Nuclear Physics B167 (1980) 479–499

REGULARIZATION BY DIMENSIONAL REDUCTION OF SUPERSYMMETRIC AND NON-SUPERSYMMETRIC GAUGE THEORIES

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Received 27 November 1979

We apply the technique of dimensional reduction to both supersymmetric and non-supersymmetric theories. Explicit one- and two-loop calculations show that in the latter case the technique is a viable alternative to conventional dimensional regularization, while in the former it preserves the Slavnov–Taylor identities of both supersymmetry and gauge invariance.

1. Introduction

Despite considerable work in the field of supersymmetry, it has always been necessary to postulate the existence of a suitable regularization technique for supersymmetric gauge theories. This is particularly relevant, for example, to proofs of ultraviolet finiteness of the S-matrix at the two-loop level in certain theories of supergravity. Recently Siegel [1] has proposed a modified version of dimensional regularization [2] based on dimensional reduction [3]. In this paper we give a detailed account of this technique and its relationship to conventional dimensional regularization. Explicit one- and two-loop calculations are performed which demonstrate that both supersymmetry and gauge invariance are maintained for all n (the number of space-time dimensions).

In the past, conventional dimensional regularization has been conjectured to give problems for supersymmetric theories. The reasons often given are:

(i) in n dimensions, Fierz rearrangements use more covariants than in four dimensions;

(ii) an action which is supersymmetric in four dimensions is not necessarily so in n dimensions;

(iii) the relative numbers of bosonic and fermionic degrees of freedom vary with n. These arguments are, of course, closely related. Nevertheless, Townsend and van Nieuwenhuizen [4] have shown in an explicit two-loop calculation that dimensional regularization respects supersymmetric Ward identities in the massless spin $(0, \frac{1}{2})$ Weiss–Zumino model [5]. In order to obtain this result, it was necessary to impose the relations $\mathrm{Tr}\,\mathbb{1} = 4$ and $\{\gamma_5, \gamma_\mu\} = 0$ (for all μ) in the γ-matrix algebra. This result was extended by Sezgin [4] to the massive case. The same algebraic rules were used in this model for a calculation of the two-loop β-function [6]. In this article we show that these results can be extended to models involving spin-1 fields, providing a modified form of dimensional regularization (called by Siegel "regularization by dimensional reduction") is employed.

The dimensional reduction technique proposed by Siegel consists of continuing in the number of space-time dimensions from 4 to n, *where n is less than 4*, but keeping the numbers of components of all other tensors fixed. In superfield language it can be seen that this procedure is equivalent to performing all the θ algebra in supergraphs [7] in four dimensions, and evaluating the resulting scalar Feynman integrals in n dimensions.

In the component field language, it is convenient for calculational purposes to split the four-dimensional vector field A_μ into an n-dimensional vector A_i *plus* a field $A_\sigma (n \leqq \sigma \leqq 4)$ which, as we shall see, transforms as ε scalars under gauge transformations ($\varepsilon = 4 - n$). Fermions, on the other hand, remain as four-component spinors. It is clear that this is only possible if we continue down in n from $n = 4$. Continuing to $n > 4$ would only be possible in theories which are dimensionally reduced forms of supersymmetric theories in higher dimension. Even then it is only possible to continue a finite distance in n before one "runs out of spinors", whereas one can formally go to all $n < 4$.

As we show, dimensional reduction is a viable alternative to dimensional regularization for ordinary gauge theories such as Yang–Mills theory and gravity. The infinities in the subtraction constants are actually different, but the Ward identities are satisfied. For supersymmetric theories dimensional reduction can be used, either when the full supersymmetry group is unbroken, in which case superfields [8] are an appropriate language as stressed by Siegel, or, remarkably, when the supersymmetry is partially fixed by a particular choice of gauge. As an example of the latter case, we consider the supersymmetry Ward identity for the spin $(\frac{1}{2}, 1)$ Yang–Mills system [9] in the Wess–Zumino gauge, and show that dimensional reduction respects this identity but dimensional regularization does not. We compare our results with observations recently made by Majumdar, Poggio and Schnitzer [10] on the same Ward identity.

Other methods for regularizing supersymmetric theories have been proposed. Delbourgo and Ramón Medrano [11] considered the extension of superfields $\phi(x, \theta)$ to n dimensions, with $2^{n/2}$ θ-components. This scheme is the natural extension of dimensional regularization to the superfield formalism but is difficult to apply in actual calculations. Clark *et al.* [12] have used the BPHZ scheme to regularize n-loop graphs; the same comment applies. Another well-known method is the higher-derivative scheme of Slavnov [13]. This does not regularize all the divergences in gauge theories and must hence be complemented with another technique.

The paper is organized as follows. In sect. 2 we begin with two very simple examples which do not use superfields and clearly show the difference between ordinary dimensional regularization and dimensional reduction. In sect. 3 we analyze pure Yang–Mills theory and its coupling to scalars and fermions. This clearly shows that the new regularization scheme is a valid alternative to dimensional regularization in ordinary gauge theories. Although Z-factors are quite different, the β-function is unmodified up to two loops, as it should be, being an observable. Furthermore, we verify that the 3-point vertices and the two-loop W self-energy satisfy their Ward identities.

In sect. 4 we show that using a supersymmetric gauge-fixing term [14] in supersymmetric quantum electrodynamics [15], the Ward identities do not contain extra terms due to the gauge artefacts. The same Ward identity for the spin 0 and spin $\frac{1}{2}$ as considered by Townsend *et al.*, now contains one contribution due to exchange of a photon which is explicitly n-dependent in dimensional regularization and would violate the Ward identity, but the extra ε-scalar contribution corrects the n to 4.

In sect. 5 we show that use of 4-dimensional gamma matrices as required by dimensional reduction allows one to avoid a well-known problem in supersymmetric Yang–Mills theory. The model is globally supersymmetric in four dimensions due to the identity [16]

$$(\bar{\lambda}^a\gamma^u\lambda^b)(\bar{\varepsilon}\gamma_\mu\lambda^c)f^{abc}=0\,.$$

If one were to go up to $n>4$, this identity would no longer hold, resulting in violation of Ward identities. But using dimensional reduction with four-dimensional gamma matrices, this identity clearly remains valid. In sect. 6 we consider superloop calculations [7] and show in some examples why our component analysis justifies the supergraph manipulations in four dimensions.

2. Two simple examples

Consider scalar quantum electrodynamics with lagrangian density in four dimensions

$$\mathscr{L}=-\tfrac{1}{4}(\partial_\mu A_\nu-\partial_\nu A_\mu)^2-(\partial_\mu\phi^*+ieA_\mu\phi^*)(\partial_\mu\phi-ieA_\mu\phi)\,. \tag{2.1}$$

Going down from 4 to n dimensions, the derivatives ∂_μ become n-dimensional derivatives ∂_i, and all fields depend on n, rather than 4, space-time coordinates. It is convenient to split the range of the index μ of A_μ into $0\leqq i\leqq n$ and $n\leqq\sigma\leqq 4$. Thus we obtain

$$\begin{aligned}\mathscr{L}=&-\tfrac{1}{2}(\partial_jA_i)^2+\tfrac{1}{2}(\partial_jA_j)^2-\partial_j\phi^*\partial_j\phi-ieA_j\phi^*\overleftrightarrow{\partial_j}\phi\\&-e^2A_i^2\phi^*\phi-\tfrac{1}{2}(\partial_jA_\sigma)^2-e^2A_\sigma^2\phi^*\phi\,.\end{aligned} \tag{2.2}$$

Clearly the last two terms describe new ε-scalars, as we shall call them. The model is gauge invariant under the n-dimensional gauge transformations

$$\delta A_i = \partial_i \Lambda \,, \qquad \delta A_\sigma = 0 \,. \tag{2.3}$$

In an ε-scalar loop, we have $\delta_{\sigma\sigma'}\delta_{\sigma'\sigma} = \delta_{\sigma\sigma} = \varepsilon$, so that the one-loop counterterms without external ε-scalars are the same as in dimensional regularization. For one-loop scalar–scalar scattering there is a finite difference if one uses minimal subtractions, and at the two-loop level the infinite parts of the Z-factors differ. At the one-loop level we also find new counter terms with external ε-scalars which are of course absent from dimensional regularization. Presumably, it can be shown to all loop orders that finite recalibrations (i.e., finite additions to the Z-factors) suffice to make the S-matrices the same in both regularization schemes.

Our second example concerns ordinary quantum electrodynamics:

$$\mathscr{L} = -\tfrac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \bar{\lambda}\gamma^\mu(\partial_\mu - ieA_\mu)\lambda \,. \tag{2.4}$$

Going down to n dimensions, the matrices γ_μ split into n matrices γ_j and ε matrices γ_σ, and the lagrangian density becomes

$$\begin{aligned}\mathscr{L} = &-\tfrac{1}{4}(\partial_j A_i - \partial_i A_j)^2 - \bar{\lambda}\gamma_j(\partial_j - ieA_j)\lambda \\ &-\tfrac{1}{2}(\partial_j A_\sigma)^2 + ie(\bar{\lambda}\gamma_\sigma\lambda)A_\sigma \,.\end{aligned} \tag{2.5}$$

All γ^i and γ^σ matrices are still considered to be four-dimensional. Note that this prevents awkward terms ln 2 in the Z-factors. The fermion self-energy receives a finite local correction due to a fermion-ε-scalar loop since the identity $\{\gamma_\sigma, \gamma_i\} = 0$ leads to

$$\gamma_\sigma \not{k} \gamma_\sigma = -\varepsilon \not{k} \,. \tag{2.6}$$

On the other hand, the ε-scalar self-energy due to a fermion loop contains a trace of the form

$$\mathrm{tr}\,(\gamma_\sigma \not{k} \gamma_{\sigma'}(\not{k} + \not{p})) = -\delta_{\sigma\sigma'}\,\mathrm{tr}\,(\not{k}(\not{k} + \not{p})) \,. \tag{2.7}$$

Thus, in this case one obtains the same result, up to a sign, as from a Yukawa coupling of the form $A_\sigma\bar{\lambda}\lambda$ or $B_\sigma\bar{\lambda}i\gamma_5\lambda$.

In fact, the ε-scalars behave in some ways as Faddeev–Popov ghosts: we only consider S-matrix elements without external ε-scalars and a closed ε-loop acquires an extra factor ε. We do not expect problems with unitarity since the four-dimensional theory is unitary.

3. Yang–Mills theory

In this section we consider pure Yang–Mills theory and Yang–Mills fields coupled to fermions and scalars at the one- and two-loop level. We compare the dimensional reduction technique with that of conventional dimensional regularization in a number of cases. First, we consider the lagrangian density (for vector field W^a_μ and ghost field C^a)

$$\mathscr{L} = -\tfrac{1}{4}(G^a_{\mu\nu})^2 - \tfrac{1}{2}(\partial_\mu W^a_\mu)^2 + C^{*a}\partial_\mu D^{ab}_\mu C^b\,, \tag{3.1}$$

where

$$G^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g f^{abc} W^b_\mu W^c_\nu\,,$$

$$D^{ab}_\mu = \partial_\mu \delta^{ab} - g f^{abc} W^c_\mu\,.$$

f_{abc} are the totally antisymmetric structure constants of the underlying semi-simple Lie group. The gauge-fixing term has been chosen so as to give a vector meson propagator of the form $\delta_{\mu\nu}\delta_{ab}(k^2)^{-1}$ (the Feynman gauge). Applying the dimensional reduction technique we obtain a lagrangian $\mathscr{L}^n + \mathscr{L}^\varepsilon$ where

$$\mathscr{L}^n = -\tfrac{1}{4}(G^a_{ij})^2 - \tfrac{1}{2}(\partial_i W^a_i)^2 + C^{*a}\partial_i D^{ab}_i C^b\,, \tag{3.2}$$

$$\begin{aligned}\mathscr{L}^\varepsilon = & -\tfrac{1}{2}(\partial_i W^a_\sigma)^2 - g f^{abc} W^b_i W^c_\sigma \partial_i W^a_\sigma \\ & -\tfrac{1}{2} g^2 f^{abc} f^{ade} W^b_i W^c_\sigma W^d_i W^e_\sigma \\ & -\tfrac{1}{4} g^2 f^{abc} f^{ade} W^b_\sigma W^c_{\sigma'} W^d_\sigma W^e_{\sigma'}\,.\end{aligned} \tag{3.3}$$

The lagrangian $\mathscr{L}^n$ is precisely of the form employed in normal dimensional regularization. $\mathscr{L}^\varepsilon$ consists of the additional terms produced by the dimensional reduction technique, involving interactions of the ε-scalars. The ε-scalars couple to the gauge fields W^a_i like scalars transforming as the adjoint representation. This is easily understood from the dimensionally reduced form of the gauge transformation on the fields W^a_μ:

$$\begin{aligned}\delta W^a_i &= \partial_i \Lambda^a + g f^{abc} W^b_i \Lambda^c\,, \\ \delta W^a_\sigma &= g f^{abc} W^b_\sigma \Lambda^c\,.\end{aligned} \tag{3.4}$$

There is, in addition, an ε-scalar 4-point coupling which is a relic of the $n=4$ theory. In the calculations to follow this term does not contribute at the level of perturbation theory considered.

We begin by briefly considering the relation between dimensional regularization and the dimensional reduction technique. Consider the one-loop W self-energy. Only the W-loop yields an n-dependent integrand given by [17]

$$\begin{aligned}\int \frac{\mathrm{d}^n k}{k^2(k-p)^2}[&(4n-6)k_\mu k_\nu + (n-6)p_\mu p_\nu + (3-2n)(k_\mu p_\nu + k_\nu p_\mu) \\ &+ \delta_{\mu\nu}(2k^2 - 2k\cdot p + 5p^2)]\,.\end{aligned} \tag{3.5}$$

This is the result for dimensional regularization. If one were to replace the explicit n inside the square brackets by 4, the difference is

$$\varepsilon \int \frac{\mathrm{d}^n k}{k^2(k-p)^2}[-4k_\mu k_\nu - p_\mu p_\nu + 2k_\mu p_\nu + 2k_\nu p_\mu]\,. \tag{3.6}$$

This is, however, precisely the contribution of an ε-scalar loop, which yields a numerator $(2k-p)_\mu(2k-p)_\nu$. Thus at the one-loop level the ε-scalars convert the dimensional regularization result to the result which would be obtained by simply performing the numerator algebra in four dimensions. It should be remarked, however, that care must be taken if one wishes to extend this prescription to higher orders. It is probable that if one carefully distinguishes between 4-dimensional $\delta_{\mu\nu}$ resulting from $W_\mu W_\mu$ contractions and n-dimensional $\delta_{\mu\nu}$ resulting from $k_\mu k_\nu$ momentum integrals, then one ends up with the dimensional reduction technique. In other words, there are two ways of carrying out "dimensional" regularization: either spinors and vectors are in n-dimensions and then tr $\mathbb{1} = 2^{n/2}$, or one keeps spinors and vectors in 4 dimensions, but only continues x^μ and q^μ to n dimensions. The latter case is what superfield techniques use. All algebra is done there in 4 dimensions by writing $\phi(x, \theta)$ and one only continues in x^μ, not in θ^α. In particular one keeps 4 gamma matrices so that in $\bar{\theta}\gamma_\mu i\gamma_5\theta A_\mu$ the index μ of A_μ is 4-dimensional. However, it is completely incorrect to consider both the $\delta_{\mu\nu}$ in $W_\mu W_\mu$ and the $\delta_{\mu\nu}$ from $k_\mu k_\nu$ integrations as 4-dimensional; this yields violations of the Ward identities from the 2-loop level on.

We now consider a Slavnov–Taylor identity relating the 3-point functions. Using the invariance of $\mathscr{L}$ with respect to the Becchi–Rouet–Stora (BRS) transformations [18],

$$\begin{aligned}\delta_{\mathrm{BRS}} W^a &= D_\mu(C^a\Lambda)\,,\\ \delta_{\mathrm{BRS}} C^{*a} &= -\Lambda\partial_\mu W^a_\mu\,,\\ \delta_{\mathrm{BRS}} C^a &= f^{abc}C^b\Lambda C^c\,,\end{aligned} \tag{3.7}$$

we derive by varying $\langle W^a_\mu(x)W^b_\nu(y)C^{*c}(z)\rangle$ the following identity for connected (not single-particle-irreducible) Green functions:

$$\begin{aligned}&\langle D_\mu C^a(x)W^b_\nu(y)C^{*c}(z) + W^a_\mu(x)D_\nu C^b(y)C^{*c}(z)\\ &\quad + W^a_\mu(x)W^b_\nu(y)\partial_\lambda W^c_\lambda(z)\rangle = 0\,.\end{aligned} \tag{3.8}$$

This identity is represented diagrammatically in fig. 1. (We multiply by $p^2q^2k^2$ and then put $p^2=q^2=k^2=0$.)

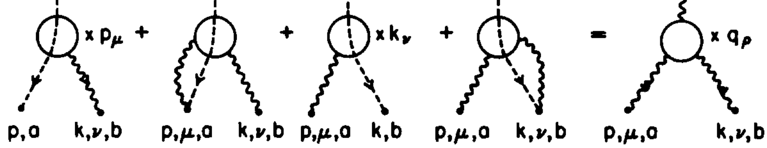


Fig. 1. The gauge Ward identity for the 3-point vertex in Yang–Mills theory [eq. (3.8)].

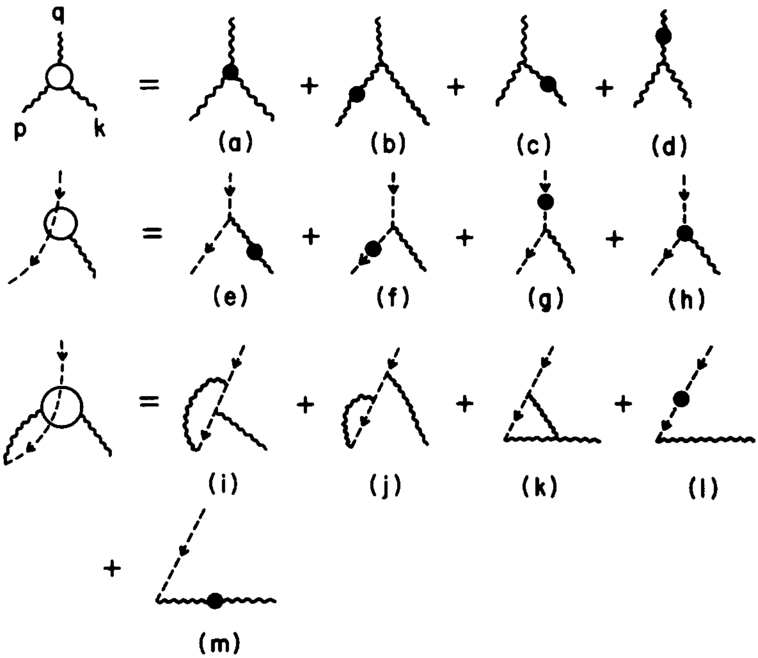


Fig. 2. Diagrams contributing to the gauge Ward identity in pure Yang–Mills theory. A blob denotes a one-loop 1PI insertion. Diagrams with the ghost line on the right have been omitted.

Consider first the case of normal dimensional regularization. It is easy to verify that the identity holds for tree graphs. In the one-loop approximation there are a number of graphs (see fig. 2), but the calculation is relatively straightforward. It is (as usual with 3-point vertices [17, 19]) convenient to take one external momentum tending to zero. We choose $k=0$. The singularities due to the $k_\mu k_\nu k^{-2}$ terms in the W self-energy in graphs (2c,e,m) cancel. We evaluate the coefficient of $p_\mu p_\nu$ in each diagram (both the infinite as well as the finite parts) and find, suppressing an overall factor $g^2 C_2(G)\pi^{n/2}(p^2)^{n/2-2} f^{abc} p_\mu p_\nu$,

$$
\begin{aligned}
a &= \left(-\frac{4}{3\varepsilon}-\frac{35}{18}\right), & b = c &= \left(\frac{10}{3\varepsilon}+\frac{31}{9}\right), & d &= 0, \\
e &= \left(\frac{10}{3\varepsilon}+\frac{31}{9}\right), & f = g &= \left(\frac{1}{\varepsilon}+1\right), & h &= \left(\frac{1}{\varepsilon}+1\right), \\
i+k &= -\tfrac{1}{2}, & j &= \left(-\frac{1}{\varepsilon}-1\right), & l = m &= 0. \qquad (3.9)
\end{aligned}
$$

Adding the ε^{-1} and ε^{0} terms, we find that the identity (3.8) is satisfied.

In order to see whether the Ward identity is also satisfied using the dimensional reduction technique, the additional graphs generated by $\mathcal{L}^{\varepsilon}$ must be investigated and

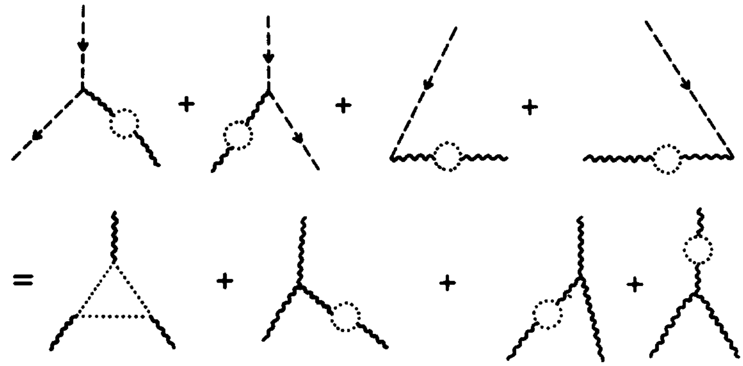


Fig. 3. The contributions of the ε-scalars to the gauge Ward identity for the three-point vertex. They independently satisfy the identity.

shown to separately satisfy the identity. The ε-scalars do not couple to ghosts, and so there are few such graphs. The resulting identity is shown diagrammatically in fig. 3. (Graphs involving the vertex $\dot{W}_\sigma W_\sigma W_i \dot{W}_i$ are not shown. They vanish because of the identity

$$\int d^n k \, (2k-q)_\mu [(k-q)^2 k^2]^{-1} = 0 \, .)$$

Note that these graphs are finite as $n \to 4$ because of the factor of ε arising from the ε-scalar loop. One could show that the identity implied by fig. 3 is identically satisfied; alternatively, it is easy to see that the only modification to (3.7) is

$$a = -\frac{4}{3\varepsilon} + \frac{35}{18} + \frac{1}{3}, \qquad b = c = e = \frac{10}{3\varepsilon} + \frac{31}{9} - \frac{1}{3}. \tag{3.10}$$

The three-point Ward identity in (3.6) is also satisfied if one couples the Yang–Mills fields to fermions and includes fermion loops. Since the graphs with fermion loops satisfy separately the Ward identity, it does not matter *in this case* whether we take $\mathrm{tr}\,\mathbb{1} = 4$ or $\mathrm{tr}\,\mathbb{1} = 2^{n/2}$. For the $p_\mu p_\nu$ terms the $(p^2)^{-1}$ and $(k^2)^{-1}$ terms cancel again and one now finds the following extra contributions due to fermion loops (suppressing a factor $g^2 T(R) \pi^{n/2} (p^2)^{n/2-2} f^{abc} p_\mu p_\nu$)

$$-a = b = c = e = -\frac{8}{3\varepsilon} - \frac{20}{9}. \tag{3.11}$$

Thus for a Yang–Mills theory with or without fermions, the Ward identity of eq. (3.8) is preserved at the one-loop level both by conventional dimensional regularization and by dimensional reduction.

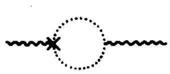
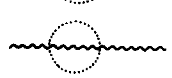
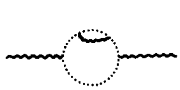
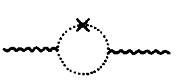
As a second example, we turn to the calculation of the Callan–Symanik function $\beta(g)$. This has already been calculated to two loops, using conventional dimensional regularization with minimal subtraction [19]. We now repeat this calculation with the lagrangian $\mathscr{L}^n + \mathscr{L}^\varepsilon$, again employing minimal subtraction. As the β function is determined from the simple poles in ε of the appropriate subtraction constants, it is clear that at the one-loop level the ε-scalars will not contribute, since any one-loop graph involving a loop of ε-scalars is finite. For the two-loop calculation we consider the $W_\mu C^* C$ vertex and hence must investigate the effect of the ε-scalars on the calculation of Z_3^{WW}, $Z_3^{\mathrm{C^*C}}$ and $Z_1^{\mathrm{C^*CW}}$.

For the calculation of Z_3^{WW} the additional graphs generated by $\mathscr{L}^\varepsilon$ contributing to the W self-energy are shown in table 1, together with the pole terms resulting from the calculation. The calculation of the contribution to Z_3^{WW} of a multiplet of scalars transforming according to an arbitrary representation of the gauge group given in ref. [20] cannot be applied without modification. This is because the factors of ε associated with the scalar loops mean that, for instance, the one-loop contribution of the scalars to the W self-energy is finite and hence requires no subtraction.

The total contribution of the ε-scalars to $\mathrm{O}(g^4)$ for the W self-energy is thus

$$\Pi^\varepsilon_{\mu\nu} = C_2(G)^2 \frac{ig^4}{(2\pi)^4} \pi^4 (p^2 \delta_{\mu\nu} - p_\mu p_\nu)(2\varepsilon)^{-1}$$
$$+ \text{terms finite as } \varepsilon \to 0 \tag{3.12}$$

TABLE 1
Contributions of the ε-scalars to the two-loop W self-energy: the dotted lines are ε-scalars, and the wavy lines are W-bosons

Diagrams	Coefficients of $p^2\delta_{\mu\nu}/\epsilon$	Coefficients of $p_\mu p_\nu/\epsilon$
	-1/3	1/3
	3/2	-3/2
	5/9	-13/18
	-4/3	4/3
	No pole terms	
	-4/3	4/3
	8/3	-8/3
	-11/9	25/18
TOTAL	1/2	-1/2

A factor $ig^4[C_2(G)]^2\, \pi^4/(2\pi)^4$ is suppressed.

Fig. 4. Graphs which vanish since $\int dk(2k-q)_\mu[k^2(k-q)^2]^{-1}=0$.

where

$$C_2(G)\delta^{ab}=f^{acd}f^{bcd}.$$

The observant reader may wonder why graphs as in fig. 4 do not contribute. The reason is that a single vector cannot couple to a scalar loop due to the identity $\int d^n k(2k-q)_\mu k^{-2}(k-q)^{-2}=0$.

$\Pi^\varepsilon_{\mu\nu}$ is transverse, as is required from the Ward identity $p_\mu \Pi_{\mu\nu}=0$. However, Z_3^{WW} is changed, i.e.

$$\tilde{Z}_3^{\mathrm{WW}}=Z_3^{\mathrm{WW}}+\tfrac{1}{2}K^2/\varepsilon. \tag{3.13}$$

Here $K=(C_2(G)/16\pi^2)u^2$, and $u=gm^{(-\varepsilon)/2}$, where m is the renormalization mass scale. We have written Z_3^{WW} for the subtraction constant as calculated in conventional dimensional regularization [19] and $\tilde{Z}_3^{\mathrm{WW}}$ for the same quantity in the dimensional reduction scheme.

Similarly we find that

$$\begin{aligned}\tilde{Z}_3^{\mathrm{C^*C}}&=Z_3^{\mathrm{C^*C}}-\tfrac{1}{4}K^2/\varepsilon,\\ \tilde{Z}_1^{\mathrm{WC^*C}}&=Z_1^{\mathrm{WC^*C}}.\end{aligned} \tag{3.14}$$

Therefore, since

$$\beta(u)=\frac{-\frac{1}{2}\varepsilon}{\dfrac{d}{du}\ln\left[uZ_1^{\mathrm{WC^*C}}/(Z_3^{\mathrm{WW}})^{1/2}Z_3^{\mathrm{C^*C}}\right]} \tag{3.15}$$

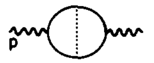
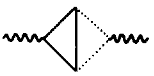
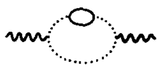
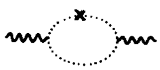
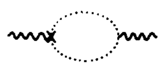
we see that up to $O(u^5)$ the calculation of $\beta(u)$ is not affected. This is a significant result in that, since the leading two terms in $\beta(u)$ should be independent of the renormalization technique, it shows that the dimensional reduction scheme, while differing non-trivially from conventional dimensional regularization, leads to the same physical results.

We now extend the above results to the case of a Yang–Mills field interacting with fermions. Thus $\mathscr{L}$ is modified as follows:

$$\begin{aligned}\mathscr{L}^n&\to\mathscr{L}^n-\bar{\psi}_p\gamma_i D_i^{pq}\psi_q\\ \mathscr{L}^\varepsilon&\to\mathscr{L}^\varepsilon+ig\bar{\psi}_p\gamma_\sigma R^a_{pq}\psi_q W^a_\sigma,\end{aligned} \tag{3.16}$$

where $D_i^{pq}=\delta^{pq}\partial_i-ig(R^a)^{pq}W_i^a$. Here R^a is the matrix representation of the ath generator of the group acting on the fermion multiplet. Thus, as described in sect. 2 for the case of QED, we see that Yukawa-type couplings appear.

TABLE 2
The ε-scalar contribution to W-self energy graphs with a fermion loop

Diagrams	Coefficient of $\delta_{\mu\nu}/\epsilon$	Coefficient of $p_\mu p_\nu/\epsilon$
	4/3	-4/3
	-8/3	8/3
	8/3	-8/3
	-4/3	4/3
	8/3	-8/3
	-8/3	8/3
TOTAL	0	0

A factor $ig^4C_2(G)T(R)\,\pi^4/(2\pi)^4$ is suppressed, where $T(R)\delta^{ab} = \mathrm{Tr}\,[R^aR^b]$.

It is easily seen that to two-loop order these couplings do not contribute to $Z_3^{C^*C}$ and $Z_1^{WC^*C}$. In the case of Z_3^{WW} graphs do exist, but as indicated in table 2, there is no net contribution. Thus for Yang–Mills theory interacting with an arbitrary multiplet of fermions, dimensional reduction gives the correct result for $\beta(g)$. The inclusion of scalar fields is also straightforward in that, as we have already seen in sect. 2, the only new coupling generated is a quartic scalar coupling which does not contribute to $\beta(g)$ to the order considered [20] (see fig. 4).

In ref. [20] $\beta(g)$ was calculated for supersymmetric Yang–Mills theory coupled to an arbitrary number of matter multiplets, by examining, as above, the C^*CW vertex. We can deduce from the above results that the same result would have been obtained for $\beta(g)$ had the dimensional reduction technique been employed. Since, as we shall see below, the dimensional reduction technique respects supersymmetry Ward identities, we conclude that the result for $\beta(g)$ in this model is correct. These remarks also apply to the O(4) model with vanishing two-loop $\beta(g)$ [21].

4. Supersymmetric QED with a supersymmetric gauge-fixing term

For supersymmetric theories with no local gauge invariance the supersymmetry Ward identities for 1PI graphs are given by

$$(\delta\Gamma/\delta\phi^k)(\delta_s\phi^k) = 0 \tag{4.1}$$

where $\delta_s\phi^k$ is the variation of the fields ϕ^k under global supersymmetry transformations. For supersymmetric gauge theories the same identity holds, provided one uses supersymmetric ghost and gauge-fixing terms. In this section we consider supersymmetric quantum electrodynamics coupled to two real scalar spin $(0, \frac{1}{2})$ multiplets [15]. In this section and sect. 6 we follow the notation of ref. [8] and the first article of ref. [7], where further details can be found. Thus the theory may be written in terms of two independent chiral superfields, $\phi_+(x, \theta)$ and $\phi_-(x, \theta)$, together with a general superfield $\psi(x, \theta)$. These can also be written in terms of component fields as

$$\phi_\pm(x, \theta) = [\exp \mp \tfrac{1}{4}\bar{\theta}\partial\!\!\!/\gamma_5\theta]\{A_\pm(x) + \bar{\theta}\psi_\pm(x) + \tfrac{1}{4}\bar{\theta}(1 \pm i\gamma_5)\theta F_\pm(x)\}, \tag{4.2}$$

$$\begin{aligned}\psi(x, \theta) = {} & A(x) + \bar{\theta}\psi(x) + \tfrac{1}{4}\bar{\theta}\theta F(x) + \tfrac{1}{4}\bar{\theta}\gamma_5\theta G(x) \\ & + \tfrac{1}{4}\bar{\theta}i\gamma_\mu\gamma_5\theta A_\mu(x) + \tfrac{1}{4}\bar{\theta}\theta\bar{\theta}\chi(x) + \tfrac{1}{32}(\bar{\theta}\theta)^2 D(x),\end{aligned} \tag{4.3}$$

where θ is a four-component Majorana spinor. The action is then

$$\begin{aligned}\int \mathscr{L}\, d^4x = \int d^4x\, \tfrac{1}{8}(\bar{D}D)^2\{ & |\phi_+|^2 \exp(2g\psi) + |\phi_-|^2 \exp(-2g\psi) \\ & + \bar{\psi}[\partial^2 + \tfrac{1}{4}(1+\alpha)(\bar{D}D)^2]\psi\},\end{aligned} \tag{4.4}$$

where

$$D_\alpha = \partial/\partial\bar{\theta}^\alpha - \tfrac{1}{2}i(\gamma^\mu\theta)_\alpha \partial/\partial x^\mu. \tag{4.5}$$

The term in the action of the form $\bar{\psi}\alpha(\bar{D}D)^2\psi$ is a supersymmetric gauge-fixing term. Under a U(1) gauge transformation the fields transform as

$$e^{2g\psi} \to \Omega_-\, e^{2g\psi}\Omega_+^{-1}, \qquad \phi_\pm \to \Omega_\pm\phi_\pm, \tag{4.6}$$

where $\Omega_+^\dagger = (\Omega_-)^{-1}$. Thus with the gauge-breaking term used in eq. (4.4), the ghosts decouple, just as in ordinary QED and the supersymmetry Ward idéntities are still given by eq. (4.1). In particular the identity derived in ref. [4] is still valid and may be written as

$$\Gamma_{A_+A_+^\dagger}(p)\delta_{\gamma\alpha} - p\!\!\!/_{\gamma\beta}\Gamma_{\bar{\psi}_{+\beta}\psi_{+\alpha}} = 0. \tag{4.7}$$

In the gauge $\alpha = -1$ the kinetic terms are given by

$$\begin{aligned}\int \mathscr{L}\, dx = \int dx\, [& \tfrac{1}{2}A\partial^2 D + \tfrac{1}{2}A_\nu\partial^2 A_\nu - \bar{\psi}\partial^2\chi - \tfrac{1}{2}F\partial^2 F \\ & - \tfrac{1}{2}G\partial^2 G - A_+^\dagger\partial^2 A_+ - A_-^\dagger\partial^2 A_- + \bar{\psi}_+ i\partial\!\!\!/\psi_+ + \bar{\psi}_- i\partial\!\!\!/\psi_- \\ & + |F_+|^2 + |F_-|^2].\end{aligned} \tag{4.8}$$

The interaction terms are more complicated but we only need those arising from the terms in eq. (4.4) of the form

$$g\int dx\, \tfrac{1}{4}(\bar{D}D)^2\{\psi(|\phi_+|^2 - |\phi_-|^2)\}. \tag{4.9}$$

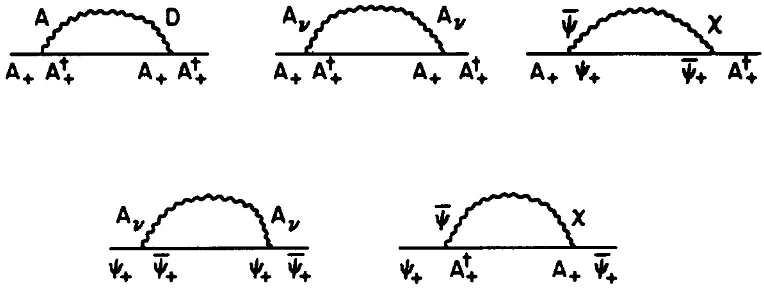


Fig. 5. One-loop contributions to the A_+ and ψ_+ self-energies.

The diagrams which contribute to one-loop in eq. (4.7) are given in fig. 5. If one uses conventional dimensional regularization, only the graph with the virtual photon A_μ is explicitly dependent on n. The n dependence arises from the equation

$$\gamma_i \not{k} \gamma_i = (2-n)\not{k}\,. \tag{4.10}$$

However, if one uses dimensional reduction, the range of the γ_μ matrices is up to 4 and one finds instead

$$\gamma_\mu \not{k} \gamma_\mu = -2\not{k}\,. \tag{4.11}$$

Explicit evaluation of the diagrams in fig. 5 shows that the Ward identity is only satisfied by the finite parts if we set $n=4$ in (4.10). Previous to the ideas set out in this paper and ref. [1], the use of (4.11) would have to be taken as an ansatz which was not derivable from a lagrangian, in much the same way as the $\mathrm{Tr}\,\mathbb{1}=4$ (rather than $2^{n/2}$) rule of ref. [4]. That we must use (4.11) rather than (4.10) can now be understood in the context of the dimensional reduction technique. It is the additional contribution from the ε-scalar which converts the n-dependent result to the $n=4$ expression.

5. Supersymmetric Yang–Mills theory

In a recent paper, Majumdar, Poggio and Schnitzer (MPS) [10] investigate supersymmetric Ward identities for supersymmetric Yang–Mills theory [9] in the Wess–Zumino gauge. They conclude that the Ward identities are violated at the one-loop level, and moreover that this failure cannot be attributed to the choice of regularization scheme (which in their case is conventional dimensional regularization). We have performed a similar investigation but our conclusion is that, at the one-loop level, the infinite parts satisfy the Ward identity whether we use conventional dimensional regularization or dimensional reduction, but the finite parts only do so if dimensional reduction is employed. We thus verify in a non-trivial example that the dimensional reduction technique, unlike conventional dimensional regularization, is compatible with supersymmetry. The lagrangian density for supersymmetric Yang–Mills theory in the Wess–Zumino gauge is

$$\begin{aligned}\mathscr{L} = &-\tfrac{1}{4}(G^a_{\mu\nu})^2 - \tfrac{1}{2}(\partial_\mu W^a_\mu)^2 + C^{*a}\partial_\mu D^{ab}_\mu C^b \\ &-\tfrac{1}{2}\bar{\lambda}^a\gamma_\mu D^{ab}_\mu\lambda^b + \tfrac{1}{2}D^2_a + J^a_\mu W^a_\mu + \bar{J}^a\lambda^a + j^a_D D^a \end{aligned} \tag{5.1}$$

where λ^a is a Majorana spinor and D^a is an auxiliary field. We have added sources J^a_μ, $\bar{J}^a$, j^a_D coupled to W^a_μ, λ^a and D^a, respectively. We define

$$Z(J^a_\mu, \bar{J}^a, j^a_D) = \int [\mathrm{d}W\,\mathrm{d}\lambda\,\mathrm{d}C\,\mathrm{d}C^*\,\mathrm{d}D]\exp i\int \mathrm{d}^4x\,\mathscr{L}. \tag{5.2}$$

The supersymmetry Ward identity is derived by considering the variation of Z under an infinitesimal supersymmetry transformation on the fields of the form

$$\begin{aligned}&\delta_s W^a_\mu = -\bar{\varepsilon}\gamma_\mu\lambda^a, \qquad \delta_s D^a = \bar{\varepsilon}i\gamma_5 \not{D}^{ab}\lambda^b, \\ &\delta_s\lambda^a = \sigma^{\mu\nu}G^a_{\mu\nu}\varepsilon + i\gamma_5 D^a\varepsilon. \end{aligned} \tag{5.3}$$

ε is a constant anticommuting Majorana spinor and $G^a_{\mu\nu}$ the Yang–Mills curvature. Since the integration measure is invariant under this transformation, we find for local $\varepsilon(x)$

$$\begin{aligned}\delta Z = 0 = &\int [\mathrm{d}W\cdots\mathrm{d}D]\int \mathrm{d}^4x\,\delta\mathscr{L}\exp i\int \mathrm{d}z\,\mathscr{L}, \\ \delta\mathscr{L} = &[\partial\cdot W^a(x)\partial_\mu(\bar{\varepsilon}(x)\gamma^\mu\lambda(x)) + (\partial_\mu C^{*a})f^{abc}\bar{\varepsilon}(x)\gamma^\mu\lambda^b(x)C^c(x) \\ &+ J^a_\mu(-\bar{\varepsilon}(x)\gamma^\mu\lambda^a(x)) + \bar{J}^a(\sigma^{\mu\nu}G^a_{\mu\nu}\varepsilon(x) + i\gamma_5 D^a\varepsilon(x)) \\ &+ j^a_D\bar{\varepsilon}(x)i\gamma_5\not{D}^{ab}\lambda^b + \bar{\varepsilon}(x)\partial_\mu S^\mu(x)]. \end{aligned} \tag{5.4}$$

S^μ is the Noether current of supersymmetry. It will not contribute to the Ward identity if ε is constant, as we shall assume from now on.

In the derivation of this result it is necessary to use the identity

$$(\bar{\lambda}^a\gamma_\mu\lambda^c)(\bar{\varepsilon}\gamma_\mu\lambda^b)f^{abc} = 0. \tag{5.5}$$

This identity is only true in four dimensions and is retained by the dimensional reduction technique. The identity also enables us to anticipate the failure of conventional dimensional regularization to preserve the Ward identity in its naive form, since there are no longer four independent γ matrices.

As in MPS, we define the generating functional $\Gamma(\hat{\phi})$ of one particle irreducible vertices in the usual way by a functional Legendre transformation [22]

$$W(J_k) = \Gamma(\hat{\phi}_k) + \int \mathrm{d}^4x\,J_k(x)\hat{\phi}_k(x), \tag{5.6}$$

where the index k denotes the fields $\hat{\lambda}$, $\hat{W}_\mu$, $\hat{D}$ and $W(J_k) = (1/i)\ln Z(J_k)$. (We distinguish the fields $\hat{\phi}_k$ from the integration variable fields ϕ_k in the path integral.)

Following (MPS), we operate on eq. (5.4) with

$$(\delta/\delta\hat{W}^c_\rho(z))(\delta/\delta\hat{\bar{\lambda}}^b(y)),$$

and generate the following identity:

$$\begin{aligned}
0 = ① &\frac{\delta J^{c'}_{\rho'}(z')}{\delta\hat{W}^c_\rho(z)}\frac{\delta\bar{J}^{b'}(y')}{\delta\hat{\bar{\lambda}}^b(y)}\langle\delta^{c'a}_{\rho'\mu}\delta^4(z'-x)(-\bar{\varepsilon}\gamma_\mu\lambda^a(x))i\lambda^{b'}(y')\rangle \\
+② &\frac{\delta\bar{J}^{b'}(y')}{\delta\hat{\bar{\lambda}}^b(y)}\frac{\delta J^{c'}_{\rho'}(z')}{\delta\hat{W}^c_\rho(z)}\langle\delta^{b'a}\delta^4(y'-x)iW^{c'}_{\rho'}(z')\sigma^{\mu\nu}G^a_{\mu\nu}(x)\varepsilon\rangle \\
+③ &\frac{\delta^2\bar{J}^{b'}(y')}{\delta\hat{W}^c_\rho(z)\delta\hat{\bar{\lambda}}^b(y)}\langle\partial\cdot W^a(x)\bar{\varepsilon}\not{\partial}\lambda^a(x)i\bar{\lambda}^{b'}(y)\rangle \\
+④ &\frac{\delta\bar{J}^{b'}(y')}{\delta\hat{\bar{\lambda}}^b(y)}\frac{\delta J^{c'}_{\rho'}(z')}{\delta\hat{W}^c_\rho(z)}\langle\partial\cdot W^a(x)\bar{\varepsilon}\not{\partial}\lambda^a(x)i\lambda^{b'}(y')iW^{c'}_{\rho'}(z')\rangle \qquad (5.7)\\
+⑤ &\frac{\delta^2\bar{J}^b}{\delta\hat{W}^c_\rho(z)\delta\hat{\bar{\lambda}}^b(y)}\langle i\lambda^{b'}(y')\partial_\mu C^{*a}(x)f^{apq}\bar{\varepsilon}\partial_\mu\lambda^p(x)C^q(x)\rangle \\
+⑥ &\frac{\delta\bar{J}^{b'}(y')}{\delta\hat{\bar{\lambda}}^b(y)}\frac{\delta J^{c'}_{\rho'}(z')}{\delta\hat{W}^c_\rho(z)}\langle i\lambda^{b'}(y')iW^{c'}_{\rho'}(z') \\
&\qquad\times(\partial_\mu C^{*a}(x))f^{apq}\bar{\varepsilon}\gamma_\mu\lambda^p(x)C^q(x)\rangle.
\end{aligned}$$

Here integration over the points y', z' and x is understood, and the notation $\langle(\cdots)\rangle$ stands for $\int d\{W\}\, d\{C^*\}\, d\{C\}\, d\{\lambda\}\, d\{D\}(\cdots)\exp i\int d^4x\,\mathscr{L}$. Thus the expressions within brackets $\langle\cdots\rangle$ represent connected (but not disconnected nor single particle irreducible) Green functions while the derivatives acting on sources give single particle irreducible Green functions, e.g.,

$$\frac{\delta J^{c'}_{\rho'}(z')}{\delta\hat{W}^c_\rho(z)} = \frac{\partial^2\Gamma}{\delta\hat{W}^{c'}_{\rho'}(z')\delta\hat{W}^c_\rho(z)} = G_{W^{c'}_{\rho'}W^c_\rho}(z', z). \qquad (5.8)$$

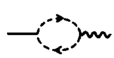
As in MPS, we have used the fact that, for example, $\delta j^a_D(z')/\delta W^c_\rho(z) = 0$ to eliminate terms involving the D field.

In its general form, the identity (5.7) is similar to that derived by MPS, but there are a few discrepancies. For example, in the sixth (last) term the derivative in $\langle\ \rangle$ acts on the C^* field while in MPS it acts on the λ field; and in the second term there exists a contribution from the term in $\sigma\cdot G$ bilinear in the W field, of which we can find no counterpart in MPS.

In the tree approximation only the 1st, 2nd and 4th terms contribute and it is easy to verify that the identity holds. Note that the 1st and 2nd terms consist essentially of the W and λ self energies, respectively, while the 4th term supplies the "missing" longitudinal part for the W self-energy as already noted by MPS.

At the one-loop level there are a number of contributing graphs. The resulting calculation is shown in table 3. (Note that the 3rd and 5th terms do not contribute,

TABLE 3
The supersymmetry Ward identity of eq. (5.7) for the spin $(\frac{1}{2}, 1)$ system in the Wess–Zumino gauge: $\mathrm{I}(p^2) = \int d^{2\omega}q(q^2(q-p)^2]^{-1}$

Diagrams	Coefficient of $p^2 \gamma_p \mathrm{I}(p^2)$	Coefficient of $\not p\, p_p \mathrm{I}(p^2)$	Correspondence to eq. 5.7
 	$\frac{(3\omega-1)}{(2\omega-1)}$	$\frac{(1-3\omega)}{(2\omega-1)}$	term 1
	$\frac{2(1-\omega)}{(2\omega-1)}$	$\frac{2(\omega-1)}{(2\omega-1)}$	term 1
	$\frac{(\omega-2)}{(2\omega-1)}$	$\frac{(2-\omega)}{(2\omega-1)}$	term 1
	$(\omega-1)$	$(1-\omega)$	term 2
	$(2-\omega)$	$(\omega-2)$	term 2
	$-\frac{3}{2}$	$\frac{3}{2}$	term 2
	$\frac{(5-8\omega)}{4(2\omega-1)}$	$\frac{3\omega-2}{2(2\omega-1)}$	term 4
	$\frac{1}{2}$	0	term 4
	$\frac{1}{4(1-2\omega)}$	$\frac{(1-\omega)}{2(2\omega-1)}$	term 6
TOTAL	0	0	

since they involve tadpole graphs). *The Ward identity is seen to be satisfied for all values of ω.* The inclusion of the ε-scalars is crucial for this result; from the table it is evident that if they are omitted, the identity is only satisfied by the pole terms in ε but not the finite parts.

Thus, contrary to MPS, we find no evidence that the supersymmetric Ward identity for the non-abelian multiplet in the Wess–Zumino gauge requires modification. The dimensional reduction technique does not violate the Ward identities, even when the supersymmetry is explicitly broken by the gauge choice and this result greatly increases one's confidence in the technique.

6. Supergraphs

Past experience has shown that a great deal of simplification can be achieved by using the supergraph technique [7] rather than working with the component fields. A single supergraph is equivalent to many component field graphs and the reason for the comparative lack of divergences in superfield theories can be easily understood in this formalism. As a simple example we first summarize massless superfield ϕ^3 theory

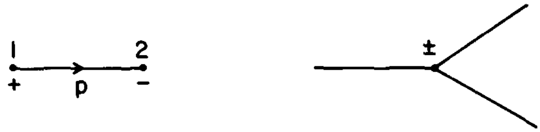


Fig. 6. Feynman rules for the $\phi_\pm^3$ theory.

[5] (i.e., the spin $[0, \frac{1}{2}]$ Wess–Zumino model). In the notation of Salam and Strathdee [8] the theory may be written in terms of chiral superfields $\phi_\pm$

$$\phi_\pm(x, \theta) = [\exp \mp\tfrac{1}{4}\bar{\theta}\not{\partial}\gamma_5\theta]\{A_\pm(x) + \bar{\theta}\psi_\pm(x) + \tfrac{1}{4}\bar{\theta}(1 \pm i\gamma_5)\theta F_\pm\}, \tag{6.1}$$

where θ is a four-component Majorana spinor. The action may be written

$$\int \mathscr{L}\, \mathrm{d}x = \int \mathrm{d}x\, \{\tfrac{1}{8}(\bar{D}D)^2(\phi_+\phi_-) - \tfrac{1}{6}g\bar{D}D(\phi_+^3 + \phi_-^3)\}, \tag{6.2}$$

where the derivatives were defined in sect. 4.

As derived in ref. [7], the corresponding superfield Feynman rules are $+-$ propagators,

$$G_{+-}(p, \theta_1, \theta_2) = -((2\pi)^4 ip^2)^{-1} \exp[\tfrac{1}{4}i\bar{\theta}_{12}\not{p}\gamma_5\theta_{12} + \tfrac{1}{2}\bar{\theta}_1\not{p}\theta_2], \tag{6.3}$$

and $\phi_\pm^3$ vertices,

$$-\tfrac{1}{6}g(2\pi)^4 i\bar{D}D. \tag{6.4}$$

These are shown diagramatically in fig. 6.

The only divergent one-loop diagram is the self-energy contribution to $+-$ propagator given in fig. 7. This propagator has the form

$$(\tfrac{4}{3}g)^2\left[\frac{1}{(2\pi)^4 i}\right]^2 \frac{1}{p^2} \exp[\tfrac{1}{4}i\bar{\theta}_{14}\not{p}\gamma_5\theta_{14} + \tfrac{1}{2}\bar{\theta}_1\not{p}\theta_4] \int \frac{\mathrm{d}^4q}{q^2(q-p)^2}. \tag{6.5}$$

In fact, this modified propagator has the same structure as that of eq. (6.3) and it is fairly straightforward to show that a counterterm of the form $(\bar{D}D)^2(\phi_+\phi_-)$ can be introduced into the lagrangian to cancel the divergent term. The structure of both the propagator and counterterm is such that supersymmetry is manifestly preserved, even for multiloop diagrams. We note that all of the supersymmetry of a typical supergraph calculation is contained in the θ-structure which in this theory neatly

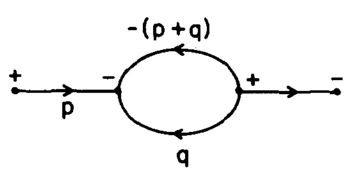
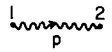
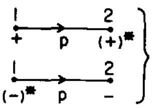
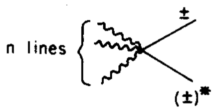


Fig. 7. The one-loop supergraph contribution to the $+-$ propagator.

TABLE 4

The Feynman rules for supersymmetric electrodynamics: the straight lines represent the matter fields ($\phi_\pm$) whilst the wavy lines represent the gauge field (ψ)

	$\dfrac{(\bar{\theta}_{12}\theta_{12})^2}{(2\pi)^4 8ip^2}$
	$\dfrac{-1}{(2\pi)^4 ip^2}\exp\{\frac{i}{4}\bar{\theta}_{12}\not{p}\gamma_5\theta_{12}+\frac{1}{2}\bar{\theta}_1\not{p}\theta_2\}$
	$\dfrac{(\pm 2g)^n (2\pi)^4\, i(\bar{D}D)^2}{8}$

factors out, leaving just a scalar integral. It is then tempting to apply conventional dimensional regularization to handle these divergent integrals. However, in the past this approach has caused some concern since the derivation of the Feynman rules relies very heavily on θ being a *four*-component Majorana spinor, together with the Fierz identity:

$$\theta_\alpha\bar{\theta}^\beta = -\tfrac{1}{4}\delta^\beta_\alpha\bar{\theta}\theta+\tfrac{1}{4}(\gamma_5)_\alpha{}^\beta\bar{\theta}\gamma_5\theta+\tfrac{1}{4}i(\gamma_\nu\gamma_5)_\alpha{}^\beta\bar{\theta}i\gamma_\nu\gamma_5\theta\,. \tag{6.6}$$

This identity is strictly four-dimensional with the γ-matrices being 4×4 and of course there are only four of them. Thus in supersymmetry theories conventional dimensional regularization cannot be implemented *via* a lagrangian, but is imposed on the integrals after manipulating the Feynman rules. This is normally a dangerous procedure which can introduce spurious anomalies in a theory.

For the ϕ^3 theory described above the fact that supergraphs preserve the supersymmetry is manifest without actually carrying out any integrals. Thus the results of ref. [4] can be understood in this formalism. For gauge theories the calculations are not so straightforward. For instance the action for supersymmetric electrodynamics [15] has been given in sect. 4. The Feynman rules which result from this section are given in table 4. It is instructive to consider the gauge self-energy shown in fig. 8,

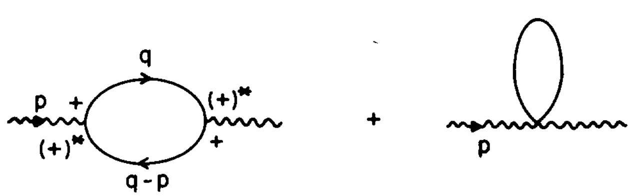


Fig. 8. One-loop supergraph contributions to the ψ self-energy.

evaluation of which leads to the expression

$$-\frac{1}{(2\pi)^8}\left(\frac{g}{32p^2}\right)^2(\bar{D}_1D_1)^2(\bar{D}_2D_2)^2\int\frac{\mathrm{d}q}{q^2(q-p)^2}$$
$$\times\psi(1)\psi(2)\exp\{-\tfrac{1}{4}i\bar{\theta}_{12}\not{p}\gamma_5\theta_{12}+\tfrac{1}{2}\bar{\theta}_1\not{p}\theta_2\}[1+\tfrac{1}{2}i\bar{\theta}_{14}\not{q}\gamma_5\theta_{14}]\,. \tag{6.7}$$

Use of the relation

$$\int \mathrm{d}q\, q^\alpha[q^2(q-p)^2]^{-1}=\tfrac{1}{2}p^\alpha\int\mathrm{d}q\,[q^2(q-p)^2]^{-1} \tag{6.8}$$

allows one to show that the counterterm required is supersymmetric without actually evaluating the integral. However, eq. (6.8) assumes that translation of the integration variable in a divergent integral is valid. This is *only* correct within the context of a regularization scheme such as dimensional regularization. Again the derivation of the Feynman rules makes essential use of the four-dimensional nature of the Majorana spinors which would appear to invalidate the use of conventional dimensional regularization. However, as Siegel observed [1], if one continues the number of space-time dimensions in an action such as in eq. (6.2) or (4.4) down from 4 to n, the dimension of the γ's and θ does not change. The supersymmetry is preserved in the lagrangian (up to total derivatives, of course.) In evaluating diagrams, the use of the Fierz identity ensures that the γ-matrix algebra is in four dimensions and that [assuming results such as (6.8)] the final integration is just a scalar integral. If one were perverse enough to do the integration before using the Fierz identity, one would have to distinguish between 4- and n-dimensional Kronecker deltas. This is essentially what happens in the component field approach where the mismatch between $\delta^{(4)}_{\alpha\beta}$ and $\delta^{(n)}_{\alpha\beta}$ can be viewed as arising from the ε-scalars. Thus the component field results reported in the previous sections can be viewed as justifying the apparently naive approach of replacing $\int \mathrm{d}^4q$ by $\int \mathrm{d}^nq$ in the final scalar integrals resulting from supergraph calculations. In particular it is important that this technique can be implemented *via* a lagrangian and thus Ward identities may be derived in the presence of the regulating parameter (n).

7. Conclusions

The dimensional reduction technique proposed by Siegel [1] leads to a regularization scheme which is as satisfactory as dimensional regularization for ordinary gauge theories, such as Yang–Mills theory and gravity. However, for globally supersymmetric theories and supergravity dimensional regularization does not maintain supersymmetry whereas the dimensional reduction technique does. Thus there exists a valid regularization scheme for supersymmetric theories. For supergravity this probably implies that the recent proofs of the finiteness of the S-matrix to two loops are correct, since these proofs assume the existence of a regularization technique which preserves supersymmetry [23].

The dimensional reduction technique is based on analytically continuing only the number of coordinates and momenta, but not the number of components of the fields. If one keeps working with four-component spinors and vectors, one still can maintain both gauge and supersymmetry Ward identities. This explains why superloop techniques can be used without violating such identities. For a clearer understanding of what happens in the component field approach, we have split the index μ of the vector field W_μ^a into our n-dimensional index i and an ε-dimensional index σ. This decomposition is a useful book-keeping device which can also be interpreted as leading to new scalars (called ε-scalars). These scalars contribute in diagrams and lead to different infinite and finite parts, such that not only the gauge but also the supersymmetry Ward identities are satisfied.

We have given a large set of examples: pure Yang–Mills theory; Yang–Mills theory coupled to scalars and fermions; supersymmetric QED and supersymmetric Yang–Mills theory. Computations were performed both at the one- and two-loop level. In all cases we found that the dimensional regularization violated supersymmetry Ward identities. We concluded by doing superloop calculations and could trace the role of dimensional reduction. This technique will evidently find important applications in both quantum gravity and quantum supergravity [24]. In this case the vierbein ε_μ^a leads to ε-vectors and ε-scalars, while the gravitino ψ_μ leads to ε-spinors.

Thus the fear, harboured for several years, that no valid regularization scheme might exist which maintains both supersymmetric and gauge Ward identities, fortunately turns out to be unfounded.

Two of the authors (DMC and DRTJ) wish to thank the Institute for Theoretical Physics at Stony Brook for hospitality and financial support while this work was performed. DMC also acknowledges a grant from the SRC, and DRTJ thanks CERN for financial assistance.

Note added in proof

Recently, Grisaru, Siegel and Roček presented a number of loop calculations using superfields [25]. They simplified supergraph Feynman rules and used their modified formalism to compute the one-loop supervertex correction, and the two-loop superpropagator correction in the $N = 4$ extended globally supersymmetric model. They point out that since all algebraic manipulations of the covariant derivatives are to be carried out first, in 4-dimensional space, whereas the momentum integration is to be subsequently carried out in D-dimensional space, supersymmetry is preserved. Their computations are the superspace counterpart of the ordinary space calculations we present. In sect. 6 we explain why superloop calculations automatically use regularization by dimensional reduction, but these questions have been explained and worked out in greater detail in ref. [25].

After this paper was submitted we received a revised version of ref. [10], in which calculations similar to those of sect. 5 are presented, (but in a general covariant gauge) with essentially identical conclusions.

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