THE CHIRAL ANOMALY AND A CLASS OF TWO-LOOP FINITE SUPERSYMMETRIC GAUGE THEORIES

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By means of the supercurrent anomaly we derive a formula for the two-loop β-function of the matter self-interactions in an arbitrary massless $N = 1$ supersymmetric gauge theory. As a trivial consequence we show that if such a theory is one-loop finite it is also two-loop finite, and we discuss whether this finiteness persists to all orders.

Recently it has been shown how the interplay between the Adler–Bardeen theorem and the supercurrent anomaly allows a determination of the gauge β-function β_g to all orders in the case of pure (no matter fields) $N = 1$ supersymmetric Yang–Mills (SSYM) [1, 2], and for $N = 2$ SSYM including an arbitrary number of hypermultiplets [2]. In a previous note [3] we showed how an extension of the analysis of the $N = 1$ case to include a general matter multiplet with arbitrary self-interactions led to a determination of β_g at the two-loop level, $\beta_g^{(2)}$, in an arbitrary supersymmetric gauge theory. Here we further show that the β-functions associated with the matter self-interactions, β_c, can be calculated by the same technique, and we derive a formula for $\beta_c^{(2)}$. Both $\beta_g^{(n)}$ and $\beta_c^{(n)}$ can be determined by an $(n-1)$ loop calculation, in the general case.

We then show that, if $\beta_g^{(1)}$ and the anomalous dimension matrix of the scalar superfields at the one-loop level $\gamma^{(1)}$ both vanish then $\beta_g^{(2)}$ and $\gamma^{(2)}$ also both vanish. (We assume supersymmetric regularisation and gauge fixing, in which case β_c is directly related to γ by the non-renormalisation theorem; note that $\gamma = 0$ is sufficient, but not necessary, to ensure $\beta_c = 0$.) We then show that a naive extension of these results to all orders fails; we are unable to show that $\beta_g^{(1)} = \gamma^{(1)} = 0$ is sufficient to ensure $\beta_g = \gamma = 0$. Nevertheless it is intriguing to speculate that there may be a set of finite theories which contains, but is larger than the $N = 2$ and $N = 4$ finite theories discussed previously [4]. Specifically, the occurrence of fermions in pairs of opposite chirality characteristic of $N = 2$ supersymmetry can then be avoided, so this class of theories may well be of interest to model-builders.

We consider $N = 1$ SSYM with a matter multiplet (ϕ) transforming according to a (in general reducible) representation R of the gauge group and with matter self-interactions described by the superpotential

$$W = \tfrac{1}{6} C^{ijk} \phi_i \phi_j \phi_k . \tag{1}$$

The supercurrent $J_{\alpha\dot\alpha}$ is given by

$$J_{\alpha\dot\alpha} = J^V_{\alpha\dot\alpha} + J^\phi_{\alpha\dot\alpha}, \tag{2}$$

where

$$J^V_{\alpha\dot\alpha} = \bar W_{\dot\alpha} W_\alpha \tag{3}$$

and

$$J^\phi_{\alpha\dot\alpha} = -\tfrac{1}{3} \bar{\mathrm{D}}_{\dot\alpha}\bar\phi\, \mathrm{D}_\alpha \phi + \tfrac{1}{3} \bar\phi\, \mathrm{i} \overleftrightarrow{\mathrm{D}}_{\alpha\dot\alpha} \phi . \tag{4}$$

J has an anomaly given by [5–15]

$$\bar{\mathrm{D}}^{\dot\alpha} J_{\alpha\dot\alpha} = -\tfrac{1}{3} (\beta_g/g)\, \mathrm{D}_\alpha [W^2] - \tfrac{1}{3} \gamma^i_j \mathrm{D}_\alpha [X^j_i], \tag{5}$$

where

$$X^j_i = \phi_i \bar{\mathrm{D}}^2 \bar\phi^i .$$

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At the one loop level we have

$$\beta_g^{(1)} = (g^3/16\pi^2)\,[T(R) - 3C_2(G)], \tag{6}$$

$$\gamma_j^{i(1)} = \tfrac{1}{2}[F_j^i - 4g^2 C_2(R_\beta)(E_\beta)_j^i], \tag{7}$$

where

$$(R^a R^a)_j^i = \sum_\beta C_2(R_\beta)(E_\beta)_j^i,$$

$$\mathrm{tr}(R^a R^b) = T(R)\,\delta^{ab} = \sum_\beta T(R_\beta)\,\delta^{ab},$$

$$C^{imn} C^*_{jmn} = F_j^i, \quad C_2(G)\,\delta^{ab} = f^{acd} f^{bcd}.$$

f^{abc} are the group structure constants, and E_β is the projector onto the irreducible representation R_β.

The matter β-function β_c^{ijk} is given by

$$\beta_c^{ijk} = \gamma_l^{(i} C^{jk)l} \tag{8}$$

and was given at the one-loop level, in, for example, ref. [16].

Now as described in ref. [2] we can define a new current J' whose first component obeys the Adler-Bardeen theorem:

$$J' = J + \Delta J, \tag{9}$$

where

$$\Delta J = \tfrac{2}{3}(\beta_g^{(1)}/g)\,J^V + 2\gamma_j^{i(1)}[(J^\phi)_i^j - \tfrac{1}{2}\bar{\phi}^j \mathrm{i} D_{\alpha\dot{\alpha}}\phi_i]. \tag{10}$$

Evaluating one loop matrix elements [10] of ΔJ and using eq. (5) at the two-loop level we obtain from the coefficient of $D_\alpha[W^2]$:

$$16\pi^2 \beta_g^{(2)} = 2\beta_g^{(1)} g^2 C_2(G) - 2\gamma_j^{i(1)} g^3 T(R_\beta)(E_\beta)_i^j/d(R_\beta) \tag{11}$$

as in ref. [3], and from the coefficient of $D_\alpha[X_i^j]$:

$$16\pi^2\,\gamma_l^{(2)k} = 2g\beta_g^{(1)}\,C_2(R_\beta)(E_\beta)_l^k$$
$$-\,\gamma_j^{(1)i}[C^{jmk}C^*_{iml} + 2g^2 (R^a)_i^k (R^a)_l^j]. \tag{12}$$

The case of $N = 2$ supersymmetry including matter fields corresponds to

$$W = 2^{-1/2} g\,\chi^j (R^a)_j^i \phi_i \xi^a, \tag{13}$$

where ϕ, χ, ξ transform according to the R, $\bar{R}$ and adjoint representations respectively. It is a straightforward exercise to verify that $\gamma_l^{(2)k} = 0$ in that case, in accordance with previous results [4]. One can also extract γ for the Wess–Zumino model by setting $g = 0$, $C^{111} = \lambda$ and we obtain

$$\gamma = \beta_\lambda/3\lambda = (\lambda^2/32\pi^2)(1 - \lambda^2/16\pi^2 + \ldots), \tag{14}$$

in accordance with the original calculation [17].

The significance of the forms of eqs. (11) and (12) is that trivially if $\beta_g^{(1)} = \gamma^{(1)} = 0$ then also $\beta_g^{(2)} = \gamma^{(2)} = 0$. Thus a supersymmetric gauge theory that is one loop finite is also two-loop finite!

This class of theories contains but is not restricted to the previously discussed $N = 2, 4$ theories. For example, the superpotential

$$W = [gN/(2N^2 - 8)^{1/2}]\, d^{abc}\phi_1^a \phi_2^b \phi_3^c$$

defines a two-loop finite theory in SU(N). (This model has the same field content as the $N = 4$ theory but with d-type instead of the f-type couplings necessary for $N = 4$ supersymmetry.) One can also construct examples with non-chirally-self-conjugate representations.

It might appear that, since in such a case we have $\Delta J = 0$, the argument extends to all orders. It is easy to see however that this is not so, naively at least. Consider the case when ϕ_i are gauge singlets: a generalized Wess–Zumino model. We would expect that $\gamma_j^i = \gamma_l^{k(1)} T_{kj}^{li}$ where T is some tensor; in other words that there are no contributions to γ from graphs with no one-loop self-energy sub-graphs. At the three loop level one sees that this is not the case [18]. Therefore in the language of our paper it seems that additional subtractions beyond ΔJ (eq. (10)) are required at the three-loop level, and we are for the moment unable to generalize the argument to all orders. Nevertheless it remains possible that the class of finite supersymmetric theories is larger than the $N = 2, 4$ classes previously discussed. This is a sufficiently interesting possibility both theoretically and because of the additional freedom it would afford finite model-builders that it deserves serious investigation. A three-loop calculation of β_g and β_c in this class of models is perfectly feasible, and will undoubtedly be performed soon.

While this paper was in preparation we received a number of relevant preprints. In ref. [19] $\beta_g^{(2)}$ is extracted from the explicit calculation in a general gauge theory of ref. [20] and it is verified by explicit computation that $\gamma^{(2)} = \beta_g^{(2)} = 0$, if $\gamma^{(1)} = \beta_g^{(1)} = 0$ in agreement with results presented here. In ref. [21] $\gamma^{(2)}$ is found by an explicit two-loop superfield calculation for a general gauge theory. We once again emphasize, however, that the derivation here of $\gamma^{(2)}$ and that of $\beta_g^{(2)}$ in ref. [3] require only one-loop calculations.

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Note added. After this paper was submitted our attention was drawn to ref. [22] which also discusses the pure (no matter) $N = 1$ case.

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