NEW CONSTRAINTS ON NEUTRINO MASSES FROM COSMOLOGY ☆

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Received 15 March 1983
Revised manuscript received 6 May 1983

We raise the lower bounds on both heavy unstable and stable neutrino masses, exploiting both the phenomena of neutrino mass mixing and possible Majorana mass terms. Stable heavy neutrinos with Majorana masses (the generic case) have an annihilation rate with is p-wave suppressed. Thus in order to have sufficient cosmic annihilation we find the previous lower bound of $m \geqslant 2$ GeV must be raised to $m_\nu \gtrsim 5$ GeV assuming estimates of the present mass density remain unchanged. Unstable neutrinos are constrained by incorporating both new data on mass mixing from experiment and the annihilation suppression described above with standard nucleosynthesis cosmology. In this case we find $m \geqslant 23$ MeV, increasing the former bound by more than an order of magnitude.

1. Introduction. In the first paper in this series [1] the gravitino mass was constrained in order that its out of equilibrium decay would not adversely affect nucleosynthesis, while photino annihilation rates constrained its mass in order that photino mass density produced via gravitino decay would not exceed allowed limits. We can now use these same mechanisms, this time combining them with new inputs from particle physics to improve limits on the mass of particles whose existence is less suspect – neutrinos.

In section 2 we briefly review the general theoretical implications of nonzero neutrino masses, concentrating on possible mass mixings in weak eigenstates, and the possibility that massive neutrinos are Majorana particles – which we demonstrate is the case in general. We then focus on annihilation mechanisms for Majorana neutrinos, motivated by a previous result regarding photinos [2]. In this case we indeed find that the annihilation rate is suppressed by p-wave momentum dependence at low energies, but this effect is countered to a degree by a reduction in the total Majorana particle density due to a reduction in the number of degrees of freedom not included in ref. [2] ‡1. Therefore, contrary to the case for photinos, where there is additional suppression due to charge effects [2], we find the remnant neutrino densities today are increased only by a factor of 3 over the case for Dirac particles [3]. However, we also find that s-wave annihilation, which depends on finite mass effects, can dominate and can sufficiently decrease the number density if $m_\nu > 5$ GeV. Finally we consider the case of unstable neutrinos, which due to mixing effects can decay before the present era. Using new results on mixing parameters, combined with our previous analysis of annihilation mechanisms, we can further constrain the mass of an unstable neutrino so that $m_\nu \gtrsim 25$ MeV.

☆ Research supported in part by the National Science Foundation under Contract Nos. PHY77-22864 and PHY82-15249, and by NSERC.

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‡1 The informed reader may wonder why this factor is much less than the suppression factor in ref. [2] for Majorana photinos. There are two reasons for this. First, ref. [2] did not include the factor of 2 reduction in η_{Thermal} for Majorana particles and antiparticles. (This omission will be corrected in the published version of ref. [2]). Next, and more significantly, in the case of photino annihilation, the rate was proportional to Q_F^4, where Q_F = charge on light fermion product. This effectively suppresses the quark decay channels and reduces the effective degree of freedom factor from ~18 to ~4, decreasing the rate by a factor of ~4 compared to the rate for neutrinos. Thus, this "charge suppression" is at least as significant as the p-wave suppression in that case.

which in turn reduces the allowed region of possible heavy unstable τ neutrino mass to include only one order of magnitude ($25 < m_\gamma < 250$ MeV).

2. Massive neutrinos – a review. The possibility that neutrinos may have large masses was considered with renewed interest with the advent of grand unified theories [4]. Two new phenomena may result from the appearance of mass terms in the lagrangian involving neutrinos which we now review.

First, the neutrino mass matrix may not be diagonal in the basis of weak eigenstates. This family mixing, similar to the Cabibbo mixing of quarks, allows for both neutrino oscillations and decays. Only the latter will interest us here. Consider, for example, a heavy neutrino mass eigenstate to be a linear combination of weak eigenstates:

$$\nu_{\mathrm{H}} \sim \tau_{\mathrm{T}} + \epsilon_2 \nu_\mu + \epsilon_1 \nu_{\mathrm{e}}. \qquad (2.1)$$

This then implies weak couplings of the heavy neutrino (suppressed by mixing angles) to the electron and muon. There then exists a decay mode into electrons, $\nu_{\mathrm{H}} \to \mathrm{e}^- \nu_{\mathrm{e}} \mathrm{e}^+$, which will mimic the muon decay mode $\mu \to \nu\, \mathrm{e}\bar{\nu}$ except for a factor $|\epsilon_1|^2$. (A similar decay involving Z^0 exchange and involving $|\epsilon_2|^2$ is GIM [5] suppressed.) Hence, using $m_\mu \sim 10^2$ MeV, $\tau_\mu \sim 10^{-6}$ s.

$$\tau_{\nu_{\mathrm{H}}}^{-1} = 10\, |\epsilon_1|^2\, (m_{\nu_{\mathrm{H}}}/10\ \mathrm{MeV})^5\ \mathrm{s}^{-1}. \qquad (2.2)$$

However, these couplings (2.1) will also effect the $\pi e\nu/\pi\mu\nu$ branching ratio. By comparing with the measured value [6], a limit on $|\epsilon_1|^2$ can be obtained as a function of possible neutrino mass. A recent analysis [7] yields the result $|\epsilon_1|^2 < \sim 10^{-4}$, $m_{\nu_{\mathrm{H}}} \sim 10$ MeV; $|\epsilon_1|^2 < 10^{-5}$, $m_\nu \sim 20$ MeV; $|\epsilon|^2 < 4 \times 10^{-6}$, $m_\nu \sim 30$ MeV, for example. It is this new data that will allow us to reconsider nucleosynthesis cosmology to refine the constraints on $m_{\nu_{\mathrm{H}}}$ in section 4.

Next, a nonzero neutrino mass must be reconciled with the peculiar structure of weak interactions, in which only two helicity states of the neutrino field (one for the neutrino and one for the antineutrino) participate. If the neutrino is massive, Lorentz transformations (boost plus rotation) can exchange helicity states and there appears to be a necessity for four helicity components (two for the neutrino and two for the antineutrino). Moreover, if the right-handed neutrino state exists, it would be a singlet under $\mathrm{SU(2)_L} \times \mathrm{U(1)}$. There is thus no symmetry which could protect its mass from being larger than the scale of weak interaction breaking, or at least which could imply that it should equal the mass of the left-handed neutrino as might be expected for a standard Dirac particle.

The solution to these problems is well known [8], and results from the realization that the neutrino can be its own antiparticle – i.e. that it be a Majorana fermion. In this case the two helicity components that are exchanged under a Lorentz transformation are what would have previously been called ν_{L} and $\bar{\nu}_{\mathrm{R}}$, and are now the two helicity components of a single Majorana fermion. Alternatively, ν_{R}, if it exists can be a component of a Majorana fermion of different mass.

A Majorana fermion can be expressed in four-component form, in which, by counting degrees of freedom, upper and lower components must be related. If one considers the most general possible mass matrix involving a four component field and its conjugate field, its diagonal mass eigenstates will be described by Majorana fields, except in the special case that the eigenstates are degenerate (i.e. $m_{\nu_{\mathrm{L}}} = m_{\nu_{\mathrm{R}}}$) in which case the two Majorana fermions can be recombined to form a Dirac fermion. Since there is no symmetry for neutrinos which would be expected to enforce this condition, we expect that in general massive neutrinos which can annihilate via weak interactions will be Majorana particles.

We now proceed to investigate the cosmological implications of these phenomena.

3. $\nu\nu$ annihilation for Majorana neutrinos. The discussions of the previous section indicate the likelihood that a massive neutrino is a Majorana particle. As recognized in ref. [2], annihilation of Majorana particles in a $J = 1$ mode can be suppressed due to fermi statistics for identical particles. Heuristically this is explained as follows. Since particle and antiparticle must be identical, then the overall wavefunction must be antisymmetric under their interchange. Thus in a $J = 1$ state, only an antisymmetric spin 1, p-wave combination is allowed. The resulting low energy annihilation cross section thus involves an extra momentum dependence.

From the point of view of remnant cosmic densities of stable massive neutrinos this annihilation suppression might be expected to have significant consequences and we therefore first calculate in detail how the above phenomena arises in neutrino annihilation processes.

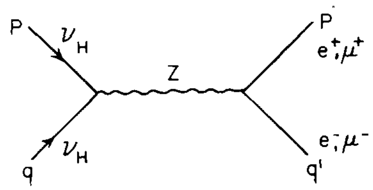

Fig. 1. Dominant annihilation mode for heavy neutrinos.

In the process we will also shed light on the annihilation suppression for photinos claimed in ref. [2].

The primary annihilation mode of stable heavy neutrinos will be that given by fig. 1. Other diagrams can be ignored, as W exchange involving light lepton end products are suppressed if heavy neutrinos are stable (i.e., mixing angles are small). Similarly, W exchange involving a possible heavy lepton weak partner of the neutrino may be ignored as we shall assume its mass is greater than m_ν.

The annihilation diagram of fig. 1 can be computed using the effective weak interaction lagrangian at energies $E \ll M_Z$:

$$\mathcal{L}_{\mathrm{I}}^{\mathrm{eff}} = -2\sqrt{G}\,[\bar{\nu}\gamma_\mu(1-\gamma_5)\nu]$$

$$\times\ \{\bar{e}\gamma^\mu[-(1-\gamma_5)/2 + 2\sin^2\theta_{\mathrm{w}}]\,e\}. \qquad (3.1)$$

In a Majorana basis, where γ matrices are pure imaginary and the Dirac equations is real, the Majorana condition (particle–antiparticle identity) is equivalent to the statement $\Psi = \Psi^*$. This in turn implies the relation $u^*(p) = v(p)$. Thus (3.1) leads to the following combination appearing in the transition element corresponding to fig. 1:

$$\bar{u}^*(p)\,\gamma_\mu(1-\gamma_5)\,u(q) - \bar{u}^*(q)\,\gamma_\mu(1-\gamma_5)\,u(p).$$

Then, using the fact that

$$\bar{u}^*(p)\left\{\begin{matrix}\gamma_\mu \\ \\ \gamma_\mu\gamma_5\end{matrix}\right\}u(q) = \bar{u}\,(q)\left\{\begin{matrix}\gamma_\mu \\ \\ -\gamma_\mu\gamma_5\end{matrix}\right\}u(p),$$

this combination reduces to $-2\,\bar{u}^*(p)\,\gamma_\mu\gamma_5\,u(q)$. It is this effective axial coupling for Majorana neutrinos that results in a momentum dependence being picked out in traces used in calculating the spin averaged cross section.

Considering first p-wave annihilation and ignoring terms down by (m_{f}/m_ν^2) (see appendix) we than calculate, in the center of mass frame of the $\nu\bar{\nu}$ system with total energy E, a total cross section in $\ell^+\ell^-$ (ℓ = light fermion);

$$\sigma_{\ell^+\ell^-} \to [G^{\mathrm{F}}\,(16-64W)/24\pi|\mathbf{v}_{\mathrm{rel}}|]\,|\boldsymbol{p}|^2,$$

where v_{rel} = relation velocity in CM frame; $W = \sin^2\theta_{\mathrm{w}}(1-2\sin^2\theta_{\mathrm{w}}) \approx 0.125$.

For neutrino masses in the range 1–10 GeV, which will concern us in this section there exist $N_{\mathrm{A}} \sim 15$–18 annihilation channels into light fermions. Hence

$$\sigma_{\mathrm{tot}} \sim [N_{\mathrm{A}}G_{\mathrm{F}}^2/3\pi|\mathbf{v}_{\mathrm{rel}}|]\,|\boldsymbol{p}|^2. \qquad (3.2)$$

Consider now the implications of the rate (3.2) for neutrino annihilation in the early universe. As described in refs. [1,3], in order not to exceed limits on the present mass density of the universe, for heavy neutrinos surviving until the present era [‡2]:

$$[\pi^2/2\xi(3)]\,[m_{\nu_{\mathrm{H}}}/40\ \mathrm{eV}]\,f_\nu(3\ \mathrm{K}) < 1, \qquad (3.3)$$

where

$$[\pi^2/2\xi(3)]\,f_\nu(T) = \frac{\text{number density of } \nu \text{ at temp. } T(\eta_\nu)}{\text{number density of radiation at temp. } T(\eta_\gamma)} \sim n_\nu(T)/T^3.$$

$f_\nu(T)$ today can be determined by integrating the Boltzmann rate equation describing the rate of change of neutrino number density with time as the universe expands, and annihilation proceeds:

$$\mathrm{d}\eta_\nu(T)/\mathrm{d}t = -(3\dot{R}/R)\,\eta_\nu - \langle\sigma v_{\mathrm{rel}}\rangle\,\eta_\nu^2(T)$$

$$+ \langle\sigma v_{\mathrm{rel}}\rangle\,\eta_\nu^{02}(T), \qquad (3.4)$$

where η^0 = equilibrium number density at temperature T, R = cosmic scale factor, $\langle\ \rangle$ = thermal average. Using (3.2) and the fact that $\langle p^2\rangle = \frac{1}{4}\,m_\nu^2\langle v_{\mathrm{rel}}^2\rangle = m_\nu^2\frac{3}{2}\,kT/m_\nu$ (as long as the neutrino energy distribution is thermal [‡3], we can cast (3.4) in a form analo-

[‡2] This constraint may be overly conservative due to changes in conventionally accepted values of the Hubble and deceleration parameters since ref. [3] was written. However, for the purposes of comparison we use the conventions of ref. [3]. Decreasing the upper bound on the present mass density by a factor α will have the effect of increasing the lower bound on m_ν by a factor $\alpha^{1/2}$.

[‡3] For $m_\nu > O(10)$ GeV the increase in $\sqrt{N_{\mathrm{F}}}$ for $x_{\mathrm{f}} \gtrsim 500$ MeV, which decreases C in (3.5) is compensated for by the fact that k in eq. (3.6) is decreased due to further photon reheating due to $\mu^+\mu^-$, etc. annihilation.

Table 1
Freezing temperature and $f(0)$ as a function of mass[a)]: case A: with Majorana suppression, case B: without suppression.

M (GeV)	Case A			Case B	
	x_f	$f(0)$	$C\mu^3/m_\nu^3$ ($\times 10^8$)	x'_f	$f'(0)$
1	0.07	5.5×10^{-7}	7.3	0.059	–
2.15	0.06	7.2×10^{-8}	7.3	0.052	–
5	0.055	9.7×10^{-9}	5.5	0.047	–
8	0.05	2.8×10^{-9}	5.3	0.044	–
10	0.049	1.53×10^{-9}	5.3	0.043	–
1.0×10^{-2}	0.5	0.033	1.55	0.4	0.01
2.0×10^{-2}	0.30	0.013	1.55	0.23	3×10^{-3}
3.0×10^{-2}	0.24	0.006	1.55	0.18	1×10^{-3}

a) $f(0)$ as shown does not include the additional factor k described in the text to account for later reheating, and should be multiplied by a factor of at least 4/11 to account for e^+e^- annihilation in order to arrive at the actual number density.

gous to that introduced in ref. [3]):

$$df/dx = C\mu^3 x(f^2 - f_0^2), \tag{3.5}$$

where

$$x = T/m_\nu, \quad f(x) = n_\nu(T)/T^3, \quad f^0(x) = \eta^0(T)/T^3,$$

and

$$C = 1.1 \times 10^8\ \mathrm{GeV}^{-3}, \quad \mu^3 = N_A m_\nu^3(\mathrm{GeV})^3/N_F(T)^{1/2},$$

and $N_F(T) = \frac{1}{2}$ (# boson spin states) + $\frac{7}{16}$ (# fermi spin states) in the radiation gas at temperature T.

The extra x dependence in (3.5) as compared to the case of Dirac neutrino annihilation implies that the neutrino annihilation rate will fall out of equilibrium faster at low temperatures, and that the present remnant abundance will be increased. The increase in the freezing temperature [the temperature at which $f(T)$ begins to exceed $f_0(T)$] due to this effect can be calculated using the approximations outlined in ref. [3] [appropriately modified to take into account the extra factor of x in (3.5)], and the results are displayed in table 1. As can be seen, the freezing temperature does not change significantly for masses in the range of 1–10 GeV which will be of interest. We will first use this fact to obtain an estimate of the effect of this p-wave suppression on raising the remnant neutrino mass density as compared to the case described in ref. [3]. As described there, once $f_0^2 \ll f^2$ (below freezing) we can ignore the second form in (3.5) and integrate the equation analytically from x_f to 0. The result is:

$$f(0) = k(C\mu^3 x_f^3)^{-1}(1 + x_f^{-1})^{-1} \tag{3.6}$$

(where the factor of k is due to further dilution due to further photon reheating, and is $\frac{4}{11}$ if only e^+e^- annihilation occurs after x_f). Now, since $x_f^{\mathrm{Maj}} \simeq x_f^{\mathrm{Dirac}}$ (see table 1) we have the result that $f^{\mathrm{Maj}}(0) \sim x_f^{-1} f^{\mathrm{Dirac}}(0)$. Plugging this expression in (3.3) and using the fact that $x_f^{-1} \sim 20$ we thus expect $m_\nu^{\mathrm{Maj}} > \sim \sqrt{20}\, m_\nu^{\mathrm{Dirac}} \sim 5 \times 2$ GeV $\sim$ O(10 GeV). However, there is another effect which works in the opposite direction. Since Majorana particles are identical to their antiparticles the final number density of both ν, and $\bar{\nu}$ is not equal to twice the density of ν alone, as in the case of Dirac fermions. This reduces the lower bound on m^{Maj} by a factor of $\sqrt{2}$ compared to m_ν^{Dirac}. Hence the net expected increase in the lower bound for massive Dirac neutrinos as compared to that for Dirac particles is $\sqrt{20}\,(\sqrt{2}^{-1}) \sim 3$.

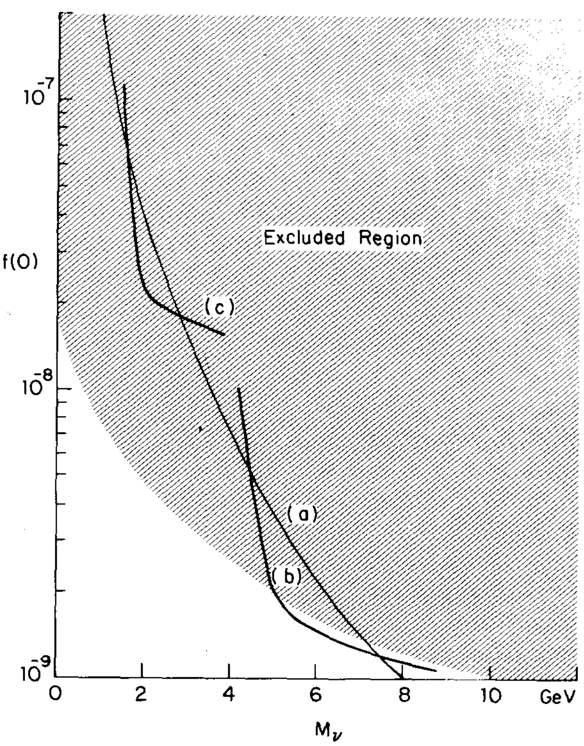

Fig. 2. $F(0)$ versus m_ν for Majorana neutrino annihilation: (a) p-wave annihilation, (b) s-wave annihilation into bottom quark, (c) s-wave annihilation into τ and charmed quark.

This approximation is confirmed by numerical integration of (3.5) which was performed for the region for $x = x_f$ to $x = 0$. The results of the analysis are displayed in fig. 2, where the maximum allowed value of f_ν (3 K) versus mass is displayed as well as the actual value. The two curves cross in the range ~7 GeV [‡3].

In the appendix, however, we demonstrate that s-wave annihilation into bottom quarks cannot only dominate in the case $m_\nu > m_{\text{bottom}} > 4.5$ GeV, but will be sufficient to reduce the neutrino number density to acceptable levels if $m_{\tilde{\nu}} > 5$ GeV. The effect of s-wave annihilation is also shown in fig. 2. We thus conclude that the lower bound on heavy neutrinos should be raised from 2 GeV in the Dirac case to 5 GeV in the Majorana case.

4. Mass mixing and annihilation suppression and heavy neutrino decays. As mentioned in ref. [1] the out of equilibrium decay of heavy unstable particles, must be constrained so that $n_{\text{baryon}}/n_{\text{photon}}$ has not changed significantly since the time of primordial nucleosynthesis in order not to affect the remnant deuterium abundance present today. There are actually two competing effects involved in the deuterium production process in the early universe, as described in refs. [9, 10]. First, an increase in n_B/n_γ at the time of helium production will increase the H^2 production at higher temperatures, resulting in more eventual conversion of H^2 to He^4. On the other hand, if the heavy particle which eventually decays has a mass density which dominates the expansion rate at the time of nucleosynthesis, this expansion rate is increased, and less ^2H is converted to He^4. Based on the detailed analysis of ref. [9], we can estimate that an increase in n_B/n_γ by a factor of $\gtrsim 7$ at the time of nucleosynthesis overcomes the increased expansion effect in the worst case of mass dominated expansion, and results in a deuterium fraction which is too small. This is the constraint we shall use to limit the possible mass of neutrinos which decay out of equilibrium. We note that this constraint (i.e. $\rho_\nu \lesssim 10\rho_{\text{radiation}}$ at the time of decay) is much stronger for the masses and decay times of interest here than the assumption that ρ_ν does not dominate the expansion rate at nucleosynthesis.

Specifically, this constraint implies:

$$m_\nu T_d^3 f(t_d) \lesssim 10\, T_d^4 . \tag{4.1}$$

In section 2 we showed that the particle physics result (2.2) constrains $\tau_{\nu H}$ in terms of $|\epsilon_1|^2$ and $m_{\nu H}$. Using the fact that $T_d \sim 10^{-3}\, t^{-1/2}$ GeV $\sim 10^{-3}\, \tau_{\nu H}^{-1/2}$ GeV [since $\tau_{\nu H} - t(kT \sim m_{\nu H}) \simeq \tau_{\nu H}$ for the cases of interest], we can then also use (4.1) to constrain $\tau_{\nu H}$ in terms of $m_{\nu H}$. Combining (4.1) and (2.2) we get a constraint relating $|\epsilon_1|^2$ and $m_{\nu H}$:

$$|\epsilon_1|^2 \geqslant (10\ \text{MeV}/m_{\nu_H})^3 f(t_d)^2 . \tag{4.2}$$

We can now use the two new results we have described here, the new limits on $|\epsilon_1|^2$, and the suppression of annihilation for Majorana particles which will find to increase $f(t_d)$ to further constrain $m_{\nu H}$ for unstable heavy neutrinos.

Turning first to table 1 we see that for masses in the range 5–30 MeV both x_f and $f(t_d)$ are increased by 50% due to suppression of annihilation, where we have

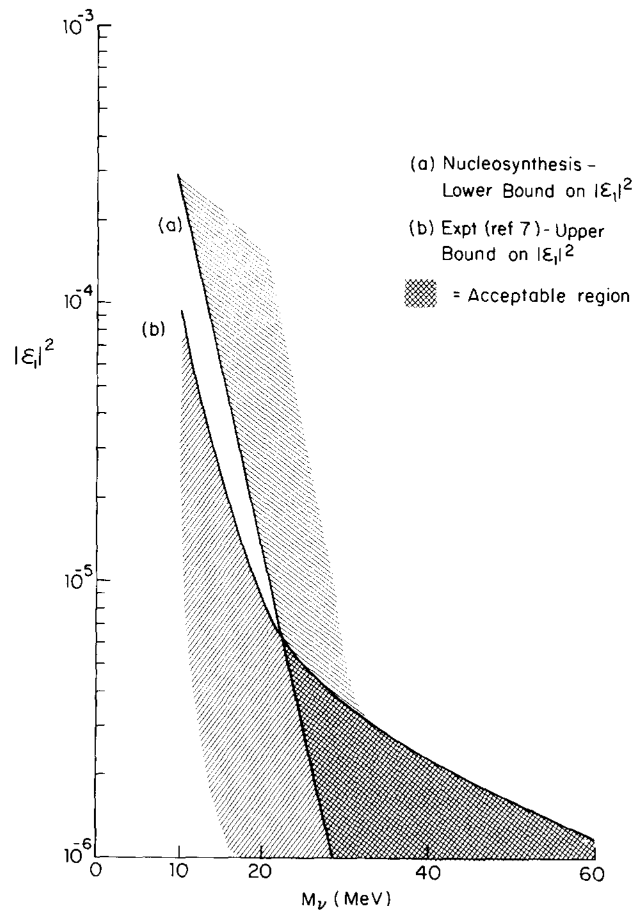

Fig. 3. Bounds on $|\epsilon_1|^2$ versus m_ν.

taken into account first the fact that at these temperatures fewer annihilation channels are available, reducing μ^3 in eq. (3.5) and also that the Majorana remnant abundance is reduced by a factor of two compared to the Dirac case. [Actually, the value of $f(t_d)$ in table 1 underestimates $f(t_d)$ slightly for the reasons discussed in footnote 4.]

In fig. 3 we display the resulting constraints on m_{ν_H} as a function of $|\epsilon_1|^2$. A curve of the present experimental limit on $|\epsilon_1|^2$ versus m_ν is displayed, along with a curve for (4.2). As demonstrated here, we can now conservatively rule out the mass range $m_\nu \lesssim 23$ MeV. This bound is is not altered significantly for Dirac particles since $f(t_d)$ remains about the same. Since experiments limit the τ neutrino to less than 250 MeV, this new result significantly reduces its possible mass range. As experimental limits on $|\epsilon_1|^2$ are improved the lower bound $m_\nu > 23$ MeV will increase [‡5].

5. Conclusions. The above results demonstrate a continued fruitful complementarity between terrestrial phenomenology, and cosmology, which can be exploited in attempts to empirically constrain the parameters of particle physics. As can be seen, we may continue to rule out possible mass regions for heavy neutrinos which may be inaccessible to direct experimental constraints for some time to come.

I would first like to acknowledge Sheldon Glashow who motivated and initiated the original effort to constrain unstable neutrino masses. As well, as usual, Joe Polchinski and Mark Wise provided large amounts of useful input. Indeed, the details of the last section were worked out in collaboration with Mark and Joe.

Appendix: S-wave annihilation. One might imagine that if $m_f^2/m_\nu^2 > N'/N\, x_i$ (where N = number of heavy fermions of mass m_F; N' = total number of decay channels), then s-wave annihilation may dominate, changing our bound on m_ν. We here analyze this situation in detail for $m_\nu \geqslant 2$ GeV.

‡4 Below ~1 MeV the massive neutrino energy distribution is no longer thermal so that $\langle v^2\rangle \sim T^2$ instead of T. This effect can be expected to increase $f(0)$ very slightly.

‡5 After this work was completed I received a communication from R. Shrock, indicating that $|\epsilon_1|^2$ could be further limited in the region of interest from double beta decay experiments. See ref. [11].

Explicitly, s-wave Majorana annihilation is suppressed relative to Dirac s-wave annihilation for a given mass m_ν by a factor depending on the mass of the decay product fermion, and the number of decay channels:

$$\langle \sigma_s^{\mathrm{maj}} v\rangle \sim (N/N')\,(m_f/m_\nu)^2\,[1-(m_f/m_\nu)^2]^{1/2}\langle\sigma_s^{\mathrm{Dirac}} v\rangle$$

$$= (N/N')\,\lambda(m)\,\langle\sigma_s^{\mathrm{Dirac}} v\rangle. \qquad (\mathrm{A}.1)$$

The function $\lambda(m)$ has a maximum value ~0.38 when $m_f/m_\nu \simeq 0.81$. First, let us consider annihilation into the τ lepton and charmed quarks which we assume are all degenerate with mass $\simeq 1.8$ GeV (hence $N = 4$).

Now Weinberg and Lee [3] demonstrated that for a Dirac particle with ~14 decay channels that (3.3) could not be satisfied unless $m_\nu > 2$ GeV. Since the s-wave Majorana annihilation is suppressed by a factor 4 (0.4)/14 = 0.11 relative to the Dirac case, it is clear that s-wave Majorana annihilation will not be sufficient to satisfy (3.3), even for a neutrino mass $m_\nu = 2.2$ GeV [which maximizes $\lambda(m)$ when $m_f = 1.8$]. Thus, even though s-wave annihilation can dominate for $m_\nu \sim 2$ GeV, it is not sufficient to affect our lower bound.

Given this reduction in cross section we can, however, calculate explicitly a lower bound on the mass of a neutrino which annihilates via the s-wave cross section in (A.1) in order that it satisfy (3.3). Assuming the heavy fermion into which it can annihilate is a quark so that $N = 3$, and assuming $\lambda(m) = 0.3$ (minimum value) we have:

$$m_{\nu_h}^2/(2\,\mathrm{GeV})^2 \approx \langle\sigma_s^{\mathrm{Dirac}} v\rangle/2\langle\sigma_s^{\mathrm{M}} v\rangle = N'/2N\lambda(m) > 6$$

$$\Rightarrow m_{\nu_h} \gtrsim 5\ \mathrm{GeV}, \qquad (\mathrm{A}.2)$$

where the factor of 2 in the denominator of (A.2) is due to the reduction in helicity states in a relativistic gas for Majorana fermions. From this, we can derive immediately that $m_f > 4$ GeV in this case. Since the bottom quark has mass ~4.5 GeV, it follows therefore from (A.2) that s-wave annihilation will be sufficient to reduce the number density for a Majorana neutrino of mass $m_{\nu_H} \gtrsim 5$ GeV. This is slightly lower than the lower bound obtained in the text from p-wave annihilation, and the result is displayed in fig. 2.

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