Comment on "Quantization of Self-Dual Field Revisited"

In a recent Letter, Srivastava[1] proposed a new quantization scheme for two-dimensional self-dual fields (chiral bosons). He introduced a linear "chiral constraint" $\partial_-\phi=0$, instead of the quadratic constraint $(\partial_-\phi)^2=0$ exploited by Siegel,[2] by using the Lagrange-multiplier method and quantizing the system via Dirac's method.[3] In this Comment we point out that, although his model has some attractive points, it does not define a theory with positivity at the quantum level.

The Lagrangian for a scalar self-dual field introduced in Ref. 1 is

$$\mathcal{L}=\tfrac{1}{2}(\partial_-\phi)(\partial_+\phi)+\lambda\partial_-\phi\,, \tag{1}$$

where $\partial_\pm=\partial_0\pm\partial_1$ and λ is a Lagrange-multiplier field which enforces $\partial_-\phi=0$. Let us consider the path-integral quantization. The generating functional is given by

$$Z[J]=\int d\phi\,d\lambda\exp\left[i\int d^2x(\mathcal{L}+J\phi)\right]. \tag{2}$$

By integrating out the λ field, we have

$$\begin{aligned}Z[J]&=\int d\phi\,\delta(\partial_-\phi)\exp\left[i\int d^2x\,J\phi\right]\\&=\int d\phi\,d\pi\exp\left[i\int d^2x(\pi\partial_-\phi+J\phi)\right].\end{aligned} \tag{3}$$

The kinetic term vanishes due to the δ functional $\delta(\partial_-\phi)$. The new multiplier field π is the momentum conjugate to ϕ from the canonical theory point of view. We obtain the Hamiltonian

$$H=\int dx^1\,\pi\partial_1\phi\,, \tag{4}$$

which coincides with that of Ref. 1.

The path integral (3) reminds us of the following generating functional of a left-handed fermion:

$$Z'[\bar\eta]=\int d\psi\,d\chi\exp\left[i\int d^2x(\chi\partial_-\psi+\bar\eta\psi)\right], \tag{5}$$

where ψ and χ are independent Grassmannian fields. It is well known that if we quantize a fermion field by using commutation relations instead of anticommutation relations (in other words, if we use ordinary fields instead of Grassmannian fields in the path integral), the vacuum (or the Dirac sea) is not stable and energy of the system is not bounded from below. This is also seen from the Hamiltonian (4), which is not positive definite for *bosonic* ϕ and π.

Therefore the model of Ref. 1 does not have positivity.

It is interesting to note that if we impose the linear chiral constraint in phase space, i.e., $\pi(x)-\phi'(x)\approx 0$,[4] we get Floreanini-Jackiw[5] chiral bosons, the Hamiltonian of which is positive definite.

This work was supported by the Alexander von Humboldt Foundation.

Koji Harada
Institut für Theoretische Physik der Universität Heidelberg
Philosophenweg 16, D-6900 Heidelberg
Federal Republic of Germany

Received 12 February 1990
PACS numbers: 11.10.Ef, 11.10.Lm

[1]P. P. Srivastava, Phys. Rev. Lett. **63**, 2791 (1989).
[2]W. Siegel, Nucl. Phys. **B238**, 307 (1984).
[3]P. A. M. Dirac, *Lectures in Quantum Mechanics* (Yeshiva Univ. Press, New York, 1964).
[4]M. Bernstein and J. Sonnenschein, Phys. Rev. Lett. **60**, 1772 (1988).
[5]R. Floreanini and R. Jackiw, Phys. Rev. Lett. **59**, 1873 (1987).

