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TOPICAL REVIEW

Finite-temperature field theory†

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1. Introduction (PVL)

1.1. Statistical mechanics

This introductory section begins with a review of the essential ingredients needed from statistical mechanics. A rather fuller review of the necessary statistical mechanics may be found in the book by Fetter and Walecka [1], which also gives a very full account of non-relativistic finite-temperature field theory. I recommend also the book by Mills [2], whose straightforward account of the finite-temperature formalism is readily generalised to the relativistic case. For a book on the relativistic formalism one must wait for the one by Kapusta, soon to be published by Cambridge University Press. This book will concentrate on the imaginary-time formalism; the best account of the real-time approach is that of Niemi and Semenoff [3].

In relativistic theories particle number is not conserved (although both lepton and baryon number are). Therefore when discussing the thermodynamics of a quantum field theory one uses the grand canonical formalism: the entropy S is maximised, keeping fixed the ensemble averages E and N of energy and lepton or baryon number. To implement these constraints two Lagrange multipliers are introduced, $\beta = 1/kT$ and μ the chemical potential.

The thermal average over the ensemble is

$$\langle Q \rangle = Z^{-1}\,\mathrm{Tr}(Q\,\mathrm{e}^{-\beta(H-\mu N)}) \tag{1.1}$$

where Z is the grand partition function defined by

$$Z = \mathrm{Tr}(\mathrm{e}^{-\beta(H-\mu N)}) \tag{1.2}$$

$$= \sum_i \langle i|\mathrm{e}^{-\beta(H-\mu N)}|i\rangle. \tag{1.3}$$

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All the thermodynamic parameters of the system are then calculable from Z. We define

$$\Omega = -\beta^{-1} \log Z \tag{1.4}$$

$$= E - TS - \mu N \tag{1.5}$$

so that

$$\mathrm{d}E = T\,\mathrm{d}S - P\,\mathrm{d}V + \mu\,\mathrm{d}N \tag{1.6}$$

$$\mathrm{d}\Omega = -S\,\mathrm{d}T - P\,\mathrm{d}V - N\,\mathrm{d}\mu \tag{1.7}$$

$$P = -\left(\frac{\partial\Omega}{\partial V}\right)_{T,\mu} \qquad S = -\left(\frac{\partial\Omega}{\partial T}\right)_{V,\mu} \qquad N = -\left(\frac{\partial\Omega}{\partial\mu}\right)_{T,V}. \tag{1.8}$$

For a large system where surface effects are negligible, $E \propto V$. From this it may be shown that $\Omega = -PV$ giving the equation of state $PV = kT \log Z$.

1.2. *Thermal Green functions*

1.2.1. Spin-$\frac{1}{2}$. Define two Green functions

$$\mathrm{i}G^{>}_{\alpha\beta}(x, y) = \langle \psi_\alpha(x)\bar{\psi}_\beta(y)\rangle \tag{1.9}$$

$$= Z^{-1}\,\mathrm{Tr}(\mathrm{e}^{-\beta(H-\mu N)}\psi_\alpha(x)\bar{\psi}_\beta(y)) \tag{1.10}$$

$$\mathrm{i}G^{<}_{\alpha\beta}(x, y) = -\langle \bar{\psi}_\beta(y)\psi_\alpha(x)\rangle \tag{1.11}$$

$$= -Z^{-1}\,\mathrm{Tr}(\mathrm{e}^{-\beta(H-\mu N)}\bar{\psi}_\beta(y)\psi_\alpha(x)) \tag{1.12}$$

where the field operators ψ and $\bar{\psi}$ are in the Heisenberg picture. For a homogenous system the two coordinates appear only through their difference:

$$G^{<>}_{\alpha\beta}(x, y) = G^{<>}_{\alpha\beta}(x - y). \tag{1.13}$$

Using these definitions together with $\mathrm{Tr}(AB) = \mathrm{Tr}(BA)$, $[H, N] = 0$, and

$$\mathrm{e}^{\beta H}\psi(t, \boldsymbol{x})\,\mathrm{e}^{-\beta H} = \psi(t - \mathrm{i}\beta, \boldsymbol{x})$$

(the last of these equations being the definition of what is meant by a field with complex time argument) we have

$$G^{<}_{\alpha\beta}(t, \boldsymbol{x}) = -\mathrm{e}^{\beta\mu}G^{>}_{\alpha\beta}(t - \mathrm{i}\beta, \boldsymbol{x}) \tag{1.14}$$

which in momentum space becomes

$$G^{<}_{\alpha\beta}(p) = -\mathrm{e}^{-\beta(p^0-\mu)}G^{>}_{\alpha\beta}(p) \tag{1.15}$$

one of the key relations of the theory.

Fourier transforming (1.9) and (1.11)

$$\mathrm{i}G^{>}_{\alpha\beta}(p) - \mathrm{i}G^{<}_{\alpha\beta}(p) = \int \mathrm{d}^4x\,\mathrm{e}^{\mathrm{i}px}\langle[\psi_\alpha(x), \bar{\psi}_\beta(0)]_+\rangle$$

$$= \rho_{\alpha\beta}(p). \tag{1.16}$$

Here ρ is the analogue of the Lehmann spectral function in zero-temperature field theory. Combining the definition of ρ with (1.14) and (1.15) gives

$$\mathrm{i}G^{<}_{\alpha\beta}(p) = -f(p^0)\rho_{\alpha\beta}(p) \tag{1.17}$$

$$iG^{>}_{\alpha\beta}(p) = (1 - f(p^0))\rho_{\alpha\beta}(p) \tag{1.18}$$

where

$$f(p^0) = (e^{\beta(p^0-\mu)} + 1)^{-1} \tag{1.19}$$

the Dirac distribution function.

For a free field the anticommutator is a c-number. So taking the thermal average then gives

$$\rho_{\alpha\beta}(p) = (\gamma p + m)_{\alpha\beta}\varepsilon(p^0)2\pi\delta(p^2 - m^2). \tag{1.20}$$

1.2.2. Spin-0. The treatment of the spin-0 case is very similar. Some sign changes occur due to the replacement of anticommutators with commutators. We now define

$$iG^{>}(x, y) = \langle\varphi(x)\varphi(y)\rangle \tag{1.21}$$

$$iG^{<}(x, y) = \langle\varphi(y)\varphi(x)\rangle. \tag{1.22}$$

It then follows that

$$G^{<}(t, \boldsymbol{x}) = e^{\beta\mu}G^{>}(t - i\beta, \boldsymbol{x}) \tag{1.23}$$

$$iG^{<}(p) = f(p^0)\rho(p) \tag{1.24}$$

$$iG^{>}(p) = (1 + f(p^0))\rho(p) \tag{1.25}$$

where now

$$f(p^0) = (e^{\beta(p^0-\mu)} - 1)^{-1} \tag{1.26}$$

and

$$iG^{>}(p) - iG^{<}(p) = \int d^4x\, e^{ipx}\langle[\varphi(x), \varphi(0)]\rangle \tag{1.27}$$

$$= \rho(p). \tag{1.28}$$

Normally there is no conserved particle number in the spin-0 case, so the chemical potential μ is zero.

1.3. Perturbation theory

1.3.1. Green functions. We define

$$\Lambda(t) = e^{iH_0 t}\, e^{-iHt} \tag{1.29}$$

$$U(t_1, t_2) = \Lambda(t_1)\Lambda^{-1}(t_2) \tag{1.30}$$

where

$$H = H_0 + H_I \tag{1.31}$$

$$H_I(t) = e^{iH_0(t)}H_I\, e^{-iH_0(t)} \tag{1.32}$$

so that

$$i\frac{\partial}{\partial t}U(t, t_0) = H_I(t)U(t, t_0) \tag{1.33}$$

where $H_I(t)$ is of course in the interaction picture.

Using $\mathrm{Tr}(AB) = \mathrm{Tr}(BA)$ and $[H_0, N] = 0$ it follows that for any value of τ we have

$$Z = Z_0 \langle U(t - \mathrm{i}\beta, \tau)\rangle_0 \tag{1.34}$$

where

$$Z_0 = \mathrm{Tr}(\mathrm{e}^{-\beta(H_0 - \mu N)}) \tag{1.35}$$

$$\langle Q\rangle_0 = \mathrm{Tr}(Q\, \mathrm{e}^{-\beta(H_0 - \mu N)}). \tag{1.36}$$

Defining interaction-picture spin-$\frac{1}{2}$ fields ψ_I, $\bar{\psi}_I$ we have similarly

$$G^{>}_{\alpha\beta}(t, t_0) = \frac{\langle U(\tau - \mathrm{i}\beta, t)\psi_{I\alpha}(t)U(t, t_0)\bar{\psi}_{I\beta}(t_0)U(t_0, \tau)\rangle_0}{\langle U(\tau - \mathrm{i}\beta, \tau)\rangle_0}. \tag{1.37}$$

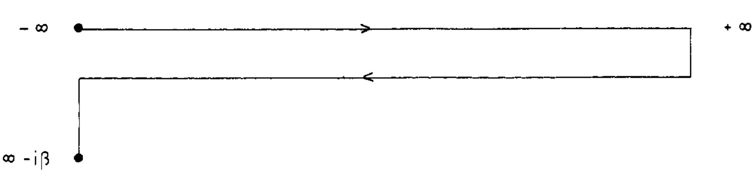
This is similar to the zero-temperature expression except we have replaced the usual time interval $(\infty, -\infty)$ with $(\tau - \mathrm{i}\beta, \tau)$ and all vacuum expectation values are replaced with thermal averages. One also finds that

$$U(t, t_0) = \sum_{n=0}^{\infty} \frac{(-\mathrm{i})^n}{n!} \int_{t_0}^{t} \mathrm{d}t_1 \ldots \mathrm{d}t_n T_C(H_I(t_1) \ldots H_I(t_n)) \tag{1.38}$$

where T_C denotes path ordering along the contour of integration.

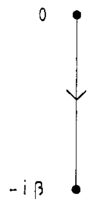
In calculating Green functions we have two extra degrees of freedom compared with the zero-temperature case. These are in the choice of τ and the contour C. Careful choice of these simplifies the relationship between Feynman rules in momentum and coordinate space. The usual choices are the real-time formalism and the imaginary-time formalism.

1.3.2. Real-time formalism. τ is chosen to be $-\infty$ and the integration contour used is



In many applications the contributions from the vertical sections of C are zero. The theory can then be written with a real field on section (i) and a different real field on section (ii), the difference being due to the imaginary-time shift of the second field.

1.3.3. Imaginary-time formalism. This has become the most popular approach amongst workers in this field. Here τ is chosen to be zero and the contour C runs straight down the imaginary axis:



t is pure imaginary with

$$-\beta < \mathrm{Im}\,(t) < \beta. \tag{1.39}$$

We define

$$G(t) = \theta(-\mathrm{Im}\,(t))G^{>}(t) + \theta(\mathrm{Im}\,(t))G^{<}(t) \tag{1.40}$$

a well defined function on the given interval and the obvious generalisation of the zero-temperature expression.

From (1.14) and (1.23) we have

$$G(t) = \pm\, \mathrm{e}^{-\beta\mu} G(t - \mathrm{i}\beta) \tag{1.41}$$

so that $\mathrm{e}^{\mathrm{i}\mu t}G(t)$ is either periodic or antiperiodic and has the Fourier expansion

$$\mathrm{e}^{\mathrm{i}\mu t}G(t) = (\mathrm{i}/\beta) \sum_{m=-\infty}^{\infty} G_m\, \mathrm{e}^{\omega_m t}. \tag{1.42}$$

$\omega_m = 2m\pi/\beta$ (bosons) and $\omega_m = (2m+1)\pi/\beta$ (fermions) are Matsubara frequencies. We have

$$G_m(p) = \int_0^{-\mathrm{i}\beta} \mathrm{d}t\, G^{>}(t,p)\, \mathrm{e}^{-(\omega_m - \mathrm{i}\mu)t} \tag{1.43}$$

$$= \int \mathrm{d}\omega\, \frac{\rho(\omega, p)}{\mathrm{i}\omega_m - (\omega - \mu)}. \tag{1.44}$$

For a free field this looks like the zero-temperature Lehmann representation except that ω_m is discrete and that p^0 is replaced by $(\mathrm{i}\omega_m + \mu)$.

The Feynman rules are similar to the zero-temperature case, with the following modifications:

$$p^0 \to (\mathrm{i}\omega_n + \mu) \tag{1.45}$$

$$(2\pi)^4 \delta^{(4)}(\Sigma_p) \to (2\pi)^3 \delta^{(3)}(\Sigma_p)(\beta/\mathrm{i})\delta_{\Sigma\omega_n, 0} \tag{1.46}$$

$$\int \frac{\mathrm{d}^4 p}{(2\pi)^4} \to \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{\mathrm{i}}{\beta} \Sigma_n. \tag{1.47}$$

(1.46) is used at each vertex, (1.47) for each loop.

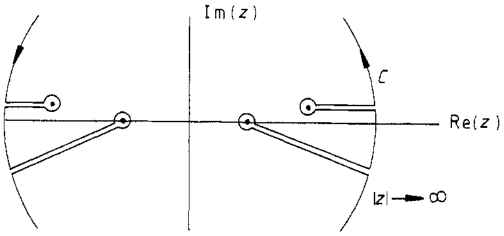


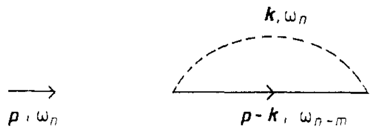
Figure 1. Contour of integration.

1.4. *Further results*

1.4.1. Example: electron self-energy. There is the analogue of the Dyson equation of zero-temperature field theory for the full electron propagator:

$$G_n(\boldsymbol{p}) = G_n^{(0)}(\boldsymbol{p}) + G_n^{(0)}(\boldsymbol{p})\,\Sigma_n(\boldsymbol{p}) G_n(\boldsymbol{p}) \tag{1.48}$$

where $\Sigma_n(\boldsymbol{p})$ is the electron self-energy and $G^{(0)}(\boldsymbol{p})$ is the free-electron propagator. To first order:

$\Sigma_n(\boldsymbol{p}) =$  (1.49)

where $\omega_{n-m} = (2\pi/\beta)[(2n+1) - 2m]$. Using the Feynman rules from the previous section, we have

$$\Sigma_n(\boldsymbol{p}) = -e^2 \int \frac{\mathrm{d}^3 \boldsymbol{k}}{(2\pi)^3} \frac{\mathrm{i}}{\beta} \sum_m \frac{\gamma^\lambda[\gamma^0(\mathrm{i}\omega_{n-m}+\mu) - \gamma^i(p^i - k^i)]\gamma_\lambda}{[(\mathrm{i}\omega_{n-m}+\mu)^2 - (\boldsymbol{p}-\boldsymbol{k})^2][(\mathrm{i}\omega_m)^2 - \boldsymbol{k}^2]} \tag{1.50}$$

in the limit of zero electron mass.

There is a standard trick for doing the frequency summation Σ_m. It expresses a general sum $\Sigma_m A(\mathrm{i}\omega_m)$ as a contour integral around the imaginary axis:

$$\sum_m A(\mathrm{i}\omega_m) = \frac{\beta}{2\pi\mathrm{i}} \oint_C \frac{\mathrm{d}z}{\mathrm{e}^{\beta z} - 1} A(z). \tag{1.51}$$

The sum comes from the residues of the integrand at the poles $z = 2m\pi\mathrm{i}/\beta$. Distorting the contour C as shown in figure 1 and using Jordan's lemma, the integral may be evaluated as the sum of the residues at all the poles of $A(z)$, or if it has branch points as an integral over the associated discontinuities.

1.4.2. Calculation of log Z. The sum of all Feynman diagrams with no external legs gives Z/Z_0. Careful combinatorics shows that $\log(Z/Z_0)$ is the sum of all connected graphs, exactly as in the zero-temperature case. The definition of Ω is in terms of $\log Z$ and each graph contributing to this has a factor $(2\pi)^3\delta^{(3)}(0)$ or the volume, V, of the system. So our calculation correctly predicts that Ω is an extensive quantity.

1.4.3. Path integral formalism. The path integral formalism at finite temperature is very similar to that at zero temperature. There are two important differences. In the action integral substitute

$$\int_{-\infty}^{\infty} \mathrm{d}x^0 \to \int_0^{-\mathrm{i}\beta} \mathrm{d}x^0 \tag{1.52}$$

and the periodicity (or antiperiodicity) of the Green functions requires that the classical integration fields have similar properties:

$$\varphi(t=0) = \pm\,\varphi(t=-\mathrm{i}\beta). \tag{1.53}$$

For bosons we have

$$Z = Z_0 \int_{\text{periodic}\,\varphi} D\varphi \exp\left(\int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3\boldsymbol{x}\, L(\tau, \boldsymbol{x})\right) \tag{1.54}$$

where $\tau = \mathrm{i}x_0$. For fermions

$$Z = Z_0 \int D\psi D\bar{\psi} \exp\left(\int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3\boldsymbol{x}(L(\tau, \boldsymbol{x}) + \mu N)\right) \tag{1.55}$$

the integration being over antiperiodic Grassman functions ψ, $\bar{\psi}$. N is the fermion number operator $\psi^\dagger\psi$, which enters because the function ψ differs from the field ψ used in (1.9) by a factor $\mathrm{e}^{\mu t/2}$. If we had stuck to the previous definition, it would be $\mathrm{e}^{\mu t/2}\psi$ that was antiperiodic and the explicit μN factor in (1.55) would be absent.

2. Field theory by perturbation (JIK)

2.1. Finite-temperature QCD

The QCD Lagrangian is, in standard notation,

$$\mathscr{L} = \bar{\psi}(\mathrm{i}\gamma^\mu\partial_\mu - M - g\gamma^\mu A^a_\mu G_a)\psi - \tfrac{1}{4}F^{\mu\nu}_a F^a_{\mu\nu} \tag{2.1}$$

$$F^{\mu\nu}_a = \partial^\mu A^\nu_a - \partial^\nu A^\mu_a - g f_{abc} A^\mu_b A^\nu_c \tag{2.2}$$

$$[G_a, G_b] = \mathrm{i} f_{abc} G_c \tag{2.3}$$

where the G_a are a set of generators of the SU(N) gauge group, with corresponding structure constants f_{abc}.

We first consider two simple cases of the theory, where there is either no quark sector or the quark mass matrix M is zero. In both these cases the gluon number is not fixed so that we have $\mu_{\text{gluon}} = 0$, and in the second we choose to set $\mu_{\text{quark}} = 0$. The only parameter in the theory is g, which is dimensionless, and so using a naive dimensional argument we can calculate the pressure P, and hence Z and all other thermodynamic quantities. We have $[P] = [T^4]$ and so there must be some function $f(g)$ with

$$P = f(g)T^4 \tag{2.4}$$

which gives

$$s = \partial P/\partial T = 4f(g)T^3 \tag{2.5}$$

$$\varepsilon = -P + Ts = 3f(g)T^4 \tag{2.6}$$

where s is the entropy density and ε the energy density. So we have the same equation of state as the ideal gas:

$$P=\tfrac{1}{3}\varepsilon. \tag{2.7}$$

However, this treatment violates our intuitive understanding of what happens. At low temperature we expect the physical system to consist of SU(N) colour singlets, a gas of pions, glueballs, etc. At high temperature these confined states would overlap and so there should be a deconfining phase transition. Our simple analysis reflects none of this. Of course the reason is that we have ignored quantum mechanics. Once we consider quantum processes dimensional transmutation occurs during renormalisation, and the QCD parameter Λ with mass dimension replaces the dimensionless coupling g. It is here that our argument breaks down.

At lowest order, we have

$$\frac{\bar{g}^2}{4\pi}=\frac{6\pi}{(11N-2N_{\mathrm{f}})\log(M/\Lambda)} \tag{2.8}$$

where $\bar{g}$ is the running coupling constant, N_{f} the number of quark flavours and M a characteristic mass which on dimensional grounds must be proportional to T. Now we see that as $T\to\infty$, $\bar{g}\to 0$, so that at high temperature the coupling becomes small and we are essentially dealing with free quarks, whilst as $T\to 0$, $\bar{g}$ gets large and perturbation theory breaks down. The large coupling at low T leads to confinement and so we see that our intuition was correct. The phase transition is entirely a quantum phenomenon [4].

There are three main methods for dealing with high-temperature field theory:

(i) phenomenological models;

(ii) perturbation theory;

(iii) lattice field theory.

Of these, (i) has difficulties because much of the theory has to be put in by hand, and whilst (iii) in principle would enable us to look at the phase transition itself, the amount of computer time required to do this is enormous.

In these lectures we will consider the second method, perturbation theory. This should be good for the high-temperature limit, since we have seen that this corresponds to small coupling, but clearly non-perturbative effects occur so results should be treated with caution when T is small.

2.2. Perturbation theory

The functional integral expression for the partition function is

$$Z=(N')^{2N_{\mathrm{g}}}\int DA_a^\mu D\bar{\psi}D\psi\,\delta(F^a)\det\left(\frac{\partial F^c}{\partial\alpha^a}\right)\exp\left(\int_0^\beta \mathrm{d}\tau\int \mathrm{d}^3\boldsymbol{x}(\mathscr{L}+\psi^\dagger\mu\psi)\right) \tag{2.9}$$

$$=\mathrm{Tr}(\mathrm{e}^{-\beta(H-\mu N)}). \tag{2.10}$$

We choose to work in the covariant gauge, so that for arbitrary $f^a(\boldsymbol{x},\tau)$ we take

$$F^a=\partial^\mu A_\mu^a-f^a(\boldsymbol{x},\tau)=0 \tag{2.11}$$

and then the standard procedure leads to

$$Z=(N')^{2N_{\mathrm{g}}}\int DA_a^\mu D\bar{\psi}D\psi D\bar{C}_b DC_b\exp\left(\int_0^\beta \mathrm{d}\tau\int \mathrm{d}^3x\,\mathscr{L}_{\mathrm{eff}}\right) \tag{2.12}$$

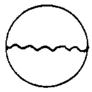
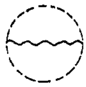
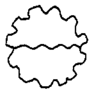
$$\mathscr{L}_{\text{eff}} = \mathscr{L} - (1/2\rho)(\partial^\mu A^a_\mu)^2 + g f^{abc} \bar{C}_a \partial_\mu A^\mu_b C_c + \psi^\dagger \mu \psi. \tag{2.13}$$

Expanding P as $\Sigma_n P_{(n)}$ where $P_{(n)}$ is the contribution of order n in g, we have after some calculation

$$P_{(0)} = \frac{\pi^2}{90} 2N_g T^4 + \sum_f N\left(\frac{7\pi^2}{180} T^4 + \frac{\mu_f^2 T^2}{6} + \frac{\mu_f^4}{12\pi^2}\right) \tag{2.14}$$

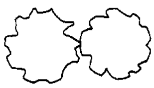
where $2N_g = 2(N^2-1)$ is the number of gluon degrees of freedom. The first term in this expression is just the Planck black-body radiation contribution from the non-interacting (at this order) gluons; the second term is the quark contribution.

The first non-trivial contribution is of order g^2. We expand $\exp(\int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3\boldsymbol{x}\, \mathscr{L}_{\text{eff}})$ in Feynman diagrams as

−1/2  − 1/2  + 1/12  + 1/8 

Each of the above diagrams is two-loop, and is slightly tricky to calculate: the result for each is a sum of four terms. The evaluation of such diagrams was explained in the previous section.

The easiest diagram to calculate is

$D =$ 

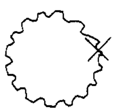
Here the momenta flowing around each loop are independent and the diagram factorises as

$$D = \left(\int \frac{\mathrm{d}p^4}{(2\pi)^4} \frac{1}{p_4^2 + \boldsymbol{p}^2} + \int \frac{\mathrm{d}p^3}{(2\pi)^3} \frac{1}{|\boldsymbol{p}|} \frac{1}{\mathrm{e}^{(\beta|\boldsymbol{p}|)} - 1}\right)^2 \tag{2.15}$$

so that the result is the following product of vacuum and matter contributions:

$$D = (\text{vacuum} + \text{matter})(\text{vacuum} + \text{matter}).$$

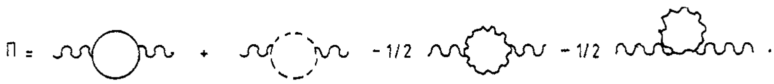
The vacuum vacuum term is cancelled by the vacuum energy renormalisation, and the two vacuum matter terms are cancelled by the self-energy renormalisation, that is by the diagram with counter term



After some further algebra we get the second-order contribution

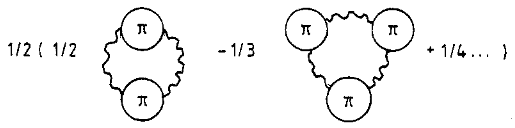
$$P_{(2)} = -\frac{g^2}{144} N N_g T^4 - \frac{g^2 N_g}{576} \sum_f \left(5T^4 + \frac{18}{\pi^2}\mu_f^2 T^2 + \frac{9}{\pi^4}\mu_f^4\right). \tag{2.16}$$

We might assume that the next contribution would be fourth order in g. This is not the case as the g^3 term is non-zero. To show this, consider the one-loop contribution to the gluon self-energy:



(2.17)

This contributes the following terms to the pressure:



$$= -\tfrac{1}{2}T\Sigma_n \int \frac{d^3p}{(2\pi)^3} \mathrm{Tr}(\ln(1 + D_0(p)\Pi(p)) - D_0(p)\Pi(p)) \tag{2.18}$$

where $D_0(p)$ is the free-gluon propagator.

Naively, the first term in this sum is of order g^4. However it is infrared divergent: the reason is that the $n=0$ contribution from the frequency sum is special. Defining the electric mass

$$m_{\mathrm{el}}^2 = -\Pi_{00}(\omega_n = 0, |\boldsymbol{p}| \to 0) = \tfrac{1}{3}g^2NT^2 + \tfrac{1}{6}g^2N_fT^2 + (g^2/2\pi^2)\sum_f \mu_f^2 \tag{2.19}$$

we have the contribution

$$\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\left(-\frac{1}{2}\left(\frac{m_{\mathrm{el}}^2}{\boldsymbol{p}^2}\right)^2 + \frac{1}{3}\left(\frac{m_{\mathrm{el}}^2}{\boldsymbol{p}^2}\right)^3 + \ldots\right) \tag{2.20}$$

to this term (the $1/\boldsymbol{p}^2$ is the gluon propagator when $\omega_n = 0$). Now although each term is divergent their sum is finite and equals $N_gTm_{\mathrm{el}}^3/12\pi$ and this contributes at order g^3 (m_{el} is of order g).

So for SU(3) without quarks

$$P = \frac{8\pi^2}{45}\left[1 - 15\left(\frac{g^2}{16\pi^2}\right) + 240\left(\frac{g^2}{16\pi^2}\right)^{3/2} + \ldots\right]. \tag{2.21}$$

In practice higher-order terms are never calculated. The coupling constant picks up a temperature dependence through

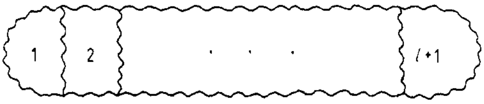
$$\frac{g^2}{16\pi^2} \to \frac{\bar{g}^2}{16\pi^2} = \frac{1}{11\ln(M^2/\Lambda_{\mathrm{mom}}^2)}. \tag{2.22}$$

A reasonable choice for M_2 in this scheme is the mean momentum squared

$$M^2 = \langle \boldsymbol{p}^2 \rangle = (3.2T)^2. \tag{2.23}$$

In figure 2 we plot $\log(P/\Lambda_{\mathrm{mom}}^4)$ against T/Λ_{mom}, which shows good convergence here for $T/\Lambda \gg 1$, but for $T/\Lambda < 1$, $P_{(0)} + P_{(2)}$ goes negative and $P_{(0)} + P_{(2)} + P_{(3)}$ has negative slope (which implies negative entropy at finite T) [5].

To be able to calculate to arbitrary order we should consider diagrams of the form



This has $(l+1)$ loops, $2l$ vertices and $3l$ propagators. The dominant infrared behaviour will come from the $n=0$ modes.

Schematically the expression for this diagram is

$$g^{2l}\left(T\int_m^T \mathrm{d}p^3\right)^{l+1} p^{2l}\frac{1}{(p^2)^{3l}}. \tag{2.24}$$

Looking at the propagator in a covariant gauge it can be decomposed:

$$\mathscr{D}^{\mu\nu}=\frac{1}{(G-p^2)}P_T^{\mu\nu}+\frac{1}{(F-p^2)}P_L^{\mu\nu}+\frac{\rho}{p^2}\frac{p^\mu p^\nu}{p^2} \tag{2.25}$$

where

$$\begin{aligned} &P_T^{00}=P_T^{0i}=P_T^{i0}=0\\ &P_T^{ij}=\delta^{ij}-(p^ip^j/\boldsymbol{p}^2)\\ &P_L^{\mu\nu}=(p^\mu p^\nu/p^2)-g^{\mu\nu}-P_T^{\mu\nu}. \end{aligned} \tag{2.26}$$

The IR cut-off m in (2.24) may be identified with $m_{\text{el}}^2=F(\omega_n=0,\boldsymbol{p}\rightarrow 0)$ or with $m_{\text{mag}}^2=G(\omega_n=0,\boldsymbol{p}\rightarrow 0)$. In any case (2.24) is of the order

$$\begin{cases} g^{2l}T^4 & l=1,2\\ g^6T^4\ln(T/m) & l=3\\ g^6T^4(g^2T/m)^{l-3} & l>3. \end{cases} \tag{2.27}$$

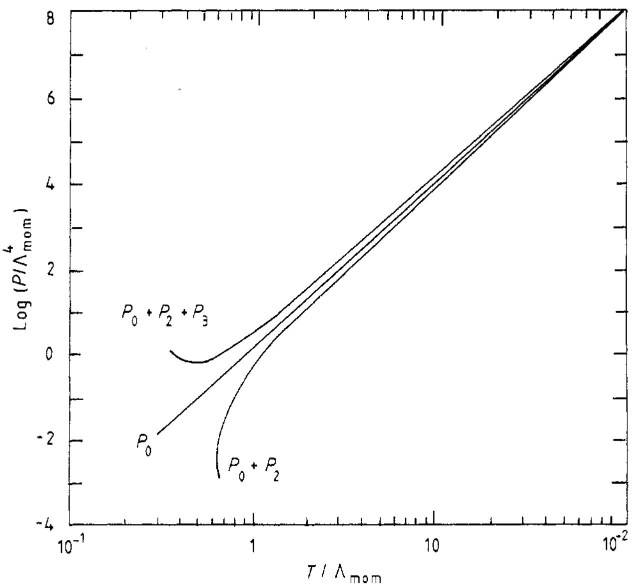
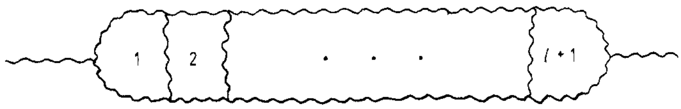


Figure 2. Plot of $\log(P/\Lambda_{\text{mom}}^4)$ against T/Λ_{mom}.

If $m=0$ and $l>2$ the diagram is IR divergent. Maybe when all diagrams of the same order are added together the divergences will cancel, but this is hard to verify. Taking $m=m_{el}\sim gT$ leads to no problems. To first order m_{mag} vanishes, to the next order it is $\sim g^2T$. This presents a difficulty because it suggests that all loops with $l>3$ contribute to order g^6; it is not known how to sum all these diagrams.

The same kind of problem arises in the gluon self-energy diagram:



When evaluated this is

$$\begin{cases} g^4T^2\ln(T/m) & l=1 \\ g^4T^2(g^2T/m)^{l-1} & l>1. \end{cases} \tag{2.28}$$

So here the problem arises at order g^4 [6].

2.3. Linear response theory

We develop this in order to probe further the structure of the QCD plasma. Consider an operator $Y(t)$; apply an external probe field $H_{ext}(t)$ and look at the change in the ensemble average of $Y(t)$ to first order in the external field. Then

$$H'(t)=H+H_{ext}(t) \tag{2.29}$$

$$\partial Y(t)/\partial t=\mathrm{i}[H'(t), Y(t)] \qquad \text{since } [H, Y(t)]=0. \tag{2.30}$$

Let $|j\rangle$ be the eigenstates of H in the Heisenberg picture, then

$$\frac{\partial}{\partial t}\langle j|Y(t)|j\rangle=\mathrm{i}\langle j|[H_{ext}(t), Y(t)]|j\rangle. \tag{2.31}$$

To first order in $H_{ext}(t)$

$$\delta\langle j|T(t)|j\rangle=\mathrm{i}\int_{t_0}^{t} \mathrm{d}t'\langle j|[H_{ext}(t'), Y(t)]|j\rangle. \tag{2.32}$$

Taking the grand canonical ensemble average

$$\delta\langle Y(t)\rangle=\frac{\Sigma_j \exp[-\beta(E_j-\mu N_j)]\delta\langle j|Y(t)|j\rangle}{\Sigma_j \exp[-\beta(E_j-\mu N_j)]} \tag{2.33}$$

$$=\mathrm{i}\int_{t_0}^{t} \mathrm{d}t' \operatorname{Tr}(\hat{\rho}[H_{ext}(t'), Y(t)]). \tag{2.34}$$

This is the basic result we shall use in the following example.

Consider the change in colour field $E_i^a(\boldsymbol{x}, t)$ due to the introduction of a classical external field $\mathscr{E}_i^a$. In the temporal axial gauge (TAG) $A_0^a=0$ and

$$E_i^a=F_{i0}^a=\partial_i A_0^a-\partial_0 A_i^a-gf^{abc}A_i^bA_0^c=-\partial_0 A_i^0 \tag{2.35}$$

$$H_{\mathrm{ext}}(\boldsymbol{x}, t) = \boldsymbol{E}_a \cdot \boldsymbol{\mathscr{E}}_a. \tag{2.36}$$

Then

$$\delta\langle E_i^a(\boldsymbol{x}, t)\rangle = -\mathrm{i}\int_{-\infty}^{\infty} \mathrm{d}t' \int \mathrm{d}\boldsymbol{x}^3\, \mathscr{E}_j^b(\boldsymbol{x}')\, \mathrm{Tr}(\hat{\rho}[E_i^a(\boldsymbol{x}, t), E_j^b(\boldsymbol{x}', t')])\Theta(t-t'). \tag{2.37}$$

This may be written in terms of the retarded propagator

$$\mathscr{D}_{\mu\nu}^{abR}(\boldsymbol{x}-\boldsymbol{x}', t-t') = \mathrm{i}\,\mathrm{Tr}(\hat{\rho}[A_\mu^a(\boldsymbol{x}, t), A_\nu^b(\boldsymbol{x}', t')])\Theta(t-t'). \tag{2.38}$$

Thus we can calculate the relation between the old and new colour fields.

Suppose that the classical field is chosen as a Coulomb-like quark–antiquark potential. When we introduce this into a gluon plasma it will be modified by a dielectric constant $\varepsilon(k)$:

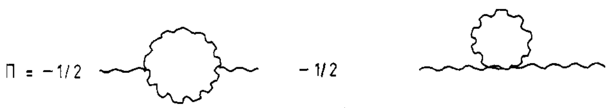
$$V(r) = Q_1 Q_2 \int \frac{\mathrm{d}k^3}{(2\pi)^3} \frac{1}{\varepsilon(\boldsymbol{k})\boldsymbol{k}^2} \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}. \tag{2.39}$$

The dielectric constant is given as a ratio of the old and modified gluon fields which is calculable from the above analysis. In TAG an exact relation is

$$\varepsilon(k) = 1 + \frac{F(k_0 = 0, \boldsymbol{k})}{\boldsymbol{k}^2} \tag{2.40}$$

$$F = -\Pi_{00} = F_{\mathrm{vac}} + F_{\mathrm{mat}}. \tag{2.41}$$

F is known only to one loop at $T > 0$.



At $T = 0$, $r \to 0$,

$$F_{\mathrm{vac}}(0, k) = \frac{11}{48\pi^2} g^2 N k^2 \ln\left(\frac{k^2}{M^2}\right) \tag{2.42}$$

and the effective coupling is

$$\bar{g}^2(k) = \frac{g^2}{\varepsilon(k)} = \frac{g^2}{1 + (11/48\pi^2) g^2 N \ln(k^2/M^2)} \tag{2.43}$$

$$= \frac{24\pi^2}{11 N \ln(k/\Lambda_{\mathrm{mom}}^{\mathrm{axial}})}. \tag{2.44}$$

This is asymptotic freedom. For $T > T_c$ and $r \to \infty$,

$$\varepsilon(k) \simeq 1 + \frac{F(0, k \to 0)}{k^2} \tag{2.45}$$

$$F(0, k \to 0) = \tfrac{1}{3} g^2 N T^2 = m_{\mathrm{el}}^2 \tag{2.46}$$

which is gauge invariant. This approximation gives the Debye potential [7]:

$$V(r) = \frac{Q_1 Q_2}{4\pi} \frac{\exp(-m_{\mathrm{el}} r)}{r}. \tag{2.47}$$

Two questions arise: how does g^2 get replaced by $\bar{g}^2(T)$, and where does the momentum dependence of F come in? The low-momentum expansion of F is

$$F_{\mathrm{mat}}(0, k) = \frac{g^2}{3} N T^2 - \frac{g^2}{4} N T k - \frac{11}{48\pi^2} g^2 N k^2 \left[\ln\left(\frac{k^2}{T^2}\right) + \frac{2}{33} + 2(\gamma - \ln 4\pi) \right]. \tag{2.48}$$

The first term is m_{el}^2, the second is gauge invariant and absent in QED and the third term gives us asymptotic freedom. Insertion of this expansion yields:

$$V(r) = -\frac{N^2-1}{2N} \frac{\bar{g}^2(T)}{4\pi r} \frac{2}{\pi} \int_0^\infty \mathrm{d}z \, \frac{z \sin(zx)}{z^2 - 2tz + 1} \tag{2.49}$$

$$\bar{g}^2(T) = \frac{24\pi^2}{11 N \ln(19.2 T/\Lambda_{\mathrm{ms}})} \tag{2.50}$$

$$m_{\mathrm{el}}^2 = \tfrac{1}{3} \bar{g}^2(T) N T^2 \tag{2.51}$$

$$x = m_{\mathrm{el}} r \qquad t = \tfrac{3}{8} m_{\mathrm{el}}/T = \tfrac{3}{8} \sqrt{\tfrac{1}{3} N} \bar{g}(T). \tag{2.52}$$

Note that g is replaced by $\bar{g}(T)$ in the last equation. From (2.49) define the screening function $S(x, t)$:

$$S(x, t) = \frac{2}{\pi} \int_0^\infty \mathrm{d}z \, \frac{z \sin(zx)}{z^2 - 2tz + 1} \tag{2.53}$$

$$= 2\left(\cos(tx) + \frac{t}{\sqrt{1-t^2}} \sin(tx) \right) \mathrm{e}^{-x\sqrt{1-t^2}} - \frac{4t}{\pi} \int_0^\infty \mathrm{d}y \, \frac{y^2 \, \mathrm{e}^{-xy}}{(1-y^2)^2 + 4t^2 y^2}. \tag{2.54}$$

For fixed x and $t \to 0$ (the high-T limit), $S \to \mathrm{e}^{-x}$. For fixed t and $x \to \infty$ (the long-distance limit):

$$S \to -\frac{8t}{\pi x^3} \qquad \text{and} \qquad V(r) \to \frac{9}{4\pi^3} \frac{(N^2-1)}{N^2} \frac{1}{T^3 r^4}. \tag{2.55}$$

This function is shown in figure 3. At large distances the Coulomb-like force is screened not exponentially but as a power law [8].

3. Improving the model (JIK)

3.1. Complex scalar field

A Lagrangian for a complex scalar field φ is

$$\mathscr{L} = \partial_\mu \varphi^* \partial^\mu \varphi + c^2 \varphi^* \varphi - \lambda(\varphi^* \varphi)^2. \tag{3.1}$$

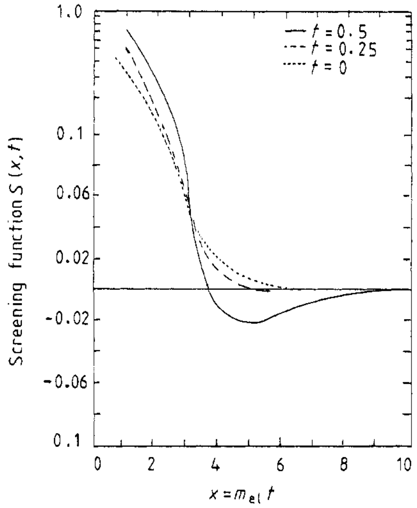


Figure 3. Screening function, $S(x, t)$.

For $\lambda = 0$ the partition function Z may be written down in path integral form, but there is an instability when the frequency $w = \sqrt{p^2 - c^2}$ is imaginary. So we take $\lambda > 0$, which corresponds to a repulsive interaction between the particles.

Guessing that the system has a condensate, we may write

$$\varphi(\boldsymbol{x}, \tau) = \xi + \chi(\boldsymbol{x}, \tau) \tag{3.2}$$

where $\chi(\boldsymbol{x}, \tau)$ is the complex dynamical field and ξ is a real constant (the U(1) symmetry allows us to choose it to be real). Then

$$\mathscr{L} = -U(\xi) + \mathscr{L}_0 + \mathscr{L}_{\mathrm{I}} \tag{3.3}$$

where

$$U(\xi) = \lambda\xi^4 - c^2\xi^2 \tag{3.4}$$

a quantity with units of energy density. Writing $\chi = (\chi_1 + \mathrm{i}\chi_2)/\sqrt{2}$ and dropping linear terms (which make no contribution to the partition function)

$$\mathscr{L}_0 = \tfrac{1}{2}(\partial_\mu \chi_1)^2 - \tfrac{1}{2}(6\lambda\xi^2 - c^2)\chi_1^2 + \tfrac{1}{2}(\partial_\mu \chi_2)^2 - \tfrac{1}{2}(2\lambda\xi^2 - c^2)\chi_2^2 \tag{3.5}$$

$$\mathscr{L}_{\mathrm{I}} = -\sqrt{2}\lambda\xi\chi_1(\chi_1^2 + \chi_2^2) - \tfrac{1}{4}\lambda(\chi_1^2 + \chi_2^2)^2. \tag{3.6}$$

Suppose $\lambda \ll 1$. Then neglecting $\mathscr{L}_{\mathrm{I}}$

$$Z = (N')^2 \int [\mathrm{d}\chi_1][\mathrm{d}\chi_2] \exp\left(-\beta V U(\xi) + \int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3\boldsymbol{x} \mathscr{L}_0 \right). \tag{3.7}$$

The thermodynamic potential is

$$\Omega(T, \xi) = -P(T, \xi) = -T \frac{\partial}{\partial V} \ln Z \tag{3.8}$$

$$= U(\chi) + \sum_{i=1}^{2} \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3} \{\tfrac{1}{2}w_i + T \ln[1 - \exp(-w_i/T)]\} \tag{3.9}$$

with $w_i = \sqrt{\bar{m}_i^2 + p^2}$, $\bar{m}_i$ being the masses in the mean-field approximation. The expression in the summation is just the contribution to the pressure from an ideal gas of massive bosons, the $\frac{1}{2}w_i$ term representing the zero-point motion.

The mean-field approximation is to neglect this term such that

$$\Omega_{\rm mf}(T=0, \xi) = U(\xi) \tag{3.10}$$

(cf figure 4). The masses corresponding to the two fields χ_1 and χ_2 are

$$\bar{m}_1^2(T=0) = 2c^2 \tag{3.11}$$

$$\bar{m}_2^2(T=0) = 0. \tag{3.12}$$

χ_2 represents a Goldstone boson.

3.2. High-temperature expansion

For $T \gg \bar{m}_i$

$$\Omega_{\rm mf}(T, \chi) = U(\chi) - \sum_{i=1}^{2} \left(\frac{\pi^2}{90} T^4 - \frac{\bar{m}_i^2}{24} T^2 + \ldots \right). \tag{3.13}$$

The quantity in the summation is the high-T expansion of the ideal-gas pressure. When T is large the T^4 term dominates. For $T \to 0$ both terms vanish, but in this limit we must include terms of higher order such as $\bar{m}_i^6/T^2$ and $\bar{m}_i^4 \ln(\bar{m}_i^2/T^2)$ which actually diverge. Keeping the first two terms in this expansion and substituting for $\bar{m}_i$,

$$\Omega_{\rm mf} = \lambda\xi^4 + (\tfrac{1}{3}\lambda T^2 - c^2)\xi^2 - (\pi^2/45)T^4 - (c^2/12)T^2. \tag{3.14}$$

This termination is a very clever one; not only does it pull out the ξ dependence but it turns out to be valid at $T=0$ also (see figure 5).

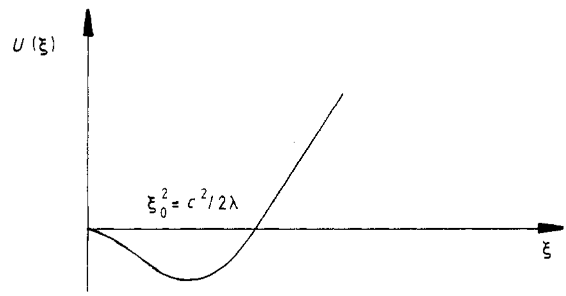


Figure 4. Potential in mean-field approximation.

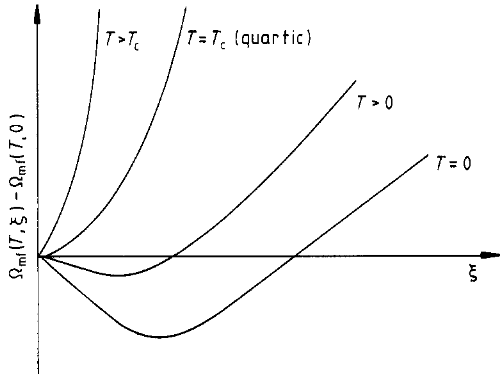


Figure 5. Behaviour of thermodynamic potential at various temperatures.

The transition temperature $T_c^2 = 3c^2/\lambda \gg 2c^2, 0$; it is greater than the boson masses and the high-temperature expansion should be valid in its vicinity. Minimising the potential

$$\xi^2(T) = \begin{cases} (c^2/2\lambda) - (T^2/6) & T<T_c \\ 0 & T>T_c \end{cases} \tag{3.15}$$

(see figure 6). Then we can calculate $P(T)$, normalising such that $P(0)=0$.

$$P(T) = \begin{cases} \left(\dfrac{\pi^2}{45} + \dfrac{\lambda}{36}\right) T^4 - \dfrac{c^2}{12} T^2 & T<T_c \\ \dfrac{\pi^2}{45} T^4 + \dfrac{c^2}{12} T^2 - \dfrac{c^4}{4\lambda} & T>T_c. \end{cases} \tag{3.16}$$

For $T \ll T_c$ this is obviously incorrect because it predicts $s=(\mathrm{d}P/\mathrm{d}T)_{T\to 0} \to -c^2T/6$, which is negative. This problem should be overcome by doing the integral (3.9) exactly instead of the high-temperature expansion.

P and $s=\partial P/\partial T$ are continuous at T_c but $\partial^2 P/\partial T^2$ is not. There is a second-order phase transition at T_c. The symmetry, broken below T_c, is restored above it by the thermal disordering of the condensate [9].

Now consider the masses as a function of temperature:

$$\bar{m}_1^2(T) = \begin{cases} 2c^2 - \lambda T^2 & T<T_c \\ -c^2 & T>T_c \end{cases} \tag{3.17}$$

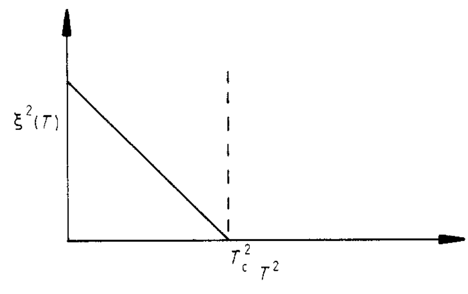


Figure 6. ξ^2 as function of T^2.

$$\bar{m}_2^2(T)=\begin{cases}-\frac{1}{3}\lambda T^2 & T<T_c\\ -c^2 & T>T_c.\end{cases} \tag{3.18}$$

There are two problems here for $T>0$; there is negative mass squared and no Goldstone boson is present. In considering the latter it is useful to construct the quantity

$$F^\mu(p)=-\mathrm{i}\xi p^\mu \mathscr{D}_2^R(p). \tag{3.19}$$

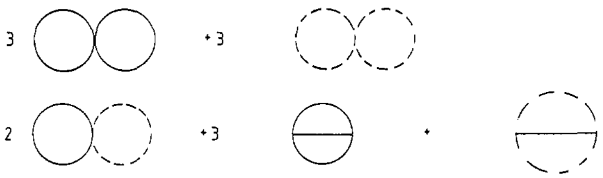
Taking the limit $\boldsymbol{p}\to 0$ one can show, in the standard way, that

$$p_\mu F^\mu(p)=-\mathrm{i}\xi. \tag{3.20}$$

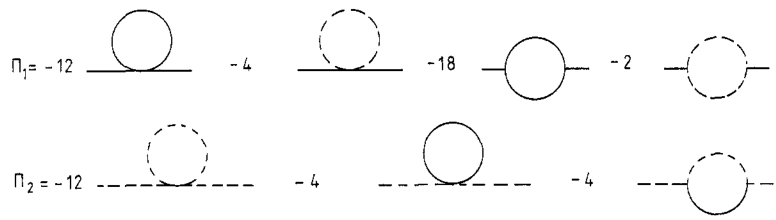
$\mathscr{D}_2^R$ has a pole at p^0 when $\boldsymbol{p}=0$. A Goldstone boson does exist and there must be something wrong with our approximation.

3.3. *Loop corrections*

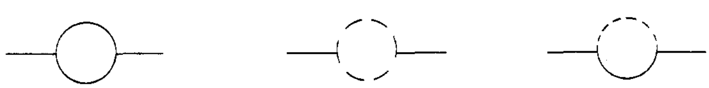
The two-loop corrections to ln Z are

$(\ln Z)_{2\text{-loop}}=$ 

A full curve represents the χ_1 propagator and a broken curve the χ_2 propagator. The four-point vertices each have a factor $-\lambda/4$ and the three-point vertices a factor $\sqrt{2}\lambda\xi\sim c\sqrt{\lambda}$. So each diagram as a whole has a λ factor. From the expression $\Pi=-2(\delta\ln Z/\delta\mathscr{D}_0)_{1\mathrm{PI}}$ the self-energy contributions are



All diagrams are naively $\mathrm{O}(\lambda T^2)$. Looking at small w, $|\boldsymbol{p}|$ and high T, the exchange diagrams



may be neglected. It is then found that

$$\Pi_1^{\text{mat}}=\Pi_2^{\text{mat}}=\tfrac{1}{3}\lambda T^2. \tag{3.21}$$

Hence

$$m_1^2=\bar{m}_1^2+\Pi_1^{\text{mat}}=\begin{cases}2c^2(1-T^2/T_c^2) & T<T_c\\ \frac{1}{3}\lambda(T^2-T_c^2) & T>T_c\end{cases} \tag{3.22}$$

$$m_2^2=\bar{m}_2^2+\Pi_2^{\text{mat}}=\begin{cases}0 & T<T_c\\ \frac{1}{3}\lambda(T^2-T_c^2) & T>T_c.\end{cases} \tag{3.23}$$

These masses have all the desired characteristics; they are always non-negative and there is a Goldstone boson for $T < T_c$. However we should really do a proper self-consistent calculation [10].

Carrying out a self-consistent calculation means expanding in terms of the mean-field and the full propagators. Consider $\Omega = \Omega(\xi, T, \mathscr{D}_1, \mathscr{D}_2)$ where

$$\mathscr{D}_1^{-1} = w_n^2 + \boldsymbol{p}^2 + \bar{m}_1^2 + \Pi_1(w_n, \boldsymbol{p}) \tag{3.24}$$

$$\mathscr{D}_2^{-1} = w_n^2 + \boldsymbol{p}^2 + \bar{m}_2^2 + \Pi_2(w_n, \boldsymbol{p}). \tag{3.25}$$

Define

$$(\bar{\mathscr{D}}_i^0)^{-1} = w_n^2 + \boldsymbol{p}^2 + \bar{m}_i^2. \tag{3.26}$$

Adding to the quadratic part of the action the term

$$-\tfrac{1}{2}\beta^2 \Sigma_n \Sigma_p [\chi_{1,-n}(-\boldsymbol{p})\Pi_1(w_n, \boldsymbol{p})\chi_{1,n}(\boldsymbol{p}) + \chi_{2,-n}(-\boldsymbol{p})\Pi_2(w_n, \boldsymbol{p})\chi_{2,n}(\boldsymbol{p})] \tag{3.27}$$

and treating it as a counter-term when we subtract it again from the interacting part, we arrive at the result:

$$\Omega = U(\xi) - \tfrac{1}{2}T\Sigma_n \int \frac{d^3p}{(2\pi)^3} [\ln(T^2\mathscr{D}_1) + \ln(T^2\mathscr{D}_2) - \mathscr{D}_1(\bar{\mathscr{D}}_1^0)^{-1} - \mathscr{D}_2(\bar{\mathscr{D}}_2^0)^{-1} + 2]$$
$$+ \sum_{l=2}^{\infty} \Omega_l(\xi, \mathscr{D}_1, \mathscr{D}_2) + \text{subtractions} \tag{3.28}$$

where l is the number of loops. Minimising with respect to ξ, $\mathscr{D}_1$ and $\mathscr{D}_2$ then gives Schwinger–Dyson equations such as [11]

$$(\mathscr{D}_1)^{-1} - (\bar{\mathscr{D}}_1^0)^{-1} = 2\sum_{l=2}^{\infty} \delta\Omega_l/\delta\mathscr{D}_1. \tag{3.29}$$

This model displays many of the problems associated with symmetry breaking at high temperature. The interested student can now proceed to study more realistic physical theories, such as the Weinberg–Salam model, and apply the results to the early universe.

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