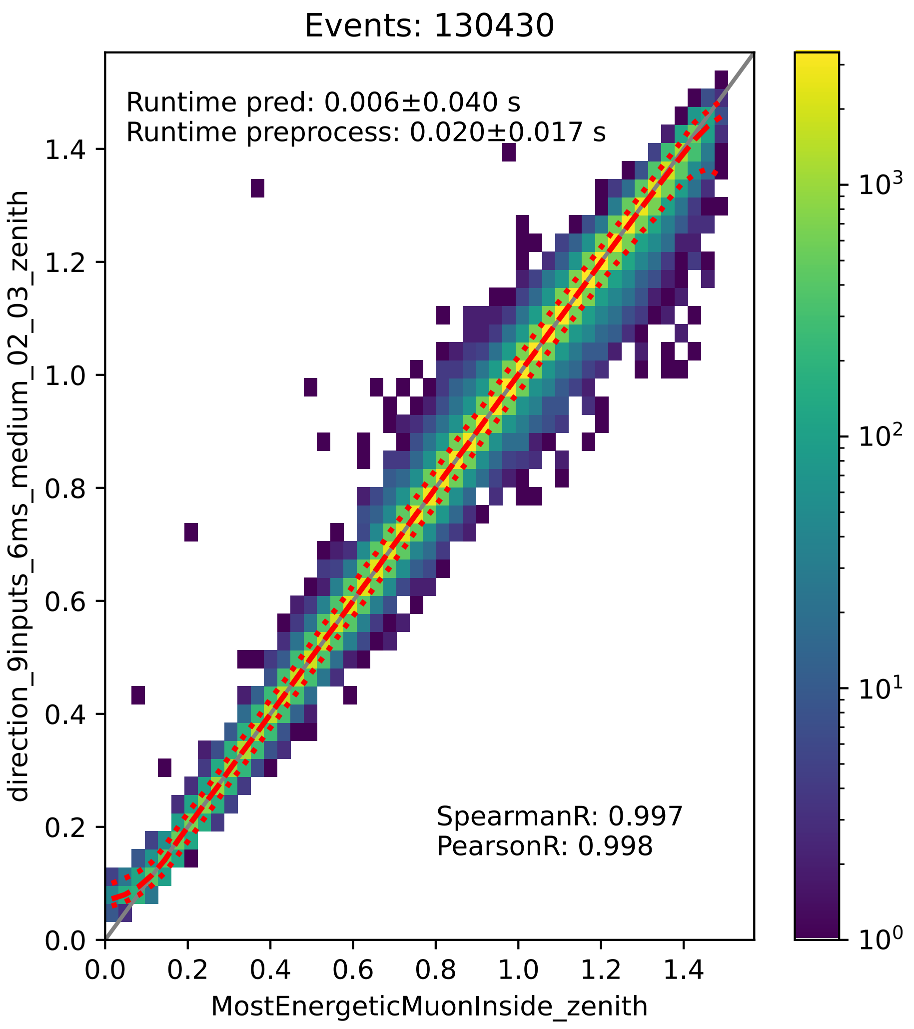
Network evaluation - Direction

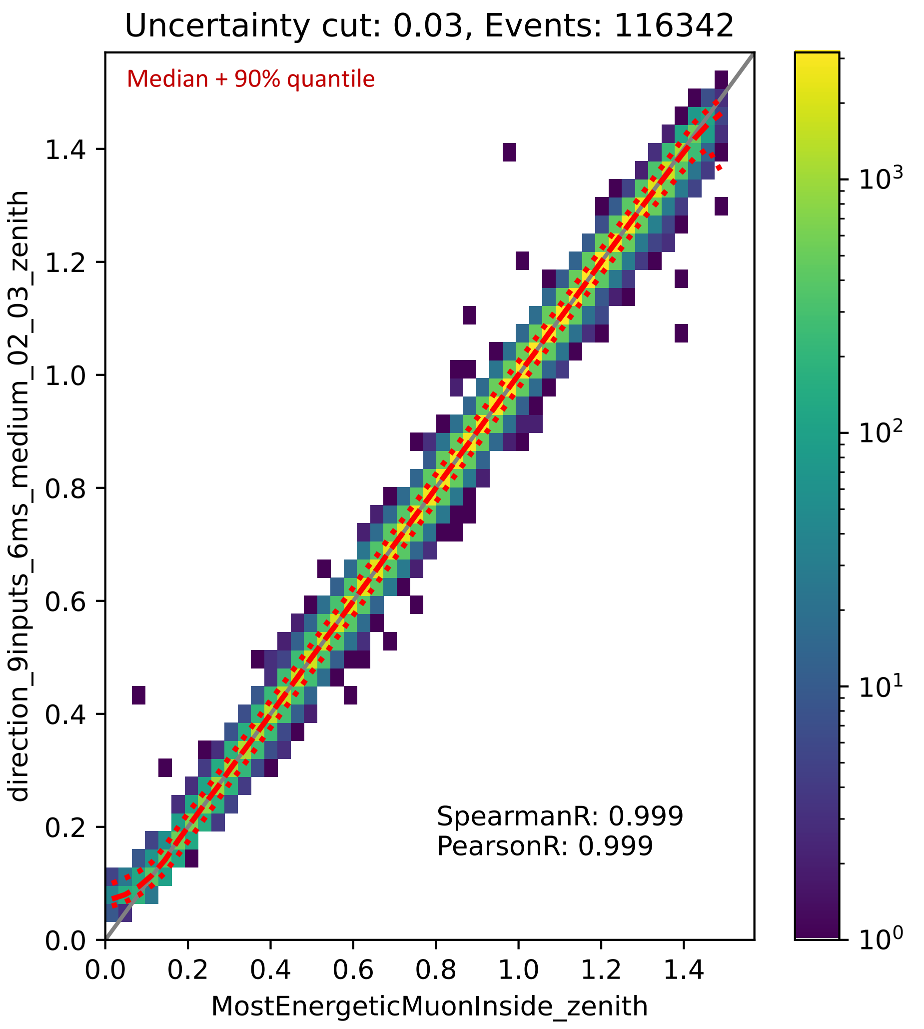
Zenith – internal cleaned pulses



Zenith – 6 ms cleaned pulses

Prediction

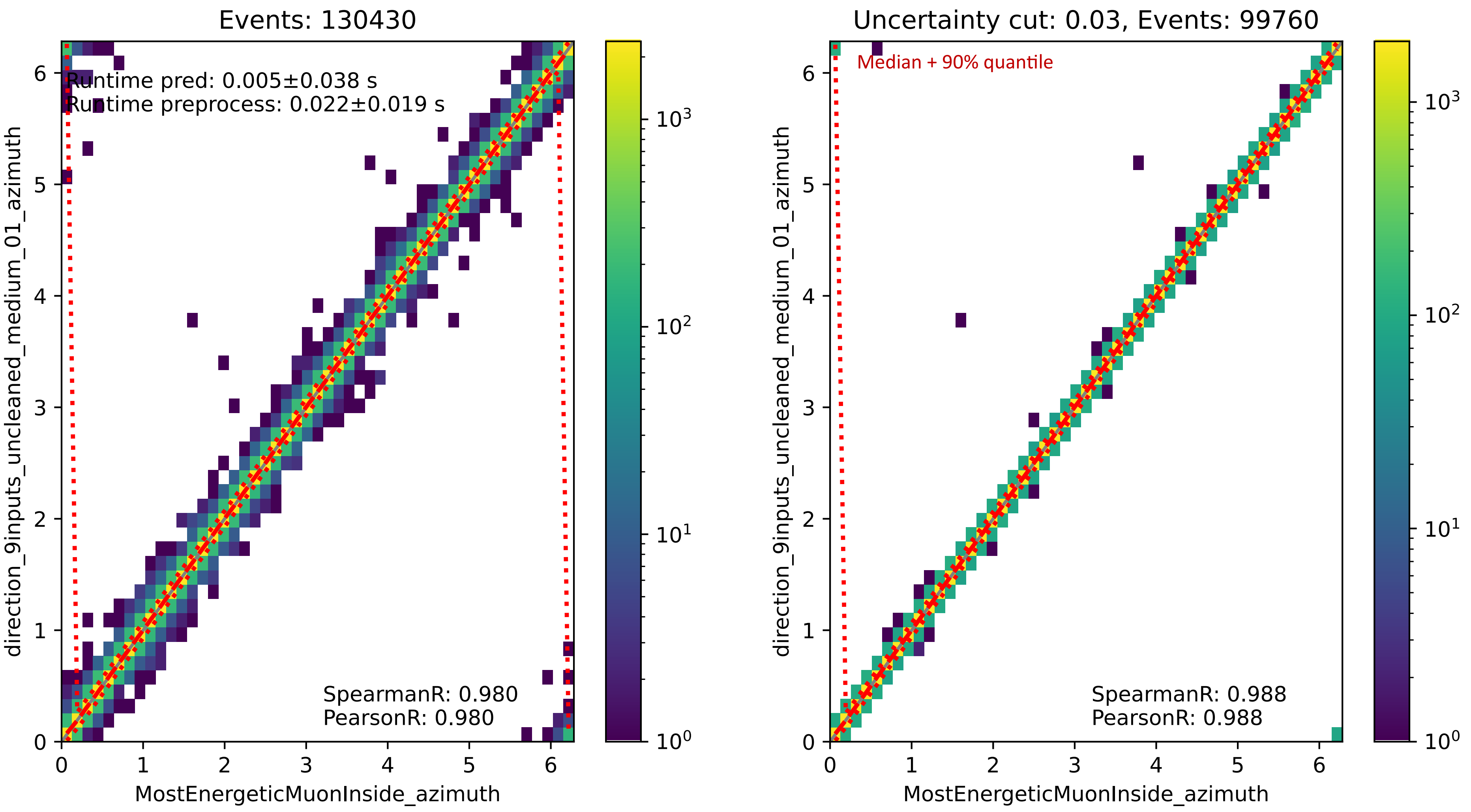




True

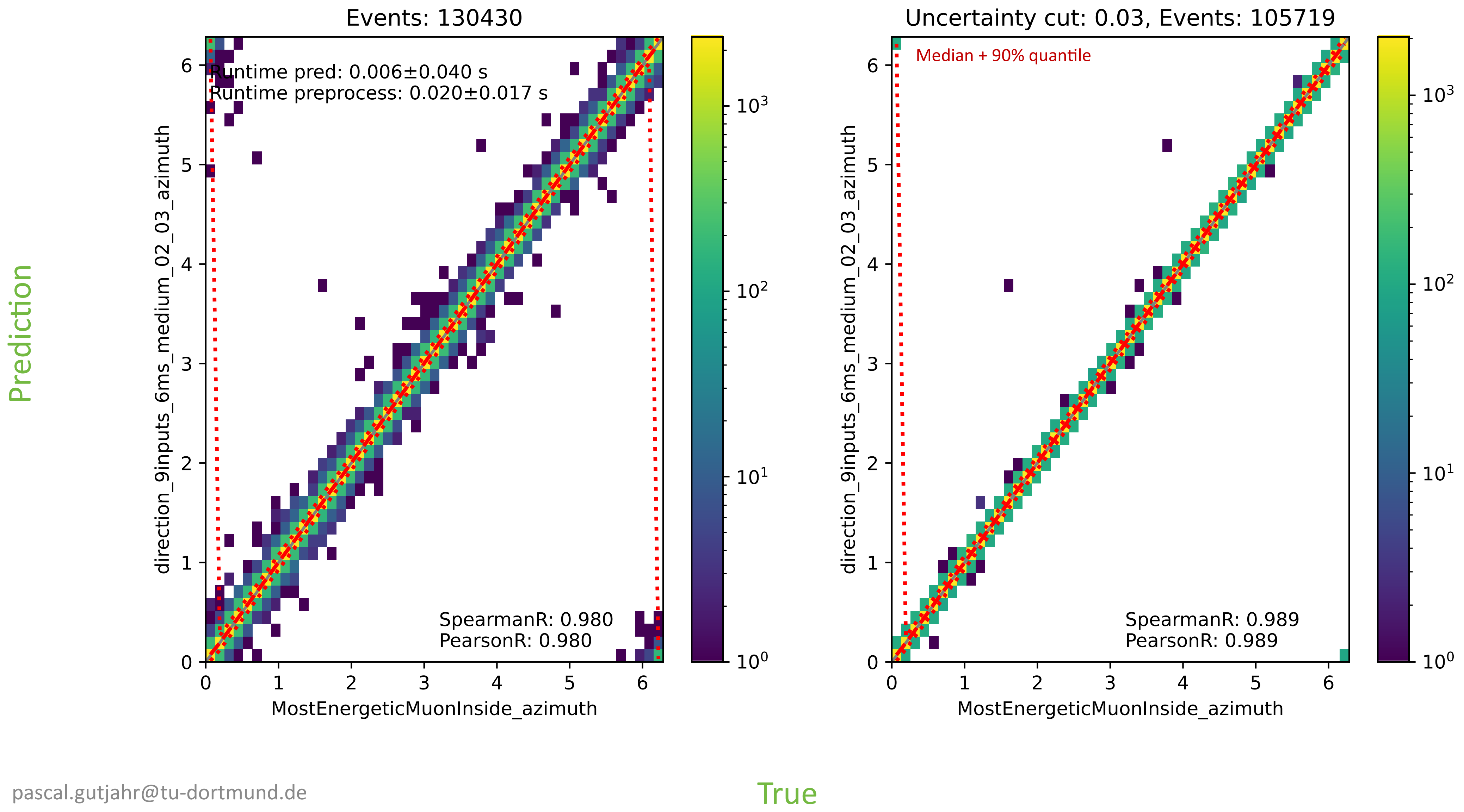
Azimuth – internal cleaned pulses

Prediction



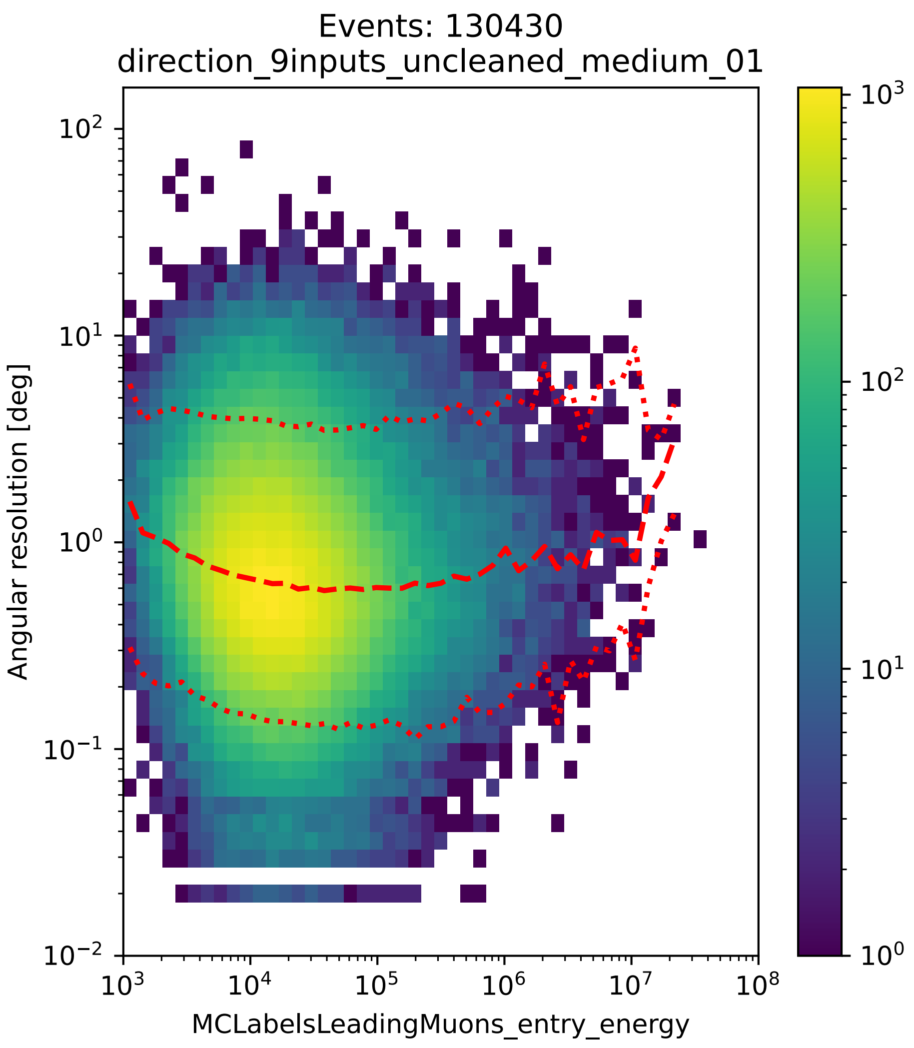
True

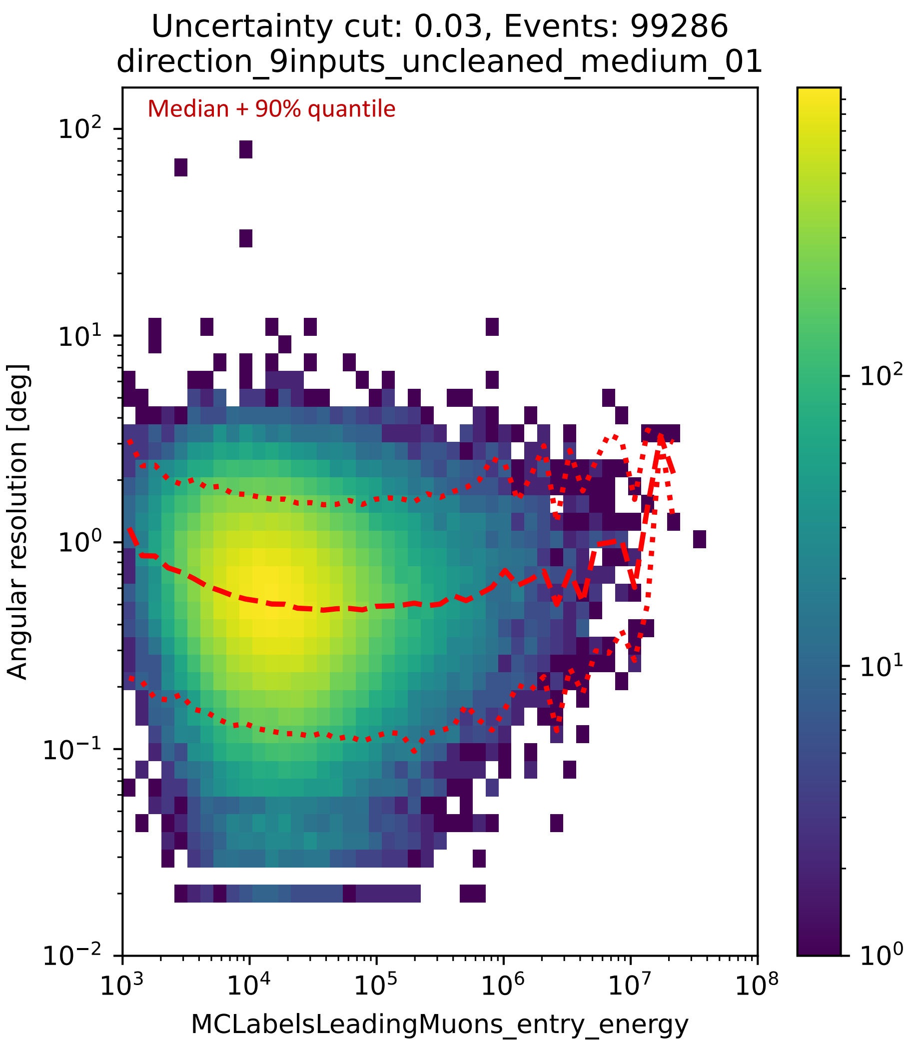
Azimuth – 6 ms cleaned pulses



Angular resolution – internal cleaned pulses

Prediction

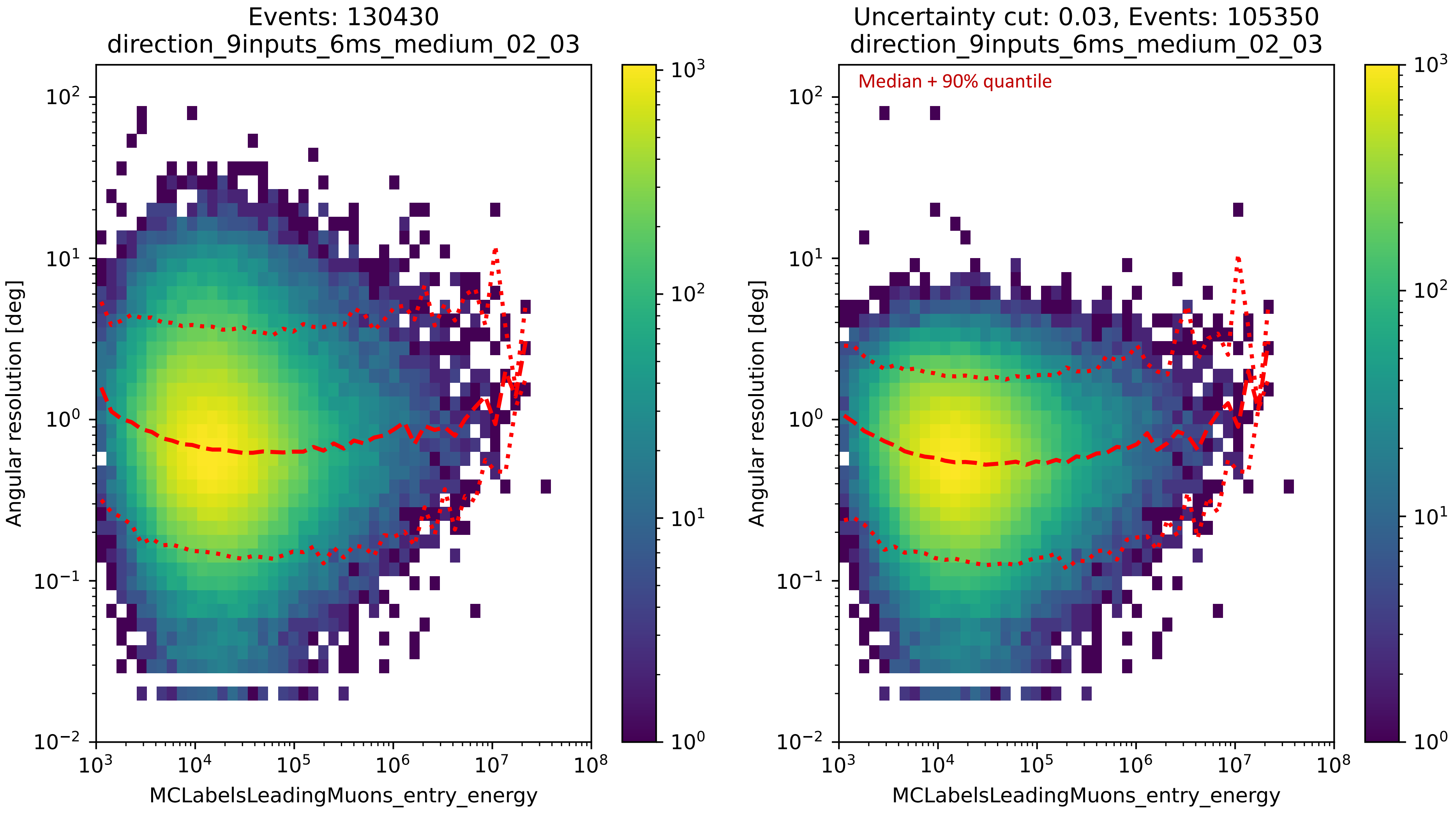




True

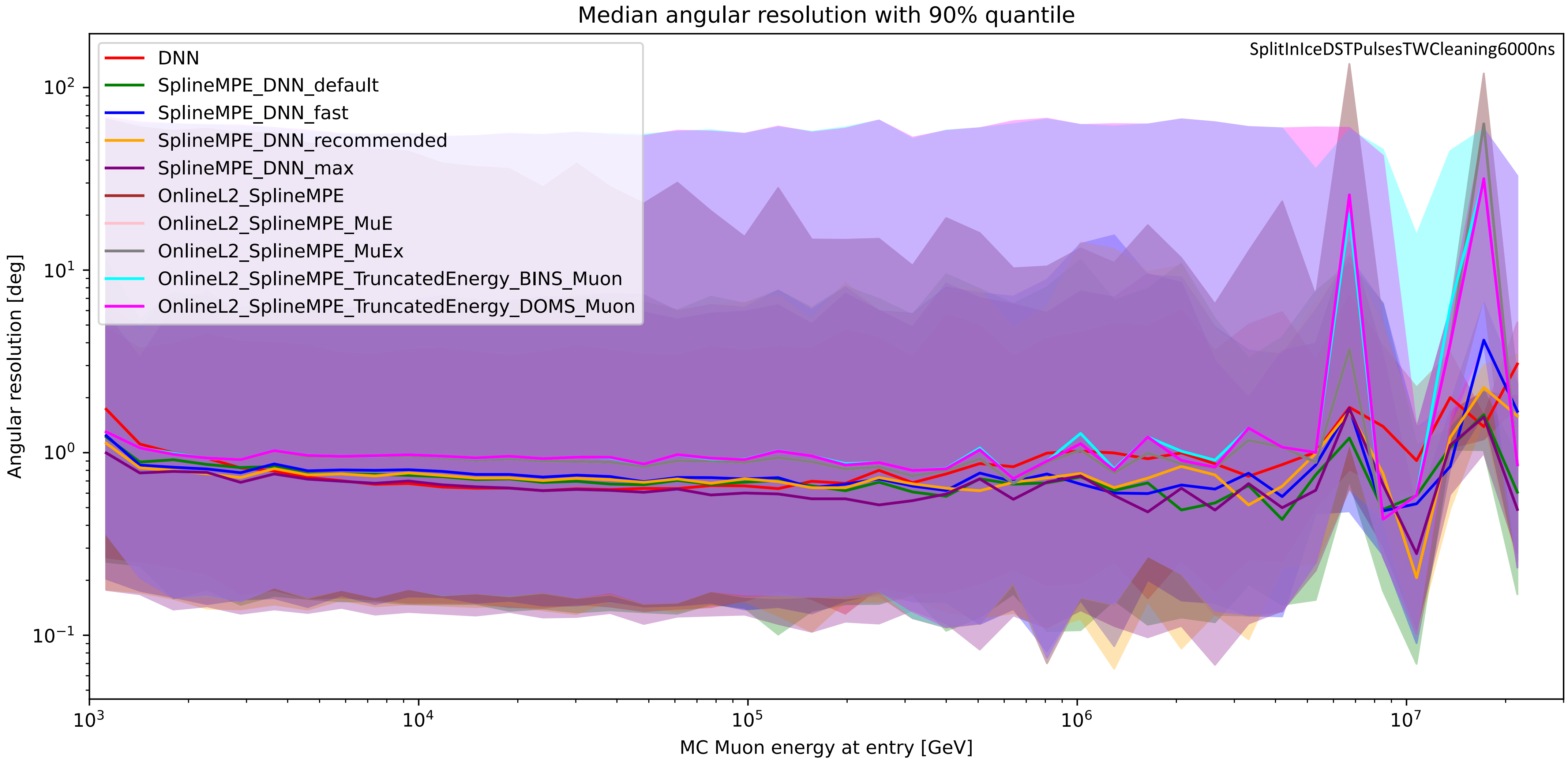
Angular resolution – 6 ms cleaned pulses

Prediction

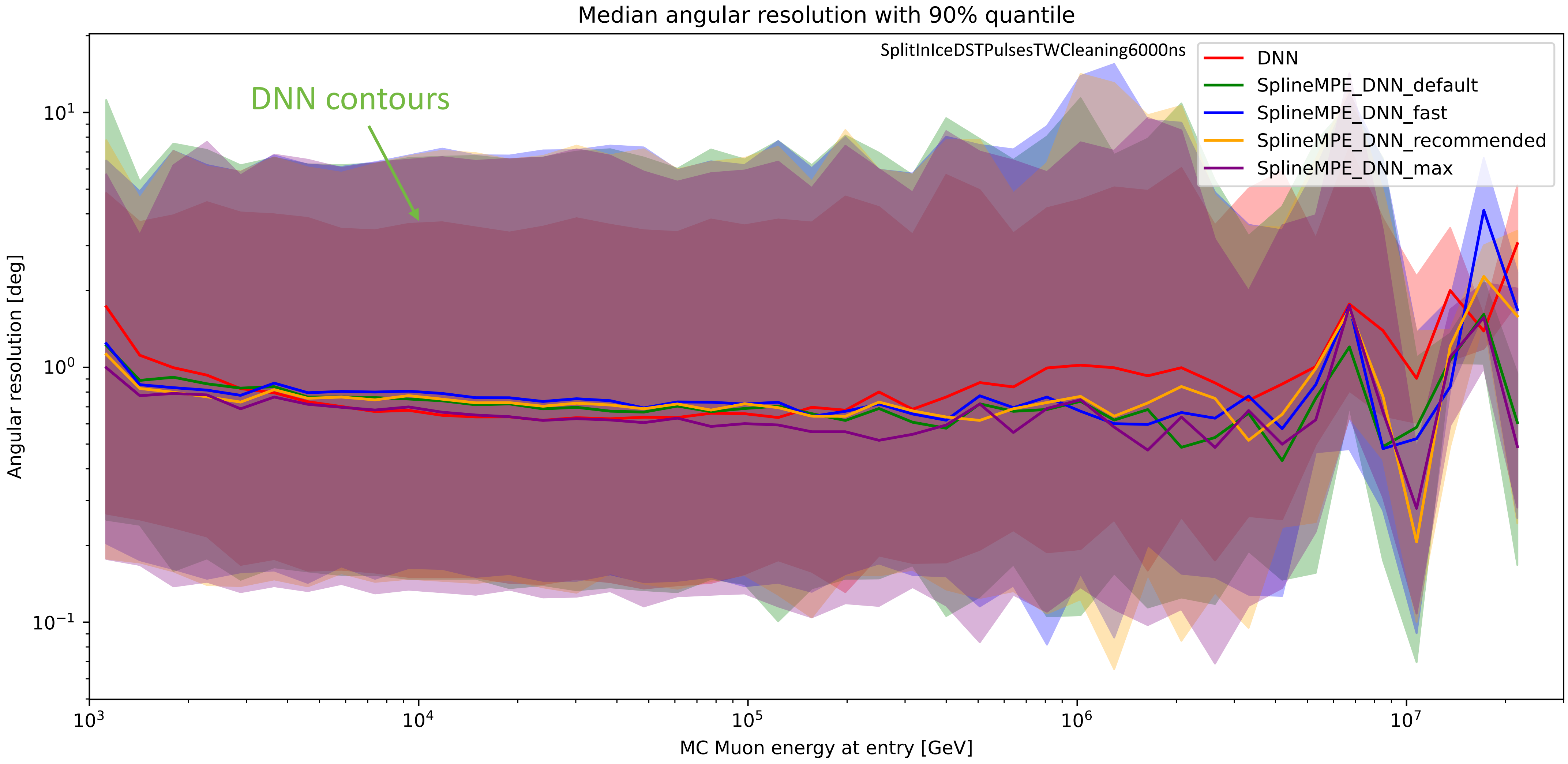


True

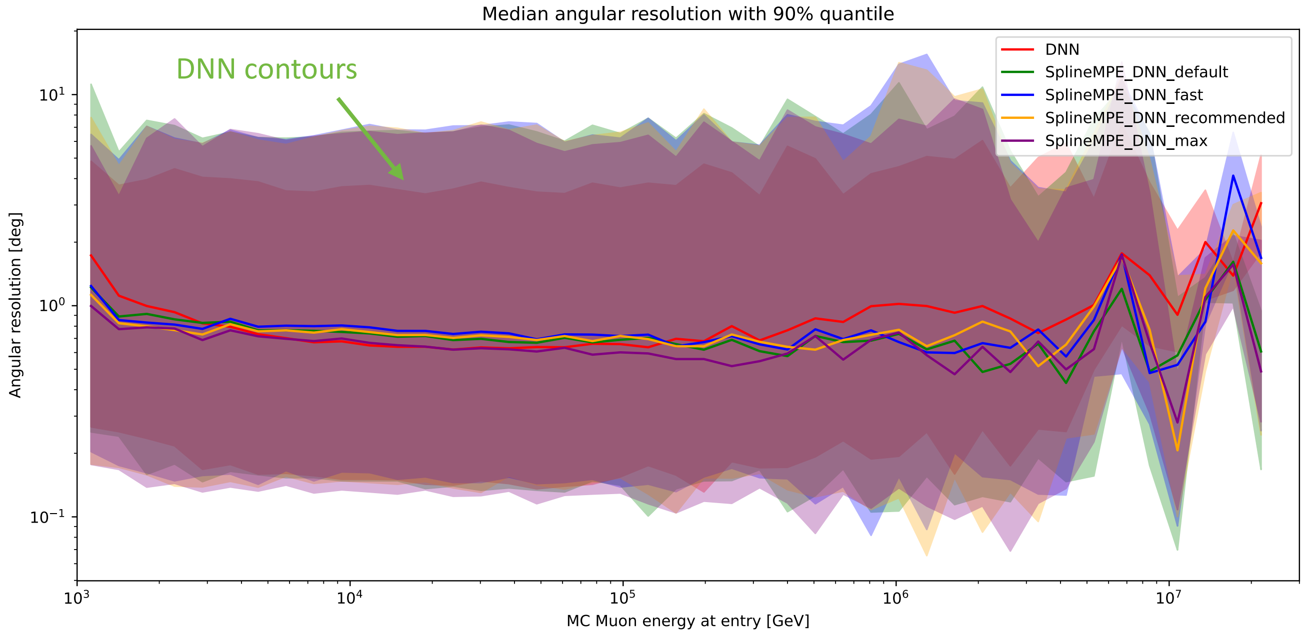
SplineMPE – DNN and conventional seeds

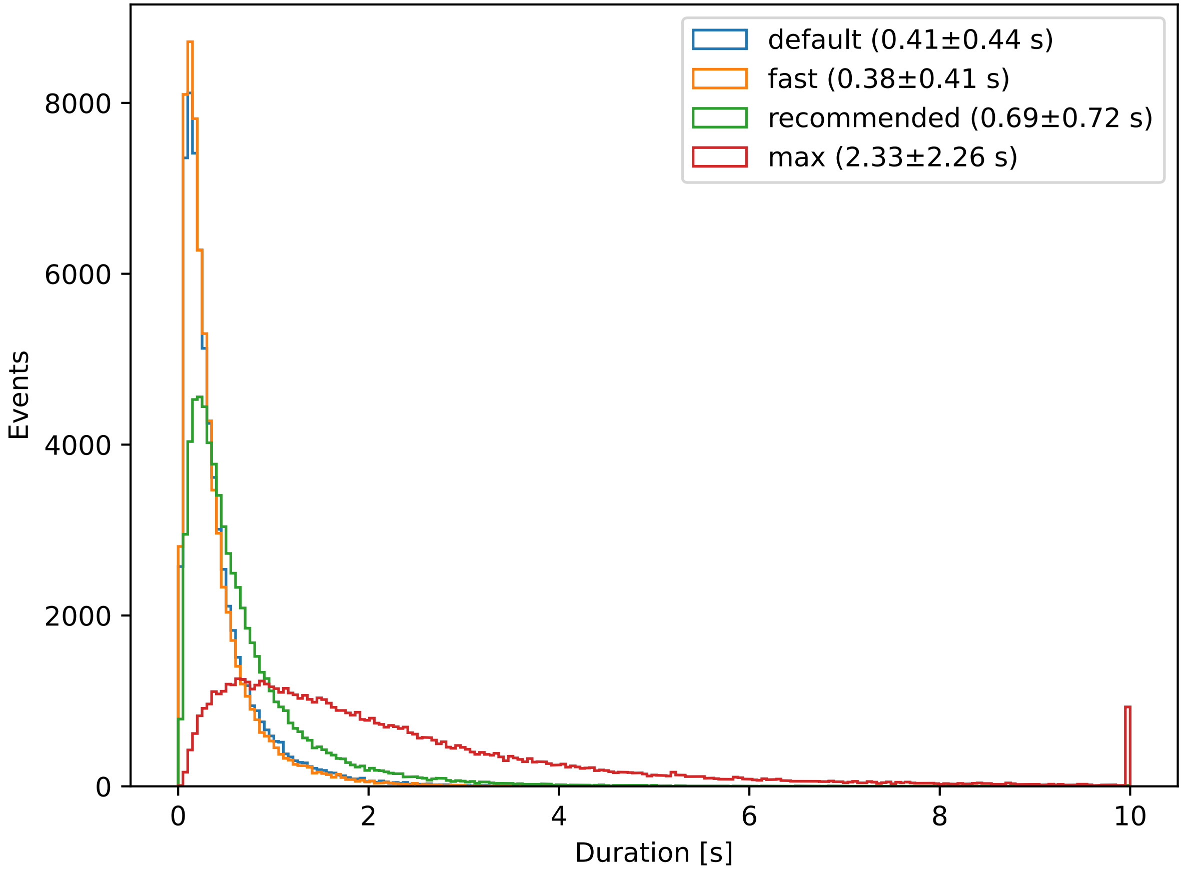


SplineMPE – DNN seeds



SplineMPE – duration per event





- Only small improvement at energies around 1 PeV
- Countours are larger
- Additional runtime

-> Use only DNN reconstruction, since we do not need the best angular resolution (zenith is more important)

Network evaluation – Track geometry

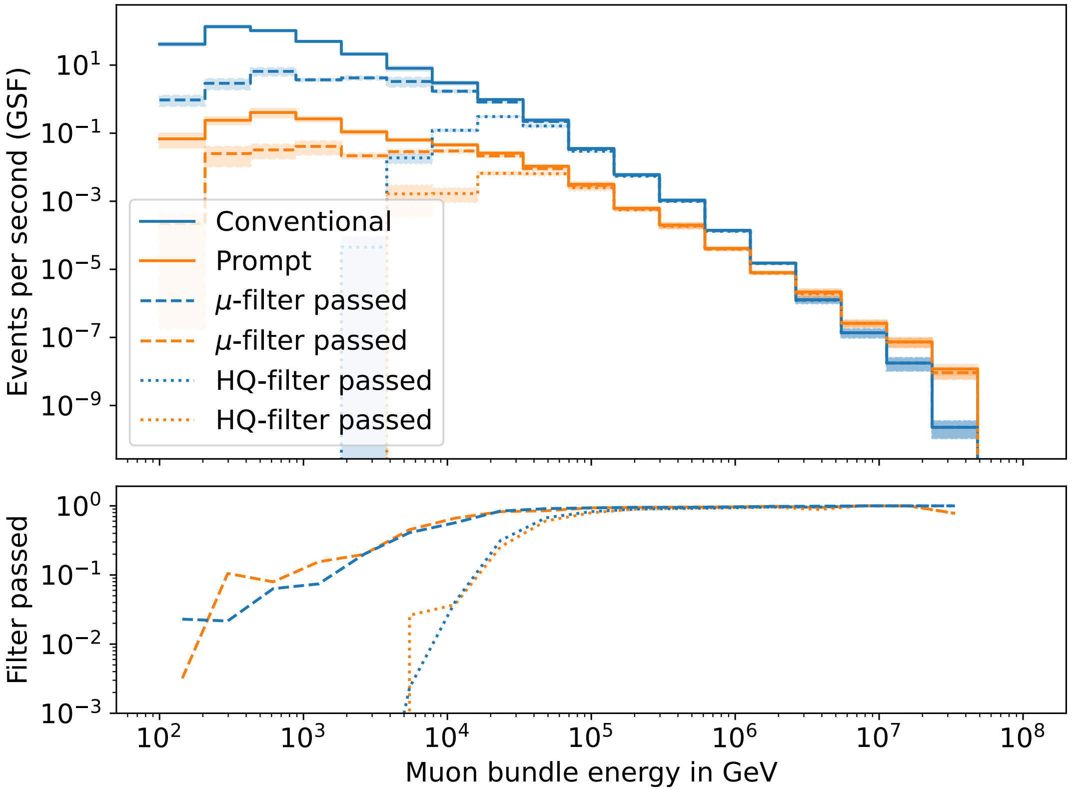
The evaluation of the track geometry works fine and can be found in the wiki

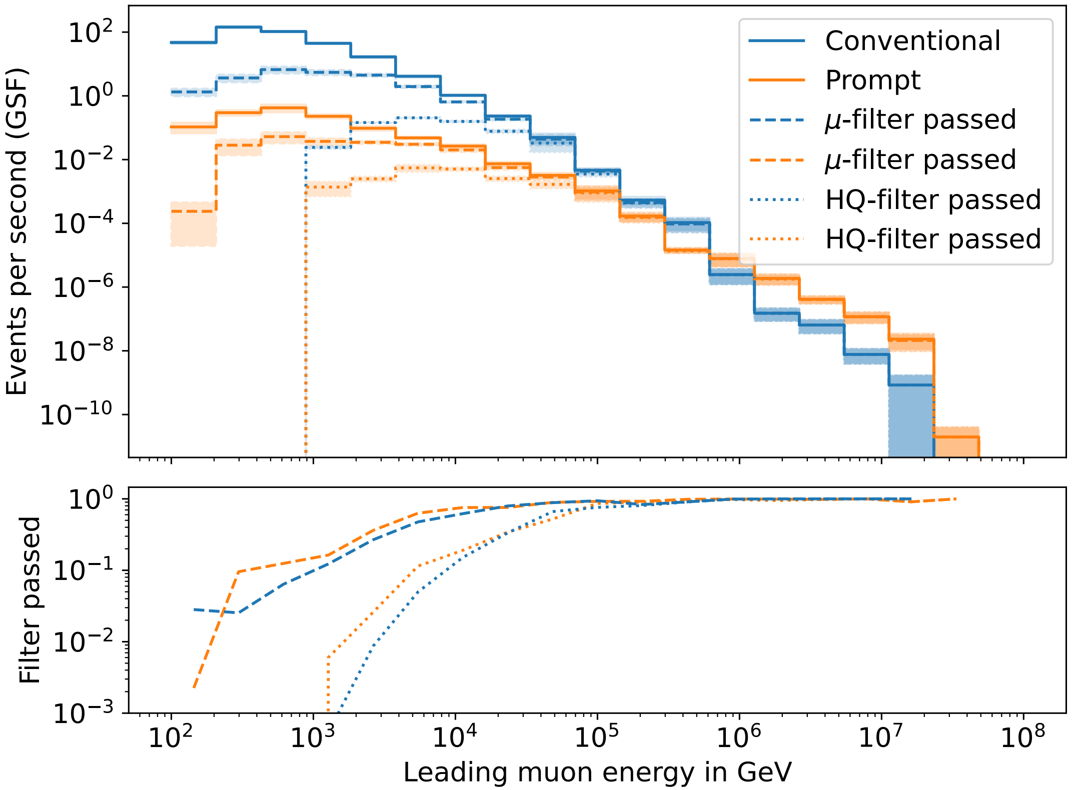
Selection – Level 3

1. L2 muon filter
2. 200 TeV bundle energy cut at surface
3. Add labels

L2 filter

Fraction rejected events	All energies	Leading energy > 10 TeV	Leading energy > 100 TeV
MuonFilter	0.93	0.28	0.06
HQFilter	0.99	0.74	0.18

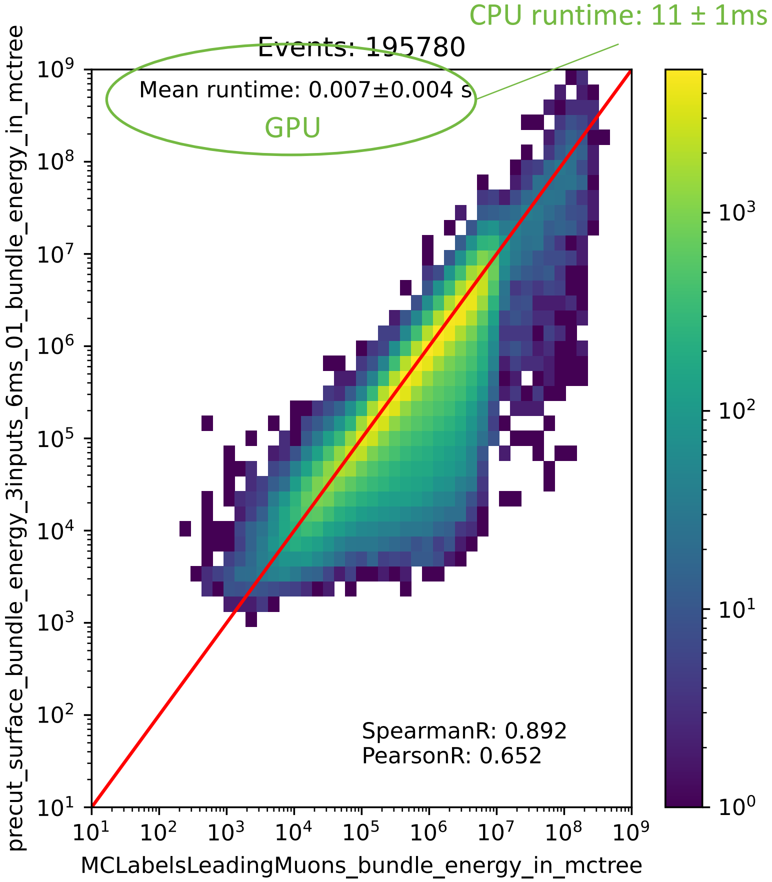


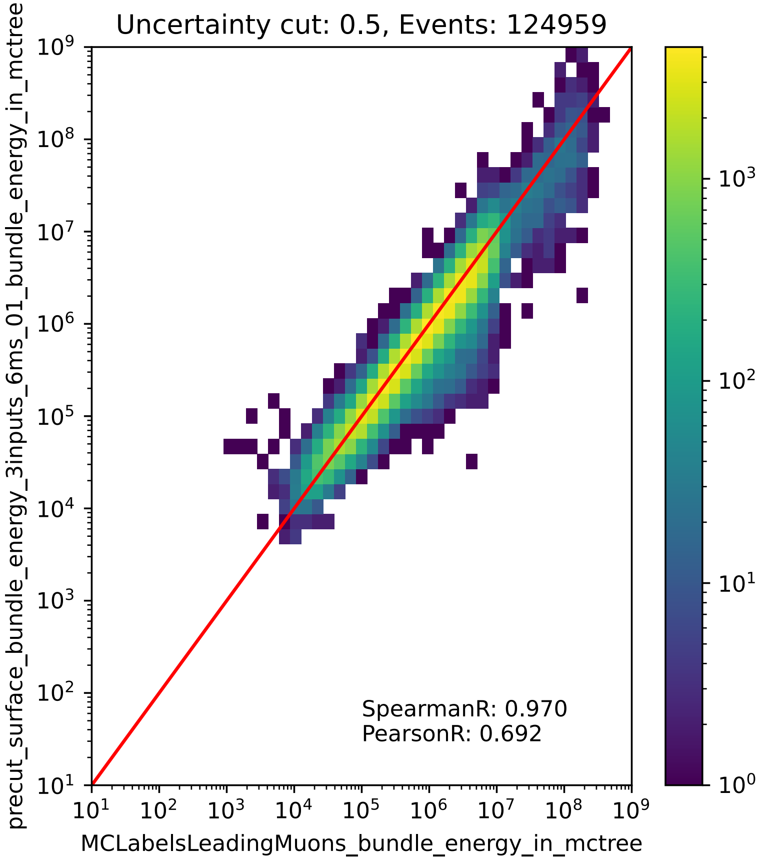


- L2 rate: 369.43 Hz
- Rate after muon filter: 24.62 Hz
- ➢ Choose muon filter to remove large amount of statistics at low energies

Bundle energy cut

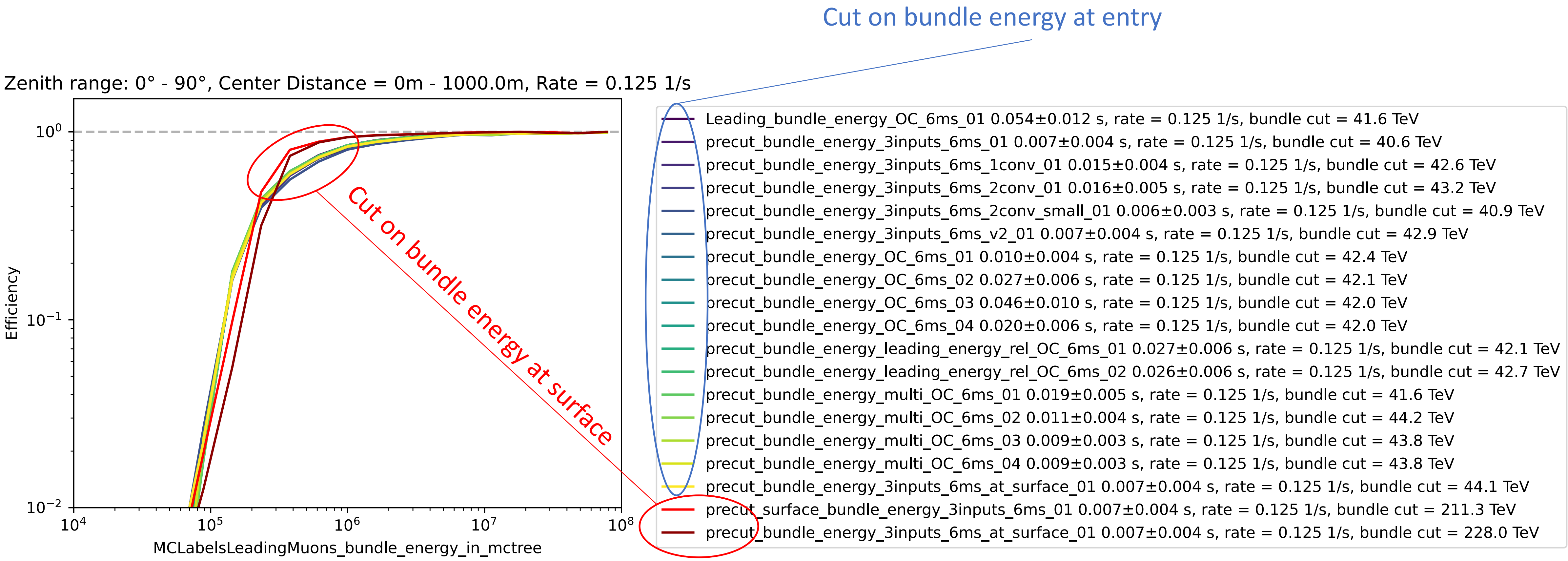
- Rate after muon filter: 24.62 Hz
- If the process of 1 event needs 1 second, 8h run takes 200h -> needs to be reduced!
- Use small, fast network to remove low energy events -> target rate 125 mHz



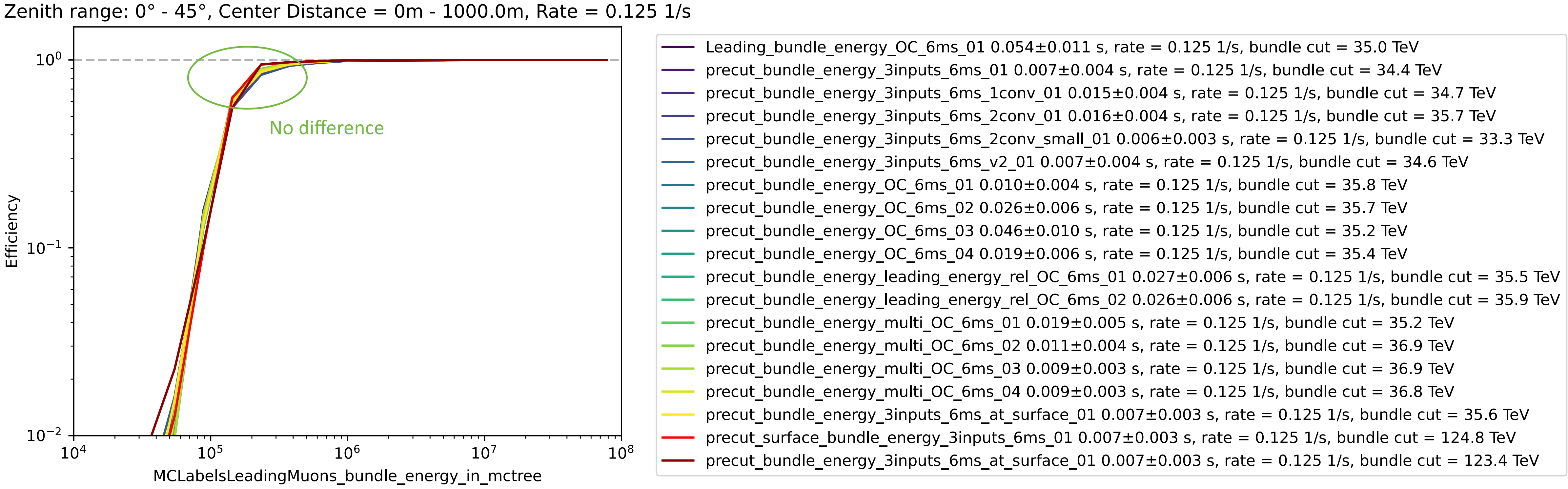


Network: precut_surface_bundle_energy_3inputs_6ms_01

Rejection efficiency- all events

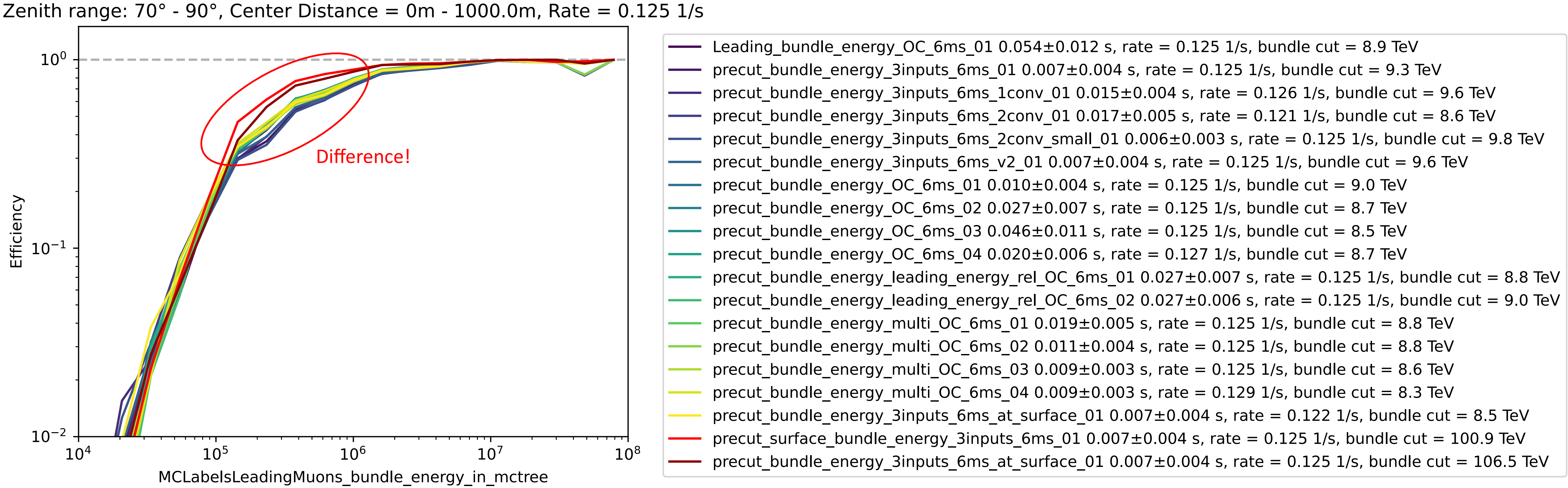


Rejection efficiency – small zenith



Rejection efficiency – high zenith

➢ At high zenith angles, events with high energies are removed with a cut on the bundle energy at entry



- Rate after muon filter: 24.62 Hz
- Rate after bundle cut: 144 mHz

➢ Perform 200 TeV cut on bundle energy at surface

Selection – Level 4

1. Add DNN network reconstructions

Add DNN networks

- Internal DNN pulse cleaning leads to slightly better results

Network	Preprocess / ms	CPU / ms	GPU / ms
direction_9inputs_uncleaned_medium_01	22 ± 20	106 ± 42	5 ± 38
leading_bundle_surface_leading_bundle_energy_OC_inputs9_large_log_02	22 ± 20	144 ± 56	3 ± 13
track_geometry_9inputs_uncleaned_01	22 ± 20	106 ± 42	3 ± 10
precut_surface_bundle_energy_3inputs_6ms_01 (added in level 3)	1 ± 1	11 ± 1	7 ± 4

What we have discussed today

- [x] Reconstruction
  - [x] Trained networks to reconstruct several properties
- [x] Selection
  - [x] Level 3 (L2MuonFilter + 200 TeV bundle energy cut at surface)
  - [x] Level 4 (add reconstructions)
- [ ] Data-MC
  - [ ] Check several properties
  - [ ] Largest mismatch in z-vertex
- [ ] Forward fit
  - [ ] Test NNMFit for analysis
- [ ] Unfolding
  - [ ] Unfold event rate
  - [ ] Calculate effective area
- [ ] New simulations
  - [ ] Preparation for large scale IceProd simulation (latest software, switch some options)
- [x] Wiki
  - [x] Created and uploaded (wiki)

Collaboration meeting

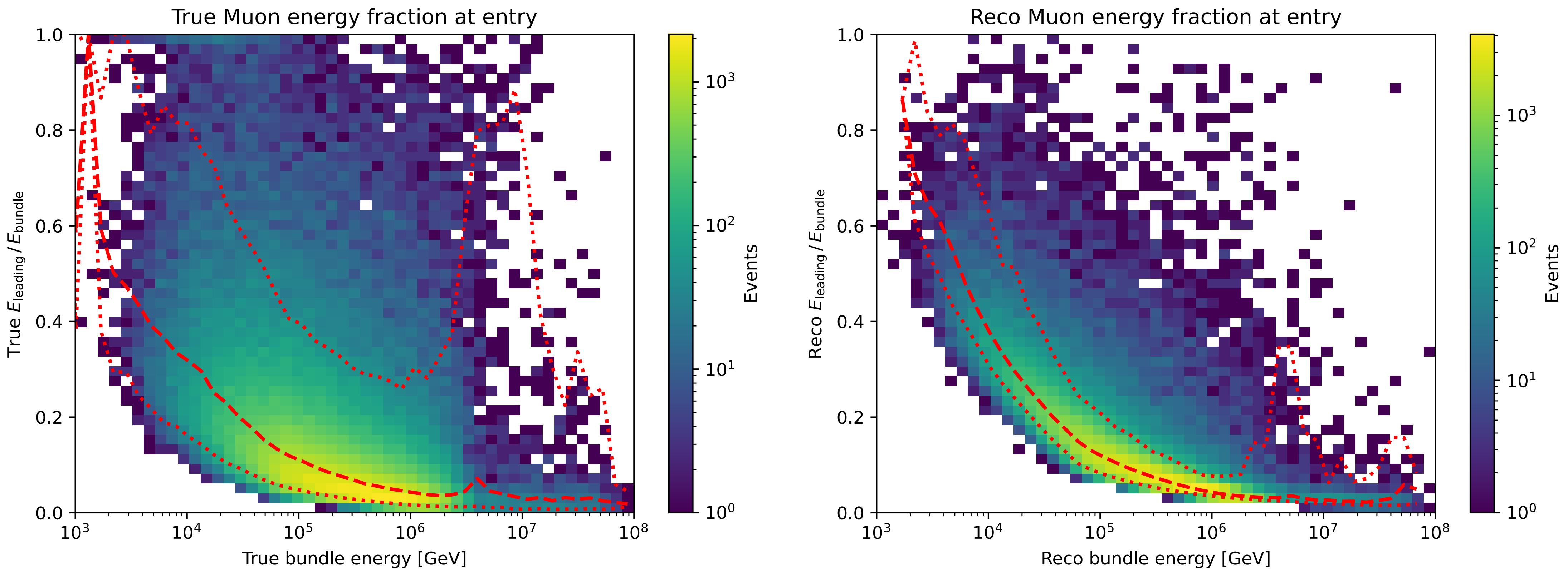
- Ask for WG Reviewer
- Storage request large-scale simulation

Thank you for your attention ☺️

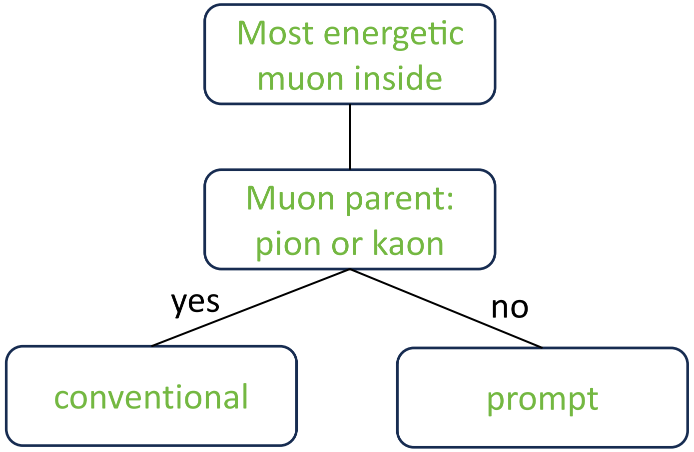
Backup

Leading muon energy fraction - leadingness

- True muon energy fraction is smeared
- Network tries to predict the median of the distribution

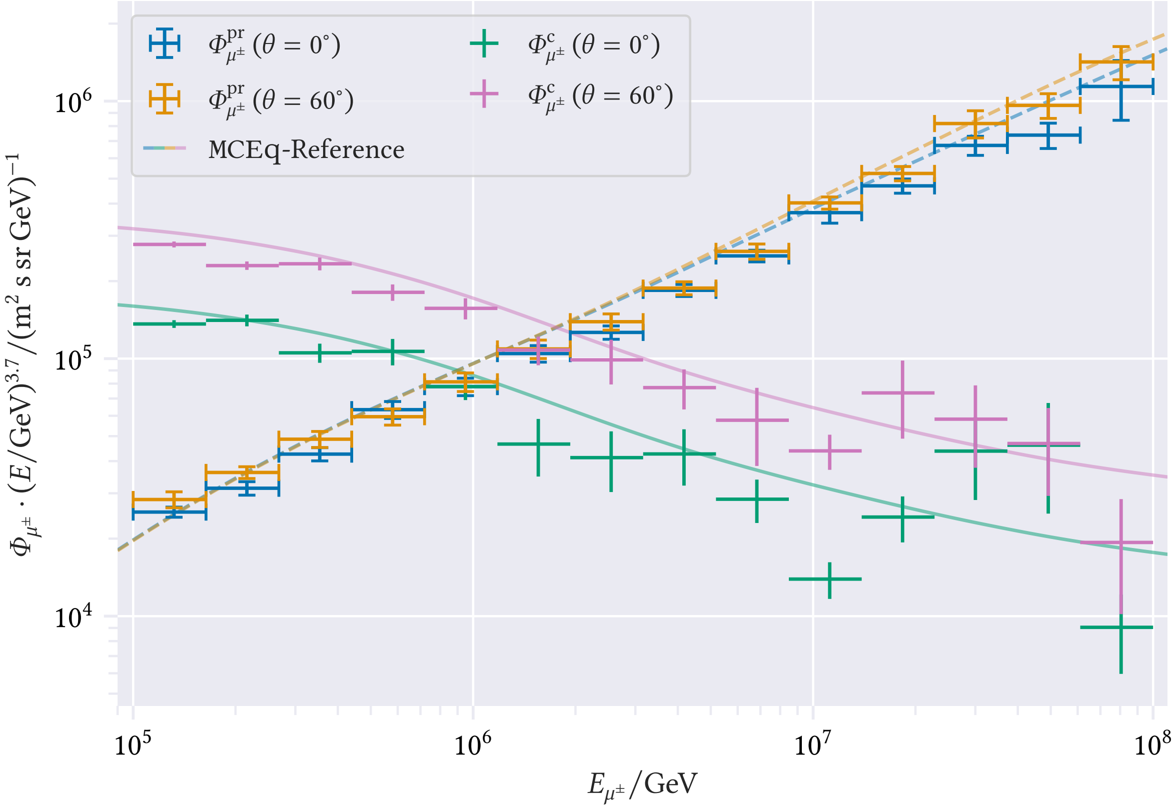


CORSIKA vs. MCEq

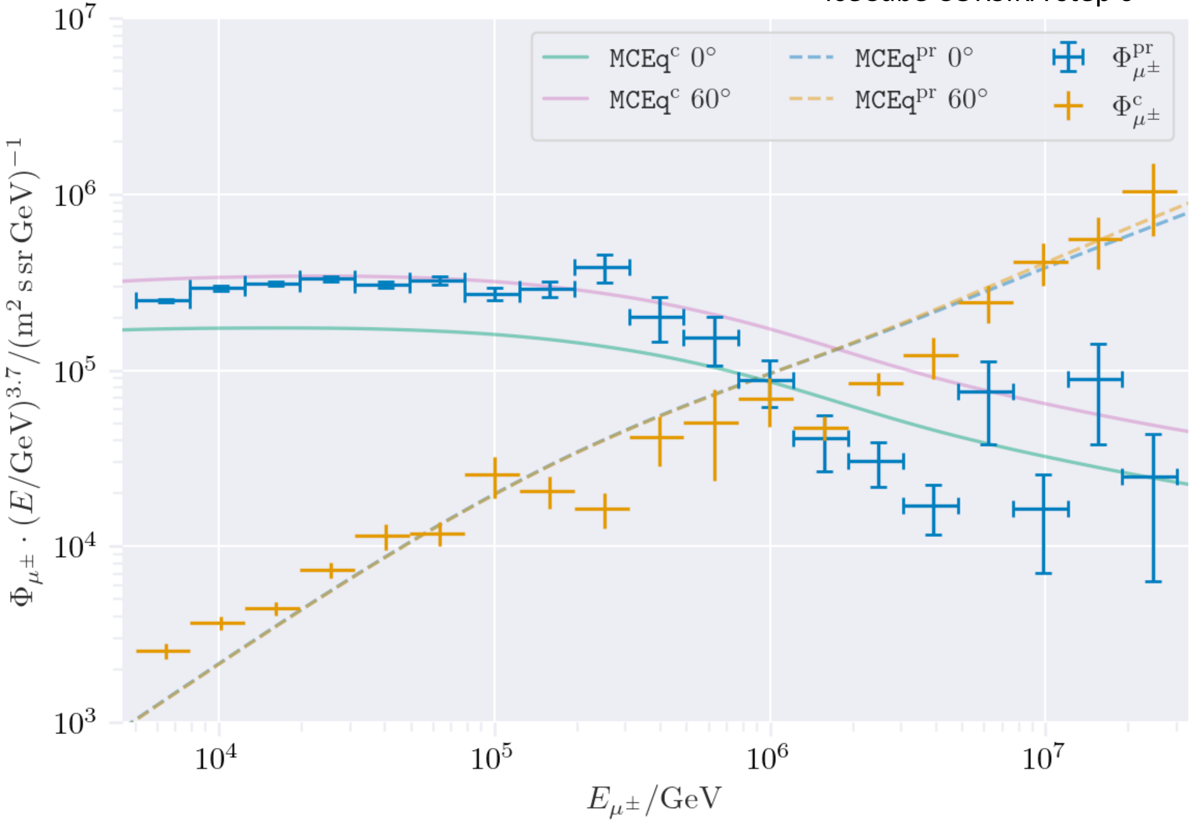


- Presented by Ludwig Neste
- Good agreement

CORSIKA standalone

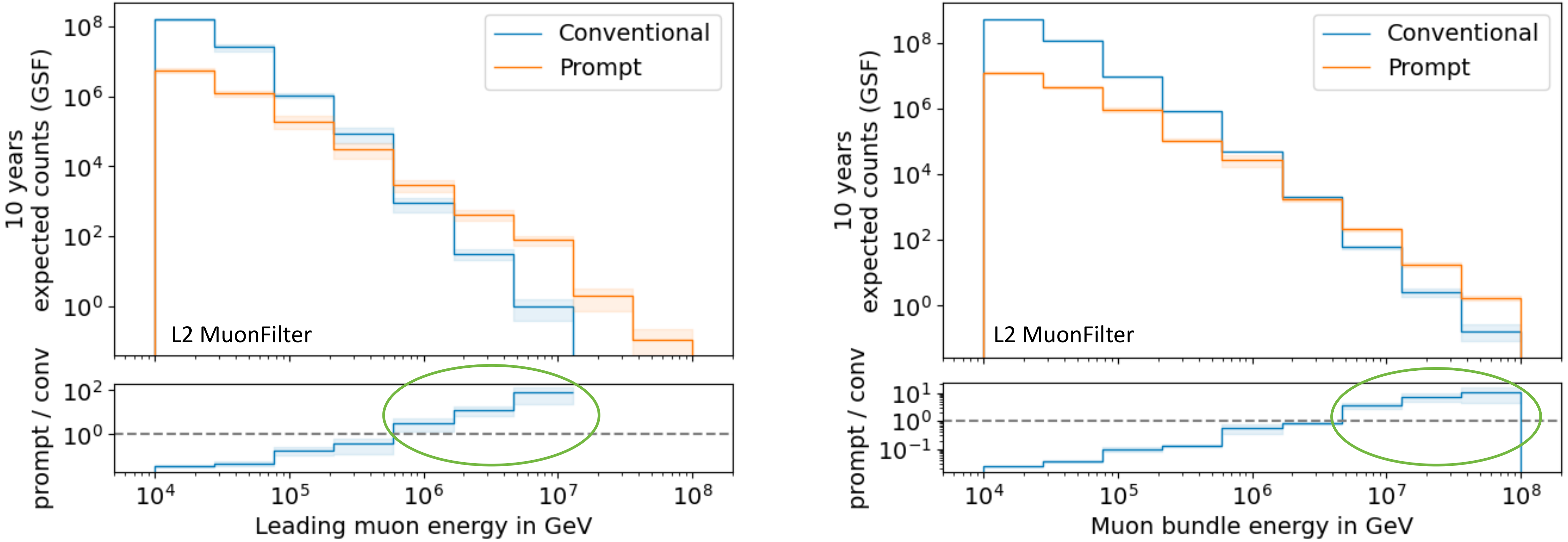


IceCube CORSIKA step 0



MC data exploration

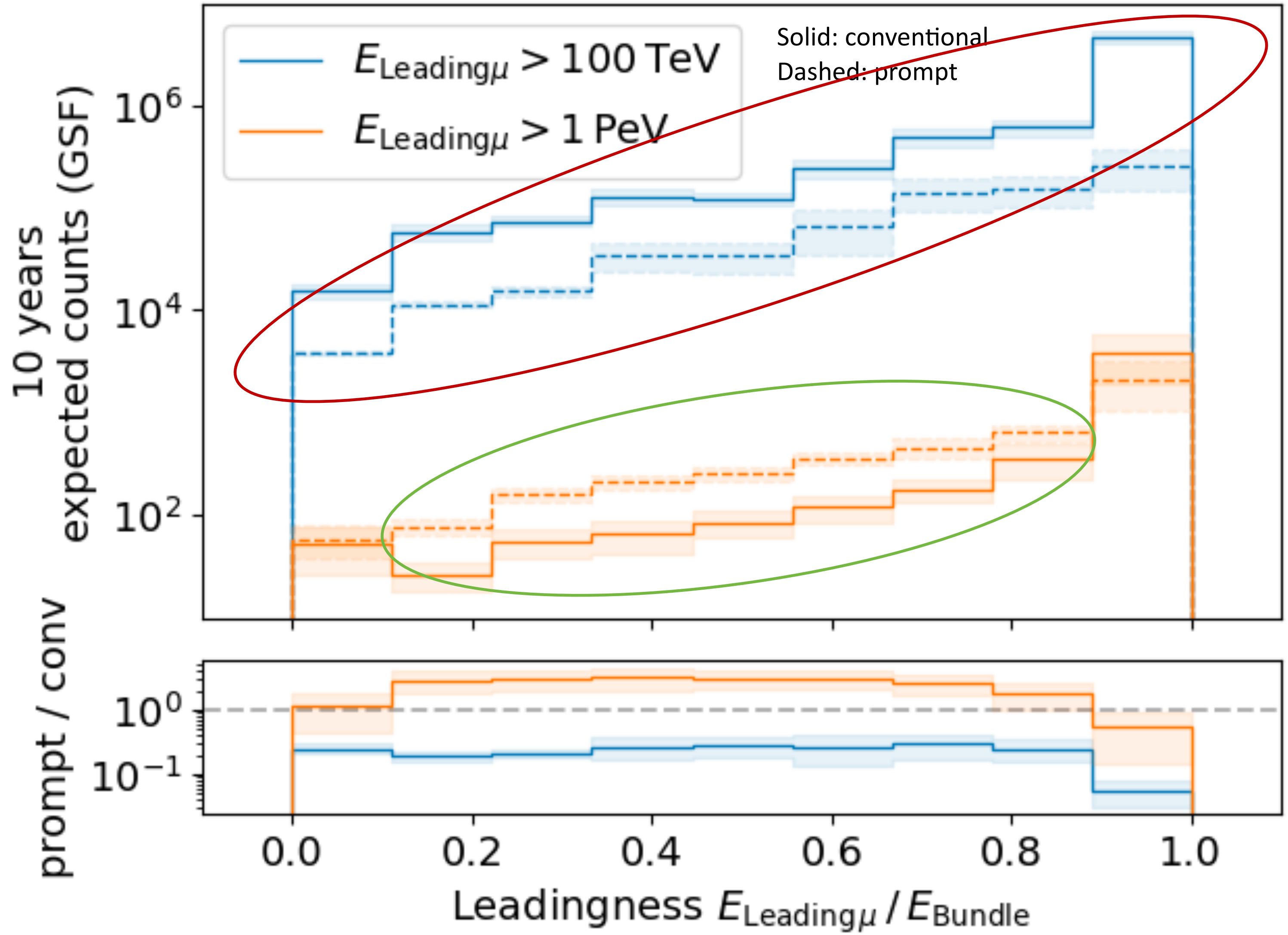
Expected muons for 10 years: leading vs. bundle energy (GSF)



- Both leading and bundle energy are sensitive to detect prompt
- Leading muon energy is more sensitive

Leading muon energy fraction

- ➢ Prompt dominates for energies > 1 PeV
- ➢ Leading energy sweet spot: 0.1 – 0.9



Leading muon contribution

Patrick Berghaus

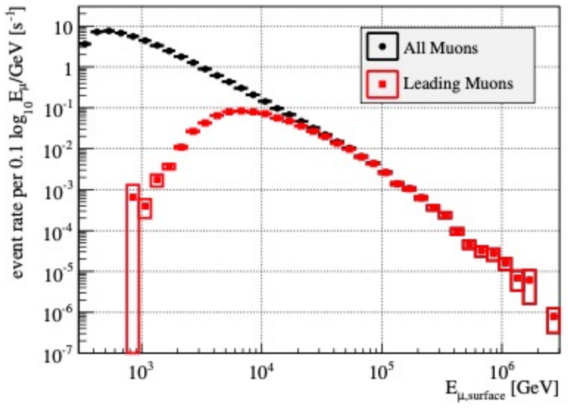
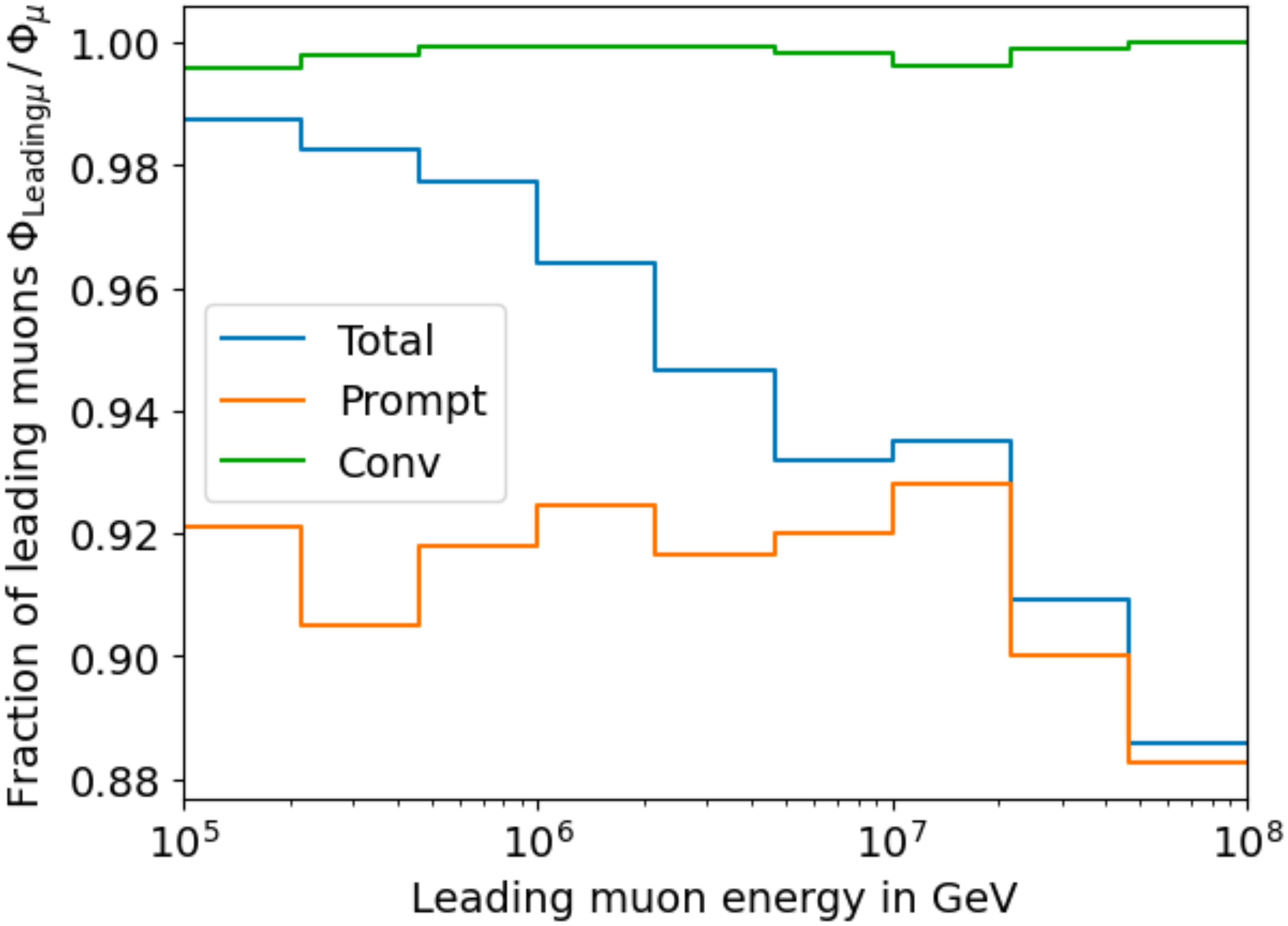


Figure 10: Surface energy distribution for all and most energetic ("leading") muons in simulated events with a total of more than 1,000 registered photo-electrons in IceCube.

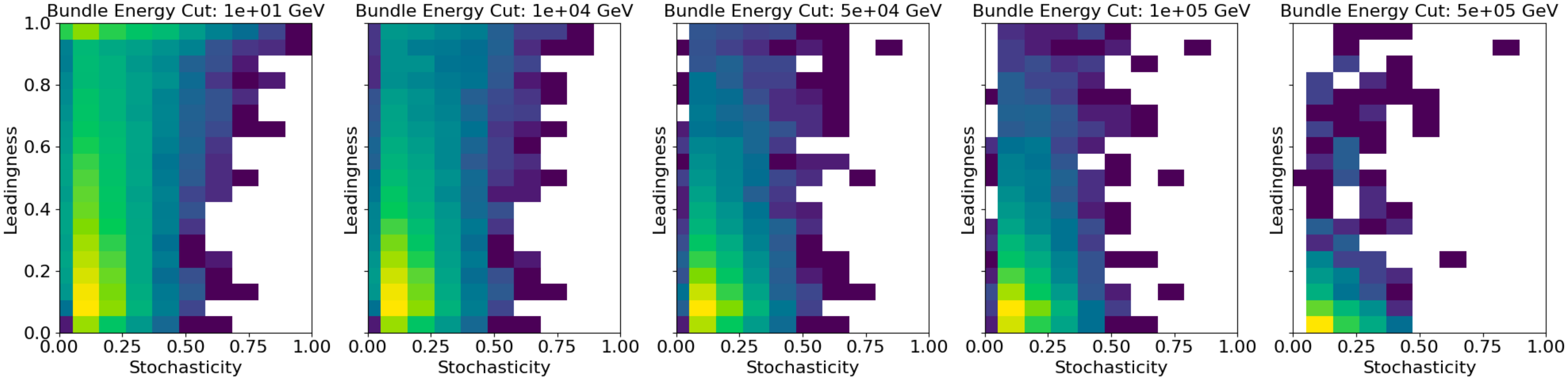
- Muons with energies between 100 TeV and 50 PeV dominate the bundle by more than 90%
- In average conventional muons are more dominant than prompt
- But: at high energies, there are more prompt than conventional events

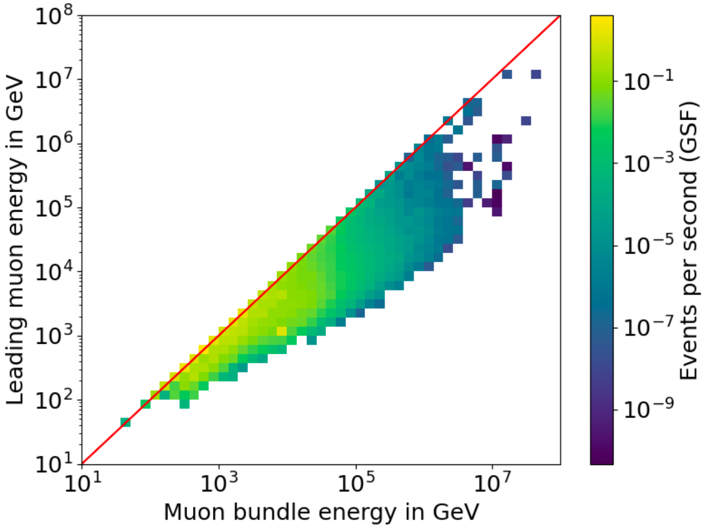
➢ High leading energy fraction does not lead to more sensitivity to detect prompt

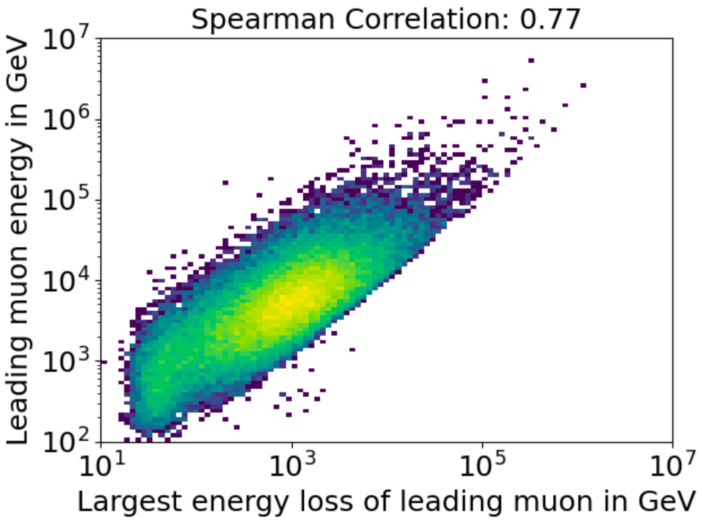


Bundle energy cut

Stochasticity



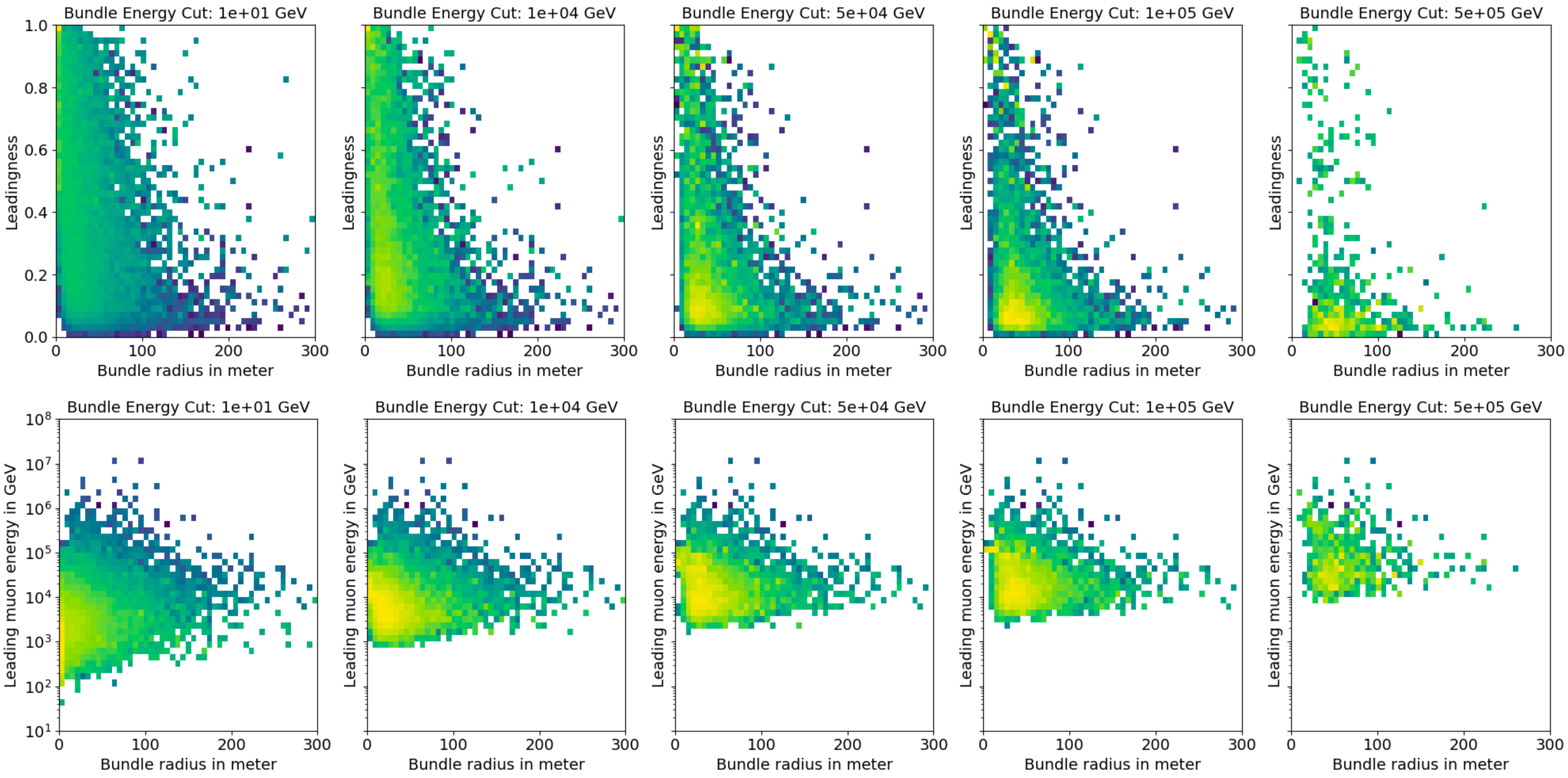




- High stochasticity leads to high leadingness, but only for a small number of events
- Leading muon energy smears out at large bundle energies
- Largest energy loss of the leading muon correlates with the leading muon energy

Bundle energy cut

Bundle radius

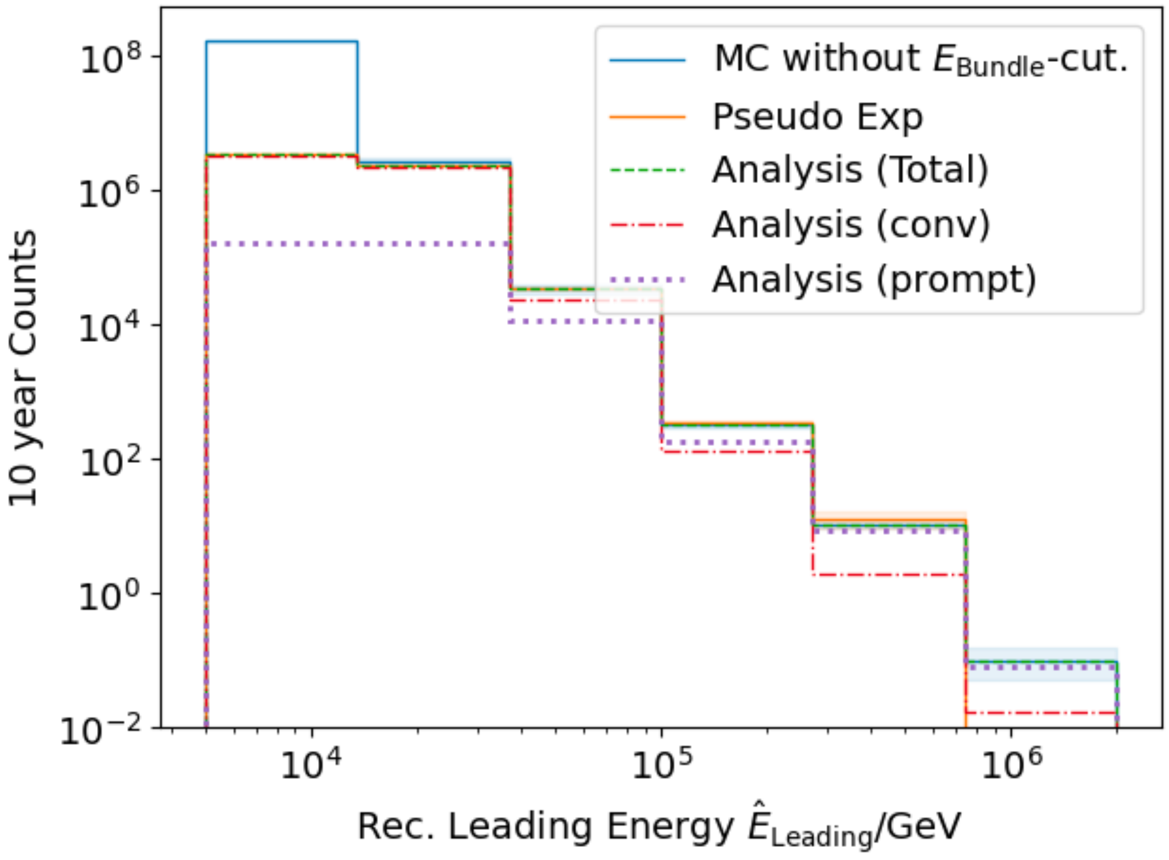


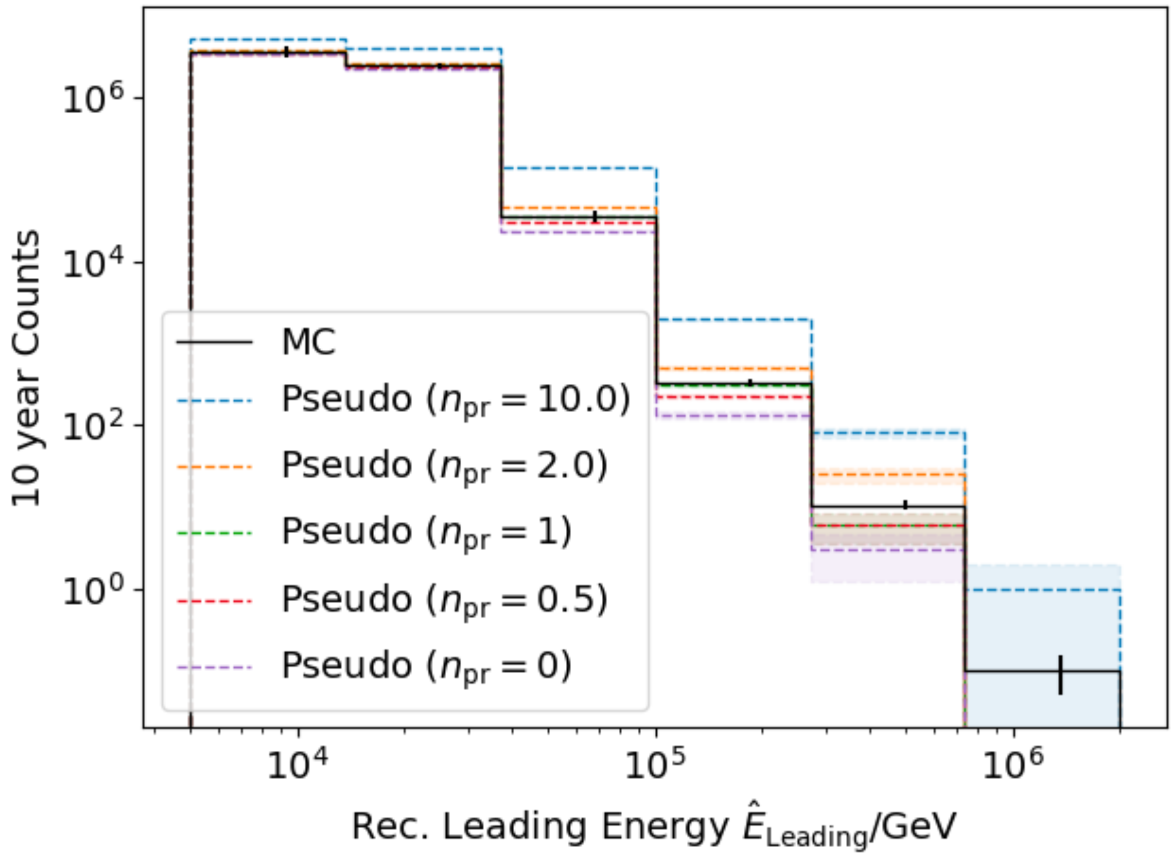
- Large bundle radius leads to low leadingness

Pseudo analysis

Pseudo data sampling

Cuts:
L2 MuonFilter
Bundle energy > 100 TeV





- Tagging allows scaling of prompt by factor n_{pr}

Poisson likelihood fit performed in leading muon energy

Prompt scaling/normalization

Conv norm = 1

MC counts per bin i

$$C_1^{\mathrm{MC}} = n_{\mathrm{pr}} C_1^{\mathrm{MC,pr}} + n_{\mathrm{conv}} C_1^{\mathrm{MC,conv}}, \dots, C_M^{\mathrm{MC}} = n_{\mathrm{pr}} C_M^{\mathrm{MC,pr}} + n_{\mathrm{conv}} C_M^{\mathrm{MC,conv}}$$

Experimental counts

$$p\left(C_i\right) = p_{\mathrm{poisson}}\left(C_i; \lambda\left(n_{\mathrm{pr}}\right) = C_i^{\mathrm{MC}}\left(n_{\mathrm{pr}}\right)\right) = \frac{\lambda\left(n_{\mathrm{pr}}\right)^{C_i} e^{-\lambda\left(n_{\mathrm{pr}}\right)}}{C_i!}$$

With a constant conv norm:
bin counts depend only on prompt norm
= expectation value per bin

Maximize likelihood

$$\mathscr{L}\left(n_{\mathrm{pr}}\right) = \prod_{i=1}^{M} p\left(C_i; n_{\mathrm{pr}}\right)$$

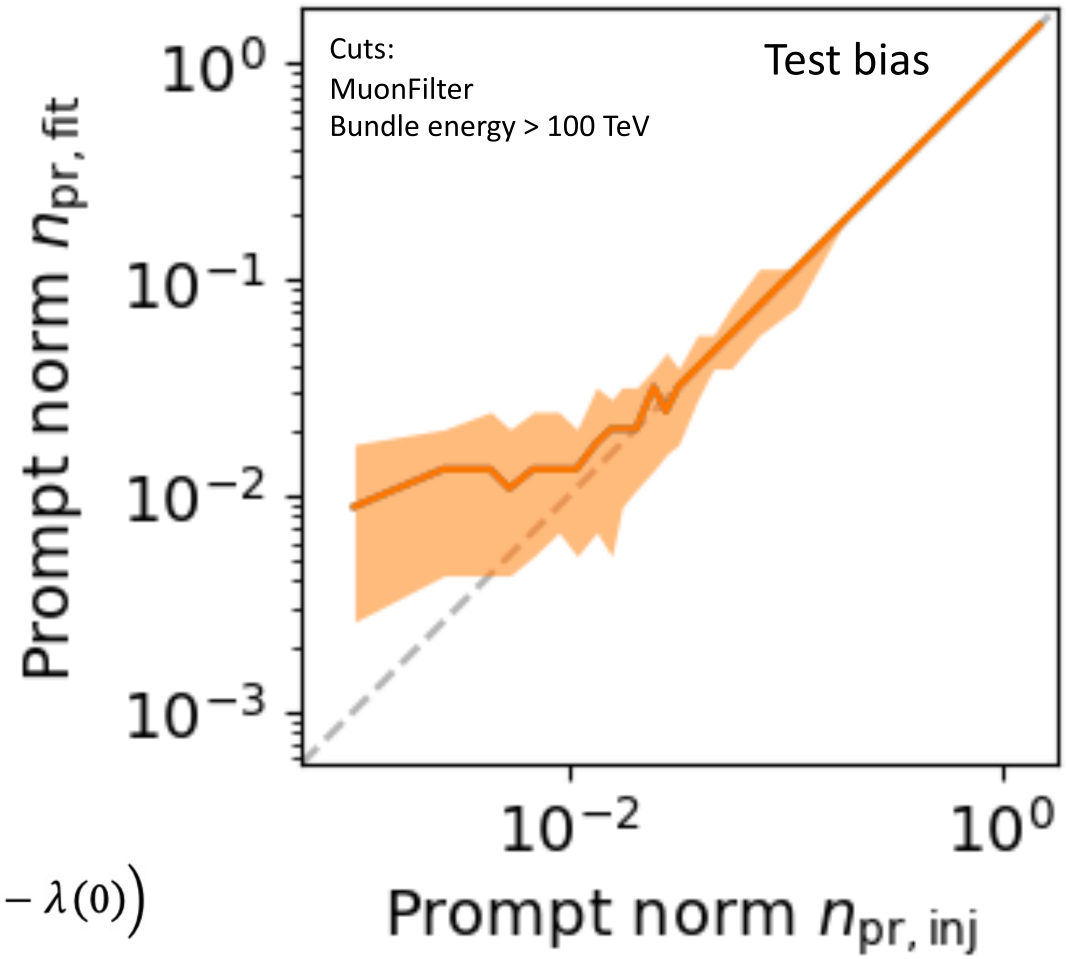
Easier:
minimize negative
log-likelihood

$$-\ln \mathscr{L} = -\sum_{i=1}^{M} C_i \ln \lambda\left(n_{\mathrm{pr}}\right) - \lambda\left(n_{\mathrm{pr}}\right) - \ln C_i!$$

$$\Lambda = -2 \ln \frac{\mathscr{L}\left(n_{\mathrm{pr}} = \hat{n}_{\mathrm{pr}}\right)}{\mathscr{L}\left(n_{\mathrm{pr}=0}\right)} = -2 \sum_{i=1}^{M} C_i \left(\ln \lambda\left(\hat{n}_{\mathrm{pr}}\right) - \ln \lambda(0)\right) - \left(\lambda\left(n_{\mathrm{pr}}\right) - \lambda(0)\right)$$

Test statistic for Wilks' theorem

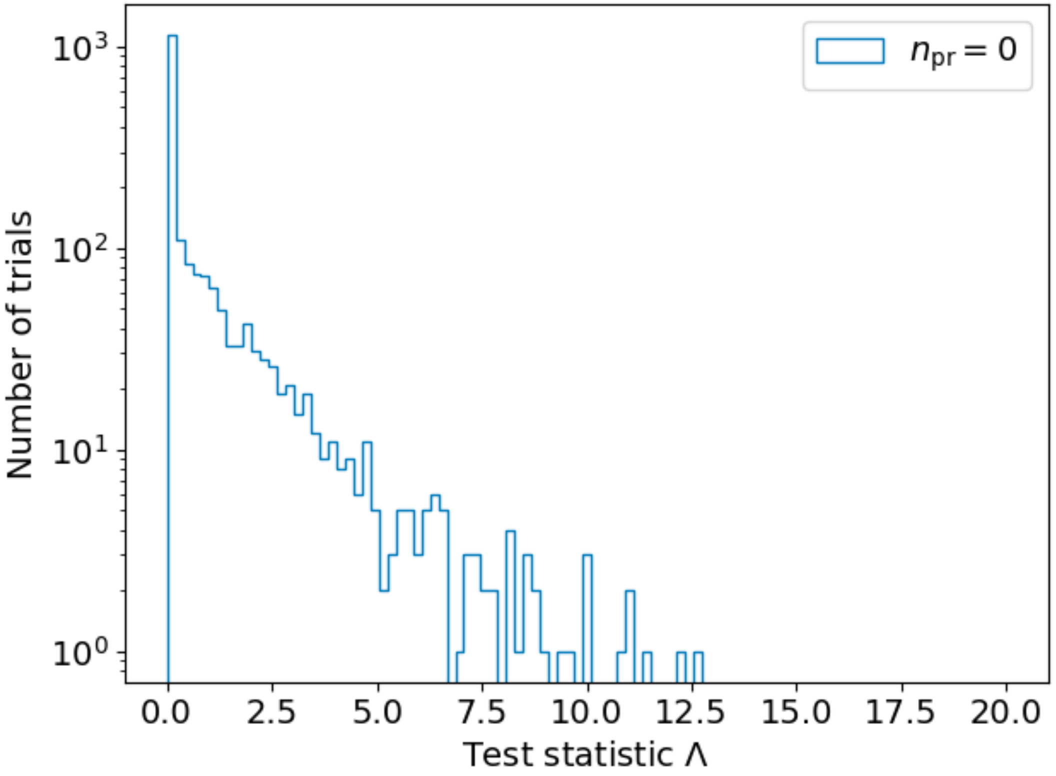
Null hypothesis: no prompt

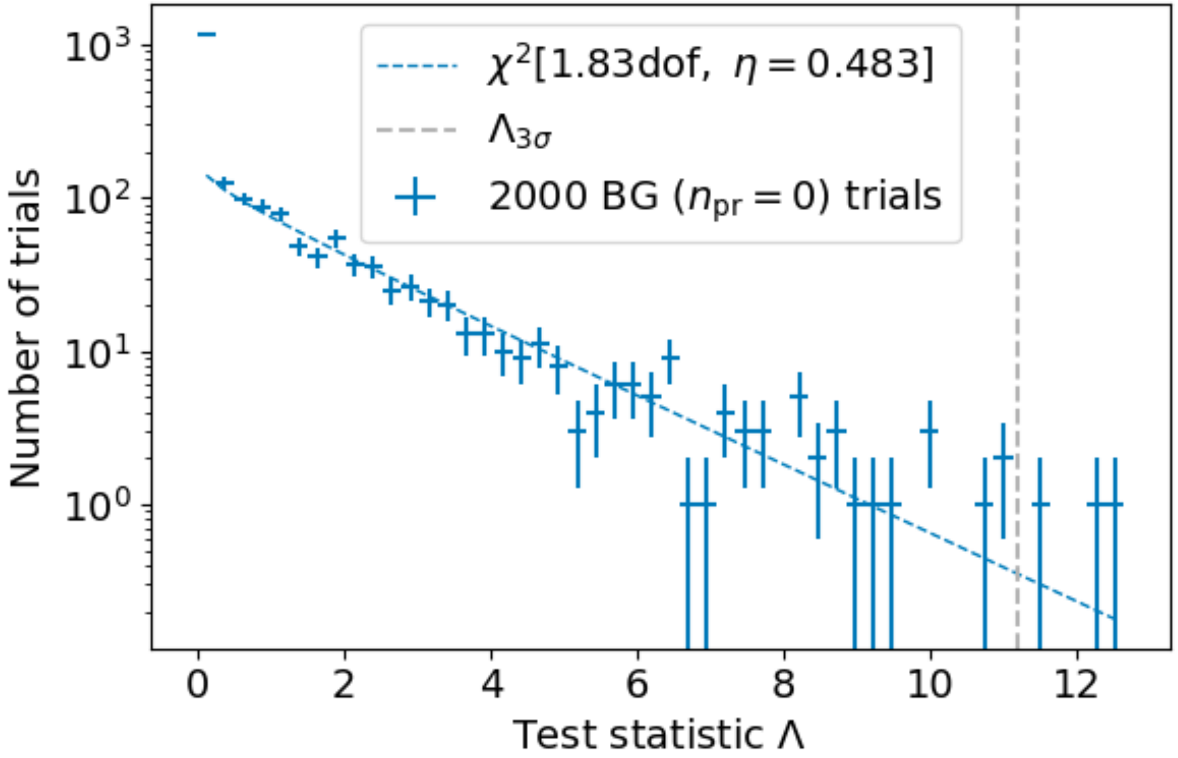


- Bias starts at a prompt normalization of 0.1

Test background statistics

Cuts:
L2 MuonFilter
Bundle energy > 100 TeV





- Background statistic is χ^2 – distributed
- Assume Wilks' theorem for test statistics

Cuts:
L2 MuonFilter
Bundle energy > 100 TeV

Discovery potential and sensitivity

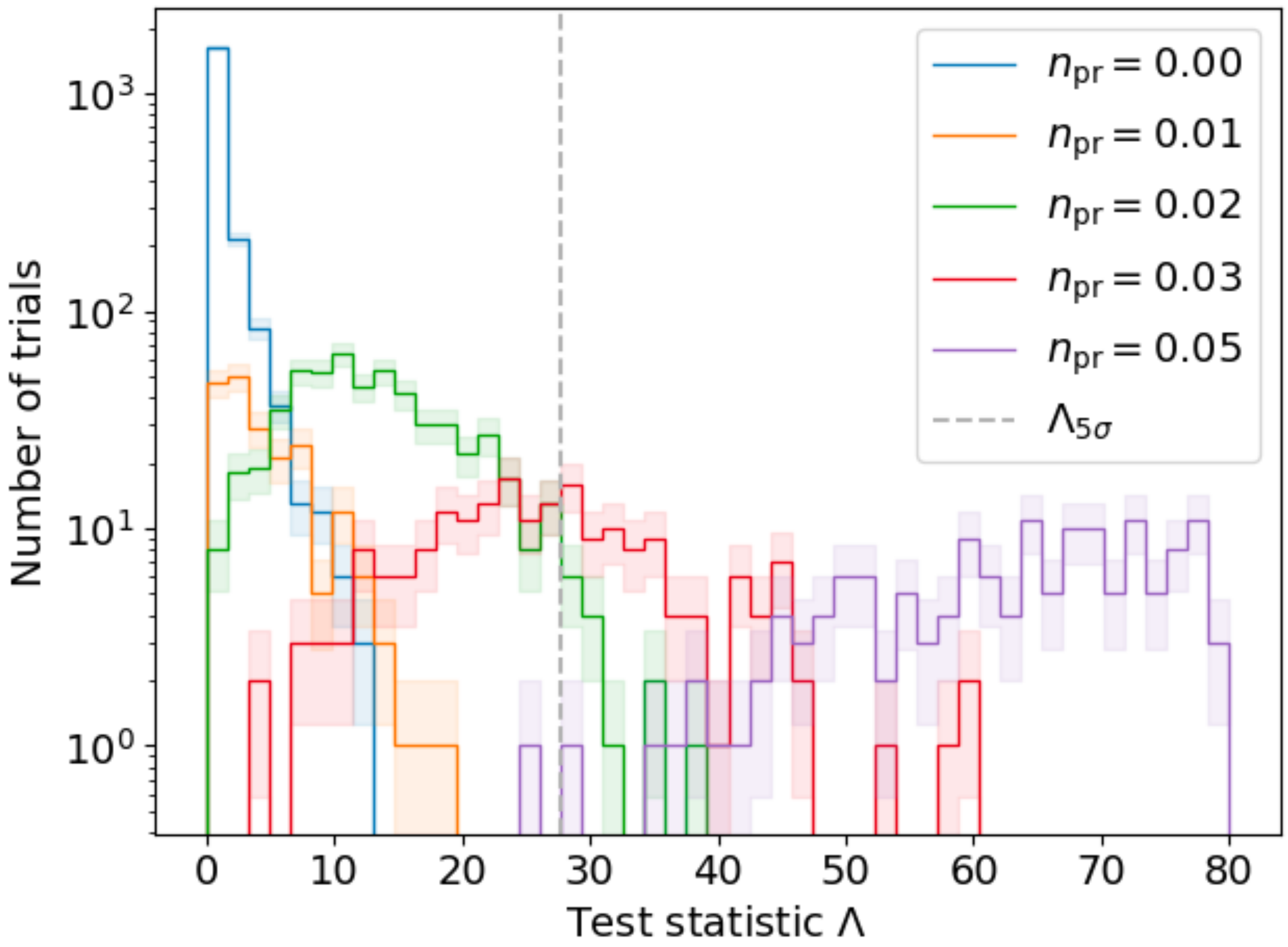
Expectation for 1 year:

- 5 sigma discovery potential: 0.102 ± 0.005
- Sensitivity: 0.024 ± 0.001

Expectation for 10 years:

- 5 sigma discovery potential: 0.032 ± 0.001
- Sensitivity: 0.007 ± 0.000

Caution:
Limited MC statistics -> events are oversampled in pseudo dataset



tu dortmund university

IceCube Neutrino Observatory

Definition of the muon flux

$$\Phi_{\text{tot}} = \Phi_{\text{conventional}} + \Phi_{\text{prompt}}$$

$\pi, K \propto$ E$^{-3.7}$

“not” $\pi, K \propto$ E$^{-2.7}$

Conventional component:

$$\underset{\propto E^{-2.7}}{p} \rightarrow \overbrace{\pi^{+}}^{\tau = 2.6 \cdot 10^{-8}\,\mathrm{s}} \rightarrow \underset{\propto E^{-3.7}}{\mu^{+}} + \nu_{\mu}$$

prompt component:

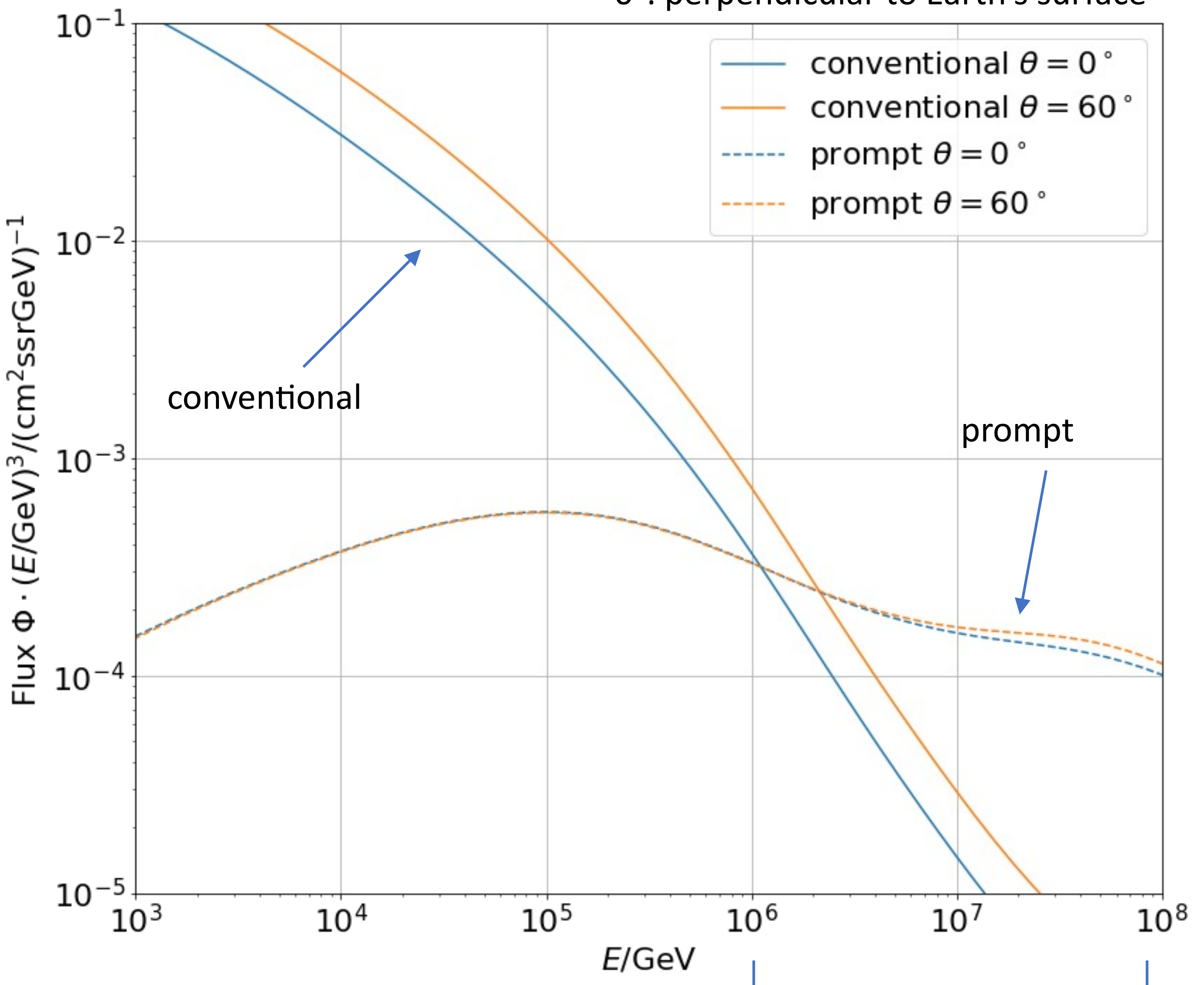
$$\underset{\propto E^{-2.7}}{p} \rightarrow \overbrace{D^{+}}^{\tau = 1.04 \cdot 10^{-12}\,\mathrm{s}} \rightarrow \bar{K}^{0} + \mu^{+} + \underset{\propto E^{-2.7}}{\nu_{\mu}}$$

Muon flux

$$\Phi_{\text{tot}} = \Phi_{\text{conv}} + \Phi_{\text{prompt}}$$

- Prompt dominates at energies larger than PeV
- Conventional particle flux depends on zenith angle

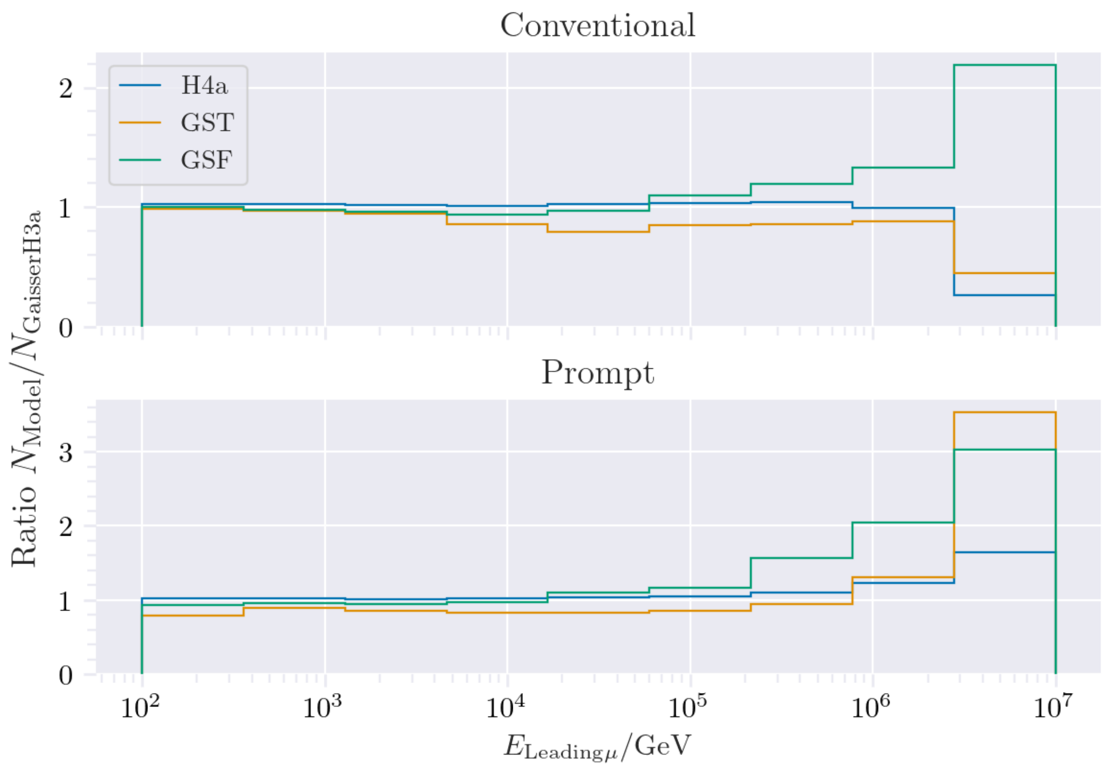
0°: perpendicular to Earth's surface



We can measure prompt muon energies from ~1 PeV to ~100 PeV

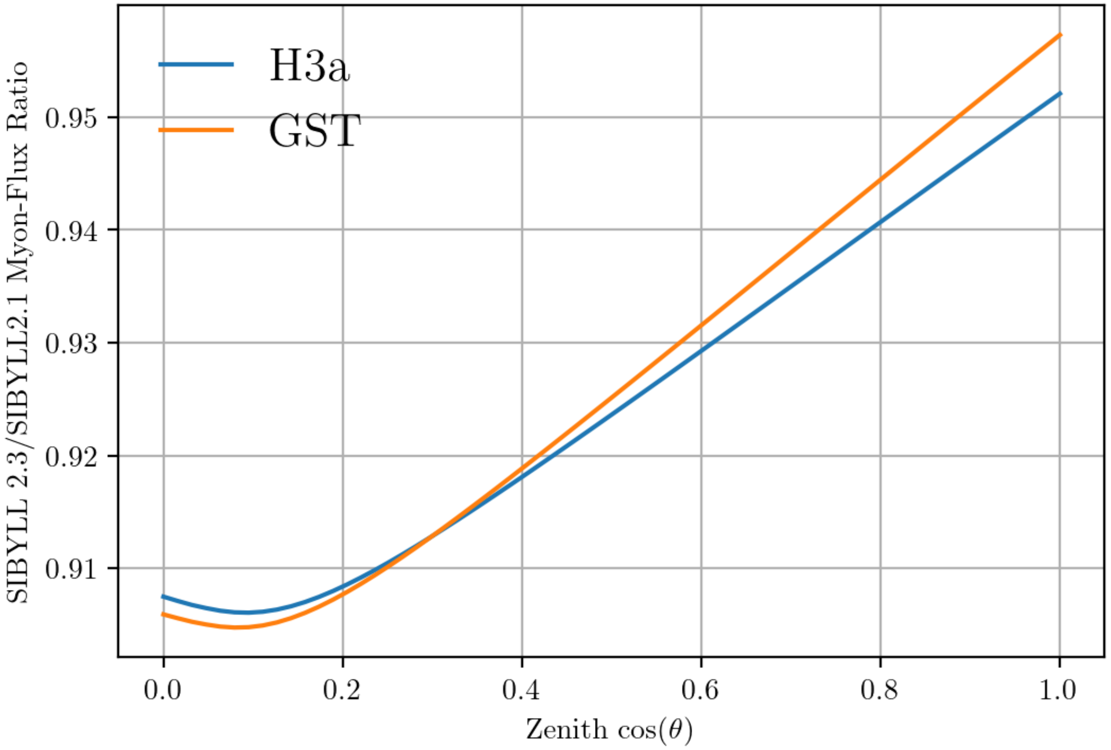
Muon production – different weightings

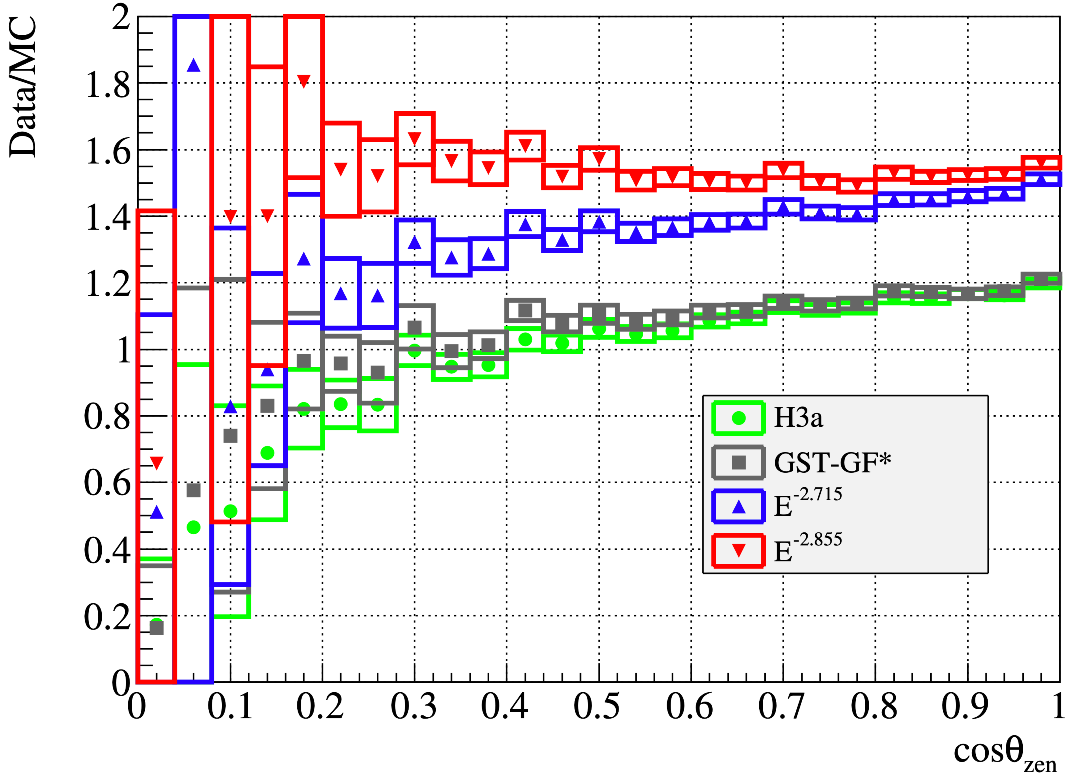
GST predicts most prompt



MCLabelsLeadingMuons_entry_energy

Solution to zenith problem?

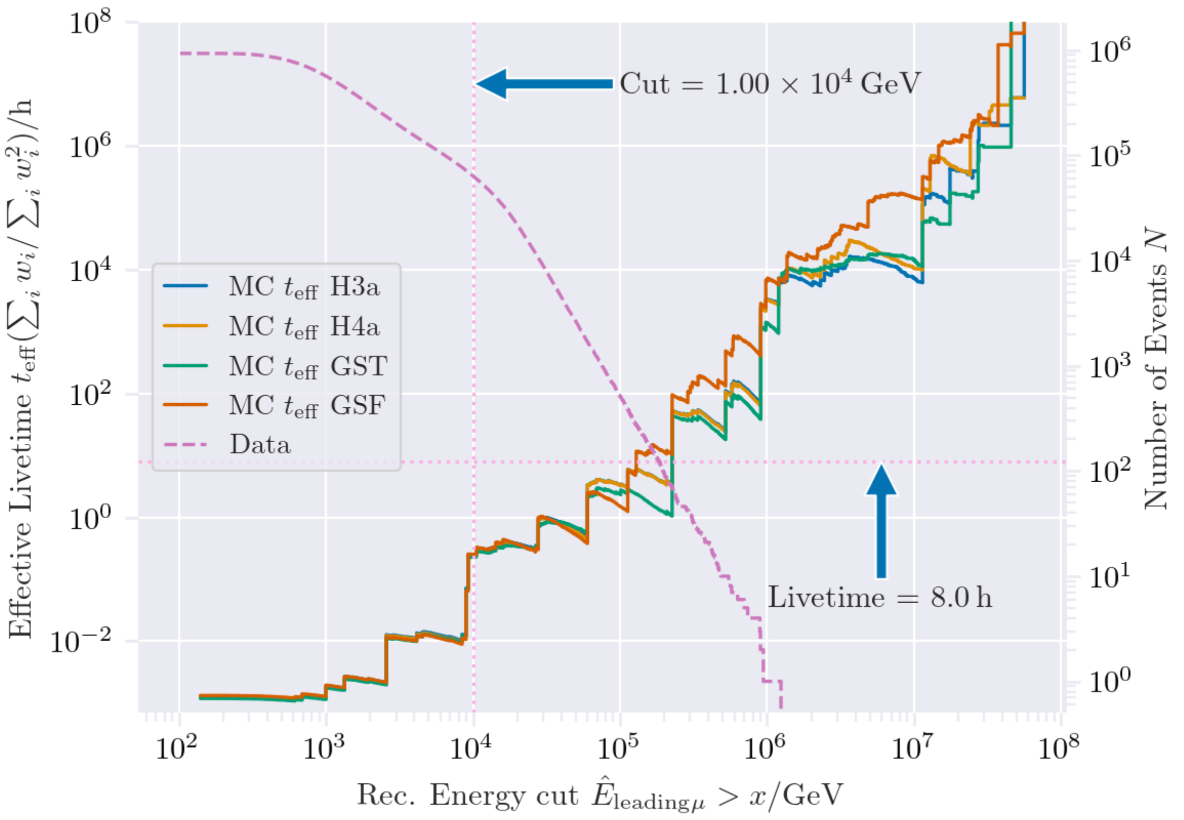




- ➢ No complete solution, but a step in the right direction

New CORSIKA extended history simulations

- CORSIKA 77420
- SIBYLL 2.3d
- Icetray 1.5.1
- 5 components (p, He, N, Al, Fe)
- Polyplopia: True
- Trimshower: True
- Ecuts1: 273 GeV (hadron min energy)
- Ecuts2: 273 GeV (muon min energy)
- Ecuts3: 10^{20} GeV (electron min energy)
- Ecuts4: 10^{20} GeV (photon min energy)
- 4 datasets:
    - 30010: 600 GeV – 1 PeV
    - 30011: 1 PeV – 100 PeV
    - 30012: 100 PeV – 1 EeV
    - 30013: 1 EeV – 50 EeV
- /data/sim/IceCube/2023/generated/CORSIKA_EHISTORY/

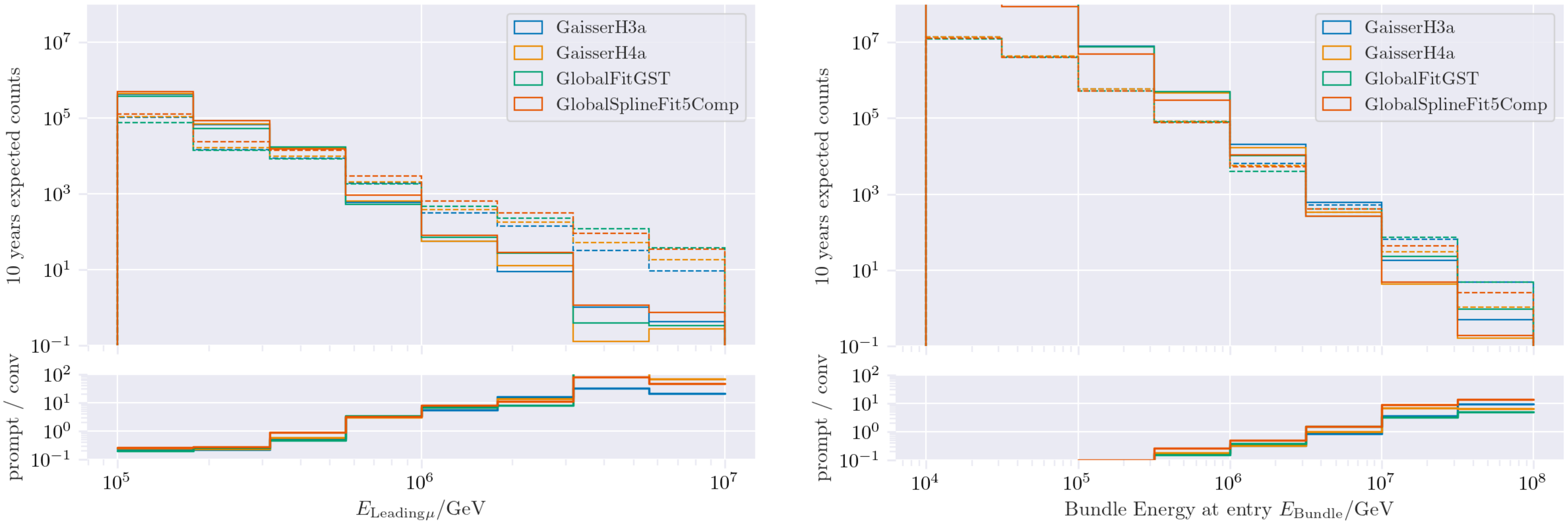


Please go ahead and test the datasets

- ➢ Sufficient statistics above 1 PeV
- ➢ Too few statistics at lower energies

Expected muons for 10 years: leading vs. bundle energy

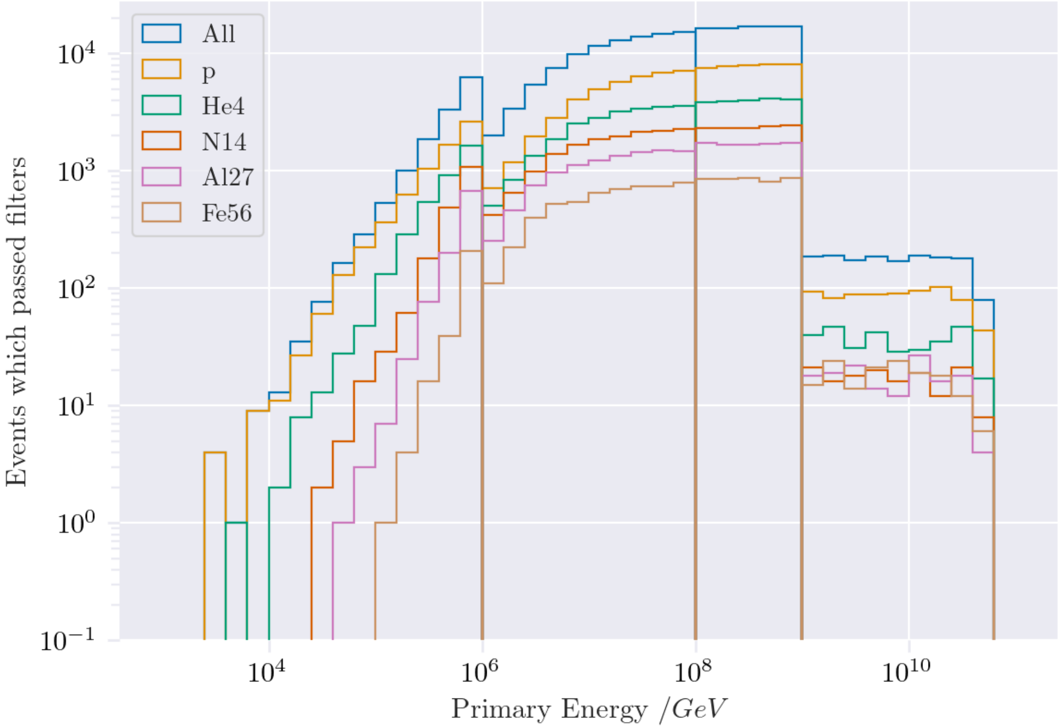
Solid: conventional
Dashed: prompt



- ➢ Different primary fluxes lead to different prompt fluxes
- ➢ Bundle energy extends to higher energies

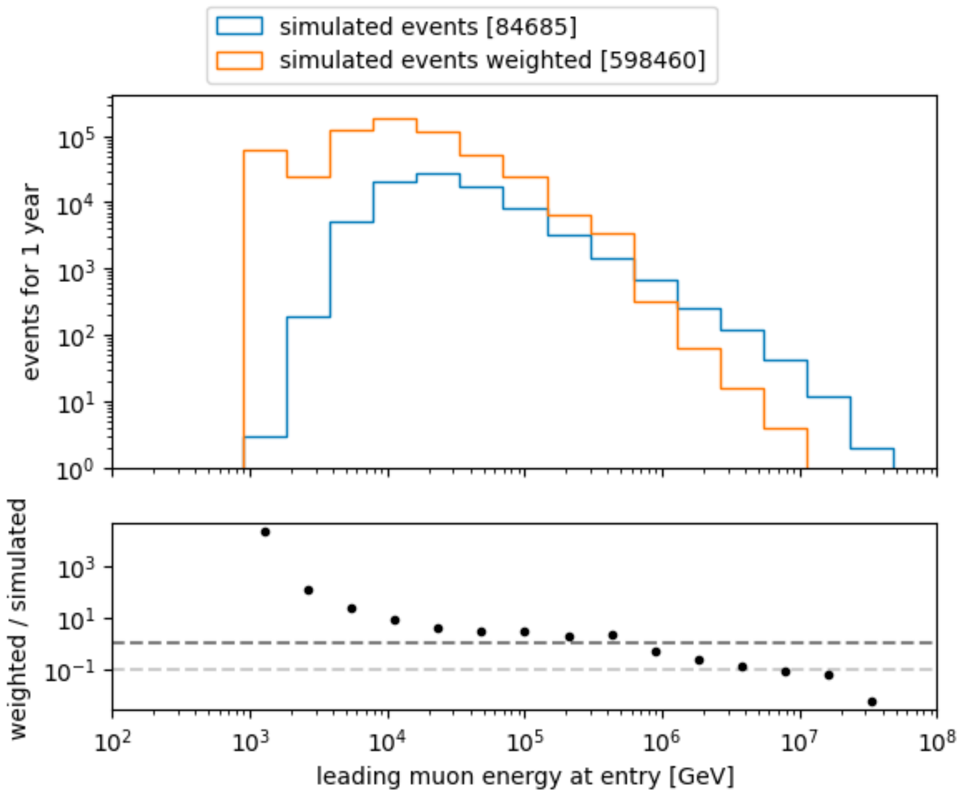
Simulated events

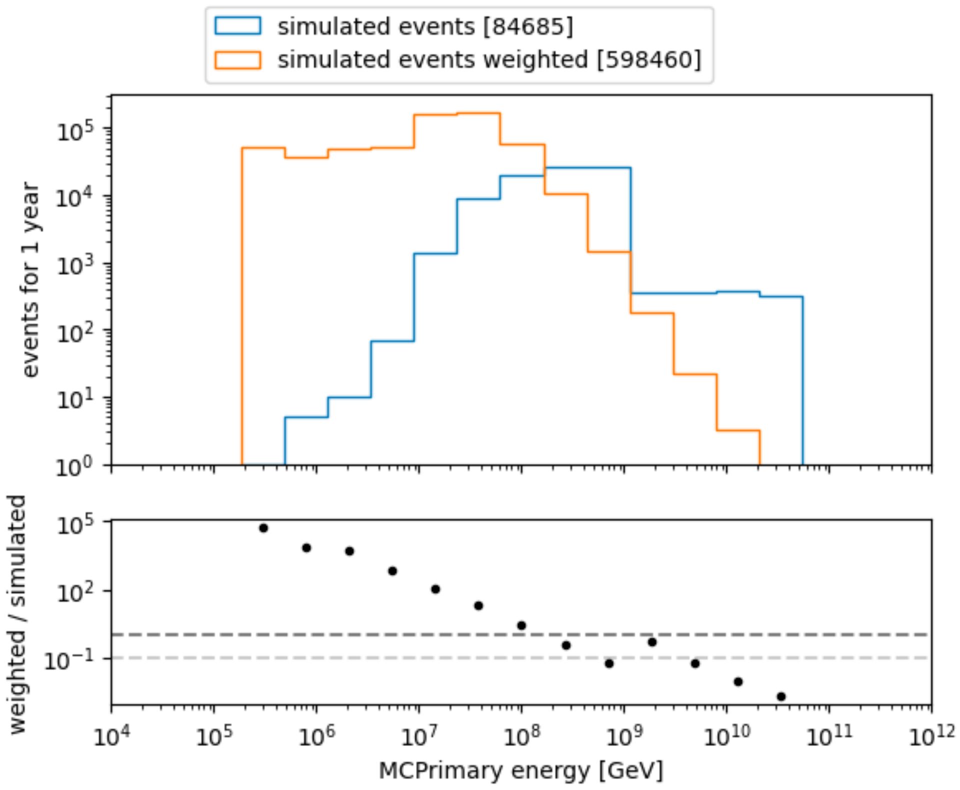
Passed muon filter



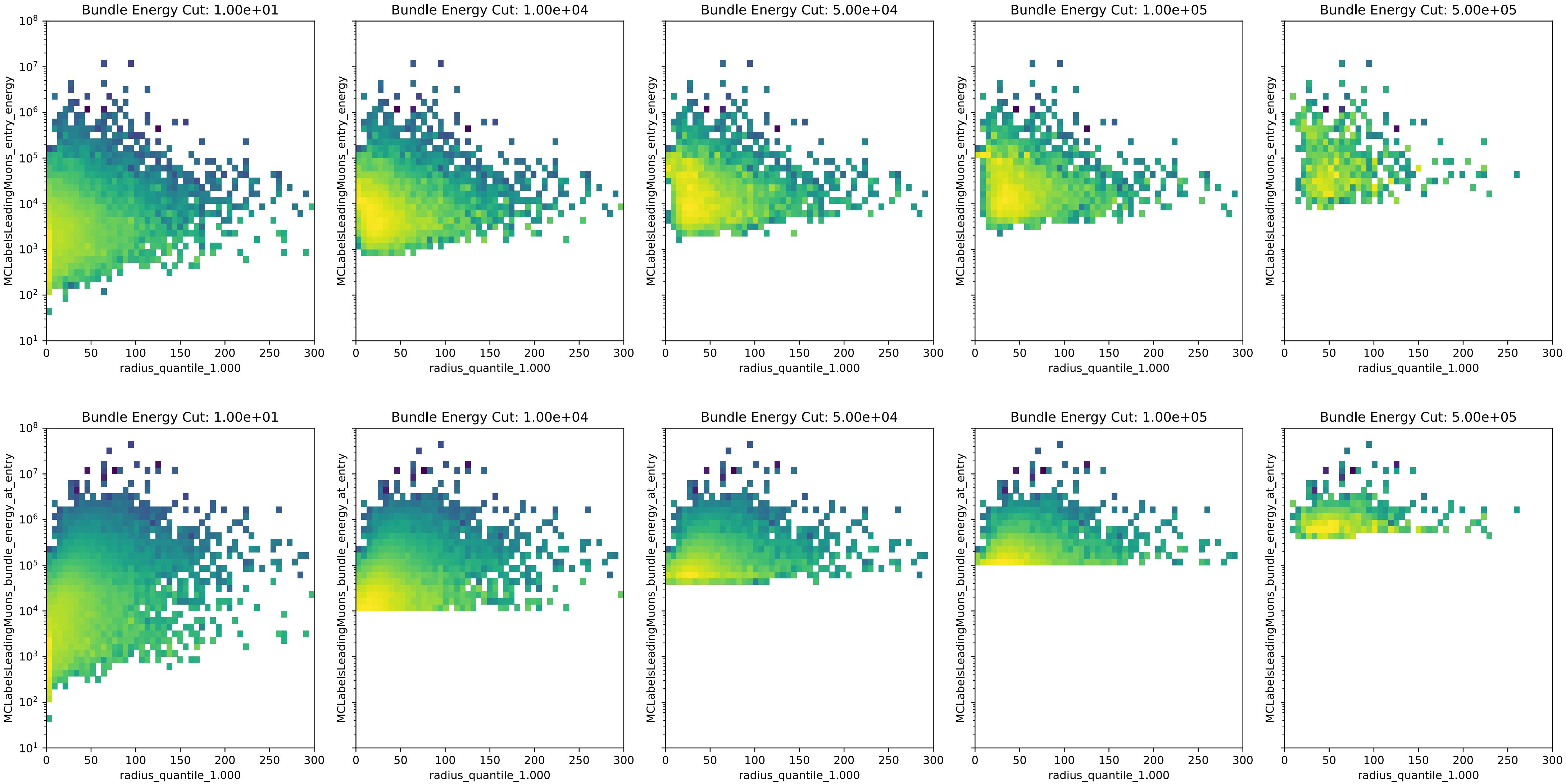
Which energies do we need to simulate?

L2 MuonFilter
Bundle energy > 100 TeV

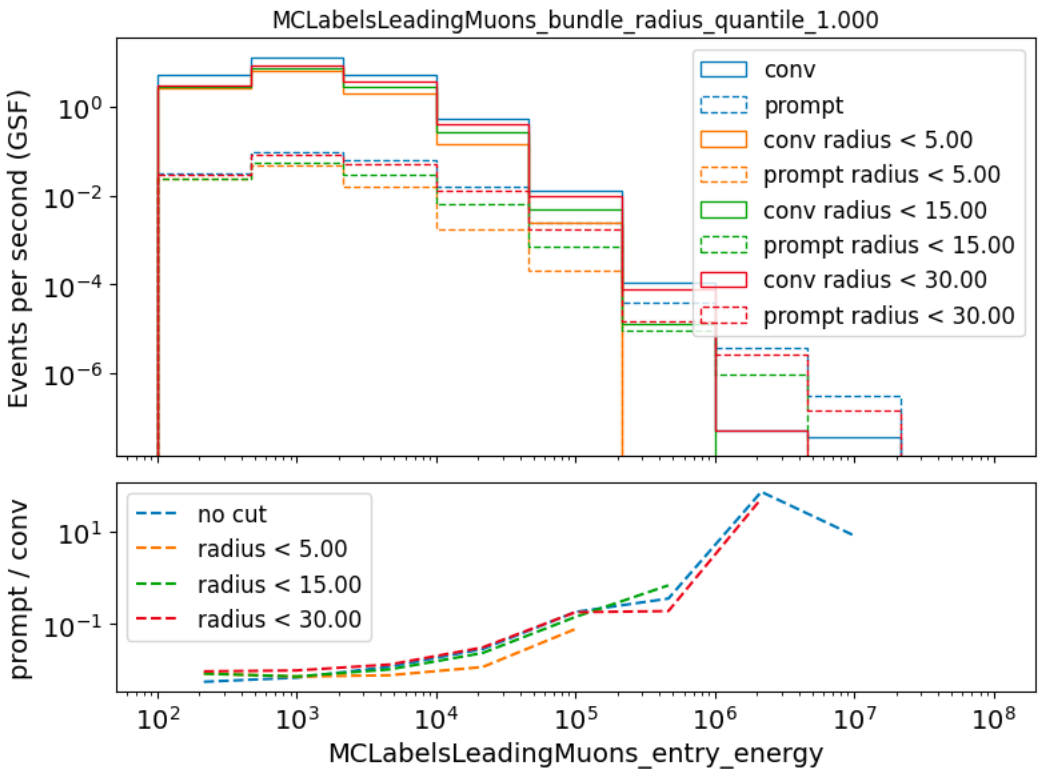




Bundle radius - energy



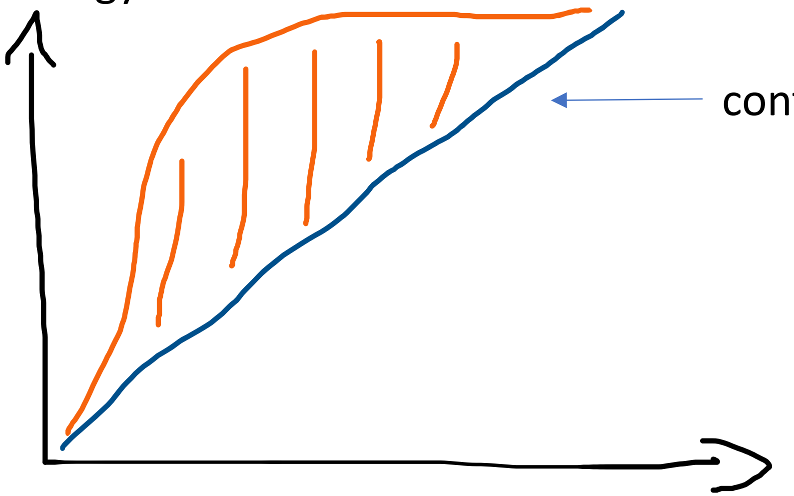
Muon flux – bundle radius



Stochasticity

muon that loses much energy at the beginning of the propagation

cumulated energy loss



continuous energy loss

track length

- calculate the area between the “continuous muon energy loss” and the accumulated muon energy losses and normalize it
- high stochasticity: area = 1
- no stochasticity: area = 0

Bundle radius

- calculate perpendicular distance between leading muon and closest point to detector center
  - reference point
- calculate the distance between the leading muon and all the other muons at the reference point
- analyze the distances weighted by their energy (100%, 99%, 95%,... energy containment)