Nuclear Physics B201 (1982) 16–100

CALCULATION OF COSMOLOGICAL BARYON ASYMMETRY IN GRAND UNIFIED GAUGE MODELS

Jeffrey A. HARVEY*,[1], Edward W. KOLB*,**,
David B. REISS*,[2] and Stephen WOLFRAM*

* *California Institute of Technology, Pasadena, CA 91125, USA*
** *Los Alamos National Laboratory, Los Alamos, NM 87545, USA*

Received 20 August 1981
(Shortened version received 9 February 1982)

Features of grand unified gauge models relevant to cosmology are discussed. Several SU(5) and SO(10) models are considered in detail. Boltzmann transport equation methods are used to calculate the development of baryon asymmetry in the early universe. Comparison with observation places constraints on possible grand unified models.

1. Introduction

Grand unified gauge models (e.g. [1]) typically attempt to combine quarks and leptons (and often also antiquarks and antileptons) as elements of the same irreducible representations of some gauge group† G (which must contain the observed low-energy symmetry group $G_{LE} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$). The gauge bosons (which transform under the adjoint representation of G) can induce transitions between any two members of an irreducible fermion representation. Hence some of them should mediate baryon (B) and lepton (L) number violating interactions, in which, for example, quarks decay into leptons and antiquarks (e.g., $uu \to \bar{d}e^+$). The limit $\gtrsim 10^{30}$ years (e.g. [1]) on the lifetime of the proton suggests, however, that any baryon-violating vector bosons should have masses $\gtrsim 10^{14}$ GeV. Direct evidence for such B-violating interactions must presumably come from observation of proton decay. However, if any B violation does indeed occur, its suppression at accessible energies due to the large masses of the intermediate bosons, should have been overcome at the extremely high temperatures which presumably existed in the very early universe (e.g. [2]). We shall discuss the

* Work supported in part by the US Department of Energy under contract no. DE-AC-03-79ER0068 and by the Fleischmann foundation.

** Work supported in part by the US Department of Energy at Los Alamos, and by the National Science Foundation (PHY-79-23638) at Caltech.

[1] Present address: Physics Department, Princeton University.

[2] Present address: Physics Department, University of Washington, Seattle.

† To obtain a theory with only one universal (bare) coupling constant for interactions of gauge bosons, the group G must be a simple Lie group (or perhaps a product of simple groups with a suitable discrete symmetry imposed).

constraints on such B-violating processes in the standard hot big bang cosmological model necessary to allow the apparent excess of baryons over antibaryons in the present universe. Even if the universe initially (say, at the Planck time) had a non-zero net baryon number (but no other net conserved quantum number), B-violating interactions at very early times should relax the asymmetry away, leaving equal numbers of baryons and antibaryons. Then, when the universe cooled to a temperature $\lesssim 50$ MeV, the baryons and antibaryons would have annihilated away* and the observed baryon number density $n_B/n_\gamma \simeq 10^{-9}$ could not be accounted for. To reconcile the possibility of rapid B-violating processes at very high temperatures with the apparent non-zero net baryon number of the universe, it is presumably necessary that a baryon asymmetry should develop from the symmetrical state present after any initial B has been erased. (The possibility of the phenomenon was suggested by Sakharov in 1967 [3].) The generation of an asymmetry of the required magnitude places severe constraints on B-violating interactions, and therefore on grand unified gauge models. The purpose of this paper is to provide a detailed and systematic description of these constraints. The basic physical phenomena involved in the generation of a baryon excess were discussed in a recent paper by two of us [4] (hereafter referred to as I), where several simple illustrative models were considered. Here, we treat more realistic and complicated gauge models, in which many of the parameters relevant to baryon number generation are determined by the basic structure of the models, rather than being arbitrary, as in the illustrative models of I.

The generation of a baryon excess from a $B = 0$ state requires B violation, CP violation and deviations from thermal equilibrium. (Without deviations from equilibrium, no "direction of time" is distinguished, and CPT invariance renders the C, CP and T violations ineffective).

Sect. 2 discusses B violation, deriving constraints on its form in grand unified models.

In sect. 3 we discuss the form of CP violation in grand unified gauge models, and the mechanisms by which it may occur.

Sect. 4 considers the statistical mechanics of baryon number generation. Subsect. 4.2 describes departures from thermal equilibrium for a single massive particle species in an expanding universe. Subsect. 4.3 shows how such a departure from equilibrium may generate asymmetries in quantum number densities associated with light particles. In the realistic models to be treated, many particle species are present: subsect. 4.4 gives the general Boltzmann equations required. Our treatment of statistical mechanics assumes the applicability of the Boltzmann equation. Subsect. 4.5 discusses the limits of validity of this approach, and considers possible extensions. Most of our calculations are performed in the context of the simplest

* Unless they have become spatially separated. This possibility is difficult to implement because of the small volume of the universe in causal contact at $T \gtrsim 50$ MeV (according to the standard cosmological model).

cosmological model in which the early universe is taken homogeneous and isotropic. Subsect. 4.6 discusses the consequences of relaxing this assumption.

In most of the models we consider, the basic process responsible for baryon number generation is the decay of superheavy bosons. *CP*-violating effects in these decays must arise from one-loop correction diagrams. Subsect. 5.1 derives the baryon asymmetry generated through such diagrams from the free decay of superheavy bosons. The consequences of these results for *CP*-violation parameters in gauge theories are described in subsect. 5.2. Unless supermassive fermions are present, no *CP* violation may occur from gauge bosons alone. *CP* violation may occur in diagrams with Higgs boson exchange in gauge vector boson decay only when several different representations of Higgs bosons coupling to fermions are present.

The simplest grand unified models are those based on the group SU(5), outlined in subsect. 6.1. Sect. 6 considers in some detail baryon number generation in several simple SU(5) models. In subsect. 6.2, we derive the Boltzmann transport equations for the "minimal" SU(5) model, involving 24_H and 5_H Higgs boson multiplets. Subsect. 6.3 demonstrates that *CP* violation in this minimal model can occur only at a high order in perturbation theory, and is thus expected to be small. Nevertheless, in subsect. 6.4 we present results on the final baryon number generated in the minimal model. We find that no acceptable choice of parameters yields an adequate baryon asymmetry. Subsect. 6.5 then considers some extensions of the minimal model involving additional Higgs multiplets. With suitable choices of parameters, these models may account for the observed baryon asymmetry.

In sect. 7 we consider grand unified models based on SO(10). Subsect. 7.1 gives a general discussion of these models, emphasizing features not present in SU(5) models. Subsect. 7.2 considers the form of B violation, and the possibility of $B-L$ violation not present in SU(5) models. Unbroken SO(10) models exhibit an exact C invariance (subsect. 7.3) whose presence would prevent generation of any baryon asymmetry. The consequences of two possible SO(10) symmetry breaking schemes are considered in subsect. 7.4 and 7.5. With suitable choices of parameters, either scheme could be responsible for the observed baryon asymmetry.

SU(5) and SO(10) models represent simple schemes for grand unification. However, it is possible that more complicated models are, in fact, necessary. The cosmological constraints on very large models will be considered further in [5]. A preliminary discussion was given in the preprint version of this paper.

2. Baryon number violation

In this section, we discuss the details of B violation. We consider constraints on the possible forms of B-violating couplings, and derive conditions under which B

TABLE 1

The particles and antiparticles in a generic light fermion family, together with their quantum numbers [SU(3) multiplicity, SU(2) multiplicity weak hypercharge $Y = T_3 - Q$]

Particles	[SU(3), SU(2), U(1)]	Antiparticles	[SU(3), SU(2), U(1)]
$\begin{pmatrix}\nu \\ E\end{pmatrix}_L$	$[1, 2, \frac{1}{2}]$	$\begin{pmatrix}\nu^c \\ E^c\end{pmatrix}_R$	$[1, 2, -\frac{1}{2}]$
E_R	$[1, 1, 1]$	E^c_L	$[1, 1, -1]$
$\begin{pmatrix}U \\ D\end{pmatrix}_L$	$[3, 2, -\frac{1}{6}]$	$\begin{pmatrix}U^c \\ D^c\end{pmatrix}_R$	$[\bar{3}, 2, \frac{1}{6}]$
U_R	$[3, 1, -\frac{2}{3}]$	U^c_L	$[\bar{3}, 1, \frac{2}{3}]$
D_R	$[3, 1, \frac{1}{3}]$	D^c_L	$[\bar{3}, 1, -\frac{1}{3}]$

and L are separately violated, but some combination (usually $B-L$) is conserved.

The generic constitution of the three known families of quarks (q) and leptons (ℓ) is summarized in table 1. In considering B, L violation at high energies, the

TABLE 2

Quantum numbers for possible spin 1 (vector) pairs of quarks and leptons to which vector bosons may couple

		[SU(3), SU(2), U(1)]	B	L	$B-L$
V_1	$\ell\bar{\ell}$, $q\bar{q}$	$[8, 3, 0]$	0	0	0
		$[8, 1, 1]$			
		$[8, 1, 0]$			
		$[1, 3, 0]$			
		$[1, 1, 1]$			
		$[1, 1, 0]$			
V_2	$q\bar{\ell}$	$[3, 3, -\frac{2}{3}]$	$\frac{1}{3}$	-1	$\frac{4}{3}$
		$[3, 1, -\frac{2}{3}]$			
V_3	$\ell\ell$	$[1, 2, \frac{3}{2}]$	0	2	-2
V_4	ℓq	$[3, 2, \frac{5}{6}]$	$\frac{1}{3}$	1	$-\frac{2}{3}$
		$[3, 2, -\frac{1}{6}]$			
V_5	qq	$[6, 2, -\frac{5}{6}]$	$\frac{2}{3}$	0	$\frac{2}{3}$
		$[6, 2, \frac{1}{6}]$			
		$[\bar{3}, 2, -\frac{5}{6}]$			
		$[\bar{3}, 2, \frac{1}{6}]$			

Quantum numbers for individual q and ℓ were given in table 1.

masses of q, ℓ may be neglected, so that the left- and right-handed components of each fermion field may be approximated as independent. Table 1 gives the $SU(3)_C$ and $SU(2)_L$ representations under which each field transforms together with the weak hypercharge $Y = T_3 - Q$ assignment which specifies the final $U(1)_Y$ transformation properties. We assume, for now, that neutrinos are described by massless Weyl fields. As indicated by present experimental results, we take all q_L, ℓ_L to transform as doublets under $SU(2)_L$ and q_R, ℓ_R to transform as singlets.

The quarks in table 1 are assigned baryon number $B = \frac{1}{3}$; the corresponding antiquarks are assigned $B = -\frac{1}{3}$. The leptons are assigned $L = +1$, and antileptons $L = -1$. The "baryon" and "lepton" numbers of other particles are determined solely by their couplings to these quarks and leptons. These couplings are required to satisfy the constraints of $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ invariance (at the high energies considered, spontaneous breaking of $SU(2)_L$ is insignificant). The couplings conserve B and L only in so far as can be arranged by assignments or B and L quantum numbers. If all the quark-lepton systems to which a given particle couples have the same B and L, then that particle may usefully be assigned a definite B and L. However, some particles may couple to several systems with different B and L, in which case no single assignment of B, L suffices, and B, L are "violated" in the interactions of the particles.

Tables 2 and 3 give the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ quantum numbers for the possible quark and lepton systems to which vector and scalar bosons may couple. Lorentz invariance requires that renormalizable vector couplings have the form $\psi_a^\dagger \sigma^\mu \psi_b V_\mu$ and that renormalizable scalar couplings have the form $\psi_a^T \sigma_2 \psi_b S$, where V_μ and S are vector and scalar fields respectively, and $\psi_{a,b}$ are spin $\frac{1}{2}$ fields (see the appendix for notation).

The standard Weinberg-Salam model together with QCD involves gauge bosons of $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$. All these bosons are of the type V_1 defined in table 2. Hence, each gauge boson may be assigned definite B and L and no B or L violation may occur. The usual Higgs scalar doublet necessary for spontaneous breaking of $SU(2)_L \otimes U(1)_Y$ to $U(1)_{em}$ is of the type S_1 defined in table 3 and again implies separate B and L conservation. In grand unified gauge theories, it is common to include both fermion and antifermion fields in the same representation of the gauge group. In these cases, bosons with couplings of types 3, 4 and 5 (S_3, V_3, . . .) may exist. A boson with couplings of type 3 must be a color singlet: it may therefore not participate in couplings 4 and 5, and may thus be assigned a definite B. On the other hand, a boson may simultaneously exhibit couplings of types 4 and 5. Such a boson therefore couples to systems with $B = \frac{1}{3}$ and $B = -\frac{2}{3}$: it may therefore be assigned no definite B, and mediates B-violating interactions between quarks and leptons. However, although the separate B and L for cases 4 and 5 differ, the combination $B - L$ is $-\frac{2}{3}$ in both cases. Thus, $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ invariance and the restriction to the observed fermion fields prevent couplings of bosons to quarks and leptons from violating $B - L$ [7–9]. At least for the purposes of these

TABLE 3

Quantum numbers for possible spin 0 (scalar) pairs of quarks and leptons, to which scalar bosons may couple

		[SU(3), SU(2), U(1)]	B	L	$B-L$
S_1	$\ell\bar{\ell}$, $q\bar{q}$	$[8, 2, \frac{1}{2}]$	0	0	0
		$[1, 2, \frac{1}{2}]$			
S_2	$q\bar{\ell}$	$[3, 2, -\frac{7}{6}]$	$\frac{1}{3}$	-1	$\frac{4}{3}$
		$[3, 2, -\frac{1}{6}]$			
S_3	$\ell\ell$	$[1, 3, 1]$	0	2	-2
		$[1, 1, 2]$			
		$[1, 1, 1]$			
S_4	ℓq	$[3, 3, \frac{1}{3}]$	$\frac{1}{3}$	1	$-\frac{2}{3}$
		$[3, 1, \frac{1}{3}]$			
		$[3, 1, \frac{4}{3}]$			
S_5	qq	$[6, 1, -\frac{4}{3}]$	$\frac{2}{3}$	0	$\frac{2}{3}$
		$[6, 1, -\frac{1}{3}]$			
		$[6, 1, \frac{2}{3}]$			
		$[6, 3, -\frac{1}{3}]$			
		$[\bar{3}, 3, -\frac{1}{3}]$			
		$[\bar{3}, 1, -\frac{4}{3}]$			
		$[\bar{3}, 1, -\frac{1}{3}]$			
		$[\bar{3}, 1, \frac{2}{3}]$			

couplings, such bosons may always be assigned a definite $B-L$. In what follows we will denote the B-violating vector bosons with quantum numbers $[3, 2, -\frac{1}{6}]$ by (X, Y) and with quantum numbers $[3, 2, \frac{5}{6}]$ by (X′, Y′). The possible B-violating scalar bosons will be denoted by S ($[3, 1, \frac{1}{3}]$), S_1 ($[3, 1, \frac{4}{3}]$), and S_2 ($[3, 3, \frac{1}{3}]$). Fermi statistics require that S_1 and S_2 couple only to pairs of fermions in different families.

Another possible scheme for B violation involves two bosons [10]: one (say χ_2) of type 2 and one (say, χ_5) of type 5. Since χ_2 and χ_5 may have the same color and electric charges, B-conserving processes such as $\chi_2 \to \chi_5 W$ or $\chi_2 \to \chi_5 H$ may occur, and give rise indirectly to B violation through the different B of the systems to which χ_2 and χ_5 couple. Similarly, $O(m_W/m_\chi)$ mixing may occur between the χ_2 and χ_5 states through their interaction with the Higgs condensate. The rate for B violation through χ exchanges is then $O(m_W^2/m_\chi^4)$: existing limits on the proton lifetime then allow m_χ as low as $\simeq 10^9$ GeV. Note that (χ_2, χ_5) exchanges conserve $B+L$, and thus violate $B-L$.

All known fermions carry non-zero color, $SU(2)_L$ or electric charges. However, there may exist massive fermions which carry no absolutely-conserved quantum numbers. Such fermions (N) may mix with their antiparticles (charge conjugates) through Majorana mass terms (of the form $m_M \overline{f^c} f$). Clearly, they may not be assigned definite B or L. If the coupling $\chi \to qN$ is present, then so may $\bar{\chi} \to \bar{q}N$ be. Thus N does not carry a definite $B-L$: production and decay of N will lead to violations of $B-L$ conservation. The types of B- and L-violating bosons allowed in this case are discussed in subsect. 7.2 in the context of SO(10) grand unified models.

3. *CP* violation

The generation of a baryon excess from an initially symmetric state requires *CP*-violating interactions. In this section, we describe the possible forms of *CP*-violating couplings between particles appearing in grand unified gauge models, and some mechanisms through which these couplings may occur.

We consider first a complex scalar field $\varphi(x, t)$. It is necessary to distinguish the field operator $\hat{\varphi}$ from the "fields" φ obtained as the expectation values of this operator in particular states. It is the q-number field operator which appears in the canonical quantization procedure, the c-number field appears in the path integral formalism.

The actions of parity (P), charge conjugation (C) and time reversal (T) on a complex scalar field are given, up to arbitrary phases, by [11]:

$$\begin{aligned} P:\quad & \varphi(x, t) \to \varphi(-x, t)\,, \\ & \hat{\varphi}(x, t) \to \hat{\varphi}(-x, t)\,; \\ C:\quad & \varphi(x, t) \to \varphi^*(x, t)\,, \\ & \hat{\varphi}(x, t) \to \hat{\varphi}^\dagger(x, t)\,; \\ T:\quad & \varphi(x, t) \to \varphi^*(x, -t)\,, \\ & \hat{\varphi}(x, t) \to \hat{\varphi}(x, -t)\,. \end{aligned}$$

The transformations P and C are represented by unitary operators, which act on $\hat{\varphi}$ just as on φ. T is an antiunitary operator, which reverses the order of factors in products of field operators. It thus interchanges the bra and ket states in an expectation value, and complex conjugates the field φ. The combined operator of CPT on $\varphi(x, t)$ yields $\varphi(-x, -t)$ and is equivalent, as usual, to a generalized Lorentz transformation.

The P, C and T transformations above are modified for particles with spin. Their action on spin $\frac{1}{2}$ fermions is outlined in the appendix. Note that separate P and C transformations interchange chirality states, while the combined CP or T transformations do not: thus massless particles with only one chirality or helicity state may have definite behavior under *CP*. We shall consider the transformations of different chirality states under *CP* independently. For spin 1 fields, *P* and *T* transformations reverse, respectively, the space and time components of the polarization vector: they may thus be considered to "raise" or "lower" the Lorentz vector index on the vector potential A_μ.

The *CP* conjugate of a particle which transforms according to a representation r of some internal symmetry group is the corresponding particle in the conjugate representation $\bar{r}$. Any U(1) factors in the internal symmetry group are associated with charges which are reversed by the action of *CP*. If the complete symmety group is an abelian product of U(1) factors, each field with non-zero charge must be complex. When the symmetry group is non-abelian the U(1) charges are generated by the Cartan subalgebra of the group. In the absence of explicit U(1) factors, the reality or complexity of fields is determined by the representations under which they transform. Three basic classes of representation may be distinguished (e.g. [12]):

Complex: $\bar{r}$ and r are completely inequivalent. The singlet representation appears in the decomposition of $r \otimes \bar{r}$, but not in $r \otimes r$.

Real: A basis exists in which the representation matrices of r are purely real, so that r and $\bar{r}$ are equivalent. The singlet representation appears in the symmetric part of $r \otimes r$.

Pseudoreal: $\bar{r}$ is unitarily equivalent to r, but there is no basis in which the representation matrices are purely real. The singlet representation appears in the antisymmetric part of $r \otimes r$. All pseudoreal representations have even dimensionality.

Real and complex representations appear in many models; pseudoreal representations are rare, since they must be used in a "doubled" form to allow construction of mass terms for scalar fields. Fermions are usually placed in complex representations; this prevents the possibility of group-invariant fermion mass terms (allowed by chiral symmetries) and avoids unobserved right-handed fermions coupled to the weak current. In most of the discussion below, complex and pseudoreal representations behave similarly: we shall usually mention only complex representations.

The adjoint representation in which gauge vector bosons appear is always real. Hence in a suitable basis, all gauge vector bosons are eigenstates of *CP*, but in general have different eigenvalues.

If a set of interactions is to conserve *CP* invariance, its lagrangian must be invariant under *CP*. The requirement that all *CPT* invariant lagrangians be hermitian places important constraints on possible *CP* violation.

Hermiticity requires each term in a lagrangian to have the form $L = g\Theta + (g\Theta)^\dagger$, where g is a coupling constant, and Θ is a product of fields. As discussed above, the action of the *CP* transformation is $CP[g\Theta] = g\Theta^\dagger$, so that $CP[L] = g\Theta^\dagger + g^\dagger\Theta$. If $\Theta \neq \Theta^\dagger$, *CP* violation occurs when $g \neq g^\dagger$. No *CP* violation is possible when $\Theta = \Theta^\dagger$.

If the lagrangian is not invariant under phase redefinitions of fields, any CP violation may formally be transferred from one term in the lagrangian to another: its physical effects nevertheless remain unchanged.

Since the gauge vector bosons transform under the real adjoint representation, cubic and quartic couplings between the gauge bosons yield real Θ, and can involve no CP violation. Similarly, cubic and quartic coupling between Higgs bosons in real representations or between those Higgs bosons and the gauge vector bosons cannot introduce CP violation. For CP violation to occur, some Higgs fields must be complex. This may occur because they appear in complex representations of a non-abelian group, or because they exhibit an additional global U(1) invariance. In the latter case, two fields which may individually be real are combined to form a complex field.

In the bare lagrangian, kinetic energy terms for all fields F have the form $F\bar{F}$. Couplings of gauge vector bosons obtained by minimal substitution from these terms have the form $gAF\bar{F}$. Since the gauge vector bosons transform under the real adjoint representation, $AF\bar{F} = (AF\bar{F})^{\dagger}$ yielding no CP violation. However, it is possible that the fermion mass matrix may contain complex entries. Gauge couplings of fermion mass eigenstates may then contain complex mixing angles and exhibit CP violation. Nevertheless, when all fermions are massless, no such mixing may occur, and CP violation is again impossible. Even with massive fermions, CP violation may not be possible in some gauge couplings. For example, if only the left-handed fermion currents which couple to light gauge bosons are considered, unitary rotations may be performed to remove CP violation unless at least three separate fermion families exist (Kobayashi–Maskawa [13] scheme). CP violation may nevertheless occur in the right-handed and B-violating currents which couple to superheavy bosons even when only one fermion family exists.

CP violation associated with couplings of Higgs bosons to fermions or to themselves may occur either as a result of explicit complex couplings ("intrinsic CP violation") or from a complex vacuum expectation of a Higgs field ("spontaneous CP violation"*). For a coupling to exhibit intrinsic CP violation, hermiticity requires that at least one of the fields involved must be complex. Spontaneous CP violation requires a CP-violating vacuum expectation value, which may be associated either with a complex field or with a pseudoscalar real field. Since spontaneous CP violation requires the presence of a Higgs condensate, it typically disappears at high temperatures (see, however [14]), while intrinsic CP violation remains unchanged. Symmetry restoration usually occurs at a temperature of the same order as the vacuum expectation value of the Higgs condensate and the mass of the corresponding Higgs boson [15, 6]. Spontaneous CP violation associated with $SU(2)_L$ breaking thus cannot survive at the temperatures of relevance to B violation. In SU(5)

* In the literature the terms "hard" and "soft" have been used for "intrinsic" and "spontaneous" CP violation. We shall reserve "hard" and "soft" to describe lagrangian terms of dimensions four and lower, respectively.

models, only the real 24_H representation attains a sufficiently large vacuum expectation value to survive at high temperatures, and thus no spontaneous *CP* violation may occur. On the other hand, in SO(10) models, complex 16_H or 126_H representations as well as real 45_H or 54_H repreentations may attain large vacuum expectation values, so that spontaneous *CP* violation at high temperatures is possible. Whenever high-temperature spontaneous *CP* violation is associated with Higgs bosons which couple to fermions, the large vacuum expectation values necessary must give large masses for some of the fermions. In general, the presence of a large vacuum expectation value for a Higgs field in a complex representation of the gauge group lowers the rank of the effective gauge symmetry by breaking at least one of the U(1) invariances associated with the Cartan subalgebra.

CP violation in couplings of W bosons to fermions may occur through complex entries in the fermion mass matrix, as mentioned above. These complex entries may arise either from intrinsic complex Yukawa couplings or spontaneously from a complex Higgs vacuum expectation value. It is possible that low-energy *CP* violation is a result of *CP*-violating Higgs boson exchanges [16] rather than of a small *CP*-violating component in W exchange [13]. The best-measured *CP*-violating effects in K decays do not distinguish between these possibilities. The magnitude of the $\Delta I = \frac{3}{2}\varepsilon'$ *CP*-violation parameter in K decays, or other potential *CP*-violating effects in b-quark decays or the neutron electric dipole moment should, however, provide evidence on these possibilities.

Although the QCD lagrangian is *CP* invariant, it is possible that instanton effects in the vacuum state may lead to a *CP*-violating term $(\vartheta/32\pi^2)\mathrm{Tr}F_{\mu\nu}\tilde{F}_{\mu\nu}$ in the effective QCD lagrangian (e.g. [17]). The absence of a measured neutron electric dipole moment then requires $\vartheta/32\pi^2 \lesssim 10^{-7}$. Two simple mechanisms which would yield $\vartheta = 0$ are not viable: the Peccei–Quinn mechanism because a light axion is not observed, and the massless u quark because of conflicts with current algebra results on quark masses (e.g. [17]). $F\tilde{F}$ terms in the effective QCD lagrangian may also arise from chiral rotations used to render the quark mass matrix real and diagonal: the coefficient of these terms is proportional to the *CP*-violating quantity arg det M_q, where M_q is the quark mass matrix. This contribution to ϑ receives corrections from higher orders in perturbation theory. If *CP* violation in the quark mass matrix occurs through soft terms with dimension 2 or 3 in the Higgs couplings, then the resulting renormalization of ϑ is finite. If it occurs by hard terms of dimension 4, ϑ may suffer formally infinite renormalization. Any spontaneous *CP* violation can contribute only to soft terms: intrinsic *CP* violation may yield either hard or soft terms. In requiring ϑ to be small, it is perhaps desirable to avoid cases of infinite renormalization, thus favoring *CP* violation in soft terms.

We have discussed above *CP* violation in models containing fundamental Higgs scalar fields. Models with dynamical symmetry breaking from composite scalar fields may also exhibit *CP* violation [18]. Since the fundamental couplings in such models are gauge couplings, all couplings must be *CP* invariant. However, the minimum of the effective potential for the composite scalar fields may correspond to complex values for the fields, and thus yield spontaneous CP violation. Another possibility is that the $q\bar{q}$ condensate may not be purely scalar, but may contain a CP-violating pseudoscalar component (c.f. [19]).

At sufficiently high temperatures, any spontaneous CP violation will usually be restored. When the universe cools below the CP-violating phase transition, the expectation values of the Higgs fields, and thus the form of the CP violation, may differ between different domains in the universe. In the simplest case, the expectation value of the Higgs field may be either $\varphi = +\varphi_0$ or $\varphi = -\varphi_0$, leading to production of baryon asymmetries with opposite signs [20]. At the temperatures $O(10^{15}\ \text{GeV})$ at which CP violation must be present in order for baryon number to be generated, no causal effects may yield correlations in fields over volumes containing more than about 10^5 particles. Fig. 1 shows schematically a section through a universe consisting of many uncorrelated cells each carrying a positive or negative Higgs field with probabilities $\frac{1}{2}$. Investigations in percolation theory (e.g. [21]) show that in three

Fig. 1. Schematic section through a universe consisting of a random mixture of equal numbers of + and − cells in which CP violation has positive and negative signs. In three dimensions, nearly all the cells are members of infinite connected domains.

dimensions, infinite connected + and − domains exist with probability one*. Numerical simulation suggests that all but about 1% of the cells lie in connected domains. Thus, even without correlations between cells, large + and − domains should exist in the early universe. At the edge of, say, a + region, the Higgs field must change sign, and therefore exhibits a non-zero derivative leading to a surface energy density on the boundary. This surface tension tends to collapse the domains. It is, however, possible that the transmission of particles through the domain walls may be sufficiently low that contraction of a domain as a result of surface tension would be opposed by pressure from the enclosed gas. In this case, large domains would tend to become spherical, and survive at least through the period of baryon number generation. Details of the domain walls determine the effect of the expansion of the universe. Constraints on the present energy density of the universe do not allow any domain walls extending over a significant fraction of the universe [22]. However, domains which were stabilized by the pressure of enclosed gas would collapse when the gas recombined. After this point, gravitational clumping and radiation pressure should suffice to hold matter and antimatter domains apart. The viability of such a scheme depends crucially on the amount of energy dissipated in the destruction of the domain walls, which must be determined by detailed calculation.

Invariance under *CPT* appears to follow from Lorentz invariance in any larangian quantum field theory obeying standard axioms. It is nevertheless conceivable that these axioms are inadequate, and that *CPT* violations could occur. With *CPT* invariance, separate violation of *CP* and *T* must be accompanied by deviation from thermal equilibrium to provide "an arrow for time" and allow baryon asymmetry generation. However, with *CPT* violation, an asymmetry may be generated even in thermal equilibrium [23]. For example, the mass of a particle and its antiparticle could differ, so that their equilibrium number densities were unequal. Without a specific model for *CPT* violation, no detailed consequences may be deduced.

4. Statistical mechanics of cosmological baryon number generation

4.1. INTRODUCTION

Having outlined above the possible forms of *B*-, *CP*-violating interactions, we now consider how these may act in the early universe to generate an excess of baryons over antibaryons from an initially symmetric state [3, 4, 6, 7, 24–30] (the destruction of a possible initial baryon number was described in sect. 4 of I, and will not be discussed here). If all particles in the universe remained in thermal equilibrium, then no direction for time would be defined, and *CPT* invariance

* For a cubic lattice, the fracton of + cells must be $\gtrsim 0.33$ for infinite + domains to exist. Other lattice geometries give slightly different percolation thresholds; all are below 0.5.

would render CP violation in the interactions irrelevant, and prevent the appearance of any baryon excess [see (I.4.1) and appendix A of I]. We shall assume that the early universe is homogeneous and isotropic (Friedmann model). Deviations from thermal equilibrium cannot occur in a homogeneous isotropic universe containing only massless particles: if massive paricles are present, such deviations may occur when $T \simeq m$.

We shall assume here that all particles obey Maxwell-Boltzmann statistics (the negligible corrections from proper use of Fermi–Dirac or Bose–Einstein statistics were discussed in subsect. 2.4 of I). In this section, we make the simplification that all particles have only one spin/color state; the correct counting of states will be included in sects. 6 and 7. In thermal equilibrium, therefore, the density of a species of particles (with mass m) in phase space is given by*

$$f_{\rm eq}(p) = {\rm e}^{-(E-\mu)/T} \equiv {\rm e}^{-(\sqrt{p^2+m^2}-\mu)/T}, \tag{4.1.1}$$

where T is the temperature of the species, and μ is a possible chemical potential. Note that, in keeping with the standard and simplest cosmological model, we assume throughout that the universe may be treated as homogeneous and isotropic. The total number density in equilibrium of the species is given from (4.1.1) by

$$n_{\rm eq} = \int \frac{{\rm d}^3 p}{(2\pi)^3} f_{\rm eq}(p) = \frac{T^3}{2\pi^2}\left(\frac{m}{T}\right)^2 K_2(m/T)\,{\rm e}^{\mu/T}, \tag{4.1.2}$$

where K_2 is a modified Bessel function (see appendix C of I). For $m \ll T$, (4.1.2) becomes

$$n_{\rm eq} \simeq {\rm e}^{\mu/T}\frac{T^3}{\pi^2}\left[1-\left(\frac{m}{2T}\right)^2 + {\rm O}\left(\left(\frac{m}{T}\right)^4 \log\left(\frac{m}{T}\right)\right)\right], \qquad (m \ll T), \tag{4.1.3}$$

while for $T \ll m$,

$$n_{\rm eq} \simeq \left(\frac{mT}{2\pi}\right)^{3/2} {\rm e}^{-(m-\mu)/T}\left[1+\frac{15T}{8m} + {\rm O}\left(\frac{T^2}{m^2}\right)\right], \qquad (m \gg T). \tag{4.1.4}$$

At early times, most of the energy density of the universe was presumably contained in essentially massless particles. Their chemical potentials were probably very small, since the universe appears to carry zero or very small net quantum numbers. Hence the energy density contributed by each species was

$$\rho_{\rm eq} \simeq \frac{3T^4}{\pi^2}; \tag{4.1.5}$$

we shall denote such species generically as γ, and take them to be ξ in number (for $10^2 \leqslant T \leqslant 10^{16}$, typical grand unified gauge models imply $\xi \simeq 100$).

* We use throughout units such that $\hbar = c = k = 1$, where k is Boltzmann's constant. Since we do not set the gravitational constant $G = 1$, the Planck mass $m_{\rm pl} = G^{-1/2} \simeq 10^{19}$ GeV appears explicitly.

The Robertson–Walker scale factor for a homogeneous universe of total energy density $\rho(t)$ expands at a rate

$$\frac{1}{R}\frac{\mathrm{d}R}{\mathrm{d}t} \equiv \frac{\dot{R}}{R} \simeq \left(\frac{8\pi\rho(t)}{3m_{\mathrm{pl}}^2}\right)^{1/2}, \tag{4.1.6}$$

where $m_{\mathrm{pl}} \simeq 10^{19}$ GeV is the Planck mass. In this expansion, the momenta of all particles are redshifted according to $|\boldsymbol{p}| \equiv p \sim 1/R$. The phase space distributions for massless particles (in thermal equilibrium, and with $\mu = 0$) remain self-similar under this rescaling, with a temperature $T \sim 1/R$. Using (4.1.5), eq. (4.1.6) then implies*

$$\begin{aligned} \frac{\dot{T}}{T} &= -\frac{\dot{R}}{R} = -\left(\frac{8\pi\rho}{3m_{\mathrm{pl}}}\right)^{1/2} \simeq -\frac{T^2}{m_{\mathrm{P}}}, \\ m_{\mathrm{P}} &= (\tfrac{1}{8}\pi)^{1/2} m_{\mathrm{pl}}/\sqrt{\xi} \simeq 8\times 10^{18}\ \mathrm{GeV}/\sqrt{\xi}. \end{aligned} \tag{4.1.7}$$

where ξ is the number of (Maxwell–Boltzmann) particle species. The equilibrium phase-space densities $f_{\mathrm{eq}}(P) = \exp(-\sqrt{p^2+m^2}/T)$ of massive particles are not, however, self-similar in the expansion $p \sim 1/R$: their forms change when $p \sim T \sim 1/R$ becomes smaller than $\sim m$. Thus the expansion of the universe may cause these phase-space densities to deviate from their equilibrium form: collisions with light particles will nevertheless restore the equilibrium form, but only after a finite relaxation time. During this temporary deviation from equilibrium B-, CP-violating processes may generate a baryon asymmetry: under certain conditions this asymmetry may survive even after the massive particles have disappeared (through decays); the time necessary to relax the asymmetry may increase faster than the age of the universe.

The Boltzmann equation will be used below to describe the effect of reactions on the densities of particle species or quantum numbers. The validity of this approach is discussed in subsect. 4.5.

4.2. EVOLUTION OF A SINGLE MASSIVE PARTICLE SPECIES

Consider a particle χ of mass m, which decays with a width Γ into two massless particles ($\gamma\gamma$). Then the number density $n = n_\gamma Y$ of χ evolves with time t in the early universe according to

$$\dot{Y} \equiv \frac{\mathrm{d}Y}{\mathrm{d}t} \simeq -\langle\Gamma\rangle\{Y - Y^{\mathrm{eq}}\} \tag{4.2.1}$$

(for the derivation of this result from the complete Boltzmann transport equation for the phase-space density $f_\chi(p)$ see subsect. 2.3 of I). The expansion of the

* Note that the effective Planck mass m_{P} given here assumes that all particles obey Maxwell–Boltzmann statistics. If instead, all were to obey Bose–Einstein statistics, m_{P} would decrease by a factor 0.96, and if Fermi–Dirac, increase by a factor of 1.03.

universe appears only implicitly in (4.2.1) through the dependence of the temperature T in Y^{eq} on the time t: the explicit dependence has been removed by consideration of the scaled quantity $Y \equiv n_\chi/n_\gamma$. The first term in (4.2.1) accounts for the decays $\chi \to \gamma\gamma$, and has the usual radioactive decay form. The second term in (4.2.1) accounts for inverse decay processes $\gamma\gamma \to \chi$, which occur when the invariant mass of the initial $\gamma\gamma$ system lies within the "χ resonance curve." Clearly such processes can occur only if T is sufficiently high that the γ energies reach m_χ: when $T \leqslant m_\chi$, the inverse processes become exponentially improbable, as exhibited by the behavior (4.1.4) of Y^{eq} in this region. The time t appearing in (4.2.1) is measured in the c.m.s. of the complete universe: in that frame, the lifetimes of χ are dilated by factors E_χ/m_χ, and hence the effective χ width $\langle\Gamma\rangle$ in (4.2.1) must be averaged over the relevant χ energy spectrum. Typically, in the χ rest frame, $\Gamma \simeq \alpha(m/4)$, where $\sqrt{\alpha}$ is a $\chi\gamma\gamma$ coupling constant. We assume, in keeping with the simplest big bang cosmology, that at sufficiently early time, all species of particles are in thermal equilibrium, so that at $t=0$, $Y = Y^{eq}$ in eq. (4.2.1).

Fig. 2 shows numerical solutions to eq. (4.2.1) with equilibrium initial conditions. Deviations from equilibrium are initiated by the expansion of the universe. They are relaxed at a rate $\sim\langle\Gamma\rangle$. Their extent depends on the relative magnitude of this relaxation rate, and the expansion rate $\sim T^2/m_P$ of the universe, at a temperature $T \simeq m_\chi$. The larger $\langle\Gamma\rangle m_P/m_\chi^2 \simeq \alpha m_P/m_\chi$ is, the smaller the maximal deviations from thermal equilibrium are. Notice that, in fig. 2, curves with αm_χ adjusted so as to give the same Γ still differ slightly, because the averaging over time-dilation factors in $\langle\Gamma\rangle$ depends on m_χ/m_P. In the high and low temperature limits, eq. (4.2.1)

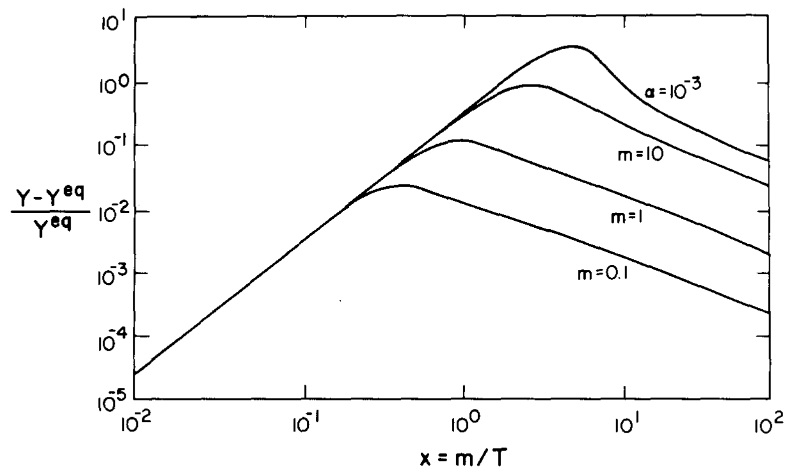


Fig. 2. Deviation of massive particle number density from equilibrium as a function of temperature T in the early universe, as calculated from eq. (4.2.1). The particle is taken to have mass m, and decay width $\Gamma = \frac{1}{4}m\alpha$. $Y = n/n_\gamma$ is the scaled number density for the particle. The expansion rate of the universe is governed by an effective Planck mass $m_P = 500\,\Pi$eV. Unless otherwise indicated, $m = 10^{15}$ GeV and $\alpha = \frac{1}{40}$.

admits of simple approximate solutions. Defining $x \equiv m/T$, eq. (4.2.1) becomes

$$\frac{\mathrm{d}Y}{\mathrm{d}x} = -m_{\mathrm{P}}\frac{\langle\Gamma\rangle}{m^2}x\{Y - Y_{\mathrm{eq}}\}. \tag{4.2.2}$$

For the averaged width $\langle\Gamma\rangle$ we approximate

$$\langle\Gamma\rangle = \left\langle\frac{m}{E}\right\rangle\Gamma \simeq \left\langle\frac{m}{E}\right\rangle_{\mathrm{eq}}\Gamma = \frac{K_1(x)}{K_2(x)}\Gamma \tag{4.2.3}$$

$$\simeq x[1+\tfrac{1}{2}x^2(\log(\tfrac{1}{2}x)+\gamma)\ldots]\Gamma \qquad (\gamma \simeq 0.5772) \qquad (x \ll 1) \tag{4.2.4a}$$

$$\simeq [1-\tfrac{3}{2}x+\cdots]\Gamma \qquad (x \gg 1)\,. \tag{4.2.4b}$$

At high temperatures, therefore, eq. (4.2.2) becomes [using eq. (4.1.3)]

$$\frac{\mathrm{d}Y}{\mathrm{d}x} \simeq -m_{\mathrm{P}}\frac{\Gamma}{m^2}x^2\{Y-(1-\tfrac{1}{4}x^2+\mathrm{O}(x^4\log x))\} \qquad (x \ll 1)\,. \tag{4.2.5}$$

The solution to this equation is

$$Y \simeq 1 - m_{\mathrm{P}}\frac{\Gamma}{20m^2}x^5 + \mathrm{O}(x^7\log x) \simeq 1 - \frac{m_{\mathrm{P}}}{m}\frac{\alpha}{80}x^5, \qquad (x \ll 1)\,, \tag{4.2.6}$$

so that

$$\frac{(Y-Y^{\mathrm{eq}})}{Y^{\mathrm{eq}}} \simeq \frac{x^2}{4}\left(1+\frac{x^2}{4}-\alpha\frac{m_{\mathrm{P}}}{m}\frac{x^3}{5}\right), \qquad (x \ll 1)\,. \tag{4.2.7}$$

It is clear that the parameter $m_{\mathrm{P}}\Gamma/m^2$ governs the magnitude of deviations from equilibrium. The numerical results of fig. 2 indicate that terms kept in eq. (4.2.7) are adequate until close to the maximum in $(Y-Y^{\mathrm{eq}})/Y^{\mathrm{eq}}$ at $x = x_{\mathrm{m}}$. Keeping only these terms suggests that x_{m} is obtained as the real root of the equation

$$8+4x^2-\alpha\frac{m_{\mathrm{P}}}{m}x^3 = 0\,, \tag{4.2.8}$$

or roughly

$$x_{\mathrm{m}} \simeq 4\frac{m}{\alpha m_{\mathrm{P}}}. \tag{4.2.9}$$

At low temperatures, (4.2.2) becomes [using eq. (4.1.4)]

$$\frac{\mathrm{d}Y}{\mathrm{d}x} \simeq -m_{\mathrm{P}}\frac{\Gamma}{m^2}x\left\{Y-(\tfrac{1}{8}\pi)^{1/2}x^{3/2}\,\mathrm{e}^{-x}\left(1+\frac{15}{8x}+\mathrm{O}\left(\frac{1}{x^2}\right)\right)\right\}, \qquad (x \gg 1)\,. \tag{4.2.10}$$

In the limit $T \to 0$ ($x \to \infty$) the expansion rate of the universe becomes negligible with respect to the rate of χ reactions, so that $Y \to Y_{eq}$. For large x, (4.2.10) may be solved with this boundary condition to give

$$Y \simeq (\tfrac{1}{8}\pi)^{1/2} x^{3/2} e^{-x}\left\{1 + \left(\frac{m^2}{m_P \Gamma} + \frac{15}{8}\right)\frac{1}{x} + O\left(\frac{1}{x^2}\right)\right\}, \qquad (x \gg 1)\,, \qquad (4.2.11)$$

$$\begin{aligned}\frac{(Y - Y^{eq})}{Y^{eq}} &\simeq \frac{m^2}{m_P}\Gamma\frac{1}{x} + O\left(\frac{1}{x^2}\right)\\ &\simeq 4\frac{m}{\alpha} m_P \frac{1}{x} + O\left(\frac{1}{x^2}\right), \qquad (x \gg 1)\,. \end{aligned} \qquad (4.2.12)$$

Again, fig. 2 shows that this approximation is numerically accurate. Note that the presence of inverse decays remains crucial in determining deviations from equilibrium even at large times. If the χ decayed freely, with no back reactions, then the second term in eq. (4.2.2) would be absent, so that at low temperatures

$$\begin{aligned} Y_{free} &= Y(x=0)\, e^{-\Gamma t}\\ &= Y(x=0) \exp\left(-m_P \frac{\Gamma}{m^2}\frac{x^2}{2}\right). \end{aligned} \qquad (4.2.13)$$

On the other hand, the rate for inverse decays is proportional to $Y^{eq} \sim \exp(-x)$, which falls much more slowly than eq. (4.2.13). The behavior of Y_{free} in (4.2.13) as a function of x depends on the expansion rate (4.1.7) of the universe. If the temperature of the universe decreased faster than $T \sim 1/t$, then the equilibrium number density $\sim\exp(-m/T)$ would fall more rapidly than the free decay probability $\sim\exp(-\Gamma t)$, and at low temperatures (large times) Y would approach (4.2.13)*.

4.3. EVOLUTION OF LIGHT PARTICLE SPECIES

In the previous section, we have assumed that neither the massive particle χ, nor its massless decay products γ carry any quantum number, so that both are charge conjugation eigenstates. This assumption is clearly inappropriate for particles which, for example, carry baryon number. The generation of a net baryon number relies on the development of a difference between the number densities of a $B \neq 0$ particle and its antiparticle. Consider a massless particle species c which carries an absolutely conserved quantum number $C = +1$. The antiparticle $\bar{c}$ carries $C = -1$. In a gas of c, $\bar{c}$ with net $C = 0$, the equilibrium distribution of c, $\bar{c}$ in phase space is $f_c(p) = f_{\bar{c}}(p) = e^{-p/T}$. However, if the gas has net $C \neq 0$, then the equilibrium distributions will involve a chemical potential μ, and become

$$f_c(p) = e^{-(p-\mu)/T}, \qquad f_{\bar{c}}(p) = e^{-(p+\mu)/T}. \qquad (4.3.1)$$

* This discussion has a potential application to thermodynamic models for hadron production in high-energy collisions. If the excited hadron material does not expand rapidly enough, the effective ρ lifetime may be increased by copious ρ production in inverse decay processes, thereby modifying correlations between final pions.

The chemical potentials of c and $\bar{c}$ are forced to be opposite by the presence of processes such as $c\bar{c} \leftrightarrow \gamma\gamma$, where γ has $C = 0$. The distributions (4.3.1) lead to a net density of the quantum number C

$$Y_C \equiv \frac{n_c - n_{\bar{c}}}{n_\gamma} = 2 \sinh\left(\frac{\mu}{T}\right) \simeq 2\frac{\mu}{T}, \tag{4.3.2}$$

where the final approximation holds so long as $(n_c - n_{\bar{c}}) \ll n_\gamma$. The characteristic rate at which a gas of c, $\bar{c}$ with an arbitrary initial configuration will relax into the equilibrium distributions (4.3.1) is governed by the total rate for interactions of the c, $\bar{c}$. If, however, some rare interactions of c violate the quantum number C, then the effective chemical potential μ may change at a rate governed by the rate for these rare interactions. In the absence of external influences (such as the expansion of the universe), a gas of c, $\bar{c}$ in the presence of C-violating interactions will eventually relax into a state of "chemical equilibrium" in which $\mu = Y_C = 0$. However, the rate of this relaxation will be much smaller than the rate at which "kinetic equilibrium" [leading to the phase-space distributions (4.3.1)] will be established. It is therefore sufficient to approximate the c, $\bar{c}$ phase-space distributions by (4.3.1) in investigating their approach to "chemical equilibrium."

Most particles which undergo B-violating interactions also participate in B-conserving interactions, some of which are mediated by light bosons (γ, G, $W^\pm$, Z^0). As discussed above, baryon number generation requires deviations from "kinetic equilibrium" which occur when the temperature of the universe falls to the masses of the heavy particles which mediate B-violating processes. In this period, the rates for B-violating reactions should be somewhat smaller than those for B-conserving reactions, primarily because of the larger masses of the mediators of B violation (another numerically significant effect is that in most models the number of possible B-violating reactions is smaller than the number of B-conserving ones by a factor between $\sim\xi$ and $\sim\xi^2$). Hence, for most light particles carrying baryon number, it should be adequate to assume distributions (4.3.1) in phase space, but with chemical potentials μ which change with time through B-violating processes. If b denotes a light particle carrying $B = 1$ (so that $\bar{b}$ has $B = -1$) then, for small $Y_B \equiv n_B/n_\gamma$,

$$Y_b \simeq e^{\mu/T} \simeq 1 + \tfrac{1}{2}Y_B + O(Y_b^2), \qquad Y_{\bar{b}} \simeq e^{-\mu/T} \simeq 1 - \tfrac{1}{2}Y_B + O(Y_B^2). \tag{4.3.3}$$

If the b, $\bar{b}$ undergo the B-violating reactions $bb \to \bar{b}\bar{b}$ and $\bar{b}\bar{b} \to bb$, then

$$\dot{Y}_B \simeq 4n_\gamma\langle\sigma v\rangle(-Y_b^2 + Y_{\bar{b}}^2) \simeq -8n_\gamma\langle\sigma v\rangle Y_B, \tag{4.3.4}$$

where $\langle\sigma v\rangle$ denotes the cross section multiplied by the relative velocity v (which cancels the $1/v$ flux factor in exothermic reactions at low incoming relative velocities) averaged over the c.m. energies for the collisions. The factor of 4 appears because these processes involve $|\Delta B| = 4$. Eq. (4.3.4) implies that any baryon excess introduced will be relaxed exponentially to zero by B-violating interactions with a characteristic time $\sim 1/(n_\gamma\langle\sigma v\rangle)$ of order of the mean free time between B-violating collisions. Note that B-conserving processes, such as $\gamma b \leftrightarrow \gamma b$ or $\gamma\gamma \leftrightarrow b\bar{b}$ do not affect $\dot{Y}_B$; they may change the momentum distribution but not the total number density of b, $\bar{b}$.

Now consider a massive particle χ which decays to two-body final states $f=(f_1, f_2)$ containing b, $\bar{b}$ and γ. First, for simplicity, assume CP invariance, so that the rates for decays and inverse decays to and from a state f are equal: $\Gamma(\chi \to f) = \Gamma(f \to \chi)$. Then the χ number density evolves [in analogy with eq. (4.2.1)] according to

$$\begin{aligned}\dot{Y}_\chi &\simeq -\sum_f \langle \Gamma(\chi\to f)\rangle\{Y_\chi - Y_\chi^{eq} Y_{f_1} Y_{f_2}\} \\ &\simeq -\sum_f \langle \Gamma(\chi\to f)\rangle\{Y_\chi - Y_\chi^{eq}(1+B_f Y_B)\} \\ &\simeq -\langle\Gamma\rangle\{Y_\chi - Y_\chi^{eq}\} + \sum_f \langle\Gamma(\chi\to f)\rangle B_f Y_\chi^{eq} Y_B\,, \end{aligned} \tag{4.3.5}$$

where B_f denotes the total baryon number of the state f. If $Y_B=0$, eq. (4.3.5) reduces to eq. (4.2.1). The evolution of the $\bar{\chi}$ number density may be obtained from (4.3.5) by charge conjugation:

$$\dot{Y}_{\bar{\chi}} \simeq -\langle\Gamma\rangle\{Y_{\bar{\chi}} - Y_\chi^{eq}\} - \sum_f \langle\Gamma(\chi\to f)\rangle B_f Y_\chi^{eq} Y_B\,, \tag{4.36}$$

where we have used CP invariance to write $\Gamma(\chi\to f) = \Gamma(\bar{\chi}\to \bar{f})$. The equilibrium distributions Y_χ^{eq} and $Y_{\bar{\chi}}^{eq}$ are equal since they depend only on $m_\chi = m_{\bar{\chi}}$. From eqs. (4.3.5) and (4.3.6) one finds

$$\dot{Y}_\chi^+ \equiv \tfrac{1}{2}(\dot{Y}_\chi + \dot{Y}_{\bar{\chi}}) = -\langle\Gamma\rangle\{Y_\chi^+ - Y_\chi^{eq}\}\,, \tag{4.3.7a}$$

$$\dot{Y}_\chi^- \equiv \tfrac{1}{2}(\dot{Y}_\chi - \dot{Y}_{\bar{\chi}}) = -\langle\Gamma\rangle Y_\chi^- + 2Y_\chi^{eq} Y_B \sum_f \langle\Gamma(\chi\to f)\rangle B_f\,. \tag{4.3.7b}$$

(In later sections, we shall often use the shortened notation $\chi_+ = Y_\chi^+$, $\chi_- = Y_\chi^-$.) If, for example, $\chi \to bb$, so that $bb \to \chi$ and $\bar{\chi} \to \bar{b}\bar{b}$, but $\chi \not\to \bar{b}\bar{b}$, then an excess of b over $\bar{b}$ ($Y_B > 0$) will result in the production of more χ than $\bar{\chi}$ in inverse decay processes, and therefore an increasing $Y_{\bar{\chi}}$. If $\sum_f \langle\Gamma(\chi\to f)\rangle B_f \neq 0$, then χ, $\bar{\chi}$ decays may affect Y_B. Such processes yield

$$\begin{aligned}\dot{Y}_B \simeq{}& Y_\chi \sum_f B_f \langle\Gamma(\chi\to f)\rangle - Y_{\bar{\chi}} \sum_f B_f\langle\Gamma(\bar{\chi}\to\bar{f})\rangle - Y_\chi^{eq}\sum_f B_f(1+B_f Y_B)\langle\Gamma(f\to\chi)\rangle \\ &+ Y_\chi^{eq}\sum_f B_f(1-B_f Y_B)\langle\Gamma(\bar{f}\to\bar{\chi})\rangle + n_\gamma \sum_{f,f'}(B_{f'}-B_f)(1+B_f Y_B)\langle v\sigma(f\to f')\rangle \\ &- n_\gamma \sum_{f,f'}(B_{f'}-B_f)(1+B_{f'} Y_B)\langle v\sigma(f'\to f)\rangle. \end{aligned} \tag{4.3.8}$$

Assuming for now CP invariance, so that $\Gamma(\chi \to \mathrm{f}) = \Gamma(\bar{\chi} \to \bar{\mathrm{f}}) = \Gamma(\mathrm{f} \to \chi) = \Gamma(\bar{\mathrm{f}} \to \bar{\chi})$, $v\sigma(\mathrm{f} \to \bar{\mathrm{f}}) = v\sigma(\bar{\mathrm{f}} \to \mathrm{f})$, this becomes

$$\dot{Y}_{\mathrm{B}} \simeq 2Y_{\chi}^{-} \sum_{\mathrm{f}} B_{\mathrm{f}} \langle \Gamma(\chi \to \mathrm{f}) \rangle - 2Y_{\chi}^{\mathrm{eq}} Y_{\mathrm{B}} \sum_{\mathrm{f}} [B_{\mathrm{f}}]^2 \langle \Gamma(\chi \to \mathrm{f}) \rangle$$
$$- Y_{\mathrm{B}} n_{\gamma} \sum_{\mathrm{f,f'}} [B_{\mathrm{f}} - B_{\mathrm{f'}}]^2 \langle v\sigma(\mathrm{f} \to \mathrm{f'}) \rangle. \tag{4.3.9}$$

In the cross sections $v\sigma(\mathrm{f} \to \mathrm{f'})$ it is necessary to remove the contribution from real intermediate χ or $\bar{\chi}$ (e.g. $\mathrm{f} \to \chi \to \bar{\mathrm{f}}$) since this is already included by iteration of the χ decay and inverse decay terms (see I, sect. 4). The first term in (4.3.9) illustrates that an excess of χ over $\bar{\chi}$ may lead to a baryon excess when the χ, $\bar{\chi}$ decay. The second two terms in (4.3.9) are manifestly negative, and cause any Y_{B} introduced to relax towards zero through B-violating interactions. It is clear from (4.3.9) and (4.3.7b) that, if CP invariance holds, any system with $Y_{\chi}^{-} = Y_{\mathrm{B}} = 0$ initially can never develop $Y_{\mathrm{B}} \neq 0$. As mentioned in sect. 1 (and at length in I), generation of a baryon excess from an initially symmetric state requires CP violation.

CPT invariance demands that

$$\Gamma(i \to j) = \Gamma(\bar{j} \to \bar{i}), \tag{4.3.10}$$

where $\bar{i}$ denotes the CP conjugate of the state i. The unitarity condition requires

$$\sum_j \Gamma(i \to j) = \sum_j \Gamma(j \to i). \tag{4.3.11}$$

Eqs. (4.3.10) and (4.3.11) then imply

$$\sum_j \Gamma(i \to j) = \sum_j \Gamma(j \to \bar{i}) = \sum_j \Gamma(j \to i) = \sum_j \Gamma(\bar{i} \to j) \tag{4.3.12}$$

(where we have interchanged the dummy labels j, $\bar{j}$ in the sums over all states). The equality of the first and last forms in (4.3.12) demonstrates the equality of the total decay widths for a particle and its antiparticle. If CP invariance were assumed, then

$$\Gamma(i \to j) = \Gamma(\bar{i} \to \bar{j}), \tag{4.3.13}$$

so that each partial width for decay into a given mode would be equal for particle and antiparticle. For baryon number generation to occur, CP invariance must be violated, and the equality (4.3.13) must fail. Using only eqs. (4.3.11) and (4.3.12), and not assuming CP invariance, eq. (4.3.8) becomes (keeping for consistency only terms of first order in both Y_{B} and CP-violating differences between rates):

$$\dot{Y}_{\mathrm{B}} \simeq (Y_{\chi}^{+} - Y_{\chi}^{\mathrm{eq}}) \sum_{\mathrm{f}} B_{\mathrm{f}} \langle \Gamma(\chi \to \mathrm{f}) - \Gamma(\bar{\chi} \to \bar{\mathrm{f}}) \rangle + Y_{\chi}^{-} \sum_{\mathrm{f}} B_{\mathrm{f}} \langle \Gamma(\chi \to \mathrm{f}) + \Gamma(\bar{\chi} \to \bar{\mathrm{f}}) \rangle$$
$$- Y_{\chi}^{\mathrm{eq}} Y_{\mathrm{B}} \sum_{\mathrm{f}} [B_{\mathrm{f}}]^2 \langle \Gamma(\chi \to \mathrm{f}) + \Gamma(\bar{\chi} \to \bar{\mathrm{f}}) \rangle - Y_{\mathrm{B}} n_{\gamma} \sum_{\mathrm{f,f'}} [B_{\mathrm{f}} - B_{\mathrm{f'}}]^2 \langle v\sigma(\mathrm{f} - \mathrm{f'}) \rangle. \tag{4.3.14}$$

The derivation of the $-Y_\chi^{eq}$ part of the first term here is somewhat subtle [as discussed in detail in I, sect. 4, and particularly eq. (2.4.9)]; it arises from a sum of contributions from inverse decay processes and from $2 \to 2$ scatterings mediated by a nearly on shell s-channel χ or $\bar{\chi}$ exchange. The first term in (4.3.14) allows generation of a baryon asymmetry by CP-, B-violating χ, $\bar{\chi}$ decays when deviations from thermal equilibrium occur, so that $Y_\chi \neq Y_\chi^{eq}$. The last two terms in (4.3.14) act to relax any asymmetry produced: the final Y_B depends critically on the size of these terms. Allowance for CP violations requires no important changes in eqs. (4.3.7) for the development of the χ, $\bar{\chi}$ number densities.

If the rate for χ interactions is much smaller than the expansion rate of the universe around the time of χ decays, then large deviations in the χ number density from its equilibrium value may occur (as illustrated in fig. 2), and the first term in eq. (4.3.14) may dominate. In this case, comparison with eq. (4.3.7a) shows that

$$\dot{Y}_B \simeq -\dot{Y}_\chi^+ \sum_f B_f \frac{\langle \Gamma(\chi \to f)\rangle - \langle \Gamma(\bar{\chi} \to \bar{f})\rangle}{\langle \Gamma \rangle}\,; \qquad (4.3.15)$$

hence, the final baryon number density generated from an initially symmetric state is simply

$$(Y_B)_{\rm final} \simeq \sum_f B_f \frac{\langle \Gamma(\chi \to f)\rangle - \langle \Gamma(\bar{\chi} \to \bar{f})\rangle}{\langle \Gamma \rangle} \equiv \varepsilon\,. \qquad (4.3.16)$$

This result is exact if the $\chi, \bar{\chi}$ decay freely, with no back reactions. At high temperatures, for zero initial baryon number, the first term in (4.3.14) always dominates, so that Y_B at early times grows like [using eq. (4.2.6)]

$$Y_B \simeq m_P \frac{\Gamma_\chi}{m_\chi^2} \frac{\varepsilon}{20} x^5 + O(x^7 \log x)\,, \qquad (x \ll 1)\,. \qquad (4.3.17)$$

This baryon asymmetry leads to an asymmetry in the number of χ and $\bar{\chi}$,

$$Y_\chi^- \simeq \left(m_P \frac{\Gamma_\chi}{m_\chi^2}\right)^2 \frac{\varepsilon}{160} x^8 - O(x)\,, \qquad (x \ll 1): \qquad (4.3.18)$$

for the single χ species considered here, this asymmetry is never important in the development of Y_B. As the temperature decreases, and Y_B increases, the third term in eq. (4.3.14) begins to counteract the first term. The ratio of these terms is roughly $R \simeq (\varepsilon/Y_B)(Y_\chi - Y_\chi^{eq})/Y_\chi^{eq}$. For small x, $R \simeq (5/x^3) m_\chi^2/(m_P \Gamma_\chi) \simeq 20 m_\chi/(\alpha m_P x^3)$. This ratio falls below one, indicating that the first term in (4.3.14) no longer dominates, at $x \simeq 1$. In the region $x \simeq 1$, the first and third terms in eq. (4.3.14) are both large, but cancel to give a net $\dot{Y}_B \simeq 0$. As the temperature decreases, R decreases rapidly according to eq. (4.2.12), and the third term in

(4.3.14) dominates, so that

$$\dot{Y}_{\mathrm{B}} \simeq -\langle \Gamma \rangle Y_\chi^{\mathrm{eq}} Y_{\mathrm{B}} , \tag{4.3.19}$$

$$\frac{\mathrm{d}Y_{\mathrm{B}}}{\mathrm{d}x} \simeq -m_{\mathrm{P}} \frac{\Gamma_\chi}{m_\chi^2} x^2 Y_\chi^{\mathrm{eq}} Y_{\mathrm{B}} .$$

The exponential fall off in Y_χ^{eq} at large x for the most part neutralizes the superficially exponential relaxation of Y_{B}; in practice Y_{B} decreases roughly as $1/x$. At very low temperatures, the presence of Y_χ^{eq} in the third term of eq. (4.3.14) renders it negligible, and only the fourth term, which results from B-violating $2 \to 2$ scatterings, makes a significant contribution to $\dot{Y}_{\mathrm{B}}$. At low energies ($\sqrt{s}$), $2 \to 2$ scattering of light particles by exchange of χ gives a cross section $v\sigma \sim \alpha^2 s/m_\chi^4$. At very low temperatures, eq. (4.3.14) thus becomes

$$\dot{Y}_{\mathrm{B}} \sim -\frac{T^3}{\pi^2} \alpha^2 \frac{T^2}{m_\chi^4} Y_{\mathrm{B}} , \tag{4.3.20}$$

$$\frac{\mathrm{d}Y_{\mathrm{B}}}{\mathrm{d}x} \sim -\frac{\alpha^2}{\pi^2} \frac{m_{\mathrm{P}}}{m_\chi} \frac{1}{x^4} Y_{\mathrm{B}} , \qquad (x \gg 1) .$$

Hence, when $2 \to 2$ scatterings dominate, Y_{B} falls like

$$Y_{\mathrm{B}} \sim \exp\left[\alpha^2 m_{\mathrm{P}}/(m_\chi x^3)\right] , \tag{4.3.21}$$

which tends to a constant non-zero value as $x \to \infty$. However, if the behavior (4.3.20) sets in at comparatively small x, the final Y_{B} will be much diminished from its maximum value, attained at $x \simeq 1$.

4.4. SEVERAL MASSIVE PARTICLE SPECIES

Above, we have considered only a single massive particle species: in realistic theories, however, there are usually several massive vector bosons, and often huge numbers of massive scalar Higgs bosons. In this section, we first discuss a simplified case in which massive bosons χ decay only to light particles (and not into other massive particles), and assume that B is the only quantum number with non-zero chemical potential carried by the light particles. Then the number densities of each species χ evolve according to eq. (4.2.1), while in eq. (4.3.14) for $\dot{Y}_{\mathrm{B}}$ a sum must be performed over the possible χ, yielding roughly

$$\dot{Y}_{\mathrm{B}} \sim \sum_\chi \langle \Gamma_\chi \rangle \{\varepsilon_\chi (Y_\chi^+ - Y_\chi^{\mathrm{eq}}) + 2 Y_\chi^- - 2 Y_\chi^{\mathrm{eq}} Y_{\mathrm{B}}\} - Y_{\mathrm{B}} n_\gamma \langle \sigma v \rangle_{\mathrm{tot}} , \tag{4.4.1a}$$

$$\dot{Y}_\chi^+ \sim \langle \Gamma_\chi \rangle \{ Y_\chi^+ - Y_\chi^{\mathrm{eq}} \} , \tag{4.4.1b}$$

$$\dot{Y}_\chi^- \sim -\langle \Gamma_\chi \rangle \{ Y_\chi^- + 2 Y_\chi^{\mathrm{eq}} Y_{\mathrm{B}} \} , \tag{4.4.1c}$$

where we have temporarily dropped numerical factors O(1) asociated with χ decay branching ratios.

Consider first the case of N identical bosons χ. If each χ decayed freely, the final baryon number generated would be $\simeq N\varepsilon_\chi$. At high temperatures, Y_B is indeed increased by a factor N; however, when $T \sim m_\chi$, the larger Y_B generated at higher temperatures and the presence of more χ bosons renders the back reaction terms in (4.4.1) more important, so that the rate of Y_B destruction is higher. Writing $y_B = Y_B/N$, eq. (4.4.1a) becomes

$$\dot{y}_B \sim \langle \Gamma_\chi \rangle \{\varepsilon_\chi (Y_\chi^+ - Y_\chi^{eq}) + 2Y_\chi^+ - 2NY_\chi^{eq} y_B\} - N y_B n_\gamma \langle \sigma v \rangle_{tot} ; \qquad (4.4.2)$$

the relaxation time for destruction of baryon number in eqs. (4.3.19) and (4.3.20) is reduced by a factor N. Fig. 3 shows the factor by which the final baryon number is modified by introduction of N identical boson species. When back reactions are unimportant, the effects of each boson add; when back reactions are important, the increase in the baryon destruction rate with N causes the final baryon number to decrease exponentially. In eq. (4.4.2) and fig. 3 we have assumed that in all cross sections, the effects of the N bosons are added incoherently: however, if the bosons are genuinely identical, their contributions add coherently, contributing a factor $\sim N^2$ instead of N. Eq. (4.4.1c) suggests that if a larger Y_B is generated at high temperatures by decays of N bosons, then a larger Y_χ^- for each boson will be produced by inverse decay processes. The typical Y_χ^- contributing to eq. (4.4.2) will therefore be a factor $\sim N$ larger than in the single boson case. Nevertheless, if the N bosons have identical masses, the Y_χ^- term will probably never be important in (4.4.2), as in the one-boson case.

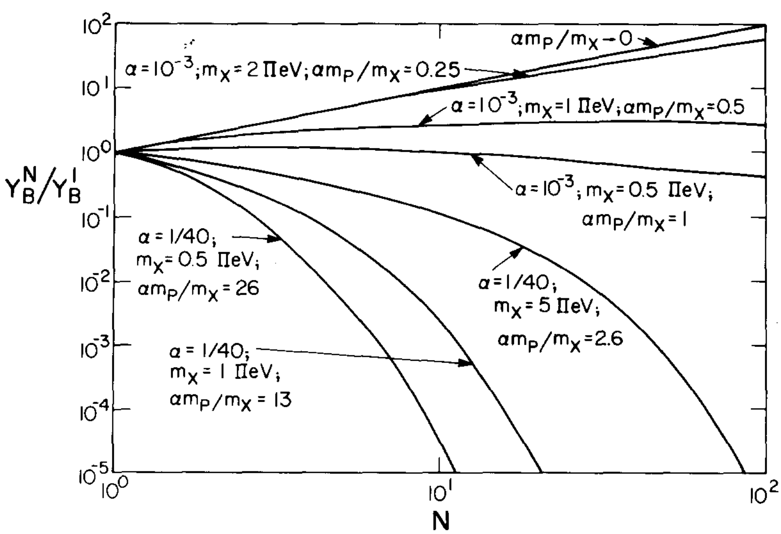


Fig. 3. Modification in final baryon number density produced by introduction of N identical massive boson species χ, as described by eq. (4.4.2). Results for various typical choices of parameters are shown.

We now consider the case of several bosons χ with different masses and couplings. At temperatures $T \gg m_\chi$, the total baryon number generated by their decays will behave as [cf., eq. (4.3.17)]

$$Y_B \simeq \frac{m_P}{80T^5} \sum_\chi \alpha_\chi \varepsilon_\chi m_\chi^4, \qquad (T \gg m_\chi), \tag{4.4.3}$$

where we have taken $\Gamma_\chi = \frac{1}{4} m_\chi \alpha_\chi$. For the minimal case (see sect. 5) of two species with $\varepsilon = \varepsilon_1 = -\varepsilon_2$, $\alpha = \alpha_1 = \alpha_2$, eq. (4.4.3) implies

$$Y_B \simeq \frac{m_P}{80T^5} \alpha \varepsilon m_1^4 \left[1 - \left(\frac{m_2}{m_1}\right)^4\right]: \tag{4.4.4}$$

if $m_1 = m_2$ then the effects of the two bosons always cancel, and no baryon excess may be generated. If m_1 and m_2 are nearly degenerate, then eq. (4.4.4) implies that the final Y_B will be smaller by a factor $\simeq [1-(m_2/m_1)^4]$ than would result if only one of the bosons were present. If $(m_2/m_1)^4 \ll 1$, then at high temperatures, Y_B should build up just as if the lighter boson χ_2 were absent. However, when the temperature falls, $(Y_2 - Y_2^{eq})$ eventually overtakes $(Y_1 - Y_1^{eq})$. When this occurs, the two "driving terms" in eq. (4.4.1) cancel. By this temperature, Y_1^{eq} is comparatively small, so that the back reaction term $-Y_1^{eq} Y_B$ is not dominant; however, Y_2^{eq} is still $\simeq 1$, so that the $-Y_2^{eq} Y_B$ is very large. This term is uncanceled when $(Y_2 - Y_2^{eq})$ compensates $(Y_1 - Y_1^{eq})$, and causes Y_B to relax exponentially to zero. Then, as the temperature decreases further, $(Y_1 - Y_1^{eq})$ becomes much smaller than $(Y_2 - Y_2^{eq})$, and baryon number is generated in the decays of the lighter χ_2 boson. If χ_1 is more than about 20% heavier than χ_2 [so that $1-(m_2/m_1)^4 \gtrsim 0.5$], then the baryon asymmetry generated by its decays is destroyed by inverse decays to χ_2, and the final Y_B is close to that obtained in the absence of χ_1. Some examples are given in fig. 4. This result suggests that the final baryon asymmetry depends only on the behavior of the lightest B-violating boson species: asymmetries generated by heavier species are destroyed by inverse decays of lighter species. The result is valid, however, only in the unusual simplified case considered here where no asymmetries may occur in quantum numbers other than B (cf. sects. 6 and 7).

Free decays of a given boson species can generate a baryon excess only if they violate CP invariance so that $\varepsilon_\chi \neq 0$. However, even if $\varepsilon_\chi = 0$, a B-violating boson species can destroy Y_B through inverse decay processes. If the lightest B-violating bosons do not violate CP invariance, then they will often destroy by inverse decays the baryon excess generated at higher temperatures by decays of heavier bosons. In some cases, however, this destruction is partly avoided by the Y_χ^- terms in $\dot{Y}_B$. When baryons are absorbed by inverse decays of lighter bosons, baryon excesses may result in $Y_\chi^- \equiv \frac{1}{2}(Y_\chi - Y_{\bar{\chi}}) \neq 0$, as suggested by eq. (4.4.1c), if the χ couples unequally to channels with opposite baryon number [see eq. (4.3.7b)]. Then, even in the absence of CP violation, the decays of unequal numbers of χ and $\bar{\chi}$ may

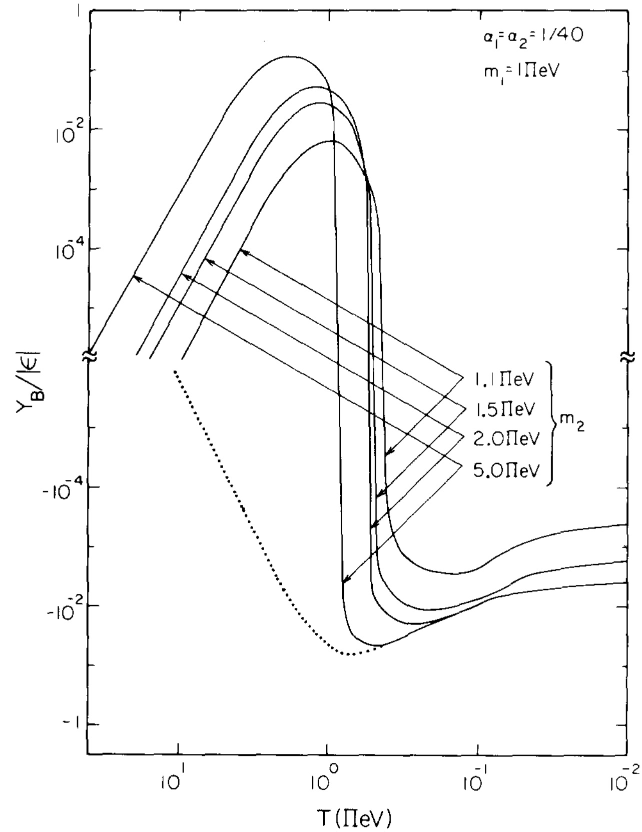


Fig. 4. Development of baryon asymmetry in the case of two supermassive bosons with masses and coupling constants m_1, m_2 and α_1, α_2. The dotted line is the result that would obtain if the contribution of boson 1 were ignored.

regenerate a baryon excess. In this way, a baryon asymmetry produced at high temperatures may be stored as a Y_χ^- of CP-conserving χ.

We have assumed above that heavy bosons may decay only to light particles. In realistic models, however, heavy bosons may usually decay to other heavy bosons as well as to light particles. At temperatures where a given heavy boson is present at sufficient density to be significant, most of its possible decay products will still be in "kinetic equilibrium" by virtue of their smaller masses. The phase-space densities of these lighter bosons may thus be approximated by their equilibrium form, but with a chemical potential, $f_\chi(p) \simeq e^{-(E-\mu)/T}$, $f_{\bar\chi}(p) \simeq e^{-(E+\mu)/T}$, so that

$$\begin{aligned} Y_\chi &\simeq Y_\chi^{\rm eq}\, e^{\mu/T} \simeq Y_\chi^{\rm eq}(1+\tfrac{1}{2}Y_\chi^-/Y_\chi^{\rm eq} + O([Y_\chi^-/Y_\chi^{\rm eq}]^2))\,, \\ Y_{\bar\chi} &\simeq Y_\chi^{\rm eq}\, e^{-\mu/T} \simeq Y_\chi^{\rm eq}(1-\tfrac{1}{2}Y_\chi^-/Y_\chi^{\rm eq} + O([Y_\chi^-/Y_\chi^{\rm eq}]^2))\,, \end{aligned} \tag{4.4.5}$$

where $Y_\chi^{\rm eq} = n_\chi^{\rm eq}/n_\gamma^{\rm eq}$ as in eq. (4.1.2), but with $\mu = 0$. Typically, reactions such as

$\gamma\chi \to \gamma\chi$ involving χ will only occur with sufficient rate to maintain the χ in "kinetic equilibrium" [and thereby validate eq. (4.4.5)] if $T \gtrsim m_\chi$: when $Y_\chi^{\text{eq}} \ll 1$, eq. (4.4.5) will become inaccurate, but the actual χ number densities in this regime will probably be so small as to be irrelevant. The inadequacy of eq. (4.4.5) at low temperatures is evident from the difference between eqs. (4.3.18) and (4.2.13). We shall use eq. (4.4.5) only in estimating the rates for inverse decays to bosons much heavier than those whose densities are approximated by eq. (4.4.5). Such rates are [cf., eq. (4,2.1)] proportional to the equilibrium number density of the heavier bosons, and are therefore negligibly small when (4.4.5) becomes inaccurate.

Using eq. (4.4.5) where appropriate, the general equation for the evolution of the number density of a species λ becomes

$$\begin{aligned}\dot{Y}_\lambda &\simeq \tfrac{1}{2}\sum_{\chi,\mathrm{f}}(Y_\chi - Y_\chi^{\text{eq}})(N_\lambda)_\mathrm{f}\langle\Gamma(\chi\to\mathrm{f})\rangle \\ &-\tfrac{1}{2}\sum_{\chi,\mu,\mathrm{f}} Y_\chi^{\text{eq}}(Y_\mu^-/Y_\mu^{\text{eq}})(N_\lambda)_\mathrm{f}(N_\mu - N_{\bar\mu})_\mathrm{f}\langle\Gamma(\chi\to\mathrm{f})\rangle \\ &-n_\gamma\sum_{\mu,\mathrm{f},\mathrm{f}'}\left\{\prod_\nu (Y_\nu^{\text{eq}})^{(N_\nu)_\mathrm{f}} - \prod_{\nu'}(Y_{\nu'}^{\text{eq}})^{(N_{\nu'})_{\mathrm{f}'}}\right\}\left(\frac{Y_\mu^-}{Y_\mu^{\text{eq}}}\right)[(N_\lambda)_\mathrm{f} - (N_\lambda)_{\mathrm{f}'}] \\ &\times[(N_\mu - N_{\bar\mu})_\mathrm{f} - (N_\mu - N_{\bar\mu})_{\mathrm{f}'}]\langle v\sigma(\mathrm{f}\to\mathrm{f}')\rangle\,, \end{aligned}\tag{4.4.6}$$

where the sums on χ, ν, ν' but not μ run over both particles and antiparticles of each species. In eq. (4.4.6) $(N_\iota)_\mathrm{f}$ denotes the number of particles of type ι in the state f. The previous results (4.3.5), (4.3.8), etc. may be derived as special cases of eq. (4.4.6). By charge conjugation and subtraction, one obtains from (4.4.6) the results [assuming, for consistency, $(Y_\mu^-/Y_\mu^{\text{eq}}) \ll 1$]

$$\begin{aligned}\dot{Y}_\lambda^- &= \tfrac{1}{2}(\dot{Y}_\lambda - \dot{Y}_{\bar\lambda}) \\ &\simeq \sum_{\chi,\mathrm{f}}(Y_\chi^+ - Y_\chi^{\text{eq}})(N_\lambda - N_{\bar\lambda})_\mathrm{f}\langle\Gamma(\chi\to\mathrm{f}) - \Gamma(\bar\chi\to\bar{\mathrm{f}})\rangle \\ &+\sum_{\chi,\mathrm{f}} Y_\chi^-(N_\lambda - N_{\bar\lambda})_\mathrm{f}\langle\Gamma(\chi\to\mathrm{f}) + \Gamma(\bar\chi\to\bar{\mathrm{f}})\rangle \\ &-\sum_{\chi,\mu,\mathrm{f}} Y_\chi^{\text{eq}}(Y_\mu^-/Y_\mu^{\text{eq}})(N_\lambda - N_{\bar\lambda})_\mathrm{f}(N_\mu - N_{\bar\mu})_\mathrm{f}\langle\Gamma(\chi\to\mathrm{f}) + \Gamma(\bar\chi\to\bar{\mathrm{f}})\rangle \\ &-n_\gamma\sum_{\mu,\mathrm{f},\mathrm{f}'}\left\{\prod_\nu (Y_\nu^{\text{eq}})^{(N_\nu)_\mathrm{f}} - \prod_{\nu'}(Y_\nu^{\text{eq}})^{(N_{\nu'})_{\mathrm{f}'}}\right\}(Y_\mu^-/Y_\mu^{\text{eq}}) \\ &\times[(N_\lambda - N_{\bar\lambda})_\mathrm{f} - (N_\lambda - N_{\bar\lambda})_{\mathrm{f}'}][(N_\mu - N_{\bar\mu})_\mathrm{f} - (N_\mu - N_{\bar\mu})_{\mathrm{f}'}]\langle v\sigma(\mathrm{f}\to\mathrm{f}')\rangle\,, \end{aligned}\tag{4.4.7}$$

where now only the sums on ν, ν' run separately over particles and antiparticles of each species.

The evolution of the density of a quantum number Q may be obtained from eq (4.4.7) using

$$Y_Q = \tfrac{1}{2}\sum_\lambda (Q_\lambda - Q_{\bar\lambda})Y_\lambda^- = \sum_\lambda Q_\lambda Y_\lambda^-\,. \tag{4.4.8}$$

Given a complete set of independent quantum numbers $\{Q_\alpha\}$ all the Y_μ^- may be written in terms of quantum number densities by inverting the corresponding relation (4.4.8) (the inversion will be singular if the Q_α are incomplete or interdependent). In our analysis of specific grand unified models below, it will be convenient to perform such an inversion, since several equations (corresponding to different flavors, etc.) then become identical, and need not be treated separately.

In addition to describing the evolution of quantities such as baryon number which are violated only in processes involving heavy bosons, eq. (4.4.7) may also in principle be used to estimate the development of quantities such as weak isospin or flavor, which are violated by light bosons. In these cases the last two terms in eq. (4.4.7) usually dominate. The magnitude of the last term is then determined by the high temperature behavior of $2 \to 2$ light boson exchange cross sections, which are difficult to define or estimate, as discussed above.

4.5. CORRECTIONS TO THE BOLTZMANN EQUATION

The Boltzmann transport equations discussed above describe a sequence of uncorrelated reactions between "free" (on-mass-shell) particles, whose cross sections are independent of the presence of other particles. In this section, we consider corrections to this picture, and discuss the limits of its validity. While important in principle, these considerations will usually be irrelevant in practice.

The basic condition for the Boltzmann equation to be valid is that the distance traveled by a particle between successive interactions should be much larger than the range of a single interaction (or than the average separation between particles) [31, 32]. This condition is satisfied only for interactions involving exchange of a heavy particle, and consequently of short range. Generation of baryon asymmetries requires deviations from thermal equilibrium, and thus typically occurs at temperatures of the same order as the masses of B-violating bosons. At such temperatures, the B-violating reactions satisfy the condition. Other reactions involving exchanges of light bosons, do not satisfy the condition; their effects on B-violating processes may nevertheless usually be approximated by effective "screened" cross sections [I].

We consider first processes in which one light particle species i is transformed into another such species j through interactions with a massive boson V. The lowest order contributing process is $ii \to jj$ with exchange of a single V in the t-channel. The range of this interaction is $\sim 1/m_V$. The rate of $i \to j$ transitions may be described by a Boltzmann equation representing a sequence of uncorrelated independent $ii \to jj$ scatterings only so long as the range of these interactions is much shorter than the distance between interactions. When this condition is violated, the "exchanged" V may undergo several interactions between its "emission" and "absorption".

Several methods may potentially be used to account for this phenomenon.

First, one could introduce "higher order corrections" to the process $ii \to jj$ involving additional incoming or outgoing particles. For example, to represent processes in which the "exchanged" V scatters once from a species k one could include the process $iik \to jjk$. This approach may be adequate for first-order corrections; however, it rapidly becomes impractical (and formally exhibits a plethora of divergences).

Second, the averaged effects of interactions with the V may be approximated as "screening" the V exchange, and may be parametrized by introducing a modified V propagator*. In the simplest approximation, the presence of additional particles reduces the mean free path for the V, and generates an effective mass of order the inverse mean free path αT. This is the approximation conventionally used in attempts to apply an effective Boltzmann equation to electron-ion plasmas. It can be adequate only if roughly equal numbers of particles with positive and negative "V charges" are present. When this is not the case (as in self-gravitating systems), genuinely long-range effects must be included, so that the form, as well as effective mass, for the V propagator must be modified. In the early universe, one is typically concerned with situations in which both positive and negative charges are present, so that the screening approximation is expected to be adequate. (The early stages of "cold" universes consisting of degenerate gas of baryons constitute an exception**).

A third approach is to consider a sequence of independent interactions involving off-mass-shell V. Then the rate for $i \to j$ transitions would be given by the rate for $i\mathrm{V} \to j$ and $i \to j\mathrm{V}$ averaged over all four-momenta for the i, j and V. The "equilibrium" distribution of particles in four-dimensional phase space then depends not only on their energy but also on their invariant mass; unlike the case of fixed invariant mass, the distribution is not determined from the Boltzmann equation without explicit assumptions for the interaction cross sections. The resulting distribution should nevertheless qualitatively have a spread in invariant mass of order αT around zero, and an average energy of order T. The Wigner function $w(k, x) \sim \int \mathrm{d}^4 y\, \mathrm{e}^{-ik\cdot y} \langle \varphi^\dagger(x+\frac{1}{2}y)\varphi(x-\frac{1}{2}y)\rangle$, where φ is a field operator and $\langle \cdots \rangle$ denotes a statistical average, may be used as a formal definition of the phase-space density when the mass-shell condition is not satisfied [33]. [The Wigner functions obey the relation $\int \mathrm{d}^3 k\, w(k, x) = n(x) = \varphi^\dagger(x)\varphi(x)$.] Extension of the derivation of the standard Boltzmann equation for on-shell particles from Wigner functions to allow for off-shell particles is, however, very difficult because interaction terms in the lagrangian cannot be neglected in comparison to kinetic energy terms.

* Formal approaches based on path integrals periodic in imaginary time are suitable for calculating static correlation functions. These may be used to deduce the V propagator, but may not be used directly in calculations on the complete time-dependent system.

** Methods used to treat gravitational clumping are even inadequate in this case; baryon number may be modified by gauge interactions involving arbitrarily small momentum transfers, so that their long-range effects are still more prevalent.

The second approach inserts corrections to amplitudes for V propagation; the third approach treats only V interaction probabilities. Interference between amplitudes for successive interactions is typically important when the distance between interactions is less than the Compton wavelengths of the interacting particles. The distance between interactions, as reflected in the average invariant mass of the particles, is probably roughly $O(1/(\alpha T))$; the Compton wavelength of the particles is $O(1/T)$, usually giving a sufficiently large ratio that quantum mechanical interference effects are small.

Most effects on say $ii \to jj$ resulting from the presence of a background gas decrease if the coupling to this gas is reduced. "Identical particle" quantum mechanical interference effects survive, however, even in the limit of zero coupling. Consider the amplitude for a particle to propagate from a point A to B. The presence of a background gas allows processes in which an on-shell particle received at B comes from the background gas, and is not the one emitted at A. (In the usual thermodynamic approximation of weak coupling, all particles in the background gas are on their mass shells.) This suggests that the complete propagation amplitude in momentum space is given by [32, 34]

$$\begin{aligned} D_t(p) &= 1/(p^2 - m^2 + i\varepsilon) \pm i\pi f(p_0)\delta(p^2 - m^2) \\ &= P \cdot 1/(p^2 - m^2) + i\pi(1 \pm f(p_0))\delta(p^2 - m^2)\,, \end{aligned} \tag{4.5.1}$$

where the upper sign is taken for bosons, and the lower for fermions. This form may be derived from the periodicity of the path integral in imaginary time when $f(p_0)$ is an equilibrium distribution; (4.5.1) is an obvious generalization to the case of an non-equilibrium (e.g. collisionless) background. Note that in the presence of spontaneously broken symmetries, the phase-space density of the condensate field may be taken as $f(p) \sim \delta^4(p)$.

All lines in a Feynman diagram carry on amplitude of the form (4.5.1). For those corresponding to "external particles" a discontinuity ("cut") is taken, and only terms proportional to $\delta(p^2 - m^2)$ survive. The presence of the background gas thus gives a correction factor $(1 \pm f_i(p_0))$ for each outgoing particle i, regardless of its couplings. These factors account as usual (see subsect 2.4.2 of I) for stimulated emission and Pauli exclusion effects, and are important only in regions of phase space with high densities. The factors are necessary to obtain the correct Fermi–Dirac and Bose–Einstein equilibrium distributions. Corrections appear not only on explicit external lines, but also for any "internal lines" whch may reach their mass shells in kinematic integrations. (This situation occurs in the calculation of *CP* violation described in sect. 5). Such factors are essential in maintaining the unitarity relation (I.A.22) for reaction rates, and enabling Boltzmann's H theorem to be proven even though the effective interaction cross sections depend on the ambient particle density.

When interactions occur in the background gas, "identical particle" effects may occur not only for on-shell particles, but for any particles with counterparts in the background gas. When the interactions are rapid, "identical particle" effects become indistinguishable from the general scattering processes discussed above.

It is impossible to give reliable numerical estimates of the corrections to the Boltzmann equation outlined above. Several simple cases are, however, amenable to treatment.

"Identical particle" corrections may be calculated in the approximation of a weakly interacting background gas (see subsect. 2.4 of I). For example, the Born approximation decay rate for a particle of mass m is modified in the presence of an equilibrium background gas with temperature T by a factor $[1 \pm e^{-m/2T}]^{-2}$. The correction is small except perhaps for a background Bose gas close to condensation.

As a next example, we consider the first corrections to the Boltzmann equation (4.2.1) for the development of the χ number density due to decay and inverse decay processes. Eq. (4.2.1) includes only $O(\alpha)$ reactions. At $O(\alpha^2)$, reactions such as $\gamma\gamma \to \chi\chi$, $\chi\chi \to \gamma\gamma$ and $\gamma\chi \to \gamma\chi$ (and perhaps $\chi\chi \to \chi\chi$) may also occur (we take here for simplicity $\chi \equiv \bar{\chi}$). The processes $\gamma\chi \to \gamma\chi$ and $\chi\chi \to \chi\chi$ serve only to redistribute the χ in phase space, without changing their total number, and thus do not contribute to $\dot{Y}$. Including the processes $\gamma\gamma \to \chi\chi$ and $\chi\chi \to \gamma\gamma$, eq. (4.2.1) becomes*

$$\dot{Y} \simeq -\langle \Gamma(\chi \to \gamma\gamma)\rangle \{Y - Y^{eq}\} - n_\gamma \langle v\sigma(\chi\chi \to \gamma\gamma)\rangle \{Y^2 - (Y^{eq})^2\}. \qquad (4.5.2)$$

For unstable particles (with, say, $\Gamma \sim \alpha m$), the second term in eq. (4.5.1) is usually irrelevant at low temperatures, since its contribution $\sim e^{-2x}$ while the first term $\sim e^{-x}$. At high temperatures, however, the second term may become large. In a formal expansion in powers of α, $\sigma(\chi\chi \to \gamma\gamma)$ in eq. (4.5.1) should be evaluated for free χ and γ. The high-energy behavior of this "free" cross section depends on the spin j of χ according roughly to (see I, sect. 3) $v\sigma \sim (\alpha^2/s)(s/m^2)^j$, where $\sqrt{s}$ is the $\chi\chi$ c.m.s. energy. (In particular cases, $v\sigma$ may decrease faster with s: for example, if χ has spin $\frac{1}{2}$, and γ spin 1, then $v\sigma \sim (\alpha^2/s) \log(s/m^2)$.) Assuming roughly equilibrium phase-space distributions, $\langle s \rangle \simeq 18T^2$ for $T \gg m$. Using the "free" cross section then implies that the second term overwhelms the first at high T. However, as discussed above, the "free" cross section is no longer adequate at large T: screening effects must be included. Except for spin zero χ, these considerably reduce $\langle v\sigma \rangle$ and suggest an effective cross-section $\langle v\sigma \rangle \sim \alpha/T^2$ rather than $\langle v\sigma \rangle \sim \alpha^2/m^2$. Taking $\Gamma = \frac{1}{4} m\alpha_1$ and $\langle v\sigma \rangle = \lambda \alpha_2^2/T^2$, so that $n_\gamma \langle v\sigma \rangle \simeq \lambda(\alpha_2/\pi)^2 T$, eq. (4.5.2) becomes

$$\frac{dY}{dx} \simeq -\left(\frac{\alpha_1}{4}\right)\left(\frac{m_P}{m}\right) x^2 \{Y - Y^{eq}\} - \lambda \left(\frac{\alpha_2}{\pi}\right)^2 \left(\frac{m_P}{m}\right) \{Y^2 - (Y^{eq})^2\}, \qquad (x \ll 1). \qquad (4.5.3)$$

* Note that if χ is stable, and can be produced or destroyed only in pairs, then the first term in (4.5.1) is absent, and the second term gives the complete Boltzmann equation for the evolution of its number density. This is approximately the case for the heavy right-handed neutrino of the SO(10) model, discussed in sect. 7 and in ref. [35]. The solution of eq. (4.5.1) for stable particles in such a case was discussed in ref. [36].

The approximate solution to this equation at high temperatures is

$$\frac{(Y-Y^{\mathrm{eq}})}{Y^{\mathrm{eq}}} \simeq \tfrac{1}{4}x^2\left\{1+\tfrac{1}{4}x^2-\tfrac{1}{6}\lambda\left(\frac{\alpha_2}{\pi}\right)^2\left(\frac{m_{\mathrm{P}}}{m}\right)x-\left(\frac{\alpha_1}{20}\right)\left(\frac{m_{\mathrm{P}}}{m}\right)x^2-\mathrm{O}(\alpha^2x^3)\right\}. \tag{4.5.4}$$

For $x=m/T \lesssim \alpha_2/\sqrt{\alpha_1}$, the $\mathrm{O}(\alpha_2^2)$ $2\to 2$ scattering term overwhelms the $\mathrm{O}(\alpha_1)1\to 2(2\to 1)$ decay (inverse decay) term. Nevertheless, at such high temperatures, the total $(Y-Y^{\mathrm{eq}})/Y^{\mathrm{eq}} \sim \alpha$, and is thus presumably small. Baryon number generation occurs predominantly when deviations from equilibrium are maximal. A rough estimate of this temperature is given by the lowest stationary point of (4.5.3) [cf., eq. (4.2.8)]: so long as $\alpha_2^2 \lesssim \alpha_1$ the decay (inverse decay) term dominates in this region. Note, however, that the coupling constants α_1, α_2 may have very different magnitudes. Any charged particles must undergo $2\to 2$ scatterings with a characteristic coupling constant $\alpha_2 \gtrsim \alpha_{\mathrm{QED}}$. Coupling constants for decays of gauge vector bosons must be of the same order: however, for Higgs bosons or heavy fermions, it is conceivable that the effective $\alpha_1 \ll \alpha_2$. It appears (see sect. 5) that for Higgs bosons, this possibility is probably not realized. Nevertheless, as discussed in ref. [35] and mentioned above, it may well occur for weakly interacting heavy fermions.

4.6. THE BOLTZMANN EQUATION IN NON-STANDARD COSMOLOGIES

In most of this paper, we approximate the early universe as homogeneous and isotropic. However, observation of structures at least up to clusters of galaxies in the present universe demonstrates that this Friedmann approximation is not exact. Most theories for the origin of galaxies suggest that any density fluctuations should nevertheless be small at the times when baryon asymmetry would be generated. In this section we discuss consequences of possible corrections to the Friedmann approximation at high temperatures.

We first discuss the form of the Boltzmann transport equation for an arbitrary cosmology. The Boltzmann equation may be written schematically in the form

$$\mathrm{L}[f]=\mathrm{C}[f]. \tag{4.6.1}$$

The right-hand collision term depends only on the phase space density f at a particular space-time point, and is therefore independent of the large-scale properties of space-time. On the other hand, the left-hand term depends on space-time derivatives of the phase space density, and is therefore potentially sensitive to the properties of space-time. The general relativistic covariant form of the Liouville operator is [41]

$$\hat{\mathrm{L}}=p^{\alpha}\frac{\partial}{\partial x^{\alpha}}+\frac{\mathrm{d}p^{\alpha}}{\mathrm{d}\tau}\frac{\partial}{\partial p^{\alpha}}-\Gamma^{\alpha}_{\beta\gamma}p^{\beta}p^{\gamma}\frac{\partial}{\partial p^{\alpha}}, \tag{4.6.2}$$

where τ is the proper time, and the Γ are the Cristoffel symbols (affine connections) which enter the covariant derivative. In a comoving frame, $\partial p^{\alpha}/\partial\tau = 0$, and the Liouville operator is

$$\hat{\mathrm{L}} = p^{\alpha}\frac{\partial}{\partial x^{\alpha}} - \Gamma^{\alpha}_{\beta\gamma}p^{\beta}p^{\gamma}\frac{\partial}{\partial p^{\alpha}}\,. \tag{4.6.3}$$

In the simplest approximation of a homogeneous isotropic universe, and setting the curvature parameter $k = 0$, the metric has the Robertson–Walker form, in which the $\Gamma^{0}_{00} = \Gamma^{0}_{0i} = 0$, $\Gamma^{0}_{ij} = R\dot{R}\hat{g}_{ij}$ where $\hat{g}_{rr} = 1$, $\hat{g}_{\vartheta\vartheta} = r^2$, $\hat{g}_{\phi\phi} = r^2\sin^2\vartheta$ and R is the Robertson–Walker scale parameter. In addition, the phase space distribution $f(\boldsymbol{p}, \boldsymbol{x}, t)$ depends only on the magnitude of $\boldsymbol{p}$ (or equivalently E), and the time t. In this case, the Boltzmann equation thus becomes

$$\frac{1}{E}\mathrm{L}[f(E,t)] = \frac{\partial f}{\partial t} - \frac{|\boldsymbol{p}|^2}{E}R\dot{R}\frac{\partial f}{\partial E} = \frac{1}{E}\mathrm{C}[f(E,t)]\,. \tag{4.6.4}$$

In a comoving frame, $P = R|\boldsymbol{p}|$, so that

$$\frac{1}{E}\mathrm{L}[f(E,t)] = \frac{\partial f}{\partial t} - \frac{\dot{R}}{R}\frac{|\boldsymbol{P}|^2}{E}\frac{\partial f}{\partial E} = \frac{1}{E}\mathrm{C}[f(E,t)]\,. \tag{4.6.5}$$

The total number density is obtained as an integral over momenta $n = \int f(E, t)\,\mathrm{d}^3P$. Inserting this into eq. (4.6.3), integrating by parts, and dropping the $E = 0$ and $E = \infty$ surface terms, one obtains

$$\frac{\mathrm{d}n}{\mathrm{d}t} - \frac{3\dot{R}}{R}n = \int \mathrm{C}[f]\frac{\mathrm{d}^3P}{E}\,. \tag{4.6.6}$$

This result has the simple physical interpretation that the expansion of the universe affects the number density only by increasing the volume containing a fixed number of particles.

The simplest non-standard modification to the Friedmann–Robertson–Walker cosmology consists in allowing anisotropy but retaining spatial homogeneity. The Bianchi classification [2] gives the possible metrics in such cases. For example, in Bianchi I cosmologies $\mathrm{d}s^2 = \mathrm{d}t^2 - [a_1^2\,\mathrm{d}x_1^2 + a_2^2\,\mathrm{d}x_2^2 + a_3^2\,\mathrm{d}x_3^2]$. (When all the scale parameters $a_i = R$, this metric reduces to the $k = 0$ Robertson–Walker metric.) The derivation of the Boltzmann equation in this case is analogous to that given above, except that the volume expansion term $3\dot{R}/R$ is replaced by $\dot{V}/V$ where $V = a_1a_2a_3$. It appears in fact that in all homogeneous cosmologies, expansion of the volume element is the sole effect of the expansion of the universe on the form of the Boltzmann transport equation. The rate of expansion of the volume is given in general by $\vartheta = u^{\alpha}{}_{;\alpha}$, where u^{α} is the fluid velocity in a comoving frame, and the semicolon denotes covariant differentiation. In a comoving frame, the complete velocity 4-tensor $u_{\alpha;\beta}$ at fixed time may be decomposed into a traceless symmetric part $\sigma_{\alpha\beta}$ (the shear tensor), an antisymmetric part $\omega_{\alpha\beta}$ (the vorticity tensor) and a trace term ϑ (the volume expansion rate). The Einstein equations may be used to relate these quantities to the fluid energy density ρ and the local Ricci curvature scalar $^*R$ at fixed times (on spacelike hypersurfaces orthogonal to the fluid flow with $\omega = 0$) [38]

$$\left(\frac{\vartheta}{3}\right)^2 = \frac{8\pi}{3}\frac{\rho}{m_{\rm pl}^2} + \frac{\sigma^2}{3} + \frac{\Lambda}{3} - \frac{^*R}{6}, \tag{4.6.7}$$

where $\sigma = \frac{1}{2}(\sigma_{\alpha\beta}\sigma^{\alpha\beta})^{1/2}$, and Λ is a possible cosmological constant. With the standard assumptions $\sigma = \omega = \Lambda = 0$, this equation reduces to the usual Robertson–Walker result $\vartheta = 3\dot{R}/R$, and $^*R = -6kR^{-2}L^{-2}$, where R is the Robertson–Walker scale parameter, and L is the curvature scale inserted to scale the curvature constant $k = \pm 1.0$. The fact that this curvature term is negligible in the present universe shows that it may be entirely ignored in the early universe. Although the cosmological constant is now small, it is possible that restoration of spontaneously-broken symmetries in the very early universe could have allowed it to be important then. This possibility has been discussed at length elsewhere [43, 44].

Changes in the expansion rate ϑ which enters the Boltzmann equation may be parametrized by modifying the temperature-time relation, and may often be accounted for simply by changing the effective Planck mass. The consequences of such changes were discussed briefly in I and treated in detail in [38]. While they affect the rates of baryon number production, the changes cannot lead to deviations from thermal equilibrium in situations where such deviations would otherwise not occur.

In the discussion above, we have made the assumption of homogeneity which implies that all phase space densities depend only on $\boldsymbol{p}$ and t and not on $\boldsymbol{x}$. In the generic case, one must allow inhomogeneity. Unless any inhomogeneity initially present or generated in the early universe is rapidly damped out, it will evolve into a universe far more inhomogeneous than is observed. Nevertheless, for a brief period, perhaps around the time of a symmetry-breaking phase transition, inhomogeneity may have existed. In such a case, the Boltzmann equation in general involves the spatial and momentum derivatives of the phase-space densities. These additional terms lead to deviations from thermal equilibrium even for massless particle densities. However, in the ideal gas approximation of infinitely rapid collisions, the modification to all massless boson and all massless fermion densities will be identical. For baryon number generation to occur, it is necessary not only that there should be deviations from equilibrium phase space densities, but also that these deviations should be different for different species of particles. It is possible that the deviations may be different for massless bosons and fermions, so that baryon number production may occur as a result of interactions involving both bosons and fermions.

5. Basic parameters for baryon number generation

5.1. GENERAL RESULTS

In this section we describe the calculation of the parameters which govern the generation of a baryon asymmetry from the basic couplings in a grand unified gauge model.

The basic parameter which enters the Boltzmann transport equations of sect. 4 is the average baryon number produced in the free decays of an equal mixture of particles χ and their antiparticles $\bar{\chi}$:

$$R_\chi \equiv \sum_{\mathrm{f}} B_{\mathrm{f}}\left\{\frac{\Gamma(\chi\to \mathrm{f})}{\Gamma_\chi}-\frac{\Gamma(\bar{\chi}\to \bar{\mathrm{f}})}{\Gamma_{\bar{\chi}}}\right\}. \tag{5.1.1}$$

Here, as in sect. 4, $\Gamma(\chi\to \mathrm{f})$ denotes the partial width for decay of χ to the final state f, Γ_χ is the total χ decay width and B_{f} is the baryon number of the state f (so that $B_{\mathrm{f}}=-B_{\bar{\mathrm{f}}}$).

In treating the statistical mechanics of baryon number production it is convenient to choose a basis so that the χ are mass eigenstates. We assume that the χ are not CP eigenstates (which is assured if χ and $\bar{\chi}$ have distinct conserved quantum numbers*). Hence the decay process itself must exhibit CP violation in order for R_χ to be non-zero. As discussed below (and proved in general in appendix B of I), this requires interference between the Born amplitude for the decay and a one loop correction with an absorptive part [6]. In addition, the couplings of the particles participating in the decay must be relatively complex.

We consider first the simplest non-trivial case: two massive bosons, X and Y, coupled to four fermion species i_1, i_2, i_3 and i_4, through the vertices of fig. 5 and their CP conjugates**. In the Born approximation, these vertices lead to the decay processes $\mathrm{X}\to \bar{i}_1 i_2$, $\mathrm{X}\to \bar{i}_3 i_4$, $\mathrm{Y}\to \bar{i}_3 i_1$, $\mathrm{Y}\to \bar{i}_4 i_2$ and the corresponding CP conjugated processes. We denote the coupling in, for example, the vertex fig. 5a by $\langle i_2|X|i_1\rangle$ so that the CP-conjugate coupling becomes $\langle i_2|X|i_1\rangle^* = \langle i_1|X^\dagger|i_2\rangle$. The quantity X here may be considered as a matrix of couplings in the space of possible fermion states i_j. Note that the set of vertices in fig. 5 is invariant under the combined transformations $\mathrm{X}\leftrightarrow \mathrm{Y}$ and $i_1\leftrightarrow i_4$. This invariance will be used below to obtain results for $\mathrm{Y}(\bar{\mathrm{Y}})$ decays from those for $\mathrm{X}(\bar{\mathrm{X}})$ decays. The couplings $\langle i_j|X|i_k\rangle$ do not include Lorentz structure which determines, for example, which helicity states of the fermions i_j may contribute.

* This is the case whenever χ decays into several light fermion states with different baryon numbers, as discussed in sect. 2.

** These vertices may be represented schematically by the interaction lagrangian

$$L \sim i_2^\dagger X i_1 + i_4^\dagger X i_3 + i_1^\dagger Y i_3 + i_3^\dagger Y i_4 + \text{h.c.}\,,$$

where all Lorentz structure has been suppressed.

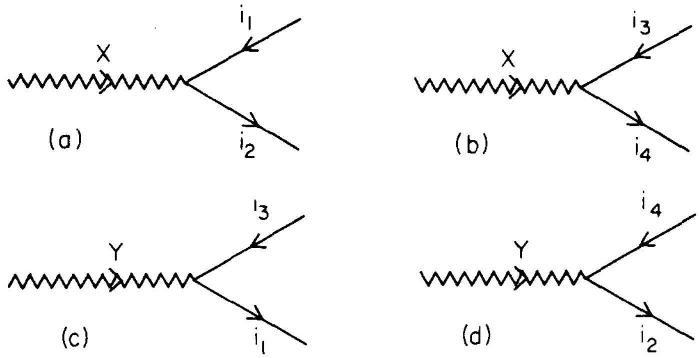


Fig. 5. Couplings of bosons X and Y to fermion species i_j in the simplest case for which B generation is possible. These couplings correspond to possible decays of X and Y in the Born approximation.

Born approximations to the X and Y decay rates may be obtained directly from the vertices of fig. 5. For example

$$\begin{aligned} \Gamma(\mathrm{X}\to i_2\bar{i}_1)_{\mathrm{Born}} &= I_{\mathrm{X}}^{12}\,|\langle i_2|X|i_1\rangle|^2 \\ &\equiv I_{\mathrm{X}}^{12}\,\langle i_2|X|i_1\rangle\langle i_1|X^\dagger|i_2\rangle\,. \end{aligned} \tag{5.1.2}$$

Here I_{X}^{12} accounts for the kinematic structure of the process $\mathrm{X}\to i_2\bar{i}_1$; it gives the complete result if all couplings are set to one. From eq. (5.1.2) it is evident that $\Gamma(\mathrm{X}\to i_2\bar{i}_1)_{\mathrm{Born}} = \Gamma(\bar{\mathrm{X}}\to \bar{i}_2 i_1)_{\mathrm{Born}}$, and hence R_{X} vanishes in this approximation. To obtain a non-zero result for R_{X}, one must include corrections arising from interference of the one-loop contributions shown in fig. 6 with the Born amplitudes of fig. 5. Consider, for example, the interference of the diagrams of fig. 5b and fig. 6a. The resulting terms in the squared amplitude is shown as fig. 7a. There the dotted line is a "unitarity cut"; each cut line represents a physical on-mass-shell particle. The amplitude for the diagram fig. 7a is then given by

$$\begin{aligned} &I_{\mathrm{XY}}^{1234}\,[\langle i_3|Y^\dagger|i_1\rangle\langle i_4|X|i_3\rangle\langle i_2|Y|i_4\rangle][\langle i_2|X|i_1\rangle]^* \\ &\quad = I_{\mathrm{XY}}^{1234}\,\langle i_3|Y^\dagger|i_1\rangle\langle i_4|X|i_3\rangle\langle i_2|Y|i_4\rangle\langle i_1|X^\dagger|I_2\rangle\,, \end{aligned} \tag{5.1.3}$$

where the kinematic factor I_{XY}^{1234} accounts for integration over the final-state phase space of i_2 and $\bar{i}_1$ and over the momenta of the internal i_4 and $\bar{i}_3$. The complex conjugate diagram, fig. 7b, has the complex conjugate amplitude

$$\begin{aligned} &(I_{\mathrm{XY}}^{1234})^*[\langle i_3|Y^\dagger|i_1\rangle\langle i_4|X|i_3\rangle\langle i_2|Y|i_4\rangle]^*\langle i_2|X|i_1\rangle \\ &\quad = (I_{\mathrm{XY}}^{1234})^*\langle i_2|X|i_1\rangle\langle i_4|Y^\dagger|i_2\rangle\langle i_3|X^\dagger|i_4\rangle\langle i_1|Y|i_3\rangle\,. \end{aligned} \tag{5.1.4}$$

Introducing notations for quadratic and quartic combinations of the couplings of fig. 5,

$$\begin{aligned} &\Xi_{jk}^{\chi} = (\Xi_{jk}^{\chi})^\dagger \equiv |\langle i_k|\chi|i_j\rangle|^2 = \langle i_k|\chi|i_j\rangle\langle i_j|\chi^\dagger|i_k\rangle\,, \\ &\Omega_{1234} = \langle i_3|Y^\dagger|i_1\rangle\langle i_1|X^\dagger|i_2\rangle\langle i_2|Y|i_4\rangle\langle i_4|X|i_3\rangle\,, \end{aligned} \tag{5.1.5}$$

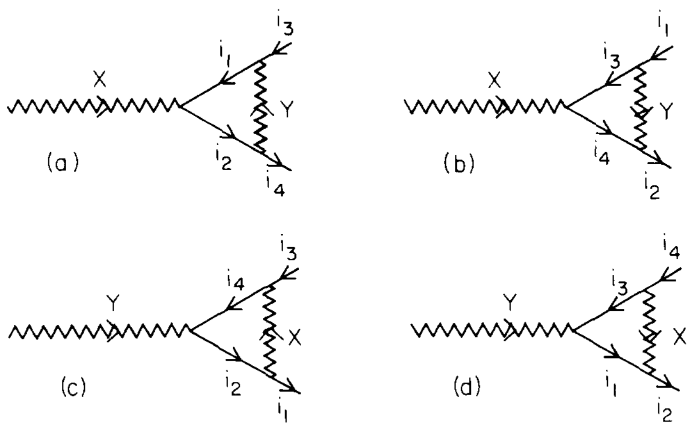


Fig. 6. One-loop corrections to the decay amplitudes for the bosons X and Y. The couplings of X and Y to the fermions i_j are shown in fig. 5.

one may write the one loop approximation to the $X \to i_2 \bar{i}_1$ decay rate obtained by adding the results (5.1.2), (5.1.3) and (5.1.4) as

$$\Gamma(X \to i_2 \bar{i}_1) = I_X^{12} \Xi_{12}^X + I_{XY}^{1234} \Omega_{1234} + (I_{XY}^{1234} \Omega_{1234})^* . \tag{5.1.6}$$

In the Born approximation, the kinematic factors I_X are always real. However, the kinematic factors I_{XY} for loop diagrams may have an imaginary part whenever any internal lines have sufficiently small masses that they may propagate on their mass shells in the intermediate state (and thereby sample the $1/i\varepsilon$ piece of the

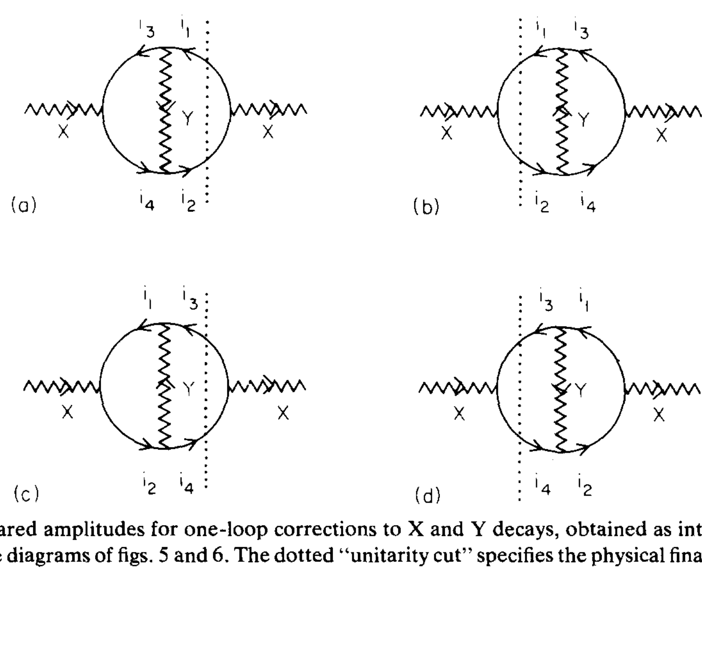


Fig. 7. Squared amplitudes for one-loop corrections to X and Y decays, obtained as interference terms between the diagrams of figs. 5 and 6. The dotted "unitarity cut" specifies the physical final state fermions.

propagator $1/(p^2-m^2+i\varepsilon)$). In the one-loop diagrams of fig. 7, this occurs when the threshold conditions

$$m_X \geqslant m_3+m_4\,, \tag{5.1.7}$$

$$m_X \geqslant m_1+m_2\,. \tag{5.1.8}$$

are satisfied. With light intermediate fermions, I_{XY} thus always exhibits an imaginary part.

We now consider the *CP*-conjugate decay $\bar{X}\to \bar{i}_2 i_1$. To obtain the *CP*-conjugate amplitude all couplings must be complex conjugated. The kinematic factors I are, however, unaffected by the *CP* conjugation (this is manifest in the fact that reversal of the direction of fermion lines in a closed loop does not affect the associated Dirac trace). Thus, to one-loop order, the complete result for $\Gamma(\bar{X}\to \bar{i}_2 i_1)$ becomes

$$\Gamma(\bar{X}\to \bar{i}_2 i_1)=I_X^{12}\Xi_{12}^X+I_{XY}^{1234}\Omega_{1234}^*+(I_{XY}^{1234})^*\Omega_{1234}\,. \tag{5.1.9}$$

The diagrams for the decays $X\to i_4\bar{i}_3$ and $\bar{X}\to \bar{i}_4 i_3$ are shown in figs. 7c and d, respectively. The loop diagrams differ from those for the decays $X\to i_2\bar{i}_1$ and $\bar{X}\to \bar{i}_2 i_1$ only in that the unitary cut is taken through the i_3 and i_4 rather than the i_1 and i_2 lines. In analogy with eqs. (5.1.8) and (5.1.9), we thus obtain

$$\Gamma(X\to i_4\bar{i}_3)=I_X^{34}\Xi_{34}+I_{XY}^{3412}\Omega_{1234}^*+(I_{XY}^{3412})^*\Omega_{1234}\,, \tag{5.1.10}$$

$$\Gamma(\bar{X}\to \bar{i}_4 i_3)=I_X^{34}\Xi_{34}+I_{XY}^{3412}\Omega_{1234}+(I_{XY}^{3412}\Omega_{1234})^*\,. \tag{5.1.11}$$

Using the results of eqs. (5.1.7) and (5.1.11) together with eq. (5.1.1) we can compute the average baryon number produced in the free decays of an equal number of X's and $\bar{X}$'s. The one-loop contribution to this asymmetry from the $i_1\bar{i}_2$ and $\bar{i}_1 i_2$ final states is given by

$$\begin{aligned}
R_X^{12} &= (B_{i_2}-B_{i_1})\frac{[\Gamma(X\to i_2\bar{i}_1)-\Gamma(\bar{X}\to \bar{i}_2 i_1)]}{[\Gamma(X\to i_2\bar{i}_1)+\Gamma(X\to i_4\bar{i}_3)]}\\
&= (B_{i_2}-B_{i_1})\frac{[I_{XY}^{1234}\Omega_{1234}+(I_{XY}^{1234}\Omega_{1234})^*-I_{XY}^{1234}\Omega_{1234}^*-(I_{XY}^{1234})^*\Omega_{1234}]}{[I_X^{12}\Xi_{12}^X+I_X^{34}\Xi_{34}^X]}\\
&= -4\frac{(B_{i_2}-B_{i_1})}{\Gamma_X}\,\mathrm{Im}\,[I_{XY}^{1234}]\,\mathrm{Im}\,[\Omega_{1234}]\,.
\end{aligned} \tag{5.1.12}$$

The analogous result for the 34 final state is

$$\begin{aligned}
R_X^{34} &= -4\frac{(B_{i_4}-B_{i_3})}{\Gamma_X}\,\mathrm{Im}\,[I_{XY}^{3412}]\,\mathrm{Im}\,[\Omega_{1234}^*]\\
&= 4\frac{(B_{i_4}-B_{i_3})}{\Gamma_X}\,\mathrm{Im}\,[I_{XY}^{3412}]\,\mathrm{Im}\,[\Omega_{1234}]\,.
\end{aligned} \tag{5.1.13}$$

The kinematic factors $\mathrm{Im}\,[I_{XY}^{1234}]$ and $\mathrm{Im}\,[I_{XY}^{3412}]$ are obtained from diagrams involving two unitarity cuts (as in fig. 8): one through the i_1 and i_2 lines and the

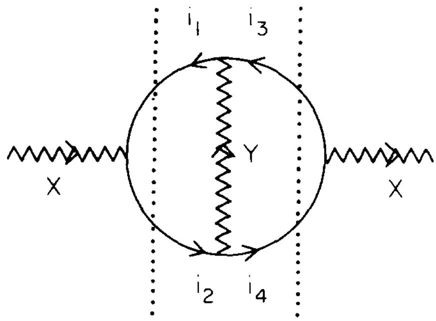


Fig. 8. "Double-cut" diagram representing the CP-violating combination of amplitudes for X decay. The dotted lines denote "unitarity cuts".

other through the i_3 and i_4 lines. The resulting quantities are invariant under the combined interchanges $i_1 \leftrightarrow i_3$ and $i_2 \leftrightarrow i_4$ and consequently are equal:

$$\mathrm{Im}\,[I_{XY}^{1234}] = \mathrm{Im}\,[I_{XY}^{3412}]\,. \tag{5.1.14}$$

Hence $R_X^{12}/R_X^{34} = (B_{i_1} - B_{i_2})/(B_{i_4} - B_{i_3})$, as expected. Notice that, if all intermediate fermions have zero mass, then the I_{XY}^{1234} are completely independent of their upper indices; corrections from fermion mass differences are of order $(m_f/m_X)^2$.

Upon adding the contributions (5.1.12) and (5.1.13) we obtain the final result

$$R_X = \frac{4}{\Gamma_X}\,\mathrm{Im}\,[I_{XY}^{1234}]\,\mathrm{Im}\,[\Omega_{1234}][B_{i_4} - B_{i_3} - (B_{i_2} - B_{i_1})]\,. \tag{5.1.15}$$

The conditions for the kinematic factor $\mathrm{Im}\,[I_{XY}^{1234}]$ to be non-vanishing were given in eqs. (5.1.5) and (5.1.6). A further condition for R_X to be nonvanishing is that both X and Y interactions violate baryon number. If X couplings were B-conserving, the two possible final states in X decay would have the same baryon number, so that

$$B_{i_2} - B_{i_1} = B_{i_4} - B_{i_3}\,, \tag{5.1.16}$$

and R_X would vanish. Similarly, if Y couplings were B-conserving,

$$B_{i_2} - B_{i_4} = B_{i_1} - B_{i_3}\,, \tag{5.1.17}$$

and R_X would again vanish. Thus both X and Y couplings must be B-violating to obtain a non-vanishing R_X. This is as implied by the general theorem given in appendix B of I. Notice that for (5.1.15) to be nonvanishing, at least two of the i_j must be distinct.

The asymmetry R_Y produced in Y and $\bar{Y}$ decays may be obtained from (5.1.15) by the transformation $X \leftrightarrow Y$, $i_3 \leftrightarrow i_4$, yielding

$$R_Y = \frac{4}{\Gamma_Y} \mathrm{Im}\ [\Omega^*_{1234}]\ \mathrm{Im}\ [I^{3142}_{YX}][B_{i_4} - B_{i_3} - (B_{i_2} - B_{i_1})]$$

$$= -\frac{4}{\Gamma_Y} \mathrm{Im}\ [\Omega_{1234}]\ \mathrm{Im}\ [I^{3142}_{YX}][B_{i_4} - B_{i_3} - (B_{i_2} - B_{i_1})]\,, \tag{5.1.18}$$

and so

$$R_X/R_Y = -\mathrm{Im}\ (I^{1234}_{XY})/\mathrm{Im}\ (I^{3142}_{YX})\,. \tag{5.1.19}$$

It follows that the average baryon number produced in the free decay of an equal number of X, $\bar{X}$, Y and $\bar{Y}$ is

$$R_{X+Y} = 4\left\{\frac{\mathrm{Im}\ [I^{1234}_{XY}]}{\Gamma_X} - \frac{\mathrm{Im}\ [I^{3142}_{YX}]}{\Gamma_Y}\right\} \mathrm{Im}\ [\Omega_{1234}][B_{i_4} - B_{i_3} - (B_{i_2} - B_{i_1})]\,. \tag{5.1.20}$$

Even if the R_X and R_Y are non-vanishing on their own, for the total to be non-zero the terms in the brace must not cancel. This requires that the particles X and Y be distinct either in mass or in the Lorentz structure of their couplings (e.g. one vector and one scalar) and that $\Gamma_X \neq \Gamma_Y$. The brace typically vanishes if X and Y are in the same irreducible representation of an unbroken symmetry group.

If more than the minimal set of four fermion species are present, the result (5.1.20) must be summed over all possible contributing $\{i_j\}$. It must also be summed over all possible (X, Y) pairs. Whenever particles have equal masses on the scale of m_X, the corresponding kinematic factors may be factored out of the summation.

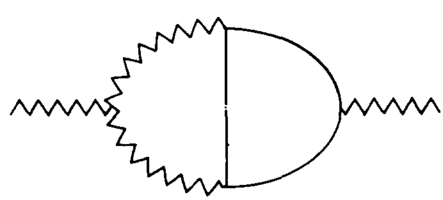


Fig. 9. A diagram involving three-boson coupling potentially contributing to *CP*-violating X decays.

The individual baryon asymmetry parameters R_X for X decays enter the complete Boltzmann transport equations discussed in sect. 4. These parameters alone determine the final baryon asymmetry only if back reactions (inverse decays) and $2 \to 2$ scatterings are ignored. The total contribution to the baryon asymmetry from decays of two species X and Y of bosons is thus not in general a simple sum of their corresponding parameters R_X and R_Y: if X and Y have different masses, the extent of back reactions is different in the two cases. If, however, X and Y are degenerate in mass, the sum given in eq. (5.1.20) represents their total contribution.

In the derivation of eq. (5.1.15) the particles i_j were assumed to be light fermions of definite baryon number. The result nevertheless remains approximately valid for any particles i_j so long as their masses are much smaller than m_X. Some of the i_j may for example be bosons, which enter through a three-boson coupling vertex, as illustrated in fig. 9. The B_j in eq. (5.1.15) should usually be replaced by the average baryon numbers generated in the decays of the corresponding i_j.

The discussion above concerns the one-loop contibutions to baryon asymmetry. In the generic case, an asymmetry occurs at this order if it is to occur at any order. However, in some simple models (such as the minimal SU(5) model treated in subsect. 6.2) the one-loop contribution vanishes, but there are higher loop contributions which are non-zero: in such cases the detailed analysis given above must be suitably generalized by summing over all possible unitarity cuts through the intermediate lines.

5.2. CONSEQUENCES FOR GAUGE MODELS

In this section, we give some general results on the value of the *CP*-violating parameter Im $[\Omega]$ defined by eq. (5.1.12) in gauge models.

As demonstrated in sect. 3, the couplings of gauge vector bosons to fermions may always be taken real and diagonal. Couplings of Higgs bosons to fermions and to each other may, however, be complex and induce mixing. After spontaneous symmetry breaking, these couplings may give rise to *CP* violation and mixing in the fermion and Higgs boson mass matrices. If fermion masses are neglected, diagrams involving only fermions and gauge vector bosons (fig. 10) can therefore yield no *CP* violation. For *CP* violation to occur in the decays of superheavy bosons, it is thus necessary for either explicit Higgs bosons or superheavy fermions with complex mixing angles to be present.

Some *CP*-violating effects involving Higgs bosons may be investigated before spontaneous symmetry breakdown. If a particular set of Higgs bosons allows *CP* violation in the unbroken theory, this *CP* violation will remain possible in the broken theory.

Consider first the case of scalar boson (S) exchange in vector boson (V) decay, as illustrated in fig. 11. The diagonal nature of the gauge couplings requires that the fermions i_1 and i_2 lie in the same irreducible representation f_1 of the gauge

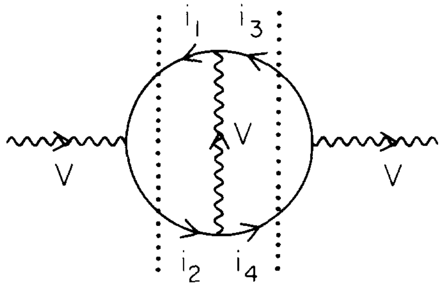


Fig. 10. Diagram for vector (gauge) boson exchange in vector boson decay.

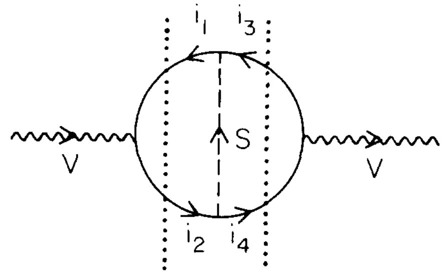


Fig. 11. Diagram for scalar (Higgs) boson exchange in vector (gauge) boson decay.

group (and i_3 and i_4 in f_2). Scalar bosons contributing to fig. 11 must lie in irreducible representations s_α such that

$$\bar{\mathrm{f}}_1 \otimes \mathrm{f}_3 \supset \mathrm{s}_\alpha\,. \tag{5.2.1}$$

In the absence of mixing between scalar bosons the exchanged S propagator is diagonal. Hence in the notation of subsect. 5.1, the coupling $\langle i_2|S|i_4\rangle$ at one end of the exchanged S line is simply the hermitian conjugate of the coupling $\langle i_3|S^\dagger|i_1\rangle$ at the other end: the product of these couplings is thus real, and no *CP* violation may occur.

CP violation may be introduced into fig. 11 through mixing terms in the S propagator arising from mixing which then causes the exchanged mass eigenstate scalar boson S to become in general a linear combination of several components with the same conserved charges. These components may occur within the same irreducible representation of the gauge group, or in different irreducible representations s_α. [Examples of both kinds appear in the illustrative SO(10) models considered in sect. 7.] If a model contains only a single B-violating Higgs boson no such mixing is possible, and *CP* violation cannot occur at the one-loop level through scalar boson exchange in vector boson decay. This is the case for the minimal SU(5) model discussed in subsect. 6.3. In the general case, we decompose the mass eigenstate field S into its unbroken group eigenstate components according to:

$$S = \alpha_1 S_1 + \alpha_2 S_2 + \cdots \tag{5.2.2}$$

We shall assume for now that just two components are present; the generalization to an arbitary number will be immediate. In this case,

$$\begin{aligned}\mathrm{Im}\,[\Omega^{1234}] &= \mathrm{Im}\,[\mathrm{Tr}\,[\langle i_3|S^\dagger|i_1\rangle\langle i_2|S|i_4\rangle]] \\ &= \mathrm{Im}\,[\mathrm{Tr}\,[(\alpha_1^*\langle i_3|S_1^\dagger|i_1\rangle + \alpha_2^*\langle i_3|S_2^\dagger|i_1\rangle) \\ &\quad\times(\alpha_1\langle i_2|S_1|i_4\rangle + \alpha_2\langle i_2|S_2|i_4\rangle)]]\,,\end{aligned} \tag{5.2.3}$$

where we have dropped the real factor corresponding to the gauge boson couplings, and the trace represents a sum over all fermion representations (usually "families"). Since $i_1, i_2 \subset f_1$ and $i_3, i_4 \subset f_2$, the couplings $\langle i_2|S_\alpha|i_4\rangle$ and $\langle i_1|S_\alpha|i_3\rangle$ are related by a real Clebsch–Gordan coefficient:

$$\langle i_2|S_\alpha|i_4\rangle = C_\alpha \langle i_1|S_\alpha|i_3\rangle \tag{5.2.4}$$

Hence

$$\begin{aligned}\mathrm{Im}\,[\Omega] &= \mathrm{Im}\,[\mathrm{Tr}\,[(\alpha_1^*\langle i_3|S_1|i_1\rangle\alpha_2^*\langle i_3|S_2^\dagger|i_1\rangle(C_1\alpha_1\langle i_1|S_1|i_3\rangle + C_2\alpha_2\langle i_1|S_2|i_3\rangle)]]\\ &= \mathrm{Im}\,[\mathrm{Tr}\,[(C_2\alpha_1^*\alpha_2\langle i_3|S_1^\dagger|i_1\rangle\langle i_1|S_2|i_3\rangle + C_1\alpha_1\alpha_2^*\langle i_3|S_2^\dagger|i_1\rangle\langle i_1|S_1|i_3\rangle)]]\\ &= (C_2 - C_1)\,\mathrm{Im}\,[\mathrm{Tr}\,[\alpha_1^*\alpha_2\langle i_1|S_1|i_3\rangle\langle i_3|S_2^\dagger|i_1\rangle]]\,.\end{aligned} \tag{5.2.5}$$

Thus, if $C_1 = C_2$, $\mathrm{Im}\,[\Omega]$ vanishes. This effect occurs when all Higgs bosons coupling to fermions have identical group charges, and are distinguished only by a "family" index. This is inevitable if all relevant Higgs bosons lie in replications of the same irreducible representation of the gauge group, and if this representation contains only one B-violating component. Examples of cases in which $C_1 \neq C_2$ are the SU(5) model with a 5_H and a 45_H (case B in subsect. 6.4) and the SO(10) model with a 10_H and a 120_H or a 126_H. In these models, *CP* violation may occur at the one-loop level from scalar boson exchange in vector boson decay. Notice that since in the absence of spontaneous symmetry breakdown, only one of the α_i is non-zero, the result (5.2.5) yields no *CP* violation in this case.

The case of vector boson exchange in scalar boson decay (illustrated in fig. 12) is exactly analogous to the case of scalar exchange in vector decay discussed above. When fig. 12 contributes, it is often important by virtue of large value of the vector couplings relative to the scalar ones.

We now consider *CP* violation arising from scalar boson (S′) exchange in scalar (S) boson decay, as illustrated in fig. 13. If only one B-violating Higgs boson is present, then the decaying and exchanged boson must be identical, and the results of subsect. 5.1 show that fig. 13 can give no *CP* violation. This is the case for the

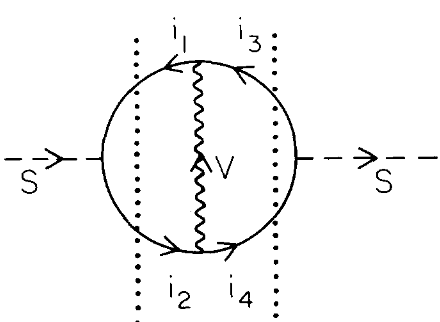


Fig. 12. Diagram for vector (gauge) boson exchange in scalar (Higgs) boson decay.

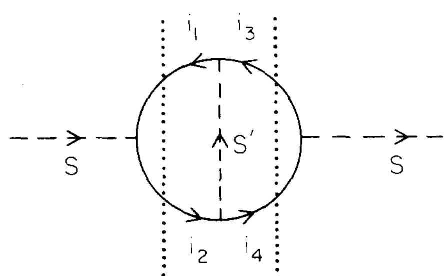


Fig. 13. Diagram for scalar (Higgs) boson exchange in scalar boson decay.

minimal SU(5) model. (However, as described in subsect. 6.3, *CP* violation may occur in higher order diagrams.) We consider for now the case in which all fermions are effectively massless. Then, in analogy with (5.2.1), the contributing scalar bosons must appear in representations s_α such that

$$f_1 \otimes \bar{f}_2 \supset \bar{s}_\alpha\,, \qquad f_4 \otimes \bar{f}_3 \supset s_\alpha\,, \qquad f_2 \otimes \bar{f}_4 \supset s'_\alpha\,, \qquad f_3 \otimes \bar{f}_1 \supset \bar{s}'_\alpha\,. \tag{5.2.6}$$

If all the left-handed fermions lie in the same complex irreducible representation, f, (or sequence of such identical representations), then $f_1 = \bar{f}_2 = \bar{f}_3 = f_4$ and these constraints become

$$f \otimes f \supset s_\alpha, s'_\alpha, \bar{s}_\alpha, \bar{s}'_\alpha\,. \tag{5.2.7}$$

For low-dimensionality representations, this requires s_α and s'_α to be real representations. hence in SO(10) models where all fermions lie in the 16 representation, only 10_H or 120_H may contribute to fig. 13; the 126_H which appears in $16_f \otimes 16_f$ is complex. [For high-dimensional fermion representations, some complex Higgs representations may satisfy (5.2.7): an example is the 126_H occurring in the symmetric product $144_f \otimes 144_f$ of SO(10).] After spontaneous symmetry breakdown, mixing between scalar bosons may occur, and the constraints (5.2.6) are no longer applicable. Thus in both SU(5) models with several Higgs representations coupling to fermions, and in SO(10) models, fig. 13 can yield *CP* violation.

The discussion above has assumed that all relevant fermion species are effectively massless. With gauge groups such as SO(10) or E(6), it is common for fermions with SU(2) singlet and thus potentially large mass terms to exist. The effect of such fermions in intermediate states of figs. 10 through 13 is always suppressed by $O(m_f^2/m_X^2)$. If only a single massive fermion exists [as in SO(10) models], then it can introduce no *CP*-violating effects into fig. 10; a single massive fermion is, however, sufficient to generate *CP* violation in figs. 11 and 12 even when (5.2.5) vanishes.

6. SU(5) models*

6.1. INTRODUCTION

SU(5) [42] is the simplest group (and the only one of rank 4) which contains as a subgroup the observed low-energy symmetry group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. The gauge vector bosons transform according to the adjoint representation 24_V. Fermions come in families, each consisting of 15 left-handed fields, transforming under the reducible representation $\bar{5} + 10$. Each family has the generic form

$$\bar{5}_f = \{\mathbf{D}^c_L, \nu_L, E_L\}$$

$$10_f = \{\mathbf{U}_{L'}, \mathbf{D}_L, \mathbf{U}^c_L, E^c_L\}\,. \tag{6.1.1}$$

* Some of the results of this section are summarized in ref. [41].

where the vector (bold face) indicates transformation as an SU(3) triplet. The superfix c denotes charge conjugation (see the appendix).

The SU(5) symmetry is spontaneously broken to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ by a 24_H representation of Higgs bosons. The $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ singlet members of this representation are postulated to attain a large vacuum expectation value $\sim 10^{15}$ GeV, and all surviving members of the representation receive large masses.

Lorentz and SU(5) invariance constrain the possible Higgs representations which may couple to fermions to be those appearing in the decompositions [cf. (5.2.1)]

$$\begin{aligned} \bar{5} \otimes \bar{5} &= \overline{10} + \overline{15}\,, \\ \bar{5} \otimes 10 &= 5 + 45\,, \qquad\qquad (6.1.2) \\ 10 \otimes 10 &= (\bar{5} + \overline{50})_S + (\overline{45})_A\,. \end{aligned}$$

With the quantum number assignments (6.1.1) only the 5, 15, and 45 representations contain a neutral member which could receive a vacuum expectation value and hence contribute to fermion masses. The decompositions (6.1.2) show that 5_H and 45_H Higgs repesentations lead to Dirac masses for the charged fermions. The presence of a 15_H would generate a Majorana mass for ν_L which cannot "naturally" be kept small: the 15_H is thus usually excluded. The minimal set of Higgs representations for an SU(5) model is thus 24_H together with 5_H or 45_H. Additional 5_H and 45_H may be added as required to obtain a suitable fit to the observed fermion masses.

The reducibility of the fermion representation implies that even with a single 5_H Higgs representation two independent Yukawa couplings exist: one to $\bar{5}_f \otimes 10_f$, giving D and E masses (equal at unification energies for the case of a single 5_H), and one to $10_f \otimes 10_f$, giving the U mass. The complete lagrangian for couplings to fermions may be written for the minimal model in the form

$$\begin{aligned} L = \sqrt{\tfrac{1}{2}}g\, 24_V \cdot [(\bar{5}_f)_i \cdot (5_f)_i + (\overline{10}_f)_i \cdot (10_f)_i] \\ + (h_U)_{ij} \cdot (10_f)_i^T \cdot (10_f)_j \cdot 5_H + (h_D)_{ij} \cdot (\bar{5}_f)_i \cdot (10_f)_j \cdot \bar{5}_H\,, \qquad (6.1.3) \end{aligned}$$

where g is the SU(5) gauge coupling consant, h_U and h_D are the two Higgs Yukawa couplings, and i and j are fermion family indices. The couplings embodied in this lagrangian are illustrated in fig. 14.

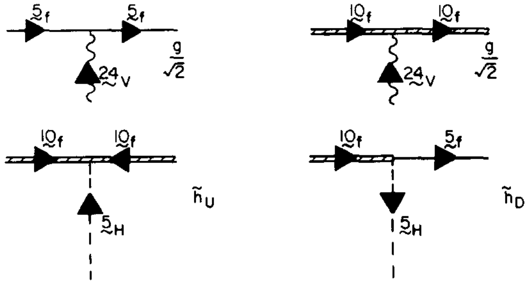


Fig. 14. Couplings of fermions (f) to gauge (V) and Higgs (H) bosons in the minimal SU(5) model.

The SU(5) representations discussed here may be decomposed according to the embedding $\mathrm{SU}(5) \supset \mathrm{SU}(3)_{\mathrm{C}} \otimes \mathrm{SU}(2)_{\mathrm{L}} \otimes \mathrm{U}(1)_{Y}$ as

$$\begin{aligned}
5 &= [3, 1, \tfrac{1}{3}] + [1, 2, -\tfrac{1}{2}]\,,\\
10 &= [3, 2, -\tfrac{1}{6}] + [\bar{3}, 1, \tfrac{2}{3}] + [1, 1, -1]\,,\\
15 &= [6, 1, \tfrac{2}{3}] + [3, 2, -\tfrac{1}{6}] + [1, 3, -1]\,,\\
24 &= [8, 1, 0] + [\bar{3}, 2, -\tfrac{5}{6}] + [3, 2, \tfrac{5}{6}] + [1, 1, 0] + [1, 3, 0]\,,\\
45 &= [1, 2, -\tfrac{1}{2}] + [8, 2, -\tfrac{1}{2}] + [\bar{6}, 1, \tfrac{1}{3}] + [\bar{3}, 1, -\tfrac{4}{3}] + [\bar{3}, 1, \tfrac{1}{3}] + [3, 3, \tfrac{1}{3}] + [\bar{3}, 2, \tfrac{7}{6}]\,,\\
\overline{50} &= [6, 3, -\tfrac{1}{3}] + [8, 2, \tfrac{1}{2}] + [3, 2, -\tfrac{7}{6}] + [\bar{6}, 1, \tfrac{4}{3}] + [\bar{3}, 1, -\tfrac{1}{3}] + [1, 1, -2]\,,
\end{aligned} \tag{6.1.4}$$

where, in $[i, j, k]$, i denotes the SU(3) representation, j the SU(2) representation, and k the U(1) charge Y, given by $Y = T_3 - Q$ [where Q is the electric charge and T_3 is the diagonal generator of SU(2) normalized so that $\mathrm{Tr}[T_3]^2 = \frac{1}{2}$ in the 5 representation of SU(5)].

In subsect. 6.2 we use the quantum number assignments (6.1.4) to determine the minimal set of independent number densities; we then derive the Boltzmann transport equations which these satisfy. In subsect. 6.3 we consider CP violation in the minimal SU(5) model; subsect. 6.4 gives results for baryon number generation in this model. CP violation in SU(5) models with additional Higgs representations is analyzed in subsect. 6.5, and results for baryon number generation in some sample models are given.

6.2. BOLTZMANN EQUATIONS FOR THE MINIMAL SU(5) MODEL

Given the rates for all reactions in an SU(5) model, one could in principle use the general Boltzmann equations (4.4.6) to determine separately the evolution of the number densities of each of the hundred or so particle species which appear. However, in realistic models where there are conserved or partially conserved global symmetries (see below), or in models where there are cancellations in the contributions to the baryon number from various species as would be the case in C symmetric models, it is unnecessary and difficult to calculate numerically the asymmetry in each fermion field and add them together to find the baryon number. For instance in C symmetric theories there may be large $\mathrm{O}(\alpha)$ asymmetries in individual fermion fields that must exactly cancel to give zero baryon number. If the asymmetry in the individual fermion fields were calculated numerically, a numerical accuracy of one part in 10^8 would be necessary to deduce that the final baryon number was less than 10^{-10}. Therefore, we incorporate the conservation laws and calculate directly only combinations of independent asymmetries. Since many of these number densities are, however, related through conservation laws, in the minimal model it is eventually necessary to trace only seven independent combinations of number densities. Note that here, as below, we use the shortened

TABLE 4

Quantum number densities corresponding to asymmetries in particle species of the minimal SU(5) model

	[SU(3), SU(2), U(1)]	Q	I_3	Π	B	L	$B-L$
$U_- \equiv U_L - U_R^c$	$[3, 2, -\frac{1}{6}]$	2	$\frac{3}{2}$	0	1	0	1
$U_-^c \equiv U_L^c - U_R$	$[\bar{3}, 1, \frac{2}{3}]$	−2	0	0	−1	0	−1
$D_- \equiv D_L - D_R^c$	$[3, 2, -\frac{1}{6}]$	−1	$-\frac{3}{2}$	0	1	0	1
$D_-^c \equiv D_L^c - D_R$	$[\bar{3}, 1, -\frac{1}{3}]$	1	0	−3	−1	0	−1
$E_- \equiv E_L - E_R^c$	$[1, 2, \frac{1}{2}]$	−1	$-\frac{1}{2}$	−1	0	1	−1
$E_-^c \equiv E_L^c - E_R$	$[1, 1, -1]$	1	0	0	0	−1	1
$\nu_- \equiv \nu_L - \nu_R^c$	$[1, 2, \frac{1}{2}]$	0	$\frac{1}{2}$	−1	0	1	−1
$X_- \equiv X - \bar{X}$	$[3, 2, -\frac{5}{6}]$	4	$\frac{3}{2}$	0			2
$Y_- \equiv Y - \bar{Y}$	$[3, 2, -\frac{5}{6}]$	1	$-\frac{3}{2}$	0			2
$S_- \equiv S - \bar{S}$	$[3, 1, -\frac{1}{3}]$	1	0				2

The field name represents the reduced number density of the field (summed over color): e.g. $U_L \equiv n_{U_L}/n_\gamma$.

notation $\sigma_\pm = Y_\sigma^\pm = (n_\sigma \pm n_{\bar{\sigma}})/n_\gamma$ for number densities of particle species, and similarly denote the reduced density of a quantum number by the name of that quantum number. For massless fermions, it is sufficient to consider σ_-; σ_+ is always close to one, and has no effect on the baryon asymmetry we discuss. Table 4 gives quantum number densities corresponding to asymmetries in each particle species. In this table, asymmetries of colored species have been summed over color. (The asymmetries are, however, not summed over the spin states of the vector bosons.) The quantum number Π is taken to be 1 for fermions laying in the 5 representation of SU(5), and zero otherwise [43].

At the temperatures of concern in baryon number generation, $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ may be taken as an exact gauged symmetry. In the standard cosmology, the net value of any quantum number associated with a long-range gauge interaction must always vanish. Thus the total electric charge of the universe should be zero. The universe should not only be a color singlet, but also have zero eigenvalues of the commuting generators τ_3 and τ_8 of $SU(3)_C$. Similarly, the universe should have $T_3 = 0$. In addition to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ invariance, the SU(5) model exhibits a further global U(1) invariance corresponding to $B-L$ conservation. There is no necessity for the total $B-L$ of the universe to vanish, but we shall usually take it to do so for simplicity. (In some of the SO(10) models discussed in sect. 7, $B-L$ is violated, and a non-zero net $B-L$ density may develop.) We shall further assume below that B and L separately are initially zero. The consequences of large initial B and L are considered in ref. [55].

As mentioned in sect. 4, the rate for exchanges of light bosons (γ, W, gluons) should be much larger than the rate for B-violating reactions induced by exchanges of heavy bosons. Processes such as $u\bar{u} \to \gamma^* \to d\bar{d}$ or $u\bar{d} \to W^* \to c\bar{s}$ serve to maintain all species in equilibrium distributions (kinetic equilibrium), but cannot affect asymmetries between particles and their antiparticles. However, other $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ reactions are relevant in that they quickly share asymmetries generated in one species among a set of species. For example, if an asymmetry developed in "red" u quarks, it would immediately be shared among all colors of u quarks by transitions from the "red" ones by gluon interactions. The asymmetry in the three color components may thus always be taken equal. Similar effects occur through W interactions, and serve to share asymmetries equally between all weak fermion isodoublets*, so that

$$U_- = D_- , \qquad \nu_- = E_- , \qquad X_- = Y_- \qquad ,\ldots . \tag{6.2.1}$$

For quarks, W interactions connect not only members of a single isodoublet, but also, through Cabibbo mixing, different families. An asymmetry generated in one quark species is thus shared with all other quarks, regardless of their weak isospin or family. As discussed below, the rates for reactions depend on Yukawa couplings of Higgs to fermions, which differ between the families. The heaviest family has the largest coupling, so that the fastest changes occur in this family. These changes are nevertheless immediately shared equally among all families. Thus, to a good approximation, one may effectively account for all families by considering only the heaviest one. Finally it is convenient to use any partially conserved quantum number as an independent asymmetry. The partially conserved quantum numbers may be found by finding the zero eigenmodes associated with exchanges of a particular boson χ [43]. Let $\boldsymbol{F}$ be a set of fermion and boson asymmetries, and let $\boldsymbol{Q}$ be a set of independent quantum number densities B, L, etc. related to $\boldsymbol{F}$ by $\boldsymbol{Q} = H\boldsymbol{F}$. The thermalization of a quantum number Q_i through reactions of a particular boson χ is given from eq. (4.3.14) by

$$\dot{Q}_i = \sum_{\chi} \chi_+^{\mathrm{eq}} M_{ij}^{\chi} Q_j ,$$

where

$$M_{ij}^{\chi} = \sum_{k,l} \Delta Q_i(\chi \to \mathrm{f}^k \mathrm{f}^l) \langle \Gamma(\chi \to \mathrm{f}^k \mathrm{f}^l) \rangle (H_{kj}^{-1} + H_{lj}^{-1})$$

and $\Delta Q_i(\chi \to \mathrm{f}^k \mathrm{f}^l)$ represents the change in the value of Q_i through the reaction $\chi \to \mathrm{f}^k \mathrm{f}^l$. Boltzmann's H theorem requires that the eigenvalues of M^{χ} are all real and non-positive. Any zero eigenvalues reveal additional symmetries; the corresponding eigenvector of number densities is then conserved in χ reactions [e.g. Π in vector boson exchanges in SU(5)]. If this eigenvector is conserved in the reactions of all χ species, then it represents a globally conserved quantum number [e.g. $B - L$ in SU(5)] and results in a further reduction in the number of independent Q_i.

* Assuming that no families with vanishing mixing angles exist.

Using the constraints discussed above, the asymmetries listed in table 4 may be written in terms of the independent set

$$\{B, \Pi, \nu_-, X_-, S_-\},$$

as

$$\begin{aligned} U_- &= D_- = \tfrac{1}{3}(B - \Pi - \nu_- + X_- - S_-)\,, \\ U^c_- &= \tfrac{1}{3}(-B - \Pi + 2X_- - 2S_-)\,, \\ D^c_- &= \tfrac{1}{3}(-\Pi - 2\nu_-)\,, \\ E_- &= \nu_-\,, \\ E^c_- &= (-B + 2\nu_- - 4X_- - 2S_-)\,, \\ Y_- &= X_-\,. \end{aligned} \tag{6.2.2}$$

The time development of number densities in SU(5) models may be obtained by substituting explicit decay branching ratios and scattering cross sections into the coupled Boltzmann equations (4.4.6) and (4.4.7). Table 5 gives the branching ratios

TABLE 5
B-violating boson decays in the minimal SU(5) model

Boson	Decay mode	Partial width	B	Π
X	$E^c_R D^c_L$	ζ_V	$-\frac{1}{3}$	0
	$U_R U_L$	$2\zeta_V$	$\frac{2}{3}$	0
	$D^c_R E^c_L$	ζ_V	$-\frac{1}{3}$	0
	$S\varphi^+$	$\kappa_X \zeta_V$		0
Y	$U^c_R E^c_L$	ζ_V	$-\frac{1}{3}$	0
	$U_R D_L$	$2\zeta_V$	$\frac{2}{3}$	0
	$\nu^c_R D_L$	ζ_V	$-\frac{1}{3}$	0
	$S\varphi^0$	$\kappa_X \zeta_V$		0
S	$U^c_R E^c_R$	$h^2_U \zeta$	$-\frac{1}{3}$	1
	$D^c_R \nu^c_R$	$h^2_U \zeta$	$-\frac{1}{3}$	1
	$U_R D_R$	$2h^2_U \zeta$	$\frac{2}{3}$	1
	$U^c_L E^c_L$	$h^2_D \zeta$	$-\frac{1}{3}$	0
	$U_L D_L$	$2h^2_D \zeta$	$\frac{2}{3}$	0
	$X\varphi^+$	$\alpha\kappa_S \zeta/2$		0
	$Y\varphi^0$	$\alpha\kappa_S \zeta/2$		0

$\zeta = [(4h^2_U + 3h^2_D) + \alpha\kappa_S]^{-1}$, $\zeta_V = [4 + \kappa_X]^{-1}$.
All final states have $B - L = \frac{2}{3}$. The Yukawa coupling constants are such that $h_U = \sqrt{\tfrac{1}{2}} g m_U/m_W$, $h_D = \sqrt{\tfrac{1}{2}} g m_D/m_W$. The parameters κ_X and κ_S are roughly given by $\kappa_X = \vartheta(m_X - m_S)$, $\kappa_S = \vartheta(m_S - m_X)$.

for B-violating boson decays in SU(5) models, averaged over the boson and antiboson in each case. CP violation appears in the differences

$$R(\chi \to \mathrm{f}) = \frac{\Gamma(\chi \to \mathrm{f}) - \Gamma(\bar{\chi} \to \bar{\mathrm{f}})}{\Gamma_\chi} \tag{6.2.3}$$

between boson and antiboson partial decay widths, whose magnitudes are discussed in subsect. 6.3. If they are energetically possible, the decays $\mathrm{X} \to \mathrm{S}\varphi$ or $\mathrm{S} \to \mathrm{X}\varphi$ proceed at a rate proportional to the SU(5) gauge coupling constant g. Their rates are parametrized by

$$\kappa_\mathrm{X} = \left(1 - \frac{m_\mathrm{X}^2}{m_\mathrm{S}^2}\right)^2 \left(1 - \frac{m_\mathrm{S}^2}{m_\mathrm{X}^2}\right)^{1/2} \vartheta(m_\mathrm{X} - m_\mathrm{S}) \,, \tag{6.2.4}$$

$$\kappa_\mathrm{S} = \left(1 - \frac{m_\mathrm{S}^2}{m_\mathrm{X}^2}\right)^2 \left(1 - \frac{m_\mathrm{X}^2}{m_\mathrm{S}^2}\right)^{1/2} \vartheta(m_\mathrm{S} - m_\mathrm{X}) \,. \tag{6.2.5}$$

Decays of X to fermions are also proportional to g, but for S they involve Yukawa couplings, and are $O(h_\mathrm{U}, h_\mathrm{D})$. If $m_\mathrm{X} \gg m_\mathrm{S}$, nearly all X, Y decays are to fermions. If $m_\mathrm{S} \gg m_\mathrm{X}$, however, then only a fraction $\sim(4h_\mathrm{U}^2 + 3h_\mathrm{D}^2)/\alpha$ of S decays are to fermions. Taking $m_\mathrm{t} \sim 25$ GeV suggests a branching ratio $\sim\frac{1}{2}$ for S decays to fermions. Since inclusion of ζ_V modifies the branching ratio to fermions by only 20%, we shall assume for simplicity below that $\kappa_\mathrm{X} = 0$ and hence $\zeta_\mathrm{V} = \frac{1}{4}$.

Inserting the branching ratios of table 5 together with relations (6.2.2) into eqs. (4.4.6) and (4.4.7) give the complete Boltzmann transport equations for the evolution of number densities in the minimal SU(5) model:

$$\begin{aligned}
\dot{X}_+ &= -\langle\Gamma_\mathrm{X}\rangle(X_+ - X_+^{\mathrm{eq}}) \,, \\
\dot{X}_- &= -\langle\Gamma_\mathrm{X}\rangle[X_- - \tfrac{1}{8}X_+^{\mathrm{eq}}(5X_- + S_-)] \,, \\
\dot{S}_+ &= -\langle\Gamma_\mathrm{S}\rangle(S_+ - S_+^{\mathrm{eq}}) \,, \\
\dot{S}_- &= -\langle\Gamma_\mathrm{S}\rangle[S_- - \zeta S_+^{\mathrm{eq}}[h_\mathrm{U}^2(\Pi + S_- - X_-) + h_\mathrm{D}^2(\tfrac{1}{3}\nu_- - \tfrac{5}{6}\Pi - 2S_- - X_-)]] \,, \\
\dot{B} &= \langle\Gamma_\mathrm{X}\rangle[(X_+ - X_+^{\mathrm{eq}})(-R(\mathrm{X} \to \mathrm{E}_\mathrm{R}^\mathrm{c}\mathrm{D}_\mathrm{L}^\mathrm{c}) + 2R(\mathrm{X} \to \mathrm{U}_\mathrm{R}\mathrm{U}_\mathrm{L}) - R(\mathrm{X} \to \mathrm{D}_\mathrm{R}^\mathrm{c}\mathrm{E}_\mathrm{L}^\mathrm{c}) \\
&\quad - R(\mathrm{Y} \to \mathrm{U}_\mathrm{R}^\mathrm{c}\mathrm{E}_\mathrm{L}^\mathrm{c}) + 2R(\mathrm{Y} \to \mathrm{U}_\mathrm{R}\mathrm{D}_\mathrm{L}) - R(\mathrm{Y} \to \nu_\mathrm{R}^\mathrm{c}\mathrm{D}_\mathrm{L}^\mathrm{c})) \\
&\quad - X_+^{\mathrm{eq}}(2B - \nu_- + \tfrac{3}{2}X_- + \tfrac{3}{2}S_-) + 2X_-] \\
&\quad - 16n_\mathrm{b}\langle v\sigma'_\mathrm{X}\rangle(2B - \nu_- + \tfrac{11}{14}X_- + \tfrac{7}{4}S_-) \\
&\quad - \langle\Gamma_\mathrm{S}\rangle[(S_+ - S_+^{\mathrm{eq}})(R(\mathrm{S} \to \mathrm{U}_\mathrm{R}^\mathrm{c}\mathrm{E}_\mathrm{R}^\mathrm{c}) + R(\mathrm{S} \to \mathrm{D}_\mathrm{R}^\mathrm{c}\nu_\mathrm{R}^\mathrm{c}) - 2R(\mathrm{S} \to \mathrm{U}_\mathrm{R}\mathrm{D}_\mathrm{R}) \\
&\quad - 2R(\mathrm{S} \to \mathrm{U}_\mathrm{L}\mathrm{D}_\mathrm{L}) + R(\mathrm{S} \to \mathrm{U}_\mathrm{L}^\mathrm{c}\mathrm{E}_\mathrm{L}^\mathrm{c})) + S_-\zeta(4h_\mathrm{U}^2 + 6h_\mathrm{D}^2) \\
&\quad + 2S_+^{\mathrm{eq}}\zeta(h_\mathrm{U}^2(2\nu_- + B + \Pi + S_- - X_-) + h_\mathrm{D}^2(-\tfrac{7}{3}\nu_- + 2B - \tfrac{7}{6}\Pi + 3X_-))] \qquad (6.2.6) \\
&\quad - 2n_\mathrm{b}\langle v\sigma'_\mathrm{S}\rangle[B(4h_\mathrm{U}^4 + 11h_\mathrm{U}^2h_\mathrm{D}^2 + 6h_\mathrm{D}^4) + \nu_-(8h_\mathrm{U}^4 - 4h_\mathrm{U}^2h_\mathrm{D}^2 - 8h_\mathrm{D}^4)
\end{aligned}$$

$$+\Pi(2h_U^4-3h_U^2h_D^2-h_D^4)+S_-(2h_U^4+4h_U^2h_D^2+6h_D^4)$$
$$+X_-(-2h_U^4+14h_U^2h_D^2-12h_D^4)]\,,$$
$$\dot{\nu}_-=-\langle\Gamma_X\rangle[3(X_+-X_+^{eq})R(Y\to\nu_R^cD^c)+X_+^{eq}\tfrac{1}{4}(5\nu_-+\Pi)+\tfrac{3}{2}X_-]$$
$$-4n_b\langle v\sigma'_X\rangle[5\nu_-+\Pi-\tfrac{15}{4}X_--\tfrac{3}{4}S_-]-\langle\Gamma_S\rangle[3(S_+-S_+^{eq})R(S\to D_R^c\nu_R^c)$$
$$+6h_U^2S_-\zeta-h_U^2S_+^{eq}(-2\nu_--B+\Pi+S_--X)]-n_b\langle v\sigma'_S\rangle[\nu_-(8h_U^2h_D^2+8h_U^4)$$
$$+B(3h_U^2h_D^2+4h_U^4)+\Pi(-8h_U^2h_D^2+2h_U^4)+X_-(-3h_U^2h_D^2-2h_U^4)$$
$$+S_-(-15h_U^2h_D^2+2h_U^4)]-n_b\langle v\sigma_\varphi\rangle(3\nu_--B)\,,$$
$$\dot{\Pi}=\langle\Gamma_S\rangle[3(S_+-S_+^{eq})(R(S\to U_R^cE_R^c)+R(S\to D_R^c\nu_R^c)+R(S\to U_RD_R))$$
$$+24h_U^2S_+-6h_U^2\zeta S_+^{eq}(\Pi+S_--X_-)]$$
$$-n_b\langle v\sigma'_S\rangle h_U^2h_D^2(38\Pi-8\nu_-+6X_-+66S_-)-4\langle v\sigma_\varphi\rangle n_b\Pi\,,$$

where

$$\zeta=[(4h_U^2+3h_D^2)+\alpha\kappa_S]^{-1}\,,$$
$$\Gamma_X=\tfrac{1}{3}\alpha m_X\,,$$
$$\Gamma_S=m_S/16\pi\zeta\,,$$
$$|v|\sigma'_X=\pi\alpha^2\Big[\frac{s}{m_X^2(s+m_X^2)}+\frac{1}{3}\frac{s(s-m_X^2)^2}{[(s-m_X^2)^2+m_X^2\Gamma_X^2]^2}+\frac{2}{s}+\frac{1}{m_X^2}$$
$$-2\Big(\frac{s+m_X^2}{s^2}\Big)\log\Big(\frac{s+m_X^2}{m_X^2}\Big)\Big]\,,$$
$$|v|\sigma'_S=\pi\frac{\alpha^2}{2s^2}\Big[s+m_S^2-\frac{m^4}{(s+m_S^2)}-2m_S^2\log\Big(\frac{s+m_S^2}{m_S^2}\Big)+\frac{1}{2}\frac{s^3(s-m_S^2)^2}{[(s-m_S^2)^2+m_S^2\Gamma_S^2]^2}\Big]\,, \tag{6.2.7}$$

$$|v|\sigma_\varphi=\frac{3\alpha}{4s}h_U^2\,,$$
$$\alpha=g^2/4\pi\,,$$
$$h_U=\sqrt{\tfrac{1}{2}}gm_U/m_W\,,$$
$$h_D=\sqrt{\tfrac{1}{2}}gm_D/m_W\,,$$

where g is the SU(5) gauge coupling constant, and $m_{U,D}$ are effective masses for the quarks in the heaviest family (evaluated at an energy scale $\sim m_X$). With $m_\tau=1.8$ GeV and $m_t=25$ GeV, $h_U\approx0.1$ and $h_D\approx0.01$. In the cross sections σ'

given above, $\sqrt{s}$ is the c.m. energy, taken to be averaged over thermal equlibrium distributions for the incoming particles. The cross sections given ignore the presence of background gas: its effects were discussed in subsect. 4.5, and will be mentioned in subsect. 6.4 below. The Γ_X and Γ_S decay widths are also averaged over equilibrium energy distributions.

In the absence of Higgs interactions $\dot{\Pi}=0$. Ignoring these interactions, and setting $\Pi=0$, eq. (6.2.6) simplifies significantly to become

$$\begin{aligned}
\dot{X}_+ &= -\langle\Gamma_X\rangle(X_+ - X_+^{eq})\,,\\
\dot{B} &= \langle\Gamma_X\rangle[(X_+ - X_+^{eq})\varepsilon_B - X_+^{eq}(2B-\nu_-)] - 16 n_b\langle v\sigma'_X\rangle(2B-\nu)\,,\\
\dot{\nu}_- &= \langle\Gamma_X\rangle[(X_+ - X_+^{eq})\varepsilon_V - \tfrac{5}{4}X_+^{eq}\nu_-] - 20 n_b\langle v\sigma'_X\rangle\nu_-,
\end{aligned} \qquad (6.2.8)$$

where ε_B and ε_ν are the relevant CP-violation parameters corresponding to eq. (6.2.6).

6.3. *CP* VIOLATION IN THE MINIMAL SU(5) MODEL

In this section, we discuss the magnitude of the CP-violation parameters appearing in the Boltzmann transport equations (6.2.8) for the minimal SU(5) model. We shall show that CP violation can occur only in high-order diagrams, and is thus suppressed [26, 44, 45].

As discussed in subsect. 5.1, CP-violating decay amplitudes must result from interference of higher order corrections to decays. The lowest order such interference diagrams for the minimal SU(5) model are shown in fig. 15. As discussed in sect. 5, corrections to gauge vector boson decays involving only vector bosons cannot give rise to CP violation at any order if all fermions are massless. According

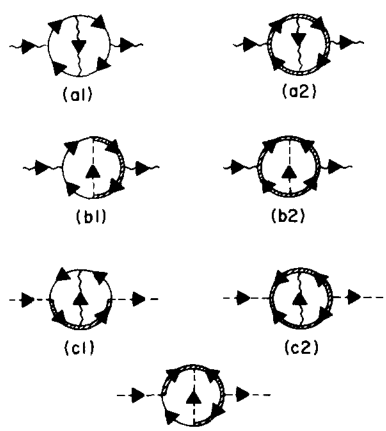


Fig. 15. First-order corrections to boson decay amplitudes in the minimal SU(5) model. None of the diagrams can give rise to the CP-violation required for baryon number generation.

to the minimal SU(5) model, a single 5_{H} Higgs scalar boson couples to fermions. This representation contains just one B-violating component. The result (5.2.5) then shows that no *CP* violation may occur in one-loop diagrams of the form illustrated in fig. 11 involving scalar boson exchange in vector boson decay or vector exchange in scalar decay. Application of eq. (5.1.20) shows that no *CP* violation may arise to one-loop order from scalar boson exchange in scalar boson decay (as illustrated in fig. 12). Another potential source of *CP* violation at lowest order is from diagrams involving three-boson couplings. Since the 24_{H} is a real representation, its vacuum expectation value must be real; in addition, any phase in the vacuum expectation value of 5_{H} may be arranged not to appear in couplings of the B-violating component of 5_{H}. Hence, three-boson couplings cannot exhibit *CP* violation in the minimal SU(5) model. These results demonstrate that no *CP* violation occurs in the one-loop approximation for the minimal SU(5) model.

We now discuss the possibility of *CP* violation in higher order diagrams for the minimal SU(5) model. Since the couplings of gauge vector bosons to fermions and bosons are purely real, addition of further such couplings to the diagrams of fig. 15 cannot yield *CP* violation. Similarly, addition of three- or four-boson couplings cannot introduce *CP* violation. Any *CP* violation must thus occur first in diagrams involving only fermions and 5_{H} Higgs bosons coupling to them. Such *CP* violation requires complex phases in the Higgs Yukawa coupling matrices defined in eq. (6.1.3). In terms of these couplings, the *CP* violation parameter Ω of (5.1.5) arising from the lowest order diagram fig. 15d may be written as $\mathrm{Tr}\,[(h_{\mathrm{D}})^{\dagger}(h_{\mathrm{D}})(h_{\mathrm{U}})^{\dagger}(h_{\mathrm{U}})]$, where the trace is taken over the fermion family indices implicitly carried by the Yukawa couplings h_i. As mentioned above, this quantity is real, so that no *CP* violation may result from the diagram of fig. 15d. At the next (three-loop) order, investigation of possible diagrams shows that all yield purely real Ω, and thus cannot introduce *CP* violation. For example, the diagram of fig. 16 gives

$$\mathrm{Tr}\,[h_{\mathrm{D}}^{\dagger}h_{\mathrm{U}}h_{\mathrm{D}}^{\dagger}h_{\mathrm{D}}h_{\mathrm{U}}^{\dagger}h_{\mathrm{D}}]\,, \tag{6.3.1}$$

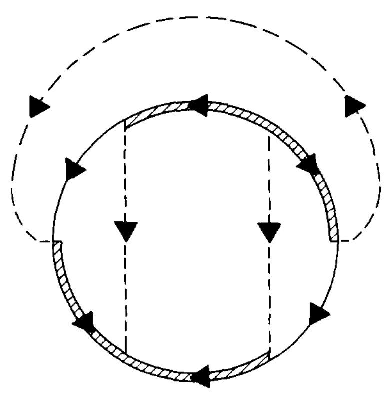


Fig. 16. Example of a second-order correction to scalar boson decay in the minimal SU(5) model. No diagrams at this order can introduce *CP* violation in the minimal SU(5) model.

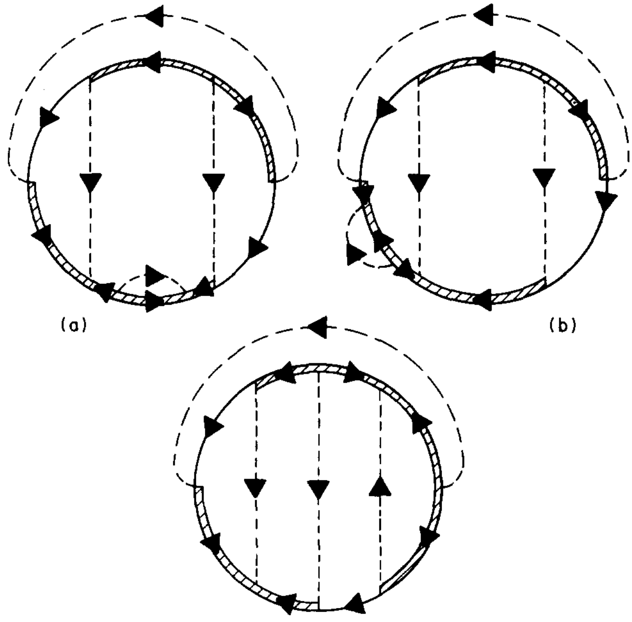


Fig. 17. Examples of a class of third-order corrections to scalar boson decay in the minimal SU(5) model which may potentially give rise to the *CP* violation necessary for baryon number production.

which is manifestly real. However, in the next order, traces such as

$$\mathrm{Tr}\,[h_D^\dagger h_U h_U^\dagger h_U h_D^\dagger h_D h_U^\dagger h_D] \tag{6.3.2}$$

corresponding to the class of diagrams illustrated in fig. 17 need not be real, and may give rise to *CP* violation. These diagrams may give rise to small but non-zero values for the *CP*-violation parameters $R(\mathrm{S}\to ij)$ appearing in the Boltzmann equations (6.2.11). The result for these parameters may be written as

$$R \simeq \tfrac{4}{7}\,\mathrm{Im}\,[I]\Big[\sum_i (h_{U_i}+h_{D_i})\Big]^6 \varepsilon\,, \tag{6.3.3}$$

where ε is a *CP*-violation parameter with $|\varepsilon|\leq 1$. Approximating the Higgs Yukawa couplings by their value for the heaviest fermion family, and taking for the momentum-space factor I the volume of available phase space, one obtains the rough estimate

$$R \sim 10^{-6} h^6 \varepsilon\,. \tag{6.3.4}$$

With the usual mechanism for spontaneous symmetry breakdown, $h \leq 1$ so that $R \leq 10^{-6}$. In the next section we shall show that unless this inequality is saturated, the baryon asymmetry which may be generated in the minimal SU(5) model is entirely inadequate to explain observational results.

6.4. BARYON NUMBER GENERATION IN THE MINIMAL SU(5) MODEL

In this section, we discuss the generation of a baryon asymmetry in the minimal SU(5) model, using the Boltzmann equations derived in subsect. 6.2, and the *CP*-violation parameters discussed in subsect. 6.3. Three basic parameters appear in these calculations: the mass of the gauge vector boson m_X, the mass of the 5_H Higgs bosons m_S, and the *CP*-violation parameter R in the Higgs decays. For the latter parameter, we use the rough estimate (6.3.3) and consider the following cases:

$$h = 1.2\,, \qquad R = 3\times10^{-6}\varepsilon\,, \tag{6.4.1a}$$

$$h = 0.4\,, \qquad R = 4\times10^{-9}\varepsilon\,. \tag{6.4.1b}$$

The X boson mass is taken as

$$m_X = 5\times10^{14}\,\mathrm{GeV} = 0.5\,\Pi\mathrm{eV}\,, \tag{6.4.2}$$

in keeping with theoretical and phenomenological estimates. The stability of spontaneous symmetry breakdown in the SU(5) model appears to require

$$10 \gtrsim m_S/m_X \gtrsim 0.1\,. \tag{6.4.3}$$

Fig. 18 shows results for the final baryon number density produced in the minimal SU(5) as a function of m_S/m_X for the cases (6.4.1a) and (6.4.1b). Explanation of the observed baryon asymmetry would require production of $|B| \gtrsim 10^{-8}$ at this epoch: fig. 18 shows that such a result is possible in the minimal SU(5) model only if $h \gg 1$ which would invalidate perturbative methods. (The naive expectation that h increases at low energy scales may be invalid if $h > g$, since the renormalization group equation for h then receives both positive and negative contributions of roughly equal size.) Thus one may conclude that the minimal SU(5) model is unable to account for the observed baryon asymmetry. It is nevertheless instructive to consider the origins of the complicated behavior seen in fig. 18.

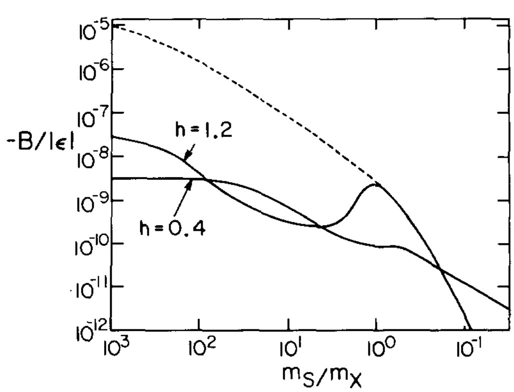


Fig. 18. Final baryon number generated in the minimal SU(5) model.

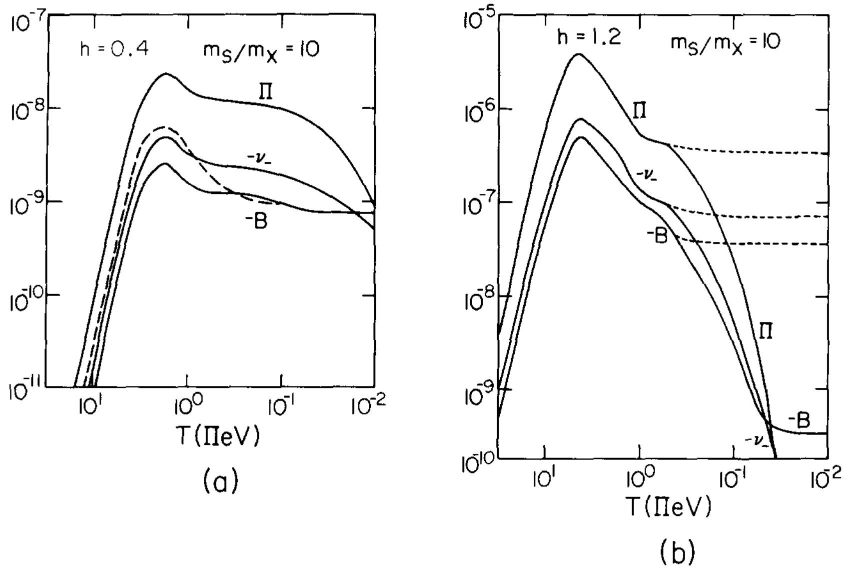


Fig. 19. Time development of quantum number densities in the minimal SU(5) model. The dashed curve in (a) shows results with a modified form for the high-energy $2 \to 2$ scattering cross-section. The dotted curves in (b) show results if light Higgs boson exchange reactions are ignored.

We first consider the case $m_S/m_X = 10$. The time development of quantum number densities for this case with $h = 0.4$ and $h = 1.2$ are shown in fig. 19. At high temperatures, S decays generate asymmetries in Π, ν_- and B as described by eq. (6.2.8). The primary effect which reduces these asymmetries at lower temperatures is $2 \to 2$ scattering. At high temperatures, the effective magnitude of $2 \to 2$ scattering cross-sections are uncertain (as discussed in subsect. 4.5). The two extreme cases $\sigma \sim \alpha^2/m_X^2$ and $\sigma \sim \alpha^2/T^2$ are shown as solid and dashed lines for B in fig. 19a. Although different at high temperatures, they yield identical final results. By virtue of their larger couplings, vector boson exchanges are usually more important than scalar boson exchanges in $2 \to 2$ scattering cross sections. However, in SU(5) models, vector boson exchanges conserve Π. Hence only Higgs boson exchanges can reduce a non-zero Π density (Π_0) generated through S decays at high temperatures. If only vector boson exchanges are considered, then from eq. (6.2.8)

$$\dot{B} \sim (5\nu_- + \Pi_0) , \qquad \dot{\nu}_- \sim (2B - \nu_-) , \tag{6.4.4}$$

which implies a final equilibrium state with non-zero baryon number

$$B = -\tfrac{1}{10}\Pi_0 , \qquad \nu_- = -\tfrac{1}{5}\Pi_0 . \tag{6.4.5}$$

The dotted lines in fig. 19b show results in this case. Changes in B require exchange of heavy vector or scalar bosons; however, changes in Π may occur through light as well as heavy boson exchanges. Typically, vector boson exchanges enforce the relations (6.4.4) during the period when Π_0 is reduced through heavy Higgs exchanges. At very low temperatures, heavy vector boson exchanges become ineffective, so that B remains constant, while light Higgs boson exchanges continue to reduce Π and ν_-. The final value of B is essentially determined through the relation (6.4.4) by the value of Π at the temperature whiere heavy vector boson exchanges become rare. Note that the larger Higgs Yukawa couplings in the case of fig. 19b cause a much more rapid decrease in Π than in fig. 19a.

The discussion of the previous paragraph applies to the case $m_S > m_X$, in which asymmetries are initially generated in S decays, and subsequently thermalized by X exchanges. If $m_S < m_X$, then the S decays responsible for generation of the asymmetries occur at a sufficiently low temperature that X exchange cannot provide significant thermalization. The presence of the X is thus essentially irrelevant, and the final baryon density behaves as it would with only one particle [I]. The dotted curve in fig. 18 shows the final B density if all X exchanges are artificially set to zero. The enhancement around $m_S/m_X = 1$ is a transition phenomenon: X exchanges are sufficiently unimportant that B is no longer constrained to be proportional to Π, and thus does not suffer the destruction experienced by Π as a result of light Higgs exchanges. Since m_X is held fixed, the decrease in B for small m_S/m_X simply reflects the increasing importance of back reactions when m_S is reduced. The similar conclusions of ref. [46] were based on a calculation which ignored non-thermalizing modes and took the final baryon number to have a simple power law dependence on h/m_S. Figs. 18 and 19 illustrate the inapplicability of these assumptions.

6.5. BARYON NUMBER GENERATION IN EXTENDED SU(5) MODELS

The results of sect. 6.4 demonstrate that no viable choice of parameters allows adequate baryon number generation in the minimal SU(5) model. In this section, we consider two simple extensions of the minimal SU(5) model, which can account for the observed baryon asymmetry with suitable choices for parameters.

In the minimal SU(5) model, a single 5_H Higgs representation is taken to couple to fermions. This representation contains a single B-violating Higgs boson (denoted S_1) with $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ quantum numbers $[3, 1, \frac{1}{3}]$. We consider two extensions of this minimal model: in model A [12], a second 5_H is introduced, and in model B a 45_H is added. In both cases we denote the $[3, 1, \frac{1}{3}]$ component of the additional Higgs representation by S_2. For the 45_H, further B violating bosons occur; we shall however assume that S_2 can be arranged to include their effects. The bosons S_1 and S_2 may in general mix; we denote the resulting mass eigenstate mixtures by S and S′, where $m_{S'} \geqslant m_S$. Similarly, we take the light Higgs boson mass eigenstates as φ and φ'. We write the Yukawa couplings of S and S′ (or φ

and φ') as h_U, h_D and h'_U, h'_D, respectively. These Yukawa couplings satisfy

$$\begin{aligned} h_U + h'_U &= \sqrt{\tfrac{1}{2}} g m_U / m_W , \\ h_D + h'_D &= \sqrt{\tfrac{1}{2}} g m_D / m_W , \end{aligned} \tag{6.5.1}$$

but are not individually determined. For simplicity we shall, however, take $h_U = h'_U$, $h_D = h'_D$.

The absence of mixing forbids decays of the form $S' \to SV$, where V is a gauge vector boson. However, in analogy to the case of the minimal SU(5) model discussed in subsect. 6.2, the decays $S' \to X\varphi'$ and $S \to X\varphi$ may occur if they are energetically possible. These decays dilute any *CP* violation arising from decays to fermions.

We consider first model A. As discussed in subsect. 5.2, this model allows no *CP* violation at first order in gauge vector boson decays; *CP* violation may, however, appear in S and S' decays through S' and S exchanges respectively. The magnitude of *CP* violation in S' decays is then given in terms of the parameter ε_S (such that $|\varepsilon_S| < 1$) by

$$\begin{aligned} R(S' \to U_R^c E_R^c) &= R(S' \to D_R^c \nu_R^c) = -R(S' \to U_L^c E_L^c) = -\tfrac{1}{2} R(S' \to U_L D_L) \\ &= 32 \zeta h_U^2 h_D^2 \,\mathrm{Im}\,[I_{S'S}] \varepsilon_S \\ &= \frac{2}{\pi} h_U^2 h_D^2 \zeta \left[1 - \left(\frac{m_S}{m_{S'}}\right)^2 \log\left(1 + \left(\frac{m_S}{m_{S'}}\right)^2\right)\right] \varepsilon_S \\ &= \frac{2}{\pi} h_U^2 h_D^2 \zeta \varepsilon_S , \qquad m'_S \gg m_S . \end{aligned} \tag{6.5.2}$$

Results for S decays are obtained by exchanging S and S', h_i and h'_i and taking $\varepsilon_{S'} = \varepsilon_S$. For $m_S \ll m_{S'}$ one then obtains

$$R_S = R(S \to U_R^c E_R^c) = \frac{1}{\pi} h_U^2 h_D^2 \zeta \left(\frac{m_S}{m_{S'}}\right)^2 \varepsilon_S , \qquad m_S \ll m_{S'} , \tag{6.5.3}$$

which goes to zero as expected when $m_{S'}$ goes to infinity. *CP* violation may occur in S' decays not only through S exchanges, but also through exchanges of light Higgs φ. As discussed in subsect. 5.1, *CP* violation in boson decays may yield an asymmetry in a particular quantum number density only if that quantum number is violated by the exchanged boson in fig. 8. Thus φ exchanges cannot lead to asymmetries in B; they can, however, contribute to ν_- and Π asymmetries. The magnitude of these contributions is given approximately by

$$\begin{aligned} R_\varphi &= R(S' \to U_R^c E_R^c) = R(S' \to D_R^c \nu_R^c) = \tfrac{1}{4} R(S' \to U_R D_R) \\ &= 16 \zeta h_U^2 h_D^2 \varepsilon_S \,\mathrm{Im}\,[I_{\varphi S'}] \\ &= \frac{1}{\pi} h_U^2 h_D^2 \zeta \varepsilon_S . \end{aligned} \tag{6.5.4}$$

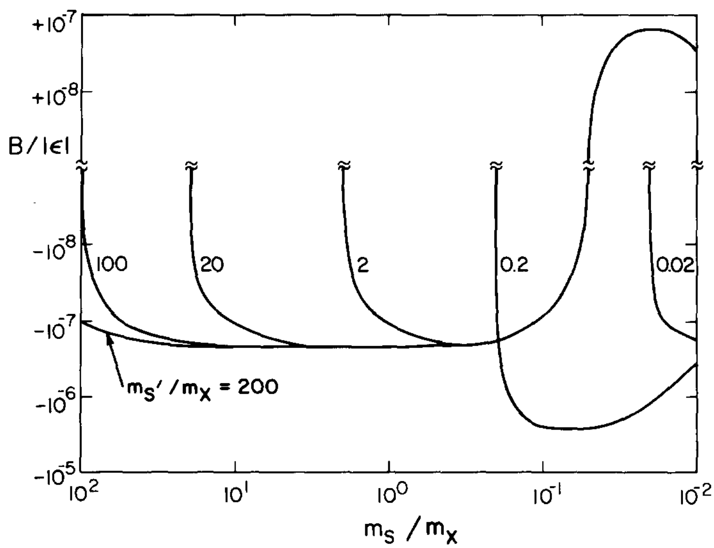


Fig. 20. Final baryon number generated in an extension of the minimal SU(5) model containing an additional 5_{H} multiplet of Higgs bosons (model A). S and S′ are the two mass eigenstate B-violating Higgs bosons.

Combining eqs. (6.5.3) and (6.5.4) yields complete quantum number densities generated by free S′ decays

$$\begin{aligned} \delta_{\mathrm{B}} &= \sum_{\mathrm{f}} B_{\mathrm{f}} R(\mathrm{S}' \to \mathrm{f}) = -R_{\mathrm{S}}, \\ \delta_{\Pi} &= \sum_{\mathrm{f}} \Pi_{\mathrm{f}} R(\mathrm{S}' \to \mathrm{f}) = 3R_{\mathrm{S}} + 3R_{\varphi}, \\ \delta_{\nu_-} &= \sum_{\mathrm{f}} (\nu_-)_{\mathrm{f}} R(\mathrm{S}' \to \mathrm{f}) = -R_{\mathrm{S}} - R_{\varphi}. \end{aligned} \tag{6.5.5}$$

(Results for S decays are obtained by the interchanges $\mathrm{S} \leftrightarrow \mathrm{S}'$, $\varphi \leftrightarrow \varphi'$, $h_j \leftrightarrow h_j'$.) The Boltzmann transport equations for model A are now obtained by replacing every occurrence of S in (6.2.10) with a suitable sum of S and S′, and inserting the parameters (6.5.5). Equations for $\dot{S}'$ analogous to those for $\dot{S}$ must also be added. Finally, the $2 \to 2$ scattering cross-section from S exchange must be supplemented with an analogous term for S′ exchange and with an SS′ interference term.

Fig. 20 shows the final baryon number density generated in model A. Note that when $m_{\mathrm{S}} = m_{\mathrm{S}'}$ our assumption $h_j = h_j'$ implies that $R_{\mathrm{S}} = R_{\mathrm{S}'}$: the ensuing cancellation allows no baryon number generation, as expected from eq. (4.4.4). Fig. 21 shows the time development of quantum number densities in model A for various choices of m_{S} and $m_{\mathrm{S}'}$.

We consider first the region $m_{\mathrm{S}'}/m_{\mathrm{X}} \gg 1$, $m_{\mathrm{S}'}/m_{\mathrm{X}} > m_{\mathrm{S}}/m_{\mathrm{X}} \gtrsim 0.1$ illustrated in fig. 21a. The contributions of various terms in the Boltzmann equation (6.2.8) for this

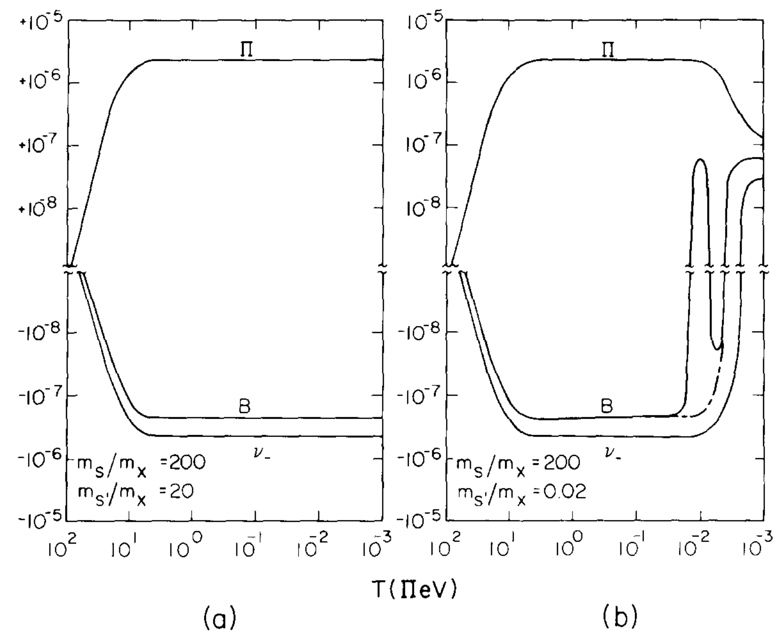


Fig. 21. Time development of quantum number densities in extended SU(5) model A.

case are shown in fig. 22a. By virtue of eq. (6.5.3), $R_S \ll R_{S'}$, so that asymmetries generated in S decay are negligible compared to those generated in S′ decay. In addition, no asymmetries may be generated by X decays in model A. S′ decays at high temperatures generate asymmetries in B, Π and ν_-. $2\to 2$ reactions involving X exchange then "thermalize" these number densities to the values (6.4.5); S exchange is unimportant since S is both more massive than X and has smaller couplings. The second region of parameters for model A is $m_{S'}/m_X \gg 1$; $m_{S'}/m_X \lesssim 0.1$. The time development of quantum number densities for this region is shown in fig. 21b; contributions to the Boltzmann equation for this case are given in fig. 22b. The initial production of quantum numbers and their thermalization through $2\to 2$ X exchange reactions here is as in the case discussed above. At low temperatures, however, reactions involving S become important, and determine the final values of the quantum number asymmetries. Notice that the sign of B changes at low temperatures in fig. 21b. This is a consequence of the fact that terms proportional to S in the Boltzmann transport equation involve combinations of Π, ν_- and B in which B may appear with a negative sign. The dominant term governing the time evolution of B for $T \sim m_S$ is $\dot{B} \sim S_+^{eq} \langle \Gamma_S \rangle (14\nu_- - 12B + 7\Pi)$ with similar equations for $\dot{\nu}_-$ and $\dot{\Pi}_-$. Since $\Pi > 0$, $\Pi > \nu_-$ and $\Pi > B$, this term tends to drive B positive. In general there are three linear combinations of B, ν_- and Π which decrease as pure exponentials until cut off at temperatures below m_S. B is a linear

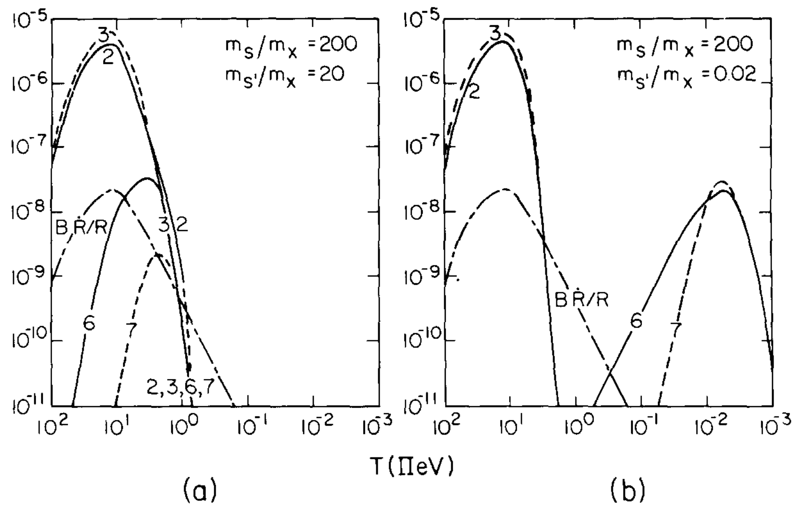


Fig. 22. Contributions to $\dot{B}$ in extended SU(5) model A from various terms in the Boltzmann transport equation (6.2.10). Positive contributions are shown as solid lines; negative ones as dashed lines. The terms of sufficient magnitude to appear are: 2: 2→2 X exchange; 3: $(S'_+ - S'^{\text{eq}}_+)$; 6: S^{eq}_+ term; 7: S_- term. The effect of the expansion of the universe included in the left-hand side of the Boltzmann equation is shown as $B\dot{R}/R$.

combination of these three exponentials, and its final value depends sensitively on the initial values of Π, ν_- and B. For this reason it is not adequate to assume that B is produced and damped in successive independent stages as in simple models which treat only one quantum number. For both $m_{S'} < m_X$ and $m_S < m_X$ inverse decays into S are no longer able to change the sign of the negative B produced through S′ decays and hence the final B is negative. The results of fig. 21 are for large values of $m_{S'}$. Most qualitative features remain unchanged when $m_{S'}$ is reduced. However, since S′ decays then occur at lower temperature, back reactions are more important, and the final asymmetries generated tend to be reduced.

Fig. 20 demonstrates that with suitable choices for undetermined parameters, extended SU(5) model A can account for the observed baryon asymmetry. The sign of the final baryon asymmetry cannot, however, be related directly to the sign of the CP-violation parameter ε without detailed knowledge of other parameters.

We now consider model B. The couplings of the 5_H and 45_H to fermions are $\{E_L(U_L)_a + \nu_L(D_L)_a + (D^c_L)_b(U^c_L)_c\varepsilon^{abc}\}(\bar{5}_H)_a$ and $\{E_L(U_L)_a + \nu_L(D_L)_a - (D^c_L)_b(U^c_L)_c\varepsilon^{abc}\}(\overline{45}_H)_b$, respectively. The difference in Clebsch-Gordan coefficients between these couplings implies that the CP violation parameter for scalar-vector diagrams may be non-zero [cf. eq. (5.2.5)]. Hence in this model, asymmetries may be generated through vector boson exchange in scalar boson decay, and in vector boson decay through scalar boson exchange, as well as through scalar boson exchange in scalar boson decay. The relevant *CP*-violation parameters in this model are then given by

$$
\begin{aligned}
&R(\mathrm{S}\to \mathrm{U_R^cE_R^c}) = R(\mathrm{S}\to \mathrm{D_R^c}\nu_\mathrm{R}^\mathrm{c}) = 32\zeta[2\pi\alpha h_\mathrm{D}^2\ \mathrm{Im}\,[I_\mathrm{VS}] + h_\mathrm{U}^2h_\mathrm{D}^2\ \mathrm{Im}\,[I_\mathrm{S'S}]]\varepsilon\,,\\
&R(\mathrm{S}\to \mathrm{U_RD_R}) = 32\zeta[-4\pi\alpha h_\mathrm{D}^2\ \mathrm{Im}\,[I_\mathrm{VS}] + h_\mathrm{U}^2h_\mathrm{D}^2\ \mathrm{Im}\,[I_\mathrm{S'S}]]\varepsilon\,,\\
&R(\mathrm{S}\to \mathrm{U_L^cE_L^c}) = 32\zeta[4\pi\alpha h_\mathrm{U}^2\ \mathrm{Im}\,[I_\mathrm{VS}] - h_\mathrm{U}^2h_\mathrm{D}^2\ \mathrm{Im}\,[I_\mathrm{S'S}]]\varepsilon\,,\\
&R(\mathrm{S}\to \mathrm{U_LD_L}) = 32\zeta[-4\pi\alpha h_\mathrm{U}^2[I_\mathrm{VS}] - 2h_\mathrm{U}^2h_\mathrm{D}^2[I_\mathrm{S'S}]]\varepsilon\,, \qquad (6.5.6)\\
&R(\mathrm{X}\to \mathrm{E_R^cD_L^c}) = R(\mathrm{Y}\to \nu_\mathrm{R}^\mathrm{c}\mathrm{D_L^c}) = 4h_\mathrm{D}^2[\mathrm{Im}\,[I_\mathrm{SV}] - \mathrm{Im}\,[I_\mathrm{S'V}]]\varepsilon\,,\\
&R(\mathrm{X}\to \mathrm{E_L^cD_R^c}) = R(\mathrm{Y}\to \mathrm{E_L^cU_R^c}) = 4h_\mathrm{U}^2[\mathrm{Im}\,[I_\mathrm{SV}] - \mathrm{Im}\,[I_\mathrm{S'V}]]\varepsilon\,,\\
&R(\mathrm{X}\to \mathrm{U_RU_L}) = R(\mathrm{Y}\to \mathrm{U_RD_L}) = -4(h_\mathrm{U}^2 + h_\mathrm{D}^2)[\mathrm{Im}\,[I_\mathrm{SV}] - \mathrm{Im}\,[I_\mathrm{S'V}]]\varepsilon\,.
\end{aligned}
$$

Terms accounting for light Higgs φ exchange must be added as before. Inserting these parameters into the Boltzmann transport equations, we obtain the results shown in fig. 23 for the final baryon asymmetry. The possibility of *CP* violation in processes involving vector bosons renders these results still more complicated than for model A. In discussing fig. 23, we consider first the region $m_{\mathrm{S'}} > m_\mathrm{S} > m_\mathrm{X}$. *CP* violation in X decays is suppressed here, since it involves exchange of scalar bosons heavier than X. After asymmetries are initially generated in S and S′ decays, they are thermalized by $2\to 2$ X exchange reactions to the values (6.4.5). The resulting

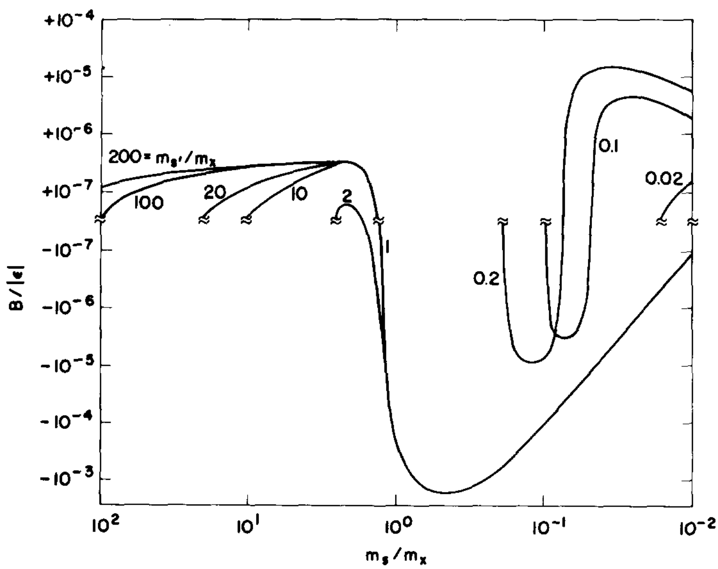


Fig. 23. Final baryon number generated in an extension of the minimal SU(5) model involving a 45_H as well as a 5_H Higgs multiplet (model B). S and S′ are the two mass eigenstate B-violating Higgs bosons considered.

B is then proportional to the original Π generated. Although the B generated by free S and S′ decays would be large since it receives contributions from vector boson exchange, only scalar exchanges can contribute to Π. Hence the final value of B is similar to that obtained in model A. We now consider the region $m_{S'} > m_X > m_S$. In this region, the final B is dominated by asymmetries generated in S decays; any B generated in X or S′ decays is destroyed by processes involving S. B generated in S decays is damped by inverse S decay processes; the final B obtained varies roughly as m_S^{-3}: a factor m_S^{-2} from the CP-violation parameter (6.5.6), and m_S^{-1} from the effects of inverse reactions. In the region $m_S < m_{S'} < m_X$ several sources of baryon number contribute with roughly equal weight, and no simple qualitative explanation of the final results is possible. Fig. 24 gives some examples of the development of the quantum numbers for this model.

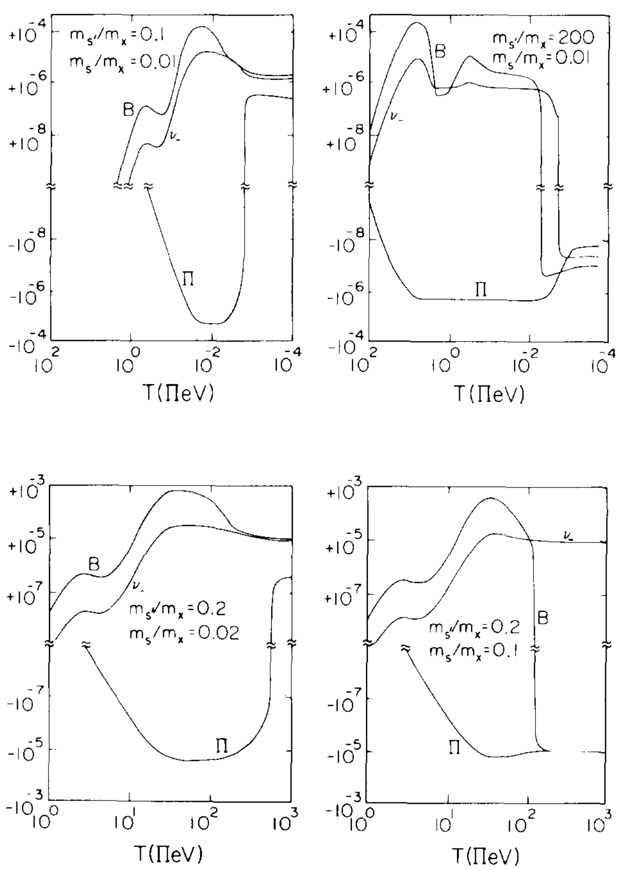


Fig. 24. Time development of quantum number densities in some sample cases of extended SU(5) model B.

7. SO(10) models*

7.1. INTRODUCTION

Although SU(5) grand unified models contain the fewest fundamental fields, they exhibit a number of seemingly undesirable features which may be avoided in models based on larger gauge groups. The first of these features is the assignment of fermions to a reducible representation of SU(5). This has the consequence that particles belong to different irreducible representations from their antiparticles. In addition, axial anomalies cancel only between different irreducible fermion representations, in a seemingly accidental manner. A further feature is the presence of a global conserved $B-L$ quantum number which has no basis in a local gauge invariance. In this section, we consider models based on SO(10) [49], in which these features are removed.

SO(10) symmetry is ultimately broken down to the low-energy $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ symmetry. This breaking may occur through a sequence of stages at different mass scales. Baryon number generation may occur at an intermediate stage in this sequence, when the effective symmetry is larger than $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. Typical subgroups of SO(10) which may represent intermediate effective symmetries are:

$$SO(10) \supset SU(4) \otimes SU(2)_L \otimes SU(2)_R\,, \tag{7.1.1a}$$

$$SO(10) \supset SU(4) \otimes SU(2)_L \otimes U(1)_R\,, \tag{7.1.1b}$$

$$SO(10) \supset SU(3)_C \otimes SU(2)_L \otimes U(1)_Y\,. \tag{7.1.1c}$$

Table 6 lists decompositions of relevant representations of SO(10) in terms of these subgroups. Transformation properties under these decompositions will be denoted as follows. (m, n_L, n_R) will denote representations of SU(4), $SU(2)_L$, and $SU(2)_R$, respectively. $\langle m, n_L, Q_R\rangle$ will denote representation of SU(4), $SU(2)_L$ and $U(1)_R$ respectively. Q_R is the $U(1)_R$ charge operator T_{3R}, defined to be $\pm\frac{1}{2}$ when acting on an $SU(2)_R$ doublet. Finally, as in previous sections, $[m, n_L, Q_Y]$ will denote representations of $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ [12, 50]. The hypercharge Y is defined so that $Y = T_{3L} - Q = -\frac{1}{2}(B-L) - T_{3R}$, where Q is the electric charge operator.

Unbroken SO(10) symmetry exhibits a local gauge invariance associated with $B-L$ (containing the global $B-L$ invariance of SU(5) models as a subgroup). When this invariance is destroyed by spontaneous symmetry breakdown, $B-L$ violating processes may occur. With effective symmetries smaller than SO(10), $B-L$ invariance is preserved whenever a $U(1)_R$ invariance exists. $B-L$ violation may thus occur when the effective symmetry is that of (7.1.1c), but not those of (7.1.1a) or (7.1.1b).

* Many of the results in this section are also presented in refs. [47, 48].

TABLE 6

Decompositions of some representations of SO(10) in terms of the subgroups $SU(5)\otimes U(1)$ and $SU(4)\otimes SU(2)_L\otimes SU(2)_R$

SO(10)	$SU(5)\otimes U(1)$	$SU(4)\otimes SU(2)_L\otimes SU(2)_R$
10	$5(2)+\bar{5}(-2)$	$(6,1,1)+(1,2,2)$
16	$1(-5)+\bar{5}(3)+10(-1)$	$(4,2,1)+(\bar{4},1,2)$
45	$1(0)+10(4)+\overline{10}(-4)+24(0)$	$(6,2,2)+(15,1,1)+(1,3,1)+(1,1,3)$
54	$15(4)+\overline{15}(-4)+24(0)$	$(6,2,2)+(20',1,1)+(1,3,3)+(1,1,1)$
120	$5(2)+\bar{5}(-2)+10(-6)$ $+\overline{10}(6)+45(2)+\overline{45}(-2)$	$(15,2,2)+(6,3,1)+(6,1,3)$ $+(\overline{10},1,1)+(10,1,1)+(1,2,2)$
126	$1(-10)+\bar{5}(-2)+10(-6)$ $+\overline{15}(6)+45(2)+\overline{50}(-2)$	$(15,2,2)+(10,1,3)+(\overline{10},3,1)$ $+(6,1,1)$

In the first case, the U(1) charge is given in parentheses, and carries an arbitrary normalization factor. Decompositions of SU(5) representations in terms of $SU(3)_C\otimes SU(2)_L\otimes U(1)_Y$ are given in subsect. 6.1.

The gauge vector bosons in SO(10) models are taken to transform according to the adjoint 45_V representation. This representation contains the $[3, 2, \frac{5}{6}]$ B-violating bosons (X, Y) of SU(5) models, together with an additional $[3, 2, -\frac{1}{6}]$ pair of B-violating bosons (X′, Y′). It also contains a further B-conserving $[3, 1, \frac{2}{3}]$ multiplet denoted V and the B-conserving gauge bosons of $SU(2)_R$, $[1, 1, -1]$, $[1, 1, 0]$, and $[1, 1, 1]$, which we will denote by W, W^0, and $\bar{W}$ respectively.

In SO(10) models each family of left-handed fermions is assigned to the 16-dimensional complex spinor representation, while the corresponding right-handed CP conjugate fields are assigned to $\overline{16}$. With this assignment of fermions, no axial anomalies appear*.

The 16 representation may be decomposed in terms of SU(5) representations as (see table 6)

$$16 = 10 + \bar{5} + 1\,, \tag{7.1.2}$$

and thus contains the usual SU(5) fermion fields, together with an additional SU(5) singlet field denoted N_L^c. The N_L^c field may be taken as a charge conjugate partner for the ν_L. This pairing allows definition of a charge conjugation operator C; no such operator may be defined in SU(5) models because ν_L has no suitable partner. Charge conjugation invariance is in general spontaneously broken. It remains unbroken only with the effective symmetry of (7.1.1a), or with the $SU(3)\otimes SU(2)_L\otimes SU(2)_R\otimes U(1)$ subgroup of (7.1.1a) in which case C interchanges $SU(2)_L$ and $SU(2)_R$ and conjugates representations in SU(4).

* This is a consequence of the fact that the symmetric product of the adjoint representation with itself does not contain the adjoint, and thus that the d coefficients to which anomalies would be proportional, must vanish.

The pairing of N_L^c with ν_L provides the possibility of a $\Delta I_W=\frac{1}{2}$ Dirac mass for the ν_L. To avoid this disastrous phenomenon, one must introduce a large ($\Delta I_W=0$) Majorana mass m_N for the N_L^c, so that the neutral lepton mass matrix takes the form

$$\begin{pmatrix} 0 & m_q \\ m_q^T & m_N \end{pmatrix}, \tag{7.1.3}$$

where m_q is of the order of quark masses [51]. For $m_N \gg m_q$ one eigenvalue of this matrix is $O(m_N)$ and represents the physical N, and the other is $O(m_q^2/m_N)$ and represents a neutrino with small but finite mass. This neutrino mass and the resulting neutrino oscillations are an important prediction of SO(10) models. The Majorana mass term for the N_L^c has $\Delta L=2$, $\Delta B=0$; it is thus permitted only if $B-L$ is violated. Hence the N_L^c must remain massless until the effective symmetry is reduced to (7.1.1c).

The Higgs content of SO(10) models is dictated by the need to break SO(10) down to $SU(3)_C \otimes U(1)_{em}$ and by the desire to obtain the observed masses and mixing angles of the fermions. Higgs fields which couple to fermions must appear in the product

$$16 \otimes 16 = (10+126)_S + (120)_A, \tag{7.1.4}$$

where S (A) indicate the symmetric (antisymmetric) product. The 10 contains the usual light $SU(2)_L$ doublet φ, together with a $[3, 1, \frac{1}{3}]$ B-violating boson denoted S. It also includes the antiparticles $\bar{\varphi}$ and $\bar{S}$. With a 10_H alone, one obtains the tree approximation mass relations

$$m_\nu = m_U = m_D = m_E, \tag{7.1.5}$$

at a mass scale $O(m_X)$ for each family. Attempts to fit the observed mass spectrum more accurately generally lead to models with a rather baroque Higgs sector [52, 53]. Inclusion of 120_H and 126_H will be discussed in subsects. 7.2 and 7.3. Since the Higgs structure of SO(10) models is considerably more complicated than in SU(5) models, we shall below analyze only a few specific cases. Other choices for Higgs structure are expected to yield qualitatively similar results.

As in SU(5) models, Higgs representations which do not couple to fermions must also appear in order to achieve the desired symmetry breakdown. Representations commonly used for this purpose are 45_H or a 54_H.

In SU(5) models, there are no particles with masses between the O(300 GeV) scale of $SU(2)_L$ breaking and the $O(10^{15}$ GeV) scale of SU(5) breaking. In SO(10) models, intermediate mass scales are possible, associated with the intermediate symmetries of eq. (7.1.1). If SO(10) breaks first to $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ or to

$SU(4)\otimes SU(2)_L\otimes U(1)_R$ before breaking to $SU(3)_C\otimes SU(2)_L\otimes U(1)_Y$, then fits to the weak mixing angle suggest that mass scales as low as 10^{10} GeV may exist. Take M_V and M_S to be the masses of typical B-violating vectors and scalars respectively, and M_R and M_C the scales at which $SU(2)_R$ and $SU(4)$ break to $U(1)_R$ and $SU(3)_C\otimes U(1)$, respectively. Then non-observation of proton decay requires that $M_V\gtrsim 4\times 10^{14}$ GeV and $M_S\gtrsim 2\times 10^{12}$ GeV. Experimental constraints on M_R and M_C are much less stringent: non-observation of muon and electron number violating decays such as $K^0\to\mu^-e^+$ gives a lower bound of about 10^4 GeV for M_C while limits on the strength of right-handed weak currents require only that $M_R\gtrsim 200$–300 GeV. Theoretical fits to α_s and the weak mixing angle give a minimum value for M_C of about 10^{10} GeV with typical values being 10^{12} GeV for SO(10) broken first to $SU(4)\otimes SU(2)_L\otimes U(10)_R$. If SO(10) breaks first to $SU(4)\otimes SU(2)_L\otimes SU(2)_R$ then typical values are $M_C\sim 10^{12}$ GeV and $M_R\sim 10^{10}$ GeV.

The presence of these intermediate mass scales for spontaneous symmetry breakdown could allow an unbroken gauge symmetry larger $SU(3)_C\otimes SU(2)_L\otimes U(1)_Y$ to exist even at the temperatures of relevance to B production. In the following sections we discuss the production of baryon number for models in which the effective symmetries of (7.1.1a), (7.1.1b) and (7.1.1c) are present.

7.2. B AND $B-L$ VIOLATION IN SO(10) MODELS

In SO(10) models, the SU(5) singlet [1, 1, 0] fermion N_L^c is present. If the effective symmetry still contains an unbroken $B-L$ invariance, then the N_L^c may be assigned a definite conserved $B-L=1$, and no new boson couplings are possible. However, when the effective symmetry is that of eq. (7.1.1c), $B-L$ is broken and the N_L^c may acquite a Majorana mass. The additional classes of boson couplings introduced by the presence of N in this case are listed in table 7. No new $SU(3)_C\otimes SU(2)_L\otimes U(1)_Y$ assignments for B-violating bosons appear in this list. However, comparison of table 7 with tables 2 and 3 shows that several bosons may violate $B-L$ through couplings to N. Two such $B-L$ violating bosons are the S and X′ discussed above. There are in addition $B-L$ violating vector bosons which transform as $[3, 1, -\frac{2}{3}]$ and [1, 1, 1] and which represent gauge bosons for the SU(4) and $SU(2)_R$ subgroups of SO(10). There are also several further $B-L$ violating scalar bosons. These appear in SO(10) representations capable of coupling to fermions as follows:

$$\begin{aligned}&[1,2,\tfrac{1}{2}]\subset 10, 120, 126\,,\\ &[3,2,-\tfrac{1}{6}]\subset 126\,,\\ &[1,1,1]\subset 120, 126\,.\end{aligned}\tag{7.2.1}$$

TABLE 7

Quantum numbers for possible vector and scalar pairs of singlet fermions N and quarks q or leptons ℓ

		[SU(3), SU(2), U(1)]	B	L	$B-L$
V_6, S_6	qN	$[3, 2, -\frac{1}{6}]$	$\frac{1}{3}$	0	$\frac{1}{3}$
		$[3, 1, -\frac{2}{3}]$			
		$[3, 1, \frac{1}{3}]$			
V_7, S_7	ℓN	$[1, 2, \frac{1}{2}]$	0	1	−1
		$[1, 1, 1]$			
V_8, S_8	NN	$[1, 1, 0]$	0	0	0

We will also consider baryon number generation when the effective symmetry is that of eq. (7.1.1b). Then, in analogy to the discussion of subsect 6.2, the unbroken $SU(4) \otimes SU(2)_L \otimes U(1)_R$ symmetry enforces relations between fermions number density asymmetries:

$$\begin{aligned} U_{L-} = \nu_{L-} = E_{L-} = D_{L-} &\equiv (n_{D_L} - n_{D_R^c})/n_\gamma, \\ E^c_{L-} = D^c_{L-} &\equiv (n_{D_L^c} - n_{D_R})/n_\gamma, \\ N^c_{L-} = U^c_{L-} &\equiv (n_{U_L^c} - n_{U_R})/n_\gamma, \end{aligned} \tag{7.2.2}$$

with n_i the number density per color of species i. Although $SU(2)_L$ symmetry implies $U_{L-} = D_{L-}$ we retain the two separately to exhibit the role of the charge conjugation invariance discussed in subsect. 7.3. In terms of the asymmetries (7.2.2), the baryon number density becomes

$$B = D_{L-} + U_{L-} - D^c_{L-} - U^c_{L-}\,. \tag{7.2.3}$$

It is also convenient to introduce a quantum number Θ, defined as the total asymmetry in (left-handed) fermion fields:

$$\Theta = D_{L-} + U_{L-} + D^c_{L-} + U^c_{L-}\,. \tag{7.2.4}$$

Because of their chiral structure, all gauge vector boson interactions conserve Θ. It may nevertheless be violated by Higgs scalar interactions. [In this respect it is analogous to the Π quantum number introduced for SU(5) models in sect. 6.]

Tables 2, 3 and 7 give possible classes of couplings which respect $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ invariance. If the effective symmetry is larger than $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, then relations may exist between some of these couplings. It turns out that with an effective symmetry $SU(4) \otimes SU(2)_L \otimes U(1)_R$, the only couplings affected in our considerations are those of the (10, 3, 1) component of

126_H, which are prevented from exhibiting B-violating couplings with this effective symmetry.

7.3. C AND CP VIOLATION IN SO(10) MODELS

The generation of a net baryon number from symmetric initial conditions requires the presence of both C and CP violation. In $SU(2)_L \otimes U(1)_Y$ weak interaction models and SU(5) grand unified models no C operator may be defined since there is no left-handed antineutrino to form the charge conjugate partner of the left-handed neutrino. In larger models, such as SO(10) or E(6), each fermion has a potential charge-conjugate partner or is an eigenstate of C and a C operation may be defined which is a symmetry of the unbroken theory [51]. The production of a C-odd quantum number (such as B or L) in these models therefore depends on the interplay between the sources of C violation and the processes which violate the quantum number under consideration.

The lack of B production in a C-symmetric theory may be seen by considering the decays of B-violating bosons χ and their antiparticles $\bar{\chi}$ as well as the decays of their charge conjugate partners χ^c and $\overline{\chi^c}$. The B produced by the decays of an equal mixture of χ and $\bar{\chi}$ into the specific final state $i_1 i_2$ and the charge conjugate decays of χ^c and $\overline{\chi^c}$ into the state $i_1^c i_2^c$ is proportional to the quantity [see eq. (5.1.12)]

$$R_\chi^{12} + (R_\chi^{12})^c = \mathrm{Im}\,[I]\,\mathrm{Im}\,[\Omega](B_{i_2} - B_{i_1}) + \mathrm{Im}\,[I^c]\,\mathrm{Im}\,[\Omega^c](B_{i_2^c} - B_{i_1^c})\,. \qquad (7.3.1)$$

I represents an integral over the intermediate momenta and final state phase space for the decay and Ω is a product of the relevant couplings. The lowest order contributions to I and Ω are discussed in sect. 5. I^c and Ω^c are the corresponding quantities for the charge conjugate reaction. In a C-symmetric theory, $I = I^c$ and $\Omega = \Omega^c$, while since B is C-odd, $B_{i_2} = -B_{i_2^c}$ and $B_{i_1} = -B_{i_1^c}$ causing $R_\chi^{12} + (R_\chi^{12})^c$ to vanish.

As discussed in subsect. 7.1, with the intermediate effective symmetry $SU(4) \otimes SU(2)_L \otimes SU(2)_R$, or the $SU(3) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)$ subgroup of this symmetry, charge conjugation invariance remains unbroken. While this symmetry exists, no baryon number generation may thus occur. Although B production requires C violation, production of quantum numbers such as Π, ν_- or Θ which are not odd under C, may occur even when C invariance is unbroken. These asymmetries may then be converted into B asymmetries at lower temperatures by C and B violating reactions. For a final non-zero B to result, the second stage must occur at sufficiently high temperatures ($\gtrsim 10^{12}$ GeV) that the effects of B-violating bosons are still important. Thus if SO(10) symmetry breaking occurs through $SO(10) \to SU(4) \otimes SU(2)_L \otimes SU(2)_R$ or $SO(10) \to SU(3) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)$, the intermediate symmetry may not persist to temperatures below $\sim 10^{12}$ GeV if an adequate B is to be produced [54].

C invariance may be broken either by the presence of different masses for the ν_L and N_L^c, or through mass splittings between bosons and their charge conjugate antibosons. A non-zero m_N can occur only with the effective symmetry (7.1.1c). In this case, any of the diagrams in figs. 9–11 may yield C and CP violating contributions proportional to m_N^2/m_χ^2.

Under the effective symmetry $SU(4)\otimes SU(2)_L\otimes SU(2)_R$ the 45_V adjoint representation of gauge vector bosons has the decomposition given in table 6. The color triplet $SU(2)_L$ doublet B-violating bosons (X, Y) and (X′, Y′) and their antiparticles combine to form the (6, 2, 2) representation. With our conventions the (X, Y) have electric charge $(-\frac{4}{3}, -\frac{1}{3})$ and the (X′, Y′) have electric charge $(-\frac{1}{3}, \frac{2}{3})$. Charge conjugation takes $X\to\bar{X}$, $Y\to\overline{X'}$, $X'\to\bar{Y}$, and $Y'\to\overline{Y'}$. B production through vector boson reactions therefore requires a mass splitting between the (X, Y) and (X′, Y′) doublets. This will in general be present if SO(10) is broken to $SU(3)_C\otimes SU(2)_L\otimes U(1)_Y$. However, if SO(10) is broken only to $SU(4)\otimes SU(2)_L\otimes U(1)_Y$, then the (6, 2, 2) splits into $\langle 6, 2, \frac{1}{2}\rangle$ and the CP conjugate state $\langle 6, 2, -\frac{1}{2}\rangle$ and as a result there is no mass splitting. The B-violating vector bosons will therefore be unable to produce a net B in their decays or to convert an asymmetry in Θ into an asymmetry in B.

The $SU(4)\otimes SU(2)_L\otimes SU(2)_R$ content of the 10_H, 120_H and 126_H Higgs representations coupling to fermions are given in table 6. When $SU(2)_R$ is broken to $U(1)_R$ some of these bosons may acquire C-violating mass splittings. The usual B-violating color triplet S appears along with its antiparticle in (6, 1, 1), and is thus an eigenstate of C. The 126_H contains (10, 3, 1) and $(\overline{10}, 3, 1)$. However, as discussed in subsect. 7.1, these bosons may violate B only when the effective symmetry is $SU(3)_C\otimes SU(2)_L\otimes U(1)_Y$ rather than $SU(4)\otimes SU(2)_L\otimes U(1)_R$. The (6, 3, 1) and (6, 1, 3) appearing in the 120_H may nevertheless both violate B and acquire masses which differ between particles and their corresponding antiparticles, and thus violate C. After the breaking $SU(2)_R\to U(1)_R$, these representations may be decomposed as $\langle 6, 1, \pm 1\rangle$ (denoted $\tilde{S}_1$), $\langle 6, 1, 0\rangle$ (denoted $\tilde{S}$) and $\langle 6, 3, 0\rangle$ (denoted $\tilde{S}^c$). The requirement of C violation therefore requires the presence of a 120_H in order for B to be produced with effective symmetry $SU(4)\otimes SU(2)_L\otimes U(1)_R$. If the effective symmetry is $SU(3)_C\otimes SU(2)_L\otimes U(1)_Y$, then since the usual B-violating color triplet scalar boson S is transformed into its antiparticle under C and hence may not have any C-violating mass splitting, B production through S decays must be proportional to the C-violating mass splitting between ν_L and N_L^c so that

$$R_S^{12}+(R_S^{12})^c=O((m_N/m_S)^2)\,, \tag{7.3.2}$$

as can be seen by expanding the relevant phase-space integrals in powers of m_N/m_S [54]. In the sequel we shall assume that this contribution to B production is negligible compared to the contribution from conversion of other asymmetries whose production is not restricted by C invariance. Note that more than one family must be present to allow the antisymmetric coupling of the 120_H to $16_f\otimes 16_f$.

7.4. B GENERATION FOR SO(10) MODELS WITH $SU(3) \otimes SU(2)_L \otimes U(1)_Y$ EFFECTIVE SYMMETRY

In this section we describe the calculation of baryon number generation in SO(10) models where $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ is the effective gauge symmetry at temperatures relevant to baryon number production. We shall assume that all B production occurs in this phase; the equations derived may nevertheless also be used with suitable initial conditions to describe the development in the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ phase of a B generated at temperatures where a larger effective symmetry exists. If SO(10) breaks first to SU(5) and then to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, then fits to the weak mixing angle suggest that $m_X \cong 0.5\ \Pi\mathrm{eV}$ but do not constrain the values of $m_{X'}$, m_W or m_V. Below we shall usually choose the values $m_W = m_V = m_{X'} = 10\ \Pi\mathrm{eV}$.

N decays are potentially an important source of B and L asymmetries in SO(10) models. The N have two distinct types of decay modes. The first are two body decays

$$\mathrm{N} \to \mathrm{e}\varphi,\ \bar{\mathrm{e}}\bar{\varphi}\,, \tag{7.4.1}$$

with $\mathrm{e} = \binom{\nu}{\mathrm{e}}$, $\varphi = \binom{\varphi^0}{\varphi^+}$ where φ is the usual $SU(2)_L \otimes U(1)_Y$ weak doublet. The width for this decay mode is

$$\Gamma_{\mathrm{N}\to\mathrm{e}\varphi} \simeq \alpha (m_{\mathrm{q}}/m_W)^2 m_{\mathrm{N}}\,, \tag{7.4.2}$$

where m_{q} is the mass of the relevant charge $\frac{2}{3}$ quark and m_W is the mass of the usual weak boson. The N may also undergo three body decays

$$\mathrm{N} \to \mathrm{qqq},\ \mathrm{q}\bar{\mathrm{q}}\ell, \ldots, \tag{7.4.3}$$

mediated by exchange of a supermassive gauge boson Ξ coupling to the N (and thus not contained in the 24_V of SU(5)). These decays have typical widths given in analogy to μ decay by

$$\Gamma_{\mathrm{N}\to\mathrm{qqq}} \sim \alpha^2 \frac{m_{\mathrm{N}}^5}{384\pi} m_{\Xi}^{-4}. \tag{7.4.4}$$

When $m_{\mathrm{N}} \leqslant m_{\Xi}$, two-body decays dominate. These decays violate lepton number, but do *not* violate baryon number. They may therefore give rise to an asymmetry in L but not in B. In models where (7.4.3) dominates (as in the unusual case $m_{\mathrm{N}} \geqslant m_{\Xi}$), N decays may violate baryon number. However, in this case, stringent lower bounds on m_{N} exist to ensure that N decays should not generate excessive entropy to dilute any B produced [35]. We shall not consider N decays below.

We shall consider a specific but presumably typical SO(10) model, in which two 10_H couple to fermions. The mass eigenstate B-violating Higgs bosons will be denoted by S and S′. We shall include *CP* violation only for exchanges of S in S′ decay and vice-versa. In analogy to the case of SU(5) models discussed in subsect. 6.2, we consider the development of the independent combinations of quantum number densities B, $B-L$, Π and ν_-. Table 8 gives the values of these quantum numbers

TABLE 8

Quantum numbers for particles contributing to baryon number production in SO(10) models with $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ effective symmetry

Particle	$SU(3) \otimes SU(2)_L \otimes U(1)_Y$	B	$B-L$	Π	ν_-
U_L^c	$[\bar{3}, 1, \frac{2}{3}]$	$-\frac{1}{3}$	$-\frac{1}{3}$	0	0
D_L, U_L	$[3, 2, -\frac{1}{6}]$	$\frac{1}{3}$	$\frac{1}{3}$	0	0
D_L^c	$[\bar{3}, 2, -\frac{1}{3}]$	$-\frac{1}{3}$	$-\frac{1}{3}$	-1	0
E_L, ν_L	$[1, 2, \frac{1}{2}]$	0	-1	-1	1
N_L^c	$[1, 1, 0]$	0	0	0	0
X, Y	$[3, 2, \frac{5}{6}]$		$-\frac{2}{3}$	0	
X′, Y′	$[3, 2, -\frac{1}{6}]$				
V	$[3, 1, -\frac{2}{3}]$	$\frac{1}{3}$			
W	$[1, 1, -1]$	0			0
S	$[3, 1, \frac{1}{3}]$				
φ	$[1, 2, -\frac{1}{2}]$	0			

TABLE 9

Quantum numbers and partial widths for supermassive boson decay modes in SO(10) models with $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ effective symmetry

Boson	Decay mode	Partial width	B	$B-L$	Π	ν_-
X, Y	E_LD_R, ν_LD_R	$\frac{1}{4}$	$\frac{1}{3}$	$-\frac{2}{3}$	0	1
	$U_L^cU_R^c, U_L^cD_R^c$	$\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{2}{3}$	0	0
	D_LE_R, U_LE_R	$\frac{1}{4}$	$\frac{1}{3}$	$-\frac{2}{3}$	0	0
X′, Y′	E_LU_R, ν_LU_R	$\frac{1}{4}$	$\frac{1}{3}$	$-\frac{2}{3}$	-1	1
	D_LN_R, U_LN_R	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{3}$	0	0
	$D_L^cU_R^c, D_L^cD_R^c$	$\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{2}{3}$	-1	0
V	$D_RE_L^c$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{4}{3}$	1	0
	$\nu_R^cU_L, E_R^cD_L$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{4}{3}$	1	-1
	$U_RN_L^c$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{3}$	0	0
W	$U_RD_L^c$	$\frac{3}{4}$	0	0	-1	0
	$N_RE_L^c$	$\frac{1}{4}$	0	1	0	0
S	$D_L\nu_L, U_LE_L$	$\frac{1}{4}$	$\frac{1}{3}$	$-\frac{2}{3}$	-1	1
	E_RU_R	$\frac{1}{8}$	$\frac{1}{3}$	$-\frac{2}{3}$	0	0
	$U_R^cD_R^c$	$\frac{1}{4}$	$-\frac{2}{3}$	$-\frac{2}{3}$	0	0
	$U_L^cD_L^c$	$\frac{1}{4}$	$-\frac{2}{3}$	$-\frac{2}{3}$	-1	0
	D_RN_R	$\frac{1}{8}$	$\frac{1}{3}$	$\frac{1}{3}$	1	0

for the various fields under consideration. Note that since the X, Y and X′, Y′ form $SU(2)_L$ doublets, the asymmetries $X_-^{(\prime)} = Y_-^{(\prime)}$. Using the decay rates for the X, X′, W, V, S, and S′ bosons given in table 9 one may derive the following set of Boltzmann equations for the evolution of the independent number densities

$$
\begin{aligned}
\dot{X}_+ &= -\langle\Gamma_X\rangle(X_+ - X_+^{eq})\,, \qquad (7.4.5)\\
\dot{X}_- &= -\langle\Gamma_X\rangle[X_- - \tfrac{1}{8}X_+^{eq}(X'_- - 5X_- + W_- + 2V_- - S_- - S'_-)]\,,\\
\dot{X}'_+ &= -\langle\Gamma_{X'}\rangle(X'_+ - X_+'^{eq})\,,\\
\dot{X}'_- &= -\langle\Gamma_{X'}\rangle[X'_- - \tfrac{1}{8}X_+'^{eq}(-X'_- + X_- - W_- - 2V_- + S_- + S'_- - (B-L))]\,,\\
\dot{V}_+ &= -\langle\Gamma_V\rangle(V_+ - V_+^{eq})\,,\\
\dot{V}_- &= -\langle\Gamma_V\rangle[V_- - \tfrac{1}{8}V_+^{eq}(4X_- + (B-L))]\,,\\
\dot{W}_+ &= -\langle\Gamma_W\rangle(W_+ - W_+^{eq})\,,\\
\dot{W}_- &= -\langle\Gamma_W\rangle[W_- - \tfrac{1}{8}W_+^{eq}(-2X'_- + 6X_- - 2W_- - 4V_- + 2S_- + 2S'_- - (B-L))]\,,\\
\dot{S}_+ &= -\langle\Gamma_S\rangle(S_+ - S_+^{eq})\,,\\
\dot{S}_- &= -\langle\Gamma_S\rangle[S_- - \tfrac{1}{8}S_+^{eq}(-2X_- - \tfrac{1}{2}(B-L))]\,,\\
\dot{S}'_+ &= -\langle\Gamma_{S'}\rangle(S'_+ - S_+'^{eq})\,,\\
\dot{S}'_- &= -\langle\Gamma_{S'}\rangle[S'_- - \tfrac{1}{8}S_+'^{eq}(-2X_- - \tfrac{1}{2}(B-L))]\,,\\
\dot{B} &= \langle\Gamma_X\rangle[(X_+ - X_+^{eq})R_X^B - 2X_-\\
&\quad + \tfrac{1}{2}X_+^{eq}(X'_- + 3X_- + W_- + 6V_- - S_- - S'_- + \nu_- + 2(B-L) - 4B)]\\
&\quad + \langle\Gamma_{X'}\rangle[(X'_+ - X_+'^{eq})R_{X'}^B - 2X'_-\\
&\quad - \tfrac{1}{2}X_+'^{eq}(X'_- - X_- - S_- - S'_- + W_- + \nu_- + (B-L) + 2B)]\\
&\quad + \langle\Gamma_S\rangle[(S_+ - S_+^{eq})R_S^B - S_- + \tfrac{1}{8}S_+^{eq}(4X_- + 6V_- + (B-L) - 6B)]\\
&\quad + \langle\Gamma_{S'}\rangle[(S'_+ - S_+'^{eq})R_{S'}^B - S'_- + \tfrac{1}{8}S_+'^{eq}(4X_- + 6V_- + (B-L) - 6B)]\\
&\quad + 4n_b\langle v\sigma'_X\rangle[X'_- + 11X_- + W_- + 10V_- - S_- - S'_- + 2\nu_- + 4(B-L) - 8B]\\
&\quad - 4n_b\langle v\sigma'_{X'}\rangle[X'_- - X_- - S_- - S'_- + W_- - 2V_- + 2\nu_- + (B-L) + 4B]\\
&\quad + 12h^4 n_b\langle v\sigma'_{S+S'}\rangle[4X_- + 4V_- + (B-L) - 4B]\,,\\
(\dot{B} - \dot{L}) &= \langle\Gamma_{X'}\rangle[(X'_+ + X_+'^{eq})R_{X'}^{B-L} - 5X'_-\\
&\quad - \tfrac{1}{2}X_+'^{eq}(3X'_- - 3X_- - 3S_- - 3S'_- + 3W_- + 5V_- - \Pi - \tfrac{1}{2}\nu_- + 3(B-L) + B)]
\end{aligned}
$$

$$+\langle\Gamma_{\mathrm{V}}\rangle[(V_+ - V_+^{\mathrm{eq}})R_{\mathrm{V}}^{B-L} + \tfrac{13}{2}V_-$$

$$-\tfrac{1}{4}V_+^{\mathrm{eq}}(2X'_- + 14X_- + 2W_- + 5V_- - 2S_- - 2S'_- - \Pi + 6(B-L) - B)]$$

$$+\langle\Gamma_{\mathrm{X}}\rangle[(W_+ - W_+^{\mathrm{eq}})R_{\mathrm{W}}^{B-L} + \tfrac{1}{2}W_- - \tfrac{1}{4}W_+^{\mathrm{eq}}(4X_- + V_- + \nu_- + (B-L) - B)]$$

$$+\langle\Gamma_{\mathrm{S}}\rangle[(S_+ - S_+^{\mathrm{eq}})R_{\mathrm{S}}^{B-L} - S_- + 3S_+^{\mathrm{eq}}(4X_- + 6V_- + (B-L) - 6B)]$$

$$+\langle\Gamma_{\mathrm{S'}}\rangle[(S'_+ - S_+'^{\mathrm{eq}})R_{\mathrm{S'}}^{B-L} - S'_- + 3S_+'^{\mathrm{eq}}(4X_- + 6V_- + (B-L) - 6B)]$$

$$-2n_{\mathrm{b}}\langle v\sigma'_{\mathrm{X'}}\rangle[7X'_- - 7X_- - 7S_- - 7S'_-$$

$$+7W_- + 10V_- - 4\Pi - 2\nu_- + 7(B-L) + 4B]$$

$$-n_{\mathrm{b}}\langle v\sigma'_{\mathrm{V}}\rangle[8X'_- + 4X_- + 8W_- + 20V_-$$

$$-8S_- - 8S'_- - 4\Pi + 11(B-L) - 4B]$$

$$-2n_{\mathrm{b}}\langle v\sigma'_{\mathrm{W}}\rangle[2X'_- + 10X_- + 2W_- + 8V_-$$

$$-2S_- - 2S'_- + 4\nu_- + 5(B-L) - 4B]$$

$$-n_{\mathrm{b}}h^4\langle v\sigma'_{\mathrm{S+S'}}\rangle[12X_- + 8\Pi + 8\nu_- + 3(B-L)] + n_{\mathrm{b}}h^2\langle v\sigma_{\varphi}\rangle\nu_-\,,$$

$$\dot{\nu}_- = \langle\Gamma_X\rangle[(X_+ - X_+^{\mathrm{eq}})R_{\mathrm{X}}^{\nu_-} + 3X_- - \tfrac{1}{4}X_+^{\mathrm{eq}}(2\Pi + 5\nu_-)]$$

$$+\langle\Gamma_{\mathrm{x'}}\rangle[(X'_+ - X_+'^{\mathrm{eq}})R_{\mathrm{X'}}^{\nu} + 3X'_- + \tfrac{1}{2}X_+'^{\mathrm{eq}}(2X'_- - 2X_- - 2S_- - 2S'_-$$

$$+2W_- + 5V_- - \Pi - \tfrac{3}{2}\nu_- + 2(B-L) - B)]$$

$$+\langle\Gamma_{\mathrm{V}}\rangle[(V_+ - V_+^{\mathrm{eq}})R_{\mathrm{V}}^{\nu} + \tfrac{3}{2}V_-$$

$$+\tfrac{1}{2}V_+^{\mathrm{eq}}(X'_- - X_- - S_- - S'_- + W_- + V_- - \Pi - 2\nu_- + (B-L) + B)]$$

$$+\langle\Gamma_{\mathrm{S}}\rangle[(S_+ - S_+^{\mathrm{eq}})R_{\mathrm{S}}^{\nu} + \tfrac{3}{2}S_-$$

$$-\tfrac{1}{4}S_+^{\mathrm{eq}}(X'_- - X_- - S_- - S'_- + W_- + V_- - \Pi + \nu_- + (B-L) - B)]$$

$$+\langle\Gamma_{\mathrm{S'}}\rangle[(S'_+ - S_+'^{\mathrm{eq}})R_{\mathrm{S'}}^{\nu_-} + \tfrac{3}{2}S'_-$$

$$-\tfrac{1}{4}S_+^{\mathrm{eq}}(X'_- - X_- - S_- - S'_- + W_- + V_- - \Pi + \nu_- + (B-L) + B)]$$

$$+2n_{\mathrm{b}}\langle v\sigma'_{\mathrm{X}}\rangle[3X'_- - 15X_- + 3W_- + 6V_- - 3S_- - 3S'_- - 4\Pi - 10\nu_-]$$

$$+2n_{\mathrm{b}}\langle v\sigma'_{\mathrm{X'}}\rangle[5X'_- - 5X_- - 5S_- - 5S'_-$$

$$+5W_- + 14V_- - 4\Pi - 6\nu_- + 5(B-L) - 4B]$$

$$+2n_{\mathrm{b}}\langle v\sigma'_{\mathrm{V}}\rangle[4X'_- - 4X_- - 4S_- - 4S'_-$$

$$+4W_- + 7V_- - 4\Pi - 5\nu_- + 4(B-L) + B]$$

$$-2n_{\mathrm{b}}h^4\langle v\sigma_{\mathrm{S+S'}}\rangle[8X'_- + 4X_- + 8V_-$$

$$-8S_- - 8S'_- - 8\Pi + 8\nu_- + 11(B-L) + 8B]$$

$$-2n_b h^2 \langle v\sigma_\varphi\rangle [4\nu_- + (B-L) - B],$$

$$\dot{\Pi} = \langle\Gamma_X\rangle[(X'_+ - X'^{eq}_+)R^\Pi_X - 9X'_- - \tfrac{1}{2}X'^{eq}_+(4X'_- - 4X_- - 4S_- - 4S'_-$$
$$+4W_- - 7V_- - \Pi - \tfrac{1}{2}\nu_- + 4(B-L) + B)]$$
$$+\langle\Gamma_V\rangle[(V_+ - V^{eq}_+)R^\Pi_V + \tfrac{9}{2}V_- - \tfrac{1}{4}V^{eq}_+(2X'_- + 10X_- + 2W_-$$
$$+5V_- - 2S_- - 2S'_- - \Pi + 5(B-L) - B)]$$
$$+\langle\Gamma_W\rangle[(W_+ - W^{eq}_+)R^\Pi_W - \tfrac{3}{2}W_- - \tfrac{1}{4}W^{eq}_+(2X'_- - 2X_- - 2S_-$$
$$-2S'_- + 2W_- + 5V_- + \nu_- + 2(B-L) - B)]$$
$$+\langle\Gamma_S\rangle[(S_+ - S^{eq}_+)R^\Pi_S - \tfrac{9}{4}S_- + \tfrac{1}{8}S^{eq}_+(6X'_-$$
$$-6X_- - 6S_- - 6S'_- + 6W_- + 12V_- - 7\Pi - \nu_- + 6(B-L))]$$
$$+\langle\Gamma_{S'}\rangle[(S'_+ - S'^{eq}_+)R^\Pi_{S'} - \tfrac{9}{4}S'_- + \tfrac{1}{8}S'^{eq}_+(6X'_- - 6X_- - 6S'_-$$
$$+6W_- + 12V_- - 7\Pi - \nu_- + 6(B-L))]$$
$$-2n_b\langle v\sigma'_{X'}\rangle[7X'_- - 7X_- - 7S_- - 7S'_-$$
$$+7W_- + 10V_- - 4\Pi - 2\nu_- + 7(B-L) + 4B]$$
$$-n_b\langle v\sigma'_V\rangle[8X'_- + 4X_- + 8W_- + 20V_- - 8S_-$$
$$-8S'_- - 4\Pi + 11(B-L) - 4B]$$
$$-2n_b\langle v\sigma'_W\rangle[2X'_- + 10X_- + 2W_- + 8V_-$$
$$-2S_- - 2S'_- + 4\nu_- + 5(B-L) - 4B]$$
$$+n_b h^4\langle v\sigma'_{S+S'}\rangle[32X'_- 4X_- + 32W_- + 80V_-$$
$$-32S_- - 32S'_- - 40\Pi + 4(B-L) - 16B]$$
$$-4n_b h^2\langle v\sigma_\varphi\rangle[\Pi - (B-L) - \tfrac{1}{2}\nu_-].$$

The averaged widths and cross sections appearing in these equations are given in subsects. 6.2 and 6.6. The effective *CP*-violation parameters, R^Q_χ, are given by a sum of the decay modes for each χ, weighted by the value of Q created in the decay and multiplied by an overall factor corresponding to the multiplicity of the decaying boson:

$$R^B_X = 2R(X \to E_L D_R) - 4R(X \to U^c_L D^c_R) + 2R(X \to D_L E_R),$$

$$R^B_{X'} = 2R(X' \to E_L U_R) - 4R(X' \to D^c_L D^c_R) + 2R(X' \to D_L N_R),$$

$$R^B_S = 2R(S \to D_L \nu_L) + R(S \to E_R U_R)$$
$$-2R(S \to U^c_R D^c_R) - 2R(S \to U^c_L D^c_L) + R(S \to D_R N_R),$$

$$R_{X'}^{B-L} = -4R(X' \to E_L U_R) + 2R(X' \to D_L N_R) - 4R(X' \to D_L^c D_R^c)\,,$$
$$R_V^{B-L} = 4R(V \to D_R E_L^c) + 4R(V \to E_R^c D_L) + R(V \to U_R N_L^c)\,,$$
$$R_W^{B-L} = R(W \to N_R E_L^c)\,,$$
$$R_S^{B-L} = -4R(S \to D_L \nu_L) - 2R(S \to E_R U_R)$$
$$\quad -2R(S \to U_R^c D_R^c) - 2R(S \to U_L^c D_L^c) + R(S \to D_R N_R)\,,$$
$$R_X^{\nu_-} = 6R(X \to E_L D_R)\,, \tag{7.4.6}$$
$$R_{X'}^{\nu_-} = 6R(X' \to E_L U_R)\,,$$
$$R_V^{\nu_-} = -3R(V \to E_R^c D_L)\,,$$
$$R_S^{\nu_-} = 6R(S \to D_L \nu_L)\,,$$
$$R_{X'}^{\Pi} = -6R(X' \to E_L U_R) - 6R(X' \to D_L^c D_R^c)\,,$$
$$R_V^{\Pi} = 3R(V \to D_R E_L^c) + 3R(V \to E_R^c D_L)\,,$$
$$R_W^{\Pi} = -R(W \to U_R D_L^c)\,,$$
$$R_S^{\Pi} = -6R(S \to D_L \nu_L) - 3R(S \to U_L^c D_L^c) + 3R(S \to D_R N_R)\,.$$

We take all the R_X, $R_{X'}$, R_W, and R_V to be zero. Neglecting m_N compared to m_S, we obtain

$$R(S \to E_R U_R) = R(S \to D_R N_R) = \tfrac{1}{2}R(S \to U_R^c D_R^c)$$
$$= -\tfrac{1}{2}R(S \to U_L^c D_L^c) = -R(S \to D_L \nu_L)\,, \tag{7.4.7}$$

so that as expected R_S^B and $R_{S'}^B$ vanish in this approximation. Eq. (7.4.7) then yields

$$R_S^{B-L} = -3R(S \to D_L \nu_L)\,,$$
$$R_S^{E} = 6R(S \to D_L \nu_L)\,, \tag{7.4.8}$$
$$R_S^{\Pi} = -15R(S \to D_L \nu_L)\,,$$

with corresponding relations for S′. In terms of the single *CP*-violation parameter ε these may be written

$$R(S \to D_L \nu_L) = \tfrac{1}{2}\pi\alpha\left(\frac{m_f}{m_W}\right)\varepsilon \,\mathrm{Im}\, I_{SS'}\,,$$
$$\tag{7.4.9}$$
$$R(S' \to D_L \nu_L) = -\tfrac{1}{2}\pi\alpha\left(\frac{m_f}{m_W}\right)\varepsilon \,\mathrm{Im}\, I_{S'S}\,,$$

where as in sect. 6 $\alpha \sim \frac{1}{40}$ is the gauge coupling constant, $m_W \sim 80$ GeV is the mass of the weak gauge boson and m_f is the effective mass of the heaviest family at the unification scale. We take $m_f/m_W = \frac{1}{20}$.

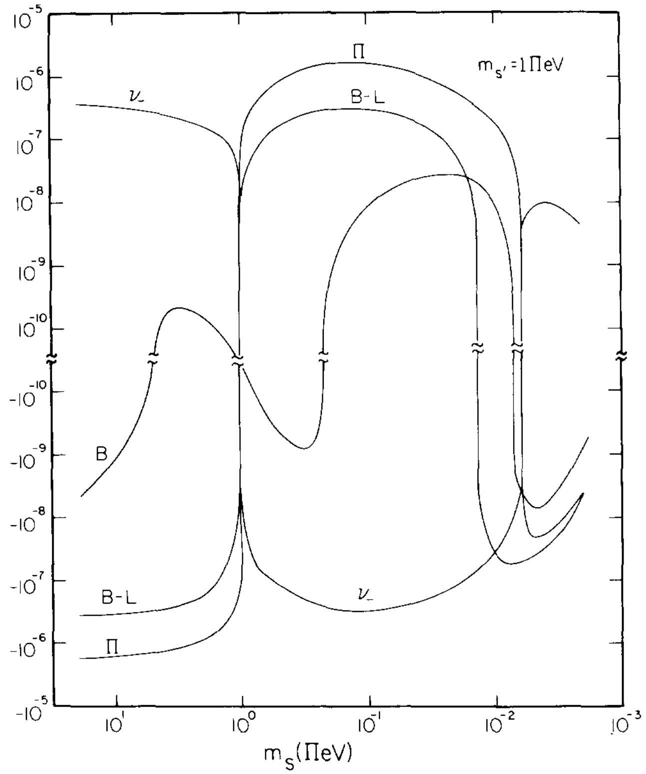


Fig. 25. Final quantum number densities (scaled by the *CP*-violation parameter ε) generated in an SO(10) model with no intermediate effective symmetry larger than $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. Results for Π and ν_- are obtained neglecting effects of light Higgs boson exchange at low temperatures. S and S′ are mass eigenstate 10_H Higgs bosons.

In subsect. 7.3 we showed that the SO(10) model discussed here can generate directly only asymmetries in $B-L$, ν_- and Π; asymmetries in B may arise only indirectly through conversion of these quantum numbers by inverse decay and $2 \rightarrow 2$ scattering processes. Fig. 25 shows the final baryon number and $B-L$ generated in this model, together with the values of ν_- and Π obtained ignoring low temperature light Higgs boson exchanges. The results assume $m_{S'} = 1\,\Pi\,\mathrm{eV}$. For $m_S > m_{S'}$, $B-L$, Π, and E are produced dominantly through the *CP*-violating decays of the S with their signs and magnitudes determined by the relations (7.4.8). When $m_S = m_{S'}$ the contributions from S decay and S′ decay exactly cancel and no asymmetries are produced. For $0.1\,\Pi\,\mathrm{eV} \leq m_S \leq m_{S'}$, S′ decays dominate and since $R(\mathrm{S} \rightarrow \mathrm{D_L E_L})$ is opposite in sign to $R(\mathrm{S'} \rightarrow \mathrm{D_L E_L})$ the values of the quantum numbers produced differ in sign from the case $m_S > m_{S'}$. For $m_S < 0.1\,\Pi\,\mathrm{eV}$, inverse decays into S tend to damp the asymmetries produced through S′ decay. The final values of the quantum numbers in this case depend sensitively on the values initially produced through S′ decay. A similar phenomenon was noted in subsect. 6.4. For $m_S \gtrsim 5\,\Pi\,\text{eV}$ B production in fig. 25 is dominated by inverse decay and $2 \to 2$ scattering processes mediated by X′. Inspection of the X′ inverse decay terms in (7.4.5) (or of the decay modes given in table 9) reveals that the combination of quantum numbers $(B-L)-B+\nu_-$ is conserved in these processes. Hence if only X′ exchange occurred, the final equilibrium values of quantum numbers would be non-zero and given by

$$B = \tfrac{1}{2}\Pi_0\,, \qquad E = -\tfrac{1}{2}\Pi_0\,, \qquad (B-L) = 0\,, \tag{7.4.10}$$

where Π_0 is the initial value of Π generated through S decays. Since $\Pi_0 < 0$, the X′ processes tend to produce a negative B. For $0.2\,\Pi\text{eV} < m_S < 5\,\Pi\text{eV}$, asymmetries are produced through S and S′ decays at temperatures below the X′ mass where X′ reactions are negligible. In this case B is dominantly produced through processes involving the X boson. Conservation of Π in X reactions leads to the equilibrium values [cf. eq. (6.4.5)]

$$E = -\tfrac{1}{5}\Pi_0\,, \qquad B = -\tfrac{1}{10}\Pi_0 + \tfrac{1}{2}(B-L)_0\,. \tag{7.4.11}$$

Since $R_S^{\Pi} = 5R_S^{B-L}$ the contributions to B tend to cancel and the resulting B is small. The fact that the X and X′ tend to produce B of opposite sign is a consequence of charge conjugation symmetry. As discussed in subsect. 7.3, unbroken C invariance would yield $m_X = m_{X'}$ and would cause the contributions of X and X′ to B production to cancel. For $m_S < 0.2\,\Pi\,\text{eV}$, B production is dominated by inverse decays into S. When m_S is sufficiently small, all asymmetries are reduced to zero.

If both the S and S′ are sufficiently light then B may also be produced directly since in this case the cancellation due to the charge conjugation symmetry is less effective.

The results of fig. 25 demonstrate that for m_S sufficiently light, the model considered in this section can generate sufficient B to accord with present observations, even though no B is produced directly through CP-violating decays. The magnitude and sign of the resulting baryon number depend sensitively, however, on the Higgs structure and the masses of the B-violating bosons. If m_N is comparable to m_S, then the B given in fig. 25 is an underestimate since then B production through CP-violating S decays may not be neglected.

7.5. B GENERATION IN SO(10) MODELS WITH $SU(4) \otimes SU(2)_L \otimes U(1)_R$ EFFECTIVE SYMMETRY

As described in subsect. 7.3, the production of baryon number in SO(10) models with $SU(4) \otimes SU(2)_L \otimes U(1)_R$ effective symmetry requires the presence of a 120_H with a C-violating mass splitting between two of its B-violating components. Since the 120_H cannot on its own account for observed fermion masses*, we include also a 10_H. We shall consider only those components in 120_H which may attain a C-violating mass splitting, and may thus contribute directly to B production.

The equations presented here may also be used to track the evolution of asymmetries produced in earlier stages. In particular, with effective $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ symmetry no B may be produced due to the unbroken charge conjugation symmetry. This restriction does not apply to asymmetries in Θ. The equation used here may also be used to treat the subsequent conversion of Θ to B when C is broken. With effective $SU(4) \otimes SU(2)_L \otimes U(1)_R$ symmetry, B may be produced directly through decays of $\tilde{S}$ and $\tilde{S}^c$. C symmetry implies that S decays may produce no net B (since $S \to S$ under C), while the B produced through $\tilde{S}$ decays must be opposite in sign to that produced in $\tilde{S}^c$ decays. To illustrate the conversion of Θ to B we will suppose that no B is produced directly through boson decays. This would be the case if asymmetries are produced dominantly through S decays but thermalized by the $\tilde{S}$ and $\tilde{S}^c$ bosons.

The quantum number assignments for the various fields are given in table 10. In this table a field stands for the asymmetry per member of an irreducible multiplet of $SU(4) \otimes SU(2)_L$. We will assume that the total charge associated with $U(1)_R$ is zero.

Using the decay modes for X, W_R, S, $\tilde{S}$, $\tilde{S}_1$, and $\tilde{S}^c$ listed in table 11, we obtain the following Boltzmann equations for the development of the independent

TABLE 10

Quantum numbers for particles contributing to baryon number production in SO(10) models with $SU(4) \otimes SU(2)_L \otimes U(1)_R$ effective symmetry

Particle	$SU(4) \otimes SU(2)_L \otimes U(1)_R$	B	Θ
D_L, U_L	$\langle 4, 2, 0\rangle$	$\frac{1}{4}$	$\frac{1}{4}$
D^c_L	$\langle \bar{4}, 1, \frac{1}{2}\rangle$	$-\frac{1}{4}$	$\frac{1}{4}$
U^c_L	$\langle \bar{4}, 1, \frac{1}{2}\rangle$	$-\frac{1}{4}$	$\frac{1}{4}$
X	$\langle 6, 2, -\frac{1}{2}\rangle$		0
W	$\langle 1, 1, 1\rangle$	0	0
S	$\langle 6, 1, 0\rangle$		
$\tilde{S}_1$	$\langle 6, 1, 1\rangle$		
$\tilde{S}$	$\langle 6, 1, 0\rangle$		
$\tilde{S}^c$	$\langle 6, 3, 0\rangle$		
φ	$\langle 1, 2, \frac{1}{2}\rangle$	0	

* Since the 120_H couples antisymmetrically to fermions, it must yield an antisymmetric fermion mass matrix with a zero eigenvalue for at least one out of three families.

TABLE 11

Quantum numbers and partial widths for supermassive boson decay modes in SO(10) models with $SU(4)\otimes SU(2)_L\otimes U(1)_R$ effective symmetry

Boson	Decay mode	Partial width	B	Θ
X	$D_R^c U_L^c$	$\frac{1}{2}$	$-\frac{1}{2}$	0
	$D_R U_L$	$\frac{1}{2}$	$\frac{1}{2}$	0
W	$D_L^c U_R$	1	0	0
S	$D_L U_L$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$
	$D_L^c U_L^c$	$\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{2}$
	$D_R^c U_R^c$	$\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{2}$
	$D_R U_R$	$\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{2}$
$\tilde{S}_1$	$D_L^c D_L^c$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$
	$U_R U_R$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
$\tilde{S}$	$D_L^c U_L^c$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$
	$D_R U_R$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
$\tilde{S}^c$	$D_L D_L$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{2}$
	$U_L U_L$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{2}$
	$D_L U_L$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{2}$
	$D_R^c D_R^c$	$\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{2}$
	$U_R^c U_R^c$	$\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{2}$
	$D_R^c U_R^c$	$\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{2}$

quantum number densities B and Θ:

$$
\begin{aligned}
\dot{X}_+ &= -\langle\Gamma_X\rangle(X_+ - X_+^{eq})\,,\\
\dot{S}_+ &= -\langle\Gamma_S\rangle(S_+ - S_+^{eq})\,,\\
\dot{\tilde{S}}_{1+} &= -\langle\Gamma_{\tilde{S}_1}\rangle(\tilde{S}_{1+} - \tilde{S}_{1+}^{eq})\,,\\
\dot{\tilde{S}}_+ &= -\langle\Gamma_{\tilde{S}}\rangle(\tilde{S}_+ - \tilde{S}_+^{eq})\,,\\
\dot{\tilde{S}}^c_+ &= -\langle\Gamma_{\tilde{S}^c}\rangle(\tilde{S}^c_+ - \tilde{S}_+^{ceq})\,,
\end{aligned}
\tag{7.5.1}
$$

$$
\begin{aligned}
\dot{B} = {}& -3\langle\Gamma_X\rangle X_+^{eq}B - \tfrac{3}{4}\langle\Gamma_S\rangle S_+^{eq}B - \tfrac{9}{4}\langle\Gamma_{\tilde{S}^c}\rangle\tilde{S}_+^{ceq}(\Theta+B)\\
& + [\tfrac{6}{4}\langle\Gamma_{\tilde{S}^+}\rangle\tilde{S}_{1+}^{eq} + \tfrac{3}{4}\langle\Gamma_{\tilde{S}^0}\rangle\tilde{S}_+^{eq}](\Theta-B) - 12n_b\langle v\sigma'_X\rangle B\\
& - 12n_b\langle v\sigma'_S\rangle B + 6[n_b\langle v\sigma'_{\tilde{S}_1}\rangle + n_b\langle v\sigma'_{\tilde{S}}\rangle](\Theta-B) - 12n_b\langle v\sigma'_{\tilde{S}^c}\rangle(\Theta+B),
\end{aligned}
$$

$$\begin{aligned}\dot{\Theta} = &\langle\Gamma_{\rm S}\rangle(S_+ - S_+^{\rm eq})R_{\rm S}^{\Theta} + \langle\Gamma_{\tilde{\rm S}^+}\rangle(\tilde{S}_{1+} - \tilde{S}_{1+}^{\rm eq})R_{\tilde{\rm S}_1}^{\Theta} + \langle\Gamma_{\tilde{\rm S}}\rangle(\tilde{S}_+ - \tilde{S}_+^{\rm eq})R_{\tilde{\rm S}^0}^{\Theta} \\ &+ \langle\Gamma_{\tilde{\rm S}^c}\rangle(\tilde{S}_+^{\rm c} - \tilde{S}_+^{\rm ceq})R_{\tilde{\rm S}^c}^{\Theta} - \tfrac{3}{4}\langle\Gamma_{\rm S}\rangle S_+^{\rm eq}\Theta \\ &- \tfrac{9}{4}\langle\Gamma_{\tilde{\rm S}^c}\rangle\tilde{S}_+^{\rm ceq}(\Theta + B) - [\tfrac{6}{4}\langle\Gamma_{\tilde{\rm S}_1}\rangle\tilde{S}_{1+}^{\rm eq} + \tfrac{3}{4}\langle\Gamma_{\tilde{\rm S}}\rangle\tilde{S}_+](\Theta - B) \\ &- 12 n_{\rm b}\langle v\sigma_{\rm S}'\rangle\Theta - 6[n_{\rm b}\langle v\sigma_{\tilde{\rm S}_1}'\rangle + n_{\rm b}\langle v\sigma_{\tilde{\rm S}}'\rangle](\Theta - B) \\ &- 12 n_{\rm b}\langle v\sigma_{\tilde{\rm S}^c}'\rangle(\Theta - B) - 4 n_{\rm b}\langle v\sigma_{\varphi}\rangle\Theta\,.\end{aligned}$$

Note that since all bosons are equally likely to decay to states with opposite values of B and Θ, the asymmetries χ_- in the boson fields do not enter into these equations. The W bosons conserve both B and Θ and thus also do not contribute to the development of B or Θ.

The total widths and cross sections appearing in these equations are given in subsect. 6.5. The effective *CP*-violation parameters are

$$\begin{aligned} R_{\rm S}^{\Theta} &= 3R({\rm S}\to{\rm D_L U_L}) + 3R({\rm S}\to{\rm D_L^c U_L^c}) - 3R({\rm S}\to{\rm D_R^c U_R^c}) - 3R({\rm S}\to{\rm D_R U_R})\,, \\ R_{\tilde{\rm S}}^{\Theta} &= 3R(\tilde{\rm S}\to{\rm D_L^c U_L^c}) - 3R(\tilde{\rm S}\to{\rm D_R U_R})\,, \\ R_{\tilde{\rm S}_1}^{\Theta} &= 3R(\tilde{\rm S}_1\to{\rm D_L^c D_L^c}) - 3R(\tilde{\rm S}_1\to{\rm U_R U_R})\,, \\ R_{\tilde{\rm S}^c}^{\Theta} &= 3R(\tilde{\rm S}^c\to{\rm D_L D_L}) + 3R(\tilde{\rm S}^c\to{\rm U_L U_L}) + 3R(\tilde{\rm S}^c\to{\rm D_L U_L}) \\ &\quad - 3R(\tilde{\rm S}^c\to{\rm D_R^c D_R^c}) - 3R(\tilde{\rm S}^c\to{\rm U_R^c U_R^c}) - 3R(\tilde{\rm S}^c\to{\rm D_R^c U_R^c})\,, \end{aligned} \tag{7.5.2}$$

which may be written using the partial widths from table 8 as

$$\begin{aligned} R_{\rm S}^{\Theta} &= 12R({\rm S}\to{\rm D_L U_L})\,, \\ R_{\tilde{\rm S}}^{\Theta} &= 6R(\tilde{\rm S}\to{\rm D_L^c U_L^c})\,, \\ R_{\tilde{\rm S}_1}^{\Theta} &= 6R(\tilde{\rm S}_1\to{\rm D_L^c U_L^c})\,, \\ R_{\tilde{\rm S}^c}^{\Theta} &= 12R(\tilde{\rm S}^c\to{\rm D_R^c D_R^c})\,. \end{aligned} \tag{7.5.3}$$

Since light Higgs φ exchange violates Θ, it presumably dominates these *CP*-violation parameters. Taking the Yukawa couplings of the $10_{\rm H}$ and $120_{\rm H}$ to be equal in magnitude and given by $\frac{1}{2}\sqrt{\frac{1}{2}}gm_{\rm f}/m_{\rm W}$, we then obtain

$$\begin{aligned} R({\rm S}\to{\rm D_L U_L}) &= \pi\alpha\frac{m_{\rm f}}{m_{\rm W}}\,\varepsilon\,{\rm Im}\,I_{{\rm S}\varphi}\,, \\ R(\tilde{\rm S}\to{\rm D_L^c U_L^c}) &= -2\pi\alpha\frac{m_{\rm f}}{m_{\rm W}}\,\varepsilon\,{\rm Im}\,I_{\tilde{\rm S}\varphi}\,, \\ R(\tilde{\rm S}_1\to{\rm D_L^c D_L^c}) &= -2\pi\alpha\frac{m_{\rm f}}{m_{\rm W}}\,\varepsilon\,{\rm Im}\,I_{\tilde{\rm S}_1\varphi}\,, \\ R(\tilde{\rm S}^c\to{\rm D_R^c D_R^c}) &= -\tfrac{2}{3}\pi\alpha\frac{m_{\rm f}}{m_{\rm W}}\,\varepsilon\,{\rm Im}\,I_{\tilde{\rm S}^0\varphi}\,. \end{aligned} \tag{7.5.4}$$

We will take $\tilde{S}_1$ to be degenerate in mass with $\tilde{S}$ in what follows. Since B is determined by the mass splitting between $\tilde{S}$ and $\tilde{S}^c$, this choice should have little effect on the final results.

Fig. 26a shows the final baryon number generated in this model, for a variety of values of m_S, $m_{\tilde{S}}$ and $m_{\tilde{S}^c}$. Figs. 26b, c show the development of B and Θ in

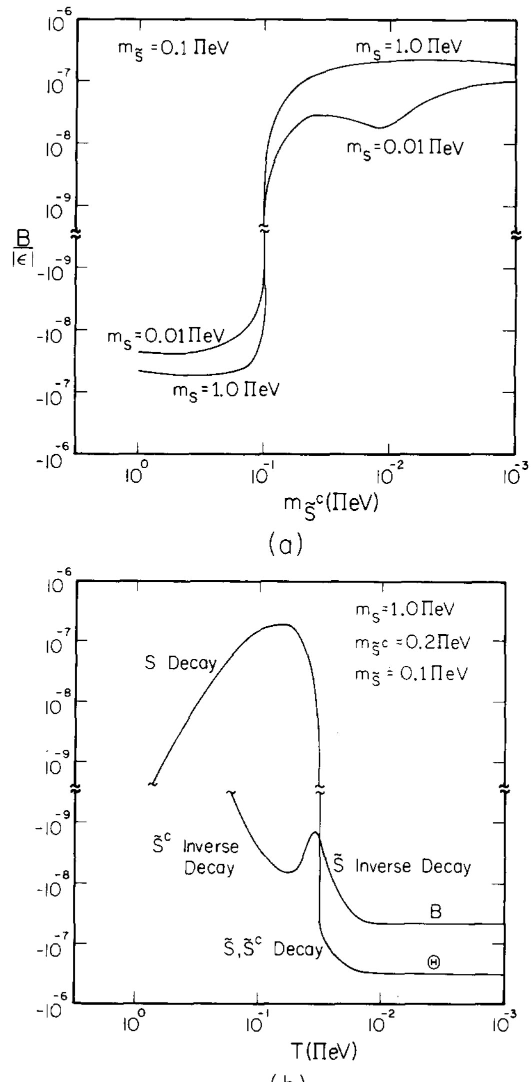
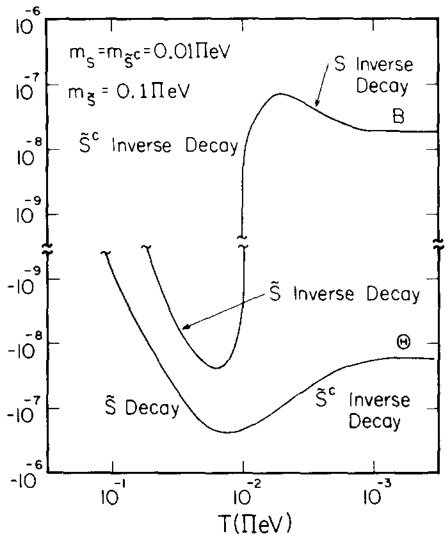


Fig. 26. Quantum number densities (scaled by the CP-violation parameter ε) generated in an SO(10) model with an $SU(4) \otimes SU(2)_L \otimes U(1)_R$ intermediate effective symmetry. $\tilde{S}$ and $\tilde{S}^c$ are mass eigenstate Higgs bosons occurring in 120_H, while S is a Higgs boson from 10_H. (a) shows the final baryon number generated for a range of S, $\tilde{S}$ and $\tilde{S}^c$ masses. (b) and (c) show the development of the independent quantum number densities Θ and B for two characteristic cases.



(c)

Fig. 26 (*cont'd.*)

two characteristic cases, and indicate the dominant processes in each temperature range. An asymmetry in Θ is produced by S, $\tilde{S}$ and $\tilde{S}^c$ decays. Asymmetry in B must then be generated by conversion of this asymmetry. Only $\tilde{S}$ and $\tilde{S}^c$ interactions violate C and thus may contribute to B.

In fig. 26b, $m_S > m_{\tilde{S}^c} > m_{\tilde{S}}$, so that $\tilde{S}^c$ inverse decays first convert positive Θ produced in S decay into a negative B. As the temperature falls below the $\tilde{S}$ mass, inverse decays into $\tilde{S}$ dominate and B is driven positive. When Θ is driven negative by $\tilde{S}$ and $\tilde{S}^c$ decays the $\tilde{S}$ inverse decays drive B negative again yielding a negative final baryon number. For $m_{\tilde{S}^c} < m_{\tilde{S}}$, the roles of $\tilde{S}$ and $\tilde{S}^c$ are reversed and the final baryon number is positive.

Fig. 26c shows the development of Θ and B when $m_{\tilde{S}} > m_S = m_{\tilde{S}^c}$. The final B produced is positive since $m_{\tilde{S}^c} < m_{\tilde{S}}$. B produced in $\tilde{S}^c$ inverse decays is reduced by S inverse decays. For $m_{S^c} < m_S$, B is produced after the effects of S inverse decays are important and as a result the final B is larger than in the previous case.

Appendix

NOTATION FOR FERMION FIELDS

We describe spin-$\frac{1}{2}$ fermions by two-component fields of definite chirality: left-handed fields are denoted ψ_L and right-handed fields ψ_R. For massless fermions, chirality and helicity are equivalent and the two chirality states are independent. Only one of the states need therefore be present (for massless neutrinos ν_R is absent).

For the two-component fields, ψ_L^c denotes the left-handed antiparticle of ψ_R, while ψ_R^c denotes the right-handed antiparticle of ψ_L. For fields in which both helicity states are present, parity (P) serves to interchange L and R components, while charge conjugation (C) interchanges particles with antiparticles, according to

$$\begin{aligned}
&P: && \psi_L \to \psi_R\,, && \psi_R \to \psi_L\,,\\
&C: && \psi_L \to \psi_L^c = \sigma_2 \psi_R^*\,, && \psi_R \to \psi_R^c = -\sigma_2 \psi_L^*\,,\\
&CP: && \psi_L \to -\sigma_2 \psi_L^*\,, && \psi_R \to \sigma_2 \psi_R^*\,,
\end{aligned}$$

where σ_2 is the Pauli matrix. These transformations are summarized in fig. 27. Note the important feature that while the separate operations of C and P interchange L and R components, the combined CP transformation does not modify the helicity state. Hence while the definition of individual C and P transformation properties require the presence of both L and R states, CP transformation properties may be defined for massless particles with only a single helicity state.

The two-component fermion fields may be collected into a four-component Dirac spinor describing a fermion of arbitrary helicity: $\Psi = \binom{\psi_L}{\psi_R}$. It is convenient to take the Dirac gamma matrices which act on this spinor in the Weyl representation:

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{pmatrix}, \qquad \gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

with σ^i $(i = 1, 2, 3)$ the usual Pauli matrices. (This representation differs from the more usual Dirac representation simply by the interchange $\gamma^0 \leftrightarrow \gamma^5$.)

The kinetic energy term in the fermion lagrangian is given by

$$\bar{\Psi} \not{\partial} \Psi = \psi_L^\dagger \sigma^\mu \partial_\mu \psi_L + \psi_R^\dagger \bar{\sigma}^\mu \partial_\mu \psi_R\,,$$

with $\sigma^\mu = (1, \sigma^i)$, $\bar{\sigma}^\mu = (1, -\sigma^i)$.

Fermion fields for which both helicity states are present may give a Dirac mass term

$$m\bar{\Psi}\Psi = m(\psi_R^\dagger \psi_L + \psi_L^\dagger \psi_R)\,.$$

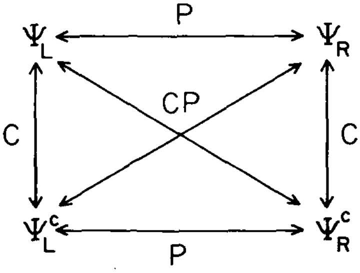


Fig. 27. Action of C and P transformations on two-component fermion fields.

If only one helicity is present, say ψ_L, no Dirac mass term may be constructed, but a Majorana mass term is still possible:

$$m\overline{\Psi^c}\,\frac{(1+\gamma_5)}{2}\,\Psi = m\psi_L^T\sigma_2\psi_L\,.$$

Here the charge-conjugate four-component spinor Ψ^c is given by

$$\Psi^c = \begin{pmatrix}\psi_L^c \\ \psi_R^c\end{pmatrix} = \begin{pmatrix}\sigma_2\psi_R^* \\ -\sigma_2\psi_L^*\end{pmatrix}.$$

For a fermion field with only a single helicity state, it is sometimes convenient to define a four-component Majorana spinor

$$\Psi_M = \begin{pmatrix}\psi_L \\ -\sigma_2\psi_L^*\end{pmatrix}$$

in terms of which the Majorana mass term becomes $\frac{1}{2}m\bar{\Psi}_M\Psi_M$.

Note that fields with Majorana mass terms may not carry any $U(1)_Q$ charges since the mass term is not invariant under gauge transformations $\psi_L \to e^{i\alpha Q}\psi_L$.

References

[1] P. Langacker, Phys. Reports, 72 (1981) 185

[2] S. Weinberg, Gravitation and cosmology (Wiley, New York, 1972);
C. Misner, K. Thorne and J. Wheeler, Gravitation (Freeman, San Francisco 1973);
L. Landau and E. Lifshitz, The classical theory of fields (Pergamon, Oxford, 1975) 4th ed

[3] A.D. Sakharov, ZhETF Pis'ma 5 (1967) 32

[4] E.W. Kolb and S. Wolfram, Nucl. Phys. B172 (1980) 224 (I); Phys. Lett. 91B (1980) 217

[5] J.A. Harvey, E.W. Kolb and S. Wolfram, Cosmology with very large gauge models, preprint in preparation

[6] D. Nanopoulos and S. Weinberg, Phys. Rev. D20 (1979) 2484

[7] S. Weinberg, Phys. Rev. Lett. 43 (1979) 1566;

[8] F. Wilczek and A. Zee, Phys. Rev. Lett. 43 (1979) 1571;

[9] H. Lipkin, Phys. Rev. Lett. 45 (1980) 311

[10] A. Hurlbert and F. Wilczek, Phys. Lett. 92B (1980) 95

[11] V.B. Berestetskii, E.M. Lifshitz and L.P. Pitaevskii, Relativistic quantum theory (Pergamon, Oxford, 1971);
J.D. Bjorken and S.D. Drell, Relativistic quantum mechanics (McGraw-Hill, New York, 1964)

[12] R. Slansky, Phys. Reports 79 (1981) 1

[13] M. Kobayashi and K. Maskawa, Prog. Theor. Phys. 49 (1975) 652

[14] R. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 42 (1979) 1651; Phys. Rev. D20 (1979) 3390; Phys. Lett. 89B (1979) 57

[15] L. Dolan and R. Jackiw, Phys. Rev. D9 (1974) 3320

[16] S. Weinberg, Phys. Rev. Lett. 37 (1976) 657

[17] R. Peccei, Proc. 19th Int. Conf. on High-energy physics, ed. S. Homma, M. Kawaguchi and H. Miyazawa, Phys. Soc. Japan, Tokyo (1979)

[18] E. Eichten, K. Lane and J. Preskill, Phys. Rev. Lett. 45 (1980) 225

[19] T.D. Lee, Phys. Rev. D8 (1973) 1226; Phys. Reports 9 (1974) 143

[20] K. Sato, Phys. Lett. 99B (1981) 66;
G. Senjanovic and F. Stecker, Phys. Lett. 96B (1980) 285;
R. Brown and F. Stecker, Phys. Rev. Lett. 43 (1979) 315

[21] J.W. Essam, Rep. Prog. Phys. 43 (1980) 833
[22] I. Kobzarev, L. Okun and Ya. Zeldovich, Phys. Lett. 50B (1974) 340;
Ya. Zeldovich, I. Kobzarev and L. Okun, ZhETF (USSR) 67 (1974) 3
[23] R. Brout and F. Englert, Nucl. Phys. B180 [FS2] (1981) 181
[24] M. Yoshimura, Phys. Rev. Lett. 41 (1978) 281 (E42 (1979) 746)
[25] S. Dimopoulos and L. Susskind, Phys. Rev. D18 (1978) 4500; Phys. Lett. 81B (1979) 416
[26] J. Ellis, M. Gaillard and D. Nanopoulos, Phys. Lett. 80B (1979) 360 (E82B (1979) 464)
[27] J. Fry, K. Olive and M. Turner, Phys. Rev. D22 (1980) 2953, 2977; Phys. Rev. Lett. 45 (1980) 2074
[28] A. Ignatev et al., Phys. Lett. 87B (1979) 114
[29] S. Weinberg, Phys. Rev. Lett. 42 (1979) 850
[30] G. Segre and H. Weldon, Phys. Rev. Lett. 44 (1980) 1737
[31] D. Montgomery and D. Tidman, Plasma kinetic theory (McGraw-Hill, New York, 1964);
T-Y. Wu, Kinetic equations of gases and plasmas (Addison-Wesley, New York, 1966)
[32] L. Kadanoff and G. Baym, Quantum statistical mechanics (Benjamin, Reading, 1962)
[33] S.R. de Groot, W.A. van Leeuwen and Ch.G. van Weert, Relativistic kinetic theory (North-Holland, 1980), part B
[34] A. Fetter and J. Walecka, Quantum theory of many-particle systems (McGraw-Hill, New York, 1971)
[35] J. Harvey, E. Kolb, D. Reiss and S. Wolfram, Nucl. Phys. B177 (1981) 456
[36] S. Wolfram, Phys. Lett. 82B (1979) 65.
[37] A. Guth, Phys. Rev. D23 (1980) 347
[38] J. Bond, E. Kolb and J. Silk, Astrophys. J., to be published
J. Barrow and M. Turner, Grand unified theories and galaxy formation, preprint (1981)
[39] J. Kristian, A. Sandage and J. Westphal, Ap. J. 221 (1978) 383
[40] M. Davis, J. Geller and J. Huchra, Ap. J. 221 (1978) 1
[41] J. Harvey, E. Kolb, D. Reiss and S. Wolfram, Phys. Rev. Lett. 47 (1981) 391;
E. Kolb, Astrophysics and grand unification, Los Alamos preprint LA-UR-81-1243 (Jan. 1981), Proc. 1981 Coral Gables Conf., to be published; Grand unification and cosmology – an environmental impact statement, Los Alamos preprint LA-UR-81-1855 (May, 1981), Proc. 5th Johns Hopkins Workshop
[42] H. Georgi and S.L. Glashow, Phys. Rev. Lett. 32 (1974) 438
[43] S. Treiman and F. Wilczek, Phys. Lett. 95B (1980) 222
[44] S. Barr, G. Segre and H. Weldon, Phys. Rev. D20 (1979) 2494
[45] A. Yildiz and P. Cox, Phys. Rev. D21 (1980) 906
[46] G. Segre and M. Turner, Phys. Lett. 99B (1981) 399
[47] J.A. Harvey, Baryon number generation and mass relations in SO(10) unified models, PhD thesis, Caltech (1981)
[48] D.B. Reiss, Some topics in grand unified models and the cosmological baryon asymmetry, PhD thesis, Caltech (1981)
[49] H. Georgi, *in* Particles and fields (AIP, New York, 1975);
H. Fritzsch and P. Minkowski, Ann. of Phys. 93 (1975) 193
[50] M. Gell-Mann and R. Slansky, unpublished;
R. Slansky, Charge conjugation and its violation in unified models, in *First* Workshop on Grand unification, ed. P.H. Frampton, S.L. Glashow and A. Yildiz (Math. Sci. Press, 1980)
[51] M. Gell-Mann, P. Ramond and R. Slansky, *in* Supergravity, ed. P. van Nieuwenhuizen and D.Z. Freedman (North-Holland, 1979)
[52] H. Georgi and D.V. Nanopoulos, Nucl. Phys. B154 (1979) 161
[53] J. Harvey, P. Ramond and D. Reiss, Phys. Lett. B92 (1980) 309
[54] V.A. Kuzmin and M.E. Shaposhnikov, Phys. Lett. B92 (1980) 115
[55] J.A. Harvey and E.W. Kolb, Phys. Rev. D24 (1981) 2090