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FERMI–BOSE HYPERSYMMETRY

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A new algebra, combining supersymmetry and internal symmetry, is presented. A massless vector hypermultiplet contains a vector, an isodoublet of left-handed Dirac spinors, and a complex scalar. These can be used as generalized gauge fields. Abelian as well as non-Abelian gauge theories are studied, and the Higgs mechanism is extended in a hypersymmetric way. We present, also, a (non-realistic) SU(2) × U(1) model; gauge invariance and hypersymmetry are spontaneously broken; two Goldstone spinors appear. Hypersymmetry allows one to define "electronic" and "muonic" numbers, and suggests that a weakly interacting scalar particle ω_γ is associated with the photon and the two neutrinos.

1. Introduction

Supersymmetry generators [1] are charges transforming under the Lorentz group as the components of a Majorana spinor. Any linear representation contains both fermions and bosons, which have equal masses if the symmetry is unbroken.

Gauge invariance has been generalized to such theories [2]. Supersymmetry can be spontaneously broken, thus providing a Goldstone spinor [3]. It is tempting to consider the (electron) neutrino as the massless Goldstone particle. In ref. [4], we constructed a model of weak and electromagnetic interactions for the electron sector. The "gauge fields" associated with the unbroken electromagnetic gauge invariance are the vector V^μ and the Majorana spinor λ_γ; the former describes the photon, the latter, which is the Goldstone spinor, the (electron) neutrino.

We defined, recently, a self-interaction for a vector multiplet; this supersymmetric extension of the Higgs model depended only on two parameters [5]. If they vanish, we are left with a vector V^μ, two Majorana spinors λ and p, and a complex scalar ω, all free and massless; we suggested that they could be used to represent the photon, the two neutrinos, and a weak-interacting scalar particle, with electronic number 1, muonic number 1.

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Now, we have to define an interaction. In order that the two neutrinos play the same role, we introduce an internal symmetry group SU(2); $\binom{p_L}{\lambda_L}$ is an isodoublet of left-handed Dirac spinors.

The mixing of supersymmetry and internal symmetry has already been realized [6], but massive representations contain a high number of particles, and it is not clear whether one can construct a renormalizable theory [7]. To avoid these problems, we introduce a new structure.

In sect. 2 of this paper, we give a general definition of "hypersymmetry"; it combines, in a new way, supersymmetry with an internal invariance group $\mathcal{G}$. In our examples, we shall choose SU(2) or one of its U(1) subgroups; in this case, the hypersymmetry algebra contains two ordinary isomorphic supersymmetry algebras. A vector hypermultiplet is composed of a vector and a scalar supermultiplet, both massless; it contains the isoscalar antisymmetric tensor $V^{\mu\nu}$, the isodoublet of left-handed Dirac spinors $\binom{p_L}{\lambda_L}$, and the complex isoscalar scalai ω, as physical fields. We also define scalar hypermultiplets.

In sect. 3, we construct a hypersymmetric extension of quantum electrodynamics, where $(V^\mu; \lambda, p; \omega)$ appears as a vector hypermultiplet of U(1) "gauge fields" interacting with "matter" hypermultiplets.

In sect. 4, U(1) gauge invariance is spontaneously broken but hypersymmetry is not: a massless vector hypermultiplet and a massless scalar one join together into a single massive vector hypermultiplet (this was studied in detail, for supermultiplets, in ref. [5]); the latter describes one vector, two Dirac spinors, and five real scalar fields, with equal masses. Two conserved quantum numbers can be defined.

We show in sect. 5 that the interaction [8] between a Yang-Mills vector supermultiplet $\mathcal{V}_i$, and a (massless) scalar supermultiplet $\mathcal{N}_i$, belonging to the regular representation of the gauge group SU(N), is indeed hypersymmetric: the model describes the self-interaction of a Yang-Mills vector hypermultiplet.

We present briefly, in sect. 6, an example of spontaneous hypersymmetry breaking: the SU(2) $\times$ U(1) gauge-invariant model of ref. [9] is made hypersymmetric. SU(2) $\times$ U(1) is spontaneously broken as usual, together with both supersymmetry invariances: the Goldstone spinors are λ_γ and p_γ, associated with the photon γ and the scalar particle ω_γ in a vector hypermultiplet. This unifies the results obtained in refs. [4,9]: in the former, λ_γ was the Goldstone spinor, and (γ, λ_γ) a vector supermultiplet; in the latter p_γ was the Goldstone spinor, and $(p_\gamma, \omega_\gamma)$ a scalar supermultiplet.

The main purpose of the paper is to show that hypersymmetry allows the construction of theories invariant under supersymmetry and (global) internal symmetry, combined in a non-trivial way; two conserved quantum numbers can be defined.

Finally, we state some remarks about the possible application of hypersymmetry to weak and electromagnetic interactions theory. In a realistic model, neutrinos cannot be Goldstone particles (but those could exist as still-unobserved massless particles). The electron and muon neutrinos might be associated with the photon, and a weak-interacting scalar particle ω_γ, in a vector hypermultiplet.

2. The hypersymmetry algebra

2.1. The supersymmetry algebra $\mathcal{A}(\mathcal{S})$

Let $M_{\mu\nu}$ and P_μ be the Poincaré group generators. Q_α are four charges transforming under the Lorentz group as the components of a Majorana spinor, and satisfying:

$$\{Q_\alpha, \bar{Q}_\beta\} = -2\gamma^\mu_{\alpha\beta} P_\mu \ ,$$

$$[Q_\alpha, P_\mu] = [P_\lambda, P_\mu] = 0 \ , \tag{1}$$

where

$$\bar{Q} = \tilde{Q}\gamma^0 \ .$$

The supersymmetry algebra $\mathcal{A}(\mathcal{S})$ is the graded Lie algebra associated with the fourteen operators $M_{\mu\nu}$, Q_α and P_μ. They commute with $P_\mu P^\mu$, and all particles in an irreducible representation have the same mass.

2.2. General definition of the hypersymmetry algebra

Let $\mathcal{G}$ be a group of internal transformations $\mathcal{M}$:

$$[\mathcal{M}, M_{\mu\nu}] = [\mathcal{M}, P_\mu] = 0 \ . \tag{2}$$

Its Lie algebra $\mathcal{A}(\mathcal{G})$ is generated by hermitian scalar charges T_l, obeying the relations:

$$[T_l, M_{\mu\nu}] = [T_l, P_\mu] = 0 \ ,$$

$$[T_l, T_m] = i c^n_{lm} T_n \ . \tag{3}$$

Let $Q_{\alpha\uparrow}$ be four charges obeying relations (1): $M_{\mu\nu}$, $Q_{\alpha\uparrow}$ and P_μ generate the supersymmetry algebra $\mathcal{A}_\uparrow(\mathcal{S})$. For any internal transformation $\mathcal{M}$ we define the supersymmetry algebra

$$\mathcal{A}_{\mathcal{M}}(\mathcal{S}) = \mathcal{M}\, \mathcal{A}_\uparrow(\mathcal{S}) \mathcal{M}^{-1} \ . \tag{4}$$

It is generated by $M_{\mu\nu}$, P_μ, and the four charges

$$Q_{\alpha\mathcal{M}} = \mathcal{M}\, Q_{\alpha\uparrow}\, \mathcal{M}^{-1} \ , \tag{5}$$

obeying the same algebra as $M_{\mu\nu}$, P_μ and $Q_{\alpha\uparrow}$.

The set of hermitian operators $Q_{\alpha\mathcal{M}}$ represent the internal symmetry group $\mathcal{G}$. Let $Q_{\alpha k}$ be a basis of the corresponding vector space; for any specific example, one can write the commutation relations of $Q_{\alpha k}$ with T_l (or its transformation properties under discrete transformations of $\mathcal{G}$).

The hypersymmetry algebra $\mathcal{A}(\mathcal{S}, \mathcal{G})$ is defined as the graded Lie algebra * associated with the operators $M_{\mu\nu}$, T_l; $Q_{\alpha k}$; and all their successive commutators or anticommutators, restricted by:

$$\{Q_{\alpha k}, \bar{Q}_{\beta k}\} = -2\gamma^{\mu}_{\alpha\beta} P_{\mu} \ ,$$

$$[Q_{\alpha k}, P_{\mu}] = [P_{\lambda}, P_{\mu}] = 0 \ , \tag{6}$$

together with Jacobi identities.

Note that the anticommutators of spinorial charges $Q_{\alpha k}$ and $\bar{Q}_{\beta k'}$ are not restricted for $k' \neq k$: they define new operators; the algebra includes, also, successive (anti)-commutators. On the contrary, in ref. [6] any anticommutator of two spinorial charges was proportional to P_{μ}, and a basis of the graded Lie algebra $\mathcal{A}_0(\mathcal{S}, \mathcal{G})$ was explicitly known **.

Since $P^{\mu}P_{\mu}$ commutes with all these operators, all particles in an irreducible representation have the same mass.

2.3. SU(2) internal symmetry

Let $Q_{\alpha\uparrow}$ and $Q_{\alpha\downarrow}$ be two Majorana spinors satisfying relations (1). They are associated with the two supersymmetry algebras $\mathcal{A}_{\uparrow}(\mathcal{S})$ and $\mathcal{A}_{\downarrow}(\mathcal{S})$. We define the doublet of left-handed Dirac spinors:

$$Q_{\mathrm{L}} = \begin{cases} Q_{\uparrow \mathrm{L}} = \frac{1}{2}(1 - i\gamma_5) Q_{\uparrow} \\ Q_{\downarrow \mathrm{L}} = \frac{1}{2}(1 - i\gamma_5) Q_{\downarrow} \ . \end{cases} \tag{7}$$

Under an internal transformation $\mathcal{M}$ associated with the SU(2) matrix M, we have:

$$\mathcal{M} \begin{pmatrix} Q_{\uparrow \mathrm{L}} \\ Q_{\downarrow \mathrm{L}} \end{pmatrix} \mathcal{M}^{-1} = M \begin{pmatrix} Q_{\uparrow \mathrm{L}} \\ Q_{\downarrow \mathrm{L}} \end{pmatrix} \tag{8}$$

Let

$$\Sigma = \exp(i\pi I_2) \tag{9}$$

be the internal transformation associated with the real SU(2) matrix

$$M_0 = i\sigma_2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} . \tag{10}$$

* If we used groups, we would define the generalized Lie group $\mathcal{G}(\mathcal{S}, \mathcal{G})$. Its elements are hypersymmetry transformations, obtained as ordered products of supersymmetry with internal transformations.

** The algebra $\mathcal{A}_0(\mathcal{S}, \mathcal{G})$ has been enlarged in ref. [10] to accommodate central charges. There exists an homomorphism $\mathcal{A} \to \mathcal{A}_0$ and any representation of $\mathcal{A}_0$ is also a representation of $\mathcal{A}$.

Instead of the full SU(2), we can choose for internal symmetry the U(1) subgroup generated by I_2, or the finite subgroup generated by Σ. We have:

$$[\Sigma, M_{\mu\nu}] = [\Sigma, P_\mu] = 0\,,$$

$$\Sigma Q_\uparrow \Sigma^{-1} = Q_\downarrow\,,$$

$$\Sigma Q_\downarrow \Sigma^{-1} = -Q_\uparrow\,,$$

$$\{Q_{\alpha\uparrow}, \overline{Q}_{\beta\uparrow}\} = \{Q_{\alpha\downarrow}, \overline{Q}_{\beta\downarrow}\} = -2\gamma^\mu_{\alpha\beta} P_\mu\,,$$

$$[Q_{\alpha\uparrow}, P_\mu] = [Q_{\alpha\downarrow}, P_\mu] = [P_\lambda, P_\mu] = 0\,, \tag{11}$$

while no restriction is imposed on $\{Q_{\alpha\uparrow}, \overline{Q}_{\beta\downarrow}\}$, as explained earlier.

We study, now, some examples of representations, and at first, the massless vector hypermultiplet, describing a real spin-one, a complex spin-zero, and a doublet of left-handed Dirac spinor fields.

2.4. *The massless vector hypermultiplet*

It contains as physical fields, those of a vector and a real scalar multiplet with respect to the supersymmetry algebra $\mathcal{A}_\uparrow(\mathcal{S})$; both are massless. The former, $\mathcal{V}$, contains a spin-one field $V_{\mu\nu}$ ★, a Majorana spinor λ (and a real auxiliary scalar field D). The latter, $\mathcal{N}$, contains a Majorana spinor p, a complex scalar $\omega = -\sqrt{\frac{1}{2}}\,i(A - iB)$, (and a complex auxiliary scalar field $H = -\sqrt{\frac{1}{2}}\,i(F + iG)$).

The (anti) commutation relations of the fields with the charges $Q_{\alpha\uparrow}$ can be written:

$$[Q_\uparrow, A] = ip\,,$$

$$[Q_\uparrow, B] = i\gamma_5 p\,,$$

$$[Q_\uparrow, V_{\mu\nu}] = i(\partial_\mu\gamma_\nu - \partial_\nu\gamma_\mu)\lambda\,,$$

$$\{Q_{\alpha\uparrow}, \bar{p}_\beta\} = [\partial\!\!\!/(-A + \gamma_5 B) + F + \gamma_5 G]_{\alpha\beta}\,,$$

$$\{Q_{\alpha\uparrow}, \bar{\lambda}_\beta\} = [\tfrac{1}{2} V_{\mu\nu}\gamma^\mu\gamma^\nu + \gamma_5 D]_{\alpha\beta}\,,$$

$$[Q_\uparrow, F] = i\partial\!\!\!/ p\,,$$

$$[Q_\uparrow, G] = i\gamma_5 \partial\!\!\!/ p\,,$$

$$[Q_\uparrow, D] = i\gamma_5 \partial\!\!\!/ \lambda\,. \tag{12}$$

We demand that (p, λ) transform as $(Q_\uparrow, Q_\downarrow)$ under internal symmetry:

$$N_\mathrm{L} = \begin{pmatrix} p_\mathrm{L} \\ \lambda_\mathrm{L} \end{pmatrix}, \qquad N^\mathrm{c}_\mathrm{R} = \begin{pmatrix} \lambda_\mathrm{R} \\ -p_\mathrm{R} \end{pmatrix}, \tag{13}$$

★ $V_{\mu\nu}$ is an antisymmetric tensor obeying the equation $\epsilon^{\mu\nu\rho\sigma}\partial_\rho V_{\mu\nu} = 0$, which is preserved by supersymmetry.

are SU(2) isodoublets, while A, B and $V_{\mu\nu}$ are isosinglets. At least for a free vector hypermultiplet, we can take the auxiliary fields F, G and D relative to $\mathcal{A}_{\uparrow}(\mathcal{S})$ transforming among themselves under internal symmetry:

$$\vec{C}\begin{cases} C_1 = -G \\ C_2 = -F \\ C_3 = D \end{cases} \tag{14}$$

is an isotriplet of auxiliary fields.

The transformation Σ (rotation of π along the second axis in the "isospin" space) acts on the fields in the following way:

$$\begin{cases} \Sigma A \Sigma^{-1} = A\,, \\ \Sigma B \Sigma^{-1} = B\,, \\ \Sigma V_{\mu\nu} \Sigma^{-1} = V_{\mu\nu}\,, \end{cases} \qquad \begin{cases} \Sigma p \Sigma^{-1} = \lambda\,, \\ \Sigma \lambda \Sigma^{-1} = -p\,; \end{cases} \qquad \begin{cases} \Sigma G \Sigma^{-1} = -G\,; \\ \Sigma F \Sigma^{-1} = F\,, \\ \Sigma D \Sigma^{-1} = -D\,. \end{cases} \tag{15}$$

From (12) and (15), we obtain the (anti) commutation relations of the fields with the charges $Q_{\downarrow} = \Sigma Q_{\uparrow} \Sigma^{-1}$:

$$\begin{aligned} &[Q_{\downarrow}, A] = i\lambda\,, \\ &[Q_{\downarrow}, B] = i\gamma_5\lambda\,, \\ &[Q_{\downarrow}, V_{\mu\nu}] = -i(\partial_\mu\gamma_\nu - \partial_\nu\gamma_\mu)p\,, \\ &\{Q_{\alpha\downarrow}, \bar{\lambda}_\beta\} = [\partial\!\!\!/(-A + \gamma_5 B) + F - \gamma_5 G]_{\alpha\beta}\,, \\ &\{Q_{\alpha\downarrow}, \bar{p}_\beta\} = [-\tfrac{1}{2} V_{\mu\nu}\gamma^\mu\gamma^\nu + \gamma_5 D]_{\alpha\beta}\,, \\ &[Q_{\downarrow}, F] = i\partial\!\!\!/\lambda\,, \\ &[Q_{\downarrow}, G] = -i\gamma_5\partial\!\!\!/\lambda\,, \\ &[Q_{\downarrow}, D] = i\gamma_5\partial\!\!\!/ p\,. \end{aligned} \tag{16}$$

Relations (12) and (16) can be written in a more compact form, manifestly SU(2) invariant:

$$\begin{aligned} &[Q_{\mathrm{L}}, A] = iN_{\mathrm{L}}\,, && \{\bar{N}_{\mathrm{L}\beta}, Q_{\mathrm{L}\alpha}\} = -2(\Delta\partial\!\!\!/)_{\alpha\beta}(A + iB)\,, \\ &[Q_{\mathrm{L}}, B] = -N_{\mathrm{L}}\,, && \{\bar{N}_{\mathrm{L}\beta}\vec{\sigma}Q_{\mathrm{L}\alpha}\} = 0\,, \\ &[Q_{\mathrm{L}}, V_{\mu\nu}] = i(\partial_\mu\gamma_\nu - \partial_\nu\gamma_\mu)N^{\mathrm{c}}_{\mathrm{R}}, && \{\bar{N}^{\mathrm{c}}_{\mathrm{R}\beta}, Q_{\mathrm{L}\alpha}\} = (\Delta\gamma^\mu\gamma^\nu)_{\alpha\beta}V_{\mu\nu}\,, \\ &[Q_{\mathrm{L}}, \vec{C}] = -\partial\!\!\!/\vec{\sigma}N^{\mathrm{c}}_{\mathrm{R}}\,, && \{\bar{N}^{\mathrm{c}}_{\mathrm{R}\beta}\vec{\sigma}Q_{\mathrm{L}\alpha}\} = 2i\Delta_{\alpha\beta}\vec{C}\,, \end{aligned} \tag{17}$$

where Δ stands for the left-handed projector $\frac{1}{2}(1 - i\gamma_5)$.

We recall that $\vec{C}$ is an isotriplet of auxiliary fields, whereas $V_{\mu\nu}$, $\binom{p_L}{\lambda_L}$, A and B are the physical fields *. They could be used to represent the electromagnetic field, the two neutrinos, and the ω_γ particle (see ref. [5] and sect. 7 of this paper). For the moment being, they obey free generalized Maxwell equations, compatible with (17) ($\vec{C} = 0$):

$$\begin{aligned} \epsilon^{\mu\nu\rho\sigma}\partial_\rho V_{\mu\nu} &= 0 \,, \\ \partial^\mu V_{\mu\nu} &= 0 \,, \\ \not{\partial}\begin{pmatrix} p_L \\ \lambda_L \end{pmatrix} &= 0 \,, \\ \Box\,(A - iB) &= 0 \,, \end{aligned} \tag{18}$$

Interactions are defined in the following sections, using generalized gauge invariance. Physical fields transform as before under SU(2) internal symmetry; but the auxiliary fields F, G and D, relative to $\mathcal{A}_\uparrow(\mathcal{S})$, do not necessarily transform among themselves, although they still do so for the U(1) gauge model of sect. 3.

2.5. *The scalar hypermultiplet*

It contains the same physical fields as a complex scalar supermultiplet of $\mathcal{A}_\uparrow(\mathcal{S})$: this one includes a Dirac spinor ψ, two complex scalars φ' and φ'' (and two complex auxiliary scalar fields H' and H''):

$$\begin{aligned} \varphi' &= -i\sqrt{\tfrac{1}{2}}(A + iB) \,, & H' &= -i\sqrt{\tfrac{1}{2}}(F + iG) \,, \\ \varphi'' &= -i\sqrt{\tfrac{1}{2}}(A - iB) \,, & H'' &= -i\sqrt{\tfrac{1}{2}}(F - iG) \,. \end{aligned} \tag{19}$$

We demand that

$$\Phi = \begin{pmatrix} -i\varphi' \\ \varphi'' \end{pmatrix}, \qquad \Phi^c = \begin{pmatrix} \varphi''^\dagger \\ -i\varphi'^\dagger \end{pmatrix} , \tag{20}$$

be SU(2) isodoublets, while ψ is an isosinglet. Under an internal symmetry transformation the auxiliary fields H' and H'' generate the SU(2) isodoublets:

$$\begin{pmatrix} H' \\ \Sigma H' \Sigma^{-1} \end{pmatrix}, \qquad \begin{pmatrix} -\Sigma H'' \Sigma^{-1} \\ H'' \end{pmatrix} . \tag{21}$$

* This set of fields was already found in ref. [11], by means of a superfield formalism [6]. Using hypersymmetry we shall succeed in defining a renormalizable interaction, and treating the non-Abelian case.

$\Sigma H'\Sigma^{-1}$ and $\Sigma H''\Sigma^{-1}$ are the auxiliary fields relative to the supersymmetry algebra $\mathcal{A}_{\downarrow}(\mathcal{S})$. They are determined by equations of motion. In the simple case of a free massive scalar hypermultiplet, we have:

$$\Sigma H'\Sigma^{-1} = iH'' ,$$
$$\Sigma H''\Sigma^{-1} = iH' . \tag{22}$$

The scalar hypermultiplet, which can be massive, describes an isoscalar Dirac spinor, and an isodoublet of complex scalar fields *. The vector and scalar hypermultiplets are the simplest representations of the hypersymmetry algebra, but one can easily imagine other ones.

3. Hypersymmetric extension of QED

We now construct hypersymmetric renormalizable field theories. We start from the example of ref. [4], but we first restrict ourselves to a U(1) gauge group, instead of SU(2) X U(1): the model describes the gauge-invariant interaction of the vector supermultiplet $\mathcal{V}$ **, the real scalar one $\mathcal{N}$ and the complex scalar one $\mathcal{S}$. We shall make from $\mathcal{V}$ and $\mathcal{N}$ a vector hypermultiplet, from $\mathcal{S}$ a scalar one. The physical fields are:

for $\mathcal{V}$, the vector V^{μ} and the Majorana spinor λ;
for $\mathcal{N}$, the Majorana spinor p and the complex scalar $\omega = -i\sqrt{\tfrac{1}{2}}(a - ib)$;
for $\mathcal{S}$, the Dirac spinor ψ and the complex scalars

$$\varphi' = -i\sqrt{\tfrac{1}{2}}(A + iB) , \qquad \varphi'' = -i\sqrt{\tfrac{1}{2}}(A - iB) , \tag{23}$$

all charged.

Let θ be an element of a Grassmann algebra of anticommuting two-component complex spinors, $\bar{\theta}$ its conjugate. $\mathcal{V}$ is represented by the real superfield V, $\mathcal{N}$ by the left-handed superfield N. $\mathcal{S}$ is represented by the superfields S, left-handed, and T, right-handed, associated, respectively, with the physical components $(\psi_L ; \varphi'')$ and $(\psi_R ; \varphi')$. The transformation

$$V \to -V , \qquad S \to T ,$$
$$N \to N^* , \qquad T \to S , \tag{24}$$

is used to define parity.

* The fields of a scalar hypermultiplet (which was also a doublet of a SU(2) gauge group) were already used in ref. [12], together with the transformations Q:

$$Q\psi Q^{-1} = \psi , \quad Q\varphi' Q^{-1} = e^{-i\alpha}\varphi' , \quad Q\varphi'' Q^{-1} = e^{i\alpha}\varphi'' .$$

The model of ref. [12] was related to supersymmetry, as explained in sects. 2 and 3 of ref. [4]. A transformation Q appears, now, as a rotation in the "isospin" space, generated by I_3.

** Now, we consider the full vector supermultiplet, composed of a vector V_{μ}, two Majorana spinors λ and χ, and scalar fields C, M, N, D. The auxiliary components χ, C, M, N vanish as soon as the special gauge of Wess and Zumino [2] is chosen.

The most general Lagrangian density compatible with supersymmetry, gauge and parity invariance, is ★:

$$\mathcal{L} = \mathcal{L}_0 + [S^* \exp(2eV)\, S + T^* \exp(-2eV)\, T + N^*N]_{D\ \text{component}}$$
$$+ [4hT^*SN + sN + uN^2 + vN^3]_{F\ \text{component}}\,, \tag{25}$$

where $\mathcal{L}_0$ stands for the kinetic energy term of the vector multiplet. In the special gauge of Wess and Zumino, all components of V vanish, except V^μ, λ and D, and the Lagrangian density becomes polynomial.

To make this model hypersymmetric, we have to demand invariance under an internal transformation $\mathcal{M}$. Choosing the Wess-Zumino gauge and considering the special transformation Σ defined in sect. 2 ($p \to \lambda$, $\lambda \to -p$), we find immediately that the coefficients u and v must vanish: R-invariance of ref. [4] is obtained here as a consequence of internal symmetry.

The Lagrangian density (25) becomes:

$$\mathcal{L} = \mathcal{L}_0 + [S^* \exp(2eV)\, S + T^* \exp(-2eV)T + N^*N]_{D\ \text{component}}$$
$$+ [4hT^*SN + sN]_{F\ \text{component}}\,. \tag{26}$$

Its expression in the special gauge can be obtained from formula (34) of ref. [4], with the substitutions:

$$\tfrac{1}{2}g \to 0, \quad \tfrac{1}{2}g' \to e, \quad \tfrac{1}{2}h \to h, \quad \xi \to 0\,,$$

$$\mathcal{L} = -\tfrac{1}{4}V_{\mu\nu}V^{\mu\nu} - \tfrac{1}{2}i\bar{\lambda}\not{\partial}\lambda - \tfrac{1}{2}i\bar{p}\not{\partial}p - \tfrac{1}{2}\partial_\mu a\partial^\mu a - \tfrac{1}{2}\partial_\mu b\,\partial^\mu b$$
$$- i\bar{\psi}\not{D}\psi - D_\mu\varphi'^\dagger D^\mu\varphi' - D_\mu\varphi''^\dagger D^\mu\varphi'' + ie\sqrt{2}\,[(\bar{\psi}_R\varphi' + \bar{\psi}_L\varphi'')\lambda$$
$$+ \bar{\lambda}(\varphi'^\dagger\psi_R + \varphi''^\dagger\psi_L)] + h\sqrt{2}[(\bar{\psi}_R\varphi'' + \bar{\psi}_L\varphi')p$$
$$- \bar{p}(\varphi''^\dagger\psi_R + \varphi'^\dagger\psi_L)] - ih\bar{\psi}(a - \gamma_5 b)\psi - \mathcal{U}\,. \tag{27}$$

The covariant derivative D_μ is defined by

$$iD_\mu = i\partial_\mu - eV_\mu\,, \tag{28}$$

where we can suppose $e > 0$. $\mathcal{U}$, the potential of the scalar fields, is given by

$$-\mathcal{U} = \tfrac{1}{2}(D^2 + f_0^2 + g_0^2) + sf_0 + H'^\dagger H' + H''^\dagger H''$$
$$+ eD(\varphi''^\dagger\varphi'' - \varphi'^\dagger\varphi') + h(f_0 + ig_0)\,\varphi'^\dagger\varphi'' + h(f_0 - ig_0)\,\varphi''^\dagger\varphi'$$
$$+ h(a + ib)\,(H'^\dagger\varphi' + \varphi''^\dagger H'') + h(a - ib)\,(H''^\dagger\varphi'' + \varphi'^\dagger H')\,. \tag{29}$$

★ We have not demanded R invariance of ref. [4], and thus, we wrote the term $[uN^2 + vN^3]_F$; a possible mass term for the scalar multiplet $\mathcal{S}$, proportional to $[T^*S]_F$, has been eliminated by means of a translation on the a component of N.

We recall that

$$N_L = \begin{pmatrix} p_L \\ \lambda_L \end{pmatrix}, \quad N_R^c = \begin{pmatrix} \lambda_R \\ -p_R \end{pmatrix}, \quad \Phi = \begin{pmatrix} -i\varphi' \\ \varphi'' \end{pmatrix}, \quad \Phi^c = \begin{pmatrix} \varphi''^\dagger \\ -i\varphi'^\dagger \end{pmatrix}, \tag{30}$$

transform as SU(2) isodoublets. Invariance under internal symmetry transformations implies *:

$$e = h\,, \qquad s = 0\,, \tag{31}$$

and the equations of motion show that:

$$\vec{C} = \begin{pmatrix} -g_0 \\ -f_0 \\ D \end{pmatrix}. \tag{32}$$

transforms as an SU(2) isotriplet.

The potential of the scalar fields, $\mathcal{U}$, is now given by:

$$-\mathcal{U} = \tfrac{1}{2}|\vec{C} - e\Phi^\dagger\vec{\sigma}\Phi^2|^2 + |H' + e(a+ib)\varphi'|^2 + |H'' + e(a-ib)\varphi''|^2$$
$$- \tfrac{1}{2}e^2(\Phi^\dagger\vec{\sigma}\Phi)^2 - e^2(a^2+b^2)\,\Phi^\dagger\Phi\,. \tag{33}$$

After elimination of auxiliary fields, we obtain simply

$$\mathcal{U} = \tfrac{1}{2}e^2(\Phi^\dagger\Phi)^2 + e^2(a^2+b^2)\,\Phi^\dagger\Phi \tag{34}$$

with

$$\Phi^\dagger\Phi = \varphi'^\dagger\varphi' + \varphi''^\dagger\varphi''\,. \tag{35}$$

The potential computed at the tree approximation has a vanishing minimum for $\varphi' = \varphi'' = 0$: both hypersymmetry and gauge invariance are conserved. Using R-invariance, we can always choose $\langle b\rangle = 0$, and parity is preserved.

Higher order computations are necessary to determine the vacuum expectation value of the a field. Possibly, it vanishes (conserved R-invariance), and the theory remains massless.

If not, we translate the a field and restore the mass term for the scalar hypermultiplet (then b is the Goldstone boson associated with spontaneous breaking of R-invariance). The final expression of the Lagrangian density is written in the manifestly SU(2) invariant way:

$$\mathcal{L} = -\tfrac{1}{4}V_{\mu\nu}V^{\mu\nu} - i\bar{N}_L\,\partial\!\!\!/ N_L - \tfrac{1}{2}\partial_\mu a\partial^\mu a - \tfrac{1}{2}\partial_\mu b\partial^\mu b - i\bar{\psi}D\!\!\!\!/\,\psi - D_\mu\Phi^\dagger D^\mu\Phi$$

* The choice $e = h$ in the U(1) version of the model of ref. [4] has already been made by Salam and Strathdee [13] in order to define a conserved parity; however, such a choice is not necessary to this end: see for example the annex of ref. [5].

$$+ ie\sqrt{2}\,[\Phi^{c\dagger}\bar{\psi}(N_R^c - iN_L) + \overline{N_R^c - iN_L}\,\psi\Phi^c] - i\bar{\psi}[m + e(a - \gamma_5 b)]\psi$$

$$-\tfrac{1}{2}e^2(\Phi^\dagger\Phi)^2 - [(m + ea)^2 + e^2b^2]\Phi^\dagger\Phi\,. \qquad (36)$$

This expression describes the interaction of $(V^\mu; \lambda, p; \omega)$, a massless vector multiplet of "gauge fields", with the scalar hypermultiplet $(\psi; \varphi', \varphi'')$.

One can easily consider the interaction with several scalar hypermultiplets. All their mass terms cannot be eliminated by means of a translation on the a field (unless we choose equal masses). The Lagrangian density can be written immediately.

4. Self-interacting vector hypermultiplet

We now break the internal symmetry group explicitly, but softly: by terms of dimension two at most in the Lagrangian density. If we add, to the Lagrangian density, any linear combination (denoted $\vec{\Gamma}\cdot\vec{C}$) of the auxiliary fields f_0, g_0 and D, both gauge invariance and the supersymmetry algebra $\mathcal{A}_\uparrow(\mathcal{S})$, are preserved. Performing a global SU(2) transformation, we find that all supersymmetry algebras $\mathcal{A}_{\mathcal{M}}(\mathcal{S}) = \mathcal{M}\mathcal{A}_\uparrow(\mathcal{S})\mathcal{M}^{-1}$ are also preserved.

The new term added to the Lagrangian density reduces the internal symmetry group from SU(2) to the U(1) subgroup of isospin rotations around $\vec{\Gamma}$. If $\vec{\Gamma}$ is parallel to the second axis, the term sf_0 is added; φ' and φ'' take equal vacuum expectation values, and parity is conserved (see subsect. 5.4 in ref. [4]). Since the model does not depend on the direction chosen for $\vec{\Gamma}$, a conserved parity operator can also be defined if $\vec{\Gamma}$ points along the third axis; in this case, we add the term ξD *, and the Lagrangian density is:

$$\mathcal{L} = \mathcal{L}_0 + [S^*\exp(2eV)S + T^*\exp(-2eV)T + N^*N]_{D\,\text{component}}$$

$$+ [4eT^*SN]_{F\,\text{component}} + \xi D\,. \qquad (37)$$

The results of sect. 5 of ref. [4] apply immediately, with the substitutions:

$$\tfrac{1}{2}g \to 0; \quad \tfrac{1}{2}g' = \tfrac{1}{2}h \to e, \quad s \to 0\,. \qquad (38)$$

We find for the potential of scalar fields:

$$\mathcal{U}(\varphi', \varphi'', a, b) = \tfrac{1}{2}e^2(\varphi'^\dagger\varphi' + \varphi''^\dagger\varphi'')^2 + \xi e(\varphi''^\dagger\varphi'' - \varphi'^\dagger\varphi') + \tfrac{1}{2}\xi^2$$

$$+ e^2(a^2 + b^2)(\varphi'^\dagger\varphi' + \varphi''^\dagger\varphi'')\,. \qquad (39)$$

* If we add the terms sf_0, the Lagrangian density is parity and R invariant (with the usual definition of parity). If we add the term ξD, it is Q- and R-invariant; Q- and R-invariances have been defined in ref. [4].

We can choose $\xi e < 0$; φ'', only, takes a non-vanishing vacuum expectation value; in the convenient gauge, it is the positive real $\sqrt{\frac{1}{2}}v$, with

$$\xi + \tfrac{1}{2}ev^2 = 0\,. \tag{40}$$

Let us write

$$\varphi' = \sqrt{\tfrac{1}{2}}\,(\varphi_1' + i\varphi_2')\,, \qquad \varphi'' = \sqrt{\tfrac{1}{2}}\,(\varphi_1'' + i\varphi_2'')\,. \tag{41}$$

φ_2'' is the Goldstone boson, eliminated by the Higgs mechanism as the vector particle becomes massive.

The potential has a vanishing minimum; all auxiliary fields f_0, g_0, D, H', H'' have vanishing vacuum expectation values, and the supersymmetry algebra $\mathcal{A}_\uparrow(\mathcal{S})$ is conserved; similarly, all supersymmetry algebras $\mathcal{A}_{\mathcal{M}}(\mathcal{S})$ are conserved.

We construct the hypersymmetry algebra from the Lorentz generators $M_{\mu\nu}$, the Majorana charges $Q_{\mathcal{M}}$, and their successive commutators or anti-commutators, among which is the energy momentum P_μ.

Hypersymmetry is conserved, but gauge invariance is spontaneously broken. The massless vector hypermultiplet $(V^\mu; \lambda, p; \omega)$ and the massless scalar one $(\psi; \varphi', \varphi'')$ join together into one massive vector hypermultiplet: this is the "hypersymmetric extension of the Higgs mechanism", exactly as the model of ref. [5] was the "supersymmetric extension of the Higgs mechanism": there the massless vector supermultiplet $(V^\mu; \lambda)$ and the massless scalar one $(\psi_L; \varphi)$ joined together into one massive vector supermultiplet, including a vector, a Dirac spinor and a real scalar.

The fields of the massive vector hypermultiplet * are:

$$\begin{array}{ll} \text{the vector} & W^\mu\,; \\ \text{the Dirac spinors} & \left\{\begin{array}{l} e = ip_L + \psi_R\,, \\ M = \psi_L - \lambda_R\,, \end{array}\right. \\ \text{the complex scalar} & \omega = -i\sqrt{\tfrac{1}{2}}(a - ib)\,, \\ \text{the three real scalars} & \varphi_1'', \varphi_1', \varphi_2'\,. \end{array} \tag{42}$$

They all have the same mass $m = ev$ (as stated in ref. [13]) owing to hypersymmetry conservation.

The result of spontaneous breaking of gauge invariance is to "add" multiplets; this can be viewed as:

addition of supermultiplets of $\mathcal{A}_\uparrow(\mathcal{S})$

$$(V^\mu; \lambda) \cup (\psi_L; \varphi'') \to (W^\mu; M; \varphi_1'')\,,$$

$$(p; \omega) \cup (\psi_R; \varphi') \to (e; \omega, \varphi')\,; \tag{43}$$

* We apply formulas of ref. [4], subsect. 5.3, with substitutions (38). The mixing angle δ vanishes. The multiplets $(Z; E_0; z)$ and $(e_0; \omega, \varphi)$ of that paper become here $(W; M; \varphi_1'')$ and $(e; \omega, \varphi')$ respectively.

addition of supermultiplets of $\mathcal{A}_{\downarrow}(\mathcal{S})$

$$(V^{\mu}; p) \cup (\psi_{\mathrm{R}}; \varphi'') \to (W^{\mu}; e; \varphi_1'') ,$$
$$(\lambda; \omega) \cup (\psi_{\mathrm{L}}; \varphi') \to (M; \omega, \varphi') ; \tag{44}$$

addition of hypermultiplets

$$(V^{\mu}; \lambda, p; \omega) \cup (\psi; \varphi', \varphi'') \to (W_{\mu}; e, M; \omega, \varphi_1'', \varphi_1', \varphi_2') . \tag{45}$$

Although this model is no more invariant under the internal symmetry group SU(2), we can define two conserved quantum numbers. The "leptonic number", denoted $L_{\mathrm{e}} + L_{\mu}$, has been studied in detail in ref. [4]. It is +1 for e; -1 for M; 2 for ω; and it vanishes for other fields.

Let I_3 be the generator of "isospin" rotations around axis three. Its non-vanishing eigenvalues are $+\frac{1}{2}$ for p_{L}, λ_{R} and φ'; $-\frac{1}{2}$ for φ''. Both I_3 and the charge Y are invariances of the initial theory. They are spontaneously broken, but the combination

$$L_{\mathrm{e}} - L_{\mu} = 2I_3 + Y . \tag{46}$$

is preserved. It defines a conserved quantum number: +1 for p_{L}, λ_{R} and ψ (and thus for the Dirac spinors e and M); 2 for φ'; 0 for φ'', ω and W^{μ}.

Finally, the two conserved quantum numbers L_{e} and L_{μ} * are given in table 1.

Table 1

Field	W^{μ}	e	M	ω	φ'	φ_1''
L_{e}	0	1	0	1	1	0
L_{μ}	0	0	-1	1	-1	0

This can be interpreted easily, using superfields: the Lagrangian density (37) is invariant under the transformations **

$$LV(x, \theta, \bar{\theta})L^{-1} = V(x, \theta\, \mathrm{e}^{-i\alpha}, \bar{\theta}\, \mathrm{e}^{i\alpha}) ,$$
$$LN(x, \theta, \bar{\theta})L^{-1} = \mathrm{e}^{i(\beta+\alpha)}\, N(x, \theta\, \mathrm{e}^{-i\alpha}, \bar{\theta}\, \mathrm{e}^{i\alpha}) ,$$
$$LS(x, \theta, \bar{\theta})L^{-1} = S(x, \theta\, \mathrm{e}^{-i\alpha}, \bar{\theta}\, \mathrm{e}^{i\alpha}) ,$$
$$LT(x, \theta, \bar{\theta})L^{-1} = \mathrm{e}^{i(\beta-\alpha)}\, T(x, \theta\, \mathrm{e}^{-i\alpha}, \bar{\theta}\, \mathrm{e}^{i\alpha}) . \tag{47}$$

* These quantum numbers already existed in the model of sect. 5 of ref. [4] for $s = 0$. $L_{\mathrm{e}} + L_{\mu}$ is associated with R-invariance. The model is also invariant under Q transformations (generated by I_3); if U is a global gauge transformation, $X = UQ$ (generated by $2I_3 + Y$) defines $L_{\mathrm{e}} - L_{\mu}$.

** L unifies the R- and X-invariances used in ref. [4], when $s = 0$. It has been introduced for a free vector hypermultiplet in sect. 6 of ref. [5].

Under this transformation, we have:

$$V^\mu \to V^\mu \quad \begin{cases} \omega \to e^{i(\beta+\alpha)}\,\omega \\ \varphi' \to e^{i(\beta-\alpha)}\varphi' \\ \varphi'' \to \qquad \varphi'' \end{cases} \qquad \begin{cases} e \to e^{i\beta}\, e \\ M \to e^{-i\alpha} M \end{cases} \tag{48}$$

L-invariance is preserved by the breaking, and we find, again, the two conserved quantum numbers given in table 1.

Equations of motion are invariant under the hypersymmetry algebra, which may have an infinite number of generators. This does not prevent the S-matrix to be invariant under a smaller algebra with a finite number of generators, since the model describes only massive particles, in finite number [10].

Summarizing, this model describes the hypersymmetric parity-invariant self-interaction of a "massive vector hypermultiplet". It allows the definition of two conserved quantum numbers. It is obtained by spontaneous breaking of gauge symmetry, and expected to be renormalizable in a way compatible with these invariances.

5. Hypersymmetric Yang-Mills theories

One does not need to know supersymmetry to write down the interaction between a Yang-Mills vector field V_i^μ and a (massless) Majorana spinor λ_i belonging to the regular representation of the gauge group (here we choose SU(N)). However, this interaction turns out to be supersymmetric [8].

Similarly, one does not have to know hypersymmetry to write down the interaction between a Yang-Mills vector supermultiplet $\mathcal{V}_i$ and a (massless) scalar supermultiplet $\mathcal{N}_i$ belonging to the regular representation of SU(N); this was done in ref. [8].

From the massless vector and scalar supermultiplets, we make a single Yang-Mills vector hypermultiplet. It describes an ordinary Yang-Mills vector field V_i^μ, the Majorana spinors λ_i and p_i, and the spinless fields a_i and b_i; they belong to the regular representation of SU(N), and are written as traceless hermitian $N \times N$ matrices ($V^\mu = V_i^\mu T^i$, etc.).

To show that the interaction is indeed hypersymmetric, we just have to establish invariance under the SU(2) group of internal symmetry transformations: they leave V_i^μ, a_i and b_i unchanged, but mix together the spinor fields λ_i and p_i;

$$\begin{pmatrix} p_{iL} \\ \lambda_{iL} \end{pmatrix}, \quad \begin{pmatrix} \lambda_{iR} \\ -p_{iR} \end{pmatrix}, \tag{49}$$

are SU(2) isodoublets.

The Lagrangian density is proportional to:

$$\mathrm{Tr}\{-\tfrac{1}{4} V_{\mu\nu} V^{\mu\nu} - \tfrac{1}{2} i\bar{\lambda}\not{D}\lambda - \tfrac{1}{2} i\bar{p}\not{D}p - \tfrac{1}{2} D_\mu a D^\mu a - \tfrac{1}{2} D_\mu b D^\mu b$$

$$+ \tfrac{1}{2} g\bar{\lambda}[a + \gamma_5 b, p] + \tfrac{1}{8} g^2 [a, b]^2\}\,, \tag{50}$$

Table 2

Field	V_i^μ	p_{iL}	λ_{iL}	ω_i
L_e	0	1	0	1
L_μ	0	0	1	1

The difference:

$$L_e - L_\mu = 2I_3 \tag{56}$$

where the covariant derivative D_μ is defined by:

$$D_\mu\lambda = \partial_\mu\lambda + \tfrac{1}{2}ig[V_\mu, \lambda]\ , \qquad \text{etc.} \tag{51}$$

We rewrite in a different way the interaction terms involving fermions:

$$\begin{aligned}\mathrm{Tr}(\bar{\lambda}[\not{V}, \lambda] + \bar{p}[\not{V}, p]) &\sim (\bar{\lambda}_i \not{V}_j \lambda_k + \bar{p}_i \not{V}_j p_k) f^{ijk} \\ &\sim (\bar{\lambda}_{iL} \not{V}_j \lambda_{kL} + \bar{p}_{iL} \not{V}_j p_{kL}) f^{ijk}\ .\end{aligned} \tag{52}$$

Similarly, we have

$$\begin{aligned}&\mathrm{Tr}(\bar{\lambda}[a + \gamma_5 b, p]) \sim \{\bar{\lambda}_{iR}(a_j + ib_j)\, p_{kL} + \bar{\lambda}_{iL}(a_j - ib_j)\, p_{kR}\} f^{ijk} \\ &\quad \sim \{(a_j + ib_j)(\bar{\lambda}_{iR} p_{kL} - \bar{p}_{iR}\lambda_{kL}) + (a_j - ib_j)(\bar{\lambda}_{iL} p_{kR} - \bar{p}_{iL}\lambda_{kR})\} f^{ijk}.\end{aligned} \tag{53}$$

The Lagrangian density (50) is now clearly invariant under the internal symmetry group SU(2). According to the analysis of sect. 2, the model is hypersymmetric.

The existence of conserved quantum numbers follows immediately from internal SU(2) invariance *; the analysis of sect. 4 can be applied immediately. Under a transformation $L(\alpha, \beta)$ defined by:

$$\begin{aligned}L\, V_i(x, \theta, \bar{\theta}) L^{-1} &= \qquad V_i(x, \theta\, \mathrm{e}^{-i\alpha}, \bar{\theta}\, \mathrm{e}^{i\alpha})\ , \\ L N_i(x, \theta, \bar{\theta}) L^{-1} &= \mathrm{e}^{i(\beta+\alpha)}\, N_i(x, \theta\, \mathrm{e}^{-i\alpha}, \bar{\theta}\, \mathrm{e}^{i\alpha})\ ,\end{aligned} \tag{54}$$

we have:

$$\begin{aligned}&V_i^\mu \to V_i^\mu\ , &&p_{iL} \to \mathrm{e}^{i\beta}\, p_{iL}\ , \\ &\omega_i \to \mathrm{e}^{i(\beta+\alpha)}\, \omega_i\ , &&\lambda_{iL} \to \mathrm{e}^{i\alpha}\, \lambda_{iL}\ .\end{aligned} \tag{55}$$

The two conserved quantum numbers L_e and L_μ are given in table 2.

* The one already exhibited in ref. [8] can now be interpreted as a result of invariance under "isospin" rotations generated by I_2. Here we prefer to use I_3.

allows the definition of a conserved "fermionic" number, which is 0 for bosons, 1 for Dirac fermions.

In this section, we have been concerned, only, with the self-interaction of a Yang-Mills vector hypermultiplet of "gauge fields". In the next section, we describe briefly interaction with scalar hypermultiplets, for an example in which spontaneous hypersymmetry breaking occurs.

6. Spontaneous hypersymmetry breaking

In refs.[4, 5] supersymmetry was spontaneously broken and the Goldstone spinor was the Majorana spinor λ_γ associated with the photon γ. In ref. [9] it was the Majorana spinor p_γ associated with the ω_γ particle. In the previous sections we learned how to associate λ and p spinors by means of SU(2) internal symmetry. Our purpose is to unify our previous examples of spontaneous supersymmetry breaking: we shall construct a hypersymmetric model which will provide two Goldstone spinors λ_γ and p_γ, belonging to a vector hypermultiplet $(\gamma;\, \lambda_\gamma, p_\gamma;\, \omega_\gamma)$. In this paper, we expose the model only very briefly.

6.1. The model

It is the one we presented in ref. [9] with gauge invariance realized locally. It described the supersymmetric, $SU(2)_G \times U(1)$, R- and parity-invariant interaction of a triplet $\mathcal{V}$ and a singlet $\mathcal{V}'$ of vector multiplets; a triplet $\mathcal{N}$ and a singlet $\mathcal{N}'$ of real scalar multiplets; a doublet $\mathcal{D}$ of complex scalar multiplets. Four coupling constants were involved: the gauge coupling constants g and g', and the coupling constants of the scalar multiplets $\mathcal{N}$ and $\mathcal{N}'$ with $\mathcal{D}$, called h and h'. The mass scale was fixed by the parameter s, the coefficient of the component f_0' of $\mathcal{N}'$ in the Lagrangian density.

From this model, we build now a hypersymmetric one: $(\mathcal{V}, \mathcal{N})$ and $(\mathcal{V}', \mathcal{N}')$ are a triplet and a singlet of vector hypermultiplets, $\mathcal{D}$ a doublet of scalar hypermultiplets ⋆. We first suppose $s = 0$. The isodoublets of the internal symmetry group $SU(2)_I$ are:

$$N_L = \begin{pmatrix} p_L \\ \lambda_L \end{pmatrix}, \qquad N_L' = \begin{pmatrix} p_L' \\ \lambda_L' \end{pmatrix}, \qquad \Phi = \begin{pmatrix} -i\varphi'^- \\ \varphi'' \end{pmatrix}, \tag{57}$$

while all other physical fields are isosinglets.

The Lagrangian density is similar to that of sect. 3 [formulas (27) and (29)]. Since the gauge group is not Abelian, terms proportional to $(\bar{\boldsymbol{\lambda}}, \boldsymbol{V\!\!\!/}, \boldsymbol{\lambda}) + (\bar{\boldsymbol{p}}, \boldsymbol{V\!\!\!/}, \boldsymbol{p})$, $(\bar{\boldsymbol{\lambda}}, (\boldsymbol{a} + \gamma_5 \boldsymbol{b}), \boldsymbol{p})$

⋆ "Singlet", "doublet" and "triplet" are relative to the gauge subgroup $SU(2)_G$, whereas "isosinglet", "isodoublet" and "isotriplet" are relative to the internal symmetry group $SU(2)_I$.

appear, as in sect. 5. A complete study of the Lagrangian density shows that hypersymmetry is realized for:

$$g = h, \quad g' = h' . \tag{58}$$

This model is the $SU(2)_G \times U(1)$ analogue of the U(1) gauge invariant hypersymmetric model of sect. 3.

Let us add to the Lagrangian density a linear function of the auxiliary fields f_0', g_0' and D'. As in sect. 3 (formula (32)), these transform as the components of an $SU(2)_I$ isotriplet. Performing an internal symmetry transformation, we can cast it into the special form sf_0' (and the sign of s is chosen so that $s/g' < 0$). The internal symmetry group is now explicitly broken, and reduced to the subgroup of "isospin" rotations around the second axis. The hypersymmetry algebra is defined as in sect. 4.

Equalities (58) constraining the model of ref. [9] allow us to identify the angle θ, defined there by $\mathrm{tg}\,\theta = h'/h$, with the usual $SU(2)_G \times U(1)$ mixing angle, defined by $\mathrm{tg}\,\theta = g'/g$.

As in [4], we define the rotated vector multiplets:

$$\begin{cases} \mathcal{V}_z = \mathcal{V}_3 \cos\theta + \mathcal{V}' \sin\theta \\ \mathcal{V}_\gamma = -\mathcal{V}_3 \sin\theta + \mathcal{V}' \cos\theta \end{cases} \qquad \mathcal{V}_- = \sqrt{\tfrac{1}{2}}(\mathcal{V}_1 + i\mathcal{V}_2) \tag{59a}$$

and as in [9], the rotated scalar multiplets:

$$\begin{cases} \mathcal{N}_z = \mathcal{N}_3 \cos\theta + \mathcal{N}' \sin\theta \\ \mathcal{N}_\gamma = -\mathcal{N}_3 \sin\theta + \mathcal{N}' \cos\theta \end{cases} \qquad \mathcal{N}_- = \sqrt{\tfrac{1}{2}}(\mathcal{N}_1 + i\mathcal{N}_2) . \tag{59b}$$

6.2. Spontaneous symmetry breaking

Let us separate the potential of scalar fields into two parts, $\mathcal{U}_1$ and $\mathcal{U}_2$, expressed in terms of auxiliary fields:

$$\mathcal{U}_1 = \tfrac{1}{2}|f + ig|^2 + \tfrac{1}{2}|f_0' + ig_0'|^2 ,$$

$$\mathcal{U}_2 = \tfrac{1}{2}(D^2 + D'^2) + H'^\dagger H' + H''^\dagger H'' , \tag{60}$$

with

$$f + ig + g\varphi''^\dagger \boldsymbol{\tau}\, \varphi' = 0 ,$$

$$f_0' + ig_0' + s + g'\varphi''^\dagger \varphi' = 0 . \tag{61}$$

The potential is strictly positive; one, at least, of the auxiliary fields has a non-vanishing vacuum expectation value, and hypersymmetry is spontaneously broken (if s, g and g' are non-zero).

$\mathcal{U}_1$ is minimum when φ' and φ'' take non-vanishing vacuum expectation values, which are in the convenient gauge [9]

$$\langle\varphi'\rangle = \begin{bmatrix} v'/\sqrt{2} \\ 0 \end{bmatrix}, \qquad \langle\varphi''\rangle = \begin{bmatrix} v''/\sqrt{2} \\ 0 \end{bmatrix}. \tag{62}$$

v' and v'' are two positive real numbers related by:

$$v'v'' = -\frac{2g's}{g^2 + g'^2}. \tag{63}$$

It is still possible to realize $\mathcal{U}_2 = 0$; this fixes:

$$v' = v'',$$

$$\langle a_-\rangle = \langle b_-\rangle = \langle a_z\rangle = \langle b_z\rangle = 0. \tag{64}$$

By means of an R transformation, we can always choose:

$$\langle b_\gamma\rangle = 0. \tag{65}$$

It follows from (62), (64) and (65) that parity is conserved. But $\langle a_\gamma\rangle$ is not determined at the tree approximation; as in ref. [9], higher order computations are necessary to determine whether R-invariance is conserved or not.

$SU(2)_G \times U(1)$ gauge invariance is spontaneously broken, and reduced to the U(1) subgroup of quantum electrodynamics.

$$A^\mu = -V_3^\mu \sin\theta + V'^\mu \cos\theta \tag{66}$$

is the massless "photon" field, whereas Z^μ and $W_{\pm}^\mu$ are the fields of heavy vector bosons of masses:

$$M_Z = |g's|^{1/2},$$

$$M_W \geqslant M_Z \cos\theta. \tag{67}$$

The equality is realized if R invariance is conserved.

6.3. Two Goldstone spinors

As in ref. [9], f_γ is the only auxiliary field with respect to $\mathcal{A}_\uparrow(\mathcal{S})$, which takes a non-vanishing vacuum expectation value. Using the Σ transformation, we find the auxiliary fields with respect to $\mathcal{A}_\downarrow(\mathcal{S})$. Among all anticommutators of the spinorial charges $Q_{\alpha\uparrow}$ and $Q_{\alpha\downarrow}$ with spinor fields, two have non-vanishing vacuum expectation values: they are:

$$\langle\{Q_{\alpha\uparrow}, \bar{p}_{\gamma\beta}\}\rangle = \langle\{Q_{\alpha\downarrow}, \bar{\lambda}_{\gamma\beta}\}\rangle = \langle f_\gamma\rangle\,\delta_{\alpha\beta}. \tag{68}$$

Both supersymmetry algebras $\mathcal{A}_\uparrow(\mathcal{S})$ and $\mathcal{A}_\downarrow(\mathcal{S})$ are spontaneously broken. p_γ and λ_γ are the corresponding Goldstone spinors.

The "photon" γ is massless from gauge invariance, the Majorana spinors p_γ and λ_γ from Goldstone theorem; the ω_γ particle is massless at zeroth order.

6.4. Two conserved quantum numbers

The conserved quantum number $L_e - L_\mu$ was defined in sects. 3 and 4 of this paper, as a linear combination of I_3, a generator of "isospin" rotations, and Y, charge associated with the U(1) gauge group [eq. (46)]. Here it is a linear combination of I_2, a generator of "isospin" rotations, and Y, "hypercharge" associated with the U(1) factor in the $SU(2)_G \times U(1)$ gauge group.

If the vacuum state preserves R invariance, as in sect. 4, we can also define the conserved quantum number $L_e + L_\mu$. As previously, the ω_γ particle has "electronic" number 1 and "muonic" number 1.

Thus, we have obtained a model where spontaneous breaking of hypersymmetry provides two Goldstone spinors p_γ and λ_γ, associated with the "photon" γ and the ω_γ-particle in a vector hypermultiplet. Parity, and the U(1) gauge group of quantum electrodynamics are preserved. Two conserved quantum numbers can be defined if the "good" vacuum state is chosen.

7. Weak and electromagnetic interactions and hypersymmetry

Although very attractive, the idea that the two neutrinos are the two Goldstone particles arising from spontaneous hypersymmetry breaking seems incompatible with phenomenology, because of low-energy theorems [14]. In a realistic theory, Goldstone particles, if not eliminated while higher spin fields acquire masses, could remain as still-unobserved massless particles, distinct from the two observed neutrinos.

We would like to conserve the association between photon and neutrino; since both an electron and a muon neutrino exist, hypersymmetry seems very relevant: it associates naturally the photon γ, two neutrino fields $\binom{p_{\gamma L}}{\lambda_{\gamma L}}$ transforming like an $SU(2)_I$ isodoublet, and a weak-interacting complex scalar ω_γ. This last particle, which might be light, would have electronic number one, muonic number one; thus, it cannot be coupled to known charged leptons only, and there is no important effect at low energies (see a more detailed discussion in subsect. 6.3 of ref. [5]).

8. Conclusion

Abandoning the superfield formalism of ref. [6], we defined the hypersymmetry algebra; we succeeded in constructing theories invariant under transformations mixing supersymmetry and internal symmetry in a non-trivial way; they are expected to be renormalizable in a way compatible with these invariances.

Abelian as well as non-Abelian gauge models have been studied; now, gauge particles are vectors, spinors and scalars. The Higgs mechanism has been extended in a hypersymmetric way, and we gave an example of spontaneous breaking, providing two Goldstone spinors. Hypersymmetry gives a good frame for separate conservation of two quantum numbers.

Applications to strong interaction physics come to mind, but we give, now, some features that a hypersymmetric theory of weak and electromagnetic interactions would possibly have.

A massless vector hypermultiplet contains, as physical fields, a vector, an isodoublet of left-handed Dirac spinors, and a complex scalar; hypersymmetry associates naturally to the photon, two neutrinos, and a weak-interacting scalar particle; the latter, neutral, has electronic number one and muonic number one.

A piece of the puzzle is, perhaps, given by fig. 1, in which the action of the invariances of the theory ($SU(2)_G \times U(1)$ as a possible gauge group, $SU(2)_I$ as the internal symmetry group, and the two supersymmetry algebras) is represented:

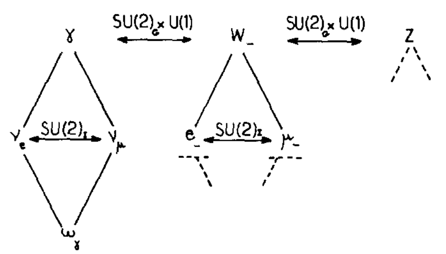

Fig. 1. Possible relations between fundamental particles in weak and electromagnetic interaction theory.

Hypersymmetry may be helpful to describe electron-muon universality, and gives the hope of a very large unification of particles.

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Note added in proof

The only difference between the original and revised versions of this paper is that we made more explicit in the latter the fact that ν_e and ν_μ should not be Goldstone particles. But what happens to the Goldstone spinors? Let us indicate some possible answers:

they are eliminated by a generalized Higgs mechanism involving spin-$\frac{3}{2}$ fields [15];

Goldstone particles cannot be produced significantly in known physical processes, because of low-energy theorems themselves. They might appear as right-handed neutrinos [16];

there exist limits of spontaneously broken supersymmetric, or hypersymmetric, theories for which Goldstone spinors decouple [17]. As an example, this occurs in the model of Section 6, when the limit $g' \to 0$, $s \to \infty$, with g and sg' fixed, is considered (in this limit the two Goldstone spinors become p' and λ', which decouple).

It still remains to construct a realistic model along these lines.

References

[1] J. Wess and B. Zumino, Nucl. Phys. B70 (1974) 39.
[2] J. Wess and B. Zumino, Nucl. Phys. B78 (1974) 1.
[3] P. Fayet and J. Iliopoulos, Phys. Letters 51B (1974) 461.
[4] P. Fayet, Nucl. Phys. B90 (1975) 104.
[5] P. Fayet, Nuovo Cimento 31A 4 (1976) 626.
[6] P.H. Dondi and M. Sohnius, Nucl. Fhys. B81 (1974) 317;
A. Salam and J. Strathdee, Nucl. Phys. B80 (1974) 499.
[7] D.M. Capper and G. Leibbrandt, Nucl. Phys. B85 (1975) 503;
J. Wess, Acta Physica 0/3462 (1975).
[8] S. Ferrara and B. Zumino, Nucl. Phys. B79 (1974) 413;
A. Salam and J. Strathdee, Phys. Letters 51B (1974) 353; Nucl. Phys. B87 (1975) 85.
[9] P. Fayet, Phys. Letters 58B (1975) 67.
[10] R. Haag, J.T. Łopuszański and M. Sohnius, Nucl. Phys. B88 (1975) 257.
[11] J. Wess, Lectures given at the Bonn Summer School 1974.
[12] P. Fayet, Nucl. Phys. B78 (1974) 14.
[13] A. Salam and J. Strathdee, Nucl. Phys. B97 (1975) 293.
[14] W.A. Bardeen, unpublished;
B. de Wit and D.Z. Freedman, Phys. Rev. Letters 35 (1975) 827.
[15] D.V. Volkov and V.A. Soroka, JETP Letters 18 (1973) 312.
[16] J. Iliopoulos, Invited talk given at the ISR Meeting held at Flaine, Hte Savoie, France, February 1976.
[17] P. Fayet, PTENS (76/10) preprint, Phys. Rev. D, to be published.
P. Fayet, PTENS (76/14) preprint, Phys. Letters B, to be published.