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Chirally Symmetric Phase of Supersymmetric  
Gluodynamics

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**Abstract**

We argue that supersymmetric gluodynamics (theory of gluons and gluinos) has a condensate-free phase. Unlike the standard phase, the discrete axial symmetry of the Lagrangian is unbroken in this phase, and the gluino condensate does not develop. Extra unconventional vacua are supersymmetric and are characterized by the presence of (bosonic and fermionic) massless bound states. A set of arguments in favor of the conjecture includes: (i) analysis of the effective Lagrangian of the Veneziano-Yankielowicz type which we amend to properly incorporate all symmetries of the model; (ii) consideration of an unsolved problem with the Witten index; (iii) interpretation of a mismatch between the strong-coupling and weak coupling instanton calculations of the gluino condensate detected previously. Impact on Seiberg's results is briefly discussed.

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# 1 Introduction

In this work, supersymmetric gluodynamics, theory of gluons and gluinos with no matter, is revisited. Our primary task is investigating the modes of realization of the discrete chiral invariance in supersymmetric gluodynamics. As is well-known from the early days of supersymmetry, this theory possesses a discrete symmetry  $Z_{2T(G)}$  where  $T(G)$  is (one half) of the Dynkin index for the given gauge group  $G$ . In  $SU(N)$  supersymmetric gluodynamics  $T(G) = N$ . Since the  $Z_{2T(G)}$  invariance is the (non-anomalous) remnant of the anomalous axial symmetry generated by the phase rotations of the gluino field, the gluino condensate  $\langle\lambda\lambda\rangle$  is the order parameter<sup>1</sup>. Usually it is believed that a non-vanishing gluino condensate develops, spontaneously breaking  $Z_{2T(G)}$  down to  $Z_2$ . Then the space of vacua consists of  $T(G)$  points. All these vacua are physically equivalent and correspond to confining dynamics qualitatively similar to that of non-supersymmetric gluodynamics. In particular, a mass gap develops, and massless excitations present at the Lagrangian level disappear from the physical spectrum.

The picture seems perfectly self-consistent; yet, two unsolved issues have clouded it for over a decade. First, the number of vacua,  $T(G)$ , does not match the value of the Witten index [1] for orthogonal and exceptional groups. (Say, for the  $O(N)$  groups with even  $N$  the index is predicted [1] to be  $(N/2) + 1$  while  $T(G) = N - 2$ ). Second, the value of the gluino condensate, calculated in the weak-coupling regime and analytically continued to the strong-coupling regime by using holomorphy [2] does not match  $\langle\lambda\lambda\rangle$  calculated directly in the strong-coupling regime [3]. More exactly, the direct calculation refers to

$$\langle\lambda\lambda(x)\lambda\lambda(0)\rangle, \quad (G = SU(2)) \quad (1)$$

and is carried out *via* instantons, plus cluster decomposition [4]. The weak-coupling regime is achieved by adding extra matter fields and working in the Higgs phase, with the subsequent limit  $m \rightarrow \infty$  where  $m$  is the matter mass term.

In this work we suggest a somewhat unexpected solution which seems to eliminate both difficulties. We will argue that extra vacuum states, with unbroken  $Z_{2T(G)}$  symmetry and vanishing gluino condensate, exist. The gauge dynamics in this unbroken phase is very peculiar. In particular, although no symmetry is spontaneously broken, it should contain massless excitations of both bosonic and fermionic type.

The above conclusion is based on two sets of arguments. First, the existence of extra vacuum states follows from the analysis of the so-called Veneziano-Yankielowicz (VY) effective Lagrangian [5, 6]. A technical problem one immediately encounters is the absence of  $Z_{2T(G)}$  degeneracy in the original VY expression. We show that this expression is incomplete, and explain how it must be amended to become compatible with all symmetries of supersymmetric gluodynamics. The corrected expression

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<sup>1</sup>The gluino field is treated in the Weyl representation; note that the condensate under discussion is  $\lambda^2$  rather than  $\bar{\lambda}\lambda$ .

exhibits  $T(G)$  minima of the scalar potential corresponding to  $Z_{2T(G)} \rightarrow Z_2$  breaking, plus an additional minimum at the origin where the gluino condensate vanishes. We then discuss the occurrence of this extra state in relation with the Witten-index problem. The  $\langle \lambda\lambda \rangle = 0$  state presumably does not contribute to the Witten index counting for the unitary gauge groups, since it is accompanied by a “fermion” zero energy state. It may contribute, however, in the case of the orthogonal groups.

Finally, a mismatch between the direct instanton calculation of the gluino condensate and an indirect derivation through the Higgs phase is interpreted as a signature of the  $\langle \lambda\lambda \rangle = 0$  vacuum contribution in the instanton calculation. A few remarks concerning infrared dynamics in the  $\langle \lambda\lambda \rangle = 0$  vacuum and the possible impact of the inclusion of light matter conclude the paper.

## 2 Veneziano-Yankielowicz Effective Lagrangian

In this section we discuss effective Lagrangians and the manifold of vacua in supersymmetric Yang-Mills theory without matter<sup>2</sup>. The Lagrangian of the model at the fundamental level is

$$\mathcal{L} = \frac{1}{g_0^2} \left[ -\frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a + i \lambda_\alpha^\dagger D^{\dot{\alpha}\beta} \lambda_\beta \right] + \frac{\vartheta}{32\pi^2} G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a, \quad (2)$$

where it is assumed, for simplicity, that the gauge group  $G$  is  $SU(N)$ . This model possesses a discrete global  $Z_{2N}$  symmetry, a residual non-anomalous subgroup of the anomalous chiral  $U(1)$ .

One of the aspects of our consideration is based on the effective Lagrangian approach. Some of the symmetries present in the theory (2) at the classical level are anomalous. It was suggested long ago that simple Lagrangians for some effective fields can summarize all information on the anomalous Ward identities.

Thus, in pure (non-supersymmetric) Yang-Mills theory the trace of the energy-momentum tensor  $\theta_{\mu\nu}$  has anomaly. Correspondingly, one can write a Lagrangian for the dilaton field (interpolating the operator of the trace of the energy-momentum tensor) which codes all  $n$ -point functions implied by this anomaly [7]. In supersymmetric gluodynamics the anomalous operators are  $\theta_\mu^\mu$ ,  $\gamma^\mu S_\mu$  and  $\partial_\mu J^\mu$  where  $S_\mu$  is the supercurrent and  $J^\mu$  is the gluino current. They form a supermultiplet. The Lagrangian realizing the anomalous Ward identities can be naturally constructed [5] in terms of the chiral superfield

$$S = \frac{3}{32\pi^2} W^2 \equiv \frac{3}{32\pi^2} \text{Tr } W^2 \quad (3)$$

where

$$W_\alpha(x_L, \theta) \equiv \frac{1}{8} \bar{D}^2 \left( e^{-V} D_\alpha e^V \right),$$

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<sup>2</sup>This theory is referred to as supersymmetric gluodynamics. The theory where the light matter fields in the fundamental representation are included will be referred to as SUSY QCD.

and the color trace above is in the fundamental representation. The lowest component of the superfield  $-W^2$  is  $\lambda\lambda$  while the  $F$  component is nothing but the original SUSY Yang-Mills Lagrangian,  $G^2 + iG\tilde{G} + i\lambda_\alpha^\dagger D^{\dot{\alpha}\beta}\lambda_\beta$ . The construction was carried out <sup>3</sup> in Ref. [5] (see also [6]); the corresponding Lagrangian is

$$\mathcal{L} = \left(\bar{S}S\right)^{1/3}\Big|_D + \left(\frac{1}{3}S\ln(S^N/\sigma^N)\right)\Big|_F + \text{h.c.} \ , \quad (4)$$

where  $\sigma$  is a numerical parameter,

$$\sigma = e\Lambda^3 e^{i\vartheta/N} \ ,$$

$\Lambda$  is the scale parameter, a positive number of dimension of mass which we will set equal to unity in the following. Finally,  $\vartheta$  is the vacuum angle. Other numerical constants irrelevant for our purposes are set equal to unity.

The derivation of Eq. (4) is pretty straightforward. The kinetic term is obviously invariant under the scale transformations and the  $R$  rotations,

$$S \rightarrow Se^{2i\beta} \ , \ \theta \rightarrow \theta e^{i\beta} \ . \quad (5)$$

The potential term is not invariant, however. For instance, under the  $R$  rotations,

$$\delta\mathcal{L} \propto \beta \left( \int d^2\theta S - \int d^2\bar{\theta} \bar{S} \right) \ , \quad (6)$$

which is exactly the anomalous Ward identity for the chiral rotations. All other anomalies are then automatically reproduced because of the supersymmetry of the VY Lagrangian.

After an appropriate rescaling, making the kinetic term canonical, one gets

$$\mathcal{L} = \left(\bar{\Phi}\Phi\right)\Big|_D + \left(\Phi^3\ln\Phi^N\right)\Big|_F + \text{h.c.} \ , \ S = \Phi^3 \ . \quad (7)$$

Up to a redefinition of the superfield this seems to be the only Lagrangian which faithfully represents the anomalous Ward identities.

This topic was in a dormant state for over a decade. The interest to this approach was revived recently in connection with the so called “integrating in” procedure in supersymmetric gauge theories with matter (see e.g. Ref. [8]).

A remark is in order here to explain in what sense Eq. (4) is an effective Lagrangian. Clearly it is not a genuine low-energy effective Lagrangian in the Wilsonian sense. There are no Goldstone bosons corresponding to the anomalous symmetries, and apart from  $W^2$ , other fields in the theory may interpolate particles with masses of the same order of magnitude as the ones retained in Eq. (4). This Lagrangian, therefore, does not arise after integrating out of the heavy modes. Rather it is an effective Lagrangian in the sense that it is a generating functional for vertex

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<sup>3</sup>The vacuum angle  $\vartheta$  in these works was set equal to zero, and the  $\vartheta$  dependence was not discussed.

functions of the field components of  $W^2$ . Of course, Eq. (4) should then be understood only as the first two terms in the derivative expansion. The higher derivative corrections to this expression should be generically large, and therefore Eq. (4) can not be expected to give a reasonable approximation to the (on shell) particle interaction vertices. The effective potential part of this Lagrangian (namely the Lagrangian evaluated on constant field configurations), however, should be exact since it is determined unambiguously by all anomalous Ward identities of the theory (it includes all relevant  $n$ -point functions evaluated at zero momenta). Therefore, the Lagrangian is suitable for examining the vacuum states of the theory.

In fact, the last remark requires some qualification. Although the Lagrangian (4) has some appealing features, even a brief examination shows that it can not be complete. First, the scalar potential following from Eq. (4) is not a single-valued function of the field. If we start, say, at  $S = \Lambda^3$  and travel continuously in the complex  $S$  plane, the value of the scalar potential at  $S = e^{2\pi i} \Lambda^3$  will be different from that at  $S = \Lambda^3$ . This is of course unacceptable<sup>4</sup>. Second, the discrete  $Z_{2N}$  symmetry inherent to the original theory (2) is not reflected in (4). These unsatisfactory features were pointed out, e.g. in Ref. [9].

Our task here is to provide a natural modification to this Lagrangian, which will cure these two problems, while preserving the correct transformation properties under the anomalous symmetries.

The key observation is as follows. Since  $S$  is supposed to be equivalent to  $W^2$ , it must satisfy a global constraint ensuring that the integral  $(1/32\pi^2) \int d^4x G\tilde{G}$  (in the Euclidean space) can only take integer values. This constraint can be imposed in an explicitly supersymmetric manner at the Lagrangian level by introducing an integer-valued Lagrange multiplier variable  $n$

$$\mathcal{L} = \left( \bar{S}S \right)^{1/3} \Big|_D + \left( \frac{1}{3} S \ln(S^N/\sigma^N) \Big|_F + \text{h.c.} \right) + \frac{2\pi i n}{3} (S - \bar{S}) \Big|_F. \quad (8)$$

Note that the variable  $n$  takes only integer values and is not a local field. It does not depend on the space-time coordinates and, therefore, integration over it<sup>5</sup> imposes only a global constraint on the topological charge. It is easy to see that (after the Euclidean rotation) the constraint does indeed take the form

$$\frac{1}{32\pi^2} \int d^4x G\tilde{G} = Z.$$

Alternatively, one can say that, in calculating the correlation functions through the functional integral with the action (4), one must sum over all branches of the logarithm. It is perfectly clear that all anomalous Ward identities are kept intact.

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<sup>4</sup>Below the lowest component of  $S$  is denoted by  $\phi$ . Sometimes, when no confusion can arise, we will still use the letter  $S$  for the lowest component of superfield.

<sup>5</sup>More exactly, one must sum over  $n$  in the partition function. Similar integer-valued Lagrange multiplier appears in the bosonized version of the Schwinger model [10].

Moreover, this prescription naturally restores the equivalence of all branches of the logarithm lost in the original construction [5]. The modification suggested is crucial.

The extra term we have added to the Lagrangian is clearly supersymmetric and is also invariant under all global symmetries of the original theory. Now both the single-valuedness of the potential and the  $Z_N$  invariance are restored. The chiral phase rotation by the angle  $2\pi k/N$  with integer  $k$  just leads to the shift of  $n$  by  $k$  units. Since  $n$  is summed over in the functional integral, the resulting Lagrangian for  $S$  is indeed  $Z_N$  invariant <sup>6</sup>.

Information we are interested in is contained in the scalar potential that follows from the effective Lagrangian (8), since it is the minima of the scalar potential that determine the vacua of the theory. It is instructive to see how the change we propose is reflected in the scalar potential. The chiral superfield  $S$  for the purpose of calculating the effective potential can be written as

$$S = \phi + \theta^2 \frac{1}{\sqrt{2}}(A + iB). \quad (9)$$

For the spatially-constant fields the Euclidean action takes the form

$$\mathcal{A}_E = \left\{ -\frac{1}{9}(\phi\phi^*)^{-2/3} \frac{1}{2}(A^2 - B^2) - \frac{N}{3}\sqrt{2}A \ln |\phi| + \frac{iN}{3}\sqrt{2}B\alpha \right\} V \quad (10)$$

where  $\alpha = \text{Arg}\phi$  and the quantization condition enforced by the summation over  $n$  in Eq. (8) is

$$\frac{\sqrt{2}}{3}BV = b, \quad b = \text{integer}. \quad (11)$$

Here  $V$  is the full space-time volume and we have set the vacuum angle  $\vartheta = 0$  for the time being. If the quantization condition is ignored, elimination of the auxiliary fields  $A$  and  $B$  leads to the original VY scalar potential

$$U(\phi) = N^2(\phi^*\phi)^{2/3} \ln \phi \ln \phi^* \equiv N^2(\phi^*\phi)^{2/3} (\ln^2 |\phi| + \alpha^2). \quad (12)$$

As was mentioned, the result is neither single-valued nor has it correct periodicity in  $\alpha$ . To calculate the corrected effective potential we have to take into account the quantization condition (11). The variable  $A$  is unconstrained and can be integrated over in the usual way, and for the variable  $B$  integration should be substituted by summation over integers,

$$\int dB \rightarrow \frac{3}{\sqrt{2}}V^{-1} \sum_{b=0,\pm 1,\dots}.$$

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<sup>6</sup>The explicit invariance here is  $Z_N$  rather than the complete  $Z_{2N}$  of the original SUSY gluodynamics, since we have chosen to write our effective Lagrangian for the superfield which is invariant under  $\lambda \rightarrow -\lambda$ .

After the field  $A$  is eliminated, as before, we obtain the following expression for the effective potential

$$U(\phi) = -V^{-1} \ln \left[ \sum_{b=0, \pm 1, \dots} \exp \left\{ -VN^2(\phi^* \phi)^{2/3} \ln^2 |\phi| - \frac{1}{4}(\phi^* \phi)^{-2/3} \frac{b^2}{V} - iN\alpha b \right\} \right] \quad (13)$$

We pause here to discuss some general features of the effective potential (13). If  $b$  could be considered as a continuous variable and the summation over  $b$  could be replaced by integration, we would recover the old potential of Eq. (12). For small phase angles,  $\alpha \ll 1/N$ , this is a valid approximation, since the coefficients of  $b^2$  and  $b$  terms are small, and the exponent is a function of  $b$  which varies very slowly. Therefore, in the vicinity of the real axis, the new effective potential is close to the old one. In fact, if  $\alpha = 0$ , the corrected potential coincides exactly with that of Veneziano and Yankielowicz. It has a minimum at  $\phi = 1$ . However, unlike the Veneziano-Yankielowicz potential,  $Z_N$  invariance of (13) is explicit: all points  $\alpha = 2\pi k/N$  are obviously equivalent, and, in particular, there are  $N$  degenerate minima at

$$\phi = e^{i2\pi \frac{k}{N}}, \quad k = 0, 1, \dots, N-1. \quad (14)$$

This means that away from the line  $\alpha = 0$ , the potential (12) gets corrections.

Expanding the exponent in (13) at small  $\alpha$  it is possible to conclude that the small- $\alpha$  expansion of  $U(|\phi|, \alpha)$  coincides with (12). This means that at small  $\alpha$

$$U(|\phi|, \alpha) \propto [\ln^2 |\phi| + \alpha^2 + \mathcal{O}(\exp(-C/\alpha^2))] .$$

Consider now the derivative of the potential with respect to  $\alpha$  at fixed value  $|\phi|$  and  $\alpha \neq 0$ ,

$$\frac{\partial U}{\partial \alpha} = iNV^{-1} \frac{\sum_{b=0, \pm 1, \dots} b \exp \left\{ -VN^2(\phi^* \phi)^{2/3} \ln^2 |\phi| - \frac{1}{4}(\phi^* \phi)^{-2/3} \frac{b^2}{V} - iN\alpha b \right\}}{\sum_{b=0, \pm 1, \dots} \exp \left\{ -VN^2(\phi^* \phi)^{2/3} \ln^2 |\phi| - \frac{1}{4}(\phi^* \phi)^{-2/3} \frac{b^2}{V} - iN\alpha b \right\}} . \quad (15)$$

This expression has the meaning of the average density of instantons minus the average density of anti-instantons in the Yang-Mills theory on the real axis but with the shifted value of the vacuum angle  $\vartheta = -N\alpha$  (see Eq. (16) below). Clearly, this expression is finite for any value of  $\alpha$ . The effective potential is therefore a continuous function of the field. Consider now the rays  $\alpha = \frac{\pi}{N}k$  where  $k$  is odd. These rays are exactly in the middle between the “valleys”  $\alpha = \frac{2\pi}{N}k$ . For these values of the angle the weight in the sum over  $b$  in the numerator of Eq. (15) is symmetric under  $b \rightarrow -b$ . The average in the numerator of Eq. (15), therefore, vanishes<sup>7</sup>. Thus, along the directions  $\text{Arg} \phi = \frac{\pi}{N}k$  where  $k$  is odd, the effective potential has the topography of a ridge.

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<sup>7</sup>There can be no “spontaneous breaking” of the symmetry  $b \rightarrow -b$  since Eq. (15) is a simple sum over  $b$  rather than a functional integral.

We conclude this section by briefly discussing the  $\vartheta$  dependence. From Eq. (4) it is clear that  $\vartheta$  enters in the scalar potential only through the combination

$$U_{\vartheta}(|\phi|, \alpha) = U(|\phi|, \alpha - \frac{\vartheta}{N}). \quad (16)$$

When  $\vartheta$  continuously varies from 0 to  $2\pi$ , the “mountain ridge” picture rotates by  $2\pi/N$ : the first valley becomes the second, and so on, cyclically. Such a picture was predicted from a general consideration [2].

### 3 The Vacuum State without Gluino Condensate

The following feature of the scalar potential (13) is important for our consideration. In addition to  $N$  minima of Eq. (14) it exhibits an unexpected solution at  $\phi = 0$ . To reveal the extra solution it may be convenient to proceed from the superfield  $S$  to the superfield  $\Phi$ , whose kinetic term has the canonical form. These superfields are related,  $S = \Phi^3$ ; the same relation holds for the lowest components,  $\phi = \varphi^3$  where  $\varphi$  is the lowest component of  $\Phi$ . The zero  $\langle \Phi \rangle$  solution corresponds to the vanishing gluino condensate, and its interpretation has been never discussed previously. This zero energy state at  $\langle \Phi \rangle = 0$  reflects a phase of the supersymmetric gluodynamics with no breaking of the  $Z_{2N}$  symmetry and vanishing gluino condensate.

The occurrence of the condensate-free phase may sound suspicious, since superficially this statement contradicts the Witten-index argument [1]. Indeed, the Witten index for the  $SU(N)$  group is  $N$ , which is exactly equal to the number of the  $\langle \lambda\lambda \rangle \neq 0$  states of vanishing energy one obtains from Eq. (14). So, the only way to reconcile the existence of the extra state at  $\phi = 0$  with this result is that it should not contribute to the Witten index.

Surprisingly, it is very difficult to rule out this possibility, and this may indeed be the case. For this to happen there must exist an equal number of  $F = \text{even}$  and  $F = \text{odd}$  states at  $\Phi = 0$ . The Lagrangian (8) does imply the existence of the massless fermion mode in the condensate-free regime. Usually, in the Wess-Zumino type models one can always introduce the mass term to the chiral superfield considered, all massless modes are eliminated, and the zero-energy states concentrated near the zeros of the superpotential are all of the bosonic type. In which case, they certainly contribute to the Witten index. That is not true for the effective Lagrangian (8). Here its form is rigid, since it merely reflects the anomalous Ward identities as well as the discrete non-anomalous symmetries of the underlying theory. The mass term is forbidden – it would explicitly violate the Ward identities, and the vacuum structure obtained in this way will have nothing to do with that of the underlying SUSY gluodynamics. For instance, it would break explicitly the  $Z_{2N}$  symmetry and eliminate all  $Z_{2N}$  breaking vacua in Eq. (14).

If the excitation modes are strictly massless, in general it is very difficult to decide which state is  $F$ -even and which is  $F$ -odd in the case of the unbroken supersymmetry, when the supercharge acts trivially on the vacuum. Therefore on the basis of the



effective potential alone we are unable to determine what is the contribution of the  $\phi = 0$  states to the Witten index. We will argue later that, in fact, the  $(-1)^F$  counting of the zero-energy states may depend strongly on the nature of higher derivative terms in the effective action, which we have neglected so far and which are not determined by the anomalous Ward identities.

Having argued that the problem with the Witten index need not be an obstruction, let us present now a positive, although subtle, argument in favor of the existence of the additional  $\langle \lambda\lambda \rangle = 0$  vacuum state in SUSY gluodynamics. To this end we need to make a digression and recall a puzzle with the dynamical calculation of the gluino condensate.

Calculation of the gluino condensate [4] was the first application of instantons in supersymmetric gluodynamics in the strong coupling regime. Consider for simplicity the  $SU(2)$  gluodynamics. In this case there are four gluino zero modes in the instanton field and, hence, there is no direct instanton contribution to the gluino condensate  $\langle \lambda\lambda \rangle$ . At the same time the instanton does contribute to the correlation function

$$\langle \lambda_\alpha^a(x) \lambda^{a\alpha}(x), \lambda_\beta^b(0) \lambda^{b\beta}(0) \rangle, \quad (17)$$

Here  $a, b = 1, 2, 3$  are the color indices and  $\alpha, \beta = 1, 2$  are the spinor ones. An explicit instanton calculation [4] shows that the correlation function (17) is equal to a non-vanishing constant.

At first sight this result might seem like supersymmetry-breaking since the instanton does not generate any boson analog of Eq. (17). Supersymmetry does not forbid, however, a non-vanishing result for Eq. (17) provided that this two-point function is actually an  $x$  independent constant.

Three elements are crucial for the proof of the above assertion: (i) the supercharge  $\bar{Q}^{\dot{\beta}}$  acting on the vacuum state annihilates it; (ii)  $\bar{Q}^{\dot{\beta}}$  commutes with  $\lambda\lambda$ ; (iii) the derivative  $\partial_{\alpha\dot{\beta}}(\lambda\lambda)$  is representable as the anticommutator of  $\bar{Q}^{\dot{\beta}}$  and  $\lambda^\beta G_{\beta\alpha}$ . The second and the third point follow from the fact that  $\lambda\lambda$  is the lowest component of the chiral superfield  $W^2$ , while  $\lambda^\beta G_{\beta\alpha}$  is its middle component.

Now, we differentiate Eq. (17), substitute  $\partial_{\alpha\dot{\beta}}(\lambda\lambda)$  by  $\{\bar{Q}^{\dot{\beta}}, \lambda^\beta G_{\beta\alpha}\}$  and obtain zero. Thus, supersymmetry requires the  $x$  derivative of (17) to vanish [4]. This is exactly what happens if the correlator (17) is a constant.

If so, one can compute the result at short distances where it is presumably saturated by small-size instantons, and, then, the very same constant is predicted at large distances,  $x \rightarrow \infty$ . On the other hand, due to the cluster decomposition property, which must be valid in any reasonable theory, the correlation function (17) at  $x \rightarrow \infty$  reduces to  $\langle \lambda\lambda \rangle^2$ . Extracting the square root we arrive at a (double-valued) prediction for the gluino condensate.

Instead of exploiting strong coupling calculations (which is always suspect), one can use a fully controllable approach in the weak coupling regime [2]. Namely, one extends the theory by adding one flavor (two chiral matter superfields, doublets with respect to the gauge  $SU(2)$ ), carries out the calculation in the weakly coupled

Higgs phase (i.e. assuming a large value of the Higgs field), and then returns back to SUSY gluodynamics by exploiting the holomorphy of the condensate in the mass parameter [2].

The result for  $\langle\lambda\lambda\rangle$  obtained in the strong coupling regime (i.e. by following the program outlined after Eq. (17)) does *not* match  $\langle\lambda\lambda\rangle$  calculated in this indirect but fully controllable way. As a matter of fact, as was shown in Ref. [3],

$$\langle\lambda\lambda\rangle_{\text{scr}}^2 = \frac{4}{5}\langle\lambda\lambda\rangle_{\text{wcr}}^2, \quad (18)$$

where the subscripts scr and wcr mark the strong and weak coupling regime calculations. Since the weak coupling regime calculation seems to be flawless, suspicion naturally falls on the strong coupling analysis. But where concretely is the loophole?

A tentative answer might be found in the hypothesis put forward by Amati *et al.* [11]. It was assumed that, instead of providing us with the expectation value of  $\lambda\lambda$  in the given vacuum, instantons in the strong coupling regime yield an average value of  $\langle\lambda\lambda\rangle$  in all possible vacuum states. In the weak coupling regime, we have a marker: a large classical VEV of the Higgs field tells us in what particular vacuum we do our instanton calculation. In the strong coupling regime, such a marker is absent.

This hypothesis by itself, however, does not explain the discrepancy (18), if there are only two vacua characterized by  $\langle\lambda\lambda\rangle = \pm\Lambda^3$ . The gluino condensate is not affected by the averaging over these two vacuum states, since the contributions of these two vacua to Eq. (17) are equal. If, however, there exist extra zero-energy states with  $\langle\lambda\lambda\rangle = 0$  which are involved in the averaging, the final result in the strong coupling regime is naturally different from that obtained in the weak coupling regime in the *given* vacuum. Moreover, the value of the condensate calculated in the strong coupling approach should be smaller, consistently with Eq. (18).

Note that the approach of Ref. [2] has nothing to say about possible states at the origin. If there is a minimum at the vanishing expectation value of the Higgs field (and vanishing gluino condensate) the theory is always in the strong coupling regime near this minimum. Introduction of the matter fields does not help to push the theory in the weak coupling regime.

Now it is in order to return to the Witten-index argument, in a bid to use it in a positive aspect. The existence of extra vacua can potentially resolve another long-standing paradox in SUSY gauge theories. It has been known for a long time that in  $SO(M)$  SUSY gauge theories the Witten index does not coincide with the number of different  $\langle\lambda\lambda\rangle \neq 0$  states of vanishing energy one obtains from the instanton calculations in the weak coupling regime [2]. The former is rank +1 while the latter is  $M - 2$  for the orthogonal groups. The anomalous and non-anomalous symmetry structure of these models is similar to that of  $SU(N)$  gauge theories. The effective potential would, therefore, have the same form as Eq. (13) with the parameter  $N$  substituted by  $M - 2$ . It would, therefore, still exhibit a minimum at  $\phi = 0$ . It is possible that for orthogonal groups the number of fermionic states at this point is

larger than the number of bosonic ones, and the difference precisely makes up for the difference between the Witten index (rank +1) and the number of  $Z_{M-2}$ -breaking bosonic minima with the non-vanishing gluino condensate.

## 4 Dynamical Consequences

In this section we present some speculations as to the nature of the  $Z_{2N}$ -symmetric vacuum. First, the energy of this state vanishes and therefore supersymmetry is unbroken. Second, the effective Lagrangian indicates the existence of massless fermions. This, as noted before, is a necessary condition to avoid the contradiction with the Witten-index calculation. The scalar field  $\varphi$  also is massless near this point, and obviously is the superpartner of the massless fermion. The existence of strictly massless particles in this phase of supersymmetric gluodynamics is remarkable since they are not Goldstone modes associated with the spontaneous breaking of some symmetry. As a matter of fact, no symmetry is spontaneously broken, and still the massless modes are present. The situation is somewhat reminiscent of conformally invariant phases of some other SUSY gauge theories (with matter) which figure so prominently in recent work on electric-magnetic duality [12]. In fact the similarity may well be even closer.

Taken at its face value, the effective potential (13) would lead one to conclude that the scalar quartic self-coupling near  $\varphi = 0$  diverges logarithmically at zero momentum. However, as pointed out earlier, one can not use this effective potential to reliably establish interactions between the particles. This is especially true near the point  $\varphi = 0$ . The reason is obvious. For non-constant fields one has to take into account higher derivative terms in the effective action. The anomalous Ward identities, while unable to determine these higher derivative terms completely, do impose some restrictions on their form. In particular, to preserve the Ward identity following from the dilatational symmetry, every extra derivative should be accompanied roughly by the factor  $(\bar{S}S)^{-1/6}$ . Example of this type of terms is

$$\frac{(\partial_\mu \bar{S}^{1/3} \partial_\mu S^{1/3})}{(\bar{S}S)^{1/3}}. \quad (19)$$

Near the point  $\phi = 0$ , starting already at this low order in derivatives, the corrections explode. The derivative expansion is not valid even for very low momenta. This suggests that the Green's functions of the fields  $S$  and  $\bar{S}$  will have non-analytic behavior in momentum. With the knowledge of the existence of massless excitations, we are lead to conjecture that at  $\varphi = 0$  at low momenta, the theory is conformal.

The precise nature of this conformal theory (anomalous dimensions, etc.) can not be determined on the basis of the effective potential alone. It is natural to expect that counting of the number of fermionic and bosonic vacua at zero would depend heavily on the properties of the conformal theory. For example, for  $SO(N)$  and  $SU(N-2)$  gauge theories, the effective potentials would be the same, but the

conformal theories at zero could be completely different. That could explain the difference in the number of the fermionic and bosonic vacua in the two cases needed to comply with the Witten-index calculation.

If the chirally invariant phase of pure gluodynamics does indeed exist, it has drastic consequences for supersymmetric theories with massless (or light) matter. Indeed, at  $N_f < N_c - 1$ , the Affleck-Dine-Seiberg (ADS) superpotential [13] (see also Ref. [6];  $M_i^j$  is an  $N_f \times N_f$  moduli matrix)

$$\mathcal{W} \propto \left( \frac{\Lambda^{3N_c - N_f}}{\det M} \right)^{\frac{1}{N_c - N_f}} \quad (20)$$

is generated through the gluino condensation in the unbroken strongly coupled  $SU(N_c - N_f)$  gluodynamics. If the latter has the condensate-free phase, the original SUSY QCD will have a branch with no superpotential generated. Previously, a similar phenomenon (an additional branch with no superpotential) was observed [14] in the  $SO(N)$  theories with  $N_f = N - 4$ . For  $M_i^j \neq 0$  the gauge symmetry is broken down to  $SU(2) \times SU(2)$ . It was noted [14] that the gaugino condensates corresponding to different  $SU(2)$ 's can have opposite signs, so that the sum vanishes, implying vanishing superpotential.

The existence of two inequivalent branches can be argued from a different perspective. Assume we introduce the mass term

$$\Delta \mathcal{L}_{\text{tree}} = m \text{Tr } M|_F \quad (21)$$

to the superpotential of the theory. For simplicity, the mass parameters for all flavors are equal. Then the matter mass term breaks  $U(1)_R$  explicitly, but the discrete  $Z_{2N}$  remains unbroken at the Lagrangian level. When  $m$  is very large, we return back to supersymmetric gluodynamics, with the solutions <sup>8</sup>

$$\langle \lambda \lambda \rangle \sim m^{N_f/N_c} \Lambda^{(3N_c - N_f)/N_f}, \quad \langle M_i^j \rangle \sim \delta_i^j m^{(N_f - N_c)/N_c} \Lambda^{(3N_c - N_f)/N_f}, \quad (22)$$

for the first branch and

$$\langle \lambda \lambda \rangle = 0, \quad \langle M_i^j \rangle = 0, \quad (23)$$

for the second. The functional  $m$  dependence is actually known exactly, for all  $m$  [2] (see also [16]), due to its holomorphic nature. This allows one to analytically continue the results to small  $m$ . Two solutions indicated in Eqs. (22) and (23) correspond to two different branches of SUSY QCD.

In the massless limit, i.e.  $m \rightarrow 0$ , at the origin ( $\langle \lambda \lambda \rangle = M_i^j = 0$ ), the theory possesses an unbroken chiral symmetry, the  $R$  symmetry. Therefore, the anomalous AVV triangles must be matched – they must be the same at the fundamental and composite levels [15]. The matching implies the existence of a rich spectrum of

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<sup>8</sup>Note that at large  $m$  the combination  $m^{N_f/N_c} \Lambda^{(3N_c - N_f)/N_f}$  is a proper scale parameter of supersymmetric gluodynamics.

massless baryons, which goes far beyond one massless fermion residing in the superfield  $W^2$  in the condensate-free phase of supersymmetric gluodynamics. Another possibility is that the  $R$ -symmetry is spontaneously broken, implying the existence of the massless “ $\eta$ ” and its fermion superpartner. The problem with the latter scenario is that no appropriate order parameter for the  $R$  symmetry breaking can be immediately found, since the obvious candidates,  $\langle\lambda\lambda\rangle$  and  $M_i^j$ , vanish. It is possible, that a non-chiral field condenses, for instance the lowest component of  $\overline{W}^2 M_i^j$ . This expectation value would break both, the  $R$  symmetry and the axial  $SU(N_f)$ .

If  $N_f = N_c - 1$ , the ADS superpotential is generated by instantons. If  $\det M \neq 0$  one can carry out calculation of the superpotential in the weak coupling regime, where the result is unambiguous (and coincides, of course, with that of Affleck *et al.*). In this way one arrives at the standard picture of a run-away vacuum, with the zero energy state at  $\det M = \infty$ . The superpotential, however, is not defined at  $\det M = 0$ . The condensate-free phase of supersymmetric gluodynamics implies a supersymmetric vacuum solution of SUSY QCD at  $\det M = 0$ .

Finally, at  $N_f = N_c$  the superpotential is not generated. A continuous manifold of degenerate vacua exists [17],

$$\det M - B\tilde{B} = \Lambda^{2N}. \quad (24)$$

At large  $\det M$  the gauge symmetry is completely broken, the theory is in the weak coupling regime, the instanton calculation is unambiguous, and one can explicitly check, by doing the instanton calculation, that the quantum moduli space is indeed described by Eq. (24). We suggest that there is an extra point in the vacuum manifold, characterized by

$$\det M = 0, \quad B = \tilde{B} = 0, \quad (25)$$

that cannot be continuously reached from the weak coupling regime.

To illustrate this, we again switch on the matter mass term (21), with the intention of continuing analytically from the limit of the large mass (pure gluodynamics) to the limit of the zero mass (massless SUSY QCD). The holomorphy in mass implies in this theory that the gluino condensate is proportional to  $m$  while  $M_i^j$  and  $B, \tilde{B}$  are proportional to  $m^0$ . As a result, the conventional Seiberg solution corresponds to the point  $B = \tilde{B} = 0$  and  $\det M = \Lambda^{2N}$  on the manifold (24). If a branch of gluodynamics with the vanishing gluino condensate exists, then  $B = \tilde{B} = 0$  and  $\det M = 0$  on this branch. Continuing analytically in  $m$  to the massless limit we observe the extra point (25) on the vacuum manifold, disconnected from Seiberg’s solution.

## 5 Conclusions

An elegant picture of supersymmetric gauge dynamics which was gradually evolving in the eighties and took a much more complete form after a recent breakthrough

[17], is still not free from question marks. We suggest an unorthodox solution eliminating all of them, completely. The key element of our consideration is the existence of the condensate-free phase of supersymmetric gluodynamics, with the unbroken  $Z_{2T(G)}$  symmetry. This additional vacuum is supersymmetric, and can be compatible with other known aspects of SUSY gauge dynamics only provided that massless excitation modes, bosonic and fermionic, are present in this vacuum. Corresponding modifications must also take place in the theories with the massless matter (SUSY QCD).

Although our conjecture seems compelling, keeping in mind the non-trivial nature of our consideration, which cannot be tested in the weak coupling regime, it is natural to be cautious. Independent confirmation and a more thorough understanding of the dynamics of unconventional massless bound states characteristic of the condensate-free phase is highly desirable. In particular, it is desirable to learn how to calculate the number of the  $F$ -even and  $F$ -odd zero-energy states in the extra minimum. A simpler task would be to detect a mismatch between the strong-coupling and weak-coupling calculations for gauge groups other than  $SU(2)$ . Studying this mismatch as a function of the group constants may reveal a pattern of “leakage” in the  $Z_{2T(G)}$  unbroken phase.

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