BOUNDS ON QUARK MIXING ANGLES FROM THE DECAY $K_L \to \mu\bar{\mu}$

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We determine bounds on quark mixing angles in the standard six-quark model from an analysis of the decay $K_L \to \mu\bar{\mu}$. When combined with previous bounds from a Cabibbo fit and study of the $K^0 \leftrightarrow \bar{K}^0$ transition amplitude, these can yield an improved upper limit on one of the angles.

The mixing angles which determine how quarks couple to each other via the charged weak current are of fundamental significance. It is thus of considerable interest to ascertain which values of these quark mixing angles are consistent with present experimental data. In the standard six-quark Kobayashi–Maskawa (KM) [1] version of the Weinberg–Salam (WS) [2] $SU(2) \times U(1)$ gauge theory, which generalizes the Glashow–Iliopoulos–Maiani (GIM) [3] mechanism, the quark mixing matrix V depends on four angles. Three of these angles, θ_i, $i = 1, 2, 3$, are *CP*-conserving, while one, denoted δ, is *CP*-violating, Extending previous estimates [4], a recent analysis utilized a generalized Cabibbo fit [5] to obtain a precise value for $|c_1| \equiv |\cos\theta_1|$ and a bound on $|s_3| \equiv |\sin\theta_3|$. These results were then combined with constraints pertaining to the $K_L K_S$ mass difference Δm and *CP* violation in the neutral kaon mass matrix to yield a complete set of bounds on the quark mixing angles [6].

In this note we shall examine a further constraint on these angles arising from an upper bound on the short distance contribution to the $K_L \to \mu\bar{\mu}$ decay rate. This constraint is reasonably restrictive and, when combined with the results of refs. [5,6], can yield, especially for large values of the t-quark mass, an improved upper bound on $|s_2|$. Moreover, the calculation of the short-distance contribution has the merit that it involves only the single current matrix element

$$\langle 0|\bar{s}\gamma_\alpha\gamma_5 d + \bar{d}\gamma_\alpha\gamma_5 s|K_L\rangle$$
$$= \sqrt{2}\langle 0|\bar{s}\gamma_\alpha\gamma_5 u|K^+\rangle = \sqrt{2} f_K p_\alpha\,, \tag{1}$$

where p_α is the momentum of the K, and f_K is well known from $K^+ \to \mu^+\nu_\mu$ decay. In contrast, the calculation of the $K^0 \leftrightarrow \bar{K}^0$ transition amplitude requires an estimate of the two-current matrix element $\langle\bar{K}^0|(\bar{s}_L\gamma_\alpha d_L)^2|K^0\rangle$ [7].

The decay $K_L \to \mu\bar{\mu}$ has been the subject of extensive study in the past [8]. There is a reliably calculable contribution to the rate from the absorptive part of the two-photon intermediate state, i.e., the process $K_L \to \gamma\gamma \to \mu\bar{\mu}$. This can be determined by the use of unitarity and gives [9]

$$\Gamma(K_L \to \mu\bar{\mu})_{\rm abs} = 1.2 \times 10^{-5}\,\Gamma(K_L \to \gamma\gamma)$$
$$= (5.9 \pm 0.6) \times 10^{-9}\,\Gamma(K_L \to {\rm all})\,. \tag{2}$$

The contributions of other intermediate states such as $\pi\pi$ and $\pi\pi\gamma$ to the absorptive part of the $K_L \to \mu\bar{\mu}$ amplitude have been computed and found to be small relative to that of the $\gamma\gamma$ state [8,10]. The present

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measured value of the $K_L \to \mu\bar{\mu}$ branching ratio is [11]

$$B(K_L \to \mu\bar{\mu})_{exp} = (9.1 \pm 1.8) \times 10^{-9}, \quad (3)$$

and hence the dispersive part is

$$B(K_L \to \mu\bar{\mu})_{disp} = (3.2 \pm 2.4) \times 10^{-9}. \quad (4)$$

In addition to these low-mass exclusive intermediate states, the short-distance contribution to the (dispersive part of the) $K_L \to \mu\bar{\mu}$ amplitude has been studied in connection with the GIM suppression of $|\Delta S| = 1$ neutral currents in the four quark model [12]. It was found to be quite small. However, in the six-quark WS–KM model this is in general no longer the case. The (U-gauge) graphs which contribute at the one-loop level to the short distance part of the $K_L \to \mu\bar{\mu}$ amplitude in the quasi-free quark model are shown in fig. 1. (Actually the calculations of ref. [12] were performed in the general R_ξ or 't Hooft–Feynman gauge.) The results of ref. [12] can easily be generalized to the six-quark case to give the short-distance (sd) amplitude

$$\mathcal{M}(d\bar{s} \to \mu\bar{\mu})_{sd} = (G_F^2/2\pi^2)$$
$$\times \left(\sum_{i=c,t} V_{is}^* V_{id} m_i^2\right) [\bar{\mu}\gamma_\alpha\gamma_5\mu]\ [\bar{s}_L\gamma^\alpha d_L]\ , \quad (5)$$

where the quark mixing matrix V is defined via the charged current $J_\alpha = \bar{q}_{iL}(2/3)\gamma_\alpha V_{ij} q_{jL}(-1/3)$. (We shall use the KM parametrization of V as in refs. [5,6].)

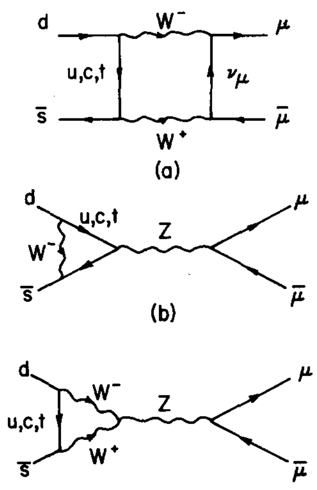

Fig. 1. One-loop diagrams contributing (in U-gauge) in the WS–KM model to the short-distance part of the $K_L \to \mu\bar{\mu}$ amplitude.

This yields a contribution to the rate, conveniently normalized relative to the $K^+ \to \mu^+\nu_\mu$ rate, of

$$B(K_L \to \mu\bar{\mu})_{sd} = \frac{G_F^2}{2\pi^4} \frac{(1 - 4m_\mu^2/m_K^2)^{1/2}}{(1 - m_\mu^2/m_K^2)^2} \quad (6)$$
$$\times \frac{(\Sigma_{i=c,t} \mathrm{Re}(V_{is}^* V_{id}) m_i^2)^2}{|V_{us}|^2} B(K^+ \to \mu^+\nu_\mu) \frac{\tau(K_L)}{\tau(K^+)}.$$

Two remarks are in order here. First, in addition to this short-distance term there is, of course, the contribution of the $\gamma\gamma$ intermediate state to $B(K_L \to \mu\bar{\mu})_{disp}$. The short-distance part of this contribution has been calculated in the four-quark model [13]. Although formally of two-loop order, it significantly modifies the c-quark part of $\mathrm{Amp}(K_L \to \mu\bar{\mu})_{disp}$. However, in the case of the t-quark part, it is negligible if $m_t \gtrsim 10$ GeV, as assumed here. The sum of the long-distance portion of the dispersive $\gamma\gamma$ term (which has also been roughly estimated [13]) and the c-quark contribution is not likely to exceed $\sim 0.5|\mathrm{Amp}(K_L \to \mu\bar{\mu})_{abs}|$. Secondly, gluon corrections slightly increase [14] the free quark contribution (5) to the rate and would thus slightly lower the upper bound on the mixing angles. However, the correction factor is close to unity for appropriate values of $\bar{\alpha}_s$, and accordingly we shall not include it here.

Our bound on quark mixing angles follows from the condition

$$B(K_L \to \mu\bar{\mu})_{sd} \leqslant B(K_L \to \mu\mu)_{disp} \leqslant 5.6 \times 10^{-9}. \quad (7)$$

Because of the presence of long-distance contributions to $B(K_L \to \mu\bar{\mu})_{disp}$, the use of eq. (6) to infer a lower bound on $B(K_L \to \mu\bar{\mu})_{sd}$ would be subject to considerable uncertainty, and consequently we will not try to analyze such a possible lower bound here. Eq. (7) implies the inequality

$$|c_2(c_1c_2 - s_2t_3c_\delta)m_c^2$$
$$+ s_2(c_1s_2 + c_2t_3c_\delta)m_t^2| < 57\ \mathrm{GeV}^2 \quad (8)$$

(where $t_i \equiv \tan\theta_i$). The first term $\propto m_c^2$ is negligible relative to the right-hand side of eq. (8); since we will use this equation to obtain an upper bound on the t-quark contribution, we will therefore drop the m_c^2 term. In our numerical analysis we utilize the value

$|c_1| = 0.9737 \pm 0.0025$, the bound $10^{-3} \lesssim |s_3| \lesssim 0.5$, and the fact that $|s_\delta|^2 \ll 1$ except for extremely small $|s_3|$ [5,6].

The resulting upper bound on $|s_2|$ is plotted in fig. 2 as a function of $|s_3|$ for the typical values m_t = (a) 15 GeV and (b) 30 GeV, and for $\xi \equiv \mathrm{sgn}(t_2 t_3 c_\delta c_1^{-1}) = \mathrm{sgn}(c_\delta) = \pm 1$. The qualitative behaviour of the curves can be explained as follows: as $|s_3|$ increases the term $|c_1 s_2 + c_2 t_3 c_\delta|$ increases (decreases) for $\xi = 1$ ($\xi = -1$). To compensate for this change $|s_2|_{\max}$ must therefore decrease (increase) in the two respective cases. For comparison the upper bound on $|s_2|$ from the Δm constraint used in ref. [6], viz. $\zeta^{-1}(\Delta m)_{\exp} < (\Delta m)_{\mathrm{KM}} < \zeta(\Delta m)_{\exp}$, where $\zeta = 2$, is also shown in the figure. (The ϵ constraint in ref. [6] did not set the most restrictive upper bound on $|s_2|$.) For m_t = 15 GeV the Δm constraint is somewhat more stringent; however, it is interesting that as m_t increases, the upper bound on $|s_2|$ due to the $K_L \to \mu\bar{\mu}$ constraint decreases more rapidly than that due to the Δm constraint and becomes smaller for $m_t \gtrsim 22$ GeV. This general feature is a result of the fact that for the case $m_c^2/m_t^2 \ll 1$ of physical relevance the $K_L \to \mu\bar{\mu}$ bound is approximately of the form $|\mathrm{Re}(V_{ts}^* V_{td})/V_{us}| < \mathrm{const}/m_t^2$, whereas the Δm bound is roughly of the form $(\mathrm{Re}(V_{ts}^* V_{td}))^2 < \mathrm{const}'/m_t^2$. One important effect of a reduction in $|s_2|$ is to diminish the magnitude of the t–d coupling, independent of the values of $|s_2|$, $|s_\delta|$, and ξ.

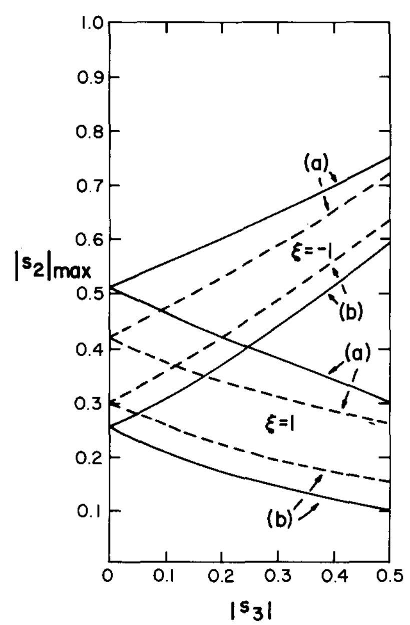

Fig. 2. Upper bounds on $|s_2|$ from $K_L \to \mu\bar{\mu}$ decay (solid curves) plotted as a function of $|s_3|$ for m_t = (a) 15 GeV (b) 30 GeV and $\xi = \pm 1$. For comparison the corresponding upper bounds from the Δm constraint are also shown (dashed curves).

Parenthetically, we note that it is unlikely that the decays $K \to \pi e\bar{e}$ and $K_L \to \gamma\gamma$ can be used to derive very reliable bounds on quark mixing angles. This is a consequence of the sensitivity of the free quark amplitudes for these decays to the low momentum region of the respective loop integrations. Furthermore, the $K_L \to \gamma\gamma$ amplitude is insensitive to the contributions of heavy quarks (apart from their obvious connection with the unitarity of V), and hence also insensitive to their mixing angles with light quarks. This last characteristic is the reverse of the situation with the $K^0 \leftrightarrow \bar{K}^0$, $K_L \to \mu\bar{\mu}$, and $K \to \pi e\bar{e}$ amplitudes.

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