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SUPERFIELD SUPERGRAVITY *

Warren SIEGEL and S. James GATES Jr. **
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

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A constraint-free superfield formalism for supergravity is described. We expand the superfield action in component fields to obtain the first derivation of the standard component-field supergravity action from a superfield formalism. The superfield formalism of supergravity is analogous to the superfield formalism of globally supersymmetric gauge theories, the gauge group here being the superspace translation group.

1. Introduction

In previous papers [1–4] one of the authors found a constraint-free superfield action for supergravity, thus providing the first complete superfield formalism for supergravity. The supergravity supervector superfield $U^M(x, \theta, \bar{\theta})$, the gauge field of the supertranslation group [1], contains no fields with spin >2, allowing the action to be expressed in terms of it without constraints [2]. The formalism is analogous to that of globally supersymmetric gauge theories in that the supergravity superfield always occurs in the form of an exponential $\exp(2U^M i\partial_M)$, where the partial derivatives $i\partial_M$ act as the generators of the superspace translation group. Upon choosing a special gauge, the spinorial translations ∂_μ pick up extra terms to become the usual supersymmetry generators $Q_\mu = \partial_\mu - \bar{\theta}^{\dot{\nu}} \sigma^m_{\mu\dot{\nu}} i\partial_m + ...$, just as in globally supersymmetric gauge theories $Q_\mu = \partial_\mu - \bar{\theta}^{\dot{\nu}} \sigma^m_{\mu\dot{\nu}} i\partial_m$ become $Q_\mu = \partial_\mu - \bar{\theta}^{\dot{\nu}} \sigma^m_{\mu\dot{\nu}}(i\partial_m + v_m)$ in a special gauge [5].

In sect. 2 we review the superfield formalism of globally supersymmetric gauge theories in a form in which chirality conditions and supergauge transformation laws are more transparent. We show that x-derivatives always occur explicitly in the form $e^{2\langle U\rangle}$, where $\langle U\rangle = \theta^\mu \bar{\theta}^{\dot{\nu}} \sigma^m_{\mu\dot{\nu}} i\partial_m$, and that these derivatives are made supergauge covariant simply by adding the gauge superfield V to $\langle U\rangle$. Then we describe the covariant derivatives ∇_A [6] in terms of which the gauge superfield's own action has a simple form.

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** Junior Fellow, Harvard Society of Fellows.

Sect. 3 describes the superfield supergravity formalism. We work in close analogy to sect. 2, giving the superfield form of the local supersymmetry (and local superconformal) group, the special gauge, the covariant derivatives, and the action. Actually, we find an entire class of supergravity actions, parametrized by a real number n, corresponding to choosing the vierbein multiplied by various powers of its determinant as the fundamental spin-2 field. Only for $n = -\frac{1}{3}$ do we find the minimal set of auxiliary fields [7]; for other n we have Breitenlohner's set of auxiliary fields [8]. In this section we also discuss the coupling to the chiral supermultiplet [3], including the extension to a locally superconformal theory with a cosmological term [9]. In addition, we give the general solution [10] to the constraints in Wess and Zumino's superfield supergravity formalism [6,11], and find the resulting form of their action is identical to the $n = -\frac{1}{3}$ case of our superconformally-extended action. We then generalize their covariant derivatives and constraints to arbitrary n, and use them to construct the actions for the vector supermultiplet and conformal supergravity in terms of our unconstrained superfields [10].

The $n = -\frac{1}{3}$ action is explicitly expanded in component fields in sect. 4, and their explicit supersymmetry transformations are given: the result is standard supergravity [12], with auxiliary fields [7]. We also compute the cosmological term in auxiliary-field form [13]. The $n \neq -\frac{1}{3}$ actions are found by use of the linear approximation and invariance arguments. The $n = -\frac{1}{3}$ linearized action in superfield form is exactly the Ogievetsky-Sokatchev action [14,15].

There are three appendices, discussing notational conventions and useful identities, some intermediate steps in the expansion of the action in component fields, and the relation of our formalism to that of Breitenlohner and that of Brink et al. [16] (which are equivalent to the case $n = -1$).

2. Global supersymmetry

2.1. The exponential

We will find the supergravity formalism by working in as close analogy as possible to the superfield formulation of globally supersymmetric gauge theories. In particular, this means that our *fundamental* fields will not be in the form of a supervierbein or superconnection, but in the form of an exponential $\exp(2U^M i\partial_M)$. As for non-supersymmetric gauge theories, we begin by generalizing a global internal symmetry of a matter action to a local symmetry. The kinetic action for a chiral superfield can be written as (see appendix A)

$$S = \int d^4x\, d^4\theta\, \bar{\phi}(x, \bar{\theta})\, e^{2\langle U\rangle} \phi(x, \theta)\,, \qquad \langle U\rangle = \theta^\mu \bar{\theta}^{\dot{\nu}} \sigma^m_{\mu\dot{\nu}} i\partial_m\,. \tag{2.1}$$

The chirality of ϕ is manifest in that it depends on θ^μ and not $\bar{\theta}^{\dot{\mu}}$ (chiral representation). For ϕ being an N-component vector, the global internal symmetry is the $U(N)$

transformation

$$\phi' = e^{i\Lambda}\phi\ , \qquad \Lambda = \bar{\Lambda} = \Lambda^i G_i\ , \tag{2.2}$$

where G_i are the hermitian matrix representation of the U(N) generators in the fundamental representation.

Since eq. (2.1) is the form of a Hilbert-space inner product with Hilbert-space metric $e^{2\langle U\rangle}$, the most general invariance is of the form

$$\phi'(x,\theta) = e^{iH}\phi(x,\theta)\ , \qquad e^{2\langle U\rangle}H = H^\dagger e^{2\langle U\rangle}\ , \qquad \bar{\partial}_{\dot\alpha} H \bar{\partial}^2 = 0\ , \tag{2.3}$$

of which eq. (2.2) is a special case. Another case is the global supersymmetry transformation: it is basically a θ-translation for ϕ, and so it has a piece $\epsilon^\alpha i\partial_\alpha$; to this we add the modified hermitian conjugage $e^{-2\langle U\rangle}(\epsilon^\alpha i\partial_\alpha)^\dagger\, e^{2\langle U\rangle}$ as prescribed by eq. (2.3) to obtain

$$\phi' = e^{i(\epsilon^\alpha iQ_\alpha + \bar{\epsilon}_{\dot\alpha} iQ^{\dot\alpha})}\,\phi\ , \qquad Q_\alpha = \partial_\alpha\ ,$$

$$Q^{\dot\alpha} = e^{-2\langle U\rangle}\bar{\partial}^{\dot\alpha}\, e^{2\langle U\rangle} = \bar{\partial}^{\dot\alpha} + 2i\theta^\beta \sigma_\beta^{a\dot\alpha}\,\partial_a\ . \tag{2.4}$$

Q_α and $Q^{\dot\alpha}$ are therefore the flat-space chiral-representation supersymmetry generators. (Of course, x-translations are generated by ∂_a, as usual.)

In order to introduce local internal symmetry, *we gauge-covariantize by adding a gauge superfield V to the derivative* $\langle U\rangle$:

$$e^{2\langle U\rangle} \to e^{2(\langle U\rangle + V)}\ , \qquad V(x,\theta,\bar\theta) = V^i G_i = \bar{V}\ , \tag{2.5a}$$

$$\phi'(x,\theta) = e^{i\Lambda(x,\theta)}\phi(x,\theta)\ , \qquad e^{2(\langle U\rangle + V')} = e^{i\bar\Lambda}\, e^{2(\langle U\rangle+V)}\, e^{-i\Lambda}\ ,$$

$$\Lambda = \Lambda^i G_i\ . \tag{2.5b}$$

(Λ is a chiral superparameter in order to maintain the chirality of ϕ.) By making all x-derivatives explicit, we have clarified the nature of V as a gauge field. In particular, the $\theta^\mu\bar\theta^{\dot\nu} v_{\mu\dot\nu}$ part of V occurs as $\theta^\mu\bar\theta^{\dot\nu}\sigma^m_{\mu\dot\nu}(i\partial_m + v_m)$, showing that *the ordinary gauge field v_m appears only in the form of the familiar covariant derivative of non-supersymmetric gauge theories.* In more conventional notation (the vector representation), V is rewritten as $e^{2(\langle U\rangle+V)} = e^{\langle U\rangle}\, e^{2\tilde V} e^{\langle U\rangle}$ (where $\tilde V$ is still of the form $V^i G_i$ by the Baker-Hausdorff theorem), and the $e^{\langle U\rangle}$'s are absorbed into ϕ and $\bar\phi$, making both the chirality condition on ϕ and the derivatives in the supergauge transformation law more obscure.

By eq. (2.4), V must transform under global supersymmetry transformations as

$$e^{2(\langle U\rangle+V')} = \exp[i(\bar\epsilon_{\dot\alpha} i\bar\partial^{\dot\alpha} + \epsilon^\alpha e^{2\langle U\rangle} i\partial_\alpha e^{-2\langle U\rangle})]\ e^{2(\langle U\rangle+V)}$$

$$\times \exp[-i(\epsilon^\alpha i\partial_\alpha + \bar\epsilon_{\dot\alpha} e^{-2\langle U\rangle} i\bar\partial^{\dot\alpha} e^{2\langle U\rangle})]\ . \tag{2.6}$$

Using the identity

$$\delta\, e^X = Y e^X + e^X Z \rightarrow \delta X = L_{X/2}\,[(Z - Y) + (\coth L_{X/2})(Z + Y)]\ ,$$

$$L_X Y \equiv [X, Y]\ , \tag{2.7}$$

we therefore have the infinitesimal transformation

$$\delta V = -[\epsilon^\alpha(\partial_\alpha - i\bar\theta^{\dot\beta}\sigma^a_{\alpha\dot\beta}\partial_a) + \bar\epsilon_{\dot\alpha}(\bar\partial^{\dot\alpha} + i\theta^\beta\sigma^{a\dot\alpha}_\beta\partial_a)]\ V$$

$$+ (L_{\langle U\rangle+V}\coth L_{\langle U\rangle+V} - 1)\ 2i(\epsilon^\alpha\bar\theta^{\dot\beta} - \bar\epsilon^{\dot\beta}\theta^\alpha)\ \sigma^a_{\alpha\dot\beta}\partial_a\ . \tag{2.8}$$

If we had used the standard gauge superfield $\widetilde{V}$, we would have only the first term. Global supersymmetry transformations are the only feature of the vector representation which is simpler than in the chiral representation; however, when we extend supersymmetry to a local symmetry, even the supersymmetry transformations will be simpler in the chiral representation, since they will then also be supergauge transformations similar to those in eq. (2.5).

The exponential $e^{2(\langle U\rangle+V)}$ (and therefore all actions which depend on V) becomes polynomial in the Wess-Zumino gauge [5]. In this gauge the lowest order of θ in V is $\theta\bar\theta$, so $V^3 = 0$. This can be seen from the infinitesimal supergauge transformation (eqs. (2.5) and (2.7))

$$\delta V = -\tfrac{1}{2}iL_{\langle U\rangle+V}[(\Lambda + \bar\Lambda) + \coth(L_{\langle U\rangle+V})(\Lambda - \bar\Lambda)] \tag{2.9}$$

$$= -\tfrac{1}{2}i\{(\Lambda - \bar\Lambda) + [\langle U\rangle + V, \Lambda + \bar\Lambda] + \tfrac{1}{3}[\langle U\rangle + V, [\langle U\rangle + V, \Lambda - \bar\Lambda]] + \ldots\}\ .$$

The simplest gauge is thus obtained by using $-\frac{1}{2}i(\Lambda - \bar\Lambda)$ to gauge away the corresponding parts of V (those which are coefficients of $1, \theta, \bar\theta, \theta^2$, and $\bar\theta^2$). More precisely, Λ can be expressed as a power series in V, and the preceding sentence describes how the lowest (i.e., first) order in V is determined, while higher orders are determined inductively [17]. The remaining gauge freedom is then given simply by

$$\Lambda(x, \theta) = \bar\Lambda(x, \bar\theta) = \lambda(x) = \bar\lambda(x)\ , \qquad \delta V = -i[V, \lambda] - i\langle U\rangle\lambda\ . \tag{2.10}$$

The usual non-supersymmetric gauge invariance is now obvious.

2.2. *Covariant derivatives*

To discuss the action for the gauge superfield V itself, it is convenient to introduce a covariant derivative ∇_A [6]. The only way to construct covariant quantities from $e^{2(\langle U\rangle+V)}$ is to combine it with derivatives as

$$e^{-2(\langle U\rangle+V)}\partial_A\, e^{2(\langle U\rangle+V)} = \exp(-2L_{\langle U\rangle+V})\,\partial_A\ .$$

Due to the transformation law of eq. (2.5), we see that only

$$\nabla_\alpha \equiv e^{-2(\langle U\rangle+V)}\partial_\alpha\, e^{2(\langle U\rangle+V)}$$

will transform simply, as $\nabla'_\alpha = e^{i\Lambda}\nabla_\alpha e^{-i\Lambda}$. Since $\nabla_{\dot\alpha} \equiv \bar\partial_\alpha$ is also invariant under this transformation, we will use ∇_α, $\nabla_{\dot\alpha}$, and their graded commutators to construct covariant operators. In particular, we choose $\nabla_a \equiv \frac{1}{4}i\sigma_a^{\alpha\dot\beta}\{\nabla_\alpha, \nabla_{\dot\beta}\}$, which is the simplest choice, has the right dimension, and becomes the ordinary derivative ∂_a in empty space. We now have

$$\nabla_\alpha = e^{-2(\langle U\rangle+V)}\partial_\alpha e^{2(\langle U\rangle+V)}\,,\quad \nabla_{\dot\alpha} = \bar\partial_{\dot\alpha}\,,\quad \nabla_a = \tfrac{1}{4}i\sigma_a^{\alpha\dot\beta}\{\nabla_\alpha, \nabla_{\dot\beta}\}\,, \tag{2.11a}$$

$$\nabla_A = D_A + i\Gamma_A\,,\quad (i\nabla)_A^\dagger = e^{2(\langle U\rangle+V)}i\nabla_A e^{-2(\langle U\rangle+V)}\,,\quad \Gamma_A = \Gamma_A^i G_i\,, \tag{2.11b}$$

$$[D_A, D_B\} = T_{AB}{}^C D_C\,,\quad [\nabla_A, \nabla_B\} = T_{AB}{}^C\nabla_C + iF_{AB}\,,\quad F_{AB} = F_{AB}{}^i G_i\,, \tag{2.11c}$$

$$T_{\alpha\dot\beta}{}^c = T_{\dot\beta\alpha}{}^c = -2i\sigma^c{}_{\alpha\dot\beta}\,,\quad \text{other components} = 0\,,\quad F_{\alpha\beta} = F_{\alpha\dot\beta} = 0\,, \tag{2.11d}$$

$$S = \frac{1}{g^2}\,\mathrm{tr}\int d^4x\, d^2\theta\, F_{\dot\beta a}F^{\dot\beta a} + \int d^4x\, d^4\theta\, \bar\phi\, e^{2(\langle U\rangle+V)}\phi\,,$$

$$F_{\dot\beta a} = \tfrac{1}{4}i\sigma^{\alpha}_{a\dot\beta}W_\alpha\,,\quad \bar\partial_{\dot\beta}W_\alpha = 0\,, \tag{2.11e}$$

$$\nabla_A{}' = e^{i\Lambda}\nabla_A e^{-i\Lambda}\,, \tag{2.11f}$$

where D_A are empty-space covariant derivatives in the chiral representation (see appendix A). Note that in the chiral representation we have the modified hermiticity condition $\bar f = e^{2(\langle U\rangle+V)}f e^{-2(\langle U\rangle+V)}$, which is also the condition which would be satisfied by a non-gauge "real" superfield (although V satisfies $V = \bar V$). The gauge superfield's action has been constructed from the covariants F_{AB} by dimensional arguments: Since θ has dimensions of $(\text{mass})^{-1/2}$ and the lower dimensional quantities $F_{\alpha\beta}$ and $F_{\alpha\dot\beta}$ vanish, we must construct the action using $\int d^2\theta$ (instead of $\int d^4\theta$; $d\theta \sim \partial/\partial\theta \sim (\text{mass})^{1/2}$), and therefore use the chiral object $F_{\dot\beta a}$.

The covariant derivatives can also be derived from a system of constraints in the manifestly hermitian vector representation:

$$(i\tilde\nabla)_A^\dagger = i\tilde\nabla_A\,,\quad \tilde F_{\alpha\beta} = \tilde F_{\alpha\dot\beta} = 0\,,\quad \tilde\nabla_{\dot\alpha}\tilde\phi(x,\theta,\bar\theta) = 0\,,\quad T_{AB}{}^C \text{ as above}\,, \tag{2.12a}$$

$$S = \frac{1}{g^2}\,\mathrm{tr}\int d^4x\, d^2\theta\, \tilde F_{\dot\beta a}\tilde F^{\dot\beta a} + \int d^4x\, d^4\theta\, \bar{\tilde\phi}\tilde\phi\,,\quad \tilde\nabla_{\dot\beta}\tilde W_\alpha = 0\,, \tag{2.12b}$$

$$\tilde\nabla_A{}' = e^{iK}\tilde\nabla_A e^{-iK}\,,\quad \tilde\phi' = e^{iK}\tilde\phi\,,\quad K = \bar K\,, \tag{2.12c}$$

which have the solution

$$\tilde\nabla_\alpha = e^{-\langle U\rangle - W}\partial_\alpha e^{\langle U\rangle+W}\,,\quad \tilde\nabla_{\dot\alpha} = e^{\langle U\rangle+\bar W}\bar\partial_{\dot\alpha}e^{-\langle U\rangle-\bar W}\,,$$

$$\tilde\nabla_a = \tfrac{1}{4}i\sigma_a^{\beta\dot\gamma}\{\tilde\nabla_\beta, \tilde\nabla_{\dot\gamma}\}\,,\quad W \neq \bar W\,, \tag{2.13a}$$

$$e^{\langle U\rangle + W'} = e^{i\bar{\Lambda}} e^{\langle U\rangle + W} e^{-iK} , \quad \Lambda = \Lambda(x, \theta) ,$$

$$\tilde{\phi} = e^{\langle U\rangle + \bar{W}} \phi(x, \theta) , \quad \phi' = e^{i\Lambda} \phi . \tag{2.13b}$$

This is analogous to describing a photon in terms of the field strength F_{mn} with the constraint $\partial_{[m}F_{np]} = 0$, with the constraint's solution $F_{mn} = \partial_{[m}A_{n]}$ introducing the new invariance $\delta A_m = \partial_m \lambda$. (If we describe QED by a "gauge-invariant" path-dependent formalism [18], $e^{\langle U\rangle + W}$ is analogous to the path-dependent phase factor $\exp(i \int_P dx^m A_m)$ [19], Λ is analogous to the usual gauge transformations, and K is analogous to a gauge transformation in the form of a path transformation.) We can now transform to the chiral representation by the non-unitary transformation generated by $e^{-\langle U\rangle - \bar{W}}$:

$$\phi = e^{-\langle U\rangle - \bar{W}} \tilde{\phi} , \quad \nabla_A = e^{-\langle U\rangle - \bar{W}} \tilde{\nabla}_A e^{\langle U\rangle + \bar{W}} ,$$

$$e^{\langle U\rangle + W} e^{\langle U\rangle + \bar{W}} = e^{2(\langle U\rangle + V)} , \quad V = \bar{V} . \tag{2.14}$$

W now occurs in the theory only as V, and all quantities are unaffected by K transformations. Essentially, V is the result of using up the K transformations to gauge away the imaginary part of W. In fact, we could have stayed in the vector representation in the gauge $W = \bar{W} = V$, using only Λ-determined K transformations found from the equation analogous to eq. (2.9) with

$$K = \tfrac{1}{2}[(\Lambda + \bar{\Lambda}) + \tanh(\tfrac{1}{2}L_{\langle U\rangle + V})(\Lambda - \bar{\Lambda})] . \tag{2.15}$$

However, the chiral representation makes many properties more manifest:

(a) the matter Lagrangian, which in the vector representation is $\tilde{\bar{\phi}}\tilde{\phi}$, is now $\bar{\phi}\, e^{2(\langle U\rangle + V)} \phi$, so ϕ has no dependence on $\bar{\theta}$ or V and no derivatives;

(b) the supergauge transformation laws are directly in terms of Λ, instead of being expressed indirectly in terms of Λ through the less simple expression eq. (2.15); and

(c) the gauge field action of eq. (2.11e) is manifestly $\bar{\theta}$-independent, whereas in the vector representation one needs to know

$$L_{\text{vector}} = e^{\langle U\rangle + \bar{W}} L_{\text{chiral}}\, e^{-\langle U\rangle - \bar{W}} ,$$

$$\text{tr} \int d^4x\, e^{\langle U\rangle + \bar{W}} f\, e^{-\langle U\rangle - \bar{W}} = \text{tr} \int d^4x f .$$

3. Superfield formalism for supergravity

3.1. The exponential

We identify supergravity as the supersymmetric gauge theory of the superspace translation group by choosing the group generators $G_M = i\partial_M$ and rewriting $\langle U\rangle + V^M i\partial_M$ as $U = U^M i\partial_M$. (We do not introduce the Lorentz group independently because Lorentz transformations are already included as θ-dependent spinorial translations generated by $\theta_{(\alpha}\partial_{\beta)}$.) This is analogous to covariantizing a flat-space Lagrangian for a spinless field by the replacement

$$\partial_a \to \partial_a + h_a{}^m \partial_m = (\langle e_a{}^m \rangle + h_a{}^m)\, \partial_m = e_a{}^m \partial_m \ ,$$

where $e_a{}^m$ is the vierbein. To make the ϕ Lagrangian a scalar density, we will also need an extra factor (analogous to $(\det e_a{}^m)^{-1}$), but the explicit form of this factor need not concern us at this point. (It will be given in subsect. 3.3.) Of course, the action for U^M itself will differ greatly from the action for V above. First, we must examine the group structure, analogous to that of eq. (2.5):

$$\phi' = \mathrm{e}^{i\Lambda}\phi \ , \qquad \mathrm{e}^{2U'} = \mathrm{e}^{i\bar{\Lambda}}\, \mathrm{e}^{2U}\, \mathrm{e}^{-i\Lambda} \ , \qquad U(x,\theta,\bar{\theta}) = U^M i\partial_M = \bar{U}^M i\partial_M \ ,$$

$$\Lambda = \Lambda^M i\partial_M = \Lambda^m(x,\theta)\, i\partial_m + \Lambda^\mu(x,\theta)\, i\partial_\mu + \Lambda_{\dot{\mu}}(x,\theta,\bar{\theta})\, i\bar{\partial}^{\dot{\mu}} \ ,$$

$$\bar{\Lambda} = \bar{\Lambda}^M i\partial_M \ . \tag{3.1}$$

Note that although Λ^m and Λ^μ are again chiral, $\Lambda_{\dot{\mu}}$ is not restricted, since $\bar{\partial}_{\dot{\mu}}\phi(x,\theta) = 0$. In particular, general x-coordinate, local Lorentz, and local supersymmetry transformations can be realized by $\Lambda^m = \lambda^m(x) = \bar{\lambda}^m$, $\Lambda^\mu = \bar{\Lambda}^\mu = \Omega^{(\mu\nu)}\theta_\mu\partial_\nu$, and $\Lambda^\mu = \bar{\Lambda}^\mu = \epsilon^\mu(x)$, respectively. (For example, for general x-coordinate transformations:

$$\delta f = \lambda^m \partial_m f \ , \qquad (\delta f^m)\, \partial_m = [\lambda^m \partial_m, f^m \partial_m] = (\lambda^n \partial_n f^m - f^n \partial_n \lambda^m)\, \partial_m \ .)$$

Actually, the group of eq. (3.1) is the local superconformal group: it contains also Weyl, local chiral, and local S-supersymmetry transformations, as $\Lambda^\mu = \bar{\Lambda}^\mu = \theta^\mu\zeta(x)$ $(\zeta = \bar{\zeta})$, $i\theta^\mu\xi(x)$ $(\xi = \bar{\xi})$, and $\theta^2\eta^\mu(x)$, respectively. The only subgroups which exclude Weyl transformations and include local supersymmetry (as determined by examining the group in the Wess-Zumino gauges, as defined below) are given by the additional condition

$$(3n+1)\, \bar{\partial}^{\dot{\mu}}\Lambda_{\dot{\mu}} = (n+1)(\partial_m\Lambda^m - \partial_\mu\Lambda^\mu) \ . \tag{3.2}$$

(This particular parametrization of the coefficients will be found to be convenient later.) n could in general be complex, with $|3n+1| \neq |n+1|$ unless $n = 0$, but we will find an action below only for real n. For the special case $n = 0$ local chiral invariance remains, but this case will also be excluded below.

The Wess-Zumino gauges are determined by eq. (2.9), with V and $\langle U\rangle + V$ replaced by U. For $n \neq -\frac{1}{3}$ we can again gauge away the $1, \theta, \bar{\theta}, \theta^2$, and $\bar{\theta}^2$ part of U, and more, since $\Lambda_{\dot{\mu}}$ is not greatly restricted by eq. (3.2). For $n = -\frac{1}{3}$ the Wess-Zumino gauge will be of a slightly different form [3], since eq. (3.2) then includes a restriction on the θ^2 part of Λ^m, but since $\Lambda_{\dot{\mu}}$ is (for that value of n only) completely arbitrary, *U^μ (and $U_{\dot{\mu}}$) can be completely gauged away*. For this reason, $n = -\frac{1}{3}$ will be the simplest case for computational purposes, since it has the fewest auxiliary fields.

Then U in the Wess-Zumino gauge reduces to

$$U^m = (\theta^2 B^m + \bar{\theta}^2 \bar{B}^m) + \theta^\mu \bar{\theta}^{\dot{\nu}} \sigma^m_{\mu\dot{\nu}} + (\bar{\theta}^2 \theta^\mu \Psi^m_\mu + \theta^2 \bar{\theta}_{\dot{\mu}} \bar{\Psi}^{m\dot{\mu}}) + \theta^2 \bar{\theta}^2 \dot{A}^m \ . \quad (3.3)$$

The remaining gauge invariance is given by

$$\Lambda^\mu = \epsilon^\mu(x) + \theta_\nu \Omega^{(\mu\nu)}(x) + \tfrac{1}{2}\theta^\mu \partial_m \lambda^m(x) - i\theta^2 \partial_m (2\epsilon^\mu B^m + \bar{\epsilon}^{\dot{\nu}} \sigma^{m\mu}_{\dot{\nu}}) \ ,$$

$$\Lambda_{\dot{\mu}} = e^{-2U} \bar{\Lambda}_{\dot{\mu}} \, e^{2U} \ ,$$

$$\Lambda^m = \lambda^m + 2i\theta^\mu (2\epsilon_\mu B^m + \bar{\epsilon}^{\dot{\nu}} \sigma^m_{\mu\dot{\nu}}) + \theta^2 b^m(x) \ , \quad \lambda = \bar{\lambda} \ , \quad \partial_m b^m = 0 \ . \quad (3.4)$$

After a suitable redefinition of fields, B^m will occur in the action only in the form $\partial_m B^m \equiv B$, due to $\delta B^m \sim b^m$. From eq. (3.4) we can see that the usual supersymmetry transformation is obtained in flat space, and that component fields are given an additional weight factor of $(\det e_a{}^m)^{-1/2}$ per each θ or $\bar{\theta}$. (For general n, the weight factor is $(\det e_a{}^m)^{-(n+1)/4(2n+1)}$.)

3.2. *The action*

We will now define a semicovariant derivative by eq. (2.11a) (again replacing $\langle U \rangle + V \to U$), and then use the semicovariant derivative to define a semicovariant *supervierbein*:

$$\hat{E}_{\dot{\alpha}} = \bar{\partial}_{\dot{\alpha}} \ , \quad \hat{E}_\alpha = e^{-2U} \partial_\alpha \, e^{2U} \ , \quad \hat{E}_a = \tfrac{1}{4} i \sigma_a^{\beta\dot{\gamma}} \{\hat{E}_\beta, \hat{E}_{\dot{\gamma}}\} \ , \quad \hat{E}_A = \hat{E}_A{}^M \partial_M \ . \ (3.5)$$

(The flat-space index "a" on $\sigma_a^{\beta\dot{\gamma}}$ indicates they are the flat-space Pauli matrices, as opposed to the curved-space $\sigma^m_{\mu\dot{\nu}}$ in eq. (3.3), which are the *vierbein*.) Interpreting the semicovariant derivative as the semicovariant supervierbein is analogous to the treatment of general relativity as the gauge theory of translations [20], where we have the semicovariant derivative $\nabla_a = e_a{}^m \partial_m = \partial_a + h_a{}^m \partial_m$, which is covariant with respect to general coordinate, but not local Lorentz, transformations. In the case of supergravity, however, general coordinate and local Lorentz transformations are unified into the same group, and the transformation laws for $\hat{E}_A$ are not quite covariant in general:

$$\delta \hat{E}_A = [i\Lambda, \hat{E}_A] + \hat{L}_A{}^B \hat{E}_B \ , \quad (3.6a)$$

$$\hat{L}_{\dot{\alpha}}{}^{\dot{\beta}} = -\bar{\partial}_{\dot{\alpha}} \Lambda^{\dot{\beta}} \ , \quad \hat{L}_\alpha{}^\beta = -e^{-2U} \partial_\alpha \bar{\Lambda}^\beta \, e^{2U} \ ,$$

$$\hat{L}_a{}^{\dot{\beta}} = \tfrac{1}{4} i \sigma_a^{\gamma\dot{\delta}} \, \hat{E}_\gamma L_{\dot{\delta}}{}^{\dot{\beta}} \ , \quad \hat{L}_a{}^\beta = \tfrac{1}{4} i \sigma_a^{\gamma\dot{\delta}} \, \hat{E}_{\dot{\delta}} \, L_\gamma{}^\beta \ ,$$

$$\hat{L}_a{}^b = \tfrac{1}{2} \sigma_a^{\alpha\dot{\beta}} \sigma^b_{\gamma\dot{\delta}} (-\delta^\gamma_\alpha L_{\dot{\beta}}{}^{\dot{\delta}} + \delta^{\dot{\delta}}_{\dot{\beta}} L_\alpha{}^\gamma) \ , \quad \text{other } \hat{L}_A{}^B = 0 \ . \quad (3.6b)$$

Unlike globally supersymmetric gauge theories, we have non-covariance due to the non-chirality of $\Lambda_{\dot{\mu}}$. In subsect. 4.4 we will show that by modifying the *supervierbein* they can be made to transform completely covariantly, with $L_A{}^B$ taking the form of a $\Lambda_{\dot\mu}$ dependent Lorentz transformation [10]. We also note that $\hat{E}_A$ become the usual chiral representation covariant derivatives D_A in flat space ($U \to \langle U\rangle$; see appendix A).

By dimensional analysis, the Lagrangian L is dimensionless, so the simplest choice is $\hat{E}^{-1} \equiv (\det \hat{E}_A{}^M)^{-1} = \det \hat{E}_M{}^A$ (the superdeterminant was defined in ref. [21]). However, since $\hat{E}_A{}^M$ is not truly covariant, $\hat{E}^{-1}$ is not a true scalar density, so we may also try an arbitrary real (non-zero) power $\hat{E}^n$. The only modification necessary is that L satisfy the scalar density form $\bar{L} = L\, e^{2\overleftarrow{U}}$ ($\to \bar{S} = S$) of the chiral-representation hermiticity condition, which for scalars is $\bar{f} = e^{2U} f\, e^{-2U}$. Using the identities

$$\delta\hat{E}_A = [u, \hat{E}_A] \to \delta\hat{E}^{-1} = \hat{E}^{-1}\overleftarrow{u}\ , \tag{3.7a}$$

$$\overline{(i\hat{E})_A} = e^{2U} i\hat{E}_A\, e^{-2U} \to \bar{\hat{E}}^{-1} = \hat{E}^{-1}\, e^{2\overleftarrow{U}} = (1\cdot e^{2\overleftarrow{U}})(e^{2U}\hat{E}^{-1}\, e^{-2U})\ , \tag{3.7b}$$

$$1 = [(1\cdot e^{2\overleftarrow{U}})\, e^{-2\overleftarrow{U}}][(1\cdot e^{-2\overleftarrow{U}})\, e^{2\overleftarrow{U}}] = (1\cdot e^{2\overleftarrow{U}})[e^{2U}(1\cdot e^{-2\overleftarrow{U}})\, e^{-2U}]$$

$$\to (1\cdot e^{-2\overleftarrow{U}})^\dagger = (1\cdot e^{2\overleftarrow{U}}) = e^{2U}(1\cdot e^{-2\overleftarrow{U}})^{-1}\, e^{-2U}\ , \tag{3.7c}$$

(where the arrows indicate the derivatives in $U^M i\overleftarrow{\partial}_M$ act to the left); we find the hermiticity condition is satisfied by $L = (1\cdot e^{-2\overleftarrow{U}})^{(n+1)/2}\hat{E}^n$. We have used only $(1\cdot e^{-2\overleftarrow{U}})$ and $\hat{E}$, and not $(1\cdot e^{-2\overleftarrow{U}})^\dagger$ and $\bar{\hat{E}}$, because we want L to transform as $\delta L = i\Lambda L + f(U, \Lambda, \bar\Lambda)\, L$, and so we do not use quantities which transform as $\delta h = i\bar\Lambda h + f(U, \Lambda, \bar\Lambda)\, h$. From the transformation properties

$$\delta\hat{E}^{-1} = -(-1)^m \partial_M(\Lambda^M\hat{E}^{-1}) - (-1)^a L_A{}^A\hat{E}^{-1}$$

$$= -(-1)^m \partial_M(\Lambda^M\hat{E}^{-1}) - (L_\alpha{}^\alpha + L^{\dot\alpha}{}_{\dot\alpha})\,\hat{E}^{-1}\ , \tag{3.8a}$$

$$\delta(1\cdot e^{-2\overleftarrow{U}}) = -i\cdot\overleftarrow{\bar\Lambda}\, e^{-2\overleftarrow{U}} + 1\cdot e^{-2\overleftarrow{U}}\, i\overleftarrow{\Lambda}$$

$$= -(-1)^m \partial_M[\Lambda^M(1\cdot e^{-2\overleftarrow{U}})] + [e^{-2U}(-1)^m(\partial_M\bar\Lambda^M)\, e^{2U}](1\cdot e^{-2\overleftarrow{U}})\ ; \tag{3.8b}$$

we have the final result (the $1/n$ normalizes $\hat{E}^n \approx (1+\theta^4\mathcal{L})^n = 1 + n\theta^4\mathcal{L}$):

$$S = \frac{1}{n\kappa^2}\int d^4x\, d^4\theta\, L\ ,\quad L = (1\cdot e^{-2\overleftarrow{U}})^{(n+1)/2}\hat{E}^n\ ,\quad \bar{L} = L\, e^{2\overleftarrow{U}}\ ,$$

$$\delta L = -(-1)^m\partial_M(\Lambda^M L) + \tfrac{1}{2}\{[(n+1)(\partial_m\Lambda^m - \partial_\mu\Lambda^\mu) - (3n+1)\,\bar\partial^{\dot\mu}\Lambda_{\dot\mu}]$$

$$+ e^{-2U}[(n+1)(\partial_m\Lambda^m - \partial_\mu\Lambda^\mu) - (3n+1)\,\bar\partial^{\dot\mu}\Lambda_{\dot\mu}]^\dagger\, e^{2U}\}\, L\ . \tag{3.9}$$

Invariance of this action requires the condition stated above:

$$(3n+1)\,\bar\partial^{\dot\mu}\Lambda_{\dot\mu} = (n+1)(\partial_m\Lambda^m - \partial_\mu\Lambda^\mu)\ . \tag{3.2}$$

For any $n \neq 0$ or ∞, this action describes supergravity. However, particular values of n have distinguishing characteristics: $n = -\frac{1}{3}$ has the minimal set of auxiliary fields, $n = -1$ has the most geometrical appearance, $n = -\frac{1}{2}$ is not generally covariant in Wess-Zumino gauges (where eq. (3.2) implies $\partial_m \lambda^m = 0$ in the equation corresponding to eq. (3.4); therefore, for this n the Wess-Zumino gauge conditions include the condition $\sqrt{-g} = 1$), and for positive integral n the action is polynomial in the *vierbein* as well as the other fields in the Wess-Zumino gauge. That many forms of the action exist is not surprising, since they correspond to different choices

$$\sigma_a^m = (\det e_a{}^m)^{-(n+1)/2(2n+1)}\, e_a{}^m$$

for the spin-2 field in terms of the canonical *vierbein*. However, *only* $n = -\frac{1}{3}$ *has the minimal set of auxiliary fields*; all other n's have Breitenlohner's set.

3.3. Chiral superfields

The superfield action for a massive, self-interacting chiral superfield coupled to supergravity is simplest for the case $n = -\frac{1}{3}$:

$$S = \int d^4x\, d^4\theta\, (1 \cdot e^{-2\overleftarrow{U}})^{1/3} \hat{E}^{-1/3} (e^{2U}\phi)^\dagger \phi + \left[\int d^4x\, d^2\theta f(\phi) + \text{h.c.}\right] , \qquad (3.10)$$

where f is an arbitrary function. This action has the scalar field coupled to gravity in the improved-energy-momentum-tensor form [4,9]. In particular, when $f(\phi) = a\phi^3$, S is locally superconformally invariant with

$$\delta\phi = i\Lambda\phi - \tfrac{1}{3}(\partial_m \Lambda^m - \partial_\mu \Lambda^\mu)\,\phi\,, \qquad (3.11)$$

and δU as usual, but dropping the condition of eq. (3.2). (For f arbitrary, eq. (3.2) implies the second term of eq. (3.11) is zero, so ϕ transforms as an ordinary scalar superfield.) Other possible actions are [1]:

$$S_{\tilde{n}} = \int d^4x\, d^4\theta\, (1 \cdot e^{-2\overleftarrow{U}})^{(\tilde{n}+1)/2} \hat{E}^{\tilde{n}} (e^{2U}\phi)^\dagger \phi\,,$$

$$\delta_{\tilde{n}}\phi = i\Lambda\phi - \tfrac{1}{2}\left[(\tilde{n}+1)(\partial_m \Lambda^m - \partial_\mu \Lambda^\mu) - (3\tilde{n}+1)(\bar{\partial}^{\dot{\mu}} \Lambda_{\dot{\mu}})\right]\phi$$

$$(\to S_0 = \int d^4x\, d^4\theta\, \bar{\phi}\, e^{U+U^\dagger}\phi\,, \quad \delta_0\phi = \tfrac{1}{2} i(\Lambda + \bar{\Lambda}^\dagger)\,\phi)\,, \qquad (3.12)$$

(where $\tilde{n}$ is an arbitrary real number, independent of n), but only the case $n = \tilde{n} = -\frac{1}{3}$ of eq. (3.10) allows arbitrary self-interaction terms of the simple form in eq. (3.10) (see subsect. 3.4). These actions are all locally superconformally invariant, but for $n \neq -\frac{1}{3}$ the group has the constraint $\bar{\partial}^2 \Lambda_{\dot{\mu}} = 0$. (This condition does not reduce the number of invariances of the Wess-Zumino gauge.)

For $\tilde{n} = n$, S in eq. (3.12) (with the opposite sign for $n < 0$) is the superfield form of supergravity extended to a locally superconformal theory: the action of eq. (3.9) is obtained by gauging ϕ to $1/\sqrt{|n|}\kappa$, using up the gauge freedom in $(n+1)(\partial_m \Lambda^m - \partial_\mu \Lambda^\mu) - (3n+1)\,\bar{\partial}^{\dot{\mu}} \Lambda_{\dot{\mu}}$. We can now choose a generally-covariant Wess-Zumino gauge for the case $n = -\frac{1}{2}$: we choose U^M to have the Breitenlohner set of auxiliary fields, as for $n \neq -\frac{1}{2}$ or $-\frac{1}{3}$ when ϕ is gauged to 1, but now we choose the gauge $\phi = J(x)^{-1/4}$, where J is an additional real scalar field. The remaining symmetry is now enlarged to include Weyl transformations $\Lambda_\mu = \zeta\theta_\mu$ ($\zeta = \bar{\zeta}$), but they can be used to gauge σ to J (J itself is invariant under the Weyl transformations). We can therefore identify $\phi = \sigma^{-1/4}$ and obtain the usual set of fields and invariances as for $n \neq -\frac{1}{2}$ or $-\frac{1}{3}$ (see also subsect. 4.2).

For the case $n = -\frac{1}{3}$, we can also introduce a cosmological term in the simple form $a\phi^3$ [9] and superconformally covariantize the action of a chiral matter superfield χ:

$$S = \int d^4x\, d^4\theta\, (1 \cdot e^{-2\bar{U}})^{1/3} \hat{E}^{-1/3} (e^{2U}\phi)^\dagger \phi \left[-\frac{3}{\kappa^2} + (e^{2U}\chi)^\dagger \chi \right]$$

$$+ \left[\int d^4x\, d^2\theta\, \phi^3 (a + k\chi + m\chi^2 + h\chi^3 + \ldots) + \text{h.c.} \right] , \tag{3.13}$$

where χ transforms as an ordinary scalar superfield. For $a = 0$ we can gauge ϕ to 1 to obtain the action of eq. (3.9) plus that of eq. (3.10) (in terms of χ instead of ϕ). For $a \neq 0$ we cannot gauge ϕ to 1 in the action, just as in ordinary gravity with a cosmological term $\sqrt{-g}$ we cannot gauge $\sqrt{-g}$ to 1 in the action. ϕ^3 is the analog of $\sqrt{-g}$ for the 6-dimensional chiral superspace of x^m and θ^μ: ϕ^3 is the weight factor in all chiral actions, and $\phi^3 = 1$ ($\rightarrow \partial_m \Lambda^m - \partial_\mu \Lambda^\mu = 0$) is the gauge choice analogous to $\sqrt{-g} = 1$ ($\rightarrow \partial_m \lambda^m = 0$). Therefore, for $n = -\frac{1}{3}$ the gauge choice $\phi = 1$ destroys the chiral general supercoordinate invariance $\delta\chi(x,\theta) = -[\Lambda^m(x,\theta)\,\partial_m + \Lambda^\mu(x,\theta)\,\partial_\mu]\,\chi$ (though not ordinary general x-coordinate invariance $\delta\chi = -\lambda^m(x)\,\partial_m\chi$).

Instead of using the gauge $\phi = 1$ in eq. (3.13), we can instead choose a gauge such that the field B (see eq. (3.3)) occurs in ϕ instead of U^m, and σ_a^m and Ψ_α^m are the canonical spin-2 and spin-$\frac{3}{2}$ fields. We then have

$$U^\mu = U_{\dot{\mu}} = 0 , \quad U^m = \theta^\mu \bar{\theta}^{\dot{\nu}} \sigma^m_{\mu\dot{\nu}} + (\bar{\theta}^2 \theta^\mu \Psi^m_\mu + \theta^2 \bar{\theta}_{\dot{\mu}} \bar{\Psi}^{m\dot{\mu}}) + \theta^2 \bar{\theta}^2 A^m , \tag{3.14a}$$

$$\phi = \sigma^{-1/3}(1 - \tfrac{2}{3}\theta^\mu \sigma_{m\mu\dot{\nu}} \bar{\Psi}^{m\dot{\nu}} + \tfrac{2}{3} i\theta^2 B) , \tag{3.14b}$$

$$\Lambda^m = \lambda^m + 2i\bar{\epsilon}_{\dot{\nu}}(\theta^\mu \sigma^{m\dot{\nu}}_\mu + \theta^2 \bar{\Psi}^{m\dot{\nu}}) , \quad \Lambda^\mu = \epsilon^\mu + \theta_\nu \Omega^{(\mu\nu)} + \theta^\mu(\zeta + i\xi) + \theta^2 \eta^\mu ,$$

$$\Lambda_{\dot{\mu}} = e^{-2U} \bar{\Lambda}_{\dot{\mu}}\, e^{2U} , \tag{3.14c}$$

$$\zeta = 0 , \quad \xi = \tfrac{1}{2} i \sigma_a^{\mu\dot{\nu}} (\epsilon_\mu \bar{\Psi}^a_{\dot{\nu}} + \bar{\epsilon}_{\dot{\nu}} \Psi^a_\mu) ,$$

$$\eta_\mu = i\sigma^m_{\mu\dot{\nu}} \partial_m \bar{\epsilon}^{\dot{\nu}} + \tfrac{2}{3} \sigma_{a\mu\dot{\nu}} \bar{\epsilon}^{\dot{\nu}} (A^a - \tfrac{1}{2} \epsilon^{abcd} \sigma_{dn}\, \sigma^p_b \partial_p\, \sigma^n_c) + \tfrac{2}{3} i\epsilon_\mu B$$

$$- \tfrac{2}{9} (\sigma_a^{\alpha\dot{\beta}} \epsilon_\alpha \bar{\Psi}^a_{\dot{\beta}})\, \sigma_{b\mu\dot{\nu}} \bar{\Psi}^{b\dot{\nu}} - \tfrac{1}{3} (\epsilon_\alpha \bar{\Psi}^a_{\dot{\beta}} - \bar{\epsilon}_{\dot{\beta}} \Psi^a_\alpha)\, \sigma_b^{\alpha\dot{\beta}} \sigma_{a\mu\dot{\nu}} \bar{\Psi}^{b\dot{\nu}} . \tag{3.14d}$$

The Weyl + local chiral and S-supersymmetry transformations $\zeta + i\xi$ and η_μ have been used to choose the 1 and θ parts of ϕ, while the b^m transformation (of eq. (3.4), not restricted by $\partial_m b^m = 0$ for local superconformal invariance) has been used to gauge B^m from U^m into ϕ (as $B = \partial_m B^m$, the rest being gauged away). This gauge is the most convenient for comparison to the standard supergravity formalism. It also allows us to evaluate the cosmological term (since B is in ϕ).

In order to treat the $n \neq -\frac{1}{3}$ superconformally-extended theory in closer analogy to $n = -\frac{1}{3}$, we note that (from eq. (3.12) for $\tilde{n} = n$)

$$\Upsilon \equiv \phi^{2/(3n+1)} \to \delta\Upsilon = -(\Lambda^m \partial_m + \Lambda^\mu \partial_\mu)\, \Upsilon$$
$$- \frac{n+1}{3n+1} (\partial_m \Lambda^m - \partial_\mu \Lambda^\mu)\, \Upsilon + \bar{\partial}^{\dot{\mu}} (\Lambda_{\dot{\mu}} \Upsilon)\,. \tag{3.15}$$

Therefore, if we replace $\bar{\partial}_{\dot{\mu}}\phi = 0$ with $\bar{\partial}^2\Upsilon = 0$, we can allow $\Lambda_{\dot{\mu}}$ to be arbitrary, as for superconformal $n = -\frac{1}{3}$. We can still choose the gauge $\Upsilon = \phi = 1$, but now we can also choose the gauge $U^\mu = 0$ with U^m containing only $\sigma^m_{\mu\dot{\nu}}$, Ψ^m_μ, and A^m, as in the gauge of eq. (3.14), with all other auxiliary fields contained in ϕ. The action is still given by eq. (3.12).

The non-superconformal kinetic actions for the chiral multiplet [22] can be obtained by replacing the $-3/\kappa^2 + (e^{2U}\chi)^\dagger\chi$ of eq. (3.13) with

$$-(3/\kappa^2) \exp\{-\tfrac{1}{3}\Omega[\kappa^2(e^{2U}\chi)^\dagger\chi]\}$$

(with Ω as in ref. [22]). To cancel the non-minimal scalar field-curvature couplings, one chooses the gauge

$$\phi = \phi_0(U) \exp[\tfrac{1}{6}\Omega(\kappa^2\bar{\chi}(x, 0)\, \chi(x, 0))]\ ,$$

where $\chi(x, 0)$ is the complex scalar field in the chiral multiplet. One could, of course, also choose a more general kinetic action containing $f[\kappa(e^{2U}\chi)^\dagger, \kappa\chi]$ with

$$\phi = \phi_0(U) f^{-1/2} [\kappa\bar{\chi}(x, 0), \kappa\chi(x, 0)]\ .$$

3.4. Covariant derivatives

In ordinary gravity it is convenient to introduce a covariant derivative to couple the vierbein to fields with non-zero spin. In sect. 2 we found that covariant derivatives were again convenient in globally supersymmetric gauge theories when treating actions more complicated than that for the coupling of a gauge superfield to a chiral superfield. We will now construct covariant derivatives for supergravity, and show their utility for constructing actions more complicated (in terms of superfields) than those described earlier in this section.

In ref. [11] Wess and Zumino proposed a superfield action for supergravity in terms of a constrained covariant derivative. We will give the solution to the constraints: it is just the superconformally-extended supergravity action for $n = -\frac{1}{3}$, as described in subsect. 3.3. Next, we will show how to use the covariant derivative to build actions. Then we will generalize Wess and Zumino's covariant derivative to all n.

Wess and Zumino's fotmalism is given by (compare eq. (2.12))

$$\nabla_A = E_A{}^M \partial_M + \tfrac{1}{2}\Phi_A{}^{bc} M_{bc}\ , \quad [\nabla_A, \nabla_B\} = T_{AB}{}^C \nabla_C + \tfrac{1}{2} R_{AB}{}^{cd} M_{cd}\ , \tag{3.16a}$$

$$T_{\alpha\beta}{}^{C} = T_{\alpha\dot\beta}{}^{C} + 2i\sigma^{c}_{\alpha\dot\beta} = T_{AB}{}^{c} + 2i(\sigma^{c}_{\alpha\dot\beta} + \sigma^{c}_{\beta\dot\alpha}) = 0\ , \quad S = \int d^4x\, d^4\theta\, E^{-1}\ , \qquad (3.16b)$$

$$\nabla'_A = e^{iK}\nabla_A\, e^{-iK}\ , \quad K = K^{M}i\partial_M + \tfrac{1}{2}K^{ab}iM_{ab} = \bar K\ , \qquad (3.16c)$$

where $\Phi_A{}^{bc}, M_{bc}, T_{AB}{}^{C}$, and $R_{AB}{}^{cd}$ are the superconnection, Lorentz generators (acting on all flat-space indices), supertorsion, and supercurvature, respectively. The solution to the constraints on the supertorsion is (in the vector representation):

$$E_\alpha = N_\alpha{}^{\beta}\Psi\hat E_\beta\ , \quad \det N_\alpha{}^{\beta} = 1\ ,$$

$$\Psi = \phi^{1/2}\bar\phi^{-1}(1\cdot e^{-\overleftarrow{W}})^{-1/3}(1\cdot e^{\overleftarrow{\bar W}})^{1/6}\hat E^{-1/6}\ , \quad E_{\dot\alpha}\phi = 0\ , \qquad (3.17a)$$

$$E_a = N_a{}^{b}\,\tfrac{1}{4}i\,[\sigma^{\alpha\dot\beta}_{b}\{\Psi\hat E_\alpha, \bar\Psi\hat E_{\dot\beta}\} - \tfrac{1}{2}\Psi\bar\Psi\sigma^{\alpha\dot\beta}_{[b}(\hat C_\alpha{}^{c}{}_{c]}\hat E_{\dot\beta} + \hat C_{\dot\beta}{}^{c}{}_{c]}\hat E_\alpha)]\ ,$$

$$N_a{}^{b} = -\tfrac{1}{2}\sigma^{\alpha\dot\beta}_{a}\sigma^{b}_{\gamma\dot\delta}N_\alpha{}^{\gamma}\bar N_{\dot\beta}{}^{\dot\delta}\ , \quad [E_A, E_B\} = C_{AB}{}^{C}E_C\ , \qquad (3.17b)$$

$$\Phi_{\alpha\beta\gamma} = -\tfrac{1}{2}(C_{\alpha\beta\gamma} + C_{\gamma[\alpha\beta]})\ , \quad \Phi_{\alpha\dot\beta\dot\gamma} = -C_{\alpha\dot\beta\dot\gamma} \quad \Phi_{abc} = -\tfrac{1}{2}(C_{abc} + C_{c(ab)})\ ,$$

$$\Phi_{Ab}{}^{c} = \tfrac{1}{2}\sigma^{\beta\dot\gamma}_{b}\sigma^{c}_{\delta\dot\epsilon}(\delta^{\dot\epsilon}_{\dot\gamma}\Phi_{A\beta}{}^{\delta} - \delta^{\delta}_{\beta}\Phi_{A\dot\gamma}{}^{\dot\epsilon})\ , \qquad (3.17c)$$

$$S = \int d^4x\, d^4\theta\, \phi\bar\phi(1\cdot e^{-\overleftarrow{W}})^{1/3}(1\cdot e^{\overleftarrow{\bar W}})^{1/3}\hat E^{-1/3}\ , \qquad (3.17d)$$

$$e^{W'} = e^{i\bar\Lambda}\, e^{W}\, e^{-iK}\ , \quad \delta\Psi = iK\Psi + \tfrac{1}{2}(e^{-W}\partial_\alpha\bar\Lambda^{\alpha}\, e^{W})\,\Psi\ ,$$

$$\delta\phi = iK\phi - \tfrac{1}{3}[e^{\bar W}(\partial_m\Lambda^{m} - \partial_\mu\Lambda^{\mu})\, e^{-\bar W}]\,\phi\ ,$$

$$\delta N_\alpha{}^{\beta} = K^{\gamma}_{\alpha}N_\gamma{}^{\beta} - \tfrac{1}{2}N_\alpha{}^{\gamma}(e^{-W}\partial_{(\gamma}\bar\Lambda^{\beta)}\, e^{W})\ ,$$

$$K^{b}_{a} = \tfrac{1}{2}\sigma^{\alpha\dot\beta}_{a}\sigma^{b}_{\gamma\dot\delta}\,(\delta^{\dot\delta}_{\dot\beta}K^{\gamma}_{\alpha} - \delta^{\gamma}_{\alpha}K^{\dot\delta}_{\dot\beta})\ , \quad \bar\partial_{\dot\alpha}\Lambda^{m} = \bar\partial_{\dot\alpha}\Lambda^{\mu} = 0\ ,$$

$$\Lambda^{\dot\mu}\ \text{arbitrary}\ , \qquad (3.17e)$$

where $\hat E_A$ and S are as in eqs. (3.5) and (3.12) (for $n = -\frac{1}{3}$) in the vector representation (see eq. (2.13)):

$$\hat E_\alpha = e^{-W}\partial_\alpha\, e^{W}\ , \quad \hat E_{\dot\alpha} = e^{\bar W}\bar\partial_{\dot\alpha}\, e^{-\bar W}\ , \quad \hat E_a = \tfrac{1}{4}i\sigma^{\alpha\dot\beta}_{a}\{\hat E_\alpha, \hat E_{\dot\beta}\}\ , \qquad (3.18a)$$

$$S = \int d^4x\, d^4\theta\, \phi\bar\phi\,[(1\cdot e^{-\overleftarrow{W}})(1\cdot e^{\overleftarrow{\bar W}})]^{(n+1)/2}\hat E^{n}((1\cdot e^{\overleftarrow{\bar W}}) = (1\cdot e^{-\overleftarrow{W}})^{\dagger})\ . \qquad (3.18b)$$

In eq. (3.17) we have the standard relation between an antisymmetric tensor K_{ab} and a symmetric spinor $K_{\alpha\beta}$ (and similarly between Φ_{Abc} and $\Phi_{A\beta\gamma}$).

The chiral representation is obtained as in sect. 2: the effect is to make the replacements $W \to 2U$, $\bar W \to 0$, $\phi \to \phi(x, \theta)$, $\bar\phi \to (e^{2U}\phi)^{\dagger}$ (corresponding to K^{M} gauge freedom); and we choose the simplest Lorentz gauge (K^{ab} gauge freedom) $N_\alpha{}^{\beta} = \delta^{\beta}_{\alpha}$. E_A (and similarly, $\Phi_A{}^{bc}$) now transforms with a Λ-dependent Lorentz transformation:

$$\delta E_A = [i\Lambda, E_A] + L_A{}^B E_B \,, \tag{3.19a}$$

$$L_{\dot\alpha}{}^{\dot\beta} = -\tfrac{1}{2}\bar\partial_{(\dot\alpha}\Lambda^{\dot\beta)} \,, \quad L_\alpha{}^\beta = \mathrm{e}^{-2U}\bar L_\alpha{}^\beta\,\mathrm{e}^{2U} \,,$$

$$L_a{}^b = \tfrac{1}{2}\sigma_a^{\alpha\dot\beta}\sigma^b_{\gamma\dot\delta}(\delta^{\dot\delta}_{\dot\beta}L_\alpha{}^\gamma - \delta^\gamma_\alpha L_{\dot\beta}{}^{\dot\delta}) \,, \quad \text{other } L_A{}^B = 0 \,, \tag{3.19b}$$

$$\mathrm{e}^{2U'} = \mathrm{e}^{i\bar\Lambda}\,\mathrm{e}^{2U}\,\mathrm{e}^{-i\Lambda} \,, \quad \delta\phi = i\Lambda\phi - \tfrac{1}{3}(\partial_m\Lambda^m - \partial_\mu\Lambda^\mu)\,\phi \,. \tag{3.19c}$$

We see that this $L_A{}^B$ is the "Lorentz part" of the $\hat L_A{}^B$ in eq. (3.6) for $\hat E_A$. In fact, E_A can be derived by first finding the factor Ψ which cancels the $\hat L_\alpha{}^\alpha$ part of $\delta E_\alpha = \delta(\Psi\hat E_\alpha)$, and then finding the terms to add to $E_a = \Psi(\mathrm{e}^{2U}\Psi)^\dagger\hat E_a + \ldots$ which cancel the $\hat L_a{}^\beta$ (and $\hat L_a{}^{\dot\beta}$) part of δE_a.

The role of the covariant derivatives ∇_A is the same as that of $\nabla_a = e_a{}^m\partial_m + \frac{1}{2}\phi_a{}^{bc}M_{bc}$ in general relativity: we start with the fundamental superfields U^M and ϕ, analogous to the vierbein $e_a{}^m$ in general relativity. We then define ∇_A (i.e., $\Phi_A{}^{bc}$ *and* $E_A{}^M$), analogous to ∇_a (i.e., $\phi_a{}^{bc}$) in general relativity, in terms of the fundamental superfields. This can be accomplished either by solving covariant constraints or by searching for quantities which transform covariantly. The covariant derivatives are then used in the construction of actions, while the fundamental superfields are used in explicit calculations and in quantization.

To translate the action for the chiral superfield χ of eq. (3.13) into the notation of covariant derivatives, we first note the identities

$$\{\nabla_{\dot\alpha}, \nabla_{\dot\beta}\} = \tfrac{1}{2}RM_{\dot\alpha\dot\beta} \,,$$

$$R \equiv \bar\partial^2(\mathrm{e}^{2U}\Psi)^{\dagger 2} = \phi^{-3}\bar\partial^2 E^{-1} \rightarrow \quad \mathrm{d}^4x\,\mathrm{d}^4\theta\, E^{-1} = -\tfrac{1}{4}\int \mathrm{d}^4x\,\mathrm{d}^2\theta\,\phi^3 R \,,$$

$$S_\mathrm{c} = \int \mathrm{d}^4x\,\mathrm{d}^2\theta\,\phi^3 L_\mathrm{c} = \int \mathrm{d}^4x\,\mathrm{d}^2\theta\,(R^{-1}\bar\partial^2 E^{-1})\,L_\mathrm{c}$$

$$= -4\int \mathrm{d}^4x\,\mathrm{d}^4\theta\, E^{-1}R^{-1}L_\mathrm{c} \,. \tag{3.20}$$

($M_{\alpha\beta}$ is the symmetric spinor form of M_{ab}.) We thus have R acting as the chiral Lagrangian for pure supergravity. The last expression for S_c can be used in all representations, although we have derived it by use of the chiral representation. We therefore have the general rule: *chiral actions are to be integrated by* $-4\int \mathrm{d}^4x\mathrm{d}^4\theta E^{-1}R^{-1}$ ($\rightarrow \int \mathrm{d}^4x\,\mathrm{d}^2\theta$ in the chiral representation, in the gauge $\phi = 1$). The action of eq. (3.13) is thus, in the vector representation,

$$S = \int \mathrm{d}^4x\,\mathrm{d}^4\theta\, E^{-1}\left(-\frac{3}{\kappa^2} + \bar\chi\chi\right) + \left(-4\int \mathrm{d}^4x\,\mathrm{d}^4\theta\, E^{-1}R^{-1}f(\chi) + \text{h.c.}\right) . \tag{3.21}$$

In particular, variation by ϕ produces from the cosmological term $f(\chi) = a$, the field equation $R = -4\kappa^2 a$, as found by Brink, Gell-Mann, Ramond and Schwarz in their extension of Wess and Zumino's field equations to include a cosmological term [16].

We can also derive the chiral action rule as follows:

$$\nabla_{\dot\alpha} L_c = 0 \to L_c = -\tfrac{1}{4}(\nabla_{\dot\alpha}\nabla^{\dot\alpha} + R) f \to -4\int d^4x\, d^4\theta\, E^{-1} R^{-1} L_c$$
$$= \int d^4x\, d^4\theta\, E^{-1} f (\nabla_{\dot\alpha}\nabla^{\dot\alpha} + R) R^{-1} = \int d^4x\, d^4\theta\, E^{-1} f\,, \tag{3.22}$$

where we have used the chirality of R and integration by parts [11]: $\int d^4x\, d^4\theta\, E^{-1} \nabla^A X_A = 0$. Since $-\frac{1}{4}(\nabla_{\dot\alpha}\nabla^{\dot\alpha} + R)$ corresponds to a covariantized $\int d^2\bar\theta = -\frac{1}{4}\bar\partial^2$, we see that the chiral action rule is a natural generalization to curved superspace. Of course, in the chiral representation it is always simpler to use $\int d^4x\, d^2\theta\, \phi^3 L_c$. (The ϕ^3 measure has been discovered independently by Ferrara and van Nieuwenhuizen [23], but only in a gauge-like eq. (3.14). This is a result of their using the Wess-Zumino gauge with the canonical spin-2 and spin-$\frac{3}{2}$ fields.)

We can also treat the vector multiplet in analogy to eqs. (2.12) and (2.13) in terms of the supergauge covariant derivative $\mathcal{D}_A$:

$$\mathcal{D}_A = \nabla_A + i\Gamma_A\,,$$

$$[\mathcal{D}_A, \mathcal{D}_B\} = T_{AB}{}^C \mathcal{D}_C + \tfrac{1}{2} R_{AB}{}^{cd} M_{cd} + iF_{AB} \quad [\Gamma_A = \Gamma^i_A G_i, \text{etc.}]\,, \tag{3.23a}$$

$$\text{constraint} \quad F_{\alpha\beta} = 0 \to \mathcal{D}_\alpha = \nabla_\alpha\,,$$

$$\text{with } W = W^M i\partial_M \text{ replaced by } W^M i\partial_M + W^i G_i\,, \tag{3.23b}$$

$$\text{constraint} \quad F_{\alpha\dot\beta} = 0 \to \Gamma_a = \tfrac{1}{4}\sigma_a^{\alpha\dot\beta}(\nabla_{\dot\beta}\Gamma_\alpha + \nabla_\alpha\Gamma_{\dot\beta} + i\{\Gamma_\alpha, \Gamma_{\dot\beta}\})\,, \tag{3.23c}$$

$$S = \frac{1}{g^2}\,\mathrm{tr}\int d^4x\, d^2\theta\, E^{-1} R^{-1} F_{\dot\beta a} F^{\dot\beta a}\,, \quad F_{\dot\beta a} = \tfrac{1}{4} i\sigma^{\alpha}_{a\dot\beta} W_\alpha\,, \tag{3.23d}$$

$$e^{W'} = e^{i\bar\Lambda} e^{W} e^{-iK}\,, \quad K = \bar K = K^M i\partial_M + \tfrac{1}{2}K^{ab} iM_{ab} + K^i G_i\,,$$

$$\Lambda = \Lambda^M i\partial_M + \Lambda^i G_i\,, \quad \bar\partial_{\dot\mu}\Lambda^i = 0\,. \tag{3.23e}$$

The coupling to the chiral multiplet is also accomplished by the replacement $W^M i\partial_M \to W^M i\partial_M + W^i G_i$ in the chiral multiplet's kinetic term (as in sect. 2). In the Abelian case, in the chiral representation, using the identities $T_{\dot\beta a}{}^{\alpha} = \frac{1}{8} i\sigma^{\alpha}_{a\dot\beta} R$ and

$$F_{AB} = \nabla_{[A}\Gamma_{B)} + i[\Gamma_A, \Gamma_B\} - iT_{AB}{}^C\Gamma_C$$

(from eq. (3.23a)), we then have ($e^{2(U+V)} \equiv e^U e^{2\tilde V} e^U$)

$$F_{\dot\beta a} = \tfrac{1}{4} i\sigma^{\alpha}_{a\dot\beta}(\nabla_{\dot\gamma}\nabla^{\dot\gamma} + R)\,\nabla_\alpha \tilde V \to S \sim \frac{1}{g^2}\,\mathrm{tr}\int d^4x\, d^4\theta\, E^{-1} R^{-1} W^\alpha W_\alpha\,,$$

$$W_\alpha = (\nabla_{\dot\gamma}\nabla^{\dot\gamma} + R)\,\nabla_\alpha \tilde V\,. \tag{3.24}$$

This is identical to the vector-multiplet action proposed by Wess and Zumino [11], since $W^\alpha W_\alpha = (\nabla_{\dot\beta}\nabla^{\dot\beta} + R)(W^\alpha \nabla_\alpha \tilde{V})$ (see eq. (3.22)). Since this action is ϕ-independent (using eq. (3.17)), it remains locally superconformally invariant when ϕ is gauged to 1. In particular, by using the gauge of eq. (3.14) we see that A^m is the only auxiliary supergravity field appearing in it. Similarly, since the conformal supergravity field strength $W_{\alpha\beta\gamma}$ [15,6] is chiral, we can write the action of conformal supergravity, which is analogous to the vector multiplet's action ($G_i \to M_{ab}$, $W_\alpha \to W_{\alpha\beta\gamma}$, $F_{\dot\beta a}{}^i \to R_{\dot\beta a}{}^{bc}$:

$$S = \int \mathrm{d}^4x\, \mathrm{d}^4\theta\, E^{-1} R^{-1} R_{\dot\beta abc} R^{\dot\beta abc} \sim \int \mathrm{d}^4x\, \mathrm{d}^4\theta\, E^{-1} R^{-1} W^{\alpha\beta\gamma} W_{\alpha\beta\gamma} \,. \tag{3.25}$$

It is easy to modify the action of the vector multiplet coupled to supergravity in order that the gauge vector couples to the gravitino and spin-$\frac{1}{2}$ particle as the gauge field of chiral transformations [24]. The action is

$$S = -\frac{3}{\kappa^2}\int \mathrm{d}^4x\, \mathrm{d}^4\theta\, (1\cdot \mathrm{e}^{-2\overleftarrow{U}})^{1/3} \hat{E}^{-1/3} (\mathrm{e}^{2(U+V)}\phi)^\dagger \phi + \frac{1}{g^2}\int \mathrm{d}^4x\, \mathrm{d}^2\theta\, \phi^3 W^\alpha W_\alpha \,,$$

$$\delta\, \mathrm{e}^{2U} = i\overline{\Lambda}\, \mathrm{e}^{2U} - \mathrm{e}^{2U} i\Lambda \,,$$

$$\delta\, \mathrm{e}^{2(U+V)} = i(\overline{\Lambda} + \hat{\overline{\Lambda}})\, \mathrm{e}^{2(U+V)} - \mathrm{e}^{2(U+V)} i(\Lambda + \hat{\Lambda}) \,,$$

$$\delta\phi = [i(\Lambda + \hat{\Lambda}) - \tfrac{1}{3}(\partial_m \Lambda^m - \partial_\mu \Lambda^\mu)]\, \phi \,, \tag{3.26}$$

(in the chiral representation) where $\Lambda = \Lambda^M i\partial_M$ and $\hat{\Lambda}$ is the Λ^i for the U(1) group which V gauges. In the Wess-Zumino gauge (where $\hat{\Lambda}(x,\theta) = \hat{\lambda}(x) = \hat{\overline{\lambda}}(x)$) with $\phi = 1$, we then have

$$\tfrac{1}{3}(\partial_m \Lambda^m - \partial_\mu \Lambda^\mu) = i\hat{\lambda} \to \Lambda_\mu = \Lambda_\mu(\hat{\lambda} = 0) - \tfrac{3}{2} i\hat{\lambda}\theta_\mu \,,$$

so $\hat{\lambda}$ is obviously a local chiral transformation. (To obtain the action in the form of deWit and van Nieuwenhuizen, we should redefine $\mathrm{e}^{2(U+V)} \equiv \mathrm{e}^U(1 + 2\tilde{V})\,\mathrm{e}^U$, so S is linear in $\tilde{V}$ in its $1/\kappa^2$ part and non-quadratic in its $1/g^2$ part.) A cosmological term $\int \mathrm{d}^4x\, \mathrm{d}^2\theta\, \phi^3$ would thus no longer be invariant.

To generalize ∇_A to all n, it is only necessary to modify the expression for Ψ in eq. (3.17a). (Of course, this also modifies the constraints.) The procedure is simple: Construct a quantity with the same transformation law as Ψ (so ∇_A is still covariant) in terms of the ϕ which has the n-dependent transformation law of eq. (3.12) ($\tilde{n} = n$). For $n \neq -\frac{1}{3}$, we thus use the semichiral $\phi = \Upsilon^{(3n+1)/2}$ of eq. (3.15), so $\Lambda_{\dot\mu}$ is arbitrary. The result is:

$$\Psi = \phi^{-(n+1)/4n}\, \overline{\phi}^{-(n-1)/4n} (1\cdot \mathrm{e}^{-\overleftarrow{W}})^{-(n^2-1)/8n} (1\cdot \mathrm{e}^{\overleftarrow{\overline{W}}})^{-(n+1)^2/8n}\, \hat{E}^{-(n+1)/4} \,,$$

$$S = \int \mathrm{d}^4x\, \mathrm{d}^4\theta\, E^{-1} = \int \mathrm{d}^4x\, \mathrm{d}^4\theta\, \phi\overline{\phi}\, [(1\cdot \mathrm{e}^{-\overleftarrow{W}})(1\cdot \mathrm{e}^{\overleftarrow{\overline{W}}})]^{(n+1)/2}\, \hat{E}^n \,. \tag{3.27}$$

However, we then have, in the chiral representation,

$$R = \overline{\partial}^2 (\mathrm{e}^{2U}\Psi)^{\dagger 2} = \overline{\partial}^2 \phi^{1/n} E^{(n+1)/2n} \,. \tag{3.28}$$

Therefore, the chiral-action rule of eq. (3.20) can be used for all n *except* -1, since $R = \bar{\partial}^2\Upsilon = 0$ for $n = -1$. In fact, the covariant derivative for $n = -1$ can be derived from the constraints of eq. (3.16b) if we separate the constraint $T_{\alpha a}{}^b = 0$ into its two parts

$$T_{\alpha a}{}^b - \tfrac{1}{4}\sigma_{a\alpha\dot\gamma}\sigma^{b\beta\dot\gamma}T_{\beta c}{}^c = 0 \quad \text{and} \quad T_{\alpha a}{}^a = 0\,,$$

and replace $T_{\alpha a}{}^a = 0$ with $R_{\alpha\beta}{}^{\gamma\delta} = 0$. For general $n \neq -\frac{1}{3}$, we have

$$T_{\alpha a}{}^b \neq 0\,, \quad \text{but} \quad T_{\alpha a}{}^b = \tfrac{1}{4}\sigma_{a\alpha\dot\gamma}\sigma^{b\beta\dot\gamma}T_{\beta c}{}^c$$

$$\text{and} \quad R + \left(\frac{n+1}{3n+1}\right)^2 (T_{\dot\alpha a}{}^a)^2 - \frac{n+1}{3n+1}\nabla_{\dot\alpha}T^{\dot\alpha}{}_a{}^a = 0\,. \tag{3.29}$$

The last constraint is equivalent to $\bar{\partial}^2\Upsilon = 0$, where Υ is expressed in terms of ϕ by eq. (3.15), and ϕ is expressed in terms of Ψ by inverting eq. (3.27). Using eq. (3.29) we can now define a new chiral-action rule for $n \neq -\frac{1}{3}$ which is the same as the old one for $n \neq -1$, but has a well-defined limit for $n = -1$. It is: *chiral actions are to be integrated by*

$$2\,\frac{3n+1}{n}\int d^4x\,d^4\theta\,E^{-1}\left[\nabla_{\dot\alpha}T^{\dot\alpha}{}_a{}^a - \frac{n+1}{3n+1}(T_{\dot\alpha a}{}^a)^2\right]^{-1} \quad (n \neq -\tfrac{1}{3})$$

$$= 2\,\frac{n+1}{n}\int d^4x\,d^4\theta\,E^{-1}R^{-1} \quad (n \neq -1)\,. \tag{3.30}$$

In the chiral representation, the chiral measure (i.e., the measure for $\int d^4x\,d^2\theta$) is therefore in general

$$-\frac{3n+1}{2n}\left[\nabla_{\dot\alpha}T^{\dot\alpha}{}_a{}^a - \frac{n+1}{3n+1}(T_{\dot\alpha a}{}^a)^2\right]^{-1}\bar{\partial}^2E^{-1} \quad (n \neq -\tfrac{1}{3})$$

$$= -\frac{n+1}{2n}R^{-1}\bar{\partial}^2E^{-1} \quad (n \neq -1\,; = \phi^3 \text{ for } n = -\tfrac{1}{3})\,. \tag{3.31}$$

4. Superfield supergravity in components

4.1. $n = -\frac{1}{3}$

We will first describe the simplest possible evaluation of the superfield action in a form directly comparable to the standard component field form. This is accomplished by using the locally superconformal extension of $n = -\frac{1}{3}$ supergravity in eq. (3.13) in the gauge of eq. (3.14). After a lengthy calculation (see appendix B for some intermediate steps) we obtain

$$S = \frac{1}{\kappa^2}\int d^4x\,\sigma^{-1}(L_0(\sigma_a^m, \Psi_a^\alpha) - \tfrac{4}{3}\hat{\bar{B}}\hat{B} - \tfrac{4}{3}\hat{A}^a\hat{A}_a)\,, \tag{4.1a}$$

$$\hat{B} = B + \tfrac{1}{12} i \bar{\Psi}^{a\dot{\alpha}} (3\eta_{ab}\epsilon_{\dot{\alpha}\dot{\beta}} + \sigma_a{}^{\gamma}{}_{\dot{\alpha}} \sigma_{b\gamma\dot{\beta}}) \bar{\Psi}^{b\dot{\beta}} ,$$

$$\hat{A}^a = A^a + \tfrac{1}{8}\epsilon^{abcd} c_{bcd} - \tfrac{1}{8}(4\eta^{ac}\eta^{bd} + 2\eta^{a(b}\eta^{d)c} - i\epsilon^{abcd}) \bar{\Psi}_b^{\dot{\beta}} \sigma_{c\alpha\dot{\beta}} \Psi_d^{\alpha} ,$$

$$(c_{ab}{}^c \equiv e_m^c e_{[a}^n \partial_n e_{b]}^m) , \tag{4.1b}$$

where L_0 is the standard Lagrangian for supergravity in terms of the *vierbein* and a Weyl spinor-vector (see appendix A). The other terms are auxiliary fields which equal zero by their field equations, but should not be eliminated before introducing Fadeev-Popov terms. The form of the action in terms of σ_a^m, Ψ_a^α, $\hat{B}$, and $\hat{A}_a$ is obvious before its calculation, since B and A_a are known to be auxiliary fields by their dimension (one: the component-field Lagrangian is dimension two without the $1/\kappa^2$), and the action is invariant under local supersymmetry transformations. What require calculation are the shifts in B and A_a to obtain $\hat{B}$ and $\hat{A}_a$, which are needed to calculate the transformations of the latter fields from eq. (3.14).

In this gauge the transformation law of eq. (3.1) simplifies to

$$\delta U = -\tfrac{1}{2} i \{(\Lambda - \bar{\Lambda}) + [U, \Lambda + \bar{\Lambda}]\}$$

$$\to \delta U^m = -\tfrac{1}{2} i (\Lambda^m - \bar{\Lambda}^m) + \tfrac{1}{2} U^n \partial_n (\Lambda^m + \bar{\Lambda}^m) - \tfrac{1}{2}(\Lambda^N + \bar{\Lambda}^N) \partial_N U^m$$

$$= -\tfrac{1}{2} i [(1 + iU^n\partial_n)\Lambda^m - (1 - iU^n\partial_n)\bar{\Lambda}^m] - \tfrac{1}{2}(\Lambda^n + \bar{\Lambda}^n)\partial_n U^m$$

$$- \{[(1 + iU^n\partial_n)\Lambda^\mu]\partial_\mu + [(1 - iU^n\partial_n)\bar{\Lambda}_{\dot{\mu}}]\bar{\partial}^{\dot{\mu}}\} U^m . \tag{4.2}$$

From eqs. (3.14) and (4.1) we then find that σ_a^m, Ψ_a^α, $\hat{B}$, and $\hat{A}_a$ transform in the usual way under general coordinate and local Lorentz transformations, as expected, and local supersymmetry transformations take the form (in agreement with ref. [7])

$$\delta\sigma_m^a = \sigma_{\mu\dot{\nu}}^a (\epsilon^\mu \bar{\Psi}_m^{\dot{\nu}} - \bar{\epsilon}^{\dot{\nu}} \Psi_m^\mu) ,$$

$$\delta\Psi_m^\alpha = -2i\mathbb{D}_m\epsilon^\alpha - 2(\delta_\beta^\alpha \delta_m^n - \tfrac{1}{3}\sigma_m^{\alpha\dot{\gamma}}\sigma_{\beta\dot{\gamma}}^n)\hat{A}_n\epsilon^\beta - \tfrac{2}{3} i \sigma_m^{\alpha\dot{\beta}} \hat{\bar{B}} \bar{\epsilon}_{\dot{\beta}} ,$$

$$\delta\hat{B} = \bar{\epsilon}_{\dot{\beta}} (i\bar{\Psi}_a^{\dot{\beta}} \hat{A}^a + \sigma_a^{\alpha\dot{\beta}} \Psi_\alpha^a \hat{B} - \tfrac{1}{2} i \epsilon^{abcd} \sigma_a^{\alpha\dot{\beta}} \sigma_{b\alpha\dot{\gamma}} \mathbb{D}_c \bar{\Psi}_d^{\dot{\gamma}}) ,$$

$$\delta\hat{A}_m = \epsilon^\alpha [\tfrac{1}{2} i \Psi_{m\alpha} \hat{B} + \tfrac{1}{4}(3\delta_m^n \delta_\alpha^\gamma - \sigma_{m\alpha\dot{\beta}} \sigma^{n\dot{\gamma}\beta}) \epsilon^{abcd} \sigma_{na} \sigma_{b\gamma\dot{\delta}} \mathbb{D}_c \bar{\Psi}_d^{\dot{\delta}}$$

$$+ (\sigma_{\alpha\dot{\beta}}^n \bar{\Psi}_m^{\dot{\beta}} - \tfrac{1}{2}\delta_m^n \sigma_{\alpha\dot{\beta}}^p \bar{\Psi}_p^{\dot{\beta}} - \tfrac{1}{4} i \epsilon^{abcd} \sigma_{ma} \sigma_b^n \sigma_{c\alpha\dot{\beta}} \bar{\Psi}_d^{\dot{\beta}}) \hat{A}_n] + \text{h.c.} , \tag{4.3}$$

where $\mathbb{D}_a$ is the general-relativistic covariant derivative (with torsion), but which is defined not to act on the curved-vector index of Ψ_m^α (see appendix A). We have added in the ϵ-dependent Lorentz transformation

$$\Omega_{\alpha\beta}(\epsilon) = \tfrac{1}{2}\sigma_{a(\alpha}{}^{\dot{\gamma}} (\epsilon_{\beta)} \bar{\Psi}_{\dot{\gamma}}^a - \bar{\epsilon}_{\dot{\gamma}} \Psi_{\beta)}^a) , \tag{4.4}$$

in order to compare with the standard transformation laws. Otherwise we would have transformations such as

$$\delta \sigma_m^a = \sigma_{m\mu\dot\nu}(\epsilon^\mu \bar\Psi^{a\dot\nu} - \bar\epsilon^{\dot\nu}\Psi^{a\mu})$$

(which are, of course, also an invariance).

We also find the cosmological term (eq. (3.13))

$$S_a = 4ia \int d^4x\, \sigma^{-1}(\bar B - \tfrac{1}{8}i\bar\Psi^{a\dot\alpha}\sigma_{[a}{}^{\gamma}{}_{\dot\alpha}\sigma_{b]\gamma\dot\beta}\bar\Psi^{b\dot\beta}) + \text{h.c.}\,, \tag{4.5}$$

in agreement with the form found by Ferrara, Grisaru and van Nieuwenhuizen by non-superfield methods.

The calculations are somewhat messier in the gauge $\phi = 1$ (eqs. (3.3), (3.4), and (3.9)), where we find

$$S = \frac{1}{\kappa^2}\int d^4x\, e^{-1}(L_0(e_a{}^m, \hat\Psi_a^\alpha) - \tfrac{4}{3}\hat{\bar B}\hat B - \tfrac{4}{3}\hat A^a \hat A_a)\,, \tag{4.6a}$$

$$e_a{}^m = \sigma^{-1/3}\sigma_a^m\,, \quad \hat\Psi_a^\alpha = \sigma^{-1/6}(\Psi_a^\alpha - \tfrac{1}{3}\sigma_a^{\alpha\dot\beta}\sigma_{\gamma\dot\beta}^b\Psi_b^\gamma)\,,$$

$$\hat B = \sigma^{-1/3}[B + \tfrac{1}{12}i\bar\Psi^{a\dot\alpha}(3\eta_{ab}\epsilon_{\dot\alpha\dot\beta} - \sigma_a{}^{\gamma}{}_{\dot\alpha}\sigma_{b\gamma\dot\beta})\,\bar\Psi^{b\dot\beta}]\,,$$

$$\hat A^a = \sigma^{-1/3}[A^a + i\sigma_m^a(B^n\partial_n\bar B^m - \bar B^n\partial_n B^m) + \tfrac{1}{8}\epsilon^{abcd}c_{bcd}$$

$$- \tfrac{1}{96}(10\eta^{ac}\eta^{bd} + 2\eta^{a(b}\eta^{d)c} - 5i\epsilon^{abcd})\,\bar\Psi_b^{\dot\beta}\sigma_{c\alpha\dot\beta}\Psi_d^\alpha]\,. \tag{4.6b}$$

(Curved-space indices on fields appearing in eq. (3.3) have been changed to flat-space by σ_a^m.) The fields $e_a{}^m$, $\hat\Psi_a^\alpha$, $\hat B$, and $\hat A^a$ are invariant under b_m transformations.

4.2. Other n

To determine the action for $n \neq -\frac{1}{3}$ (except $n = -\frac{1}{2}$), it is sufficient to examine the linearized form of the action of eq. (3.9). We first obtain the linearized action in superfield form by substituting $e^{2U} = e^{\langle U\rangle}\, e^{2H}\, e^{\langle U\rangle}$ ($H = H^A D_A$) into $L_{\text{vector}} = e^{\langle U\rangle} L_{\text{chiral}}\, e^{-\langle U\rangle}$, and keeping only up to $O(H^2)$. The result is

$$\frac{1}{n}L \to -\tfrac{1}{6}(G_a{}^a G_b{}^b + 3G_a{}^b G_b{}^a)$$

$$+ (3n+1)\{\tfrac{1}{6}[G_a{}^a + 3i(D_\alpha H^\alpha - \bar D^{\dot\alpha} H_{\dot\alpha})]^2 + \frac{1}{2n}[(-1)^a D_A H^A]^2\}\,,$$

$$G_a{}^b \equiv \tfrac{1}{4}\sigma_a^{\alpha\dot\beta}[D_\alpha, \bar D_{\dot\beta}]\, H^b\,, \tag{4.7}$$

where D_A are the vector-representation flat-space covariant derivatives. For the case $n = -\frac{1}{3}$, which is obviously the simplest, H^α does not occur in the action, and the action is identical to that proposed by Ogievetsky and Sokatchev. The remaining invariance for that case is given by

$$[\bar{D}_{\dot\beta}, \Lambda^a \partial_a + \Lambda^\alpha Q_\alpha] = 0$$

$$\to \Lambda^a \partial_a + \Lambda^\alpha Q_\alpha = \{\bar{D}_{\dot\beta}, [\bar{D}^{\dot\beta}, L^\alpha D_\alpha]\} = (-4i\sigma^a_{\alpha\dot\beta}\bar{D}^{\dot\beta} L^\alpha)\, \partial_a + (\bar{D}^2 L^\alpha)\, D_\alpha$$

$$\to \delta H^a = 2\sigma^a_{\alpha\dot\beta}(D^\alpha \bar{L}^{\dot\beta} - \bar{D}^{\dot\beta} L^\alpha)\,, \quad 0 = Q_\alpha \Lambda^\alpha - \partial_a \Lambda^a = \bar{D}^2 D_\alpha L^\alpha\,, \tag{4.8}$$

where Q_α are the vector-representation flat-space supersymmetry generators (see appendix A). When the restriction $\bar{D}^2 D_\alpha L^\alpha = 0$ (i.e., eq. (3.2) for $n = -\frac{1}{3}$) is lifted, one obtains the linearized version of the local superconformal group [15]. Also, if one linearizes the action of eq. (3.10), one finds that H^a couples to the supercurrent superfield [4]. Note that for $n \neq -\frac{1}{3}$, H^α appears only as $D_\alpha H^\alpha$, which is equivalent to $-\frac{1}{2}i(\bar{\Upsilon} - 1)$ (after introducing Υ, H^α appears only as $\bar{\Upsilon} - 1 - 2iD_\alpha H^\alpha$).

Expansion of the Lagrangian in eq. (4.7) is simple in comparison to the calculations of subsect. 4.1. For $n \neq -\frac{1}{3}$ or $-\frac{1}{2}$, we can gauge away all of H except

$$H^a = \theta^\alpha \bar\theta^{\dot\beta} h^a_{\alpha\dot\beta} + (\bar\theta^2 \theta^\alpha \Psi^a_\alpha + \theta^2 \bar\theta_{\dot\alpha} \bar\Psi^{a\dot\alpha}) + \theta^2 \bar\theta^2 A^a\,,$$

$$H^\alpha = \theta^\alpha \bar\theta_{\dot\beta} \bar\rho^{\dot\beta} + \bar\theta^2 \theta^\alpha \bar{B} + \theta^2 \bar\theta_{\dot\beta}(v + iw)^{\alpha\dot\beta} + \theta^2 \bar\theta^2 \beta^\alpha\,. \tag{4.9}$$

We then find for the action

$$S = \frac{1}{\kappa^2}\int d^4x\, e^{-1}\, [L_0(e_a{}^m, \hat\Psi^\alpha_a) + \frac{4}{n}(3n+1)^2 \hat{\bar{B}}\hat{B} - \tfrac{4}{3}\hat{A}^a \hat{A}_a + \frac{4}{n}(3n+1)\, \hat{v}^a \hat{v}_a$$

$$+ 12(3n+1)\, \hat{w}^a \hat{w}_a + \frac{4}{n}(3n+1)^2 (\hat\beta^\alpha \hat\rho_\alpha + \hat{\bar\beta}_{\dot\alpha} \hat{\bar\rho}^{\dot\alpha})]\,, \tag{4.10a}$$

$$e_a{}^m = \sigma^{-(n+1)/2}\, \sigma^m_a = \delta^m_a(1 - \tfrac{1}{2}(n+1)\, h_b{}^b) + h_a{}^m\,,$$

$$\hat\Psi^\alpha_a = \Psi^\alpha_a - \tfrac{1}{2}\sigma^{\alpha\dot\beta}_a \sigma^b_{\gamma\dot\beta} \Psi^\gamma_b\,, \quad \hat{B} = B\,,$$

$$\hat{A}^a = A^a + \tfrac{1}{8}\epsilon^{abcd} c_{bcd}\,, \quad \hat{v}_a = v_a + \partial_m \sigma^m_a - \tfrac{1}{4} n\, \sigma^m_a \partial_m \ln \sigma\,,$$

$$\hat{w}^a = w^a - \tfrac{1}{6}A^a + \tfrac{1}{24}\epsilon^{abcd} c_{bcd}\,,$$

$$\hat\rho^\alpha = \rho^\alpha - \frac{in}{2(3n+1)}\, \sigma^{\alpha\dot\beta}_a \bar\Psi^a_{\dot\beta}\,,$$

$$\hat\beta^\alpha = \beta^\alpha + \tfrac{1}{2} i \sigma^{a\alpha\dot\beta} \sigma^m_a \partial_m \bar\rho_{\dot\beta} - \frac{1}{2(3n+1)}\, \sigma^m_a \partial_m (\Psi^{a\alpha} - \tfrac{1}{2} n\, \sigma^{\alpha\dot\beta}_b \sigma^a_{\gamma\dot\beta} \Psi^{b\gamma}$$

$$- \tfrac{1}{4} n\, \sigma^{a\alpha\dot\beta} \sigma_{b\gamma\dot\beta} \Psi^{b\gamma})\,. \tag{4.10b}$$

We have introduced the full non-linearity into the linearized solution where convenient ($\sigma_a^m = \delta_a^m + h_a{}^m$). As a result, *the action as written in eq. (4.10a) is the exact non-linear action*, although the relations of the fields appearing there to those in eq. (4.9) pick up extra terms and factors in the full non-linear form. The only significant difference in the action for various n (except $-\frac{1}{3}$ and $-\frac{1}{2}$) is that the signs of some of the auxiliary fields differ between the regions $n < -\frac{1}{3}$, $-\frac{1}{3} < n < 0$, and $n > 0$. In comparison with $n = -\frac{1}{3}$ (eq. (4.6)), we also notice the interesting discontinuity in the shifts for $\hat{\Psi}$: $\hat{\Psi} \sim \Psi - \frac{1}{3}\sigma\sigma \cdot \Psi$ for $n = -\frac{1}{3}$, but $\hat{\Psi} \sim \Psi - \frac{1}{2}\sigma\sigma \cdot \Psi$ for all other n. This is due to the contribution of ρ to Ψ's action. For $n = -\frac{1}{2}$, the action in eq. (4.10a) is also the correct action, although we now have $e_a{}^m = J^{-1}\sigma^{-1/4}\sigma_a^m$ (and other J-dependences in the shifts), where $-i\theta^\alpha \ln J$ has been added to H^α. Also, for this case we can gauge σ to 1 (e.g., $e = J^{-4}$, independent of σ).

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Appendix A

Conventions and identities

The following is a list of notational conventions and useful identities. Capital Latin letters are used for superspace indices, small Latin for vector, small Greek for two-spinor (dotted and undotted). Letters from the beginning of each alphabet are used for flat-space indices, the middle for curved-space. Bars and daggers indicate hermitian conjugation, except for operators $X = X^M i\partial_M$, where $\bar{X} \equiv \bar{X}^M i\partial_M$, as compared to the hermitian conjugate with the reverse operator-ordering $X^\dagger = (-1)^m i\partial_M \bar{X}^M$.

$$\epsilon_{\alpha\beta} = \epsilon^{\alpha\beta} = \epsilon_{\dot{\alpha}\dot{\beta}} = \epsilon^{\dot{\alpha}\dot{\beta}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{A.1}$$

$$\theta_\alpha = \epsilon_{\alpha\beta}\theta^\beta , \quad \theta^2 = \theta^\alpha\theta_\alpha = \epsilon_{\alpha\beta}\theta^\alpha\theta^\beta , \quad \theta^\alpha\theta_\beta = \tfrac{1}{2}\theta^2\delta_\beta^\alpha , \quad \partial_\alpha\theta^\beta = \delta_\alpha^\beta , \tag{A.2}$$

$$\bar{\theta}^{\dot{\alpha}} = (\theta^\alpha)^\dagger , \quad \bar{\theta}_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\bar{\theta}^{\dot{\beta}} , \quad \bar{\theta}^2 = (\theta^2)^\dagger = \bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}} = -\epsilon_{\dot{\alpha}\dot{\beta}}\bar{\theta}^{\dot{\alpha}}\bar{\theta}^{\dot{\beta}} ,$$

$$\bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\beta}} = \tfrac{1}{2}\bar{\theta}^2\delta_{\dot{\alpha}}^{\dot{\beta}} ; \quad \bar{\partial}^{\dot{\alpha}}\bar{\theta}_{\dot{\beta}} = \delta_{\dot{\beta}}^{\dot{\alpha}} , \tag{A.3}$$

$$\sigma_a^{\alpha\dot{\beta}} = \sigma_{\beta\dot{\alpha}}^a = (1, \vec{\sigma})_{a\alpha\beta} , \quad \sigma_{\alpha\dot{\beta}}^a\sigma_b^{\alpha\dot{\beta}} = 2\delta_b^a , \quad \sigma_{\gamma\dot{\delta}}^a\sigma_a^{\alpha\dot{\beta}} = 2\delta_\gamma^\alpha\delta_{\dot{\delta}}^{\dot{\beta}} , \tag{A.4}$$

$$\sigma_a^{\alpha\dot{\beta}}\sigma_{b\gamma\dot{\beta}}\sigma_c^{\gamma\dot{\delta}} = [(\eta_{ab}\eta_{cd} + \eta_{ad}\eta_{bc} - \eta_{ac}\eta_{bd}) + i\epsilon_{abcd}]\ \sigma^{d\alpha\dot{\delta}} ,$$

$$\eta_{ab} = (+---) , \quad \epsilon_{0123} = 1 , \tag{A.5}$$

$$X^M Y_M = X^m Y_m + X^\mu Y_\mu + X_{\dot{\mu}} Y^{\dot{\mu}} ,$$

$$(-1)^m X_M Y^M = X_m Y^m - X_\mu Y^\mu - X^{\dot\mu} Y_{\dot\mu} , \tag{A.6}$$

$$[X_a, Y_b\} = [X_a, Y_b] , \quad [X_a, Y_\beta\} = [X_a, Y_\beta] , \quad [X_\alpha, Y_\beta\} = \{X_\alpha, Y_\beta\} , \tag{A.7}$$

$$\sigma^m_{\alpha\dot\beta} = \sigma^a_{\alpha\dot\beta}\sigma^m_a , \quad \sigma = \det \sigma^m_a , \tag{A.8}$$

$$\int d^2\theta = \tfrac{1}{2}\int d\theta^2 \int d\theta^1 = \tfrac{1}{2}\partial_2\partial_1 = -\tfrac{1}{4}\partial^2 ,$$
$$\int d^2\theta\, \theta^2 = 1 , \quad \int d^2\bar\theta = -\tfrac{1}{4}\bar\partial^2 . \tag{A.9}$$

Dirac spinors: $\Psi = \begin{pmatrix} \eta_\alpha \\ \bar\chi^{\dot\alpha} \end{pmatrix}$, $\quad \bar\Psi = (\chi^\alpha \bar\eta_{\dot\alpha})$,

Majorana: $\Psi = \begin{pmatrix} \eta_\alpha \\ \bar\eta^{\dot\alpha} \end{pmatrix}$,

$$\gamma_a = \begin{pmatrix} 0 & \sigma_{a\alpha\dot\beta} \\ \sigma_a^{\beta\dot\alpha} & 0 \end{pmatrix} , \quad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} , \quad C = \begin{pmatrix} \epsilon_{\alpha\beta} & 0 \\ 0 & -\epsilon^{\dot\alpha\dot\beta} \end{pmatrix} , \tag{A.10}$$

flat-space covariant derivatives: D_A ($D_{A\ \mathrm{vector}} = e^{\langle U\rangle} D_{A\ \mathrm{chiral}}\, e^{-\langle U\rangle}$) ;

chiral representation: $D_{\dot\alpha} = \bar\partial_{\dot\alpha}$, $\quad D_\alpha = \partial_\alpha + 2i\bar\theta^{\dot\beta}\sigma^a_{\alpha\dot\beta}\partial_a = e^{-2\langle U\rangle}\partial_\alpha\, e^{2\langle U\rangle}$,
$D_a = \partial_a$,

vector representation: $D_{\dot\alpha} = \bar\partial_{\dot\alpha} - i\theta^\beta\sigma^a_{\beta\dot\alpha}\partial_a = e^{\langle U\rangle}\bar\partial_{\dot\alpha}\, e^{-\langle U\rangle}$,

$$D_\alpha = \partial_\alpha + i\bar\theta^{\dot\beta}\sigma^a_{\alpha\dot\beta}\partial_a = e^{-\langle U\rangle}\partial_\alpha\, e^{\langle U\rangle} , \quad D_a = \partial_a ; \tag{A.11}$$

flat-space supersymmetry generators: Q_A ($Q_{A\ \mathrm{vector}} = e^{\langle U\rangle} Q_{A\ \mathrm{chiral}}\, e^{-\langle U\rangle}$) ;

chiral representation: $Q_\alpha = \partial_\alpha$, $\quad Q_{\dot\alpha} = \bar\partial_{\dot\alpha} + 2i\theta^\beta\sigma^a_{\beta\dot\alpha}\partial_a = e^{-2\langle U\rangle}\bar\partial_{\dot\alpha}\, e^{2\langle U\rangle}$,
$Q_a = \partial_a$,

vector representation: $Q_\alpha = \partial_\alpha - i\bar\theta^{\dot\beta}\sigma^a_{\alpha\dot\beta}\partial_a = e^{\langle U\rangle}\partial_\alpha\, e^{-\langle U\rangle}$,

$$Q_{\dot\alpha} = \bar\partial_{\dot\alpha} + i\theta^\beta\sigma^a_{\beta\dot\alpha}\partial_a = e^{-\langle U\rangle}\bar\partial_{\dot\alpha}\, e^{\langle U\rangle} , \quad Q_a = \partial_a . \tag{A.12}$$

$$\det X_{AB} = \left(\int dz\, d\bar z\, e^{-\bar z^A \delta_{AB} z^B}\right)/\left(dz\, d\bar z\, e^{-\bar z^A X_{AB} z^B}\right)$$
$$= \det X_{ab}/\det[X_{\alpha\beta} - X_{\alpha a}(X^{-1})^{ab}X_{b\beta}]$$
$$= \det[X_{ab} - X_{a\alpha}(X^{-1})^{\alpha\beta}X_{\beta b}]/\det X_{\alpha\beta} , \tag{A.13}$$

where δ_{AB} is just a Kronecker delta, the integration is over a complex supervector $z^A = \sqrt{\tfrac{1}{2}}(x^A + iy^A)$, and $(X^{-1})^{ab}$ and $(X^{-1})^{\alpha\beta}$ refer to the usual inverses of X_{ab} and $X_{\alpha\beta}$, not X_{AB}.

$$L_0 = -\tfrac{1}{2}\mathcal{R} + \epsilon^{abcd}\bar{\Psi}^{\dot\beta}_a\sigma_{b\alpha\dot\beta}\mathcal{D}_c\Psi^\gamma_d\ , \qquad \mathcal{D}_a\Psi^\alpha_b = e^m_a e^n_b(\delta^\alpha_\beta\partial_m + \tfrac{1}{2}\sigma_{cd}{}^\alpha{}_\beta\omega^{cd}_m)\,\Psi^\beta_n\ ,$$

$$e = \det e^m_a\ , \qquad \mathcal{R} = e_a{}^m e_b{}^n\mathcal{R}_{mn}{}^{ab} = e_a{}^m e_b{}^n(\partial_{[m}\omega^{ab}_{n]} + \omega^{ac}_{[m}\omega_{n]c}{}^b)\ ,$$

$$\sigma_{ab}{}^\alpha{}_\gamma = \tfrac{1}{4}\sigma^{\alpha\dot\beta}_{[a}\sigma_{b]\gamma\dot\beta}\ ,$$

$$\omega^{bc}_m = e_{ma}\tfrac{1}{2}(-f^{abc} + f^{bca} - f^{cab})\ ,$$

$$f_{abc} = e_{mc}e^n_{[a}\partial_n e^m_{b]} + \tfrac{1}{2}i\bar{\Psi}^{\dot\beta}_{[a}\sigma_{c\alpha\dot\beta}\Psi^\alpha_{b]}\ , \tag{A.14}$$

$$(\bar{\Psi}^{a\dot\alpha}\sigma^b_{\beta\dot\alpha}\Psi^{c\beta})(\bar{\Psi}^{d\dot\gamma}\sigma^e_{\delta\dot\gamma}\Psi^{f\delta}) = -\tfrac{1}{2}(\eta^{bg}\eta^{eh} + \eta^{bh}\eta^{eg} - \eta^{be}\eta^{gh} + i\epsilon^{begh})$$
$$\times\,(\bar{\Psi}^{d\dot\alpha}\sigma_{g\beta\dot\alpha}\Psi^{c\beta})(\bar{\Psi}^{a\dot\gamma}\sigma_{h\delta\dot\gamma}\Psi^{f\delta})\ . \tag{A.15}$$

Appendix B

Expansion of the action

Here we list some intermediate steps in the expansion of the supergravity action in terms of component fields. For the method of calculation used to derive eq. (4.6) (gauge $\phi = 1$), we have the following equations:

$$\hat{E}_{\dot\alpha}{}^M = \delta^M_{\dot\alpha} \to \hat{E}_{\dot\mu}{}^A = \delta^A_{\dot\mu}\ , \tag{B.1}$$

$$U^\mu = 0 \to \hat{E}_A{}^\mu = \delta^\mu_A \to \hat{E} = \det \hat{E}_a{}^m\ , \tag{B.2}$$

$$\hat{E}_\alpha{}^m = 2i\partial_\alpha U^m + 2(U^n\partial_n\partial_\alpha U^m - U^m\overleftarrow{\partial}_n\partial_\alpha U^n)$$
$$\to \hat{E}_a{}^m = \tfrac{1}{2}\sigma^{\beta\dot\gamma}_a[\partial_\beta\bar\partial_{\dot\gamma}U^m + i\bar\partial_{\dot\gamma}(U^n\partial_n\partial_\beta U^m - U^m\overleftarrow{\partial}_n\partial_\beta U^n)]\ , \tag{B.3}$$

$$(1\cdot \mathrm{e}^{-2\overleftarrow{U}})^{1/3} = 1 - \tfrac{2}{3}i\partial_m U^m - [\tfrac{2}{3}U^n\partial_n\partial_m U^m + \tfrac{2}{3}(\partial_m U^m)^2]\ , \tag{B.4}$$

$$\hat{E}_a{}^m = \sigma^m_b(\delta^b_a + Y_a{}^b)\ , \qquad Y^5 = 0 \to [\det(1 + Y)]^{-1/3} = \mathrm{e}^{-\mathrm{tr}\,\ln(1+Y)/3}$$
$$= 1 - \tfrac{1}{3}\mathrm{tr}\,Y + \tfrac{1}{18}[(\mathrm{tr}\,Y)^2 + 3\,\mathrm{tr}\,Y^2] - \tfrac{1}{162}[(\mathrm{tr}\,Y)^3 + 9(\mathrm{tr}\,Y)(\mathrm{tr}\,Y^2)$$
$$+ 18\,\mathrm{tr}\,Y^3] + \tfrac{1}{1944}[(\mathrm{tr}\,Y)^4 + 18(\mathrm{tr}\,Y)^2(\mathrm{tr}\,Y^2) + 27(\mathrm{tr}\,Y^2)^2$$
$$+ 72(\mathrm{tr}\,Y)(\mathrm{tr}\,Y^3) + 162\,\mathrm{tr}\,Y^4]\ , \tag{B.5}$$

$$\hat{E}_\alpha{}^\mu = \delta^\mu_\alpha\,, \quad \hat{E}_{\dot\alpha}{}^{\dot\mu} = \delta^{\dot\mu}_{\dot\alpha}\,, \quad \hat{E}_\alpha{}^{\dot\mu} = \hat{E}_{\dot\alpha}{}^{\mu} = \hat{E}_{\dot\alpha}{}^{m} = \hat{E}_a{}^{\mu} = \hat{E}_a{}^{\dot\mu} = 0\,,$$

$$\hat{E}_\alpha{}^m = 4i\theta_\alpha B^m + 2i\bar\theta^{\dot\nu}\sigma^m_{\alpha\dot\nu} + 2i\bar\theta^2\Psi^m_\alpha + 4i\theta_\alpha\bar\theta_{\dot\nu}\bar\Psi^{m\dot\nu}$$
$$+ 4\bar\theta^2\theta_\beta[\delta^\beta_\alpha(iA^m + \bar B^n\partial_n B^m - B^n\partial_n\bar B^m) + \tfrac14\sigma^{a\beta\dot\gamma}\sigma^b_{\alpha\dot\gamma}\sigma^n_{[a}\partial_n\sigma^m_{b]}]$$
$$+ 4\theta^2\bar\theta^{\dot\nu}\sigma^a_{\alpha\dot\nu}(B^n\partial_n\sigma^m_a - \sigma^n_a\partial_n B^m)$$
$$+ 2\theta^2\bar\theta^2[2(B^n\partial_n\Psi^m_\alpha - \Psi^n_\alpha\partial_n B^m) + \sigma^a_{\alpha\dot\beta}(\sigma^n_a\partial_n\bar\Psi^{m\dot\beta} - \bar\Psi^{n\dot\beta}\partial_n\sigma^m_a)]\,,$$

$$\hat{E}_a{}^m = \sigma^m_b\{\delta^b_a + \sigma^{\beta\dot\gamma}_a(\theta_\beta\bar\Psi^b_{\dot\gamma} - \bar\theta_{\dot\gamma}\Psi^b_\beta) + 2i\theta^2\sigma^b_n(\sigma^p_a\partial_p B^n - B^p\partial_p\sigma^n_a)$$
$$+ \theta^\alpha\bar\theta^{\dot\beta}\sigma_{e\alpha\dot\beta}\sigma^b_n[4\delta^e_a(A^n + i(B^p\partial_p\bar B^n - \bar B^p\partial_p B^n)) + \epsilon^{cd}{}_a{}^e\sigma^p_c\partial_p\sigma^n_d$$
$$+ i\sigma^p_{[a}\partial_p\sigma^{ne]}] + i\theta^2\bar\theta_{\dot\gamma}\sigma^{\beta\dot\gamma}_a\sigma^b_n[\sigma^c_{\beta\dot\mu}(\sigma^p_c\partial_p\bar\Psi^{n\dot\mu} - \bar\Psi^{p\dot\mu}\partial_p\sigma^n_c)$$
$$+ 2(B^p\partial_p\Psi^n_\beta - \Psi^p_\beta\partial_p B^n)]\,, \tag{B.6}$$

$$(1\cdot e^{-2\bar U})^{1/3} = 1 - \tfrac23 i(\theta^2 B + \bar\theta^2\bar B) - \tfrac23 i\theta^\mu\bar\theta^{\dot\nu}\sigma^a_{\mu\dot\nu}\partial_m\sigma^m_a$$
$$- \tfrac23 i(\bar\theta^2\theta^\mu\partial_m\Psi^m_\mu + \theta^2\bar\theta_{\dot\nu}\partial_m\bar\Psi^{m\dot\nu}) - \tfrac23\theta^2\bar\theta^2[i\partial_m A^m + (\bar B^m\partial_m B + B^m\partial_m\bar B)$$
$$+ \tfrac23\bar B B + \tfrac12\sigma^m_a\partial_m(\partial_n\sigma^{na}) + \tfrac16(\partial_m\sigma^m_a)(\partial_n\sigma^{na})]\,. \tag{B.7}$$

For the other method of calculation in sect. 4, we set B^m (and $B \equiv \partial_m B^m$) to zero in the above equations, as B now occurs only in ϕ. We then have the additional factor in the Lagrangian $(e^{2U}\phi)^\dagger\phi$, where

$$\sigma^{2/3}(e^{2U}\phi)^\dagger\phi = 1 + \tfrac23\sigma^{\beta\dot\gamma}_a(\bar\theta_{\dot\gamma}\Psi^a_\beta - \theta_\beta\bar\Psi^a_{\dot\gamma}) + \tfrac23 i(\theta^2 B - \bar\theta^2\bar B)$$
$$+ \tfrac23\theta^\alpha\bar\theta^{\dot\beta}(\tfrac23\sigma^a_{\alpha\dot\gamma}\sigma^b_{\delta\dot\beta}\bar\Psi^{\dot\gamma}_a\Psi^\delta_b + i\sigma^m_{\alpha\dot\beta}\partial_m\ln\sigma)$$
$$+ \tfrac23 i\theta^2\bar\theta_{\dot\beta}(\bar\Psi^{m\dot\beta}\partial_m\ln\sigma - \tfrac13\sigma^a_{\alpha\dot\gamma}\bar\Psi^{\dot\gamma}_a\sigma^{m\alpha\dot\beta}\partial_m\ln\sigma + \tfrac23\sigma^{\alpha\dot\beta}_a\Psi^a_\alpha B)$$
$$+ \tfrac23 i\bar\theta^2\theta^\alpha(\Psi^m_\alpha\partial_m\ln\sigma - \tfrac13\sigma^{\beta\dot\gamma}_a\Psi^a_\beta\sigma^m_{\alpha\dot\gamma}\partial_m\ln\sigma + \sigma^{\beta\dot\gamma}_a\sigma^m_{\alpha\dot\gamma}\partial_m\Psi^a_\beta + \tfrac23\sigma_{a\alpha\dot\beta}\bar\Psi^{a\dot\beta}\bar B)$$
$$+ \tfrac23 i\theta^2\bar\theta^2[A^m\partial_m\ln\sigma - \tfrac13\sigma^{\alpha\dot\beta}_a\bar\Psi^m_{\dot\beta}\Psi^a_\alpha\partial_m\ln\sigma + \sigma^{\alpha\dot\beta}_a\bar\Psi^m_{\dot\beta}\partial_m\Psi^a_\alpha$$
$$- \tfrac23\sigma^{\alpha\dot\beta}_a\bar\Psi^a_{\dot\beta}\Psi^m_\alpha\partial_m\ln\sigma + \tfrac29\sigma^{\alpha\dot\beta}_a\sigma^{\gamma\dot\delta}_b\bar\Psi^a_{\dot\beta}\Psi^b_\gamma\sigma^m_{\alpha\dot\delta}\partial_m\ln\sigma - \tfrac23\sigma^{\alpha\dot\beta}_a\sigma^{\gamma\dot\delta}_b\bar\Psi^a_{\dot\beta}\sigma^m_{\alpha\dot\delta}\partial_m\Psi^b_\gamma$$
$$- \tfrac12 i\sigma^m_a\partial_m(\sigma^{na}\partial_n\ln\sigma) + \tfrac16(\sigma^m_a\partial_m\ln\sigma)(\sigma^{na}\partial_n\ln\sigma) - \tfrac23 i\bar B B]\,. \tag{B.8}$$

Appendix C

Formalisms of Breitenlohner and Brink et al.

In order to relate our formalism to that of Breitenlohner, we will first describe a way of introducing a superconnection which is analogous to supergauging the Lorentz group (in analogy to sect. 2). Essentially this is done simply by the generalization [25]

$$U = U^M i\partial_M \to \widetilde{U} = U^M i\partial_M + \tfrac{1}{2}U^{ab} iM_{ab} , \tag{C.1}$$

where the Lorentz generators M_{ab} act on all flat-space indices *and* on curved-space spinorial indices, *including* those on θ^μ, $\bar{\theta}_{\dot{\mu}}$, ∂_μ, and $\bar{\partial}^{\dot{\mu}}$. M_{ab} may be chosen to act in other ways, but this way is most convenient in terms of the transformation law for $\widetilde{U}$ and the expressions for the supertorsion and supercurvature in terms of the supervierbein and superconnection. We will start with the complex superfield $\widetilde{W}$ instead of $\widetilde{U}$, in analogy with sect. 2, to allow the Lagrangian to be written also in the vector representation. Also, this will show not only the semi-chiral $\widetilde{\Lambda}$ transformations but also the real $\widetilde{K}$ transformations. We then have the following equations in the vector representation:

$$\nabla_\alpha = e^{-\widetilde{W}}\partial_\alpha e^{\widetilde{W}} , \quad \nabla_{\dot{\alpha}} = e^{\bar{\widetilde{W}}}\bar{\partial}_{\dot{\alpha}} e^{-\bar{\widetilde{W}}} , \quad \nabla_a = \tfrac{1}{4} i\sigma_a^{\beta\dot{\gamma}}\{\nabla_\alpha, \nabla_{\dot{\beta}}\} , \tag{C.2a}$$

$$\nabla_A = E_A{}^M \partial_M + \tfrac{1}{2}\Phi_A{}^{bc} M_{bc} , \tag{C.2b}$$

$$[\nabla_A, \nabla_B\} = T_{AB}{}^C \nabla_C + \tfrac{1}{2}R_{AB}{}^{cd} M_{cd} \to T_{\alpha\beta}{}^C = T_{\alpha\dot{\beta}}{}^C + 2i\sigma^c_{\alpha\dot{\beta}}$$

$$= R_{\alpha\beta}{}^{cd} = R_{\alpha\dot{\beta}}{}^{cd} = 0 , \tag{C.2c}$$

$$e^{\widetilde{W}'} = e^{i\bar{\widetilde{\Lambda}}} e^{\widetilde{W}} e^{-i\widetilde{K}} , \quad \widetilde{\Lambda} = \Lambda^M i\partial_M + \tfrac{1}{2}\Lambda^{ab} iM_{ab} ,$$

$$\widetilde{K} = \bar{\widetilde{K}} = K^M i\partial_M + \tfrac{1}{2}K^{ab} iM_{ab} , \tag{C.2d}$$

$$L = [(1 \cdot e^{-\overleftarrow{\widetilde{W}}})(1 \cdot e^{-\overleftarrow{\widetilde{W}}})^\dagger]^{(n+1)/2} E^n . \tag{C.2e}$$

Λ^M satisfies the same condition as before (eq. (3.2)), but Λ^{ab} *is completely arbitrary, so* W^{ab} *can be completely gauged away.* In fact, $(1 \cdot e^{-\overleftarrow{\widetilde{W}}})$ and E are separately invariant under arbitrary Λ^{ab} transformations. In terms of the $\widetilde{K}$ transformations (which are the only transformations appearing in formalisms expressed in terms of only ∇_A explicitly) $(1 \cdot e^{-\overleftarrow{\widetilde{W}}})$ and E^{-1} are both scalar densities of weight 1, so hermiticity and weight determine the form of L in terms of them. However, Λ invariance is needed to show that only one value of n may be used: we cannot use a superposition of L's for different n's. We can transform to the chiral representation by

$$\int d^4x\, d^4\theta\, L(W, \bar{\widetilde{W}}) = \int d^4x\, d^4\theta\, L(\widetilde{W}, \bar{\widetilde{W}})\, e^{-\overleftarrow{\bar{\widetilde{W}}}} = \int d^4x\, d^4\theta\, L(2\widetilde{U}, 0) , \tag{C.3}$$

which gets us back to the action of eq. (3.9) with the substitution of eq. (C.1). If we use the Λ^{ab} invariance, we can gauge away all of U^{ab}, and therefore all of $\Phi_A{}^{bc}$, leaving us with the original action of eq. (3.9) in terms of U^M only.

The formalism of Breitenlohner is equivalent to the case $n = -1$ (where the component fields are all tensors instead of tensor densities) of the $\widetilde{U}$ formalism in Wess-Zumino gauges. As in the $\widetilde{U}$ formalism, in Breitenlohner's formalism *the connection supermultiplet can be completely gauged away.* The translation, supersymmetry, and Lorentz generators in Breitenlohner's supermultiplet correspond to ∂_m, ∂_μ, and M_{ab}, respectively, in $\widetilde{U}$, and his supermultiplet corresponds to the $\theta\bar{\theta}$, $\bar{\theta}^2\theta$ (and $\theta^2\bar{\theta}$), and $\theta^2\bar{\theta}^2$ parts of $\widetilde{U}$ (the rest being gauged away in Wess-Zumino gauges). Breitenlohner's additional (to local supersymmetry) gauge freedom is the gauge freedom in Λ^{ab} and the additional gauge freedom in $\Lambda_{\dot{\mu}}$. The fact that the connection can be gauged away means that the action is independent of it (except for gauge-breaking terms), and therefore Breitenlohner's formalism is a second-order formalism (as is ours), and contains terms explicitly quadratic in x-derivatives (as can be seen in his terms quadratic in the torsion).

Since Breitenlohner's connections can be gauged away and are not useful in the construction of covariant actions, it is more convenient to drop them from the formalism entirely. One should therefore not use the "covariant" derivatives of eq. (C.2) (which are no more covariant than the semicovariant derivatives $\hat{E}_A$), but instead use the true covariant derivatives of eq. (3.17) as modified by eq. (3.27) for $n \neq -\frac{1}{3}$. Of course, as far as component-field calculations are concerned, it is more convenient to work with the $n = -\frac{1}{3}$ minimal set of auxiliary fields instead of Breitenlohner's fields.

We can also include the semichiral superfield Υ in the formalism for $n = -1$ by modifying only the ∇_α (and $\nabla_{\dot{\alpha}}$) equation in eq. (C.2a) to

$$\nabla_\alpha = \mathrm{e}^{-\widetilde{W}}[\bar{\Upsilon}^{1/2}\partial_\alpha - (\partial^\beta\bar{\Upsilon}^{1/2})\, M_{\alpha\beta}]\, \mathrm{e}^{\widetilde{W}} . \tag{C.4}$$

The formalism of Brink et al., for non-zero cosmological parameter a is then obtained by the substitution

$$\Upsilon \to \Upsilon + \kappa^2 a\bar{\theta}^2 , \tag{C.5}$$

which follows from solving $R = \bar{\partial}^2\Upsilon = -4\kappa^2 a$ as a constraint (in the chiral representation). In the gauge $\Upsilon = 1$, the condition $\bar{\partial}^{\dot{\mu}}\Lambda_{\dot{\mu}} = 0$ now becomes

$$\bar{\partial}^{\dot{\mu}}[(1 + \kappa^2 a\bar{\theta}^2)\,\Lambda_{\dot{\mu}}] = 0 \to \Lambda_{\dot{\mu}} = (1 - \kappa^2 a\bar{\theta}^2)\,\Lambda_{\dot{\mu}}(a = 0) . \tag{C.6}$$

Although the derivatives in eq. (C.2) are not covariant , we can choose a "covariant gauge condition" such that the remaining invariances of the action transform the derivatives covariantly:

$$T_{\alpha a}{}^b - \tfrac{1}{4}\sigma_{a\alpha\dot{\beta}}\sigma^{b\gamma\dot{\beta}}T_{\gamma d}{}^d = 0 . \tag{C.7}$$

This "gauge condition" is equivalent to modifying the constraints of eq. (3.16b), as adapted for arbitrary n by eq. (3.29), by merely replacing $T_{ab}{}^c = 0$ with $R_{\alpha\beta}{}^{cd} = 0$.

(This can be done for all n. For $n = -1$, we replace $R = 0$ with $R = -4\kappa^2 a$ to include the cosmological constant.) The resulting covariant derivatives are the same as the old ones, *except* that $\phi_a{}^{bc}$ has been changed:

$$\phi_a{}^{bc} = \tfrac{1}{4} i\sigma_a^{\alpha\dot\beta}(E_\alpha\phi_{\dot\beta}{}^{bc} + E_{\dot\beta}\phi_\alpha{}^{bc} + \phi_\alpha{}^{d[b}\phi_{\dot\beta}{}^{c]}{}_d + \phi_{\alpha\dot\beta\dot\gamma}\phi^{\dot\gamma bc} + \phi_{\dot\beta\alpha}{}^{\gamma}\phi_\gamma{}^{bc}) . \tag{C.8}$$

The gauge condition (C.7) is the superspace form of Breitenlohner's gauge choices for his connection supermultiplet, and is one of the set of constraints + field equations of Brink et al. We can thus consider the resulting covariant derivatives as either a covariant gauge choice for Breitenlohner's original set of supergravity fields, or, more conveniently, as a choice of covariant derivatives for Breitenlohner's reduced set of fields (as in eq. (4.10a)), which is the same as that of sect. 3 except for a "non-minimal coupling" in ∇_a, due to a tensor being added to $\phi_a{}^{bc}$. (Or, conversely, we could consider ∇_a of sect. 2 as the one with the non-minimal coupling, depending on whether $T_{ab}{}^c = 0$ or $R_{\alpha\dot\beta}{}^{cd} = 0$ is considered more fundamental.) For the matter multiplets considered in this article, the two couplings are identical.

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