Nuclear Physics B187 (1981) 149–183

SUPERGRAPHITY
(I). Background field formalism

M.T. GRISARU[1]

Department of Physics, Brandeis University, Waltham, Massachusetts 02154, USA

W. SIEGEL[2]

California Institute of Technology, Pasadena, California 91125, USA

Received 10 February 1981

We describe the background field formalism in superfield supergravity. This formalism makes calculations of one-loop quantities no more difficult than corresponding calculations in QCD. Unlike component methods, which violate manifest global supersymmetry with non-supersymmetric gauge conditions, our techniques, in conjunction with those described in previous papers, also allow the straightforward calculation of higher loops.

1. Introduction

It is well known that perturbation calculations in quantum gravity are exceedingly difficult because of the tensorial nature and complicated couplings of the graviton field. In component supergravity the calculations are just as difficult, and although the final results are expected to be simpler due to supersymmetry, no simplifications are apparent in the intermediate steps. Consequently, very few calculations have actually been performed in gravity and supergravity: four-point tree amplitudes [1], one-loop divergences [2–4], some one-loop propagator corrections [2], and a scalar-loop contribution to graviton-graviton scattering [5]. Many of these calculations required the use of computers for the algebraic manipulations. Even with the use of computers, the calculation of, e.g., the one-loop graviton-graviton scattering amplitude in pure gravity seems very difficult.

In general, supergraph calculations are easier both because superfields carry fewer Lorentz indices than their component fields, and because cancellations due to supersymmetry are automatic. In supergravity the basic superfield is a (axial) vector superfield, and calculating with it is simpler than with a second-rank tensor and a spinor-vector. Recently, methods were developed [6] which allow for easy evaluation

[1] Research supported in part by NSF grant no. PHY79-20801.

[2] Research supported in part by the US Department of Energy under contract no. DE-AC-03-79ER0068 and by the Fleischmann Foundation.

of loop amplitudes of scalar superfields [6, 7]. The same methods can be applied to vector superfields. In ref. [6] it was also shown that background-field methods further simplify the handling of the scalar gauge superfield (i.e., the Yang-Mills vector multiplet). The background-field method will lead to even more simplifications when applied to the vector gauge superfield of supergravity, since one can avoid (at least at one loop) non-covariant renormalizations, such as those which also appear in ordinary gravity in non-background gauges [8].

In this paper we will set up a procedure for doing loop calculations in superfield supergravity which leads to easy and efficient evaluation of on-shell amplitudes. This is achieved by using background field quantization and the methods of ref. [6] for supergraph evaluation. The on-shell condition for the background fields (which appear only through covariant derivatives and superfield strengths) leads to considerable additional simplifications. Among applications of our methods we mention a calculation of the one-loop four-particle S-matrix in supergravity, and an understanding of the supersymmetric nature of anomalies in supergravity. We are also hopeful that a calculation of the three-loop divergence in supergravity is possible.

We formulate our work in terms of the exponential form of the unconstrained supervector superfield [9], which has already been used for setting up the superfield Feynman rules [10], but not in a background gauge. (Different field parametrizations have been used by Namazie and Storey [11], and Taylor [12], but with drawbacks which will be described below.)

Our paper is organized as follows: In sect. 2 we describe the splitting between background and quantum supergravity superfields. The quantum fields are a real vector H^a and a chiral scalar χ, as in ordinary-gauge quantization [10], while the background fields appear only through the background-field covariant derivatives (and their field strengths), in analogy to the background fields of super-Yang-Mills [6]. In sect. 3 the supergravity action is expanded to second order in the quantum fields, which is sufficient for one-loop calculations. Sect. 4 discusses the general methods used for quantization in background field gauges. Although many of these methods are relevant to other gauge theories, they are necessary in superfield supergravity. These procedures are: (a) a general rule for ghost counting, a useful check on the quantization procedure, (b) "catalyst" ghosts, a technique for introducing extra fields to eliminate spurious non-localities in the action and $1/p^4$ propagators, (c) "hidden" ghosts [13], which appear when constrained gauge-fixing functions are used, (d) a rule to choose gauge parameters in accordance with the Faddeev-Popov rules of quantization, (e) compensating fields, which are necessary in superfield supergravity for Faddeev-Popov quantization and cosmological terms, and (f) what kinds of superfields are allowed in a general supergravity background. These methods are then applied specifically to superfield supergravity in sect. 5 to obtain the appropriate gauge fixing and ghosts. In order to simplify the procedure, we restrict ourselves to on-shell background fields, which are sufficient to calculate S-matrix elements. We obtain not only the usual "first-generation" ghosts, but also "second-generation" ghosts associated with the gauge invariance of the former ghosts. (The latter ghosts have no couplings to the quantum fields, and therefore contribute only at one loop.) Our conclusions are contained in sect. 6. For completeness, we have given brief reviews of the background-field formalism for super-Yang-Mills in appendix A and of classical superfield supergravity in appendix B. There is also an appendix C containing details of the calculation of the ghost action. (An introduction to parts of this work, as well as a more pedagogical introduction to superfields and superfield supergravity, may be found in the separate contributions of M. Roček and each of the authors to the Proceedings of the Nuffield Supergravity Workshop [14].)

2. Background-quantum splitting

In conventional field theory the background formalism has proven quite valuable, particularly in discussing divergences in gauge theories and certain formal questions connected with the renormalizability of such theories. However, its use as a tool in computing scattering amplitudes has been limited to phase shifts [15]. In supersymmetry it was first discussed by Honerkamp, Schlindwein, Krause, and Scheunert [16], but only for supersymmetric QED. In non-abelian (Yang-Mills) theories it was shown [6] that the method can be quite powerful for the computation of (loop) Green functions: the calculations are simplified by suitably choosing a (background-covariant) gauge-fixing term which cancels some of the interaction terms. We will find similar simplifications in supergravity. Furthermore, the possibility of maintaining some gauge invariance in the process of quantization (background-covariant gauge-fixing terms) gives an added measure of control in the calculations.

In non-supersymmetric Yang-Mills, the background-quantum splitting can be described by expressing the covariant derivatives (and thus the field strengths) in terms of background-field covariant derivatives and the quantum fields. This makes the background gauge invariance manifest. The prescription for the splitting is to take the covariant derivative ∇_a and simply replace the empty-space derivative ∂_a by the background-field covariant derivative $\mathcal{D}_a$, and the gauge field by the quantum gauge field. Since the covariant derivative is linear in the gauge field, this is equivalent to merely replacing the gauge field with the sum of the background and quantum fields: the replacement

$$\nabla_a = \partial_a - iA_a \to \nabla_a = \mathcal{D}_a - iA_a^{(\mathrm{Q})} = \left(\partial_a - iA_b^{(\mathrm{B})}\right) - iA_a^{(\mathrm{Q})}$$

is equivalent to the replacement $A_a \to A_a^{(\mathrm{Q})} + A_a^{(\mathrm{B})}$.

In superfield supergravity (as well as super-Yang-Mills: see appendix A), we again use the prescription to replace the flat-space covariant derivatives D_A by the background-field covariant derivatives $\mathcal{D}_A$. However, this no longer corresponds to a linear splitting of the gauge field, since the covariant derivative is now expressed non-linearly in terms of the unconstrained fields H^a and χ. Also, we desire the background-field covariant derivative to be in its vector representation, where its transformation law and hermiticity condition are simpler, while the unconstrained quantum fields should be in their chiral representation, where superfluous fields are eliminated (see appendices A and B).

We therefore seek a splitting which has the result that the chiral-representation covariant derivatives ∇_A, which are of the form ($\nabla_A = E_A{}^B D_B + \frac{1}{2}\phi_{Ab}{}^c M_c{}^b$: see appendix B):

unsplit ∇_A:

$$
\begin{aligned}
\overline{\nabla}_{\dot\alpha} &= \bar\psi\,(H,\chi)\left[\bar D_{\dot\alpha} + \tfrac{1}{2}\hat{\bar\omega}_{\dot\alpha a}{}^b(H,\chi)M_b{}^a\right],\\
\nabla_\alpha &= \mathrm{e}^{-H}\psi(H,\chi)\left[D_\alpha + \tfrac{1}{2}\hat\omega_{\alpha a}{}^b(H,\chi)M_b{}^a\right]\mathrm{e}^H,\\
\nabla_a &= -i\sigma_a^{\alpha\dot\beta}\left\{\nabla_\alpha, \overline{\nabla}_{\dot\beta}\right\},\\
H &= H^a i\partial_a, \qquad \bar D_{\dot\alpha}\chi = 0,
\end{aligned}
\tag{2.1}
$$

are transformed into

background-gauge ∇_A:

$$
\begin{aligned}
\overline{\nabla}_{\dot\alpha} &= \bar\psi\,(H,\chi;\mathcal{D})\left[\bar{\mathcal{D}}_{\dot\alpha} + \tfrac{1}{2}\bar\omega_{\dot\alpha a}{}^b(H,\chi;\mathcal{D})M_b{}^a\right],\\
\nabla_\alpha &= \mathrm{e}^{-H}\psi(H,\chi;\mathcal{D})\left[\mathcal{D}_\alpha + \tfrac{1}{2}\omega_{\alpha a}{}^b(H,\chi;\mathcal{D})M_b{}^a\right]\mathrm{e}^H,\\
\nabla_a &= -i\sigma_a^{\alpha\dot\beta}\left\{\nabla_\alpha, \overline{\nabla}_{\dot\beta}\right\},\\
H &= H^a i\mathcal{D}_a, \qquad \bar{\mathcal{D}}_{\dot\alpha}\chi = 0,
\end{aligned}
\tag{2.2}
$$

where H^a and χ are now the quantum fields, and the corresponding background fields now appear only implicitly through the background-field covariant derivatives $\mathcal{D}_A$. In actual calculations of the effective action, one never needs to expand $\mathcal{D}_A$ in terms of the unconstrained background fields, and only works directly with its two parts, the background field supervierbein $\mathcal{E}_A{}^B$ and superconnection $\varphi_{Ab}{}^c$ ($\mathcal{D}_A = \mathcal{E}_A + \frac{1}{2}\varphi_{Ab}{}^c M_c{}^b, \mathcal{E}_A = \mathcal{E}_A{}^B D_B$). This not only makes background invariance more manifest in the calculations (by avoiding the unconstrained background superfields, which transform non-linearly), but considerably simplifies the algebra (avoiding the explicit non-linear expansion of $\mathcal{D}_A$ in terms of the unconstrained fields) and limits the types of terms which can be expected to contribute to the effective action (since, e.g., $\mathcal{D}_A$ have higher dimensions than the unconstrained superfields).

The above transformation can be accomplished by the following non-linear splitting of the unconstrained supervector superfield:

$$\exp\left(H^A iD_A\right) = \exp\left(W_{(\mathrm{B})}{}^A iD_A\right)\exp\left(H_{(\mathrm{Q})}{}^A i\mathcal{E}_A\right)\exp\left(\overline{W}_{(\mathrm{B})}{}^A iD_A\right),$$

or equivalently

$$\exp\left(W^A iD_A\right) = \exp\left(W_{(\mathrm{B})}{}^A iD_A\right)\exp\left(W_{(\mathrm{Q})}{}^A i\mathcal{E}_A\right), \tag{2.3}$$

where $e^H = e^W e^{\overline{W}}$ relates the chiral representation H and vector representation W (see appendix B). We also split ϕ $(= 1 + \chi)$ as $\phi = e^{-\overline{W}_{(\mathrm{B})}}\phi_{(\mathrm{B})}\phi_{(\mathrm{Q})}$ $(\overline{D}_{\dot\alpha}\phi = \overline{\mathcal{D}}_{\dot\alpha}\phi_{(\mathrm{B})} = \overline{\mathcal{D}}_{\dot\alpha}\phi_{(\mathrm{Q})} = 0)$. The splitting is as in super-Yang-Mills (see appendix A), except that the background fields now also appear in the exponential with $H_{(\mathrm{Q})}$ as $H_{(\mathrm{Q})}{}^A\mathcal{E}_A$, in order to make $H_{(\mathrm{Q})}{}^A$ a tangent-space supervector under background transformations. As usual, we will choose the gauge $H_{(\mathrm{Q})}{}^\alpha = 0$. As in super-Yang-Mills, we then go to a new representation by making a non-unitary transformation with $\overline{W}_{(\mathrm{B})}$ on (2.1) (or equivalently with $-\overline{W}_{(\mathrm{Q})}$ on the vector representation ∇_A), but also making a non-unitary Lorentz transformation to covariantize $H_{(\mathrm{Q})}{}^A i\mathcal{E}_A$ into $H_{(\mathrm{Q})}{}^A i\mathcal{D}_A$:

$$\begin{aligned}\nabla_A{}' = {}& \exp\left(-L_\alpha{}^\beta M_\beta{}^\alpha\right)\exp\left(\overline{W}_{(\mathrm{B})}{}^A iD_A\right)\nabla_A\exp\left(-\overline{W}_{(\mathrm{B})}{}^A iD_A\right)\\ &\times\exp\left(L_\alpha{}^\beta M_\beta{}^\alpha\right).\end{aligned} \tag{2.4}$$

$L_\alpha{}^\beta$ is determined by use of the following identities:

$$\begin{aligned}&\text{for any } A = A^A\mathcal{D}_A + \tfrac{1}{2}A_a{}^b M_b{}^a = A^A\mathcal{E}_A + \tfrac{1}{2}A'_a{}^b M_b{}^a,\\ &B_a{}^b, \text{ and } C_a{}^b,\\ &\text{and for some } B'_a{}^b,\ C'_a{}^b, \text{ and } C''_a{}^b,\\ &[A, \tfrac{1}{2}B_a{}^b M_b{}^a] = \tfrac{1}{2}B'_a{}^b M_b{}^a, \text{ and therefore}\\ &\exp(A + \tfrac{1}{2}C_a{}^b M_b{}^a) = \exp(A)\exp(\tfrac{1}{2}C'_a{}^b M_b{}^a)\\ &\qquad\qquad = \exp(\tfrac{1}{2}C''_a{}^b M_b{}^a)\exp(A).\end{aligned} \tag{2.5}$$

In order to obtain (2.2) from (2.1) [using the splitting (2.3)], we apply the transformation (2.4), with $L_\alpha{}^\beta$ given by $C'_\alpha{}^\beta$ in (2.5) when we choose $A = H_{(\mathrm{Q})}{}^A i\mathcal{E}_A$ and $C_a{}^b = H_{(\mathrm{Q})}{}^C i\varphi_{Ca}{}^b$ $(\frac{1}{2}X_a{}^b M_b{}^a \equiv X_\alpha{}^\beta M_\beta{}^\alpha + X_{\dot\alpha}{}^{\dot\beta}\overline{M}_{\dot\beta}{}^{\dot\alpha})$. (From now on, H refers only to $H_{(\mathrm{Q})}{}^A i\mathcal{D}_A$.)

The explicit form of ψ and ω in (2.2) can most easily be determined by re-solving the appropriate constraints on the full covariant derivatives ∇_A [rather than explicitly performing (2.4) and using the ψ and $\hat{\omega}$ of (2.1) given by (B.18b) and (B.20)]. ψ is found by solving parts of $T_{ab}{}^c = T_{\alpha\dot\beta}{}^{\dot\gamma} = 0$ on the full torsion [17] ($E = \det E_A{}^B$, $\mathcal{E} = \det \mathcal{E}_A{}^B$):

$$0 = \left(T_{\dot\alpha b}{}^b - T_{\dot\alpha\beta}{}^\beta\right) - \bar\psi\left(T_{\dot\alpha b}{}^b - T_{\dot\alpha\beta}{}^\beta\right)_{(B)} = \overline{\nabla}_{\dot\alpha}\ln\left(\bar\psi^2 E\mathcal{E}^{-1}\right). \tag{2.6a}$$

Therefore the argument of the logarithm is background-chiral and can be expressed as

$$\bar\psi^2 E\mathcal{E}^{-1} = \phi^{-3}, \qquad \overline{\mathcal{D}}_{\dot\alpha}\phi = 0, \qquad \phi = 1 + \chi, \tag{2.6b}$$

which, after some algebra, leads to [cf. (B.18b)]

$$\bar\psi = \phi^{-1}\left(e^{-H}\bar\phi\right)^{1/2}\left(1\cdot e^{-\bar H}\right)^{1/6}\hat{E}^{-1/6}\mathcal{E}^{1/6}\left(e^{-H}\mathcal{E}\right)^{-1/6}. \tag{2.6c}$$

We have used the equations ($\hat{E} = \det \hat{E}_A{}^B$)

$$\hat\nabla_A \equiv \left(\overline{\mathcal{D}}_{\dot\alpha}, e^{-H}\mathcal{D}_\alpha e^H, -i\sigma_a^{\beta\dot\gamma}\{\hat\nabla_\beta, \hat\nabla_{\dot\gamma}\}\right) \equiv \hat{E}_A{}^B\mathcal{D}_B + \tfrac{1}{2}\hat\phi_{Ab}{}^c M_c{}^b$$

$$\to E = \left(\bar\psi e^{-H}\psi\right)^2\hat{E}\mathcal{E},$$

$$\overline{\mathcal{E}} = \mathcal{E}, \qquad \bar{E}^{-1} = E^{-1}e^{-\bar H}, \qquad \bar{\hat{E}}^{-1} = \hat{E}^{-1}e^{-\bar H}. \tag{2.6d}$$

Therefore the background-quantum splitting for the action is

$$S = -\frac{6}{\kappa^2}\int d^4x\, d^4\theta\, E^{-1} = -\frac{6}{\kappa^2}\int d^4x\, d^4\theta\, \mathcal{E}^{-1}\hat{E}^{-1/3}\left(1\cdot e^{-\bar H}\right)^{1/3}\phi e^{-H}\bar\phi, \tag{2.7}$$

which depends on the background fields through $\mathcal{E}$ and $\mathcal{D}_A$, and on the quantum fields explicitly and through $\hat{E}$. The explicit solution for $\omega_{\alpha a}{}^b$ will not be needed here. It can be found straightforwardly by the same method as in ref. [17].

The use of $H = H^A i\mathcal{D}_A$ instead of $H^A i\mathcal{E}_A$ not only makes background covariance more manifest but also simplifies algebraic manipulations by giving torsions and curvatures directly from $[\mathcal{D}_A, \mathcal{D}_B\}$ instead of indirectly from $[\mathcal{E}_A, \mathcal{E}_B\}$. In order to facilitate such manipulations it is convenient to make the following definitions: for any $A = A^A\mathcal{D}_A + \frac{1}{2}A_a{}^b M_b{}^a$,

$$\langle A\rangle \equiv A^A\mathcal{D}_A, \qquad \langle e^A\rangle \equiv e^{\langle A\rangle}. \tag{2.8}$$

We then have identities such as

$$\langle[\langle A\rangle,\langle B\rangle]\rangle=\langle[A,B]\rangle, \tag{2.9a}$$

and, applying (2.5), we have the equivalence

$$\exp\left(A^A\mathcal{E}_A\right)\exp\left(B^A\mathcal{E}_A\right)=\exp\left(C^A\mathcal{E}_A\right)$$
$$\leftrightarrow\langle\exp\left(A^A\mathcal{D}_A\right)\exp\left(B^A\mathcal{D}_A\right)\rangle=\exp\left(C^A\mathcal{D}_A\right). \tag{2.9b}$$

The background transformations are given completely by the vector-representation parameter K (since we never see an explicit $W_{(\mathrm{B})}$), while the quantum transformations are given completely by the chiral-representation parameter Λ. These transformations are, respectively [cf. (A.9), (B.15), and (B.21)]

$$(1)\quad \mathcal{D}_A{}'=e^{iK}\mathcal{D}_Ae^{-iK},\qquad H'=e^{iK}He^{-iK},\qquad \phi'=e^{iK}\phi,$$
$$K=\bar{K}=K^Ai\mathcal{D}_A+\tfrac{1}{2}K_a{}^biM_b{}^a; \tag{2.10a}$$

$$(2)\quad \mathcal{D}_A{}'=\mathcal{D}_A,\qquad e^{H'}=\langle e^{i\bar{\Lambda}}e^{H}e^{-i\Lambda}\rangle,$$
$$\phi'=\exp\left[i\Lambda-\tfrac{1}{3}\left(\mathcal{D}_a\Lambda^a-\mathcal{D}_\alpha\Lambda^\alpha-iG_a\Lambda^a\right)\right]\phi,$$
$$\Lambda=\Lambda^Ai\mathcal{D}_A\neq\bar{\Lambda}. \tag{2.10b}$$

(R, G_a, and $W_{\alpha\beta\gamma}$ refer to the background field strengths, found from $\mathcal{D}_A$ as by (B.23a).) Eq. (2.9b) can be used to write an equivalent form of the quantum transformation law for H:

$$\exp\left(H'^Ai\mathcal{E}_A\right)=\exp\left(-\bar{\Lambda}^A\mathcal{E}_A\right)\exp\left(H^Ai\mathcal{E}_A\right)\exp\left(\Lambda^A\mathcal{E}_A\right). \tag{2.11}$$

To maintain the chirality of ϕ and [using (2.5)] to preserve the form of ∇_α, Λ must satisfy the chirality condition [cf. (B.4)]

$$\left[\bar{\mathcal{D}}_{\dot\alpha},\Lambda\right]\phi=0\rightarrow\begin{cases}(\mathrm{i})\quad 0=\left[\bar{\mathcal{D}}_{\dot\alpha},\Lambda^b\mathcal{D}_b+\Lambda^\beta\mathcal{D}_\beta\right]^c=\bar{\mathcal{D}}_{\dot\alpha}\Lambda^c-\tfrac{1}{2}i\sigma^c_{\beta\dot\alpha}\Lambda^\beta,\\(\mathrm{ii})\quad 0=\left[\bar{\mathcal{D}}_{\dot\alpha},\Lambda^b\mathcal{D}_b+\Lambda^\beta\mathcal{D}_\beta\right]^\gamma=\bar{\mathcal{D}}_{\dot\alpha}\Lambda^\gamma+\tfrac{1}{2}i\sigma_b{}^\gamma{}_{\dot\alpha}R\Lambda^b,\end{cases} \tag{2.12a}$$

which implies, for some spinor superfield parameter L_α [cf. (B.5)],

$$\Lambda_{\alpha\dot\beta}=-2i\bar{\mathcal{D}}_{\dot\beta}L_\alpha,\qquad \Lambda_\alpha=\bar{\mathcal{D}}^2L_\alpha. \tag{2.12b}$$

(We have used $T_{\dot{\alpha}b}{}^{c} = T_{\dot{\alpha}\beta}{}^{\gamma} = 0$, $T_{\dot{\alpha}\beta}{}^{c} = \frac{1}{2} i\sigma^{c}_{\beta\dot{\alpha}}$, and $T_{\dot{\alpha}b}{}^{\gamma} = \frac{1}{2} i\sigma_{b}{}^{\gamma}{}_{\dot{\alpha}}R$. Also, the solution can be found from (i) alone: (ii) is redundant.) Due to our working in the gauge $H^{\alpha} = 0$, we also have

$$\Lambda^{\dot{\alpha}} = \mathrm{e}^{-H}\mathcal{D}^2\bar{L}^{\dot{\alpha}} \tag{2.12c}$$

[in analogy to (B.6)].

3. Expansion of supergravity action

In this section we prepare the way for loop calculations by expanding the action in powers of the quantum fields. We shall restrict ourselves here to expansion up to second order in these fields, which is sufficient for one-loop calculations. Higher-order terms are not difficult to obtain, but we shall postpone exhibiting them to a future publication where higher-loop calculations will be presented.

We must expand the exponentials and the determinant $\hat{E}^{-1/3}$ in (2.7) in powers of H. We first define Δ_A by

$$\hat{E}_A = \hat{E}_A{}^B\mathcal{D}_B = \left(\delta_A^B + \Delta_A{}^B\right)\mathcal{D}_B = \mathcal{D}_A + \Delta_A\,. \tag{3.1}$$

The expansion of $\hat{E}^{-1/3}$ is then, to quadratic order in Δ (and H):

$$\begin{aligned}\hat{E}^{-1/3} &= \left[\det(1+\Delta)\right]^{-1/3} = \exp\left(-\tfrac{1}{3}\mathrm{tr}\ln(1+\Delta)\right)\\ &= 1 - \tfrac{1}{3}\mathrm{tr}\Delta + \tfrac{1}{18}(\mathrm{tr}\Delta)^2 + \tfrac{1}{6}\mathrm{tr}(\Delta^2)\\ &= 1 - \tfrac{1}{3}(-1)^a\Delta_A{}^A + \tfrac{1}{18}\left[(-1)^a\Delta_A{}^A\right]^2 + \tfrac{1}{6}(-1)^a\Delta_A{}^B\Delta_B{}^A\,.\end{aligned} \tag{3.2}$$

We next expand the exponentials in $\hat{\nabla}_A$ in (2.6d) to second order in H:

$$\begin{aligned}\Delta_A = \Big\langle\Big(0_{\dot{\alpha}}, [\mathcal{D}_\alpha, H] + \tfrac{1}{2}[[\mathcal{D}_\alpha, H], H],\\ -i\sigma_a^{\beta\dot{\gamma}}\Big(\big\{\bar{\mathcal{D}}_{\dot{\gamma}}, [\mathcal{D}_\beta, H]\big\} + \tfrac{1}{2}\big\{\bar{\mathcal{D}}_{\dot{\gamma}}, [[\mathcal{D}_\beta, H], H]\big\}\Big)\Big)\Big\rangle\,.\end{aligned} \tag{3.3}$$

$\Delta_A{}^B$ is found by comparing the coefficients of $\mathcal{D}_A$ on both sides, using (B.23a) to express $[\mathcal{D}_A, \mathcal{D}_B\}$ in terms of the background R, G_a, and $W_{\alpha\beta\gamma}$. All explicit M_{ab} terms (i.e., excluding the implicit superconnection terms in $\mathcal{D}_A$) can immediately be dropped: they do not contribute to $\langle \mathrm{e}^{-H}\mathcal{D}_\alpha \mathrm{e}^{H}\rangle$ due to the identity (for $A = A^A\mathcal{D}_A + \frac{1}{2}A_a{}^bM_b{}^a$, $B_A = B_A{}^B\mathcal{D}_B + \frac{1}{2}B_{Ab}{}^cM_c{}^b$)

$$\langle[\langle A\rangle, \langle B_A\rangle]\rangle = \langle[\langle A\rangle, B_A]\rangle\,, \qquad (\neq\langle[A, B_A]\rangle)\,, \tag{3.4}$$

since $\langle H\rangle = H$. Also, we can ignore M_{ab} terms in $e^{-H}\mathcal{D}_\alpha e^H$ occurring in $-i\sigma_a^{\alpha\dot\beta}\langle\{\bar{\mathcal{D}}_{\dot\beta}, e^{-H}\mathcal{D}_\alpha e^H\}\rangle$, since they will contribute terms only to $\Delta_a{}^{\dot\beta}$, which does not contribute to $\hat{E}$ (to any order in H) due to $\Delta_{\dot\alpha}{}^B = 0$ [to all orders: see (3.2)]. For example, to find $\Delta_\alpha{}^B$ to lowest order in H:

$$[\mathcal{D}_\alpha, H] = [\mathcal{D}_\alpha, H^a i\mathcal{D}_a] = i[\mathcal{D}_\alpha, H^a]\mathcal{D}_a + iH^a[\mathcal{D}_a, \mathcal{D}_\alpha]$$

$$= i(\mathcal{D}_\alpha H^a)\mathcal{D}_a - \tfrac{1}{2}H^a\sigma_{a\alpha}{}^{\dot\beta}\left(\bar{R}\bar{\mathcal{D}}_{\dot\beta} - G^\gamma{}_{\dot\beta}\mathcal{D}_\gamma\right) + \text{curvature terms}$$

$$\to \Delta_\alpha{}^\beta = -\tfrac{1}{2}H_{\alpha\dot\gamma}G^{\beta\dot\gamma}, \qquad \Delta_\alpha{}^b = i\mathcal{D}_\alpha H^b. \tag{3.5}$$

(Again, we can ignore $\Delta_\alpha{}^{\dot\beta}$.) We thus find, to appropriate order in $H\,(H\cdot\mathcal{D} \equiv H^a\mathcal{D}_a)$:

$$\Delta_\alpha{}^\beta = -\tfrac{1}{2}(1 - \tfrac{1}{2}iH\cdot\mathcal{D})H_{\alpha\dot\gamma}G^{\beta\dot\gamma} - \tfrac{1}{2}\left(\mathcal{D}_\alpha H^{\gamma\dot\delta}\right)H_{\epsilon\dot\zeta}$$

$$\times\left[\tfrac{1}{4}\delta_\gamma^\epsilon\bar{\mathcal{D}}_{(\dot\delta}G^{\beta\dot\zeta)} - \delta_{\dot\delta}^{\dot\zeta}\left(-\tfrac{1}{2}W_\gamma{}^{\epsilon\beta} + \tfrac{1}{4}\delta_{(\gamma}^\beta\mathcal{D}^{\epsilon)}R\right)\right]$$

$$+ \tfrac{1}{8}H_{\alpha\dot\delta}G^{\gamma\dot\delta}H_{\gamma\dot\epsilon}G^{\beta\dot\epsilon} - \tfrac{1}{8}\delta_\alpha^\beta R\bar{R}H^2,$$

$$\Delta_\alpha{}^b = i\mathcal{D}_\alpha H^b,$$

$$\Delta_a{}^\beta = -i\sigma_a^{\gamma\dot\delta}\left(-\tfrac{1}{2}\bar{\mathcal{D}}_{\dot\delta}H_{\gamma\dot\epsilon}G^{\beta\dot\epsilon} + \tfrac{1}{2}R\mathcal{D}_\gamma H^\beta{}_{\dot\delta}\right),$$

$$\Delta_a{}^b = -i\sigma_a^{\gamma\dot\delta}\left(i\bar{\mathcal{D}}_{\dot\delta}\mathcal{D}_\gamma H^b + \tfrac{1}{2}i\sigma^b_{\epsilon\dot\delta}\Delta_\gamma{}^\epsilon\right). \tag{3.6}$$

We have used the fact that, for the part of the action quadratic in H, we need only the linear parts of $\Delta_\alpha{}^b$ and $\Delta_a{}^\beta$ [see (3.2)], and can also drop any total derivatives of quadratic terms. After plugging (3.6) into (3.2), we find (again dropping unimportant terms):

$$\hat{E}^{-1/3} = 1 - \tfrac{1}{3}\Big\{\bar{\mathcal{D}}_{\dot\beta}\mathcal{D}_\alpha H^{\alpha\dot\beta} - (1 - \tfrac{1}{2}iH\cdot\mathcal{D})G\cdot H - \tfrac{1}{8}\left(\mathcal{D}_\alpha H^{\gamma\dot\delta}\right)H_{\epsilon\dot\zeta}$$

$$\times\left[\delta_\gamma^\epsilon\bar{\mathcal{D}}_{(\dot\delta}G^{\alpha\dot\zeta)} - \delta_{\dot\delta}^{\dot\zeta}\left(-2W_\gamma{}^{\epsilon\alpha} + \delta_{(\gamma}^\alpha\mathcal{D}^{\epsilon)}R\right)\right]$$

$$+ \tfrac{1}{4}\left[2(G\cdot H)^2 - G^2H^2\right] - \tfrac{1}{4}R\bar{R}H^2\Big\} + \tfrac{1}{18}\left(\bar{\mathcal{D}}_{\dot\beta}\mathcal{D}_\alpha H^{\alpha\dot\beta} - G\cdot H\right)^2$$

$$+ \tfrac{1}{6}\Big\{\left(\bar{\mathcal{D}}_{\dot\beta}\mathcal{D}_\alpha H^{\gamma\dot\delta} - \tfrac{1}{2}\delta_{\dot\beta}^{\dot\delta}H_{\alpha\dot\epsilon}G^{\gamma\dot\epsilon}\right)\left(\bar{\mathcal{D}}_{\dot\delta}\mathcal{D}_\gamma H^{\alpha\dot\beta} - \tfrac{1}{2}\delta_{\dot\delta}^{\dot\beta}H_{\gamma\dot\zeta}G^{\alpha\dot\zeta}\right)$$

$$- \left(\bar{\mathcal{D}}_{\dot\delta}H_{\gamma\dot\epsilon}G^{\beta\dot\epsilon} - R\mathcal{D}_\gamma H^\beta{}_{\dot\delta}\right)\mathcal{D}_\beta H^{\gamma\dot\delta} - \tfrac{1}{2}\left[2(G\cdot H)^2 - G^2H^2\right]\Big\}. \tag{3.7a}$$

We also have the relevant terms of (expanding $\phi = 1 + \chi$):

$$(1\cdot \mathrm{e}^{-\bar{H}})^{1/3} = 1 - \tfrac{1}{3} i \mathcal{D}\cdot H + \tfrac{1}{9}(\mathcal{D}\cdot H)^2, \tag{3.7b}$$

$$\phi \mathrm{e}^{-H}\bar{\phi} = 1 + (\chi + \bar{\chi}) + \chi\bar{\chi} - iH\cdot\mathcal{D}\bar{\chi}. \tag{3.7c}$$

Finally, we obtain the quadratic part of the lagrangian by multiplying together (3.7a–c) to obtain [see (2.7)]:

$$\begin{aligned}
\mathcal{E}E^{-1} = {} & (\chi + \bar{\chi} + \tfrac{1}{3}G\cdot H) + \chi\bar{\chi} - \tfrac{1}{3}i(\chi - \bar{\chi})\mathcal{D}\cdot H + \tfrac{1}{3}(\chi + \bar{\chi})G\cdot H + \tfrac{1}{12}R\bar{R}H^2 \\
& + \tfrac{1}{18}(G\cdot H)^2 + \tfrac{1}{12}(\mathcal{D}\cdot H)^2 - \tfrac{1}{36}\Big(\big[\bar{\mathcal{D}}_{\dot\beta}, \mathcal{D}_\alpha\big] H^{\alpha\dot\beta}\Big)^2 \\
& - \tfrac{1}{18}(G\cdot H)\big[\bar{\mathcal{D}}_{\dot\beta}, \mathcal{D}_\alpha\big] H^{\alpha\dot\beta} + \tfrac{1}{6}R(\mathcal{D}^\alpha H^{\beta\dot\gamma})(\mathcal{D}_\beta H_{\alpha\dot\gamma}) \\
& + \tfrac{1}{24}(\mathcal{D}_\alpha H^{\gamma\dot\delta})H_{\epsilon\dot\zeta}\Big[\delta_\gamma^{\epsilon}\bar{\mathcal{D}}_{(\dot\delta}G^{\alpha\dot\zeta)} - \delta_{\dot\delta}^{\dot\zeta}\big(-2W_\gamma{}^{\epsilon\alpha} + \delta_{(\gamma}^{\alpha}\mathcal{D}^{\epsilon)}R\big)\Big] \\
& - \tfrac{1}{6}H^{\alpha\dot\beta}\big(\mathcal{D}_\alpha\bar{\mathcal{D}}_{\dot\beta}\bar{\mathcal{D}}_{\dot\delta}\mathcal{D}_\gamma + \mathcal{D}_\gamma\bar{\mathcal{D}}_{\dot\delta}\bar{\mathcal{D}}_{\dot\beta}\mathcal{D}_\alpha\big)H^{\gamma\dot\delta}.
\end{aligned} \tag{3.8}$$

By using the identity (with $\square \equiv \mathcal{D}^a\mathcal{D}_a$)

$$\begin{aligned}
\mathcal{D}_\alpha\bar{\mathcal{D}}_{\dot\beta}\bar{\mathcal{D}}_{\dot\delta}\mathcal{D}_\gamma + \mathcal{D}_\gamma\bar{\mathcal{D}}_{\dot\delta}\bar{\mathcal{D}}_{\dot\beta}\mathcal{D}_\alpha = {} & \tfrac{1}{2}R\bar{R}M_{\alpha\gamma}\bar{M}_{\dot\beta\dot\delta} - \tfrac{1}{2}(\mathcal{D}_{(\alpha}R)\mathcal{D}_{\gamma)}\bar{M}_{\dot\beta\dot\delta} \\
& + \tfrac{1}{4}C_{\alpha\gamma}C_{\dot\beta\dot\delta}\Big\{\{\mathcal{D}^2, \bar{\mathcal{D}}^2\} - \square \\
& \qquad - \Big[-\bar{R}\bar{\mathcal{D}}^{\dot\epsilon} + G^{\zeta\dot\epsilon}\mathcal{D}_\zeta + (\mathcal{D}^\zeta G^{\eta\dot\epsilon})M_{\zeta\eta} \\
& \qquad\quad + \bar{W}^{\dot\epsilon}{}_{\dot\zeta}{}^{\dot\eta}\bar{M}_{\dot\eta}{}^{\dot\zeta}, \bar{\mathcal{D}}_{\dot\epsilon}\Big]\Big\},
\end{aligned} \tag{3.9}$$

we can rewrite (3.8) as

$$\begin{aligned}
\mathcal{E}E^{-1} = {} & \{\chi + \bar{\chi} + \tfrac{1}{3}G\cdot H\} + \Big\{\chi\bar{\chi} - \tfrac{1}{3}i(\chi - \bar{\chi})\mathcal{D}\cdot H + \tfrac{1}{12}H\cdot\square H \\
& + \tfrac{1}{12}(\mathcal{D}\cdot H)^2 + \tfrac{1}{36}\Big(\big[\bar{\mathcal{D}}_{\dot\beta}, \mathcal{D}_\alpha\big]H^{\alpha\dot\beta}\Big)^2 \\
& - \tfrac{1}{6}\Big[\big(\bar{\mathcal{D}}^2 + \tfrac{3}{2}R\big)H\Big]\cdot\Big[\big(\mathcal{D}^2 + \tfrac{3}{2}\bar{R}\big)H\Big]\Big\} + \Big\{\tfrac{1}{3}(\chi + \bar{\chi})G\cdot H + \tfrac{1}{18}(G\cdot H)^2 \\
& + \tfrac{1}{3}R\bar{R}H^2 + \tfrac{1}{8}(\mathcal{D}^2R + \bar{D}^2\bar{R})H^2 - \tfrac{1}{12}H\cdot G^{\alpha\dot\beta}\big[\bar{\mathcal{D}}_{\dot\beta}, \mathcal{D}_\alpha\big]H \\
& - \tfrac{1}{18}(G\cdot H)\big[\bar{\mathcal{D}}_{\dot\beta}, \mathcal{D}_\alpha\big]H^{\alpha\dot\beta} + \tfrac{1}{24}H^{\alpha\dot\beta}\Big[\big(\mathcal{D}_{(\alpha}G^{\dot\delta}_{\gamma)}\big)\bar{\mathcal{D}}_{\dot\delta}H^{\gamma}{}_{\dot\beta} \\
& - \big(\bar{\mathcal{D}}_{(\dot\beta}G^{\delta}{}_{\dot\gamma)}\big)\mathcal{D}_\delta H_\alpha{}^{\dot\gamma}\Big] + \tfrac{1}{12}H^{\alpha\dot\beta}\big(W_\alpha{}^{\gamma\delta}\mathcal{D}_\gamma H_{\delta\dot\beta} + \bar{W}_{\dot\beta}{}^{\dot\gamma\dot\delta}\bar{\mathcal{D}}_{\dot\gamma}H_{\alpha\dot\delta}\big)\Big\},
\end{aligned} \tag{3.10}$$

where we have also used (B.23b). The expression in the first set of braces is linear in the quantum fields and cancels with the source terms in the action, as usual in the background-field formalism. (Note that

$$\int d^4x\, d^4\theta\, \mathcal{E}^{-1}\chi = \int d^4x\, d^2\theta\, e^{-\bar{W}_{(B)}}(\phi_{(B)})^3(\bar{\mathcal{D}}^2 + R)\chi = \int d^4x\, d^2\theta\, e^{-\bar{W}_{(B)}}(\phi_{(B)})^3 R\chi .$$

Therefore, variation with respect to χ and H^a gives $R = J$ and $G_a = J_a$, respectively.) The expression in the second set of braces is the direct covariantization of the flat-background expression [10] (where we have covariantized the gauge-fixing function $\bar{\mathcal{D}}^{\dot\beta} H_{\alpha\dot\beta}$, using $\bar{\mathcal{D}}_{\dot\beta}(\bar{\mathcal{D}}^{\dot\gamma} H_{\alpha\dot\gamma}) = -\frac{1}{2}(\bar{\mathcal{D}}^2 + \frac{3}{2}R)H_{\alpha\dot\beta}$). The actual quantity which enters in the action is obtained by multiplying the right hand side of (3.10) by $-(6/\kappa^2)\mathcal{E}^{-1}$.

4. General methods for quantization

In this section we describe the quantization methods used for the classical action previously discussed. In order to avoid possible difficulties with infrared divergences [18], we will restrict ourselves to gauges with propagators which go as $1/p^2$. Our only other restriction is that we put the background fields on shell to simplify calculations. While the latter condition may seem severe, we note that it still allows us to compute the S-matrix, which is the only quantity that is free of divergences through at least two loops (except in special gauges where the Green functions are finite as well [4]; in particular, one might expect the supersymmetric Fermi-Feynman gauge to be such a gauge, since off-shell convergence properties are improved in this gauge for supersymmetric Yang-Mills theories [6]). In any case, our methods for quantization are sufficiently general so that they can be modified for off-shell quantization without much difficulty.

Besides using only propagators with $1/p^2$ behavior (in contrast to refs. [10–12]), we have avoided many problems occurring in previous globally supersymmetric treatments of quantizing supergravity: (1) Our gauge-fixing and ghost terms in the lagrangian are local. (The non-local terms in ref. [11], although canceled in the *linearized* action in certain gauges, will remain in the interaction terms. In ref. [12] the non-local and non-analytic operator $\Box^{1/2}$ appears in a ghost kinetic term.) (2) We have included all "hidden ghosts" [13] which occur due to use of constrained gauge-fixing functions, and are essential in background-field formalisms (but were overlooked in the background-field treatment of ref. [11]). (3) We have chosen our gauge parameters in a way such that naive Faddeev-Popov quantization gives results consistent with quantization in ghost-free (axial) gauges. (This is not true for refs. [11,12].) (4) We have included the compensating chiral scalar superfield ϕ, which is necessary for proper gauge fixing and for handling cosmological terms (although we do not consider such terms here). ϕ was not included in the treatments of refs.

[11,12]. The necessity of including it among the quantum fields is basically due to the fact that ϕ can be completely gauged away only by a differential transformation, which introduces ghosts. (5) We have used only ghost fields which can be defined in an arbitrary (off-shell) background. (The background-field treatment of ref. [11] uses chiral vectors and chiral dotted spinors, which do not exist in a general background [19].) We will discuss each of these points below in greater detail.

4.1. GHOST COUNTING

Before we go into detail about the ghosts and gauge-fixing, we will apply a simple counting argument [13] to determine what the ghost structure must be. Our original lagrangian is of the form (ignoring interactions) $L_0 = H\Box\tilde{\Pi}H + \bar{\chi}\chi$, where $\tilde{\Pi}$ is a sum of projection operators. (There are also $H\chi$ crossterms.) Gauge fixing (which cancels the crossterms), for the Fermi-Feynman gauge, changes this to $L_1 = H\Box H + \bar{\chi}\chi$. The (linearized) gauge invariance $\delta H_{\alpha\dot{\beta}} \sim \bar{D}_{\dot{\beta}}L_\alpha - D_\alpha\bar{L}_{\dot{\beta}}, \delta\chi \sim \bar{D}^2D^\alpha L_\alpha$ indicates that gauge modes have been introduced into the lagrangian. They can be canceled by a term obtained by substituting for H and χ the expressions $H'_{\alpha\dot{\beta}} \sim \bar{D}_{\dot{\beta}}\psi_\alpha - D_\alpha\bar{\psi}_{\dot{\beta}}, \chi' \sim \bar{D}^2D^\alpha\psi_\alpha$, with ψ_α a spinor ghost. The resulting lagrangian can be put in the form $L_2 = H\Box H + \bar{\chi}\chi + \bar{\psi}\Box\not{\partial}\tilde{\Pi}'\psi$. Here $\tilde{\Pi}'$ is a new projection operator reflecting the fact that ψ itself has a gauge invariance: $\delta\psi_\alpha = \Lambda_\alpha$, where Λ_α is chiral, obviously leaves H' and χ' invariant. Repeating the above procedure, we introduce a "second-generation" chiral ghost ϕ_α with *normal* statistics by $\psi'_\alpha = \phi_\alpha$, and obtain $L_3 = H\Box H + \bar{\chi}\chi + \bar{\psi}\Box\not{\partial}\psi + \bar{\phi}\Box\not{\partial}\phi$ (ϕ_α has no further gauge invariances).

We note that since $\Box\not{\partial} = (\not{\partial})^3$, the above ψ_α is equivalent to three such ghosts with ordinary kinetic terms. Similar analysis can be applied to ϕ_α: we first note that $\bar{\phi}\Box\not{\partial}\phi$ is equivalent to $\phi\Box D^2\phi + \text{h.c.}$ This can be shown by treating ϕ_α and $\bar{\phi}_{\dot{\alpha}}$ as independent fields, performing a shift of the form $\phi_\alpha \to \phi_\alpha + ai\partial_{\alpha\dot{\beta}}\bar{D}^2\Box^{-1}\bar{\phi}^{\dot{\beta}}$ and $\bar{\phi}_{\dot{\beta}} \to \bar{\phi}_{\dot{\beta}}$, which has jacobian 1, and then a second shift $\phi_\alpha \to \phi_\alpha, \bar{\phi}_{\dot{\beta}} \to \bar{\phi}_{\dot{\beta}} + bi\partial_{\alpha\dot{\beta}}D^2\Box^{-1}\phi^\alpha$. Choosing $ab = -\frac{1}{2}$ gives the desired transformation. (Equivalently, we note that $\bar{\phi}\not{\partial}\phi$ and $\phi D^2\phi + \text{h.c.}$ are both gauge-fixed lagrangians for a chiral gauge spinor [20] in different gauges: $L = (D^\alpha\phi_\alpha - \bar{D}^{\dot{\alpha}}\bar{\phi}_{\dot{\alpha}})^2 + \alpha(D^\alpha\phi_\alpha + \bar{D}^{\dot{\alpha}}\bar{\phi}_{\dot{\alpha}})^2$, with $\alpha = \pm 1$, so that they must be equivalent.) Now we use $\int d^4\theta(\phi\Box D^2\phi + \text{h.c.}) = \int d^2\theta\phi\Box^2\phi + \text{h.c.}$ Since $\int d^2\theta\phi\Box^2\phi$ gives the same one-loop contribution to the functional integral as $\int d^2\theta(\phi_1\Box\phi_1 + \phi_2\Box\phi_2) = \int d^4\theta(\phi_1D^2\phi_1 + \phi_2D^2\phi_2)$, we can write our final lagrangian as $L_4 = H\Box H + \bar{\chi}\chi + \Sigma_{i=1}^3\bar{\psi}_i\not{\partial}\psi_i + \Sigma_{i=1}^2(\phi_iD^2\phi_i + \text{h.c.})$. (As before, $\phi D^2\phi + \text{h.c.}$ is equivalent to $\bar{\phi}\not{\partial}\phi$.) Our final set of fields thus consists of one real vector (H_a), one chiral scalar (χ), three complex first-generation spinor ghosts (ψ_α), and two chiral second-generation spinor ghosts (ϕ_α). We may also find other ghosts which cancel among themselves (at one loop). This set of ghosts will be found by direct calculation in sect. 5 (including interaction with the on-shell background). Note that we have expressed L_4 in a form where all fields have propagators with no worse infrared behavior than $1/p^2$.

4.2. CATALYST GHOSTS

In order to cancel unwanted terms in the lagrangian, including those which would give $1/p^4$ terms in a propagator or non-local terms in an interaction (which were themselves introduced in the process of canceling other unwanted terms), it is sometimes necessary to introduce "catalyst" fields which cancel the unwanted terms through their gauge fixing, but are themselves canceled by their ghosts. For example, consider a lagrangian of the form $L_0 = A\Box[(1-\Pi)+\alpha\Pi]A$ ($\Pi^2=\Pi$; $\alpha\neq 0,1$). In some cases it would be more convenient to have simply $L_0' = A\Box A$: e.g., if Π were a differential operator, L_0 could give unwanted $1/p^4$ propagators. The simplest way to solve the problem would be by a field redefinition $A' = [(1-\Pi)+\alpha^{-1/2}\Pi]A$, which would give $L_0 = A'\Box A'$, but if Π were a non-local operator this would introduce non-localities into the interaction terms. By introducing catalyst fields we will be able to achieve a similar result. The catalyst fields will cancel with their ghosts at one loop. However, in the case where the original field A has quantum interactions (which would be important at higher loops), the catalyst fields will also have quantum interactions, not canceled by their ghosts (which will not have quantum interactions).

The catalyst fields are introduced as follows: we add a new field B to the lagrangian which gives no contribution to the functional integral (either by giving it a propagator $= 1$, or by adding in a ghost with the same propagator but opposite statistics). We then introduce shifts $A \to A + \mathcal{O}B$, $B \to B + \mathcal{O}'A$ which cancel the unwanted A-terms yet give no A-B crossterms in the lagrangian. For the above example (considering for simplicity the case where Π is a matrix, to avoid hidden ghosts), we choose a B such that $\Pi B = B$ (and an opposite-statistics field B' with $\Pi B' = B'$), and add it to L_0 to give $L_1 = L_0 + \beta B\Box B + B'\Box B'$ ($\beta = \pm 1$). We then make a first shift $B \to B + a\Pi A$, followed by a second shift $A \to A + bB$. By choosing $a = -\beta b = \pm\sqrt{\beta(1-\alpha)}$ (with $\beta = +1$ for $\alpha < 1$, and $\beta = -1$ for $\alpha > 1$), we find $L_1 \to A\Box A + \beta\alpha B\Box B + B'\Box B'$. Note that this procedure is *equivalent* to introducing B by means of the shift $A \to A + B$ only, with no separate terms for B in the lagrangian, and applying the usual gauge-fixing procedures for the invariance $\delta A = \lambda$, $\delta B = -\lambda$ ($\Pi\lambda = \lambda$): adding a gauge-fixing term $(\alpha^2/(1-\alpha))(B-((1-\alpha)/\alpha)\Pi A)\Box(B-((1-\alpha)/\alpha)\Pi A) + B'\Box B'$, we end up with $L = A\Box A + (\alpha/(1-\alpha))B\Box B + B'\Box B'$. (The Faddeev-Popov first and second ghosts have propagator $= 1$ in this example, and have been dropped; B' is a "third ghost" [21] introduced to cancel the $\Box$ in the averaging function of the 't Hooft trick. In more general cases, the first and second ghosts are canceled by hidden ghosts.)

To illustrate the characteristics of catalyst fields when applied particularly to superfields, let us consider a real scalar field V in the presence of an on-shell background field, with lagrangian

$$L_0 = V\left(-2\nabla^\alpha\bar{\nabla}^2\nabla_\alpha + a\{\nabla^2,\bar{\nabla}^2\}\right)V. \tag{4.1a}$$

For $a \neq 0$ this lagrangian has no gauge invariance, and we only consider this case. For $a \neq 1$, V has $1/p^4$ terms in its propagator (and complicated vertices). However, the contribution $-\frac{1}{2}\ln\det(-2\mathcal{D}^\alpha\overline{\mathcal{D}}^2\mathcal{D}_\alpha + a\{\mathcal{D}^2, \overline{\mathcal{D}}^2\})$ of this lagrangian after V-integration is *independent of a* (for $a \neq 0$), since it is of the form

$$L = V\Box\left[\Pi_{1/2} + a(\Pi_{0+} + \Pi_{0-})\right]V, \tag{4.1b}$$

where the Π's are the scalar projection operators, generalized to include on-shell background:

$$\Pi_{0+} = \Box^{-1}\overline{\mathcal{D}}^2\mathcal{D}^2, \qquad \Pi_{0-} = \Box^{-1}\mathcal{D}^2\overline{\mathcal{D}}^2,$$
$$\Pi_{1/2} = -2\Box^{-1}\mathcal{D}^\alpha\overline{\mathcal{D}}^2\mathcal{D}_\alpha, \tag{4.2}$$

which satisfy the usual conditions $\Pi_i\Pi_j = \delta_{ij}\Pi_j$ (not summed), $\Pi_i^\dagger = \Pi_i$. One is thus essentially evaluating an expression of the form

$$\ln\det\begin{pmatrix} A & 0 \\ 0 & aB \end{pmatrix} = \ln\det A + \ln\det B + \text{constant}.$$

To obtain this result explicitly by use of catalyst fields, we perform the shift

$$V \to V + (\eta + \bar{\eta}), \qquad \overline{\mathcal{D}}_{\dot\beta}\eta = 0, \tag{4.3}$$

to introduce our catalyst field η. This produces the gauge invariance

$$\delta V = \Lambda + \bar{\Lambda}, \qquad \delta\eta = -\Lambda, \qquad \overline{\mathcal{D}}_{\dot\beta}\Lambda = 0. \tag{4.4}$$

Gauge fixing for this invariance is equivalent to introducing a lagrangian for η which has no dynamics (as in the previous example). Choosing the gauge-fixing function

$$F = \overline{\mathcal{D}}^2(V + \bar{b}\bar{\eta}), \qquad \overline{\mathcal{D}}_{\dot\beta}F = 0, \tag{4.5}$$

and gauge-fixing term

$$L_1 = 2(1-a)F\bar{F}, \tag{4.6}$$

we obtain

$$L_0 + L_1 = V\Box V + 2\frac{a}{1-a}\eta\Box\bar{\eta}, \qquad \text{for } b = -\frac{a}{1-a}. \tag{4.7a}$$

Just as a term $\psi\Box^2\psi$ for a general scalar ψ would be equivalent to $\psi_1\Box\psi_1 + \psi_2\Box\psi_2$, since $\ln\det\Box^2 = 2\ln\det\Box$, $\eta\Box\bar{\eta}$ is equivalent to $\eta_1\bar{\eta}_1 + \eta_2\bar{\eta}_2 + \eta_3\bar{\eta}_3$. Since η is chiral, we must consider $\int d^2\theta$ of chiral lagrangians to see this: (from now on, $\mathcal{E}^{-1}$ is implicit for $\int d^4\theta$, and $e^{-\overline{W}_{(B)}}(\phi_{(B)})^3$ for $\int d^2\theta$)

$$\int d^2\theta\, \eta'\eta + \text{h.c.}$$

gives 0,

$$\int d^2\theta\, \eta'\bar{\mathcal{D}}^2\bar{\eta} + \text{h.c.} = \int d^4\theta\, \eta'\bar{\eta} + \text{h.c.}$$

gives $-\ln\det\Delta$, where

$$\Delta \equiv \begin{pmatrix} 0 & \bar{\mathcal{D}}^2 \\ \mathcal{D}^2 & 0 \end{pmatrix} \text{ on } \begin{pmatrix} \eta \\ \bar{\eta} \end{pmatrix},$$

$$\int d^2\theta\, \eta'\bar{\mathcal{D}}^2\mathcal{D}^2\eta + \text{h.c.} = \int d^2\theta\, \eta'\Box\eta + \text{h.c.} = \int d^4\theta\, \eta'\mathcal{D}^2\eta + \text{h.c.}$$

gives $-\ln\det\Delta^2 = -2\ln\det\Delta$,

$$\int d^2\theta\, \eta'\bar{\mathcal{D}}^2\mathcal{D}^2\bar{\mathcal{D}}^2\bar{\eta} + \text{h.c.} = \int d^4\theta\, \eta'\Box\bar{\eta} + \text{h.c.}$$

gives $-\ln\det\Delta^3 = -3\ln\det\Delta$, etc. The $-\ln\det\Delta$ terms can be thought of as arising from the jacobians of successive field redefinitions

$$\Delta\begin{pmatrix} \eta \\ \bar{\eta} \end{pmatrix} \to \begin{pmatrix} \eta \\ \bar{\eta} \end{pmatrix}.$$

We can thus write (4.7a) (by scaling $\bar{\eta} \to \bar{\eta}, \eta \to \frac{1}{2}((1-a)/a)\eta$) as

$$L_0 + L_1 = V\Box V + \sum_{i=1}^{3} \eta_i\bar{\eta}_i. \tag{4.7b}$$

However, the gauge fixing [(4.5), (4.6)] also introduces three ghosts, just as in the case of a gauge vector multiplet [6]: two Faddeev-Popov ghosts from varying F in (4.5), and a third ghost η'_3 to normalize the averaging $\int \mathcal{D}\chi\mathcal{D}\bar{\chi}\mathcal{D}\eta'_3\mathcal{D}\bar{\eta}'_3\exp(\chi\bar{\chi} + \eta'_3\bar{\eta}'_3)\delta(F-\chi)\delta(\bar{F}-\bar{\chi})$ so that the integral without the delta-functionals gives 1. We thus obtain for the total lagrangian:

$$L = V\Box V + \sum_{i=1}^{3} (\eta_i\bar{\eta}_i + \eta'_i\bar{\eta}'_i), \tag{4.8a}$$

where η_i' have the opposite statistics to η_i. Therefore, η_i' cancel η_i, and we have simply

$$L = V\Box V. \tag{4.8b}$$

The catalyst fields have thus canceled themselves out, serving merely to perform the nonlocal redefinition which transforms (4.1) into (4.8b). However, if V had also quantum interactions, so would η_i (whereas η_i' still would not), and we would not be able to ignore the catalysts in higher-loop calculations. In that case, catalyst fields would still allow us to transform the V part of the lagrangian to (4.8b), while avoiding explicit nonlocal interactions that a nonlocal field redefinition would introduce. Of course, integrating out the η_i and η_i' would effectively reintroduce non-localities, but there are many advantages to working with a local lagrangian.

4.3. HIDDEN GHOSTS

Hidden ghosts were described in ref. [13] as arising whenever one applies the 't Hooft gauge-averaging trick to a constrained gauge-fixing function. The arguments of that paper can also be applied here, but we will give an independent argument here which is simpler when applied to superfields. As a simple example, consider the chiral spinor gauge superfield ϕ_α [20] with gauge invariance $\delta\phi_\alpha = \bar{D}^2 D_\alpha K$ ($K = \bar{K}$) and gauge-fixing function $F = \bar{F} = D^\alpha\phi_\alpha + \bar{D}^{\dot\alpha}\bar{\phi}_{\dot\alpha}$ satisfying (because of the chirality of ϕ_α) the constraint $\bar{D}^2 F = 0$, so that F is a linear superfield. In the functional integral the usual averaging procedure $\int \mathfrak{D}f \exp(\int d^4x d^4\theta f^2)\delta(F-f)$ will run into difficulties because $\bar{D}^2 F = 0$ implies now $\bar{D}^2 f = 0$. Since functional integration is only defined with respect to unconstrained superfields or chiral ones, we must either define averaging over a linear superfield F (as in ref. [13]), or "complete" F to an unconstrained superfield. Choosing the latter course, we must complete F, which has no undifferentiated terms with θ^2 or $\bar{\theta}^2$ factors, by adding a superfield of the form $\bar{\theta}^2\eta + \text{h.c.}$, where η is chiral. The only way to do this supersymmetrically is by shifting $F \to \hat{F} = F + (D^2\Box^{-1}\eta + \text{h.c.})$ (since $D^2/\Box = \bar{\theta}^2 +$ derivative-terms) and integrating over η. This will trivially take care of the components of $\hat{F}$ which receive no contribution from F due to the constraint $\bar{D}^2 F = 0$: e.g., the component of $\hat{F}$ defined by $A \equiv \bar{D}^2\hat{F}|_{\theta=0}$ is just equal to $\bar{D}^2 D^2\Box^{-1}\eta|_{\theta=0} = \eta|_{\theta=0} \equiv B$, so the $\bar{\theta}^2$ component of $\delta(\hat{F})$ is just the delta functional $\delta(B)$ of the θ-independent component of η. (Operating on superfields with D_α's, instead of ∂_α's, and evaluating at $\theta = 0$ is a covariant way of defining the components of a superfield [22].) This component thus gives a trivial contribution to the functional integral. Components of F which are pure gauge without derivatives (i.e., components of ϕ_α which are zero in a Wess-Zumino gauge) are treated in the usual way by $\hat{F}$. The only remaining component of F is the gauge-fixing function $\partial^b A_{ab}$ for the antisymmetric-tensor component gauge field A_{ab}. The corresponding component of $\hat{F}$ is $\partial^b A_{ab} + \partial_a\Box^{-1}C$, where C is a component of η. By supersymmetry, we would expect that, since all other components of $\hat{F}$ have the correct form, this term would also. In fact, if one applies the 't Hooft trick to this term only, one obtains gauge-fixing terms $(\partial^b A_{ab})^2 - C\Box^{-1}C$ in the effective lagrangian: this C (or equivalently a field C' of opposite statistics appearing as $C'\Box C'$) is exactly the hidden ghost described in ref. [13], so our argument based on supersymmetry agrees with the analysis of that reference. In sect. 5, we will consider the similar case of supergravity, where the gauge-fixing function $F_\alpha = \bar{\mathfrak{D}}^{\dot\beta}(H_{\alpha\dot\beta} + \cdots)$ also satisfies $\bar{\mathfrak{D}}^2 F_\alpha = 0$, and so the gauge is fixed with $\hat{F}_\alpha = F_\alpha + \mathfrak{D}^2\Box_+^{-1}\phi_\alpha$, where ϕ_α is a chiral spinor.

4.4. CHOICE OF GAUGE PARAMETERS

The standard Faddeev-Popov procedure is a convenient method of performing quantization in a gauge-invariant way. However, unitarity is not obvious, since one could add an arbitrary gauge-invariant contribution to the functional integral and still preserve gauge-invariance while destroying unitarity. One way to check unitarity is to compare with canonical quantization in an axial (ghost-free) gauge (see, e.g., ref. [23]). In supersymmetric gauge theories, the analog of the axial gauge is the Wess-Zumino gauge (with the remaining ordinary component-field gauge invariance fixed by the usual component-field axial gauge). This is because no extra ghosts are introduced in going to a Wess-Zumino gauge, since the fields which do not appear in ordinary component-field formulations are gauged away by nonderivative, algebraic transformation laws (e.g., $\delta A = a$: no spacetime derivatives). This last point is crucial: *The superfield parametrization of the gauge transformation must allow the elimination of the superfluous component fields by non-derivative transformation laws.* Otherwise, the superfield formulation would produce extra ghosts, and therefore a result inconsistent with the axial gauge (i.e., Wess-Zumino gauge) result. To illustrate, let us consider the analogous parametrization ambiguity in non-supersymmetric Yang-Mills: suppose that instead of parametrizing the transformation law as $\delta A_a = \nabla_a\lambda$, we write it as $\delta A_a = \nabla_a\Box\lambda'$ ($\Box = \nabla^a\nabla_a$). This is classically equivalent (with the appropriate modification of boundary conditions). However, direct application of the Faddeev-Popov procedure will produce the ghost term $c'\partial\cdot\nabla\Box c$ instead of the usual $c'\partial\cdot\nabla c$: thus, an extra gauge-invariant factor $\det\Box$ will be introduced into the functional integral, destroying unitarity. Therefore, $\delta A_a = \nabla_a\lambda$ is the only parametrization which gives the correct, unitary result, unless special care is taken to modify the Faddeev-Popov prescription. Similarly, in the case of supergravity, the L_α parametrization used above (originated in ref. [24]) is the only one (with flat or general background) which agrees with the axial gauge. (We do not consider parameters with higher spin, such as $\delta H_{\alpha\dot\beta} = \bar{D}_{\dot\beta} D^\gamma K_{(\alpha\gamma)} + \text{h.c.}$ for $\chi = 0$, because they lead to an infinite set of ghosts: $\delta K_{(\alpha\beta)} = D^\gamma K_{(\alpha\beta\gamma)}$, etc. Without background, the Λ parametrization of ref. [9] for the transformation of the superfield U^M is the correct one, but in that case perturbation theory would not be manifestly globally-supersymmetric.) This can be seen by component expansion of the linearized (B.3), (B.5) and (B.6) to obtain a gauge similar to (B.1).

4.5. THE COMPENSATING FIELD

It is necessary to have the field ϕ in the classical theory (though one may choose a quantum gauge where it vanishes) in order to use the L_α parametrization. Thus, one must start in a gauge with both the fields H_a and ϕ present in order to perform naive Faddeev-Popov quantization. If ϕ were eliminated from the classical theory, L_α would have to be constrained. The solution of this constraint would express L_α in terms of derivatives of other superfield parameters (as in refs. [25,11,12]). However, as explained above, some of the component-field transformation laws used to obtain the Wess-Zumino gauge would contain extra spacetime derivatives: therefore, naive Faddeev-Popov quantization would introduce spurious extra ghosts for these derivatives, the quantization would not be equivalent to axial-gauge quantization, and unitarity would be lost. The necessity of keeping ϕ is related to the fact that its θ^2 component is gauged away (into H_a) by $\delta B = \partial_a b^a$, which introduces ghosts.

4.6. FIELDS ALLOWED IN A BACKGROUND

The only types of chiral superfields we will use will be scalars and undotted spinors, which are defined in a general background [19]. On the other hand, refs. [11,12] use chiral vectors, which do not exist in a general background [19]: due to (B.23a), for any vector $\phi_{\alpha\dot\beta}$ satisfying the chirality condition $\bar{\mathscr{D}}_{\dot\gamma}\phi_{\alpha\dot\beta} = 0$, we have

$$0 = \left\{\bar{\mathscr{D}}_{\dot\gamma}, \bar{\mathscr{D}}_{\dot\delta}\right\}\phi_{\alpha\dot\beta} = -R\bar{M}_{\dot\gamma\dot\delta}\phi_{\alpha\dot\beta} = -\tfrac{1}{2}RC_{\dot\beta(\dot\gamma}\phi_{\alpha\dot\delta)}. \tag{4.9}$$

Therefore, for the background $R \neq 0$, $\phi_{\alpha\dot\beta}$ must itself vanish, so the formalisms of refs. [11,12] must be modified to be applicable in a general background. Although in this paper we will only apply our formalism to on-shell background, which does have $R = 0$, our methods and choice of ghosts are such as to allow straightforward generalization to off-shell background.

5. Gauge fixing and ghosts

We will now apply the techniques of sect. 4 to superfield supergravity. In order to fix the gauge of the action (2.7) [with linearized expansion (3.10)], we first express the quantum transformation laws (2.10b) in terms of L_α, as given by (2.12):

$$\delta H_{\alpha\dot\beta} = -2\left(\bar{\mathscr{D}}_{\dot\beta}L_\alpha - \mathscr{D}_\alpha\bar{L}_{\dot\beta}\right) + \mathrm{O}(H),$$

$$\delta\phi = -(\bar{\mathscr{D}}^2 + R)L^\alpha\mathscr{D}_\alpha\phi - \tfrac{1}{3}\left[(\bar{\mathscr{D}}^2 + R)\mathscr{D}^\alpha L_\alpha\right]\phi, \qquad \text{exactly}, \tag{5.1}$$

in an arbitrary background [using also (B.23)]. We therefore also have

$$\delta\phi^3 = -(\bar{\mathscr{D}}^2 + R)\mathscr{D}^\alpha L_\alpha\phi^3. \tag{5.2}$$

From now on we will work only with on-shell background fields [i.e., the field equations (B.24) are satisfied], so the action quadratic in H and χ becomes (in units $\kappa = 1$)

$$S = \int d^4x d^4\theta \mathcal{E}^{-1}\Big[-6\chi\bar{\chi} + 2i(\chi - \bar{\chi})\mathcal{D}\cdot H - \tfrac{1}{2}H\cdot\Box H - \tfrac{1}{2}(\mathcal{D}\cdot H)^2$$

$$-\tfrac{1}{6}\Big(\big[\bar{\mathcal{D}}_{\dot\beta}, \mathcal{D}_\alpha\big] H^{\alpha\dot\beta}\Big)^2 + (\bar{\mathcal{D}}^2 H)\cdot(\mathcal{D}^2 H)$$

$$-\tfrac{1}{2}H^{\alpha\dot\beta}\Big(W_\alpha{}^{\gamma\delta}\mathcal{D}_\gamma H_{\delta\dot\beta} + \bar{W}_{\dot\beta}{}^{\dot\gamma\dot\delta}\bar{\mathcal{D}}_{\dot\gamma} H_{\alpha\dot\delta}\Big)\Big]. \tag{5.3}$$

In order to cancel the $H\chi$ crossterms, we choose the following modification of the gauge-fixing function $\bar{D}^{\dot\beta} H_{\alpha\dot\beta}$ of ref. [10]:

$$F_\alpha = \bar{\mathcal{D}}^{\dot\beta}\Big(H_{\alpha\dot\beta} + i\bar{a}\mathcal{D}_{\alpha\dot\beta}\Box_-^{-1}\bar{\phi}^3\Big) \tag{5.4}$$

(for some constant a), where

$$\Box_+ = \Box + 2W^\alpha{}_\beta{}^\gamma\mathcal{D}_\alpha M_\gamma{}^\beta, \qquad \Box_- = \Box + 2\bar{W}^{\dot\alpha}{}_{\dot\beta}{}^{\dot\gamma}\bar{\mathcal{D}}_{\dot\alpha}\bar{M}_{\dot\gamma}{}^{\dot\beta} \tag{5.5a}$$

is the chiral form of $\Box \equiv \mathcal{D}^a\mathcal{D}_a$, as follows from the identities

$$\Box_+\phi_{\alpha\cdots\beta} = \bar{\mathcal{D}}^2\mathcal{D}^2\phi_{\alpha\cdots\beta}, \qquad \Box_-\bar{\phi}_{\dot\alpha\cdots\dot\beta} = \mathcal{D}^2\bar{\mathcal{D}}^2\bar{\phi}_{\dot\alpha\cdots\dot\beta}, \tag{5.5b}$$

when acting on a chiral superfield $\phi_{\alpha\cdots\beta}$ with only undotted spinor indices. In the special case of a chiral scalar ϕ [as in (5.4)], $\Box_+\phi = \Box\phi$, but the distinction will be non-trivial when considering chiral spinor ghosts ϕ_α below. We have used $\bar{\phi}^3 = (1+\bar{\chi})^3$ in (5.4) instead of just $\bar{\chi}$ for the following reason: by redefining L_α (or just the corresponding Faddeev-Popov ghosts) as

$$L_\alpha \to \phi^{-3}L_\alpha \tag{5.6a}$$

($= L_\alpha - 3\chi L_\alpha + \cdots$, which doesn't affect the arguments of subsect. 4.4), (5.1) and (5.2) become

$$\delta H_{\alpha\dot\beta} = -2\big(\bar{\mathcal{D}}_{\dot\beta}L_\alpha - \mathcal{D}_\alpha\bar{L}_{\dot\beta}\big) + O(H) + O(\chi),$$

$$\delta\chi = -\tfrac{1}{3}\bar{\mathcal{D}}^2\mathcal{D}^\alpha L_\alpha + O(\chi),$$

$$\delta\phi^3 = -\bar{\mathcal{D}}^2\mathcal{D}^\alpha L_\alpha, \qquad \text{exactly}, \tag{5.6b}$$

so that now Faddeev-Popov quantization using F_α of (5.4) will contribute non-local terms to *only the kinetic terms* of the ghosts, and not their quantum interactions. Non-local kinetic terms, but not non-local quantum-interaction terms, can be made local by the use of catalyst ghosts, as described in subsect. 4.2 and this will be shown explicitly below.

We fix the gauge with the following contribution to the functional integral:

$$\int \mathcal{D}\zeta\mathcal{D}\bar{\zeta}\mathcal{D}\phi\mathcal{D}\bar{\phi}\,\delta\left(F_\alpha + \mathcal{D}^2\Box_+^{-1}\phi_\alpha - \zeta_\alpha\right)\delta\left(\bar{F}_{\dot\alpha} + \bar{\mathcal{D}}^2\Box_-^{-1}\bar{\phi}_{\dot\alpha} - \bar{\zeta}_{\dot\alpha}\right)\cdot\exp\int \mathrm{d}^4x\,\mathrm{d}^4\theta\,\mathcal{E}^{-1}$$
$$\times\left[-\tfrac{1}{2}\left(\mathcal{D}^\alpha\zeta_\alpha - \bar{\mathcal{D}}^{\dot\alpha}\bar{\zeta}_{\dot\alpha}\right)^2 - \tfrac{1}{6}\left(\mathcal{D}^\alpha\zeta_\alpha + \bar{\mathcal{D}}^{\dot\alpha}\bar{\zeta}_{\dot\alpha}\right)^2 + 2\left(\mathcal{D}^\alpha\bar{\zeta}^{\dot\beta}\right)\left(\bar{\mathcal{D}}_{\dot\beta}\zeta_\alpha\right)\right], \tag{5.7}$$

where ϕ_α is the hidden ghost which completes the linear spinor F_α ($\bar{\mathcal{D}}^2 F_\alpha = 0$) to a general spinor $\hat{F}_\alpha = F_\alpha + \mathcal{D}^2\Box_+^{-1}\phi_\alpha$, as described in subsect. 4.3. The terms in the exponent have been chosen to cancel the non-$\Box$, non-W, $\mathrm{O}(H^2)$ terms in (5.3), and we also choose

$$a = \tfrac{2}{5} \tag{5.8}$$

to cancel the $H\chi$ crossterms, after performing the ζ-integration. We also find that (5.7) contributes no $H_a\phi_\alpha$ or $\chi\phi_\alpha$ crossterms, and the ϕ_α kinetic term (with $\int \mathrm{d}^4\theta\,\mathcal{E}^{-1}$)

$$\bar{\phi}^{\dot\beta}\Box_+^{-1}\bar{\mathcal{D}}^2\mathcal{D}_{\alpha\dot\beta}\mathcal{D}^2\Box_+^{-1}\phi^\alpha = \bar{\phi}^{\dot\beta}\Box_+^{-1}\mathcal{D}_{\alpha\dot\beta}\phi^\alpha = \bar{\phi}^{\dot\beta}\mathcal{D}_{\alpha\dot\beta}\Box_+^{-1}\phi^\alpha \tag{5.9a}$$

becomes (with the chiral background-covariant measure $\int \mathrm{d}^2\theta\,\mathrm{e}^{-\bar{W}_{(\mathrm{B})}}\phi_{(\mathrm{B})}^3$)

$$\int \mathrm{d}^2\theta\,\mathrm{e}^{-\bar{W}_{(\mathrm{B})}}\phi_{(\mathrm{B})}^3\phi^\alpha\phi_\alpha + \text{h.c.} \tag{5.9b}$$

after the successive "rotations" (with unit jacobian)

$$(1)\qquad \phi_\alpha \to \phi_\alpha + ib\mathcal{D}_{\alpha\dot\beta}\bar{\mathcal{D}}^2\Box_-^{-1}\bar{\phi}^{\dot\beta}, \qquad \bar{\phi}_{\dot\beta} \to \bar{\phi}_{\dot\beta},$$
$$(2)\qquad \phi_\alpha \to \phi_\alpha, \qquad \bar{\phi}_{\dot\beta} \to \bar{\phi}_{\dot\beta} + ic\mathcal{D}_{\alpha\dot\beta}\mathcal{D}^2\Box_+^{-1}\phi^\alpha, \tag{5.10}$$

with $bc = -\frac{1}{2}$, as described in subsect. 4.1. We have used the identity

$$\mathcal{D}^\alpha\Box_+\chi_\alpha = \Box\mathcal{D}^\alpha\chi_\alpha, \qquad \text{when } \bar{\mathcal{D}}_{\dot\beta}\chi_\alpha = 0. \tag{5.11}$$

(Note that $\Box_+^{-1}\phi_\alpha$ is also chiral.) Thus the hidden ghost ϕ_α decouples, and can be dropped in this case. ($\phi_{(\mathrm{B})}$ can be eliminated from (5.9b) either by redefinition $\phi_\alpha \to \phi_{(\mathrm{B})}^{-3/2}\phi_\alpha$ or by background gauge choice $\phi_{(\mathrm{B})} = 1$. $\mathrm{e}^{-\bar{W}_{(\mathrm{B})}}$ cancels the $\bar{W}_{(\mathrm{B})}$ dependence in $\phi_{(\mathrm{B})}$ and ϕ_α.)

Due to the fact that the exponent of (5.7) has background-covariant derivatives, it is not a true averaging over the gauges chosen by the delta-functionals, and so we must include a third ghost [21,6] $\psi_{3\alpha}$ with abnormal statistics (ζ_α has normal statistics) in addition to the usual Faddeev-Popov first and second ghosts $\psi_{1\alpha}, \psi_{2\alpha}$. Its contribution to the quantum lagrangian is thus

$$-\tfrac{1}{2}\left(\mathcal{D}^\alpha\psi_{3\alpha} - \bar{\mathcal{D}}^{\dot\alpha}\bar\psi_{3\dot\alpha}\right)^2 - \tfrac{1}{6}\left(\mathcal{D}^\alpha\psi_{3\alpha} + \bar{\mathcal{D}}^{\dot\alpha}\bar\psi_{3\dot\alpha}\right)^2 + 2\left(\mathcal{D}^\alpha\bar\psi_3^{\dot\beta}\right)\left(\bar{\mathcal{D}}_{\dot\beta}\psi_{3\alpha}\right). \quad (5.12)$$

As described in subsect. 4.2, there are two ways to put this into the standard form $\bar\psi^{\dot\beta}\mathcal{D}_{\alpha\dot\beta}\psi^\alpha$: by a non-local redefinition, or by a shift

$$\psi_{3\alpha} \to \psi_{3\alpha} + \bar{\mathcal{D}}^2\mathcal{D}_\alpha\psi_3 + \mathcal{D}_\alpha\phi_3, \qquad \left(\bar{\mathcal{D}}_{\dot\beta}\phi_3 = 0\right), \quad (5.13)$$

which introduces catalyst fields. As described in subsect. 4.2, and in more detail for this case in appendix C, gauge fixing for the catalyst fields ψ_3, ϕ_3 cancels them out of the action completely, while simultaneously allowing (5.12) to be recast into the desired form.

We now apply the Faddeev-Popov procedure to the gauge-fixing function F_α of (5.4) and (5.8), under gauge transformations (5.6b). As discussed above, the quantum interaction terms will be local. The kinetic terms (i.e., terms with background but without quantum interaction) are (after some algebra)

$$\begin{aligned}&\left(\bar{\mathcal{D}}^{\dot\beta}\psi_2^\alpha - \mathcal{D}^\alpha\bar\psi_2^{\dot\beta}\right)\left(\bar{\mathcal{D}}_{\dot\beta}\psi_{1\alpha} - \mathcal{D}_\alpha\bar\psi_{1\dot\beta}\right) + \tfrac{1}{5}\Big[\left(\bar{\mathcal{D}}^2\mathcal{D}^\alpha\psi_{2\alpha}\right)\Box_-^{-1}\left(\mathcal{D}^2\bar{\mathcal{D}}^{\dot\beta}\bar\psi_{1\dot\beta}\right)\\ &\qquad + \left(\mathcal{D}^2\bar{\mathcal{D}}^{\dot\beta}\bar\psi_{2\dot\beta}\right)\Box_+^{-1}\left(\bar{\mathcal{D}}^2\mathcal{D}^\alpha\psi_{1\alpha}\right)\Big].\end{aligned} \quad (5.14)$$

To eliminate the non-local terms and obtain the desired $\bar\psi^{\dot\beta}\mathcal{D}_{\alpha\dot\beta}\psi^\alpha$ form, we again introduce catalyst ghosts:

$$\psi_{1\alpha} \to \psi_{1\alpha} + \mathcal{D}_\alpha\psi_1, \qquad \psi_{2\alpha} \to \psi_{2\alpha} + \mathcal{D}_\alpha\psi_2. \quad (5.15)$$

Unlike the case for $\psi_{3\alpha}$, ψ_1 and ψ_2 will *not* decouple, but their one-loop contributions (which do not involve quantum interactions) will still be canceled (see appendix C). Also, $\psi_{1\alpha}$ and $\psi_{2\alpha}$ have not only the gauge invariance introduced by ψ_1 and ψ_2, but also the gauge invariance due to their appearing only as $\bar{\mathcal{D}}_{\dot\beta}\psi_\alpha$:

$$\begin{aligned}\delta\psi_{1\alpha} &= \Lambda_{1\alpha} + \mathcal{D}_\alpha L_1, \qquad \delta\psi_1 = -L_1, \qquad \bar{\mathcal{D}}_{\dot\beta}\Lambda_{1\alpha} = 0,\\ \delta\psi_{2\alpha} &= \Lambda_{2\alpha} + \mathcal{D}_\alpha L_2, \qquad \delta\psi_2 = -L_2, \qquad \bar{\mathcal{D}}_{\dot\beta}\Lambda_{2\alpha} = 0.\end{aligned} \quad (5.16)$$

Besides the ghosts which cancel ψ_1 and ψ_2 at one loop, we will thus also obtain chiral spinors $\phi_{1\alpha}$ and $\phi_{2\alpha}$ as the *second* Faddeev-Popov ghosts ("anti-ghosts") of $\psi_{1\alpha}$ and

$\psi_{2\alpha}$, whereas the corresponding *first* Faddeev-Popov ghosts and *third* ghosts will be found to decouple after various field redefinitions. Also, various shifts are needed which mix the chiral spinor ghosts with the scalar ghosts which cancel ψ_1 and ψ_2. (Since these ghosts have no quantum interactions, we could instead perform non-local field redefinitions.) The details may be found in appendix C.

The total result for the quantum action in an on-shell background to second order in the quantum fields is

$$S=\int d^4x d^4\theta \mathcal{E}^{-1}\left[-\tfrac{1}{2}H\cdot\Box H-\tfrac{1}{2}H^{\alpha\dot\beta}\left(W_\alpha{}^{\gamma\delta}\mathcal{D}_\gamma H_{\delta\dot\beta}+\bar W_{\dot\beta}{}^{\dot\gamma\dot\delta}\bar{\mathcal{D}}_{\dot\gamma}H_{\alpha\dot\delta}\right)\right.$$
$$\left.-\tfrac{18}{5}\chi\bar\chi+\sum_{i=1}^{3}\bar\psi_i^{\dot\beta}i\mathcal{D}_{\alpha\dot\beta}\psi_i^\alpha+\sum_{i=1}^{2}\left(\phi_i^\alpha\mathcal{D}^2\phi_{i\alpha}+\text{h.c.}\right)\right], \quad (5.17)$$

where only $\psi_{i\alpha}$ has abnormal statistics. The ghosts ψ_i and ψ_i' ($i=1,2$), with opposite statistics, will also be relevant in higher-loop calculations: they have kinetic terms $\Sigma_{i=1}^{2}(\bar\psi_i\Box\psi_i+\bar\psi_i'\Box\psi_i')$, and ψ_i (the one with abnormal statistics) will also have quantum interactions. Note that (5.17) is just the covariantization of the action found by counting arguments in subsect. 4.1, plus the W terms. A similar situation occurs in (off-shell) background quantization of the non-abelian vector multiplet [6].

An equivalent action with a different set of fields can be obtained by replacing ϕ with a real scalar [26] by the redefinition (in an arbitrary background):

$$\phi^3=1-(\bar{\mathcal{D}}^2+R)V, \quad (5.18)$$

where V transforms as [after the redefinition of (5.6a)]

$$\delta V=\mathcal{D}^\alpha L_\alpha+\bar{\mathcal{D}}^{\dot\alpha}\bar L_{\dot\alpha}, \qquad \text{exactly}. \quad (5.19)$$

F_α of (5.4) can then be replaced by a local expression of the form

$$F_\alpha=\bar{\mathcal{D}}^{\dot\beta}H_{\alpha\dot\beta}+b\mathcal{D}_\alpha V. \quad (5.20)$$

After taking into account the other resultant modifications (e.g., in (5.16) $\Lambda_{1\alpha}$ is replaced by $i(\bar{\mathcal{D}}^2+R)\mathcal{D}_\alpha K$, $K=\bar K$), it can be shown (by the methods of sect. 4) that (5.17) is replaced by

$$S=\int d^4x d^4\theta \mathcal{E}^{-1}\left[-\tfrac{1}{2}H\cdot\Box H-\tfrac{1}{2}H^{\alpha\dot\beta}\left(W_\alpha{}^{\gamma\delta}\mathcal{D}_\gamma H_{\delta\dot\beta}+\bar W_{\dot\beta}{}^{\dot\gamma\dot\delta}\bar{\mathcal{D}}_{\dot\gamma}H_{\alpha\dot\delta}\right)+V\Box V\right.$$
$$\left.+\sum_{i=1}^{3}\bar\psi_i^{\dot\beta}i\mathcal{D}_{\alpha\dot\beta}\psi_i^\alpha+\sum_{i=1}^{3}V_i\Box V_i+\sum_{i=1}^{7}\chi_i\bar\chi_i\right], \quad (5.21)$$

where $\psi_{i\alpha}$ and χ_i have abnormal statistics. The real scalars V_i appear as second-generation ghosts, and χ_i as third generation. Again, there are also catalyst ghosts, relevant at higher loops.

In general, the total field content is the following: (1) the physical fields, H^a and χ (or V), which contribute at all loops; (2) the first-generation Faddeev-Popov ghosts, $\psi_{1\alpha}$ and $\psi_{2\alpha}$, which contribute at all loops; (3) the first-generation third ghost $\psi_{3\alpha}$ and *all* higher-generation ghosts, which contribute *only* at *one* loop; (4) the catalyst ghosts ψ_1 and ψ_2 (for $\psi_{1\alpha}$ and $\psi_{2\alpha}$), which contribute *only* at *more than one* loop (being canceled by ψ_1' and ψ_2' at one loop). If we had not worked in a background field gauge, the fields of type 3 would decouple, and the Feynman rules following from the actions corresponding to (5.17) and (5.21) would be identical to each other.

6. Conclusions

Expression (5.17) [or (5.21)] is the end point of our manipulations and the starting point of calculations we shall present in a sequel to this paper. The quadratic action is sufficient to perform one-loop calculations with the background fields on shell, and among possible applications we mention two: an explicit expression of the one-loop four-particle S-matrix in supergravity [27], and a calculation of the one-loop (triangle) anomalies (chiral, trace and supercurrent) in a manifestly supersymmetric manner, with external supergravity fields and internal matter or supergravity fields [28].

Extension for higher-loop calculations is straightforward although algebraically cumbersome. Self-interactions of the quantum supergravity fields are obtained by expanding the action (2.7) in higher powers of H and χ. In addition, the quantum interactions of $\psi_{1\alpha}$ and $\psi_{2\alpha}$ and their catalysts ψ_1 and ψ_2 with H and χ are obtained by expanding (2.10b) and (2.12) for substitution into (5.4) [and using the redefinitions (5.6a) and (5.15)]. The resulting action will be essentially a background covariantization of the action which would have been obtained in a non-background gauge. (Note that even the non-background gauge action obtained by our methods is an improvement over that of ref. [10], due to the elimination of $1/p^4$ ghost propagators by catalysts.)

We thank the members of DAMTP for thier hospitality, and especially S.W. Hawking and M. Roček, organizers of the Nuffield Supergravity Workshop, where part of this work was done. During part of this work W. Siegel was also at the Physics Department of Brandeis University (supported by NSF grant PHY78-09644), and at the Institute for Advanced Study (supported by DoE contract DE-AC02-76ER02220).

Appendix A

BACKGROUND FORMALISM FOR SUPER-YANG-MILLS

This appendix is a brief description of the background field formalism for supersymmetric non-abelian gauge theories. The motivations have been explained in sect. 2; applications have been presented in ref. [6]. Here we merely give an analog to the supergravity formalism of sect. 2: the basic steps are the same in super-Yang-Mills as in supergravity, but the equations are simpler, so the details are easier to follow. (Similar methods have also been discussed by Pervushin, Kazakov and Pushkin [29] for non-supersymmetric scalar field theories with non-linear invariances.)

Our notation is as follows: we use two-component spinors and work with *euclidean* (i.e., Wick-rotated) spacetime coordinates. Small Greek letters denote spinor indices, small Latin letters vector indices and capital Latin letters superspace indices: e.g., $A=(\alpha,\dot\alpha,a)$. (For further details, see ref. [6].)

Super-Yang-Mills theories are described by covariant derivatives $\nabla_A = D_A - i\Gamma_A$ of the form [30]:

$$\overline{\nabla}_{\dot\alpha} = \mathrm{e}^{\overline{W}}\overline{D}_{\dot\alpha}\mathrm{e}^{-\overline{W}}, \qquad \nabla_\alpha = \mathrm{e}^{-W}D_\alpha \mathrm{e}^{W}, \qquad \nabla_a = -i\sigma_a^{\alpha\dot\beta}\left\{\nabla_\alpha, \overline{\nabla}_{\dot\beta}\right\};$$

$$(i\nabla_A)^\dagger = (-1)^a i\nabla_A; \tag{A.1a}$$

in the vector representation, and in the chiral representation:

$$\overline{\nabla}_{\dot\alpha} = \overline{D}_{\dot\alpha}, \qquad \nabla_\alpha = \mathrm{e}^{-V}D_\alpha \mathrm{e}^{V}, \qquad \nabla_a = -i\sigma_a^{\alpha\dot\beta}\left\{\nabla_\alpha, \overline{\nabla}_{\dot\beta}\right\};$$

$$(i\nabla_A)^\dagger = \mathrm{e}^{V}\left[(-1)^a i\nabla_A\right]\mathrm{e}^{-V}, \qquad V^\dagger = V, \qquad \mathrm{e}^{V} = \mathrm{e}^{W}\mathrm{e}^{\overline{W}}. \tag{A.1b}$$

In this paper we use

$$\overline{D}_{\dot\alpha} \equiv -\tfrac{1}{2}i\left(\bar\partial_{\dot\alpha} + \theta^\beta i\partial_{\beta\dot\alpha}\right), \qquad D_\alpha \equiv \tfrac{1}{2}i\left(\partial_\alpha + \bar\theta^{\dot\beta} i\partial_{\alpha\dot\beta}\right), \qquad D_a \equiv \partial_a,$$

a slight (but very convenient) modification from the notation of ref. [6]. (There are also representations with $\overline{D}_{\dot\alpha}, D_\alpha \to -\frac{1}{2}i\bar\partial_{\dot\alpha}, \frac{1}{2}i\partial_\alpha$ and $V \to 2\theta^\alpha\bar\theta^{\dot\beta}i\partial_{\alpha\dot\beta} + V$ or $W \to \theta^\alpha\bar\theta^{\dot\beta}i\partial_{\alpha\dot\beta} + W$ [9], but these are less useful for globally supersymmetric perturbation theory, so we will not discuss them.) The vector representation is manifestly hermitian, whereas the chiral representation uses fewer fields (the real V instead of the complex W). Eq. (A.1b) can be obtained from (A.1a) by the transformation

$$\nabla'_A = \mathrm{e}^{-\overline{W}}\nabla_A \mathrm{e}^{\overline{W}}. \tag{A.2}$$

The gauge transformation laws relevant to the vector and chiral representations are respectively:

$$\text{vector:} \qquad \nabla'_A = \mathrm{e}^{iK}\nabla_A \mathrm{e}^{-iK}, \qquad \mathrm{e}^{W'} = \mathrm{e}^{i\overline{\Lambda}}\mathrm{e}^{W}\mathrm{e}^{-iK};$$

$$\text{chiral:} \qquad \nabla'_A = \mathrm{e}^{i\Lambda}\nabla_A \mathrm{e}^{-i\Lambda}, \qquad \mathrm{e}^{V'} = \mathrm{e}^{i\overline{\Lambda}}\mathrm{e}^{V}\mathrm{e}^{-i\Lambda};$$

where

$$K=\bar{K}, \qquad \bar{D}_{\dot{\alpha}}\Lambda=0. \tag{A.3}$$

We perform the background-quantum splitting by

$$e^{W}=e^{W_{(B)}}e^{W_{(Q)}} \tag{A.4a}$$

$$\rightarrow e^{V}=e^{W_{(B)}}e^{V_{(Q)}}e^{\bar{W}_{(B)}}. \tag{A.4b}$$

The form of (A.4b) shows that the quantum field is naturally in the chiral representation, while the background field is naturally in the vector representation. We make the splitting as $e^{W}=e^{W_{(B)}}e^{W_{(Q)}}$, and not $e^{W_{(Q)}W_{(B)}}$, so that $W_{(B)}$ will sit next to D_{α} in $\nabla_{\alpha}=e^{-W}D_{\alpha}e^{W}$. The gauge transformation on W from (A.3) is then

$$e^{W'_{(B)}}e^{W'_{(Q)}}=e^{i\bar{\Lambda}}(e^{W_{(B)}}e^{W_{(Q)}})e^{-iK}$$

$$=\begin{cases}(e^{i\bar{\Lambda}}e^{W_{(B)}}e^{-iK})(e^{iK}e^{W_{(Q)}}e^{-iK})\\ e^{W_{(B)}}\left[(e^{-W_{(B)}}e^{i\bar{\Lambda}}e^{W_{(B)}})e^{W_{(Q)}}e^{-iK}\right].\end{cases} \tag{A.5}$$

Thus, with the usual definitions that background transformations transform the background field as the original gauge field and the quantum field as a covariant non-gauge field, whereas the quantum transformations leave the background field invariant, we obtain the following transformation laws:

$$\text{background:} \qquad e^{W'_{(B)}}=e^{i\bar{\Lambda}}e^{W_{(B)}}e^{-iK}, \qquad W'_{(Q)}=e^{iK}W_{(Q)}e^{-iK}; \tag{A.6a}$$

$$\text{quantum:} \qquad W'_{(B)}=W_{(B)}, \qquad e^{W'_{(Q)}}=e^{i\bar{\tilde{\Lambda}}}e^{W_{(Q)}}e^{-iK},$$

$$\tilde{\Lambda}=e^{\bar{W}_{(B)}}\Lambda e^{-\bar{W}_{(B)}}. \tag{A.6b}$$

In order to obtain the background-vector, quantum-chiral representation, we start from either (A.1a) or (A.1b) and make the corresponding one of the following non-unitary similarity transformations (which take the form of complex gauge transformations):

$$\text{from the vector:} \qquad \nabla'_{A}=e^{-\bar{W}_{(Q)}}\nabla_{A}e^{\bar{W}_{(Q)}},$$

$$\text{from the chiral:} \qquad \nabla'_{A}=e^{\bar{W}_{(B)}}\nabla_{A}e^{-\bar{W}_{(B)}}. \tag{A.7}$$

The covariant derivatives then take the form:

$$\bar{\nabla}_{\dot\alpha} = \bar{\mathcal{D}}_{\dot\alpha}, \qquad \nabla_\alpha = e^{-V}\mathcal{D}_\alpha e^{V}, \qquad \nabla_a = -i\sigma_a^{\alpha\dot\beta}\{\nabla_\alpha, \bar{\nabla}_{\dot\beta}\},$$

$$\bar{\mathcal{D}}_{\dot\alpha} = e^{\bar{W}}\bar{D}_{\dot\alpha}e^{-\bar{W}}, \qquad \mathcal{D}_\alpha = e^{-W}D_\alpha e^{W}, \qquad \mathcal{D}_a = -i\sigma_a^{\alpha\dot\beta}\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot\beta}\},$$

$$(i\nabla_A)^\dagger = e^{V}\left[(-1)^a i\nabla_A\right]e^{-V}, \qquad (i\mathcal{D}_A)^\dagger = (-1)^a i\mathcal{D}_A, \tag{A.8}$$

where W now refers only to the background field and V only to the quantum. Since it is never necessary to expand $\mathcal{D}_A$ in terms of W in calculating the effective action (but only to separate out the empty-space part, as $\mathcal{D}_A = D_A - i\Gamma_A^{(\mathrm{B})}$), we will no longer discuss W, and only work with $\mathcal{D}_A$ and V. For these fields, the transformation laws (A.6) become:

$$\text{background:} \qquad \mathcal{D}'_A = e^{iK}\mathcal{D}_A e^{-iK}, \qquad V' = e^{iK}Ve^{-iK}, \qquad K = \bar{K}; \tag{A.9a}$$

$$\text{quantum:} \qquad \mathcal{D}'_A = \mathcal{D}_A, \qquad e^{V'} = e^{i\bar{\tilde\Lambda}}e^{V}e^{-i\tilde\Lambda}, \qquad \bar{\mathcal{D}}_{\dot\alpha}\tilde\Lambda = 0. \tag{A.9b}$$

The constraint $\bar{\mathcal{D}}_{\dot\alpha}\tilde\Lambda = 0$ is equivalent to the definition of $\tilde\Lambda$ in (A.6b) with constraint $\bar{D}_{\dot\alpha}\Lambda = 0$. Note that the background transformations now use only K (not Λ), while the quantum transformations use only $\tilde\Lambda$.

A covariantly chiral matter superfield η, satisfying $\bar{\nabla}_{\dot\alpha}\eta = 0$, becomes, after the transformation (A.7), a background-covariantly chiral superfield $\tilde\eta$, satisfying $\bar{\mathcal{D}}_{\dot\alpha}\tilde\eta = 0$. From the transformation laws corresponding to (A.3):

$$\text{vector:} \qquad \eta' = e^{iK}\eta,$$

$$\text{chiral:} \qquad \eta' = e^{i\Lambda}\eta; \tag{A.10}$$

we obtain the transformation laws corresponding to (A.9):

$$\text{background:} \qquad \tilde\eta' = e^{iK}\tilde\eta,$$

$$\text{quantum:} \qquad \tilde\eta' = e^{i\tilde\Lambda}\tilde\eta. \tag{A.11}$$

Appendix B

CLASSICAL SUPERFIELD SUPERGRAVITY

In this appendix we present a review of the superspace formulation of supergravity sufficient for following the details of our work and extending it. However, for a discussion of the motivation, proofs and comparison with components, consult the references cited below. Just as in ordinary gravity, there are two facets to the formalism: (1) a simple set of unconstrained fundamental fields [9], in terms of which quantization must be performed, and (2) a set of covariant derivatives [31], in terms of which manifestly invariant actions can easily be constructed. The unconstrained fields appear upon solving the constraints on the covariant derivatives [17]; conversely, covariant derivatives can be constructed from the fundamental fields by invariance arguments [32]. For simplicity of presentation, and because it suffices for our purpose, we will restrict ourselves to the minimal auxiliary field case ($n=-\frac{1}{3}$ in ref. [9], where unconstrained superfields and covariant derivatives are discussed for the more general case).

The unconstrained superfields and their transformation laws are most simply represented in the chiral representation. In this representation, supergravity is described in terms of two superfields: a real (axial) supervector H^A (appearing as $H=H^A iD_A$) and a compensating chiral scalar $\phi=1+\chi$. It is usually convenient to choose a gauge where H^α (and $H^{\dot\alpha}$) $=0$ (so that H^A becomes an ordinary vector H^a), and we will use such a gauge. (No ghosts are required for such a gauge choice, even when a background is introduced, although the remaining gauge invariance becomes slightly more complicated.) ϕ plays the same role as the compensating scalar field in the conformal (Weyl)-invariant formulation of Einstein gravity, but in supergravity it is also needed for the description of a cosmological term, and for proper treatment of gauge fixing (see subsect. 4.5). Our action will thus be locally superconformally invariant. In the absence of a cosmological term, it is possible to choose a gauge where $\chi=0$ (with non-trivial ghosts), but we will find it more convenient not to do so. We can also go to Wess-Zumino-type gauges where the superfields have expansions such as

$$H^a(x,\theta,\bar\theta\,)=\theta^\alpha\bar\theta^{\dot\beta}\sigma^b_{\alpha\dot\beta}h_b{}^a+\left(\bar\theta^2\theta^\alpha\psi^a_\alpha+\theta^2\bar\theta^{\dot\alpha}\bar\psi^a_{\dot\alpha}\right)+\theta^2\bar\theta^2A^a,$$

$$\chi(x,\theta)=\tfrac{1}{3}\theta^2B, \tag{B.1}$$

with $h_b{}^a(\sim e_b{}^a-\delta^a_b)$, ψ^a_α, A^a and B the vierbein, gravitino, and axial vector and scalar-pseudoscalar auxiliary fields of component supergravity (up to redefinitions). However, such gauges are not supersymmetric, and will not be used in this paper.

We define chiral-representation "semicovariant" derivatives by:

$$\hat E_{\dot\alpha}=\bar D_{\dot\alpha},\qquad \hat E_\alpha=\mathrm{e}^{-H}D_\alpha\mathrm{e}^{H},\qquad \hat E_a=-i\sigma_a^{\alpha\dot\beta}\left\{\hat E_\alpha,\hat E_{\dot\beta}\right\}. \tag{B.2}$$

This is the direct analog of the super-Yang-Mills covariant derivatives of (A.1b). (It is also analogous to the "chiral-representation" D_A, defined by

$$\bar D_{\dot\alpha}=-\tfrac{1}{2}i\bar\partial_{\dot\alpha},\qquad D_\alpha=\mathrm{e}^{-2U_0}\tfrac{1}{2}i\partial_\alpha\mathrm{e}^{2U_0},\qquad D_a=\partial_a=-i\sigma_a^{\alpha\dot\beta}\left\{D_\alpha,\bar D_{\dot\beta}\right\},$$

where $U_0 = \theta^\alpha \bar{\theta}^{\dot\beta} \sigma^a_{\alpha\dot\beta} i\partial_a$. One could then replace $2U_0 \to 2U = 2U_0 + H$ [9], which is analogous to $e_a{}^b = \delta_a^b + \frac{1}{2} h_a{}^b$. However, in this representation global supersymmetry is not manifest, so we will not use it here. We actually use $e^{2U} = e^{U_0} e^H e^{U_0}$, so U_0 is a "flat" background: cf. (2.3).)

The general supercoordinate transformations also take a form analogous to super-Yang-Mills [see (A.10) and (A.3)], with χ a chiral density (with weight such that $\delta \int d^4x\, d^2\theta \phi^3 = 0$):

$$\eta' = e^{i\Lambda}\eta,$$

$$e^{H'} = e^{i\bar\Lambda} e^H e^{i\Lambda}, \qquad \chi' = \exp\left[i\Lambda - \tfrac{1}{3}(\partial_a \Lambda^a - D_\alpha \Lambda^\alpha)\right](1+\chi),$$

$$\Lambda = \Lambda^A iD_A = \Lambda^a i\partial_a + \Lambda^\alpha iD_\alpha + \Lambda^{\dot\alpha} i\bar{D}_{\dot\alpha}, \qquad \bar\Lambda = \bar\Lambda^A iD_A. \tag{B.3}$$

[Note that $\Lambda^{\dot\alpha} \neq \bar\Lambda^{\dot\alpha} \equiv (\Lambda^\alpha)^\dagger$.] In order to solve the chirality condition on the matter field η,

$$\left[\bar{D}_{\dot\alpha}, \Lambda\right]\eta = 0, \qquad \text{when } \bar{D}_{\dot\alpha}\eta = 0, \tag{B.4}$$

we express Λ^a and Λ^α in terms of a new parameter L^α:

$$\Lambda^a i\partial_a + \Lambda^\alpha iD_\alpha = \left\{\bar{D}^{\dot\beta}, \left[\bar{D}_{\dot\beta}, L^\alpha iD_\alpha\right]\right\}$$

$$\to \Lambda^a = -i\sigma^a_{\alpha\dot\beta}\bar{D}^{\dot\beta}L^\alpha, \qquad \Lambda^\alpha = \bar{D}^2 L^\alpha, \qquad \Lambda^{\dot\alpha} \text{ arbitrary}. \tag{B.5}$$

Due to $\delta H^\alpha \sim \bar\Lambda^\alpha + \cdots$, we can use up $\Lambda^{\dot\alpha}$ to gauge away H^α (and $H^{\dot\alpha}$) without introducing ghosts (see subsect. 4.4). We then have a restricted set of gauge transformations:

$$\delta e^H = i\bar\Lambda e^H - e^H i\Lambda \text{ has no spinor part} \to \Lambda^{\dot\alpha} = e^{-H}\bar\Lambda^{\dot\alpha} = e^{-H} D^2 \bar{L}^{\dot\alpha}. \tag{B.6}$$

By invariance arguments the action is found to be

$$S = -\frac{6}{\kappa^2}\int d^4x\, d^4\theta (1 \cdot e^{-\overleftarrow{H}})^{1/3} \hat{E}^{-1/3}(1+\chi) e^{-H}(1+\bar\chi), \tag{B.7a}$$

where now $H = H^a i\partial_a$ and

$$\hat{E}_A = \left(\hat{E}_{\dot\alpha}, \hat{E}_\alpha, \hat{E}_a\right) = \hat{E}_A{}^B D_B, \qquad \hat{E} = \det \hat{E}_A{}^B. \tag{B.7b}$$

($\overleftarrow{H} = H^A i\overleftarrow{D}_A$ means that the derivative acts to the left.) Due to our choice of weight for χ, the supersymmetric cosmological term $\lambda \int d^4x\, d^2\theta \phi^3 + \text{h.c.}$ has the same form as a self-interaction term for chiral matter. When expanded out in a Wess-Zumino gauge as in (B.1), the action (B.7a) (and the cosmological term) agrees with the usual supergravity action in terms of component fields. For quantization in a supersymmetric gauge, supersymmetric gauge-fixing and Faddeev-Popov ghost terms are added, and the action (B.7a) is expanded in powers of H^a and χ to obtain the supergraph rules [10].

The manifestly hermitian vector-representation semicovariant derivatives are defined as [cf. (A.1a)]:

$$\hat{E}_{\dot{\alpha}} = e^{\overline{W}} \overline{D}_{\dot{\alpha}} e^{-\overline{W}}, \qquad \hat{E}_{\alpha} = e^{-W} D_{\alpha} e^{W}, \qquad \hat{E}_a = -i\sigma_a^{\alpha\dot{\beta}}\left\{\hat{E}_{\alpha}, \hat{E}_{\dot{\beta}}\right\}. \tag{B.8}$$

In this representation ϕ is covariantly chiral:

$$\hat{E}_{\dot{\alpha}}\phi = 0 \to \phi = e^{\overline{W}}\hat{\phi}, \qquad \overline{D}_{\dot{\alpha}}\hat{\phi} = 0. \tag{B.9}$$

As in super-Yang-Mills, we have $e^H = e^W e^{\overline{W}}$, expressing e^W as the "square root" of e^H, analogous to the vierbein being the square root of the metric. The analogy goes further: the vierbein has a larger group of transformations than the metric (local Lorentz transformations in addition to general coordinate transformations). Similarly here, the transformation law for W,

$$e^{W'} = e^{i\overline{\Lambda}} e^{W} e^{-i\hat{K}}, \qquad \eta' = e^{i\hat{K}}\eta,$$

$$\chi' = \exp\left\{i\hat{K} - \tfrac{1}{3}\left[e^{\overline{W}}(\partial_a\Lambda^a - D_\alpha\Lambda^\alpha)\right]\right\}(1+\chi),$$

$$\hat{K} = \overline{\hat{K}} = \hat{K}^A iD_A, \tag{B.10}$$

is compatible with (B.3) and extends it to include the $\hat{K}$ transformation. In fact, $\hat{K}$ can be used to gauge away the imaginary part of W, in which gauge $W = \frac{1}{2}H$. A simple recipe is that the chiral representation can be obtained from the vector representation by making the replacements $W \to H, \overline{W} \to 0, \chi \to \chi, \overline{\chi} \to e^{-H}\overline{\chi}$. In the vector representation the action (B.7a) takes the form

$$S = -\frac{6}{\kappa^2}\int d^4x\, d^4\theta \left(1 \cdot e^{-\overleftarrow{W}}\right)^{1/3}\left(1 \cdot e^{\overleftarrow{\overline{W}}}\right)^{1/3} \hat{E}^{-1/3}(1+\chi)(1+\overline{\chi}). \tag{B.11}$$

The covariant derivatives are expressed in terms of the supervierbein $E_A{}^B$ and superconnection $\phi_{Ab}{}^c$:

$$\nabla_A = E_A + \tfrac{1}{2}\phi_{Ab}{}^c M_c{}^b, \qquad E_A = E_A{}^B D_B. \tag{B.12}$$

Here the Lorentz generators $M_a{}^b$ act on tangent-space indices as

$$\left[\tfrac{1}{2}X_b{}^c M_c{}^b, f_a\right] = X_a{}^b f_b, \qquad \left[\tfrac{1}{2}X_b{}^c M_c{}^b, f_\alpha\right] = X_\alpha{}^\beta f_\beta, \tag{B.13a}$$

for arbitrary antisymmetric X_{ab}, with

$$X_\alpha{}^\beta = \tfrac{1}{4}\sigma^a_{\alpha\dot\gamma}\sigma_b^{\beta\dot\gamma}X_a{}^b, \qquad \tfrac{1}{2}X_a{}^b M_b{}^a = X_\alpha{}^\beta M_\beta{}^\alpha + X_{\dot\alpha}{}^{\dot\beta}\overline{M}_{\dot\beta}{}^{\dot\alpha}. \tag{B.13b}$$

We also use

$$\left[E_A, E_B\right\} = C_{AB}{}^C E_C, \qquad \left[\nabla_A, \nabla_B\right\} = T_{AB}{}^C\nabla_C + \tfrac{1}{2}R_{ABc}{}^d M_d{}^c, \tag{B.14}$$

to define $C_{AB}{}^C$, the supertorsion $T_{AB}{}^C$, and the supercurvature $R_{ABc}{}^d$. In the vector representation (the only representation available before the constraints are solved), the derivatives transform covariantly under general-supercoordinate and superlocal-Lorentz transformations with real superfield parameters:

$$\nabla'_A = e^{iK}\nabla_A e^{-iK}, \qquad K = \overline{K} = K^A iD_A + \tfrac{1}{2}K_a{}^b iM_b{}^a. \tag{B.15}$$

The action is expressed in terms of the determinant of the supervierbein:

$$S = -\frac{6}{\kappa^2}\int d^4x\, d^4\theta\, E^{-1}. \tag{B.16}$$

The covariant derivatives satisfy the (modified) Wess-Zumino constraints

$$T_{\alpha\beta}{}^\gamma = T_{\alpha\beta}{}^{\dot\gamma} = T_{\alpha\dot\beta}{}^{\dot\gamma} = T_{\alpha\beta}{}^c = T_{\alpha b}{}^c = R_{\alpha\dot\beta c}{}^d = 0, \qquad T_{\alpha\dot\beta}{}^c = \tfrac{1}{2}i\sigma^c_{\alpha\dot\beta}, \tag{B.17}$$

as well as further constraints which follow from Bianchi identities $[\nabla_{[A}, [\nabla_B, \nabla_{C)}\}\} = 0$. The constraints are solved by expressing the covariant derivatives in terms of three superfields: the complex supervector W^A, the covariantly chiral scalar ϕ, and a bispinor $N_\alpha{}^\beta$. (Like the imaginary part of W^A, $N_\alpha{}^\beta$ is pure gauge: the former is gauged away by K^A, the latter by $K_a{}^b$. In fact, only H^a and ϕ cannot be completely gauged away by non-derivative transformation laws.) The covariant supervierbeine are given by

$$E_\alpha = N_\alpha{}^\beta\psi\hat{E}_\beta, \qquad E_{\dot\alpha} = \overline{N}_{\dot\alpha}{}^{\dot\beta}\bar\psi\hat{E}_{\dot\beta},$$

$$E_a = -iN_a{}^b\left[\sigma_b^{\alpha\dot\beta}\left\{\psi\hat{E}_\alpha, \bar\psi\hat{E}_{\dot\beta}\right\} - \tfrac{1}{2}\psi\bar\psi\sigma_{[b}^{\alpha\dot\beta}\left(\hat{C}_\alpha{}^c{}_{c]}\hat{E}_{\dot\beta} + \hat{C}_{\dot\beta}{}^c{}_{c]}\hat{E}_\alpha\right)\right], \tag{B.18a}$$

where $\hat{C}_{AB}{}^{C}$, ψ and $N_a{}^b$ are defined by

$$\left[\hat{E}_A, \hat{E}_B\right\} = \hat{C}_{AB}{}^{C}\hat{E}_C,$$

$$\psi = \phi^{1/2}\bar{\phi}^{-1}\left(1\cdot e^{-\bar{W}}\right)^{-1/3}\left(1\cdot e^{\bar{\bar{W}}}\right)^{1/6}\hat{E}^{-1/6},$$

$$N_a{}^b = \tfrac{1}{2}\sigma_a^{\alpha\dot{\beta}}\sigma^b_{\gamma\dot{\delta}}N_\alpha{}^\gamma\bar{N}_{\dot{\beta}}{}^{\dot{\delta}}, \qquad \det N_\alpha{}^\beta = 1. \tag{B.18b}$$

[$\hat{E}_A$ and ϕ are the vector-representation quantities described above: see (B.8) and (B.9).] Substituting into (B.16), we get the action of (B.11). In addition to the transformation laws (B.10) of W and ϕ, we have

$$\delta N_\alpha{}^\beta = i\hat{K}N_\alpha{}^\beta - K_\alpha{}^\gamma N_\gamma{}^\beta - N_\alpha{}^\gamma\left(\tfrac{1}{2}e^{-W}D_{(\gamma}\bar{\Lambda}^{\beta)}\right). \tag{B.19}$$

The superconnections are given by

$$\phi_{\alpha bc} = -C_{\alpha[bc]}, \qquad \phi_{abc} = -i\sigma_a^{\alpha\dot{\beta}}\left(E_\alpha\phi_{\dot{\beta}bc} + E_{\dot{\beta}}\phi_{\alpha bc} + \phi_{\dot{\alpha}[b|}{}^d\phi_{\beta d|c]}\right). \tag{B.20}$$

The supertorsions and supercurvatures can be computed from their definitions (B.14). In the chiral representation (with $W \to H$, $\bar{W} \to 0$, and also $N_\alpha{}^\beta \to \delta_\alpha^\beta$, $\bar{N}_{\dot{\alpha}}{}^{\dot{\beta}} \to \delta_{\dot{\alpha}}^{\dot{\beta}}$), the covariant derivatives transform as

$$\delta\nabla_A = [i\Lambda, \nabla_A] - L_A{}^B\nabla_B,$$

$$L_{\dot{\alpha}}{}^{\dot{\beta}} = \tfrac{1}{2}\bar{D}_{(\dot{\alpha}}\Lambda^{\dot{\beta})}, \qquad L_\alpha^\beta = \tfrac{1}{2}\left(e^{-H}D_{(\alpha}\bar{\Lambda}^{\beta)}\right), \qquad L_a{}^b = \tfrac{1}{2}\sigma_a^{\alpha\dot{\beta}}\sigma^b_{\gamma\dot{\delta}}L_\alpha{}^\gamma L_{\dot{\beta}}{}^{\dot{\delta}}, \qquad \text{rest} = 0. \tag{B.21}$$

$L_\alpha{}^\beta$ has thus replaced $K_\alpha{}^\beta$ as a Λ-dependent Lorentz transformation, satisfying a chiral-representation hermiticity condition of the form

$$\bar{f} = e^H f e^{-H}, \tag{B.22}$$

as does the chiral-representation ∇_A.

The Bianchi identities of the theory imply relations between the various parts of $T_{AB}{}^C$ and $R_{ABc}{}^d$. When solved [33] they allow all of $T_{AB}{}^C$ and $R_{ABc}{}^d$ to be expressed in terms of three basic tensor superfields—R (chiral), G_a (real) and $W_{\alpha\beta\gamma}$ (chiral and totally symmetric)—which can in turn be expressed in terms of W^A, ϕ, and $N_\alpha{}^\beta$. The solution to the Bianchi identities can be written as

$$\{\nabla_\alpha, \nabla_\beta\} = -\bar{R} M_{\alpha\beta},$$

$$\{\nabla_\alpha, \bar{\nabla}_{\dot\beta}\} = \tfrac{1}{2} i \nabla_{\alpha\dot\beta},$$

$$\left[\nabla_\alpha, \tfrac{1}{2} i \nabla_{\beta\dot\gamma}\right] = -\tfrac{1}{2} C_{\alpha\beta}\left[-\bar{R}\bar{\nabla}_{\dot\gamma} + G^{\delta}{}_{\dot\gamma}\nabla_\delta + \left(\nabla^\delta G^{\varepsilon}{}_{\dot\gamma}\right) M_{\delta\varepsilon} + \bar{W}_{\dot\gamma\dot\delta}{}^{\dot\varepsilon}\bar{M}_{\dot\varepsilon}{}^{\dot\delta}\right]$$
$$-\tfrac{1}{2}\left(\bar{\nabla}_{\dot\gamma}\bar{R}\right) M_{\alpha\beta}, \qquad \text{(B.23a)}$$

where

$$\nabla_{\alpha\dot\beta} = \sigma^a_{\alpha\dot\beta}\nabla_a, \qquad G_{\alpha\dot\beta} = \sigma^a_{\alpha\dot\beta} G_a, \qquad M_\alpha{}^\beta = \tfrac{1}{4}\sigma^a_{\alpha\dot\gamma}\sigma_b^{\beta\dot\gamma} M_a{}^b.$$

$[\nabla_{\alpha\dot\beta}, \nabla_{\gamma\dot\delta}]$ can be found straightforwardly by substituting $-2i\{\nabla_\alpha, \bar{\nabla}_{\dot\beta}\}$ for $\nabla_{\alpha\dot\beta}$ and applying (B.23a). We also have the additional Bianchi identities

$$\bar{\nabla}^{\dot\beta} G_{\alpha\dot\beta} = \nabla_\alpha R, \qquad \nabla^\alpha W_{\alpha\beta\gamma} = \tfrac{1}{2} i \nabla_{(\beta}{}^{\dot\alpha} G_{\gamma)\dot\alpha}. \qquad \text{(B.23b)}$$

The field equations are

$$R = G_a = \nabla^\alpha W_{\alpha\beta\gamma} = 0. \qquad \text{(B.24)}$$

In order to perform quantum calculations, one needs to express all quantities in terms of H^a and χ. The actual steps are as follows: (1) compute $\hat{E}_A$ (to desired order in H^a) from (B.2), and thus $\hat{E}_A{}^B$ and $\hat{E}$ from (B.7b), (2) compute $\hat{C}_{AB}{}^C$ and ψ from (B.18b), (3) compute E_A from (B.18a) ($N_\alpha{}^\beta = \delta_\alpha^\beta$), (4) compute $C_{AB}{}^C$ from (B.14), (5) compute $\phi_{Ab}{}^c$ from (B.20), and (6) compute R, $G_{\alpha\dot\beta}$ and $W_{\alpha\beta\gamma}$ from (B.23a) [with ∇_A given by (B.12)].

Appendix C

CALCULATION OF THE GHOST LAGRANGIAN

In this appendix we present more details of the calculations of the ghost lagrangian presented in sect. 5. The calculations consist primarily of introducing many catalyst ghosts, most of which cancel each other out (at all loop orders).

In the case of the first-generation third ghost $\psi_{3\alpha}$, performing the shift (5.13) introduces the gauge invariances

$$\delta\psi_{3\alpha} = \bar{\mathcal{D}}^2\mathcal{D}_\alpha L + \mathcal{D}_\alpha\Lambda, \qquad \delta\psi_3 = -L, \qquad \delta\phi_3 = -\Lambda\left(\bar{\mathcal{D}}_{\dot\beta}\Lambda = 0\right). \qquad \text{(C.1)}$$

For our gauge-fixing functions we choose

$$F_1 = \mathcal{D}^\alpha\left[\psi_{3\alpha} + \bar{\mathcal{D}}^2\mathcal{D}_\alpha\left(a\psi_3 + b\bar{\psi}_3\right) + c\mathcal{D}_\alpha\phi_3\right] + \bar{\mathcal{D}}^2\mathcal{D}^2\left(d\psi_3 + e\bar{\psi}_3\right),$$

$$F_2 = \bar{\mathcal{D}}^2\left(f\psi_3 + g\bar{\psi}_3\right), \tag{C.2}$$

so that F_1 is a general, complex superfield and F_2 is chiral. After performing the shift (5.13) on (5.12), we add all possible gauge-fixing terms constructed from F_1 and F_2 which are of the right dimension. This includes nonlocal terms constructed from the scalar projection operators of (4.2). For example, we have terms $(F_1)^2, F_1\bar{F}_1, \bar{F}_2\Box F_2, F_1\mathcal{D}^2F_2, F_1\Pi_{0+}F_1$. In introducing all such terms with arbitrary constants, and with the arbitrary constants introduced in (C.2), we have more than enough parameters to fix the gauges (C.1) so that the lagrangian takes the form

$$-i\bar{\psi}_3^{\dot\beta}\mathcal{D}_{\alpha\dot\beta}\psi_3^\alpha + \tfrac{5}{8}\phi_3\Box\bar{\phi}_3 - \tfrac{1}{8}\left(\psi_3\Box^2\psi_3 + \bar{\psi}_3\Box^2\bar{\psi}_3\right), \tag{C.3a}$$

or, after trivial redefinitions and renormalizations of the fields,

$$\bar{\psi}_3^{\dot\beta}i\mathcal{D}_{\alpha\dot\beta}\psi_3^\alpha + \sum_{i=1}^{4} V_{3i}\Box V_{3i} + \sum_{i=1}^{3} \phi_{3i}\bar{\phi}_{3i}, \qquad \left(V_{3i} = \bar{V}_{3i}\right). \tag{C.3b}$$

One now obtains "second-generation" ghosts: first, second and third ghosts arising from the gauge fixing of the invariances (C.1) of the first-generation ghosts. These ghosts are general (complex) and chiral scalars, corresponding to L and Λ in (C.1), and F and F_2 in (C.2). In order to simplify the second-generation lagrangians, one performs shifts which mix the general and chiral ghosts (to eliminate crossterms) in a way consistent with dimensional analysis and chirality conditions, and introduces shifts as in (4.3). One thus obtains third-generation ghosts from introducing these last catalyst fields. Since each generation has maximum spin $\frac{1}{2}$ less than the previous generation, the third generation is the last. The net result, after putting all of the lagrangian which is derived from $\psi_{3\alpha}$ in the canonical form $\bar{\psi}_3^{\dot\beta}i\mathcal{D}_{\alpha\dot\beta}\psi_3^\alpha + \Sigma V\Box V + \Sigma\phi\bar{\phi}$, is that all the catalysts V and ϕ cancel, as did the catalysts η in the example (4.8a).

The simplification of the lagrangian for $\psi_{1\alpha}$ and $\psi_{2\alpha}$ of (5.14) is performed similarly, except now we also have chiral spinor ghosts ϕ_α, arising from the gauge invariance $\Lambda_{1\alpha}$ and $\Lambda_{2\alpha}$ of (5.16). All other ghosts are catalysts. In order to eliminate crossterms between the ϕ_α's and catalyst scalars ψ, we make shifts of the form $\psi \to \psi + a\bar{\mathcal{D}}^{\dot\alpha}\bar{\phi}_{\dot\alpha}$ or $\psi \to \psi + a\Box^{-1}\bar{\mathcal{D}}^{\dot\alpha}\bar{\phi}_{\dot\alpha}$, and $\phi_\alpha \to \phi_\alpha + b\bar{\mathcal{D}}^2\mathcal{D}_\alpha\bar{\psi}$ or $\phi_\alpha \to \phi_\alpha + b\Box_+^{-1}\bar{\mathcal{D}}^2\mathcal{D}_\alpha\psi$ (depending on the dimensionalities of the ghosts), which merely serve to "rotate" our ϕ_α-ψ basis so that the kinetic term is diagonal. After performing these shifts and those of the kind applied to $\psi_{3\alpha}$, and fixing gauges for all generations appropriately, we find that all scalar catalysts cancel, leaving $(\bar{\psi}_2^{\dot\beta}i\mathcal{D}_{\alpha\dot\beta}\psi_1^\alpha$

$+ \text{h.c.}) + \Sigma_{i=1}^{2}(\phi_i^{\alpha}\mathcal{D}^2\phi_{i\alpha} + \text{h.c.})$, or, equivalently, $\Sigma_{i=1}^{2}[\bar{\psi}_i^{\dot{\beta}} i\mathcal{D}_{\alpha\dot{\beta}}\psi_i + (\phi_i^{\alpha}\mathcal{D}^2\phi_{i\alpha} + \text{h.c.})]$ for the part of the ghost lagrangian derived from $\psi_{1\alpha}$ and $\psi_{2\alpha}$. (We have also dropped trivial terms of the form $\int d^4\theta\, \phi^{\alpha}\mathcal{D}^2\Box_{+}^{-1}\phi_{\alpha} = \int d^2\theta\, \phi^{\alpha}\phi_{\alpha}$ which do not depend on the background fields.) The projection-operator argument applied to (4.1b) also explains the cancellation of the catalysts for $\psi_{1\alpha}$ and $\psi_{2\alpha}$, except that here we have $\Sigma a_i \Pi_i$ with not all $a_i \neq 0$, corresponding to the gauge invariance $\delta\psi_{\alpha} = \Lambda_{\alpha}$, which results in the ghosts ϕ_{α}. However, since $\psi_{1\alpha}$ and $\psi_{2\alpha}$ have quantum interactions, we must keep the scalars ψ_1 and ψ_2 of (5.15) when performing higher-loop calculations (although their one-loop contributions are canceled by the corresponding ghosts ψ_1' and ψ_2' of opposite statistics).

References

[1] C.F. Cooke, Ph.D. thesis, University of North Carolina (1964);
A. Chester, Phys. Rev. 143 (1966) 1275;
B.S. DeWitt, Phys. Rev. 162 (1967) 1239;
D.J. Gross and R. Jackiw, Phys. Rev. 166 (1968) 1287;
F.A. Berends and R. Gastmans, Nucl. Phys. B88 (1975) 99;
M.T. Grisaru, P. van Nieuwenhuizen and C.C. Wu, Phys. Rev. D12 (1975) 397;
M.T. Grisaru and H.N. Pendleton, Nucl. Phys. B124 (1977) 81

[2] D.M. Capper, G. Leibbrandt and M. Ramón Medrano, Phys. Rev. D8 (1973) 4320;
M.R. Brown, Nucl. Phys. B56 (1973) 194;
D.M. Capper, M.J. Duff and L. Halpern, Phys. Rev. D10 (1974) 461;
D.M. Capper and M.J. Duff, Nucl. Phys. B82 (1974) 147;
M. Nouri-Moghadam and J.G. Taylor, J. Phys. A8 (1975) 334;
D.M. Capper, Nuovo Cim. A25 (1975) 29;
D.M. Capper and M.A. Namazie, Nucl. Phys. B142 (1978) 535;
D.M. Capper, J. Phys. A13 (1980) 199

[3] G. 't Hooft and M. Veltman, Ann. Inst. H. Poincaré 20 (1974) 69;
S. Deser and P. van Nieuwenhuizen, Phys. Rev. Lett. 32 (1974) 245; Nuovo Cim. Lett. 11 (1974) 218; Phys. Rev. D10 (1974) 401, 411;
S. Deser, H.-S. Tsao and P. van Nieuwenhuizen, Phys. Lett. 50B (1974) 491; Phys. Rev. D10 (1974) 3337;
M.T. Grisaru, P. van Nieuwenhuizen and C.C. Wu, Phys. Rev. D12 (1975) 1813, 3203;
P. van Nieuwenhuizen, Spin and torsion in quantum gravity, *in* Relativity, strings and gravity, Caracas (1975);
M. Nouri-Moghadam and J.G. Taylor, Proc. Roy. Soc. A344 (1975) 87;
P. van Nieuwenhuizen and J.A.M. Vermaseren, Phys. Lett. 65B (1976) 263;
M.T. Grisaru, P. van Nieuwenhuizen and J.A.M. Vermaseren, Phys. Rev. Lett. 37 (1976) 1662;
P. van Nieuwenhuizen and J.A.M. Vermaseren, Phys. Rev. D16 (1977) 298;
S.M. Christensen and M. Duff, Phys. Lett. 76B (1978) 571; Nucl. Phys. B154 (1979) 301;
M. Fischler, Phys. Rev. D20 (1979) 396;
E. Sezgin and P. van Nieuwenhuizen, Phys. Rev. D22 (1980) 301;
D. Atkatz, Phys. Lett. 93B (1980) 318;
S.M. Christensen, M.J. Duff, G.W. Gibbons and M. Roček, Phys. Rev. Lett. 45 (1980) 161;
M.J. Duff and P. van Nieuwenhuizen, Phys. Lett. 94B (1980) 179;
M.K. Fung, P. van Nieuwenhuizen and D.R.T. Jones, Phys. Rev. D22 (1980) 2995

[4] R.E. Kallosh, O.V. Tarasov and I.V. Tyutin, Nucl. Phys. B137 (1978) 145

[5] J. Zak, Brandeis Ph.D. thesis (1980);
M.T. Grisaru and J. Zak, Phys. Lett. 90B (1980) 237

[6] M.T. Grisaru, W. Siegel and M. Roček, Nucl. Phys. B159 (1979) 429

[7] L.F. Abbott and M.T. Grisaru, Nucl. Phys. B169 (1980) 415;
M.T. Grisaru, M. Roček and W. Siegel, Phys. Rev. Lett. 45 (1980) 1063;
W. E. Caswell and D. Zanon, Phys. Lett. 100B (1981) 152;
M. Roček and P.K. Townsend, Phys. Lett. 96B (1980) 72;
W.E. Caswell and D. Zanon, Nucl. Phys. B182 (1981) 125;
M.T. Grisaru, M. Roček and W. Siegel, Nucl. Phys. B183 (1981) 141
[8] D.M. Capper and M. Ramón Medrano, Phys. Rev. D9 (1974) 1641;
K.S. Stelle, Phys. Rev. D16 (1976) 953
[9] W. Siegel, Supergravity superfields without a supermetric, Harvard preprint HUTP-77/A068 (November, 1977); The superfield supergravity action, Harvard preprint HUTP-77/A080 (December, 1977); A polynomial action for a massive, self-interacting chiral superfield coupled to supergravity, Harvard preprint HUTP-77/A077 (December, 1977); A derivation of the supercurrent superfield, Harvard preprint HUTP-77/A089 (December, 1977);
W. Siegel and S.J. Gates, Jr., Nucl. Phys. B147 (1979) 77
[10] W. Siegel, Phys. Lett. 84B (1979) 197
[11] M.A. Namazie and D. Storey, Nucl. Phys. B157 (1979) 170
[12] J.G. Taylor, Quantization of superfield supergravity, King's College preprint (February, 1979; revised May, 1979)
[13] W. Siegel, Phys. Lett. 93B (1980) 170
[14] M.T. Grisaru, Background superfield method, *in* Superspace and supergravity, ed. S.W. Hawking and M. Roček (Cambridge University Press, Cambridge 1981);
M. Roček, An introduction to superspace and supergravity, *ibid*;
W. Siegel, The wraiths of graphs, *ibid*.
[15] G. Ecker and J. Honerkamp, Nucl. Phys. B52 (1973) 211; 62 (1973) 509
[16] J. Honerkamp, M. Schlindwein and M. Scheunert, Nucl. Phys. B95 (1975) 397
[17] W. Siegel, Nucl. Phys. B142 (1978) 301;
S.J. Gates, Jr. and W. Siegel, Nucl. Phys. B163 (1980) 519
[18] O. Piguet and A. Rouet, Nucl. Phys. B108 (1976) 265
[19] M. Roček, private communication;
M. Fischler, Phys. Rev. D20 (1979) 1842
[20] W. Siegel, Phys. Lett. 85B (1979) 333
[21] N.K. Nielsen, Nucl. Phys. B140 (1978) 499;
R.E. Kallosh, Nucl. Phys. B141 (1978) 141
[22] J. Wess and B. Zumino, Phys. Lett. 79B (1978) 394
[23] S. Coleman, Secret symmetry, *in* Laws of hadronic matter, Erice, 1973 (Academic Press, New York, 1975), p. 139
[24] S. Ferrara and B. Zumino, Nucl. Phys. B134 (1978) 301
[25] V. Ogievetsky and E. Sokatchev, Nucl. Phys. B124 (1977) 309
[26] S.J. Gates, Jr., Nucl. Phys. B184 (1981) 381
[27] M.T. Grisaru and W. Siegel, Supergraphity, part II, in preparation
[28] M.J. Duff, M.T. Grisaru and W. Siegel, in preparation
[29] V.N. Pervushin, Theor. Math. Phys. 22 (1975) 203;
D.I. Kazakov, V.N. Pervushin and S.V. Pushkin, Theor. Math. Phys. 31 (1977) 389;
D.I. Kazakov and S.V. Pushkin, Lett. Math. Phys. 2 (1978) 195
[30] J. Wess, Supersymmetry-supergravity, *in* Topics in quantum field theory and gauge theories, Salamanca, 1977 (Springer-Verlag, Berlin, 1978), p. 81
[31] V.P. Akulov, D.V. Volkov and V.A. Soroka, JETP Lett. 22 (1975) 187;
J. Wess and B. Zumino, Phys. Lett. 66B (1977) 361;
R. Grimm, J. Wess and B. Zumino, Phys. Lett. 73B (1978) 415;
J. Wess and B. Zumino, Phys. Lett. 74B (1978) 51
[32] V.I. Ogievetsky and E. Sokatchev, The gravitational axial superfield and the formalism of differential geometry, JINR preprint E2-12511 (June, 1979), submitted to Yad. Fiz.
[33] R. Grimm, J. Wess and B. Zumino, Nucl. Phys. B152 (1979) 255