Evaluation of the chiral anomaly in gauge theories with γ_5 couplings

Kazuo Fujikawa
Research Institute for Theoretical Physics, Hiroshima University, Takehara, Hiroshima 725, Japan
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A straightforward application of various regularization schemes in perturbation theory generally leads to physically unacceptable results in the evaluation of the chiral anomaly if the gauge theory contains γ_5 couplings. In gauge theory the anomaly-free properties should be imposed on all the gauge vertices, regardless of whether they are vector or axial-vector, and thus the (nongauge) vector vertex of a triangle diagram could contain an anomaly if the gauge couplings involve γ_5. This is illustrated by the fermion number (vector) current in the Weinberg-Salam theory. It is then shown that the path-integral formalism naturally resolves those complications and gives rise to a unique gauge-invariant result. The Hermiticity of eigenvalue equations in Euclidean gauge theory plays an important role in the path-integral formalism. A path-integral treatment of gauge theory with S and P Higgs couplings is also described.

I. INTRODUCTION

There is a long history in the perturbation evaluation of the chiral (Adler-Bell-Jackiw) anomaly, and a general calculational scheme appears to be well established.[1,2] Recently, however, McKay and Young[3] and also Einhorn and Jones[4] pointed out that a straightforward perturbative evaluation of triangle diagrams on the basis of conventional regularization schemes gives rise to the chiral anomaly which differs from the result of the path-integral derivation[5,6] in the case of gauge theories with γ_5 couplings. These authors took the standard perturbative evaluation of triangle diagrams[1,2] as granted and emphasized the (possible) subtlety in the path-integral method.

In the present note, we show that the origin of the possible discrepancy between the perturbative calculation and the path-integral method should rather be traced to the arbitrariness in perturbation theory. Our basic observation is very simple. In gauge theory, we impose the anomaly-free properties on *all* the gauge vertices, regardless of whether they are vector or axial-vector. Otherwise the gauge theory is inconsistent. As a result, even the (nongauge) vector current could contain an anomaly if the gauge theory involves γ_5 couplings. The naive perturbative evaluation of triangle diagrams, which often preserves the conservation property of vector vertices, thus gives rise to *physically unacceptable* results in these cases.

We illustrate this property by considering the fermion number [U(1)-vector] current in the Weinberg-Salam theory,[7] since this current is known to lead to the physically interesting quark (or lepton) number nonconservation[8] in the presence of instantons.[9] We then present a detailed path-integral treatment of gauge theory with γ_5 couplings, the essence of which was briefly described elsewhere.[6] It is shown that the path-integral method gives rise to a unique gauge-invariant result. It is also pointed out that the modified path-integral prescription suggested by Einhorn and Jones[4] leads to the non-Hermitian operators for eigenvalue equations in Euclidean gauge theory and to a gauge-noninvariant result.

II. FERMION NUMBER CURRENT IN THE WEINBERG-SALAM THEORY

To illustrate the possible arbitrariness in the perturbative evaluation of the chiral anomaly, we consider the anomaly associated with the U(1)-*vector* current in the Lagrangian

$$\mathscr{L}=\bar{\psi}_R i\partial\!\!\!/\psi_R+\bar{\psi}_L iD\!\!\!\!/\,\psi_L-\tfrac{1}{2}\,\mathrm{Tr}F^{\mu\nu}F_{\mu\nu}\ , \tag{1}$$

where ψ_L and ψ_R are the left- and right-handed chiral doublets of fermions, $D\!\!\!\!/\equiv\gamma^\mu(\partial_\mu+igA_\mu)$, and $A_\mu\equiv A_\mu^aT^a$ with T^a the generators of SU(2),

$$[T^a,T^b]=if^{abc}T^c,\ \ \mathrm{Tr}T^aT^b=\tfrac{1}{2}\delta^{ab}\ . \tag{2}$$

After the Wick rotation $x^0\to -ix^4$ and $A_0\to iA_4$ to Euclidean space-times, the operator $D\!\!\!\!/$ becomes a Hermitian operator,

$$\begin{aligned}D\!\!\!\!/&=i\gamma^0D_4+\gamma^kD_k\\ &\equiv\gamma^4D_4+\gamma^kD_k\ ,\end{aligned} \tag{3}$$

where our γ-matrix convention follows that of Bjorken and Drell:[10] γ^0 is Hermitian and γ^k ($k=1,2,3$) are anti-Hermitian. The Hermitian γ_5 matrix is defined by

$$\gamma_5\equiv i\gamma^0\gamma^1\gamma^2\gamma^3=\gamma^4\gamma^1\gamma^2\gamma^3=-\gamma^1\gamma^2\gamma^3\gamma^4\ . \tag{4}$$

After the Wick rotation, the metric becomes $g_{\mu\nu}=(-1,-1,-1,-1)$. The model (1) may be regarded as a prototype of the Weinberg-Salam theory;[7] the possible Higgs couplings do not modify the essential aspect of the following discussions (this point is clarified later). As is well known, the SU(2) gauge couplings of (1) are anomaly-free [see also Eq. (31)] and thus the model (1) is consistent as a gauge theory.[11,6]

The subtlety mentioned above in the chiral anomaly arises when one calculates the triangle diagrams involving the fermion number current [i.e., U(1)-*vector* current] in (1). The correct result is, in the operator language (note that A_μ in Ref. 6 corresponds to $-igA_\mu$ here),

$$\partial_\mu j^\mu(x) = -i\frac{g^2}{16\pi^2}\mathrm{Tr}\,{}^*F^{\mu\nu}F_{\mu\nu}\ , \tag{5}$$

which arises from the two relations

$$\begin{aligned} &\partial_\mu j_L^\mu(x) = -i\frac{g^2}{16\pi^2}\mathrm{Tr}\,{}^*F^{\mu\nu}F_{\mu\nu}\ ,\\ &\partial_\mu j_R^\mu(x) = 0 \end{aligned} \tag{6}$$

with

$$\begin{aligned} &j^\mu(x) \equiv j_L^\mu(x) + j_R^\mu(x)\ ,\\ &j_L^\mu(x) \equiv \bar\psi(x)\gamma^\mu\left[\frac{1-\gamma_5}{2}\right]\psi(x)\ ,\\ &j_R^\mu(x) \equiv \bar\psi(x)\gamma^\mu\left[\frac{1+\gamma_5}{2}\right]\psi(x)\ ,\\ &{}^*F^{\mu\nu} \equiv \tfrac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta},\quad \epsilon^{1234} = \epsilon^{1230} = 1\ . \end{aligned} \tag{7}$$

The results (5) and (6) are implicit in the instanton paper by 't Hooft,[8] and they are obtained in a natural manner by the path-integral method[6] if one adopts the *chiral* basis to deal with two noncommuting operators $[\gamma_5, \not{D}] \neq 0$.

In perturbative calculations, however, one obtains the normal divergence for the vector current

$$\partial_\mu j^\mu(x) = 0 \tag{8}$$

if the dimensional regularization is naively applied. Moreover, one obtains[3]

$$\begin{aligned} &\partial^\mu j_L^\mu(x) = \frac{-i}{3}\left[\frac{g^2}{16\pi^2}\right]\mathrm{Tr}\,{}^*F^{\mu\nu}F_{\mu\nu}\ ,\\ &\partial_\mu j_R^\mu(x) = \frac{i}{3}\left[\frac{g^2}{16\pi^2}\right]\mathrm{Tr}\,{}^*F^{\mu\nu}F_{\mu\nu} \end{aligned} \tag{9}$$

if one constructs $j_L^\mu(x)$ and $j_R^\mu(x)$ from the naive evaluation of (AVV) and (AAA) *triangle* diagrams, namely, if one adjusts the values of box diagrams to those of independently evaluated triangle diagrams so that the final result is non-Abelian gauge invariant. It should be emphasized that this matching between the triangle and box diagrams becomes highly nontrivial if one allows the anomaly for axial-vector *gauge* vertices (and thus breaks the local gauge invariance), as is the case in the present examples (8) and (9).

In perturbative calculations, one may of course adopt regularization schemes other than the dimensional one. One may, for example, use the point-splitting method which allows one to treat vector and axial-vector vertices symmetrically.[2] One then obtains[3] (again by adjusting the box diagrams suitably) instead of (8) and (9) (in the operator language)

$$\begin{aligned} &\partial_\mu j^\mu(x) = \frac{-i}{3}\left[\frac{g^2}{16\pi^2}\right]\mathrm{Tr}\,{}^*F^{\mu\nu}F_{\mu\nu}\ ,\\ &\partial_\mu j_L^\mu(x) = \frac{-i}{3}\left[\frac{g^2}{16\pi^2}\right]\mathrm{Tr}\,{}^*F^{\mu\nu}F_{\mu\nu}\ ,\\ &\partial_\mu j_R^\mu(x) = 0\ . \end{aligned} \tag{10}$$

These three sets of results, (5)—(6), (8)—(9), and (10), are all different. (The conventional Pauli-Villars regularization is not appropriate here since the fermion-mass term breaks the *local* gauge invariance explicitly. See also Sec. IV.)

The origin of the above discrepancy is traced to the difference between the gauge theory and the nongauge theory, and eventually to the arbitrariness in the perturbative evaluation of triangle diagrams in nongauge theories. The basic observation here is that one *imposes* the anomaly-free properties on all the gauge vertices in gauge theory, regardless of whether they are vector or axial-vector. Otherwise, the gauge theory is inconsistent. One of course needs suitable sets of fermion multiplets to ensure this property for a general gauge group.[11]

Once one imposes the normal properties on all the gauge vertices, the triangle (and box) diagrams appearing in the calculation of (5) are uniquely specified. As a result, even the vector current (5) can contain the anomaly if the gauge coupling involves γ_5. In general, therefore, one has to treat the three vertices of the axial-vector-current correlation function $\langle 0 | T^* j_5^\mu j_5^\nu j_5^\alpha | 0\rangle$, for example, in a quite *asymmetrical* manner depending on whether they are gauge vertices or not. The *ad hoc* symmetric treatment of all the three vertices of $\langle 0 | T^* j_5^\mu j_5^\nu j_5^\alpha | 0\rangle$ is not allowed.

If one keeps these points in mind when evaluating triangle (and box) diagrams in a carefully defined perturbation theory (as in the paper of Rosenberg[12]), one can reproduce the path-integral results (5) and (6). The selection rule of the fermion number nonconservation[8] is correctly specified by (5). When one evaluates the axial-vector-current correlation function $\langle 0 | T^* j_5^\mu j_5^\nu j_5^\alpha | 0\rangle$ *without* referring to the gauge coupling, on the other hand, the treatment of the three vertices becomes quite arbitrary. In this case, the path-integral method gives rise to just one of the possible results.

The moral of this simple discussion is that a straightforward application of various regularization schemes is often dangerous, and it is the *physical requirement* which dictates how to evaluate triangle diagrams. In particular, these regularization schemes, which were devised in times when the gauge couplings generally meant vector couplings, should be treated with due care in the case of gauge theories containing γ_5 couplings. The only reliable calculational scheme in the conventional perturbation theory thus appears to be the original method of Rosenberg[12] with suitable local gauge invariance imposed on all the gauge vertices.

III. PATH INTEGRAL FOR GAUGE THEORIES WITH γ_5 COUPLINGS

The path-integral treatment of the chiral anomaly in the case of gauge theory with γ_5 couplings has been briefly explained in Ref. 6. In view of the recent controversy,[3,4] it may be appropriate to present a more detailed path-integral prescription.

Before we get involved in detailed calculations, we note that the arbitrariness encountered in perturbative calculations of triangle diagrams may be viewed as a manifestation of the failure of the *naive* unitary transformation from the Heisenberg picture to the interaction picture. In other words, the Feynman rules in the interaction picture do *not* contain sufficient information to completely specify how to evaluate those diagrams involving anomalies. In the framework of perturbation theory, the evaluation of triangle diagrams, for example, thus becomes quite arbitrary and crucially depends on the specific regularization one employs.

In the framework of the path integral, this failure of the naive unitary transformation manifests itself as the basis dependence of the anomaly factor.[5,6] To cope with this problem, the covariant-derivative operator $\not{D}$ (or the "Euclidean energy operator") has been used as a basic ingredient of the path-integral formalism. In the case of gauge theory with γ_5 couplings, the choice of the proper basis vectors becomes more involved since

$$[\not{D},\gamma_5]\neq 0 \tag{11}$$

and, as a result, the Hermiticity of the eigenvalue equations could be spoiled unless one carefully chooses the basic operators; the non-Hermiticity could in turn spoil the unitary transformation mentioned above.

To illustrate these considerations, we use the specific $SU(2)_L\times SU(2)_R$ gauge theory (suitably extended to the Euclidean metric) in the following:

$$\begin{aligned}\mathscr{L}=&\bar{\psi}_L i\not{D}(A)\psi_L+\bar{\psi}_R i\not{D}(B)\psi_R+G\bar{\psi}_L H(x)\psi_R\\&+G^*\bar{\psi}_R H(x)^\dagger\psi_L-\tfrac{1}{2}\,\mathrm{Tr}F^{\mu\nu}(A)F_{\mu\nu}(A)\\&-\tfrac{1}{2}\,\mathrm{Tr}F^{\mu\nu}(B)F_{\mu\nu}(B)+\mathscr{L}_{\text{Higgs}}\ ,\end{aligned}\tag{12}$$

where

$$\begin{aligned}&\not{D}(A)\equiv\gamma^\mu(\partial_\mu+igA_\mu^aT^a)\ ,\\&\not{D}(B)\equiv\gamma^\mu(\partial_\mu+ig'B_\mu^aT^a)\ ,\\&F_{\mu\nu}(A)\equiv(\partial_\mu A_\nu^a-\partial_\nu A_\mu^a+gf^{abc}A_\mu^bA_\nu^c)T^a\ ,\end{aligned}\tag{13}$$

and similarly for $F_{\mu\nu}(B)$ with T^a the generator of SU(2) defined in (2). The fermion fields $\psi_L(x)$ and $\psi_R(x)$ stand for the $SU(2)_L$ and $SU(2)_R$ doublets, respectively, and the Higgs field $H(x)$ transforms as (2,2) under $SU(2)_L\times SU(2)_R$. A detailed form of the Higgs Lagrangian $\mathscr{L}_{\text{Higgs}}$ is not important in the following discussions.

The Lagrangian (12) represents a general class of renormalizable gauge theories with *V*, *A*, *S*, and *P* couplings. Based on the standard analysis, the $SU(2)_L\times SU(2)_R$ *gauge* couplings are anomaly-free [see Eq. (31)] and (12) is consistent as a gauge theory. The extension of this discussion to more complicated cases such as the Georgi-Glashow SU(5) model[13] is straightforward.

To reflect the basic dynamics in (12) to the definition of the path integral as much as possible, we expand the fermionic variables ψ_L and ψ_R in terms of the eigenvectors of *Hermitian* operators $\not{D}(A)$ and $\not{D}(B)$.

We start with the Euclidean eigenvalue equation with a fixed gauge-field configuration

$$\not{D}(A)\varphi_n(x)=\lambda_n\varphi_n(x),\quad \int\varphi_n(x)^\dagger\varphi_m(x)d^4x=\delta_{m,n} \tag{14}$$

and note that $\gamma_5\varphi_n(x)$ belongs to the eigenvalue $-\lambda_n$. We thus define the "chiral" orthonormal set[6]

$$\begin{aligned}\varphi_n^L(x)&\equiv\left[\frac{1-\gamma_5}{\sqrt{2}}\right]\varphi_n(x)\ \text{ if }\lambda_n>0\ ,\\&\equiv\left[\frac{1-\gamma_5}{2}\right]\varphi_n(x)\ \text{ if }\lambda_n=0\ ,\end{aligned}\tag{15}$$

$$\begin{aligned}\varphi_n^R(x)&\equiv\left[\frac{1+\gamma_5}{\sqrt{2}}\right]\varphi_n(x)\ \text{ if }\lambda_n>0\ ,\\&\equiv\left[\frac{1+\gamma_5}{2}\right]\varphi_n(x)\ \text{ if }\lambda_n=0\ .\end{aligned}\tag{16}$$

[To render the eigenvalue problem in (14) well defined, one may apply, for example, a periodic boundary condition in a four-dimensional box.] We then expand

$$\begin{aligned}\psi_L(x)&=\sum_{\lambda_n\geq 0}a_n\varphi_n^L(x)\equiv\sum_{\lambda_n\geq 0}a_n\langle x\mid n\rangle_L\ ,\\\bar{\psi}_L(x)&=\sum_{\lambda_n\geq 0}\bar{b}_n\varphi_n^R(x)^\dagger\equiv\sum_{\lambda_n\geq 0}\bar{b}_n\langle n\mid x\rangle_R\end{aligned}\tag{17}$$

with a_n and $\bar{b}_n$ the elements of the Grassmann algebra. The transformations from ψ_L and $\bar{\psi}_L$ to a_n and $\bar{b}_n$ are formally unitary due to the orthonormality of φ_n^L and φ_n^R. The path-integral measure (for the left-handed fermion part) becomes

$$\begin{aligned}\prod_x\mathscr{D}\psi_L(x)\mathscr{D}\bar{\psi}_L(x)=&\det(\langle x\mid n\rangle_L)^{-1}\det(\langle n\mid x\rangle_R)^{-1}\\&\times\prod_n da_n d\bar{b}_n\ ,\end{aligned}\tag{18}$$

where $\det(\langle x\mid n\rangle_L)$, for example, stands for a determinant of an (infinite-dimensional) matrix specified by the indices *n* and *x*. Formally $\det(\langle x\mid n\rangle_L)=1$ up to a phase factor due to the orthonormality of basis vectors, but in any case the chiral Jacobian factor below can be evaluated without referring to $\det(\langle x\mid n\rangle_L)$. We take the right-hand side of (18) as our primary definition of the path-integral measure. The action for the left-handed fermion part is diagonalized as

$$\int d^4x\,\bar{\psi}_L i\not{D}(A)\psi_L=\sum_{\lambda_n\geq 0}i\lambda_n\bar{b}_na_n\ . \tag{19}$$

We note that the present prescription corresponds to the decomposition

$$\begin{aligned}&\det[\not{D}(A)(1-\gamma_5)/2]\\&\quad=\det(\langle n\mid x\rangle_R)^{-1}\prod_{\lambda_n\geq 0}\lambda_n\det(\langle x\mid n\rangle_L)^{-1}\\&\quad=\det(\langle x\mid n\rangle_R)\prod_{\lambda_n\geq 0}\lambda_n\det(\langle n\mid x\rangle_L)\end{aligned}$$

with the phase factor in (18).

Under the *localized* (infinitesimal) chiral transformation

$$\psi_L(x)\to e^{i\alpha(x)\alpha_5}\psi_L(x)=e^{-i\alpha(x)}\psi_L(x)\ ,$$
$$\bar\psi_L(x)\to\bar\psi_L(x)e^{i\alpha(x)\gamma_5}=\bar\psi_L(x)e^{i\alpha(x)}\ , \tag{20}$$

the Lagrangian (12) changes as

$$\mathscr{L}\to\mathscr{L}+\partial_\mu\alpha(x)\bar\psi_L(x)\gamma^\mu\psi_L(x)$$
$$+\alpha(x)[iG\bar\psi_L(x)H(x)\psi_R(x)+\text{H.c.}]\ . \tag{21}$$

The path-integral measure (18) then transforms as

$$\prod_x \mathscr{D}\psi_L(x)\mathscr{D}\bar\psi_L(x)\to\prod_x\mathscr{D}\psi_L(x)\mathscr{D}\bar\psi_L(x)\exp\left[i\int\alpha(x)\sum_{\lambda_n\ge 0}[\varphi_n^L(x)^\dagger\varphi_n^L(x)-\varphi_n^R(x)^\dagger\varphi_n^R(x)]dx\right]$$
$$=\prod_x\mathscr{D}\psi_L(x)\mathscr{D}\bar\psi_L(x)\exp\left[-i\int\alpha(x)A(x)d^4x\right] \tag{22}$$

with

$$A(x)=\sum_{\text{all}\,\lambda_n}\varphi_n(x)^\dagger\gamma_5\varphi_n(x)$$
$$\equiv\lim_{M\to\infty}\sum_{\lambda_n}\varphi_n(x)^\dagger\gamma_5 e^{-\lambda_n{}^2/M^2}\varphi_n(x)$$
$$=\lim_{M\to\infty}\sum_{\lambda_n}\varphi_n(x)^\dagger\gamma_5 e^{-\not D(A)^2/M^2}\varphi_n(x)$$
$$=\lim_{M\to\infty}\text{Tr}\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\gamma_5 e^{-\not D(A)^2/M^2}e^{ikx}$$
$$=\frac{g^2}{16\pi^2}\text{Tr}\,{}^*F^{\mu\nu}(A)F_{\mu\nu}(A)\ , \tag{23}$$

if one follows the standard evaluation,[5,6] which corresponds to a local version of the Atiyah-Singer index theorem.[14] In (23) we performed a *carefully defined* unitary transformation of basis vectors from $\varphi_n(x)$ to the plane-wave basis. The final result of (23) is independent of the regularization in the sense that $\exp(-\lambda_n{}^2/M^2)$ may be replaced by *any* smooth function $f(\lambda_n{}^2/M^2)$ with $f(0)=1$ and

$$f(\infty)=f'(\infty)=f''(\infty)=\cdots=0\ ,$$

and still the same result is obtained.[6]

From (20)—(23), we obtain the Ward-Takahashi (WT) identity by the variational-derivative method,[5,6] which corresponds to the operator relation

$$\partial_\mu[\bar\psi_L(x)\gamma^\mu\psi_L(x)]=iG\bar\psi_L(x)H(x)\psi_R(x)+\text{H.c.}$$
$$-i\frac{g^2}{16\pi^2}\text{Tr}\,{}^*F^{\mu\nu}(A)F_{\mu\nu}(A)\ . \tag{24}$$

Similarly, we construct the chiral basis starting with

$$\not D(B)\phi_n(x)=\lambda_n\phi_n(x),\quad \int\phi_n(x)^\dagger\phi_m(x)d^4x=\delta_{n,m} \tag{25}$$

and expand

$$\psi_R(x)=\sum_{\lambda_n\ge 0}c_n\phi_n^R(x)\ ,$$
$$\bar\psi_R(x)=\sum_{\lambda_n\ge 0}\bar d_n\phi_n^L(x)^\dagger\ . \tag{26}$$

We then obtain another WT identity corresponding to the right-handed chiral transformation $\psi_R(x)\to e^{i\alpha(x)}\psi_R(x)$ and $\bar\psi_R(x)\to\bar\psi_R(x)e^{-i\alpha(x)}$ (written in the operator language):

$$\partial_\mu[\bar\psi_R(x)\gamma^\mu\psi_R(x)]=-iG\bar\psi_L(x)H(x)\psi_R(x)+\text{H.c.}$$
$$+i\frac{g'^2}{16\pi^2}\text{Tr}\,{}^*F^{\mu\nu}(B)F_{\mu\nu}(B)\ . \tag{27}$$

The relations (24) and (27) stand for the general chiral identities for gauge theories with γ_5 couplings. As a special case of these relations one can recover the results in (5) and (6).

In the above treatment, we use the *Hermitian* operators $\not D(A)$ and $\not D(B)$ as basic operators to define the basis vectors by taking the local version of the Atiyah-Singer index theorem[14] as a guiding principle. One may, however, want to estimate the possible effects of the Higgs coupling on the chiral anomaly. For this purpose, one may rewrite the Lagrangian (12) as

$$\mathscr{L}=\bar\psi i\mathscr{D}\psi+\mathscr{L}_{\text{YM}}+\mathscr{L}_{\text{Higgs}} \tag{28}$$

with

$$\mathscr{D}\equiv\not D(A)\left[\frac{1-\gamma_5}{2}\right]+\not D(B)\left[\frac{1+\gamma_5}{2}\right]$$
$$-iGH(x)\left[\frac{1+\gamma_5}{2}\right]-iG^*H(x)^\dagger\left[\frac{1-\gamma_5}{2}\right]\ , \tag{29}$$

which may be used as a basic operator to define the basis vectors. The operator $\mathscr{D}$ is, however, *not* Hermitian in the Euclidean theory,

$$\mathscr{D}^\dagger=\not D(A)\left[\frac{1+\gamma_5}{2}\right]+\not D(B)\left[\frac{1-\gamma_5}{2}\right]$$
$$+iG^*H(x)^\dagger\left[\frac{1+\gamma_5}{2}\right]+iGH(x)\left[\frac{1-\gamma_5}{2}\right]$$
$$\ne\mathscr{D}\ , \tag{30}$$

and, as is explained in Sec. IV, one has to consider $\mathscr{D}^\dagger\mathscr{D}$ and $\mathscr{D}\mathscr{D}^\dagger$ as basic operators to define the basis vectors in the path integral. One can then confirm that the Higgs couplings do not modify the chiral anomaly [see Eq. (61)] and our prescription in this section gives rise to the most general chiral WT identities.

We finally comment on the anomaly-free criterion in the present formalism. As was explained in Ref. 6, one obtains the divergence equation for the (external) current associated with the *gauge coupling* as

$$D_\mu(A)(\bar{\psi}_L\gamma^\mu T^a\psi_L)=iG\bar{\psi}_L(x)T^aH(x)\psi_R(x)+\text{H.c.}-i\frac{g^2}{16\pi^2}\text{Tr}T^a{}^*F^{\mu\nu}(A)F_{\mu\nu}(A) \quad (31)$$

if one considers, instead of (20),

$$\psi_L(x)\to e^{i\alpha(x)T^a\gamma_5}\psi_L(x)\,,\qquad \bar{\psi}_L(x)\to\bar{\psi}_L(x)e^{i\alpha(x)T^a\gamma_5} \quad (32)$$

with other variables *fixed.* In (31), $D_\mu(A)$ stands for the covariant derivative for the adjoint representation of SU(2). Thus, the anomaly-free criterion, i.e., the vanishing of the last term in (31), becomes identical to that in perturbation theory.[11] This is natural if one remembers that the result (31) corresponds to the perturbative result when one imposes the anomaly-free condition on all the gauge vertices of triangle and box diagrams except for the vertex corresponding to $\bar{\psi}_L\gamma^\mu T^a\psi_L$. The anomaly-free criterion in perturbation theory means that one can impose the anomaly-free condition on *all* the gauge vertices of triangle and box diagrams, and consequently, the vanishing of the last term in (31) gives rise to the same criterion as in the perturbative treatment. From the path-integral viewpoint, the appearance of the anomaly in (31) means that the fermionic functional measure is not invariant under the local gauge transformation (32). [See Ref. 15 for the complications associated with global topological properties which should be taken into account when one performs integration over gauge fields. These complications are closely related to the nontrivial phase factor $\det(\langle x\mid n\rangle_L)^{-1}\det(\langle n\mid x\rangle_R)^{-1}$ in the chiral fermionic measure in (18).]

IV. ALTERNATIVE PATH-INTEGRAL PRESCRIPTION

In this section we would like to describe an alternative path-integral prescription,[16] which is equivalent to the prescription in Sec. III as far as chiral identities are concerned. This alternative prescription is, however, more flexible in some other respects.

We first examine the modified path-integral prescription suggested by Einhorn and Jones[4] and show that their modified prescription leads to non-Hermitian eigenvalue equations in Euclidean theory. We then describe how to maintain the Hermiticity and thus the local gauge invariance of the underlying theory. In Ref. 4 they consider a model corresponding to the axial-vector U(1) gauge theory defined by

$$\mathscr{L}=\bar{\psi}i\gamma^\mu(\partial_\mu+igA_\mu\gamma_5)\psi-\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}\,. \quad (33)$$

According to our prescription in Sec. III, this should be rewritten as

$$\mathscr{L}=\bar{\psi}_Li\gamma^\mu(\partial_\mu-igA_\mu)\psi_L+\bar{\psi}_Ri\gamma^\mu(\partial_\mu+igA_\mu)\psi_R-\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} \quad (34)$$

and consequently (in the operator language)

$$\partial_\mu(\bar{\psi}_L\gamma^\mu\psi_L)=-i\frac{g^2}{16\pi^2}{}^*F^{\mu\nu}F_{\mu\nu}\,,\qquad \partial_\mu(\bar{\psi}_R\gamma^\mu\psi_R)=i\frac{g^2}{16\pi^2}{}^*F^{\mu\nu}F_{\mu\nu}\,. \quad (35)$$

In particular, Eq. (35) gives rise to

$$\partial_\mu(\bar{\psi}\gamma^\mu\gamma_5\psi)=i\frac{g^2}{8\pi^2}{}^*F^{\mu\nu}F_{\mu\nu}\,. \quad (36)$$

As was pointed out by McKay and Young,[3] the result (36) *disagrees* (by a factor of $\frac{1}{3}$) with the conventional perturbative evaluation of the three axial-vector-current correlation function

$$\text{Fourier transform of }\partial_\rho\langle 0\mid T^*j_5^\rho(x)j_5^\mu(y)j_5^\nu(z)\mid 0\rangle=-\frac{i}{6\pi^2}\epsilon^{\mu\alpha\nu\beta}p_\alpha q_\beta\,. \quad (37)$$

This discrepancy is not unexpected, since our result (36) corresponds to the case where one imposes the anomaly-free condition on the gauge vertices (consequently, the coefficient of the anomaly is identical to that of the customary AVV triangle diagram), whereas all the three axial-vector vertices are treated *symmetrically* in (37), as was explained in Sec. II.

Incidentally, the Abelian model (33) contains an anomaly in the triple gauge coupling and thus it is *not* consistent as a gauge theory. Also the decomposition into left and right components as in (34) is more or less required in the case of non-Abelian theory, since the generators γ_5T^a do not form a closed algebra, $[\gamma_5T^a,\gamma_5T^b]\neq if^{abc}\gamma_5T^c$.

Aside from these complications, we want to examine the interesting observation of Einhorn and Jones.[4] They pointed out that the conventional perturbative result (37) is obtained by the path-integral method if one uses

$$\tilde{\not{D}}(A)\equiv\gamma^\mu(\partial_\mu+igA_\mu\gamma_5)=\gamma^\mu\tilde{D}_\mu(A) \quad (38)$$

as the basic operator to define the path integral. We are thus confronted with an apparent difficulty; it appears that the path-integral prescription is quite arbitrary. In the following we want to show that this arbitrariness is naturally resolved if one imposes the Hermiticity condition on the basic operators involved.

We first recall that the inner product of basis vectors is defined in the Euclidean theory by

$$\int\Psi(x)^\dagger\Phi(x)d^4x\,, \quad (39)$$

which gives rise to

$$\int\Psi(x)^\dagger\tilde{\not{D}}(A)\Phi(x)d^4x=\int[\tilde{\not{D}}(-A)\Psi(x)]^\dagger\Phi(x)d^4x \quad (40)$$

if one remembers that γ^μ is anti-Hermitian and γ_5 is Hermitian ($\gamma_5{}^2=1$) in our convention. Namely,

$$\tilde{\not{D}}(A)^\dagger=\tilde{\not{D}}(-A)\neq\tilde{\not{D}}(A)\,. \quad (41)$$

Therefore, a naive eigenvalue equation for $\tilde{\not{D}}(A)$ does not generally lead to an orthonormal basis (with real eigenvalues) and the formal unitary transformations such as (17) could be spoiled. Consequently, the mere use of $\tilde{\not{D}}(A)$ does not guarantee the (local) gauge invariance of the resulting path-integral formula. One can confirm that the path-integral evaluation of the anomaly factor on the basis of $\tilde{\not{D}}(A)$ corresponds to the use of the Pauli-Villars regularization in perturbation theory. The massive fermionic regulator, however, explicitly breaks the axial-vector *gauge* invariance. [The formal gauge invariance of the final result for the anomaly (37) in this perturbative calculation is somewhat accidental, arising from the special properties of Abelian theory and the antisymmetric tensor $\epsilon^{\mu\nu\alpha\beta}$. This point becomes clear when one considers the matching between triangle and box diagrams in non-Abelian theory.]

General treatment of non-Hermitian operators

A general prescription to deal with a non-Hermitian operator in the path integral, which can be readily applied to $\tilde{\not{D}}(A)$ in (38), has been described in Ref. 16 (see, in particular, the Appendix there) in connection with the relativistic string theory in the manner of Polyakov.[17] We first define two (positive-semidefinite) Hermitian operators[16]

$$H_\psi \equiv \tilde{\not{D}}(A)^\dagger \tilde{\not{D}}(A) = \tilde{\not{D}}(-A)\tilde{\not{D}}(A) \,, \qquad H_{\bar\psi} \equiv \tilde{\not{D}}(A)\tilde{\not{D}}(A)^\dagger = \tilde{\not{D}}(A)\tilde{\not{D}}(-A) \tag{42}$$

and define the orthonormal sets $\{\tilde\varphi_n\}$ and $\{\tilde\phi_n\}$ for a fixed gauge-field configuration by

$$H_\psi \tilde\varphi_n(x) = \lambda_n{}^2 \tilde\varphi_n(x), \quad \int \tilde\varphi_n(x)^\dagger \tilde\varphi_m(x) d^4x = \delta_{n,m} \,, \qquad H_{\bar\psi}\tilde\phi_n(x) = \lambda_n{}^2 \tilde\phi_n(x), \quad \int \tilde\phi_n(x)^\dagger \tilde\phi_m(x) d^4x = \delta_{n,m} \,. \tag{43}$$

We then expand $\psi(x)$ and $\bar\psi(x)$ as

$$\psi(x) = \sum_n a_n \tilde\varphi_n(x) \,, \qquad \bar\psi(x) = \sum_n \bar b_n \tilde\phi_n(x)^\dagger \,. \tag{44}$$

The fermionic part of the action is then diagonalized as (with $\lambda_n \geq 0$)

$$\int \bar\psi(x) i\tilde{\not{D}}(A)\psi(x) d^4x = \sum_n i\lambda_n \bar b_n a_n \,, \tag{45}$$

since $\tilde\phi_n(x) = (1/\lambda_n)\tilde{\not{D}}(A)\tilde\varphi_n(x)$ for $\lambda_n \neq 0$ if one suitably chooses the phase of $\tilde\varphi_n$ and $\tilde\phi_n$, and $\tilde{\not{D}}(A)\tilde\varphi_n = 0$ for $\lambda_n = 0$ due to the inner product (39). To be precise, λ_n are the *singular values* of $\tilde{\not{D}}(A)$ which are generally different from the eigenvalues of $\tilde{\not{D}}(A)$. In general,

$$\prod_{n=1}^{N} \lambda_n \neq \prod_{n=1}^{N} |\lambda_n^{(0)}|$$

with $\lambda_n^{(0)}$ the eigenvalues of $\tilde{\not{D}}(A)$; only in the limit $N \to \infty$ (i.e., the large-cutoff limit), do these two expressions formally become equal and give rise to the absolute value of the determinant of $\tilde{\not{D}}(A)$. The phase factor of $\det\tilde{\not{D}}(A)$ is included in $\det[\tilde\varphi_n(x)]^{-1}\det[\tilde\phi_n(x)^\dagger]^{-1}$ of the path-integral measure in (47) below. namely,

$$\det\tilde{\not{D}}(A) = \det[\tilde\phi_n(x)^\dagger]^{-1} \prod_n \lambda_n \det[\tilde\varphi_n(x)]^{-1} = \det[\tilde\phi_n(x)] \prod_n \lambda_n \det[\tilde\varphi_n(x)^\dagger] \,.$$

This corresponds to the diagonalization of a general matrix M by two unitary matrices U and V, $M = V\Lambda U^\dagger$, with Λ a diagonal matrix.[16]

Under the localized chiral transformation

$$\psi(x) \to e^{i\alpha(x)\gamma_5}\psi(x) \,, \qquad \bar\psi(x) \to \bar\psi(x) e^{i\alpha(x)\gamma_5} \,, \tag{46}$$

one obtains the Jacobian factor for fermions:

$$d\mu \equiv \prod_x \mathscr{D}\psi(x)\mathscr{D}\bar\psi(x) = \det[\tilde\varphi_n(x)]^{-1}\det[\tilde\phi_n^\dagger(x)]^{-1} \prod_n da_n d\bar b_n \to d\mu \exp\left[-i\int \alpha(x) \sum_n [\tilde\varphi_n(x)^\dagger \gamma_5 \tilde\varphi_n(x) + \tilde\phi_n(x)^\dagger \gamma_5 \tilde\phi_n(x)] d^4x \right] \,. \tag{47}$$

This Jacobian factor is estimated by the standard procedure[5,6]

$$\begin{aligned}\sum_n \tilde\varphi_n(x)^\dagger \gamma_5 \tilde\varphi_n(x) &\equiv \lim_{M\to\infty} \sum_n \tilde\varphi_n(x)^\dagger \gamma_5 e^{-\lambda_n{}^2/M^2} \tilde\varphi_n(x) = \lim_{M\to\infty} \sum_n \tilde\varphi_n(x)^\dagger \gamma_5 e^{-H_\psi/M^2} \tilde\varphi_n(x) \\ &= \lim_{M\to\infty} \mathrm{Tr} \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \gamma_5 e^{-H_\psi/M^2} e^{ikx} \\ &= \frac{g^2}{16\pi^2} {}^*F^{\mu\nu}F_{\mu\nu} \,. \end{aligned} \tag{48}$$

Similarly, the result for $\sum \tilde\phi_n \gamma_5 \tilde\phi_n$ is obtained from (48) by the replacement $A_\mu \to (-A_\mu)$. In evaluating (48), it is convenient to use the relation

$$H_\psi = \tilde D_\mu(A)\tilde D^\mu(A) + \frac{ig}{4}\gamma_5[\gamma^\mu,\gamma^\nu]F_{\mu\nu}(A) \tag{49}$$

with $\tilde D_\mu(A)$ defined in (38). We note that (49) is manifestly local gauge covariant, whereas $[\tilde{\not{D}}(A)]^2$ is *not*. The result (48) combined with the one for $\sum \tilde\phi_n \gamma_5 \tilde\phi_n$ gives rise to our previous result (36).

This agreement between (36) and (48) is also seen more directly as follows. We first observe that

$$H_\psi = \not{D}(-A)^2\left[\frac{1-\gamma_5}{2}\right]+\not{D}(A)^2\left[\frac{1+\gamma_5}{2}\right],$$
$$H_{\bar\psi} = \not{D}(-A)^2\left[\frac{1+\gamma_5}{2}\right]+\not{D}(A)^2\left[\frac{1-\gamma_5}{2}\right], \tag{50}$$

which follows from

$$\tilde{\not{D}}(A)\equiv\not{D}(-A)\left[\frac{1-\gamma_5}{2}\right]+\not{D}(A)\left[\frac{1+\gamma_5}{2}\right] \tag{51}$$

with

$$\not{D}(A)\equiv\gamma^\mu(\partial_\mu+igA_\mu)\ . \tag{52}$$

Consequently, the sets of basis vectors for H_ψ and $H_{\bar\psi}$ in (43) are expressed as

$$\{\tilde\varphi_n\}=\{\varphi_n^L(-A)\}+\{\varphi_n^R(A)\}\ ,$$
$$\{\tilde\phi_n\}=\{\varphi_n^R(-A)\}+\{\varphi_n^L(A)\}\ , \tag{53}$$

where φ_n^L and φ_n^R are defined in (14)—(16) with $\not{D}(A)$ there replaced by (52). Therefore,

$$\sum_n\tilde\varphi_n(x)^\dagger\gamma_5\tilde\varphi_n(x)=\sum_n[-\varphi_n^L(-A)^\dagger\varphi_n^L(-A)+\varphi_n^R(A)^\dagger\varphi_n^R(A)]\ ,$$
$$\sum_n\tilde\phi_n(x)^\dagger\gamma_5\tilde\phi_n(x)=\sum_n[\varphi_n^R(-A)^\dagger\varphi_n^R(-A)-\varphi_n^L(A)^\dagger\varphi_n^L(A)]\ . \tag{54}$$

Thus, the Jacobian factor in (47) becomes

$$\sum_n\{[\varphi_n^R(A)^\dagger\varphi_n^R(A)-\varphi_n^L(A)^\dagger\varphi_n^L(A)]+[\varphi_n^R(-A)^\dagger\varphi_n^R(-A)-\varphi_n^L(-A)^\dagger\varphi_n^L(-A)]\}\ , \tag{55}$$

which is precisely the chiral Jacobian factor for (34) if one adopts the prescription in Sec. III.

The path-integral prescription in this section turns out to be convenient when one estimates the (possible) effects of the Higgs coupling on the chiral anomaly. To illustrate this, we generalize (33) by adding two real scalar fields $\phi_1(x)$ and $\phi_2(x)$ as

$$\mathscr{L}=\bar\psi i\gamma^\mu(\partial_\mu+igA_\mu\gamma_5)\psi+G\bar\psi[\phi_1(x)+i\gamma_5\phi_2(x)]\psi-\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}+\mathscr{L}_{\text{Higgs}}\ , \tag{56}$$

which is formally axial-vector-gauge invariant if one transforms the scalar field as

$$\phi_1(x)+i\phi_2(x)\to e^{-2i\alpha(x)\gamma_5}[\phi_1(x)+i\gamma_5\phi_2(x)] \tag{57}$$

simultaneously with (46) and a gauge transformation of A_μ. The basic operator in the Euclidean theory becomes

$$\mathscr{D}\equiv\gamma^\mu(\partial_\mu+igA_\mu\gamma_5)-iG[\phi_1(x)+i\gamma_5\phi_2(x)]\ , \tag{58}$$

which is not Hermitian. We thus define

$$H'_\psi\equiv\mathscr{D}^\dagger\mathscr{D},\quad H'_{\bar\psi}\equiv\mathscr{D}\mathscr{D}^\dagger \tag{59}$$

instead of (42). We note that

$$H'_\psi=\tilde D_\mu(A)\tilde D^\mu(A)+\frac{ig}{4}\gamma_5[\gamma^\mu,\gamma^\nu]F_{\mu\nu}(A)+G^2[\phi_1{}^2+\phi_2{}^2]-iG\gamma^\mu[(\partial_\mu-2igA_\mu\gamma_5)(\phi_1+i\gamma_5\phi_2)] \tag{60}$$

which is manifestly gauge covariant if one remembers (57).

Repeating the procedure from (43) to (48), we have the chiral Jacobian factor [with $\tilde\varphi_n(x)$ now defined for H'_ψ]

$$\sum_n\tilde\varphi_n(x)^\dagger\gamma_5\tilde\varphi_n(x)\equiv\lim_{M\to\infty}\sum_n\tilde\varphi_n(x)^\dagger\gamma_5e^{-H'_\psi/M^2}\tilde\varphi_n(x)=\lim_{M\to\infty}\text{Tr}\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\gamma_5e^{-H'_\psi/M^2}e^{ikx}$$
$$=\frac{g^2}{16\pi^2}{}^*F^{\mu\nu}F_{\mu\nu}\ . \tag{61}$$

One can thus confirm that the chiral anomaly is not influenced by the Higgs couplings.[1,2] It is not difficult to extend this analysis to the more general case (29).

We finally comment on the connection of the present path-integral prescription with the perturbative calculation. This prescription corresponds to the regularization of the *bare* (exact) fermion propagator before the integration over gauge and Higgs fields as (see also Ref. 16)

$$\langle 0|T^*\psi_\alpha(x)\bar\psi_\beta(y)|0\rangle_{\text{reg}}\equiv\left[\mathscr{D}(x)^\dagger\frac{1}{\mathscr{D}\mathscr{D}^\dagger(x)}f(\mathscr{D}\mathscr{D}^\dagger(x))\right]_{\alpha\beta}\delta(x-y)=\sum_n\tilde\varphi_n(x)_\alpha\frac{1}{\lambda_n}f(\lambda_n{}^2)\tilde\phi_n(y)^\dagger_\beta\ , \tag{62}$$

where we used $\tilde\varphi_n=(1/\lambda_n)\mathscr{D}^\dagger\tilde\phi_n$ valid for $\lambda_n\neq0$ if the phase conventions are suitably chosen; $\lambda_n\neq0$ for all n may hold for the generic Higgs-field configuration in (29) and (58). One may also use

$$\mathscr{D}^\dagger(\mathscr{D}\mathscr{D}^\dagger)^{-1}=\mathscr{D}^\dagger(\mathscr{D}^\dagger)^{-1}\mathscr{D}^{-1}=\mathscr{D}^{-1}$$

in (62). The regulator function $f(x)$ may be *any* smooth function which rapidly vanishes at $x=+\infty$ with $f(0)=1$ and

$$f(\infty)=f'(\infty)=f''(\infty)=\cdots=0\ .$$

A convenient choice of $f(x)$ for perturbative calculations may be

$$f(x)=\left[\frac{M^2}{x+M^2}\right]^n \tag{63}$$

with $n=1$ or 2 and M the regulator mass which should be set to ∞ at the end. If one expands (62) in powers of the gauge and Higgs fields, one obtains a (gauge invariantly regularized) perturbative series which gives rise to the same result for the anomaly as in our path-integral prescription. The regulator (62) is convenient to evaluate closed fermion loops, for which the conventional regularization such as the dimensional one has certain difficulties. [We note that the regularization (62) is *not* defined in the Lagrangian level.]

V. CONCLUSION

We have shown that the perturbative calculation of the chiral anomaly should be carefully performed in gauge theories with γ_5 couplings. In general, it is the physical requirement which dictates how to evaluate triangle diagrams in conventional perturbation theory.

As for the derivation of chiral WT identities in the path-integral formalism, the original path-integral prescription[6] gives rise to the unique local gauge-invariant result in gauge theories with γ_5 couplings. The Hermiticity of the basic operator to define orthonormal basis vectors becomes quite important in the Euclidean gauge theory, and it ensures the local gauge invariance of the underlying theory. This association of the Hermiticity with the local gauge invariance is an interesting aspect of the path-integral formulation of the anomaly. An extension of the path-integral treatment to the cases with S and P Higgs couplings was described, and it was confirmed that the chiral anomaly is independent of these nongauge couplings. This shows that the chiral Jacobian factor, when suitably evaluated, contains all the information on the chiral anomaly in renormalizable gauge theories.[18]

Note added in proof. A. Andrianov and L. Bonora [Report. No. TH 35-39-CERN, and IFPD Padova Report No. 16/83 (unpublished)] suggest that the continuation $A_\mu \to iA_\mu$ can define a Hermitian operator, for example, for the Abelian axial-vector gauge theory in Eq. (33) in this paper. This continuation, however, *explicitly* breaks the local gauge invariance. A simultaneous continuation in A_μ and the coordinate variable x^μ, as in the case of the Wick rotation, is required to maintain the manifest local gauge invariance; the Hermiticity is not acquired in this case.

Recently various authors discussed the chiral anomaly from the viewpoint of the Wess-Zumino consistency condition [see B. Zumino, Y.-S. Wu, and A. Zee, University of Washington report, 1983 (unpublished); R. Zackiw, Les Houches lectures, MIT report, 1983 (unpublished) and references therein]. This method is based on the functional derivative of (unspecified) det$\mathscr{D}$ with respect to the gauge field A_μ^a. It turns out that the gauge-noninvariant expression for the anomaly on the basis of the naive application of, e.g., the Pauli-Villars regulator to parity-violating gauge theories, satisfies this consistency condition. This is presumably due to the fact that all the gauge vertices are treated on an equal footing in these calculations. In the path-integral method, the same expression for the anomaly is obtained by using the prescription of Einhorn and Jones in Ref. 4. [In arbitrary space-time dimensions, see R. Delbourgo and P. Jarvis, Tasmania report, 1983 (unpublished); see also S.-K. Hu, B.-L. Young, and D. McKay, Iowa report, 1983 (unpublished).] In any case, the consistency method leads to the same anomaly-free criterion as the one given by the path-integral prescription in this paper; this agreement is physically quite natural as was explained in the text.

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