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ALGEBRA OF STRONG AND ELECTROWEAK INTERACTIONS

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An algebraic approach to a description of electroweak and strong interactions is examined in the context of binary geometric physics based on the principles of the Fokker–Feynman direct interparticle interaction theories, the Kalutsa–Klein multidimensional geometrical models, and the theory of physical structures. It is demonstrated that in this approach, the electroweak and strong interactions of elementary particles through intermediate vector bosons are characterized by subtypes of the algebraic classification of complex 3×3 *matrices suggested by A. Z. Petrov for the classification of the Einstein's spaces.*

INTRODUCTION

In the present paper, the theory of electroweak and strong interactions, alternative to generally accepted calibration theory, is developed within the framework of *binary geometric physics* [1–4] based on the concept of long-range interaction, a number of multidimensional geometrical models [5], and the S-matrix approach of quantum theory. In this approach, there is no *a priori* set space-time, and hence the method of localization of internal symmetry groups used in calibration theory does not hold. The given approach is based on *the theory of binary systems with complex relations* being a generalization of the Kulakov theory of binary structures [6]. The principles of binary geometric physics have been considered in detail in the special literature (for example, see [1–4]). Here we only recall the principles employed in the present work aimed to demonstrate that in the given approach, the well-known channels of electroweak and strong interactions (through intermediate vector bosons) are characterized by different subtypes of the algebraic classification of complex 3×3 matrices of the so-called internal parameters comprising three elements and describing elementary particles (quarks and leptons).

1. PRINCIPLES OF BINARY GEOMETRIC PHYSICS

Two sets of elements corresponding to initial and final states of a physical system provide the basis for binary geometric physics (binary systems with complex relations), and pair relations (complex numbers) between any pair of elements from the two sets are specified. *An algebraic law*, that is, a universal function with arguments being *pair relations* $u_{i\alpha}$ among r elements of one set and s elements of another set is postulated. The numbers r and s specify *the rank* (r, s) *of the binary system with complex relations* (BSCR). The rank (r, r) describes physical interactions. *The basis* (being an analog of the classical reference frame) comprising $r - 1$ elements is then introduced, and each element of the set is characterized by $r - 1$ parameters specifying relations with the basis elements. *The basic symmetry* implying the fulfillment of the law for arbitrary elementary bases is postulated.

In the BSCR with rank (r, r), a particular algebraic expression – the so-called *basic* $r \times r$ *relation* that comprised antisymmetrically r elements of each set and is invariant with respect to the $SL(r-1, C)$ transformation group – is unambiguously determined. In binary geometric physics this expression is the inverse image of the action (Lagrangian) of the particle interaction and simultaneously the inverse image of the multidimensional metric (in the geometrical approach) as well as the inverse image of the S-matrix of quantum theory.

The basic representation of the $SL(r - 1, C)$ group is realized in the space of BSCR element parameters;

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for this reason, the parameters characterizing each individual element are the Finsler $(r-1)$-spinor [7]. It was demonstrated that for a description of space-time geometry and physical interactions, it is sufficient to put the BSCR rank equal to (6,6) (that is, to set $r = 6$).

Elementary particles (leptons and quarks) in binary geometric physics are described by 3 BSCR elements on which specific conditions are imposed. Since each BSCR element with rank (r, r) is characterized by 5 parameters, the elementary particle in each state (initial or final) will be described by a complex 3×5 matrix of the parameters of these elements. According to the Kalutsa–Klein theory, the parameters are subdivided into two external parameters that characterize the spins and momenta of particles and three internal parameters that determine the particle charges for the electroweak and strong interactions. In fact, the initial BSCR with rank (6,6) is subdivided into two BSCRs with ranks (3,3) describing the momenta and with ranks (4,4) describing the particle charges. Recall that additional components of the momenta in the Kalutsa–Klein multidimensional geometric models also determine the charges of elementary particles. Thus, the 3×3 charge matrices characterizing the interaction of particles are separated from the 3×5 matrices describing particles. Exactly these 3×3 charge matrices are studied in the present paper.

The three-component columns of the charge 3×3 matrices are generalizations of the two-component spinors. They are subjected to the $SL(3, C)$ group of 16-parametric transformations. A cubic form

$$\begin{vmatrix} i^1 & k^1 & j^1 \\ i^2 & k^2 & j^2 \\ i^3 & k^3 & j^3 \end{vmatrix} = i^1k^2j^3 + i^3k^1j^2 + i^2k^3j^3 - i^3k^2j^1 - i^2k^1j^3 - i^1k^3j^2, \tag{1}$$

defined for three 3-component columns is invariant with respect to these transformations. The coefficients C^l_s of transformations of the $SL(3, C)$ group meet the condition

$$C^1_1C^2_2C^3_3 + C^3_1C^1_2C^2_3 + C^2_1C^3_2C^1_3 - C^3_1C^2_2C^1_3 - C^1_1C^3_2C^2_3 - C^2_1C^1_2C^3_3 = 1, \tag{2}$$

generalizing the condition $C^1_1C^2_2 - C^1_2C^2_1 = 1$ well known in the theory of 2-component spinors subjected to transformations of the $SL(2, C)$ group. For this reason, the 3-columns are referred to as the *3-component Finsler spinors* [7].

The basic 6×6 relations describe interactions of a pair of particles. Two particles can interact only when they have different matrices of internal parameters; otherwise, interaction will not occur, because the characteristic determinants of the basic relation will vanish. In other words, particles must have different internal states. Two states are introduced: the so-called ground or U-state with the nondegenerate matrix of internal parameters and a number of excited X-states (for example, in quantum chromodynamics hadrons interact through 8 different gluons). *In this approach, interactions are exchange in character*, interacting particles exchange their U- and X-states, and the structure of their 3×3 matrices determines the interaction type.

2. DESCRIPTION OF STRONG QUARK INTERACTIONS

In the case of strong interactions, the U-state is characterized by the 3×3 matrix of the form

$$U = [\boldsymbol{c}_{(1)}, \boldsymbol{c}_{(2)}, \boldsymbol{c}_{(3)}], \tag{3}$$

where the columns $\boldsymbol{c}_{(i)}$ are linearly independent by definition; they are the Finsler 3-spinors [8], and their components are the internal parameters of the particle. In this case, the nondegeneracy of the U-state is postulated: $\det U \neq 0$.

Let us consider 3×3 matrices of the internal quark parameters in the X-state in the case of strong interactions. The so-called A- and B-channels are responsible for interactions through neutral gluons corresponding to the third and eighth Hell–Mann diagonal matrices – generators of the $SU(3)$ group. In [2] it was demonstrated that these channels of strong interactions can be described if we assume that the matrix of the neutral X-state is formed by three collinear columns $\boldsymbol{c}'_{(i)}$, each being proportional to the nonzero vector-column $\boldsymbol{c}'$ with the coefficient C'_i:

$$X = [C'_1\boldsymbol{c}', C'_2\boldsymbol{c}', C'_3\boldsymbol{c}']. \tag{4}$$

The column $\boldsymbol{c}'$ can be resolved into three noncoplanar columns $\boldsymbol{c}_{(i)}$ of the U-matrix, and (considering the symmetry) the coefficients can be put identical, for example, equal to unity:

$$\boldsymbol{c}' = \sum_{i=1} \boldsymbol{c}_{(i)}. \tag{5}$$

In [2] it was demonstrated that from the 6×6 basic relation, the inverse image of the action of the strong interaction of two quarks through neutral gluons is written in the form

$$S_{\text{int}} = \frac{1}{2} D^2_{(S)} \times \sum_{s,r=1} (u^{\mu}_{1(s)} u_{2(r)\mu}) c_{(s)(r)}, \tag{6}$$

where $D_{(S)} = \det U$, $u_{1(s)\mu}$ is the four-current of the sth element (quark) to within a constant, and the quantities $c_{(s)(r)}$ form the 3×3 symmetrical matrix whose components are the above-introduced coefficients C'_i. These coefficients are included as pair differences, and hence only two of them are independent. Let us proceed to new coefficients C' and $\widetilde{C}'$:

$$C'_2 - C'_3 = C' + \widetilde{C}', \quad C'_3 - C'_1 = -C' + \widetilde{C}', \quad C'_1 - C'_2 = -2\widetilde{C}'. \tag{7}$$

As can be seen from Eq. (6), the interaction is current-current in character in agreement with the generally accepted principles. The A- and B-channels differ by values of the coefficients C' and $\widetilde{C}'$: $\widetilde{C}' = 0$ and $C' \neq 0$ for the A-channel, whereas $\widetilde{C}' \neq 0$ and $C' = 0$ for the B-channel.

As demonstrated in [2, 4], to describe strong interactions through charged gluons, it is sufficient to construct a matrix of the X-state from the matrix of the U-state by changing the sign of one or two columns (there are 6 variants corresponding to the number of charged gluons). The inverse of the interaction Lagrangian has the form analogous to Eq. (6).

In the case of transitions between physically equivalent bases (reference systems), the columns of matrices of U- and X-states are transformed as the Finsler 3-spinors, that is, through of the basic representation of the $SL(3,C)$ group. It is important that C'_i (and hence C' and $\widetilde{C}'$) remain invariant under the $SL(3,C)$ transformations.

3. ELECTROWEAK QUARK INTERACTIONS

Within the framework of binary geometric physics, electroweak interactions are described by another particular case of 3×3 matrices of X-states of elementary particles [2, 4], when one row of the matrix of internal particle parameters is fixed and its components are set equal to a constant $c_3 \neq 0$. Thus, for the U-state we have

$$U^{(s)} = \begin{pmatrix} c^1_{(1)} & c^1_{(2)} & c^1_{(3)} \\ c^2_{(1)} & c^2_{(2)} & c^2_{(3)} \\ c^3_{(1)} & c^3_{(2)} & c^3_{(3)} \end{pmatrix} \to U^{(w)} = \begin{pmatrix} c^1_{(1)} & c^1_{(2)} & c^1_{(3)} \\ c^2_{(1)} & c^2_{(2)} & c^2_{(3)} \\ c_3 & c_3 & c_3 \end{pmatrix}. \tag{8}$$

The matrices of excited X-states are subjected to the same modification with the replacement $c_3 \to c'_3$.

In this case, the general $SL(3,C)$ group of the Finsler 3-spinor transformations is truncated to the $SL(2,C)$ group (which is further reduced to the $SU(2)$ group of electroweak interactions), acting already in the space of 2-component columns:

$$\boldsymbol{c}_{(i)} = \begin{pmatrix} c^1_{(i)} \\ c^2_{(i)} \end{pmatrix}, \qquad i = 1, 2, 3. \tag{9}$$

The structure of X-matrices can be described by resolving the 2-component columns into two linearly independent complex vectors $\boldsymbol{q}$ and $\boldsymbol{l}$: $\boldsymbol{c}'_{(i)} = C'_i \boldsymbol{q} + K'_i \boldsymbol{l}$. Furthermore, by virtue of the fact that the coefficients C'_i and K'_i enter the inverse image of the Lagrangian of electroweak interactions as pair differences, it is expedient to proceed to an ensemble of independent quantities $\{C'_i\} \to \{C', \widetilde{C}'\}$ and $\{K'_i\} \to \{K', \widetilde{K}'\}$ using the formulas analogous to Eq. (7).

The A- and B-channels of the electroweak interaction are determined by analogy with the strong interaction. The values $\widetilde{C}' = \widetilde{K}' = 0$, $C' \neq 0$, and $K' \neq 0$ correspond to the A-channel, whereas $C' = K' = 0$, $\widetilde{C}' \neq 0$, and $\widetilde{K}' \neq 0$ correspond to the B-channel. Here we consider only neutral channels, and the A- and B-channels are interpreted as an exchange by photons and Z-bosons, respectively.

Thus, the universal principle of describing the strong and electroweak interactions is suggested in binary geometric physics.

4. NINE-DIMENSIONAL VECTORS AND BASIC INTERACTIONS

The above-described neutral channels of strong and electroweak interactions can be interpreted in terms of vectors of the effective 9-dimensional vector space. To this end, we note that the real *components of the 9-dimensional vector* are determined by analogy with the 4-dimensional vectors constructed from quadratic combinations of the 2-component spinors (bispinors) of the BSCR with rank $(4,4)$ in terms of three pairs of complex conjugate parameters (i^s, α^s), (k^s, β^s), and (j^s, γ^s):

$$
\begin{aligned}
B^0_{(3)} &\equiv P^0 = \frac{1}{2}(i^1\alpha^1 + i^2\alpha^2 + k^1\beta^1 + k^2\beta^2 + j^1\gamma^1 + j^2\gamma^2),\\
B^1_{(3)} &\equiv P^1 = \frac{1}{2}(i^1\alpha^2 + i^2\alpha^1 + k^1\beta^2 + k^2\beta^1 + j^1\gamma^2 + j^2\gamma^1),\\
B^2_{(3)} &\equiv P^2 = \frac{i}{2}(i^1\alpha^2 - i^2\alpha^1 + k^1\beta^2 - k^2\beta^1 + j^1\gamma^2 - j^2\gamma^1),\\
B^3_{(3)} &\equiv P^3 = \frac{1}{2}(i^1\alpha^1 - i^2\alpha^2 + k^1\beta^1 - k^2\beta^2 + j^1\gamma^1 - j^2\gamma^2),\\
B^4_{(3)} &\equiv P^4 = \frac{1}{2}(i^1\alpha^3 + i^3\alpha^1 + k^1\beta^3 + k^3\beta^1 + j^1\gamma^3 + j^3\gamma^1),\\
B^5_{(3)} &\equiv P^5 = \frac{i}{2}(i^1\alpha^3 - i^3\alpha^1 + k^1\beta^3 - k^3\beta^1 + j^1\gamma^3 - j^3\gamma^1),\\
B^6_{(3)} &\equiv P^6 = \frac{1}{2}(i^2\alpha^3 + i^3\alpha^2 + k^2\beta^3 + k^3\beta^2 + j^2\gamma^3 + j^3\gamma^2),\\
B^7_{(3)} &\equiv P^7 = \frac{i}{2}(i^2\alpha^3 - i^3\alpha^2 + k^2\beta^3 - k^3\beta^2 + j^2\gamma^3 - j^3\gamma^2),\\
B^8_{(3)} &\equiv P^8 = i^3\alpha^3 + k^3\beta^3 + j^3\gamma^3.
\end{aligned}
\tag{10}
$$

From the law of $SL(3,C)$ transformations of the parameters (3-component spinors), the vector law of transformations of the components B^A in the 9-dimensional manifold $B'^A = L^A_{\cdot B} B^B$ can be easily derived, where the coefficients of linear transformations $L^A_{\cdot B}$ are expressed through quadratic combinations of the coefficients C^r_s and C^{*r}_s.

The *basic* 3×3 *relation* constructed from the pair relations of the form $u_{i\alpha} = i^1\alpha^1 + i^2\alpha^2 + i^3\alpha^3$, important in the theory of BSCR with rank (4,4), can be expressed as a cube of the 9-dimensional vector with components specified by Eq. (10):

$$
\begin{aligned}
\begin{vmatrix} u_{i\alpha} & u_{i\beta} & u_{i\gamma} \\ u_{k\alpha} & u_{k\beta} & u_{k\gamma} \\ u_{j\alpha} & u_{j\beta} & u_{j\gamma} \end{vmatrix} = \begin{vmatrix} i^1 & k^1 & j^1 \\ i^2 & k^2 & j^2 \\ i^3 & k^3 & j^3 \end{vmatrix} \times \begin{vmatrix} \alpha^1 & \beta^1 & \gamma^1 \\ \alpha^2 & \beta^2 & \gamma^2 \\ \alpha^3 & \beta^3 & \gamma^3 \end{vmatrix} &= G_{ABC} B^A_{(3)} B^B_{(3)} B^C_{(3)} \\
= B^8\left((B^0)^2 - (B^1)^2 - (B^2)^2 - (B^3)^2\right)& \\
+2B^1\left(B^4B^6 + B^5B^7\right) - B^0\left((B^4)^2 + (B^5)^2 + (B^6)^2 + (B^7)^2\right)& \\
+2B^2\left(B^5B^6 - B^4B^7\right) + B^3\left((B^4)^2 + (B^5)^2 - (B^6)^2 - (B^7)^2\right).&
\end{aligned}
\tag{11}
$$

This expression comprising cubes of vector components is substituted for the 4-dimensional relativistic invariant quadratic form $g_{\mu\nu}B^\mu B^\nu$. This invariant is generally nonzero for the 9-dimensional vectors defined on three pairs of elements.

When the vector $B^A_{(2)}$ is defined on two pairs of conjugate elements, invariant (11) is identically equal to zero. Therefore, these vectors can be referred to as (singly) *isotropic* in the theory of BSCR with rank (4,4). Its 4-dimensional invariant corresponding to the first row of Eq. (11) is nonzero.

When the vector $B^A_{(1)}$ is defined on one pair of conjugate elements, the invariant is again equal to zero. It is a sum of two quantities each being equal to zero. In particular, this is the 4-dimensional invariant of the first row of Eq. (11). For this reason, the 9-dimensional vector constructed from one pair of conjugate elements can be referred to as *doubly isotropic*.

Based on the foregoing, we can state that *the neutral channels of strong interactions are described by* 3×3*-matrices of the* X*-state of particles, and the doubly isotropic 9-dimensional vector is constructed from the Finsler* 3*-spinors of these matrices. The charged channels of strong and electroweak interactions are described by* 3×3*-matrices of the* X*-state, and the nonisotropic vectors are constructed from these matrices, whereas the neutral channels of electroweak interactions are characterized by* 3×3*-matrices of the* X*-states resulting in the singly isotropic* 9*-dimensional vectors.*

5. ALGEBRAIC CLASSIFICATION OF QUADRATIC MATRICES

The essence of the above-described channels of strong and electroweak interactions can be understood in the context of the algebraic classification for 3×3-matrices of the X-states of particles. To this end, we recall the main points of the algebraic classification of square complex matrices. There are several methods of its presentation. We took advantage of the method used by Petrov [8] when he analyzed Einstein's spaces. This method is reduced to the algebraic classification of the 3×3-matrix comprising a complex combination of the Riemann–Christoffel or Weyl tensor components. The well-known Penrose–Petrov diagram illustrates this classification (Fig. 1).

The so-called λ-matrices are classified. Recall that the λ-matrix is taken to mean the matrix the components of which are polynomials of a certain, generally speaking, complex parameter λ. Any such matrix by the so-called *elementary* transformations (including multiplication of a matrix row by a nonzero number or addition of other rows multiplied by an arbitrary polynomial) can be reduced to the canonical form in which this matrix will comprise λ-polynomials only along the main diagonal, and each subsequent polynomial is divided into the preceding one (or is equal to zero). These polynomials are called *invariant dividers* of the matrix and are denoted by $E_k(\lambda)$. (Hereinafter we deal with 3×3-matrices, so that $k = 1, 2, 3$.) Each invariant divider can be resolved into binomial factors (each ith root of the polynomial $E_k(\lambda)$ is the eigenvalue of the λ-matrix of multiplicity α_{ki}):

$$E_k = (\lambda - \lambda_1)^{\alpha_{k1}} \dots (\lambda - \lambda_r)^{\alpha_{kr}} = \prod_{i=1}^{r} (\lambda - \lambda_i)^{\alpha_{ki}}. \tag{12}$$

In the case of three-row matrices, there are 6 canonical forms that cannot be transformed into each other using elementary transformations; therefore, 6 subtypes of λ-matrices are identified, and each subtype is completely characterized by its canonical form. Six subtypes form three algebraic types T_1, T_2, and T_3 corresponding to three columns in Fig. 1. Each subtype has the following representation:

$$[(\alpha_{11} \dots \alpha_{k1} \dots \alpha_{N1}) \dots (\alpha_{1r} \dots \alpha_{kr} \dots \alpha_{Nr})], \tag{13}$$

where

$$\sum_{k=1}^{N} \sum_{i=1}^{r} \alpha_{ki} = N, \tag{14}$$

N is the order of the λ-matrix, and r is the number of eigenvalues. Each subtype is designated by special symbols indicated in the Penrose–Petrov diagram (Fig. 1).

The problem of algebraic classification of the matrix A is reduced to its comparison with the characteristic λ-matrix ($(A(\lambda)_{ik} = A_{ik} - \lambda \delta_{ik}$) and finding its canonical form.

$$
\begin{tikzcd}
& & \mathrm{I}[(1)(1)(1)] \arrow[dl] \arrow[d] \\
& \mathrm{II}[(2)(1)] \arrow[dl] \arrow[d] & \mathrm{D}[(1,1)(1)] \arrow[l] \arrow[dl] \arrow[d] \\
\mathrm{III}[(3)] & \mathrm{N}[(1,2)] \arrow[l] & \mathrm{O}[(1,1,1)] \arrow[l]
\end{tikzcd}
$$

(T_3) (T_2) (T_1)

Fig. 1. The Penrose–Petrov diagram.

The invariant dividers can be found by different methods. In particular, one of the methods involves the construction of a series of polynomials $D_k(\lambda)$, each being the greatest common divisor of all kth order minors of the matrix $A(\lambda)$. The theorem of linear algebra states that any invariant divider can be represented in the form

$$E_k(\lambda) = \frac{D_k(\lambda)}{D_{k-1}(\lambda)}, \qquad D_0(\lambda) = 1. \tag{15}$$

The invariant dividers $E_k(\lambda)$ completely determine the canonical form of the matrix $A(\lambda)$ and hence the type of the initial matrix A.

In our case, of interest are 3×3-matrices from which the 3×3 basic relations of the particle in X- and U-states are constructed; therefore, the λ-matrix is defined as follows:

$$X_{ik}(\lambda) = X_{ik} - \lambda U_{ik}, \tag{16}$$

where the matrix components of the nondegenerate state U, rather than the unit matrix components (as in the case for the conventional characteristic matrix), are the coefficients of λ.

6. ALGEBRAIC CLASSIFICATION AND STRONG INTERACTIONS

Since $X_N^{(s)}$ matrices of strong interactions determine the doubly isotropic 9-dimensional vectors, that is, have the rank equal to unity, λ-matrix (16) for strong interactions has the form

$$X_N^{(s)}(\lambda) = X_N^{(s)} - \lambda U^{(s)} = \begin{pmatrix} C'_1 q_1 - \lambda U_{11}^{(s)} & C'_2 q_1 - \lambda U_{12}^{(s)} & C'_3 q_1 - \lambda U_{13}^{(s)} \\ C'_1 q_2 - \lambda U_{21}^{(s)} & C'_2 q_2 - \lambda U_{22}^{(s)} & C'_3 q_2 - \lambda U_{23}^{(s)} \\ C'_1 q_3 - \lambda U_{31}^{(s)} & C'_2 q_3 - \lambda U_{32}^{(s)} & C'_3 q_3 - \lambda U_{33}^{(s)} \end{pmatrix}. \tag{17}$$

To determine an algebraic type of this λ-matrix, the greatest common divisors of minors of this matrix must be calculated, and its canonical form must be constructed by the well-known rules. Our analysis demonstrates that the canonical form depends on the constant $C_0 = C'_1 + C'_2 + C'_3$. In accordance with the foregoing, when $C_0 = 0$, we obtain the canonical matrix

$$X_A^{(s)}(\lambda) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda^2 \end{pmatrix} \tag{18}$$

of the subtype N.

When $C_0 \neq 0$, we obtain the canonical matrix

$$X_B^{(s)}(\lambda) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda(\lambda - C_0) \end{pmatrix} \tag{19}$$

of the subtype D.

According to Petrov, two neutral channels (A- and B-channels) of strong interactions are manifested in both algebraic subtypes (D and N).

For the charged channel of strong interactions, when, for example, the sign of the first column is changed, the corresponding λ-matrix has the form

$$X_C^{(s)}(\lambda) = \begin{pmatrix} -(\lambda+1)c^1_{(1)} & (1-\lambda)c^1_{(2)} & (1-\lambda)c^1_{(3)} \\ -(\lambda+1)c^2_{(1)} & (1-\lambda)c^2_{(2)} & (1-\lambda)c^2_{(3)} \\ -(\lambda+1)c^3_{(1)} & (1-\lambda)c^3_{(2)} & (1-\lambda)c^3_{(3)} \end{pmatrix}. \tag{20}$$

By the standard method, it is reduced to the following canonical form:

$$X_{C\mathrm{can}}^{(s)}(\lambda) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & (\lambda-1) & 0 \\ 0 & 0 & (\lambda-1)(\lambda+1) \end{pmatrix}, \tag{21}$$

corresponding to the algebraic subtype D.

The joint consideration of a number of peculiarities of strong interactions allows us to conclude that *the neutral B-channel and the charged channels of strong interactions are characterized by the algebraic subtype D, whereas the neutral A-channel is characterized by the algebraic subtype N.*

7. ALGEBRAIC CLASSIFICATION IN ELECTROWEAK INTERACTIONS

For neutral channels of electroweak quark interactions, the above-considered properties of 3×3-matrices of the X-state yield the following form of the λ-matrices:

$$X_N^{(w)}(\lambda) = X_N^{(w)} - \lambda U^{(w)}$$
$$= \begin{pmatrix} c'_3 - \lambda c_3 & c'_3 - \lambda c_3 & c'_3 - \lambda c_3 \\ (C'_1 q_1 + K'_1 l_1) - \lambda(C_1 q_1 + K_1 l_1) & (C'_2 q_1 + K'_2 l_1) - \lambda(C_2 q_1 + K_2 l_1) & (C'_3 q_1 + K'_3 l_1) - \lambda(C_3 q_1 + K_3 l_1) \\ (C'_1 q_2 + K'_1 l_2) - \lambda(C_1 q_2 + K_1 l_2) & (C'_2 q_2 + K'_2 l_2) - \lambda(C_2 q_2 + K_2 l_2) & (C'_3 q_2 + K'_3 l_2) - \lambda(C_3 q_2 + K_3 l_2) \end{pmatrix}. \tag{22}$$

Proceeding to the independent coefficients C, $\widetilde{C}$, K, and $\widetilde{K}$ and analogous coefficients with primes and taking into account that the matrix $X^{(w)}$ is degenerate, we find the roots of the characteristic equation:

$$\lambda_1 = 0, \qquad \lambda_2 = \frac{c'_3}{c_3}, \qquad \lambda_3 = \frac{(C\widetilde{K}' - K\widetilde{C}') + (C'\widetilde{K} - K'\widetilde{C})}{C\widetilde{K} - K\widetilde{C}}. \tag{23}$$

Using the standard methods, we obtain that for the electroweak quark interactions, the canonical form of the matrix $X^{(w)}(\lambda)$ in the general form can be written as follows:

$$X_B^{(w)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \lambda(\lambda-\lambda_2)(\lambda-\lambda_3) \end{pmatrix}, \tag{24}$$

which corresponds to algebraic subtype I.

In the particular case of $\lambda_3 = 0$, the canonical form of this λ-matrix is

$$X_A^{(w)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \lambda^2(\lambda-\lambda_2) \end{pmatrix}, \tag{25}$$

that is, it corresponds to algebraic subtype II.

Another particular case of $\lambda_2 = \lambda_3$ corresponds to the algebraic subtype D; however, it is not realized in theory.

An analysis demonstrates that *for the neutral B-channel* (*weak Z-interactions*) $\lambda_3 \neq 0$, that is, *algebraic subtype I is realized. For the A-channel* (*of the electromagnetic interaction*), *the condition* $\lambda_3 = 0$ *is satisfied, that is, according to the Petrov classification this channel is characterized by algebraic subtype II.*

It can be easily demonstrated that *electroweak interactions of quarks through charged W-bosons, as well as the charged channels of strong interactions, are characterized by the algebraic subtype D.*

CONCLUSIONS

The results of algebraic classification of neutral channels obtained in this work are tabulated in Table 1 below.

TABLE 1

Interactions	A-channel	B-channel
Electroweak	II	I
Strong	N	D

From the foregoing, we can conclude the following:

1) According to the Petrov classification, the neutral A-channels of strong and electroweak interactions are described by the same second (T_2) algebraic type, and the neutral B-channels are described by the same first (T_1) algebraic type.

2) The difference between the strong and weak interactions consists in the subtypes of the above-indicated types, namely, the strong interactions have more specific algebraic subtypes.

3) The charged channels of strong and electroweak interactions are characterized by the same subtype D of the first algebraic type.

We specially emphasize that in this approach to the description of the basic physical interactions based on binary geometric physics no concepts associated with the classical space-time were used. The transition to the conventional relations of quantum field theory and calibration approach, strictly speaking, is possible only after macroscopic averaging described in [4–6].

It should also be noted that this consideration does not involve the gravitational interaction. Within the framework of binary geometric physics, gravitation is induced (by electroweak interactions) in character, and is manifested only if we take into account the effect of the surrounding particles, that is, it is directly connected with the Mach principle. The discussion of this problem considered in detail in [4, 6] is beyond the scope of this work.

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