Rep. Prog. Phys., Vol. 42, 1979. Printed in Great Britain

Phase transitions in gauge theories and cosmology

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Abstract

In this review we discuss phase transitions in super-dense matter, which consists of particles interacting in accordance with the unified gauge theories of weak, strong and electromagnetic interactions. It is shown that at a sufficiently large temperature a phase transition takes place after which almost all elementary particles in the hot super-dense matter become massless and weak interactions become long-range like electromagnetic interactions. Analogous phenomena may take place with an increase of fermion density in cold dense matter, and also in the presence of external fields and currents. Phase transitions in gauge theories lead to a time dependence of the masses of particles, of coupling constants and of the cosmological term in the expanding Universe, to the appearance of a domain structure of vacuum, to substance energy non-conservation, to a possibility of obtaining the 'hot' Universe starting with a 'cold' one, and to some other unusual effects important for cosmology and for elementary particle physics.

This review was received in April 1978.

0034-4885/79/030389+49 $05.00

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1. Introduction

The discovery of the unified gauge theories of weak, strong and electromagnetic interactions at the beginning of the 1970s has opened up a new phase of development of the theory of elementary particles and of quantum field theory. For brevity, and in accordance with the accepted terminology, we shall call these theories SBGT (spontaneously broken gauge theories). A consistent description of weak and strong interactions was performed first by means of SBGT, and at present the quantum field theory treatment of weak and strong interactions is carried out almost exclusively in the framework of SBGT.

Therefore, it was natural to investigate which consequences SBGT lead to when describing matter at high temperature and density, when the effects connected with weak and strong interactions should be taken into account. The results of this investigation appear to be rather unexpected and interesting.

It appears that in super-dense matter, which consists of the particles interacting in accordance with SBGT, various phase transitions should occur, which lead to the restoration of the originally broken symmetry between weak, strong and electromagnetic interactions. As a result, almost all elementary particles become massless, weak and strong interactions become long-range like electromagnetic interactions, the energy of substance is not conserved due to the 'pumping' of energy from the non-observable classical scalar field, and a number of other striking and unusual effects take place. Being interesting in themselves, these effects are also very important for an understanding of the physical processes at early stages in the evolution of the Universe. In some cases the phase transitions in the early Universe may influence strongly the large-scale structure of the Universe and even the properties of elementary particles at the present time.

The existence of various phase transitions in super-dense matter may also appear to be important for the investigation of the properties of matter in the cores of neutron stars, for the theory of quark stars, and also for the investigation of the processes of multiparticle production in high-energy particle collisions.

In the present article we shall give a review of the results concerning phase transitions in gauge theories. Since many of the effects discussed in this review may seem rather unusual, we shall try to outline everywhere when possible the derivations of the main results, but for the details of calculations usually we shall refer the reader to the original literature. Before beginning the presentation of the main content of the review we shall remind readers of some general features of SBGT.

1.1. *Gauge theories with spontaneous symmetry breaking*

Before the discovery of SBGT the most popular theory of weak interactions was the theory involving the intermediate vector meson. Weak interactions in this theory were mediated by a heavy vector W_μ meson (figure 1(*a*)), whereas electromagnetic interactions were mediated by the massless vector (electromagnetic) field A_μ (figure 1(*b*)). This similarity stimulated many attempts to consider weak and electromagnetic interactions from a unique point of view by means of a unification of these interactions in some general symmetry group.

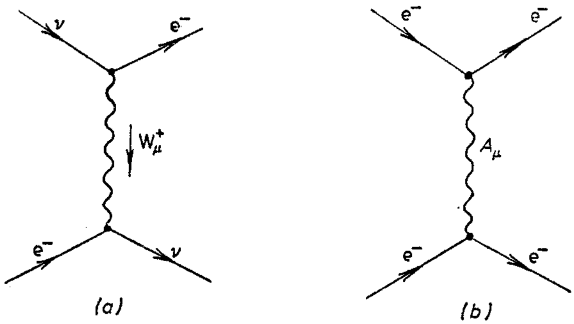

Figure 1. (*a*) Weak interaction between neutrino and electron mediated by the heavy W_μ meson. (*b*) Electromagnetic interaction between electrons mediated by the electromagnetic field A_μ.

However, this programme seemed to be unrealisable since the photons A_μ were massless while the W_μ mesons were massive. The same property also leads to the non-renormalisability of the theory of weak interactions. The reason for this non-renormalisability is that the Green function of the vector particle W_μ with the mass m and momentum k:

$$G_{\mu\nu}{}^{W}(k)=\left(\delta_{\mu\nu}+\frac{k_\mu k_\nu}{m^2}\right)\frac{1}{k^2+m^2} \tag{1.1}$$

(everywhere throughout this review we use Euclidean Green functions, $k^2=k_0{}^2+\boldsymbol{k}^2$) does not vanish at $k\to\infty$.

This leads to additional ultraviolet divergences, increasing with each new order of the perturbation theory. In quantum electrodynamics (QED) the corresponding difficulties are absent just due to the vanishing of $G_{\mu\nu}{}^{A}(k)$ at $k\to\infty$. For example, in the transverse gauge $\partial_\mu A_\mu=0$ in QED:

$$G_{\mu\nu}{}^{A}(k)=\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)\frac{1}{k^2}. \tag{1.2}$$

Nevertheless not only weak and electromagnetic, but also strong, interactions have been unified and the resulting models of weak, strong and electromagnetic interactions prove to be renormalisable. This success is connected with the use of theories with spontaneous symmetry breaking.

A simple example of a theory with spontaneous symmetry breaking is the theory of a scalar field ϕ with the Lagrangian:

$$L=\tfrac{1}{2}(\partial_\mu\phi)^2+\tfrac{1}{2}\mu^2\phi^2-\tfrac{1}{4}\lambda\phi^4. \tag{1.3}$$

For simplicity we suppose everywhere the coupling constant λ to be small (weak coupling): $0<\lambda\ll 1$. The 'wrong' sign of the mass term $\frac{1}{2}\mu^2\phi^2$ makes the Lagrangian (1.3) similar to that of the hypothetic superluminal particles—tachyons. This means that the ordinary solution of the Lagrange equation in the theory (1.3), which could be obtained from perturbation theory by expanding ϕ near $\phi=0$, is unstable. The simplest way to understand this is to consider the expression for the energy (for the effective potential, see §2) of a constant field $\phi=\sigma$, which at the classical level is given by:

$$V(\sigma)=-\tfrac{1}{2}\mu^2\sigma^2+\tfrac{1}{4}\lambda\sigma^4. \tag{1.4}$$

It is seen that due to the 'wrong' sign of the mass term $-\frac{1}{2}\mu^2\sigma^2$ the energy $V(\sigma)$

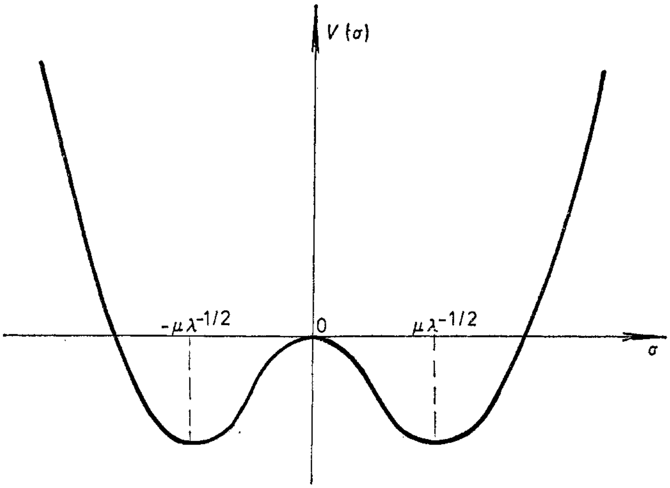

Figure 2. Effective potential $V(\sigma)$ in the model (1.3).

has a minimum not at $\sigma=0$ but at $\sigma=\pm\mu\lambda^{-1/2}$ (figure 2). Therefore, even if the system initially was in the symmetric state $\sigma=0$, very rapidly a transition to the state $\sigma=\pm\mu\lambda^{-1/2}$ should take place. This effect is just the so-called spontaneous symmetry breaking. In the language of the field operators this means that after the spontaneous symmetry breaking the vacuum expectation value $\langle 0|\phi|0\rangle$ does not vanish:

$$\langle 0|\phi|0\rangle \equiv \sigma = \pm\mu\lambda^{-1/2} \neq 0.$$

To return to the usual creation and annihilation operators with vanishing vacuum expectation values one should perform a shift:

$$\phi \to \phi + \sigma \tag{1.5}$$

where σ is the classical constant scalar field:

$$\sigma = \pm\mu\lambda^{-1/2} \tag{1.6}$$

and after the shift one has as usual:

$$\langle 0|\phi|0\rangle = 0.$$

The Lagrangian (1.3) after the shift is given by:

$$\begin{aligned} L(\phi+\sigma) &= \tfrac{1}{2}(\partial_\mu(\phi+\sigma))^2 + \tfrac{1}{2}\mu^2(\phi+\sigma)^2 - \tfrac{1}{4}\lambda(\phi+\sigma)^4 \\ &= \tfrac{1}{2}(\partial_\mu\phi)^2 - \tfrac{1}{2}(3\lambda\sigma^2-\mu^2)\phi^2 - \lambda\sigma\phi^3 - \tfrac{1}{4}\lambda\phi^4 + \tfrac{1}{2}\mu^2\sigma^2 - \tfrac{1}{4}\lambda\sigma^4 - \sigma(\lambda\sigma^2-\mu^2)\phi. \end{aligned} \tag{1.7}$$

Taking equation (1.6) into account the last term in equation (1.7) vanishes and the mass term in equation (1.7) has a correct sign and corresponds to the ϕ particles with the mass squared:

$$m_\phi{}^2 = 2\mu^2. \tag{1.8}$$

One can obtain the same result by a somewhat more accurate method often used in what follows. Let us consider the vacuum average of the Lagrange equation for the theory (1.3):

$$\langle 0|\delta L/\delta\phi|0\rangle = \langle 0|(\Box\phi + \mu^2\phi - \lambda\phi^3)|0\rangle = 0. \tag{1.9}$$

After the shift (1.5) this equation becomes:

$$\Box\sigma-(\lambda\sigma^2-\mu^2)\sigma-3\lambda\sigma\langle 0|\phi^2|0\rangle-\lambda\langle 0|\phi^3|0\rangle=0. \tag{1.10}$$

In the lowest order in λ the average $\langle 0|\phi^3|0\rangle$ vanishes, and the term $-3\lambda\sigma\langle 0|\phi^2|0\rangle$, which appears due to the vacuum fluctuations of the field ϕ, can be removed by the mass renormalisation (i.e. by adding the counter-term $\delta m^2\phi^2$ to the Lagrangian (1.3)). In this case equation (1.8) yields:

$$\Box\sigma+\sigma(\mu^2-\lambda\sigma^2)=0 \tag{1.11}$$

(for the quantum corrections to this equation see §2). Then, supposing $\sigma=$ constant (translationally invariant vacuum state), one obtains two possible solutions: $\sigma=0$ and $\sigma=\pm\mu\lambda^{-1/2}$. To obtain the excitation spectrum in the theory (1.3) one should perform in equation (1.10) an infinitesimal shift $\sigma\to\sigma+\delta\sigma$, where σ is one of the two possible constant solutions of equation (1.8). At $\sigma=0$ the equation for $\delta\sigma$ obtained in such a way is:

$$\Box\delta\sigma+\mu^2\delta\sigma=0 \tag{1.12}$$

which corresponds to the existence of tachyons with the mass squared $-\mu^2$. Therefore, the fluctuations $\delta\sigma(\boldsymbol{k})$ with the momentum $\boldsymbol{k}$ near $\sigma=0$ will grow with time as $\exp\,[(\mu^2-\boldsymbol{k}^2)^{1/2}t]$ (unstable vacuum state). Meanwhile, at $\sigma=\pm\mu\lambda^{-1/2}$ the excitation spectrum is given by the equation:

$$\Box\delta\sigma-2\mu^2\delta\sigma=0 \tag{1.13}$$

which corresponds to ordinary particles with the positive mass squared $2\mu^2$ (equation (1.8)) (stable vacuum).

Let us try to understand now how one can make vector mesons massive by means of spontaneous symmetry breaking without loss of renormalisability. For this purpose we shall consider here the Higgs model (Higgs 1964a, b, 1966, Kibble 1967, Guralnik *et al* 1964, Englert and Brout 1964), which describes the interaction of a massless vector field A_μ with a complex scalar field χ with a 'wrong' sign of the mass term $\mu^2\chi^*\chi$:

$$L=-\tfrac{1}{4}(\partial_\mu A_\nu-\partial_\nu A_\mu)^2+(\partial_\mu+\mathrm{i}eA_\mu)\chi^*(\partial_\mu-\mathrm{i}eA_\mu)\chi+\mu^2\chi^*\chi-\lambda(\chi^*\chi)^2. \tag{1.14}$$

This Lagrangian is invariant under the Abelian U(1) group of the gauge transformations:

$$A_\mu(x)\to A_\mu(x)+\frac{1}{e}\,\partial_\mu\zeta(x) \tag{1.15}$$

$$\chi(x)\to\chi(x)\,\exp\,(\mathrm{i}\zeta(x))$$

and, in particular, under the global rotation:

$$\chi\to\chi\mathrm{e}^{\mathrm{i}\theta} \tag{1.16}$$

where $\theta=$ constant. Therefore, without loss of generality one may regard that after spontaneous symmetry breaking in the Higgs model (1.14) it is only the real component of the field χ that acquires some positive constant classical part σ:

$$\chi(x)=\frac{1}{\sqrt{2}}\,(\chi_1(x)+\mathrm{i}\chi_2(x))\to\frac{1}{\sqrt{2}}\,(\chi_1(x)+\sigma+\mathrm{i}\chi_2(x)) \tag{1.17}$$

and after the shift (1.17) $\langle 0|\chi_i|0\rangle = 0$. According to the Goldstone theorem (Goldstone 1961, Goldstone *et al* 1962) in the gauges, which do not break the invariance of the Lagrangian (1.14) under the rotation (1.16), massless scalar particles should appear after spontaneous symmetry breaking (1.17). To verify it let us perform the shift (1.17) in the Lagrangian (1.14). After the shift the Lagrangian (1.14) looks as follows:

$$\begin{aligned} L = & -\tfrac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \tfrac{1}{2}(\partial_\mu \chi_1)^2 + \tfrac{1}{2}(\partial_\mu \chi_2)^2 + \tfrac{1}{2}e^2((\chi_1+\sigma)^2 + \chi_2{}^2)A_\mu{}^2 \\ & - e\sigma A_\mu \partial_\mu \chi_2 + e(\chi_2\partial_\mu\chi_1 - \chi_1\partial_\mu\chi_2)A_\mu - \tfrac{1}{2}(3\lambda\sigma^2 - \mu^2)\chi_1{}^2 - \tfrac{1}{2}(\lambda\sigma^2-\mu^2)\chi_2{}^2 \\ & - \lambda\sigma\chi_1(\chi_1{}^2+\chi_2{}^2) - \tfrac{1}{4}\lambda(\chi_1{}^2+\chi_2{}^2)^2 - \sigma(\lambda\sigma^2-\mu^2)\chi_1 + \tfrac{1}{2}\mu^2\sigma^2 - \tfrac{1}{4}\lambda\sigma^4 \\ & + \frac{1}{2\alpha}(\partial_\mu A_\mu)^2 \end{aligned} \tag{1.18}$$

where the last term is added to fix the gauge. It is seen that at the classical level the energy of the field σ is given by:

$$V(\sigma) = -\tfrac{1}{2}\mu^2\sigma^2 + \tfrac{1}{4}\lambda\sigma^4 \tag{1.19}$$

just like in the theory (1.3) (see (1.4)). Therefore the classical field σ, which appears after the spontaneous symmetry breaking, is equal to $\sigma = \mu\lambda^{-1/2}$ as in (1.3). The propagators of the fields A_μ and χ_i ,which correspond to the Lagrangian (1.18), are of a manifestly 'renormalisable' type. For example, in the transverse gauge $\partial_\mu A_\mu = 0$ ($\alpha \to 0$ in (1.18)) these propagators are given respectively by:

$$G_{\mu\nu}{}^A = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{1}{k^2+e^2\sigma^2} \tag{1.20}$$

$$G_{\chi_1} = \frac{1}{k^2+2\lambda\sigma^2} \tag{1.21}$$

$$G_{\chi_2} = \frac{1}{k^2}. \tag{1.22}$$

From (1.22) it is seen that after the symmetry breaking the field χ_2 becomes massless in accordance with the Goldstone theorem. The propagator of the field A_μ can be represented in the form:

$$G_{\mu\nu}{}^A = \left(\delta_{\mu\nu} + \frac{k_\mu k_\nu}{e^2\sigma^2}\right)\frac{1}{k^2+e^2\sigma^2} - \frac{k_\mu k_\nu}{k^2e^2\sigma^2}. \tag{1.23}$$

The first part of this Green function is the propagator of a vector particle with the mass $e\sigma = e\mu\lambda^{-1/2}$ (compare with (1.1)) while the second part of (1.23) corresponds to the propagation of a longitudinally polarised massless particle with indefinite metric. It can be shown that the contribution of this last particle to all physical processes is cancelled exactly by the corresponding contribution of the Goldstone particle χ_2. To illustrate this statement let us perform a change of variables in the Lagrangian (1.14):

$$\begin{aligned} \chi(x) & \to \frac{1}{\sqrt{2}}(\phi(x)+\sigma)\exp(\mathrm{i}\zeta(x)/\sigma) \\ A_\mu(x) & \to A_\mu(x) - \frac{1}{e\sigma}\partial_\mu\zeta(x) \end{aligned} \tag{1.24}$$

instead of the previously used shift (1.17). The Lagrangian (1.14) at $\sigma=\mu\lambda^{-1/2}$ is then transformed into:

$$L=-\tfrac{1}{4}(\partial_\mu A_\nu-\partial_\nu A_\mu)^2+\tfrac{1}{2}e^2\sigma^2A_\mu{}^2+\tfrac{1}{2}e^2\phi^2A_\mu{}^2+e^2\sigma\phi A_\mu+\tfrac{1}{2}(\partial_\mu\phi)^2-\lambda\sigma^2\phi^2$$
$$-\lambda\sigma\phi^3-\tfrac{1}{4}\lambda\phi^4+\mu^2/4\lambda \qquad (1.25)$$

from which the auxiliary field $\zeta(x)$ has been completely transformed away.

The propagators of the fields A_μ and ϕ in this case are given by:

$$G_{\mu\nu}{}^A=\left(\delta_{\mu\nu}+\frac{k_\mu k_\nu}{e^2\sigma^2}\right)\frac{1}{k^2+e^2\sigma^2} \qquad (1.26)$$

$$G_\phi=\frac{1}{k^2+2\lambda\sigma^2} \qquad (1.27)$$

and no Goldstone mesons and the particles with indefinite metric are present.

It can be seen that the Lagrangian (1.25) is equal to the Lagrangian (1.18) (without the gauge term $(1/2\alpha)(\partial_\mu A_\mu)^2$) in the gauge $\chi_2=0$. This means that both the Goldstone field χ_2 and the longitudinal field with indefinite metric can be transformed away by choosing the gauge $\chi_2=0$. Therefore, after spontaneous symmetry breaking in the Higgs model instead of the massless vector field A_μ (two degrees of freedom) and the complex field χ with negative mass squared $-\mu^2$ (two degrees of freedom), one deals with the massive vector field A_μ (three degrees of freedom) with the mass $m_A=e\sigma=e\mu\lambda^{-1/2}$ and with the real scalar field ϕ with the mass $m_\phi=\sqrt{2\lambda}\sigma=\sqrt{2}\,\mu$ (one degree of freedom). The mechanism of the vector meson mass generation discussed above (the Higgs mechanism) serves as a basis for SBGT. Note that in the R gauges specified by the term $(1/2\alpha)(\partial_\mu A_\mu)^2$ in (1.18) one can show that the theory is renormalisable. However, in these gauges it is difficult to prove the unitarity of the theory in the subspace of the physical states (without the Goldstone field χ_2 and the field with indefinite metric). Meanwhile in the gauge $\chi_2=0$ (1.25) the unitarity is manifest, and therefore this gauge is called 'unitary gauge' (U gauge), but the renormalisability of the theory in this gauge is not obvious. However, the theories (1.18) and (1.25) are equivalent since the corresponding Lagrangians differ in the gauge conditions only and, consequently, the theory is both renormalisable and unitary. A more accurate formulation and the proof of this important statement were given by 't Hooft (1971), Slavnov (1972), Taylor (1971), Lee (1972), Lee and Zinn-Justin (1972), 't Hooft and Veltman (1972), Tyutin and Fradkin (1974), Kallosh and Tyutin (1973) and Ross and Taylor (1973).

Fermion masses in most of SBGT also appear due to spontaneous symmetry breaking. As an example we shall consider here a simplified version of the linear σ model, which is often used for the description of strong interactions (see, for example, Lee 1972). The Lagrangian of this model is given by:

$$L=\tfrac{1}{2}(\partial_\mu\phi)^2+\tfrac{1}{2}\mu^2\phi^2-\tfrac{1}{4}\lambda\phi^4+\bar\psi(\mathrm{i}\partial_\mu\gamma_\mu-g\phi)\psi=L_\phi+\bar\psi(\mathrm{i}\partial_\mu\gamma_\mu-g\phi)\psi \qquad (1.28)$$

where ψ is the massless fermion field interacting with the scalar field ϕ and the Lagrangian L_ϕ is given by (1.3). After spontaneous symmetry breaking in the theory (1.28) the field ϕ, as in (1.3), acquires a non-vanishing constant classical part $\sigma=\pm\mu\lambda^{-1/2}$ (1.6). This leads to the appearance of the term $g\sigma\bar\psi\psi$ in the Lagrangian (1.28), which means that after the symmetry breaking the fermions acquire a mass

$$m_\psi=|g\sigma|=g\mu\lambda^{-1/2}.$$

The first realistic SBGT of weak and electromagnetic interactions has been suggested by Weinberg (1967) and Salam (1968). According to this model all vector particles and fermions before symmetry breaking are massless. After symmetry breaking only photons and neutrinos remain massless, whereas the intermediate vector mesons responsible for weak interactions acquire very large masses (~ 80 GeV). Even this first comparatively simple model proves to be fairly successful in the description of weak and electromagnetic interactions. However, the real interest in theories of this type appeared only at the beginning of the 1970s when it was shown that SBGT are renormalisable and therefore all quantum corrections in these theories can be computed unambiguously.

Further investigations have shown that some of the SBGT have serious advantages not only over the old theories of weak and strong interactions (which usually were unrenormalisable), but even over the ordinary quantum electrodynamics. More than twenty years ago it was argued that in most of the models in quantum field theory (and in particular in quantum electrodynamics) effective coupling constants become infinite at large momenta and the Green functions contain tachyon poles (Landau and Pomeranchuk 1955, Fradkin 1955). This leads to the vacuum instability with respect to spontaneous generation of infinitely strong classical fields (Linde 1977a, Kirzhnits and Linde 1978a,b), and therefore the models with such a pathological behaviour of coupling constants and Green functions are physically unacceptable. Fortunately a class of the non-Abelian asymptotically free gauge theories has been discovered (Gross and Wilczek 1973, Politzer 1973), in which effective coupling constants vanish at large momenta. In some of these theories tachyons are absent or disappear after symmetry breaking, and therefore such theories are free from the difficulties mentioned above (Voronov and Tyutin 1975). Another advantage of the asymptotically free SBGT is that the asymptotic freedom may serve as an explanation of approximate Bjorken scaling in strong interactions. All these (and many other) beautiful properties of gauge theories, together with the experimental discovery of neutral currents (of weak interactions mediated by neutral vector mesons) and of 'charmed' particles, which naturally appear in SBGT, have led most physicists into a belief that just the principle of spontaneous breaking of gauge invariance should be a basis for a theory of all fundamental interactions. For a detailed discussion of the properties of SBGT one can refer to excellent reviews by Abers and Lee (1973), Weinberg (1974b) and Fradkin and Tyutin (1974).

In recent years many attempts have been made to consider the theories in which some of the symmetries are broken not spontaneously, but dynamically (i.e. due to quantum effects), or even remain unbroken. The most interesting theory of this type is quantum chromodynamics (QCD), in which the Yang–Mills vector fields responsible for strong interactions are supposed to be massless, but the resulting 'strong' forces are believed to be short-range due to the effects connected with infrared instability and some special topological properties of the massless non-Abelian theories (see, for example, Wilson 1974, Polyakov 1977, 1978, Callan *et al* 1977, 't Hooft 1977). Unfortunately, in spite of a large number of different suggestions neither QCD nor realistic four-dimensional theories with dynamical symmetry breaking have been completely constructed so far. (As for the dynamical symmetry breaking in a very interesting approach of Coleman and Weinberg (1973), it was shown that this dynamical symmetry breaking is actually equivalent to a spontaneous one (Linde 1976a,d).) Therefore, in this review we shall give some comments concerning the possible phase transitions in QCD and in the models with dynamical symmetry breaking but the main content of the review will be concerned with gauge theories with spontaneous symmetry breaking.

1.2. *Phase transitions in* SBGT

The idea of spontaneous symmetry breaking which appears so useful in applications to quantum field theory actually was used long ago in solid-state physics and in quantum statistics applied to the theories of such phenomena as ferromagnetism, superfluidity, superconductivity, etc. In fact, any phase transition is a process with a change of symmetry which before (or after) the phase transition is spontaneously broken. The most instructive examples in this respect are the theories of Bose condensation and superconductivity.

Let us consider, for example, the occupation numbers in the Bose gas:

$$n_p = \langle a_p^+ a_p \rangle \tag{1.29}$$

which correspond to the number of particles with the momentum $\boldsymbol{p}$ in a system under consideration. Here a_p^+ and a_p are operators of creation and annihilation for a particle with the momentum $\boldsymbol{p}$, and $\langle \ldots \rangle$ is the Gibbs average (see, for example, Landau and Lifshitz 1964, Abrikosov *et al* 1964, Fradkin 1965):

$$\langle \ldots \rangle = \frac{Sp[\exp(-H/T) \ldots]}{Sp[\exp(-H/T)]} \tag{1.30}$$

where H is the Hamiltonian of the system. At low temperatures almost all particles of the Bose gas are condensed in the state with $\boldsymbol{p}=0$, i.e.:

$$\langle a_0^+ a_0 \rangle \sim N$$

where N is the overall number of particles in the system, $N \gg 1$. This means that:

$$[a_0 a_0^+] = 1 \ll \langle a_0^+ a_0 \rangle.$$

Therefore in the computation of thermodynamical (Gibbs) averages the non-commutativity of the operators a_0 and a_0^+ is inessential, and in the limit $N \to \infty$ one can deal with these operators as with ordinary C numbers. From this point of view one can treat the appearance of a classical (C-number) part σ of the field ϕ in SBGT as a Bose condensation of the particles of the field ϕ in a state with vanishing four-momentum $\boldsymbol{p}=0$. We should note, however, that Bose condensation occurs in SBGT in spite of the non-conservation of the number of scalar particles. At the same time a condensation of an ideal Bose gas is caused by an 'overcrowding' of energy levels but it is impossible if the number of particles in the gas is not fixed. This is exactly the reason why Bose condensation of the photon gas does not take place. Bose condensation in the theories with spontaneous symmetry breaking occurs due to interaction between the scalar particles, and in this sense a more accurate analogue to SBGT is superconductivity theory. This analogy becomes particularly clear if one compares equation (1.14) with the expression for the energy of a superconductor in the phenomenological superconductivity theory of Ginzburg and Landau (1950) (see also the textbooks by De Gennes (1966) and Saint-James *et al* (1969)):

$$E = E_0 + \tfrac{1}{2}H^2 + \frac{|(\nabla - 2ieA)\psi|^2}{2m} - \alpha|\psi|^2 + \beta|\psi|^4. \tag{1.31}$$

Here E_0 is the energy of a normal metal without the magnetic field H, ψ is the classical Cooper-pair field, $2m$ is the mass of the Cooper pair, and α and β are some phenomenological parameters.

By the comparison of (1.14) and (1.31) it is seen that the Higgs model is nothing but a covariant generalisation of the phenomenological Ginzburg–Landau theory. As in the Higgs model, the 'wrong' sign of α leads to instability of the symmetric ('disordered' in the terminology of quantum statistics) state $\psi=0$, and to the appearance of the superfluid Bose condensate of the Cooper pairs (of the 'order parameter' $\psi\neq 0$). After the symmetry breaking in expression (1.30) there arises the mass term of the field A, analogous to the term $\frac{1}{2}e^2\sigma^2A_\mu{}^2$ in (1.25). This fact is just responsible for the exponential decrease of the magnetic field inside superconductors (Meissner effect).

The analogy between superconductivity theory and SBGT appears to be extremely useful in the study of macroscopic consequences of SBGT. It is known, for example, that heating destroys superconductivity, since at sufficiently high temperatures the Bose condensate of the Cooper pairs 'evaporates'. Proceeding from the analogy between SBGT and superconductivity theory Kirzhnits (1972) (see also Kirzhnits and Linde 1972) has suggested that in the thermodynamic equilibrium systems of elementary particles, interacting in accordance with SBGT, at a sufficiently high temperature the Bose condensate of scalar particles disappears. This leads to the restoration of initial symmetry between weak, strong and electromagnetic interactions. As a result of this phase transition all particles, which acquire their masses due to spontaneous symmetry breaking, become massless again, and weak and strong interactions become long-range like electromagnetic interactions. These conclusions and the estimates of the critical temperature T_c of the phase transition have been further confirmed by the work of Weinberg (1974a), Dolan and Jackiw (1974) and Kirzhnits and Linde (1974a, b, 1976a).

As is known, superconductivity can be destroyed not only by heating but also by external currents and magnetic fields. Therefore, it was natural to expect that analogous effects should occur in SBGT as well (Kirzhnits 1972, Kirzhnits and Linde 1972). And indeed it proves that symmetry restoration in SBGT takes place in the presence of large external currents (Linde 1975a, Kirzhnits and Linde 1976a, Krive 1976) and in strong massive vector 'quasimagnetic' (see §4) fields (Kirzhnits and Linde 1974b, 1976a, Krive *et al* 1976b).

In the papers by Salam and Strathdee (1974, 1975) it was suggested that not only quasimagnetic fields but also an ordinary magnetic field can lead to symmetry restoration in SBGT. A further analysis of this problem has shown, however, that in most of the models with neutral currents symmetry restoration takes place not due to a magnetic field but due to the quasimagnetic fields created simultaneously by the magnetic-field sources (Kirzhnits and Linde 1976b, Linde 1976b).

Extreme conditions, which are necessary for all these effects to occur, can be achieved in cores of neutron stars, in processes of multiparticle production and also at early stages of the evolution of the Universe. In this review we shall be concerned mainly with the consequences of SBGT for cosmology; it will be seen that taking account of the phase transitions in SBGT leads to a substantial reconsideration of the standard viewpoint on the physical processes in the early Universe.

The contents of the present review is organised as follows. In §2 we consider in a more detailed way than in the introduction (by means of the effective potential method) the problem of spontaneous symmetry breaking in quantum field theory.

In §3 a theory of high-temperature symmetry restoration in SBGT is presented, and the infrared problem in quantum statistics of massless gauge fields is discussed. In §4 symmetry behaviour in external fields is investigated. In §5 we discuss the effects connected with the fermion charge density increase. Finally in §6 some consequences of the investigated phenomena for cosmology and for elementary particle physics are discussed.

2. Effective potential and spontaneous symmetry breaking in quantum field theory

As is mentioned in the introduction, spontaneous generation of the classical part (non-vanishing vacuum expectation) σ of the field ϕ takes place only if it is energetically advantageous. At the classical level the potential energy of the 'ordered' state $\sigma \neq 0$ in the theories (1.3), (1.14) and (1.28) is actually less than the energy of the disordered state $\sigma = 0$ (see, for example, (1.4)). It should be determined, however, whether quantum corrections may invalidate this conclusion.

To answer this question one should consider the so-called 'effective potential' $V(\sigma)$, the quantity which has the meaning of the potential energy of the field σ taking into account all the quantum corrections (Jona-Lasinio 1964, Coleman and Weinberg 1973, Jackiw 1974). For this purpose we shall consider first the simplest model (1.3) and represent $L(\phi+\sigma)$ (1.7) as a sum $L_0 + L_{\text{int}}$, where:

$$L_0 = \tfrac{1}{2}(\partial_\mu(\phi+\sigma))^2 - \tfrac{1}{2}(3\lambda\sigma^2 - \mu^2)\phi^2 + \tfrac{1}{2}\mu^2\sigma^2 - \tfrac{1}{4}\lambda\sigma^4.$$

To make the state with arbitrary σ equilibrium we shall add to L_{int} an external source term $I(x)(\phi(x)+\sigma(x))$ and introduce the generating functional of the connected Green functions $W(I)$ determined by the equation:

$$W(I) = W_0 + \ln Z(I).$$

Here $-W_0$ is the energy of the field σ in the classical approximation:

$$-W_0 = \tfrac{1}{2}(\dot{\sigma})^2 + \tfrac{1}{2}(\nabla\sigma)^2 - \tfrac{1}{2}\mu^2\sigma^2 + \tfrac{1}{4}\lambda\sigma^4 \tag{2.1}$$

and $Z(I)$ is the vacuum expectation value of the S matrix $S(I)$, where:

$$S(I) = T\exp\{\textstyle\int \mathrm{d}x[L_{\text{int}}(x) + I(x)(\phi(x)+\sigma(x))]\} \tag{2.2}$$

and the integration is performed over Euclidean four-dimensional space. It is well known that the quantity $-W(I)$ is equal to the total energy of the system in the presence of an external current I (see, for example, Coleman and Weinberg 1973, Abers and Lee 1973). To separate from $-W(I)$ the part $V(\sigma)$, which corresponds to the energy of the field σ, one should subtract from $-W(I)$ the source energy $-\int \mathrm{d}x\sigma(x)I(x)$:

$$V(\sigma) = -W(I) + \textstyle\int \mathrm{d}xI(x)\sigma(x). \tag{2.3}$$

For the time-independent $\sigma(x)$ ($\sigma(x) = \sigma(\boldsymbol{x})$) this quantity has the meaning of the potential energy of the field σ and is called the effective potential. Generally speaking, the most energetically favourable state may appear to be a state with a spatially inhomogeneous field $\sigma(\boldsymbol{x})$ (e.g. some periodic (crystalline) structure). Fortunately it can be shown that this is not the case for most of the models with weak coupling considered in the present review (see, however, §5.4).

Returning to the point-independent quantities I and σ one can obtain the following equations for the derivatives of the effective potential (Jona-Lasinio 1964, Coleman and Weinberg 1973, Jackiw 1974):

$$\mathrm{d}V/\mathrm{d}\sigma = I$$

$$\mathrm{d}^2V/\mathrm{d}\sigma^2 = G_\phi^{-1}(0)$$

where $G_\phi(k)$ is the Green function of the field ϕ with the momentum k. Therefore (as one could expect) in the absence of external current I the system may be only in the state which corresponds to an extremum of the potential energy of the field σ:

$$\mathrm{d}V/\mathrm{d}\sigma = 0. \tag{2.4}$$

This state will be stable only if:

$$\mathrm{d}^2V/\mathrm{d}\sigma^2 = G_\phi^{-1}(0) \geqslant 0. \tag{2.5}$$

This last inequality has a simple physical meaning. Up to higher-order corrections:

$$G_\phi^{-1}(0) = m_\phi^2 \tag{2.6}$$

where m_ϕ is the mass of the field ϕ. In this sense inequality (2.5) implies that only the states without tachyons can be stable. (In fact, this statement can be proved without any recourse to perturbation theory (Kirzhnits and Linde 1978b).)

Graphically the effective potential $V(\sigma)$ (2.3) is given by a set of all one-particle irreducible vacuum diagrams, corresponding to the Lagrangian $L(\phi+\sigma)$ (1.7) without the terms linear in ϕ (Jackiw 1974) (see figure 3). In this review we shall present the results of the calculation of $V(\sigma)$ in the one-loop approximation only (figure 3(a) and (b)) and analyse in which cases these results are reliable.

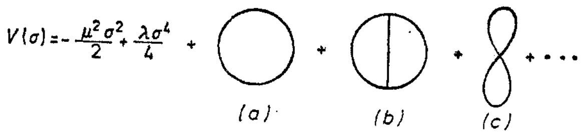

Figure 3. Diagrams for $V(\sigma)$ in the model (1.3).

According to the above-mentioned rules in the one-loop approximation one obtains:

$$V(\sigma) = -\tfrac{1}{2}\mu^2\sigma^2 + \tfrac{1}{4}\lambda\sigma^4 + \frac{1}{2(2\pi)^4}\int \mathrm{d}^4k \ln(k^2 + m^2(\sigma)) \tag{2.7}$$

where $m^2(\sigma) = 3\lambda\sigma^2 - \mu^2$. The meaning of this expression for $V(\sigma)$ becomes particularly clear after the integration over k_0 in (2.7). The result (up to an infinite constant, which can be removed by the vacuum energy renormalisation) is given by:

$$V(\sigma) = -\tfrac{1}{2}\mu^2\sigma^2 + \tfrac{1}{4}\lambda\sigma^4 + \frac{1}{(2\pi)^3}\int \mathrm{d}^3k(k^2 + m^2(\sigma))^{1/2}. \tag{2.8}$$

Thus in the one-loop approximation the effective potential in (1.3) can be considered as a sum of the 'classical' potential energy of the field (1.4) and of the σ-dependent shift in the vacuum energy density due to the zero-point oscillations of the field ϕ. (In this sense equation (2.8) for the effective potential $V(\sigma)$ in the theory (1.3) resembles the well-known Heisenberg–Euler effective Lagrangian of electromagnetic field (Heisenberg and Euler 1936).) The integral corresponding to the energy of the zero-point oscillations diverges. For the renormalisation of $V(\sigma)$ one can add to $L(\phi+\sigma)$ the counter-terms of the type $C_1(\partial_\mu(\phi+\sigma))^2$, $C_2(\phi+\sigma)^2$, $C_3(\phi+\sigma)^4$ and C_4. Actually, in our case, only the last three counter-terms are needed. The constants C_2 and C_3 will be determined by imposing the following normalisation conditions on $V(\sigma)$:

$$\left.\frac{\mathrm{d}V}{\mathrm{d}\sigma}\right|_{\sigma=\mu\lambda^{-1/2}}=0$$
$$\left.\frac{\mathrm{d}^2V}{\mathrm{d}\sigma^2}\right|_{\sigma=\mu\lambda^{-1/2}}=2\mu^2. \tag{2.9}$$

These normalisation conditions imply that the position of the minimum of $V(\sigma)$ at $\sigma\neq 0$ and the curvature $\mathrm{d}^2V/\mathrm{d}\sigma^2$ in this minimum remain the same as in the classical theory. The constant C_4 fixes the vacuum energy at some given value of σ. Of course, one could use some other normalisation conditions (see, for example, Coleman and Weinberg 1973), but all the physical results should be equivalent (Linde 1976a).

The final expression for $V(\sigma)$ in (1.3), which can be obtained from (2.8) taking account of the conditions (2.9), is given by (Kirzhnits and Linde 1976a):

$$V(\sigma)=-\frac{\mu^2}{2}\sigma^2+\frac{\lambda}{4}\sigma^4+\frac{\lambda}{64\pi^2}(3\lambda\sigma^2-\mu^2)\ln\frac{3\lambda\sigma^2-\mu^2}{2\mu^2}+\frac{21\lambda\mu^2}{64\pi^2}\sigma^2-\frac{27\lambda^2}{128\pi^2}\sigma^4.$$

It is seen that with $\lambda\ll 1$ the quantum corrections become considerable at asymptotically large values of σ only (at $\lambda\ln\lambda\sigma^2/\mu^2\gtrsim 1$), when an account of all higher orders of the perturbation theory in λ becomes necessary. Therefore one may think that in (1.3) at $\lambda\ll 1$ everywhere up to the asymptotically large values of σ one can use the classical expression (1.4) for the effective potential. (Of course, in the case $\lambda\gtrsim 1$ quantum corrections may become essential even at small σ (Chang 1975, 1976, Marguder 1976).)

A much more interesting situation appears in the theories with several different coupling constants. As an example we shall consider first the Higgs model (1.14). In this case the effective potential $V(\sigma)$ in the transverse gauge $\partial_\mu A_\mu=0$ is given by the diagrams shown in figure 4. At $e^2\ll\lambda$ the contribution of vector particles to

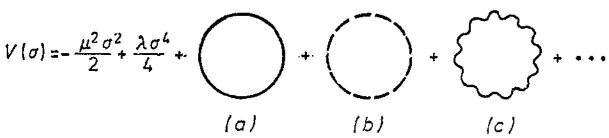

Figure 4. Diagrams for $V(\sigma)$ in the Higgs model. Full, broken and wavy lines correspond to the fields χ_1, χ_2 and A_μ, respectively.

$V(\sigma)$ (figure 4(*c*)) can be neglected, and the situation becomes similar to that considered above, but at $e^2\gg\lambda$, when the contribution of the scalar particles (figures 4(*a*) and (*b*)) can be neglected, the expression for $V(\sigma)$ takes the form (Linde 1976a):

$$V(\sigma)=-\frac{\mu^2\sigma^2}{2}\left(1-\frac{3e^4}{16\pi^2\lambda}\right)+\frac{\lambda\sigma^4}{4}\left(1-\frac{9e^4}{32\pi^2\lambda}\right)+\frac{3e^4\sigma^4}{64\pi^2}\ln\frac{\lambda\sigma^2}{\mu^2}. \tag{2.10}$$

From this equation it is seen that at $\lambda<3e^4/16\pi^2$ the effective potential acquires an additional ('dynamical') minimum at $\sigma=0$, and at $\lambda<3e^4/32\pi^2$ this minimum becomes even deeper than the ordinary ('classical') minimum at $\sigma=\mu\lambda^{-1/2}$ (see figure 5). Thus at $\lambda<3e^4/32\pi^2$ quantum corrections lead to the dynamical symmetry restoration in the Higgs model. Unlike in (1.3) these effects take place not due to the large logarithmic factor $\lambda \ln \lambda\sigma^2/\mu^2 \gtrsim 1$, but due to some particular relations between coupling constants λ and e^2 ($\lambda \sim e^4$), when 'classical' terms in expression (2.9) for the effective potential $V(\sigma)$ are of the same order as the lowest-order quantum corrections in e^2. The higher-order corrections are proportional to λ^2 and e^6 and can be neglected compared with the terms taken into account in equation (2.10). This means that the results discussed above are reliable. One can also

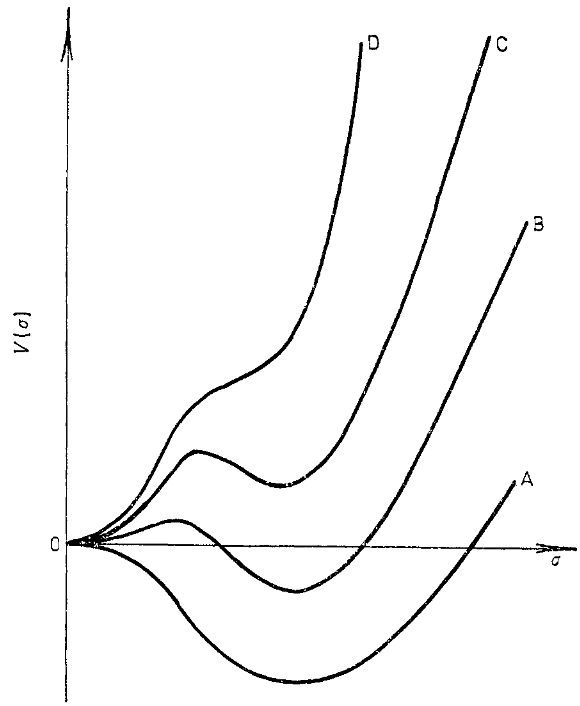

Figure 5. Effective potential $V(\sigma)$ in the Higgs model. A, $\lambda>3e^4/16\pi^2$; B, $3e^4/16\pi^2>\lambda>3e^4/32\pi^2$; C, $3e^4/32\pi^2>\lambda>0$; D, $\lambda=0$.

show that with the normalisation conditions (2.9) quantum corrections to the masses m_A and m_ϕ at $\sigma=\mu\lambda^{-1/2}$ are small:

$$m_A{}^2=e^2\sigma^2(1+O(e^2))$$
$$m_\phi{}^2=2\lambda\sigma^2(1+O(e^2)).$$

Therefore the results discussed above imply that symmetry breaking in the Higgs model is energetically favourable only if the Higgs meson mass m_ϕ is sufficiently large (Linde 1976a, Weinberg 1976):

$$m_\phi{}^2>\frac{3e^2}{16\pi^2}\,m_A{}^2. \tag{2.11}$$

We shall return to the discussion of this result in §6.

Whereas quantum corrections connected with the zero-point oscillations of the vector fields enforce dynamical symmetry restoration, the effects connected with fermions are quite the opposite. For example, in the simplified σ model (1.28) at large σ the effective potential in the one-loop approximation is given by (Krive and Linde 1976):

$$V(\sigma)=-\frac{\mu^2}{2}\sigma^2+\frac{\lambda}{4}\sigma^4+\frac{9\lambda^2-4g^4}{64\pi^2}\sigma^4\ln\frac{\lambda\sigma^2}{\mu^2}. \tag{2.12}$$

It is seen that at large σ the fermion contribution is negative, and at $3\lambda<2g^2$ the effective potential is unbounded from below (see figure 6).

Of course, for $\sigma\to\infty$ (at max (λ, g^2) ln $\lambda\sigma^2/\mu^2\gtrsim 1$) the one-loop results become unreliable. Nevertheless, at $\lambda\ll g^2$ there exists some region of σ $[\sigma^2\sim(\mu^2/\lambda)\exp(\lambda/g^4)]$ for which $V(\sigma)<V(\mu\lambda^{-1/2})$ and the one-loop approximation is still reliable. Therefore, in the σ model (1.28) at $\lambda\ll g^2$, instead of the ordinary spontaneous symmetry breaking a much stronger dynamical symmetry breaking should take place (Krive and Linde 1976).

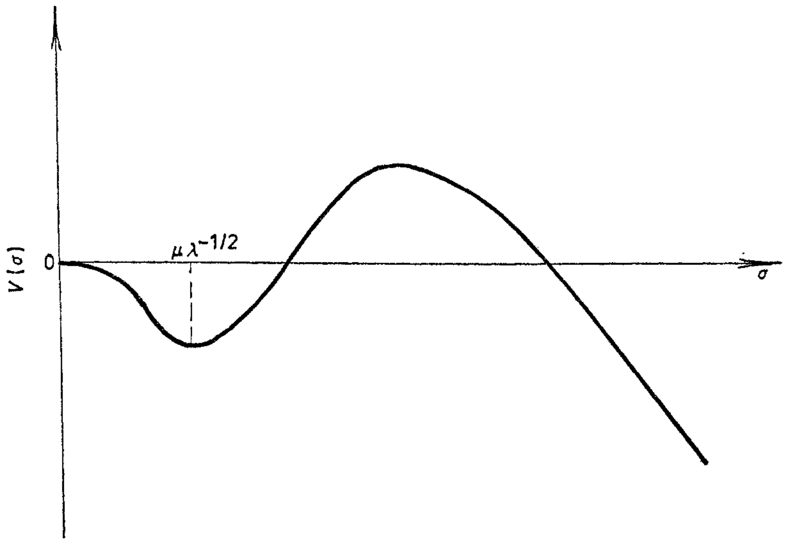

Figure 6. Effective potential $V(\sigma)$ in the σ model (1.28) at $\lambda\ll g^2$.

The above consideration shows that the quantum effects may be of crucial importance for the symmetry behaviour in gauge theories. (For a more detailed discussion of the dynamical effects in SBGT, see Linde (1976d).) However, these effects take place only at certain particular relations between coupling constants. Therefore, everywhere in this review (except for some cases which we shall mention explicitly) we shall consider the models with the relations between coupling constants for which the dynamical effects are small ($\lambda>e^4/16\pi^2$ in the Higgs model, $\lambda\gtrsim g^2$ in the σ-model, etc) and the effective potential is given by the 'classical' expressions of the type (1.4).

3. Symmetry restoration at high temperatures

3.1. Elementary theory of the phase transition

Now, after discussion of the main features of spontaneous symmetry breaking in quantum field theory, we shall begin an investigation of the symmetry behaviour in thermodynamic equilibrium systems of particles interacting in accordance with SBGT (Kirzhnits 1972, Kirzhnits and Linde 1972, 1974a, b, 1976a, Weinberg 1974a, Dolan and Jackiw 1974). We shall consider first a thermodynamic equilibrium system of the scalar particles ϕ with the Lagrangian (1.3):

$$L = \tfrac{1}{2}(\partial_\mu\phi)^2 + \tfrac{1}{2}\mu^2\phi^2 - \tfrac{1}{4}\lambda\phi^4.$$

These particles have no conserved charge and their number is not conserved as well. Therefore the only parameter characterising the thermodynamic equilibrium system of the ϕ particles in the temperature (T) of the system, which characterises the density of the particles in the momentum space:

$$n_p = \frac{1}{\exp(\omega_p/T) - 1} \tag{3.1}$$

where $\omega_p = (p^2 + m^2)^{1/2}$ is the energy of the particle with momentum p and mass m. According to (3.1) all the particles in the ground state at $T = 0$ disappear and we return to the situation discussed in the previous section.

At a non-vanishing temperature all physically interesting quantities (thermodynamic potentials, Green functions, etc) in the system under consideration are given not by vacuum averages as in the field theory, but by the Gibbs averages defined by (1.20):

$$\langle \ldots \rangle = \frac{Sp[\exp(-H/T)\ldots]}{Sp[\exp(-H/T)]}$$

where H is the Hamiltonian of the system. In particular, the symmetry breaking parameter (density of the Bose condensate of the field ϕ) in the system is given not by the vacuum expectation value $\sigma = \langle 0|\phi|0\rangle$, but by the temperature-dependent quantity $\sigma(T) = \langle\phi\rangle$.

In order to investigate symmetry behaviour in (1.3) at $T \neq 0$, let us consider the Lagrange equation for the field ϕ in this theory:

$$(\Box + \mu^2 - \lambda\phi^2)\phi = 0 \tag{3.2}$$

and take the Gibbs average of (3.2) as has been done in the introduction for the case $T = 0$. The resulting equation is (compare with (1.10)):

$$\Box\sigma(T) - (\lambda\sigma^2(T) - \mu^2)\sigma(T) - 3\lambda\sigma(T)\langle\phi^2\rangle - \lambda\langle\phi^3\rangle = 0. \tag{3.3}$$

In the lowest order in λ the quantity $\langle\phi^3\rangle$ vanishes as in field theory. To calculate $\langle\phi^2\rangle$ one should take into account that:

$$\langle\phi^2\rangle = \frac{1}{(2\pi)^3}\int \frac{d^3p}{2\omega_p}(2\langle a_p{}^+ a_p\rangle + 1).$$

Then using equations (1.29) and (3.1) and discarding the temperature-independent term $(2\pi)^{-3}\int d^3p/2\omega_p$, which can be eliminated by the mass renormalisation at $T = 0$, one finds that:

$$\langle\phi^2\rangle = F(T, m_\phi) \equiv \frac{1}{2\pi^2}\int_0^\infty \frac{p^2\, dp}{(p^2 + m_\phi{}^2)^{1/2}[\exp(p^2 + m_\phi{}^2)^{1/2}/T - 1]} \tag{3.4}$$

where m_ϕ is the mass of the field ϕ.

As will be seen, all interesting effects take place only at $T \gg m_\phi$, when one can neglect m_ϕ in (3.4). In this case:

$$\langle\phi^2\rangle = F(T, 0) = T^2/12 \tag{3.5}$$

and equation (3.3) looks like:

$$\Box\sigma(T) - (\lambda\sigma^2(T) - \mu^2 + \tfrac{1}{4}\lambda T^2)\sigma(T) = 0. \tag{3.6}$$

Supposing, as before, that $\sigma(T) = \text{constant}$ we get:

$$\sigma(T)(\lambda\sigma^2(T) - \mu^2 + \tfrac{1}{4}\lambda T^2) = 0. \tag{3.7}$$

This equation at a sufficiently low temperature T has two solutions:

$$\sigma(T) = 0 \qquad \sigma^2(T) = \frac{\mu^2}{\lambda} - \frac{T^2}{4}. \tag{3.8}$$

The second solution disappears at temperatures greater than the critical temperature:

$$T_c = 2\mu\lambda^{-1/2} = 2\sigma(0). \tag{3.9}$$

To obtain the excitation spectrum in (1.3) at $T \neq 0$ one should perform in equation (3.3) an infinitesimal shift $\sigma \to \sigma + \delta\sigma$, where σ is one of the two possible solutions of equation (3.7) (see the introduction). At $\sigma(T) = 0$ the corresponding equation for the fluctuations of the field σ is:

$$\Box\delta\sigma - (-\mu^2 + \tfrac{1}{4}\lambda T^2)\delta\sigma = 0$$

which implies that the mass of the scalar field excitations at $\sigma = 0$ is given by:

$$m_\phi{}^2 = -\mu^2 + \tfrac{1}{4}\lambda T^2. \tag{3.10}$$

The value of $m_\phi{}^2$ (3.10) at $T < T_c$ is negative, and thus the disordered solution $\sigma(T) = 0$ is unstable at $T < T_c$.

Analogously it can be shown that for the solution $\sigma^2(T) = \mu^2/\lambda - T^2/4$:

$$m_\phi{}^2 = 3\lambda\sigma^2(T) - \mu^2 + \tfrac{1}{4}\lambda T^2 = 2\lambda\sigma^2(T). \tag{3.11}$$

For both solutions $\sigma(T) = 0$ and $\sigma(T) \neq 0$ the masses (3.10) and (3.11) vanish at $T = T_c$ and the mass squared of the field ϕ in the disordered state $\sigma(T) = 0$ becomes positive (the state $\sigma(T) = 0$ becomes stable) at the same temperature T_c, at which the ordered solution $\sigma(T) \neq 0$ disappears. This means that at the critical temperature $T = T_c$ (3.9) the phase transition with the symmetry restoration in (1.3) takes place.

Equations (3.8), (3.10) and (3.11) for $\sigma(T)$ and $m_\phi(T)$ are illustrated by figures 7(*a*) and (*b*). One should note that, with an increase of temperature, the symmetry breaking parameter $\sigma(T)$ in (1.3) decreases continuously, which corresponds to the second-order phase transition.

3.2. *Effective potential at* $T \neq 0$

The same results can also be obtained in another way by the generalisation of the effective potential method for the case $T \neq 0$. This generalisation proves to be very simple (Dolan and Jackiw 1974, Kirzhnits and Linde 1974a). In quantum statistics at $T \neq 0$ the effective potential $V(\sigma, T)$ is given by the same one-particle irreducible 'vacuum' diagrams as in field theory. The only difference is that at

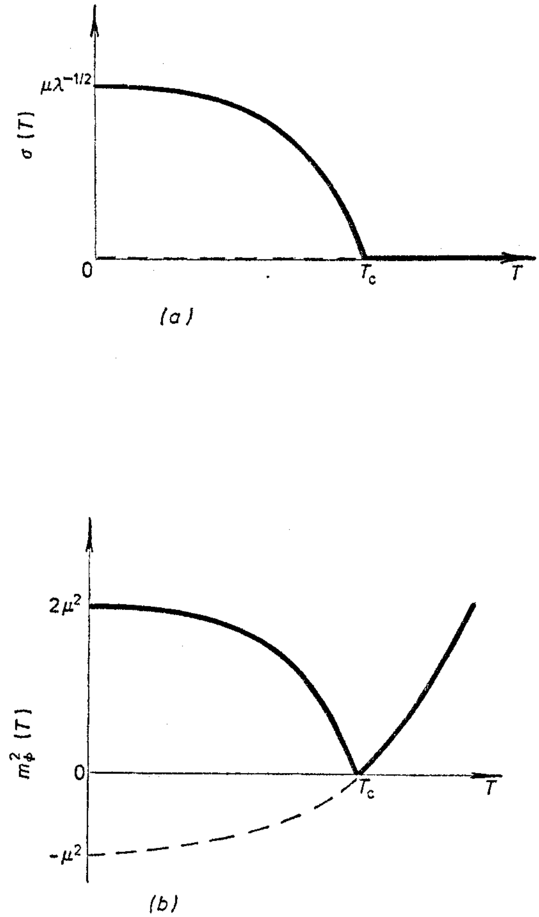

Figure 7. Temperature dependence of σ and m_ϕ in the model (1.3). Broken lines correspond to an unstable state $\sigma=0$ at $T<T_c$.

$T\neq 0$ the component k_0 of the momentum in all Euclidean integrals corresponding to these diagrams should be replaced by $2\pi nT$ for bosons and by $(2n+1)\pi T$ for fermions, and instead of the integration over k_0 one should perform a summation over all integer n: $\int dk_0 \to 2\pi T \Sigma_{n=-\infty}^{\infty}$. For example, the expression (2.6) for $V(\sigma)$ in (1.3) is transformed at $T\neq 0$ to the following:

$$V(\sigma, T) = -\frac{\mu^2}{2}\sigma + \frac{\lambda}{4}\sigma^4 + \frac{T}{2(2\pi)^3}\sum_{n=-\infty}^{\infty}\int d^3k \ln\left[(2\pi nT)^2 + k^2 + m^2(\sigma)\right] \tag{3.12}$$

where $m^2(\sigma) = 3\lambda\sigma^2 - \mu^2$. For the renormalisation of this expression one can use the same counter-terms as in the field theory. Performing summation and integration in (3.12) one obtains the following expression for the renormalised effective potential $V(\sigma, T)$ at $T \gg m$, the terms of the higher orders in λ being neglected (Weinberg 1974a, Dolan and Jackiw 1974, Kirzhnits and Linde 1974a, b, 1976a):

$$V(\sigma, T) = -\frac{\mu^2}{2}\sigma^2 + \frac{\lambda}{4}\sigma^4 - \frac{\pi^2}{90}T^4 + \frac{m^2(\sigma)}{24}T^2. \tag{3.13}$$

It can easily be seen that the equation $dV/d\sigma=0$ (2.4), which determines the equilibrium value of σ, in this case exactly coincides with (3.7). On the other hand, up to the higher orders in λ, $G_\phi^{-1}(0)=m_\phi^2$ as in the field theory. From this equation and equation (2.5) one can again obtain equations (3.10) and (3.11) for m_ϕ^2 and show that only the solutions with $m_\phi^2\geqslant 0$ are stable according to the stability condition $d^2V/d\sigma^2\geqslant 0$ (2.5).

By means of the 'energetical' approach used above one can give a very simple description of the high-temperature phase transition in the theory (1.3); namely, at $T=0$ the effective potential has a minimum at $\sigma=\mu\lambda^{-1/2}$. With an increase of temperature the energy difference between the minimum of $V(\sigma, T)$ at $\sigma\neq 0$ and the maximum at $\sigma=0$ decreases. At $T=T_c$ this energy difference vanishes

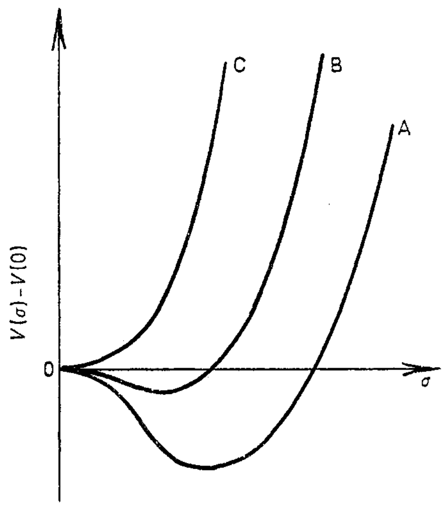

Figure 8. Effective potential $V(\sigma, T)$ in the model (1.3) at $\sigma>0$. A, $T=0$; B, $0<T<T_c$; C, $T>T_c$.

together with the minimum at $\sigma\neq 0$, and at $T>T_c$ the effective potential $V(\sigma, T)$ has its minimum at $\sigma=0$, i.e. the symmetry is restored (see figure 8).

3.3. Higher orders of perturbation theory

The investigation of the high-temperature symmetry restoration in SBGT performed above was based on the use of the lowest order of perturbation theory in λ. One may ask therefore whether the results obtained by this method are reliable.

This question is not quite trivial. For example, in the higher-order corrections to the effective potential at $T\neq 0$, besides small terms $\sim\lambda^n T^4$, $\lambda^n T^2 m^2(\sigma)$ the terms proportional to $m_\phi^{-n}(T)$ also appear. At small $m_\phi(T)$ such terms become large and should be taken into account.

To analyse this problem in a more detailed way let us consider the diagrams of the Nth order in λ for $V(\sigma, T)$ in (1.3) at $\sigma=0$ $(T>T_c)$. To obtain the correct behaviour of $V(\sigma, T)$ at small $m_\phi(T)$ one should use the self-consistent approximation in which the zero-temperature mass $m(\sigma)=(3\lambda\sigma^2-\mu^2)^{1/2}$ is replaced by the temperature-dependent mass $m_\phi(T)$ (Kirzhnits and Linde 1974a, 1976a). In this approximation:

$$V_{(0,\,T)}{}^N \sim (2\pi T)^{N+1}\lambda^N \int d^3p_1 \ldots d^3p_{N+1} \sum_{n_i=-\infty}^{\infty} \prod_{K=1}^{2N} [(2\pi r_K T)^2 + q_K{}^2 + m_\phi{}^2(T)]^{-1} \tag{3.14}$$

where q_K is a uniform linear combination of p_i, r_K is the corresponding combination of n_i, $i=1, \ldots, N+1$, $K=1, \ldots, 2N$. At $m_\phi \to 0$ the leading term in the sum over n_i is the term with all $n_i=0$ (and, consequently, $r_K=0$), since the factors, which contain $(2\pi r_K T)^2$, are not singular at $m_\phi \to 0$, $q_K \to 0$. The leading term is thus given by:

$$\tilde{V}_{(0,\,T)}{}^N \sim (2\pi T)^{N+1}\lambda^N \int d^3p_1 \ldots d^3p_{N+1} \prod_{K=1}^{2N} (q_K{}^2 + m_\phi{}^2(T))^{-1}$$
$$\sim \lambda^3 T^4 \left(\frac{\lambda T}{m_\phi(T)}\right)^{N-3}. \tag{3.15}$$

It is seen that near T_c when $m_\phi \to 0$ the role of higher orders of perturbation theory can be significant even at small λ. Analogous terms singular at $m_\phi \to 0$ exist also at $\sigma \neq 0$ $(T<T_c)$. Therefore, near T_c one should take into account all higher orders of perturbation theory, or use such well-known methods as ϵ expansion, $1/N$ expansion, etc (Wilson and Kogut 1974, see also Baym and Grinstein 1977). Fortunately, by the use of (3.10) and (3.11) it can be shown that such terms are important only in the small region near the critical temperature, in which:

$$|T-T_c| \lesssim \lambda T_c. \tag{3.16}$$

Everywhere except this small region near T_c higher-order corrections are small and our results concerning the phase transition in the model (1.3) are reliable.

3.4. The phase transition in the Higgs model

The methods developed above can be easily applied to more complicated theories with spontaneous symmetry breaking. For example, in the Higgs model (1.14) in the transverse gauge $\partial_\mu A_\mu = 0$, instead of equation (3.3) for the constant classical scalar field $\sigma(T)$ the following equation holds:

$$\left\langle \frac{\delta L}{\delta \phi} \right\rangle = \sigma(T)[\mu^2 - \lambda(\sigma^2(T) + 3\langle \chi_1{}^2 \rangle + \langle \chi_2{}^2 \rangle) + e^2 \langle A_\mu{}^2 \rangle] = 0. \tag{3.17}$$

Let us first suppose that, as in (1.3), the phase transition in the Higgs model takes place at $T \gg m_{\chi_i}$, m_A. Using the same methods which have been used in the investigation of the high-temperature symmetry behaviour in (1.3) one can show that at $T \gg m_{\chi_i}$, m_A:

$$\langle \chi_1{}^2 \rangle = \langle \chi_2{}^2 \rangle = -\tfrac{1}{3}\langle A_\mu{}^2 \rangle = T^2/12 \tag{3.18}$$

(compare with (3.4) and (3.5)). In this case equation (3.14) looks like:

$$\sigma\left(\lambda\sigma^2 - \mu^2 + \frac{4\lambda + 3e^2}{12} T^2\right) = 0 \tag{3.19}$$

from which it follows that the critical temperature in the Higgs model is given by (Weinberg 1974a, Dolan and Jackiw 1974, Kirzhnits and Linde 1974b, 1976a):

$$T_{c_1}{}^2 = \frac{12\mu^2}{4\lambda + 3e^2}. \tag{3.20}$$

According to (3.19) the value of $\sigma(T)$ depends on T continuously, i.e. we again deal with the second-order phase transition as in (1.3).

However, one can verify that our initial assumption that $T \gg m_{\chi_i}$, m_A in the region near the critical temperature is valid only at $\lambda \gg e^4$ (Kirzhnits and Linde 1974b, 1976a). It can be shown that at $\lambda \lesssim e^4$ $m_A(T_{c_1}) \approx e\mu\lambda^{-1/2} \gtrsim T_{c_1}$, and therefore at $T \sim T_{c_1}$ the vector particle contribution to (3.14), being proportional to $\langle A_\mu{}^2 \rangle = -3F(T, m_A)$ (compare with (3.4)), is very small. In this case the theory of the phase transition in the Higgs model becomes more complicated than in (1.3) (Kirzhnits and Linde 1974b, 1976a, Iliopoulos and Papanicolaou 1976); namely, in the temperature interval $T_{c_1} < T < T_{c_2}$ (where T_{c_2} is of the order of $m_A(T=0) = e\mu\lambda^{-1/2}$) there exist three different possible values of the field $\sigma(T)$: $\sigma_1(T) \neq 0$, $\sigma_2(T) \neq 0$ ($\sigma_1 > \sigma_2$) and $\sigma = 0$ (see figure 9). Correspondingly the effective potential

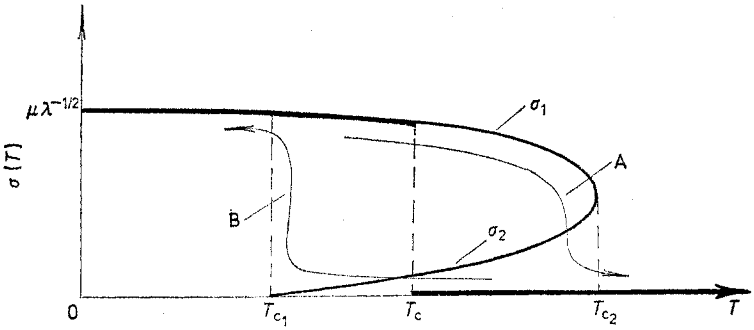

Figure 9. Temperature dependence of σ in the Higgs model at $3e^4/16\pi^2 < \lambda \lesssim e^4$. The thick line corresponds to a stable state of the system. Arrows show the behaviour of σ with an increase (A) and with a decrease (B) of temperature.

$V(\sigma, T)$ at $T_{c_1} < T < T_{c_2}$ has three extrema: two minima at $\sigma = \sigma_1(T)$ and at $\sigma = 0$ and a maximum at $\sigma = \sigma_2(T)$ (see figure 10). The state $\sigma = \sigma_2(T)$ is unstable at all temperatures. The state $\sigma = \sigma_1(T)$ is stable at low temperatures and becomes metastable at $T > T_c$, where the critical temperature T_c can be obtained from the equation:

$$V(\sigma_1(T_c), T_c) = V(0, T_c) \tag{3.21}$$

(see figure 10). Therefore at the temperature $T = T_c$ ($T_c > T_{c_1}$) the first-order (discontinuous) phase transition to the state $\sigma = 0$ takes place (figure 9).

The theory of this first-order phase transition is particularly simple at $3e^4/16\pi^2 \ll \lambda \ll e^4$. In this case it proves that $m_A(T_c) \gg T_c$ for the solution $\sigma = \sigma_1(T_c)$. As a result the vector particles give no contribution to $V(\sigma_1(T_c), T_c)$ and up to higher-order corrections in e^2, $\sigma_1(T_c) = \sigma_1(0) = \mu\lambda^{-1/2}$. Now let us take into account that at sufficiently high temperatures ($T \gg m$) every degree of freedom gives the contribution $-(\pi^2/90)T^4$ to $V(\sigma, T)$ (see (3.13)). At $\sigma = 0$ the massless vector field and the complex scalar field contribute (four degrees of freedom), and therefore $V(0, T_c) = -(4\pi^2/90)T^4$. On the other hand, at $\sigma = \sigma_1(T)$ the contribution $\sim T^4$

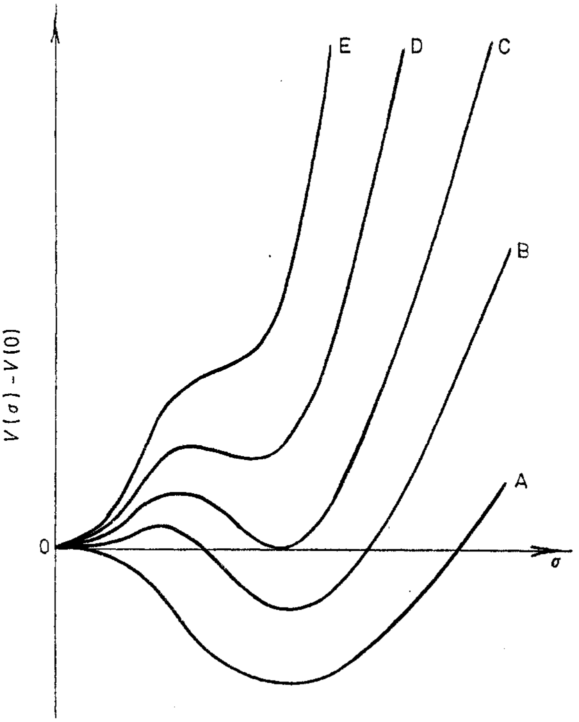

Figure 10. Effective potential $V(\sigma, T)$ in the Higgs model at $3e^4/16\pi^2 < \lambda \leqslant e^4$. A, $0 < T < T_{c1}$; B, $T_{c1} < T < T_c$; C, $T = T_c$: D, $T_c < T < T_{c2}$; E, $T > T_{c2}$.

is given only by the real scalar particles (one degree of freedom), i.e. $V(\sigma_1(T_c), T_c) = -\mu^4/4\lambda - \pi^2 T^4/90$. Then from equation (3.21) it follows that the critical temperature of the phase transition is given by (Kirzhnits and Linde 1974b, 1976a):

$$T_c = \left(\frac{15\lambda}{2\pi^2}\right)^{1/4} \mu. \tag{3.22}$$

At $\lambda \lesssim 3e^4/16\pi^2$ radiative corrections to $V(\sigma, T)$ become essential even at $T=0$ (see §2). As a result at $3e^4/32\pi^2 < \lambda < 3e^4/16\pi^2$ the curves for $V(\sigma, T)$ and $\sigma(T)$ acquire the form represented in figures 11 and 12.

Note that at $\lambda \to 3e^4/32\pi^2$ the critical temperature $T_c \to 0$ and at $\lambda < 3e^4/32\pi^2$ the symmetry in the Higgs model, as was already mentioned in §2, is restored even at $T=0$. Therefore, one could expect that we have a good chance of observing the effects connected with the symmetry restoration in SBGT in a laboratory if the critical temperature T_c is not too high. Unfortunately this is not quite true because of effects connected with the kinetics of the first-order phase transitions in SBGT.

These phase transitions proceed by the spontaneous formation of bubbles filled with matter in a new phase. The walls of the bubbles spread up with a velocity almost equal to that of light, and very soon all the Universe becomes filled with matter in the new phase. However, the formation of the bubbles is a barrier tunnelling process. Such a process may take place due to quantum fluctuations (Voloshin *et al* 1974, Coleman 1977) or due to thermodynamic ones (Linde 1977b), but in both cases it proves that the time necessary for the creation of at least one such bubble in all the Universe is usually much greater than the age of the Universe.

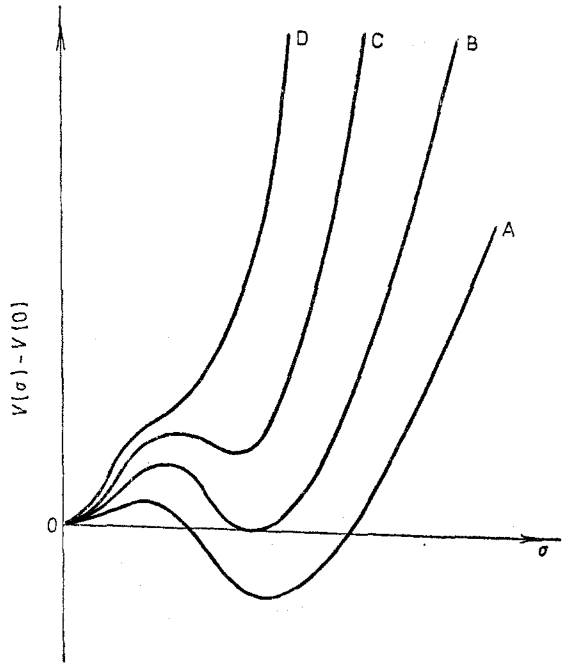

Figure 11. Effective potential $V(\sigma, T)$ in the Higgs model at $3e^4/32\pi^2 < \lambda < 3e^4/16\pi^2$. A, $0 < T < T_c$; B, $T = T_c$; C, $T_c < T < T_{c2}$; D, $T > T_{c2}$.

As a result, the first-order phase transitions in the gauge theories appear to be possible only in cases when the energetical barrier between two local minima of $V(\sigma, T)$ becomes extremely small. Therefore, the first-order phase transition in the Higgs model actually takes place at $T \approx T_{c_1}$ when the temperature decreases and at $T \approx T_{c_2}$ when the temperature increases (see figure 9). As a result, to realise the investigated phase transition 'in the laboratory' one should heat the system not up to the temperature T_c, which may be very small, but up to the relatively large temperature $T_{c2} \sim m_A(T = 0)$.

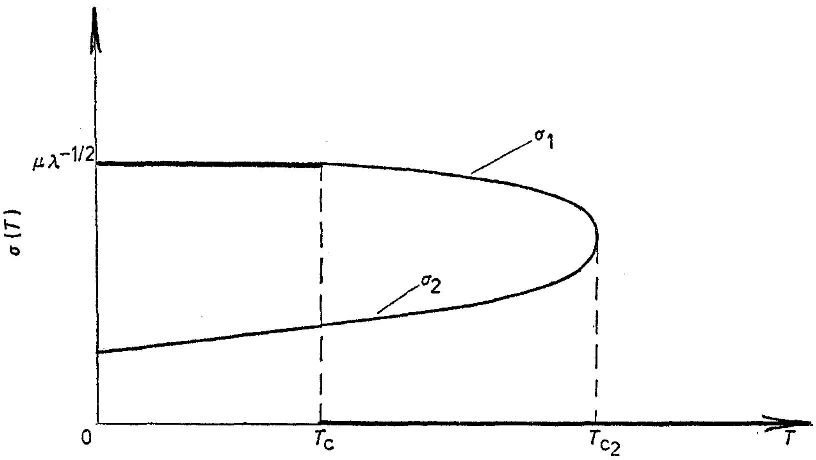

Figure 12. Temperature dependence of σ in the Higgs model at $3e^4/32\pi^2 < \lambda < 3e^4/16\pi^2$. The thick line corresponds to a stable state of the system.

The methods used above for the investigation of the symmetry behaviour in the Higgs model can be easily extended to more realistic theories with non-Abelian gauge fields and fermions. The estimates of the value of T_c in most models of weak and electromagnetic interactions give the critical temperature a value of the order of 10–1000 GeV (10^{14}–10^{16} K). Such temperatures have existed only in the early stages of the evolution of the Universe. In theories of strong interactions the critical temperature may be much less: $T_c \sim 100$ MeV–1 GeV (10^{12}–10^{13} K). Temperatures of this order can exist not only in the early Universe, but also inside the 'fireballs' which are created in the process of multiparticle production in the collisions of high-energy elementary particles†.

Strictly speaking, all these conclusions only concern the calculations in the lowest order of perturbation theory. As for the Higgs model, by the same methods as in §3.3, it can be shown that our results are actually reliable everywhere except in some small region near T_c. Massless vector particles, which appear after the symmetry restoration in this model, cause no difficulties of the type mentioned in §3.3 since these particles do not interact directly with each other. However, in the non-Abelian gauge theories of the Weinberg–Salam model type the reliability of the lowest-order results at $T \geqslant T_c$ is much less obvious. The reason is that after the symmetry restoration in non-Abelian theories the self-interacting Yang–Mills fields become massless. If the temperature corrections do not produce a mass for these fields (this question will be discussed in the next subsection), the corresponding infrared divergences may invalidate all perturbative results at $T \geqslant T_c$. In particular, one cannot be sure that the symmetry at $T \geqslant T_c$ is completely restored. In any case, however, it can be shown that at $T \geqslant T_c$ the value of $\sigma(T)$ is greatly diminished. For example, in the non-Abelian theories with $\lambda \sim g^2$ (where g is the constant of the self-interaction of the Yang–Mills fields) it can be shown that $\sigma(T_c) \lesssim g\sigma(0)$, $\sigma(T > T_c) \lesssim gT$ (Kirzhnits and Linde 1976b).

In conclusion, we would also like to note that the phase transitions of the type discussed above may take place not only in SBGT, but in the theories with dynamical symmetry breaking as well: see, for example, an investigation of the high-temperature phase transition in the two-dimensional Gross–Neveu model (Jacobs 1974, Hiro-O-Wada 1974, Harrington and Yildiz 1975, Dashen *et al* 1975).

3.5. The infrared problem in quantum statistics of gauge fields

Infrared divergences, which appear in the non-Abelian gauge theories with spontaneous symmetry breaking at $T > T_c$, exist also in quantum chromodynamics (QCD), in which the Yang–Mills fields responsible for strong interactions are supposed to be massless. One could hope, however, that due to the temperature corrections massless Yang–Mills fields in both the theories acquire some sufficiently large mass ($m \gg g^2T$, see below). This would solve the infrared problem in quantum statistics

† In the papers by Hagedorn (1965) and Frautschi (1971) it was argued that there may exist a limiting temperature $T_0 \sim 150$ MeV. These papers have been repeatedly criticised and the arguments in favour of the existence of a limiting temperature, in our opinion, are rather unconvincing, but in any case it can be shown that the same reasons which could lead to the existence of T_0 (an assumption concerning the exponential spectrum of elementary particles) lead also to the symmetry restoration in SBGT at some $T_c < T_0$ (Linde 1975b, Kirzhnits and Linde 1976a). Moreover, after the symmetry restoration the particle spectrum changes and there are no reasons to expect that in the theory with this new spectrum any limiting temperature exists (Cabibbo and Parisi 1975, Kirzhnits and Linde 1976b).

of gauge fields. To understand whether the massless Yang–Mills fields actually may become massive due to the temperature corrections we shall remind ourselves first of the corresponding results concerning the photon mass at $T \neq 0$.

As is well known, the photon Green function $G_{\mu\nu}(k)$ at $T \neq 0$ has a singular point at $k_0 = eT/3$, $\boldsymbol{k} = 0$ (Fradkin 1965). One could think (see, for example, Kislinger and Morley 1976a) that this means that photons at $T \neq 0$ have a mass $m = eT/3$, which serves as an infrared cutoff in the theory. Actually, however, analytic properties of the Green function $G_{\mu\nu}(k)$ at $T \neq 0$ are much more complicated than at $T = 0$ and the existence of a singularity at $k_0 \neq 0$, $\boldsymbol{k} = 0$ (as well as a singularity of $G_{\mu\nu}(k)$ at $k_0 = |\boldsymbol{k}| \neq 0$ (Shuryak 1978) is irrelevant to the problem of infrared divergences at $T \neq 0$.

Indeed, as is shown in §3.3, the leading infrared divergences in quantum statistics of the Bose particles are connected with the 'static' limit $k_0 = 0$, $\boldsymbol{k} \to 0$ of the corresponding Green functions (the term with all $r_i = 0$ in (3.14)). Properties of $G_{\mu\nu}(k)$ in quantum electrodynamics in this limit are well known. For example, in the Coulomb gauge $\partial_i A_i = 0$ the photon Green function $G_{\mu\nu}$ at $k_0 = 0$ has the following structure (Fradkin 1965):

$$\begin{aligned} G_{00}(\boldsymbol{k}) &= \frac{1}{\boldsymbol{k}^2 + \Pi_{00}(\boldsymbol{k})} \\ G_{i0}(\boldsymbol{k}) &= G_{0i}(\boldsymbol{k}) = 0 \\ G_{ij}(\boldsymbol{k}) &= \left(\delta_{ij} - \frac{k_i k_j}{\boldsymbol{k}^2}\right) \frac{1}{\boldsymbol{k}^2 + A(\boldsymbol{k})} \end{aligned} \tag{3.23}$$

where $i, j = 1, 2, 3$. The quantity $\Pi_{00}(\boldsymbol{k})$ (which is equal to the corresponding component of the polarisation operator $\Pi_{\mu\nu}(k_0 = 0, \boldsymbol{k})$) does not vanish at $\boldsymbol{k} \to 0$. At a sufficiently low plasma density, $\Pi_{00}(0)$ is given by:

$$\Pi_{00}(0) = \left(\frac{n_e}{4\pi e^2 T^2}\right)^2 \tag{3.24}$$

where n_e is the density of pairs of charged particles at the temperature T (Landau and Lifshitz 1964). In the high-temperature limit (Fradkin 1965):

$$\Pi_{00}(0) = e^2 T^2/3. \tag{3.25}$$

Thus, according to (3.23)–(3.25), $G_{00}(k_0 = 0, \boldsymbol{k} \to 0) = \Pi_{00}^{-1}(0) \neq \infty$, and in this sense the component $G_{00}(k)$ at $k_0 = 0$, $\boldsymbol{k} \to 0$ behaves as the Green function of a massive field with the mass squared $m^2 = \Pi_{00}(0)$.

On the other hand, it can be shown that for all orders of perturbation theory $A(\boldsymbol{k}) \sim \boldsymbol{k}^2$ at $\boldsymbol{k} \to 0$ (Fradkin 1965). Therefore at $k_0 = 0$, $\boldsymbol{k} \to 0$ the Green function $G_{ij}(k)$ behaves like that of massless particles, $G_{ij} \sim 1/\boldsymbol{k}^2$.

The physical meaning of such a difference between the low-momentum behaviour of $G_{00}(k)$ and $G_{ij}(k)$ is very simple. One can easily verify that in the Coulomb gauge at $k_0 = 0$ the Green function of the electric field $\langle E_i E_j \rangle$ is proportional to G_{00}, and the Green function of the magnetic field $\langle H_i H_j \rangle \sim G_{ij}$. Electrostatic forces in a gas of charged particles become short-range due to the Debye screening ($G_{00}(0) = \Pi_{00}^{-1}(0) = \lambda_D^2$, where λ_D is the Debye length). If there are no magnetic charges in the theory, the magnetic forces cannot be screened and remain long-range, in accordance with the above results.

Note, however, that in some SBGT the monopoles actually exist ('t Hooft 1974, Polyakov 1974). In such theories magnetic forces also become screened. The theory of the magnetic screening in the low-density monopole–antimonopole plasma is completely analogous to that of electrostatic Debye screening; in particular, it can be shown that:

$$A(0)=\left(\frac{n_{\mathrm{M}}}{4\pi q^2T^2}\right)^2 \tag{3.26}$$

where n_{M} is the density of the monopole–antimonopole pairs at $T\neq 0$ and q is the magnetic charge of the monopole (compare with (3.24)). In the dilute gas of monopoles:

$$n_{\mathrm{M}}=\frac{1}{4\pi^2}\int_0^\infty n_k k^2\,\mathrm{d}k=\frac{1}{4\pi^2}\int_0^\infty\frac{k^2\mathrm{d}k}{\exp\,[(k^2+m_{\mathrm{M}}{}^2)^{1/2}T^{-1}]}=\frac{cm_{\mathrm{M}}{}^2}{4\pi^2}\exp\left(-\frac{m_{\mathrm{M}}}{T}\right) \tag{3.27}$$

where n_k is the density of monopoles with momentum $\boldsymbol{k}$, m_{M} is the mass of the monopole and c is some constant ($c\sim 1$). Finally one obtains the following expression for the 'mass' of the magnetic field at $T\neq 0$, $T\ll m_{\mathrm{M}}$ (Polyakov 1978, Linde 1978a):

$$m=\frac{cm_{\mathrm{M}}}{16\pi^3q^2}\exp\left(-\frac{m_{\mathrm{M}}}{T}\right). \tag{3.28}$$

Therefore, in the presence of monopoles at $T\neq 0$ photons actually become massive. However, in most SBGT the mass of the monopole is extremely large, and therefore the photon mass becomes appreciable only at $T\sim T_{\mathrm{c}}$, when m_{M} vanishes. Moreover, at $T>T_{\mathrm{c}}$ the monopoles of the type discovered by 't Hooft and Polyakov disappear, and the photon Green function $G_{ij}(k)$ again behaves like that for massless particles in the limit $k_0=0$, $\boldsymbol{k}\to 0$. Fortunately, the infrared problem in quantum electrodynamics (QED) can be easily solved since in QED there are no self-interacting massless Bose fields.

Now let us return to the quantum statistics of massless Yang–Mills fields. For this case one can also show that in the Coulomb gauge the Green function of the Yang–Mills field has the same structure as the Green function for photons (3.23) (except for the extra factor δ^{ab}, where a, b are isotopical indices). As in QED, it can be shown that in perturbation theory $\Pi_{00}(\boldsymbol{k}\to 0)\neq 0$, $A(\boldsymbol{k}\to 0)\sim \boldsymbol{k}^2$ *if the diagrams for the polarisation operator* $\Pi_{\mu\nu}{}^{ab}(k)$ *are not singular at* $k_0=0$, $\boldsymbol{k}\to 0$. In QED this requirement is always satisfied, but in the quantum statistics of massless Yang–Mills fields at $T\neq 0$ infrared divergences are so strong that the diagrams for $\Pi_{\mu\nu}{}^{ab}$ actually become singular at $\boldsymbol{k}\to 0$. In the lowest order in g^2 the polarisation operator has only logarithmic singularity, and therefore $A(\boldsymbol{k}\to 0)\sim \boldsymbol{k}^2\ln \boldsymbol{k}^2$, i.e. as in QED the 'magnetic' part of the Yang–Mills field remains massless at $T\neq 0$. Dimensional estimates show that to the order g^4 the polarisation operator $\Pi_{\mu\nu}{}^{ab}$ may become singular enough to produce a 'mass' $m=A^{1/2}(0)$ of the Yang–Mills field, which would serve as an infrared cutoff in the theory. This mass, however, would be very small ($m\sim g^2T$), and order-by-order calculations in quantum statistics in this case would remain unreliable. Indeed, by the methods developed in §3.3 one can show that at $m\lesssim g^2T$ higher orders of perturbation theory become of the same order or greater than the lower ones. It is not excluded that the mass of the Yang–Mills fields at $T\neq 0$ can be generated by some non-perturbative effects of the type shown by the photon mass generation in the monopole–antimonopole plasma.

Unfortunately, at present it is not quite clear if the mass generated by the non-perturbative effects can be sufficiently large (greater than g^2T).

As was mentioned at the beginning of this subsection, the massless Yang–Mills fields exist not only in SBGT at $T > T_c$, but also in quantum chromodynamics (QCD). In recent years the main interest in the macroscopic consequences of QCD was connected with the possibility of quark liberation in super-dense matter and with the possible existence of super-dense but relatively cold quark stars, which were supposed to consist of free-quark matter (Collins and Perry 1975). Note that if one could obtain the thermodynamic potential and the effective gauge coupling constant g in QCD at some density, and if one could show that at this density the coupling constant g becomes small ($g \ll 1$), then it would be possible to use the renormalisation group equation and to prove that in super-dense matter $g \to 0$ and quarks behave as an ideal relativistic gas (Collins and Perry 1975, Kislinger and Morley 1976b). The only problem here is whether one can reliably obtain thermodynamic characteristics of a system containing massless Yang–Mills fields at any non-vanishing density and temperature.

At exactly zero temperature infrared divergences are absent from the diagrams for the thermodynamical potential of the quark matter in QCD. Therefore one could expect that many of the results concerning the cold quark matter (see, for example, Chapline and Nauenberg 1977, Freedman and McLerran 1977, Baluni 1978) are actually reliable. However, at any non-vanishing temperature the infrared divergences appear again. Moreover, the propagator of the massless Yang–Mills field at low momenta is drastically modified by the higher-order (and non-perturbative) effects. The true low-momentum behaviour of the improved propagator is now absolutely unknown even in quantum field theory (i.e. at vanishing density and temperature). As a result the calculation of the diagrams for the thermodynamic potential in QCD with the improved propagators of the Yang–Mills fields becomes ambiguous even at zero temperature.

A very interesting approach to the infrared problem in quantum statistics of gauge fields is contained in a recent paper by Polyakov (1978). In this paper it is concluded that at a sufficiently high temperature quarks in QCD become free and massless Yang–Mills fields acquire some relatively small mass (which was suggested to be $\sim g^2T$). This important statement has been obtained by means of a non-perturbative method which effectively takes into account all leading infrared divergences in quantum statistics of massless Yang–Mills fields. Unfortunately this method cannot be extended to the investigation of the relatively cold quark matter inside quark stars. Moreover, in his treatment of massless Yang–Mills fields, Polyakov has neglected non-leading infrared divergences. Such an approximation is sufficiently good for the study of massless infrared-stable theories of the type given by the theory $\lambda\phi^4$ (Wilson and Kogut 1974), but the applicability of this method to the infrared-unstable Yang–Mills theory does not seem quite clear.

To summarise our discussion we note that the existence of singularities of $G_{\mu\nu}^{ab}(k)$ at $k_0 \neq 0$ (Kislinger and Morley 1976a, Shuryak 1978) does not prove the existence of an infrared cutoff in quantum statistics of massless Yang–Mills fields, and *in this sense* massless Yang–Mills fields do not become massive due to lowest-order temperature corrections. Nevertheless the Yang–Mills fields may acquire some non-vanishing 'mass' $m = A^{1/2}(0)$ (which has the meaning of an infrared cutoff) due to higher-order corrections or due to non-perturbative effects. If, as one may expect, this mass is small ($m \lesssim g^2T$), then the higher-order corrections are large and all the lowest-order results concerning the quantum statistics of the gauge fields are unreliable. In this case some more elaborate methods should be used for the investigation of QCD at $T \neq 0$ and of SBGT at $T > T_c$ (see also the discussion at the end of §3.4).

Some results concerning quantum statistics of the massless non-Abelian gauge fields obtained in recent years by the usual methods (and in particular an equation of state $p = \frac{1}{3}\epsilon$ for the super-dense matter (Collins and Perry 1975)) seem very natural and may be true. However, one should recognise that these results will be finally proved (or disproved) only when the infrared problem in the quantum statistics of gauge fields will be solved. A solution of this problem may appear rather unexpected, for example the Yang–Mills fields in super-dense matter may prove not to be in a gaseous state, but in a crystalline one (Akhiezer *et al* 1978, Linde 1978b) (see also §5.4 of the present review). We hope that future investigations will shed some light on the questions touched upon in this section.

4. Symmetry behaviour in external fields

4.1. Quasimagnetic massive vector fields

As is mentioned in the introduction, most of the high-temperature effects in SBGT can be anticipated from the analogy between SBGT and superconductivity theory (Kirzhnits 1972, Kirzhnits and Linde 1972). This analogy also proves to be very useful for the investigation of symmetry behaviour in SBGT in the presence of external fields.

It is known that an external magnetic field destroys superconductivity. Symmetry restoration in an external quasimagnetic field $\mathscr{H} = \text{rot}\ \boldsymbol{A}$ in the Higgs model would be an exact analogue of this effect, but it is impossible to create a homogeneous external quasimagnetic field since the field A_μ in the Higgs model is massive. Fortunately the only fact which should be taken into account in the investigation of the behaviour of a superconductor in a magnetic field H is that some external currents or fields exist which should create the field H inside the superconductor *after* the destruction of superconductivity. Here we shall investigate an analogous situation in the Higgs model. We shall analyse the Higgs model in the presence of such external currents, which at $\sigma = 0$ should create the quasimagnetic field $\mathscr{H}$ in some spatial domain. The simplest way to realise this situation is to consider an ordinary magnetic coil with an electric current, which is also a neutral weak current according to the Weinberg–Salam model; at $\sigma = 0$ this coil created not only an ordinary magnetic field, but also a quasimagnetic field $\mathscr{H}$, corresponding to weak interactions: $\mathscr{H} = \text{rot}\ \boldsymbol{Z}$, where Z_μ is a heavy neutral intermediate vector meson (Weinberg 1967, Salam 1968).

For brevity and using the same terminology as in superconductivity theory we shall speak of the symmetry behaviour 'in an external quasimagnetic field $\mathscr{H}$'. One should recognise, however, that the quasimagnetic field actually appears inside the system only *after* the phase transition with symmetry restoration. The investigation of such a phase transition in SBGT is completely analogous to the behaviour of a superconductor in an external magnetic field, and here we shall present only the results of this investigation concerning the Higgs model (Kirzhnits and Linde 1974b, 1976a, see also Harrington and Shepard 1976).

In the Higgs model there exist three different critical fields, $\mathscr{H}_c$, $\mathscr{H}_{c_1}$, $\mathscr{H}_{c_2}$.

The thermodynamic critical field $\mathscr{H}_c$ is determined by the relation:

$$\mathscr{H}_c^2/2 = \mu^4/4\lambda \tag{4.1}$$

and has the meaning of a maximum field for which a homogeneous ordered phase $\sigma = \mu\lambda^{-1/2}$ without a quasimagnetic field has lower energy than the disordered phase $\sigma = 0$ with the quasimagnetic field inside the system. From (4.1) it follows that:

$$\mathscr{H}_c = \mu^2/\sqrt{2\lambda}. \tag{4.2}$$

The upper critical field $\mathscr{H}_{c_2}$ determines the lower boundary of the region in which the disordered state $\sigma = 0$ can be stable or metastable. At $\mathscr{H} < \mathscr{H}_{c_2}$, $\sigma = 0$ scalar particles have a tachyon spectrum. In the Higgs model (1.14):

$$\mathscr{H}_{c_2} = \mu^2/e. \tag{4.3}$$

From (4.2) and (4.3) it follows that:

$$\mathscr{H}_c/\mathscr{H}_{c_2} = e/\sqrt{2\lambda} = m_A/m_\phi. \tag{4.4}$$

Like superconductors, the Higgs model may be of the first type ($e^2 > 2\lambda$, $m_A > m_\phi$) or of the second type ($e^2 < 2\lambda$, $m_A < m_\phi$). In the first-type models, $\mathscr{H}_c > \mathscr{H}_{c_2}$ and the phase transition to the disordered state $\sigma = 0$ takes place at $\mathscr{H} = \mathscr{H}_c$. In the second-type models ($\mathscr{H}_{c_2} > \mathscr{H}_c$) two different phase transitions take place with an increase of electric current in a magnetic coil. The first phase transition occurs at the current which at $\sigma = 0$ would create inside the magnetic coil the quasimagnetic field:

$$\mathscr{H}_{c_1} = \frac{e^2}{2\lambda}\,\mu^2\left(\ln\frac{\sqrt{2\lambda}}{e} + 0{\cdot}08\right). \tag{4.5}$$

However, actually at $\mathscr{H} = \mathscr{H}_{c_1}$ the phase transition takes place not to the state $\sigma = 0$ but to some inhomogeneous state. Inside the magnetic coil there appear quasimagnetic flux tubes, each of which contains a quasimagnetic flux quantum $\Phi_0 = 2\pi/e$. Finally at $\mathscr{H} = \mathscr{H}_{c_2}$ the phase transition to the disordered state $\sigma = 0$ takes place and the quasimagnetic field $\mathscr{H}$ completely penetrates into the system.

Analogous results have been obtained in the non-Abelian gauge theories as well (Krive *et al* 1976b). Unfortunately the critical strength of the quasimagnetic field $\mathscr{H}$ in most of the models is very large. Let us take for example $\sigma \sim 100$ MeV, $m_\phi \sim 1$ GeV (strong interactions), then $\mathscr{H}_c \sim 2 \times 10^{18}$ G; for the weak interaction models $\mathscr{H}_c$ is even much greater.

4.2. *Magnetic and electric fields*

In our investigation of the symmetry behaviour inside a magnetic coil we have taken into account the quasimagnetic field, which is absent until the phase transition takes place, but we have neglected an ordinary magnetic field created by the same magnetic coil. The reason is that the quasimagnetic field $\mathscr{H}$ interacts with the condensate at the classical level (the quasimagnetic field acquires mass $\sim \sigma$), whereas the magnetic field affects the condensate only due to radiative corrections. Therefore, in most of the realistic models with neutral currents (which create the quasimagnetic fields) magnetic fields actually can be neglected in the analysis of the symmetry restoration (Linde 1975c, 1976b).

It is not excluded, of course, that there exist some realistic theories with neutral currents in which, for some reasons, the effects connected with the quasimagnetic fields are small. To get an idea of what we shall deal with in this case Salam and Strathdee have considered some models without neutral currents, and their first estimates of the critical magnetic field have been very encouraging: $H_c \sim 10^6$–10^{16} G (Salam and Strathdee 1974, 1975). These estimates have stimulated some attempts to discover the phase transitions in SBGT experimentally since the magnetic field inside heavy nuclei may be of the order of $H \sim 10^{15}$ G. Moreover, the parameter σ of the symmetry breaking depends only on the invariants $E^2 - H^2$ and $\boldsymbol{E}.\boldsymbol{H}$. Therefore, the electric field may also affect symmetry breaking, and in heavy nuclei this field may be even greater than the magnetic one: $E \sim 10^{16}$–10^{17} G.

Unfortunately a further analysis of this question has shown that the estimates $H_c - E_c \sim 10^6$–10^{16} G are too optimistic (Linde 1975c, 1976b). To verify it let us discuss again the first model considered by Salam and Strathdee (1974). The Lagrangian of this model is:

$$L = -\tfrac{1}{4}(G_{\mu\nu}{}^a)^2 + \tfrac{1}{2}(\nabla_\mu \phi^a)^2 + \tfrac{1}{2}\mu^2(\phi^a)^2 - \tfrac{1}{4}\lambda((\phi^a)^2)^2 \tag{4.6}$$

where the scalar and vector fields ϕ^a and $A_\mu{}^a$ are triplets with respect to O(3) symmetry ($a = 1, 2, 3$), and:

$$\begin{aligned} \nabla_\mu \phi^a &= \partial_\mu \phi^a + e\epsilon^{abc} A_\mu{}^b \phi^c \\ G_{\mu\nu}{}^a &= \partial_\mu A_\nu{}^a - \partial_\nu A_\mu{}^a + e\epsilon^{abc} A_\mu{}^b A_\nu{}^c. \end{aligned} \tag{4.7}$$

The symmetry being broken spontaneously to O(2), the component ϕ^3 of the scalar field acquires a non-zero vacuum expectation value:

$$\langle 0|\phi^3|0\rangle = \sigma = \mu\lambda^{-1/2}. \tag{4.8}$$

The corresponding component of the vector field $A_\mu{}^3$ does not acquire mass and can be identified with the electromagnetic potential.

To investigate symmetry behaviour in the model (4.6) in the presence of an external magnetic field $\boldsymbol{H} = \mathrm{rot}\ \boldsymbol{A}^3$ one should introduce as before an effective potential $V(\sigma, H)$.

Let us suppose for simplicity that $e^2 \ll \lambda$. In this case one can neglect the contribution from the vector particles. To obtain the contribution from charged scalar particles with the mass m one should use the propagator:

$$G(k_0, k_H, n) = \frac{1}{k_0{}^2 - k_H{}^2 + (2n+1)eH + m^2} \tag{4.9}$$

instead of the ordinary propagator $G(k) = (k^2 + m^2)^{-1}$. Here k_H is the projection of the particle momentum $\boldsymbol{k}$ onto the direction of the field $\boldsymbol{H}$, $n = 0, 1, 2, \ldots$. The integral $\int \mathrm{d}^4k$ in the one-loop expression for $V(\sigma)$ should be replaced by

$$2\pi eH \int_{-\infty}^{\infty} \mathrm{d}k_0\, \mathrm{d}k_H \sum_{n=0}^{\infty}.$$

The effect of all this is in the one-loop correction to $\mathrm{d}V/\mathrm{d}\sigma$, which at $H \gg m$ after renormalisation is given by:

$$\frac{\mathrm{d}V^1(\sigma, H)}{\mathrm{d}\sigma} = -\frac{\lambda e\sigma H}{8\pi^2} \int_0^\infty \frac{\mathrm{d}x}{x^2}\left(1 - \frac{x}{\sinh x}\right) \equiv -\frac{\lambda e \sigma H}{8\pi^2} B \tag{4.10}$$

where $B \sim 1$. The equation for the equilibrium value of σ at $H \neq 0$ in this case has the following form (Linde 1976a):

$$\frac{\mathrm{d}V}{\mathrm{d}\sigma} = 0 = \sigma\left(\lambda\sigma^2 - \mu^2 - \frac{\lambda e H}{8\pi^2} B\right) \tag{4.11}$$

from which it is seen that an increase of H *increases* the symmetry breaking parameter σ.

One can easily verify that the H-dependent contribution from fermions, which could be added to the model (4.6), also increases symmetry breaking in this model. Only the vector particle contribution may lead to the symmetry restoration in (4.6) at a sufficiently large H.

Following from (4.10), a characteristic strength of the magnetic field which can substantially modify the symmetry breaking parameter σ in the model (4.6) is of the order of:

$$H_{\mathrm{c}} \sim \frac{8\pi^2\mu^2}{\lambda e} = \frac{8\pi^2\sigma^2(H=0)}{e}. \tag{4.12}$$

If $\sigma(0) \sim 250$ GeV as in the Weinberg–Salam model, the characteristic field is of the order of $H_{\mathrm{c}} \sim 10^{27}$ G. To make an estimate, which can serve as a lowest bound for H_{c}, let us take $\sigma(0) \sim 100$ MeV, as in some theories of strong interactions. In this case $H_{\mathrm{c}} \sim 10^{19}$–$10^{20}$ G (Linde 1975c, 1976b). This result is in agreement numerically with the result of some other model calculations by Salam and Strathdee (1976). The same estimate can also be obtained for the characteristic value of the electric field E. This means that the fields which can exist inside nuclei ($H \sim 10^{15}$ G, $E \sim 10^{16}$–10^{17} G) are still insufficient to give a considerable modification of symmetry breaking in most of the gauge theories.

Generally speaking, there exists some possibility of the phase transitions in SBGT at relatively small values of external fields. Such a phase transition could take place if for some special reasons the effective potential $V(\sigma)$ at $H = 0$ has several different local minima, and the values of $V(\sigma)$ at these minima are almost equal. As was shown in §2, these conditions are satisfied, e.g. in the Higgs model at $\lambda \sim 3e^4/32\pi^2$. In this case the first-order phase transition between these local minima of $V(\sigma)$ may take place in the presence of very weak external fields. Unfortunately, as was mentioned in §3, the time which is necessary for a first-order phase transition in SBGT to take place is usually much greater than the age of the Universe. Therefore, one may expect that the most interesting effects connected with the phase transitions in external fields could take place only in the early Universe when such strong fields may actually have existed.

5. Effects connected with the fermion density increase

5.1. *Theories without neutral currents (σ model)*

In the previous section we have considered high-temperature symmetry behaviour in quantum field theory, the chemical potentials of all particles being equal to zero. In that case, at all temperatures the system under investigation contains equal amounts of particles and antiparticles, and at $T = 0$ all particles disappear.

One may wonder, however, which physical effects take place in super-dense *cold* matter in SBGT. To examine this problem we shall consider below a dense gas of fermions with a chemical potential $\alpha \neq 0$. Following Lee and Wick (1974) we shall consider first the simplified σ model (1.28) with the Lagrangian:

$$L = \tfrac{1}{2}(\partial_\mu \phi)^2 + \tfrac{1}{2}\mu^2\phi^2 - \tfrac{1}{4}\lambda\phi^4 + \bar{\psi}(i\partial_\mu\gamma_\mu - g\phi)\psi.$$

At a low density of fermions ($\alpha \to 0$) the lowest energy state in the cold matter will be as before the state with $\langle\phi\rangle = \sigma \approx \mu\lambda^{-1/2}$. At large density it becomes important that with an increase of σ the fermion masses increase, which is energetically disadvantageous. Therefore, at a sufficiently large density the state with $\sigma = 0$ becomes energetically favourable and symmetry restoration in (1.28) takes place.

To describe this effect quantitatively one should calculate the one-loop corrections to the effective potential due to the existence of the dense gas of fermions with the chemical potential α. The calculation of $V(\sigma)$ in this case goes like that in §2. The only difference is that one should add $i\alpha$ to the component k_0 of the fermion momentum in the corresponding Euclidean integrals (see, for example, Fradkin 1965).

As will be seen, at $\lambda \sim g^2 \ll 1$ symmetry restoration in (1.28) takes place at $\alpha \sim \sigma \gg m_\phi$, m_ψ. In this case the one-loop correction to $dV/d\sigma$ can be easily calculated (Lee and Wick 1974, Harrington and Yildiz 1974) and is given by:

$$\frac{dV^1}{d\sigma} = \tfrac{1}{2}g^2 \left(\frac{9j^2}{\pi^2}\right)^{1/3} \sigma \tag{5.1}$$

where $j^2 = j_0{}^2 - \boldsymbol{j}^2$, $j_\mu = \langle\bar{\psi}\gamma_\mu\psi\rangle$ is the fermion current, and the fermion density j_0 in the rest frame of the medium ($\boldsymbol{j} = 0$) is equal to (Landau and Lifshitz 1964):

$$j_0 = \langle\bar{\psi}\gamma_0\psi\rangle = \alpha^3/3\pi^2.$$

The equation $dV/d\sigma = 0$ in this case looks like:

$$\frac{dV}{d\sigma} = 0 = \sigma\left[\lambda\sigma^2 - \mu^2 + \tfrac{1}{2}g^2\left(\frac{9j^2}{\pi^2}\right)^{1/3}\right]. \tag{5.2}$$

From this equation it can be easily obtained that at $\boldsymbol{j} = 0$, $j_0 = j_0{}^c$, where:

$$j_0{}^c = \frac{2\sqrt{2}\,\pi}{3}\left(\frac{\mu}{g}\right)^3 \tag{5.3}$$

is the critical fermion density, a second-order phase transition with the symmetry restoration takes place. In the terminology of Lee and Wick this symmetry restoration is the phase transition from the 'normal' nuclear matter ($\sigma \neq 0$) to the 'abnormal' matter ($\sigma = 0$).

At $\lambda \lesssim g^4$ this phase transition becomes the first-order one (like the high-temperature phase transition in the Higgs model at $\lambda \lesssim e^4$). Moreover, at $\lambda \lesssim g^4$ no external pressure is needed for this phase transition to occur. In this case it becomes energetically advantageous for fermions to collapse into 'super-nuclei' inside which $\sigma = 0$ and the fermions are massless (Lee and Wick 1974, Lee and Margulies 1975).

The last result is particularly interesting, and it was even suggested that it is possible to produce such super-nuclei in heavy-ion collisions. A similar idea of local symmetry restoration inside a super-nucleus in application to quark matter was used in the SLAC bag model of quark confinement (Bardeen *et al* 1975). Unfortunately the problem of the Lee–Wick super-nuclei is not quite so clear. Indeed, as was mentioned in §2, at $\lambda \ll g^2$ the vacuum in (1.28) is unstable with respect to spontaneous generation of an extremely large classical field σ. At $g \ll 1$ from inequality $\lambda \lesssim g^4$ (which is needed for the super-nuclei formation) it follows that $\lambda \ll g^2$. This means that at $g \ll 1$, instead of the symmetry restoration obtained by Lee and Wick an extremely strong symmetry breaking should take place. Therefore the super-nuclei suggested by Lee and Wick may be stable only at $g \gtrsim 1$, when the lowest-order approximation used in their work is inapplicable. Moreover, one may argue (Krive and Chudnovsky 1978) that even if the super-nuclei could be formed, they would be unstable with respect to the pion condensation. From our point of view, however, the more important fact is that the symmetry restoration in the cold dense matter takes place only in the theories without neutral currents and neutral vector mesons of the type given by (1.28), or under some special circumstances for which the effects connected with the neutral currents are small (see the next section). The effects connected with the neutral vector mesons are actually small in 'normal' nuclear matter at a sufficiently low density, when the short-range interactions mediated by the relatively heavy vector mesons can be neglected. However, in the dense 'normal' matter as well as in the 'abnormal' matter, in which the interactions mediated by the vector mesons become long-range, the vector mesons should be taken into account. As will be shown in the next subsection, the effects connected with the neutral vector mesons are opposite to those considered above and lead to an increase of symmetry breaking in cold dense matter (Linde 1975a, 1976c).

5.2. *Theories with neutral currents* (*Weinberg–Salam model*)

High-density symmetry restoration in (1.28), discussed above, may seem rather unexpected from the point of view of the analogy between SBGT and superconductivity theory. Indeed, it is well known that an increase of an electric current $\boldsymbol{j}$ leads to the symmetry restoration in superconductivity theory (see, for example, De Gennes 1966). Therefore, one could expect that an increase of an external fermion current should lead to symmetry restoration in SBGT. In gauge theories the symmetry breaking parameter σ is a function of $j^2 = j_0{}^2 - \boldsymbol{j}^2$, and therefore an increase of a fermion charge density j_0 should lead to an *increase* of symmetry breaking.

The reason why we have obtained an opposite result in the study of the simplified σ model (1.28) is that this model is not a gauge theory and the current $j_\mu = \langle \bar{\psi} \gamma_\mu \psi \rangle$ in this model does not interact with a neutral vector field as distinct from the electric current in a superconductor. In the theories in which there exist some currents of the type $j_\mu = \langle \bar{\psi} \gamma_\mu \psi \rangle$ interacting with neutral vector mesons ('theories with neutral currents') an increase of the fermion charge density j_0 actually leads to an increase of symmetry breaking.

As an example we shall consider the Higgs model (1.14), extended by the inclusion of fermions:

$$L = -\tfrac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + (\partial_\mu + \mathrm{i}eA_\mu)\chi^*(\partial_\mu - \mathrm{i}eA_\mu)\chi + \mu^2 \chi^* \chi - \lambda(\chi^* \chi)^2 + \bar{\psi}(\mathrm{i}\partial_\mu \gamma_\mu - m)\psi - e\bar{\psi}\gamma_\mu \psi A_\mu. \quad (5.4)$$

Let us suppose that there exists a non-vanishing fermion current density $j_\mu = \langle \bar{\psi} \gamma_\mu \psi \rangle = \text{constant} \neq 0$. The current being constant, there is no reason to expect translational invariance breaking in this simple model (see however below (§5.4)). Therefore, we shall suppose the classical part of the physical fields χ_1 and A_μ to be constant in space and time. This means that in the transverse gauge $\partial_\mu A_\mu = 0$

we shall try to find a solution corresponding to (5.4) at $j_\mu \neq 0$ of the form:

$$\chi(x)=\frac{1}{\sqrt{2}}(\chi_1(x)+\sigma+\mathrm{i}\chi_2(x))$$
$$A_\mu(x)=B_\mu(x)+C_\mu \tag{5.5}$$

where $\langle\chi_i\rangle=\langle B_\mu\rangle=0$ and the classical fields σ and C_μ are space–time-independent. In this case at the classical level the Lagrange equations for the fields σ and C_μ are:

$$\left\langle\frac{\delta L}{\delta\chi_1}\right\rangle=0=-\sigma(\lambda\sigma^2-\mu^2)+e^2C_\mu{}^2\sigma$$
$$\left\langle\frac{\delta L}{\delta A_\mu}\right\rangle=0=e^2C_\mu\sigma^2-ej_\mu. \tag{5.6}$$

The same equations could also be obtained by varying over σ and C_μ the *effective* Lagrangian L_{eff}, which is equal to the Lagrangian (5.4), the two last terms in (5.4) being replaced by $-ej_\mu A_\mu$. These equations are nothing but a covariant generalisation of the Ginzburg–Landau equations in the theory of superconductivity (De Gennes 1966). From (5.6) it follows that:

$$\sigma(\lambda\sigma^2-\mu^2)-j^2/\sigma^3=0 \tag{5.7}$$

where $j^2=j_0{}^2-\boldsymbol{j}^2$. Equation (5.7) implies that, like in the theory of superconductivity, an increase of $\boldsymbol{j}$ leads to the symmetry restoration in the Higgs model (5.4), whereas an increase of the fermion charge density j_0 increases the symmetry breaking (Linde 1975a, 1976c).

If a Lagrangian of the type (5.4) contained a term $\sim g\bar{\psi}\psi\phi$ as in (1.27), then on the left-hand side of equation (5.7) a term $\sim g^2\sigma\,(j^2)^{1/3}$ would appear, promoting symmetry restoration with an increase of j^2 (see (5.2)). However, at $g^2 \ll 1$ this term can be neglected compared with the term j^2/σ^3 in (5.7). Moreover, even at $g^2 \gtrsim 1$ the term j^2/σ^3, which appears in equation (5.7) due to the existence of neutral currents (the term $-\bar{\psi}\gamma_\mu\psi A_\mu$ in (5.4)), becomes the leading one at large density, i.e. the density increase at large j^2 always leads to an increase in symmetry breaking. This result, which was first obtained for the extended Higgs model (5.4) (Linde 1975a, 1976c, Kirzhnits and Linde 1976a) has also been confirmed by the investigation of some other gauge theories with neutral currents (Sato and Nakamura 1976, Krive and Chudnovsky 1976a, b, Källman 1977). In particular, it has been shown that in the Weinberg–Salam model an equation for σ coincides exactly with (5.7) if the quantity j_μ in (5.7) is understood as a neutrino current $j_\mu=\frac{1}{2}\langle\bar{\nu}\gamma_\mu(1+\gamma_5)\nu\rangle$ (Linde 1976c). Therefore, both in the Higgs model (5.4) and in the Weinberg–Salam model at sufficiently large j^2:

$$\lambda\sigma^6=j^2. \tag{5.8}$$

Let us now consider the case $\boldsymbol{j}=0$. Following from (5.7), the characteristic fermion density at which the parameter σ increases substantially is:

$$j_0\sim\mu^3/\lambda=\sqrt{\lambda}\,\sigma^3(0) \tag{5.9}$$

where $\sigma(0)\equiv\sigma(j_\mu=0)=\mu\lambda^{-1/2}$. To estimate the characteristic density j_0 we shall take $\sigma(0)\sim 250$ GeV as in the Weinberg–Salam model. In this case $j_0\sim\sqrt{\lambda}\times10^{48}$ cm^{-3}. For $\sigma(0)\sim 100$ MeV, $\mu\sim 1$ GeV (strong interactions) and $j_0\sim 10^{39}$ cm^{-3}. The last value of fermion density is of the same order as the density in the cores of neutron stars (Zeldovich and Novikov 1971).

5.3. *Symmetry behaviour at a simultaneous increase of fermion density and temperature*

In the previous subsections it was shown that high-temperature effects lead to symmetry restoration in SBGT, whereas an increase of a fermion charge density usually leads to a further increase of the symmetry breaking. Therefore, to study symmetry behaviour in hot dense matter one should take into account the two opposed factors (temperature and fermion charge density) simultaneously. For this purpose we shall consider again the extended Higgs model (5.4) and take into account temperature corrections to the Lagrange equations (5.7) for σ and C_μ. In the lowest order in λ and e^2 the corresponding equations take the form:

$$\left\langle\frac{\delta L}{\delta\chi_1}\right\rangle = -\sigma[\lambda\sigma^2-\mu^2+\lambda(3\langle\chi_1{}^2\rangle+\langle\chi_2{}^2\rangle)-e^2C_\mu{}^2-e^2\langle B_\mu{}^2\rangle]+2e^2C_\mu\langle B_\mu\chi_1\rangle=0 \tag{5.10}$$

$$\left\langle\frac{\delta L}{\delta A_\mu}\right\rangle = e^2C_\mu\sigma^2+e^2C_\mu(\langle\chi_1{}^2\rangle+\langle\chi_2{}^2\rangle)+2e^2\sigma\langle B_\mu\chi_1\rangle+e\langle\chi_2\partial_\mu\chi_1-\chi_1\partial_\mu\chi_2\rangle-ej_\mu=0. \tag{5.11}$$

In a rough approximation one may neglect the non-diagonal terms in (5.10) (Linde 1976c). However, to get more detailed information concerning symmetry behaviour in the model (5.4) one should take into account that the non-diagonal terms in (5.10) and (5.11) at $j_\mu\neq 0$ do not vanish, since after the shift of variables (5.5) the Lagrangian (5.4) contains the non-diagonal terms $e^2\sigma C_\mu B_\mu\chi_1$ and $eC_\mu(\chi_2\partial_\mu\chi_1-\chi_1\partial_\mu\chi_2)$. For simplicity we shall suppose here that $\lambda\gg e^4$. In this case it can be shown that all the interesting effects take place at $T\gg m_{\chi_i}$, m_A, $(j^2)^{1/6}$, when:

$$\langle\chi_1{}^2\rangle=\langle\chi_2{}^2\rangle=-\tfrac{1}{3}\langle B_\mu{}^2\rangle=T^2/12 \tag{5.12}$$

(compare with (3.17)) and the non-diagonal terms in (5.10) can actually be neglected. As for the non-diagonal terms in (5.11), one can verify that they, together with the term $e^2C_\mu(\langle\chi_1{}^2\rangle+\langle\chi_2{}^2\rangle)$, are proportional to the polarisation operator of the field B_μ at zero momentum ($k_0=0$, $\boldsymbol{k}\to 0$). In this limit the only non-vanishing component of the polarisation operator is $\Pi_{00}(0)=\frac{1}{3}e^2T^2$ (Fradkin 1965). As a result equations (5.10) and (5.11) at $\boldsymbol{j}=0$ take the form:

$$\begin{gathered}\sigma[\lambda\sigma^2-\mu^2-e^2C_0{}^2+\tfrac{1}{12}T^2(3e^2+4\lambda)]=0\\ j_0-eC_0(\sigma^2+\tfrac{1}{3}T^2)=0\end{gathered} \tag{5.13}$$

from which it follows that:

$$\sigma\left(\lambda\sigma^2-\mu^2-\frac{j_0{}^2}{(\sigma^2+\frac{1}{3}T^2)^2}+\frac{3e^2+4\lambda}{12}\,T^2\right)=0. \tag{5.14}$$

At $T=0$ this equation coincides with equation (5.7), whereas at $j_0=0$ (5.14) coincides with equation (3.19). Let us consider the high-density limit, which will be most important for the cosmological applications of the above results. In this case the critical temperature T_c, at which symmetry restoration takes place in (5.4), is given by:

$$T_c{}^6 = \frac{108 j_0{}^2}{3e^2 + 4\lambda}. \tag{5.15}$$

In the Weinberg–Salam model the corresponding equation for σ is slightly more complicated than (5.14):

$$\sigma\left[\lambda\sigma^2 - \mu^2 - \frac{j_0{}^2}{\sigma^2 + \frac{1}{3}T^2(1 + 8\cos^4\theta_W)} + \left(4\lambda + \frac{3e^2(1 + 2\cos^2\theta_W)}{\sin^2 2\theta_W}\right)\frac{T^2}{12}\right] = 0. \tag{5.16}$$

Here j_0 is an excess of the neutrino density n_ν over the antineutrino density $n_{\bar\nu}$:

$$j_0 = n_\nu - n_{\bar\nu} = \tfrac{1}{2}\langle \bar\nu\gamma_0(1+\gamma_5)\nu\rangle \tag{5.17}$$

and θ_W is the Weinberg angle, which characterises the relative strength of weak and electromagnetic interactions (see, for example, Weinberg 1972); according to recent experimental data $\sin^2\theta_W \approx 0{\cdot}24$ (Holder *et al* 1977). In the high-density limit equation (5.16) yields the following expression for the critical temperature in the Weinberg model:

$$T_c{}^6 \approx \frac{j_0{}^2}{\lambda + 3e^2}. \tag{5.18}$$

For further applications it is convenient to express temperature T through the photon density n_γ at this temperature (Landau and Lifshitz 1964):

$$n_\gamma = \frac{2\zeta(3)}{\pi^2}T^3 \approx 0{\cdot}244\,T^3. \tag{5.19}$$

Thus from (5.18) and (5.19) it follows that at large j_0 symmetry restoration in the Weinberg–Salam model takes place when the photon density n_γ becomes greater than the critical density $n_\gamma{}^c$, where:

$$n_\gamma{}^c \approx \frac{j_0}{4(\lambda + 3e^2)^{1/2}} = \frac{n_\nu - n_{\bar\nu}}{4(\lambda + 3e^2)^{1/2}}. \tag{5.20}$$

It is seen therefore that the phase transition in the Weinberg model takes place only at some definite relationship between the densities of photons and neutrinos. This fact will be important for our discussion of the cosmological consequences of the phase transitions in SBGT (see §6).

5.4. Condensation of the Yang–Mills fields in super-dense matter

In the previous subsections we have assumed that super-dense matter is spatially homogeneous, i.e. that super-dense matter should be in a gaseous or a liquid, but not in a crystalline state. The reason is that in those theories without Yang–Mills fields inhomogeneous states of super-dense matter consisting of ultra-relativistic particles are energetically unfavourable at the classical level. Therefore, crystallisation of super-dense matter usually becomes possible only in some special cases when radiative corrections are sufficiently large, see, for example, a theory of pion condensation (Migdal 1977) or a theory of vector field condensation in super-dense matter (Akhiezer *et al* 1978).

However, in the non-Abelian gauge theories crystallisation becomes possible even at the classical level (Linde 1978b). To outline the main features of this effect we shall consider here as an example the simplest (though non-realistic) non-Abelian O(3)-symmetric theory (4.6) without scalar fields but in the presence of fermions with a non-vanishing electric charge density $j_0{}^3$. The corresponding effective Lagrangian L_{eff} (see §5.2) is:

$$L_{\text{eff}} = -\tfrac{1}{4}(G_{\mu\nu}{}^a)^2 - ej_0{}^3A_0{}^3 \tag{5.21}$$

where $a = 1, 2, 3$, $A_\mu{}^3$ is the electromagnetic potential (see §4.2).

One can easily verify that the classical Lagrange equations in (5.21) possess a standing wave solution:

$$\begin{aligned} A_1{}^1 &= A_1{}^2 = C \sin mZ \\ A_2{}^1 &= A_2{}^2 = C \cos mZ \\ A_3{}^a &= A_i{}^3 = A_0{}^{1,2} = 0 \end{aligned} \tag{5.22}$$

where

$$2eA_0{}^3C^2 = j_0{}^3 \qquad m = eA_0{}^3. \tag{5.23}$$

Note that the covariant divergence of the current $j_0{}^3$ vanishes in the solution (5.22). The energy-momentum tensor $T_{\mu\nu}$ of the field (5.22) has time-independent gauge-invariant components $T_{12} = T_{21} = -2m^2C^2 \sin 2mZ$, which are periodic in one of the space coordinates. Therefore, the solution (5.22) and (5.23) corresponds to a one-dimensional Yang–Mills crystal. The electric charge density of this crystal is equal to $-2eA_0{}^3C^2$ and exactly compensates the fermion charge density $j_0{}^3$ (see equation (5.23)).

Analogous Yang–Mills crystals may also exist in those theories with spontaneous symmetry breaking. However, we have discussed above only one solution (5.22) of the non-linear Yang–Mills equations. To analyse whether the Yang–Mills fields in super-dense matter should actually be in a crystalline state or that some other type of the vector field condensation takes place, one should try to find other (periodic or non-periodic) solutions. An investigation of this problem is now in progress.

6. SBGT and cosmology

6.1. *Symmetry behaviour in the early Universe*

In this section we shall discuss briefly the most important consequences of the phase transitions in gauge theories.

The most evident (though the least investigated) possibility for using the results concerning phase transitions in SBGT is connected with superconductivity theory. This possibility is based on the often-used analogy between superconductivity theory and SBGT. Phenomenological properties of superconductors are well known, and one can use them to get many correct predictions of the macroscopic effects in SBGT. On the other hand, the theory of symmetry breaking and restoration in SBGT is in fact much simpler than that in superconductivity theory. For example, only about twenty years after the discovery of the microscopic theory of superconductivity (Bardeen *et al* 1957) it was shown that the transition from a superconductive to a normal state in certain cases is not a second-order phase transition, but a first-order one (Halperin *et al* 1974). In SBGT the same result has been independently obtained in one of the first papers on phase transitions in gauge theories (Kirzhnits and Linde 1974b).

In gauge theories, due to their relative simplicity, some effects have been obtained which are absent or have not been discovered so far in superconductivity theory. It was shown in particular that, under certain conditions, quantum fluctuations and external factors may substantially *increase* symmetry breaking in SBGT. If analogous possibilities should be discovered in superconductivity theory, this would lead to some progress in solving the problem of high-temperature superconductivity.

Another interesting possibility of using the results discussed above is connected with the theory of multiparticle production in collisions of high-energy elementary particles. In such collisions fireballs filled with elementary particles in a thermodynamically equilibrium state are created, and then these fireballs decay into many separate elementary particles. According to the statistical theory of multiparticle production (see, for example, a review of this theory by Feinberg (1972)) the temperature inside the fireballs may exceed several hundred MeV, and therefore this temperature may be sufficient for the phase transitions in the theory of strong interactions (Eliezer and Weiner 1976, Krive *et al* 1977).

However, the most interesting consequences of the phase transitions in gauge theories are connected with cosmology. According to the hot Universe theory, the Universe has been expanding and gradually cooling from a state with infinite temperature and density (Zeldovich and Novikov 1975, Weinberg 1972). Therefore, at a sufficiently small time t from the beginning of the expansion of the Universe the extreme conditions necessary for all phenomena considered in this review have been actually realised. As a result, at $t \to 0$ symmetry behaviour in SBGT was determined by the relation between the effects connected with an increase of temperature and the opposed effects connected with an increase of fermion charge density (Linde 1976a, Krive *et al* 1976a).

To illustrate this general statement let us consider symmetry behaviour in the Weinberg–Salam model at $t \to 0$. As follows from the discussion in §5.3 symmetry behaviour in the Weinberg–Salam model depends on the relative magnitude of the photon density n_γ and the fermion charge density j_0. In the course of the Universe evolution the ratio of the photon density n_γ to the charge density j_0 of leptons and baryons (specific entropy) remains practically constant (Weinberg 1972, Zeldovich and Novikov 1975). Therefore, in order to solve the problem whether the symmetry at $t \to 0$ was restored or not it is sufficient to compare the present fermion 'weak' charge density with the photon density. The baryon charge density $n_{\mathrm{B}} - n_{\bar{\mathrm{B}}} \approx n_{\mathrm{B}}$ at present is of the order of:

$$n_{\mathrm{B}} \sim 10^{-8} n_\gamma \tag{6.1}$$

and this is the reason why the effects connected with the baryon charge asymmetry of the Universe do not influence symmetry restoration in the Weinberg–Salam model at $t \to 0$. The lepton charge density of all known types of leptons except neutrinos is of the same order as n_{B} and therefore it also does not affect the symmetry restoration. Meanwhile, an excess of neutrinos over antineutrinos $j_0 = n_\nu - n_{\bar{\nu}}$ may be very large. The strongest constraint on the value of j_0 for the electron neutrinos follows from the theory of helium production in the early Universe: $j_0 \lesssim 10^3 \text{ cm}^{-3}$ (Zeldovich and Novikov 1975). The value of j_0 for the muon neutrinos may be a few orders greater. On the other hand, the photon density in the Universe at present is of the order of $n_\gamma \sim 4 \times 10^2$ cm^{-3}. If one neglects variation of n_γ/j_0 in the course of the evolution of the Universe (i.e. if specific entropy actually is a constant) then according to (5.20) one may conclude that at $t \to 0$ symmetry in the Weinberg–Salam model has not been restored only if at present:

$$j_0 \gtrsim 4(\lambda + 3e^2)^{1/2}\, n_\gamma. \tag{6.2}$$

If one takes, for example, $\lambda \sim e^2 \sim 10^{-1}$, the inequality (6.2) implies that:

$$j_0 \gtrsim 2{\cdot}5\, n_\gamma. \tag{6.3}$$

Such a possibility does not contradict present cosmological constraints on the value of j_0. Therefore, further on we shall study both possibilities $j_0 \gtrsim 2{\cdot}5\, n_\gamma$ and $j_0 \lesssim 2{\cdot}5\, n_\gamma$, which correspond respectively to symmetry restoration or symmetry breaking in the Weinberg–Salam model at $t \to 0$.

From (6.1) it follows that inequality (6.3) is equivalent to the following relation between the lepton (L) and baryon (B) charges of the Universe:

$$L \gtrsim 10^8\, B. \tag{6.4}$$

Thus, if the Universe is not so enormously charge-asymmetric, then the symmetry in the Weinberg–Salam model at $t \to 0$ was restored and all particles except the Higgs mesons were massless. The time from the beginning of the expansion of the Universe, at which the phase transition in the Weinberg–Salam model has taken place, is of the order of:

$$t \sim 10^{-7}\text{–}10^{-10}\ \text{s}.$$

If the inequalities (6.3) and (6.4) hold (strong charge asymmetry of the Universe), the symmetry was always broken, and at $t \to 0$ masses of all particles except photons and neutrinos were infinitely growing.

Now let us discuss some consequences of the phase transitions and of the unusual properties of super-dense matter in the early Universe.

6.2. *Quarks in the Universe*

Attempts to explain the reasons why free quarks up to now have not been discovered can be approximately divided into three large groups.

(i) Free quarks have infinite energy (permanent confinement of quarks): see, for example, papers by Wilson (1974), Polyakov (1977, 1978), Callan *et al* (1977), 't Hooft (1977) and Nambu (1974).

(ii) Free quarks have some extremely large but finite energy (partial confinement of quarks). The most known model of this type is the SLAC bag model (Bardeen *et al* 1975).

(iii) Free quarks decay immediately into leptons, but baryons built up from these quarks are practically stable (Pati and Salam 1973, 1974).

Permanent confinement of quarks may be achieved due to the infrared instability and some special topological properties of the massless Yang–Mills field theory ('electric confinement') (see Wilson 1974, Polyakov 1977, 1978, Callan *et al* 1977, 't Hooft 1977) or due to the existence of quasimagnetic flux tubes, which connect 'quasimonopoles' identified with quarks ('magnetic' confinement) (see Nambu 1974, Polyakov 1975, Mandelstam 1975, Linde 1976e). The 'magnetic' mechanism of quark confinement works only if symmetry in SBGT is broken, and therefore at a sufficiently high temperature $T > T_c$ the quarks become free. It was argued (Polyakov 1978) that 'electric' confinement also does not work at sufficiently large temperatures. This point, however, does not seem quite clear (see §3.5). In both cases, however, at the present time free quarks should be completely absent and therefore permanent confinement of quarks does not lead to any difficulties of the type discussed below.

In cases (ii) and (iii) at a sufficiently high temperature quarks should no doubt become free. As was shown by Zeldovich *et al* (1965) some of the stable quarks of type (ii), which were free in the early Universe, have not enough time to meet each other and form hadrons. These quarks would remain free at the present time and a lot of them would be discovered, for example, in the cosmic rays. On the other hand, free unstable quarks of type (iii) would almost completely decay into leptons, and as a result an extremely small amount of baryons would remain in the Universe (Okun' and Zeldovich 1976).

One cannot exclude, of course, that due to some special circumstances some of the theories of types (ii) and (iii) will not lead to these difficulties. However, at present we can see only one possibility of avoiding the problems peculiar to theories (ii) and (iii), namely, if the Universe is sufficiently strongly charge-asymmetric, then due to an increase of the fermion density in the early Universe the quark masses have been increasing as $t \to 0$ at the same rate as the temperature. As a result, if the ratio of the quark masses to the temperature was sufficiently large, the quarks never have been free and all the difficulties mentioned above disappear (Linde 1976c). Note, however, that the necessity to use such exotic assumptions as a strong charge asymmetry of the Universe to save theories (ii) and (iii) may serve as an argument against these theories in favour of the theories with permanent confinement of quarks.

6.3. *Domain structure of vacuum*

The kinetics of the process of symmetry breaking in the cooling Universe may be very complicated. Let us consider, for example following Zeldovich *et al* (1974), the process of symmetry breaking in the simplest model (1.3). In sufficiently far removed (causally unconnected) domains of the Universe the phase transition with the symmetry breaking may proceed from the disordered state $\sigma = 0$ *into two different states*: into the state with the field $\sigma = +\mu\lambda^{-1/2}$ or into the state with $\sigma = -\mu\lambda^{-1/2}$. The domains with the different signs of the field σ are separated from each other by thin walls ('kinks') inside which the field σ varies from $-\mu\lambda^{-1/2}$ to $+\mu\lambda^{-1/2}$. Such a domain structure is energetically unfavourable and the walls collapse or spread up to infinity with the velocity almost equal to the velocity of light c. Therefore the size of the domains filled with the constant homogeneous field σ of one of the two signs is of the order of ct (the so-called radius of the horizon), which is extremely large. However, the domain walls have such a large surface energy density that if at least one such a wall existed at present inside the horizon (and this seems to be unavoidable), the observable part of the Universe would be greatly anisotropic.

This result implies that most of the theories with spontaneous breaking of a discrete symmetry of the type (1.3) (which is symmetric with respect to the change $\phi \to -\phi$) contradict cosmological data (Zeldovich *et al* 1974). This conclusion is very important since it rules out many theories with spontaneous breaking of (discrete) *CP* invariance. One should note, however, that in some models with a discrete symmetry breaking it is possible to avoid the difficulties connected with the appearance of the domain walls. For this purpose one should assume, as in the preceding subsection, that the Universe is greatly charge-asymmetric (6.4). As was shown in §6.1, for some theories this may lead to the absence of the phase transition in the early Universe and, consequently, to the absence of the undesirable domain walls.

It is worth noting also that the same mechanism, which provides the domain wall formation after the phase transition with a discrete symmetry breaking, leads also to the vortex tube formation after the U(1) symmetry breaking in the Higgs model and to the creation of monopoles after the non-Abelian symmetry breaking (Kibble 1976). It can be shown that the formation of the vortex tubes does not lead to any considerable cosmological effects. However, some consequences of the creation of monopoles in the early Universe would be in contradiction with the cosmological data (see Zeldovich and Khlopov 1978).

6.4. *Substance energy non-conservation and the time-dependent cosmological term*

One of the most unexpected effects connected with the phase transitions in gauge theories is the substance energy non-conservation due to energy 'pumping' from the non-observable Bose condensate in the processes under consideration (Kirzhnits and Linde 1974a, 1976a).

The physical meaning of this effect may be easily understood if one takes into account that all matter in SBGT at a non-vanishing temperature (for simplicity we take here all chemical potentials and external fields equal to zero) can be uniquely divided into two parts: a set of interacting particles and the Bose condensate $\sigma(T)$. Note that the condensate is a Poincaré-invariant object, constant in space and time, which does not fix any preferred reference frame or preferred direction in space–time, and which does not influence a test particle since $\partial_\mu \sigma_\mu = 0$. Therefore the condensate $\sigma(T)$ at a given temperature can manifest itself only through its influence on the space–time curvature connected with a non-zero energy-momentum tensor of the condensate $g_{\mu\nu}\epsilon(\sigma)$, and in this sense the condensate does not differ from the ordinary vacuum of a quantum field theory. This is the reason why the division of all matter into a set of interacting particles and the condensate has the direct meaning of the division of matter into the observable part (i.e. substance) and non-observable part (vacuum).

In those theories without spontaneous symmetry breaking such a division is fixed once and forever. In our case the characteristics of the vacuum state are temperature-dependent. This fact is an inevitable consequence of spontaneous symmetry breaking and is connected with the appearance of the classical temperature-dependent field $\sigma(T)$ in the Lagrangian.

Let us consider, for example, the energy-momentum tensor, corresponding to the simplest theory (1.3):

$$\theta_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \tfrac{1}{2}g_{\mu\nu}[(\partial_\alpha\phi)^2 - 2\sigma(\lambda\sigma^2-\mu^2)\phi - (3\lambda\sigma^2-\mu^2)\phi^2 - 2\lambda\sigma\phi^3 - \tfrac{1}{2}\lambda\phi^4]$$
$$+ g_{\mu\nu}[\tfrac{1}{4}\lambda\sigma^4 - \tfrac{1}{2}\mu^2\sigma^2 + \epsilon(0)] \qquad (6.5)$$

where $\epsilon(0)$ is an arbitrary constant which can be subtracted from the Lagrangian (1.3) to fix the vacuum energy at $\sigma = 0$.

Averaging of $T_{\mu\nu}$ (where $T_{\mu\nu}$ is the operator part of $\theta_{\mu\nu}$) in the lowest order of perturbation theory gives a quantity with the obvious physical meaning of the energy-momentum tensor of interacting particles (some function of the occupation numbers n_p, vanishing at $n_p \to 0$). The C-number part of $\theta_{\mu\nu}$ gives the energy-momentum tensor of the non-observable Bose condensate $g_{\mu\nu}\epsilon(\sigma)$, and thus the overall energy-momentum tensor of matter $\langle\theta_{\mu\nu}\rangle$ can be divided into two parts:

$$\langle\theta_{\mu\nu}\rangle = \langle T_{\mu\nu}\rangle + g_{\mu\nu}\epsilon(\sigma) \tag{6.6}$$

where $\langle T_{\mu\nu}\rangle$ is the energy-momentum tensor of substance, and the condensate energy is given by:

$$\epsilon(\sigma) = \epsilon(0) + \tfrac{1}{4}\lambda\sigma^4(T) - \tfrac{1}{2}\mu^2\sigma^2(T). \tag{6.7}$$

Due to the dependence of $\epsilon(\sigma)$ on the condensate density $\sigma(T)$, the energy-momentum tensor of the condensate (of vacuum), $g_{\mu\nu}\epsilon(\sigma)$, varies in the processes in which the condensate density $\sigma(T)$ changes. As a result, in such processes only the overall energy-momentum tensor $\langle\theta_{\mu\nu}\rangle$ is conserved, but not the energy-momentum tensors of substance and vacuum separately. This means that in the processes where temperature changes with time, the first law of thermodynamics applied to substance is violated, i.e. the energy of the observable part of matter is non-conserved due to energy 'pumping' from the non-observable condensate (Kirzhnits and Linde 1974a, 1976a).

The physical meaning of what has been said becomes particularly clear if we consider a first-order phase transition in the Higgs model at $\lambda \ll e^4$. As is noted in §3.3 the first-order phase transition with symmetry breaking in the Higgs model takes place with a decrease of temperature near the point T_{c_1} (see figure 9). After the phase transition at $T \approx T_{c_1}$ the Bose condensate $\sigma(T) \approx \sigma(0) = \mu\lambda^{-1/2}$ appears. The substance energy density $\sim T^4$ at $T = T_{c_1}$ is much less than the energy density $\mu^4/4\lambda$ released due to the condensate formation (or, in other words, due to the vacuum reconstruction). Since before the phase transition the Bose condensate is absent and the substance energy is relatively small, an observer would see the creation of most of the substance at the point of the phase transition practically 'from nothing'. Note, however, that the effect of the non-conservation of the substance energy has nothing in common with stationary cosmology in which the creation of matter from nothing was postulated (Bondi and Gold 1948, Hoyle 1948). In our case the total energy-momentum tensor of matter $\langle\theta_{\mu\nu}\rangle$ is exactly conserved.

From the point of view of gravity theory the division of $\langle\theta_{\mu\nu}\rangle$ into $\langle T_{\mu\nu}\rangle$ and $g_{\mu\nu}\epsilon(\sigma)$ corresponds to the representation of the total energy-momentum tensor as a sum of the energy-momentum tensor of substance and the cosmological term (Linde 1974, Veltman 1974, 1975, Dreitlein 1974). The results discussed above imply in particular that in theories with spontaneous symmetry breaking the cosmological term $g_{\mu\nu}\epsilon(\sigma)$ is temperature-dependent (Linde 1974), and at $T > T_c$ the cosmological term becomes equal to $g_{\mu\nu}\epsilon(0)$. According to the present cosmological data $\epsilon(\sigma(T=0)) = \epsilon(\mu\lambda^{-1/2}) \lesssim 10^{-29}$ g cm^{-3}. On the other hand, in the Weinberg–Salam model $\sigma(T=0) \approx 250$ GeV, and if, for example, $\lambda \sim 10^{-2}$, then $\epsilon(0) = \frac{1}{4}\lambda\sigma^4(T=0) \sim 10^{25}$ g cm^{-3}. This means that in the early Universe the cosmological term was at least 10^{50} times greater than at present (Linde 1974, see also Bludman and Ruderman 1977). Of course, the energy of substance in the early Universe was also extremely large. Nevertheless, in certain cases the temperature-dependent cosmological term may become greater than $\langle T_{\mu\nu}\rangle$. In particular, near the point of the first-order phase transition with symmetry breaking in the Higgs model at

$\lambda \lesssim e^4$ the cosmological term is actually much greater than the substance energy-momentum tensor. In some cases the effects connected with the substance energy non-conservation and the time-dependence of the cosmological term in the expanding Universe may lead to important cosmological consequences (see §6.6).

6.5. Boiling of vacuum and an interplay between symmetry breaking in gauge theories and cosmology

In §2 it was pointed out that at certain relations between masses and coupling constants radiative corrections lead to symmetry restoration in the Higgs model (Linde 1976a). Analogous effects also take place in more realistic theories such as the Weinberg–Salam model. It proves (Weinberg 1976, Linde 1976b, d) that for the Weinberg angle $\sin^2 \theta_W \approx 0{\cdot}24$ the effective potential in the Weinberg–Salam model acquires an additional 'dynamical' minimum at $\sigma = 0$ if the Higgs meson mass m_ϕ is less than 9·31 GeV. At $m_\phi < 6{\cdot}55$ GeV this minimum becomes even deeper than the ordinary minimum at $\sigma = \mu\lambda^{-1/2}$ (see figure 5), i.e. the vacuum state with the broken symmetry becomes metastable. However, as was stressed in §3.3 the lifetime of such metastable vacuum states in SBGT usually exceeds the age of the Universe. Therefore a decay of a metastable vacuum state in SBGT (boiling of vacuum, see §3.3) may take place only at some special relations between masses and coupling constants at which the energy barrier between the stable and metastable vacua becomes very small (Voloshin *et al* 1974, Coleman 1977, Linde 1977b).

This important observation leads to some rather unexpected conclusions concerning symmetry breaking in the Weinberg–Salam model. Let us first suppose that the Universe is indeed greatly charge-asymmetric ($L \gtrsim 10^8 B$, see equation (6.4)). This implies that at $t \to 0$ the symmetry in the Weinberg–Salam model was broken. With a decrease of neutrino density in the course of the expansion of the Universe a 'dynamical' minimum of the effective potential has appeared at $\sigma = 0$. If the Higgs meson mass is sufficiently small ($m_\phi < 6{\cdot}55$ GeV), this minimum at the present time should become deeper than the minimum at $\sigma = \mu\lambda^{-1/2}$. However, an investigation of the phase transition from the state $\sigma = \mu\lambda^{-1/2}$ to the state $\sigma = 0$ shows that it is a tunnelling process which occurs in the course of the Universe evolution only if $m_\phi \lesssim 450$ MeV (Linde 1977b).

On the other hand, if the Universe is not so greatly charge-asymmetric, then initially the symmetry in the Weinberg–Salam model has been restored, and the new minimum of $V(\sigma)$ at $\sigma \neq 0$ appeared only after the cooling of the Universe. In this case the transition from the state $\sigma = 0$ to the state $\sigma = \mu\lambda^{-1/2}$ would proceed only if $m_\phi > 9{\cdot}3$ GeV (Linde 1977b).

At the present time the symmetry in the theory of weak and electromagnetic interactions is broken. This fact, together with the results discussed above, leads us to the following conclusions concerning the Weinberg–Salam model.

(i) The Higgs meson mass in the Weinberg model should exceed 450 MeV.

(ii) If, as one may expect, the Universe is not enormously charge-asymmetric, then $m_\phi > 9{\cdot}3$ GeV.

(iii) If the Higgs meson mass is in the range 450 MeV $< m_\phi < 9{\cdot}3$ GeV, then the symmetry in the Weinberg model may be broken at the present time only if the Universe is greatly charge-asymmetric. In other words, for certain relations between masses and coupling constants, *symmetry breaking in the Weinberg model is completely determined by the charge asymmetry of the Universe.* The discovery of the Higgs meson with a mass in the interval between 450 MeV and 9·3 GeV would imply the existence of a 'neutrino sea' in the Universe with an excess of neutrinos over antineutrinos $j_0 = n_\nu - n_{\bar{\nu}} \gtrsim 10^3\ \mathrm{cm}^{-3}$. Such a correspondence between symmetry breaking in the Weinberg–Salam model and the charge asymmetry of the Universe seems to be a rather unexpected example of an interplay between micro- and macrophysics.

From the results obtained above one can draw a general conclusion concerning symmetry breaking in more complicated theories, namely that, if the effective potential corresponding to some theory has a number of deep enough local minima, then due to the large lifetime of the metastable vacuum states in quantum field theory the solution of the problem in which of the local minima the Universe should be at the present time is determined not by energetical considerations but by the physical processes at the early stages of the Universe evolution.

6.6. *Cold Universe?*

In order to demonstrate how much the existence of the phase transitions in SBGT may modify the present theory of the Universe evolution we shall consider below (Chibisov and Linde 1978) symmetry behaviour in cold dense matter, which consists of leptons and baryons with mutually compensated weak charge densities. In this case (charge-symmetric Universe) the effects connected with neutral currents (see §5.2) disappear and an increase of fermion density leads to symmetry restoration as in the σ model.

Let us suppose, contrary to the usual belief, that initially the Universe was cold. Nevertheless, due to an increase of fermion density at $t \to 0$ symmetry in the early Universe has been restored, and in the course of the Universe expansion a phase transition with symmetry breaking has taken place. As was mentioned in §5.1, at certain relations between coupling constants this phase transition is the first-order one. In this case, after the decay of the metastable vacuum $\sigma = 0$ all its energy $\epsilon(0)$ has been transformed into the thermal energy of substance, and therefore we again obtain an ordinary hot Universe, but only at $t > t_c$, where t_c is the time of the phase transition.

By an appropriate choice of the coupling constants in SBGT one may arrange things so that the phase transition with symmetry breaking will take place at a comparatively small fermion density. In this case, before the phase transition almost all the energy is concentrated in the metastable vacuum state $\sigma = 0$. Therefore, after the decay of the metastable vacuum almost all the substance energy appears 'from nothing' (substance energy non-conservation) and the ratio of the photon density n_γ to the baryon density in the hot matter (specific entropy) determined by the choice of the coupling constants in SBGT may be made arbitrarily large. In particular, at a certain choice of coupling constants one may get an experimental value of the specific entropy $s \sim 10^8$.

One should note that the first-order phase transition in SBGT proceeds by bubble formation (see §3.3) which, in principle, could lead to large inhomogeneities in the energy density in the early Universe and consequently to the creation of a large number of black holes. This may lead to a contradiction with the present cosmological data (Vainer and Naselsky 1977). However, in the theories with superstrong symmetry breaking (see, for example, Georgi and Glashow 1974) the phase transition with symmetry breaking takes place soon after the Planck time $t_{\mathrm{Pl}} \sim 10^{-43}$ s. In this case all the black holes formed after the phase transition were extremely small and very soon they had evaporated due to the Hawking effect (Hawking 1975).

Since the phase transition takes place during the very early stages of the Universe evolution, all the observational consequences of our cold Universe model are the same as in the ordinary hot Universe theory.

In recent years there have been some other attempts to obtain all the observational consequences of the hot Universe theory starting from a cold Universe (see, for example, Zeldovich and Starobinsky 1976, Carr 1977). In these papers it is supposed that the cold Universe becomes hot due to some effects connected with primordial black holes, which are formed from initial density fluctuations in the early Universe. However, in this approach one needs to make several assumptions concerning the spectrum of the initial density fluctuations in the early Universe, which may seem rather unnatural, and some of the predictions of these models differ from those of the hot Universe theory (Carr 1977). In our case all the effects are determined by the properties of the elementary particle theory only, and no special assumptions concerning initial density fluctuations are needed.

Of course, all the effects discussed in this section take place only at certain conditions (weak charge symmetry of the Universe) and at some definite relations between coupling constants. In any case, however, the possibility of such a complete reconsideration of the theory of the Universe evolution seems to be a very interesting example of the new perspectives opened now in cosmology in connection with phase transitions in gauge theories.

7. Conclusions

The most interesting phenomena discovered in solid-state physics and quantum statistics, such as ferromagnetism, superfluidity and superconductivity, are connected with spontaneous symmetry breaking. It is not surprising, therefore, that the study of phase transitions in SBGT, being in the boundary region between elementary particle theory, quantum statistics and cosmology, leads to such non-trivial results as a time dependence of particle masses, coupling constants and of the cosmological term in the expanding Universe, to the appearance of a domain structure of vacuum, to substance energy non-conservation, to the correspondence between symmetry breaking in elementary particle theory and in cosmology, to the possibility of a description of all observed properties of the hot Universe supposing that initially the Universe was cold, etc. Note that all these effects do not appear as a consequence of some exotic hypotheses and assumptions. As is mentioned in the introduction, spontaneous symmetry breaking is a basic principle for practically all theories of weak, strong and electromagnetic interactions considered at the present time.

All the results discussed in the present review have been obtained only a few years ago and one may expect that the most interesting results concerning phase transitions in gauge theories are still to be obtained. In our review we have not discussed possible consequences of the phase transitions in SBGT for the theory of multiparticle production in collisions between high-energy elementary particles (Eliezer and Weiner 1976, Krive *et al* 1977). Everywhere throughout this review only the weak coupling case has been considered, whereas in the strong coupling regime a lot of new interesting effects may appear, such as the crystallisation of scalar and of Abelian vector fields, spontaneous generation of magnetic fields (ferromagnetism of super-dense matter), etc (Akhiezer *et al* 1978). Crystallisation of the non-Abelian gauge fields, which may take place even in the weak coupling case (Linde 1978b), also deserves further investigation. Some unsolved and very interesting problems arise in the study of the quantum statistics of massless Yang–Mills fields (see §3.5). Many questions are connected with the cosmological consequences of phase transitions in gauge theories. For example, one may wonder whether the large density fluctuations, which appear after the phase transitions in SBGT, could lead to galaxy formation. Another problem to be analysed is the possible connection between the phase transitions in SBGT and black hole formation in the early Universe. One may hope that further investigation of the macroscopic consequences of SBGT will be useful for a better understanding of the physical structure of gauge theories and will give us the possibility of looking from a new point of view at some other problems in theoretical physics.

Acknowledgments

A considerable part of this review is based on the results obtained in a collaboration with D A Kirzhnits. It is a pleasure to express my deep gratitude to D A Kirzhnits for his help and for many enlightening discussions. I am also thankful to G Chapline, E M Chudnovsky, E S Fradkin, R E Kallosh, I V Krive, A M Polyakov, A Salam, J Strathdee, E V Shuryak, I V Tyutin and Ya B Zeldovich for useful discussions of various problems touched upon in this review.

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