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Invariant Regularization of Supersymmetric Chiral Gauge Theory

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We formulate a manifestly supersymmetric gauge covariant regularization of supersymmetric chiral gauge theories. In our scheme, the effective action in the superfield background field method *above* one-loop is always supersymmetric and gauge invariant. The gauge anomaly has a covariant form and can emerge only in one-loop diagrams with all the external lines being the background gauge superfield. We also present several illustrative applications in the one-loop approximation: the self-energy part of the chiral multiplet and of the gauge multiplet; the super-chiral anomaly and the superconformal anomaly; as the corresponding anomalous commutators, the Konishi anomaly and an anomalous supersymmetric transformation law of the supercurrent (the "central extension" of $N = 1$ supersymmetry algebra) and of the R-current.

§1. Introduction

Obtaining a regularization scheme which is invariant under preferred symmetries (such as the gauge symmetry) is important. As a matter of principle, the existence of such a regularization directly shows that these symmetries have no intrinsic quantum anomaly. Even at the practical level, use of such a regularization (at least conceptually) simplifies various calculations, because it avoids the introduction of non-invariant counter-terms necessary to recover the Ward-Takahashi identity. Such a regularization also automatically leads to correct physical predictions in view of the preferred or imposed symmetries; a well-known example is the anomalous divergence of the axial current (the Adler-Bell-Jackiw anomaly) which would not appear without imposing the gauge invariance.

In this paper, we formulate an invariant regularization of supersymmetric *chiral* gauge theories. As a recent review of the issues relating to regularization of supersymmetric theories, see Ref. 1) and references cited therein. For definiteness, we set up a regularization scheme for the effective action in the background field method.[2)-4)] Once this is done, S-matrix elements can also be constructed.[3)] Although our scheme is perturbative in nature, it possesses the following properties. (1) It manifestly preserves supersymmetry at every step, being formulated in terms of the superfield[5),6)] in an exactly four-dimensional spacetime. In particular, there is no ambiguity associated with γ_5 and the totally anti-symmetric tensor, unlike the dimensional reduction.[7)] In this respect, our scheme is similar to the supersymmetric

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higher covariant derivative regularization.[8] However our scheme regularizes one-loop diagrams as well. (2) It is manifestly gauge covariant, unlike the conventional Pauli-Villars regularization (for the Pauli-Villars regularization in supersymmetric theories, see Ref. 9)); it is related to the generalized Pauli-Villars regularization.[10] Here, by "gauge covariance", we mean covariance under the background gauge transformation[6] and not under the BRST transformation. In fact, in our scheme, the effective action *above* one-loop is always gauge invariant, and a possible breaking of the gauge symmetry due to the gauge anomaly[11] can emerge only in one-loop diagrams in which all the external lines are the background gauge superfield. The one-loop diagrams on the other hand are regularized gauge covariantly.[12,13] Hence the gauge anomaly has a covariant form.[12-15] When the gauge representation is free of the gauge anomaly, however, the gauge invariance is restored and our scheme as it stands provides a gauge invariant regularization.

The organization of this paper is as follows. In §2, the superfield background field method is summarized in a somewhat different representation from Ref. 6). This representation is more suitable for our purpose. On the basis of this method, we diagonalize a part of the action which is quadratic in quantum fields in the presence of the background gauge field. This gives us a "partially diagonalized" propagator of quantum fields (the first half of §3). In the second half of §3, our regularization scheme is formulated by using the propagators thus obtained. It is explained how the supersymmetry and the background gauge invariance or covariance are respected in the process of regularization. Sections 4 and 5 are devoted to illustrative applications in the one-loop approximation. In §4, we present a somewhat detailed evaluation of the two-point one particle irreducible (1PI) functions, the self-energy part of the chiral multiplet and of the gauge multiplet (the vacuum polarization tensor). In §5, the super-chiral anomaly[15-17] and the superconformal anomaly[18-22] are evaluated. Since our scheme also provides a supersymmetric gauge covariant definition of composite operators, an evaluation of anomalies in the form of an anomalous supersymmetric transformation law[23-25] as well as in the form of current non-conservation is straightforward and transparent. Section 6 is devoted to a conclusion. Details concerning the evaluation of anomalous factors are summarized in the Appendix.

Our convention is basically that of Ref. 26), unless otherwise stated. In particular, the signature of the metric is $(-+++)$. For simplicity of presentation, we assume the gauge representation R of the chiral multiplet, which will be denoted as T^a, is irreducible. The normalization of the gauge generator is $[T^a, T^b] = it^{abc}T^c$, $\mathrm{tr}\, T^aT^b = T(R)\delta^{ab}$, $(T^a)_{ij}(T^a)_{jk} = C(R)\delta_{ik}$ and $t^{acd}t^{bcd} = C_2(G)\delta^{ab}$.

§2. Superfield background field method

We set up our scheme on the basis of the background field method.[2-4] The reason is that the background field method allows us to treat the gauge field and the matter field on an equal footing in view of the gauge covariance. Moreover, to make the supersymmetry manifest, we utilize the superfield background field method. Its basic framework is presented in Ref. 6) (see also Ref. 27)). However, in view of the "full-chiral" representation[26] that is commonly used, it is rather convenient to work directly with the following "quantum-chiral background-chiral" representation. This representation differs from the quantum-chiral background-vector representation adapted in Ref. 6). Therefore, it seems helpful to summarize this representation while explaining our original notation.

We consider the most general renormalizable supersymmetric model, whose classical action is given by[*), 26)]

$$S=\frac{1}{2T(R)}\int d^6z\ \mathrm{tr}\,W^\alpha W_\alpha+\int d^8z\,\Phi^\dagger e^V\Phi+\int d^6z\left(\frac{1}{2}\Phi^T m\Phi+\frac{1}{3}g\Phi^3\right)+\text{h.c.} \tag{2·1}$$

In our quantum-chiral background-chiral representation,[**)] the vector and the chiral superfields are split as

$$e^V=e^{V_B}e^{V_Q},\qquad \Phi=\Phi_B+\Phi_Q. \tag{2·2}$$

Here, the subscripts B and Q represent the background field and the quantum field, respectively. We shall regard V_B as a vector superfield, and thus V_Q is *not* a vector superfield. Instead, its conjugate is given by

$$V_Q^\dagger=e^{V_B}V_Qe^{-V_B}. \tag{2·3}$$

In this quantum-chiral background-chiral representation, the original gauge transformation[26]

$$e^{V'}=e^{-i\Lambda^\dagger}e^Ve^{i\Lambda},\qquad \Phi'=e^{-i\Lambda}\Phi, \tag{2·4}$$

where $\Lambda=T^a\Lambda^a$ is a chiral superfield $\overline{D}_{\dot\alpha}\Lambda=0$, is realized in the following two different ways. (i) By the quantum field transformation:

$$V_B'=V_B,\qquad e^{V_Q'}=\left(e^{-V_B}e^{-i\Lambda^\dagger}e^{V_B}\right)e^{V_Q}e^{i\Lambda},\qquad \Phi'=e^{-i\Lambda}\Phi. \tag{2·5}$$

(ii) By the background field transformation:

$$e^{V_B'}=e^{-i\Lambda^\dagger}e^{V_B}e^{i\Lambda},\qquad V_Q'=e^{-i\Lambda}V_Qe^{i\Lambda},\qquad \Phi'=e^{-i\Lambda}\Phi. \tag{2·6}$$

In both transformations, the gauge parameter Λ is simply a *chiral* superfield, whence the name of the representation.

Next, we introduce the background covariant spinor derivative symbol:

$$\nabla_\alpha\equiv e^{-V_B}D_\alpha e^{V_B}\quad\text{and}\quad \overline{D}_{\dot\alpha}. \tag{2·7}$$

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*) The generic coordinate of the superspace is denoted as $z=(x^m,\theta^\alpha,\overline{\theta}_{\dot\alpha})$. The full superspace integration measure and the chiral superspace measure are abbreviated as $d^8z=d^4xd^2\theta d^2\overline{\theta}$ and $d^6z=d^4xd^2\theta$, respectively.

**) The quantum fields V_Q in this representation and V^Q in the quantum-chiral background-vector representation[6)] are related as $V_Q=e^{-g\overline{W}^B}gV^Qe^{g\overline{W}^B}$, where W^B is the background field in Ref. 6). Our background field V_B is given by $e^{V_B}=e^{gW^B}e^{g\overline{W}^B}$.

Since the gauge parameter Λ in (2·6) is chiral, both of these operators transform as $\nabla' = e^{-i\Lambda}\nabla e^{i\Lambda}$ under the background field transformation. The vector covariant derivative symbol is also defined by the anti-commutator:

$$\{\nabla_\alpha, \overline{D}_{\dot\alpha}\} \equiv -2i\sigma^m_{\alpha\dot\alpha}\nabla_m. \tag{2·8}$$

Then, on the gauge representation R, the covariant derivative is defined by

$$\mathcal{D}_\alpha \Phi \equiv \nabla_\alpha \Phi, \qquad \mathcal{D}_m \Phi \equiv \nabla_m \Phi, \tag{2·9}$$

where Φ is a generic field in the representation R. Clearly these operations have a covariant meaning under the background field transformation (2·6). On the other hand, the quantum field V_Q transforms as the adjoint representation under the background transformation (2·6). In such a representation, the covariant derivative is defined by

$$\mathcal{D}_\alpha V \equiv [\nabla_\alpha, V\}, \qquad \mathcal{D}_m V \equiv [\nabla_m, V], \tag{2·10}$$

where a(n) (anti-)commutator is used when V is Grassmann-even(-odd). It is again clear that (2·10) has a background gauge covariant meaning.

Expressions become even simpler with use of the adjoint gauge representation matrix, which is defined by

$$(\mathcal{T}^a)^{bc} \equiv -it^{abc}, \qquad \mathrm{tr}\, \mathcal{T}^a\mathcal{T}^b = C_2(G)\delta^{ab}. \tag{2·11}$$

With this convention, the covariant derivative in the adjoint representation (2·10) can be written as

$$\mathcal{D}_\alpha V = T^a(\widetilde{\nabla}_\alpha V)^a, \qquad \mathcal{D}_m V = T^a(\widetilde{\nabla}_m V)^a, \tag{2·12}$$

where a component of the covariant derivative is defined by

$$(\widetilde{\nabla}_\alpha V)^a \equiv (e^{-\mathcal{V}_B})^{ab} D_\alpha (e^{\mathcal{V}_B})^{bc} V^c, \qquad \{\widetilde{\nabla}_\alpha, \overline{D}_{\dot\alpha}\} = -2i\sigma^m_{\alpha\dot\alpha}\widetilde{\nabla}_m, \tag{2·13}$$

and $\mathcal{V}_B$ is the background gauge superfield in the adjoint representation:

$$\mathcal{V}_B \equiv \mathcal{T}^a V_B^a. \tag{2·14}$$

The similarity of (2·13) and the covariant derivative (2·7) and (2·9) is obvious.

Now the essence of the background field method [2)-4)] is to use the gauge fixing condition which is covariant under the background gauge transformation (2·6). Therefore, as usual, we impose the Lorentz-type gauge fixing condition and its conjugate:

$$\overline{D}^2 V_Q = f, \qquad \mathcal{D}^2 V_Q = e^{-V_B} f^\dagger e^{V_B}. \tag{2·15}$$

Note that the gauge fixing function f is a chiral superfield: $\overline{D}_{\dot\alpha} f = 0$. Then the standard procedure [28)] gives rise to the gauge fixing term and the ghost-anti-ghost term:

$$S' = -\frac{\xi}{8T(R)} \int d^8z\, \mathrm{tr}(\overline{D}^2 V_Q)(\mathcal{D}^2 V_Q)$$

$$+\frac{1}{T(R)}\int d^8z\ \mathrm{tr}(e^{-V_B}c'^\dagger e^{V_B}+c')$$
$$\times\mathcal{L}_{V_Q/2}\cdot\left[(c+e^{-V_B}c^\dagger e^{V_B})+\coth(\mathcal{L}_{V_Q/2})\cdot(c-e^{-V_B}c^\dagger e^{V_B})\right]$$
$$+\frac{1}{T(R)}\int d^8z\ \mathrm{tr}\, e^{-V_B}b^\dagger e^{V_B}b, \qquad (2\cdot16)$$

where ξ is the gauge parameter. By construction, this action is invariant under the background field transformation, (2·6) and $b'=e^{-i\Lambda}be^{i\Lambda}$ etc. Note that, since the parameter Λ of the quantum field transformation (2·5) is chiral and the gauge fixing function f in (2·15) is also chiral, all the ghost c, anti-ghost c', and Nielsen-Kallosh ghost b[29] are simply chiral superfields: $\overline{D}_{\dot\alpha}c=\overline{D}_{\dot\alpha}c'=\overline{D}_{\dot\alpha}b=0$.

To carry out perturbative calculations, we expand the total action $S_T\equiv S+S'$ in powers of the quantum fields as $S_T=S_{T0}+S_{T1}+S_{T2}+S_{T3}+\cdots$. The zeroth order action has the same form as the classical action S,

$$S_{T0}=\frac{1}{2T(R)}\int d^6z\ \mathrm{tr}\, W_B^\alpha W_{B\alpha}+\int d^8z\,\Phi_B^\dagger e^{V_B}\Phi_B$$
$$+\int d^6z\left(\frac{1}{2}\Phi_B^T m\Phi_B+\frac{1}{3}g\Phi_B^3\right)+\text{h.c.}, \qquad (2\cdot17)$$

where the background field strength $W_{B\alpha}$ has been defined by

$$W_{B\alpha}\equiv-\frac{1}{4}\overline{D}^2(e^{-V_B}D_\alpha e^{V_B})=-\frac{1}{4}[\overline{D}_{\dot\alpha},\{\overline{D}^{\dot\alpha},\nabla_\alpha\}]. \qquad (2\cdot18)$$

The last expression is convenient for various calculations, and it shows that $W_{B\alpha}$ is in fact a gauge covariant object. The first order action S_{T1} is given by

$$S_{T1}=-\frac{1}{T(R)}\int d^8z\ \mathrm{tr}\, V_Q\mathcal{D}^\alpha W_{B\alpha}$$
$$+\int d^8z\,(\Phi_B^\dagger e^{V_B}V_Q\Phi_B+\Phi_Q^\dagger e^{V_B}\Phi_B+\Phi_B^\dagger e^{V_B}\Phi_Q)$$
$$+\int d^6z\,(\Phi_Q^T m\Phi_B+g\Phi_Q\Phi_B^2)+\text{h.c.} \qquad (2\cdot19)$$

Since the first order action does not contribute to 1PI diagrams, S_{T1} can be neglected in the following discussion.

Let us study the action quadratic in the quantum fields, S_{T2}. We decompose it as $S_{T2}=S_{T2}^{\text{gauge}}+S_{T2}^{\text{chiral}}+S_{T2}^{\text{mix}}+S_{T2}^{\text{ghost}}$. The part made purely from the gauge superfield S_{T2}^{gauge} is given by

$$S_{T2}^{\text{gauge}}=\frac{1}{T(R)}\int d^8z$$
$$\times\,\mathrm{tr}\,V_Q\left[\frac{1}{8}\mathcal{D}^\alpha\overline{D}^2\mathcal{D}_\alpha+W_B^\alpha\mathcal{D}_\alpha+\frac{1}{2}(\mathcal{D}^\alpha W_{B\alpha})-\frac{\xi}{16}(\mathcal{D}^2\overline{D}^2+\overline{D}^2\mathcal{D}^2)\right]V_Q$$
$$=\int d^8z\,V_Q^a\left[\frac{1}{8}\widetilde{\nabla}^\alpha\overline{D}^2\widetilde{\nabla}_\alpha+\frac{1}{2}\mathcal{W}_B^\alpha\widetilde{\nabla}_\alpha-\frac{\xi}{16}(\widetilde{\nabla}^2\overline{D}^2+\overline{D}^2\widetilde{\nabla}^2)\right]^{ab}V_Q^b, \quad (2\cdot20)$$

where we have shifted to the adjoint representation in the last line. The field strength in the adjoint representation is defined by

$$\mathcal{W}_{B\alpha} \equiv \mathcal{T}^a W^a_{B\alpha} = -\frac{1}{4}\overline{D}^2(e^{-\mathcal{V}_B} D_\alpha e^{\mathcal{V}_B}) = -\frac{1}{4}[\overline{D}_{\dot\alpha}, \{\overline{D}^{\dot\alpha}, \tilde\nabla_\alpha\}],$$
$$\overline{\mathcal{W}}'_{B\dot\alpha} \equiv \mathcal{T}^a \overline{W}'^a_{B\dot\alpha} = \frac{1}{4}e^{-\mathcal{V}_B} D^2(e^{\mathcal{V}_B}\overline{D}_{\dot\alpha}e^{-\mathcal{V}_B})e^{\mathcal{V}_B} = \frac{1}{4}[\tilde\nabla^\alpha, \{\tilde\nabla_\alpha, \overline{D}_{\dot\alpha}\}], \quad (2\cdot21)$$

and in this paper we define the conjugate of the background field strength as

$$\overline{W}'_{B\dot\alpha} \equiv e^{-V_B}\overline{W}_{B\dot\alpha}e^{V_B} = \frac{1}{4}[\nabla^\alpha, \{\nabla_\alpha, \overline{D}_{\dot\alpha}\}] \quad \text{and} \quad \overline{W}_{B\dot\alpha} \equiv \frac{1}{4}D^2(e^{V_B}\overline{D}_{\dot\alpha}e^{-V_B}). \quad (2\cdot22)$$

Then by noticing the identity

$$\nabla^2\overline{D}^2 + \overline{D}^2\nabla^2 - 2\nabla^\alpha\overline{D}^2\nabla_\alpha = 16\nabla^m\nabla_m + 8\overline{W}'_{B\dot\alpha}\overline{D}^{\dot\alpha} + 4(\overline{D}_{\dot\alpha}\overline{W}'^{\dot\alpha}_B), \quad (2\cdot23)$$

which holds in an arbitrary representation, we find in the super-Fermi-Feynman gauge $\xi = 1$,

$$S^{\text{gauge}}_{T2} = \int d^8z\, V^a_Q \left(-\tilde\nabla^m\tilde\nabla_m + \frac{1}{2}\mathcal{W}^\alpha_B\tilde\nabla_\alpha - \frac{1}{2}\overline{\mathcal{W}}'_{B\dot\alpha}\overline{D}^{\dot\alpha}\right)^{ab} V^b_Q. \quad (2\cdot24)$$

The part of the quadratic action S_{T2} quadratic in the quantum chiral superfield is given by

$$S^{\text{chiral}}_{T2} = \int d^8z\, \Phi^\dagger_Q e^{V_B}\Phi_Q + \int d^6z\, \frac{1}{2}\Phi^T_Q m\Phi_Q + \text{h.c.} \quad (2\cdot25)$$

We also have mixing terms between the gauge and the chiral superfields:

$$S^{\text{mix}}_{T2} = \int d^8z \left(\Phi^\dagger_B e^{V_B}V_Q\Phi_Q + \Phi^\dagger_Q e^{V_B}V_Q\Phi_B + \frac{1}{2}\Phi^\dagger_B e^{V_B}V^2_Q\Phi_B\right)$$
$$+ \int d^6z\, g\Phi_B\Phi^2_Q + \text{h.c.}, \quad (2\cdot26)$$

where we have included the Yukawa term in S^{mix}_{T2} for later convenience.

To second order in quantum fluctuations, the ghost action becomes

$$S^{\text{ghost}}_{T2} = \frac{1}{T(R)}\int d^8z\, \text{tr}(c'^\dagger e^{V_B}ce^{-V_B} + c^\dagger e^{V_B}c'e^{-V_B} + b^\dagger e^{V_B}be^{-V_B})$$
$$= \int d^8z \left[c'^{\dagger a}(e^{\mathcal{V}_B})^{ab}c^b + c^{\dagger a}(e^{\mathcal{V}_B})^{ab}c'^b + b^{\dagger a}(e^{\mathcal{V}_B})^{ab}b^b\right]. \quad (2\cdot27)$$

The similarity of this expression to the action of the chiral superfield Φ_Q (2·25) is obvious. As a consequence, one-loop quantum effects of ghost fields are simply obtained by substituting $T^a \to \mathcal{T}^a$ and that of Φ_Q by multiplying -3 (with $m = 0$ and $g = 0$). Recall that the ghost fields are Grassmann-odd chiral superfields.

For one-loop level calculations, which we consider in later sections, it is sufficient to retain only quadratic actions, S^{gauge}_{T2}, S^{chiral}_{T2}, S^{ghost}_{T2} and S^{mix}_{T2}. Although we need S_{T3}, S_{T4} and so on for higher loop calculations, further expansion is straightforward.

§3. Supersymmetric gauge covariant regularization

In formulating our regularization scheme, we need a formal propagator of quantum fields in the presence of the background gauge field V_B. In the first half of this section, therefore, we explain how to obtain these propagators. These propagators are defined by inverting the kinetic operators in $S_{T2}^{\rm gauge}$, $S_{T2}^{\rm chiral}$ and $S_{T2}^{\rm ghost}$. In other words, we formally diagonalize these parts of the quadratic action. The remaining parts of the action, $S_{T2}^{\rm mix}$, S_{T3} and so on, are regarded as the perturbation. By organizing the perturbative expansion in this way, we can preserve the background gauge covariance in the regularized theory, as will be explained in the second half of this section.

3.1. *Propagators*

It is straightforward to find the propagator of the quantum gauge superfield V_Q by formally diagonalizing the quadratic term $S_{T2}^{\rm gauge}$ (2·24). The Schwinger-Dyson equation corresponding to $S_{T2}^{\rm gauge}$ is*)

$$2\left(-\widetilde{\nabla}^m\widetilde{\nabla}_m+\frac{1}{2}\mathcal{W}_B^\alpha\widetilde{\nabla}_\alpha-\frac{1}{2}\overline{\mathcal{W}}'_{B\dot\alpha}\overline{D}^{\dot\alpha}\right)^{ab}\left\langle T^*V_Q^b(z)V_Q^c(z')\right\rangle=i\delta^{ac}\delta(z-z'). \quad (3\cdot1)$$

In deriving this expression, we have used the reality constraint, $\mathcal{D}^\alpha\mathcal{W}_{B\alpha}\equiv\{\widetilde{\nabla}^\alpha,\mathcal{W}_{B\alpha}\}=\overline{D}_{\dot\alpha}\overline{\mathcal{W}}_B'^{\dot\alpha}$.[26] By formally solving the relation (3·1), we have

$$\left\langle T^*V_Q^a(z)V_Q^b(z')\right\rangle=\frac{i}{2}\left(\frac{1}{-\widetilde{\nabla}^m\widetilde{\nabla}_m+\mathcal{W}_B^\alpha\widetilde{\nabla}_\alpha/2-\overline{\mathcal{W}}'_{B\dot\alpha}\overline{D}^{\dot\alpha}/2}\right)^{ab}\delta(z-z'). \quad (3\cdot2)$$

Hereafter, the brackets $\langle\cdots\rangle$ represent an expectation value in an unconventional interaction picture in which $S_{T2}^{\rm gauge}$, $S_{T2}^{\rm chiral}$ and $S_{T2}^{\rm ghost}$ are regarded as the "unperturbative part".

A derivation of the propagator of the quantum chiral superfield Φ_Q, on the other hand, is somewhat tricky due to the chirality constraint. We start with the Schwinger-Dyson equation derived from $S_{T2}^{\rm chiral}$ (2·25):

$$\begin{aligned}
&-\frac{1}{4}\overline{D}^2e^{V_B^T}\left\langle T^*\Phi_Q^{\dagger T}(z)\Phi_Q^\dagger(z')\right\rangle+m\left\langle T^*\Phi_Q(z)\Phi_Q^\dagger(z')\right\rangle=0,\\
&-\frac{1}{4}D^2e^{V_B}\left\langle T^*\Phi_Q(z)\Phi_Q^\dagger(z')\right\rangle+m^\dagger\left\langle T^*\Phi_Q^{\dagger T}(z)\Phi_Q^\dagger(z')\right\rangle=-\frac{i}{4}D^2\delta(z-z'). \quad (3\cdot3)
\end{aligned}$$

We first multiply $m^\dagger$ from the left of the first relation in (3·3). Then by noting the fact that m is a constant matrix which satisfies $m^\dagger T^{aT}=-T^am^\dagger$ for the gauge invariance of the mass term, we find

$$\begin{aligned}
&\left(\frac{1}{16}\overline{D}^2\nabla^2-m^\dagger m\right)\left\langle T^*\Phi_Q(z)\Phi_Q^\dagger(z')\right\rangle\\
&=\left[\nabla^m\nabla_m-\frac{1}{2}W_B^\alpha\nabla_\alpha-\frac{1}{4}(\mathcal{D}^\alpha W_{B\alpha})-m^\dagger m\right]\left\langle T^*\Phi_Q(z)\Phi_Q^\dagger(z')\right\rangle
\end{aligned}$$

*) The delta function in the full superspace is denoted as $\delta(z-z')=\delta(x-x')\delta(\theta-\theta')\delta(\bar\theta-\bar\theta')$.

$$= \frac{i}{16}\overline{D}^2\nabla^2 e^{-V_B}\delta(z-z'), \tag{3·4}$$

where the second relation of (3·3) has been used. In going from the first line to the second line in (3·4), we have used the identity

$$\overline{D}^2\nabla^2 + \nabla^2\overline{D}^2 - 2\overline{D}_{\dot\alpha}\nabla^2\overline{D}^{\dot\alpha} = 16\nabla^m\nabla_m - 8W_B^\alpha\nabla_\alpha - 4(\mathcal{D}^\alpha W_{B\alpha}), \tag{3·5}$$

and the fact that Φ_Q is chiral ($\overline{D}_{\dot\alpha}\Phi_Q = 0$). Since $\overline{D}^2$ and ∇^2 have no inverse, it is generally dangerous to invert the first line of (3·4). (This is obvious by considering the massless case.) Therefore, we invert the *second* line of (3·4) instead to yield

$$\left\langle T^*\Phi_Q(z)\Phi_Q^\dagger(z')\right\rangle = \frac{i}{16}\overline{D}^2\frac{1}{\nabla^m\nabla_m - W_B^\alpha\nabla_\alpha/2 - (\mathcal{D}^\alpha W_{B\alpha})/4 - m^\dagger m}\nabla^2 e^{-V_B}\delta(z-z'). \tag{3·6}$$

Here it is important to note that $\overline{D}^2$ and the inverse operator in (3·6) commute with each other, as a result of the identity

$$\begin{aligned}\frac{1}{16}\overline{D}^2\nabla^2\overline{D}^2 &= \overline{D}^2\left[\nabla^m\nabla_m - \frac{1}{2}W_B^\alpha\nabla_\alpha - \frac{1}{4}(\mathcal{D}^\alpha W_{B\alpha})\right] \\ &= \left[\nabla^m\nabla_m - \frac{1}{2}W_B^\alpha\nabla_\alpha - \frac{1}{4}(\mathcal{D}^\alpha W_{B\alpha})\right]\overline{D}^2,\end{aligned} \tag{3·7}$$

which follows from (3·5). Our derivation of the chiral propagator (3·6), and especially the step from the first line to the second line in (3·4), might appear *ad hoc*. However, our expression (3·6) satisfies the following three criterion: (1) It is indeed a solution of the Schwinger-Dyson equation, (3·3). (2) It is manifestly chiral in the z variable and anti-chiral in the z' variable. (3) It reduces to the Grisaru-Roček-Siegel propagator [6] when $V_B = 0$. Note that the Schwinger-Dyson equation (3·3) itself does not ensure the chirality, and instead the chirality constraint must be supplemented by hand. In this sense, our expression (3·6) is the unique solution.

We see below that it is convenient to abbreviate the denominator of (3·6) as

$$\frac{1}{\nabla^m\nabla_m - W_B^\alpha\nabla_\alpha/2 - (\mathcal{D}^\alpha W_{B\alpha})/4 - m^\dagger m} \to \frac{1}{\nabla^2\overline{D}^2/16 - m^\dagger m}, \tag{3·8}$$

in the sense that

$$\begin{aligned}&\overline{D}^2\frac{1}{\nabla^m\nabla_m - W_B^\alpha\nabla_\alpha/2 - (\mathcal{D}^\alpha W_{B\alpha})/4}\frac{\nabla^2\overline{D}^2}{16} \\ &\qquad = \frac{1}{\nabla^m\nabla_m - W_B^\alpha\nabla_\alpha/2 - (\mathcal{D}^\alpha W_{B\alpha})/4}\frac{\overline{D}^2\nabla^2}{16}\overline{D}^2 = \overline{D}^2,\end{aligned} \tag{3·9}$$

where we have used the identity (3·5). The general abbreviation rule turns to be

$$\nabla^m\nabla_m - \frac{1}{2}W_B^\alpha\nabla_\alpha - \frac{1}{4}(\mathcal{D}^\alpha W_{B\alpha}) \leftrightarrow \begin{cases}\nabla^2\overline{D}^2/16 & \text{on the right of } \overline{D}^2, \\ \overline{D}^2\nabla^2/16 & \text{on the left of } \overline{D}^2.\end{cases} \tag{3·10}$$

In what follows, this rule is always understood. For example, a formal manipulation such as

$$\overline{D}^2 \frac{1}{\nabla^2 \overline{D}^2} \nabla^2 \overline{D}^2 = \frac{1}{\overline{D}^2 \nabla^2} \overline{D}^2 \nabla^2 \overline{D}^2 = \overline{D}^2, \tag{3·11}$$

is legitimate. This kind of manipulation is especially useful in a calculation of anomalies (see §5). However, note that

$$\overline{D}^2 \frac{1}{\nabla^2 \overline{D}^2} \nabla^2 \neq 1, \tag{3·12}$$

because $\overline{D}^2$ and ∇^2 themselves have no inverse. With the abbreviation rule (3·10), the propagator of the chiral superfield (3·6) may be written as

$$\left\langle T^* \Phi_Q(z) \Phi_Q^\dagger(z') \right\rangle = \frac{i}{16} \overline{D}^2 \frac{1}{\nabla^2 \overline{D}^2/16 - m^\dagger m} \nabla^2 e^{-V_B} \delta(z - z'). \tag{3·13}$$

This form of the propagator is found in the literature (see Ref. 30) for example). As we have explained, however, this expression must be used with care. In a similar way, we find the "Majorana-mass inserted part",

$$\left\langle T^* \Phi_Q(z) \Phi_Q^T(z') \right\rangle = \frac{i}{4} \overline{D}^2 \frac{1}{\nabla^2 \overline{D}^2/16 - m^\dagger m} m^\dagger \delta(z - z'). \tag{3·14}$$

As was noted below (2·27), the propagator of ghost fields which follows from S_{T2}^{ghost} can be obtained by simply replacing $T^a \to \mathcal{T}^a$ in (3·13) (with $m = 0$):

$$\begin{aligned}\left\langle T^* c^a(z) c'^{\dagger b}(z') \right\rangle &= \left\langle T^* c'^a(z) c^{\dagger b}(z') \right\rangle = \left\langle T^* b^a(z) b^{\dagger b}(z') \right\rangle \\ &= i \left(\overline{D}^2 \frac{1}{\tilde{\nabla}^2 \overline{D}^2} \tilde{\nabla}^2 e^{-\mathcal{V}_B} \right)^{ab} \delta(z - z'). \end{aligned} \tag{3·15}$$

This completes our derivation of propagators which are obtained by diagonalizing S_{T2}^{gauge}, S_{T2}^{chiral} and S_{T2}^{ghost}.

3.2. *Regularization scheme*

We are now ready to explain our regularization scheme. What we do is basically an unconventional perturbative expansion in which the parts of the quadratic action, S_{T2}^{gauge}, S_{T2}^{chiral} and S_{T2}^{ghost}, are regarded as the "unperturbative part." The remaining parts of the total action, S_{T2}^{mix}, S_{T3} and so on, are regarded as the perturbation.*) Then the regularization is implemented by substituting for the propagators, which diagonalize the unperturbative part, with modified ones.

For example, the propagator of the vector multiplet (3·2) is replaced by

$$\left\langle T^* V_Q^a(z) V_Q^b(z') \right\rangle$$

*) When the background scalar superfield Φ_B has a vacuum expectation value, it is necessary to diagonalize the quadratic action including S_{T2}^{mix}. This generalization should be straightforward.

$$\rightarrow \frac{i}{2}\left[f\left((-\widetilde{\nabla}^m\widetilde{\nabla}_m + \mathcal{W}_B^\alpha\widetilde{\nabla}_\alpha/2 - \overline{\mathcal{W}}'_{B\dot\alpha}\overline{D}^{\dot\alpha}/2)/\Lambda^2\right) \right.$$
$$\left. \times \frac{1}{-\widetilde{\nabla}^m\widetilde{\nabla}_m + \mathcal{W}_B^\alpha\widetilde{\nabla}_\alpha/2 - \overline{\mathcal{W}}'_{B\dot\alpha}\overline{D}^{\dot\alpha}/2} \right]^{ab} \delta(z-z'), \qquad (3\cdot16)$$

where Λ is the cutoff mass parameter and $f(t)$ is a regulating factor which decreases sufficiently rapidly:*)

$$f(0)=1, \qquad f(\infty)=f'(\infty)=f''(\infty)=\cdots=0. \qquad (3\cdot17)$$

Clearly, the prescription (3·16) is equivalent to the proper-time cutoff in the proper-time representation[31] of the propagator:

$$\frac{i}{2}\left\{\int_0^\infty d\tau\, g(\Lambda^2\tau)\exp\left[-\tau\left(-\widetilde{\nabla}^m\widetilde{\nabla}_m + \frac{1}{2}\mathcal{W}_B^\alpha\widetilde{\nabla}_\alpha - \frac{1}{2}\overline{\mathcal{W}}'_{B\dot\alpha}\overline{D}^{\dot\alpha}\right)\right]\right\}^{ab}\delta(z-z'), \qquad (3\cdot18)$$

where $g(x)$ is the inverse Laplace transformation of $f(t)/t$. For example, $g(x) = \theta(x-1)$ for $f(t)=e^{-t}$. By modifying the propagator in this way, the ultraviolet behavior of Feynman integrals is tamed, and simultaneously the gauge covariance of the propagator under the background gauge transformation is preserved. In fact, it can easily be seen that the propagator, even with the modification (3·16), transforms covariantly under the background gauge transformation (2·6):

$$\left\langle T^* V_Q^a(z)V_Q^b(z')\right\rangle' = \left[e^{-i\widetilde{\Lambda}(z)}\right]^{ac}\left\langle T^* V_Q^c(z)V_Q^d(z')\right\rangle\left[e^{i\widetilde{\Lambda}(z')}\right]^{db}, \qquad (3\cdot19)$$

where $\widetilde{\Lambda}\equiv T^a\Lambda^a$ is the gauge parameter in the adjoint representation.

Similarly, the propagator of the chiral multiplet (3·13) is replaced by

$$\left\langle T^*\Phi_Q(z)\Phi_Q^\dagger(z')\right\rangle$$
$$\rightarrow \frac{i}{16}f(-\overline{D}^2\nabla^2/16\Lambda^2)\overline{D}^2\frac{1}{\nabla^2\overline{D}^2/16-m^\dagger m}\nabla^2 e^{-V_B}\delta(z-z'), \qquad (3\cdot20)$$

and (3·14) is replaced by

$$\left\langle T^*\Phi_Q(z)\Phi_Q^T(z')\right\rangle \rightarrow \frac{i}{4}f(-\overline{D}^2\nabla^2/16\Lambda^2)\overline{D}^2\frac{1}{\nabla^2\overline{D}^2/16-m^\dagger m}m^\dagger\delta(z-z'). \qquad (3\cdot21)$$

It is easy to see that the propagator (3·20) transforms as

$$\left\langle T^*\Phi_Q(z)\Phi_Q^\dagger(z')\right\rangle' = e^{-i\Lambda(z)}\left\langle T^*\Phi_Q(z)\Phi_Q^\dagger(z')\right\rangle e^{i\Lambda^\dagger(z')}, \qquad (3\cdot22)$$

under the background gauge transformation (2·6). In (3·20) and (3·21), we did not include the mass term in the regulating factor $f(t)$. Although it would be

*) We assume $f(t)$ has a power series expansion in t.

natural to also include the mass term in the regulating factor with the proper-time cutoff prescription, such as (3·18), it is not required from the viewpoint of the gauge covariance (3·22). Therefore we omit it from $f(t)$ for simplicity. The propagators of the ghost fields (3·15) are similarly replaced by

$$\left\langle T^* c^a(z) c'^{\dagger b}(z')\right\rangle = \left\langle T^* c'^a(z) c^{\dagger b}(z')\right\rangle = \left\langle T^* b^a(z) b^{\dagger b}(z')\right\rangle$$
$$\to i\left[f(-\overline{D}^2\widetilde{\nabla}^2/16\Lambda^2)\overline{D}^2\frac{1}{\widetilde{\nabla}^2\overline{D}^2}\widetilde{\nabla}^2 e^{-\mathcal{V}_B}\right]^{ab}\delta(z-z'). \qquad (3\cdot23)$$

Note that the substitutions in (3·20), (3·21) and (3·23) preserve the correct chirality.

Now we have to distinguish two cases in evaluating the effective action or 1PI Green's functions:

(I) *Most* 1PI Green's functions are obtained by simply connecting vertices in $S_T-(S_{T2}^{\text{gauge}}+S_{T2}^{\text{chiral}}+S_{T2}^{\text{ghost}})$ by the modified propagators of the quantum fields, (3·16), (3·20), (3·21) and (3·23). Recall that we have formally diagonalized S_{T2}^{gauge}, S_{T2}^{chiral} and S_{T2}^{ghost} in constructing the propagators, and thus those parts of the action must be subtracted from the perturbation. Symbolically, we evaluate the 1PI part of an expansion of

$$\left\langle \exp\left\{i\left[S_T-(S_{T2}^{\text{gauge}}+S_{T2}^{\text{chiral}}+S_{T2}^{\text{ghost}})\right]\right\}\right\rangle, \qquad (3\cdot24)$$

where it is understood that the modified propagators are used. This defines the first part of the effective action, $\Gamma_{\text{I}}[V_B,\Phi_B]$.

(II) However, the above case does not exhaust all. The exception is 1PI diagrams made only from vertices in S_{T2}^{gauge}, S_{T2}^{chiral} and S_{T2}^{ghost} in the language of the conventional perturbative expansion. Note that these are necessarily one-loop diagrams in which all the external lines are the background gauge superfield. This part of the effective action, which will be denoted as $\Gamma_{\text{II}}[V_B]$, corresponds to the logarithm of the one-loop determinant arising from the Gaussian integration of S_{T2}^{gauge}, S_{T2}^{chiral} and S_{T2}^{ghost} in our perturbative expansion. However, as is well-known, the determinant factor cannot be expressed as a one-loop diagram made from the propagator.*) Therefore we are naturally led to consider a *variation* of the effective action instead which can be expressed by a one-loop diagram with a composite operator inserted:

$$\frac{\delta\Gamma_{\text{II}}[V_B]}{\delta V_B^a(z)}\equiv\langle J^a(z)\rangle, \qquad J^a(z)\equiv\frac{\delta}{\delta V_B^a(z)}(S_{T2}^{\text{gauge}}+S_{T2}^{\text{chiral}}+S_{T2}^{\text{ghost}}), \qquad (3\cdot25)$$

where we have introduced the *gauge current superfield* $J^a(z)$ as a variation of the quadratic action.**) Of course, the modified propagators have to be used in (3·25). We again emphasize that (3·25) consists purely of *one-loop diagrams in which all the external lines are the background gauge superfield.* Therefore, the second part of the effective action $\Gamma_{\text{II}}[V_B]$ defined by (3·25), if it exists (see below), is one-loop exact

*) A naive ansatz such as $\left\langle S_{T2}^{\text{chiral}}\right\rangle$, that is, closing the propagator by another S_{T2}^{chiral} to form a loop, does not give the correct combinatorics.

**) Note that this is a partial gauge current defined with respect to S_{T2}^{gauge}, S_{T2}^{chiral} and S_{T2}^{ghost}, and not the full gauge current corresponding to the total action S_T.

and depends only on the background vector superfield V_B. The total effective action is given by a sum: $\Gamma[V_B,\Phi_B] = \Gamma_{\rm I}[V_B,\Phi_B] + \Gamma_{\rm II}[V_B]$.

In addition to the above prescription we have the following: (III) A Green's function with a certain composite operator $O(z)$ inserted is computed by

$$\left\langle O(z)\exp\left\{i\left[S_T - (S_{T2}^{\rm gauge} + S_{T2}^{\rm chiral} + S_{T2}^{\rm ghost})\right]\right\}\right\rangle, \qquad (3\cdot26)$$

as usual. Again the modified propagators are used.

First of all, it is obvious that with the above prescription, all the Green's functions are made finite by choosing a sufficiently rapidly decreasing function $f(t)$. The high momentum part of the momentum integration in all the internal loops is suppressed as $\sim f(k^m k_m)^n$, where n is the number of internal lines in a loop. In what follows, furthermore, we show that our scheme preserves the supersymmetry, the background gauge invariance (for case (I)), and the background gauge covariance (for cases (II) and (III)).

Supersymmetry: Since our formulation is expressed entirely by the superfield in an exactly four dimensional spacetime, the supersymmetry is manifest at every step. For example, with the above prescription, one can prove the $N = 1$ non-renormalization theorem[6),26),28)] in the form that the first part of the effective action $\Gamma_{\rm I}[V_B,\Phi_B]$ is a $d^4\theta$ integral of a product of superfields whose Grassmann coordinates are common, θ and $\overline{\theta}$. For cases (II) and (III), the expectation value of the gauge current $\langle J^a(x,\theta,\overline{\theta})\rangle$, or of the composite operator $\langle O(x,\theta,\overline{\theta})\rangle$, is a product of superfields whose Grassmann coordinates are common, θ and $\overline{\theta}$. These statements follow from the fact that the modified propagators (3·16), (3·20), (3·21) and (3·23) are expressed by the full superspace delta function $\delta(z-z')$ and the fact that the modified propagator of the chiral multiplet (3·20) has the $\overline{D}^2$ factor on the z variable and the D^2 factor on the z' variable. The latter fact allows one to rewrite interaction vertices in the superpotential as an integral over the full superspace.[6),26),28)] Then proof of the $N = 1$ non-renormalization theorem[6),26),28)] can be repeated even with new unconventional interactions arising from the regulating factor. This theorem implies, in particular, that $\Gamma_{\rm I}[V_B,\Phi_B]$ is supersymmetric invariant, and the composite operators, $\langle J^a(z)\rangle$ or $\langle O(z)\rangle$, are in fact superfield. Therefore, if $\Gamma_{\rm II}[V_B]$ exists as a functional integration of $\langle J^a(z)\rangle$ with respect to $V_B(z)$ (see below), $\Gamma_{\rm II}[V_B]$ is supersymmetric invariant too, and consequently the total effective action is supersymmetric invariant.

Gauge covariance: To see how the background gauge invariance and covariance are preserved with the above prescription, suppose that we perform the background gauge transformation (2·6) on V_B and Φ_B in a perturbative expansion of (3·24) $\sim$ (3·26). Then, at each vertex to which the interaction term, say S_{T3}, is attached, the background gauge transformation (2·6) on V_B and Φ_B induces the gauge transformation on V_Q and Φ_Q, because the interaction term is invariant under the background gauge transformation. Note that each term of the action, $S_{T2}^{\rm mix}$, S_{T3}, S_{T4} and so on, is individually invariant under the background gauge transformation. Then, those induced transformations on V_Q and Φ_Q are canceled by the transformation at the ends of each propagator line which is induced by a covariant transformation law such as (3·19) and (3·22). This cancellation does not occur only at a vertex where a certain operator with a non-trivial gauge representation is inserted, as (3·25). In this way, we see that (at each order of the loop expansion) $\Gamma_{\rm I}[V_B, \Phi_B]$ is *invariant* under the gauge transformation, and the gauge current (3·25) transforms gauge *covariantly*:

$$\langle J^a(z)\rangle' = \frac{\partial V_B^b(z)}{\partial V_B'^a(z)} \left\langle J^b(z)\right\rangle . \tag{3·27}$$

A similar conclusion holds for (3·26): If $O(z)$ has non-trivial gauge indices, it transforms as expected from the classical transformation law. In particular, a gauge singlet operator such as the superconformal current [18] is regularized gauge *invariantly*. The crucial ingredient in the above demonstration of the gauge covariance is the covariance property of the modified propagators, (3·19) and (3·22); we have designed the regulating factors so that this property holds.

For our scheme and the standard proper-time cutoff regularization (see, for example, the first two references in Ref. 32)), there is an important difference in the treatment of one-loop diagrams. In our scheme, the second part of the effective action $\Gamma_{\rm II}[V_B]$ is defined through its *variation* $\langle J^a(z)\rangle$ in (3·25), instead of a direct definition, such as the proper-time cutoff in the proper-time representation of the one-loop *determinant*. This apparently small difference in treatment, however, has a significant consequence. The point is that one cannot in general define one-loop diagrams of a chiral fermion in a gauge invariant way, because of the existence of the gauge anomaly. [11),32),33)] One thus has to break at some step the gauge symmetry. How to break this is crucial for the regularization properties.

As we have seen in (3·27), the gauge current $\langle J^a(z)\rangle$ in our scheme transforms covariantly under the background gauge transformation. This can be interpreted as implying that, in a one-loop diagram, the gauge symmetry at all the vertices, *except* that of $J^a(z)$, is preserved. Putting a different way, a possible breaking of the gauge symmetry due to the anomaly is forced on the $J^a(z)$-vertex. In this way, the gauge symmetry is "maximally" preserved in this treatment. On the other hand, in a "Bose symmetric" treatment [32)] such as the proper-time cutoff of the one-loop determinant or the conventional Pauli-Villars, a breaking of the gauge symmetry is partitioned equally on every vertex. The problem with the latter treatment is that it often breaks the gauge symmetry "too much".*) For example, the conventional Pauli-Villars does not give the gauge invariant vacuum polarization tensor even for anomaly free representations ($\operatorname{tr} T^a\{T^b, T^c\} = 0$) for which a gauge invariant regularization would be possible.

Our prescription (3·25), on the other hand, respects the gauge symmetry as much as possible to an extent which does not contradict with the gauge anomaly. Moreover, when the gauge representation is free of the gauge anomaly, it is possible to impose the gauge invariance also on the $J^a(z)$-vertex and, eventually, the full gauge invariance is restored. In fact, it is possible to show that the gauge current $\langle J^a(z)\rangle$

*) The important exception is the generalized Pauli-Villars regularization, which is closely related to our treatment of one-loop diagrams. (See the last reference in Ref. 12) and Okuyama and Suzuki in Ref. 10).)

has the covariant gauge anomaly:

$$-\frac{1}{4}\overline{D}^2\left\{\mathcal{L}_{V_B/2}\cdot\left[\coth(\mathcal{L}_{V_B/2})-1\right]\cdot T^b\right\}^a\left\langle J^b(z)\right\rangle \overset{\Lambda\to\infty}{=} -\frac{1}{64\pi^2}\operatorname{tr}T^aW_B^\alpha W_{B\alpha}. \tag{3·28}$$

This is in accord with the gauge covariance property (3·27). We note that this is the *exact* expression of the gauge anomaly in our scheme,*) because only one-loop diagrams contribute to $\langle J^a(z)\rangle$ in (3·25), and higher-loop diagrams are regularized gauge invariantly. The left-hand side of (3·28) is a supersymmetric generalization of the gauge covariant divergence of the gauge current. This might be interpreted as a gauge non-invariance of the effective action, $-i\delta\Gamma_{\rm II}[V_B^\Lambda]/\delta\Lambda^a(z)$, in view of the identification (3·25). Equation (3·28) shows that the gauge symmetry at the $J^a(z)$-vertex is restored when $\operatorname{tr}T^a\{T^b,T^c\}=0$. Therefore, the full gauge invariance is automatically restored for anomaly-free cases. This is one of advantages of our scheme. Note that the gauge multiplet and the ghost multiplet do not contribute to (3·28) because $\operatorname{tr}\mathcal{T}^a\{\mathcal{T}^b,\mathcal{T}^c\}=0$.

Our prescription (3·25), in place of the manifest gauge covariance, however, sacrifices the manifest Bose symmetry among gauge vertices in a one-loop diagram. As a consequence, the second part of the effective action $\Gamma_{\rm II}[V_B]$ whose variation reproduces the gauge current $\langle J^a(z)\rangle$ may not exist. It is easy to see that the gauge anomaly must vanish for such a functional $\Gamma_{\rm II}[V_B]$ to exist: When the effective action exists, the gauge anomaly must satisfy the Wess-Zumino consistency condition,[14] and thus the gauge anomaly has the consistent form.[32),33] Our anomaly (3·28), on the other hand, has a covariant form. The only possible way out is a zero covariant anomaly which trivially satisfies the consistency condition.

The reverse is non-trivial: When the gauge anomaly vanishes, is the Bose symmetry restored, and is it possible to "integrate" the gauge current $\langle J^a(z)\rangle$ to obtain the effective action $\Gamma_{\rm II}[V_B]$? We expect the answer is yes, as is suggested from a restoration of the gauge symmetry at all the vertices in anomaly-free cases. In fact, our prescription (3·25) is a natural supersymmetric generalization of the gauge covariant regularization of non-supersymmetric chiral gauge theories.[12),13] In the non-supersymmetric theory, when the gauge anomaly vanishes, it *is* possible to write down a formal expression of the gauge invariant effective action whose variation reproduces the covariant gauge current (in the limit $\Lambda\to\infty$).[13] Although we postpone to a separate publication[34] a detailed analysis of the "integrability" of (3·25) (with a detailed account of (3·28)), we may expect that the integrability must be restored, because the supersymmetry should be irrelevant with regard to this point, and otherwise it would eventually be impossible to define a gauge invariant effective action even for anomaly-free cases.

If the integrability is restored in anomaly-free cases, then the effective action $\Gamma_{\rm II}[V_B]$ is supersymmetric and gauge invariant, because $\langle J^a(z)\rangle$ is a gauge covariant superfield (3·27). In particular, a part of the effective action $\Gamma_{\rm II}[V_B]$ corresponding to one-loop diagrams of the gauge multiplet and the ghost multiplet should always

*) By introducing an external gauge field which couples to the global axial current, it might be possible to give a simple proof of the Adler-Bardeen theorem in a similar way.

exist, because the adjoint representation is anomaly-free. The non-integrability can emerge only in one-loop diagrams of the chiral multiplet Φ_Q.*)

In summary, we have observed the following properties of our regularization scheme. The first part of the effective action $\Gamma_{\rm I}[V_B, \Phi_B]$ (3·24), which contains *all* the 1PI *multi-loop* diagrams, is always supersymmetric and gauge invariant. A classically gauge covariant superfield composite operator, such as the gauge current $\langle J^a(z)\rangle$ in (3·25), remains a gauge covariant superfield. The breaking of the Bose symmetry and the non-integrability of (3·25) associated with the gauge anomaly will be restored for anomaly-free gauge representations (analysis of this point will be reported elsewhere[34]). When the Bose symmetry is restored, we have a supersymmetric invariant, gauge invariant effective action $\Gamma_{\rm II}[V_B]$. This complication associated with the gauge anomaly, however, occurs only in one-loop diagrams of Φ_Q in which all the external lines are the background gauge superfield V_B.

§4. Illustrative calculations

In this section we present a somewhat detailed calculation of one-loop 1PI two-point functions to illustrate how our regularization scheme works. We believe this demonstration is useful because our scheme, through the regulating factor, produces new interaction vertices which do not appear in the conventional super-Feynman rule. These new vertices ensure the gauge covariance. Since our main concern in this section is the supersymmetry and the gauge symmetry, we neglect the effect of the superpotential by setting $m = g = 0$.

To carry out actual calculations, we have to choose a form of the regulating factor, which should satisfy (3·17) and must be $O(1/t^\alpha)$ with $\alpha > 1$ for $t \to \infty$ to regulate tadpole diagrams (see below). In this section, we choose as a simple choice**)

$$f(t) \equiv \frac{1}{(t+1)^2}. \tag{4·1}$$

Our first example is the self-energy part of the chiral multiplet. Since the external lines of this function are the background chiral superfield Φ_B, this case matches category (I) in the previous section, (3·24). At the one-loop level, the self-energy part is given in configuration space by

$$\left\langle \frac{\delta^2 S_{T2}^{\rm mix}}{\delta\Phi_B^\dagger(z)\delta\Phi_B(z')}\right\rangle\Bigg|_{V_B=0} + i\left\langle T^* \frac{\delta S_{T2}^{\rm mix}}{\delta\Phi_B^\dagger(z)}\frac{\delta S_{T2}^{\rm mix}}{\delta\Phi_B(z')}\right\rangle\Bigg|_{V_B=0}$$
$$= \frac{1}{2}T^aT^b\left(-\frac{\overline{D}'^2}{4}\right)\left[-\frac{D^2}{4}\delta(z-z')\right]\left\langle V_Q^a(z')V_Q^b(z')\right\rangle\Big|_{V_B=0}$$

*) In the example of a two-point function of V_B in the next section, the non-integrability associated with the gauge anomaly does not appear, because, as is clear from (3·28), the gauge anomaly is $O(V_B^2)$ in $\langle J^a(z)\rangle$, which corresponds to a triangle or a higher-point diagram of V_B.

**) If one can neglect the contribution of tadpoles for some reason, even use of $f(t) = 1/(t+1)$ may be made. This considerably simplifies the calculation.

$$+i\left(-\frac{D^2}{4}\right)\left(-\frac{\overline{D}'^2}{4}\right)T^a\left\langle T^*\Phi_Q(z)\Phi_Q^\dagger(z')\right\rangle\Big|_{V_B=0}T^b\left\langle T^*V_Q^a(z)V_Q^b(z')\right\rangle\Big|_{V_B=0}. \tag{4·2}$$

In diagrammatical language, the first term on the right-hand side is a tadpole. According to the substitution (3·16), we see that the tadpole contribution identically vanishes:

$$\begin{aligned}\left\langle V_Q^a(z')V_Q^b(z')\right\rangle\Big|_{V_B=0} &\to \frac{i}{2}\delta^{ab}\lim_{w\to z'}f(-\Box/\Lambda^2)\frac{1}{-\Box}\delta(z'-w)\\ &= \frac{i}{2}\delta^{ab}\lim_{w\to z'}\frac{\Lambda^4}{(-\Box+\Lambda^2)^2}\frac{1}{-\Box}\delta(z'-w)\\ &= -\frac{1}{2}\delta^{ab}\int\frac{d^4k}{i(2\pi)^4}\frac{\Lambda^4}{(k^2+\Lambda^2)^2}\frac{1}{k^2}\delta(\theta'-\theta')\delta(\overline{\theta}'-\overline{\theta}')\\ &= 0,\end{aligned} \tag{4·3}$$

where we have used $\delta(x-y)=\int d^4k\,e^{ik(x-y)}/(2\pi)^4$. As is well-known, this cancellation is a consequence of the supersymmetry. However, note that the quadratic divergence is regularized in (4·3), and a subtlety such as $\infty\times 0$ does not arise.

On the other hand, the second term on the right-hand side of (4·2) leads to

$$\begin{aligned}&\to iC(R)\left(-\frac{D^2}{4}\right)\left(-\frac{\overline{D}'^2}{4}\right)\\ &\quad\times\left\{\left[\frac{i}{16}f(-\Box/\Lambda^2)\overline{D}^2\frac{1}{\Box}D^2\delta(z-z')\right]\left[\frac{i}{2}f(-\Box/\Lambda^2)\frac{1}{-\Box}\delta(z-z')\right]\right\}\\ &=\frac{i}{2}C(R)\left(-\frac{D^2}{4}\right)\left(-\frac{\overline{D}'^2}{4}\right)\delta(\theta-\theta')\delta(\overline{\theta}-\overline{\theta}')\left[f(-\Box/\Lambda^2)\frac{1}{-\Box}\delta(x-x')\right]^2,\end{aligned} \tag{4·4}$$

where we have used

$$\delta(\theta-\theta')\delta(\overline{\theta}-\overline{\theta}')\frac{\overline{D}^2D^2}{16}\delta(\theta-\theta')\delta(\overline{\theta}-\overline{\theta}')=\delta(\theta-\theta')\delta(\overline{\theta}-\overline{\theta}'). \tag{4·5}$$

Finally, by going to momentum space, we have (recall (4·1))

$$\begin{aligned}&-\frac{1}{2}C(R)\left(-\frac{D^2}{4}\right)\left(-\frac{\overline{D}'^2}{4}\right)\delta(\theta-\theta')\delta(\overline{\theta}-\overline{\theta}')\\ &\quad\times\int\frac{d^4p}{(2\pi)^4}e^{ip(x-x')}\int\frac{d^4k}{i(2\pi)^4}\frac{\Lambda^4}{[(k-p)^2+\Lambda^2]^2}\frac{\Lambda^4}{(k^2+\Lambda^2)^2}\frac{1}{(k-p)^2}\frac{1}{k^2}\\ &\overset{\Lambda\to\infty}{=}-\frac{1}{32\pi^2}C(R)\left(-\frac{D^2}{4}\right)\left(-\frac{\overline{D}'^2}{4}\right)\delta(\theta-\theta')\delta(\overline{\theta}-\overline{\theta}')\\ &\qquad\qquad\times\int\frac{d^4p}{(2\pi)^4}e^{ip(x-x')}\left(\ln\frac{\Lambda^2}{p^2}-\frac{5}{6}\right).\end{aligned} \tag{4·6}$$

In terms of the one-loop effective action, (4·6) implies

$$\Gamma_{\rm I}^{(1)}[V_B=0,\Phi_B]=-\frac{1}{32\pi^2}C(R)\int d^4\theta\int d^4x\,d^4x'\,\Phi_B^\dagger(x,\theta,\bar\theta)\Phi_B(x',\theta,\bar\theta)$$
$$\times\int\frac{d^4p}{(2\pi)^4}\,e^{ip(x-x')}\left(\ln\frac{\Lambda^2}{p^2}-\frac{5}{6}\right)+\cdots. \quad (4·7)$$

Apart from the non-universal constant $-5/6$, which depends on the precise form of the regulating factor $f(t)$, (4·7) coincides with the well-known one-loop result.[28] In fact, since we *know* that the first part of the effective action $\Gamma_{\rm I}[V_B,\Phi_B]$ is always gauge invariant, we can covariantize the local part of the effective action (which is proportional to $\ln\Lambda^2$) as $\int d^8z\,\Phi_B^\dagger e^{V_B}\Phi_B$, in accord with the background gauge invariance.

Next, let us consider the vacuum polarization tensor, a one-loop 1PI two-point function of V_B ($\Phi_B=0$). This is a typical example belonging to category (II), (3·25). We first study the contribution of the chiral multiplet. From (2·25) and (3·20), the regularized gauge current is given by

$$\langle J^a_{\rm chiral}(z)\rangle\equiv\left\langle\frac{\delta S^{\rm chiral}_{T2}}{\delta V_B^a(z)}\right\rangle$$
$$=\lim_{w\to z}\operatorname{tr}\left\langle T^*\Phi_Q(z)\Phi_Q^\dagger(w)\right\rangle\left\{T^a+\frac{1}{2}[T^aV_B(w)+V_B(w)T^a]+O(V_B^2)\right\}$$
$$\to i\lim_{w\to z}\operatorname{tr}f(-\bar D^2\nabla^2/16\Lambda^2)\bar D^2\frac{1}{\nabla^2\bar D^2}\nabla^2e^{-V_B}$$
$$\times\left[T^a+\frac{1}{2}(T^aV_B+V_BT^a)+O(V_B^2)\right]\delta(z-w). \quad (4·8)$$

Since we need another V_B-line to form the vacuum polarization tensor, it is sufficient to expand (4·8) in powers of V_B to $O(V_B)$. In doing so, we must first recall that the abbreviation rule (3·10) is assumed in (4·8). Then, the expansion becomes easy by noting the relation (3·9). As a result, we have

$$\bar D^2\frac{1}{\nabla^2\bar D^2}=\bar D^2\left[\frac{1}{16\Box}-\frac{1}{16\Box}(-V_BD^2\bar D^2+D^2V_B\bar D^2)\frac{1}{16\Box}+O(V_B^2)\right], \quad (4·9)$$

and, for the present choice (4·1),

$$f(-\bar D^2\nabla^2/16\Lambda^2)\bar D^2$$
$$=\left[\frac{\Lambda^4}{(-\Box+\Lambda^2)^2}+\frac{\Lambda^4}{(-\Box+\Lambda^2)^2}\frac{-\bar D^2V_BD^2+\bar D^2D^2V_B}{16\Lambda^2}\frac{\Lambda^2}{-\Box+\Lambda^2}\right.$$
$$\left.+\frac{\Lambda^2}{-\Box+\Lambda^2}\frac{-\bar D^2V_BD^2+\bar D^2D^2V_B}{16\Lambda^2}\frac{\Lambda^4}{(-\Box+\Lambda^2)^2}+O(V_B^2)\right]\bar D^2. \quad (4·10)$$

Substituting (4·9) and (4·10) into (4·8), we have

$$\langle J^a_{\rm chiral}(z)\rangle=i\lim_{w\to z}\operatorname{tr}T^a\frac{\Lambda^4}{(-\Box+\Lambda^2)^2}\frac{1}{\Box}\frac{\bar D^2D^2}{16}\delta(z-w)$$

$$+i\lim_{w\to z}\operatorname{tr}T^a\left[-\frac{\Lambda^4}{(-\Box+\Lambda^2)^2}\frac{1}{\Box}C+\frac{\Lambda^4}{(-\Box+\Lambda^2)^2}C\frac{1}{-\Box+\Lambda^2}\right.$$
$$\left.+\frac{1}{-\Box+\Lambda^2}C\frac{\Lambda^4}{(-\Box+\Lambda^2)^2}\right]\frac{\overline{D}^2D^2}{16}\delta(z-w)+O(V_B^2), \qquad (4\cdot11)$$

where the combination C has been defined by

$$C\equiv\frac{\overline{D}^2D^2}{16}V_B\frac{1}{\Box}-V_B. \qquad (4\cdot12)$$

Then after some standard spinor algebra, we have in configuration space,

$$\langle J^a_{\rm chiral}(z)\rangle=i\lim_{y\to x}\operatorname{tr}T^a\frac{\Lambda^4}{(-\Box+\Lambda^2)^2}\frac{1}{\Box}\delta(x-y)$$
$$+i\lim_{y\to x}\operatorname{tr}T^a\left[-\frac{\Lambda^4}{(-\Box+\Lambda^2)^2}\frac{1}{\Box}C'+\frac{\Lambda^4}{(-\Box+\Lambda^2)^2}C'\frac{1}{-\Box+\Lambda^2}\right.$$
$$\left.+\frac{1}{-\Box+\Lambda^2}C'\frac{\Lambda^4}{(-\Box+\Lambda^2)^2}\right]\delta(x-y)+O(V_B^2), \qquad (4\cdot13)$$

where the new combination C' has been defined by

$$C'\equiv\left[\frac{1}{16}(\overline{D}^2D^2V_B)-\frac{i}{2}(\overline{D}\overline{\sigma}^mDV_B)\partial_m\right]\frac{1}{\Box}. \qquad (4\cdot14)$$

The first line of (4·11) and (4·13) is $O(V_B^0)$, and thus is a tadpole diagram. By going to momentum space, we find a quadratically divergent Fayet-Iliopoulos D-term:[35]

$$\langle J^a_{\rm chiral}(z)\rangle|_{V_B=0}=\operatorname{tr}T^a\Lambda^4\int\frac{d^4k}{i(2\pi)^4}\frac{1}{(k^2+\Lambda^2)^2}\frac{1}{k^2}=\frac{1}{16\pi^2}\operatorname{tr}T^a\Lambda^2. \qquad (4\cdot15)$$

Unlike dimensional reduction, in which quadratic divergences always vanish, our regularization produces this term. However, note that this term vanishes in conventional models such as the supersymmetric QED, in which $\operatorname{tr}T^a=0$, so that the gauge-gravitational mixed anomaly disappears.

The vacuum polarization tensor is given by the $O(V_B)$ term in (4·13). In momentum space, we have

$$\left.\frac{\delta\langle J^a_{\rm chiral}(z)\rangle}{\delta V_B^b(z')}\right|_{V_B=0}=\operatorname{tr}T^aT^b\int\frac{d^4p}{(2\pi)^4}e^{ip(x-x')}$$
$$\times\Lambda^4\int\frac{d^4k}{i(2\pi)^4}\left(\frac{1}{16}\overline{D}^2D^2+\frac{1}{2}\overline{D}\overline{\sigma}^mDk_m\right)\delta(\theta-\theta')\delta(\overline{\theta}-\overline{\theta}')\frac{1}{k^2}$$
$$\times\left\{\frac{1}{[(k+p)^2+\Lambda^2]^2}\frac{1}{(k+p)^2}\right.$$
$$\left.+\frac{1}{[(k+p)^2+\Lambda^2]^2}\frac{1}{k^2+\Lambda^2}+\frac{1}{(k+p)^2+\Lambda^2}\frac{1}{(k^2+\Lambda^2)^2}\right\}. \qquad (4\cdot16)$$

In this expression, the vector derivative ∂_m in D_α and $\overline{D}_{\dot\alpha}$ is understood as ip_m. Note that if the $\Lambda \to \infty$ limit is taken in the integrand, only the first term in the curly bracket survives; this term is what one would have in the conventional superdiagram calculation. The remaining terms in (4·16) (the last line) are specific to our scheme, and these terms ensure the gauge invariance of this two-point function. The momentum integration of (4·16) is straightforward in the limit $\Lambda \to \infty$, and finally we find

$$\left.\frac{\delta\left\langle J^a_{\text{chiral}}(z)\right\rangle}{\delta V^b_B(z')}\right|_{V_B=0}$$
$$\overset{\Lambda\to\infty}{=} \frac{1}{64\pi^2}T(R)\delta^{ab}\frac{1}{4}D^\alpha\overline{D}^2D_\alpha\delta(\theta-\theta')\delta(\overline{\theta}-\overline{\theta}')\int\frac{d^4p}{(2\pi)^4}\,e^{ip(x-x')}\left(\ln\frac{\Lambda^2}{p^2}+1\right). \tag{4·17}$$

This is the contribution of the chiral multiplet to the vacuum polarization tensor.

Next let us consider the contribution of the gauge multiplet and the ghost multiplet. As is well-known,[6] however, the gauge multiplet cannot contribute to this function at the one-loop level, because the number of spinor derivatives is not sufficient, as seen from the form of the propagator (3·2) (it requires at least four V_B). This is one of advantages of the superfield background field method. This is also the case even with our modification (3·16). Therefore we do not have to evaluate the V_Q-loop. No further calculation is needed also for the ghost's loop, because the contribution of the ghosts is precisely obtained by $T^a \to \mathcal{T}^a$ and by multiplying the contribution of the chiral multiplet (4·17) by -3, as was noted below (2·27). Therefore, from (4·17), we have the expression

$$\Gamma_{\text{II}}[V_B] = \frac{1}{16\pi^2}\Lambda^2\int d^8z\ \text{tr}\,V_B + \frac{1}{64\pi^2}[T(R)-3C_2(G)]\frac{1}{2T(R)}\int d^4\theta\int d^4x\,d^4x'$$
$$\times\,\text{tr}\,V_B(x,\theta,\overline{\theta})\left[\frac{1}{4}D^\alpha\overline{D}^2D_\alpha V_B(x',\theta,\overline{\theta})\right]\int\frac{d^4p}{(2\pi)^4}\,e^{ip(x-x')}\left(\ln\frac{\Lambda^2}{p^2}+1\right)+O(V_B^3), \tag{4·18}$$

completely in terms of the effective action. As discussed in the previous section, the non-integrability of the gauge current $\langle J^a(z)\rangle$ associated with the gauge anomaly does not emerge at this order of the expansion in V_B; actually the effective action has been obtained to $O(V_B^2)$ in (4·18). Once the effective action is obtained, we know that it is gauge invariant, and thus we may covariantize the local term proportional to $\ln\Lambda^2$ as $\int d^6z\ \text{tr}\,W_B^\alpha W_{B\alpha}$. Of course, when the matter content has the gauge anomaly ($\text{tr}\,T^a\{T^b,T^c\}\neq 0$) there may exist a finite $O(V_B^2)$ piece in $\langle J^a(z)\rangle$ which cannot be expressed as a variation of the effective action. Equation (4·18) again coincides with the supersymmetric gauge invariant one-loop result,[28] apart from the non-universal constant $+1$. The coefficient of $\ln\Lambda^2$ gives, as is well-known, the one-loop β-function of the gauge coupling constant.

In summary, we have observed that our general discussion on the supersymmetry and the gauge covariance in the previous section actually holds in simple but explicit examples. We hope that the above examples also convinced the reader that our scheme is not too complicated and that it actually has practical applicability.

§5. Super-chiral and superconformal anomalies

Since our scheme gives a supersymmetric gauge covariant definition of composite operators, it also provides a simple and reliable way to compute quantum anomalies. In this section, we present several examples in the one-loop approximation. Throughout this section, we assume that the background chiral superfield Φ_B and the Yukawa coupling g vanish, for simplicity of analysis.

The first example is the super-chiral anomaly, [15)-17)] which is defined as a breaking of the naive Ward-Takahashi identity:

$$-\frac{1}{4}\overline{D}^2\left\langle\Phi^\dagger e^V\Phi(z)\right\rangle+\left\langle\Phi^T m\Phi(z)\right\rangle=0. \tag{5·1}$$

This identity is associated with the chiral symmetry of the massless action, $\Phi(z)\to e^{i\alpha}\Phi(z)$, and its explicit breaking by the mass term. Let us evaluate this anomaly on the basis of our regularization scheme. We first take in (5·1) the quadratic terms in the quantum fields (i.e., the one-loop approximation). Then, as explained in (3·26), the regularized super-chiral current is defined by

$$\left\langle\Phi_Q^\dagger e^{V_B}\Phi_Q(z)\right\rangle\equiv\frac{i}{16}\lim_{z'\to z}\operatorname{tr}f(-\overline{D}^2\nabla^2/16\Lambda^2)\overline{D}^2\frac{1}{\nabla^2\overline{D}^2/16-m^\dagger m}\nabla^2\delta(z-z'), \tag{5·2}$$

and similarly, from (3·21),

$$\left\langle\Phi_Q^T m\Phi_Q(z)\right\rangle\equiv\frac{i}{4}\lim_{z'\to z}\operatorname{tr}f(-\overline{D}^2\nabla^2/16\Lambda^2)\overline{D}^2\frac{1}{\nabla^2\overline{D}^2/16-m^\dagger m}m^\dagger m\delta(z-z'). \tag{5·3}$$

We then directly apply $-\overline{D}^2/4$ on the composite operator in (5·2). In so doing, it is important to note that the derivative acts not only on the z variable but also on the z' variable, because the equal-point limit is taken prior to the differentiation. Then, by noting the chirality of (5·2) with respect to the z variable and the relation (3·11), we find

$$\begin{aligned}
&-\frac{1}{4}\overline{D}^2\left\langle\Phi_Q^\dagger e^{V_B}\Phi_Q(z)\right\rangle+\left\langle\Phi_Q^T m\Phi_Q(z)\right\rangle\\
&\quad=-\frac{i}{4}\lim_{z'\to z}\operatorname{tr}f(-\overline{D}^2\nabla^2/16\Lambda^2)\overline{D}^2\delta(z-z')\\
&\quad\overset{\Lambda\to\infty}{=}-\frac{1}{64\pi^2}\operatorname{tr}W_B^\alpha W_{B\alpha}(z),
\end{aligned} \tag{5·4}$$

which reproduces the well-known form of the super-chiral anomaly. [15)-17)] The details of the calculation of (5·4) [15)] are reviewed in the Appendix. We note that the expression (5·4) holds even in *chiral* gauge theories, and in this sense (5·4) may be viewed as a supersymmetric version of the fermion number anomaly. [36)] One might notice that our calculation of the super-chiral anomaly (5·4) has resulted in the Fujikawa method of anomaly evaluation.[12),15)] In fact, the covariant regularization[12)] was originally abstracted from the Fujikawa method.

Since we have defined the regularized composite operator in (5·2) and (5·3), an anomalous supersymmetric commutation relation associated with the super-chiral anomaly, the Konishi anomaly,[23)] can be derived straightforwardly. First we note in the Wess-Zumino gauge,[26)]

$$\Phi^\dagger e^V \Phi = A^\dagger A + \sqrt{2}\,\overline{\theta}\overline{\psi} A + \cdots , \tag{5·5}$$

and thus classically,

$$\sqrt{2}\,\overline{\psi}^{\dot{\alpha}} A = \overline{D}^{\dot{\alpha}} \left(\Phi^\dagger e^V \Phi\right)\Big|_{\theta=\overline{\theta}=0} . \tag{5·6}$$

Therefore we may define the supersymmetric transformation of the composite operator $\overline{\psi}^{\dot{\alpha}} A$ as*)

$$\begin{aligned}
\frac{1}{2\sqrt{2}} \left\langle \{\overline{Q}_{\dot{\alpha}}, \overline{\psi}^{\dot{\alpha}}_Q A_Q(x)\} \right\rangle &\equiv \frac{1}{4} \overline{Q}_{\dot{\alpha}} \overline{D}^{\dot{\alpha}} \left\langle \Phi^\dagger_Q e^{V_B} \Phi_Q(z) \right\rangle \Big|_{\theta=\overline{\theta}=0} \\
&= \frac{1}{4} \overline{D}^2 \left\langle \Phi^\dagger_Q e^{V_B} \Phi_Q(z) \right\rangle \Big|_{\theta=\overline{\theta}=0} \\
&= \left\langle A^T_Q m A_Q(x) \right\rangle - \frac{1}{64\pi^2} \operatorname{tr} \lambda^\alpha_B \lambda_{B\alpha}(x).
\end{aligned} \tag{5·7}$$

This is the Konishi anomaly[23)] (recall that we set $\Phi_B = 0$). Our derivation (5·7) which might appear almost identical to that in Ref. 15), however, has a conceptual advantage: The point is that we have first defined the *regularized composite operator*. In this respect, our approach is similar to the original derivation by Konishi,[23)] in which the composite operator is defined by the gauge invariant point splitting. However, we regularized the composite operator in terms of the *superfield*. Therefore the supersymmetric transformation of the regularized composite operator can be performed by one stroke of the differential operator, $\overline{Q}_{\dot{\alpha}} = -\partial/\partial\overline{\theta}^{\dot{\alpha}} + i\theta^\alpha \sigma^m_{\alpha\dot{\alpha}} \partial_m$.[26)] Recall that we have shown in the previous section that a regularized composite operator (3·26) is in fact a superfield. In this way, the relation of the Konishi anomaly (5·7) to the super-chiral anomaly (5·4) can be made transparent.

As our next example of the one-loop anomaly, let us consider the superconformal anomaly[18)-22)] emerging from the chiral matter loop. It is a breaking of the Ward-Takahashi relation:

$$\overline{D}^{\dot{\alpha}} \langle R_{\alpha\dot{\alpha}}(z) \rangle - 2 \left\langle \Phi^T m e^{-V} D_\alpha e^V \Phi(z) \right\rangle + \frac{2}{3} D_\alpha \left\langle \Phi^T m \Phi(z) \right\rangle = 0. \tag{5·8}$$

The superconformal current $R_{\alpha\dot{\alpha}}$ is defined by $R_{\alpha\dot{\alpha}} = R^{\rm chiral}_{\alpha\dot{\alpha}} + R^{\rm gauge}_{\alpha\dot{\alpha}}$, where[18),22)]

$$\begin{aligned}
R^{\rm chiral}_{\alpha\dot{\alpha}} &= -\overline{D}_{\dot{\alpha}}(\Phi^\dagger e^V) e^{-V} D_\alpha e^V \Phi - \frac{1}{3}[D_\alpha, \overline{D}_{\dot{\alpha}}](\Phi^\dagger e^V \Phi) \\
&= -\overline{D}_{\dot{\alpha}}(\Phi^\dagger_Q e^{V_B}) \nabla_\alpha \Phi_Q - \frac{1}{3}[D_\alpha, \overline{D}_{\dot{\alpha}}](\Phi^\dagger_Q e^{V_B} \Phi_Q) + \cdots ,
\end{aligned} \tag{5·9}$$

*) The normalization of the bracket $\{,\}$ must be regarded as that of the (classical) Poisson bracket. For the (anti-)*commutator*, one has to multiply the right-hand side of (5·7), (5·23) and (5·26) by i.

and [18), 20)]

$$R_{\alpha\dot\alpha}^{\rm gauge}=-\frac{2}{T(R)}\,{\rm tr}\,W_\alpha e^{-V}\overline{W}_{\dot\alpha}e^V. \tag{5·10}$$

In what follows, we consider only quantum effects of the Φ_Q-loop and regard the gauge superfield V as a classical field. In other words, we shall use the (classical) equation of motion of the gauge superfield V.

From (5·9), the regularized superconformal current of the chiral multiplet is defined to the one-loop level by

$$\begin{aligned}
&\left\langle R_{\alpha\dot\alpha}^{\rm chiral}(z)\right\rangle\\
&\equiv-\frac{i}{16}\lim_{z'\to z}{\rm tr}\,\nabla_\alpha f(-\overline{D}^2\nabla^2/16\Lambda^2)\overline{D}^2\frac{1}{\nabla^2\overline{D}^2/16-m^\dagger m}\nabla^2\overline{D}_{\dot\alpha}\delta(z-z')\\
&\quad-\frac{1}{3}\frac{i}{16}[D_\alpha,\overline{D}_{\dot\alpha}]\lim_{z'\to z}{\rm tr}\,f(-\overline{D}^2\nabla^2/16\Lambda^2)\overline{D}^2\frac{1}{\nabla^2\overline{D}^2/16-m^\dagger m}\nabla^2\delta(z-z').
\end{aligned}\tag{5·11}$$

To deal with this expression, we note the identity

$$D_\alpha\lim_{z'\to z}{\rm tr}\,A(z)\delta(z-z')=\lim_{z'\to z}{\rm tr}[\nabla_\alpha,A(z)\}\delta(z-z'), \tag{5·12}$$

where $A(z)$ is an arbitrary operator. Then by repeatedly using this identity and noting the chirality, we find

$$\begin{aligned}
&\left\langle R_{\alpha\dot\alpha}^{\rm chiral}(z)\right\rangle\\
&=\frac{1}{3}\frac{i}{16}\lim_{z'\to z}{\rm tr}\Bigg[-\nabla_\alpha f(-\overline{D}^2\nabla^2/16\Lambda^2)\overline{D}^2\frac{1}{\nabla^2\overline{D}^2/16-m^\dagger m}\nabla^2\overline{D}_{\dot\alpha}\\
&\qquad+f(-\overline{D}^2\nabla^2/16\Lambda^2)\overline{D}^2\frac{1}{\nabla^2\overline{D}^2/16-m^\dagger m}\nabla^2\overline{D}_{\dot\alpha}\nabla_\alpha\\
&\qquad+\overline{D}_{\dot\alpha}\nabla_\alpha f(-\overline{D}^2\nabla^2/16\Lambda^2)\overline{D}^2\frac{1}{\nabla^2\overline{D}^2/16-m^\dagger m}\nabla^2\Bigg]\delta(z-z').
\end{aligned}\tag{5·13}$$

By applying $\overline{D}^{\dot\alpha}$ further, we have

$$\begin{aligned}
&\overline{D}^{\dot\alpha}\left\langle R_{\alpha\dot\alpha}^{\rm chiral}(z)\right\rangle\\
&=\frac{1}{3}\frac{i}{16}\lim_{z'\to z}{\rm tr}\Bigg[\nabla_\alpha f(-\overline{D}^2\nabla^2/16\Lambda^2)\overline{D}^2\frac{1}{\nabla^2\overline{D}^2/16-m^\dagger m}\nabla^2\overline{D}^2\\
&\qquad-f(-\overline{D}^2\nabla^2/16\Lambda^2)\overline{D}^2\frac{1}{\nabla^2\overline{D}^2/16-m^\dagger m}\nabla^2\overline{D}_{\dot\alpha}\nabla_\alpha\overline{D}^{\dot\alpha}\\
&\qquad-\overline{D}^2\nabla_\alpha f(-\overline{D}^2\nabla^2/16\Lambda^2)\overline{D}^2\frac{1}{\nabla^2\overline{D}^2/16-m^\dagger m}\nabla^2\Bigg]\delta(z-z').
\end{aligned}\tag{5·14}$$

Then we use identities

$$\begin{aligned}
\nabla^2\overline{D}_{\dot\alpha}\nabla_\alpha\overline{D}^{\dot\alpha}&=-\frac{1}{2}\nabla^2\overline{D}^2\nabla_\alpha-2\nabla^2W_{B\alpha},\\
\overline{D}^2\nabla_\alpha\overline{D}^2&=-4W_{B\alpha}\overline{D}^2
\end{aligned}\tag{5·15}$$

to yield

$$\begin{aligned}
&\overline{D}^{\dot{\alpha}}\left\langle R^{\rm chiral}_{\alpha\dot{\alpha}}(z)\right\rangle - 2\left\langle \Phi_Q^T m \nabla_\alpha \Phi_Q(z)\right\rangle + \frac{2}{3}D_\alpha\left\langle \Phi_Q^T m \Phi_Q(z)\right\rangle \\
&\quad = \frac{i}{2}\lim_{z'\to z}\operatorname{tr}\nabla_\alpha f(-\overline{D}^2\nabla^2/16\Lambda^2)\overline{D}^2\delta(z-z') \\
&\quad\quad -\frac{i}{6}D_\alpha\lim_{z'\to z}\operatorname{tr} f(-\overline{D}^2\nabla^2/16\Lambda^2)\overline{D}^2\delta(z-z') \\
&\quad\quad +2\frac{i}{16}\lim_{z'\to z}\operatorname{tr} W_{B\alpha} f(-\overline{D}^2\nabla^2/16\Lambda^2)\overline{D}^2\frac{1}{\nabla^2\overline{D}^2/16-m^\dagger m}\nabla^2\delta(z-z'), \quad (5\cdot16)
\end{aligned}$$

where we have used (5·12) again. In this expression, the first two lines on the right-hand side are regarded as the anomaly, and the last line can be interpreted as a composite operator. From (A·6) and (A·9) in the Appendix, we finally obtain

$$\begin{aligned}
&\overline{D}^{\dot{\alpha}}\left\langle R^{\rm chiral}_{\alpha\dot{\alpha}}(z)\right\rangle - 2\left\langle \Phi_Q^T m \nabla_\alpha \Phi_Q(z)\right\rangle + \frac{2}{3}D_\alpha\left\langle \Phi_Q^T m \Phi_Q(z)\right\rangle \\
&\overset{\Lambda\to\infty}{=} -\frac{1}{8\pi^2}\left[\Lambda^2\int_0^\infty dt\, f(t)+\frac{1}{6}\Box\right]\operatorname{tr} W_{B\alpha}(z)+\frac{1}{192\pi^2}D_\alpha \operatorname{tr} W_B^\beta W_{B\beta}(z) \\
&\quad + 2\operatorname{tr} W_{B\alpha}(z)\left\langle \Phi_Q(z)\Phi_Q^\dagger(z)\right\rangle e^{V_B(z)}. \quad (5\cdot17)
\end{aligned}$$

The presence of the last term is expected, because the classical equation of motion of the gauge multiplet V leads to

$$\overline{D}^{\dot{\alpha}} R^{\rm gauge}_{\alpha\dot{\alpha}} = -2\operatorname{tr} W_\alpha \Phi\Phi^\dagger e^V. \quad (5\cdot18)$$

The right-hand side of this classical expression, when defined to one-loop accuracy, cancels the last term of (5·17). Therefore, the superconformal anomaly becomes

$$\begin{aligned}
&\overline{D}^{\dot{\alpha}}\left\langle R_{\alpha\dot{\alpha}}(z)\right\rangle - 2\left\langle \Phi_Q^T m \nabla_\alpha \Phi_Q(z)\right\rangle + \frac{2}{3}D_\alpha\left\langle \Phi_Q^T m \Phi_Q(z)\right\rangle \\
&\overset{\Lambda\to\infty}{=} -\frac{1}{8\pi^2}\left[\Lambda^2\int_0^\infty dt\, f(t)+\frac{1}{6}\Box\right]\operatorname{tr} W_{B\alpha}(z)+\frac{1}{192\pi^2}D_\alpha \operatorname{tr} W_B^\beta W_{B\beta}(z), \quad (5\cdot19)
\end{aligned}$$

when only the one-loop quantum effect of the chiral multiplet is considered.

We note that although the first term of the superconformal anomaly (5·19) is perhaps not familiar, it is perfectly consistent with the anomaly multiplet structure of the superconformal anomaly;[18] in particular it belongs to the linear anomaly multiplet.[19),37] It is interesting to note that this term exists only when the gauge-gravitational mixed anomaly exists, i.e., when $\operatorname{tr} T^a \neq 0$.

Now, as we have done for the Konishi anomaly in (5·7), we may derive from the superconformal anomaly (5·19) the anomalous "central extension" of the $N=1$ supersymmetry algebra which has recently been advocated by Shifman and co-workers.[25] An analysis of this problem from the viewpoint of path integrals and the Bjorken-Johnson-Low prescription is given in Ref. 38). We first note the definition of the supercharge:

$$\overline{Q}_{\dot{\alpha}} = \int d^3x\, \overline{J}^0_{\dot{\alpha}}, \qquad \overline{J}^m_{\dot{\alpha}} = -\frac{1}{2}\overline{\sigma}^{m\dot{\beta}\beta}\overline{J}_{\dot{\alpha}\beta\dot{\beta}}. \quad (5\cdot20)$$

The *improved* supercurrent $\overline{J}_{\dot{\beta}\dot{\alpha}\alpha}$ is related to the superconformal current $R_{\alpha\dot{\alpha}}$ as[18]

$$R_{\alpha\dot{\alpha}} = R^{(0)}_{\alpha\dot{\alpha}} - i\overline{\theta}_{\dot{\beta}} \left(\overline{J}^{\dot{\beta}}{}_{\dot{\alpha}\alpha} - \frac{2}{3}\delta^{\dot{\beta}}_{\dot{\alpha}}\overline{J}^{\dot{\gamma}}{}_{\dot{\gamma}\alpha}\right) + \cdots, \tag{5·21}$$

where the first component $R^{(0)}_{\alpha\dot{\alpha}}$ is the R-current. From these relations, we have classically

$$\overline{J}^0_{\dot{\alpha}} = -\frac{i}{2}\overline{\sigma}^{0\dot{\beta}\beta}\left(\overline{D}_{\dot{\alpha}}R_{\beta\dot{\beta}} - 2\varepsilon_{\dot{\alpha}\dot{\beta}}\overline{D}^{\dot{\gamma}}R_{\beta\dot{\gamma}}\right)\Big|_{\theta=\overline{\theta}=0}. \tag{5·22}$$

Therefore we may define the supersymmetric transformation of the supercurrent operator as

$$\begin{aligned}\left\langle\{\overline{Q}_{\dot{\alpha}}, \overline{J}^0_{\dot{\beta}}(x)\}\right\rangle &\equiv -\frac{i}{2}\overline{\sigma}^{0\dot{\gamma}\gamma}\overline{Q}_{\dot{\alpha}}\left[\overline{D}_{\dot{\beta}}\left\langle R_{\gamma\dot{\gamma}}(z)\right\rangle - 2\varepsilon_{\dot{\beta}\dot{\gamma}}\overline{D}^{\dot{\delta}}\left\langle R_{\gamma\dot{\delta}}(z)\right\rangle\right]\Big|_{\theta=\overline{\theta}=0} \\ &= \frac{i}{2}\overline{\sigma}^{0\dot{\gamma}\gamma}(\varepsilon_{\dot{\alpha}\dot{\beta}}\overline{D}_{\dot{\gamma}} + 2\varepsilon_{\dot{\beta}\dot{\gamma}}\overline{D}_{\dot{\alpha}})\,\overline{D}^{\dot{\delta}}\left\langle R_{\gamma\dot{\delta}}(z)\right\rangle\Big|_{\theta=\overline{\theta}=0} \\ &= -\frac{1}{48\pi^2}\overline{\sigma}^{0m}_{\dot{\alpha}\dot{\beta}}\partial_m \operatorname{tr}\lambda^{\alpha}_{B}\lambda_{B\alpha}(x),\end{aligned} \tag{5·23}$$

where we have used the superconformal anomaly (5·19).*) The "central extension" of the $N=1$ supersymmetry algebra[25] is obtained by integrating (5·23) over the spatial coordinate x. (Recall that we have taken into account only the chiral matter loop in (5·19).) In deriving the second line from the first line in (5·23), we have used the identity (see the last reference of 25))

$$\overline{D}_{\dot{\alpha}}\overline{D}_{\dot{\beta}}X_{\gamma\dot{\gamma}}\Big|_{\theta=\overline{\theta}=0} = -\varepsilon_{\dot{\alpha}\dot{\beta}}\,\overline{D}_{\dot{\gamma}}\overline{D}^{\dot{\delta}}X_{\gamma\dot{\delta}}\Big|_{\theta=\overline{\theta}=0}, \tag{5·24}$$

which may be confirmed component by component. From the above derivation, it is clear that use of the improved supercurrent (5·21) and the presence of the superconformal anomaly (5·19) are crucial to have the anomalous term. Although our derivation in (5·23) is apparently almost identical to that in Ref. 25), it is conceptually quite transparent, as we have explained for the Konishi anomaly (5·7).

In a similar way, we may study another anomalous supersymmetric transformation law, the commutator between the supercharge and the R-current (or charge).[24] The R-current is defined from the superconformal current (5·21) by

$$\begin{aligned}R^{(0)m} &= -\frac{1}{2}\overline{\sigma}^{m\dot{\alpha}\alpha}R^{(0)}_{\alpha\dot{\alpha}} \\ &= -\frac{1}{2}\overline{\sigma}^{m\dot{\alpha}\alpha}\,R_{\alpha\dot{\alpha}}\big|_{\theta=\overline{\theta}=0}.\end{aligned} \tag{5·25}$$

Then the supersymmetric transformation of the R-current is related to the superconformal anomaly as

$$\left\langle[\overline{Q}_{\dot{\alpha}}, R^{(0)0}(x)]\right\rangle \equiv -\frac{1}{2}\overline{\sigma}^{0\dot{\beta}\beta}\,\overline{Q}_{\dot{\alpha}}\left\langle R_{\beta\dot{\beta}}(z)\right\rangle\Big|_{\theta=\overline{\theta}=0}$$

*) We have set $m=0$ for simplicity.

$$= -i\left\langle \overline{J}^0_{\dot\alpha}(x)\right\rangle - \overline{\sigma}^{0\dot\beta\beta}\varepsilon_{\dot\alpha\dot\beta}\,\overline{D}^{\dot\gamma}\,\langle R_{\beta\dot\gamma}(z)\rangle\Big|_{\theta=\overline{\theta}=0}$$

$$= -i\left\langle \overline{J}^0_{\dot\alpha}(x)\right\rangle + \frac{i}{8\pi^2}\left[\Lambda^2\int_0^\infty dt\, f(t) + \frac{1}{6}\Box\right]\operatorname{tr}\lambda^\alpha_B(x)\sigma^0_{\alpha\dot\alpha}$$

$$-\frac{i}{48\pi^2}\operatorname{tr}\lambda^\alpha_B(x)\left[\frac{1}{2}\sigma^0_{\alpha\dot\alpha}D_B(x) + \sigma^m_{\alpha\dot\alpha}v^{+0}_{B\ \ m}(x)\right], \qquad (5{\cdot}26)$$

where we have used the relation (5·22) and the superconformal anomaly (5·19).*) In the last line, v^+_{mn} is the self-dual part of the field strength:

$$v^+_{mn} \equiv \frac{1}{2}\left(v_{mn} + \frac{i}{2}\varepsilon_{mnkl}v^{kl}\right). \qquad (5{\cdot}27)$$

In Ref. 24), an anomalous term in the commutator (5·26) is analyzed in the supersymmetric pure Yang-Mills theory with use of the gauge invariant point splitting regularization. Although the structure of our result (5·26) is not quite the same as that of Ref. 24) (the anomalous term is given by the dual of the field strength instead of the self-dual part), the discrepancy seems to originate from the definition of the regularized R-current (see also Ref. 38)).

Similar calculations are possible also for the superconformal anomaly emerging from the gauge multiplet loop.[16),39),40)] We will report on this part of analysis elsewhere.[34)]

§6. Conclusion

In this paper, we have formulated a manifestly supersymmetric gauge covariant regularization of supersymmetric chiral gauge theories. As we have shown, this scheme has many desired features which are, we believe, not shared by other single regularization schemes proposed to this time.**) Certainly, we must admit that we did not mention the unitarity of the S-matrix which, with the BRST (or quantum gauge) symmetry, is not manifest in our regularization scheme. This point is concerned with the issue of how the effective action in the background field method and the S-matrix are related[2),3)] *at the regularized level.* We are at present unable to make a general statement on this point. However, it is obvious that the background gauge invariance of the effective action (or 1PI diagrams) is certainly a *necessary condition* for the unitarity of the S-matrix, because otherwise the longitudinal mode does not decouple from the S-matrix. We emphasize that our scheme, to our knowledge, is the first attempt which fulfills this necessary condition for supersymmetric *chiral* gauge theories.

We have presented several illustrative applications, but only in the one-loop approximation. The situation for multi-loop diagrams in our scheme, on the other

*) We have again set $m = 0$ for simplicity.

**) Strictly speaking, the integrability of (3·25) in anomaly-free cases remains to be proven to definitely conclude the existence of the effective action in such cases (although we are almost sure of this point from experience in non-supersymmetric theories[13)]). This analysis regarding the integrability will be reported elsewhere.[34)] hand, is rather simple, because the supersymmetry and the gauge invariance of the effective action are always ensured. Although this property is similar to that of the supersymmetric higher covariant derivative regularization,[8] our calculation rule is reasonably simple compared to the higher derivative regularization. Therefore it is of great interest to see how actual higher loop calculations proceed in our scheme. We hope to come back this problem in the near future.

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Appendix

In this appendix, we explain the evaluation of the anomalous factors,

$$\lim_{z'\to z} \operatorname{tr} \begin{Bmatrix} 1 \\ \nabla_\alpha \end{Bmatrix} f(-\overline{D}^2\nabla^2/16\Lambda^2)\overline{D}^2\delta(z-z'). \tag{A·1}$$

Our calculation is basically a generalization of that in Ref. 15). However, the actual calculation is somewhat quicker, thanks to our manifestly gauge covariant treatment. First, let us recall that the abbreviation rule (3·10) is assumed in (A·1). Next, we note the momentum representation of the delta function and

$$\begin{aligned} e^{-ikx}\overline{D}_{\dot\alpha}e^{ikx} &= \overline{D}_{\dot\alpha} + \theta^\alpha\sigma^m_{\alpha\dot\alpha}k_m, \\ e^{-ikx}\nabla_m e^{ikx} &= \nabla_m + ik_m, \\ e^{-ikx}\nabla_\alpha e^{ikx} &= \nabla_\alpha - \sigma^m_{\alpha\dot\alpha}\overline{\theta}^{\dot\alpha}k_m, \end{aligned} \tag{A·2}$$

and thus (A·1) can be expressed as

$$\begin{aligned} &\operatorname{tr}\Lambda^4 \int \frac{d^4k}{(2\pi)^4} \begin{Bmatrix} 1 \\ \nabla_\alpha - \sigma^m_{\alpha\dot\alpha}\overline{\theta}^{\dot\alpha}\Lambda k_m \end{Bmatrix} \\ &\times f\Bigg(k^n k_n - 2ik^n\nabla_n/\Lambda - W_B\sigma^n\overline{\theta}k_n/(2\Lambda) \\ &\qquad -\nabla^n\nabla_n/\Lambda^2 + W^\beta_B\nabla_\beta/(2\Lambda^2) + (\mathcal{D}^\beta W_{B\beta})/(4\Lambda^2)\Bigg) \\ &\times(\overline{D}_{\dot\gamma} + \theta^\gamma\sigma^k_{\gamma\dot\gamma}\Lambda k_k)(\overline{D}^{\dot\gamma} + \theta^\delta\sigma^{l\dot\gamma}_\delta\Lambda k_l)\ \delta(\theta-\theta')\delta(\overline{\theta}-\overline{\theta}')\Big|_{\theta=\theta',\overline{\theta}=\overline{\theta}'}, \end{aligned} \tag{A·3}$$

where we have rescaled the momentum variable as $k_m \to \Lambda k_m$. Then we expand $f(\cdots)$ in inverse powers of Λ. At this step, it should be noted that at least two spinor derivatives have to act on the delta function to survive:

$$\overline{D}^2\ \delta(\overline{\theta}-\overline{\theta}')\Big|_{\overline{\theta}=\overline{\theta}'} = -4, \qquad \nabla_\alpha\nabla_\beta\ \delta(\theta-\theta')|_{\theta=\theta'} = -2\varepsilon_{\alpha\beta}. \tag{A·4}$$

But also note that three are too many:

$$\nabla_\alpha \nabla_\beta \nabla_\gamma = 0. \tag{A·5}$$

Then it is easy to see that the first case of (A·3) gives[15)]

$$\begin{aligned}
&\lim_{z'\to z} \operatorname{tr} f(-\overline{D}^2\nabla^2/16\Lambda^2)\overline{D}^2\delta(z-z') \\
&\quad \overset{\Lambda\to\infty}{=} -\int \frac{d^4k}{(2\pi)^4} f''(k^m k_m) \operatorname{tr} W_B^\alpha W_{B\alpha} \\
&\quad = -\frac{i}{16\pi^2} \operatorname{tr} W_B^\alpha W_{B\alpha},
\end{aligned} \tag{A·6}$$

where we have used the value of the momentum integration

$$\int \frac{d^4k}{(2\pi)^4} \left\{ \begin{array}{c} f'(k^m k_m) \\ f''(k^m k_m) \\ f'''(k^m k_m) k^n k^k \end{array} \right\} = \frac{i}{16\pi^2} \left\{ \begin{array}{c} -\int_0^\infty dt\, f(t) \\ 1 \\ -\eta^{nk}/2 \end{array} \right\}, \tag{A·7}$$

which is useful also for the second case in (A·3).

On the other hand, the necessary expansion is somewhat lengthy in the second case of (A·3), and we do not reproduce it here. It might be useful, however, to note the following: The straightforward expansion of (A·3) contains terms explicitly proportional to $\overline{\theta}^2$ and $\overline{\theta}_{\dot\alpha}$ which, at first glance, do not seem to vanish. If these terms survive, the right-hand side of (A·3) cannot be a superfield, because the lowest component ($\theta = \overline{\theta} = 0$ term) of these terms vanishes (and thus the higher components must vanish if it is a superfield[26)]). In fact, it can be confirmed that the terms explicitly proportional to $\overline{\theta}^2$ and $\overline{\theta}_{\dot\alpha}$ vanish by noting relations such as

$$\operatorname{tr} W_{B\alpha} W_B^\beta W_{B\beta} = 0, \qquad \nabla_m W_{B\alpha} W_{B\beta} - \nabla_m W_{B\beta} W_{B\alpha} = \varepsilon_{\alpha\beta} \nabla_m W_B^\gamma W_{B\gamma}, \tag{A·8}$$

where the first relation follows from the cyclic property of the trace.

Finally, after some rearrangements, we obtain

$$\begin{aligned}
&\lim_{z'\to z} \operatorname{tr} \nabla_\alpha f(-\overline{D}^2\nabla^2/16\Lambda^2)\overline{D}^2\delta(z-z') \\
&\quad \overset{\Lambda\to\infty}{=} \frac{i}{4\pi^2}\left[\Lambda^2 \int_0^\infty dt\, f(t) + \frac{1}{6}\Box\right] \operatorname{tr} W_{B\alpha} - \frac{i}{32\pi^2} D_\alpha \operatorname{tr} W_B^\beta W_{B\beta}.
\end{aligned} \tag{A·9}$$

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Note added: By using the technique presented in the Appendix, it is easy to obtain the *divergent part* of the gauge current (3·25) for an *arbitrary* $f(t)$:

$$\begin{aligned} &\Lambda\frac{d}{d\Lambda}\langle J^a(z)\rangle \\ &\overset{\Lambda\to\infty}{=}\frac{\delta}{\delta V^a(z)}\left[\frac{\Lambda^2}{8\pi^2}\int_0^1 dt\, f(t)\int d^8z\ \mathrm{tr}\, V_B+\frac{T(R)-3C_2(G)}{64\pi^2}\int d^6z\ \mathrm{tr}\, W_B^\alpha W_{B\alpha}\right]. \end{aligned}$$

This shows that the divergent part is always (i.e., whether or not the gauge anomaly is present) integrable, and the one-loop β-function is actually independent of $f(t)$.