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Review of The $K_L \to \mu^+\mu^-$ Puzzle

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It is recalled that the conventional mechanisms for weak and electromagnetic interactions and for observed particles lead via, unitarity and CPT, to a predicted value for the branching ratio of $K_L \to \mu^+\mu^-$ in contradiction with the experimental upper limit. We review the models suggested as a solution to this puzzle which introduce new particles and/or new interactions; the models which preserve CP invariance appear unattractive or seem dubious from the experimental point of view; the models which incorporate large CP violations, predict an anomalously large value for the ($K_S \to \mu^+\mu^-$)-branching ratio which requires experimental confirmation.

I. INTRODUCTION

The decay of K_L into $\mu^+\mu^-$ has aroused considerable interest in recent years both because it provides a sensitive test for the existence of neutral lepton currents and/or for the structure of second order weak interactions, which yield contributions to the $K_L \to \mu^+\mu^-$ decay a priori competitive with those of first order weak–fourth order electromagnetic interactions, and because the experimental configuration of this decay makes its detection quite simple. The hope was for a large experimental branching ratio which would give direct evidence for the existence of neutral lepton currents (direct or induced), and thus various authors [1–5] have computed for various pseudoscalar decays into a pair of leptons the branching ratio expected from the contribution of the first (or zero) order weak–fourth order electromagnetic interactions. These calculations have taken into

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account only the contribution of the two photon intermediate state, which is believed to be the predominant contribution, and were performed under the assumption of CPT and CP invariance in the framework of different models. It was first noted by Berman and Geffen [2] in the similar context of $\pi^0 \to e^+e^-$ decay, by Geffen and Young [6] for the $\eta \to \mu^+\mu^-$ decay and later by several authors [5, 7, 8] in the context of $K_L \to \mu^+\mu^-$ decay that the contribution of the two photon intermediate state to the absorptive part of the amplitude $A(X \to l^+l^-)$, is model independent. Thus this provides a lower limit for the branching ratio $\Gamma(K_L \to \mu^+\mu^-)/\Gamma(K_L \to \gamma\gamma)$:

$$\Gamma(K_L \to \mu^+\mu^-)/\Gamma(K_L \to \gamma\gamma) \geqslant 1.2 \times 10^{-5}. \tag{I.1}$$

This lower limit is at present violated by the experimental data for the branching ratio $\Gamma(K_L \to \mu^+\mu^-)/\Gamma(K_L \to \text{all})$ [9]:

$$\Gamma(K_L \to \mu^+\mu^-)/\Gamma(K_L \to \text{all}) < 1.8 \times 10^{-9} \quad (90\,\% \text{ confidence level}), \tag{I.2}$$

which together with an average of the experimental values for the branching ratio $\Gamma(K_L \to \gamma\gamma)/\Gamma(K_L \to \text{all})$ [10–16][1]:

$$\Gamma(K_L \to \gamma\gamma)/\Gamma(K_L \to \text{all}) = (4.9 \pm 0.4) \times 10^{-4}, \tag{I.3}$$

TABLE I

Experimental Values for the $K_L \to \gamma\gamma$ Branching Ratio

Experiment	Value
Todoroff [10]	6.7 ± 2.2
Kunz [11]	5.5 ± 1.1
Enstrom [13]	4.5 ± 1.0
Arnold [12]	5.3 ± 1.5
Banner (1968) [16]	4.7 ± 0.6
Barmin [14]	4.55 ± 0.92
Repellin [15]	5.0 ± 1.0
Banner (1972) [16]	4.32 ± 0.55
Average	4.7 ± 0.3

[1] See Table I for experimental results on $K_L \to \gamma\gamma$ decays. Taking into account the latest result of Banner et al. [16] the average becomes 4.7 ± 0.3.

yields the following upper limit for $\Gamma(K_L \to \mu^+\mu^-)/(K_L \to \gamma\gamma)$:

$$\Gamma(K_L \to \mu^+\mu^-)/\Gamma(K_L \to \gamma\gamma) < 0.4 \times 10^{-5}. \tag{I.4}$$

Renewed interest in the $K_L \to \mu^+\mu^-$ decay evidently arose from this puzzling situation, and as no one was willing to abandon unitarity or CPT invariance, the contributions to the $K_L \to \mu^+\mu^-$ decay from other known intermediate states, for which Martin, de Rafael, and Smith had given order of magnitude estimates, were calculated [17, 18] for the $\pi\pi\gamma$ state in a rather model independent fashion and for the 3π intermediate state with the help of soft pion techniques [19, 20]. The contribution of the $2\pi\gamma$ state can at most change the unitarity limit of Eq. (I.1) by 10 % and that of the 3π state, even if the soft pion techniques are not suited for the calculation of this contribution, is too small by several orders of magnitude to affect this limit. Assuming that CPT invariance[2] and unitarity remain valid principles, theoretical efforts have been developed essentially along two directions. A first class of models which preserve CP invariance puts the burden on some new dynamics which are introduced either in the form of yet nonobserved light particles [23, 24] or through an anomalously strong coupling of the 3π state to the $\mu^+\mu^-$ state [25] and which provide somewhat accidentally the necessary cancellation of the contribution of the two photon state to the absorptive part of the amplitude $A(K_L \to \mu^+\mu^-)$. A second class of models puts the burden on CP violation which shows up both through the mixing of CP even and odd states in the K_L and K_S state and through a large CP violation in at least one of the transition amplitudes $K \to \mu^+\mu^-$ or $K \to \gamma\gamma$. These ingredients were indeed shown in [26] to be necessary, if the discrepancy between the theoretical lower bound [Eq. (I.1)] and the experimental upper bound [Eq. (I.4)] is to be attributed to CP violation. In the presence of CP violation, new unitarity relations between the decays of K_L and K_S into $\mu^+\mu^-$ and $\gamma\gamma$ were derived first by Christ and Lee [27] and then by Gaillard [26] under more general assumptions and by Dass and Wolfenstein [28–30] in the framework of specific models. These relations, together with the experimental upper limit for the branching ratio of the decays $K_L \to \mu^+\mu^-$ [Eq. (I.2)] and $K_S \to \gamma\gamma$ and the experimental value for the branching ratio of the $K_L \to \gamma\gamma$ decay [Eq. (I.3)], yield very large values for the branching ratio $\Gamma(K_S \to \mu^+\mu^-)/\Gamma(K_S \to \text{all})$ as compared with computed theoretical values. More precisely the smallest value compatible with Eq. (I.2) and unitarity, under the assumption that the contribution of the 3π intermediate state or of any unknown particles to the $K_L \to \mu^+\mu^-$ decay is negligible, is [26]

$$\Gamma(K_S \to \mu^+\mu^-)/\Gamma(K_S \to \text{all}) \geqslant 2 \times 10^{-7},$$

[2] The possibility of a violation of CPT has been discussed by Bogomolny et al. [21] (a maximal CPT violation in the $(K_L - \mu^+\mu^-)$ coupling leads to extremely small effects for other observed phenomena and would therefore be hard to detect) and by Achiman [22].

whereas the branching ratio computed in [7, 8] is

$$\Gamma(K_S \rightarrow \mu^+\mu^-)/\Gamma(K_S \rightarrow \text{all}) \simeq 10^{-11}.$$

Such a large value for the branching ratio of $K_S \rightarrow \mu^+\mu^-$ is not at present excluded by experimental data [31]:

$$\Gamma(K_S \rightarrow \mu^+\mu^-)/\Gamma(K_S \rightarrow \text{all}) < 4 \times 10^{-7}.$$

This may be easily understood however only through the presence of large neutral currents and large CP violation in $K \rightarrow 2\gamma$ [30] or through an anomalously strong coupling of the $\pi\pi$ state to the $\mu^+\mu^-$ state [25].

In Section II, we briefly review the derivations of the unitarity limits for the decay $K_L \rightarrow \mu^+\mu^-$ under the assumption of CP invariance and for $K_S \rightarrow \mu^+\mu^-$ in the presence of a large CP violation. Section III is devoted to the discussion of the various assumptions which are necessary for the derivation of the unitarity limits. In Section IV we give a brief discussion of the models of class I (CP conservation), and we review the characteristics of the models of class II (CP violation). Section V is devoted to the conclusions.

II. Unitarity Limits for the K_L and K_S Decays Into $\mu^+\mu^-$

The derivations of the unitarity lower limits are briefly sketched first for the $K_L \rightarrow \mu^+\mu^-$ decay in the framework of a CP invariant theory and then for both the K_L and the K_S decays into $\mu^+\mu^-$ in the presence of appreciable CP violation. The assumptions necessary for these derivations will be discussed in some detail in the next section, since the desire to justify seriously these assumptions has led to a complete study of the contributions of intermediate states other than the two photon state. Thus the assumptions are just quoted here.

1. CP *Invariance and the Unitarity Limit for the $K_L \rightarrow \mu^+\mu^-$ Decay*

The ingredients used in the derivation are:

(a) CPT invariance;

(b) unitarity;

(c) CP invariance;

(d) dominance of the two photon intermediate contribution to the absorptive part of the amplitude $A(K_L \rightarrow \mu^+\mu^-)$;

(e) vanishing of the absorptive part of the amplitude $A(K_L \rightarrow \gamma\gamma)$.

Unitarity relates the absorptive part of an amplitude, $T - T^{\dagger}$, to the contribution of all possible intermediate states of mass-shell particles: $-i(T - T^{\dagger}) = TT^{\dagger} = T^{\dagger}T$ where T denotes the scattering matrix. CPT invariance relates the transition matrix elements of the K^o and $\bar{K}^o$:

$$\langle n \mid T \mid K^o \rangle = \langle \bar{K}^o \mid T \mid \bar{n} \rangle = \langle \bar{n} \mid T^{\dagger} \mid \bar{K}^o \rangle^*,$$

where $\mid \bar{n} \rangle$ denotes the CPT conjugate of the state $\mid n \rangle$ and where the conventions of [27] are followed:

$$\text{CPT} \mid K^o \rangle = \mid \bar{K}^o \rangle \qquad \text{and} \qquad \text{CP} \mid K^o \rangle = \mid \bar{K}^o \rangle.$$

CP invariance simply reads

$$\langle n \mid T \mid K^o \rangle = \langle n' \mid T \mid \bar{K}^o \rangle,$$

where $\mid n' \rangle$ denotes the CP transform of the state $\mid n \rangle$. Let us introduce the CP odd and even states,

$$\mid K_2 \rangle \equiv 1/2^{1/2}[\mid K^o \rangle - \mid \bar{K}^o \rangle] \qquad \text{and} \qquad \mid K_1 \rangle \equiv 1/2^{1/2}[\mid K^o \rangle + \mid \bar{K}^o \rangle],$$

and the transition amplitudes of K^o and $\bar{K}^o$ to the CP even and odd states $\mu^+\mu^-$, namely the states ${}^3P_o$ and ${}^1S_o$, respectively,

$$\begin{aligned} A(K^o \to \mu^+\mu^-, \text{CP} = \pm 1) &= b_{\pm} + ia_{\pm}, \\ A(\bar{K}^o \to \mu^+\mu^-, \text{CP} = \pm 1) &= \pm(b_{\pm}^* + ia_{\pm}^*), \end{aligned} \tag{II.1}$$

where this parametrization is the most general one. Now CP invariance requires:

$$b_+ + ia_+ = b_+^* + ia_+^* \qquad \text{and} \qquad b_- + ia_- = b_-^* + ia_-^*.$$

Therefore $b_{\pm}$ and $a_{\pm}$ are real. Neglecting the CP impurity, the states K_S and K_L are identified with K_1 and K_2, respectively. Then the decay amplitude for K_L into $(\mu^+\mu^-)\text{CP} = +1$ is forbidden and the amplitude for $K_L \to (\mu^+\mu^-)\text{CP} = -1$ takes the form

$$A(K_2 \to \mu^+\mu^-, {}^1S_o) = 2^{1/2}[b_- + ia_-],$$

and a_- is determined by unitarity and CPT invariance

$$\begin{aligned} &-i\langle \mu^+\mu^-, \text{CP} = -1 \mid T - T^{\dagger} \mid K_2 \rangle \\ &\quad = -i\langle \mu^+\mu^-, \text{CP} = -1 \mid T \mid K_2 \rangle + i\langle \mu^+\mu^-, \text{CP} = -1 \mid T \mid K_2 \rangle^* = 2\sqrt{2}\, a_-, \end{aligned}$$

$$\begin{aligned} 2\sqrt{2}\, a_- = -\tfrac{1}{2} \sum_n [&\langle \mu^+\mu^-, \text{CP} = -1 \mid T \mid n \rangle \langle \bar{n} \mid T \mid K_2 \rangle^* \\ &+ \langle \mu^+\mu^-, \text{CP} = -1 \mid T \mid \bar{n} \rangle^* \langle n \mid T \mid K_2 \rangle], \end{aligned}$$

where the sum over n is restricted to CP odd and total angular momentum $J = 0$ states. Similarly for a_+, one obtains:

$$2\sqrt{2}\, a_+ = \tfrac{1}{2} \sum_n [\langle \mu^+\mu^-, \mathrm{CP} = +1 \mid T \mid n\rangle\langle \bar{n} \mid T \mid K_1\rangle^* + \langle \mu^+\mu^-, \mathrm{CP} = +1 \mid T \mid \bar{n}\rangle^* \langle n \mid T \mid K_1\rangle],$$

where the sum is now restricted to CP even and angular momentum $J = 0$ states. For the $K_S \to \mu^+\mu^-$ decay, appreciable contributions to the absorptive amplitude arise from at least three intermediate states, namely the $\pi\pi$, $\gamma\gamma$, and $\pi\pi\gamma$ states. This prevents one from establishing rigourous lower bounds for this decay. For the $K_L \to \mu^+\mu^-$ decay, on the contrary, it is believed that the main contribution to the absorptive amplitude arises from the two photon intermediate state and that furthermore the absorptive part of the amplitude $A(K_L \to \gamma\gamma)$ is negligible with respect to the dispersive part; this yields

$$A(K_2 \to \gamma\gamma, \mathrm{CP} = -1) = \sqrt{2}\, d_-, \quad d_- \text{ real (CP invariance)}.$$

$$\sqrt{2}\, a_- = \tfrac{1}{2} \sqrt{2}\, d_- \operatorname{Re}\langle \mu^+\mu^-, \mathrm{CP} = -1 \mid T \mid \gamma\gamma, \mathrm{CP} = -1\rangle.$$

The amplitude for the transition $\gamma\gamma \to \mu^+\mu^-$, where the photons are on the mass-shell, can be computed in second order perturbation theory of quantum electrodynamics; the only graph is:

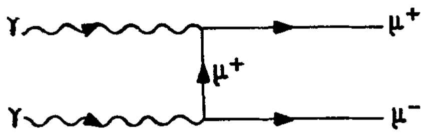

FIGURE 1

the graph of Fig. 1 gives

$$\varphi_- \equiv [\tfrac{1}{2} \operatorname{Re}\langle \mu^+\mu^-, \mathrm{CP} = -1 \mid T \mid \gamma\gamma, \mathrm{CP} = -1\rangle]^2 = 1.2 \times 10^{-5}.$$

This calculation, together with the known experimental value of the branching ratio for the $K_L \to \gamma\gamma$ decay [10–16], yields a lower limit for the branching ratio of the $K_L \to \mu^+\mu^-$ decay, derived first by Quigg and Jackson [5]:

$$\Gamma(K_L \to \mu^+\mu^-)/\Gamma(K_L \to \text{all}) \geqslant 6.0 \times 10^{-9}.$$

2. *Unitarity Bounds in the Presence of* CP *Violation*

The previous theoretical bound is at present violated by experimental data [9] and thus the question arises whether this contradiction can be attributed to

CP violation. Two preliminary remarks may be made [26] under quite general assumptions[3]; CP violation cannot reconcile the theoretical lower bound and the experimental upper limit:

(a) either if the mixing of CP even and odd states in the composition of the states K_L and K_S is ignored; or

(b) if the transition amplitudes for K_1 and K_2 to the channels $\gamma\gamma$ and $\mu^+\mu^-$ are CP invariant. We will sketch the derivation given in [26] under the following assumptions:

(1) dominance of the 2γ contribution to the absorptive part of the amplitudes $A(K_1 \to \mu^+\mu^-, \mathrm{CP} = -1)$, $A(K_2 \to \mu^+\mu^-, \mathrm{CP} = -1)$ and $A(K_2 \to \mu^+\mu^-, \mathrm{CP} = +1)$; and

(2) vanishing absorptive part of the amplitudes $A(K_1 \to \gamma\gamma, \mathrm{CP} = -1)$, $A(K_2 \to \gamma\gamma, \mathrm{CP} = -1)$ and $A(K_2 \to \gamma\gamma, \mathrm{CP} = +1)$.

The transition amplitudes $a_\pm$ and $b_\pm$, introduced in Eqs. (II.1) which describe K_1 and K_2 decays into $\mu^+\mu^-$, are now complex since we allow for CP violation:

$$\begin{aligned}
A(K_1 \to \mu^+\mu^-, \mathrm{CP} = -1) &= \sqrt{2}\,[-\mathrm{Im}\, a_- + i\, \mathrm{Im}\, b_-],\\
A(K_1 \to \mu^+\mu^-, \mathrm{CP} = +1) &= \sqrt{2}\,[\mathrm{Re}\, b_+ + i\, \mathrm{Re}\, a_+],\\
A(K_2 \to \mu^+\mu^-, \mathrm{CP} = -1) &= \sqrt{2}\,[\mathrm{Re}\, b_- + i\, \mathrm{Re}\, a_-],\\
A(K_2 \to \mu^+\mu^-, \mathrm{CP} = +1) &= \sqrt{2}\,[-\mathrm{Im}\, a_+ + i\, \mathrm{Im}\, b_+].
\end{aligned} \tag{II.2}$$

Defining similarly to Eq. (II.1) the amplitudes $c_\pm$ and $d_\pm$ which describe the K^o and $\bar{K}^o$ decays into $\gamma\gamma$ with respectively to $\mathrm{CP} = \pm 1$, we may write the K_1 and K_2 decays into $\gamma\gamma$ in the following form:

$$\begin{aligned}
A(K_1 \to \gamma\gamma, \mathrm{CP} = -1) &= \sqrt{2}\,[-\mathrm{Im}\, c_- + i\, \mathrm{Im}\, d_-],\\
A(K_1 \to \gamma\gamma, \mathrm{CP} = +1) &= \sqrt{2}\,[\mathrm{Re}\, d_+ + i\, \mathrm{Re}\, c_+],\\
A(K_2 \to \gamma\gamma, \mathrm{CP} = -1) &= \sqrt{2}\,[\mathrm{Re}\, d_- + i\, \mathrm{Re}\, c_-],\\
A(K_2 \to \gamma\gamma, \mathrm{CP} = +1) &= \sqrt{2}\,[-\mathrm{Im}\, c_+ + i\, \mathrm{Im}\, d_+].
\end{aligned} \tag{II.3}$$

CPT invariance and unitarity imply that $c_\pm$ are the absorptive parts of these amplitudes, and by virtue of assumption (2) they satisfy

$$c_- = 0 \qquad \text{and} \qquad \mathrm{Im}\, c_+ = 0.$$

[3] These assumptions are strongly supported either by theoretical arguments or by experimental data, as we shall see in Section III.

Finally unitarity and CPT invariance together with assumption (1) yield:

$$\begin{aligned}\sqrt{2}\,\mathrm{Re}\, a_- &= \sqrt{2}\,(\mathrm{Re}\, d_-)\,\varphi_-^{1/2},\\ \sqrt{2}\,\mathrm{Im}\, a_- &= \sqrt{2}\,(\mathrm{Im}\, d_-)\,\varphi_-^{1/2},\\ \sqrt{2}\,\mathrm{Im}\, a_+ &= \sqrt{2}\,(\mathrm{Im}\, d_+)\,\varphi_+^{1/2},\end{aligned} \tag{II.4}$$

where φ_+ denotes

$$[\tfrac{1}{2}\,\mathrm{Re}\langle \mu^+\mu^-, \mathrm{CP} = +1 \mid T \mid \gamma\gamma, \mathrm{CP} = +1\rangle]^2,$$

and where the value of φ_+ is computed from lowest order perturbation theory in quantum electrodynamics:

$$\varphi_+ = 0.96 \times 10^{-5}.$$

Now the apparatus is ready for the derivation of the preliminary remarks. If we neglect the mixing of CP even and odd states in the composition of the K_L state:

$$K_L = K_2\,,$$

the amplitudes for K_L decay into $\mu^+\mu^-$ CP $= \pm 1$ add incoherently

$$\Gamma(K_L \to \mu^+\mu^-) = 2[(\mathrm{Re}\, b_-)^2 + (\mathrm{Re}\, a_-)^2 + (\mathrm{Im}\, a_+)^2 + (\mathrm{Im}\, b_+)^2],$$

which is larger than

$$2[(\mathrm{Re}\, a_-)^2 + (\mathrm{Im}\, a_+)^2].$$

The latter is related to the $K_L \to \gamma\gamma$ decay rate by Eqs. (II.4),

$$2[(\mathrm{Re}\, a_-)^2 + (\mathrm{Im}\, a_+)^2] = 2[\varphi_-(\mathrm{Re}\, d_-)^2 + \varphi_+(\mathrm{Im}\, d_+)^2],$$

which is larger than

$$2\varphi_+[(\mathrm{Re}\, d_-)^2 + (\mathrm{Im}\, d_+)^2] = \varphi_+\Gamma(K_L \to \gamma\gamma).$$

Thus a new unitarity bound arises,

$$\Gamma(K_L \to \mu^+\mu^-)/(K_L \to \gamma\gamma) \geqslant 0.96 \times 10^{-5},$$

not very different from the unitarity limit derived under the assumption of CP conservation, namely, 1.2×10^{-5}, and which is also incompatible with present experimental data [9–16].

The second remark concerns the implications of CP conservation for the transition amplitudes $K_{1,2} \to \mu^+\mu^-$, $\gamma\gamma$ which is a prediction of superweak theory taken in a strict sense. Under this assumption, K_1 and K_2 decay respectively only into CP $= +1$ and CP $= -1$ states[4], which are orthogonal states; thus the decay rates for $K \to \mu^+\mu^-$ and $K \to \gamma\gamma$ contain no interference terms between K_1 and K_2 decay amplitudes:

$$\Gamma(K_L \to \mu\mu) = \Gamma(K_2 \to \mu^+\mu^-) + |\epsilon|^2\, \Gamma(K_1 \to \mu^+\mu^-), \tag{II.5}$$

$$\begin{aligned} \Gamma(K_L \to \gamma\gamma) &= \Gamma(K_2 \to \gamma\gamma) + |\epsilon|^2\, \Gamma(K_1 \to \gamma\gamma) \\ &= \Gamma(K_2 \to \gamma\gamma)[1 - |\epsilon|^4] + |\epsilon|^2\, \Gamma(K_S \to \gamma\gamma). \end{aligned} \tag{II.6}$$

The assumptions (1) and (2) concerning, respectively, the absorptive amplitudes for $K_2 \to \mu^+\mu^-$, CP $= -1$ and $K_2 \to \gamma\gamma$, CP $= -1$ yield:

$$\begin{aligned} \Gamma(K_2 \to \mu^+\mu^-) &= 2\{(\mathrm{Re}\, b_-)^2 + \varphi_-(\mathrm{Re}\, d_-)^2\} \\ \Gamma(K_2 \to \gamma\gamma) &= 2(\mathrm{Re}\, d_-)^2 \Rightarrow \varphi_-\Gamma(K_2 \to \gamma\gamma) \leqslant \Gamma(K_2 \to \mu^+\mu^-). \end{aligned} \tag{II.7}$$

This unitarity bound together with Eq. (II.5) and the experimental limit for the decay rate of $K_L \to \mu^+\mu^-$ [9] allow one to set an upper bound on $\Gamma(K_2 \to \gamma\gamma)$:

$$\varphi_-\Gamma(K_2 \to \gamma\gamma) \leqslant \Gamma(K_2 \to \mu^+\mu^-) \leqslant \Gamma(K_L \to \mu^+\mu^-) < 1.8 \times 10^{-9}\Gamma_L\,. \tag{II.8}$$

Using the experimental values

$$\Gamma(K_L \to \gamma\gamma) = (4.9 \pm 0.4) \times 10^{-4}\Gamma_L$$

and the following relation (II.6):

$$\Gamma(K_S \to \gamma\gamma) = |\epsilon|^{-2}[\Gamma(K_L \to \gamma\gamma) - \Gamma(K_2 \to \gamma\gamma)(1 - |\epsilon|^4)],$$

we obtain from the bound on $\Gamma(K_2 \to \gamma\gamma)$ (II.8):

$$\Gamma(K_S \to \gamma\gamma) \geqslant 3.5 \times 10^{-4}\, |\epsilon|^{-2}\, \Gamma_L = 14 \times 10^{-2}\Gamma_S\,. \tag{II.9}$$

The lack of interference effects implies that a suppression of the rate for $K_L \to \mu^+\mu^-$ relative to $\varphi_-\Gamma(K_L \to \gamma\gamma)$ can only arise if it is the K_1 rather than the K_2 contribution which dominates the rate for $K_L \to \gamma\gamma$. This in turn implies a relative

[4] In particular for the decays $K \to \mu^+\mu^-$ and $K \to \gamma\gamma$ and with our conventions this is equivalent to the reality property of the amplitudes $a_\pm$, $b_\pm$, $c_\pm$, and $d_\pm$.

enhancement of the rate for $K_S \to \gamma\gamma$ by a factor of the order of $|\epsilon|^{-2}$. The limit (II.9) is again in conflict with the most recent experimental bound [16]:[5]

$$\Gamma(K_S \to \gamma\gamma)/\Gamma(K_S \to \text{all}) \leqslant 0.5 \times 10^{-4} \qquad [16].$$

Consequently, abandoning the assumptions that CP violation can be neglected either in the states K_L and K_S themselves or in the transition amplitudes, several authors have derived, after the initial paper by Christ and Lee [27], unitarity lower limits for the branching ratio of the decay $K_S \to \mu^+\mu^-$:

$$\Gamma(K_S \to \mu^+\mu^-)/\Gamma(K_S \to \text{all}) \geqslant 5.6 \times 10^{-7} \quad \text{(Christ and Lee [27])}, \tag{II.10}$$

$$\left.\begin{aligned}\Gamma(K_S \to \mu^+\mu^-)/\Gamma(K_S \to \text{all}) &\geqslant 2 \times 10^{-7}\\ \Gamma(K_S \to \mu^+\mu^-)/\Gamma(K_S \to \text{all}) &\geqslant 2.8 \times 10^{-7}\end{aligned}\right\} \text{(Gaillard [26])}^6, \tag{II.11}$$

$$\left.\begin{aligned}\Gamma(K_S \to \mu^+\mu^-)/\Gamma(K_S \to \text{all}) &\geqslant 5.2 \times 10^{-7}\\ \Gamma(K_S \to \mu^+\mu^-)/\Gamma(K_S \to \text{all}) &\geqslant 10.4 \times 10^{-7}\end{aligned}\right\} \text{(Dass and Wolfenstein [28–30])},^6 \tag{II.12}$$

whereas the best experimental upper limit for this branching ratio, a result given by Wahl at Brookhaven [31] is

$$\Gamma(K_S \to \mu^+\mu^-)/\Gamma(K_S \to \text{all}) \lesssim 4 \times 10^{-7}.$$

This is indeed not very far from the theoretical lower limits quoted previously, so there remains some hope that in a not too remote future, the large theoretical values for the branching ratio may be either confirmed or invalidated by experiment, yielding a clear answer to the question: can the small branching ratio for $(K_L \to \mu^+\mu^-)$ decay be attributed to CP violating effects?

The limits quoted in Eqs. (II.10–12) depend on the assumptions made; those of Dass, as well as those of Dass and Wolfenstein [30] which are not quoted here, depend on the way in which CP violation is realized and will be discussed in Section IV. The weakest are those of [26] which were stated at the beginning of this subsection; we will therefore restrict ourselves here to the derivation of [26].

Let us consider the decay amplitudes for K_1 and K_2 into $\mu^+\mu^-$ and $\gamma\gamma$ introduced

[5] The authors of [16] quote the result, $\Gamma(K_S \to \gamma\gamma)/\Gamma(K_S \to \text{all}) = (-1.9 + 2.4) \times 10^{-4}$. The negative result is meaningful if there is a destructive interference between K_S and K_L decays into two photons. Assuming no interference, they obtain the limit, $\Gamma(K_S \to \gamma\gamma)/\Gamma(K_S \to \text{all}) \lesssim 0.5 \times 10^{-4}$; if the interference is maximal, this limit becomes $\Gamma(K_S \to \gamma\gamma)/\Gamma(K_S \to \text{all}) \lesssim 7 \times 10^{-4}$, both at 90% confidence level.

[6] The exact value depends on interference effects in $K \to 2\gamma$ decays for [26] and on assumptions about absorptive amplitudes for [30], as will be discussed in detail below.

in Eqs. (II.2) and (II.3). We recall that the assumptions made imply the following equations for the absorptive parts of K_1 and K_2 decays into $\gamma\gamma$:

$$c_- = \mathrm{Im}\, c_+ = 0,$$

and the following unitarity relations between the absorptive amplitudes for $K \to \mu^+\mu^-$ and the dispersive amplitudes for $K \to \gamma\gamma$:

$$\begin{aligned} \sqrt{2}\, \mathrm{Re}\, a_- &= \sqrt{2}\, (\mathrm{Re}\, d_-)\, \varphi_-^{1/2} \\ \sqrt{2}\, \mathrm{Im}\, a_- &= \sqrt{2}\, (\mathrm{Im}\, d_-)\, \varphi_-^{1/2} \\ \sqrt{2}\, \mathrm{Im}\, a_+ &= \sqrt{2}\, (\mathrm{Im}\, d_+)\, \varphi_+^{1/2}. \end{aligned}$$

However, the states K_L and K_S have a small admixture of CP-even and -odd states, respectively,

$$| K_L\rangle = | K_2\rangle + \epsilon \,| K_1\rangle \qquad \text{and} \qquad | K_S\rangle = | K_1\rangle + \epsilon\, | K_2\rangle.$$

These relations are valid to order $O(|\,\epsilon\,|^2)$ and we shall neglect second order terms in $|\,\epsilon\,|$ throughout this discussion. It is now a trivial matter to write the decays of K_L and K_S into $\mu^+\mu^-$ and to derive the following unitarity relations:

$$\begin{aligned} &\mathrm{Im}\, A(K_L \to \mu^+\mu^-, \mathrm{CP} = -1) - i(\mathrm{Re}\, \epsilon)\, A^*(K_1 \to \mu^+\mu^-, \mathrm{CP} = -1) \\ &\quad = (\varphi_-)^{1/2}\, A(K_L \to \gamma\gamma, \mathrm{CP} = -1), \end{aligned} \tag{II.13}$$

$$\begin{aligned} &\mathrm{Re}\, A(K_L \to \mu^+\mu^-, \mathrm{CP} = +1) - \mathrm{Re}[\epsilon A(K_1 \to \mu^+\mu^-, \mathrm{CP} = +1)] \\ &\quad = i(\varphi_+)^{1/2}[A(K_L \to \gamma\gamma, \mathrm{CP} = +1) - \epsilon A(K_1 \to \gamma\gamma, \mathrm{CP} = +1)], \end{aligned} \tag{II.14}$$

where K_1 can be replaced by K_S to first order in ϵ. This yields the unitarity lower bound for the decay $K_S \to \mu^+\mu^-$:

$$\begin{aligned} &|\,\epsilon\,|\, [\Gamma(K_S \to \mu^+\mu^-)]^{1/2} \\ &\quad \geqslant [\varphi_- \,|\, A(K_L \to \gamma\gamma, \mathrm{CP} = -1)|^2 + \varphi_+ \,|\, A(K_L \to \gamma\gamma, \mathrm{CP} = +1) \\ &\qquad - \epsilon A(K_S \to \gamma\gamma, \mathrm{CP} = +1)|^2]^{1/2} - [\Gamma(K_L \to \mu^+\mu^-)]^{1/2}, \end{aligned} \tag{II.15}$$

for which one has used the following bounds:

$$\begin{aligned} |(\mathrm{Re}\, \epsilon)\, A^*(K_S \to \mu^+\mu^-, \mathrm{CP} = -1)| &\leqslant |\,\epsilon A(K_S \to \mu^+\mu^-, \mathrm{CP} = -1)|, \\ |\,\mathrm{Re}[\epsilon A(K_S \to \mu^+\mu^-, \mathrm{CP} = +1)]| &\leqslant |\,\epsilon A(K_S \to \mu^+\mu^-, \mathrm{CP} = +1)|. \end{aligned}$$

This bound is improved if one makes the further assumption, implicit in the derivation of Christ and Lee[7], that the absorptive amplitude $\mathrm{Im}\, A(K_1 \to \mu^+\mu^-, \mathrm{CP} = +1)$ is also dominated by the two photon intermediate state contribution:

$$\mathrm{Im}\, A(K_1 \to \mu^+\mu^-, \mathrm{CP} = +1) = (\varphi_+)^{1/2}\, \mathrm{Re}\, A(K_1 \to \gamma\gamma, \mathrm{CP} = +1)$$

and that the absorptive amplitude $\mathrm{Im}\, A(K_1 \to \gamma\gamma, \mathrm{CP} = +1)$ is negligible $\mathrm{Re}\, c_+ = 0 \Rightarrow c_+ = 0$. Equation (II.14) is thus replaced by the following equation:

$$\begin{aligned}&\mathrm{Re}\, A(K_L \to \mu^+\mu^-, \mathrm{CP} = +1) - (\mathrm{Re}\, \epsilon)\, A^*(K_1 \to \mu^+\mu^-, \mathrm{CP} = +1)\\ &\quad = i(\varphi_+)^{1/2}\, A(K_L \to \gamma\gamma, \mathrm{CP} = +1).\end{aligned}$$

Combining this equation with Eq. (II.13) one obtains

$$\begin{aligned}&(\mathrm{Re}\, \epsilon)[\Gamma(K_S \to \mu^+\mu^-)]^{1/2}\\ &\quad \geqslant [\varphi_- \mid A(K_L \to \gamma\gamma, \mathrm{CP} = -1)|^2 + \varphi_+ \mid A(K_L \to \gamma\gamma, \mathrm{CP} = +1)|^2]^{1/2}\\ &\qquad - [\Gamma(K_L \to \mu^+\mu^-)]^{1/2}\\ &\quad \geqslant \varphi_+[\Gamma(K_L \to \gamma\gamma)]^{1/2} - [\Gamma(K_L \to \mu^+\mu^-)]^{1/2}. \qquad \text{(II.15)}\end{aligned}$$

Thus the rate for $K_S \to \gamma\gamma$ drops out and $|\, \epsilon\, |$ is replaced by $\mathrm{Re}\, \epsilon$ which is empirically very close to $|\, \epsilon\, |/\sqrt{2}$. This gives the bound quoted in Eq. (II.10).

The last point we wish to discuss in this section concerns the numerical value of the lowest bound quoted for the rate of the $K_S \to \mu^+\mu^-$ decay [(Eq. (II.11)]. Since only the CP even channel of $K_S \to \gamma\gamma$ appears in the unitarity relations, this value reaches its minimum in the presence of large CP violation in the $K_L \to \gamma\gamma$ decay and of interference effects between K_L and K_S decays into the CP $= +1$ $\gamma\gamma$ state; maximal CP violation corresponds to

$$A(K_L \to \gamma\gamma, \mathrm{CP} = -1) = 0,$$

and

$$|\, A(K_L \to \gamma\gamma, \mathrm{CP} = +1)|^2 = \Gamma(K_L \to \gamma\gamma),$$

and maximal destructive interference occurs when

$$\begin{aligned}&|\, A(K_L \to \gamma\gamma, \mathrm{CP} = +1) - \epsilon A(K_S \to \gamma\gamma, \mathrm{CP} = +1)|\\ &\quad = |\, A(K_L \to \gamma\gamma, \mathrm{CP} = +1)| - |\, \epsilon A(K_S \to \gamma\gamma, \mathrm{CP} = +1)|,\end{aligned}$$

[7] Explicitly they assume that only the two-photon state contributes to the absorptive part of $K_L \to \mu^+\mu^-$ and that $K_L \to \gamma\gamma$ is purely dispersive. If $(\pi\pi \to \mu^+\mu^-) \simeq \alpha^2$ then the two pion contribution can modify the absorptive part of $K_L \to \mu^+\mu^-$ by 7 % [34] and this already lowers the bound by 30 %.

where the last term amounts at most up to 5% of the previous term; under these conditions the numerical value of the bound is

$$\Gamma(K_S \to \mu^+\mu^-)/\Gamma(K_S \to \text{all}) \geqslant 2 \times 10^{-7}.$$

This value was obtained by introducing for $|A(K_S \to \gamma\gamma, \text{CP} = +1)|$ the maximal value allowed by experimental data,[8] namely,

$$\Gamma(K_S \to \gamma\gamma)/\Gamma(K_S \to \text{all}) < 7 \times 10^{-4}.$$

However, it has been argued [30] that if the amplitude $A(K_S \to \gamma\gamma, \text{CP} = +1)$ were so large, this amplitude would be predominantly dispersive, as no known intermediate state could give rise to such a large absorptive amplitude; thus, the essentially real amplitude $A(K_S \to \gamma\gamma, \text{CP} = +1)$ could not interfere maximally with $A(K_L \to \gamma\gamma, \text{CP} = +1)$ and the value for the lower bound of the branching ratio for the $K_S \to \mu^+\mu^-$ decay would be slightly higher. Let us now consider the opposite situation where no interference effect occurs, either because the value of $|A(K_S \to \gamma\gamma, \text{CP} = +1)|$ is too small, or because the K_L and K_S amplitudes are out of phase; in this case the numerical value for the bound on the branching ratio for $K_S \to \mu^+\mu^-$ becomes

$$\begin{aligned}\Gamma(K_S \to \mu^+\mu^-)\,|\,\epsilon\,|^2 &\geqslant \{(\varphi_+)^{1/2}[\Gamma(K_L \to \gamma\gamma)]^{1/2} - [\Gamma(K_L \to \mu^+\mu^-)]^{1/2}\}^2 \\ &\Rightarrow \Gamma(K_S \to \mu^+\mu^-)/\Gamma(K_S \to \text{all}) \geqslant 2.8 \times 10^{-7}.\end{aligned}$$

The unitarity lower bounds for the branching ratio of the decay of $K_S \to \mu^+\mu^-$ clearly must be tested experimentally. The experimental violation of the lowest bound, namely 2×10^{-7}, would force us to introduce yet unknown particles or a new interaction with anomalously strong coupling of the 3π state to the $\mu^+\mu^-$ state. In the next section, devoted to the discussion of the assumptions introduced in the derivation of the unitarity bounds, the contributions to the absorptive part of known intermediate states other than the two photon state are carefully studied under the assumption that the coupling of the $\mu^+\mu^-$ final state to these intermediate states obeys usual quantum electrodynamics.

III. Contribution to the Unitarity Relations from Intermediate States other than the two Photon States

In this section we examine first the possible second order weak contributions to the absorptive part of the amplitude $A(K_L \to \mu^+\mu^-)$ arising from the exchange of

[8] The older value 1.6×10^{-7} was obtained from the previous experimental value for $K_S \to 2\gamma$ decays of [15, 16, 32, 33].

charged W-vector bosons. In a second step, it will be seen that the contribution of the $\pi\pi$ intermediate state to the absorptive parts of the amplitudes

$$A(K_{1,2} \to \mu^+\mu^-, \mathrm{CP} = -1) \qquad \text{and} \qquad A(K_{1,2} \to \gamma\gamma, \mathrm{CP} = -1)$$

vanish due to parity conservation in quantum electrodynamics; similarly the contribution from the $\pi\pi$ intermediate state to the absorptive parts of the amplitudes $A(K_2 \to \mu^+\mu^-, \mathrm{CP} = +1)$ and $A(K_2 \to \gamma\gamma, \mathrm{CP} = +1)$ is shown to be negligible. Finally we shall give the estimates for the contributions of the $\pi\pi\gamma$ state [18] and of the 3π intermediate state [8, 19].

1. *Second Order Weak Contributions*

We are searching for possible intermediate states which give second order weak contributions to the unitarity relation for the decay $K_L \to \mu^+\mu^-$. This excludes intermediate states containing photons which cannot be coupled to K_L at zero order in the electromagnetic constant α. On the other hand all known decays of K_L are already first order weak decays. Thus the transition from the intermediate state i, which is a decay channel of K_L, to the $\mu^+\mu^-$ state should be a first order weak transition which corresponds to the exchange of a single vector boson. In the absence of neutral currents, semileptonic decay channels must contain at least one pion; these states cannot convert to a dimuon state without photon exchange as well as $W^\pm$ exchange. Transitions from purely hadronic decay states are forbidden at first order in the weak interactions. A strangeness changing neutral current coupled to leptons with the usual strength of first order weak interactions is of course ruled out by the $K_L \to \mu^+\mu^-$ rate itself.[9] Finally a theory which contains only strangeness conserving neutral currents [35] would allow for the transition $3\pi \to \mu^+\mu^-$ with an amplitude of the order of the Fermi coupling constant G, but this contribution to the absorptive amplitude for $K_L \to \mu^+\mu^-$ cannot be expected to be much larger than the electromagnetic contribution of the 3π-intermediate state ($\sim \alpha^2$) which is discussed in Section III.4.

2. *Contributions from the $\pi\pi$ State*

Returning to absorptive amplitudes which are of weak-electromagnetic origin, we note following [18], that the transitions $\pi\pi \to \mu^+\mu^-$, $\pi\pi \to \gamma\gamma$, and $\gamma\gamma \to \mu^+\mu^-$ for total angular momentum $J = 0$ are CP conserving, even if the electromagnetic interactions violate charge conjugation invariance, as was suggested in the model of Bernstein, Feinberg, and Lee [36]. In fact, parity conservation alone implies

[9] The possible role of a weaker direct $K_L \to \mu\mu$ coupling will be discussed in Section IV.

CP conservation for these processes, as $\pi\pi$, $\gamma\gamma$, and $\mu^+\mu^-$ ($J = 0$) are eigenstates of charge conjugation with eigenvalue $+1$:

$C(\pi^+\pi^-) = (-1)^{l+I}$, since $J = l = 0$ and in view of the Bose statistics for the pions,

$C(\pi^o\pi^o) = +1$,

$C(\gamma\gamma) = (-1)^2 = +1$,

$C(\mu^+\mu^-) = (-1)^{l+s} = +1$, since $\mathbf{J} = \boldsymbol{l} + \mathbf{S}$ and $\mathbf{J}^2 = 0 \Rightarrow l = S$.

This implies that the contributions of the $\pi\pi$ intermediate state, which has $\mathrm{CP} = +1$, to the absorptive amplitudes $A(K_{1,2} \to \mu^+\mu^-, \mathrm{CP} = -1)$ and $A(K_{1,2} \to \gamma\gamma, \mathrm{CP} = -1)$ vanish.

Let us now consider the contributions of the $\pi\pi$ intermediate state to the absorptive amplitudes $A(K_2 \to \mu^+\mu^-, \mathrm{CP} = +1)$ and $A(K_2 \to \gamma\gamma, \mathrm{CP} = +1)$. The phases of the states $| K^o\rangle$ and $| \overline{K}^o\rangle$ can be changed by a strangeness gauge transformation which leaves the electromagnetic and strong interaction transition amplitudes invariant. By virtue of unitarity and CPT invariance, the amplitudes $A(K^o \to 2\pi, I)$ and $A(\overline{K}^o \to 2\pi, I)$ are related by:

$$A(\overline{K}^o \to 2\pi, I) = e^{2i\delta_I} A^*(K^o \to 2\pi, I)$$

where I denotes the isospin of the (2π)-state and δ_I the $\pi\pi$-strong interaction phase shift in the isospin I channel. Thus conventionnally the phase of the state K^o is adjusted so that the phase of the transition amplitude $A(K^o \to \pi\pi, I = 0)$ is just the strong interaction $\pi\pi$ phase shift δ_o in the $l = 0$ channel; then by virtue of CPT invariance,

$$A(K^o \to \pi\pi, I = 0) = A(\overline{K}^o \to \pi\pi, I = 0).$$

Consequently, $A(K_2 \to \pi\pi, I = 0)$ vanishes and we need to consider only the transition $(K_2 \to \pi\pi, I = 2)$, which is characterized by the parameter ϵ':

$$\epsilon' = (i/\sqrt{2})\{\mathrm{Im}[A(K^o \to \pi\pi, I = 2)\, e^{-i\delta_2}]/A(K^o \to \pi\pi, I = 0)\} \times e^{i\delta_2}.$$

The parameter ϵ', which measures the departure from superweak theory, seems from experiment to be small with respect to the CP impurity parameter ϵ:

$$\epsilon'/\epsilon \leqslant 0.2 \qquad [37].$$

The neglect of ϵ' which appears as a reasonable approximation leads to the assumption that the contributions from the $\pi\pi$-intermediate state to the absorptive amplitudes for the decays $(K_2 \to \mu^+\mu^-, \mathrm{CP} = +1)$ and $(K_2 \to \gamma\gamma, \mathrm{CP} = +1)$

vanish. Thus the assumptions of [26] concerning the $\pi\pi$ intermediate state are strongly motivated.

3. *Contributions from the $\pi\pi\gamma$ States*

To the relevant order in electromagnetic interactions the contribution of the $\pi\pi\gamma$ intermediate state to the absorptive parts of the amplitudes $A(K_{1,2} \to \mu^+\mu^-)$ may be summarized by the following graph:[10]

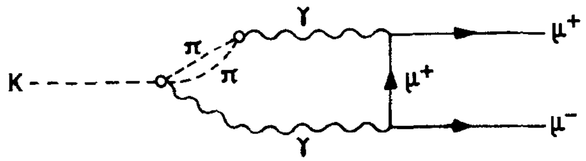

FIGURE 2

As was noted above, the $\gamma\gamma \to \mu^+\mu^-$ transition amplitude for total angular momentum $J = 0$ is CP conserving, furthermore the annihilation of the dipion system into a photon, which is described by the matrix element $\langle 0 \mid J_\mu \mid \pi\pi \rangle$, conserves CP: the vacuum and the dipion state are CP $= +1$ eigenstates and thus the contribution of a current J_μ, which is odd under the CP transformation, vanishes. Consequently the contribution from the $\pi\pi\gamma$ intermediate state to the absorptive amplitudes $A(K_{1,2} \to \mu^+\mu^-, \mathrm{CP} = \eta)$ arises only from the $\pi\pi\gamma$ state with a CP eigenvalue equal to η.

There are several estimates of the contribution of the $\pi\pi\gamma$ contribution to the decay $(K_L \to \mu^+\mu^-)$ [8, 17, 18]. In [18] the graph of Fig. 2 is computed under the assumption that the $(K \to \pi\pi\gamma)$ amplitude has a negligible absorptive part,[11] and using a linear parametrization in the dipion squared mass, s, for the form factors in the $K \to \pi\pi\gamma$ amplitude and for the pion electromagnetic form factor. The parametrization of the form factors has subsequently been generalized [38] by allowing for nonlinear form factors (e.g. pole or multipole terms). The dependence on $t = (p_K - p_{\pi+})^2$ is at most quadratic and its range of variation is less than s; this variation can safely be neglected. The result of [18] remains unchanged [38], namely, the maximum interference effect between the $\gamma\gamma$ and the $\pi\pi\gamma$ contributions to the absorptive part of the amplitude $A(K_L \to \mu^+\mu^-, \mathrm{CP} = -1)$ is of the order of 10 % of the $\gamma\gamma$ contribution and with a priori reasonable form factors $\leqslant 3$ %. This

[10] This state cannot contribute to the absorptive part of the $(K \to \gamma\gamma)$ amplitude to lowest order in α.

[11] In fact the introduction of the 3π intermediate state contribution to the absorptive part of $A(K_L \to \pi\pi\gamma)$ would only lower the contribution of the $\pi\pi\gamma$ intermediate state to the absorptive part of $A(K_L \to \mu^+\mu^-)$.

limit is obtained using the experimental upper limit for the branching ratio $\Gamma(K_L \to \pi\pi\gamma)/\Gamma(K_L \to \text{all}) \leqslant 4 \times 10^{-4}$ [39]; in the calculation of [38] the detection efficiency has also been taken into account. This does not alter significantly the unitarity limit for $(K_L \to \mu^+\mu^-)$ decay and may thus be neglected. The calculation has been done assuming K_L decays into a CP odd final state; it is doubtful that the assumption of a CP even final state would give a significantly larger contribution; this could however be explicitly checked by repeating the calculations of [38]. This would justify the assumption of [26] that $\pi\pi\gamma$ does not contribute to $A(K_2 \to \mu^+\mu^-, \text{CP} = +1)$. The similar estimate for the contribution of the $\pi\pi\gamma$ intermediate state to the absorptive part of the amplitude $A(K_1 \to \mu^+\mu^-, \text{CP} = -1)$ is necessary in order to justify completely the neglect of these contributions, which are assumed negligible in [26]. The experimental value for the branching ratio for the $(K_S \to \pi\pi\gamma)$ decay is quite large [40]:

$$\Gamma(K_S \to \pi\pi\gamma, E_\gamma > 50 \text{ MeV}) = (2.3 \pm 0.8) \times 10^{-3}.$$

However, most of this rate is due to the contribution of the Bremsstrahlung process which leads to a (CP = +1) final state. By a study of the photon spectrum, the direct emission amplitude was found [41] to be less than 35 % of the Bremsstrahlung amplitude under the assumption of CP conservation, this corresponds to an amplitude 20 times larger than that for $K_L \to \pi\pi\gamma$. The experiment has not been analyzed for the presence of a CP = −1 transition; however, the absence of interference in this case, ruling out the possibility of cancellations, generally allows one to set better limits on the magnetic (CP = −1) emission rate. This is the case for $K^+ \to \pi^0\pi^+\gamma$ where the analysis has been performed including both terms [42–44]. If we use the above limit of 35 % as an upper bound for

$$|A(K_S \to \pi\pi\gamma, \text{CP} = -1)|/|A(K_S \to \pi\pi\gamma)|,$$

and take into account the factor ϵ, then the contribution of the amplitude $A(K_S \to \pi\pi\gamma, \text{CP} = -1)$ to the unitarity relation for $K_L \to \mu^+\mu^-$ is negligible:[12]

$$\begin{aligned} &|\epsilon A(K_S \to \pi\pi\gamma, \text{CP} = -1)\, A^*(\pi\pi\gamma \to \mu^+\mu^-)| \\ &\quad < 4 \times 10^{-2}\, |A(K_L \to \pi\pi\gamma)\, A^*(\pi\pi\gamma \to \mu^+\mu^-)|_{\max} \\ &\quad \lesssim 2 \times 10^{-3}\, |A(K_L \to \gamma\gamma)\, A^*(\gamma\gamma \to \mu^+\mu^-)|. \end{aligned}$$

4. *Contribution from the* 3π *State*

Let us review finally the situation for the contribution of the 3π intermediate state to the absorptive parts of $A(K_2 \to \mu^+\mu^-, \text{CP} = -1)$ and $A(K_2 \to \gamma\gamma, \text{CP} = -1)$.

[12] The limit on the absorptive amplitude for $K_L \to \mu^+\mu^-$ arising from the $\pi\pi\gamma$ state is taken from [18].

For the first contribution, an order of magnitude estimate was given by Martin et al. [8]:

$$\mathrm{Abs}^{3\pi}(K_2 \to \mu^+\mu^-, \mathrm{CP} = -1)/\mathrm{Abs}^{\gamma\gamma}(K_2 \to \mu^+\mu^-, \mathrm{CP} = -1) \simeq 1\,\%.$$

This contribution was computed by Adler et al. [19] on the basis of a soft pion model [45, 46],[13] which has recently given remarkable results for the Ke_4 form factors. Using the experimental decay rate for $(K_L \to \gamma\gamma)$ decay, the result is:

$$\mathrm{Abs}^{3\pi}(K_2 \to \mu^+\mu^-, \mathrm{CP} = -1)/\mathrm{Abs}^{\gamma\gamma}(K_2 \to \mu^+\mu^-, \mathrm{CP} = -1) \leqslant 6 \times 10^{-5}.$$

Although this computation should perhaps not be taken too literally, as the amplitude $A(3\pi \to \gamma\gamma)$ vanishes for strictly soft pions, it gives some idea of the suppression of the 3π intermediate state contribution which is essentially due to the 3π phase space factor: $(\varphi_{3\pi})^{1/2} \simeq 10^{-3}$. Similarly the result of [19] for the 3π contribution to the absorptive part of $A(K_2 \to \gamma\gamma, \mathrm{CP} = -1)$ is of the order 5×10^{-5} with respect to the experimentally known value for $|A(K_L \to \gamma\gamma)|$. Similar arguments would hold for the contribution of the 3π intermediate state to the absorptive parts of $A(K_1 \to \mu^+\mu^-, \mathrm{CP} = -1)$ and $A(K_1 \to \gamma\gamma, \mathrm{CP} = -1)$ except that the branching ratio for $(K_S \to \gamma\gamma)$ decay is not yet experimentally known. As a conclusion, although it seems very unlikely that the 3π contribution to the absorptive amplitudes are significant, further investigation of the $(3\pi \to \gamma\gamma)$ transition amplitude, which is used as an input for the computation of the $(3\pi \to \mu^+\mu^-)$ transition amplitude, would help to clarify the situation; in particular direct experimental information on the $(3\pi \to \gamma\gamma)$ transition would either confirm or invalidate the theoretical arguments.

The previous arguments are not valid in the context of specific models suggested in order to reconcile the experimental data for $(K_L \to \mu^+\mu^-)$ decay and unitarity; this will be seen in the next section for models of class *I*.

IV. Models for $K \to \mu\bar{\mu}$ Decay

Essentially the models may be cast into two categories. The models of the first category preserve CP invariance in the $(K_L \to \mu^+\mu^-)$ decay by introducing new light particles or new interactions which strongly enhance the contribution to the absorptive part of the $(K_2 \to \mu^+\mu^-)$ decay amplitude from intermediate CP odd states of known particles, the cancellation between the new absorptive contributions and the absorptive contribution from the two photon intermediate state, which is at this level purely accidental, accounts for the experimental low branching ratio

[13] Estimates were made also in [20] by Pratap et al. using the same model.

of the $(K_L \to \mu^+\mu^-)$ decay. The models of the second category, on the other hand, attribute this low branching ratio to CP violating effects; however, as we have seen in Section II, this forces a large branching ratio for the $(K_S \to \mu^+\mu^-)$ decay, which is very hard to understand on the basis of known interactions, and large CP violation either in the $(K_1 \to \mu\bar{\mu})$ decay or in the $(K_2 \to \gamma\gamma)$ decay; thus the models under consideration suggest new dynamics which might explain these effects. This section is devoted to a schematic review of the models of category I, which are examined in subsection 1 and of the models of category II which are analyzed in subsection 2.

1. CP *Conserving Models*

Among the CP conserving models, the model of Barshay [25] is certainly the most conservative; it introduces only an enhanced $(3\pi - \mu\bar{\mu})$ coupling. Barshay has studied the consequences of this interaction on μ–p scattering, the muonic atoms, the anomalous magnetic moment of the muon and μ-pair production in hadronic reactions and found them to be negligible. However his method of calculation is not unique; Chen et al. [48] find on the contrary that the necessary $3\pi - \mu\bar{\mu}$ coupling is too strong to be compatible with present limits on the muon magnetic moment and μ-pair production by hadrons. One difference is the following: Barshay considers the $3\pi - \mu\bar{\mu}$ coupling to be the basic one and obtains an effective $\pi - \mu$ coupling via a dispersion integral which contains the 3π phase space, while Chen et al. relate the $\pi - \mu\bar{\mu}$ coupling to $3\pi \to \mu\bar{\mu}$ by soft pion techniques.

The other CP conserving models assume the existence of undetected light particles which give rise to new intermediate states in the $(K_L \to \mu^+\mu^-)$ decay. Alles and Pati [24] introduce a new boson, χ_o, which is shown necessarily to be a vector particle and is assumed to be an isoscalar. As a matter of fact, the non observation of the decays $\eta \to \chi_o\chi_o$ and $K^+ \to \pi^+\chi_o$ rule out a significant contribution from the state $\chi_o\chi_o$ and $\pi^o\chi_o$ to the absorptive part of the amplitude $A(K_L \to \mu^+\mu^-, \text{CP} = -1)$, thus the significant intermediate state must be $\chi_o\gamma$, and, furthermore, the meson χ_o must convert into a photon in order that the absorptive contribution from the state $\chi_o\gamma$ be first order weak–fourth order electromagnetic as is the absorptive contribution of the two photon intermediate.

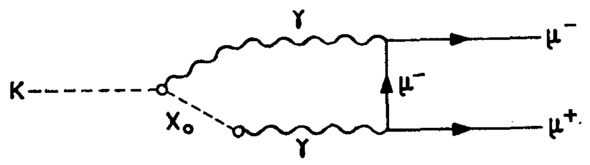

FIGURE 3

The previous requirements restrict the quantum numbers of the meson χ_o to $C = -1$ $J^P = 1^-$

$$350 < m_{\chi_0} < 490 \text{ MeV}.$$

The necessary large decay rate for $K^o \to \chi_o \gamma$:

$$\Gamma(K_L \to \chi_o \gamma)/\Gamma(K_L \to \text{all}) \simeq 10^{-2},$$

and the contribution from the χ_o to the anomalous magnetic moment of the muon, which is experimentally bounded, yield further constraints on the mass of the χ_o:

$$350 \leqslant m_{\chi_0} \leqslant 425 \text{ MeV}.$$

However, results of a recent experiment [49] which looked for $(\pi\mu)$ resonance in the $(K \to \pi\mu\nu)$ decay rule out masses of the χ_o larger than 370 MeV. Aside from direct searches for $K_L \to l\bar{l}\gamma$ a crucial test for the χ_o meson would come from the electron–positron collisions between 350 and 370 MeV, where a clear peak should be detected at the mass of the χ_o meson. An outside theoretical motivation for the introduction of the χ_o is in the context of a larger (badly broken) symmetry group than SU_3. This would imply a new conserved quantum number, and hence a new conserved current to which the χ_o would couple as ρ, ω, φ couple to the usual currents. However, recently we received results [50] of a search for $K_L \to \chi_o\gamma$ ($\chi_o \to \pi^o\gamma$ or e^+e^-) which appears to rule out definitively this model.

Finally, Sehgal [22] has suggested the existence of light fermions, denoted λ and $\bar{\lambda}$; a new intermediate state, $\lambda\bar{\lambda}$, gives a contribution to the absorptive part of $A(K_2 \to \mu^+\mu^-, \text{CP} = -1)$ which is assumed comparable to that of the two photon intermediate state. The coupling of the state $\lambda\bar{\lambda}$ to the state $\mu\bar{\mu}$ and e^+e^- is chosen pseudoscalar, so that the contribution from this coupling to $K \to \pi l^+ l^-$ vanishes; this leads however to a prediction for $(K_L \to e^+e^-)$ decay:

$$\Gamma(K_L \to e^+e^-)/\Gamma(K_L \to \text{all}) \simeq 10^{-9};$$

and this prediction could be relaxed by breaking $\mu - e$ universality. A more disturbing prediction of this model, however, is the contribution from a $\lambda\bar{\lambda}$ state to the anomalous magnetic moment of the muon; for a reasonable value of the cut-off parameter ~ 1 GeV (the contribution is quadratically divergent):

$$\Delta(g-2) \simeq 9 \times 10^{-7},$$

which is at the limit of presently acceptable values (see [51])

$$\Delta(g-2) < 9 \times 10^{-7}, \quad \text{at two standard deviations.}$$

Chen et al. have analyzed in detail the possibility, previously suggested by Martin et al. [24 of 48], that the solution to the discrepancy may lie in the existence of a new pseudoscalar meson of zero strangeness, nearly degenerate with the kaon and which decays into two photons. This will contribute to the absorptive parts of both $K_L \to \gamma\gamma$ and $K_L \to \mu^+\mu^-$ and can thus provide the necessary cancellations if the width of this particle is comparable to the difference between its mass and the K-mass. However, the absence of evidence for this particle implies strong couplings reduced by a factor 10 with respect to η-couplings, for example. Furthermore, its weak coupling to the kaon must be four orders of magnitude smaller than what is usually assumed for the $\eta - K$-coupling. For these reasons the authors conclude that this solution is highly unattractive.

In conclusion, the introduction of new particles or interactions has no serious motivation except as a solution to the $(K_L \to \mu^+\mu^-)$ decay puzzle; the value of the branching ratio for this decay is lowered accidentally by cancellation of the two photon contribution and of the contribution from the new phenomenon. Except for Chen's model [48] they are at the limit of what is experimentally acceptable.

2. *Christ–Lee Models*

Christ and Lee [27] have first pointed out that the $(K_L \to \mu^+\mu^-)$ puzzle could be solved with the help of CP violation; the models to be discussed are suggestions for dynamical mechanisms which could give rise to the minimal requirements (see Section II) of the Christ–Lee phenomena:

(a) a large branching ratio for $(K_S \to \mu^+\mu^-)$ decay

$$\Gamma(K_S \to \mu^+\mu^-)/\Gamma(K_S \to \text{all}) \geqslant 2 \times 10^{-7};$$

(b) a large CP violation at least in one of the decay amplitudes

$$A(K_{1,2} \to \mu^+\mu^-) \qquad \text{and} \qquad A(K_{1,2} \to \gamma\gamma).$$

In fact the usual mechanisms [7, 8] lead to estimates for the $K_S \to \mu^+\mu^-$ branching ratio of the order of 10^{-11} which is four orders of magnitude lower than the value necessary for the Christ–Lee phenomena. Similarly specific models for CP violation are needed in order to account for large violations in the transition amplitudes which describe neutral K-decay to $\gamma\gamma$ and/or $\mu^+\mu^-$:

$$|A(\text{CP violation})|/|A(\text{CP conservation})| \sim 1.$$

Since the strong decay channels of neutral K-mesons appear to be almost superweak (for example, the transition amplitudes for $K^o \to 2\pi$ are compatible with no CP violation $\epsilon'/\epsilon \leqslant 0.2 \times 10^{-3}$ [37]), no theory of CP violation in the transition amplitudes has as yet been accessible to experimental verification. Finally, a further constraint is imposed on the models by the extremely low experimental limit on the neutron electric dipole moment [52]; this rules out a large CP violating coupling of the $(\pi\pi)$ state to the $(\mu^+\mu^-, \text{CP} = -1)$ state which could, in principle yield a large CP violating amplitude for the transition $(K_1 \to \mu^+\mu^-, \text{CP} = -1)$. These constraints restrict CP violation to the K-decay transition amplitudes which are of weak-electromagnetic origin since, as we have seen in Section III, the relevant electromagnetic interactions cannot violate CP because of parity conservation.

Following the discussion of Dass and Wolfenstein [30], we cast the various Christ–Lee models into three classes, denoted hereafter A, B, and C. The main feature of the class A-models is a large CP violating and dispersive amplitude $\text{Im}\, A(K_1 \to \mu^+\mu^-, \text{CP} = -1)$:

$$\text{Re}\, A(K_1 \to \mu^+\mu^-, \text{CP} = -1) \quad \text{and} \quad |A(K_1 \to \mu^+\mu^-, \text{CP} = +1)|$$

are negligible with respect to

$$\text{Im}\, A(K_1 \to \mu^+\mu^-, \text{CP} = -1).$$

A destructive interference between the K_1 and K_2 transition amplitudes to the state $(\mu^+\mu^-, \text{CP} = -1)$ accounts for the low experimental branching ratio for $(K_L \to \mu^+\mu^-)$ decay:

$$\begin{aligned} &A(K_L \to \mu^+\mu^-, \text{CP} = -1) \\ &\quad = A(K_2 \to \mu^+\mu^-, \text{CP} = -1) + i\epsilon\, \text{Im}\, A(K_1 \to \mu^+\mu^-, \text{CP} = -1), \end{aligned}$$

where the absorptive imaginary part of $A(K_2 \to \mu^+\mu^-, \text{CP} = -1)$ is given by the contribution of the two photon real intermediate state and where the decay amplitude $A(K_L \to \mu^+\mu^-, \text{CP} = +1)$ is negligible due to the absence of CP violation in the decay amplitudes $A(K_{1,2} \to \gamma\gamma)$ and of a large $(K_1 \to \mu^+\mu^-, \text{CP} = +1)$ decay amplitude. These models are particularly economical, since one needs only one new CP violating interaction for the $(K_1 \to \mu^+\mu^-, \text{CP} = -1)$ transition amplitude. However they lead to a very large $(K_S \to \mu^+\mu^-)$ branching ratio as compared to the lowest compatible with the Christ–Lee phenomena:

$$\Gamma(K_S \to \mu^+\mu^-)/\Gamma(K_S \to \text{all}) \geqslant 10 \times 10^{-7}. \tag{IV.1}$$

Dass [28] has in fact shown that this limit holds in any model in which the transition amplitudes for $K_{1,2} \to \gamma\gamma$ are CP conserving as long as the absorptive part of the amplitude $A(K_2 \to \mu^+\mu^-, \text{CP} = -1)$ is dominated by the two photon state. Then the absorptive amplitude for $K_1 \to \mu^+\mu^-, \text{CP} = -1$ vanishes as the transition

$K_1 \to \gamma\gamma$, CP $= -1$ is forbidden by hypothesis and if the muons couple only through the parity conserving electromagnetic interactions (see Section III.2). The above prediction [Eq. (IV.1)] would be lowered by a factor of 2 if the absorptive part $\mathrm{Re}\, A(K_1 \to \mu^+\mu^-, \mathrm{CP} = -1)$ is comparable to the dispersive part $\mathrm{Im}\, A(K_1 \to \mu^+\mu^-, \mathrm{CP} = -1)$. However, as discussed above, an anomalous CP violating $\mu\mu - \pi\pi$ coupling is ruled out by the small limit on the neutron dipole moment. Thus a nonvanishing absorptive part for $K_1 \to \mu^+\mu^-$, CP $= -1$ would require the introduction of a new (weakly coupled) CP-violating decay channel, which appears highly unlikely. Thus Eq. (IV.1) can be taken as a prediction of class A-models. Another prediction of class A-models is also a consequence of the reality property of $A(K_1 \to \mu^+\mu^-, \mathrm{CP} = -1)$, namely:

$$\mathrm{Re}\, A(K_2 \to \mu^+\mu^-, \mathrm{CP} = -1)/\mathrm{Im}\, A(K_2 \to \mu^+\mu^-, \mathrm{CP} = -1) \simeq -0.45,$$

where the minus sign is in contradiction with the usual estimates [8]; this contradiction might be resolved by the introduction of form factors. However, this class of models seems definitely ruled out by the latest result for $K_S \to \mu^+\mu^-$ [31].

Wolfenstein [29] has suggested a simple realization for a class-A model. He introduces a neutral CP violating current coupled to the muon current $\bar{\psi}\gamma_\mu\gamma_5\psi$. This neutral hadronic current arises quite naturally in the Okubo–Bace theory [53] where the semiweak W-hadron coupling (order g) violates CP. The usual lowest order weak interactions which proceed via emission and absorption of a virtual charged W-boson and, which are of order g^2, are CP invariant. However the inclusion of neutral currents and trilinear strong self-couplings of W-bosons allows CP violating transitions of order g^3.

In the class-B models, the CP violating effect occurs in $A(K_2 \to \gamma\gamma, \mathrm{CP} = +1)$, which is assumed to dominate the CP conserving amplitude $A(K_2 \to \gamma\gamma, \mathrm{CP} = -1)$ and gives rise to large CP violation in $(K_2 \to \mu^+\mu^-)$ decay. Simultaneously one introduces a new CP conserving mechanism which yields a large dispersive amplitude $A(K_1 \to \mu^+\mu^-, \mathrm{CP} = +1)$, for example a neutral hadronic CP conserving current coupled to the muon current:

$$\mathrm{Im}\, A(K_1 \to \mu^+\mu^-, \mathrm{CP} = +1) \ll \mathrm{Re}\, A(K_1 \to \mu^+\mu^-, \mathrm{CP} = +1).$$

The Christ–Lee phenomenon goes as follows:

(a) $A(K_L \to \mu^+\mu^-, \mathrm{CP} = -1)$ is small because it is dominated by the contribution of the two photon (CP $= -1$) state which is lowered by the large CP violation in $(K_2 \to \gamma\gamma)$ decay.

(b) $A(K_L \to \mu^+\mu^-, \mathrm{CP} = +1)$ is small due to destructive interference between $A(K_2 \to \mu^+\mu^-, \mathrm{CP} = +1)$ given by the large contribution of the two photon (CP $= +1$) state, which arises from large CP violation in ($K_2 \to \gamma\gamma$) decay, and $\epsilon A(K_1 \to \mu^+\mu^-, \mathrm{CP} = +1)$ whose large absolute value is due to the neutral current. The predictions of these models for the branching ratio of ($K_S \to \mu^+\mu^-$) decay and for the dispersive amplitude, Im $A(K_2 \to \mu^+\mu^-, \mathrm{CP} = +1)$, are

$$\Gamma(K_S \to \mu^+\mu^-)/\Gamma(K_S \to \text{all}) \geqslant 6 \times 10^{-7},$$

$$R = \mathrm{Im}\, A(K_2 \to \mu^+\mu^-, \mathrm{CP} = +1)/\mathrm{Re}\, A(K_2 \to \mu^+\mu^-, \mathrm{CP} = +1) \simeq 0.39.$$

The prediction for the dispersive amplitude, Im $A(K_2 \to \mu^+\mu^-, \mathrm{CP} = +1)$, is obtained by considering the experimental upper limit for $\Gamma(K_L \to \mu^+\mu^-)$ and the theoretically allowed lower limit for $\Gamma(K_S \to \mu^+\mu^-)$:

$$\begin{aligned}&(\mathrm{Re}\,\epsilon) \times A(K_S \to \mu^+\mu^-, \mathrm{CP} = +1)\\ &\quad = \mathrm{Re}\, A(K_L \to \mu^+\mu^-, \mathrm{CP} = +1) - \mathrm{Re}\, A(K_2 \to \mu^+\mu^-, \mathrm{CP} = +1),\end{aligned}$$

$$\begin{aligned}&A(K_L \to \mu^+\mu^-, \mathrm{CP} = +1)\\ &\quad = \mathrm{Re}\, A(K_2 \to \mu^+\mu^-, \mathrm{CP} = +1) + i\, \mathrm{Im}\, A(K_2 \to \mu^+\mu^-, \mathrm{CP} = +1)\\ &\qquad + \mathrm{Re}\,\epsilon(1 + i)\, \mathrm{Re}\, A(K_S \to \mu^+\mu^-, \mathrm{CP} = +1),\end{aligned}$$

and

$$\begin{aligned}A(K_L \to \mu^+\mu^-, \mathrm{CP} = +1) &\simeq \mathrm{Re}\, A(K_L \to \mu^+\mu^-, \mathrm{CP} = +1)\\ \Rightarrow R &= \frac{\mathrm{Im}\, A(K_2 \to \mu^+\mu^-, \mathrm{CP} = +1)}{\mathrm{Re}\, A(K_2 \to \mu^+\mu^-, \mathrm{CP} = +1)}\\ &= -\mathrm{Re}\,\epsilon \times \frac{A(K_S \to \mu^+\mu^-, \mathrm{CP} = +1)}{i(\varphi_+)^{1/2}\, A(K_2 \to \gamma\gamma, \mathrm{CP} = +1)},\end{aligned}$$

where the contribution of $A(K_2 \to \gamma\gamma, \mathrm{CP} = -1)$ to the ($K_2 \to \gamma\gamma$) decay rate is neglected. In contradistinction to class A-models, the sign of R is now in agreement with usual estimates and no further ingredient is necessary for these models. The B-class models are suggested by the ideas of Bernstein, Feinberg, and Lee [36] who attribute CP violation of the weak interactions to C-violation in electromagnetic interactions of hadrons or of Gell-Mann [54] who suggested CP violation in the self interaction of W-bosons. This leads to CP violation in processes involving both photon and W-exchange.

This class of models also appears to be ruled out by the result of [31]. However it should be noted that both theoretical and experimental uncertainties have been neglected.[14]

[14] The value of [31]: $\Gamma(K_S \to \mu^+\mu^-)/\Gamma(K_S \to \text{all}) < 4 \times 10^{-7}$ depends on experimental cuts.

In class C-models, the assumption that the CP conserving amplitude is predominantly dispersive is relaxed and one allows for

$$\text{Abs}\, A(K_1 \to \mu^+\mu^-, \text{CP} = +1) \simeq \text{Disp}\, A(K_1 \to \mu^+\mu^-, \text{CP} = +1)$$

whereas the CP violating effect still occurs in the $(K_2 \to \gamma\gamma)$ decays:

$$| A(K_2 \to \gamma\gamma, \text{CP} = +1)| \gg | A(K_2 \to \gamma\gamma, \text{CP} = -1)|.$$

As an immediate consequence, the lower bounds for $\Gamma(K_S \to \mu^+\mu^-)$ are reduced by a factor 2 with respect to the bounds of class B-models [30]:

(a) $\Gamma(K_S \to \mu^+\mu^-)/\Gamma(K_S \to \text{all}) \geqslant 3 \times 10^{-7}$,

in the absence of destructive interference between $A(K_L \to \mu^+\mu^-, \text{CP} = +1)$ and $\epsilon A(K_S \to \mu^+\mu^-, \text{CP} = +1)$;

(b) $\Gamma(K_S \to \mu^+\mu^-)/\Gamma(K_S \to \text{all}) \geqslant 2 \times 10^{-7}$,

in the presence of maximal destructive interference effects (see previous class B-models).

No prediction concerning the ratio R of the dispersive part and of the absorptive part of the amplitude $A(K_2 \to \mu^+\mu^-, \text{CP} = +1)$ can be made in the presence of an absorptive part for $A(K_1 \to \mu^+\mu^-, \text{CP} = +1)$. A realization of class C-models was proposed by Barshay [25], who suggested the existence of an enhanced CP conserving $\pi\pi - \mu\bar{\mu}$ (CP $= +1$) coupling; this effectively gives rise to large absorptive and dispersive amplitude for $(K_S \to \mu\bar{\mu})$ decay, whereas CP violation in $(K_2 \to \gamma\gamma)$ amplitude is, as in class B-models, attributed to C violation in electromagnetic interactions of hadrons or in the conjunction of weak and electromagnetic interactions. According to Barshay the corrections due to this enhanced $(\pi\pi - \mu\bar{\mu})$ coupling to the anomalous magnetic moment of the muon are smaller than the presently accepted deviation from the experimental value [51].[15]

This ends the present discussion of the various models suggested in the framework of the $(K_L \to \mu^+\mu^-)$ puzzle.

V. Conclusion

The $K_L \to \mu^+\mu^-$ puzzle rests on the experimental upper limit for the branching ratio of this decay [9]: $\Gamma(K_L \to \mu^+\mu^-)/\Gamma(K_L \to \text{all}) < 1.8 \times 10^{-9}$. It would of course be desirable for this experiment to be repeated independently. However,

[15] However a particular mechanics suggested for this coupling by Barshay [55] involving a ν^* [56] exchange is incompatible with the data on the anomalous magnetic moment of the muon.

if the result is indeed confirmed, it may open the door to new physics. The possibility that $K \to \mu^+\mu^-$ decays may lead to an understanding of the origin of CP violation is particularly appealing. However the simplest models proposed to explain this effect are already excluded. If the solution indeed lies in CP violation, there must be in particular strong effects in $K \to 2\gamma$ decays. Another possible new phenomenon includes the existence of new pion–muon interactions. In this respect, new experiments are needed: further measurements of $(K_S \to \mu^+\mu^-)$ decay, photon polarization or K_L, K_S interference in $K \to \gamma\gamma$ as a measure of CP violation in this decay, the production cross-section for three pions in electron–positron colliding beams in a kinematic configuration where two photon exchange is expected to be predominant, precision measures of the levels in muonic atoms. An optimistic point of view is the hope that these studies will lead to clues for the understanding of higher order weak interactions or for electromagnetic interactions of hadrons.

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