# NON-GUT BARYOGENESIS

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*Contents:*



*Abstract:*
A review is presented of different scenarios of baryogenesis mostly developed after the GUT era. The lepton and electric asymmetries of the Universe are also discussed.

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## 1. Introduction

The notion of symmetry is probably a central one in modern physics. All modern theories of elementary particles use this concept extensively. Moreover, only due to some exact symmetries observed in Nature is our life possible. A well known example is the general covariance of gravitational interactions ensuring the vanishing of the graviton mass and consequently long-range gravitational forces. However, in most situations just the opposite is true, and in fact our world is predominantly a world of broken symmetries. Breaking at least of one of them is crucial for the creation of our Universe, for the formation of visible matter and ultimately for the existence of life. This is the celebrated baryon asymmetry of the Universe.

We know that there is no antimatter around us despite the symmetry between particles and antiparticles expressed by the *CPT*-theorem. Astronomical observations show that at least our Galaxy consists of usual matter, that is, of protons, neutrons and electrons, while an inconsiderable amount of their antiparticles are observed. Strictly speaking it is not known whether all the visible part of the Universe consists of matter or if there might be some spatially separated regions formed by antibaryons and positrons. What is definite is that all the observed galaxies consist either of matter or antimatter but not of both. Groups of interacting galaxies or several galaxies in a common cloud of intergalactic gas are known and they definitely consist of the same form of matter; otherwise energetic $\gamma$-quanta from the chain of reactions

$$p\bar{p} \to \cdots \pi^0 \cdots \to 2\gamma \tag{1.1}$$

would be observed. A low $\gamma$-flux in the appropriate energy region makes us conclude that baryons dominate over antibaryons over all the Universe or that they are separated by a large distance (for more details see Steigman [1976], Stecker [1985]),

$$l_B \gg l_{gal}\,, \tag{1.2}$$

where $l_B$ is a characteristic scale at which baryonic charge density changes sign and $l_{gal} \approx 100$ kpc is the galactic size. The challenge is clearly either to find a model of particle–antiparticle asymmetry of the Universe or a mechanism of matter–antimatter separation at the huge scale $l_B$.

Only 25 years ago the excess of matter over antimatter in the Universe was considered as one of the initial conditions of the Friedman cosmology and the value of the ratio of baryonic charge density to number density of relic photons,

$$\beta_0 = (N_B - N_{\bar{B}})/N_\gamma \approx 3 \times 10^{-10}\,, \tag{1.3}$$

was believed to be one of the fundamental cosmological constants imposed "externally" at the creation of the Universe. (To be more exact, it was assumed that the baryon number density per comoving volume was conserved, which for adiabatic expansion coincides with the inverse entropy per baryon.) The philosophy has changed completely after the famous paper by Sakharov [1967], where it was argued that the particle–antiparticle asymmetry can arise dynamically from a charge symmetric or even from an arbitrary initial state. This is a consequence of three physical postulates,

(i) baryonic charge nonconservation,
(ii) breaking of charge ($C$) and combined ($CP$) symmetries,
(iii) deviation from thermal equilibrium.

The last two postulates are pretty well established, while the first one is predicted by modern particle theory. In 1967 it was the weakest point of the model because it was generally believed that proton stability implied baryonic charge conservation. The existence of a 10 billion year old baryonic Universe was considered as a strong argument in favor of $B$-conservation. Now the attitude is just the opposite and the existence of the charge asymmetric Universe is believed to prove $B$-nonconservation. However, as has become known relatively recently, neither of the above mentioned postulates are obligatory for baryogenesis and a charge asymmetric universe could evolve without any of them (for the details see sections 4–7).

Another approach was advocated by Omnès [1969, 1970], who assumed that there is a repulsive interaction between baryons and antibaryons leading to their spatial separation. However, no mechanism of dynamical matter–antimatter separation on a galactic scale has been found. As we will see in what follows, this approach in a very different form, combined with the above three principles by Sakharov, was revitalized when the idea of exponential Universe expansion came into being [Sato 1981].

After the paper by Kuzmin [1970], who proposed a different realization of the model than Sakharov, the topic was forgotten for almost a decade. With the advent of Grand Unification models, which predicted baryon nonconservation [Pati and Salam 1974; Georgi and Glashow 1974] it became clear that all the necessary ingredients of baryogenesis are supplied by the theory and the papers by Ignatiev et al. [1978] and by Yoshimura [1978] initiated a stream of papers which has remained deep and wide up until now. References to and discussion of the earlier models which are mostly based on Grand Unification can be found in the reviews by Dolgov and Zel'dovich [1981] and by Kolb and Turner [1983]. Here we give a brief overview of the older scenarios, concentrating mostly on new ideas developed during the last decade.

## 2. Three cornerstones of baryogenesis

*2.1. Baryonic charge conservation* always looked mysterious. In contrast to electric charge conservation, which is closely connected with the gauge symmetry of electrodynamics and the zero mass of the photon, no long-range forces generated by baryonic charge are found. Such forces would look like an additional (anti)gravitational interaction depending on the chemical content of the sample. The validity of the equivalence principle puts a very restrictive limit on the strength of this interaction. The analysis made by Lee and Yang [1955] and by Okun [1969] results in the following upper bounds on baryonic and leptonic coupling constants:

$$\alpha_B < 10^{-44}, \qquad \alpha_L < 10^{-48}. \tag{2.1}$$

These numbers are based on the tests of the equivalence principle with an accuracy of $10^{-12}$ found for the acceleration to the Sun [Braginsky and Panov 1971]. The upper bounds (2.1) make one believe that either "baryonic photons" do not exist at all or their mass is nonvanishing. Both options do not imply baryonic charge nonconservation but at least make allowance for it. Moreover the development of particle physics during the last 15 years made baryonic charge nonconservation favorable from a theoretical point of view.

There are many theories predicting a transformation of baryons into other particles. For example, grand unification models with characteristic energy scale $E_{\Delta B}$ of about $10^{15}$ GeV, supersymmetric models in which $B$-nonconserving processes could proceed between $10^{16}$ GeV and $10^2$ GeV depending upon the concrete model, and electroweak theory with $E_{\Delta B} \approx 10^3$–$10^2$ GeV. The last case is especially interesting because at the level of the classical Lagrangian baryonic charge is conserved and $B$-nonconservation arises as a result of quantum corrections ['t Hooft 1976a, b]. What makes this latter proposal especially sound is that in contrast to the previous possibilities the electroweak theory is verified by experiment. It is not quite as speculative (at the moment) in nature as GUT and SUSY are.

As for experiment, neither proton or nucleus decay nor (n–$\bar{\text{n}}$)-oscillations have been observed up to now and the only "experimental" evidence in favour of $B$-nonconservation is presented by the baryon asymmetry of the Universe. The inflationary Universe model makes the cosmological arguments very strong. Indeed, if $B$ is conserved, sufficiently long inflation with $H_I t_I > 70$ could not have occurred. The density of any conserved charge is inversely proportional to the volume so that in the course of exponential expansion it should go down as $\exp(-3H_I t)$. Since the inflationary stage to be successful must last at least 65 Hubble times, the baryonic charge density in the pre-inflationary stage should have been exp(200) times larger than it was at the end of inflation. By itself such an initial condition is not logically excluded. However, the arguments based on the energy density evolution makes it impossible. Indeed, the baryonic energy density also evolves as $\exp(-3H_I t)$ if the baryons are nonrelativistic. The energy density of relativistic baryons goes down even faster, $\rho_B \sim \exp(-4H_I t)$. Experimental expansion can be realized only if the total energy density $\rho_{tot}$ is (approximately) constant. For conserved baryons this can be true, as follows from eq. (1.3), only for 8 or even 6 Hubble times, i.e., $H_I t_I < 8$. At earlier time the baryonic charge density should dominate and so the total energy density could not be constant. It would destroy inflation. Since we do not know any other way to create our Universe without inflation, baryonic charge nonconservation seems to be pretty well established. There are, however, some rather exotic counterexamples to this statement allowing baryogenesis with conserved baryonic charge (see sections 4 and 5).

2.2. The status of *charge symmetry breaking* is in some sense opposite to that of $B$-nonconservation. The decay $K_L^0 \to 2\pi$ and the different branching ratios,

$$K_L^0 \to \pi^+ e^- \bar{\nu} / K_L^0 \to \pi^- e^+ \nu = 1 - 0.003 ,$$

observed in experiment (see, e.g., Particle Data Group [1990]) unambiguously show that particles and antiparticles are really different, or in other words that $C$- and $CP$-symmetries are broken. So in contrast to baryonic charge, breaking of the particle–antiparticle symmetry is experimentally established.

As for the theory, there are several models of $CP$-violation but none of them can be considered as established. The simplest one is explicit $CP$-violation induced by complex masses or coupling constants in the Lagrangian like, e.g., complex mixing angles in the Kobayashi–Maskawa quark mass matrix or complex Yukawa coupling constants of the Higgs fields. Cosmologically interesting are models of spontaneous $C$ (or $CP$)-violation [Lee 1973] when the Lagrangian is $C(CP)$-symmetric and charge invariance is broken by a condensate of a complex scalar field. In these models the Universe is asymmetric only locally while at large scales the average charge asymmetry vanishes. The inflationary stage during which microscopically small regions expanded up to astronomically large ones makes spontaneous $C(CP)$-violation realistic for baryogenesis. Without inflation the characteristic size of regions with a definite (positive or negative) sign of the baryonic charge density would be much smaller than the observed one, which is at least of the galactic size. Charge symmetry in the early Universe could also be broken by a complex classical (scalar) field which was out of the minimum of its potential and relaxed down to it only at a later stage of evolution of the Universe. Such a mechanism can be realized in particular by an axionic field. In fact any complex scalar field with mass smaller than the Hubble parameter during inflation develops a stochastic classical condensate due to the rising quantum fluctuations and thus breaks $C(CP)$-symmetry. In such models the amplitude of $C(CP)$-violation in the early and contemporary Universe might be quite different. A review of $C(CP)$-violation with application to baryogenesis has been given by Peccei [1989], where appropriate references can be found (see also the following sections).

*2.3.* The necessity of *deviation from thermal equilibrium* (this was emphasized by Okun and Zel'dovich [1976]) for baryogenesis follows from the fact that equilibrium distribution functions are determined by the particle energy $E$ and its chemical potential $\mu$ only,

$$n_{\mathrm{eq}} = (\mathrm{e}^{(E-\mu)/T} \pm 1)^{-1} . \tag{2.2}$$

If charge is not conserved the corresponding chemical potential vanishes in equilibrium, $\mu = 0$. Masses of particles and antiparticles should be the same because of the $CPT$-theorem. So the equilibrium number densities of particles and antiparticles should be equal,

$$N = \bar{N} = \int \frac{\mathrm{d}^3 p}{(2\pi)^3}\, n_{\mathrm{eq}} , \tag{2.3}$$

and no charge asymmetry arises.

It is noteworthy that the form (2.2) for $n_{\mathrm{eq}}$ is known to make the collision integral vanish in the kinetic equation because of the condition of detailed balance. If $CP$ and, due to the $CPT$-theorem, $T$ are broken the condition of detailed balance is not fulfilled. Still it can be shown that $S$-matrix unitarity ensures the standard form of $n_{\mathrm{eq}}$ even in the absence of time reversal invariance [Dolgov 1979; Toussaint et al. 1979; Weinberg 1979; Dolgov and Zeldovich 1981].

One more comment deserves attention. It is usually assumed that the vanishing of, say, baryonic charge in equilibrium is achieved by the interplay of direct and inverse reactions which compensate the excess of particles over antiparticles. This is not the case, however, if detailed balance is broken. To see this let us consider the following simple example. Let baryonic charge be nonconserved in heavy particle decays, $\mathrm{X} \to \mathrm{qq}$ and $\bar{\mathrm{X}} \to \bar{\mathrm{q}}\bar{\mathrm{q}}$, where q and $\bar{\mathrm{q}}$ are quarks and antiquarks, respectively. The decay rates are given by the expressions

$$\Gamma(\mathrm{X} \to \mathrm{qq}) = (1 + \tfrac{1}{2}\varepsilon)\Gamma_0 , \qquad \Gamma(\bar{\mathrm{X}} \to \bar{\mathrm{q}}\bar{\mathrm{q}}) = (1 - \tfrac{1}{2}\varepsilon)\Gamma_0 , \tag{2.4}$$

where $\varepsilon$ is nonzero because of $C$- and $CP$-violation. Of course the total decay widths of particles and antiparticles are the same by the $CPT$-theorem so to obtain $\varepsilon \neq 0$ other decay channels, $\mathrm{X} \to \mathrm{f}_1\mathrm{f}_2$, in addition to qq must be open.

By $CPT$-transformation of eq. (2.4) one gets

$$\Gamma(\bar{q}\bar{q}\rightarrow\bar{X}) = (1+\tfrac{1}{2}\varepsilon)\Gamma_0\,, \qquad \Gamma(qq\rightarrow X) = (1-\tfrac{1}{2}\varepsilon)\Gamma_0\,. \tag{2.5}$$

Thus the direct and inverse reactions push $\Delta B$ in the same direction. Nevertheless $N_B = N_{\bar{B}}$ in equilibrium. The destruction of the baryonic charge is achieved by inelastic two-body scattering $f_1f_2 \rightarrow qq$ and so on (for details see Dolgov [1980a] or Dolgov and Zel'dovich [1981]).

In the following sections we will see how these principles work to generate $\langle B\rangle \neq 0$ in the Universe and also discuss some models which are effective even if some of the conditions (i), (ii), (iii) are not fulfilled.

## 3. Heavy particle decays

The model of baryogenesis based on out-of-equilibrium decay of massive particles is the oldest and probably the most popular one. It incorporates all three basic ingredients of baryogenesis discussed above and naturally fits many new theories of elementary particles as well as new cosmological scenarios. Essential features of the model remain the same even in very elaborate approaches to particle physics. That is why in what follows we will not dwell on technicalities, which can be found in earlier reviews or original papers, but discuss mostly the underlying basic ideas.

Baryonic charge nonconservation is inherent for all unification models where quarks and leptons belong to a common symmetry multiplet. Charge symmetry breaking is easily introduced because of the large number of particle species. Note, however, that $CP$-violation is not obligatory to the theory (in contrast to $B$-nonconservation) but is introduced rather arbitrarily. Decaying massive particles may either be vector gauge or scalar Higgs bosons of the unification scheme. However, other options are also open. Originally simple grand unification theories based on SU(5), SO(10) or some other groups were very popular (baryosynthesis in those frameworks is reviewed by Dolgov and Zel'dovich [1981] and in more detail by Kolb and Turner [1983]), later on supersymmetric and supergravity models came into play and relatively recently superstring inspired phenomenology appeared on the stage. The number of possible underlying particle physics models is pretty large and their predictions are not particularly certain. That is why in some papers a new attitude was accepted, namely, not to restrict oneself to a particular particle physics model but, conversely, to find a reasonable baryogenesis scenario and to test its implication for particle physics.

Particle decay widths and masses in different models may differ by several orders of magnitude and correspondingly the characteristic temperature of baryogenesis may be almost as large as the Planck mass, $m_{Pl} \simeq 1.22\times10^{19}$ GeV, or as low as O(MeV) but the essential features of the scenario do not change.

It what follows we distinguish the models by the mechanism of thermal equilibrium breakdown. The simplest (and the most natural) source of the latter is just expansion of the Universe, which is not adiabatic with respect to massive particles. To see that let us consider the kinetic equation in the homogeneous and isotropic case of the Friedman–Robertson–Walker background,

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \frac{\partial n}{\partial t} + \frac{\partial n}{\partial p}\frac{\mathrm{d}p}{\mathrm{d}t} = \frac{\partial n}{\partial t} - Hp\frac{\partial n}{\partial p} = S\,, \tag{3.1}$$

where $n = n(p, t)$ is the particle occupation number, $p$ is the particle momentum, $H$ is the Hubble parameter and $S$ is the collision integral. For massless particles this equation is solved by the equilibrium expression (2.2) with adiabatically decreasing temperature, $\dot{T} = -HT$. That is why the Planck spectrum of electromagnetic background radiation is not distorted by expansion of the Universe. Massive particles are always out of equilibrium and the "driving force" induced by the expansion of the Universe is proportional to $m^2$. However, the higher their interaction strength the smaller is the deviation from equilibrium [see eq. (3.2) below].

In short, the process of baryonic charge accumulation proceeds as follows. A heavy particle X decays into channels with different baryonic charge $B$. In the case of gauge bosons of Grand Unification those decay channels are $X \to qq$ and $X \to \bar{q}\bar{\ell}$. The antiparticle $\bar{X}$ decays into charge conjugate channels but due to $C$- and $CP$-parity nonconservation the partial widths of the charge conjugate channels are different [see eqs. (2.4) and (2.5)]. In thermal equilibrium all the possible reactions exactly compensate each other and no charge asymmetry arises. Moreover, any pre-existing charge asymmetry is washed out exponentially fast if the corresponding charge is not conserved. This is a consequence of $S$-matrix unitarity and the $CPT$-theorem (see, e.g., Dolgov and Zel'dovich [1981]). If, however, thermal equilibrium is broken the different reactions do not compensate each other and some net charge asymmetry can arise. A very simple example of this kind is the decay of massive particles with far from equilibrium number density into a plasma with $T \ll m$. To create such a state, special mechanisms are necessary, which are discussed below; here we consider a nonsophisticated case of primeval plasma which is almost adiabatically cooled down by the expansion of the Universe. Many models of baryogenesis, especially the early ones, are based on this scenario.

It is intuitively evident that the baryon asymmetry is determined by the ratio of the decay width $\Gamma_X$ to the expansion rate of the Universe at the moment when the temperature of the primeval quasi-equilibrium plasma is equal to the decaying boson mass,

$$Q = \Gamma_X / H(m_X) = 0.6 m_{Pl} \Gamma_X / \sqrt{K}\, m_X^2 . \tag{3.2}$$

At smaller temperatures the deviation from equilibrium is larger but the boson number density is much smaller.

The last equation follows from the well known expressions for the energy density in the Universe,

$$(3/8\pi) H^2 m_{Pl}^2 = \tfrac{1}{30} \pi^2 K T^4 . \tag{3.3}$$

Here the l.h.s. presents the critical energy density (closure density) and the r.h.s. is the energy density of an equilibrium relativistic plasma with $K = \Sigma\,(g_b + \frac{7}{8} g_f)$, where $g_{b(f)}$ are the number of bosonic (fermionic) spin states and the sum is taken over all relativistic particle species. Typical $K \approx 10^2$.

The kinetic equations governing the generation of baryons can be solved analytically in the large-$Q$ limit [Dolgov 1980a]. In the case of kinetic (in contrast to chemical) equilibrium of the decay products the result is

$$\beta = (10^{-1}\text{–}10^{-2}) \frac{\varepsilon}{Q \ln Q} , \tag{3.4}$$

where $\beta$ is defined by expression (1.3) and $\varepsilon = (\Gamma_{qq} - \Gamma_{\bar{q}\bar{q}})/\Gamma_{tot}$. The numerical coefficient in eq. (3.4) depends upon the concrete model of particle physics. If kinetic equilibrium with respect to the decay products is not restored the value of $\beta$ is $\sqrt{\ln Q}$ larger. These analytic results match the low-$Q$ limit [Weinberg 1979] and for large $Q$ agree with numerical calculations by Harvey et al. [1982].

Expressions (3.4) and (3.2) show in particular that light particles ($m_X \ll m_{Pl}$) would generate a baryon asymmetry by the described mechanism only if their decay width is sufficiently low,

$$\Gamma_X / m_X < 3 \times 10^9 \varepsilon (m_X / m_{Pl}) . \tag{3.5}$$

Even the gauge bosons of Grand Unification with $m_X \simeq 10^{15}$ GeV are not particularly good for baryosynthesis, in particular because the relative charge asymmetry $\varepsilon$ in their decays is usually very small.

Possibly a more appropriate candidate for the role of the creators of our world are heavy Higgs bosons H [Barr et al. 1979; Nanopoulos and Weinberg 1979]. For the Higgs bosons generically $Q \leq 1$ and the charge asymmetry might be two orders of magnitude larger. For this mechanism it is necessary that $m_X > m_H$, otherwise the charge asymmetry generated by H-decays would be washed out by X-bosons. Note that in this sense Higgs bosons of Grand Unification models are very natural for baryosynthesis. In many other schemes special care should be taken to ensure $Q \leq 1$. However, even for the Higgs bosons of Grand Unification too strong Yukawa couplings (or, in other words, too heavy quarks) are excluded as was argued by Enquist and Nanopoulos [1987].

There is one more appealing feature of the baryosynthesis scenario based on Grand Unification models. A long lifetime of the proton is easily incorporated into the model. It is proportional to $m_X^4$ and for $m_X > 10^{15}$ GeV is compatible with the existing experimental limit, $\tau_p > 10^{32}$ yr [Particle Data Group 1990]. Unfortunately in simple unification models $m_X$ is about 3 times smaller and the predicted proton lifetime is already below the existing limit. Supersymmetric models could lift the scale of baryonic charge nonconservation $E_{\Delta B}$ up to $10^{16}$ GeV but simultaneously they open other ways of proton decay with a milder suppression in $m_X$. So special care should be taken regarding possible proton decay especially in low temperature scenarios of baryogenesis (see below). As we shall see in section 9, baryonic charge might be strongly nonconserved at the electroweak scale $E_{\Delta B} = E_W = \mathrm{O(TeV)}$. In that case, however, the amplitude is rescaled to lower energies exponentially, which makes $B$-nonconservation at $E \ll E_W$ practically unobservable.

In the simplest version of the SU(5) model, $C$- and $CP$-nonconservation manifests itself only at a high order in the gauge coupling constant and correspondingly $\varepsilon$ is too small. So additional Higgs multiplets apart from 5- and 24-plets should be introduced or the group SO(10) should be explored. In this way the necessary value of $\beta$ (eq. 1.3) can be obtained. A detailed analysis of this problem and a list of references can be found in the paper by Harvey et al. [1982].

The characteristic feature of baryogenesis at the Grand Unification scale is the large temperature $T_B = m_{GUT} \geq 10^{14}$ GeV and correspondingly the early time, $t_B \leq 10^{-34}$ s. After that any considerable entropy rise in the Universe is forbidden, otherwise the ratio $N_B/N_\gamma$ would be too small because the result (1.3) in Grand Unification models is practically the maximal possible one. In particular in this scenario first order phase transitions with large supercooling or matter dominated phases are forbidden for $t > 10^{-34}$ s. These constraints are not easy to satisfy with almost any new particle physics model, and so one becomes inclined to look for a lower temperature scenario of baryogenesis. Moreover there are at least three other reasons in favor of late baryogenesis.

First, the reheating temperature after inflation, $T_{rh}$, is typically much smaller than $m_{GUT}$. The former is easily evaluated as

$$T_{rh} \approx 0.1 \sqrt{\Gamma_i m_{Pl}} , \tag{3.6}$$

where $\Gamma_i$ is the decay rate of the coherent inflaton field into usual matter. The inflaton is known to be very weakly coupled, otherwise its quantum fluctuations would give rise to unacceptable density perturbations (see, e.g., the review by Blau and Guth [1987]). Correspondingly $T_{rh}$ can hardly be larger than $3 \times 10^{12}$ GeV, and in some models it is even several orders of magnitude lower [down to O(MeV)]. In the extended inflation model [La and Steinhardt 1989] $T_{rh}$ may be higher.

Second, a high reheating temperature is not compatible with supergravity models with a relatively light gravitino, $m_g \leq 10$ TeV. Gravitinos could be abundantly produced at early stages of evolution of the Universe and later on would destroy the nice standard primordial nucleosynthesis scenario by the products of their decay or/and change the thermal history of the Universe in particular producing too much entropy [Weinberg 1982]. Of course inflation would dilute practically to zero any pre-existing amount of gravitinos [Ellis et al. 1982a] but they could be copiously produced after reheating. To avoid too much production of gravitinos, the reheating temperature should be less than $10^{10}$ GeV $(m_g/100\ \text{GeV})$ [Khlopov and Linde 1984; Ellis et al. 1984].

Third, electroweak processes at $t = O(\text{TeV})$ might completely wash out an earlier generated baryon asymmetry with initially vanishing $B - L$, where $L$ is leptonic charge (see section 9).

Any of these reasons is not completely compelling by itself but taken together they make low temperature baryogenesis attractive. The earliest paper on late baryogenesis by Barbieri et al. [1981] appeared during the "GUT era" (see also Masiero and Mohapatra [1981], Masiero and Yanagida [1982], and Masiero et al. [1982]), when the above mentioned problems were not so pressing (in fact some of them were not known at that time).

Subsequent papers on late baryogenesis can be roughly divided into two groups. In one of them the authors go from cosmology to particle physics [Claudson et al. 1984; Kosower et al. 1985; Dimopoulos and Hall 1987] just assuming that baryogenesis proceeds at low temperature, derive general constraints on the model and investigate the consequences for particle physics. The considered range of $T_B$ is 1 TeV–1 MeV, which would make observation of $B$-violating processes in direct accelerator experiments possible, in particular in $e^+e^-$ collisions [Dimopoulos and Hall 1987]. Proton stability despite a low $E_{\Delta B}$ may be achieved by leptonic charge conservation.

Another group uses the more traditional approach from particle physics to cosmology. With the development from GUT to supersymmetric GUT and further to superstrings great expectations arose that it might be possible to describe the high energy physics responsible for baryogenesis in terms of a small number of, or even without, unknown parameters. Unfortunately these expectations have failed and at least at the moment we do not have a well defined Theory of Everything. Still cosmological applications have revealed many interesting properties which a theory should have, in particular, suppression of $d = 5$ operators inducing fast proton decay, the type of phase transition in the course of cooling down, possible mechanisms of thermal equilibrium breaking, etc. The literature on this particular subject is immense and should be reviewed in a separate paper. We mention here only a few papers where further references can be found: Coughlan et al. [1985a, b], Lazarides et al. [1986a, b, 1988], Mahajan [1986], Mohapatra and Valle [1987], Yamamoto [1987], Campbell et al. [1987a, 1988], Ellis et al. [1988b] and Panagiotakopoulos [1990, 1991].

To successfully achieve low temperature baryogenesis one has to take care of a proper deviation from thermal equilibrium. If nonequilibrium is induced by a nonzero particle mass in a relativistic equilibrium plasma the particles should be sufficiently weakly interacting [see eq. (3.5)]. Not only their decay rate is to be small but also their reaction and in particular annihilation rates must be low. This definitely excludes gauge bosons with normal coupling $\alpha \approx 10^{-2}$ from the role of sources of matter in our world if the baryogenesis scale $T_{\Delta B}$ is below $m_{\text{GUT}}$. In better shape from this point of view is the

Majorana neutrino, which exists in some unification schemes and which was first proposed for baryogenesis by Yanagida and Yoshimura [1980] or other heavy weakly interacting fermions (see, e.g., Barbieri et al. [1981]).

Subsequent theoretical developments reveal new ways of creating nonequilbrium conditions in the early Universe. A very natural one is reheating of the Universe by inflaton decay when inflation ends [Abbot et al. 1982; Albrecht et al. 1982; Dolgov and Linde 1982]. The evolution of the inflaton field $\Phi$ is as follows. Initially it monotonically rolls down to the potential origin where the latter can be approximated by the harmonic term, $U(\Phi) = m_\Phi^2 \Phi^2/2$. When the expansion rate $H$ of the Universe becomes of the order of the inflaton mass, $m_\Phi$, coherent oscillations of $\Phi$ near the origin begin,

$$\Phi(t) = \frac{m_{\mathrm{Pl}}}{\sqrt{3\pi}} \frac{\sin m_\Phi(t + t_{\mathrm{i}})}{m_\Phi(t + t_{\mathrm{i}})} . \tag{3.7}$$

The decay rate can be evaluated perturbatively if the oscillation frequency, $m_\Phi$, exceeds the masses of produced particles and when the latter are large, a quasiclassical approximation is valid. It is noteworthy that even the production of massless particles is suppressed if the amplitude of the field oscillations $m_{\mathrm{Pl}}/\sqrt{3\pi}\, m_\Phi(t + t_i)$ is larger than $m_\Phi/g$, which $g$ is the coupling constant to those light particles, $\mathscr{L}_{\mathrm{int}} = g\Phi\bar{\psi}\psi$. This suppression is associated with the fact that $g\Phi$ behaves as the effective particle mass, but in contrast to the constant mass case, where the suppression is exponential, the oscillating effective mass results only in power law suppression because the particles are mostly produced when the value of their mass approaches zero during the oscillations [Dolgov and Kirilova 1989]. It is the same phenomenon as the power law suppression of the integral from $\exp(-\omega t)$ when the lower bound in $t$ is zero. In any case the suppressed decay rate results in a smaller reheating temperature and thus favors cold baryogenesis. The energy spectrum of the particles created by the external field (3.7) is evidently not a thermal one and if baryogenesis proceeds faster than thermalization, the suppression (3.4) can be avoided. The necessary conditions for realization of a successful inflationary scenario with proper baryogenesis and without the gravitino problem were investigated by Steinhardt and Turner [1984] and Ovrut and Steinhardt [1984].

Note that in an initially out-of-equilibrium state the charge asymmetry can be developed not only by particle decays but also by inelastic scattering of light particles [Dodelson 1987]. This mechanism, though less effective than decays, might be essential if production of heavy particles is suppressed.

The same mechanism of thermal equilibrium breakdown takes place if $B$-violating particles are produced by any other nonequilibrium sources like, e.g., macroscopic topological defects in the early Universe: domain walls, strings, monopoles (see section 8). Baryogenesis by domain walls can operate in particular in inflationary cosmology based on the Brans–Dicke theory, which was proposed by Mathiazhagen and Johri [1984] and recently put forward by La and Steinhardt [1989], who gave it the name "extended inflation". In this model inflation ends with nucleation and collision of bubbles formed by the inflaton field. Later on the energy contained in the bubble walls transforms into particles. The created plasma is initially far from thermal equilibrium and if baryon asymmetry is produced during this stage it might be considerable. In the model of baryogenesis in the extended inflationary scenario discussed by Barrow et al. [1991a] the baryon asymmetry comes from decays of supermassive bosons of Grand Unification and the essential physics is the same as is discussed in the beginning of this section.

There is, however, a danger for the baryogenesis in the extended inflationary scenario coming from the Brans–Dicke scalar field $\phi_{\mathrm{BD}}$ [Yoshimura 1991]. To successfully implement the model of extended inflation the Brans–Dicke parameter $\omega$ should be bounded from above, $\omega < 30$ [Weinberg 1989]. On the other hand, astronomical data constrain it from below, $\omega > 500$ (see, e.g., the review by Will [1984]). The restriction, however, is valid only for a massless Brans–Dicke field or, to be more exact, for a field with a Compton wave length larger than the size of the solar system. Thus we have to conclude that the field $\phi_{BD}$ cannot be strictly massless. The lower bound on its mass which follows from this argument is $m_{BD} > 10^{-18}$ eV. There is no special reason to expect $\phi_{BD}$ to be massless. While a zero mass graviton is maintained by general covariance and a zero mass photon is maintained by electromagnetic gauge invariance, there is no known symmetry ensuring the vanishing of $m_{BD}$. Even if $m_{BD} = 0$ classically, quantum corrections generically should induce a nonzero mass. A massive Brans–Dicke field would create a cosmological disaster if it were stable or long-lived. Its energy density is determined by the Planck epoch when the amplitude of the field was of the order of $m_{Pl}$. If the decay was not essential the energy density of $\phi_{BD}$ would by now grossly exceed the critical one for $m_{BD} > 10^{-26}$ eV [Yoshimura 1991]. Note that this is already below the above mentioned limit on $m_{BD}$. A much heavier Brans–Dicke field is, however, permitted because of a smaller lifetime. It is coupled to matter as

$$\mathscr{L}_{BD} = \beta \phi_{BD} T^{\mu}_{\mu} / m_{Pl} , \tag{3.8}$$

where $\beta = \text{const.} = O(1)$ and $T^{\mu}_{\mu}$ is the trace of the energy–momentum tensor of matter. The coupling is typically of gravitational strength and so the cosmology of $\phi_{BD}$ resembles that of the gravitino. Because of the weak coupling the lifetime of the heavy Brans–Dicke field is large so that at some stage of the evolution of the Universe the heavy quanta of $\phi_{BD}$ might dominate the energy density. This could be dangerous for the baryon asymmetry because of the large amount of entropy produced by their late decays or annihilation. In the case considered the released entropy can be as large as $10^8$. Such a huge value of $S$ jeopardizes an early baryogenesis scenario. On the other hand, the Brans–Dicke field can be turned from the enemy of baryosynthesis into an ally if its decay creates baryon nonconserving particles out of thermal equilibrium and this gives rise to baryon asymmetry [Yoshimura 1991]. This proceeds essentially along the same lines as in the case of inflaton decay [Abbot et al. 1982; Dolgov and Linde 1982].

Somewhat similar is the model of gravitino induced baryogenesis proposed by Cline and Raby [1991]. To realize it one needs a nonvanishing number density of gravitinos produced at the post-inflationary epoch and baryon nonconservation in decays of superpartners. A rather high reheating temperature, close to the Grand Unification scale, is necessary for an abundant production of gravitinos after inflation. Since gravitinos are very weakly interacting they are naturally out of thermal equilibrium when $T < m_g$. With gravitino mass $m_g = O(10\,\text{TeV})$ the temperature of decay in this model is about 1 MeV, so it is really late baryogenesis almost coinciding in time with nucleosynthesis. Energetic particles produced by gravitino decays could distort the good agreement of the standard primordial nucleosynthesis model with observations, which is one of the cornerstones of modern cosmology. Analysis by Scherrer et al. [1991] shows, however, that for $m_g > 50\,\text{TeV}$ the agreement with the observed light element abundances survives.

Another supersymmetry inspired model of low temperature baryogenesis, which, in contrast to the previous one, does not require a high reheating temperature, was considered recently by Molerach and Roulet [1991]. In their model the baryon asymmetry is generated by $C$-odd, $B$-nonconserving decays of gluinos, which in turn originated from the decays of the superpartner of the axion, the axino, at $T \approx 1$ GeV. Since the axinos have a stronger interaction than the gravitinos they can be abundantly produced at relatively low reheating temperatures, $T_{rh} \geq 10^4$ GeV. Axions, which are obligatory for the model, are the natural candidates for the dark matter in the Universe.

Interesting scenarios of low temperature baryosythesis similar in their main features have been proposed by Lazarides et al. [1986a], by Yamamoto [1986, 1987], and by Mohapatra and Valle [1987]. These scenarios can be naturally realized in superstring models. They are based on out-of-equilibrium decays of heavy (in the sense that $m \gg T$) particles $\chi$. Thermal equilibrium can be strongly broken as a result of a phase transition in the course of the cooling down of the Universe as can be seen in the following example. Let $\chi$ interact with a scalar Higgs field $\phi$ as is given by the Lagrangian

$$L_{\text{int}} = \lambda\chi^*\chi\phi^*\phi . \tag{3.9}$$

At low temperatures the potential of $\phi$ has a minimum at $\phi \neq 0$. This gives rise to condensate formation at $\langle\phi\rangle = \sigma$ and a contribution to the mass of $\chi$ equal to $\delta m_\chi^2 = \lambda^2\sigma^2$. At higher temperatures the minimum is shifted to $\phi = 0$ and the condensate is destroyed. Correspondingly the mass of $\chi$ vanishes or becomes small, $m_\chi < T$. At that period thermal equilibrium is established and the number density of $\chi$ is close to the photon number density, $n_\chi \sim T^3$. The model is organized in such a way that the field $\phi$ quickly rolls down to the minimum point $\sigma$ so that $\chi$-particles acquire a mass of the order of $\lambda\sigma \gg T$ in a time which is short in comparison with their annihilation rate. Hence their number density, which practically did not change during this process, became much larger than the equilibrium one,

$$N_\chi \gg N_{\chi_{\text{eq}}} = (m_\chi T/2\pi)^{3/2}\exp(-m_\chi/T) . \tag{3.10}$$

This ensures one of the conditions necessary for successful baryosynthesis. The excess of baryons produced in the decays of $\chi$ is not washed out if the plasma temperature is low enough so that all the processes with $B$-nonconservation are effectively frozen.

When the field $\phi$ reaches the minimum of the potential it starts to oscillate around it, producing particles. The corresponding entropy increase could dilute $\beta = N_B/N_\gamma$ by about $10^6$ [Lazarides et al. 1986a]. This demands a very large baryon excess in the decays of $\chi$ or a stimulation of the phase transition to the minimum of $U(\phi)$ so that the supercooling becomes negligible.

A possible way of stimulating the phase transition was proposed by Srednicki [1982] and Nanopoulos and Tamvakis [1982]. They argued that for some values of the Higgs field amplitude $\phi$ and temperature $T$, the asymptotically free gauge coupling constant becomes strong and a condensate of particular bilinear combinations of fermionic fields may develop, in the same way as quark field vacuum condensates $\langle\bar{\psi}\psi\rangle$ are formed in QCD. As was shown by Witten [1981], such condensates do indeed stimulate the phase transition because the Yukawa-type coupling $g\phi\bar{\psi}\psi$ for nonzero $\langle\bar{\psi}\psi\rangle$ corresponds to a linear term in the potential $U(\phi)$, which creates a nonvanishing force at the origin driving $\phi$ away to another equilibrium point. This mechanism was used by Campbell et al. [1987a, b, 1988] to suppress large entropy production by phase transitions after baryosynthesis.

Dannenberg and Hall [1987] considered a model of baryogenesis at the weak phase transition where all the above mentioned ideas are realized. Their goal was to construct a model in which the value of baryon asymmetry can be expressed in terms of particle physics parameters measurable in direct laboratory experiments. It was assumed that the source of the asymmetry is top quark decays. Proton stability in the model is ensured by leptonic charge conservation. Decays of heavy nuclei and neutron–antineutron oscillations are suppressed by a weak coupling to light generations. The electroweak phase transition was assumed to be of first order so that t-quarks quickly get their mass and get out of thermal equilibrium as was discussed above. To prevent too large a supercooling and consequently a large entropy production after the phase transition, account was taken of the linear term in the potential of the Higgs field induced by the quark condensate.

This scenario, however, is on the verge of contradiction with experiment because, in order to get the observed value of $\beta$ the model needs a light Higgs boson with a mass around 1 GeV and a top quark mass close to 80 GeV. In more complicated versions of the model the prediction of a light Higgs boson can be avoided. Anyway the possibility to test baryogenesis theory in laboratory experiments looks exciting.

A low temperature baryogenesis scenario based on a specific superstring model has been considered by Panagiotakopoulos [1990, 1991]. Baryon asymmetry is generated by out-of-equilibrium decays of heavy ($m \approx 10^3$ GeV) leptons after an intermediate-scale phase transition. The initial state is far from thermal equilibrium with a rather low energy density so that the temperature of the thermalized plasma after heavy particle decays is below O(1 GeV). An interesting feature of the model is a low $(B-L)$-breaking scale.

Let us note also the very surprising result that the sign of baryon excess generated by heavy particle decays is not prearranged by the sign of $CP$-symmetry breaking but is also determined by the kinetics of the processes [Yokoyama et al. 1987; Yokoyama et al. 1988]. As has been found by numerical calculations, the baryon asymmetry could change sign in the evolution of the Universe and depending upon the relation between the rates of the reactions and the expansion rate can be either positive or negative with the same $CP$-odd amplitudes. In more complicated versions of the model this could even give rise to B or $\bar{\text{B}}$ dominated worlds in different parts of the Universe.

In conclusion, the good old baryogenesis scenario based on out-of-equilibrium decays of massive particles is still in a good shape. First simple models based on the groups SU(5) or SO(10) most probably give too low a value for the asymmetry but more elaborate models with new mechanisms of equilibrium breaking can describe the observed world. Those models are compatible with new unified schemes based on supersymmetric or superstring theories but unfortunately our hope to calculate the value of $\beta$ exactly still remains only a hope because of a large number of unknown relevant parameters such as particle masses, decay widths, amplitudes of $C$- and $CP$-violation, etc.

## 4. Black hole evaporation

Almost 20 years ago Beckenstein [1972a, b] noticed that baryons could disappear in a black hole apparently demonstrating baryonic charge nonconservation in outer space. It is impossible to draw any conclusion regarding the baryonic number of a black hole by measuring all kinds of fields outside it. This is connected with the absence of long-range forces created by the baryonic charge (see section 2) and is formulated in other words as the absence of baryonic hairs on a black hole. The importance of this result was very much emphasized by Hawking's [1974, 1975a] discovery of black hole evaporation. This means that a black hole formed by baryonic matter could disappear into nothing leaving behind charge symmetric radiation. This is a striking new phenomenon of baryon nonconservation induced by a non-trivial topology of space–time while baryonic charge is formally conserved in the Lagrangian.

It was understood immediately that this effect together with charge symmetry breaking could produce the observed baryon asymmetry of the universe from an arbitrary initial state [Hawking 1975b; Carr 1976]. It was shown, however, that in the simple model of evaporation of noninteracting particles, baryon asymmetry is not generated if baryonic charge is conserved [Toussaint et al. 1979]. The arguments are closely connected with the thermal character of black hole evaporation and with the absence of charge asymmetry in a thermally equilibrium state. Particle interactions, however, invalidate the statement and permit black holes to evaporate asymmetrically even if baryonic charge is conserved.

The concrete mechanism of the process has been proposed by Zel'dovich [1976a] and detailed calculations of the effect have been done by Dolgov [1980b, 1981].

Another possible model of the generation of baryon asymmetry in the Universe by black hole evaporation is more traditional and consists of heavy particle radiation by a disappearing black hole with subsequent $B$-nonconserving and charge asymmetric decay of those particles [Toussaint et al. 1979; Turner and Schramm 1979; Turner 1979; Barrow 1980; Barrow and Ross 1981; Barrow et al. 1991b]. Baryogenesis in this scenario goes essentially along the same lines as in the previous section. Black holes act as a source, which naturally creates a nonequilibrium number density of heavy particles.

Before going further let us remind the reader of some basic facts about black hole evaporation. For more details one can refer, e.g., to the book by Novikov and Frolov [1989]. A black hole with mass $M_{\mathrm{BH}}$ radiates from its gravitational radius $r_g = 2M_{\mathrm{BH}}/m_{\mathrm{Pl}}^2$ as a black body with temperature [Hawking 1975a]

$$T_{\mathrm{BH}} = m_{\mathrm{Pl}}^2/8\pi M_{\mathrm{BH}} . \tag{4.1}$$

A remarkable fact is that the smaller $M_{\mathrm{BH}}$ is, the larger the temperature is. $T_{\mathrm{BH}}$ is equal to the Grand Unification energy $10^{15}$ GeV for $M_{\mathrm{BH}} \approx 10^{-2}$ g.

The energy flux radiated by a body with temperature (4.1) is

$$\dot{M}_{\mathrm{BH}} = -\frac{\pi^2}{30} K T_{\mathrm{BH}}^4 \frac{4\pi \cdot 4M_{\mathrm{BH}}^2}{m_{\mathrm{Pl}}^4} \approx -4\times 10^{-5} K \frac{m_{\mathrm{Pl}}^4}{M_{\mathrm{BH}}^2} \equiv -\gamma \frac{m_{\mathrm{Pl}}^4}{M_{\mathrm{BH}}^2} , \tag{4.2}$$

where $K$ is the effective number of light particle species ($m < T_{\mathrm{BH}}$).

After particles are produced they propagate in the gravitational field of the black hole with the interaction depending on their mass and spin. Some of them can even be captured back. Numerical solution of the corresponding wave equations [Page 1976] gives practically the result (4.2) with a slightly different numerical coefficient $\gamma$.

Integrating eq. (4.2) one easily gets

$$M_{\mathrm{BH}}(t) = M_{\mathrm{i}}[1 - (t - t_{\mathrm{i}})/\tau_{\mathrm{BH}}]^{1/3} , \tag{4.3}$$

where $M_{\mathrm{i}}$ is the black hole mass at initial moment $t_{\mathrm{i}}$ and

$$\tau_{\mathrm{BH}} = \frac{1.33\times 10^5}{K m_{\mathrm{Pl}}} \left(\frac{M_{\mathrm{i}}}{m_{\mathrm{pl}}}\right)^3 \tag{4.4}$$

is the lifetime of the black hole with initial mass $M_{\mathrm{i}}$. The dependence of $K$ on the black hole temperature is neglected in eq. (4.3) but this is justified for an order-of-magnitude evaluation. The lifetime of a black hole with initial temperature $10^{15}$ GeV is approximately $10^{-28}$ s (we take here $K = 10^2$).

A possible baryogenesis scenario without baryonic charge nonconservation looks as follows. Let there exist a massive boson A decaying through the channels $\mathrm{A} \to \bar{\mathrm{L}}\mathrm{H}$ and $\mathrm{A} \to \mathrm{L}\bar{\mathrm{H}}$, where L and H are light and heavy baryons, respectively, and $\bar{\mathrm{L}}$ and $\bar{\mathrm{H}}$ are their antiparticles. Once more note that baryonic charge in the decays is conserved. If charge symmetry is broken the partial widths of the decays should be different,

$$\Gamma(\mathrm{A}\to\mathrm{L}\bar{\mathrm{H}}) - \Gamma(\mathrm{A}\to\mathrm{H}\bar{\mathrm{L}}) = \Gamma_{\mathrm{A}}\Delta\,, \tag{4.5}$$

where $\Gamma_{\mathrm{A}}$ is the total decay width. The destinies of the light and heavy baryons produced in the decay depend on their masses. The probability of inverse capture of a heavy particle is larger than that of a light one. So the flux of heavy baryons and antibaryons (H and $\bar{\mathrm{H}}$) at large distance from the black hole should be smaller than that of L and $\bar{\mathrm{L}}$. If $\Delta > 0$ light baryons are more abundantly produced than light antibaryons (for H and $\bar{\mathrm{H}}$ the situation is the opposite). So in the process of evaporation of the black hole baryonic charge is accumulated in our Universe and an equal amount of antibaryonic charge is concealed in black holes.

The flux of baryonic charge produced by a black hole into external space is proportional to the product $\Delta(W_{\mathrm{L}} - W_{\mathrm{H}})$, where $W_{\mathrm{L(H)}}$ is the probability of escape of a light (heavy) baryon from the gravitational field of the black hole. To calculate $W$ one has to know the wave equation describing particle propagation in the field of the black hole with account of the decay. Referring for the technical details to Dolgov [1980b] we present here the final result. If black holes with mass $M$ had relative energy density $\kappa = \rho_{\mathrm{BH}}/\rho_{\mathrm{tot}}$ before evaporation, then in the most favorable case of $m_{\mathrm{A}} \approx m_{\mathrm{H}} \gg m_{\mathrm{L}}$ and $\mu_{\mathrm{A}} = m_{\mathrm{A}} r_{\mathrm{g}} \gg 1$ (here $r_{\mathrm{g}} = 2M/m_{\mathrm{Pl}}^2$ is the gravitational radius of the black hole) the baryon asymmetry of the Universe is

$$\beta = \frac{N_{\mathrm{B}}}{\rho_{\mathrm{tot}}/T} \approx 0.1 K^{-3/4} \kappa \frac{\Gamma_{\mathrm{A}}}{m_{\mathrm{A}}} \left(\frac{m_{\mathrm{Pl}}}{M}\right)^{1/2} \Delta\,, \tag{4.6}$$

where $M$ is the initial mass of the black hole and $K$ is the number of particle species in the primeval plasma [see eq. (3.3)]. The quantities $\Gamma_{\mathrm{A}}/m_{\mathrm{A}}$ and $\Delta$ are small in comparison with 1 so that the ratio $m_{\mathrm{Pl}}/M$ should not be too small. For example, for $\Delta = 10^{-4}$ and $\Gamma_{\mathrm{A}}/m_{\mathrm{A}} = 10^{-2}$ the black holes should be rather light, $M \leq 10^6 m_{\mathrm{Pl}}$, to generate $\beta$ close to the observed value. The lifetime of such a black hole is about $\tau_{\mathrm{BH}} \leq 10^{-22}$ s and the evaporation temperature $T_{\mathrm{BH}} = m_{\mathrm{Pl}}^2/8\pi M \geq 10^{12}$ GeV. Note that in order to realize the considered mechanism the corresponding particles should be very heavy in accordance with the condition

$$\mu_{\mathrm{A}} = m_{\mathrm{A}} r_{\mathrm{g}} = 2 m_{\mathrm{A}} M / m_{\mathrm{Pl}}^2 \geq 1\,. \tag{4.7}$$

This gives $m_{\mathrm{A}} > m_{\mathrm{H}} \geq m_{\mathrm{Pl}}^2/M$.

Though this mechanism does not look particularly natural it still deserves consideration because it presents an example of generation of the baryon asymmetry of the Universe when baryonic charge is strictly conserved in particle physics. Still, even if this is the case the proton is not absolutely stable. It should decay into a positron plus neutral particles by the formation of an intermediate microscopic black hole [Zel'dovich 1976b, 1977]. The proton lifetime with respect to this process is about $10^{45}$ yr.

It is noteworthy that there is an interesting analogy between the nonconservation of baryons in the process of black hole evaporation connected with a nontrivial topology of space–time and the nonconservation of baryons in electroweak processes connected with a nontrivial topology in gauge field space (see sections 8 and 9).

For the calculation of the baryon asymmetry of the Universe produced by black hole evaporation, one needs to know the number density and mass distribution of primordial black holes. Unfortunately, from the first paper by Zel'dovich and Novikov [1966] advocating the possibility of "small" black hole formation in the early Universe up to the most recent paper by Hsu [1990] progress in this direction was rather slow. In the latter paper, black hole formation in extended inflation has been considered. As we have already mentioned, extended inflation ends with the formation and coalescence of bubbles of the new phase inside the old metastable one. A large part of the energy in this process is concentrated in the bubble walls. This gives rise to large energy density inhomogeneities and thus facilitates primordial black hole formation.

Baryogenesis via primordial black holes in extended inflation has been considered by Barrow et al. [1991b]. In their calculations the authors adopted a simple procedure of parametrizing the relative energy density of black holes by a constant $\beta$ [$\kappa$ in our notation, see eq. (4.6)] and considered different scenarios implied by different values of $\beta$ (or $\kappa$) and by different lifetimes or masses of the black holes.

The most favorable case for baryogenesis is that of large $\kappa$ when black holes dominate the energy density of the Universe from the very beginning, i.e., from the moment when inflation ends. Black hole masses should be sufficiently low so that the black hole temperature would be larger than the masses of particles creating baryon asymmetry. If these two conditions are fulfilled the magnitude of the asymmetry would be approximately the same as in the case of out-of-equilibrium production of heavy particles by bubble wall collisions (see section 3). With smaller $\kappa$ and/or larger $M$ the amount of "useless" entropy diluting the asymmetry becomes larger and so $N_B/N_\gamma$ becomes smaller. Corresponding expressions for different cases are presented in the paper by Barrow et al. [1991b]. The calculation is straightforward so the reader could either derive them him(her)self or refer to the original paper. It is argued by the authors that this mechanism can in principle produce the desired value of the baryon asymmetry though less easily than that based on direct heavy particle decays. There is one exception, however, when the reheating temperature after inflation, $T_{rh}$, is smaller than $T_{\Delta B}$. In this case the standard mechanism does not operate while black holes still can produce an asymmetry because in the process of evaporation they ultimately reach high temperatures $T_{BH} > T_{\Delta B}$.

In conclusion one comment is in order. Possible magnetic monopole evaporation by black holes was considered as a way to restrict the black hole number density [Barrow and Ross 1981; Barrow et al. 1991a, b]. It seems, however, that the production of monopoles even by black holes with a temperature exceeding the monopole mass is suppressed because the monopole size is much larger than its Compton wave length.

$$l_M = 1/\alpha m_M \gg 1/m_M , \tag{4.8}$$

while the size of a black hole with $T_{BH} = m_M$ is of the order of $r_g = 1/4\pi m_M$. Evidently it is not easy to produce a large object by a small source. This invalidates the upper bound on the number density of small black holes and permits a more effective baryogenesis.

## 5. Baryosymmetric baryogenesis

Another model of baryogenesis with conserved baryonic charge but without concealment of the latter inside vaporizing black holes was proposed recently by Dodelson and Widrow [1990a, b, c]. In short their basic assumption is that except for quarks there exist some other (weakly interacting) baryons b and the asymmetry between quarks and antiquarks is exactly compensated by the asymmetry between $\bar{b}$ and b. The asymmetry between quarks and antiquarks (accompanied by the opposite asymmetry between b and $\bar{b}$) can be generated, e.g., in heavy particle decays due to $C$- and $CP$-violation even if baryonic charge is conserved. In fact this idea is realized in any previously known baryogenesis scenario with conserved $B - L$, where $B$ and $L$ are baryonic and leptonic charges, respectively. One could just claim that baryonic and antileptonic charges are identical and the immediate formal conclusion would be that our Universe is baryosymmetric. The proton is known to be unstable in these models and after a while all the protons will decay into positrons plus radiation. Thus the hidden baryonic (or leptonic) symmetry becomes explicit. The model by Dodelson and Widrow is essentially the same with the difference that the proton (but not the positron or antineutrino) is the lightest baryon and so it is absolutely stable while the new baryon b is heavier than p and so ultimately should decay. Except for the explanation of proton stability, the model has another nice feature of identifying dark matter in the Universe with hidden antibaryons $\bar{\mathrm{b}}$. Moreover, if the mass of b is not very much different from the mass of the proton, the energy density of dark matter formed by b-particles should be rather close to that of nucleonic matter,

$$\Omega_{\mathrm{b}} = \Omega_{\mathrm{B}} m_{\mathrm{B}} / |b| m_{\mathrm{N}} . \tag{5.1}$$

Here the subscripts B and b refer to usual (protons and neutrons) and invisible (b-particles) baryons, respectively, $\Omega$ is as usual the ratio of the energy density to the critical (closure) one and $b$ is the magnitude of the baryonic charge of a b-particle.

Dodelson and Widrow did not pursue this simple scenario of baryogenesis with conserved baryonic charge, though this possibility was mentioned in a recent paper by Dodelson et al. [1990]. Instead they considered the model of spontaneously broken baryonic $\mathrm{U}(1)_B$ symmetry at high temperatures with symmetry restoration at low temperatures. In this model baryon asymmetry is generated during the high temperature phase when baryonic charge is not conserved. It proceeds essentially along the same lines as is described in section 3. The difference is that equal baryonic charge but of opposite sign is accumulated in the vacuum in the form of the zero energy Goldstone mode. When symmetry is restored, antibaryonic charge stored in the vacuum manifests itself as some heavy and long-lived (by assumption) antibaryons $\bar{\mathrm{b}}$. So the Universe is formally baryosymmetric.

This model of symmetry restoration at low temperature looks rather unusual at first sight but it has long been known in field theory at nonzero temperature [Weinberg 1974; Mohapatra and Senjanovic 1979a, b]. A simple model of this kind is realized with the following interaction potential of two scalar fields $\chi_1$ and $\chi_2$:

$$U(\chi_1, \chi_2) = \lambda_1 |\chi_1|^4 + \lambda_2 |\chi_2|^4 - \lambda_{12} |\chi_1|^2 |\chi_2|^2 + m_1^2 |\chi_1|^2 + m_2^2 |\chi_2|^2 , \tag{5.2}$$

with positive coupling constants $\lambda_j$. The temperature corrections are effectively reduced to the substitutions

$$\chi^2 \to \langle \chi_{\mathrm{q}}^2 \rangle + b_1 T^2 ,$$

where $\chi_{\mathrm{q}}^2$ is the average value of the corresponding quantum operator over the vacuum while the temperature dependent term comes from averaging over the thermal bath. The constants $b_j$ are positive and of order unity. They are determined by the number density of the particles in the plasma and in lowest order do not depend on the coupling constants. Usually the interaction potential is taken in the form $\lambda |\chi|^4$. Its contribution for temperature corrections to the effective mass squared is positive, $\delta m^2(T) = \lambda b_2 T^2$, and this results in the standard picture of symmetry restoration at high temperatures. If, however, $\lambda_{12}$ is sufficiently larger than $\lambda_1$ (but still $\lambda_1 \lambda_2 > \lambda_{12}^2$ so that the potential remains bounded from below), the term $-\lambda_{12} b_2 T^2$ dominates at high enough temperatures and the effective mass of $\chi_1$, $m^2_{\rm eff}(T) = m_1^2 - \lambda_{12} b_2 T^2$, becomes negative inducing symmetry violation. At small temperatures the mass is evidently positive and the symmetry is restored.

Thus let us assume that the Lagrangian is invariant with respect to U(1) transformations,

$$\psi_j \to \psi_j \exp(\mathrm{i} b_j \theta)\,, \tag{5.3}$$

where $b_j$ is the baryonic charge of the field $\psi_j$. This invariance results in baryonic current conservation,

$$\partial^\mu J^b_\mu = 0\,. \tag{5.4}$$

Among the fields $\psi_j$ there is a scalar field $\phi$ which has baryonic charge, say, equal to 1 and which acquires a nonzero vacuum expectation value $v$ at high temperatures, $v^2 = \mathrm{O}(m^2/\lambda)$, inducing spontaneous symmetry breaking. Small fluctuations of $\phi$ near the new vacuum state can be written as follows:

$$\phi = \frac{v+\rho}{\sqrt{2}}\, \mathrm{e}^{\mathrm{i}\theta/v}\,. \tag{5.5}$$

Here $\rho$ is a real massive field with mass of the order of $m_j^2$ in expression (5.2) while $\theta$ is a massless scalar field which always appears when a global symmetry is spontaneously broken [Goldstone 1961].

The low energy Lagrangian for $\theta$ has the form

$$\mathcal{L}_\theta = \tfrac{1}{2}\, \partial^\mu\theta\, \partial_\mu\theta + \frac{1}{v}\, \partial_\mu\theta \cdot j^\mu\,, \tag{5.6}$$

where $j_\mu$ is the baryonic current of all particles except $\theta$. The corresponding equation of motion is

$$\partial^\mu (v\, \partial_\mu \theta + j_\mu) = 0\,. \tag{5.7}$$

This is in fact the conservation law (5.4) rewritten in the broken symmetry phase. One sees that nonconservation of the baryonic current of ordinary matter fields is exactly compensated by the Goldstone mode. Further interpretation is infrared ambiguous, however. Integrating eq. (5.7) over the three-volume $V$ one gets

$$\frac{\mathrm{d}}{\mathrm{d}t}\,(Q_{\rm B} + Q_{\rm b}) = \int \mathrm{d}s^i\, (v\, \partial_i\theta + j_i^{\rm B})\,, \tag{5.8}$$

where $Q_{\rm B} = \int \mathrm{d}^3V\, j_t^{\rm B}$ is the baryonic charge of particles inside the volume $V$ and $Q_{\rm b} = v \int \mathrm{d}^3V\, \partial_t\theta$ is the baryonic charge carried by the Goldstone bosons. If a process which changes $Q_{\rm b}$ takes place in a finite spatial volume, the surface $S$ can be chosen so that the integral on the r.h.s. of eq. (5.8) vanishes and the sum $(Q_{\rm B} + Q_{\rm b})$ is conserved. If the space is infinite, Goldstone bosons might be radiated away leaving behind a nonzero baryonic charge density of ordinary matter. In a closed universe, however, the r.h.s. of eq. (5.8) vanishes if the integral is taken over the volume of the whole Universe and so the total charge does not change. The same might also be true for a finite but large part of a homogeneous and isotropic Universe.

Still the problem deserves further clarification because the concrete mechanism ensuring the proper production of the Goldstone field accompanying a baryon nonconserving decay remains unknown. It was argued by Dodelson and Widrow [1990c] that soft Goldstone bosons are produced during this decay in the same way as an infinite number of infrared photons are produced in electrodynamics. The analogy is not exact, however. Photons are produced by a diagonal electromagnetic current and because of that the amplitude of the process has an infrared singularity. Typically the matrix element of the electromagnetic current is of the form

$$\langle j_\mu \rangle = p_{1\mu}/p_1 k - p_{2\mu}/p_2 k ,$$

where $k$ is the photon four-momentum and $p_1$ and $p_2$ are those of charged particles. The difference of two terms is a result of gauge invariance. It makes the singularity softer, avoiding an infrared catastrophe in electrodynamics.

In the case of Goldstone particles there is no infrared singularity at all. Indeed, the coupling (5.6) can be rewritten in the form $\mathscr{L}_{\mathrm{int}} = \theta\, \partial_\mu j^\mu / v$. This means that there is no diagonal coupling for Goldstone bosons which remains nonvanishing in the infrared limit. For example, the most general form of the matrix element of the current of two scalar particles in the momentum representation has the form

$$\langle 1 | j_\mu | 2 \rangle = a(p_1 + p_2)_\mu + b(p_1 - p_2)_\mu .$$

Evidently in the diagonal case when $p_1^2 = p_2^2$ the matrix element of $\partial_\mu j^\mu$ contains only one term proportional to $k^2$, where $k$ is the four-momentum of the Goldstone boson. It is easy to repeat these arguments for spinor particles and in fact for particles with any spin. This is another formulation of the statement that Goldstone bosons do not create long-range forces which fall off as $1/r^2$.

Hence the pole of the propagator is not reached in the physical region for zero-energy Goldstones and the classical field, which is necessary for the balance of the baryonic charge, seems not to be created.

Anyway, baryosymmetric baryogenesis can be realized, if not by this mechanism, then by the one with a heavy long-lived baryon. Note that in the version of the model with spontaneous symmetry breaking, one also needs heavy long-lived baryons which are produced when the symmetry is restored and the baryonic charge accumulated in the vacuum reappears in the form of those (anti)baryons. So the phenomenological consequences of both versions of the scenario are the same. It is predicted that dark matter in the Universe consists, at least in part, of new weakly interacting antibaryons. The upper bound for the mass of these baryons is determined by the condition $\Omega_{\mathrm{b}} < 1$ and depends upon their baryonic charge. Thus it is reasonable to assume that $m_{\mathrm{b}} < 100\,\mathrm{GeV}$. These particles, b, must be unstable, decaying into antiprotons and also producing high energy $\gamma$-rays. From the observational bounds on the fluxes of $\gamma$ and $\bar{\mathrm{p}}$ it follows that the lifetime of b-baryons should be large, $\tau_{\mathrm{b}} > 10^{10} t_{\mathrm{u}}$, where $t_{\mathrm{u}} \simeq 10^{10}$ yr is the age of the Universe [Dodelson and Widrow 1990c]. Such a huge lifetime means that their interactions are very weak. This in turn implies that the number density of relic b- and $\bar{\mathrm{b}}$-particles should be large. The latter can be written as follows (see, e.g., Dolgov and Zel'dovich [1981]):

$$r_{\mathrm{b}} \equiv \frac{N_{\mathrm{b}}}{N_\gamma} = \frac{1}{\sigma_{\mathrm{b}\bar{\mathrm{b}}} m_{\mathrm{Pl}} m_{\mathrm{b}}} = g_{\mathrm{b}}^{-2} \frac{m_{\mathrm{b}}}{m_{\mathrm{Pl}}} , \tag{5.9}$$

where $\sigma_{b\bar{b}}$ is the cross-section of $b\bar{b}$-annihilation. It is parametrized as $\sigma_{b\bar{b}} = g_b^2/m_b^2$ with constant $g$. To get the number density of relic $b\bar{b}$-pairs below the number density corresponding to the baryon asymmetry we should have $g_b^2 > 10^{-8}$. Those simultaneous conditions of a small decay rate and a rather large cross-section makes the model rather unnatural though it is formally not excluded that annihilation is strong while decay is weak. The example considered above of $B \equiv \bar{L}$ satisfies this requirement. Natural frameworks of the realization of the model remain to be found but the presented scenario of baryogenesis with conserved baryonic charge is interesting by itself.

An interesting though probably unrealistic theoretical problem is what would happen if $U(1)_B$ were realized locally. It might, however, be realistic for the gauge invariance of electrodynamics. In this case an electric charge asymmetry of the Universe could be generated if $U(1)_{em}$ was broken. The consequences of this assumption as well as a possible magnitude of this asymmetry are discussed in section 12.

## 6. Baryogenesis by baryonic charge condensate

Supersymmetry has opened a new possibility for baryogenesis due to the existence of scalar fields with nonzero baryonic or/and leptonic charge. It is known that a condensate of a scalar field which does not possess a conserving charge survives during inflation. Moreover, quantum fluctuations of a scalar field increase at the inflationary stage and, together with inflationary stretching of short waves, these fluctuations can create a scalar field condensate if it did not exist before. These effects permit the realization of a beautiful scenario of baryogenesis proposed by Affleck and Dine [1985].

According to their scenario, the baryon excess may result from the decay of the condensate of a scalar superpartner of a colorless and electrically neutral combination of quark and lepton fields $\langle \chi \rangle$. It is conjectured that in the history of the Universe there was a period when the squark $\tilde{q}$ and the slepton $\tilde{\ell}$ fields had nonzero vacuum expectation value $\langle \chi \rangle \neq 0$. The condensate $\langle \chi \rangle$ could form during the inflationary stage if $B$ and $L$ were not conserved, owing to an enhancement of the quantum fluctuations of the $\chi$ field [Linde 1985]. If $B$ is conserved the baryonic charge density would go down as $\exp(-3H_I t)$. For nonconserved $B$, the quantum fluctuations of $\chi$ during inflation create a baryon charge density of the order of $H_I^3$, where $H_I$ is the Hubble parameter during the inflationary stage. Baryon asymmetry of the Universe emerges as a result of the decay of $\chi$ into fermions, when inflation is over and the baryon charge of the condensate is transferred to the baryon charge of quarks; this latter charge is conserved during the subsequent evolution.

This scenario can solve both the problems caused by the low post-inflation reheating temperature and those due to the $B$-violating processes during the electroweak phase transition. In addition, this model has an especially attractive feature: neither explicit nor spontaneous charge symmetry violation is needed. The charge symmetry may be stochastically broken by quantum fluctuations. As a result of inflation, domains with a given $B$ become macroscopically large. If there is no explicit charge symmetry breaking, the universe would consist of baryonic and antibaryonic parts with voids in between. The size of these domains could be smaller than the visible part of the universe, so in principle they might be observable. The separation of the baryonic and antibaryonic matter could be forced by the pressure produced by $B\bar{B}$ annihilation and by gravitational clumping of regions with higher baryonic and antibaryonic number density. A universe of this type was discussed by Brown and Stecker [1979] and Sato [1981] in a model with spontaneous $CP$-violation. However, spontaneous breaking of a discrete symmetry leads to the domain wall problem [Zel'dovich et al. 1974]; one has somehow to get rid of these walls. The virtue of the scenario presented below is that matter and antimatter domains can be formed without domain walls. Another interesting feature of the model is a natural realization of biasing in large scale structure formation, since the distribution of the baryonic density does not necessarily coincide with dark matter. Last but not the least, the model permits the generation of a large lepton asymmetry (of the order of one) together with a reasonably small baryon asymmetry if $B - L$ is not conserved [Dolgov and Kirilova 1991; Dolgov 1990].

In the original Affleck–Dine model the baryon asymmetry $\beta = (N_B - N_{\bar{B}})/N_\gamma$ was found to be considerably greater than 1. However, as was argued by Linde [1985], the baryonic charge to entropy ratio cannot exceed unity. To advocate this Linde proposed a specific model which indeed possesses this property. In fact this is a general statement in the conditions of the Affleck and Dine [1985] model [Dolgov and Kirilova 1989] and thus the generated asymmetry could be at most of the order of unity. Still, even that is too large and to reduce it to the observed value scenarios with large entropy generation after baryogenesis were considered (see, e.g., Ellis et al. [1987a, 1988a]). A natural source of entropy might be the decays of inflatons if they are sufficient long-lived.

Specific versions of Affleck–Dine mechanism in superstring inspired models have been considered by Campbell et al. [1987b], Ellis et al. [1987b], and by Connors et al. [1989]. It is shown that superstring theories can provide a reasonable value of the baryon asymmetry without accompanying problems of gravitino or other fields in the hidden sector of the theory. As was argued by Enqvist et al. [1988], extra scalar fields different from the one which carries the baryonic charge might present a danger to baryogenesis because their decay would heat up the Universe and destroy the baryonic charge condensate.

Higher-order corrections to the self-interaction potential of $\chi$ were discussed by Ng [1989], who considered the terms of the form $\chi^3\chi^{*3}$. They destroy the valleys in the potential along which $\chi$ could rise and diminish the baryon asymmetry.

The bulk of the papers dedicated to the Affleck and Dine mechanism deal with the problem of how to suppress the originally huge ($\beta \approx 1$) baryon asymmetry. We will argue, however, that small values of $\beta$ are more natural for this scenario and that only with a particular choice of parameters one could get $\beta$ of the order of unity. The basic idea is the following. The scalar field condensate during inflation is developed along the so called flat directions of the potential where the latter does not increase. Baryonic charge is connected with the phase degree of freedom of $\chi$ and thus corresponds to "motion" in the transverse direction $\chi_t$. The potential should not be flat in that orthogonal direction since otherwise baryonic charge would be conserved. Thus one would expect that $\chi_t$ should oscillate around the "bottom of the valley". Field oscillations should give rise to particle production, which in turn damps the oscillations. It was believed that due to a large value of the $\chi$-condensate along the valley the masses of particles coupled to $\chi$ is to be large too. However, as we will see in what follows, the oscillation frequency is generically of the same order of magnitude as the effective masses of the particles so their production is essential. Consequently the oscillations are damped and correspondingly the baryonic charge stored in the condensate is also damped.

To demonstrate how all this works let us consider a toy model with the following self-interaction potential of the field $\chi$:

$$U(\chi) = m^2|\chi|^2 + \tfrac{1}{2}\lambda_1|\chi|^4 + \tfrac{1}{4}\lambda_2(\chi^4 + \chi^{*4}) . \tag{6.1}$$

The concrete form of the potential is not important. The essential feature is baryonic current nonconservation induced by the last term,

$$\partial_\mu j^\mu_B \equiv \partial_\mu (i\chi^* \partial^\mu \chi - i\,\partial^\mu \chi^* \chi) = i\lambda_2(\chi^{*4} - \chi^4)\,. \tag{6.2}$$

We assume that the field $\chi$ is massless in the symmetric high-temperature phase and acquires a mass of about $10^2$–$10^3$ GeV after symmetry breaking. At any rate $m \ll H_I$. In supersymmetric theories the coupling constants $\lambda_i$ are of the order of the gauge coupling constant $\alpha$. We will consider the specific case $\lambda_1 = -\lambda_2 = \lambda > 0$. In this case the potential $U(\chi)$ possesses directions in which it does not rise, the so called valleys, and the quantum fluctuations of the field $\chi$ grows along these valleys. Note that the existence of valleys is a generic feature of supersymmetric grand unified theories (see, e.g., Affleck and Dine [1985]). A valley in the potential $U(\chi)$, where $\chi$ is a multicomponent scalar field, means that there is a direction along which some component of $\chi$ can be considered as a free massless field. It is found that quantum fluctuations of the latter are unstable in the de Sitter background [Vilenkin and Ford 1982; Linde 1982] and rise with time as

$$\langle \chi^2 \rangle \simeq H_I^3 t/4\pi^2\,. \tag{6.3}$$

This rise is stopped by the potential term (if the direction is not exactly flat) and the limiting value can be evaluated by the virial theorem equating the kinetic and potential energy of the field fluctuations,

$$U(\chi_{max}) \simeq H_I^4\,. \tag{6.4}$$

For the case of a real free massive field, when $U(\chi) = m^2\chi^2/2$, this problem was solved by Bunch and Davies [1978], who found

$$\langle \chi_m^2 \rangle \simeq 3H^4/8\pi^2 m^2\,. \tag{6.5}$$

For the case of a quartic potential, $U = \lambda\chi^4$, the solution is not known (at least to me) but eq. (6.4) permits the estimate

$$\langle \chi_\lambda^2 \rangle = O(H_I^2/\sqrt{\lambda})\,. \tag{6.6}$$

The equations of motion for a spatially homogeneous field $\chi = \chi_1 + i\chi_2$ have the form

$$\ddot\chi_1 + 3H\dot\chi_1 + (m^2 + 4\lambda\chi_2^2)\chi_1 = 0\,, \tag{6.7a}$$

$$\ddot\chi_2 + 3H\dot\chi_2 + (m^2 + 4\lambda\chi_1^2)\chi_2 = 0\,. \tag{6.7b}$$

When inflation is over, the growth of the quantum fluctuations of the $\chi$ field in a curved space–time is less effective than the influence of the expansion of the Universe. As a result, the $\chi$ field amplitude must decrease. The creation of light particles by $\chi(t)$ can also lead to a decrease in its amplitude. Let us specify a flat direction corresponding to $\chi_2 = 0$. Along this valley, $\chi_1 \equiv \mathrm{Re}\,\chi$ increases up to the value $\chi_1^{max} = H_I(H_I\tau)^{1/2}/2\pi$ (provided $H_I\tau < H_I^2/m^2$). Here $\tau$ is the duration of inflation. For larger $\tau$, the maximum value of $\chi$ is $\chi_1^{max} \approx H_I^2/(2\pi m)$. The quantum fluctuations of $\chi_2$ are bounded by the steep walls of the valley $(\langle \chi_2^2 \rangle)^{1/2} \approx H_I^2/(\chi_1^{max}\lambda^{1/2})$. These values of $\chi_1$ and $\chi_2$ constitute the initial conditions at the onset of the Friedman phase after the inflation phase is over. The initial value of $\dot\chi$ is obtained from the condition $\rho_{kin} = \dot\chi^2 \simeq H_I^4$, that is, $\dot\chi \propto H_I^2$. With these initial values, the baryon density is

$$N_B = j_0^B = 2(\dot{\chi}_{10}\chi_{20} - \dot{\chi}_{20}\chi_{10}) = O(H_I^2\chi_{10}) \geq H_I^3\,, \tag{6.8}$$

that is, barring the case of accidental smallness, each quantum of the $\chi$ field carries a baryon charge of order 1.

The following mechanical analogy is instructive for the analysis of the evolution of the baryon charge. The field equations (6.7) are equivalent to the mechanical equation of motion of a point-like particle in the two-dimensional plane $\{\chi_1, \chi_2\}$, with a potential $U(\chi)$ and a coefficient of "liquid" friction $H$. In these mechanical terms, the baryon charge corresponds to the angular momentum of the particle, and its nonconservation is caused by the nonsphericity of potential (6.1). If the potential has almost flat directions, the particle oscillates between the walls of the valley, slowly approaching the origin (in the case of small $m^2 \neq 0$). Hence, the angular momentum or, correspondingly, the baryon charge oscillates, periodically reversing its sign. At this stage, $\langle B \rangle = 0$ but $\langle B^2 \rangle \neq 0$ and is not small. Near the point $\chi_1 = \chi_2 = 0$, oscillations are replaced by a rotation around the origin with nearly constant angular momentum. The sign of the latter (i.e., the sign of $B$) is stochastically determined by the initial oscillations of $\chi$ during inflation. Particle creation by $\chi$ oscillations transverse to the flat direction leads to a considerable additional frictional force and, as a result, to damping of the oscillations. Subsequently, a moving point-like particle reaches the origin with a vanishingly small momentum. This is a qualitative explanation of the small value of the baryon asymmetry of the Universe in this scenario.

We assume that, when inflation ends, the coherent oscillations of the inflaton give rise to the MD-type of expansion with $H = 2/3t$. In this case eq. (6.7a) has, for small $\chi_2^2 = H_I^4/\lambda\chi_1^2$, the solution

$$\chi_1 = \chi_{10} + C/t\,, \tag{6.9}$$

where $C = O(1)$ and the initial value of $t$ is $t_0 = H_I^{-1}$. This solution is valid up to $t = m^{-1}$ when the oscillations around the origin of the potential begin and the solution takes the form $\chi_1 = \chi_{10}\sin(mt)/mt$. For (almost) constant $\chi_1$ the solution of eq. (6.7b) is easily found,

$$\chi_2 = \frac{H_I}{\sqrt{\lambda}\,\chi_{10}t}\cos(2\sqrt{\lambda}\,\chi_{10}t + \delta)\,, \tag{6.10}$$

where $\delta$ is a constant phase determined by the initial conditions.

The average energy density of this solution is of the order of $H_I^2/t^2$. It is negligible in comparison with the inflaton energy density $\rho_I \sim m_{Pl}^2/t^2$, as has been implicitly assumed. Note that at this stage the baryonic charge density oscillates changing its sign. The fast oscillations of the field $\chi_2$, eq. (6.10), give rise to particle production due to the coupling $g\chi\bar{\psi}_1\psi_2$, where $g^2/4\pi = \alpha$. The rate of this process can be calculated perturbatively and is equal to

$$\Gamma_2 = 2(g^2/4\pi)\sqrt{\lambda}\,\chi_{10}\,. \tag{6.11}$$

Note that it is much larger than the decay width of real $\chi$-mesons, $\Gamma_\chi = g^2/4\pi m_\chi$, because $\chi_0 \gg m_\chi$. The particles produced at this stage have zero average baryonic charge because $\chi_2$ decays into particles and antiparticles with equal probability if $C$ or $CP$ is conserved. As a result of the decay, the amplitude of the field decreases as $\chi_2 \to \chi_2\exp(-\Gamma_2 t)$ and the baryonic charge contained in the classical field $\chi$ is diminished by the factor $\exp(-\Gamma_2/m)$. So the baryon asymmetry of the Universe cannot be created in this version of the scenario with the parameters of the Affleck–Dine supersymmetric model.

Now let us consider the case when there are no valleys in the potential. In the toy model (6.1) this corresponds to $\lambda_1 > \lambda_2$. Now $\chi$ does not reach a very large value at the inflationary stage and $\chi_1$ and $\chi_2$ are generically of the same order of magnitude, $\chi_1 \sim \chi_2 \sim \lambda^{-1/4} H_I$. The equations of motion in this case have oscillating behavior too and the rate of particle production is given by the expression [Dolgov and Kirilova 1991]

$$\Gamma = |\dot{\rho}_\chi / \rho_\chi| \equiv \dot{N}_f \bar{E}_f / \rho_\chi = (g^2/4\pi) \lambda^{1/4} H_I (H_I t)^{-2/3} . \tag{6.12}$$

Here $\rho_\chi$ is the energy density of the oscillating $\chi$-field, $N_f$ and $\bar{E}_f$ are number density and average energy of the produced fermions, respectively, which are coupled to $\chi$ by $\mathscr{L}_{int} = g\chi\bar{\psi}_f\psi_f$. Note that the decay rate is a function of time $t$.

The characteristic damping time is determined by the condition

$$\int_{t_0}^{t_1} \Gamma \, dt = \frac{3g^2}{4\pi} \lambda^{1/4} (H_I t_1)^{1/3} = 1 . \tag{6.13}$$

Thus, for $H_I = 10^{14}$ GeV, $m = 10^3$ GeV, and $g^2/4\pi \approx \alpha \approx 10^{-2}$ the field $\chi$ transfers practically all its energy to relativistic particles at the moment $t = m^{-1}$. So efficient baryogenesis could proceed only if $m \gg 10^3$ GeV or for very small $\lambda$ and $g$.

Now let us consider the case when the mass term starts to operate earlier than the $\chi$ decays, that is, for $(H_I t_1)^{4/3} (m/H_I)^2 > \lambda^{1/2}$ or $(m/H_I) > 10\alpha^2\lambda^{3/4}$. In this case the baryonic charge contained in $\chi$ survives up to the epoch of conservation when $\chi$ oscillates harmonically with frequency $m$,

$$\chi = \chi_0(\varepsilon)(C_1 e^{imt} + C_2 e^{-imt}) , \tag{6.14}$$

and produces quarks with nonzero baryonic charge. The rate of production of light particles is equal to $\Gamma_\chi = g^2 m/4\pi$ if $g\chi_0(t) < m_\chi$ and is equal to $4\pi^{-5/2}\Gamma_\chi (m/g\chi_0)^{1/2}$ if $g\chi_0(t) > m_\chi$ [Dolgov and Kirilova 1990]. For $t \geq \Gamma_\chi^{-1}$ all baryonic charge contained in $\chi$ goes into the baryonic charge of quarks. The baryonic charge of the classical field (6.14) is determined by the ratio of the efficients $C_1$ and $C_2$. For $|C_1| = |C_2|$, $B_\chi = 0$ and for $C_2 = 0$, $B_\chi = N_\chi$, that is, each quantum of the condensate has $B = 1$. In this case the plasma of elementary particles produced after thermalization has a large chemical potential corresponding to the baryonic charge. For this reason the naive estimate of the plasma temperature,

$$T \approx [\rho_\chi (t = \Gamma_\chi^{-1})]^{1/4} \gg m , \tag{6.15}$$

may be invalid.

Note that in the models discussed in the references quoted above the expression for the decay width of $\chi$ is $\Gamma_\chi \sim m_\chi^3/\chi^2$. This result is valid for $\chi \gg m_\chi$ if the coupling of $\chi$ to light fermions is realized only because of radiation corrections and the coupling constant is proportional to $m_\chi/\chi$. However, $\chi$ is directly coupled to would-be heavy particles which acquire a large mass when the $\chi$-condenstate is developed. As we have seen above, the frequency of $\chi$-oscillations perpendicular to the flat direction is

$$\omega \sim \sqrt{\lambda}\, \chi_1 , \tag{6.16}$$

whereas the effective mass of the particles directly coupled to $\chi$ is

$$m_{\mathrm{eff}} \sim g\chi_1 \,. \tag{6.17}$$

Depending upon the relation between $g$ and $\lambda$ the particle production rate is either exponentially suppressed or rather large, as was considered above. Correspondingly the baryon asymmetry would be either large, $\beta = \mathrm{O}(1)$ as in the original paper by Affleck and Dine [1985], or small and close to observations. Probably in supersymmetric models the particle production rate is too high and $\beta$ is too small.

In view of that the version of the model with large damping of $B$ because of particle production (but not as large as with supersymmetric coupling constants) seems most appealing. It has a very interesting property. Depending on the form of the fluctuations of $\chi$ there are regions with different value and sign of the baryonic charge. Their characteristic size can be evaluated as follows. During inflation the fluctuations of $\chi$ on the scale $L_0 = H_{\mathrm{I}}^{-1}$ are of the order of magnitude

$$\delta\chi = \chi(0) - \chi(L_0) = H_{\mathrm{I}}/2\pi \,. \tag{6.18}$$

This scale inflates up to $L = H_{\mathrm{I}}^{-1}\exp(H_{\mathrm{I}}\tau)$ in a time $\tau$ and the fluctuation amplitude during this time increases as

$$\Delta\chi = \chi(0) - \chi(L) \approx (H_{\mathrm{I}}/2\pi)\sqrt{H_{\mathrm{I}}\tau} \tag{6.19}$$

[see eq. (6.3)]. The field would change by the order of unity over a distance $L_1 = H_{\mathrm{I}}^{-1}\exp[(2\pi\chi_0/H_{\mathrm{I}})]$. As we have already noted the characteristic amplitude of the field $\chi$ at the inflationary stage is determined by the condition $U(\chi) = (H_{\mathrm{I}}/2\pi)^4$; hence

$$L_1 = H_{\mathrm{I}}^{-1}\exp(\lambda^{-1/2}) \,. \tag{6.20}$$

Now since the initial baryonic charge of $\chi$ is given by expression (6.8) with

$$\dot{\chi} = -(3H)^{-1}\,\partial U/\partial\chi \tag{6.21}$$

(this follows from the equations of motion in the case of large $H$), we obtain that $B$ changes by the order of unity at the same scale $L_1$.

This means that the Universe consists of regions filled either by baryons or by antibaryons with characteristic size $L_1$. The value of the ratio $N_{\mathrm{B}}/N_\gamma$ should be different in these regions because it is determined by initial stochastic fluctuations of the field. These regions are separated by a region with a small density of baryons or antibaryons. In order to have same sign of the baryon asymmetry in all the visible part of the Universe the constant $\lambda$ should be smaller than $10^{-4}$. Still the existence of regions filled by antimatter in our Universe is not excluded and the observed $\gamma$-background probably confirms this [Stecker 1985].

In conclusion let us summarize the interesting features of the discussed mechanism. First, for its realization charge symmetry breaking is not necessary. The sign of the baryon asymmetry is determined by quantum fluctuations during the inflationary stage. Domains of matter and antimatter should be formed in the model but without domain walls. These domains are separated from each other by regions with low baryonic density. The value of the baryon asymmetry is very sensitive to the coupling constants of $\chi$ to fermions and to its self-coupling $\lambda$ and may vary from 1 to $10^{-15}$. This permits in particular, if $B-L$ is not conserved, the construction of natural models in which $\beta$ is close to the observed value $10^{-9}$–$10^{-10}$ while the leptonic asymmetry is large [Dolgov and Kirilova 1991; Dolgov 1990]. It would give rise to a large chemical potential of electronic neutrinos and influence the primordial nucleosynthesis. It is possible in particular that the characteristic size of regions with a definite leptonic charge is small in comparison with the visible size of the Universe. This would lead to systematic variation of the abundances of light elements and especially of $^4$He in different regions in the sky. Last but not the least, the model permits low temperature generation of baryon asymmetry.

## 7. Baryogenesis in thermal equilibrium

There are two possible ways to generate charge asymmetry in thermal equilibrium. The first is an explicit breaking of the sacred *CPT*-theorem, which is not particularly appealing because there is no consistent model of *CPT*-violation. We consider this here only for completeness. Another possibility is an effective *CPT*-invariance breaking caused by expansion of the Universe which breaks *T*-invariance [Cohen and Kaplan 1987, 1988]. No sacred principles are violated in this case, moreover it looks rather natural. It is better, however, to describe this not as a breaking of the *CPT*-theorem but as an induction of an effective time-dependent baryonic chemical potential by expansion of the Universe.

There is no known way now to describe *CPT*-violation consistently. Still it seems reasonable that even in the *CPT*-broken world the standard statistical mechanics should be valid because the usual form of the equilibrium distribution functions is ensured by diagonal elements of the *S*-matrix unitarity condition without referring to *CPT*-invariance (see, e.g., Dolgov and Zel'dovich [1981]).

One of the possible consequences of *CPT*-breaking is inequality of particle–antiparticle masses. Hence even in thermal equilibrium an excess of particles over antiparticles should exist,

$$\delta N_{\mathrm{B}} = N_{\mathrm{B}} - N_{\bar{\mathrm{B}}} = gB \int \frac{\mathrm{d}^3 p}{2\pi^3} \left[\exp(-\sqrt{p^2+m^2}/T) - \exp(-\sqrt{p^2+\bar{m}^2}/T)\right]$$
$$\approx \frac{gB}{4\pi^2}\frac{\delta m^2}{T^2} T^3 \int_{m/T}^{\infty} \mathrm{d}y\, \mathrm{e}^{-y}\sqrt{y^2-(m/T)^2}\,, \tag{7.1}$$

where $\delta m^2 = m^2 - \bar{m}^2 = 2m\,\delta m$ is the difference of particle–antiparticle masses caused by *CPT*-breaking and by assumption $\delta m^2 \ll T^2$, $g$ is the number of spin states of baryons, and $B$ is their baryonic charge. For simplicity the Boltzmann distribution function has been used here. It is easy to see that quantum Bose or Fermi corrections are not essential. Note the absence of the chemical potential corresponding to the baryonic charge because of nonconservation of the latter.

The excess of baryons is determined by eq. (7.1) up to the moment corresponding to the temperature $T_{\Delta B}$ below which the $B$-nonconserving processes effectively stop. After that, the baryonic charge in a comoving volume remains constant. The relative baryonic charge for $m \ll T_{\Delta B}$ is

$$\beta \equiv \frac{N_{\mathrm{B}}}{N_\gamma} = \frac{gB}{\pi^2}\frac{\delta m^2}{T_{\Delta B}^2}\,. \tag{7.2}$$

If baryons are not conserved at the electroweak scale, that is $T_{\Delta B} = 1\,\mathrm{TeV}$, then for a t-quark mass of 100 GeV the relative mass difference should be $\delta m_t/m_t > 10^{-6}$ to generate the observed baryon asymmetry. Such a *CPT*-violation does not contradict experiment especially because the t-quark has not yet been discovered. Still this possibility does not look appealing because it is impossible to break *CPT*-invariance in the framework of conventional analytical Lorentz-invariant field theory with positive definite energy.

A possibility of *CPT*-violation at an early stage of the evolution of the Universe has been considered by Barshay [1981], who proposed a time-dependent contribution to the particle masses which is different for particles and antiparticles.

Another phenomenological manifestation of possible *CPT*-breaking is a difference in the lifetimes of particles and antiparticles. This can explain the observed baryon asymmetry only in the case of nonequilibrium processes. In this sense we return to the situation described in sections 2 and 3.

Effective *CPT*-violation connected with expansion of the Universe does not demand, as was the case above, a rejection of any sacred principles of field theory and is permitted in the standard model. A nonzero baryonic charge can be generated in this case in thermal equilibrium due to interaction with an external nonstationary field. A model of this kind has been proposed by Cohen and Kaplan [1987, 1988]. In their model a neutral scalar field $\phi$ is coupled to the baryonic current as

$$L_{\mathrm{I}} = \Lambda^{-1}\,\partial_\mu \phi\, j^\mu_{\mathrm{B}}\,, \tag{7.3}$$

where $\Lambda$ is a constant with the dimension of mass. This type of coupling arises, e.g., when global baryonic U(1) symmetry is spontaneously broken. (See also section 9, where some mechanisms to create an effective interaction of this form are discussed.) If a classical homogeneous scalar field $\phi$ existed in the Universe in the past and if its variation with time was slow enough so that thermal equilibrium was not broken, then the interaction (6.3) was equivalent to a nonzero baryonic chemical potential, $\mu_{\mathrm{B}} = \dot{\phi}/\Lambda$,

$$L_{\mathrm{I}} = (\Lambda^{-1}\dot{\phi})N_{\mathrm{B}} = \mu_{\mathrm{B}} N_{\mathrm{B}}\,, \tag{7.4}$$

where $N_{\mathrm{B}}$ is the baryonic charge density. This means that in thermal equilibrium there exists a baryon asymmetry, the corresponding charge density being

$$N_{\mathrm{B}} = \tfrac{1}{6} B_{\mathrm{q}} \mu_{\mathrm{B}} T^2 (1 + \mu_{\mathrm{B}}^2/\pi^2 T^2)\,, \tag{7.5}$$

where $B_{\mathrm{q}}$ is the baryonic charge of a quark (it is assumed that only quarks contribute to $j^\mu_{\mathrm{B}}$). This expression remains valid as long as equilibrium with respect to $B$-nonconserving processes is maintained. When these processes are switched off the baryonic charge in a comoving volume becomes constant,

$$\beta = \mu_{\Delta B}/T_{\Delta B}\,. \tag{7.6}$$

Given the initial value of $\dot{\phi}$ it is easy to calculate the evolution of the baryonic charge density. If $\phi$ is a Goldstone field it is convenient to introduce a dimensionless phase variable $\theta = g\phi/\Lambda$ described by the Lagrangian

$$L_\theta = f^2\,\partial_\mu \theta\, \partial^\mu \theta - V(\theta) + g\,\partial_\mu \theta\, j^\mu_{\mathrm{B}}\,, \tag{7.7}$$

where $g$ is a coupling constant and the dimensional parameter $f$ defines the energy scale at which the $U(1)_B$ symmetry is spontaneously broken. There might also be an explicit breaking of the symmetry and for that reason the potential term $V(\theta)$ is present in the Lagrangian. Near its minimum the potential $V(\theta)$ can be approximated by the usual expression

$$V(\theta) = \tfrac{1}{2}m^2 f^2 \theta^2 . \tag{7.8}$$

The equation of motion of a homogeneous field $\theta$ in the Robertson–Walker metric has the form

$$\ddot{\theta} + 3H\dot{\theta} + m^2\theta = -(g/f^2)(\dot{N}_B + 3HN_B) . \tag{7.9}$$

In thermal equilibrium, when $N_B$ is given by eq. (7.5) with $\mu = g\dot{\theta}$, the following equation for $\theta$ is valid:

$$(\partial_t + 3H)[\dot{\theta}(1 + \tfrac{1}{6}g^2 B_q H/H_0)] + m^2\theta = 0 , \tag{7.10}$$

where $H_0$ is the initial value of the Hubble constant. If the phase transition took place after inflation was over, the temperature $T_0$ at the initial moment should be of the order of $f$. This case, however, contradicts observations because, if it were true, the size of the domains with a definite sign of $\dot{\theta}$ (and of $N_B$) would be much smaller than the present horizon and even the galactic size. Hence the phase transition should took place before the end of inflation. This permits an exponentially large increase in the domain size. The initial value of the Hubble constant coincides in this case with that at the end of inflation, $H_0 = H_I$, and the initial temperature is equal to the reheating temperature.

At earlier stages when $H > m$ the last term in this equation can be neglected and the solution takes the form

$$\dot{\theta} = \dot{\theta}_0(1 + \tfrac{1}{6}g^2 B_q)(1 + g^2 B_q H/6H_0)^{-1}(T/T_0)^3 . \tag{7.11}$$

The rate of variation of $\mu$, $\dot{\mu}/\mu = \ddot{\theta}/\dot{\theta} \approx 3H$, should be small in comparison with the rate of $B$-nonconserving reactions so that thermal equilibrium is maintained. At this time the baryonic asymmetry is determined by the expression

$$\beta = \frac{N_B}{N_\gamma} = \frac{\pi^2 B_q g}{12\zeta(3) T_0} \frac{\dot{\theta}_0}{T_0} \left(\frac{T}{T_0}\right)^2 , \tag{7.12}$$

which is effectively valid until $T = T_{\Delta B}$ when $B$-nonconservation is broken. Here $\zeta(3) \approx 1.2$ and terms of the order of $g^2$ have been neglected. The initial value $\dot{\theta}_0$ is crucial for the realization of this scenario. One would expect that $\dot{\theta}_0 \simeq gH_I^2/f$. This follows from the statement that the kinetic energy of quantum fluctuations of the scalar field $\phi = \theta\Lambda/g$ during inflation is of the order of $H_I^4$. We can express $H_I$ through the reheating temperature using the relations

$$H_I^2 \simeq \rho/m_{Pl}^2 \simeq T_{rh}^4/m_{Pl}^2 , \tag{7.13}$$

and find for the baryon asymmetry

$$\beta \sim g^2(T_{rh}/f)(T_{rh}/m_{Pl})^2 . \tag{7.14}$$

This version of the scenario definitely demands a very high reheating temperature.

A more realistic mechanism of creating a nonzero $\dot{\theta}$ is connected with explicit symmetry breaking, which gives rise to the potential $V(\theta)$. In this case, the initial value of $\dot{\theta}$ may be even zero. At the inflationary stage, the classical field $\theta$ remains practically constant due to the large Hubble friction. This constant value generically does not coincide with $\theta = 0$ where the potential has a minimum. When inflation is over, the Hubble parameter starts to decrease and $\theta$ begins to roll down towards the minimum of $V(\theta)$. The rate of rolling down can be evaluated as $\dot{\theta} \approx (m^2/H)\theta$. The baryon asymmetry for this case was evaluated by Cohen and Kaplan [1988], who give the result

$$\beta \simeq \theta_0 \left( \frac{m_\theta}{H_{\Delta B}} \right)^2 \frac{T_{\Delta B}}{m_{\mathrm{Pl}}} \frac{S_{\Delta B}}{S_{\mathrm{f}}} , \tag{7.15}$$

where the last factor accounts for the entropy production subsequent to the freezing of baryon nonconserving interactions.

This expression is valid if the Hubble parameter at the moment when the $B$-nonconserving interaction is switched off exceeds the mass of the $\theta$-field, $H_{\Delta B} > m_\theta$. In the opposite case $\theta$ starts to oscillate around zero when $B$-nonconservation is still in equilibrium and this results in a suppression of the baryon asymmetry.

An interesting problem in connection with this mechanism of baryogenesis is the effect of quantum fluctuations of $\theta$, which might be rather large at the end of the inflationary stage. The characteristic length of these fluctuations can be evaluated in the same way as has been done in section 6 [see eq. (6.20)]. As was noted by Turner et al. [1989], these fluctuations could naturally give rise to isocurvature baryon number fluctuations, which might be essential for the formation of large scale structure in the Universe (see section 11).

One can see that this scenario of baryogenesis and that considered in section 6 have much in common. Isocurvature density fluctuations are a natural result of both. Another similarity is that neither of these scenarios requires explicit violation of charge symmetry ($C$ and $CP$) for a realization of baryogenesis. In the model we consider here, the choice between baryons and antibaryons is made stochastically by the sign of the initial value of $\dot{\theta}$. The average baryon charge of the Universe should be equal to zero as in any model without explicit charge symmetry breaking. However, as in the scenario of section 6, the size of regions with a definite sign of $B$ can be exponentially inflated and become of astronomical scale compatible with observations.

Recently the approach considered here has been applied to baryogenesis at the electroweak scale [Dine et al. 1991; Cohen et al. 1991]. We will discuss this below in section 9.

## 8. Topological defects and baryogenesis

There are three types of stable topological defects which can exist in our three-dimensional space. They are two-dimensional domain walls, one-dimensional strings or vortices and point-like monopoles. A detailed review of their properties was given by Vilenkin [1985]. Topological analysis can be found in papers by Kibble [1976, 1980]. Recently the cosmological implications of three-dimensional topological defects, so called textures, have been considered [Davies 1987; Turok 1989]. Unlike the lower-dimensional ones, the latter are unstable but may still be cosmologically interesting. There are two possible mechanisms by which topological defects can contribute to baryogenesis: First, by production of baryon nonconserving heavy particles in the course of their collapse or mutual collision and, second, by catalyzing baryon nonconservation [Rubakov 1981, 1982; Callan 1982]. In what follows we briefly review the properties of the topological defects and discuss their effect on baryogenesis.

We start from simpler two-dimensional structures. Domain walls are formed when a discrete symmetry is broken. Their properties can be studied on the example of a real scalar field with the Lagrangian

$$L = \tfrac{1}{2}(\partial_\mu \phi)^2 - \tfrac{1}{4}\lambda(\phi^2 - \eta^2)^2 . \tag{8.1}$$

This model possesses $Z_2$ symmetry generated by the transformation $\phi \to -\phi$. The system described by Lagrangian (8.1) has a doubly degenerate ground state corresponding to $\langle \phi \rangle = \pm\eta$. One can conclude even without solving the equation of motion that there should be a stable solution $\phi(x)$ interpolating between $\phi = +\eta$ and $\phi = -\eta$. Such a field combination is called a domain wall. In the case considered the solution is explicitly known,

$$\phi_W(x) = \eta \tanh(mx/2) , \tag{8.2}$$

where $m = \sqrt{2\lambda}\,\eta$ is the mass of the $\phi$ quanta over the stable vacuum $|\langle \phi \rangle| = \eta$. The thickness of the wall is $2/m$, which is microscopically small*) while its longitudinal size can be macroscopic.

The surface energy density of the wall is

$$\sigma = \int T^{00}\,dx = 2\sqrt{2\lambda}\,\eta^3/3 . \tag{8.3}$$

For $\eta = 10^3$ GeV the mass of 1 cm$^2$ of the wall is $m = \sigma \cdot 1\ \text{cm}^2 \approx 10^{13}$ g and for the characteristic Grand Unification scale $m \approx 10^{49}\ \text{g} \approx 10^{16}\ M_\odot$.

Let $M$ be the manifold of different possible vacuum values of $\phi$. It is easy to understand that domain walls appear when $M$ is not a connected manifold or, in other words, there exist parts which are separated from each other. This is formulated in mathematical language by the statement that the homotopic group $\pi_0(M)$ is nontrivial. In the example considered $\pi_0(M) = Z_2$. In some models $M$ is more complicated so that $\pi_0(M) = Z_N$ (as, e.g., in some axionic models). In that case the vacuum is $N$-fold degenerate and the domain walls may form a foam-like pattern resembling the observed structure of the Universe.

Domain walls appear in particular in the models with spontaneous breaking of $CP$-invariance [Lee 1973; Zel'dovich et al. 1974]. Models of this kind are of interest for baryogenesis. The vacuum condensate in such models is complex and the states $\langle \chi \rangle = \eta \exp(i\alpha)$ and $\langle \chi \rangle = \eta \exp(-i\alpha)$ are degenerate because of $CP$-invariance of the Lagrangian. A simple model of this kind is given by the following self-interaction potential of $\chi$:

$$U(\chi) = -m^2\chi^*\chi + \lambda_1(\chi^*\chi)^2 + \lambda_2(\chi^3\chi^*) + \lambda_2^*(\chi^{*3}\chi) . \tag{8.4}$$

This potential is invariant with respect to the substitution $\chi \leftrightarrow \chi^*$ (that is, particle–antiparticle transformation) if the constant $\lambda_2$ is real, $\lambda_2 = \lambda_2^*$. We assume that this is true and the theory is

*) Recently the cosmological implications of macroscopically thick domain walls have been considered [Hill et al. 1989].

$C$-invariant. The invariance is spontaneously broken, however, if $\lambda_2 > 0$. Indeed, the potential has a minimum at

$$\langle \chi \rangle = \frac{m \exp(\mathrm{i}\alpha)}{\sqrt{2\lambda_1 + 4\lambda_2 \cos 2\alpha}} \equiv \rho \exp(\mathrm{i}\alpha) , \tag{8.5}$$

where $\alpha = \pm\pi/2$ if $\lambda_2 > 0$ and $\alpha = 0$, $\pi$ if $\lambda_2 < 0$. The complex vacuum condensate of $\chi$ generates complex coupling constants and gives rise to $C$- and $CP$-violation. To make the effects observable, a coupling of $\chi$ with other fields is necessary, e.g., $\chi\bar{\psi}_i\psi_j + \chi^*\bar{\psi}_j\psi_i$.

There are two $CP$-odd degenerate vacua in this model, $\langle \chi \rangle = \pm \mathrm{i}\rho$. The amplitudes of $CP$-violation over these vacua differ in sign. With additional scalar fields it is possible to build a model in which not only the signs but the values of the $CP$-odd amplitudes are different. In particular, along with the $CP$-odd solution there could exist a $CP$-even one.

The domains with different values of $\langle \chi \rangle$ are separated by walls of the type considered above. Such domain walls in the Universe would lead to a cosmological disaster [Zel'dovich et al. 1974]. We shall see in what follows (section 11), however, that there are ways to overcome the difficulty and permit spontaneous $CP$-violation and consequently a charge symmetric universe as a whole.

Besides the topologically stable walls there could exist walls between the real and false vacua if the latter is metastable. Such walls generically move with acceleration resulting either in expansion or collapse of one phase inside the other.

Let us consider now a complex scalar field described by the Lagrangian

$$\mathscr{L} = (\partial_\mu \phi^*)(\partial_\mu \phi) - \lambda(|\phi|^2 - \eta^2)^2 . \tag{8.6}$$

The theory is invariant with respect to the phase transformations $\phi \to \exp(\mathrm{i}\alpha)\, \phi$, which form the U(1) group. The vacuum of this system is infinitely degenerate because of the arbitrariness in the phase of the condensate $\langle \phi \rangle = \eta \exp(\mathrm{i}\alpha)$. It is easy to realize a case in which the phase of the condensate in different causally disconnected regions is different and such that the phase variation on a closed contour is nonzero,

$$\Delta\alpha = 2\pi n . \tag{8.7}$$

Here $n$ is an integer to ensure the single-valuedness of the condensate on that closed contour.

Evidently the absolute value of the condensate should vanish when the contour is shrunk to zero. Otherwise the field $\phi$ becomes discontinuous due to the discontinuity in the phase $\alpha$. So we conclude that the equations of motion permit the existence of strings that are closed or infinite lines on which $\langle \phi \rangle = 0$. Such field configurations possess nonzero energy and are stable because of their topology.

Let us consider the case of an infinite straight string directed along the $Z$-axis. The field can be written in the form $\phi = f(\rho) \exp(\mathrm{i}\theta)$. Here $\rho = (x^2 + y^2)^{1/2}$ is the transverse distance from the string and $\theta$ is the polar angle in the $xy$-plane. We take $n = 1$ [see eq. (8.7)]. Generalization to arbitrary $n$ is trivial. The function $f(\rho)$ satisfies the equation

$$\left(\partial_\rho^2 + \frac{1}{\rho}\partial_\rho - \frac{1}{\rho^2}\right) f - 2\lambda(f^2 - \eta^2) f = 0 . \tag{8.8}$$

We will look for solutions of this equation for which $f(\rho \to \infty) \to \eta$. This corresponds to the vacuum state at large distance. It can be easily checked that the solution can be written in the form $f = \eta Z(\sqrt{2\lambda}\,\eta\rho)$, where $Z(x) = 1 - 1/2x^2 + 3/8x^4 + \cdots + O(\exp(-\sqrt{2}\,x)/\sqrt{x})$ when $x \to \infty$.

The solution of eq. (8.8) regular at small $x$ behaves as $Z(x) \sim x$. The natural question is whether the solution discussed above which is regular at infinity matches the solution regular at the origin. The affirmative answer is given by the variational principle. Indeed, solutions which are singular at the origin have higher energy than test functions with the right behavior at zero and infinity.

The energy density per unit of string length is

$$\mu = \tfrac{1}{2}\eta^2 \int_0^R \mathrm{d}x\, x[(z^2-1)^2 + 4z'^2 + 4z^2/x^2] \to 2\pi\eta^2 \ln(R\eta\sqrt{\lambda})\,. \tag{8.9}$$

The energy density logarithmically diverges as a function of the upper integration limit $R$. This is connected with the long-range action created by massless Goldstone bosons.

This divergence is connected with the last term in eq. (8.9), which originates from $|\partial\phi/\partial\theta|^2 = f^2/r^2$. If two antiparallel strings are considered, the variation of the phase along the encircling contour is evidently zero. So for this configuration $r^2|\partial\phi/\partial\theta|^2 \to 0$ as $r \to \infty$ and the linear energy density of two antiparallel strings is finite. Analogously the mass of the closed string of radius $R$ is also finite, $E \sim R\eta^2 \ln(R\eta\sqrt{\lambda})$.

It is known that Goldstone degrees of freedom hide in the longitudinal components of the vector fields if a local symmetry breaks down so that massless particles do not appear. Thus one may expect that strings in theories with local symmetry breaking are different from those with global symmetry. In particular they should have finite energy per unit of length.

Formally, the long-range string action comes from the contribution of $(r^{-1}\,\partial\phi/\partial\theta)^2$ to the energy density. This term slowly decreases when $r \to \infty$ because of the nontrivial dependence of $\phi$ and $\theta$. This results in a logarithmic divergence of the energy density (8.9). In gauge theories the energy density is determined not by the usual derivative $\partial_\mu\phi$ but by the covariant one $D_\mu\phi = (\partial_\mu + \mathrm{i}gA_\mu)\phi$. Now it proves possible to find a solution with $A_\mu$ such that $D_\mu\phi$ tends to zero sufficiently fast so that the finiteness of the energy is ensured. The corresponding potential $A_\mu \sim r^{-1}$ for $r \to \infty$. The energy of the vector field, which is proportional to $(\partial A)^2$, is also finite.

The vortex line solution for the case of spontaneously broken local symmetry was found by Nielsen and Oleson [1973]. The vector field potential has the form $A(\rho,\theta) = A(\rho)\boldsymbol{n}_\theta$, and the asymptotic behavior of the field amplitude is

$$f(\rho) \approx \eta[1 - \exp(-2\sqrt{\lambda}\,\eta\rho)]\,, \qquad A(\rho) \approx (g\rho)^{-1}[1 - 2\exp(-\sqrt{2}\,g\eta\rho)]\,. \tag{8.10}$$

The rate of approach to the vacuum values is determined by the masses of the scalar, $2\sqrt{\lambda}\,\eta$, and vector, $\sqrt{2}\,g\eta$, bosons.

In contrast to the global case, the energy density of local strings is well localized. The string is a piece of the false vacuum of the field $\phi$ with transverse size $(2\sqrt{\lambda}\,\eta)^{-1}$ and a tube of magnetic field with radius $(\sqrt{2}\,g\eta)^{-1}$. The flux of the magnetic field is calculated by the Stokes theorem,

$$\int \boldsymbol{B}\,\mathrm{d}s = \oint \boldsymbol{A}\cdot \mathrm{d}\boldsymbol{l} = 2\pi/g\,. \tag{8.11}$$

The linear energy density of a string $\mu$ is composed of the energy densities of the vector and scalar fields. Each of them is of the order of $\eta^2$. In Grand Unification models with $\eta = 10^{15}$ GeV the energy density of a string is about $10^{20}$ g/cm.

As we have seen on this simple example, a string is formed if there exist nonshrinkable contours on the manifold $M$ of vacuum values of $\phi$. In the case considered $M$ is a one-dimensional sphere $S^1$, $|\phi| = \eta$. The function $\phi(\rho \to \infty, \theta)$ realizes the mapping of $S^1$ in the physical three-dimensional space into $M = S^1$ in the internal space of fields. All these transformations can be divided into classes corresponding to a definite number of windings of $\phi$ on $S^1$ when $\theta$ changes from 0 to $2\pi$. This is just the number $n$ in eq. (8.7). These classes are called homotopy classes. They form the homotopy group $\pi_1$ of the manifold $M$. The elements of the group are the topologically different nonshrinkable contours on $M$. In the case considered $\pi_1(M) = Z$, where Z is the manifold of integers.

In the general case when the group $G$ is spontaneously broken to a subgroup $H$, the manifold $M$ is the factor space of $G/H$. It is the manifold of values of $\phi$ in which the values of $\phi$ connected by transformations from $H$ are identified. Strings are formed if $\pi_1(G/H)$ is nontrivial.

We formulate without proof the recipe which often permits one to determine $\pi_k(G/H)$. Let us consider the sequence (this is called the exact sequence of homotopy groups)

$$\cdots \to \pi_k(G) \to \pi_k(G/H) \to \pi_{k-1}(H) \to \pi_{k-1}(G) \to \cdots , \tag{8.12}$$

where $\pi_k(G)$ is the $k$th homotopy group of the manifold $G$ determined by the number of topologically different nonshrinkable spheres $S^k$ on this manifold. If two terms in this sequence with numbers $i$ and $(i+3)$ are trivial then the homotopy groups of the terms with numbers $(i+1)$ and $(i+2)$ coincide. The proof of this statement in a digestible form for physicists can be found, e.g., in the book by Dubrovin et al. [1979]. A short and good introduction into the problem is presented by Olshanetsky [1982].

The convenience of using sequence (8.12) is based on the fact that many known groups are connected and simply connected. For example,

$$\pi_1(\mathrm{SU}(N)) = 0 , \quad \text{for } N \geq 2 ; \qquad \pi_1(\mathrm{Spin}(N)) = 0 . \tag{8.13}$$

Spin($N$) is the so called covering group of the rotation group SO($N$), i.e., the rotation group in $N$-dimensional space in which spinor representations are permitted. A covering group $\bar{G}$ is by construction simply connected and so $\pi_1(\bar{G}) = 0$. If $K$ is the center of group $\bar{G}$, i.e., the subgroup of $\bar{G}$ commuting with all the elements of $\bar{G}$, then it can be shown that $\pi_1(G/K) = K'$, where $K'$ is a subgroup of $K$. In particular, since $\mathrm{SO}(N) = \mathrm{Spin}(N)/Z_2$, then

$$\pi_1(\mathrm{SO}(N)) = Z_2 \quad (N > 2) .$$

Thus strings are formed when connected $[\pi_0(G) = 0]$ and simply connected $[\pi_1(G) = 0]$ groups $G$ are broken if the unbroken subgroup is not simply connected, $\pi_1(H) \neq 0$.

An interesting property of cosmic strings is that they may be superconducting [Witten 1985]. The mechanism of superconductivity looks simpler when the charged particles which carry electric current are bosons. Let us consider the model possessing $U_{em}(1) \times U_R(1)$ symmetry. $U_{em}(1)$ is the usual gauge symmetry of electromagnetic interactions. It is unbroken in vacuum while $U_R(1)$ is spontaneously broken. The breaking of the latter produces strings. Let there be two scalar fields in the model, $\phi$ with $Q = 1$ and $R = 0$ and $\sigma$ with $Q = 0$ and $R = 1$. Here $Q$ is the electric charge and $R$ is the gauge charge corresponding to the group $U_R(1)$. The scalar field potential is chosen in the form

$$U(\phi, \sigma) = \tfrac{1}{2}\lambda_1(|\phi^2| - \eta_1^2)^2 + \tfrac{1}{2}\lambda_2|\sigma|^4 + \lambda_3|\sigma|^2(|\phi|^2 - \eta_2^2) . \tag{8.14}$$

It can easily be shown that there is an interval of values of the parameters $\lambda$ and $\eta$ for which it is energetically favorable to have $|\langle\phi\rangle| = \eta$ and $\langle\sigma\rangle = 0$ outside of the string and $\langle\phi\rangle = 0$ and $\langle\sigma\rangle = \sigma_0 \neq 0$ in the center of the string. By order of magnitude $\sigma_0^2 \sim (\lambda_3/\lambda_2)\eta_2^2$. The energy of the system in this case is minimized by a classical field $\sigma(x, y)$ such that $\sigma \to 0$ when $\rho \to 0$, where $x$ and $y$ are the transverse coordinates and $\rho^2 = x^2 + y^2$. Evidently, $\exp(\mathrm{i}\theta)\,\sigma(x, y)$ with a constant phase $\theta$ also minimizes the energy. This means that there exist massless perturbations

$$\sigma(x, y, z, t) = \mathrm{e}^{\mathrm{i}\theta(z,t)}\sigma(x, y) ,$$

which gives rise to superconductivity.

The effective action for the field $\theta$ is obtained from the kinetic term $|(\partial_\mu + \mathrm{i}A_\mu)\sigma|^2$. Making the natural assumption that $A$ slowly changes on the transverse size of the string we can write

$$A_\mu(x, y, z, t) \approx A_\mu(0, 0, z, t)$$

in the region where $\sigma \neq 0$. In this approximation the action has the form

$$S_\theta = K \int \mathrm{d}z\, \mathrm{d}t\, [(\partial_\theta + eA_t)^2 - (\partial_z\theta + eA_z)^2] , \tag{8.15}$$

where $K = \int \mathrm{d}x\, \mathrm{d}t\, |\sigma(x, y)|^2$. Using the value of $\sigma_0$ estimated above and the value of the mass of $\sigma$ inside the string $m_\sigma \sim \sqrt{\lambda_3}\,\eta_2$, we get $K \approx \lambda_2^{-1}$. The electromagnetic current inside the string is equal to

$$J_i(z, t) = \delta S_\theta / \delta A_i(z, t) = 2Ke(\partial_i\theta + eA_i) . \tag{8.16}$$

String superconductivity is due to the fact that the total variation of $\theta$ along a closed string might be nonzero (multiple of $2\pi$). The corresponding topological invariant can be defined as

$$N = \frac{1}{2\pi} \oint \mathrm{d}l\, \frac{\mathrm{d}\theta}{\mathrm{d}l} .$$

Here $N$ is an integer. If there is no quantum tunneling transition on the string such that $\sigma$ passes through zero, $N$ is conserved. The probability of such a transition for a sufficiently small current is proportional to $\exp(-c/\lambda_1)$ because the field $\phi$ determines the height of the potential barrier between the inner part of the string where $\sigma \neq 0$ and the external space where $\sigma = 0$. For small $\lambda_1$ the tunneling transition can be neglected and $N$ is conserved. If the current in the string is large the probability of quantum jumps increases. This results in current saturation at the value $J_{\max} = e\eta/\lambda_1^{1/2}$ [Witten 1985].

The equations of motion for the electromagnetic vector potential $A_i$ ($i = x, y$) are obtained from the action

$$S = -\frac{1}{4} \int \mathrm{d}^4x\, F_{\mu\nu}^2 + S_\theta .$$

If the gauge condition $\partial_i A_i = 0$ is imposed they have the form $\partial^2 A_i = J_i$. Recall that $J_i$ depends on $A$ (eq. 8.16). In the thin string approximation the equation is easily solved,

$$J \simeq \frac{2Ke}{1 + (Ke^2/\pi)\ln(R/R_0)} \frac{N}{R}, \tag{8.17}$$

where $R$ is the curvature radius of the string and $R_0$ is its transverse size.

Using eq. (8.16) one can find the law of current variation in an external electric field $E_i = -\partial_t A_i$. It is easy to see that the following equation is valid:

$$\frac{\mathrm{d}}{\mathrm{d}t} \int \mathrm{d}l\, J = 2Ke^2 \int \mathrm{d}l\, E .$$

Hence for a current that is homogeneous along the string we get

$$\dot{J} = 2Ke^2 E . \tag{8.18}$$

Note that, in contrast to Ohm's law, $J = \sigma E$, the time derivative of the current and not the current itself is proportional to the electric field. This is a characteristic feature of superconductors.

It was argued by Witten [1985] that superconducting cosmic strings could exist also in the case when the current is carried by fermions which get their mass in the vacuum due to the condensate of $\phi$. Inside the string they are massless. In essence the model is reduced to the bosonic case by the substitution

$$\bar{\psi}\gamma_i\psi = \varepsilon^{ij}\, \partial_i \sigma/\sqrt{\pi} ,$$

where $\psi$ is the operator of the fermion field. In this model eq. (8.18) is locally valid with the substitution $q^2 \leftrightarrow 2\pi Ke^2$, where $q$ is the charge of the fermions. In this case the current is saturated because of the Fermi exclusion principle. Indeed, if the Fermi momentum of the current carriers is larger than their mass in vacuum then it is energetically favorable for them to escape from the string. This results in the upper bound

$$J < qm/2\pi . \tag{8.19}$$

For fermion masses at the Grand Unification scale $J_{\max} = 10^{22}$ A, at the electroweak scale $J_{\max} = 10^8$ A, and for electrons $J_{\max} = 10$ A. When the current approaches the maximal value the string starts to produce particles with a rate proportional to the external electric field.

There is, however, an important difference between fermions and bosons, which makes fermionic superconductivity questionable. For the former there is no topological conservation law which ensures the constant value of the current. As a result dissipation processes are essential and the superconductivity is destroyed. The dissipation is connected with quantum scattering processes, which are not accounted for in the classical limit. In particular processes of the type $\bar{\mathrm{f}}\mathrm{f} \to 2\gamma$ and $\bar{\mathrm{f}}\mathrm{f} \to \mathrm{e}^+\mathrm{e}^-$ would result in current damping according to the equation

$$\partial J = 2e^2 KE - \alpha^2 J^2 . \tag{8.20}$$

Indeed, if $j = J/S$ is the current density in the superconductor, particle elimination from the current is governed by the equation $\partial_t j = -\sigma j^2$, where $\sigma$ is the scattering or annihilation cross-section. By order of magnitude it is $\sigma \simeq \alpha^2/m^2$, where $m$ is the characteristic mass scale of the theory and $\alpha$ is the gauge coupling constant. The string cross-section is $S \simeq m^{-2}$. Hence the small quantity $m^{-2}$ does not enter eq. (8.20). In the absence of an external field the current falls as $J \sim (\alpha^2 t)^{-1}$ and if $E \neq 0$ the equation has the stationary point $J = (2e^2 KE/\alpha^2)^{1/2}$. Thus cosmologically interesting currents do not arise in these strings. A more detailed analysis is given by Barr and Matheson [1987], who discovered this phenomenon.

Monopoles may appear if a continuous nonabelian group is spontaneously broken. We start from the following simple example. Let us consider the group SO(3) broken down to SO(2) ~ U(1) by the isotopic vector scalar field $\phi^a$ $(a = 1, 2, 3)$. The Lagrangian of the field $\phi$ has the form (8.6) with the substitution $|\phi|^2 \to \phi^2 = \phi_a \phi^a$. When the symmetry breaks down, two of the three gauge bosons become massive and one remains massless. In the vacuum state $\phi_a \phi^a = \eta^2$. So the manifold of vacuum values of $\phi_a$ is a two-dimensional sphere $S^2$. We assume as usual that energy density goes down at large distances and so

$$\phi^a(r \to \infty, \theta, \Phi) \to \eta(r^a/r) . \tag{8.21}$$

This ansatz determines the mapping of $S^2$ in the physical space on $M$. It is easy to see that such a mapping cannot be continuously transformed into the trivial one and hence the corresponding configuration of $\phi$ is stable. In topological language this means that the group $\pi_2(M)$ is nontrivial. Indeed,

$$\pi_2(\mathrm{SO}(3)/\mathrm{SO}(2)) = \pi_2(\mathrm{S}^2) = \mathrm{Z} .$$

There exist infinitely many mappings, which are characterized by the integer $n$, i.e., the number of windings of the sphere in the physical space on the sphere in the space of $\phi_a$. Depending upon the direction of the winding one can speak of monopoles or antimonopoles.

When a group $G$ is broken down to a group $H$ monopoles arise only if $\pi_2(G/H) \neq 0$. For any Lie group $\pi_2$ is trivial,

$$\pi_2(G) = 0 \tag{8.22}$$

(see, e.g., Dubrovin et al. [1979]). This makes it easy to use the sequence (8.12) to find $\pi_2(G/H)$. In particular if a simply connected group $G$ $[\pi_1(G) = 0]$ is broken down to $H = H' \times \mathrm{U}(1)$, monopoles necessarily appear. Indeed,

$$\pi_1(H' \times \mathrm{U}(1)) = \pi_1(H') + \pi_1(\mathrm{U}(1)) = \pi_1(H') + \mathrm{Z} .$$

The unbroken symmetry in particle physics is just of this form, $H = \mathrm{SU}(3)_c \times \mathrm{U}(1)_{em}$, so the status of monopoles in Grand Unification models is much firmer than that of strings or domain walls. If, however, the original symmetry group is $G = \mathrm{SU}(3)_c \times \mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$ and no Grand Unification exists, monopoles do not arise. Indeed, it can be shown that when $\mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$ is broken to $\mathrm{U}(1)_{em}$ the factor group $G/H$ is topologically equivalent to SU(2) and $\pi_2(G/H) = 0$. Monopoles disappear because of the mixing of the Z-boson and the photon.

Let us turn now to configuration (8.21). As we know it is impossible to construct a finite energy configuration in the three-dimensional space using only scalar fields [Derrick 1964]. Indeed, the contribution to the energy from the angular derivatives of the field linearly diverges as a function of the upper limit of integration,

$$E \sim \int^{R} \mathrm{d}^3 r \left(\frac{1}{r^2}\,\frac{\partial\phi}{\partial\theta}\right)^2 \sim R\,. \tag{8.23}$$

Such a field configuration is called a global monopole. They can arise during phase transitions in the early Universe. The energy of a monopole–antimonopole pair is finite and proportional to the distance between them. This would result in their fast annihilation if the system is not somehow stabilized, e.g. by rotation.

In gauge theories single monopoles with finite energy can exist. Such solutions of the equations of motion have been found by 't Hooft [1974] and Polyakov [1974]. They possess a nonzero magnetic charge, that is, they are the famous magnetic monopoles which have been searched for during several centuries. It is noteworthy that originally at the Lagrangian level there is no magnetic charge in the model in contrast to the Dirac approach. We start from the Lagrangian formalism of gauge theory. It is known that the standard Lagrangian theory does not let us introduce monopoles as fundamental objects. As we will see below, in nonabelian gauge field theory there exist stable configurations of scalar and vector fields which at large distances from the center coincide with the field of a magnetic monopole. In contrast to the elementary magnetic monopoles these configurations are not singular when $r \to \infty$.

We have already seen that the divergence of the energy of a topologically nontrivial configuration of a scalar field can be compensated by the appropriate vector field ensuring a sufficiently fast decrease of the covariant derivative $D_\mu \phi = (\partial_\mu + \mathrm{i}gA_\mu)\phi$. According to that and to the asymptotic condition (8.21), we look for a solution of the equations of motion in the form

$$\phi^a = \frac{r^a}{r}\,\eta f(r)\,, \qquad A_i^a = \frac{r^j}{gr^2}\,\varepsilon_{aij} F(r)\,, \qquad A_t^a = 0\,, \tag{8.24}$$

with the boundary conditions $F(\infty) = f(\infty) = 1$ and $F(0) = f(0) = 0$. Here $\phi^a$ and $A_j^a$ are isotopic vectors with respect to the group O(3), $a = 1,2,3$ is the group index and $i$, $j = 1,2,3$ are the space indices. The characteristic scales which determine the approach to the asymptotic expressions (8.24) are the masses of the scalar and vector fields, respectively. The existence of such a solution can be justified by the variational principle.

The monopole size, which is of the order of the inverse mass of bosons, is large in comparison with the Compton wave length of the monopole, $\lambda = m_{\mathrm{M}}^{-1}$. This reflects the fact that the monopole is a classical, not quantum field configuration. The mass of the monopole can be evaluated by the energy of its magnetic field,

$$m_{\mathrm{M}} \sim \int_{m_A^{-1}}^{\infty} g^{-2} r^{-4}\,\mathrm{d}^3 r \sim m_A g^{-2} = \eta g^{-1}\,. \tag{8.25}$$

The next type of topologically nontrivial field structure is the so called texture, which may arise if the third homotopy group $\pi_3$ of the vacuum manifold $M$ is nontrivial. This is always the case if the symmetry group $G = SU(N)$, $SO(N)$, $Sp(N)$ is broken down completely so that $H$ and correspondingly $\pi_3(H)$ and $\pi_2(H)$ are trivial. It follows from the exact sequence (8.12) that $\pi_3(M) \sim \pi_3(G)$ while for the above-mentioned groups $\pi_3(G) = Z$ + a finite group (see, e.g., Fomenko and Fuchs [1989], p. 228).

A very well known example of textures comes from nonabelian gauge theories. Indeed, the purely gauge potential

$$A_\mu(x) = -(\mathrm{i}/g) u_q(x)\, \partial_\mu u_q^{-1}(x)\,, \quad u_q(x) = \exp\{\mathrm{i}\pi q[1 + \boldsymbol{x}\cdot\boldsymbol{\tau} f(x)/\sqrt{x^2+\rho^2}]\}\,, \tag{8.26}$$

realizes a mapping of $S^3$ on the group $G$. Here $\boldsymbol{\tau}$ are the Pauli matrices, $q = 0, 1, 2, \ldots$, and $f(x) \to 1$ when $x \to \infty$. These field configurations have, however, zero energy and correspond to different vacuum states. The factor $q$ is called the topological charge of the vacuum and in some models it can be interpreted as a baryonic charge. Vacuum states corresponding to different $q$ cannot be continuously transformed into each other by any gauge transformation. This means in particular that they are separated in field space by some potential barriers. These barriers are finite and penetrable. The probability of barrier penetration was found by Belavin et al. [1975]. The trajectory in field space in imaginary time determining the penetration is called an instanton and the action on an instanton is

$$S_\mathrm{i} = 8\pi^2/g^2\,. \tag{8.27}$$

The vanishing of the energy of textures is a common feature of theories with locally realized symmetry even in the presence of Higgs fields (see, e.g., Turok [1989]). It can be seen on the following simple example of a texture formed by the four-component scalar field $\phi^a$ ($a = 1, 2, 3, 4$):

$$\phi^a = \eta(\sin\chi \sin\theta \cos\Phi, \sin\chi \sin\theta \sin\Phi, \sin\chi\cos\theta, \cos\chi)\,. \tag{8.28}$$

Here $\theta$ and $\Phi$ are the usual polar angles and $\chi(r, t)$ is a function which describes the shape of the texture, $\chi(0, t) = 0$, $\chi(r \to \infty, t) \to \pi$. One sees that $\phi^2 = \phi_a\phi^a = \eta^2 = \text{const}$. Therefore the potential energy of $\phi$, $U(\phi) = \lambda(\phi^2 - \eta^2)^2$, vanishes and the only contribution to the energy density $\rho$ of the texture can come from the kinetic part. In the case of a locally realized symmetry the contribution to $\rho$ from the derivative term $\partial\phi$ can be compensated by a pure gauge field and so $\rho$ vanishes identically. Thus textures with nonzero energy can exist only if a global nonabelian symmetry is spontaneously broken. Such objects are, however, unstable and the knot in $\phi$-space is rewound in the course of their evolution. In the example considered here, eq. (8.28), the field evolves so that $\chi \to \pi$ over all space except for a small region near $r = 0$ [Turok 1989]. When the size of the core becomes smaller than $m_\chi^{-1}$, $\chi$ can jump over the potential barrier from 0 to $\pi$. So the texture disappears.

There are two physical phenomena which make topological defects interesting for baryogenesis: The catalysis of baryon nonconservation by monopoles and especially by cosmic strings and catastrophic processes during their evolution giving rise to the production of heavy baryon nonconserving particles. A detailed discussion of the monopole catalysis of proton decay would lead us too far, so in what follows we present only the basic points of the theory. A good discussion of the problem can be found in the review by Matveev et al. [1988].

In the standard $SU(3) \times SU(2) \times U(1)$ theory baryonic current is conserved at the classical Lagrangian level,

$$\partial_\mu j^\mu_{\mathrm{B(cl)}} = 0\,. \tag{8.29}$$

There might be some nonconservation at the Grand Unification scale but at the moment let us forget about that. (It is impossible to forget about that completely because there are no monopoles in the standard model and they appear only in the framework of some unification scheme.)

The conservation law (8.29) is valid only classically. Quantum corrections are known to destroy it. This is the celebrated chiral anomaly [Adler 1969; Bell and Jackiw 1969; Bardeen 1969]: Quantum corrections do not respect the symmetry of the classical system under $U(1)_B$ transformations. Taking them into account results in a nonvanishing r.h.s. of the divergence equation,

$$\partial_\mu j^\mu_{\mathrm{B}} = (g^2\gamma/16\pi^2)G\tilde{G}\ , \tag{8.30}$$

where $g$ is the gauge coupling constant, $\gamma$ is a numerical coefficient depending upon the type and number of virtual particles contributing to the famous triangle Feynman diagram, and $G_{\mu\nu}$ is the gauge field strength,

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu]\ . \tag{8.31}$$

Nonconservation of the baryonic current leads of course to nonconservation of the baryonic charge and in particular to proton decay and to generation of baryon excess in the Universe. However, the way from nonconservation of the anomalous current to observational consequences is not as simple as it may look. The point is that the r.h.s. of eq. (8.29) is a total derivative, $G\tilde{G} = \partial_\mu K^\mu$, where

$$K^\mu = 2\varepsilon^{\mu\nu\alpha\beta}(A_\nu\, \partial_\alpha A_\beta + \tfrac{2}{3}\mathrm{i}gA_\nu A_\alpha A_\beta)\ , \tag{8.32}$$

and so all its matrix elements vanish in perturbation theory.

Using expression (8.26) one can check that $K_0$ is the density of the vacuum topological charge. Thus a change in baryonic charge is accompanied, due to eq. (8.30), by an equal change in topological charge of the vacuum. The latter can proceed only by a quantum tunneling transition through the barrier separating vacua with different topological charge. The amplitude of this transition in the quasiclassical approximation is proportional to the exponentially small factor

$$\exp(-S_{\mathrm{i}}) = \exp(-8\pi^2/g^2) \tag{8.33}$$

[see eq. (8.27)]. As a result baryon nonconservation is strongly suppressed. In particular for electroweak interactions the exponential factor is $\exp(-2\pi \sin^2\theta_{\mathrm{W}}/\alpha_{\mathrm{em}}) \approx 10^{-80}$.

The possible nonconservation of baryonic charge due to the quantum anomaly was first noted by 't Hooft [1976a, b], who showed that in electroweak interactions baryonic charge can change by two units. The size of the effect is fantastically small but nevertheless the result is very important: Nonconservation of baryonic charge is not originally introduced into the theory but the dynamics necessarily leads to it. Moreover, as we will see in what follows, exponential suppression can be absent in some cases and the effect can be considerable.

Strong nonconservation of baryonic charge can be induced by the magnetic monopole [Rubakov 1981, 1982; Callan 1982]. The cross-section of the reaction $\mathrm{p} + \mathrm{M} \to \mathrm{M} + \mathrm{X}$ is of the order of $m_{\mathrm{p}}^{-2}v^{-1}$, where X denotes the products of proton decay, $m_{\mathrm{p}}$ is the proton mass, and $v$ is the relative velocity of the proton and monopole. Qualitatively this phenomenon can be understood as follows. Let us consider S-wave scattering of a massless charged fermion on the monopole. Due to the interaction of the magnetic moment of the fermion with the strong magnetic field of the monopole the moment is aligned along the field. Thus the helicity of the incoming particle must have a definite value and that of the outgoing one must have the opposite helicity. So the scattering proceeds with a helicity flip. This was observed by Kazama et al. [1977], who have found the solution of the Dirac equation in the monopole field. Particles with the proper value of the helicity are attracted by the singular magnetic field of the monopole and the cross-section of the process is large, $\sigma \sim E^{-2}$. The field of the 't Hooft–Polyakov monopole is not singular at $r = 0$ but is smoothed down at $r \approx m_X^{-1}$. This means that the fermion is attracted down to a distance of this order. Here $m_X$ is the mass of the gauge boson of the Grand Unification group. The strong attraction occurs for the S-wave only, for higher waves there is no helicity flip. The seeming contradiction with the law of chirality conservation is resolved by the chiral anomaly,

$$\partial_\mu j_5^\mu = \text{const.}\, \boldsymbol{E} \cdot \boldsymbol{H} \,,$$

where the right-hand side is different from zero because of the overlap of the magnetic field of the monopole and the electric field of the incoming fermion. If the chiral anomaly were not known, consideration of the scattering on the monopole would show that the axial current is not conserved.

In this example we have omitted for simplicity some essential details. One has to fix the gauge group in which monopoles appear and the representation to which the scattered fermions belong. Apart from that, care should be taken about the absence of the anomaly in the physical currents. These conditions specify which charges are not conserved in the scattering on the monopole.

The same physical picture is true for proton–monopole scattering. Since in Grand Unification models, in which monopoles exist, baryonic current is anomalous one should expect baryonic charge nonconservation in the process of scattering while color and electric charge are conserved. For example, on the quark level the process

$$\mathrm{u_R} + \mathrm{M} \to \bar{\mathrm{u}}_\mathrm{L} + \bar{\mathrm{d}}_\mathrm{L} + \mathrm{e}_\mathrm{L}^+ + \mathrm{M} \tag{8.34}$$

is permitted and its cross-section is of the order of $\sigma \sim E^{-2}$, where $E$ is the quark energy. This greatly exceeds the naive estimation of the cross-section of $B$-nonconserving reactions on the monopole. Generally speaking one should expect $B$-nonconservation in reactions with monopoles of Grand Unification even in the framework of perturbation theory. In the monopole core not only a magnetic field exists but also fields of heavy gauge or Higgs bosons of Grand Unification which do not conserve $B$ in reactions with quarks and leptons. So, if a scattered particle penetrates the core it could induce a $B$-nonconserving process. It is evident, however, that the cross-section of such a process should be inversely proportional to $m_{\mathrm{GUT}}^2$.

Baryon nonconservation catalyzed by magnetic monopoles might be essential for cosmology because this could erase a pre-existing baryon asymmetry. Moreover, monopole–antimonopole annihilation into heavy gauge or Higgs bosons of a Grand Unification model might copiously produce them far from thermal equilibrium. The decays of those bosons could create the baryon asymmetry of the Universe [Nussinov 1982]. However, the number density of monopoles in the Universe should be very small, otherwise they would overclose the Universe. For the Grand Unification monopoles with mass $m_\mathrm{M} \simeq 10^{16}$ GeV the number density should be about 25 orders of magnitude smaller than that of photons. With such a small number density, their influence on the baryon asymmetry would be unnoticeable [Ellis et al. 1982b; Nussinov 1982]. To overcome this difficulty Nussinov proposed that monopole annihilation is greatly enhanced in comparison with the standard approach so that their number density in the early Universe might be large without contradiction with contemporary bounds. Several mechanisms for reducing the monopole number density were discussed in the literature (for references see the paper by Nussinov [1982]) prior to the proposal of the inflationary Universe model.

An analogous model of baryogenesis from monopole–antimonopole annihilation has been considered recently by Dixit and Sher [1992]. They assumed that the gauge invariance of electrodynamics was broken by the mechanism of Langacker and Pi [1980] below the electroweak phase transition. The gauge invariance was restored at some smaller temperature but when it was broken the Universe was superconducting and the monopoles rapidly annihilated. As a result the monopole number density was reduced to an acceptable level. Heavy particles produced by monopole annihilation could generate a baryon asymmetry. Unfortunately the magnitude of $\beta$ in this scenario is a few orders of magnitude below the observed value. Moreover, taking monopole catalysis of baryonic charge nonconservation into account would further reduce the effect. An interesting feature of the scenario is an early matter domination stage in the Universe. This can influence in particular the electroweak phase transition.

In the standard model cosmic strings should be more effective than monopoles since their number density in the early Universe might be high enough. Unlike monopoles, which are rather reluctant to annihilate [Zel'dovich and Khlopov 1978; Preskil 1979], cosmic strings would disappear at later stages by formation of closed loops in the process of their intersection. Later on the loops would collapse emitting gravitational radiation (for the details see Vilenkin [1985]). Because of that the formation of cosmic strings after the inflationary stage is permitted and so their number density, though within the observational limits, might be noticeable. On the other hand, the monopoles must be diluted by inflation and so their number density is exponentially suppressed. One would expect that the cross-section of $B$-nonconserving processes on a cosmic string should be of the order of $m_{\mathrm{GUT}}^{-2}$ [Brandenberger et al. 1988, 1989], since there is no long-range force created by the string and the size of the region where $B$-nonconserving fields are excited is of the order of $m_{\mathrm{GUT}}^{-1}$. This is not the case, however, if the fermionic charges in the model are noninteger multiples of the Higgs field charge [Alford and Wilczek 1989; Alford et al. 1989; Perivolaropoulos et al. 1990]. If this is the case, the amplitude of fermion scattering on the string due to the Aharonov–Bohm effect [1959] becomes nonvanishing.

In short this effect consists of the following. The phase of the wave function of a charged particle moving in an electromagnetic field from point $x_1$ to point $x_2$ changes by

$$\Delta\phi = e\int_{x_1}^{x_2} A_\mu \, \mathrm{d}x^\mu \,, \tag{8.35}$$

where $A_\mu$ is the electromagnetic vector potential. This is a consequence of the gauge invariance of the theory. For a closed contour the change of the phase is expressed through the magnetic field flux inside this contour,

$$\Delta\phi = e\oint A_\mu \, \mathrm{d}x^\mu = e\int \mathrm{d}\boldsymbol{s}\cdot\boldsymbol{H} \equiv e\Phi \,. \tag{8.36}$$

This results in an observable change of the phase of the wave function even if the particle trajectory goes in the region where the magnetic field is absent.

Local cosmic strings are known to be very thin solenoids with magnetic field concentrated at a distance of the order of $m^{-1}$, where $m$ is the scale of the symmetry breaking. The flux along the string is quantized, $\Phi_n = 2\pi n/e$, where $e$ is the charge of the Higgs field. Hence for fermions with charges which are integer multiples of $e$ the Aharonov–Bohm effect is unobservable. For an arbitrary value of the fermion charge $e_f$ this mechanism gives rise to fermion scattering with the following cross-section per unit of string length [Alford and Wilczek 1989]:

$$\frac{d\sigma}{d\theta\, dl} = \frac{\sin^2\pi(e_F/e)}{2\pi k \sin^2(\theta/2)} . \tag{8.37}$$

Here $k$ is the fermion momentum.

If gauge fields in the string core are nonabelian, inelastic fermion scattering becomes also possible [Perivolaropoulos et al. 1990]. The eigenvectors of the Aharonov–Bohm scattering operator do not generally coincide with the eigenstates of the free Hamiltonian and this evidently results in inelastic scattering. If bosonic fields in the string core do not conserve baryonic charge the string would catalyze baryon nonconserving scattering. The cross-section of this process is of the same order of magnitude as (8.37). Note the difference with nonconservation of baryons in electroweak interactions, where it is absent in the original Lagrangian and arises only because of the quantum anomaly.

We can evaluate the rate $\Gamma_s$ of the baryon nonconserving processes induced by strings in the following way:

$$\Gamma_s = \frac{d\sigma}{de} l N_s , \tag{8.38}$$

where $l$ is the average string size, $N_s$ is the string number density, and $d\sigma/dl$ is the $B$-nonconserving cross-section per unit of string length. In accordance with eq. (8.37) $d\sigma/dl \approx 1/T$, where $T$ is the scattered particle temperature. It is reasonable to assume that the string size and the distance between strings are approximately the same at the moment of their formation, so $lN_s \approx l^{-2}$. The value of $l$ is determined by the correlation length during the phase transition which produced strings. The correlation length $\xi$ is equal to the inverse effective mass of the Higgs field. Formally at the moment of the phase transition it vanishes as

$$m_{\rm eff}^2(T) = \lambda(T_c^2 - T^2) . \tag{8.39}$$

However, the phase transition does not stop abruptly at $T = T_c$. The formation and destruction of the new phase go on down to a smaller temperature $T_G$, the so called Ginzburg temperature. The latter can be determined as follows. In thermal equilibrium the probability of a certain field configuration is determined by the free energy $F = -T \ln Z$. The free energy of the fluctuation corresponding to the transition from one phase to another in a region of size $\xi = m_{\rm eff}^{-1}$ is equal to

$$\Delta F \approx \xi^3 m^4(T)/\lambda = m(T)/\lambda \tag{8.40}$$

(here $m/\lambda$ is the height of the potential barrier). Such fluctuations effectively disappear when $\Delta F > T$. So $T_G$ is determined by the condition $T_c^2 - T_G^2 = \lambda T_G^2$. Hence for $\lambda \ll 1$ we find

$$\xi_G \simeq (\lambda T_c)^{-1} . \tag{8.41}$$

Thus the crucial ratio of the reaction rate to the expansion rate of the universe is

$$\Gamma_s/H \approx \lambda^2(45/4\pi^3 K)^{1/2} m_{Pl}/T_c\,, \tag{8.42}$$

where $K = O(10^2)$ is the number of particle species in the primeval plasma [see eq. (3.3)].

Rather soon after string formation their distribution in the Universe approaches the so called scaling regime when the correlation length becomes of the order of $H^{-1}$ and their number density $N_s \sim H^3$. One can check that in this regime $(H/\Gamma_s)$ becomes smaller than that given by eq. (8.42).

Thus the influence of strings on the early created baryon asymmetry of the Universe crucially depends upon the values of the coupling constant $\lambda$ of the Higgs field self-interaction and the critical temperature. For large $\lambda$ and/or small $T_c$ strings could catalyze down to zero all pre-existing baryon asymmetry. Fortunately this could not happen if $T_{\Delta B} < T_c$.

Topological defects do not necessarily destroy the baryon asymmetry, they can also contribute to its creation. The scenario is more or less like the standard one with heavy particle decays. The only difference is that the source of those particles is not the quasi-equilibrium primeval plasma at high temperature but the production of the particles either by the final stage of collapse of string loops [Bhattacharjee et al. 1982; Brandenberger et al. 1991] or by singular points on the string moving with the speed of light, the so called cusps [Kawasaki and Maeda 1988]. Brandenberger et al. [1991] considered also baryogenesis by collapsing textures or global monopole annihilation but found them less effective than the collapse of string loops. The latter could give a sufficiently large value of $\beta$; moreover, the catalysis of $B$-nonconserving scattering on the strings is shown to be rather weak and unable to wash down the baryon asymmetry.

An interesting possibility not yet explored is the baryogenesis by superconducting cosmic strings. A new phenomenon may take place with them which does not exist for ordinary strings. If two superconducting strings come into contact they start to align due to the very strong magnetic interaction of their currents. This would result in a catastrophic gluing of the strings. If one of the strings forms a closed loop the other would wind along this loop forming a "thick" string. The electric field strength at this process might be high enough for heavy particle production and after that the baryogenesis scenario goes by the well known way. This catastrophic winding of superconducting strings may be a source of other explosive processes in the Universe.

Baryogenesis by the heavy particles produced in domain wall collisions was considered by Holdom [1983] in a field theoretical model with $SU(2)_L \otimes SU(2)_R$ symmetry and by Barrow et al. [1991a] in the extended inflation scenario. The latter is briefly discussed in section 3. It can be realized in the standard GUT framework, that means in particular high temperatures. However, with one or other trick discussed above the relevant temperature can be made much smaller.

In the model by Holdom [1983] baryonic charge nonconservation is completely anomalous but the exponential smallness of the amplitudes (8.33) is not very small because of a large value of the gauge coupling constant $g_R$ of the $SU(2)_R$ group at large temperatures. To suppress proton decay $g_R$ should be small at zero temperature. It implies a rather unusual type of renormalization of $g_R$ in a nonabelian gauge theory, but makes possible sufficient baryogenesis at low temperature.

## 9. Electroweak baryogenesis

This is at present the most widely discussed model of baryogenesis. The majority of papers on the subject are devoted to this particular model. A very nice feature of it is that the baryon asymmetry of the Universe can be generated at least in principle in the framework of the standard $SU(3) \times SU(2) \times$

U(1) theory without any crucial additional assumptions. It presents all three ingredients of the standard baryogenesis scenario: baryonic charge, $C$- and $CP$-nonconservation (this is a contribution from particle physics), and the possibility of deviation from thermal equilibrium in cosmological conditions. Unfortunately in the minimal Standard Model the value of the asymmetry proves to be too small, but very simple modifications of the latter like, e.g., introduction of additional Higgs fields can make it operative. Different versions of the scenario with respect to possible mechanisms of $C$- and/or $CP$-violation and thermal equilibrium breaking have been proposed but the basic point remains the same. This is baryonic charge nonconservation induced by the quantum chiral anomaly [Adler 1969; Bell and Jackiw 1969; Bardeen 1969] and a topologically nontrivial vacuum structure in nonabelian gauge theories [Belavin et al. 1975]. As is discussed in the previous section, these two properties of the theory give rise to baryonic charge nonconservation ['t Hooft 1976a, b] though the latter is conserved in the classical Lagrangian. As we have already mentioned the effect is exponentially suppressed [see eq. (8.33)] since it originates from the quantum tunneling transition between states with different topological charges (8.26) separated by a potential barrier.

We know from quantum mechanics that the amplitude of barrier penetration in the quasiclassical approximation is determined by the factor

$$\exp\left(-2\int_{x_1}^{x_2} \mathrm{d}x\,\sqrt{U(x)-E}\right), \tag{9.1}$$

where the integral is taken over the classically forbidden region where the potential energy $U(x)$ exceeds the total energy $E$. Evidently with increasing $E$ the suppression becomes weaker and for $E > U(x)$ the exponential suppression disappears. The particle can classically go over the barrier. If there are particles in the heat bath with temperature $T$ their energy distribution is given by the Boltzmann exponent, $\exp(-E/T)$, and for sufficiently high temperature the transition over the barrier might be more favorable than a quantum tunneling transition. These simple considerations make one think that at high temperatures baryonic charge possibly is strongly nonconserved, though the situation is not as simple as in quantum mechanics. The crucial difference is that a transition between states with different topological charges can be achieved only by particular topologically nontrivial field configurations like the textures discussed in section 8.

The first papers (to my knowledge) where this mechanism was mentioned in connection with baryogenesis are those by Linde [1977] and by Dimopoulos and Susskind [1978]. Baryonic charge nonconservation in cosmological conditions induced by a nontrivial topology of space–time with nonvanishing $R\tilde{R}$ was considered by Gibbons [1979] (here $R$ is the Riemann tensor and $\tilde{R}$ is the dual one). Note in this connection that there is not only a contribution to anomalous current nonconservation from vector gauge fields (eq. 8.30) but also from gravitons. The latter was found by Kimura [1969], Delbourgo and Salam [1972] and by Eguchi and Freund [1976], who corrected an error of a factor of two in the paper by Delbourgo and Salam. In the paper by Christ [1980] anomalous baryon number nonconservation at high energy was discussed and the author noted that there should exist a static solution of the classical equations of motion of the vector and Higgs fields which corresponds to the saddle point in the potential barrier separating vacua with different topological charges. This solution had already been known for some time [Dashen et al. 1974] but its relation to $B$-nonconserving transitions over the barrier was not understood. It was later rediscovered by Manton [1983] (see also Klinkhamer and Manton [1984]) and by Forgacs and Horvath [1984] and received the name sphaleron.

The topological charge of the sphaleron was found to be 1/2, so it is situated just between two vacua with $q = 0$ and $q = 1$. In the SU(2) model with a doublet Higgs field the sphaleron configuration has the form

$$A_k^{\mathrm{S}} = \mathrm{i}\, \frac{\varepsilon_{klm} x^l \tau^m}{r^2}\, f(\xi)\,, \qquad \phi^{\mathrm{S}} = \frac{\mathrm{i}\phi_0}{\sqrt{2}}\, \frac{\tau^l x^l}{r} \begin{pmatrix} 0 \\ 1 \end{pmatrix} h(\xi)\,. \tag{9.2}$$

Here $\xi = g\phi_0 r$, $g$ is the gauge coupling constant, $r = (\Sigma\, x_j^2)^{1/2}$, and the functions $f(\xi)$ and $h(\xi)$ vanish at $\xi = 0$ and tend to 1 when $\xi \to \infty$.

It is evident that the sphaleron energy is equal to the height of the barrier between vacua with different topological charges,

$$E_{\mathrm{S}} = (2M_{\mathrm{W}}/\alpha_{\mathrm{W}}) f(\lambda/g^2)\,, \tag{9.3}$$

where $M_{\mathrm{W}}$ is the mass of the gauge boson of the SU(2) group (W-boson), which is nonzero in the phase of broken symmetry, $\alpha_{\mathrm{W}} = g^2/4\pi$, and $\lambda$ is the quartic self-interaction coupling constant of the Higgs field. The function $f(y)$ monotonically changes between 1.56 and 2.72 when $y$ runs from zero to infinity. Note that in electroweak theory $E_{\mathrm{S}}$ is about 10 TeV. It roughly determines the scale at which baryonic charge might be strongly nonconserved.

These phenomena form the basis for the picture of the baryon number violation model proposed by Kuzmin et al. [1985] (see also the review paper by Matveev et al. [1988]). An essential assumption of the scenario is that the primeval plasma is in thermal equilibrium with a possible exception for $B$-nonconserving processes. If this is the case the probability of a field state with energy $E$ is given by the Boltzmann exponent $\exp(-E/T)$ up to a less essential pre-exponential factor. It is valid in particular for the sphaleron configuration which determines the probability of fluctuations changing the baryonic charge of the system. To evaluate the probability of $B$-nonconserving processes induced by the sphalerons in thermal equilibrium one should take into account the dependence of the W-boson mass on the temperature [Kirzhnits 1972] (see also the review by Linde [1979]) as well as that of $\alpha_{\mathrm{W}}$. The temperature dependence of the W-boson mass is induced by that of the Higgs field condensate and in the case of a second order phase transition is given by the expression

$$m_{\mathrm{W}}^2(T) = m_{0\mathrm{W}}^2(1 - T^2/T_{\mathrm{c}}^2)\,, \tag{9.4}$$

where $m_{0\mathrm{W}}$ is the zero temperature value of the W mass and $T_{\mathrm{c}} = \mathrm{O}(10^2\ \mathrm{GeV})$ is the critical temperature of the phase transition; $m_{\mathrm{W}}(T) = 0$ for $T > T_{\mathrm{c}}$ up to temperature corrections to the mass (like the plasma frequency in the case of electrodynamics).

The dependence of the coupling constant on $T$ comes from logarithmic renormalization and in fact is contained in the pre-exponential factor. The latter has been calculated by Arnold and McLerran [1987]. With all these corrections taken into account the probability of $B$-nonconserving thermal fluctuations is [Arnold and McLerran 1987]

$$\Gamma_{\Delta B} = \frac{T^4 \omega_-^{\mathrm{S}}}{M_{\mathrm{W}}(T)} \left(\frac{\alpha_{\mathrm{W}}}{4\pi}\right)^4 N_{\mathrm{tr}} N_{\mathrm{rot}} \left(\frac{2M_{\mathrm{W}}(T)}{\alpha_{\mathrm{W}} T}\right)^7 \exp\!\left(-\frac{E^{\mathrm{S}}(T)}{T}\right) \tilde{f}(\lambda/g^2)\,, \tag{9.5}$$

where $\omega_-^{\mathrm{S}} \simeq M_{\mathrm{W}}(T)$ is the value of the only one-sphaleron negative mode (it reflects the fact that the sphaleron is unstable and determines its lifetime); for $\lambda = g^2$, $N_{tr} = 26$ and $\tilde{N}_{rot} = 5.3 \times 10^3$; these factors come from integration over the translation and rotation zero modes; $\tilde{f} = O(1)$ is a slowly varying function of the ratio $\lambda / g^2$, and $E_S(T)$ is the sphaleron energy with the account of the temperature dependence of $M_W$.

A numerical estimate [Arnold and McLerran 1987] of the ratio of $\Gamma_{\Delta B}$ to the expansion rate of the Universe gives

$$\Gamma_{\Delta B}/H \approx 10^{24}[m_W(T)/T]^2 \exp[-120 c m_W(T)/T] , \tag{9.6}$$

where $c$ is a numerical constant of the order of one.

This result is valid in the temperature range $M_W(T) < T < E_S(T)$. At small temperature quantum tunneling transitions described by instantons are more effective, while at large temperature the saddle point approximation is evidently incorrect.

Expression (9.5) shows that $\Gamma_{\Delta B}/H \gg 1$ for $T$ above several hundred GeV and quickly drops practically to zero in that region. Hence a baryon asymmetry which could be generated at high temperature should be washed out by the electroweak interactions operating at $T \gg 100$ GeV and when $B$-nonconserving processes go out of equilibrium they do that so fast that no asymmetry can be generated. This is true, however, only if initially the difference of baryonic and leptonic charges of the Universe was zero, $B - L = 0$ (to be more exact, all three nonanomalous combinations of the quantum numbers $L_j - B/3$, where $L_j$ is the leptonic charge of the $j$th generation, should vanish, see, e.g., Kuzmin et al. [1987]). It is known that in the standard electroweak theory the $B - L$ current is not anomalous so $B - L$ is conserved. This opens the possibility to save baryons despite electroweak "mass murder". To this end one requires a high temperature baryo- (and lepto)genesis generating nonzero $B - L$, such as, e.g., is possible in the SO(10) Grand Unification model. A particular scenario of this kind has been proposed by Fukugita and Yanagida [1986], who considered a model with a heavy Majorana lepton which generated lepton asymmetry of the Universe at $T > T_W = O(1\ \text{TeV})$ (see section 10). After that electroweak interactions more or less equally share the lepton asymmetry generated by the Majorana fermion between leptons and baryons.

Another way out of the problem may be presented by scenarios of low temperature baryogenesis such that $T_{\Delta B} < T_W$. We will not dwell here on these possibilities, which are discussed in the previous sections (see also section 12, where the possibility is considered that the electroweak symmetry was never restored), but instead discuss the models of baryon asymmetry generation by the same electroweak processes that erased it initially. First of all one has to abandon the assumption of a second order electroweak phase transition since otherwise the departure from thermal equilibrium would be too small [see eq. (9.6)].*) Second, a mechanism for large $CP$-violation should be found. The standard model with Kobayashi–Maskawa $CP$-breaking phases gives a negligible amplitude due to the symmetry between different generations of quarks and leptons.

A first proposal to overcome these difficulties has been made by Shaposhnikov [1986, 1987, 1988], later in collaboration with Bochkarev and Khlebnikov [Bochkarev et al. 1990a]. A crucial assumption of this approach is the existence of a Chern–Simons condensate density

$$N_{CS} = \frac{g^2}{16\pi^2}\, \varepsilon^{ijk}\, \mathrm{Tr}(F_{ij}A_k + \tfrac{2}{3} i g A_i A_j A_k) \tag{9.7}$$

*) It has been argued recently by Brandenberger and Davis [1992] that in the presence of electroweak strings baryogenesis could proceed with a second order phase transition.

at high temperatures. As we have seen in section 8, this quantity coincides with the time component of the vector $K_\mu$ which enters the anomaly equations (8.30), (8.32). It determines the topology in gauge field space. For pure gauges (that is, for potentials such that $F_{\mu\nu} = 0$) it determines the topological charge of the vacuum. Creation of a condensate of $N_{CS}$ might be induced by instability of the high temperature nonabelian plasma with respect to formation of domains of "magnetic" field of strength $g^3T^2$ with characteristic size $(g^2T)^{-1}$. This hypothesis was advocated by Linde [1980] and by Kalashnikov et al. [1982] on the basis of the infrared instability of the high temperature theory with massless gauge bosons. If this is the case the density of the Chern–Simons number inside a domain would be

$$\langle N_{CS}^2 \rangle^{1/2} \approx \alpha_W^3 T^3 . \tag{9.8}$$

When at smaller temperatures the electroweak symmetry is broken and gauge bosons become massive the condensate is destroyed and due to the anomaly equation (8.30) is transformed into baryons.

Note that baryonic charge generation in this scenario proceeds at this stage not by crossing the barrier separating vacua with different topological charges but by "moving" along the barrier when the Chern–Simons number of the gauge fields is transformed into baryonic charge. Of course at an earlier stage processes with topological change are necessary, otherwise baryonic charge cannot be generated.

Thus one could expect a baryonic charge density inside a domain of the order of

$$N_B \sim \alpha_W^3 N_\gamma . \tag{9.9}$$

Of course, when averaged over a region large with respect to the domain size, the net baryonic charge, $\bar{N}_B$, should be zero if charge invariance is respected. The charge asymmetry effects in the standard model are generically too weak to produce a considerable value of $\bar{N}_B$ with possibly a single exception [Shaposhnikov 1986, 1987], when the effective potential as a function of the Chern–Simons number is exactly flat at the bottom up to some rather large value of $N_{CS}$. If this is the case then even a very small, $CP$-breaking term could induce a complete transition to a state with definite sign of $N_{CS}$.

In this case the baryon asymmetry does not depend on the amplitude of $CP$-violation and is given by the expression

$$\beta = N_B/N_\gamma = \alpha_W^3/S , \tag{9.10}$$

where $S$ is the entropy dilution factor coming from the subsequent evolution of the plasma.

At the moment this is the only known possibility to generate a considerable baryonic excess in the framework of the minimal standard model.

In this case the theory rather accurately fixes the mass of the Higgs meson because it should be ensured that the entropy dilution factor is not too large and the $B$-nonconserving fluctuations after the phase transition do not erase the obtained asymmetry. If the coupling constant $\lambda$ of the Higgs field self-interaction is large, $\lambda \geq g_W^2$, the phase transition is of second order (see, e.g., Linde [1979]), the deviations from thermal equilibrium are tiny and the baryon asymmetry is negligible. If $\lambda$ is small the phase transition is of first order with strong supercooling and with subsequent large entropy production. Since the Higgs boson mass, $m_H$, receives a positive contribution from $\lambda\langle\phi^2\rangle$ (where $\langle\phi\rangle$ is the amplitude of the Higgs field condensate), the bounds on $\lambda$ can be translated into bounds on $m_H$. For a successful implementation of the scenario one should have $m_H \approx 45$ GeV [Shaposhnikov 1986, 1987; Bochkarev and Shaposhnikov 1987], which is on (or maybe now already beyond) the verge of contradiction with experiment. Recently Shaposhnikov [1992] claimed that the higher order loop corrections to the temperature dependent terms in the Higgs boson potential made the bound on $m_H$ less restrictive, raising it up to 65 GeV, but this conclusion was criticized by Dine et al. [1992a, b]. Moreover, the existence of the Chern–Simons condensate is not proven so it is worthwhile to look for models of baryogenesis without the latter. This can be done at the expense of an extension of the standard model. A straightforward possibility is a nonminimal Higgs sector with more than one doublet.

Bounds on $m_H$ in the two-doublet model have been derived by Bochkarev et al. [1990b, 1991] and by Turok and Zadrozny [1992]. They turned out to be approximately a factor of two less restrictive than those in the minimal model. In the two-doublet model it is possible to have $m_H = 80$ GeV and a first order phase transition. The two-doublet model is also much less restrictive with respect to *CP*-violation and permits a *CP*-breaking amplitude large enough to generate baryon asymmetry without the Chern–Simons condensate [Turok and Zadrozny 1990, 1991; McLerran et al. 1991].

Turok and Zadrozny [1990, 1991] proposed a scenario of baryogenesis based on the effective action induced by the fermionic triangle diagram with one Higgs field and two gauge field legs,

$$S_0 = -\int \frac{g_W^2 N_f}{24\pi^2}\, \theta(x) F\tilde{F}\, d^4x\,, \tag{9.11}$$

where $N_f$ is the number of quark generations and $\theta(x)$ is the relative phase of the two Higgs fields. Taking into account that $F\tilde{F}$ is a total derivative [see eq. (8.32)] of the vector $K^\mu$, which is conserved together with the baryonic current (8.30), one can rewrite this action in the form (7.3). In other words, there is a linear term in the effective potential of the Chern–Simons number proportional to $\dot{\theta} N_{CS}$. Hence to generate baryon asymmetry the dynamics of the model should ensure dominance of a particular sign of $\dot{\theta}$. An interaction potential of the two Higgs fields satisfying this demand can easily be constructed. Let the value of $\theta$ minimizing the potential energy depend upon the amplitudes of the Higgs fields so that the value of $\theta$ in the vacuum of the broken symmetry phase, $\theta_0$, is different from $\theta_1$ which minimizes the height of the barrier between symmetric and nonsymmetric phases. If the phase transition is of first order then there is a period of phase coexistence when bubbles of the broken symmetry phase expand inside the symmetric phase. In the process of wall propagation $\theta$ changes predominantly in one direction from $\theta_1$ to $\theta_0$ and correspondingly the sign of $\dot{\theta}$ is fixed. So, if there is a nonvanishing density of the Chern–Simons number in the high temperature phase then after collision with the wall it would be changed asymmetrically thus inducing an asymmetric production of baryons.

High temperature effects lead to modification and suppression of the action (9.11) [McLerran et al. 1991]. In the particular case of a homogeneous Higgs field and the gauge condition $A_0 = 0$ it has the form

$$S_T = -\mathrm{i}\tfrac{7}{4}\zeta(3)\left(\frac{m_t}{\pi T}\right)^2 \frac{2}{v_1^2} \int dt\, (\phi_1^+\dot{\phi}_1 - \dot{\phi}_1^+\phi_1)\tilde{N}_{CS} \equiv O\tilde{N}_{CS}\,, \tag{9.12}$$

where $\zeta$ is the Rieman $\zeta$-function, $v_1$ is the vacuum expectation value of the Higgs field $\phi_1$, $m_t$ is the mass of the t-quark and $\tilde{N}_{CS}$ is the part of the Chern–Simons number (9.7) that is quadratic in the gauge field. The contributions of lighter quarks are proportional to their mass squared and small. This term exists also in the minimal electroweak theory but without the second Higgs field one cannot obtain a systematic sign of $\dot{\phi}_1$ producing effective *CP*-violation.

The action (9.12) vanishes in the symmetric phase since there $m_t = 0$. It vanishes also in a broken symmetry phase because gauge bosons are massive there and the Chern–Simons number of excitations should be zero at the minimum of the potential. The rate of $B$-nonconserving sphaleron transitions on the walls depends upon the direction in which $\tilde{N}_{CS}$ changes and for that reason baryon asymmetry is generated. In this scenario baryons are produced on the domain walls separating two phases when they propagate in the primeval plasma. In the broken symmetry phase, where gauge bosons are massive, both instanton and sphaleron transitions are negligible while in the symmetric phase the large rate of $B$-nonconserving processes keeps $B$ and $L$ equal to zero if initially $B - L = 0$ (see the next section). Breaking of equilibrium together with charge symmetry violation induced by a particular sign of $\dot{\phi}$ are effective only on the walls.

The rate of sphaleron transitions per unit time and unit volume at high temperatures can be estimated as [Arnold and McLerran 1987; Khlebnikov and Shaposhnikov 1988]

$$\Gamma_S \approx (\alpha_W T)^4 . \tag{9.13}$$

This result can be intuitively understood as follows. The characteristic sphaleron size is of the order of $(\alpha_W T)^{-1}$. (Note that it is large in comparison with the coherence length $\xi = T^{-1}$ in high temperature plasma.) This means that the number density of sphalerons in "sphaleron saturated" plasma is $(\alpha_W T)^3$. The rate of sphaleron decay is $\alpha_W T$. Combining these two factors we get the above estimate. There is a subtle point in these arguments connected with the sphaleron number density. One would expect that it is not easy to form a coherent field state on a scale much larger than the inverse temperature. Numerical simulations by Ambjørn et al. [1990, 1991], however, support the above presented estimate. It would be interesting in this connection to find the dependence of the sphaleron production rate on the lattice size at fixed temperature because the presumed suppression of classical field formation might be connected with the infinite number of degrees of freedom. If the result (9.13) is correct then the baryon asymmetry produced by the sphaleron transitions on the part of the domain walls where the sphaleron energy is small enough can be estimated as follows [McLerran et al. 1991]. The rate of baryonic charge production per unit time and per surface area of the bubble wall is given by

$$\mathrm{d}B/\mathrm{d}t\,\mathrm{d}\sigma \approx (\alpha_W T)^4 l \langle O \rangle , \tag{9.14}$$

where $l$ is the effective thickness of the domain wall and $\langle O \rangle$ is the average value inside the wall of the operator $O$ defined by eq. (9.12). Since the bubbles of broken symmetry phase ultimately occupy the whole volume $V$ of the Universe the factor $\mathrm{d}t\,\mathrm{d}\sigma$ can be replaced by $\mathrm{d}V$. The effective thickness $l$ of the wall separating symmetric and nonsymmetric phases is of the order of $1/T$. Thus for the ratio of the baryonic charge density to that of the entropy $S = (2\pi^2/45)K_{\mathrm{eff}}T^3$ we obtain

$$\beta = N_B/S \approx \alpha_W^4 \langle O \rangle / K_{\mathrm{eff}} , \tag{9.15}$$

where $K_{\mathrm{eff}} \approx 10^2$ at the temperature of the electroweak phase transition. The average value of $O$ inside the domain wall was evaluated by McLerran et al. [1991] as $10^{-2}$–$10^{-3}$. This gives for $\beta$ a result very close to the observed one.

Another mechanism proposed by Ambjørn et al. [1987, 1989] and by Turok and Zadrozny [1991, 1992] can also be operative. It is argued that there exist field fluctuations with nonzero Chern–Simons number in the high temperature phase. In the broken symmetry phase they are destroyed by the nonvanishing mass of the gauge bosons. These fluctuations are in fact gauge field textures discussed in the previous section. Their number density can be naively estimated in the same way as the number density of sphalerons [see discussion following eq. (9.13)]. Numerical simulations by Ambjørn et al. [1987, 1989] confirm those estimates and give

$$N_{\mathrm{Q}} \approx 3(\alpha_{\mathrm{W}} T)^3 . \tag{9.16}$$

Another kind of estimate applicable to $N_{\mathrm{Q}}$ was made by Turok [1989] for the global texture case (see section 8). It gives numerically the same result as eq. (9.16). The interaction of those gauge textures with the domain wall is $P$- (and $CP$-)asymmetric due to the action (9.12). Hence asymmetry in the Chern–Simons number can be generated. The model by Turok and Zadrozny [1991, 1992] gives essentially the same result for $\beta$ as given by eq. (9.15).

The validity of this scenario crucially depends upon the efficiency of formation of winding field configurations at high temperatures. A considerable amount of work in this direction is necessary for a better understanding of the phenomenon.

Another possible realization of sphaleron induced baryogenesis was considered by McLerran [1989]. Large $CP$-violation in that model comes from the well known QCD $\theta$-term, $\theta F\tilde{F}$, which is effective at high temperatures only. $CP$-violation in low energy physics is assumed to be erased by an invisible axion. This scenario of high temperature breaking of charge symmetry was considered by Yoshimura [1983] and consists of the following. After the Peccei–Quinn symmetry is broken the axion potential remains practically flat down to temperatures of the order of 1 GeV. The value of the phase $\theta$ during this stage is most probably of the order of unity. At $T \leq 1$ GeV the axion mass becomes effectively nonzero and the field relaxes to the equilibrium value $\theta = 0$ which corresponds to $CP$-conservation.

$CP$-odd effects induced by the $\theta$-term are known to be observable because of the instanton transitions between vacua with different topological charge. The arguments are essentially the same as in the case of $B$-nonconserving processes discussed in section 8. However, at high temperatures strong interaction instantons are suppressed (see, e.g., Gross et al. [1981]). This supression is mostly due to the asymptotic freedom of QCD and the corresponding decrease of the QCD coupling constant $g_{\mathrm{S}}(T)$ at high temperature. Subsequently the instanton action $S_{\mathrm{i}} = 8\pi^2/g^2(T)$ becomes larger. This mechanism also supresses the axion mass, which might be nonzero already at the electroweak scale when quarks become massive. Thus it seems that the same mechanism which keeps a large $CP$-violating amplitude suppresses its possible physical manifestations. At high temperatures, however, transitions over the barrier become more favorable than quantum mechanical barrier penetration. The former are governed by the strong interaction sphalerons [Cornwall and Tiktopoulos 1986] permitting sufficiently large $CP$-violation [McLerran 1989].

Baryon asymmetry generation in this model proceeds by the interference of the $CP$-even process induced by an electroweak $B$-nonconserving sphaleron with the $CP$-odd one arising due to final state interaction with a strong helicity nonconserving sphaleron. Helicity nonconservation comes from the same triangle quantum anomaly as $B$-nonconservation with substitution of the chiral current $j_\mu^{\mathrm{L}} - j_\mu^{\mathrm{R}}$ for the baryonic one. The helicity flip amplitude, however, does not interfere with the nonflip amplitude and to cure that another helicity flip is necessary. This can be achieved by radiation of scalar particles from quarks in the final state. These new scalars should have a large mass $m_{\mathrm{H}} > m_{\mathrm{W}}$ to avoid a flavor-changing neutral current and relatively strong Yukawa couplings to quarks to create the observed baryon asymmetry. This makes the model interesting for experiments on the next generation of accelerators.

The above mentioned suppression of strong $CP$-violation at the electroweak scale could be avoided if the electroweak phase transition was very much delayed and took place near the QCD phase transition at $T = O(100\,\mathrm{MeV})$. Such a possibility was recently advocated by Kuzmin et al. [1992]. It could be realized if the Higgs potential was very flat at the origin like, e.g., the Coleman–Weinberg potential, $U = \lambda\phi^4 \ln \phi^2/\sigma^2$. Unfortunately the dilution of the baryon asymmetry by the entropy released in the course of the phase transition, is acceptably low only for a very light Higgs boson, $m_H < 100\,\mathrm{keV}$, which is excluded by experiment. However, in extended versions of the model this restriction might be not valid. As a by-product the model predicts a reasonable amount of axionic dark matter in the Universe.

Another kind of standard model extension which permits a proper electroweak baryogenesis was considered by Cohen et al. [1990a, b]. They proposed that neutrinos are massive and $CP$-violation comes from their mixing angles. The known light neutrinos were assumed to receive their masses through couplings to new heavy neutrinos, which are weak interaction singlets. It was also assumed that leptonic charge conservation was spontaneously broken at the electroweak phase transition while above the phase transition $L$ was conserved of course up to anomalous effects. The same phase transition gives nonzero masses to neutrinos. An appropriate framework for realization of this scenario is the singlet Majoron model by Chikashige et al. [1981].

The charge asymmetry generation proceeds as follows. In the course of the electroweak phase transition (which is assumed to be of first order) bubbles of new phase are formed and expand in the old phase. Neutrinos, which are massive inside the bubbles and massless outside them, are reflected by the bubble walls (especially the heavy ones). Since leptonic charge on the wall is not conserved and $CP$-invariance is broken, reflections of neutrinos and antineutrinos might be different and this could give rise to a nonzero flux of leptonic charge from domain walls into the old phase. At that stage the mechanism of creation of (leptonic) charge asymmetry is essentially the same as heavy particle decays. In the symmetric phase $L$-nonconservation by the above described mechanism is not operative but $B + L$ is strongly nonconserved. So leptonic asymmetry is transformed into a mixture of leptonic and baryonic asymmetries. In this respect the scenario resembles that proposed by Fukugita and Yanagida [1986].

For the ratio of baryonic charge to entropy Cohen et al. [1990a, b] obtained the following simple expression:

$$\beta \approx A\, m_\nu^2 G_F / K_{\mathrm{eff}}\,, \tag{9.17}$$

where $m_\nu$ is the neutrino mass, $G_F = 10^{-5}/m_N^2$ is the Fermi coupling constant, $K_{\mathrm{eff}} \approx 100$ is the number of degrees of freedom in the plasma at the phase transition and $A$ is a factor connected with the neutrino mixing angles. For $A$ of the order of unity it would be possible to obtain the required value of $\beta$ in the three generation model if the mass of the heaviest neutrino, that is $\nu_\tau$, is about 30 MeV. This value is dangerously close to the experimental upper bound (see Particle Data Group [1990]). Such a heavy neutrino is compatible with cosmology only if it is unstable and decays into a lighter neutrino and a Majoron. If that heavy $\tau$-neutrino is excluded by experiment the scenario could be realized in a model with extra generation(s) of quarks and leptons. The latter should be heavier than 45 GeV so that they are not copiously produced by $Z^0$ decays. This in turn demands a small mixing angle factor $A$ in eq. (9.17). The model can explain the observed baryon asymmetry and is accessible to direct experimental verification.

As we mentioned above [see eq. (9.11)], the triangle anomaly gives rise to an effective interaction of the form

$$\mathcal{L}_6 = \partial_\mu |\phi|^2\, j_B^\mu / m^2\,. \tag{9.18}$$

This resembles expression (7.3), which was exploited by Cohen and Kaplan [1987, 1988] in their model of spontaneous baryogenesis. Thus it is tempting to apply this approach to baryogenesis at the electroweak scale. This idea was realized by Dine et al. [1991]. Apart from the dimension-six operator (9.18) they proposed the existence of the dimension-five operator (7.3). The latter can be generated in a supersymmetric extension of the standard model by one-loop diagrams with virtual gauginos or higgsinos. In this case $\phi$ is an electroweak singlet field which will later on be denoted as $s$ to keep agreement with the notations of Dine et al. [1991].

The value of the baryon asymmetry in this scenario essentially depends upon the rate of baryon nonconservation $\Gamma_{\Delta B}$. It was assumed that for a small field amplitude, $g\phi < \alpha_W T$, the rate is given by eq. (9.13) while for larger $\phi$ it abruptly falls down to zero. This is in agreement with the arguments presented at the beginning of this section.

The kinetic equation for the baryon number density can be written as follows:

$$\frac{\mathrm{d}N_B}{\mathrm{d}t} = -\frac{\Gamma_{\Delta B} C}{T^3}\left(N_B - \frac{T^2}{6m^2}\frac{\mathrm{d}|\phi|^2}{\mathrm{d}t}\right), \tag{9.19}$$

where $C = O(1)$ is a numerical coefficient and $N_B$ is the baryonic charge density. The second term on the r.h.s. appears due to the effective chemical potential produced by the interaction (9.18) [see also (7.4)]. In equilibrium one would expect $N_B = (T^2/6m^2)\,\mathrm{d}|\phi|^2/\mathrm{d}t$. The equilibrium is not reached, however, if the phase transition is of first order and $\phi$ changes fast in comparison with the expansion of the Universe. In that case $N_B$ can be neglected on the r.h.s. of eq. (9.19) and we obtain for the baryon asymmetry

$$\beta \approx (\alpha_W^6/K_{\mathrm{eff}} g^2)(T/m)^2\,, \tag{9.20}$$

where $m = O(100\,\mathrm{GeV})$ and this expression is valid up to $T = T_C = O(100\,\mathrm{GeV})$.

In the case of the interaction (7.3) the source term is given by the expression $(T^2/6\Lambda)\,\partial_t s$. The baryon asymmetry can be found analogously and is equal to

$$\beta \approx (\alpha_W^5/K_{\mathrm{eff}} g)(T/\Lambda)\,. \tag{9.21}$$

If $\phi$ (or $s$) changes slowly in time as, e.g., in the case of a second order phase transition when $\dot\phi \approx H\phi$, one should take into account in the kinetic equation the expansion of the Universe, $(-3HN_B)$, as well as the variation of temperature with time. In this case the deviations from thermal equilibrium are small and the first term on r.h.s. of eq. (9.19) cannot be neglected. For $N_B$ the following estimate is now valid:

$$\beta \approx T/m_{\mathrm{Pl}}\,, \tag{9.22}$$

which is definitely too low.

This scenario was reconsidered by Cohen et al. [1991] and Nelson et al. [1991], who noticed that not only baryonic charge but any nonconserved charge which is not orthogonal to the baryonic one could produce baryon asymmetry. To this end they considered the two Higgs doublet model and found that a "chemical potential" for fermionic hypercharge arose analogously to that for baryonic charge. In contrast to baryonic charge, however, the former could be generated by tree level diagrams and correspondingly the effect might be larger. It can be shown that in electrically and $B - L$ neutral plasma a nonvanishing hypercharge gives rise to a nonvanishing baryonic charge (see the next section) and the corresponding baryon asymmetry as estimated by Nelson et al. [1991] in the most favorable case can be even 3–4 orders of magnitude larger than the observed value. However, the magnitude of the effect was rather overestimated. Weak hypercharge is known to be conserved at high temperatures when the electroweak symmetry is unbroken, while the baryonic charge is, just the opposite, conserved in the low temperature phase and is not conserved at high temperatures. Thus the discussed mechanism is operative if both phases coexist. Nonzero hypercharge might be generated in the high temperature phase due to an interaction with the bubbles of broken symmetry phase. Reflection of left-handed fermions at the bubble wall turns them into right-handed ones (and vice versa) and a considerable flux of hypercharge into the symmetric high temperature phase should be produced if there is a reasonable $CP$-violation on the wall, as was argued by Cohen et al. However, the hypercharge is strictly conserved from the point of view of the observer in the high temperature phase. This means that the bubbles must acquire a hypercharge equal in magnitude and opposite in sign to that accumulated in the high temperature plasma where the symmetry was unbroken. Long-range Coulomb like forces, created by hypercharge in the symmetric phase, prevent hypercharge separation or, in other words, there should be a fast Debye screening of bubbles of broken phase and very little hypercharge could participate in driving baryogenesis. As was estimated by A. Cohen (private communication), this effect would diminish the magnitude of the baryon asymmetry by approximately two orders of magnitude.

We conclude that the standard electroweak theory possesses all the properties which are necessary for baryogenesis. The minimal model, however, gives a very low result due to the smallness of the $CP$-violating amplitude. The only exception is Shaposhnikov's [1986, 1987] proposal with degeneracy in the Chern–Simons number. The latter remains questionable and, what is more, the mass of the Higgs field necessary for the realization of the scenario should be too low, $m_H < 45$ GeV. This is practically excluded by the LEP experiments.

Scenarios based on extensions of the minimal model can do better. They rather naturally may generate the observed value of the baryon asymmetry if $B$-nonconservation is not suppressed at high temperatures. The potential barrier preventing formation of winding field configurations possessing nonzero Chern–Simons number is absent at $T > T_c$ and that makes it plausible that the rate of their production might be high enough so that they reach the equilibrium number density. Numerical simulations support this conclusion but still a more extensive study of their production is desirable, in particular of the dependence of the production rate on the number of points in the numerical simulation. Unfortunately a reliable analytical treatment of the production of a classical coherent field state by particle collisions in a thermal bath is not known now. In this connection the example of annihilation and production of magnetic monopoles may be instructive. The cross-section of monopole–antimonopole annihilation is assumed to be $\sim g_m^2/M_m^2$, where $g_m$ is the magnetic charge and $M_m$ is the monopole mass. If this is true then by the detailed balance condition the rate of monopole production is given by the same quantity. This would lead to the surprising conclusion that the production of monopole–antimonopole pairs in multiparticle collisions is neither exponentially nor otherwise strongly suppressed. The other logically permitted possibility is that monopole–antimonopole annihilation is strongly suppressed in comparison with the above given cross-section. This looks even more surprising, although the reluctance of soliton annihilation in the $(1+1)$-dimensional sine-Gordon model supports this conclusion. The infinite number of conservation laws in the latter case make this support not especially persuasive. Moreover, monopoles and antimonopoles are strongly attracting and are likely to form bound states with subsequent quick annihilation. Thus a more natural conclusion is that monopole annihilation is not suppressed. This makes one feel that the analogous formation of winding fields in high temperature plasma is also not suppressed and baryonic charge is indeed strongly nonconserved above the electroweak phase transition. Still one more open logical possibility, as was noted by S. Nussinov (private communication) is that monopole–antimonopole annihilation leads to a state which is otherwise not easily created in the primeval plasma. This means that, although the rate of monopole–antimonopole annihilation might be high, the rate of their production in the primeval plasma is low even at $T < M_{\mathrm{m}}$.

## 10. Lepton asymmetry of the Universe

The lepton asymmetry of the Universe (LAU) is not directly observable as is the baryonic one. Baryons are visible in the sky while neutrinos are not. There is also a difference between baryons and leptons in that $N_{\bar{\mathrm{B}}} \ll N_{\mathrm{B}}$ while $N_\nu \approx N_{\bar{\nu}}$. Some leptonic asymmetry is contained in the observable electron sector but it is constrained by the electrical neutrality of the Universe (see section 12) and thus should not exceed the baryon asymmetry. A bound on the LAU though very weak can be obtained from the upper bound on the total energy density of the Universe [Freese et al. 1983]. Since the energy density of a degenerate neutrino gas with chemical potential $\mu_\nu$ is $\rho_\nu = \mu_\nu^4/8\pi^2$, it follows from the condition $\rho_\nu < \rho_{\mathrm{c}}$ that $\mu_\nu < 10^2\sqrt{h_{100}}$ eV. Here $h_{100} = H/100\,\mathrm{km/s\,Mpc}$. In the course of the evolution of the Universe $\mu_\nu$ scales as the temperature and in terms of the photon temperature $\xi_\nu = (\mu_\nu/T_\gamma) < 42\sqrt{h_{100}}$.

A stronger limit on $\mu_\nu$ comes from the consideration of primordial nucleosynthesis [Rana 1984; Kolb and Turner 1987; Terasawa and Sato 1988; Olive et al. 1991] (for a review and earlier references see Dolgov and Zel'dovich [1981]). As was first noticed by Schwartsman [1969], primordial nucleosynthesis permits one to restrict the number of neutrino flavors. According to modern data, the theory is incompatible with observations if the number of neutrino flavors is larger than three (see, e.g., Olive et al. [1990] or Schramm [1990]). In short the arguments are the following. The energy density of relativistic particles in the Universe is given by the expression

$$\rho = \frac{\pi^2}{30} KT^4 = \frac{3}{32\pi} \frac{m_{\mathrm{Pl}}^2}{t^2}, \tag{10.1}$$

where $K$ counts the bosonic and 7/8 of the fermionic degrees of freedom.

The abundances of primordially produced light elements crucially depend upon the neutron to proton ratio $r = N_{\mathrm{n}}/N_{\mathrm{p}}$. The latter is determined by thermal equilibrium until the reactions $\mathrm{e}^- + \mathrm{p} \leftrightarrow \mathrm{n} + \bar{\nu}$ and $\mathrm{e}^+ + \mathrm{n} \leftrightarrow \mathrm{p} + \bar{\nu}$ keep pace with the expansion of the Universe. At $T = T_{\mathrm{f}} \approx 0.7\,\mathrm{MeV}$ the expansion rate $H$ becomes larger than the weak interaction rate $\Gamma_{\mathrm{W}} \sim G^2T^5$ and $r$ freezes at the constant value $\exp(-\Delta m/T_{\mathrm{f}})$, where $\Delta m \approx 1.3\,\mathrm{MeV}$ is the neutron–proton mass difference. Because of eqs. (10.1) the value of the freezing temperature depends on the number of degrees of freedom, $T_{\mathrm{f}} \sim K^{1/6}$, and that is how the number of neutrino species comes into play.

If neutrinos of a certain flavor are strongly degenerate, $\xi_\nu = \mu_\nu/T > 1$, their contribution to the energy density is $\rho_\nu = \mu_\nu^4/8\pi^2$ instead of $7\pi^2T^4/120$. In order to preserve good agreement of the standard big-bang nucleosynthesis theory with observations we have to assume that $|\xi_\nu| < 3$.

A much stronger bound is valid for the chemical potential of the electron neutrino. In contrast to other neutrino flavors $\mu_{\nu_{\mathrm{e}}}$ directly affects the frozen n/p ratio,

$$N_{\mathrm{n}}/N_{\mathrm{p}} = \exp(-\Delta m/T_{\mathrm{f}} - \mu_{\nu_{\mathrm{e}}}).$$

This does not permit $|\xi_{\nu_e}|$ to be larger than a few per cent. It is possible to get better agreement of the theory with observations with a nonzero $\xi_{\nu_e}$ [Terasawa and Sato 1988; Olive et al. 1991]. The recent analysis by Starkman [1992], however, questioned these results. By simultaneous variation of several parameters he found that the theory can well describe the data with $\mu_{\nu_\mu} \approx \mu_{\nu_\tau} = (5\text{–}25)T$, $\mu_{\nu_e} \approx T$, and $\Omega_B h_{100}^2$ as large as 0.1–1.

Correspondingly the asymmetry with respect to the electronic charge ($L_e/N_\gamma$) should be smaller than a few times $10^{-2}$ (though $L_e/N_\gamma = O(1)$ is not excluded) while the asymmetry with respect to muonic and tauonic charges ($L_\mu/N_\gamma$ and $L_\tau/N_\gamma$) is permitted to be of the order of 10. In the framework of Grand Unification Theories it is natural to expect that the lepton asymmetry is of the same order as the baryonic one [Turner 1980]. In the SU(5) model they are exactly the same because $B-L$ is conserved in this model. This is not true for the group SO(10) but it is still difficult to get $L \gg B$ in this framework though formally it is not excluded [Harvey and Kolb 1981].

A large value of the lepton asymmetry can be naturally obtained in the version of Affleck and Dine [1985] scenario considered by Dolgov and Kirilova [1991] (see also Dolgov [1990]), which is discussed in section 6.*) The model opens an interesting possibility of variation of $L$ on the galactic scale. This effect could explain the observed spatial variation of the $^4$He abundance. To be meaningful the generation of a lepton asymmetry should proceed below the electroweak epoch when (and "if") the baryon and lepton numbers were strongly nonconserved. Otherwise all the pre-existing $B - L$ charge asymmetry would be more of less equally shared between $B$ and $L$. If there are no $B$ and/or $L$ nonconserving processes below the electroweak scale one would expect $B$ and $L$ to be approximately of the same size, i.e., $\xi_\nu \approx 10^{-9}\text{–}10^{-10}$.

The problem of the lepton asymmetry of the Universe is especially sound if electroweak processes only wash out the pre-existing asymmetry but do not create a new one. This would be the case, if, e.g., the electroweak phase transition is of second order. If baryonic charge is conserved below the electroweak scale then the baryon asymmetry could be proportional to the earlier generated $B - L$ asymmetry. There is some confusion in the literature about the proportionality coefficient so that we derive it below. Although the calculations are rather simple, our result does not coincide with many results presented in different papers [Kuzmin et al. 1985, 1987; Aoki 1986; Kolb and Turner 1987; Matveev et al. 1988; Harvey and Turner 1990; Nelson and Barr 1990], which are moreover in mutual contradiction with each other. The numerical difference is not large but it is worthwhile to have a correct expression. That is why we make this simple exercise below in some detail and hopefully obtain the correct result.**)

To describe a charge asymmetric particle distribution the notion of chemical potential $\mu_j$ for particles of type $j$ is introduced. Their equilibrium phase space distribution is given by the well known expression

$$n_j = \{\exp[(E - \mu_j)/T] \pm 1\}^{-1}, \tag{10.2}$$

where "+" stands for fermions and "−" stands for bosons. It is evident that the bosonic chemical potential should not exceed the particle mass, otherwise $n$ would not be positive definite. If the charge asymmetry in the bosonic sector is so large that even the maximum possible value of the chemical potential $\mu_j = m_j$ is not sufficient to get the appropriate value of the asymmetry, a Bose condensate is formed and $n_j$ can be written in the form

$$n_{jc} = (2\pi)^3 N_c \delta^{(3)}(p) + \{\exp[(E - m)/T] - 1\}^{-1}. \tag{10.3}$$

*) The possibility of creating $B - L$ asymmetry via the Affleck–Dine mechanism has been considered recently also by Morgan [1991].

**) I am grateful to K.-I. Aoki for a discussion of this point.

Thermal equilibrium with respect to a reaction $i \leftrightarrow f$ demands

$$\sum \mu_i = \sum \mu_f , \tag{10.4}$$

where the summation is over all particles participating in the reaction. The statements (10.3) and (10.4) can be easily verified by the kinetic equation.

If certain particles are not conserved their chemical potential vanishes. In particular, the chemical potential of photons in equilibrium is zero because different numbers of photons can be produced by the same initial state. This implies in particular that the chemical potentials of particles and antiparticles have opposite signs, $\mu + \bar{\mu} = 0$, if they are in equilibrium with respect to annihilation into photons.

The fermionic charge asymmetry is given by the expression

$$N_f - \bar{N}_f = g_s \int_0^\infty \frac{d^3p}{(2\pi)^3} (\{\exp[(E-\mu)/T]+1\}^{-1} - \{\exp[(E+\mu)/T]+1\}^{-1}) , \tag{10.5}$$

where $g_s$ is the number of spin states. For the physically interesting case of small masses and chemical potentials (in comparison with temperature) we obtain

$$N_f - N_{\bar{f}} = \frac{g_s T^3}{2\pi^2} \left( 4\xi \int_0^\infty \frac{dy\, y}{e^y+1} + \int_{-\xi}^0 \frac{dy\,(y+\xi)^2}{e^y+1} + \int_0^\xi \frac{dy\,(y-\xi)^2}{e^y+1} \right) \approx \tfrac{1}{6} T^2 \mu . \tag{10.6}$$

Here $\xi = \mu/T$.

A similar derivation for bosons is slightly more complicated because we cannot put $m=0$ at the very beginning. In the case of strictly massless bosons, however, the asymmetry is trivially given by $N_c$ from eq. (10.3). In the general case (but still with $m/T \ll 1$, $\mu/T \ll 1$) we obtain

$$N_b - N_{\bar{b}} = \tfrac{1}{3} g_s T^2 \mu . \tag{10.7}$$

For $\mu = m$ the contribution from $N_c$ should be taken into account too. Note the factor of two difference with respect to fermions with the same chemical potential.

Now we can evaluate the charge asymmetries with respect to different charges when electroweak processes are in equilibrium. In what follows we use the particle symbols for the corresponding chemical potentials to simplify the notation. For example, $\mu_{W^+} \equiv W^+$, $\mu_{u_L} \equiv u_L$, etc.

Equilibrium with respect to charge currents implies

$$W^+ = u_{Lj} - d_{Lj} = \nu_k - l_{Lk} , \tag{10.8}$$

where $j$ and $k$ are flavor indices. It is assumed that the chemical potentials do not depend upon color because of fast color changing processes. Moreover, the quark chemical potentials do not depend upon flavor, $q_j = q$ ($q = u, d$), due to charged flavor changing currents. This might not be true for leptons, so we will distinguish between $l_e$, $l_\mu$ and $l_\tau$ (and the corresponding $\nu_j$). Moreover, the chemical potentials of left-handed and right-handed particles should be the same (with possible exceptions for neutrinos) because of the high rate of $R \leftrightarrow L$ transformation, $\Gamma_{LR} = (m_{eff}/T)^2 \Gamma_{LL}$, where $\Gamma_{LL}$ is the rate of reactions with left-handed particles. At $T = 10^2$–$10^3$ GeV, $\Gamma_{LL}$ is so much larger than $H$ that $\Gamma_{LR} > H$ even for electrons.

Electroweak interactions are believed (see section 9) to produce fast $B$- and $L$-nonconserving but $B-L$ conserving transitions when quarks from all the generations, taken by three from each generation (we assume that the number of colors is three), are transformed into leptons with corresponding electric charge. With account of eq. (10.8) this results in the following relation:

$$3N(u+d)+\sum_j (l_j+\nu_j)=0\,, \tag{10.9}$$

where $N$ is the number of quark generations.

We assume that there are $M$ Higgs doublets in the model and no other extra charged fields. The chemical potential of the charged component of the Higgs doublet is

$$H^+=u-d\,. \tag{10.10}$$

Electric neutrality of the plasma imposes the following relation between the chemical potentials:

$$Q_{\rm el}\sim 2\cdot 3\cdot N(\tfrac{2}{3}u-\tfrac{1}{3}d)+2\cdot 3\cdot(u-d)+2(M-1)(u-d)-2\sum l_j=0\,. \tag{10.11}$$

Here the factors corresponding to the color and spin degrees of freedom are explicitly written down. Note that bosons have an extra factor of two in comparison with fermions [compare eqs. (10.6) and (10.7)]. We are going to use these relations below the phase transition so that we have written the contribution from three spin components of the W-bosons while the number of charged Higgs fields is $M-1$. One combination of the Higgs fields is known to turn into the longitudinal component of W.

The baryonic charge density is proportional to

$$B\sim 3\cdot 2N(\tfrac{1}{3}u+\tfrac{1}{3}d)\,, \tag{10.12}$$

and leptonic charge density is

$$L\sim 2\sum l_j+\sum \nu_j\,. \tag{10.13}$$

It is assumed here that only one spin state of neutrinos is excited. If some neutrinos are sufficiently massive, eq. (10.13) should be modified correspondingly. An evident modification should be also made in eqs. (10.11) and (10.12) if the t-quark mass is not negligible in comparison with the temperature $T_{\Delta B}$ at which the $B$-nonconserving electroweak processes are frozen.

Now it is a matter of simple algebra to get

$$B=\frac{8N+4(M-1)+12}{24N+13(M-1)+39}(B-L)\,,\qquad L=-\frac{16N+9(M-1)+27}{24N+13(M-1)+39}(B-L)\,. \tag{10.14}$$

For $M=1$ it coincides with the results presented by Nelson and Barr [1990]. Note that $B+L\neq 0$ [Aoki 1986; Harvey and Turner 1990] contrary to earlier claims. There are three nonanomalous currents conserved by electroweak interactions, $j^a_\mu=j^{\rm B}_\mu-3j^{{\rm L}a}_\mu$ with $a={\rm e},\mu,\tau$. All the charges in the plasma should be described in terms of those three. It happens, however, as we saw above, that the baryonic and total leptonic charges $L=L_{\rm e}+L_\mu+L_\tau$ are expressed through $B-L$. This result is evident because of the symmetry between charges.

We see that the baryon asymmetry survives after equilibrium electroweak processes if initially there was a nonzero $B-L$ asymmetry. A small value of $L\approx -2B$ does not necessarily exclude large individual electronic, muonic and tauonic asymmetries if there is an accidental cancellation between them [Kolb and Turner 1987], though this does not look particularly natural.

An easy way to generate a leptonic and $B-L$ asymmetry, which could give an appropriate baryon asymmetry after electroweak mixing, is presented by the decays of a heavy Majorana neutrino as was advocated by Fukugita and Yanagida [1986]. A different realization of this idea in the framework of inflationary cosmology was put forward recently by Lazarides and Shafi [1991], who assumed that the inflaton decays into one or several species of heavy Majorana neutrinos. The latter generate a leptonic asymmetry by their subsequent decays. The essential features of this type of scenario are discussed in section 3. Baryogenesis via leptogenesis by a heavy right-handed Majorana neutrino $\nu_{\mathrm{MR}}$ was considered recently by Luty [1992], who obtained reasonable values of $\beta$ in the mass range of $\nu_{\mathrm{MR}}$ from $10^3$ GeV up to $10^{19}$ GeV.

For earlier work on the role of Majorana neutrinos in baryo- and leptogenesis see Yanagida and Yoshimura [1980, 1981], Barbieri et al. [1981], Fukugita et al. [1981], and Langacker et al. [1982]. In the first of them it was argued that in the framework of the group SO(10) the mass of the Majorana neutrino should be larger than $10^{10}$ GeV to generate the observed baryon asymmetry. In the last one a different problem was considered, namely the reduction of pre-existing lepton asymmetry by light Majorana neutrinos. This issue attracted much attention recently in connection with electroweak processes [Fukugita and Yanagida 1990; Harvey and Turner 1990; Nelson and Barr 1990; Campbell et al. 1991; Fischler et al. 1991]. The point is that separate leptonic (or baryonic) charge nonconservation together with sphaleron induced $B-L$ conserving processes would completely wash out both baryon and lepton asymmetries. The condition that this has not happened permits one in particular to put an upper bound on the Majorana mass term of light neutrinos. As is shown by Fischler et al. [1991],

$$m_{\mathrm{M}} < 4\ \mathrm{eV}/(T_{B-L}/10^{10}\ \mathrm{GeV})^{1/2}\ , \tag{10.15}$$

where $T_{B-L}$ is the scale where $B-L$ asymmetry is generated.

An exhaustive study of the bounds imposed by electroweak physics on the properties of $B$ and $L$ nonconserving interactions in different extensions of the standard model has been performed by Campbell et al. [1991]. In particular their results on neutron–antineutron oscillations and $R$-parity breaking are stronger than those obtained in laboratory experiments. One should keep in mind, however, that all these are valid if electroweak processes do not create baryon asymmetry and if there is no baryogenesis below the electroweak scale.

*Note added.* Dreiner and Ross [1992] obtained recently the surprising result that the baryon asymmetry is not washed out and the constraints mentioned above can be avoided for nonzero masses of leptons.

## 11. Baryogenesis and large scale structure of the Universe

The first scenarios of baryogenesis based on Grand Unification Theories predicted that the ratio $N_{\mathrm{B}}/N_{\gamma}$ is a universal constant depending only on particle physics parameters. Indeed, the models generically gave a result of the form (3.4), where the decay asymmetry $\varepsilon$ was determined by the explicit $CP$-violation, which in turn was defined by the phases of the coupling constants in the Lagrangian. It was advocated by Brown and Stecker [1979] that in the case of spontaneous breaking of charge symmetry [Lee 1973] $\varepsilon$ would differ in sign in causally disconnected regions while being of the same absolute value and that would give rise to a Universe on average equally populated by baryons and antibaryons. The characteristic size of the regions with a definite sign of the baryon asymmetry is of the order of the size of a causally connected region during baryogenesis. In the case of the Friedman expansion the latter is equal to

$$L_{\Delta B} \approx (90/32\pi^3 K) m_{\rm Pl}/T_{\Delta B}^2 , \tag{11.1}$$

where $T_{\Delta B}$ is the baryogenesis temperature. At the present time this scale is expanded up to

$$L_0 = L_{\Delta B}\, T_{\Delta B}/T_{\gamma 0} \approx 5 \times 10^{16}\ \text{cm}(1\ \text{GeV}/T_{\Delta B}) , \tag{11.2}$$

where $T_{\gamma 0} \approx 2.7$ K is the present-day temperature of the electromagnetic background radiation. $L_0$ is definitely negligibly small in comparison with the galactic size. It makes this simple model out of the question. A resolution of this difficultly was proposed by Sato [1981], who noticed that the Universe might expand exponentially if the symmetry breaking in Grand Unified Theory proceeded through a first order phase transition with large supercooling. Spontaneous $CP$-violation should occur prior to this inflationary stage, which makes possible an arbitrarily large size of domains with a definite sign of the baryon asymmetry. This scenario was further elaborated by Stecker [1985], who advocated the notion of a baryon-symmetric Universe with a characteristic size of the domains of the order of the scale of galaxy clusters. The domain wall problem would be solved if the charge symmetry is restored at smaller temperature [Kuzmin et al. 1981], or if the degeneracy between two different $CP$-odd vacua is lifted dynamically [Mohanty and Stecker 1984], or if it was not exact from the very beginning due to a small explicit $CP$-breaking.

The idea of inflation has proven to be extremely fruitful for inventing different scenarios of baryogenesis. In particular it made possible the construction of the island Universe model [Dolgov and Kardashev 1986; Dolgov et al. 1987], in which islands of baryons and antibaryons float in a sea of invisible matter and background radiation. The sizes of those islands can be arbitrary (but astronomically large). Especially interesting is the case where the size of our island is smaller than the present day horizon and in particular corresponds to a red-shift $z = 5$–$10$. In that case there is a chance to prove the validity of the model if there is no visible matter beyond this distance. An attractive feature of the model is that it permits small angular variations of the background radiation temperature, $\Delta T/T$, without contradiction with the possibility of large scale structure formation. The point is that the background radiation we observe now comes from the region outside of the island and so its variations are not connected with baryonic density fluctuations as is the case in the standard scenario. In any case the model is interesting from a theoretical point of view since it demonstrates the possible existence of a very inhomogeneous Universe at large scales without contradiction with the observed homogeneity and isotropy at relatively small distances. The Universe in this model is on the average charge symmetric, which is definitely more attractive than a world in which the symmetry is shifted towards baryons.

In short, the scenario leading to the insular structure can be realized as follows. First, the charge symmetry should be spontaneously broken [Lee 1973] and the phase transition to the $CP$-odd phase should be of first order with supercooling and formation of bubbles of new phase inside the quasi-stable $CP$-symmetric phase. Second, there should be a rather long period of exponential expansion after the phase transition. Otherwise the size of the $CP$-odd bubbles is too small. If the phase transition took place before the end of inflation but not far from it, the island size could be of the order of the present horizon size but with a suitable tuning of the parameters still slightly smaller than the latter. When inflation ends and the Universe is (re)-heated, an excess of particles over antiparticles or vice versa is generated inside of the bubble by any mechanism of baryogenesis discussed above. Outside of the bubbles the charge symmetry is unbroken and the baryonic charge density is equal to zero. In these regions the number density of baryons is negligibly small. After sufficient cooling the process of baryosynthesis stops. Starting from this moment, the walls of the bubble separate from the island border because they propagate in the false vacuum almost with the speed of light and the island expands in accordance with the Friedman law $a \sim t^{1/2}$ or $a \sim t^{2/3}$ (for $\Omega = 1$). A very similar mechanism of the creation of cosmic bubbles but of slightly smaller size has been considered by Kofman et al. [1987].

The formation of a condensate of the complex scalar field $\phi$ which generates $CP$-violation is equally probable in reflection symmetric directions so that on the average in a large volume the symmetry between particles and antiparticles is conserved. This is a generic feature of any theory with spontaneous, in contrast to explicit, charge symmetry breaking.

We assume that there is a potential barrier between the charge symmetric false vacuum $\phi = 0$ and the charge asymmetric real vacuum, where $\phi \neq 0$. This barrier ensures quasi-stability of the false vaccuum and a phase transition of first order to the real vacuum.

As a toy model let us consider the following potential:

$$v(\phi) = m_1^2|\phi|^2 + \tfrac{1}{2}m_2^2(\phi^2 + \phi^{*2}) + \tfrac{1}{2}\lambda|\phi|^4 \ln(|\phi|^2/\eta^2)\,. \tag{11.3}$$

For sufficiently large $\eta$ the potential has a global minimum at $\phi = \phi_{\text{min}} \neq 0$. If $m_1^2 > m_2^2 > 0$ it has also a local minimum at $\phi = 0$ separated from the global one by a potential barrier. The term proportional to $m_2^2$ makes the potential nonsymmetric with respect to a phase rotation of $\phi$. The shape of $v(\phi)$ as a function of $\beta = \arg\phi$ is given by the expression $m_1^2 + m_2^2 \cos 2\beta$. Hence for $|\langle\phi\rangle| \neq 0$ the absolute minimum of the potential is reached at $\cos 2\beta = -1$. So there are four degenerate minima in the model. Two of them correspond to the $CP$-considering phase while the other two are $CP$-nonconserving. Our Universe is assumed to be formed in the bubble of $CP$-odd phase.

The probability of barrier penetration from the false vacuum state $\langle\phi\rangle = 0$ to the real $CP$-nonconserving one is exponentially suppressed for small coupling constant $\lambda$ but the coupling of $\phi$ to the inflaton field $\Phi$ could facilitate the phase transition and, what is more, could make the duration of inflation, after the bubble of $CP$-odd phase is formed, relatively short. This would permit the formation of bubbles with the present-day size smaller than the horizon.

We assume that the inflationary expansion is generated by the potential energy $V(\Phi)$ when $\Phi$ is far from the equilibrium value $\Phi_0$. In this region the potential $V(\Phi)$ is almost flat and $\Phi$ slowly rolls down to the equilibrium. When $\Phi$ approaches $\Phi_0$ the curvature of the potential increases, $\Phi$ moves faster and starts to oscillate around the equilibrium point. The oscillations of $\Phi$ lead to particle production and to heating of the Universe. Later or simultaneously baryogenesis starts.

Let the coupling between $\phi$ and $\Phi$ be of the general renormalizable form

$$L_{\text{int}} = \lambda'|\phi|^2(\Phi - \Phi_1)^2\,, \tag{11.4}$$

where $\Phi_1 < \Phi_0$ and $\lambda' > 0$. This interaction leads to an effective time dependent mass of $\phi$,

$$\Delta m^2(t) = \lambda'[\Phi(t) - \Phi_1]^2\,.$$

We assume that the parameters are such that

$$m_1^2 + \Delta m^2(t) > m_2^2 > 0$$

when $1 - \Phi/\Phi_1 \geq O(1)$ but $m_2^2 > m_1^2$. In this case the state $\phi = 0$ is almost always classically stable with respect to small fluctuations and only when $\Phi$ is close to $\Phi_1$ is there a period of instability. Quantum fluctuations of $\phi$ at that time increase and, if they exceed a critical value $\phi_c$, until moment when the condition $m_1^2 + \Delta m^2(t) > m_2^2$ becomes valid again they do not return to the false vacuum state but rise up to $\pm\phi_{min}$. Thus bubbles of *CP*-odd vacuum can be formed. The average bubble size $d$ and the distance $l$ between them are very much model dependent. In particular the value of $l$ can vary from 0 to infinity.

A serious problem to baryonic island cosmology might be presented by the domain walls separating the false vacuum from the one or two (four?) degenerate real vacua with different signs (and amplitudes) of the *CP*-violating amplitude. Fortunately several ways are known in the literature to get rid of these walls, which we have mentioned in the beginning of this section.

The different chemical content of matter inside and outside the bubble gives rise to different expansion laws and this in turn produces anisotropy of the background radiation. A position of an observer such that the far island boundary is beyond the horizon while the close boundary is seen is excluded by the data. On the other hand, if all the parts of the island are seen, the asymmetry proves to be rather small. For example, if the red-shift of the far and close boundaries are, respectively, 8 and 3, the dipole anisotropy of the background radiation is $\Delta T/T = 3 \times 10^{-8}$ [Dolgov et al. 1987].

Above we have considered the scenario in which *CP* was not broken outside the island borders during baryogenesis. This means that the world outside is baryon symmetric with negligibly small nucleon and antinucleon number density, $N_N = N_\gamma(\sigma_{ann} m_{Pl} m_N)^{-1} \approx 10^{-19} N_\gamma$ (see, e.g., Zel'dovich and Novikov [1975] or Dolgov and Zel'dovich [1981]). If this is the case then the annihilation on the island boundary is practically unobservable. It is possible, however, that two mechanisms of *CP*-breaking are operative: spontaneous and explicit. If in particular the sign of explicit *CP*-violation is such that the space outside of our island is predominantly antibaryonic then the annihilation at the boundary can be the source of a considerable $\gamma$-ray background [Dolgov et al. 1987].

The island Universe model naturally predicts a quasi-periodic distribution of baryonic matter inside the island [Dolgov et al. 1987]. This result, however, is not specific for the island model but is generic for many models of baryogenesis combined with inflation [Chizhov and Dolgov 1992] (see also Dolgov [1990]). The following simple and natural assumptions are sufficient for generating a (quasi)periodic distribution of baryonic matter in our Universe:

1. The existence of a complex scalar field $\phi$ whose mass $m$ is small in comparison with the Hubble parameter during inflation, $H_I$.
2. An essential nonharmonic term in the self-interaction potential of $U(\phi)$ like, e.g., $\lambda|\phi|^4$.
3. Formation during the inflationary stage of a classical field condensate $\langle\phi\rangle = \phi_{cl}(r, t)$ which is not strictly constant in space.

The first two assumptions are absolutely innocent while the third one, though it seems natural, might need an explanation. There are at least two mechanisms to form a space dependent classical field. The first one is the rise of quantum fluctuations of $\phi$ during the inflationary stage, which is discussed in section 6 [see eqs. (6.3)–(6.6) and (6.18)–(6.20)]. The second one is a first order phase transition which $\phi$ might undergo jumping from the false vacuum at $\phi = 0$ to a nonzero value forming a bubble of new phase. Generically $\phi$ jumps to the slope of the potential barrier, which can be far away from the bottom of the potential well at $\phi = \phi_{min}$. The initial shape of the bubble is determined by the form of the potential $U(\phi)$ and can be rather flat like, e.g., $\phi_i(r) = \phi_0(1 + r^2/r_0^2)^{-1}$ or some other similar form with the characteristic scale in $r$ equal to $r_0$. The concrete form of $\phi(r)$ is not essential. During the inflationary stage $\phi(r)$ remains constant (in terms of comoving coordinates $r$) because of large Hubble friction. The equation of motion has the form

$$\partial_t^2\phi - \frac{1}{a^2(t)}\,\partial_i^2\phi + 3H_\mathrm{I}\,\partial_t\phi + U'(\phi) = 0\,, \tag{11.5}$$

and if $H_\mathrm{I}$ is large $\partial_t\phi \approx 0$. The physical scale of spatial variation of $\phi$ becomes exponentially large, $l = r_0\exp(H_\mathrm{I}\tau)$, where $\tau$ is the duration of inflation after the bubble $\phi_i(r)$ was formed. Starting from a microscopically small $r_0$ we can easily get in this way cosmologically interesting values of $l$.

An interesting possibility would be realized if $\phi$ coincides with the inflaton $\Phi$. Of course in this case $\Phi$ should be a complex field. In this model, the end of inflation and the onset of oscillations of $\phi$ near equilibrium are naturally connected.

We do not want to specify further the details of possible mechanisms of the formation of the classical field $\phi$. If not generic it is at least very natural. It is also natural that $\phi$ is a function of the space coordinates and that the characteristic scale $r_0$ of its variation has been inflated up to a cosmologically interesting size. We assume that only the first terms in the Taylor expansion of $\phi(r, t)$ in $r$ are essential,

$$\phi_\mathrm{i}(r) \equiv \phi(r, t_\mathrm{i}) = \phi_0\left(1 + \frac{\boldsymbol{r}\cdot\boldsymbol{n}}{r_0} + \frac{r_i r_j c_{ij}}{r_0^2} + \cdots\right), \tag{11.6}$$

where $\boldsymbol{n}$ is the unit vector in the direction of the gradient of $\phi$ and $t_\mathrm{i}$ is the initial time.

During the inflationary stage $\phi(r, t)$ slowly moves to the equilibrium point $\phi_\mathrm{min}$ because of the large Hubble friction. When inflation is over and the Hubble parameter goes down as $t^{-1}$, $\phi$ starts to oscillate around $\phi_\mathrm{min}$ with a decreasing amplitude. The decrease is connected with the expansion of the Universe and possibly with particle production by the oscillating field. If the potential of $\phi$ is not strictly harmonic a monotonic behavior in $r$ is transformed into spatial oscillations of $\phi$. This is due to the fact that only in a harmonic potential, $U_\mathrm{h} = m^2|\phi|^2$, the period of oscillations does not depend on the amplitude. Anharmonicity results in different time periods in different points of space (because the initial amplitude depends on $r$) and so in nonmonotonic behaviour of $\phi$ in space. To illustrate this statement let us consider a simple toy model with

$$U(\phi) = \lambda|\phi|^{2n}/n\,. \tag{11.7}$$

We neglect for the time being the expansion of the Universe and the space derivative terms in the equation of motion. As for the former, it is essential and can be easily taken into account. The neglect of the space derivative term is well justified because inflation has made it very small in comparison with the time derivative. The space dependence of $\phi$ is taken into account adiabatically.

The equation of motion of $\phi$ in the potential (11.7) possesses the conservation law

$$|\dot\phi|^2 = \lambda(|\phi_\mathrm{i}|^{2n} - |\phi|^{2n})/n\,. \tag{11.8}$$

It has been assumed that initially at $t = t_\mathrm{i}$, $\dot\phi = 0$ and so $\phi_\mathrm{i}(r)$ is the initial value of $\phi$. Making the substitutions $t = \lambda^{-1/2}n^{1/2}\phi_\mathrm{i}^{1-n}y$ and $\phi = \phi_\mathrm{i}Z(y)$ we see that $Z(y)$ satisfies the equation $(Z')^2 = 1 - Z^{2n}$, so it is a periodic function of $y$ with period of the order of 1. Hence

$$\phi(r, t) = \phi_0(r) Z(\lambda^{1/2} n^{-1/2} \phi_i(r)^{n-1} t) . \tag{11.9}$$

Thus, if we retain only linear terms in the expansion of $\phi_0(r)$, eq. (11.6), the function $\phi(r, t)$ is an oscillating and almost periodic function of $r$ with slowly changing amplitude and with period $\Delta r$ such that

$$\Delta r / r_0 = [\lambda^{1/2} n^{-1/2} (n-1) t \phi_0^{n-1}]^{-1} \mathrm{O}(1) . \tag{11.10}$$

Since $\phi$ is a complex field a nonzero value of it should generally give rise to $C$- and $CP$-violating effects. It can be understood as a slowly varying Higgs field generating complex fermion masses. In particular, if $\phi$ has not reached the equilibrium point at the moment of baryosynthesis $t_B$, and if the characteristic time scale of the latter is small in comparison with $\phi/\dot{\phi}$, then baryogenesis would make a snapshop of $\phi(r, t)$ because the generated baryonic charge density is proportional to the amplitude of $CP$-violation, which in turn is proportional to $\phi(r, t_B)$. There are two physically different cases. The first one is that the equilibrium value of $\phi$ is zero, so that the vacuum state is $CP$-even. $CP$-violation effects are proportional to the deviation of $\phi$ from the equilibrium point. This kind of $CP$-violation can be called stochastic because the original nonvanishing $\phi$ has been generated stochastically and $CP$-odd amplitudes vanish at equilibrium.

Since $\phi$ oscillates around zero, baryonic layers in this model are alternating with antibaryonic ones. This results in large isocurvature fluctuations. For a flat spectrum of the fluctuations there should be unacceptable angular fluctuations of the temperature of the microwave background radiation [Efstathiou and Bond 1987]. In our case, however, the spectrum is not flat and the problem deserves further consideration. Moreover, in the island Universe model, if the red-shift of the island boundary is smaller than the red-shift of hydrogen recombination, $z_r = 10^3$, the expected angular variation of the background radiation temperature could be very small because in this case the radiation reaches us now from baryon-empty space. Too strong electromagnetic radiation produced by the annihilation in the neighboring layers can be avoided since these layers should be spatially separated due to the pressure created by the annihilation and due to the gravitational clumping of layers with high baryon (antibaryon) number density.

The second case corresponds to oscillations of $\phi$ around $\phi_0 \neq 0$. The periodic distribution of baryons is now superimposed on a constant baryonic background and relative baryonic density perturbations may be small. A similar picture arises if together with $CP$-violation induced by $\phi$ there is explicit $CP$-violation so that the net result has a definite sign. If $CP$ is broken spontaneously there is a symmetric equilibrium point $\phi = -\phi_0$. In the spatial regions where $\phi_0$ is changed into $-\phi_0$ antibaryons are mostly produced but these parts of the Universe can be can easily beyond the present-day horizon. In a realistic cosmological background the characteristic scale of variation of the baryon number density $\Delta r$ is different from (11.10) and is given by the expression [Chizhov and Dolgov 1992]

$$\Delta r / r_0 \approx g^{2/3} \lambda^{-1/4} , \tag{11.11}$$

where $\lambda$ is the quartic self-interaction coupling constant and $g$ is the Yukawa coupling constant of the inflaton to light fermions. The latter determines the reheating temperature when inflation is over. For $\lambda = g^2 = 10^{-2}$ we get $\Delta r / r_0 \approx 0.1$.

This mechanism might explain the observed periodicity in the large scale structure of the Universe [Broadhurst et al. 1990] if $r_0$ is approximately equal to 1 Gpc. If higher order terms in the expansion

(11.6) are essential there would be no periodicity but a more complicated and still oscillating distribution of baryons.

In a realistic model of baryogenesis, the time scale of the $\phi$ oscillations around $\phi_{\min}$ is of the same order as the duration of baryogenesis. This results in the suppression of baryon generation by a factor $\sim 10^{-3}$. Still, since the *CP*-odd amplitude may be large in this model, a sufficient baryon asymmetry can be produced.

Note that this kind of model gives rise to the so called isocurvature baryon number fluctuations. This means that the variation of the total energy density is zero or negligibly small but the chemical content of matter is different. Only when baryons become nonrelativistic are the variations in the chemical content transformed into variations in the total energy density. In the earlier models of baryogenesis, so called adiabatic fluctuations, when $N_B/N_\gamma = \text{const.}$ and the total energy density fluctuates, were typical while it was rather hard to get isocurvature fluctuations. An exception is presented by the model of Yoshimura [1983], where the spatial variation of the *CP*-odd amplitude is induced by a varying axion field.

A similar mechanism of generating isocurvature baryon density perturbations exists in the spontaneous baryogenesis scenario [Turner et al. 1989; Mollerach 1990] and in the Affleck and Dine scenario [Dolgov and Kirilova 1991]. In those models the value of the baryon asymmetry is determined by the amplitude of a scalar field which either itself has a nonzero baryonic charge or interacts with a (nonconserved) baryonic current (see sections 6 and 7). An essential assumption necessary for the realization of this mechanism is that the relevant processes proceeded during inflationary stage. The latter is capable of stretching microscopically short waves up to cosmologically significant size and of amplification of the amplitude of quantum fluctuations (section 6). These two phenomena create a classical stochastic field from the quantum fluctuations of a scalar field in the De Sitter background. The fluctuation spectrum is known to be generically of the Harrison–Zel'dovich form, $d^3k/k^3$ (see, e.g., the reviews by Blau and Guth [1987] or by Goncharov et al. [1987]). The fluctuation amplitude as found by Turner et al. [1989] is

$$(\delta N_B/N_B)_k \approx H_I/2\pi f\theta_0 , \tag{11.12}$$

where $f$ and $\theta$ are defined by eq. (7.7) and $\theta_0$ is the initial value of the field $\theta$. The factor $1/\theta_0$ appears because it is assumed that the baryon asymmetry is proportional to $\theta_0$. Of course a flat spectrum of fluctuations is not obligatory. As we have seen in the previously considered example, it might even have a strong peak at some particular wave length (11.11) [Dolgov et al. 1987].

An analysis of the isocurvature fluctuations produced by baryogenesis has been done recently by Yokoyama and Suto [1991] and by Sasaki and Yokoyama [1991]. They used the model of baryogenesis (or maybe it is better to say leptogenesis) through the decays of heavy Majorana leptons. An essential assumption of the scenario is the simultaneous operation of two mechanisms of *CP*-violation: an explicit one directly introduced into the Lagrangian and a stochastic one induced by spatially varying the massless Majoron field. The last one is analogous to that proposed by Yoshimura [1983] with the substitution of the Majoron for the axion. This corresponds to a complex scalar field condensate discussed above. In the case of the Majoron field, however, the phase of the Higgs field is unobservable because of the underlying U(1) symmetry (though spontaneously broken) and in order to make this mechanism operative a hard breaking of the U(1) is necessary. The explicit *CP*-breaking produces the homogeneous part of the asymmetry while that connected with the Majoron is responsible for the fluctuations around the average value. If the fluctuations are relatively small, the sign of the asymmetry would remain the same all over the Universe. The fluctuation spectrum depends upon the realization of the inflationary scenario and is not necessarily of the Harrison–Zel'dovich form. In a particular case of power law inflation the model is able to describe the observed structure.

A different mechanism of generation of almost isothermal density perturbations, which does not rely on inflation, has been proposed by Fukugita and Rubakov [1986]. It is based on the observation that $B$-nonconserving electroweak processes (see section 9) are very sensitive to the plasma temperature. So if the process of baryogenesis was accompanied by reheating of the Universe and the temperature of reheating was below but close to the temperature of the electroweak phase transition then small fluctuations in the reheating temperature would induce much larger fluctuations in the baryon number.

Note also that baryogenesis in the framework of the extended inflation scenario [Barrow et al. 1991a] rather naturally gives rise to isocurvature perturbations, which are created because the baryonic charge to entropy ratio depends upon the curvature radius of percolating bubbles when inflation ends.

Recently a model with very strong baryogenesis at small scales and with the usual low amplitude at large scales was considered by Dolgov and Silk [1992a]. The model permits one to explain all the dark matter in the Universe by baryons (and antibaryons). It resembles the island Universe model discussed in the beginning of this section with the difference that the average size and mass of baryonic and antibaryonic islands are much smaller than in the previous case. Outside these islands baryogenesis proceeds in the usual way, giving the known value $\beta = 10^{-9}$–$10^{-10}$, while inside them it is more effective, producing $\beta$ several orders of magnitude higher [up to $\beta = O(1)$]. This can be achieved by spontaneous $CP$-breaking in the first order phase transition during inflation or by the Affleck and Dine [1985] mechanism specially modified so that it is operative only in a tiny fraction of space. This can be realized if there is a coupling of the form (11.4) of the Affleck–Dine field $\chi$ (see section 6) to the inflaton field $\Phi$ with the rest of the potential of the form (11.3) plus (6.1). As a result $\chi$ can develop a large vacuum expectation value only when $\Phi$ is close to $\Phi_1$. The parameters of the model can be chosen so that the probability for $\chi$ to go over the potential barrier and to acquire a large value, is small and correspondingly strong baryogenesis proceeds only in relatively small bubbles. Strong baryogenesis is charge symmetric so there should be an equal number of baryonic and antibaryonic bubbles. Their mass spectrum is given by the expression

$$\frac{\mathrm{d}n}{\mathrm{d}M} = \frac{1}{M_0^4} \exp[-\alpha - \gamma \log^2(M/M_0)] , \tag{11.13}$$

where $M_0$, $\alpha$, and $\gamma$ are arbitrary parameters. It was assumed that $M_0 = 100M_\odot$, $\gamma = O(1)$ and $\alpha$ is normalized so that the total mass of the bubbles is equal to the hidden mass of the Universe. It can be shown that the bulk of the bubbles have formed black holes at a very early stage of the evolution of the Universe. These black holes might be the dominant constituents of the dark matter in the Universe. Very heavy black holes with $M = (10^7$–$10^8)M_\odot$ could form quasars or galactic nuclei. Objects of small mass did not collapse into black holes but would predominantly form stars or antistars. These stars as well as the black holes were formed very early, prior to the large scale structure formation. The stars of the usual baryonic matter would be dead by now while those of antimatter might be observed as cosmological sources of γ-radiation created by the annihilation. If they radiate at the Eddington limit their lifetime should be of the order of 5 billion years. Another possible consequence of this almost baryosymmetric cosmological model is the existence of antimatter (antinuclei) in cosmic rays.

This scenario has an evident shortcoming that the masses of the visible baryonic matter and the dark baryonic matter are not connected (at least in the simple version of the scenario) and they can differ by several orders of magnitude. This is a common shortcoming of all traditional baryogenesis scenarios. An attempt to solve the problem has been made by Dodelson and Widrow in a model of baryogenesis with conserved baryonic charge (section 5). Another approach to the problem has been put forward by Griest and Seckel [1987], Barr et al. [1990], Barr [1992], and Kaplan [1992]. These authors assumed that, except for baryons, there exist new stable massive particles X, possessing a new (quasi)conserved quantum number. In contrast to baryons, these new particles do not interact with light so that they can indeed form the dark matter. If the charge asymmetry in the sector of these particles is approximately of the same size as the baryon asymmetry, the mass of the dark matter is given by the relation

$$\Omega_{DM} = \Omega_B(m_X/m_N)(\beta_X/\beta_B) . \tag{11.14}$$

The model was further detailed by the assumption that there is a new $U(1)_X$ symmetry, which is broken by the chiral anomaly, in the same way as $U(1)_B$, and that the $X$-asymmetry is generated by the electroweak processes along with the baryon asymmetry. This naturally implies similar values of both asymmetries.

In summary, the models of baryogenesis are not only successful in explaining the spatially averaged value of the baryon asymmetry but can also help to solve the long standing problem of creation of initial density inhomogeneities necessary for the formation of large scale structure of the Universe and even to support the idea of baryonic (and antibaryonic) dark matter in the Universe.

## 12. Electric asymmetry of the Universe

The baryon and lepton asymmetries of the Universe considered in the previous sections have the common feature that the corresponding charges are not coupled to massless vector bosons [see eqs. (2.1)]. These charges are presumably not conserved and hence they cannot be coupled to massless vector bosons. This is not the case for electric charge. It is commonly assumed that the electromagnetic gauge invariance is unbroken and correspondingly electric current is strictly conserved and the mass of the photon is zero. This conjecture is supported by the very strong upper limit on the photon mass,

$$m_\gamma < 10^{-28}\ \mathrm{eV} , \tag{12.1}$$

which follows from the existence of coherent galactic fields on scales of about 10 kpc (for a discussion of the bounds on $m_\gamma$ and the corresponding references see, e.g., the review by Dolgov and Zel'dovich [1981]).

If the electric charge is conserved (and was conserved in the past during earlier stages of the evolution of the Universe) there is no way to create or destroy an electric charge asymmetry. The above mentioned mechanisms of baryon asymmetry generation with conserved baryonic charge are not operative in this case just because of long-range forces associated with the massless photons. Thus, if there is a net electric charge in the Universe, it could be created only from the "very beginning" by specific initial conditions. In that sense we return to the old philosophy which existed with respect to the baryon asymmetry when it was believed that baryonic charge was strictly conserved and the Universe had been created with an excess of baryons over antibaryons.

A nonvanishing mean electric charge density, $N_e$, in the Universe is definitely out of the question since it gives rise to an infinitely large energy density. Indeed, from the equation

$$\operatorname{div} E = 4\pi e N_e \tag{12.2}$$

it follows that $E$ is a linearly rising function of the space coordinates and the energy density of the field tends to infinity, $\rho \sim E^2 \to \infty$. Of course in such a model both isotropy and homogeneity are destroyed. If the Universe is closed, $E^2$ would remain finite but the Gauss theorem demands the vanishing of the total charge of a closed Universe. It is really a striking conclusion meaning that a single noncompensated electron in a huge universe would prevent it from closing. It has been argued by Kim and Lee [1990] that a charged closed universe can exist since the gauge invariance of electromagnetism must be broken in these conditions and the photon must acquire a nonzero mass. However, the mechanism of $U(1)_{em}$ breaking remains unclear.

Although when averaged over all the Universe, a nonzero electric charge density is forbidden, it is not excluded that locally $N_e \neq 0$. The sizes of such regions should be larger than the mean free path of charged particles in the cosmic plasma, otherwise the charge excess would be quickly neutralized. The mean free path can be evaluated as

$$l_{\text{free}} = (m_e/T)^{1/2}/\sigma_{\text{Th}} N_\gamma \approx 3.5 \times 10^{25}\ \text{cm}\ (3\ \text{K}/T)^{7/2} \approx 10\ \text{Mpc}\ (3\ \text{K}/T)^{7/2} . \tag{12.3}$$

Here $\sigma_{\text{Th}} = 8\pi\alpha^2/3m_e^2$ is the Thompson cross-section, $N_\gamma = 0.24T^3$ is the photon number density in the background radiation and $m_e$ is the electron mass.

Observational data permit us to put very strong bounds on the local electric charge density. A straightforward limit on the electric charge of an astronomical object follows from the condition that the electric repulsive force should exceed that of gravitational attraction,

$$N_B m_B m_e / m_{Pl}^2 > \alpha N_e , \tag{12.4}$$

where $N_B$ and $N_e$ are, respectively, the baryon and electric charge densities in the object, $m_e$ is the mass of the particles which create the charge excess, $m_B \approx 1$ GeV is the baryon mass, and $\alpha = 1/137$ is the fine structure constant. This results in a very strong limit,

$$N_e/N_B < 10^{-36} m_e/m_B . \tag{12.5}$$

It is valid for any gravitationally bound object in the equilibrium (electrostatic) state. If, however, an electric charge excess is created by inequality of the charges of the electron and the proton, then a much milder bound can be deduced [Lyttleton and Bondi 1959]

$$(N_B m_B)^2/m_{Pl}^2 > \alpha N_e^2 . \tag{12.6}$$

This gives

$$N_e/N_B < 10^{-18} . \tag{12.7}$$

The assumption of unequal charges of the electron and the proton seems to be rather exotic, so in what follows we will dwell on the more conservative case of equal charges.

The strict electrostatic bound (12.5) is not valid if one permits electric currents flowing at galactic, or even larger scales, up to those given by eq. (12.3). The gravitationally bound matter is assumed to be neutral while nonzero $N_e$ is realized by a flow of electrons or protons. A nonzero electric charge density would create, in accordance with eq. (12.2), an electric field which would destroy the observed isotropy in the Universe. A relatively recent analysis of the data, giving upper limits on $N_e$, has been done by Orito and Yoshimura [1985] (there one can find references to earlier papers on the subject). An electric field that is spatially constant on the scale $l_e$ would give rise to anisotropic expansion and in particular to a dipole asymmetry of the electromagnetic background radiation. The latter is known to be $(\Delta T/T)_{\text{dipole}} \approx 2 \times 10^{-3}$. This gives

$$N_e/N_B \leq 4 \times 10^{-20}(10^{28}\ \text{cm}/l_e)\,. \tag{12.8}$$

A much stronger bound can be obtained from the observed isotropy of cosmic rays, which should be distorted by an electric field uniform on the galactic scale. Hence one gets

$$N_e/N_B < 10^{-29}\,. \tag{12.9}$$

In terms of the ratio of the charge density to the entropy density it reads

$$\beta_e = N_e/N_\gamma < 10^{-38}\,. \tag{12.10}$$

Let us now turn to possible mechanisms of generating electric charge asymmetry. If one rejects the idea of eternal charge asymmetry or, maybe better, asymmetry imposed by the initial conditions, one has to adopt the assumption that the gauge invariance of electrodynamics was broken at some stage of evolution of the Universe. During this stage an electric charge asymmetry of the Universe should be generated because the theory possesses all the necessary ingredients for that: current nonconservation, $C$- and $CP$-violation, and expansion of the Universe inducing nonstationarity. There are two possible mechanisms to break the gauge invariance: a spontaneous and an explicit one. Assume first that $\mathrm{U}(1)_{\text{em}}$ was broken spontaneously at high temperatures as has been proposed by Langacker and Pi [1980] for a solution of the magnetic monopole problem. In the case of a spontaneously broken symmetry, the charge asymmetry generated in the sector of the physical particles is exactly compensated by a charge of opposite sign concealed in the vacuum. To demonstrate this, let us consider the equation of motion for the electromagnetic and the charged Higgs field,

$$\partial_\mu F_{\mu\nu} = \mathrm{i}e(\partial_\nu \phi^* \phi - \phi^* \partial_\nu \phi) - 2e^2|\phi|^2 A_\nu + j_\nu^{\text{ext}} \equiv j_\nu^{\text{tot}}\,, \tag{12.11}$$

$$\partial_\mu^2 \phi - \mathrm{i}e(\partial_\mu A_\mu)\phi - 2\mathrm{i}eA_\mu\, \partial_\mu \phi - e^2 A_\mu^2 + \partial U/\partial\phi^* = 0\,, \tag{12.12}$$

where $j_\nu^{\text{ext}}$ is an external electric current created, say, by fermions. Note that the total current $j_\nu^{\text{tot}}$ is automatically conserved since $\partial_\mu \partial_\nu F_{\mu\nu} \equiv 0$. This is a generic feature of a theory with a spontaneously broken gauge symmetry. Because of the conservation of $j_\nu^{\text{tot}}$ the total charge remains zero even after a charge asymmetry in the fermionic sector has developed. When the symmetry is restored the compensating charge reappears from the vacuum in the form of the physical Higgs bosons. The corresponding solution of the equations of motion has the form

$$A_\mu = 0\,, \qquad \phi = \phi_0 \exp(\mathrm{i}mt)\,, \tag{12.13}$$

where the amplitude $\phi_0$ is determined by the value of the charge asymmetry in the fermionic sector $\beta_{ef}$ so that the total charge asymmetry should be identically zero, $\beta_{ef} + \beta_{e\phi} = 0$. Consequently there would be no electric asymmetry even locally if $\beta_{ef}$ were homogeneous. But this is definitely not the case and $\beta_{ef}$ should generically be a function of space due to the mechanisms considered in section 11. Thus after the symmetry restoration the distribution of the heavy Higgs bosons in the Universe should be inhomogeneous. If the mean free path of the energetic decay products of $\phi$ is larger than the size of the inhomogeneities, they stream out of the region of higher $\phi$-density creating a local charge asymmetry. The maximum possible value of the latter is determined by the condition that the corresponding electrostatic potential should be smaller than the energy of the decay products, $E = O(m_\phi)$. This gives

$$\beta_e(R) \le (m_\phi/T)(TR)^{-2} \approx 10^{-25}(m_\phi/m_{Pl})(1\,\mathrm{MeV}/T)(R_{gal}/R)^2 . \tag{12.14}$$

Here $R_{gal}$ is the galactic size and the temperature $T$ corresponds to the moment when the characteristic size of the region with an excess of charged Higgs bosons, $R$, is smaller than the horizon. It is assumed that $R$ scales as $T^{-1}$ so that the quantity $TR$ is constant. The created charge asymmetry should be neutralized by the slower process of discharge in the primeval plasma. Those secondary currents are carried by the thermal particles with a short mean free path. Both the primary and the secondary currents would generate chaotic magnetic fields on astronomical scales. These fields could be large enough to seed the observed magnetic fields in galaxies [Dolgov and Silk 1992b].

Let us now turn to the case of explicit breaking of the gauge invariance. First consider the case when the electromagnetic current is conserved and the breaking of $U(1)_{em}$ manifests itself only in a nonzero mass of the photon. The cosmology of a charged universe with a nonzero mass of the photon was considered by Barnes [1979]. The Maxwell equations in this case are modified as

$$\partial_\mu F_{\mu\nu} + m_\gamma^2 A_\nu = 4\pi j_\nu . \tag{12.15}$$

Now the uniform electric charge density, $j_0 = eN_e$, does not destroy, in contrast to standard electrodynamics, the isotropy and homogeneity of the Universe. Indeed, eq. (12.14) has the solution

$$A_0 = 4\pi j_0/m_\gamma^2 , \tag{12.16}$$

corresponding to zero electric field. So, if the photon has even a tiny but a nonzero mass satisfying condition (12.1), the Universe could have a nonvanishing electric charge density and, what is more, a closed Universe may be electrically charged. The bounds (12.4)–(12.10) are now applicable only to the inhomogeneous part of the charge asymmetry and on the scale below $m_\gamma$, while the homogeneous part of $\beta_e$ may be well above these limits.

A possible way to introduce a nonzero $m_\gamma$ is a nonminimal coupling to the curvature scalar, $\xi R A_\mu^2$. This coupling has been discussed in the literature for a long time and in particular recently by Turner and Widrow [1988] as a driving force for the creation of the cosmic magnetic fields.

An upper limit for the electric charge asymmetry of the Universe can be derived from the condition that the energy density of the electromagnetic potential

$$\rho_A = (4\pi)^2 j_0^2/2m_\gamma^2 \tag{12.17}$$

does not exceed the closure density $\rho_c = m_{Pl}^2/6\pi t^2$. Here $m_\gamma = \xi R = \xi T_\mu^\mu/8\pi m_{Pl}^2$ and $T_\nu^\mu$ is the energy–momentum tensor. $T^\mu_\mu = \rho_c$ at the matter-dominated (MD) stage. Correspondingly the relative energy density is

$$\Omega_A = \frac{\rho_A}{\rho_c} = \frac{2304\pi^5}{\xi} \frac{N_\gamma^2 t^4}{m_{Pl}^2} \beta_e^2 . \tag{12.18}$$

Here $\beta_e$ is the electric asymmetry defined in the usual way as the ratio of the electric charge density to the entropy density, $\beta_e = j_0/N_\gamma$. At the MD stage (and for $\Omega = 1$)

$$N_\gamma^2 t^4/m_{Pl}^2 \approx 10^{52}(t_U/5\times 10^{17}\ \mathrm{s}) = \text{const.} ,$$

where $t_U$ is the age of the Universe. Correspondingly we get $\Omega_A \approx 10^{58}\beta_e^2\xi^{-1}$. Hence for $\beta_e \approx 10^{-29}$ and $\xi = O(1)$ the energy density associated with the electric charge asymmetry could provide $\Omega = 1$. This would be a rather unusual form of dark matter.

Note that at the MD stage $R \sim T^\mu_\mu \sim a^{-3}$ and $\rho_A \sim j_0^2 R^{-1} \sim a^{-3}$, where $a$ is the scale factor in the Robertson–Walker metric. A massive vector field with a constant mass is known [Zel'dovich 1961] to create the stiffest equation of state $p = \rho$. This results in $\rho \sim a^{-6}$ and $a \sim t^{1/3}$. This is not true in the case considered since $m_\gamma^2 \sim R \to 0$ and the expansion regime is matter dominated, $p = 0$. This can be seen in particular from the Einstein equations with $(\xi R A_\mu^2 + j_\mu A^\mu)$-term taken into account.

As we have mentioned above, an electric charge asymmetry could be generated if the current is not conserved due to an explicit (in contrast to spontaneous) breaking of the $U(1)_{em}$ symmetry. A mechanism for that was considered by Orito and Yoshimura [1985] in a cosmological model based on a multidimensional theory of the Kaluza–Klein type. The extra dimensions in this theory are supposed to compactify and the geometrical symmetry of the compactified space manifests itself in our world of large space dimensions as a gauge symmetry. As was assumed by Orito and Yoshimura, the extra dimensions were not compactified at an early stage (or alternatively, the manifold of extra space dimensions was not as symmetric as it is now), and because of that the $U(1)_{em}$ symmetry was explicitly broken. Electric charge asymmetry could be generated during this stage. If it does not vanish prior to the restoration of the gauge invariance, the arguments presented above show that $U(1)_{em}$ can never be restored and the photon remains massive. Electric current nonconservation, which is possible in principle in that case, should completely die out. Otherwise there would be serious theoretical problems due to nonrenormalizability of the theory. As was argued by Voloshin and Okun [1978] the strong infrared (ultraviolet?) catastrophe in the emission probability of the longitudinal photons by a nonconserved current would result in an exponential suppression of the corresponding amplitudes, $A \sim \exp(-cE_\gamma^2/m_\gamma^2)$, where $c$ is a constant. This would restore current conservation for photon energies above $m_\gamma$. Current nonconservation might be effective only for $E \lesssim m_\gamma$.

## 13. Conclusion

As we have seen there are plenty of models in the market satisfying a variety of tastes. All of them could give $\beta = 10^{-9}$–$10^{-10}$ with large proton lifetime, $\tau_p > 10^{32}$ yr. Low temperature baryogenesis is now in fashion so that the constraints imposed on the (re)heating temperature in inflationary models are satisfied. Models without $B$-nonconservation, or without $C$- and $CP$-violation, or without breaking of thermal equilibrium are possible.

Inflation opens a new possibility of a charge symmetric Universe as a whole with B and $\bar{B}$ separated by large distances. Early baryosynthesis models with spontaneous or stochastic $C$ and $CP$-breaking at the inflationary stage naturally predict a (quasi)periodic distribution of matter in the Universe. The characteristic scale is model dependent and the recently observed structure might be compatible with it. A more natural and more exciting possibility is that of alternating baryonic and antibaryonic layers. If this is the case, care should be taken of angular variation of the microwave background temperature. The latter would be sufficiently small in the spherically symmetric case if the Earth is in the center of the world as the archaic people believed. An interesting question in this connection is whether it is possible to distinguish antibaryonic from baryonic matter at a distance of about 100 Mpc. In any case new models of baryosynthesis naturally give rise not only to adiabatic (as was always the case) but also to isocurvature perturbations.

A prediction testable by observations, indirectly connected with baryogenesis, is the galactic scale variation of the leptonic asymmetry, which could give rise to spatial variations of the primordial abundances of light elements. Except for these striking possibilities there are seemingly no chances to distinguish between the above discussed models by astronomical observations. Particle physics might slightly help in this respect if baryonic charge nonconservation at the TeV energy scale is observed.

The fact that we have a large number of models has its dark side, meaning that we do not know which one is indeed operative. It is even possible that the Occam razor principle is violated and several periods of baryosynthesis existed in the course of the evolution of the Universe. This of course does not make things easier for theorists. Probably the model of electroweak baryogenesis is in the best shape though a few points of the model deserve clarification. Electroweak baryogenesis has the nice quality that it permits at least in principle the creation of baryon asymmetry remaining in the framework of the standard $SU(3) \times SU(2) \times U(1)$ model. Unfortunately the experimental bounds on the Higgs boson mass tend to exclude the minimal standard model and an extension of the latter is necessary. So in some sense baryon asymmetry of the Universe is an indication of physics beyond the standard model.

The latter issue is often discussed at particle physics conferences but at least up to now all experimental data for elementary particles can be explained by the minimal standard model. What presents very strong arguments in favor of physics beyond the standard model is cosmology. With the possible exception of baryogenesis there are several cosmological phenomena which are not compatible with the standard model. First of all it is inflation. If the electroweak phase transition was of first order some inflation could be realized even in the standard model but a satisfactory inflationary scenario in this framework is not known (at least to me). Another cosmological problem which demands new physics is the problem of the hidden mass. The latter might be explained by a nonzero neutrino mass, $m_\nu = O(10\,\text{eV})$ but the distribution of the visible as well as of the dark matter resists this simple hypothesis. The theory of the formation of large scale structure of the Universe cannot describe the observed picture neither in a two-component baryonic–photonic Universe nor with the addition of massive neutrinos.

In the last decade there has been a great deal of progress in our understanding of the physics of initial density inhomogeneities. It has been found in particular that physical processes during baryogenesis might play an important role in the formation of the inhomogeneities. Still, an extension of the standard model to this end is necessary. A related problem is that of early quasar creation. The existence of a number of quasars near $z = 5$ is not easily swallowed with any present model based on standard physics. Hopefully the quasar energetics can be explained by standard physics (black hole accretion) but there are also some unresolved problems.

Last but not the least in this list is the cosmological constant problem, which demonstrates a singular example of 100(!) orders of magnitude discrepancy between theoretical expectations and astronomical data. This is probably the central problem of both cosmology and (quantum?) field theory, which definitely cannot be resolved in the standard model.

Ten years ago in our review with Zel'dovich, we put the origin of the primordial density fluctuations as one of the most important cosmological mysteries. It has been resolved now at least in principle. It will be interesting to find out whether any of the above mentioned problems is resolved in the next decade and in particular if a single and commonly accepted model of baryogenesis is found.

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