Nuclear Physics B108 (1976) 265–274
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SUPERSYMMETRIC BPHZ RENORMALIZATION
(II). Supersymmetric extension of the pure Yang-Mills model

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Received 25 February 1976

We generalize the supersymmetric way of performing the BPHZ renormalization introduced in paper (I) to cases including massless fields. This is used to prove the Slavnov identity in the supersymmetric extension of the pure Yang-Mills theory.

1. Introduction

In a preceding paper [1], referred to as (I), we introduced a supersymmetric way of performing the BPHZ renormalization; we applied it to the renormalization of of the supersymmetric extension of QED. We want here to deal with the extension of a non-Abelian gauge theory, specifically the supersymmetric extension of the SU(2) Yang-Mills model. For such a model, involving massless fields, the BPHZ scheme is not available. However, Lowenstein and Zimmermann [2] have recently given a modified version of the BPHZ scheme adapted to theories involving massless fields.

This version is generalized here to the supersymmetric case exactly in the same way as the usual BPHZ scheme was extended in paper (I). In sect. 2 the tree approximation of the supersymmetric extension of the SU(2) Yang-Mills model is recalled. Sect. 3 deals with the renormalization scheme. Applying it, the Slavnov identity is proved in sect. 4.

2. Supersymmetric extension of the SU(2) Yang-Mills model in the classical approximation

Let us recall here the results given in [3,4]. The notations are those of [1,4].

$V(x, \theta, \bar{\theta})$ will denote a triplet of vector superfields V_i, $C_\pm(x, \theta, \bar{\theta})$ and $\bar{C}_\pm(x, \theta, \bar{\theta})$ the corresponding triplet of anticommuting Faddeev-Popov ghosts chiral superfields. From now on we shall use the notation $\tilde{\phi} \equiv (\phi \text{ or } \bar{\phi})$ for any object like $\theta, D, C_\pm$.

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Under a gauge transformation,

$$e^{gV} \to e^{-ig\Lambda^+} e^{gV} e^{ig\Lambda} , \tag{1}$$

where Λ is a triplet of chiral fields, Λ^+ its hermitian conjugate.

The infinitesimal version of (1) will be written:

$$\delta V = iR\Lambda - i\bar{R}\bar{\Lambda} , \tag{2}$$

with

$$R_i{}^j = \delta_i{}^j - \tfrac{1}{2}\epsilon_{ijk} V_k + O(V^2) , \qquad \bar{R}_i{}^j = \delta_i{}^j + \tfrac{1}{2}\epsilon_{ijk} V_k + O(V^2) . \tag{3}$$

The supersymmetric action consists of a gauge-invariant part, a gauge breaking part, and a Faddeev-Popov ($\phi\pi$) part:

$$I = I_{\text{inv.}} + I_{\text{gauge}} + I_{\phi\pi} ,$$

$$I_{\text{inv.}} = \int \mathrm{d}S \frac{1}{g^2} \operatorname{Tr} W^\alpha W_\alpha + \text{h.c.} , \tag{4}$$

with

$$W_\alpha = \bar{D}\bar{D}(e^{-gV} D_\alpha e^{gV}) ,$$

$$I_{\text{gauge}} = \frac{2}{\alpha} \int \mathrm{d}V \operatorname{Tr} DDV\bar{D}\bar{D}V ,$$

$$I_{\phi\pi} = \frac{2}{\alpha} \int \mathrm{d}V (R_{ij} C_{+j} - \bar{R}_{ij} \bar{C}_{+j})(\bar{D}\bar{D}\bar{C}_{-i} + DDC_{-i}) .$$

This action is invariant under the Slavnov transformations [4,5]

$$\delta V = \lambda(RC_+ - \bar{R}\bar{C}_+) \equiv \lambda Q ,$$

$$\delta C_+ = -\tfrac{1}{2}\lambda i\, C_+ \times C_+ \equiv \lambda P ,$$

$$\delta C_- = \lambda \bar{D}\bar{D} V ,$$

where λ is an anticommuting infinitesimal parameter and is also invariant under the discrete charge conjugation-type transformation

$$V \leftrightarrow -V , \quad C_\pm \leftrightarrow \pm\bar{C}_\pm , \quad D_\alpha \leftrightarrow \bar{D}_{\dot{\alpha}} . \tag{6}$$

Let us introduce external fields coupled to Q and P

$$I_{\text{ext}} = \int \mathrm{d}V \rho Q + \int \mathrm{d}S\, ZP + \int \mathrm{d}\bar{S}\, \bar{Z}\bar{P} . \tag{7}$$

The external fields ρ and Z have the following properties: ρ has dimension 1, is odd by charge conjugation, and carries the Faddeev-Popov charge -1; Z has dimension 1, is even by charge conjugation, and carries the Faddeev-Popov charge -2.

We are now in a position to write the Slavnov identity [4,5] for the proper Green functions $\Gamma(V, C_\pm, \rho, Z)$

$$\mathcal{S}(\Gamma) = \int \mathrm{d}V\left[-\frac{\delta\Gamma}{\delta V}\frac{\delta\Gamma}{\delta\rho} + V\left(\frac{\delta\Gamma}{\delta C_-}+\frac{\delta\Gamma}{\delta\bar{C}_-}\right)\right] + \int \mathrm{d}S\,\frac{\delta\Gamma}{\delta C_+}\frac{\delta\Gamma}{\delta Z} + \int \mathrm{d}\bar{S}\,\frac{\delta\Gamma}{\delta\bar{C}_+}\frac{\delta\Gamma}{\delta\bar{Z}} = 0\,. \tag{8}$$

The iterated Slavnov identity can be written:

$$\mathcal{S}^2(\Gamma) = \int \mathrm{d}V\frac{\delta\Gamma}{\delta\rho}\left(\frac{\delta\Gamma}{\delta C_-} + \frac{\delta\Gamma}{\delta\bar{C}_-}\right) = 0\,. \tag{9}$$

3. Renormalization of supergraphs

3.1. Power counting

Feynman rules superfields may be found in [4], where further references are given. For the model considered in this paper, the free propagators are

$$\Delta_{VV}(k,\theta_1,\bar\theta_1,\theta_2,\bar\theta_2) = \frac{-i\,e^{\bar\theta_1\gamma\theta_2 k}}{4^4(1-s)^2M^2}\left[\frac{1}{k^2+\alpha(1-s)^2M^2} - \frac{1}{k^2+(1-s)^2M^2}\right]$$
$$-\frac{i}{4^4}\theta_{12}\theta_{12}\bar\theta_{12}\bar\theta_{12}\left[\frac{1}{k^2+(1-s^2)M^2} + \frac{\alpha}{k^2+\alpha(1-s)^2M^2}\right]$$

$$\Delta_{C_+C_-}(k,\theta_1,\bar\theta_1,\theta_2,\bar\theta_2) = \frac{-i\,e^{\bar\theta_1\gamma\theta_2 k}\theta_{12}\theta_{12}}{2\cdot 4^3[k^2+\alpha(1-s)^2M^2]}\,,$$

$$\Delta_{\bar{C}_+\bar{C}_-}(k,\theta_1,\bar\theta_1,\theta_2,\bar\theta_2) = \frac{-i\,e^{\bar\theta_1\gamma\theta_2 k}\bar\theta_{12}\bar\theta_{12}}{2\cdot 4^3[k^2+\alpha(1-s)^2M^2]}\,,$$

$$\Delta_{C_\pm\bar{C}_\mp} = 0\,, \tag{10}$$

where

$$\theta_{ij} \equiv \theta_i - \theta_j\,,$$

$$\bar\theta_1\gamma^\mu\theta_2 \equiv \theta_1\sigma^\mu\bar\theta_2 - \theta_2\sigma^\mu\bar\theta_1\,,$$

$$\theta_1\theta_1 = \theta_1^\alpha\theta_{1\alpha}\,, \qquad \bar\theta_1\bar\theta_1 = \bar\theta_{1\dot\alpha}\bar\theta_1^{\dot\alpha}\,, \ldots$$

The s dependence of the denominators was implemented, following Lowenstein and Zimmerman [2], by adding to the free part of the action the Slavnov non-invariant mass term

$$-16(1-s)^2M^2\,\mathrm{Tr}\left[\int \mathrm{d}V\,V^2+\frac{2}{\alpha}\int \mathrm{d}S\,C_+C_- -\frac{2}{\alpha}\int \mathrm{d}\bar{S}\,\bar{C}_+\bar{C}_-\right]. \tag{11}$$

The physical value of s is $s = 1$. Before going any further let us remark that if α is not equal to one in the tree approximation, a term $1/k^4$ appears in the free propagator at $s = 1$ which brings infrared divergences which cannot be removed. The same occurs if α is not equal to one in the upper orders: then a bilinear term in V which is not $\int \mathrm{d}V V\Box V$ appears (for instance

$$\mathrm{d}VV\,\frac{\partial}{\partial\theta}\frac{\partial}{\partial\theta}\frac{\partial}{\partial\bar\theta}\frac{\partial}{\partial\bar\theta}V),$$

and such counterterms imply the same infrared divergences as a term $1/k^4$ in the propagator. So, from now on, we make the assumption that the only quadratic term in V in the effective Lagrangian is $\int \mathrm{d}VV\Box V$.

Note: No specific problem occurs due to trilinear couplings in V: charge conjugation invariance together with the SU(2) structure implies that they are at least proportional to the impulsion.

Let us consider a one-particle irreducible (1PI) graph Γ, and let us call L its number of loops, V its number of VV lines, C its number of C_+C_- and $\bar{C}_+\bar{C}_-$ lines, N_V its number of external legs $V, N_\pm$ its number of external legs $C_\pm$ and $\bar{C}_\pm$. Its vertices are of the

$$V\text{ type:}\qquad \int \mathrm{d}V(1-s)^{l_i}s^{l'_i}\mathcal{D}^{\mu_i}V^{t_i}\tilde{C}_+^{p_i^+}\tilde{C}_-^{p_i^-}\,,\qquad i=1,\dots,m_V\,,$$

$$S\text{ type:}\qquad \int \mathrm{d}\tilde{S}(1-s)^{p_i}s^{l'_i}\tilde{C}_+^{p_i^+}\tilde{C}_-^{p_i^-}\,,\qquad i=1,\dots,m_S\,, \tag{12}$$

where $\tilde{C}_\pm = C_\pm$ or $\bar{C}_\pm$, $\mathrm{d}\tilde{S} = \mathrm{d}S$ or $\mathrm{d}\bar{S}$ and so on, and $\mathcal{D}^{\mu_i}$ is a covariant differential operator of order μ_i acting on some fields inside the monomial.

We also consider non-integrated vertices ("normal products"), numbered by $j = 1, \dots, M_V(M_S)$ for the V type (S type) ones. Let us still define their ultraviolet (UV) and infrared (IR) dimensions by

$$d_i = l_i + l'_i + \tfrac{1}{2}\mu_i + p_i^+\,,\qquad r_i = l_i + \tfrac{1}{2}\mu_i + p_i^+\,. \tag{13}$$

The Feynman rules in configuration space for 1PI graphs are [4]:

(i) To each V line or ghost line from vertex i to vertex j attach the corresponding propagator.

(ii) To each external leg i of the V (S) type connected to a vertex j attach the supersymmetric "δ function" $\delta_V(i,j)$ ($\delta_{\tilde{S}}(i,j)$).

(iii) To each vertex i attach the differential operator $\mathcal{D}^{\mu_i}$ and the factor $(1-s)^{l_i}s^{l'_i}$.

(iv) Perform the vector or chiral integrations according to the type of the vertex.
These rules give the following structure for the integrand in momentum space, written in a shortened way:

$$I_\Gamma(p,k,\tilde{\theta}) = e^{E(p,\theta,\bar{\theta})} \sum_{n=0}^{n_{\max}} \tilde{\theta}^n I_{\Gamma,n}(p,k)\,, \tag{14}$$

where p, k summarized the external and internal momenta, $\tilde{\theta}$ stands for the θ_i's, $\bar{\theta}_i$'s associated to the external legs, $E(p, \theta, \bar{\theta})$ is an homogeneous polynomial of dimension 0 in the p_i's and $\tilde{\theta}_i$'s ($\tilde{\theta}$ being of dimensions $-\frac{1}{2}$), $\tilde{\theta}^n$ is a monomial of degree n (hence of dimension $-\frac{1}{2}n$) in the $\tilde{\theta}_{ij}$'s, and, finally,

$$n_{\max} = 4N_V + 2N_+ + 2N_- + 4M_V + 2M_S - 4 \tag{15}$$

is the maximum number of independent components of θ_{ij}, $\bar{\theta}_{ij}$, the V type (S type), of the external legs and of the normal products being taken into account.

The IR and UV superficial divergence degrees of the coefficients $I_{\Gamma,n}(p,k)$ are defined, as in ref. [7], by

$$\begin{aligned} r_n(\Gamma) &= \underline{\deg}_{s-1,p,k} I_{\Gamma,n}(p,k) + 4L\,, \\ d_n(\Gamma) &= \overline{\deg}_{s,p,k} I_{\Gamma,n}(p,k) + 4L\,, \end{aligned} \tag{16}$$

where $\underline{\deg}_x f(x)$, respectively $\overline{\deg}_x f(x)$, means the asymptotic power for x tending to zero, respectively to infinity. The UV divergence degree was computed in paper (I):

$$\begin{aligned} d_n(\Gamma) = 4 - 2N_V - N_+ - 2N_- &+ \sum_{i=1}^{m_V} (d_i - 2) \\ + \sum_{i=1}^{m_S} (d_i - 3) + \sum_{j=1}^{M_V+M_S} (d_j - 4) + \tfrac{1}{2}n\,. \end{aligned} \tag{17}$$

We calculate now the IR degree $r_0(\Gamma)$ of $I_{\Gamma,0}(p,k)$; since $\dim(\theta) = -\frac{1}{2}$, we shall have

$$r_n(\Gamma) = r_0(\Gamma) + \tfrac{1}{2}n\,. \tag{18}$$

By inspection of the propagators (10) and vertices (12), one finds

$$r_0(\Gamma) = 4L - 4V - 2C + \sum_{i=1}^{m_V+m_S+M_V+M_S} l_i + x\,; \tag{19}$$

x is the total dimension of the derivatives $\tilde{D}$ which remain after all θ_{ij}'s have been absorbed. Before the absorption of the θ_{ij}'s there were μ_i derivatives at each vertex, plus 4 (2) of them at each integrated vertex of the V (S) type; we had to count 2 θ_{ij}'s for each ghost line, 4 (2) of them for each external V ($\widetilde{C}_\pm$) leg. The dimension of $\widetilde{D}$ being $\frac{1}{2}$, we have thus

$$x = \tfrac{1}{2} \sum_{\text{verticles}} \mu_i + 2m_V + m_S - C - 2N_V - N_+ - N_- \,. \tag{20}$$

Using the topological relations

$$L = V + C - m_V - m_S - M_V - M_S + 1 \,, \tag{21}$$

$$\sum_{\text{vertices}} p_i^+ = C + N_- \,, \tag{22}$$

we find

$$r_0(\Gamma) = 4 - 2N_V - N_+ - 2N_- + \sum_{i=1}^{m_V} (r_i - 2)$$

$$+ \sum_{i=1}^{m_S} (r_i - 3) + \sum_{j=1}^{M_V+M_S} (r_i - 4) \,, \tag{23}$$

where the IR degree r_i of vertex i is defined by (13).

The IR (respectively UV) degree of divergence is not allowed to decrease (respectively increase) arbitrarily with the order of perturbation series. In our purely massless theory this restricts the couplings in the effective action to have IR and UV dimensions

$$r_i = d_i = 2 \,, \qquad \text{for } V \text{ type coupling} \,,$$

$$r_i = d_i = 3 \,, \qquad \text{for } S \text{ type coupling} \,. \tag{24}$$

Assigning to "normal products" effective IR and UV degrees ρ_i, δ_i restricted by

$$0 \leqslant \rho_i \leqslant r_i \,, \qquad \delta_i \geqslant d_i \,, \tag{25}$$

we define the effective degrees for $I_{\Gamma,n}(p,k)$:

$$\rho_n(\Gamma) = 4 - \mathcal{N}(\Gamma) + \sum_{\text{norm. prod.}} (\rho_i - 4) + \tfrac{1}{2}n \,,$$

$$\delta_n(\Gamma) = 4 - \mathcal{N}(\Gamma) + \sum_{\text{norm. prod.}} (\delta_i - 4) + \tfrac{1}{2}n \,, \tag{26}$$

$n = 0, \ldots, n_{\max}$ where $n_{\max}$ is given by (15) and

$$\mathcal{N}(\Gamma) = 2N_V + N_+ + 2N_- \,. \tag{27}$$

For the subtraction rules, which we define below, to give well-defined Green func tions in the Euclidean region it is sufficient that, due to Lowenstein [7], the following criterion be satisfied.

Let $\hat{\Gamma}$ be the augmented graph obtained from Γ by drawing special lines (q lines) which carry the external momenta p and which meet in a new internal vertex V_0. The propagator assigned to a q line is chosen θ independent, free of infrared singularity and decreasing fast enough, for p tending to infinity, in order that each 1PI subgraph γ containing the vertex V_0 has a negative $\delta_n(\gamma)$. Let $\{\gamma_k, k = 1, ..., f\}$ be a set of 1PI subgraphs of $\hat{\Gamma}$ such that the contracted graph $\overline{\Gamma} = \Gamma/\{\gamma_n\}$ contains no q line.

Let us now consider the contribution of the terms I_{γ_k, n_k} of the subgraphs γ_k to $I_{\hat{\Gamma},n}(p, k)$, $\rho_n(\overline{\Gamma})$ being the IR degree of the corresponding contracted integrand. Lowenstein's criterion, generalized for our purpose, consists of the fulfillment of the inequality:

$$\tau_n \equiv \rho_n(\overline{\Gamma}) + \sum_{k=1}^{f} \max\{0, \rho_{n_k}(\gamma_k)\} > 0 , \tag{28}$$

for any

$$n \in [0, n_{\max}(\Gamma)] , \qquad n_k \in [0, n_{\max}(\gamma_k)] .$$

Applying eq. (26) we get the relations:

$$\rho_n(\overline{\Gamma}) = 4 + \sum_{k=1}^{f} [\mathcal{N}(\gamma_k) - 4 - \tfrac{1}{2}n_k] + \tfrac{1}{2}n ,$$

$$\rho(\gamma_k) \geqslant 4 - \mathcal{N}(\gamma_k) + \tfrac{1}{2}n_k . \tag{29}$$

In the last equation the sign $\geqslant$ takes into account the fact that one of the γ_i contains the q lines which improve the counting. Then

$$\tau_n \geqslant \rho_n(\overline{\Gamma}) + \sum_{k=1}^{f} \rho(\gamma_k) \geqslant 4 + \tfrac{1}{2}n > 0 , \tag{30}$$

which proves the fulfillment of the criterion eq. (28).

3.2. Subtraction rules

Each term of the expansion (14) of the integrand of a 1PI subgraph γ will be subtracted according to the prescriptions of Lowenstein [7]. For this purpose we introduce, as in paper (I), operators τ_z^{ω} acting on functions $f(z, \theta)$, where $z = (z_1, ..., z_n)$ and $\theta = (\theta_1, ... \theta_m, \overline{\theta}_1, ..., \overline{\theta}_m)$, defined by the two properties

$$\tau_z^{\omega} f(z) = t_z^{\omega} f(z) ,$$

$$\tau_z^{\omega} \widetilde{\theta}_i f(z, \theta) = \widetilde{\theta}_i \tau_z^{\omega + 1/2} f(z, \theta) , \tag{31}$$

where t_z^{ω} is the Taylor expansion in z around $z = 0$ up to order ω, and is taken to be zero if $\omega < 0$.

The subtracted integrand $R_\Gamma(p, k, \theta)$ of a 1PI supergraph Γ is obtained from the unsubtracted one by applying to it Zimmermann's forest formula [8]; the subtraction operator T_γ, acting on the subintegrand associated to subgraph γ, is defined by

$$1 - T_\gamma = (1 - \tau^{\rho_0(\gamma)-1}_{p^\gamma, s^\gamma - 1})(1 - \tau^{\delta_0(\gamma)}_{p^\gamma, s^\gamma}) , \tag{32}$$

where p^γ, s^γ are the external momenta and s parameters of subgraph γ, and the IR and UV subtraction degrees $\rho_0(\gamma)$ and $\delta_0(\gamma)$ are given by (26). It is clear from the rules (31) that each term $I_{\gamma,n}(p^\gamma, k^\gamma)$ of the expansion (14) of $I_\gamma(p^\gamma, k^\gamma, \theta^\gamma)$ will be subtracted with the appropriate degrees $\rho_n(\gamma)$ and $\delta_n(\gamma)$.

Normal products $N^\rho_\delta[Q]$ are taken into account in the definitions above and the power counting formula (26), their IR and UV degrees ρ and δ being subjected to the restrictions (25) and the condition [7]

$$0 \leqslant \rho \leqslant \delta . \tag{33}$$

4. Slavnov identity

The proof of the Slavnov identity follows exactly [5]: due to the action principle we can write

$$\mathcal{S}(\Gamma) = \Delta\Gamma , \tag{34}$$

where Δ has dimension 3 if of V type, 4 if of chiral type, carries the Faddeev-Popov charge +1, and is even under charge conjugation. $\Delta\Gamma$ denotes the generating functional of the proper Green functions with insertions Δ. Δ can be decomposed in a part depending on the external fields:

$$\Delta^{\text{ext}} = \rho\Delta_\rho + Z\Delta_Z \tag{35}$$

and a part Δ^{int} which does not depend on these external fields.

In fact it is easy to see that $\Delta_Z = 0$ due to SU(2) invariance, the only possible term being $\int dS(C \times C)(C \times Z)$ + c.c., which is zero. Introducing a term $\beta\Delta$ in the Lagrangian, where β is an external field, Δ can be proved to satisfy the consistency condition:

$$\mathcal{S}^\Delta(\mathcal{L}) + \mathcal{S}(\Delta) + O(\hbar\Delta) = \mathcal{S}^2(R^\Delta) , \tag{36}$$

where R^Δ has the same dimension and quantum numbers as the Lagrangian. $\mathcal{S}^\Delta(\mathcal{L})$ is induced through the renormalization of $\mathcal{S}$ due to the term $\beta\Delta^{\text{ext}}$ in $\mathcal{S}$ which changes the ρ coupling.

$$\mathcal{S}^\Delta(\mathcal{L}) = \int dV \rho \frac{\delta Q}{\delta V} \Delta_\rho . \tag{37}$$

Let us write the most general possible form of $\rho\Delta_\rho$:

$$\rho\Delta_\rho = \int dV\{F_2^0(C_+ \times C_+ - \bar{C}_+ \times \bar{C}_+)\cdot\rho$$
$$+ \sum_{p\ \mathrm{even} \geq 2} [F_1^p[(C_+ \times \rho)\cdot V(C_+ \cdot V)V^{p-2} - (\bar{C}_+ \times \rho)\cdot V(\bar{C}_+ \cdot V)V^{p-2}]$$
$$+ F_2^p(C_+ \times C_+ - \bar{C}_+ \times \bar{C}_+)\cdot \rho\, V^p + F_3^p(C_+ \times C_+ - \bar{C}_+ \times \bar{C}_+)\cdot V(\rho\cdot V)V^{p-2}$$
$$+ G_1^p[(C_+ \times \rho)\cdot V(\bar{C}_+ \cdot V)V^{p-2} - (\bar{C}_+ \times \rho)\cdot V(C_+ \cdot V)V^{p-2}]]$$
$$+ \sum_{p\ \mathrm{odd}} [F^p[(C_+ \cdot \rho)(C_+ \cdot V) + (\bar{C}_+ \cdot \rho)(\bar{C}_+ \cdot V)]V^{p-1}$$
$$+ G^p[(C_+ \cdot \rho)(\bar{C}_+ \cdot V) - (\bar{C}_+ \cdot \rho)(C_+ \cdot V)]V^{p-1}]\}\,.$$

The shorthand notation $V^{2n} = (V\cdot V)^n$ is used.

At first we see that the first term is the naive variation of $F_2^0\rho(C_+ - \bar{C}_+)$ and so can be absorbed.

We shall now consider the terms which depend on ρ in the consistency condition recursively in V. Let us first remark that no derivative can appear in the ρ-dependent part of the left-hand side for dimensional reasons; then, due to the structure of $\mathcal{S}^2$ (eq. (9)), there is no contribution of the right-hand side to the terms in ρ. Let us suppose that we have eliminated any term in Δ_ρ containing a power of V less than m. Then the contribution of $\mathcal{S}^\Delta(\mathcal{L})$ starts at order m, but a part of the contribution of $\mathcal{S}(\Delta)$, due to the Slavnov variation of V, starts at order $m-1$.

The consistency condition reduces to

$$\mathcal{S}(\Delta_\rho^m) = O(V^m) + O(\hbar\Delta) + \text{terms not depending on } \rho\,. \tag{39}$$

It is straightforward to solve this equation using eq. (38). The result is:

$$F_3^m = 0\,,$$
$$F_1^m = -G_1^m = F_2^m m!\,,$$
$$F^m = -G^m\,, \tag{40}$$

which implies recursively in V that $\rho\Delta_\rho$ is a naive variation and can be absorbed.

After elimination of the external fields in the anomaly Δ, eqs. (34) and (36) reduce to:

$$\mathcal{S}(\Gamma) = \Delta^{\mathrm{int.}}\,, \qquad \mathcal{S}(\Delta^{\mathrm{int.}}) = \mathcal{S}^2(R^\Delta)\,. \tag{41}$$

Moreover, we restricted the terms of the Lagrangian with the UV and IR degree; specifically at $s = 1$ all the possible terms are of dimension 2 if of V type and of dimension 3 if of chiral type.

Then it comes from the general form of the equations of motion [6] that at $s = 1$ Δ is also of maximum dimension, i.e. dimension 3 for V type and 4 for chiral type. Moreover, if a quadratic term occurs, it has necessarily the form $V\Box(C^+ - \bar{C}^+)$ (a term of another form, for instance

$$V \frac{\partial}{\partial\theta}\frac{\partial}{\partial\theta}\frac{\partial}{\partial\bar{\theta}}\frac{\partial}{\partial\bar{\theta}}(C_+ + \bar{C}_+) ,$$

would yield infrared divergences).

From now on we enforce $s = 1$. As usual Δ^{int} can be decomposed in

$$\Delta^{\mathrm{int}} = \Delta^{\natural} + \Delta^{\flat} , \quad \mathcal{S}(\Delta^{\natural}) = 0 , \qquad \mathcal{S}(\Delta^{\flat}) \neq 0 . \tag{42}$$

The consistency condition implies that there exist possible counterterms for the Lagrangian such that $\Delta^{\flat} = \mathcal{S}\hat{\mathcal{L}} + O(\hbar\Delta)$, and $\Delta^{\flat}$ can be absorbed by modifying the effective Lagrangian.

The $\Delta^{\natural}$ part is constrained by $\mathcal{S}(\Delta^{\natural}) = 0$ and, therefore, $\mathcal{S}^2(\Delta^{\natural}) = 0$. It is easy to check that these conditions imply $\Delta^{\natural} = 0$, which proves the Slavnov identity

$$\mathcal{S}(\Gamma)\big|_{s=1} = 0 . \tag{43}$$

We want to thank T. Clark, J.H. Lowenstein and K. Sibold for fruitful discussions.

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