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IMPROVED METHODS FOR SUPERGRAPHS

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We introduce some new techniques into superfield perturbation theory which allow considerable simplifications in calculations. As a result, we show that all contributions to the effective action can be written as integrals over a single $d^4\theta$. We also give the background field formalism for supersymmetric non-abelian gauge theories. To illustrate our methods, we give examples of loop calculations: in particular, we show that in O(4) extended supersymmetric non-abelian gauge theories all one-loop propagator corrections cancel identically (both infinite and finite parts) and that these theories, at one loop, are finite and have no renormalizations (in the Fermi-Feynman gauge).

1. Introduction

The potential value of supergraphs [1–4] for perturbation theory was recognized at an early stage in the development of supersymmetry: the supersymmetry Ward identities are manifest, and the "miraculous" cancellations in component field calculations [5] are simple consequences of the superfield Feynman rules. However, relatively few superfield calculations were carried out [6–10]. Although simpler than conventional calculations, they were nonetheless cumbersome because the methods did not fully exploit the inherent advantages of supergraphs as a manifestly supersymmetric formalism. It was necessary to manipulate expressions involving explicit spinor variables, and to rely on complicated algebraic identities to obtain simple final results.

Two recent developments motivate another look at supergraphs. First, the superspace theory of supergravity has developed to a point where one can do superfield perturbation theory calculations in locally supersymmetric theories. In particular,

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the quantization procedure and Feynman rules have been written down for the supergravity superfields [4], and 1- and higher-loop calculations seem tractable. Secondly, in global supersymmetry, component field calculations have revealed that the O(4) extended supersymmetric gauge theory [11] has vanishing 1- [12] and 2-loop [13] β-function. Clearly this result could be obtained more simply in the context of a superfield calculation, and the 3-loop situation can be investigated. Again, a simplification of supergraph methods is desirable if such a program is to be carried out.

In this paper we will describe some techniques that simplify supergraph calculations. These techniques make greater use of supersymmetry than previous ones and allow us to quickly rederive a number of known results, in addition to obtaining some new ones. In sect. 2 we give the Feynman supergraph rules for chiral and vector supermultiplets in a form with no explicit θ's: all θ's are implicit in covariant derivatives (D_α and $\bar{D}_{\dot{\alpha}}$) and $\delta^4(\theta_{12})$ factors. We then prove (by explicit construction) that any supergraph contribution to the effective action is "local" in θ (i.e., contains only one $\int d^4\theta$). In sect. 3, to illustrate our methods, we give some explicit loop calculations of propagator corrections and the first supergraph calculations of vertex corrections in renormalizable theories. We also comment on some additional cancellations in the O(4) extended vector multiplet. In sect. 4 we introduce the background field formalism for non-abelian vector multiplets, which has the interesting property that the quantum gauge parameter is a chiral superfield (chiral representation) while the background gauge parameter is a real superfield (vector representation). We also show that the calculations of sect. 3 simplify further in background covariant gauges. Sect. 5 contains our conclusions. Appendix A contains our notational conventions and some useful formulae. Appendix B contains some details about the vector-multiplet action.

2. Supergraph rules

We will consider general actions for interacting chiral and vector multiplets (with gauge-breaking term, ghosts, and sources, in euclidean x-space; see appendix B)

$$
\begin{aligned}
S = {} & \int d^4x\, d^4\theta\, \bar{\phi}\, e^{gV} \phi - \left[\int d^4x\, d^2\theta \left(\tfrac{1}{2} m\phi^2 + \frac{1}{3!}\lambda\phi^3 \right) + \text{h.c.} \right] \\
& + \frac{1}{64 g^2}\, \mathrm{tr} \int d^4x\, d^2\theta\, W^\alpha W_\alpha - \tfrac{1}{16}\, \mathrm{tr} \int d^4x\, d^4\theta\, (D^2 V)(\bar{D}^2 V) \\
& + \mathrm{tr} \int d^4x\, d^4\theta\, (\bar{c}' - c')\, L_{gV/2} \left[(\bar{c} + c) + (\coth L_{gV/2})(c - \bar{c}) \right] \\
& + \left[\int d^4x\, d^2\theta\, J\phi + \text{h.c.} \right] + \mathrm{tr} \int d^4x\, d^4\theta\, \mathcal{J} V\,, \qquad (2.1)
\end{aligned}
$$

where

$$V=V^iG_i\,,\quad [G_i,G_j]=if_{ij}{}^kG_k\,,\quad W_\alpha=\bar{D}^2(\mathrm{e}^{-gV}D_\alpha\,\mathrm{e}^{gV})\,,\quad L_xY\equiv[X,Y]\,;$$

$$f_{ikl}f_{jkl}=k_1\delta_{ij}\,,\quad \mathrm{tr}\,G_iG_j=k_2\delta_{ij}\,,\quad G_iG_i=k_3I \tag{2.2}$$

(for each factor group and each of its irreducible representations); with $k_2=1$ for the G's appearing in the gauge action. (ϕ is a column vector; m and λ in general are matrices.) The tr $\int \mathrm{d}^4x\mathrm{d}^2\theta\; W^\alpha W_\alpha$ term is real by itself, up to surface terms (such as instanton contributions, which we will ignore, since we are only concerned with perturbation theory; see appendix B). By not explicitly adding in the hermitian conjugate, we have reduced the number of terms at purely vector vertices by a factor of 2. We use the Fermi-Feynman gauge to avoid infrared problems [14].

The major simplification we introduce is in the Feynman rules for chiral superfields [1], which will be treated in a way similar to vector superfields. Spinor coordinates θ will never appear explicitly, but only implicitly in the covariant derivatives D_α and $\bar{D}_{\dot\alpha}$, $\delta^4(\theta_{12})$'s, and $\int\mathrm{d}^4\theta$'s, so that supersymmetry is manifest in our Feynman rules. Thus, we find simplifications in many calculations. The only algebra will be that of the covariant derivatives, involving integration by parts, commutators, and "transfer" across a propagator through the relation

$$D_\alpha(\theta_1,p)\,\delta^4(\theta_{12})=-D_\alpha(\theta_2,-p)\,\delta^4(\theta_{12})\,.$$

We will also use the identities (see appendix A; $z_{12}=z_1-z_2$, where $z=(x,\theta,\bar\theta)$)

$$\frac{\delta}{\delta J(z_1)}J(z_2)=-\tfrac{1}{4}\bar{D}_1^2\delta^8(z_{12})\,, \tag{2.3a}$$

$$\int\mathrm{d}^4x\mathrm{d}^2\theta(-\tfrac{1}{4}\bar{D}^2f)=\int\mathrm{d}^8z\,f\,, \tag{2.3b}$$

$$\int\mathrm{d}^4x\mathrm{d}^2\theta f=\int\mathrm{d}^8z(-\tfrac{1}{4}D^2/\Box)\,f\,, \tag{2.3c}$$

where J is a chiral source. The propagators are obtained from the free part of the chiral multiplet action in (2.1) by rewriting it as

$$\begin{aligned}S_0&=\int\mathrm{d}^8z\,[\bar\phi\phi-\tfrac{1}{2}m\phi(-\tfrac{1}{4}D^2/\Box)\,\phi-\tfrac{1}{2}m\bar\phi(-\tfrac{1}{4}\bar{D}^2/\Box)\,\bar\phi\\&\quad+J(-\tfrac{1}{4}D^2/\Box)\,\phi+\bar{J}(-\tfrac{1}{4}\bar{D}^2/\Box)\,\bar\phi]\\&=\int\mathrm{d}^8z\left[\tfrac{1}{2}(\phi\bar\phi)\begin{pmatrix}\tfrac{1}{4}mD^2/\Box & 1\\ 1 & \tfrac{1}{4}m\bar{D}^2/\Box\end{pmatrix}\begin{pmatrix}\phi\\ \bar\phi\end{pmatrix}+(\phi\bar\phi)\begin{pmatrix}(-\tfrac{1}{4}D^2/\Box)\,J\\ (-\tfrac{1}{4}\bar{D}^2/\Box)\,\bar{J}\end{pmatrix}\right]\\&\equiv\int\mathrm{d}^8z(\tfrac{1}{2}\psi^{\mathrm{T}}A\psi+\psi^{\mathrm{T}}B)\,.\end{aligned} \tag{2.4}$$

Performing the functional integral

$$Z_0(J) = \int \mathcal{D}\phi \mathcal{D}\bar{\phi}\, \mathrm{e}^S = \exp(-\tfrac{1}{2}\int \mathrm{d}^8 z\, B^{\mathrm{T}} A^{-1} B)\,,$$

as usual, where

$$A^{-1} = \begin{pmatrix} -\frac{1}{4} m\bar{D}^2/(\Box - m^2) & 1 + m^2 \bar{D}^2 D^2/16\Box(\Box - m^2) \\ 1 + m^2 D^2 \bar{D}^2/16\Box(\Box - m^2) & -\frac{1}{4} m D^2/(\Box - m^2) \end{pmatrix}, \tag{2.5}$$

leads to

$$-\tfrac{1}{2}\int \mathrm{d}^8 z B^{\mathrm{T}} A^{-1} B = \int \mathrm{d}^8 z \Bigg[\bar{J}\frac{1}{-\Box + m^2} J + \tfrac{1}{2} J\left(\frac{\frac{1}{4} m D^2}{-\Box(-\Box + m^2)}\right) J + \tfrac{1}{2}\bar{J}\left(\frac{\frac{1}{4} m \bar{D}^2}{-\Box(-\Box + m^2)}\right)\bar{J}\Bigg]\,, \tag{2.6}$$

from which the free propagators can be read. The vertices can be obtained from the usual functional formula

$$Z(J) = \exp[S_{\mathrm{int}}(\delta/\delta J)]\, Z_0(J)\,,$$

where

$$S_{\mathrm{int}}(\delta/\delta J)\, J(z_1) J(z_2) J(z_3) = -\lambda \frac{1}{3!}\, [\int \mathrm{d}^4 x_4 \mathrm{d}^2\theta_4 (\delta/\delta J(z_4))^3]\, J(z_1) J(z_2) J(z_3)$$

$$= -\lambda \int \mathrm{d}^8 z_4\, \delta^8(z_{14})[-\tfrac{1}{4}\bar{D}_2^2 \delta^8(z_{24})]\,[-\tfrac{1}{4}\bar{D}_3^2 \delta^8(z_{34})]\,. \tag{2.7}$$

Thus, a ϕ^3 vertex has a factor $-\frac{1}{4}\bar{D}^2$ acting on 2 of the 3 propagators entering it, with an integral $\mathrm{d}^4 x\, \mathrm{d}^4\theta$, while the $\phi\bar{\phi}$ and $\phi\phi$ propagators are

$$\frac{1}{p^2 + m^2}\,\delta^4(\theta_{12}) \quad \text{and} \quad \frac{1}{p^2(p^2 + m^2)}\,\tfrac{1}{4} m D^2(p, \theta_1)\,\delta^4(\theta_{12})\,,$$

respectively. Instead, one could integrate $\mathrm{d}^4 x\, \mathrm{d}^2\theta$ and associate a factor $-\frac{1}{4}\bar{D}^2$ with a ϕ-end of each propagator: the $\phi\bar{\phi}$ and $\phi\phi$ propagators would then respectively become

$$(-\tfrac{1}{4}\bar{D}_1^2)(-\tfrac{1}{4}D_2^2)\frac{1}{p^2 + m^2}\,\delta^4(\theta_{12})$$

$$= \frac{1}{p^2 + m^2}\exp[(2\theta_1\bar{\theta}_2 - \theta_1\bar{\theta}_1 - \theta_2\bar{\theta}_2)^{\alpha\dot{\beta}}\sigma^a_{\alpha\dot{\beta}} p_a]\,,$$

$$(-\tfrac{1}{4}\bar{D}_1^2)(-\tfrac{1}{4}\bar{D}_2^2)\frac{1}{p^2(p^2 + m^2)}\,\tfrac{1}{4} m D_1^2\,\delta^4(\theta_{12}) = \frac{m}{p^2 + m^2}\,(-\tfrac{1}{4}\bar{D}_1^2)\,\delta^4(\theta_{12})\,, \tag{2.8}$$

which are the propagators in the usual treatment. However, it is more convenient to use $\int d^4x d^4\theta$, and associating the $-\frac{1}{4}\bar{D}^2$ factors with vertices makes the chiral-multiplet Feynman rules more analogous to those for the vector multiplet (see below). Now *all* renormalizable vertices have 4 D's, and massless propagators are $\pm(1/p^2)\,\delta^4(\theta_{12})$ (+ for chiral multiplets, − for vector; the $1/p^2$ pole in the massive $\phi\phi$ propagator is cancelled by vertex factors, as seen in (2.8)).

In order to read off the vector-multiplet vertices from S, we rewrite its action as

$$\frac{1}{64g^2}\mathrm{tr}\int d^4x d^2\theta\, W^\alpha W_\alpha = -\frac{1}{16g^2}\mathrm{tr}\int d^8z (e^{-gV}D^\alpha e^{gV})\,\bar{D}^2(e^{-gV}D_\alpha e^{gV})\,,$$

$$\begin{aligned}(e^{-gV}D_\alpha e^{gV}) &= [\exp(-L_{gV})\,D_\alpha]\cdot 1\\ &= (D_\alpha + [-gV, D_\alpha] + \tfrac{1}{2}[-gV,[-gV,D_\alpha]] + \ldots)\cdot 1\\ &= gD_\alpha V - \tfrac{1}{2}g^2[V,(D_\alpha V)] + \ldots\,.\end{aligned} \tag{2.9}$$

Up to fourth order, this piece of the action is thus, with the gauge-breaking term (see appendix B):

$$\mathrm{tr}\int d^4x d^4\theta [-\tfrac{1}{2}V\Box V + \tfrac{1}{16}gV\{D^\alpha V, \bar{D}^2 D_\alpha V\} - \tfrac{1}{16}g^2(\tfrac{1}{4}[V, D^\alpha V]\,\bar{D}^2[V, D_\alpha V]$$
$$+ \tfrac{1}{3}(D^\alpha V)\,\bar{D}^2[V,[V,D_\alpha V]])]\,. \tag{2.10}$$

To the same order, the ghost terms are

$$\mathrm{tr}\int d^4x d^4\theta (\bar{c}'c + c'\bar{c} + \tfrac{1}{2}g(\bar{c}' - c')[V, \bar{c} + c]$$
$$+ \tfrac{1}{12}g^2(\bar{c}' - c')[V,[V, c - \bar{c}]])\,. \tag{2.11}$$

We can now summarize our Feynman rules for the effective action.

(i) Massless propagators are $\pm(1/p^2)\,\delta^4(\theta_{12})$, with + for ($\phi\bar{\phi}$ propagators of) chiral superfields (scalar and ghost multiplets) and − for real superfields (vector multiplets); the massive scalar multiplet has

$$+\frac{1}{p^2+m^2}\delta^4(\theta_{12}) \quad \text{and} \quad \frac{m}{p^2(p^2+m^2)}\tfrac{1}{4}D^2(p,\theta_1)\,\delta^4(\theta_{12})$$

for its $\phi\bar{\phi}$ and $\phi\phi$ propagators, respectively.

(ii) Vertices are read directly from S, with an extra $-\frac{1}{4}\bar{D}^2$ (or $-\frac{1}{4}D^2$) for each chiral (or antichiral) superfield, but omitting one $-\frac{1}{4}\bar{D}^2$ (or $-\frac{1}{4}D^2$) for converting $d^2\theta$ (or $d^2\bar{\theta}$) into $d^4\theta$.

(iii) There are the usual combinatoric factors, and −1 for each ghost loop.

(iv) The resulting amputated one-particle-irreducible graphs should have each amputated external line multiplied by the appropriate superfield, with a $-\frac{1}{4}\bar{D}^2$ (or $-\frac{1}{4}D^2$) omitted at a vertex for each external chiral (or antichiral) superfield.

(v) There are the usual integrations

$$\int \Big[\prod_{\text{loops}} \mathrm{d}^D p(2\pi)^{-D}\Big]\Big[\prod_{\text{ext}} \mathrm{d}^4 p(2\pi)^{-4}\Big](2\pi)^4\delta^4\Big(\sum_{\text{ext}} p\Big)\,, \quad \int \prod_{\text{vert}} \mathrm{d}^4\theta$$

($\mathrm{d}^D p$ indicates supersymmetric dimensional regularization [10]; see below).

As a result of this manifestly supersymmetric formulation of the Feynman rules we can prove a theorem on the θ dependence of the effective action: *each term in the effective action can be expressed as an integral over a single* $\mathrm{d}^4\theta$; i.e., the integrand is, in general, local in θ (a function of only one θ^α and $\bar\theta^{\dot\alpha}$), although non-local in x (a function of many x's). This is shown as follows: consider any loop in a one-particle-irreducible diagram. To it corresponds a cycle $\delta^4(\theta_{12})\,\delta^4(\theta_{23})\,...\,\delta^4(\theta_{n1})$ with derivatives $D, \bar D$, acting on the δ-functions. One can integrate by parts to remove the D_1's and D_2's from $\delta^4(\theta_{12})$ (and have them act on $\delta^4(\theta_{23})$, $\delta^4(\theta_{n1})$, or on lines external to the loop), and integrate over θ_2 (or θ_1), removing $\delta^4(\theta_{12})$ and replacing θ_2 by θ_1 everywhere. The process can be continued until the cycle is reduced to a single δ-function, and one has an expression for the graph of the form

$$\int \mathrm{d}^4\theta_1\,\mathrm{d}^4\hat\theta_i\, f(\theta_1, \hat\theta_i)[D\,...\,D\delta^4(\theta_{n1})]\,|_{\theta_n=\theta_1}$$

(p-dependence suppressed), where $\hat\theta_i$ refers to θ's of vertices not in the loop. The expression in brackets is easily evaluated: a product $D\,...\,D$ can be reduced to 4 or fewer D's by using the commutation relations and $(D)^3 = (\bar D)^3 = 0$. But $[D\,...\,D\delta^4(\theta_{n1})]\,|_{\theta_n=\theta_1} = 0$ for fewer than 4 D's, since $\delta^4(\theta_{n1}) = (\theta_n-\theta_1)^2(\bar\theta_n-\bar\theta_1)^2$. Thus, the only non-zero contributions are of the form (after appropriate commutations) $[D^2\bar D^2\delta^4(\theta_{n1})]\,|_{\theta_n=\theta_1} = 16$. The graph then takes the form $\int \mathrm{d}^4\theta_1\,\mathrm{d}^4\hat\theta_i \tilde f(\theta_1, \hat\theta_i)$: the entire loop has been contracted to a point in θ-space. This procedure is now carried out loop by loop until the entire graph is a point in θ-space: i.e., it is of the form $\int \mathrm{d}^4\theta\,\mathrm{d}^4 p_i F(\theta, p_i)$.

The procedure we have obtained for proving the theorem is in fact the easiest way to actually evaluate the θ integrals, as we shall illustrate by explicit calculations in sect. 3. Power-counting rules are now easily found: from the Feynman rules given above, propagators go as $1/p^2$ at large momenta (except massive $\phi\phi$ propagators which go as $D^2/p^4 \sim 1/p^3$), *all* renormalizable vertices go as $D^4 \sim p^2$, and external chiral lines go effectively as $1/D^2 \sim 1/p$. Finally, from our θ-integration method we see that loops go as $\mathrm{d}^4p/D^4 \sim p^2$, since four D's are used to cancel a loop's final $\delta^4(\theta_{n1})$. The power counting is then trivial.

We observe that, having reduced the θ integrations to a single one, the supersymmetry Ward identities are also trivial: the effective action is manifestly supersymmetric, having the form

$$\int \mathrm{d}^4x_1\,...\,\mathrm{d}^4x_n\,\mathrm{d}^4\theta\, F_1(x_1, \theta)\,...\,F_n(x_n, \theta)\, G(x_1, ..., x_n)\,,$$

where the F's are products of superfields and their derivatives, while G is translationally invariant, so that the effective action is invariant under

$$\theta^\alpha \to \theta^\alpha + \epsilon^\alpha\,,\quad \bar\theta^{\dot\alpha} \to \bar\theta^{\dot\alpha} + \bar\epsilon^{\dot\alpha}\,,\quad x_i^a \to x_i^a - i(\epsilon^\alpha\bar\theta^{\dot\beta} + \bar\epsilon^{\dot\beta}\theta^\alpha)\,\sigma^a_{\alpha\dot\beta}\,.$$

It is clear then that supersymmetry is preserved by the following dimensional-regularization procedure [10]: all the algebraic manipulations of the covariant derivatives are to be carried out first, in 4-dimensional space, whereas the momentum integration is to be subsequently carried out in D-dimensional space.

3. Loop calculations

We present in this section a number of examples of loop calculations which illustrate the simplicity of our Feynman rules. More complicated calculations will be discussed elsewhere. Previous loop calculations (besides some in non-renormalizable ϕ^4 theory) have been for propagator corrections: the 1-loop correction [6,7] and a 2-loop correction [7] for the ϕ^3 model; 1-loop corrections for interacting chiral and (massive) abelian vector multiplets [8] (Ward identities were used to help evaluate the vector-multiplet propagator corrections); the 1-loop corrections for the non-abelian vector multiplet [9] (although the variable $g\psi = e^{gV} - 1$ was used, which does not generally exist as an unconstrained variable: e.g., for O(N) or SU(N), tr $V = 0$ implies tr ψ is a non-zero dependent variable); and the one-loop correction to the vierbein-multiplet propagator due to a massless chiral multiplet [10]. We will repeat some of these propagator calculations, give some calculations of 1-loop vertex graphs, and comment on applications to O(4) extended vector multiplets (including the 1- and 2-loop β-function).

The simplest non-zero, but somewhat trivial, 1-loop graph is the massless chiral-multiplet propagator correction in fig. 1. We are calculating a contribution to the effective action, and according to our Feynman rules we obtain

$$\Gamma(\phi,\bar\phi) = \tfrac{1}{2}\lambda^2 \int \frac{d^4p}{(2\pi)^4}\,\frac{d^Dk}{(2\pi)^D}\,\frac{1}{k^2(p+k)^2} \int d^4\theta_1\, d^4\theta_2\, \bar\phi(-p,\theta_1)\,\phi(p,\theta_2)$$

$$\times\, \delta^4(\theta_{12})\, \tfrac{1}{16}\bar D^2 D^2(k,\theta_2)\, \delta^4(\theta_{12})\,. \tag{3.1}$$

The interaction

$$-\lambda \frac{1}{3!} \int d^4x\, d^2\theta\, \phi^3$$

in principle would introduce three factors $-\frac{1}{4}\bar D^2$, but one was replaced by an external field and a second one converted $\int d^2\theta$ into $\int d^4\theta$. Finally, by integration by parts the $\bar D^2$ and the D^2 (from the $\bar\phi^3$ vertex) were arranged to act on the same propagator. The θ integration is immediate and we find (recall

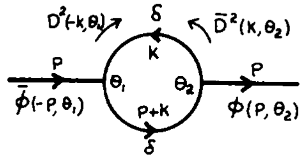

Fig. 1. A trivial propagator correction: massless ϕ^3.

$[\bar{D}^2 D^2 \delta^4(\theta_{12})]|_{\theta_1=\theta_2} = 16)$

$$\Gamma(\phi, \bar{\phi}) = \tfrac{1}{2}\lambda^2 \int \frac{d^4p}{(2\pi)^4} A(p) [\ d^4\theta\ \bar{\phi}(-p,\theta)\, \phi(p,\theta)] \ , \tag{3.2}$$

$$A(p) = \int \frac{d^Dk}{(2\pi)^D} \frac{1}{k^2(p+k)^2} = \frac{1}{(4\pi)^{D/2}} \Gamma(2 - \tfrac{1}{2}D) \frac{[\Gamma(\frac{1}{2}D - 1)]^2}{\Gamma(D-2)} (p^2)^{D/2-2} \ .$$

A less trivial calculation is of the massless chiral-multiplet contribution to the vector propagator, as shown in fig. 2. After using relations such as

$$\bar{D}^2(-k,\theta_1)\, D^2(k,\theta_2)\, \delta^4(\theta_{12}) = D^2(k,\theta_2)\, \bar{D}^2(-k,\theta_1)\, \delta^4(\theta_{12})$$

$$= D^2(k,\theta_2)\, \bar{D}^2(k,\theta_2)\, \delta^4(\theta_{12}) \ , \tag{3.3}$$

we have

$$\Gamma(V) = g^2 k_2 \operatorname{tr} \int \frac{d^4p}{(2\pi)^4} \frac{d^Dk}{(2\pi)^D} \frac{1}{k^2(p+k)^2} \int d^4\theta_1 d^4\theta_2\, V(-p,\theta_1)\, V(p,\theta_2)$$

$$\times [\tfrac{1}{16}\bar{D}^2 D^2(-p-k,\theta_2)\, \delta^4(\theta_{12})]\, [\tfrac{1}{16} D^2 \bar{D}^2(k,\theta_2)\, \delta^4(\theta_{12})] \ . \tag{3.4}$$

We integrate the $\bar{D}^2 D^2$ derivatives in the first bracket by parts and obtain the following non-zero terms (suppressing the arguments (k, θ_2) for D on $\delta^4(\theta_{12})$ and (p, θ_2) for D on V):

$$\Gamma(V) = g^2 k_2 \operatorname{tr} \int \frac{d^4p}{(2\pi)^4} \frac{d^Dk}{(2\pi)^D} \frac{1}{k^2(p+k)^2} \int d^4\theta_1 d^4\theta_2\, V(-p,\theta_1)\, \delta^4(\theta_{12})$$

$$\times \tfrac{1}{256} \{ [D^2\bar{D}^2 D^2 \bar{D}^2 \delta^4(\theta_{12})] - 4 [D^\alpha \bar{D}^{\dot\beta} D^2 \bar{D}^2 \delta^4(\theta_{12})]\, D_\alpha \bar{D}_{\dot\beta}$$

$$+ [D^2 \bar{D}^2 \delta^4(\theta_{12})]\, D^2 \bar{D}^2 \}\ V(p,\theta_2)$$

$$= g^2 k_2 \operatorname{tr} \int \frac{d^4p}{(2\pi)^4} \frac{d^Dk}{(2\pi)^D} \frac{1}{k^2(p+k)^2}$$

$$\times \int d^4\theta\ V(-p,\theta) [-k^2 - \tfrac{1}{2} k^{\alpha\dot\beta} D_\alpha \bar{D}_{\dot\beta} + \tfrac{1}{16} D^2 \bar{D}^2]\ V(p,\theta) \ . \tag{3.5}$$

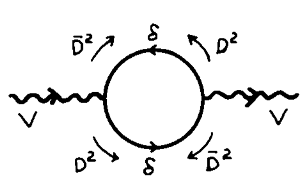

Fig. 2. A less simple propagator correction: massless ϕ loop for V propagator.

We then use the D-dimensional integrals

$$\int d^D k \frac{1}{(p+k)^2} = 0 \,, \qquad \int d^D k \frac{(2k+p)_a}{k^2(p+k)^2} = 0 \,, \tag{3.6}$$

to obtain (with projection operators $\Pi_0 = -\{D^2, \bar{D}^2\}/16p^2$, $\Pi_{1/2} = D^\alpha \bar{D}^2 D_\alpha / 8p^2$)

$$\Gamma(V) = g^2 k_2 \operatorname{tr} \int \frac{d^4 p}{(2\pi)^4} A(p) \int d^4\theta \; V(-p,\theta)\left[\tfrac{1}{4}p^{\alpha\dot{\beta}} D_\alpha \bar{D}_{\dot{\beta}} + \tfrac{1}{16} D^2 \bar{D}^2\right] V(p,\theta)$$

$$= g^2 k_2 \operatorname{tr} \int \frac{d^4 p}{(2\pi)^4} A(p) \left[\tfrac{1}{2} \int d^4\theta \; V(-p,\theta)\, p^2 \Pi_{1/2} V(p,\theta)\right] . \tag{3.7}$$

In the abelian case this can be rewritten as

$$g^2 k_2 \operatorname{tr} \int \frac{d^4 p}{(2\pi)^4} A(p) \left[\frac{1}{64 g^2} \int d^2\theta \; W^\alpha(-p,\theta)\, W_\alpha(p,\theta)\right] . \tag{3.8}$$

In the non-abelian case the graph gives only the part of this expression quadratic in the fields.

Other 1-loop propagator corrections are calculated in a similar manner; we simply quote the results. The corrections to the ghost propagators ($\bar{c}'c$ and $c'c$ in figs. 3a,b, respectively) are zero due to cancellations between graphs which follow directly from the signs of terms in (2.11). The vector correction to the massless chiral-multiplet propagator (fig. 3c), the ghost correction to the vector-multiplet propagator (fig. 3d), and the vector correction to its own propagator (fig. 3e) are respectively given by

$$-g^2 k_3 \int \frac{d^4 p}{(2\pi)^4} A(p) \left[\int d^4\theta \; \bar{\phi}(-p,\theta)\, \phi(p,\theta)\right] ,$$

$$g^2 k_1 \operatorname{tr} \int \frac{d^4 p}{(2\pi)^4} A(p) \left[\tfrac{1}{2} \int d^4\theta \; V(-p,\theta)\, p^2 (-\tfrac{1}{2}\Pi_{1/2} - \tfrac{1}{2}\Pi_0)\, V(p,\theta)\right] ,$$

$$g^2 k_1 \operatorname{tr} \int \frac{d^4 p}{(2\pi)^4} A(p) \left[\tfrac{1}{2} \int d^4\theta \; V(-p,\theta)\, p^2 (-\tfrac{5}{2}\Pi_{1/2} + \tfrac{1}{2}\Pi_0)\, V(p,\theta)\right] . \tag{3.9}$$

Adding the last two expressions, the longitudinal (Π_0) parts cancel, thus maintaining the Ward identity for the vector-multiplet propagator. The lengthiest of the calcula-

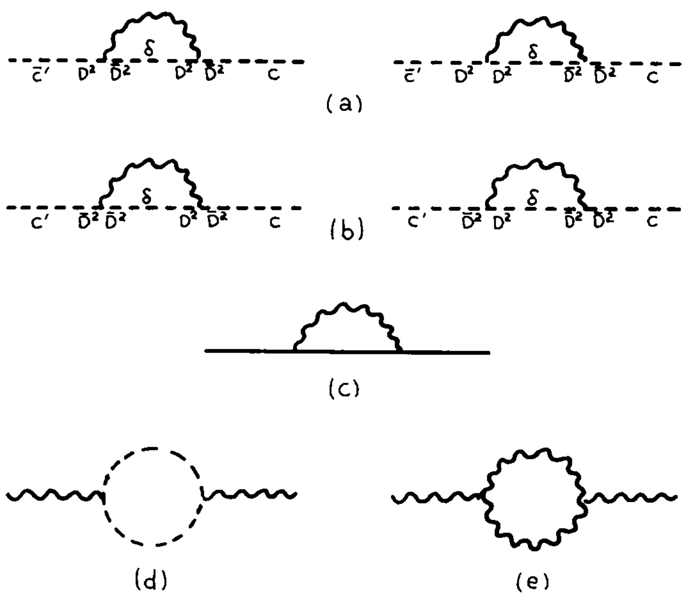

Fig. 3. Other one-loop propagator corrections.

tions is that for fig. 3e: the 6 terms at each vertex (from $\mathrm{tr}\int \mathrm{d}^4x\mathrm{d}^4\theta\, V\{D^\alpha V, \bar{D}^2 D_\alpha V\}$) produce 36 terms. Due to symmetry between the two lines and the two vertices, there are only 7 separate calculations with at least 4 D's in the internal loop (recall that 4 D's are needed for a non-zero contribution).

At this point we look at the 1-loop propagator corrections for the O(4) extended vector multiplet [11] with the following action (which has also recently been discussed by Fayet [15]) in terms of 1 real and 3 chiral (unextended) superfields, all in the adjoint representation:

$$S = \mathrm{tr}\Big(\int \mathrm{d}^4x\mathrm{d}^4\theta\; \mathrm{e}^{-gV}\bar{\phi}_i \mathrm{e}^{gV}\phi_i + \frac{1}{64g^2}\int \mathrm{d}^4x\mathrm{d}^2\theta\; W^\alpha W_\alpha$$
$$+\, ig\frac{1}{3!}\int \mathrm{d}^4x\mathrm{d}^2\theta\; \epsilon_{ijk}\phi_i[\phi_j,\phi_k] + ig\frac{1}{3!}\int \mathrm{d}^4x\mathrm{d}^2\bar{\theta}\; \epsilon_{ijk}\bar{\phi}_i[\bar{\phi}_j,\bar{\phi}_k]\Big)\,, \tag{3.10}$$

plus the usual gauge-breaking and ghost terms (i, j, k label the 3 chiral multiplets: $\phi_i = \phi_i{}^j G_j$). The O(4) symmetry (which determines the value of the ϕ^3 coupling) is not manifest in (3.10), but is manifest (though the supersymmetry is not) when S is expressed in terms of component fields. The corrections to the ghost, chiral-multiplet and vector-multiplet propagators are (from (3.2), (3.7) and (3.9)), with a factor

$$g^2 k_1\, \mathrm{tr}\int \frac{\mathrm{d}^4p}{(2\pi)^4} A(p) \int \mathrm{d}^4\theta\;;$$

$$\text{fig. 3a} = \text{fig. 3b} = 0\,,$$

$$\text{fig. 1} + \text{fig. 3c} = \bar{\phi}_i(-p,\theta)[(+1)+(-1)]\,\phi_i(p,\theta) = 0\,,$$

$$\begin{aligned}&\text{fig. 2} + \text{fig. 3d} + \text{fig. 3e} \\ &\quad = \tfrac{1}{2}V(-p,\theta)\,p^2\,[(+3\Pi_{1/2}) + (-\tfrac{1}{2}\Pi_{1/2} - \tfrac{1}{2}\Pi_0) + (-\tfrac{5}{2}\Pi_{1/2} + \tfrac{1}{2}\Pi_0)]\;V(p,\theta) = 0.\end{aligned} \tag{3.11}$$

Therefore, all one-loop propagator corrections vanish identically in this theory. The cancellation in the ghost propagator already occurs in the unextended vector multiplet, but the cancellation in the chiral-multiplet propagator is due to the value of the ϕ^3 coupling constant, and the cancellation in the vector-multiplet propagator is due to the multiplicity (3) of the chiral multiplets. The fact that the finite parts cancel along with the infinite parts is a result of the manifest supersymmetry, since each graph has momentum dependence proportional to $A(p) = (4\pi)^{-2}(2-\frac{1}{2}D)^{-1} + \dots$. This result should be compared with the component-field calculation, where, although $\beta(g)$ vanishes at one loop [12], not even the infinite parts of the propagator corrections $(Z-1)$ vanish at one loop (except for the vector component-field in a background gauge, since $\beta(g)$ can be determined from that Z alone).

The one-loop three-point vertex corrections of fig. 4 give, with a factor

$$g^3 k_1\,\mathrm{tr}\int \mathrm{d}^4p\,\mathrm{d}^4q\,(2\pi)^{-8}\left[\int \mathrm{d}^4k\,(2\pi)^{-4}k^{-2}(k+p)^{-2}(k+q)^{-2}\right]\,,$$

$$-i\frac{1}{3!}\epsilon_{ijk}\int \mathrm{d}^2\theta\;\phi_i(-p,\theta)[\phi_j(q,\theta),\phi_k(p-q,\theta)]\,[0+\tfrac{1}{2}p^2+\tfrac{1}{2}q^2+\tfrac{1}{2}(p-q)^2]\,, \tag{3.12}$$

and for fig. 5 ($k^{\alpha\dot\beta} \equiv k^a\sigma_a^{\alpha\dot\beta}$), with the same factor,

$$\begin{aligned}&\int \mathrm{d}^4\theta\,[\phi_i(-p,\theta),\bar{\phi}_i(q,\theta)]\,\{[(k+p)^2 - \tfrac{1}{2}(k+p)^{\alpha\dot\beta}D_\alpha\bar{D}_{\dot\beta} - \tfrac{1}{16}D^2\bar{D}^2] \\ &\quad + [\tfrac{1}{2}(k+q)^2 + \tfrac{1}{4}(k+q)^{\alpha\dot\beta}D_\alpha\bar{D}_{\dot\beta} - \tfrac{1}{32}D^2\bar{D}^2] + [0] + [-\tfrac{3}{4}(k+p)^2 - \tfrac{3}{4}(k+q)^2] \\ &\quad + [\tfrac{1}{32}D^\alpha\bar{D}^2D_\alpha + \tfrac{1}{8}k^{\alpha\dot\beta}[D_\alpha,\bar{D}_{\dot\beta}]]\}\;V(p-q,\theta) \\ &= \int \mathrm{d}^4\theta\,[\phi_i(-p,\theta),\bar{\phi}_i(q,\theta)](-\tfrac{1}{16})\{D^\alpha\bar{D}^2D_\alpha + (p+q)^{\alpha\dot\beta}[D_\alpha,\bar{D}_{\dot\beta}]\}\;V(p-q,\theta)\end{aligned} \tag{3.13}$$

in the O(4) extended theory. (We have used the identity $f_{ilm}f_{jmn}f_{knl} = \frac{1}{2}(f_{ilm}f_{jmn} + f_{ljm}f_{imn})f_{knl} = -\frac{1}{2}f_{jim}f_{lmn}f_{knl} = \frac{1}{2}k_1 f_{ijk}$.) In fig. 4, each diagram is convergent, while the sum of the diagrams in fig. 5 is convergent (implying, with (3.11), the vanishing of $\beta(g)$ at one loop). Therefore, *at one loop this theory is finite and has no renormalizations.* (The renormalizations of all other vertices are determined from these two and the propagator renormalizations.) This suggests the possibility of some supergravity theory being finite, if calculations are made with supergraphs [4], in spite of the existence of actions which could serve as counterterms.

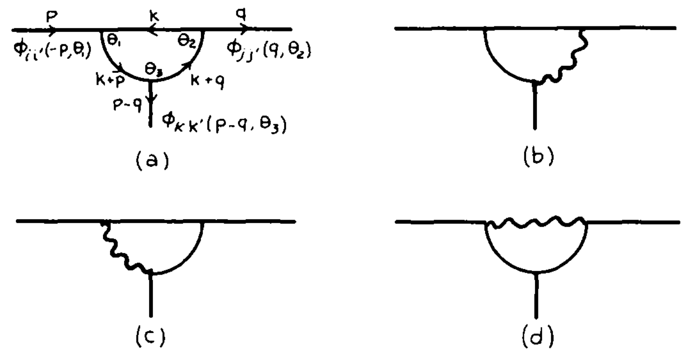

Fig. 4. One-loop vertex corrections in the O(4) extended theory.

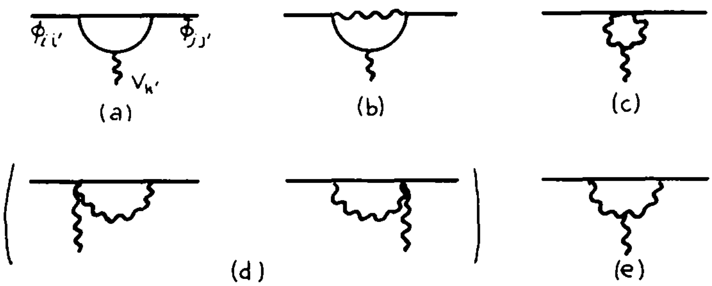

Fig. 5. More one-loop vertex corrections in the O(4) extended theory.

We can look now at the 2-loop β-function. Since the ϕ^3 term in the effective action is unrenormalized to all orders by power counting, we only need to calculate the 2-loop chiral-propagator correction. This can be obtained by using the Schwinger-Dyson equations depicted in fig. 6. Using the results of the 1-loop calculations of figs. 4,5, (half of) 5d, and 5c in figs. 6a,b,c,d, respectively, (see eqs. (3.12) and (3.13)) we readily obtain

$$\begin{aligned}
&g^4 k_1^2 \operatorname{tr}\int \mathrm{d}^4p(2\pi)^{-4}\,\bar{\phi}_{\dot{\iota}}(-p,\theta)\,\phi_{\dot{\iota}}(p,\theta)\{-\tfrac{1}{2}[p^2+q^2+(p+q)^2] \\
&\quad +\tfrac{3}{4}q^2+\tfrac{1}{4}q^2+0+0\} \\
&\quad \times\int \mathrm{d}^4q\,\mathrm{d}^4k(2\pi)^{-8}q^{-2}k^{-2}(p+q)^{-2}(p+k)^{-2}(q-k)^{-2} \\
&\quad = g^4 k_1^2 \operatorname{tr}\int \mathrm{d}^4p(2\pi)^{-4}\,\bar{\phi}_{\dot{\iota}}(-p,\theta)\,\phi_{\dot{\iota}}(p,\theta)(-\tfrac{1}{2}p^2) \\
&\quad \times\int \mathrm{d}^4q\,\mathrm{d}^4k(2\pi)^{-8}q^{-2}k^{-2}(p+q)^{-2}(p+k)^{-2}(q-k)^{-2}\,. \qquad (3.14)
\end{aligned}$$

Therefore, the two-loop propagator correction is finite, which is in agreement with

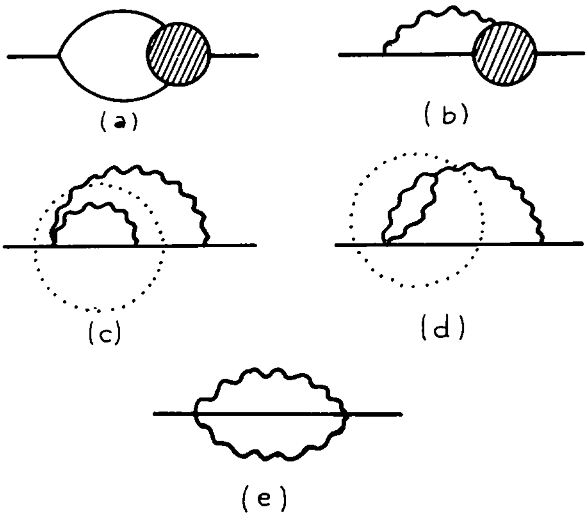

Fig. 6. Two-loop $\phi\bar{\phi}$ propagator corrections.

the vanishing of the 2-loop β-function [13]. The calculation of the 3-loop β-function is straightforward, and will be reported in a future publication.

4. The background field method

In this section we will discuss the background field method as applied to gauge superfields. As we will see, besides its familiar advantages, the method leads to considerable simplifications in actual calculations of individual graphs.

The background field method has already been applied in ref. [2] to models with chiral fields and abelian gauge fields. The procedure used there was the standard one [16,17]: in the Green function functional integral one writes the fields as a *sum* of classical (background) fields and quantum fields ($A = A^{\mathrm{B}} + A^{\mathrm{Q}}$) and re-expresses the functional in terms of the classical fields (instead of the sources). An invariance of the theory under *linear* (gauge) transformations $\delta A_i = \lambda_i + M_i{}^j A_j$ becomes an invariance of the background field functional under $\delta A_i{}^{\mathrm{B}} = \lambda_i + M_i{}^j A_j{}^{\mathrm{B}}$, with the quantum fields transforming covariantly: $\delta A_i{}^{\mathrm{Q}} = M_i{}^j A_j{}^{\mathrm{Q}}$.

In the case of non-abelian gauge superfields this procedure has to be modified, primarily because the gauge transformations are non-linear. In order to explain the procedure we review the description of supersymmetric gauge theories in vector and chiral representations. We start by defining covariant derivatives [18] (in euclidean x-space; see appendix A)

$$\nabla_A = D_A - i\Gamma_A \,, \quad \Gamma_A = \Gamma_A{}^i G_i \,, \quad (i\nabla_A)^\dagger = (-1)^a i\nabla_A \,,$$

$$\nabla_a = -\tfrac{1}{4} i \sigma_a^{\alpha\dot{\beta}} \{\nabla_\alpha, \overline{\nabla}_{\dot{\beta}}\} \,, \tag{4.1}$$

which transform as

$$\nabla'_A = e^{igK}\nabla_A\, e^{-igK}\,, \qquad K = K^i G_i = K^\dagger\,. \tag{4.2}$$

We define covariantly chiral superfields ϕ:

$$\bar\nabla_{\dot\alpha}\phi = 0\,, \tag{4.3}$$

which transform as

$$\phi' = e^{igK}\phi\,. \tag{4.4}$$

As a consequence of (4.3), we have

$$\{\bar\nabla_{\dot\alpha}, \bar\nabla_{\dot\beta}\}\,\phi = 0 \rightarrow \{\bar\nabla_{\dot\alpha}, \bar\nabla_{\dot\beta}\} = 0\,, \tag{4.5}$$

and this constraint has the solution

$$\nabla_\alpha = e^{-gW} D_\alpha\, e^{gW}\,, \quad \bar\nabla_{\dot\alpha} = e^{g\bar W}\bar D_{\dot\alpha}\, e^{-g\bar W}\,, \quad W = W^i G_i \neq \bar W\,. \tag{4.6}$$

W transforms as (see (4.2))

$$e^{gW'} = e^{ig\bar\Lambda}\, e^{gW}\, e^{-igK}\,, \quad \Lambda = \Lambda^i G_i\,, \quad \bar D_{\dot\alpha}\Lambda = 0\,, \tag{4.7}$$

where the Λ transformation is a consequence of the freedom in defining W: it leaves ∇_A invariant. Note that one can choose a K-gauge where $W = \bar W$.

The more familiar description is obtained by means of a (non-unitary) transformation to the "chiral" representation (with respect to W, not U, see appendix A):

$$\hat\phi = e^{-g\bar W}\phi\,, \quad \hat\nabla_A = e^{-g\bar W}\nabla_A\, e^{g\bar W}\,, \quad (i\hat\nabla_A)^\dagger = e^{gV}(-1)^a i\nabla_A\, e^{-gV}\,,$$

$$\hat\nabla_\alpha = e^{-gV} D_\alpha\, e^{gV}\,, \qquad \hat{\bar\nabla}_{\dot\alpha} = \bar D_{\dot\alpha}\,, \qquad \bar D_{\dot\alpha}\hat\phi = 0\,. \tag{4.8}$$

The gauge field appears now only in the real combination

$$e^{gV} \equiv e^{gW}\, e^{g\bar W}\,, \qquad e^{gV'} = e^{ig\bar\Lambda}\, e^{gV}\, e^{-ig\Lambda}\,. \tag{4.9}$$

In the background field method we wish to make manifest the background field invariance by having the quantum field transform covariantly, and the background field appear only in the form of background covariant derivatives. This is achieved by defining

$$e^{gW} = e^{gW^{\mathrm{B}}}\, e^{gW^{\mathrm{Q}}}\,, \tag{4.10}$$

or equivalently

$$e^{gV} = e^{gW^{\mathrm{B}}}\, e^{gW^{\mathrm{Q}}}\, e^{g\bar W^{\mathrm{Q}}}\, e^{g\bar W^{\mathrm{B}}} = e^{gW^{\mathrm{B}}}\, e^{gV^{\mathrm{Q}}}\, e^{g\bar W^{\mathrm{B}}}\,. \tag{4.11}$$

The covariant derivatives become

$$\nabla_\alpha = e^{-gW^{\mathrm{Q}}}\, e^{-gW^{\mathrm{B}}} D_\alpha\, e^{gW^{\mathrm{B}}}\, e^{gW^{\mathrm{Q}}} \equiv e^{-gW^{\mathrm{Q}}}\,\mathcal{D}_\alpha\, e^{gW^{\mathrm{Q}}}\,. \tag{4.12}$$

It is thus convenient to go to a representation which is quantum-chiral and background-vector by (compare (4.8))

$$\hat{\phi} = \mathrm{e}^{-g\overline{W}^{\mathrm{Q}}}\,\phi\,, \qquad \hat{\nabla}_A = \mathrm{e}^{-g\overline{W}^{\mathrm{Q}}}\,\nabla_A\,\mathrm{e}^{g\overline{W}^{\mathrm{Q}}}\,,$$

$$\hat{\nabla}_\alpha = \mathrm{e}^{-gV^{\mathrm{Q}}}\,\mathcal{D}_\alpha\,\mathrm{e}^{gV^{\mathrm{Q}}}\,, \qquad \hat{\overline{\nabla}}_{\dot\alpha} = \overline{\mathcal{D}}_{\dot\alpha}\,, \qquad \overline{\mathcal{D}}_{\dot\alpha}\hat{\phi} = 0\,. \tag{4.13}$$

W^{B} now appears only in the background covariant derivatives $\mathcal{D}_A$ (and in $\hat{\phi} = \mathrm{e}^{g\overline{W}^{\mathrm{B}}}\check{\phi}$, $\bar{D}_{\dot\alpha}\check{\phi} = 0$), and W^{Q} appears only in V^{Q}. (The background-field splitting for $\hat{\phi}$ is now simply $\hat{\phi} = \hat{\phi}^{\mathrm{B}} + \hat{\phi}^{\mathrm{Q}}$.)

We observe now that the transformation laws,

$$(\mathrm{e}^{gW^{\mathrm{B}}}\,\mathrm{e}^{gW^{\mathrm{Q}}})' = \mathrm{e}^{ig\bar{\Lambda}}\,\mathrm{e}^{gW^{\mathrm{B}}}\,\mathrm{e}^{gW^{\mathrm{Q}}}\,\mathrm{e}^{-igK}$$

$$= \mathrm{e}^{gW^{\mathrm{B}}}(\mathrm{e}^{-gW^{\mathrm{B}}}\,\mathrm{e}^{ig\bar{\Lambda}}\,\mathrm{e}^{gW^{\mathrm{B}}})\,\mathrm{e}^{gW^{\mathrm{Q}}}\,\mathrm{e}^{-igK}\,, \tag{4.14}$$

can be interpreted either as (i) quantum field transformations:

$$W^{\mathrm{B}\prime} = W^{\mathrm{B}}\,, \quad \mathrm{e}^{gW^{\mathrm{Q}\prime}} = \mathrm{e}^{ig\bar{\tilde{\Lambda}}}\,\mathrm{e}^{gW^{\mathrm{Q}}}\,\mathrm{e}^{-igK}\,, \quad \hat{\phi}' = \mathrm{e}^{ig\tilde{\Lambda}}\,\hat{\phi}\,,$$

$$\tilde{\Lambda} = \mathrm{e}^{g\overline{W}^{\mathrm{B}}}\,\Lambda\,\mathrm{e}^{-g\overline{W}^{\mathrm{B}}}\,, \qquad \overline{\mathcal{D}}_{\dot\alpha}\tilde{\Lambda} = 0\,; \tag{4.15a}$$

or (ii) background field transformations:

$$\mathrm{e}^{gW^{\mathrm{B}}} = \mathrm{e}^{ig\bar{\Lambda}}\,\mathrm{e}^{gW^{\mathrm{B}}}\,\mathrm{e}^{-igK}\,, \quad W^{\mathrm{Q}\prime} = \mathrm{e}^{igK}\,W^{\mathrm{Q}}\,\mathrm{e}^{-igK}\,, \quad \hat{\phi}' = \mathrm{e}^{igK}\,\hat{\phi}\,. \tag{4.15b}$$

Therefore, for the quantities $\mathcal{D}_\alpha$, V^{Q}, and $\hat{\phi}$ which appear explicitly in the action, we have the two sets of transformations:

$$\text{(i)} \quad \mathcal{D}_\alpha{}' = \mathcal{D}_\alpha\,, \quad \mathrm{e}^{gV^{\mathrm{Q}\prime}} = \mathrm{e}^{ig\bar{\tilde{\Lambda}}}\,\mathrm{e}^{gV^{\mathrm{Q}}}\,\mathrm{e}^{-ig\tilde{\Lambda}}\,, \quad \hat{\phi}' = \mathrm{e}^{ig\tilde{\Lambda}}\,\hat{\phi}\,; \tag{4.16a}$$

$$\text{(ii)} \quad \mathcal{D}_\alpha{}' = \mathrm{e}^{igK}\,\mathcal{D}_\alpha\,\mathrm{e}^{-igK}\,, \quad V^{\mathrm{Q}\prime} = \mathrm{e}^{igK}\,V^{\mathrm{Q}}\,\mathrm{e}^{-igK}\,, \quad \hat{\phi}' = \mathrm{e}^{igK}\,\hat{\phi}\,. \tag{4.16b}$$

We thus see that only the covariantly chiral parameter $\tilde{\Lambda}$ appears in the quantum transformations, whereas only the real parameter K appears in the background transformations. This is a result of our working in a background-vector, quantum-chiral representation. The vector representation for W^{B} is necessary in order to have $\bar{V}^{\mathrm{Q}} = V^{\mathrm{Q}}$, rather than the hermiticity condition $\bar{V}^{\mathrm{Q}} = \mathrm{e}^{gV^{\mathrm{B}}}V^{\mathrm{Q}}\,\mathrm{e}^{-gV^{\mathrm{B}}}$ which would occur in a background-chiral representation (and similarly for $(i\mathcal{D}_A)^\dagger$, in analogy to (4.8)). The chiral representation for V^{Q} is convenient because it eliminates the second (quantum) K transformations which act on W^{Q} (see (4.15a)).

The feature which distinguishes the background field formalism from the conventional method is that one can choose for the quantum gauge fields a background covariant gauge-fixing term, and thus obtain a gauge-invariant effective action. For general gauge fields A with gauges $f(A) = a$, gauge-fixing terms are introduced by a modification of the 't Hooft trick [19]

$$\delta[f(A) - a] \to \int \mathcal{D}a\mathcal{D}b \exp[-\int (a\hat{O}a + b\hat{O}b)]\, \delta[f(A) - a]$$
$$= \int \mathcal{D}b \exp[-\int (f\hat{O}f + b\hat{O}b)] \ , \tag{4.17}$$

where b is a *third ghost* (the same type of field as a but with opposite statistics) to normalize

$$\int \mathcal{D}a\mathcal{D}b \exp[-\int (a\hat{O}a + b\hat{O}b)] = 1 \ .$$

In ordinary gauge theories $\hat{O} = 1$ and b is unconstrained, so $\int \mathcal{D}b \exp[-\int b\hat{O}b] = 1$ and b decouples from the rest of the theory. An example where b does not decouple is in component supergravity, when one uses a gravitationally background covariant gauge-fixing term for the gravitino, so $\hat{O}$ is a differential operator which depends on the background vierbein [20].

The background-covariantized form of the gauge-breaking term in (2.1) is (dropping the Q on V^{Q})

$$S_{\mathrm{GB}} = -\tfrac{1}{16}\mathrm{tr}\int \mathrm{d}^4x\mathrm{d}^4\theta (\mathcal{D}^2 V)(\bar{\mathcal{D}}^2 V) \ , \tag{4.18}$$

which comes from

$$\int \mathcal{D}a\mathcal{D}\bar{a}\mathcal{D}b\mathcal{D}\bar{b} \exp[-\int \mathrm{d}^4x\mathrm{d}^4\theta(\bar{a}a + \bar{b}b)]\, \delta(\bar{\mathcal{D}}^2 V - a)\, \delta(\mathcal{D}^2 V - \bar{a}) \, . \tag{4.19}$$

Therefore, a and b are background chiral: $\bar{\mathcal{D}}_{\dot{\alpha}} a = \bar{\mathcal{D}}_{\dot{\alpha}} b = 0$ (since $\bar{\mathcal{D}}_{\dot{\alpha}}\bar{\mathcal{D}}^2 V = 0$). Hence, b is W^{B} dependent: in fact, the b action is just the usual action for a chiral superfield coupled to W^{B} (with opposite statistics), and its contribution cannot be ignored. The complete background-covariantized form of (2.1) is therefore (dropping $\hat{}$ on $\hat{\phi}$):

$$S = \int \mathrm{d}^4x\mathrm{d}^4\theta\, \bar{\phi}\, \mathrm{e}^{gV}\phi - \left[\int \mathrm{d}^4x\mathrm{d}^2\theta \left(\tfrac{1}{2}m\phi^2 + \frac{1}{3!}\lambda\phi^3\right) + \mathrm{h.c.}\right]$$
$$- \frac{1}{16g^2}\,\mathrm{tr}\int \mathrm{d}^4x\mathrm{d}^4\theta (\mathrm{e}^{-gV}\mathcal{D}^\alpha \mathrm{e}^{gV})\, \bar{\mathcal{D}}^2 (\mathrm{e}^{-gV}\mathcal{D}_\alpha \mathrm{e}^{gV})$$
$$- \tfrac{1}{16}\mathrm{tr}\int \mathrm{d}^4x\mathrm{d}^4\theta (\mathcal{D}^2 V)(\bar{\mathcal{D}}^2 V) + \mathrm{tr}\int \mathrm{d}^4x\mathrm{d}^4\theta\, \bar{b}b$$
$$+ \mathrm{tr}\int \mathrm{d}^4x\mathrm{d}^4\theta (\bar{c}' - c')\, L_{gV/2}[(\bar{c} + c) + (\coth L_{gV/2})(c - \bar{c})] \ ,$$
$$\bar{\mathcal{D}}_{\dot{\alpha}}(\phi, b, c, c') = 0 \ . \tag{4.20}$$

In order to examine one-loop calculations, we look at the terms quadratic in the quantum fields (suppressing commutators):

$$S_2 = \int \mathrm{d}^4x\,\mathrm{d}^4\theta\,(\bar{\phi}^{\mathrm{Q}}\phi^{\mathrm{Q}} + g\bar{\phi}^{\mathrm{B}} V\phi^{\mathrm{Q}} + g\bar{\phi}^{\mathrm{Q}} V\phi^{\mathrm{B}})$$
$$- \left[\int \mathrm{d}^4x\,\mathrm{d}^2\theta\,(\tfrac{1}{2}m\phi^{\mathrm{Q}2} + \tfrac{1}{2}\lambda\phi^{\mathrm{B}}\phi^{\mathrm{Q}2}) + \text{h.c.}\right]$$
$$+ \operatorname{tr}\int \mathrm{d}^4x\,\mathrm{d}^4\theta\,[-\tfrac{1}{2}V(\mathcal{D}^a\mathcal{D}_a - \mathcal{W}^\alpha\mathcal{D}_\alpha + \overline{\mathcal{W}}^{\dot\alpha}\overline{\mathcal{D}}_{\dot\alpha})V + \bar{b}b + \bar{c}'c + c'\bar{c}]\,. \quad (4.21)$$

(Here $\mathcal{W}_\alpha$ is the background field strength.) Since ϕ, b, c and c' are background chiral, they are W^{B} dependent: $\phi^{\mathrm{Q}} = \mathrm{e}^{gW^{\mathrm{B}}}\check{\phi}^{\mathrm{Q}}, \bar{D}_{\dot\alpha}\check{\phi} = 0$, etc. The Feynman rules of sect. 2 must be applied to $\check{\phi}^{\mathrm{Q}}, \check{b}, \check{c}$, and $\check{c}'$, although the effective action should be expressed in terms of only ϕ^{B} (not $\check{\phi}^{\mathrm{B}}$) and $\mathcal{D}_A$ (including $\mathcal{W}_\alpha$) to avoid explicit W^{B}'s. To obtain (3.21) from (4.20) we have used the identity

$$\mathcal{D}^\alpha\overline{\mathcal{D}}^2\mathcal{D}_\alpha - \tfrac{1}{2}(\mathcal{D}^2\overline{\mathcal{D}}^2 + \overline{\mathcal{D}}^2\mathcal{D}^2) - \mathcal{W}^\alpha\mathcal{D}_\alpha - \tfrac{1}{2}(\mathcal{D}^\alpha\mathcal{W}_\alpha)$$
$$= -8\mathcal{D}^a\mathcal{D}_a - \mathcal{W}^\alpha\mathcal{D}_\alpha + \overline{\mathcal{W}}^{\dot\alpha}\overline{\mathcal{D}}_{\dot\alpha}\,. \quad (4.22)$$

The fact that (4.21) contains only one spinor derivative leads to a startling conclusion: since a loop needs four spinor derivatives to give a non-zero result, all V-loops with fewer than four vertices vanish! In particular, in the O(4) extended theories the one-loop corrections to the vector-multiplet propagator and three-point function are automatically zero without computation: the V-loop contribution is zero, and the three ghost contributions are identical in form and opposite in sign to the three scalar-multiplet contributions. What in ordinary gauges were the hardest calculations have become the simplest in the background gauge.

The other vertex corrections in the O(4) model can also be recalculated. The calculation of fig. 4 is the same, and only the last 2 diagrams of fig. 5 give changed contributions, leading to the result (in place of (3.13))

$$-\tfrac{3}{32}\int \mathrm{d}^4\theta\,[\phi_i(-p,\theta), \bar{\phi}_i(q,\theta)]\,[D^\alpha\bar{D}^2D_\alpha V(p-q,\theta)]\,, \quad (4.23)$$

where all the fields are background fields, and $D^\alpha\bar{D}^2D_\alpha V$ may be covariantized to $\{\nabla^\alpha, \mathcal{W}_\alpha\}$. (The calculation is simpler in the gauge $W = \bar{W} = \frac{1}{2}V$, where only the last diagram of fig. 5 gives a changed contribution).

Since the supergravity superfields also occur in exponentials [21], with covariant derivatives similar to those for supersymmetric non-abelian gauge theories, the background field methods of this section can be generalized for supergravity.

5. Conclusions

We hope to have convinced the reader that supergraph techniques are simple and lead to useful results. We have shown in sect. 2 that our Feynman rules imply that the effective action is local in θ, supersymmetry is manifest, and supersymmetric dimensional regularization can easily be applied. The calculations in sect. 3 in addition to illustrating our rules, reveal some remarkable properties of the O(4) extended model: not only is the one-loop β-function zero, but so are (the infinite and finite parts of) the propagator corrections (in the supersymmetric Fermi-Feynman gauge). In sect. 4, we demonstrated that the background field method can be applied to non-abelian gauge superfields, and leads to further calculational simplifications.

It is clear that, since the one-loop results for the O(4) model are so simple, two-loop calculations are not very difficult, and three-loop ones possible. Our methods also open the way to extensive superfield calculations in supergravity. We expect the increase in difficulty in going from global to local supersymmetry to be less than that in going from QED to gravity. We hope to report our results on these topics soon.

Appendix A

Conventions

We define partial derivatives $\partial_A = (\partial_a, \partial_\alpha, \bar{\partial}_{\dot\alpha})$ for the superspace coordinates $z^A = (x^a, \theta^\alpha, \bar{\theta}^{\dot\alpha})$:

$$\partial_A z^B = \delta_A^B\,, \quad (\partial_a x^b = \delta_a^b\,, \quad \partial_\alpha \theta^\beta = \delta_\alpha^\beta\,, \quad \bar{\partial}_{\dot\alpha}\bar{\theta}^{\dot\beta} = \delta_{\dot\alpha}^{\dot\beta})\,. \tag{A.1}$$

The natural index-contraction convention is therefore [22]:

$$(z \cdot \partial)\, z^A = z^A \to z \cdot \partial = z^A \partial_A = z^a \partial_a + z^\alpha \partial_\alpha + \bar{z}^{\dot\alpha}\bar{\partial}_{\dot\alpha}\,; \tag{A.2a}$$

$$\begin{aligned} X \cdot Y &= X^A Y_A = Y^A X_A = X^a Y_a + X^\alpha Y_\alpha + X^{\dot\alpha} Y_{\dot\alpha} \\ &= (-1)^a X_A Y^A = X_a Y^a - X_\alpha Y^\alpha - X_{\dot\alpha} Y^{\dot\alpha}\,; \end{aligned} \tag{A.2b}$$

$$\begin{aligned} &X_A = (X \cdot \eta)_A = X^B \eta_{BA}\,, \\ &X^A = \eta^{AB} X_B \to X \cdot X = X^A X^B \eta_{BA} = \eta^{BA} X_A X_B \to \eta_{AB} = (-1)^{ab}\eta_{BA}\,, \\ &\eta^{AB}\eta_{BC} = (-1)^a \delta_C^A\,, \quad \partial_A z_B = \eta_{AB}\,; \end{aligned} \tag{A.2c}$$

$$\begin{aligned} &\eta_{AB} = (\eta_{ab}, \epsilon_{\alpha\beta}, \epsilon_{\dot\alpha\dot\beta})\,, \quad \eta^{AB} = (\eta^{ab}, \epsilon^{\alpha\beta}, \epsilon^{\dot\alpha\dot\beta}) \to \eta_{ab} = \eta_{ba}\,, \\ &\epsilon_{\alpha\beta} = -\epsilon_{\beta\alpha}\,, \quad \epsilon_{\dot\alpha\dot\beta} = -\epsilon_{\dot\beta\dot\alpha}\,, \\ &\eta^{ab}\eta_{bc} = \delta_c^a\,, \quad \epsilon^{\alpha\beta}\epsilon_{\beta\gamma} = -\delta_\gamma^\alpha\,, \quad \epsilon^{\dot\alpha\dot\beta}\epsilon_{\dot\beta\dot\gamma} = -\delta_{\dot\gamma}^{\dot\alpha}\,; \end{aligned} \tag{A.2d}$$

$$\begin{aligned} &\theta^2 = \theta^\alpha\theta_\alpha = \theta^\alpha\theta^\beta\epsilon_{\beta\alpha}\,, \quad \theta^\alpha\theta_\beta = \tfrac{1}{2}\delta_\beta^\alpha\theta^2\,, \\ &\theta_\alpha\theta_\beta = -\tfrac{1}{2}\epsilon_{\alpha\beta}\theta^2\,, \quad \theta^\alpha\theta^\beta = -\tfrac{1}{2}\epsilon^{\alpha\beta}\theta^2\,. \end{aligned} \tag{A.2e}$$

(Actually, due to dimensionalities, e.g., $\theta \sim x^{1/2}$, one never uses $X^A X^B \eta_{BA}$, but only $X^a X^b \eta_{ba}$, $X^\alpha X^\beta \epsilon_{\beta\alpha}$, or $X^{\dot\alpha} X^{\dot\beta} \epsilon_{\dot\beta\dot\alpha}$. Also, note that, from (A.1), δ_A^B acts as a tensor with indices ordered as $\delta_A{}^B$, which is useful in remembering identities such as $\epsilon^{\alpha\beta}\epsilon_{\beta\gamma} = \delta^\alpha{}_\gamma = -\delta_\gamma{}^\alpha = -\delta_\gamma^\alpha$.) We then have the hermiticity conditions

$$z^A = \bar z^A \,, \quad (\text{i.e.}, x^a = \bar x^a \,, \quad \theta^\alpha = (\bar\theta^{\dot\alpha})^\dagger) \;; \tag{A.3a}$$

$$\delta_A^B = \bar\delta_A^B = (\partial_A z^B)^\dagger = [\partial_A, z^B\}^\dagger = -(-1)^a(\partial_A)^\dagger z^B \to (i\partial_A)^\dagger$$
$$= (-1)^a (i\partial_A) \;; \tag{A.3b}$$

$$X^A = \bar X^A \,, \quad (X \cdot Y)^\dagger = X \cdot Y \to Y_A = (-1)^a \bar Y_A \;; \tag{A.3c}$$

$$\theta^2 = (\bar\theta^2)^\dagger \to \epsilon_{\dot\alpha\dot\beta} = -(\epsilon_{\alpha\beta})^* \to \theta_\alpha = -(\bar\theta_{\dot\alpha})^\dagger \,. \tag{A.3d}$$

($[,\}$ is the usual graded commutator.)

The covariant derivatives and supersymmetry generators are defined by

$$[D_A, D_B\} = T_{AB}{}^C D_C \,, \quad [Q_A, Q_B\} = -T_{AB}{}^C Q_C \,, \quad [D_A, Q_B\} = 0 \,,$$
$$(iD_A)^\dagger = (-1)^a (iD_A) \,, \quad (iQ_A)^\dagger = (-1)^a (iQ_A) \,,$$
$$T_{AB}{}^C = (2i\sigma^c_{\alpha\dot\beta}, 2i\sigma^c_{\beta\dot\alpha}) \;; \tag{A.4}$$

with the solution

$$D_a = Q_a = \partial_a \,, \quad D_\alpha = \mathrm{e}^{-U}\partial_\alpha \mathrm{e}^U \,, \quad \bar D_{\dot\alpha} = \mathrm{e}^U \bar\partial_{\dot\alpha} \mathrm{e}^{-U} \,,$$
$$Q_\alpha = \mathrm{e}^U \partial_\alpha \mathrm{e}^{-U} \,, \quad \bar Q_{\dot\alpha} = \mathrm{e}^{-U}\bar\partial_{\dot\alpha}\mathrm{e}^U \,,$$
$$U = \theta^\alpha \bar\theta^{\dot\beta} \sigma^a_{\alpha\dot\beta} i\partial_a \to D_\alpha = \partial_\alpha + i\bar\theta^{\dot\beta}\sigma^a_{\alpha\dot\beta}\partial_a \,, \quad \text{etc.} \tag{A.5}$$

(For convenience, we work in a vector U-representation. The chiral U-representation is obtained by $\hat D_A = \mathrm{e}^{-U} D_A \mathrm{e}^U$, etc., so, e.g., $\hat D_\alpha = \mathrm{e}^{-2U}\partial_\alpha \mathrm{e}^{2U}$, $\hat{\bar D}_\alpha = \bar\partial_{\dot\alpha}$.) We work in euclidean x-space ($\eta_{ab} = \eta^{ab} = \delta_{ab}$; to transform to Minkowski space, $\delta_{ab} \to -\eta_{ab}$, $\sigma^a_{\alpha\dot\beta} \to +\sigma^a_{\alpha\dot\beta}$, $\sigma_a^{\alpha\dot\beta} \to -\sigma_a^{\alpha\dot\beta}$, $\partial_a \to +\partial_a$, $\partial^a \to -\partial^a$, etc.), where the σ's satisfy ($\epsilon_{0123} = \epsilon^{0123} = i$)

$$\sigma^a_{\alpha\dot\beta}\sigma_b^{\alpha\dot\beta} = 2\delta^a_b \,, \qquad \sigma^a_{\alpha\dot\beta}\sigma_a^{\gamma\dot\delta} = 2\delta^\gamma_\alpha \delta^{\dot\delta}_{\dot\beta} \,,$$
$$\sigma^a_{\alpha\dot\beta}\sigma^{b\gamma\dot\beta}\sigma^c_{\gamma\dot\delta} = [(\delta^{ab}\delta^{cd} + \delta^{ad}\delta^{bc} - \delta^{ac}\delta^{bd}) + i\epsilon^{abcd}]\,\sigma_{d\alpha\dot\delta} \,. \tag{A.6}$$

From the covariant derivatives one can construct the projection operators for a scalar superfield [1]:

$$\Pi_{0+} = \bar D^2 D^2/16\Box \,, \quad \Pi_{0-} = D^2 \bar D^2/16\Box \,, \quad (\Pi_0 \equiv \Pi_{0+} + \Pi_{0-}) \;;$$
$$\Pi_{1/2} = -D^\alpha \bar D^2 D_\alpha/8\Box = -\bar D^{\dot\alpha} D^2 \bar D_{\dot\alpha}/8\Box \;; \tag{A.7a}$$

$$\Pi_i \Pi_j = \delta_{ij} \Pi_j \text{ (not summed)}\,, \quad \sum \Pi_i = 1\,;$$

$$\Pi^\dagger_{1/2} = \Pi_{1/2}\,, \qquad \Pi^\dagger_{0\pm} = \Pi_{0\mp}\,. \tag{A.7b}$$

For an arbitrary real superfield V, $\Pi_{0+} V$ is an arbitrary chiral superfield (and $\Pi_{0-} V$ its hermitian conjugate), while $\Pi_{1/2} V$ is the transverse part of the vector multiplet (thus, for the vector multiplet, $\Pi_0 V$ can be arbitrarily gauged: $\Pi_0 V = 0$ in the supersymmetric Landau gauge). $\Pi_0 \Box$ has a "square root":

$$[\tfrac{1}{4}(D^2 + \bar{D}^2)]^2 = \Pi_0 \Box\,, \tag{A.8}$$

analogous to $(\not{\partial})^2 = \Box$. Our conventions for spinor integration are

$$\int \mathrm{d}^2\theta\, \theta^2 = 1\,, \quad \int \mathrm{d}^2\bar{\theta}\, \bar{\theta}^2 = 1 \to \int \mathrm{d}^2\theta = -\tfrac{1}{4}D^2\,, \quad \int \mathrm{d}^2\bar{\theta} = -\tfrac{1}{4}\bar{D}^2\,. \tag{A.9}$$

(The latter identities are true only after integration by parts with $\int \mathrm{d}^4x$.) In most equations D^2 and $\bar{D}^2$ are accompanied by factors of $\frac{1}{4}$ (due not only to our convention (A.9), but basically to the normalization in (A.8): e.g., for the kinetic term $\int \mathrm{d}^4x\, \mathrm{d}^4\theta\, \phi\bar{\phi}$, the conveniently normalized mass term $-\int \mathrm{d}^4x\, \mathrm{d}^2\theta\, \frac{1}{2} m\phi^2$ + h.c. requires $\int \mathrm{d}^2\theta = \pm\frac{1}{4}D^2$).

Appendix B

Gauge action

We will first prove the reality condition

$$\frac{1}{64g^2} \operatorname{tr} \int \mathrm{d}^4x\, \mathrm{d}^2\theta\, W^\alpha W_\alpha = \frac{1}{64g^2} \operatorname{tr} \int \mathrm{d}^4x\, \mathrm{d}^2\bar{\theta}\, \bar{W}^{\dot{\alpha}} \bar{W}_{\dot{\alpha}}\,. \tag{B.1}$$

Using the identity [23]

$$\operatorname{tr} \int \mathrm{d}^4x\, \partial_\alpha f = \operatorname{tr} \int \mathrm{d}^4x\, D_\alpha f = \operatorname{tr} \int \mathrm{d}^4x\, [\nabla_\alpha, f]\,, \tag{B.2}$$

we have

$$\operatorname{tr} \int \mathrm{d}^4x\, \mathrm{d}^2\theta\, W^\alpha W_\alpha = -\tfrac{1}{4} \operatorname{tr} \int \mathrm{d}^4x \{\nabla^\beta, [\nabla_\beta, W^\alpha W_\alpha]\}\,. \tag{B.3}$$

The Bianchi identities

$$\{\nabla^\alpha, W_\alpha\} = \{\bar{\nabla}^{\dot{\alpha}}, \bar{W}_{\dot{\alpha}}\}\,, \quad \{\nabla_{(\alpha}, W_{\beta)}\} = -8\sigma^{a\dot{\gamma}}_{\alpha} \sigma^{b}_{\beta\dot{\gamma}} F_{ab}\,, \quad \{\bar{\nabla}_{\dot{\beta}}, W_\alpha\} = 0 \tag{B.4}$$

(where $[\nabla_a, \nabla_b] \equiv -iF_{ab}$), along with the constraint $\{\nabla_\alpha, \nabla_\beta\} = 0$, then imply that

$$\{\nabla^\alpha, [\nabla_\alpha, W^\beta W_\beta]\} = -2(\{\nabla^\alpha, W^\beta\}\{\nabla_\alpha, W_\beta\} - W^\beta[\nabla^\alpha, \{\nabla_\alpha, W_\beta\}]) \tag{B.5}$$

$$= -(\{\nabla^\alpha, W_\alpha\})^2 + 128(F^{ab}F_{ab} + \tfrac{1}{2}i\epsilon^{abcd}F_{ab}F_{cd}) + 8i\sigma^a_{\alpha\dot{\beta}} W^\alpha [\nabla_a, \bar{W}^{\dot{\beta}}]$$

is real after taking the trace and integrating over d^4x.

The form (B.3) for the gauge action, with the expansion (B.5), also explicitly shows the component field content of S [23], since one simply evaluates the integrand of $\int d^4x$ at $\theta = \bar{\theta} = 0$ to obtain $W_\alpha \to$ spinor field, $\{\nabla^\alpha, W_\alpha\} \to$ auxiliary scalar field, $F_{ab} \to$ gauge-field strength, $\nabla_a \to$ gauge-covariant derivative.

The quadratic part of (B.1) is

$$\tfrac{1}{16}\mathrm{tr}\int d^4x d^4\theta\, V D^\alpha \bar{D}^2 D_\alpha V = -\mathrm{tr}\int d^4x d^4\theta\, \tfrac{1}{2} V \Pi_{1/2} \Box V\,. \tag{B.6}$$

$\Pi_{1/2} V$ is the "transverse" part of V; the gauge-breaking term contains the longitudinal part $\Pi_0 V$:

$$S_{\mathrm{GB}} = -\frac{1}{\alpha}\,\mathrm{tr}\int d^4x d^4\theta\, \tfrac{1}{2} V \Pi_0 \Box V = -\frac{1}{\alpha}\,\frac{1}{16}\,\mathrm{tr}\int d^4x d^4\theta (D^2 V)(\bar{D}^2 V)\,, \tag{B.7}$$

and we set $\alpha = 1$ (Fermi-Feynman gauge) to avoid infrared problems. The form of the Faddeev-Popov ghost term in (2.1) follows from the transformation law

$$e^{gV'} = e^{ig\bar{\Lambda}} e^{gV} e^{-ig\Lambda} \to \delta V = -i L_{gV/2}\left[(\bar{\Lambda} + \Lambda) + \coth(L_{gV/2})(\Lambda - \bar{\Lambda})\right]\,. \tag{B.8}$$

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