Cosmological baryon and lepton number in the presence of electroweak fermion-number violation

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In the presence of rapid fermion-number violation due to nonperturbative electroweak effects certain relations between the baryon number of the Universe and the lepton numbers of the Universe are predicted. In some cases the electron-neutrino asymmetry is exactly specified in terms of the baryon asymmetry. Without introducing new particles—beyond the usual quarks and leptons—it is necessary that the Universe possess a nonzero value of $B-L$ prior to the epoch of fermion-number violation if baryon and lepton asymmetries are to survive. Contrary to intuition, even though electroweak processes violate $B+L$, a nonzero value of $B+L$ persists after the epoch of rapid fermion-number violation. If the standard model is extended to include lepton-number violation, for example, through Majorana neutrino masses, then electroweak processes will reduce the baryon number to zero even in the presence of an initial $B-L$ unless $20M_L \gtrsim \sqrt{T_{B-L} m_{\rm Pl}}$ where M_L sets the scale of lepton-number violation and T_{B-L} is the temperature at which a $B-L$ asymmetry is produced. In some models this implies that neutrinos must be so light that they cannot contribute appreciably to the mass density of the Universe.

I. INTRODUCTION

Baryogenesis is one of the most attractive and compelling ideas to emerge from the study of early Universe cosmology. Nonequilibrium processes that violate B, C, and CP allow the Universe to develop a net baryon number, usually at very early times, $t \lesssim 10^{-34}$ sec.[1] The possibility that electroweak processes involving sphaleron configurations might lead to very rapid baryon- and lepton-number violation shortly after the electroweak phase transition [$T \sim 100$–300 GeV (Ref. 2)] has raised the specter that the baryon asymmetry of the Universe might be washed away, rendering baryogenesis impotent. Electroweak baryon-number violation arises because baryon number is anomalous; because $B-L$ is anomaly-free, a baryon number that is produced can be "protected" if it has a nonzero projection onto $B-L$. This point has been discussed in Ref. 3.

Since sphaleron processes conserve $B-L$ and violate $B+L$, one might have thought that they would leave the Universe with $B+L=0$, thereby making the simple prediction that

$$B = \frac{(B-L)_i}{2}, \quad L = -B = -\frac{(B-L)_i}{2} ,$$

where subscript i refers to the asymmetry that existed before fermion-number violation becomes important. (This is basically the conclusion reached in Ref. 3.) In fact, the situation is more complicated: Sphaleron processes only involve the left-handed fields, and the charge neutrality of the Universe must be preserved. Steps toward taking these facts into account were made in Refs. 4 and 5, but we believe their analysis is still incomplete.

In addition, it has also been argued that fermion-number-violating processes are also important at temperatures above the electroweak symmetry-breaking scale, although they are no longer amenable to a semiclassical analysis in terms of sphaleron configurations.[6] Moreover, if the $B-L$ asymmetry needed to ensure that a baryon asymmetry survives is generated dynamically, rather than being imposed as an initial condition, then there may be residual $(B-L)$-violating interactions at energies below the scale at which the $B-L$ asymmetry is initially produced. When considered along with electroweak violation of $B+L$, such interactions will drive the baryon number rapidly to zero. Since the dominant $(B-L)$-violating operator at low energies is the dimension-five operator responsible for Majorana neutrino masses,[7] this can impose a very interesting and stringent constraint on neutrino masses and the scale at which the $B-L$ asymmetry is generated.[8]

In this paper we will derive the equilibrium relations between B and L that arise in the presence of rapid fermion-number violation, at temperatures both above and below that of electroweak symmetry breaking. We will also discuss the constraint that rapid fermion-number violation places on neutrino masses.

II. EXERCISE IN EQUILIBRIUM THERMODYNAMICS

Particle asymmetries are most conveniently expressed in terms of chemical potentials. For simplicity we will assume that all species can be treated as being ultrarela-

tivistic. At temperatures above the electroweak phase transition ($T \gtrsim T_C \sim 300$ GeV), this should be an excellent approximation. At temperatures below the transition, a more careful analysis would include mass effects and the precise temperature dependence of sphaleron processes.[9] We nonetheless expect our results to be at least qualitatively correct, and that they will serve to illustrate the salient points. The crucial relation between the excess of particle over antiparticle and the particle's chemical potential is given by

$$n_+ - n_- = \frac{gT^3}{3}\left[\frac{\mu}{T}\right] \quad \text{(bosons)}, \tag{1a}$$

$$n_+ - n_- = \frac{gT^3}{6}\left[\frac{\mu}{T}\right] \quad \text{(fermions)}, \tag{1b}$$

$$\frac{n_+ - n_-}{s} = \frac{15g}{2\pi^2 g_*}\left[\frac{\mu}{T}\right] \quad \text{(bosons)}, \tag{1c}$$

$$\frac{n_+ - n_-}{s} = \frac{15g}{4\pi^2 g_*}\left[\frac{\mu}{T}\right] \quad \text{(fermions)}, \tag{1d}$$

where n_+ is the equilibrium number density of the particle species, n_- that of the CP-conjugate species, μ is the chemical potential of the particle species ($-\mu$ is the chemical potential of the CP-conjugate species), g counts the internal degrees of freedom, $s = 2\pi^2 g_* T^3/45$ is the entropy density, g_* counts the total number of relativistic degrees of freedom, and we have assumed that $|\mu/T| \ll 1$ (nondegenerate limit; baryon and lepton chemical potentials are expected to be of order $|\mu| \sim 10^{-10}T$) and $|\mu| < m$ (no Bose condensation). Provided that there is no significant entropy production, the entropy density s is proportional to R^{-3}; therefore, the ratio of any number density to s corresponds to the particle number per comoving volume. For example, the baryon number is defined as $B = (n_b - n_{\bar{b}})/s$, where n_b ($n_{\bar{b}}$) is the number density of baryons (antibaryons). From this point forward we will deal exclusively with chemical potentials; number densities can be obtained from chemical potentials via Eqs. (1).

The original analysis of nonperturbative, electroweak violation of baryon number was carried out for temperatures high compared to m_W but low compared to the electroweak phase-transition temperature ($T_C \sim 300$ GeV $\gtrsim T \gtrsim m_W \simeq 80$ GeV) by a semiclassical expansion about the sphaleron configuration.[2] Although there have been many technical objections to this procedure, there seems to be a concensus emerging that the general picture is indeed valid and that there should be a period of rapid fermion-number violation (rate $\Gamma \gg H$; H is the expansion rate of the Universe) at temperatures of order 100–300 GeV. In addition, it seems likely that rapid baryon-number-violating processes occur at temperatures above the electroweak phase-transition temperature as well; however, they are not amenable to the same semiclassical analysis.[6] This idea has received recent support from real-time numerical simulations of high-temperature, electroweak theory.[10] We will therefore analyze the equilibrium distribution of quantum numbers

both above and below the electroweak phase transition. We first restrict ourselves to the standard model, that is, no $B - L$ violation, and then later discuss the effects of including ($B - L$)-violating processes.

The standard model consists of N generations of quarks and leptons, m complex Higgs doublets, and the usual gauge fields of $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. For our purposes the relevant fields are N left-handed quark doublets (u_{iL}, d_{iL}, $i = 1$ to N), N left-handed lepton doublets (ν_i, e_{iL}, $i = 1$ to N), N right-handed quark singlet fields (u_{iR}, d_{iR}, $i = 1$ to N), N right-handed charged lepton fields (e_{iR}, $i = 1$ to N), W^+, and m complex Higgs doublets (ϕ_i^+, ϕ_i^0 and ϕ_i^{0*}, ϕ_i^-, $i = 1$ to m). The eight gluon fields and W^0 and B^0 fields have vanishing chemical potential and can be ignored for this exercise. The vanishing chemical potentials for the gluon fields assures equal chemical potentials for all colors of quarks. At temperatures above the electroweak phase transition, the W^+ will also have a vanishing chemical potential which imposes equality of the chemical potentials for fields in the same electroweak multiplet.

The chemical potentials are assigned as follows: μ_W for W^-, μ_0 for all m ϕ^0 Higgs fields, μ_- for all ϕ^- Higgs fields; μ_{uL} for all the left-handed up-quark fields, μ_{uR} for all the right-handed up-quark fields; μ_{dL} for all the left-handed down-quark fields, μ_{dR} for all the right-handed down-quark fields, μ_i for the left-handed neutrino fields, μ_{iL} for the left-handed charged lepton fields, and μ_{iR} for the right-handed charged lepton fields. Cabibbo mixing should maintain the equality of the various up and down-quark states, respectively. We also assume that mixing between the m Higgs doublets maintains the equality of their chemical potentials. In the absence of flavor-mixing neutrino interactions (e.g., due to neutrino masses), the lepton generations will not in general have equal chemical potentials; if there are rapid flavor-mixing interactions, then $\mu_{iL} \equiv \mu_L$, $\mu_{iR} \equiv \mu_R$, and $\mu_i \equiv \mu/3$. In all, there are $7 + 3N$ chemical potentials; fortunately thermodynamical equilibrium imposes a number of relations between them.

Recall that whenever a reaction is occurring rapidly—in the early Universe the criterion is $\Gamma \gg H$—the sum of the chemical potentials of the incoming particles is equal to that of the outgoing particles. Rapid electroweak interactions in the early Universe enforce the following equilibrium relations among the chemical potentials:

$$\mu_W = \mu_- + \mu_0 \quad (W^- \leftrightarrow \phi^- + \phi^0), \tag{2a}$$

$$\mu_{dL} = \mu_{uL} + \mu_W \quad (W^- \leftrightarrow \bar{u}_L + d_L), \tag{2b}$$

$$\mu_{iL} = \mu_i + \mu_W \quad (W^- \leftrightarrow \bar{\nu}_{iL} + e_{iL}), \tag{2c}$$

$$\mu_{uR} = \mu_0 + \mu_{uL} \quad (\phi^0 \leftrightarrow \bar{u}_L + u_R), \tag{2d}$$

$$\mu_{dR} = -\mu_0 + \mu_W + \mu_{uL} \quad (\phi^0 \leftrightarrow d_L + \bar{d}_R), \tag{2e}$$

$$\mu_{iR} = -\mu_0 + \mu_W + \mu_i \quad (\phi^0 \leftrightarrow e_{iL} + \bar{e}_{iR}). \tag{2f}$$

By the use of these relations we can express all of the chemical potentials in terms of $3 + N$ chemical potentials: μ_W, μ_0, μ_{uL}, and μ_i. In addition, it will prove convenient

to define the sum of the neutrino chemical potential: $\mu \equiv \sum_i \mu_i$. The electroweak $B + L$ anomaly implies the existence of processes that correspond to the creation of a "$n_L \nu$" state from each generation out of the vacuum. Here $n_L \nu$ is shorthand for $u_L d_L d_L \nu_L$. We will refer to such vacuum transitions as "sphaleron processes," even if they occur at high temperatures and do not involve the true sphaleron saddle-point solution. So long as sphaleron interactions are rapid, the following relation among the chemical potentials is enforced:

$$N(\mu_{uL} + 2\mu_{dL}) + \sum_i \mu_i = 0 , \tag{3a}$$

or, equivalently,

$$3N\mu_{uL} + 2N\mu_W + \mu = 0 . \tag{3b}$$

Now let us express the baryon, lepton, charge, and I_3 (third component of weak isospin) number densities in terms of our chosen set of $3 + N$ chemical potentials:

$$B = N(\mu_{uL} + \mu_{uR}) + N(\mu_{dL} + \mu_{dR})$$
$$= 4N\mu_{uL} + 2N\mu_W , \tag{4}$$

$$L = \sum_i (\mu_i + \mu_{iL} + \mu_{iR}) = 3\mu + 2N\mu_W - N\mu_0 , \tag{5}$$

$$Q = 2N(\mu_{uL} + \mu_{uR}) - N(\mu_{dL} + \mu_{dR})$$
$$- \sum_i (\mu_{iL} + \mu_{iR}) - 4\mu_W - 2m\mu_- ,$$
$$= 2N\mu_{uL} - 2\mu - (4N + 2m + 4)\mu_W + (4N + 2m)\mu_0 , \tag{6}$$

$$Q_3 = \frac{3N}{2}(\mu_{uL} - \mu_{dL}) + \frac{1}{2}\sum_i (\mu_i - \mu_{iL})$$
$$- 4\mu_W - m(\mu_0 + \mu_-) ,$$
$$= -(2N + m + 4)\mu_W . \tag{7}$$

A. $T \gtrsim T_C$

Above the critical temperature both Q and Q_3 must be zero; the latter immediately implies that $\mu_W = 0$.[11] Further, by using $Q = 0$ and the relation implied by sphaleron transitions, both B and L can be expressed in terms of a single chemical potential (taken to be μ_{uL}):

$$B = 4N\mu_{uL}, \quad L = -\frac{14N^2 + 9Nm}{2N + m}\mu_{uL} . \tag{8}$$

From these the following relations can be derived:

$$B + L = -\frac{6N + 5m}{22N + 13m}(B - L) , \tag{9a}$$

$$B = \frac{8N + 4m}{22N + 13m}(B - L) , \tag{9b}$$

$$L = -\frac{14N + 9m}{22N + 13m}(B - L) . \tag{9c}$$

We have expressed the various asymmetries in terms of $B - L$ because $B - L$ is conserved by all electroweak in-

teractions including those of sphalerons. It is interesting that in spite of the fact that sphaleron transitions violate $B + L$, thermodynamic equilibrium requires a nonzero value of $B + L$.

B. $T \lesssim T_C$

Below the critical temperature, Q must still be zero, but it is no longer necessary for Q_3 to be zero since $SU(2)_L$ has been broken. However, because of the vacuum condensate of ϕ^0 Higgs bosons, μ_0 must be equal to zero. Again, it follows that both B and L can be expressed in terms of a single chemical potential (again taken to be μ_{uL}):

$$B = \frac{8N^2 + 4N(m + 2)}{m + 2}\mu_{uL} ,$$
$$L = -\frac{16N^2 + 9N(m + 2)}{m + 2}\mu_{uL} . \tag{10}$$

From these the following relations can be derived:

$$B + L = -\frac{8N + 5(m + 2)}{24N + 13(m + 2)}(B - L) , \tag{11a}$$

$$B = \frac{8N + 4(m + 2)}{24N + 13(m + 2)}(B - L) , \tag{11b}$$

$$L = -\frac{16N + 9(m + 2)}{24N + 13(m + 2)}(B - L) . \tag{11c}$$

The relation between B and L for temperatures above the electroweak transition [cf. Eqs. (9)] is the same as a relation appearing in Ref. 12 if we take $m = 1$ [cf. Eq. (4.24)]. However, in Ref. 12 it was argued that this relation holds *below* the phase transition also. We would argue that the relation between B and L in this regime is given by Eqs. (11) for the reasons discussed above.

To summarize, at high temperatures, $T \gtrsim T_C$, B, L, $B - L$, and $B + L$ are related by Eqs. (9), provided that electroweak fermion-number-violating interactions are indeed rapid. At low temperatures, $T \lesssim T_C$, a slightly different set of relations are predicted [cf. Eqs. (11)], again provided that fermion-number violation is rapid. In both instances thermal equilibrium requires that $B + L \propto B - L$, in spite of the fact that sphaleron transitions violate $B + L$.

The relationship between B and L today depends upon the history of electroweak fermion-number violation. In the simplest scenario, for $T \gtrsim T_C$, baryon and lepton numbers are related by Eqs. (9), and for $T \lesssim T_C$ they are related by Eqs. (11). Here, of course, we are assuming that sphaleron processes are rapid both above and below the electroweak phase transition. If sphaleron processes are *not* rapid after the electroweak phase transition, e.g., if the electroweak phase transition is strongly first order or if they are suppressed by some mechanism, then the baryon and lepton numbers are today given by Eqs. (9).

In the absence of flavor-changing interactions in the lepton sector, e.g., due to neutrino masses, there is no prediction about the individual lepton numbers L_i. This is because sphaleron interactions (and high-temperature weak interactions that violate fermion number) involve transitions that do not mix the different generations, so

that $B/3 - L_i$ is conserved.[3] While the total lepton number L must be of the same magnitude as the baryon number B, it is still possible that two (or more) of the lepton numbers could be much larger; for example, $L_\mu, L_\tau \gg B$, with $L_\mu = -L_\tau$.[13] In terms of chemical potentials, the N independent neutrino chemical potentials $(\mu_i, i = 1, \ldots, N)$ determine the individual lepton numbers (and the total lepton number). As a practical matter, the N independent chemical potentials might better be chosen to be μ and μ_i, $i = 1, \ldots, N - 1$.)

If we assume that $L_i = L/3$, as would be the case if there are rapid flavor-changing interactions in the lepton sector, then it is possible to predict the lepton asymmetries of the Universe. For example, suppose that the final relationship between B and L is given by Eqs. (11), with $N = 3$ and $m = 1$. Then we have

$$L_i = \tfrac{1}{3} L = -\tfrac{25}{36} B \ .$$

At temperatures much less than T_C, sphaleron transitions become impotent (rate $\Gamma \ll H$), and thereafter B and L are effectively conserved. However, weak interactions can still interconvert protons and neutrons, and electrons and neutrinos: e.g., $n + \nu_e \leftrightarrow p + e^-$. Around the time of primordial nucleosynthesis, $t \sim 1$ sec and $T \sim 1$ MeV, the weak interactions freeze out, when the neutron-to-proton ratio is about $\tfrac{1}{7}$. To a good approximation that ratio stays constant thereafter.[14] This implies that $B_n/B_p \simeq \tfrac{1}{7}$, where $B = B_n + B_p$, and B_n is the baryon number residing in neutrons and B_p that in protons. The lepton number $L_1 = L_e + L_{\nu_e}$, and charge neutrality requires that $L_e = B_p$. From these we can solve, of the asymmetry between electron neutrinos and antielectron neutrinos,

$$L_{\nu_e} = \left[\frac{L/B}{3} - \frac{1}{1 + B_n/B_p} \right] B \simeq -1.6 B \ . \quad (12)$$

Similar relations hold for other values of N, m, and for $T \gtrsim T_C$. Whether such a prediction can ever be tested remains to be seen.

III. BEYOND THE STANDARD MODEL

We have so far assumed that $B - L$ is absolutely conserved. However, if the $B - L$ asymmetry needed to ensure the survival of a baryon number today is to be generated, rather than imposed as an initial condition, then $B - L$ must also be violated at some scale. In fact, in many grand unified theories, such as SO(10) or E_6, $B - L$ is promoted to a gauge symmetry which is then spontaneously broken at some energy scale. Since $B - L$ is conserved at temperatures above the scale of $B - L$ breaking, it is not possible for a baryon asymmetry to be produced above the scale of $B - L$ breaking since in the presence of rapid sphaleron processes, $B, L \propto B - L$, which must be zero. Once $B - L$ is spontaneously broken, a nonzero $B - L$ asymmetry can be produced, for example, by the standard "drift and decay" scenario.[1]

In many models a significant source of L violation arises from the existence of Majorana neutrino masses.
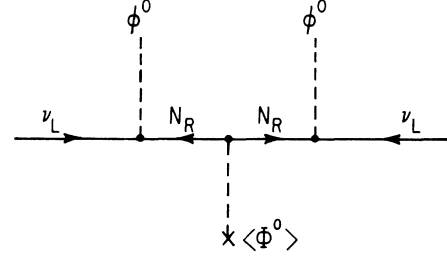

FIG. 1. Feynman diagram that leads to the dimension-five operator which violates lepton number. N_R indicates the superheavy right-handed neutrino, ν_L the light left-handed neutrino, ϕ^0 the neutral electroweak Higgs boson, and Φ^0 a superheavy Higgs boson.

For example, in SO(10), the breaking of $B - L$ will result in a Majorana mass for the right-handed neutrino field on the order of the symmetry-breaking scale. The Feynman diagram shown in Fig. 1 will lead to a dimension-five L-violating operator in the low-energy theory given by

$$\mathcal{L}_{\Delta L = 2} = \frac{m_\nu}{v^2} l_L l_L \phi\phi + \text{H.c.} \ , \quad (13)$$

where l_L refers to a left-handed lepton doublet, ϕ is the electroweak Higgs doublet, m_ν is the mass acquired by the left-handed neutrino after electroweak symmetry breaking, and $v \simeq 248$ GeV is the expectation value of the Higgs doublet. If the L-violating interactions induced by this operator occur at a rate faster than the expansion rate, then any $B - L$ asymmetry, and hence any B asymmetry, will be quickly reduced to zero by the combination of this L violation and sphaleron transitions. This can be shown very easily by adding the condition

$$\mu_0 + \mu_i = 0 \quad (14)$$

to the previous conditions on the chemical potentials [cf. Eqs. (2), (3), (6), and (7)]. It then follows that thermal equilibrium requires vanishing chemical potential for *all* particle species.

Arranging that rapid sphaleron transitions together with rapid L violation do not reduce any $B - L$ asymmetry to zero places a stringent constraint on the scale at which L is violated and the scale at which $B - L$ is produced. The dimension-five operator in Eq. (13) leads to L violation at a rate given by

$$\Gamma_{\Delta L = 2} = \sum_i \langle n_i \sigma_i \rangle \simeq \frac{1}{\pi^3} \frac{T^3 m_\nu^2}{v^4} \ , \quad (15)$$

through the processes $\nu_L \nu_L \leftrightarrow \phi^{0*}\phi^{0*}$ and $\nu_L \phi^0 \leftrightarrow \bar{\nu}_R \phi^{0*}$. Here $\sigma_i = m_\nu^2/2\pi v^4$ is the cross section (which is the same for either process) and $n_i = n_\phi = n_\nu \simeq T^3/\pi^2$. Demanding that this rate be less than the expansion rate $H = 1.67 g_*^{1/2} T^2/m_{\text{Pl}} \simeq 20 T^2/m_{\text{Pl}}$ (where $g_* \simeq 100$) at temperatures below that at which the $B - L$ asymmetry is produced implies

$$m_\nu \lesssim 20 \frac{v^2}{(T_{B-L} m_{\text{Pl}})^{1/2}} \simeq \frac{4 \text{ eV}}{(T_{B-L}/10^{10} \text{ GeV})^{1/2}} \ . \quad (16)$$

If we write $m_v = v^2/M_L$, where M_L is the scale of lepton-number violation, then this gives the constraint

$$20M_L \gtrsim \sqrt{T_{B-L} m_{\text{Pl}}} \; . \tag{17}$$

In the simplest scenarios we might expect $B - L$ to be produced through the decay of a Higgs (or gauge) boson that becomes massive when $B - L$ is spontaneously broken. We would then expect M_L and T_{B-L} to be comparable, in which case the previous condition can be satisfied only if M_L is within one or two orders of magnitude of the Planck scale. This would imply extremely small masses for the left-handed neutrinos and would preclude a neutrino species from contributing significantly to the mass density of the Universe. If the scale M_L is sufficiently small for neutrino masses to be in the 20–100-eV range for the heaviest neutrino species, then there must exist some mechanism for producing $B - L$ at relatively low temperatures $T_{B-L} \sim 10^8$ GeV.

We should be a little more precise about the implications for the masses of left-handed neutrinos. Recall that there are N independent neutrino chemical potentials that determine the individual lepton numbers; for $N = 3$, we can take them to be μ, μ_e, and μ_τ, or μ_e, μ_μ, and μ_τ. In the absence of rapid flavor-changing interactions in the lepton sector, the individual lepton numbers are conserved separately. The result that $B = 0$ only follows if $\mu \equiv \sum_i \mu_i = 0$; we can only be confident that $\mu = 0$ if the individual $\mu_i = 0$.

The dimension-five operator that leads to lepton-number violation induces transitions between the neutrino-mass eigenstates and their CP conjugates. If the neutrino-mass eigenstates and weak-interaction eigenstates coincide, then the lepton-number-violating interactions will not mix the individual lepton numbers, and μ will only be zero if the individual chemical potentials are all driven to zero. In this limiting case, the constraint to the neutrino mass [Eq. (17)] applies to the *lightest* neutrino species, because its lepton-number-violating interactions will be the slowest, and therefore the limiting rate.

On the other hand, if the neutrino-mass eigenstates and weak-interaction eigenstates are "well mixed," i.e., that mixing angles between the two sets of eigenstates that are of order unity or if there are other rapid interactions that mix the different neutrino species, then the dimension-five operator corresponding to the heaviest neutrino species—possibly supplemented by other flavor-changing interactions—will be sufficient to drive all the neutrino chemical potentials, and hence both μ and B, to zero. In this case constraint (17) applies to the *heaviest* neutrino species. [Of course, it could also be that the flavor and mass eigenstates coincide, and that the mechanism for baryogenesis only involves primarily the heaviest generation, in which case initially $|\mu_e|, |\mu_\mu| \ll |\mu_\tau|$, and μ_τ would be driven to zero by the dimension-five operator. In such a scenario, constraint (17) would apply to the *heaviest* neutrino species.]

To summarize, our constraint on the left-handed neutrino masses [Eq. (17)] is model dependent. In all cases it applies to the *lightest* neutrino species and could, depending on the potency of flavor-changing interactions in the lepton sector, apply to the *heaviest* neutrino species.

(The authors of Ref. 8 derived a much less stringent bound, $m_{v_l} \lesssim 50$ keV, based upon the same argument. This is because they assumed that electroweak fermion-number violation was rapid only at temperatures of order 100–300 GeV—that is, only below the electroweak phase transition. By considering the likely possibility that electroweak fermion-number violation is rapid at temperatures $T \gtrsim T_C$ also, we have derived our much more stringent bound.)

IV. SUMMARY

In conclusion, we have seen that contrary to naive expectations, rapid electroweak fermion-number-violating processes do not lead to a zero value of $B + L$, but rather predict a definite nonzero value related to the $B - L$ asymmetry. Moreover, provided that there are rapid lepton flavor-mixing interactions, a definite relationship between the electron-neutrino asymmetry and baryon asymmetry is predicted. In order to produce a baryon asymmetry that survives the epoch(s) of rapid fermion-number violation, a $B - L$ asymmetry must be produced. In theories with lepton-number violation in the form of Majorana neutrino masses, sphaleron processes together with L violation are likely to reduce any $B - L$ asymmetry, and hence any B asymmetry, to zero unless the left-handed neutrino masses are small.

[1]For a review of baryogenesis, see E. W. Kolb and M. S. Turner, Annu. Rev. Nucl. Part. Sci. **33**, 645 (1983).

[2]V. A. Kuzmin, V. A. Rubakov, and M. E. Shaposhnikov, Phys. Lett. **155B**, 36 (1985); for a recent review of the subject, see V. A. Matveev, V. A. Rubakov, A. N. Tavkhelidze, and M. E. Shaposhnikov, Usp. Fiz. Nauk. **156**, 253 (1988); [Sov. Phys. Usp. **31**, 916 (1988)]; M. E. Shaposhnikov, in *Proceedings of the XXIV International Conference on High Energy Physics*, Munich, West Germany, 1988, edited by R.

Kotthaus and J. Kuhn (Springer, Berlin, 1988); in *Proceedings on the Workshop on Sphalerons and Related Topics*, edited by L. McLerran (University of Minnesota, Minneapolis, 1988).

[3]E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, Redwood City, CA, 1990), pp. 188–190; Mod. Phys. Lett. A **2**, 285 (1987); V. A. Kuzmin, V. A. Rubakov, and M. E. Shaposhnikov, Phys. Lett. B **191**, 171 (1987); P. Arnold and L. McLerran, Phys. Rev. D **36**, 581 (1987).

[4]S. M. Barr, R. S. Chivukula, and E. Farhi, UCSB-ITP Report

No. NSF-ITP-90-27, 1990 (unpublished).

[5]S. Yu. Khlebnikov and M. E. Shaposhnikov, Nucl. Phys. **B308**, 885 (1988).

[6]P. Arnold and L. McLerran, Phys. Rev. D **36**, 581 (1987); S. Yu. Khlebnikov and M. E. Shaposhnikov, Nucl. Phys. **B308**, 885 (1988); M. Dine, O. Lechtenfeld, B. Sakita, W. Fischler, and J. Polchinski, *ibid.* **B342**, 381 (1990).

[7]S. Weinberg, Phys. Rev. Lett. **43**, 1566 (1979); also see F. Wilczek and A. Zee, *ibid.* **43**, 1571 (1979).

[8]M. Fukugita and T. Yanagida, Phys. Rev. D **42**, 1285 (1990). In this paper the authors only consider the case where rapid fermion-number violation occurs at temperatures just below the electroweak phase transition.

[9]The effect of nonzero particle masses leads to corrections of order m^2/T^2 and is discussed by V. A. Kuzmin, V. A. Rubakov, and M. E. Shaposhnikov, Phys. Lett. B **191**, 171 (1987), and by A. I. Bochkarev, S. Yu. Khlebnikov, and M. E. Shaposhnikov, Nucl. Phys. **B329**, 493 (1990).

[10]D. Yu. Grigoriev, V. A. Rubakov, and M. E. Shaposhnikov, Phys. Lett. B **216**, 172 (1989); J. Ambjorn, T. Askgaard, H. Porter, and M. E. Shaposhnikov, *ibid.* **244**, 479 (1990).

[11]One might worry about the consequences of assigning Bose species a chemical potential, e.g., that if $\mu \neq 0$, Bose condensation is possible (in which case μ should be zero). Here this concern is not justified. At temperatures below T_C, all the relevant gauge and Higgs bosons are massive, and so long as $|\mu| < m$, there can be no Bose condensate. Since $|\mu/T|$ is expected to be of order 10^{-10}, this condition is easily satisfied. At temperatures above T_C, the $W^{\pm}$ gauge bosons are massless, but thermal equilibrium dictates that $\mu_W = 0$.

[12]S. Yu. Khlebnikov and M. E. Shaposhnikov, Nucl. Phys. **B308**, 885 (1988).

[13]The possibility of producing a "large" lepton number ($L \gtrsim 1$) and a "small" baryon number ($B \sim 10^{-10}$) has previously been discussed by M. S. Turner, Phys. Lett. **98B**, 145 (1980), and by J. A. Harvey and E. W. Kolb, Phys. Rev. D **24**, 2090 (1981).

[14]During nucleosynthesis, essentially all the neutrons present become incorporated into ^{4}He, with the resulting mass fraction of ^{4}He being $Y \simeq 2(B_n/B_p)/(1+B_n/B_p) \simeq 0.24$. After primordial nucleosynthesis, weak interactions will modify the neutron-to-proton ratio during stellar nucleosynthesis since stars make additional ^{4}He and heavier elements. Today, stars have increased the mass fraction of ^{4}He from the primordial value of about 24% to about 30%; in addition, they have converted a baryon-mass fraction of about 2% into elements heavier than ^{4}He. We have ignored the small change in B_n/B_p since primordial nucleosynthesis.