out. We know virtually nothing about the ρ' but again might expect a $1^- \to 1^-$ transition. The ρ would also be expected to diffract up to the g meson. We have discussed the 1^- family of particles because of familiarity. These might better be done with photons incident rather than pions. The sort of diffractive cascading described might occur in any sort of mixed particle states $(f-f')$.

DYNAMICAL MODEL OF DUAL AMPLITUDES*

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We re-examine Nielsen's interpretation of the Veneziano amplitudes from the point of view of the functional formulation. It is shown that the sum of a large number of "fishnet" Feynman graphs of very high order can be approximated by generalized Veneziano amplitudes.

It has been argued by Nielsen,[1] and more recently by Olesen,[2] that the integrand of the Koba-Nielsen integral representation[3] of the N-particle Veneziano amplitude may be derived from an nth-order planar Feynman diagram in the limit $n \to \infty$. Although such a dynamical model of dual amplitudes[4] is appealing, their arguments for the model are not entirely convincing. It is the purpose of this Letter to show that a sum of a large number of planar fishnet Feynman diagrams with N external particles is indeed approximated by an N-particle Veneziano amplitude, and more complicated Feynman diagrams are approximated by corresponding Feynman-like dual amplitudes.[5]

Recently we have formulated the dual amplitudes in terms of functional integrations.[6] Since this formulation is essential to our discussion, let us first present the N-particle Veneziano amplitude $V_N(k_1 \cdots k_N)$ in this formulation:

$$(2\pi)^4 \delta^{(4)}(\textstyle\sum_i k_i) V_N(k_1 \cdots k_N) = C^{-1} \int_0^{2\pi} d\theta_N \int_0^{\theta_N} d\theta_{N-1} \cdots \int_0^{\theta_2} d\theta_1 \Pi_i |z_i - z_{i-1}|^c \times \langle \exp[(2\pi)^{1/2} i \textstyle\sum_j k_j \cdot \varphi(z_j)] \rangle, \tag{1}$$

where

$$c = \alpha(0) - 1, \quad z_j = e^{i\theta_j} \; (j = 1, 2, \cdots, N) \tag{2}$$

and $\langle \cdots \rangle$ indicates the functional average over $\varphi_\mu(x,y)$ $(\mu = 1, 2, 3, 4)$, which are functions defined on a unit disk D. More precisely we write

$$\langle \exp[(2\pi)^{1/2} i \textstyle\sum_j k_j \cdot \varphi(z_j)] \rangle \propto \int \mathfrak{D}^{(4)} \varphi(x,y) \exp[\int_D \int dx dy \, \mathfrak{L}(\varphi) + i(2\pi)^{1/2} \textstyle\sum_j k_j \cdot \varphi(z_j)], \tag{3}$$

where $\mathfrak{L}(\varphi)$ is given by

$$\mathfrak{L}(\varphi) = -\frac{1}{2}\left[\left(\frac{\partial \varphi}{\partial x}\right)^2 + \left(\frac{\partial \varphi}{\partial y}\right)^2\right] \equiv -\tfrac{1}{2}(\nabla\varphi)^2. \tag{4}$$

The proof of (1) is carried out by proving that the functional integral (3) is proportional to

$$\prod_{i \neq j} |z_i - z_j|^{k_i k_j}$$

so that (1) becomes the Koba-Nielsen integral representation of V_N.

Let us now consider a fishnet Feynman graph with N external lines as shown in Fig. 1. Although the main argument does not depend on details of the model, to make the problem definite let us assume that the internal and external lines represent neutral pseudoscalar mesons with $\lambda\varphi^4$ coupling. We map a given fishnet graph to the

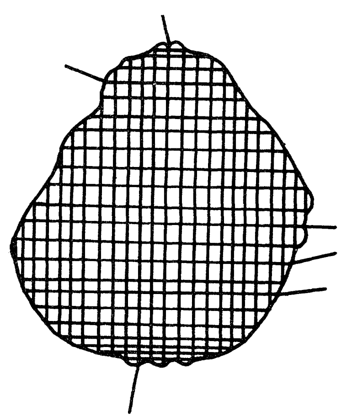

FIG. 1. Fishnet diagram.

equally spaced graph as indicated in Fig. 1. If n is sufficiently large the graphs of order n are classified in terms of the shape S of the boundary and by the place of attachment l_i of the external lines. Thus, if we denote the corresponding Feynman amplitudes by $A_n(S, l_i, k_i)$ the total amplitude due to the "fishnet" Feynman graphs is given by

$$A = \sum_n \sum_S \sum_{l_i} A_n(S, l_i, k_i), \tag{5}$$

where $\sum_S \sum_{l_i}$ implies a sum of all possible graphs of order n, which for large n becomes the sum over different shapes of the boundary and the integration along the boundary.

Let the area of the unit square be ϵ^2; then the area $\mathfrak{A}$ inside the boundary is

$$\mathfrak{A} \approx \epsilon^2 n.$$

So the limit $n \to \infty$ is equivalent to $\epsilon \to 0$ for finite $\mathfrak{A}$. Let us denote vertices of the graph by a set of coordinates (a, b); then the Feynman amplitude $A_n(S, l_i, k_i)$ is given by

$$A_n = \lambda^n \int \cdots \int \prod_{a,b} d^4x(a, b) \exp[i \sum_{j=1}^{N} k_j \cdot x(l_j)] P([x(a, b) - x(a+1, b)]^2) P([x(a, b) - x(a, b+1)]^2), \tag{6}$$

where $P((x_1 - x_2)^2)$ is the propagator from x_1 to x_2. Since

$$\prod_{a,b} P([x(a, b) - x(a+1, b)]^2) = \exp\{\sum_{a,b} \ln P([x(a, b) - x(a+1, b)]^2)\},$$

and in the limit $n \to \infty$ we may replace the sum in the exponent by an integral, we can approximate (6) by the following functional integration:

$$C_n \int \mathfrak{D}^{(4)}\varphi(x, y) e^{i(2\pi)^{1/2} \sum_j k_j \cdot \varphi(l_j)} \exp\left[\int_{D_S} \int \frac{dxdy}{\epsilon^2} \left\{ \ln P\left(2\pi\epsilon^2 \left(\frac{\partial\varphi}{\partial x}\right)^2\right) + \ln P\left(2\pi\epsilon^2 \left(\frac{\partial\varphi}{\partial y}\right)^2\right)\right\}\right], \tag{7}$$

where C_n is a constant which depends only on n, and D_S is the domain inside the boundary S. To make the formula resemble (3) we made the following replacement for the integration variables:

$$x_\mu(a, b) \to (2\pi)^{1/2} \varphi_\mu(x, y).$$

Next let us assume that the propagator P does not have light-cone singularities. Although this assumption is contrary to the ordinary Feynman propagator of a spinless meson, in view of the expected modifications of the propagator the assumption would not be so unreasonable.[7] Then, we expand the exponent of (7) in powers of ϵ^2. The first term merely provides a constant factor and the second term gives the form of the Lagrangian (4), so that for large n we obtain

$$A_n(S, l_i, k_i) \approx C_n' \int \mathfrak{D}^{(4)}\varphi(x, y) \exp[i(2\pi)^{1/2} \sum_j k_j \cdot \varphi(l_j)] \exp[\kappa \int_{D_S} \int dxdy \mathfrak{L}(\varphi)], \tag{8}$$

where

$$\kappa = -2\pi P'(0)/P(0). \tag{9}$$

We then insert (8) into (5) and replace $\sum_{l_i}$ by the integral along the boundary. Since $dxdy\mathfrak{L}(\varphi)$ is invariant under conformal transformations $z = x + iy \to z' = x' + iy'$ and since the measure $\mathfrak{D}^{(4)}\varphi(x, y)$ is defined to be invariant, we can make a conformal transformation such that the boundary S maps onto a unit circle:

$$A \approx \sum_n C_n' \int_0^{2\pi} d\theta_N \cdots \int_0^{\theta_2} d\theta_1 \sum_S \frac{\partial(l_1 \cdots l_N)}{\partial(\theta_1 \cdots \theta_N)} \int \mathfrak{D}^{(4)}\varphi(x, y) \exp[\kappa \int_D \int dxdy \mathfrak{L}(\varphi) + i(2\pi)^{1/2} \sum_j k_j \cdot \varphi(z_j)]. \tag{10}$$

Since the conformal transformation depends on S, the Jacobian also depends on S. But when it is summed over all possible shapes, $\sum_S \partial(l_1 \cdots l_N)/\partial(\theta_1 \cdots \theta_N)$ becomes independent of θ_i, so we obtain the form (1) and (3) with

$$c = \alpha(0) - 1 = 0. \tag{11}$$

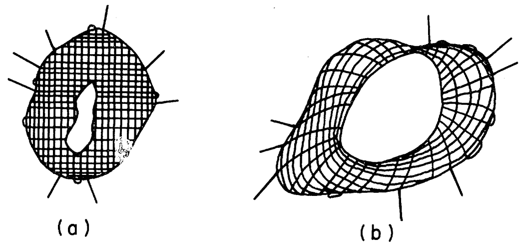

FIG. 2. More complicated diagrams, (a) orientable and (b) nonorientable.

It is not very difficult to see that the slope of the trajectory is given by

$$\alpha'(0)=1/\kappa; \tag{12}$$

this number depends on the details of the dynamics [see Eq. (9)] but for reasonable form of propagators $\kappa>0$.

We have shown that a part [large n] of the "fishnet" Feynman amplitude is approximated by a generalized Veneziano amplitude with $\alpha(0)=1$. Since the total amplitude is supposed to satisfy casuality, the tachyon pole at $j=0$ due to $\alpha(0)=1$ may not be real since the approximation may be bad near the pole.

Up to now we discussed planar "fishnet" Feynman graphs, which define a simply connected surface as shown in Fig. 1.[8] It is easy to extend the discussion to more complicated graphs such as the ones indicated in Fig. 2. The only modification to be made is to extend the sum over S to all possible boundaries of two-dimensional surfaces of constant area.

We have investigated the problem by the model of $\lambda\varphi^4$ coupling with modified propagator. But the same result would be obtained from other models such as a quark model in which the quark lines are along the boundaries of graphs and the gluons [mesons], which have Yukawa interaction with quarks and $\lambda\varphi^4$ interaction among themselves, provide a net. Another interesting class of models worth mentioning are nonpolynomial Lagrangian models.[9] If one considers the vertices of the net are due to major coupling and the propagators are those of supergraphs which are modified by minor couplings, the assumption of absence of light-cone singularities may be justified in this model. We would expect the result does not depend on the details of the model.

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[1]H. B. Nielsen, "A Physical Model for the n-Point Veneziano Model" (to be published).

[2]P. Olesen, "A Parton View on Dual N Point Functions" (to be published).

[3]Z. Koba and H. B. Nielsen, Nucl. Phys. B12, 517 (1969).

[4]We would like to note that a similar attempt has been made by H. Suura (private communication) from a slightly different viewpoint. One of us (B.S.) thanks Professor Suura for fruitful discussions.

[5]K. Kikkawa, B. Sakita, and M. A. Virasoro, Phys. Rev. 184, 1701 (1969); K. Kikkawa, S. Klein, B. Sakita, and M. A. Virasoro, Phys. Rev. (to be published).

[6]C. S. Hsue, B. Sakita, and M. A. Virasoro, Wisconsin Report No. COO-277 (to be published).

[7]Even if we include the light-cone singularities, we can separate them from the rest and the discussion can be applied to the rest.

[8]We believe that the result is valid not only for "fishnet" graphs but also for all planar graphs and semiplanar graphs based on the following argument. Let us map the vertices of the given Feynman graph to corners of an equally spaced square graph, then subdivide the graph into m sections such that in the limit of $n\to\infty$ the number of sections m as well as the number of vertices in a section $q=n/m$ become infinite, i.e., $m\to\infty$ and $q\to\infty$. The semiplanar graph is defined as a class of graphs in which nonplanar lines appear within a section. Using the same approximation used for (7) we approximate the product of propagators in a section by

$$\text{const}\times\exp\left[-\tfrac{1}{2}\iint_d dxdy\left\{\mu(x,y)\left(\frac{\partial\varphi}{\partial x}\right)^2+\nu(x,y)\left(\frac{\partial\varphi}{\partial y}\right)^2+\lambda(x,y)\left(\frac{\partial\varphi}{\partial x}\right)\cdot\left(\frac{\partial\varphi}{\partial y}\right)\right\}\right],$$

where the integration is on the section d. The third term in the exponent appears from propagators which run diagonally in x,y axis. Since the sign of $(\partial\varphi/\partial x)(\partial\varphi/\partial y)$ is positive for the propagators which run 1st and 3rd quadrant direction, and negative for 2nd and 4th, the average value of $\lambda(x,y)$ is 0 [number of Feynman diagrams with $\overline{\lambda}=0 \gg$ number of Feynman diagrams with $\overline{\lambda}\neq 0$]. Similarly $\overline{\mu}=\overline{\nu}$. Therefore, the product of propagators in a given section is approximated in the limit of $q\to\infty$ [$d\to 0$] by const $\times \exp[\overline{\mu}\iint_d dxdy\ \mathcal{L}(\varphi)]$.

[9]A. Salam, in Proceedings of the Seventh Coral Gables Conference on Symmetry Principles at High Energies, Univ. of Miami, 1970 (to be published).

NEW CONSTRAINTS IN HIGH-ENERGY ELECTRON-POSITRON ANNIHILATION INTO HADRONS*

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We consider relations between the total cross section $\sigma_T(e^+ + e^- \to$ hadrons) and the differential cross section for $e^+ + e^- \to H +$ anything, where H is a hadron. We obtain new restrictions on inelastic form factors in the timelike region. One of our results is to show that field algebra is incompatible with scale invariance *à la* Bjorken and present experimental data.

It has been known[1] for a long time that high-energy hadron production in electron-positron collisions, in the single-photon exchange approximation, contains direct information about the constitution of the hadronic current in the region of timelike momentum transfers. The asymptotic behavior of the total cross section $\sigma_T(e^+ + e^- \to$ hadrons) allows us to distinguish[2] among the different kinds of current algebra and to know whether or not there is a finite hadronic contribution to the electric change, etc.

A second method[3,4] of studying the structure of the electromagnetic current has also been discussed: The differential cross section $d\sigma(e^+ + e^- \to H +$ anything) with respect to the energy of the hadron H can probe the electromagnetic current for the timelike momentum transfer if scale invariance *à la* Bjorken[5] is valid.

In this paper we show how the properties of the electromagnetic current can be explored in greater detail by examining the relationship between $\sigma_T(e^+ + e^- \to$ hadrons) and the processes $e^+ + e^- \to H +$ anything. New restrictions are deduced which the different versions of current algebra must fulfill. One of our results is to show that the field algebra, introduced by Knoll, Lee, Weinberg, and Zumino[6] is inconsistent with scale invariance *à la* Bjorken[5] and present experimental data.[7] This result, obtained in the timelike region, can also be obtained, under a weaker form, in the spacelike region using a new sum rule recently given by Jackiw, Van Royen, and West.[8] Here our discussion will be general, and we will consider Bjorken asymptotics[3-5] only as a particular case.

Let us consider the kinematics first. The second-rank tensor

$$P_{\mu\nu} = N\sum_n (2\pi)^3\delta^{(4)}(q-P-P_n)\langle 0|j_\mu(0)|n, H(P)\text{out}\rangle\langle \text{out} H(P), n|j_\nu(0)|0\rangle$$
$$\equiv \overline{W}_1^H(q^2,\nu)\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)+\frac{\overline{W}_2^H(q^2,\nu)}{m_H^2}\left(P_\mu-\frac{\nu}{q^2}q_\mu\right)\left(P_\nu-\frac{\nu}{q^2}q_\nu\right), \quad (1)$$

where j_μ is the electromagnetic current, P and q are the momenta of the hadron H and the virtual photon, respectively, $\nu = P\cdot q$, and $N=2P_0$ for the boson H ($N=P_0/m_H$ for the fermion H), is directly related[3,4] to the differential cross section $d\sigma(e^+ + e^- \to H +$ anything). Throughout we imply a spin sum if H has a spin. On the other hand, the spectral function of the photon propagator $\Pi(q^2)$ is defined by[9]

$$\Pi_{\mu\nu} = \sum_z (2\pi)^3\delta^{(4)}(q-P_z)\langle 0|j_\mu(0)|z\rangle\langle z|j_\nu(0)|0\rangle = (-g_{\mu\nu}q^2+q_\mu q_\nu)\Pi(q^2). \quad (2)$$

Let the maximum multiplicity of the particle H in the state z, for a given mass squared q^2, be $n_H(q^2)$; then energy conservation requires

$$q_0 \geqslant m_H n_H(q^2). \quad (3)$$

Considering the positive definiteness of every contribution of intermediate states, we can obtain the