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Effective potential for the order parameter of gauge theories at finite temperature

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SU(n) gauge theories at finite temperature $T=\beta^{-1}$ are analyzed in terms of the spontaneous breakdown of a $Z(n)$ symmetry corresponding to the order parameter $L(\vec{x})=(1/n)\mathrm{Tr}\,P\,\exp[ig\int_0^\beta A_0(\vec{x},t)dt]$. An "effective potential" for L is evaluated in the one-loop approximation for both the continuum and the lattice gauge theories. It is shown that the $Z(n)$ symmetry is broken, so that the continuum theory does not confine for high temperatures. Similarly the lattice theory does not confine for sufficiently weak coupling if the number of time sites N_t is finite. It is argued that as $N_t\to\infty$ the $Z(n)$ symmetry is restored and the theory will confine for all values of the coupling.

INTRODUCTION

Gauge theories at finite temperature (in fact, field theories in general) can be analyzed in terms of a Euclidean path integral over a finite "time" interval $0\leq t\leq\beta$. In the case of SU(n) gauge theories the partition function is written[1]

$$Z(\beta)=N\int dA_\mu^a\exp(-\tfrac{1}{4}F_{\mu\nu}^aF_{\mu\nu}^a)\,. \tag{1}$$

Here A_μ^a are the potentials and $F_{\mu\nu}^a$ are the field strengths. The path integral (1) is over all A_μ^a which obey the periodic boundary condition $A_\mu^a(t=0)=A_\mu^a(t=\beta)$.

The phase structure of the theory can be studied[2] by means of the order parameter

$$L(\vec{x})=\frac{1}{n}\mathrm{Tr}P\exp\left[ig\int_0^\beta A_0(\vec{x},t)dt\right], \tag{2}$$

where $A_0=A_0^a\lambda_a$ and where λ^a are generators of SU(n). $\langle L(\vec{x})\rangle$ represents the partition function in the presence of an infinitely massive external source [in the representation of SU(n) determined by λ] at the spatial point $\vec{x}$. If $\langle L(\vec{x})\rangle=0$ the theory is in a confining phase whereas if $\langle L(\vec{x})\rangle\neq 0$ it is in a nonconfining phase. The energy of separation of two sources can be probed by means of the correlation function $\langle L(\vec{x})L(\vec{y})\rangle$.

$L(\vec{x})$ is invariant under *periodic* gauge transformations, i.e., those which satisfy the relation $U(t=0)=U(t=\beta)$. However, there exist gauge transformations $U(\vec{x},t)$ which are not periodic for which $U(t=\beta)=e^{2\pi ik/n}U(t=0)$ with $k=0,1,2,\ldots,n-1$. $U(t=\beta)$ is an element of SU(n) since $e^{2\pi ik/n}$ is an element of $Z(n)$, the center of SU(n). Under such a gauge transformation, the periodic boundary conditions on A_μ^a are maintained and the Hamiltonian is invariant.

However, the order parameter L is not invariant. In fact $L(\vec{x})\to e^{-2\pi ik/n}L(\vec{x})$. It follows that $\langle L(\vec{x})\rangle=e^{-2\pi ik/n}\langle L(\vec{x})\rangle$ for $k=0,\ldots,n-1$. Thus $\langle L(\vec{x})\rangle=0$. So, unless the $Z(n)$ symmetry is spontaneously broken, the theory will confine.

In this paper I present the results of a computation of the effective potential for $L(\vec{x})$ at the one-loop level.[3] The calculation is performed in the continuum theory at a finite temperature $T=\beta^{-1}$ and in the Wilson lattice theory with finite number of time sites, N_t. The effective potential has a minimum when $L(\vec{x})=e^{2\pi ik/n}$ indicating that in perturbation theory the $Z(n)$ symmetry is spontaneously broken. The one loop calculation is reliable at high temperature. One can thus establish that SU(n) gauge theories lose their confining property at high temperature. The results of the calculation suggest mechanisms for the restoration of the symmetry at low temperatures.

The same result holds in the lattice calculation. The lattice theory does not confine for sufficiently small g^2 when the number of time sites N_t is finite. However, as $N_t\to\infty$ the effective potential flattens indicating that the $N_t=\infty$ theory confines for all values of g^2 which, in turn, implies confinement in the continuum theory.[4,5]

In the next section, the effective potential in the continuum SU(2) theory with no fermions is computed. A more general version of this calculation was done by Gross, Pisarski and Yaffe[5] in a somewhat different context. The subsequent sections contain the results in the Wilson lattice theory and a summary.

THE EFFECTIVE POTENTIAL FOR SU(2) IN THE CONTINUUM

Consider the $T=0$ SU(2) gauge theory in the $A_0=0$ gauge (with no fermions). The physical sector of the Hilbert space of states consists only of gauge-invariant states. The projection operator into these states is given by

$$P_{\mathrm{phys}}=\int\mathfrak{D}\lambda^a(\vec{x})\exp\left(i\int d^3x\,D_iE_i^a\lambda_a/g\right), \tag{3}$$

where the integral is over all possible gauge functions λ which are strictly functions of $\vec{x}$. The partition function is given by a sum only over states in the physical sector of the Hilbert space:

$$Z(\beta)=\sum_n \langle n|e^{-\beta H}P_{\mathrm{phys}}|n\rangle\,, \tag{4}$$

where H is the $A_0=0$ gauge Hamiltonian for SU(2) gauge theories. Using Eq. (3),

$$Z(\beta)=\int \mathfrak{D}\lambda^a(\vec{x})\sum_n\langle n|e^{-\beta\hat{H}}|n\rangle\,, \tag{5}$$

where

$$\hat{H}=H-i\int d^3x\,\mathfrak{D}_iE_i^a(\lambda^a/\beta g)\,.$$

A path-integral expression for $Z(\beta)$ can be derived by the usual method[6] using the Hamiltonian $\hat{H}$ instead of H. The result is

$$Z(\beta)=N\int \mathfrak{D}(g\beta A_0)dA_i^a(\vec{x},t)\exp\left(-\tfrac{1}{2}\int_0^\beta dt\int d^3x[(\partial_0A_i-\partial_iA_0+gA_0\times A_i)^2+B^2]\right), \tag{6}$$

where $A_0^a(\vec{x})=\lambda^a(\vec{x})/\beta g$ depends strictly on $\vec{x}$, and where $A_i^a(t=0)=A_i^a(t=\beta)$. A_0 thus appears as a Lagrange multiplier which ensures that we stay in the physical sector of the Hilbert space.

At finite temperature we *cannot* eliminate A_0 by going to the $A_0=0$ gauge; such a transformation on the functional integral would force A_i to violate the periodic boundary conditions. We still have the freedom of performing time-independent gauge transformations. In fact we can work in a gauge[7] where

$$A_0^a(\vec{x})=\delta_{a3}\phi(\vec{x})\,. \tag{7}$$

In such a gauge Eq. (6) can be written[8]

$$\begin{aligned}Z(\beta)=N\int &\{1-\cos[g\beta\phi(\vec{x})]\}d\phi(\vec{x})dA_i^a(\vec{x},t)\\ &\times\exp\left(-\tfrac{1}{2}\int_0^\beta dt\int d^3x[(\nabla\phi)^2+(\partial_0A_i+g\phi\hat{3}\times A_i)^2+B^2]\right),\end{aligned} \tag{8}$$

where $\hat{3}_a=\delta_{a3}$. The factor $1-\cos g\beta\phi$ arises as the invariant measure factor for SU(2).[9] $L(\vec{x})$ is totally determined by ϕ:

$$L(\vec{x})=\cos\left(\frac{\beta g\phi(\vec{x})}{2}\right). \tag{9}$$

We shall evaluate the effective potential for ϕ rather than that for L.

It is clear that the action in (8) will be minimized when $\nabla\phi=0$. Let us then look at a constant $g\phi(\vec{x})=C$. If $C=0$ then $L=1$ and if $C=2\pi/\beta$ then $L=-1$. For any value of C the action is minimized when $A_i^a=0$, in which case $S=0$. (The proof is that for any small δA_i, $S>0$.)

It is important to note that this result holds *only* in the gauge $\partial_0A_0=0$ and $A_0^a=\phi\delta_{a3}$. We see that at the tree (or classical) level, all values of C have equal effective action, though the $Z(n)$ symmetry only implies equal action for $C=0$ and $C=2\pi/\beta$.

We can now proceed to the one-loop calculation where we shall see that this degeneracy is lifted.

The one-loop effective potential[10] is evaluated by writing $\phi(\vec{x})=C/g+\delta\phi(\vec{x})$ and $A_i^a(\vec{x},t)=\delta A_i^a(\vec{x},t)$. The measure term $\prod\{1-\cos[g\beta\phi(\vec{x})]\}$ is a ghost term and can be written[11]

$$\exp\left(\int d^3x\ln\{1-\cos[g\beta\phi(\vec{x})]\}\int d^3k/(2\pi)^3\right). \tag{10}$$

The action (including the ghost term) is now

$$\begin{aligned}S=\tfrac{1}{2}\int_0^\beta dt\int d^3x[&(\nabla\delta\phi)^2+(\partial_0\delta A_i+C\hat{3}\times\delta A_i)^2\\ &+\tfrac{1}{2}(\partial_i\delta A_j-\partial_j\delta A_i)^2]\\ &-\int\frac{d^3k}{(2\pi)^3}d^3x\ln(1-\cos\beta C)+O(g^2)\,.\end{aligned} \tag{11}$$

$Z(\beta)$ is now in the form of a Gaussian integral. Writing S in the general form

$$S=\mathrm{const}+(\delta A)^TM(\delta A)+(\delta\phi)^TN(\delta\phi) \tag{12}$$

implies

$$Z(\beta)=[(\det M)(\det N)]^{-1/2}\,; \tag{13}$$

$\det M\times\det N$ is simply the product of the eigenvalues of M and N.

It is straightforward to diagonalize M and N by Fourier transforming δA and $\delta\phi$. Since $0\le t\le\beta$ we have $\omega=2\pi n/\beta$ with integer n, $-\infty<n<\infty$. A short calculation yields the following eigenvalues for each ω and $\vec{K}$: K^2 from $\delta\phi$; $(\omega\pm C)^2+K^2$, and $(\omega\pm C)^2+K^2$ from the transverse components of $A_i^{(1)}$ and $A_i^{(2)}$; $(\omega\pm C)$ from the longitudinal components of $A_i^{(3)}$; ω^2 from the longitudinal components of $A_i^{(3)}$. The longitudinal components are gauge artifacts and, as we shall see, they are canceled by the ghost terms. Using the fact that $(\prod$ eigenvalues$)^{-1/2}=\exp[-\frac{1}{2}\sum\ln(\text{eigenvalues})]$ and using the definition of the free energy $Z(\beta)=e^{-\beta F(\beta)}$ we find

$$F(\beta)=\frac{1}{\beta}\frac{V}{2}\sum_{n=-\infty}^{\infty}\int\frac{d^3K}{(2\pi)^3}\{\ln K^2+2\ln[(\omega+C)^2+K^2]+2\ln[(\omega-C)^2+K^2]+2\ln(\omega^2+K^2)+\ln(\omega+C)^2+\ln(\omega-C)^2+\ln\omega^2\}$$
$$-\frac{1}{\beta}V\int\frac{d^3k}{(2\pi)^3}\ln(1-\cos\beta C)\,, \tag{14}$$

where $V=\int d^3x$ is the volume of space, $\omega=2\pi n/\beta$, and the last term in (14) is the ghost term.

The value of $F(\beta)$ at $C=0$ has been studied extensively.[12] It is simply the free energy of an ideal gluon gas at temperature β^{-1}. We wish to expose the C dependence of F by evaluating $F(\beta,C)-F(\beta,C=0)$.

We shall start by evaluating

$$G=\sum_{n=-\infty}^{\infty}\int d^3K\{\ln[(\omega+C)^2+K^2]-\ln[\omega^2+K^2]\}. \tag{15}$$

To this end let

$$A=\sum_{n=-\infty}^{\infty}\{\ln[(\omega+C)^2+K^2]-\ln[\omega^2+K^2]\}. \tag{16}$$

This is evaluated by taking $\partial A/\partial K^2$:

$$\frac{\partial A}{\partial K^2}=\sum_{n=-\infty}^{\infty}\left(\frac{1}{(\omega+C)^2+K^2}-\frac{1}{\omega^2+K^2}\right). \tag{17}$$

This sum can be evaluated by standard contour integral techniques[6] with the result that

$$\frac{\partial A}{\partial K^2}=\frac{\beta}{2K}\left(\frac{\sinh\beta K}{\cosh\beta K-\cos\beta C}-\frac{\sinh\beta K}{\cosh\beta K-1}\right). \tag{18}$$

This can be integrated to obtain

$$A=\ln(\cosh\beta K-\cos\beta C)-\ln(\cosh\beta K-1) \tag{19}$$

(since $A=0$ at $C=0$). From Eq. (15),

$$G(C)=\int d^3K\ln(1-2\cos\beta Ce^{-\beta K}+e^{-2\beta K})$$
$$+\text{terms independent of } C. \tag{20}$$

Notice that Eq. (20) is symmetric[13] under $C\to -C$ and that at $C=0$ it reduced to the usual expression for the free energy of a massless ideal Bose gas.

Returning to Eq. (14) we evaluate

$$\sum_n\int d^3K[\ln(\omega+C)^2-\ln\omega^2]$$
$$=\left(\int d^3K\right)\sum_n[\ln(\omega+C)^2-\ln\omega^2]$$
$$=\int d^3K\ln(1-\cos\beta C)+\text{terms independent of } C\,, \tag{21}$$

where Eq. (19) has been used with $K=0$.

Finally, putting together Eqs. (14), (20), and (21) we find that the free energy per unit volume (the effective potential for C) is given by

$$V_{\rm eff}(C)=-\frac{F(\beta,C)}{V}$$
$$=\frac{1}{2\beta(2\pi)^3}\int d^3K\{4\ln(1-2\cos\beta Ce^{-\beta k}+e^{-2\beta k})$$
$$+2\ln(1-\cos\beta C)\}$$
$$-\frac{1}{\beta}\int\frac{d^3k}{(2\pi)^3}\ln(1-\cos\beta C)$$
$$+\text{terms independent of } C\,. \tag{22}$$

The ghost term cancels the longitudinal gluon piece (as advertised) and we obtain

$$V_{\rm eff}(C)=\frac{1}{2\beta}\int\frac{d^3k}{(2\pi)^3}4\ln(1-2\cos\beta Ce^{-\beta k}+e^{-2\beta k})$$
$$=\frac{1}{2(2\pi)^2\beta^4}\int_0^\infty dx\,4\ln(1-2\cos\beta Ce^{-x}+e^{-2x})\,. \tag{23}$$

The integral in Eq. (23) can be evaluated[15] by writing $\ln(1-2\cos\beta Ce^{-x}+e^{-2x})=\mathrm{Re}\ln(1-e^{-x}e^{i\beta C})$, expanding the logarithm, integrating, and resumming the series to obtain

$$V_{\rm eff}(C)=-\frac{2\pi^2}{\beta^4}\left(\frac{1}{45}-\frac{1}{24}\left\{1-\left[\left(\frac{\beta C}{\pi}\right)_{\rm mod 2}-1\right]^2\right\}^2\right). \tag{24}$$

This is the key result of this paper. $V_{\rm eff}(C)$ is sketched in Fig. 1. At the one-loop level $V(C)$ is independent of g^2 and has a trivial dependence on β. The overall curve scales as $1/\beta^4$ and C has minima at $2n\pi/\beta$. V has the $Z(n)$ symmetry $C\to C+2\pi/\beta$ and the symmetry $C\to -C$.[13]

At the one-loop level the $Z(n)$ symmetry is spon-

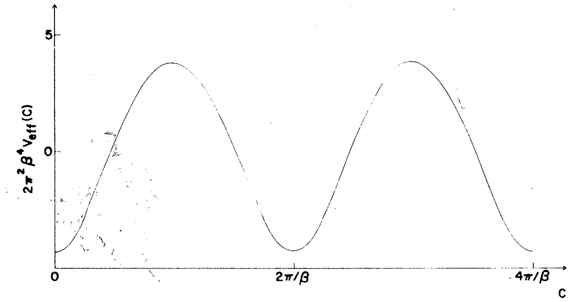

FIG. 1. The effective potential for $C=\beta A_0$ in the continuum SU(2) gauge theory.

taneously broken. Since perturbative calculations are reliable at high temperature we expect that the theory will not confine at high T. This symmetry is expected to be restored at low temperature, even as $g^2 \to 0$. A possible mechanism for this is the formation of $Z(n)$ bubbles.[14] If we are in the vacuum $C=0$ one can imagine forming bubbles of the alternate vacuum $C=2\pi/\beta$. Such bubbles can restore the $Z(n)$ symmetry. It turns out that these bubbles are related to finite-temperature instantons.[15,16] Notice that the minima in C are separated by $2\pi T$. As $T\to 0$ they become closer, whereas quantum fluctuations affect ϕ directly ($\phi = C/g + \delta\phi$). This may provide a mechanism whereby the symmetry, though broken at high temperature (quark liberation), is restored at small temperature (quark confinement).

LATTICE CALCULATION OF V_{eff} [FOR SU(2)]

The continuum calculation discussed above carries with it one particular hazard. Despite the fact that if $\langle L\rangle \neq 0$ the theory cannot confine heavy-quark sources, it is not necessarily true that $\langle L\rangle = 0$ implies confinement. $\langle L\rangle = 0$ implies only that the free energy of an isolated quark is infinite. This could be either an infrared or an ultraviolet infinity.[17] To settle this issue it is useful to regularize the theory by working on a lattice in which case $\langle L\rangle = 0$ necessarily implies confinement.

The lattice calculation is very analogous to the continuum calculation and I shall only sketch it in this paper. The calculation is done in the Wilson lattice gauge theory with a finite number of time sites N_t. The continuum finite-temperature theory can be analyzed by studying the behavior of the theory for fixed bare coupling constant and for large N_t.[14] $N_t=\infty$ is the usual lattice gauge theory and it is believed that this theory confines for all values of g^2. For finite N_t it is believed that the theory confines only for couplings greater than some critical coupling $g_{\text{cr}}^2(N_t)$. At $g_{\text{cr}}^2(N_t)$ the theory loses its confining property. If we introduce a lattice spacing a and let $\beta = N_t a$; then as $a \to 0$ g^2 is renormalized in the usual fashion[18] by letting $g^2 \equiv g^2(a)$. If $g^2(a)$ and $g_{\text{cr}}^2(N_t) = g_{\text{cr}}^2(\beta/a)$ have the same behavior as $a\to 0$ the continuum theory will have a phase transition at finite β. This procedure for taking the continuum limit at finite β and for studying the critical properties of the theory is discussed at length in Ref. 14.

In this paper the theory is analyzed for small g^2 and for all finite N_t. The functional integral is

$$Z = \int \mathfrak{D}U \exp\left[-\frac{1}{g^2}\sum_{\text{plaquettes}}\left(1-\tfrac{1}{2}\operatorname{Tr}UUUU\right)\right], \tag{25}$$

where the system has N_s space sites and N_t time sites and where the spatial variables satisfy $U(t=0)=U(t=N_t)$ (periodic boundary conditions). As in the continuum we study the order parameter

$$L(\vec{x}) = \tfrac{1}{2}\operatorname{Tr}\prod_{t=0}^{N_t-1} U_t(t,t+1) = \cos\alpha(\vec{x}), \tag{26}$$

where U_t is a link variable in the time direction. There is a $Z(2)$ symmetry which implies $\langle L\rangle = 0$ unless there is spontaneous symmetry breaking.

We calculate the effective potential by working in a gauge where U_t is constant in time and where U_t is diagonal (in analogy with the continuum conditions $\partial_0 A_0 = 0;\, A_0^a \alpha\, \delta_{a3}$). Let $U_t(\vec{x}) = e^{i\alpha(\vec{x})\sigma_3/N_t}$. Then

$$L(\vec{x}) = \cos[\alpha(\vec{x})]\,. \tag{27}$$

In our gauge the action is minimized when $\nabla\alpha = 0$ and when all spatial links are set equal to 1. The "free energy" $F=-(\ln Z)/N_t$ is evaluated for $\alpha(\vec{x}) = \alpha + \delta\alpha(\vec{x})$ in the one-loop approximation by diagonalizing the inverse propagator in momentum space. The details of the calculation will be presented in a later paper.[19] The result for the free energy per unit volume in the limit $N_s \to \infty$ is

$$V_{\text{eff}}(\alpha) = \frac{F}{N_s^3} = -\frac{\ln Z}{N_t} = \frac{1}{2}\frac{4}{N_t}\int_{-\pi}^{\pi}\frac{d^3K}{(2\pi)^3}\ln(1-2\cos 2\alpha\, e^{-2N_t h} + e^{-4N_t h}) + \text{terms independent of } \alpha\,, \tag{28}$$

where

$$h = \ln[\mathfrak{D} + (\mathfrak{D}^2+1)^{1/2}]$$

and

$$\mathfrak{D}^2 = \sin^2(K_x/2) + \sin^2(K_y/2) + \sin^2(K_z/2)\,. \tag{29}$$

This is the key result of the lattice calculation.

As $N_t\to\infty$, $h \to K/2$ and Eq. (27) becomes

$$V_{\text{eff}}^{\text{lattice}}(\alpha) \underset{N_t\to\infty}{\sim} \frac{1}{(2\pi)^2}\frac{1}{2}\frac{4}{N_t^4}\int_0^\infty x^2 dx\, \ln(1-2\cos 2\alpha\, e^{-x} + e^{-2x}) + \text{ terms independent of } \alpha\,. \tag{30}$$

This has the same shape as sketched in Fig. 1. Thus for finite N_t the symmetry is spontaneously broken at small g^2. This seems to be a firm result of this calculation. Any nonperturbative effects should be small as $g^2 \to 0$. At most they will cause a phase transition at some finite value $g_{cr}^2(N_t)$. Since we know that the theory confines at large g^2 it follows that lattice gauge theories at finite N_t have a deconfinement transition at some value of g^2.

It is suggestive that $V_{eff}(\alpha) \to \text{const}$ as $N_t \to \infty$. This may lead to symmetry restoration at $N_t = \infty$ in agreement with the expected result. It is essential to see whether a two-loop calculation also shows that $V_{eff} \to \text{const}$ as $N_t \to \infty$.

SUMMARY

The effective potential for the order parameter L (actually for $\alpha = \text{arc}\cos L$) has been evaluated for SU(2) gauge theories both in the continuum and on the lattice in the one-loop approximation. The effective potential has a minimum at $L=1$ and a symmetric minimum at $L=-1$. This implies spontaneous breaking of the $Z(2)$ symmetry which, in turn, implies that the quarks are not confined. This one-loop calculation should be reliable at high temperatures. We can thus conclude that quarks are not confined at high temperature. At lower temperature, higher-order corrections and topological effects become increasingly important. The distance between the minima of A_0 is proportional to T so that these effects could easily cause restoration of the $Z(2)$ symmetry and confinement at low temperature.

The lattice calculation is equally encouraging. For weak coupling and finite number of time sites the calculation establishes the spontaneous breakdown of the theory thus implying a phase transition at some coupling $g_{cr}^2(N_t)$. However, as $N_t \to \infty$ both the height of the barrier between the two vacuums and the "distance" between them ($U = e^{i\alpha\sigma_3/N_t}$ so $\Delta\alpha/N_t \to 0$) tends to zero and the symmetry may be restored. This would imply that the lattice theory ($N_t = \infty$) confines for all g^2 and thus that the continuum theory is confining.

The calculations are easily carried over to SU(n) gauge theories. The results will be presented in a later publication.

When fermions are included into the theory the $Z(2)$ symmetry is no longer present and $V_{eff}(C)$ no longer has a symmetry $C \to C + 2\pi/\beta$. Instead it only has the symmetry $C \to C + 4\pi/\beta$. The symmetric minimum at $C = 2\pi/\beta$ in Fig. 1 becomes only a local minimum. These calculations have been done and the results will also be presented in a later paper in which the effect of this minimum on quark confinement with fermions will be studied.

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[1]See, for example, C. Bernard, Phys. Rev. D 9, 3312 (1974).

[2]L. Susskind, Phys. Rev. D 20, 2610 (1979); A. Polyakov, Phys. Lett. 73B, 477 (1978).

[3]Actually the effective potential for the Casimir invariants of A_0 is computed. This will be discussed in detail in the text.

[4]For a review of lattice gauge theories see John B. Kogut, Rev. Mod. Phys. 51, 657 (1979).

[5]For a discussion of the lattice theory at finite temperature see L. McLerran and B. Svetitsky, Phys. Lett. 98B, 195 (1981); J. Kuti, J. Polonyi, and K. Szlachanyi, *ibid.* 98B, 199 (1981); Nathan Weiss, UBC report, 1980 (unpublished). Another comprehensive discussion of finite-temperature gauge theories is given by D. Gross, R. D. Pisarski, and L. G. Yaffe, Rev. Mod. Phys. 53, 43 (1981).

[6]This will discussed in more detail in a later publication.

[7]There are some "hedgehog" field configurations (such as $\phi^a = x^a$) for which such a transformation would be singular. We neglect the effect of such configurations, though they may be significant in studying θ vacuums.

[8]To arrive at this result, integrate the term $(\partial_0 A_i)\partial_i\phi$ by parts and use the fact that $\partial_0\phi = 0$. Note also that $\hat{3}\cdot(\hat{3}\times A_i) = 0$.

[9]See, for example, E. Wigner, *Group Theory* (Academic, New York, 1959), p. 152.

[10]R. Jackiw, Phys. Rev. D 9, 1686 (1973).

[11]The factor $\int d^3k/(2\pi)^3$ is the volume of phase space. If we introduce a lattice spacing a then $\sum_{\vec{x}} f(\vec{x}) = (1/a^3)a^3 \times \sum_{\vec{x}} f(\vec{x}) = (1/a^3)\int d^3x f(\vec{x})$ with $1/a^3 = \int d^3k/(2\pi)^3$.

[12]One paper on this subject is L. Dolan and R. Jackiw, Phys. Rev. D 9, 3320 (1974).

[13]It is in fact generally true in our gauge that there is a symmetry $\phi \to -\phi$. This is simply a gauge rotation by an angle π about the $\hat{1}$ axis. This keeps A_0 in the $\hat{3}$ direction but changes the sign of ϕ.

[14]Nathan Weiss, Ref. 5.

[15]D. Gross *et al.*, Ref. 5.

[16]N. Snyderman, Report No. SLAC-PUB-2636, 1980 (unpublished); B. Svetitsky, private communication.

[17]In fact in QED $V_{\rm eff}(C)=$constant and $\langle L\rangle=0$ due to ultraviolet infinities. When fermions are included as dynamical degrees of freedom such a problem is avoided by mass renormalization. However, when only external sources are present, care must be taken in interpreting the fact that $\langle L\rangle=0$.

[18]For a review see Ref. 4. Also see, for example, M. Creutz, Phys. Rev. D 21, 2308 (1980); L. McLarren and B. Svetitsky (Ref. 5), and J. Kuti *et al.* (Ref. 5).

[19]The reader who wishes to check the calculation may need the fact that

$$\sum_{n=0}^{N-1}\left\{\sin^2\pi\left[\frac{(n-\gamma)}{N}\right]+C^2\right\}^{-1}=\frac{N}{2C}\,\frac{1}{\cosh h}\,\frac{\sinh 2Nh}{\sin^2\pi\gamma+\sinh^2 Nh}\,,$$

where $h=\sinh^{-1}C$.