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BOUND-STATE APPROACH TO STRANGENESS IN THE SKYRME MODEL*

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We show that baryons carrying heavy flavors, such as strangeness and charm, can be described by bound states of the corresponding heavy mesons in the background field of the basic SU(2) skyrmion. This method is quantitatively successful to $O(N_c^0)$, in the sense of the large-N_c expansion, but at $O(1/N_c)$ it experiences problems associated with our lack of knowledge of higher-derivative terms in the Skyrme action. We derive a model-independent mass relation for strange baryons which is in excellent agreement with experiment.

1. Introduction

Although the Skyrme soliton approach to low-energy hadron physics works very well in describing non-strange baryons [1], attempts to apply it to strange, or heavier, baryons run into some difficulties [2–4]. The problem is that flavor quantum numbers are usually associated with collective coordinate rotations of the soliton, while the collective coordinates themselves arise from unbroken flavor symmetries. When the relevant flavor symmetries are broken badly enough, the rotator treatment fails and some other dynamics must be found to describe the heavy flavors. Since flavor SU(3) is not too badly broken, this problem may perhaps be circumvented for strangeness, but it is unavoidable for heavy flavors, like charm, for which there is not even an approximate associated rotator coordinate.

The large-N_c limit of QCD, to which the Skyrme model is just an approximation, gives a clear suggestion on how to incorporate heavy flavors. The large-N_c lagrangian actually contains only meson fields, with baryon number arising from topological properties of those meson fields which live on a compact manifold [5] (typically the Goldstone pions of unbroken $SU(2)\times SU(2)$). Heavy-flavor quantum numbers are simply carried directly by the appropriate non-topological heavy mesons such as K's, D's and F's. The basic baryon is a topological soliton mainly built out of the Goldstone pions. The simplest way for it to carry strangeness, etc., is for the baryon soliton to bind a meson carrying the appropriate quantum number. Whether or not,

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and for what quantum number channels, this happens depends on the pion-heavy meson interactions implied by the details of the large-N_c lagrangian.

In this paper we will show that the above "bound-state" picture of flavor works quite well in describing strange baryons. The crucial point is that since strangeness is not too badly broken, we can plausibly use SU(3) symmetry to determine the kaon-pion couplings from the known pion-pion couplings. The question of the possible existence of kaon bound states in the background field of a skyrmion can then be studied in detail. We find that the requisite bound states exist in precisely those channels needed to reproduce the quark model quantum numbers of strange baryons. As a consistency check, we show that as the kaon mass (the SU(3) symmetry-breaking parameter) goes to zero, the bound state goes smoothly into a zero-energy state which can be identified with the SU(3) collective coordinate zero mode.

Upon collective coordinate quantization, the kaon bound state carries spin $\frac{1}{2}$ and no isospin. These are precisely the quantum numbers of the strange quark. Thus, in the quark model language, we have added a heavy quark to $(N_c - 1)$ light quarks described by the soliton. The bound-state mode tells us the distribution of strangeness. It is the analog of the heavy-quark wave function in the non-relativistic quark model.

In calculating baryon masses to $O(N_c^0)$ we ignore the rotation of the skyrmion, whose angular velocity is $O(1/N_c)$. To this order we compute the binding energy of the meson-skyrmion system, as well as the splitting, arising from the Wess-Zumino term, between normal (quark model) and exotic baryons. Both of these quantities come out in good agreement with experiment. As we turn to $O(1/N_c)$ corrections, which are crucial for the baryon spectroscopy, we find that they are also easy to calculate in our treatment. In the quark model, the $O(1/N_c)$ contributions to baryon masses are produced by spin-spin interactions. The mass formulae we derive are in qualitative agreement with quark model expectations. We also derive one "model-independent" relation between baryon masses which depends solely on our collective coordinate quantization scheme, but is independent of the detailed form of the action. This relation between strange baryon masses holds to 3% – an accuracy comparable to that of the model-independent relations for non-strange baryons found by other authors [1].

The coefficients in our mass formulae depend on what terms are used to stabilize the skyrmion. We find that, with the Skyrme term alone, the sign of the spin-spin interaction between a heavy and a light quark comes out wrong. We argue that, with the addition of a suitable set of higher-derivative terms in the Skyrme lagrangian, this quantity can easily be arranged to have the right sign. The point is that $O(1/N_c)$ corrections to strange baryon masses appear to be very sensitive to the detailed higher-derivative structure of the action, about which, unfortunately, very little is now known. This problem, which is quite apparent in our calculation, was not as obvious in the rotator treatment of SU(3) skyrmions, where the quark masses could be included only perturbatively. In fact, the rotator treatment has its own difficulties in achieving a satisfactory fit to strange and non-strange masses simultaneously [3], and this may be due to similar causes.

These difficulties do not appear to affect the accuracy of the $O(N_c^0)$ predictions of the model. Our $O(N_c^0)$ treatment seems to give better results than the rotator treatment in that the mass of the Λ comes out well when the parameters of the Skyrme lagrangian, F_π and e, are chosen to fit the N and Δ masses exactly.

To summarize, our results strongly suggest that the strange quark mass is large enough that the bound-state picture of strangeness is physically more correct than the collective coordinate one. This picture also makes it possible to examine the fascinating questions of the existence and stability of strange nuclear matter [6] and the hypothetical H-dibaryon [7] from a completely new perspective. Although these matters will be the subject of further publications, we make some preliminary observations at the end of this paper.

2. A model lagrangian

To test the idea that strange baryons can be described as bound states of kaons in the field of a Skyrme soliton, we need an action to describe the interaction of kaons with nonlinear pion fields. We assume that in the limit of unbroken SU(3), the meson lagrangian is the obvious extension of the usual Skyrme model [8], i.e.,

$$L_{\text{Skyrme}} = \tfrac{1}{16}F_\pi^2 \operatorname{tr}\left(\partial_\mu U^+ \partial^\mu U\right) + \frac{1}{32e^2}\operatorname{tr}\left[\partial_\mu U U^+, \partial_\nu U U^+\right]^2, \tag{2.1}$$

where $U(\boldsymbol{x}, t)$ takes values in SU(3). When SU(3) is broken by a large kaon mass term, the physically relevant field configurations will be small fluctuations into kaonic directions about the purely pionic soliton configurations needed to describe the baryons. By carrying out this expansion we can identify the kaon-pion interactions needed to discuss the bound-state question. In first approximation we need only concern ourselves with terms up to second order in kaon fields. Higher-order terms describe kaon self-interactions and are irrelevant to the question of the existence of bound states. The basic notion here is that the kaon interactions are given accurately enough by the unbroken-symmetry lagrangian and that symmetry breaking can be included accurately enough by adding a simple kaon mass term.

To proceed, we choose a standard form for a configuration which has kaon fields excited and is close to a given purely pionic configuration. The parametrization suggested by studies of non-linear realizations of chiral symmetry [9] is

$$U = \sqrt{U_\pi}\, U_{\mathrm{K}} \sqrt{U_\pi}\,, \tag{2.2}$$

where

$$U_\pi = \exp\left(i \frac{2}{F_\pi} \boldsymbol{\tau} \cdot \boldsymbol{\pi} \right), \tag{2.3}$$

$$U_K = \exp\left(i \frac{2}{F_\pi} \lambda_a K_a \right) \tag{2.4}$$

are built in the obvious way out of the kaonic and pionic generators of SU(3), respectively. To get our interaction lagrangian, we substitute this form for U in the Skyrme lagrangian and expand in powers of K, keeping only terms up to second order (actually, when we expand about the soliton imbedded in SU(2), terms first order in K will vanish as well). The reasonably simple end result of this rather painful exercise is

$$\begin{aligned}
&L_{\mathrm{Skyrme}}(U_\pi) + (D_\mu K)^+ D_\mu K - m_K^2 K^+ K \\
&\quad - \tfrac{1}{8} K^+ K \left\{ \mathrm{tr}\left(\partial_\mu U_\pi^+ \partial^\mu U_\pi \right) + \frac{1}{e^2 F_\pi^2} \mathrm{tr}\left[\partial_\mu U_\pi U_\pi^+, \partial_\nu U_\pi U_\pi^+ \right]^2 \right\} \\
&\quad - \frac{1}{e^2 F_\pi^2} \Big\{ 2 (D_\mu K)^+ D_\nu K \,\mathrm{tr}(A^\mu A^\nu) + \tfrac{1}{2} (D_\mu K)^+ D^\mu K \,\mathrm{tr}\left(\partial_\nu U_\pi^+ \partial^\nu U_\pi \right) \\
&\qquad\qquad - 6 (D_\nu K)^+ [A^\nu, A^\mu] D_\mu K \Big\},
\end{aligned} \tag{2.5}$$

where

$$A_\mu = \tfrac{1}{2}\left(\sqrt{U_\pi^+}\, \partial_\mu \sqrt{U_\pi} - (U_\pi \to U_\pi^+) \right),$$

$$V_\mu = \tfrac{1}{2}\left(\sqrt{U_\pi^+}\, \partial_\mu \sqrt{U_\pi} + (U_\pi \to U_\pi^+) \right)$$

and the kaon covariant derivative is defined by

$$D_\mu K = \partial_\mu K + V_\mu K. \tag{2.6}$$

This lagrangian has no unknown parameters if we think of F_π and e as having been determined by the fit of the SU(2) Skyrme soliton to the masses of the nucleon and delta. A more general approach is to think of it as a particular case of the nonlinear realization of SU(2) × SU(2) chiral symmetry on a linear kaon field. All of the terms in eq. (2.5) are SU(2) × SU(2) invariants of the kind described in ref. [9] and in the general approach would have arbitrary coupling constants which would have to be extracted from experiment. In the case at hand, all arbitrary parameters have been eliminated by invoking SU(3) symmetry. This is possible and reasonable for a discussion of strange mesons and baryons, but would not be if we attempted to study charmed baryons.

3. Bound-state mechanics

The bound-state problem of interest to us arises when we set the pion field in eq. (2.5) equal to the field of a static skyrmion,

$$U_\pi = \exp(iF(r)\boldsymbol{\tau}\cdot\hat{r}),$$

with $F(r)$ equal to the profile which minimizes the energy in the baryon number equal to one sector. The resulting lagrangian for the kaon field is time translation-invariant and quadratic in the kaon fields. It will have energy eigensolutions

$$K = K(\boldsymbol{r})\exp(-iEt),$$

in terms of which we will expand the kaon field operator. Bound states will correspond to those solutions for which $E < m_{\mathrm{K}}$.

Since the background field $U(\boldsymbol{r})$ is invariant under combined spatial and isospin rotations, $\boldsymbol{T} = \boldsymbol{I} + \boldsymbol{L}$, one can perform a partial wave analysis of the kaon modes. The usual arguments show that a complete commuting set of observables is T, L and T_z and one can write the general kaon mode as the product of a radial function $k(r)$ and a generalized spherical harmonic, Y_{TLT_z}. Substituting this into the lagrangian, we obtain, after some calculation, the following action for the radial function (from here on it is convenient to measure times and distances in units of $1/eF_\pi$; energy naturally comes out in units of eF_π):

$$-L = h(r)k^{*\prime}k' + k^*k\left(\mu_{\mathrm{K}}^2 - E^2 f(r) + V_{\mathrm{eff}}(r)\right), \tag{3.1}$$

where $f(r) = 1 + 2s(r) + d(r)$, $h(r) = 1 + 2s(r)$, $d(r) = (F')^2$, $s(r) = (\sin F/r)^2$, $c(r) = (\sin\frac{1}{2}F)^2$,

$$V_{\mathrm{eff}}(r) = -\tfrac{1}{4}(d+2s) - 2s(s+2d) + \frac{1+d+s}{r^2}\left(l(l+1) + 2c^2 + 4c\boldsymbol{I}\cdot\boldsymbol{L}\right)$$
$$+\frac{6}{r^2}\left\{s(c^2 + 2\boldsymbol{I}\cdot\boldsymbol{L}c - \boldsymbol{I}\cdot\boldsymbol{L}) + \frac{\mathrm{d}}{\mathrm{d}r}\left((c+\boldsymbol{I}\cdot\boldsymbol{L})F'\sin F\right)\right\}$$

and $\boldsymbol{I} = \frac{1}{2}\boldsymbol{\tau}$ is the kaon isospin operator. This action leads via the usual variational principle, to an ordinary differential equation eigenvalue problem for the energy E:

$$-\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(h(r)r^2\frac{\mathrm{d}}{\mathrm{d}r}k\right) - E^2 f(r)k + \left(m_{\mathrm{K}}^2 + V_{\mathrm{eff}}(r)\right)k = 0. \tag{3.2}$$

It differs in an essential way from the usual Schrödinger eigenvalue problem through the appearance of the function $f(r)$, which is strictly greater than 1, multiplying the eigenvalue E^2. It is this feature, rather than attractive potential energy terms, which is most directly responsible for the appearance of bound-state eigenvalues.

As is usual in a problem with spherical symmetry, there is a repulsive centrifugal barrier term in the potential which determines which partial wave lies lowest. Expanding the equation satisfied by $F(r)$ [1] near the origin, it is easy to show that

$$F(r) \to \pi - ar + br^3 .$$

It follows, after some manipulation, that in the neighborhood of $r = 0$, eq. (3.2) reduces to

$$-\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}}{\mathrm{d}r}k\right) + \frac{l_{\mathrm{eff}}(l_{\mathrm{eff}}+1)}{r^2}k = 0 ,$$

where

$$l_{\mathrm{eff}}(l_{\mathrm{eff}}+1) = l(l+1) + 4\boldsymbol{I}\cdot\boldsymbol{L} + 2$$

and l_{eff} takes on the values

$$l_{\mathrm{eff}} = \begin{cases} l+1, & \text{if } t = l + \frac{1}{2} \\ l-1, & \text{if } t = l - \frac{1}{2} . \end{cases} \tag{3.3}$$

Roughly speaking, we expect the lowest-lying modes to correspond to $l_{\mathrm{eff}} = 0$, and this expectation is borne out by direct computation. An absolutely crucial point is that $l_{\mathrm{eff}} = 0$ corresponds to $l = 1$. This means that kaons are most strongly bound to the skyrmion in positive parity p-wave states. This fact is crucial in obtaining the correct quark model spin-parity systematics of strange-baryon spectroscopy.

We have studied this eigenvalue problem numerically for the lowest few partial waves and find that for the skyrmion background there is only one bound state, lying in the $l_{\mathrm{eff}} = 0$ partial wave. We will shortly use its detailed properties to make predictions of the masses of the strange baryons but want to immediately point out that it satisfies an important consistency check on our model: in the limit of unbroken SU(3) symmetry, we expect the skyrmion to possess a normalizable zero mode corresponding to the existence of a collective coordinate for strangeness. It is easy to show that this zero mode should appear in the $l_{\mathrm{eff}} = 0$ partial wave. In other words, our bound state should go over smoothly into a zero-energy bound state as the kaon mass goes to zero. This is precisely what happens (see fig. 1).

At this point it should be emphasized that we are solving an eigenvalue problem for E^2 and that our eigensolutions are effectively doubled. The significance of this will be discussed shortly. For the moment, let us observe that in a general background there is nothing which prevents the E^2 eigenvalue from becoming negative,

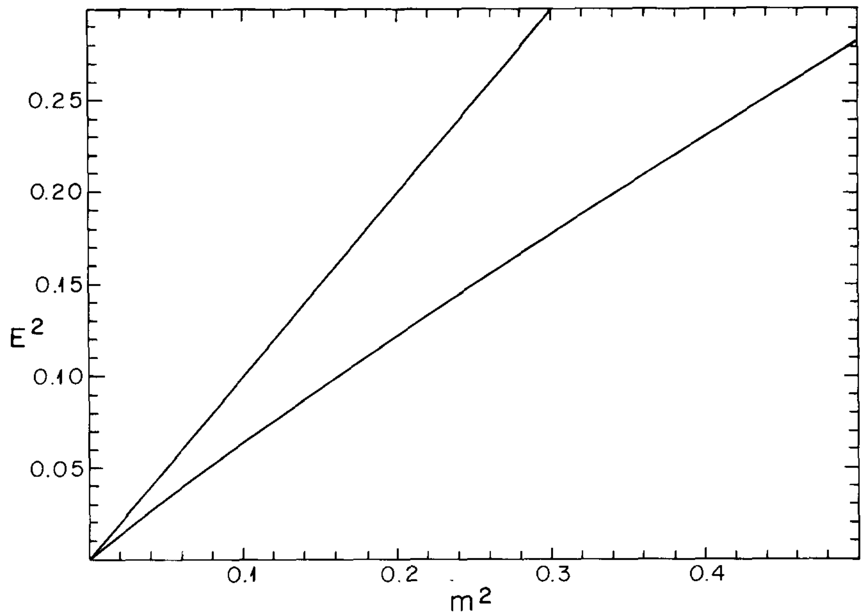

Fig. 1. Graph of bound-state energy-squared versus massive meson mass-squared. Energies are expressed in units of eF_π. The straight line shows the threshold $E^2 = m^2$.

so that the above test is quite non-trivial. We will eventually find cases in which the eigenvalue does become negative, and indicate a very interesting interpretation of that phenomenon.

4. The Wess-Zumino term

As things now stand, the kaon energy eigenvalues come in pairs of opposite sign, which means that kaon and anti-kaons have the same spectrum. This is not desirable as it would mean that for every $S = -1$ bound state there is an $S = +1$ bound state, contrary to the quark model and experience! The problem is that we have yet to include the effect of the Wess-Zumino term, a defect we now remedy. We will find, as usual, that this term plays a crucial role in getting the quantum numbers of the system right, although the mechanics are rather different than in the rotator approach to strangeness.

We will treat the Wess-Zumino term exactly as we have treated the other parts of the action: we expand the general SU(3) expression [5]

$$-\frac{i}{240\pi^2}\int \mathrm{d}^5x\, \varepsilon^{ijklm}\mathrm{tr}\left(U^+\partial_i UU^+\partial_j UU^+\partial_k UU^+\partial_l UU^+\partial_m U\right)$$

to second order in kaon fields about the SU(2) skyrmion, obtaining the simple expression

$$L_{\mathrm{WZ}} = i\frac{2N_c}{5F_\pi^2} B^\mu\left((D_\mu K)^+ K - K^+ D_\mu K\right), \tag{4.1}$$

where B_μ is the SU(2) baryon number current. We first concern ourselves with the leading, $O(N_c^0)$, contribution of this term. To that order we may neglect time derivatives of the background skyrmion field and drop terms involving space components of the baryon current. The kaon lagrangian then becomes

$$L = f(r)\dot{K}^+\dot{K} + i\lambda(r)(\dot{K}^+K - K^+\dot{K}) + L'(K), \tag{4.2}$$

where L' is the piece involving no time derivatives, $f(r)$ is defined by eq. (3.1), and

$$\lambda(r) = \frac{2N_c}{5F_\pi^2} B^0. \tag{4.3}$$

The crucial point is that the new terms are of odd order in kaon time derivatives and break the symmetry $E \to -E$. Unfortunately, the hamiltonian we will derive from this action, while still only quadratic in kaon fields, will not be exactly soluble and we will have to resort to approximate methods.

In terms of rescaled fields $G = \sqrt{f}K$ the lagrangian is

$$L = \dot{G}^+\dot{G} + i\frac{\lambda}{f}(\dot{G}^+G - G^+\dot{G}) + L'\left(\frac{G}{\sqrt{f}}\right).$$

The resulting hamiltonian is conveniently broken into two pieces,

$$H = H_0 + H_1,$$

where

$$H_0 = \int d^3x\left\{\Pi^+\Pi + \left(\frac{\lambda}{f}\right)^2 G^+G - L'\right\},$$

$$H_1 = i\int d^3x\,\frac{\lambda}{f}(G^+\Pi - \Pi^+G), \tag{4.4}$$

and Π is the momentum conjugate to G^+. H_0 is the hamiltonian we would get in the absence of the Wess-Zumino interaction, with a small change – the addition of a term proportional to λ^2 to the effective potential. H_0 defines the same kind of eigenvalue problem as discussed in the previous section. The eigenvalue ω and eigenfunction $g(r)$ are found by solving the following equation for a rescaled function $k(r)=g(r)/\sqrt{f}(r)$:

$$-\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(h(r)r^2\frac{\mathrm{d}}{\mathrm{d}r}k\right)-\omega^2 f(r)k+\left(m_{\mathrm{K}}^2+V_{\mathrm{eff}}+\frac{\lambda^2}{f}\right)k=0. \tag{4.5}$$

This differs from the eigenvalue problem of the previous section by the addition of the λ^2/f term to the effective potential. In practice, this has a small, but significant, effect on the numerical results. We use these eigenfunctions to expand the canonical fields in a conventional way,

$$G=\sum_n \frac{1}{\sqrt{2\omega_n}}\left(a_n g_n+b_n^+ g_n^{\mathrm{c}}\right),$$

$$\Pi=-i\sum_n\sqrt{\tfrac{1}{2}\omega_n}\left(a_n g_n-b_n^+ g_n^{\mathrm{c}}\right),$$

where the charge conjugation is defined by

$$g_n^{\mathrm{c}}=\tau_2 g_n^*,$$

and then express the pieces of the hamiltonian in terms of the creation and annihilation operators so defined:

$$:H_0:=\sum_n\omega_n\left(a_n^+a_n+b_n^+b_n\right),$$

$$:H_1:=\sum_{nm}C_{nm}\left(a_n^+a_m-b_n^+b_m\right), \tag{4.6}$$

where

$$C_{nm}=\int\mathrm{d}^3x\, g_n^+ g_m\frac{\lambda}{f}.$$

By construction, of course, H_0 is diagonal, but H_1 is not. It is easy to construct the strangeness operator for this theory and show that the a_n are annihilation operators for particles of $S=+1$, while the b_n are annihilation operators for particles of $S=-1$.

The net result of all this is that the energies of $S=1$ states, which are annihilated by a's, are given by the eigenvalues of the matrix

$$\omega_n\delta_{nm}+C_{nm}, \tag{4.7}$$

while the energies of $S=-1$ states, which are annihilated by b's, are given by the eigenvalues of

$$\omega_n \delta_{nm} - C_{nm} . \tag{4.8}$$

When λ/f is independent of $\boldsymbol{r}$, C_{nm} is diagonal, and the net effect is that all $S=+1$ ($S=-1$) states are pushed up (down) by a constant amount. This would be the situation in a background of dense nuclear matter. Unfortunately, in the case of interest, λ/f does depend on the radial variable r, and there is, as far as we know, no simple method for determining the eigenvalues of the above matrices. The best we know how to do is to use first-order perturbation theory, which appears to be reasonably accurate in the cases of interest to us. In that approximation, the energies of the $S=-1$ and $S=+1$ bound states are

$$\omega_0 - C_{00} , \tag{4.9}$$

$$\omega_0 + C_{00} , \tag{4.10}$$

respectively. The normal state becomes more bound, and the exotic, non-quark model state gets pushed up (into the continuum, in fact).

It is well known that in the SU(3) rotator treatment of skyrmions the Wess-Zumino term imposes a constraint on quantization [10], which is crucial to the appearance of the observed particle multiplets. In our "bound-state" picture of flavor the Wess-Zumino term, once again, plays a crucial role: this time, the breaking of the degeneracy between normal and exotic baryons.

5. Collective coordinate quantization

We have now determined the hamiltonian to $O(N_c^0)$. This hamiltonian approximately gives the energy of the lowest-lying member of a tower of states of increasing spin and isospin. To get the quantum numbers and energy splittings of these states, we must examine the $O(1/N_c)$ hamiltonian describing the effect of rotations of the background skyrmion. To do this, we define the SU(2) rotator coordinate, $A(t)$, corresponding to a general time-dependent configuration according to

$$\begin{aligned} U(\boldsymbol{r}, t) &= A(t) U_0(\boldsymbol{r}) A^{-1}(t) , \\ K(\boldsymbol{r}, t) &= A(t) \tilde{K}(\boldsymbol{r}, t) . \end{aligned} \tag{5.1}$$

By this definition, $\tilde{K}$ is the kaon field as observed in the "rest frame" of the background skyrmion.

Under an isospin rotation, these variables transform as

$$\begin{aligned} A &\to BA , \\ \tilde{K} &\to \tilde{K} , \end{aligned} \tag{5.2}$$

since the isospin is entirely carried by the factor of A in $K = A\tilde{K}$. In other words, $\tilde{K}$ has zero isospin. Under a spatial rotation,

$$U_0 \to \exp(i\boldsymbol{\alpha}\cdot\boldsymbol{L})U_0\exp(-i\boldsymbol{\alpha}\cdot\boldsymbol{L}) = \exp(-i\boldsymbol{\alpha}\cdot\boldsymbol{I})U_0\exp(i\boldsymbol{\alpha}\cdot\boldsymbol{I}),$$

$$\tilde{K} \to \exp(i\boldsymbol{\alpha}\cdot\boldsymbol{L})\tilde{K} = \exp(-i\boldsymbol{\alpha}\cdot\boldsymbol{I})\exp(i\boldsymbol{\alpha}\cdot\boldsymbol{T})\tilde{K}, \tag{5.3}$$

where $\boldsymbol{T} = \boldsymbol{I} + \boldsymbol{L}$, and we have used the invariance of the spherically symmetric skyrmion U_0 under $\boldsymbol{T}$. The above transformation rules tell us that the effect of a spatial rotation on U and K is to transform A and $\tilde{K}$ in the following way:

$$A \to A\exp(-i\boldsymbol{\alpha}\cdot\boldsymbol{I}),$$

$$\tilde{K} \to \exp(i\boldsymbol{\alpha}\cdot\boldsymbol{T})\tilde{K}. \tag{5.4}$$

Therefore, the spin operator for $\tilde{K}$ is $\boldsymbol{T}$, the operator used in sect. 2 to classify partial waves.

We can now classify the (I, J) eigenvalues available to kaons bound to the skyrmion rotator. The only genuine bound state lies in the $T = \frac{1}{2}$ partial wave, and each occupation of it contributes $(I, J) = (0, \frac{1}{2})$. These are precisely the quantum numbers of a heavy quark, bound in an s-wave! In view of this we identify our bound state mode with the s-wave non-relativistic quark wave function. The next partial wave, which has $T = \frac{1}{2}$ and parity minus describes a heavy quark bound in a p-wave. Thus, our model provides yet another example where the skyrmion treatment leads to structure identical to that found in the quark model.

The rotator is known to have $(I, J) = (I, I)$ with I taking on either all integral or all half-integral values. To get half-integral total angular momentum, the rotator must be half-integrally quantized for even bound-state occupation number and integrally quantized for odd bound-state occupation number. Since the bound state has $S = -1$, this gives states with integral isospin for odd strangeness and half-integral isospin for even strangeness, exactly as required by the quark model.

More specifically, the states with $S = -1$ come from occupying the bound state once. Their allowed total quantum numbers are $(I, J) = (i, i) + (0, \frac{1}{2})$ with $i = 0, 1, 2, \ldots$. These states correspond to the Λ, Σ, Σ^* and higher, probably unphysical states. Multiple bound-state occupations give the higher strangeness baryons of the octet and decuplet. These states all have parity plus because, as mentioned earlier, the bound state itself has parity plus. As mentioned before, there is also a state with $T = \frac{1}{2}$ and parity minus ($l_{\text{eff}} = 1$), whose energy lies close to threshold. A single occupation of this state would give a set of negative parity partners of the Λ, Σ, etc. Such states have in fact been observed.

The above states are split from each other by terms of $O(1/N_c)$. The splitting hamiltonian must be derived from the terms in the lagrangian which depend on the rotator velocities

$$A^{-1}\dot{A} = i\dot{\alpha}_a\tau_a . \tag{5.5}$$

The result of a calculation to collect all terms of $O(1/N_c)$ is

$$2\Omega(\dot{\alpha}_a)^2 + \left\{ \int d^3x \left(if\dot{\alpha}_a \dot{K}^+ M_a K + \lambda\dot{\alpha}_a K^+ M_a K - i\lambda\varepsilon_{ibc}\dot{\alpha}_b r_c K^+ D_i K \right.\right.$$
$$\left.\left. + 2i\dot{\alpha}_a \dot{K}^+ D_i K \operatorname{tr}(A_i P_a) - 6i\dot{\alpha}_a \dot{K}^+ [P_a, A_i] D_i K \right) \right\} + \{\text{c.c.}\} , \tag{5.6}$$

where

$$M_a = \tfrac{1}{2}\left(\sqrt{U_0^+}\,\tau_a\sqrt{U_0} + (U_0 \to U_0^+)\right),$$

$$P_a = \tfrac{1}{2}\left(\sqrt{U_0^+}\,\tau_a\sqrt{U_0} - (U_0 \to U_0^+)\right)$$

and Ω is the skyrmion moment of inertia. Including the new terms and computing the hamiltonian correct to $O(1/N_c)$, we find

$$H = M + H_0 + H_1 + H_2 + O(1/N_c^2) .$$

H_0 and H_1 are the expressions presented in sect. 4, and

$$H_2 = \frac{1}{2\Omega}\left(S_i + \int d^3x\, Q_i \right)^2 , \tag{5.7}$$

where $\boldsymbol{S}$ is the rotator spin operator and Q_i are lengthy expressions depending on $G = \sqrt{f}(r)K$ and Π – the canonical coordinates and momenta. We now expand G, Π in terms of partial wave creation and annihilation operators for kaons and antikaons and construct H in terms of them. The results for H_0 and H_1 are the same as in sect. 4. The contribution of the bound-state mode creation and annihilation operators to : $\int d^3x\, Q_i$: turns out to be

$$\tfrac{1}{2}(a_m^+ \tau_i^{mn} a_n)(c_1 - c_2) + \tfrac{1}{2}(b_m^+ \tau_i^{mn} b_n)(c_1 + c_2) , \tag{5.8}$$

where m and n run over two T_z components of the bound state (to $O(N_c^0)$ the lagrangian does not depend on T_z), and

$$c_1 = \int \mathrm{d}r\, r^2 g^* g \left\{ \sin^2 \tfrac{1}{2} F - \tfrac{1}{3} \cos^2 \tfrac{1}{2} F + \frac{2}{f r^2} \left(\frac{\mathrm{d}}{\mathrm{d}r} \left(r^2 \frac{\mathrm{d}F}{\mathrm{d}r} \sin F \right) - \tfrac{4}{3} \sin^2 F \cos^2 \tfrac{1}{2} F \right) \right\},$$

$$c_2 = \int \mathrm{d}r\, r^2 \frac{\lambda}{f \omega_0} g^* g \left\{ \tfrac{4}{3} \cos^2 \tfrac{1}{2} F + \frac{2}{f r^2} \left(\frac{\mathrm{d}}{\mathrm{d}r} \left(r^2 \frac{\mathrm{d}F}{\mathrm{d}r} \sin F \right) - \tfrac{4}{3} \sin^2 F \cos^2 \tfrac{1}{2} F \right) \right\}, \quad (5.9)$$

where $g(r)$ is the bound-state radial function. We recognize $\frac{1}{2} a^+ \tau a$ as the effective spin operator for the $S = 1$ meson, and $\frac{1}{2} b^+ \tau b$ as the effective spin operator for the $S = -1$ meson. It follows that the $O(1/N_c)$ corrections to the energies of the $S = -1$ baryons are neatly expressed as

$$\frac{1}{2\Omega} \left(\boldsymbol{J}_{\mathrm{rot}} + (c_1 + c_2) \boldsymbol{J}_{\mathrm{mes}} \right)^2. \quad (5.10)$$

Similar formulae can be derived for exotic baryons of various types.

There is a whole tower of $S = -1$ baryon states with splittings of $O(1/N_c)$ built upon our bound state. As mentioned earlier, their quantum numbers are $(I, J) = (I, I \pm \frac{1}{2})$, where $I = 0, 1, \ldots$ Of course, only the two lowest values of isospin are present at $N_c = 3$, and they correspond to the Λ, Σ and Σ^*. According to the above analysis, our predictions for their masses are

$$\Lambda = M_{\mathrm{cl}} + \omega_0 - C_{00} + \frac{3}{8\Omega} c^2,$$

$$\Sigma = \Lambda + \frac{1}{\Omega}(1 - c),$$

$$\Sigma^* = \Lambda + \frac{1}{\Omega}\left(1 + \tfrac{1}{2} c\right), \quad (5.11)$$

where $c = c_1 + c_2$.

Unfortunately, c turns out to be negative when the kaon mass is set equal to its observed value. This leads to marked disagreements with baryon spectroscopy: for example, Σ is predicted to be heavier than Σ^*. We can understand the significance of this by studying what happens in the limit of infinitely heavy quark mass, where our treatment becomes exact. We recall that our eigenvalue problem is:

$$-\frac{1}{r^2} \frac{\mathrm{d}}{\mathrm{d}r} \left(h(r) r^2 \frac{\mathrm{d}}{\mathrm{d}r} k \right) - E^2 \left\{ (F')^2 + 2 \left(\frac{\sin F}{r} \right)^2 \right\} k + V_{\mathrm{eff}}(r) k = \left(E^2 - m_{\mathrm{K}}^2 \right) k.$$

It is clear that, as $m_K^2 \to \infty$, $E^2 \to \infty$. In this limit $V_{\text{eff}}(r)$ is negligible compared to $-E^2\{(F')^2 + 2(\sin F/r)^2\}$. Therefore, a very heavy meson will be localized at the minimum of $(F')^2 + 2(\sin F/r)^2$. In the Skyrme model, the profile function $F(r)$ is such that a heavy meson is localized at the origin. As a result, c depends only on the behavior of $F(r)$ near the origin:

$$F(r) \to \pi - \alpha r .$$

From eq. (5.9) it follows that, for very heavy quarks

$$c = 1 - \frac{6\alpha^2}{1 + 3\alpha^2} . \tag{5.12}$$

Numerically, α turns out to be ≈ 1, which leads to $c \approx -0.5$, in drastic disagreement with the quark model result that c is positive for any quark masses [11].

The problem is that the quantity c is very sensitive to the detailed form of the skyrmion profile. Our disaster would not have happened if α had been small compared to one or, equivalently, if the background skyrmion profile had been less sharply peaked at the origin. In fact, such a smoothing-out of the profile can easily be achieved by adding higher-derivative terms to the action. The large-N_c QCD lagrangian of course contains a multitude of such terms and there is no a priori reason to neglect them. They have been ignored here, and elsewhere, only for reasons of computational simplicity. The problem we have encountered suggests to us that a satisfactory quantitative treatment of the $O(1/N_c)$ physics of strange baryons requires a serious understanding of the relative importance of the possible higher-derivative terms in the Skyrme action. Fortunately, it appears that the $O(N_c^0)$ physics of baryons is not sensitive to this issue.

6. Numerical fits

The parameters c and Ω, that enter our mass formulae (5.11), depend on the form of the assumed mesonic action. If terms with six and more derivatives are included, these parameters can change appreciably, as explained in the previous section. On the other hand, although the model-dependent quantities (like c and the moments of inertia) are not guaranteed to be given correctly by the Skyrme model, the general form of our mass formulae (5.11) depends only on the quantization scheme we have adopted. We note that from these formulae, along with the SU(2) relation [1]

$$\Delta - N = \frac{3}{2\Omega} ,$$

we can eliminate c and Ω to derive a mass relation

$$\Delta - N = \Sigma^* + \tfrac{1}{2}\Sigma - \tfrac{3}{2}\Lambda\,. \tag{6.1}$$

Experimentally, the right-hand side of this equation $= 293$ MeV, and the left-hand side $= 304$ MeV. The accuracy of this relation ($\approx 3\%$) is comparable to the accuracy of SU(2) model-independent relations [1]. By comparison, some SU(3) "model-independent" relations based on rotator quantization are in rather poor agreement with experiment [12, 13]. This is due primarily to the crudeness of perturbative treatment of quark masses. It is satisfying to note that the bound-state approach to strangeness leads to a model-independent relation between masses of strange baryons, which is as accurate as SU(2) model-independent relations.

Since the relation (6.1) is independent of the interactions between heavy and light mesons and follows only from the quantization scheme we have adopted, we expect the analog of this relation to hold for charmed baryons, i.e.

$$\Delta - N = \Sigma_c^* + \tfrac{1}{2}\Sigma_c - \tfrac{3}{2}\Lambda_c\,. \tag{6.2}$$

Experimentally, $\Sigma_c - \Lambda_c = 166$ MeV. Therefore, we predict $\Sigma_c^* - \Sigma_c = 44$ MeV, which is several times smaller than the $\Sigma^* - \Sigma$ splitting. This is consistent with the quark model prediction that the strength of the spin-spin coupling between two quarks is roughly inversely proportional to the quark masses. We hope that the relation (6.2) will be upheld by experimental data, when Σ_c^* is observed.

In the quark model our mass formulae (5.11) and the ensuing relations (6.1) and (6.2) follow simply from the existence of one-gluon exchange between quarks, resulting in the spin-spin interactions which can account for all the baryon splittings. It has been shown that a skyrmion description of baryons consisting of light quarks is equivalent to large-N_c quark model [2]. Our approach indicates that heavy flavors can be incorporated into the Skyrme model with results, once again, equivalent to the quark model results.

Although the numerical values of the $O(1/N_c)$ corrections for baryons with heavy quarks are not predicted correctly by the Skyrme model, our treatment to $O(N_c^0)$ produces reasonable results. If we choose the parameter values that fit the N and Δ masses exactly ($F_\pi = 129$ MeV, $e = 5.45$ and $m_K = 495$ MeV), we are in a position to calculate, approximately, the mass of the Λ and the splitting between normal and exotic baryons. They turn out to be 1140 MeV and 250 MeV respectively. If the non-zero pion mass is taken into account, the mass of the Λ comes out even closer to its observed value. The fact that the basic scale of strange baryon masses comes out correctly, without any readjustment of the parameters needed to fit the non-strange baryons, is very encouraging.

Another interesting prediction of our model is the tower of states built on the $L = 0$, $T = \frac{1}{2}$ mode ($l_{\text{eff}} = 1$), which lies close to threshold. These states have the same

(I, J) quantum numbers as the lowest multiplet, but parity $-$. The $(0, \frac{1}{2})$ of this mirror multiplet has been observed – it is the lowest excitation of Λ. Its mass (1405 MeV) is indeed very close to the NK threshold.

7. Comments and conclusions

This calculation was prompted, in part, by the failure of the SU(3) rotator treatment to reproduce the correct masses of low-lying baryons. In the rotator approach, the quark masses can be included only perturbatively. Our physical assumption was that the strange quark is heavy enough to treat it on a separate footing. To implement this plan, we have expanded the SU(3) Skyrme action to second order in kaon fields. This approximation becomes exact only in the limit of infinite kaon mass. As our calculation has demonstrated, it also produces good results when the kaon mass is set equal to its observed value. The bound states of strange mesons and skyrmions exist, and their quantum numbers are precisely those of the strange baryons. We have also shown how a simple collective coordinate quantization scheme leads to correct spin-isospin quantum numbers and produces $O(1/N_c)$ corrections to baryon masses, which can be identified with hyperfine splittings in the quark model. Unfortunately, the sign of the spin-spin interaction between a heavy and a light quark comes out wrong. We see this not as a problem with our approach, but rather a problem with the mesonic action that has been adopted. Omitting various higher-derivative terms from the action is responsible both for some failures of the rotator approach and the failure of hyperfine splittings in our treatment. This conjecture is supported by the fact that SU(2) model-independent relations typically hold to a few per cent, while the model-dependent ones are satisfied, at best, with a 30% accuracy. To describe correctly all the properties of strange baryons, a more realistic meson action is needed.

If one attempts to describe baryons with one of the heavy flavors, like charm, the "bound-state" approach is the only plausible treatment. However, in this case it is clearly wrong to extract the couplings of heavy mesons from the SU(3) symmetric action. The coefficients of various $SU(2)\times SU(2)$-invariant terms have to be determined experimentally. Since there is little experimental information about the relevant interactions, no interesting comparisons can be made. Strangeness, on the other hand, provides an interesting intermediate case, where there is an appreciable quark mass difference, but we are justified in extracting the kaon couplings from an SU(3) symmetric action.

There are a few interesting applications, which we have yet to investigate in detail, but to which our approach is well-suited: these are the problems of the existence of strange nuclear matter and the existence, stability and formation of the H-particle. Consider the properties of kaon bound states in a background pion field corresponding to nuclear matter of variable density [14]. It is possible that as the density increases, the kaon bound-state energy-squared eventually goes negative. This would

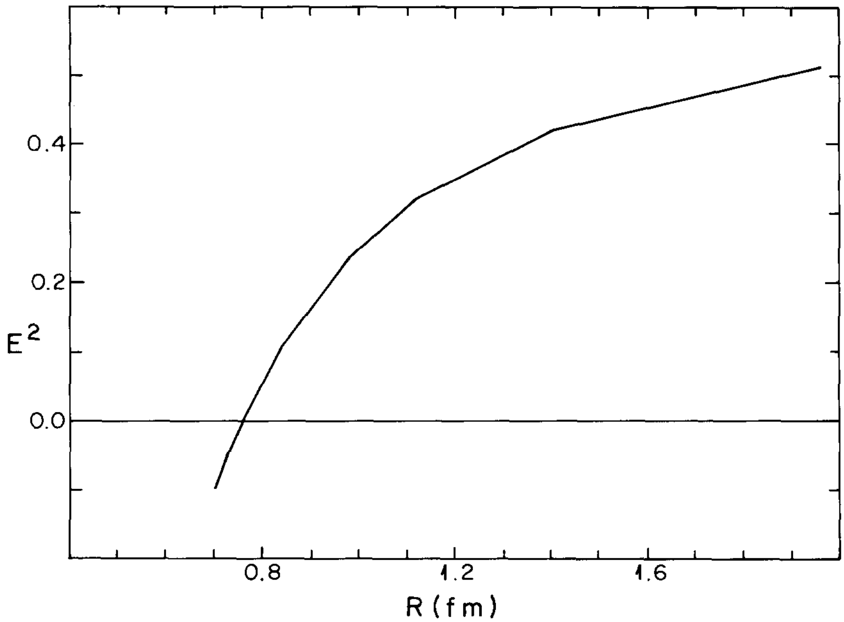

Fig. 2. Graph of bound-state energy-squared versus skyrmion radius R. Energy is expressed in units of kaon mass.

actually signal an instability of the purely pionic configuration to a non-zero expectation value of strange meson fields, i.e. to strange nuclear matter! A toy calculation in the background of a skyrmion constrained to have all its baryon number within a radius R which can then be varied shows that such a zero crossing does occur at densities which are not much higher than ordinary nuclear density (see fig. 2). In this calculation, the SU(2) skyrmion in a spherical box of radius R was found by solving the equation for the profile function $F(r)$ [1] with the boundary condition $F(R)=0$. On the other hand, the kaon mode was only required to vanish at ∞. This roughly corresponds to the physical situation inside dense baryonic matter.

One might be led in a similar way to the existence of a particle like the H: consider a purely pionic $B=2$ configuration like the deuteron. It may very well be that this configuration is sufficiently strongly deformed from the asymptotic two-skyrmion configuration that there are negative energy-squared kaon modes indicating an instability to configurations with strange field expectation values. This is more or less the essential content of the Skyrme version of the H-particle which has been discussed recently [15]. Our picture is presumably quantitatively more reliable and, if it works as conjectured above, would allow us to follow, in a semiclassical sense, the formation of the H from widely-separated baryons and permit an analysis of formation cross sections and channels. We hope to report on studies of these and related questions in the near future.

References

[1] G. Adkins, C. Nappi and E. Witten, Nucl. Phys. B228 (1983) 552
[2] A. Manohar, Nucl. Phys. B248 (1984) 19
[3] M. Praszalowicz, CERN-TH-4114/85
[4] M. Chemtob, CEN preprint 1984, Z. Phys. C, to be published
[5] E. Witten, Nucl. Phys. B223 (1983) 422, 433
[6] E. Witten, Phys. Rev. D30 (1984) 72
[7] R.L. Jaffe, Phys. Rev. Lett. 38 (1977) 195
[8] T.H.R. Skyrme, Nucl. Phys. 31 (1962) 550, 556; Proc. Roy. Soc. 260 (1961) 127
[9] C. Callan, S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177 (1969) 2239, 2247
[10] E. Guadagnini, Nucl. Phys. B236 (1984) 35
[11] A. De Rujula, H. Georgi, and S.L. Glashow, Phys. Rev. D12 (1975) 147
[12] G. Adkins and C. Nappi, Nucl. Phys. B249 (1985) 507
[13] J. Bijnens, H. Sonoda and M. Wise, Phys. Lett. 140B (1984) 421
[14] I. Klebanov, Princeton preprint (1985)
[15] A.P. Balachandran, A. Barducci, F. Lizzi, V.G.J. Rogers and A. Stern, Phys. Rev. Lett. 52 (1984) 887