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CHIRAL QUARKS AND THE NON-RELATIVISTIC QUARK MODEL*

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We study some of the consequences of an effective lagrangian for quarks, gluons and goldstone bosons in the region between the chiral symmetry breaking and confinement scales. This provides an understanding of many of the successes of the non-relativistic quark model. It also suggests a resolution to the puzzle of the hyperon non-leptonic decays.

1. Introduction

Why does the non-relativistic quark model work? This has remained a mystery since the harmonic oscillator model was used in the early 1960's to calculate hadronic properties [1]. Things were better understood after the introduction of quantum chromodynamics [2], but it still seemed remarkable that the quarks inside a proton could be treated as non-relativistic particles. We shall concentrate on the properties of hadrons made up of the three light flavors (u, d, s) in what follows, and try and understand the successes of the non-relativistic quark model in terms of an effective chiral quark theory.

The strong interactions of the u, d and s quarks are described by the QCD lagrangian

$$\mathcal{L} = i\bar{\psi}_{\mathrm{L}} \not{D} \psi_{\mathrm{L}} + i\bar{\psi}_{\mathrm{R}} \not{D} \psi_{\mathrm{R}} - \bar{\psi}_{\mathrm{L}} M \psi_{\mathrm{R}} - \bar{\psi}_{\mathrm{R}} M \psi_{\mathrm{L}} , \tag{1.1}$$

$$\psi_{\mathrm{L,R}} = \begin{bmatrix} u \\ d \\ s \end{bmatrix}_{\mathrm{L,R}} , \tag{1.2}$$

$$D_\mu = \partial_\mu + igG_\mu , \tag{1.3}$$

$$M = \begin{bmatrix} m_{\mathrm{u}} & & \\ & m_{\mathrm{d}} & \\ & & m_{\mathrm{s}} \end{bmatrix} , \tag{1.4}$$

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g is the gluon coupling constant and G_μ is the gluon field. The covariant derivative is diagonal in flavour space. All colour indices have been suppressed. If we neglect the mass term, then $\mathcal{L}$ has an $SU(3)_L \times SU(3)_R$ chiral flavour symmetry. (A more complete discussion including Z_3 factors is given in sect. 5.) Two non-perturbative effects are alleged to take place in this theory: confinement and chiral symmetry breaking. The $SU(3)_L \times SU(3)_R$ chiral symmetry is spontaneously broken to an $SU(3)_v$ symmetry at some scale which we call $\Lambda_{\chi SB}$. This implies that there must be an octet of goldstone bosons: the π's, K's and η. We shall denote the confinement scale by Λ_{QCD}. There is no reason for these two scales to be the same, and we will argue later that $\Lambda_{\chi SB}$ is in fact greater than Λ_{QCD}, so that we can study the effective field theory in the intermediate region. It is not obvious that this is a sensible thing to do in the real world, where $\Lambda_{\chi SB}$ and Λ_{QCD} are not widely separated (we will argue that $\Lambda_{\chi SB} \simeq 1$ GeV and $\Lambda_{QCD} \simeq 100$–300 MeV). This perverse point of view, however, leads to a consistent picture which explains the successes of the non-relativistic quark model. Some of the calculations described here have originally been performed 20 years earlier [1]; we simply reinterpret them in this new language.

The effective lagrangian in the region between $\Lambda_{\chi SB}$ and Λ_{QCD} has fundamental quark and gluon fields, because these particles are not bound into colour-singlet hadrons at such short distances. Furthermore the quark-gluon interaction will still be described by an $SU(3)_{colour}$ gauge theory. Since the $SU(3)_L \times SU(3)_R$ global chiral symmetry is spontaneously broken, there is also an octet of pseudoscalar goldstone bosons, which are put in as fundamental fields. These goldstone boson fields are essential if the lagrangian is to consistently reproduce the effects of a spontaneously broken global symmetry. All the other hadrons are obtained as qqq or $q\bar{q}$ bound states. The main drawback of this approach is that it treats the ρ and π very differently; the π is a fundamental field in the lagrangian whereas the ρ is a $q\bar{q}$ bound state. One might have hoped that the only difference between the ρ and the π was the internal spin structure of the quarks. There is also the problem of double counting: there is a pseudoscalar $q\bar{q}$ bound state as well as a fundamental π field, and there are qqq baryons as well as baryonic solitons. We will discuss all these issues in what follows.

2. The effective lagrangian

The effective lagrangian below the chiral symmetry breaking scale involves quark, gluon, and goldstone boson fields. The spontaneously broken $SU(3)_L \times SU(3)_R$ chiral symmetry is realized non-linearly. As is well known, all such realizations are physically equivalent, and we choose a convenient one [3,4].

The goldstone boson dynamics is conveniently described in terms of a 3×3 matrix field $\Sigma(x)$ taking values in SU(3), which transforms under $SU(3)_L \times SU(3)_R$ as

$$\Sigma \to L \Sigma R^\dagger . \tag{2.1}$$

The goldstone boson fields

$$\pi = \frac{1}{\sqrt{2}} \begin{bmatrix} \sqrt{\frac{1}{2}}\,\pi^0 + \sqrt{\frac{1}{6}}\,\eta & \pi^+ & K^+ \\ \pi^- & -\sqrt{\frac{1}{2}}\,\pi^0 + \sqrt{\frac{1}{6}}\,\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{bmatrix}, \tag{2.2}$$

are defined *in terms of* Σ by

$$\Sigma = e^{2i\pi/f}. \tag{2.3}$$

With this normalization convention

$$f \simeq 93\ \text{MeV}. \tag{2.4}$$

A field ξ is also defined in terms of Σ by

$$\xi = e^{i\pi/f}, \quad \Sigma = \xi\xi. \tag{2.5}$$

ξ transforms under $SU(3)_L \times SU(3)_R$ as

$$\xi \to L\xi U^\dagger = U\xi R^\dagger, \tag{2.6}$$

where (2.6) defines U as an implicit function of the chiral transformation (L, R) *and* the π fields.

We also have a set of colour triplet Dirac fermions

$$\psi = \begin{bmatrix} u \\ d \\ s \end{bmatrix}, \tag{2.7}$$

which transform as

$$\psi \to U\psi. \tag{2.8}$$

The effective lagrangian can then be written down as the most general possible lagrangian which is invariant under chiral SU(3) [eqs. (2.1)–(2.8)] and conserves $\mathcal{C}$, $\mathcal{P}$, and $\mathcal{T}$. The last requirement follows because the lagrangian (1.1) has these discrete symmetries, which are not spontaneously broken by the QCD dynamics. The first few terms of the effective lagrangian are

$$\begin{aligned} \mathcal{L} = \bar{\psi}\,(i\not{D} + \not{V})\psi + g_A \bar{\psi} \not{A} \gamma_5 \psi - m\bar{\psi}\psi \\ + \tfrac{1}{4} f^2 \operatorname{tr} \partial^\mu \Sigma^\dagger \partial_\mu \Sigma - \tfrac{1}{2} \operatorname{tr} F_{\mu\nu} F^{\mu\nu} + \cdots, \end{aligned} \tag{2.9}$$

where

$$V_\mu = \tfrac{1}{2}\left(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger\right), \tag{2.10}$$

$$A_\mu = \tfrac{1}{2} i\left(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger\right), \tag{2.11}$$

are combinations of goldstone boson fields which are vector and axial vector respectively. They transform under chiral SU(3) as

$$V_\mu \to U V_\mu U^\dagger + U \partial_\mu U^\dagger , \tag{2.12}$$

$$A_\mu \to U A_\mu U^\dagger , \tag{2.13}$$

$$D_\mu = \partial_\mu + i g G_\mu , \tag{2.14}$$

is the usual covariant derivative, where G_μ is the gluon field,

$$F_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu + i g \left[G_\mu , G_\nu \right] , \tag{2.15}$$

is the field-strength tensor and the gluons are normalized as

$$G_\mu = G_\mu^A T^A , \tag{2.16}$$

$$\operatorname{tr} T^A T^B = \tfrac{1}{2} \delta^{AB} . \tag{2.17}$$

m represents a contribution to the constituent quark masses due to chiral symmetry breaking and is approximately 350 MeV. This is the so-called "soft" mass [5]

$$m \simeq g^2 \frac{\langle \bar{\psi} \psi \rangle}{q^2} , \tag{2.18}$$

produced by chiral symmetry breaking. It becomes soft (i.e. falls off with q^2) only at momenta large compared to $\Lambda_{\chi\mathrm{SB}}$ and Λ_{QCD} where the operator product expansion leading to (2.18) is valid. For momenta small compared to $\Lambda_{\chi\mathrm{SB}}$, this term can be treated as a "hard" quark mass term.

Since the lagrangian we have written is for an effective field theory, it will include non-renormalizable terms of arbitrarily high dimension. We therefore need a power counting argument to determine what dimensionful parameter suppresses these non-renormalizable terms.

All these non-renormalizable terms involve more derivatively coupled goldstone boson fields. At low energies (small compared to our dimensionful parameter), these terms are suppressed by a small ratio of pion momentum to this dimensionful parameter. For example, we know that in the QCD lagrangian below M_{W}, all higher-dimension terms induced by integrating out the W and Z are suppressed by the appropriate power of $\Lambda_{\mathrm{weak}} \simeq 100$ GeV needed to produce a dimension-four lagrangian term. To motivate the general formula (2.29) below, consider, e.g. the π-π scattering diagram [6, 4] (Fig. 1).

Each vertex is produced by expanding the kinetic term

$$\tfrac{1}{4} f^2 \operatorname{tr} \partial^\mu \Sigma^\dagger \partial_\mu \Sigma , \tag{2.19}$$

in powers of π/f. Each vertex is therefore of the form

$$\frac{p^2\pi^4}{f^2}, \tag{2.20}$$

where p represents derivatives acting on the π fields in various combinations. Since this is going to be a naive power counting argument, we do not need a more detailed form for this term. (This can always be obtained by a careful derivation of (2.20) from (2.19) keeping track of matrix ordering, etc.) One particular piece of the π-π scattering diagram has all the derivatives (i.e. p's) acting on the external lines. The value of this piece is then

$$\simeq \frac{p^4\pi^4}{f^4}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{1}{(k^2)^2} \tag{2.21}$$

$$\simeq \frac{p^4\pi^4}{f^4}\frac{1}{(4\pi)^2}\log\left(\Lambda^2_{\chi\mathrm{SB}}/\kappa^2\right), \tag{2.22}$$

where we have evaluated the loop integral using $\Lambda_{\chi\mathrm{SB}}$ as an ultraviolet cutoff, because our effective theory is not valid at higher energies. κ is some random renormalization point for the external momenta. A term of the form (2.22) can also be produced in an expansion of a higher-dimension term in the effective lagrangian such as

$$\frac{f^2}{\Lambda^2_{\chi\mathrm{SB}}}\operatorname{tr}\left(\partial^\mu\Sigma\partial^\nu\Sigma^\dagger\partial_\mu\Sigma\partial_\nu\Sigma^\dagger\right), \tag{2.23}$$

where $\Lambda_{\chi\mathrm{SB}}$ has been introduced to make it a dimension four term. Changes in the renormalization point κ can then be absorbed into a redefinition of the coefficient of (2.23). If there is no accidental fine-tuning of parameters, we would expect the coefficient of (2.23) to be at least as large as the coefficient induced by a rescaling of order 1 in the renormalization point κ of the π-π scattering diagram. This implies that

$$\frac{f^2}{\Lambda^2_{\chi\mathrm{SB}}} \gtrsim \frac{1}{(4\pi)^2}, \tag{2.24}$$

or

$$\Lambda_{\chi\mathrm{SB}} \lesssim 4\pi f. \tag{2.25}$$

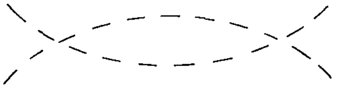

Fig. 1.

This suggests that we use

$$\Lambda_{\chi\mathrm{SB}} = 4\pi f, \tag{2.26}$$

as an estimate of the chiral symmetry breaking scale and as the dimensionful parameter that suppresses non-renormalizable terms in $\mathcal{L}$. What we have actually found is an inequality for the various coefficients; that they be at least as big as those obtained using (2.26) and dimensional analysis. We are going to assume that these coefficients are actually of the same order of magnitude as the values obtained using dimensional analysis. This has no a priori justification, but often turns out to be useful in practice. We shall call this procedure "naive dimensional analysis".

Next, consider the most general possible vertex involving quark, gluon and goldstone boson fields. A coefficient consistent with naive dimensional analysis is

$$(2\pi)^4\delta^4(\Sigma p_i)\left(\frac{\pi}{f}\right)^A\left(\frac{\psi}{f\sqrt{\Lambda}}\right)^B\left(\frac{gG_\mu}{\Lambda}\right)^C\left(\frac{p}{\Lambda}\right)^D f^2\Lambda^2_{\chi\mathrm{SB}}. \tag{2.29}$$

Here δ^4 is the δ-function for momentum conservation at the vertex, and p denotes derivatives acting on either the π, ψ or G_μ fields. This vertex must, of course, be produced by an expansion of a chiral-invariant, gauge invariant and Lorentz invariant term in $\mathcal{L}$ that conserves $\mathcal{C}$, $\mathcal{P}$, and $\mathcal{T}$. To prove this assertion, consider an arbitrary Feynman diagram involving a total of V vertices of the form (2.29) with (A, B, C, D) values equal to (A_i, B_i, C_i, D_i), $i = 1 \cdots V$. Consider the contribution of this diagram to the term (2.29). There are two categories of diagrams which contribute to (2.29): those with $\Sigma C_i > C$ and those with $\Sigma C_i = C$. The first kind have internal gluon lines, and so their contributions to (2.29) are suppressed by powers of $\alpha_s/4\pi$, and are therefore negligible because $\alpha_s \simeq 0.28$, as we will see later. Hence the dominant contribution to a term of the form (2.29) comes from loop diagrams with only external gluons, so that $\Sigma C_i = C$. The diagram is (we use Λ for $\Lambda_{\chi\mathrm{SB}}$ in what follows for notational simplicity)

$$\begin{gathered}(2\pi)^4\delta^4(\Sigma p_i)\left(\frac{\pi}{f}\right)^A\left(\frac{\psi}{f\sqrt{\Lambda}}\right)^B\left(\frac{gG_\mu}{\Lambda}\right)^C\left(\frac{p}{\Lambda}\right)^D f^2\Lambda^2 \\ f^{A+B+2V-2-\Sigma(A_i+B_i)}\Lambda^{B/2+C+D+2V-2-\Sigma(B_i/2+C_i+D_i)} \\ g^{\Sigma C_i - C}\, k^{\Sigma D_i - D}\,(2\pi)^{4(V-1)}\left[\delta^{(4)}(\Sigma p_i)\right]^{V-1} \\ \int\left[\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{1}{k^2}\right]^{I_\pi}\int\left[\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{1}{k}\right]^{I_\psi},\end{gathered} \tag{2.30}$$

where I_π and I_ψ are the number of internal pion and quark lines respectively. We have picked out that part of the diagram which has D derivatives acting on external lines (denoted by p). The remaining derivatives all act on internal lines and are denoted by k. All the other factors are trivial. For example, consider powers of f. We put together V vertices of the form (2.29) so the net "f-factor" is

$$f^{-\Sigma(A_i+B_i)+2V}, \tag{2.31}$$

which is the same as the "f-factor"

$$f^{-A-B+2}\left[f^{A+B+2V-2-\Sigma(A_i+B_i)}\right], \tag{2.32}$$

in (2.30). We now proceed to evaluate the diagram using naive dimensional analysis. The identities

$$\Sigma A_i = A + 2I_\pi, \tag{2.33}$$

$$\Sigma B_i = B + 2I_\psi, \tag{2.34}$$

$$\Sigma C_i = C, \tag{2.35}$$

(conservation of ends), and

$$L = I - V + 1 \tag{2.36}$$

$$= I_\pi + I_\psi - V + 1, \tag{2.37}$$

(L = number of loops) will be required. We first use (2.33)–(2.37) to simplify the exponents of f and Λ:

$$\begin{gathered}(2\pi)^4\delta^4(\Sigma p_i)\left(\frac{\pi}{f}\right)^A\left(\frac{\psi}{f\sqrt{\Lambda}}\right)^B\left(\frac{gG_\mu}{\Lambda}\right)^C\left(\frac{p}{\Lambda}\right)^D f^2\Lambda^2\\ f^{-2L}\Lambda^{D+2V-2-I_\psi-\Sigma D_i}(2\pi)^{4(V-1)}\left[\delta^{(4)}(\Sigma p_i)\right]^{V-1}\\ k^{\Sigma D_i-D}\int\left[\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{1}{k^2}\right]^{I_\pi}\int\left[\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{1}{k}\right]^{I_\psi}.\end{gathered} \tag{2.38}$$

There are a total of $I_\pi + I_\psi - (V-1) = L$ loop integrals to be performed. Each of these produces a factor

$$\frac{1}{16\pi^2} = \frac{1}{(4\pi)^2}. \tag{2.39}$$

Since all integrals are cutoff at Λ, we can estimate them by replacing all internal momenta by Λ. Thus

$$k \to \Lambda, \tag{2.40}$$

$$\left[\int \frac{\mathrm{d}^4 k}{(2\pi)^4}\right]^L \to \frac{\Lambda^{4L}}{(4\pi)^{2L}}, \tag{2.41}$$

and we obtain

$$(2\pi)^4 \delta^4(\Sigma p_i)\left(\frac{\pi}{f}\right)^A \left(\frac{\psi}{f\sqrt{\Lambda}}\right)^B \left(\frac{gG_\mu}{\Lambda}\right)^C \left(\frac{p}{\Lambda}\right)^D f^2\Lambda^2\left[f^{-2L}(4\pi)^{-2L}\Lambda^{2L}\right]. \tag{2.42}$$

Using our favorite identity $\Lambda = 4\pi f$, we find that the factor in square brackets is 1, so that we have generated a term of exactly the same form as (2.29).

We have therefore proved that the prescription (2.29) produces a set of coefficients for the higher-dimension operators consistent with naive dimensional analysis. The coefficients generated by (2.29) are compatible with gauge invariance, for (2.29) says that ∂_μ and gG_μ both have a coefficient $1/\Lambda$, and hence so does $D_\mu = \partial_\mu + igG_\mu$. As an example, the terms $\bar{\psi}\xi^\dagger \not{\partial}\xi\psi$, $\operatorname{tr} \partial^\mu \Sigma^\dagger \partial_\mu \Sigma$, $\operatorname{tr} \partial^\mu \Sigma^\dagger \partial^\nu \Sigma \partial_\mu \Sigma^\dagger \partial_\nu \Sigma$, $\operatorname{tr} F^{\mu\nu}F_{\mu\nu}$ are of order

$$\bar{\psi}\xi^\dagger \not{\partial}\xi\psi\left(\frac{1}{f\sqrt{\Lambda}}\right)^2 f^2\Lambda^2 = \bar{\psi}\xi^\dagger \not{\partial}\xi\psi, \tag{2.43}$$

$$\operatorname{tr} \partial^\mu \Sigma^\dagger \partial_\mu \Sigma \frac{1}{\Lambda^2} f^2\Lambda^2 = f^2 \operatorname{tr} \partial^\mu \Sigma^\dagger \partial_\mu \Sigma, \tag{2.44}$$

$$\operatorname{tr} \partial^\mu \Sigma^\dagger \partial^\nu \Sigma \partial_\mu \Sigma^\dagger \partial_\nu \Sigma \frac{1}{\Lambda^4} f^2\Lambda^2 = \frac{f^2}{\Lambda^2} \operatorname{tr} \partial^\mu \Sigma^\dagger \partial^\nu \Sigma \partial_\mu \Sigma^\dagger \partial_\nu \Sigma, \tag{2.45}$$

$$\operatorname{tr} F^{\mu\nu}F_{\mu\nu}\left[\left(\frac{g}{\Lambda}\right)^2 \frac{1}{\Lambda^2} f^2\Lambda^2\right] = \frac{g^2 f^2}{\Lambda^2} \operatorname{tr} F^{\mu\nu}F_{\mu\nu}. \tag{2.46}$$

Note that the power counting argument produced the correct normalizations for the kinetic terms in (2.43)–(2.44). (2.46) is a little unexpected. It says that the gluon kinetic energy has a coefficient $\simeq 1/(4\pi)^2$ instead of $\frac{1}{2}$. This is an example where the coefficient in the chiral lagrangian is larger than one might have guessed by pure dimensional analysis, because of the presence of a dimensionless coupling constant. The same thing would have occurred if we had included photons; we would have expected

$$\frac{e^2 f^2}{\Lambda^2} F^{\mu\nu} F_{\mu\nu}, \tag{2.47}$$

which has a coefficient $\sim \frac{1}{2}$ only if $e \sim 4\pi$! It was precisely because g (or e) was much less than 4π, that internal gluon effects were negligible compared with internal pion and quark loops.

We might also want to consistently count powers of m, the effective quark mass. In that case, it is clear that one multiplies the general vertex (2.29) by

$$\left(\frac{m}{\Lambda_{\chi\mathrm{SB}}}\right)^{|\Delta\chi|/2}, \tag{2.48}$$

where $\Delta\chi$ is the chirality violation at the vertex. This is simply because each m produces a $\Delta\chi = 2$. The factor in the denominator is a $\Lambda_{\chi\mathrm{SB}}$ because extra mass insertions do not produce any factors of 4π as they do not increase the number of loops. Alternatively, a mass is just like an external field which gets a vacuum expectation value, and so has a factor of $\Lambda_{\chi\mathrm{SB}}$, just like the external gluon fields. Thus $\bar{\psi}\psi$ has $\Delta\chi = 2$, and so has one factor of m in front of it, etc. It is worth keeping in mind that all the arguments above have been based on dimensional analysis, and cannot be trusted to any great numerical accuracy. They should, however, provide a qualitative guide as to what is going on.

MATCHING CONDITIONS

One would like to do better than these naive power counting arguments for the various coefficients. In particular, one would like to calculate the dimensionless parameters such as g_A and α_s in the effective theory. Since the quark fields in the effective theory are not the same as the ones in the QCD lagrangian above $\Lambda_{\chi\mathrm{SB}}$ there is no reason for these coefficients to be the same in the two theories. Let us first look at g_A. The $\mathrm{SU}(2)_L$ current in the effective lagrangian is

$$j_L^{\mu A} = \bar{\psi}\gamma^\mu(1 - g_A\gamma_5)T^A\psi + \text{terms involving } \pi, \tag{2.49}$$

and this is the current which couples to the W boson. We can now use (2.49) to calculate hyperon semileptonic decay amplitudes and find g_A by fitting to experiment. For example, the G_A/G_V ratio in β-decay can be calculated using (2.49) and non-relativistic quark model wave functions. One finds

$$G_A/G_V = \left(\tfrac{5}{3}\right) g_A, \tag{2.50}$$

which gives

$$g_A = 0.7524, \tag{2.51}$$

using the experimental value of 1.255 for G_A/G_V. This is the first example of a matching condition. The current

$$j_{\mu L}^{QCD} = \bar{\psi}\gamma^{\mu}(1-\gamma_5)T^A\psi, \tag{2.52}$$

in the QCD lagrangian *above* $\Lambda_{\chi SB}$ is the same as (2.49) *below* $\Lambda_{\chi SB}$ in the sense that this is the current associated with the $SU(3)_L$ chiral charge. Since we know $\mathcal{L}_{QCD}$, we should be able to calculate the form of (2.49), and in particular the value of g_A. This is a hard non-perturbative calculation which we have no idea how to perform. Presumably one could calculate it using a lattice Monte Carlo.

For example, one could pick any matrix element of (2.52) that was convenient, and compute it on a lattice. The same matrix element can also be computed using (2.49) in the effective theory. By equating the two we get g_A. One can then use g_A and calculate, for example, the hyperon semileptonic decay amplitudes using a quark model. It is no longer necessary to calculate the matrix elements of the current (2.52) in all the hyperon states using lattice methods. This hybrid: lattice calculation of a matching condition plus a chiral lagrangian cum quark model calculation of matrix elements should be a useful way of attacking the baryons. Thus in the effective lagrangian approach, all the complicated dynamics of chiral symmetry breaking have been subsumed into these calculations of various matching conditions. Our ignorance of the dynamics is reflected in the large (infinite) number of parameters in the effective lagrangian.

The value of α_s in the effective theory can be computed by looking at the ratio of colour and electromagnetic hyperfine splittings of the baryon spectrum [7]. One finds

$$\alpha_s \simeq 0.28, \tag{2.53}$$

which is much smaller than one might have naively guessed. Since momentum transfers inside a baryon are comparable with Λ_{QCD}, one might have expected an $\alpha_s > 1$. This is another example of a coefficient that has to be matched across the boundary between QCD and our effective theory. This small value of α_s in the effective theory explains many of the successes of the NRQM. Since α_s is quite small, the gluon-exchange binding force is relatively weak, and the baryons are not as strongly bound as one might have expected. Thus the NRQM works simply because these effective quarks *are* non-relativistic, with

$$\beta = v/c \simeq \alpha_s \sim 0.3. \tag{2.54}$$

Can we understand why α_s is smaller in this effective theory? There are qualitative arguments which suggest that this might happen.

The chiral symmetry breaking occurs at a scale $\Lambda_{\chi SB}$ because $\alpha_s(q^2 = \Lambda_{\chi SB}^2)$ becomes very large, and drives $\bar{\psi}\psi$ to get a non-zero vacuum expectation.

Thus the large value of α_s destabilizes the original vacuum with respect to the one where the chiral symmetry is spontaneously broken. Since the reason for the instability was a large α_s, it is clear that this broken-symmetry vacuum will have a smaller value of α_s. Otherwise a large α_s would cause $\bar{\psi}\psi$ to get a vacuum expectation value which would increase α_s which would increase $\langle\bar{\psi}\psi\rangle$ and the whole theory would bootstrap itself into inconsistency. Thus spontaneous symmetry breaking must reduce the value of α_s in the effective theory.

The various interactions we have been discussing so far are not the complete story. The QCD lagrangian is not simply $i\bar{\psi}\not{D}\psi$, but has explicit quark mass terms which we have neglected, as well as higher-dimension operators which produce weak decays, baryon number violating decays, etc. Our effective lagrangian must therefore contain additional terms responsible for these interactions. Any operator $\mathcal{O}$ transforming as (R_1, R_2) under $SU(3)_L \times SU(3)_R$ will produce *all* possible operators in the effective theory with the same chiral transformation properties. The coefficients of the induced operators are in principle calculable in terms of the coefficient of $\mathcal{O}$. This computation is yet another example of a matching condition*. It is worth repeating that $\mathcal{O}$ produces all operators which transform as (R_1, R_2) under chiral SU(3). Even if $\mathcal{O}$ could be written as the product of two currents, there is no reason for the induced operators to have this form. E.g. The explicit $SU(3)_L \times SU(3)_R$ breaking mass term

$$M = \begin{bmatrix} m_u & & \\ & m_d & \\ & & m_s \end{bmatrix}, \tag{2.55}$$

transforms as $M \to LMR^\dagger$ under $SU(3) \times SU(3)$. The effective theory will thus contain

$$\tfrac{1}{2} f^2 \operatorname{tr} \mu M \Sigma^\dagger + \text{h.c.}, \tag{2.56}$$

where μ is a parameter with the dimension of a mass. This produces a mass for the pseudoscalar mesons. We will discuss the other operators in greater detail in sect. 4.

One could have done an analysis very similar to that presented above using a chiral lagrangian with mesons and baryons instead of mesons and quarks. There are several advantages to the quark chiral lagrangian. It has fewer parameters, and hence greater predictive power. For example, the quark lagrangian predicts a D/F ratio of $\frac{3}{2}$ for baryon semileptonic decays. In the baryon lagrangian, both D and F would have been free parameters. We will return to the baryon lagrangian in sect. 4 when we discuss the hyperon non-leptonic decays.

* We will discuss this in more detail in sect. 4.

3. Baryon magnetic moments

Our chiral quarks have a Dirac magnetic moment

$$\mu = q\left(\frac{e}{2m}\right)\boldsymbol{\sigma}, \tag{3.1}$$

where q is the quark charge in units of e, and m is the mass of the quark. m is around 360 MeV for the u and d and 540 MeV for the strange quark. This gives

$$\mu_{\mathrm{d}} \simeq -0.9 \text{ n.m. (nuclear magnetons)}, \tag{3.2}$$

$$\mu_{\mathrm{s}} \simeq -0.6 \text{ n.m.}, \tag{3.3}$$

$$\mu_{\mathrm{u}} = -2\mu_{\mathrm{d}} \simeq 1.8 \text{ n.m.} \tag{3.4}$$

Thus we would conclude that the baryon magnetic moments should be calculated in terms of the quark moments (3.2)–(3.4) using the non-relativistic quark model wave functions. This is the standard analysis which we know works to better than 20 percent. Let us look at higher-dimension terms in the chiral lagrangian to see what are the other contributions to the baryon magnetic moments. There is a term

$$\frac{me}{\Lambda_{\chi\mathrm{SB}}^2}\bar{\psi}\sigma^{\mu\nu}F_{\mu\nu}(aQ+b)\psi, \tag{3.5}$$

$$Q = \begin{bmatrix} +\frac{2}{3} & & \\ & -\frac{1}{3} & \\ & & -\frac{1}{3} \end{bmatrix}, \tag{3.6}$$

whose coefficient was determined using the arguments of sect. 2. The form $aQ+b$ comes from V-spin invariance. This produces anomalous magnetic moments

$$\mu \simeq \left(\frac{m^2}{\Lambda_{\chi\mathrm{SB}}^2}\right) q\left(\frac{e}{2m}\right)\boldsymbol{\sigma}, \tag{3.7}$$

which are 10 percent of the values (3.1). The part of (3.5) proportional to the unit matrix (in flavour space) no longer preserves the relation

$$\mu_{\mathrm{u}} = -2\mu_{\mathrm{d}}. \tag{3.8}$$

We therefore expect a violation of this relation at the 10 percent level. There are also four-quark operator contributions to the baryon magnetic moments. These cannot be treated as effective quark moments, because they do not act independently on each of the quarks inside a baryon. There is, however, reason to believe that matrix elements of four-quark operators between baryon states are suppressed relative to those of two-quark operators. The reason for this will be discussed in sect. 4 on hyperon non-leptonic decays. There are also relativistic corrections to the baryon moments which are of order $v^2/c^2 \simeq \alpha_s^2 \simeq 0.1$ which are also 10 percent of the lowest order result. This makes the analysis of the moments to better than 10 percent accuracy complicated, with many parameters. We will not redo the analysis of the relativistic corrections here. The result is that [8]

$$\mu_{\mathrm{B}} = \mu_{\mathrm{B}}^{(0)} + \varepsilon_{\mathrm{B}}\left(\frac{Q_1}{m_1} + \frac{Q_2}{m_2} - \frac{2Q_3}{m_3}\right), \tag{3.9}$$

$\mu_{\mathrm{B}}^{(0)}$ is the lowest order $((v/c)^0)$ moment,

$$\mu_{\mathrm{B}}^{(0)} = \tfrac{4}{3}\mu_1 - \tfrac{1}{3}\mu_2, \quad B = N, \Sigma, \Xi, \tag{3.10}$$

$$\mu_{\Lambda}^{(0)} = \mu_{\mathrm{s}}, \tag{3.11}$$

and ε_{B} is a parameter that depends upon the quark masses, and so takes on different values for $\mathrm{B} = \mathrm{N}, \Sigma, \Lambda, \Xi$. 1 and 2 refer to the two "identical" quarks (i.e. the ones with respect to which the wave function is symmetric or antisymmetric) and 3 is the other quark. The experimental values for the moments are listed in table 1. Using these, we can determine the various parameters. Using the values

$$m_{\mathrm{u}} = m_{\mathrm{d}} = 360\ \mathrm{MeV}, \tag{3.12}$$

$$m_{\mathrm{s}} = 540\ \mathrm{MeV}, \tag{3.13}$$

TABLE 1
Baryon magnetic moments

Baryon	Experimental value [9] (in nuclear magnetons)
p	2.7928456 (11)
n	−1.91304184 (88)
Λ	-0.6138 ± 0.0047
Σ^+	2.357 ± 0.012
Σ^-	-1.151 ± 0.021
Ξ^0	-1.253 ± 0.014
Ξ^-	-0.69 ± 0.04

obtained from the mass spectrum, we find

$$\mu_{\mathrm{u}} = 1.98 \pm 0.02 \text{ n.m.}, \tag{3.14}$$

$$\mu_{\mathrm{d}} = -1.10 \pm 0.02 \text{ n.m.}, \tag{3.15}$$

$$\mu_{\mathrm{s}} = -0.75 \pm 0.03 \text{ n.m.}, \tag{3.16}$$

$$\varepsilon_{\mathrm{N}} = -0.042, \quad \varepsilon_{\Lambda} = 0.069, \tag{3.17}$$

$$\varepsilon_{\Sigma} = -0.116, \quad \varepsilon_{\Xi} = -0.089.$$

From this, one can get a parameter free prediction for the Ω^- moment from (3.9). This is because the coefficient of ε_{Ω} vanishes so there are no new parameters required. Using the above values, we find

$$\mu_{\Omega^-} = -2.26 \pm 0.08 \text{ n.m.} \tag{3.18}$$

Note that $-\mu_{\mathrm{u}}/2\mu_{\mathrm{d}} = 0.9$, and as expected, is close to one. Unfortunately, there are so many free parameters in this analysis that we have virtually no predictive power. In a specific potential model, one can do a more detailed analysis and determine the values of ε_{B}.

The chiral quark theory has provided an explanation for the scale of the baryon magnetic moments, as well as an estimate of the accuracy of the lowest order fit to the moments.

4. Hyperon non-leptonic decays

In this section, we discuss the $\Delta S = 1$ hyperon non-leptonic decays in the context of a chiral quark theory. The $\Delta S = 1$ transitions come from terms of the form

$$\mathcal{L}^{\Delta S=1} = \bar{u}\gamma^{\mu}(1-\gamma_5)s\bar{d}\gamma_{\mu}(1-\gamma_5)u, \tag{4.1}$$

in the QCD lagrangian. These terms transform as $(8,1)\oplus(27,1)$ under $\mathrm{SU(3)_L} \times \mathrm{SU(3)_R}$. We will concentrate on the octet interaction assuming that it is enhanced by some combination of short distance QCD and the matching onto the effective chiral theory. The effective lagrangian therefore contains all possible interactions transforming as $(8,1)$ which have the quantum numbers of $\bar{\mathrm{d}}\mathrm{s}$. The easiest way to find these is to introduce a matrix

$$h = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \tag{4.2}$$

which transforms as

$$h \to L h L^{\dagger}, \tag{4.3}$$

and write down all possible chiral SU(3) invariant interactions involving one power of h.

$$\mathcal{L}^{\Delta S=1} = \rho m \bar{\psi} \xi^{\dagger} h \xi \psi + \sigma m \bar{\psi} \xi^{\dagger} h \xi \gamma_5 \psi + \lambda f^2 \operatorname{tr} h \partial_{\mu} \Sigma \partial^{\mu} \Sigma^{\dagger} + \cdots , \tag{4.4}$$

where λ, ρ, σ are unknown constants which are determined by the matching conditions. The m's and f's have been introduced in a manner consistent with the general power counting analysis of sect. 2, so that λ, ρ, σ are dimensionless.

The power counting analysis of sect. 2 was for terms in the effective lagrangian. What we want here is an extension of that analysis to the coefficient of $\Delta S = 1$ operators in the weak lagrangian. The analysis is very similar to that for terms in $\mathcal{L}$. We obtain the $\Delta S = 1$ weak lagrangian $\mathcal{L}^{\Delta S=1}$ at a scale M_W by integrating out the W. This is then scaled down to $\Lambda_{\chi SB}$ using the standard perturbative QCD analysis [10]. This produces $\mathcal{L}^{\Delta S=1}$ as a sum of coefficients times operators. Any given operator gets matched across the boundary to a sum of all possible operators with the same chiral transformation properties. The coefficients of the octet operators presumably get enhanced in this matching condition. What we want here is an extension of our assumption of naive dimensional analysis to this matching condition. The extension is clear. Any operator induced by $\mathcal{L}^{\Delta S=1}$ is going to have a very small coefficient typical of any weak interaction operator. Hence the dominant contribution to weak-interaction operators through loop diagrams comes from diagrams where only one vertex is from $\mathcal{L}^{\Delta S=1}$, and all other vertices are from $\mathcal{L}$. The analysis is then exactly as before. The general rule for obtaining the coefficients is then: use the arguments of sect. 2 to write down the coefficients of the operator. Multiply it by a small dimensionless number which will characterize the strength of the weak interactions. As before, this argument is basically dimensional analysis, and some of the coefficients could turn out to be much larger than those guessed at above.

The invariant amplitudes for non-leptonic decays can be written as [11]

$$M = G m_{\pi}^2 \bar{u}_{\mathrm{f}}(p_{\mathrm{f}})[A + B\gamma_5] u_{\mathrm{i}}(p_{\mathrm{i}}), \tag{4.5}$$

where m_{π} is the π^+ mass and $G \equiv 10^{-5}\,(m_{\text{proton}})^{-2}$. The A amplitude describes the parity violating s-wave decays, the B amplitude the parity conserving p-wave decays. These decays can be calculated using the chiral quark lagrangian (4.4). Before we do that, however, it will be helpful to discuss the decay amplitudes using a chiral baryon theory. There are 2 $\Delta S = 1$ baryon operators whose matrix elements are not proportional to momenta or symmetry breaking. These are

$$\mathcal{L}_{\mathrm{B}} = \eta \operatorname{tr} \bar{B} \left[d \{ \xi^{\dagger} h \xi, B \} + f [\xi^{\dagger} h \xi, B] \right]. \tag{4.6}$$

There are other terms which involve no explicit derivatives, such as

$$\operatorname{tr} \bar{B}\gamma_5 \xi^\dagger h \xi B . \tag{4.7}$$

But their matrix elements vanish in the symmetry limit because of the γ_5. They contain an implicit factor of the symmetry breaking, or pion momentum.

It is reasonable to suppose that (4.6) gives the dominant contribution to the baryon non-leptonic decays (assuming octet enhancement), with the effect of operators like (4.7) suppressed by powers of $p_\pi/\Lambda_{\chi SB}$. This assumption gives the standard current algebra results for the non-leptonic decays. It works very well for the s-wave amplitudes. For example, with

$$d = -\tfrac{1}{2}, \quad f = \tfrac{3}{2}, \tag{4.8}$$

the result for the s-wave decay amplitudes is shown below, along with the experimental value (where the signs have been chosen consistent with the conventions used in the particle data book).

No fit has been attempted here, but it is clear that everything works to 10 percent or so, which is all we can expect from the effective theory.

The theoretical predictions exhibit five relations among the seven amplitudes which follow from (4.6). Three of these are $\Delta I = \frac{1}{2}$ relations

$$\begin{aligned} A(\Lambda^0_-) &= -\sqrt{2} A(\Lambda^0_0), \\ A(\Xi^-_-) &= \sqrt{2} A(\Xi^0_0), \\ \sqrt{2} A(\Sigma^+_0) + A(\Sigma^+_+) &= A(\Sigma^-_-). \end{aligned} \tag{4.9}$$

One is the Lee-Sugawara relation

$$\sqrt{3} A(\Sigma^+_0) + A(\Lambda^0_-) = 2A(\Xi^-_-), \tag{4.10}$$

TABLE 2

	A (Th)	A (exp)
Λ^0_-	1.63	1.47 ± 0.01
Λ^0_0	-1.15	-1.07 ± 0.01
Σ^+_+	0	0.06 ± 0.01
Σ^+_0	1.41	1.48 ± 0.05
Σ^-_-	2.00	1.93 ± 0.01
Ξ^0_0	1.44	1.55 ± 0.03
Ξ^-_-	2.04	2.04 ± 0.01

and one is the simple relation

$$A(\Sigma_+^+) = 0\,. \tag{4.11}$$

All are satisfied as well as we could expect (or better).

Unfortunately, we can now take (4.8) from the s-waves and predict the p-waves. The p-wave decays, in this picture, arise from so-called "pole diagrams" in which the hyperon changes strangeness through the $\bar{B}B$ term in (4.6) before or after a pion is emitted. The pion emission is accompanied by a factor of pion momentum, proportional to symmetry breaking. But this is compensated by the pole in the baryon propagator which is inversely proportional to the symmetry breaking, so the contribution is large. The only trouble is that the predictions for the p-wave decay look absolutely nothing like the data. This has been a puzzle for nearly twenty years. Is this a general problem for the effective lagrangian idea, or is there something more specifically wrong with the assumption that (4.6) dominates the non-leptonic decays?

Can we find our way out of this difficulty in the chiral quark model? As in the baryon theory, there are octet operators in the quark theory which do not involve derivatives or μM. With two quark fields there is a unique operator,

$$\bar{\psi}\xi^\dagger h \xi \psi\,. \tag{4.12}$$

With four quark fields, there are a variety of operators, such as

$$\bar{\psi}\xi^\dagger h \xi \gamma^\mu T^A \psi \bar{\psi} \gamma_\mu T^A \psi\,, \tag{4.13}$$

where the T^A are color SU(3) generators.

We can estimate the relative importance of the baryon matrix elements of the two- and four-quark operators using the quark chiral lagrangian. The power counting analysis says that the operators (4.12)–(4.13) should have coefficients

$$\chi m \bar{\psi}\xi^\dagger h \xi \psi\,, \tag{4.14}$$

$$\frac{\chi}{f^2}\left(\bar{\psi}\xi^\dagger h \xi \gamma^\mu T^A \psi\right)\left(\bar{\psi}\gamma_\mu T^A \psi\right), \tag{4.15}$$

where χ is a small dimensionless number.

The matrix element of (4.14) between baryon states

$$\chi m \langle B | \bar{\psi}\xi^\dagger h \xi \psi | B' \rangle \simeq \chi m\,, \tag{4.16}$$

from the Feynman diagram in Fig. 2.

The diagram for the matrix element of (4.15) is given in fig. 3 so that

$$\frac{\chi}{f^2}\langle B | \left(\bar{\psi}\xi^\dagger h \xi \gamma^\mu T^A \psi\right)\left(\bar{\psi}\gamma_\mu T^A \psi\right) | B' \rangle \simeq \chi \frac{|\psi(0)|^2}{f^2}\,, \tag{4.17}$$

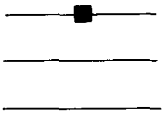

Fig. 2.

where $\psi(0)$ is the amplitude to find two quarks in a baryon at the same point. We know from various quark model estimates that [12]

$$|\psi(0)|^2 \simeq (100 \text{ MeV})^3 \simeq f^3 . \tag{4.18}$$

Hence the 4-quark operator matrix elements are suppressed by

$$f/m \simeq \tfrac{1}{3} . \tag{4.19}$$

The most penetrating way to analyze the chiral quark theory is to use it to construct a chiral baryon theory by comparing the matrix elements of the corresponding interactions between various states. For example, the matrix elements which give rise to the s-wave amplitudes in the quark theory from the two-quark operator, (4.12), are obtained by acting with $\bar{\psi}[h, \pi]\psi$ on each of the quark lines. Any s-quark can emit a π^0 or π^- and turn into a d or a u. This produces a baryon matrix element like (4.6), but because the quark flavor dependence is like that of an SU(3) generator, the matrix is pure f. Thus (4.12) makes a contribution to (4.6) with

$$d = 0 .$$

The four-quark operators like (4.13) produce similar terms but with both f and d contributions. Thus we expect a slight suppression of the d amplitude, by a factor of about $\frac{1}{3}$ (from (4.19)) which is just what is observed (in (4.8)).

Now consider the contribution of the two-quark operator to p-wave decays. The relevant pieces of the interaction lagrangian are:

$$\bar{\psi}\left(\rho m \xi^\dagger h \xi + \rho m \xi^\dagger h^\dagger \xi + g_A \not{A} \gamma_5 - m - \tfrac{1}{2}\xi^\dagger M \xi^\dagger - \tfrac{1}{2}\xi M \xi\right)\psi , \tag{4.20}$$

where

$$M = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix} . \tag{4.21}$$

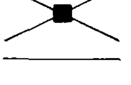

Fig. 3.

Expanding in powers of π, we get

$$-\bar{\psi}m\psi - \bar{\psi}S\psi - g_A\bar{\psi}\not{P}\gamma_5\psi + \cdots,$$

$$S = M - \rho m(h + h^\dagger) - i\rho m[h + h^\dagger, \pi]/f,$$

$$P_\mu = \partial_\mu \pi, \tag{4.22}$$

where the terms not written explicitly involve more powers of the pion field. The S term contains the terms in (4.12) which give rise to the two quark contribution to the s-wave decay. But it also contains an off diagonal contribution to the quark mass, $\rho m(h + h^\dagger)$. We can diagonalize the quark mass matrix by making a V-spin rotation which to lowest order in ρ has the form

$$\psi \to \psi' = R\psi,$$

$$R = 1 + \rho m(h - h^\dagger)/\Delta m,$$

$$\Delta m = m_s - m_d. \tag{4.23}$$

When we rewrite (4.22) in terms of the mass eigenstate fields, it is (to first order in ρ)

$$-\bar{\psi}'m\psi' - \bar{\psi}'S'\psi' - \bar{\psi}'\not{P}'\gamma_5\psi',$$

$$S' = RSR^\dagger = M - i\rho m[h + h^\dagger, \pi]/f,$$

$$P' = RP_\mu R^\dagger = \partial_\mu \pi + \rho m\left[h - h^\dagger, \partial_\mu \pi\right]/\Delta m. \tag{4.24}$$

In this mass eigenstate basis, the two-quark operators no longer produce any pole diagrams. Instead the effects of the pole diagrams are produced by the second term in P'. The $1/\Delta m$ enhancement arises because the angle of the V-spin rotation (4.23) is proportional to $1/\Delta m$. In this language, the s-wave and p-wave contributions are related because the s-wave amplitudes determine ρ which fixes the V-spin rotation R which in turn produces the extra term in P'_μ.

Now when we use this quark theory to construct a baryon chiral lagrangian, we can treat S and P_μ as small. All the terms involve either derivatives, the chiral symmetry breaking mass M or the small parameter ρ. Thus we can work to first order in S and P_μ and the relevant terms are

$$\operatorname{tr}\bar{B}\left(f[S, B] + d\{S, B\}\right) + \operatorname{tr}\bar{B}\left(F\left[P_\mu, \gamma^\mu\gamma_5 B\right] + D\left\{P_\mu, \gamma^\mu\gamma_5 B\right\}\right). \tag{4.25}$$

In the non-relativistic limit, $d = 0$ and $D/F = \frac{3}{2}$. The point is that (4.25) produces s-wave and p-wave decays which are related in the usual way, because it is simply

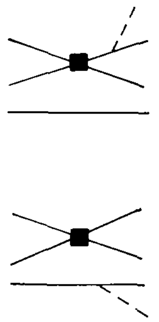

Fig. 4.

the usual baryon lagrangian. We can obtain the p-wave amplitudes either by calculating pole diagrams or by removing the off-diagonal terms in the *baryon* mass matrix with a V-spin rotation ($B' = RBR^{\dagger}$ where R is given by (4.23)). These are equivalent and yield the standard p-wave predictions which look nothing like the data. The trouble is that the structure of the two-quark operator is too much like that of the chiral symmetry breaking mass term.

The situation for the four-quark operators is different. These cannot be absorbed into a mass term. They contribute to the p-wave decays through diagrams such as the one in fig. 4. These contributions are inversely proportional to Δm because there is always a virtual quark which is off-shell by an amount proportional to Δm. But here there is no reason to believe that these contributions can be related to corresponding s-wave processes (as in fig. 5). The momenta carried by the internal quarks are quite different. Thus, in general we expect such diagrams to produce terms in $\mathcal{L}_{\mathrm{B}}$ in addition to (4.25), such as

$$\frac{1}{\Delta m} \operatorname{tr} \bar{B} \not{A} \gamma_5 B \xi^{\dagger} h \xi . \tag{4.26}$$

Note that these have no term independent of π because A_μ begins at $\mathrm{O}(\pi)$. (4.16) gives a p-wave amplitude coming from the one-pion piece,

$$\frac{1}{\Delta m} \operatorname{tr} \bar{B} \gamma^{\mu} \gamma_5 \partial_{\mu} \pi B h . \tag{4.27}$$

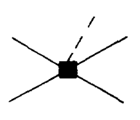

Fig. 5.

One can now count the number of such invariants. There are four octets ($\bar{B}$, B, A^{μ} and h), and so 8 independent SU(3) invariants. Of these, three are irrelevant because

$$\pi h = 0, \tag{4.28}$$

if we are only interested in the $\pi^{\pm}$, π^{0} in π. (i.e. πh depends only on the K's and η). There are therefore five parameters for the p-wave amplitudes, and so one can fit the experimental data, since the data has only four independent amplitudes. The rest are trivially related by isospin which is preserved in our analysis.

Why did operators involving $\not{A}$ like

$$\frac{1}{\Delta m} \operatorname{tr} \bar{B} \not{A} B \xi^{\dagger} h \xi, \tag{4.28}$$

not get induced for the s-waves?

The s-wave four-quark operator diagram looks like the one in fig. 5 where the vertex is, e.g.

$$\bar{\psi}\,[h, \pi]\gamma^{\mu} T^{A} \psi \bar{\psi} \gamma_{\mu} T^{A} \psi . \tag{4.29}$$

There is an analogous diagram, fig. 6, which comes from the π-independent piece of (4.13). If we treat the dot in fig. 6 as an operator insertion, then there is no difference between the baryon matrix elements of fig. 5 and fig. 6. They can be summed into a term which contains

$$h + \frac{i}{f}[h, \pi] + \cdots . \tag{4.30}$$

But $A_{\mu} = -(1/f)\partial_{\mu}\pi + \cdots$ and so *has no π independent piece*. Thus terms involving $\not{A}$ are not induced for the s-waves, so that our two-parameter analysis was all that was required.

To summarize, the chiral quark model has resolved the problem associated with the hyperon non-leptonic decays. There are only two independent terms in the baryon chiral lagrangian for the s-waves and so we have two predictions (in addition to isospin relations) (4.10)–(4.11) which agree with experiment. There are five independent terms for the p-wave decays so all predictive power is lost, but there is no longer any contradiction with the world. If one were willing to work harder, then

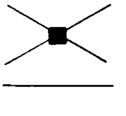

Fig. 6.

one could calculate some of these coefficients by taking the quark chiral theory and using a quark model to evaluate various matrix elements. This would increase the predictive power. For now, we will content ourselves with the negative result that the p-wave amplitudes cannot be simply predicted.

5. Problems

Let us look in greater detail at the topology of the chiral lagrangian. $\Sigma(x)$ takes values in SU(3). The lowest-order lagrangian for the goldstone boson fields is

$$\mathcal{L} = \tfrac{1}{4} f^2 \operatorname{tr} \partial^\mu \Sigma^\dagger \partial_\mu \Sigma . \tag{5.1}$$

This forces $\Sigma(x)$ to take on a constant value at infinity for any finite energy solution. Thus the static, finite energy solutions are characterized by maps

$$\Sigma : S^3 \to SU(3) . \tag{5.2}$$

Since

$$\pi_3(SU(3)) = \mathbb{Z} ,$$

these maps fall into homotopy classes characterized by an integral winding number. These are the well-known solitons [13]. [Note that since the lower homotopy groups are trivial, we have no domain walls or strings in the chiral theory.] It is not possible to review the details of some of the recent work on this subject; we just state the main results here. There is a term (the Wess-Zumino-Witten [14] term) in the chiral theory which reproduces the effects of the anomalous triangle diagram. When this term is included in the effective lagrangian, it gives a contribution to the baryon-number current which is the topological winding number of Σ. Soliton number is baryon number, and the soliton is a baryon. Our chiral theory has fundamental quark fields, so there are baryons which are quark bound states. Have we double-counted the baryons?

The soliton is not stable if the lagrangian has only the kinetic term (5.1). This follows from Derrick's theorem [15]. To stabilize the soliton, we need to add higher-dimension terms such as

$$\alpha \left[\operatorname{tr} \partial^\mu \Sigma^\dagger \partial_\mu \Sigma \right]^2 , \tag{5.3}$$

to the lagrangian. The mass of the soliton will then depend upon α. The value of α in an effective theory with quarks is not the same as the value of α when the quarks are integrated out. Thus the quark theory could have a value of α which either made the soliton very heavy ($\gg \Lambda_{\chi SB}$), or did not stabilize the soliton because it had the wrong sign. In either case, there is no double counting of the baryons.

The next question is the double counting of pions. There are fundamental pions in our lagrangian, as well as light q$\bar{q}$ bound states. Our theory is different from the usual potential models since it has a fundamental pion that must be included in the inter-quark potential. In the pseudoscalar q$\bar{q}$ bound state sector, there is an additional s-channel annihilation diagram, fig. 7, which is not present in usual potential models. This causes the fundamental π and q$\bar{q}$ bound state of mix. Since the π is massless, this pushes the q$\bar{q}$ state to a higher mass. We use the same techniques as those used by De Rújula, Georgi and Glashow [2] in studying the $\pi\eta\eta'$ mixing problem. The annihilation diagram fig. 7 which causes the q$\bar{q}$-π mixing is proportional to p^2 (where p is the pion momentum), since the pions are derivatively coupled. Let m be the mass of the q$\bar{q}$ state obtained from a potential model calculation.

The q$\bar{q}$ and π can be written as the eigenvectors of the equation

$$\begin{bmatrix} p^2 & 0 \\ 0 & p^2 \end{bmatrix} \psi(p^2) = \begin{bmatrix} m^2 & 0 \\ 0 & 0 \end{bmatrix} \psi(p^2), \tag{5.4}$$

with the eigenvalues of p^2 being the masses. Fig. 7 introduces off-diagonal terms εp^2 on the left-hand side. The equation is now

$$\begin{bmatrix} p^2 & \varepsilon p^2 \\ \varepsilon p^2 & p^2 \end{bmatrix} \psi(p^2) = \begin{bmatrix} m^2 & 0 \\ 0 & 0 \end{bmatrix} \psi(p^2), \tag{5.5}$$

which has eigenvalues

$$p^2 = 0, \tag{5.6}$$

$$p^2 = \frac{m^2}{1-\varepsilon^2}, \tag{5.7}$$

As expected, the pion is massless, and the q$\bar{q}$ mass is pushed up. One could in principle calculate ε and hence determine the q$\bar{q}$ mass. Since the pion-quark

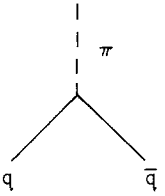

Fig. 7.

coupling is $\partial_\mu \pi/f$, it will involve some combination of f and the wave-function at the origin. Since $|\psi(0)|^2 \sim (100\ \text{MeV})^3 \sim f^3$ we expect $\varepsilon \sim 1$. However, as mentioned in sect. 4, we do not know $\psi(0)$ to any great accuracy, and so cannot actually calculate ε. There are now two possibilities for the other mass. If ε is close to one, it gets a mass $\gg \Lambda_{\chi\text{SB}}$, and so is no longer in the effective theory. ε could be such that it is pushed up the mass of the π', the next pseudoscalar state. The other possibility is that $\varepsilon = 1$. In that case the $q\bar{q}$ state gets pushed to infinite energy, and so is no longer part of the physical Hilbert space. In either case there is no double counting in the effective theory, and as far as the effective theory is concerned, these two possibilities are indistinguishable.

Another problem with the effective theory is that of light glueballs. The strong-coupling constant has a value $\alpha_s \sim 0.28$ when renormalized at a typical hadronic momentum scale, say 500 MeV. This implies that $\Lambda^{\text{eff}}_{\text{QCD}}$ in the effective theory is very small. It would seem that there should be very light glueball states with masses of order $\Lambda^{\text{eff}}_{\text{QCD}}$. It is not clear what causes these states to be pushed up to around 1 GeV.

S. Weinberg has been saying much of this for years. We are merely taking him seriously. We are also very grateful to S. Coleman for his attempts to keep us honest.

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