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THE ENERGY-MOMENTUM TENSOR IN A NON-ABELIAN QUARK GLUON THEORY

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A finite energy-momentum tensor is constructed in an asymptotically free non-Abelian gauge theory with massive fermions (quarks). An explicit formula is found for the trace when inserted into a gauge-invariant Green function, and its properties at short distances are discussed.

The relevance of asymptotically free gauge theories [1] to high-energy physics makes the energy-momentum tensor of such theories an interesting object, since it in principle determines the hadronic mass spectrum. The present note is dedicated to the constrcution of an energy-momentum tensor which is finite order by order in perturbation theory, as well as its trace, and to a discussion of some of the properties of the latter.

We use dimensional regularisation [2]. The effective Lagrangian generating the renormalized quark gluon field theory with gauge group SU(2) and one isodoublet of quarks (these restrictions are, of course, inessential) is

$$\begin{aligned}\mathcal{L}(x) = &-\tfrac{1}{4}Z_3 F_{\mu\nu}F_{\mu\nu}(x) - \frac{1}{2\xi}(\partial_\mu A_\mu)^2(x) + c^*\partial_\mu(\widetilde{Z}_3\partial_\mu c \\ &+ g\mu^{\epsilon/2}\widetilde{Z}_1 A_\mu \times c)(x) + Z_2\Big(-\tfrac{1}{2}\overline{\psi}\gamma\overleftrightarrow{\partial}\psi - (m+\delta m)\overline{\psi}\psi \\ &+ ig\mu^{\epsilon/2}\frac{\widetilde{Z}_1}{\widetilde{Z}_3}\overline{\psi}\gamma_\mu \tfrac{1}{2}\tau A_\mu\psi(x)\Big)\,, \qquad (1)\end{aligned}$$

with

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g\frac{\widetilde{Z}_1}{\widetilde{Z}_3}\mu^{\epsilon/2}A_\mu \times A_\nu\,, \qquad (2)$$

and where A_μ, ψ and c are the gluon, quark, and Faddeev-Popov [3] ghost fields,

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respectively, while g is the coupling constant, m the quark mass, ξ the gauge-fixing parameter, μ an auxiliary mass unit [2], and $4-\epsilon$ the number of space-time dimensions. The renormalizations constants $\delta m, \widetilde{Z}_1, Z_2, Z_3$ and $\widetilde{Z}_3$ are chosen so as to make the perturbation series finite. We also introduce the effective action

$$S = \int \mathrm{d}^4 x\, \mathcal{L}(x)\,. \tag{3}$$

With the effective Lagrangian (1), the symmetrized energy-momentum tensor $\Theta_{\mu\nu}$ becomes (cf. [4])

$$\begin{aligned}\Theta_{\mu\nu} = &-Z_3 F_{\mu\lambda}F_{\nu\lambda} + \frac{1}{\xi} A_\mu \partial_\nu \partial_\lambda A_\lambda + \frac{1}{\xi} A_\nu \partial_\mu \partial_\lambda A_\lambda - \partial_\mu c^*(\widetilde{Z}_3 \partial_\nu c \\ &+ g\mu^{\epsilon/2}\widetilde{Z}_1 A_\nu \times c) - \partial_\nu c^*(\widetilde{Z}_3\partial_\mu c + g\mu^{\epsilon/2}\widetilde{Z}_1 A_\mu \times c) \\ &- \tfrac{1}{4} Z_2 \overline{\psi}(\gamma_\mu \overleftrightarrow{\partial}_\nu + \gamma_\nu \overleftrightarrow{\partial}_\mu)\psi + \tfrac{1}{2} i g \mu^{\epsilon/2} \frac{\widetilde{Z}_1}{\widetilde{Z}_3} Z_2 \overline{\psi}(\gamma_\nu A_\mu + \gamma_\mu A_\nu)\tfrac{1}{2}\tau\psi \\ &- \delta_{\mu\nu}\left(-\tfrac{1}{4} Z_3 F_{\lambda\rho}F_{\lambda\rho} - Z_2\overline{\psi}\left(\tfrac{1}{2}\gamma\overleftrightarrow{\partial} + m + \delta m - i g\mu^{\epsilon/2}\frac{\widetilde{Z}_1}{\widetilde{Z}_3}\gamma_\lambda A_\lambda\, \tfrac{1}{2}\tau\right)\psi\right. \\ &\left. + \frac{1}{2\xi}(\partial_\lambda A_\lambda)^2 + \frac{1}{\xi} A_\lambda \partial_\lambda \partial_\rho A_\rho - \partial_\lambda c^*(\widetilde{Z}_3 \partial_\lambda \widetilde{c} + g\mu^{\epsilon/2}\widetilde{Z}_1 A_\lambda \times c)\right).\end{aligned} \tag{4}$$

The expression (4) qualifies as an energy-momentum tensor since it has the following divergence, found *without* use of the field equations:

$$\begin{aligned}\partial_\mu \Theta_{\mu\nu} = &-\frac{\delta S}{\delta A^a_\lambda}\partial_\nu A^a_\lambda - \frac{\delta S}{\delta c^a}\partial_\nu c^a - \frac{\delta S}{\delta c^*}\partial_\nu c^{*a} - \frac{\delta S}{\delta\psi}\partial_\nu\psi - \frac{\delta S}{\delta\overline{\psi}}\partial_\nu\overline{\psi} \\ &\times \partial_\mu\left(\frac{\delta S}{\delta A^a_\mu} A^a_\nu + \frac{\delta S}{\delta\psi}\tfrac{1}{8}[\gamma_\mu,\gamma_\nu]\psi + \frac{\delta S}{\delta\overline{\psi}}\overline{\psi}\,\tfrac{1}{8}[\gamma_\nu,\gamma_\mu]\right),\end{aligned} \tag{5}$$

so *if* we use the field equations, we see that $\Theta_{\mu\nu}$ is conserved. Also, with

$$X = \prod_{i=1}^{m} A^{a_i}_{\mu_i}(x_i) \prod_{j=1}^{n} c^{b_j}(y_j) c^{*c_j}(z_j) \prod_{k=1}^{o} \psi(u_k)\overline{\psi}(v_k)\,, \tag{6}$$

the following Ward identity is a consequence of eq. (5):

$$\frac{\partial}{\partial x_\mu}\langle 0|T\Theta_{\mu\nu}(x)X|0\rangle = -i\left\{\sum_{i=1}^{m}\delta(x-x_i)\frac{\partial}{\partial x_{i\nu}} + \sum_{j=1}^{n}\left(\delta(x-y_j)\frac{\partial}{\partial y_{j\nu}}\right.\right.$$

$$+\,\delta(x-z_j)\frac{\partial}{\partial z_{j\nu}}\Bigg)+\sum_{k=1}^{o}\left(\delta(x-\mu_k)\frac{\partial}{\partial u_{k\nu}}+\delta(x-v_k)\frac{\partial}{\partial v_{k\nu}}\right)\Bigg\}\langle 0|TX|0\rangle$$

$$+\,i\frac{\partial}{\partial x_\mu}\{\sum_{i=1}^{m}\delta(x-x_i)\delta_{\mu\mu_i}\langle 0\,|TX(A^{a_i}_{\mu_i}(x_i)\to A^{a_i}_{\nu}(x_i))|0\rangle$$

$$+\sum_{k=1}^{o}\,[\delta(x-u_k)\langle 0|TX(\psi(u_k)\to\tfrac{1}{8}[\gamma_\mu,\gamma_\nu]\psi(u_k))|0\rangle$$

$$+\,\delta(x-v_k)\langle 0|TX(\overline{\psi}(v_k)\to\overline{\psi}(v_k)\tfrac{1}{8}[\gamma_\nu,\gamma_\mu])|0\rangle\}\,, \tag{7}$$

expressing the translational invariance of the theory and showing furthermore that $\Theta_{\mu\nu}$ is correctly normalized.

The finiteness of $\Theta_{\mu\nu}$ in perturbation theory follows from eq. (7) and the general theory on renormalisation of composite operators in gauge theories [5]. The right-hand of eq. (7) is finite, so the Green functions of $\Theta_{\mu\nu}$ must also have this properly, apart from terms with vanishing divergence, and the counterterms of $\Theta_{\mu\nu}$ must therefore have the form

$$(\partial_\mu\partial_\nu-\delta_{\mu\nu}\Box)P(A,c,c^*)\,, \tag{8}$$

where P is a scalar polynomial in the fields with mass dimension 2. Also $\Theta_{\mu\nu}$ consists of terms which are either gauge invariant or which can be obtained from some other expression through a Slavnov transformation [6,7], apart from a term $-\frac{1}{2}\delta_{\mu\nu}(\delta S/\delta c^{*a})c^{*a}$ which is manifestly finite by the ghost field equation. This implies [5] that the counterterms of $\Theta_{\mu\nu}$ either must be invariant under a generalization of the Slavnov transformation also involving the sources (in the sense of functional integration) of the composite operators arising under the Slavnov transformation, or they must arise from some other expression through this generalised Slavnov transformation. But no polynomial in the fields of the form (8) and having these properties is available, so no renormalization counterterms can be constructed, and $\Theta_{\mu\nu}$ is therefore finite.

Having established that $\Theta_{\mu\nu}$ is correctly normalized and is finite, we turn to its trace $\Theta_{\mu\mu}$ which determines the way in which dilatational and conformal invariance are broken. When calculating $\Theta_{\mu\mu}$ one should have in mind that though the expression as a whole is finite, it has constituents which only make sense through dimensional regularization. Therefore one must use the value $4-\epsilon$ for the trace of $\delta_{\mu\nu}$ instead of 4. Doing so, one gets from (4)

$$\Theta_{\mu\mu}=\epsilon\left[-\tfrac{1}{4}Z_3F_{\lambda\rho}F_{\lambda\rho}-\frac{1}{2\xi}\,(\partial_\lambda A_\lambda)^2\right]-(2-\epsilon)\,\frac{\delta S}{\delta c^{*a}}\,c^{*a}$$

$$+ (2-\epsilon)\partial_\lambda \left[c^*(\widetilde{Z}_3 \partial_\lambda c + g\widetilde{Z}_1 \mu^{\epsilon/2} A_\lambda \times c) - \frac{1}{\xi} A_\lambda \partial_\rho A_\rho \right]$$

$$+ Z_2(m+\delta m)\overline{\psi}\psi - (3-\epsilon)\frac{1}{2}\left(\frac{\delta S}{\delta\psi}\psi + \frac{\delta S}{\delta\overline{\psi}}\overline{\psi}\right), \tag{9}$$

When $\Theta_{\mu\mu}$ is inserted into a Green function, the second and fourth term on the right-hand side of eq. (9) give rise to contact terms which in the dilatational Ward identity exactly produce the canonical scaling dimensions of the ghost and quark fields. The term with a factor ϵ in front can be reduced to a linear combination of finite composite fields [8], where some of the coefficients are known,from the renormalization group analysis [9,10]; this point will be commented upon later on.

Eqs. (4) and (9) simplify considerably when $\Theta_{\mu\nu}$ and $\Theta_{\mu\mu}$ occur inside a Green function of gauge invariant operators because of the Slavnov-Taylor identity [6] and the ghost field equation. Eq. (4) becomes

$$\Theta_{\mu\nu}\Big|_{\text{gauge inv.}} = -Z_3 F_{\mu\lambda}F_{\nu\lambda} - \tfrac{1}{4}Z_2\overline{\psi}(\gamma_\mu \overleftrightarrow{\partial}_\nu + \gamma_\nu \overleftrightarrow{\partial}_\mu)\psi$$

$$+ \tfrac{1}{2} i g \mu^{\epsilon/2} \frac{\widetilde{Z}_1}{\widetilde{Z}_3} Z_2 \overline{\psi}(\gamma_\nu A_\mu + \gamma_\mu A_\nu)\tfrac{1}{2}\tau\psi - \delta_{\mu\nu}\Big(-\tfrac{1}{4}Z_3 F_{\lambda\rho}F_{\lambda\rho}$$

$$- Z_2\overline{\psi}\Big(\tfrac{1}{2}\gamma\overleftrightarrow{\partial} + m + \delta m - i g\mu^{\epsilon/2}\frac{\widetilde{Z}_1}{\widetilde{Z}_3}\gamma_\lambda A_\lambda \tfrac{1}{2}\tau\Big)\psi\Big); \tag{10}$$

which is the formula expected from classical field theory, while from eq. (9) one gets

$$\Theta_{\mu\mu}\Big|_{\text{gauge inv.}} = -\epsilon\,\tfrac{1}{4}Z_3 F_{\lambda\rho}F_{\lambda\rho} + Z_2(m+\delta m)\overline{\psi}\psi$$

$$- (3-\epsilon)\tfrac{1}{2}\left(\frac{\delta S}{\delta\psi}\psi + \frac{\delta S}{\delta\overline{\psi}}\overline{\psi}\right). \tag{11}$$

The reduction of the term $-\epsilon\frac{1}{4}Z_3 F_{\lambda\rho}F_{\lambda\rho}$ referred to in the preceding paragraph also becomes much simpler here. It has been carried out in ref. [8] to which we refer for details; here only the result will be stated.

What happens is that the two renormalization group functions $\beta(g,\epsilon)$ and $\delta(g,\epsilon)$ turn up instead of ϵ. They are independent of ξ and can be found from

$$\left(\beta(g,\epsilon) + \tfrac{1}{2}\epsilon g + \beta(g,\epsilon) g\frac{\partial}{\partial g}\right)\frac{\widetilde{Z}_1}{\widetilde{Z}_3 Z_3^{1/2}} = 0\,, \tag{12}$$

$$\left(\beta(g,\epsilon)\frac{\partial}{\partial g} + \delta(g,\epsilon)\right)Z_4 = O\,; \qquad Z_4 = \frac{m+\delta m}{m}. \tag{13}$$

Also they have finite limits for $\epsilon \to 0$.

The analysis of ref. [8] was carried out for insertions of $-\epsilon\frac{1}{4}Z_3 F_{\lambda\rho}F_{\lambda\rho}$ in Green functions of the gauge-invariant composite operator $Z_2 Z_4 \overline{\psi}\psi$. Here we shall be more interested in Green functions of the electromagnetic current J_μ:

$$J_\mu = iqZ_2 \overline{\psi}\gamma_\mu \psi \ , \tag{14}$$

where q is the electric charge of the quark measured in units of the electron charge; but the results of [8] are easily adapted to this case also.

The formula accomplishing the required transformation of eq. (11) is the equivalent of ref. [8] eq. (4.9),

$$\begin{aligned} -\epsilon\tfrac{1}{4}Z_3 F_{\lambda\rho}F_{\lambda\rho} &= \beta(g,\epsilon)\left\{\frac{\partial}{\partial g}\,[\mathcal{L}(x) + \mathcal{A}_\mu(x) J_\mu(x)]\right\} \\ &+ \frac{1}{g}\left(1 + g\frac{\partial}{\partial g}\ln\frac{\widetilde{Z}_1}{\widetilde{Z}_3}\right)\partial_\lambda\,[Z_3 A_\mu F_{\mu\lambda} + \widetilde{Z}_3 c^* \partial_\lambda c] \\ &- \delta(g,\epsilon)(m+\delta m) Z_2 \overline{\psi}\psi \ , \end{aligned} \tag{15}$$

valid when inserted into the path integral generating Green functions of $J_\mu(x)$, with $\mathcal{A}_\mu(x)$ the source of $J_\mu(x)$. Eq. (15) is convenient for the conversion of the dilatational Ward identity into the renormalization group equation for a Green function of J_μ's in which case one encounters $\int d^4x \Theta_{\mu\mu}(x)$, so the total differential drops out. For the unintegrated form of (15) it is seen from ref. [8] that the two operators multiplying $\beta(g,\epsilon)$ and $\delta(g,\epsilon)$ are both finite for $\epsilon \to 0$ because of the presence of the total differential.

The operator multiplying $\beta(g,\epsilon)$ in eq. (15) can be cast in a somewhat more elegant form by means of ref. [8] eq. (3.12); eq. (15) then becomes

$$\begin{aligned} -\tfrac{1}{4}\epsilon Z_3 F_{\lambda\rho}F_{\lambda\rho} &= \beta(g,\epsilon)\,\frac{1}{g}\left\{\left(1 + g\frac{\partial}{\partial g}\ln\frac{\widetilde{Z}_1}{\widetilde{Z}_3 Z_3^{1/2}}\right)\tfrac{1}{2}Z_3 F_{\lambda\rho}F_{\lambda\rho} \right. \\ &\left. - g\,\frac{\partial \ln Z_4}{\partial g}\,Z_2(m+\delta m)\overline{\psi}\psi\right\} - \delta(g,\epsilon) Z_2(m+\delta m)\overline{\psi}\psi \ . \end{aligned} \tag{16}$$

This relation can also be found directly from (12) and (13), but then one obtains no information on the finiteness for $\epsilon \to 0$ of the operator multiplying $\beta(g,\epsilon)$.

We are now able to obtain one of our main results. When eq. (16) is inserted into eq. (11) and the limit $\epsilon \to 0$ is performed, one gets

$$\Theta_{\mu\mu}\Big|_{\text{gauge inv.}} \underset{\epsilon\to 0}{\to} \frac{\beta(g,0)}{g}\ \tfrac{1}{2}F_{\lambda\rho}F_{\lambda\rho}\Big|_{\text{ren.}} + (1 - \delta(g,0))m\overline{\psi}\psi\Big|_{\text{ren.}}$$

$$-\tfrac{3}{2}\left(\frac{\delta S}{\delta \psi}\psi + \frac{\delta S}{\delta \overline{\psi}}\overline{\psi}\right) . \tag{17}$$

In eq. (17) the trace of the energy momentum tensor $\Theta_{\mu\nu}$ in four dimensions and after insertion into a Green function of J_μ's is expressed as a linear combination of finite composite operators, and the coefficients are functions of the coupling constant known from the renormalization group analysis. The finite composite operators $\frac{1}{2}F_{\lambda\rho}F_{\lambda\rho}\,|_{\mathrm{ren.}}$ and $m\overline{\psi}\psi\,|_{\mathrm{ren.}}$ are defined by

$$\tfrac{1}{2}F_{\lambda\rho}F_{\lambda\rho}\Big|_{\mathrm{ren.}} = \lim_{\epsilon\to 0}\left\{\left(1 + g\frac{\partial}{\partial g}\ln\frac{\widetilde{Z}_1}{\widetilde{Z}_3 Z_3^{1/2}}\right)\tfrac{1}{2}Z_3 F_{\lambda\rho}F_{\lambda\rho} - g\frac{\partial \ln Z_4}{\partial g} Z_2 \right.$$

$$\left. \times\, (m + \delta m)\overline{\psi}\psi\right\} , \tag{18}$$

$$m\overline{\psi}\psi\Big|_{\mathrm{ren.}} = \lim_{\epsilon\to 0}\{(m + \delta m)Z_2\overline{\psi}\psi\} . \tag{19}$$

If gauge-invariant operators other than J_μ are present in the Green function it becomes necessary to modify the definition of $\frac{1}{2}F_{\lambda\rho}F_{\lambda\rho}\,|_{\mathrm{ren.}}$ (cf. ref. [8] eqs. (3.12) and (4.9)), and there will be extra terms in eq. (17) which in the dilatational Ward identity make the operators acquire their anomalous dimension. For Green functions of gauge-dependent operators one has a much more complicated situation; the detailed analysis can be carried out by the methods of ref. [8] but is too long to be given here. What happens is that $\Theta_{\mu\mu}$ besides having terms with the same structure as (17) but with a more involved definition of $\frac{1}{2}F_{\lambda\rho}F_{\lambda\rho}\,|_{\mathrm{ren.}}$ (essentially the right-hand side of ref. [8] eq. (3.13)) as well as terms producing the anomalous dimensions of the fields in the dilatational Ward identity, also contains finite composite operators of mass dimension 4 *without* a factor $\beta(g, 0)$ in front (cf. ref. [11]). The number of such operators increases if the momentum transfer of $\Theta_{\mu\mu}$ is non-zero, as can be seen already from the third terms on the right-hand side of (9). This is related to the fact that the usual simple connection between the breaking of dilatational and conformal invariance no longer exists when gauge dependent operators occur in the Green function [8].

A couple of other comments are in order here:

(i) Eq. (17) carries over to quantum electrodynamics, if the fields and their coupling are changed appropriately and the following substitution is made:

$$\frac{\widetilde{Z}_1}{\widetilde{Z}_3} \to 1 . \tag{20}$$

(ii) Eq. (18) shows that the operator $\frac{1}{2}Z_3 F_{\lambda\rho}F_{\lambda\rho}$ even inside a gauge-invariant Green function is not multiplicatively renormalizable but also needs a subtractive renormalization.

The factor $\beta(g, 0)$ in front of the first term of (17) means that if g is chosen such that β is zero than $\Theta_{\mu\mu}$ is asymptotically "soft" in the sense familiar from the scalar ϕ^4 theory [12]. The present theory is, however, known to be asymptotically free [1], and it is thus of more interest to know what happens when β is non-zero but g is in the domain of attraction of the ultraviolet-stable fix point at the origin.

This question can be studied by means of the renormalization group equation [9,10]. The energy-momentum tensor $\Theta_{\mu\nu}$ as well as the electromagnetic current J_μ are not renormalized and therefore have no anomalous dimensions, so we can immediately write the solution of the renormalization group equation for Green function with n currents and one insertion of $\Theta_{\mu\nu}$ (here λ is a real variable):

$$\langle 0 | T\Theta_{\mu\nu}\left(\frac{x}{\lambda}\right) J_{\mu_1}\left(\frac{x_1}{\lambda}\right) \dots J_{\mu_n}\left(\frac{x_n}{\lambda}\right) |0\rangle \Big|_{m,g}$$

$$= \lambda^{4+3n} \langle 0 | T\Theta_{\mu\nu}(x) J_{\mu_1}(x_1) \dots J_{\mu_n}(x_n) |0\rangle \Big|_{m(\lambda), g(\lambda)} , \tag{21}$$

with:

$$\lambda \frac{\mathrm{d}g(\lambda)}{\mathrm{d}\lambda} = \beta(g(\lambda), 0); g(1) = g , \tag{22}$$

$$\lambda \frac{\mathrm{d}m(\lambda)}{\mathrm{d}\lambda} = -(1 - \delta(g(\lambda)))m(\lambda); m(1) = m . \tag{23}$$

The factor λ^{4+3n} gives the canonical scaling behaviour of the Green function, and the subscripts indicate which values of the parameters m and g are appropriate for the evaluation of the Green function in question. Note that we need not specify the value of the gauge parameter ξ in (21); in fact, it can be shown by means of the Slavnov-Taylor identity [6] that the Green functions occurring here are independent of ξ. In the case $n = 2$ there will be an extra contact term in the renormalization group equation, and eq. (21) is only valid when this is neglected, i.e. when x does not coincide with x_1 and x_2.

From eq. (21) the corresponding relation for the trace immediately follows, and from eq. (17) one then finds that the Green function $\langle 0 | T\Theta_{\mu\mu}(x) J_{\mu_1}(x_1) \dots J_{\mu_n}(x_n) |0\rangle$ at short distances at least is suppressed by a factor $\beta(g(\lambda))/g(\lambda)$ compared with the canonical behaviour, when contact terms are disregarded. With

$$\beta(g) = -b_0 g^3 + O(g^5) , \qquad b_0 = \frac{5}{12\pi^2} > 0 , \tag{24}$$

one has for large λ

$$\frac{2\beta(g(\lambda))}{g(\lambda)} \simeq -\frac{1}{\ln \lambda} , \tag{25}$$

so the suppression is only logarithmical. But this is sufficient to make possible the analysis of the trace anomaly of $\langle 0 | T\Theta_{\mu\mu}(x) J_{\mu_1}(x_1) J_{\mu_2}(x_2) | 0 \rangle$ due to Crewther, Chanowitz and Ellis [13] to the extent that we can define the amplitude (independent of ϵ):

$$F(0) = -\tfrac{1}{3}\pi \int_{R(\epsilon)} \mathrm{d}^4x \mathrm{d}^4y \, xy \langle 0 | T\Theta_{\mu\mu}(x) J_\lambda(0) J_\lambda(y) | 0 \rangle + O\left(\frac{1}{\ln \epsilon}\right) , \tag{26}$$

(where the integrand has been continued analytically to Euclidean space and the subscript $R(\epsilon)$ means that the integration variables are restricted by $|x| > \epsilon$, $|y| > \epsilon$, or $|x - y| > \epsilon$), and arrive at the conclusion

$$F(0) = \frac{2q^2}{3\pi} . \tag{27}$$

If several quarks are present one must substitute here for q^2 the sum of the squares of the charges.

Had we been interested in a Green function containing also gauge-invariant operators having would-be anomalous dimensions then there would be a competing logarithmic factor, and the question of asymptotic behaviour would not be so clear-cut as in the example treated above.

The situation is different if the physical coupling constant is so large that it is in the domain of attraction of a non-trivial ultraviolet first-order fixed point g_∞. Eq. (22) then implies that for large λ:

$$\beta(g(\lambda)) \propto \lambda^{\beta'(g_\infty)} , \tag{28}$$

where $\beta'(g_\infty) < 0$, and $\Theta_{\mu\mu}$ will thus, apart from contact terms, have an effective scaling dimension $4 + \beta'(g_\infty)$ when inserted into a gauge-invariant Green function, leading to a suppression by a power relative to the canonical scaling law.

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Note added in proof. After this work was completed, we learned that essentially identical results have been obtained by Adler et al. [14] for QED, and by Collins et al. [15] for the non-Abelian theory. Also, a term of $\Theta_{\mu\mu}$ similar to the first term of eq. (17) has been determined by Minkowski [16] by a different method.

References

[1] G. 't Hooft, Nucl. Phys. B62 (1973) 444;
H.D. Politzer, Phys. Rev. Letters 30 (1973) 1346;
D.J. Gross and F. Wilczek, Phys. Rev. Letters 30 (1973) 1343.

[2] G. 't Hooft and M. Veltman, Nucl. Phys. B44 (1972) 189;
C.G. Bollini and J.J. Giambiagi, Phys. Letters 40B (1972) 566;
J.F. Ashmore, Nuovo Cimento Letters 4 (1972) 289;
G. 't Hooft, Nucl. Phys. B61 (1973) 455.
[3] L.D. Faddeev and V.N. Popov, Phys. Letters 25B (1967) 29.
[4] D.Z. Freedman, I.J. Muzinich and E.J. Weinberg, Ann. of Phys. 87 (1974) 95;
D.Z. Freedman and E.J. Weinberg, Ann. of Phys. 87 (1974) 354.
[5] J.A. Dixon and J.C. Taylor, Nucl. Phys. B78 (1974) 552;
H. Kluberg-Stern and J.B. Zuber, Phys. Rev. D12 (1975) 467, 482, 3159;
S.D. Joglekar and B.W. Lee, Ann. of Phys. 97 (1976) 160.
[6] A.A. Slavnov, Theor. Math. Phys. 10 (1972) 99;
J.C. Taylor, Nucl. Phys. B33 (1971) 436;
J. Zinn-Justin, Lecture notes in physics, vol. 37 (Springer, Berlin, 1975).
[7] C. Becchi, A. Rouet and R. Stora, Comm. Math. Phys. 42 (1975) 127; Ann. of Phys. 98 (1976) 287.
[8] N.K. Nielsen, Nucl. Phys. B97 (1975) 527.
[9] M. Gell-Mann and F.E. Low, Phys. Rev. 95 (1954) 130;
C.G. Callan, Phys. Rev. D2 (1970) 227;
K. Symanzik, Comm. Math. Phys. 18 (1970) 227.
[10] M.J. Holwerda, W.L. van Neerwen and R.P. van Royen, Nucl. Phys. B75 (1974) 302;
J.C. Collins and A.J. Macfarlane, Phys. Rev. D10 (1974) 1201.
[11] S. Sarkar, Phys. Letters 50B (1974) 499; Nucl. Phys. B83 (1974) 108.
[12] B. Schroer, Nuovo Cimento Letters 2 (1971) 867.
[13] R.J. Crewther, Phys. Rev. Letters 28 (1972) 1421;
M.S. Chanowitz and J. Ellis, Phys. Rev. D7 (1973) 2490.
[14] S.L. Adler, J.C. Collins and A. Duncan, Energy-momentum-tensor trace anomaly in spin-$\frac{1}{2}$ quantum electrodynamics, Princeton preprint (1976).
[15] J.C. Collins, A. Duncan and S.D. Joglekar, Trace and dilatation anomalies in gauge theories, preprint, COO 2220-88 (1976).
[16] P. Minkowski, On the anomalous divergence of the dilatation current in gauge theories, Bern preprint (1976).