CPT violation and baryogenesis

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Abstract

We examine the effects on baryogenesis of spontaneous CPT violation in a string-based scenario. Under suitable circumstances, certain CPT-violating terms can produce a large baryon asymmetry at the grand-unified scale that reduces to the observed value via sphaleron or other dilution mechanisms. © 1997 Elsevier Science B.V.

1. Introduction

Mechanisms for generation of the baryon asymmetry of the Universe naturally link cosmology with ideas from particle physics. Simultaneous conditions that are sufficient for baryogenesis are the violation of baryon number, the violation of C and CP symmetries, and the existence of nonequilibrium processes [1]. These conditions can be met in a grand-unified theory (GUT) through the decay of heavy states at high energy [2–6], through the decay of states in supersymmetric or superstring-inspired models at somewhat lower energies [7–11], or via the thermalization of the vacuum energy of supersymmetric states [12]. The conditions can also be met in the electroweak model through sphaleron-induced transitions between inequivalent vacua above the electroweak phase transition [5,13]. Under suitable circumstances, such transitions can dilute baryon asymmetries generated at higher energies [14].

A mechanism is known by which certain string theories may spontaneously break CPT symmetry [15]. If CPT and baryon number are violated, a baryon asymmetry could arise in thermal equilibrium [16,17]. This mechanism for baryogenesis would have the advantage of being otherwise independent of C- and CP-violating processes, which in a GUT are typically contrived to match the observed baryon asymmetry and are unrelated to the experimentally measured CP violation in the standard model.

In this work, we investigate the consequences for baryogenesis of certain CPT-violating terms arising in a string-based framework. The basic effects are determined in Section 2, while dilution mechanisms are considered in Section 3. We show that the observed baryon asymmetry could be reproduced via this scenario.

2. CPT violation and baryogenesis

For definiteness, we assume the source of baryon-number violation is one or more processes mediated by heavy leptoquark bosons of mass M_X in a GUT, possibly supersymmetric. The details of this theory play no essential role in what follows. Baryon-number

violation in the early Universe from the leptoquarks is assumed to be negligible below some temperature T_D. However, we do *not* take $T_D \sim M_X$ *a priori*. Instead, we estimate the value of T_D needed to reproduce the observed baryon asymmetry via CPT-violating interactions. Verification that T_D is large and of order M_X therefore provides a consistency check.

We take the CPT-violating interactions to arise from a string-based scenario, via couplings between Lorentz tensors T and fermions χ, ψ in the low-energy four-dimensional effective lagrangian [15]. Suppressing Lorentz indices for simplicity, these have the schematic form $\mathcal{L}_I \supset \lambda M^{-k} T \cdot \overline{\psi} \Gamma (i\partial)^k \chi + \text{h.c.}$, where λ is a dimensionless coupling constant, M is a large mass scale (presumably within roughly an order of magnitude of the Planck mass), Γ denotes a gamma-matrix structure, and $(i\partial)^k$ represents the action of a four-derivative at order $k \geq 0$. The CPT violation appears when appropriate components of T acquire nonzero expectation values $\langle T \rangle$.

For simplicity, we limit the scope of the present work to the subset of these CPT-violating terms leading directly to a momentum- and spin-independent energy shift of particles relative to antiparticles. Terms of this type can produce effects in neutral-meson systems that could be observed in laboratory experiments [15,18]. These terms are diagonal in the fermion fields and involve expectation values $\langle T \rangle$ of only the zero components of T:

$$\mathcal{L}_I \supset \frac{\lambda \langle T \rangle}{M^k} \overline{\psi} (\gamma^0)^{k+1} (i\partial_0)^k \psi + \text{h.c.}. \tag{1}$$

Since no large CPT violation is observed in nature, the expectation $\langle T \rangle$ must be suppressed in the effective theory relative to the low-energy scale m_l. The suppression factor is presumably some (non-negative) power l of the ratio of the low-energy scale to M: $\langle T \rangle \sim (m_l/M)^l M$. Since each factor of $i\partial_0$ also acts to provide a low-energy suppression, the condition $k+l = 2$ determines the dominant terms [15]. In what follows, we consider the various values of k and l in turn.

In the context of baryogenesis, we assume each fermion ψ represents a standard-model quark of mass m_q and baryon number $1/3$. The energy splitting between a quark and its antiquark arising via Eq. (1) can be viewed as a contribution to an effective chemical potential μ that drives the production of baryon number in thermal equilibrium.

To begin, consider a CPT-violating coupling for a single quark field. The equilibrium phase-space distributions of quarks q and antiquarks $\bar{q}$ at temperature T are $f_q(p) = (1 + e^{(E-\mu)/T})^{-1}$ and $f_{\bar{q}}(p) = (1 + e^{(E+\mu)/T})^{-1}$, respectively, where p is the momentum and $E = \sqrt{m_q^2 + p^2}$. If g is the number of internal quark degrees of freedom, then the difference between the number densities of quarks and antiquarks is

$$n_q - n_{\bar{q}} = \frac{g}{(2\pi)^3} \int d^3 p \, [f_q(p) - f_{\bar{q}}(p)]$$

$$= \frac{g}{2\pi^2} \int_{m_q}^{\infty} dE \, E \sqrt{E^2 - m_q^2}$$

$$\times \left[\frac{1}{1 + e^{(E-\mu)/T}} - \frac{1}{1 + e^{(E+\mu)/T}} \right]. \tag{2}$$

The contribution to the baryon-number asymmetry per comoving volume is given by $(n_q - n_{\bar{q}})/3s$, where the entropy density s of relativistic particles is

$$s(T) = \frac{2\pi^2}{45} g_s(T) T^3 ,$$

$$g_s(T) = \sum_B g_B \left(\frac{T_B}{T} \right)^3 + \frac{7}{8} \sum_F g_F \left(\frac{T_F}{T} \right)^3 . \tag{3}$$

In this expression, the number of degrees of freedom of relativistic bosons B and fermions F forming the plasma are taken to be g_B and g_F, respectively. Their component temperatures are denoted T_B and T_F, to allow for possible decoupled particles. The photon and quark gases have the same temperature T.

Consider first the case $k = 0$ with $l = 2$. This generates via Eq. (1) an effective chemical potential of $\mu \sim m_l^2/M \simeq 10^{-17} m_l$. Substitution into Eq. (2) and use of the condition $\mu \ll T$, which holds for any reasonable decoupling temperature T_D, gives a contribution to the baryon number per comoving volume of

$$\frac{n_q - n_{\bar{q}}}{3s} \sim \frac{15g}{2\pi^4 g_s(T)} \frac{\mu}{T} I_0(m_q/T) , \tag{4}$$

where

$$I_0(r) = \int_r^{\infty} dx \, x \sqrt{x^2 - r^2} e^x (1 + e^x)^{-2} . \tag{5}$$

The integral obeys the condition $I_0(r) < I_0(0) = \pi^2/6$.

With two spins and three colors, $g = 6$ for a given quark flavor. The result (4) applies for each flavor. In GUT models, $g_s \gtrsim 10^2$ for $T \gtrsim 100$ MeV. Disregarding possible cancellations among contributions from different flavors, the net baryon number per comoving volume produced in this way with three generations of standard-model particles is therefore $n_B/s \sim (10^{-2}\mu/T)I_0(m_q/T) \sim (10^{-19}m_l/T)I_0(m_q/T)$. This is far too small to reproduce the observed value $n_B/s \simeq 10^{-10}$. Note that choices of $l \geq 3$ would produce even smaller values. We can therefore exclude baryogenesis with standard-model quarks via $k = 0$ CPT-violating couplings.

Consider next the cases with $k \geq 1$. These have CPT-violating couplings of the type in Eq. (1) involving at least one time derivative. In thermal equilibrium, it is a good approximation to replace each time derivative with a factor of the associated quark energy. This produces energy-dependent contributions to the effective chemical potential, given by

$$\mu \sim \left(\frac{m_l}{M}\right)^l \frac{E^k}{M^{k-1}} . \tag{6}$$

Using Eq. (2), we find that each quark generates a contribution to the baryon number per comoving volume of

$$\frac{n_q - n_{\bar{q}}}{3s} \sim \frac{15g}{4\pi^4 g_s(T)} I_k(m_q/T) , \tag{7}$$

where

$$I_k(r) = \int\limits_r^\infty dx\, x \sqrt{x^2 - r^2}\, \frac{\sinh(\lambda_k x^k)}{\cosh x + \cosh(\lambda_k x^k)} \tag{8}$$

and

$$\lambda_k = \left(\frac{m_l}{M}\right)^l \left(\frac{T}{M}\right)^{k-1} . \tag{9}$$

If $k = 1$, the dominant contribution arises when $l = 1$. Then, $\lambda_1 = m_l/M \ll 1$ and we have

$$I_1(r) \approx \frac{m_l}{M} \int\limits_r^\infty dx\, \frac{x^2 \sqrt{x^2 - r^2}}{1 + \cosh x} . \tag{10}$$

It can be shown that $I_1 < 12m_l/M$. This means that the contribution to n_B/s from the $k = 1$ terms is again too small to reproduce the known baryon asymmetry.

If $k \geq 2$, the dominant contribution appears when $l = 0$. This gives $\lambda_k = (T/M)^{k-1}$. Assuming the decoupling temperature T_D is well below the scale M, the integral I_k has integrand peaking near $x \sim 1$ and exponentially suppressed in the region $1 \ll x < M/T$. It diverges for $x > M/T$. Physically, different values of x allow for contributions of fermions of different energies $E = xT$ to the processes generating baryon number. The divergence of the integrals for $E > M$ is evidently an unphysical artifact of the low-energy approximation. Since few particles have energy near M at temperatures much less than M, the integrands can safely be truncated above the region $T \ll E < M$. The integrals become

$$I_k(r) \approx \left(\frac{T}{M}\right)^{k-1} \int\limits_r^\infty dx\, \frac{x^{k+1}\sqrt{x^2 - r^2}}{1 + \cosh x} . \tag{11}$$

This shows that baryogenesis is more suppressed as k increases from the value $k = 2$.

If $k = 2$ is assumed, then $\lambda_2 = T/M$. A good estimate of the value of the integral $I_2(m_q/T)$ can be obtained by setting m_q/T to zero, since the fermion mass either vanishes or is much smaller than the decoupling temperature T_D. We obtain $I_2(m_q/T) \approx I_2(0) \simeq 7\pi^4 T/15M$. Combining this with Eq. (7) produces for six quark flavors a baryon asymmetry per comoving volume given by

$$\frac{n_B}{s} \sim \frac{21g}{2g_s(T)} \frac{T}{M} \simeq \frac{3}{5} \frac{T}{M} . \tag{12}$$

For an appropriate value of the decoupling temperature T_D, it follows that the observed baryon asymmetry can be matched provided the interactions violating baryon number are still in thermal equilibrium at this temperature. In estimating the value of T_D, the effects of dilution mechanisms must be taken into account. We do this in the next section. Note that for $k \geq 3$ the extra suppression by powers of T/M further raises the decoupling temperature T_D required.

3. Dilution mechanisms

A potentially important source of baryon-asymmetry dilution is the occurrence of sphaleron transitions, which violate baryon number. These processes are

expected to be unsuppressed at temperatures above the electroweak phase transition [13].

Denote the total baryon- and lepton-number densities by B and L, respectively. We assume that the GUT conserves the quantity $B - L$. Sphaleron-induced baryon-asymmetry dilution occurs when $B - L$ vanishes [14]. The dilution can be estimated by calculating the expectation of the baryon number density using standard model fields in thermal equilibrium at the temperature T_S where the sphaleron transitions freeze out.

Consider N generations of quarks with masses m_{q_i} and leptons with masses m_{l_i}, $i = 1, \ldots, N$. The free energy in a unit volume for the system in equilibrium at temperature T is given by

$$\mathcal{F} = 6 \sum_{i=1}^{2N} F(m_{q_i}, \mu) + \sum_{i=1}^{N} [2F(m_{l_i}, \mu_i) + F(0, \mu_i)] \,, \tag{13}$$

where the parameters μ and μ_i are the chemical potentials of the quarks and the ith lepton, respectively. Note that these are true chemical potentials here, unlike the effective chemical potential used in the preceding section. In the expression (13), the free energy in a unit volume for each constituent fermion field of mass m and chemical potential μ is given by the standard expression

$$F(m, \mu) = -T \int \frac{d^3k}{(2\pi)^3} \\ \times \left[\ln(1 + e^{-(E-\mu)/T}) + (\mu \to -\mu) \right] \,, \tag{14}$$

where E is the energy of a fermion with momentum k.

Sphaleron transitions preserve the N quantities $L_i = l_i - N^{-1}B$, where the individual lepton-number densities are denoted l_i. In thermal equilibrium, this leads to the relation $\mu = -\sum_i \mu_i/3N$. Since the sphaleron freeze-out temperature T_S is larger than any fermion mass, the free energy in a unit volume can be well approximated by

$$F(m, \mu) \approx F(m, 0) - \frac{1}{12} \mu^2 T^2 \left(1 - \frac{3}{2\pi^2} \frac{m^2}{T^2} \right) \,. \tag{15}$$

The conserved number densities L_i are therefore given by

$$L_i = -\frac{\partial \mathcal{F}}{\partial \mu_i} \approx -\frac{\mu T^2}{3N} \alpha + \frac{\mu_i T^2}{2} \beta_i \,, \tag{16}$$

where

$$\alpha \equiv 2N - \frac{3}{2\pi^2} \sum_{i=1}^{2N} \frac{m_{q_i}^2}{T^2} \,, \quad \beta_i \equiv 1 - \frac{1}{\pi^2} \frac{m_{l_i}^2}{T^2} \,. \tag{17}$$

Solving for μ_i and summing over i leads to the expression

$$\mu = -\frac{6}{T^2} \left(\sum_{i=1}^{N} \frac{L_i}{\beta_i} \right) \left(9N + \frac{2\alpha}{N} \sum_{j=1}^{N} \frac{1}{\beta_j} \right)^{-1} \,. \tag{18}$$

Since each quark carries baryon number $1/3$, the expectation of baryon density is [14]

$$B = -2 \sum_{i=1}^{2N} \frac{\partial F(m_{q_i}, \mu)}{\partial \mu}$$

$$= -2\alpha \left(\sum_{i=1}^{N} \frac{L_i}{\beta_i} \right) \left(9N + \frac{2\alpha}{N} \sum_{j=1}^{N} \frac{1}{\beta_j} \right)^{-1}$$

$$\approx \begin{cases} -\dfrac{4}{13\pi^2} \displaystyle\sum_{i=1}^{N} L_i \dfrac{m_{l_i}^2}{T^2} \,, & B - L = 0 \,, \\ \dfrac{4}{13} (B - L) \,, & B - L \neq 0 \,. \end{cases} \tag{19}$$

In the last step, only the leading-order contribution has been kept.

Consider first the case where $B - L = 0$ initially. Taking the leptoquark decays to be dominated by the heaviest lepton of mass m_L [14], it follows from Eq. (19) that the baryon- and lepton-number densities are diluted through sphaleron effects by a factor of approximately $4(N - 1)m_L^2/13\pi^2 N T_S^2$. Combining this result with Eq. (12) produces at the present epoch a net contribution from three generations to the magnitude of the baryon-number asymmetry per comoving volume of

$$\frac{n_B}{s} \sim \frac{28g}{13\pi^2 g_s(T_D)} \frac{m_L^2 T_D}{T_S^2 M} \,. \tag{20}$$

Taking the heaviest lepton to be the tau and the freeze-out temperature T_S to be the electroweak scale, this means the baryon asymmetry produced via GUT processes is diluted by a factor of about 10^{-6}. Thus, the

observed value of the baryon asymmetry can be re-produced if, in a GUT model conserving $B - L$ with $B - L = 0$ initially, baryogenesis takes place via $k = 2$ CPT-violating terms at a decoupling temperature $T_D \simeq 10^{-4}M$, followed by sphaleron dilution. This value of T_D is close to the GUT scale and leptoquark mass M_X, as is required for consistency.

Note that in obtaining Eq. (20) we have used the estimate of the baryon asymmetry obtained in Eq. (12) of section 2, which neglects any possible effects from sphalerons occurring at the GUT scale. The sphaleron transition rate at high temperatures T is $\Gamma \approx \alpha_W^4 T^4$ [19], where α_W is the electroweak coupling constant. This implies that the rate of baryon-number violation exceeds the expansion rate of the Universe

$$H \approx \sqrt{g_s} T^2 / M \qquad (21)$$

for temperatures below $\alpha_W^4 M \simeq 10^{12}$ GeV [20]. Sphaleron effects at the GUT scale can therefore safely be disregarded.

If instead we examine the case $B - L \neq 0$, Eq. (19) shows that essentially no sphaleron dilution occurs. A less attractive possibility then could be countenanced: baryogenesis at T_D as above, but with the asymmetry introduced via initial conditions. In this case dilution might occur through other mechanisms, such as the decay of the dilaton in string theories [21,22].

If Eq. (20) is to hold, then at the GUT scale the leptoquark interactions that violate baryon number must still be in thermal equilibrium with respect to the expansion rate of the Universe. Suppose baryon number is violated via (direct and inverse) leptoquark decays and scattering, occurring with gauge-coupling strength α_X. Then, the rates Γ_D for decay and Γ_S for scattering at temperature $T > M_X$ are (see, for instance, ref. [4]):

$$\Gamma_D \approx g_s \frac{\alpha_X M_X^2}{\sqrt{T^2 + M_X^2}}, \quad \Gamma_S \approx g_s \frac{\alpha_X^2 T^5}{(T^2 + M_X^2)^2}. \qquad (22)$$

These rates are to be compared with the expansion rate H of the Universe, Eq. (21). With the above decoupling temperature T_D and a reasonable coupling α_X, both Γ_D / H and Γ_S / H exceed one and so the decay and scattering processes are indeed in thermal equilibrium at the GUT scale.

As an aside, we remark that for $k = 2$ the decoupling temperature T_D is low enough for baryogenesis

to be compatible with primordial inflationary models of the chaotic type and possibly also with new inflationary models. The examples given in refs. [23,24] are also consistent with COBE bounds on the primordial energy-density fluctuations and with the upper bound on the reheating temperature that avoids the overproduction of gravitinos [24,25].

4. Summary

In this work, we have explored the possibility that baryogenesis involves spontaneous CPT breaking arising in a string-based framework. In the presence of interactions that violate baryon number, the CPT-breaking terms with $k = 2$ appearing in Eq. (1) can generate a large baryon asymmetry with the Universe in thermal equilibrium at the GUT scale. If the interactions preserve $B - L = 0$, the subsequent sphaleron dilution reproduces the observed value of the baryon asymmetry.

Acknowledgements

References

[1] A.D. Sakharov, JETP Lett. 5 (1967) 24.
[2] M. Yoshimura, Phys. Rev. Lett. 41 (1978) 281; Phys. Lett. B 88 (1979) 294.
[3] A.Yu. Ignatiev, N.V. Krasnikov, V.A. Kuzmin and A.N. Tavkhelidze, Phys. Lett. B 76 (1978) 436.
[4] S. Weinberg, Phys. Rev. Lett. 42 (1979) 850.
[5] A.D. Linde, Phys. Lett. B 70 (1977) 306.
[6] S. Dimopoulos and L. Susskind, Phys. Rev. D 18 (1978) 4500.
[7] M. Claudson, L.J. Hall and I. Hinchliffe, Nucl. Phys. B 241 (1984) 309.
[8] K. Yamamoto, Phys. Lett. B 168 (1986) 341.
[9] O. Bertolami and G.G. Ross, Phys. Lett. B 183 (1987) 163.
[10] A. Cline and S. Raby, Phys. Rev. D 43 (1991) 1781.
[11] S. Mollerach and S. Roulet, Phys. Lett. B 281 (1992) 303.

[12] I. Affleck and M. Dine, Nucl. Phys. B 249 (1985) 361.

[13] V.A. Kuzmin, V.A. Rubakov and M.E. Shaposhnikov, Phys. Lett. B 155 (1985) 36.

[14] V.A. Kuzmin, V.A. Rubakov and M.E. Shaposhnikov, Phys. Lett. B 191 (1987) 171.

[15] V.A. Kostelecký and R. Potting, Nucl. Phys. B 359 (1991) 545; Phys. Rev. D 51 (1995) 3923; Phys. Lett. B 381 (1996) 389.

[16] A.D. Dolgov and Ya.B. Zeldovich, Rev. Mod. Phys. 53 (1981) 1.

[17] A.G. Cohen and D.B. Kaplan, Phys. Lett. B 199 (1987) 251; Nucl. Phys. B 308 (1988) 913.

[18] D. Colladay and V.A. Kostelecký, Phys. Lett. B 344 (1995) 259; Phys. Rev. D 52 (1995) 6224;

V.A. Kostelecký and R. Van Kooten, Phys. Rev. D 54 (1996) 5585.

[19] J. Ambjorn and A. Krasnitz, Phys. Lett. B 362 (1995) 97.

[20] V.A. Rubakov and M.E. Shaposhnikov, Uspekhi Fiz. Nauk 166 (1996) 493 (hep-ph/9603208).

[21] M. Yoshimura, Phys. Rev. Lett. 66 (1991) 1559.

[22] M.C. Bento, O. Bertolami and P.M. Sá, Mod. Phys. Lett. A 7 (1992) 911.

[23] A.S. Goncharov and A.D. Linde, Phys. Lett. B 139 (1984) 27; Class. Quantum Gravity 1 (1984) L75.

[24] G.G. Ross and S. Sarkar, Nucl. Phys. B 461 (1996) 597.

[25] M.C. Bento and O. Bertolami, Phys. Lett. B 365 (1996) 59.