CLASSICAL YANG–MILLS SOLUTIONS, CONDENSATION OF W MESONS AND SYMMETRY OF COMPOSITION OF SUPERDENSE MATTER

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It is shown that an increase of the density of cold matter leads to the condensation of W mesons. As a consequence of this phase transition, the densities of all types of right and left leptons (baryons) in superdense matter become equal to each other.

In recent years there has been considerable interest in the investigation of the properties of superdense matter from the point of view of gauge theories of weak, strong and electromagnetic interactions [1–3]. One of the most interesting results of this investigation is that the density of the Bose condensate of the scalar Higgs mesons, responsible for asymmetry breaking in gauge theories, depends crucially on temperature and density of matter. As a consequence, an increase of density and temperature leads to a number of phase transitions, which modify substantially the properties of superdense matter [1,2,4]. These phase transitions may have important consequences for cosmology and even for elementary particle physics, see a review of phase transitions in gauge theories in ref. [5].

In all the above mentioned papers, however, the effects connected with the Bose condensation of *scalar* fields only have been considered. In a recent paper [6] it was shown that the condensation of abelian vector fields similar to pion condensation [7] also may take place in superdense matter, but only due to radiative corrections in the strong coupling case, when the usual perturbation theory results are not quite reliable.

In the present paper (see also refs. [5,8]) it will be shown that in non-abelian theories the condensation of vector fields appears at the classical level and may take place at any value of the coupling constants. It will be shown also that as a consequence of the non-abelian vector field condensation, the densities of right and left leptons (baryons) with increasing overall lepton density become equal to each other, independently of the initial relations between the densities of different leptons (baryons) in the low density matter.

As a preliminary step we shall consider here the SU(2)-symmetric theory of a Yang–Mills field A_μ^a interacting with a multiplet of scalar fields φ^a, $a = 1,2,3$:

$$L = -\tfrac{1}{4}(G_{\mu\nu}^a)^2 + \tfrac{1}{2}(\nabla_\mu\varphi^a)^2 + \tfrac{1}{2}m^2(\varphi^a)^2 - \tfrac{1}{4}\lambda((\varphi^a)^2)^2 \,. \tag{1}$$

Here

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + e\epsilon^{abc} A_\mu^b A_\nu^c, \tag{2}$$

$$\nabla_\mu\varphi^a = \partial_\mu\varphi^a + e\epsilon^{abc} A_\mu^b\varphi^c \,.$$

After spontaneous symmetry breaking in the theory (1) the component φ^3 of the scalar field φ^a acquires a nonvanishing classical part $\varphi_{\mathrm{cl}}^3 \equiv \sigma = m/\sqrt{\lambda}$. The components $A_\mu^{1,2}$ acquire a mass $M_A = e\sigma$, whereas the component A_μ^3 does not acquire mass and can be identified with the electromagnetic potential.

Now let us consider fermions with a nonvanishing electric charge density $j_0^3 = \mathrm{const}$. To describe effectively the influence of the electric charge density j_0^3 on A_μ^a, one should add the term $-ej_0^3A_0^3$ to the lagrangian (1) (for a manifestly gauge-invariant description of this situation see below). One can easily verify that the Lagrange equations in the theory (1) with the external source j_0^3 possess in the Coulomb gauge ($\partial_i A_i^a = 0, A_0^a = 0$ for $j_\mu^a = 0$) an exact standing wave solution, satisfying the consistency check $\nabla_\mu j_\mu^a = 0$:

$$A_1^1 = C\cos kz, \quad A_2^1 = C\sin kz,$$

$$A_3^a = A_i^{2,3} = A_0^{1,2} = 0, \quad A_0^3 = \pm(k^2/e^2 + \sigma^2)^{1/2}\ , \tag{3}$$

where the parameters C and σ are determined by the equations

$$eA_0^3C^2 = j_0^3, \quad m^2 - \lambda\sigma^2 + 2e^2C^2 = 0\ . \tag{4}$$

The energy of the system is equal to

$$E = (\lambda\sigma^2 - m^2)^2/4\lambda - \tfrac{1}{2}e^2(A_0^3)^2C^2 + \tfrac{1}{2}k^2C^2$$
$$+ e^2C^2\sigma^2 + ej_0^3A_0^3 = e^4C^4/\lambda + ej_0^3A_0^3\ . \tag{5}$$

An elementary analysis of eq. (5) shows that the energetically most favourable solution (3) is (as one could expect) the solution with $k = 0$. This solution corresponds to a spatially homogeneous condensate of the massive vector field A_1^1 with electric charge density $-e^2A_0^3C^2$, which exactly compensates the fermion charge density ej_0^3, see eq. (4).

One could think that the situation we are considering is unrealistic, since at present the fermion electric charge density is zero. However, as will be shown below, in superdense matter it usually is energetically favourable to have a nonvanishing fermion electric charge density compensated by the electric charge density of the vector meson condensate.

As an example we shall consider here cold dense matter composed of electrons e and neutrinos ν_e, described by the Weinberg–Salam model of weak and electromagnetic interactions [9]. In this case at a sufficiently large lepton density $n_\ell > n_\ell^c$ a phase transition takes place with condensation of the $W^\pm$ mesons. The physical reason for this phase transition is the following. At low density neutrinos alone can exist in the system, $n_\nu = n_\ell$, since the total electric charge density of the system is equal to the electron charge density and should be zero (see footnote 1) (for simplicity we neglect here all other fermions except e and ν_e, see, however, below). With increasing density it becomes energetically more and more favourable to distribute the conserved lepton charge density equally between neutrinos and left and right electrons, since this would provide a minimum of the Fermi energy of the leptons. This distribution of n_ℓ takes place at a sufficiently large neutrino density $n_\nu > n_\ell^c$ together with the creation of the W meson condensate, the electric charge density of the condensate being exactly equal (with opposite sign) to the charge density of the electrons which appear in the system after the phase transition.

For a quantitative description of the dense electron–neutrino matter one should, as usual, add to the lagrangian of the Weinberg–Salam model [9] the term μl, where l is the lepton charge density operator,

$$l = \bar{e}\gamma_0 e + \tfrac{1}{2}(\bar{\nu}_e\gamma_0(1-\gamma_5)\nu_e), \tag{6}$$

and μ is the corresponding chemical potential (the lepton charge density is the only conserved quantity, which characterizes our system). Note, that l is a gauge-invariant operator, and therefore the addition of μl to the lagrangian does not spoil the gauge invariance of the initial theory.

An analog of the vector meson condensate (2) in the Weinberg–Salam model for the energetically most favourable case $k = 0$ in the Coulomb gauge looks as follows:

$$W_1^\pm = C\ , \quad W_{0,2,3}^\pm = A_i^3 = 0, \quad A_0^3 = \pm\tfrac{1}{2}\sigma\ , \tag{7}$$

where the parameters C and σ are determined by the equations

$$\langle \delta L/\delta A_0\rangle = 2e^2C^2A_0 + e(n_{e_L} + n_{e_R}) = 0\ , \tag{8}$$

$$\langle \delta L/\delta\sigma\rangle = m^2 - \lambda\sigma^2 + \frac{e^2Z_0^2}{\sin^2 2\theta} + \frac{e^2C^2}{2\sin^2\theta} = 0, \tag{9}$$

$$\langle \delta L/\delta B_0\rangle = \frac{e^2\sigma^2 Z_0}{4\sin\theta\cos^2\theta}$$
$$+ \frac{2}{2\cos\theta}(2n_{e_R} + n_{e_L} + n_{\nu_e}) = 0. \tag{10}$$

Here, as usual

$$W_\mu^\pm = 2^{-1/2}(A_\mu^1 \mp A_\mu^2)\ ,$$

$$Z_\mu = B_\mu\sin\theta - A_\mu^3\cos\theta,$$

$$A_\mu = B_\mu\cos\theta + A_\mu^3\sin\theta, \tag{11}$$

σ is the classical part of the Higgs meson field φ, θ is the Weinberg angle, $\langle\ \rangle$ denotes the thermodynamical average [10]. Everywhere in these equations the effects connected with the electron mass are neglected. The densities n_ν, n_{e_R} and n_{e_L} are, as usual, proportional to the third degree of the constant coefficients of the terms $\frac{1}{2}(\bar{\nu}_e\gamma_0(1-\gamma_5)\nu_e)$, $\frac{1}{2}(\bar{e}\gamma_0(1+\gamma_5)e)$ and $\frac{1}{2}(\bar{e}\gamma_0(1-\gamma_5)e)$ in the lagrangian (to the third degree of the effective chemical potentials of ν_e, e_R and e_L) [10]:

$$n_\nu = (1/6\pi^2)\,(\mu + (e/\sin 2\theta)Z_0)^3\,, \tag{12}$$

$$n_{e_R} = (1/6\pi^2)\,(\mu + e\,\mathrm{tg}\,\theta\, Z_0 + eA_0)^3,$$

$$n_{e_L} = (1/6\pi^2)\,(\mu - e\,\mathrm{ctg}\,2\theta\, Z_0 + eA_0)^3.$$

Note that the condensate (7) may appear only at a sufficiently large density $n_\nu > n_\ell^c$, whereas at all densities the trivial solution of the Yang–Mills equations, $W_\mu^\pm = 0$, is possible. Let us determine now the critical density n_ℓ^c. In the analysis of eqs. (8)–(12) we shall suppose for simplicity that $e^4 \ll \lambda$, since in the case of $e^4 \gtrsim \lambda$ one should take into account radiative corrections in e^2 to eqs. (8)–(12) [11]. One more useful fact is that the field Z_0 at densities $\sim n_\ell^c$ is a quantity of higher order in e^2 as compared with A_0^3 and C. Therefore, one may omit Z_0 in eqs. (8)–(12), which makes the further analysis very simple.

Namely, at low densities only the trivial solution $W_\mu^\pm = 0$ is possible, and therefore according to eqs. (8)–(12)[‡1]

$$A_0^3 = A_0 \sin\theta = -(\mu/e)\sin\theta\,,$$

$$n_\nu = \mu^3/6\pi^2, \quad n_{e_R} = n_{e_L} = 0.$$

Starting with $A_3^0 = -(\mu/e)\sin\theta = -\frac{1}{2}\sigma$ the condensate solution (7) becomes possible. At this point

$$n_\nu = n_\ell^c = \frac{1}{6\pi^2}\left(\frac{e\sigma}{2\sin\theta}\right)^3 = \frac{M_W^3}{6\pi^2}\,,$$

where M_W is the W meson mass.

With a further increase of the lepton density n_ℓ the condensate solution becomes energetically favourable, i.e. n_ℓ^c is the critical density of the second-order phase transition to the state with the W meson condensate. To prove this let us analyse the expression for the energy of the system under consideration (cf. eq. (5)):

$$E = \frac{(\lambda\sigma^2 - m^2)^2}{4\lambda} - \frac{e^2 Z_0^2 \sigma^2}{2\sin^2 2\theta} + \tfrac{3}{4}(6\pi^2)^{1/3}(n_{\nu_e}^{4/3} + n_{e_R}^{4/3} + n_{e_L}^{4/3}) = E_{\mathrm{Bose}} + E_{\mathrm{Fermi}}.$$

By use of eqs. (8)–(12) it can be shown easily that $E_{\mathrm{Bose}} = \mathrm{O}(\lambda, e^2, e^4/\lambda)E_{\mathrm{Fermi}}$. Therefore $E \approx E_{\mathrm{Fermi}}$ and the minimum of the energy E at fixed $n_\ell = n_\nu + n_{e_R} + n_{e_L}$ corresponds to the most "symmetric" relation between n_ν, n_{e_R} and n_{e_L} (the best relation would be $n_{e_R} = n_{e_L} = n_\nu$). W meson condensation makes this "symmetrization" possible and therefore W meson condensation in superdense matter is indeed energetically favourable.

Note that for $n_\ell \to \infty$ the quantities eZ_0 and eA_0 are of order $\mathrm{O}(\lambda, e^2, e^4/\lambda)\mu \ll \mu$. This means that according to eq. (12) W meson condensation in superdense matter leads to the energetically most favourable relation between the densities of electrons and neutrinos: $n_{\nu_e} = n_{e_R} = n_{e_L}$.

It can be shown, that analogous relations should be valid for all other particles:

$$n_{\nu_\mu} = n_{\mu_R} = n_{\mu_L}, \quad n_{p_R} = n_{p_L} = n_{n_R} = n_{n_L}\,, \quad \text{etc.}$$

The general rule is that the baryon charge density b, the electron lepton charge density l_e and the muon lepton charge density l_μ in superdense matter are equally distributed between left and right baryons (quarks) and left and right leptons. This result seems to be rather general and model independent, since the only reason that the above mentioned symmetry of matter composition is usually broken (the condition of fermion electroneutrality) disappears after the emergence of the charged condensate of the Yang–Mills fields, the energy of the condensate being much lower than that of the relativistic Fermi gas $\sim n_\ell^{4/3}$. An analogous result is valid also for superdense matter *at any nonvanishing temperature*:

$$n_{\nu_e} - n_{\tilde{\nu}_e} = n_{e_R} - n_{\tilde{e}_R} = n_{e_L} - n_{\tilde{e}_L}\,, \quad \text{etc.}$$

One may compare this result with the well-known relation, valid for the relativistic Fermi gas in the high-temperature limit [10]:

‡1 To be more accurate, even at low density n_{e_R} and $n_{\tilde{e}_L} \equiv -n_{e_L}$ do not vanish: from eqs. (8), (12) it follows that in the absence of W meson condensate ($C = 0$)

$$n_{e_R} = n_{\tilde{e}_L} \doteq \frac{1}{6\pi^2}\left(\frac{eZ_0}{2\sin 2\theta}\right)^3 \neq 0.$$

Therefore, even at zero temperature right electrons and left positrons do not annihilate each other (!) and can coexist in thermodynamic equilibrium. However, the densities n_{e_R} and $n_{\tilde{e}_L}$ are very small ($n_{e_R} = n_{\tilde{e}_L} \ll n_\nu$) and vanish in our approximation, in which all the effects connected with Z_0 are neglected.

$n_{\nu_e} = n_{\tilde{\nu}_e} = n_{e_R} = n_{\tilde{e}_R} = n_{e_L} = n_{\tilde{e}_L} = \ldots$.

In conclusion it is worth noting that taking the strong interactions into account the theory of charged meson condensation becomes even more interesting. Namely, in the theory of weak and electromagnetic interactions [9] the phase transition with breaking of the fermion electroneutrality condition occurs at $n_\nu = (1/6\pi^2)(M_W)^3$, i.e., roughly speaking, when the average distance between the neutrinos becomes $\sim M_W^{-1}$. In the unified theories of weak, strong and electromagnetic interactions this phase transition should take place much earlier (at $n_\nu = (1/6\pi^2)\, m_{\pi^\pm}^3$) due to $\pi^\pm$ meson condensation. The theory of this effect is completely analogous to the theory of $W^\pm$ meson condensation considered above. One may expect that with increasing density a large number of different phase transitions with the condensation of charged scalar and vector mesons should actually take place, enforcing the asymptotic symmetry of the composition of superdense matter considered in the present paper. We hope to discuss these questions as well as the possible cosmological consequences of the charged meson condensation in a separate publication.

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