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Abstract

One-loop effective potential of the $SU(2)\times U(1)$ electroweak interactions in a constant electromagnetic field is calculated by using the background-field method. Our result shows that the effective potential is independent of the certain gauge condition. A combination of the above result and the grand unified theory suggests the possibility that the strong magnetic field may restore the symmetry which is broken spontaneously. Also, the critical field of the first-order phase transition is calculated when the value of the potential at the true minimum is close to that at the origin.

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1. Introduction

In particle physics the notion of the vacuum whose symmetry is broken spontaneously[1] is now very widely accepted. The most successful example by using this notion is the unified theory of the electroweak interactions, namely the standard model based on the spontaneously broken $SU(2)\times U(1)$ gauge symmetry.[2] All the weak interactions mediated by charged and neutral currents are extensively studied and all the experimental data can be fitted so far by using a single parameter of the theory,[3] the so called Weinberg angle $\sin^2\theta_W = 0.23\pm0.02$. However, little is known about the Higgs mechanism, owing to the elusiveness of the Higgs boson. Only the lower bound of the mass of the Higgs boson is theoretically known.[4),5]

It is also convincing that the strong interactions can be described by the quantum chromodynamics (QCD) based on color $SU(3)$ unbroken symmetry.[6] It is then probable that the above three fundamental interactions can be incorporated into a simple gauge group. The most simple model among such grand unified theories (GUTS) is the $SU(5)$ theory.[7]

In order to look into the notions of the ordered and the disorderd states of the elementary systems, it is very popular to use the effective functional method.[8),9] A few years ago Salam and Strathdee suggested from an analogy of the solid state physics that the strong electromagnetic environment may restore the broken symmetry[10] which generates the Cabibbo angle and/or CP violation. They demonstrated the above idea by using $O(3)$ and $SU(2)$ gauge theories. Since unfortunately their model is far from reality, it is not clear what will happen in the real world. The purpose of this paper is two fold. (i) One is to study this problem in a more realistic model based on $SU(2)\times U(1)$ gauge group and also SU(5) gauge group as the GUT. (ii) The other is to introduce a new powerful method to calculate the effective potential due to quantum corrections.

The plan of this paper is as follows. In sec. 2 we obtain the one-loop effective potential of the spin 0, $\frac{1}{2}$, 1 fields of the Weinberg-Salam model in the presence of a external uniform electromagnetic field. Our calculation is done by using "'t Hooft-Feynman" gauge. We also compare our results with those which were obtained by other people in different contexts. To the leading order in $Tr(F^2) = 2(E^2-H^2)$, this expression can be approximated to be (by neglecting the fermion and the Higgs contributions):

$$V_{eff.}(h_c)=V_0(h_c) + \frac{3}{64\pi^2}[2m_W^4 \ell n m_W^2 + m_Z^4 \ell n m_Z^2] - \frac{7\alpha}{16\pi} Tr(F^2)\ell n m_W^2, \qquad (1.1)$$

where

$$V_0(h_c) = -\frac{1}{4}\nu^2 h_c^{\ 2} + \frac{1}{16}\lambda h_c^{\ 4}. \qquad (1.2)$$

In this formula V_0 is the tree-approximated classical potential expressed as a function of neutral scalar fields h_c. The functions $m_W(h_c)$ and $m_Z(h_c)$ are the masses associated with particles of W and Z bosons in the constant and classical background field h_c. The second term is the one-loop correction in the absence of external electromagnetic field, which was first calculated by Coleman and Weinberg.[11] The last term is the one-loop correction

in the presence of the external electromagnetic field, and it becomes the same form in a certain limit as that given by Salam and Strathdee.[10] It can also be identified with the h_c-dependent part of the photon wave function renormalization.

In sec. 3 we consider the phase transition of the Weinberg-Salam model by using the effective potential derived in sec. 2. In this case details of the symmetry breaking of the SU(5) model are used. In SU(5) we assume the following symmetry breaking pattern.

$$SU(5) \rightarrow SU(3)\times U(1)\times U(1) \rightarrow SU(3)\times U(1) \qquad (1.3)$$

The minimal Higgs system involves an adjoint $\underline{24}$ representation Φ for the super strong breaking, at the same time $SU(2)\times U(1)$ is assumed to be broken weakly. This system also involves a spinorial $\underline{5}$ representation h for the breaking of $SU(2)\times U(1)$ down to U(1). We can evaluate the critical field $Tr(F_c^{\ 2})$ only when the symmetry of the electroweak interactions is broken weakly. In this case $Tr(F_c^{\ 2})$ is given by

$$Tr(F_c^{\ 2}) = -\frac{\pi}{7\alpha}\frac{1}{\ell n\left(\frac{\sigma^2+\Delta^2}{\Delta^2}\right)}(\lambda_0-\lambda_c)\sigma^4 \qquad (1.4)$$

where $h_c = \sigma$ is the minimum point of the potential in the absence of the external field, and at $\lambda_0 = \lambda_c$ the value of the potential at $h_c = 0$ is equal to that at $h_c = \sigma$, which corresponds to the lower bound of the Higgs mass evaluated by Linde[4] and Weinberg.[5] Δ^2 is a parameter associated with that part of $SU(2)\times U(1)$ breaking which comes from the Higgs fields of an adjoint $\underline{24}$ representation.

Sec. 4 is devoted to some discussion.

2. Effective Potential

In this section we calculate the one-loop effective potential in the presence of the constant electromagnetic field. The one-loop effect of the vector meson to the effective potential was first calculated by Salam and Strathdee.[10] Their method is to sum up all the zero-point oscillations of the charged vector field in a constant magnetic field. When the external field is the electric field, the charged particle is accelerated as the time passes. The charged particle is not in a stationary state. Therefore, their method of summing up the harmonic oscillations can not be applied there. We should like to calculate the potential in a constant electromagnetic field in a Lorentz covariant manner.

In our approach, we employ the background-field method which was developed by DeWitt.[9),12)]

A. The simple example: scalar QED

Before examining the effective potential of the Weinberg-Salam model, let us consider, as a guidance, the effective potential of the scalar electrodynamics. We restrict our consideration only to the case where the momentum transfer of the external electromagnetic fields to the charged particles vanish.

Let us consider the following classical action of scalar electrodynamics, where the mass of the charged particle is given by the vacuum expectation value of some neutral scalar via Higgs mechanism.

$$S = \int d^4x \mathcal{L}^{(0)}, \tag{2.1}$$

$$\mathcal{L}^{(0)} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + |D_\mu\phi|^2 - m^2\phi^*\phi - V(\chi), \tag{2.2}$$

where $D_\mu = \partial_\mu - i\, e\, A_\mu$ is the covariant derivative, ϕ is the charged field and χ is the scalar field. The mass of the charged field m^2 can be written by the classical field χ_c of the neutral field:

$$m^2 = g^2 \chi_c{}^2. \tag{2.3}$$

We are interested in the effective potential of the charged field. The corresponding Lagrangian L is given by

$$L = -\phi^*[D_\mu \cdot D^\mu + m^2]\phi. \tag{2.4}$$

The one-loop effective Lagrangian is given by the functional integral over the quantum fields,

$$\exp[\frac{i}{\hbar} \int d^4x \mathcal{L}^{(1)}] = \int [d\phi^* d\phi] \exp[\frac{i}{\hbar} \int d^4x\, L], \tag{2.5}$$

where N is a normalization constant determined by the requirement of the boundary condition.

One may solve for $\mathcal{L}^{(1)}$ explicitly if one can obtain exact solutions to the Green's function equation

$$[D_x{}^2 + m^2]G(x,y) = -\delta^4(x-y), \tag{2.6}$$

where

$$-\frac{\hbar}{i} G(x,y) = \langle 0|T\{\phi^*(x), \phi(y)\}|0\rangle \tag{2.7}$$

$$\equiv \frac{\int [d\phi^* d\phi]\phi^*(x)\phi(y)\exp[\frac{i}{\hbar}\int d^4xL]}{\int [d\phi^* d\phi]\exp[\frac{i}{\hbar}\int d^4xL]} \tag{2.8}$$

Before solving eq. (2.6), we show how we can obtain the effective Lagrangian by the solution of it. Differenciating both sides of eq. (2.5) with respect to m^2, we have

$$\frac{\partial \mathcal{L}^{(1)}}{\partial m^2} = \langle 0|T\{\phi^*(x)\phi(x)\}|0\rangle$$
$$= -\frac{\hbar}{i} G(x,x). \tag{2.9}$$

Then the whole procedure devolves upon determination of

$$G(x,x) = \int \frac{d^n p}{(2\pi\hbar)^n} G(x,p), \tag{2.10}$$

where we extend the space-time dimension to n, in order to extract the finite part by the use of the dimensional regularization.[13)]

Now, we will solve eq. (2.6). G(x,y) can also be regarded as a function of x and x-y, and Fourier transformation is performed only for x-y part,

$$G(x,y) = \tilde{G}(x,x-y)$$
$$= \int \frac{d^4 p}{(2\pi\hbar)^4} e^{-ip\cdot(x-y)/\hbar} G(x,p). \tag{2.11}$$

For a while, we will show $\hbar$ explicitly. From (2.6), G(x,p) obeys

$$[q^2-m^2]G(x,p)+2i\hbar(\partial_\mu A^\mu)G(x,p)+2i\hbar q_\mu\partial^\mu G(x,p)-\hbar^2\partial^2 G(x,p) = 1, \tag{2.12}$$

where $\quad q_\mu = (p+eA)_\mu. \tag{2.13}$

Equivalently we can write

$$G(x,p) = \frac{1}{q^2-m^2}\{1-i\hbar[g(\partial^\mu A_\mu)+2q_\mu\partial^\mu]G(x,p)+\hbar^2\partial^2 G(x,p)\}. \tag{2.14}$$

We can solve G(x,p) by iteration method. For a constant electromagnetic field, we can write the vector potential of the form

$$A_\mu = -\frac{1}{2} F_{\mu\nu}x^\nu, \tag{2.15}$$

by an appropriate choice of the gauge, where $F_{\mu\nu}$ is a constant antisymmetric tensor. Note that

$$\partial^\mu A_\mu = 0, \tag{2.16}$$

$$q^\mu\partial_\mu(q^2-m^2) = 0. \tag{2.17}$$

Therefore, (2.14) reduces to

$$G(x,p) = \frac{1}{K}[1+\hbar^2\partial^2 G(x,p)], \tag{2.18}$$

where

$$K = q^2-m^2. \tag{2.19}$$

Starting from

$$G^{(0)} = \frac{1}{K}, \tag{2.20}$$

we can obtain $G^{(2)}$ in the form

$$G^{(2)} = \frac{1}{K}[1+\hbar^2\partial^2 G^{(0)}]$$
$$= \frac{1}{K} + \frac{(\hbar e)^2}{2}\mathrm{Tr}(F^2)\left[\frac{1}{K^3} - \frac{4}{n}\frac{q^2}{K^4}\right]. \tag{2.21}$$

Putting this $G^{(2)}$ back into (2.18), we obtain

$$G^{(4)} = \frac{1}{K} + \frac{(\hbar e)^2}{2}\,\mathrm{Tr}(F^2)\left[\frac{1}{K^2} - \frac{4}{n}\cdot\frac{q^2}{K^4}\right]$$
$$+ (\hbar e)^4[\mathrm{Tr}(F^2)]^2\left[\frac{3}{4}\cdot\frac{1}{K^5} - \frac{10}{n}\cdot\frac{q^2}{K^6} + \frac{40}{n(n+2)}\cdot\frac{(q^2)^2}{K^7}\right]$$
$$+ (\hbar e)^4\mathrm{Tr}(F^4)\left[\frac{1}{K^5} - \frac{16}{n}\cdot\frac{q^2}{K^6} + \frac{80}{n(n+2)}\cdot\frac{(q^2)^2}{K^7}\right] \tag{2.22}$$

where, in anticipation of the integration over q, we make the following replacement:

$$q^\mu q^\nu \to \frac{q^2}{n}\, g^{\mu\nu} \tag{2.23}$$

and

$$q^\sigma q^\rho q^\xi q^\eta \to \frac{(q^2)^2}{n(n+2)}\,[g^{\sigma\rho}g^{\xi\eta}+g^{\sigma\xi}g^{\rho\eta}+g^{\sigma\eta}g^{\rho\xi}]. \tag{2.24}$$

Our method for solving $G(x,p)$ by a power series expansion in $\hbar$ is meaningful since the loop expansion is equivalent to the power-series expansion in $\hbar$.[14] Fortunately in our case, expansion in $\hbar$ is equal to the perturbation expansion in e. The effective Lagrangian $\mathcal{L}^{(1)}$ is obtained from (2.9) by integrating $G(x,x)$ with respect to m^2,

$$\mathcal{L}^{(1)} = \frac{i\hbar}{(2\pi\hbar)^n}\int d^n p\int dm^2\, G^{(4)}(x,p)$$
$$= \frac{1}{32\pi^2\hbar^3}\,[m^4\ln m^2 + \frac{(\hbar e)^2}{6}\,\mathrm{Tr}(F^2)\ln m^2]$$
$$+ \frac{\hbar}{90m^4}\left(\frac{e^2}{4\pi}\right)^2\left\{\frac{1}{4}\,\mathrm{Tr}(F^4) + \frac{5}{16}\,[\mathrm{Tr}(F^2)]^2\right\} \tag{2.25}$$

The first term coincides with the result given by Salam and Strathdee[10] when we replace $\mathrm{Tr}(F^2)$ by $-2H^2$. The second term is the same result as that first given by Schwinger.[8]

B. Fermion Case

For the sake of completeness, we will consider the effects of the fermion loop to the effective potential in the presence of the external uniform electromagnetic field. The Lagrangian is given by

$$\mathcal{L} = \bar\psi[i\partial\!\!\!/-m]\psi. \tag{2.26}$$

The propagator in the presence of the external field is given by

$$[i\partial\!\!\!/-m]S_F(x,y) = \delta^4(x-y). \tag{2.27}$$

By making a Fourier transformation

$$S_F(x,y) \equiv \tilde S_F(x,x-y)$$
$$= \int\frac{d^4p}{(2\pi\hbar)^4}\, e^{-ip\cdot(x-y)/\hbar}S_F(x,p), \tag{2.28}$$

we rewrite eq. (2.27) as

$$S_F(x,p) = \frac{1}{q\!\!\!/-m}\,[1-i\hbar\partial\!\!\!/S_F(x,p)], \tag{2.29}$$

where

$$q\!\!\!/ = p\!\!\!/ + eA\!\!\!/ . \tag{2.30}$$

We can solve eq. (2.29) by an iteration method. Eq. (2.15) is used as the gauge condition of the electromagnetic field. Starting from

$$S_F^{(0)} = \frac{1}{q\!\!\!/-m}, \tag{2.31}$$

we obtain

$$S_F^{(1)} = \frac{1}{\not{q}-m}\,[1 - \frac{1}{2}\, ie\hbar F_{\sigma\mu}\gamma^{\mu}\frac{1}{\not{q}-m}\,\gamma^{\sigma}\frac{1}{\not{q}-m}\,], \tag{2.32}$$

$$\begin{aligned} S_F^{(2)} = \frac{1}{\not{q}-m}\,[1 &- \frac{1}{2}\, ie\hbar F_{\sigma\mu}\gamma^{\mu}\frac{1}{\not{q}-m}\,\gamma^{\sigma}\frac{1}{\not{q}-m} \\ &-\frac{1}{4}(e\hbar)^2(F_{\alpha\sigma}F_{\beta\mu}+F_{\alpha\beta}F_{\sigma\mu}+F_{\alpha\mu}F_{\sigma\beta})\gamma^{\sigma}\frac{1}{\not{q}-m}\gamma^{\alpha}\frac{1}{\not{q}-m}\gamma^{\mu}\frac{1}{\not{q}-m}\gamma^{\beta}\frac{1}{\not{q}-m}\,]. \end{aligned} \tag{2.33}$$

The trace of $S_F^{(2)}$ is given by

$$\mathrm{Tr}[S_F^{(2)}] = 2^{\frac{n}{2}}\, m[\,\frac{1}{\not{q}-m} - \frac{1}{2}(e\hbar)^2\mathrm{Tr}(F^2)\frac{4}{n}\,\frac{q^2}{(q^2-m^2)^4}\,], \tag{2.34}$$

where, as the normalization of the n-dimension, we used

$$\mathrm{Tr}(I) = 2^{\frac{n}{2}}. \tag{2.35}$$

The effective potential by the fermion loop is given by

$$\begin{aligned} \mathcal{L}_f^{(1)} &= -\frac{\hbar}{i}\int dm\ \mathrm{Tr}[S_F^{(2)}(x,x)] \\ &= \frac{2}{32\pi^2\hbar^3}\,[m^4\ell nm^2 - \frac{(\hbar e)^2}{3}\,\mathrm{Tr}(F^2)\ell nm^2], \end{aligned} \tag{2.36}$$

which coincides with the results of Salam-Strathdee[10] if we replace $\mathrm{Tr}(F^2)$ by $-2H^2$.

C. Weinberg-Salam Model with No Fermions

We are now in a position to calculate the effective potential of the Weinberg-Salam model with one scalar doublet. For simplicity, we neglect the fermion contribution, which was calculated in the preceding section. If necessary, we can add these contributions to the effective potential. The Lagrangian is

$$= -\frac{1}{4}\vec{F}_{\mu\nu}\vec{F}^{\mu\nu}-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}+|(\partial_\mu - \frac{ig'}{2}B_\mu - \frac{ig}{2}\vec{\tau}\cdot\vec{A}_\mu)h|^2 - V(h^\dagger h), \tag{2.37}$$

where

$$\vec{F}_{\mu\nu} = \partial_\mu\vec{A}_\nu - \partial_\nu\vec{A}_\mu + g\vec{A}_\mu\times\vec{A}_\nu, \tag{2.38a}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \tag{2.38b}$$

$$h = \begin{pmatrix} 0 \\ \frac{h_c}{\sqrt{2}} \end{pmatrix} + \begin{pmatrix} \phi^+ \\ \frac{\eta+i\zeta}{\sqrt{2}} \end{pmatrix}, \tag{2.39}$$

$$V(h^\dagger h) = -\frac{1}{2}\,\nu^2 h^\dagger h + \frac{\lambda}{4}\,(h^\dagger h)^2. \tag{2.40}$$

The vector bosons of the mass eigen states are

$$W_\mu^\pm = \frac{A_\mu^1 \mp iA_\mu^2}{\sqrt{2}} \tag{2.41a}$$

$$A_\mu = \cos\theta_W B_\mu + \sin\theta_W A_\mu^3 \tag{2.41b}$$

$$Z_\mu = \sin\theta_W B_\mu - \cos\theta_W A_\mu^3 \tag{2.41c}$$

To make the discussion parallel to the proceding sections, it is appropriate to take the covariant R_ξ gauge.[15] Putting then

$$A_\mu = -\frac{1}{2}\,F_{\mu\nu}x^\nu \tag{2.42}$$

$$\partial^\mu Z_\mu + \frac{1}{2\xi}\,h_c\sqrt{g^2+g'^2}\ \zeta = f_z(x) \tag{2.43}$$

$$\sqrt{2}D^\mu W_\mu^+ - i\,\frac{1}{\sqrt{2}\xi}\,gh_c\phi^+ = f_+(x), \tag{2.44}$$

we have the following gauge fixing term

$$\mathcal{L}_{G.F.} = -\frac{\xi}{2}\,[\partial^\mu Z_\mu + \frac{1}{2\xi}\,h_c\sqrt{g^2+g'^2}\,\zeta]^2$$
$$-\frac{\xi}{2}\,[\sqrt{2}D^{*\mu}W_\mu^- + i\frac{1}{\sqrt{2}\xi}\,gh_c\phi^-][\sqrt{2}D^\nu W_\nu^+ - i\frac{1}{\sqrt{2}\xi}\,gh_c\phi^+]. \qquad (2.45)$$

The Faddeev-Popov (F.P.) gauge-conpensating term[10] is

$$\mathcal{L}_{F.P.} = -i\overline{C^-}[D^{*2} + \frac{1}{4\xi}\,g^2h_c^2]C^-$$
$$-i\overline{C^+}[D^2 + \frac{1}{4\xi}\,g^2h_c^2]C^+$$
$$-i\overline{C^Z}[\partial^2 + \frac{1}{4\xi}\,(g^2+g'^2)h_c^2]C^Z$$
$$+\mathcal{L}_{F.P.\ int.} \qquad (2.46)$$

where use has been made of the Hermiticity assignment of the F.P. ghost,[17] i.e.,

$$\overline{C}^\dagger = \overline{C} \quad \text{and} \quad C^\dagger = C. \qquad (2.47)$$

Here, C^-, C^+ and C^Z are F.P. ghosts associated with W^-, W^+ and Z, and are defined associated with eqs. (2.41) by

$$C^\pm = \frac{C_1 \mp iC_2}{\sqrt{2}} \qquad (2.48a)$$

$$C^A = \cos\theta_W C_B + \sin\theta_W C_3 \qquad (2.48b)$$

$$C^Z = \sin\theta_W C_B - \cos\theta_W C_3 \qquad (2.48c)$$

Since $\mathcal{L}_{F.P.\ int.}$ contains no A_μ terms, it does not contribute to the one-loop correction. It contributes only in higher loops. Thus we finally obtain the effective Lagrangian from eqs. (2.37), (2.45), and (2.46),

$$\mathcal{L}_{eff.} = \mathcal{L}_0 + \Delta\mathcal{L} \qquad (2.49)$$

where

$$\mathcal{L}_0 = W_\mu^-[D^2g^{\mu\nu} - (1-\xi)D^\nu D^\mu + \frac{1}{4}\,g^2h_c^2g^{\mu\nu} - 2ie\hbar F^{\mu\nu}]W_\nu^+$$
$$-\phi^-[D^2 + \frac{1}{4\xi}\,g^2h_c^2]\phi^+$$
$$-i\overline{C^+}[D^2 + \frac{1}{4\xi}\,g^2h_c^2]C^+$$
$$-i\overline{C^-}[D^{*2} + \frac{1}{4\xi}\,g^2h_c^2]C^-$$
$$+\frac{1}{2}\,\zeta[\partial^2 + \frac{1}{4\xi}\,(g^2+g'^2)h_c^2]\zeta$$
$$-i\overline{C^Z}[\partial^2 + \frac{1}{4\xi}\,(g^2+g'^2)h_c^2]C^Z$$
$$-\frac{1}{2}\,\eta[\partial^2 + m_\eta^2]\eta \qquad (2.50)$$

and

$$\Delta\mathcal{L} = \mathcal{L}_{int.} + \mathcal{L}_{F.P.int.} + V(\tfrac{1}{2}\,h_c^2)$$
$$+\frac{1}{3!}\,\phi'^3\,\frac{\partial^3}{\partial h_c^3}\,V(\tfrac{1}{2}\,h_c^2) + \frac{1}{4!}\,\phi'^4\,\frac{\partial^4}{\partial h_c^4}\,V(\tfrac{1}{2}\,h_c^2). \qquad (2.51)$$

Here, $\mathcal{L}_{int.}$ is the rest part of eq. (2.37) which appears when we rewrite eq. (2.37) in a covariant form. This term does not contain such terms, $\phi^-A_\mu W^{+\mu}$ that contribute to the one-loop approximation, but it contains the electromagnetic interaction terms such as $W^+_\mu W^-_\nu Z^\mu A^\nu$ that does not contribute to the one-loop potential but does contributes to the higher order loops. We expand the gauge invariant potential $V(h^\dagger h)$ around the classical field. The quadratic part of $\frac{1}{2!}\phi'^2 \frac{\partial^2}{\partial h_c^2} V$ is absorbed into the mass term of the scalar field.

The Lagrangian induced by the one-loop effects, $\mathcal{L}^{(1)}$, is given by the functional integral over the quantum field.

$$\begin{aligned}
&\exp[\tfrac{i}{\hbar}\int d^n x \mathcal{L}^{(1)}] \\
&= N\int [dW^+dW^-d\phi^+d\phi^-d\overline{c}^+dc^+d\overline{c}^-dc^-dZd\overline{c}^Z dc^Z d\eta]\exp[\tfrac{i}{\hbar}\int d^n x \mathcal{L}_0] \\
&= \{N_W\int [dW^+dW^-]\exp[\tfrac{i}{\hbar}\int d^n x \mathcal{L}_W]\}\times\cdots\times\{\tfrac{1}{N_\eta}\int [d\eta]\exp[\tfrac{i}{\hbar}\int d^n x \mathcal{L}_\eta]\}, \qquad (2.52)
\end{aligned}$$

where

$$\mathcal{L}_W = W^-_\mu[D^2 g^{\mu\nu} - (1-\xi)D^\nu D^\mu + \tfrac{1}{4}g^2 h_c^2 g^{\mu\nu} - 2ie\hbar F^{\mu\nu}]W^+_\nu$$

etc. (2.53)

The factorization of the right-hand side of eq. (2.52) indicates that $\mathcal{L}^{(1)}$ is the sum of $\mathcal{L}^{(1)}_W$, $\mathcal{L}^{(1)}_\phi$, etc., i.e.,

$$\mathcal{L}^{(1)} = \mathcal{L}^{(1)}_W + \mathcal{L}^{(1)}_\phi + \cdots + \mathcal{L}^{(1)}_\eta, \qquad (2.54)$$

where $\mathcal{L}^{(1)}_W$ is defined by

$$\exp[\tfrac{i}{\hbar}\int d^n x \mathcal{L}^{(1)}_W] = N\int [dW^+dW^-]\exp[\tfrac{i}{\hbar}\int d^n x \mathcal{L}_W]. \qquad (2.55)$$

We can solve $\mathcal{L}^{(1)}_W$ in the same way as in the preceding sections. Therefore, we do not repeat the calculation. Instead, we only

show the results. For simplicity we calculate by setting $\xi = 1$. Then, the results are

$$\mathcal{L}^{(1)}_W = -4\cdot\frac{1}{32\pi^2}[m_W^4 + \frac{2\pi\alpha}{3}\mathrm{Tr}(F^2)]\ln m_W^2 + \frac{16\alpha}{32\pi}\mathrm{Tr}(F^2)\ln m_W^2, \qquad (2.56)$$

$$\mathcal{L}^{(1)}_\phi = -\frac{1}{32\pi^2}[m_W^4 + \frac{2\pi\alpha}{3}\mathrm{Tr}(F^2)]\ln m_W^2, \qquad (2.57)$$

$$\mathcal{L}^{(1)}_{C^+} = \frac{1}{32\pi^2}[m_W^4 + \frac{2\pi\alpha}{3}\mathrm{Tr}(F^2)]\ln m_W^2, \qquad (2.58)$$

$$\mathcal{L}^{(1)}_{C^-} = \frac{1}{32\pi^2}[m_W^4 + \frac{2\pi\alpha}{3}\mathrm{Tr}(F^2)]\ln m_W^2, \qquad (2.59)$$

$$\mathcal{L}^{(1)}_Z = -2\cdot\frac{1}{32\pi^2}m_Z^4 \ln m_Z^2, \qquad (2.60)$$

$$\mathcal{L}^{(1)}_\zeta = -\frac{1}{2}\cdot\frac{1}{32\pi^2}m_Z^4 \ln m_Z^2, \qquad (2.61)$$

d $$\mathcal{L}^{(1)}_{C^Z} = \frac{1}{32\pi^2}m_Z^4 \ln m_Z^2. \qquad (2.62)$$

In the above results, we neglect the η contribution by assuming that m_η is smaller than W and Z boson masses. By adding eqs. (2.56) $\sim$(2.62) to the classical potential $V(\frac{1}{2}h_c^2)$, we obtain the effective potential

$$\begin{aligned}
V_{\text{eff.}} = &-\frac{1}{4}\nu^2 h_c^2 + \frac{1}{16}\lambda h_c^4 + \frac{3}{64\pi^2}[2m_W^4 \ln m_W^2 + m_Z^4 \ln m_Z^2] \\
&- \frac{7\alpha}{16\pi}\mathrm{Tr}(F^2)\ln m_W^2. \qquad (2.63)
\end{aligned}$$

In the absence of the external electromagnetic field, our result coincides with that of Coleman and Weinberg,[11] which was worked out in the Landau gauge. The last term coincides with that given by Salam and Strathdee[10] which was calculated in the unitary gauge, if we replace $\mathrm{Tr}(F^2)$ by $-2H^2$. As is well known, Green's functions are unrenormalizable in the unitary gauge. Therefore, our approach is not applicable to the evaluation of the potential in this gauge.

Hence our one-loop effective potential seems to be independent of the gauge.[18] It is thus meaningful to discuss the phase transition of the electromagnetic field by using (2.63).

3. Phase Transition of the Electroweak Interactions

A. Model

It is the most attractive trial to include the all interactions in a compact gauge group. The most simple example is the SU(5),[7] which includes strong, electromagnetic and weak interactions. The SU(5) requires at least adjoint <u>24</u> ϕ and spinorial <u>5</u> h representations which reveal the following symmetry breaking pattern:[19]

$$SU(5) \xrightarrow{\phi} SU(3)\times SU(2)\times U(1) \xrightarrow{h} SU(3)\times U(1) \qquad (3.1)$$

Since nothing is known about the symmetry hierarchy, we may consider a somewhat different hierarchy. First consider the superstrong breaking by adjoint representation. We introduce the matrix notation

$$\phi = \frac{1}{\sqrt{2}} \lambda_i \phi_i \ . \qquad (3.2)$$

We assume the following potential form

$$V(\phi) = -\frac{1}{2}\mu^2 \mathrm{Tr}(\phi^2) + \frac{1}{4} a[\mathrm{Tr}(\phi^2)]^2 . \qquad (3.3)$$

This form allows ϕ to be in an arbitrary direction as far as the $\mathrm{Tr}(\phi^2)$ is fixed at $\frac{\mu^2}{a}$. Let us assume that the following Higgs vacuum expectation value is preferred

$$\langle 0|\phi|0\rangle = \begin{bmatrix} v & & & & \\ & v & & & \\ & & v & & \\ & & & (-\frac{3}{2}-\frac{1}{2}\epsilon)v & \\ & & & & (-\frac{3}{2}+\frac{1}{2}\epsilon)v \end{bmatrix}, \qquad (3.4)$$

where v is determined by

$$-\mu^2 + \frac{a}{2} v^2[15 + \epsilon^2] = 0 . \qquad (3.5)$$

We suppose that ϵ^2 is very small in order to be compatible with the phenomenology. If ϵ^2 is finite but small, then SU(2)×U(1) breaks weakly to U(1)×U(1). The spinorial <u>5</u> representation h,

$$h = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{h_c}{\sqrt{2}} \end{bmatrix} + \begin{bmatrix} h^1 \\ h^2 \\ h^3 \\ \phi^+ \\ \frac{\eta + i\zeta}{\sqrt{2}} \end{bmatrix}$$

breaks U(1)×U(1) down to U(1). There appears Goldstone bosons owing to the extra symmetry of ϕ fields provided the cross-coupling of the ϕ and h fields are excluded. The introduction of the general (ϕ,h) coupling terms such as

$$V(\phi,h) = \alpha(h^\dagger h)\mathrm{Tr}(\phi^2) + \beta h^\dagger \phi^2 h$$

destroys the extra symmetry, and Goldstone bosons appear no more.[19]

Hence our symmetry breaking pattern is.[20)]

$$SU(5) \to SU(3)\times U(1)\times U(1) \to SU(3)\times U(1).$$

In this case, masses of the gauge bosons are given by

$$m_W^2 = \frac{1}{4} g^2 (h_c{}^2+\Delta^2),$$
$$m_Z^2 = \frac{1}{4} (g^2+g'^2) h_c{}^2, \tag{3.8}$$

where

$$\Delta^2 = 2\epsilon^2 v^2. \tag{3.9}$$

We assume Δ^2 is approapriately small, i.e.,

$$\sigma^2 e^{-\frac{4\pi}{7\alpha}} < \Delta^2 < \sigma^2 \tag{3.10}$$

where $\sigma \neq 0$ is the absolute minimum point of the one-loop effective potential $V_{eff.}(\frac{1}{2} h_c{}^2)$ in the absence of the external field. Since the order of v is 10^{14} GeV or so, we assume that, in the energy range of $O(10^2$ GeV), v does not depend on the electromagnetic field.

B. Renormalization and critical field

In the tree approximation, the potential is

$$V_{tree}(h_c) = -\frac{1}{4} \nu_0{}^2 h_c{}^2 + \frac{1}{16} \lambda_0 h_c{}^4.$$

The classical potential $V_{tree}(h_c)$ has a minimum at the point where $h_c = \sigma \equiv \sqrt{2\nu_0{}^2/\lambda_0}$. We also impose upon the effective potential $V_{eff.}(h_c,F)$ the renormalization condition[4)] at the point where $h_c = \sqrt{2\nu_0{}^2/\lambda_0}$. Then,

$$\left.\frac{\partial V_{eff.}}{\partial h_c}\right|_{h_c=\sigma} = 0, \qquad \left.\frac{\partial^2 V_{eff.}}{\partial h_c{}^2}\right|_{h_c=\sigma} = \nu_0{}^2,$$
$$Z_3(\sigma) = 1, \tag{3.11}$$

where Z_3 is the wave function renormalization of the photon field. The result is

$$V_{eff.}(h_c,F) = V_{eff.}(h_c) - \frac{7\alpha}{16\pi} \mathrm{Tr}(F^2) \ln\left\{\frac{h_c{}^2+\Delta^2}{\sigma^2+\Delta^2}\right\}. \tag{3.12}$$

where

$$V_{eff.}(h_c) = -\frac{1}{4} (\nu_0{}^2+\tilde{\nu}^2) h_c{}^2 + \frac{1}{16} (\lambda_0+\tilde{\lambda}) h_c{}^4$$
$$+ \frac{3}{64\pi^2} \left[\frac{1}{8} g^4 (h_c{}^2+\Delta^2)^2 \ln\left(\frac{h_c{}^2+\Delta^2}{\sigma^2+\Delta^2}\right) + \frac{1}{16} (g^2+g'^2)^2 h_c{}^4 \ln\left(\frac{h_c{}^2}{\sigma^2}\right)\right], \tag{3.13}$$

$$\tilde{\nu}^2 = -\frac{3}{64\pi^2} \left[g^4 (\sigma^2 - \frac{1}{2} \Delta^2) + \frac{1}{2} (g^2+g'^2) \sigma^2\right], \tag{3.14}$$

$$\tilde{\lambda} = -\frac{3}{64\pi^2} \left[3g^4 + \frac{3}{2} (g^2+g'^2)^2\right]. \tag{3.15}$$

We first consider $V_{eff.}(h_c)$. We look for the necessary condition for the local minimum at $h_c = \sigma$ is deeper than the minimum at $h_c = 0$. From eq. (3.13), the symmetry breaking occurs only if

$$\lambda_0 > \lambda_c, \tag{3.16}$$

where

$$\lambda_c = \frac{3g^4}{64^2} \left[1-2 \frac{\Delta^2}{\sigma^2} + \frac{1}{2} \frac{(g^2+g'^2)^2}{g^4} + 2 \frac{\Delta^4}{\sigma^4} \ln\left(\frac{\sigma^2+\Delta^2}{\Delta^2}\right)\right]. \tag{3.17}$$

This gives the lower bound of the Higgs boson mass.

$$m_\eta^2 = \frac{1}{2}\lambda_0\sigma^2$$

$$\geq \frac{3}{16\pi^2\sigma^2}\left[2\left(M_W^2 - \frac{1}{4}g^2\Delta^2\right)^2 + M_Z^2 - g^4\sigma^2\Delta^2 + g^4\Delta^4\ln\left(\frac{\sigma^2+\Delta^2}{\Delta^2}\right)\right], \tag{3.18}$$

where

$$M_W^2 = \frac{1}{4}g^2(\sigma^2+\Delta^2),$$
$$M_Z^2 = \frac{1}{4}(g^2+g'^2)\sigma^2, \tag{3.19}$$

corresponding to eq. (3.8). Our result coincides with the result given by Weinberg[5] in the limit $\Delta \to 0$.

To evaluate the critical field, we must solve for the following two equations.

$$\frac{\partial}{\partial h_c} V_{eff}(h_c,F_c)\Big|_{h_c=\sigma_c} = 0, \tag{3.20}$$

$$V_{eff}(0, F_c) = V_{eff}(\sigma_c,F_c), \tag{3.21}$$

where $h_c = \sigma_c$ is the local minimum point in the presence of the external electromagnetic field, which is expressed by the first equation. The phase transition occurs when the local minimum at $h_c = 0$ becomes the true minimum. Hence the critical field is given by solving the above two equations. In general it is hard to solve eqs. (3.20) and (3.21) analytically. Since $Tr(F_c^2)$ vanishes when λ_0 tends to λ_c, $Tr(F_c^2)$ can be expanded by a power series in $(\lambda_0-\lambda_c)$. No constant term exists. We can slove these two equations only when $V_{eff.}(\sigma)\sim V_{eff.}(0)$. In this case, the restoration of the symmetry occurs at a rather weak electromagnetic field,

$$Tr(F_c^2) = -\frac{\pi}{7\alpha}\frac{1}{\ln\left(\frac{\sigma^2+\Delta^2}{\Delta^2}\right)}(\lambda_0-\lambda_c)\sigma^4. \tag{3.22}$$

The above equation is valid only when higher powers of $Tr(F_c^2)$ can safely be neglected. This condition is given by

$$\frac{\pi}{7}\cdot\frac{1}{\ln\left(\frac{\sigma^2+\Delta^2}{\Delta^2}\right)}\cdot(\lambda_0-\lambda_c)\cdot\left(\frac{\sigma}{\Delta}\right)^4 < 1. \tag{3.23}$$

The negative sign of $Tr(F_c^2)$ shows that the first-order phase transition occurs in an environment of the magnetic field. We will show the above considerations schematically.

One of the most interesting things is the case where the second derivative of the effective potential at the origin vanishes, i.e.,

$$\frac{\partial^2}{\partial h_c^2} V_{eff.}(h_c)\Big|_{h_c=0} = 0. \tag{3.24}$$

The attractiveness of the above condition is that there may be the possibility of solving the gauge hierarchy problem.[21] In this case, the critical magnetic field is given by

$$H_c^2 \simeq \frac{3}{112\pi\alpha}\cdot\frac{1}{\ln\left(\frac{\sigma^2+\Delta^2}{\Delta^2}\right)}[2M_W^4 + M_Z^4]. \tag{3.25}$$

Here, we assumed the condition of eq. (3.23). By putting $M_W \simeq M_Z$, and by using perturbative constraint on Z_3,

$$1 \geq \frac{7\alpha}{4\pi}\ln\left(\frac{\sigma^2+\Delta^2}{\Delta^2}\right), \tag{3.26}$$

we obtain the lower bound of the critical magnetic field,

$$H_c \gtrsim \frac{3}{8\pi} M_W^2 = 1.10\times 10^{22} \text{ gauss}, \tag{3.26}$$

for $M_W = 80$ GeV. In this case it is difficult to check the symmetry restoration by the magnetic field even in a heavy nucleus.

4. Discussion

We have studied the one-loop effective potential in the environment of the strong electromagnetic field. The new method enables us to calculate the effective potential due to quantum corrections in a Lorentz covariant manner. Our calculation suggests that the effective potential is gauge independent. The explicit calculation also shows that up to order α there appears only one scalar quantity $\mathrm{Tr}(F^2) = F_{\mu\nu}F^{\nu\mu}$ in the effective Lagrangian, the other (pseudo) scalar $\epsilon_{\alpha\beta\gamma\delta}F^{\alpha\beta}F^{\gamma\delta}$ do not appear in the Lagrangian as has been anticipated.

So far we have neglected the contribution of the fermion loops. When the symmetry is restored, the fermion masses go to zero and we are worried about the infrared divergences, which spoils the perturbation calculation. We have also neglected the momentum transfer of the classical fields. Therefore, the possible pair production by a background field can not be described. We

hope that these problems should be resolved simultaneously. Although our result shows that it is hard to find the symmetry restoration by experiments, it may be applicable to the very early universe.

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Figure Caption

Fig. 1. Schematical representation of the effective potential. Cases A, B, and C correspond to $Tr(F^2) < 0$, $Tr(F^2) = 0$, $Tr(F^2) > 0$ respectively. The arrows represent the true minimum of the potential.

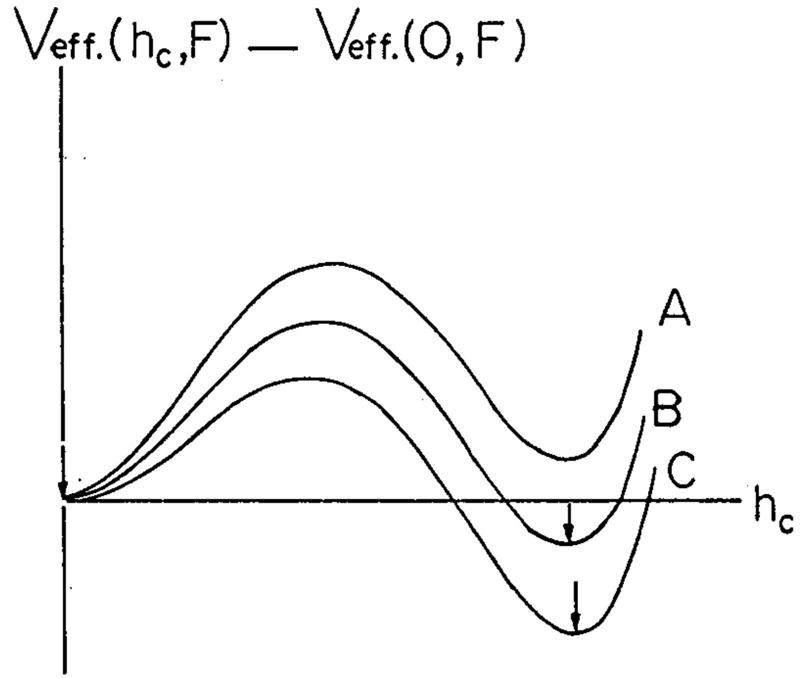

Fig. I