THE YUKAWA β-FUNCTION IN $N = 1$ RIGID SUPERSYMMETRIC THEORIES

P. WEST
Mathematics Department, King's College, London, WC2, UK

Received 1 December 1983

The two-loop β function for the "Yukawa" coupling constant is evaluated in an arbitrary rigid $N = 1$ supersymmetric theory.

In a recent paper [1] it was shown that a rigid $N = 1$ supersymmetric theory was finite at two loops if it was finite at one loop. To obtain this result the two-loop β function for the gauge coupling in an arbitrary supersymmetric theory was found and it was shown that for the case of a one-loop finite supersymmetry theory the two-loop β function for the Yukawa coupling constant vanished [1]. In this paper the two-loop β function for the "Yukawa" coupling constant in an arbitrary rigid $N = 1$ supersymmetric theory is calculated. Taken with the results of ref. [1] this determines the two-loop renormalization group equations for an arbitrary rigid $N = 1$ supersymmetric theory.

The most general renormalizable $N = 1$ supersymmetric action containing particles of spin one and less and invariant under a gauge group G is of the form

$$A = \int d^4x\, d^4\theta\, [\bar{\varphi}_a (e^{gV})^a{}_b \varphi^b]$$

$$+ \int d^4x\, d^2\theta\, \frac{\mathrm{Tr}(W^\alpha W_\alpha)}{64 g^2 C_2(G)}$$

$$+ \left(\int d^4x\, d^2\theta\, \frac{1}{3!}\, d_{abc}\varphi^a\varphi^b\varphi^c + \text{h.c.}\right)$$

$$+ \text{ghost} + \text{gauge fixing}\,, \tag{1}$$

where

$$W_\alpha = \bar{D}^2 (e^{-gV} D_\alpha\, e^{gV})\,. \tag{2}$$

Possible mass terms are ignored for the present as they do not affect the β functions. The a index runs over irreducible representations A and members of a given irreducible representation s (i.e. $a = \{A, s\}$) and $V^a{}_b = V^i (T_i)^a{}_b$. Here $(T_i)^a{}_b = (T_i{}^A)^s{}_t$ are the generators of the group G in the irreducible representation A.

The potential is invariant under G provided

$$d_{abc}(T_i)^c{}_d + d_{dac}(T_i)^c{}_b + d_{bdc}(T_i)^c{}_a = 0\,, \tag{3}$$

where d_{abc} is totally symmetric in a, b and c. We will call d_{abc} the Yukawa coupling constant.

Let us denote the wave function renormalization constants for the chiral fields φ_a and V by $Z_a{}^{a'}$ and Z respectively and that for d_{abc} by $Z_{abc}{}^{a'b'c'}$. As a result of the non-renormalization theorem [2], we find that

$$Z_{abc}{}^{efg} Z_a^{1/2\,a} Z_{b'}^{1/2\,b} Z_{c'}^{1/2\,c} = \delta_{a'}{}^{(e}\delta_{b'}{}^{f}\delta_{c'}{}^{g)}\,. \tag{4}$$

We will evaluate the infinite part of the two loop $\bar{\varphi}\varphi$ propagator and use eq. (4) to determine $Z_{abc}{}^{a'b'c'}$ to two loops. The two-loop super Feynman graphs which contribute to the $\bar{\varphi}\varphi$ propagator are given in fig. 1. In fig. 1 a blob denotes the relevant one-loop one-particle irreducible graph including any one-loop counter term that may be required. Using super Feynman rules [3] in the formulation given in ref. [4] we find that graphs (a), (b), (c), (d) and (e) give the infinite result

$$2 \int \bar{d}^4p\, d^4\theta\, \bar{\varphi}_a(-p,\theta)\, \varphi^b(p,\theta)\, G_{(1)}(p)$$

$$\times\, [C_A (C_A - S_B^A)\, \delta_b^a]\,, \tag{5}$$

0.370-2693/84/$ 03.00 © Elsevier Science Publishers B.V. (North-Holland Physics Publishing Division)

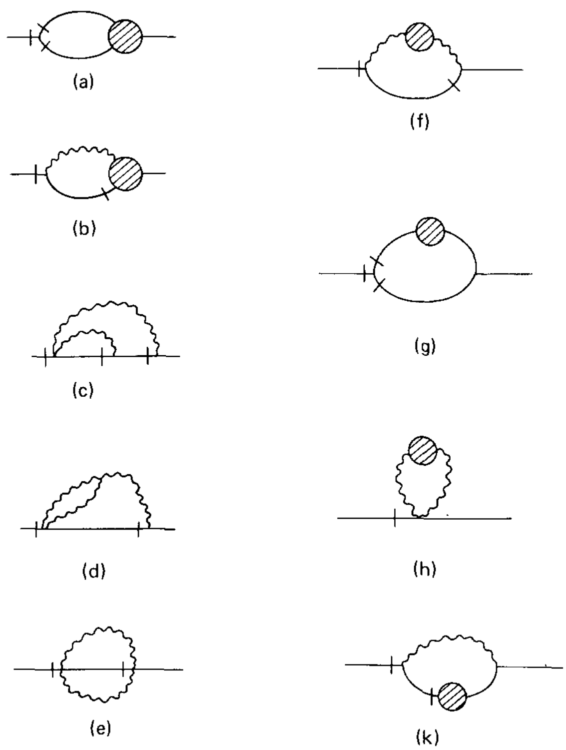

Fig. 1. The two-loop $\bar{\varphi}\varphi$ propagator.

while graphs (f), (g), (h) and (k) give rise to the infinite contribution

$$\int \bar{d}^4 p\, d^4\theta\, \bar{\varphi}_a(-p,\theta)\,\varphi^b(p,\theta)\, G_{(1)}(p) \Big[-\delta^a_b R^A_B + \bar{d}^{acd} C_D d_{cbd} + \tfrac{1}{2} C_A \Big(-3C_2(G) + \sum_A T_A \Big) \delta^a_b + \delta^a_b C_A [S^A_B - C_A] \Big] . \qquad (6)$$

In the above equations, we have used the following definitions

$$f_{ikl} f_{jkl} = \delta_{ij} C_2(G) , \quad (T^i)^a{}_b (T^i)^b{}_c = C_A \delta^a_c ,$$

$$d_{ace} d^{bce} = 2S_a{}^b = 2\delta^b_a S^B_A , \quad (T^j)^a{}_b (T^i)^b{}_a = \sum_A T_A \delta^{ij} ,$$

$$\delta^a_b R^A_B = d^{acd} S_d{}^e d_{ceb} , \qquad (7)$$

$$G(p) = \mu^{4\epsilon} \int \bar{d}^n k\, \bar{d}^n q\, [q^2 k^2 (q+k)^2 (p+q)^2]^{-1} , \qquad (8)$$

and

$$G_{(1)}(p) = G(p) + \mu^{2\epsilon} \int \frac{\bar{d}^n q\, A_1(q)}{q^2 (p+q)^2} , \qquad (9)$$

where A_1 is the divergent part of

$$A(q) = \mu^{2\epsilon} \int \frac{\bar{d}^n q}{q^2 (p+q)^2} \qquad (10)$$

and $n = 4 - 2\epsilon$.

Adding the contributions of eqs. (5) and (6) together we find that the infinite part of the $\bar{\varphi}\varphi$ propagator at two loops is given by

$$- \int \bar{d}^4 p\, d^4\theta\, \bar{\varphi}_a(-p,\theta)\,\varphi^b(p,\theta)\, T^a{}_b\, G_{(1)}(p) , \qquad (11)$$

where

$$-T^a{}_b = -\delta^a_b R^A_B + \bar{d}^{acd} C_D d_{cbd} + \tfrac{1}{2} \delta^a_b C_A \Big(-3C_2(G) + \sum_A T_A \Big) + \delta^a_b C_A (C_A - S^A_B) . \qquad (12)$$

Using the fact that

$$G(p) = -(4\pi)^{-4} \{ -\tfrac{1}{2}\epsilon^{-2} - \tfrac{1}{2}\epsilon^{-1} [5 - 2\gamma_E - 2\ln(p^2/4\pi\mu^2)] + \text{constant} + O(\epsilon) \ldots \} , \qquad (13)$$

and

$$A(p) = (4\pi)^{-2} [\epsilon^{-1} + 2 - \gamma_E - \ln(p^2/4\pi\mu^2) + O(\epsilon)] , \qquad (14)$$

we find that the required contribution of $Z^A{}_B$ to the two-loop counterterm $(Z^A{}_B - \delta^A{}_B)\,\bar{\varphi}_A \varphi^B$ is

$$Z^A{}_B = \delta^A{}_B + Z_1{}^A{}_B + Z_2{}^A{}_B + \ldots , \qquad (15)$$

where

$$Z_1{}^A{}_B = -(4\pi)^{-2} \epsilon^{-1} (S^A{}_B - C_A \delta^A{}_B) ,$$

and

$$Z_2{}^A{}_B = (4\pi)^{-4} (\tfrac{1}{2}\epsilon^{-1} - \tfrac{1}{2}\epsilon^{-2})\, T^A{}_B . \qquad (16)$$

In the above procedure we have used dimensional reduction [5] and minimal subtraction. To use an alternative subtraction scheme one simply changes $A_{(1)}$.

Let us define the anomalous dimension $\gamma_A{}^B$ of the chiral field by

$$\gamma_A{}^B = Z^{-1/2}{}_A{}^C \mu \frac{\partial}{\partial\mu} Z^{1/2}{}_C{}^B , \tag{17}$$

and the β and β_{efg} functions for the couplings g and d_{efg} respectively by

$$\beta(g, d, \epsilon) = \mu \frac{\partial}{\partial\mu} g , \qquad \beta_{efg}(g, d, \epsilon) = \mu \frac{\partial}{\partial\mu} d_{efg} . \tag{18}$$

Following a similar argument to that found in ref. [6] we find that the ϵ dependence of the above functions is given by

$$\beta(g, d, \epsilon) = \beta(g, d) - \epsilon g ,$$

$$\beta_{efg}(g, d, \epsilon) = \beta_{efg}(g, d) - \epsilon d_{efg} . \tag{19}$$

The dependence of the renormalization constants on ϵ is of the form

$$Z = 1 + \sum_{i=1}^{\infty} Z^{(i)}/(\epsilon)^i \tag{20}$$

as well as similar equations for $Z_A{}^B$ and $Z_{abc}{}^{a'b'c'}$. Using eqs. (17)–(20) we find that

$$\beta = g\left(g \frac{\partial}{\partial g} + d_{efg} \frac{\partial}{\partial d_{efg}}\right) Z^{(1)} , \tag{21}$$

$$\beta_{abc} = \left(g \frac{\partial}{\partial g} + d_{efg} \frac{\partial}{\partial d_{efg}}\right) Z^{(1)}_{abc}{}^{a'b'c'} d_{a'b'c'} , \tag{22}$$

and

$$\gamma_B{}^C = -\frac{1}{2}\left(g \frac{\partial}{\partial g} + d_{efg} \frac{\partial}{\partial d_{efg}}\right) Z^{(1)}{}_B{}^C . \tag{23}$$

Eq. (4) then implies the relation

$$\beta_{abc} = 3\gamma_{(a}{}^e d_{bc)e} , \tag{24}$$

which extends a similar result [7] already known the Wess–Zumino model.

Utilizing the calculated renormalization constants of eq. (16) we find that

$$\gamma_B{}^C = (4\pi)^{-2}(S_B{}^C - C_B \delta^C{}_B) - (4\pi)^{-4} T_B{}^C , \tag{25}$$

and that

$$\beta_{abc} = 3(4\pi)^{-2}[(S_{(a}{}^e - C_A \delta_{(a}{}^e)$$

$$- (4\pi)^{-2} T_{(a}{}^e]\, d_{bc)e} . \tag{26}$$

We notice that the β_{efg} function vanishes at two loops in the case of an $N = 2$ supersymmetric theory as predicted by general arguments and is in agreement with ref. [8], where the above calculation was performed for the special case of $N = 2$ supersymmetric theories. The reader who wishes to understand the general structure of such a calculation in more detail is referred to this last reference. The result also agrees with the result for the Wess–Zumino model [9] which is obtained by setting $d_{111} = \lambda$, $C_2(G) = \Sigma_A T_A = C_A = 0$. It would be interesting to study the relationship between eq. (20) and the two-loop Yukawa coupling in a general renormalizable theory which was calculated using dimensional regularization in ref. [10].

Given a knowledge of the two-loop β functions of an arbitrary $N = 1$ rigid supersymmetric theory it is possible to plot the evolution of the couplings at the two-loop level, in particular one may examine which of the theories possess only asymptotically free coupling constants [11].

The author would like to thank M. Gell-Mann for encouragement and A. Parkes and P. Howe for discussions.

References

[1] A. Parkes and P. West, Finiteness in rigid supersymmetric theories, King's College, London, preprint.

[2] J. Wess and B. Zumino, Phys. Lett. 49B (1974) 52; J. Iliopoulos and B. Zumino, Nucl. Phys. B76 (1974) 310; S. Ferrara, J. Iliopoulos and B. Zumino, Nucl. Phys. B77 (1974) 41.

[3] A. Salam and J. Strathdee, Phys. Rev. D11 (1975) 1521; S. Ferrara and O. Piguet, Nucl. Phys. B93 (1975) 261.

[4] M.T. Grisaru, M. Rocek and W. Siegel, Nucl. Phys. B159 (1979).

[5] W. Siegel, Phys. Lett. 84B (1979) 193.

[6] D.J. Gross, in: Methods in field theory, eds. C.R. Balian and J. Zinn-Justin (North-Holland, Amsterdam, 1976).

[7] S. Ferrara, J. Iliopoulos and B. Zumino, Nucl. Phys. B77 (1974) 413.

[8] P. Howe and P. West, The two-loop β function in extended rigid supersymmetric theories, King's College, London, preprint.

[9] L. Abbott and M. Grisaru, Nucl. Phys. B169 (1980) 415.

[10] M. Machacek and M. Vaughn, The two-loop renormalization group equations in a general quantum field theory II; Yukawa couplings, Northeastern University preprint.

[11] M. Gell-Mann, private discussion.