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MANIFEST LORENTZ INVARIANCE SOMETIMES REQUIRES NON-LINEARITY*

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Manifestly Lorentz-invariant actions for self-dual antisymmetric-tensor gauge fields are found by using cubic Lagrange-multiplier terms. The Lagrange-multiplier fields are not dynamical, and can be eliminated to obtain a quadratic light-cone action.

1. Introduction

Auxiliary fields (fields which have trivial dynamics but are not gauge degrees of freedom) are important in field theory for (i) manifesting invariances (such as Lorentz invariance of supersymmetry), (ii) constructing invariant actions (with kinetic and interaction terms which are separately invariant), (iii) deriving power-counting rules, (iv) performing perturbation theory conveniently, (v) allowing continuation to euclidean space (in the case of auxiliary fields which manifest Lorentz invariance), especially for studying non-perturbative phenomena, and (vi) studying bound states (see e.g. ref. [1]). (For a review of auxiliary fields in supersymmetric and nonsupersymmetric theories, see ref. [2].)

Unfortunately, there has been some difficulty in finding auxiliary fields for theories where the field strengths satisfy self-duality conditions: specifically, ten-dimensional supersymmetric Yang-Mills (and its reduction to lower dimensions) [3] and twice-odd-dimensional self-dual antisymmetric tensors [4]. Based only on the assumption of a quadratic action (for the free theories), it has been shown that auxiliary fields do not exist which manifest supersymmetry for the former theories or Lorentz invariance for the latter.

However, similar problems have been solved by the use of non-linear actions. In non-linear σ-models, the full (off-shell) invariance cannot be realized in the quadratic part of the action (except in a contracted form), and can be realized linearly only when compensating fields are introduced. (For a review of compensating fields, see ref. [5].) The manifestly Lorentz-covariant action for a supersymmetric particle [6] (and also a supersymmetric string [7]) is non-linear even in the free case, and can be

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made quadratic only by going to a light-cone formalism, where Lorentz invariance is not manifest.

In this paper we provide a solution for the case of the self-dual antisymmetric tensor. Our approach is analogous to CP(n) models. There, a complex n-vector, on which SU($n+1$) is non-linearly realized, is obtained from a linear ($n+1$)-vector representation ϕ by a cubic Lagrange-multiplier term $\lambda(\phi^\dagger\phi - 1)$ [8,1]. Here, we provide a similar Lagrange-multiplier term for the square of the anti-self-dual part of the field strength of the antisymmetric-tensor gauge field. (Such an approach to $N=4$ Yang-Mills was suggested in ref. [3].) Using a term quadratic in the field strength, instead of linear, avoids generating dynamics for the Lagrange multiplier.

2. $D = 2$

2.1. FREE THEORY

In general Minkowski space-times of dimension $D = 4n+2$, an anti-symmetric-tensor gauge field of rank $2n$ can have a self-duality condition imposed on its field strength (rank $2n+1$) [4]. We first consider the simplest example: $D=2$, $n=0$. The "antisymmetric tensor" is then just a scalar A with *global* invariance and field strength

$$\delta A = \text{constant},$$
$$F_a = \partial_a A \rightarrow \delta F_a = 0\,. \tag{2.1}$$

The self-dual and anti-self-dual parts of the field strength are

$$F^{(\pm)}{}_a \equiv \tfrac{1}{2}\left(F_a \pm \varepsilon_a{}^b F_b\right). \tag{2.2a}$$

Working with light-cone coordinates $x^{\pm} = \sqrt{\tfrac{1}{2}}\,(x^1 \pm x^0)$, with

$$\varepsilon_{ab} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

($\varepsilon_{+-} = -\varepsilon^{+-} = 1$) and metric

$$\eta_{ab} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} (\eta_{+-} = 1)\,,$$

we have

$$F^{(\pm)}{}_{\pm} = \partial_{\pm} A\,, \qquad F^{(\pm)}{}_{\mp} = 0\,. \tag{2.2b}$$

Self-duality ($F_a = \varepsilon_a{}^b F_b \rightarrow F^{(-)}{}_a = 0$) thus implies the usual Klein-Gordon equation:

$$F^{(-)}{}_a \sim \partial_- A = 0 \rightarrow \partial_a F^a = 2\partial_a F^{(-)a} = \Box A = 2\partial_+ \partial_- A = 0\,. \tag{2.3}$$

However, the self-dual field strength describes half as many modes as the usual one ($\partial_- A = 0 \to A = A(x^+)$, while $\partial_+ \partial_- A = 0 \to A = A^{(+)}(x^+) + A^{(-)}(x^-)$). Since F_+ and F_- (and the $\pm$ components of any vector index) transform independently under the 2D Lorentz group (as seen e.g. in (2.2)), from now on we will use explicit $\pm$ indices.

The lagrangian describing the self-dual field strength is

$$L = -(\partial_+ A)(\partial_- A) + \tfrac{1}{2}\lambda(\partial_- A)^2 . \tag{2.4}$$

The Lagrange multiplier λ is the doubly self-dual part λ_{++} of a (traceless) symmetric tensor λ_{ab}. The field equations following from variation of the action $S = \int d^2x\, L$ are

$$0 = \delta S/\delta\lambda = \tfrac{1}{2}(\partial_- A)^2 \to \partial_- A = 0 ,$$

$$0 = \delta S/\delta A = 2\partial_+ \partial_- A - \partial_-(\lambda \partial_- A) \to 0 = 0 . \tag{2.5}$$

The field equations are thus redundant: λ drops out of the field equations, so it must be a gauge degree of freedom. The gauge transformation which leaves the action invariant is

$$\delta A = \varepsilon \partial_- A , \qquad \delta\lambda = 2\partial_+ \varepsilon + \varepsilon \overleftrightarrow{\partial}_- \lambda . \tag{2.6}$$

We can thus gauge λ to 0 using $\delta\lambda \approx 2\partial_+\varepsilon$. However, with suitable boundary conditions λ cannot be gauged away for all x^+ (e.g. on a torus $\delta\int dx^+\, \lambda \approx 0$). Then $\delta S/\delta\lambda = 0$ at some value of x^+ implies $\partial_- A = 0$ at that x^+, and $\delta S/\delta A = 0$ everywhere further implies $\partial_- A = 0$ everywhere. (Physically, the field equations are gauge-independent by definition, so $\partial_- A = 0$ must be obtainable from the action, even when λ is almost gauged away.)

2.2. INTERACTING THEORY

In general space-times of dimension $D = 4n + 2$, the self-dual rank-$2n$ antisymmetric tensor A can be coupled to a complex rank-n antisymmetric tensor B (see ref. [9] for $D = 10$, in a light-cone formalism). The essential modification is that A transforms non-trivially under the gauge transformation of B: $\delta B \sim \partial\zeta$, $\delta A \sim i(\bar{B}\partial\zeta - B\partial\bar{\zeta})$ (with antisymmetrization on all indices). As a result, the field strength of A obtains an extra term: $F \sim \partial A + i\bar{B}\overleftrightarrow{\partial}B$. Here we will consider only $D = 2$ (and $D = 6$ in subsect. 3.3; generalization to arbitrary D is straightforward). A and B are both scalars, and their gauge transformations $\partial\zeta$ are replaced by constants a and b (as in (2.1)):

$$\delta A = a - \tfrac{1}{2}i(\bar{b}B - b\bar{B}) , \qquad \delta B = b . \tag{2.7}$$

The invariant field strengths are

$$F_a = \partial_a A + \tfrac{1}{2} i \bar{B} \overleftrightarrow{\partial}_a B, \qquad G_a = \partial_a B. \tag{2.8}$$

We now look for an invariant action for which the self-duality of F_a again implies its Klein-Gordon type equation, which now includes interactions. The latter equation is dual to the Bianchi identity. We thus find

$$\partial_{[a} F_{b]} = i \bar{G}_{[a} G_{b]}, \qquad F_a = \varepsilon_a{}^b F_b \to \partial_a F^a = i \varepsilon^{ab} \bar{G}_a G_b. \tag{2.9}$$

([ ] is antisymmetrization with terms of weight 1: e.g. $\varepsilon_{ab}\varepsilon^{cd} = -\delta_{[a}{}^c \delta_{b]}{}^d$.) Therefore, we first find an action which gives the last equation from $\delta S/\delta A = 0$, and then append the Lagrange-multiplier term $\frac{1}{2}\lambda(F_-)^2$. The result is (for coupling k^2):

$$k^2 L = -\tfrac{1}{2}(F_a)^2 - \bar{G}^a G_a - i A \varepsilon^{ab} \bar{G}_a G_b + \tfrac{1}{2}\lambda(F_-)^2. \tag{2.10}$$

The third term can be shown to be invariant under (2.7) by integration by parts (in the action). Due to the presence of this term, the action is invariant under the interacting generalization of (2.6):

$$\delta A = \varepsilon F_-, \qquad \delta B = 0, \qquad \delta\lambda = 2\partial_+ \varepsilon + \varepsilon \overleftrightarrow{\partial}_- \lambda. \tag{2.11}$$

2.3. STRINGS AND SUPERSYMMETRY

The lagrangian (2.4) is similar to that of a relativistic string (see e.g. ref. [10]). The lagrangian for the string can be written as

$$L = \tfrac{1}{2}\sqrt{-g}\, g^{mn} (\partial_m X)\cdot(\partial_n X), \tag{2.12}$$

where g_{mn} is a 2D metric and X is a 2D scalar, but with many (26) components. The lagrangian of (2.4) can be obtained by choosing X to have only one component A, and restricting the metric by $g^{++} = 0$ so that

$$\sqrt{-g}\, g^{mn} = \begin{pmatrix} 0 & 1 \\ 1 & -\lambda \end{pmatrix}. \tag{2.13}$$

Locally this is just a gauge choice, but enforcing it at the boundaries as well eliminates one of the usual string constraints. Also, the product $X \cdot X$ in (2.12) is a Minkowski product $(- + \cdots +)$, which changes the nature of the constraint for multicomponent X. The gauge transformations (2.6) follow from the 2D general-coordinate invariance of (2.12), where ε represents the surviving reparametrization invariance for x^- once (2.13) (which fixes x^+, up to conformal transformations) has been imposed.

The action of (2.4) can be generalized to a supersymmetric model: A appears in a 2D $N=1$ real scalar superfield ϕ, with a 2-component real anticommuting coordinate $\theta^{\alpha}=(\theta^{+},\theta^{-})$, while λ appears in a real superfield Λ which has helicity $-\frac{3}{2}(\Lambda^{a\alpha}\to\Lambda^{--})$:

$$\phi = A + \theta^{\alpha}\chi_{\alpha} + \theta^{2}C = A + \theta^{+}\chi_{+} + \theta^{-}\chi_{-} + i\theta^{+}\theta^{-}C,$$

$$\Lambda = \eta + \theta^{+}\lambda + \theta^{-}\xi + \theta^{2}\psi,$$

$$D_{\pm} = (\partial/\partial\theta^{\pm}) + \theta^{\pm}i\partial_{\pm}\,. \tag{2.14}$$

The action, which includes (2.4), is

$$S = \int d^{2}x\, d^{2}\theta \left[-i(D_{+}\phi)(D_{-}\phi) + i\Lambda(D_{-}\phi)(\partial_{-}\phi)\right]. \tag{2.15}$$

The field equations are

$$0 = \delta S/\delta\Lambda = i(D_{-}\phi)(\partial_{-}\phi),$$

$$0 = \delta S/\delta\phi = -iD_{+}D_{-}\phi - i\partial_{-}(\Lambda D_{-}\phi) + iD(\Lambda\partial_{-}\phi)\,. \tag{2.16}$$

It would appear that Λ does not drop out of the field equations, since the Λ equation could imply just $\partial_{-}\phi=0$, which is weaker than the supersymmetric self-duality condition $D_{-}\phi=0$ (since $D_{-}(D_{-}\phi)=i\partial_{-}\phi$), and the ϕ equation contains both $\Lambda D_{-}\phi$ and $\Lambda\partial_{-}\phi$. Nevertheless, the action (2.15) is invariant under a gauge transformation which allows Λ to be gauged away:

$$\delta\phi = \varepsilon\partial_{-}\phi - \tfrac{1}{2}i(D_{-}\varepsilon)D_{-}\phi,$$

$$\delta\Lambda = D_{+}\varepsilon + \varepsilon\overleftrightarrow{\partial}_{-}\Lambda - \tfrac{1}{2}i(D_{-}\varepsilon)D_{-}\Lambda\,. \tag{2.17}$$

The action and invariances also follow from a truncation (in analogy to (2.13)) of (conformal) supergravity coupled to a scalar multiplet (as obtained e.g. by truncation of the corresponding 3D $N=1$ model [11], or from a form of the action for the 10D string). Thus, the $\delta S/\delta\phi=0$ equation in (2.16) is just a (supergravitational) covariantization of $-iD_{+}D_{-}\phi=0$ (which it becomes in the gauge $\Lambda=0$). Furthermore, $D_{+}(D_{-}\phi)=0$ and $i\partial_{-}\phi=D_{-}(D_{-}\phi)=0$ *together* imply $D_{-}\phi=0$.

This model is equivalent on-shell to the following "light-cone" superfield formalism, using superfields which depend on *only* θ^{+}:

$$S = \int d^{2}x\, d\theta^{+}\left[i(D_{+}\hat{\phi})(\partial_{-}\hat{\phi}) - \tfrac{1}{2}i\hat{\Lambda}(\partial_{-}\hat{\phi})^{2}\right]. \tag{2.18}$$

The equivalence is due to the fact that in the previous model $D_-\phi = \partial_-\phi = 0$ implies $(\partial/\partial\theta^-)\phi = 0$, so $\phi \to \hat{\phi}$, where $\hat{\phi}$ satisfies only $\partial_-\hat{\phi} = 0$. The local invariance is

$$\delta\hat{\phi} = \hat{\varepsilon}\partial_-\hat{\phi}, \qquad \delta\hat{\Lambda} = 2D_+\hat{\varepsilon} + \hat{\varepsilon}\overleftrightarrow{\partial}_-\hat{\Lambda}. \tag{2.19}$$

In six dimensions the analogous property would be that an $N=2$ superfield formulation would be equivalent on-shell to an $N=1$ (but *not* light-cone) formulation. (Note that in $D=6$ a symmetric "real" bispinor $F_{(\alpha\beta)}$ already represents a self-dual field strength: the natural inclusion of the anti-self-dual one $F^{(\alpha\beta)}$ would imply the use of a second spinor coordinate θ_α in addition to θ^α, in analogy to θ^+ and θ^-, χ_+ and χ_- in $D=2$, as in (2.14).)

The generalization to $N=2$ supersymmetry can be obtained by truncating $N=2$ supergravity coupled to an $N=2$ scalar multiplet (which itself follows from truncating $N=1$ in $D=4$ [12], or the string with critical dimension 2 [13]):

$$D_\pm = (\partial/\partial\theta^\pm) + \tfrac{1}{2}\bar{\theta}^\pm i\partial_\pm, \qquad \bar{D}_\pm = (\partial/\partial\bar{\theta}^\pm) + \tfrac{1}{2}\theta^\pm i\partial_\pm,$$

$$H = \Lambda i\partial_-, \qquad \{\bar{D}_-, e^{-H}D_-e^H\} = iE\partial_-,$$

$$S = \int d^2x\, d^4\theta\, E^{-1}\phi e^{-H}\bar{\phi} = \int d^2x\, d^4\theta\, [\phi\bar{\phi} + \Lambda(D_-\phi)(\bar{D}_-\bar{\phi}) + O(\Lambda^2)],$$

$$e^{H'} = e^{i\bar{\omega}}e^He^{-i\omega}, \qquad \phi' = e^{i\omega}\phi,$$

$$\omega = (-\bar{D}_+\varepsilon)\partial_- + (i\bar{D}_-\bar{D}_+\varepsilon)D_- + (e^{-H}iD_-D_+\bar{\varepsilon})\bar{D}_-$$

$$\to \delta\phi = \bar{D}_+\bar{D}_-\varepsilon D_-\phi,$$

$$\delta\Lambda = \bar{D}_+\varepsilon - \tfrac{1}{2}i(\bar{D}_+\varepsilon)\overleftrightarrow{\partial}_-\Lambda - \tfrac{1}{2}(\bar{D}_-\bar{D}_+\varepsilon)D_-\Lambda + \text{h.c.} + O(\Lambda^2). \tag{2.20}$$

These superfields are functions of complex $\theta^\pm$ (and their hermitian conjugates $\bar{\theta}^\pm$): ϕ is a chiral scalar ($\bar{D}_\pm\phi = 0$), Λ is a real scalar, and ε is a complex (one-component) spinor.

3. $D=6$

3.1. FREE THEORY

To study the properties of self-dual theories where the antisymmetric tensors are true gauge fields (as opposed to just Goldstone scalars, as in $D=2$), it is sufficient to consider the case $D=6$. (Generalization to $D=10, 14, \ldots$ is straightforward.) The gauge transformation and invariant field strengths are

$$\delta A_{ab} = \partial_{[a}\zeta_{b]},$$

$$F_{abc} = \tfrac{1}{2}\partial_{[a}A_{bc]},$$

$$F^{(\pm)}{}_{abc} = \tfrac{1}{2}\left(F_{abc} \pm \tfrac{1}{6}\varepsilon_{abcdef}F^{def}\right). \tag{3.1}$$

From the metric analogy of subsect. 2.3, we couple the Lagrange multiplier to the part of the energy-momentum tensor of A containing only $F^{(-)}$ ($FF = F^{(+)}F^{(+)} + F^{(-)}F^{(-)}$):

$$L = -\tfrac{1}{12}F^{abc}F_{abc} + \tfrac{1}{4}\lambda^{ab}F_a^{(-)cd}F^{(-)}{}_{bcd}. \tag{3.2}$$

λ^{ab} is symmetric and traceless (corresponding to conformal gravity). The field equations are

$$0 = \delta S/\delta\lambda^{ab} = \tfrac{1}{4}F_a^{(-)cd}F^{(-)}{}_{bcd} \to F^{(-)}{}_{abc} = 0,$$

$$0 = \delta S/\delta A^{ab} = \partial^c F^{(-)}{}_{abc} - \tfrac{1}{4}\partial^c\lambda^d{}_{[a}F^{(-)}{}_{bc]d} \to 0 = 0. \tag{3.3}$$

(We have used the Bianchi identity $\partial^c F_{abc} = 2\partial^c F^{(-)}{}_{abc}$.) The fact that the λ^{ab} equation implies $F^{(-)} = 0$ can be seen from the λ^{00} component: it is the (positive definite) sum of squares of $F^{(-)}{}_{0ij}$, implying $F^{(-)}{}_{0ij} = 0$, but then $F^{(-)}{}_{ijk} = \tfrac{1}{2}\varepsilon_{0ijklm}F^{(-)0lm}$ also vanishes.

Corresponding to the absence of λ^{ab} from the field equations, we again have a general-coordinate-like invariance

$$\delta A_{ab} = \varepsilon^c F^{(-)}{}_{abc} + \tfrac{1}{12}\varepsilon^c\lambda^d{}_{[a}F^{(-)}{}_{bcd]} + O(\lambda^2),$$

$$\delta\lambda_{ab} = \partial_{(a}\varepsilon_{b)} + \varepsilon^c\partial_c\lambda_{ab} + \tfrac{1}{2}\lambda_{c(a}\left(\partial_{b)}\varepsilon^c - \partial^c\varepsilon_{b)}\right) - \text{trace} + O(\lambda^2). \tag{3.4}$$

In order to derive this invariance, we used the following identities, obtained by comparing $F^{(-)}F^{(-)}$ and $F^{(-)}\overleftrightarrow{\partial}F^{(-)}$ to the same expressions with all $F^{(-)}{}_{abc} \to -\tfrac{1}{6}\varepsilon_{abcdef}F^{(-)def}$:

$$F^{(-)abc}F^{(-)}{}_{abc} = 0,$$

$$F^{(-)abe}F^{(-)}{}_{cde} = \tfrac{1}{4}\delta_{[c}{}^{[a}F^{(-)b]ef}F^{(-)}{}_{d]ef},$$

$$F^{(-)acd}\overleftrightarrow{\partial}_e F^{(-)}{}_{bcd} = 0,$$

$$F^{(-)abc}\overleftrightarrow{\partial}_g F^{(-)}{}_{def} = \tfrac{1}{8}\delta_{[d}{}^{[a}F^{(-)bc]h}\overleftrightarrow{\partial}_g F^{(-)}{}_{ef]h}. \tag{3.5}$$

These identities also follow from just group theory: in terms of SU(4) ~ SO(6) representations (actually SU*(4) ~ SO(5,1)), the symmetric part of $\underline{10} \otimes \underline{10}$ is $\underline{20} \oplus \underline{35}$, and the antisymmetric part is $\underline{45}$ ($F^{(-)}$ is a $\underline{10}$, $F^{(+)}$ a $\overline{\underline{10}}$). Unfortunately, (3.4) is the largest invariance for which $\delta\lambda$ (or any other possible Lagrange multiplier of $F^{(-)}F^{(-)}$) has a field-independent term ($\partial\varepsilon$), so it is not clear how to quantize this theory in a covariant gauge. (There is an additional local invariance of the form $\delta A_{ab} = 0$, $\delta\lambda_{ab} = \varepsilon^{[cd]}{}_{(a}F^{(-)}{}_{b)cd}$, where $\varepsilon^{[ab]}{}_b = \varepsilon_{[abc]} = 0$, so $\varepsilon^{[ab]}{}_c$ is a $\underline{64}$, but it is not clear how to use this invariance for gauge-fixing.)

3.2. LIGHT CONE

However, the action (3.2) can be quantized easily in a light-cone formalism. We first choose a ζ-gauge for (3.1):

$$A_{+a} = 0. \tag{3.6}$$

In this gauge, $F^{(-)}{}_{abc}$ has the independent components (expanding the indices as longitudinal + transverse: $a = (+\,-\,i)$, $x^{\pm} = \sqrt{\tfrac{1}{2}}\,(x^1 \pm x^0)$, $\varepsilon_{+-ijkl} = \varepsilon_{ijkl}$):

$$\begin{aligned}
F^{(-)}{}_{+-i} &= \tfrac{1}{2}\left(\partial_+ A_{-i} - \tfrac{1}{2}\varepsilon_{ijkl}\partial_j A_{kl}\right) = -\tfrac{1}{6}\varepsilon_{ijkl}F^{(-)}{}_{jkl},\\
F^{(-)}{}_{+ij} &= \partial_+ \tfrac{1}{2}\left(A_{ij} - \tfrac{1}{2}\varepsilon_{ijkl}A_{kl}\right) = \partial_+ A^{(-)}{}_{ij},\\
F^{(-)}{}_{-ij} &= \tfrac{1}{2}\left[\left(\partial_- A_{ij} - \partial_{[i}A_{-j]}\right) + \tfrac{1}{2}\varepsilon_{ijkl}\left(\partial_- A_{kl} - \partial_{[k}A_{-l]}\right)\right],
\end{aligned} \tag{3.7}$$

where

$$A^{(\pm)}{}_{ij} \equiv \tfrac{1}{2}\left(A_{ij} \pm \tfrac{1}{2}\varepsilon_{ijkl}A_{kl}\right). \tag{3.8}$$

Note that only $F^{(-)}{}_{-ij}$ contains ∂_-, the "time" derivative for light-cone formalisms corresponding to the gauge (3.6). Next, we expand the Lagrange-multiplier term:

$$\begin{aligned}
\tfrac{1}{4}\lambda^{ab}F_a^{(-)cd}F^{(-)}{}_{bcd} = {}& \tfrac{1}{4}\lambda_{++}F^{(-)}{}_{-ij}F^{(-)}{}_{-ij} + \tfrac{1}{4}\lambda_{--}F^{(-)}{}_{+ij}F^{(-)}{}_{+ij}\\
&- \lambda_{+-}F^{(-)}{}_{+-i}F^{(-)}{}_{+-i} + \lambda_{+i}F^{(-)}{}_{+-j}F^{(-)}{}_{-ij}\\
&- \lambda_{-i}F^{(-)}{}_{+-j}F^{(-)}{}_{+ij}\\
&- \hat{\lambda}_{ij}\left(F^{(-)}{}_{+-i}F^{(-)}{}_{+-j} - \tfrac{1}{2}F^{(-)}{}_{+ik}F^{(-)}{}_{-jk}\right),
\end{aligned} \tag{3.9}$$

where $\hat{\lambda}_{ij}$ is the traceless part of λ_{ij} ($\lambda_a{}^a = 0 \to \lambda_{ij} = \hat{\lambda}_{ij} - \tfrac{1}{2}\delta_{ij}\lambda_{+-}$). We have used (3.7) to relate $F^{(-)}{}_{ijk}$ to $F^{(-)}{}_{+-i}$, and the fact that $F^{(-)}{}_{-ij}$ is four-dimensionally self-dual (as in (3.8)) while $F^{(-)}{}_{+ij}$ is anti-self-dual.

The invariance of (3.4) is more than sufficient to choose the ε-gauge

$$\lambda_{++}=0. \tag{3.10}$$

Since λ_{--} and λ_{+-} appear in terms without ∂_-'s, their variation gives only kinematic constraints, not dynamical field equations. Therefore, we eliminate them, as auxiliary fields, by their equations of motion:

$$\begin{aligned}
&0=\delta S/\delta\lambda_{--}=\tfrac{1}{4}F^{(-)}{}_{+ij}F^{(-)}{}_{+ij}\to F^{(-)}{}_{+ij}=0\to A^{(-)}{}_{ij}=0,\\
&0=\delta S/\delta\lambda_{+-}=-F^{(-)}{}_{+-i}F^{(-)}{}_{+-i}\to F^{(-)}{}_{+-i}=0\\
&\quad\to A_{-i}=(\partial_+)^{-1}\tfrac{1}{2}\varepsilon_{ijkl}\partial_j A_{kl}=(\partial_+)^{-1}\partial_j A^{(+)}{}_{ij}.
\end{aligned} \tag{3.11}$$

(As usual in light-cone formalisms, we neglect contributions from $\partial_+=0$.) As a result, *all* Lagrange-multiplier terms in (3.9) vanish (the first by (3.10), the next two by functionally integrating out λ_{--} and λ_{+-} to obtain the two constraints, the rest by applying the constraints). The result of applying the constraints to the remainder of the lagrangian (3.2) is

$$L=\tfrac{1}{4}A^{(+)}{}_{ij}\Box A^{(+)}{}_{ij}, \tag{3.12}$$

the light-cone action of ref. [4]. (All components of A_{ab} except $A^{(+)}{}_{ij}$ are determined by (3.6) and (3.11).)

3.3. INTERACTING THEORY

The same procedure can be applied to the interacting theory generalizing the 2D theory of subsect. 2.2. Corresponding to eqs. (2.7)–(2.10),

$$\begin{aligned}
&\delta A_{ab}=\tfrac{1}{2}i\left[(\partial_{[a}\bar{\zeta})B_{b]}-(\partial_{[a}\zeta)\bar{B}_{b]}\right]\quad\text{or }\tfrac{1}{2}i(\bar{\zeta}G_{ab}-\zeta\bar{G}_{ab}),\qquad \delta B_a=\partial_a\zeta,\\
&F_{abc}=\tfrac{1}{2}\partial_{[a}A_{bc]}+\tfrac{1}{2}i\bar{B}_{[a}\overleftrightarrow{\partial}_b B_{c]},\qquad G_{ab}=\partial_{[a}B_{b]},\\
&\tfrac{1}{6}\partial_{[a}F_{bcd]}=\tfrac{1}{4}i\bar{G}_{[ab}G_{cd]},\qquad F^{(-)}=0\to\partial^c F_{abc}=\tfrac{1}{4}i\varepsilon_{abcdef}\bar{G}^{cd}G^{ef},\\
&k^2L=-\tfrac{1}{12}(F_{abc})^2-\tfrac{1}{2}\bar{G}^{ab}G_{ab}-\tfrac{1}{8}i\varepsilon^{abcdef}A_{ab}\bar{G}_{cd}G_{ef}+\tfrac{1}{4}\lambda^{ab}F^{(-)}{}_a{}^{cd}F^{(-)}{}_{bcd}.
\end{aligned} \tag{3.13}$$

(The two forms of δA_{ab} are equivalent modulo a ζ_a transformation from (3.1).) Eq. (3.4) is unmodified, with $\delta B_a=0$. The light-cone action can be derived by choosing the gauge $B_+=0$, in addition to (3.6) and (3.10), and using the procedure of (3.11) to determine $A^{(-)}{}_{ij}$ and A_{-i}, as well as $\delta S/\delta \bar{B}_- = 0$ to determine the auxiliary field B_-, giving an action in terms of just $A^{(+)}{}_{ij}$ and B_i.

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