THERMALIZATION OF BARYON ASYMMETRY

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In an expanding and cooling universe, baryon asymmetry generated through baryon- and *CP*-violating interactions in the early high temperature stages will tend at later stages to thermalize toward a situation of reduced baryon asymmetry We analyze this thermalization effect in the context of the SU(5) model of grand unification. The thermalization is characterized by distinctive eigenmodes. For SU(5) we observe that there is one particular mode that does not thermalize at all.

The ideas of Grand Unification make it plausible that baryon conservation is only an approximation, which holds very well at familiar energy scales but which breaks down completely at high energies ($E \gtrsim 10^{15}$ GeV). If this is so, and if the early universe attains correspondingly high temperatures, then the observed cosmic asymmetry between matter and antimatter content may be understood to be a relic produced in the early, hot stages of the universe and frozen in ever since [1]. One pictures the asymmetry as arising from baryon number and *CP* violating processes involving interactions and decays of very heavy bosons (mass scale $M_X \approx 10^{15}$ GeV), in the era where these particles are going out of thermal equilibrium. Quantitative estimates of the asymmetry, as measured by the present ratio of baryon to photon number densities, are highly uncertain: the expectations are sensitive to details concerning the nature and strength of *CP* violation, the Higgs structure, and other features of the microscopic model of grand unification [2].

However, there is one important piece of any complete analysis which is less sensitive to many of these details, in particular to the role of *CP* violation. Suppose some net baryon number has been generated, by whatever means (heavy boson decays, perhaps Hawking radiation from small black holes), in the early stages where *CP* violating effects are important. Later on, as the universe cools and these effects become suppressed, the baryon asymmetry will begin to diminish as a result of baryon violating but *CP* conserving interactions among the lighter particles – quarks and leptons. Here the heavy bosons enter as virtual particles, which mediate the interactions among fermions. Given enough time, these processes might be expected to "thermalize" the baryon asymmetry, reducing it toward the equilibrium value zero. In an expanding universe, the surviving asymmetry depends on the relative rates of thermalization and expansion. The temperature dividing line between these two stages is sensitive to details. But once the thermalization stage is underway, small residual effects of *CP* violation can reasonably be ignored. it is a good approximation here to treat the baryon violating interactions among fermions to lowest order, where *CP* violation makes no appearance and where Higgs exchange effects can presumably be ignored. Thus, given a grand unification model which is definite with respect to gauge boson and fermion content, and assuming an expanding universe of the standard Friedmann type, one can compute precisely the rate of decrease of baryon number in the thermalization stage. We shall do just this, for the SU(5) grand unification model [3]. These matters have been discussed previously in the literature, in rough treatments which lump together the various species of fermions [4]. As we shall discover, however, interesting new features emerge where one proceeds more carefully. Thermalization takes the form of an eigenmode problem.

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There are different modes with different thermalization rates; and, in particular, one mode which does not thermalize at all.

In the SU(5) model the fermions group into several different families: for example, the family u_L, u_R, d_L, d_R, e_L, e_R, ν_e, where the neutrino is purely left-handed and where each quark comes in three colors (indices suppressed). Let f be the number of such families ($f = 3$ at latest count). We suppose that the population of any quark species is primordially independent of color, in which case this remains so in all subsequent eras. We concern ourselves with the epoch following the disappearance of the superheavy gauge bosons and Higgs particles, so temperature $T \lesssim M_X$, but we suppose that T is large compared to all fermion masses and masses of the SU(2) × U(1) gauge and Higgs bosons. Thus we deal with populations of effectively massless particles. The fermions interact via baryon conserving processes mediated by exchange of color gluons, photons and SU(2) × U(1) gauge bosons; and via baryon violating processes mediated by exchange of superheavy gauge bosons. Exchange of superheavy or light Higgs bosons will be ignored, on the usual expectation that Higgs couplings are proportional to small fermion/gauge boson mass ratios. The fermion, vector boson vertices are of comparable strength for all classes of reactions; but once T is at all appreciably smaller than M_X the baryon violating processes are suppressed in rate relative to the others by a factor of $(T/M_X)^4$.

We assume that kinetic equilibrium is maintained at each instant by virtue of the stronger, baryon conserving collisions; so that all species have the same temperature. For the same reason, if z_i is the fugacity of particles of species i, $\bar{z}_i$ the fugacity of the corresponding antiparticles, we suppose that $\bar{z}_i = 1/z_i$ at each instant. Moreover, since fermion masses are being ignored so too can we ignore fermion mixing angles (i.e., generalizations of the Cabibbo angle); but masslessness also entails that one must keep track separately of left-handed and right-handed fermions.

Collision reactions will change the relative populations of various species and, in particular, the baryon violating processes will change the relative numbers of particles and antiparticles. Let N'_a ($\bar{N}'_a$) be the number density of fermions (antifermions) of species a:

$$N'_a = \int \frac{d^3p}{(2\pi)^3}\, n_a(E), \quad n_a = \frac{1}{1 + z_a^{-1}\, e^{E/T}}\,; \tag{1}$$

$$\bar{N}'_a = \int \frac{d^3p}{(2\pi)^3}\, \bar{n}_a(E), \quad \bar{n}_a = \frac{1}{1 + z_a\, e^{E/T}}\,. \tag{1'}$$

For small asymmetry between particles and antiparticles the fugacity z_a will be close to unity and we therefore expand the above expressions to first order in $z_a - 1$. Thus

$$N'_a = N_0 + \tfrac{1}{2}N_a, \quad \bar{N}'_a = N_0 - \tfrac{1}{2}N_a, \tag{2}$$

where

$$N_a = \tfrac{1}{6}T^3(z_a - 1) \tag{3}$$

and N_0 is the same for fermions and antifermions of all species.

The structure of our problem may be exemplified by consideration of a particular, related set of baryon violating reactions: (i) $a + b \rightleftarrows \bar{c} + \bar{d}$ (e.g. $u_L + u_R \rightleftarrows \bar{d}_R + \bar{e}_\nu$) and (ii) $\bar{a} + \bar{b} \rightleftarrows c + d$. We ignore small *CP* violating effects so that the forward and backward reactions are described by the same invariant matrix element $\mathcal{M}$. The reactions (i) contribute, e.g., to dN'_a/dt; reactions (ii) to $d\bar{N}'_a/dt$. Taking the difference, we find

$$\begin{aligned}\frac{dN_a}{dt} = -\frac{(2\pi)^4}{(2\pi)^9} \int \frac{d^3p_a}{2E_a}\frac{d^3p_b}{2E_b}\frac{d^3p_c}{2E_c}\frac{d^3p_d}{2E_d} \\ \times\, \delta(p_a + p_b - p_c - p_d)\, |\mathcal{M}|^2 \\ \times\, [n_a n_b(1-\bar{n}_c)(1-\bar{n}_d) - \bar{n}_a\bar{n}_b(1-n_c)(1-n_d) \\ - \bar{n}_c\bar{n}_d(1-n_a)(1-n_b) + n_c n_d(1-\bar{n}_a)(1-\bar{n}_b)]\,.\end{aligned} \tag{4}$$

This expression depends on the fugacities. Expanding to first order about $z = 1$ we have

$$dN_a/dt = -\gamma(z_a + z_b + z_c + z_d - 4), \tag{4'}$$

or from eq. (3)

$$dN_a/dt = -(6\gamma/T^3)(N_a + N_b - N_c - N_d), \tag{4''}$$

where the temperature-dependent coefficient γ depends on the matrix element $\mathcal{M}$ and follows from evaluation of the integral in eq. (4). One now generalizes eq. (4) to allow for all baryon violating and conserving reactions involving species a (e.g., the reactions $a + c \rightleftarrows \bar{b} + \bar{d}$, etc.). The net result is a coupled set of linear differential equations in $7 \times f$ population densities N_i:

$$dN_i/dt = -\Gamma_{ij} N_j\,. \tag{5}$$

Even for $f = 3$ (three families) the problem looks to be rather unwieldy. Fortunately, there is a series of reductions which enable us to simplify the problem to a situation involving only two coupled equations. The reductions are as follows:

(i) The exchange of SU(5) vector bosons is invariant under permutations of the families. As a consequence, if we write the matrix Γ in 7×7 block form organized by family we find the structure

$$\Gamma = \begin{pmatrix} A & B & B & \dots & B \\ B & A & B & \dots & B \\ \vdots & \vdots & \vdots & & \vdots \\ B & . & . & \dots & A \end{pmatrix}. \tag{6}$$

The eigenvectors of such a block matrix are easy to analyze. They are of two types.

Type I:

$$(v \;\; v \;\; \dots \;\; v)^{\mathrm{T}}. \tag{7}$$

Here v is an eigenvector of the 7×7 matrix $A + (f - 1)B$.

Type II: Here the block eigenvector contains only two non-vanishing entries, w and $-w$, in any position, e.g.

$$(w \;\; -w \;\; 0 \;\; 0 \;\; \dots)^{\mathrm{T}}, \quad (0 \;\; w \;\; 0 \;\; -w \;\; 0 \;\; \dots)^{\mathrm{T}}, \tag{8}$$

etc., where w is an eigenvector of the 7×7 matrix $A - B$.

The type II eigenvectors are of no interest in the present problem since they carry no net baryon number. Baryons in one family are compensated by antibaryons in another. Thus, we are interested only in type I eigenvectors, so only in the eigenvalue problem for the 7×7 matrix $A + (f - 1)B$.

(ii) In the SU(5) model vector boson exchanges conserve not only charge Q and baryon minus lepton number $B - L$ but also a quantity, we call it R, which represents the number of particles in the 5-dimensional representation. Thus, the following quantities are conserved.

$$Q = \tfrac{2}{3}(u_{\mathrm{L}} + u_{\mathrm{R}}) - \tfrac{1}{3}(d_{\mathrm{L}} + d_{\mathrm{R}}) - (e_{\mathrm{L}} + e_{\mathrm{R}}) + \dots,$$
$$B - L = \tfrac{1}{3}(u_{\mathrm{L}} + u_{\mathrm{R}} + d_{\mathrm{L}} + d_{\mathrm{R}}) - (e_{\mathrm{L}} + e_{\mathrm{R}} + \nu_{\mathrm{e}}) + \dots, \tag{9}$$
$$R = d_{\mathrm{R}} - e_{\mathrm{L}} - \nu_{\mathrm{L}} + \dots,$$

where, e.g., u_{L} devotes the difference in number of fermions and antifermions of given color for species u_{L}, etc; and the dots ... represent obvious, similar contributions from the other families.

Corresponding to each of these conserved quantities there is an eigenvector of Γ [hence of $A + (f - 1)B$] with zero eigenvalue. In searching for the other, nonvanishing eigenvalues we may then restrict ourselves to the subspace with $Q = B - L = R = 0$. This allows us to eliminate these variables and thereby reduce ourselves to a 4×4 matrix problem. This technical simplification aside, however, the physically important feature of SU(5) is the conservation of R. We will discuss the implications later on.

(iii) Finally we observe that the ordinary baryon *conserving* weak interactions conserve the quantities $u_{\mathrm{L}} + d_{\mathrm{L}}$ and $e_{\mathrm{L}} + \nu_{\mathrm{e}}$ (as well as the corresponding quantities in the other families). These baryon conserving reactions proceed more rapidly than the baryon violating ones once the temperature is appreciably smaller than M_{X}. For this regime, therefore, we can impose $u_{\mathrm{L}} - d_{\mathrm{L}} = e_{\mathrm{L}} - \nu_{\mathrm{e}} = 0$ as instantaneous conditions, treating $u_{\mathrm{L}} + d_{\mathrm{L}}$ and $e_{\mathrm{L}} + \nu_{\mathrm{e}}$ as slowly varying quantities. This reduces us to a 2×2 matrix eigenvalue problem.

The above reductions are all straightforward, but tedious. We shall merely outline the main elements and then quote final results:

(i) For the reactions $\mathrm{a} + \mathrm{b} \rightleftarrows \bar{\mathrm{c}} + \bar{\mathrm{d}}$ the squared matrix element is proportional to the square of $s = (p_{\mathrm{a}} + p_{\mathrm{b}})^2$ whenever a and b have the same helicity, and proportional to the square of $t = (p_{\mathrm{a}} - p_{\mathrm{c}})^2$ or $u = (p_{\mathrm{a}} - p_{\mathrm{d}})^2$ in other helicity situations. In fact we always encounter the same combination

$$|\mathcal{M}|^2 = (g^2/M_{\mathrm{X}}^2)^2(s^2 + t^2 + u^2). \tag{10}$$

(ii) We next have to carry out the integration in eq. (4) in order to extract the coefficient γ defined there. In the term proportional to s^2 in $|\mathcal{M}|^2$ we could reduce the problem analytically to a two-fold integral, which we then evaluated numerically with fair precision. As for the t^2 and u^2 terms it is easy to see that they contribute equally and that their sum makes a contribution which is bounded above by that coming from the s^2 term. However, here we could not find any easy analytic reductions and our final numerical estimates are correspondingly inexact. Very roughly, we estimate that $\langle t^2 + u^2 \rangle / \langle s^2 \rangle = 0.5$. Thanks to the bound, however, we can be certain that our final result for γ is correct to within 50%. Given all the other uncertainties in the baryon asymmetry problem, this seems quite tolerable and we could see no reason to carry out a more exact, four-fold numerical integration to improve the estimate.

(iii) Collecting all our results, we now work out the 7×7 matrix $A + (f-1)B$ and then carry out the reductions described above. This leads to a 2×2 matrix problem in two independent quantities, which we take to be $\epsilon \equiv u_L + d_L$ and $\eta = d_R$ (the symbols u_L, d_L, d_R, recall, denote the difference in number density of particles and antiparticles of the indicated type). With $\phi = \binom{\epsilon}{\eta}$ we have

$$d\phi/dt = -\lambda \Gamma^{(2)} \phi, \tag{11}$$

where

$$\Gamma^{(2)} = \begin{pmatrix} 38 & 4 \\ 2 & 12 \end{pmatrix} + (f-1)\begin{pmatrix} 22 & 4 \\ 2 & 12 \end{pmatrix}, \tag{12}$$

and

$$\lambda = 2(96/\pi^3)(\alpha_X^2/M_X^4)T^5. \tag{13}$$

Here $\alpha_X = g^2/4\pi \approx \frac{1}{50}$ is the unification coupling constant parameter. The factor of 2 in eq. (13) reflects our numerical estimate of the integral of eq. (4); as discussed, it is correct to within about 50%.

The matrix $\Gamma^{(2)}$ has two eigenmodes. For the case $f = 3$ the eigenvalues are $\Lambda \approx 83.6$ and $\Lambda \approx 34.4$. Clearly, the smaller the eigenvalue the longer the survival time of baryon asymmetry in that mode. For a given mode ϕ, with eigenvalue Λ, we have

$$d\phi/dt = -\lambda \Lambda \phi. \tag{14}$$

(iv) Finally, we convert from time to temperature using the equations for cosmological expansion:

$$T^{-1}\, dT/dt = -(\tfrac{8}{3}\pi G\rho)^{1/2}, \tag{15}$$

$$\rho = \tfrac{1}{30}\pi^2(\eta_B + \tfrac{7}{8}\eta_F)T^4 \equiv \tfrac{1}{30}\pi^2 \eta T^4, \tag{16}$$

where η_B and η_F are, respectively, the number of boson and fermion helicity states.

Any distribution among species in the differences between fermion and antifermion populations can be expressed in terms of the amplitudes corresponding to the various eigenmodes. Let $A(T_I)$ be the amplitude for a particular eigenmode, with eigenvalue Λ, at some initial temperature T_I; and let $A(0)$ be the amplitude at a much later time (e.g., now) when the temperature is reduced essentially to zero. From eqs. (13)–(16) we then have

$$A(0) = A(T_I)\, e^{-W}, \tag{17}$$

$$W = 2(96/\pi^4)(5/4\pi\eta)^{1/2} \Lambda \alpha_X^2 (T_I/M_X)^3 (M_p/M_X), \tag{18}$$

where M_p is the Planck mass. In the case of three families, the smaller of our non-vanishing eigenvalues is $\Lambda \approx 34.4$. For the rest, typical values are $\eta \approx 70$, $\alpha_X \approx 1/50$, $M_X \approx 10^{15}$ GeV. This gives, roughly, $W \approx 20(T_I/M_X)^3$.

Clearly, the decrease in baryon asymmetry in the thermalization stage occurs predominantly in the highest temperature era. Much depends, therefore, on how far back we can go in time (how high in temperature) within the framework of the above analysis. With $M_X \approx 10^{15}$ GeV any baryon asymmetry that survives down to $T_\pm \approx \frac{1}{3}M_X$ will survive essentially unchanged thereafter. The delicacy in all of this is that transition from baryon generating stage to the thermalization stage occurs, presumably, just in the vicinity of $T \approx M_X$.

However, for the SU(5) model we have observed that there is an eigenmode with zero eigenvalue, reflecting conservation of the quantity R defined in eq. (9). That is, there is a mode carrying non-vanishing baryon number which does not thermalize at all. Thus, if a non-vanishing value of R arises in the earlier stages where baryon asymmetry is generated then there exists a safe channel for survival of baryon asymmetry in the subsequent thermalization regime. Similarly safe channels exist for the conserved quantities Q and $B - L$, of course, but these latter are of no interest: both Q and $B - L$ are presumably zero at primordial times and remain zero for all later times. However, even within the SU(5) model R is not strictly conserved. Conservation of R holds only in the approximation, relevant to the thermalization regime, where one can ignore fermion masses and Higgs particle effects. However, in the stage where baryon asymmetry is being generated by heavy boson decay, *CP*-violating Higgs particle effects can play an essential role [2]. Even if R is primordially zero, a non-vanishing value can arise through these Higgs effects. If a non-vanishing value of R is present at the onset of the thermalization stage, it will survive essentially unchanged thereafter.

To recapitulate: For the non-vanishing thermalization modes, the decrease in baryon asymmetry in the thermalization stage has been quantitatively estimated and seen to be marginally worrisome. Our general procedures provide the tools that could be employed for other, competing grand unification models. The orders of magnitude found here are probably representative of a wide class of models. The non-thermalizing mode associated with approximate conservation of the quantity R is special to SU(5).

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