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TEN-DIMENSIONAL SUPERSYMMETRIC YANG-MILLS THEORY IN TERMS OF FOUR-DIMENSIONAL SUPERFIELDS*

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We show that the usual formulation of the $N = 4$ supersymmetric Yang-Mills theory in terms of $N = 1$ superfields can be generalized to describe the full ten-dimensional theory.

1. Introduction

The $N = 4$ supersymmetric Yang-Mills theory (SSYM) [1, 2] is very interesting for several reasons. It is the maximally supersymmetric interacting theory not containing gravity, and as such is unique. Like $N = 8$ supergravity [3], it is a limit of a ten-dimensional superstring theory [4]. It is invariant under a very large symmetry group, the $N = 4$ superconformal group. Finally, and most importantly, its perturbation expansion appears to be free of ultraviolet divergences [5], even after the addition of suitable mass terms breaking the symmetries of the theory [6]. While finiteness is not presently regarded as a crucial feature for a renormalizable theory, it offers some hope that supersymmetry may eliminate the nonrenormalizable ultraviolet divergences of gravity in the framework of the $N = 8$ supergravity theory, or in the framework of the theory of superstrings [7].

The original component formulation of $N = 4$ SSYM was obtained from the $N = 1$ SSYM theory in ten dimensions [1, 2], a theory consisting simply of a Majorana-Weyl spinor in the adjoint representation of a gauge group, minimally coupled to a Yang-Mills boson. The ten-dimensional theory is reduced to four dimensions by assuming independence of the fields on six of the ten space-time coordinates. The resulting component theory consists of a vector, four Weyl spinors and six scalars, all in the adjoint representation of the gauge group. Correspondingly, the simple ten-dimensional supersymmetry breaks into four four-dimensional supersymmetries, and the Lorentz group breaks into the direct product of the four-dimensional Lorentz group and a global internal $SU(4) \approx SO(6)$ group. In the component formulation the SU(4) symmetry is manifest, but the four supersymmetries are not.

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As the main problem with making supersymmetries manifest is the existence of auxiliary fields for the off-shell closure of the supersymmetry algebra, one approach is to forego the use of auxiliary fields altogether, eliminate all the gauge and auxiliary degrees of freedom, and formulate the theory in terms of light-cone superfields [5, 8]. The Bose and Fermi degrees of freedom then match even off shell, but manifest Lorentz covariance is lost. Each covariant supersymmetry spinor generator splits into two parts, one of which is a manifest symmetry of the resulting action. The action also has a manifest internal SO(4), or even the full SU(4) [8, 9] invariance. However, only an E_2 subgroup of the Lorentz group is linearly realized on the fields.

A formulation of $N = 4$ SSYM in terms of $N = 1$ covariant superfields is also known [10, 11]. In this case the Lorentz symmetry, one of the supersymmetries and an $SU(3) \otimes U(1)$ subgroup of the SU(4) are manifest, but the extra supersymmetries and the $SU(4)/(SU(3) \otimes U(1))$ symmetries are realized as complicated non-linear transformations of the superfields. Recent progress in understanding the superspace formulation of $N = 1$ SSYM in six dimensions [12, 13] (which yields $N = 2$ SSYM in four dimensions upon dimensional reduction) also allows a formulation in terms of $N = 2$ superfields. The ultimate goal in this kind of approach would be a formulation in terms of $N = 4$ superfields. This would also be of interest for the superspace formulation of $N = 4$ supergravity, where $N = 4$ SSYM would enter as a compensator. The formulation in terms of $N = 4$ superfields, if it exists, is expected to possess uncommon features, in order to circumvent, for instance, the counting argument of ref. [14], which suggests that a set of auxiliary fields leading to closure of the supersymmetry algebra cannot be found for $N = 4$ SSYM, or for $N = 1$ SSYM in ten dimensions. In this context, a clear sign of trouble would be finding ultraviolet divergences in six dimensions at two loops, as suggested by superstring counting arguments [4], but excluded by superspace counting arguments based on the assumption that an $N = 4$ superfield formulation exists [15]. This problem is currently under investigation.

A possible way to attack the auxiliary field problem is to first seek an off-shell formulation of the ten-dimensional theory in terms of covariant superfields, and then derive the four-dimensional $N = 4$ superfield formulation by dimensional reduction in superspace, in analogy with what was originally done in components. For this approach, it is interesting to investigate what our present, incomplete understanding of the four-dimensional problem can teach us about the ten-dimensional theory. This requires undoing a dimensional reduction (an operation that might be called dimensional oxidation!), and is the subject of the present paper. Here we show that it is possible to extend the known formulation of $N = 4$ SSYM in terms of $N = 1$ four-dimensional superfields to provide an interesting, if somewhat unusual, description of the ten-dimensional theory. While several symmetries of the action are not manifest, this formalism is the only one known in which *all the fields are geometrical objects*. This theory can be dimensionally reduced, in the normal manner, to give a four-dimensional superspace formulation in any $4 \leqslant D \leqslant 10$. This is the first instance in which four-dimensional superfields have been used to describe a higher-dimensional theory. For the time being, this result may be regarded as a curious feature of four-dimensional unextended superspace. Our hope, however, is that it may also serve as a useful starting point in the search for the complete ten-dimensional action. In fact, the formalism does suggest one tensor auxiliary field that should be included in the complete theory.

The plan of this paper is as follows. In sect. 2 we construct covariant derivatives and field strengths, and derive the equations of motion of the ten-dimensional theory. In sect. 3 we construct the action. In sect. 4 we describe the global symmetries of the four-dimensional action, and the corresponding symmetries of the ten-dimensional action. In sect. 5 we discuss quantization and calculate ultraviolet divergent one-loop corrections to the S matrix. A discussion of our results is presented in sect. 6, and the appendices contain a few comments about our notation and conventions, and the derivation of a formula used in proving the gauge invariance of the ten-dimensional action.

2. Geometry and field equations

As stated in the introduction, the component form of $N=4$ SSYM in four dimensions [1,2] consists of a vector, four Weyl spinors and six scalars, all in the adjoint representation of a gauge group, and interacting via a single coupling constant. The corresponding formulation in terms of $N=1$ superfields [10,11] fits the six scalars and three of the spinors in an SU(3) triplet of chiral superfields, and the remaining spinor and the vector in a real scalar superfield. The action is:

$$S_4 = \mathrm{tr}\int \mathrm{d}^4x \left\{ \int \mathrm{d}^4\theta\, \mathrm{e}^{-gV}\bar{\varphi}^i \mathrm{e}^{gV}\varphi_i - \frac{1}{g^2}\int \mathrm{d}^2\theta\, W^\alpha W_\alpha \right.$$

$$\left. + \frac{ig}{3!}\int \mathrm{d}^2\theta\, \varepsilon^{ijk}\varphi_i[\varphi_j, \varphi_k] + \frac{ig}{3!}\int \mathrm{d}^2\bar{\theta}\, \varepsilon_{ijk}\bar{\varphi}^i[\bar{\varphi}^j, \bar{\varphi}^k] \right\}, \quad (2.1)$$

where $W_\alpha = \bar{D}^2(\mathrm{e}^{-gV}\mathrm{D}_\alpha \mathrm{e}^{gV})$ is the chiral field strength of the real superfield, and the trace is over the group indices, with $\mathrm{tr}(T^aT^b) = \delta^{ab}$. The action (2.1) is invariant under the gauge transformations

$$\delta_\Lambda \mathrm{e}^{gV} = i(\bar{\Lambda}\,\mathrm{e}^{gV} - \mathrm{e}^{gV}\Lambda), \quad (2.2a)$$

$$\delta_\Lambda \varphi_i = i[\Lambda, \varphi_i], \quad (2.2b)$$

with Λ an infinitesimal Lie-algebra valued chiral parameter. In the rest of this paper we shall, for simplicity, set $g = 1$.

The chiral superfields transform as matter fields under the four-dimensional gauge transformations in eqs. (2.2). In the ten-dimensional theory, however, the $\theta = 0$ parts of φ_i are components of the ten-dimensional vector, and are thus gauge fields. This suggests, as a first step, that the gauge transformation of φ_i in eq. (2.2b) be modified by the addition of terms involving derivatives of the gauge parameter with respect to the extra coordinates. The chirality of φ_i and SU(3) covariance then lead uniquely to:

$$\delta_\Lambda e^V = i(\bar{\Lambda} e^V - e^V \Lambda), \qquad \delta_\Lambda \varphi_i = \partial_i \Lambda + i[\Lambda, \varphi_i]. \tag{2.3}$$

Here the derivatives with respect to the extra six spatial dimensions have been grouped into a $\underline{3}$ of SU(3), in analogy with the grouping of the six four-dimensional scalars into the $\theta = 0$ parts of the chiral superfields.

With *every* field now being a gauge field, it is of interest to consider the covariant derivatives $\bar{\nabla}_{\dot{\alpha}}$, ∇_α, ∇_i and ∇^i. As usual, $\nabla_{\alpha\dot{\alpha}}$ is defined to be the anticommutator of the spinorial covariant derivatives. Working in the chiral representation, one demands that the covariant derivatives all transform as

$$\nabla_A \to e^{i\Lambda} \nabla_A e^{-i\Lambda} \tag{2.4}$$

under a gauge transformation, with Λ a chiral parameter. The covariant derivatives are therefore:

$$\begin{aligned}
\bar{\nabla}_{\dot{\alpha}} &\equiv \bar{D}_{\dot{\alpha}} - i\bar{\Gamma}_{\dot{\alpha}} = \bar{D}_{\dot{\alpha}}, \\
\nabla_\alpha &\equiv D_\alpha - i\Gamma_\alpha = e^{-V} D_\alpha e^V, \\
\nabla_i &\equiv \partial_i - i\Gamma_i = \partial_i - i\varphi_i, \\
\nabla^i &\equiv \bar{\partial}^i - i\Gamma^i = e^{-V}(\bar{\partial}^i - i\bar{\varphi}^i) e^V.
\end{aligned} \tag{2.5}$$

Taking commutators of covariant derivatives then generates field strengths, which by construction transform covariantly under gauge transformations. The *non-vanishing* field strengths are:

$$\begin{aligned}
W_\alpha &\equiv [\bar{\nabla}^{\dot{\alpha}}, \{\nabla_\alpha, \bar{\nabla}_{\dot{\alpha}}\}] = \bar{D}^2(e^{-V} D_\alpha e^V), \\
W_{\dot{\alpha}} &\equiv [\nabla^\alpha, \{\bar{\nabla}_{\dot{\alpha}}, \nabla_\alpha\}] = e^{-V} D^2 (e^V \bar{D}_{\dot{\alpha}} e^{-V}) e^V, \\
F_{\alpha i} &\equiv [\nabla_\alpha, \nabla_i] = -iD_\alpha \varphi_i - \partial_i (e^{-V} D_\alpha e^V) - i[(e^{-V} D_\alpha e^V), \varphi_i], \\
F_{\dot{\alpha}}{}^i &\equiv [\bar{\nabla}_{\dot{\alpha}}, \nabla^i] = \bar{D}_{\dot{\alpha}}(e^{-V} \bar{\partial}^i e^V) - i\bar{D}_{\dot{\alpha}}(e^{-V} \bar{\varphi}^i e^V), \\
F_{ij} &\equiv [\nabla_i, \nabla_j] = -i(\partial_i \varphi_j - \partial_j \varphi_i - i[\varphi_i, \varphi_j]), \\
F^{ij} &\equiv [\nabla^i, \nabla^j] = -i e^{-V}(\bar{\partial}^i \bar{\varphi}^j - \bar{\partial}^j \bar{\varphi}^i - i[\bar{\varphi}^i, \bar{\varphi}^j]) e^V, \\
F_i^j &\equiv [\nabla_i, \nabla^j] = \partial_i (e^{-V} \bar{\partial}^j e^V) - i\partial_i (e^{-V} \bar{\varphi}^j e^V) + i\bar{\partial}^j \varphi_i \\
&\quad + i[(e^{-V} \bar{\partial}^j e^V), \varphi_i] + [(e^{-V} \bar{\varphi}^j e^V), \varphi_i].
\end{aligned} \tag{2.6}$$

W_α, $W_{\dot{\alpha}}$, $F_{\alpha i}$ and $F_{\dot{\alpha}}{}^i$ have dimension $\frac{3}{2}$, whereas F_{ij}, F^{ij} and F_i^j have dimension 2.

It should be noted that the form (2.5) of the covariant derivatives follows from the constraints

$$F_{\alpha\beta} \equiv \{ \nabla_\alpha, \nabla_\beta \} = 0 ,$$

$$F_{\dot\alpha\dot\beta} = \{ \overline{\nabla}_{\dot\alpha}, \overline{\nabla}_{\dot\beta} \} = 0 ,$$

$$F_{\alpha\dot\alpha} \equiv \{ \nabla_\alpha, \overline{\nabla}_{\dot\alpha} \} - \tfrac{1}{2} i \nabla_{\alpha\dot\alpha} = 0 , \tag{2.7a}$$

$$F_\alpha^{\ i} \equiv [\nabla_\alpha, \nabla^i] = 0 , \qquad F_{\dot\alpha i} \equiv [\overline{\nabla}_{\dot\alpha}, \nabla_i] = 0 , \tag{2.7b}$$

where, in the chiral representation, we choose $\overline{\nabla}_{\dot\alpha} = \overline{D}_{\dot\alpha}$. The new constraints are easily understood by examining the field strengths. At $\theta = \bar\theta = 0$, F_{ij}, F^{ij} and $F_i^{\ j}$ contain the part of the non-abelian field strength $F_{\hat\mu\hat\nu}$ corresponding to the extra six dimensions, and W_α, $W_{\dot\alpha}$, $F_{\alpha i}$ and $F_{\dot\alpha}^{\ i}$ contain the components of the ten-dimensional spinor. The new constraints (2.7b) simply state that the field strengths $F_\alpha^{\ i}$ and $F_{\dot\alpha i}$ vanish. This is necessary as at $\theta = \bar\theta = 0$ they would contain 3 *new* physical spinors, which do not exist in the component theory.

We can now ask ourselves what covariant equations of motion we can write for the ten-dimensional theory using the field strengths in eqs. (2.6). Interestingly, the answer is almost uniquely determined by dimensionality and SU(3) covariance. Indeed, the equation of motion of V can only be

$$2\{ \nabla^\alpha, W_\alpha \} + \gamma F_i^{\ i} = 0, \tag{2.8}$$

and the equation of motion for φ_i can only be

$$i\{ \nabla^\alpha, F_{\alpha i} \} + \delta \varepsilon_{ijk} F^{jk} = 0 . \tag{2.9}$$

The constants γ and δ are then fixed by comparison with the four-dimensional theory to be $\gamma = -1$, $\delta = -\frac{1}{2} i$. They could also be determined by demanding the covariance of the field equations under the non-manifest symmetries of the theory.

3. The action

The next problem is to construct an action that yields these equations of motion. To this end we notice that, under arbitrary variations of the fields, the ten-dimensional action must vary as:

$$\begin{aligned} \Delta S_{10} = \operatorname{tr} \int \mathrm{d}^{10} x \Big\{ & \int \mathrm{d}^4\theta \, \Delta V \big(2\{ \nabla^\alpha, W_\alpha \} - F_i^{\ i} \big) \\ & + \int \mathrm{d}^2\bar\theta \big(\mathrm{e}^{-V} \delta\bar\varphi^i \mathrm{e}^V \big) \big(i\{ \nabla^\alpha, F_{\alpha i} \} - \tfrac{1}{2} i \varepsilon_{ijk} F^{jk} \big) \\ & + \int \mathrm{d}^2\theta \, \delta\varphi_i \big(i\{ \overline{\nabla}^{\dot\alpha}, F_{\dot\alpha}^{\ i} \} - \tfrac{1}{2} i \varepsilon^{ijk} F_{jk} \big) \Big\} , \end{aligned} \tag{3.1}$$

where $\Delta V \equiv e^{-V}\delta e^{V}$ is the gauge covariant variation of V, and $e^{-V}\delta\bar{\varphi}^i e^{V}$ is the chirally covariant variation of $\bar{\varphi}^i$. Reconstructing the action from this variation is not straightforward, as the equations of motion mix the various field strengths and involve the field strengths themselves, not only their derivatives. To proceed further, it is convenient to introduce in eq. (3.1) the explicit form of the field strengths in terms of the fields. One obtains:

$$\begin{aligned}\Delta S_{10} = \operatorname{tr}\int d^{10}x \Big\{ \int d^4\theta\, \Delta V\Big(2\{e^{-V}D^{\alpha}e^{V}, \bar{D}^2(e^{-V}D_{\alpha}e^{V})\} - \partial_i(e^{-V}\bar{\partial}^i e^{V}) \\ + i\big(\partial_i(e^{-V}\bar{\varphi}^i e^{V}) - e^{-V}\bar{\partial}^i(e^{V}\varphi_i) - i[\varphi_i, e^{-V}\bar{\varphi}^i e^{V}]\big)\Big) \\ + \int d^2\theta\, \delta\varphi_i\big(i\bar{D}^2(e^{-V}\bar{\partial}^i e^{V} - ie^{-V}\bar{\varphi}^i e^{V}) \\ - \tfrac{1}{2}\varepsilon^{ijk}(\partial_j\varphi_k - \partial_k\varphi_j - i[\varphi_j, \varphi_k])\big) \\ + \int d^2\bar{\theta}\, \delta\bar{\varphi}^i i\big(iD^2(e^{V}\partial_i e^{-V} - ie^{V}\varphi_i e^{-V}) \\ - \tfrac{1}{2}\varepsilon_{ijk}(\bar{\partial}^j\bar{\varphi}^k - \bar{\partial}^k\bar{\varphi}^j - i[\bar{\varphi}^j, \bar{\varphi}^k])\big)\Big\}. \end{aligned} \tag{3.2}$$

Most of the terms in the action can indeed be guessed, using eq. (3.2) and comparing with the four-dimensional action given in eq. (2.1). One is thus led to consider

$$\begin{aligned} S_{10} = \operatorname{tr}\int d^{10}x \Big\{ \int d^4\theta\, e^{-V}\bar{\varphi}^i e^{V}\varphi_i - \int d^2\theta\, W^{\alpha}W_{\alpha} + \frac{i}{3!}\int d^2\theta\, \varepsilon^{ijk}\varphi_i[\varphi_j, \varphi_k] \\ + \frac{i}{3!}\int d^2\bar{\theta}\, \varepsilon_{ijk}\bar{\varphi}^i[\bar{\varphi}^j, \bar{\varphi}^k] - \tfrac{1}{2}\int d^2\theta\, \varepsilon^{ijk}\varphi_i\partial_j\varphi_k - \tfrac{1}{2}\int d^2\bar{\theta}\, \varepsilon_{ijk}\bar{\varphi}^i\bar{\partial}^j\bar{\varphi}^k \\ + i\int d^4\theta\,(\partial_i e^{-V})\bar{\varphi}^i e^{V} - i\int d^4\theta\, e^{V}\varphi_i(\bar{\partial}^i e^{-V}) \\ + \tfrac{1}{2}\int d^4\theta\,(e^{-V}\bar{\partial}^i e^{V})(e^{-V}\partial_i e^{V})\Big\}. \end{aligned} \tag{3.3}$$

This action in fact yields the correct equations of motion for φ_i and $\bar{\varphi}^i$. In the vector equation, however, the terms with two ∂_i's are not reproduced correctly, as the last term in eq. (3.3) varies as

$$-\tfrac{1}{2}\int d^{10}x\, d^4\theta\, \Delta V\big(\partial_i(e^{-V}\bar{\partial}^i e^{V}) + \bar{\partial}^i(e^{-V}\partial_i e^{V})\big), \tag{3.4}$$

whereas the corresponding term in eq. (3.2) is

$$-\int \mathrm{d}^{10}x\,\mathrm{d}^4\theta\,\Delta V \partial_i (\mathrm{e}^{-V}\bar{\partial}^i \mathrm{e}^V). \tag{3.5}$$

One therefore needs to find a new term to be added to the action that varies into

$$-\tfrac{1}{2}\int \mathrm{d}^{10}x\,\mathrm{d}^4\theta\,\Delta V\big(\partial_i(\mathrm{e}^{-V}\bar{\partial}^i\mathrm{e}^V) - \bar{\partial}^i(\mathrm{e}^{-V}\partial_i\mathrm{e}^V)\big). \tag{3.6}$$

Such a term must be very similar in structure to the last term in eq. (3.3), but must be odd under the interchange of ∂_i and $\bar{\partial}^i$, and *cannot be written in terms of the potential* e^V *only, but must also contain the prepotential* V *explicitly.* In order to complete the construction of the action, it is convenient to expand the last term in eq. (3.3) in powers of V using

$$\mathrm{e}^{-V}\partial_i\mathrm{e}^V = \frac{1-\mathrm{e}^{-L_V}}{L_V}(\partial_i V), \tag{3.7}$$

where $L_V X \equiv [V, X]$. The result is

$$\mathrm{tr}\int \mathrm{d}^{10}x\,\mathrm{d}^4\theta(\bar{\partial}^i V)\frac{\cosh L_V - 1}{L_V^2}(\partial_i V), \tag{3.8}$$

and the missing term is

$$\mathrm{tr}\int \mathrm{d}^{10}x\,\mathrm{d}^4\theta(\bar{\partial}^i V)\frac{\sinh L_V - L_V}{L_V^2}(\partial_i V), \tag{3.9}$$

which contains the odd function of L_V corresponding to (3.8), and is thus odd under the interchange of ∂_i and $\bar{\partial}^i$*. A proof that varying this term yields (3.6) can be found in appendix B.

It is interesting to note that the term in eq. (3.9) can be also recovered from eq. (3.6) using a prescription recently given by Koller [13]. One reconstructs the term from its variation simply by replacing V with tV, ΔV with V, and by integrating over t from zero to one. This trick replaces functional integrations with respect to the fields with integrations over scalar parameters, thus undoing the combinatorics of functional differentiation. In our case, the method works particularly simply if we start from eq. (3.5). The proceedure above yields

* Note that $\mathrm{tr}(AL_V^n B) = (-1)^n \mathrm{tr}(BL_V^n A)$.

$$\operatorname{tr}\int \mathrm{d}^{10}x\,\mathrm{d}^4\theta\,(\bar{\partial}^i V)\int_0^1 \mathrm{d}t\,\frac{e^{tL_V}-1}{L_V}(\partial_i V). \tag{3.10}$$

Performing the t integration then clearly leads to the sum of (3.8) and (3.9).

We have thus found that

$$\begin{aligned} S_{10} = \operatorname{tr}\int \mathrm{d}^{10}x \Big\{ & \int \mathrm{d}^4\theta\, e^{-V}\bar{\varphi}^i e^V \varphi_i - \int \mathrm{d}^2\theta\, W^\alpha W_\alpha + \frac{i}{3!}\int \mathrm{d}^2\theta\, \varepsilon^{ijk}\varphi_i[\varphi_j,\varphi_k] \\ & + \frac{i}{3!}\int \mathrm{d}^2\bar{\theta}\, \varepsilon_{ijk}\bar{\varphi}^i[\bar{\varphi}^j,\bar{\varphi}^k] - \tfrac{1}{2}\int \mathrm{d}^2\theta\,\varepsilon^{ijk}\varphi_i\partial_j\varphi_k - \tfrac{1}{2}\int \mathrm{d}^2\bar{\theta}\,\varepsilon_{ijk}\bar{\varphi}^i\bar{\partial}^j\bar{\varphi}^k \\ & + i\int \mathrm{d}^4\theta\,(\partial_i e^{-V})\bar{\varphi}^i e^V - i\int \mathrm{d}^4\theta\, e^V\varphi_i(\bar{\partial}^i e^{-V}) \\ & + \tfrac{1}{2}\int \mathrm{d}^4\theta\,(e^{-V}\bar{\partial}^i e^V)(e^{-V}\partial_i e^V) \\ & + \int \mathrm{d}^4\theta\,(\bar{\partial}^i V)\frac{\sinh L_V - L_V}{L_V^2}(\partial_i V)\Big\} \end{aligned} \tag{3.11}$$

yields the equations of motion (2.8) and (2.9) (with $\gamma = -1$ and $\delta = -\frac{1}{2}i$) and is invariant under the gauge transformations in eqs. (2.3). We wish to emphasize that the action in eq. (3.11) yields gauge covariant equations of motion, even though it is not expressible in terms of field strengths and covariant derivatives only. The purely chiral (or antichiral) terms are obviously, if not manifestly, gauge invariant, as they have the form of a gauge invariant mass term for a three-dimensional non-abelian gauge theory. The lack of manifest gauge invariance will be a common feature of all extended superspace formulations of supersymmetric theories, as increasing the number of anticommuting coordinates lowers the dimensionality of the volume element, and does not leave room for squares of curvatures, which at least have dimension 3.

Starting from eq. (3.11), one can recover the usual component form of the ten-dimensional theory as follows. First, for simplicity, one goes to a Wess-Zumino gauge eliminating the chiral and antichiral parts of V and reducing the action to a polynomial function of V. Then one integrates out the θ's, replacing the integrals by spinorial derivatives and using the definitions of the component fields in terms of the superfields given in appendix A. The following changes of notation are then required. First of all, the spinors are grouped together into a ten-dimensional spinor. Then the four-dimensional spinor indices are eliminated in favor of four-component notation. Finally, the complex SU(3) triplets of spatial derivatives ∂_i and of field components A_i are regrouped according to the conventional SO(6) vector notation. The result of these manipulations is

$$S_{10} = \operatorname{tr}\int d^{10}x \Big\{ -\tfrac{1}{4}F^{\hat\mu\hat\nu}F_{\hat\mu\hat\nu} - \tfrac{1}{2}i\bar\lambda\gamma^{\hat\mu}D_{\hat\mu}\lambda$$
$$+\|\bar F^i - \tfrac{1}{2}i\varepsilon^{ijk}\big(\partial_j A_k - \partial_k A_j - i[A_j, A_k]\big)\|^2$$
$$+\big(D - \tfrac{1}{2}i\big(\bar\partial\cdot A - \partial\cdot\bar A - i[\bar A^i, A_i]\big)\big)^2\Big\}, \tag{3.12}$$

which is the usual component form of the ten-dimensional action, together with extra terms that vanish when the equations of motion for the auxiliary fields are used. It is interesting to note that the equations of motion for the auxiliary fields are

$$F_i = \tfrac{1}{2}i\varepsilon_{ijk}\big(\bar\partial^j\bar A^k - \bar\partial^k\bar A^j - i[\bar A^j, \bar A^k]\big), \tag{3.13a}$$

$$D = \tfrac{1}{2}i\big(\bar\partial\cdot A - \partial\cdot\bar A - i[\bar A^i, A_i]\big). \tag{3.13b}$$

The right-hand sides of eqs. (3.13a) and (3.13b) are, respectively, the SU(3) singlet and triplet parts of the $\underline{15}$ of SO(6)

$$G_{IJ} = \partial_I A_J - \partial_J A_I - \sqrt{\tfrac{1}{2}}\, i[A_I, A_J]. \tag{3.14}$$

This suggests that the bosonic auxiliary fields F_i, $\bar F^i$ and D would appear in the complete ten-dimensional action together with extra auxiliary fields completing a $\underline{45}$ of SO(9, 1), $G_{\hat\mu\hat\nu}$. There should, of course, be other auxiliary fields, as the number of bosonic auxiliary fields must exceed the number of fermionic auxiliary fields by only 7, for the off-shell equality of Bose and Fermi degrees of freedom.

4. Global symmetries of the ten-dimensional action

The four-dimensional action in eq. (2.1), besides being gauge invariant, possesses several global symmetries [16]. It is invariant under the direct product of the four-dimensional Lorentz group and an SO(6) ≈ SU(4) group corresponding to spatial rotations in the extra dimensions. Moreover, it is invariant under four global supersymmetries. Indeed, as stated in the introduction, the four-dimensional action possesses the full $N = 4$ superconformal symmetry. This, however, does not concern us, as we are interested in symmetries that generalize to the ten-dimensional theory.

Consider first the supersymmetry that corresponds to the $N = 1$ superspace coordinates. Its parameter fits into an x-independent real scalar superfield ζ, which also contains, in its non-gauge part, the parameters of four-dimensional translations and of the U(1) subgroup of the SU(4) realized as combined chiral rotations of the fermionic superspace coordinates and of the chiral superfields (R transformations). These transformations correspond to shifts of the superspace coordinates. In four dimensions, by adding a gauge transformation of parameter

$$\Lambda = -\bar{D}^2\left[(D^\alpha\zeta)(e^{-V}D_\alpha e^V)\right], \tag{4.1}$$

they can be written in the covariant form

$$\begin{aligned} \Delta V &= i\left[(\nabla^\alpha\zeta)W_\alpha - (\bar{\nabla}^{\dot\alpha}\zeta)W_{\dot\alpha}\right], \\ \delta\varphi_i &= -i\bar{\nabla}^2\left[(\nabla^\alpha\zeta)(\nabla_\alpha\varphi_i) + \tfrac{1}{3}(\nabla^2\zeta)\varphi_i\right]. \end{aligned} \tag{4.2}$$

The modified transformations in eq. (4.2), however, are not a symmetry of the ten-dimensional action as they stand. A signal of this is that they are not gauge invariant (even up to a gauge transformation) in $D > 4$, in contradiction with the commutativity of supersymmetry transformations and gauge transformations. Moreover, invariance of the fifth and sixth terms in eq. (3.11) demands that the transformations be modified by the addition of orbital pieces. The correct transformations are:

$$\begin{aligned} \Delta V &= i\left[(\nabla^\alpha\zeta)W_\alpha - (\bar{\nabla}^{\dot\alpha}\zeta)W_{\dot\alpha}\right] + \tfrac{1}{3}i(\bar{\nabla}^2\nabla^2\zeta)e^{-V}(\bar{x}\cdot\bar{\partial} - x\cdot\partial)e^V, \\ \delta\varphi_i &= -i\bar{\nabla}^2\left[(\nabla^\alpha\zeta)iF_{\alpha i} + \tfrac{1}{3}(\nabla^2\zeta)\varphi_i\right] + \tfrac{1}{3}i(\bar{\nabla}^2\nabla^2\zeta)(\bar{x}\cdot\bar{\partial} - x\cdot\partial)\varphi_i. \end{aligned} \tag{4.3}$$

Apart from the orbital pieces, the changes amount only to the replacement of the noncovariant quantity $\nabla_\alpha\varphi_i$ with the field strength $F_{\alpha i}$.

Next we consider the three extra four-dimensional supersymmetries. Their parameters, together with the parameters of central charge transformations Z_i and the parameters of SU(4)/(SU(3) ⊗ U(1)) transformations ω_i, fit into an SU(3) triplet of x-independent chiral superfields χ_i

$$\chi_i = Z_i + \theta^\alpha\varepsilon_{\alpha i} + \theta^2\omega_i. \tag{4.4}$$

The four-dimensional action is indeed invariant under

$$\begin{aligned} \Delta V &= i\left(e^{-V}\chi_i\bar{\varphi}^i e^V - \bar{\chi}^i\varphi_i\right), \\ \delta\varphi_i &= \varepsilon_{ijk}\bar{\nabla}^2\left(\bar{\chi}^j e^{-V}\bar{\varphi}^k e^V\right) + 2i(\nabla^\alpha\chi_i)W_\alpha. \end{aligned} \tag{4.5}$$

In finding the corresponding transformations for the ten-dimensional action, it is useful to note that the central charge transformations become translations in the extra dimensions. Moreover, the SU(4)/(SU(3) ⊗ U(1)) transformations are Lorentz transformations in the extra dimensions and, as such, acquire orbital parts. The correct transformations for the ten-dimensional action are:

$$\Delta V = i\left(e^{-V}\chi_i\bar{\varphi}^i e^V - \bar{\chi}^i\varphi_i\right) - e^{-V}(\bar{\chi}\cdot\partial + \chi\cdot\bar{\partial})e^V$$
$$-\varepsilon_{ijk}\left(\bar{\nabla}^2\bar{\chi}^j\right)x^k e^{-V}\bar{\partial}^i e^V - \varepsilon^{ijk}\left(\nabla^2\chi_j\right)\bar{x}_k e^{-V}\partial_i e^V,$$
$$\delta\varphi_i = \varepsilon_{ijk}\bar{\nabla}^2\left[\bar{\chi}^j e^{-V}(\bar{\varphi}^k + i\bar{\partial}^k)e^V\right] + 2i\left(\nabla^\alpha\chi_i\right)W_\alpha - \chi\cdot\bar{\partial}\varphi_i$$
$$-\varepsilon_{jkl}\left(\bar{\nabla}^2\bar{\chi}^j\right)x^k\bar{\partial}^l\varphi_i - \varepsilon^{jkl}\left(\nabla^2\chi_j\right)\bar{x}_k\partial_l\varphi_i. \tag{4.6}$$

By adding a gauge transformation of parameter

$$\Lambda = -\varepsilon_{ijk}\bar{\nabla}^2\left[\bar{\chi}^j x^k e^{-V}(i\bar{\partial}^i + \bar{\varphi}^i)e^V\right] - \varepsilon^{ijk}\left(\nabla^2\chi_j\right)\bar{x}_k\varphi_i, \tag{4.7}$$

the χ transformations can be cast in a more elegant form, involving the field strengths of eqs. (2.6):

$$\Delta V = i\left(e^{-V}\chi\cdot\bar{\varphi}e^V - \bar{\chi}\cdot\varphi\right) - e^{-V}(\chi\cdot\bar{\partial} + \bar{\chi}\cdot\partial)e^V + 2\varepsilon_{ijk}x^k\left(\bar{\nabla}^{\dot{\alpha}}\bar{\chi}^j\right)F_{\dot{\alpha}}^{\ i}$$
$$-2\varepsilon^{ijk}\bar{x}_k\left(\nabla^\alpha\chi_j\right)F_{\alpha i} + \varepsilon_{ijk}\bar{\chi}^j x^k\{\bar{\nabla}^{\dot{\alpha}}, F_{\dot{\alpha}}^{\ i}\} - \varepsilon^{ijk}\chi_j\bar{x}_k\{\nabla^\alpha, F_{\alpha i}\},$$
$$\delta\varphi_i = i\varepsilon_{jkl}\bar{\nabla}^2\left(\bar{\chi}^j F_i^{\ l}\right)x^k + 2i\left(\nabla^\alpha\chi_i\right)W_\alpha + \varepsilon^{jkl}\left(\nabla^2\chi_j\right)\bar{x}_k F_{li} - \chi\cdot\bar{\partial}\varphi_i. \tag{4.8}$$

This result, unlike the ζ transformations, contains, as well as covariant terms, non-covariant ones which cannot be eliminated because of the chirality of φ_i. As a consequence, χ transformations commute with gauge transformations only up to a gauge transformation of parameter $-\chi\cdot\bar{\partial}\Lambda$.

Finally, we consider the remaining Lorentz transformations, corresponding to SO(9, 1)/SU(4). Of these, the purely four-dimensional Lorentz transformations are an obvious symmetry of the action, manifest in the way the spinor indices are contracted together, with the superfields V, φ_i and $\bar{\varphi}^i$ transforming as scalars under them. On the other hand, the "off-diagonal" Lorentz transformations which rotate the four-dimensional coordinates into those of the extra six dimensions, and therefore have no analogue in the four-dimensional theory, are not an obvious symmetry and require direct investigation. It is natural to try to fit the corresponding parameters $\lambda_i^{\alpha\dot{\alpha}}$ into an SU(3) triplet of complex x-independent superfields. This, however, does not lead to a symmetry of the action, which is not surprising, as the extra parameters do not correspond to symmetries of the component action. Restricting the complex superfields to be of the form $\lambda_i = \lambda_i^{\alpha\dot{\alpha}}\theta_\alpha\bar{\theta}_{\dot{\alpha}}$, i.e., demanding that they only have a non-vanishing $\theta_\alpha\bar{\theta}_{\dot{\alpha}}$ component, is indeed enough to ensure that the transformations

$$\Delta V = 2(\mathrm{e}^{-V}\lambda\cdot\bar{\varphi}\mathrm{e}^{V} + \bar{\lambda}\cdot\varphi) + \bar{\lambda}^{j\alpha\dot{\alpha}}\mathrm{e}^{-V}\big[(x_{\alpha\dot{\alpha}} - i\theta_\alpha\bar{\theta}_{\dot{\alpha}})\partial_j - \bar{x}_j\partial_{\alpha\dot{\alpha}}\big]\mathrm{e}^{V}$$
$$+\lambda_j^{\alpha\dot{\alpha}}\mathrm{e}^{-V}\big[(x_{\alpha\dot{\alpha}} + i\theta_\alpha\bar{\theta}_{\dot{\alpha}})\bar{\partial}^j - x^j\partial_{\alpha\dot{\alpha}}\big]\mathrm{e}^{V},$$
$$\delta\varphi_i = 4\bar{D}^2\big[(D^\alpha\lambda_i)(\mathrm{e}^{-V}D_\alpha\mathrm{e}^{V})\big] + 2i\varepsilon_{ijk}\bar{D}^2\big[\bar{\lambda}^j\mathrm{e}^{-V}(\bar{\varphi}^k + i\bar{\partial}^k)\mathrm{e}^{V}\big]$$
$$+\lambda_j^{\alpha\dot{\alpha}}\big[(x_{\alpha\dot{\alpha}} + i\theta_\alpha\bar{\theta}_{\dot{\alpha}})\bar{\partial}^j - x^j\partial_{\alpha\dot{\alpha}}\big]\varphi_i + \bar{\lambda}^{j\alpha\dot{\alpha}}\big[(x_{\alpha\dot{\alpha}} + i\theta_\alpha\bar{\theta}_{\dot{\alpha}})\partial_j - \bar{x}_j\partial_{\alpha\dot{\alpha}}\big]\varphi_i \tag{4.9}$$

be an invariance of the ten-dimensional action. We note that the chirality of φ_i demands that the four-dimensional coordinates $x_{\alpha\dot{\alpha}}$ appear in the orbital parts of these transformations only in the chiral combinations $(x_{\alpha\dot{\alpha}} \pm i\theta_\alpha\bar{\theta}_{\dot{\alpha}})$, and thus the explicit $\theta_\alpha\bar{\theta}_{\dot{\alpha}}\lambda_j^{\alpha\dot{\alpha}}$'s cannot be absorbed in a general superfield λ_j.

5. Quantization and Feynman rules

We now turn to the problem of quantization. Following the standard procedure for quantizing gauge theories, we must add a gauge-fixing term and the corresponding Faddeev-Popov ghost action. To this end, we notice that the lagrangian in eq. (3.11) contains quadratic terms mixing the vector and chiral multiplets, a situation similar to that of spontaneously broken Yang-Mills theory, where the kinetic terms mix scalar and vector fields. The non-local gauge-fixing term

$$S_{\mathrm{GF}} = -\operatorname{tr}\int \mathrm{d}^{10}x\,\mathrm{d}^4\theta\left(\bar{D}^2V + i\frac{\bar{D}^2}{\Box_4}\partial\cdot\bar{\varphi}\right)\left(D^2V - i\frac{D^2}{\Box_4}\bar{\partial}\cdot\varphi\right). \tag{5.1}$$

generalizes the four-dimensional Feynman-type gauge and diagonalizes the kinetic term in an SU(3) covariant way. This is the gauge-fixing term associated with the gauge

$$D^2\bar{D}^2V + i\partial\cdot\bar{\varphi} = 0. \tag{5.2}$$

The Faddeev-Popov ghost lagrangian is then determined to be:

$$S_{\mathrm{FP}} = -\operatorname{tr}\int \mathrm{d}^{10}x\,\mathrm{d}^4\theta\bigg\{(c'+\bar{c}')\big[L_{V/2}(c+\bar{c}) + L_{V/2}\coth L_{V/2}(c-\bar{c})\big]$$
$$-c'\frac{\Box_6}{\Box_4}\bar{c} + i(\partial_i c')\frac{1}{\Box_4}\big[\bar{c},\bar{\varphi}^i\big] + \bar{c}'\frac{\Box_6}{\Box_4}c - i(\bar{\partial}^i\bar{c}')\frac{1}{\Box_4}\big[c,\varphi_i\big]\bigg\}, \tag{5.3}$$

which also contains non-local terms. The non-localities are only introduced by our gauge choice, and it turns out that rearranging the covariant derivatives according to the standard rules of superfield perturbation theory [10] always cancels the non-local terms in Green functions not involving external ghosts.

The propagators are obtained by inverting the quadratic part of the gauge-fixed lagrangian

$$\operatorname{tr}\int d^{10}x\, d^4\theta \left\{ \bar{\varphi}^i\varphi_i - \tfrac{1}{2}V\square_{10}V - \tfrac{1}{2}\varepsilon^{ijk}\varphi_i\partial_j\frac{D^2}{\square_4}\varphi_k - \tfrac{1}{2}\varepsilon_{ijk}\bar{\varphi}^i\bar{\partial}^j\frac{\bar{D}^2}{\square_4}\bar{\varphi}^k \right.$$

$$\left. -(\partial\cdot\bar{\varphi})\frac{1}{\square_4}(\bar{\partial}\cdot\varphi) + \bar{c}'c + \bar{c}'\frac{\square_6}{\square_4}c - c'\bar{c} - c'\frac{\square_6}{\square_4}\bar{c} \right\}. \tag{5.4}$$

There is now a $\varphi - \varphi$ propagator of the form

$$-i\varepsilon_{ijk}\frac{\bar{p}^k}{p_{10}^2}\frac{D^2}{p_4^2}\delta(\theta_1-\theta_2), \tag{5.5}$$

and a corresponding $\bar{\varphi}-\bar{\varphi}$ propagator, resembling those of a massive chiral multiplet. The other propagators differ from those of the four-dimensional theory in the corresponding Feynman-type gauge only by the obvious replacement of $\square_4$ with $\square_{10}$. There are also some additional vertices in the theory coupling vectors to a single chiral superfield, additional purely vector vertices and new couplings of chiral fields to ghosts. The propagators and cubic vertices of the theory are shown in fig. 1. It

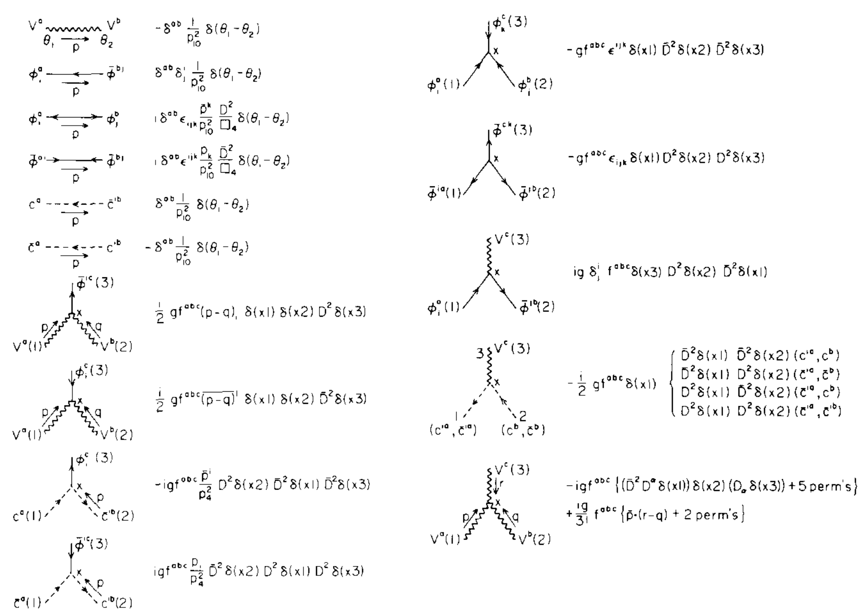

Fig. 1. Propagators and cubic vertices.

should be noted that the new contributions to the purely vector vertex do not involve spinorial derivatives, and therefore usually do not contribute to loop diagrams.

In order to compute quantum corrections, one must, as usual, regularize the theory to localize and control the infinities of the Feynman diagrams. As one wants the regularization scheme to preserve as many of the symmetries of the theory as possible, one can use the only freedom in the theory: the fact that it can be written in any space-time dimension $D \leqslant 10$ by dimensional reduction. We should point out that here we use two different dimensional reductions. The first is a classical procedure. If one wants to work in $D < 10$ dimensions, one must set some of the ∂_i's to 0* (i.e. the fields are taken to be independent of some of the x^i). Thus, in $D = 6$, for example, one would set $\partial_2 = \partial_3 = 0$. The second is adapted from the dimensional reduction scheme of ref. [17], in which one keeps the indices of the fields and of the D operators fixed, while varying the range of the indices of the momenta. In our case, as we encounter terms where a ∂_i is contracted with an ε tensor, we shall keep the SU(3) indices running over an integral number of values, and let the 4-dimensional momenta become $4 - 2\varepsilon$ dimensional. Thus, in $D = 6$, for example, we would work with ∂_1, $\bar{\partial}^1$ and ∂_μ, with $\mu = 1, \ldots, 4 - 2\varepsilon$.

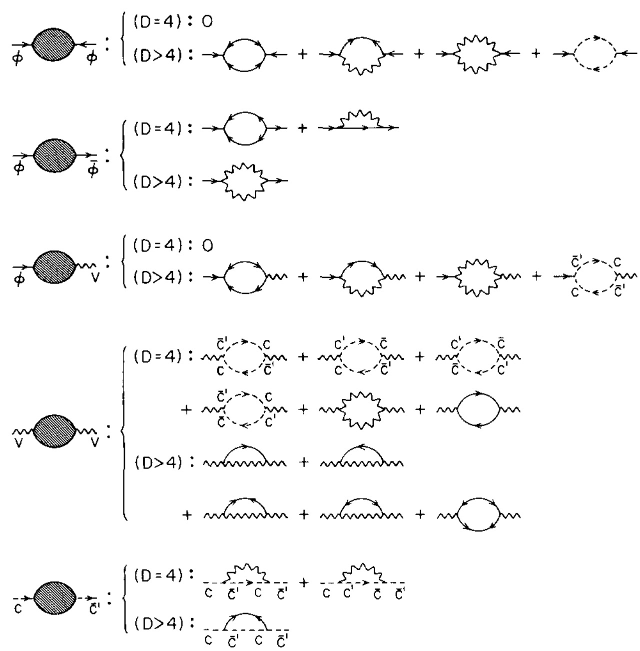

Fig. 2. One-loop propagator diagrams.

* In odd dimensions, because of our complex SU(3) notation, it is necessary to set one $\partial_i = \bar{\partial}^i$.

Calculations with this model parallel those in four dimensions. They are somewhat more laborious, however, because the addition of the new vertices, and especially the presence of the new chiral propagators, increases considerably the number of diagrams contributing to a given process. As an example, consider the one-loop corrections to the propagators in $D > 4$. The relevant diagrams are shown in fig. 2, where we have taken care to distinguish between diagrams contributing in four dimensions and extra diagrams introduced by the new vertices of the ten-dimensional action. In $D > 4$ the diagrams containing vertices of the four-dimensional theory only do not separately add up to zero, because the D algebra generates terms like $k_4^2/(k_D^2(k+p)_D^2)$, which only vanish in four dimensions, where they are massless tadpoles. However, when the new diagrams are added, one finds that, as in the corresponding gauge in four dimensions, all the one-loop propagator corrections *vanish identically* in this theory in the gauge (5.2) for any $D \leqslant 10$.

In four dimensions, all three particle vertices are finite, as suggested by superfield power counting rules and $N = 4$ supersymmetry. As with the propagators, one might hope that this feature would persist in higher dimensions. The situation for the higher-point Green functions, however, is somewhat more complicated. For example, the triple chiral vertex is corrected by the appropriate diagrams in fig. 3 that, in six dimensions, give the divergent contribution

$$\frac{i}{4(4\pi)^3\varepsilon}\int d^6x\, d^4\theta\, \varepsilon^{ijk}\left(D^2\varphi_i\right)[\varphi_j, \varphi_k], \tag{5.6}$$

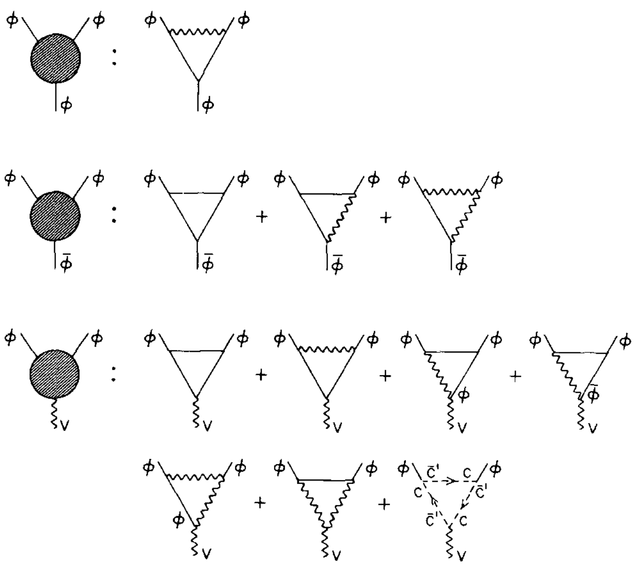

Fig. 3. One-loop $\varphi\varphi\varphi$, $\varphi\varphi\bar{\varphi}$ and $\varphi\varphi V$ diagrams.

which must be removed by adding a counterterm. It thus appears that the theory is already divergent at this level. As Green functions are gauge dependent, however, we must check whether this divergence is a physical one or not. One's first thought may be that, as in $D > 4$ there is no phase space for massless three-particle interactions, divergences in three-point functions are irrelevant. This is clearly not true, however, as they can contribute in higher point non-1PI contributions to S-matrix amplitudes. It is thus necessary to study whether the counterterm of (5.6) contributes as an insertion in S-matrix amplitudes.

A well known example of a "harmless" divergence, familiar from ordinary renormalizable field theories, is that of wave-function renormalization. In non-renormalizable theories the existence of dimensionful coupling constants allows this concept to be generalized to arbitrary non-linear field redefinitions. Generically, if we shift a field Ψ by $\Psi \to \Psi + \hbar\Delta\Psi$, the action transforms as

$$S[\Psi] \to S[\Psi] + \hbar\frac{\delta S}{\delta\Psi}\Delta\Psi, \tag{5.7}$$

where $\delta S/\delta\Psi$ can be recognized as the equation of motion for Ψ. Divergences proportional to equations of motion can therefore be absorbed at one loop by field redefinitions which, as is well known, do not affect the S-matrix [18]. These infinities are familiar from the case of pure gravity at one loop [19], and are the only kind of divergences allowed in non-renormalizable theories.

The counterterm of (5.6) does not appear to vanish when the equation of motion of φ_i is used, as the linearized equation of motion for φ_i relates it to V and $\bar{\varphi}^m$. However, when the divergent parts of the other three-point functions are added, (5.6) mixes with suitable contributions from the $\varphi\varphi\bar{\varphi}$ and $\varphi\varphi V$ vertex corrections in fig. 3 to become

$$\frac{i}{4(4\pi)^3\varepsilon}\int \mathrm{d}^6x\,\mathrm{d}^4\theta\,\varepsilon^{ijk}[\varphi_j,\varphi_k]\big((D^2\varphi_i) - \varepsilon_{ilm}\bar{\partial}^l\bar{\varphi}^m - iD^2\partial_i V\big), \tag{5.8}$$

which is proportional to the linearized equation of motion of $\bar{\varphi}^i$. This can be eliminated by the field redefinition

$$\bar{\varphi}^i \to \bar{\varphi}^i - \frac{i}{4(4\pi)^3\varepsilon}\varepsilon^{ijk}D^2[\varphi_j,\varphi_k]. \tag{5.9}$$

Similarly, it can be shown that all the one-loop infinities of the three-point functions in $D > 4$ are field redefinitions. It may be noted that, while field redefinitions do not occur in four dimensions for the three-point functions, they do occur for the four-point functions in superfield SSYM theory [20].

The next step is to consider the 1PI four-point Green functions, the least divergent of which is the $\varphi\varphi\varphi\varphi$ amplitude. The diagrams contributing to this amplitude are

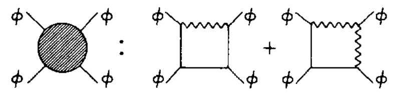

Fig. 4. One-loop $\varphi\varphi\varphi\varphi$ diagrams.

shown in fig. 4. Superfield power counting now shows that the amplitude becomes divergent in eight dimensions, and the one-loop $\varphi\varphi\varphi\varphi$ S matrix is thus finite in six dimensions. In eight dimensions, however, the divergence of this amplitude is not a gauge artifact, and the S matrix itself now diverges. This can be seen as, after the contribution from the 1PI amplitude is added to eq. (5.8) and the non-linear field equations are used, the resulting divergent $\varphi\varphi\varphi\varphi$ terms do not vanish. Therefore, we conclude that at one loop the S matrix starts to be ultraviolet divergent in eight dimensions. This result agrees with the superstring calculations of ref. [4].

We conclude by drawing to the attention of the reader the remarkable ghost-free gauge $\varphi_1 = 0$*, that in six dimensions reduces the action to

$$S_{10} = \mathrm{tr}\int \mathrm{d}^{10}x \left\{ \int \mathrm{d}^4\theta\, \mathrm{e}^{-V}\bar{\varphi}^i \mathrm{e}^V \varphi_i - \int \mathrm{d}^2\theta\, W^\alpha W_\alpha + \int \mathrm{d}^4\theta\, (\bar{\partial} V)\frac{\sinh L_V - L_V}{L_V^2}(\partial V) \right.$$

$$\left. + \tfrac{1}{2}\int \mathrm{d}^4\theta\, (\mathrm{e}^{-V}\bar{\partial}\mathrm{e}^V)(\mathrm{e}^{-V}\partial \mathrm{e}^V) \right\}, \tag{5.10}$$

where i is now an SU(2) index. Many of the interactions involving chiral fields have disappeared, leaving only a minimal coupling of the scalar superfield to the remaining chiral superfields φ_2 and φ_3. The price for this, however, is a complicated vector propagator:

$$\frac{1}{\Box_6}\left\{1 + 2\frac{D^\alpha \bar{D}^2 D_\alpha}{\Box_{10}}\right\}. \tag{5.11}$$

This propagator involves four spinorial derivatives, which considerably complicates the evaluation of graphs.

6. Discussion

We have shown that the $N = 1$ superfield formulation of $N = 4$ supersymmetric Yang-Mills theory possesses a non-trivial extension to higher dimensions. This reflects, in the framework of superspace, the close connection existing between

* It may appear puzzling that one can set both φ_1 and $\bar{\varphi}^1$ to zero. However, a linear combination of $\bar{\varphi}^1$ and φ_1 is transferred to the lower components of V.

four-dimensional theories with extended supersymmetry and their higher-dimensional analogues. The result is a theory containing gauge fields only which, despite its lacking manifest gauge covariance, yields gauge covariant equations of motion expressible in terms of field strengths and covariant derivatives only. The theory has a somewhat peculiar look. The free part contains off-diagonal terms, and the interactions cannot be expressed in terms of the potential e^V only, but require the appearance of the prepotential V as well. Some of the invariances are manifest, such as one four-dimensional supersymmetry, the four-dimensional Lorentz symmetry and the SU(3) symmetry, but the extra Lorentz symmetries and the remaining parts of the ten-dimensional supersymmetry are realized as complicated non-linear transformations of the superfields. As remarked in the introduction, our ten-dimensional action may provide a starting point for the construction (if possible) of the full superspace ten-dimensional action, from which it would follow after performing some θ integrations and eliminating some of the auxiliary fields. This theory also suggests that the ten-dimensional auxiliary fields should include a bosonic antisymmetric tensor. We have also presented some examples of one-loop calculations, reproducing in particular the superstring result [4] that the one-loop corrections to the S matrix of dimensionally continued $N=4$ SSYM become divergent at $D=8$ dimensions.

A corresponding formulation of 11-dimensional supergravity in terms of four-dimensional $N=1$ superfields should also be possible. This, however, would require, as a starting point, that $N=8$ supergravity be written in terms of $N=1$ superfields, and is far beyond our present knowledge, since to date even $N=2$ supergravity has not been fully written in terms of $N=1$ superfields.

One of us (A.S.) would like to thank J.L. Mañes for his kind hospitality during his stay at Berkeley.

Appendix A

We use two-component notation for the four-dimensional indices throughout. Our conventions are those of ref. [21], so that our spinorial covariant derivatives are

$$D_\alpha = \tfrac{1}{2}i\left(\partial_\alpha + i\bar{\theta}^{\dot{\alpha}}\partial_{\alpha\dot{\alpha}}\right), \qquad \bar{D}_{\dot{\alpha}} = -\tfrac{1}{2}i\left(\bar{\partial}_{\dot{\alpha}} + i\theta^{\alpha}\partial_{\alpha\dot{\alpha}}\right), \tag{A.1}$$

satisfying

$$\{D_\alpha, \bar{D}_{\dot{\alpha}}\} = \tfrac{1}{2}i\partial_{\alpha\dot{\alpha}}. \tag{A.2}$$

The derivatives with respect to the extra six coordinates are grouped into the three complex quantities ∂_i and their complex conjugates $\bar{\partial}^i$. For example:

$$\partial_1 = \frac{\partial}{\partial x^4} + i\frac{\partial}{\partial x^5}. \tag{A.3}$$

It follows that

$$\bar{\partial}^i \partial_i = \square_6 , \tag{A.4}$$

where $\square_6$ denotes the part of the D'Alembertian operator corresponding to the extra six coordinates. The definition we use for the components of vectors differs from (A.3) by a normalization factor so that, for example

$$A_1 = \sqrt{\tfrac{1}{2}}\,(A_4 + iA_5), \tag{A.5}$$

and one goes from SO(6) vector indices to SU(3) indices according to

$$A^I B_I = \bar{A}^i B_i + A_i \bar{B}^i , \qquad A^I \partial_I = \sqrt{\tfrac{1}{2}}\,(\bar{A}^i \partial_i + A_i \bar{\partial}^i). \tag{A.6}$$

Ten-dimensional vector indices are denoted by hatted Greek letters.

The component fields are defined in terms of the covariant derivatives and field strengths as

$$\begin{aligned} A_{\alpha\dot{\alpha}} &= \sqrt{2}\,\Gamma_{\alpha\dot{\alpha}}, \qquad \lambda_\alpha = \sqrt{2}\,W_\alpha , \qquad D = \{\nabla^\alpha, W_\alpha\}, \\ A_i &= \Gamma_i , \qquad \lambda_{\alpha i} = i\sqrt{2}\,F_{\alpha i}, \qquad F_i = i\{\nabla^\alpha, F_{\alpha i}\}, \end{aligned} \tag{A.7}$$

at $\theta = \bar{\theta} = 0$. Here $\Gamma_{\alpha\dot{\alpha}}$ is the connection in the anticommutator of ∇_α and $\bar{\nabla}_{\dot{\alpha}}$ and Γ_i is the connection in ∇_i. In a Wess-Zumino gauge these definitions become

$$\begin{aligned} A_{\alpha\dot{\alpha}} &= \sqrt{2}\,[\bar{D}_{\dot{\alpha}}, D_\alpha] V, \qquad \lambda_\alpha = \sqrt{2}\,\bar{D}^2 D_\alpha V, \qquad D = D^\alpha \bar{D}^2 D_\alpha V, \\ A_i &= \varphi_i , \qquad \lambda_{\alpha i} = \sqrt{2}\,D_\alpha \varphi_i , \qquad F_i = D^2 \varphi_i . \end{aligned} \tag{A.8}$$

Appendix B

We want to show that the variation of the term in eq. (3.9) is indeed (3.6). To this end, it is convenient to rewrite (3.9), using an exponential parametrization, as:

$$\tfrac{1}{2}\mathrm{tr}\int \mathrm{d}^{10}x\,\mathrm{d}^4\theta \int_0^1 \mathrm{d}x \int_0^x \mathrm{d}y\,(\bar{\partial}^i V)(\mathrm{e}^{yL_V} - \mathrm{e}^{-yL_V})(\partial_i V). \tag{B.1}$$

Then, in order to perform the variation, all one needs is the following formula for varying the exponential of a commutator

$$(\delta \mathrm{e}^{\sigma L_V}) A = \mathrm{e}^{\sigma L_V}\left[\frac{1 - \mathrm{e}^{-\sigma L_V}}{L_V}\delta V, A\right]. \tag{B.2}$$

The variation of (B.1) can be written, using another exponential parametrization, as

$$\tfrac{1}{2}\int \mathrm{d}^{10}x \int \mathrm{d}^4\theta \int_0^1 \mathrm{d}x \int_0^x \mathrm{d}y \int_0^y \mathrm{d}z\, \delta V$$

$$\times \left\{ \mathrm{e}^{yL_V}\left[(\partial_i V), \mathrm{e}^{-zL_V}(\bar\partial^i V)\right] + \mathrm{e}^{zL_V}\left[(\partial_i V), \mathrm{e}^{-yL_V}(\bar\partial^i V)\right] \right.$$

$$+ \mathrm{e}^{yL_V}\left[\mathrm{e}^{-zL_V}(\partial_i V), (\bar\partial^i V)\right] + \mathrm{e}^{zL_V}\left[\mathrm{e}^{-yL_V}(\partial_i V), (\bar\partial^i V)\right.$$

$$\left. + \left[\mathrm{e}^{-zL_V}(\partial_i V), \mathrm{e}^{-yL_V}(\bar\partial^i V)\right] + \left[\mathrm{e}^{-yL_V}(\partial_i V), \mathrm{e}^{-zL_V}(\bar\partial^i V)\right] \right\}, \quad \text{(B.3)}$$

or, using the symmetry of the integrand above under the interchange of y and z,

$$\tfrac{1}{2}\mathrm{tr}\int \mathrm{d}^{10}x\, \mathrm{d}^4\theta \int_0^1 \mathrm{d}x \int_0^x \mathrm{d}y \int_0^x \mathrm{d}z \left\{ \mathrm{e}^{yL_V}\left[(\partial_i V), \mathrm{e}^{-zL_V}(\bar\partial^i V)\right] \right.$$

$$\left. + \mathrm{e}^{yL_V}\left[\mathrm{e}^{-zL_V}(\partial_i V), (\bar\partial^i V)\right] + \left[\mathrm{e}^{-yL_V}(\partial_i V), \mathrm{e}^{-zL_V}(\bar\partial^i V)\right] \right\}. \quad \text{(B.4)}$$

The answer then follows after performing the y and z integrations and using the identity

$$\mathrm{e}^{\sigma L_V}(AB) = (\mathrm{e}^{\sigma L_V}A)(\mathrm{e}^{\sigma L_V}B), \quad \text{(B.5)}$$

which is a direct consequence of $\mathrm{e}^{L_V}A = \mathrm{e}^V A \mathrm{e}^{-V}$, and the identity

$$\left[\frac{1}{L_V}A, \frac{1}{L_V}B\right] = \frac{1}{L_V}\left[A, \frac{1}{L_V}B\right] + \frac{1}{L_V}\left[\frac{1}{L_V}A, B\right], \quad \text{(B.6)}$$

which is just a convenient rewriting of the Jacobi identity.

References

[1] F. Gliozzi, J. Scherk and D.I. Olive, Nucl. Phys. B122 (1977) 253
[2] L. Brink, J.H. Schwarz and J. Scherk, Nucl. Phys. B121 (1977) 77
[3] E. Cremmer and B. Julia, Nucl. Phys. B159 (1979) 141
[4] M.B. Green, J.H. Schwarz and L. Brink, Nucl. Phys. B198 (1982) 444
[5] S. Mandelstam, U.C. Berkeley preprint UCB-PTH-82/10 (1982); Nucl. Phys. B213 (1983) 149; L. Brink, O. Lindgren and B.E.W. Nilsson, Nucl. Phys. B212 (1983) 401
[6] A. Parkes and P.C. West, Nucl. Phys. B222 (1983) 269
[7] See J.H. Schwarz, Phys. Reports 89 (1982) 225
[8] L. Brink, O. Lindgren and B.E.W. Nilsson, Nucl. Phys. B212 (1983) 401
[9] J.H. Schwarz, unpublished
[10] M. Grisaru, W. Siegel and M. Roček, Nucl. Phys. B159 (1979) 429
[11] P. Fayet, Nucl. Phys. B149 (1979) 137
[12] J. Koller, Nucl. Phys. B222 (1983) 319; P.S. Howe, K.S. Stelle and P.K. Townsend, Nucl. Phys. B214 (1983) 519

[13] J. Koller, Caltech preprint CALT-68-981 (1982), Phys. Lett. 124B (1983) 324
[14] W. Siegel and M. Roček, Phys. Lett. 105B (1981) 275
[15] M.T. Grisaru and W. Siegel, Nucl. Phys. B201 (1982) 292
[16] See S.J. Gates, M.T. Grisaru, M. Roček and W. Siegel, Superspace (New York, Addison-Wesley, 1983)
[17] W. Siegel, Phys. Lett. 84B (1979) 193
[18] B.W. Lee, in Methods in field theory, eds. R. Balian and J. Zinn-Justin (New York, North-Holland, 1975)
[19] G.'t Hooft and M. Veltman, Ann. Inst. H. Poincaré 20 (1974) 69
[20] J. W. Juer and D. Storey, Phys. Lett. 119B (1982) 125
[21] M. Roček, in Superspace and supergravity, eds. S. W. Hawking and M. Roček (Cambridge University Press, 1981)