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COSMOLOGICAL BARYON GENERATION AT LOW TEMPERATURES*

Mark CLAUDSON

Lawrence Berkeley Laboratory and Department of Physics, University of California, Berkeley, CA94720, USA

Lawrence J. HALL

Lyman Laboratory of Physics, Harvard University, Cambridge, MA02138, USA

Ian HINCHLIFFE

Lawrence Berkeley Laboratory, University of California, Berkeley, CA94720, USA

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Mechanisms for generating the net baryon number of the universe which do not involve grand unification are considered. Detailed calculations in a simple extension of the standard SU(3)$\times$ SU(2)$\times$U(1) model are used to show that physics at temperatures of order 1 TeV can account for the observed baryon to entropy ratio. Some comments regarding alternatives to this simple model and some speculations concerning the implications for cosmology are offered.

1. Introduction

In this paper we will consider mechanisms for generating the net baryon number of the universe which do not rely on grand unified theories or on the existence of superheavy particles with masses of order 10^{14} GeV. Instead we will concentrate on mechanisms by which particles in the 1 TeV mass range could be responsible for the presently observed baryon number.

In the context of the standard big bang cosmology [1], the present baryon density n_{B} can be determined experimentally by two methods [2]. In the first of these, galactic masses and the deceleration parameter are used to determine lower and upper limits, respectively, for the mass density ρ of the universe. Assuming that baryons dominate the mass of the universe and using the observed photon density n_γ of the 3 K background radiation, these limits correspond to the constraint $n_{\mathrm{B}}/n_\gamma = 10^{-8.9\pm1.6}$. The large uncertainty here is due to uncertainties in the Hubble constant ($H_0 = 50$–$100\ \mathrm{km}\cdot\mathrm{sec}^{-1}\,\mathrm{Mpc}^{-1}$) and in the ratio $\Omega = \rho/\rho_c$ ($\Omega = 0.005$–2)

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of the mass density to the critical density $\rho_c = 3H_0^2/8\pi G_N$ (where G_N denotes Newton's constant). Recent observations [3] suggest that ρ is near ρ_c with much of the mass in a form other than baryons. For this reason, we will use the value $n_B/n_\gamma = 10^{-9.9\pm0.9}$ obtained from big bang nucleosynthesis [4]. The ratio n_B/n_γ is not constant as the universe cools since the density of photons is increased by processes such as e^+e^- annihilation and since baryons and antibaryons were both roughly as abundant as photons at temperatures greater than 1 GeV. However, if baryon number is approximately conserved, the ratio $\Delta n_B/s$ of the net baryon number density $\Delta n_B = n_B - n_{\bar{B}}$ to the entropy density s will be constant unless phase transitions or non-equilibrium reactions generate an appreciable amount of entropy. At the present, $s = 7.02 n_\gamma$ so that, neglecting the present density of antibaryons, $\Delta n_B/s = 10^{-10.8\pm0.9}$.

If baryon number (B) is absolutely conserved, the net baryon number of the universe must be chosen as an initial condition, and the present value of $\Delta n_B/s$ is inexplicable. However, if baryon number is not conserved, a universe with zero initial excess of baryons over antibaryons may evolve into one with $\Delta n_B \neq 0$ provided that two additional criteria are satisfied [2, 5–7]. Firstly, charge conjugation (C) and time reversal (CP by CPT) must be violated, and the violations of these symmetries must be manifest in asymmetries in the cross sections or decay rates for baryon number violating reactions. Secondly, some species of particle which participates in $\Delta B \neq 0$ reactions must be out of thermodynamic equilibrium since Δn_B vanishes in equilibrium when baryon number is violated. Indeed, in the absence of appreciable deviations from equilibrium, the dominant effect of $\Delta B \neq 0$ reactions is to damp any non-zero initial value for Δn_B. These particles must interact rather weakly to prevent decays or annihilations from maintaining equilibrium as the temperature falls below their mass. In particular, the reaction rates for these processes must be less than the expansion rate $\dot{R}/R = -\dot{T}/T$ at temperatures T of order the particle mass. (Here R is the Robertson–Walker scale factor and dots denote time derivatives.) The assumption that $\Delta n_B = 0$ prior to the era in which CP and B violating asymmetries generate baryon number will be justified if any primordial baryon number is diluted by entropy generating phenomena or damped by $\Delta B \neq 0$ reactions. Following this era, $\Delta B \neq 0$ interactions will decouple so that $\Delta n_B/s$ remains roughly constant between the time of baryon generation and the present.

Shortly after the introduction of grand unified theories [8] (GUTs), it was realized that these theories had all the features needed to account for the observed baryon number, and several authors have explored this possibility [2, 5–7, 9]. For example, the simplest GUT (SU(5) with 24- and 5-dimensional Higgs representations and three generations) has baryon number violation in the decays of superheavy (10^{14} GeV) gauge bosons and Higgs scalars and CP violation from phases in the Yukawa couplings of the Higgs. Unfortunately the baryon excess obtained in this minimal model ($\Delta n_B/n_\gamma \sim 10^{-16}$) is too small to account for that observed [10].

However, more complicated GUTs with additional fermion or Higgs representations can produce a sufficiently large baryon excess.

In this paper we adopt a radically different approach and explore mechanisms for generating Δn_B which do not involve superheavy particles. Although GUTs can account for the observed excess, there are several reasons to consider alternative mechanisms in which baryon number is generated at lower temperatures than in GUTs. Because baryon generation in GUTs takes place at very high temperatures, the baryon excess produced by GUTs is vulnerable to dilution by entropy generating phenomena which may occur after the GUT phase transition. For example, if inflation [11] were to occur after the GUT phase transition, the baryon excess generated by GUTs would be diluted, and it would be necessary to create the observed excess at lower temperatures. Other potentially dangerous sources of entropy include non-inflationary phase transitions or non-equilibrium decays of particles at intermediate mass scales, for example, in technicolor theories [12]. Also, we note that in some supersymmetric models of inflation, the phase transition does not reheat the universe sufficiently to produce superheavy particles abundantly [13]. In such models, baryons must be generated by a mechanism which does not involve superheavy particles. Finally, given the failure to observe proton decay [14], the experimental evidence for GUTs is limited to the prediction of the weak mixing angle [15]. With this in mind, it is worthwhile to consider alternatives to GUT baryon generation, especially since the existence of these alternatives may open new possibilities for the evolution of the universe.

We will now consider the general features needed for baryon generation at a temperature of order 1 TeV. In the standard big bang cosmology, the expansion rate is given by [1]

$$\frac{\dot{R}}{R}=\sqrt{\tfrac{8}{3}\pi G_N \rho}\,. \tag{1.1}$$

The energy density ρ is assumed to be dominated by relativistic species so that, at a temperature T and in units where Boltzmann's constant equals one,

$$\rho=\tfrac{1}{30}\pi^2 g_* T^4\,. \tag{1.2}$$

Here g_* is an effective number of degrees of freedom equal to the number of bosonic degrees of freedom plus $\frac{7}{8}$ of the number of fermionic degrees of freedom. In the standard $SU(3)\times SU(2)\times U(1)$ model with a single Higgs doublet and three generations of quarks and leptons, we have $g_*=106.75$ and

$$\dot{R}/R=-\dot{T}/T=T^2/K\,, \tag{1.3}$$

with $K=7.15\times 10^{17}$ GeV. Particles with a mass $M\sim 1$ TeV which are not neutral under $SU(3)\times SU(2)\times U(1)$ have reaction rates for annihilation or decay which are at least of order $\alpha^2 M$ at temperatures of order their mass. Because these rates are much larger than the expansion rate for $T\sim 1$ TeV, any model which hopes to generate Δn_{B} at this temperature must contain particles with no $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ gauge interactions. (This explains the failure of the model of ref. [16].) In the next section, we will consider the generation of baryon number in an extension of the standard model which contains neutral singlet Majorana fermions.

Of course, to generate a baryon excess, baryon number must be violated, and the presence of $\Delta B\neq 0$ interactions at low energies is severely constrained by the failure to observe any $\Delta B\neq 0$ processes in nature. Although the strongest of these constraints is that from the proton lifetime [14], this constraint is easily satisfied by making the proton absolutely stable. In the models considered here, lepton number conservation is imposed as a symmetry, and all particles beyond those present in the standard model have masses greater than 1 GeV. All final states for $|\Delta B|=1$ nucleon decay are then forbidden by either lepton number or kinematics. This solution is somewhat unattractive, both because it involves imposing a global symmetry by hand and because the presence of baryon number violation without lepton number violation is rather unnatural from the perspective of grand unification. However there are examples of GUTs in which only one of baryon and lepton numbers appears as a global symmetry of the effective theory at energies much less than the GUT scale [17].

In addition to $|\Delta B|=1$ nucleon decays, the presence of baryon number violation may lead to potentially disastrous rates for $|\Delta B|=2$ transitions such as neutron-antineutron oscillations, $|\Delta B|=2$ nucleus decays and $\mathrm{pp}\to\mathrm{K}^+\mathrm{K}^+$. Such processes are not forbidden by lepton number and proceed through six-quark operators of the generic form $C(\bar{q}^{\mathrm{c}}q)^3$ where C is a model-dependent coefficient. The failure to observe free neutron–antineutron transitions with an oscillation time less than 10^7 sec (ref. [18]) imposes the constraint

$$C\langle\bar{n}|(\bar{q}^{\mathrm{c}}q)^3|n\rangle\leq 7\times 10^{-32}\,\mathrm{GeV}\,. \tag{1.4}$$

In the bag model, the matrix element appearing here has been estimated to be $\langle n|(\bar{q}^{\mathrm{c}}q)^3|n\rangle\approx 10^{-5}\,\mathrm{GeV}^6$ with the exact value depending on the spin structure of the operator [19], and the resulting bound on the coefficient is $C\leq 7\times 10^{-27}\,\mathrm{GeV}^5$. The failure to observe $|\Delta B|=2$ decays of nuclei may also be interpreted as a constraint on the coefficients of six-quark operators. Present data [20] limit the lifetime for such a decay to be greater than 10^{32} years and lead to the constraint $C\leq 10^{-27}\,\mathrm{GeV}^5$ with some uncertainty arising from nuclear wave functions.

In the next section, we consider the generation of baryon number in a simple extension of the standard model. A detailed discussion of this model leads to rate equations for the evolution of Δn_{B}, and these equations are numerically integrated to show that the observed value of $\Delta n_{\mathrm{B}}/s$ can be obtained from processes occurring at temperatures of order 1 TeV. In sect. 3, we discuss alternatives to the model of sect. 2 with emphasis on the use of supersymmetry to motivate the appearance of new particles. Finally, sect. 4 summarizes the conclusions and speculates on possible implications for cosmology.

2. Baryon generation in a simple model

In this section, we consider the generation of the cosmological baryon number in a simple extension of the standard $SU(3)\times SU(2)\times U(1)$ model which contains a single weak doublet Higgs field and three generations of quarks and leptons. As noted in the introduction, the generation of baryon number requires the existence of additional particles and interactions not present in the standard model. In particular, baryon number violating interactions can produce a net baryon number at temperatures of order 1 TeV only if some particle species is neutral with respect to $SU(3)\times SU(2)\times U(1)$. Here this requirement will be met by the inclusion of two massive Majorana fermions N_r $(r=1,2)$ which have no gauge interactions. To accommodate the need for baryon number violation, we also include a scalar field Φ transforming as an SU(3) triplet, SU(2) singlet with weak hypercharge $-\frac{1}{3}$. In a two-component spinor notation, the new fields N_r and Φ have the Yukawa couplings

$$\begin{aligned}\mathscr{L}_{N\Phi} &= A_{ra}N_r D_{a\alpha}\Phi^{\alpha} + B_{ab}U_{a\alpha}D_{b\beta}\Phi^{*}_{\gamma}\varepsilon^{\alpha\beta\gamma}\\ &\quad+\tfrac{1}{2}C_{ab}Q_a^{\alpha m}Q_b^{\beta n}\Phi^{\gamma}\varepsilon_{\alpha\beta\gamma}\varepsilon_{mn}\,,\end{aligned} \tag{2.1}$$

in which a, b are generation indices, while α, β, γ and m, n are SU(3) and SU(2) indices, respectively. Also, Q denotes the left-handed quark (SU(2) doublet) fields, while U and D are the left-handed antiquark (SU(2) singlet) fields. Finally, the coupling constants A, B and C are arbitrary complex matrices. In addition to these interactions, we include Majorana masses M_r of order 1 TeV for the N_r along with a positive mass-squared M_Φ^2 of order $(10^2\text{–}10^3\ \text{TeV})^2$ and quartic interactions for the color triplet scalar Φ. When the interactions of the standard model are also included, the action is the most general consistent with all symmetries (including lepton number) and renormalizability.

The Yukawa couplings (2.1) admit a baryon number symmetry under which the baryon number of the left-handed N_r fields is 1 while that of Φ is $-\frac{2}{3}$. This symmetry acts in a chiral manner on the four-component Majorana fields N_r and is broken by the Majorana mass terms for these fields. Because the one-particle states for massive Majorana fields do not form eigenstates of chirality, it is more convenient to assign the N_r baryon number zero. Baryon number is then violated by the Yukawa couplings (2.1). The existence of this renormalizable baryon number violation at low energies is restricted by the failure to observe any $\Delta B\neq 0$ processes. In the present model, $|\Delta B|=1$ nucleon decay does not occur since all kinematically allowed final states are forbidden by lepton number conservation. On the other hand, graphs such as that shown in fig. 1 contribute to $|\Delta B|=2$ processes. Taking the couplings A, B and C to be of order λ, the limits on $|\Delta B|=2$ processes discussed in the introduction lead to the constraint

$$\frac{\lambda^4}{M_1 M_\Phi^4}\leq 10^{-27}\ \text{GeV}^{-5}\,, \tag{2.2}$$

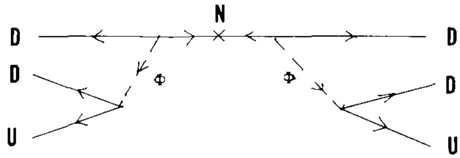

Fig. 1. A typical lowest order graph for the $|\Delta B| = 2$ transition $3q \leftrightarrow 3\bar{q}$. Arrows on (solid) fermion lines refer to chirality while those on (dashed) scalar lines refer to the flow of color triplet charge. The cross denotes an insertion of the Majorana N mass.

in which M_1 is the lighter N mass. Since only λ/M_Φ is relevant in what follows, we fix M_Φ to be 300 TeV. The constraint ($\lambda^4/M_1 \leq 10^{-5}\,\mathrm{GeV}^{-1}$) is then satisfied for an interesting range of values for λ and M_r.

The model considered here contains *CP* violation from phases in the Yukawa couplings (2.1) and also from phases in the quark mass matrix [21]. Contributions to the processes considered below which involve quark masses m_q are suppressed by powers of m_q/M_r and can be neglected. When quark masses are neglected, the couplings B and C can be chosen real and diagonal by field redefinitions. However, taking the Majorana masses M_r to be real, the phases in A cannot be removed when the number of N_r is greater than one. These remaining phases produce *CP* and B violating asymmetries in the decay rates and scattering cross sections of the N_r. In the following, the total rates and the asymmetries for these processes are computed in the limit of large M_Φ and zero quark masses. Also, we neglect the mass M_1 of the lighter Majorana fermion (N_1) in processes which involve both N_1 and N_2.

The presence of $\Delta B \neq 0$ interactions in this model is manifest in the lowest order decays of the N_r. As shown in fig. 2, the final states for these decays can have $B = \pm 1$ or $B = 0$. Adding figs. 2a and 2b, the total rate for $\Delta B \neq 0$ decays of N_r is

$$\begin{aligned} \Gamma_r &= \Gamma_{N_r \to qqq} + \Gamma_{N_r \to \bar{q}\bar{q}\bar{q}} \\ &= \frac{M_r^5}{512\pi^3 M_\Phi^4} \sum_{abc} \left[|A_{ra}B_{bc}|^2 + \tfrac{1}{2} A_{ra}B_{bc}A_{rc}^* B_{ba}^* + |A_{ra}C_{bc}^*|^2 \right]. \end{aligned} \tag{2.3}$$

In addition, the rate for the $\Delta B = 0$ decay of fig. 2c is

$$\Gamma_{12} = \Gamma_{N_2 \to N_1 q\bar{q}} = \frac{M_2^5}{1024\pi^3 M_\Phi^4} \sum_{ab} |A_{2a}A_{1b}^*|^2 . \tag{2.4}$$

Neglecting the mass of N_1, the lowest order asymmetry in the $\Delta B \neq 0$ decay of N_2 is due to the graph shown in fig. 3a. As required by unitarity for contributions to *CP* violating asymmetries in decay rates or cross sections, this graph contains a loop with a physical (on-shell) intermediate state. Permutation of the identical particles in the final state produces the two distinct contributions shown schematically in figs. 3b and 3c to the interference between the amplitude of fig. 3a and the lower

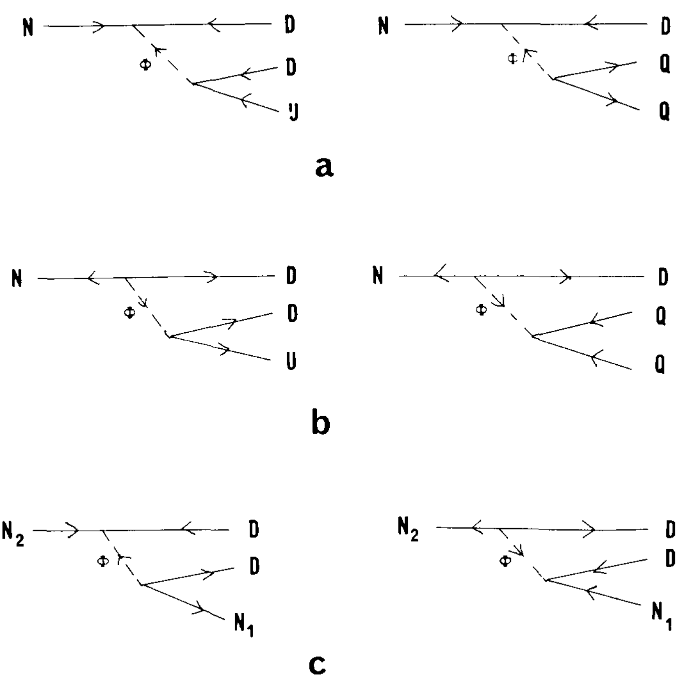

Fig. 2. Lowest order contributions to N-decay. The final states have baryon number (a) $B=1$, (b) $B=-1$ and (c) $B=0$. The crossed graphs for (a) and (b) are not shown.

order amplitude of fig. 2a. After summing over generations, the product of couplings appearing in fig. 3b is real; and for this reason, fig. 3b contributes equally to $\Delta B=1$ and $\Delta B=-1$ decays and cancels in the asymmetry. However, the combination of couplings from fig. 3c has a non-zero imaginary part provided that the number of generations exceeds one. Evaluating the contribution of fig. 3c, we find that

$$\Delta\Gamma = \Gamma_{\mathrm{N}_2\to\mathrm{qqq}} - \Gamma_{\mathrm{N}_2\to\bar{\mathrm{q}}\bar{\mathrm{q}}\bar{\mathrm{q}}}$$

$$= -\frac{M_2^7}{40960\pi^4 M_\Phi^6}\sum_{abcd}\mathrm{Im}\,(A_{2a}^* A_{2b} B_{cb}^* B_{cd} A_{1d}^* A_{1a})\,. \tag{2.5}$$

The magnitude of this asymmetry can be relevant to cosmological baryon generation ($\Delta\Gamma/\Gamma_2 \geqslant 10^{-10}$) for a range of parameters in which the couplings A, B and C are of order $\frac{1}{10}$ while the N_2 mass is of order 1 TeV. In obtaining this asymmetry, the existence of more than a single species of Majorana fermion was crucial both to the presence of CP violating phases and to the presence of a physical intermediate state in the graph of fig. 3a. Indeed, the lightest N has no CP violating asymmetry in its $\Delta B \neq 0$ decays at this order.

In addition, we must consider scattering processes which involve the Majorana fermions, because additional asymmetries occur in $\Delta B \neq 0$ scattering and also

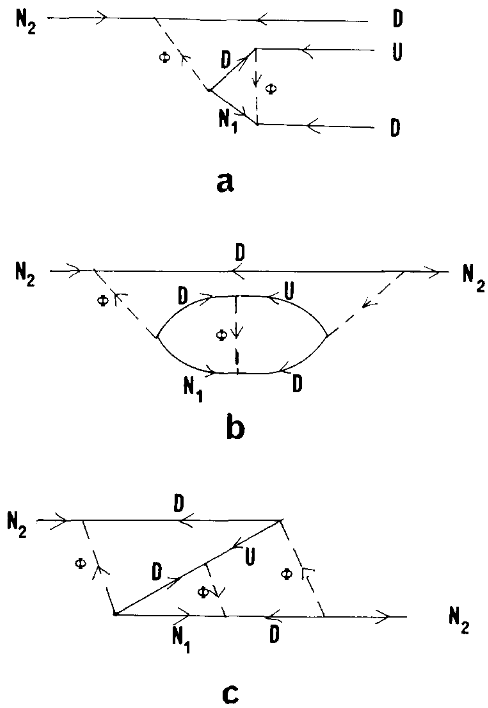

Fig. 3. Lowest order graph which contributes to the CP violating asymmetry $\Delta\Gamma$ in N_2 decay. Shown are (a) the contribution to the amplitude, (b) a contribution to the interference term which cancels in the asymmetry, and (c) the crossed contribution which produces the asymmetry.

because these processes play an important role in governing the deviations from equilibrium which are obtained as the temperature drops below the N masses. The lowest order graphs for the relevant $\Delta B=0$ processes appear in fig. 4, and the resulting cross sections (weighted by the relative velocity) for these processes are

$$\sigma_r v=(\sigma v)_{\mathrm{N}_r\mathrm{N}_r\to \mathrm{q}\bar{\mathrm{q}}}=\frac{s-4M_r^2}{16\pi M_\Phi^4}\sum_{ab}|A_{ra}A_{rb}^*|^2\,,$$

$$\sigma_{12} v=(\sigma v)_{\mathrm{N}_1\mathrm{N}_2\to \mathrm{q}\bar{\mathrm{q}}}=\frac{1}{32\pi M_\Phi^4}\,\frac{s(2s+M_2^2)}{s+M_2^2}\sum_{ab}|A_{2a}A_{1b}^*|^2\,,$$

$$\begin{aligned}\sigma_{\mathrm{N}_\mathrm{q}} v&=(\sigma v)_{\mathrm{N}_2\mathrm{q}\to \mathrm{N}_1\mathrm{q}}+(\sigma v)_{\mathrm{N}_2\bar{\mathrm{q}}\to \mathrm{N}_1\bar{\mathrm{q}}}\\ &=\frac{1}{576\pi M_\Phi^4}\,\frac{s(8s+M_2^2)}{s+M_2^2}\sum_{ab}|A_{2a}A_{1b}^*|^2\,,\end{aligned}\tag{2.6}$$

where s denotes the square of the total center-of-mass energy. In averaging over initial states in (2.6), we have assumed that, at temperatures of order 1 TeV, quark masses and SU(2) gauge interactions maintain equilibrium between the various

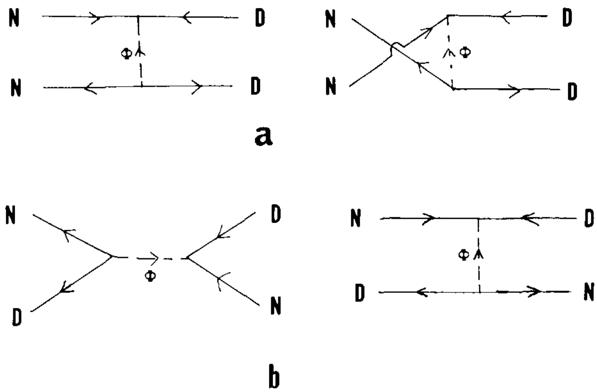

Fig. 4. Lowest order contributions to (a) N-annihilation and (b) Nq scattering with $\Delta B = 0$.

species of quarks so that right-handed charge $-\frac{1}{3}$ quarks account for $\frac{1}{4}$ of the total number of quarks. In a similar manner, the $\Delta B \neq 0$ scatterings shown in fig. 5 combine to give

$$\sigma_{rq} v = (\sigma v)_{N_r \bar{q} \to qq} + (\sigma v)_{N_r q \to \bar{q}\bar{q}}$$
$$= \frac{1}{144\pi M_\Phi^4} \frac{s(5s+M_r^2)}{s+M_r^2} \sum_{abc} [|A_{ra}B_{bc}|^2 + \tfrac{1}{2}A_{ra}B_{bc}A_{rc}^* B_{ba}^* + |A_{ra}C_{bc}^*|^2] \,. \tag{2.7}$$

The last quantity of interest is the asymmetry in the $\Delta B \neq 0$ scattering of the N_r shown in fig. 6. The portion of the figure to the left of the dotted line represents a

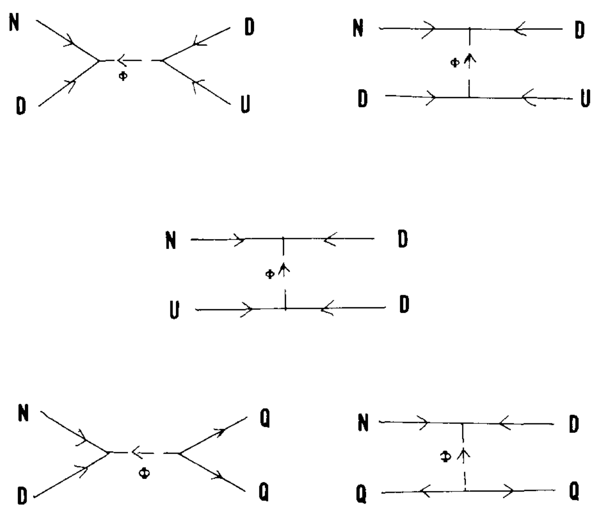

Fig. 5. Lowest order graphs for the $\Delta B = 1$ reaction $N\bar{q} \to qq$. Those for the $\Delta B = -1$ reaction $Nq \to \bar{q}\bar{q}$ are related by *CP*.

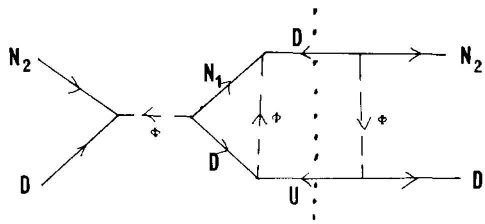

Fig. 6. Lowest order contribution to the asymmetry $\Delta\sigma_2$ in N_2q scattering with $\Delta B \neq 0$. The portion of the figure to the left of the dotted line is a contribution to the amplitude while the figure as a whole represents the contribution to the interference term. The asymmetry in N_1q scattering is given by interchanging N_1 and N_2.

contribution to the amplitude for $N_2q \to qq$, while the figure as a whole gives the contribution to the interference between this and the lowest order amplitude. Evaluating this contribution, we find that

$$\Delta\sigma_2 v = (\sigma v)_{N_2\bar{q}\to qq} - (\sigma v)_{N_2 q\to \bar{q}\bar{q}}$$

$$= -\frac{1}{1536\pi^2 M_\Phi^6} \frac{s^3}{s+M_2^2} \sum_{abcd} \mathrm{Im}\,(A_{2a}^* A_{2b} B_{cb}^* B_{cd} A_{1d}^* A_{1a})\,. \tag{2.8}$$

Similarly, the asymmetry in $\Delta B \neq 0$ scattering of N_1 is

$$\Delta\sigma_1 v = -\frac{s^2 - M_2^4}{s^2}\left(1 - \frac{M_2^2}{s}\right)\Delta\sigma_2 v\,, \tag{2.9}$$

in which the change in sign comes from the antisymmetry of the imaginary part of the couplings in eq. (2.8), while the factors result from differences in the energy denominators $1/2p_N^0 2p_q^0$ and in the integral over physical intermediate states.

To reduce the number of independent parameters, we will assume that A, B and C are all of order λ. In summing over generations, it is not assumed that B and C have been diagonalized. When the terms of the sum are not positive, we take the phases to be randomly distributed so that the square root of the number of terms is obtained. Specifically, we take

$$\sum_{ab} |A_{ra} A_{sb}^*|^2 = 9\lambda^4\,,$$

$$\sum_{abc} \{|A_{ra} B_{bc}|^2 + \tfrac{1}{2} A_{ra} B_{bc} A_{rc}^* B_{ba}^* + |A_{ra} C_{bc}^*|^2\} = \tfrac{117}{2}\lambda^4\,,$$

$$\sum_{abcd} \mathrm{Im}\,(A_{2a}^* A_{2b} B_{cb}^* B_{cd} A_{1d}^* A_{1a}) = -9\lambda^6\,. \tag{2.10}$$

Of course, this procedure would fail in the case of a single generation for which the last of these sums vanishes. The use of other procedures such as diagonalizing B and C before computing the sums or assuming that only the heaviest two generations have significant couplings does not alter the asymmetries $\Delta\Gamma/\Gamma_2$ and $\Delta\sigma_2/\sigma_{2q}$ by more than a factor of two.

To determine the reaction rates for the processes considered above, it is necessary to thermally average the decay widths and cross sections. The treatment of this in the appendix makes use of two important simplifying assumptions. Firstly, we assume that deviations from thermodynamic equilibrium occur only in the total number of particles of each species so that the phase space density of each species is proportional to its equilibrium distribution. Because the N_r interact weakly at these temperatures, this assumption is not necessarily justified. For example, the N_1 produced in N_2 decay might fail to thermalize prior to their decay. Secondly, the true Fermi–Dirac distributions have been replaced by their low temperature Maxwell–Boltzmann approximations, and accordingly the calculations have been done using the zero-temperature propagators. This is partially justified since the main interest is in temperatures below the masses of the N_r. Although not justified for ultra-relativistic quarks, this simplification should not affect the numerical results significantly. For example, the equilibrium number of quarks in Fermi–Dirac statistics differs from its Maxwell–Boltzmann value by only a factor of $\frac{3}{4}\zeta(3) \approx 0.90$ (where $\zeta(p)$ denotes Riemann's ζ-function).

Using the thermally averaged widths and cross sections (eqs. (A.8) and (A.12) of the appendix), we may write rate equations for the time evolution of the number densities n_r of the N_r and of the net baryon number density Δn_B. Neglecting the asymmetries, we have

$$\dot{n}_1 + \frac{3\dot{R}}{R} n_1 = -[\langle\Gamma_1\rangle + \langle\sigma_{1q}v\rangle\bar{n}_q](n_1 - \bar{n}_1) + [\langle\Gamma_{12}\rangle + \langle\sigma_{Nq}v\rangle\bar{n}_q]\left(n_2 - \frac{\bar{n}_2}{\bar{n}_1} n_1\right)$$
$$- \langle\sigma_{12}v\rangle(n_1 n_2 - \bar{n}_1\bar{n}_2) - 2\langle\sigma_1 v\rangle(n_1^2 - \bar{n}_1^2)\,, \tag{2.11}$$

$$\dot{n}_2 + \frac{3\dot{R}}{R} n_2 = -[\langle\Gamma_2\rangle + \langle\sigma_{2q}v\rangle\bar{n}_q](n_2 - \bar{n}_2) - [\langle\Gamma_{12}\rangle + \langle\sigma_{Nq}v\rangle\bar{n}_q]\left(n_2 - \frac{\bar{n}_2}{\bar{n}_1} n_1\right)$$
$$- \langle\sigma_{12}v\rangle(n_1 n_2 - \bar{n}_1\bar{n}_2) - 2\langle\sigma_2 v\rangle(n_2^2 - \bar{n}_2^2)\,. \tag{2.12}$$

Here $\bar{n}_q = 36/\pi^2\beta^3$ is the equilibrium density of quarks at inverse temperature β, while $\bar{n}_r = M_r^2/\pi^2\beta K_2(\beta M_r)$ is the equilibrium density of N_r. ($K_n(x)$ denotes the modified Bessel functions.) The terms here refer successively to N-decay and Nq scattering with $\Delta B \neq 0$, N-decay and Nq scattering with $\Delta B = 0$, N_1N_2 annihilation and, finally, N_1N_1 or N_2N_2 annihilation. The terms involving the equilibrium densities $\bar{n}_r$ represent the inverse reactions which have been included using the *CPT* argument of the appendix.

For the net baryon number density, we retain only the terms linear in the small quantities $\Delta\Gamma$, $\Delta\sigma_r$ and Δn_B and obtain

$$\Delta\dot{n}_B + \frac{3\dot{R}}{R}\Delta n_B = \langle\Delta\Gamma\rangle(n_2 - \bar{n}_2) + \langle\Delta\sigma_2 v\rangle\bar{n}_q\left(n_2 - \frac{\bar{n}_2}{\bar{n}_1} n_1\right)$$
$$- \frac{3\Delta n_B}{2\bar{n}_q}\sum_{r=1}^{2}[3\langle\Gamma_r\rangle\bar{n}_r + \langle\sigma_{rq}v\rangle\bar{n}_q(n_r + 2\bar{n}_r)]\,. \tag{2.13}$$

The terms here represent the generation of baryon number through the asymmetries $\Delta\Gamma$ and $\Delta\sigma_r$ and its destruction through the lowest order $\Delta B \neq 0$ contributions to inverse N-decay and to Nq and N$\bar{\text{q}}$ scattering and inverse scattering. As noted on general grounds in ref. [7], the term $-\langle\Delta\Gamma\rangle\bar{n}_2$, needed so that the reaction rates cancel in equilibrium, includes the asymmetry in inverse N_2 decay (which contributes $\langle\Delta\Gamma\rangle\bar{n}_2$ by *CPT*) and also the asymmetry between the reactions $3\text{q}\to 3\bar{\text{q}}$ and $3\bar{\text{q}}\to 3\text{q}$ which contributes $-2\langle\Delta\Gamma\rangle\bar{n}_2$. Finally, we note that the contribution from the difference between the cross sections for $\bar{\text{q}}\bar{\text{q}}\to\text{Nq}$ and $\text{qq}\to\text{N}\bar{\text{q}}$ cancels by the *CPT* argument of the appendix when the relationship between $\langle\Delta\sigma_1 v\rangle$ and $\langle\Delta\sigma_2 v\rangle$ [eq. (A.12)] is used.

For numerical purposes, it is convenient to use β as the independent variable and to write equations for the ratios $r_r = n_r/n_\gamma$ and $\Delta r_\text{B} = \Delta n_\text{B}/n_\gamma$ of the number densities to the photon number density $n_\gamma = 2/\pi^2\beta^3$. In the standard cosmology, eq. (1.3) leads to

$$\frac{\mathrm{d}}{\mathrm{d}\beta}\left(\frac{n}{n_\gamma}\right) = \frac{K\beta}{n_\gamma}\left(\dot{n} + \frac{3\dot{R}}{R}n\right) \tag{2.14}$$

for any density n. Here $K = [45/4\pi^3 G_\text{N} g_*]^{1/2}$ with $g_* = 106.75$ in the present model. The evolution equations then takes the form

$$\begin{aligned}
\frac{\mathrm{d}r_1}{\mathrm{d}\beta} = -K\beta\Bigg\{&\left[\langle\Gamma_1\rangle + \frac{36\langle\sigma_{1\text{q}}v\rangle}{\pi^2\beta^3}\right](r_1 - \bar{r}_1) - \left[\langle\Gamma_{12}\rangle + \frac{36\langle\sigma_{\text{Nq}}v\rangle}{\pi^2\beta^3}\right]\left(r_2 - \frac{\bar{r}_2}{\bar{r}_1}r_1\right) \\
&+ \frac{2\langle\sigma_{12}v\rangle}{\pi^2\beta^3}(r_1 r_2 - \bar{r}_1\bar{r}_2) + \frac{4\langle\sigma_1 v\rangle}{\pi^2\beta^3}(r_1^2 - \bar{r}_1^2)\Bigg\}, \\
\frac{\mathrm{d}r_2}{\mathrm{d}\beta} = -K\beta\Bigg\{&\left[\langle\Gamma_2\rangle + \frac{36\langle\sigma_{2\text{q}}v\rangle}{\pi^2\beta^3}\right](r_2 - \bar{r}_2) + \left[\langle\Gamma_{12}\rangle + \frac{36\langle\sigma_{\text{Nq}}v\rangle}{\pi^2\beta^3}\right]\left(r_2 - \frac{\bar{r}_2}{\bar{r}_1}r_1\right) \\
&+ \frac{2\langle\sigma_{12}v\rangle}{\pi^2\beta^3}(r_1 r_2 - \bar{r}_1\bar{r}_2) + \frac{4\langle\sigma_2 v\rangle}{\pi^2\beta^3}(r_2^2 - \bar{r}_2^2)\Bigg\}, \\
\frac{\mathrm{d}\Delta r_\text{B}}{\mathrm{d}\beta} = K\beta\Bigg\{&\langle\Delta\Gamma\rangle(r_2 - \bar{r}_2) + \frac{36\langle\Delta\sigma_2 v\rangle}{\pi^2\beta^3}\left(r_2 - \frac{\bar{r}_2}{\bar{r}_1}r_1\right) \\
&- \frac{\Delta r_\text{B}}{12}\sum_{r=1}^{2}\left[3\langle\Gamma_r\rangle\bar{r}_r + \frac{36\langle\sigma_{r\text{q}}v\rangle}{\pi^2\beta^3}(r_r + 2\bar{r}_r)\right]\Bigg\},
\end{aligned} \tag{2.15}$$

in which $\bar{r}_r = \bar{n}_r/n_\gamma = \frac{1}{2}(\beta M_r)^2 K_2(\beta M_r)$. Assuming that the universe is in equilibrium with $\Delta n_\text{B} = 0$ at temperatures above the N masses, the initial conditions for eqs. (2.15) are $r_1 = r_2 = 1$ and $\Delta r_\text{B} = 0$ near $\beta = 0$.

We have numerically integrated the evolution eqs. (2.15) for $M_\Phi = 300$ TeV and for various values of λ, M_1 and M_2. The behavior of r_1, r_2 and Δr_B as functions of β is shown in fig. 7 for the values $\lambda = 0.1$, $M_1 = 400$ GeV and $M_2 = 1000$ GeV. As the temperature drops below the masses of N_1 and N_2, the density ratios r_1 and r_2

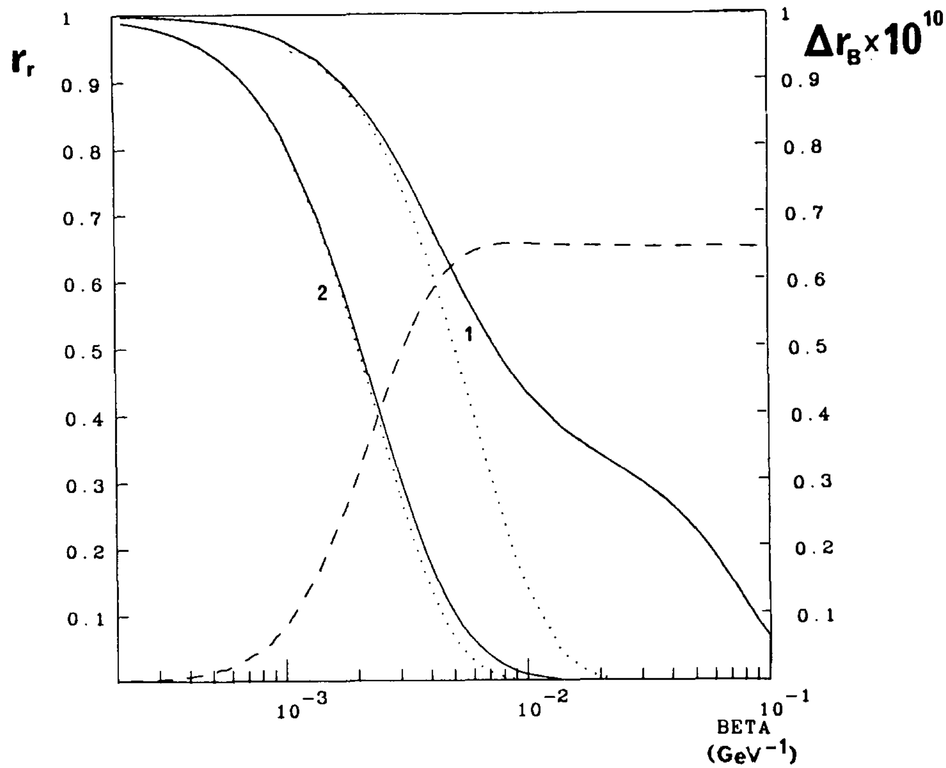

Fig. 7. Evolution of the density ratios $r_r = n_r/n_\gamma$ for $r = 1$ and 2 as functions of the inverse temperature β (solid lines). Also shown are the equilibrium ratios $\bar{r}_r$ (dotted lines) and the baryon to photon ratio Δr_B in units of 10^{-10} (dashed line). The figure is for $\lambda = 0.1$, $M_1 = 400$ GeV and $M_2 = 1$ TeV.

for these particles (solid lines) deviate from their equilibrium behavior (dotted lines). These deviations are larger for N_1 than for N_2, partially because the reaction rates are smaller for N_1 and partially because some N_1 are produced in the reactions which deplete the number of N_2. The baryon excess is generated when the N_2 density deviates from equilibrium, and most of this excess (~80%) is produced by the scattering asymmetry $\Delta\sigma_2$ at temperatures between M_2 and M_1 for which $n_2/\bar{n}_2 > n_1/\bar{n}_1$. Most of the N_2 are removed by the reactions $N_2\bar{q} \to qq$ and $N_2 q \to \bar{q}\bar{q}$, and at these temperatures the fractional asymmetry between these reactions is of order $1–5 \times 10^{-9}$. The density ratio r_2 deviates from its equilibrium value by roughly 0.01–0.04, and these deviations produce the final value $\Delta n_B/n_\gamma = 6.7 \times 10^{-11}$ which is reached when the number of N_2 becomes negligible. Finally, we note that only a small fraction (<1%) of the baron excess is lost in scattering with N_1 after the era of baryon generation.

The importance of scattering processes seen here is in sharp contrast with the conventional picture of baryon generation in grand unified theories [2, 5, 6, 7, 9, 10]. In many GUTs, including the simplest examples, $\Delta B \neq 0$ scattering of superheavy particles is higher order in perturbation theory than $\Delta B \neq 0$ decay, and the CP violating asymmetries in cross sections are negligible. In such models, decays are the dominant processes in depleting the abundances of superheavy particles, and the baryon number is generated through the asymmetries in these decays. However, scattering may be an important source of baryon number in models containing superheavy fermions since decay and scattering can occur at the same order for these particles [22].

Immediately after the era of baryon generation, the ratio of the photon number density to the entropy density is

$$\left(\frac{n_\gamma}{s}\right)_{\rm gen} = \frac{45\zeta(3)}{\pi^4 g_*} \approx \tfrac{1}{192} \tag{2.16}$$

for $g_* = 106.75$. After the decay of the residual N_1, the universe will expand isentropically, and $\Delta n_B/s$ will remain constant unless a significant amount of entropy is produced by the weak interaction phase transition. However, most of the baryon excess is generated prior to the decay of the N_1, and these decays can produce entropy since they occur out of equilibrium. To estimate this effect [23], we assume that all of the N_1 escape annihilation and that they decay instantaneously when the temperature falls to $1/\beta_1$. At this point, the energy density is

$$\rho = \frac{\pi^2 g_*}{30\beta_1^4}\left[1 + \frac{45\zeta(3)}{\pi^4 g_*}\beta_1 M_1\right], \tag{2.17}$$

in which we have included the contributions of the relativistic species and of the non-relativistic N_1. Since the energy density is conserved in an instantaneous process, the decay of N_1 reheats the relativistic species to a temperature $1/\beta_r$ given by

$$\frac{\beta_1}{\beta_r} = \left[1 + \frac{45\zeta(3)}{\pi^4 g_*}\beta_1 M_1\right]^{1/4}, \tag{2.18}$$

and the entropy is increased by the factor $(\beta_1/\beta_r)^3$. Collecting the factors, the value $(\Delta n_B/s)_0$ of the baryon to entropy ratio at the present time is related to the value $(\Delta n_B/n_\gamma)_{\rm gen}$ of the baryon to photon ratio immediately after baryon generation by

$$\left(\frac{\Delta n_B}{s}\right)_0 \approx \left(\frac{\Delta n_B}{n_\gamma}\right)_{\rm gen} \frac{1}{192}\left[1 + \frac{\beta_1 M_1}{192}\right]^{-3/4}. \tag{2.19}$$

The value of β_1 may be estimated by finding the temperature at which n_1/n_γ has fallen to $1/e$.

These effects are incorporated in fig. 8 which shows the present ratio $(\Delta n_B/s)_0$ as a function of λ for three choices of the Majorana masses. For small values of λ, the asymmetries $\Delta\Gamma/\Gamma_2$ and $\Delta\sigma_2/\sigma_{2q}$ are small, and the rate Γ_1 of N_1 decay is so small that the entropy produced by these decays dilutes the baryon excess significantly. This dilution is never significant for the values of λ which produces

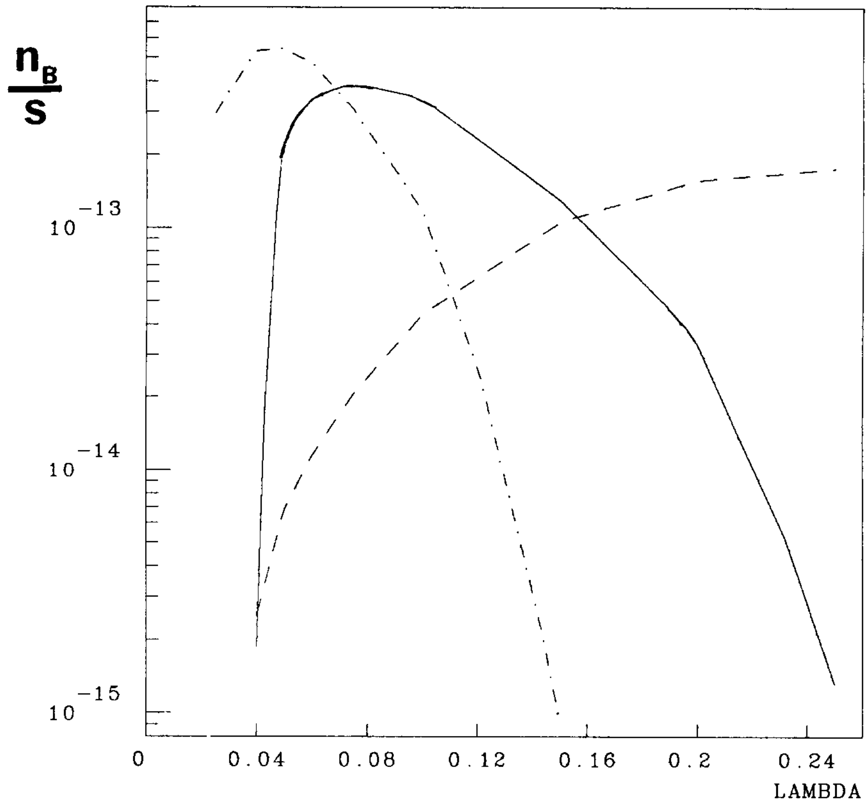

Fig. 8. The present baryon to entropy ratio $(\Delta n_B/s)_0$ as a function of λ for three choices of the N masses: $M_1 = 80$ GeV and $M_2 = 200$ GeV (dashed line), $M_1 = 400$ GeV and $M_2 = 1$ TeV (solid line) and $M_1 = 800$ GeV and $M_2 = 2$ TeV (dot-dashed line).

the largest values of $(\Delta n_B/s)_0$ shown in fig. 8. For large values of λ, the deviations from equilibrium for N_2 are small, and little baryon number is generated. As the Majorana masses increase, the asymmetries $\Delta\Gamma/\Gamma_2$ and $\Delta\sigma_2/\sigma_{2q}$ rise, and a larger excess is generated. However, the total reaction rates also increase with M_1 and M_2. This reduces the deviations from equilibrium so that a large excess can be obtained only for small values of λ. Finally, we note that the baryon excess is relatively insensitive to the mass ratio M_1/M_2, although for very small values of M_1, the excess can be diluted by the entropy produced in N_1 decay.

A rough understanding of the dependence of the baryon excess on λ and M_2 can be obtained by comparing the reaction rates with the cosmological expansion rate. Because of phase space and combinatorial factors, the depletion of N_2 is dominated by scattering with $\Delta B \neq 0$. This reaction proceeds at the rate/volume

$$\langle\sigma_{2q}v\rangle \bar{n}_q n_2 \sim \frac{\lambda^4 M_2^2}{\pi M_\Phi^4} T^3 n_2 \tag{2.20}$$

at temperatures T less than M_2. Comparing this with the volume expansion term $3\dot{R}/Rn_2$, we find that this reaction becomes ineffective at reducing the number of N_2 when the universe cools below the decoupling temperature

$$T_{\rm d} \sim \frac{3\pi M_\Phi^4}{K\lambda^4 M_2^2}$$

$$\approx 1\ {\rm TeV}\left(\frac{1}{10\lambda}\,\frac{M_\Phi}{300\ {\rm TeV}}\right)^4\left(\frac{1\ {\rm TeV}}{M_2}\right)^2 . \tag{2.21}$$

For $T_{\rm d} < M_2$, the deviations from equilibrium will be suppressed by the Boltzmann factor $e^{-M_2/T_{\rm d}}$. On the other hand, as $T_{\rm d}$ is increased beyond M_2, the growth of these deviations is slow and is offset by the decrease of $\Delta\sigma_2/\sigma_{2q}$. It follows that the optimum value of λ is that for which $T_{\rm d} \sim M_2$, that is

$$\lambda \approx \frac{1}{10}\left(\frac{M_\Phi}{300\ {\rm TeV}}\right)\left(\frac{1\ {\rm TeV}}{M_2}\right)^{3/4} . \tag{2.22}$$

As long as $T_{\rm d} \sim M_2$, the deviations from equilibrium obtained as the temperature falls below $M_2(\Delta n_2/n_\gamma \approx 0.02$ from fig. 7) will be roughly constant, and the magnitude of the baryon excess will be largely governed by the size of the asymmetry

$$\frac{\langle \Delta\sigma_2\rangle}{\langle \sigma_{2q}\rangle} \sim \frac{\lambda^2 M_2^2}{8\pi M_\Phi^2} \approx (4\times 10^{-9})\left[10\lambda\,\frac{300\ {\rm TeV}}{M_\Phi}\,\frac{M_2}{1\ {\rm TeV}}\right]^2 \tag{2.23}$$

at temperatures of order M_2. Using eq. (2.16), we may estimate the present baryon excess as

$$\left(\frac{\Delta n_{\rm B}}{s}\right)_0 \sim \left(\frac{n_\gamma}{s}\right)_{\rm gen}\left(\frac{\Delta n_2}{n_\gamma}\right)\left(\frac{\Delta\sigma_2}{\sigma_{2q}}\right) \approx 4\times 10^{-13}\left(\frac{M_2}{1\ {\rm TeV}}\right)^{1/2} \tag{2.24}$$

when λ is given by eq. (2.22). The estimates (2.22) and (2.24) are reasonably reliable guides to the locations and heights of the maxima in fig. 8.

The maximum value of the baryon excess shown on the curves of fig. 8 is $(\Delta n_{\rm B}/s)_0 = 5.5\times 10^{-13}$ for $\lambda = 0.05$, $M_1 = 800$ GeV and $M_2 = 2$ TeV. The rough estimate (2.24) suggests that this may be increased by a factor of three by increasing M_2 by a factor of ten, and in fact numerical integration of the evolution equations for $\lambda = 0.01$, $M_1 = 8$ TeV and $M_2 = 20$ TeV produces the value $(\Delta n_{\rm B}/s)_0 = 1.7\times 10^{-12}$. This value is the same as the experimental lower limit on $\Delta n_{\rm B}/s$, and the values obtained for $M_2 \sim 1$–2 TeV are not far below this limit. To conclude this section, we note that the calculation of the baryon excess is subject to several uncertainties, especially since the couplings A, B and C involve many unconstrained parameters. For example, if we abandon eq. (2.10) and instead assume that only two generations have appreciable couplings in eq. (2.1), the value of $(\Delta n_{\rm B}/s)_0$ typically increases by a factor of two. Alternatively, if it is assumed that the phases for each generation add constructively, a factor of three increase would be expected.

Also, the minimal choice of only two species of Majorana fermion has been made merely for convenience. If the model were the low-energy residue of a GUT, we might expect the presence of three species of N, one for each generation. Naively, we would expect the baryon excess to be roughly 2–3 times larger when a third N is included. However, the actual increase might be larger since the processes which deplete the density of the third N could drive the density of N_2 farther from equilibrium. Finally, if the couplings A_{1a} of the lighter Majorana fermion are taken to be of order λ_1 while the other couplings A_{2a}, B and C are still of order λ, the asymmetries can be increased by the factor $(\lambda_1/\lambda)^2$ without significantly affecting the total reaction rates of N_2. Of course, λ_1 cannot be increased arbitrarily, both because of the limits on $|\Delta B| = 2$ processes and because processes such as $N_2 q \to N_1 q$ will eventually dominate the reaction rates of N_2. For Majorana masses of order 1 TeV, the latter constraint is the more stringent; but even this constraint allows the asymmetries to be increased by an order of magnitude. For these reasons, the true lower limit on the masses of the Majorana fermions is probably of order 1 TeV.

3. Alternative low-energy models for baryon generation

In the previous section, it was shown that the cosmological net baryon number can be produced at temperatures of order 1 TeV in a simple extension of the standard model. In this section, we will consider some alternative mechanisms for baryon generation at low temperatures and argue that the model of sect. 2 is minimal for this purpose, although not necessarily unique. The emphasis of this section is largely on supersymmetric models, because supersymmetry offers a convenient motivation for the introduction of the additional particles (neutral Majorana fermions and color triplet scalars) discussed in sect. 2.

To begin however, we examine a non-supersymmetric alternative to the model of sect. 2. At temperatures of order 1 TeV, the deviations from equilibrium needed to generate an excess of baryons can be obtained only if the model contains particles with no $SU(3) \times SU(2) \times U(1)$ gauge interactions. If the gauge group is not extended, the only alternative to the neutral fermions of sect. 2 is the inclusion of one or more neutral singlet scalars X. Although these scalars have no Yukawa couplings to the fermions of the standard model, there are three species of color triplet, weak singlet scalars Φ^α with hypercharges $-\frac{4}{3}$, $-\frac{1}{3}$ and $\frac{2}{3}$ which can couple to quarks. The most general such couplings which preserve lepton number admit a baryon number symmetry under which the Φ^α have baryon number $-\frac{2}{3}$; however this symmetry could be broken by $|\Delta B| = 2$ terms such as $\Phi^\alpha \Phi^\beta \Phi^\gamma \varepsilon_{\alpha\beta\gamma}$ and $X\Phi^\alpha \Phi^\beta \Phi^\gamma \varepsilon_{\alpha\beta\gamma}$ in the scalar potential. In such a model, the neutral scalars can decay or annihilate into $B = 0$ final states through operators such as $X\Phi^*_\alpha \Phi^\alpha$ or $X^2 \Phi^*_\alpha \Phi^\alpha$, respectively, and these processes will maintain thermodynamic equilibrium unless they are suppressed by small couplings or by powers of M_X/M_Φ for $M_X \ll M_\Phi$. Because these $\Delta B = 0$ processes are lower order than are those with $\Delta B \neq 0$, it is difficult to obtain large

CP violating asymmetries in the reaction rates for the neutral scalars without making extremely artificial choices for the parameters. In this respect, the model of sect. 2 is very different in that $\Delta B \neq 0$ processes dominate the reaction rates of the Majorana fermions N_r present in that model.

It is presumably possible to construct a supersymmetric model which incorporates the mechanism of sect. 2 for baryon generation by promoting the fields of that section to superfields and by including terms such as (2.1) in the superpotential. However it is interesting to consider the supersymmetric case separately since supersymmetry can be used to motivate the appearance of Majorana fermions and color triplet scalars. Indeed, any supersymmetric extension of the standard model necessarily contains a neutral singlet fermion and color triplet scalars—these are the U(1) gauge fermion (bino) and the scalar quarks (squarks), respectively. If the superpotential contains all terms which preserve lepton number (including the $|\Delta B|=1$ term $f_{abc}U_{a\alpha}D_{b\beta}D_{c\gamma}\varepsilon^{\alpha\beta\gamma}$), the model will have couplings similar to those considered in sect. 2. Specifically, the gauge fermion Yukawa couplings $g'\lambda D_{a\alpha}A^{*\alpha}_{Da}$ replace the couplings of the N_r in eq. (2.1), while $\Delta B \neq 0$ couplings such as $f_{abc}U_{a\alpha}D_{b\beta}A_{Dc\gamma}\varepsilon^{\alpha\beta\gamma}$ replace the otther terms of (2.1). Here λ denotes the bino field, A_X is the scalar component of the chiral superfield X, and the same symbol is used for both chiral superfields and their fermion components. This supersymmetric model is then analogous to the model of sect. 2 when the bino and the squarks are taken to correspond to the N_r and Φ fields of sect. 2, respectively. Because of this correspondence, it is interesting to ask whether the observed cosmological baryon number can be generated when $\Delta B \neq 0$ interactions, but no new fields, are added to the minimal supersymmetric extension of the standard model [23]. Of course, squark masses similar to the Φ mass of sect. 2 (300 TeV) are rather unnatural. However, even setting this problem aside, we will argue that there are serious objections to this approach.

One objection is that the supersymmetric model has fewer *CP* violating phases than the model of sect. 2 since the bino interacts through the real gauge coupling g' and since field redefinitions suffice to remove the phases from f_{abc} unless the number of generations exceeds two. More importantly, the supersymmetric model contains only a single bino, while the model of sect. 2 contained at least two neutral fermion fields. For these reasons, the graphs analogous to those (figs. 3 and 6) which produced *CP* violating asymmetries in sect. 2 either involve real combinations of couplings or have no physical intermediate state; and therefore these do not produce asymmetries in the supersymmetric model. Of course, the limitation to a single bino is removed in extended ($N=2$) supersymmetric models. However, in view of the restricted nature of the couplings in such models, it would be rather difficult to obtain a satisfactory asymmetry.

The problems can be understood by the following argument. The minimal supersymmetric extension of the standard $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ model, including soft operators which break supersymmetry and with baryon (B) and lepton (L) numbers conserved, possesses a discrete R-parity invariance

$$R_p = (-1)^{3B+L+2S}, \tag{3.1}$$

in which S denotes spin. When the $\Delta B \neq 0$ coupling f_{abc} is included and lepton number conservation is retained to ensure the stability of the proton, $(-1)^{3B}R_p$ survives as an unbroken symmetry. If the bino is the lightest R_p odd particle, the lowest order bino decays must be into final states with $B = \pm 1$. Using CPT and unitarity, it can be shown that contributions to CP and B violating asymmetries in decay rates must involve a $\Delta B \neq 0$ rescattering of the final state; and in the present context, this rescattering must involve $|\Delta B| = 2$ operators such as that induced by graphs analogous to fig. 1. Unfortunately, the asymmetry obtained in this way is far too small to account for the observed baryon number. This is because $|\Delta B| = 2$ rescattering amplitudes are constrained to be small, both by the experimental limits on $|\Delta B| = 2$ processes and, more importantly, since the interactions mediated by squarks must be suppressed if deviations from equilibrium are to be obtained. A similar argument may be used to restrict asymmetries in cross sections for $\Delta B \neq 0$ scattering in the supersymmetric model. This argument might be circumvented if some R_p odd particle, not necessarily a neutral singlet, were lighter than the bino. For example, if an SU(2) gauge fermion (wino) were the lightest R_p odd particle, graphs analogous to figs. 3 and 6 with N_2 and N_1 respectively replaced by the bino and wino would have a physical intermediate state. These graphs might produce a sufficiently large asymmetry provided that an adequate source of CP violation were introduced. At a minimum, this would require the inclusion of additional generations with quark masses large enough for CP violation to be obtained from the quark mass matrix.

4. Conclusions

In this paper, we have explored mechanisms for generating the net baryon number of the universe at low temperatures of order 1 TeV. In sect. 2 it was shown that the cosmological baryon number could be produced by physics occurring at these temperatures in a simple extension of the standard $SU(3) \times SU(2) \times U(1)$ model. Some alternative models were considered in sect. 3, and it was argued that the mechanism of sect. 2 was at least minimal, if not unique. Models which incorporate a mechanism similar to that of sect. 2 are rather heretical in that they contain renormalizable baryon number violation at energies much less than the usual scale of grand unification (10^{14} GeV). Although this feature can be made phenomenologically acceptable by imposing lepton number as a global symmetry, it also makes these models somewhat difficult to embed into conventional schemes of grand unification. Since we are not entirely willing to forsake either the attractiveness of unification or the successful GUT prediction of $\sin^2 \theta_W$ (ref. [15]), it would be interesting to construct a GUT which has a model such as that of sect. 2 as its low energy residue. Although it is possible for GUTs to lead to low energy theories which contain baryon number violation without lepton number violation [17], we will at present simply regard this complication as the price to be paid in any model which generates baryon number at low temperatures. In addition, we note that in sect. 2 we were led to introduce scalars with mass of order 300 TeV. This is both interesting and potentially troublesome, since it is not clear what, if any, physics is to be associated with this scale.

In seeking alternative mechanisms for cosmological baryon number generation, we were not motivated by any particular outstanding problem with baryon generation in GUTs; and in fact, GUT baryon generation certainly does provide one possible explanation for the observed baryon number. Instead, given the lack of evidence for baryon number violation at superheavy scales [14], we regard it as worthwhile to consider alternative and perhaps more testable mechanisms, expecially since these alternatives allow a wider range of cosmological scenarios. In particular, baryon generation at low temperatures may revive some models of inflation which do not reheat sufficiently to produce the observed baryon number through mechanisms involving superheavy particles. For example, in some, although not all, of the supersymmetric inflation models, the reheating temperature can be as small as 1 MeV (ref. [13]). Although models in which the reheating temperature is this small are probably inconsistent with baryon generation through any mechanism, it is interesting that the alternatives to GUT baryon generation considered here allow for reheating temperatures much less than those allowed by mechanisms involving superheavy particles. Moreover, mechanisms for baryon generation at low temperatures can accommodate previously unexplored scenarios such as inflation in phase transitions occurring at scales much below that of GUT symmetry breaking. In the model of sect. 2, the reheating temperature for an inflationary phase transition would probably be required to be somewhat larger than the mass of the heaviest Majorana fermion in order to abundantly repopulate the numbers of these particles. Although this does not allow inflation at the weak scale, it does allow for inflation at intermediate scales far below the GUT scale. While there is no compelling reason to have inflation at scales below that of GUTs, the existence of this possibility is rather intriguing.

Note added

After completion of this work, we were informed that the generation of baryon number from Majorana fermions with mass 10^7–10^{12} GeV has been considered by Masiero and Yanagida [25]. In that paper, baryon generation is associated with an intermediate scale (10^7 GeV) in the context of a GUT. We thank R. Peccei for pointing this out.

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Appendix

THERMALLY AVERAGING WITH CLASSICAL STATISTICS

In this appendix, we review the finite temperature averaging of decay rates and cross sections using classical (Maxwell–Boltzmann) statistics [7] and present the results of this averaging for the processes of interest in sect. 2. In Maxwell–Boltzmann statistics, the density of the ith species of particle in phase space is taken to be

$$f_i(p_i) = g_i \frac{n_i}{\bar{n}_i} e^{-\beta \cdot p_i} , \tag{A.1}$$

in which the integer g_i counts the spin and internal quantum number multiplicity of the species, while p_i denotes the 4-momentum. Also, n_i is the actual number density of the species, and $\bar{n}_i$ is its density in thermodynamic equilibrium. This choice reflects the assumption that all species are in kinetic equilibrium at a common temperature ($\beta^0 = 1/T$ in a comoving frame), but that the species may not be in chemical equilibrium. In chemical equilibrium, the density is then

$$\bar{n}_i = g_i \int \frac{d^3 p_i}{(2\pi)^3} e^{-\beta \cdot p_i} = \frac{g_i m_i^2}{2\pi^2 \beta} K_2(\beta m_i) , \tag{A.2}$$

in which m_i is the mass of the ith species, and $K_n(x)$ denotes the modified Bessel functions. For massless particles, we have $m_i^2 K_2(\beta m_i) \to 2/\beta^2$ so that $\bar{n}_i \to g_i/\pi^2\beta^3$.

For a general reaction $A_1 \cdots A_N \to B_1 \cdots B_M$, the use of classical statistics allows the reaction rate/volume to be written as

$$W_{A \to B} = \frac{1}{s_i s_f} \int \left[\prod_{i=1}^{N} f_i(p_i)\, d\omega_{p_i} \right] \left[\prod_{j=1}^{M} d\omega_{q_j} \right] (2\pi)^4 \delta^4 \left(\sum_{i=1}^{N} p_i - \sum_{j=1}^{M} q_j \right) |M_{A \to B}|^2 . \tag{A.3}$$

Here s_i and s_f are statistical factors obtained by including a factor of $n!$ for every set of identical particles in the initial and final states, respectively. Also, $d\omega_k = d^3k/(2\pi)^3 2k^0$, while $|M_{A \to B}|^2$ is the square of the Lorentz invariant transition amplitude averaged and summed over initial and final quantum numbers, respectively.

In the following, we consider explicitly the cases of decay ($N = 1$) and scattering ($N = 2$). This will be sufficient since the reaction rates for the inverse reactions can then be determined by the CPT relation

$$\left(\prod_{j=1}^{M} g_{\bar{j}}\right)|M_{\bar{B}\to\bar{A}}|^2=\left(\prod_{i=1}^{N} g_i\right)|M_{A\to B}|^2\,, \tag{A.4}$$

in which the factors arise from the averaging over initial quantum numbers in $|M|^2$ and in which $\bar{j}$ labels the CP conjugate of the jth species. Since the masses and multiplicities of CP conjugate species are equal, we may use momentum conservation to show that

$$W_{\bar{B}\to\bar{A}}=\left(\prod_{i=1}^{N}\frac{\bar{n}_i}{n_i}\right)\left(\prod_{j=1}^{M}\frac{n_{\bar{j}}}{\bar{n}_{\bar{j}}}\right)W_{A\to B}\,. \tag{A.5}$$

When CP is conserved, this is equivalent to the principle of detailed balance. When CP is not conserved, *detailed* balance may not apply. However, unitarity may still be used to show that the total rates for transitions to and from any given species cancel in equilibrium.

For decays $A\to B_1\cdots B_M$, the reaction rate (A.3) reduces to

$$W_{A\to B}=2m_A\Gamma_{A\to B}\int d\omega_{p_A}f_A(p_A)=n_A\langle\Gamma_{A\to B}\rangle\,, \tag{A.6}$$

in which $\Gamma_{A\to B}$ is the partial width for the decay and where the averaged partial width is

$$\begin{aligned}\langle\Gamma_{A\to B}\rangle&=\frac{4\pi^2\beta}{m_A K_2(\beta m_A)}\Gamma_{A\to B}\int d\omega_{p_A}\,e^{-\beta\cdot p_A}\\&=\Gamma_{A\to B}\frac{K_1(\beta m_A)}{K_2(\beta m_A)}\,.\end{aligned} \tag{A.7}$$

The extra factor is simply the thermal average of the time dialation factor for the decay of a moving particle. Applying this to the decay rates (2.3–2.5) of the model of sect. 2 and making use of the simplifications in (2.10), we find

$$\begin{aligned}\langle\Gamma_r\rangle&=\frac{117\lambda^4M_r^5}{1024\pi^3M_\Phi^4}\frac{K_1(\beta M_r)}{K_2(\beta M_r)}\,,\\ \langle\Gamma_{12}\rangle&=\frac{9\lambda^4M_2^5}{1024\pi^3M_\Phi^4}\frac{K_1(\beta M_2)}{K_2(\beta M_2)}\,,\\ \langle\Delta\Gamma\rangle&=\frac{9\lambda^6M_2^7}{40960\pi^4M_\Phi^6}\frac{K_1(\beta M_2)}{K_2(\beta M_2)}\,.\end{aligned} \tag{A.8}$$

Similarly, for a general scattering process $A_1A_2\to B_1\cdots B_M$, the reaction rate/volume is

$$\begin{aligned}W_{A\to B}&=\frac{1}{s_i}\int d\omega_{p_1}\,d\omega_{p_2}f_{A_1}(p_1)f_{A_2}(p_2)[2p_1^0 2p_2^0(\sigma v)_{A\to B}]\\&=n_{A_1}n_{A_2}\langle(\sigma v)_{A\to B}\rangle\,,\end{aligned} \tag{A.9}$$

in which $(\sigma v)_{A\to B}$ is the conventionally defined cross section. The quantity in square brackets depends only on $s=(p_1+p_2)^2$, and the averaged cross section $\langle(\sigma v)_{A\to B}\rangle$ may thus be given as

$$\langle(\sigma v)_{A\to B}\rangle=\left[\prod_{i=1}^{2}\frac{2\pi^2\beta}{m_i^2K_2(\beta m_i)}\right]\frac{1}{16\pi^2}\int ds\,d\omega_Q\,e^{-\beta\cdot Q}\bar{\sigma}(s)$$
$$=\frac{\beta}{16}\left[\prod_{i=1}^{2}\frac{1}{m_i^2K_2(\beta m_i)}\right]\int ds\,\sqrt{s}K_1(\beta\sqrt{s})\bar{\sigma}(s)\,, \tag{A.10}$$

where $Q^2=s$ and

$$\bar{\sigma}(s)=\frac{1}{s_i}[2p_1^0 2p_2^0(\sigma v)_{A\to B}]8\pi\int d\omega_{p_1}\,d\omega_{p_2}(2\pi)^4\delta^4(Q-p_1-p_2)\,. \tag{A.11}$$

This result may be regarded as averaging over phase space integrals for the incoming particles. Again using eq. (2.10), the averaged cross sections in the model of sect. 2 are

$$\langle\sigma_r V\rangle=\frac{27\lambda^4M_r^2}{128\pi M_\Phi^4}\left[\left(\frac{K_3(\beta M_r)}{K_2(\beta M_r)}\right)^2-\left(\frac{K_1(\beta M_r)}{K_2(\beta M_r)}\right)^2\right],$$

$$\langle\sigma_{12}v\rangle=\frac{9\lambda^4M_2^2}{64\pi M_\Phi^4}\left[2\frac{K_4(\beta M_2)}{K_2(\beta M_2)}+1\right],$$

$$\langle\sigma_{\mathrm{Nq}}v\rangle=\frac{\lambda^4M_2^2}{128\pi M_\Phi^4}\left[8\frac{K_4(\beta M_2)}{K_2(\beta M_2)}+1\right],$$

$$\langle\sigma_{r_q}v\rangle=\frac{13\lambda^4M_r^2}{64\pi M_\Phi^4}\left[5\frac{K_4(\beta M_r)}{K_2(\beta M_r)}+1\right],$$

$$\langle\Delta\sigma_2 v\rangle=-\frac{\bar{n}_1}{\bar{n}_2}\langle\Delta\sigma_1 v\rangle$$
$$=\frac{3\lambda^6M_2^4}{1024\pi^2M_\Phi^6}\left[\frac{K_4(\beta M_2)}{K_2(\beta M_2)}+\frac{6}{\beta M_2}\frac{K_4(\beta M_2)}{K_2(\beta M_2)}\right]. \tag{A.12}$$

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