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SUPERCONDUCTING STRINGS

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It is known that certain spontaneously broken gauge theories give rise to stable strings or vortex lines. In this paper it is shown that under certain conditions such strings behave like superconducting wires whose passage through astrophysical magnetic fields would generate a variety of striking and perhaps observable effects. The superconducting charge carriers may be either bosons (if a charged Higgs field has an expectation value in the core of the string) or fermions (if charged fermions are trapped in zero modes along the string, as is known to occur in certain circumstances). They might be observable as synchrotron sources or as sources of high-energy cosmic rays. If the charge carriers are ordinary quarks and leptons, the strings have important baryon number violating interactions with magnetic fields; such a string, traversing a galactic magnetic field of 10^{-6} G, creates baryons (or antibaryons) at a rate of order 10^{12} particles/cm of string per second.

1. Introduction

Of the various topological objects that can arise in spontaneously broken gauge theories, the first to be uncovered were the strings or vortex lines [1]. In recent years, there has been considerable interest in the possibility that strings which originated in symmetry breaking at a very high mass scale may have served as seeds for galaxy formation [2].

All proposals to date for observing strings have been based on their gravitational interactions. A string that originates in symmetry breaking at a mass scale $\Lambda \simeq 10^{16}$ GeV (a convenient value in grand unification) has a mass per unit length of order 10^{22} g/cm. Such a heavy string might be seen as a gravitational lens [3] and would be a source of long-wavelength gravitational radiation [4] which might be detectable by observations of pulsar timing [5]. On the other hand, there might exist string of $\Lambda \ll 10^{16}$ GeV; the possibility of observing such strings through their gravitational interactions would be very slim. As an extreme example, if strings arise near the electroweak scale so $\Lambda \sim 1$ TeV (perfectly possible in the O(10) model we will consider later), then the mass per unit length is only about 10^{-6} g/cm. Such a string, if it has the length of a galaxy (radius 10^{22} cm) has a mass of only 10^{16} g - comparable to the mass of a comet. Obviously, such strings would not be observed by their gravitational interactions.

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In this paper, it will be suggested that - under certain assumptions about gauge symmetry breaking - strings might behave as superconducting wires whose motion through astronomical magnetic fields would produce spectacular effects. The search for such effects gives a chance of observing strings if Λ is small and can supplement other observations if Λ is large. Indeed, observation of the effects to be discussed in this paper would yield a novel way to study astrophysical fields and plasmas. As we will see, the physics and astrophysics of superconducting strings is rather complicated and model dependent, and in this paper we will make only some general and preliminary observations.

As we will see, the charge carriers on a superconducting string may be either bosons or fermions. There are bosonic charge carriers if a charged Higgs field has a vacuum expectation value in the core of the string. In that case, roughly speaking, electric currents are carried up and down the string by Golstone bosons. Alternatively, string superconductivity with Fermi charge carriers can arise if there are femion zero modes in the field of the vortex line. The possibility of such zero modes have been known for some time [6]. The case of interest here was discussed in detail by Jackiw and Rossi [7]; the relevant index theorem was analyzed by E. Weinberg [8].

The phenomena that will be discussed here are, in certain respects, somewhat reminiscent of the Callan-Rubakov effect [9] in the field of a magnetic monopole. The string, no matter how thin it may be, produces macroscopic effects in its interaction with electromagnetic fields.

Unlike magnetic monopoles, strings are not a geneal feature of grand unified theories, but arise only under certain assumptions concerning the Higgs structure [16]. And even if strings exist, whether they are superconducting depends on details of their microscopic structure.

2. Bose charge carriers

The present work began with a study of the possible implications of Fermi zero modes along a string. However, it turns out that superconductivity with Bose charge carriers is also possible. As the theory is simpler in that case, we will consider it first. The case of Bose charge carriers is important because, as we will see, in any theory with strings which also has charged scalar fields, the strings are superconducting in a reasonably large range of values of the Higgs parameters*.

For illustrative purposes, consider a $U(1)\times U(\tilde{1})$ gauge theory with an unbroken gauge symmetry Q (electromagnetism) and a broken gauge symmetry R responsible for the existence of string. Assume that the theory has a Higgs field φ of $Q=0$,

* The discussion in this section was partly suggested by a comment by J. Harvey about the possible behaviour of the ordinary electroweak Higgs doublet near the core of a magnetic monopole (1983, unpublished). Harvey pointed out that with a suitable Higgs potential scalar fields of zero or very low expectation value in vacuum can have a large expectation value near the core of a monopole. The same effect can also arise due to interaction with fermions (E. Witten, 1983, unpublished).

$R = 1$, and a second scalar field σ of $Q = 1$, $R = 0$. The kinetic part of the lagrangian is

$$L_{\mathrm{kin}} = -\tfrac{1}{4}F_{\mu\nu}^2 - \tfrac{1}{4}R_{\mu\nu}^2 + D_\mu\sigma^* D^\mu\sigma + D_\mu\varphi^* D^\mu\varphi\,, \tag{1}$$

where $F_{\mu\nu} = \partial_\mu\partial_\nu - \partial_\nu A_\mu$, $R_{\mu\nu} = \partial_\mu R_\nu - \partial_\nu R_\mu$, $D_\mu\sigma = (\partial_\mu + ieA_\mu)\sigma$, $D_\mu\varphi = (\partial_\mu + igR_\mu)\varphi$; A_μ and R_μ and the U(1) and U($\tilde{1}$) gauge fields, and e and g are the respective gauge couplings.

The theory also has a scalar potential. The general gauge invariant quartic potential can be parametrized

$$V(\sigma, \varphi) = \tfrac{1}{8}\lambda(|\varphi|^2 - \mu^2)^2 + \tfrac{1}{4}\tilde{\lambda}|\sigma|^4 + f|\sigma|^2|\varphi|^2 - m^2|\sigma|^2\,. \tag{2}$$

The range of parameters of interest to us is $\mu^2, m^2 > 0$, $f|\sigma|^2 - m^2 > 0$, plus an additional condition that will be imposed later. In this range, the minimum of the potential is at $\langle\sigma\rangle = 0$, $|\langle\varphi\rangle| = \mu$, so electromagnetism is unbroken but R is spontaneously broken.

The breakdown of R leads [1] to the existence of vortex lines (fig. 1). In the vortex field, φ is independent of two coordinates, say z and t; φ vanishes at $x = y = 0$ and changes in phase by 2π in making any closed circuit around the string. In the core of the string there is a U($\tilde{1}$) magnetic field with an integrated flux of $2\pi/g$. The mass per unit length of the string is of order μ^2/λ.

Although with the parameters we have assumed, $\langle\sigma\rangle = 0$ in vacuum, it is not necessarily true that $\langle\sigma\rangle = 0$ in the core of the string. In fact, since $\varphi = 0$ in the string core, and we assume $m^2 > 0$ in eq. (2), the potential energy favors $\sigma \neq 0$ near the core. The kinetic energy tends to resist this tendency, since σ must vanish at large distances from the string.

We must explore the balance between kinetic and potential energy. There certainly exists a string solution with $\sigma = 0$ and ($A_\mu = 0$) everywhere; it is the string solution of the $U(\tilde{1})$ theory. In this solution $\varphi(x, y)$ has an absolute value $|\varphi|$ which increases monotonically from $|\varphi| = 0$ at $r = 0$ to $|\varphi| = \mu$ at $r = \infty$. Let us ask whether this solution is stable. The equation for small fluctuations in σ around a φ background is

$$\ddot{\sigma} - \nabla^2\sigma + (f|\varphi|^2 - m^2)\sigma = 0\,. \tag{3}$$

Looking for a solution of the form $\sigma(x, y, z, t) = e^{-i\omega t}\sigma_0(x, y)$, this reduces to

$$\left(-\frac{\mathrm{d}^2}{\mathrm{d}x^2} - \frac{\mathrm{d}^2}{\mathrm{d}y^2}\right)\sigma_0 + V(r)\sigma_0 = \omega^2\sigma_0\,, \tag{4}$$

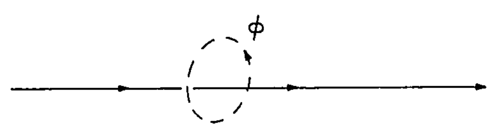

Fig. 1. The string or vortex line that arises from spontaneous breaking of a U(1) symmetry. The Higgs field changes in phase by 2π in circling the string.

where $V(r)=f|\varphi|^2-m^2$. This is a two-dimensional Schrödinger equation with potential $V(r)$. If there is a normalizable bound state solution with $\omega^2<0$, then the string with $\sigma=0$ is unstable, and will relax to a lower energy state of $\sigma\neq 0$. The potential $V(r)$ is attractive near $r=0$, $V(0)=-m^2$, and increases monotonically as r increases to $V(\infty)=f\mu^2-m^2$. There is definitely an allowed range of parameters in which (4) has a bound state solution. For instance, if $m^2=f\mu^2$ (corresponding to σ having zero bare mass) then $V(r)$ is negative definite, with $V(\infty)=0$. It is known that in two dimensions the Schrödinger equation with a negative definite potential always has a bound state*, so there is certainly a bound state solution if $m^2=f\mu^2$. By continuity, there is also a bound state solution in a continuous range of m^2 for $m^2<f\mu^2$. The effects of interest to us arise in this range. (Perhaps it is worth while to note that the mass of the massive component of φ is $M^2=\lambda\mu^2$, while m^2 is a mass parameter of σ. It is perfectly natural to have $m^2\sim M^2$, and $f\sim\lambda$ is also natural, since f and λ are quartic scalar couplings, so $m^2\sim f\mu^2$ is a natural range.)

Assuming $\sigma\neq 0$ in the minimum energy string solution, let $\sigma_0(x, y)$ be the value of the σ-field that minmizes the energy. Asymptotically, for large r, $\sigma_0\sim e^{-m'r}$, where $m'=\sqrt{\lambda\mu^2-m^2}$ is the σ-mass in vacuum. The fact that $\sigma\neq 0$ in the string means, roughly, that electromagnetism is spontaneously broken in the core of the string, and there will be Goldstone bosons carrying charge up and down the string.

If $\sigma_0(x, y)$ minimizes the energy of the string, so does $e^{i\vartheta}\sigma_0(x, y)$ for any real constant ϑ. This merely amounts to a charge rotation of the core of the string. More generally, we must treat ϑ as a sort of collective coordinate - or in this case, a collective field. The string has low-energy excitations of the form

$$\sigma(x, y, z, t)=e^{i\vartheta(z,t)}\sigma_0(x, y)\,, \tag{5}$$

where $\vartheta(z, t)$ is an arbitrary slowly varying function. These excitations will be responsible for making the string into a superconducting wire.

To find the suitable effective action for ϑ one proceeds as follows. The ϑ-dependent terms of the lagrangian come entirely from the terms $|(\partial_0+ieA_0)\sigma|^2-|(\partial_z+ieA_z)\sigma|^2$ in the kinetic energy. In evaluating those terms, we use ansatz (5) for σ, and we assume A_μ is slowly varying on the scale of the string, so that we can set $A_\mu(x, y, z, t)=A_\mu(0, 0, z, t)$ whenever $\sigma\neq 0$. For convenience we write $A_\mu(0, 0, z, t)$ as $A_\mu(z, t)$. On this basis the effective action for ϑ becomes

$$I_\vartheta=K\int dz\, dt\,[(\partial_0\vartheta(z, t)+eA_0(z, t))^2-(\partial_z\vartheta(z, t)+eA_z(z, t))^2]\,, \tag{6}$$

where

$$K=\int dx\, dy\,|\sigma_0(x, y)|^2\,. \tag{7}$$

(6) is gauge invariant, as it must be, under $\vartheta\to\vartheta+\lambda$, $A_\mu\to A_\mu+(1/e)\partial_\mu\lambda$.

* The binding energy is exponentially small if the potential is weak and short range, but that is not so here. In fact, the range of the potential is of order $1/\sqrt{f\mu^2}$, and the dimensionless product (range)^2 (strength of potential) is of order one.

To estimate K, let us note that if $f\mu^2 \sim m^2$ in (2), then $\sigma_0 \neq 0$ in a region of radius of order $1/m$, and in this region $\sigma_0 \sim m/\sqrt{\tilde{\lambda}}$. (That is so because if an instability occurs due to a bound state solution of eq. (4), the instability will be shut off by the $\tilde{\lambda}\sigma^4$ term in the potential at $\sigma_0 \sim m/\sqrt{\tilde{\lambda}}$.) So $K \sim 1/\tilde{\lambda}$.

To describe the long-wavelength interactions of strings and electromagnetic fields, we must add to (6) the standard electromagnetic action. We thus have

$$I = I_A + I_\vartheta = -\tfrac{1}{4}\int \mathrm{d}^4x\, F_{\mu\nu}F^{\mu\nu} + K\int \mathrm{d}z\, \mathrm{d}t(\partial_i\vartheta + eA_i)^2\,, \tag{8}$$

where it will be understood that Greek indices μ, ν run over all four values but Latin indices i, j take only two values (t and z) appropriate for the string.

Being quadratic in A_μ and ϑ, (8) is exactly soluble, and phenomena such as the scattering of light by the string or the interaction of the string with a static magnetic field can be described completely. We will however, postpone a treatment of these matters until later; after deriving an action analogous to (8) for strings with Fermi charge carriers, we will treat the two cases together.

Here we will make a few brief remarks aimed at showing that the string under discussion is indeed superconducting.

We compute the electromagnetic current as $J_i = -\delta I_\vartheta/\delta A_i$. It is

$$J_i(z, t) = 2Ke(\partial_i\vartheta + eA_i)\,. \tag{9}$$

Given this expression, we would like to see that there are current carrying states of the string that do not relax.

Consider a large closed string (fig. 2). The basic reason that the string can carry persistent currents is that it is possible to define a topological invariant

$$N = \frac{1}{2\pi}\oint \mathrm{d}l\frac{\mathrm{d}\vartheta}{\mathrm{d}l}\,, \tag{10}$$

where l is a parameter along the string. Despite being the integral around a closed loop of the derivative of ϑ, N need not vanish. As ϑ was introduced by the formula $\sigma = \mathrm{e}^{i\vartheta}\sigma_0$, it is defined only module 2π. In general, ϑ can change by any multiple of 2π in circling the string, so N can be any integer.

What we will show is that in a sector of non-zero N there is non-zero current flowing around the string (although this current is much less than one might naively

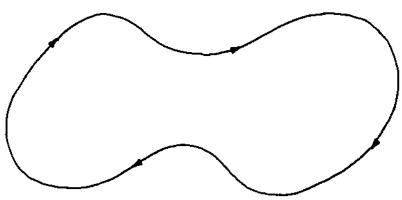

Fig. 2. A large closed string of characteristic scale R.

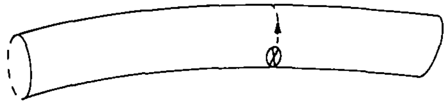

Fig. 3. The current I on an ordinary metal superconducting loop can relax by one unit in a process in which a vortex line (shown as an ⊗) nucleates on the inside and migrates to the outside.

guess). In a sector of non-zero N, the ground state of the string is current carrying so the currents persist.

Before getting into the details of computing the current as a function of N, let us note that it is not strictly true that N is conserved. In an ordinary, superconducting metal loop (fig. 3), there are in principle quantum (or thermal) tunneling processes in which a magnetic flux tube is created on one side of the wire, migrates across it, and is discharged on the other side. The process changes the analogue of N by one unit, but in practice such a process is extremely unlikely, and the supercurrents seem to continue forever. The analogue for our superconducting strings is as follows. Since ϑ was essentially the phase of the σ-field, ϑ would be ill-defined if $\sigma=0$. In a process in which σ passes through zero at some point on the string at some time, N could change (by one unit if σ has a simple zero). Since it is energetically favored to have $\sigma\neq 0$, a process in which σ passes through zero is an exponentially suppressed tunneling process, with a rate of order $e^{-1/\lambda}$ (λ being a φ^4 coupling). Only when the current flowing in the string becomes so large that its energy content is comparable to the energy needed to set $\sigma=0$ do the processes in which N changes occur at a reasonable rate; at that point the current carried by the string saturates, and it ceases to behave like a superconducting wire. This only occurs, however, when the current is of order $e\mu/\sqrt{\sigma}$ (μ^2/λ being the mass per unit length of the string).

Let us now compute the current carried by a string (not necessarily circular) of circumference $2\pi R$ in its lowest energy state of fixed N (fig. 2). Since the current is conserved and can only be carried on the string and since the state of lowest energy for given N will be time independent, it is obvious that the tangential component of the current is constant on the string regardless of the precise geometry. The equation for the vector potential

$$\nabla^2 A_i - \partial_i(\nabla\cdot A) = J_i \tag{11}$$

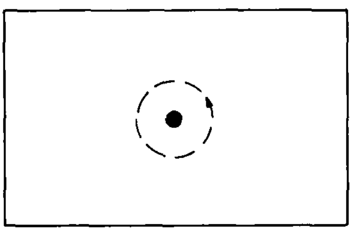

Fig. 4. The process analogous to that of fig. 3 for superconducting strings. The Higgs field σ vanishes somewhere on the world sheet of the string (shown by a dot) and changes in phase by 2π in circling this point.

is not quite as trivial as it appears since J_i depends on A_i. Nonetheless, J_i is gauge invariant, and in the gauge $\nabla \cdot \boldsymbol{A} = 0$, (11) has the simple solution

$$A_i(\boldsymbol{x}) = -\frac{1}{4\pi}\oint \mathrm{d}l \frac{1}{|\boldsymbol{x}-\boldsymbol{x}(l)|} J_i(\boldsymbol{x}(l))\,. \tag{12}$$

To evaluate the current, we need to know A_i on the string. However, if $\boldsymbol{x}$ is on the string, the integral in (12) diverges logarithmically when l is such that $\boldsymbol{x} = \boldsymbol{x}(l)$. The divergence arises because we have assumed that the string is a mathematical line of zero thickness. If we bear in mind that the string has in fact some tiny but non-zero width $1/\Lambda$, the "divergent" part of the integral in (12) is $2\ln \Lambda$. On dimensional grounds, the integral is $2\ln(\Lambda R)$ plus a piece that is finite as $\Lambda R \to \infty$. (That piece is negligible because in our appplications $\Lambda R \gg e^{100}$.) So along the string

$$A_i(\boldsymbol{x}(l)) = -\frac{\ln(\Lambda R)}{2\pi} J_i(\boldsymbol{x}(l)). \tag{13}$$

Now, let us denote the components of J_i and A_i along the string as J and A, respectively. Eq. (13) says $A = -\ln(\Lambda R)J/2\pi$. Since (by current conservation) J is a constant along the string, the same is true for A.

By eq. (9),

$$J = 2Ke\left(\frac{\mathrm{d}\vartheta}{\mathrm{d}l} + eA\right). \tag{14}$$

Since J and A are constants along the string in this gauge, the same is true of $\mathrm{d}\vartheta/\mathrm{d}l$.

As for the value of this constant, since

$$N = \frac{1}{2\pi}\oint \mathrm{d}l \frac{\mathrm{d}\vartheta}{\mathrm{d}l}, \tag{15}$$

we see that on a string of circumference $2\pi R$, the value of $\mathrm{d}\vartheta/\mathrm{d}l$ is

$$\frac{\mathrm{d}\vartheta}{\mathrm{d}l} = \frac{N}{R}. \tag{16}$$

Combining eqs. (13), (14) and (16), we find finally that in a sector of $N \neq 0$, the string carries a current

$$J = \frac{2Ke}{1 + Ke^2\ln(\Lambda R)/\pi}\,\frac{N}{R}. \tag{17}$$

For $K \gg 1$ (since we had $K \sim 1/\tilde{\lambda}$) and $\ln(\Lambda R) \gg 1$ (since R will be an astronomical distance, and $1/\Lambda$ is an elementary particle length) this reduces to

$$J = \frac{2\pi}{\ln(\Lambda R)}\,\frac{\hbar c^2}{e}\,\frac{N}{R}, \tag{18}$$

where $\hbar$ and c have been restored. This is very much less than the value we would estimate if we naively drop the second term in in eq. (14). Despite the large logarithm in the denominator, the current in (18) is big enough to have astronomical effects, as we will see. It should perhaps be noted that in applications, N scales like R^2 (the magnetic flux the string may cross) and N/R is very large.

This completes the demonstration that the string is superconducting. But before leaving the subject, if it is interesting to work out an exact formula for the change in the average current on the string in an arbitrary time dependent process. We have

$$\oint J\,\mathrm{d}l = 2Ke\oint \mathrm{d}l\left(\frac{\mathrm{d}\vartheta}{\mathrm{d}l}+eA\right)$$

$$=4\pi KeN + 2Ke\oint \mathrm{d}l\,A_i\frac{\mathrm{d}x^i}{\mathrm{d}l}\,. \tag{19}$$

Here N is time independent and $\oint \mathrm{d}l\,A_i\,\mathrm{d}x^i/\mathrm{d}l = \Phi$, Φ being the magnetic flux through any surface spanning the string. So

$$\frac{\mathrm{d}}{\mathrm{d}t}\oint J\,\mathrm{d}l = 2Ke^2\frac{\mathrm{d}\Phi}{\mathrm{d}t}\,. \tag{20}$$

One night expect that by the Meissner effect, magnetic flux lines could not cross the string and $\mathrm{d}\Phi/\mathrm{d}t$ would vanish. This is not so, in essence because we are dealing not with a macroscopic superconductor but with a thin string whose thickness is comparable to the magnetic penetration length. We will return to that point in sect. 4 after developing the theory of strings with Fermi charge carriers.

The discussion in this section is quite general, in the sense that almost any string that arises in a realistic theory with elementary charged scalars can be superconducting in the way we have described. It is disappointing, however, that this phenomenon depends on quantitative details of the Higgs structure which are hard to predict. There is, in fact, a situation in which string superconductivity would arise for general topological reasons. This is the case of "Alice strings" which have the property that a physical system that loops around the string comes back charge conjugated [11, 12]. Such strings cannot exist in nature, since charge conjugation is not a symmetry. Were they to exist, their properties would be quite remarkable. (For instance, looping around an ordinary star, the Alice string turns it into an anti-matter star.) It has been shown [12] that for topological reasons a charged field always has an expectation value in the core of an Alice string. So, by virtue of our comments, Alice strings are supconducting. The strings we have considered in this section are tame compared to Alice strings but have the virtue that they might exist in nature. Like an ordinary metal wire at low temperature, they are superconducting not for topological reasons but merely because the superconducting state is energetically favored.

3. Fermi charge carriers

We have seen that string superconductivity can arise from the expectation value of a charged Higgs field in the string core. We will now see that superconductivity can also arise from the zero modes of fermions moving in the field of the string [6]. These zero modes, in the situation of interest here have been discussed in detail by Jackiw and Rossi [7] and analyzed in terms of index theorems by Weinberg [8]. The role of charged fermions in our problem is somewhat reminiscent of their behaviour in the field of a magnetic monopole [9]. (The analogy is closely related to the logical possiblity that the vortex line we are studying could terminate in a magnetic monopole of a higher gauge group.)

To introduce the role of fermions, return to the $\mathrm{U}(1)\times \mathrm{U}(\tilde{1})$ gauge theory of sect. 2 with an unbroken gauge symmetry Q (electromagnetism) and a broken gauge symmetry R. Breakdown of R occurs due to the expectation value of a scalar field φ of $Q=0$, $R=1$. As a result, we obtain the vortex lines* (fig. 1) of our previous discussion; φ changes in phase by 2π in any closed path around the vortex line and vanishes in the vortex core. However, in contrast to sect. 2, we will here assume that all charged Higgs fields vanish in the vortex core, as well as outside.

The role of fermions becomes interesting if there are fermions which obtain their masses from coupling to φ. To this end, we introduce a two-component, left-handed spinor field $\psi_{\alpha \mathrm{L}}$ of $Q=q$, $R=r$, and a second spinor field $\chi_{\alpha\mathrm{L}}$ of $Q=-q$, $R=-r-1$. (These fields are left-handed in the sense that $i\gamma^0\gamma^1\gamma^2\gamma^3\psi=-\psi$, $i\gamma^0\gamma^1\gamma^2\gamma^3\chi=-\chi$.) With a suitable Yukawa coupling,

$$L=\bar{\psi}i\not{D}\psi+\bar{\chi}i\not{D}\chi-\lambda(\varphi\varepsilon^{\alpha\beta}\psi_{\alpha\mathrm{L}}\chi_{\beta\mathrm{L}}+\mathrm{h.c.})\,, \tag{21}$$

the ψ- and χ-fields get masses from their coupling to φ.

Let us discuss the Dirac equation for the ψ- and χ-fields coupled to the vortex line. Let us first discuss solutions that are independent of z and t. It follows [6-8] from an index theorem that there is precisely one solution of the Dirac equation $(\psi,\chi)=(\beta(x,y),\tilde{\beta}(x,y))$ that is independent of z and t and normalizable in the $x-y$ plane in the sense that $\int \mathrm{d}x\,\mathrm{d}y(|\psi|^2+|\chi|^2)<\infty$**. Moreover, this solution is an eigenstate of the operator $i\gamma^1\gamma^2$ (which Jackiw and Rossi call σ^3 or "particle conjugation"). In a vortex field the transverse zero mode obeys $i\gamma^1\gamma^2\beta=\beta$, $i\gamma^1\gamma^2\tilde{\beta}=\tilde{\beta}$. In an anti-vortex field (in which φ changes in phase by -2π in making a circuit) it obeys $i\gamma^1\gamma^2\beta=-\beta$, $i\gamma^1\gamma^2\tilde{\beta}=-\tilde{\beta}$.

Now we look for four-dimensional solutions of the Dirac equation in the special form $\psi(x,y,z,t)=\alpha(z,t)\beta(x,y)$, $\chi(x,y,z,t)=\alpha(z,t)\tilde{\beta}(x,y)$. The Dirac operator is $\not{D}=(\gamma^0\,\partial/\partial t+\gamma^3\,\partial/\partial z)+\not{D}_T$ where $\not{D}_T$ is the transverse Dirac operator which has

* Multiple vortex lines in which φ changes by $2\pi n$ are also of interest, but for simplicity we will discuss mainly the minimal case. Likewise, we will mainly discuss fermions whose masses violate R by $\Delta R=\pm 1$, though more general cases could arise in some realistic models. The more general Dirac equation was studied in refs. [7, 8].

** In ref. [7], Jackiw and Rossi considered precisely our lagrangian (1) except that they set $r=-\frac{1}{2}$ and set $\psi=\chi$. Relaxing the condition $r=-\frac{1}{2}$ cannot change the index because r is a parameter that can be continuously adjusted. As for the condition $\psi=\chi$, relaxing it means that in eq. (2.8) and subsequent equations of ref. [7], ψ and χ^* are independent variables rather than complex conjugates of one another. This would slightly simplify the analysis of ref. [7].

β and $\tilde{\beta}$ as zero modes. So α must obey

$$\left(\gamma^0\frac{\partial}{\partial t}+\gamma^3\frac{\partial}{\partial z}\right)\alpha=0\,. \tag{22}$$

Also, since $i\gamma^0\gamma^1\gamma^2\gamma^3\psi=-\psi$ and (in a vortex field) $i\gamma^1\gamma^2\beta=+\beta$, we need $\gamma^0\gamma^3\alpha=-\alpha$. With this requirement, (22) reduces to

$$\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial z}\right)\alpha=0\,, \tag{23}$$

and the solution is $\alpha(z,t)=f(z+t)$. Thus, fermions which are trapped in the transverse zero modes travel in the $-z$ direction at the speed of light. From the point of view of a one-dimensional observer who lives on the string, there is a massless chiral fermion (an eigenstate of $\gamma^0\gamma^3$) living on the string.

How would we obtain fermions trapped on the string and traveling in the $+z$ direction? To achieve this, we must introduce left-handed fermions that get masses from coupling to φ^* instead of φ. So we introduce Weyl spinors $\hat{\psi}$ of $Q=\hat{q}$, $R=\hat{r}$, and $\hat{\chi}$ of $Q=-\hat{q}$, $R=-\hat{r}+1$. These obtain masses from Yukawa coupling to φ^*.

$$\Delta L=-\gamma(\varphi^*\varepsilon^{\alpha\beta}\hat{\psi}_\alpha\hat{\chi}_\beta+\text{c.c.})\,. \tag{24}$$

Coupling to φ^* instead of φ means that the vortex looks like an anti-vortex. As a result, the transverse zero mode has opposite eigenvalue of $i\gamma^1\gamma^2$, and travels in the opposite direction.

Now let us discuss the question of cancellation of anomalies. The (3+1)-dimensional theories (1) and (4) have QQR, QRR and RRR triangle anomalies, all of which must cancel for the sake of consistency.

The coefficient of the QQR anomaly from ψ and χ is $q^2r+(-q)^2(-r-1)=-q^2$. More generally, we may consider a theory with several pairs (ψ_i,χ_i) getting masses from coupling to φ; if the ψ_i have charges (q_i, r_i), the total contribution of the (ψ_i,χ_i) to the QQR anomaly is $-\sum q_i^2$.

Now consider the pair $(\hat{\psi},\hat{\chi})$ whose mass comes from coupling to φ^*. They contribute $\hat{q}^2\hat{r}+(-\hat{q})^2(-\hat{r}+1)=+\hat{q}^2$ to the QQA anomaly. More generally, if there are several pairs $(\hat{\psi}_i,\hat{\chi}_i)$ of this type, with $\hat{\psi}_i$ having charges $(\hat{q}_i,\hat{r}_i)$, the total contribution to the QQR anomaly is $+\sum\hat{q}_i^2$. Including pairs of both types, the equation

$$\sum q_i^2=\sum\hat{q}_i^2 \tag{25}$$

is the condition for cancellation of the QQR anomaly.

Eq. (25) is of fundamental importance for string physics. Each multiplet (ψ_i,χ_i) that gets mass from φ gives rise to a massless fermion of charge $Q=q_i$ traveling on the string in the $-z$ direction. Each multiplet $(\hat{\psi}_i,\hat{\chi}_i)$ that gets mass from φ^* gives rise to a massless fermion of charge $Q=\hat{q}_i$ traveling on the string in the $+z$ direction. For charged fermions in 1+1 dimensions, consistency of the gauge couplings (cancellation of the $(1+1)$-dimensional analogue of the triangle anomaly) requires [13]

$$\sum_{\text{left-movers}} q_i^2 = \sum_{\text{right-movers}} \hat{q}_i^2 \,. \tag{26}$$

This is precisely eq. (25) for cancellation of the QQA anomaly in the underlying theory. Therefore, the effective field theory on a string is always anomaly free if the underlying theory was anomaly free. A closely analogous principle in Kaluza-Klein theory ensures that anomaly free theories remain anomaly free after any dimensional reduction [14].

Cancellation of the QRR anomaly in the underlying theory also has an important implication for string physics, which we will discuss briefly. The vortex field is not invariant under rotations about the z-axis. It is also, of course, not invariant under R, which is spontaneously broken. However, the classical vortex field is invariant under $\tilde{R} = R + J_z$ - simultaneous space and R-rotations. It can be shown [7] that for the pair (ψ, χ) which gets mass from φ, the transverse zero mode has $\tilde{R} = r + \frac{1}{2}$; for the pair $(\tilde{\psi}, \tilde{\chi})$ which gets mass from φ^*, the tranverse zero mode has $\tilde{R} = \hat{r} - \frac{1}{2}$. Is $\tilde{R}$ really conserved in string physics? This requires cancelllation of the $\tilde{R}Q$ anomaly in the effective two-dimensional theory. The cancellation condition is

$$\sum_{\text{left-movers}} q_i(r_i + \tfrac{1}{2}) = \sum_{\text{right-movers}} \hat{q}_i(\hat{r}_i - \tfrac{1}{2}) \,. \tag{27}$$

It can easily be seen that this equation coincides with the condition for cancellation of the QRR anomaly in the underlying $(3+1)$-dimensional theory. Therefore, $\tilde{R}$ is always realized as a bona fide global symmetry in string physics. This will not be very important in the relatively simple models we will consider in this paper, because in those models all the transverse zero modes have $\tilde{R} = 0$. However, conservation of $\tilde{R}$ may be important in other cases, especially if $\tilde{R}$ is reallized as a non-vector-like symmetry.

Let us now discuss the effective field theory describing charged fermions trapped on a string in transverse zero modes. In the simplest anomaly free situation, we may have a mode of charge q traveling in the $+z$ direction and another mode of charge q traveling in the $-z$ direction. (Perhaps it should be stressed that the antiparticles, of charge $-q$, always travel in the same direction as the particles. This follows from the CPT theorem or from the fact that if a field ψ is a function of $z - t$ then the complex conjugate ψ^* is likewise a function of $z - t$, not $z + t$.) We can describe this situation by introducing a $(1+1)$-dimensional Dirac field $\psi(z, t)$ which lives on the string, interacting with the electromagnetic potential A_μ which is a function of all four coordinates. If we adopt convention that Greek letters μ, ν, etc. range from 0 to 3 while Latin letters i, j, k take only two values tangent to the world sheet of the string (so $i, j = 0$ or 3 if the string runs in the z-direction), then the action is

$$I = -\tfrac{1}{4} \int \mathrm{d}^3x \, \mathrm{d}t \, F_{\mu\nu} F^{\mu\nu} + \int \mathrm{d}z \, \mathrm{d}t \, \bar{\psi}(z, t) i\gamma^i D_i \psi(z, t) \,, \tag{28}$$

where $D_i\psi = (\partial_i + ieA_i)\psi$. This model is very similar to the effective field theory that arises in the Callan-Rubakov process. Happily, it can be exactly solved by the same technique of bosonization that makes possible the solution of the Schwinger model [15]. One introduces a scalar field $\varphi(z, t)$ living on the string such that $\bar{\psi}\gamma^i\psi = (1/\sqrt{\pi})\varepsilon^{ij}\,\partial_j\varphi$. (We take $\varepsilon^{03} = +1$.) (28) is then equivalent to

$$I = -\tfrac{1}{4}\int \mathrm{d}^3x\,\mathrm{d}t\,F_{\mu\nu}F^{\mu\nu} + \int \mathrm{d}z\,\mathrm{d}t\left(\tfrac{1}{2}\dot{\varphi}^2 - \frac{1}{2}\left(\frac{\partial\varphi}{\partial z}\right)^2 - \frac{q}{\sqrt{\pi}}A_i\varepsilon^{ij}\,\partial_j\varphi\right)$$

$$= -\tfrac{1}{4}\int \mathrm{d}^3x\,\mathrm{d}t\,F_{\mu\nu}F^{\mu\nu} + \int \mathrm{d}z\,\mathrm{d}t\left(\tfrac{1}{2}\dot{\varphi}^2 - \frac{1}{2}\left(\frac{\partial\varphi}{\partial z}\right)^2 - \frac{q}{\sqrt{\pi}}\varphi E\right), \qquad (29)$$

where $E = \varepsilon^{ij}\,\partial_i A_j$ is the component of the electric field tangent to the string.

A slight generalization of this is important for applications. Suppose there are not one but k fermi modes $\psi_1, \psi_2, \ldots, \psi_k$, of charges $q_1, q_2, \ldots, q_k$ trapped on the string. Then (29) would be replaced by

$$\hat{I} = -\tfrac{1}{4}\int \mathrm{d}^3x\,\mathrm{d}t\,F_{\mu\nu}F^{\mu\nu} + \sum_i \int \mathrm{d}z\,\mathrm{d}t\left(\tfrac{1}{2}\dot{\varphi}_i^2 - \frac{1}{2}\left(\frac{\partial\varphi_i}{\partial z}\right)^2 - \frac{1}{\sqrt{\pi}}q_i\varphi_i E\right). \qquad (30)$$

(30) can easily be reduced to (29) as in the many-flavor Schwinger model [15]. Choose a new orthonormal basis of scalar fields with $\bar{\varphi} = (1/\sqrt{\sum e_i^2})\sum e_j\varphi_j$ and $k-1$ other fields φ_α orthogonal to $\bar{\varphi}$. Then the φ_α do not couple to electromagnetism, and the coupling of $\bar{\varphi}$ is

$$\hat{I} = -\tfrac{1}{4}\int \mathrm{d}^3x\,\mathrm{d}t\,F_{\mu\nu}F^{\mu\nu} + \int \mathrm{d}z\,\mathrm{d}t\left(\frac{1}{2}\left(\frac{\partial\bar{\varphi}}{\partial t}\right)^2 - \frac{1}{2}\left(\frac{\partial\bar{\varphi}}{\partial z}\right)^2 - \frac{1}{\sqrt{\pi}}\bar{q}\bar{\varphi}E\right), \qquad (31)$$

where $\bar{q} = \sqrt{\sum_i q_i^2}$. Thus the string with a single trapped mode of charged fermions is equivalent to the multi-mode case, with the replacement $q^2 \to \sum_i q_i^2$*.

Being an exactly soluble, quadratic lagrangian, (29) (or (31)) is very similar to the effective lagrangian (8) that we found for the string with Bose charge carriers. In fact, the two are not just similar but equivalent (except for the values of the parameters). One way to see the equivalence is to write the Euler-Lagrange equation for φ that follows from (29):

$$\partial_i\partial^i\varphi + \frac{q}{\sqrt{\pi}}E = 0. \qquad (32)$$

With $E = \varepsilon^{ij}\,\partial_i A_j$, this says

$$\partial_i\left(\partial^i\varphi + \frac{q}{\sqrt{\pi}}\varepsilon^{ij}A_j\right) = 0, \qquad (33)$$

* It is a peculiarity of gauge couplings in $1+1$ dimensions that the large scale interaction of the string depend on $\sum q_i^2$ and not on the individual q_i. More generally, one could consider a case where the left-movers and right-movers have independent charges q_i and $\hat{q}_j$. Then $\sum q_i^2 = \sum \hat{q}_j^2$ whenever the underlying theory makes sense, as discussed in the text, and the string interactions can be shown to still depend only on $\sum q_i^2$.

so the current $X^i=\partial^i\varphi+(q/\sqrt{\pi})\varepsilon^{ij}A_j$ is conserved. In two dimensions a conserved current such as X^i can always be written as the derivative of a scalar, $X^i=\varepsilon^{ij}\,\partial_j\gamma$ for some γ. γ obeys $\partial_i\gamma=-\varepsilon_{ij}X^j$ so $\partial_i\,\partial^i\gamma=-\partial^i\varepsilon_{ij}X^j=-\partial_i\varepsilon^{ij}(\partial_j\varphi+(q/\sqrt{\pi})\varepsilon_{jk}A^k)=-q/\sqrt{\pi}\,\partial_iA^i$. But the equation $\partial_i(\partial^i\gamma+(q/\sqrt{\pi})A^i)=0$ comes from the lagrangian $\frac{1}{2}(\partial_i\gamma+(q/\sqrt{\pi})A_i)^2$, which is quite reminiscent of eq. (8). In fact, if we write

$$\tilde{I}=-\tfrac{1}{4}\int \mathrm{d}^3x\,\mathrm{d}t\,F_{\mu\nu}F^{\mu\nu}+\int \mathrm{d}z\,\mathrm{d}t\,\tfrac{1}{2}\left(\partial_i\gamma+\frac{q}{\sqrt{\pi}}A_i\right)^2, \tag{34}$$

then the Euler-Lagrange equations for γ and A_μ derived from (34) are equivalent to those for φ and A_μ derived from (29), with the connection

$$\partial^i\varphi+\frac{q}{\sqrt{\pi}}\varepsilon^{ij}A_j=\varepsilon^{ij}\,\partial_j\gamma \tag{35}$$

indicated by the above discussion.

But eq. (34) is our old lagrangian (8), with the charge of variable $\gamma=\sqrt{2K}\,\vartheta$ and the substitution $q\to\sqrt{2\pi K}e$. Therefore, the interaction with electromagnetic fields of strings with Bose or Fermi charge carriers are described by equivalent lagrangian (31) or (8), with the couplings related by

$$\sum q_i^2\leftrightarrow 2\pi Ke^2\,. \tag{36}$$

Since $K\sim 1/\tilde{\lambda}$, as we saw in sect. 2, the effective coupling is much bigger in the case of Bose charge carriers. But we will see in sect. 4 that this affects the phenomenology much less than one might expect. The main differences in phenomenology will turn out to result from anomalous particle production which occurs in the Fermi case, and which we will now begin to explore.

There is a useful, elementary way of seeing, again, that the string is superconducting. We will use the form (29) of the lagrangian. Consider the behavior of the φ-field of eq. (29) in the presence of a prescribed (z-independent) E-field. The equation for φ is

$$\ddot{\varphi}+\frac{q}{\sqrt{\pi}}E=0\,. \tag{37}$$

Since the electromagnetic current along the string is $J=-q\bar{\psi}\gamma^3\psi=-(q/\sqrt{\pi})\dot{\varphi}$, (37) is equivalent to

$$\frac{\mathrm{d}J}{\mathrm{d}t}=\frac{q^2}{\pi}E\,. \tag{38}$$

This equation means that the string is superconducting. If an electric field E is applied for a time T, a current q^2ET/π builds up. This current remains even if the electric field is turned off after time T. By contrast, for a wire of finite conductivity σ, the current is $J=\sigma E$ and vanishes (after a certain characteristic time) if E is turned off.

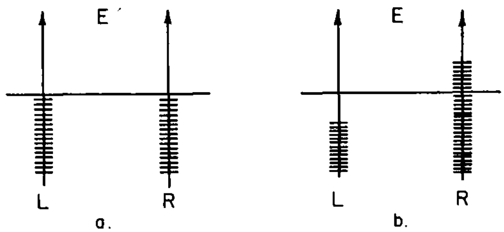

Fig. 5. (a) In the ground state of the string, both left-movers and right-movers (L and R in the figure) have zero Fermi energy. (b) An applied electric field shifts the Fermi energies of left- and right-movers in opposite directions.

One may wonder whether the occurrence of persistent currents is an artifact of our approximation of keeping only the fermion zero modes. Can the currents relax in some higher approximation? For strings with Bose charge carriers, it was conservation of N (defined in eq. (10)), which guaranteed that the persistent currents were persistent. For strings with Fermi charge carriers, it will turn out that an analogous role is played by quantum numbers such a baryon and lepton number. When we consider realistic models in the latter part of this paper, we will find that the current carrying state of the string carries quantum numbers such as baryon and lepton number and is the minimum energy state of given quantum numbers. This will prevent the persistent currents from relaxing until large values of the current are reached at which (as discussed later) eq. (38) finally breaks down.

Eq. (38) has a simple and important microscopic explanation in terms of the original, unbosonized form of the theory. Consider the string before the electric field is applied. In the ground state (fig. 5a) the negative energy fermion states are filled and the positive energy states are empty; this is so for both right-movers and left-movers. Now apply an electric field E along the string (in the $+z$ direction). It can be represented by an electrostatic potential $V=-Ez$. In the presence of the field, the right- or left-moving fermions continue to travel to the right or left at the speed of light. However, they gain or lose a kinetic energy qEL in traveling to the right (or left) a distance L. This means that after a time t the Fermi momentum of the right-moving charges is no longer zero but is $p_F = qEt$. Since the density of states per unit length in $1+1$ dimensions is $p_F/2\pi$, the increase in p_F from zero to qEt corresponds to a creation of $qEt/2\pi$ right-moving fermions per unit length. Since each fermion has charge q and moves at the speed of light, the right-movers carry a current $J_R = q^2Et/2\pi$ at time t.

As for the left-movers, their Fermi energy at time t is $-qEt$. This corresponds to the creation of *anti*particles with charge $-q$ and $p_F = qEt$. Since the antiparticles have charge $-q$ but are traveling in the $-z$ direction, they again carry a current $J_L = q^2Et/2\pi$. The total current is $J_L + J_R = q^2Et/\pi$, as we computed before.

The moral of the story is that when an electric field is applied, right-moving particles and left-moving antiparticles are both created at the rate

$$\frac{d^2 N_R}{dt\,dz} = \frac{d^2 N_L}{dt\,dz} = \frac{qE}{2\pi\hbar}. \tag{39}$$

Here $d^2N/dt\,dz$ is the rate of particle creation per unit time per unit length of string, and we have restored Planck's constant. The Fermi momentum of both right-moving particles and left-moving antiparticles builds up at a rate

$$\frac{dp_F}{dt} = qE. \tag{40}$$

The similarity of eq. (40) to the Lorentz force law $dp/dt = qE$ for a particle in an electric field is no accident; eq. (40) is the Lorentz force law applied to the particles at the top of the Fermi sea. (The magnetic field is not relevant because particles in transverse zero modes cannot be deflected off the string.) If there are several species of charge carrier on the string, (39) and (40) hold for each separately.

The dramatic import of eq. (39) will become apparent when we consider models in sects. 6 and 7. For the time being, let us simply note that the right-moving particles and left-moving antiparticles which are created (according to eq. (39)) when the string sits in an electric field are not nameless and faceless electric charges. They will in any real model carry baryon number, lepton number, or other quantum numbers. There is no reason that the net baryon and lepton numbers of the particles created according to equation (39) need vanish, and in real models this will not be the case.

We can now answer the question: at what point does the current-carrying ability saturate for a string with Fermi charge carriers? For a given charge carrier of charge q, whose mass in vacuum is m, the Fermi momentum along the string cannot exceed m; for $p_F > m$, it is energetically favored for fermions to jump out of the transverse zero mode and into a normal state outside the string, of energy m. For $p_F = m$, the number of left- or right-moving charge carriers in the transverse zero mode is $m/2\pi$. So the maximum current that can be carried in a single left- or right-moving zero mode by a fermion of charge q and mass m is

$$J_{\max} = q\frac{mc^2}{2\pi\hbar}. \tag{41}$$

In any real model several left- and right-moving modes will contribute to the maximum current.

If the charge carriers are electrons, eq. (41) indicates that the maximum current they can carry (in a single zero mode) is about 20 amperes. On the other hand, if the charge carriers are superheavy fermions of mass, say 10^{16} GeV, the maximum current in a single zero mode is about 4×10^{20} A.

The sensational current-carrying ability of strings with superheavy fermions trapped in zero modes is matched by strings with Bose charge carriers. In sect. 2 we noted that insofar as the phase of the charged Higgs field σ is well-defined, strings with Bose charge carriers are perfect superconductors. Processes in which the phase cannot be defined (since σ goes through zero somewhere on the world sheet of the string) are tunneling processes that occur at a significant rate only when $J \sim eMc^2/\hbar$, M being the mass scale of the symmetry breaking in which the string formed. So for grand unified strings with Bose charge carriers, the maximum current is again 10^{20} A or above.

4. Strings in magnetic fields

So far, we have discussed the behavior of strings in somewhat artificial situations. For instance, in sect. 3 we considered the behavior of the string in a given electric field. In a real problem, an applied electric field will excite currents on the string which will create magnetic fields which (via Maxwell's equations) will induce electric fields. It is the total electric field, not the externally applied field, to which the current on the string responds.

To gain a more realistic insight into the behavior of superconducting strings, consider a simple but crucial problem: the scattering of light by the string. We will consider the problem with Fermi charge carriers and the lagrangian of eq. (29), but the other case is essentially the same.

For simplicty (fig. (6)), we consider an electromagnetic wave that is incident at 90° to the string, with the polarization such that the electric field vector is parallel to the string (which we assume to run in z-direction).

We can describe this problem in a gauge with $A_t = A_x = A_y = 0$, and since the problem is z-independent, we may assume $A_z = A(x, y, t)$, $\varphi = \varphi(t)$. The equations of motion are

$$\ddot{A} - \nabla^2 A - \frac{q}{\sqrt{\pi}} \delta^2(x) \dot{\varphi} = 0 ,$$

$$\ddot{\varphi} + \frac{q}{\sqrt{\pi}} \dot{A}_z(x = y = 0) = 0 , \tag{42}$$

where ∇^2 is the two-dimensional laplacian, and $\delta^2(x)$ is the transverse delta function.

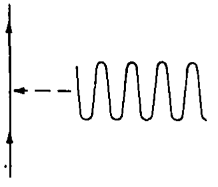

Fig. 6. An electromagnetic wave normally incident on a string.

If $A(x, y, t) = A(x, y)\,\mathrm{e}^{-i\omega t}$, $\varphi = \varphi\,\mathrm{e}^{-i\omega t}$ then (42)reduces to

$$\left(-\nabla^2 + \frac{q^2}{\pi}\delta^2(x)\right)A(x, y) = \omega^2 A(x, y)\,. \tag{43}$$

This is the non-relativistic Schrödinger equation for scattering from a delta function potential in two dimensions*. The scattering solution obeys

$$\begin{aligned} A(x) &= \mathrm{e}^{ik\cdot x} - \int \mathrm{d}^2x'\, G(x, x')\frac{q^2}{\pi}\,\delta^2(x')A(x') \\ &= \mathrm{e}^{ik\cdot x} - \frac{q^2}{\pi}G(x, 0)A(0)\,, \end{aligned} \tag{44}$$

where $G(x, x')$ is the Green function

$$G(x, x') = \int \frac{\mathrm{d}^2 \boldsymbol{k}}{(2\pi)^2}\,\frac{\mathrm{e}^{ik\cdot(x-x')}}{\boldsymbol{k}^2 - \omega^2 - i\varepsilon}\,. \tag{45}$$

(45) implies that

$$A(0) = \frac{1}{1 + q^2 G(0, 0)/\pi}\,. \tag{46}$$

However, this needs to be interpreted with care because $G(0, 0)$ is infinite. The integral in (45) is ultraviolet divergent, the divergent term being $(1/2\pi)\ln(\Lambda/\omega)$, where Λ is a cutoff. The divergence is closely related to the one we met in sect. 2, and arises from assuming the string has zero size. If we allow for the fact that the current is carried by modes of non-zero spatial extent, the delta function in equation (43) would be replaced by a short-range but non-singular potential. We interpret (46) to mean

$$A(0) = \frac{1}{1 + (q^2/2\pi^2)\ln(\Lambda/\omega)}\,, \tag{47}$$

with a cut-off Λ that depends on the structure of the string.

The electric field at the position of the string is proportional to $A(0)$. Since we took the incident wave to be $\mathrm{e}^{ik\cdot x}$, the incident wave corresponded to $A(0) = 1$. The fields induced by the currents excited on the string reduce $A(0)$ by a factor

$$\eta = \frac{1}{1 + (q^2/2\pi^2)\ln(\Lambda/\omega)}\,. \tag{48}$$

In principle, as $\Lambda \to \infty$, $\eta \to 0$ and there are no currents excited on the string (and also no scattered wave; see eq. (44)). In fact, however, for realistic values of the parameters, η is not extremely small. An extreme upper bound on Λ might be

* Of course, this equation must also emerge in the alternative, equivalent form (8) of the theory of strings and electromagnetic fields. It is easily derived in that case by choosing a gauge $\vartheta = 0$.

$\Lambda = 10^{19}$ GeV (if the string thickness is the Planck length). A lower bound on values of ω of practical interest is $\omega/2\pi \gtrsim 10^{-10}\,\mathrm{y}^{-1}$ (since we cannot observe processes with longer duration than the present age of the universe). With these values $\ln(\Lambda/\omega) \simeq 140$. If we set q to be the charge of the electron, then with these values one gets $n \simeq 0.6$.

Actually, η can be smaller than this if there are several fermion zero modes trapped on the string. As we saw in relating eq. (29) to eq. (31), the effect of that is just to replace q^2 by $\sum q_i^2$, so in general

$$\eta = \frac{1}{1 + (\sum q_i^2/2\pi^2)\ln(\Lambda/\omega)}\,. \tag{49}$$

For example, in the models that we will consider in sects. 6 and 7, $\sum q_i^2 = 4e^2$ (if thee are three generations), and $\eta \simeq \frac{1}{3}$ for relevant values of ω. In any case, η is a number moderately less than one, and depends on ω only weakly in the frequency range of interest. When we need to make a numerical evaluation we will set $\eta = \frac{1}{3}$.

Returning to the scattering problem, the solution is

$$A(x) = \mathrm{e}^{ik\cdot x} - \sum \frac{q^2 i}{\pi}\eta G(x,0)\,. \tag{50}$$

The asymptotic behavior of the Green function in two dimensions is $G(x,0) \underset{|x|\to\infty}{\longrightarrow} \sqrt{i/8\pi\omega|x|}\,\mathrm{e}^{i\omega|x|}$. The scattering is isotropic, with scattering amplitude

$$f = -\frac{\sum q_i^2\eta}{\pi}\sqrt{\frac{i}{8\pi\omega}}\,, \tag{51}$$

and the total cross section per unit length of string, integrated over scattering angles, is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z} = \frac{(\sum q_i^2)^2\eta^2}{8\pi^3}\lambda\,, \tag{52}$$

where $\lambda = 2\pi/\omega$ is the wavelength of the incident radiation. For instace, if $\sum q_i^2 = 4e^2$ (as in our later models) and $\eta = \frac{1}{3}$, then $\mathrm{d}\sigma/\mathrm{d}z \simeq 6\times 10^{-5}\lambda$.

It is easy to adapt these results for the case the current is carried by a boson of charge e. One just replaces $\sum q_i^2$ by $2\pi K e^2$, as in eq. (35). Making this replacement in η also, the scattering amplitude is in this case

$$f = -Ke^2\frac{1}{1 + (Ke^2/\pi)\ln(\Lambda/\omega)}\sqrt{\frac{i}{8\pi\omega}}\,. \tag{53}$$

In practice, with $Ke^2/\pi \gtrsim 1$ (since we had $K \sim 1/\tilde{\lambda}$), and $\ln(\Lambda/\omega) \gg 1$, this is

$$f = -\frac{2\pi}{\ln(\Lambda/\omega)}\sqrt{\frac{i}{8\pi\omega}}\,. \tag{54}$$

The scattering cross section per unit length of string is then

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z}=\frac{\pi}{2(\ln(\Lambda/\omega))^2}\lambda\,. \tag{55}$$

If, say, $\ln(\Lambda/\omega)=100$, then $\mathrm{d}\sigma/\mathrm{d}z=1.6\times10^{-4}\,\lambda$, comparable to the result with Fermi charge carriers.

These results are reminiscent of the Callan–Rubakov effect in that the apparent size of the string as measured by the scattering cross section is (except for a slowly varying logarithm) almost independent of its physical dimensions. If the string is probed by visible light, it appears to have a thickness of a few tenths of an angstrom. If it is probed by a light wave of wavelength 30 000 light years (galactic dimensions) it appears to have a thickness of a few light years.

We can now refine some previous formulas. The rate of current buildup when an electric field is applied to the string is given by (37) if E is the total electric field along the string. If by E we mean an applied, external field, then the total field on the string in ηE, so the current builds up at a rate

$$\frac{\mathrm{d}J}{\mathrm{d}t}=\eta E\frac{c}{\pi\hbar}\sum q_i^2\,, \tag{56}$$

where $\hbar$ and c have been restored. (If the current is carried by bosons, one replaces $\sum q_i^2$ by $2\pi Ke^2$ in (56) and in the definition of η.)

As for particle production by the string, here too the E-field of eq. (38) is to be interpreted as ηE, E being the externally applied field. So for any mode of charge q trapped on the string, right-moving particles (or left-moving antiparticles) are created at a rate

$$\frac{\mathrm{d}^2N}{\mathrm{d}t\,\mathrm{d}z}=\frac{q\eta E}{2\pi\hbar}\,. \tag{57}$$

In eqs. (56) and (57), it must be noted that once the current carried by the string saturates (when $p_{\mathrm{F}}\sim m$, so that further produced particles are ejected off the string in modes of non-zero energy), the string no longer generates fields that couteract an applied field, so we can set $\eta=1$ in (56) and (57).

Let us now think of strings in their astrophysical setting. Strings typically oscillate with moderately relativistic velocities (so $v/c\sim1$, but not the ultrarelativistic regime $1/\sqrt{1-v^2/c^2}\gg1$). Consider a portion of string that is traversing a galactic or intergalactic magnetic field with velocity v (fig. 7). In view of the Meissner effect, a *macroscopic* superconductor could not be crossed by magnetic field lines and therefore would push the field lines upstream. In our case, we are dealing with a *microscopic* superconducting wire which, in effect, is as thin as the magnetic penetration length.

As seen by the oscillating string, the galactic or intergalactic magnetic field is similar to a light wave of large wavelength. With Fermi charge carriers, eq. (51)

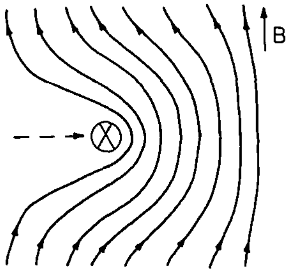

Fig. 7. A top view. A superconducting string (shown as an ⊗) crossing galactic magnetic field lines. Via the Meissner effect, it tends to push them upstream.

shows that the scattering amplitude is of order $\alpha = e^2/4\pi$, so only a fraction of order α or about 10^{-2} of the magnetic field lines are scattered upstream by the moving string. With Bose charge carriers, the fraction of scattered field lines is roughly the same because of the factor $1/(\ln \Lambda/\omega))$ in eq. (54).

If we (naively) ignore the ubiquitous galactic and intergalactic plasma, the scattered field lines would simply escape as an outgoing electromagnetic wave, corresponding to the scattering cross section estimated in eqs. (52) or (55). In actuality, the magnetic field lines are tied in the plasma, at least on large scales. An oscillating string in an astrophysical system containing a magnetic field plus plasma would drive a dynamo effect, tangling field lines and building up the magnetic field. If a closed string has radius L at maximum extension, the time scale for its oscillations is $\tau \simeq L/c$. Since the string pulls with it a fraction of order α of the field lines on each oscillation, the time scale for dynamo activity by an oscillating string might naively be of order $\alpha^{-1}\tau \simeq \alpha^{-1}L/c$ if the repeated oscillations cause the magnetic field to build up coherently, or of order $\alpha^{-2}L/c$ if the process is more like a random walk.

By contrast, galaxies rotate with $v/c \simeq 10^{-3}$. The rotating plasma draws along *all* of the magnetic field lines, and the time scale for dynamo activity associated with the galactic rotation is $2\pi R/v \simeq 10^4 R/c$, where R is the radius of the galaxy. If a string is oscillating in a galaxy, the dynamo activity driven by the string might be hard to distinguish from the conventional galactic dynamo if $L \sim R$. However, for $L \ll R$, the string dynamo might rapidly produce strong small-scale irregularities in the galactic magnetic field. One might also look for strings that are oscillating and generating fields outside of galaxies; such an oscillating string, if it generates a strong field, might be visible due to synchroton radiation by ambient charges.

More impressive than the general dynamo activity due to oscillating strings is the very intense magnetic field that surrounds the string in its motion. If a string moves at velocity v across a field B_0, then in the rest frame of the string there is an applied electric field $E = v/cB_0$. The rate at which the current builds up is given by eq. (56). The string will travel a distance d in a time $t = d/v$, and after traveling that distance the current on the string (assuming it has not had a chance to relax by processes discussed briefly in sect. 3) is $J = \eta B_0(d/\pi\hbar)\sum_i q_i^2$. A wire carrying a current J is surrounded by an azimuthal magnetic field whose strength, at distance r from the string, is $B(r) = J/2\pi rc$, or in this case

$$B(r) = \frac{d}{r} B_0 \eta \sum \frac{q_i^2}{2\pi^2 \hbar c}\,. \tag{58}$$

If, for instance, $\eta = \frac{1}{3}$ and $\sum q_i^2 = 4e^2$, this can be written

$$B(r) = (1.9 \times 10^{14}\ \mathrm{G})\left(\frac{1\ \mathrm{cm}}{r}\right)\left(\frac{d}{10^4\ \mathrm{persec}}\right)\left(\frac{B_0}{10^{-6}\ \mathrm{G}}\right). \tag{59}$$

In eq. (59) the values of 10^4 parsecs and 10^{-6} G for d and B_0 correspond roughly to the radius of the luminous portion of our galaxy and the strength of the coherent component of the galactic magnetic field. If a coherent field of order 10^{-6} G extends over a distance of 10^6 parsec (roughly the intergalactic distance) one can contemplate that a string passing through a galaxy picks up a magnetic field $B(r) \sim (2 \times 10^{16}\ \mathrm{G})(1\ \mathrm{cm}/r)$. Similar values emerge for a very long string that has traveled a cosmological distance if we assume that a plausible intergalactic magnetic field of 10^{-9} G has a coherent component that extends over a cosmological distance of 10^9 parsecs.

Evidently, the magnetic field near a string can be larger even than the fields in pulsars. Even with the "modest" value $d = 10^4$ parsec, the magnetic field of the string dominates the ambient field B_0 out to a distance $r = 1.9 \times 10^{20}\ \mathrm{cm} \simeq 10$ parsecs.

We also will want quantitative estimates for the rate at which the Fermi momentum of a charge carrier increases and the rate at which charge carriers are created. With $E = vB_0/c$, we find from (22) the Fermi momentum of a charge carrier of charge q builds up at a rate

$$\begin{aligned}\frac{\mathrm{d}p_{\mathrm{F}}}{\mathrm{d}t} &= \eta q \frac{v}{c} B_0 \\ &= 3.0\,\frac{\mathrm{MeV}}{\mathrm{sec}}\left(\frac{q}{e}\right)\left(\frac{v}{c}\right)\left(\frac{B_0}{10^{-6}\ \mathrm{G}}\right).\end{aligned} \tag{60}$$

Thus in crossing a galaxy (radius 10^4 parsecs or 3×10^4 light years; crossing time 6×10^4 y or 2×10^{12} sec) a charge carrier of $q = e$ builds up a Fermi momentum of order 6×10^9 GeV! This is valid for superheavy fermions whose mass m is bigger than 6×10^9 GeV. Otherwise, as discussed at the end of sect. 3, the Fermi momentum levels off at $p_{\mathrm{F}} = m$, and further crossing of field lines results in ejection of fermions from the string (as particles occupy modes of non-zero tranverse energy) rather than build-up of p_{F}.

As for the rate at which particles are created per unit length per unit time, it is

$$\begin{aligned}\frac{\mathrm{d}^2 N}{\mathrm{d}t\,\mathrm{d}z} &= \frac{\eta q}{2\pi\hbar}\,\frac{v}{c} B_0 \\ &= 2.2 \times 10^{11}\ \mathrm{cm}^{-1}\,\mathrm{sec}^{-1}\left(\frac{q}{e}\right)\left(\frac{\eta}{\frac{1}{3}}\right)\left(\frac{v}{c}\right)\left(\frac{B_0}{10^{-6}\ \mathrm{G}}\right).\end{aligned} \tag{61}$$

This is clearly a macroscopic rate, about 1 mole of particles/km of string per year. Again, we may set $\eta = 1$ once the current on the string saturates.

Formulas of this section that are written for strings with Fermi charge carriers can, of course, be easily adapted for strings with Bose charge carriers. We replace $\sum q_i^2$ by $2\pi K e^2$; for realistic values of K it is equivalent to make the substitution

$$\eta \sum q_i^2 \leftrightarrow \frac{2\pi^2}{\ln(\Lambda/\omega)}, \tag{62}$$

where Λ is a cutoff depending on the string size and ω is a frequency characteristic of the string motion.

5. Bose phenomenology

Model building for strings with Bose charge carriers is easy to describe. Almost any theory with strings in which there are elementary charged bosons can behave in the way described in sect. 2. This is possible for the various grand unified theories with strings that have been discussed in the literature [16]. However, this phenomenon always occurs only for a certain range of the Higgs parameters.

Since anomalous particle production processes do not occur for strings with Bose charge carriers, such strings must be detected via their magnetic effects or via the release of the magnetic energy they accumulate. The magnetic field at a distance r from a string that has traveled a distance d in a magnetic field B_0 can be found by adapting (58) in the usual way. We get (see eq. (62))

$$\begin{aligned} B(r) &= \frac{d}{r} B_0 \frac{1}{\ln(\Lambda/\omega)} \\ &= 3\times 10^{14}\,\mathrm{G}\left(\frac{1\ \mathrm{cm}}{r}\right)\left(\frac{d}{10^4\ \mathrm{parsec}}\right) \\ &\quad \times \left(\frac{\mathrm{B}_0}{10^{-6}\ \mathrm{G}}\right)\left(\frac{100}{\ln(\Lambda/\omega)}\right), \end{aligned} \tag{63}$$

which is comparable to the result with Fermi charge carriers.

One possibility is that the string will be visible as a synchrotron radiation source due to charged particles spiralling in the very strong field (63). Since the Bose string (unlike the string with Fermi charge carriers) does not create its own plasma via anomalous particle production processes, this would have to occurr through interaction of the string with astrophysical plasma. An analysis of such interaction will not be attempted here.

Another possibliity is that the very large magnetic energy represented by (63) may somehow be discharged in a luminous form. The magnetic energy is $\frac{1}{2}\int_0^{2\pi} \mathrm{d}\varphi \int_0^\infty \mathrm{d}r\, rB(r)^2$. The r-integral diverges logarithmically; the divergence can be interpreted as $\ln(\Lambda/\omega)$. So after traveling a distance d in an ambient magnetic field

B_0, the string has magnetic energy per unit length

$$\frac{\mathrm{d}E}{\mathrm{d}z}=\frac{\pi d^2 B_0^2}{\ln(\Lambda/\omega)}. \tag{64}$$

If the string is traveling at velocity v, then $d = vt$, and the rate of build-up of magnetic energy is

$$\frac{\mathrm{d}^2E}{\mathrm{d}z\,\mathrm{d}t}=\frac{2\pi d B_0}{(\ln(\Lambda/\omega))}v$$
$$=2\times10^{38}\,\text{ergs parsec}^{-1}\,\text{sec}^{-1}$$
$$\times\left(\frac{v}{c}\right)\left(\frac{B_0}{10^{-6}\,\mathrm{G}}\right)^2\left(\frac{d}{10^4\,\text{parsec}}\right)\left(\frac{100}{\ln(\Lambda/\omega)}\right). \tag{65}$$

This a respectable rate of energy build-up. The luminosity of the sum is about 4×10^{33} ergs/sec, so a string that crosses a galaxy is accumulating magnetic energy at the rate of roughly 10^4 times the solar luminosity per parsec.

By adapting eq. (59), we can determine the current carried by the string after traveling a distance d in a magnetic field B_0. If we set $\eta\sum q_i^2/\hbar=2\pi^2/\ln(\Lambda/\omega)$, $E=vB/c$ and $t=d/v$, we get

$$J=\frac{2\pi}{\ln(\Lambda/\omega)}B_0 dc$$
$$=1.9\times10^{16}\,\text{amperes}\left(\frac{B_0}{10^{-6}\,\mathrm{G}}\right)\left(\frac{d}{10^4\,\text{parsec}}\right)\left(\frac{100}{\ln(\Lambda/\omega)}\right). \tag{66}$$

(65) and (66) combine into

$$\frac{\mathrm{d}^2E}{\mathrm{d}z\,\mathrm{d}t}=JB_0\frac{v}{c}. \tag{67}$$

This simply says that the change in energy of the string can be interpreted as coming from the electromagnetic force $\boldsymbol{J}\cdot\boldsymbol{E}=JB_0v/c$ on the current it carries.

The current (66) must be compared to the maximum current the string can carry. As we noted earlier this is of order $\hat{J}(M)=eMc^2/\hbar$, where M is some mass characterizing the string. In practical units

$$\hat{J}(M)=\frac{eMc^2}{\hbar}=2.5\times10^{21}\,\text{amperes}\left(\frac{M}{10^{15}\,\text{GeV}}\right). \tag{68}$$

As the current approaches its limiting value, tunneling processes in which the quantum number N of sect. 2 changes become rapid. In such processes, sparks fly. A unit of magnetic flux escapes from the string as a pulse of electromagnetic radiation, with characteristic frequency determined by the microphysics of the string and therefore very large. If a string with Bose charge carriers somehow manages to travel far enough in a strong enough field as to saturate its current carrying ability, it would become a source of ultra-high-energy gamma rays with luminosity (65), as tunneling processes balance the accreted energy.

It may be that there are other ways that part of the large available energy (65) can be converted to an observable form. Perhaps part of this energy is released through instabilities, or encounters with unusual astrophysical environments, or when a string which has been "spun up" suddenly meets a patch in which the magnetic field is oppositely oriented. The available energy is large enough that release of a fraction of it might make the string observable.

6. An O(10) model with Fermi charge carriers

We now will turn to some grand unified theories which can have superconducting strings with Fermi charge carriers. Let us first, however, consider some simple facts about stable and unstable strings. In sects. 2 and 3, we introduced the idea of superconducting strings with a simple $\mathrm{U}(1)\times\tilde{\mathrm{U}}(1)$ gauge theory, where U(1) (which we called Q) is the unbroken electromagnetic gauge symmetry, and $\tilde{\mathrm{U}}(1)$ (which we called R) is spontaneously broken. The $\mathrm{U}(1)\times\tilde{\mathrm{U}}(1)$ gauge theory, with spontaneous breaking of $\tilde{U}(1)$, always has stable strings. When $\mathrm{U}(1)\times\tilde{\mathrm{U}}(1)$ is embedded in a unified group G, however, the strings remain stable only if the symmetry breaking leaves unbroken a discrete subgroup of $\tilde{\mathrm{U}}(1)$ [2, 10]. If $\mathrm{U}(1)\times\tilde{\mathrm{U}}(1)$ is embedded in G and broken in such a way that there is no such unbroken discrete symmetry, then the strings that are stable at the $\mathrm{U}(1)\times\tilde{\mathrm{U}}(1)$ level are destabilized by the embedding in G. They can decay by the nucleation of monopole-antimonopole pairs [17]. It is possible, however for the nucleation rate to be negligibly small. If G is broken to $\mathrm{U}(1)\times\tilde{\mathrm{U}}(1)$ at a scale M, and $\mathrm{U}(1)\times\tilde{\mathrm{U}}(1)$ is broken to $\tilde{\mathrm{U}}(1)$ at a smaller scale $\tilde{M}$, then the monopole nucleation rate (per unit length of string) is of order $M^2\exp[-(1/\alpha)(M/\tilde{M})^2]$, which can be very small even if $\tilde{M}$ is only moderately less than M*. For instance, if $\tilde{M}=0.1M$, a string stretching across our horizon might have a lifetime of 10^{10^4} years. In what follows we will consider two models: an O(10) model in which the strings can be (but need not be) absolutely stable, and an E_6 model in which the strings are unstable but could easily have lifetimes of 10^{10^4}–$10^{10^{30}}$ years. In the O(10) model that we will consider, an elaboration of the Higgs structure is required in order for the strings to be superconducting. In the E_6 case this occurs with the most obvious choice of the Higgs structure.

Considering first O(10), this group has an $\mathrm{SU}(5)\times\tilde{\mathrm{U}}(1)$ subgroup, where SU(5) contains the known interactions and $\tilde{\mathrm{U}}(1)$ is a linear combination of $B-L$ (baryon minus lepton number), Q (electromagnetism), and Y (weak hypercharge). It is breaking of $\tilde{\mathrm{U}}(1)$ that will lead to the existence of strings.

* A very analogous process was treated in ref. (18).

O(10) can be broken to $SU(5)\times\tilde{U}(1)$ (or $SU(3)\times SU(2)\times U(1)\times\tilde{U}(1)$; for our purposes it does not matter) by the expectation value of a Higgs field φ_{45} in the 45 or adjoint representation of O(10). The $SU(5)\times\tilde{U}(1)$ content of some O(10) representations is as follows. The fundamental 10 of O(10) is

$$10=5^{1}\oplus\bar{5}^{-1}, \tag{69}$$

where 5 and $\bar{5}$ label the SU(5) representations and ±1 are the $\tilde{U}(1)$ charges. The standard $SU(2)\times U(1)$ Higgs doublet can be placed in the 10 of O(10). Quarks and leptons fit conveniently into the 16 of O(10), which is one of the two spinor representations of O(10). Under $SU(5)\times\tilde{U}(1)$ it transforms as

$$16=1^{-5/2}\oplus\bar{5}^{3/2}\oplus10^{-1/2}. \tag{70}$$

The other O(10) representations that will interest us are the 126 (which is a fifth-rank antisymmetric tensor φ_{ijklm} that obeys a self-duality condition $\varphi_{ijklm}=(i/5)\varepsilon_{ijklmnopqr}\varphi^{nopqr}$ and the 210 (which is a fourth-rank antisymmetric tensor φ_{ijkl}). Their $SU(5)\times\tilde{U}(1)$ decompositions, which are easily constructed from their descriptions as antisymmetric tensors, are rather lengthy; the portions of interest to us are

$$\begin{aligned}126&=1^{5}\oplus\cdots,\\210&=5^{-4}\oplus\bar{5}^{4}\oplus\cdots.\end{aligned} \tag{71}$$

As far as phenomenology is concerned, the scale of $\tilde{U}(1)$ breaking could be anywhere from about 1 TeV to the Planck mass (the most stringent lower bound probably comes from the smallness of the electron neutrino mass). Regardless of the scale of $\tilde{U}(1)$ breaking, phenomenology requires that $\tilde{U}(1)$ be broken by an SU(5) singlet (or at least $SU(3)\times SU(2)\times U(1)$ singlet) with non-zero $\tilde{U}(1)$ quantum numbers. This can most simply be the $1^{-5/2}$ component of the 16 or the 1^{5} component of the 126.

If $\tilde{U}(1)$ is broken by a φ_{16}, there are no truly stable strings. There are nonetheless strings that are stable for all practical purposes if $\langle\varphi_{16}\rangle<0.1\langle\varphi_{45}\rangle$, as we discussed earlier. If $\tilde{U}(1)$ is broken by a φ_{126}, there are topologically stable strings [10]. The strings are stable because of an unbroken Z_2 gauge symmetry, namely the symmetry under a 2π rotation in the O(10) group. Under that symmetry tensors such as the 10,126 and 210 are invariant while spinorial representations such as the 16 change sign. If $\langle\varphi_{16}\rangle\neq0$, this symmetry is broken and there are no absolutely stable strings. For our purposes it does not matter very much if $\tilde{U}(1)$ is broken by a 16 or 126; to be definite we will assume it is a 126 that breaks $\tilde{U}(1)$.

With $\tilde{U}(1)$ breaking, strings form. On passing around the strings, the 1^{5} component of φ_{126} changes in phase by 2π. In the core of the string there is a magnetic flux tube. The magnetic field in the string core is not necessarily in the direction of the generator R of $\tilde{U}(1)$. In general the magnetic field is in the direction of $K\sim\lambda(R+X)$

where X is some SU(5) generator. Thus, in the fundamental representation of O(10),

$$K=\frac{1}{5}\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

$$+\begin{pmatrix} 0 & X_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -X_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & X_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -X_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & X_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -X_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & X_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -X_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & X_5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -X_5 & 0 \end{pmatrix}, \qquad (72)$$

where the first term is $\frac{1}{5}$ of the $\tilde{\mathrm{U}}(1)$ generator R and in the second term X we require $\sum_{i=1}^{5} X_i=0$ so that X is in SU(5). (The factor of $\frac{1}{5}$ in (72) will be explained shortly.) The Higgs fields change in phase by $2\pi K$ in circling the string, and therefore the basic topological condition governing K is that (with proper definition of how K is related to the magnetic field in the string core) all fields with vacuum expectation values have integer eigenvalues of K. The factor $\frac{1}{5}$ multiplying R in (72) corresponds to the minimal non-trivial solution of this condition, since the relevant component of the φ_{126} has $R=5$. At the level of grand unified and O(10) breaking, the topological condition does not restrict the X_i, since X leaves the vacuum invariant anyway. The X_i are to be found on energetic grounds. If the relevant energy is mainly magnetic, the energy is minimized if $X_i=0$, since the $\tilde{\mathrm{U}}(1)$ and SU(5) magnetic energies are additive.

Prior to SU(2)×U(1) breaking, the only component of the standard fermion 16-plet to get mass is the right-handed neutrino. Its mass comes from a $\varphi_{126}\varphi_{10}\psi_{10}$ coupling, and since the φ_{126} changes phase by 2π in going around the string, a zero mode of right-handed neutrinos is trapped on the string [19]. As neutrinos are electrically neutral, the string does not become superconducting at this stage.

Now, at energies of a few hundred GeV, we turn on the $SU(2)\times U(1)$ breaking. As mentioned earlier, the usual $SU(2)\times U(1)$ Higgs doublet lies in the 10 of O(10). In a conventional arrangement, it is the last two components of the 10 which receive vacuum expectation values. However, complications arise when this low-energy Higgs field meets a string. If the X_i of eq. (72) are zero, the ordinary Higgs components of φ_{10} will change in phase by $\pm\frac{2}{5}\pi$ in circling the string. (The φ_{126} changes in phase by 2π in circling the string, but the R quantum number of the relevant components of φ_{10} arc $\pm\frac{1}{5}$ as big.) This $\frac{2}{5}\pi$ phase change of φ_{10} is incompatible with φ_{10} having a vacuum expectation value. To remedy the situation, X_5 in eq. (33) must be non-zero, with $\frac{1}{5}+X_5 = integer$. We must minimize the energy, subject to this condition and the constraint $\sum X_i = 0$. The magnetic energy is roughly* proportional to $\sum X_i^2$, so if magnetic energy is the main component, the minimum energy choice is $X_5 = -\frac{1}{5}$, $X_1 = X_2 = X_3 = X_4 = 0.05$. The SU(5) generator described by those values of the X_i is a linear combination of electromagnetism and weak hypercharge.

In this picture, the structure of the string is as follows (fig. 8). In an inner region with radius of order $1/e\langle\varphi_{126}\rangle$, there is a $\tilde{U}(1)$ magnetic flux tube. In an outer region with radius of order $1/e\langle\varphi_{10}\rangle$, there is an electroweak magnetic flux tube.

In making a tiny circuit around the inner core of the string, the φ_{10} vary by approximately a $\frac{2}{5}\pi$ change in phase. This is incompatible with a non-zero value of φ_{10}, so it vanishes near the inner core of the string. However, in circling around the *whole* string, including the outer core, φ_{10} has no change in phase, and so it need not vanish outside the string. In particular, since φ_{10} does not change in phase in circling the string, there are no zero modes of ordinary quarks and leptons which get their masses from φ_{10}, and the string is not superconducting.

Now let us discuss some alternative scenarios that lead to a different answer. We will consider three possibilities: (i) a theory that is not unified in O(10); (ii) multiple vortex lines; (iii) an elaboration of the Higgs structure. Of these, the first two are easier to discuss, but the last may be the most interesting.

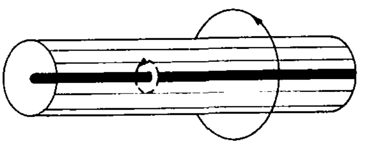

Fig. 8. With the minimal Higgs structure, the O(10) string has an inner core of $\tilde{U}(1)$ magnetic flux and a much larger outer core with electroweak fields. The $SU(2)\times U(1)$ doublet changes phase by $\frac{2}{5}\pi$ in circling the inner core (dotted line) but not in circling the whole structure (solid line).

* This ignores the fact that the SU(2) and U(1) couplings are renormalized differently from the grand unified scale to the electroweak scale. For present purposes we may ignore that complication.

6.1. DISPENSING WITH GRAND UNIFICATION

Our theory required an $SU(3)\times SU(2)\times U(1)$ singlet $\tilde{\varphi}$ that breaks $\tilde{U}(1)$. $O(10)$ group theory requires that such a singlet have a $\tilde{U}(1)$ charge R that is a multiple of $\frac{5}{2}$ (in units where the usual $SU(2)\times U(1)$ doublet has $R=1$). If the strings are to be stable in $O(10)$, then $O(10)$ group theory requires that the $\tilde{U}(1)$ charge of $\tilde{\varphi}$ must be a multiple of 5; this was our choice above. If we were willing to dispense with unification in $O(10)$ and consider an $SU(5)\times\tilde{U}(1)$ or $SU(3)\times SU(2)\times U(1)\times\tilde{U}(1)$ theory, we could introduce an $SU(5)$ singlet $\tilde{\varphi}$ of $R=1$. If this field breaks $\tilde{U}(1)$ and changes in phase by 2π in circling the string, then (since it has the same $\tilde{U}(1)$ charge as the electroweak doublet) the electroweak doublet would also change in phase by 2π in circling the string. As a result the string would trap quark and lepton zero modes and would be superconducting (with a phenomenology that we will consider later). In this picture the string would not contain an electroweak flux line in its core*.

6.2. MULTIPLY CHARGED STRINGS

Returning to the $O(10)$ picture, if the Higgs field φ_{126} that breaks $\tilde{U}(1)$ is lighter than the $\tilde{U}(1)$ gauge field, then the $\tilde{U}(1)$ symmetry breaking is similar to that in a "type I" superconductor. The interaction between $\tilde{U}(1)$ vortex lines is attractive, and the theory has strings or vortex lines of multiple topological charge n. The strings of $n>1$ are not strictly stable when embedded in $O(10)$, but as we noted before they can have immense lifetimes.

In going around a multiple vortex line, φ_{126} changes in phase $2\pi n$. The change in phase of the ordinary Higgs doublet in circumnavigating the string is $2\pi(\frac{1}{5}n+X_5)$. The requirement is that $\frac{1}{5}n+X_5=$ *integer*. For large enough n, the state of the string that minimizes the magnetic energy is superconducting. For instance, if $n=3$, the solution of minimum magnetic energy is $X_5=0.4$, $X_1=X_2=X_3=X_4=-0.1$. With this solution the electroweak doublet has a phase change of 2π in circling the string, leading to the superconducting phenomenology that we will discuss shortly. If $\tilde{U}(1)$ is broken by a $\tilde{\varphi}_{16}$, the condition on X_5 is $\frac{2}{5}n+X_5=$ *integer*, and for all $|n|\geq 2$ the magnetic energy is minimized by a superconducting configuration with $\frac{2}{5}n+X_5\neq 0$.

6.3. THE HIGGS STRUCTURE

Finally, and perhaps most important, we must explore the possibility that the structure of the string is not determined only by minimizing the magnetic energy. An additional Higgs component of the energy may be decisive.

* As an alternative to considering a non-unified $SU(5)\times\tilde{U}(1)$ gauge theory, one may consider the case in which $SU(5)$ is a gauge symmetry but $\tilde{U}(1)$ is a global symmetry. Breaking of $\tilde{U}(1)$ leads to "global strings" discussed by Everett and Villenkin [16]. These $\tilde{U}(1)$ global strings have just the superconducting phenomenology we are considering, since the index theorem for strings does not distinguish global and local strings [7, 8].

Suppose that the theory contains a scalar multiplet transforming as the 210 of O(10). Such a field cannot couple directly to the usual fermions, so its influence on phenomenology is indirect.

Some components of the φ_{210} transform under $\mathrm{SU}(5)\times\tilde{\mathrm{U}}(1)$ as $5^{-4}\oplus\bar{5}^{4}$. The 5^4 contains two components $\alpha=\binom{\alpha^+}{\alpha^0}$ with the $\mathrm{SU}(2)\times\mathrm{U}(1)$ quantum numbers of the electroweak doublet. The neutral component α^0 has zero or very low expectation value in vacuum compared to the scale of grand unification since $\mathrm{SU}(2)\times\mathrm{U}(1)$ is weakly broken. The charged component α^+ has zero expectation value in vacuum.

We saw in sect. 2 that the Higgs couplings may be such that it is energetically favored for a scalar with zero (or very small) vacuum expectation value to have a large expectation in the core of the string. The possibility that a charged scalar has a big expectation value in the core was considered in sect. 2. What will happen if it is favored for the *neutral* scalar α^0 to have a large expectation value in the string core?

The phase change of α^0 in circling the string is $2\pi(-\frac{4}{5}+X_5)$ (since α^0 has $R=-4$). So if α^0 is to be non-zero at $r=0$, the string must choose $X_5=\frac{4}{5}$ (and probably $X_1=X_2=X_3=X_4=-\frac{1}{5}$, if there are no other complicating factors) already at the grand unified scale. Taking $X_5=\frac{4}{5}$ is costly in magnetic energy, but this will be overcome if the Higgs potential gives a strong enough tendency for $\alpha^0\neq 0$ in the string core*.

What is the low-energy physics if $X_5=\frac{4}{5}$ at the grand unified scale? The ordinary Higgs multiplet breaks up as $5^1\oplus\bar{5}^{-1}$ under $\mathrm{SU}(5)\times\tilde{\mathrm{U}}(1)$. The usual electroweak doublet φ, is in the 5^1, and its neutral component φ^0 differs by five units of R from the field α^0 just considered. If $X_5=\frac{4}{5}$, its phase change in circling the string is $2\pi(\frac{1}{5}+\frac{4}{5})=2\pi$, and this is the situation of interest to us.

One way or another, let us suppose that the O(10) string has such a structure that the electroweak doublet φ changes in phase by 2π in circumnavigating it. Let us now discuss the phenomenology which is largely independent of the details of how the 2π phase change in φ comes about.

In O(10), up quarks get masses from Yukawa coupling to φ, while down quarks and electrons get masses from couplings to φ^* (or another doublet that transforms like φ^*). Therefore (fig. 9), in view of our previous remarks in sect. 3, the zero modes of u and $\bar{u}$ travel in one direction on the string - say to the right - while the

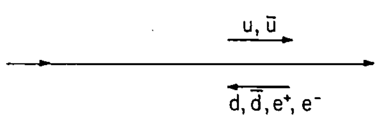

Fig. 9. In the superconducting form of the O(10) string, u- and ū-quarks travel in one direction while d, d̄, e⁺ and e⁻ travel in the other direction.

* The magnetic and Higgs energies are of order $1/e^2$ and $1/\lambda$ respectively, so the Higgs energy can overcome the magnetic energy if $\lambda<e^2$.

zero modes of d, $\bar{d}$, e^- and e^+ travel in the opposite direction on the string. We can check that this structure is compatible with the general remarks in sect. 3 on cancellation of anomalies. The sum of the squared charges of three colors of right-moving up quarks is $3(\frac{2}{3})^2=\frac{4}{3}$, while the sum of the squared charges of left-moving electrons and three colors of down quarks is $(-1)^2+3(-\frac{1}{3})^2=\frac{4}{3}$.

Now let the string pass through a magnetic field. By virtue of the discussion in sect. 3, right-moving positive charges and left-moving negative charges are produced (or vice versa if the string passes through the field in the opposite direction). The rate at which electrons are produced was given in eq. (64) and is

$$\frac{d^2N_e}{dt\,dz}=2.2\times10^{11}\ \mathrm{cm}^{-1}\ \mathrm{sec}^{-1}\left(\frac{v}{c}\right)\left(\frac{B}{10^{-6}\ \mathrm{G}}\right)\left(\frac{\eta}{\frac{1}{3}}\right). \tag{73}$$

With three colors of down quark of one-third the electron charge, the total rate of creation of down quarks equals the rate of reaction of electrons. As for up quarks, with three colors and $-\frac{2}{3}$ the electron charge (but traveling on the string in the opposite direction relative to electrons), they are created at twice the rate of electrons. The quantum numbers of the produced particles are *uude* - conserving electric charge (as must be the case) and also $B-L$. As far as quantum numbers are concerned, this represents the creation of 2.2×10^{11} hydrogen atoms/cm of string per second.

In fact, though, we should not restrict ourselves to u, d and e^-. Heavier fermions that get masses from coupling to the electroweak doublet are produced at a rate proportional to their electric charge. Ths string creates u, c and t; d, s and b; and e, μ and τ at identical rates. This is quite similar, in fact, to the 't Hooft weak instanton process [20].

If the fermions fit into N_g standard generations, the total rate of production of baryons and leptons (or antibaryons and antileptons if the string travels through the field in the opposite direction) is

$$\frac{d^2N_B}{dt\,dz}=\frac{d^2N_L}{dt\,dz}=6.6\times10^{11}\ \mathrm{cm}^{-1}\ \mathrm{sec}^{-1}$$
$$\times(\tfrac{1}{3}N_q)\left(\frac{v}{c}\right)\left(\frac{B}{10^{-6}\ \mathrm{G}}\right)\left(\frac{\eta}{\frac{1}{3}}\right). \tag{74}$$

What will happen when the string crosses a galactic magnetic field? We previously saw (eq. (63)) that in reasonable field of 10^{-6} G, the quark and lepton (or antiquark and antilepton) Fermi momenta build up at a rate of several MeV per second.

In a fraction of a second, the electron Fermi momentum reaches the electron rest mass, 511 keV, and it would be energetically favored for electrons to escape from the string as free particles. It does not seem that that occurs at once, though, because it is prevented by conservation of longitudinal momentum along the string. Whether that is so or not, within several minutes the quark and lepton Fermi momenta reach a value of several hundred MeV. At this point particles can certainly escape from the string via reactions such as $e_L^- d_L u_R u_R \to e_{free}^- p_{free}$(here e_L^- is a left-moving electron on the string and similarly for d_L, u_R; e_{free}^- and p_{free} are an electron and proton that have escaped from the string).

The Fermi momenta therefore saturate at several hundred MeV (and perhaps less for electrons). This is true for heavy flavors as well as for u, d and e because quarks and leptons of the second and third generations can turn into u, d and e by weak processes. Beyond this point the passage of the string through the galactic field does not result in the build up of larger currents or Fermi momenta in the string. It results in the manufacture (and ejection into the world outside the string) of plasma (or anti-plasma) at the rate of about 2×10^{12} ions and electrons/cm of string per second. (Recall that once the current saturates we may set $\eta = 1$.)

As it passes through the field, the string comes to be surrounded with a plasma of its own making; we will not attempt to explore the properties of that plasma.

When the string passes through a region of reversed magnetic field it begins to produce antimatter rather than matter (or vice versa). Annihilation processes might become important.

The order of magnitude of the energy of the particles ejected from the string is 2×10^{12} GeV/cm/sec, or about 10^{28} ergs/parsec/sec. Compared to the solar luminosity of 3.9×10^{33} ergs/sec the luminosity of the string is not very impressive. However, it may be that while the string is in an antimatter mode and is surrounded by a dense antimatter plasma, it manufactures complex antimatter nuclei that may be obervable in cosmic rays. Or perhaps part of the energy is released in rare or sporadic events that might be observable. Or perhaps strings dominate the production of low-energy antiprotons in the universe.

The microscopic origin of the baryon violating processes considered here is the triangle anomaly. If B, Q and Y are baryon number, electron charge, and weak hypercharge, there are BQY and $BQ\tilde{\mathrm{U}}(1)$ triangle anomalies. The core of the string contains $\tilde{\mathrm{U}}(1)$ and Y magnetic fields parallel to the string. As the string moves through the galactic magnetic field, there is an ordinary electric field parallel to the string. All conditions are satisfied, therefore, for anomalous production of baryons, as in weak instantons [20].

Although the processes we have considered violate baryon number, there is a quantity which they conserve. It is

$$\hat{B} = B + \frac{eN_g}{2\pi\hbar c}\sum_i \Phi_i , \tag{75}$$

where N_g is the number of generations, the sun runs over all strings, and Φ_i is the magnetic flux through the ith string. (Φ_i is the *total* magnetic flux, including ambient magnetic fields and fields due to the current circulating in the string.) $\hat{B}$ is conserved by ordinary interactions of particles and strings, though it is violated by other processes such as weak instantons and superheavy gauge boson exchange. Conserva-

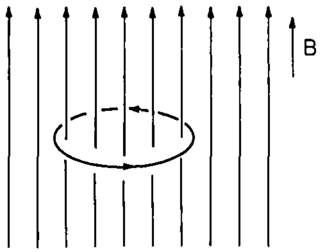

Fig. 10. A circular string in a constant magnetic field.

tion of $\hat{B}$ expresses the fact that the string creates or destroys baryons only insofar as it crosses magnetic field lines. Eq. (75) is the Fermi analogue of our old eq. (20).

A consequence of the processes considered here is that strings in magnetic fields do not oscillate freely if they are too large and their tension is too low. Consider, for instance, a circular string (fig. (10)) embedded in a constant magnetic field B. Let σ be the energy per unit length of the string. Initially the string shrinks under its tension. As it crosses magnetic field lines the quark and lepton Fermi energies reach the value at which further passage across field lines will cause baryons to be ejected from the string. Let the string radius at this point be R.

If the string shrinks further in radius from R to $R-\delta R$, its length is diminished by $2\pi\delta R$, saving an energy $2\pi\delta R\sigma$. However, the area of the string is reduced by $2\pi R\delta R$, and the magnetic flux through it is reduced by $\delta\Phi = 2\pi RB\delta R$. The number of baryons (or anti-baryons) emitted in this process is $N_g e\delta\varphi/2\pi\hbar c$ or $N_g eBR\delta R/\hbar c$. The cost in energy is $N_g eBR\delta R m_p c/\hbar$, where m_p is the mass of the proton. The condition for this process to be energetically favorable is that $2\pi\sigma > eBN_g Rm_p c/\hbar$ or

$$R < \frac{2\pi\hbar\sigma}{eBN_g m_p c}$$

$$= (2\ \text{parsecs})\left(\frac{3}{N_g}\right)\left(\frac{10^{-6}\,\text{G}}{B}\right)\left[\frac{\sigma}{(1\ \text{TeV})^2}\right]. \tag{76}$$

Thus, if strings are generated at the weak interaction scale, so $\sigma \sim (1\ \text{TeV})^2$, then only rather small strings are capable of oscillating freely; weak interaction strings have a tension which is too feeble to drag a string more than a few parsec long across the magnetic field lines. Strings that violate eq. (76) will be dragged around by the motion of the magnetic field lines (which in turn are tied to plasma motions in our galaxy). However, if $\sigma \gtrsim (10^8\ \text{GeV})^2$, then even a string that stretches all the way across our current horizon obeys the inequality of eq. (76). It should be mentioned, perhaps, that the effect which prevents the shrinkage of the string in eq. (76) has an alternative explanation which does not explicitly involve the production of baryons. It is a result of the $J\times B$ force of the magnetic field B on the current J carried by the string; this force tends to counteract the string tension and can overcome the string tension if the current is too large and the string too long. In this language, it is clear that the trapping of strings in magnetic field lines can also occur if the charge carriers are bosons.

7. An E_6 model

In this section we will consider another model of superconducting strings in which the charge carriers are fermions, based on the gauge group E_6. We will consider a symmetry breaking pattern in E_6 that leads to strings that are very long lived, 10^{10^4} years or more, but not strictly stable. However [16], strictly stable strings can also arise in E_6. Compared to the model considered in the last section, the one to be considered here is exotic, with superheavy fermions playing an important role, and has the advantage that the Higgs structure required to trap fermions on the string is simple and straightforward.

A Higgs field φ_{78} in the adjoint representation of E_6 can break E_6 to $O(10)\times\overline{U(1)}$ (or to $H\times\overline{U(1)}$, where H is a subgroup of O(10); for us the difference is not relevant). Let us assume that this occurs at the grand unified scale M_{GUT}. Under $O(10)\times\overline{U(1)}$, the fundamental 27 of E_6 transforms as

$$27 = 1^1 \oplus 10^{-1/2} \oplus 16^{1/4}\,. \tag{77}$$

Standard fermions can be placed in the 27 of E_6; the $16^{1/4}$ is an ordinary generation, while the 1^1 and $10^{-1/2}$ can get very large masses after $\overline{U(1)}$ breaking.

We assume that there is also a Higgs field φ_{27} in the same representation of E_6 with a $\varphi_{27}\psi_{27}\psi_{27}$ coupling to fermions (allowed by E_6 group theory). At a mass scale $\bar{M}$, the 1^1 component of φ_{27} gets a vacuum expectation value, breaking $\overline{U(1)}$. Let us call this field σ. Breaking of $\overline{U(1)}$ gives rise to strings, with σ changing in phase by 2π in circling the string. The strings are not strictly stable when embedded in E_6, but they are stable for all practical purposes if $\bar{M} \leqslant 0.1\,M_{GUT}$.

We noted that the fermion multiplet ψ_{27} contains a standard generation and exotic fermions 1^1 and $10^{-1/2}$. The 1^1, having no $SU(3)\times SU(2)\times U(1)$ interactions, is all but invisible. The $10^{-1/2}$ gets a mass of order $\bar{M}$ from coupling to σ, as a result of the $\varphi_{27}\psi_{27}\psi_{27}$ interaction. Since σ changes in phase by 2π in circling the string, the $10^{-1/2}$ fermions, which we will refer to E, have a zero mode along the string. This zero mode travels in a definite direction on the string, say to the right. Since some of the components of H are electrically charged, the string will be superconducting. However, a string with right-moving charge carriers only is anomalous; the anomaly must be cured by ordinary fermions.

Ordinary fermions, transforming as $16^{1/4}$, can get mass from coupling to a $10^{-1/2}$. We therefore assume that the $10^{-1/2}$ gets a vacuum expectation value at the electroweak scale. In fact, the $10^{-1/2}$ contains two $SU(2)\times U(1)$ doublets, φ and $\tilde{\varphi}$. One of them, say φ, is in a 5 of SU(5) and gives mass to u-quarks while the other, $\tilde{\varphi}$, is in a $\bar{5}$ of SU(5) and gives mass to d-quarks and electrons.

Since σ, which has $\overline{\mathrm{U}(1)}=1$, changes in phase by 2π in circling the E_6 string, φ and $\tilde{\varphi}$, which have $\overline{\mathrm{U}(1)}=-\frac{1}{2}$, would change in phase by $-\pi$ in circling the string if the core of the string contains $\overline{\mathrm{U}(1)}$ magnetic fields only. Since this is unacceptable if φ and $\tilde{\varphi}$ have vacuum expectation values, the core of the string must contain additional O(10) magnetic fields, which may be a combination of weak hypercharge Y or the field we called $\tilde{\mathrm{U}}(1)$ in discussing the O(10) string. Our two electroweak doublets φ and $\tilde{\varphi}$ are independent fields but transform under $\mathrm{SU}(5)\times\tilde{\mathrm{U}}(1)$ as complex conjugates of each other, so their Y and $\tilde{\mathrm{U}}(1)$ quantum numbers are equal and opposite. If, therefore, O(10) magnetic fields cause φ to change in phase by an amount x in circling the string, they will cause $\tilde{\varphi}$ to change in phase by $-x$.

Altogether, φ and $\tilde{\varphi}$ change in phase by $-\pi+x$ and $-\pi-x$, respectively in circling the string. Since the magnetic core of the string must be such that each field changes by an integer multiple of 2π, we see that x must be an odd multiple of π. Which multiple occurs must be found by minimizing the energy, and depends on details of the Higgs interactions. We will consider only the two minimal solutions, $x=\pm\pi$.

If $x=\pi$, then $\tilde{\varphi}$ changes phase by -2π in circling the string, and d-quarks and electrons, which get masses from $\tilde{\varphi}$, will be trapped on the string in left-moving zero modes. If instead $x=-\pi$, it is φ that has a -2π phase change around the string, and it is u-quarks that will be trapped on the string in left-moving states. The two possibilities are sketched in fig. 11. In either case, the string has left-moving ordinary matter and right-moving heavy E-particles. One may easily see that both patterns in fig. 11 are anomaly-free.

The phenomenology of this system is rather odd and depends on details we have not yet specified. As an example, suppose that the scale $\bar{M}$ of $\overline{\mathrm{U}(1)}$ breaking is very large, perhaps $\bar{M}\sim 10^{10}-10^{17}$ GeV, (but $\bar{M}\leq 0.1 M_{\mathrm{GUT}}$, so that the strings are long-lived). The E-particles obviously cannot decay by $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ interactions since complete $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ multiplets of E-particles are superheavy. However, there is no reason for the E-particles to be stable. They may decay by Higgs interactions, and they will certainly decay by interactions analogous to proton decay. Even in the latter case, the E-particles are short-lived because of their large mass and the fact that the proton lifetime scales like m_{p}^2. However, low-energy E-particles on the string could be extremely long-lived. This is the case we wish to consider.

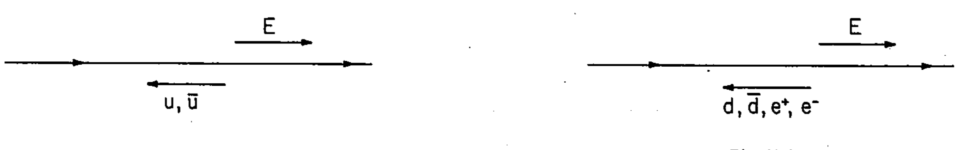

Fig. 11. The two minimal forms of the E_6 string. Superheavy E-particles travel in one direction, while normal particles travel in the opposite direction. The normal particles may be u, ū (as in (a)), or d, d̄, e^+, e^- (as in (b)).

Imagine the E_6 string crossing galactic magnetic field lines. Consider a string of the type in Fig. 11a. As it crosses field lines, left-moving u-quarks and right-moving negatively charged E-particles are created (or left-moving ū-quarks and positively charged E-particles, if the string moves in the opposite direction). After a few minutes u-quarks begin to escape from the string. They leave behind a negatively charge string containing E-particles. Three u-quarks escaping from the string have the quantum numbers of Δ^{++}, which decays $\Delta^{++} \to p\pi^+ \to pe^+ +$ neutrinos. So the matter outside the string forms a positively charged proton-*positron* plasma. This plasma cannot escape from the string, being electrically bound to the negatively charged string. Were the string moving in the opposite direction, it would be surrounded by a bound electron-antiproton plasma. In either case this plasma builds up at our canonical rate of 10^{12} particles/cm of string per second.

Now we must ask what happens to the E-particles. The most fascinating possibility is that the E-particles may be able to turn into normal matter and escape from the string only by interactions similar to proton decay. If m_p is the proton mass, τ_p the proton lifetime, and p_F the Fermi momentum of E-quarks, the time scale for interactions of proton decay type is crudely $t = \tau_p(p_F/m_p)^5$. We want $T \leqslant 10^4$ y (the galactic crossing time) for these interactions to be important. If, say, $\tau_p = 10^{40}$ years, then $T = 10^4$ years for $p_F = 10^7$ GeV. Such large values of p_F are readily attained; we saw earlier from eq. (63) that fermions reach $p_F \sim 10^{10}$ GeV as a string crosses a galaxy. If the string really behaves in this way, it is a very interesting cosmic ray source. The ejected particles would be hadrons, photons and leptons (including neutrinos) with energies of order 10^7 GeV. At that emitted energy, the rate of 10^{12} particles/cm/sec corresponds to a luminosity of 10^{35} ergs/parsec/sec in cosmic rays, so one parsec of string would have roughly 25 times the luminosity of the sum.

Even more exotic, perhaps, would be a string in which superheavy fermions carry current in *both* directions and can escape only by hard scattering processes. This might be achievable by adding additional fermion multiplets of the O(10) or E_6 strings. Such a string would be a naturally occurring superconducting collider of remarkable energy and luminosity.

8. Outlook

Superconducting strings are rather interesting objects for particle physics and astrophysics. If they exist, there are reasonable hopes that they will eventually be observable. Their main drawback is that string superconductivity depends on detailed properties of models, so that if superconducting strings can be shown *not* to exist, this will provide only limited information about particle physics.

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and J. Peebles. While this manuscript was under preparation, a new paper appeared on fermion zero modes in O(10) vortex lines [21].

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