A PARTON VIEW ON DUAL AMPLITUDES

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We present simple arguments based on a parton picture of hadrons which show that the limit of sums of very high order, unrenormalized planar Feynman diagrams lead to dual N point functions. The parton mass turns out to be of the order of one GeV.

The idea [1] that hadrons are composed of partons has been very useful inelastic electron-hadron scattering. Recently Bjorken and Paschos [2] proposed a model where the nucleon is identified as a three-quark structure accompanied by an infinite sea quarks-antiquarks. In view of the interesting results obtained in these simple models, it is tempting to go a step further and investigate the strong interactions from a parton point of view. That is to say, we would like to imagine that hadron-hadron collisions are the effects of interactions between some constituent partons.

In this paper we present a simple parton model of the strong interactions leading to the N point dual amplitude [3]. Our technical assumptions are not identical to those in refs. [1] and [2], but ideologically our approach is similar.

For simplicity we assume that the partons are scalar particles, although this assumption is probably not essential. Otherwise our main assumptions are the following.

(a) - All hadrons are composed of an infinite sea of virtual partons. The interactions between the different hadrons can be described by very high order, unrenormalized planar Feynman diagrams* involving an infinite number of partons as virtual particles.

(b) - The masses of the virtual partons are large (of the order of one GeV).

We shall then give a simple argument that these assumptions (which are discussed in more detail in the following) lead to dual N point functions [3] (or to generalizations of these functions).

We mention that previously both of us have given arguments [4, 5] in favour of such a dynamical model of dual amplitudes. These arguments were, however, not very transparent or rigorous. Later on Sakita and Virasoro [6] have given arguments in favour of these dynamical model, but these arguments are also not quite convincing since the authors in ref. [6] had to ignore, e.g., singularities on the light-cone in Feynman propagators. The arguments presented in the present paper are, in our opinion, more transparent (and perhaps also more rigorous) than previous arguments.

We imagine that the interaction between N external particles in fig. 1a proceeds through the interaction of a very large number n of partons (at the end we shall perform the limit $n \to \infty$). The interactions between the partons is described by Feynman diagrams with propagators and vertices in the usual way#. All diagrams are assumed to be planar. Using Feynman parameters and a scaling, the expression for the Feynman diagram fig. 1a can be written in the well-known form [7].

$$F_n \sim (-1)^{n-2k} \int_0^\infty \frac{\prod_{j=1}^{n} \mathrm{d}\alpha_j}{\Delta_n(\alpha)^2} \exp \sum_{j=1}^{n} \alpha_j (k_j(\alpha)^2 - m_j^2). \quad (1)$$

* To give meaning to the unrenormalized diagrams we introduce regulators.

\# It is unimportant for the lowwing arguments which type of Lagrangian one assumes. For the sake of definiteness one can, e.g., think of a ϕ^4 type of interaction.

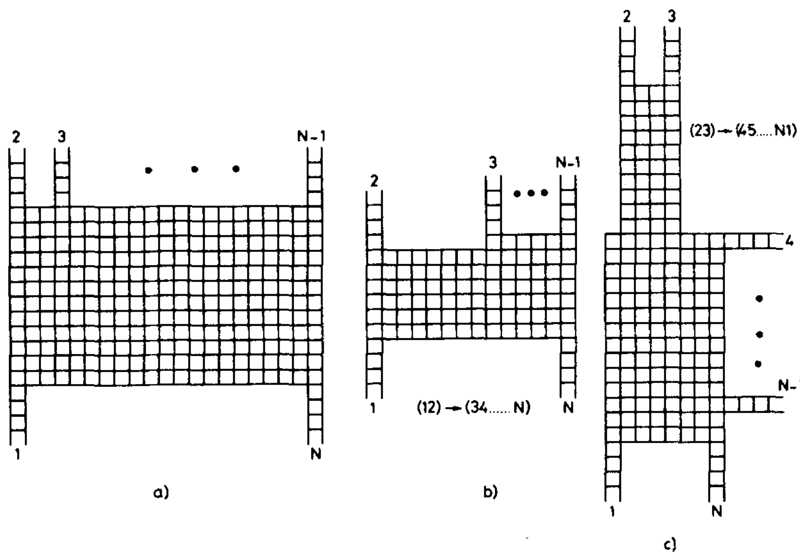

Fig. 1.

Since the diagram is planar we can always find a region with no singularities, and the exponent in (1) is therefore negative [7]. The loop integration has performed in (1), and the momenta are determined by Kirchhoff's laws (for the notation, see Bjorken and Drell [7]). $\Delta_n(\alpha)$ is a determinant depending only on the α's. In the electric circuit analogy [7] the k_j are the internal currents flowing in the circuit and α_j is the resistance of the jth line. Kirchhoff's laws then state that the potential drop around any closed loop is xero, and the sum of currents entering a vertex is zero. The k_j therefore depend on the α's. Since the expression (1) is unrenormalized, some regulators are needed. In order not to complicate the notation we have tacitly understood that (1) has been cut off in some way.

So far we have used assumption (a) only. We now invoke assumption (b), and for the sake of the argument we assume that $m_j^2 \to \infty$. Later on we shall discuss "how large" infinity has to be. Now, since we have a strong exponential damping for large values of α_i the integrals receive their main contributions from small values of α_i, $\alpha_i = \bar{\alpha}_i$ say. To a good approximation we therefore have

$$F_n \sim \exp\Big(\sum_{j=0}^{n} \bar{\alpha}_j k_j(\bar{\alpha})^2\Big) \times \int_0^\infty \frac{\prod_{j=1}^{n} \mathrm{d}\alpha_j}{\Delta_n(\alpha)^2} \exp\Big(-\sum_{j=1}^{n} \alpha_j m_j^2\Big) \; . \tag{2}$$

From the point of view of physics we would of course prefer m_j^2 to be finite, instead of having $m_j^2 \to \infty$. Therefore we have to estimate how large m_j^2 is in order that the above argument is valid. It is clear that the condition

$$m_j^2 \gg |k_j^2(\bar{\alpha})| \tag{3}$$

is necessary for the argument leading from eq. (1) to eq. (2) to be a good approximation, since then the term in the exponent involving the masses dominates entirely over the other term. Now [7]

$$\sum_{j=1}^{n} \bar{\alpha}_j k_j(\bar{\alpha})^2 = \sum_{i,k} R_{ik}(\bar{\alpha}) p_i p_k \quad , \tag{4}$$

where p_i and p_k are the external momenta and $R_{ik}(\bar{\alpha}) > 0$ are the equivalent resistance of the lumped circuit corresponding to fig. 1a. In eq. (4), n is very large, and from the point of view of orders of magnitude we have

$$|k_j(\bar{\alpha})| \sim O\left(\frac{p_{\text{external}}}{\sqrt{n}}\right) \; , \tag{5}$$

provided the sum on the right-hand side of eq. (4) is finite in the limit $n \to \infty$. We shall later see that this is indeed the case. The inequality (3) is therefore satisfied (no matter how large the external momenta are) in the limit $n \to \infty$. Of course, in order that $\bar{\alpha}_j$ is small, one still has to require that m_j^2 is "large", but as shown by (3) and (5), "large" can mean, e.g., a few GeV. The exact magnitude of m_j^2 depends of course on the details of the dynamics. Note that $\bar{\alpha} \sim O(1/m^2)$.

To proceed we use eq. (4) and obtain

$$F_n \sim C_n \exp\left[\sum_{i,j} R_{ij}(\bar{\alpha}) p_i p_j\right] , \tag{6}$$

$$C_n \sim \int_0^\infty \frac{\prod_{j=1}^n \mathrm{d}\alpha j}{\Delta_n(\alpha)^2} \exp\left(-\sum_{j=1}^n \alpha_j m_j^2\right) \quad ,$$

where C_n is a constant which is infinite in the limit where the regulator masses go to infinity. The problem of calculating F_n is thus reduced to the problem of calculating the resistance R_{ij} between the two points z_i and z_j, where the external "currents" (momenta) p_i and p_j enter the diagram#. In the limit $n \to \infty$ we replace the resistive network by a continuous distribution, and it is well known that ##

$$R_{ij} \propto \log|z_i - z_j| \quad . \tag{7}$$

Finally we have to sum over all possible graphs of order n. In the limit $n \to \infty$ this is equivalent to integrating over all possible positions of the points z_i and z_j. We therefore obtain‡

$$\sum_{\substack{\text{all graphs}\\ \text{of very high}\\ \text{order}}} F_n \xrightarrow[n\to\infty]{} C_\infty \int \prod_{j=1}^n \mathrm{d}\theta_j \times \tag{8}$$

$$\times \prod_{i<j} |z_i - z_j|^{2\alpha' p_i p_j} \quad , \qquad \alpha' = \bar{\alpha}/\pi \quad ,$$

where $z_i = \exp(\mathrm{i}\theta_i)$. We have assumed here that all parton masses are equal, $m_j^2 = m^2$, so that $\bar{\alpha} j = \bar{\alpha}$. Eq. (8) is the N point dual amplitude with trajectory intercept $\alpha(0) = 1$ and slope $\alpha' = \bar{\alpha}/\pi$. Eq. (8) is only determined up to the constant C_∞. In order to have a finite result we must take C_∞ proportional to

$$\left[\int_0^{2\pi} \mathrm{d}\theta_1 \int_0^{\theta_1} \mathrm{d}\theta_2 \int_0^{\theta_2} \mathrm{d}\theta_3 \frac{1}{|z_1 - z_2||z_2 - z_3||z_3 - z_1|}\right]^{-1} . \tag{9}$$

The specific resistance is given by $\bar{\alpha}$.

This equation explains why the sum on the right-hand side of eq. (4) is finite in the limit $n \to \infty$.

‡ Here we integrate over the unit circle. The domain over which we integrate is unimportant because the total head generation is invariant under a conformal mapping of one surface into another surface [the exponent in eq. (6) is the total heat generation].

Therefore we have strictly speaking only shown that planar Feynman diagrams approach the N point function in the limit $n \to \infty$ up to an infinite constant. Since the parton model is an unrenormalized field theory this does not disturb us.

We conclude with the following comments.

- In order to avoid unnecessary criticism we mention that the usual statement that complicated Feynman graphs are generally unimportant (e.g., single ρ exchange is usually much more important than the exchange of a million ρ's); is not relevant here. In the parton model all hadrons are composed of an infinite sea of partons. Thus, diagrams with only a few parton lines do not make sense. In the framework of an unrenormalized theory the compositeness of all particles are not unnatural.
- Duality is easily understood in the parton framework. One can, e.g., see the resonance in channel (12) in fig. 1a in the way shown in fig. 1b. However, contrary to the harmonic oscillator formalism (at least with multiperipheral variables) one can also see the resonance in channel (23) in a natural way, as shown in fig. 1c.
- Since one knows how to introduce spin in Feynman diagrams the above arguments might show how to write down dual N point functions with some particles having half-integer spins.
- It has recently been shown by many authors [8] in different formulations that the dynamical picture of the Veneziano amplitude proposed above can be made to include dual loops if one cuts a hole in the planar net of Feynman diagrams.
- Intuitively our assumption about the large parton mass is similar to a model proposed by Susskind [9], in which the Bethe-Salpeter equation is used. There are also some similarities to the work of Nabu [10]. However, the details are different. One would also expect intuitively that there is a connection between assumption (b) and the neglect of light-cone singularities in ref. [6], since the light-cone singularities correspond to very high energies.
- In the ϕ^4 model with fish-net diagrams used to derive eq. (8), we have seen from that the slope is given by $\alpha' = \bar{\alpha}/\pi$. It then follows from the experimental fact $\alpha' \approx 1\ \mathrm{GeV}^{-2}$ that the parton mass m is of the order of 1 GeV, since $\bar{\alpha} \sim O(1/m^2)$. The exact magnitude of $\bar{\alpha}$ depends on the details of the dynamics. The relation between α' and $\bar{\alpha}$ also depends on the dynamical details, and a ϕ^3 theory with bee-cell diagrams gives $\alpha' = \sqrt{3}\bar{\alpha}/\pi$. We shall not discuss this point further, since the main result (8) is valid no mat-

ter what the Langrangian is. The basic assumption is that $m \sim 1$ GeV makes the limit $m \to \infty$ a reasonable dynamical approximation.

- It seems that partons with mass of the order 1 GeV should have been seen experimentally. It is, however, difficult to imagine that free partons should have any properties distinguishing them easily from hadrons, and it is a possibility that partons can be considered superpositions of hadron states. The partons do probably not exists as simple particles because of their (strong) interactions *.

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References

[1] R.P.Feynman, nupublished.
[2] J.D.Bjorken and E.A.Paschos, Phys. Rev. 185 (1969) 1975.
[3] See, e.g., the reviw by Chan Hong-Mo-CERN Preprint TH. 1057 (1969).
[4] H.B.Nielsen, An almost physical interpretation of the n-point Veneziano model integrand, NORDITA Preprint (1969), to be presented at the Kiev Conference.
[5] P.Olesen, A parton view on dual n-point functions (1970) unpublished manuscript.
[6] B.Sakita and M.A.Virasoro, University of Wisconsin Preprint COO-280 (1970).
[7] J.D.Bjorken and S.D.Drell, Relativistic quantum fields (McGraw Hill, New York, 1965); R.J.Eden, P.V.Landshoff, D.I.Olive and J.C. Polkinghorne - The analytic S matrix, (Cambridge University Press, 1966).
[8] D.Fairlie and H.B.Nielsen, NORDITA Preprint (1970); G.Frye and L.Susskind, Yeshiva Preprint (1970); C.S.Hsue, B.Sakita and M.A.Virasoro, Wisconsin Preprint COO-277 (1970).
[9] L.Susskind, Yeshiva Preprint (1969).
[10] Y.Nambu, Chicago Preprint (1969).

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