Nuclear Physics B343 (1990) 689–704
North-Holland

COVARIANT QUANTIZATION OF CHIRAL BOSONS AND OSP(1, 1|2) SYMMETRY

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Received 12 February 1990
(Revised 24 April 1990)

The covariant quantization of a chiral boson is carried out on an extended phase space. To convert the second-class constraint $\partial_+\phi(\sigma,\tau)=0$ into first-class ones, à la Faddeev and Shatashvili, infinitely many auxiliary scalars are introduced. We have explored the OSp(1, 1|2) symmetry, which is a group theoretical extension of the BRST symmetry, to prove the no-ghost theorem and the chirality of the physical subspace. The infinite product of determinants from the ghosts and auxiliary scalars in path integral are regularized in an OSp invariant way, leading to the desired partition function. The formulation holds good in the presence of static background gauge fields.

1. Introduction

World-sheet chiral bosons, which propagate only in one direction, are the building blocks of string theories [1]. They were first used in the construction of heterotic-type strings [2, 3] which are phenomenologically interesting because of the built-in left–right asymmetry.

Much attention has been attracted to the covariant quantization of chiral bosons [4]. The problem is challenging because of the difficulty related directly to the fact that the chiral constraint $\partial_+\phi(x)=0$ $(x=\sigma,\tau)$ is a second-class constraint. As is well known, a first-class constraint can be incorporated into the action by introducing a Lagrange multiplier $\lambda(x)$ so that the action acquires a gauge symmetry. The equation of motion of $\lambda(x)$ gives the constraint and $\lambda(x)$ itself can be gauged away. In such a formulation, covariant quantization can be carried out either in the BRST or path-integral formalism. For a second-class constraint, such a scheme does not work, since the action incorporating the constraint does not have the desired gauge symmetry which can be used to gauge away the Lagrange multiplier. A natural thought to overcome this difficulty is to replace the second-class constraint by a first-class one. A trick for chiral bosons was invented by Siegel [5], who squared $\partial_+\phi(x) = 0$ to obtain a first-class constraint $(\partial_+\phi(x))^2 = 0$. However, this squared constraint is (infinitely) reducible, as emphasized in ref. [6]. Therefore, the usual one-step gauge-fixing procedure does not completely fix the Siegel symmetry and leads to anomaly and other pathological features [7]. It seems that a proper BRST covariant quantization of Siegel's action needs infinitely many generations of ghosts. To cancel the Siegel anomaly one may introduce the so-called "notons" or "non-movers" [8], but it still remains to see whether this trick works in the presence of curved backgrounds.

In this paper, we will present an alternative covariant quantization for chiral bosons in both BRST and path-integral approaches, avoiding the use of Siegel's action. For the world-sheet scalar field $\phi_0(x)$ and its conjugate $\pi_0(x)$, we will replace the second-class constraint $\pi_0 + \phi_0' = 0$ ($\phi_0' = \partial_\sigma \phi_0$), in the spirit of ref. [9], with an infinite set of irreducible first-class constraints $\pi_{n-1} + \phi_{n-1}' - (\pi_n - \phi_n') = 0$ which involve infinitely many auxiliary scalar fields $\pi_n(x), \phi_n(x)$ $(n > 0)$. This procedure can not be truncated at finite n, otherwise to reproduce the original chirality the last constraint still has to be second class. The central issue is then how to deal with the infinite number of auxiliary scalars and the ghosts associated with the infinite set of constraints. In this paper we will explore an OSp(1, 1|2) symmetry in the extended phase space to address this issue.

The OSp(1, 1|2) symmetry is a group theoretical extension of the covariant BRST symmetry on the extended phase space. The BRST generator is one of the four nilpotent anticommuting generators of the OSp(1, 1|2) algebra. For general discussions of the relations between BRST and OSp algebras, see for example ref. [10]. The states in the quantum Hilbert space can be classified (or characterized) by their transformation properties (or representation contents) under the OSp group. This greatly simplifies the analysis of the physical subspace, and is particularly helpful for our formalism which involves an infinite number of auxiliary fields and ghosts. By exploring this symmetry it becomes a simple matter to prove the absence of all unwanted degrees of freedom (including ghosts, auxiliary fields and left-moving part of (ϕ_0, π_0)) in the physical subspace. Also it is possible to give an OSp invariant regularization for the infinite product of determinants from the ghosts and auxiliary fields to obtain the desired partition function for the chiral boson.

In sect. 2, we show that our infinite set of first-class constraints really lead to free chiral bosons at the classical level. In sect. 3, we carry out a covariant BRST quantization on the extended phase space. We also introduce the OSp(1,1|2) transformations for the variables in the extended phase space and give the construction of an OSp invariant effective action. The no-ghost theorem and the chirality of the physical subspace is proved by exploring the OSp symmetry in parallel to a previous analysis by two of us [6] for a non-covariant BRST quantization. In sect. 4, we present a path-integral analysis for the formulation established in sect. 3. We regularize the path integral in a way which respects a residual OSp(1,1|2) symmetry and show that the partition function is exactly what we expect for a chiral boson. In sect. 5, we generalize our formulation to chiral bosons in the presence of static background gauge fields. Sect. 6 is devoted to discussions and conclusions.

2. Classical chiral constraints

A free boson, with the action $S_0 = \int d^2x\, \partial_+\phi_0\partial_-\phi_0$ ($\partial_\pm = 1/\sqrt{2}(\partial_\tau \pm \partial_\sigma)$), becomes a chiral boson by imposing the second-class constraint $\pi_0 + \phi_0' = 0$ with $\pi_0 = \dot{\phi}_0$ ($\dot{\phi}_0 = \partial_\tau\phi_0$). A la Faddeev and Shatashvili [9], we turn this constraint into a first-class one, $\pi_0 + \phi_0' - (\pi_1 - \phi_1') = 0$, by introducing new variables π_1 and ϕ_1 which are conjugate to each other. To recover the original chirality one may put $\pi_1 - \phi_1' = 0$. But this is of second class, which we wanted to avoid. So again we put $\pi_1 + \phi_1' - (\pi_2 - \phi_2') = 0$ with new conjugate variables π_2 and ϕ_2. We have to go on forever and can not stop after a finite number of steps, because otherwise the last constraint would still be second class. In this way we introduce infinitely many irreducible first-class constraints

$$T_m = \pi_{m-1} + \phi_{m-1}' - (\pi_m - \phi_m') = 0 \qquad (m = 1,2,3,\dots)\,. \tag{2.1}$$

The action for the ϕ_n's is

$$S = \int d^2x \sum_{n=0}^{\infty} (-1)^n \partial_+\phi_n\partial_-\phi_n\,, \tag{2.2}$$

which implies

$$\pi_n = (-1)^n \dot{\phi}_n \tag{2.3}$$

and the hamiltonian is

$$H = \tfrac{1}{2}\int d^2x \sum_{n=0}^{\infty} \left(\pi_n^2 + \phi_n'^2\right). \tag{2.4}$$

With the Poisson bracket

$$[\pi_n(x), \phi_l(x')] = -\delta_{nl}\delta^2(x-x'), \tag{2.5}$$

one can easily verify that T_m are really first-class constraints:

$$[T_m(x), T_k(x')] = 0,$$
$$[T_m(x), H] = (-1)^{m+1}T_m'(x). \tag{2.6}$$

To put the theory in a covariant form, we incorporate (2.1) into the action through Lagrange multipliers $\lambda_n(x)$ and obtain

$$S_\lambda = \int d^2x \sum_{n=0}^{\infty} \pi_n\dot{\phi}_n - H + \int d^2x \sum_{n=1}^{\infty} \lambda_n T_n. \tag{2.7}$$

It is invariant under the following gauge transformations:

$$\delta\phi_n = -\epsilon_{n+1} + \epsilon_n,$$
$$\delta\pi_n = -\epsilon_{n+1}' - \epsilon_n',$$
$$\delta\lambda_n = \dot{\epsilon}_n + (-1)^n\epsilon_n', \tag{2.8}$$

where $\epsilon_n(x)$ are infinitesimal gauge parameters with $\epsilon_0 = 0$. So S_λ has infinitely many (abelian) gauge symmetries. The equations of motion of the Lagrange multipliers λ_n give the constraints $T_n = 0$, while the λ_n themselves can be gauged away.

Our aim is to get a chiral boson, i.e. only the right-moving part of (π_0, ϕ_0). But the constraints $T_n = 0$ do not manifestly lead to this result. We show in the following that at the classical level, eq. (2.7) really describes a chiral boson after fixing all the gauge symmetries.

Let us first note that normally a covariant gauge choice always has some residual symmetries. For the gauge invariant action (2.7), setting $\lambda_n = 0$ does not fix all the gauge symmetries (2.8). The residual gauge transformations are

$$\delta\phi_{2m} = -\epsilon_{2m+1}(\sigma^+) + \epsilon_{2m}(\sigma^-),$$
$$\delta\phi_{2m+1} = -\epsilon_{2m+2}(\sigma^-) + \epsilon_{2m+1}(\sigma^+),$$
$$\delta\pi_{2m} = -\epsilon_{2m+1}'(\sigma^+) - \epsilon_{2m}'(\sigma^-),$$
$$\delta\pi_{2m+1} = -\epsilon_{2m+2}'(\sigma^-) - \epsilon_{2m+1}'(\sigma^+),$$
$$\delta\lambda_n = 0, \tag{2.9}$$

where $\sigma^{\pm} = 1/\sqrt{2}(\tau \pm \sigma)$. The gauge-fixing conditions for (2.9) can be taken to be

$$s_m = \pi_m - \phi'_m = 0 \qquad (m = 1, 2, 3, \dots). \tag{2.10}$$

Then all the gauge degrees of freedom in (2.7) are exhausted. Combining eqs. (2.1), (2.10) and $\lambda_m = 0$ together, we conclude that the only physical degree of freedom possessed by (2.7) is the right-moving part of (π_0, ϕ_0).

3. BRST quantization and OSp(1, 1|2) symmetry

We have already had in our phase space the world-sheet scalars ϕ_n, their canonical momenta π_n and the Lagrange multipliers λ_m. To carry out the BRST quantization, we need to extend this phase space. Treating λ_m as coordinates, we associate to them an equal number of momenta ρ_m whose Poisson bracket with λ_m are

$$[\rho_m(x), \lambda_k(x')] = -\delta_{mk}\delta^2(x - x'). \tag{3.1}$$

Corresponding to the bosonic pairs (ρ_m, λ_m) and those degrees of freedom that should be constrained in (π_n, ϕ_n), we introduce conjugate pairs of ghosts and antighosts $(\bar{b}_m, c_m)$ and $(\bar{c}_m, b_m)$ $(m > 0)$ which obey fermionic Poisson brackets (we will not indicate the Grassmann parity of them when there is no confusion)

$$[\bar{b}_m(x), c_k(x')] = -\delta_{mk}\delta^2(x - x'),$$
$$[\bar{c}_m(x), b_k(x')] = -\delta_{mk}\delta^2(x - x'). \tag{3.2}$$

The BRST operator is given by

$$\Omega = \int d^2x \sum_{n=1}^{\infty} (c_n T_n - b_n \rho_n), \tag{3.3}$$

which is nilpotent ($\Omega^2 = 0$). Under the BRST operation,

$$\begin{aligned} &[\Omega, \phi_n] = -c_{n+1} + c_n, && [\Omega, \pi_n] = -c'_{n+1} - c'_n, \\ &[\Omega, \lambda_n] = b_n, && [\Omega, \rho_n] = 0, \\ &[\Omega, \bar{b}_n] = -T_n, && [\Omega, c_n] = 0, \\ &[\Omega, \bar{c}_n] = \rho_n, && [\Omega, b_n] = 0. \end{aligned} \tag{3.4}$$

The general BRST invariant hamiltonian is then given by

$$H_{\text{eff}} = H + \int d^2x \sum_{n=1}^{\infty} (-1)^n \bar{b}_n c_n - [\Omega, \Psi], \tag{3.5}$$

where Ψ is an arbitrary fermionic gauge-fixing function in the extended phase space. The physical subspace satisfying

$$\Omega|\text{phys}\rangle = 0 \tag{3.6}$$

is independent of the choice of Ψ. The effective action on the extended phase space (setting $\rho_0 = \lambda_0 = c_0 = \bar{b}_0 = \bar{c}_0 = b_0 = 0$)

$$S_{\text{eff}} = \int d^2x \sum_{n=0}^{\infty} \left(\pi_n \dot{\phi}_n + \rho_n \dot{\lambda}_n + \dot{c}_n \bar{b}_n + \dot{b}_n \bar{c}_n\right) - H_{\text{eff}} \tag{3.7}$$

is also BRST invariant.

Although this invariance holds for any Ψ, S_{eff} is not automatically OSp(1,1|2) invariant for a generic Ψ. However, we can find a particular Ψ such that S_{eff} (also H_{eff}) becomes OSp(1,1|2) invariant.

At this point, let us recall some basic facts about OSp(1,1|2). First write down the following multiplets:

$$\begin{aligned} P_{An} &= \left(T_n, -\lambda_n, b_n, \bar{b}_n\right), \\ Q_n^A &= \left(S_n, \rho_n, \bar{c}_n, c_n\right), \end{aligned} \tag{3.8}$$

with $A = (+, -, \theta, \bar{\theta})$. Here S_n in eq. (3.8) is determined by the Poisson bracket requirement

$$\left[P_{An}(x), Q_m^B(x')\right] = -\delta_A^B \delta_{nm} \delta^2(x - x') \tag{3.9}$$

as

$$S_n(x) = \tfrac{1}{2}\left[\int^{\sigma} \pi_n(\sigma', \tau)\, d\sigma' - \phi_n(x)\right]. \tag{3.10}$$

The multiplets Q_n^A and P_{An} transform as covariant vectors and countravariant vectors under the graded Lie group OSp(1,1|2) generated by

$$M^{AB} = \int d^2x \sum_{n=1}^{\infty} \left[Q_n^A P_n^B - (-)^{ab} Q_n^B P_n^A\right], \tag{3.11}$$

where the indices on P_n have been raised by the OSp invariant metric,

$$\eta^{AB}=\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&0&-1\\0&0&1&0\end{pmatrix},$$

and a,b denote the Grassmann parity of A,B. The quadratic combination $X^AX^B\eta_{BA}$ of a vector X^A is an OSp(1, 1|2) scalar. The OSp(1, 1|2) group has eight generators. The BRST operator Ω given by eq. (3.3) is one of the four nilpotent generators:

$$\Omega=M^{\bar{\theta}-}. \tag{3.12}$$

One of the others is the anti-BRST operator:

$$\bar{\Omega}=M^{\theta-}=\int d^2x\sum_{n=1}^{\infty}\left(\bar{c}_nT_n+\rho_n\bar{b}_n\right). \tag{3.13}$$

The remaining two are the conjugate of Ω and $\bar{\Omega}$ (with $A=-\to A=+$):

$$M^{\bar{\theta}+}=\int d^2x\sum_{n=1}^{\infty}\left(-c_n\lambda_n-S_nb_n\right),$$

$$M^{\theta+}=\int d^2x\sum_{n=1}^{\infty}\left(-\bar{c}_n\lambda_n+S_n\bar{b}_n\right). \tag{3.14}$$

The generators

$$M^{\theta\theta}=-2\int d^2x\sum_{n=1}^{\infty}\bar{c}_n\bar{b}_n,$$

$$M^{\bar{\theta}\bar{\theta}}=2\int d^2x\sum_{n=1}^{\infty}c_nb_n, \tag{3.15}$$

interchange Ω with $\bar{\Omega}$, $M^{\bar{\theta}+}$ with $M^{\theta+}$ and vice versa. Since this group has rank 2, in the representation realized by the multiplets (3.8), the two simultaneously diagonalized generators are the ghost number

$$N=M^{\bar{\theta}\theta}=\int d^2x\sum_{n=1}^{\infty}\left(-c_n\bar{b}_n+\bar{c}_nb_n\right), \tag{3.16}$$

and the W-number (nonzero for $A = +, -$ components)

$$W = M^{-+} = -\int d^2x \sum_{n=1}^{\infty} (S_n T_n + \rho_n \lambda_n). \tag{3.17}$$

Now we choose

$$\Psi = \int d^2x \sum_{n=1}^{\infty} (-1)^n \bar{c}_n \lambda'_n. \tag{3.18}$$

The effective action becomes

$$S_{\text{eff}} = \int d^2x \sum_{n=0}^{\infty} \left[\pi_n \dot{\phi}_n + \rho_n \dot{\lambda}_n + \dot{c}_n \bar{b}_n + \dot{b}_n \bar{c}_n - \tfrac{1}{2}(-1)^n \left(\pi_n^2 + \phi_n'^2\right) \right.$$
$$\left. + (-1)^n \left(c'_n \bar{b}_n + b'_n \bar{c}_n + \rho_n \lambda'_n\right) \right]. \tag{3.19}$$

It is straightforward to check that this S_{eff} (also H_{eff}) commutes with all the generators of OSp(1, 1|2). So we have achieved an OSp invariant action (3.19).

Now we are ready to show, by exploring the OSp(1, 1|2) symmetry, that the non-trivial physical states satisfying eq. (3.6) and $|\text{phys}\rangle \neq \Omega|\text{something}\rangle$ are just the right-moving chiral bosons of the field (ϕ_0, π_0). Upon quantization, we replace all Poisson brackets [,] by commutators or anticommutators i[,]. All the field and ghost operators are grouped into the OSp vectors P_n^A, Q_n^A and the OSp scalar $\pi_0 - \phi'_0$. Thus all the states can be classified in terms of their OSp representation contents. They are labeled by the ghost number n and the W-number w. Each representation is labeled by M, the highest value of n. Note that

$$[\Omega, N] = -\Omega, \qquad [\Omega, W] = \Omega, \tag{3.20}$$

which imply that the BRST operator increases the ghost number by one and decreases the W-number by one. Therefore the states $|\chi\rangle = |n, w, M\rangle$ with either the highest n value or the lowest w value are annihilated by Ω, i.e. they are physical states. But all these states except for those with $M = 0$ are of the form $\Omega|\zeta\rangle$ with $|\zeta\rangle = |n-1, w+1, M\rangle$. So they are trivial. The only non-trivial physical states are OSp scalars. They belong to $M = 0$ representations. Further, all the states are contained in the tensor products of the OSp representations. An arbitrary tensor product of two representations M_1 and M_2 contains each representation M between $|M_1 - M_2|$ and $M_1 + M_2$ once and only once. Thus only $|M\rangle \otimes |M\rangle$ contains a scalar $|\psi\rangle$. $G_{ij} = \langle i, M|j, M\rangle$ gives an invariant and graded symmetric metric on $|M\rangle$, where i stands for the pair (n, w). $|\psi\rangle$ is equal to $G^{ij}|j, M\rangle \otimes |i, M\rangle$ with G^{ij} being the inverse of G_{ij} defined by $(-)^k G_{ik} G^{kj} = \delta_i^j$ ($(-)^k = +1$ (-1) for Bose (Fermi) states). A state $\langle\varphi|$ which has nonzero inner product with $|\psi\rangle$ must be of the form $\langle M| \otimes \langle M|$ and $\langle\psi|$ is the only non-trivial physical state of this form. But by applying the fact that the supertrace of OSp(1,1|2) unity vanishes for $M \neq 0$, $|\psi\rangle$ has zero norm unless it is of the form $|0\rangle \otimes |0\rangle = |\pi_0 - \phi_0'\rangle|\pi_0 - \phi_0'\rangle$. It follows by induction that non-trivial physical N-particle states are of the form $|\pi_0 - \phi_0'\rangle_1|\pi_0 - \phi_0'\rangle_2 \dots |\pi_0 - \phi_0'\rangle_N$ [6]. They are the desired right-moving chiral bosons.

4. Partition function and OSp(1,1|2) symmetry

We turn to the path-integral analysis of a chiral boson with the action S_{eff}. We expect that the right-moving part and the left-moving part of the boson decouple, and the contributions from the left-moving part and other non-physical parts get completely cancelled. The partition function is

$$Z = \int \mathrm{D}\phi\, \mathrm{D}\pi\, \mathrm{D}\lambda\, \mathrm{D}\rho\, \mathrm{D}c\, \mathrm{D}\bar{b}\, \mathrm{D}b\, \mathrm{D}\bar{c} \exp(iS_{\text{eff}})\,. \tag{4.1}$$

Since the OSp(1,1|2) transformation is canonical, it does not change the measure $\mathrm{D}\phi\, \mathrm{D}\pi\, \mathrm{D}\lambda\, \mathrm{D}\rho\, \mathrm{D}c\, \mathrm{D}\bar{b}\, \mathrm{D}b\, \mathrm{D}\bar{c}$ where all the subscripts and superscripts are suppressed. Therefore, the partition function (4.1) is OSp invariant.

To evaluate it, we first integrate over π_n. It contributes to (4.1) a factor $\exp[\frac{1}{2}i(-1)^n\dot{\phi}_n^2]$. Then integrate over the pairs (λ_n, ρ_n), $(c_n, \bar{b}_n)$ (or equivalently $(\lambda_n, \rho_n), (b_n, \bar{c}_n)$). The contributions cancel with each other. Eq. (4.1) now becomes

$$Z = \int \mathrm{D}\phi\, \mathrm{D}b\, \mathrm{D}\bar{c} \exp\big(i\tilde{S}_{\text{eff}}\big)\,,$$

$$\tilde{S}_{\text{eff}} = \int \mathrm{d}^2x \sum_{n=0}^{\infty} \left[\tfrac{1}{2}(-1)^n\big(\dot{\phi}_n^2 - \phi_n'^2\big) + \dot{b}_n\bar{c}_n + (-1)^n b_n'\bar{c}_n\right], \tag{4.2}$$

or explicitly,

$$\begin{aligned}\tilde{S}_{\text{eff}} = \int \mathrm{d}^2x\big(&\partial_+\phi_0\partial_-\phi_0 - \sqrt{2}\,\bar{c}_1\partial_-b_1 - \partial_+\phi_1\partial_-\phi_1 - \sqrt{2}\,\bar{c}_2\partial_+b_2 \\ &+\partial_+\phi_2\partial_-\phi_2 - \sqrt{2}\,\bar{c}_3\partial_-b_3 - \partial_+\phi_3\partial_-\phi_3 + \dots\big)\,.\end{aligned} \tag{4.3}$$

The path integral over ϕ_n contributes $\mathrm{Det}^{-1/2}\partial_+\partial_- = \mathrm{Det}^{-1/2}\partial_+ \cdot \mathrm{Det}^{-1/2}\partial_-$; the integral over $(\bar{c}_{2m}, b_{2m})$ contributes $\mathrm{Det}\,\partial_+$ and that over $(\bar{c}_{2m+1}, b_{2m+1})$ contributes $\mathrm{Det}\,\partial_-$. So the evaluation of (4.2) involves an infinite product of determinants. It needs a regularization. Here we propose to use an OSp invariant regularization.

We have emphasized the OSp symmetry of S_{eff} before. So, not surprisingly, after integrating over π_n, λ_n, ρ_n, c_n and $\bar{b}_n$, $\tilde{S}_{\mathrm{eff}}$ still has a kind of residual OSp symmetry. We will regularize the path integral in a way which respects this symmetry. The generators of the residual OSp(1, 1|2) group are

$$N=\int \mathrm{d}^2x \sum_{n=1}^{\infty} \bar{c}_n b_n ,$$

$$\bar{\Omega}=\int \mathrm{d}^2x \sum_{n=1}^{\infty} \bar{c}_n T_n ,$$

$$M^{\bar{\theta}+}=-\int \mathrm{d}^2x \sum_{n=1}^{\infty} S_n b_n ,$$

$$W=-\int \mathrm{d}^2x \sum_{n=1}^{\infty} S_n T_n . \tag{4.4}$$

The parameters associated with other four generators of the OSp(1, 1|2) group are set to zero. To show the residual OSp transformation laws, let ϕ_n formally consist of its right-moving part R_n and left-moving part L_n. (In the path-integral approach, in which ϕ_n is off shell, we do not have an explicit decomposition for R_n and L_n. But we may formally define that the path integral over R_n contributes $\mathrm{Det}^{-1/2}\partial_+$ to the partition function and that path integral over L_n contributes $\mathrm{Det}^{-1/2}\partial_-$; this gives a sense to the right-moving part R_n and left-moving part L_n in the path integral.) We define the OSp transformation laws for R_n and L_n so that $[\ ,\phi_n]=[\ ,R_n]+[\ ,L_n]$. With such a definition, it makes sense to discuss the OSp property of $\tilde{S}_{\mathrm{eff}}$ in terms of the OSp properties of R_n and L_n. By definition, the right-moving part of ϕ_0 is physical, so we let R_0 transform as an OSp scalar, in agreement with the rigorous analysis in the last section. Under $\bar{\Omega}$ and $M^{\bar{\theta}+}$, L_0 transforms to b_1 and $\bar{c}_1$. They are left-moving in the sense that the path integral over each of them all contributes $\mathrm{Det}\,\partial_-$. We choose the variation of L_1 to compensate the ones of L_0, b_1 and $\bar{c}_1$ in $\tilde{S}_{\mathrm{eff}}$ such that L_0, b_1, $\bar{c}_1$ and L_1 form a left-multiplet. And so on. Thus all the variables in $\tilde{S}_{\mathrm{eff}}$ are grouped into the following multiplets characterized by k:

$$\begin{array}{lcccccccccccc}
 & \phi_0 & \bar{c}_1 & b_1 & \phi_1 & \bar{c}_2 & b_2 & \phi_2 & \bar{c}_3 & b_3 & \phi_3 & \dots & \dots \\
k=0 & R_0 & & & & & & & & & & & \\
k=1 & L_0 & \bar{c}_1 & b_1 & L_1 & & & & & & & & \\
k=2 & & & & R_1 & \bar{c}_2 & b_2 & R_2 & & & & & \\
k=3 & & & & & & & L_2 & \bar{c}_3 & b_3 & L_3 & & \\
\dots & \dots & \dots & & & & & & & & & &
\end{array}$$

In summary, by definition under the residual OSp group we have consistently

$$
\begin{aligned}
&[\bar{\Omega}, R_{2m}] = \bar{c}_{2m}, && [\bar{\Omega}, L_{2m}] = -\bar{c}_{2m+1}, \\
&[\bar{\Omega}, R_{2m+1}] = -\bar{c}_{2m+2}, && [\bar{\Omega}, L_{2m+1}] = \bar{c}_{2m+1}, \\
&[\bar{\Omega}, \bar{c}_n] = 0, && [\bar{\Omega}, b_n] = -T_n,
\end{aligned}
\tag{4.5}
$$

$$
\begin{aligned}
&[N, R_n] = 0, && [N, L_n] = 0, \\
&[N, \bar{c}_n] = -\bar{c}_n, && [N, b_n] = b_n,
\end{aligned}
\tag{4.6}
$$

$$
\begin{aligned}
&[M^{\bar{\theta}+}, L_{2m}] = 0, && [M^{\bar{\theta}+}, R_{2m}] = \tfrac{1}{2}\int_\sigma b_{2m}(\sigma', \tau)\,\mathrm{d}\sigma', \\
&[M^{\bar{\theta}+}, R_{2m+1}] = 0, && [M^{\bar{\theta}+}, L_{2m+1}] = \tfrac{1}{2}\int_\sigma b_{2m+1}(\sigma', \tau)\,\mathrm{d}\sigma', \\
&[M^{\bar{\theta}+}, \bar{c}_n] = S_n, && [M^{\bar{\theta}+}, b_n] = 0,
\end{aligned}
\tag{4.7}
$$

$$
\begin{aligned}
&[W, L_{2m}] = S_{2m+1}, && [W, R_{2m}] = -S_{2m} + \tfrac{1}{2}\int_\sigma T_{2m}(\sigma', \tau)\,\mathrm{d}\sigma', \\
&[W, R_{2m+1}] = S_{2m+2}, && [W, L_{2m+1}] = -S_{2m+1} + \tfrac{1}{2}\int_\sigma T_{2m+1}(\sigma', \tau)\,\mathrm{d}\sigma', \\
&[W, \bar{c}_n] = 0, && [W, b_n] = 0.
\end{aligned}
\tag{4.8}
$$

It is easy to check that the part of $\tilde{S}_{\mathrm{eff}}$ involving each multiplet labeled by k possesses the residual OSp symmetry. Say, for $k = 2m + 1$,

$$
\delta^L_{\mathrm{OSp}}\left[\int \mathrm{d}^2x\left(\partial_+\phi_{2m}\partial_-\phi_{2m} - \sqrt{2}\,\bar{c}_{2m+1}\partial_- b_{2m+1} - \partial_+\phi_{2m+1}\partial_-\phi_{2m+1}\right)\right] = 0, \tag{4.9}
$$

where only L_{2m}, L_{2m+1} in ϕ_{2m}, ϕ_{2m+1} vary; for $k = 2m$,

$$
\delta^R_{\mathrm{OSp}}\left[\int \mathrm{d}^2x\left(-\partial_+\phi_{2m-1}\partial_-\phi_{2m-1} - \sqrt{2}\,\bar{c}_{2m}\partial_+ b_{2m} + \partial_+\phi_{2m}\partial_-\phi_{2m}\right)\right] = 0, \tag{4.10}
$$

where only R_{2m-1}, R_{2m} in ϕ_{2m-1}, ϕ_{2m} vary. The sum of the variations for one and the same term $(\delta^L + \delta^R)_{\mathrm{OSp}}(\int \mathrm{d}^2x\,\partial_+\phi_{2m}\partial_-\phi_{2m})$ in eqs. (4.9) and (4.10) is equal to the usual OSp variation of the ϕ_{2m} term. This is sufficient to prove the OSp invariance of $\tilde{S}_{\mathrm{eff}}$.

To regularize the path integral (4.2) in an OSp invariant way, we group the path integrals over each OSp multiplet together. Eq. (4.2) then turns out to be

$$
\begin{aligned}
Z &= \left(\mathrm{Det}^{-1/2}\partial_+\right)\left(\mathrm{Det}^{-1/2}\partial_-\cdot\mathrm{Det}\,\partial_-\cdot\mathrm{Det}^{-1/2}\partial_-\right) \\
&\quad\times\left(\mathrm{Det}^{-1/2}\partial_+\cdot\mathrm{Det}\,\partial_+\cdot\mathrm{Det}^{-1/2}\partial_+\right)\left(\mathrm{Det}^{-1/2}\partial_-\cdot\mathrm{Det}\,\partial_-\cdot\mathrm{Det}^{-1/2}\partial_-\right)\dots \\
&= \mathrm{Det}^{-1/2}\partial_+\,,
\end{aligned}
\tag{4.11}
$$

as desired. This formulation avoids a non-local determinant in the path-integral measure which has appeared in ref. [11].

5. Chiral bosons in background gauge fields

We now generalize our formalism in earlier sections to the case that a chiral boson couples to a background gauge field A_+ and A_-. We expect to be able to isolate out the contribution from the right-moving chiral boson with a coupling to A_+.

In the presence of the background (abelian) gauge field, the action for ϕ_n is

$$
S^{(A)} = \int \mathrm{d}^2x \sum_{n=0}^{\infty}\left[(-1)^n\partial_+\phi_n\partial_-\phi_n + 2iA_+\partial_-\phi_n - 2iA_-\partial_+\phi_n\right], \tag{5.1}
$$

which implies

$$
\pi_n = (-1)^n\dot{\phi}_n + \sqrt{2}\,i(A_+ - A_-)\,, \tag{5.2}
$$

and the corresponding hamiltonian is

$$
\begin{aligned}
H^{(A)} = \int \mathrm{d}^2x \sum_{n=0}^{\infty}(-1)^n\Big[&\tfrac{1}{2}\left(\pi_n - \sqrt{2}(A_+ - A_-)\right)^2 \\
&+ \tfrac{1}{2}\left(\phi_n' + (-1)^n\sqrt{2}\,i(A_+ + A_-)\right)^2 + (A_+ + A_-)^2\Big].
\end{aligned}
\tag{5.3}
$$

The first-class constraints $T_m^{(A)}$ remain the same as (2.1). This system is covariantly described on the extended phase space by the effective action

$$
\begin{aligned}
S_{\mathrm{eff}}^{(A)} = \int \mathrm{d}^2x \sum_{n=0}^{\infty}\Big[&\left(\pi_n\dot{\phi}_n + \rho_n\dot{\lambda}_n + \dot{c}_n\bar{b}_n + \dot{b}_n\bar{c}_n\right) \\
&+ (-1)^n\left(c_n'\bar{b}_n + b_n'\bar{c}_n + \rho_n\lambda_n'\right)\Big] - H^{(A)}.
\end{aligned}
\tag{5.4}
$$

To maintain the OSp(1, 1|2) symmetry for (5.4), we modify eq. (3.10) to

$$S_{2m}(x) = \tfrac{1}{2}\int^{\sigma} \pi_{2m}(\sigma', \tau)\, \mathrm{d}\sigma' - \tfrac{1}{2}\phi_{2m}(x) - \sqrt{2}\, i\int^{\sigma} A_{+}(\sigma', \tau)\, \mathrm{d}\sigma',$$

$$S_{2m+1}(x) = \tfrac{1}{2}\int^{\sigma} \pi_{2m+1}(\sigma', \tau)\, \mathrm{d}\sigma' - \tfrac{1}{2}\phi_{2m+1}(x) + \sqrt{2}\, i\int^{\sigma} A_{-}(\sigma', \tau)\, \mathrm{d}\sigma'. \quad (5.5)$$

With the same OSp analysis as in sect. 3, we conclude that the physical subspace consists of OSp scalars, of the form

$$|\pi_0 - \phi_0' - 2\sqrt{2}\, iA_{+}\rangle_1 |\pi_0 - \phi_0' - 2\sqrt{2}\, iA_{+}\rangle_2 \dots |\pi_0 - \phi_0' - 2\sqrt{2}\, iA_{+}\rangle_N .$$

We remark here that on the physical subspace the consistency of $T_n = S_n = 0$ with the equations of motion $\partial_{+}\partial_{-}\phi_n + (-1)^n i(\partial_{-}A_{+} - \partial_{+}A_{-}) = 0$ requires $\dot{A}_{+} = \dot{A}_{-} = 0$. We have used this condition in the proof of the OSp invariance of (5.4).

The partition function in the presence of the background gauge field is

$$Z^{(A)} = \int \mathrm{D}\phi\, \mathrm{D}\pi\, \mathrm{D}\lambda\, \mathrm{D}\rho\, \mathrm{D}c\, \mathrm{D}\bar{b}\, \mathrm{D}b\, \mathrm{D}\bar{c}\, \exp\left(iS_{\text{eff}}^{(A)}\right). \quad (5.6)$$

The integration over π_n contributes to (5.6) a factor $\exp[\frac{1}{2}i(-1)^n\dot{\phi}_n^2 - \sqrt{2}(A_{+} - A_{-})\dot{\phi}_n]$. The integrations over the pairs (λ_n, ρ_n) and $(c_n, \bar{b}_n)$ cancel with each other as before. The explicit effective action then reads

$$\tilde{S}_{\text{eff}}^{(A)} = \int \mathrm{d}^2x \big[\partial_{+}\phi_0\partial_{-}\phi_0 + 2i(A_{+}\partial_{-}\phi_0 - A_{-}\partial_{+}\phi_0) - \sqrt{2}\,\bar{c}_1\partial_{-}b_1$$

$$-\partial_{+}\phi_1\partial_{-}\phi_1 + 2i(A_{+}\partial_{-}\phi_1 - A_{-}\partial_{+}\phi_1) - \sqrt{2}\,\bar{c}_2\partial_{+}b_2 + \dots\big], \quad (5.7)$$

with

$$Z^{(A)} = \int \mathrm{D}\phi\, \mathrm{D}b\, \mathrm{D}\bar{c}\, \exp\left(i\tilde{S}_{\text{eff}}^{(A)}\right). \quad (5.8)$$

The integration over ϕ_n contributes

$$\mathrm{Det}^{-1/2}\partial_{+}\cdot \mathrm{Det}^{-1/2}\partial_{-}\cdot \exp\left[(-1)^n i\int \mathrm{d}^2x\left(A_{+}\frac{\partial_{-}}{\partial_{+}}A_{+} + A_{-}\frac{\partial_{+}}{\partial_{-}}A_{-} - 2A_{+}A_{-}\right)\right]$$

to eq. (5.8) while the integrations over $(\bar{c}_{2m}, b_{2m})$ and $(\bar{c}_{2m+1}, b_{2m+1})$ contribute $\mathrm{Det}\,\partial_{+}$ and $\mathrm{Det}\,\partial_{-}$ respectively. Again, we need a regularization for the path integrals to deal with the infinite product of determinants. The residual OSp analysis in sect. 4 is still applicable here. That is, for the multiplet $L_{2m}, \bar{c}_{2m+1}, b_{2m+1}$ and L_{2m+1},

$$\delta^L_{\mathrm{Osp}}\left[\int d^2x\big(\partial_+\phi_{2m}\partial_-\phi_{2m} + 2i(A_+\partial_-\phi_{2m} - A_-\partial_+\phi_{2m}) - \sqrt{2}\,\bar{c}_{2m+1}\partial_-b_{2m+1}\right.$$
$$\left. -\partial_+\phi_{2m+1}\partial_-\phi_{2m+1} + 2i(A_+\partial_-\phi_{2m+1} - A_-\partial_+\phi_{2m+1})\big)\right] = 0, \quad (5.9)$$

and for the multiplet $R_{2m-1}, \bar{c}_{2m}, b_{2m}$ and R_{2m},

$$\delta^R_{\mathrm{OSp}}\left[\int d^2x\big(-\partial_+\phi_{2m-1}\partial_-\phi_{2m-1} + 2i(A_{+-}\phi_{2m-1} - A_-\partial_+\phi_{2m-1})\right.$$
$$\left. -\sqrt{2}\,\bar{c}_{2m}\partial_+b_{2m} + \partial_+\phi_{2m}\partial_-\phi_{2m} + 2i(A_+\partial_-\phi_{2m} - A_-\partial_+\phi_{2m})\big)\right] = 0, \quad (5.10)$$

where again we have used the condition $\dot{A}_+ = \dot{A}_- = 0$ which we obtained previously for consistency. So we can regularize (5.8) by grouping the contributions from members of each OSp multiplet together. Neglecting the local counterterm $2A_+A_-$, eq. (5.8) becomes

$$\begin{aligned}
Z^{(A)} = &\left[\mathrm{Det}^{-1/2}\partial_+ \exp\left(i\int d^2x A_+(\partial_-/\partial_+)A_+\right)\right]\\
&\times\left[\mathrm{Det}^{-1/2}\partial_- \exp\left(i\int d^2x A_-(\partial_+/\partial_-)A_-\right)\right.\\
&\left.\times \mathrm{Det}\,\partial_-\ \mathrm{Det}^{-1/2}\partial_- \exp\left(-i\int d^2x A_-(\partial_+/\partial_-)A_-\right)\right]\\
&\times\left[\mathrm{Det}^{-1/2}\partial_+ \exp\left(-i\int d^2x A_+(\partial_-/\partial_+)A_+\right)\right.\\
&\left.\cdot \mathrm{Det}\,\partial_+\ \mathrm{Det}^{-1/2}\partial_+ \exp\left(i\int d^2x A_+(\partial_-/\partial_+)A_+\right)\right]\\
&\times \dots\\
= &\ \mathrm{Det}^{-1/2}\partial_+ \exp\left(i\int d^2x A_+(\partial_-/\partial_+)A_+\right),
\end{aligned} \quad (5.11)$$

as desired.

Similarly, in the presence of background gravity, we have the desired conformal anomaly from the right-moving boson only; the contributions from the left-moving part and other non-physical parts are completely cancelled too. The details are left to another publication [12].

6. Discussions and conclusions

We have carried out a covariant BRST quantization for a single chiral boson and the corresponding path-integral analysis. The treatment can be directly extended to many chiral bosons.

One important feature of our formulation is the introduction of infinitely many auxiliary scalars and ghosts. The appearance of infinitely many auxiliary fields is the price paid for converting the second-class chiral constraint into irreducible first-class constraints, and one needs an infinite set of ghosts because the irreducible first-class constraints are infinite in number.

Another feature is the use of the $OSp(1,1|2)$ symmetry, which is essential in our formulation for dealing with the infinite number of ghosts and auxiliary fields. From the algebraic point of view, the OSp algebra incorporates both the BRST and anti-BRST operators and their conjugates. From the representation-theory point of view, it gives a more complete characterization of the states in the BRST quantization. Its powerfulness lies in its ability to greatly simplify the proof of the no-ghost theorem and to provide a simple characterization of non-trivial physical states in terms of representation-theory analysis. These advantages turn out to be crucial in our formalism, which involves an infinite set of ghosts and auxiliary fields. On the one hand we have explored the OSp symmetry to show that the physical subspace contains only chiral bosons but no any unwanted degrees of freedom. On the other hand we have proposed an OSp invariant regularization for the infinite product of determinants from ghosts and auxiliary scalars in path integral, which immediately results in the desired partition function for a chiral boson.

An immediate interesting application of our present work is, of course, to string theories, especially to the covariant quantization of the bosonic formulation of heterotic strings. However, that needs to incorporate the gravity backgrounds and supersymmetry and to show the cancellation of conformal or superconformal anomaly. It is too long to discuss all these in this paper, so it is left to another publication [12]. It is also hoped that our formulation would be helpful in formulating a covariant string field theory for heterotic-type strings.

Feng Yu thanks the School of Natural Sciences, the Institute of Advanced Study for its warm hospitality. This work was supported in part by U.S. NSF grant PHY-8706501 and by Monell Foundation through the Institute for Advanced Study.

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