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SOFT BREAKING OF SUPERSYMMETRY

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Using superfield methods we discuss systematically explicit soft breaking of global supersymmetry. We find that, in a component field language, dimension-two operators are soft while, in general, dimension-three operators are not and introduce quadratic divergences not present in the unbroken theory. At the one-loop level we give a parallel discussion based on the effective potential.

1. Introduction

Recently there has been some interest in studying models with explicit softly broken supersymmetry with possible phenomenological applications in mind [1,2]. One of the reasons for this interest stems from the fact that in supersymmetric theories certain divergences, in particular quadratic ones, are absent and consequently such theories enjoy certain naturalness properties not shared by general renormalizable theories.

In theories with spontaneously broken supersymmetry the quadratic divergences are still absent, but such theories are notoriously rigid and offer little hope for building realistic models. For example the mass formula, generally valid in such cases [3], $\Sigma(-1)^{2J}(2J+1)m_J^2=0$ makes it difficult to construct a spontaneously broken supersymmetry GUT model where the unwanted supersymmetric partners of the particles of the low-energy regime have decoupled by acquiring large masses. On the other hand, explicit breaking, via addition of large mass terms for the unwanted particles, evidently bypasses the mass formula and leads to the construction of some interesting models [1,2]. The question has been raised, however, whether such

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breaking is soft, in the sense of not generating unwanted quadratic divergences, and for special cases the answer is yes [3–5]. Obviously the general situation deserves investigation and in particular the following questions require answers: (i) In a general globally supersymmetric theory what explicit breakings are soft, in the sense of not generating quadratic divergences? (ii) For a given explicit breaking what new (logarithmically infinite) terms are generated in the effective action so as to require introduction of new counterterms and hence new parameters in the lagrangian?

In this paper we attempt to answer these questions. We believe that the natural setting for studying the problem is in a superfield formalism. Indeed, in such a formalism the answers are almost trivial to obtain to all orders of perturbation theory and provide us with a supersymmetric version of Symanzik's criterion [6]. The main observation is that in a superfield language supersymmetry is equivalent to translational invariance in superspace, the space of x's and θ's. Giving some superfield $\Psi(x,\theta,\bar{\theta})$, a fixed value is equivalent to breaking this invariance. This then suggests the following procedure: Given a supersymmetric (superfield) action, we couple to the quantum fields, in a manifestly supersymmetric fashion, some external (spurion) superfields. Supersymmetry breaking is achieved by giving these superfields suitable fixed values. Soft breaking is achieved by only allowing couplings which are consistent with the (power counting) renormalizability criteria of superfield perturbation theory so that no new *types* of divergences are introduced. New induced infinities are read off from the conventional counterterms involving products of the quantum and spurion fields. Generically we find that symmetry-breaking terms of dimension two (masses for spin-zero fields) are soft, but *terms of dimension three are not*. For example adding an arbitrary A^3 or a $\bar{\psi}\psi$ term (where A, ψ are the scalar and spinor of a scalar multiplet) does not break the supersymmetry softly: quadratic divergences are generated. On this point we disagree with the authors of ref. [5]. The only allowed dimension-three terms are the particular combination $A^3 - 3AB^2$ for a scalar multiplet and a mass term $\bar{\lambda}\lambda$ for the spinor of a vector multiplet.

Our paper is organized as follows: In sect. 2 we give our results in component field form. In sect. 3 we justify our results by examining their superfield counterparts. Sect. 4 gives an alternative way of understanding the results, at the one-loop level, in terms of the effective potential.

2. Soft breaking of supersymmetry: component results

A typical renormalizable globally supersymmetric lagrangian contains the spin-zero fields A_i, B_i and spin-$\frac{1}{2}$ fields ψ_i of a scalar multiplet, and the spin-$\frac{1}{2}$ fields λ_j and spin-one fields $V_{\mu j}$ of a vector multiplet as well as the auxiliary fields F_i, G_i, D_j. If F_i and G_i have been eliminated, one obtains (common) mass terms, and cubic, Yukawa and quartic interactions for the particles of scalar multiplets, all governed by a common coupling constant [7]. If D_j has been eliminated, one finds, in addition to gauge self-couplings of $V_{\mu j}$ and λ_j (in the non-abelian case), also couplings to the fields of the scalar multiplet and a quartic A_i, B_i interaction all governed by the same gauge coupling constants g_j. Supersymmetry may be spontaneously broken if some F_i or D_j acquires a vacuum expectation value, but this does not affect our arguments; nor are they affected by breaking of some internal symmetry. If the theory is regularized in a supersymmetric fashion (or Ward identities are imposed) one only finds certain logarithmic divergences except for a possible quadratically divergent linear D term [if associated with a U(1) gauge group]. In the following we shall omit internal symmetry labels and use a symbolic notation, e.g., $A^2 = c_{ij}A_iA_j$, etc.

We give now a list of possible soft breakings of supersymmetry which we believe to be exhaustive. We add breaking terms to the lagrangian and exhibit the new infinities they generate. We need not discuss breaking terms linear in the fields. The case of a linear A term has been studied in the classic paper of Iliopoulos and Zumino [8], while linear F or D terms do not lead to explicit breaking. In the following we do not include in the discussion renormalizations of the breaking terms which correspond to an overall (common to the multiplet) wave-function renormalization, and which are always to be understood.

(a)
$$\mathcal{L}_{\text{break}} = \mu^2(A^2 + B^2), \tag{1}$$

a common mass term for the spin-zero fields of a scalar multiplet. This case together with the following has been discussed recently in ref. [5] by means of Ward identities. There is now a logarithmic divergence which requires a renormalization of the parameter μ. In addition a logarithmically divergent A term is present and a logarithmically divergent D term may be generated if the scalar multiplet to which A, B belongs couples to a U(1) gauge superfield.

(b)
$$\mathcal{L}_{\text{break}} = \mu^2(A^2 - B^2), \tag{2}$$

giving equal and opposite additions to the masses of the spin-zero fields of a scalar multiplet. A logarithmically divergent F term may be generated.

(c)
$$\mathcal{L}_{\text{break}} = \mu\bar{\lambda}\lambda, \tag{3}$$

giving a mass term to the fermion of a vector multiplet. One finds a logarithmic divergence which requires a renormalization of μ and in addition logarithmically divergent A, F, $FA - GB$ and $A^2 + B^2$ terms may be generated for some scalar multiplets.

(d)
$$\mathcal{L}_{\text{break}} = \gamma(A^3 - 3AB^2), \tag{4}$$

a non-supersymmetric interaction term. Induced logarithmically divergent terms are

$A, F, FA - GB, A^2 + B^2$ and $\bar{\lambda}\lambda$. We note that cases (b) and (d) correspond to adding to the lagrangian terms of the form $\mathrm{Re}\, f(A + iB)$, where $f(z) = z^2$ or z^3, respectively. It has already been observed [3] that these are cases of explicit breaking which maintain the mass formula of spontaneously broken supersymmetry. $\mathcal{L}_{\text{break}} = FA - GB$ is a linear combination of (a) and (d).

This list essentially exhausts all cases of soft supersymmetry breaking. Linear combinations may be possible. We now discuss some cases of breaking which are *not soft*.

(a′)
$$\mathcal{L}_{\text{break}} = \mu\bar{\psi}\psi, \tag{5}$$

giving a mass term to the fermion of a scalar multiplet. Quadratic divergences are induced, for example in a linear A term, (which will lead to a quadratic renormalization of μ) as well as logarithmically divergent mass terms for all members of the scalar multiplet.

(b′)
$$\mathcal{L}_{\text{break}} = \alpha A^3, \tag{6}$$

or any other non-supersymmetric interaction term (except for the case discussed above). Again quadratic divergences are induced, for example in a linear A term.

Case (a′) might appear surprising at first. It is certainly allowed to break the mass equality between bosons and fermions by adding a term $\pm\mu^2(A^2 + B^2)$ to the scalars so as to end up with masses $\sqrt{m^2 \pm \mu^2}$, $\sqrt{m^2 \pm \mu^2}$ and m, but not by adding a term $\mu\bar{\psi}\psi$ to the fermions. The point is that in general m also appears in some of the interactions, e.g., $mgA(A^2 - B^2)$ and it is the relation of fermion mass to coupling that keeps divergences under control. If it is destroyed quadratic divergences appear, for example when the mass pattern is $m, m, m + \mu$.

3. Soft breaking: the superspace picture

A conventional renormalizable globally supersymmetric lagrangian has the form [9] (we use two-component spinor notation)

$$\mathcal{L} = \int d^4\theta \left[\bar{\phi}_i e^V \phi_i + V\right] + \int d^2\theta \left[W^\alpha W_\alpha + P(\phi_i)\right] + \text{h.c.} \tag{7}$$

Here

$$\phi = \sqrt{\tfrac{1}{2}}\,(A + iB) + \theta^\alpha\psi_\alpha + \sqrt{\tfrac{1}{2}}\,\theta^2(F - iG) = z + \theta^\alpha\psi_\alpha + \theta^2 h \tag{8}$$

is a chiral (left-handed) superfield (scalar multiplet) in chiral representation. In the same representation $\bar{\phi} = \exp(-2i\theta\sigma\cdot\partial\bar{\theta})\phi^*$. $P(\phi_i)$ is a polynomial at most cubic in the ϕ's and (in Wess-Zumino gauge)

$$V = \theta\sigma^\mu\bar{\theta}V_\mu + \theta^2\bar{\theta}^{\dot{\alpha}}\bar{\lambda}_{\dot{\alpha}} + \bar{\theta}^2\theta^\alpha\lambda_\alpha + \tfrac{1}{2}\theta^2\bar{\theta}^2 D \tag{9}$$

is a general real gauge superfield (vector multiplet). For any gauge group with generators T_α, $V=gV^\alpha T_\alpha$ and ϕ_i may carry a representation of the group. Finally $W_\alpha=\bar{D}^2[\mathrm{e}^{-V}, D_\alpha \mathrm{e}^V]$ is the (chiral) field strength where $D_\alpha, \bar{D}_{\dot\alpha}$ are superspace covariant derivatives. The internal symmetry may dictate the absence of the linear V term or certain terms in $P(\phi_i)$.

Superfield Feynman rules use the propagators [10,11]

$$\langle\bar{\phi}\phi\rangle=\frac{1}{p^2+m^2}\delta^4(\theta-\theta'),$$

$$\langle\phi\phi\rangle=\frac{mD^2}{p^2(p^2+m^2)}\delta^4(\theta-\theta'),$$

$$\langle VV\rangle=-\frac{1}{p^2}\delta^4(\theta-\theta'). \tag{10}$$

Each vertex has associated with it an integral $\mathrm{d}^4\theta$ and factors $\bar{D}^2$ (D^2) acting on the propagators for each ϕ ($\bar{\phi}$) line leaving the vertex (but omitting one such factor if the corresponding term in the Lagrangian contains only a $\mathrm{d}^2\theta$ integral). Other, explicit factors of $D_\alpha, \bar{D}_{\dot\alpha}$ appearing in the lagrangian (e.g., from W_α) act on the propagators as well. What makes the lagrangian in eq. (7) renormalizable is the fact that in any graph, *at most* four factors of $D_\alpha, \bar{D}_{\dot\alpha}$ are present at each vertex. Finally it can be shown [10] that the whole effective action can be written as an expression *local* in θ:

$$\Gamma(\phi,\bar{\phi},V)=\int\mathrm{d}^4x_i\mathrm{d}^4\theta\, G(\phi,\bar{\phi},V,D_\alpha\phi\ldots;x_i,\theta). \tag{11}$$

Power counting leads to the following result [11]: The degree of divergence of any graph is

$$d=2-E_\mathrm{c}-M_\mathrm{c} \qquad (-1 \text{ for a graph with only chiral or antichiral external lines}),$$

where E_c is the number of external chiral and antichiral lines, and M_c is the number of internal (massive) $\langle\phi\phi\rangle$ and $\langle\overline{\phi\phi}\rangle$ propagators. This result and its implications for possible divergences are easy to understand by means of the following dimensional argument: Divergent graphs correspond to local terms in the effective action of the form

$$\Gamma_\infty(\phi,\bar{\phi},V)=\int\mathrm{d}^4x\mathrm{d}^4\theta\,\mathcal{P}(\phi,\bar{\phi},V,D_\alpha\phi,\ldots), \tag{12}$$

where $\mathcal{P}$ is a polynomial in the fields and their derivatives and the integration is *always* $\mathrm{d}^4\theta$ (never $\mathrm{d}^2\theta$ or $\mathrm{d}^2\bar{\theta}$) [10]. Since $\mathrm{d}^4\theta$ has dimension two, $\mathcal{P}$ must also have dimension 2. ϕ has dimension one, D_α has dimension one-half, while V is dimensionless. Thus, graphs with more than two external ϕ's are convergent. A $\langle\phi\phi\rangle$ or $\langle\overline{\phi\phi}\rangle$ propagator produces a numerator factor of m which contributes to the dimension of $\mathcal{G}$. If $\mathcal{G}$ is made up of only chiral fields the θ-integration gives zero unless some D_α's (at least two then) are present to make the integrand non-chiral, and again the D_α's contribute to the dimension of $\mathcal{G}$ reducing the number of fields that can be present. Finally, *if* gauge invariance requires V to appear through its field strength $W_\alpha \sim \bar{D}^2 D_\alpha V$, this again limits the possible divergences involving V fields. (We are oversimplifying; one must use the full machinery of Slavnov-Taylor identities, at least in the non-abelian case [12].) The net result of the analysis is to establish that the only divergent terms in the effective action have the form (we omit some factors of D_α, $\bar{D}_{\dot\alpha}$)

$$\int d^4\theta\, \bar{\phi}\phi, \qquad \int d^4\theta\, \bar{\phi} V \phi,$$
$$\int d^4\theta\, VV, \qquad \int d^4\theta\, VVV, \qquad \int d^4\theta\, V. \tag{13}$$

(Ghosts are described by chiral superfields which follow the same rules.) The divergences are all logarithmic, except for the last one which is quadratic. Terms proportional to ϕ^2 or ϕ^3 are finite.

We shall break supersymmetry softly by coupling to a supersymmetric system external spurion superfields in a manner consistent with the power counting criteria for renormalizability: No more than four factors of D_α or $\bar{D}_{\dot\alpha}$ should appear at any vertex where the spurion field is inserted. Supersymmetry will be broken by giving these superfields fixed, x-independent, θ-dependent values since this destroys the translational invariance in super-space: $\Psi(x,\theta,\bar\theta) \to \Psi(x+\varepsilon\sigma\bar\theta+\theta\sigma\bar\varepsilon, \theta+\varepsilon, \bar\theta+\bar\varepsilon)$. The list of possibilities is quite short. We now discuss each case, correlating it to the cases of the previous section. The lagrangians given there are obtained from the ones below by doing the θ integration. In addition to the original divergences containing only the quantum fields we will generate new ones, involving the spurion fields as well and they are the ones we are interested in. We will not consider divergences involving the spurion fields only which correspond to insertions into vacuum diagrams.

(a) $$\mathcal{L}_{\text{break}} = \int d^4\theta\, U\bar{\phi}\phi = \tfrac{1}{2}\mu^2(A^2+B^2), \tag{14}$$

where $U = \mu^2\theta^2\bar\theta^2$ is a dimension zero general superfield. Then the power counting rules and the dimensionality of U indicate that a renormalization of the term itself (i.e., of μ^2) is needed, a logarithmically divergent $U\phi$ term may be generated (if some chiral superfield is massive) and, if a U(1) gauge field is present, a logarithmically divergent (because of gauge invariance) UV term is produced:

$$\Delta\mathcal{L} \sim \int \mathrm{d}^4\theta\, U\phi + \int \mathrm{d}^4\theta\, UD^\alpha W_\alpha + \text{h.c.} \sim \mu^2 mA + \mu^2 D \tag{15}$$

may be induced.

(b)
$$\mathcal{L}_{\text{break}} = \int \mathrm{d}^2\theta\, \chi\phi^2 + \text{h.c.} = \mu^2(A^2 - B^2), \tag{16}$$

where $\chi = \mu^2\theta^2$ is a dimension one chiral superfield. New (logarithmic) divergences correspond to

$$\Delta\mathcal{L} \sim \int \mathrm{d}^4\theta\, \bar{\chi}\phi + \text{h.c.} \sim F, \tag{17}$$

so that a linear F term is induced.

Since χ is neutral under whatever internal symmetry group is present (hence ϕ^2 must stand for some invariant chiral superfield product) no infinities involving gauge superfields are generated.

(c)
$$\mathcal{L}_{\text{break}} = \int \mathrm{d}^2\theta\, \eta W^\alpha W_\alpha + \text{h.c.}, \tag{18}$$

where $\eta = \mu\theta^2$ is a neutral, *dimension-zero* chiral superfield. Since $W_\alpha = \lambda_\alpha + \theta$-dependent terms, this just generates a fermion mass term. While this is not a situation which one encounters in global supersymmetry (η is like the compensating field of supergravity [13]), it gives vertices with just four factors of D_α (after using one $\bar{D}^2$ from W_α to convert the $\mathrm{d}^2\theta$ to a $\mathrm{d}^4\theta$ integral) and hence it is admissible. Since η is dimensionless the following logarithmically divergent terms may be generated (in addition to $\mathcal{L}_{\text{break}}$ itself):

$$\Delta\mathcal{L} \sim \int \mathrm{d}^4\theta\, \bar{\eta}\phi + \text{h.c.} \sim F, \tag{19}$$

$$\Delta\mathcal{L} \sim \int \mathrm{d}^4\theta\, \eta\bar{\phi}\phi + \text{h.c.} \sim FA - GB, \tag{20}$$

$$\Delta\mathcal{L} \sim \int \mathrm{d}^4\theta (\eta\bar{\eta}\phi + \text{h.c.}) \sim A \tag{20$'$}$$

(at the two loop level first, and only if a massive chiral superfield is present) and

$$\Delta\mathcal{L} \sim \int \mathrm{d}^4\theta\, \bar{\eta}\eta\bar{\phi}\phi \sim A^2 + B^2. \tag{21}$$

(d)
$$\mathcal{L}_{\text{break}} = \int \mathrm{d}^2\theta\, \eta\phi^3 + \text{h.c.} = \sqrt{\tfrac{1}{2}}\,\mu(A^3 - 3AB^2). \tag{22}$$

In general one induces the same logarithmically divergent terms as in case (c) above. $\mathcal{L}_{\text{break}} = \int d^4\theta(\eta + \bar{\eta})\phi\bar{\phi}$ can be reduced by a field redefinition to a linear combination of (a), (b) and (d).

This completes the discussion of soft breaking possibilities. We now comment on some hard breaking cases:

$$\text{(a}'\text{)} \qquad \mathcal{L}_{\text{break}} = \int d^4\theta\, U D^\alpha\phi D_\alpha\phi + \text{h.c.}, \tag{23}$$

with $U = \mu\theta^2\bar{\theta}^2$, corresponds to a mass term $\mu\bar{\psi}\psi$ for the spinor of a scalar multiplet since $D_\alpha\phi = \psi_\alpha + \theta$-dependent terms. However, the vertices contain six factors of D_α, $\bar{D}_{\dot\alpha}$ and lead to quadratic divergences. Similarly

$$\text{(b}'\text{)} \qquad \mathcal{L}_{\text{break}} = \int d^4\theta\, U(\phi + \bar{\phi})^3 = 2\sqrt{2}\,\gamma A^3, \tag{24}$$

where $U = \gamma\theta^2\bar{\theta}^2$. This gives vertices with six factors of D_α, $\bar{D}_{\dot\alpha}$ and leads, for example to quadratic divergences such as

$$\Delta\mathcal{L} \sim \int d^4\theta\, U\phi + \text{h.c.} \sim A. \tag{25}$$

We conclude this section by remarking that all of these results can be easily checked by using superfield perturbation theory and examining some supergraphs appropriate to each case. One important fact to keep in mind is that certain terms are not induced because the corresponding graphs require $\langle\phi\phi\rangle$ or $\langle\bar{\phi}\bar{\phi}\rangle$ propagators which, as mentioned earlier, introduce mass factors which reduce the degree of divergence. For example, in case (c) or (d) a term $\sim \bar{\eta}\phi^2$ cannot be produced since the diagrams must contain two mass factors and hence are convergent.

4. Soft breaking: the effective potential picture

It is quite instructive to examine the situation, at the one-loop level at least, as revealed by the divergence structure of the effective potential. In a renormalizable theory the only possible quadratic divergences correspond to terms linear or quadratic in scalar fields i.e. tadpoles and self-energy terms (aside from a field-independent vacuum energy–cosmological term divergence) so that we may restrict ourselves to the dependence of the one-loop effective potential V_1 on such fields. It is well known that this dependence is described by [14] (in Landau gauge and with Majorana fermions)

$$V_1(\Psi_0) \sim \sum_J (-1)^{2J}(2J+1)\,\text{Tr}\int d^4k \ln\left[k^2 + M_J^2(\Psi_0)\right], \tag{26}$$

where the trace is over internal indices and $M_J(\Psi_0)$ is the tree level mass matrix for particles of spin J (described by fields Ψ_J)

$$(M_J)_{ij} = -\frac{\delta^2 \mathcal{L}}{\delta\Psi_{Ji}\delta\Psi_{Jj}}\bigg|_{\partial_\mu\Psi=0} \tag{27}$$

If a cutoff Λ is introduced in the above integral it is easy to verify that the $O(\Lambda^2)$ part of V_1 is proportional to the expression

$$\sum(-1)^{2J}(2J+1)\operatorname{Tr} M_J^2(\Psi_0). \tag{28}$$

It is this expression which controls the presence and form of the quadratic divergences [3,15]. It vanishes in supersymmetric theories and also in (most) models with spontaneously broken supersymmetry for any value of Ψ [3]. (In a slightly different context Veltman [16] has analyzed the absence of quadratic divergences which leads him to the same expression.)

For concreteness let us consider a supersymmetric theory with N' (matter) chiral or antichiral (left- and right-hand) multiplets $\phi_i = (z_i, \psi_{Li}, h_i)$, $\bar{\phi}_k = (z_k^*, \psi_{Rk}, h_k^*)$ and a chiral U(N') symmetry. Vector multiplets $V^j = (V_\mu^j, \lambda^j, D^j)$ can be introduced to gauge subgroups G_j, $\Pi_j G_j = G \subseteq U(N')$. The V^j are hermitian matrices and we include the coupling constants g_j in their definition. Whenever convenient we group the ϕ_i into a single N' vector Φ and the V^j into a single matrix V. When we need to be explicit we denote by $T_{j\alpha}$ the αth generator of G_j.

The lagrangian for such a theory is a sum of four terms:

$$\mathcal{L}_G = \sum_j \operatorname{Tr}\left(-\tfrac{1}{4}V_{\mu\nu}^2 - i\bar{\lambda}_L \not{\partial}\lambda_R + \tfrac{1}{2}D^2\right)_j, \tag{29}$$

$$\mathcal{L}_K = (D_\mu Z)^2 + i\Psi_L \not{D}\Psi_R + i\sqrt{2}\left(Z^\dagger\lambda_R\Psi_R + \Psi_L\lambda_L Z\right) + H^\dagger H + Z^\dagger D Z, \tag{30}$$

$$\mathcal{L}_V = P'^a H_a + H^{\dagger a}P_a'^* - \tfrac{1}{2}(\Psi_L)_a P''^{ab}(\Psi_L)_b - \tfrac{1}{2}(\Psi_R)_a P''^{*ab}(\Psi_R)_b, \tag{31}$$

$$\mathcal{L}_D = \sum_i \xi_i D^i. \tag{32}$$

Here $P(z, z^*)$ is a G-invariant polynomial of degree not higher than three, $P_a' = \partial P/\partial z_a$ while in $\mathcal{L}_D$ the sum only includes terms D^i corresponding to abelian G_i.

After elimination of auxiliary fields D^j, H_k the mass matrices can be calculated according to eq. (27) and we find [17]

$$M_0^2 = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, \tag{33}$$

$$A^a{}_b = P''^{ac}P_{bc}''^* + \left(T_\alpha^j Z\right)_b\left(Z^\dagger T_\alpha^j\right)^a + \left(T_\alpha^j\right)_b^a\left(Z^\dagger T_\alpha^j Z + \xi^j\right), \tag{34a}$$

$$B^{ab}=P'''^{abc}P_c'^{*}+\left(Z^{\dagger}T_{\alpha}^{j}\right)^{a}\left(Z^{\dagger}T_{\alpha}^{j}\right)^{b}, \tag{34b}$$

$$C_{ab}=(B_{ab})^{*}, \tag{34c}$$

$$D_a{}^b=(A^a{}_b)^{*}, \tag{34d}$$

$$M_{1/2}=\left(\begin{array}{cc|c} 0 & P''^{*}_{ab} & \sqrt{2}\,(T_{\beta}Z)_a \\ P''^{ab} & 0 & \sqrt{2}\left(Z^{\dagger}T_{\beta}\right)^{a} \\ \hline \sqrt{2}\left(Z^{\dagger}T_{\alpha}\right)^{b} & \sqrt{2}\,(T_{\alpha}Z)_b & 0 \end{array}\right), \tag{35}$$

$$M_1^2=Z^{\dagger}\left\{T_{\alpha}^{j},T_{\beta}^{j}\right\}Z \tag{36}$$

(summation over repeated indices)

It is easy to see that (no factor of two for the fermions since left- and right handed parts are explicitly separated)

$$\mathrm{Tr}\left(M_0^2-M_{1/2}^2+3M_1^2\right)=2\left(Z^{\dagger}T_{\alpha}^{i}Z+\xi^{i}\right)\mathrm{Tr}\,T_{\alpha}^{i}=2D_{\alpha}^{i}\mathrm{Tr}\,T_{\alpha}^{i}, \tag{37}$$

where we have reintroduced the auxiliary fields D^i by means of their equations of motion. Therefore the only quadratically divergent terms are linear in the D^i and are present only if we have some gauged U(1) groups G_i and n chiral multiplets ϕ_r with

$$\mathrm{Tr}\,T^{i}=\sum_{r=1}^{n}g_r^{i}\neq 0. \tag{38}$$

From eq. (37) we see that such terms are equivalent to quadratically divergent self-energy corrections for the corresponding spin-zero fields. This failure of the mass formula is well known for the case of the U(1) Fayet model [18,3]. We note that eq. (37) is valid whether or not supersymmetry or some internal symmetry is spontaneously broken. It is clear that a U(1) D-term is absent if the U(1) is at some scale unified in a semisimple grand unification group [19,20].

In this framework we discuss now the explicit breaking situation. For simplicity we shall assume that our groups G_j are semisimple so that the right-hand side of eq. (37) vanishes. Our soft breaking will be G-invariant. The chiral fields ϕ transform under G_j, and we also indicate explicitly chiral singlets (s,Σ,τ) which are neutral under G_j. The explicit symmetry-breaking terms of the previous sections are given by the following list:

(a) $\mu_1^2[z_a^{*}z^{a}+s^{*}s]$,

(b) $\mu_2^2\mathrm{Re}[z_az_a+s^2]$,

(c) $\mu_3[\lambda_L\lambda_L+\lambda_R\lambda_R]$,

(d) $\gamma[\mathrm{Re}(s^3+s^{*3})]$,

(a′) $\mu_4[\psi_L\psi'_L+\psi_R\psi'_R+\Sigma_L\Sigma_L+\Sigma_R\Sigma_R]$,

(b′) $\nu[(s+s^*)^3]$. (39)

These terms give additional contributions to the mass matrices. We observe that

$$\mathrm{Tr}\,M_0^2=-2\frac{\delta\mathcal{L}}{\delta z_a\delta z^{*a}}, \tag{40}$$

so that only (a) and (b′) contribute. The fermion mass matrix receives contributions from (c) and (b′) so that symbolically

$$\Delta M_{1/2}=\left(\begin{array}{cc|cc} 0 & \mu_4 & 0 & 0 \\ \mu_4 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & \mu_3 \\ 0 & 0 & \mu_3 & 0 \end{array}\right). \tag{41}$$

Finally M_1^2 remains unchanged. We find then

$$\mathrm{Tr}\left(M_0^2-M_{1/2}^2+3M_1^2\right)=2c_1\mu_1^2+6\nu(s+s^*)+c_2\left[2\mu_4^2+\mu_4(s+s^*)\right]+2c_3\mu_3^2, \tag{42}$$

where c_1, c_2, c_3 are positive constants.

We immediately recover the results of the previous sections: cases (a),(b),(c),(d) lead at most to field independent quadratic divergences. The only field-dependent quadratic divergences are induced by terms (a′),(b′) and appear in linear $s+s^*=A$ terms. We conclude by observing that in principle one can also study in this fashion induced logarithmic divergences which, as can be shown from eq. (26), involve the quantities $\mathrm{Tr}\,M_J^4$.

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Note added. Some cases of soft breaking have also been discussed by D.M. Capper [21]. Formula (37) and its relation to U(1) charges together with their implication on the cancellation of the quadratic divergences have been independently obtained by S. Ferrara and F. Palumbo.

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