Strong Coupling Constant with Flavor Thresholds at Four Loops in the Modified Minimal-Subtraction Scheme

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We present in analytic form the matching conditions for the strong coupling constant $\alpha_s^{(n_f)}(\mu)$ at the flavor thresholds to three loops in the modified minimal-subtraction scheme. Taking into account the recently calculated coefficient β_3 of the Callan-Symanzik beta function of quantum chromodynamics, we thus derive a four-loop formula for $\alpha_s^{(n_f)}(\mu)$ together with appropriate relationships between the asymptotic scale parameters $\Lambda^{(n_f)}$ for different numbers of flavors n_f. [S0031-9007(97)04062-3]

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The strong coupling constant $\alpha_s^{(n_f)}(\mu) = g_s^2/(4\pi)$, where g_s is the gauge coupling of quantum chromodynamics (QCD), is a fundamental parameter of the standard model of elementary particle physics; its value $\alpha_s^{(5)}(M_Z)$ is listed among the constants of nature in the Review of Particle Physics [1]. Here, μ is the renormalization scale, and n_f is the number of active quark flavors q, with mass $m_q \ll \mu$. The μ dependence of $\alpha_s^{(n_f)}(\mu)$ is controlled by the Callan-Symanzik beta function of QCD,

$$\frac{\mu^2 d}{d\mu^2}\frac{\alpha_s^{(n_f)}(\mu)}{\pi} = \beta^{(n_f)}\left(\frac{\alpha_s^{(n_f)}(\mu)}{\pi}\right) = -\sum_{N=0}^{\infty}\beta_N^{(n_f)}\left(\frac{\alpha_s^{(n_f)}(\mu)}{\pi}\right)^{N+2}. \quad (1)$$

The calculation of the one-loop coefficient $\beta_0^{(n_f)}$ about 25 years ago [2] has led to the discovery of asymptotic freedom and to the establishment of QCD as the theory of strong interactions. In the class of schemes where the beta function is mass independent, which includes the minimal-subtraction (MS) schemes of dimensional regularization [3], $\beta_0^{(n_f)}$ and $\beta_1^{(n_f)}$ [4] are universal. The results for $\beta_2^{(n_f)}$ [5] and $\beta_3^{(n_f)}$ [6] are available in the modified MS ($\overline{\mathrm{MS}}$) scheme [7]. For the reader's convenience, $\beta_N^{(n_f)}$ ($N = 0, \ldots, 3$) are listed for the n_f values of practical interest in Table I.

In MS-like renormalization schemes, the Appelquist-Carazzone decoupling theorem [8] does not in general apply to quantities that do not represent physical observables, such as beta functions or coupling constants; i.e., quarks with mass $m_q \gg \mu$ do not automatically decouple. The standard procedure to circumvent this problem is to render decoupling explicit by using the language of effective field theory. As an idealized situation, consider QCD with $n_l = n_f - 1$ massless quark flavors and one heavy flavor h, with mass $m_h \gg \mu$. Then, one constructs an effective n_l-flavor theory by requiring consistency with the full n_f-flavor theory at the heavy-quark threshold $\mu^{(n_f)} = \mathcal{O}(m_h)$. This leads to a nontrivial matching condition between the couplings of the two theories. Although $\alpha_s^{(n_l)}(m_h) = \alpha_s^{(n_f)}(m_h)$ at leading and next-to-leading order, this relation does not generally hold at higher orders in the $\overline{\mathrm{MS}}$ scheme. If the μ evolution of $\alpha_s^{(n_f)}(\mu)$ is to be performed at $N + 1$ loops, i.e., with the highest coefficient in Eq. (1) being $\beta_N^{(n_f)}$, then consistency requires that the matching conditions be implemented in terms of N-loop formulas. Then, the residual μ dependence of physical observables will be of order $N + 2$. A pedagogical review of the QCD matching conditions at thresholds to two loops may be found in Ref. [9].

The literature contains two conflicting results on the two-loop matching condition for $\alpha_s^{(n_f)}(\mu)$ in the $\overline{\mathrm{MS}}$ scheme [10,11]. The purpose of this Letter is to settle this issue by an independent calculation and to take the next step, to three loops. As a consequence, Eq. (9.7) in the encyclopedia by the Particle Data Group [1] will be corrected and extended by one order. We shall also add the four-loop term in the formula (9.5a) for $\alpha_s^{(n_f)}(\mu)$ in Ref. [1].

In order to simplify the notation, we introduce the couplant $a^{(n_f)}(\mu) = \alpha_s^{(n_f)}(\mu)/\pi$ and omit the labels μ and n_f wherever confusion is impossible. Integrating Eq. (1) leads to

$$\ln\frac{\mu^2}{\Lambda^2} = \int\frac{da}{\beta(a)} = \frac{1}{\beta_0}\left[\frac{1}{a} + b_1\ln a + (b_2 - b_1^2)a + \left(\frac{b_3}{2} - b_1 b_2 + \frac{b_1^3}{2}\right)a^2\right] + C, \quad (2)$$

TABLE I. $\overline{\mathrm{MS}}$ values of $\beta_N^{(n_f)}$ for variable n_f.

n_f	$\beta_0^{(n_f)}$	$\beta_1^{(n_f)}$	$\beta_2^{(n_f)}$	$\beta_3^{(n_f)}$
3	$\frac{9}{4}$	4	$\frac{3863}{384}$	$\frac{445}{32}\zeta(3) + \frac{140\,599}{4608}$
4	$\frac{25}{12}$	$\frac{77}{24}$	$\frac{21\,943}{3456}$	$\frac{78\,535}{5184}\zeta(3) + \frac{4\,918\,247}{373\,248}$
5	$\frac{23}{12}$	$\frac{29}{12}$	$\frac{9769}{3456}$	$\frac{11\,027}{648}\zeta(3) - \frac{598\,391}{373\,248}$
6	$\frac{7}{4}$	$\frac{13}{8}$	$-\frac{65}{128}$	$\frac{11\,237}{576}\zeta(3) - \frac{63\,559}{4608}$

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where $b_N = \beta_N/\beta_0$ ($N = 1,2,3$), Λ is the so-called asymptotic scale parameter, and C is an arbitrary constant. The second equation of Eq. (2) is obtained by expanding the integrand. The conventional $\overline{\mathrm{MS}}$ definition of Λ, which we shall adopt in the following, corresponds to choosing $C = (b_1/\beta_0)\ln\beta_0$ [7,12].

Iteratively solving Eq. (2) yields

$$a = \frac{1}{\beta_0 L} - \frac{b_1 \ln L}{(\beta_0 L)^2} + \frac{1}{(\beta_0 L)^3}[b_1^2(\ln^2 L - \ln L - 1) + b_2] + \frac{1}{(\beta_0 L)^4}\left[b_1^3\left(-\ln^3 L + \frac{5}{2}\ln^2 L + 2\ln L - \frac{1}{2}\right) - 3b_1 b_2 \ln L + \frac{b_3}{2}\right], \quad (3)$$

where $L = \ln(\mu^2/\Lambda^2)$ and terms of $\mathcal{O}(1/L^5)$ have been neglected. Equation (3) extends Eq. (9.5a) of Ref. [1] to four loops.

The particular choice of C [7,12] in Eq. (2) is predicated on the grounds that it suppresses the appearance of a term proportional to (const$/L^2$) in Eq. (3). For practical applications, it might be more useful to define C by equating the one- and two-loop expressions of $\alpha_s(\mu)$, i.e., by nullifying the $\mathcal{O}(1/L^2)$ term in Eq. (3), at some convenient reference scale μ_0 [13], e.g., at $\mu_0 = M_Z$. By contrast, in the standard approach, one has $\mu_0 = \sqrt{e}\,\Lambda$, which is in the nonperturbative regime. This would lead to the choice $C = (b_1/\beta_0)\ln[\beta_0\ln(\mu_0^2/\Lambda^2)]$. The advantage of this convention would be that the values of Λ would be considerably more stable under the inclusion of higher-order corrections. Another interesting alternative is to adjust C in such a way that Λ becomes n_f independent [14].

It is interesting to quantitatively investigate the impact of the higher-order terms of the beta function in Eq. (1) on the μ dependence of $\alpha_s^{(n_f)}$ for fixed n_f. For illustration, we consider, as an extreme case, the evolution of $\alpha_s^{(5)}(\mu)$ from $\mu = M_Z$ down to scales of the order of the proton mass. Specifically, we employ the four-loop formula (3) and its N-loop approximations, with $N = 1,2,3$, which emerge from Eq. (3) by discarding the terms of $\mathcal{O}(1/L^{N+1})$. In each case, we determine $\Lambda^{(5)}$ from the condition that $\alpha_s^{(5)}(M_Z) = 0.118$ [1] be exactly satisfied. For comparison, we also consider the exact solution of Eq. (1) with all known beta-function coefficients included. In Fig. 1, the various results for $1/\alpha_s^{(5)}(\mu)$ are plotted versus μ/M_Z using a logarithmic scale on the abscissa. Consequently, the one-loop result appears as a straight line. All curves precisely cross at $\mu = M_Z$, outside the figure. We observe that, for N increasing, the expanded N-loop results of Eq. (3) gradually approach the exact four-loop solution of Eq. (1) in an alternating manner. Down to rather low scales, the two-loop result already provides a remarkably useful approximation to the exact four-loop result, while the one-loop result is far off.

Next, we outline the derivation of the three-loop matching condition. In the following, unprimed quantities refer to the full n_f-flavor theory, while primed objects belong to the effective theory with $n_l = n_f - 1$ flavors. Furthermore, bare quantities are labeled by the superscript 0. We wish to derive the decoupling constant ζ_g in the relation $g_s' = \zeta_g g_s$ between the renormalized couplings g_s and g_s'. Exploiting knowledge [6] of the coupling renormalization constant Z_g within either theory, this task is reduced to finding $\zeta_g^0 = \zeta_g Z_g'/Z_g$. The Ward identity $\zeta_g^0 = \tilde\zeta_1^0/(\tilde\zeta_3^0\sqrt{\zeta_3^0})$, where

$$G_\mu^{0\prime} = \sqrt{\zeta_3^0}\,G_\mu^0, \qquad c^{0\prime} = \sqrt{\tilde\zeta_3^0}\,c^0, \qquad \Lambda_\mu^{0\prime} = \tilde\zeta_1^0\tilde\zeta_3^0\sqrt{\zeta_3^0}\,\Lambda_\mu^0\,, \quad (4)$$

with G_μ, c, and Λ_μ being the fields of the gluon and the Faddeev-Popov ghost, and the $G\bar c c$ vertex, respectively, then leads us to consider the heavy-quark contributions to the corresponding vacuum polarizations and vertex correction, $\Pi_G^h(q_G^2)$, $\Pi_c^h(q_c^2)$, and $\Gamma_\mu^h(q_c, q_{\bar c})$. Specifically, we have

$$\zeta_3^0 = 1 + \Pi_G^{0h}(0), \qquad \tilde\zeta_3^0 = 1 + \Pi_c^{0h}(0)\,, \qquad \tilde\zeta_1^0 = 1 + \left.\frac{q^\mu\Gamma_\mu^{0h}(q,-q)}{q^2}\right|_{q=0}. \quad (5)$$

In total, we need to compute 3 + 1 + 5 two-loop and 189 + 25 + 228 three-loop Feynman diagrams. The 5 two-loop diagrams pertinent to $\tilde\zeta_1^0$ add up to zero. Typical three-loop specimens are depicted in Fig. 2. In order to cope with the enormous complexity of the problem at hand, we make successive use of powerful symbolic manipulation programs. We generate and compute the relevant diagrams with the packages QGRAF [15] and MATAD [16], respectively. The cancellation of the ultraviolet

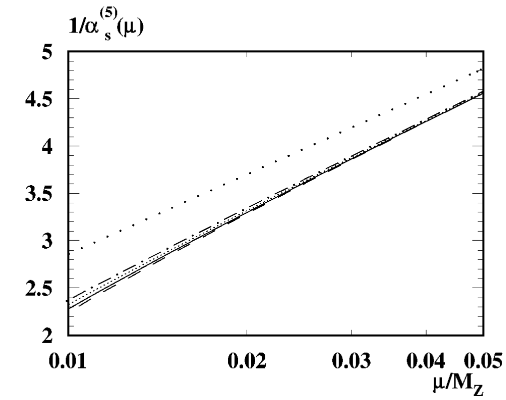

FIG. 1. μ dependence of $\alpha_s^{(5)}(\mu)$ calculated from $\alpha_s^{(5)}(M_Z) = 0.118$ using Eq. (3) at one (coarsely dotted), two (dashed), three (dot-dashed), and four (solid) loops. The densely dotted line represents the exact solution of Eq. (1) at four loops.

singularities, the gauge-parameter independence, and the renormalization-group (RG) invariance serve as strong checks for our calculation.

If we measure the matching scale $\mu^{(n_f)}$ in units of the RG-invariant $\overline{\mathrm{MS}}$ mass $\mu_h = m_h(\mu_h)$, our result for the ratio of $a' = a^{(n_l)}(\mu^{(n_f)})$ to $a = a^{(n_f)}(\mu^{(n_f)})$ reads

$$\frac{a'}{a} = 1 - a\,\frac{\ell_h}{6} + a^2\left(\frac{\ell_h^2}{36} - \frac{19}{24}\,\ell_h + c_2\right) + a^3\left[-\frac{\ell_h^3}{216} - \frac{131}{576}\,\ell_h^2 + \frac{\ell_h}{1728}\,(-6793 + 281\,n_l) + c_3\right], \tag{6}$$

where $\ell_h = \ln[(\mu^{(n_f)})^2/\mu_h^2]$ and

$$c_2 = \frac{11}{72}, \qquad c_3 = -\frac{82\,043}{27\,648}\,\zeta(3) + \frac{564\,731}{124\,416} - \frac{2633}{31\,104}\,n_l\,. \tag{7}$$

Here, ζ is Riemann's zeta function, with values $\zeta(2) = \pi^2/6$ and $\zeta(3) \approx 1.202\,057$. Our result for c_2 agrees with Ref. [11], while it disagrees with Ref. [10]. For the convenience of those readers who prefer to deal with the pole mass M_h, we list here a simple formula [17] for μ_h in terms of M_h and $A = a^{(n_f)}(M_h)$, which incorporates the well-known two-loop relation between $m_h(M_h)$ and M_h [18]. It reads

$$\frac{\mu_h}{M_h} = 1 - \frac{4}{3}\,A + A^2\left\{\frac{\zeta(3)}{6} - \frac{\zeta(2)}{3}\,(2\ln 2 + 7) - \frac{2393}{288} + \frac{n_f}{3}\left[\zeta(2) + \frac{71}{48}\right]\right\}. \tag{8}$$

Using a similar relation, with A expressed in terms of a and $\mathcal{L}_h = \ln[(\mu^{(n_f)})^2/M_h^2]$, we may rewrite Eq. (6) as

$$\frac{a'}{a} = 1 - a\,\frac{\mathcal{L}_h}{6} + a^2\left(\frac{\mathcal{L}_h^2}{36} - \frac{19}{24}\,\mathcal{L}_h + C_2\right) + a^3\left[-\frac{\mathcal{L}_h^3}{216} - \frac{131}{576}\,\mathcal{L}_h^2 + \frac{\mathcal{L}_h}{1728}\,(-8521 + 409 n_l) + C_3\right], \tag{9}$$

where

$$C_2 = -\frac{7}{24}, \qquad C_3 = -\frac{80\,507}{27\,648}\,\zeta(3) - \frac{2}{3}\,\zeta(2)\left(\frac{1}{3}\,\ln 2 + 1\right) - \frac{58\,933}{124\,416} + \frac{n_l}{9}\left[\zeta(2) + \frac{2479}{3456}\right]. \tag{10}$$

Going to higher orders, one expects, on general grounds, that the relation between $\alpha_s^{(n_l)}(\mu')$ and $\alpha_s^{(n_f)}(\mu)$, where $\mu' \ll \mu^{(n_f)} \ll \mu$, becomes insensitive to the choice of $\mu^{(n_f)}$ as long as $\mu^{(n_f)} = \mathcal{O}(m_h)$. This has been checked in Ref. [9] for three-loop evolution in connection with two-loop matching. Armed with our new results, we are in a position to explore the situation at the next order. As an example, we consider the crossing of the bottom-quark threshold. In particular, we wish to study how the $\mu^{(5)}$ dependence of the relation between $\alpha_s^{(4)}(M_\tau)$ and $\alpha_s^{(5)}(M_Z)$ is reduced as we implement four-loop evolution with three-loop matching. Our procedure is as follows. We first calculate $\alpha_s^{(4)}(\mu^{(5)})$ with Eq. (3) by imposing the condition $\alpha_s^{(4)}(M_\tau) = 0.36$ [9], then obtain $\alpha_s^{(5)}(\mu^{(5)})$ from Eq. (9), and finally compute $\alpha_s^{(5)}(M_Z)$ with Eq. (3). For consistency, N-loop evolution must be accompanied by $(N-1)$-loop matching; i.e., if we omit terms of $\mathcal{O}(1/L^{N+1})$ in Eq. (3), we need to discard those of $\mathcal{O}(a^N)$ in Eq. (9) at the same time. In Fig. 3, the variation of $\alpha_s^{(5)}(M_Z)$ with $\mu^{(5)}/M_b$ is displayed for the various levels of accuracy, ranging from one-loop to four-loop evolution. For illustration, $\mu^{(5)}$ is varied rather extremely, by almost 2 orders of magnitude. While the leading-order result exhibits a strong logarithmic behavior, the analysis is gradually getting more stable as we go to higher orders. The four-loop curve is almost flat. Besides the $\mu^{(5)}$ dependence of $\alpha_s^{(5)}(M_Z)$, its absolute normalization is also significantly affected by the higher orders. At the central scale $\mu^{(5)} = M_b$, we again encounter an alternating convergence behavior.

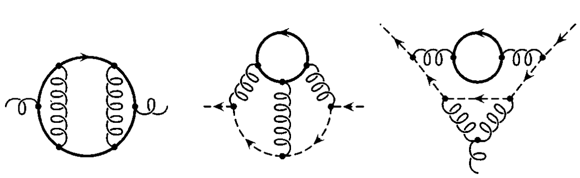

FIG. 2. Typical three-loop diagrams pertinent to $\Pi_G^h(q_G^2)$, $\Pi_c^h(q_c^2)$, and $\Gamma_\mu^h(q_c, q_{\bar{c}})$. Looped, dashed, and solid lines represent gluons G, Faddeev-Popov ghosts c, and heavy quarks h, respectively.

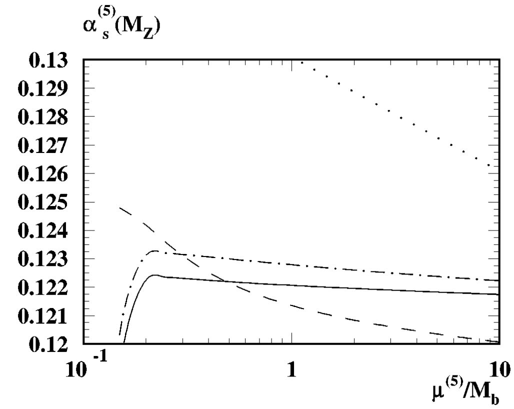

FIG. 3. $\mu^{(5)}$ dependence of $\alpha_s^{(5)}(M_Z)$ calculated from $\alpha_s^{(4)}(M_\tau) = 0.36$ and $M_b = 4.7$ GeV using Eq. (3) at one (dotted), two (dashed), three (dot-dashed), and four (solid) loops in connection with Eq. (9) at the respective order.

As we have learned from Fig. 3, in higher orders, the actual value of $\mu^{(n_f)}$ does not matter as long as it is comparable to the heavy-quark mass. In the context of Eq. (6), the choice $\mu^{(n_f)} = \mu_h$ [1] is particularly convenient, since it eliminates the RG logarithm ℓ_h. With this convention, we obtain from Eqs. (2), (3), and (6) a simple relationship between $\Lambda' = \Lambda^{(n_l)}$ and $\Lambda = \Lambda^{(n_f)}$, viz.,

$$\beta_0' \ln \frac{\Lambda'^2}{\Lambda^2} = (\beta_0' - \beta_0) l_h + (b_1' - b_1) \ln l_h - b_1' \ln \frac{\beta_0'}{\beta_0} + \frac{1}{\beta_0 l_h}[b_1(b_1' - b_1) \ln l_h + b_1'^2 - b_1^2 - b_2' + b_2 + c_2] + \frac{1}{(\beta_0 l_h)^2}\left\{-\frac{b_1^2}{2}(b_1' - b_1)\ln^2 l_h + b_1[-b_1'(b_1' - b_1) + b_2' - b_2 - c_2]\ln l_h + \frac{1}{2}(-b_1'^3 - b_1^3 - b_3' + b_3) + b_1'(b_1^2 + b_2' - b_2 - c_2) + c_3\right\}, \tag{11}$$

where $l_h = \ln(\mu_h^2/\Lambda^2)$. The $\mathcal{O}(1/l_h^2)$ term of Eq. (11) represents a new result. Leaving aside this term, Eq. (11) disagrees with Eq. (9.7) of Ref. [1]. This disagreement may partly be traced to the fact that the latter equation is written with the c_2 value obtained in Ref. [10], which differs from the value listed in Eq. (7). Furthermore, in the same equation, the terms involving β_2 should be divided by 4. Equation (11) represents a closed three-loop formula for $\Lambda^{(n_l)}$ in terms of $\Lambda^{(n_f)}$ and μ_h. For consistency, it should be used in connection with the four-loop expression (3) for $\alpha_s^{(n_f)}(\mu)$ with the understanding that the underlying flavor thresholds are fixed at $\mu^{(n_f)} = \mu_h$. The inverse relation that gives $\Lambda^{(n_f)}$ as a function of $\Lambda^{(n_l)}$ and μ_h emerges from Eq. (11) via the substitutions $\Lambda \leftrightarrow \Lambda'$; $\beta_N \leftrightarrow \beta_N'$ for $N = 0, \ldots, 3$; and $c_N \to -c_N$ for $N = 2, 3$. The on-shell version of Eq. (11), appropriate to the choice $\mu^{(n_f)} = M_h$, is obtained by substituting $l_h \to L_h = \ln(M_h^2/\Lambda^2)$ and $c_n \to C_N$ for $N = 2, 3$. Analogously to the case of $\mu^{(n_f)} = \mu_h$, its inverse, which gives $\Lambda^{(n_f)}$ in terms of $\Lambda^{(n_l)}$ and M_h, then follows through the replacements $\Lambda \leftrightarrow \Lambda'$; $\beta_N \leftrightarrow \beta_N'$ for $N = 0, \ldots, 3$; and $C_N \to -C_N$ for $N = 2, 3$.

In conclusion, we have extended the standard description of the strong coupling constant in the $\overline{\text{MS}}$ renormalization scheme to include four-loop evolution and three-loop matching at the quark-flavor thresholds. As a by-product of our analysis, we have settled a minor discrepancy in the literature regarding the two-loop matching conditions [10,11]. These results will be indispensible in order to relate the QCD predictions for different observables at next-to-next-to-next-to-leading order. Meaningful estimates of such corrections already exist [19].

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