ANOMALOUS SUPERSYMMETRY TRANSFORMATION OF SOME COMPOSITE OPERATORS IN SQCD

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The requirement that local products of fields be defined as appropriate limits of regularized operators, in a way compatible both with gauge invariance and with supersymmetry, leads to an anomalous supersymmetry transformation law of some (gauge-invariant) composite operators. We discuss an example of this and its application in supersymmetric QCD in four dimensions.

Local products of fields play as fundamental a role in supersymmetric theories as in conventional quantum field theories. Because of short-distance singularities, local products of operators are not well defined usually. In attempting to define consistently these quantities as suitable local limits of better defined (regularized) products, naïve Ward–Takahashi identities involving composite operators are sometimes forced to fail. The axial anomaly [1] is a famous example among these.

In this note, we discuss what appears to be an anomalous supersymmetry transformation law of certain (gauge-invariant) local operator products in supersymmetric gauge theories. These arise as follows. We require that local products of two field operators be defined, as in the conventional field theories, as local limits of appropriately regularized operators. We further require that this must be done in a way compatible both with gauge invariance and with supersymmetry. This can be realized for instance by first constructing non-local superfields which are invariant under the generalized gauge transformations and which, in a naïve local limit, reduce to the local products we are interested in. They contain the supersymmetric version of usual path-ordered integrals.

Correct local limits can then be studied in each component by sending the point-splitting distance ϵ^μ to zero. Extra contribution of the path-ordered integral is superficially of order $O(\epsilon)$, and would vanish if the limit $\epsilon \to 0$ would encounter no singular behaviour. In reality, it turns out that because of the interactions, the relevant operator products do become singular linearly, and consequently finite terms arise from the extra contributions coming from the path-ordered integral.

As an independent check, the analysis is repeated by using the Pauli–Villars regularization method. The result of this analysis confirms the answer obtained by the point-splitting method as outlined above.

To be definite, we consider the supersymmetric version of quantum chromodynamics with SU(N) gauge group in four dimensions (SQCD) [2]. Gauge multiplet is described by a vector superfield V, and left-handed matter fields are two chiral superfields ($i = 1, ..., N$),

$$\Phi^i(x, \theta, \bar\theta) = (\phi(x) + \sqrt{2}\theta\psi(x) + \theta\theta F_\phi(x) + ...)^i , \qquad \chi_i(x, \theta, \bar\theta) = (\eta(x) + \sqrt{2}\theta\chi(x) + \theta\theta F_\eta(x) + ...)_i , \tag{1}$$

transforming as N and N^* colour representations, respectively [‡1].

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[‡1] We follow the convention and notation of ref. [3] except that we use the Bjorken–Drell metric. Flavour structure plays no essential role in the following discussion, and hence flavour indices will be suppressed.

The example of the anomalous transformation law we discuss below, is ($\bar{Q}_{\dot{\alpha}}$ = a generator of supersymmetry transformation)

$$\{\bar{Q}_{\dot{\alpha}}, \bar{\psi}^{\dot{\alpha}}(x)\,\phi(x)\}/2\sqrt{2} = -m\eta(x)\,\phi(x) + (g^2/32\pi^2)\,\lambda(x)\,\lambda(x)\,, \tag{2}$$

where all products are colour-singlet (e.g., $\eta\phi \equiv \eta_i\phi^i$; $\lambda\lambda \equiv \Sigma_{a=1}^{N^2-1}\lambda^a\lambda^a$). $\lambda(x)$ is the fermion partner (gluino) of the gauge boson field. (In the Wess–Zumino gauge, $V = \theta\sigma^\mu\bar{\theta}v_\mu(x) - \mathrm{i}\bar{\theta}\bar{\theta}\theta\lambda + \ldots$.) Eq. (2) is rather surprising in that the superpartner of a matter composite field $\bar{\psi}(x)\,\phi(x)$ contains an anomalous piece made of the gluino fields. Eq. (2), being an operator equation, can be used to derive an infinite number of Ward–Takahashi-like identities among various n-point functions.

As a simplest application of eq. (2), let us take the vacuum expectation value of both sides. Supersymmetry is not spontaneously broken for $m \neq 0$ [4], hence one finds a relation

$$m\langle\eta\phi\rangle = \langle(g^2/32\pi^2)\,\lambda\lambda\rangle\,. \tag{3}$$

(The same relation holds for each flavour, in the multi-flavour case.) Notice that if it were not for the anomalous right-hand side, one would have concluded that $\langle\eta\phi\rangle = 0$ for all non-vanishing mass m [‡2]. Since this order parameter is related through a generalized Dashen's formula due to Veneziano [5] [‡3] to the masses and couplings of pseudo-Goldstone particles, the implications of $\langle\eta\phi\rangle = 0$ would be very peculiar.

Actually, relation (3) has been obtained first in an effective lagrangian approach to SQCD [6]. Requiring that $\mathcal{L}_{\mathrm{eff}}$ reproduce correctly anomalous chiral and R symmetry properties, they find that the f term ($\mathcal{L}_{\mathrm{eff}} = \int \mathrm{d}^2\theta f +$ h.c. + kinetic terms) must satisfy [6]

$$T\partial f/\partial T = S - mT\,, \tag{4}$$

where the superfields S and T are given as $S \equiv -(g^2/32\pi^2)\,W^\alpha W_\alpha = (g^2/32\pi^2)\{\lambda\lambda + \ldots\}$, $T \equiv \chi\Phi = \eta\phi + \ldots$. The lowest component of eq. (4), computed at the minimum of the potential, is precisely eq. (3) [‡4]. This fact suggests a close connection between the anomaly in eq. (2) and axial (and, most likely, also supercurrent) anomaly. It implies also that the anomalous term in eq. (2) should be really there for consistency.

The proof of eq. (2) proceeds as follows. In order to ensure that we deal with gauge-invariant quantities, we first construct a non-local superfield $O(x; u; \theta, \bar{\theta})$,

$$O(x;u;\theta,\bar{\theta}) \equiv \chi_i(x,\theta,\bar{\theta})\,U^i{}_j(x,u;\theta,\bar{\theta})\,\Phi^j(u,\theta,\bar{\theta})\,, \tag{5a}$$

$$U(x,u;\theta,\bar{\theta}) \equiv \mathrm{P}\exp\left(\frac{\mathrm{i}}{4}\int_u^x \mathrm{d}z^\mu\,\bar{\sigma}_\mu^{\dot{\beta}\alpha}\bar{D}_{\dot{\beta}}\,\mathrm{e}^{-V}D_\alpha\,\mathrm{e}^V\right). \tag{5b}$$

(P ≡ path ordering; in the abelian case one replaces $\bar{D}_{\dot{\beta}}\,\mathrm{e}^{-V}D_\alpha\,\mathrm{e}^V$ by $\bar{D}_{\dot{\beta}}D_\alpha V$ [‡5].) Under the generalized local gauge transformation,

$$\Phi \to \mathrm{e}^{-\mathrm{i}\Lambda}\Phi\,, \qquad \chi^{\mathrm{T}} \to \chi^{\mathrm{T}}\,\mathrm{e}^{\mathrm{i}\Lambda}\,, \qquad \mathrm{e}^V \to \mathrm{e}^{-\mathrm{i}\Lambda^\dagger}\,\mathrm{e}^V\,\mathrm{e}^{\mathrm{i}\Lambda}\,, \tag{6}$$

where $\Lambda(x,\theta,\bar{\theta})$ is an arbitrary chiral superfield, $\bar{D}_{\dot{\beta}}\mathrm{e}^{-V}D_\alpha\,\mathrm{e}^V$ transforms as

$$\bar{D}_{\dot{\beta}}\mathrm{e}^{-V}D_\alpha\,\mathrm{e}^V \to \mathrm{e}^{-\mathrm{i}\Lambda}(\bar{D}_{\dot{\beta}}\mathrm{e}^{-V}D_\alpha\,\mathrm{e}^V + \bar{D}_{\dot{\beta}}D_\alpha)\,\mathrm{e}^{\mathrm{i}\Lambda}\,. \tag{7}$$

‡2 The existence of this naïve argument, which triggered the present study, was taught to me by G. Veneziano.

‡3 We have checked that the derivation of the formulas of ref. [5] goes through after taking account of the anomalies discussed here. It is indeed the left-hand side of eq. (3) which enters the generalized Dashen's formula.

‡4 Eq. (3) can also be obtained more directly [7] by making use of anomalous (R-type) and non-anomalous [combination of $U_{\mathrm{R}}(1)$ and $U_{\mathrm{A}}(1)$] Ward–Takahashi identities.

‡5 String operators similar to U of eq. (5b) appeared in the literature [8].

Using the fact that $\bar{\sigma}_{\mu}^{\dot{\beta}\alpha}\bar{D}_{\dot{\beta}}D_{\alpha} \Rightarrow 4\mathrm{i}\partial_{\mu}$ when acting on a chiral superfield, it can be easily shown that $O(x;u;\theta,\bar{\theta})$ is invariant under the general local transformation, eq. (6).

Now we study the lowest component of O in the Wess–Zumino gauge. The exponent of eq. (5b) becomes

$$\bar{\sigma}_{\mu}^{\dot{\beta}\alpha}\bar{D}_{\dot{\beta}}\,\mathrm{e}^{-V}D_{\alpha}\mathrm{e}^{V}|_{\theta=\bar{\theta}=0} = -\bar{\sigma}_{\mu}^{\dot{\beta}\alpha}\sigma_{\alpha\dot{\beta}}^{\nu}v_{\nu} = -2v_{\mu}(z)\,. \tag{8}$$

Thus one finds

$$O(x;u;\theta=\bar{\theta}=0)|_{\mathrm{WZ\ gauge}} = \eta(x)\left[\mathrm{P}\exp\left(-\frac{\mathrm{i}}{2}\int_{u}^{x}\mathrm{d}z^{\mu}v_{\mu}(z)\right)\right]\phi(u)\,. \tag{9}$$

We recognize here the ordinary path-ordered integral for gauge-invariant non-local operators. In fact, O is just the non-local version of the local meson-like superfield $T \sim \chi\Phi$.

Let us then define the local product $\eta(x)\,\phi(x)$ [and similarly for other products appearing in eq. (2)] as

$$\eta(x)\,\phi(x) \equiv \lim_{\epsilon\to 0} O(x+\epsilon;x-\epsilon;\theta=\bar{\theta}=0)\,. \tag{10}$$

One finds (by denoting $U(x,u) \equiv \mathrm{P}\exp[-(\mathrm{i}/2)\int_u^x \mathrm{d}z^{\mu}v_{\mu}]$,

$$m\eta(x)\,\phi(x) = -\lim_{\epsilon\to 0} F^{*}(x+\epsilon)\,U(x+\epsilon,x-\epsilon)\,\phi(x-\epsilon) \tag{11a}$$

$$= -\lim_{\epsilon\to 0}\{\bar{Q},\bar{\psi}\}\,U\phi/2\sqrt{2} \tag{11b}$$

$$= -\lim_{\epsilon\to 0}[\{\bar{Q},\bar{\psi}U\phi\} - \epsilon^{\mu}\bar{\psi}^{\dot{\alpha}}(x+\epsilon)\,\epsilon_{\dot{\alpha}\dot{\beta}}\bar{\sigma}_{\mu}^{\dot{\beta}\alpha}\lambda_{\alpha}(x)\,\phi(x-\epsilon)]/2\sqrt{2}\,, \tag{11c}$$

where use was made of the equation of motion, and of the supersymmetry transformation properties, $\bar{Q}_{\dot{\alpha}}v_{\mu} = -\mathrm{i}\epsilon_{\dot{\alpha}\dot{\beta}}\bar{\sigma}_{\mu}^{\dot{\beta}\alpha}\lambda_{\alpha}$ and $\bar{Q}_{\dot{\alpha}}\phi = 0$.

The second term of eq. (11c) is apparently of order $\mathrm{O}(\epsilon)$: however, it is easy to show that the product $\bar{\psi}(x+\epsilon)\,\lambda(x)\,\phi(x-\epsilon)$ contains a linear singularity, $\sim(\epsilon^{\nu}/\epsilon^{2})$. Indeed, the lagrangian contains a Yukawa interaction vertex, $\mathrm{i}\phi^{\dagger}\lambda\psi/\sqrt{2}$, so that $\bar{\psi}(x+\epsilon)\,\lambda(x)\,\phi(x-\epsilon)$ behaves as (as $\epsilon\to 0$, $\epsilon^{0}>0$),

$$\lambda(x)\int \mathrm{d}^4z\,\langle T\psi(z)\,\bar{\psi}(x+\epsilon)\rangle\,\lambda(z)\langle T\phi(x-\epsilon)\,\phi^{\dagger}(z)\rangle \sim \lambda^2(x)\,\epsilon^4\epsilon^{-3}\epsilon^{-2} \sim \lambda^2(x)\,\epsilon^{-1}\,. \tag{12}$$

The exact form of the second term of eq. (11c) can be worked out by going to momentum space ‡6, and the final step is

$$-\tfrac{1}{4}\mathrm{i}g^2\epsilon_{\dot{\alpha}\dot{\beta}}\bar{\sigma}_{\mu}^{\dot{\beta}\alpha}\lambda_{\alpha i}^{j}(x)\,\lambda^{\beta i}{}_{j}(x)\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{\partial}{\partial k^{\mu}}\left[\frac{\sigma^{\nu}k_{\nu}}{(k^2-m^2)^2}\right]_{\beta}^{\dot{\alpha}} = (g^2/32\pi^2)\,\lambda^a(x)\,\lambda^a(x)\,. \tag{13}$$

[In eq. (13), we have rescaled the gauge multiplet, $V\to gV$.] (See fig. 1.) This proves eq. (2).

It is of crucial importance that we started with an operator which is invariant under the general gauge transformation, eq. (6) ‡7. In this way, one ensures that the result is not a gauge artefact.

The same result can be obtained by considering the $\bar{\theta}^2$ component of (a suitable non-local version of) the product $\Phi^{*}\mathrm{e}^{V}\Phi$ rather than $\chi\Phi$.

Up to now, we have adopted the point-splitting method in order to define properly the local products appearing

‡6 In fact, we follow closely the point-splitting method used in deriving the correct axial current divergence (see Jackiw [1]).

‡7 It seems to be more difficult to justify arguments [9] such as $\langle\bar{Q}(\lambda^{\alpha}\sigma_{\alpha\dot{\beta}}^{\mu}v_{\mu})\rangle \propto \langle\lambda\lambda\rangle = 0$, which does not satisfy this requirement. Indeed, the result of ref. [10] is a counter example in which supersymmetry is unbroken but $\langle\lambda\lambda\rangle \neq 0$.

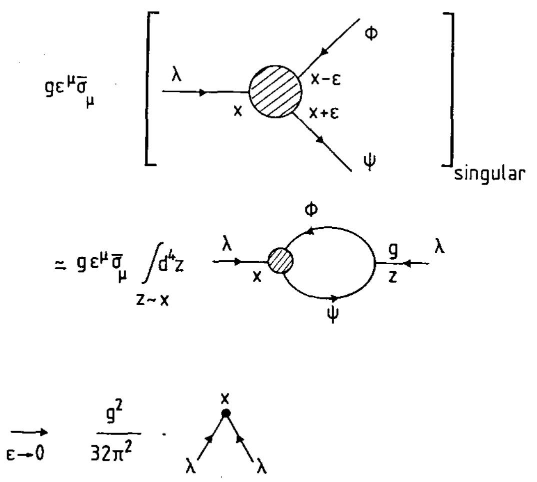

Fig. 1. Loop diagram which illustrates the emergence of the local operator $(g^2/32\pi^2)\ \lambda\lambda(x)$.

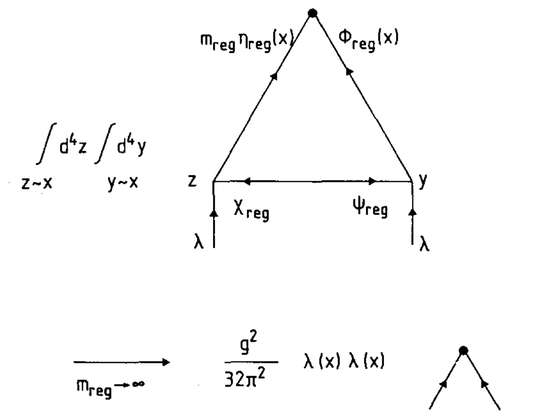

Fig. 2. The diagram which gives a finite contribution proportional to $(g^2/32\pi^2)\ \lambda\lambda(x)$ in $\lim_{m_{\mathrm{reg}}\to\infty} G_{\mathrm{reg}}$ in the Pauli–Villars method.

in eq. (2). The same result, if correct, must be obtained in any other regularization method which respects both gauge invariance and supersymmetry.

As one such example, we next consider the Pauli–Villars regularization. Suppose that the theory is regularized by introducing a set of regulator chiral superfields, Φ_{reg} and χ_{reg}, corresponding to Φ and χ of eq. (1), with a large mass m_{reg}. We are interested in computing general Green's functions involving the insertion of a composite operator, $F^*(x)\,\phi(x) = -m\eta(x)\,\phi(x)$.

In this regularized theory, for each given n-point function

$$G = \langle T\{[-m\eta(x)\,\phi(x)]\,A_1(z_1)\,A_2(z_2)\ldots\}\rangle \tag{14}$$

[the $A_i(z_i)$ are arbitrary fields], there exists a corresponding contribution of the regulator fields,

$$G_{\mathrm{reg}} = -\langle T\{[-m_{\mathrm{reg}}\eta_{\mathrm{reg}}(x)\,\phi_{\mathrm{reg}}(x)]\,A_1(z_1)\,A_2(z_2)\ldots\}\rangle\,. \tag{15}$$

This, compared to eq. (14), contains extra propagators of the regulator fields with large mass m_{reg}, and may be disregarded in the limit $m_{\mathrm{reg}}\to\infty$, unless the matrix element [the coefficient of m_{reg} in eq. (15)] behaves as m_{reg}^{-1}. In close analogy with the case of axial anomaly, this is precisely what happens here. Indeed, the diagram of fig. 2 gives by a simple calculation

$$G_{\mathrm{reg}} \underset{m_{\mathrm{reg}}\to\infty}{=} \langle T\{[(g^2/32\pi^2)\,\lambda(x)\,\lambda(x)]\,A_1(z_1)\,A_2(z_2)\ldots\}\rangle\,. \tag{16}$$

Thus the correct local operator product involves the combination

$$-m\eta(x)\,\phi(x) + (g^2/32\pi^2)\,\lambda(x)\,\lambda(x)\,,$$

which appeared in eq. (2).

The result discussed so far can be best formulated by saying that the $\bar\theta^2$ component of the (correctly defined) supermultiplet $\Phi^*\,\mathrm{e}^V\Phi$ is not given by the naïve answer, $F_\phi^*\phi = -m\eta\phi$, but by the sum including $(g^2/32\pi^2)\ \lambda\lambda$.

Other components of $\Phi^*\,\mathrm{e}^V\Phi$ can be studied in a similar manner [‡8], using the Pauli–Villars regularization. The

[‡8] I thank T.R. Taylor and G. Veneziano for collaboration in this part of the work.

final answer is

$$\Phi^* e^V \Phi = (\Phi^* e^V \Phi)_{\text{naïve}} - (g^2/32\pi^2)[\bar{\theta}^2 W^\alpha W_\alpha + \theta^2 \bar{W}_{\dot\alpha} \bar{W}^{\dot\alpha} - \tfrac{1}{2}\theta^2\bar{\theta}^2 (WW|_{\theta^2} + \bar{W}\bar{W}|_{\bar{\theta}^2})] \tag{17}$$

(in Wess–Zumino gauge). A similar result holds for the product $\chi^* e^{-V}\chi$.

The right-hand side of eq. (17) contains the anomalous terms which appear to break supersymmetry. In fact, they are just what is needed to restore the supermultiplet structure of local product, $\Phi^* e^V \Phi$.

Notice that lowest components of $\Phi^* e^V \Phi$ have no anomalous pieces. This is related to the fact that there exist no gauge-invariant operators made of gauge fields with dimension less than three.

If renormalization of local operator products is performed by using a momentum space subtraction scheme [11], $(\Phi^* e^V \Phi)_{\text{naïve}}$ of eq. (17) should be regarded as the normal product defined without over subtractions. One needs then to add an anomalous second term to construct a multiplet. (On the other hand, one could define [11] normal products of different kinds in which more subtractions are performed than is required to render Green's functions finite. In such a case, the second term would appear automatically as an effect of renormalization. This way of interpreting anomaly is, of course, possible also in the case of axial current divergence.)

The $\theta^2\bar{\theta}^2$ component of eq. (17) is of particular interest. The right-hand side contains a term proportional to $\mathcal{L}_{\text{gauge}} = \frac{1}{4}(W^\alpha W_\alpha|_{\theta^2} + \text{h.c.})$. It might thus look as if $\mathcal{L}_{\text{matter}}$, if properly defined, already contains a part proportional to $\mathcal{L}_{\text{gauge}}$. Actually, this is not the case. $\mathcal{L}_{\text{matter}}$ is a sum

$$\Phi^* e^V \Phi|_{\theta^2\bar{\theta}^2} + (\Phi \leftrightarrow \chi,\ V \leftrightarrow -V) + m\chi\Phi|_{\theta^2} + \text{h.c.}$$

After taking account of the contributions coming from $m\chi\Phi|_{\theta^2}$ + h.c., one finds that anomalous terms proportional to $\mathcal{L}_{\text{gauge}}$ cancel exactly.

In conclusion, the requirement that local product of operators be defined as limits of suitably regularized quantities, in a manner compatible both with gauge invariance and with supersymmetry, leads to anomalous relationship among superpartners of certain composite operators. Further implications of this will be discussed elsewhere.

It is a great pleasure to express my thanks to G. Veneziano for generously imparting his knowledge and insights to me through numerous discussions on the subject. Useful discussions with S. Ferrara, K. Shizuya and T.R. Taylor are also gratefully acknowledged.

Note added. After finishing this work, I was informed that Clark and others had obtained in supersymmetric QED a result [12] which reads, in our notation,

$$\tfrac{1}{4}\bar{D}^2(\Phi^* e^V \Phi) = m\chi\Phi + (g^2/32\pi^2)\, W^\alpha W_\alpha\,.$$

This can be obtained by applying $\bar{D}^2$ on our eq. (17). In this case the equation of motion involving $\Phi^* e^V \Phi$ looks anomalous. I believe that our methods of derivation, the extension of the result to the non-abelian case, the interpretation of the result (as an anomaly in the supermultiplet structure of the composite operator $\Phi^* e^V \Phi$), and the realization that eqs. (2) or (17) might have possible applications in strongly interacting theories [as in the discussion of eq. (3)], are new. The work of Clark et al., furthermore, suggests that eqs. (2) and (17) have no radiative corrections of higher order. I thank T.R. Taylor for bringing ref. [12] to my attention.

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