

# TeV astrophysics constraints on Planck scale Lorentz violation

Ted Jacobson,<sup>\*</sup> Stefano Liberati,<sup>†</sup> and David Mattingly<sup>‡</sup>

Department of Physics, University of Maryland, College Park, Maryland 20742–4111

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We analyze observational constraints from TeV astrophysics on Lorentz violating nonlinear dispersion for photons and electrons without assuming any *a priori* equality between the photon and electron parameters. The constraints arise from thresholds for vacuum Čerenkov radiation, photon decay and photo-production of electron-positron pairs. We show that the parameter plane for cubic momentum terms in the dispersion relations is constrained to an order unity region in Planck units. We find that the threshold configuration can occur with an asymmetric distribution of momentum for pair creation, and with a hard photon for vacuum Čerenkov radiation.

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There are several reasons to suspect that Lorentz invariance may be only a low energy symmetry. This possibility is suggested by the ultraviolet divergences of local quantum field theory, as well as by tentative results in various approaches to quantum gravity and string theory [1–5]. Moreover, Lorentz symmetry can only ever be verified up to some finite observationally accessible velocity, which leaves untested an infinite volume of the supposed symmetry group.

The possibility of Lorentz violation can be studied, without a particular fundamental theory in hand, by considering its manifestation in dispersion relations for particles. If rotational invariance is preserved, it is natural to assume that deviations from the Lorentz invariant dispersion relation  $E^2(p) = m^2 + p^2$  can be characterized at low energies by an expansion with integral powers of momentum,  $E^2 = m^2 + p^2 + \sum_{n=1}^{\infty} a_n p^n$ , where the  $a_n$  are coefficients with mass dimension  $2 - n$  which might be positive or negative. (Throughout this Rapid Communication  $p$  denotes the absolute value of the 3-momentum vector  $\mathbf{p}$ , and we use units with the low energy speed of light in vacuum equal to unity.) Different approaches to quantum gravity suggest different leading order Lorentz violating terms. The terms with  $n \leq 4$  have mostly been considered so far. Observations limit the coefficients  $a_{1,2}$  to be extremely small (see e.g. [6–8] and references therein). In this Rapid Communication we shall assume they are precisely zero.

The cubic and higher order coefficients have negative mass dimension, so if the Lorentz violation descends from the Planck scale  $M_P = (\hbar c^5/G)^{1/2} \approx 1.22 \times 10^{19}$  GeV,  $a_n$  would be expected to be of order  $M_P^{2-n}$ . In this case the Lorentz violation is naturally suppressed, and the lowest order term would dominate all the higher ones as long as the momentum is less than  $M_P$ , hence we shall restrict to a single Lorentz violating term. We thus consider the constraints that high energy observations impose on dispersion relations of the form

$$E_a^2 = p_a^2 + m_a^2 + \eta_a p_a^n / M^{n-2}, \quad (1)$$

where  $a$  labels different fields and  $n \geq 3$ . We have introduced the energy scale  $M = 10^{19}$  GeV  $\sim M_P$  so that the coefficients  $\eta_a$  are dimensionless. If the Lorentz violation comes from quantum gravity effects, one would expect  $\eta_a$  to be within a few orders of magnitude of unity. In the absence of a fundamental theory one has no reason to expect any particular relation between the coefficients  $\eta_a$  for different particles, except perhaps that they should all be of the same order of magnitude. Since the dispersion relation (1) is not Lorentz invariant, it can only hold in one reference frame. We assume along with all previous authors that this frame coincides with that of the cosmic microwave background. The velocity of the earth relative to this frame is negligible for our purposes.

Observational consequences of the  $\eta$  term in Eq. (1) may seem out of reach because of the Planck scale suppression. However this is not so. Dispersion relations like Eq. (1) produce kinematic relations from energy-momentum conservation that differ from the usual Lorentz invariant case. As a result reactions can take place that are normally forbidden, and thresholds for reactions are modified. One can expect deviations from standard threshold kinematics when the last two terms of Eq. (1) are of comparable magnitude. Assuming  $\eta$  is of order unity this yields the condition  $p_{\text{dev}} \sim (m_a^2 M^{n-2})^{1/n}$ , which is  $\sim (m_a/m_e)^{2/3} \times 10$  TeV for  $n=3$  and  $\sim (m_a/m_e)^{1/2} \times 10^4$  TeV for  $n=4$ , where  $m_e$  is the electron mass. Although these energies are currently not achievable in particle accelerators (except in the case of massive neutrinos which however are too weakly coupled to provide constraints) they are in the range of current astrophysical observations. In fact, it has been suggested by several authors [7,9–15] (see also [16] and references therein) that we may already be observing deviations from Lorentz invariance via the possibly missing Greisen-Zatsepin-Kuzmin (GZK) cutoff on cosmic ray protons with ultrahigh energy greater than  $7 \times 10^{19}$  eV [17], and the possible overabundance of gamma rays above 10 TeV from the blazar system Markarian 501 [15,18]. Here we shall mostly not consider the constraints imposed by asking Lorentz violation to explain these puzzles. Instead we restrict our attention to constraints imposed by consistency with known phenomena (or lack thereof).

*Observational constraints.* Several studies of observational limits on Lorentz violating dispersion relations have

<sup>\*</sup>Electronic address: jacobson@physics.umd.edu

<sup>†</sup>Electronic address: liberati@physics.umd.edu

<sup>‡</sup>Electronic address: davemm@physics.umd.edu

already been carried out [7–14,19–21], with various different assumptions about the coefficients. Our study focuses on purely QED interactions involving just photons and electrons. We assume  $n=3$ , since the  $n=4$  terms are suppressed by another inverse power of  $M$ . Unlike other studies, no *a priori* relation between the coefficients  $\eta_\gamma$  and  $\eta_e$  is assumed, and we combine all the different constraints in order to determine the allowed region in the parameter plane. To eliminate the subscript  $a$  we introduce  $\xi := \eta_\gamma$ ,  $\eta := \eta_e$ , and  $m := m_e$ .

The modified dispersion relations for photons and electrons in general allow two processes that are normally kinematically forbidden: vacuum Čerenkov radiation,  $e^- \rightarrow e^- \gamma$ , and photon decay,  $\gamma \rightarrow e^+ e^-$ . In addition the threshold for photon annihilation,  $\gamma \gamma \rightarrow e^+ e^-$ , is shifted. The vacuum Čerenkov process is extremely efficient, leading to an energy loss rate that goes like  $E^2$  well above threshold. Similarly the photon decay rate goes like  $E$ . Thus any electron or photon known to propagate must lie below the corresponding threshold.

We consider constraints that follow from three considerations: (i) Electrons of energy  $\sim 100$  TeV are believed to produce observed x-ray synchrotron radiation coming from supernova remnants [22], and to also produce multi-TeV photons by inverse-Compton scattering with these x rays [23,24]. Assuming these electrons are actually present, vacuum Čerenkov radiation must not occur up to that energy.<sup>1</sup> (ii) Gamma rays up to  $\sim 50$  TeV arrive on earth from the Crab nebula [25], so photon decay does not occur up to this energy. (iii) Cosmic gamma rays are believed to be absorbed in a manner consistent with photon annihilation off the infrared (IR) background with the standard threshold [26]. Observation (iii) is not model independent, so the corresponding constraint is tentative and subject to future verification.

*Modified kinematics.* The processes  $e^- \rightarrow e^- \gamma$  and  $\gamma \rightarrow e^+ e^-$  correspond to the basic QED vertex, but are normally forbidden by energy-momentum conservation together with the standard dispersion relations. When the latter are modified, these processes can be allowed. To see this, let us denote the photon 4-momentum by  $k_4 = (\omega_k, \mathbf{k})$ , and the electron and positron 4-momenta by  $p_4 = (E_p, \mathbf{p})$  and  $q_4 = (E_q, \mathbf{q})$ . For the two reactions energy-momentum conservation then implies  $p_4 = k_4 + q_4$  and  $k_4 = p_4 + q_4$ , respectively. In both cases, we have  $(p_4 - k_4)^2 = q_4^2$ , where the superscript “2” indicates the Minkowski squared norm. Using the Lorentz breaking dispersion relation Eq. (1) this becomes

$$\xi k^3 + \eta p^3 - \eta q^3 = 2M(E_p \omega_k - p k \cos \theta), \quad (2)$$

<sup>1</sup>The competing energy loss by synchrotron radiation is irrelevant for this constraint. The rate of energy loss from a particle of energy  $E$  due to the vacuum Čerenkov effect goes like  $-e^2 E^2$ , while that from synchrotron emission goes like  $-e^4 B^2 E^2 / m^4$  (using units where  $c = \hbar = 1$ ). For a magnetic field of about one micro Gauss (as those involved in supernova remnants) the synchrotron emission rate is 40 orders of magnitude smaller than the vacuum Čerenkov rate.

where  $\theta$  is the angle between  $\mathbf{p}$  and  $\mathbf{k}$ . In the standard case the coefficients  $\xi$  and  $\eta$  are zero and the right-hand side (RHS) of Eq. (2) is always positive, hence there is no solution. It is clear that non-zero  $\xi$  and  $\eta$  can change this conclusion and allow these processes.

To derive the observational constraints one needs to determine the threshold for each process, i.e. the lowest energy for which the process occurs. Assuming monotonicity of all the dispersion relations (for the relevant momenta  $\ll M$ ) one can show [27] that all thresholds for processes with two particle final states occur when the final momenta are parallel. Moreover, for two particle initial states the incoming momenta are antiparallel. This implies that at a threshold  $\theta=0$  in Eq. (2) and that in the corresponding formula for the photon annihilation we shall consider antiparallel photons and parallel leptons. These geometries have been assumed in previous works but to our knowledge they were not shown to be necessary. In fact they are not necessary if the dispersion relations are not monotonic. Details concerning the determination of the thresholds are reported in [28].

*Vacuum Čerenkov radiation.* We find that an electron can emit Čerenkov radiation in the vacuum if  $\eta > 0$  or if  $\eta < 0$  and  $\xi < \eta$ . Depending on the values of the parameters, the threshold configuration can occur with a zero-energy photon or with a finite energy photon. These two cases correspond to the following two threshold relations, respectively:

$$p_{\text{th}} = \left( \frac{m^2 M}{2\eta} \right)^{1/3} \quad \text{for } \eta > 0 \text{ and } \xi \geq -3\eta, \quad (3)$$

$$p_{\text{th}} = \left[ -\frac{4m^2 M(\xi + \eta)}{(\xi - \eta)^2} \right]^{1/3} \quad \text{for } \xi < -3\eta < 0, \quad (4)$$

or  $\xi < \eta \leq 0$ .

The reaction is not allowed in the region where  $\xi > \eta$  and  $\eta < 0$ . Note that if  $\xi = \eta$  only the solution (3) yields a finite threshold.

Electrons of energy  $\sim 100$  TeV are indirectly observed via x-ray synchrotron radiation coming from supernova remnants [22]. Thus for example in the region of the parameter plane where Eq. (3) holds we obtain the constraint  $\eta < m^2 M / 2p_{\text{th}}^3 \sim 10^{-3}$ .

*Photon decay.* A photon can spontaneously decay into an electron-positron pair provided  $\xi$  is sufficiently great for any given  $\eta$ . Contrary to Lorentz-invariant kinematics of pair creation thresholds, we find that the two particles of the pair do not always have equal momenta. Photon decay is allowed above a broken line in the  $\eta$ - $\xi$  plane given by  $\xi = \eta/2$  in the quadrant  $\xi, \eta > 0$  and by  $\xi = \eta$  in the quadrant  $\xi, \eta < 0$ . Above this line, the threshold is given by

$$k_{\text{th}} = \left( \frac{8m^2 M}{2\xi - \eta} \right)^{1/3} \quad \text{for } \xi \geq 0, \quad (5)$$

$$k_{\text{th}} = \left[ \frac{-8m^2 M \eta}{(\xi - \eta)^2} \right]^{1/3} \quad \text{for } \eta < \xi < 0. \quad (6)$$

The first relation (5) arises when the electron and positron momenta are equal at threshold. The second relation (6) applies in the case of asymmetric distribution of momenta. Note that if  $\xi = \eta$ , the asymmetric threshold disappears, leaving just the symmetric one.

The constraint we impose is that the threshold is above 50 TeV, the highest energy of observed gamma rays from the Crab nebula [25]. The strength of the constraint is determined by the smallness of the quantity  $m^2 M/k_{\max}^3$ . For  $k_{\max} = 50$  TeV one gets  $m^2 M/k_{\max}^3 \approx 0.02$ .

**Photon annihilation.** The standard threshold for a gamma ray to annihilate with an IR background photon of energy  $\epsilon$  is  $k_s = m^2/\epsilon$ . In the presence of dispersion the threshold relations take approximately the same form as for photon decay, Eqs. (5),(6), with the replacement  $\xi \rightarrow \xi'$ , where  $\xi' \equiv \xi + 4\epsilon M/k_{\text{th}}^2$ . (Here we have used the fact that  $\epsilon$  is much smaller than any other scale in the problem.) However, now these relations correspond respectively to cubic and quartic polynomial equations for  $k_{\text{th}}$  (since  $\xi'$  is itself a function of  $k_{\text{th}}^2$ ), and the condition that determines whether the threshold is given by the symmetric (5) or asymmetric (6) relation is more complicated. The detailed analysis can be found in [28]. Here we merely state the result. Rather than fixing  $\eta$ ,  $\xi$  and  $\epsilon$  and solving the relations for  $k_{\text{th}}$ , we fix  $\epsilon$  and  $k_{\text{th}}$  and use the threshold relations to solve for  $\xi$  as a function of  $\eta$ . When  $k_{\text{th}} < 1.5k_s$  the symmetric threshold applies for  $\xi' > 0$  and the asymmetric one applies for  $\eta < \xi' < 0$ . When  $k_{\text{th}} > 1.5k_s$  there is no symmetric threshold, and the asymmetric one applies only for  $\xi$  below the symmetric  $k_{\text{th}} = 1.5k_s$  line,  $\xi = \eta/2 - (16/27)(\epsilon^3 M/m^4)$ . In the case  $\xi = \eta$  the threshold configuration is never asymmetric [28].

For the observational consequences it is important to recognize that the threshold shifts are much more significant at higher energies than at lower energies. To exhibit this dependence, it is simplest to fix a gamma ray energy  $k$  and to solve for the corresponding soft photon threshold energy  $\epsilon_{\text{th}}$ . Taking the ratio with the usual threshold  $\epsilon_{\text{th},0}$ , we find a dependence on  $k$  at least as strong as  $k^{3/2}$ . Introducing  $k_{10} \equiv k/(10 \text{ TeV})$ , we have

$$\frac{\epsilon_{\text{th}}}{\epsilon_{\text{th},0}} = 1 + \frac{(\eta - 2\xi)}{20} k_{10}^3 \quad \text{for } \xi' \geq 0, \quad (7)$$

$$\frac{\epsilon_{\text{th}}}{\epsilon_{\text{th},0}} = \frac{(\eta - \xi)}{10} k_{10}^3 + \sqrt{-\frac{\eta}{5}} k_{10}^3 \quad \text{for } \eta < \xi' < 0. \quad (8)$$

High energy TeV gamma rays from the blazars Markarian 421 and Markarian 501 have been detected out to 17 TeV and 24 TeV, respectively [29,30]. Although the sources are not well understood, and the intergalactic IR background is also not fully known, detailed modeling shows that the data are consistent with some absorption by photon annihilation off the IR background (see e.g. [18,26,29] and references therein). However, while the inferred source spectrum for Markarian 501 is consistent with expectations for energies less than around 10 TeV, above this energy there have been claims [15,18] that far more photons than expected are detected. Nevertheless, recent analysis based on a more de-

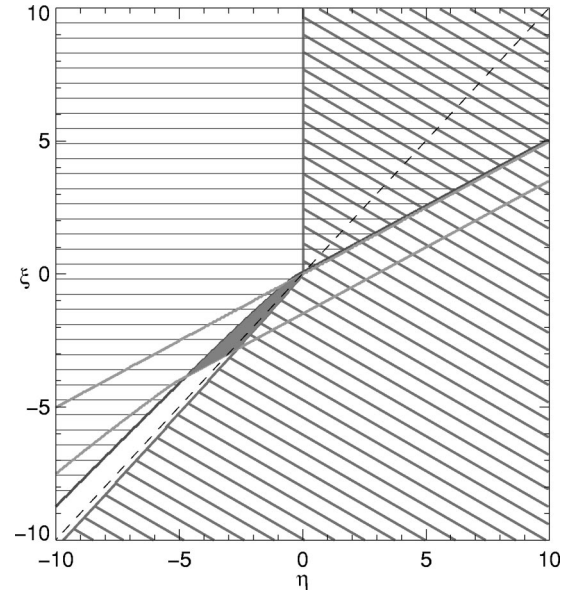


FIG. 1. Combined constraints on the dimensionless photon and electron parameters for the case  $n=3$  [ $\eta, \xi=1$  corresponds to the dimensionful coefficient  $M^{-1}=10^{-19} \text{ GeV}^{-1}$  of the  $n=3$  term in Eq. (1)]. The regions excluded by the photon decay and Čerenkov constraints are lined horizontally and diagonally, respectively. The region between the two diagonal lines corresponds to a threshold between one and two times the standard threshold (which is 10 TeV for photon annihilation with an IR photon of energy 0.025 eV). The upper line corresponds to the unmodified threshold. The shaded patch is the part of the allowed region that falls between these photon annihilation thresholds. The dashed line is  $\xi = \eta$ .

tailed reconstruction of the IR background do not seem to corroborate this point of view [26].

Due to these uncertainties sharp constraints from photon annihilation are currently precluded. Instead, we just determine the range of parameters  $\xi, \eta$  for which the threshold  $k_{\text{th}}$  lies between 10 TeV and 20 TeV for an IR photon of energy 0.025 eV with which a 10 TeV photon would normally be at threshold. Based on current observations it seems unlikely that the threshold could lie far outside this range. [It has previously been proposed [9] that raising this threshold by a factor of two could explain the potential overabundance of photons over 10 TeV. Given the strong energy dependence of the threshold shift in Eqs. (7) and (8) this threshold raising would not be obviously in disagreement with current observations below 10 TeV.]

**Combined constraints.** Putting together all the constraints and potential constraints we obtain the allowed region in the  $\eta$ - $\xi$  plane (see Fig. 1). The photon decay and Čerenkov constraints exclude the horizontally and diagonally shaded regions, respectively. The allowed region lies in the lower left quadrant, except for an exceedingly small sliver near the origin with  $0 < \eta \leq 10^{-3}$  and a small triangular region ( $-0.16 \leq \eta < 0, 0 < \xi \leq 0.08$ ) in the upper left quadrant. The range of the photon annihilation threshold previously discussed falls between the two roughly parallel diagonal lines. The upper diagonal line corresponds to the standard threshold  $k_s = 10$  TeV and the lower diagonal line to not more than twice that threshold. If future observations of the blazar



fluxes and the IR background confirm agreement with standard Lorentz invariant kinematics, the region allowed by the photon annihilation constraint will be squeezed toward the upper line ( $k_{\text{th}}=k_s$ ). This would close off *all* the available parameter space except for a region much smaller than unity around the Lorentz-invariant values  $\xi=0=\eta$ .

**Conclusions.** We have shown that astrophysical observations put strong constraints on the possibility of Lorentz-violating Planck scale cubic modifications to the electron and photon dispersion relations. The constraints arise due to the effect these modifications have on thresholds for various reactions. We have also seen that the threshold configurations with a final state electron-positron pair sometimes involve unequal momenta for the pair, unlike what occurs for all Lorentz-invariant decays. This can happen if  $\xi \neq \eta$  and  $\xi, \eta < 0$ .

The allowed region in the  $\eta$ - $\xi$  plane includes  $\xi=\eta=-1$ , which has been a focus of previous work [5,9,12,13]. The negative quadrant has most of the allowed parameter range. Note that in this quadrant all group velocities are less than the low energy speed of light.

To further constrain the cubic case will require new observations. Finding higher energy electrons would not help much, while finding higher energy undecayed photons would squeeze the allowed region onto the line  $\xi=\eta$ . To shrink the allowed segment of this line using the reactions we have considered would require observations confirming the usual threshold for photon annihilation to higher precision.

Perhaps other processes could be used as well. One might have hoped that observations comparing the time of flight of photons of different frequencies from distant sources such as gamma ray bursts and active galactic nuclei would help constrain the absolute value of  $\xi$  (see e.g. [2,31,32]). Unfortunately current observations just yield  $|\xi| \leq 122$  for  $n=3$ . This

is an interesting constraint but it is not competitive with the other ones already considered here. [However the Milagro Gamma Ray Observatory [33] and the forthcoming Gamma Ray Large Area Space Telescope (GLAST) mission [34] may provide more stringent constraints of this type.] Another idea is to exploit the fact that the reaction  $\gamma \rightarrow 3\gamma$  is kinematically allowed with finite phase space and nonzero amplitude in the presence of modified dispersion, unlike in the standard case. This photon decay channel occurs at *all energies* if  $\xi > 0$ , i.e. it has *no threshold*, so it might be thought to provide a very powerful constraint on positive  $\xi$ . Unfortunately, however, the amplitude for this reaction is far too small to provide any useful constraint [28].

It is interesting to consider the case of the possibly missing GZK cutoff [17]. If the cutoff is really missing, it has been proposed to explain this using Lorentz violating dispersion [7,9]. The relevant protons are at such a high energy—over  $10^{19}$  eV—that it takes only tiny Lorentz violating parameters  $\eta_a$  in Eq. (1) to increase the threshold by an amount of order unity or more. In particular, if one assumes all coefficients  $\eta_a$  are equal, this only requires  $\eta$  negative and  $|\eta| \geq m_p^2 M^{n-2}/p^n \sim 10^{-38+9n}$ . For  $n=3$  this is  $10^{-11}$ , and for  $n=4$  it is still only  $10^{-2}$ . Thus for both the  $n=3$  and  $n=4$  cases only very small values of  $\eta$  are needed to dramatically modify the GZK cutoff, so a shifted cutoff could be explained by Lorentz-violating constants with our constraints. However recent data [35,36] strongly support the existence of the GZK cutoff at its expected (Lorentz invariant) value. If this is confirmed, the above analysis shows that the GZK reaction provides very good constraints for modifications up to  $n=4$  [28].

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