Standard Model of Elementary Particle Interactions

L. N. Lipatov
Petersburg Nuclear Physics Institute
Gatchina, 188300, St. Petersburg, Russia

Abstract

The Electro-Weak Theory is reviewed.

1 Introduction

Elementary particles can participate in strong, electromagnetic and weak interactions. The gravity is essential only for very massive objects and at very high energies which can not be obtained in the laboratory experiments. All known hadrons are constructed from 6 types of quarks grouped in the three doublets (u, d), (c, s) and (t, b). Each doublet together with the corresponding doublets of leptons (ν_e, e), (ν_μ, μ) and (ν_τ, τ) belong to a multiplet known as a generation. The fermion masses (measured in GeV) are rather different:

$$\begin{pmatrix} \nu_e & e & u & d \\ 0? & 5\,10^{-4} & 3\,10^{-3} & 6\,10^{-3} \end{pmatrix},$$

$$\begin{pmatrix} \nu_\mu & \mu & c & s \\ 0? & 0.1 & 1.5 & 0.1 \end{pmatrix},$$

$$\begin{pmatrix} \nu_\tau & \tau & t & b \\ 0? & 1.8 & 175 & 4 \end{pmatrix}.$$

and according to the electro-weak theory they appear as a result of the Yukawa interactions of the corresponding massless fermions with the Higgs boson H. The electroweak forces are caused by an exchange of the vector bosons W, Z and photons.

In 1998 there were performed underground experiments in Super Kamiokande (Japan) devoted to the measurement of the neutrino content of atmospheric showers. According to these experiments the various neutrinos (presumably ν_μ and ν_τ) mix one with another. This mixing implies in particular that neutrinos have non-zero masses. The analogous mixing of ν_e with other neutrinos can explain the deficit of the solar neutrinos ν_e at the underground experiments in comparison with predictions of the theory of nuclear reactions inside our Sun. Moreover, recently other neutrinos (presumably ν_μ or ν_τ) were discovered in the flow of particles going from the Sun, which can be considered as an evidence for the mixing of the electronic neutrinos. Further, the results of a new Japan experiment with the beam of neutrinos obtained from the accelerator in KEK and measured in Super Kamiokande (situated in several hundred kilometers from KEK) are similar to those obtained with the atmospheric neutrinos.

The quarks participate also in strong interactions. They exist in three different colour states and their colour charge causes the emission and absorption of 8 massless gluons. The theory of strong interactions - Quantum Chromodynamics (QCD) is based on the Yang-Mills model with the gauge group $SU(3)$. Gluons are quanta of the gauge field A^a_μ ($a = 1, 2, ...8$). They are transformed according to the adjoint representation of $SU(3)$. The quarks u, d, c, s, t and b belong to the fundamental representation of this group. Their interaction Lagrangian is $L^{QCD}_{int} = g_s\, j^a_\mu\, A^a_\mu$, where $j^a_\mu = \overline{\psi}\lambda^a\gamma_\mu\psi$ is the vector current, constructed in an universal way from the quark fields, and g_s is the QCD coupling constant.

Due to the effect of running the QCD coupling constant $\alpha_s \sim 1/\ln\frac{q}{\Lambda_{QCD}}$ one can use the perturbation theory for the theoretical description of hard processes occurring at large momentum transfers $q \gg \Lambda_{QCD}$, where $\Lambda_{QCD} \simeq 200\,MeV$ is the mass scale in QCD for which $\alpha_s = g_s^2/(4\pi) \sim 1$. The growth of g_s in the region of small q (large distances) leads to the confinement of quarks and gluons and causes a rearrangement of the QCD vacuum state resulting in an appearance of vacuum condensates of gluon and quark fields. The quarks u

and d are practically massless in comparison with the QCD scale Λ_{QCD}, which is a reason of an approximate isotopic and γ_5-invariance. The existence of a non-vanishing vacuum condensate $< 0|\overline{\psi}\psi|0 >\sim 100\, MeV$ violates the γ_5-invariance and leads to an appearance of the Goldstone particle - π-meson whose interaction is universal.

The charged quarks and leptons participate also in electromagnetic interactions described by the Quantum Electrodynamics (QED) based on the Abelian gauge theory with the interaction Lagrangian $L_{int}^{QED} = e\; j_\mu^{el} A_\mu$. The vector potential A_μ describes photons, e denotes the electron charge ($\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$) and $j_\mu^{el} = \sum_i Q_i \overline{\psi}_i \gamma_\mu \psi_i$ is the electromagnetic current constructed from a linear combination of the fermion fields $\overline{\psi}_i$, ψ_i with coefficients Q_i proportional to their electric charges measured in units of the electron charge e. The effective charge $e(q^2)$ grows at large momentum transfers q, which leads to the well-known Landau-Pomeranchuck problem of vanishing the physical charge in the local limit.

The first phenomenological theory of weak interactions was suggested in 1934 by the Italyn physicist E. Fermi for the nuclear β-decay. It is based on the Lagrangian $L_{int}^{we} = \frac{G}{\sqrt{2}} j_\mu^w j_\mu^w$ where $G \simeq 10^{-5}/GeV^2$. The charged weak currents j_μ^w are responsible in particular for the transitions $e \to \nu$ or $n \to p$. In 1956 the american theorists T.D. Lee and C.N. Yang assumed, that the parity is not conserved in the weak interactions. This hypothesis was verified soon experimentally by C.S. Wu with collaborators. In 1964 J. Cronin with colleagues discovered a non-conservation of the combined charge and space parity (CP) in K_0-decays.

The theory of electro-weak interactions, based on the Yang-Mills theory with the gauge group $SU(2) \times U(1)$, was constructed in 60-s by S. Glashow, S. Weinberg and A. Salam. They predicted the existence of the intermediate vector bosons $W^\pm$ and Z responsible for the weak processes induced in L_{int} by the product of charged or neutral currents. These vector bosons were discovered experimentally in 1984 by C. Rubbia with collaborators. Now the properties of the intermediate bosons are thoroughly investigated. The heaviest predicted fermion - the t-quark was discovered only in 1995.

Now the Standard Model of elementary particle interactions is in a good agreement with all experimental data, although in its formulation in future there could be some modifications. Nevertheless, the Higgs particle important for the self-consistency of the theory is not yet discovered. Its mass according to the CERN experimental data should be larger than 110 GeV. Moreover, from the radiative corrections to the electroweak processes the Higgs mass is estimated also around 100 GeV.

2 Massless particles in quantum field theory

In the Standard Model the fermions and vector bosons are considered initially as massless particles, because in the opposite case the theory will not be self-consistent. The mechanisms responsible for the conservation of this massless property are different for elementary particles with different spins. The simplest way to guarantee that the scalar particle will not obtain a non-zero mass in upper orders of the perturbation theory, is to use the Goldstone mechanism. For example, one can consider the isotopically invariant theory of the self-interacting complex scalar field φ transforming as an iso-doublet

$$\varphi \to e^{i\frac{1}{2}\overrightarrow{\chi}\overrightarrow{\tau}}\varphi\,,$$

where $\overrightarrow{\tau}$ are the Pauli matrices and $\overrightarrow{\chi}$ are the parameters of isotopic rotations. The corresponding Lagrangian is

$$L_\varphi = |\partial_\mu \varphi|^2 - \frac{1}{2}\,\lambda^2 \left(|\varphi|^2 - \frac{1}{2}\,\eta^2 \right)^2 .$$

The minimum of the energy in the classical approximation appears at

$$\varphi_{cl} = \frac{|\,\eta|}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

and we extract this solution among many others by choosing an appropriate phase. Indeed, if one will parametrize the scalar field as follows

$$\varphi = e^{i\,\frac{1}{2}\overrightarrow{\chi}\,\overrightarrow{\sigma}} \begin{pmatrix} 0 \\ \frac{|\,\eta|}{\sqrt{2}} + \frac{H}{\sqrt{2}} \end{pmatrix} ,$$

its potential energy does not depend on the angles $\overrightarrow{\chi}$:

$$L_\varphi = |\partial_\mu e^{i\,\frac{1}{2}\overrightarrow{\chi}\,\overrightarrow{\sigma}} \begin{pmatrix} 0 \\ \frac{|\eta|}{\sqrt{2}} + \frac{H}{\sqrt{2}} \end{pmatrix} |^2 - \frac{1}{2}\,\lambda^2 H^2 \left(|\,\eta| + \frac{H}{2} \right)^2 .$$

and therefore the fields $\chi^a(x)$ describe the massless scalar particles in an accordance with the Goldstone theorem claiming, that in the case of the spontaneous violation of the continuous symmetry there appears a massless particle restoring this symmetry.

In the gauge theories the vector fields interact with the conserved vector current $\overline{\psi}\gamma_\mu\psi$ or axial one $\overline{\psi}\gamma_\mu\gamma_5\psi$, which is invariant under the multiplication of the spinor fields by the matrix γ_5 corresponding to the helicity conservation. This invariance guarantees the massless property of the corresponding fermions in all orders of the perturbation theory. The interaction of the fermions with scalar or pseudoscalar particles also leave them massless in the perturbation theory. In the Standard Model the fermions obtain their masses only as a consequence of the spontaneous appearance of the scalar field condensate η.

The massless vector particle has only two degrees of freedom corresponding to the polarization vectors $l^\xi_\mu = (\delta_{\mu 1} + \xi \delta_{\mu 2})/\sqrt{2}$ with two helicities $\xi = \pm 1$ being the projection of the spin $\overrightarrow{s}$ on the direction of the particle momentum $\overrightarrow{k}$ (chosen in such way that $k_1 = k_2 = 0$). Nevertheless, to describe the vector particle in a Lorentz-invariant way one should introduce the vector field W_μ ($\mu = 0, 1, 2, 3$) with four degrees of freedom. On the mass shell $k^2 = 0$ we can impose the Lorentz condition $k_\mu\, l_\mu = 0$ for the polarization vector l_μ of the massless vector particle and remove its longitudinal component by an appropriate gauge transformation $\delta l_\mu \sim k_\mu$. Therefore the gauge invariance is important for reducing l_μ to two physical degrees of freedom for the massless particle. In the Abelian gauge theory-QED one can introduce the photon mass without violating the renormalizability of the theory, but in the Yang-Mills model only the Higgs mechanism allows one to provide a non-zero mass to the vector bosons.

3 Gauge symmetry of the electro-weak theory

Lagrangian of the Electro-Weak Theory is

$$L = -\frac{1}{2} tr\, G^2_{\mu\nu} - \frac{1}{4} F^2_{\mu\nu} + i \sum_k \overline{\psi}_k\, \widehat{D}\, \psi_k + |D_\mu \varphi|^2 - \frac{1}{2}\,\lambda^2 \left(|\varphi|^2 - \frac{1}{2}\,\eta^2 \right)^2 + L_{Yu} ,$$

where the Yukawa Lagrangian is

$$L_{Yu} = -\sum_k f_k^{(-)} \left(\overline{\psi^a}_{k\,L}\, \psi_{k\,R}^{(-)}\, \varphi^a + \overline{\psi}_{k\,R}^{(-)}\, \psi_{k\,L}^a\, \varphi^{a*} \right) - \sum_k f_k^{(+)} \left(\overline{\psi^a}_{k\,L}\, \psi_{k\,R}^{(+)}\, \widetilde{\varphi}^a + \overline{\psi}_{k\,R}^{(+)}\, \psi_{k\,L}^a\, \widetilde{\varphi}^{a*} \right).$$

The symbol tr means the trace in the matrix 2×2 describing the weak isotopic spin of the W-bosons. Only left fermion fields ψ_{kL} with the helicity $\xi = -\frac{1}{2}$ interact with the Yang-Mills field W_μ. and fermions with both helicities - left (L) and right (R) interact with the Abelian field B_μ. The spinors $\psi_{k\,R}^{(-)}$ and $\psi_{k\,R}^{(+)}$ describe the right fermions corresponding respectively the low and upper components of the left doublets. The intensities $F_{\mu\nu}$ and $G_{\mu\nu}$ for the vector fields W_μ and B_μ are given below

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu\,,\ G_{\mu\nu} = \overrightarrow{G_{\mu\nu}}\,\overrightarrow{t} = \partial_\mu W_\nu - \partial_\nu W_\mu - ig\,[W_\mu\,, W_\nu]\,.$$

Here the weak isospin group generators $\overrightarrow{t} = \frac{1}{2}\overrightarrow{\tau}$ satisfy the commutation relations

$$\left[t^a, t^b\right] = i\varepsilon^{abc} t^c\,,\ tr\,(t^a t^b) = \frac{1}{2}\delta^{ab}$$

and are expressed in terms of the Pauli matrices

$$\tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\,,\ \tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\,,\ \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\,,$$

acting on the isospin indices a of the spinors ψ_k^a.

The gauge fields B_μ and $W_\mu = \overrightarrow{W_\mu}\,\overrightarrow{t}$ describe the vector bosons B and W^a responsible for the electro-weak interactions. As it was said above, the fields $\psi_{k\,L}^a$ correspond to the massless fermions with the helicity $\xi = -\frac{1}{2}$:

$$\psi_{k\,L}^a = \frac{1+\gamma_5}{2}\,\psi_k^a = \begin{pmatrix} \chi_L^a \\ -\chi_L^a \end{pmatrix}\,,\ \gamma_5 = -\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}\,.$$

The spinor χ_L^a for the real particle with the momentum $\overrightarrow{p}$ satisfies the Dirac equation

$$(\overrightarrow{\sigma}, \overrightarrow{p})\,\chi_L^a = -\,|\overrightarrow{p}|\,\chi_L^a\,,\ \ \chi_L^a = (2\,|\overrightarrow{p}|\,(|\overrightarrow{p}| + p_z))^{-\frac{1}{2}} \begin{pmatrix} -p_x + ip_y \\ |\overrightarrow{p}| + p_z \end{pmatrix} \chi^a,$$

where χ^a is a normalized weak iso-spinor: $|\chi^a|^2 = 1$.

The fields $\psi_{k\,L}^a$ for quarks (q) and leptons (l) belong to fundamental representations of the gauge $SU(2)$-group described for the first generation by the iso-spinors

$$\psi_{qL} = \begin{pmatrix} \psi_L^{(u)} \\ \psi_L^{\prime\,(d)} \end{pmatrix}\,,\ \psi_{lL} = \begin{pmatrix} \psi_L^{\prime(\nu)} \\ \psi_L^{(e)} \end{pmatrix}\,,$$

where $\psi_L^{\prime\,(d)}$ is a linear combination (unitary transformation) of the fields describing the physical d, s and b quarks (see below the discussion of the Cabbibo-Kobayashi-Maskava matrix). Analogously $\psi_L^{\prime(\nu)}$ are linear combinations of the neutrino states with definite masses. The spinors $\psi_R^{(+)} = \psi_R^{(u,\nu)}$ and $\psi_R^{(-)} = \psi_R^{(d,e)}$ for the right fermions with $\xi = 1/2$ are transformed as singlets under the action of the group $SU(2)$. Therefore the mass term for fermions in the Lagrangian is impossible because it would lead to the violation of the invariance under the gauge group $SU(2)$.

The complex scalar doublet φ^a is introduced to provide masses to fermions $\psi^{(i)}$ and vector bosons with the use of the Higgs mechanism related to the appearance of its vacuum condensate $v = \sqrt{1/2}\,\eta$ as a result of the spontaneous symmetry breaking of the $SU(2)$-symmetry. The field $\widetilde{\varphi}$ is constructed from the components of the field φ as follows

$$\widetilde{\varphi} = i\tau_y \varphi^*$$

to provide masses to fermions $\psi^{(d)}$. One can easily verify, that due to the known properties of the Pauli matrices τ_i both φ and $\widetilde{\varphi}$ are transformed as isospinors under the action of the group $SU(2)$:

$$\varphi' = e^{i\overrightarrow{t}\,\overrightarrow{\chi}}\varphi\,, \;\; \widetilde{\varphi}' = e^{i\overrightarrow{t}\,\overrightarrow{\chi}}\widetilde{\varphi}\,.$$

The parameters $f_k^{(+)}$ and $f_k^{(-)}$ are the Yukawa coupling constants and λ is the corresponding self-coupling for the Higgs fields.

The quantities D_μ are the covariant derivatives:

$$D_\mu = \partial_\mu - ig\,\frac{\overrightarrow{\tau}}{2}\,\overrightarrow{W_\mu} - ig'\frac{Y}{2}\,B_\mu\,,$$

where g and g' are the gauge coupling constants for the non-Abelian and Abelian interactions correspondingly. $\overrightarrow{T} = \frac{\overrightarrow{\tau}}{2}$ is the weak isotopic spin and Y is the weak hypercharge which is different for left and right fermions. For all particles these quantum numbers are related by the Gell-Mann - Nishijima formula:

$$Q = T^3 + \frac{Y}{2}\,,$$

where Q is the electric charge. This formula will be derived below. Because inside each multiplet there are particles with opposite values of T^3, the quantum number $\frac{Y}{2}$ is equal to an average electric charge of the multiplet.

For the first generation the quantum numbers of the right and left neutrino ν^e, electron e, up- (u) and down- (d) quarks are

particle	$\nu_L'^e$	e_L	e_R	u_L	d_L'	u_R	d_R
T^3	$1/2$	$-1/2$	0	$1/2$	$-1/2$	0	0
Y	-1	-1	-2	$1/3$	$1/3$	$4/3$	$-2/3$
Q	0	-1	-1	$2/3$	$-1/3$	$2/3$	$-1/3$

In the second and third generations the corresponding fermions are μ- and τ-leptons, corresponding neutrinos ν^μ and ν^ι, strange-(s), charm- (c), beauty- (b) and top- (t) quarks with the quantum numbers:

particle	$\nu_L'^\mu$	μ_L	μ_R	c_L	s_L'	c_R	s_R
T^3	$1/2$	$-1/2$	0	$1/2$	$-1/2$	0	0
Y	-1	-1	-2	$1/3$	$1/3$	$4/3$	$-2/3$
Q	0	-1	-1	$2/3$	$-1/3$	$2/3$	$-1/3$

and

particle	ν'^{τ}_L	τ_L	τ_R	t_L	b'_L	t_R	b_R
T^3	$1/2$	$-1/2$	0	$1/2$	$-1/2$	0	0
Y	-1	-1	-2	$1/3$	$1/3$	$4/3$	$-2/3$
Q	0	-1	-1	$2/3$	$-1/3$	$2/3$	$-1/3$

The left quarks with $T^3 = -1/2$ enter in the weak doublets with the corresponding upper quarks u, c and t in the mixed states

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \end{pmatrix},$$

where V is the unitary Cabbibo-Kobayashi-Maskawa matrix (an analogous matrix exists for neutrinos). Note, that each quark can be in three different colour states.

In a general case the Yukawa contribution written in terms of the doublets of left fields ψ'^a_{kL} and corresponding right fields ψ'_{kR} is constructed in a gauge-invariant way as follows

$$-\sum_{k,r}\left(f^{(-)}_{kr}\overline{\psi'^a}_{k\,L}\,\psi'^{(-)}_{rR}\,\varphi^a + f^{(-)*}_{kr}\overline{\psi'}^{(-)}_{k\,R}\,\psi'^a_{rL}\,\varphi^{a*}\right) - \sum_{k,r}\left(f^{(+)}_{kr}\overline{\psi'^a}_{k\,L}\,\psi'^{(+)}_{r\,R}\,\widetilde{\varphi}^a + f^{(+)*}_{kr}\overline{\psi}'^{(+)}_{k\,R}\,\psi'^a_{r\,L}\,\widetilde{\varphi}^{a*}\right),$$

where $f^{(\pm)}_{kr}$ are arbitrary matrices (not necessary hermitian). These matrices can be diagonalized for quarks and leptons by the use of different unitary transformations for the left and right fields $\psi'^a_{rL} = U_{rs}\psi^a_{sL}$, $\psi'^a_{rR} = V_{rs}\psi^a_{sR}$. It is related to the fact, that due to the symmetry properties of the QCD and electro-weak theory these unitary transformations are independent. For quarks in the gauge where $\varphi = \begin{pmatrix} 0 \\ \varphi_0 \end{pmatrix}$ the upper components of the iso-spinors $\overline{\psi'^a}_{k\,L}$ can be chosen to coincide with the physical fields ψ^a_s in terms of which the mass matrix is diagonal. On the contrary for leptons the low components of the iso-spinor can be chosen to coincide with the physical particles e, μ and τ. In this case the upper components will be mixtures of the massive neutrinos, which would lead to neutrino oscillations in an agreement with the Super-Kamiokande experiments.

The W-bosons have a vanishing hypercharge Y and can be in three states W^+, W^- and W^0 with their electric charges $Q = 1$, -1 and 0. The fields

$$W_{\mp} = \frac{1}{\sqrt{2}}(W^1 \mp iW^2)$$

describe the production of corresponding particles and the annihilation of antiparticles. Indeed, their interaction with fermions can be written as follows

$$g\overline{\psi}_{kL}\frac{\overrightarrow{\tau}}{2}\,\overrightarrow{W_\mu}\gamma_\mu\,\psi'_{kL} = g\overline{\psi}_{kL}\left(\frac{\tau_3}{2}\,W^3_\mu + \frac{\tau_+W_{-\mu} + \tau_-W_{+\mu}}{\sqrt{2}}\right)\gamma_\mu\,\psi'_{kL}$$

and leads due to the properties of the Pauli matrices

$$\tau_- = \frac{\tau_1 - i\tau_2}{2} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad \tau_+ = \frac{\tau_1 + i\tau_2}{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

to the processes of the type $e + W^+ \to \nu$ going with the conservation of the electric charge Q and the hypercharge Y.

The quantum numbers of the Higgs bosons

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}, \quad \varphi^* = \begin{pmatrix} \varphi^- \\ \overline{\varphi}^0 \end{pmatrix}$$

are the following:

boson	φ^+	φ^0	φ^-	$\overline{\varphi}^0$
T^3	$1/2$	$-1/2$	$-1/2$	$1/2$
Y	1	1	-1	-1
Q	1	0	-1	0

.

The fields φ describe the annihilation of corresponding particles and production of antiparticles. One can verify, that in all processes, induced by the above Lagrangian, the electric charge and hypercharge are conserved together with the third component of the weak isotopic spin. For example, the contributions, proportional to f_k^- and f_k^+, give transitions $d_R + \varphi^+ \to u_L$ and $u_R + \varphi^- \to d_L$ correspondingly.

The conservation of the weak isospin and the hypercharge is very important for the symmetry properties of the Standard Model. Namely, the theory is invariant under the gauge transformations:

$$\varphi \to \exp(i\frac{g}{2}\overrightarrow{\tau}\overrightarrow{\chi})\,\exp(i\frac{g'}{2}\,Y\,\chi)\,\varphi\,,\ \psi \to \exp(i\frac{g}{2}\overrightarrow{\tau}\overrightarrow{\chi})\,\exp(i\frac{g'}{2}\,Y\,\chi)\,\psi,$$

$$W_\mu \to \left(\exp(i\frac{g}{2}\overrightarrow{\tau}\overrightarrow{\chi})\left(W_\mu + ig^{-1}\partial_\mu\right)\exp(-i\frac{g}{2}\overrightarrow{\tau}\overrightarrow{\chi})\right),$$

$$B_\mu \to \left(\exp(i\frac{g'}{2}Y\,\chi)\left(B_\mu + i\frac{2}{Y}\,g'^{-1}\partial_\mu\right)\exp(-i\frac{g'}{2}Y\,\chi)\right),$$

because the covariant derivatives and intensities of gauge fields are transformed in this case as follows

$$D_\mu \to \exp(i\frac{g}{2}\overrightarrow{\tau}\overrightarrow{\chi})\,\exp(i\frac{g'}{2}\,Y\,\chi)\;D_\mu\exp(-i\frac{g}{2}\overrightarrow{\tau}\overrightarrow{\chi})\exp(-i\frac{g'}{2}\,Y\,\chi)$$

$$G_{\mu\nu} \to \exp(i\frac{g}{2}\overrightarrow{\tau}\overrightarrow{\chi})G_{\mu\nu}\exp(-i\frac{g}{2}\overrightarrow{\tau}\overrightarrow{\chi})\,,\ F_{\mu\nu} \to F_{\mu\nu}\,.$$

As one can easily verify, the field $\widetilde{\varphi} = i\tau_2\varphi^*$ is transformed under the gauge transformation in the same way as φ:

$$\widetilde{\varphi} \to \exp(i\frac{g}{2}\overrightarrow{\tau}\overrightarrow{\chi})\,\exp(i\frac{g'}{2}\,Y\,\chi)\,\widetilde{\varphi}\,,$$

which is important for the gauge invariance of the Lagrangian. Because only left components of fermions interact with the W-bosons, we can not include in the Lagrangian the Dirac mass term $m\overline{\psi}\psi = m\left(\overline{\psi}_R\psi_L + \overline{\psi}_L\psi_R\right)$ without violating the gauge invariance. It is the reason, why initially the fermions are considered to be massless and their mass appears as a result of the Higgs mechanism (see below).

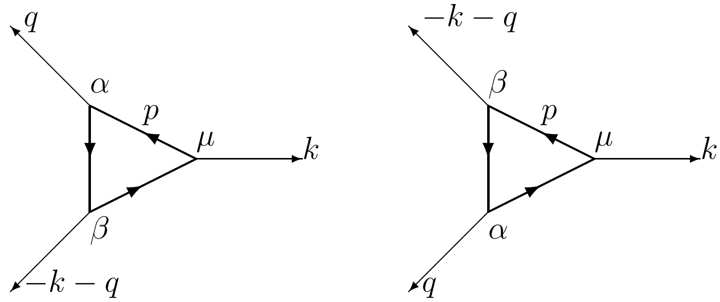

Figure 1: Triangle anomaly diagrams

4 Quantum anomalies in the Standard Model

The γ_5-invariance of the theory could be spoiled by the quantum anomalies destroying the gauge invariance and as a consequence one could lost the renormalizability of the Standard Model. The bosons $\overrightarrow{W}$ and B interact with the operators $\overrightarrow{j_\mu}$ and j_μ being the sums of vector and axial currents. They are conserved on the classical level, but on the one-loop level this conservation is violated in the triangle diagrams, constructed from the closed fermion line which connects three vector boson vertices. Let us consider for example the Adler-Bell-Jackiw diagrams for the interaction of two vector and one axial currents (see Fig. 1). Their contribution can be written as follows

$$\Gamma_{\mu\alpha\beta} = -\int \frac{d^4p}{(2\pi)^4}\, tr\, \left(\gamma_\mu\gamma_5\, \frac{1}{\widehat{p}+\widehat{k}}\, \gamma_\beta\, \frac{1}{\widehat{p}-\widehat{q}}\, \gamma_\alpha\, \frac{1}{\widehat{p}} + \gamma_\mu\gamma_5\, \frac{1}{\widehat{p}+\widehat{k}}\, \gamma_\alpha\, \frac{1}{\widehat{p}+\widehat{k}+\widehat{q}}\, \gamma_\beta\, \frac{1}{\widehat{p}}\right),$$

where the momenta of external photons attached to the vector currents j_α and j_β are q and -$k-q$ correspondingly. The momentum of the vector boson attached to the axial current $j_{5\mu}$ is k. Obviously, this contribution is Bose-symmetric under the simultaneous permutation of the Lorentz indices α, β and momenta of two photons $q, -k-q$. Because the above integrals are divergent at large p as $\int (d^4p)/p^3$, their contribution has an uncertainty related to the possible shift of the integration variable $p \to p-l$. Indeed, one can verify by the Wick rotation to the Euclidean space the following relation

$$I_\gamma = \int \frac{d^4p}{(2\pi)^4} \left(\frac{p_\mu}{p^4} - \frac{(p-l)_\mu}{(p-l)^4}\right) = l_\sigma \int_{|p|<\Lambda} \frac{i\, d^3p\, dp_4}{(2\pi)^4} \frac{\partial}{\partial p_\sigma} \frac{p_\mu}{p^4} = i\, \frac{1}{32\pi^2}\, l_\mu\,.$$

Therefore taking into account the Bose-symmetry the uncertainty in the calculation of $\Gamma_{\mu\alpha\beta}$ is

$$\Delta\Gamma_{\mu\alpha\beta} = c\,\varepsilon_{\mu\alpha\beta\sigma}(2q+k)_\sigma\,,$$

for an arbitrary factor c. The appearance of the antisymmetric tensor $\varepsilon_{\mu\alpha\beta\sigma}$ is related to the presence of γ_5 in the trace of the γ-matrices due to the relation

$$tr\, (\gamma_5\gamma_\mu\gamma_\alpha\gamma_\beta\gamma_\sigma) = 4\, i\, \epsilon_{\mu\alpha\beta\sigma}\,.$$

On the other hand, we can calculate the above Feynman integrals for their combinations related to the Ward identities for $\Gamma_{\mu\alpha\beta}$:

$$q_\alpha \Gamma_{\mu\alpha\beta} = 4\,\varepsilon_{\mu\beta\rho\gamma} \int_{|p|<\Lambda} \frac{d^3p\, d\,p_4}{(2\pi)^4} \left(\frac{(p+k)^\rho (p-q)^\gamma}{(p+k)^2 (p-q)^2} - \frac{(p+k+q)^\rho p^\gamma}{(p+k+q)^2 p^2} \right) =$$

$$4\,\varepsilon_{\mu\beta\rho\gamma}(-q_\sigma) \int_{|p|<\Lambda} \frac{d^3p\, d\,p_4}{(2\pi)^4} \frac{\partial}{\partial\, p_\sigma} \frac{k^\rho p^\gamma}{(p+k)^2 p^2} = -\frac{1}{8\pi^2} \varepsilon_{\mu\beta\rho\gamma}\, k_\rho\, q_\gamma\,,$$

$$k_\mu \Gamma_{\mu\alpha\beta} = 4\,\varepsilon_{\alpha\beta\rho\gamma} \int_{|p|<\Lambda} \frac{d^3p\, d\,p_4}{(2\pi)^4} \times$$

$$\left(\frac{(p+k+q)^\rho p^\gamma}{(p+k+q)^2 p^2} - \frac{(p+k)^\rho (p-q)^\gamma}{(p+k)^2 (p-q)^2} + \frac{(p+k)^\rho (p+k+q)^\gamma}{(p+k)^2 (p+k+q)^2} - \frac{(p-q)^\rho p^\gamma}{(p-q)^2 p^2} \right) =$$

$$4\,\varepsilon_{\alpha\beta\rho\gamma} \int_{|p|<\Lambda} \frac{d^3p\, d\,p_4}{(2\pi)^4} \left(q_\sigma \frac{\partial}{\partial p_\sigma} \frac{(p+k)^\rho p^\gamma}{(p+k)^2 p^2} + (k+q)_\sigma \frac{\partial}{\partial\, p_\sigma} \frac{(p-q)^\rho p^\gamma}{(p-q)^2 p^2} \right) = \frac{1}{4\pi^2} \varepsilon_{\alpha\beta\rho\gamma}\, k_\rho\, q_\gamma\,.$$

Therefore for our choice of the integration variables we lost the conservation of both vector and axial currents. Using the above uncertainty of adding $\Delta\Gamma_{\mu\alpha\beta}$ to the contribution of $\Gamma_{\mu\alpha\beta}$ we can redefine it as follows:

$$\widetilde{\Gamma}_{\mu\alpha\beta} = \Gamma_{\mu\alpha\beta} + \frac{1}{8\pi^2} \varepsilon_{\mu\alpha\beta\gamma}\,(2q+\,k)_\gamma$$

to provide the conservation of the vector current:

$$q_\alpha\, \widetilde{\Gamma}_{\mu\alpha\beta} = 0\,,$$

but in this case the Ward identity for the axial current is not fulfilled:

$$k_\mu \Gamma_{\mu\alpha\beta} = \frac{1}{2\pi^2} \varepsilon_{\alpha\beta\rho\gamma}\, k_\rho\, q_\gamma\,.$$

In the operator form this anomalous relation looks as follows

$$\partial_\mu\, j_{5\mu} = \frac{\alpha}{4\pi}\, F_{\rho\gamma}\, \widetilde{F}_{\rho\gamma}\,,$$

where $\widetilde{F}_{\rho\gamma} = \varepsilon_{\rho\gamma\delta\sigma} F^{\delta\sigma}$ is the dual intensity and $\alpha = \frac{e^2}{4\pi}$ is the fine structure constant. To verify it one can calculate the matrix element from the left and right hand sides of this equality between the photon states in the first non-zero order of the perturbation theory.

In the Standard Model there could be three dangerous anomalies.

The first anomaly is related to the transition of the B boson to two gluons through intermediate quarks. To cancel it we should impose the following condition for the hypercharges of the right (R) and left (L) quarks giving the anomalous contributions of opposite signs:

$$\sum_{q\in R} Y_q = \sum_{q\in L} Y_q\,.$$

This relation is fulfilled in each generation: $1/3 + 1/3 = 4/3 - 2/3$, which means, that the averaged values of the electric charge $\overline{Q}$ for the left and right quarks coincide. Note, that the absence of the gravitation anomaly in the decay of the B boson to two gravitons is a consequence of the equality of the averaged electric charge for left and right fermions in each generation.

The second anomaly appears in the transition of the B boson into the W^+W^--system through the intermediate left quarks and leptons. To cancel it the following equality between hypercharges of the left leptons and quarks should be fulfilled:

$$\sum_{l\in L} Y_l + \sum_{q\in L} Y_q = 0\,.$$

This relation is also fulfilled with taking into account the quark colour degrees of freedom: $2(-1)+2\,(1/3)\,3 = 0$. It is equivalent to the requirement, that the total electric charge of the left fermions is zero in each generation, which can be considered as a theoretical argument, why the electric charge of quarks is proportional to 1/3 of the electron charge.

The third relation is related with the cancellation of the anomaly in three B boson interactions appearing due to the the triangle diagrams having lepton and quark loops:

$$\sum_{l\in R} Y_l^3 + \sum_{q\in R} Y_q^3 = \sum_{l\in L} Y_l^3 + \sum_{q\in L} Y_q^3\,.$$

This relation is also valid for each generation: $(-2)^3+3(4/3)^3+3(-2/3)^3 = 2(-1)^3+2(1/3)^3 3$. It is fulfilled due to the fact, that the charges of the u and d quarks are $2/3$ and $-1/3$ correspondingly.

Thus, the quantum numbers of the fermions in the Standard Model are such, that all anomalies are cancelled and the theory turns out to be renormalizable.

5 Higgs phenomenon

The Hamiltonian of the electroweak theory has the minimum at

$$|\varphi_{cl}| = v = \frac{\eta}{\sqrt{2}}\,.$$

We can perform the non-Abelian gauge transformations in such way to remove from the doublet φ the upper component and the phase from the lower component:

$$\varphi = \begin{pmatrix} 0 \\ v + \frac{H}{\sqrt{2}} \end{pmatrix}.$$

In this gauge the scalar field H describing quantum fluctuations near the vacuum condensate v is real. Other three components of φ are transformed in the fields, corresponding to longitudinal polarizations of the three massive vector bosons, as it will be demonstrated below. The Higgs Lagrangian L_φ in the above parametrization takes the form:

$$L_\varphi = -\frac{m_\varphi^2}{2}\,H^2\left(1+\frac{H}{2\eta}\right)^2$$

where the physical mass m_φ of the Higgs particle is

$$m_\varphi^2 = 2\lambda^2 v^2 = \lambda^2 \eta^2.$$

The kinetic part of the Lagrangian for the field φ expanded near $\varphi = v$ can be written as follows after the diagonalization of the quadratic form

$$L_\varphi^{kin} = |D_\mu \varphi|^2 = \frac{1}{2}(\partial_\mu H)^2 + (1 + \frac{H}{\eta})^2 \left(m_w^2 W_{-\mu} W_{+\mu} + \frac{m_z^2}{2} Z_\mu^2 \right),$$

where

$$W_- = \frac{W^1 - iW^2}{\sqrt{2}}\,,\ W_+ = \frac{W^1 + iW^2}{\sqrt{2}}\,,\ W_+ = (W_-)^*$$

and W^+ describes the annihilation of the W^+-bosons and the production of W^--bosons with the mass

$$m_w^2 = \frac{1}{2}\, g^2\, v^2 = \frac{1}{4} g^2 \eta^2.$$

As for fields W_μ^3 and B_μ, it turns out that the mass

$$m_z^2 = \frac{1}{2}\,(g^2 + g'^2)\, v^2 = \frac{1}{4}(g^2 + g'^2)\eta^2$$

appears only for the following linear combination

$$Z_\mu = \frac{g W_\mu^3 \ - g'\, B_\mu}{\sqrt{g^2 + g'^2}} = \cos\theta_w\ W_\mu^3 \ - \sin\,\theta_w\ B_\mu\,,$$

corresponding to the physical particle - Z-boson. Here θ_w is the Weinberg angle, defined by the relations

$$\tan\,\theta_w = \frac{g'}{g}\,,\ \cos\,\theta_w = \frac{m_w}{m_z}\,.$$

The orthogonal combination of W_μ^3 and B_μ corresponds to the photon

$$A_\mu = \frac{g' W_\mu^3 \ + g B_\mu}{\sqrt{g^2 + g'^2}} = \cos\theta_w\ B_\mu \ + \sin\,\theta_w\ W_\mu^3$$

remaining massless. Experimentally the masses of the W and Z bosons are

$$m_w \simeq 80.33\, GeV\,,\ \ m_z \simeq 91.187\, GeV$$

and therefore the Weinberg angle in radians is

$$\theta_w \simeq \arccos\frac{m_w}{m_z} \simeq 0.\,4930$$

which corresponds to $\sin^2\theta_w \simeq 0.224$. The above Lagrangian is gauge invariant only under the transformations of the photon field $\delta A_\mu = \partial_\mu \chi$. The propagators of the W and Z bosons satisfy the equations

$$-\left(k_\mu k_\sigma - (k^2 - m^2)\delta_{\mu\sigma}\right) D_{\sigma\nu} = \delta_{\mu\nu}$$

and correspond to the physical gauge

$$D_{\mu\nu}(k) = \frac{\Lambda_{\mu\nu}(k)}{k^2 - m^2}\,,\ \Lambda_{\mu\nu}(k) = -\delta_{\mu\nu} + \frac{k_\mu\, k_\nu}{m^2}\,,$$

where $\Lambda_{\mu\nu}(k)$ on the mass shell ($k^2 = m^2$) is the projector on physical states

$$\Lambda_{\mu\nu}(k) = \sum_{i=1,2,L} e_\mu^{i*}(k)\, e_\nu^i(k)\,,\ e_\mu(k)\, k_\mu\ = 0\,,\ e_\mu^{i*}(k)\, e_\mu^j(k) = -\delta^{ij}\,.$$

Here $e_0^{1,2} = e_3^{1,2} = 0$ and $e_{1,2}^L = 0$, $k_0 e_0^L = k_3 e_3^L$. In other gauges the phases of the Higgs field φ should be considered as dynamical fields having the propagator $D(k)$. For example, in the renormalizable R_ξ-gauge of t'Hooft the corresponding propagators are

$$D_{\mu\nu}(k) = -\frac{\delta_{\mu\nu} - (1-\xi)\frac{k_\mu\, k_\nu}{k^2 - m^2\xi}}{k^2 - m^2}\,,\ D(k) = \frac{1}{k^2 - m^2\xi}\,.$$

The poles at $k^2 = m^2\xi$ are non-physical and are cancelled in scattering amplitudes.

The kinetic part of the Lagrangian for the fermions interacting with the gauge fields $W_\mu^\pm$, Z_μ and A_μ, can be written as follows

$$L_\psi^{kin} = i\sum_k \overline{\psi}_k\, \widehat{D}\, \psi_k = \sum_k (\overline{\psi}_k i\hat{\partial}\, \psi_k + L_k^W + L_k^A + L_k^Z)\,,$$

where

$$\begin{aligned} L_k^W &= \frac{g}{\sqrt{2}}\overline{\psi}_k\left(\tau_+\widehat{W_-} + \tau_-\widehat{W_+}\right)\psi_k\,,\ L_k^A = e\,\overline{\psi}_k \hat{A} Q\,\psi_k\,,\\ L_k^Z &= \frac{g}{\cos\theta_w}\,\overline{\psi}_k \hat{Z}\left(T^3 - Q\,\sin^2\theta_w\right)\psi_k\,. \end{aligned}$$

The elementary electric charge e turns out to be expressed in terms of the gauge coupling constants as follows

$$e = g\,\sin\theta_w = g'\cos\theta_w = \sqrt{g^2 + g'^2}\sin\theta_w\,\cos\theta_w$$

and the operator Q is related to T^3 and Y with the Gell-Mann-Nishijima formula. The matrices $\tau_\pm$ are given below

$$\tau_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\,,\ \tau_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\,.$$

For example, the term containing τ_+ describes the amplitude for the transition $e_L \to \nu^e W^-$:

$$M = \frac{g}{\sqrt{2}}\,\overline{u}(p_{\nu^e})\,\gamma_\sigma\,\frac{1+\gamma_5}{2}\,u(p_e)\,l_\sigma^*(p_w)\,,$$

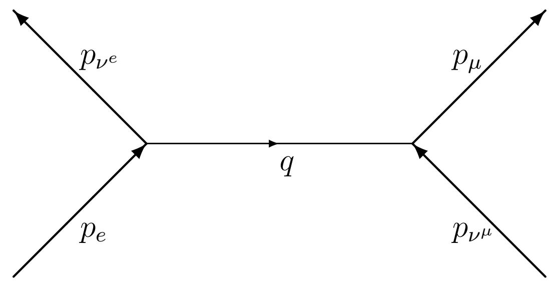

Figure 2: Feynman graph for the transition $e\nu^\mu \to \nu^e\mu$ with the W-exchange

where $u(p_{\nu^e})$ and $u(p_e)$ are the spinors of the neutrino and electron correspondingly and $l_\mu(p_w)$ is the polarization vector of the produced W^--boson. According to the Feynman rules the more complicated process of the transition $e_L\nu^\mu \to \nu^e\mu_L$ is described by the amplitude (see Fig. 2)

$$M = \left(\frac{g}{\sqrt{2}}\right)^2\, \overline{u}(p_{\nu^e})\,\gamma_\sigma\,\frac{1+\gamma_5}{2}\,u(p_e)\,\frac{1}{q^2-m_w^2}\,\overline{u}(p_\mu)\,\gamma_\sigma\,\frac{1+\gamma_5}{2}\,u(p_{\nu^\mu})\ .$$

Here $q = p_e - p_{\nu^e}$ is the momentum transfer. For low energies, when $q^2 \ll m_w^2$, this amplitude can be constructed from the corresponding term of the Fermi Hamiltonian

$$L_{Fermi} = -\frac{G}{\sqrt{2}}\,\overline{\psi}_{\nu^e}\,\gamma_\sigma\,(1+\gamma_5)\psi_e\,\overline{\psi}_\mu\,\gamma_\sigma\,(1+\gamma_5)\psi_{\nu^\mu}\,,$$

if

$$G = \sqrt{2}\,\frac{g^2}{8}\,\frac{1}{m_w^2} = \frac{\pi}{\sqrt{2}}\,\frac{\alpha_e}{\sin^2\theta_w}\,\frac{1}{m_w^2} = \frac{7.\,235\times 10^{-2}}{m_w^2}\,,$$

where $\alpha_e = e^2/(4\pi)$ is the fine structure constant. The known value of G equal to $10^{-5}/m_p^2$ is in an agreement with this formula. This agreement is an important achievement of the unified electro-weak theory.

The terms of the initial Lagrangian constructed only from the vector boson fields can be written as follows

$$-\frac{1}{2}tr\,G_{\mu\nu}^2\ -\frac{1}{4}F_{\mu\nu}^2 = L_W^{kin} + L_Z^{kin} + L_A^{kin} + L_{int}\,.$$

Here L_i^{kin} are the kinetic terms for the vector fields

$$L_W^{kin} = -\frac{1}{2}\ |D_\mu W_{-\nu} - D_\nu W_{-\mu}|^2\ ,\ D_\mu = \partial_\mu - i\,e(A_\mu + \cot\theta_w\,Z_\mu)\ ,$$

$$L_Z^{kin} = -\frac{1}{4}\left(F_{\mu\nu}^Z\right)^2\ ,\ F_{\mu\nu}^Z = \partial_\mu Z_\nu - \partial_\nu Z_\mu\,,$$

$$L_A^{kin} = -\frac{1}{4}\left(F_{\mu\nu}^A\right)^2 \ , \ F_{\mu\nu}^A = \partial_\mu A_\nu - \partial_\nu A_\mu$$

and L_{int} are the terms describing their self-interaction

$$L_{int} = \frac{ig}{2}\left(F_{\mu\nu}^{W^3}\right)(W_{-\mu}W_{+\nu} - W_{+\mu}W_{-\nu}) + g^2 (W_{-\mu}W_{+\nu} - W_{+\mu}W_{-\nu})^2 \ .$$

$$F_{\mu\nu}^{W^3} = \partial_\mu W_\nu^3 - \partial_\nu W_\mu^3 = F_{\mu\nu}^Z + F_{\mu\nu}^A \, .$$

Note, that L_{int} includes in particular the interaction of a photon with the magnetic momentum of the W-boson which does not appear in the minimal procedure of the obtaining the Lagrangian for the electro-magnetic interactions.

At last the Yukawa interaction responsible for the non-zero fermion masses m_k can be written as follows

$$L_{Yuk} = -\sum_k m_k\left(1 + \frac{H}{\eta}\right)\overline{\psi_k}\ \psi_k \ ,$$

where the sum goes over all fermions including neutrinos. In the case of massive neutrinos the upper components of lepton doublets interacting with the W - fields are linear combinations of the physical neutrino fields:

$$\begin{pmatrix} \nu_L^{e\prime} \\ \nu_L^{\mu\prime} \\ \nu_L^{\tau\prime} \end{pmatrix} = \widetilde{V}\ \begin{pmatrix} \nu_L^{e} \\ \nu_L^{\mu} \\ \nu_L^{\tau} \end{pmatrix} ,$$

where $\widetilde{V}$ is the matrix similar to the Cabbibo-Kobayashi-Maskawa matrix for quarks.

The total Lagrangian of the Electroweak Theory written in terms of the fields W_μ and Z_μ in the physical gauge is

$$L = -\frac{1}{2}\ |D_\mu W_{-\nu} - D_\nu W_{-\mu}|^2 - \frac{1}{4}\left((F_{\mu\nu}^Z)^2 + (F_{\mu\nu}^A)^2\right) + \left(m_w^2\ |W_{-\mu}|^2 + \frac{m_z^2}{2}Z_\mu^2\right)(1 + \frac{H}{\eta})^2 +$$

$$\frac{ig}{2}\left(F_{\mu\nu}^{W^3}\right)(W_{-\mu}W_{+\nu} - W_{+\mu}W_{-\nu}) + g^2 (W_{-\mu}W_{+\nu} - W_{+\mu}W_{-\nu})^2 +$$

$$\sum_k \overline{\psi}_k \left(i\widehat{\partial}\ - m_k(1 + \frac{H}{\eta})\right)\ \psi_k + \frac{1}{2}(\partial_\mu H)^2 - \frac{m_\varphi^2}{2}H^2\,(1 + \frac{H}{2\eta})^2 +$$

$$e\sum_k\ \overline{\psi}_k\widehat{A}Q\,\psi_k + \frac{g}{\sqrt{2}}\sum_k \overline{\psi}_k\left(\tau_+\widehat{W_-} + \tau_-\widehat{W_+}\right)\psi_k + \frac{g}{\cos\theta_w}\sum_k\ \overline{\psi}_k\widehat{Z}\left(T^3 - Q\ \sin^2\theta_w\right)\ \psi_k \, .$$

6 Cabbibo-Kobayashi-Maskawa matrix

The low components of the quark doublets are superpositions of the states with the different masses and the interaction of quarks with the W-bosons can be written in the form

$$L_{qW} = \frac{g}{\sqrt{2}} \begin{pmatrix} \overline{u} & \overline{c} & \overline{t} \end{pmatrix} \gamma_\mu \frac{1+\gamma_5}{2} W^+_\mu V \begin{pmatrix} d \\ s \\ b \end{pmatrix} + h.c. \ ,$$

where

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

is the Cabbibo-Kobayashi-Maskawa (CKM) matrix. A similar matrix should exist for neutrinos, if they mix each with others. It is very probable, that such mixing was observed recently experimentally in Super-Kamiokande.

The phases of the quark fields can be redefined without any physical consequences, because all other terms of the Lagrangian do not depend on these phases. We can use this freedom to simplify the CKM matrix. Because this matrix is unitary, it has n^2 real parameters in a general case of n generations. By redefining the phases of quark fields one can remove $2n-1$ parameters taking into account, that the phase, proportional to the unit matrix, can be included in ψ or $\overline{\psi}$. Therefore the total number of the physical parameters is $(n-1)^2$. For three generations their number is 4 and in accordance with the parametrization of Kobayashi and Maskava we can use three Euler angles and one phase to present this matrix as follows:

$$V_{KM} = \Omega_z(\vartheta_2)\, \Omega_x(\vartheta_1) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix} \Omega_z(\vartheta_3) =$$

$$V_{KM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\vartheta_2 & \sin\vartheta_2 \\ 0 & -\sin\vartheta_2 & \cos\vartheta_2 \end{pmatrix} \begin{pmatrix} \cos\vartheta_1 & \sin\vartheta_1 & 0 \\ -\sin\vartheta_1 & \cos\vartheta_1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\vartheta_3 & \sin\vartheta_3 \\ 0 & -\sin\vartheta_3 & \cos\vartheta_3 \end{pmatrix} .$$

The phase factor $e^{i\delta}$ is very important, because it will lead to the CP-nonconservation. By multiplying the above three matrices we obtain

$$V_{KM} = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - e^{i\delta} s_2 s_3 & c_1 c_2 s_3 + e^{i\delta} s_2 c_3 \\ s_1 s_2 & -c_1 s_2 c_3 - e^{i\delta} c_2 s_3 & -c_1 s_2 s_3 + e^{i\delta} c_2 c_3 \end{pmatrix} ,$$

where $s_i = \sin\vartheta_i$, $c_i = \cos\vartheta_i$. In the parametrization of Maiani we have three subsequent rotations around axes x, y, z with certain phase multiplications between them:

$$V_M = \begin{pmatrix} c_\beta\, c_\theta & c_\beta\, s_\theta & s_\beta\, e^{-i\delta} \\ -c_\gamma\, s_\theta - \, c_\theta\, s_\beta\, s_\gamma\, e^{i\delta} & -c_\gamma\, c_\theta - \, s_\theta\, s_\beta\, s_\gamma\, e^{i\delta} & c_\beta\, s_\gamma \\ s_\gamma\, s_\theta - \, c_\theta\, s_\beta\, c_\gamma\, e^{i\delta} & -s_\gamma\, c_\theta - \, s_\theta\, s_\beta\, c_\gamma\, e^{i\delta} & c_\beta\, c_\gamma \end{pmatrix} .$$

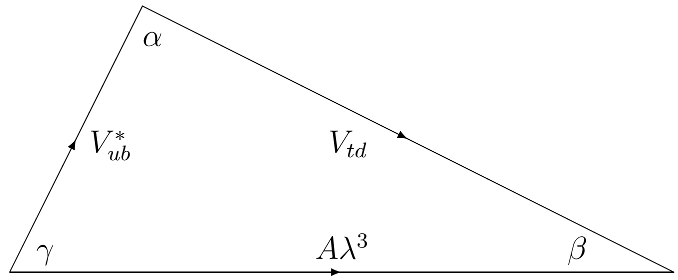

Figure 3: Unitarity triangle

Experimentally the non-diagonal components of V are small and therefore this matrix can be parametrized approximately in the Wolfenstein form:

$$V_W = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\,\lambda^3\,(\rho - i\,\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\,\lambda^2 \\ A\,\lambda^3\,(1-\rho-i\,\eta) & -A\,\lambda^2 & 1 \end{pmatrix},$$

where in fact λ is the Cabbibo angle and

$$\lambda = 0.22\,, \;\; A = 1 \pm 0.2\,, \;\; \rho^2 + \,\eta^2 \leq 0.3\,.$$

From the unitarity constraints we obtain the orthogonality of columns of the CKM matrix. It leads in particular to the relation

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0\,.$$

Using that $V_{ud} \simeq V_{tb} \simeq 1$ and

$$V_{cd}V_{cb}^* \simeq -A\lambda^3 \simeq -(1.1 \pm 0.2)10^{-2}\,,$$

we have

$$V_{ub}^* + V_{td} = (1.1 \pm 0.2)10^{-2}$$

and therefore one can consider the complex numbers V_{ub}^* and V_{td} as the vectors on a two-dimensional plane corresponding to the sides of a triangle having the third side lying on the real axis with its length equal to $A\lambda^3$ (see Fig. 3). Only the position of the vertex opposite to this side is not known well. The lengths of the sides of this triangle are related directly to the decay probabilities for the heavy quarks and its angles determine the effects of the CP violation. From various experiments we obtain, that V_{ub}^* and V_{td} are approximately orthogonal ($\alpha \approx \pi/2$) and

$$|V_W| = \begin{pmatrix} 0.9722 - 0.9748 & 0.216 - 0.223 & 0.002 - 0.005 \\ 0.209 - 0.228 & 0.959 - 0.976 & 0.037 - 0.043 \\ 0 - 0.09 & 0 - 0.16 & 0.07 - 0.993 \end{pmatrix}.$$

Other parameters of the Standard Model are known better. The Fermi constant is

$$G = 1.16637(1)\,10^{-5}\,GeV^{-2}\,.$$

Therefore the parameter η related to the vacuum expectation $v = \frac{\eta}{\sqrt{2}}$ of the Higgs field is

$$\eta = \left(\sqrt{2}G\right)^{-\frac{1}{2}} \simeq 246\, GeV\,.$$

If we assume, that the Higgs mass $m_\varphi > 114\, GeV$, as it follows from the CERN experiments, we obtain for the Higgs self-interaction coupling constant λ the value

$$\lambda = \frac{m_\varphi}{\eta} > 0.463\,.$$

The scattering amplitude a of two Higgs particles $a = 3\lambda^2$ is of the order of unity and can lead to a strong interaction in the Higgs sector.

Moreover, because the lepton and quark masses are

$$m_e = 0.51100\, MeV,\ m_\mu = 105.658\, MeV,\ m_\tau = 1777.0\, MeV\,,$$

$$m_u = 1 - 5\, MeV,\ m_d = 3 - 9\, MeV,\ m_s = 75 - 170\, MeV\,,$$

$$m_c = 1.15 - 1.35\, GeV,\ \ m_b = 4.0 - 4.4\, \Gamma eV,\ \ M_t = 170 - 179\, GeV\,,$$

the Yukava constants g are also rather large. For example, for t-quark we have $g = m_t/\eta = 0.71$. It means, that the perturbation theory is broken at least for t-quark interactions. Note, that the large errors in the values of fermion masses are related in particular with the use of different renormalization schemes for their definitions.

For the calculation of radiative corrections to the physical observables in the Standard Model it is important to know the QED fine structure constant at the scale $q^2 \sim M_Z^2$. Taking into account its renormalization due to the light leptons and hadrons, we obtain

$$\alpha(M_Z)^{-1} = \alpha^{-1}(1 - \Delta\alpha) = 127.938 \pm 0.027.$$

Note, that with the use of the Gell-Mann-Low equation for the QCD running coupling constant

$$\frac{d\,\alpha_c}{d\ln\mu} = -\alpha_c\left(\beta_0\frac{\alpha_c}{2\pi} + \beta_1\frac{\alpha_c^2}{4\pi^2} + \beta_2\frac{\alpha_c^3}{64\pi^3} + ...\right)\,,$$

where

$$\beta_0 = 11 - \frac{2}{3}n_f\,,\ \beta_1 = 51 - \frac{19}{3}n_f\,,\ \beta_2 = 2857 - \frac{5033}{9}n_f + \frac{325}{27}n_f^2\,,$$

we can calculate α_c at arbitrary μ using the experimental information, that

$$\alpha_c(M_z) = 0.118 \pm 0.002\,.$$

7 Neutrino's masses and oscillations

Direct measurements of the neutrino energies and momenta in the experiments of the type of the β-decay of nuclei give the following restrictions on their masses

$$m(\nu^e) < 15eV\,,\ m(\nu^\mu) < 170keV\,,\ m(\nu^\tau) < 18.2MeV\,.$$

From the measurement of the neutrino oscillations one can obtain some information about the quantity

$$\delta m_{ij}^2 = m_{\nu^i}^2 - m_{\nu^j}^2 \,.$$

Indeed, let us consider for simplicity the mixing of two neutrinos ν_1 and ν_2 with the masses m_1 and m_2, respectively. We denote by ν_1' and ν_2' the states entering in the upper components of the corresponding weak doublets:

$$\begin{pmatrix} \nu_1' \\ \nu_2' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} .$$

In the experimental situation, where ν_1' is produced and ν_2' is measured in the distance L from the source, the amplitude at the observation point is

$$\psi = e^{ip(t-L)} \left(\nu_1 e^{im_1^2 \frac{t}{2p}} \cos\theta + \nu_2 e^{im_2^2 \frac{t}{2p}} \sin\theta \right) ,$$

where $\nu_{1,2}$ are the states for the corresponding neutrino states and we used the expansion $\varepsilon - p \simeq m^2/(2p)$ for relativistic particles. Therefore, the probability to find the state ν_2' is

$$P(\nu_1' \to \nu_2') = |\langle \nu_2' | \psi \rangle|^2 = \sin^2(2\theta)\, \sin^2 \left(\frac{\delta m_{12}^2}{4} \frac{L}{E} \right)$$

or numerically

$$P(\nu_1' \to \nu_2') = \sin^2(2\theta)\, \sin^2 \left(1.267 \, \frac{\delta m_{12}^2}{(eV)^2} \, \frac{L/(M)}{E/(MeV)} \right)$$

providing, that δm_{12}, E and L are measured correspondingly in eV, MeV and meters.

In the detector of the atmospheric neutrinos situated deeply in the earth at Super-Kamiokande it was discovered, that their number depends on the angle ϑ between their momenta and the vertical direction. If we denote the number of neutrinos moving from the Earth $(-1 < \cos\vartheta < -0.2)$ by U and their number moving to the Earth $(0.2 < \cos\vartheta < 1)$ by D, the quantity

$$A = \frac{U - D}{U + D}$$

is different for the electron and muon neutrinos: $A_e = -0.036, \;\; A_\mu = -0.296$.

According to the existing theory, the neutrinos are obtained as products of the decay of π and K mesons, appearing from the nuclear interaction of cosmic protons with the atoms in the atmosphere. The main decays are

$$\pi^+,\, K^+ \to \mu^+ \nu_\mu \to e^+ \nu_e \overline{\nu_\mu} \nu_\mu, \;\; \pi^-,\, K^- \to \mu^- \overline{\nu_\mu} \to e \overline{\nu_e} \overline{\nu_\mu} \nu_\mu \,.$$

Therefore we should have the relation between numbers of μ and e neutrinos

$$\frac{\nu_\mu + \overline{\nu_\mu}}{\nu_e + \overline{\nu_e}} = 2$$

independently from the azimuthal angle ϑ. Indeed, the number of ν_e does not depend on this angle, but the number of neutrinos ν_μ moving from the Earth is significantly less than their number moving to the Earth. It can be interpreted as a manifestation of the transition of neutrinos ν_μ produced in the atmosphere to other neutrinos during their trip through the Earth. Apparently these other neutrinos are not ν_e, because the flow of ν_e does not depend practically on the angle ϑ. The most probable possibility is the mixing between ν_μ and ν_τ. Everything can be explained if the length L of the oscillations is around $500km$. The best fit of the experimental data corresponds to

$$\delta m^2 = 0.5\ 10^{-2} eV^2\ ,\ \sin^2(2\theta) = 1\,.$$

If the masses of the neutrino ν_μ and its partner completely different, we obtain for the mass of heavier neutrinos $m_\nu = 0.07\ eV$.

There are also some problems with the solar neutrinos, which should appear in various nuclear reactions on the Sun of the type

$$4p \to He^4 + 2e^+ + 2\nu_e\,.$$

They are measured in many underground laboratories including Gran Sasso (Italy) and Super-Kamiokande (Japan) for different intervals of energies E_ν. As a rule, their number turns out be approximately two times smaller than it is predicted by the theory. The natural explanation is the oscillation of ν_e with other neutrinos during their trip from the Sun to the Earth. If it happens, than

$$0.5\ 10^{-10} eV < \delta m^2 < 0.8\ 10^{-10} eV\,,\ \sin^2\theta > 0.65\,.$$

Moreover, the non-electronic neutrinos (probably ν_τ) flying from the Sun were discovered in August of 2001 by canadian physicists at the Sudbury Neutrino Observatory by measuring their scattering off the atomic nucleus through the Z-exchange. The neutrino can oscillate also in strong electro-magnetic fields inside the Sun (the Mikheev-Smirnov effect). Now some experiments are planned to investigate the neutrino mixing on the existing accelerators. For example, it is planned to produce the neutrino beam in CERN (Switzerland) and to measure it in Gran Sasso (Italy). Analogous experiments are planned in the Fermilab (Chicago) with the neutrino measurement in Canada (detector Sudan) and in KEK (Japan) with the detector Super-Kamiokande. In these experiments there will be enough bases ($L > 300\,km$) to observe the neutrino oscillations. The experimentalists attempt also to discover the neutrino masses m_ν directly in the various weak processes of the type $n \to p + e + \overline{\nu_e}$ by measuring the electron spectrum $\frac{d\Gamma}{dE} \sim \sqrt{(\Delta - E)^2 - m_\nu^2} = p_\nu$ near the upper kinematical boundary $E_0 = \Delta = m_n - m_p$.

8 Decay of vector bosons

According to Quantum Mechanics the Green function of a composite state has a pole in the energy E at the resonance mass:

$$G \sim \frac{1}{E - M + i\frac{1}{2}\Gamma}$$

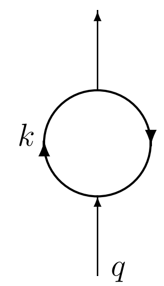

Figure 4: Polarization operator for the photon

Because the vector bosons are unstable particles, their Green function also has the pole singularity in the physical gauge

$$D_{\mu\nu}(q) = (\delta_{\mu\nu} - \frac{q_\mu q_\nu}{M^2})\, D(q)\,,\;\; D(q) = \frac{1}{q^2 - M^2 + \Pi(q^2)} \sim \frac{1}{E - M + i\frac{1}{2}\Gamma}\,,$$

where $\Pi(q^2)$ is the renormalized polarization operator and therefore we obtain the following relation between its imaginary part $\Im\,\Pi(q^2)$ and the resonance width Γ:

$$\Gamma = \frac{\Im\,\Pi(M^2)}{M}\,.$$

Let us begin with the decay of the W^- - boson to the system of the leptons e and $\bar{\nu}$. In this case we can neglect the electron mass in comparison with the W-boson mass. If we compare the polarization operator Π_W of the W-boson with the corresponding operator Π_γ for the virtual photon in the massless QED (see Fig. 4), apart from the substitution $e \to g$ we have the additional factors

$$\frac{1+\gamma_5}{2\sqrt{2}}$$

for each of two vertices. One should also take into account, that the linear term in γ_5 does not give any contribution after integration over the fermion momentum k and the product of two γ_5 matrices is unity. It gives effectively

$$\Pi_W = \frac{1}{4}\frac{g^2}{e^2}\Pi_\gamma = -\frac{1}{4}\frac{g^2}{12\pi^2}\, q^2\, \ln(-\frac{q^2}{m_e^2} - i\epsilon)\,,$$

because the running coupling constant in QED at large q^2 is

$$\alpha(q^2) = \frac{\alpha}{1 - \frac{\alpha}{3\pi}\,\ln(-\frac{q^2}{m_e^2})}\;,\;\; \alpha = \frac{e^2}{4\pi}\,.$$

Thus, using the relation $\Im\,\ln(-q^2 - i\epsilon) = -\pi$, we can obtain the partial width for the decay $W^- \to e\,\overline{\nu^e}$

$$\Gamma_{e\,\overline{\nu^e}} = \frac{g^2}{48\,\pi}\, M_W = \frac{G}{6\,\sqrt{2}\,\pi}\, M_W^3 \simeq 226\, MeV\,.$$

The same result is valid for the W-decay to the $\mu\,\overline{\nu^\mu}$ and $\tau\,\overline{\nu^\tau}$. As for the decay to the quark system $u_i\,\overline{d_j}$, we obtain in analogous way

$$\Gamma_{u_i\,\overline{d_j}} = C\,\frac{G}{6\,\sqrt{2}\,\pi}\,M_W^3\ \ |V_{ij}|^2 \simeq |V_{ij}|^2\ 705\,MeV\,, \tag{1}$$

where V_{ij} is the corresponding element of the CKM matrix. Due to its unitarity it is cancelled after summation over j. The factor

$$C = 3\,\left(1+\frac{\alpha_s(M_W^2)}{\pi}+1.409\,\left(\frac{\alpha_s(M_W^2)}{\pi}\right)^2-12.77\,\left(\frac{\alpha_s(M_W^2)}{\pi}\right)^3\right)$$

for quarks takes into account their 3 colour degrees of freedom and the QCD interaction with the strong running coupling constant $\alpha_s = g_s^2/(4\pi) \sim 0.2$ calculated at the scale equal to M_W^2.

Taking into account, that the t-quark has the mass $m_t \simeq 175\,MeV$ and therefore the W-boson does not have the decay channels with this quark, we obtain, that the total width of the W-boson is

$$\Gamma_{tot}^W = 3\,\Gamma_{e\,\overline{\nu^e}} + 2\,\left(\Gamma_{u\bar{d}} + \Gamma_{u\bar{s}} + \Gamma_{u\bar{b}}\right) \simeq 2.09\,GeV$$

in a good agreement with the experimental data. Note, that because the quarks do not exist in the free state, the theoretically obtained Γ_{tot}^W is compared with the decay in leptons and hadrons. Therefore, the agreement of the experimental data with the above formulas means, that the hadronization effects for quarks are not very essential, although for light quarks they can be significant.

Let us consider now the Z-boson decays. Its interaction Lagrangian can be presented as follows

$$L^Z = \frac{g}{2\cos\theta_w}\sum_k\,\overline{\psi}_k\hat{Z}\,(g_v+g_a\gamma_5)\ \psi_k\,,$$

where

$$g_v = T_3 - 2\,Q\sin^2\theta_w\,,\ \ g_a = T_3\,.$$

Using the arguments similar to the case of the W-decays, we obtain the relation between the corresponding partial widths of the Z-boson:

$$\Gamma_Z = \Gamma_W\ (g_v^2+g_a^2)\,\frac{M_z}{M_w\,\cos^2\theta_w} = \Gamma_W\ (g_v^2+g_a^2)\,\frac{M_z^3}{M_w^3}\,.$$

We put approximately $\sin\,\theta_w = \frac{1}{2}$ and obtain for the $\nu\overline{\nu}$-channel

$$\Gamma_{\nu\overline{\nu}} = \Gamma_{e\overline{\nu^e}}\ (\frac{1}{4}+\frac{1}{4})\,\frac{M_z^3}{M_w^3} = \frac{1}{2\cos^3\theta_w}\,\Gamma_{e\overline{\nu^e}} \simeq 167\,MeV,$$

the $e\overline{e}$-channel

$$\Gamma_{e\overline{e}} = \Gamma_{e\overline{\nu^e}}\; ((-\frac{1}{2}+2\,\frac{1}{4})^2+\frac{1}{4})\,\frac{M_z^3}{M_w^3} \simeq 84\,MeV,$$

the $u\overline{u}$-channel

$$\Gamma_{u\overline{u}} = C\,\Gamma_{e\overline{\nu^e}}\; ((\frac{1}{2}-2\,\frac{2}{3}\,\frac{1}{4})^2+\frac{1}{4})\,\frac{M_z^3}{M_w^3} \simeq 300\,MeV$$

and the $d\overline{d}$-channel

$$\Gamma_{d\overline{d}} = C\,\Gamma_{e\overline{\nu^e}}\; ((-\frac{1}{2}+2\,\frac{1}{3}\,\frac{1}{4})^2+\frac{1}{4})\,\frac{M_z^3}{M_w^3} \simeq 380\,MeV\,.$$

Finally for the total width of the Z-boson we have

$$\Gamma^Z_{tot} = 3\,\Gamma_{\nu\,\overline{\nu}} + 3\,\Gamma_{e\overline{e}} + 2\,\Gamma_{u\overline{u}} + 3\,\Gamma_{d\overline{d}} \simeq 2.49\,7GeV$$

with an excellent agreement with the experimental data.

9 Production of vector bosons in electron-positron collisions

The Z and W bosons can be produced in the $e\overline{e}$ colliding beams and at high energy hadron collisions. To begin with, let us consider the Z-boson production at the e^-e^+ collisions. The total cross-section according to the optical theorem is proportional to the discontinuity of the elastic scattering amplitude in the forward direction. At large energies s we have

$$\sigma_{tot} = \frac{1}{2s}\,\frac{1}{i}\,\Delta\;A(s,0)\,,\;\;\Delta\,A(s,0) = A(s+i\epsilon,0) - A(s-i\epsilon,0)\;.$$

Using the Feynman rules for the Standard Model we can write the elastic amplitude with the virtual Z-boson in the s channel as follows (see Fig. 5)

$$A(s,0) = -\left(\frac{g}{2\cos\theta_w}\right)^2 \frac{\frac{1}{4}Tr\,(\gamma_\sigma(g_v+g_a\gamma_5)\widehat{p_e}\,\gamma_\sigma\,(g_v+g_a\gamma_5)\,\widehat{p_{\overline{e}}})}{s-M_Z^2+i\,\Gamma_{tot}\;M_Z}\,,$$

where g_v and g_a are at large s one can neglect the electron mass.

Therefore the total cross-section is

$$\sigma_Z = (g_v^2+g_a^2)\left(\frac{g}{2\cos\theta_w}\right)^2 \frac{\Gamma_{tot}}{4\,M_Z\,(E_{nr}^2+\frac{1}{4}\Gamma_{tot}^2)}\,,$$

where $E_{nr} = W - M_Z$ ($W=\sqrt{s}$) is the non-relativistic energy of the initial particles and we consider the region near the resonance: $E_{nr} \ll M_Z$. We can rewrite it in the simple form

$$\sigma_Z(s) = \frac{3\pi}{M_Z^2}\;\;\frac{\Gamma_{e\overline{e}}\,\Gamma_{tot}}{E_{nr}^2+\frac{1}{4}\Gamma_{tot}^2}$$

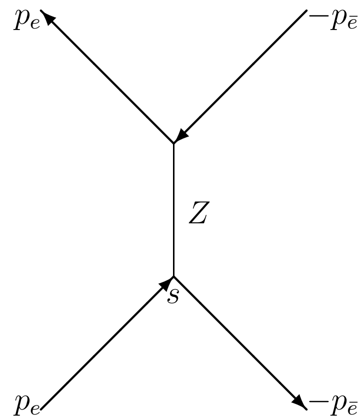

Figure 5: Feynman graph for the $e\bar{e}$ forward scattering through the Z-production

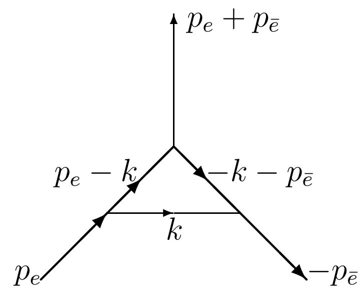

Figure 6: One loop diagram for the Sudakov vertex

with the use of the above expression for the $e\bar{e}$ width of the Z -boson decay

$$\Gamma_{e\bar{e}} = g^2 \, \frac{(g_v^2 + g_a^2)}{48\,\pi} \frac{M_z}{\cos^2\theta_w} \; .$$

It turns out, that the electromagnetic corrections are essential for the Z production in $e\bar{e}$ collisions. Let us consider the electromagnetic vertex at high external photon virtuality $s \gg m_e^2$, where m_e is the mass of the electron (see Fig. 6). We use the Sudakov variables for the momentum of the virtual photon with the mass λ (introduced for the infrared regularization)

$$k = \alpha\, p_e' + \beta\, p_{\bar{e}}' + k_\perp \,, \;\; (k_\perp, p_{e,\bar{e}}) = 0 \,, \;\; d^4k = \frac{s'}{2}\, d\,\alpha\, d\,\beta\, d^2k_\perp \,.$$

Here p_e' and $p_{\bar{e}}'$ are light-cone momenta ($p_i'^2 = 0$) constructed as linear combinations of momenta p_e and $p_{\bar{e}}$ of the colliding electron and positron:

$$p_e' = p_e - \frac{m_e^2}{2(p_e p_{\bar{e}})} p_{\bar{e}} \,, \;\; p_{\bar{e}}' = p_{\bar{e}} - \frac{m_e^2}{2(p_e p_{\bar{e}})} p_e \;, \; s' = 2p_e' \, p_{\bar{e}}' \simeq s \,.$$

Taking into account, that in the essential region of the integration the Sudakov parameters are small

$$|\alpha| \ll 1\,,\;\; |\beta| \ll 1\,,\;\; \overrightarrow{k_\perp}^2 = -k_\perp^2 \ll \sqrt{s}\,,$$

we can simplify the spin structure in the matrix element:

$$\overline{v}(-p_{\overline{e}})\,\gamma_\sigma(-\widehat{p}_{\overline{e}} - \widehat{k} - m_e)\,\gamma_\mu\,(\widehat{p}_e - \widehat{k} - m_e)\,\gamma_\sigma\,u(p_e) \simeq -2s\,\overline{v}(-p_{\overline{e}})\,\gamma_\mu\;u(p_e)\,.$$

Therefore the relative correction to the vertex can be written in the Sudakov variables as follows

$$\Delta\gamma \simeq \int \frac{\frac{is^2\,e^2}{(2\pi)^4} d\,\alpha\,d\,\beta\,d^2\overrightarrow{k_\perp}}{s\alpha\beta - \overrightarrow{k_\perp}^2 - \lambda^2 + i\varepsilon}\,\frac{1}{s\alpha(\beta-1) - \overrightarrow{k_\perp}^2 - m_e^2\beta + i\varepsilon}\,\frac{1}{s\beta(\alpha+1) - \overrightarrow{k_\perp}^2 + m_e^2\alpha + i\varepsilon}$$

$$\simeq -\frac{e^2}{8\pi^2}\int_0^1\int_0^1 \frac{d\,\alpha\,d\beta\;\;\theta(s\alpha\beta - \lambda^2)}{\left(\alpha + \frac{m_e^2}{s}\beta\right)\left(\beta + \frac{m_e^2}{s}\alpha\right)}\,,$$

where we used the fact, that with a double-logarithmic accuracy only the integration region $|\alpha|\,,\,|\beta| \ll 1$ is essential. With the same accuracy in the integral over $\overrightarrow{k_\perp}^2$ we substituted (neglecting a small contribution from the principal value prescription)

$$\frac{1}{s\alpha\beta - \overrightarrow{k_\perp}^2 - \lambda^2 + i\varepsilon} \to -i\pi\,\theta(s\alpha\beta - \lambda^2)\;\delta(s\alpha\beta - \overrightarrow{k_\perp}^2)\,.$$

It means, that in the essential region of integration the photon with the momentum k is on its mass shell. In the last integral we can introduce the logarithmic variables

$$\xi = \ln\,\frac{s\,\alpha}{m_e^2}\,,\;\; \eta = \ln\,\frac{s\,\beta}{m_e^2}\,,\;\; \rho = \ln\,\frac{s}{m_e^2}\,,\; L = \ln\,\frac{m_e^2}{\lambda^2}$$

and obtain (see Fig. 7)

$$\Delta\gamma \simeq -\frac{\alpha}{2\pi}\int_{-\infty}^{\rho} d\xi\int_{-\infty}^{\rho} d\eta\,\theta(\xi{+}\eta{-}\rho{+}L)\,\theta(\xi{-}\eta{+}\rho)\,\theta(\eta{-}\xi{+}\rho) = -\frac{\alpha\ln\frac{s}{m_e^2}}{2\pi}\left(\frac{\ln\frac{s}{m_e^2}}{2} + \ln\frac{m_e^2}{\lambda^2}\right)\,.$$

The photon momentum is collinear to momenta p_e or $p_{\overline{e}}$ providing that $\beta \gg \alpha$ or $\alpha \gg \beta$ correspondingly. Therefore we can write the result for the Sudakov vertex in one loop in another form

$$\Delta\gamma = -2\frac{\alpha}{\pi}\int_\lambda^{\sqrt{s}}\frac{d\omega}{\omega}\int_{m/\sqrt{s},\,\lambda/\omega}^{1}\frac{d\theta}{\theta} = -2\frac{\alpha}{\pi}\left(\ln\frac{m}{\lambda}\ln\frac{\sqrt{s}}{m} + \frac{1}{2}\ln^2\frac{\sqrt{s}}{m}\right)\,,$$

where $\omega = \beta\sqrt{s}$ and $\theta = |k_\perp|/\omega$ are the photon frequency and its emission angle. The final result for the Sudakov vertex in the leading logarithmic approximation is

$$\gamma = e^{\Delta\gamma}$$

and can be obtained for example with the use of the evolution equation in λ. In the case of the Z-boson production at $W - M \sim \Gamma_z$ we should take into account apart from the

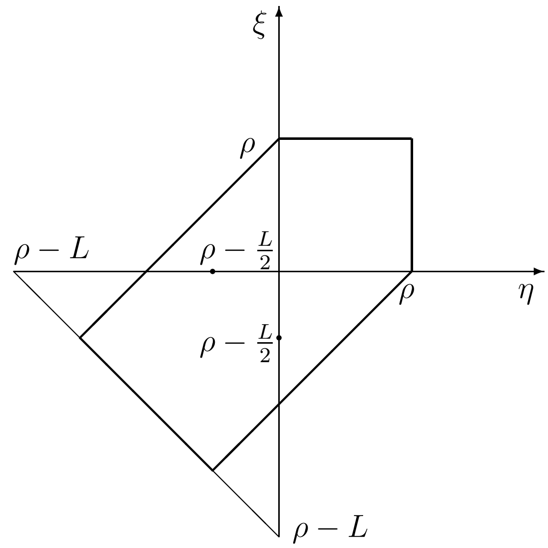

Figure 7: DL region of integration in the Sudakov vertex

virtual photon contributions $\sim \gamma^2$, leading to the suppression of the cross-section in the resonance region, also the photon bremsstrahlung cancelling infrared divergences at $\lambda \to 0$ in the Sudakov form-factor:

$$\sigma = e^{2\,\Delta\gamma} \sum_{n=0}^{\infty} \left(4\frac{\alpha}{\pi}\right)^n \frac{1}{n!} \prod_{r=1}^{n} \left(\int_{\lambda}^{\sqrt{s}} \frac{d\omega_r}{\omega_r} \int_{m/\sqrt{s},\, \lambda/\omega_r}^{1} \frac{d\theta_r}{\theta_r} \right) \int_0^{\infty} d\omega\ \delta(\omega - \sum_{l=1}^{n} \omega_l)\, \sigma_Z((W-\omega)^2)\,.$$

It leads to the increasing of the cross-section for the energies $W - M_Z \gg \Gamma_Z$ because the emission by e^+ and e^- of a photon with the frequency $\omega \simeq W - M_Z$ returns the initial particles to the resonance region where the cross-section is large $\sigma(W^2) \sim \frac{\alpha}{\Gamma}(W - M_Z)^{-1}$. This effect results in an appearance of the radiative tail for σ at $W > M_Z$. In the particular case when the energy $W - M_Z \ll M_Z$ with the use of the representation

$$2\,\pi\,\delta(\omega - \sum_{l=1}^{n} \omega_l) = \int_{-\infty}^{\infty} d\,t\, e^{it\,(\omega - \sum_{l=1}^{n} \omega_l)}$$

one can write the cross-section for the inclusive production of the Z -boson in the simple form

$$\sigma \simeq \int_0^{\infty} \sigma_Z((W-\omega)^2)\, d\omega \int_{-\infty}^{\infty} \frac{d\,t}{2\pi}\, e^{it\,\omega} \left(\min\left(\frac{1}{|t| M_Z},\, 1 \right) \right)^{\frac{4\alpha}{\pi} \ln \frac{W}{m_e}} \simeq$$

$$\int_0^{M_Z} \sigma_Z((W-\omega)^2)\, d\omega\, \frac{d}{d\omega} \left(\frac{\omega}{M_Z} \right)^{\frac{4\alpha}{\pi} \ln \frac{W}{m_e}} = \int_0^{M_Z} \sigma((W-\omega)^2)\, \frac{d\omega}{\omega}\, \frac{4\alpha}{\pi}\, \ln \frac{W}{m_e} \left(\frac{\omega}{M_Z} \right)^{\frac{4\alpha}{\pi} \ln \frac{W}{m_e}},$$

where $\omega \ll M_Z$ is the total energy of the produced photons. Effectively this result corresponds to the multiplication of σ_Z by the factor γ^2 in which instead of the condition $|k_\perp| > \lambda$ for the infrared cut-off of the virtual photon momenta we use the condition $\omega = \sqrt{s}(\alpha + \beta) > 1/|t|$ (see Fig. 8).

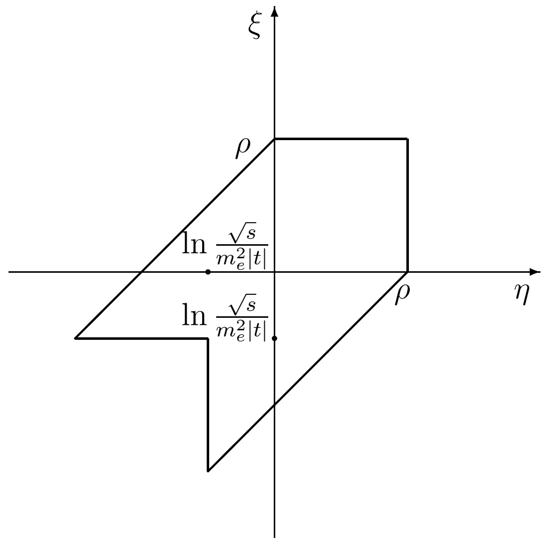

Figure 8: DL region of integration in the Z-production cross-section

Let us introduce the momenta $x_1 p_e$ and $x_2 p_{\bar{e}}$ of the bare electrons and positrons - partons incide the physical electrons and positrons respectively. Then the expression for σ can be written as

$$\sigma = \int_0^1 dx_1 \int_0^1 dx_2 \, \sigma_Z(sx_1x_2) \, n(x_1) \, n(x_2) \,, \;\; sx_1x_2 \simeq W^2 \left(1 - (2 - x_1 - x_2)\right) \simeq W^2 - 2W\omega \,,$$

where $n(x_1)$ and $n(x_2)$ are the inclusive densities of the corresponding bare particles inside physical ones. It corresponds to the physical picture, in which the Z-boson is produced in the collision of the bare electron and positron (see Fig. 9).

The density of the number of bare electrons inside the physical electron for $x \to 1$ is given by the expression

$$n(x) = \frac{2\alpha}{\pi} \ln \frac{W}{m_e} \, (1-x)^{-1 + \frac{2\alpha}{\pi} \ln \frac{W}{m_e}} .$$

The equivalence of two above expressions for σ is a consequence of the relation for small $\frac{2\alpha}{\pi} \ln \frac{W}{m_e}$

$$\int_0^\epsilon d\epsilon_1 \, n(1-\epsilon_1) \, n(1-\epsilon+\epsilon_1) \simeq \frac{4\alpha}{\pi} \ln \frac{W}{m_e} \, \epsilon^{-1 + \frac{4\alpha}{\pi} \ln \frac{W}{m_e}} .$$

The density $n(x)$ satisfies the sum rule

$$\int_0^1 d\,x \, n(x) = 1 \,,$$

meaning that in our approximation ($x \to 1$) there is only one bare electron inside the physical one. The partonic expression for the cross-section σ in terms of the product of $n(x_{1,2})$ is valid also in the case when $W - M_Z \sim M_Z$, but in this case $n(x)$ is more complicated and can be found from the evolution equations for partonic distributions in QED (see below).

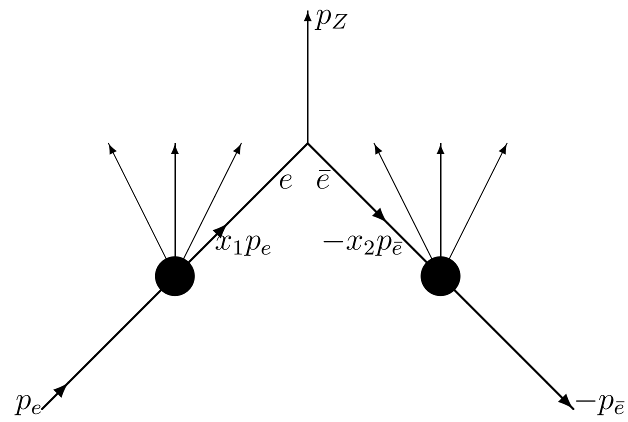

Figure 9: Partonic description of the Z-production in $e\bar{e}$ collisions

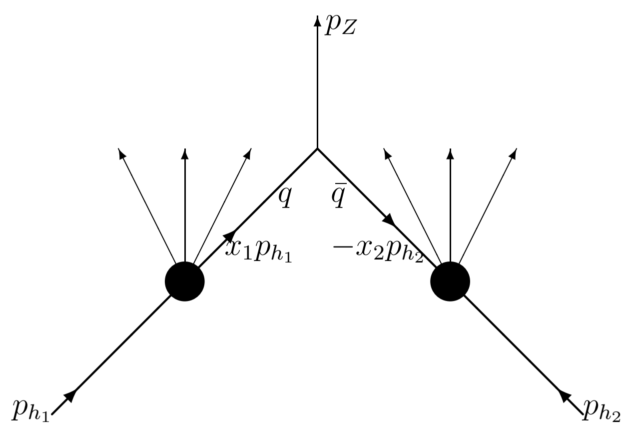

Figure 10: Partonic description of the Z-production in $h_1 h_2$ collisions

Similar formulas are valid in the framework of the parton model for the W and Z production in the hadron-hadron collisions but in this case the suppression of the resonance production is related with the contribution of virtual gluons.

10 Parton model

It is known, that hadrons are composite states of the point-like bare particles - partons having quantum numbers of quarks and gluons. In the framework of the parton model the cross-section for Z production in hadron-hadron collisions is expressed in terms of the product of the inclusive probabilities $D^q_{h_1}(x_1)$, $D^{\overline{q}}_{h_2}(x_2)$ to find the quark and anti-quark with the energies $x_{1,2}\sqrt{s/4}$ inside of the colliding hadrons h_1, h_2 and the Born cross-section $\sigma_{q\overline{q}\to Z}\,(s\,x_1x_2)$ for the boson production in the quark-anti-quark collisions. This expression is integrated over the Sudakov components $x_{1,2}$ of momenta of the quark and anti-quark (see Fig. 10)

$$\sigma_{h_1h_2\to Z} = \frac{1}{3}\sum_q \int dx_1 dx_2 \, \left(D_{h_1}^q(x_1)\, D_{h_2}^{\overline{q}}(x_2) + D_{h_1}^{\overline{q}}(x_1)\, D_{h_2}^q(x_2) \right) \sigma_{q\overline{q}\to Z}\,(sx_1x_2)\,,$$

where the factor $\frac{1}{3}$ is related with the fact, that in the quark and anti-quark distributions $D_h^{q,\overline{q}}(x)$ the sum over three colour states of the quark is implied, but Z is produced in annihilation of quarks and antiquarks with an opposite colour.

Initially the parton model was applied to the description of the deep-inelastic scattering of leptons off hadrons. For example, the differential cross-section of the inclusive scattering of electrons with the initial and final momenta p_e and p'_e respectively off the hadron with its momentum p_h can be written as follows (see Fig. 11)

$$d\sigma_\gamma = \frac{\alpha^2}{\pi}\frac{1}{q^4} L^{\mu\nu} W_{\mu\nu} \frac{d^3p'_e}{(pp_e)E'}\;,\; E' = |p'_e|\;,$$

where only the exchange of the photon with the moment $q = p_e - p'_e$ was taken into account (for large q^2 one should also take into account the Z-boson exchange, which leads in particular to the parity non-conservation effects proportional to $g_a \sim T_3$). The electron tensor $L^{\mu\nu}$ is

$$L^{\mu\nu} = \frac{1}{2} tr\,(\widehat{p'_e}\gamma_\mu \widehat{p_e}\gamma_\nu) = 2\,(p'^\mu_e p^\nu_e + p'^\nu_e p^\mu_e - \delta^{\nu\mu}(p'_e p_e))$$

and the hadronic tensor

$$W_{\mu\nu} = \frac{1}{4}\sum_n \langle p \left| J^{el}_\mu(0) \right| n\rangle \, \langle n \left| J^{el}_\nu(0) \right| p\rangle \, (2\pi)^4 \delta^4(p+q-p_n)$$

can be written using the properties of the gauge-invariance and the parity conservation as follows

$$\frac{1}{\pi} W_{\mu\nu} = -\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1(x,Q^2) + \left(p_\mu - \frac{q_\mu (pq)}{q^2}\right)\left(p_\nu - \frac{q_\nu (pq)}{q^2}\right) \frac{F_2(x,Q^2)}{pq}\,.$$

Here the Bjorken variables Q^2 and x are

$$Q^2 = -q^2\,,\;\, x = \frac{Q^2}{2pq}\,.$$

The structure functions $F_{1,2}(x,Q^2)$ do not depend on Q^2 in the framework of the Bjorken-Feynman parton model, which corresponds to the Bjorken scaling:

$$\lim_{Q\to\infty} F_{1,2}(x,Q^2) = F_{1,2}(x)\,.$$

In this model the structure functions can be calculated in the impulse approximation as a sum of the structure functions for the charged partons averaged with the partonic distributions (see Fig. 11)

Providing that the charged partons are fermions (quarks) with the momentum k using

$$|n\rangle\,\langle n| = \int \frac{d^4p_n}{(2\pi)^3}\;\delta(p_n^2 - m^2)$$

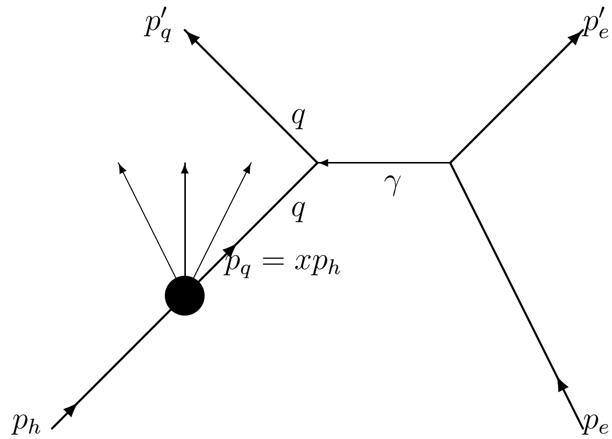

Figure 11: Partonic description of the deep-inelastic eh scattering

one can calculate their structure functions

$$\frac{1}{\pi}W^{f}_{\mu\nu} = \frac{Q_q^2}{2}\delta\left((k+q)^2\right)\frac{1}{2}tr\,(\widehat{k}\,\gamma_\mu(\widehat{q}+\widehat{k})\gamma_\nu) = -Q_q^2\frac{x}{2}\,\delta^{\perp}_{\mu\nu}\,\delta(\beta - x)\,,$$

where β is the Sudakov variable $\beta = \frac{kq}{pq}$ and $x = \frac{Q^2}{2pq}$. With the use of the identity

$$-\delta^{\perp}_{\mu\nu} = -\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + \left(p_\mu - \frac{q_\mu(pq)}{q^2}\right)\left(p_\nu - \frac{q_\nu(pq)}{q^2}\right)\frac{2x}{pq}$$

we obtain, that in the framework of the quark-parton model the Callan-Gross relation between F_1 and F_2 is valid

$$F_2(x) = 2xF_1(x)$$

and

$$F_2(x) = \sum_q e_q^2\, n^q(x)\,.$$

Here $n^q(x)$ is the quark distribution in the hadron, normalized in such way, that the electric charge conservation takes the form

$$1 = \sum_{\overline{q}} e_{\overline{q}} \int_0^1 dx\, n^{\overline{q}}(x)\,.$$

The structure functions can be expressed in terms of the cross-sections σ_t and σ_l for the scattering of the virtual photons with the transverse (t) and longitudinal (l) polarization. In the quark-parton model $\sigma_l = 0$ in an accordance with the Callan-Gross relarion between \$$F_1$\$ and \$$F_1$\$.

Another important deep-inelastic process is the scattering of neutrinos and anti-neutrinos off hadrons. The first contribution is induced by the Z-boson exchange (a neutral current contribution). In this case the expressions for inclusive cross-sections are similar to the case of the electron scattering. Another contribution is related with the t-channel exchange of the W-boson. The cross-section of the corresponding inclusive process, in which in the final state only produced electron or positron with the momentum p'_e are measured, is given below

$$d\sigma^{\nu,\overline{\nu}} = \frac{g^4}{16\pi^3}\left(\frac{1}{2\sqrt{2}}\right)^2\left(\frac{1}{q^2-M_W^2}\right)^2 L^{\mu\rho}_{\nu,\overline{\nu}} W^{\nu,\overline{\nu}}_{\mu\rho}\frac{d^3p'_e}{(pp_{\nu,\overline{\nu}})E'}\,,$$

where, taking into account, that initial neutrino or anti-neutrino have the fixed helicities, we have

$$L^{\mu\rho}_{\nu,\overline{\nu}} = tr(\widehat{p'_e}\gamma^\mu(1\pm\gamma_5)\widehat{p_{\nu,\overline{\nu}}}\gamma^\rho(1\pm\gamma_5)) =$$
$$8\left(p'^\mu_e p^\rho_{\nu,\overline{\nu}} + p'^\rho_e p^\mu_{\nu,\overline{\nu}} - \delta^{\rho\mu}(p'_e p_{\nu,\overline{\nu}}) \pm i\epsilon^{\mu\rho\lambda\delta}p^\lambda_{\nu,\overline{\nu}}\,p'^\delta_e\right)\,.$$

For the hadronic tensor we have

$$\frac{1}{\pi}W^{\nu,\overline{\nu}}_{\mu\rho} = -\left(\delta_{\mu\rho}-\frac{q_\mu q_\rho}{q^2}\right)F_1^{\nu,\overline{\nu}} + \left(p_\mu - \frac{q_\mu(pq)}{q^2}\right)\left(p_\rho-\frac{q_\rho(pq)}{q^2}\right)\frac{F_2^{\nu,\overline{\nu}}}{pq} + i\epsilon_{\mu\rho\lambda\delta}\,p^\lambda\,q^\delta\frac{F_3^{\nu,\overline{\nu}}}{2pq}\,,$$

and in the framework of the parton model for the scattering off proton one can obtain

$$F_1^\nu = \frac{1}{2x}F_2^\nu = d(x)\cos^2\theta_c + \overline{u}(x) + s(x)\,\sin^2\theta_c\,,$$
$$\frac{1}{2}F_3^\nu = d(x)\cos^2\theta_c - \overline{u}(x) + s(x)\,\sin^2\theta_c\,,$$
$$F_1^{\overline{\nu}} = \frac{1}{2x}F_2^{\overline{\nu}} = u(x) + \overline{d}(x)\cos^2\theta_c + \overline{s}(x)\,\sin^2\theta_c\,,$$
$$\frac{1}{2}F_3^{\overline{\nu}} = u(x) - \overline{d}(x)\cos^2\theta_c - \overline{s}(x)\,\sin^2\theta_c\,.$$

Here $u(x)$, $d(x)$, $s(x)$, $\overline{u}(x)$, $\overline{d}(x)$, $\overline{s}(x)$ are the parton distributions of the corresponding quarks in the proton and θ_c is the Cabbibo angle - the parameter of the Cabbibo-Kobayashi-Maskawa matrix (we neglect the presence of the heavier quarks in the proton). The elements of the CKM matrix appear in the vertex of the quark interaction with the W-boson.

The simplest process, in which one can measure the distribution of the hadrons with the relative momentum z inside quarks $n_q^h(z)$ is the inclusive annihilation of the e^+e^- -pair into hadrons where only the momentum p of one hadron h is measured. Its differential cross-section in pure QED (neglecting the Z-boson contribution) is (see Fig. 16)

$$d\sigma_\gamma = \frac{4\alpha^2}{\pi}\frac{1}{s^3}\overline{L}^{\mu\nu}\overline{W}_{\mu\nu}\frac{d^3p}{E}\,,\; E=|p|\;\;,\;\; s=q^2\,,$$

where

$$\overline{L}^{\mu\nu} = \frac{1}{4}tr\left(\widehat{p_e}\gamma^\mu\widehat{p_{\overline{e}}}\gamma^\nu\right) = p_e^\mu p_{\overline{e}}^\nu + p_e^\nu p_{\overline{e}}^\mu - \delta^{\mu\nu}(p_e p_{\overline{e}})$$

and

$$\overline{W}_{\mu\nu} = \frac{1}{8}\sum_n \langle 0\left|J^{el}_\mu(0)\right|n,h\rangle\,\langle n,h\left|J^{el}_\nu(0)\right|0\rangle\,(2\pi)^4\delta^4(q-p_n-p_h)\,.$$

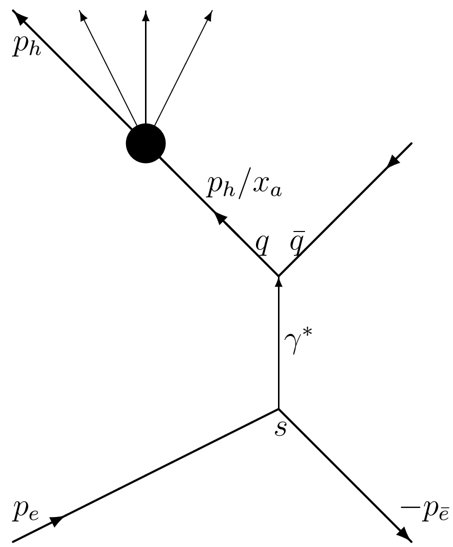

Figure 12: Inclusive $e\bar{e}$-annihilation to hadrons in the parton model

We have from the gauge invariance and the parity conservation

$$\frac{1}{\pi}\overline{W}_{\mu\nu} = -\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\overline{F}_1(x_a) + \left(p_\mu - \frac{q_\mu(pq)}{q^2}\right)\left(p_\nu - \frac{q_\nu(pq)}{q^2}\right)\frac{\overline{F}_2(x_a)}{pq},$$

where

$$x_a = \frac{2pq}{q^2} < 1\,,\; q^2 = s.$$

In the parton model the inclusive annihilation $e^+e^- \to h + ...$ is described as the process, in which initially e^+ and e^- produce the pair $q\overline{q}$ and later q or $\overline{q}$ transform into the hadron system with an extracted particle h. For the structure functions $\overline{F}_1(x_a, q^2)$ and $\overline{F}_2(x_a, q^2)$ we obtain in this model:

$$\overline{F}_1(z) = -\frac{z}{2}\overline{F}_2(z) = \frac{3}{z}\sum_q e_q^2(h_q(z) + h_{\overline{q}}(z))\,,$$

where $h_q(z)$ and $h_{\overline{q}}(z)$ are distributions of hadrons h inside the corresponding partons q and $\overline{q}$ (they are called also the fragmentation functions). The factor 3 is related to the number of coloured quarks in the fundamental representation of the gauge group $SU(3)$.

In QCD the Bjorken scaling for the structure functions is violated and the quark and hadron distributions depend logarithmically on Q^2 in an accordance with the evolution equations.

11 Evolution equations

In the framework of the parton model one can introduce the wave function of the hadron in the infinite momentum frame $|\overrightarrow{p}| \to \infty$ with the following normalization condition:

$$1 = \|\Psi\|^2 \equiv \sum_n \int \prod_{i=1}^{n} \frac{d\beta_i d^2 k_{\perp i}}{(2\pi)^2} \, |\Psi(\beta_1, k_{\perp 1}; \beta_2, k_{\perp 2}; ...; \beta_n, k_{\perp n})|^2 \, \delta(1 - \sum_{i=1}^{n} \beta_i) \, \delta^2(\sum_{i=1}^{n} k_{\perp i}) \, .$$

The wave function $\Psi(\beta_1, k_{\perp 1}; \beta_2, k_{\perp 2}; ...; \beta_n, k_{\perp n})$ satisfies the Schrödinger equation and contains in the perturbation theory the energy propagators

$$\left(E(p) - \sum_{i=1}^{n} E(k_i) \right)^{-1} = 2\,|\overrightarrow{p}| \left(m^2 - \sum_{i=1}^{n} \frac{m_i^2 + \overrightarrow{k}_{i\perp}^2}{\beta_i} \right)^{-1} .$$

For the renormalized field theories in the right hand side of the normalization condition for Ψ there are ultraviolet divergences. We regularize them by introducing the ultraviolet cut-off Λ^2 over the integrals in $k_{i\perp}$:

$$\left| k_{i\perp}^2 \right| < \Lambda^2 \, .$$

The amplitudes of the physical processes do not depend on Λ due to the renormalizability - the possibility to compensate the ultraviolet divergences by the renormalization - choosing an appropriate bare coupling constant depending on Λ to obtain a finite renormalized coupling constant in the limit $\Lambda \to \infty$. Therefore in a hard process of the type of the deep-inelastic ep -scattering one can fix Λ as follows

$$\Lambda^2 = Q^2 \, .$$

In this case we obtain the usual formulas of the parton model for the physical amplitudes. In the gauge theories QED and QCD to conserve the partonic picture one should use a physical gauge in which the virtual particles contain only the states with the positive norm. It is convenient to chose the light-cone gauge

$$A_\mu \, q_\mu^{'} = 0 \, , \; q^{'} = q - \frac{q^2}{2pq} \, p \, , \; q'^2 = 0 \, ,$$

where the propagator of the gauge boson is

$$D_{\mu\nu}(k) = - \frac{\delta_{\mu\nu} - \frac{k_\mu q_\nu^{'} + k_\nu q_\mu^{'}}{kq'}}{k^2} = \frac{\sum_{i=1,2} e_\mu^i(k') \, e_\nu^{i*}(k')}{k^2} + \frac{q'_\mu q'_\nu}{(kq')^2} \, , \qquad (2)$$

containing on the mass shell $k'^2 = 0$ ($k' = k - \frac{k^2}{xs} q'$) only the physical polarization vectors $e_\mu^i(k')$ satisfying the constraints

$$e_\mu^i(k') \, k'_\mu = e_\mu^i(k') \, q'_\mu = 0 \, .$$

They have the following Sudakov expansion

$$e^i = e_\perp^i - \frac{k e_\perp^i}{kq'} q' \, .$$

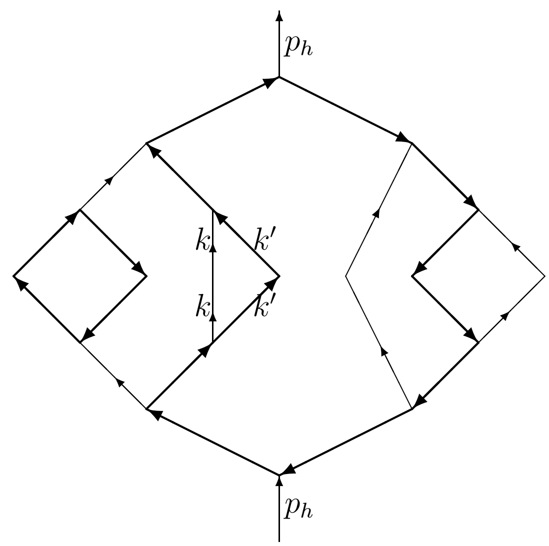

Figure 13: Normalization condition for the partonic wave function

The Λ-dependence of the partonic wave functions expressed in terms of the physical charge is determined by the renormalization group:

$$|\Psi(\beta_1, k_{\perp 1}; \beta_2, k_{\perp 2}; ...; \beta_n, k_{\perp n}|^2 \sim \prod_r Z_r^{n_r}\,.$$

Here $\sqrt{Z_r} < 1$ are the renormalization constants for the wave functions of the corresponding fields and Z_r is the probability to find the physical particle r in the corresponding one-particle bare state. Further, n_r is the number of the bare particles r in the state of n partons. For finding the Λ-dependence of the limits of integration in $k_{\perp i}$ one should take into account, that the largest contribution in the normalization condition occurs from the momentum configuration of the type of the Russian "matrëshka", when the constituent particles in the initial hadron consist of two partons, each of these partons again consists of two other partons and so on. In each step of this parton branching the transverse momenta of the particles grow and only one of the last partons in this chain of decays reaches the largest possible value $|k_\perp| = \Lambda$. It is related to the fact, that only for such configuration the number of the energy propagators with large denominators is minimal. Moreover, the quantum-mechanical interference of amplitudes with different schemes of decays is not important in the normalization condition within the leading logarithmic accuracy in $\ln\Lambda$ (see Fig. 13).

Therefore if we shall differentiate the normalization condition in Λ, the most essential contribution will appear from the limits of the integrals over $k_\perp^2 \simeq k_\perp'^2$ for two partons p, p' produced in the end of the decay chain (see Fig. 13) and one can obtain from this differentiation the equation

$$0 = \sum_r \overline{n_r} \left(\frac{d\ln Z_r}{d\ln(\Lambda^2)} + \gamma_r \right)\,,\; \gamma_r = \sum_{p,p'} \frac{d\left\|\Psi_{r\to pp'}\right\|^2}{d\ln(\Lambda)^2}\,.$$

Here $\|\Psi_{r\to pp'}\|^2 \sim g^2$ is the one-loop contribution to the norm of the wave function of the parton r related to its transition to the two particles p and p'. The quantity

$$\overline{n_r} = \sum_n n_r \int \prod_{i=1}^{n} \frac{d\beta_i d^2 k_{\perp i}}{(2\pi)^2} \, |\Psi(\beta_1, k_{\perp 1}; \beta_2, k_{\perp 2}; ...; \beta_n, k_{\perp n}|^2 \, \delta(1 - \sum_{i=1}^{n} \beta_i) \, \delta^2(\sum_{i=1}^{n} k_{\perp i})$$

is an averaged number of partons r in the hadron. Because this number is different in various hadrons, we obtain

$$\frac{dZ_r}{d\ln(\Lambda^2)} = -\gamma_r Z_r \,,$$

which coincides with the Callan-Simanzik equation for the renormalization constants and γ_r is the anomalous dimension for the corresponding field.

In an analogous way, after the differentiation of the partonic expression for the density of the number of partons k in the hadron

$$n_r(x) = \sum_n \int \prod_{i=1}^{n} \frac{d\beta_i d^2 k_{\perp i}}{(2\pi)^2} \, |\Psi(\beta_1, k_{\perp 1}; ...; \beta_n, k_{\perp n}|^2 \, \delta(1 - \sum_{i=1}^{n} \beta_i) \, \delta^2(\sum_{i=1}^{n} k_{\perp i}) \sum_{i \in r} \delta(\beta_i - x) \,,$$

and the use of the Callan-Simanzik equation we shall obtain for $n_r(x)$ the evolution equations of Dokshitzer, Gribov, Lipatov, Altarelli and Parisi (DGLAP).

In QCD the DGLAP equations have the form

$$\frac{d}{d\xi} n_k(x) = -w_k \, n_k(x) + \sum_r \int_x^1 \frac{dy}{y} \, w_{r\to k}\left(\frac{x}{y}\right) \, n_r(y) \,, \tag{3}$$

where w_r is proportional to γ_r:

$$w_r = \sum_k \int_0^1 dx \, x \, w_{r\to k}(x) \,, \tag{4}$$

which can be obtained from the energy conservation

$$\sum_k \int_0^1 d\,x \, x \, n_k(x) = 1 \,.$$

The variable ξ

$$\xi = -\frac{2N_c}{\beta_s} \ln \frac{\alpha(Q^2)}{\alpha_\mu}$$

is related to the QCD running constant

$$d\xi = \frac{\alpha(Q^2) \, N_c}{2\pi} \, d\ln Q^2 \,, \;\; \alpha(Q^2) = \frac{\alpha_\mu}{1 + \beta_s \frac{\alpha_\mu}{4\pi} \ln \frac{Q^2}{\mu^2}} \,, \; \beta_s = \frac{11}{3} N_c - n_f \frac{2}{3} \,. \tag{5}$$

By integrating over x one can obtain simpler evolution equations for an averaged number of partons:

$$\frac{d}{d\xi} n_k = -w_k \, n_k + \sum_r w_{r\to k} \; n_r \,, \;\; w_{r\to k} = \int_0^1 dx \, w_{r\to k}\,(x) \;. \tag{6}$$

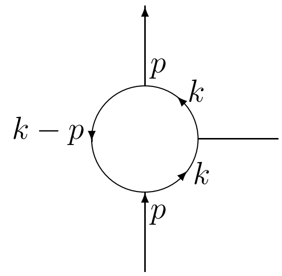

Figure 14: Matrix element of the conserved current or energy-momentum tensor

for the average number of partons

$$n_k = \int_0^1 d\,x\, n_k(x)\,.$$

For the electric charge and other additive quantum numbers we have the conservation law

$$\frac{d}{d\xi}\sum_k Q_k\, n_k = 0 \tag{7}$$

due to the following property

$$\sum_k Q_k\, w_{r\to k} = w_r\, Q_r$$

of the transition probabilities. The energy conservation

$$\frac{d}{d\xi}\sum_k \int_0^1 dx\, x\, n_k(x) = 0 \tag{8}$$

is valid due to the following property of the splitting kernels

$$\sum_k \int_0^1 dx\, x\, w_{r\to k}(x) = w_r\,. \tag{9}$$

In turn, these sum rules can be applied for finding the splitting kernels $w_{r\to i}(x)$. For this purpose on one hand we should use the Feynman diagram approach to calculate the matrix elements of the conserved currents $j_\mu(z)$ or energy-momentum stress tensor $T_{\mu\nu}(z)$ in one-loop approximation (see Fig. 14). On the other hand with the use of the Sudakov parametrization

$$k = \alpha q' + xp + k_\perp$$

of momenta of the virtual particles we can write for these matrix elements

$$\xi \int_0^1 dx\; w_{r\to i}(x) = i\int \frac{|s'|\, d\alpha\, dx\, d^2k_\perp}{2\,(2\pi)^4}\, \frac{g^2\,\sum_j\, |\gamma_{r\to i,j}|^2}{-s(1-x)\alpha - \overrightarrow{k}_\perp^{2} + i\varepsilon}\, \frac{x}{\left(sx\alpha - \overrightarrow{k}_\perp^{2} + i\varepsilon\right)^2}$$

in the case, when the particles r and k have the same conserved quantum number Q, and

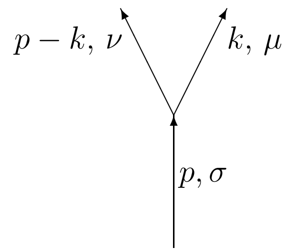

Figure 15: Yang-Mills vertex

$$\xi\int_0^1 dx\,x\,w_{r\to i}(x) = i\int\frac{|s'|\,d\alpha\,dx\,d^2k_\perp}{2(2\pi)^4}\,\frac{g^2\sum_j\,|\gamma_{r\to i,j}|^2}{-s(1-x)\alpha-\overrightarrow{k}_\perp^2+i\varepsilon}\,\frac{x^2}{\left(sx\alpha-\overrightarrow{k}_\perp^2+i\varepsilon\right)^2}\,.$$

for the vertex of the energy-momentum stress tensor. Here $\gamma_{r\to k,l}$ is the corresponding transition QCD amplitude calculated in the helicity basis, which is convenient, because the helicity is conserved for the corresponding matrix elements. We find below the amplitudes $\gamma_{r\to k,l}$ for all possible parton transitions. In the above expressions the integrals over α are non-zero only if $0<x<1$ and they can be calculated by resudies giving finally

$$w_{r\to k}(x) = \sum_j\frac{|\gamma_{r\to i,j}|^2}{N_c}\,\frac{x(1-x)}{2\left|\overrightarrow{k}_\perp\right|^2}\,. \tag{10}$$

11.1 Integral kernels for gluon-gluon transitions

We start with the discussion of the splitting kernels $w_{r\to k}\left(\frac{x}{y}\right)$ for the transition from gluon to gluon. The gluon Yang-Mills vertex for the transition $(p,\sigma)\to(k,\mu)+(p-k,\nu)$ can be written as follows (see Fig. 15)

$$\gamma_{\sigma\mu\nu} = (p-2k)_\sigma\,\delta_{\mu\nu}+(p+k)_\nu\,\delta_{\sigma\mu}+(k-2p)_\mu\,\delta_{\nu\sigma} \tag{11}$$

up to the colour factor f_{abc} being the structure constant of the colour group $SU(N_c)$ entering in the commutation relations of the generators:

$$[T_a,T_b] = if_{abc}\,T_c\,. \tag{12}$$

On the mass shell the numerators of the gluon propagators in the light-cone gauge coincide with the projectors to physical states. The corresponding polarization vectors satisfying also the Lorentz condition $ek'=0$ have the following Sudakov representation

$$e_\sigma(p)=e^\perp_\sigma\,,\ e_\mu(k')=e^\perp_\mu-\frac{ke^\perp}{kq'}\,q'_\mu\,,\ e_\nu(p-k')=e^\perp_\nu-\frac{(p-k)e^\perp}{(p-k)q'}\,q'_\nu\,,\ k'=k-\frac{k^2}{2kq'}q'\,. \tag{13}$$

After the multiplication of the Yang-Mills vertex $\gamma_{\sigma\mu\nu}$ with these polarization vectors one can introduce the tensor with transverse components according to the definition

$$\gamma_{\sigma\mu\nu} e^{\sigma}(p) e^{\mu}(k') e^{\nu}(p-k') = \gamma^{\perp}_{\sigma\mu\nu} e^{\sigma}_{\perp}(p) e^{\mu}_{\perp}(k') e^{\nu}_{\perp}(p-k') \,, \tag{14}$$

where

$$\gamma^{\perp}_{\sigma\mu\nu} = -2k^{\perp}_{\sigma}\, \delta^{\perp}_{\mu\nu} + \frac{2}{1-x}\, k^{\perp}_{\nu}\, \delta^{\perp}_{\sigma\mu} + \frac{2}{x}\, k^{\perp}_{\mu}\, \delta^{\perp}_{\nu\sigma} \,, \;\; x = \frac{kp}{q'p} \,. \tag{15}$$

The states with definite helicities

$$e^{\pm}_{\perp} = \frac{1}{\sqrt{2}}(e^1 \pm i e^2) \,. \tag{16}$$

We put the helicity λ of the initial gluon with the momentum p equal $+1$, because for $\lambda = -1$ the results can be obtained from the obtained expressions by changing the signs of helicities of the final particles. Then the non-zero matrix elements

$$\gamma_{\lambda_1\lambda_2} = \gamma^{\perp}_{\sigma\mu\nu}\, e^{\sigma}_{+}(p) e^{*\mu}_{\lambda_1}(k') e^{*\nu}_{\lambda_2}(p-k') \tag{17}$$

are

$$\gamma_{++} = \sqrt{2}\frac{k^*}{x(1-x)} \,, \; \gamma_{+-} = \sqrt{2}\frac{x}{1-x}\, k \,, \;\; \gamma_{+-} = -\sqrt{2}\frac{1-x}{x}\, k \,, \tag{18}$$

where $k = k_1 + ik_2$, $k^* = k_1 - ik_2$.

The dimensionless quantities

$$w_{1^+\to i}(x) = \frac{x(1-x)}{2\,|k|^2} \sum_k |\gamma_{ik}|^2 \,, \tag{19}$$

are the splitting kernels (we cancelled the colour factor $f_{abc}f_{a'bc} = N_c\delta_{aa'}$). They equal

$$w_{1^+\to 1^+}(x) = \frac{1+x^4}{x(1-x)} \,, \;\; w_{1^+\to 1^-}(x) = \frac{(1-x)^4}{x(1-x)} \,. \tag{20}$$

The contribution to $w_g = w_{g\to gg} + w_{g\to q\bar{q}}$ from the transition to gluons is

$$w_{g\to gg} = \frac{1}{2}\int_0^1 \frac{1+x^4+(1-x)^4}{x(1-x)}\; dx \,, \tag{21}$$

where we substituted x by $\frac{1}{2}$ in the integrand due to its symmetry to the substitution $x \to 1-x$. The divergency of $w_{g\to gg}$ at $x = 0, 1$ is cancelled in the evolution equations. There is also the contribution to w_g from the quark-antiquark state (see below):

$$w_{g\to q\bar{q}} = \frac{n_f}{2N_c}\int_0^1 \left(x^2 + (1-x)^2\right)\; dx \; = \frac{n_f}{3N_c} \,. \tag{22}$$

The matrix elements for the anomalous dimension matrix describing gluon-gluon transitions can be written as follows

$$w^{j}_{r\to k} = \int_0^1 dx\; w_{r\to k}(x)\, \left(x^{j-1} - x\,\delta_{rk}\right) - \frac{n_f}{3N_c}\, \delta_{rk} \,. \tag{23}$$

Thus, we obtain

$$w^{j}_{1^+\to 1^+} = 2\psi(1) - 2\psi(j-1) - \frac{1}{j+2} - \frac{1}{j+1} - \frac{1}{j} - \frac{1}{j-1} + \frac{11}{6} - \frac{n_f}{3N_c} \quad . \tag{24}$$

and

$$w^{j}_{1^+\to 1^-} = -\frac{1}{j+2} + \frac{3}{j+1} - \frac{3}{j} + \frac{1}{j-1} \, . \tag{25}$$

The vector current anomalous dimension for the gluodynamics

$$w^{jv}_{1\to 1} = 2\psi(1) - 2\psi(j-1) - \frac{2}{j+2} + \frac{2}{j+1} - \frac{4}{j} + \frac{11}{6} - \frac{n_f}{3N_c} \tag{26}$$

The anomalous dimension for the axial current containing the operator $G_{\mu\sigma}\widetilde{G}_{\mu\nu}$ in the gluodynamics

$$w^{ja}_{1\to 1} = 2\psi(1) - 2\psi(j) - \frac{4}{j+1} + \frac{2}{j} + \frac{11}{6} - \frac{n_f}{3N_c}. \tag{27}$$

The energy conservation sum rule for $j = 2$

$$w^{2v}_{1\to 1} + w^{2v}_{1\to 1/2} = 0. \tag{28}$$

is fulfilled as it can be verified from the expression for $w^{jv}_{1\to 1/2}$ obtained in the next section. Because the contribution $w^{jv}_{1\to 1/2}$ is proportional to n_f, one can verify that $w^{2v}_{1\to 1} = 0$ at $n_f = 0$.

11.2 Transition from gluon to quark

The propagator of the massless fermion can be written in the form

$$G(k) = \frac{\widehat{k}}{k^2}\,, \;\; \widehat{k} = \sum_{\lambda=\pm} u^\lambda(k')\,\overline{u^\lambda(k')} + \frac{k^2}{2kq'}\,\widehat{q}'\,,\;\; k' = k - \frac{k^2}{xs}q'\,,$$

where $k'^2 = 0$. The last contribution in $\widehat{k}$ is absent on the mass shell or providing that the vertex neighbouring to the propagator G is $\widehat{q}'$. The massless fermion with the momentum $\overrightarrow{k}$ and the helicity $\lambda/2$ is described by the spinor

$$u^\lambda(\overrightarrow{k}) = \sqrt{k_0}\begin{pmatrix} \varphi^\lambda \\ \lambda\varphi^\lambda \end{pmatrix}\,, \;\; \overline{u}\,\gamma_\mu\, u = 2\,k_\mu\,,\; \widehat{k} = \sum_\lambda u^\lambda \overline{u}^\lambda\,, \tag{29}$$

where the Pauli spinor φ satisfies the equation

$$\overrightarrow{\sigma}\,\overrightarrow{k}\,\varphi^\lambda - \lambda\,k_0\,\varphi^\lambda = \begin{pmatrix} k_3 - \lambda\,k_0 & k_1 - ik_2 \\ k_1 + ik_2 & -k_3 - \lambda\,k_0 \end{pmatrix}\begin{pmatrix} \varphi^\lambda_1 \\ \varphi^\lambda_2 \end{pmatrix} = 0\,,\;\; k_0 = \left|\overrightarrow{k}\right|. \tag{30}$$

In the light-cone frame $p = p_3 \to \infty$ we have

$$\varphi^+ \simeq \begin{pmatrix} 1 \\ \frac{k}{2\,x\,p} \end{pmatrix}\,,\;\; \varphi^- \simeq \begin{pmatrix} \frac{-k^*}{2\,x\,p} \\ 1 \end{pmatrix}\,. \tag{31}$$

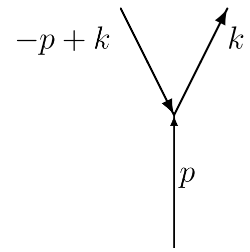

Figure 16: Gluon-quark-anti-quark vertex

The massless anti-fermion with the momentum $\overrightarrow{p}-\overrightarrow{k}$ and the helicity $\lambda'/2$ is described by the spinor

$$v^{\lambda'}(-\overrightarrow{p}+\overrightarrow{k})=\sqrt{p_0-k_0}\left(\begin{array}{c}\chi^{\lambda'}\\ -\lambda'\chi^{\lambda'}\end{array}\right), \tag{32}$$

where the Pauli spinor χ satisfies the equation

$$\overrightarrow{\sigma}(\overrightarrow{p}-\overrightarrow{k})\chi^{\lambda'}+\lambda'(p_0-k_0)\,\chi^{\lambda'}=0\,. \tag{33}$$

In the light-cone frame we have

$$\chi^{-}\simeq\left(\begin{array}{c}1\\ \frac{-k}{2(1-x)\,p}\end{array}\right),\ \chi^{+}\simeq\left(\begin{array}{c}\frac{k^*}{2\,(1-x)\,p}\\ 1\end{array}\right). \tag{34}$$

Therefore for the matrix element of the vertex for the transition of the gluon with its momentum $\overrightarrow{p}$ and helicity 1 into a pair of fermions (see Fig. 16) we obtain

$$\gamma^{\lambda\lambda'}=\overline{u}^{\lambda}(\overrightarrow{k})\,\frac{\gamma^1+i\gamma^2}{\sqrt{2}}\,v^{\lambda'}(-\overrightarrow{p}+\overrightarrow{k})=$$

$$2\,\sqrt{(p_0-k_0)\,k_0}\lambda\,\delta_{\lambda,-\lambda'}\ \ \sqrt{2}\varphi^{\lambda *}\left(\begin{array}{cc}0&1\\0&0\end{array}\right)\chi^{\lambda'}\,. \tag{35}$$

Thus, we obtain for the non-zero matrix elements

$$\gamma^{+-}=-\sqrt{2}\,k\sqrt{\frac{x}{1-x}},\ \ \gamma^{+-}=-\sqrt{2}\,k\sqrt{\frac{1-x}{x}}\,. \tag{36}$$

Using the representation for the splitting kernels similar to the pure gluonic case

$$w_{1^+\to q}(x)=\frac{n_f}{2N_c}\frac{x(1-x)}{2\,|k|^2}\,\left|\gamma^{+q}\right|^2, \tag{37}$$

where $\frac{n_f}{2N_c}$ is the colour factor for the loops with n_f quarks, because we included the factor N_c in the definition of ξ. Thus, we obtain

$$w_{1^+\to q^+}(x)=w_{1^+\to\overline{q}^+}(x)=\frac{n_f}{2N_c}x^2\,,\ w_{1^+\to q^-}(x)=w_{1^+\to\overline{q}^-}(x)=\frac{n_f}{2N_c}(1-x)^2\,. \tag{38}$$

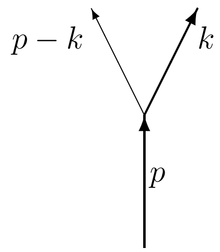

Figure 17: Quark-gluon-quark vertex

The total probability for the gluon transition to quark is

$$w_{1\to q,\bar{q}} = \frac{n_f}{2N_c}\int_0^1 \left(x^2+(1-x)^2\right)\ dx\ = \frac{n_f}{3N_c}\,, \tag{39}$$

For QED the colour factor $\frac{n_f}{2N_c}$ is absent and we obtain $w_{1\to\frac{1}{2}\frac{\bar{1}}{2}} = \frac{2}{3}$. Taking into account, that in the perturbation theory $\xi = \frac{\alpha}{2\pi}\ln\Lambda^2$ for QED, we obtain, that the probability for the photon to be in the e^+e^- state is $\frac{\alpha}{3\pi}\ln\Lambda^2$ in an agreement with the known result for the charge renormalization in this theory.

The non-vanishing matrix elements for the anomalous dimension matrix are

$$w^j_{1^+\to q^+} = \frac{n_f}{2N_c}\frac{1}{j+2}\,,\ w^j_{1^+\to q^-}(x) = \frac{n_f}{2N_c}\left(\frac{1}{j}-\frac{2}{j+1}+\frac{1}{j+2}\right) \tag{40}$$

and these elements for the vector and axial current are

$$w^{jv}_{1\to 1/2} = \frac{n_f}{N_c}\left(\frac{1}{j}-\frac{2}{j+1}+\frac{2}{j+2}\right)\,,\ w^{ja}_{1\to 1/2}(x) = \frac{n_f}{N_c}\left(-\frac{1}{j}+\frac{2}{j+1}\right)\,, \tag{41}$$

where we added the gluon transitions to the quark and antiquark.

11.3 Transition from quark to quark and gluon

The amplitude for the transition of a quark with the helicity + to quark and gluon (see Fig. 17) can be written as follows in the light cone gauge for the gluon polarization vector:

$$\overline{u}^+(k)\,(\widehat{e^*}^\lambda_\perp + \frac{k_\perp e^{*\lambda}}{(p-k,q')}\,\widehat{q}')\,u^+(p) = \sqrt{k_0p_0}2\varphi^{+*}\overrightarrow{\sigma}\overrightarrow{e^{*\lambda}}\varphi^+ + 2\,k_\perp e^{*\lambda}\,\frac{\sqrt{x}}{1-x}\,. \tag{42}$$

Therefore we have

$$\overline{u}^+(k)\,(\widehat{e}^{+*}_\perp + \frac{k_\perp e^{+*}}{(p-k,q')}\,\widehat{q}')\,u^+(p) = \sqrt{2}k^*\frac{1}{\sqrt{x}(1-x)}\,,$$

$$\overline{u}^+(k)\,(\widehat{e}^{-*}_\perp + \frac{k_\perp e^{-*}}{(p-k,q')}\,\widehat{q}')\,u^+(p) = \sqrt{2}k\frac{\sqrt{x}}{(1-x)}\,. \tag{43}$$

Thus, the splitting kernels are

$$w_{1/2^+\to 1^+}(x) = \frac{c}{x}\,,\; w_{1/2^+\to 1^-}(x) = c\frac{(1-x)^2}{x}\,,\; w_{1/2^+\to 1/2^+}(x) = c\frac{1+x^2}{1-x}\,, \tag{44}$$

where

$$c = \frac{N_c^2-1}{2N_c^2} \tag{45}$$

is the colour factor for the corresponding loop (note, that N_c is included in ξ)

The total contribution to $w_{1/2}$ is

$$w_{1/2} = \frac{c}{2}\int_0^1 \left(\frac{1+x^2}{1-x} + \frac{1+(1-x)^2}{x}\right)\ dx\ . \tag{46}$$

The corresponding anomalous dimensions are

$$w^j_{1/2^+\to 1^+} = \frac{c}{j-1}\,,\; w^j_{1/2^+\to 1^-} = c\left(\frac{1}{j-1} - \frac{2}{j} + \frac{1}{j+1}\right)\,, \tag{47}$$

$$w^j_{1/2^+\to 1/2^+} = c\int_0^1 \frac{1+x^2}{1-x}\,(x^{j-1}-1)\ dx = c\left(2\psi(1) - 2\psi(j) - \frac{1}{j+1} - \frac{1}{j} + \frac{3}{2}\right)\,. \tag{48}$$

The vector and axial contributions are

$$w^{jv}_{1/2\to 1} = c\left(\frac{2}{j-1} - \frac{2}{j} + \frac{1}{j+1}\right)\,,\; w^{ja}_{1/2\to 1} = c\left(\frac{2}{j} - \frac{1}{j+1}\right)\,. \tag{49}$$

We have the sum rules:

$$w^{1v}_{1/2\to 1/2} = 0\,,\; w^{2v}_{1/2\to 1} + w^{2v}_{1/2\to 1/2} = 0, \tag{50}$$

expressing the conservation of the baryonic charge and the energy in the quark decay.

As it is seen from the above formulae, there are two relations among the matrix elements of the anomalous dimension matrix for $n_f = N_c$

$$\frac{1}{c}\left(w^{jv}_{1/2\to 1} + w^{jv}_{1/2\to 1/2}\right) = w^{jv}_{1\to 1} + \ w^{jv}_{1\to 1/2}\ = 2\psi(1) - 2\psi(j-1) - \frac{3}{j} + \frac{3}{2}\,,$$

$$\frac{1}{c}\left(w^{ja}_{1/2\to 1} + w^{ja}_{1/2\to 1/2}\right) = w^{ja}_{1\to 1} + w^{ja}_{1\to 1/2} = 2\psi(1) - 2\psi(j) - \frac{2}{j+1} + \frac{1}{j} + \frac{3}{2}\,.$$

These relations are consequences of the supersymmetry (SUSY), where the SUSY multiplet consists of the gluon and its partner - gluino being a Maiorano fermion belonging to the adjoint representation of the gauge group . Indeed, in this theory the total probability to find gluon and gluino with a given Feynman parameter x inside gluon and gluino should be the same for these two particles. On the other hand, in the transition of the gluon to the $q\bar{q}$ pair in QCD we have the extra factor $2n_f/(2N_c)$ in comparison with its transition to two gluinos. Analogously, in the probability of the transition of the quark to the gluon and quark we have the factor c in comparison with the corresponding gluino transition.

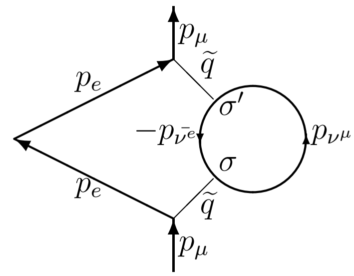

Figure 18: μ-meson polarization operator

12 Heavy lepton decays

The amplitude for the μ-meson decay in the Born approximation of the Standard Model is given below

$$A = \frac{G}{\sqrt{2}}\, \overline{u}(p_{\nu^\mu})\, \gamma_\sigma\, (1+\gamma_5)\, u(p_\mu)\, \frac{-m_w^2}{q^2 - m_w^2}\, \overline{u}(p_e)\, \gamma_\sigma\, (1+\gamma_5)\, v(-p_{\overline{\nu^e}}) \ ,$$

where $v(-p_{\overline{\nu^e}})$ describes the electronic anti-neutrino with the momentum $p_{\overline{\nu^e}}$ and $q = p_\mu - p_{\nu^\mu} = p_e + p_{\overline{\nu^e}}$. It is convenient to perform its Fiertz transformation:

$$A = -\frac{G}{\sqrt{2}}\, \overline{u}(p_{\nu^\mu})\, \gamma_\sigma\, (1+\gamma_5)\ v(-p_{\overline{\nu^e}})\, \frac{-m_w^2}{q^2 - m_w^2}\, \overline{u}(p_e)\, \gamma_\sigma (1+\gamma_5)\, u(p_\mu).$$

If we write the Green function of the μ-meson in the form

$$G(p_\mu) = \frac{1}{\widehat{p_\mu} - m_\mu + \Sigma(p_\mu)} \ ,$$

where $\Sigma(p_\mu)$ is the corresponding polarization operator, the total width of the μ-meson is given below

$$\Gamma^\mu_{tot} = 2\, \Im\, \Sigma(m_\mu)\, .$$

Here it is implied, that in the coordinate system where the μ-meson is in rest one should use the Dirac equation for its wave function

$$\widehat{p_\mu} \to m_\mu\, .$$

Taking into account, that $\Sigma(p_\mu)$ can be written in the local limit $q^2 \ll m_W^2$ as follows (see Fig. 18)

$$\Im\Sigma(p_\mu) = -\frac{G^2}{2} \int \frac{d^3 p_e}{(2\pi)^3 2E_e}\ M_{\sigma\sigma'}\ \Im\Pi_{\sigma\sigma'}\,(\widetilde{q})\ ,\ \widetilde{q} = p_\mu - p_e$$

where

$$M_{\sigma\sigma'} = \bar{u}(p_e)\gamma_\sigma\,(1+\gamma_5)\,\left(\widehat{p_e}+m_e\right)\gamma_{\sigma'}(1+\gamma_5)\,u(p_\mu)$$

and

$$\Pi_{\sigma\sigma'}\,(\,\widetilde{q}) = -2\left(\,\widetilde{q}^2\delta_{\sigma\sigma'} - \,\widetilde{q}_\sigma\,\widetilde{q}_{\sigma'}\right)\frac{1}{12\pi^2}\ \ \ln(-\frac{\widetilde{q}^2}{m_\nu^2} - i\epsilon)\ .$$

The expression for $\Pi_{\sigma\sigma'}\,(\,\widetilde{q})$ coincides with the asymptotics of the photon polarization operator up to the factor $2/e^2$ related to the additional contribution from γ_5 terms. Thus, we obtain

$$M_{\sigma\sigma'}\ \Im\Pi_{\sigma\sigma'}\,(\widetilde{q})\ \rightarrow \bar{u}(p_e)\frac{\widetilde{q}^2\,\gamma_\sigma\,\widehat{p_e}\,\gamma_\sigma - \widehat{\widetilde{q}}\,\widehat{p_e}\,\widehat{\widetilde{q}}}{3\pi}u(p_\mu) \rightarrow \frac{2m_\mu\,m_e^2 - 3E_e(m_\mu^2+m_e^2) + 4m_\mu E_e^2}{3\pi}\,.$$

where we used that $\gamma_\sigma\gamma_\rho\gamma_\sigma = -2\gamma_\rho$ and

$$\widetilde{q}^2 = m_\mu^2 + m_e^2 - 2m_\mu E_e\,,\quad 2(\widetilde{q}\,p_e) = 2m_\mu\,E_e - 2m_e^2\,,\ \ \widehat{p_e} = E_e,\ \ \widehat{\widetilde{q}} = m_\mu - E_e\,.$$

By integrating over angles we obtain for the total width of the μ-meson:

$$\Gamma_{tot}^\mu = -G^2\int_{m_e}^{\frac{m_e}{2}(\frac{m_\mu}{m_e}+\frac{m_e}{m_\mu})}\ \frac{p_e dE_e}{4\pi^2}\ M_{\sigma\sigma'}\ \Im\Pi_{\sigma\sigma'}\,(\widetilde{q})\ .$$

With the use of the new integration variable x introduced as follows

$$E_e = \frac{m_e}{2}(x+\frac{1}{x})\,,\ \ p_e = \frac{m_e}{2}(x-\frac{1}{x})$$

we obtain

$$\Gamma_{tot}^\mu = \frac{G^2 m_\mu^5}{192\pi^3}\ F(\varepsilon)\,,$$

where ε is the small parameter

$$\varepsilon = \frac{m_e}{m_\mu}$$

and

$$F(\varepsilon)\ = -4\varepsilon^2\int_1^{\varepsilon^{-1}}\ \frac{(x-\frac{1}{x})^2}{x}\ \left(2\varepsilon^2 - \frac{3\varepsilon}{2}(x+\frac{1}{x})(1+\varepsilon^2) + \varepsilon^2(x+\frac{1}{x})^2\right)dx =$$

$$1 - 8\varepsilon^2 - 24\varepsilon^4\ln\varepsilon + 8\varepsilon^6 - \varepsilon^8\,.$$

Provided that one wants to calculate the correlations between momentum of the produced electron and its spin, the γ-matrix structure $M_{\sigma\sigma'}$ should be substituted by the expression

$$M'_{\sigma\sigma'} = \frac{1}{4m_\mu}tr\,(\widehat{p_\mu}+m_\mu)(1-\gamma_5\widehat{s})\gamma_\sigma\,(1+\gamma_5)\ \left(\widehat{p_e}+m_e\right)\gamma_{\sigma'}(1+\gamma_5),\ (sp_\mu)=0,\ s^2=-1$$

where s_σ is the Lorentz vector. Indeed, the wave function of the μ-meson in rest is the eigenfunction of the operator $\overrightarrow{\sigma}\overrightarrow{s}$ with its eigen value equal to 1 and the corresponding projector to this state is

$$\frac{1}{2}\frac{1}{2m_\mu}(\widehat{p_\mu}+m_\mu)(1-\gamma_5\widehat{s}) = \frac{1}{2}\begin{pmatrix} 1+\overrightarrow{\sigma}\overrightarrow{s} & 0 \\ 0 & 0 \end{pmatrix} .$$

In this case we have

$$M'_{\sigma\sigma'}\ \Im\Pi_{\sigma\sigma'}(\widetilde{q}) \to (1-\widehat{s})\frac{-\widetilde{q}^2\ \widehat{p_e}\ -2\ \widetilde{q}\,p_e\ \widehat{\widetilde{q}}}{3\pi} \to$$

$$\frac{2m_\mu m_e^2 - 3E_e(m_\mu^2+m_e^2) + 4m_\mu E_e^2 + \overrightarrow{s}\,\overrightarrow{p_e}(-m_\mu^2 - 3m_e^2 + 4m_\mu E_e)}{3\pi}$$

and the distribution in the angle ϑ between the vectors $\overrightarrow{s}$ and $\overrightarrow{p_e}$ is

$$\frac{d\Gamma^\mu_{tot}}{d\cos\vartheta} = \frac{G^2 m_\mu^5}{192\pi^3}\ (F(\varepsilon)+G(\varepsilon)\cos\vartheta)\ ,$$

where $F(\varepsilon)$ was calculated above and $G(\varepsilon)$ is given below:

$$G(\varepsilon)\ = -4\varepsilon^2\int_1^{\varepsilon^{-1}}\ \frac{(x-\frac{1}{x})^2}{x}\,\frac{\varepsilon}{2}\,(x-\frac{1}{x})\ \left(-1-3\varepsilon^2+2\varepsilon\,(x+\frac{1}{x})\right)dx =$$

$$-\frac{1}{3} - 30\varepsilon^4 + \varepsilon^8 - \frac{40}{3}\varepsilon^6 + \frac{32}{3}\varepsilon^3 + 32\varepsilon^5\,.$$

The linear dependence from $\cos\vartheta$ is a consequence of the parity non-conservation in weak interactions. The electron is produced more probably in the direction opposite to the vector $\overrightarrow{s}$ because due to the structure of the weak current the massless electron should have the negative helicity and the direction of its spin coincides approximately with that of the μ-meson.

One can obtain the similar distribution in the μ-neutrino energy. By neglecting m_e we find from the previous formulas

$$\Gamma^\mu_{tot} \simeq -G^2 m_\mu\int_0^{m_\mu/2}\ (-3m_\mu+4E_\nu)\ \frac{E_\nu^2 dE_\nu}{12\pi^3} \simeq \frac{1}{192}G^2\frac{m_\mu^5}{\pi^3}.$$

Now we can calculate the correction

$$2\frac{q^2}{M_W^2} = 2m_\mu\frac{m_\mu - 2E_\nu}{M_W^2}$$

from two W-boson propagators to the μ-meson width related with the finite mass of the W-boson:

$$\Delta\Gamma^\mu_{tot} \simeq -G^2\frac{2m_\mu^2}{M_W^2}\int_0^{m_\mu/2}(m_\mu-2E_\nu)\,(-3m_\mu+4E_\nu)\ \frac{E_\nu^2 dE_\nu}{12\pi^3} = \Gamma^\mu_{tot}\,\frac{3}{5}\,\frac{m_\mu^2}{M_W^2}\,.$$

Taking into account also the electromagnetic corrections to the total width we obtain

$$\Gamma^{\mu}_{tot} = \frac{G^2 m_\mu^5}{192\pi^3}\, F(\varepsilon)\, \left(1 + \frac{3}{5}\,\frac{m_\mu^2}{M_W^2}\right) \left(1 + \frac{\alpha_e(m_\mu)}{\pi}(\frac{25}{8} - \frac{\pi^2}{2}) + C\, \left(\frac{\alpha_e(m_\mu)}{\pi}\right)^2\right)\,,$$

$$C = \frac{156815}{5184} - \frac{518}{81}\,\pi^2 - \frac{895}{36}\,\zeta(3) + \frac{67}{720}\pi^4 + \frac{53}{6}\pi^2 \ln 2$$

and the running fine structure constant is

$$\alpha_e(m_\mu) \simeq \alpha + \frac{\alpha^2}{3\pi}(\ln\frac{m_\mu^2}{m_e^2} - \frac{1}{2})\,.$$

From the comparison with the total width one can obtain the most precise value for the Fermi constant:

$$G = (1.16639 \pm 0.00002)10^{-5} GeV^{-2}.$$

The similar formulas can be obtained also for the partial width of the τ-lepton decay to the systems $e\nu^\tau\overline{\nu^e}$ and $\mu\nu^\tau\overline{\nu^\mu}$:

$$\Gamma^{\tau}_{leptons} \simeq 2\,\frac{G^2 m_\tau^5}{192\pi^3}\ .$$

In the total width one should take into account also the transition to the quark systems $d\overline{u}$ and $s\overline{u}$. If one neglect the masses of quarks in comparison with the τ-lepton mass, the dependence from the Weinberg angle is cancelled in this sum due to the unitarity of the CKM matrix. Taking into consideration the colour degrees of freedom, we obtain for the total width of the τ-lepton:

$$\Gamma^{\tau}_{tot} \simeq 5\,\frac{G^2 m_\tau^5}{192\pi^3}\ .$$

In the quark channel the interaction of quarks in the final state is important because the effective coupling constant in this case is not very small. Approximately this interaction can be taken into account in the form of radiative corrections in the perturbative QCD. To calculate the partial widths of the τ-lepton decay to different hadron channels one can use the methods developed for the semi-leptonic interactions.

13 Semileptonic hadron decays

We start with the well-investigated process of the neutron decay $n \to pe\overline{\nu}$ described by the phenomenological Fermi amplitude

$$A = \frac{G\, \cos\vartheta_c}{\sqrt{2}}\ \overline{u}(p_p)\,\gamma_\sigma\,(1 + g_a\,\gamma_5)\,u(p_n)\ \ \overline{u}(p_e)\,\gamma_\sigma\,(1+\gamma_5)\,v(-p_{\overline{\nu^e}})\ ,$$

where $\vartheta_c \approx 0.2$ is the Cabbibo angle, related to the element of the CKM matrix responsible for mixing $d' = d\cos\vartheta_c + s\sin\vartheta_c$ the d and s quarks entering in the weak doublet together with

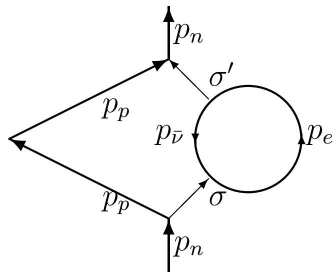

Figure 19: Neutron decay

the u quark provided that the contribution of the b quark is negligible. Really the Standard Model gives the amplitude only for the transition $d \to ue\overline{\nu}$ of the constituent quark inside the proton containing also at least two u quarks. The matrix element of the charged quark current between the nucleon states is renormalized due to the strong interactions. There appear all possible spin structures with the corresponding form-factors. However, for the case of the neutron decay the energy transfer

$$\Delta = m_n - m_p \simeq 1.29\, MeV$$

from hadrons to leptons is small in comparison with the essential scale $\sim m_\rho$ of the strong interactions and therefore one can neglect the q^2-dependence of the form-factors. Moreover, it is possible to omit all other spin structures in this matrix element except the vector and axial currents j^v_σ and j^a_σ correspondingly. The quantity j^v_σ is the charged component of the conserved hadronic isotopic current $\overrightarrow{j_\sigma}$ and therefore for $q^2 = 0$ it is not renormalized ($g_v = 1$). As for the axial current, its renormalization is not large. Namely, it turns out, that

$$g_a \simeq 1.26\,.$$

The width of the neutron decay for small Δ can be written analogously to the case of the μ-meson as follows (see Fig. 19)

$$\Gamma^n_{tot} \simeq \frac{G^2}{2} \cos^2\vartheta_c \int_0^\Delta \frac{d^3p_p}{(2\pi)^3}\; R_{\sigma\sigma'}\; 2\,\Im\Pi_{\sigma\sigma'}\,(q)\,,$$

where in the non-relativistic limit $|\overrightarrow{p}| \sim \Delta \ll m_n$ the spin tensor for the polarized neutron equals

$$R_{\sigma\sigma'} = \frac{1}{8m_nm_p} Tr\; (\widehat{p_n} + m_n)(1 - \gamma_5\,\widehat{s_n})\,\gamma_\sigma\,(1 + g_a\gamma_5)\;\; (\widehat{p_p} + m_p)\,\gamma_{\sigma'}\,(1 + g_a\gamma_5) \simeq$$

$$\delta_{\sigma 0}\delta_{\sigma' 0}(1 + g_a^2) - \delta_{\sigma\sigma'}\,g_a^2 - g_a(s_{n\sigma}\delta_{\sigma' 0} + s_{n\sigma'}\delta_{\sigma 0})$$

and the discontinuity of the lepton polarization operator is

$$2\,\Im\Pi_{\sigma\sigma'}\,(q)=\int\frac{d^3p_{\overline{\nu}}}{(2\pi)^2\,|p_{\overline{\nu}}|}\,\delta(p_e^2-m_e^2)\widetilde{M}_{\sigma\sigma'}\,.$$

Here

$$\widetilde{M}_{\sigma\sigma'}=\frac{1}{2}Tr\,\left(\widehat{p_{\overline{\nu}}}\,\gamma_\sigma\,(1+\gamma_5)\,\;(\widehat{p_e}+m_e)\,\gamma_{\sigma'}\,(1+\gamma_5)\right)=4(p_{\overline{\nu}\sigma}\,p_{e\sigma'}+p_{\overline{\nu}\sigma'}\,p_{e\sigma}-\delta_{\sigma\sigma'}\,(p_{\overline{\nu}}\,p_e))\,.$$

Therefore

$$R_{\sigma\sigma'}\widetilde{M}_{\sigma\sigma'}=4E_eE_{\overline{\nu}}\left(1+\overrightarrow{v_e}\overrightarrow{v_{\overline{\nu}}}+g_a^2\,(3-\overrightarrow{v_e}\,\overrightarrow{v_{\overline{\nu}}})+2g_a\,(\overrightarrow{s_n}\overrightarrow{v_{\overline{\nu}}}+\overrightarrow{s_n}\,\overrightarrow{v_e})\right),$$

where $\overrightarrow{v_e}=\overrightarrow{p_e}/E_e$ and $\overrightarrow{v_{\overline{\nu}}}=\overrightarrow{p_{\overline{\nu}}}/E_{\overline{\nu}}$ are the lepton velocities and $\overrightarrow{s_n}$ is the unit vector in the direction of the neutron spin. The distribution of the lepton momenta in the neutron decay is given below

$$4E_eE_{\overline{\nu}}\frac{d^6\Gamma^n_{tot}}{d^3p_ed^3p_{\overline{\nu}}}=G^2\cos^2\vartheta_c\;\frac{1}{(2\pi)^5}\;R_{\sigma\sigma'}\widetilde{M}_{\sigma\sigma'}\,\delta(\Delta-E_e-E_{\overline{\nu}})\,.$$

The momentum distribution of electrons is

$$\frac{d^3\Gamma^n_{tot}}{d^3p_e}=G^2\cos^2\vartheta_c\;\frac{1}{8\pi^4}\;(\,\Delta-E_e)^2\left(1+3\,g_a^2+2g_a\,\overrightarrow{s_n}\,\overrightarrow{v_e}\right).$$

At last the total width of the neutron decay is

$$\Gamma^n_{tot}=G^2\cos^2\vartheta_c\;\frac{1+3g_a^2}{2\pi^3}\;\int_{m_e}^{\Delta}(\,\Delta-E_e)^2\sqrt{E_e^2-m_e^2}\,E_e\,d\,E_e=$$

$$C\,(\frac{m_e}{\Delta})\,\frac{G^2\Delta^5}{60\pi^3}\;\cos^2\vartheta_c\,(1+3g_a^2)\,,$$

where

$$C(\varepsilon)=30\int_\varepsilon^1(\,1-x)^2\sqrt{x^2-\varepsilon^2}\,x\,d\,x=$$

$$\sqrt{(1-\varepsilon^2)}-\frac{9}{2}\varepsilon^2\sqrt{(1-\varepsilon^2)}+\frac{15}{2}\varepsilon^4\ln\left(1+\sqrt{(1-\varepsilon^2)}\right)-4\varepsilon^4\sqrt{(1-\varepsilon^2)}-\frac{15}{2}\varepsilon^4\ln\varepsilon\,.$$

For the known values of the electron mass $m_e\simeq 0.511\,MeV$ and $\Delta=1.294\;MeV$ we obtain

$$C(\varepsilon)=0.473\,,\;\Gamma^n_{tot}\simeq 6,8\times 10^{-25}MeV\,,$$

which corresponds to the neutron life time

$$t_n=\frac{m_p}{\Gamma^n_{tot}}\,0.7\,10^{-24}\,\sec\simeq 900\,\sec\;.$$

We can use another approach to the neutron decay problem by calculating initially the discontinuity of the lepton polarization operator

$$2\,\Im\Pi_{\sigma\sigma'}(q) = C_1(q^2)\frac{q_\sigma q_{\sigma'}}{q^2} + C_2(q^2)\left(\delta_{\sigma\sigma'} - \frac{q_\sigma q_{\sigma'}}{q^2}\right)$$

where in the c.m. coordinate system, where $|\overrightarrow{p_e}| = \frac{q^2-m^2}{2q}$ and $E_e = \frac{q^2-m^2}{2q}$, we have after calculating the trace of γ-matrices and integrating over the angles

$$C_1(q^2) = \frac{1}{\pi}\frac{|\overrightarrow{p_{\overline{\nu}}}|}{|q_0|}\left(E_e E_{\overline{\nu}} + \overrightarrow{p_e}\overrightarrow{p_{\overline{\nu}}}\right) = \frac{1}{2\pi}\frac{(q^2-m_e^2)^2}{q^4}\;m_e^2\,,$$

$$C_2(q^2) = \frac{1}{\pi}\frac{|\overrightarrow{p_{\overline{\nu}}}|}{|q_0|}\left(-E_e E_{\overline{\nu}} + \frac{1}{3}\overrightarrow{p_e}\overrightarrow{p_{\overline{\nu}}}\right) = \frac{1}{2\pi}\frac{(q^2-m_e^2)^2}{q^4}\left(-\frac{2}{3}q^2 - \frac{1}{3}m_e^2\right).$$

Using for the case of the non-polarized neutron the relation

$$R_{\sigma\sigma'}2\,\Im\Pi_{\sigma\sigma'}(q) = (1+g_a^2)\left(C_1(q^2)\frac{\Delta^2}{q^2} + C_2(q^2)\left(1-\frac{\Delta^2}{q^2}\right)\right) - g_a^2\left(C_1(q^2)+3C_2(q^2)\right),$$

we derive for the total width by introducing the new integration variable $x = |\overrightarrow{q}|/\Delta$ and $Q = q/\Delta = \sqrt{1-x^2}$

$$\Gamma^n_{tot} \simeq \frac{G^2\Delta^5}{(2\pi)^3}\;\cos^2\vartheta_c\int_0^{\sqrt{1-\varepsilon^2}} x^2\;\frac{(1-x^2-\varepsilon^2)^2}{(1-x^2)^2}\;f\,(x)\,d\,x\,,$$

where

$$f(x) = \frac{2\pi q^4}{(q^2-m_e^2)^2}R_{\sigma\sigma'}2\,\Im\Pi_{\sigma\sigma'}(q) = \frac{2}{3}x^2 + \varepsilon^2\frac{1}{3}\frac{3+x^2}{1-x^2} + g_a^2\left(2 - \frac{4}{3}x^2 + \varepsilon^2\frac{1}{3}\frac{3+x^2}{1-x^2}\right).$$

One can easily verify, that the above result for the integrated width is obtained again, because

$$\frac{15}{2}\int_0^{\sqrt{1-\varepsilon^2}} x^2\;\frac{(1-x^2-\varepsilon^2)^2}{(1-x^2)^2}\;f(x)\,d\,x = (1+3g_a^2)\,C(\varepsilon)\,.$$

Let us consider now the π^--meson decay to the lepton system $l\overline{\nu_l}$, where l can be e or μ. The corresponding amplitude is given by the expression

$$A = \frac{G\cos\vartheta_c}{\sqrt{2}}\;\left\langle 0\,|j^a_\sigma|\,\pi^-\right\rangle\;\;\overline{u}(p_l)\,\gamma_\sigma\,(1+\gamma_5)\,v(-p_{\overline{\nu_l}})\,,$$

where due to the parity conservation in the strong interactions only the matrix element of the axial current $j^a_\sigma = \overline{\psi_u}\gamma_\sigma\;\gamma_5\,\psi_d$ is non-zero. With the use of the Lorenz invariance its matrix element can be parametrized as follows

$$\left\langle 0\,|j^a_\sigma|\,\pi^-\right\rangle = f_\pi\,q_\sigma\;,$$

where q_σ is the π-meson momentum and f_π is a phenomenological constant with the dimension of mass (proportional to the wave function of the π-meson at small relative distance between the quark and anti-quark). Therefore the width of the π^- with respect to its decay to $l\overline{\nu_l}$ is expressed

$$\Gamma_{e\overline{\nu_e}} = |f_\pi|^2 \frac{G^2 \cos^2\vartheta_c}{2} \frac{1}{2m_\pi} \; q_\sigma \, q_{\sigma'} \, 2\,\Im\Pi_{\sigma\sigma'}(q)$$

in terms of the discontinuity $2\,\Im\Pi_{\sigma\sigma'}(q)$ of the polarization operator for the W-boson for $q^2 = m_\pi^2$. According to the above formulas we obtain

$$\Gamma_{l\overline{\nu_l}} = |f_\pi|^2 \frac{G^2 \cos^2\vartheta_c}{2} \frac{m_\pi}{2} \; C_1(m_\pi^2) = |f_\pi|^2 \frac{G^2 \cos^2\vartheta_c}{8\pi} \left(1 - \frac{m_l^2}{m_\pi^2}\right)^2 m_\pi \, m_l^2 \, .$$

The ration for the π-meson partial widths for $e\overline{\nu}_e$ and $\mu\overline{\nu}_\mu$ channels is small

$$\frac{\Gamma_{e\overline{\nu}_e}}{\Gamma_{\mu\overline{\nu}_\mu}} = \frac{\left(1 - \frac{m_e^2}{m_\pi^2}\right)^2 m_e^2}{\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 m_\mu^2} \simeq 1.3 \; 10^{-4} \, .$$

From the comparison with the experimental data one can obtain $f_\pi \simeq 130\, MeV$.

The matrix element of the axial current, entering in the amplitude of the β-decay of the neutron, can be written in the form, corresponding to its conservation

$$A^\alpha = -g_a \overline{u}_p \gamma^\alpha \gamma_5 u_n + g_a \, \frac{q^\alpha}{q^2} \, \overline{u}_p \widehat{q} \gamma_5 u_n \, ,$$

where the contribution of the second term to the neutron width is negligible, because q^α acting on the lepton current gives a small value proportional to the electron mass. In this term the factor $1/q^2$ can be interpreted as the π-meson pole $1/q^2$, in which the mass m_π is neglected. For the corresponding diagram we obtain with the use of the above definition of f_π

$$g_a \, \frac{q^\alpha}{q^2} \, \overline{u}_p \widehat{q} \gamma_5 u_n = f_\pi \frac{1}{q^2} \frac{\sqrt{2}}{f} q^\alpha \, \overline{u}_p \widehat{q} \gamma_5 u_n \, .$$

Here $\sqrt{2}$ is the corresponding Clebsch-Gordan coefficient for the π-meson-nucleon interaction isotopically invariant which can be written in the gradient form

$$\frac{1}{f} \, \overline{u} \, \overrightarrow{\tau} \widehat{q} \, \gamma_5 u \, \overrightarrow{\varphi} = g \; \overline{u} \, \overrightarrow{\tau} \gamma_5 u \, \overrightarrow{\varphi} \, , \quad g = -\frac{2m_N}{f} \, ,$$

where $f = 140\, MeV$ corresponding to $\frac{g^2}{4\pi} = 15$. Thus, from the above formulas with the use of the equality

$$\overrightarrow{\tau} \, \overrightarrow{\varphi} = \tau_3 \, \varphi_3 + \sqrt{2} \, (\tau_+ \, \varphi_- + \tau_- \, \varphi_+)$$

we obtain the Goldberger-Treiman relation

$$g_a = f_\pi \, \frac{\sqrt{2}}{f} \simeq 1.3 \, ,$$

which is in an agreement with the experimental value of $g_a \simeq 1.26$. This relation is a consequence of the hypothesis of the partial conservation of the axial current (PCAC). This hypothesis is used also together with the current algebra to relate the matrix elements with the different number of π-mesons.

There are many other semi-leptonic processes to which one can apply the similar technique. In the case of the strangeness changing processes with $|\Delta S| = 1$ we have the following rule

$$\Delta Q = \Delta S$$

for the change of the electric charge between the initial and final hadronic states. It is related with the fact, that the underlying process is the quark decay $s \to ul\bar{l}$. In particular, because $K^+ = u\overline{s}$, $K^0 = d\overline{s}$ there are transitions $K^+ \to l^+\nu$ (K^+_{l2} decay), $K^+ \to \pi^0 l^+\nu$ (K^+_{l3} decay), $K^0 \to \pi^- l^+\nu$ (K^0_{l3} decay), $K^+ \to \pi^+\pi^- l^+\nu$ (K^+_{l4} decay), $K^+ \to \pi^0\pi^0 l^+\nu$ ($K^{+\prime}_{l4}$ decay), $K^0 \to \pi^0\pi^- l^+\nu$ (K^0_{l4} decay), $\Lambda \to pl^-\overline{\nu}$, $\Sigma^- \to nl^-\overline{\nu}$, $\Xi^- \to \Lambda l^-\overline{\nu}$, $\Xi^- \to \Sigma^0 l^-\overline{\nu}$, $\Xi^0 \to \Sigma^+ l^-\overline{\nu}$, $\Omega^- \to \Xi^0 l^-\overline{\nu}$, but the transitions $K^0 \to \pi^+ l^-\overline{\nu}$, $\overline{K}^0 \to \pi^- l^+\nu$, $K^+ \to \pi^+\pi^+ l^-\overline{\nu}$ are not observed. In a similar way the transitions $\Xi^- \to nl^-\overline{\nu}$, $\Xi^0 \to pl^-\overline{\nu}$, $\Omega^- \to \Lambda^0 l^-\overline{\nu}$, $\Omega^- \to \Sigma^0 l^-\overline{\nu}$, $\Omega^- \to nl^-\overline{\nu}$ are impossible in the lowest order of the perturbation theory because they contradict to the rule $|\Delta S| = 1$.

The decays K_{l2} and K_{l3} are comparatively simple. In the first case the amplitude for the transition $K^+ \to l^+\overline{\nu}_l$ is similar to the amplitude for the decay $\pi^+ \to l^+\overline{\nu}_l$ and its width is

$$\Gamma_{l\overline{\nu_l}} = |f_K|^2 \, \frac{G^2 \sin^2 \vartheta_c}{8\pi} \left(1 - \frac{m_l^2}{m_K^2}\right)^2 m_K \, m_l^2 \, .$$

Here $f_K \simeq 1.27\, f_\pi$ and ϑ_c is the Cabbibo angle. The matrix element of the K_{l3}-decay $K^+ \to l^+\overline{\nu}_l\pi^0$ is related with the matrix element of the conserved vector current, which simplifies considerably its tensor structure.

In the semi-leptonic decays one can attempt to use the parton model ideas by expressing the hadron width in terms of the quark widths. But for the hadrons constructed from the light quarks u, d, s there are difficulties in this approach, related with the absence of a good theory for compound states of these quarks. Even the mass m_q of the quark in the hadron is the parameter, which can be different from the mass of the "free" quark, but this parameter is important in the parton model approach, because the total width is proportional to m_q^5.

The investigation of the hadron decays with the transition between heavier quarks $b \to c$, $b \to u$, $t \to b$, $t \to s$, $t \to d$ allows one to find from the experimental data the elements of the Cabbibo-Kobayashi-Mascawa matrix. The hadrons, constructed from heavy quarks, are similar to the atoms in QED, because their interaction is Coulomb-like due to the fact, that their relative distances are small. It gives a possibility to derive the heavy quark effective theory (HQET) allowing to express the wave functions of different quarkonia in terms of several universal functions. Here one can use the perturbative QCD due to its asymptotic freedom. The operator product expansion and the ideas of the parton models are used also in HQET. It allows to extract the elements of the Cabbibo-Kobayashi-Maskawa matrix from the hadronic width with a good accuracy. The value of these parameters is very important for the self-consistency of the Standard Model.

14 Non-leptonic weak interactions

There are weak hadron-hadron interactions according to the Standard Model because the weak currents responsible for the emission and absorption of the $W^{\pm}$ and Z bosons contain the quark contributions

$$j_q^{\pm} = \frac{g}{\sqrt{2}} \overline{\Psi_q} \gamma_\mu \frac{1+\gamma_5}{2} V \ \Psi_q \,, \ j_q = \frac{g}{\cos\theta_w} \overline{\Psi_q} \widehat{Z} \left(T^3 - Q \sin^2\theta_w \right) \Psi_q \,.$$

However, for the processes, in which the quantum numbers of quarks are conserved this interaction is difficult to measure because the hadrons participate also in the strong interactions. Nevertheless, already in 1964 in the experiments of Yu. Abov, P. Krupchitsky, V. Lobashov and V. Nazarenko the parity non-conservation in the electromagnetic transitions between the nuclear energy levels was discovered. It is difficult to calculate the corresponding probabilities in the Standard Model because the dynamics of the nuclear interactions is rather complicated. The experiments with baryons and mesons would be more clean for the theoretical interpretation, but here the predicted effects of the parity non-conservation are very small to be measured.

The simplest pure hadronic process is the weak decay of hadrons with the strangeness change $|\Delta S| = 1$. Because it goes on the quark level due to the transition $s \to ud\overline{u}$ the difference of the total isotopic spins ΔT of the hadrons in the initial and final states can be $1/2$ and $3/2$, but experimentally the amplitudes of the transitions with $\Delta T = 3/2$ are much less in comparison with the amplitudes satisfying the rule

$$\Delta T = 1/2 \,.$$

The theoretical explanation of this phenomenon is based on the idea, that the strong interactions can enhance the amplitudes with $\Delta T = 1/2$ and suppress those with $\Delta T = 3/2$. The arguments supporting this idea are based on perturbative QCD. To begin with, let us write the effective low energy Hamiltonian for the strangeness changing transitions in the form

$$H = \sqrt{2} G \cos\theta_c \sin\theta_c \left(a_3 I_3 + a_6 I_6 \right) + h.c. \ ,$$

where θ_c is the Cabbibo angle, I_3, I_6 are the operators constructed from the quark fields with the left (L) helicity

$$I_3 = \left(\overline{d}_L \gamma_\mu u_L \right) \left(\overline{u}_L \gamma_\mu s_L \right) - \left(\overline{u}_L \gamma_\mu u_L \right) \left(\overline{d}_L \gamma_\mu s_L \right) \ ,$$

$$I_6 = \left(\overline{d}_L \gamma_\mu u_L \right) \left(\overline{u}_L \gamma_\mu s_L \right) + \left(\overline{u}_L \gamma_\mu u_L \right) \left(\overline{d}_L \gamma_\mu s_L \right) \ .$$

In the electro-weak theory $a_3 = a_6 = 1$, but these parameters are renormalized due to the strong interactions of quarks at small distances, where one can use the perturbative QCD.

Such form of the effective Hamiltonian is related to the fact, that with the use of the Fiertz transformation for the first term in the right-hand sides of expressions for I_r and anti-commutativity of the quark fields one can verify, that I_3 and I_6 describe the quark transitions $us \to ud$ in the states $\overline{3}$ and 6 which are the tensors $\Psi^{i_1 i_2}$ respectively anti-symmetric

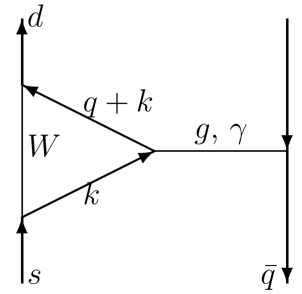

Figure 20: Penguin diagrams

and symmetric under the transmutation of the quark colour indices. These states are the eigenstates of the colour group Kazimir operators and are renormalized in a multiplicative way. With the use of the renormalization group in the leading logarithmic approximation of QCD the corresponding form-factors a_3 and a_6 can be easily calculated:

$$a_3 = \left(\frac{\alpha(\mu^2)}{\alpha(M_W^2)}\right)^{\frac{4}{b}} \simeq 3\,,\;\; a_6 = \left(\frac{\alpha(\mu^2)}{\alpha(M_W^2)}\right)^{-\frac{2}{b}} \simeq 0.6\,,$$

where

$$\alpha(Q^2) = \frac{4\pi}{b \ln \frac{Q^2}{\Lambda^2}}\,,\;\; b = 11 - \frac{2}{3}n_f$$

and the numerical estimates correspond to $\Lambda \simeq 200MeV$, $n_f = 3$ and $\alpha(\mu^2) \simeq 1$. It is seen from these relations, that indeed in the perturbation theory one can obtain the relative suppression $\frac{0.6}{3} = 0.2$ for the weak transitions with $\Delta T = 3/2$ in a comparison with the transitions with $\Delta T = 1/2$, but experimentally the suppression is much larger: $0.02 - 0.05$. Presumably it is related with significant effects of the renormalization of the corresponding amplitudes from the non-perturbative region where $\alpha_s \sim 1$. This hypothesis can be verified in the framework of a non-perturbative approach based on the QCD sum rules or by the direct calculation of the corresponding matrix elements of the weak quark Hamiltonian on computers with the use of the lattice formulation of gauge theories suggested by A. Polyakov and t'Hooft.

In an analogous way one can obtain the renormalized effective action for the transitions related to heavier quark interactions contained in the Cabbibo-Kobayashi-Maskava matrix. Moreover, there is another effect caused by the interference between the weak and other interactions of quarks. Namely, the W-boson can be emitted and absorbed by the same quark with a simultaneous exchange of the photon or gluon with other quarks in the hadron. It corresponds to the so-called penguin diagrams (see Fig. 20). The existence of these diagrams is related to the fact, that the hadrons are composite states of several quarks participating in the electromagnetic and strong interactions. The weak interaction with the virtual W-boson can appear as a radiative correction to the gluon or photon vertex leading to the transition between different quarks. For example for the transition $s \to d$ in the gluon vertex Γ^a_σ (a is the colour index of the gluon and q is its momentum) the corresponding one-loop correction is (see Fig. 20)

$$\Gamma^a_\sigma = -\frac{G}{\sqrt{2}}\int\frac{d^4k}{(2\pi)^4 i}\sum_{i=u,c,t}V_{is}V^*_{id}\,\overline{u}_d\gamma_\mu(1+\gamma_5)\frac{\widehat{k}+m_i}{k^2-m_i^2}\gamma_\sigma t^a\frac{\widehat{k}+\widehat{q}+m_i}{(k+q)^2-m_i^2}\gamma_\mu(1+\gamma_5)u_s\,,$$

where we used the effective Fermi theory, because the essential momenta $k^2 \le M_W^2$ due to the fact, that for $k^2 \gg m_i^2$ the integrand tends to zero at least as $1/k^4$ due to the unitarity constraint $\sum_{i=u,c,t}V_{is}V^*_{id} = 0$ for the Kobayashi-Maskawa matrix V_{ij}. Further, m_i are the masses of the virtual quarks and t^b are the generators of the colour group $SU(3)$ in the fundamental representation. Using the Fiertz transformations for the γ_μ matrices one can reduce the above expression to the quark contribution of the polarization operator $\Pi_{\sigma\sigma'}(q)$ of the gluon. Due to the gauge invariance this operator has the transversal form

$$\Pi^{aa'}_{\sigma\sigma'}(q) = \frac{1}{2}\delta_{aa'}\left(q^2\delta_{\mu\mu'} - q_\mu q_{\mu'}\right)\Pi(q^2)\,,$$

where with the logarithmic accuracy we obtain $\Pi(q^2) \sim \frac{1}{12\pi^2}\ln\frac{q^2}{m_c^2}$. Further, because the essential region of integration is $m_u^2 \ll k^2 \ll m_c^2$, we can neglect all matrix elements V_{ij} except the Cabbibo contributions $V_{ud} = \cos\theta_c$ and $V_{us} = \sin\theta_c$. Thus, the final expression for the effective four-fermion Lagrangian resulting from the calculation of the gluon penguin diagram is

$$L = -\sqrt{2}G\sin\theta_c\cos\theta_c\frac{\alpha_s(\mu^2)}{3\pi}\ln\frac{m_c^2}{\mu^2}\,\overline{d}_L t^a\gamma_\sigma s_L\,\left(\overline{u}\,t^a\gamma_\sigma\,u + \overline{d}\,t^a\gamma_\sigma\,d\right)\,.$$

Here the gluon propagator $1/q^2$ is cancelled by the factor q^2 appearing in the polarization operator $\Pi^{aa'}_{\sigma\sigma'}(q)$. We substituted also q^2 in the argument of the logarithm by μ^2 which is chosen in such a way, that $\alpha_s(\mu^2) = 1$. Even in this case the numerical value of the penguin diagram contributions turns out to be less than the above term proportional to a_6 responsible for the violation of the rule $\Delta T = 1/2$. Nevertheless, sometimes (for example, in the the problem of the CP violation) the penguin diagrams lead to interesting physical effects.

After obtaining the effective quark-quark Hamiltonian with the use of the renormalization of the bare Fermi interaction the problem is reduced to the calculation of its matrix elements between the hadron states. To solve it several non-perturbative approaches are used. One of them called the vacuum dominance method corresponds to the calculation of the product of two currents by inserting between them instead of the complete set of states only the vacuum state. Because the matrix element of the current between the vacuum state and a meson can be expressed sometimes in terms of the phenomenological constants f_π or f_K, this method allows estimate the transition amplitudes in these cases. Another approach is based on effective chiral Lagrangians having the symmetry properties of the effective quark interaction. In the framework of this approach one can relate different weak processes in terms of several phenomenological parameters. The most direct method of calculating the matrix elements uses the approximate functional integration on the lattice, but up to now this method can be applied only to simple cases.

15 CP violation

The neutral mesons K^0 and $\overline{K^0}$ have the different quark content: $d\overline{s}$ and $\overline{d}s$, which means, in particular, that they have different strong interactions with the matter. However, in weak interactions strangeness and isotopic spin are not conserved quantum numbers. Therefore, the eigenstates of the weak Hamiltonian should be linear superpositions of these mesons. Being pseudoscalar ($P = -1$), $|K^0\rangle$ and $\left|\overline{K^0}\right\rangle$ are mixed under the simultaneous transformations of the charge conjugation and parity

$$CP \left|K^0\right\rangle = \left|\overline{K^0}\right\rangle \, , \; CP \left|\overline{K^0}\right\rangle = \left|K^0\right\rangle \, ,$$

where we have chosen definite relative phases of the particle and anti-particle states. If the weak interactions would be invariant under the CP-conjugation, the eigenstates of their Hamiltonian should be fixed as follows

$$\left|K_1^0\right\rangle = \frac{|K^0\rangle + \left|\overline{K^0}\right\rangle}{\sqrt{2}} \, , \; \left|K_2^0\right\rangle = \frac{|K^0\rangle - \left|\overline{K^0}\right\rangle}{\sqrt{2}} \, .$$

They are also the eigenfunctions of the operator CP with the eigenvalues ± 1 correspondingly. Due to the effects of the CP violation in weak interactions these states are also mixed each with another, but we neglect initially these small effects. Let us consider the decay of the particles K_1^0 and K_2^0 in states consisting from two and three π-mesons. Because the spin of the K-meson is zero, the total angular momentum of the final particles should be also zero. Therefore the states $\pi^0\pi^0$, $\pi^+\pi^-$ have $CP = 1$ and the states $\pi^0\pi^0\pi^0$, $\pi^0\pi^+\pi^-$ have $CP = -1$, because in the case of three π-mesons one can neglect the configurations in which the relative angular momenta l_{ij} are non-zero due to their suppression by small factors $(k_{ij}/m_K)^{l_{ij}}$, where $k_{ij} \sim m_K - 3m_\pi$. It means, that under the condition of the CP-conservation there could be only the following decays

$$K_1^0 \to \pi^0\pi^0, \, \pi^+\pi^- \, ; \quad K_2^0 \to \pi^0\pi^0\pi^0 \, , \; \pi^0\pi^+\pi^- \, .$$

The isotopic invariance allows us to fix the relative weights of the different channels. Indeed, the general structure of the amplitude $K \to \pi\pi$, combined with the Bose statistics and isotopic symmetry, contains only the contributions A_0 and A_2 from the isospin $T = 0$ and $T = 2$ states correspondingly and with the use of the Clebsch-Gordan coefficients we obtain

$$A_{K_1^0 \to \pi^+\pi^-} = \sqrt{\frac{2}{3}} \, A_0 \, e^{i\delta_0} + \sqrt{\frac{1}{3}} \, A_2 \, e^{i\delta_2} \, ,$$

$$A_{K_1^0 \to \pi^0\pi^0} = \sqrt{\frac{1}{3}} \, A_0 \, e^{i\delta_0} - \sqrt{\frac{2}{3}} \, A_2 \, e^{i\delta_2} \, ,$$

$$A_{K^+ \to \pi^+\pi^0} = \frac{1}{2}\sqrt{3} \, A_2 \, e^{i\delta_2},$$

where δ_T are the well known phases of the $\pi\pi$-scattering in the S-wave in the final state with the isotopic spin T. Using these isotopic relations from the experimental data on the partial widths for decays in different $\pi\pi$-channels we obtain the small ratio for the amplitudes A_2 and A_0

$$\frac{A_2}{A_0} \simeq 4.5\,10^{-2}\,,$$

which is a phenomenological consequence of the rule $\Delta T = 1/2$ in the hadron weak decays. By the use of the same rule for the K-decays to the $\pi\pi\pi$-channels we can leave only the isotopic state with $T = 1$ compatible with the Bose statistics of the π-mesons. In such way one can obtain the following relations among the decay amplitudes:

$$|A_{K^+\to\pi^+\pi^+\pi^-}|^2 : \left|A_{K_2^0\to\pi^0\pi^0\pi^0}\right|^2 : \left|A_{K_2^0\to\pi^0\pi^+\pi^-}\right|^2 : |A_{K^+\to\pi^+\pi^0\pi^0}|^2 = 4:3:2:1$$

which are fulfilled experimentally for the corresponding partial widths after taking into account the differences in the masses of π^0 and $\pi^\pm$-mesons with the relative accuracy $\sim 1/10$ compatible with the approximate rule $\Delta T = 1/2$.

The states K_1^0 and K_2^0 decaying into the systems $\pi\pi$ and $\pi\pi\pi$ correspondingly, have the different mean life times

$$\tau_1 \simeq 0.893\;10^{-10}\,\mathrm{sec}\;, \quad \tau_2 \simeq 5.17\;10^{-8}\,\mathrm{sec}\;.$$

Moreover, they have the different masses

$$\Delta\,m = m_1 - m_2 \simeq -3.49\;10^{-12} MeV\;.$$

Their total widths $\Gamma_{tot} = 1/\tau$ are

$$\Gamma_{tot}^{K_1^0} = 7.37\times 10^{-12} MeV\,,\;\Gamma_{tot}^{K_2^0} = 1.274\times 10^{-14} MeV\;.$$

Both $\Delta\,m$ and Γ_{tot} are determined by the values of the polarization operators $\Pi_{1,2}(M^2)$ for the mesons $K_{1,2}^0$

$$\Delta\,m = \frac{1}{2M}\,Re\,\left(\Pi_2(M^2) - \Pi_1(M^2)\right)\;,\;\Gamma_{tot}^{K_r^0} = \frac{1}{M}\,Im\,\Pi_r(M^2)\,,$$

but the real and imagine parts of $\Pi_r(M^2)$ are related by the dispersion relations and therefore in the case of a rapid convergency of the dispersion integral we should obtain $\Delta\,m \simeq \Gamma_{tot}^{K_1^0}$. However, a non-zero value of $\Delta\,m$ could be even in the case, when K_1^0 and K_2^0 would be stable.

If we introduce the effective Hamiltonian H of the weak interactions, then

$$\Delta\,m = \left\langle K_1^0\right|H\left|K_1^0\right\rangle - \left\langle K_2^0\right|H\left|K_2^0\right\rangle = \left\langle\overline{K^0}\right|H\left|K^0\right\rangle + \left\langle K^0\right|H\left|\overline{K^0}\right\rangle.$$

Let us consider now the quantity

$$i\mu_{21} = \frac{1}{2}\left(\left\langle\overline{K^0}\right|H\left|K^0\right\rangle - \left\langle K^0\right|H\left|\overline{K^0}\right\rangle\right).$$

Due to the property of hermicity of the Hamiltonian μ_{21} is real and the masses of particles and particles coincide. We can rewrite the last relation as follows

$$i\mu_{21} = \left\langle K_1^0 \right| H \left| K_2^0 \right\rangle = -\left\langle K_2^0 \right| H \left| K_1^0 \right\rangle .$$

Therefore a non-vanishing value of μ_{21} would lead to the transitions $K_2 \to K_1$ meaning the non-conservation of CP.

Such effects were observed experimentally firstly by J. Christenson, J. Cronin, V. Fitch and R. Turlay in 1964. They measured the oscillations of the K^0- and $\overline{K^0}$-mesons as a function of the distance from the source. An arbitrary quantum mechanical state in this case can be written as a superposition

$$\Psi = c_1 \left| K^0 \right\rangle + c_2 \left| \overline{K^0} \right\rangle ,$$

where the coefficients satisfy the Schroedinger equation

$$i \frac{d\, c_i}{d\, t} = h_{ik}\, c_k .$$

Because the K^0- and $\overline{K^0}$-mesons are unstable, h_{ik} contains the anti-hermitian contribution

$$h_{ik} = M_{ik} - i\Gamma_{ik} .$$

The CP transformation is equivalent to transposition $1 \longleftrightarrow 2$ of indices and the T reversion corresponds to the transposition of h_{ik}. Therefore CPT invariance leads to the relations

$$M_{11} = M_{22}\, ,\ \ \Gamma_{11} = \Gamma_{22}$$

and the approximate CP symmetry means

$$M_{12} \simeq M_{21}\, ,\ \Gamma_{12} \simeq \Gamma_{21}$$

and therefore M_{12} and Γ_{12} are almost real numbers. The eigenvalues of the matrix h_{ik} can be written as follows

$$\delta\, E_S = \delta\, m_S - \frac{i}{2}\gamma_S \ = h_{11} + \sqrt{h_{12}h_{21}}\, ,\ \delta\, E_L = \delta\, m_L - \frac{i}{2}\gamma_L \ = h_{11} - \sqrt{h_{12}h_{21}}$$

and the corresponding eigenfunctions for the short- and long-living states are

$$\left| K_S^0 \right\rangle = \frac{|K_1^0\rangle - \varepsilon\, |K_2^0\rangle}{\sqrt{1+\varepsilon^2}}\ ,\ \ \left| K_L^0 \right\rangle = \frac{|K_2^0\rangle + \varepsilon\, |K_1^0\rangle}{\sqrt{1+\varepsilon^2}}\, ,$$

where

$$\varepsilon = \frac{\sqrt{h_{21}} - \sqrt{h_{12}}}{\sqrt{h_{21}} + \sqrt{h_{12}}}$$

is a small parameter responsible for the CP non-conservation. It is complex and according to the experimental data equals

$$\varepsilon \simeq 0.23\ 10^{-2}\, e^{i\delta}\,,\ \ \delta \simeq \pi/4\,.$$

The evolution of the wave function of the K^0-meson, produced at the time $t = 0$, is given by the expression

$$\Psi = \frac{1}{\sqrt{2(1+\varepsilon^2)}}\Big((1-\varepsilon)\left|K_S^0\right\rangle\, e^{(-i\delta\, m_S - \frac{1}{2}\gamma_S)\,t} + (1+\varepsilon)\left|K_L^0\right\rangle\, e^{(-i\delta\, m_L - \frac{1}{2}\gamma_L)\,t}\Big)$$

and therefore, providing the CP parity is violated, after the decay of the short-living component in the beam there is a small mixture $\sim \varepsilon$ of the K_1^0 state decaying to the system $\pi\pi$, which was discovered experimentally in 1964. It was found, that

$$B = \frac{\Gamma(K_L \to \pi^+\pi^-)}{\Gamma(K_L \to \pi\,\pi\,\pi)} \simeq 2\,10^{-3}.$$

From this result one can obtain

$$|\varepsilon| = \sqrt{\frac{\Gamma(K_L \to \pi^+\pi^-)}{\Gamma(K_S \to \pi^+\pi^-)}} \simeq 2.3\ 10^{-3}\,.$$

Here we used the known ratio for the total widths of the K_L and K_S

$$\frac{\Gamma(K_L \to \pi\,\pi)}{\Gamma(K_S \to \pi\,\pi\,\pi)} \simeq 0.55\ 10^3$$

and the relation

$$\frac{\Gamma(K_L \to \pi^+\pi^-)}{\Gamma(K_L \to \pi\,\pi\,)} = \frac{2}{3}$$

which is a consequence of the fact. that the $\pi\pi$ system is in the $T = 0$ isotopic state due to its Bose symmetry and the approximate rule $\Delta T = 1/2$ for the hadronic decays.

Initially the CP violation was interpreted in the framework of the Wolfenstein model. In this model the parameter ε is expressed in terms of the above introduced matrix element $i\mu_{21} = \langle K_1^0|\, H\,|K_2^0\rangle$ appearing in the second order of the perturbation theory as follows (see Fig. 21)

$$\varepsilon = \frac{i\mu_{21}}{m_L - m_S - \frac{i}{2}(\Gamma_L - \Gamma_S)}\,.$$

From the experimental values for $|\varepsilon|$ and

$$m_L - m_S \simeq 3.49\ 10^{-12} MeV\,,\ \Gamma_L - \Gamma_S \simeq -7.36 \times 10^{-12} MeV$$

we obtain

$$\mu_{21} = |\varepsilon|\ \sqrt{|m_L - m_S|^2 + (\Gamma_L - \Gamma_S)^2/4} \simeq 1.2\ 10^{-14}\, MeV.$$

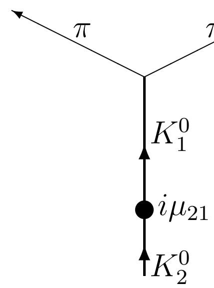

Figure 21: Wolfenstein mechanism of the CP violation

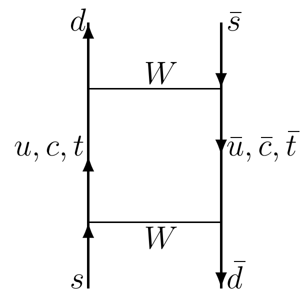

Figure 22: Quark box diagram for $\Delta S = 2$ transitions

This value of μ_{21} is very small and is observed experimentally only because it is enhanced by the energy propagator with a small denominator. In accordance with this smallness the Wolfenstein mechanism is known as a superweak model. In the framework of this model one can predict the phase of the parameter ε

$$\varepsilon = |\varepsilon| \; e^{i\delta} \,, \; \tan \, \delta \; = -2 \, \frac{m_L - m_S}{\Gamma_L - \Gamma_S} \simeq 0.95$$

Therefore, $\delta \simeq \pi/4$. The experimental information concerning δ is obtained from the measurement of the charge asymmetry in the semi-lepton decays of the K_L mesons. This asymmetry is in the agreement with the predictions of the Wolfenstein model

$$\frac{\Gamma(K_L \to l^+ \, \pi^- \, \nu) - \Gamma(K_L \to l^- \, \pi^+ \, \overline{\nu})}{\Gamma(K_L \to l^+ \, \pi^- \, \nu) + \Gamma(K_L \to l^- \, \pi^+ \, \overline{\nu})} = \frac{|1+\varepsilon|^2 - |1-\varepsilon|^2}{|1+\varepsilon|^2 + |1-\varepsilon|^2} \simeq 2 \, Re \; \varepsilon \simeq 2 \; |\varepsilon| \cos \delta \,.$$

In the framework of the Standard Model both $\Delta \, m$ and $i \, \mu_{21}$ are obtained from the calculation of the matrix element of the scattering amplitude for the transition $s \, \overline{d} \to \overline{s} \, d$ appearing in the second order of the perturbation theory corresponding to the box diagram Fig. 22. After emission of the W^--boson the s-quark is transformed in the u-, c- or t-quarks with the subsequent transition to the d-quark. Analogously after absorption of the W^--boson the $\overline{d}$-quark is transformed in the $\overline{u}$-, $\overline{c}$- or $\overline{t}$- quarks with the subsequent transition to the $\overline{s}$-quark. The vertices of the interaction of the W-boson with quarks are proportional to the corresponding elements of the Cabbibo-Kobayashi-Maskawa matrix V_{ik} and the scattering amplitude for the transition $s \, \overline{d} \to \overline{s} \, d$ is

$$A = \frac{G^2}{2} \int \frac{d^4p}{(2\pi)^4 i} T_{\mu\nu}(p\,)\, \widetilde{T}_{\mu\nu}(p)\,,$$

where

$$T_{\mu\nu}(p\,) \;= \overline{u}_d\, \gamma_\mu\, (1+\gamma_5) \sum_{i=u,c,t} V_{is}V_{id}^* \,\frac{\widehat{p}+m_i}{p^2-m_i^2}\, \gamma_\nu\, (1+\gamma_5)\, u_s$$

and

$$\widetilde{T}_{\mu\nu}(p\,) \;= \overline{v}_{\overline{d}}\, \gamma_\nu\, (1+\gamma_5) \sum_{i=u,c,t} V_{is}V_{id}^* \,\frac{\widehat{p}+m_i}{p^2-m_i^2}\, \gamma_\mu\, (1+\gamma_5)\, v_{\overline{s}}\,.$$

If we would leave only the lightest quark u in two above sums, the integral in p would be quadratic divergent as $\int d^4p\, p^{-2}$ for $p \to \infty$. Due to the unitarity of the CKM matrix we have

$$\sum_{i=u,c,t} V_{is}V_{id}^* = 0\,.$$

It leads to the convergency of the integral in the region of the order of masses of the heavy quarks, which corresponds to the well-known mechanism of the Glashow, Illiopoulos and Maiani. We can neglect the mass of the u-quark and obtain using the unitarity condition

$$T_{\mu\nu}(p\,) \;= 2\, \overline{u}_d\, \gamma_\mu\, (1+\gamma_5)\; \widehat{p}\, \gamma_\nu\, u_s \sum_{i=c,t} V_{is}V_{id}^* \;\frac{m_i^2}{p^2(p^2-m_i^2)}\;,$$

$$\widetilde{T}_{\mu\nu}(p\,) \;= 2\, \overline{v}_{\overline{d}}\, \gamma_\nu\, (1+\gamma_5)\, \widehat{p}\, \gamma_\mu\, v_{\overline{s}} \sum_{i=u,c,t} V_{is}V_{id}^* \,\frac{m_i^2}{p^2(p^2-m_i^2)}.$$

After averaging in the direction of the momentum p and simplifying the spin structure with the use of the relation

$$\overline{u}_d\, \gamma_\mu\, (1+\gamma_5)\; \widehat{p}\, \gamma_\nu\, u_s\; \overline{v}_{\overline{d}}\, \gamma_\nu\, (1+\gamma_5)\, \widehat{p}\, \gamma_\mu\, v_{\overline{s}} \to p^2\, \overline{u}_d\, \gamma_\sigma\, (1+\gamma_5)\; u_s\; \overline{v}_{\overline{d}}\, \gamma_\sigma\, (1+\gamma_5)\; v_{\overline{s}}$$

we calculate the integrals and obtain

$$A = -\frac{G^2}{8\pi^2}\; \overline{u}_d\, \gamma_\sigma\, (1+\gamma_5)\; u_s\; \overline{v}_{\overline{d}}\, \gamma_\sigma\, (1+\gamma_5)\; v_{\overline{s}}\; M^2,$$

where

$$M^2 = m_c^2\; (V_{cs}V_{cd}^*)^2 + m_t^2\; (V_{ts}V_{td}^*)^2 + 2\frac{m_c^2 m_t^2}{m_t^2 - m_c^2} \ln \frac{m_t^2}{m_c^2}\, V_{cs}V_{cd}^*\, V_{ts}V_{td}^*\,.$$

Because the matrix elements V_{ts} and V_{td} are very small, we can estimate

$$Re\, M^2 \simeq m_c^2\, \cos^2\vartheta_c\, \sin^2\vartheta_c\,,$$

where ϑ_c is the Cabbibo angle. The imaginary part of M^2 responsible for the CP violation is related with the phase δ appearing in V_{ts} and V_{td} for the parametrization of Kobayashi and Maskawa.

The effective Hamiltonian for the quark interactions in the one-loop approximation can be written as follows

$$H = G_2\, j_\sigma\, j_\sigma\,,\ j_\sigma = \overline{\psi}_d\, \gamma_\sigma\, (1+\gamma_5)\, \psi_s\,,\ G_2 = \frac{G^2}{16\pi^2}\, M^2\,,$$

where we took into account, that the above scattering amplitude according to the Wick theorem should contain the extra factor 2 in comparison with H, which is related with the Fermi statistics of the quark fields. When we calculate the matrix element of H between the states $|K^0\rangle$ and $\left|\overline{K^0}\right\rangle$ two different contributions should be taken into account. Namely, with taking into account the colour indices i,j of the quarks and with the use the vacuum dominance hypothesis for the intermediate states, one can obtain

$$\left\langle\overline{K^0}\right| H \left|K^0\right\rangle = G_2\, \left\langle\overline{K^0}\right| (j_\sigma)_i^j\, |0\rangle\, \langle 0|\, (j_\sigma)_{i'}^{j'} \left|K^0\right\rangle \left(\delta_{j'}^{i'}\delta_j^i + \delta_j^{i'}\delta_{j'}^i\right) = \frac{4}{3}\, G_2\, \left\langle\overline{K^0}\right| j_\sigma\, |0\rangle\, \langle 0|\, j_\sigma \left|K^0\right\rangle,$$

where the term containing $\delta_j^{i'}\delta_{j'}^i$ and appearing after the Fierz transformation gives the contribution proportional $\frac{1}{3}$ from the common factor $\frac{4}{3} = 1 + \frac{1}{3}$ after its projecting into the singlet colour state. We used the expression

$$\langle 0|\, j_\sigma \left|K^0\right\rangle = f_K\, p_\sigma\,,$$

where $f_K \simeq 1.2\, f_\pi$ is the constant similar to f_π entering in the lepton partial width of the K-meson

$$\Gamma_{K\to l\overline{\nu_e}} = |f_K\,|^2\, \frac{G^2 \sin^2\vartheta_c}{8\pi} \left(1 - \frac{m_l^2}{m_K^2}\right)^2 m_K\, m_l^2\,.$$

It gives approximately

$$\Delta\, m = \left\langle\overline{K^0}\right| H \left|K^0\right\rangle + \left\langle K^0\right| H \left|\overline{K^0}\right\rangle \simeq \frac{4m_c^2\cos^2\vartheta_c}{3\pi m_\mu^2}\, \Gamma_{K\to\mu\overline{\nu_e}}$$

in an agreement with the experimental data. As for the Wolfenstein parameter μ_{21}, in accordance with the above formulas it is expressed through the imaginary part of G_2 related with the phase δ in the Kobayashi-Maskava parametrization of the CKM matrix

$$\varepsilon = \frac{i\mu_{21}}{\Delta\, m + \frac{i}{2}(\Gamma_S - \Gamma_L)} =$$

$$\frac{e^{i\varphi}}{\sqrt{2}} s_2 s_3 \sin\delta\, \frac{1 + \zeta s_2(s_3\cos\delta - s_2) - \ln\zeta}{1 + \zeta s_2^2(s_2^2 + 2s_2 s_3\cos\delta + s_3^2\cos(2\delta)) + 2s_2(s_3 + s_2\cos\delta)\ln\zeta}\,,$$

where $\varphi \simeq \pi/4$ and $\zeta = m_t^2/m_c^2$. It is possible to verify, that the CP-violating effects are absent if one of the parameters s_i, δ is zero. Because these parameters are not known well, the experimental value of ε allows to fix only a certain combination of them.

In the Standard Model there is an additional source of the CP violation in the K-decays apart from the Wolfenstein mechanism related with the parameter ε in the K_L-meson wave function. Namely, there can be the direct decay of the K_2-meson in the $\pi^+\pi^-$ and $\pi^0\pi^0$-channels which is characterized by the parameter ε'. If we introduce for these channels the quantities

$$\eta_{+-} = \frac{\langle \pi^+\pi^- | \, K_L \rangle}{\langle \pi^+\pi^- | \, K_S \rangle} \; , \; \eta_{00} = \frac{\langle \pi^0\pi^0 | \, K_L \rangle}{\langle \pi^0\pi^0 | \, K_S \rangle} \; ,$$

they are parametrized by ε and ε' as follows

$$\eta_{+-} = \varepsilon + \varepsilon' \, , \;\; \eta_{00} = \varepsilon - 2\varepsilon' .$$

In the case if we impose the constraint $Im \, A_0 = 0$ on the amplitude of the transition $K \rightarrow \pi\pi$ with the π-meson state isospin $T = 0$ (corresponding to the so called Wu-Yang gauge), then ε' is proportional to $Im \, A_2$, which demonstrates, that the factors 1 and -2 in front of ε' in the above expressions for η_{+-} and η_{00} are related to the corresponding Clebsch-Gordan coefficients. The phase φ' of the parameter ε is expressed in terms of the phases δ_0 and δ_2 for the $\pi\pi$ scattering amplitudes with the isospins $T = 0$ and $T = 2$ as follows

$$\varphi' = \delta_0 - \delta_2 + \frac{\pi}{2} \simeq \frac{\pi}{4}$$

and therefore the ratio ε'/ε is a real quantity. Recently it was measured experimentally

$$\frac{\varepsilon'}{\varepsilon} \simeq (2.1 \pm 0.5) \, 10^{-3} \, .$$

In the Standard Model the parameter ε' appears from the penguin diagrams (see Fig. 20) leading to the direct transition of K_2^0 in $\pi\pi$. There is some uncertainty in the calculation of the matrix elements of these diagrams between the hadron states, because the big contributions from different graphs cancel. The theoretical prediction for ε' turns out to be slightly less than its measured value, but it is not considered now seriously as a drawback of the Standard Model.

16 Higgs interactions

The Higgs boson H is the last undiscovered particle of the Standard Model. In 2000 it was announced by the CERN experimentalists working at the large electron-positron collider (LEP) with the total energy $\sqrt{s} = 210 \; GeV$ that this particle was detected and it has the mass around $114 \, GeV$. But the physical community considers this value only as a low estimate for the Higgs mass. The reaction, in which the Higgs particle has been searched, was

$$e^+e^- \rightarrow Z^* \rightarrow Z + H \, ,$$

where Z^* is the virtual Z-boson. The decay $H \rightarrow b\bar{b}$ was used for the Higgs identification.

There are some restrictions on the Higgs mass from below and above. Note, that according to the Standard Model its mass is related to the Higgs self-coupling coupling constant λ as

$$m_H = \lambda\,\eta\,,$$

where $\eta/\sqrt{2}$ is the mean value of the Higgs field φ.

The low boundary for m_H is obtained from the fact, that the Higgs potential in one loop approximation is

$$V(\varphi) = \mu^2\,|\varphi|^2 + \gamma\,|\varphi|^4\,\ln\left(|\varphi|^2/m^2\right)\,,$$

where

$$\gamma = \frac{3\sum m_v^4 + \sum m_s^4 - 4\sum m_f^4}{16\,\pi^2\eta^4}$$

is expressed in terms of the masses $m_v = \frac{g\eta}{2}, m_s = \lambda\eta, m_f\, = f\,\eta/\sqrt{2}$ of virtual vector bosons v, scalars s (including the Higgs particles) and fermions f which are proportional to their coupling constants with the Higgs particles. This result can be obtained by calculating the renormalization of the coupling constant λ due to the contribution of corresponding loop diagrams with a subsequent substitution of the ultraviolet cut-off Λ by the external field φ.

From the condition, that the potential energy has a non-trivial minimum at $\varphi = \eta/\sqrt{2}$ we have

$$V'(\varphi) = 0\,.$$

Furthermore, in accordance with the stability requirement for the non-trivial vacuum the value of $V(\varphi)$ at $\varphi = \eta/\sqrt{2}$ should be negative, which leads to the restriction (neglecting fermions and scalar particles)

$$m_H^2 > \frac{3(m_Z^4 + 2m_W^4)}{16\pi^2\eta^2} \simeq 7,3\,GeV\,.$$

On the other hand, the Higgs mass can not be bigger than 1 TeV because in the last case the coupling constant λ will be large and there will be a strong interaction in the Higgs sector. Moreover, the amplitudes of the W-boson scattering with the longitudinal polarization will be also large. In particular, the cross-section for scattering the vector bosons with longitudinal polarizations will grow $\sim s$ in some region of energy, which is not compatible with the unitarity restrictions.

The Higgs boson can be discovered at hadron-hadron high energies collisions in the reaction, where the colliding hadrons produce two gluons (or photons) annihilated subsequently in the Higgs particle. The most probable transition of two gluons into the Higgs particles takes place through the triangle diagram with the heavy quark t because the coupling of the Higgs boson with particles is proportional to their masses. The effective lagrangian for the transition $gg \to H$ can be easily calculated from the one-loop correction to the kinetic part of the gluon interactions:

$$L_{Higgs} = \frac{\alpha_s}{12\pi}\,\frac{\chi}{\eta}\,G^a_{\mu\nu}G^a_{\mu\nu}\,,$$

where χ is the Higgs field. This result can be obtained easily from the known formula $-\frac{\alpha}{3\pi}\ln\frac{\Lambda^2}{m^2}$ for the one-loop correction to the photon Lagrangian $\frac{1}{4}F^2_{\mu\nu}$ in QED. Indeed, one should only take into account the colour factor $\frac{1}{2}\delta_{ab} = tr\, t^a t^b$ for the quark contribution to the strong coupling constant and the fact, that in the external Higgs field m^2 in the argument of the logarithm should be substituted by $m^2(1+\frac{\chi}{\eta})^2$ with the subsequent expansion in χ.

The interaction of the nucleon N with the Higgs can be written in the form

$$L_{NH} = \frac{M}{\eta}\,\chi\,\overline{\psi}\psi\,,$$

where the value of M in the chiral limit $m_u, m_d \to 0$ is related to the matrix element of the trace of the energy-momentum tensor of the gluon field

$$\theta^g_{\mu\mu} = -\frac{b\,\alpha_s}{8\pi}\,G^a_{\mu\nu}G^a_{\mu\nu}\,,$$

appearing as a quantum anomaly in the one-loop approximation due to the renormalization of the strong coupling constant α_s ($b = 11 - \frac{2}{3}\,n_f \simeq 9$). Indeed, for the nucleon mass in the chiral limit we obtain

$$m_N\,\overline{\psi}\psi = \left\langle N \left| \theta^g_{\mu\mu} \right| N \right\rangle$$

and therefore

$$L_{NH} = \langle N\, |L_{Higgs}|\, N\rangle = \frac{1}{\eta}\,\frac{2\,m_N}{3\,b}\,\overline{\psi}\psi,$$

where we considered only t as a heavy quark. The last relation means, that $M \simeq -\frac{2m_N}{27}$.

In the Standard Model the observables can be expressed in terms of the bare parameters: the coupling constants g, $\overline{g} = \sqrt{g^2 + g'^2}$, the condensate η of the Higgs fields, the Yukawa coupling constants and the matrix elements of the Cabbibo-Kobayashi-Maskawa matrix. However it is convenient to write these observables in terms of a set of quantities which are well measured Such well known parameters are the electromagnetic fine structure constant $\alpha = e^2/(4\pi)$, the Fermi constant G_μ and the mass m_Z of the Z-boson related with the Weinberg angle according to the definition

$$\sin^2\theta_W\,\cos^2\theta_W\; = \frac{\pi\overline{\alpha}}{\sqrt{2}G_\mu m_Z}\,,$$

where $\overline{\alpha}^{-1} = \alpha^{-1}(m_Z) \simeq 128.94$ These quantities in the perturbation theory can be written as functions of the bare values of the fundamental parameters g, g', η One can use these relations to express other physical quantities as functions of α, G_μ, and m_Z. It will give a possibility to obtain accurate restrictions on unknown parameters of the type of the Higgs mass which enter in the loop corrections for the observables.

For example with a sufficient accuracy we can write the W-boson mass m_W and the parameters g_A and g_V entering in the amplitude of the decay $Z \to e^+e^-$ in one-loop approximation as follows

$$\frac{m_W}{m_Z} = \cos\theta_W + \frac{\cos^3\theta_W}{2(\cos^2\theta_W - \sin^2\theta_W)} \left(\frac{\Pi_Z(0)}{m_Z} - \frac{\Pi_W(0)}{m_W} \right),$$

$$g_A = -\frac{1}{2} - \frac{1}{4} \left(\frac{\Pi_Z(0)}{m_Z} - \frac{\Pi_W(0)}{m_W} \right),$$

$$g_V = -1 - 4\sin^2\theta_W + \frac{4\sin^2\theta_W \, \cos^2\theta_W}{\cos^2\theta_W - \sin^2\theta_W} \left(\frac{\Pi_Z(0)}{m_Z} - \frac{\Pi_W(0)}{m_W} \right).$$

Here $\Pi_Z(0)$ and $\Pi_W(0)$ are the renormalized polarized operators for Z- and W- bosons, calculated for $q^2 = 0$. In the right hand side one can substitute $\frac{m_W}{m_Z}$ by its Born value $\cos\theta_W$.

To regularize the Feynman integrals in the gauge theories it is convenient to consider them in the D-dimensional space

$$\int \frac{d^4p}{(2\pi)^4} \to \mu^{-D+4} \int \frac{d^Dp}{(2\pi)^D}, \; D - 4 = -2\varepsilon \to 0,$$

where μ is the normalization parameter for the coupling constant g_μ. In the minimal subtraction scheme $\overline{MS}$ the dependence of the bare coupling constant g_0 entering in the initial Lagrangian is constructed as an infinite series in the renormalized coupling g_μ with the coefficients which are the polynomials in $(D-4)^{-1}$ chosen from the condition of the cancellation of all poles at $D \to 4$ in the physical quantities.

Using the dimensional regularization we can calculate the expression appearing in the polarization operator:

$$\Pi^{\psi}_{\mu\nu}(q^2) = -i\mu^{-D+4} \int \frac{d^Dp}{(2\pi)^D} \frac{Tr\,\gamma_\mu(\gamma_5)(\widehat{p} + m_1)\gamma_\nu(\gamma_5)(\widehat{p} + \widehat{q} + m_1)}{(p^2 + m_1^2)((p+q)^2 + m_2^2)},$$

where (γ_5) means the contribution of the axial current. For $\varepsilon \to 0$ one can obtain

$$4\pi^2\,\Pi^{\psi}_{\mu\nu}(q^2) = \left(\frac{1}{\varepsilon} - \gamma + \ln\,4\pi \right) \left(\frac{q^2\delta_{\mu\nu} - q_\mu q_\nu}{3} - \frac{\delta_{\mu\nu}}{2} \left(m_1^2 + m_2^2 \mp 2m_1m_2 \right) \right) +$$

$$\int_0^1 dx \, \left(2x(x-1)(q^2\delta_{\mu\nu} - q_\mu q_\nu) + \delta_{\mu\nu} \left(m_1^2(1-x) + m_2^2x \mp m_1m_2 \right) \right) \ln \frac{m_1^2(1-x) + m_2^2x - q^2x(1-x)}{\mu^2},$$

where the upper (lower) sign corresponds to the vector (axial) current; γ is the Euler constant.

Using this expression we obtain the difference of the polarization operators for the contribution of the doublet (t, b) (in the limit $m_b = 0$)

$$\frac{\Pi^{\psi}_Z(0)}{m_Z} - \frac{\Pi^{\psi}_W(0)}{m_W} = \frac{\overline{g}^2}{16} \frac{\Pi^{\psi}_A(m_t, m_t)}{m_Z^2} - \frac{g^2}{8} \frac{\Pi^{\psi}_A(m_t, 0) + \Pi^{\psi}_V(m_t, 0)}{m_W^2} =$$

$$\frac{3g^2m_t^2}{64\pi^2m_W^2} = \frac{3\overline{\alpha}}{16\pi^2\cos^2\theta_W\,\sin^2\theta_W} \left(\frac{m_t}{m_Z} \right)^2.$$

Here the factor 3 appears from three colours. Thus, the contribution of the t quark in the radiative correction is proportional to m_t^2, which allowed to predict its mass before the discovery.

Let us go now to the contribution of the Higgs particle to the observables. Again only its coupling with the vector bosons is essential. We have for the polarization operator of the W-boson

$$\Pi^H_{\mu\nu}(q^2) = -ig^2\mu^{-D+4}\int \frac{d^Dp}{(2\pi)^D}\frac{m_W^2\delta_{\mu\nu}-p_\mu p_\nu}{p^2-m_W^2}\,\frac{1}{(p-q)^2-m_W^2}\,.$$

Neglecting the small contribution proportional to $q_\mu q_\nu$ one obtains for the coefficient in front of $\delta_{\mu\nu}$

$$\Pi^H = \frac{g^2}{16\pi^2}\left(\frac{1}{\varepsilon}-\gamma\right)\left(\frac{5}{4}m_W^2+\frac{1}{4}m_H^2-\frac{1}{12}q^2\right)+\frac{g^2}{16\pi^2}m_W^2(2\ln 2+\ln\pi)-$$
$$-\frac{g^2}{16\pi^2}\int_0^1 dx\,\Big((1-x)m_W^2+xm_H^2+q^2(x^2-x)\Big)\left(\ln 2+\frac{\ln x}{2}+\frac{1}{2}\right)+$$
$$\frac{g^2}{32\pi^2}\int_0^1 dx\,\Big(-(1-x)m_W^2+xm_H^2+q^2(x^2-x)\Big)\ln\frac{(1-x)m_W^2+xm_H^2+q^2(x^2-x)}{\mu^2}\,.$$

In the observables the contribution proportional to $1/\varepsilon$ is cancelled and for the finite contribution depending on m_H we have

$$\Pi^H = \frac{g^2}{32\pi^2}\int_0^1 dx\,\Big(-(1-x)m_W^2+xm_H^2+q^2(x^2-x)\Big)\ln\frac{(1-x)m_W^2+xm_H^2+q^2(x^2-x)}{\mu^2}\,.$$

The analysis of the dependence of m_W and g_A, g_V from the Higgs mass uses the above expression and for $m_t = 175\pm 5GeV$ we obtain, that the best values for these physical quantities is obtained if

$$m_H = 79^{+47}_{-29}\,GeV\,,\ \alpha_s(m_Z) = 0,1182\pm 0.0027\,,$$

where α_s is the strong fine structure constant. It is obvious, that the CERN value $m_H \geq 114GeV$ is almost in a conflict with the above estimates, obtained from the radiative corrections. Note, that providing that $m_H\,, m_t \gg m_{W,Z}$ the theoretical expressions for the observables are significantly simplified:

$$\frac{\Pi_W(0)}{m_W^2}-\frac{\Pi_W(m_W^2)}{m_W^2} = \frac{g^2}{32\pi^2}\,\frac{1}{6}\,\ln\frac{m_H^2}{m_Z^2}\,,$$

$$\frac{\Pi_Z(m_Z^2)}{m_Z^2}-\frac{\Pi_W(m_W^2)}{m_W^2} = -\frac{\overline{g}^2}{32\pi^2}\sin^2\theta_w\,\frac{5}{3}\,\ln\frac{m_H^2}{m_Z^2}+\frac{3\overline{g}^2}{64\pi^2}\,\ln\frac{m_t^2}{m_Z^2}\,,$$

$$\Pi'_Z(m_Z^2) = -\frac{\overline{g}^2}{32\pi^2}\,\frac{1}{6}\,\ln\frac{m_H^2}{m_Z^2}\,.$$

17 Grand Unification

Let us introduce the fine structure constants of the Standard Model by the definition

$$\overline{\alpha_i} = \frac{\alpha_i}{4\pi}\,,\;\; \alpha_i = \frac{g_i^2}{4\pi}\,, \tag{51}$$

where g_i for $i = 1, 2, 3$ are the coupling constants for the vector bosons of the gauge groups $U(1), SU(2)$ and $SU(3)$ of the Standard Model. The Grand Unification Theory (GUT) of the electro-weak and strong interactions is based on the idea, that at large boson virtualities Q^2 the effective coupling constant increases in the Abelian gauge model and decreases in the Yang-Mills theory. Therefore at some large $Q^2 = M_{GUT}^2$ all three couplings could coincide and at higher Q^2 the evolution of the unified coupling constant is determined by the unified Yang-Mills theory with an extended gauge symmetry ($SU(5)$ or higher).

Note, that in this unified theory the interaction of the gauge bosons with the fermions has the form

$$L_{int} = g_{GUT}\,\overline{\psi} T_a \widehat{V^a} \psi\,,$$

where g_{GUT} is the GUT coupling constant and T_a are its gauge group generators normalized as usual

$$tr\;(T_a T_b) = \frac{1}{2}\,\delta_{ab}\,.$$

Here tr implies the summation over the particles inside each generation. Because with taking into account the colour degrees of freedom the number of weak doublets in each generation is $n_w = 1 + 3 = 4$, we have for the $SU(2)$ coupling constant

$$g = \frac{g_{GUT}}{2}\,.$$

Analogously, the number of coloured triplets, corresponding to the fundamental representation of the $SU(3)$ group in each generation, is 4 (corresponding to the u and d quarks having two helicities) and therefore

$$g_s = \frac{g_{GUT}}{2}\,.$$

Further, the correctly normalized generator T_Y in the unified group $SU(5)$, corresponding to the hypercharge Y, is

$$T_Y = c\,\frac{Y}{2}\,,$$

where the constant c is obtained from the relation

$$c^2 \sum_{fam} \frac{Y^2}{4} = \frac{c^2}{4}\left(2^2 + 2 + 3\left(\left(\frac{4}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + 2\left(\frac{1}{3}\right)^2\right)\right) = c^2\frac{10}{3} = \frac{1}{2}\,.$$

Therefore the coupling constant g' for the B-boson is

$$g' = \frac{g_{GUT}}{2}\sqrt{\frac{3}{5}}\,.$$

It means, that we can chose

$$\overline{\alpha_2} = \frac{g^2}{16\pi^2}\,,\ \overline{\alpha_3} = \frac{g_s^2}{16\pi^2}\,,\ \overline{\alpha_1} = \frac{5}{3}\,\frac{g'^2}{16\pi^2}$$

with the property:

$$\lim_{Q^2\to M_{GUT}^2}\ \overline{\alpha_i} = \frac{g_{GUT}^2}{64\pi^2}\,.$$

The renormalization group equation for $\overline{\alpha_i(t)}$ as a function of $t = \ln(M_{GUT}^2/Q^2)$ is

$$\frac{d\,\overline{\alpha_i(t)}}{d\,t} = -b_i\,\overline{\alpha_i(t)}^2\,, \tag{52}$$

where b_i are the sums of contributions of the gauge particles, fermions and the Higgs bosons.

$$b_1 = c_1^F\,N_F + c_1^H\,N_H\,,$$

$$b_2 = -2\,\frac{11}{3} + c_2^F\,N_F + c_2^H\,N_H\,,$$

$$b_3 = -3\,\frac{11}{3} + c_3^F\,N_F + c_3^H\,N_H\,. \tag{53}$$

The first contribution in b_2 and in b_3 proportional to $11/3$ is related with the pure gauge boson self-interaction. The second term $\sim N_f$ is determined by the interaction of the vector bosons with fermions (N_F is the number of generations). The third contribution is related with the virtual Higgs particles (N_H is their number).

The electron contribution to the QED running fine structure constant is well known

$$b_{QED} = \frac{4}{3}\,. \tag{54}$$

For QCD the contribution of fermions contains the additional factor $1/2$ related with the normalization of the colour generators $tr(t^a t^b) = \frac{1}{2}\delta^{ab}$ and the factor 2 because in each generation we have 2 quarks of a different type

$$c_3^F = \frac{4}{3}\,. \tag{55}$$

For the case of the weak gauge group $SU(2)$ we have the factor $1/2$ corresponding to the normalization of the weak isospin generators, the factor 4, related with the four weak multiplets in the each generation (including three coloured quarks and a lepton) and the additional factor $1/2$ due to the fact, that only left fermions interact with the W-bosons and therefore

$$c_2^F = \frac{4}{3}\,. \tag{56}$$

As for the interaction with the B-boson, we should take into account the change of the normalization of the corresponding coupling constant g' and the fact, that the particles with different helicities have different hypercharges, which again gives

$$c_1^F = \frac{4}{3}\,. \tag{57}$$

The equality $c_1^F = c_2^F = c_3^F$ is related to the fact, that gluons, W and B - bosons belong to the same multiplet of GUT.

The Higgs bosons do not give any contributions to b_3 because they are not coloured objects

$$c_3^H = 0\,. \tag{58}$$

In the scalar electrodynamics the contribution of the charged scalar particle to the β-function is

$$b_{SED} = \frac{1}{3}\,, \tag{59}$$

because the corresponding polarization operator equals

$$i\int \frac{d^4k}{(2\pi)^4}\,\frac{(2k-q)_\mu\,(2k-q)_\nu}{k^2(k-q)^2} \simeq \frac{1}{16\pi^2}\,\frac{1}{3}\left(q^2\delta_{\mu\nu} - q_\mu q_\nu\right)\,\ln\frac{\Lambda^2}{q^2}\,.$$

Therefore, taking into account the factor 1/2 from the normalization of the isospin generators, we obtain

$$c_2^H = \frac{1}{6}\,. \tag{60}$$

At last because for the doublet of the Higgs particles we have the extra factor $2(|Y|/2)^2 = 1/2$ apart from the normalization multiplier 3/5 one obtains

$$c_1 = \frac{1}{10}\,. \tag{61}$$

The solution of the renormalization group equations can be written as follows

$$\frac{1}{\overline{\alpha_i(t)}} - \frac{1}{\overline{\alpha_i(0)}} = b_i\; t\,. \tag{62}$$

If we shall assume, that all three $\overline{\alpha_i(t)}$ coincide at $t=0$ corresponding to the scale $Q^2 = M_{GUT}^2$ of the Great Unification, three right lines in the plane $(1/\overline{\alpha_i(t)},\; t\;)$ should intersect in one point. If indeed it takes place, we can calculate the important parameter M_{GUT}^2 related with the masses of the heavy vector bosons responsible in particular for the transition between quarks and leptons. Now the experimental accuracy of the measurement of $\overline{\alpha_i(t)}$ is high enough to verify, that these three lines do not intersect in the same point (see Fig. 23). Therefore the naive realization of the Great Unification idea does not work.

However, one can consider the supersymmetric ($SUSY$) generalization of the Standard Model, in which for each boson and fermion we have a superpartner with an opposite statistics. For example, for gluons and vector bosons such new particles are Majorano fermions: gluinos, W-bosinos and B-bosinos described by real fields. Superpartners of quarks and leptons are scalar particles - squarks and sleptons. The supersymmetric generalization of the

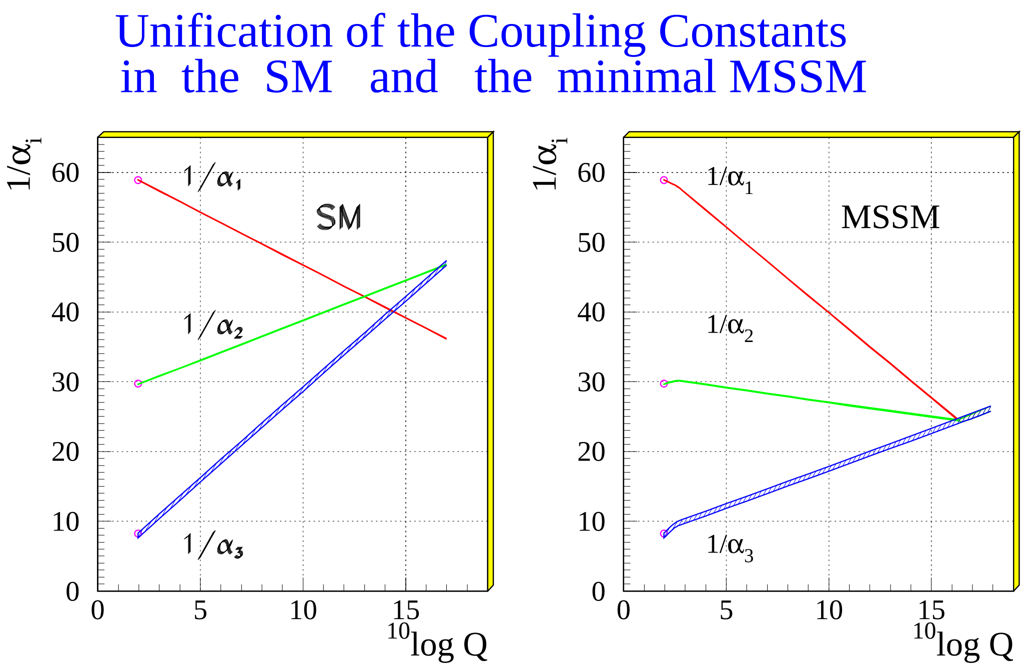

Figure 23: Evolution of the inverse of the three coupling constants in the Standard Model (left) and in the supersymmetric extension of the SM (MSSM) (right). Only in the latter case unification is obtained. The SUSY particles are assumed to contribute only above the effective SUSY scale M_{SUSY} of about 1 TeV, which causes a change in the slope in the evolution of couplings. The thickness of the lines represents the error in the coupling constants.

Standard Model have at least one additional parameter - mass m_{SUSY} of the superpartners. If the energy is less, than this mass, one can neglect the existence of the supersymmetry and the renormalization group equations are the same, as in the Standard Model, but if the energies are bigger than m_{SUSY}, one should take into account the superpartners of the usual particles and we have for the β-functions b_i:

$$
\begin{aligned}
b_1^{susy} &= 2\,N_F + \frac{3}{10} N_H\,, \\
b_2^{susy} &= -6 + 2\,N_F + \frac{1}{2}\,N_H\,, \\
b_3^{susy} &= -9 + 2\,N_F\,.
\end{aligned}
\tag{63}
$$

Indeed, because the gluino belongs to the adjoint representation of the $SU(3)$ group and is a real particle, its contribution to b_3^{susy} equals $\frac{4}{3}\,3\,\frac{1}{2} = 2$. Analogous contribution of W-bosinos to b_2^{susy} is $\frac{4}{3}\,2\,\frac{1}{2} = \frac{4}{3}$. Squarks give the additional contribution $\frac{1}{3}\,\frac{1}{2}4\,N_F = \frac{2}{3}\,N_F$ to b_3^{susy} because the corresponding loop diagram has the colour factor $tr\,(t^a t^b){=}\frac{1}{2}$ and they can be in four states corresponding to two helicities of quarks u and d. Squarks and sleptons give the total contribution $\frac{1}{3}\,\frac{1}{2}4\,N_F = \frac{2}{3}\,N_F$ to b_2^{susy} because they are superpartners for quarks and leptons with two helicities. These particles give the same contribution $\frac{2}{3}\,N_F$ to b_2^{susy} because their hypercharges coincide with those for quarks and leptons. The contribution of the Higgsinos to b_2^{susy} is $\frac{4}{3}\frac{1}{2}\frac{1}{2}N_H = \frac{1}{3}N_H$ because it belongs to the fundamental representation of the gauge group $SU(2)$ and its helicity is $-1/2$. At last due to the same arguments the Higgsino gives the contribution $\frac{3}{5}\frac{4}{3}\frac{1}{2}\frac{1}{2}N_H = \frac{1}{5}N_H$ in b_2^{susy}. The additional contribution of the sleptons, squarks and Higgsinos in the quantum anomalies is cancelled inside of each generation only if there are two Higgs supermultiplets. The necessity of the existence of two Higgs particles in SUSY also is a consequence of the fact, that in an opposite case we can not provide the non-zero masses for the upper and lower components of the weak fermion doublets.

Thus, by adjusting the parameter m_{SUSY} we can impose on the renormalization group lines $(1/\alpha_i(t),\ t\ )$ the property for them to intersect in the same point (see Fig. 23). It gives a possibility to find that $m_{SUSY} \simeq 10^4 GeV$. Therefore one obtain the prediction for the experiments at future accelerators concerning the possible existence of the supersymmetric particles in this region of the energy. Another argument, supporting such estimate for the masses of superpartners, is related with the solution of the so-called hierarchy problem. Indeed the mass m_H of Higgs particle is much smaller than M_{GUT}, but the one-loop correction to the Higgs mass in SUSY gives $\delta m_H^2 \sim \lambda\, m_{SUSY}^2$ and therefore m_{SUSY} can not be bigger than $10^4 GeV$.

The next problem is the choice of the unified gauge group and its representations to place all existing particles with their superpartners in corresponding super-multiplets. It turn out, that the GUT predicts a number of interesting effects. Because the quarks and leptons in these theories are placed in the same gauge multiplets, there can be processes, in which the baryonic charge is not conserved due to the quark transitions $uu \to e^+\overline{d},\ ud \to \overline{\nu}\overline{d},\ ud \to e^+\overline{u}$. The decay of the proton, which is one of the most wide spread particle of the Universe, is not observed yet, but the existing experimental data do not contradict to the predictions of GUT based on the supersymmetric generalization of the unified theory with the gauge group $SU(5)$ group.

Another interesting consequence of GUT is a natural explanation of the fact, that the quarks have the electric charges proportional to $1/3$ of the electron charge. To clarify it, let us consider for example GUT based on the $SU(5)$ gauge group. In this case the fermions in each generation belong to two representations of the group: fundamental Q_{aR} with 5 components and anti-symmetric in the group indices D_{ab}.

The content of the fundamental representation is

$$Q_{aR} = \Big(d_1,\, d_2,\, d_3,\, e^+,\, \overline{\nu}_e\Big)_R\ ,$$

where the subscript R corresponds to the fermions with the right helicity $+1/2$ and the indices $1, 2, 3$ describe the different colour states of d-quarks. The content of the anti-symmetric representation is

$$D_{ab} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \overline{u}^3 & -\overline{u}^2 & -u_1 & -d_1 \\ -\overline{u}^3 & 0 & \overline{u}^1 & -u_2 & -d_2 \\ \overline{u}^2 & -\overline{u}^1 & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -e^+ \\ d_1 & d_2 & d_3 & e^+ & 0 \end{pmatrix}_L\ .$$

The anti-particles belong to the hermitially conjugated representations. The right neutrino (if it exists) belong to the singlet representation of the $SU(5)$ group.

One of the generators of the gauge group $SU(5)$ is the electric charge. Because the trace of all generators in this group should be equal to zero, we obtain for the fundamental representation

$$3Q_d + 1 = 0$$

in accordance with the known electric charge $Q_d = -1/3$ of the d quark.

In the case, when the gauge group of GUT is $SO(10)$, the spinor representation contains 16 component and therefore all fermions in the generation, including the right neutrino, can be placed in the same multiplet. In this model also the life time of the proton is too short to be in the agreement with the experimental data.

The supersymmetric generalizion of the Standard Model is considered now as the most attractive possibility, because in $SUSY$ the symmetry of the physical space-time is extended. Apart from the usual coordinates x_μ here the anti-commuting coordinates θ_i , $\overline{\theta}_i$ are also introduced. They are two-component spinors. Index i takes generally N values. N is called the rank of the supersymmetry. The Poincare group containing translations P_μ and rotations $M_{\mu\nu}$ (including the Lorentz transformations) is extended. We have now the supergenerators Q_i, $\overline{Q}_i$ containing the derivatives over the coordinates θ_i , $\overline{\theta}_i$.

In the case of $N = 1$ the anti-commutation relations for the supergenerators are

$$\left\{Q_\alpha, \overline{Q}_\beta\right\} = 2\,\sigma^\mu_{\alpha\beta} P_\mu \,,\; P_\mu = i\partial_\mu \,,$$

$$\{Q_\alpha, Q_\beta\} = 0 \;,\; .\{\overline{Q}_\alpha, \overline{Q}_\beta\} = 0 \,.$$

The supergenerators Q_α, $\overline{Q}_\alpha$, corresponds to the infinitesimal supertranslations

$$x_\mu \to x_\mu + i\theta\sigma_\mu\overline{\varepsilon} - i\varepsilon\sigma_\mu\overline{\theta} \,,\; \theta \to \theta + \varepsilon \,,\; \overline{\theta} \to \overline{\theta} + \overline{\varepsilon} \,,$$

where $\varepsilon, \overline{\varepsilon}$ are anti-commuting parameters. They can be realized on the scalar superfields $F(x,\, \theta,\, \overline{\theta})$ as the differential operators

$$Q_\alpha = \frac{\partial}{\partial\theta} - i\sigma^\mu_{\alpha\beta}\overline{\theta}^\beta\partial_\mu \,,\; \overline{Q}_\alpha = \frac{\partial}{\partial\overline{\theta}_\alpha} + i\theta^\beta\sigma^\mu_{\beta\alpha}\partial_\mu \;.$$

Note, that the supergravity can be obtained as the field theory invariant under the supertranslations with the space-time dependent parameters $\varepsilon, \overline{\varepsilon}$.

Due to the anti-commutativity of θ , $\overline{\theta}$ the Taylor expansion of the superfields in θ , $\overline{\theta}$ contains a finite number of terms. The coefficients of this expansion produce the supermultiplets including simultaneously bosonic and fermionic fields. To diminish their number one can impose some additional condition on $F(x,\, \theta,\, \overline{\theta})$. In particular the chiral superfield satisfy the following constraint

$$\overline{D}\,\Phi = 0 \,,$$

where the covariant derivative is defined as follows

$$\overline{D}_\alpha = -\frac{\partial}{\partial\overline{\theta}_\alpha} - i\theta^\beta\sigma^\mu_{\beta\alpha}\partial_\mu \,.$$

This superfield has the following Taylor expansion

$$\Phi(x,\,\theta,\,\overline{\theta}) = A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) =$$

$$A(y) + i\theta\sigma^\mu\overline{\theta}\partial_\mu A(x) + \frac{1}{4}\theta\theta\overline{\theta\theta}\partial^2_\mu A(x) + \sqrt{2}\theta\psi(x) - \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\psi(x)\sigma^\mu\overline{\theta} + \theta\theta F(x)\,,$$

where

$$y = x + i\theta\sigma\overline{\theta}\,.$$

The components of the chiral superfield are the complex scalar field A and the Weyl spinor fields having 2 degrees of freedom. The field F is an non-physical auxiliary field, because the superfield does not contain its derivatives and therefore it can be excluded with the use of the equations of motion. The supersymmetric transformation of the superfield in the component form is

$$\delta_\varepsilon A = \sqrt{2}\varepsilon\psi\,,\ \ \delta_\varepsilon\psi = i\sqrt{2}\sigma^\mu\overline{\varepsilon}\partial_\mu A + \sqrt{2}\varepsilon F\,,\ \ \delta_\varepsilon F = i\sqrt{2}\overline{\varepsilon}\sigma^\mu\partial_\mu\psi\,.$$

The variation of F is the total derivative vanished providing we shall integrate F over x.

The antichiral superfield Φ^+ can be constructed by imposing the constraint

$$D\,\Phi = 0\,,\ \ D_\alpha = -\frac{\partial}{\partial\theta_\alpha} + i\sigma^\mu_{\alpha\beta}\overline{\theta}\partial_\mu\,.$$

The powers Φ^n and $\overline{\Phi}^n$ are chiral and antichiral fields and $\Phi^+\Phi$ is a general superfield. Therefore we can construct the simplest renormalized supersummetric Lagrangian as

$$L_\Phi = \Phi^+_i\Phi_i|_{\theta\theta\overline{\theta\theta}} + \left(\left(\lambda_i\Phi_i + \frac{m_{ij}}{2}\Phi_i\Phi_j + \frac{f_{ijk}}{3}\Phi_i\Phi_j\Phi_k\right)|_{\overline{\theta}\theta} + h.c.\right)\,,$$

where $|_{\theta\theta\overline{\theta\theta}}$ and $|_{\overline{\theta}\theta}$ means the extraction of the corresponding term in the Taylor expansion of the previous expression. The action $S_\Phi = \int d^4xL_\Phi$ is invariant under the supersymmetric transformation because the super-variation of the integrand is proportional to the derivative ∂_μ. This action gives the supersymmetric generalization of the Higgs self-interaction in the Standard Model.

To construct the generalization of gauge terms of the Standard Model we need to use the vector superfield $V = V^+$.which is the general superfield

$$V(x,\theta,\overline{\theta}) = C(x) + i\theta\chi(x) - i\overline{\theta}\chi(x) + \frac{i}{2}\theta\theta\left(M(x) + iN(x)\right) - \frac{i}{2}\overline{\theta\theta}\left(M(x) - iN(x)\right)$$

$$-\theta\sigma^\mu\overline{\theta}v_\mu(x) + i\theta\theta\overline{\theta}\left(\lambda(x) + \frac{i}{2}\sigma^\mu\partial_\mu\chi(x)\right) - i\overline{\theta\theta}\theta\left(\overline{\lambda}(x) + \frac{i}{2}\sigma^\mu\partial_\mu\overline{\chi}(x)\right) + \frac{1}{2}\theta\theta\overline{\theta\theta}\left(D(x) + \frac{1}{2}\partial^2_\mu C(x)\right)\,.$$

For the real vector superfield V the physical degrees of freedom are the vector gauge field v_μ and the Majorana spinor field λ. All other fields χ, C, M, N, D are non-physical and can be eliminated with the use of equations of motion and by the gauge transformation

$$V \to V + \Phi + \Phi^+,$$

where the gauge parameters Φ and Φ^+ are chiral and antichiral fields respectively. In the Wess-Zumino gauge $\chi = C = M, = N = 0$.

We can introduce the field strength tensor W_α (similar to $F_{\mu\nu}$ in the gauge theories)

$$W_\alpha = -\frac{1}{4}\overline{D}^2 e^V D_\alpha e^{-V}\, , \, \overline{W}_\alpha = -\frac{1}{4} D^2 e^V \overline{D}_\alpha e^{-V}\, ,$$

being a polynomial in the Wess-Zumino gauge

$$W_\alpha = T^a \left(-i\lambda^a_\alpha + \theta_\alpha D^a - \frac{i}{2} \left(\sigma^\mu \overline{\sigma}^\nu \theta\right)_\alpha F^a_{\mu\nu} + \theta\theta\sigma^\mu D_\mu \overline{\lambda}^a \right)\, ,$$

where for the non-Abelian theory

$$F^a_{\mu\nu} = \partial_\mu v^a_\nu - \partial_\nu v^a_\mu + f^{abc} v^b_\mu v^c_\nu\, , \; D_\mu \overline{\lambda}^a = \partial_\mu \overline{\lambda}^a + f^{abc} v^b_\mu v^c_\nu\, .$$

Note, that W_α is a chiral field.

In the Abelian case we obtain more simple formulas

$$W_\alpha = \frac{1}{4}\overline{D}^2 D_\alpha V\, , \, \overline{W}_\alpha = \frac{1}{4} D^2 \overline{D}_\alpha V\, .$$

The gauge Lagrangian has the form

$$L_g = \frac{1}{4}\int d^2\theta\, W^\alpha W_\alpha + \frac{1}{4}\int d^2\overline{\theta} \overline{W}^\alpha \overline{W}_\alpha\, .$$

In the Wess-Zumino gauge we obtain

$$L_g = \frac{1}{2} D^2 = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} - i\lambda\sigma^\mu D_\mu \overline{\lambda}\, ..$$

Taking such Lagrangians for the gauge group $SU(2)$ and for the Abelian theory we obtain the SUSY generalization of the kinetic terms of the electroweak theory.

The chiral superfields for the matter have the following gauge transformation

$$\Phi \to e^{-ig\Lambda}\Phi\, , \; \Phi^+ \to \Phi^+ e^{ig\Lambda}\, .$$

Therefore the gauge-invariant kinetic terms in the Lagrangian for the matter field should have the form

$$L_{kt} = \Phi^+_i e^{gV} \Phi_i \big|_{\theta\theta\overline{\theta}\overline{\theta}}\, .$$

In such manner we can construct the SUSY generalization of all terms of the Standard Model.