CPT-odd leptogenesis

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We calculate the baryon asymmetry of the Universe resulting from the combination of higher-dimensional Lorentz-noninvariant CPT-odd operators and dimension five operators that induce the majorana mass for neutrinos. The strength of CPT-violating dimension five operators capable of producing the observed value of baryon abundance is directly related to neutrino masses and found to be in the trans-Planckian range $(10^{-24}–10^{-22})$ GeV^{-1}. Confronting it with observational tests of Lorentz symmetry, we find that this range of Lorentz/CPT violation is strongly disfavored by the combination of the low-energy constraints and astrophysical data.

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I. INTRODUCTION

Since the seminal paper by Sakharov [1], it is well known that the baryon asymmetry of the Universe (BAU) can be generated dynamically, through the combination of baryon number violating processes, C and CP violation, and the departure from thermal equilibrium. It turns out that the standard model (SM) has all necessary ingredients for this to happen. Notably, the $B + L$ number is violated by the high-temperature sphaleron processes [2,3]. However, the existing amount of CP-violation combined with tight constraints on the Higgs sector prevents efficient baryogenesis in the SM. Thus, BAU presents a formidable hint on physics beyond SM, and motivates new experimental searches for the extended electroweak sector and new sources of CP violation.

It has also been known for some time that CPT-odd perturbations can effectively replace two Sakharov's conditions for baryogenesis: violation of CP invariance and the deviation from thermal equilibrium [4]. Indeed, a CPT-odd shift in the "mass" of a SM fermion (e.g. top quark [5]), Δm_{CPT} would serve as an effective chemical shift between baryons and antibaryons above the scale of the electroweak phase transition. It is easy to see that the $\Delta m_{CPT}/m_t \sim O(10^{-6})$ effect for top quark would be required to generate the observed asymmetry [5]. Unfortunately, at the level of the Lagrangian it is impossible to define a consistent "CPT-odd mass" without breaking the Lorentz invariance. CPT-odd mass would have to be identified with dimension three Lorentz-noninvariant operators [6]. Given the strength of constraints on lower-dimensional CPT/Lorentz noninvariant operators [7], one has to conclude that lower-dimensional operators cannot be a source of the observed baryon asymmetry.

The problem of CPT-odd baryogenesis was readdressed in Ref. [8] and recently in Ref. [9,10]. In [8,9] among other options higher-dimensional CPT-odd operators were suggested as a source for baryon asymmetry. Suppose that a dimension five operator that shifts the dispersion relations of baryons relative to antibaryons is added on top of the SM. Let us further assume that initial value for $B - L$ is zero. Then in the temperature range from 10^{10} to 10^2 GeV where the sphaleron processes are in thermal equilibrium, the resulting baryon asymmetry will be determined by the amount of CPT violation in the theory. If CPT-violating interactions are given by a dimension five operator parametrized by $1/\Lambda_{CPT}$, the inverse energy scale of CPT violation, the resulting baryon asymmetry at the sphaleron freeze-out ($T \sim M_W$) will be given by

$$Y_b = \frac{\Delta b}{s} \sim \frac{T}{\Lambda_{CPT}} \sim \frac{M_W}{\Lambda_{CPT}}, \tag{1}$$

where s is the entropy. It is then clear that $\Lambda_{CPT} < 10^{12}$ GeV will be required to produce an observable asymmetry. Given the fact that both low-energy data and astrophysical constraints limit a typical scale Λ_{CPT} to be higher than the Planck scale, such a scenario is completely ruled out.

In this paper we explore the idea of the CPT-odd leptogenesis that is capable of enhancing estimate (1) by many orders of magnitude. The main feature of any leptogenesis scenario is the use of the lepton-number nonconservation at high temperatures that results in a nonvanishing $B - L$ number that is preserved by sphaleron processes [11]. One of the advantages of leptogenesis is that the most natural way of mediating the lepton-violating processes is through heavy majorana neutrinos, which also supply masses to the light neutrinos via the see-saw mechanism. Heavy right-handed neutrinos with mass M_R mediate lepton-number violating processes, and thus keep lepton-number violating processes in equilibrium until the temperature decreases to the point where the Hubble rate Γ_H begins to dominate over the lepton-violation rate Γ_L. In the assumption that Yukawa couplings are on the order one, this moment in the Universe's history can be determined as

$$\Gamma_L \propto \frac{T^3}{M_R^2} \sim \Gamma_H \propto \frac{T^2}{M_{\rm Pl}},$$

which gives an estimate for the temperature of the freeze-out for the $B - L$ number:

$$T_R \propto \frac{M_R^2}{M_{\rm Pl}}.$$

Therefore, in the scenarios of CPT-odd leptogenesis, one obtains the asymmetry which freezes out at $T = T_R$ rather than at $T = M_W$:

$$Y_{l(b)} \sim \frac{M_R^2}{M_{\rm Pl}\Lambda_{CPT}}.$$

Obviously, for $M_R \sim 10^{15}$ GeV one gets a great enhancement by $T_R/M_W \sim 10^9$ over the CPT baryogenesis scenarios (1) where $B - L$ is zero.

The purpose of this paper is to explore the CPT-odd leptogenesis scenario, determine the required strength of the CPT-violating operators, and confront it with the existing laboratory and astrophysics constraints. For reasons explained earlier, we concentrate on CPT-odd interactions of mass dimension five. We introduce CPT-odd operators into the fermion sector of the standard model [12]:

$$\mathcal{L}_{LV} = \sum_{i=L,E,Q,U,D} \eta_i^{\mu\nu\rho} \cdot \bar{\psi}_i \gamma_\mu \mathcal{D}_\nu \mathcal{D}_\rho \psi_i, \qquad (2)$$

which cause an asymmetric shift of the dispersion relations for fermions and antifermions. Here $\eta_i^{\mu\nu\rho}$ is a symmetric irreducible Lorentz violating spurion field that can depend on the type of the SM fermions; the summation extends over all fields that carry the lepton or baryon number. The transmutation to the lower-dimensional operators can be protected by the irreducibility condition $\eta^{\mu\nu}{}_{\nu} = 0$. A zeroth component of $\eta_i^{\mu\nu\rho}$, $\eta_i^{000} \equiv \eta_i$ in the reference frame where the primordial plasma is at rest provides an asymmetric shift in the dispersion relations for particles and antiparticles. This way positive $\eta_{\rm lepton}$ creates a surplus of antileptons over leptons in equilibrium which is maintained when the rate for the lepton-number violating processes is faster than the Hubble expansion. It is notable that such CPT-odd perturbations allow for potential leptogenesis already with one flavor of heavy majorana neutrinos, whereas conventional leptogenesis requires at least two of them [11]. In the rest of this paper, we examine closely the kinetic equations for the $L(B)$-violating processes when CPT-odd shifts (2), lepton-number violation, and sphaleron processes are taken into account. We adjust the coupling constants η in Eq. (2) in such a way as to produce the observed value of the baryon asymmetry and compare the results with the existing limits on Lorentz violating (LV) interactions. We argue that the combination of bounds on LV from observations of high-energy cosmic rays [13] and the low-energy clock comparison experiments render the CPT-odd leptogenesis scenario fine-tuned for models with operators of mass dimension five (2), but allow it for higher-dimensional operators.

II. REACTION RATES AND BOLTZMANN EQUATIONS

To demonstrate how the CPT-odd leptogenesis works, we consider a model with only one heavy majorana neutrino. Its off-shell exchange mediates lepton-number violating processes that freeze out at the temperatures well below M_R. At $T > T_R$ these processes maintain the equilibrium value for the lepton-number asymmetry. In this section we calculate the rate of the lepton-number violating processes and include it in the Boltzmann equations together with the sphaleron rate.

The mass term Lagrangian for heavy neutrinos reads as

$$\mathcal{L}_m = -\tfrac{1}{2}M_R \bar{N}_M N_M + h_a \cdot \bar{L}_a H N_M + h_a^\dagger \cdot \bar{N}_M H^\dagger L_a, \qquad (3)$$

where N_M are singlet majorana neutrinos and h_a are the Yukawa couplings. We switch to Weyl spinors for convenience, in which the Lagrangian can be rewritten as

$$\mathcal{L}_m = -\tfrac{1}{2}M_R(NN + \bar{N}\bar{N}) + h_a \cdot \bar{L}_a \bar{N} H + h_a^\dagger \cdot H^\dagger N L_a, \qquad (4)$$

where index a runs over three different generations, and

$$N_M = \begin{pmatrix} N_\alpha \\ \bar{N}^{\dot{\alpha}} \end{pmatrix}.$$

Integrating out the heavy neutrinos, one obtains an effective lepton-number violating vertex:

$$\mathcal{L}_{\rm eff} = \frac{Y_{ij}^\nu}{2M_R} H^\dagger L_i^\alpha H^\dagger L_{j\alpha} + \text{H.c.}, \qquad (5)$$

where $Y_{ab}^\nu = h_a^\dagger h_b^\dagger$. Substituting the vacuum expectation value for the Higgs field in (5) creates a majorana mass term for light neutrinos. This interaction induces lepton-number violating processes which determine the lepton asymmetry until the lepton freeze-out. Alternatively, we could step by the stage with the heavy right-handed neutrinos and postulate (5) as a starting point in our analysis while taking Y_{ab}^ν to be an arbitrary complex symmetric matrix.

Introduction of the CPT-odd interactions (2) leads to the modification of dispersion relations for the SM leptons and antileptons. Taking lepton doublets, we neglect the mass terms and find

$$E_L(p) = |\vec{p}| + \eta_L \vec{p}^2, \qquad E_{\bar{L}}(p) = |\vec{p}| - \eta_L \vec{p}^2. \qquad (6)$$

Equation (6) leads to a shift in the equilibrium number density of leptons

$$n_L^{\rm eq} = \frac{g_L}{\pi^2 \beta^3}\left(1 - \frac{12\eta_L}{\beta}\right),$$

with the opposite sign of the shift for antileptons. Here g_L is the total number of the spin, gauge and flavor degrees of freedom associated with electroweak doublets L, and β is the inverse temperature. The difference

$$n_i^{\mathrm{eq}} - n_{\bar{i}}^{\mathrm{eq}} = -24\eta_i g_i(\pi^2\beta^4)^{-1}, \tag{7}$$

where $i = L$ for now, represents an equilibrium lepton number induced by *CPT* violation in the lepton doublet sector. As stated in the introduction section, the final abundance can be roughly estimated by evaluating the equilibrium density at the temperature of the freeze-out. A more accurate answer, however, can be obtained by analyzing Boltzmann equations in the presence of sphaleron processes and lepton-number violation.

There are two types of interactions induced by the effective Lepton-Higgs vertex [14,15], shown in Fig. 1. They generate the following processes relevant for leptogenesis:

$$L + L \leftrightarrow H + H, \qquad L + H \leftrightarrow \bar{L} + H,$$

with the same set of processes for antileptons. However, since the relevant part of the *CPT*-odd interactions is time reversal invariant, the amplitudes for direct and inverse processes are equal, and we therefore have only three different amplitudes, which we call A_{LL}, $A_{\bar{L}\bar{L}}$, and A_{LH}. Denoting the corresponding reaction rates (per unit volume) by W_{LL}, $W_{\bar{L}\bar{L}}$, W_{LH}, and $\bar{W}_{LH}$, we have

$$\begin{aligned}
W_{LL} &= \int d\pi_p d\pi_q d\pi_k d\pi_r (2\pi)^4 \delta^4(p+q-k-r)|A_{LL}|^2 f_L^{\mathrm{eq}}(p) f_L^{\mathrm{eq}}(q),\\
W_{\bar{L}\bar{L}} &= \int d\pi_p d\pi_q d\pi_k d\pi_r (2\pi)^4 \delta^4(p+q-k-r)|A_{\bar{L}\bar{L}}|^2 f_{\bar{L}}^{\mathrm{eq}}(p) f_{\bar{L}}^{\mathrm{eq}}(q),\\
W_{LH} &= \int d\pi_p d\pi_q d\pi_k d\pi_r (2\pi)^4 \delta^4(p+q-k-r)|A_{LH}|^2 f_L^{\mathrm{eq}}(p) f_H^{\mathrm{eq}}(q),\\
\bar{W}_{LH} &= \int d\pi_p d\pi_q d\pi_k d\pi_r (2\pi)^4 \delta^4(p+q-k-r)|A_{\bar{L}H}|^2 f_{\bar{L}}^{\mathrm{eq}}(p) f_H^{\mathrm{eq}}(q),
\end{aligned}$$

where $f_{L,H}^{\mathrm{eq}}(p)$ are the equilibrium distribution functions for Higgs fields and lepton doublets. In a toy model where only the lepton doublets and Higgs fields are present one can immediately write the Boltzmann equations for the lepton-number density as

$$\begin{aligned}
(\partial_t + 3\Gamma_H)n_L &= -2W_{LL}\left(\frac{n_L^2}{(n_L^{\mathrm{eq}})^2} - 1\right) - W_{L\bar{H}}\left(\frac{n_L}{n_L^{\mathrm{eq}}} - \frac{n_{\bar{L}}}{n_{\bar{L}}^{\mathrm{eq}}}\right)\\
(\partial_t + 3\Gamma_H)n_{\bar{L}} &= -2W_{\bar{L}\bar{L}}\left(\frac{n_{\bar{L}}^2}{(n_{\bar{L}}^{\mathrm{eq}})^2} - 1\right) - \bar{W}_{L\bar{H}}\left(\frac{n_{\bar{L}}}{n_{\bar{L}}^{\mathrm{eq}}} - \frac{n_L}{n_L^{\mathrm{eq}}}\right).
\end{aligned} \tag{8}$$

Here the Hubble rate is $\Gamma_H = 1.66 g_*^{1/2} T^2/M_{\mathrm{Pl}}$ in terms of the total effective number of degrees of freedom g_*. The factor of 2 in the right-hand side of Eq. (8) reflects the fact that the LL processes change the number of leptons by two.

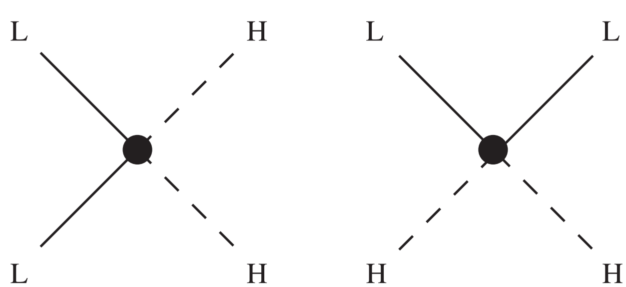

FIG. 1. $\Delta L = 2$ processes generated by the effective vertex (5).

An important thing to note is that even though we could have modified the dispersion relations for the Higgs field, its *CPT*-violating parameter would not enter the equations for the lepton-number density at tree level.

In order to generalize Eqs. (8) onto the full set of SM fields, we introduce the effective parameters of *CPT* violation in the lepton and baryon sectors:

$$\begin{aligned}
\eta_l &= \frac{g_L\eta_L + g_E\eta_E}{g_L + g_E};\\
\eta_b &= \frac{g_Q\eta_Q + g_U\eta_U + g_D\eta_D}{g_Q + g_U + g_D},
\end{aligned} \tag{9}$$

where g_i is the corresponding number of degrees of freedom in each sector. These parameters enter (7) with $i = l$, b, and $g_l = g_L + g_E$, $g_b = g_Q + g_U + g_D$.

As already mentioned, one also has to include sphaleron processes, which affects one linear combination of baryon and lepton-number densities. The main effect of sphalerons is to wash out $B + L$, while keeping $B - L$ intact. Since the processes we consider occur far above the electroweak transition, the sphaleron rate has linear dependence on temperature [3,16]. In the presence of *CPT* violation, the sphaleron contribution to the Boltzmann equation for n_l, n_b [3,17,18] should be modified for the presence of the equilibrium baryon and lepton numbers (7):

$$\partial_t(n_b + n_l) = -\Gamma_{\mathrm{sph}}(n_b - n_b^{\mathrm{eq}} + n_l - n_l^{\mathrm{eq}}), \tag{10}$$

where

$$\Gamma_{\rm sph} \simeq \omega T \quad \text{with} \quad \omega \simeq 10^{-5}.$$

Equation (10) implies that $B + L$ is washed out completely, and is somewhat simplifed relative to the realistic case. A detailed analysis shows (see e.g. Ref. [19]) that the washout is only partial, with the final value of $B + L$ controlled by a nonzero $B - L$, but we will employ the naive evolution Eq. (10), arguing that the corrections to this equation are much smaller than the uncertainty with which ω is known.

Next we make a well-justified assumption of smallness of the chemical potentials:

$$\frac{n_i}{n_i^{\rm eq}} = e^{\mu_i/T} \simeq 1 + \mu_i/T,$$

which enables us to linearize the kinetic equations in μ_i. The kinetic equations for n_l take the following form:

$$\begin{aligned}(\partial_t + 3\Gamma_H)n_l &= -(4W_{LL} + 2W_{LH})\mu_l/T \\ &\quad - \omega T(\mu_l/T + \mu_b/T) \\ (\partial_t + 3\Gamma_H)n_{\bar{l}} &= (4W_{\bar{L}\bar{L}} + 2\bar{W}_{LH})\mu_l/T \\ &\quad + \omega T(\mu_l/T + \mu_b/T).\end{aligned} \tag{11}$$

For the (anti)baryons the kinetic equations are the same except that there are no contributions from the lepton-number violating rates. A significant simplification comes from the smallness of the chemical potential. There are two possible sources for CPT-odd contributions to the reaction rates in Eq. (8): modified dispersion relations and CPT-odd modifications of thermal rates. The smallness of μ_i/T allows us to neglect any CPT-odd effects in the reaction rates in the right-hand side of Eq. (8), as they induce effects of the 2nd order in the CPT-violating parameter. Therefore, we take $W_{\bar{L}\bar{L}} = W_{LL}$ and $\bar{W}_{LH} = W_{LH}$.

From the above equations we only need their difference, the actual lepton (baryon) asymmetry. For convenience, we express the equilibrium number density in terms of the unmodified number density $n_i^0 = g_i/\pi^2 \cdot T^3$

$$n_{i,\bar{i}}^{\rm eq} = n_i^0(1 \mp 12\eta_i T), \qquad i = l, b.$$

The asymmetries Y_i then can be defined as

$$n_i - n_{\bar{i}} \equiv 2n_i^0 \cdot Y_i, \qquad Y_i = \mu_i/T - 12\eta_i T.$$

We also introduce a dimensionless parameter γ, by factoring out the dimensionful parameters T^3/M_R^2 from the rate of lepton-number violating processes,

$$4W_{LL} + 2W_{L\bar{H}} = \gamma\frac{T^6}{M_R^2},$$

so that γ scales as the fourth power of the neutrino Yukawa couplings or the sum of the squares of the eigenvalues of Y_{ab}^ν:

$$\gamma = \frac{3}{2\pi^2}\Big(\sum|h_a|^2\Big)^2 = \frac{3}{2\pi^2}\,{\rm Tr}(Y_{\rm diag}^\nu Y_{\rm diag}^{\nu\dagger}). \tag{12}$$

Expressing Eqs. (11) in terms of Y_i and changing variables from time to temperature, we get

$$\begin{aligned} g_l\frac{d}{dT}Y_l &= \frac{0.6}{g_*^{1/2}}\frac{\omega M_{\rm Pl}}{T^2}(g_l(Y_l + 12\eta_l T) + g_b(Y_b + 12\eta_b T)) \\ &\quad + \frac{0.6\pi^2}{g_*^{1/2}}\frac{\gamma M_{\rm Pl}}{M_R^2}\cdot(Y_l + 12\eta_l T) \\ g_b\frac{d}{dT}Y_b &= \frac{0.6}{g_*^{1/2}}\frac{\omega M_{\rm Pl}}{T^2}(g_l(Y_l + 12\eta_l T) \\ &\quad + g_b(Y_b + 12\eta_b T)).\end{aligned} \tag{13}$$

The quantity of the ultimate interest is the baryon asymmetry at the present time (normalized, e.g. on the photon number density, $n_\gamma = s/7.04$ [20]). Using $s = \frac{2\pi^2}{45}g_* T^3$, one can express the experimentally measured baryon to photon ratio via the asymmetry Y_b that enters (13),

$$\alpha_B = 7.04\frac{45}{\pi^4}\frac{g_b}{g_*}Y_b \simeq 0.6Y_b \equiv (6.1 \pm 0.3) \times 10^{-10}, \tag{14}$$

where we use $g_b = 18$ and $g_* = 106.75$.

Note, that in the limit when the rate of sphaleron processes is very small, $\Gamma_{\rm sph} \ll \Gamma_L$ (Γ_L is the rate of the lepton-number violating processes), one can solve the kinetic equations exactly. Taking $\omega \to 0$ in (13), we have

$$\frac{d}{dT}Y_l = \frac{\lambda M_{\rm Pl}}{M_R^2}(Y_l + 12\eta_l T). \tag{15}$$

where we have introduced $\lambda = 0.6\pi^2(g_*^{1/2}g_l)^{-1}\gamma$. A solution that corresponds to n_l close to equilibrium value at $T \gg T_R$ has the following form:

$$Y_l = -12\frac{\eta_l M_R^2}{\lambda M_{\rm Pl}} - 12\eta_l T, \tag{16}$$

which provides us with the expression for the lepton asymmetry:

$$Y_l^{\rm fr} = -12\frac{\eta_l M_R^2}{\lambda M_{\rm Pl}}. \tag{17}$$

The inclusion of sphalerons will diffuse approximately half of the lepton number yield into the baryon number [3], so that Eq. (17) is also an estimate for the BAU.

III. THE STRENGTH OF *CPT* VIOLATION DERIVED FROM BAU

In this section, we provide the numerical solutions to Eqs. (13), determine the required strength of CPT violation and confront it with existing experimental constraints.

To solve the system of kinetic equations, one has to add proper initial conditions. It is reasonable to impose these initial conditions at the temperatures where the essential part of leptogenesis begins, which we take to be $M_R = 10^{15}$ GeV:

$$Y_l|_{M_R} = Y_l^{\rm eq}, \qquad Y_b|_{M_R} = 0. \tag{18}$$

At high temperatures leptons and antileptons were in thermal and chemical equilibrium, which had a nonzero value of the lepton number defined by η_l. This choice is quite sensible since the freeze-out temperature T_R suggested by neutrino masses is sufficiently smaller than M_R. As for baryons, we impose symmetric $n_b = n_{\bar{b}}$ conditions at high temperatures (10^{15} GeV), as there are no fast processes that would bring Y_b to the equilibrium position set by η_b.

Since we chose to fix M_R, the only free parameters left are η_i and η_b parametrizing the strength of *CPT* violation, and the neutrino Yukawa couplings. For the latter there is some natural range suggested by the oscillations among the light neutrino flavors. Introducing an "effective" neutrino mass that the kinetic Eqs. (13) depend on,

$$m_\nu^{\rm eff} \equiv \left(\sum m_{\nu_i}^2\right)^{1/2} = \left(\frac{\sum |Y_{\rm diag}^\nu|^2 v^2}{2M_R}\right)^{1/2}, \tag{19}$$

we notice that $(m_\nu^{\rm eff})^2$ is larger than any of the individual Δm_{ij}^2 measured in the oscillations experiments. Thus, taking the largest of Δm_{ij}^2 suggested by the oscillation of atmospheric neutrinos, $\sqrt{\Delta m_{\rm atm}^2} \simeq 0.05$ eV [21] we find the following natural range for $m_\nu^{\rm eff}$:

$$0.05\ {\rm eV} \leq m_\nu^{\rm eff} \leq 0.65\ {\rm eV}, \tag{20}$$

where the upper limit comes from the cosmological bound on the sum of neutrino masses [22]. Defining the freeze-out temperature via relation $\Gamma_H(T_R) = \Gamma_L(T_R)$, one can translate (20) to the realistic range of T_R:

$$10^{12}\ {\rm GeV} < T_R < 10^{14}\ {\rm GeV}. \tag{21}$$

On the lower end of this range T_R overlaps with the sphaleron ignition temperature $T_{\rm sph}$, which is estimated to be of the order 10^{12} GeV [23].

The final result of our analysis is the prediction for the strength of *CPT* violation in lepton and baryons sectors. Since Eqs. (13) are linear in Y_i, it is sufficient to solve them numerically for two cases

$$\eta_l \neq 0, \qquad \eta_b = 0, \quad \text{and} \quad \eta_l = 0, \qquad \eta_b \neq 0,$$

and then using the experimental value of BAU, fix the values of η_l and η_b as functions of $m_\nu^{\rm eff}$.

Figure 2 exhibits the resulting dependence of η_i on $m_\nu^{\rm eff}$ within a phenomenologically viable range of $m_\nu^{\rm eff}$ bounded by two vertical dashed lines. One notices that η_b does not change much in the "physical" region. For η_l-dominated scenario, in contrast, the increase of η_l with $m_\nu^{\rm eff}$ is well pronounced. As expected, the lower mass $m_\nu^{\rm eff}$ leads to a *higher* freeze-out temperature T_R, and thus lower $m_\nu^{\rm eff}$ requires *lower CPT* violating parameter η_l to get an observed value of BAU. Also not surprisingly, the η_l and η_b required to reproduce BAU in our scenario have opposite signs.

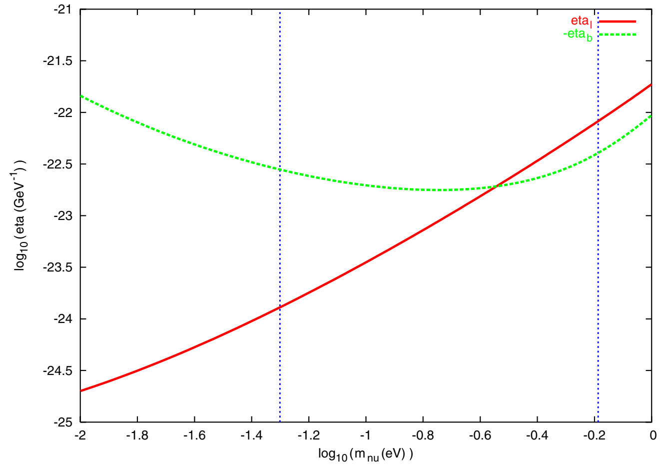

FIG. 2 (color online). *CPT*-odd parameters η_l, η_b necessary to generate the observed BAU versus the effective neutrino mass. The left vertical line indicating the value of $m_\nu^{\rm eff}$ suggested by the oscillation of atmospheric neutrinos and the right vertical line showing the cosmological upper limit on $m_\nu^{\rm eff}$ [22], bound the phenomenologically viable domain of $m_\nu^{\rm eff}$.

The lower end of the range (20) corresponds to a hierarchical scenario m_1^2, $m_2^2 \ll m_3^2$, with the tau-neutrino being the heaviest. The size of the *CPT* violation suggested by the *CPT*-odd leptogenesis in this case is found to be

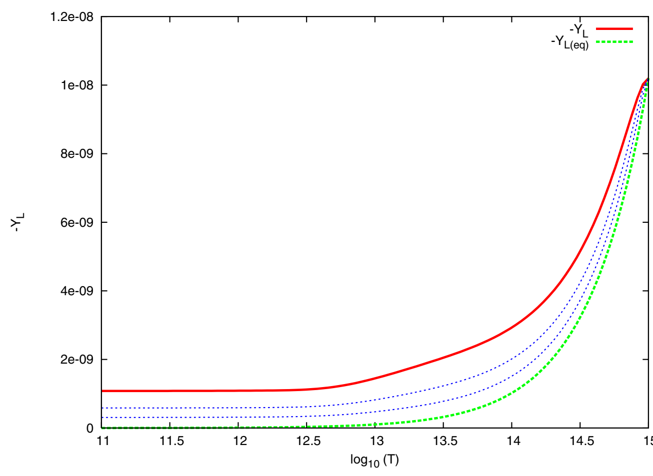

FIG. 3 (color online). Lepton asymmetry (solid line) and equilibrium lepton asymmetry (dashed line) driven by *CPT* violation in the lepton sector for $m_\nu^{\rm eff} = 0.05$ eV as function of temperature. The amount of *CPT* violation is fixed to $\eta_l = 9 \times 10^{-25}\ {\rm GeV}^{-1}$ to yield the observed value of baryon asymmetry. The final low-temperature plateau of $-Y_l$ equals to the baryon asymmetry Y_b. The dotted lines correspond to $m_\nu^{\rm eff} = 0.07$ eV and 0.10 eV, and demonstrate the approach to the equilibrium curve with the increase of mass $m_\nu^{\rm eff}$.

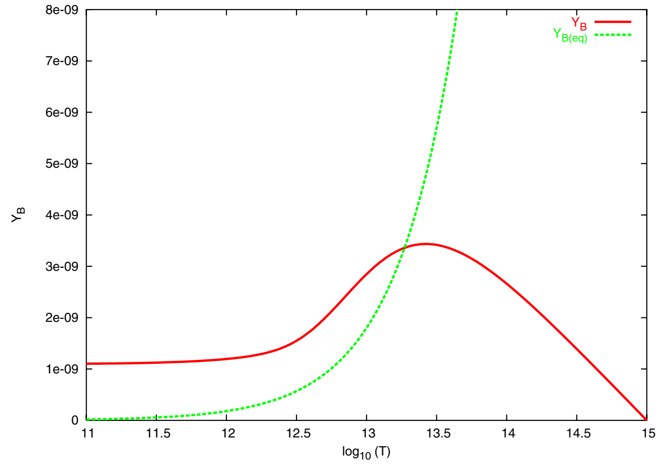

FIG. 4 (color online). Baryon asymmetry Y_b and equilibrium baryon asymmetry vs temperature with CPT violation concentrated in the baryon sector. The parameters $m_\nu^{\rm eff} = 0.05$ eV and $\eta_b = -1.5 \times 10^{-23}$ GeV^{-1} are chosen to match the observed asymmetry.

$$\eta_l = 9 \times 10^{-25}\ \mathrm{GeV}^{-1}, \qquad \eta_b = 0 \quad \text{or} \qquad \eta_b = -1.5 \times 10^{-23}\ \mathrm{GeV}^{-1}, \qquad \eta_l = 0. \tag{22}$$

This is the main prediction of our work.

Figs. 3 and 4 illustrate the case of $m_\nu^{\rm eff} = 0.05$ eV in more detail, by showing the evolution of the baryon/lepton asymmetry as a function of temperature. When CPT violation is concentrated in the lepton sector, see Fig. 3, the lepton asymmetry follows the equilibrium value of the (lepton) asymmetry at high temperatures to freeze-out below 10^{14} GeV. When CPT violation is given by η_b, the asymmetry Y_b starts from zero, overshoots the equilibrium curve just above 10^{13} GeV, to freeze-out at lower temperatures.

IV. COMPARISON WITH EXPERIMENTAL CONSTRAINTS ON *CPT* VIOLATION

Now we are ready to confront our predictions for CPT-violation (22) with the experimental constraints on it. The modification of dispersion relation by dimension five operators has been discussed at length in the literature. Below we list a set of relevant constraints on dimension five operators in the fermionic sector of the SM and comment on their applicability:

$$|\eta_d - \eta_Q - 0.5(\eta_u - \eta_Q)| < 10^{-27}\ \mathrm{GeV}^{-1}, \qquad [12,24]$$
$$|\eta_L|, |\eta_E| < 10^{-20}\ \mathrm{GeV}^{-1}, \qquad [25]$$
$$|\eta_L|, |\eta_E| < 10^{-33}\ \mathrm{GeV}^{-1}. \qquad [13]$$

The first constraint arises because the axial-vector-like combinations of η_i in the quark sector lead to the coupling of the nucleon spin to a preferred direction. In models where the preferred frame is associated with the rest frame of the cosmic microwave background, the net spin energy shift is proportional to the velocity of the lab frame $v \sim O(10^{-3})$, $\Lambda_{\rm QCD}^2 \eta_i (v \cdot s)$, which is to be compared with the experimental sensitivity 10^{-31} GeV [24,25]. The low-energy constraints on lepton operators are considerably weaker [12]. The strongest constraints on dimension five CPT-odd operators in the lepton sector come from considerations of $p \to p l \bar{l}$ processes that become energetically allowed and prevent acceleration of protons to energies of $\sim 10^{21}$ eV. It is important that constraints [13] are double-sided, which is the consequence of asymmetric modification of dispersion relation for leptons and antileptons (6).

The strength of CPT violation in the lepton sector derived from the baryon asymmetry (22) is consistent with the astrophysical bounds on CPT-violating QED [26], but appears to be grossly inconsistent with [13]. In fact, typical constraints on dimension five operators [13] derived from the existence of the high-energy cosmic rays appear to destroy any hopes for the CPT-odd baryogenesis, even if the scale of T_R is pushed all the way up to the Planck scale. It is easy to see, however, that this is not the case. If CPT-odd sources in the quark sector dominate over the lepton sources by a factor of 20–30, strong constraints on CPT violation might be avoided. If, for example, among the CPT-odd sources the right-handed up quark has the largest modification of its dispersion relation, the energetically favored process $p \to \Delta^{++}\pi^-$ allows the ultra high-energy cosmic rays to exist in the form of Δ^{++}, an option which cannot be observationally ruled out [13]. It is very important to observe that the *negative* sign of η_U suggested by BAU (22) is exactly the sign of η_U needed for $p \to \Delta^{++}\pi^-$ to happen at high energies. Nevertheless, the required size of η_U, $\eta_U \sim -(10^{-23}\text{–}10^{-22})$ GeV^{-1} appears to be in sharp conflict with the low-energy constraints [12,27], and at least 4 orders of magnitude tuning for dimension five sources is needed. This consideration shows an important complementarity between the astrophysical bounds on Lorentz violation and low-energy searches of the breakdown of rotational invariance.

We can extend our analysis to theories where CPT violation comes from operators of dimension seven, nine, etc., should for some contrived reasons lower-dimensional operators be absent. We note that to sufficient accuracy, the resulting BAU will be determined by the equilibrium lepton asymmetry at the freeze-out time, $\eta^{(7)}T_R^3$, $\eta^{(9)}T_R^5$, where $\eta^{(n)}$ parametrize the strengths of the higher-dimensional operators:

$$\mathcal{L} = \sum \eta^{(n)}_{\kappa\mu\ldots\nu} \bar{\psi} \gamma^\kappa \mathcal{D}^\mu \ldots \mathcal{D}^\nu \psi.$$

As before, the transmutation to lower-dimensional operators can be forbidden by the irreducibility of $\eta^{(n)}$ tensors.

The low-energy constraints on dimension seven and higher CPT-odd operators are totally irrelevant, as the possible influence on the nucleon spin is suppressed by extra power(s) of $(\Lambda_{\rm QCD}/\Lambda_{CPT})^2$. The constraints coming from the propagation of the high-energy cosmic rays are harder to avoid, as their relative strength scales down as $(E_{\rm max}/\Lambda_{CPT})^2$, where $E_{\rm max}$ is the maximal energy of the high-energy cosmic rays $E_{\rm max} \sim 10^{12}$ GeV. In fact, since the decoupling temperature T_R can only be marginally larger than 10^{12} GeV, the *CPT*-violating sources of dimension seven in the lepton sector allowed by the cosmic rays would not be able to produce the required size of the baryon asymmetry. However, the same loophole with the stability of Δ^{++} at high-energies exists for the dimension seven operators, and the right-handed up-quark *CPT* violation at the level of

$$\eta_U^{(7)} = -[(10^{17}\text{–}10^{18})\ \text{GeV}]^{-3} \tag{23}$$

results in the right magnitude of BAU while avoiding all experimental constraints on Lorentz and *CPT* violation.

V. DISCUSSION

We have seen that the presence of *CPT*-odd interactions is theoretically capable of replacing *two* of Sakharov's conditions of baryogenesis: nonconservation of *CP* symmetry and departure from thermodynamical equilibrium. The reason for this is that nonzero lepton (or baryon) asymmetry can develop even in thermal equilibrium if the *CPT*-violating shifts of dispersion relations for particles and antiparticles and fermion number violating processes are operative at the same time [4]. In this paper, we considered in detail the idea of leptogenesis driven by *CPT*-violating sources in the fermionic sector of the standard model. In this scenario, the generation of the $B - L$ number occurs at temperatures of about 10^{12}–10^{14} GeV, which results in a huge enhancement of the asymmetry as compared to the *CPT*-odd electroweak baryogenesis scenario, where $B - L = 0$ and the equilibrium value for $B + L$ is maintained until the electroweak breaking, $T \sim 100$ GeV. Consequently, the *CPT*-odd leptogenesis requires only trans-Planckian size of *CPT* violation, $\eta_i \sim 10^{-22}$–10^{-24} GeV^{-1}.

We believe that this is the minimal level of *CPT* violation required to reproduce the observable asymmetry. Lower levels of *CPT*-breaking may generate BAU only at the expense of raising the decoupling temperature for $B - L$ processes, to the range of the, e.g., GUT scale. Models with such a high initial temperature possess very serious cosmological problems of their own related to the overproduction of dangerous relics (monopoles, gravitinos), and are difficult to incorporate into inflation.

The most natural models of *CP*-odd leptogenesis require two heavy neutrino singlets to work. We have shown that one species is perfectly sufficient for the *CPT*-odd scenario. In fact, one could take even more conservative approach and associate the majorana masses of light neutrinos with the effective Lorentz-conserving interaction (5) without specifying its origin. The *CPT*-odd leptogenesis in this case will proceed exactly as described in the paper, as long as (5) remains unsuppressed at high energies. As a consequence of the reduced heavy sector, the connection to the phenomenology of light neutrinos becomes more direct. As shown, the rate of the lepton-number violating processes is directly proportional to the sum of the mass squared of all light neutrino species.

Confronting the predicted size of *CPT*-violation with the existing experimental and astrophysical constraints we find that both the low-energy precision searches of preferred directions and the astrophysical constraints derived from the existence of charged high-energy cosmic rays puts severe constraints on *CPT*-odd leptogenesis. The latter, being especially stringent, rules out a possibility of *CPT*-odd leptogenesis driven by η_l when $\eta_b = 0$. The inverse case, $\eta_l = 0$; $\eta_b \neq 0$ cannot be ruled out from the astrophysical considerations, as the bounds would not apply if e.g. the *CPT* violation is concentrated in the right-handed up-quark sector. In this case, however, one should expect sizable effects in the clock comparison experiments. Current sensitivity to such operators is at the level of 10^{-27} GeV^{-1}, and thus would require at least 4 orders of magnitude fine-tuning to make Eq. (22) evade the bounds.

The *CPT*-odd interactions that modify dispersion relations represent a relatively small subset of dimension five *CPT*-odd interactions [28]. Is it feasible that other operators could drive (baryo)leptogenesis while evading strong astrophysical and laboratory constraints? If physics responsible for *CPT* violation preserves supersymmetry, operators that modify dispersion relations are simply not allowed [29,30]. Instead, a different class of *CPT*-odd operators may appear:

$$\bar{L}\gamma_\mu L H^\dagger H, \qquad \bar{Q}\gamma_\mu Q H^\dagger H, \quad \text{etc.} \tag{24}$$

When the lepton or baryon number is calculated in equilibrium, such operators will create an effective chemical potential that grows with temperature, $\mu \sim T^2\zeta$, where ζ parametrizes the strength of *CPT* violation. The easiest way to see that is to consider the thermal field theory correlator between the baryon/lepton-number density and such *CPT*-odd operators. Inside a thermal loop, the Higgs field bilinear will produce T^2, and the scaling of the effective chemical potential with temperature will be exactly the same as in the case of η_i operators. Although operators (24) do not influence the propagation of the high-energy cosmic rays, they have a phenomenological "problem" of their own. Inside loops such operators create quadratic divergencies and generate dimension three *CPT*-odd operators proportional to the square of the ultraviolet cutoff. In the most UV-protected case, the role of this cutoff is assumed by the supersymmetric soft-breaking scale. Still, the strength of typical constraints is on the order of $10^{-10}M_{\rm Pl}^{-1}$ [30], making the scenario driven by Eq. (24) fine-tuned below 1 ppm level. Finally, what if *CPT*-violation is concentrated in the heavy right-handed neutrino sector? Phenomenology of such model was addressed in Ref. [31], where it was shown that loop effects reintroduce *CPT* violation in the sector of charged leptons. Upon integrating out heavy neutrino fields, one produces operators similar to Eq. (24), and therefore such possibility is also fine-tuned.

Our main conclusion is that the natural levels of *CPT*/Lorentz violation suggested by the *CPT*-odd (lepto)baryogenesis scenario are 10^{-3}–10^{-5} in the Planck mass units, which is well within the ranges already disfavored by the laboratory experiments and observations of the high-energy cosmic rays. This analysis relies on the spurion approach to *CPT* violation, which assumes that the strength of the *CPT*-odd source was essentially the same in the early Universe and today. It is of course conceivable that the dynamical effects could have been responsible for the *CPT* breaking at high temperatures, sourcing the baryogenesis, with relaxation of *CPT* sources to zero at the later stage [32].

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