NEUTRINO MASS PROBLEM AND GAUGE HIERARCHY

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Given a solution of the gauge hierarchy problem, one finds, without any further unnatural condition, a neutrino mass which is hierarchically small compared with the light-charged fermion masses.

Unified gauge theories of weak, electromagnetic and strong interactions with an underlying left–right (L $\leftrightarrows$ R) symmetry predict the existence of right-handed neutrinos [1,2]. Neutrinos therefore may obtain a Dirac mass just like leptons and quarks. No quantum number forbids this type of mass [3] since it gets induced by the masses of the charged fermions through radiative corrections (see fig. 1). In the models based on the gauge group O(10) the electron–neutrino Dirac mass is found to be equal to the u quark mass up to radiative corrections. The experimental bound for a possible electron–neutrino mass is given by

$$m_\nu < 10^{-4}\, m_e\,. \qquad (1)$$

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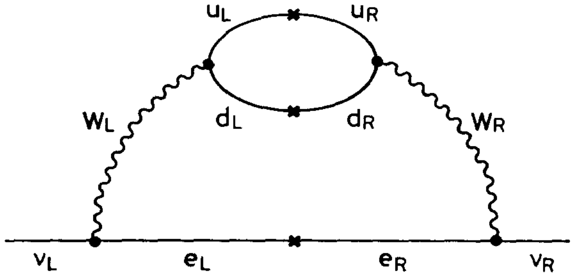

Fig. 1. Contribution to the ν Dirac mass induced at the two-loop level by charged fermion masses.

In order to overcome this apparent difficulty a Majorana mass a for the right-handed neutrino has been proposed [4]. The mass a is an SU(2) × U(1) invariant. The ratio of this mass a over a typical light-charged fermion mass can have natural values as big as the mass which characterizes the spontaneous breakdown of the grand gauge group versus the breaking scale of its electroweak subgroup SU(2) × U(1). This latter mass ratio comprises many orders of magnitude and is referred to as the gauge hierarchy.

The Majorana mass a, together with the Dirac mass b, induces through (divergent) loop corrections a Majorana mass c for the left-handed neutrino, since both a and b are non-vanishing there is no quantum number left to forbid c (see fig. 2). The Majorana mass c breaks

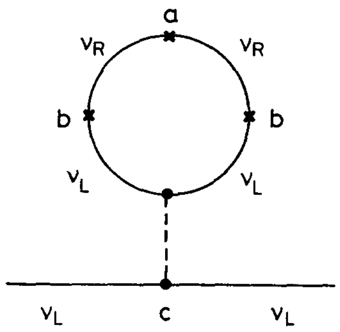

Fig. 2. Majorana mass term c for the left-handed neutrino induced by a neutrino loop with both Dirac mass insertions b and a Majorana mass a for the right-handed neutrino

the electroweak group SU(2) × U(1) and will be very small compared with a.

The lagrangian for the Fermi fields contains now a general neutrino mass term:

$$\mathcal{L}_M = (\nu_\mathrm{L}, \nu_\mathrm{L}^c)\begin{pmatrix} c & b \\ b & a \end{pmatrix}\begin{pmatrix} \nu_\mathrm{L} \\ \nu_\mathrm{L}^c \end{pmatrix}, \quad b, c \ll a . \tag{2}$$

(For simplicity we neglect the mixing between different fermion generations.) The eigenvalues of the mass matrix (2) are:

$$m_1 \approx a , \quad m_2 \approx c - b^2/a . \tag{3}$$

In the O(10) model b is caused by the vacuum expectation value (VEV) of a Higgs scalar transforming as an O(10) vector and is equal to the u type quark mass m_u, whereas a and c are given by VEV's of the SU(5) singlet and the SU(5) 15-plet pieces respectively of a **126** representation of Higgs scalars, multiplied by a Yukawa coupling constant.

In their original proposal Gell-Mann et al. assumed a vanishing Majorana mass c for the left-handed neutrino [4]. This leads to a mass relation between the essentially left-, respectively right-handed neutrino mass eigenstates and the mass of the u type quark

$$|m_2| = m_\mathrm{u}^2/m_1 . \tag{4}$$

The mass $|m_2|$ of the usual (left-handed) neutrino is then sufficiently small on the scale of charged fermion masses to agree with (1) because of the hierarchical suppression factor $b/a = m_\mathrm{u}/m_1$. This idea was criticized [5] because $c = 0$ is not a natural solution, as we have just seen. Therefore, one has to weaken the original assumption made by the authors of ref. [4] and allow for a non-vanishing but small ratio c/b. The non-vanishing parameter c spoils the neutrino–quark mass relation (4) and one stays with the general formula (3) [‡1]. The experimental bound (1) requires c/b to be hierarchically small and this now looks like a second new hierarchy problem within the final step of gauge symmetry breaking: SU(2) × U(1) → $\mathrm{U_{EM}}(1)$. In the language of VEV of SU(2) × U(1) multiplets: why is the VEV of the triplet, which causes the Majorana mass of ν_L, so much smaller than the VEV of the standard doublet which is responsible for the charged fermion masses? In this formulation the problem is no longer confined to L ⇆ R symmetric unified gauge models. It arises as soon as the model contains an SU(2) × U(1) triplet ($Y = -1$) with a non-vanishing Yukawa coupling to the left-handed lepton doublet.

Within the framework of the most general renormalizable Higgs self-interaction we show that the existence of the usual gauge hierarchy (which allows for a very small ratio b/a) automatically forces the Majorana mass c of the left-handed neutrino to be of the order

$$c = b \cdot \mathrm{O}(b/a) . \tag{5}$$

This less demanding condition for the Gell-Mann et al. solution of the neutrino mass problem therefore requires no additional constraint for the scalar self-interaction besides the gauge hierarchy. We have nothing to contribute to a solution of the latter problem.

In order to explain our statement (5) we will use only the electroweak subgroup SU(2) × U(1) of the grand gauge group. If we look at the minimum of the most general renormalizable gauge invariant Higgs interaction we obviously can restrict ourselves to those representations which contain an electrically neutral component. Among these representations three types play a crucial role for our argument:

1) the standard $Y = -1/2$ doublet $q_m^{1/2}$ which is responsible for the main breaking of SU(2) × U(1) and for the masses of the charged fermions; in the O(10) model this doublet is contained in the 10 representation;

2) the $Y = -1$ triplet q_m^1 which causes the Majorana mass of the left-handed neutrino [for the O(10) case it is in **126**];

3) SU(2) × U(1) invariants which generically will have VEV on the big symmetry scale.

The most general renormalizable gauge invariant Higgs potential contains the following piece (summation of repeated indices)

$$-A_{m_1 m_2}^{j_1 j_2}\, q^1_{-(m_1+m_2)} \hat{q}^{j_1}_{m_1} \hat{q}^{j_2}_{m_2} + \text{h.c.} + B^2 q^1_m q^{*1}_{-m}$$
$$(|j_1 - j_2| \leqslant 1 \leqslant j_1 + j_2) . \tag{6}$$

By $\hat{q}^j_m$ we denote SU(2) × U(1) representations inequivalent both to q^1 and q^{*1} such that their electrically neutral components couple to the neutral component q^1_1. An example for such a term is given by the symmetric combination of $q^{1/2} \otimes q^{1/2}$ coupled to q^1. This

‡1 This is quite different from the neutrino masses in the minimal O(10) model which were studied recently by Witten [6]. There no **126** representation is introduced at all and the left-handed (right-handed) neutrinos have calculable mass ratios with respect to the u type quark mass.

coupling cannot be forbidden since it gets induced by the radiative corrections shown in fig. 2. The coefficient $A^{1/2\ 1/2}_{1/2\ 1/2}$ is related to that SU(2) × U(1) invariant Higgs field whose VEV creates the big Majorana mass for the right-handed neutrino [in the O(10) example the SU(5) singlet piece of the **126**]. The mass term B^2 of the q^1 field contains the SU(2) × U(1) invariant fields twice.

Let us now fix all the Higgs fields except q^1_1 at their VEV's and look at the potential as a polynomial of q^1_1 only. The leading linear term in q^1_1 has a coefficient given by.

$$-\langle A^{j_1 j_2}_{m_1 m_2}\rangle\langle \hat{q}^{j_1}_{m_1}\rangle\langle \hat{q}^{j_2}_{m_2}\rangle , \quad m_1 + m_2 = -1 . \qquad (7)$$

Its magnitude is determined by one power of the big symmetry breaking scale and two powers of the small scale of the SU(2) × U(1) breaking. All other contributions to the linear term in q^1_1 contain three VEV's of non-trivial SU(2) × U(1) Higgs representations and are of third order in the small breaking scale and therefore suppressed by one order of the gauge hierarchy factor.

A similar argument holds for the VEV $\langle B^2\rangle$ in front of the q^1_1 mass term. If we assume no further unnatural constraint on the Higgs coupling parameter the various contributions to $\langle B^2\rangle$ will not cancel each other and $\langle B^2\rangle$ will be of the order of the big symmetry breaking (mass)^2. Other coupling terms, quadratic in q^1_1, but not shown in (6), are suppressed by at least one power of the gauge hierarchy factor because they involve at least one further VEV of a non-trivial SU(2) × U(1) representation. Given its huge mass term $\langle B^2\rangle$, q^1_1 can get a VEV compatible with the gauge hierarchy only for $\langle B^2\rangle$ positive. Furthermore, this mass term then dominates over the third and fourth powers of q^1_1 in the Higgs potential if $\langle q^1_1\rangle$ is consistent with the gauge hierarchy, i.e., $\langle q^1_1\rangle$ is in the range of the small symmetry breaking mass. The minimum of the Higgs potential with respect to q^1_1 is then found at

$$\langle q^1_1\rangle = \langle A^{j_1 j_2}_{m_1 m_2}\rangle\langle \hat{q}^{j_1}_{m_1}\rangle\langle \hat{q}^{j_2}_{m_2}\rangle / 2\langle B^2\rangle . \qquad (8)$$

This value is essentially the VEV of the SU(2) doublet multiplied by the gauge hierarchy factor. Assuming that ratios of Yukawa coupling constants are not several orders in magnitude this is precisely our statement (5): the left-handed Majorana mass is of the order of the light charged fermion masses times the gauge hierarchy mass ratio.

Let us now add a few comments. We kept our notion as simple as possible by considering only one $Y = -1$ triplet contributing to the ν_L Majorana mass. The argument is easily generalized to cases where this representation comes with higher multiplicity.

Non-vanishing VEV of any non-trivial SU(2) × U(1) representation besides the $Y = -1/2$ doublet gives corrections to the $\Delta I_{\text{weak}} = 1/2$ rule of the electroweak model. Even though the experimental bounds on these corrections are not very stringent, it may be interesting to note that the preceding argument can be generalized to show that VEV's of any Higgs fields carrying higher (weak) isospin are also suppressed, relative to the doublet VEV, by the gauge hierarchy factor. For a special choice of Higgs coupling parameters this suppression of correction to the $\Delta I = 1/2$ rule was known to happen in the grand unified model based on group SU(5) [7]. Buras et al. found a solution for the minimum of the Higgs potential such that the non-vanishing VEV of the SU(2) triplet which is part of the adjoint representation of SU(5) is suppressed by the gauge hierarchy ratio relative to the VEV of the SU(2) doublet.

If the symmetry breakdown of the grand gauge group down to SU(2) × U(1) is not by a single step but via a $SU_L(2) \times SU_R(2) \times U(1)$ symmetry, then the big symmetry breaking mass we often referred to may be associated with the breaking of this L ⇆ R symmetric subgroup. Renormalization group arguments give for this gauge hierarchy the following bounds [8]

$$10^4 \leqslant M_R/M_L \leqslant 10^9 , \quad \text{for } 0.25 \geqslant \sin^2\theta_W \geqslant 0.21 . \qquad (9)$$

Interestingly enough, these bounds are consistent with the small neutrino mass (1) and not very far from the experimental limit.

Finally, let us comment on the effect of higher order corrections to the Higgs potential due to all possible kinds of loop diagrams. The divergent parts of these higher order corrections renormalize the coupling parameters of the Higgs potential, thereby spoiling (in the generic case) the gauge hierarchy which was arranged to hold at the lower level of perturbation theory [9]. In order to restore the gauge hierarchy at the given order of perturbation theory, one has to readjust the Higgs coupling parameters appropriately. Having done this fine tuning of Higgs coupling parameters, the hierarchy of neutrino to charged fermion masses follows without any new constraint.

Following the Gildener–Weinberg analysis of the gauge hierarchy problem [9] it is easy to understand the suppression of the VEV's for the $j \geqslant 1$ Higgs fields. Starting with a Higgs potential in the SU(2) × U(1) unbroken phase the matrix of second derivatives has positive eigenvalues of order of the big symmetry breaking mass squared (except for the Goldstone direction). With an unnatural condition on the Higgs coupling parameters one artificially lowers the eigenvalue of a mode transforming as a doublet under SU(2) × U(1) to almost zero on the heavy mass scale. All other non-Goldstone modes remain heavy. The light mode then develops a small VEV. The other SU(2) × U(1) representations may couple linearly to the square of this VEV, and then they are pulled out of their original equilibrium position. The deviations from the old equilibrium position are, however, very small even on the scale of the doublet VEV because of the huge quadratic stabilizing mass terms. The natural suppression of VEV's for $j \geqslant 1$ Higgs fields relative to the doublet VEV is just a consequence of the unnaturalness (or in other words the difficulty) for achieving small VEV for the doublet in a situation where the generic scale of the scalar field is huge.

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Note added. After completion of this work we learned that a similar observation on the hierarchy of neutrino to charged fermion masses has been made by Barbieri and Nanopoulos [10] in the context of a discussion of a unified model based on group E_6. We thank these authors for telling us about their work and for useful discussions.

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