ELSEVIER

Nuclear Physics B 496 (1997) 231-242

NUCLEAR PHYSICS B

World-volume approach to absorption by non-dilatonic branes

Igor R. Klebanov

Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544, USA

Received 14 February 1997; accepted 2 April 1997

Abstract

We calculate classical cross sections for absorption of massless scalars by the extremal 3-branes of type IIB theory, and by the extremal 2- and 5-branes of M-theory. The results are compared with corresponding calculations in the world-volume effective theories. For all three cases we find agreement in the scaling with the energy and the number of coincident branes. For 3-branes, whose stringy description is known in detail in terms of multiple D-branes, the string theoretic absorption cross section for low-energy dilatons is in *exact* agreement with the classical gravity. This suggests that scattering from extremal 3-branes is a unitary process well described by perturbative string theory. © 1997 Elsevier Science B.V.

PACS: 04.65.+e; 04.70.Dy; 11.25.-w

Keywords: String theory; M-theory; Dirichlet branes; Self-dual threebrane; Absorption; Effective low-energy supergravity

1. Introduction

Extremal black holes with non-vanishing horizon area may be embedded into string theory or M-theory using intersecting p-branes [1-8]. These configurations are useful for a microscopic interpretation of the Bekenstein-Hawking entropy. Furthermore, a number of emission/absorption calculations agree with a simple 'effective string' model for the dynamics of the intersection [3,9-15]. Unfortunately, at the moment there is no complete derivation of this model from first principles. Thus, it is useful to examine simpler configurations which involve parallel branes only.

A microscopic interpretation of the entropy of near-extremal p-branes was first studied in Refs. [16,17]. It was found the scaling of the Bekenstein-Hawking entropy with the temperature agrees with that for a massless gas in p dimensions only for the 'non-dilatonic p-branes': namely, the self-dual 3-brane of the type IIB theory, and the 2- and

PII S0550-3213(97)00235-6

5-branes of M-theory.[1] In [20] a way of reconciling the differing scalings for the dilatonic branes was proposed. According to this 'correspondence principle' [21,20], the string theory and the semiclassical gravity descriptions are in general expected to match only at a special value of the temperature, which corresponds to the horizon curvature comparable to the string scale.[2] In [20] it was shown that, in all known cases, the stringy and the Bekenstein-Hawking entropies match at this point up to factors of order 1. Part of the ambiguity in this factor comes from knowing the matching point only approximately. However, for the non-dilatonic branes this ambiguity is absent: the matching can be achieved at any scale because the stringy and the semiclassical entropies have identical scalings with temperature. This still leaves a discrepancy - the relative factor of 4/3 - for the 3-brane entropy [16]. In [20] a qualitative explanation of this factor was attributed to strong coupling effects on the world-volume. In view of the new results that we will present here, one may wonder if there exists an exact explanation of the 3-brane entropy in terms of a weakly coupled theory (perhaps utilizing the S-duality).

The non-dilatonic branes have a number of special properties. A notable property of their extremal metrics is that the transverse part of the geometry is non-singular: instead of a singularity we find an infinitely long throat whose radius is determined by the charge (the vanishing of the horizon area is due to the longitudinal contraction). The metric describing a non-dilatonic p-brane carrying an elementary unit of charge has the spatial curvature bounded from above by a quantity of order the Planck scale. Thus, for a large number N of coincident branes, the curvature may be made arbitrarily small in Planck units. For instance, for N D3-branes, the curvature is bounded by a quantity of order

$$\frac{1}{\sqrt{N\kappa_{10}}} \sim \frac{1}{\alpha'\sqrt{Ng_{\rm str}}} .$$

Thus, to suppress the string scale corrections to the classical metric, we need to take the limit $Ng_{\rm str} \to \infty$.

The tensions of non-dilatonic branes depend on $g_{\rm str}$ and α' only through the gravitational constant κ in the appropriate dimension, which is also the only scale present in the semiclassical description. Indeed, the D3-brane tension is $\sim 1/\kappa_{10}$, the M2-brane tension is $\sim 1/\kappa_{11}^{2/3}$, and the M5-brane tension is $\sim 1/\kappa_{11}^{4/3}$. This means that we can compare the expansions of various quantities in powers of κ between the microscopic and the semiclassical descriptions. It is often said that, in the microscopic description such an expansion is not tractable because it proceeds in powers of $Ng_{\rm str}$, a quantity that has to be considered very large. We will see, however, that for the 3-brane absorption cross section the actual expansion parameter is

$$N\kappa_{10}\omega^4 \sim Ng_{\rm str}\alpha'^2\omega^4 , \tag{1}$$

where ω is the incident energy. Thus, we may consider a 'double scaling limit'

$$Ng_{\rm str} \to \infty , \qquad \omega^2\alpha' \to 0 , \tag{2}$$

[1] Some ideas on how to extend this agreement to the dilatonic branes [18] were suggested in [19].

[2] For N parallel D-branes, $Ng_{\rm str}$ is of order 1 at the matching point [20].

where the expansion parameter (1) is kept small. Moreover, the classical absorption cross section is naturally expanded in powers of $\omega^4 \times \text{curvature}^{-2}$, which is the same expansion parameter (1) as the one governing the string theoretic description of the 3-branes. Thus, the two expansions of the cross section may indeed be compared, and we will find that the leading term agrees exactly! In our opinion, this provides evidence in favor of scattering off extremal 3-branes being a unitary process, well described by perturbative string theory.

In view of the special properties mentioned above, we believe that the non-dilatonic branes admit more detailed string theory-semiclassical gravity comparisons than those allowed in general by the correspondence principle of Refs. [21,20]. Also, the world-volume dynamics is better understood here than in the case of intersecting branes, which allows for a calculation from first principles. Indeed, N parallel D3-branes are known to be described by a $U(N)$ gauge theory on the world-volume [22]. For multiple M-branes the world-volume theory is not known in detail but, with minimal assumptions about its structure, we will be able to make interesting comparisons as well.

The structure of the paper is as follows. In Section 2 we calculate the classical absorption cross sections for low-energy massless scalars incident at right angles on the non-dilatonic branes. In Section 3 we compare with the cross sections for an incident scalar to turn into a pair of massless modes on the brane moving in opposite directions. We find that the scalings with the energy and the number of branes agree in all cases. For the 3-branes, which is the only case where we are able to fix the normalizations, we find exact agreement between the string theoretic and the classical cross sections. In Section 4 we study higher partial waves. We identify the leading terms in the effective action which convert the incident scalar into $l+2$ massless world-volume modes. Cross section for this process yields agreement in scaling with the classical absorption in the lth partial wave.

2. Classical absorption by extremal branes

In this section we carry out classical absorption calculations for the three cases of interest: the 3-brane in $D = 10$, and the 2- and 5-branes in $D = 11$.

The extremal 3-brane metric [23] can be written as

$$ds^2 = A^{-1/2}\left(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2\right) + A^{1/2}\left(dr^2 + r^2 d\Omega_5^2\right) ,$$

where

$$A = 1 + \frac{R^4}{r^4} .$$

The s-wave of a minimally coupled massless scalar satisfies

$$\left[\rho^{-5}\frac{d}{d\rho}\rho^5\frac{d}{d\rho} + 1 + \frac{(\omega R)^4}{\rho^4}\right]\phi(\rho) = 0 , \tag{3}$$

where $\rho = \omega r$. Thus, we are interested in absorption by the Coulomb potential in six spatial dimensions. For small ωR this problem may be solved by matching an approximate solution in the inner region to an approximate solution in the outer region.

To approximate in the inner region, it is convenient to use the variable $z = (\omega R)^2/\rho$. Then (3) turns into

$$\left[\frac{d^2}{dz^2} - \frac{3}{z}\frac{d}{dz} + 1 + \frac{(\omega R)^4}{z^4}\right]\phi = 0. \tag{4}$$

Substituting $\phi = z^{3/2} f(z)$, we find

$$\left[\frac{d^2}{dz^2} - \frac{15}{4z^2} + 1 + \frac{(\omega R)^4}{z^4}\right] f = 0. \tag{5}$$

The last term may be ignored if $z \gg (\omega R)^2$, i.e. if $\rho \ll 1$. In this region, (5) is easily solved in terms of cylinder functions. Since we are interested in the incoming wave for small ρ, the appropriate solution is

$$\phi = i(\omega R)^4 \rho^{-2}\left[J_2\left(\frac{(\omega R)^2}{\rho}\right) + iN_2\left(\frac{(\omega R)^2}{\rho}\right)\right], \quad \rho \ll 1, \tag{6}$$

where J and N are the Bessel and Neumann functions.

Another way to manipulate (3) is by substituting $\phi = \rho^{-5/2}\psi$, which gives

$$\left[\frac{d^2}{d\rho^2} - \frac{15}{4\rho^2} + 1 + \frac{(\omega R)^4}{\rho^4}\right]\psi = 0. \tag{7}$$

Now the last term is negligible for $\rho \gg (\omega R)^2$, where (7) is solvable in terms of cylinder functions. If $\omega R \ll 1$, then the inner region ($\rho \ll 1$) overlaps the outer region ($\rho \gg (\omega R)^2$), and the approximate solutions may be matched. We find that (6) matches onto

$$\phi = \frac{32}{\pi}\rho^{-2} J_2(\rho), \quad \rho \gg (\omega R)^2. \tag{8}$$

The absorption probability may be calculated as the ratio of the flux at the throat to the incoming flux at infinity, with the result

$$\mathcal{P} = \frac{\pi^2}{16^2}(\omega R)^8.$$

In d spatial dimensions, the absorption cross section is related to the s-wave absorption probability by [24]

$$\sigma = \frac{(2\pi)^{d-1}}{\omega^{d-1}\Omega_{d-1}}\mathcal{P},$$

where $\Omega_D = 2\pi^{(D+1)/2}/\Gamma(\frac{D+1}{2})$ is the volume of a unit D-dimensional sphere. Thus, for the 3-brane we find[3]

[3] By absorption cross section we will consistently mean the cross section per unit longitudinal volume of the brane.

$$\sigma_{3\text{-brane}} = \frac{\pi^4}{8}\omega^3 R^8 . \tag{9}$$

This exercise may be easily repeated for the other two non-dilatonic branes. For the M5-brane the extremal metric is [25]

$$ds^2 = A^{-1/3}\left(-dt^2 + dx_1^2 + \ldots + dx_5^2\right) + A^{2/3}\left(dr^2 + r^2 d\Omega_4^2\right) ,$$

where $A = 1 + (R^3/r^3)$. Now the s-wave problem reduces to absorption by the Coulomb potential in five spatial dimensions,

$$\left[\rho^{-4}\frac{d}{d\rho}\rho^4\frac{d}{d\rho} + 1 + \frac{(\omega R)^3}{\rho^3}\right]\phi(\rho) = 0 .$$

The approximate solution in the inner region is

$$\phi = iy^3[J_3(y) + iN_3(y)] ,$$

where $y = 2(\omega R)^{3/2}/\sqrt{\rho}$. This matches onto

$$\phi = 24\sqrt{\frac{2}{\pi}}\rho^{-3/2}J_{3/2}(\rho)$$

in the outer region. The absorption probability is $\mathcal{P} = \pi(\omega R)^9/9$, and the absorption cross section is found to be

$$\sigma_{5\text{-brane}} = \frac{2\pi^3}{3}\omega^5 R^9 . \tag{10}$$

For the M2-brane the extremal metric is [26]

$$ds^2 = A^{-2/3}\left(-dt^2 + dx_1^2 + dx_2^2\right) + A^{1/3}\left(dr^2 + r^2 d\Omega_7^2\right) ,$$

where $A = 1 + (R^6/r^6)$. Now the s-wave problem reduces to absorption by the Coulomb potential in eight spatial dimensions,

$$\left[\rho^{-7}\frac{d}{d\rho}\rho^7\frac{d}{d\rho} + 1 + \frac{(\omega R)^6}{\rho^6}\right]\phi(\rho) = 0 .$$

Now the solution in the inner region is

$$\phi = iy^{3/2}[J_{3/2}(y) + iN_{3/2}(y)] ,$$

where $y = (\omega R)^3/\rho^2$. This matches onto

$$\phi = 48\sqrt{\frac{2}{\pi}}\rho^{-3}J_3(\rho)$$

in the outer region. The absorption probability is $\mathcal{P} = \pi(\omega R)^9/24^2$, and the absorption cross section is found to be

$$\sigma_{2\text{-brane}} = \frac{2\pi^4}{3}\omega^2 R^9 . \tag{11}$$

3. Absorption of scalars in the effective field theory

In this section we perform effective field theory calculations for the D3-branes and find complete agreement with the classical results. For the M2-branes and the M5-branes all the scaling exponents agree, but the normalizations cannot be fixed due to insufficient knowledge of the world-volume theory describing many parallel branes.

First we consider absorption of scalars by D3-branes. There are several types of fields that act as scalars from the point of view of the $D = 7$ black hole obtained by wrapping the 3-brane over T^3. For example, we could consider the gravitons with polarizations along the 3-brane, $h_{\alpha\beta}$. Another scalar is the dilaton, ϕ, which we discuss in detail here. From the low-energy effective action of type IIB theory it is clear that the dilaton is a minimally coupled massless scalar, i.e. its s-wave part satisfies the equation (3) analyzed in the previous section. Our effective action analysis will produce the absorption cross section which is in perfect agreement with the classical result (9) obtained from an analysis of (3).

The coupling of the dilaton to the quadratic terms in the D3-brane action is[4] [27]

$$S = T_3 \int d^4x \left[\frac{1}{2} \sum_{i=4}^{9} \partial_\alpha X^i \partial^\alpha X^i - \frac{1}{4} e^{-\phi} F_{\alpha\beta}^2 \right] , \tag{12}$$

where $F_{\alpha\beta}$ is the field strength for the gauge field on the 3-brane describing its longitudinal dynamics, while the six fields X^i describe its transverse oscillations.

$$T_3 = \sqrt{\pi}/\kappa_{10} \tag{13}$$

is the D3-brane tension [28]. A string theoretic calculation of all the cubic terms in (12) can be given with the methods developed in [29].

Fixing the gauge for A_α, we find two physical photons. Thus, there are two canonically normalized physical fields $\tilde{A}$, each having a cubic coupling to the dilaton given by

$$-\frac{1}{2} \int d^4x \, \phi \partial_\alpha \tilde{A} \partial^\alpha \tilde{A} .$$

The ten-dimensional effective action is given by

$$S_{\text{bulk}} = -\frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{g} [R - \tfrac{1}{2} \partial_\mu \phi \partial^\mu \phi + \dots]$$

so that the canonically normalized dilaton field is

$$\tilde{\phi} = \frac{\phi}{\sqrt{2}\kappa_{10}} .$$

Thus, the world-volume theory contains the coupling

[4] I am grateful to S. Gubser and A. Tseytlin for valuable discussions on the structure of this action.

$$-\frac{\kappa_{10}}{\sqrt{2}}\int d^4x\,\tilde{\phi}\partial_\alpha\tilde{A}\partial^\alpha\tilde{A}\,. \tag{14}$$

A scalar incident on the brane at right angles may be converted into a pair of bosonic massless world-volume modes moving in opposite directions (it is easy to see that a pair of on-shell fermions cannot be created). Calculating the amplitude for this process, we get

$$\mathcal{A} = -\frac{\kappa_{10}}{\sqrt{2}}2\frac{p_1\cdot p_2}{\sqrt{2}\omega^{3/2}} = -\frac{\kappa_{10}\sqrt{\omega}}{2}\,.$$

Note that each state has normalization factor $1/\sqrt{2E}$ and $E_1 = E_2 = \omega/2$. There is also a factor of 2 because either of the $\tilde{X}$'s can create either of the final particles. Since $\boldsymbol{p}_1 = -\boldsymbol{p}_2$, $p_1\cdot p_2 = \omega^2/2$.

Thus, each species contributes the absorption cross section

$$\frac{1}{2}\int\frac{d^3p_1}{(2\pi)^3}\int\frac{d^3p_2}{(2\pi)^3}(2\pi)^4\delta(E_1+E_2-\omega)\delta^3(\boldsymbol{p}_1+\boldsymbol{p}_2)\mathcal{A}^2\,,$$

where $1/2$ is included because the final particles are identical. Doing the integral we find that the absorption cross section due to each species of massless bosons coupling to the dilaton is

$$\frac{1}{2}\frac{\kappa_{10}^2\omega^3}{32\pi}\,. \tag{15}$$

Since there are two such species, the total absorption cross section is

$$\sigma = \frac{\kappa_{10}^2\omega^3}{32\pi}\,.$$

So far, our analysis has covered the case of a single D3-brane. Extending it to N coincident D3-branes is straightforward. Now each of the $\tilde{A}$'s is replaced by a hermitian $N\times N$ matrix, and the interaction vertex becomes [5]

$$-\frac{\kappa_{10}}{\sqrt{2}}\int d^4x\,\tilde{\phi}\,\mathrm{Tr}\,\partial_\alpha\tilde{A}\partial^\alpha\tilde{A}\,.$$

Now there are $2N^2$ possible species in the final state, each contributing (15) to the absorption cross section. Thus, the string theoretic result for the total cross section is

$$\sigma_{\text{3-brane}} = \frac{\kappa_{10}^2N^2\omega^3}{32\pi}\,. \tag{16}$$

Now we show that this is identical to the classical result (9). We equate the ADM mass per unit volume of the 3-brane,

$$\frac{2\pi^3R^4}{\kappa_{10}^2}\,,$$

[5] Terms involving $[\tilde{A}_\alpha, \tilde{A}_\beta]$ give contributions to the cross section which are suppressed by powers of κ_{10}. Perhaps they define the quantum corrections to the classical result of General Relativity.

to the corresponding quantity in the D-brane description [28], $\frac{\sqrt{\pi}}{\kappa_{10}}N$. Thus, we find [16]

$$R^4 = \frac{\kappa_{10} N}{2\pi^{5/2}} .$$

With this substitution, the classical formula (9) becomes identical to the string theory result (16). This is the first example outside the domain of validity of the effective string model where such matching works exactly!

Now we turn to the M-branes. Here we are forced to be more schematic because a world-volume description of multiple parallel branes is not yet understood. We will simply assume a minimal coupling between a scalar field and the massless world-volume modes, [6]

$$S = \frac{T_p}{2} \int d^{p+1}x \, \phi \partial_\alpha X^i \partial^\alpha X^i . \tag{17}$$

We also assume that the number of such modes scales with N in the way suggested by the near-extremal entropy, i.e. as N^3 for N coincident M5-branes, and as $N^{3/2}$ for N coincident M2-branes [17]. Introducing properly normalized fields,

$$\tilde{\phi} \sim \frac{\phi}{\kappa_{11}} , \qquad \tilde{X}^i = \sqrt{T_p} X^i ,$$

we find the cubic vertex

$$S_3 \sim \kappa_{11} \int d^{p+1}x \, \tilde{\phi} \partial_\alpha \tilde{X}^i \partial^\alpha \tilde{X}^i .$$

Calculating the cross sections, we then have

$$\sigma_{5\text{-brane}} \sim \kappa_{11}^2 \omega^5 N^3 , \tag{18}$$

$$\sigma_{2\text{-brane}} \sim \kappa_{11}^2 \omega^2 N^{3/2} . \tag{19}$$

In order to compare them with the classical results, we need the charge quantization rules. For N coincident M5-branes, we have [7,17]

$$q_5 = N\sqrt{2} \left(\frac{\pi}{2\kappa_{11}} \right)^{1/3} = \frac{3\Omega_4}{\sqrt{2}\kappa_{11}} R^3 .$$

Solving for R and substituting into the classical result (10) reduces it to (18), up to normalization. For N coincident M2-branes [7,17],

$$q_2 = N\sqrt{2} (2\pi^2 \kappa_{11})^{1/3} = \frac{6\Omega_7}{\sqrt{2}\kappa_{11}} R^6 .$$

Solving for R and substituting into the classical result (11) reduces it to (19), up to normalization.

[6] To estimate the scaling of the absorption cross section it is sufficient to leave out the world-volume gauge fields and to work with the scalars only.

4. Higher partial waves

In this section we estimate the classical cross sections for scalars incident in higher partial waves. This allows us to identify the terms in the world-volume effective actions that are responsible for these processes.

For the extremal 3-brane, the lth partial wave satisfies

$$\left[\rho^{-5}\frac{d}{d\rho}\rho^5\frac{d}{d\rho}+1+\frac{(\omega R)^4}{\rho^4}-\frac{l(l+4)}{\rho^2}\right]\phi^{(l)}=0\,.$$

In the outer region, the approximate solution is

$$\phi^{(l)}=B\rho^{-2}J_{l+2}(\rho)\,,$$

while in the inner region

$$\phi^{(l)}=i(\omega R)^4\rho^{-2}\left[J_{l+2}\left(\frac{(\omega R)^2}{\rho}\right)+iN_{l+2}\left(\frac{(\omega R)^2}{\rho}\right)\right]\,.$$

Matching the two regions, we find that $B\sim(\omega R)^{-2l}$. Therefore, the ratio of fluxes is $\sim(\omega R)^{8+4l}$, and the absorption cross section is

$$\sigma^{(l)}_{3\text{-brane}}\sim\omega^{3+4l}R^{8+4l}\sim\omega^{3+4l}(N\kappa)^{2+l}\,. \tag{20}$$

Analysis of the effective action shows that all partial waves are reproduced (at least schematically) by the leading term in the effective action

$$\frac{\sqrt{\pi}}{4\kappa_{10}}\int d^4x\,\phi(x,X)F^2_{\alpha\beta}\,.$$

The term responsible for absorbing the lth partial wave is [7]

$$\frac{\sqrt{\pi}}{4\kappa_{10}}\int d^4x\,\frac{1}{l!}(\partial_{i_1}\ldots\partial_{i_l}\phi)X^{i_1}\ldots X^{i_l}F^2_{\alpha\beta}\,. \tag{21}$$

For N coincident 3-branes, the natural non-abelian generalization of (21) is

$$\frac{\sqrt{\pi}}{4\kappa_{10}}\int d^4x\,\frac{1}{l!}(\partial_{i_1}\ldots\partial_{i_l}\phi)\,\mathrm{Tr}\,X^{i_1}\ldots X^{i_l}F^2_{\alpha\beta}\,.$$

It is not hard to see that the amplitude produced by this term scales as $\sim\kappa_{10}^{(2+l)/2}$ so that the scaling of the cross section agrees with that of the classical result (20). The number of distinct final states grows as N^{2+l}, so that the N-dependence also agrees with (20).[8] Finally, a quick estimate of the ω-dependence from the Feynman rules gives ω^{3+4l}, again in agreement with (20). Of course, once the power of κ_{10} is matched, the power of ω is guaranteed to be correct by dimensional analysis.

[7] To obtain the correct normalization of the cross section, it is probably necessary to add the fermionic terms required by supersymmetry. In this paper we restrict ourselves to analyzing the purely bosonic processes.

[8] For example, the $U(N)$ index structure of the cubic ($l=1$) vertex is $X^I_J\partial_\alpha X^J_K\partial^\alpha X^K_I$. There are three independent summations giving the group theory factor $\sim N^3$.

Now we extend this schematic analysis to the M-branes. For the M5-brane the lth partial wave satisfies

$$\left[\rho^{-4}\frac{d}{d\rho}\rho^4\frac{d}{d\rho}+1+\frac{(\omega R)^3}{\rho^3}-\frac{l(l+3)}{\rho^2}\right]\phi^{(l)}(\rho)=0\,.$$

In the inner region the approximate solution is

$$\phi^{(l)}=iy^3[J_{3+2l}(y)+iN_{3+2l}(y)]\,,\qquad y=2(\omega R)^{3/2}/\sqrt{\rho}\,,$$

which matches onto

$$\phi^{(l)}=B\rho^{-3/2}J_{(3+2l)/2}(\rho)$$

in the outer region. We find that $B\sim(\omega R)^{-3l}$, so that

$$\sigma^{(l)}_{\text{5-brane}}=\omega^{5+6l}R^{9+6l}\sim\omega^{5+6l}(N\kappa_{11}^{2/3})^{3+2l}\,. \tag{22}$$

For the M2-brane the lth partial wave satisfies

$$\left[\rho^{-7}\frac{d}{d\rho}\rho^7\frac{d}{d\rho}+1+\frac{(\omega R)^6}{\rho^6}-\frac{l(l+6)}{\rho^2}\right]\phi^{(l)}(\rho)=0\,.$$

Now the solution in the inner region is

$$\phi^{(l)}=iy^{3/2}[J_{(3+l)/2}(y)+iN_{(3+l)/2}(y)]\,,\qquad y=(\omega R)^3/\rho^2\,,$$

which matches onto

$$\phi^{(l)}=B\rho^{-3}J_{3+l}(\rho)$$

in the outer region. We find that $B\sim(\omega R)^{-3l/2}$, so that

$$\sigma^{(l)}_{\text{2-brane}}=\omega^{2+3l}R^{9+3l}\sim\omega^{2+3l}(N\kappa_{11}^{4/3})^{(3+l)/2}\,. \tag{23}$$

The scalings of (22) and (23) with respect to κ_{11} are reproduced by the action (17): we Fourier expand the scalar field $\phi(X)$ and identify the term with the lth derivative of ϕ as the one responsible for absorbing the lth partial wave. The scalings with respect to N are harder to understand. We believe that they will provide valuable clues on the symmetry structure of the effective action describing N coincident M-branes.

5. Conclusions

The self-dual 3-brane of type IIB theory is a nice laboratory for comparing string theory with semiclassical gravity. The stringy description of a macroscopic 3-brane is well understood in terms of a large number N of parallel Dirichlet branes [28,22]. The only scale present in the low-energy effective action is $\kappa_{10}\sim g_{\text{str}}\alpha'^2$, the ten-dimensional gravitational constant. Using perturbative string theory, we may expand various quantities in powers of $N\kappa_{10}\omega^4$. The result may be compared with a similar expansion generated by the semiclassical methods of General Relativity, which use the classical 3-brane geometry as the background. In this paper we have carried out such a comparison for the absorption cross section of minimally coupled massless scalars, and found *exact* agreement to leading order.

We believe that there is a number of interesting extensions of our calculations. Comparing normalizations for higher partial waves is a feasible, if somewhat technical, exercise. Another interesting extension is to incident particles of higher spin, such as the gravitons. In the effective field theory, the leading coupling of the graviton of a given polarization, say h_{67}, is given by

$$\sqrt{2}\kappa_{10} \int d^4x\, h_{67}\, \mathrm{Tr}\, \partial_\alpha \tilde{X}^6 \partial^\alpha \tilde{X}^7 ,$$

where h_{67} is a canonically normalized field. The calculation is almost identical to that given in Section 3, and we find that the graviton is absorbed with the same cross section as the scalars (16). In classical gravity, however, it is quite difficult to derive the graviton propagation equations. It would be interesting to derive this equation in the 3-brane background and compare the resulting absorption cross section with the prediction of string theory (16).

The 2- and 5-branes of M-theory bear many similarities with the 3-brane of type IIB [17]. Their geometries are non-singular, while their world-volume theories are governed by the eleven-dimensional Planck scale. Thus, it should be possible to compare the expansions in powers of κ_{11} generated by the M-theory and the semiclassical supergravity. We showed that, with minimal assumptions about the world-volume theories of many coincident M-branes, the scalar absorption cross sections agree up to normalizations. If we assume that the exact agreement must hold, then the information provided by the semiclassical methods is a valuable guide to formulating the M-theory.

If the multiple coincident D3-branes and M-branes are indeed the unitary quantum systems underlying their classical geometry, then there is a wealth of perturbative calculations, of the type carried out in Refs. [30,31,29], that may shed more light on this remarkable phenomenon.

Acknowledgements

We are grateful to C.G. Callan, S.S. Gubser, M. Krasnitz and A.A. Tseytlin for useful discussions and comments. This work was supported in part by DOE grant DE-FG02-91ER40671, the NSF Presidential Young Investigator Award PHY-9157482, and the James S. McDonnell Foundation grant No. 91-48.

References

[1] M. Cvetič and A.A. Tseytlin, Phys. Lett. B 366 (1996) 95, hep-th/9510097; Phys. Rev. D 53 (1996) 5619, hep-th/9512031.
[2] A. Strominger and C. Vafa, Phys. Lett. B 379 (1996) 99, hep-th/9601029.

[3] C.G. Callan and J.M. Maldacena, Nucl. Phys. B 472 (1996) 591, hep-th/9602043.
[4] J.M. Maldacena and A. Strominger, Phys. Rev. Lett. 77 (1996) 428, hep-th/9603060.
[5] C. Johnson, R. Khuri and R. Myers, Phys. Lett. B 378 (1996) 78, hep-th/9603061.
[6] A.A. Tseytlin, Nucl. Phys. B 475 (1996) 49, hep-th/9604035.
[7] I.R. Klebanov and A.A. Tseytlin, Nucl. Phys. B 475 (1996) 179, hep-th/9604166.
[8] V. Balasubramanian and F. Larsen, Nucl. Phys. B 478 (1996) 199, hep-th/9604189.
[9] A. Dhar, G. Mandal and S. R. Wadia, Phys. Lett. B 388 (1996) 51, hep-th/9605234.
[10] S.R. Das and S.D. Mathur, Nucl. Phys. B 478 (1996) 561, hep-th/9606185.
[11] S. Das and S.D. Mathur, Nucl. Phys. B 482 (1996) 153, hep-th/9607149.
[12] S.S. Gubser and I.R. Klebanov, Nucl. Phys. B 482 (1996) 173, hep-th/9608108.
[13] J.M. Maldacena and A. Strominger, Rutgers preprint RU-96-78, hep-th/9609026.
[14] S.S. Gubser and I.R. Klebanov, Phys. Rev. Lett. 77 (1996) 4491, hep-th/9609076.
[15] C.G. Callan, Jr., S.S. Gubser, I.R. Klebanov and A.A. Tseytlin, Nucl. Phys. B 489 (1997) 65, hep-th/9610172;
I.R. Klebanov and M. Krasnitz, Phys. Rev. D 55 (1997) 3250, hep-th/9612051.
[16] S.S. Gubser, I.R. Klebanov and A.W. Peet, Phys. Rev. D 54 (1996) 3915, hep-th/9602135;
A. Strominger, unpublished notes.
[17] I.R. Klebanov and A.A. Tseytlin, Nucl. Phys. B 475 (1996) 165, hep-th/9604089.
[18] M.J. Duff, H. Lu and C.N. Pope, Phys. Lett. B 382 (1996) 73, hep-th/9604052.
[19] H. Lu, S. Mukherji, C. Pope and J. Rahmfeld, Phys. Lett. B 389 (1996) 248, hep-th/9604127.
[20] G. Horowitz and J. Polchinski, hep-th/9612146.
[21] L. Susskind, hep-th/9309145.
[22] E. Witten, Nucl. Phys. B 460 (1996) 541.
[23] G. Horowitz and A. Strominger, Nucl. Phys. B 360 (1991) 197.
[24] S. Das, G. Gibbons and S. Mathur, hep-th/9609052.
[25] R. Güven, Phys. Lett. B 276 (1992) 49.
[26] M.J. Duff and K.S. Stelle, Phys. Lett. B 253 (1991) 113.
[27] A. Tseytlin, Nucl. Phys. B 469 (1996) 51, hep-th/9602064.
[28] J. Polchinski, Phys. Rev. Lett. 75 (1995) 4724, hep-th/9510017.
[29] A. Hashimoto and I.R. Klebanov, Phys. Lett. B 381 (1996) 437, hep-th/9604065.
[30] I.R. Klebanov and L. Thorlacius, Phys. Lett. B 371 (1996) 51, hep-th/9510200;
S.S. Gubser, A. Hashimoto, I.R. Klebanov and J.M. Maldacena, Nucl. Phys. B 472 (1996) 231, hep-th/9601057.
[31] R. Garousi and R. Myers, Nucl. Phys. B 475 (1996) 193, hep-th/9603194.