1 Orlicz Norms and its Generalization

Definition 1.1 (Orlicz Norms)**.** Let $g : [0, \infty) \to [0, \infty)$ be a non-decreasing function with $g(0) = 0$. of a real-valued random variable X is given by

$$\|X\|_g := \inf\{\eta > 0 : \mathbb{E}\left[g(|X/\eta|)\right] \le 1\}. \tag{1}$$

It follows from (1) that if g is monotone,

$$\mathbb{P}\Big(|X| \ge \eta g^{-1}(t)\Big) \le \frac{1}{t} \text{ for all } t \ge 0.$$

Definition 1.2 (Sub-Weibull Variable)**.** A random variable X is said to be sub-Weibull of order $\alpha > 0$, denoted as sub-Weibull (α), if

$$\|X\|_{\psi_\alpha} < \infty, \text{ where } \psi_\alpha(x) \ := \ \exp\left(x^\alpha\right) - 1 \text{ for } x \ge 0. \tag{2}$$

Based on this definition, it follows that if X is sub-Weibull (α), then

$$\mathbb{P}(|X| \ge t) \le 2\exp\left(-\frac{t^\alpha}{\|X\|_{\psi_\alpha}^\alpha}\right) \text{ for all } t \ge 0.$$

Typically, X is sub-gaussian when $\alpha = 2$ and is sub-exponential when $\alpha = 1$.

Definition 1.3 (Marginal Sub-Weibull Vectors)**.** A random vector $X \in \mathbb{R}^q$ is said to be marginally sub-Weibull if for every $1 \le j \le q$, $X(j)$ is sub-Weibull and the marginal sub-Weibull norm is given by

$$\|X\|_{M,\psi_\alpha} \ := \ \sup_{1 \le j \le q} \|X(j)\|_{\psi_\alpha}.$$