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THE STATISTICAL THEORY OF ANOMALOUS FERMION NUMBER NON-CONSERVATION

S.Yu. KHLEBNIKOV and M.E. SHAPOSHNIKOV

Institute for Nuclear Research of the USSR Academy of Sciences, Moscow 117312, USSR

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We present a non-equilibrium method for studying anomalous B-non-conservation in the electroweak theory at finite temperature. We derive formally the equation of B-evolution advocated previously on physical grounds. The kinetic coefficient in it is quite generally expressed through a thermodynamic average. This expression allows for an estimate of higher order corrections and is applicable both below and above the phase transition temperature.

1. Introduction

Electroweak non-perturbative physics [1] connected with the complicated vacuum structure of non-abelian gauge theories and anomalous non-conservation of fermionic currents appears to be very diverse at extreme conditions [2, 3]. For instance, strong violation of the fermionic number occurs in superdense matter [2]. Moreover, the processes with baryon number (B) non-conservation are unsuppressed at sufficiently high temperatures [3].

The existence of these processes has important cosmological consequences. First, they wash out [3] the baryon asymmetry generated by GUT interactions, unless special conditions are satisfied [4]. Second, they give rise to the possibility of creating a baryonic excess entirely within the framework of the standard electroweak theory [5, 6]. The latter, in particular, makes it possible to obtain the upper bounds on Higgs [5, 6] and top-quark [7] masses.

However, the theory of anomalous fermon number non-conservation at finite temperatures is still based mainly on intuitive grounds. Even the formal proof that there is no familiar semiclassical suppression [1] is absent. The main difficulty is related to the essentially non-equilibrium character of the process. The relevant Green functions must be computed at large *minkowskian* time and this cannot be done by analytical continuation of the *approximate* (perturbative or semiclassical) result for the euclidean Green functions [8]. The same problem arises in the description of the decay of the metastable state due to thermal fluctuations. A possible way out is to supplement the equilibrium framework with some dynamical

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assumptions. For example, in [9] the decay rate of the metastable state was identified with a probability flux across the saddle point separating the stable and metastable minima of the potential. This was calculated by means of the so-called steady-state distribution function instead of the true non-equilibrium distribution. The steady-state situation is set up by continuously replenishing the metastable state at a rate equal to the rate at which it is leaking through the saddle point. Very similar constructions are implied in later works on the subject [10, 11]. Clearly, the use of the steady-state distribution is just an additional assumption necessary to treat the essentially non-equilibrium process in the equilibrium framework. Besides, the method strongly relies on the gaussian approximation for the energy near the saddle point, accounting for non-linear corrections being problematic. Finally, the stationary approach cannot yield an explicit time dependence of the relevant quantities, so the exponential character of the process should be prescribed rather than obtained. It would be interesting in itself to describe the metastable state decay with less engaging inputs.

However, the situation is even more uncertain in the electroweak theory. The bosonic energy has an infinite set of equivalent minima, all related to the trivial one by the so-called large gauge transformations [12] surviving after the gauge fixing. As soon as fermions are taken into account the equivalence of the minima is lost. When bosonic fields vary from some values to gauge-transformed ones, as a consequence of the anomaly, one of the Dirac levels moves from the upper continuum to the lower one or vice versa [13, 14]. This level crossing leads to a change in the number of real baryons, and also leptons, each equal to the number of fermion generations.

Only at temperatures below the critical T_c the neighbouring minima are effectively divided by the energy barrier represented by the saddle point. In ref. [3] the rate of anomalous B-non-conservation at $T < T_c$ was estimated as a decay rate of the metastable state built near one of the minima. However, to gain a more quantitative understanding one must be able to treat a more general situation. In an arbitrary non-equilibrium state all the minima are filled somehow. The probability flux across a given saddle point is an individual quantity dependent on the relative filling of the neighbouring minima. The problem is how to extract from all this variety of quantities, dependent on time and initial conditions, the intrinsic rate of the B-non-conservation. Moreover, the earlier approach leaves open the question of how to handle the case $T > T_c$.

The purpose of the present paper is to elaborate direct non-equilibrium methods overcoming the above difficulties. We will be able to obtain a formal demonstration that B-non-conservation is indeed unsuppressed at sufficiently high temperature. We verify the relevance of the saddle point solution to the calculations at $T < T_c$ and obtain the kinetic equation for B with the kinetic coefficient expressed through a combination of determinants in the saddle point background. Some subtle points in this expression, obscure in more heuristic treatment [15], are clarified. The

method itself and some general equations are applicable also at $T > T_c$. However, the actual computations are spoiled by the infrared divergences of the high-temperature theory. Nevertheless, the full parametric dependence of the rate at $T > T_c$ can be found. Moreover, lattice calculation of the numerical coefficient seems possible.

The whole problem of describing B non-conservation is divided into two distinct parts. The *real-time* non-equilibrium analysis yields a variant of the fluctuation-dissipation theorem relating the above kinetic coefficient to the equilibrium correlation function. The latter is reduced to some thermodynamic average, calculated by *euclidean* technique, but with no analytical continuation needed. It is in this sense that we call our method a real-time one. Some higher order contributions, indeed necessary at $T > T_c$, can be taken into account.

Though the treatment is microscopic, an element of irreversibility is introduced by hand. However, our approach to the problem is the first one that deals with well defined formal expressions at each stage of the calculation, and also the assumptions used can be written down explicitly.

The basis of our method is the so-called reduced description of statistical systems (see [16]). When the temperature T is small compared to the height of the barrier represented by the saddle point, a hierarchy of relaxation times exists in the electroweak theory. Namely, we expect the characteristic time of B non-conservation to be much larger than the time necessary for the kinetic equilibrium to be established. Thus, it is plausible to assume that at the last stage of its evolution the distribution function depends on time only through a limited set of slowly varying parameters. We identify them with non-equilibrium averages of certain functions of dynamical variables and find kinetic equations governing their large time asymptotics. The technical tool used to carry out this program is the non-equilibrium distribution function, established by Zubarev [17]. The above kinetic equations are combined to yield a single equation describing the baryon number time dependence.

The paper is organized as follows. In sect. 2 we consider the Green functions relevant to B non-conservation. The method is given in sect. 3 taking the classical metastability problem as an example. All basic ideas and definitions are introduced there. We address the case of the electroweak theory again in sect. 4. Sect. 5 gives the conclusion.

2. The anomalous Green functions

Let us discuss the problems arising in the description of B non-conservation in more detail. The anomaly equation for the baryonic current J_μ^B in the standard electroweak theory reads

$$\partial_\mu J_\mu^B = \frac{n_f}{32\pi^2}\left(-g^2 \tilde{F}_{\mu\nu}^a F_{\mu\nu}^a + g'^2 \tilde{F}_{\mu\nu}^0 F_{\mu\nu}^0\right), \tag{2.1}$$

where n_f is the number of generations, $F^a_{\mu\nu}$, $a = 1,2,3$ and $F^0_{\mu\nu}$ are the SU(2) and U(1) field strengths, g and g' are the corresponding coupling constants, $\tilde{F}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}F_{\rho\sigma}$. The determination of the net number of real baryons created (or destroyed) during the real-time classical gauge field evolution requires some care. Define the operator J^B_μ at the initial $t = 0$ moment of time by the normal ordering procedure. Then

$$N_B(t) = \int J^B_0(x,t)\,\mathrm{d}^3x \tag{2.2}$$

at $t = 0$ simply counts the real baryons in the initial state. Now consider N_B at the final moment t. If the gauge field $A_\mu(t)$ differs from $A_\mu(0)$ by a large gauge transformation it may be proved [14] that $N_B(t)$ simply counts the real baryons in the final state, so that (2.1) predicts the net production of baryons equal to

$$\Delta N = n_f Q(t), \qquad Q(t) = \frac{g^2}{32\pi^2}\int_0^t \tilde{F}^a_{\mu\nu}F^a_{\mu\nu}\,\mathrm{d}^4x. \tag{2.3}$$

Note, that only SU(2) fields contribute, because U(1) components at moments 0 and t do not differ at all.

Of course, there are other directions in the functional space of configurations (A, φ) which lead to a nonzero value of the integral over $\tilde{F}F$. For instance, when the system moves from $A = 0$ at $t = 0$ to the abelian configuration

$$A^1_1 = a\cos\kappa x_3, \qquad A^1_2 = a\cos(\kappa x_3 + \varphi) \tag{2.4}$$

(all other components of the field A are equal to zero), the total volume density of fermionic level crossings is given again by the anomaly equation* (2.1) [2]:

$$\Delta N/V = n_f Q(t)/V = n_f \frac{g^2}{32\pi^2}a^2\kappa\sin\varphi, \tag{2.5}$$

where V is the volume of the system. However, this level crossing is not directly associated with true fermion number non-conservation. The latter takes place when there are transitions between different gauge sectors. In particular, the configurations of the type (2.4) exist already in pure abelian gauge theories, where B non-conservation is absent due to trivial vacuum structure. The main difference between the abelian direction in the functional space and that associated with large

* This fact is nontrivial, because $N_B(t)$ does not count the number of real fermions in the background field (2.4) just because the configuration (2.4) cannot be gauge transformed to zero. This means, strictly speaking, that the integrated eq. (2.1) tells nothing particular about fermionic level crossings and baryon creation [14]. Nevertheless, the *density* of created fermions is given by the anomaly, when the volume of the system goes to infinity [2].

gauge transformations is the behaviour of the energy of a pure gauge-Higgs system. Due to gauge invariance, we have degeneracy of the configurations (A, φ) and (A^u, φ^u) independently on the value of the winding number of gauge transformation u. In contrast, the free energy of the configuration (2.4) grows with $Q(t)$ for sufficiently large $Q(t)$ (for the case of small amplitudes of the field $A \sim gT$, where T is the temperature, there could be a degeneracy connected with infrared properties of the gauge-Higgs high-temperature plasma, see [5]).

We expect, therefore, that the finite-temperature equilibrium average $\langle Q^2(t)\rangle_0$ at sufficiently large t becomes a suitable characteristic of B non-conservation. Since $\langle Q(t)\rangle_0 = 0$, it measures the mean-square "topological" dispersion in the position of the system. Though $\langle Q^2(t)\rangle_0$, as discussed above, accounts also for redundant motions, having nothing to do with B-non-conservation, at large t the contribution of these is negligible, because to get larger $\langle Q^2\rangle_0$ they ought to climb higher in energy. On the contrary, the relevant trajectories pass from a minimum to another with no systematic energy growth. More precisely, a slight growth yet persists as a consequence of the real fermion number increase. This leads to $\lim_{t\to\infty}\langle Q^2(t)\rangle_0 = const$ which is somewhat inconvenient to extract the rate of B non-conservation. However, if the baryonic density B/V is small compared to T^3 as it always occurs, the rate may be calculated in a theory without fermions, where presumably

$$\left\langle Q^2(t)\right\rangle_0 = \Gamma t, \qquad t \to \infty . \tag{2.6}$$

The linear dependence on t reflects the random nature of the process and Γ is just the rate we are interested in:

$$\Gamma = V \lim_{t\to\infty} G(t),$$

$$G(t) = \left\langle Q(t) \quad g^2\tilde{F}F(0)/32\pi^2\right\rangle_0. \tag{2.7}$$

To establish (2.6) we have to calculate $G(t)$ and get convinced that it stays constant in the $t \to \infty$ limit in pure gauge-Higgs theory.

Let us stress that the euclidean semiclassical methods are of little help in doing so. $G(t)$ is related to the corresponding euclidean Green function $G_E(\tau)$, $0 \leqslant \tau < \beta = 1/T$, through analytical continuation:

$$G_E(\tau) \to G_E(it) = G(t). \tag{2.8}$$

At $\tau = 1/T$ $G_E(\tau)$ is exponentially small: $G_E(\beta) \propto \exp(-2\pi/\alpha_w(T))$. (Just this quantity was actually estimated in [18]. In fact, $\langle Q^2(-i\beta)\rangle_0$ is precisely the second derivative of the equilibrium free energy with respect to the vacuum parameter θ and the above suppression is in agreement with the results on θ-dependence [19].) Clearly, by itself this number has no relation to $G_E(it)$, $t \to \infty$, we are going to

compute. Moreover, the analytical continuation of the semiclassical or perturbative expression for $G_E(\tau)$, $0 \leqslant \tau < \beta$ is also inapplicable, since the latter contains periodic functions like $\sin \beta\tau, \cos \beta\tau$ giving rise to terms growing exponentially with t. In fact, the breakdown of the semiclassical and perturbative approximations takes place already at $t \gtrsim (\alpha_w T)^{-1}$.

The problem with secular terms in the naive perturbative expansions is in fact well-known [16]. The real-time method proposed in this paper solves it effectively for the case under discussion. However, we relate the real-time calculation of the Green function (2.7) to appendix A because it proceeds in somewhat loose terms. To construct a formal theory it turns to be much more convenient to deal not with (2.7) but instead with the non-equilibrium distribution function*. In this sense our approach has more from statistical physics than from field theory. To start with, let us consider a more simple case of the decay of the metastable state.

3. The non-equilibrium approach to the decay of the metastable state

Consider a classical system with a large number of degrees of freedom. Suppose that the Hamilton function is

$$H(p, q) = \tfrac{1}{2} \sum_n p_n^2 + U(q), \tag{3.1}$$

where the potential $U(q)$ has two local minima and a saddle point. This is the simplest case where the classical metastability problem arises. In what follows the minima will be referred to as metastable and stable ones though, when their energies are close or equal to each other, the distinction is merely a nominal one. The canonical coordinates and momenta may be chosen as representing the normal modes of the saddle point, i.e. near the saddle point

$$U(q) = \bar{E} + \tfrac{1}{2} \sum_n \lambda_n q_n^2, \tag{3.2}$$

where $\bar{E}$ is the saddle point potential energy. Let only one of λ, say λ_1, be negative. Some of the others may well be zero, reflecting the breakdown of certain hamiltonian symmetries by the saddle point.

In classical statistics the task is to find the time-dependent distribution function $\rho(p, q; t)$, satisfying the Liouville equation

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \frac{\partial \rho}{\partial t} + \{\rho, H\} = 0 \tag{3.3}$$

* Nevertheless, (2.7) may be the most suitable object for real-time computer study, especially at $T > T_c$. This was partly done in [6b].

or at least to study some of its properties. For example, in the present case we are mainly interested in the rate at which the metastable state decays. This is often identified with the probability flux through the saddle point and determined by means of a certain time-independent distribution function instead of the true distribution function ρ. The most complete theory of this kind was developed by Langer [9]. Langer's stationary distribution function is a solution to (3.3) only in the neighbourhood of the saddle point. There is implied a source of probability somewhere in the metastable region and a sink in the stable region. Clearly, such a replacement of the distribution function constitutes an important a priori assumption, necessary to treat an essentially non-equilibrium process in the equilibrium framework (see also the later works [10, 11]).

The most consistent approach would be to deal with the non-equilibrium distribution $\rho(p, q; t)$ itself. However, one can hardly treat a multidimensional system out of equilibrium unless there occurs a hierarchy of relaxational times, so that the idea of the reduced description can be implemented.

3.1. THE REDUCED DESCRIPTION

If the temperature T is small compared with the height of the barrier represented by the saddle point, we expect the evolution of the system with arbitrary initial condition to have two distinct stages. At the first one there are approximate equilibrium distributions formed near each of the minima separately. At the second stage the system evolves slowly towards complete equilibrium. According to the concept of the reduced description, at this last stage the distribution function depends on time only through a parameter, which is a non-equilibrium average of some function of canonical variables and has the meaning of a full probability of observing the system in the metastable state

$$\rho(p, q; t) = \rho(p, q; \bar{n}(t)),$$

$$\bar{n}(t) = \langle n(p, q) \rangle \equiv \int n(p, q) \rho(p, q; t) \, \mathrm{d}p \, \mathrm{d}q. \tag{3.4}$$

The representation (3.4) alone enables us to gain some general results. However, to obtain a more detailed description one needs some information on the particular shape of the function $n(p, q)$.

Note, that we proceed quite analogously to the usual method of deriving the hydrodynamic equations [20], where the parameters representing the distribution function are average values of densities of particle number, flux and energy. Note also, that the choice of this particular set of quantities is based on physical grounds rather than on a strict microscopic theory since the construction of the latter requires explicit summation of the secular terms in a formal expansion in the small parameter [16]. The same reason enforces us too to use physical arguments in determining the function $n(p, q)$ entering (3.4).

Loosely speaking, the function $n(p, q)$ is one that would remain constant if the thermal fluctuations from one phase to another have been switched off. To handle this formally let us go to $T = 0$. Consider a family of surfaces in q-space passing through the saddle point, each tangent to the gradient of the potential energy in each of its points. All these also pass through both the minima except the only one, which is orthogonal to the steepest descent direction q_1 in the saddle point. This surface divides the q-space into two half-spaces corresponding to different phases. Let its equation be $f(q) = 0$. Suppose, at some moment the system is on the "watershed" surface heading to the stable region. From the microscopic point of view, the system will accomplish an energy conserving evolution governed by the Hamilton equations and dependent on the initial momenta. However, a system with many degrees of freedom admits a statistical description in addition to a microscopic one. Most probably, it will go "downhill" to the stable minimum and then stay in its very vicinity for such a huge time as can ever be addressed. This is because the finite-energy initial perturbation disturbes considerably only a relatively small number of degrees of freedom, the others playing a role of damping environment. Surely, for the finite system there are also the initial momenta, for which the system will cross the surface once more and relax in the metastable region. However, the properly defined measure of these is expected to tend to zero as the number of variables is increased. Thus we adopt $n(p, q)$ as a step function only of q smeared out somehow near the surface $f(q) = 0$. This smearing out may be taken to be arbitrary small. Putting it zero is a manifestation of the thermodynamic limit for our case and is performed at the end of the calculation. Any details of such a regularization are easily shown to be irrelevant for the final result. Let the region $f(q) < 0$ correspond to the stable phase, while the region $f(q) > 0$ to the metastable one. Without loss of generality

$$n(p, q) = \theta(f(q)), \tag{3.5}$$

where smearing out of the θ-function is implied.

At $T \neq 0$ all degrees of freedom become excited, the total energy runs to infinity. For a chosen set of these, the others form the heat bath, not only damping large perturbations but also stimulating them, so that the probability of the phase transition is non-zero. This is just the effect we are studying and it will arise automatically in our formalism. Now, with eq. (3.5) at hand we proceed to the derivation of the kinetic equation for $\bar{n}$.

3.2. THE DERIVATION OF THE KINETIC EQUATION

As soon as the slowly varying parameter $\bar{n}(t)$ is identified with the average value of a specific function of coordinates and momenta, the derivation of the kinetic

equation for $\bar{n}$ is primarily a technical task. A variety of methods [16,17,21] can be applied, but the most convenient for our purpose is the one developed by Zubarev [17]. It allows to obtain fluctuation-dissipation theorems [21] in a formal and simple way, offering a general expression for the non-equilibrium distribution function through the parameters realizing the reduced description. In our case

$$\rho(p,q;t) = N\exp\varepsilon\int_{-\infty}^{t} e^{\varepsilon(t'-t)}(-\beta H + \mu(t')n(t'))\,dt', \qquad \varepsilon \to +0. \quad (3.6)$$

Here $\beta = 1/T$, $n(t') \equiv n(p(t'),q(t'))$ and $p(t'),q(t')$ are related to $p(t)=p$, $q(t)=q$ by the Hamilton equations of motion. At every moment of time the average value $\bar{n}$ of the function $n(p,q)$ calculated with the non-equilibrium distribution (3.6) is prescribed to coincide [17] with that calculated with the so-called local equilibrium distribution

$$\rho_l(p,q;t) = N_l\exp(-\beta H + \mu(t)n(p,q)), \quad (3.7)$$

that is

$$\bar{n}(t) = \langle n(p,q)\rangle_l \equiv \int n(p,q)\rho_l(p,q;t)\,dp\,dq. \quad (3.8)$$

This determines implicitly the parameter $\mu(t)$ through $\bar{n}(t)$.

The small positive ε serves to select the retarded solution of the Liouville equation. It is exactly the point where the irreversibility enters. At finite $\varepsilon > 0$ [17]

$$\frac{\partial \ln\rho}{\partial t} + \{\ln\rho, H\} = -\varepsilon(\ln\rho - \ln\rho_l). \quad (3.9)$$

ε is taken to zero after the thermodynamic limit is performed. At $\varepsilon \to +0$ the distribution function ρ satisfies the Liouville equation as it should.

Our derivation is based on the averaging of the continuity equation

$$\dot{n}(p,q) = \sum_n \left(\frac{\partial n}{\partial p_n}\dot{p}_n + \frac{\partial n}{\partial q_n}\dot{q}_n\right), \quad (3.10)$$

where the time derivatives $\dot{p}_n, \dot{q}_n$ are to be expressed through p, q by means of the Hamilton equations. It is the non-equilibrium distribution (3.6) that must be used for this averaging since only then, due to the Louville equation the average of a time derivative is equal to the derivative of the average. In particular

$$\dot{\bar{n}} = \langle \dot{n}(p,q)\rangle. \quad (3.11)$$

Integration by parts in the exponent of (3.6) yields:

$$\rho(p,q;t) = N\exp\Big(-\beta H + \mu(t)n(p,q) - \int_{-\infty}^{t} e^{\varepsilon(t'-t)}(\dot{\mu}n + \dot{n}\mu)\,dt'\Big). \quad (3.12)$$

In the leading order with respect to the deviation from equilibrium μ is small. $\dot{\mu}$ is completely negligible in (3.12) due to the slow rate of the process. So,

$$\rho \simeq \rho_0(1 + C - \langle C\rangle_0), \quad (3.13)$$

where

$$C = \mu n - \int_{-\infty}^{t} e^{\varepsilon(t'-t)}\mu(t')\dot{n}(t')\,dt', \quad (3.14)$$

and $\langle\cdot\rangle_0$ denotes the averaging with the equilibrium distribution

$$\rho_0 = N_0\exp(-\beta H). \quad (3.15)$$

Substituting (3.13) in (3.11) and taking into account that $\langle\dot{n}\rangle_0 = 0$ and $\langle\dot{n}n\rangle_0 = 0$ we have

$$\dot{\bar{n}} = -\mu \lim_{\varepsilon\to+0}\int_{-\infty}^{t} e^{\varepsilon(t'-t)}\langle\dot{n}(t')\dot{n}(t)\rangle_0\,dt', \quad (3.16)$$

where μ is taken out of the integral because the equilibrium correlation function in (3.16) is well localized near $t' = t$. The parameter μ is determined from eq. (3.8). In the leading order in μ it yields

$$\mu = (\bar{n} - \langle n\rangle_0)/(\langle n^2\rangle_0 - \langle n\rangle_0^2), \quad (3.17)$$

and we obtain the kinetic equation for $n' = \bar{n} - \langle n\rangle_0$:

$$\dot{n}' = -\frac{n'}{\langle n^2\rangle_0 - \langle n\rangle_0^2}\lim_{\varepsilon\to+0}\int_{-\infty}^{t} e^{\varepsilon(t'-t)}\langle\dot{n}(t')\dot{n}(t)\rangle_0\,dt'. \quad (3.18)$$

Note, that in deriving (3.18) we have not used any particular expression for n. A more detailed description can be obtained with the help of eq. (3.5). First, we observe that the main contributions to $\langle n\rangle_0$ and $\langle n^2\rangle_0$ come from the neighbourhoods of the minima where n is equal to zero or unity, so $\langle n^2\rangle_0 = \langle n\rangle_0$. The continuity equation takes the form

$$\dot{n}(t) = \delta(f(q))\sum_n \frac{\partial f}{\partial q_n}p_n, \quad (3.19)$$

where according to the previous discussion $\delta(f(q))$ is a smooth function localized in

a vicinity of an arbitrary small size ε' near the surface $f(q)=0$. The correlation function in (3.18) is non-zero only if both at the moment t and at the moment t' the system is in the above ε'-vicinity and moreover the normal component of its velocity $v_\perp = |\Sigma_n(\partial f/\partial q_n)p_n|$ is non-zero in either of these moments. Thus the time interval $t-t'$ is of order $\varepsilon'/v_\perp$, i.e. can be made infinitely small. So, the motion of the system may be regarded as a free one during this interval:

$$q_n(t') = q_n(t) + (t'-t)p_n(t),$$
$$p_n(t') = p_n(t). \tag{3.20}$$

Now the integral in (3.18) can be evaluated explicitly:

$$\dot{n}' = -\frac{n'}{\langle n\rangle_0(1-\langle n\rangle_0)}\frac{1}{2}\left\langle \delta(f(q))\left|\sum_n \frac{\partial f}{\partial q_n}p_n\right|\right\rangle_0 . \tag{3.21}$$

Restricted to the surface $f(q)=0$, the saddle point is a minimum of the energy and hence is a relevant solution to expand about. For instance, in the gaussian approximation $f(q)=q_1$ and

$$\left\langle \delta(f(q))\left|\sum_n \frac{\partial f}{\partial q_n}p_n\right|\right\rangle_0 = 2TW\prod_n{}'\left(\frac{2\pi T}{\lambda_n^{1/2}}\right)N_0\exp(-\bar{E}/T)$$
$$\equiv 2TWN_0\exp(-\bar{F}/T). \tag{3.22}$$

The factor $2T$ is due to the p_1 integration, W is due to zero modes. Schematically

$$W = \int \exp\left(-\tfrac{1}{2}\sum p_i^2\right)\prod_i \mathrm{d}p_i\,\mathrm{d}q_i, \qquad \lambda_i = 0, \tag{3.23}$$

and can be calculated by standard methods, the product $\prod'$ is taken over all positive frequences. Besides, the saddle point free energy $\bar{F}$ is introduced by definition. Similarly, near the metastable minimum

$$E = E_0 + \tfrac{1}{2}\sum_m p_m^{(0)2} + \tfrac{1}{2}\sum_m \lambda_m^{(0)}q_m^{(0)2}, \tag{3.24}$$

with all λ's assumed to be positive, and

$$\langle n\rangle_0 = N_0\prod_m\left(\frac{2\pi T}{\lambda_m^{(0)1/2}}\right)\exp(-E_0/T) \equiv N_0\exp(-F_0/T). \tag{3.25}$$

With these notations the kinetic equation takes the form

$$\dot{n}' = -\frac{n'}{1-\langle n\rangle_0} TW \exp\bigl(-(\bar{F}-F_0)/T\bigr). \tag{3.26}$$

If the stable minimum is much deeper than the metastable one, $\langle n\rangle_0$ may be neglected compared to unity and the kinetic coefficient exactly coincides with the expression for the probability flux, obtained in [9] by stationary methods.

3.3. QUANTUM EFFECTS

So far, our consideration was purely classical. However, the method is generalized to the quantum case. Use the non-equilibrium density matrix [17] which has the same form as (3.6) but with H and n replaced by the corresponding operators. The non-commutativity violates the simple expansion (3.13) and we obtain the kinetic equation of the same form as (3.18) but with

$$\left\langle \int_0^1 \mathrm{d}\tau \mathrm{e}^{\beta H\tau}\dot{n}(t')\mathrm{e}^{-\beta H\tau}\dot{n}(t)\right\rangle_0 \tag{3.27}$$

instead of $\langle \dot{n}(t')\dot{n}(t)\rangle_0$. Besides, $\langle\,\cdot\,\rangle_0$ now stands for the quantum Gibbs averaging. Physically, the modification is related to the possibility of the decay due not only to thermal fluctuations but also to quantum uncertainty. However, this uncertainty may be neglected if it is small compared to the characteristic scale of the relevant classical motions. Then, the non-equilibrium part of the previous analysis must be unaltered and eqs. (3.18) and (3.21) remain intact, the ordering of coordinates and momenta in (3.21) being inessential. For instance, in the gaussian approximation the most dangerous region is the neighbourhood of the saddle point. The typical energy change there is of order T. So, $\Delta p_{\mathrm{cl}} \sim \sqrt{T}$, $\Delta q_{\mathrm{cl}} \sim \sqrt{T/|\lambda_1|}$ and $\Delta p_{\mathrm{cl}}\Delta q_{\mathrm{cl}} \sim T/\sqrt{|\lambda_1|}$. Thus, the required condition is

$$T \gg \sqrt{|\lambda_1|} \tag{3.28}$$

Actually, the quantum corrections to (3.18) and (3.21) are $\mathrm{O}(|\lambda_1|/T^2)$ (compare ref. [11]).

Perform a (classical) canonical transformation to new variables (P, Q), so that $f(q) \equiv Q_1$, $(\partial f/\partial q)^{-2}(\partial f/\partial q \cdot p) \equiv P_1$. Then in (3.21) there appears $\langle \delta(Q_1)|P_1|\rangle_0$ (operators) which may be easily taken to the form

$$\langle \delta(Q_1)|P_1|\rangle_0 = \sqrt{2/\pi}\int \mathrm{d}Q_1 \frac{1}{Q_1}\frac{\partial G}{\partial Q_1}, \tag{3.29}$$

where

$$G(Q_1) = \langle Q_1|\mathrm{Tr}'\mathrm{e}^{-\beta H}|0\rangle, \tag{3.30}$$

$|0\rangle$ standing for the eigenstate $Q_1 = 0$ (not for vacuum), Tr′ is over the other degrees of freedom.

The gaussian case may be treated more explicitly:

$$\langle \delta(q_1)|p_1| \rangle_0 = Z^{-1} \int \delta(q_1)|p_1| e^{-ip_1(q_1' - q_1)} Z'(q_1', q_1) \frac{dq_1\, dp_1}{2\pi} dq_1', \quad (3.31)$$

where Z is the full equilibrium partition sum and Z' is the reduced one, i.e. the trace of the equilibrium density matrix over all q's, except q_1. Both admit the standard functional integral representation, in particular,

$$Z'(q_1', q_1) = \int_{q_1}^{q_1'} Dq(\tau) \exp\left[- \int_0^\beta \left(\tfrac{1}{2} \sum_n (dq_n/d\tau)^2 + U(q) \right) d\tau \right], \quad (3.32)$$

all $q_n(\tau)$ for $n \neq 1$ being periodic. The classical approximation for q_1 arises when Z' is well localized at $q_1 = q_1'$. Then

$$\begin{aligned} \langle \delta(q_1)|p_1| \rangle_0 &= Z^{-1} \int dq_1\, dp_1 |p_1| e^{-\beta p_1^2/2} \delta(q_1) \\ &\quad \times \int D'q(\tau) \exp\left[- \int_0^\beta \left(\tfrac{1}{2} \sum_{n \neq 1} \left(\frac{dq_n}{d\tau} \right)^2 + U(q) \right) d\tau \right] \\ &= 2T e^{-\bar{E}/T} Z^{-1} \int D'q(\tau) \exp\left[- \int_0^\beta \left(\tfrac{1}{2} \sum_{n \neq 1} \left(\frac{dq_n}{d\tau} \right)^2 + U(q_{n \neq 1}) \right) d\tau \right], \end{aligned} \quad (3.33)$$

the prime in the measure denoting that q_1 integration is omitted. The similar equation holds for $\langle n \rangle_0$. Thus, the quantum effects replace the products of positive eigenvalues in eqs. (3.22) and (3.25) with those of the corresponding Bose factors. Also fermions can be included in the formalism. Note, the positive eigenvalues are associated through (3.33) with the euclidean propagators. This will be important for the estimate of quantum corrections in the next section.

Eq. (3.26) can hardly be applied to first-order phase transitions in infinite volume field theory. The reason is that our basic assumption on the hierarchy of times breaks down there. The actual time of the phase transition is not the one of formation of a single bubble but mainly the time of the expansion of bubbles and their interactions with each other. The breakdown of the approximation is signalled by the volume factor appearing in the kinetic coefficient of (3.26) due to the translational zero mode so that the characteristic time of the phase transition is $1/V$ which is nonsense. The best we can do is to interpret the kinetic coefficient as the

nucleation probability per total volume. Still, eq. (3.26) may be meaningful in some other many-dimensional systems. In particular, similar equations are very useful in the investigation of the B-non-conservation where no expanding bubbles occur.

4. The anomalous electroweak baryon number non-conservation at finite temperature.

In a four-dimensional field theory we encounter some extra difficulties. The very definition of the Hamilton operator is spoiled by ultra-violet divergences. The reasonable prescription is to rely on perturbation theory, dealing with the hamiltonian defined to a given order. In fact, in calculating equilibrium properties naive perturbation theory breaks down – the powers of T may compensate for the powers of the coupling constants [22]. The perturbation theory may be partially restored. First, redefine the tree scalar potential including some quadratic term, so that the most dangerous loop contributions are subtracted [22]. Then, redefine the static charges interaction, incorporating the effect of Debye screening [19]. Finally, use the running coupling constants, taken at the point T, instead of bare ones. Choosing for the sake of definiteness the Coulomb gauge, we arrive at the effective hamiltonian ($\alpha \to 0$):

$$H = \int \Big(\tfrac{1}{2} P_i \varepsilon_{ij}^{-1} P_j + \tfrac{1}{4} F_{ij}^2 + p_{\varphi^*} p_\varphi + |D_i \varphi|^2 + \lambda(T) \big(\varphi^* \varphi - \tfrac{1}{2} v^2(T) \big)^2 + \frac{1}{2\alpha} (\partial_i A_i)^2 + \partial_i \bar{c} D_i c \Big) d^3x + \text{fermions} + \text{counterterms}. \quad (4.1)$$

Here $(A_i, \varphi, \varphi^*, \ldots) \equiv Q$, $(P_i, p_\varphi, p_{\varphi^*}, \ldots) \equiv P$ are coordinates and momenta (including fermions); $\bar{c}, c$ are ghosts, $v(T)$ is a T-dependent Higgs v.e.v. [23, 22]. ε_{ij} is the static dielectric penetrability of the electroweak plasma, accounting for the charge screening: $\varepsilon_{ij} = \delta_{ij} - m_{\text{el}}^2 \partial_i \partial_j / \partial^4$, $m_{\text{el}}^2 \propto g^2 T^2$. H should be used in the Gibbs averaging:

$$\langle \cdot \rangle_0 = \frac{\int (\cdot)\, \Pi \exp\big(\int_0^\beta (iP\, dQ/d\tau - H)\, d\tau \big)\, DP\, DQ\, D\bar{c}\, Dc}{\int \Pi \exp\big(\int_0^\beta (iP\, dQ/d\tau - H)\, d\tau \big)\, DP\, DQ\, D\bar{c}\, Dc} \quad (4.2)$$

where Π is the projection operator onto physical states:

$$\Pi = \int DA_0 \exp\Big(\int_0^\beta iA_0 (D_i P_i + J_0)\, d\tau \Big), \quad (4.3)$$

J_0 is the gauge charge density of matter fields. The boundary conditions for bosonic coordinates, A_0 and ghosts are periodic in τ, bosonic momenta are unrestricted, while all the fermionic fields are antiperiodic.

Even the modified perturbation theory based on (4.1) may fail. The reason is that at some critical temperature T_c $v(T)$ vanishes and at $T > T_c$ the parameter $v^2(T)$ becomes negative [22–24]. This leads to the well-known infrared problem in statistics of gauge fields near and above T_c [25,19]. Anyhow, (4.1) is a suitable starting point*.

Now the generalization of the formalism of the previous section is straightforward. Consider the static bosonic energy, $\partial_i A_i = 0$;

$$U(q) = \int \left(\tfrac{1}{4} F_{ij}^2 + |D_i \varphi|^2 + \lambda(T) \left(\varphi^* \varphi - \tfrac{1}{2} v^2(T) \right)^2 \right) \mathrm{d}^3 x , \tag{4.4}$$

$q \equiv (A_i$ (transverse), $\varphi, \varphi^*)$. $U(q)$ has an infinite set of energy minima q_b labelled by b, the number of real baryons B appearing as a result of the level crossing phenomenon as discussed in the Introduction, divided by the number of generations: $b = B/n_f = 0, \pm 1, \ldots$. We expect that either two of the minima, say q_b and q_{b+1}, are separated by the surface $f_b(q) = 0$ similar to the one discussed in sect. 3. Namely, in each of its points, the gradient of the static energy is orthogonal to the normal while the surface does not pass through any minimum. So $f_b(q)$ satisfies:

$$\int \left(\frac{\delta U}{\delta A} \frac{\delta f_b}{\delta A} + \frac{\delta U}{\delta \varphi} \frac{\delta f_b}{\delta \varphi^*} + \frac{\delta U}{\delta \varphi^*} \frac{\delta f_b}{\delta \varphi} \right) \mathrm{d}^3 x = 0 . \tag{4.5}$$

These surfaces definitely exist below the critical temperature T_c since then there are saddle points in q-space related by "large" gauge transformations to the sphaleron solution, obtained in [26]. At $T > T_c$ there are no saddle points, but it is intuitively clear that the surfaces with the above property still exist.

We shall verify a posteriori that the processes with B-non-conservation are indeed slow compared to the usual kinetic processes in the electroweak plasma. So the reduced description is applicable. The non-equilibrium distribution function on the last stage of its evolution is represented by an infinite number of parameters $\bar{n}_b$ replacing the single parameter $\bar{n}$ of the previous section. Each of these is a non-equilibrium average of some function n_b of bosonic coordinates and has the meaning of a full probability of observing the system in the given b-sector. With the same reasoning as in sect. 3,

$$n_b(q) = \theta(f_{b-1}) - \theta(f_b) , \tag{4.6}$$

where again the smearing out of step functions is implied when necessary. For any q

$$\sum_b n_b(q) = 1 . \tag{4.7}$$

* We stress that the use of (4.1) instead of the original hamiltonian is a matter of convenience, not of principle. Alternatively, we might have used the original expression and summed the relevant corrections in the course of the calculations (see ref. [27] for a two-dimensional example).

The non-equilibrium statistical operator is

$$\rho(P,Q;t) = N\exp\varepsilon\int_{-\infty}^{t} e^{\varepsilon(t'-t)}\Big(-\beta H + \sum_b \mu_b(t')n_b(t')\Big)\,dt', \qquad \varepsilon \to +0. \quad (4.8)$$

It would be very instructive to obtain an explicit expression for the function f_b. We have not succeeded so far (see, however, appendix B). Fortunately, the particular expression for f_b will not be essential for the derivation of some general results (subsects. 4.1 and 4.2) and for the actual calculations at $T < T_c$ (subsect. 4.3.1.). The only place where it is necessary is the calculation at $T > T_c$ (subsect. 4.3.2). The determination of f_b could be part of the lattice investigation at $T > T_c$.

4.1. FLUCTUATION-DISSIPATION FORMULA

First, let us verify the condition (3.28) which allows us to manipulate with (4.8) as with a classical distribution function in a sense explained at the end of sect. 3. Below T_c the magnitude of the saddle point negative mode is of the order $m_W(T)$, the effective T-dependent W mass. Above T_c there is no saddle point and $m_W(T) = 0$ but the motions in the vicinity of the critical surface are governed by the fundamental inverse correlation length of the order $\alpha_W(T)T$ [25,19]. (See subsects. 4.3 and 4.4 for more detail). Thus, the process becomes essentially classical at

$$T \gg m_W(T).$$

Proceeding in full analogy with sect. 3 we obtain in the leading order in the μ's:

$$\dot{\bar{n}}_b = -\sum_{b'}\mu_{b'}\lim_{\varepsilon\to+0}\int_{-\infty}^{t} e^{\varepsilon(t'-t)}\big\langle \dot{n}_{b'}(t')\dot{n}_b(t)\big\rangle_0\,dt', \quad (4.9)$$

$$\bar{n}_b = \langle n_b\rangle_0 - \sum_{b'}\mu_{b'}\langle n_b\rangle_0\langle n_{b'}\rangle_0 + \sum_{b'}\mu_{b'}\langle n_{b'}n_b\rangle_0, \quad (4.10)$$

instead of (3.16) and (3.17). Clearly, $\langle n_b n_{b'}\rangle_0 = \delta_{bb'}\langle n_b\rangle_0$.

Some care must be taken when solving (4.10) as a linear system with respect to the μ's. The mean-square deviation of B in equilibrium goes as $\sqrt{V}$, V = the total volume. With the normalization (4.7) this implies $\langle n_b\rangle_0 \sim 1/\sqrt{V}$. So, the off-diagonal coefficients $\langle n_b\rangle_0\langle n_{b'}\rangle_0$ in (4.10) are much smaller than the diagonal ones $\langle n_b\rangle_0$. Nonetheless, they might have contributed the same due to their large number. However, a fine compensation occurs. Indeed, let us verify that

$$\mu_b = n'_b/\langle n_b\rangle_0,$$

$$n'_b = \bar{n}_b - \langle n_b\rangle_0, \quad (4.11)$$

obtained by neglecting the off-diagonal terms, is a fairly good approximation. The

next iteration, extracted from (4.10) is $\mu_b = (n'_b + \langle n_b\rangle_0 \Sigma_{b'\neq b} n'_{b'})/\langle n_b\rangle_0$. Since $\Sigma_b n'_b = 0$, $\Sigma_{b'\neq b} n'_{b'} = -n'_b$ and the correction to (4.11) is indeed $1/\sqrt{V}$.

Thus, we arrive at a system of kinetic equations for $n'_b = \bar{n}_b - \langle n_b\rangle_0$:

$$\dot{n}'_b = -\sum_{b'} L_{bb'} n'_{b'}, \tag{4.12}$$

where $L_{bb'}$ is a matrix of kinetic coefficients

$$L_{bb'} = \frac{1}{\langle n_{b'}\rangle_0} \lim_{\varepsilon\to+0} \int_{-\infty}^{t} e^{\varepsilon(t'-t)} \langle \dot{n}_{b'}(t') \dot{n}_b(t)\rangle_0 \, dt'. \tag{4.13}$$

This is a manifestation of the fluctuation-dissipation theorem. In fact, $L_{bb'}$ is three-diagonal, since the correlators $\langle \dot{\theta}_{b'}(t')\dot{\theta}_b(t)\rangle_0$ with $b \neq b'$ vanish as a result of the strong suppression of large fluctuations passing through both $f_b = 0$ and $f_{b'} = 0$. So, performing the t' integration as before, we have

$$L_{bb'} = \begin{cases} (\Lambda_b + \Lambda_{b-1})/\langle n_b\rangle_0, & b' = b \\ -\Lambda_{b-1}/\langle n_{b-1}\rangle_0, & b' = b-1 \\ -\Lambda_b/\langle n_{b+1}\rangle_0, & b' = b+1 \\ 0, & \text{otherwise} \end{cases} \tag{4.14}$$

$$\Lambda_b = \tfrac{1}{2} \left\langle \delta(f_b(q)) \left| \frac{\partial f_b}{\partial q} p \right| \right\rangle_0. \tag{4.15}$$

We introduce a "derivative" with respect to the discrete index b:

$$\frac{d}{db} f_b \equiv f_b - f_{b-1}. \tag{4.16}$$

Neglecting the third and higher "derivatives" we may rewrite eq. (4.12) as a discrete version of the Fokker-Planck equation

$$\dot{\bar{n}}_b = \frac{d}{db}\left(\frac{\Gamma_b \bar{n}_b}{T} \frac{d}{db} F_b \right) + \frac{d}{db}\left(\Gamma_b \frac{d\bar{n}_b}{db} \right), \tag{4.17}$$

$$\Gamma_b = \Lambda_b/\langle n_b\rangle_0, \tag{4.18}$$

$$F_b = -T \ln(\langle n_b\rangle_0/N_0), \tag{4.19}$$

where we have used $\bar{n}_b$ instead of $n'_b = \bar{n}_b - \langle n_b\rangle_0$ because $\langle n_b\rangle_0$ satisfies eq. (4.17) by itself. Eq. (4.18) acquires its real sense in the thermodynamic limit when the

discrete nature of B becomes inessential. It turns into the genuine Fokker-Planck equation for the distribution function $\bar{n}(B) \equiv \bar{n}_b/n_f$, normalized to unity: $\int \bar{n}(B)\,\mathrm{d}B = 1$, $B = n_f b$.

The "inverse friction" $\Gamma_b \equiv \Gamma(B)$ receives a contribution from the surface $f_b(q) = 0$. Clearly only configurations local in space are relevant, otherwise the energy is infinite. Hence $\Gamma(B)$ is proportional to the total volume V as a consequence of the translational invariance. The "driving potential" $F_b \equiv F(B)$ having a meaning of the free energy is, of course, also proportional to it. This ensures a proper volume dependence of the first term in the r.h.s. of eq. (4.17) while the second, diffusion, term has an extra $1/V$ factor and thus may be dropped. Our final result reads

$$\frac{\partial \bar{n}(B)}{\partial t} = n_f^2 \frac{\partial}{\partial B}\left(\frac{\Gamma(B)\bar{n}(B)}{T} \frac{\partial F(B)}{\partial B} \right), \tag{4.20}$$

and may be regarded as a description of the Stokes movement in the potential $F(B)$:

$$\dot{B} = -n_f^2 \frac{\Gamma(B)}{T} \frac{\partial F}{\partial B} \tag{4.21}$$

with a random initial condition. Note that eq. (4.21) has exactly the form proposed in [3] on physical grounds. The equilibrium solution to (4.20) is

$$\langle n \rangle_0 = \delta(B - B_0), \tag{4.22}$$

where B_0 is the minimum of $F(B)$. Note, that passing from (4.17) to (4.20) we just leave aside the usual thermodynamic fluctuations of B suppressed by the powers of V.

Thus we have demonstrated the existence of baryon number violating processes at finite temperature. The effect is real-time and essentially classical, so we expect B-non-conservation to be unsuppressed. The precise determination of the rate requires the evaluation of the equilibrium averages in (4.18), (4.19).

4.2. CALCULATION OF $F(B)$

The driving potential $F(B)$ is determined by the vicinity of the corresponding minimum and may be calculated in perturbation theory. In the perturbative framework $F(B)$ may be viewed as a free energy of a weakly interacting gas of ordinary particles at a given baryon number. Note, that in this framework the fixation of B makes clear sense. The lowest orders perturbative calculations of $F(B)$ are reliable even near and above T_c because the infrared divergences reveal themselves only in the order $g^6(T)$ [25,19].

Neglecting all masses compared to T and all interactions we may compute $F(B)$ by standard thermodynamics. Note, that since the main contribution comes from

particles with momenta of order $T \gg M_W(T)$ we should use the spectrum of the theory with $v = 0$ rather than the Higgs one even at $T < T_c$. Then, besides the baryon number B the state is characterized by the hypercharge Y, isospin T_3, and, if there is no mixing in the leptonic sector, by the leptonic numbers $L_1 \dots L_{n_f}$, n_f the number of generations. Only Y, T_3 and $B/n_f - L_i$ are the invariants of the full interacting system. Let us confine ourselves to

$$B/V, L_i/V \ll T^3, \qquad Y = T_3 = 0,$$

the case most important to cosmology. Then

$$F(B) = \frac{3}{4} \frac{22n_f + 13}{n_f(5n_f + 3)} \frac{(B - B_0)^2}{VT^2} + \text{const}, \tag{4.23}$$

$$B_0 = \frac{8n_f + 4}{22n_f + 13} \left(B - \sum_i L_i \right). \tag{4.24}$$

This was previously computed in [4] but the contribution of bosonic fields which also turns to be B-dependent was missed there. (A similar mistake was made in [15].) Fortunately, the error is numerically negligible.

The corrections to B_0 due to masses and interactions, needed for some cosmological applications [4] may also be found. Leptons give the main contributions:

$$\Delta B_0 = -A \frac{6}{13\pi^2} \sum_{i=1}^{n_f} \frac{\overline{m}_{l_i}^2(T)}{T^2} \left(L_i - \frac{1}{n_f} B \right). \tag{4.25}$$

Here $\overline{m}_{l_i}^2$ is an average square of lepton masses of a given generation:

$$\overline{m}_l^2(T) = \tfrac{1}{3} \left(m_{l_L}^2(T) + m_{l_R}^2(T) + m_\nu^2(T) \right),$$

$A \simeq 1$ is a coefficient, slightly depending on a number of fermionic flavours. For $T > T_c$ the fermions acquire masses through radiative corrections*. Nonsymmetric on leptonic flavours part, contributing to (4.25) for $B - \sum_i L_i = 0$, is

$$\left[\frac{\overline{m}_{l_i}^2(T)}{T^2} \right]_{NS} = \tfrac{1}{6} f_i^2 = \frac{\pi \alpha_W}{3} \frac{m_{l_i}^2(0)}{m_W^2}, \tag{4.26}$$

* The conclusion of refs. [3,4] that baryon asymmetry generated in grand unified theories with $B - L$ conservation completely disappears at the moment of electroweak phase transition was obtained in the tree approximation and is spoiled by radiative corrections, see (4.26). Nevertheless, the baryon generation in GUTs with $B - L = 0$ remains highly problematic because only 10^{-6} part of the baryon asymmetry survives (in the electroweak theory with 3 fermionic generations).

whereas for $T < T_c$

$$\left[\frac{\overline{m}_i^2(T)}{T^2}\right]_{\rm NS} = \frac{1}{3}\left(\frac{f_i v(T)}{T}\right)^2 + \tfrac{1}{6} f_i^2 . \tag{4.27}$$

Here f_i is Yukawa coupling of a massive fermion.

4.3. CALCULATION OF Γ

Note, that the B-dependent term written explicitly in (4.23) is only a small correction to the leading VT^4 term included in the constant. However, it is just the one most important there because only $\mathrm{d}F/\mathrm{d}B$ enters eq. (4.17). The B-dependence of Γ is not so essential and may be neglected. Now we argue that this allows us to perform the Γ calculation in the effective three-dimensional theory, which is a classical limit of (4.1). Let us also neglect here the hypercharge interactions offering corrections in $\sin^2\theta_{\rm W}$. Then,

$$\Gamma = \tfrac{1}{2} T \frac{\int \delta(f_0(q)) \left|\frac{\partial f_0}{\partial q} p\right| \Pi \exp(-\beta H_3)\, Dq\, Dp\, D\bar{c}\, Dc}{\int n_0 \Pi \exp(-\beta H_3)\, Dq\, Dp\, D\bar{c}\, Dc}, \tag{4.28}$$

where

$$q = (A_i^a, \varphi, \varphi^*), \qquad p = (P_i^a, p_\varphi, p_{\varphi^*}),$$

$$H_3 = \int \Big(\tfrac{1}{2} P_i \varepsilon_{ij}^{-1} P_j + \tfrac{1}{4} F_{ij}^2 + p_{\varphi^*} p_\varphi + |D_i \varphi|^2 + \lambda(T)\big(\varphi^*\varphi - \tfrac{1}{2} v^2(T)\big)^2 + \frac{1}{2\alpha}(\partial_i A_i)^2 + \partial_i \bar{c} D_i c \Big) \mathrm{d}^3 x + \text{counterterms}, \qquad \alpha \to 0,$$

$$\Pi = \int DA_0 \exp \int \mathrm{d}^3 x\, i\beta A_0^a \Big(D_i P_i^a + ig\varphi^* \frac{\tau^a}{2} p_{\varphi^*} - igp_\varphi \frac{\tau^a}{2} \varphi \Big). \tag{4.29}$$

The counterterms in (4.29) make (4.28) ultraviolet-finite. Their effect is to subtract the ring graphs in the scalar propagator, which are the only divergent ones in the three-dimensional theory, since they were already incorporated in the redefinition of $v(T)$ (see also [7]).

The quantum corrections to (4.28) are

$$\mathrm{O}\big((B/V)^{2/3}/T^2, (L_i/V)^{2/3}/T^2, m^2(T)/T^2\big) \qquad \text{and} \qquad \mathrm{O}(\alpha(T)),$$

$m(T)$ and $\alpha(T)$ standing for any mass and interaction constant. Thus, they are small. Note, that if we were to calculate the real time correlation functions, we should not get off so cheap. For instance, the quantum effects modify considerably the gauge propagator in the frequency region $\omega \lesssim k \ll T$ [19]. So, it is important that we were able to represent Γ as a simple thermodynamic average, given by the euclidean functional integral (4.2). This allows to estimate the higher order corrections to Γ by mere examination of the loop contributions to euclidean propagators, say, in the saddle point background (remember remarks in subsect. 3.3). The real-time response frequencies do not appear. See the discussion of real-time formalism in appendix C.

Though this argument is quite complete we should like to comment on the role of a particular real-time effect – Landau damping in a plasma [28] since it was proposed earlier [15] as a relevant one for the determination of Γ. Consider again the simpler model of sect. 3 but now suppose that it is not isolated but rather a part of a larger interacting system. Generally, one can account for modification by simply enlarging the set of dynamical variables and using the previous formalism based on the Liouville equation. However, under certain conditions it is possible to proceed differently. Deal with a distribution function dependent on the former (smaller) number of variables but now governed by the Fokker-Planck equation with the effect of the extra degrees of freedom incorporated in the friction coefficients. What would then happen to the decay rate? This question was first addressed by Langer [9] in the gaussian approximation. His answer obtained by the steady-state method differs from the one (3.22) for the isolated system by a factor $|\kappa|/|\lambda_1|^{1/2}$, where $|\kappa|$ is the growth rate of the unstable mode at the saddle point taking friction into account. In the electroweak plasma there is also a sort of friction – Landau damping in the relaxation of the initial perturbation. One may be tempted to add a similar factor in the gaussian expression for Γ, extracting κ from the real-time propagator with the necessary loop corrections included [15]. However, there is a major difference between the two cases and the analogy is misleading. The friction discussed by Langer is due to the interaction with some external degrees of freedom. This implies a change in the microscopic equations of motion for the system (they are of Langevin type) and in the equation for the distribution function. In a sense, the damping factor, appearing in Γ replaces the determinant corresponding to these additional variables. In contrast, the electroweak plasma is perfectly isolated. (*Almost* isolated in our formalism – remember small ε in (4.8).) Landau damping as well as the very existence of the temperature distribution are provided by the presence of the huge number of degrees of freedom within the theory. The microscopic equations of motion are the hamiltonian equations deduced from (4.1). Exactly these were used to obtain the relationship between p and $\dot{q}$ when deriving (4.15). Landau damping does not affect the rate. This conclusion differs from the one suggested in ref. [15] on intuitive grounds. Below we present the estimates of Γ based on (4.28).

4.3.1. $T < T_c$. $v^2(T) > 0$ and the Higgs v.e.v. is non-zero. There is a saddle point solution to the classical equations determined from H_3. This is the sphaleron of refs. [26] with the energy $\bar{E} \propto m_W(T)/\alpha_W(T)$. Expand the numerator in (4.28) near the sphaleron. Note again, how the saddle point becomes the relevant configuration. In the gaussian approximation $f_0(q)$ is simply the coordinate corresponding to the sphaleron negative mode. Similarly to (3.22), (3.25) we have

$$\Gamma = TW \,\mathrm{Det}'\{3\} \exp(-\bar{E}/T), \tag{4.30}$$

where $\mathrm{Det}'\{3\}$ is a combination of various three-dimensional determinants in the sphaleron background. The volume factor appearing in W due to the translational zero mode is quite on the spot now as discussed earlier. The renormalization prescription for infinite products was already fixed.

The gaussian approximation leading to (4.30) is reliable only at $T \ll m_W(T)/\alpha_W(T)$. Then B-non-conservation is exponentially slow compared to the usual kinetics in the plasma. At $m_W(T) \lesssim \alpha_W T$ the infrared disasters come into play indicating the vicinity of the phase transition and the approximation breaks down.

For $\lambda(T) \sim g^2$ the r.h.s. of eq. (4.30) is easily estimated. Up to numerical constants, the three translational zero modes produce $(\bar{E}/T)^{3/2}T^3V$, the three "rotational" ones – $(\bar{E}/T)^{3/2}[T/m_W(T)]^3$. (This estimate coincides with that of [15] where numerical normalization factors can also be found.) The T-dependent contribution of positive modes is obtained with the help of the Levinson theorem, relating the asymptotics of the phase shift of the continuum to the number of discrete eigenvalues. It gives $[m_W(T)/T]^7$, where m_W stands for the characteristic continuum energy, for which the phase shift is non-zero, and 7 (= 6 zero modes and one negative) is the total number of non-positive discrete modes in the sphaleron background (the positive discrete modes, if they exist do not produce T-dependence, see [27]). So, even the negative mode of the sphaleron reveals itself by its own m_W/T factor. Note, however, that since it comes from the continuum, this factor has nothing to do with the real process of sphaleron decay. Thus, the damping of this decay discussed in [15] is irrelevant to the determination of Γ. (At this point we disagree with [15].) Finally, (4.30) gives:

$$\Gamma = \mathrm{const}\, T(\bar{E}/T)^3 (m_W(T)/T)^4 T^3 V \exp(-\bar{E}/T). \tag{4.31}$$

This expression coincides with that of ref. [15] up to damping factor unjustly included there.

4.3.2. $T > T_c$. Now the Higgs v.e.v. is zero. There is no saddle point and Γ cannot be computed in the gaussian approximation. However, the eq. (4.28) may be somewhat simplified. First, the Coulomb interaction may be neglected due to charge screening [19, 25]. This is formally achieved by taking ε^l_{ij} in (4.28) to infinity. Then P^l_i (longitudinal) and A_0 integrations are carried out cancelling the ghost determi-

nant. Moreover, at sufficiently high temperatures the scalar integrations may be dropped since scalars develop non-vanishing mass of order $g(T-T_c)^{1/2}$ already in the one-loop order, while the transverse vector fields are expected to receive only $\alpha_W T$ from higher loops. Thus, at $T - T_c \gg \alpha_W T$ we arrive at

$$\Gamma = \tfrac{1}{2}T \frac{\int \delta(f_0(A_i, \varphi = 0)) \left| \frac{\partial f_0}{\partial A_i} P_i \right| \exp(-\beta H_A)\, DA\, DP}{\int \exp(-\beta H_A)\, DA\, DP}$$

$$H_A = \int \left(\tfrac{1}{2}P_i^2 + \tfrac{1}{4}F_{ij}^2 + \frac{1}{2\alpha}(\partial_i A_i)^2 \right) d^3x, \qquad \alpha \to 0. \tag{4.32}$$

(we have introduced a new redundant gaussian integration over the longitudinal P_i).

We perform a scale transformation to dimensionless variables:

$$A = gTA', \qquad P = g^3T^2P', \qquad x = x'/g^2T, \qquad c = gTc'. \tag{4.33}$$

Then $H_A = TH_A'$, where H_A' is obtained from (4.32) by replacing the variables and putting $g = 1$. Similarly, $f_0(A) \equiv f_0(A, \varphi = 0)$ must be $f_0(A', g = 1)$ up to a constant which, clearly, cancels out from (4.32). So, the scaling determines g, T dependence of Γ:

$$\Gamma/VT = \text{const}\, g^8T^3, \tag{4.34}$$

where g^2 comes from Γ and g^6T^3 from volume scaling.

Rewrite (4.21) in terms of baryonic density $\rho = B/V$:

$$\dot{\rho} = -n_f^2 \frac{\Gamma}{VT} \frac{\partial f}{\partial \rho}, \tag{4.35}$$

where $f = F/V$ is the spatial density of the free energy. This shows that the characteristic inverse time of B-dissipation is

$$\tau^{-1} = \text{const}\, g^8T = \zeta \alpha_w^4 T, \tag{4.36}$$

i.e. the process is indeed slow compared to the kinetic scale $\alpha_W^2 T$. The contribution of some particular set of configurations to B non-conservation at $T > T_c$ has been estimated in [15] and is of the order α_W^4. Unfortunately, the explicit figures for ζ cannot be considered either as an upper, or as a lower bound on ζ.

With the knowledge of the explicit form of the function $f_0(A)$ the coefficient in front of α_W^4 can be estimated by means of Monte Carlo technique on the lattice. See the discussion of the surface $f_0(A) = 0$ in appendix B.

5. Conclusion

Let us summarize our results. The first is *the derivation* of eq. (4.21). This relates B-non-conservation at finite temperature to such basic concepts of statistical mechanics as the reduced description and the fluctuation-dissipation theorem. Valuable in itself, the above understanding also enables us to refine the actual computation of the rate. It becomes particularly clear why the sphaleron solution, though a saddle point and not a minimum, is a relevant classical configuration at $T < T_c$. Also, $\alpha_W^4 T$ estimate for the rate at $T > T_c$ is consistently obtained.

The determinant expression (4.30) was known for some time. However, its precise meaning was never well understood. Not only did we show how a single coefficient Γ absorbs the wealth of information on the B-violating transitions through a variant of the Fokker-Planck equation. We presented the expression (4.18) for Γ which is exact in a sense that it is not classical and gaussian from the very beginning. This allows us to estimate the quantum corrections.

One may still feel unsatisfied that we do not proceed exclusively from the microscopic equations of motion and invoke some hypothesis concerning the slowly-varying variables. However, one should realize that even the microscpic proof of the statistical character of the electroweak theory underlying equilibrium calculations is a matter of tremendous difficulty, hardly expected to be ever performed. So much the more this is true for non-equilibrium phenomena. In our opinion, the whole program carried out here is as much on the same level of "microscopicity" as, for instance, the standard derivation of the Boltzmann equation.

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Appendix A

Here we carry out the real-time calculation of the Green function (2.7). Our purpose is to complement the statistical theory developed in the text by intuitive, but closer related to the familiar field theory, considerations.

According to (2.7) in the classical gauge-Higgs theory the rate of B-non-conservation is given by

$$\Gamma = V \lim_{t\to\infty} Z^{-1} \int Dp\, Dq\, Q(t) \frac{g^2}{32\pi^2} F\tilde{F}(0) \exp(-\beta H), \tag{A.1}$$

where $q = (A_i^{\mathrm{tr}}, \varphi, \varphi^*)$ and $p = (P_i^{\mathrm{tr}}, p_\varphi, p_{\varphi^*})$ are coordinates and momenta taken at $t = 0$, H is the hamiltonian, Z is the equilibrium partition sum. The canonical variables at moment t are related to p, q by the classical equations of motion.

Consider for definiteness the case $T < T_c$. Then the main contribution to (A.1) comes from the trajectories starting at $t = 0$ somewhere in the vicinity of a minimum but then passing near a saddle point. At $t = \infty$ such a trajectory will most probably relax in the neighbouring minimum. Thus we can put at $t \to \infty$

$$Q(t) = \pm 1 \tag{A.2}$$

depending on the particular direction of this motion.

In what follows we confine ourselves to a single saddle point – the sphaleron of refs. [26]. Similarly, Z may be understood as taken over a single minimum. The trajectories passing near the sphaleron admit a very convenient parametrisation. Define a small disc in coordinate space, orthogonal to the sphaleron negative mode direction:

$$A(x; c_n, x_0, \omega) = A_{\text{sph}}(x - x_0, \omega) + \sum_n{}' c_n A^{(n)}(x - x_0, \omega),$$

$$\varphi(x; c_n, x_0, \omega) = \varphi_{\text{sph}}(x - x_0, \omega) + \sum_n{}' c_n \varphi^{(n)}(x - x_0, \omega), \tag{A.3}$$

where $(A^{(n)}, \varphi^{(n)})$ represent the nth positive (as indicated by prime) sphaleron normal mode, x_0 and ω are the collective coordinates, corresponding to the infinitesimal translations and rotations, $c_n \ll 1$. The relevant trajectories in phase space form a tube covered up by the disc in its real time evolution with arbitrary initial momenta. Consider a small section of the tube confined between the image of the disc at moment τ of this evolution and that at moment $\tau + d\tau$. This may be regarded as a relevant part of $t = 0$ phase space integrated over in (A.1). According to the Liouville theorem the volume of this section is the same as it was a time τ back, namely

$$Dp\,Dq = Dp_\tau\,Dq_\tau = \prod_n{}' dc_n\,dp_n\,dx_0\,dp_{x_0}\,d\omega\,dp_\omega |p_-|\,d\tau\,dp_-, \tag{A.4}$$

where p_- is the projection of the initial momentum on the normal to the disc, i.e. the momentum corresponding to the negative mode, while p_n and p_{x_0}, p_ω are momenta conjugate to the positive and zero modes coordinates. H is a conserved quantity, so it may be taken on the disc. $F\tilde{F}(0)$ is now viewed as a topological density appearing in the space origin after a time τ of the evolution with initial conditions of type (A.3) specified by a set $\{c_n, x_0, \omega; p_n, p_{x_0}, p_\omega\}$. The x_0 and τ integrations of (A.4) turn this density into the full topological charge Q' created in the decay of the sphaleron. We expect, that $Q' = \pm \frac{1}{2}$, the sign being the same as in (A.2)*. Clearly, taking into account other integrations (remember also the volume factor in (A.1)) this yields the result (4.30).

* This property can be partially checked by computer solving the classical equations of motion.

Appendix B

Though we have not yet succeeded in determining the explicit form of the function $f_b(A, \varphi)$ entering the description of B-non-conservation, let us present here a reasonable candidate. This is

$$f_b(q) = N_{\mathrm{CS}} - \left(b + \tfrac{1}{2}\right), \tag{B.1}$$

where N_{CS} is the SU(2) Chern-Simons number

$$N_{\mathrm{CS}} = \frac{g^2}{32\pi^2} \int \varepsilon_{ijk} \left(F^a_{ij} A^a_k - \tfrac{1}{3} g f^{abc} A^a_i A^b_j A^c_k \right) \mathrm{d}^3x . \tag{B.2}$$

Even at the best case (B.1) determines f_b only locally. There are surely configurations with N_{CS} half-integer which have nothing to do with the process considered. These appear, for instance, in the redundant directions discussed in sect. 2. In fact, the manifold with, say, $N_{\mathrm{CS}} = \frac{1}{2}$ is not a surface but rather a number of them. What we expect is that among these is the only one which satisfies also (4.5).

(B.1) is distinguished by the following. If we have used in (4.28) $f_0(q) = N_{\mathrm{CS}} - \frac{1}{2}$, then $\partial f / \partial q \cdot p = (g^2/32\pi^2) \int F\tilde{F} \mathrm{d}^3x$ which is the space integral of the familiar topological density, and (4.28) may be viewed as an average topological flux.

The simplest test of (B.1) is to check whether the sphaleron negative mode coincides with the normal to $N_{\mathrm{CS}} = \frac{1}{2}$. We will not attempt it here.

Appendix C

The method of this paper enables us to represent the rate of B-non-conservation as a thermodynamic average (4.18) given by the euclidean functional integral. At $T < T_c$ this may be approximated with the product of Matzubara frequencies, see (4.30), only slightly affected by the quantum corrections. Let us now comment on the quasiparticle approach to the problem [15] dealing with the real-time response frequencies of the plasma.

Consider the following representation of the gaussian expression for the rate [9, 11]:

$$\Gamma = \frac{\omega_-}{\pi T} \mathrm{Im}\, F . \tag{C.1}$$

Im F is introduced *by definition* as:

$$\mathrm{Im}\, F = \tfrac{1}{2} T \frac{2\pi T}{\omega_-} \prod_n{}' \left(\frac{2\pi T}{\omega_n} \right) W \prod_m \left(\frac{\omega^{(0)}_m}{2_\pi T} \right), \tag{C.2}$$

ω_- is the magnitude of the imaginary frequency $i\omega_-$ which corresponds to the

negative mode. Clearly, ω_- cancels from (C.1) but the notations are to resemble the theory of the false vacuum decay [29], where Im E has the full meaning of the imaginary part of the energy and is obtained by analytical continuation. Never mind, real-time or Matzubara frequencies are taken in (C.1) since in the gaussian approximation they are equal to each other. More precisely, there are two real-time poles for each excitation mode reflecting the existence of two independent solutions to the real time equations of motion, but they differ only in sign.

Now one may be tempted to account for non-gaussian effects by using in (C.1) and (C.2) the quasiparticle frequencies corresponding to the poles of the real-time propagators in the sphaleron background with some loop corrections included [15]. The latter need not be small, so these frequencies may differ considerably from the gaussian and, hence, from the Matzubara ones. However, we shall claim that the quasiparticle picture, though perfectly good for excitations of the stable state, is somewhat ambiguous for the description of metastability. The crucial point is that eq. (C.1) was in fact never derived beyond a gaussian approximation and such a generalization of it is an assumption. Thus, Im F in (C.1) has no precise definition in this case and we can only speculate on what it can be.

Note, that the "natural" definition of the non-Gaussian Im F through the product of all frequencies immediately runs into difficulties. The reason is that already in the one-loop approximation for the propagators the invariance under reflexion of time is lost, Landau damping arises [28]. Thus, the two poles corresponding to each zero or negative sphaleron mode need not be equal in magnitude. Consider for instance the translational motion of a sphaleron as a whole. On physical grounds we expect the coordinate of its center to depend on time like $C_1 + C_2\exp(-\gamma t)$, where C_1 is an arbitrary initial position, C_2 is proportional to the initial velocity, γ the friction in the medium. The corresponding poles are at $\omega = 0$ and $\omega = -i\gamma$. The first is just the translational zero frequency, which must be replaced by the collective coordinate and ensures the proper volume factor in (C.2), the second pole is irrelevant. Similarly, the unstable mode gives poles at $\omega = i\omega'_-$ and $\omega = -i\omega'_+$, the first corresponding to the growth of instability, the second – to its decrease. What should we use in Im F: ω'_-, ω'_+ or $\sqrt{\omega'_-\omega'_+}$? By the contour integral representation, used in [15] it may be proved that in the high temperature limit the squared product of Matzubara frequencies is equal up to the sign to the product of all real-time poles, both of them for each degree of freedom included. Thus, the latter of possibilities mentioned above leads to Im F coinciding with the Matzubara expression. Such a choice was adopted in [15] without motivation, while in front of Im F in (C.1) just ω'_- was used, yielding Γ largely different from the gaussian one. However, we might have chosen ω'_- instead of the square root to be included in Im F. This is even closer to our experience with the false vacuum decay [29]. Then Γ is the one obtained in the text up to a factor $\sqrt{\omega'_-\omega'_+}/\omega_-$. Most likely, this is unity, i.e. the above statement is true for non-negative modes separately. Otherwise, there are some more problems with the quasiparticle formalism.

To summarize, the quasiparticle approach to metastability at finite temperature is very uncertain as it stands now. To establish it firmly one probably needs the procedure of analytical continuation, analogous to the one used at zero temperature.

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