# A Remark on Lorentz Violation at Finite Temperature

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## Abstract

We investigate the radiatively induced Chern-Simons-like term in four-dimensional field theory at finite temperature. The Chern-Simons-like term is temperature dependent and breaks the Lorentz and CPT symmetries. We find that this term remains undetermined although it can be found unambiguously in different regularization schemes at finite temperature.

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## I. INTRODUCTION

In the last years, it has been discussed in the literature if the Lorentz and CPT symmetries are exact or not [1, 2, 3, 4, 5, 6, 7, 8, 9]. Theoretical investigations have pointed out that these symmetries can be approximate. The spontaneous breakdown of Lorentz symmetry was first studied in string theory [10]. The modern quantum field theory also admits such phenomenon from the theoretical point of view. In other words, the basic laws of nature have Lorentz and CPT symmetries, but the vacuum solution of the theory could spontaneous violate these symmetries. This mechanism is identical to Higgs mechanism for particle field theory.

The standard model extension [5] which preserves underlying properties that are renormalizability, unitarity and gauge invariance, in the vacuum state appears to be filled up by fields that select a fixed direction in spacetime, and explicitly violate Lorentz and CPT symmetries. The realization of this violation can be obtained in the QED by adding the term $\frac{1}{2}k_\mu\epsilon^{\mu\alpha\beta\gamma}F_{\alpha\beta}A_\gamma$ to the Maxwell's theory — this is a Chern-Simons-like term with a constant vector $k_\mu$ in four dimensions — and another term which is a CPT-odd kinetic term for fermions, i.e., $\bar{\psi}b_\mu\gamma^\mu\psi$ with a constant vector $b_\mu$ [1]. Such an extension of QED does not break the gauge symmetry of the action and equations of motion but it does modify the dispersion relations for different polarization of photons and Dirac's spinors.

The dynamical origin of the parameters $k_\mu$ and $b_\mu$ present in the Lorentz and CPT symmetry breaking has called attention of a great number of people and is an interesting problem to be analyzed. In this respect, a relation between $k_\mu$ and $b_\mu$ is obtained when we integrate over the fermion fields in the modified Dirac action such that radiative corrections may lead to $k_\mu = Cb_\mu$. This result introduces a modification of the Electrodynamics, which allows for the explicit violation of Lorentz and CPT symmetries. The issue has been carefully investigated in several different contexts leading to results where $C$ does vanish [4] and results where $C$ does not [7]. See Refs. [11, 12, 13, 14], for further details on such issues. In this work we will analyze the behavior of the parameter $C$ when we take temperature into account. We do this by using derivative expansion method of the fermion determinant [15, 16, 17, 18, 19] and the imaginary time formalism. By comparing with results in the literature [20, 21, 22], we find that although this term can be found unambiguously in different regularization schemes at finite temperature, it remains undetermined. We then conclude that its value can be only fixed by phenomenological constraints [14].

## II. THE MODEL

In this section we shall induce the Chern-Simons-like term [23]. CPT and Lorentz symmetries are violated in the fermionic sector as

$$\mathcal{L} = \bar{\psi}\left[i\not{\partial} - m - \gamma_5 \not{b} - e\not{A}\right]\psi, \tag{1}$$

where $b_\mu$ is a constant 4-vector which selects a fixed direction in the space-time.

The corresponding generating functional is

$$Z[A] = \int D\bar{\psi}(x) D\psi(x) \exp\left[i \int \mathcal{L} \;\; d^4x\right] \tag{2}$$

We substitute Eq.(1) into Eq.(2) and integrate over the fermion field to obtain

$$Z[b, A] = Det(i\not{\partial} - m - \gamma_5\not{b} - e\not{A}) = \exp[iS_{eff}[b, A]]. \tag{3}$$

Since we assume the field $A_\mu$ as an external field, no Legendre transformation is required to go from the connected vacuum functional to effective action. Then, the effective action takes the form

$$S_{eff}[b, A] = -iTr\ln[i\not{p} - m - \gamma_5\not{b} - e\not{A}]. \tag{4}$$

Here $Tr$ stands for the trace over Dirac matrices, trace over the internal space as well as for the integrations in momentum and coordinate spaces. We use this expression to write

$$S_{eff}[A] = S^{(0)}_{eff}[b] + S^{(1)}_{eff}[b, A]. \tag{5}$$

Since the term $S^{(0)}_{eff}[b]$ is independent of the gauge field and cannot induce Chern-Simons, we shall focus only on the second term $S^{(1)}_{eff}[b, A]$ given by

$$\begin{aligned} S^{(1)}_{eff}[b, A] &= -i\int_0^1 dz \int \frac{d^4p}{(2\pi)^4} \\ &\times\, tr\left[\frac{1}{\not{p} - m - \gamma_5\not{b} - ze\not{A}(x - i\frac{\partial}{\partial p_\mu})}\not{A}(x)\right]. \end{aligned} \tag{6}$$

Let us make use of the relation

$$\frac{1}{A - B} = \frac{1}{A} + \frac{1}{A}B\frac{1}{A} + \frac{1}{A}B\frac{1}{A}B\frac{1}{A} + \cdots$$

and consider $A = \not{p} - m - \gamma_5 \not{b}$ and $B = ze\not{A}(x - i\frac{\partial}{\partial p_\mu})$. We can manipulate the Eq.(6) to keep only first order derivative terms which are linear in $\not{b}$ and quadratic in $\not{A}$. Carrying out the integral in $z$ gives

$$\begin{aligned} S^{(1)}_{eff}[b, A] &= -\frac{e^2}{2}\int d^4x \int \frac{d^4p}{(2\pi)^4} \\ &\times \operatorname{tr}\left[\frac{1}{\not{p}-m} i\partial_\mu \not{A} \frac{\partial}{\partial p_\mu}\frac{1}{\not{p}-m}\gamma_5\not{b}\frac{1}{\not{p}-m}\,\not{A}\right. \\ &+ \left.\frac{1}{\not{p}-m}\gamma_5\not{b}\frac{1}{\not{p}-m} i\,\partial_\mu\not{A}\frac{\partial}{\partial p_\mu}\frac{1}{\not{p}-m}\not{A}\right], \end{aligned} \tag{7}$$

where we have used the relation

$$\frac{\partial}{\partial p_\mu}\frac{1}{\not{p}-m} = -\frac{1}{\not{p}-m}\gamma^\mu\frac{1}{\not{p}-m}.$$

Now taking the traces of the products of $\gamma$ matrices on relevant terms, i.e., the terms that contain $tr\,\gamma_5\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta$, the Eq.(7) takes the form

$$S^{(1)}_{eff}[b, A] = -\frac{e^2}{2}\int d^4\,x \int \frac{d^4\,p}{(2\pi)^4}\frac{N}{(p^2-m^2)^4}, \tag{8}$$

where $N$ is given by

$$\begin{aligned} N &= -4i(p^2-m^2)\left[\epsilon^{\alpha\beta\mu\sigma}\left(3m^2+p^2\right) - 4\epsilon^{\alpha\beta\mu\nu}p_\nu p^\sigma\right] \\ &\times\ b_\sigma\partial_\mu A_\alpha A_\beta. \end{aligned} \tag{9}$$

Note that by power counting the momentum integral in Eq. (8) have terms with logarithmic divergence. Let us use the relation [7, 8, 14]

$$\int \frac{d^Dq}{(2\pi)^D} q_\mu q_\nu f(q^2) = \frac{g_{\mu\nu}}{D}\int\frac{d^Dq}{(2\pi)^D} q^2 f(q^2), \tag{10}$$

that naturally removes the logarithmic divergence. Now, considering $D = 4$, the terms containing $p^2$ and $p_\nu p^\sigma$ in (9) cancel out and we find

$$N = -12m^2 i(p^2-m^2)\epsilon^{\alpha\beta\mu\sigma}b_\sigma\partial_\mu A_\alpha A_\beta. \tag{11}$$

In this way, the logarithmic divergence in (8) disappears, so that the effective action now reads

$$\begin{aligned} S^{(1)}_{eff}[b, A] &= \left[6im^2e^2\int\frac{d^4\,p}{(2\pi)^4}\frac{1}{(p^2-m^2)^3}\right] \\ &\times\ \epsilon^{\alpha\beta\mu\sigma}b_\sigma\int d^4\,x\partial_\mu A_\alpha A_\beta, \end{aligned} \tag{12}$$

which is finite by power counting. Evaluating the momentum integral in the (12) we obtain unambiguously the Chern-Simons coefficient [7]

$$k_\mu = \frac{3e^2}{16\pi^2} b_\mu. \tag{13}$$

However, if we use another regularization scheme $k_\mu$ may vanish, as for instance, in Pauli-Villars regularization scheme [5]. The fact that the value of $k_\mu$ depends on the regularization scheme, corresponds to an "ambiguity", i.e., finite values but undetermined ones [11]. This issue has been well discussed in the literature [11, 12, 13, 14]. Next we study such undetermined coefficient when we take temperature into account.

Let us now assume that the system is at thermal equilibrium with a temperature $T = 1/\beta$. In this case we can use Matsubara formalism, for fermions, which consists in taking $p_0 = (n+1/2)2\pi/\beta$ and changing $(1/2\pi)\int dp_0 = 1/\beta \sum_n$ [24]. We also change the Minkowski space to Euclidean space, by making $x_0 = -ix_4$, $p_0 = ip_4$ and $b_0 = ib_4$, such that $p^2 = -p_E^2$, $p_E^2 = \mathbf{p}^2 + p_4^2$, $d^4p = id^4p_E$ and $d^4x = -id^4x_E$. Now the Eq.(12) can be written as

$$S_{eff}^{(1)}[b, A] = 6e^2 f(m^2, \beta)\epsilon^{\alpha\beta\mu\sigma} b_\sigma \int (-i) d^4\, x_E \partial_\mu A_\alpha A_\beta, \tag{14}$$

where $f(m^2, \beta)$, is the Chern-Simons coefficient dependent on the temperature which is given by

$$\begin{aligned} f(m^2, \beta) &= \frac{m^2}{\beta} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \sum_{n=-\infty}^{\infty} \frac{1}{(\mathbf{p}^2 + p_4^2 + m^2)^3} \\ &= \frac{m^2}{2\beta} \frac{d^2}{d(m^2)^2} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \sum_{n=-\infty}^{\infty} \frac{1}{(\mathbf{p}^2 + p_4^2 + m^2)^2}. \end{aligned} \tag{15}$$

We calculate the momentum integral by adopting dimensional regularization scheme to obtain

$$f(m^2, \beta) = \frac{m^2}{2\beta} \frac{\Gamma(3 - D/2)}{(4\pi)^{D/2}} \sum_{n=-\infty}^{\infty} \frac{1}{(p_4^2 + m^2)^{3-D/2}}. \tag{16}$$

To perform summation we shall use below an explicit representation for the sum over the Matsubara frequencies [25]:

$$\begin{aligned} \sum_n [(n+b)^2 + a^2]^{-\lambda} &= \frac{\sqrt{\pi}\,\Gamma(\lambda - 1/2)}{\Gamma(\lambda)(a^2)^{\lambda - 1/2}} \\ &+ 4\sin(\pi\lambda) \int_{|a|}^{\infty} \frac{dz}{(z^2 - a^2)^\lambda} Re\left(\frac{1}{\exp 2\pi(z + ib) - 1}\right), \end{aligned} \tag{17}$$

which is valid for $1/2 < \lambda < 1$. This implies that for $\lambda = 3 - D/2$ as given in Eq.(16) we cannot apply this relation for $D = 3$, because the integral in (17) does not converge. Thus, let us perform the analytical continuation of this relation, so that we obtain

$$\int_{|a|}^{\infty} \frac{dz}{(z^2-a^2)^{\lambda}} Re\left(\frac{1}{\exp 2\pi(z+ib)-1}\right) =$$
$$= \frac{1}{2a^2}\frac{3-2\lambda}{1-\lambda}\int_{|a|}^{\infty}\frac{dz}{(z^2-a^2)^{\lambda-1}} Re\left(\frac{1}{\exp 2\pi(z+ib)-1}\right)$$
$$- \frac{1}{4a^2}\frac{1}{(2-\lambda)(1-\lambda)}\int_{|a|}^{\infty}\frac{dz}{(z^2-a^2)^{\lambda-2}}\frac{d^2}{dz^2} Re\left(\frac{1}{\exp 2\pi(z+ib)-1}\right). \tag{18}$$

Now for $D = 3$ the Eq.(16) takes the form

$$f(m^2,\beta) = \frac{1}{32\pi^2} + \frac{1}{16}F(\xi), \tag{19}$$

where $\xi = \frac{\beta m}{2\pi}$ and the function

$$F(\xi) = \int_{|\xi|}^{\infty} dz (z^2-\xi^2)^{1/2}\frac{\tanh(\pi z)}{\cosh^2(\pi z)}, \tag{20}$$

approaches the limits: $F(\xi \to \infty) \to 0$ $(T \to 0)$ and $F(\xi \to 0) \to 1/2\pi^2$ $(T \to \infty)$ — see Fig.1. Thus, we see that at high temperature the Chern-Simons coefficient is twice its value at zero temperature, i.e., $f(m^2, \beta \to 0) = 1/16\pi^2$. On the other hand, at zero temperature, one recovers the result (13).

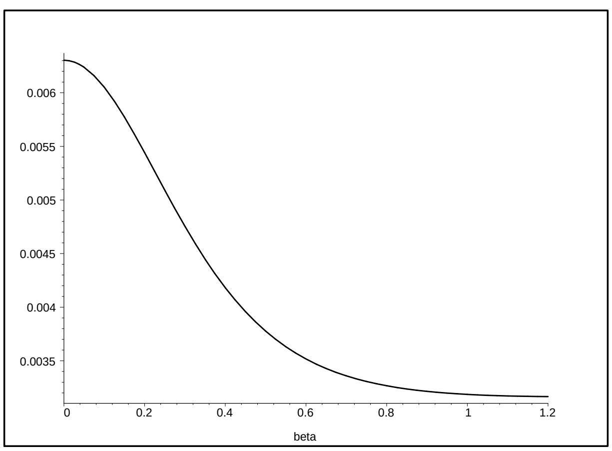


FIG. 1: The function $f(m^2, \beta)$ is diferent from zero everywhere. At zero temperature $(\beta \to \infty)$, the function tends to a nonzero value $1/32\pi^2$.

## III. CONCLUSIONS

We have studied the induction of Chern-Simons-like term at finite temperature. We adopted dimensional regularization to evaluate momentum integrals. Our result is finite, but does not fully agree with other results in the literature. We argue that this is due to different regularization schemes. We find that at zero temperature limit we recover the result found at zero temperature in Ref.[7]. In such limit our result leads to a nonzero Chern-Simons-like term, a behavior also predicted in Ref.[20]-obtained with the use of dimensional regularization- and the result in Ref.[22]- obtained with the use of cut off regularization scheme. However, it is in conflict with the result found in Ref. [21] which suggests the vanishing of the Chern-Simons-like term at zero temperature. On the other hand, at high temperature our result behaves as the result of Ref.[21]. But now, however, it conflicts with the results in Ref.[20] and in Ref.[22] which predict that the Chern-Simons-like term vanishes at high temperature. These results are all finite, and they show that the Chern-Simons-like coefficient is indeed undetermined just as it happens at zero temperature — see Ref.[11]. Our result supports the understanding that the Chern-Simons-like term can only be determined by phenomenological constraints, as already emphasized in Ref.[14].

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