SUPERSYMMETRY AND INSTANTONS

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We show that the eigenvalue equations for the fluctuation of scalars, fermions and gluon around any classical self-dual solution of the Yang–Mills theory have the same spectrum of non-zero eigenvalues. In the case of a supersymmetric Yang–Mills theory this implies that the one loop correction around any self dual instanton is just given by a counting of the zero modes of the gluon, fermion and ghost.

It has been proposed by Polyakov [1] that the existence of classical solutions of euclidean field theories may allow one to compute the large distance behavior of the corresponding theory. This can be done by expanding the partition function and the Green's functions around those classical solutions keeping only up to the quadratic terms in the field fluctuations. However the computation of the quantum corrections may become so complicated that it is very difficult to have an estimate of them.

One way out is to consider quantum field theories as the supersymmetric ones which have such an amount of symmetry that the calculation of the quantum corrections becomes very simple.

In this letter we consider the eigenvalue equations for the fluctuations of scalars, fermions and gluon around any classical self-dual solution of the Yang–Mills theory and we show that they have the same spectrum of eigenvalues (except for the zero modes) if the fields transform according to the same representation of the gauge group. As a consequence of this fact in the case of a supersymmetric theory, where the number of bosons is equal to the number of fermions, one gets a complete cancellation among the determinants that give the one-loop correction. Therefore the one-loop quantum correction is just given by an integral over the collective coordinates and by a logarithmic term containing the subtraction point μ whose coefficient can be determined by a counting of the zero modes of the gluon, fermions and ghost. It is given by $8N - 4NN_f$, where N is the Pontryagin number of the classical solution and N_f is the number of the fermion flavors.

The same considerations apply also to some two-dimensional theories as the supersymmetric ϕ^4 and sine-Gordon that have been recently constructed. One gets that the one-loop correction to the mass of the soliton is identically vanishing because of the cancellation between the bosonic and fermionic eigenfrequencies. These results generalize to the case of solitons and instantons the cancellation of the vacuum diagrams occurring in the vacuum sector in any supersymmetric theory [2].

If we expand the Yang–Mills action with scalars and fermions around any classical solution $\bar{A}_\mu$ of the self-duality equation:

$$\bar{F}_{\mu\nu} = \tilde{\bar{F}}_{\mu\nu} = \tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}\bar{F}_{\rho\sigma}, \tag{1}$$

one gets the following eigenvalue equations respectively for the fluctuation of the scalar, fermion and gluon:

$$\bar{\nabla}_\mu \bar{\nabla}_\mu \phi = -\lambda^2 \phi, \tag{2a}$$

$$i\gamma^\mu \bar{\nabla}_\mu \psi = \lambda\psi, \tag{2b}$$

$$V_{\nu\mu}a_\mu = \bar{\nabla}_\mu(\bar{\nabla}_\mu a_\nu - \bar{\nabla}_\nu a_\mu) + [\bar{F}_{\nu\mu}, a_\mu] = -\lambda^2 a_\nu, \quad (2c)$$

where $\bar{\nabla}$ is the gauge covariant derivative in the background field.

We want to show now that the fermion equation has the same spectrum of non-zero eigenvalues as the scalar provided that they transform according to the same representation of the gauge group [‡1].

If one uses the following representation for the euclidean γ-matrices:

$$\gamma^\mu = \begin{pmatrix} 0 & \alpha^\mu \\ \bar{\alpha}^\mu & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \begin{matrix} \alpha_\mu = (-\mathrm{i}\boldsymbol{\sigma}, 1), \\ \bar{\alpha}_\mu = (\mathrm{i}\boldsymbol{\sigma}, 1), \end{matrix} \quad (3)$$

and if one writes ψ as:

$$\psi = \psi_+ + \psi_-, \qquad \psi_\pm = \tfrac{1}{2}(1 \pm \gamma_5)\psi, \quad (4)$$

one gets the following equations for $\psi_\pm$:

$$T\psi_- = \mathrm{i}\alpha_\mu \nabla_\mu \psi_- = \lambda\psi_+, \quad (5a)$$

$$T^+\psi_+ = \mathrm{i}\bar{\alpha}_\mu \nabla_\mu \psi_+ = \lambda\psi_-, \quad (5b)$$

where T^+ is the adjoint of T.

Using the identities

$$\bar{\alpha}^\mu \alpha^\nu = \delta^{\mu\nu} + 2\mathrm{i}\sigma^{\mu\nu}, \qquad \alpha^\mu \bar{\alpha}^\nu = \delta^{\mu\nu} + 2\mathrm{i}\bar{\sigma}^{\mu\nu}, \quad (6)$$

with

$$\begin{aligned} \sigma_{\mu\nu} &= (1/4\mathrm{i})[\bar{\alpha}_\mu\alpha_\nu - \bar{\alpha}_\nu\alpha_\mu], \quad \sigma_{\mu\nu} = \tilde{\sigma}_{\mu\nu}, \\ \bar{\sigma}_{\mu\nu} &= (1/4\mathrm{i})[\alpha_\mu\bar{\alpha}_\nu - \alpha_\nu\bar{\alpha}_\mu], \quad \bar{\sigma}_{\mu\nu} = -\tilde{\bar{\sigma}}_{\mu\nu}, \end{aligned} \quad (7)$$

and the commutator of two covariant derivatives:

$$[\bar{\nabla}_\mu; \bar{\nabla}_\nu]^{ab} = \epsilon^{acb}\bar{F}^c_{\mu\nu}, \quad (8)$$

it is easy to show that

$$TT^+\psi_+ = \{(\nabla_\mu\nabla^\mu)^{ab} + \mathrm{i}\bar{\sigma}_{\mu\nu}\epsilon^{acb}\bar{F}^c_{\mu\nu}\}\psi^b_+ = -\lambda^2\psi^a_+, \quad (9a)$$

$$T^+T\psi_- = \{(\nabla_\mu\nabla^\mu)^{ab} + \mathrm{i}\sigma_{\mu\nu}\epsilon^{acb}\bar{F}^c_{\mu\nu}\}\psi^b_- = -\lambda^2\psi^a_-. \quad (9b)$$

But if the background field is self-dual, then because of eq. (7) $\bar{\sigma}_{\mu\nu}\bar{F}_{\mu\nu} = 0$ and eq. (9a) becomes the same equation as the eq. (2a) for the scalar field. Therefore, starting from any solution $\phi(x)$ with eigenvalue λ^2 of the scalar, eq. (2a), one can construct an eigenfunction of the fermion eq. (2b) with the same eigenvalue λ given by

$$\psi_+(x) = \phi(x)\alpha, \qquad \psi_-(x) = \lambda^{-1}T^+\psi_+(x), \quad (10)$$

where α is a constant arbitrary two-dimensional spinor. In addition the fermion eq. (9b) has also $C(T)N_\mathrm{f}N$ zero modes (N_f is the number of fermion flavors, N is the Pontryagin number, and $C(T) = \frac{2}{3}T(T+1)(2T+1)$), whose correspondent eigenfunctions have negative chirality ($\psi_+ = 0$) [4].

Starting from the fermion eigenfunction $\psi_n(x)$:

$$\psi_n(x) = (1 + \mathrm{i}\,\gamma^\mu\bar{\nabla}_\mu/\lambda_n)(\tfrac{1}{2}(1+\gamma^5))\phi_n(x)\alpha_n \quad (11)$$

with eigenvalues $\lambda_n \neq 0$ given in terms of the scalar eigenfunctions $\phi_n(x)$, it is possible to construct the fermion Green's function corresponding to the non-zero eigenvalues:

$$S_\mathrm{F}(x,y) = \sum_n \frac{\psi_n(x)\psi_n^+(y)}{\lambda_n}. \quad (12)$$

Using eq. (11) into eq. (12) and summing over both positive and negative eigenfrequencies one gets:

$$\begin{aligned} S_\mathrm{F}(x,y) &= \mathrm{i}\gamma^\mu\bar{\nabla}^x_\mu\Delta(x,y)(\tfrac{1}{2}(1+\gamma_5)) \\ &\quad + (\tfrac{1}{2}(1+\gamma_5))\Delta(x,y)\mathrm{i}\gamma_\mu\overleftarrow{\bar{\nabla}}{}^\mu_y, \end{aligned} \quad (13)$$

where

$$\Delta(x,y) = \sum_n \frac{\phi_n(x)\phi_n^+(y)}{\lambda_n^2}. \quad (14)$$

Eq. (13) agrees with the analogous result of ref. [5].

The same procedure applies to the eigenvalue equation for the vector fluctuations. The gauge is specified by adding a gauge fixing term $\frac{1}{2}(\bar{\nabla}_\mu A_\mu)^2$ to the lagrangian, and the eigenvalue eq. (2c) is then modified to:

$$V_{\nu\mu}a_\mu + S_{\nu\mu}a_\mu = -\lambda^2 a_\nu, \quad (15)$$

where $V_{\nu\mu}$ is defined in eq. (2c) and $S_{\mu\nu} = \bar{\nabla}_\mu\bar{\nabla}_\nu$. Let us take $a_\mu = b_\mu + \bar{\nabla}_\mu\alpha$ with $\bar{\nabla}_\mu b_\mu = 0$, then $S_{\nu\mu}b_\mu = 0$, $V_{\nu\mu}\bar{\nabla}_\mu\alpha = 0$ and the eigenvalue equation becomes

$$\begin{pmatrix} V_{\nu\mu}b_\mu \\ S_{\nu\mu}\bar{\nabla}_\mu\alpha \end{pmatrix} = -\lambda^2 \begin{pmatrix} b_\nu \\ \bar{\nabla}_\nu\alpha \end{pmatrix}. \quad (16)$$

Let us consider the operators:

$$T\begin{pmatrix} b \\ \bar{\nabla}\alpha \end{pmatrix} = \begin{pmatrix} \eta^{(-)}_{a\rho\sigma}\bar{\nabla}_\rho b_\sigma \\ \bar{\nabla}_\mu\bar{\nabla}_\mu\alpha \end{pmatrix}, \quad T^+\begin{pmatrix} f \\ h \end{pmatrix} = \begin{pmatrix} -\eta^{(-)}_{a\rho\sigma}\bar{\nabla}_\rho f_a \\ -\bar{\nabla}_\rho h \end{pmatrix}, \quad (17)$$

where $\eta^{(-)}_{a\rho\sigma}$ is defined in ref. [3].

[‡1] The equality of the non-zero eigenvalues of eqs. (2) has also been used by 't Hooft in the calculation of the quantum fluctuations around the $N = 1$ instanton [3].

It is easy to prove that $-T^+T$ reproduces the left-hand-side of eq. (16), while TT^+ gives

$$TT^+\begin{pmatrix} f \\ h \end{pmatrix} = -\begin{pmatrix} +\bar{\nabla}_\mu \bar{\nabla}_\mu f_a \\ \bar{\nabla}_\mu \bar{\nabla}_\mu h \end{pmatrix}. \tag{18}$$

Since TT^+ and T^+T have the same spectrum of non-zero eigenvalues, it follows from eq. (18) that the eigenvalue eq. (16) has the same non-zero eigenvalues as the scalar eq. (2a). Starting from any eigenfunction $\phi(x)$ of the scalar eq. (2a), it is possible to construct the eigenfunctions of eq. (16) corresponding to the same eigenvalue.by applying T^+ to

$$v(j) = \begin{pmatrix} f^a(j) = \delta_{ja}\phi \\ h = 0 \end{pmatrix}$$

and

$$v(0) = \begin{pmatrix} f_a = 0 \\ h = \phi(x) \end{pmatrix}.$$

One gets

$$a_\mu^{(i)} = -\lambda^{-1}\eta^{(-)}_{i\rho\mu}\bar{\nabla}_\rho\phi(x), \quad a_\mu^{(0)} = -\lambda^{-1}\bar{\nabla}_\mu\phi(x). \tag{19}$$

Each eigenvalue is therefore four times degenerate with respect to the scalar, but two components are killed in the functional integral by the Faddeev–Popov ghost (whose eigenvalue equation is the same as the scalar) so that only the two physical components of the gauge field give actually a contribution.

Using the eigenfunction (19) one can construct the propagator of the gluon

$$\begin{aligned} D_{\mu\nu}(x,y) &= \sum_n \sum_{k=0}^{3} \frac{a_n^{(k)}(x)a_n^{(k)+}(y)}{\lambda_n^2} \\ &= -[\delta_{\mu\rho}\delta_{\nu\sigma} + \eta^{(-)}_{i\mu\rho}\eta^{(-)}_{i\nu\sigma}]\, \overrightarrow{\nabla}^x_\rho \Delta^2(x,y) \overleftarrow{\nabla}^y_\sigma, \end{aligned} \tag{20}$$

which agrees with the result of ref. [5].

In order to compute the one-loop correction around any classical solution $\bar{A}_\mu$ of the partition function of the Yang–Mills theory, one must compute the determinants of the operators (2). As in ref. [6] ‡2, we use the ζ-function regularization procedure which gives the following regularized formula for the determinant of the operator A:

$$\det((\mu\alpha)^{-1}A) = (\mu\alpha)^{-Z-\zeta_0}\, e^{-\zeta_0'}, \tag{21}$$

where μ is the subtraction point, α is containing the scales of the instantons and $\zeta_T(s)$ is defined in terms of the eigenvalues of A by:

$$\zeta_s(T) = \sum_n [\lambda_n(T)]^{-s}. \tag{22}$$

Z is the number of zero modes of A and the index T refers to the isospin of the field. Using the fact that the operators (2) have the same non-zero eigenvalues, one gets:

$$\zeta_s^{\mathrm{F}}(T) = 2\zeta_s(T), \quad \zeta_s^{\mathrm{V}}(1) = 2\zeta_s(1), \tag{23}$$

where $\zeta_s(T)$ is the ζ-function corresponding to the scalar field and the factor 2 counts the number of components of a massless spinning particle. In the ζ-function corresponding to the vector we have also included the contribution of the ghost field which behaves as the scalar.

Inserting those expressions in the partition function corresponding to the Pontryagin number N one gets:

$$Z^{(N)} = \int \mathrm{d}C\, e^{-8\pi^2 N/g^2} (\mu\alpha)^A e^B, \tag{24}$$

where $\int \mathrm{d}C$ is integral over the collective coordinates and

$$A = 8N - C(T)NN_\mathrm{f} + N_\mathrm{s}\tilde{\zeta}_0(T_\mathrm{s}) + 2\tilde{\zeta}_0(1) - 4N_\mathrm{f}\tilde{\zeta}_0(T_\mathrm{f}), \tag{25}$$

$$2B = \tilde{\zeta}_0'(T_\mathrm{s})N_\mathrm{s} + 2\tilde{\zeta}_0'(1) - 4N_\mathrm{f}\tilde{\zeta}_0'(T_\mathrm{f}). \tag{26}$$

$\tilde{\zeta}_s(T)$ is the difference between the ζ-function in the background instanton field and that in the vacuum.

In particular in a supersymmetric theory ($T_\mathrm{s} = T_\mathrm{f} = 1$) one has the same number of bosons and fermions. This implies that $N_\mathrm{s} + 2 = 4N_\mathrm{f}$ and therefore ‡3

$$B = 0, \quad A = (8 - 4N_\mathrm{f})N. \tag{27}$$

Therefore the coefficient of the term containing the subtraction point μ is only given in terms of the zero modes of the gluon, fermions and ghost.

In the supersymmetric case the partition function becomes extremely simple

‡2 The ζ-function regularization procedure has also been used to compute the zero point energy of a dual string [7,13]. The one loop correction around the $N = 1$ instanton has been also computed in refs. [3] and [8].

‡3 The vanishing of B in the case of the $N = 1$ instanton in a supersymmetric theory has been also noticed in ref. [9].

$$Z^{(N)} = \int \mathrm{d}C \exp\left[-\frac{8\pi^2}{g^2} + (8 - 4N_\mathrm{f}) \log \mu\alpha\right] N. \quad (28)$$

In particular in the supersymmetric theory with an SU(4) internal symmetry containing 6 real scalars and 2 complex fermions ($N_\mathrm{f} = 2$) also the logarithmic term in eq. (28) is vanishing ‡4. This is the consequence of the fact that this particular supersymmetric theory can be obtained as the zero slope limit of the Neveu–Schwarz–Ramond (NSR) model in ten dimensions after the compactification of the six extra dimensions [11]. It is known, in fact, that the loop corrections in the string model do not give rise to any ultraviolet divergence, and in the NSR model this property seems to be preserved also in the zero slope limit (for a general review of dual models, see, e.g., refs. [12,13]). If the same property is also valid in the closed string sector of the NSR model, one would get the vanishing of the conformal anomaly in the case of the SO(4) supergravity which contains 2 spin 0, 4 real spin 1/2, 4 real spin 3/2, 6 spin 1 and 1 spin 2 [14]. Imposing the vanishing of the trace anomaly in the SO(4) supergravity, one gets the following anomaly for a Majorana spin 3/2 field ‡5:

$$T_\mu{}^\mu = \frac{1}{180(4\pi)^2}\{-165 R_{\mu\nu}R^{\mu\nu} + 38R^2 - \tfrac{143}{4} R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\}. \quad (29)$$

It would be interesting to study what happens to the Hawking effect [16] ‡6 in a theory without trace anomaly.

In the last part of this letter we discuss some two-dimensional sypersymmetric models with soliton solutions which have been recently constructed [18,19].

It has been shown in ref. [18] that the following action is supersymmetric:

$$S = \int \mathrm{d}x\, \mathrm{d}t\, \{-\tfrac{1}{2}(\partial_\mu\phi)^2 - \tfrac{1}{2}[V'(\phi)]^2 - \tfrac{1}{2}\mathrm{i}\,\bar\psi \partial\!\!\!/ \psi - \tfrac{1}{2}\mathrm{i}\,\bar\psi\psi V''(\phi)\}. \quad (30)$$

In particular, if $V'(\phi) = \lambda^{-1/2}(m^2 - \phi^2)$, one gets a supersymmetric version of the ϕ^4-theory, while if $V'(\phi) = 2m^2\lambda^{-1/2}\sin(\sqrt{\lambda}\,\phi/2m)$, one gets a supersymmetric version of the sine-Gordon equation. A time independent soliton solution satisfies the following first order equation:

$$(\mathrm{d}/\mathrm{d}x)\,\phi_\mathrm{cl}(x) = V'(\phi_\mathrm{cl}). \quad (31)$$

In order to compute the lowest quantum correction to the classical mass of the soliton one must solve the following eigenvalue equations:

$$\left\{-\frac{\mathrm{d}^2}{\mathrm{d}x^2} + [V''(\phi_\mathrm{cl})]^2 + V'(\phi_\mathrm{cl})V'''(\phi_\mathrm{cl})\right\}\eta(x) = \omega_\mathrm{B}^2\eta(x), \quad (32a)$$

$$\left\{\gamma_1\frac{\mathrm{d}}{\mathrm{d}x} + V''(\phi_\mathrm{cl})\right\}\chi(x) = \mathrm{i}\omega_\mathrm{F}\gamma_0\chi(x). \quad (32b)$$

If one writes

$$\chi(x) = \chi_+(x) + \chi_-(x), \quad \chi_\pm(x) = \tfrac{1}{2}(1 \pm \gamma_1)\chi, \quad (33)$$

eq. (32b) becomes:

$$T\chi_+ = \left\{\frac{\mathrm{d}}{\mathrm{d}x} + V''(\phi_\mathrm{cl})\right\}\chi_+ = \mathrm{i}\omega_\mathrm{F}\gamma_0\chi_-, \quad (34a)$$

$$T^+\chi_- = \left\{-\frac{\mathrm{d}}{\mathrm{d}x} + V''(\phi_\mathrm{cl})\right\}\chi_- = \mathrm{i}\omega_\mathrm{F}\gamma_0\chi_+. \quad (34b)$$

It is easy to prove that

$$T^+T\chi_+ = \left\{-\frac{\mathrm{d}^2}{\mathrm{d}x^2} + [V''(\phi_\mathrm{cl})]^2 - V'(\phi_\mathrm{cl})V'''(\phi_\mathrm{cl})\right\}\chi_+ = \omega_\mathrm{F}^2\chi_+, \quad (35a)$$

$$TT^+\chi_- = \left\{-\frac{\mathrm{d}^2}{\mathrm{d}x^2} + [V''(\phi_\mathrm{cl})]^2 + V'(\phi_\mathrm{cl})V'''(\phi_\mathrm{cl})\right\}\chi_- = \omega_\mathrm{F}^2\chi_-, \quad (35b)$$

where eq. (30) has been used.

The eigenvalue equation for χ_- is identical to the one for $\eta(x)$; therefore one gets for the fermions the

‡4 The β function of this supersymmetric theory has been recently computed and found to be vanishing up to two loops [10]. We thank S. Ferrara for communicating this result to us.

‡5 We used the values for the conformal anomalies given in ref. [15].

‡6 For a connection between the Hawking effect and the trace anomaly see ref. [17].

same spectrum of eigenvalues as in the case of the boson ($\omega_F^2 = \omega_B^2$). If $\eta(x)$ is an eigenfunction of the eq. (32a) with eigenvalue $\omega^2 \neq 0$, then one can construct the following eigenfunction of the fermion equation:

$$\chi_- = \eta(x)\alpha_-, \qquad \chi_+ = -(\mathrm{i}\omega)^{-1}\gamma^0 T^+ \chi_- . \tag{36}$$

It is also easy to see that eqs. (32a) and (35b) have a zero mode corresponding respectively to the invariance of the action under translations and supersymmetry. Eq. (35a) instead does not have any zero mode. As a consequence of the equality of the boson and fermion eigenvalues one gets that the one loop correction to the soliton mass is identically vanishing:

$$M = M_{\mathrm{cl}} + \frac{1}{2}\sum \omega_B - \frac{1}{2}\sum \omega_F = M_{\mathrm{cl}}. \tag{37}$$

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