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MAGNETIC MONOPOLES AND DYONS IN $N = 1$ SUPERSYMMETRIC THEORIES*

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We discuss the structure of magnetic monopoles and dyons in $N = 1$ supersymmetric theories which have a spontaneously broken gauge symmetry. The lowest order mass splittings between states of the monopole supermultiplet are evaluated in a model with softly broken supersymmetry.

1. Introduction

The resolution of the gauge hierarchy problem may lie in describing physics below the Planck scale by a simple ($N = 1$) supersymmetric grand unified theory. In most such theories, the spontaneous breaking of the grand unified gauge symmetry at a superheavy scale M_x leads to the presence of magnetic monopoles [1] and dyons [2] with masses of order $\alpha^{-1}M_x$. Since no supersymmetric partners for the observed particles have been detected, supersymmetry must be broken. If supersymmetry is broken at a scale much less than M_x, the monopoles and dyons must form supermultiplets of approximately degenerate boson and fermion states★.

In a quantum theory, monopoles and dyons are represented by states in which the expectation values of field operators are given by the classical monopole solution, to lowest order in the small coupling expansion [4, 5]. Because the classical solution depends on a choice of position $\boldsymbol{X}$ for the center of the monopole and on an electromagnetic charge rotation angle α★★, it is necessary to introduce states $|\boldsymbol{X}, \alpha\rangle$

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★ Some results pertaining to monopoles in $N = 2$ supersymmetric theories may be found in ref. [3].

★★ I.e., given any classical monopole solution $\varphi^{\mathrm{cl}}(\boldsymbol{x})$, we may form other classical solutions $\varphi^{\mathrm{cl}}(\boldsymbol{x}-\boldsymbol{X}, \alpha) = e^{-iQ\alpha}\varphi^{\mathrm{cl}}(\boldsymbol{x}-\boldsymbol{X})$ in which Q is the electric charge generator. Also, in realistic theories, the monopoles and dyons will carry other quantum numbers such as color. This complication is immaterial and will be ignored.

labeled by these parameters. Alternatively, we may use momentum and charge eigenstates,

$$|\boldsymbol{P}, q\rangle = \frac{1}{\sqrt{V}} \int \mathrm{d}^3 X \frac{\mathrm{d}\alpha}{2\pi} \mathrm{e}^{i\boldsymbol{P}\cdot\boldsymbol{X}} \mathrm{e}^{iq\alpha} |\boldsymbol{X}, \alpha\rangle , \tag{1}$$

which correspond to the classical monopole in the sense that

$$\langle \boldsymbol{P}, q | \varphi(\boldsymbol{x}) | \boldsymbol{P}', q' \rangle = \frac{1}{V} \int \mathrm{d}^3 X \frac{\mathrm{d}\alpha}{2\pi} \mathrm{e}^{i(\boldsymbol{P}'-\boldsymbol{P})\cdot\boldsymbol{X}} \mathrm{e}^{i(q'-q)\alpha} \varphi^{\mathrm{cl}}(\boldsymbol{x}-\boldsymbol{X}, \alpha) , \tag{2}$$

to leading order in the coupling g. Here $\varphi(\boldsymbol{x})$ denotes an arbitrary quantum field, while $\varphi^{\mathrm{cl}}(\boldsymbol{x}-\boldsymbol{X}, \alpha)$ is the classical monopole solution with center at $\boldsymbol{X}$ and charge rotation angle α. The momentum states are normalized to unity in a box of volume V.

In a supersymmetric theory, the monopole and dyon states must form representations of the supersymmetry algebra [6]

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0 , \tag{3a}$$

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\sigma^{\mu}_{\alpha\dot{\alpha}} P_\mu . \tag{3b}$$

The simplest ansatz for realizing monopoles and dyons at rest in a supersymmetric theory thus consists of a set of supermultiplets containing the states

$$|0, q\rangle , \qquad \text{with} \qquad \bar{Q}_{\dot{\alpha}} |0, q\rangle = 0 , \tag{4a}$$

$$|\tfrac{1}{2}, q\rangle = \frac{1}{\sqrt{2M}} Q_1 |0, q\rangle , \tag{4b}$$

$$|-\tfrac{1}{2}, q\rangle = \frac{1}{\sqrt{2M}} Q_2 |0, q\rangle , \tag{4c}$$

$$|\tilde{0}, q\rangle = \frac{1}{2M} Q_1 Q_2 |0, q\rangle . \tag{4d}$$

Here $|0, q\rangle$ and $|\tilde{0}, q\rangle$ are spin-0 monopole and dyon states, while $|\frac{1}{2}, q\rangle$ and $|-\frac{1}{2}, q\rangle$ are spin-$\frac{1}{2}$ monopole and dyon states with spin projection along the z axis $\pm\frac{1}{2}$, respectively. The states have zero momentum and electric charge q. States with non-zero momentum may be constructed by Lorentz transformations.

The correspondence between these states and the classical monopole solution has already been noted in eq. (2). Because the monopole mass M in eqs. (4) is of order g^{-2}, it is easy to show that the states of the ansatz all correspond to the same classical solution. It is not necessary to include larger angular momenta in the monopole and dyon supermultiplets whenever there are no spin degeneracies beyond those required by supersymmetry*. Supermultiplets containing higher spins arise as bound or scattering states of a monopole or dyon and other particles.

In this paper, we explore the monopole spectrum in a supersymmetric version of the Georgi-Glashow model. In sect. 2, we show that the states of the ansatz coincide with those inferred [4,5] from zero energy solutions of the linearized equations of motion in the presence of the classical monopole field. In sect. 3, we compute the lowest order mass splittings between states of the monopole supermultiplet which arise when supersymmetry is explicitly broken by mass terms. Concluding remarks are given in sect. 4, while two appendices contain further details and summarize the conventions.

2. Zero modes

The supersymmetric Georgi-Glashow model [9] has an SO(3) gauge group with generators T_a ($a = 1, 2, 3,$) and gauge coupling constant g. In Wess-Zumino gauge [10], the gauge superfield V_a has components G_a^μ (vector), λ_a (left-handed spinor) and D_a (auxiliary). We also include two chiral matter multiplets: a triplet Φ_a and a singlet X. These have complex scalar, left-handed fermion and auxiliary components φ_a, ψ_a and F_a and φ_x, ψ_x and F_x, respectively. The superspace potential for the model is

$$W = \tfrac{1}{2} a X \left(\Phi_a \Phi_a - \frac{m^2}{2g^2} \right), \tag{5}$$

in which the coupling constant a and mass parameter m are chosen real and positive.

In the Higgs vacuum of the model, supersymmetry is unbroken, while the SO(3) gauge symmetry is broken down to an electromagnetic U(1) by the VEVs

$$\langle \varphi_x \rangle = 0, \qquad \langle \varphi_a \rangle = \frac{m}{\sqrt{2}\, g} \delta_{ai} \hat{r}^i. \tag{6}$$

Here $\hat{r}^i = r^i/r$ is the radial unit vector. (We use radial gauge in which $\varphi_a \propto \delta_{ai} \hat{r}^i$ throughout the paper.) The model possesses a 't Hooft-Polyakov classical monopole

* In the Georgi-Glashow model, higher spin states are not needed because the classical monopole solution is rotationally invariant [7]. Higher spin states may also be associated with isospin degeneracies of the classical solution. For example, Jackiw and Rebbi [8] have exhibited a model in which the monopole has spin-$\frac{1}{2}$. In a supersymmetric version of that model, the monopole supermultiplet would contain spins $\frac{1}{2}$, 0, 1 and $\frac{1}{2}$. See also the following footnote.

solution [1] in which the non-zero fields are

$$\varphi_a^{\mathrm{cl}} = \frac{\delta_{ai}\hat{r}^i}{gr} H(mr), \tag{7a}$$

$$G_{ai}^{\mathrm{cl}} = \varepsilon_{ijk} \frac{\delta_{aj}\hat{r}^k}{gr} (1 - K(mr)), \tag{7b}$$

$$F_x^{\mathrm{cl}} = -\tfrac{1}{2} a \left(\varphi_a^{\mathrm{cl}} \varphi_a^{\mathrm{cl}} - \frac{m^2}{2g^2} \right). \tag{7c}$$

The classical monopole mass is then

$$M = \int \mathrm{d}^3x \left\{ \tfrac{1}{4}\left(G_{aij}^{\mathrm{cl}}\right)^2 + \left(D_i\varphi_a^{\mathrm{cl}}\right)^2 + \left(F_x^{\mathrm{cl}}\right)^2 \right\}. \tag{8}$$

In the Prasad-Sommerfield [11] limit ($a \to 0$), we have $M = 4\pi m/g^2$.

In quantizing around the classical monopole solution, excitations of zero energy modes provide new monopole or dyon states [4, 5]. Excitations of the zero modes associated with translations and electromagnetic charge rotations lead to the momentum and charge states of eq. (1). There are no zero modes associated with rotations of the monopole because the classical solution (7) is invariant with respect to the rotation operator $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S} + \boldsymbol{T}$ which preserves radial gauge.

The states related by supersymmetry are associated with fermionic zero modes [12]; i.e., with static solutions of the fermion equation of motion

$$\begin{pmatrix} i\bar{\sigma}_\mu D_{ab}^\mu & -\sqrt{2}\, g\varphi_c^{\mathrm{cl}}(T_c)_{ab} & 0 \\ -\sqrt{2}\, g\varphi_c^{\mathrm{cl}}(T_c)_{ab} & i\sigma_\mu D_{ab}^\mu & -\varphi_a^{\mathrm{cl}} \\ 0 & -a\varphi_b^{\mathrm{cl}} & i\bar{\sigma}_\mu \partial^\mu \end{pmatrix} \begin{pmatrix} \lambda_b \\ \bar{\psi}_b \\ \psi_x \end{pmatrix} = 0 \tag{9}$$

in the presence of the classical monopole field. Rather than solving eq. (9), we obtain the zero modes associated with supersymmetry by making an infinitesimal supersymmetry transformation followed by an infinitesimal supergauge transformation which restores Wess-Zumino gauge [13]. The rules for such transformations are derived in appendix B. Applying these to the classical solution (7) produces the static, c-number fermion fields:

$$\lambda_a = -\tfrac{1}{2} N \sigma^{\mu\nu} \zeta G_{a\mu\nu}^{\mathrm{cl}}, \tag{10a}$$

$$\bar{\psi}_a = -i\sqrt{2}\, N \bar{\sigma}^\mu \zeta D_\mu \varphi_a^{\mathrm{cl}}, \tag{10b}$$

$$\psi_x = \sqrt{2}\, N \zeta F_x^{\mathrm{cl}}. \tag{10c}$$

Here ζ is an arbitrary two-component, c-number spinor normalized by $\zeta\sigma^0\bar{\zeta} = 1$. The normalization constant $N = 1/\sqrt{2M}$ with M given by eq. (8) is chosen so that the zero modes are normalized to unit probability

$$\int d^3x \{\lambda_a \sigma^0 \bar{\lambda}_a + \psi_a \sigma^0 \bar{\psi}_a + \psi_x \sigma^0 \bar{\psi}_x\} = 1 . \tag{11}$$

It is straightforward to verify that the fermion fields (10) actually solve the "Dirac equation" (9). Indeed, zero modes constructed in this manner must exist in any supersymmetric theory which contains monopoles. The zero modes (10) form a $J = \frac{1}{2}$ doublet, and excitations of these modes provide four states (for each momentum $\boldsymbol{P}$ and charge q) which coincide with the states $|0\rangle$, $|\frac{1}{2}\rangle$, $|-\frac{1}{2}\rangle$ and $|\tilde{0}\rangle$ discussed in the introduction*. Following Jackiw and Rebbi [5], we note that eq. (4) may be used to show that the zero modes (10) are the leading $O(g^0)$ terms in off-diagonal matrix elements of the fermion fields.

3. Mass splittings

In this section, we consider the addition of a gauge invariant perturbation H' which softly breaks supersymmetry and compute the lowest order mass splittings between the members of the monopole supermultiplet. In the presence of such a term, the fermion states $|\frac{1}{2}\rangle$ and $|-\frac{1}{2}\rangle$ with charge q remain degenerate with mass $M_q^{(F)}$. In the basis $\{|0\rangle, |\tilde{0}\rangle\}$, we write the mass matrix for the spin-0 states with charge q as

$$M_q^{(B)} = \begin{pmatrix} M_q^{(F)} + \Delta_q & d_q \\ d_q^* & M_q^{(F)} - \Delta_q + 2\delta_q \end{pmatrix} . \tag{12}$$

Using eq. (4), we have, to linear order in H', that

$$\begin{aligned} \Delta_q &= \langle 0, q|H'|0, q\rangle - \langle \tfrac{1}{2}, q|H'|\tfrac{1}{2}, q\rangle \\ &= \frac{1}{4M} (\bar{\sigma}^0)^{\dot{\alpha}\alpha} \langle 0, q|\{\bar{Q}_{\dot{\alpha}}, [Q_\alpha, H']\}|0, q\rangle , \end{aligned} \tag{13}$$

* We have not addressed the question of whether eq. (9) has any further zero modes. Such zero modes would introduce extra spin degeneracies which could be removed by quantum corrections. In a one-dimensional supersymmetric model (c.f., Olive and Witten, ref. [3]), there is a unique zero mode [5] which coincides with that found by the method used here.

where the results using the $|\frac{1}{2}\rangle$ and $|-\frac{1}{2}\rangle$ states have been averaged. Similarly,

$$d_q = -\frac{1}{4M}\langle 0, q|\{Q^\alpha, [Q_\alpha, H']\}|0, q\rangle, \tag{14}$$

$$\delta_q = \frac{1}{32M^2}\langle 0, q|\{\bar{Q}_{\dot\alpha}, [Q^\alpha, \{\bar{Q}^{\dot\alpha}, [Q_\alpha, H']\}]\}|0, q\rangle. \tag{15}$$

Note that the mass-squared sum rule [14] is satisfied if and only if $\delta_q = 0$ [15].

Prior to evaluating the matrix elements (13–15) for specific choices of H', it is useful to derive selection rules which restrict the form of the mass splittings. The supersymmetric Georgi-Glashow model has two discrete symmetries: parity (P) which is described in appendix A and G parity (G) under which V_a and X are even while Φ_a is odd. Both P and G are spontaneously broken in the Higgs vacuum (6). However, the product PG is unbroken and is also a symmetry of the classical solution (7)*. Since the electric charge is odd under PG and since

$$(PG)Q_\alpha(PG)^{-1} = i\bar{Q}^{\dot\alpha}, \qquad (PG)\bar{Q}^{\dot\alpha}(PG)^{-1} = -iQ_\alpha, \tag{16}$$

we have

$$(PG)|0, q\rangle = |\tilde{0}, -q\rangle, \qquad (PG)|\tilde{0}, q\rangle = |0, -q\rangle. \tag{17}$$

Assuming H' to be PG invariant, it follows that

$$M_q^{(\mathrm{F})} = M_{-q}^{(\mathrm{F})}, \qquad d_q = d_{-q}^*, \tag{18a}$$

$$\delta_q = \delta_{-q} = \tfrac{1}{2}(\Delta_q + \Delta_{-q}). \tag{18b}$$

The first of these is obtained by applying PG to the spin-$\frac{1}{2}$ states $|\pm\frac{1}{2}\rangle$.

The supersymmetric Georgi-Glashow model also possesses an unbroken R symmetry [16] under which V_a and Φ_a have zero R character while X has R character two**. Because

$$[R, Q_a] = -Q_\alpha, \qquad [R, \bar{Q}_{\dot\alpha}] = \bar{Q}_{\dot\alpha}, \tag{19}$$

in which R denotes the R symmetry charge, the states $|0\rangle$, $|\frac{1}{2}\rangle$, $|-\frac{1}{2}\rangle$, and $|\tilde{0}\rangle$ with

* PG can be regarded as the CP operator which preserves radial gauge. In unitary or string gauge ($\varphi_a \propto \delta_{a3}$), the action of PG is supplemented by a 180° gauge rotation $e^{i\pi\varphi\cdot T}$ which reverses the sign of the VEV in the Higgs vacuum and which moves the string from the plus z axis to the minus z axis. Here φ is the azimuthal unit vector in polar coordinates.

** The R symmetry current is not anomalous because the contributions of matter and gauge fermions to this anomaly cancel.

charge q must have R charges r_q+1, r_q, r_q and r_q-1, respectively. Incidentally, since $(PG)R(PG)^{-1}=-R$, we have $r_q=-r_{-q}$. There are two interesting cases for H':

(i) $\Delta R=0$. In this case, $d_q=0$ and the $|0\rangle$ and $|\tilde{0}\rangle$ states are not mixed by the perturbation.

(ii) $|\Delta R|=2$. In this case, $\Delta_q=\delta_q=0$ to linear order in H'. The mass-squared sum rule is satisfied and the bosons have masses $M_q^{(\mathrm{F})}\pm|d_q|$.

For perturbations with any other values of ΔR, all of the corrections to the masses which are linear in H' must vanish.

In the following, we evaluate the mass splittings for three choices of the perturbation H'. For convenience, we work only with the electrically neutral monopole states. We begin with the gauge fermion mass term

$$H'_\lambda=\mu_\lambda\int \mathrm{d}^3x\,\lambda_a\lambda_a+\text{h.c.} \tag{20}$$

For real μ_λ, this perturbation is PG-invariant and has $|\Delta R|=2$. It follows that, of the matrix elements (13–15) with $q=0$, only $\mathrm{Re}(d_0)$ can be non-zero. The spin-0 monopole masses are $M_0^{(\mathrm{F})}\pm d_0$.

We can evaluate d_0 using eq. (14) and the transformation rules derived in appendix B. Alternatively, by writing H'_λ in the manifestly supergauge invariant form

$$H'_\lambda=\mu_\lambda\int \mathrm{d}^3x\,\mathrm{d}^4\vartheta\,\vartheta^2\bar{\vartheta}^2\left(W_aW_a+\bar{W}_a\bar{W}_a\right), \tag{21}$$

in which W_a is the chiral superfield strength, we can use the usual realization of supersymmetry transformations in terms of the spinor differential operators Q_α and $\bar{Q}^{\dot{\alpha}}$ of eq. (B.3). The terms of these operators which contain space-time derivatives do not contribute. Integrating the spinor derivatives by parts then gives

$$\begin{aligned} d_0&=\frac{\mu_\lambda}{M}\int \mathrm{d}^3x\,\mathrm{d}^2\vartheta\,\langle 0|W_aW_a|0\rangle \\ &=\frac{\mu_\lambda}{M}\int \mathrm{d}^3x\,\langle 0|\tfrac{1}{2}G_a^{\mu\nu}G_{a\mu\nu}-2i\bar{\lambda}_a\bar{\sigma}_\mu D^\mu\lambda_a-D_a^2+\tfrac{1}{4}i\varepsilon_{\mu\nu\kappa\lambda}G_a^{\mu\nu}G_a^{\kappa\lambda}|0\rangle . \end{aligned} \tag{22}$$

To leading order in the coupling, this may be evaluated using the classical solution (7). The result is

$$d_0=\frac{\mu_\lambda}{2M}\int \mathrm{d}^3x\left(G_{aij}^{\mathrm{cl}}\right)^2. \tag{23}$$

In the Prasad-Sommerfield limit, $d_0=\mu_\lambda$.

Next we consider adding a mass term for the triplet of scalars,

$$H'_\varphi = \mu_\varphi^2 \int d^3x\, \varphi_a^* \varphi_a = \mu_\varphi^2 \int d^3x\, d^4\vartheta\, \vartheta^2 \bar{\vartheta}^2 \Phi^* e^{2V} \Phi , \tag{24}$$

where $V = gV_aT_a$. Since H'_φ is invariant under both PG and R, the spin-0 monopoles remain degenerate and are split from the spin-$\frac{1}{2}$ monopoles by the amount $\Delta_0 = \delta_0$. Proceeding as with H'_λ, we find

$$\Delta_0 = \frac{\mu_\varphi^2}{M} \int d^3x\, d^4\vartheta\, \vartheta\sigma^0\bar{\vartheta} \langle 0|\Phi^* e^{2V}\Phi|0\rangle \tag{25a}$$

$$= -\frac{\mu_\varphi^2}{2M} \int d^3x\, \langle 0| - i\varphi_a^* \overleftrightarrow{D}^0 \varphi_a + \bar{\psi}_a \bar{\sigma}^0 \psi_a |0\rangle . \tag{25b}$$

This matrix element vanishes when evaluated using the classical monopole solution and receives its leading contribution from one-loop graphs [17]. Instead of evaluating these graphs, we compute δ_0 from

$$\delta_0 = \frac{\mu_\varphi^2}{2M^2} \int d^3x\, d^4\vartheta\, \langle 0|\Phi^* e^{2V}\Phi|0\rangle$$

$$= -\frac{\mu_\varphi^2}{2M^2} \int d^3x\, \langle 0| D_\mu\varphi_a^* D^\mu\varphi_a + i\bar{\psi}_a\bar{\sigma}_\mu D^\mu \psi_a + F_a^* F_a$$

$$- \sqrt{2}\, g\left(\varphi_a^*(T_b\psi)_a\lambda_a + \text{h.c.}\right) + D_a \varphi_b^*(T_a\varphi)_b |0\rangle . \tag{26}$$

Using the classical solution, the leading contribution is

$$\delta_0 = \frac{\mu_\varphi^2}{2M^2} \int d^3x\, |D_i \varphi_a^{\text{cl}}|^2 , \tag{27}$$

which becomes $\delta_0 = g^2\mu_\varphi^2/16\pi m$ in the Prasad-Sommerfield limit. It is somewhat surprising that the symmetries of the model relate a sum of one-loop graphs (25b) to the classical result (27). This is possible because the extra factor of M^{-1} present in δ_0 provides an extra explicit factor of g^2. For this perturbation, the lightest monopole states have spin-$\frac{1}{2}$.

Finally, when a mass term is added for the singlet scalar field,

$$H'_x = \mu_x^2 \int d^3x\, \varphi_x^* \varphi_x , \tag{28}$$

the selection rules again require the spin-0 monopoles to remain degenerate. However, in this case, the splitting

$$\Delta_0 = \delta_0 = -\frac{\mu_x^2}{2M^2}\int d^3x\, |F_x^{cl}|^2, \tag{29}$$

is negative, and the spin-0 monopole states are lighter than those with spin-$\frac{1}{2}$. This result vanishes in the Prasad-Sommerfield limit.

4. Conclusions

In sects. 1 and 2, it was shown that magnetic monopoles and dyons in $N=1$ supersymmetric theories can be consistently represented by supermultiplets containing spin-0 (smonopole) and spin-$\frac{1}{2}$ (monopolino) states. In sect. 3, we evaluated the mass splittings between smonopoles and monopolinos in a Georgi-Glashow model with softly broken supersymmetry. In general, only the lightest of these states need be stable. For example, in the Georgi-Glashow model with supersymmetry broken by a mass term for the triplet of scalars, the smonopoles may decay to a monopolino and a massless photino. In this model, the lightest dyon state for each electric charge is stable because all charged fields have masses of order m in the Higgs vacuum. However, in realistic models of this type [18], there will be light charged fermions which render the dyons unstable.

In realistic models with softly broken supersymmetry*, the scale of supersymmetry breaking is of order 1 TeV, while the superheavy scale is of order 10^{17} GeV. The mass splittings among monopole states due to gauge fermion mass terms will then be of order 1 TeV, while those due to scalar mass terms will be less than 10^{-4} eV. In view of this, we find it somewhat unlikely that the lightest monopole state is a monopolino.

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Appendix A

SPINOR CONVENTIONS AND PARITY

We write a four-component Majorana spinor

$$\Psi = \begin{pmatrix} \psi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}, \tag{A.1}$$

* Models with spontaneously broken local supersymmetry can also be represented by soft global supersymmetry breaking below the Planck scale.

in terms of a two-component left-handed spinor ψ and its right-handed complex conjugate $\bar{\psi}$. Two-valued spinor indices are raised and lowered with the antisymmetric invariants $\varepsilon_{\alpha\beta} = \varepsilon^{\alpha\beta} = \varepsilon_{\dot{\alpha}\dot{\beta}} = \varepsilon^{\dot{\alpha}\dot{\beta}}$, i.e.,

$$\psi^{\alpha} = \varepsilon^{\alpha\beta}\psi_{\beta}, \qquad \psi_{\alpha} = \psi^{\beta}\varepsilon_{\beta\alpha},$$

$$\bar{\psi}^{\dot{\alpha}} = \varepsilon^{\dot{\alpha}\dot{\beta}}\bar{\psi}_{\dot{\beta}}, \qquad \bar{\psi}_{\dot{\alpha}} = \bar{\psi}^{\dot{\beta}}\varepsilon_{\dot{\beta}\dot{\alpha}}. \tag{A.2}$$

In this basis, the Dirac matrices are

$$\gamma^{\mu} = \begin{pmatrix} 0 & (\sigma^{\mu})_{\alpha\dot{\alpha}} \\ (\bar{\sigma}^{\mu})^{\dot{\alpha}\alpha} & 0 \end{pmatrix}, \tag{A.3}$$

where $\sigma^{\mu} = (1, \boldsymbol{\sigma})$ and $\bar{\sigma}^{\mu} = (1, -\boldsymbol{\sigma})$. We also define

$$\sigma^{\mu\nu} = \tfrac{1}{2}i(\sigma^{\mu}\bar{\sigma}^{\nu} - \sigma^{\nu}\bar{\sigma}^{\mu}), \qquad \bar{\sigma}^{\mu\nu} = \tfrac{1}{2}i(\bar{\sigma}^{\mu}\sigma^{\nu} - \bar{\sigma}^{\nu}\sigma^{\mu}). \tag{A.4}$$

Parity (P) interchanges the left- and right-handed components of Ψ i.e.,

$$P\psi_{a}(x)P^{-1} = i\bar{\psi}^{\dot{\alpha}}(-x),$$

$$P\bar{\psi}^{\dot{\alpha}}(x)P^{-1} = -i\psi_{\alpha}(-x). \tag{A.5}$$

Here the phases are determined up to an overall sign by requiring the kinetic and mass terms,

$$\bar{\Psi}\gamma_{\mu}\partial^{\mu}\Psi = \psi\sigma_{\mu}\partial^{\mu}\bar{\psi} + \bar{\psi}\bar{\sigma}_{\mu}\partial^{\mu}\psi, \qquad \bar{\Psi}\Psi = \psi\psi + \bar{\psi}\bar{\psi}, \tag{A.6}$$

to be even under parity.

The parity transforms of scalars $\varphi(x)$ and vectors $G^{\mu}(x)$ are $\varphi^{*}(-x)$ and $(G^{0}(-x), -G^{i}(-x))$, respectively. From the supersymmetry transformation rules, we deduce that

$$PQ_{\alpha}P^{-1} = i\bar{Q}^{\dot{\alpha}}, \qquad P\bar{Q}^{\dot{\alpha}}P^{-1} = -iQ_{\alpha}. \tag{A.7}$$

Finally, we note that, under these rules, $P^{2} = (-1)^{F}$.

Appendix B

MAINTAINING WESS-ZUMINO GAUGE

Here we derive the supersymmetry transformation rules which preserve Wess-Zumino (WZ) gauge. We begin by considering the vector superfield in WZ gauge

$$V_{A} = \vartheta\sigma_{\mu}\bar{\vartheta}G_{A}^{\mu} + \vartheta^{2}\bar{\vartheta}\bar{\lambda}_{A} + \bar{\vartheta}^{2}\vartheta\lambda_{A} + \tfrac{1}{2}\vartheta^{2}\bar{\vartheta}^{2}D_{A}. \tag{B.1}$$

Here the index A labels the (hermitian) generators T_A of an arbitrary simple or U(1) gauge group. Under a supersymmetry transformation,

$$\delta_Q V_A = (\zeta \underset{\sim}{Q} + \overline{\zeta \underset{\sim}{Q}}) V_A, \tag{B.2}$$

where

$$\underset{\sim}{Q}_\alpha = \frac{\partial}{\partial \vartheta^\alpha} + i(\sigma_\mu \bar{\vartheta})_\alpha \partial^\mu, \qquad \bar{\underset{\sim}{Q}}^{\dot{\alpha}} = \frac{\partial}{\partial \bar{\vartheta}_{\dot{\alpha}}} + i(\bar{\sigma}_\mu \vartheta)^{\dot{\alpha}} \partial^\mu, \tag{B.3}$$

are the differential operators which realize supersymmetry transformations in superspace. Because $\delta_Q V_A$ is not in WZ gauge, we must find the compensating supergauge transformation which restores WZ gauge.

This may be done by introducing

$$\Sigma = e^{2V}, \tag{B.4}$$

where $V = g V_A T_A$ involves the gauge coupling constant g. Under supersymmetry transformations, Σ transforms like V_A in (B.2). Unlike V_A, Σ transforms covariantly under supergauge transformations; i.e., denoting the chiral superfield gauge transformation parameters by $\Omega = g\Omega_A T_A$, we have

$$\delta_G \Sigma = i\left[\Sigma\Omega - \Omega^\dagger \Sigma\right]. \tag{B.5}$$

By requiring

$$\Sigma' = \Sigma + \delta_Q \Sigma + \delta_G \Sigma = e^{2V'} \tag{B.6}$$

with V'_A in WZ gauge, we find

$$\Omega_A = e^{-i\vartheta\sigma_\mu\bar{\vartheta}\partial^\mu}\left[2i\left(\vartheta\sigma_\mu\bar{\zeta}G^\mu_A + \vartheta^2\bar{\zeta}\bar{\lambda}_A\right)\right]. \tag{B.7}$$

The resulting component field transformation rules are

$$\delta G^\mu_A = \zeta\sigma^\mu\bar{\lambda}_A - \bar{\zeta}\bar{\sigma}^\mu\lambda_A, \qquad \delta\lambda_A = \zeta D_A - \tfrac{1}{2}\sigma_{\mu\nu}\zeta G^{\mu\nu}_A, \qquad \delta\bar{\lambda}_A = \bar{\zeta} D_A + \tfrac{1}{2}\bar{\sigma}_{\mu\nu}\bar{\zeta} G^{\mu\nu}_A,$$

and

$$\delta D_A = -i\left[\zeta\sigma_\mu(D^\mu\bar{\lambda})_A + \bar{\zeta}\bar{\sigma}_\mu(D^\mu\lambda)_A\right], \tag{B.8}$$

in which D^μ denotes the gauge covariant derivative.

Similarly, for an arbitrary chiral superfield

$$\Phi = \mathrm{e}^{-i\vartheta\sigma_\mu\bar{\vartheta}\partial^\mu}\left[\varphi + \sqrt{2}\,\vartheta\psi + \vartheta^2 F\right], \tag{B.9}$$

we have

$$\delta\Phi = \left[\zeta \underset{\sim}{Q} + \bar{\zeta}\underset{\sim}{\bar{Q}} - i\Omega\right]\Phi. \tag{B.10}$$

The components of this provide us with the rules

$$\delta\varphi = \sqrt{2}\,\zeta\psi, \qquad \delta\psi = \sqrt{2}\left[\zeta F - i\sigma_\mu\bar{\zeta}D^\mu\varphi\right],$$

and

$$\delta F = -i\sqrt{2}\,\bar{\zeta}\bar{\sigma}_\mu D^\mu\psi + 2g\bar{\zeta}\bar{\lambda}_A T_A\varphi. \tag{B.11}$$

These rules were first derived by de Wit and Freedman [13].

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