COSMOLOGICAL CONSTRAINTS ON MASSES AND COUPLINGS OF LEPTOQUARKS

V.A. KUZMIN and M.E. SHAPOSHNIKOV

Institute for Nuclear Research of the Academy of Sciences of the USSR, Moscow 117312, USSR

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The evolution of the baryon asymmetry of the universe is investigated in realistic GUT's based on the gauge group SU(5) and SO(10). The factors of the macroscopic suppression of the asymmetry are found. We obtain relations between the masses of scalar and gauge leptoquarks and also between their coupling constants at which the asymmetry is maximal.

Cosmological implications of elementary particle physics have become recently of great and special interest. The unification of interactions in the framework of grand unified theories made it possible to solve in a natural and elegant manner [1] one of the most intriguing puzzles of modern cosmology – the baryon asymmetry of the universe (BAU) [2]. Due to the extremely large mass scale of the unification, $M \sim 10^{15}$ GeV, the young universe of age $t \lesssim 10^{-35}$ s and of temperature $T \gtrsim 10^{15}$ GeV seems to serve as a unique proving ground where models of grand unification have been tested [‡1]. Theories contradictory to cosmological "experiment" have to be considered as nonviable. The synthesis of any GUT and big bang cosmology has to explain in any case the origin of the BAU.

In the present paper we investigate the question on what mass relation of the gauge and scalar leptoquarks the maximal BAU would be generated. Amusingly, it happens that the necessary mass relations take place at $h \sim g^2, f \sim g, g, h$, and f being gauge, Higgs and Yukawa couplings, respectively, which are just those realized in some asymptotically free [5] and supersymmetric [6] models. In a sense we may say that, possibly, it is the (first?) indication that nature prefers indeed models of those kinds.

To investigate the problem of the mass relationship between scalar and gauge bosons, we turn to the analysis of kinetic equations which describe the evolution of the baryon asymmetry. Let us consider the system of vector and scalar leptoquarks with a total number of degrees of freedom N_{X} and N_{H} and masses m_{X} and m_{H}, respectively. In SU(5) [7] and SO(10) [8] models the structure of the interaction of the gauge bosons X transforming as (3,2) ((3,1)) with respect to the group SU(3) × SU(2) is such that the following equalities hold $\Gamma_{\mathrm{qq}} = \Gamma_{\mathrm{q}\ell}$ ($\Gamma_{\mathrm{qq}} = 0$) where Γ_{qq} and $\Gamma_{\mathrm{q}\ell}$ are the partial widths in tree approximation for leptoquark decays into diquark and quark–lepton channels, respectively. This is true also for the scalar bosons H transforming as (3,1) ((3,2)).

This fact, which we shall use now extremely simplifies the investigation of kinetic equations. One can easily see, for example, that particles with $\Gamma^i_{\mathrm{qq}} = 0$ have no influence on the evolution of the baryon number in the expanding universe. Indeed one may ascribe consistently the baryon number $B = 1/3$ to those bosons, which should be conserved in all their interactions with fermions [‡2].

We would note that the baryon excess in decays of X-and H-bosons is not symmetric in quark flavours; moreover the BAU is mainly determined by the excess of heavy quarks from the third generation [‡3]

‡1 Low-energy testable consequences of GUT's could be proton decay [3] and neutron–antineutron oscillations [4].

‡2 Non-conservation of baryon number in this case could take place only due to boson self-interaction. This effect has a small influence on the kinetics of the BAU generation.

‡3 *CP*-violation effects in X- and H-boson decays on particles from any generation are proportional to corresponding Yukawa constants, i.e. to quark masses.

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Defining the microscopic asymmetry as follows:

$$\Delta_{\mathrm{ms}}^{\mathrm{X,H}} = \frac{1}{\Gamma_{\mathrm{tot}}} \sum_{\mathrm{a}} [\Gamma(\mathrm{X,H} \to \mathrm{a}) B_{\mathrm{a}} + \Gamma(\bar{\mathrm{X}}, \bar{\mathrm{H}} \to \bar{\mathrm{a}}) B_{\bar{\mathrm{a}}}] \quad (1)$$

[$\Gamma_{\mathrm{tot}}^{\mathrm{X,H}}$ is the total width of the leptoquark, B_{a} is the baryon number of the a-th decay channel, $\Gamma(\mathrm{X,H} \to \mathrm{a})$ is the partial width for X (H) decays] and assuming charge neutrality of the universe at initial time we obtain the linearized kinetic equations [‡4]:

$$\mathrm{d}\Delta/\mathrm{d}t = \sum_i \Delta_{\mathrm{ms}}^i N_i \, \mathrm{d}Y^i/\mathrm{d}t - \Delta v_{\mathrm{B}}, \quad (2)$$

$$\mathrm{d}Y^i/\mathrm{d}t = -\Gamma_{\mathrm{tot}}^i [K_1(m_i/T)/K_2(m_i/T)] \, (Y^i - Y_{\mathrm{eq}}^i), \quad (3)$$

$$12 v_{\mathrm{B}} = \sum_i N_i \Gamma_i (m_i/T)^2 K_1(m_i/T) + \frac{1}{8\pi^2 T^2} \int_0^\infty S^{3/2} \mathrm{d}s \, \sigma'(s) \, K_1(\sqrt{s}/T), \quad (4)$$

where $K_1(x)$ and $K_2(x)$ are the modified Bessel functions of the first and second kind, respectively, N_i is the number of degrees of freedom of the X- and H-bosons including color ones, Δ is the baryon asymmetry in the unit of comoving volume, Y^i is the relative concentration of X_i, $Y_{\mathrm{eq}}^i = \frac{1}{2}(m_i/T)^2 K_2(m_i/T)$, $\sigma'(s)$ is the total (except for the resonant contribution in the s-channel) cross section of the process $\mathrm{qq} \to \bar{\mathrm{q}}\bar{\ell}$ summed over the quark and lepton flavours, $v_{\mathrm{B}}(t)$ is the rate of processes with baryon number con-conservation, Γ_i is the effective fermionic width,

$$\Gamma_{\mathrm{X}} = \tfrac{4}{3} \alpha_{\mathrm{GUT}} m_{\mathrm{X}}, \quad \Gamma_{\mathrm{H}} = 2\alpha_{\mathrm{H}} m_{\mathrm{H}}, \quad \sigma' = \sigma_{\mathrm{X}} + \sigma_{\mathrm{H}}$$
$$\sigma_{\mathrm{X}} = \tfrac{200}{3} [\pi \alpha_{\mathrm{GUT}}^2 \, s/(m_{\mathrm{X}}^2 + s)^2] \, N_{\mathrm{X}},$$
$$\sigma_{\mathrm{H}} = 88 \, [\pi \alpha_{\mathrm{H}}^2 \, s/(m_{\mathrm{H}}^2 + s)^2] \, N_{\mathrm{H}}. \quad (5)$$

Taking into account the additional heating of the relic black-body radiation due to annihilation of heavy particles (see also refs. [2,10]) we obtain for the BAU at $t \to \infty$:

$$\Delta = \frac{n_{\mathrm{B}}}{n_\gamma} = \frac{45\zeta(3)}{4\pi^4 N} \sum_i \Delta_{\mathrm{ms}}^i N_i S_i, \quad (6)$$

where N is the total number of degrees of freedom of the particles which are effectively massless at temperature T, $\zeta(x)$ is the Riemann function, S_i is the macroscopic suppression factor due to dissipation of baryon number in boson decay processes, inverse decays and scatterings of quarks and leptons,

$$S_i = \int_0^\infty \mathrm{d}t \, \frac{\mathrm{d}Y^i(t)}{\mathrm{d}t} \exp\left(-\int_t^\infty v_{\mathrm{B}}(t') \, \mathrm{d}t'\right). \quad (7)$$

The dependence of the macroscopic suppression factors of the vector (S_{X}) and scalar (S_{H}) asymmetry on m_{H} in the SO(10) model with two **10** of Higgses is shown in fig. 1. In the range of interest ($S \gtrsim 10^{-6}$) the ratio of the SU(5) to SO(10) factors is close to 1.

It is easy to understand qualitatively the meaning of the curves in fig. 1. At $m_{\mathrm{H}} \ll m_{\mathrm{X}}$ vector (scalar) particles affect in no way the baryon excess arising in decays of scalar (vector) particles. Therefore BAU is destroyed only in those various reactions with asymmetry parents taking part in it and the faster it goes the less their mass. If $m_{\mathrm{H}} \gg m_{\mathrm{X}}$ then the asymmetry arising due to scalars (vectors) is destroyed in vector (scalar) interactions.

Let us discuss the BAU generation due to gauge-boson decays. The microscopic asymmetry in decays of gauge bosons with maximal *CP*-violation, and the most favourable relation between masses (say, $m_{\mathrm{H}_1} > m_{\mathrm{X}}$, $m_{\mathrm{H}_2} < m_{\mathrm{X}}$) in the SU(5) model with two $\bar{\mathbf{5}}$ of Higgses [or in the SO(10) model with two **10** is given by (see also ref. [11]):

$$\Delta_{\mathrm{ms}}^{\mathrm{X}} = \tfrac{1}{24} \alpha_{\mathrm{H}} \, [1 - (m_{\mathrm{H}_2}/m_{\mathrm{X}})^2]^2 \, \theta(1 - m_{\mathrm{H}_2}/m_{\mathrm{X}}). \quad (8)$$

The results of the calculation of the upper bound for the overall BAU arising in decays of gauge bosons are shown in table 1. It can be seen from table 1 that the BAU generation through decays of X-bosons is possible only if $m_{\mathrm{X}} \gtrsim 8 \times 10^{14}$ GeV. In other words, in the usual SU(5) model the gauge bosons do not seem to contribute significantly to the BAU. (By the way, we would notice, that in this case the experimental observation of proton decay becomes extremely difficult, if possible. And on the other hand the observation of proton decay will presumably mean that gauge bosons do not contribute to BAU.) We find

‡4 The evolution of the baryon asymmetry in the simple model has been considered in ref. [9].

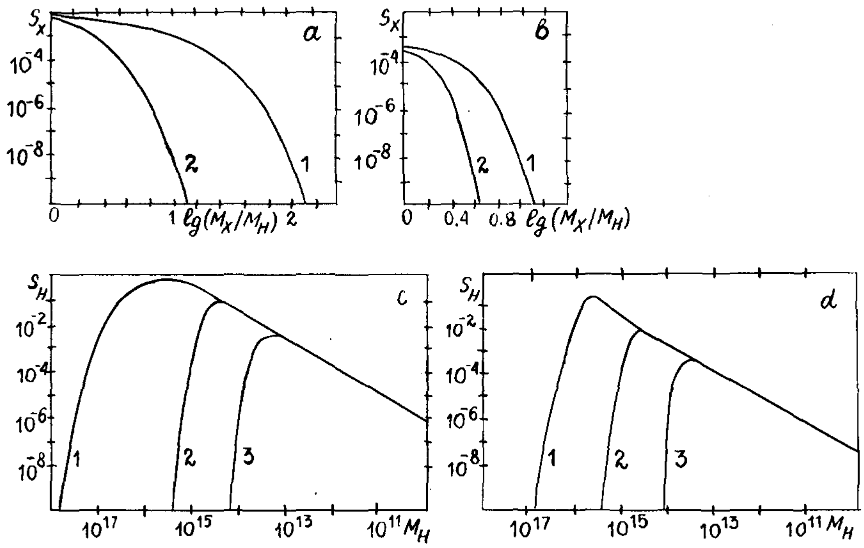

Fig. 1. The macroscopic suppression factors S_X and S_H of the BAU generation in the decays of vector (a), (b) and scalar (c), (d) bosons, respectively. The curves in (a) correspond to $m_X = 10^{16}$ GeV and in (b) to $m_X = 10^{15}$ GeV. The curves 1 in (a), (b) are calculated for $\alpha_H = 10^{-4}$, the curves 2 for $\alpha_H = 10^{-3}$. Curve 1 in (c), (d) refers to $m_X = 10^{16}$ GeV, curve 2 to $m_X = 10^{15}$ GeV, curve 3 to 10^{14} GeV. The curves in (c) are calculated at $\alpha_H = 10^{-4}$ and in (d) at $\alpha_H = 10^{-3}$.

that the overall BAU is maximal at

$$m_H/m_X = 0.7\text{–}0.9, \quad N_X\alpha_X \approx 33 N_H\alpha_H, \tag{9}$$

for $m_X = 8 \times 10^{14}\text{–}10^{16}$ GeV. Eq. (9) gives $\alpha_H = 0.2\,\alpha_X$ and $\alpha_H = 0.4\,\alpha_X$ for the SU(5) and SO(10) models, respectively.

One can see from table 1 that the maximal value of the BAU at $m_X = 8 \times 10^{14}\text{–}10^{16}$ GeV is just rather close to the observational one, hence the relations of eq. (9) should be fulfilled with good enough accuracy. Therefore one may conclude that for BAU generation through decays of gauge bosons to be possible, there

Table 1
The maximal values of the BAU generated in decays of vector leptoquarks in SU(5) and SO(10) models.

m_X (GeV)	Δ_{max} [SU(5) and SO(10)]
8×10^{14}	1×10^{-10}
1×10^{15}	2.2×10^{-10}
3×10^{15}	1.2×10^{-9}
1×10^{16}	1.2×10^{-8}

should be fixed a rather strict relation between the Yukawa and gauge coupling constants (in fact it should be $f \sim g, f^2/4\pi = \alpha_H, g^2/4\pi = \alpha_{GUT}$) and between the masses of scalar and gauge bosons or, in other words, between the Higgs and gauge coupling constants as far as $m_H = h^{1/2}V$, $m \sim gV$ (V being the vacuum expectation value), i.e. it should be $h \sim g^2$. It is just this relation $f \sim g \sim h^{1/2}$ which is realized in asymptotically free [5] and in some supersymmetric models [6]. That is why the very existence of BAU may be treated as the phenomenological argument in favour of asymptotically free and super symmetric models.

Consider now BAU generation through scalar particle decays. It is seen from fig. 1 that optimal conditions for BAU generation are realised at $m_X/m_H = 2\text{–}3$, i.e. just as in the case of gauge-boson decays at $h \sim g^2$. Now at $m_X = (5\text{–}6) \times 10^{14}$ GeV (which is one of the recent estimates of m_X from low-energy phenomenology, see, e.g. ref. [12]) we have at the extremum of S_H: $S_H = 2 \times 10^{-2}$ if $\alpha_H = 10^{-4}$ and $S_H = 2 \times 10^{-3}$ if $\alpha_H = 10^{-3}$.

It may be shown that the magnitude of the microscopic asymmetry in scalar-boson decays in the SO(10) model with two 10-plets of Higgses ϕ_1 and ϕ_2, coupled directly to fermions, is given by (for simplicity we have chosen the scalar potential to be symmetric under the reflections $\phi \leftrightarrow -\phi$):

$$\Delta^1_{\mathrm{ms}} = \tfrac{32}{7}\,\alpha_{\mathrm{GUT}}\,(m_t/m_W)(m_b/m_W)$$
$$\times \sin\beta[f(m_1, m_2, 0) - f(m_1, m_2, m_{\nu R})],$$
$$f(m_1, m_2, m_{\nu R}) = [1 + \tfrac{1}{7}(1 - m^2_{\nu R}/m_1^2)^2]^{-1}$$
$$\times (1 - m^2_{\nu R}/m_1^2)^2\,\theta(1 - m_{\nu R}/m_1)$$
$$\times \left[1 - \frac{m_2^2}{m_1^2 - m^2_{\nu R}}\ln\left(1 + \frac{m_1^2 - m^2_{\nu R}}{m_2^2}\right) + \frac{m_1^2(m_1^2 - m_2^2)}{(m_1^2 - m_2^2)^2 + \alpha_H^2 m_2^4}\right], \tag{10}$$
$$\Delta^2_{\mathrm{ms}} = -\Delta^1_{\mathrm{ms}}(m_2, m_1, m_{\nu R}),$$

where β is the *CP*-violating phase, $m_t, m_b, m_W, m_{\nu R}$ are the masses of the t-quark, the b-quark, the W-boson and the right-handed neutrino, respectively [at $m_{\nu R} \geqslant m_1, m_2$ eq. (10) is valid for the SU(5) model with two 5 also]. In the derivation of formulas (10) we have neglected Cabbibbo angles and have supposed that the vev's of the neutral components of ϕ_1 and ϕ_2 are equal. The factors $[f(m_1, m_2, 0) - f(m_1, m_2, m_{\nu R})]$ in eq. (10) correspond to the $\mathrm{CPP_{LR}}$-symmetry [13] of the SO(10) model: at $m_{W_R} \to 0$ one gets $\Delta^1_{\mathrm{ms}} \to 0$, $\Delta^2_{\mathrm{ms}} \to 0$.

From the requirement $\Delta \geqslant 10^{-10}$ follows now immediately that the masses of the scalar particle should not differ considerably: $0.5\text{–}0.3 \leqslant m_1/m_2 \leqslant 2\text{–}3$. If there is no mixing of scalars ϕ_1 and ϕ_2 near-shell then in any case the inequality $(1\text{–}3) \times 10^{13}\ \mathrm{GeV} \leqslant \min\{m_1, m_2\} < m_X$ should hold. The microscopic asymmetry reaches its maximum at $m_1^2 - m_2^2 = \alpha_H m_1^2$ which corresponds to the mixing of ϕ_1 and ϕ_2 near the mass shell. In this case with the maximal *CP*-violation $\Delta_{\max} = (80\text{–}2) \times 10^{-6}$, $m_H > (0.3\text{–}10) \times 10^9$ GeV. (The values quoted correspond to $m_t = 15\text{–}50$ GeV.)

Thus we obtained that the requirement of the quantitative explanation of the baryon asymmetry of the universe results in rather strict relations between the masses and coupling constants of scalar and vector bosons. The GUT's in which these relations are not fulfilled seem therefore to be excluded and those theories containing them in a natural way are preferable from the cosmological point of view.

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