N = 1 supersymmetric Yang–Mills theory in d = 4 and its Batalin–Vilkovisky quantization by spinor superfields

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Four dimensional $N = 1$ supersymmetric Yang–Mills theory action is written in terms of the spinor superfields in transverse gauge. This action is seemingly first order in space–time derivatives. Thus, it suggests that the generalized fields approach of obtaining Batalin–Vilkovisky quantization can be applicable. In fact, generalized fields which collect spinor superfields possessing different ghost numbers are introduced to obtain the minimal solution of its Batalin–Vilkovisky master equation in a compact form.

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1 Introduction

One of the important elements in the Berkovits formulation[1] of covariant quantization of ten dimensional relativistic superparticle and superstring theories is the spinor superfield which was introduced in [2]. Being a superconnection component, this spinor superfield possesses gauge freedom. In fact, its component fields are derived in the so called transverse gauge, where the gauge freedom depending on anticommuting coordinates is eliminated. Transverse gauge was first introduced in [3] to study some aspects of $N = 3$ supersymmetric Yang–Mills theory in four dimensional superspace. We focus on the spinor superconnection components given in [3] to deal with $N = 1$ supersymmetric Yang–Mills theory in $d = 4$. We show that spinor superconnection in transverse gauge can be used in an action which yields $N = 1$ super Yang–Mills action in component fields once the integrations over anticommuting coordinates of the superspace are performed. This action is apparently linear in space–time derivatives.

A formalism for quantization of gauge theories respecting the original symmetries is due to Batalin and Vilkovisky (BV)[4]. It is a systematic procedure of obtaining actions of gauge theories which can be used in the related path integrals. This method is formulated in terms of ghost fields and antifields. Utilizing these fields BV–master equation is introduced based on the BRST invariance of quantized gauge theory actions. Solution of BV–master equation is the action which can be employed in the related path integrals. To have a better understanding of different features of BV–method the initial gauge fields, ghosts and their antifields are gathered as components of generalized fields[5] or superfields[6][2]. We denote the latter as τ–fields for distinguishing them from the other superfields. It was shown that these formalisms are related[7]: τ–fields can be seen as a systematic way of writing generalized fields and their products. In terms of these formalisms mostly gauge theories which are not supersymmetric are considered with an exception presented in [7]. Writing four dimensional $N = 1$ supersymmetric Yang–Mills action in terms of spinor superfields in transverse gauge suggests that a generalized spinor superfield can be introduced to obtain the minimal solution of its BV–master equation. This compact form of the minimal solution of BV–master equation can be useful to understand geometrical aspects of the theory. Moreover, it constitutes an example of applicability of the generalized fields and τ–fields methods to theories which are second order in space–time derivatives, although the original form of the action is seemingly first order in the derivatives.

[2]For some applications and similar formalisms see the references in [7].

In the next section we start with recalling definitions of superconnection components of four dimensional $N=1$ supersymmetric Yang–Mills theory in transverse gauge. Afterwards, we employ them in an action bilinear in the spinor superfields and first order in covariant derivatives. We show that after integrating over anticommuting variables $N=1$ supersymmetric Yang–Mills action in component fields follows. In Section 3 we introduce ghost fields and antifields to write the appropriate generalized fields or equivalently τ–fields. By generalizing the action of Section 2 we show that BV–quantization can be achieved in a compact form utilizing these generalized fields. Lastly we discuss the results obtained.

2 Formulation of N = 1 supersymmetric Yang–Mills action by spinor superfields

We deal with the four dimensional superspace whose coordinates are $(x^{\alpha\dot{\alpha}},\ \theta^{\alpha}, \bar{\theta}^{\dot{\alpha}})$. $(\alpha, \dot{\alpha})$ are the two component spinor indices[3]. Let us briefly mention the procedure presented in [3] to find the superconnection components $\omega_\alpha, \bar{\omega}_{\dot{\alpha}}, \tilde{A}_{\alpha\dot{\alpha}}$, starting from the gauge and spinor fields $A_{\alpha\dot{\alpha}}, \chi_\alpha, \bar{\chi}_{\dot{\alpha}}$, in the transverse gauge:

$$\theta^{\alpha}\omega_{\alpha} + \bar{\theta}^{\dot{\alpha}}\bar{\omega}_{\dot{\alpha}} = 0. \tag{1}$$

Superconnection components are related by some constraint equations and Bianchi identities. Hence, they are defined to satisfy some differential equations. We deal with $N=1$ supersymmetric Yang–Mills theory, so that, one should solve the following equations given in terms of the superfields $\tilde{A}_{\alpha\dot{\alpha}}, \tilde{\bar{\chi}}_{\dot{\alpha}}, \tilde{\chi}_{\alpha}$,

$$\begin{aligned}
\mathcal{D}\tilde{A}_{\alpha\dot{\alpha}} &= -i\theta_{\alpha}\tilde{\bar{\chi}}_{\dot{\alpha}} - i\bar{\theta}_{\dot{\alpha}}\tilde{\chi}_{\alpha}, && (2)\\
\mathcal{D}\tilde{\bar{\chi}}_{\dot{\alpha}} &= 2\bar{\theta}^{\dot{\beta}}\tilde{F}_{\dot{\alpha}\dot{\beta}}, && (3)\\
\mathcal{D}\tilde{\chi}_{\alpha} &= 2\theta^{\beta}\tilde{F}_{\alpha\beta}, && (4)
\end{aligned}$$

where we introduced the operator

$$\mathcal{D} = \theta^{\alpha}\frac{\partial}{\partial\theta^{\alpha}} + \bar{\theta}_{\dot{\alpha}}\frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}}.$$

$\tilde{A}_{\alpha\dot{\alpha}}, \tilde{\bar{\chi}}_{\dot{\alpha}}, \tilde{\chi}_{\alpha}$ are the superfields whose first components are $A_{\alpha\dot{\alpha}}, \sqrt{3}\bar{\chi}_{\dot{\alpha}}, \sqrt{3}\chi_{\alpha}$. All of them take values in the gauge algebra. Here we adopt a normalization which

[3] Conventions of [8] are used and for explicit calculations in superspace see [9].

is suitable to our formalism. Selfdual and anti-selfdual field strengths are given, respectively, as

$$\tilde{F}^{\alpha\beta} = -\epsilon_{\dot{\alpha}\dot{\beta}}\left[(\partial^{\dot{\alpha}\alpha} - i\tilde{A}^{\dot{\alpha}\alpha})\tilde{A}^{\dot{\beta}\beta} - (\partial^{\dot{\beta}\alpha} - i\tilde{A}^{\dot{\beta}\alpha})\tilde{A}^{\dot{\alpha}\beta}\right], \quad (5)$$

$$\tilde{F}^{\dot{\alpha}\dot{\beta}} = -\epsilon_{\alpha\beta}\left[(\partial^{\dot{\alpha}\alpha} - i\tilde{A}^{\dot{\alpha}\alpha})\tilde{A}^{\dot{\beta}\beta} - (\partial^{\dot{\alpha}\beta} - i\tilde{A}^{\dot{\alpha}\beta})\tilde{A}^{\dot{\beta}\alpha})\right]. \quad (6)$$

Once $\tilde{A}_{\alpha\dot{\alpha}}$ is obtained, the spinor superconnections $\omega_\alpha, \bar{\omega}_{\dot{\alpha}}$ can be derived by the equations

$$(1+\mathcal{D})\omega_\alpha = 2\bar{\theta}^{\dot{\alpha}}\tilde{A}_{\alpha\dot{\alpha}}, \quad (7)$$

$$(1+\mathcal{D})\bar{\omega}_{\dot{\alpha}} = 2\bar{\theta}^{\alpha}\tilde{A}_{\alpha\dot{\alpha}}. \quad (8)$$

By solving (2)–(4) for $\tilde{\bar{\chi}}_{\dot{\alpha}}, \tilde{\chi}_\alpha$ up to the second order in $\theta_\alpha, \bar{\theta}_{\dot{\alpha}}$ and for $\tilde{A}_{\alpha\dot{\alpha}}$ up to the third order, one finds

$$\begin{aligned}
\tilde{A}_{\alpha\dot{\alpha}} &= A_{\alpha\dot{\alpha}} - i\sqrt{3}\theta_\alpha\bar{\chi}_{\dot{\alpha}} - i\sqrt{3}\bar{\theta}_{\dot{\alpha}}\chi_\alpha - i\theta_\alpha\bar{\theta}^{\dot{\beta}}F_{\dot{\alpha}\dot{\beta}} + i\theta^\beta\bar{\theta}_{\dot{\alpha}}F_{\alpha\beta} \\
&\quad -\frac{i}{\sqrt{3}}\theta^2\bar{\theta}_{\dot{\alpha}}\mathrm{D}_{\alpha\dot{\beta}}\bar{\chi}^{\dot{\beta}} - \frac{i}{\sqrt{3}}\bar{\theta}^2\theta_\alpha\mathrm{D}_{\dot{\alpha}\beta}\chi^\beta + \cdots, && (9)\\
\tilde{\chi}_\alpha &= \sqrt{3}\chi_\alpha + 2\theta^\beta F_{\alpha\beta} + \sqrt{3}\theta^2\mathrm{D}_{\alpha\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} + \sqrt{3}\theta_\alpha\theta^\beta\mathrm{D}_{\beta\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} \\
&\quad +\sqrt{3}\theta_\alpha\bar{\theta}_{\dot{\beta}}\mathrm{D}^{\dot{\beta}\beta}\chi^\beta + \cdots, && (10)\\
\tilde{\bar{\chi}}_{\dot{\alpha}} &= \sqrt{3}\chi_{\dot{\alpha}} + 2\theta^{\dot{\beta}}F_{\dot{\alpha}\dot{\beta}} + \sqrt{3}\bar{\theta}^2\mathrm{D}_{\dot{\alpha}\alpha}\chi^\alpha + \sqrt{3}\bar{\theta}_{\dot{\alpha}}\bar{\theta}_{\dot{\beta}}\mathrm{D}^{\dot{\beta}\alpha}\chi_\alpha \\
&\quad +\sqrt{3}\bar{\theta}_{\dot{\alpha}}\theta^\alpha\mathrm{D}_{\alpha\dot{\beta}}\bar{\chi}^{\dot{\beta}} + \cdots, && (11)
\end{aligned}$$

in terms of the covariant derivative $\mathrm{D}_{\alpha\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} = \partial_{\alpha\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} - i[A_{\alpha\dot{\alpha}}, \bar{\chi}^{\dot{\alpha}}]$. Now, by solving (7) and (8) we can write the spinor superconnection components as

$$\omega_\alpha = \bar{\theta}^{\dot{\alpha}}A_{\alpha\dot{\alpha}} + \frac{2i}{\sqrt{3}}\bar{\theta}^2\chi_\alpha + \frac{2i}{\sqrt{3}}\epsilon_{\alpha\beta}\theta^\beta\bar{\theta}^{\dot{\beta}}\bar{\chi}_{\dot{\beta}} - \frac{i}{2}\bar{\theta}^2\theta^\beta F_{\beta\alpha} + \frac{2i}{5\sqrt{3}}\theta^2\bar{\theta}^2\mathrm{D}_{\alpha\dot{\alpha}}\bar{\chi}^{\dot{\alpha}}, \quad (12)$$

$$\bar{\omega}_{\dot{\alpha}} = \theta^\alpha A_{\alpha\dot{\alpha}} - \frac{2i}{\sqrt{3}}\theta^2\bar{\chi}_{\dot{\alpha}} - \frac{2i}{\sqrt{3}}\epsilon_{\dot{\alpha}\dot{\beta}}\theta^\beta\bar{\theta}^{\dot{\beta}}\chi_\beta + \frac{i}{2}\theta^2\bar{\theta}^{\dot{\beta}}F_{\dot{\alpha}\dot{\beta}} + \frac{2i}{5\sqrt{3}}\theta^2\bar{\theta}^2\mathrm{D}_{\dot{\alpha}\alpha}\chi^\alpha. \quad (13)$$

We would like to utilize the superconnection components (9),(12) and (13) in transverse gauge (1) to obtain $N=1$ supersymmetric Yang–Mills theory action. To achieve this let us define the action

$$S_0 = -\frac{i}{2} < \omega^\alpha D_{\alpha\dot{\alpha}}\bar{\omega}^{\dot{\alpha}} > -\frac{i}{2} < \bar{\omega}_{\dot{\alpha}}D^{\dot{\alpha}\alpha}\omega_\alpha >, \quad (14)$$

where we suppress trace over the gauge algebra and adopt the notation

$$< \mathcal{O} > \equiv \int d^4xd^2\theta d^2\bar{\theta}\ \mathcal{O}.$$

Note that the covariant derivative act on the A fields as $D_{\alpha\dot{\alpha}} = \partial_{\alpha\dot{\alpha}} - iA_{\alpha\dot{\alpha}}$. Integrals over the anticommuting variables vanish except

$$\begin{aligned} \int d^2\theta\ \theta^\alpha\theta^\beta &= \frac{1}{2}\epsilon^{\beta\alpha}, \\ \int d^2\bar{\theta}\ \theta^{\dot{\alpha}}\theta^{\dot{\beta}} &= \frac{1}{2}\epsilon^{\dot{\alpha}\dot{\beta}}. \end{aligned}$$

Performing the integrals over the anticommuting coordinates $\theta_\alpha, \bar{\theta}_{\dot{\alpha}}$ in (14) one reaches to

$$S_0 = \frac{-1}{2}\int d^4x\ \left[\frac{1}{4}F^{\alpha\beta}F_{\alpha\beta} + \frac{1}{4}F_{\dot{\alpha}\dot{\beta}}F^{\dot{\alpha}\dot{\beta}} + i\chi^\alpha \mathrm{D}_{\alpha\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} + i\bar{\chi}_{\dot{\alpha}}\mathrm{D}^{\dot{\alpha}\alpha}\chi_\alpha\right]. \tag{15}$$

This is the $N = 1$ supersymmetric Yang–Mills theory action without auxiliary fields.

3 The BV–quantized action by generalized spinor superfields

Form of the action (14) which is seemingly first order in derivatives advocate to accomplish its BV quantization by the generalized fields approach[5] or equivalently the τ–fields method[6]. Although, in the original formulations of these methods one deals with kinetic and interaction parts of the actions separately, here, a unified treatment is preferred.

Gauge transformations of $N = 1$ supersymmetric Yang–Mills theory are irreducible. Therefore, to perform its quantization in a covariant manner we introduce the anticommuting, gauge algebra valued ghost field η. To acquire τ–fields let us also introduce the Grassmann variables τ_μ, where μ is 4 dimensional vector index. τ_μ are defined to possess ghost number 1 as the ghost field η. Then, using the covariant derivative D_μ we define the operator

$$\mathrm{D}_\tau = \tau_\mu \mathrm{D}^\mu - i[\eta,] \tag{16}$$

possessing ghost number 1. As it is announced this operator is acquainted with interactions. One introduces generalized spinor superfields which are the τ–fields carrying spinor index, $\Psi_{\tau\alpha}$ and $\Phi_{\tau\alpha}$, to write the action

$$S_1 = \frac{i}{2} < \int d^4\tau \Psi^\alpha_\tau \mathrm{D}_\tau \Phi_{\tau\alpha} > . \tag{17}$$

Trace over the gauge algebra is suppressed. A physical action is defined to possess ghost number zero and even Grassmann parity. Thus, due the ghost number attributed to τ_μ we choose $\Psi_{\tau\alpha}$ and $\Phi_{\tau\alpha}$ to have ghost numbers 0 and 3, respectively. Moreover, we let $\Psi_{\tau\alpha}$ to be Grassmann odd and $\Phi_{\tau\alpha}$ to be Grassmann even. A consistent choice for the components of τ–fields (generalized fields) is

$$\Psi_{\tau\alpha} = \Psi_{0\alpha} + \tau_\mu \Psi^\mu_{1\alpha} + \tau_\mu\tau_\nu\Psi^{\mu\nu}_{2\alpha}, \tag{18}$$

$$\Phi_{\tau\alpha} = \tau_\mu\tau_\nu\Phi^{\mu\nu}_{2\alpha} + \tau_\mu\tau_\nu\tau_\rho\Phi^{\mu\nu\rho}_{3\alpha} + \tau_\mu\tau_\nu\tau_\rho\tau_\sigma\Phi^{\mu\nu\rho\sigma}_{4\alpha}. \tag{19}$$

Ghost number N_g and Grassmann parity ϵ of the components of Ψ_τ and Φ_τ are

	Ψ_0	Ψ_1	Ψ_2	Φ_2	Φ_3	Φ_4
N_g	0	-1	-2	1	0	-1
ϵ	1	0	1	0	1	0

Negative ghost number fields are antifields of the BV formalism.

Without altering the number of linearly independent elements and taking into consideration ghost numbers we define

$$\Phi^{\mu\nu\rho\sigma}_{4\alpha} = \frac{1}{4!}\epsilon^{\mu\nu\rho\sigma}\Psi^\star_{0\alpha}, \tag{20}$$

$$\Phi^{\mu\nu\rho}_{3\alpha} = \frac{1}{3!}\epsilon^{\mu\nu\rho\sigma}\Phi_{1\alpha\sigma}, \tag{21}$$

$$\Phi^{\mu\nu}_{2\alpha} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\phi_{2\alpha\rho\sigma}, \tag{22}$$

$$\Psi^\mu_{1\alpha} = \Phi^{\star\mu}_{1\alpha}, \tag{23}$$

where star indicates the antifield as usual.

τ integrations vanish except the following one

$$\int d^4\tau\ \tau^\mu\tau^\nu\tau^\rho\tau^\sigma = \epsilon^{\mu\nu\rho\sigma}. \tag{24}$$

By plugging the generalized fields (18),(19) into the action (17) and integrating over τ_μ one finds

$$S_1 = \frac{1}{2} < -i\Psi^\alpha_0 \mathrm{D}_\mu\Phi^\mu_{1\alpha} + \Psi^\alpha_0[\eta, \Psi^\star_{0\alpha}] + i\Phi^{\star\alpha\mu}_1\mathrm{D}^\nu\phi_{2\alpha\mu\nu} + \Psi^{\alpha\mu}_1[\eta, \Phi_{1\alpha\mu}] + 2\Psi^{\alpha\mu\nu}_2[\eta, \phi_{2\alpha\mu\nu}] > . \tag{25}$$

Similarly, we introduce the action for the τ–fields (generalized fields) with dotted indices:

$$S_2 = \frac{i}{2} < \int d^4\tau \Psi_{\tau\dot\alpha}\mathrm{D}_\tau\Phi^{\dot\alpha}_\tau > . \tag{26}$$

Component fields of $\Psi_{\tau\dot\alpha}$ and $\Phi^{\dot\alpha}_\tau$ are chosen in accord with to the undotted ones (18),(19). Hence, after performing the τ_μ integrations in (26) one gets

$$S_2 = \frac{1}{2} < -i\Psi_{0\dot\alpha}\mathrm{D}_\mu\Phi_1^{\dot\alpha\mu} + \Psi_{0\dot\alpha}[\eta, \Psi_0^{\star\dot\alpha}] + i\Phi^{\star\mu}_{1\dot\alpha}\mathrm{D}^\nu\phi^{\dot\alpha}_{2\mu\nu} + \Psi^\mu_{1\dot\alpha}[\eta, \Phi^{\dot\alpha}_{1\mu}] + 2\Psi^{\mu\nu}_{2\dot\alpha}[\eta, \phi^{\dot\alpha}_{2\mu\nu}] > . \tag{27}$$

To express the component fields in terms of spinor superfields we adopt the definitions

$$\Phi^\mu_{1\alpha} = \sigma^\mu_{\alpha\dot\alpha}\Phi^{\dot\alpha}_0, \quad \Phi^{\dot\alpha}_{1\mu} = \bar\sigma^{\dot\alpha\alpha}_\mu\Phi_{0\alpha}, \tag{28}$$

$$\Phi^{\star\alpha}_{1\mu} = \Phi^\star_{0\dot\alpha}\bar\sigma^{\dot\alpha\alpha}_\mu, \quad \Phi^{\star\mu}_{1\dot\alpha} = \Phi^{\star\alpha}_0\sigma^\mu_{\alpha\dot\alpha}, \tag{29}$$

$$\Psi^{\mu\nu}_{2\alpha} = \Psi^\beta_2\sigma^{\mu\nu}_{\beta\alpha}, \quad \Psi^{\dot\alpha}_{2\mu\nu} = \Psi_{2\dot\beta}\bar\sigma^{\dot\beta\dot\alpha}_{\mu\nu}, \tag{30}$$

$$\phi^\alpha_{2\mu\nu} = \sigma^{\alpha\beta}_{\mu\nu}\phi_{2\beta}, \quad \phi^{\mu\nu}_{2\dot\alpha} = \bar\sigma^{\mu\nu}_{\dot\alpha\dot\beta}\phi^{\dot\beta}_2, \tag{31}$$

where σ_0 is (-1)identity, $\sigma_{1,2,3}$ are the Pauli matrices and

$$\sigma^{\mu\nu\beta}_\alpha = \frac{1}{4}(\sigma^\mu_{\alpha\dot\alpha}\bar\sigma^{\nu\dot\alpha\beta} - \sigma^\nu_{\alpha\dot\alpha}\bar\sigma^{\mu\dot\alpha\beta}).$$

Plugging (28)–(31) into (25) leads to

$$S_1 = \frac{1}{2} < -i\Psi^\alpha_0\mathrm{D}_{\alpha\dot\alpha}\Phi^{\dot\alpha}_0 + \Psi^\alpha_0[\eta, \Psi^\star_{0\alpha}] - \frac{3i}{2}\Phi^\star_{0\dot\alpha}\mathrm{D}^{\dot\alpha\alpha}\phi_{2\alpha} - 4\Phi^\star_{0\dot\alpha}[\eta, \Phi^{\dot\alpha}_0] + 6\Psi^\alpha_2[\eta, \phi_{2\alpha}] > . \tag{32}$$

Similarly, one can show that employing (28)–(31) in (27) yields

$$S_2 = \frac{1}{2} < -i\Psi_{0\dot\alpha}\mathrm{D}^{\dot\alpha\alpha}\Phi_{0\alpha} + \Psi_{0\dot\alpha}[\eta, \Psi^{\star\dot\alpha}_0] - \frac{3i}{2}\Phi^{\star\alpha}_0\mathrm{D}_{\alpha\dot\alpha}\phi^{\dot\alpha}_2 - 4\Phi^{\star\alpha}_0[\eta, \Phi_{0\alpha}] + 6\Psi_{2\dot\alpha}[\eta, \phi^{\dot\alpha}_2] > . \tag{33}$$

By analyzing ghost numbers of component fields one can observe that the natural identifications are

$$\Psi_{0\alpha} \equiv \omega_\alpha, \quad \Psi^\star_{0\alpha} \equiv \omega^\star_\alpha, \tag{34}$$

$$\Phi_{0\dot\alpha} \equiv \bar\omega_{\dot\alpha}, \quad \Phi^\star_{0\dot\alpha} \equiv \bar\omega^\star_{\dot\alpha}. \tag{35}$$

Here $\omega^\star$ and $\bar\omega^\star$ are given by (12) and (13) with the replacement of component fields with their antifields. However, the fields ϕ_2 and Ψ_2 possessing ghost numbers 1 and -2 are not dictated. We define them suitable to our purposes as

$$\phi^\alpha_2 \equiv \frac{4}{3}\bar\theta^2\theta^\alpha\eta, \ \Psi^\alpha_2 \equiv \frac{1}{16}\theta^\alpha\eta^\star. \tag{36}$$

Putting (32) and (33) together results in the full action

$$S = S_1 + S_2 = \frac{i}{2} < \int d^4\tau \left[\Psi_{\tau\dot{\alpha}} \mathrm{D}_\tau \Phi_\tau^{\dot{\alpha}} + \Psi_\tau^{\alpha} \mathrm{D}_\tau \Phi_{\tau\alpha} \right] > . \tag{37}$$

By performing the integrals over the anticommuting variables τ_μ and θ_α, $\bar{\theta}_{\dot{\alpha}}$ and plugging (34)–(36) into the full action (37) we get

$$S = S_0 + \int d^4x \left\{ -\frac{i}{2} A^{\star\alpha\dot{\alpha}} \mathrm{D}_{\alpha\dot{\alpha}} \eta - \frac{i}{2} A^{\star\dot{\alpha}\alpha} \mathrm{D}_{\dot{\alpha}\alpha} \eta + \frac{1}{2} \chi^{\star\alpha} [\eta, \chi_\alpha] + \frac{1}{2} \bar{\chi}^{\star}_{\dot{\alpha}} [\eta, \bar{\chi}^{\dot{\alpha}}] + \frac{1}{2} \eta^\star [\eta, \eta] \right\} . \tag{38}$$

S_0 is given in (15). Indeed, (38) is the minimal solution of the BV master equation for $N = 1$ supersymmetric Yang–Mills theory.

4 Discussions

The main achievements of this work are to write either $N = 1$ supersymmetric action (14) or its BV–quantized action (37) in very simple forms. Thus, studying some geometrical aspects like BRST cohomology should be easier. Moreover, simplifications should arise in calculations of the related path integrals.

This work is constituting an example to the vast applicability of the generalized fields or τ–fields formalisms: Although both of the methods can be applied to actions which are seemingly first order in derivatives, this dependence is only in shape. Therefore, applicability of these approaches is not restricted to the theories whose actions can be written as first order in derivatives.

Theories with higher supersymmetry are difficult to handle. Hence, generalizations of our results to $N > 1$ theories would be extremely useful for explicit calculations and to reveal their geometrical aspects.

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