DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 82-049
August 1982

INTRODUCTION TO THE ELECTROWEAK THEORY

by

Emmanuel A. Paschos

Institute of Physics, University Dortmund

NOTKESTRASSE 85 · 2 HAMBURG 52

DESY 82-049
August 1982

INTRODUCTION TO THE ELECTROWEAK THEORY

EMMANUEL A. PASCHOS

Institute of Physics

University Dortmund

INTRODUCTION TO THE ELECTROWEAK THEORY

PART I.

THE ROAD TO UNIFICATION

PART II.

THE ELECTROWEAK THEORY

PART III.

PREDICTIONS OF THE E + W THEORY

PART IV.

RENORMALIZATION

FOREWORD

These lectures were delivered at the Maria Laach school for high energy physics in the period 8 - 18 Sept, 1981. The audience of the school consisted of university students working for or having completed their diplomarbeit. The audience included both theorists and experimentalists.

The aim was to provide an introduction to the subject matter and discuss some interesting topics beyond the electroweak theory. The chapters are organized very closely to the lectures. An introductory section reviews the main motivations which led to the electroweak theory. The standard model as developed by Glashow, Salam and Weinberg is described in section II. Section III deals with the predictions of the theory concerning intermediate gauge bosons, charged currents, neutral currents and Higgs particles. The last part outlines the renormalization of the electroweak theory and the determination of the weak mixing angle, including radiative corrections. For the sake of rapid communication the last lecture dealing with extensions beyond the standard model (grand unification, technicolor, ...) is not included in the manuscript. I hope that in the near future I can rewrite and extend several of the chapters before the manuscript reaches its final form for publication.

I wish to thank Professors J.K. Bienlein and H.D. Dahmen for providing the opportunity to deliver these lectures and Professor P. Carruthers for encouraging the undertaking of the project. To R.W. Brown, U. Türke and H. Usler who read the manuscript and pointed out misprints I express my warmest thanks.

Finally, I did not prepare an exhaustive list of all the original papers, but instead included at the end of the chapters book and articles, which are very close to the presentation of the lectures. For a remedy of this deficiency, and for reference to subjects which are not included in the text, I refer to several recent excellent books and articles:

E.S. Abers and B.W. Lee, "Gauge Theories" in Physics Reports, Vol. 9C, North-Holland Publ. Comp. Amsterdam (1973)

S.L. Adler and R.F. Dashen, Current Algebras, W.A. Benjamin, Inc. N.Y. (1968)

P. Becker, M. Böhm and H. Joos, Eichtheorien, Teubner Studienbücher, Stuttgart (1981)

J.D. Bjorken and S.D. Drell, Relativistic Quantum Mechanics and Relativistic Quantum Fields, Mc Graw-Hill Comp. (1965)

L.D. Faddev and A.A. Slavnov, Gauge Fields: Introduction to Quantum Theory, The Benjamin Cummins Comp. Reading Mass. (1978)

R.P. Feynman, Photon-Hadron Interactions, W.A. Benjamin, Reading Mass. (1972)

C. Itzykson and J.B. Zuber, Quantum Field Theory, Mc Graw Hill (1980)

L.-F. Li, Introduction to Gauge Theories of Electromagnetic and Weak Interactions, Santa Barbara preprint NSF-ITP-81-25

T.D. Lee, Particle Physics and Introduction to Field Theory, Harwood Acad. Press (1981)

P. Ramond, Field Theory, A Modern Primer, The Benjamin Cummins Comp. Reading Mass. (1981)

G. Segré, Introduction to Electroweak Interactions, University Pennsylvania preprint (1982)

J.C. Taylor, Gauge Theories of Weak Interactions, Cambridge University Press (1976).

Finally, I would like to acknowledge the theses of R. Murzik, U. Türke, H. Usler and M. Wirbel, which were useful references for several chapters.

PART I

1. The Electromagnetic Current

The interaction between particles is successfuly described by forces generated with the exchange of elementary particles and fields. A typical example is the electromagnetic force, which is attributed to the exchange of photons. A typical electromagnetic process is shown in figure 1. The Feynman amplitude for the process has the form

$$\mathcal{M} = (-ie)^2 J^{\mu}(x) \frac{-i g_{\mu\nu}}{q^2 + i\epsilon} J^{\nu}(x) \tag{1.1}$$

where the J_{μ}'s are electromagnetic currents, $-ig_{\mu\nu}/(q^2+i\varepsilon)$ is the photon propagator and e is the electric charge. The structure and properties of the currents played an important role in the development of the weak interactions. We begin this book with a description of currents and their properties.

For leptonic processes the electromagnetic current describes the interaction of the photon with a charged fermion. The current is a local operator

$$J_{\mu}(x) = Q_{\ell} \overline{\psi}_{\ell}(x) \gamma_{\mu} \psi_{\ell}(x) \tag{1.2}$$

with Q_{ℓ} the charge of the fermions, $\psi_{\ell}(x)$ is the field for the lepton ℓ and γ_{μ} a Dirac matrix. The current $J_{\mu}(x)$ is a generalization of the classical concept of a current as it appears in Maxwell's theory. In classical electrodynamics $J_{\mu}(x)$ is a four-vector with components

$$J_{\mu}(x) = [\rho(x), \vec{J}(x)] \tag{1.3}$$

with $\rho(x)$ denoting the charge density and the vector $\vec{J}(x)$ the charge flow. The total charge of a particle is given by the integral

$$Q = \int d^3x\, J_0(x) \tag{1.4}$$

The current in (1.2) is an operator which transforms also as a four vector. The fields occurring above are operators which create and destroy localized particle states and satisfy canonical commutation relations.

The currents acquired special importance in particle physics because they

i) obey selection rules,

ii) satisfy commutation relations, which are non-linear relations relating the strength of interactions relative to each other and

iii) are established as useful probes for investigating the structure within hadrons.

Finally the weak and electromagnetic currents are closely related to each other, as we discuss in Chapter 2, a property which suggests that they belong to one theory which unifies the weak and electromagnetic interactions.

The simplest current is the electromagnetic current studied explicitly in textbooks of quantum electrodynamics. It is instructive, at the very beginning, to compute one electromagnetic process explicitly in order to establish the notation and several techniques which will be used later on. A process frequently studied in colliding ring experiments is

$$e^+ + e^- \rightarrow \mu^+ + \mu^-$$

This process is represented by the diagram in figure 1, which also defines the external momenta. The Feynman amplitude $\mathcal{M}$ is obtained by following standard Feynman rules described in the appendix of Bjorken and Drell, volume 1.

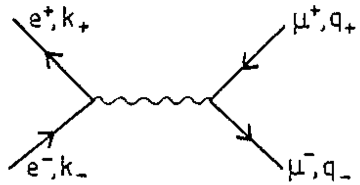


Fig. (1.1)

$$\mathcal{M} = \bar{v}(k_+)(-ie\gamma_\mu)\,u(k_-)\,\frac{-i\,g^{\mu\nu}}{q^2+i\epsilon}\,\bar{u}(q_-)(-ie\gamma_\nu)\,v(q_+) \qquad (1.5)$$

The currents introduced in (1.1) are easily identified. The transition probability is the square of the matrix element, which by standard techniques is written

$$\sum \mathcal{M}^*\mathcal{M} = \frac{e^4}{q^4}\,\mathrm{Tr}\left[\gamma_\mu \frac{\not k_+ + m_e}{2m_e}\gamma_\nu \frac{\not k_- - m_e}{2m_e}\right] \cdot$$

$$\mathrm{Tr}\left[\gamma^\mu \frac{\not q_+ - m_\mu}{2m_\mu}\gamma^\nu \frac{\not q_- + m_\mu}{2m_\mu}\right] \qquad (1.6)$$

The two traces are very similar. We compute the first one by standard techniques.

$$\mathrm{Tr}\left[\gamma_\mu \frac{\not k_+ + m_e}{2m_e}\gamma_\nu \frac{\not k_- - m_e}{2m_e}\right] = \frac{1}{m_e^2}\left[k_{+\mu}k_{-\nu} + k_{+\nu}k_{-\mu} - g_{\mu\nu}\left(k_+\cdot k_- + m_e^2\right)\right] \qquad (1.7)$$

with simple algebra we write Equ(1.6) in terms of inner products of the momenta

$$\sum_{\mathrm{spins}} |\mathcal{M}|^2 = \frac{e^4}{q^4}\,\frac{2}{m_e^2 m_\mu^2}\,\cdot$$

$$\left[k_+\cdot q_+\,k_-\cdot q_- + k_+\cdot q_-\,k_-\cdot q_+ + m_e^2\,q_+\cdot q_- + m_\mu^2\,k_+\cdot k_- + 2m_\mu^2 m_e^2\right] \qquad (1.8)$$

In colliding beam experiments the center of mass frame is also the laboratory frame. We define the total center of mass energy by 2E

$$s = q^2 = 4E^2$$

and the scattering angle between the electron and the negative muon by θ. Then, neglecting terms proportional to m_e or m_μ,

$$k_-\cdot q_- = k_+\cdot q_+ = E^2(1-\cos\theta) \qquad (1.9)$$

$$k_+\cdot q_- = k_-\cdot q_+ = E^2(1+\cos\theta) \qquad (1.10)$$

$$k_+\cdot k_- = q_+\cdot q_- = 2E^2 \qquad (1.11)$$

It now follows

$$\sum_{\mathrm{spins}} |\mathcal{M}|^2 = \frac{e^4}{q^4}\,\frac{4}{m_e^2 m_\mu^2}\,E^4\left(1+\cos^2\theta\right) \qquad (1.12)$$

The cross section has the form

$$d\sigma = \frac{1}{8}\,\frac{m_e}{E_+}\,\frac{m_\mu}{E_-}\,(2\pi)^4\,\delta^4\left(k_+ + k_- - q_+ - q_-\right)\left(\frac{m_\mu}{E_+}\right)\frac{d^3q_+}{(2\pi)^3}\left(\frac{m_e}{E_-}\right)\frac{d^3q_-}{(2\pi)^3} \qquad (1.13)$$

The factors occurringabove are standard in such calculations and we describe them in detail The first factor of 1/2 is a flux factor in the center of mass system. Because of the normalization of the spinors

$$\bar{u}(p,s)\,u(p,s) = 1 \qquad (1.14a)$$

$$\bar{v}(p,s)\,v(p,s) = -1 \qquad (1.14b)$$

summation over the spin gives the projection operators

$$\sum_s u_\alpha(p,s)\,\overline{u_\beta(p,s)} = \left(\frac{\not p + m}{2m}\right)_{\alpha\beta} \qquad (1.15a)$$

and

$$\sum_s v_\alpha(p,s)\,\overline{v_\beta(p,s)} = \left(\frac{-\not p + m}{2m}\right)_{\alpha\beta} \qquad (1.15b)$$

It also requires the introduction of a factor m/E_i for each external fermion line. The term $|\mathcal{M}|^2$ is the square of the Feynman amplitude which is special for each process. For our process $\mathcal{M}$ was computed from Equ. (1.5) to (1.8). The next two factors including the 4-dimensional δ-function represent energy-momentum conservation. The last factors are the phase space factors,

one for each particle. They are written in a Lorentz invariant form, as it follows from the identity

$$d^3q\left(\frac{m}{E}\right) = 2m\int_0^\infty dq_0\,\delta\left(q_\mu q^\mu - m^2\right)d^3q$$

$$= 2m\int_{-\infty}^{\infty} d^4q\,\delta\left(q_\mu q^\mu - m^2\right)\theta(q_0)$$

$$\text{with}\quad \theta(q_0) = \begin{cases} 1 & \text{for } q_0 > 0 \\ 0 & \text{for } q_0 < 0 \end{cases} \qquad (1.16)$$

The last step in the evaluation of any cross section involves the evaluation of phase space integrals. In the present case there is only a two body phase space, which is easy to calculate using (1.16). It is convenient also to work in the center of mass system where

$$q^2 = 4E^2 \quad , \quad q\cdot q_+ = 2EE_+$$

and E_+ is the energy of μ^+. (1.17)

$$\sigma = 2\int \frac{e^4}{q^4}\left(1+\cos^2\theta\right)\delta^4\left(k_+ + k_- - q_+ - q_-\right)\frac{2E_+}{(2\pi)^2}\,\frac{d^3q_+}{2E_+}\,d^3q_-$$

$$= 2\int \frac{e^4}{q^4}\left(1+\cos^2\theta\right)\delta\left(q_+^2 - m_\mu^2\right)\frac{2E_+}{(2\pi)^2}\,d^3q_-$$

$$= 2\,\frac{e^4}{q^4}\int\left(1+\cos^2\theta\right)\frac{1}{4E}\,\delta(\epsilon - \epsilon_-)\,\frac{2E_+}{(2\pi)^2}\,\epsilon_-^2\,d\epsilon_-\,d\Omega \qquad (1.18)$$

In intermediate steps we used

$$\int dx\, \delta(f(x)) = \left| df(x)/dx \right|^{-1}_{x=x_0}$$

with $f(x_0) = 0$. The differential cross section now follows performing the integrals and averaging over the initial spins

$$\frac{d\bar{\sigma}}{d\Omega} = \frac{1}{4}\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4q^2}\left(1+\cos^2\theta\right) \quad . \qquad (1.19)$$

The total cross section is

$$\bar{\sigma} = \frac{4\pi\alpha^2}{3q^2} \quad . \qquad (1.20)$$

<u>P r o b l e m</u>

Compute the differential cross section for

$$e^+ + e^- \longrightarrow \mu^+ + \mu^-$$

with the initial beams polarized. Consider two cases:

(i) electrons and positrons with longitudinal polarization,

(ii) electrons and positrons polarized normal to the beam plane and in opposite directions (natural polarization).

<u>The Current for Hadronic States</u>. The electromagnetic current for a proton is more complicated since protons are not point like particles, but have a measurable physical size formed by the cloud of pions and other hadrons which surround them. As a first attempt one would write the electromagnetic current for a proton as

$$J_\mu(x) = e\,\bar{\psi}_p(x)\,\gamma_\mu\,\psi_p(x) \qquad (1.21)$$

with $\psi_p(x)$ the proton field. This form is ruled out immediately by a calculation of the vertex as dictated by quantum-electrodynamics

$$\langle p'|\, J_\mu(x)\, e^{-iq\cdot x}\, |p\rangle = e\,\bar{u}(p')\,\gamma_\mu\, u(p) \int d^4x\, e^{-i(q-p'+p)\cdot x} \qquad (1.22)$$

This describes a point particle with unit charge and a Dirac magnetic moment. It obviously fails for the case of a proton which has a size and an anomalous magnetic moment.

One therefore expects a more general structure introduced by considering the hadronic current as a vector operator that satisfies general symmetry principles. We begin by writing it in the form

$$\langle p'|\, J_\mu(x)\, e^{-iq\cdot x}\, |p\rangle = e\,\bar{u}(p')\,\Gamma_\mu(p',p)\,u(p)\,(2\pi)^4\,\delta^4(p'-q-p) \qquad (1.23)$$

with Γ_μ a four vector and u(p), u(p') are spinors that satisfy the Dirac equation. Under translations in space and time $J_\mu(x)$ transform as

$$J_\mu(x) = e^{iP\cdot x} J_\mu(0)\, e^{-iP\cdot x} \tag{1.24}$$

whre P is the total energy momentum operator; thus (1.23) reduces to

$$\langle p'|\, J_\mu(0)\,|p\rangle = e\,\overline{u(p')}\,\Gamma_\mu(p',p)\,u(p) \tag{1.25}$$

A second property is current conservation

$$\frac{\partial J_\mu(x)}{\partial x_\mu} = 0 \tag{1.26}$$

which quarantees the conservation of charge

$$\frac{dQ(t)}{dt} = \int \frac{\partial J_0}{\partial x_0}(x)\, d^3x = -\int \vec{\nabla}\cdot\vec{J}(x)\, d^3x = -\int_{\text{surface}} \vec{J}\cdot\vec{n}\, dS = 0 \tag{1.27}$$

In the last integral the surface can be chosen large enough where the electromagnetic current vanishes faster than $1/_{r}2$ so that the surface integral gives zero. Lorentz invariance and current conservation limits the form of the operator Γ_μ

$$\langle p'|\, J_\mu(0)\,|p\rangle = e\,\overline{u}(p')\left[\gamma_\mu F_1(q^2) + i\sigma_{\mu\nu}q^\nu F_2(q^2)\right] u(p) \tag{1.28}$$

The functions $F_1(q^2)$ and $F_2(q^2)$ are called the electromagnetic form factors of the nucleons. They are measured in electron proton and neutron scattering. The electric form factor of the proton is normalized to 1 at zero momentum transfer, $F_1(0)=1$. The second form factor measures at $q^2=0$ the anomalous part of the magnetic moment of the target

$$F_2^a(0) = \mu^a = \left\{ \begin{array}{ll} 1.79 & \frac{e}{2M} \text{ for proton} \\ -1.91 & \frac{e}{2M} \text{ for neutron} \end{array} \right\} \tag{1.29}$$

As functions of q^2 the form factors decrease rapidly with q^2 reflecting the limited size of the hadronic cloud around the proton.

References Ch. 1

1. J.D. Bjorken and S.D. Drell,
   Relativistic Quantum Mechanics New York, McGraw-Hill, 1964.
2. S.D. Drell and F. Zachariasen,
   Electromagnetic Structure of Nucleons, Oxford Un. Press, 1961.

2. The Weak Currents and their Properties

The idea that weak decays of particles are described by a local four-fermion interaction was proposed by Fermi in 1935. Since then the effective current-current interaction has been successively enlarged to incorporate new pieces of experimental observations. At the end of the sixties the current-current interaction was of the form

$$\mathcal{L} = \frac{G}{\sqrt{2}} J_\mu(x) J^{\mu +}(x) \qquad (2.1)$$

with G the Fermi coupling constant and $J_\mu(x)$ a charged current with a leptonic and hadronic term

$$J_\mu(x) = \ell_\mu(x) + h_\mu(x) \qquad (2.2)$$

The leptonic part of the current is

$$\ell_\mu(x) = \overline{\Psi}_e(x)\gamma_\mu(1-\gamma_5)\Psi_{\nu_e}(x) + \overline{\Psi}_\mu(x)\gamma_\mu(1-\gamma_5)\Psi_{\nu_\mu}(x) \qquad (2.3)$$

with the first term corresponding to the electron and its neutrino and the second term to the muon and its neutrino. Its space-time structure has a vector part analogous to the electromagnetic current and an axial part introduced after the discovery of parity violation.[1] A direct calculation, similar to that in chapter 1, but now using the currents in (2.3), gives the μ-decay spectrum, which is in good agreement with experiment.

It also gives the decay rate of the muon as

$$\Gamma(\mu \to e + \nu + \bar{\nu}) = \frac{G^2 m_\mu^5}{192\pi^3} \qquad (2.4)$$

From the observed decay rate and the mass of the muon the constant G is determined to be

$$G = (1.16632 \pm 0.00004) \times 10^{-5}\ \mathrm{GeV}^{-2} \qquad (2.5)$$

The hadronic current consists of several parts determined by detailed analyses of hadron decays.[2] The decay of a neutron $n \to p + e^- + \nu$ is well described by a current

$$\overline{\Psi}(p)\gamma_\mu(g_V - g_A\gamma_5)\Psi(n) \qquad (2.6)$$

where g_V and g_A are again from factors describing the effects of strong interactions in the hadrons. The vector form factor was precisely determined and it is strikingly close to 1, while g_A is about 1.24. An explanation was proposed[3] that the strangeness conserving part of h_μ is an isospin current

$$V_\mu^+ - A_\mu^+ = (V_\mu^1 + iV_\mu^2) - (A_\mu^1 + iA_\mu^2) \qquad (2.7)$$

with a Vector and an Axial current; furthermore V_μ^1 and V_μ^2 are the first and second components of an isospin current. This means that the charge

$$T^i = \int V_0^i(x)\, d^3x \tag{2.8}$$

are the same isospin generators occurring in the strong interactions and are therefore conserved. This rule is called the conserved vector current hypothesis (CVC). The T^i's form a group, whose commutation relations produce a third component of isospin

$$\left[T^i, T^j\right] = i\,\epsilon^{ijk}\, T_k \tag{2.9}$$

In the late sixties T^3 was not yet observed to mediate transitions with the strength G; there was no weak neutral current. But such an operator already existed in the electromagnetic current. The electromagnetic current consisted of two parts

$$J_\mu^{em}(x) = V_\mu^3(x) + \frac{1}{\sqrt{3}} V_\mu^8(x) \tag{2.10}$$

with $V_\mu^3(x)$ being the third component of isospin and V_μ^8 an isoscalar current transforming, as the 8 component of SU(3). It is evident that there is a relation between the weak and the electromagnetic currents, since the vector part of the weak current and the isovector part of the electromagnetic current form an isotriplet. The form of the interaction in (2.1) defines a universal coupling for leptonic, semileptonic and non-leptonic decays. Once the coupling constant G is determined, as in (2.5) from the muon decay, it can be used to translate the isotriplet hypothesis into relations between electromagnetic and weak matrix elements. The previous discussion of the weak interactions

was partly motivated from the direct analogy with electromagnetism. The electromagnetic interaction (1.1) has the form

$$\mathcal{M} = i e^2\, J^\mu \frac{g_{\mu\nu}}{q^2} J^\nu \tag{2.11}$$

with an explicit photon propagator and an effective current

$$J_\mu = \ell_\mu^{em} + J_\mu^{em} \tag{2.12}$$

The hadronic current is defined (2.10) and the leptonic current has a term for each charged lepton

$$\ell_\mu^{em} = \bar{\Psi}_\ell \gamma_\mu \Psi_\ell \tag{2.13}$$

The propagator term is obviously missing in (2.1). It should have been there in the form

$$g^2 \Delta_{\mu\nu} = g^2 \frac{-i g_{\mu\nu}}{q^2 - M_W^2} \tag{2.14}$$

if the weak interaction is mediated by the exchange of a particle of mass Mw and a coupling strength g. At very low energies , however, where most of the decays take place, $q^2 \ll M_W^2$ and

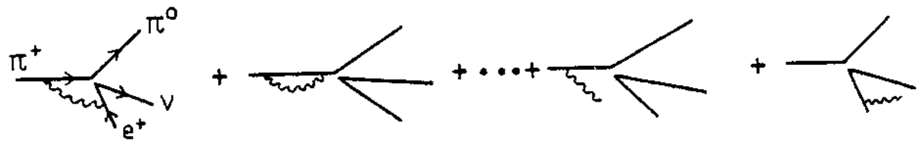
$$g^2 \Delta_{\mu\nu} \longrightarrow i\, g_{\mu\nu} \frac{g^2}{M_W^2} = i\, g_{\mu\nu} \frac{G}{\sqrt{2}} \tag{2.15}$$

thus the form in (2.1) is adequate.

The values for g_V are measured in nuclear β-decays and in the decay

$$\pi^+ \to \pi^0 + e^+ + \nu \qquad (2.16)$$

Special attention is required for the radiative corrections to this process, which we study in the following chapters. It suffices here to remark that the radiative corrections come from the diagrams



calculated with the introduction of arbitrary cutoff, which will be justified later on in the electroweak theory. The best value for

$$g_V = 0.974 = \cos(13^\circ)$$

The deviation from 1 is significant, because no reasonable values of the fitting parameters could give $g_V = 1$.

The small discrepancy of 2.6% was explained, by the observation that the hadronic currents $h_\mu^\pm$ do not generate only an SU(2) group as the lepton currents do, but contain other pieces responsible for strangeness-changing decays, like

$$\Xi^0 \to p + e^- + \bar{\nu}_e \qquad (2.17)$$

From the absence of decays like $\Sigma^+ \to n + e^+ + \nu$ one concludes that the decays satisfy the rule $\Delta S = \Delta Q$. Thus the hadronic current is the sum of a $\Delta S = 0$ and a $\Delta S = 1$ term

$$h_\mu = g_V h_\mu^{(0)} + g_S h_\mu^{(1)} \qquad (2.18)$$

The first current is the isospin current that appears in (1.7)

$$h_\mu^{(0)} = V_\mu^+ - A_\mu^+ \qquad (2.19)$$

The second term induces strangeness changing transitions and it has, in SU(3), the form

$$h_\mu^{(1)} = \left(V_\mu^4 - i\,V_\mu^5\right) - \left(A_\mu^4 - i\,A_\mu^5\right) \qquad (2.20)$$

Even though SU(3) is not an exact symmetry, the matrix element of $h_\mu^{(0)}$ and $h_\mu^{(1)}$ can be accurately estimated. The conclusion from numerous estimates in experiments give $g_S/g_V \sim 0.25$. In 1963 Cabibbo observed that $g_S^2 + g_V^2 = 1$. Therefore

$$h_\mu = \cos\theta\, h_\mu^{(0)} + \sin\theta\, h_\mu^{(1)} \qquad (2.21)$$

In this way universality is restored[4] since the sum of the squares of the hadronic couplings equals the coupling observed in muon decay. In addition the discrepancy of g_V from 1 is understood. The angle θ_c is called the Cabibbo angle and its value is determined $\sin\theta_c = 0.228 \pm 0.012$.

In terms of quark fields the charged current is given by

$$h_\mu = \bar{u}\,\gamma_\mu(1-\gamma_5)\left[d\cos\theta_c + s\sin\theta_c\right] \qquad (2.22)$$

with u,d and s the fields for the up, down and strange quarks, respectively. At this stage (2.23) is just a mnemonic that shows explicitly the transformation properties of the currents. The quark bilinears represent currents with these transformation properties. In general, the matrix element of current(s) between hadronic states has a definite Lorentz and SU(3) structure but it also involves unknown form factors, which depend on the structure of hadrons. Some of the form factors were already introduced in equations (1.23) and (2.26). For specific kinematic regions, e.g. in deep inelastic scattering, the currents probe directly the constituents within the hadrons and the quark representation becomes very powerful. We shall return to this point in the discussion of the parton model.

References Ch. 2

1. Parity violation was suggested theoretically by

T.D. Lee and C.N. Yang, Phys. Rev. 104, 254 (1956) and observed by

C.S. Wu, E. Ambler, R.W. Hayward, D.D. Hoppes and R.P. Hudson, Phys. Rev. 105, 1413 (1957)

R.L. Garwin, L.M. Lederman and M. Weinrich, Phys. Rev. 105, 1415 (1957);
V.L. Telegdi and A.M. Friedman, Phys. Rev. 105, 1681 (1957)

2. For an introduction to weak interactions

J.D. Bjorken and S.D. Drell, Relativistic Quantum Mechanics

S. Gasiorowicz, Elementary Particle Physics (John Wiley and Sons, New York 1966)

3. Ideas of CVC and V-A interactions were proposed by

R.P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958)

S.S. Gerschtein and J.R. Zeldovich JETP (USSR) 29, 698 (1955)

E.C.G. Sudarshan and R.E. Marshak, in: Proc. Padua Conf. on Mesons and Recently Discovered Particles (1957),

4. N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963).

3. The Partially conserved axial current

A second property of the weak currents deals with the axial term. The charged pion decays into $\mu\nu$ pairs, and it means that the isospin carrying current $A_\mu^\pm(x)$ has a non-vanishing expectation value between the vacuum and the one pion state

$$\langle 0 | A_\mu^\pm(x) | \pi^\pm(q) \rangle = i f_\pi(q^2)\, q_\mu\, e^{-iq\cdot x} \tag{3.1}$$

Here q_μ is the 4-momentum of the pion, f_π its decay constant. This is the general form of the matrix element consistent with Lorentz invariance. The coupling $f_\pi(q^2)$ is determined on the pion mass-shell. It is tempting to assume that the axial current is also conserved, but taking the divergence of (3.1) we find

$$q^\mu \langle 0 | A_\mu^\pm(x) | \pi^\pm(q) \rangle = i f_\pi m_\pi^2\, e^{-iq\cdot x} \tag{3.2}$$

From this relation we conclude that the divergence of the axial current is not zero, since neither f_π, nor m_π is equal to zero; The divergence of the axial current $\partial_\mu A^\mu(x)$ is known on the pion mass shell. Its extrapolation away from the mass shell is in principle a terra incognita. However, for many low enery processes which involve the exchange of the axial current with four momentum q_μ, it was found that the matrix elements of the divergence of the axial current are slowly varying functions provided $q^2 \lesssim m_\pi^2$. In terms of equations it means that we can represent

$$\partial^\mu A_\mu^i = f_\pi m_\pi^2\, \phi^i \tag{3.3}$$

with the qualification that the matrix element of the corresponding pion current is a slowly varying function of q^2 in the interval $0 < q^2 < m_\pi^2$.

A notable result is the Goldberger-Treiman[1] relation obtained by applying PCAC to the hadronic element of β-decay. Consider the matrix element of the axial current between a neutron and a proton state. It has the general form

$$\langle p | A_\mu^+ | n \rangle = \overline{u(p')} \left[\gamma_\mu \gamma_5\, g_A(q^2) + q_\mu \gamma_5\, g_P(q^2) \right] u(p) \tag{3.4}$$

Taking the divergence on both sides of this equation.

$$\langle p | \partial_\mu A_\mu^+ | n \rangle = \left[2M\, g_A(q^2) + q^2 g_P(q^2) \right] \bar{u}(p')\, \gamma_5\, u(p) \tag{3.5}$$

On the other hand from (3.3)

$$\langle p | \partial_\mu A_\mu^+ | n \rangle = f_\pi m_\pi^2 \langle p | \phi^+ | n \rangle = f_\pi m_\pi^2 \frac{1}{q^2 - m_\pi^2} \langle p | j_\pi^+ | n \rangle \tag{3.6}$$

According to PCAC the last matrix element does not change much by taking the limit $q^2 \to 0$ and gives the pion nucleon coupling constant

$$\langle p | j_\pi^+ | n \rangle = \sqrt{2}\; g_{\pi NN}\, \bar{u}(p')\, \gamma_5\, u(p) \tag{3.7}$$

From (3.5) - (3.7) it now follows

$$g_{\pi NN} f_\pi = \sqrt{2}\, M\, g_A \tag{3.8}$$

This is the Goldberger-Treiman relation. For the experimental values of the coupling constants it holds at the 10% level. It is a remarkable equation, relating the pion-nucleon coupling constant to two couplings of weak interactions.

There is another way of looking at PCAC.[2,3] The meaning of PCAC is that the actual world is not far away from the limit in which the axial currents are conserved at the expense of having zero mass pions ($m_\pi = o$, $f_\pi \neq o$). In this approach we can still define f_π and g_A through equ. (3.1) and (3.4). Because the axial current is now conserved (3.5) becomes

$$2M\, g_A(q^2) + q^2\, g_p(q^2) = 0 \qquad (3.9)$$

Now one-pion exchange produces a pole in $f_p(q^2)$

$$g_p(q^2) = \sqrt{2}\,\frac{g_{N\pi\pi} f_\pi}{q^2} + \text{non-singular terms}, \qquad (3.10)$$

which together with (3.9) gives again the Goldberger-Treiman relation.

The subjects covered in the last two chapters represent two basic topics developed long before the electroweak theory. They strongly suggest that the weak force is not an isolated phenomenon, but one intimately connected with the other forces of nature. The isotriplet hypothesis clearly states that the isovector part of the electromagnetic current and the vector part of the

weak current, $\Delta S = o$, form an isospin triplet. In addition, it states that the charges $T^{\pm}$ are the same generators as those of strong isospin. The isotriplet hypothesis also posed a problem: to explain why the neutral member of the multiplet did not also occur in the weak interactions. This question was answered with the discovery of neutral currents.

The partially conserved axial current hypothesis relates couplings of the weak interactions to the pion-nucleon coupling constant through the Goldberger-Treiman relation. PCAC, combined with equal time commutation relations, makes it possible to calculate the deviation of g_A from 1 as an integral over the pion-nucleon cross sections.[4] Consequences of PCAC hold at the 10-20% level. They are understood to hold because the mass of the pion is small in comparison with the masses of other hadrons. That is, there is an underlying symmetry which is broken by the small mass of the pion. The previous remarks provide a strong motivation to search for a closer connection between the weak, electromagnetic, and perhaps the strong interactions. The successful theory which unifies the weak and electromagnetic forces is studied in the following chapters. The electroweak theory is so far in good agreement with experiment. It makes predictions to be tested in the future. Finally, the reader should keep in mind that the theory must also provide a natural explanation of the empirical rules described so far and others to be described in in later chapters.

References Ch. 3

1. M.L. Goldberger and S.B. Treiman, Phys. Rev. 111, 354 (1958)

2. S. Coleman, Hadrons and their Interactions (School Ettore Majorana, Edited by A. Zichichi) Academic Press, Pg. 9, 1968

3. B.W. Lee, Chiral Dynamics, Gordon and Breach (1972)

4. S.L. Adler, Phys. Rev. 14, 1051 (1965)

   W.I. Weisberger, Phys. Rev. 14, 1047 (1965)

   For an introduction to Current Algebra

   S.L. Adler and R.F. Dashen, Current Algebra Benjamin Inc., New York, 1968.

PART II

4. The Yang-Mills Field

The sucessful and simple theory which unifies the weak and electromagnetic interactions is based on the group SU(2)xU(1). We develop the theory in several steps. First we describe, in this chapter, the main features of a gauge theory. Then we will describe a theory containing only electrons and the corresponding neutrinos. Finally, the theory is extended to incorporate hadrons.

The structure of a Yang-Mills theory is almost completely determined by the requirement that the internal symmetry transformations of the fields can be carried out independently at different space-time points. In other words the theory is invariant under local transformations. Let ψ be a multiplet of n Dirac fields. The multiplets belong to representations of the group SU(N). The free Dirac Lagrangian (set m=o for now) is

$$\mathcal{L} = \bar{\psi}\, i\gamma_\mu \partial^\mu \psi = i\bar{\psi}\not{\partial}\psi \qquad (4.1)$$

is invariant under the transformation

$$\begin{aligned} \psi &\to \psi' = U\psi \\ \bar{\psi} &\to \bar{\psi}' = \bar{\psi}U^\dagger \end{aligned} \qquad (4.2)$$

for $\quad U^\dagger U = UU^\dagger = 1$.

We write

$$U = e^{i\varepsilon_i \lambda_i/2} \tag{4.3}$$

with $i=1,2,\ldots,n^2-1$, λ_i the generators of the group with $\lambda_i=\lambda_i^+$ and the ε_i's real constants.

If we allow ε_i to be a function of x, then (4.1) is no longer invariant. In fact, the lagrangian transforms into

$$\mathcal{L} \longrightarrow \mathcal{L}' = \overline{\Psi}\, i\gamma_\mu \left[\partial^\mu + \frac{i}{2}\, \epsilon_i^\mu(x)\right] \Psi \tag{4.4}$$

with $\varepsilon_i^\mu(x) = \frac{\partial \varepsilon_i(x)}{\partial x\mu}$. Following Yang and Mills,[1] we introduce a set of vector fields $B_i^\mu(x)$ and couple them to the currents as follows

$$\mathcal{L} = \overline{\Psi}\, i\gamma_\mu \left[\partial^\mu + \Lambda_i B_i^\mu\right] \Psi + \mathcal{L}_B \tag{4.5}$$

where Λ_i is a set of matrices still to be determined and $\mathcal{L}_B$ is a function of the B_i^μ's only. Each vector field is characterized by a Lorentz index μ and an internal symmetry index i. We now demand that L remains invariant under the transformations (4.2) with ε_i a function of x; this will require that B_i^μ transforms in such a way as to cancel the additional term in (4.4) Let $\hat{B}_i^\mu$ be the transformed vector field. Then for L to remain invariant,

$$U^{-1} \frac{\partial}{\partial x_\mu} U + U^{-1} \Lambda_i U \hat{B}_i^\mu = \Lambda_i B_i^\mu \tag{4.6}$$

must hold. As the λ_i's form a complete set of NxN traceless matrices, we can attempt to write

$$\Lambda_k = \frac{1}{2}\, i e \lambda_k \tag{4.7}$$

the imaginary i is there because the λ_k's and the B's are hermitian. Considering infinitesimal transformations

$$U \simeq 1 + \frac{i}{2} \epsilon_i \lambda_i \tag{4.8}$$

with

$$\left[\frac{1}{2}\lambda_i, \frac{1}{2}\lambda_j\right] = i f_{ijk}\, \frac{1}{2}\lambda_k \tag{4.9}$$

For the infinitesimal transformation we can solve for $\hat{B}_i^\mu$ in (4.6). A convenient method is to rewrite (4.6) as

$$\frac{i}{2} e \lambda_i \hat{B}_i^\mu = U \frac{i}{2} e \lambda_i U^{-1} B_i^\mu - \frac{i}{2} \lambda_i \epsilon_i^\mu \tag{4.10}$$

then use (4.8) and (4.9) to obtain

$$\hat{B}_k^\mu = B_k^\mu - f_{kij}\, \varepsilon_i B_j^\mu - \frac{1}{e}\, \varepsilon_k^\mu \tag{4.11}$$

It is convenient to introduce a covariant derivative

$$D^\mu = \partial^\mu + \frac{i}{2} e \lambda_i B_i^\mu$$

and rewrite the fermion part in (4.5) as

$$\mathcal{L}_F = i \overline{\Psi} \not{D}\, \Psi \tag{4.12}$$

Next we must construct L_B. It must be Lorentz invariant and invariant under $B \to \hat{B}$. It must also contain the kinetic term of the B_μ fields. In analogy to the procedure of obtaining gauge invariant field strengths in electrodynamics, we define

$$F_i^{\mu\nu} = \partial^\nu B_i^\mu - \partial^\mu B_i^\nu + e f_{ijk} B_j^\mu B_k^\nu \qquad (4.13)$$

If we introduce the vector notation

$$\vec{B}^\mu = (B_1^\mu, B_2^\mu, \ldots, B_N^\mu), \text{ etc., where } N=n^2-1 \text{ and}$$
$$(\vec{A} \times \vec{B})_i = f_{ijk} A_j B_k \qquad (4.14)$$

we can write (4.11) and (4.13) as

$$\vec{F}^{\mu\nu} = \partial^\nu \vec{B}^\mu - \partial^\mu \vec{B}^\nu + e \vec{B}^\mu \times \vec{B}^\nu \qquad (4.15)$$

The last term in (4.15) does not occur in electrodynamics and is introduced to assure $\vec{F}^{\mu\nu}$ as a vector under gauge transformations. We can now build a scalar lagrange function for the $\vec{B}^\mu$ fields

$$\mathcal{L}_B = -\frac{1}{4} \vec{F}^{\mu\nu} \cdot \vec{F}_{\mu\nu} \qquad (4.16)$$

A theory with L_B alone is called a pure Yang-Mills theory. The currents of the original Lagrangian are given by

$$J_\alpha^\mu(x) = i \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} \lambda_\alpha \psi = i \bar{\psi}(x) \gamma^\mu \frac{\lambda_\alpha}{2} \psi(x) \qquad (4.17)$$

To sum up, we constructed a theory invariant under local gauge transformation. We found that the invariance requirements are fulfilled by introducing vector fields coupled to conserved currents. The next task is to find a gauge theory which contains the weak and electromagnetic currents described in the previous chapters. We consider a gauge model of electrons and their neutrinos. At the very beginning we must answer two questions :

(i) On which group to base the theory ? and

(ii) In which representation of the group belong the fermion fields ?

The currents of the theory must include at least the charged weak currents

$$J_\mu^+(x) = \bar{\nu}_e(x) \gamma_\mu (1-\gamma_5) e(x) \qquad (4.18)$$

its hermitian adjoint and the electromagnetic current

$$J_\mu^{em}(x) = \bar{e}(x) \gamma_\mu e(x) \qquad (4.19)$$

We need three vector fields with which to couple them. They correspond to the intermediate gauge bosons, $W^\pm$, and the photon. The smallest group is SU(2). This group however, is unacceptable because the currents (4.11) and (4.12) do not form an SU(2) algebra. This becomes evident by considering their charges and studying their commutation relations. Consider the charges

$$T^+ = \frac{1}{2} \int \nu_e^+(x) (1-\gamma_5) e(x) \quad , \quad T^- = (T^+)^+ \qquad (4.20)$$

The commutator is

$$\left[T^+, T^-\right] = \frac{1}{4}\int d^3x\, d^3y \left[\nu^+(x)(1-\gamma_5)e(x),\ e^+(y)(1-\gamma_5)\nu(x)\right]$$

$$= \frac{1}{2}\int d^3x\, d^3y \left[\nu^+(x)(1-\gamma_5)\nu(x) - e^+(x)(1-\gamma_5)e(x)\right]\delta^3(x-y)$$

$$iT^3 = \left[T^+, T^-\right] = \frac{1}{2}\int d^3x \left[\bar{\nu}(x)\gamma_0(1-\gamma_5)\nu(x) - \bar{e}(x)\gamma_0(1-\gamma_5)e(x)\right] \qquad (4.21)$$

which is not the charge corresponding to the electromagnetic current. There are now two alternative solutions :

(i) Introduce new leptons and modify the weak current $J^{\pm}_{\mu}$ so that we get the right SU(2) algebra, or

(ii) Introduce another gauge boson W_3 with its corresponding current. In this alternative there are four gauge bosons $W^{\pm}, Z, \gamma$ and the group must be enlarged to SU(2)xU(1).

Both alternatives were actively studied and it became evident only after the discovery of several new phenomena that nature prefers the second solution.

Problem

Show that under gauge transformations the field strength tensor $F^i_{\mu\nu}$ transforms as a vector on the index i. The result holds including terms linear in $\varepsilon_i(x)$.

References Ch. 4

1. C.N. Yang and R.L. Mills, Phys. Rev. Vol. 96 p. 191 (1954)

5. Construction of the Model [1]

Again we consider only electrons and its neutrino in a theory based as the group SU(2)xU(1). We must establish how they transform under SU(2) and U(1), separately. From the previous section an SU(2) algebra is generated by the weak charges

$$T_+ = \frac{1}{2}\int d^3x \left[e^+ (1-\gamma_5)\nu \right] \tag{5.1}$$

$$T_- = \left[T_+\right]^+ \tag{5.2}$$

$$T_3 = \frac{1}{2}\int d^3x \left[\nu_e^+ (1-\gamma_5)\nu_e - e^+(1-\gamma_5)e \right] \tag{5.3}$$

This means that the left-handed fields

$$e_L = \frac{1}{2}(1-\gamma_5)\,e \quad \text{and} \quad \nu_{eL} = \frac{1}{2}(1-\gamma_5)\,\nu_e$$

form a doublet

$$\Psi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \tag{5.4}$$

The charges are defined as

$$T_i = \int \Psi_L^+ \left(\frac{\tau_i}{2}\right) \Psi_L \, d^3x \tag{5.5}$$

and

$$Q = -\int e^+ e \, d^3x = -\int \left(e_L^+ e_L + e_R^+ e_R \right) d^3x \tag{5.6}$$

Since we should include Q in the group, Q must be a combination

of T_3 and the generator of U(1), denoted by Y :

$$Q - T_3 = -\frac{1}{2}\int d^3x \left[e_L^+ e_L + 2\, e_R^+ e_R + \nu_L^+ \nu_L \right] \equiv \frac{Y}{2} \tag{5.7}$$

This relation involves the difference between the charge Q and the weak isospin T_3 and it defines a new quantum number the weak isospin. It is analogous to the Gell-Mann-Nishijima formula of the strong interactions established by empirical data. With this definition the charge coincides with what we were always using as charge, but the weak isospin and hypercharge, when extended to quarks, do not always coincide with the corresponding hadronic quantum numbers. In (5.7) appear the left-handed and right-handed leptonic numbers

$$N_R = \int e_R^+ e_R \, d^3x \tag{5.8}$$

$$N_L = \int \left(e_L^+ e_L + \nu_L^+ \nu_L \right) d^3x \tag{5.9}$$

Thus the hypercharge operator in SU(2) is the unit matrix which commutes with the other generators of the group

$$\left[Y, T_i \right] = 0 \qquad \text{for } i = 1, 2, 3 \tag{5.10}$$

From the relation $Y = -(N_L + 2N_R)$ we deduce

$Y = 1$ for left-handed states and

$Y = 2$ for right-handed states.

This satisfies the original requirements of selecting a group and the representations for the fields with

$$\Psi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad \text{a weak isodoublet}$$

and $\Psi_R = e_R$ an isosinglet.

Finally, we give the parts of the lagrangian describing the fermions and gauge fields of SU (2) and by B the field of U (1). The field tensors are written as

$$F_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g\,\varepsilon^{ijk} A^j_\mu A^k_\nu \qquad (5.11)$$

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \qquad (5.12)$$

and the lagrangian

$$\mathcal{L}_B = -\frac{1}{4} F^i_{\mu\nu} F^{i,\mu\nu} - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} \qquad (5.13)$$

The gauge fields at this stage are massless.

The lagrangian for the leptons is

$$\mathcal{L}_F = i\overline{\Psi}_R \gamma^\mu \left(\partial_\mu + i g' B_\mu\right)\Psi_R + i\overline{\Psi}_L \gamma^\mu \left(\partial_\mu + \frac{i}{2} g' B_\mu + \frac{i}{2} g \tau^i A^i_\mu\right)\Psi_L \qquad (5.14)$$

We notice again that the leptons are massless because there are no $\overline{\Psi}_R\Psi_L$ and $\overline{\Psi}_L\Psi_R$ terms, which indeed are not SU(2)xU(1) invariant. Masses to both gauge mesons and leptons are introduced by scalar particles and the Higgs mechanism which we study next.

6. The Higgs mechanism

The masses of the gauge bosons and of fermions can be generated by the introduction of scalar fields.[2] We introduce a complex scalar doublet

$$\phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix} \qquad (6.1)$$

From the relation $Q = T_3 + \frac{Y}{2}$, it follows $Y = 1$ for ϕ. Each of the fields has a real and an imaginary part so that there are four independent scalar fields.

The Lagrangian is given by

$$\mathcal{L}_\phi = \left(D_\mu \phi\right)^+ \left(D^\mu \phi\right) - V\left(\phi^+\phi\right) \qquad (6.2)$$

with the covariant derivative given as

$$D_\mu = \partial_\mu - i g' B_\mu - i g \frac{\tau^i}{2} A^i_\mu \qquad (6.3)$$

and the potential

$$V(\phi^+\phi) = -\mu^2 \phi^+\phi + \lambda(\phi^+\phi)^2 \qquad (6.4)$$

The presence of an additional doublet allows the introduction of Yukawa couplings

$$\mathcal{L}_Y = -G_e\left[\overline{\Psi}_R \phi^+ \Psi_L + \overline{\Psi}_L \phi \Psi_R\right] \qquad (6.5)$$

The complete lagrangian is finally

$$\mathcal{L} = \mathcal{L}_F + \mathcal{L}_B + \mathcal{L}_\phi + \mathcal{L}_Y \qquad (6.6)$$

and it is invariant under gauge transformations of the group SU(2)xU(1).

To break the symmetry we note that the potential V (ϕ) has a locus of minima at

$$\frac{\partial V}{\partial \phi_+^*} = -\mu^2\phi_+ + 2\lambda\left(|\phi_+|^2+|\phi_0|^2\right)\phi_+ = 0 \qquad (6.7)$$

$$\frac{\partial V}{\partial \phi_0^*} = -\mu^2\phi_0 + 2\lambda\left(|\phi_+|^2+|\phi_0|^2\right)\phi_0 = 0 \qquad (6.8)$$

that is, at

$$|\phi_+|^2 + |\phi_0|^2 = \frac{\mu^2}{2\lambda} \quad . \qquad (6.9)$$

We can choose the ground state (vacuum state) at the minimum of the theory. Since it must carry vacuum quantum numbers

$$|\phi_0| = \left(\frac{\mu^2}{2\lambda}\right)^{1/2} \quad \text{and} \quad \phi_+ = 0 \qquad (6.10)$$

In the quantum theory this corresponds to $\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$ with $v = \left(\frac{\mu^2}{2\lambda}\right)^{1/2}$. In other words, one of the neutral scalar fields acquires a vacuum expectation value at the minimum of the potential.

The next step is to try to rewrite the lagrangian in terms of fields displaced relative to the minimum of the potential. By this procedure, we choose a direction in the potential thus breaking the symmetry and then define fields relative to this minimum. We parametrize the four scalar fields in terms of ξ_1, ξ_2, ξ_3 and η defined by

$$\phi(x) = U^{-1}(\vec{\xi})\begin{pmatrix} 0 \\ \frac{v+\eta}{\sqrt{2}} \end{pmatrix} \qquad (6.11)$$

where $U^{-1}(\vec{\xi})$ has the form of a gauge transformation

$$U^{-1}(\vec{\xi}) = e^{-i\vec{\xi}\cdot\vec{\tau}/2v} \quad . \qquad (6.12)$$

We can define new fields through a gauge transformation

$$\phi' = U(\vec{\xi})\,\phi \qquad (6.13)$$

$$\psi_L' = U(\vec{\xi})\,\psi_L \quad \text{and} \quad \psi_R' = \psi_R \qquad (6.14)$$

$$\frac{1}{2}\vec{\tau}\cdot\vec{A}_\mu' = \frac{1}{2}U(\vec{\xi})\,\vec{\tau}\cdot\vec{A}_\mu U^{-1}(\vec{\xi}) + \frac{i}{g}\left[\partial_\mu U^{-1}(\vec{\xi})\right]U(\vec{\xi}) \qquad (6.15)$$

This transformation has the form described in section 4 with a new feature: the fields themselves occur in the transformation. By this method we selected a gauge, known as the unitary gauge. Upon substitution in the lagrangian, the terms $\mathcal{L}_f$ and $\mathcal{L}_B$ retain the same form when expressed in terms of the new fields, but the $\mathcal{L}_\phi$ and $\mathcal{L}_Y$ are modified.

Consider first the $\mathcal{L}_\phi$ term. We set

$$\phi = U^{-1}(\vec{\xi})\begin{pmatrix} 0 \\ \frac{v+\eta}{\sqrt{2}} \end{pmatrix} = U^{-1}(\vec{\xi})\left(\frac{v+\eta}{\sqrt{2}}\right)\begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad (6.16)$$

Substituting the ϕ field in terms of the new field, it appears as if we are making a gauge transformation on the field $\begin{pmatrix} 0 \\ \frac{v+\eta}{\sqrt{2}} \end{pmatrix}$.

Therefore, $\mathcal{L}_\phi$ and $\mathcal{L}_Y$ can be written in terms of the new field only, since the gauge term $U^{-1}(\xi)$ will automatically disappear. Thus we can consider for $\mathcal{L}_\phi$ only the term

$$D_\mu\phi = \partial_\mu\eta\,\frac{\chi}{\sqrt{2}} - \frac{i}{2}\left(\frac{v+\eta}{\sqrt{2}}\right)\left(g'B_\mu + g\tau^i A_\mu^i\right)\chi \quad . \qquad (6.17)$$

$$(D_\mu \phi)^+ (D^\mu \phi) = \frac{1}{2} (\partial_\mu \eta) \partial^\mu \eta \; +$$

$$\frac{1}{8} (v+\eta)^2 \chi^+ \left\{ (g' B_\mu + g \tau^i A_\mu^i)(g' B^\mu + g \tau^i A^{i,\mu}) \right\} \chi \qquad (6.18)$$

The cross-term is purely imaginary and does not appear in the product. We study in detail the structure of the second term.

$$(g' B_\mu + g \vec{\tau} \cdot \vec{A}_\mu)(g' B^\mu + g \vec{\tau} \cdot \vec{A}^\mu) =$$

$$\left[g'^2 B^\mu B_\mu + g^2 \vec{A}_\mu \cdot \vec{A}^\mu + 2 g g' B_\mu \vec{\tau} \cdot \vec{A}^\mu \right] \qquad (6.19)$$

and between the χ states

$$\chi^+ [\; \dots \;] \chi = g'^2 B_\mu B^\mu + g^2 A_\mu^i A^{i,\mu} - 2 g g' B_\mu A^{3,\mu}$$

$$= (g' B_\mu - g A_\mu^3)^2 + g^2 A_\mu^+ A^{-,\mu} \qquad (6.20)$$

The evaluation of the term linear in $\vec{\tau}$ is most easily done using $\vec{\tau} \cdot \vec{A}_\mu = \sqrt{2} (\tau^+ A_\mu^- + \tau^- A_\mu^+) + \tau^3 A_\mu^3$ and properties of $\tau^+ \chi$.

Define new fields

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (A_\mu^1 \mp i A_\mu^2) \quad , \quad Z_\mu = \frac{g A_\mu^3 - g' B_\mu}{\sqrt{g^2 + g'^2}}$$

and

$$A_\mu = \frac{g B_\mu + g' A_\mu^3}{\sqrt{g^2 + g'^2}} \; . \qquad (6.21)$$

We note that the first fields acquired masses

$$M_W = \frac{1}{2} g v \quad , \quad M_Z = \frac{1}{2} \sqrt{g^2 + g'^2}\, v \qquad (6.22)$$

but the field A_μ remains massless. The physical correspondence for the fields is evident, with A_μ representing the photon and the other three the intermediate gauge bosons of the weak interaction.

This analysis has other interesting features to which we shall return in later sections. Among them are the generation of masses for the electron, self couplings of the Higgs particles, as well as couplings to the gauge bosons.

An interesting property is the disappearance of the ξ_1, ξ_2, ξ_3 fields from the lagrangian. These three degrees of freedom did not disappear but were transformed into longitudinal states of the vector mesons.

Finally, the simple form of the masses, relation (6.22), depends on the fact that field ϕ was a weak isodoublet. It survives even if ϕ is replaced by a finite number of isodoublet fields. It fails, however, when Higgses belonging to other representation are introduced. Consider, for instance, a theory which contains in addition a triplet field

$$\vec{\Sigma} = \begin{pmatrix} \Sigma^+ \\ \Sigma^0 \\ \Sigma^- \end{pmatrix} \qquad (6.23)$$

with $\langle \Sigma^0 \rangle = \sigma \neq 0$. Then, repeating the steps (6.17) - (6.22) and the SU(2) matrices for the three dimensional representation, we obtain the masses

$$M_{W^\pm} = \frac{1}{2} g \left(v^2 + \sigma^2\right)^{1/2} \quad , \quad M_Z = \frac{1}{2}\left(g^2 + g'^2\right)^{1/2} v \quad . \qquad (6.24)$$

Problem

Use the representation matrices

$$M_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} , \quad M_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \text{ and } M_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

for I = 1 and corresponding matrices for I = 1/2 to prove equ. (6.24).

7. Predictions in the Leptonic Sector

7.(a) General Predictions

The unification of the electric and weak force into SU(2)xU(1) depends on two coupling constants g and g'. It is convenient for the subsequent discussion to introduce the Weinberg angle

$$\tan\theta_W = g'/g \qquad (7.1)$$

The physical states for the neutral fields are the mass states Z_μ and A_μ. From (6.21) and (7.1) the physical states are written as

$$A_\mu = \cos\theta_W \, B_\mu + \sin\theta_W \, A_\mu \qquad (7.2)$$

$$Z_\mu = -\sin\theta_W \, B_\mu + \cos\theta_W \, A_\mu \qquad (7.3)$$

We can rewrite the leptonic Lagrangian

$$\mathcal{L}_f = \overline{\Psi}_R i\gamma^\mu \left(\partial_\mu + ig'\frac{Y}{2} B_\mu\right)\Psi_R + \overline{\Psi}_L i\gamma^\mu\left(\partial_\mu + i g'\frac{Y}{2} B_\mu - i g \frac{\vec{\tau}}{2}\cdot\vec{A}_\mu\right)\Psi_L \qquad (7.4)$$

in terms of the physical fields $W^\pm_\mu$, Z_μ and A_μ. Using again the identity $\vec{\tau}\cdot\vec{A}_\mu = \sqrt{2}\left(\tau^+ W^-_\mu + \tau^- W^+_\mu\right) + \tau_3 A^3_\mu$

(7.5)

We can read off the couplings of the $W^{\pm}$'s to the charged currents. The couplings Z_μ and A_μ to their respective currents follows after some algebra. We consider the B_μ and the A^3_μ terms in (7.4).

Making the substitutions

$$B_\mu = \cos\theta_W \, A_\mu + \sin\theta_W \, Z_\mu$$

$$A^3_\mu = -\sin\theta_W \, A_\mu + \cos\theta_W \, Z_\mu$$

$$g' = g \tan\theta_W$$

we may rewrite the neutral gauge boson couplings as

$$\mathcal{L}_{NC} = g' \overline{\Psi}_R \gamma^\mu \frac{Y}{2} \Psi_R + \overline{\Psi}_L \gamma^\mu \left(g' \frac{Y}{2} B_\mu + g \tau_3 A^3_\mu \right) \Psi_L$$

$$= gs \left[\overline{\Psi}_R \gamma^\mu \frac{Y}{2} \Psi_R + \overline{\Psi}_L \gamma^\mu \left(\frac{Y}{2} + \tau_3 \right) \Psi_L \right] A_\mu$$

$$+ g/_c \left[-s^2 \left(\overline{\Psi}_R \gamma^\mu \frac{Y}{2} \Psi_R + \overline{\Psi}_L \gamma^\mu \frac{Y}{2} \Psi_L \right) + c^2 \overline{\Psi}_L \gamma^\mu \tau_3 \Psi_L \right] Z_\mu \qquad (7.6)$$

with $(c, s) = (\cos\theta_W, \sin\theta_W)$.
The hypercharge $Y/_2$ can be replaced according to (5.7) by

$$\overline{\Psi}_R \, Y/_2 \, \gamma^\mu \Psi_R = \overline{\Psi}_R Q \gamma^\mu \Psi_R$$

$$\overline{\Psi}_L \frac{Y}{2} \gamma^\mu \Psi_L = \overline{\Psi}_L (Q - \tau_3) \gamma^\mu \Psi_L$$

giving at the end

$$\mathcal{L}_{NC} = gs \left[\overline{\Psi}_R \gamma^\mu Q \Psi_R + \overline{\Psi}_L \gamma^\mu Q \Psi_L \right] A_\mu$$

$$+ \frac{g}{c} \left[\overline{\Psi}_L \gamma^\mu \tau_3 \Psi_L - s^2 \left(\overline{\Psi}_L \gamma^\mu Q \Psi_L + \overline{\Psi}_R \gamma^\mu Q \Psi_R \right) \right] Z_\mu \ . \qquad (7.7)$$

Finally, substituting for $\Psi_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$ and $\Psi_R = e_R$, we obtain

$$\mathcal{L} = i \bar{e} \not{\partial} e + i \bar{\nu}_L \not{\partial} \nu_L - gs \, A^\mu J^{em}_\mu + \frac{g}{2\sqrt{2}} \left[\bar{\nu} \gamma^\mu (1-\gamma_5) e W^+_\mu + \bar{e} \gamma^\mu (1-\gamma_5) \nu W^-_\mu \right]$$

$$- \frac{g}{2\sqrt{2} c} \left[\frac{1}{2} \bar{\nu} \gamma^\mu (1-\gamma_5) \nu + \frac{1}{2} \bar{e} \gamma^\mu \gamma_5 e - \frac{1}{2} \left(1 - 4 s^2 \right) \bar{e} \gamma^\mu e \right] Z_\mu \qquad (7.8)$$

We have a gauge theory mediated by four gauge mesons. One of them is massless and its coupling is the usual electromagnetic coupling, which requires that we identify the electromagnetic charge by

$$e = g \sin\theta_W \qquad (7.9)$$

There are charge currents mediated by the $W^{\pm}$ bosons whose mass is

$$M_W = \frac{1}{2} g v \quad . \qquad (7.10)$$

The low energy data determine the Fermi coupling constant which must be indentified as

$$G_F/_{\sqrt{2}} = \frac{g^2}{8 M_W^2} \qquad (7.11)$$

The electroweak theory goes beyond the (V-A)-theory and predicts the existence of neutral currents mediated by the Z-Vector meson whose mass is

$$M_Z = \frac{1}{2} g v \frac{1}{\cos\theta_W} \qquad (7.12)$$

Thus the ratio $$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} \tag{7.13}$$

is predicted to be 1. For this follows a general feature that the overall strength of the neutral current is determined

$$\mathcal{L}_{NC} = \frac{g}{\cos\theta_W}\left[\frac{1}{2}\bar{\nu}_L \gamma_\mu \nu_L + \frac{1}{2}\bar{e}\gamma_\mu\gamma_5 e - \frac{1}{2}\left(1-4\sin^2\theta_W\right) J_\mu^{em}\right] Z^\mu \tag{7.14}$$

The lagrangian in (7.7) and (7.8) describes the weak and electromagnetic couplings of electrons and neutrinos. It constrains four unknown constants: g, $\sin^2\theta_W$ and, indirectly, the masses M_W and M_Z, which will occur in the propagators of the particles W and Z. We can use three experimental measurements to determine all of them. Electromagnetic measuments determine the fine structure constant

$$\alpha = \frac{e^2}{4\pi} = 1/137.036 \tag{7.15}$$

The muon decay rate is used to determine G_F as in (7.11). Neutral current measurements, discussed in chapters 10 and 13, are used to determine $\sin^2\theta_W = 0.22 \pm 0.01$. With these three low energy measurements we determine the other parameters

$$M_W^2 = \frac{\pi\alpha}{\sqrt{2}\, G_F \sin^2\theta_W} \tag{7.16}$$

$$M_Z^2 = \frac{M_W^2}{\cos^2\theta_W} \tag{7.17}$$

and $$v = \frac{e}{2\sqrt{2}\, G_F \sin\theta_W} \tag{7.18}$$

The Glashow-Salam-Weinberg theory makes striking new predictions

i) It foresees the existence of the neutral currents and their explicit couplings in terms of a single parameter the Weinberg angle $\sin^2\theta_W$.

ii) It foresees the existence of heavy intermediate gauge bosons, whose masses satisfy the relations (7.16) and (7.17) and therefore predicted by low energy data. In addition it predicts the coupling of W's and Z's to other particles.

iii) It is based on the existence of fundamental pseudoscalar particles, the Higgses.

These predictions lead to a rich phenomenology. Many predictions are already confirmed by experiments and others will be tested in the future.

7(b). <u>Feynman Rules.</u> The quantization of the theory follows the method of <u>Feynman Path</u> Integrals and will not be discussed here. We list instead the Feynman rules for the R_ξ-gauges. [3] In chapter six the three Higgs mesons were eliminated by selecting a special gauge: the unitary gauge. In explicit calculations it is convenient to use the covariant gauge, where the unphysical particles also appear. The physical scalar bosons (Higgs mesons) are denoted by ψ, as opposed to the unphysical scalars denoted by $\phi^\pm$ and χ. The Faddeev-Popov ghosts are denoted by $c^\pm$, c_Z and c_A.

The massive gauge boson propagator is

$$\Delta^{\mu\nu}(p) \;=\; -i\,\frac{g^{\mu\nu} + (\xi - 1)\,\frac{p^\mu p^\nu}{p^2 - \xi M^2}}{p^2 - M^2}$$

For special gauges

$\xi = 1$: $\quad \Delta^{\mu\nu} = \dfrac{-ig^{\mu\nu}}{p^2 - M^2}$ (Feynman Gauge)

$\xi = 0$: $\quad \Delta^{\mu\nu} = \dfrac{-i(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2})}{p^2 - M^2}$ (Landau Gauge)

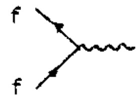
$\xi \to \infty$ $\quad \Delta^{\mu\nu} = \dfrac{-i(g^{\mu\nu} - \frac{p^\mu p^\nu}{M^2})}{p^2 - M^2}$ (Unitary Gauge)

In the Feynman gauge ($\xi = 1$) other propagators are

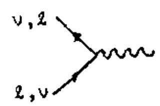
A ∿∿∿ A : $\dfrac{-ig^{\mu\nu}}{p^2}$	Z ∿∿∿ Z : $\dfrac{-ig^{\mu\nu}}{p^2 - M_Z^2}$	W ∿∿∿ W : $\dfrac{-ig_{\mu\nu}}{p^2 - M_W^2}$
ψ - - - - - ψ : $\dfrac{i}{p^2 - M_H^2}$	χ - - - - - χ : $\dfrac{i}{p^2 - M_Z^2}$	ϕ - - - - - ϕ : $\dfrac{i}{p^2 - M_W^2}$
c_A ---→--- c_A : $\dfrac{i}{p^2}$	c_Z ---→--- c_Z : $\dfrac{i}{p^2 - M_Z^2}$	$c_\pm$ ---→--- $c_\pm$: $\dfrac{i}{p^2 - M_W^2}$

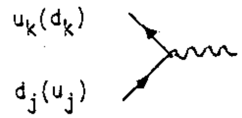
f ——→—— f $\quad = \quad \dfrac{i}{\not p - m}$

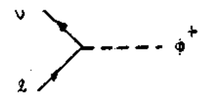
Fermion Vertices:

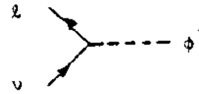

$A_\mu = -ieQ\,\gamma_\mu$

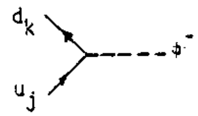

$Z_\mu = \dfrac{ig}{\cos\theta}\,\gamma_\mu(\tfrac{1}{2}I_3\gamma_- - Qs^2)$

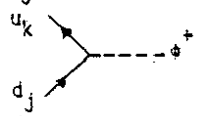

$W_\mu = \dfrac{ig}{2\sqrt{2}}\gamma_\mu\gamma_-$

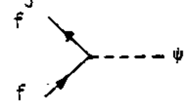

$W_\mu = \dfrac{ig}{2\sqrt{2}}U_{kj}^{(+)}\gamma_\mu\gamma_-$


$\phi^+ = \dfrac{-ig}{2\sqrt{2}}\dfrac{m_\ell}{M_W}\gamma_+$

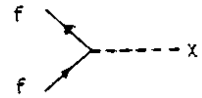

$\phi^- = \dfrac{-ig}{2\sqrt{2}}\dfrac{m_\ell}{M_W}\gamma_-$


$\phi^- = \dfrac{ig}{2\sqrt{2}}U_{kj}^{+}\left(\dfrac{m^j}{M_W}\gamma_+ - \dfrac{\tilde m^k}{M_W}\gamma_-\right)$


$\phi^+ = \dfrac{-ig}{2\sqrt{2}}U_{kj}\left(\dfrac{\tilde m^j}{M_W}\gamma_+ - \dfrac{m^k}{M_W}\gamma_-\right)$

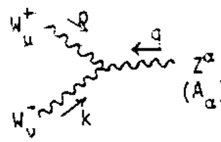

$\psi = \dfrac{-ig}{2}\dfrac{m}{M_W}$

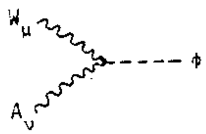
only for charged fermions

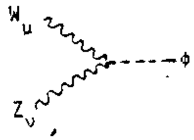

$\chi = \overset{+}{(-)}\dfrac{g}{2}\dfrac{m}{M_W}\gamma_5$

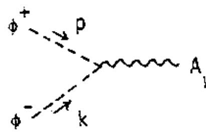
only for charged fermions
+ for leptons,
- for up quarks.

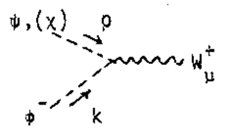
Meson Vertices:


$= -ig\cos\theta\;(ie)\;\{g^{\mu\nu}(p-k)^\alpha + g^{\nu\alpha}(k-q)^\mu + g^{\alpha\mu}(q-p)^\nu\}$

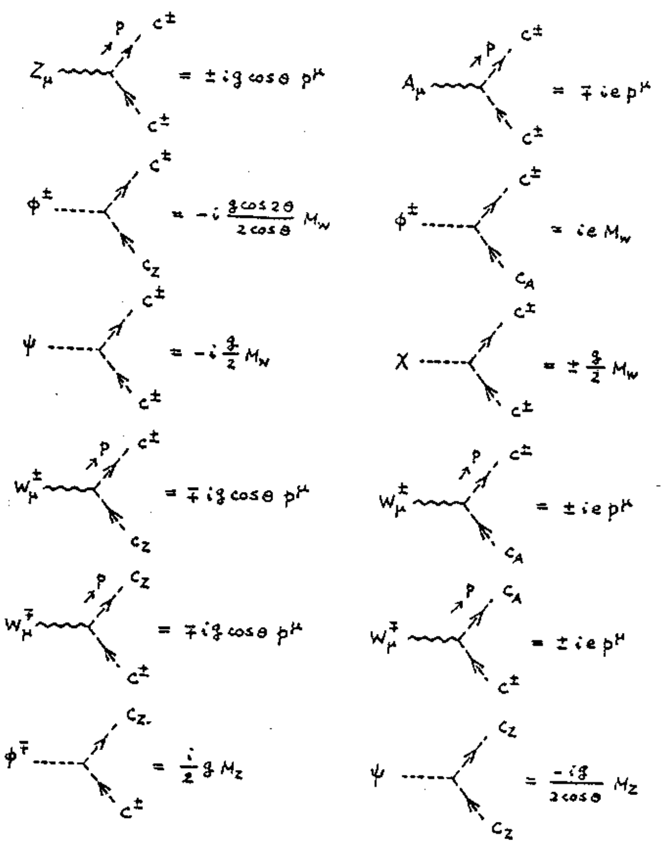

$= -ieM_W g^{\mu\nu}$


$= \dfrac{-ig\sin^2\theta}{\cos\theta}M_W g^{\mu\nu}$


$= -ie(p-k)_\mu$


$= i\,\dfrac{g}{(-1)2}(p-k)_\mu$

Faddeev-Popov Vertices:



References Ch. 5, 6 and 7

1. S.L. Glashow, Nucl. Phys. 22, 579 (1961)
   S. Weinberg, Phys. Rev. Lett. 19 (1967) 264
   A. Salam, in Elementary Particle Physics ed. by N. Svartholm (Stockholm 1967), p. 367

2. P.W. Higgs, Phys. Rev. 145, 1156 (1966)

3. Derivation of the rules and/or lists of them occur in the books mentioned in the introduction and in additional articles:
   G. 't Hooft and M.T. Veltman, Diagrammar CERN-preprint 73-9 (1973)
   B.W. Lee, Phys. Rev. D5, 823 (1972)
   P. Ramond, Field Theory, A Modern Primer
   The Benjamin/Cummings Publishing Comp., (1981)
   K. Fujikawa, B.W. Lee and A.I. Sanda, Phys.Rev. D6, 2923 (1972).

8. Incorporating the Hadrons

For quarks the situation is more complicated, since we must deal with many more fields: the massive quarks u, d, c, s, (t), b. The top quark is in parentheses since it has not been observed yet. When the Lagrangian is introduced for the hadronic sector, we group the quarks into left handed doublets

$$q^1_{oL} = \begin{bmatrix} u_o \\ d_o \end{bmatrix}_L, \quad q^2_{oL} = \begin{bmatrix} c_o \\ s_o \end{bmatrix}_L \quad \text{and} \quad q^3_{oL} = \begin{bmatrix} t_o \\ b_o \end{bmatrix}_L \tag{8.1}$$

and right-handed singlets $u_{oR}, d_{oR}, s_{oR}, c_{oR}, t_{oR}, b_{oR}$.

We assign to each quark a weak isospin and hypercharge quantum number defined by $Y/2 = Q-I_3$. It is emphasized that the weak isospin and hypercharge are different from the corresponding quantum numbers frequently used in strong interactions. The fermion part of the lagrangian is

$$\mathcal{L} = i \sum_i \bar{q}^i_{oR} \left(\partial_\mu + i g' Y/2\, B_\mu \right) \gamma^\mu q^i_{oR}$$

$$+ i \sum_i \bar{q}^i_{oL} \left(\partial_\mu + i g' \tfrac{Y}{2} B_\mu + i g \vec{\tau} \cdot \vec{A}_\mu \right) \gamma^\mu q^i_{oL} \tag{8.2}$$

At this stage there is no reference to physical states, since all quarks are massless and consequently any superposition of states will do for our considerations. This property is emphasized by the subscript o. The physical quarks are defined as eigenstates of the fermion mass matrix and will be written without subscripts.

Masses for the quarks are introduced by considering again the Higgs fields

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix} \quad \text{and} \quad \tilde{\phi} = i \sigma_2 \phi^* = \begin{pmatrix} \phi^o \\ -\phi^- \end{pmatrix} \tag{8.3}$$

Both of these fields transform like doublets under SU(2). A general Yukawa coupling invariant under SU(2)xU(1) was given in (6.5). We adopt Yukawa couplings for the quark fields

$$\mathcal{L}_Y = G_1 \left[\bar{q}^1_{oL} \tilde{\phi} u_R + h.c. \right] + G_2 \left[\bar{q}^1_{oL} \tilde{\phi} d_R + h.c. \right] + \ldots \tag{8.4}$$

Quark masses are now obtained by replacing ϕ by its vacuum expectation value:

$$\langle \phi^+ \rangle = 0 \quad \text{and} \quad \langle \phi^o \rangle = \frac{v}{2} \tag{8.5}$$

This is analogous to the procedure followed in order to generate masses for the gauge bosons.

The constants $G_1, G_2, \ldots$ are arbitrary and are adjusted to produce the masses of the quarks. In the past five years a great deal of effort was invested in order to obtain relations between the G_i's and [1] consequently relations between the quark states, but the complete solution to the fermion mass problem is still not available.

The same procedure is followed in order to develop masses for the leptons. But the absence of right-handed neutrino states leaves all neutrinos massles. This introducesgreater freedom in reactions with neutrinos since any superposition of neutrino wave functions is an admissible state. In contrast the physical quark states have different masses and a distinction of the flavor quantum numbers (strangeness, charm, ...) is meaningful.

The general mass matrix has the form

$$\mathcal{M} = \sum_{i,j} G_{ij}\, \bar{q}^i_{oL}\, v\, q^j_{oR} + h.c. \tag{8.6}$$

It generates a mass matrix for the upper quarks and a different matrix for the down quarks. We denote them by $\mathcal{M}^u$ and $\mathcal{M}^d$, respectively. These matrices are neither unitary nor hermitian, but can always be brought into diagonal form by two unitary matrices one acting to the left and the other to the right, i.e.

$$U_L \mathcal{M}^u U_R = D^u \quad \text{and}$$

$$V_L \mathcal{M}^d V_R = D^d \tag{8.7}$$

The elements of the diagonal matrices D^u and D^d are the quark masses. In general some of the eigenvalues are negative but they are still admissible as masses, because the minus sign can be eleminated by a γ_5-transformation of the fields

$$\Psi_{\substack{L\\R}} \longrightarrow \Psi'_{\substack{L\\R}} = \gamma_5 \Psi_{\substack{L\\R}} = \mp \Psi_{\substack{L\\R}} \tag{8.8}$$

The columns of the matrices U_L, V_L and U_R, V_R are eigenfunctions of the mass matrices operating on the left and

on the right. They transform the q^i_o quarks into the physical states. Define column matrices for the up and down quarks

$$q^u_o = \begin{bmatrix} u_o \\ c_o \\ t_o \end{bmatrix} \quad \text{and} \quad q^d_o = \begin{bmatrix} d_o \\ s_o \\ b_o \end{bmatrix} \tag{8.9}$$

and the physical states without subscripts. It now follows

$$q^u_L = U_L\, q^u_{oL} \quad \text{and} \quad q^d_L = V_L\, q^d_{oL} \tag{8.10}$$

The charge currents in (8.2) involve the original quarks q^i_o. When they are rewritten in terms of the physical states they involve the matrix $U_c = U_L V_L$, which mixes quarks of different flavors. Explicitly

$$J^+_\mu = \bar{q}^u_o \gamma_\mu (1-\gamma_5)\, q^d_o = \bar{q}^u \gamma_\mu (1-\gamma_5)\, U_c\, q^d \tag{8.11}$$

Thus the down quarks are coupled to the upper quarks through a unitary matrix, U_c. For the case of four quarks

$$U_c = \begin{bmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{bmatrix} \tag{8.12}$$

which produces the currents

$$J^+_\mu = \bar{u}\, \gamma_\mu (1-\gamma_5)\left[d\cos\theta_c + s\sin\theta_c \right] + \bar{c}\, \gamma_\mu (1-\gamma_5)\left[-d\sin\theta_c + s\cos\theta_c \right] \tag{8.13}$$

The first term is the Cabibbo current. The second term is a new charged current which couples down and strange quarks to the charm quark. The charged currents change flavors through the matrix (8.12), known as the GIM-matrix.[2] The angle θ_c is the Cabibbo angle and it also determines the charm quark couplings.

Next we determine the structure of the neutral currents. We begin with the lagrangian (8.2) and repeat the arguments from (7.4) to (7.7). The couplings of the quarks to neutral gauge boson is

$$\mathcal{L}_{NC} = e \sum_{i=1}^{3} \bar{q}^i Q \gamma^\mu q^i A_\mu + \frac{g}{c} \sum_{i=1}^{3} \left\{ \bar{q}_L^i \tau_3 \gamma^\mu q_L^i - s^2 \bar{q}^i Q \gamma^\mu q^i \right\} Z_\mu \tag{8.14}$$

At first sight the quarks occuring in (8.14) should have a subscript zero, since they are the lagrangian quarks occuring in (8.2). But since (8.14) is diagonal in the quark states, the unitary matrices $U_{L,R}$ and $V_{L,R}$ disappear and the omission of the subscript is justified. We introduce a convenient notation and write the neutral current couplings in the form

$$\mathcal{L}_{NC} = \frac{g}{2\sqrt{2}\,c} \left\{ \bar{q}^\alpha \gamma^\mu \left(g_V^\alpha - g_A^\alpha \gamma_5 \right) q^\alpha \right\} Z_\mu \tag{8.15}$$

with q^α standing for generic states of up or down quarks.

The couplings are given in Table (8.I). In the table

States	g_V	g_A
Up Quarks	$1/2 - 4/3 \sin^2\theta_W$	1/2
Down Quarks	$-1/2 + 2/3 \sin^2\theta_W$	$-1/2$
Neutrinos	1/2	1/2
Chargedleptons	$-1/2 + 2 \sin^2\theta_W$	$-1/2$

Table 8.1. Couplings of Quarks and Leptons to Z°.

we also include the couplings to neutrinos and charged leptons when $q^\alpha = \nu$ or e.

The careful student noted that the classification of quarks into doublets and singlets in (8.1) is indeed arbitrary. With so many quark fields it is possible to assign them to higher representations of the group. In fact this alternative was pursued actively for quite some time. An important criterion for the representation assignment of the fermion fields follows from the very restrictive bounds that exist on strangeness-changing and charm-changing neutral couplings.

In order to describe the bounds on flavor changing neutral couplings quantitatively, we introduce an effective interaction

$$\mathcal{L}_{eff.} = \frac{G}{\sqrt{2}} \Big\{ \bar{s}\, \gamma_\mu (h_V^s + h_A^s \gamma_5)\, d\ \bar{s}\, \gamma^\mu (h_V^s + h_A^s \gamma_5)\, d\ +$$

$$\bar{c}\, \gamma_\mu (h_V^c + h_A^c \gamma_5)\, u\ \bar{c}\, \gamma^\mu (h_V^c + h_A^c \gamma_5)\, u \Big\} \qquad (8.16)$$

where h_V^i and h_A^i are phenomenological constants to be determined by experiment. We note that the strength of the interaction is $G/\sqrt{2}$. Thus if flavor changing neutral couplings occur at the tree level we expect the $h_{V,A}^i$'s to be of order one. Experimentally they will produce a mass difference for the eigenstates of the K°-$\overline{K}^\circ$ system through the diagram in fig.(8.1).The observed mass difference of the physical states K_L° and K_S° give the bounds for $h_V^{\bar{s}d}$ and $h_A^{\bar{s}d}$ in Table II.

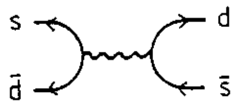


Fig. (8.1)

The bounds on the charm-changing couplings are derived from observations on the mixing properties of the D°-$\overline{D}^\circ$ system.

Consider the situation where a state D^o is produced at the initial time. As time goes on the state evolves into a superposition of $\bar{D}^o$ and D^o, the former contributing to abnormal decays. The net effect is that over long periods of time there are normal decays into

$$\ell^+ + \nu + \cdots \quad \text{or} \quad K^- + \ldots$$

and abnormal decays into

$$\ell^- + \bar{\nu} + \ldots \quad \text{or} \quad K^+ + \ldots \quad .$$

Denoting by N^- the number of abnormal decays into a specific channel or sum of channels and by N^+ the normal decays into the corresponding channels, it was found [3] that

$$\rho_o \equiv \frac{N^-}{N^- + N^+} = \frac{1}{2}\ \frac{4(\Delta m)^2 + (\Gamma_L - \Gamma_S)^2}{4(\Delta m)^2 + (\Gamma_L + \Gamma_S)^2} \qquad (8.17)$$

where Δm is the mass difference between D_L-D_S and Γ_L, Γ_S are the respective widths. Similarly, consider the production of a $D\bar{D}^o$ pair in the reaction

$$e + e^- \longrightarrow D^o + \bar{D}^o \qquad (8.18)$$

and study its time development. In the one-photon approximation the produced $D^o\bar{D}^o$ state has C = -1 and produces normal decays into $(K^+K^-h_1)$, as well as abnormal decays into $(K^+K^+h_2)$ and $K^-K^-h_3)$ where h_1, h_2, h_3 are final states with total strangeness zero. Denoting the number of decays into each system by $N^{-+} + N^{+-}$, N^{++} and N^{--}, respectively it was shown [3] that

$$\rho_1 \equiv \frac{N^{++} + N^{--}}{N^{+-} + N^{-+} + N^{++} + N^{--}} = \rho_o \qquad (8.19)$$

ρ_o being the same expression of masses and widths that occurs in (8.17). Relations (8.17) and (8.19) are general and they hold for other pairs of particles containing heavier quarks, like $B_d^o - \bar{B}_d^o$, $B_s^o - \bar{B}_s^o$, ... If an experimental upper bound ρ_1 is known, then it follows

$$\Delta m \leq \left(\frac{2\rho_1}{1-2\rho_1}\right)^{1/2} \frac{(\Gamma_L + \Gamma_S)}{2} \quad . \qquad (8.20)$$

Estimates of the widths give

$$\Gamma_{11} = \frac{1}{2}(\Gamma_L + \Gamma_S) = \frac{G^2 M_D^5}{192\pi^3} g(\varepsilon) \qquad (8.21)$$

where $\epsilon = M_s/M_c$ and

$$g(\varepsilon) = 1 - 8\varepsilon^2 - 24\varepsilon^4 \ln\varepsilon + 8\varepsilon^6 - \varepsilon^8 \quad .$$

The mass difference is estimated from the effective Lagrangian as

$$M_{12} = \frac{G}{\sqrt{2}} (h_V^2 \text{ or } h_A^2)\ 2\ f_D^2 m_D \qquad (8.22)$$

where h_V and h_A are assumed to be comparable and the reduced matrix element is approximated by

$$\langle D^0 | [\bar{c}\gamma_\mu (1-\gamma_5) u]^2 | \bar{D}^0 \rangle = 4 \frac{(f_D M_D)^2}{2M_D} \quad . \qquad (8.23)$$

For the D-decay coupling constant there are numerous estimates, using different approaches; a realistic value is f_D = 300 MeV. The above results together with the experimental bound [4]

$$\rho_1 \lesssim 5\ \% \quad \text{at} \quad 90\ \%\ \text{c.l.} \qquad (8.24)$$

give the stringent bound [5] occuring in Table II. The estimate of the reduced matrix element can be improved by a calculation in the bag model [6] ; however, the above estimate is conservative and suffices for these lectures. The reason that the bound on h_V or h_A

is so restrictive becomes evident once it is realized that M_{12} is first-order weak while Γ_{11} is second-order weak.

It is of interest to determine experimental bounds for the mixing of flavor for the heavier quark systems like $\bar{b}d$, $\bar{b}s$, $\bar{t}u$, $\bar{t}c$ ... Some of them will be forthcoming in e^+e^- experiments and I will return to this topic later on.

In gauge theories the large suppressions in Table (8.II) are incorporated by demanding that

i) Direct flavor-changing neutral couplings are suppressed to this level of accuracy. This is satisfied by constructing gauge theories where direct flavor-changing neutral couplings are absent at the tree level.

ii) Even with the above choice flavor changing effects can be induced by higher-order corrections that involve charged currents Higher order effects are of $O(G\alpha)$ and contradict the two entries in Table I. Such effects are acceptable if they occur at the level $G\alpha(\frac{m_q}{M_W})^2$.

iii) If the symmetry breaking is through Higgs' mechanism, then the resulting neutral couplings should preserve flavor to the accuracy of Table (8.II).

Couplings between quark pairs	Bound on h or h_V
$\bar{s}d$	2.5×10^{-4}
$\bar{c}u$	6.8×10^{-4}
$\bar{b}d$, $\bar{b}s$, $\bar{t}u$	?

Table (8.II)

For a gauge theory based on the group SU(2)xU(1), the bounds are satisfied if we classify [7] the quarks into representations of the group in such a way that quarks of the same charge and the same helicity have the same T (total weak isospin) and T_3 (third component of isospin).

<u>Corollary.</u> For quarks with only two charges (2/3, -1/3), there must be equal numbers of up and down quarks. Let us denote by n the number of quarks of a given charge. We can write the charged currents in terms of the matrix

$$\Psi_L = \begin{bmatrix} u \\ c \\ t \\ \vdots \\ d \\ s \\ b \end{bmatrix} \begin{matrix} \uparrow \\ n \\ \downarrow \\ \uparrow \\ n \\ \downarrow \end{matrix} \qquad \text{and the Pauli matrix} \qquad \tau^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \tag{8.23}$$

as follows

$$J_\mu^{cc}(x) = (\bar u\, \bar c\, \bar t \ldots \bar d\, \bar s\, \bar b \ldots)_L \gamma_\mu \left[\begin{array}{c|c} 0 & M \\ \hline 0 & 0 \end{array}\right] \begin{bmatrix} u \\ c \\ t \\ \vdots \\ d \\ s \\ b \\ \vdots \end{bmatrix}_L$$

$$= \bar\Psi_L \gamma_\mu \tau^+ \otimes M \Psi_L \tag{8.24}$$

with M an nxn unitary matrix.

We illustrate the implications of the result with some examples.

<u>Example 1.</u> Models with three quarks are not allowed, since they will produce strangeness changing neutral currents. To solve this problem Glashow, Iliopoulos and Maiani [2] introduced a charmed quark. The matrix M is

$$M = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{8.25}$$

and the charged current

$$J_\mu^+ = (\bar u\, \bar c\, \bar d\, \bar s) \begin{bmatrix} 0 & M \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ c \\ d \\ s \end{bmatrix} \tag{8.26}$$

<u>Example 2.</u> Five quarks u, c, d, s, b have been observed up to date. They will produce flavor changing neutral couplings. To eliminate these couplings the sixth (top)-quark is introduced. The charged current is now given by a 3 x 3 unitary matrix which involves the Euler angles and an additional phase, δ . It was introduced by Kobayashi and Maskawa,[8] who emphasized that the phase introduces CP-violation in the theory. The standard notation is

$$\begin{bmatrix} d_\theta \\ s_\theta \\ b_\theta \end{bmatrix} = \begin{bmatrix} c_1 & s_1 s_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 + s_2 s_3 e^{i\delta} & c_1 c_2 s_3 - s_2 c_3 e^{i\delta} \\ -s_1 s_2 & c_1 s_2 c_3 - c_2 s_3 e^{i\delta} & c_1 s_2 s_3 + c_2 c_3 e^{i\delta} \end{bmatrix} \cdot \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

$$= \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{tb} & V_{ts} & V_{tb} \end{bmatrix} \cdot \begin{bmatrix} d \\ s \\ b \end{bmatrix} \tag{8.27}$$

In closing, we give the quark multiplets

$$\begin{pmatrix} u \\ d_\theta \end{pmatrix}_L , \begin{pmatrix} c \\ s_\theta \end{pmatrix}_L , \begin{pmatrix} t \\ b_\theta \end{pmatrix}_L \quad \text{and} \quad u_R, c_R, \ldots, b_R \tag{8.28}$$

and the lepton multiplets

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L , \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L , \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \quad \text{and} \quad e_R, \mu_R, \tau_R \ . \tag{8.29}$$

We adopted an elegant solution to account for the suppression of flavor changing neutral couplings[9]. We did not discuss yet the couplings of the Higgs particles which we take up in the next chapters. We already introduced a new problem: the determination of the three angles θ_1, θ_2 and θ_3 and the phase δ that occur in the KM-matrix. The rich phenomenology of the electroweak theory is studied in the next chapters.

References Ch. 8

1. S. Weinberg, in Transactions of the New York Academy of Sciences. Series II. Vol. 38 (1977).

   H. Fritzsch, Phys. Lett. 73B, 317 (1978)

   T. Kitazoe and K. Tanaka, Phys. Rev. D18, 3476 (1978)

   H. Georgi and D.V. Nanopoulos, Nucl. Phys. B155, 52 (1979)

2. S. Glashow, I. Iliopoulos and L. Maiani, Phys. Rev. D2, 1285 (1970)

3. A. Pais and S.B. Treiman, Phys. Rev. D12, 2744 (1975)

   L.B. Okun, V.I. Zakharov and B.M. Pontecorvo, Nuovo Cim. Lett. 13, 218 (1975)

4. G. Feldman et.al., Phys. Rev. Lett. 38, 1313 (1977)

   G. Goldhaber et.al. Phys. Lett. 69B, 503,(1977)

   A. Barbaro-Galtieri (Private Communication)

5. E.A. Paschos, Phys. Rev. Lett. 39, 858 (1977)

   For up dated bounds see:

   R.N. Cahn and H. Harari, Nucl. Phys. B176, 135 (1980)

   I. Montray, Z. Phys. C7, 45 (1980)

   G.L. Kane and R. Thun, Phys. Lett. 94B, 513 (1980)

6. R.E. Shrock and S.B. Treiman, Phys. Rev. D19, 2148 (1979)

   S.K. Bose and E.A. Paschos, Nucl. Phys. B169, 384 (1980)

7. S. Glashow and S. Weinberg, Phys. Rev. D15, 1958 (1977)

   E.A. Paschos, Phys. Rev. D15, 1966 (1977)

8. M. Kobayashi and K. Maskawa, Progr. Theor. Phys. 49, 652 (1973)

9. Constraints on Higgs Couplings are discussed by

   R. Gatto, G. Morchio and F. Strocchi, Phys. Rev. B74, 265 (1979); B 83, 348 (1979)

   G. Segré and H.A. Weldon, Ann. Phys. 124, 37 (1980)

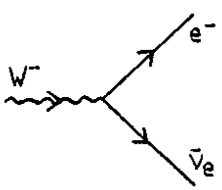
   F. Paige, E.A. Paschos, T.L. Trueman, Phys. Rev. D15, 3416 (1977)

PART III

9. Predictions for the Intermediate Vector Mesons

The masses of the $W^{\pm}$ and Z^{o} particles are determined by (7.16) and (7.17). These relations were obtained to lowest order. They are expected to be modified by a few percent by higher order corrections. We shall return to the precise determination of the parameters of the theory in chapter 17.

Decays of W's and Z's. A unique property of the gauge bosons is the absence of semileptonic decays. The decays of W's into $e\bar{\nu}$-pairs is described by the simple diagram in fig. (9.1). The matrix element is



$$\mathcal{M} = \frac{ig}{2\sqrt{2}} \varepsilon^{\mu} \bar{u}(e) \gamma_{\mu}(1-\gamma_5) v(\nu) \tag{9.1}$$

with ε^{μ} denoting the polarization of the W.

The decay width into a lepton pair is

$$\Gamma(W^- \to e\bar{\nu}) = \frac{GM_W^3}{\sqrt{2}\, 6\pi} \tag{9.2}$$

Fig. (9.1)

which is 211.3 Mev for $M_W = 80$ GeV/c^2 (235.9 MeV for $M_W = 83$ GeV/c^2). There are additional decays into $\mu\bar{\nu}_{\mu}$, $\tau\bar{\nu}_{\tau}$ and quark pairs. The total width is obtained by multiplying the leptonic width by 12: (3 lepton families) + (3 quark families) x (3colors) = 12.

$$\Gamma_{total} = \frac{2GM_W^3}{\sqrt{2}\,\pi} = 2.535 \quad \text{for } M_W = 80 \text{ GeV.} \tag{9.3}$$

The leptonic decays produce a charged lepton and an undetected neutrino. The hadronic decays can be detected through reconstruction of hadronic jets, which imitate some of the kinematic characteristics of the original quarks.

By comparison the leptonic decays of Z^{o} have the unique signature of producing charged lepton pairs with a given invariant mass. The decay into a $\nu\bar{\nu}$-pair gives the partial width

$$\Gamma(Z^{o} \to \nu\bar{\nu}) = \frac{GM_Z^3}{12\sqrt{2}\,\pi} \tag{9.4}$$

The sum of all decays modes gives

$$\Gamma_{tot}(Z) = \frac{GM_Z^3}{3\sqrt{2}\,\pi} \left\{3 - 6s^2 + 8s^4\right\} \tag{9.5}$$

$$\approx 2.5 \text{ GeV} \quad \text{for} \quad M_Z = 92 \text{ GeV}$$

$$s^2 = 0.225$$

The branching ratio into μ-pairs is

$$B(Z \to \mu^+\mu^-) = \frac{0.12}{N} \tag{9.6}$$

for N generations of fermions. In the standard model with three generations the branching ratio into muons is $\simeq 4$ %. It is worth noting that $\Gamma_{tot}(Z)$ is sensitive to the total number of neutrinos and could produce some exotic surprises.

Searches for Z's and W's. According to present plans the Z^{o} is expected to be detected first in hadronic collisions. Cross sections and counting rates were estimated by many groups [1,2,3] for the CERN $\bar{p}p$ collider, the Fermilab TEVATRON and at ISABELLE in Brookhaven. At this time, the CERN $\bar{p}p$-collider is operating with a luminosity $10^{+28}\ cm^{-2}sec^{-1}$ and it is expected that first results for Z^{o} searches will be available in 1983.

The production cross section for the reactions

$$p + p \rightarrow Z^\circ + X \quad (9.7)$$
$$\hookrightarrow \mu^+\mu^-$$

$$\bar{p} + p \rightarrow Z^\circ + X \quad (9.8)$$

was calculated using the quark-parton model and should be rather accurate. Fig. (9.2) shows the differential cross section $d\sigma/dm_{\ell^+\ell^-}$ for the reaction (9.7) and a center-of-mass energy $\sqrt{s}$ = 400 GeV.

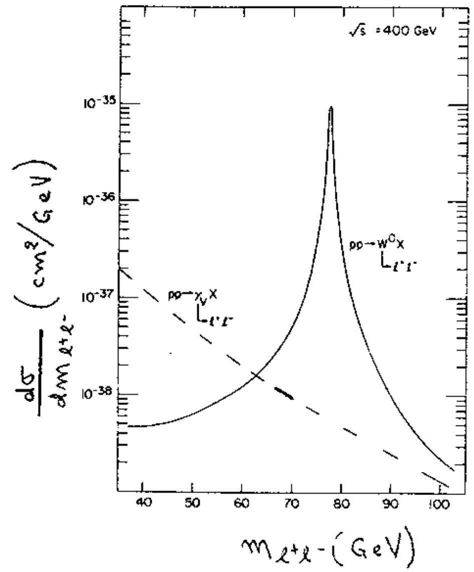


Fig. (9.2). Production cross section for Z° 's (ref. 2) .

The dashed curve in the figure is the background from direct Drell-Yan pairs. In $\bar{p}$-p reactions there is an enhancement of the cross section by a factor of 3 to 5.

The production of $W^\pm$ 'S was also estimated in the same model or with the help of scaling laws. Its detection is more difficult because it measures only the momentum of the muon. To detect $W^\pm$'s one measures the distribution of events as a function of P_T, the momentum of the muon normal to the beam direction.

Fig. (9.3) shows a Monte Carlo simulation for the detection of single muons from $W^\pm \rightarrow \mu^\pm \nu$. The figure shows the counting rate as a function of P_T. The two simulations correspond to masses 77.8 and 79.5 GeV. They correspond to a large ISABELLE detector with 1000 hours of running at an energy of $\sqrt{s}$ = 700 GeV. and a luminosity $L = 2\times10^{31}\ cm^{-2}\ sec^{-1}$.

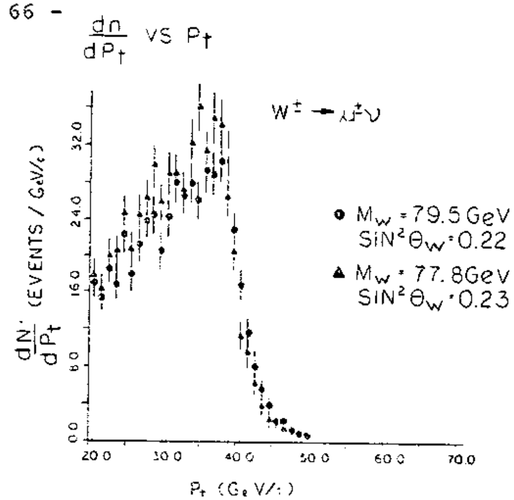


Fig. 9.3
Monte Carlo simulation of the detection of $W^\pm$'s (ref. 4).

It is very likely that the intermediate gauge bosons will be discovered in hadronic induced reactions. But the copious production of Z and detailed studies of its properties will occur in electron-positron rings, where one observes the direct formation of the Z's

$$e^+ + e^- \rightarrow Z^0 \rightarrow \text{Final states}$$

The final states include $\mu^+\mu^-$, hadronic jets, neutrinos or other states into which the Z^0 decays. The cross section for Z^0 formation and its subsequent decay in the final state F is

$$\sigma(s) = \frac{12\pi}{s} \frac{\Gamma(Z \rightarrow e^+e^-)\,\Gamma(\Gamma \rightarrow F)\, M_Z^2}{(s - M_Z^2)^2 + M_Z^2 \Gamma_{tot}^2} \quad (9.9)$$

It is expected that the beam resolution in this experiments tc be 100 MeV, much more narrow than the width of the Z. By choosing the beam energy $S = M_Z^2$ and observing in the final state μ-pairs, one measures $B\ (Z \to \mu^+\mu^-)$. The total width can be measured directly. Then by measuring each decay mode separately we can sum them up to obtain Γ-visible. The difference between $\Gamma_{tot} - \Gamma_{vis}$ gives the decay into unobserved channels, i.e. neutrinos. The formation cross section is huge as it is shown in fig. (9.5) in units of $\left(4\pi\alpha^2/3s\right)$. In these units the cross section at the peak is larger than the point cross section by a factor of 4000.

On the other hand, the production of $W^{\pm}$'s in electron-positron colliding rings requires still higher energies since they are produced in pairs. The production cross section is shown in fig. (9.5).

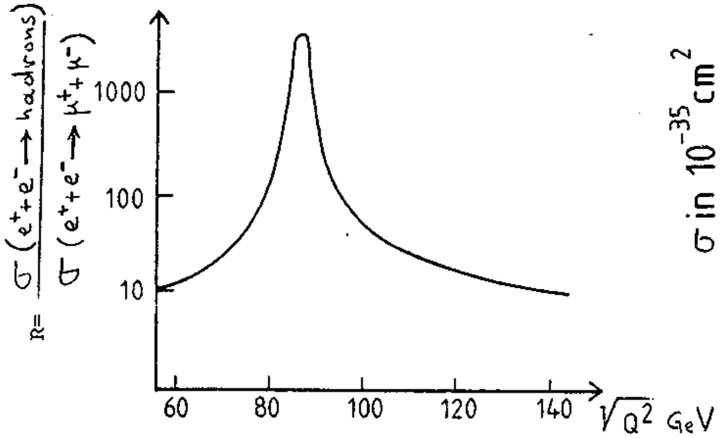


Fig. (9.4)

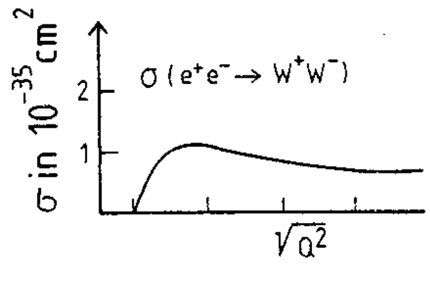


Fig. (9.5)

Finally, Z's are also produced in electron-hadron reactions.[6] Production diagrams are shown in fig. (9.6).

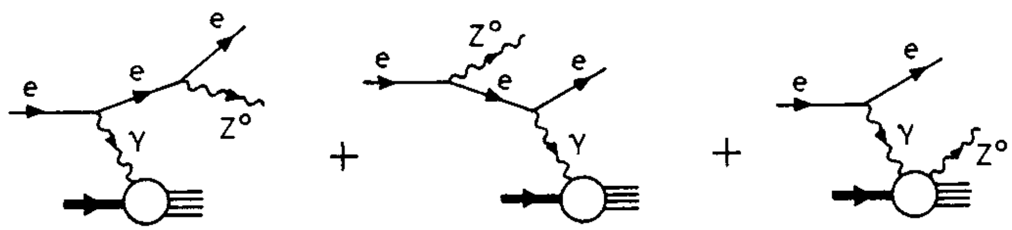


Fig. (9.6) Feynman diagrams contributing to $e + p \longrightarrow e + Z^o + X$.

The construction of an electron-proton colliding ring is considered at DESY. It is called HERA and will produce particles with the energies $E_e = 45$ GeV and $E_p = 280$ GeV for electrons and protons, respectively. The estimated cross sections for Z^o production are

$$\sigma_{tot} = 5\times10^{-37}\ \text{cm}^2 \quad \text{for } M_Z = 89 \text{ GeV and } s = 9.4\times10^4\ (\text{GeV})^2.$$

References Ch. 9

1. R.B. Palmer, E.A. Paschos, N.P. Samios and L.L. Wang, Phys. Rev. D14, 118 (1976); L.B. Okun and M.B. Voloshin, Nucl. Phys. B120, 459 (1977); J.B. Kogut and J. Shigemitsu, Nucl. Phys. B129, 461 (1977).

2. R.F. Peierls, T.L. Trueman and L.L. Wang, Phys. Rev. D16, 1397 (1977).

3. C. Quigg, Rev. Mod. Physics 49, 297 (1977); M. Abud, R. Gatto and C.A. Savoy, Ann. of Phys. 122, 219 (1979).

4. M. Chen et. al. Isabelle Workshop (Summer 1981) Vol. 2 pg. 448

5. Study of the electron-proton storage ring project Hera ECFA-report 80+42 (1980).

6. R.W. Brown et al. Phys. Rev. Lett. 33, 1119 (1974).

10. <u>NEUTRAL CURRENTS</u> (i) Leptonic Reactions.

The most striking evidence for the electroweak theory is the agreement with neutral current processes. In chapter 8 we introduce an effective neutral current interaction in Equ. (8.15). The coupling to quarks and leptons were given in Table (8.1). All the couplings involve a single parameter $\sin^2\theta_W$, the Weinberg angle. A second parameter is the strength of the neutral current interaction. In the standard model it is determined by equ. (7.11) and (7.13). At low energies, $q^2 \ll M_Z$, it is

$$\frac{g^2}{8 M_Z^2 \cos^2\theta_W} = -\frac{G}{\sqrt{2}} \qquad (10.1)$$

The simplest neutral current processes are those involving only leptons, since they are free of strong interaction complications. The first case to consider is neutrino-electron scattering. Several reactions are shown in fig. (10.1)

Neutral Current   Charged Current   Reaction

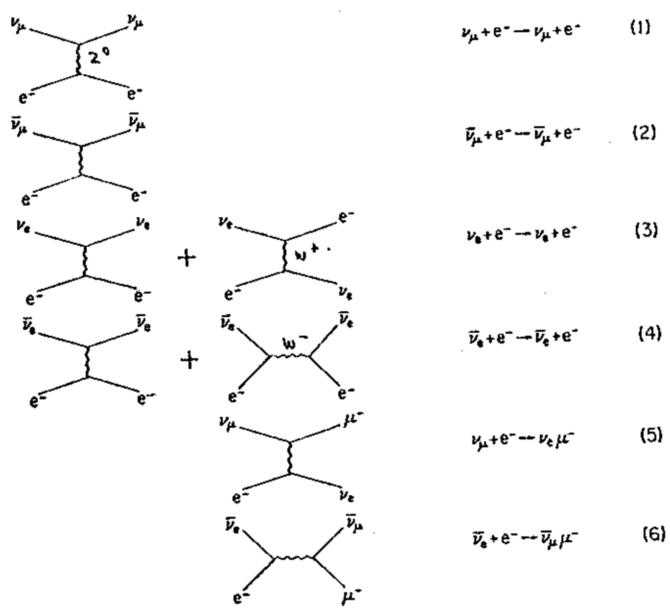


Figure (10.1)

It is emphasized that some of the reactions exist only in the presence of neutral currents. This is the case with

$$\nu_\mu + e^- \to \nu_\mu + e^- \qquad (10.2)$$

and

$$\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^- \qquad (10.3)$$

Other reactions like

$$\bar{\nu}_e + e^- \to \bar{\nu}_e + e^- , \qquad (10.4)$$

are modified by the neutral current diagrams. It is convenient to write the effective interaction for all reactions in the form

$$\mathcal{L}_{eff.} = \frac{g^2}{8(q^2 - M_Z^2)\cos^2\theta_W} \left\{ \bar{\nu}\gamma_\mu(1-\gamma_5)\nu \, \bar{e}\gamma^\mu(g_V - g_A\gamma_5)e \right\} \qquad (10.5)$$

All Feynman diagrams are not in the charge retaining form, but they are brought into this form by Fierz's reordering theorem. A special form of the theorem permits one to interchange the second and the fourth spinor. The effective leptonic couplings are given in Table (10.1).

Reaction	GWS Theory		V-A Theory	
	g_V	g_A	g_V	g_A
$\nu_\mu + e^- \to e^- + \nu_\mu$	$-\frac{1}{2} + 2\sin^2\theta_W$	$-\frac{1}{2}$	0	0
$\bar{\nu}_\mu + e^- \to e^- + \bar{\nu}_\mu$	$-\frac{1}{2} + 2\sin^2\theta_W$	$+\frac{1}{2}$	0	0
$\nu_e + e^- \to e^- + \nu_e$	$\frac{1}{2} + 2\sin^2\theta_W$	$+\frac{1}{2}$	1	1
$\bar{\nu}_e + e^- \to e^- + \bar{\nu}_e$	$\frac{1}{2} + 2\sin^2\theta_W$	$-\frac{1}{2}$	1	-1

Table (10.1)

By following the techniques of chapter 1, we calculate the differential cross section in the laboratory frame

$$\frac{d\sigma}{dE} = \frac{G^2 m}{2\pi}\left\{(g_V+g_A)^2 + (g_V-g_A)^2\left(1-\frac{E}{E_\nu}\right)^2 + \frac{mE}{E_\nu^2}\left(g_A^2-g_V^2\right)\right\} \quad (10.6)$$

with m the mass of the electron, E the energy of the final electron and E_V the energy of the incident neutrino.

Several remarks are now in order:

(i) The last term is proportional to the electron mass and can be neglected at accelerator energies.

(ii) The above formula is also useful in the description of charge current reactions. For instance

$$g_V = g_A = 1$$

for the reaction $\nu_\mu + e^- \rightarrow \mu^- + \nu_e$. (10.7)

Defining the inelasticity $y = \frac{E}{E_V}$, we write the cross section as

$$\frac{d\sigma}{dy} = \frac{G^2}{\pi}\left(2mE_\nu\right) \quad (10.8)$$

For the reaction $\nu_\mu + \bar{u} \rightarrow \mu^- + \bar{d}$ (10.9)

$$g_V = 1,\ g_A = -1$$

and

$$\frac{d\sigma}{dy} = \frac{G^2}{\pi}\left(2mE_\nu\right)\left(1-y\right)^2 \quad (10.10)$$

We shall find these cross-sections useful for the description hadronic reactions.

(iii) For leptonic reactions we can use the effective couplings from Table (10.1) and Equ. (10.6). In figure (10.2) are plotted the neutrino-electron scattering cross sections against $\sin^2\theta_W$. Since all leptonic reactions depend

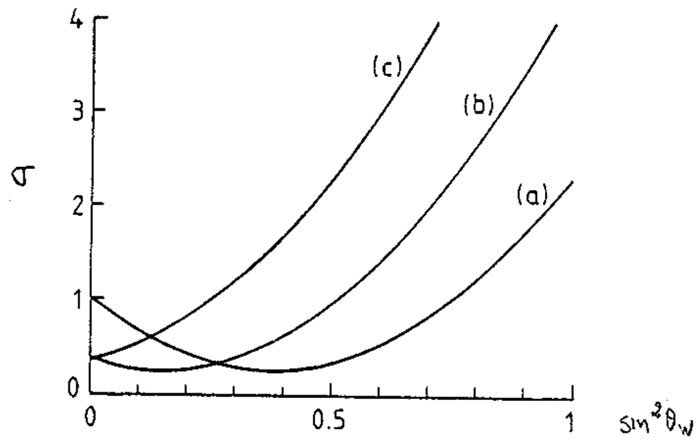


Fig. (10.2). Neutrino-electron scattering cross sections in units of $G^2 m_e E_\nu / 2\pi$ against $\sin^2\theta_W$: (a) $\nu_\mu e^-$, (b) $\bar{\nu}_\mu e^-$, (c) $\bar{\nu}_e e^-$.

on a single parameter, we can look at the experimental results and determine it. Alternatively we could leave g_V and g_A as free parameters and plot the experimentally allowed regions in the g_V - g_A plane. An exact value for the cross section corresponds to a conic section in the g_V - g_A plane.

<u>Purely Leptonic Processes:</u>

(α) The reaction $\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$

has been detected by Reines et al.[1] at the Savannah River Fission Reactor. Both charged and neutral currents contribute to this reaction. They reported the result

$$\sigma(\bar{\nu}_e e^- \to \bar{\nu}_e e^-) = (1.06 \pm 0.22) \cdot \sigma_{V-A}$$

where σ_{V-A} is the cross section predicted for this process in the V-A theory, $0.54 \times 10^{-41}\ E_\nu\ cm^2/GeV$. The experimental result is consistent with the V-A and also with the standard model with

$$\sin^2\theta_W = 0.29 \pm 0.05$$

(β) The reaction $\nu_\mu + e^- \to \nu_\mu + e^-$

has been studied in several experiments [2-7]. The average slope from all experiments is

$$\sigma/E_\nu = (1.6 \pm 0.4) \times 10^{-42}\ cm^2/GeV$$

yielding a weak angle

$$\sin^2\theta_W = 0.22 {+0.08 \atop -0.05}$$

(γ) Data on the reaction $\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^-$

are still very limited and recent experiments report [2,3,8-11] only upper bound for the slope of the cross section. The average cross section from all experiments is

$$\sigma/E_\nu = (1.3 \pm 0.6) \times 10^{-42}\ cm^2/GeV$$

yielding an angle

$$\sin^2\theta_W = 0.23 {+0.09 \atop -0.23}$$

The results from these experiments are summarized in Fig. (10.3). In the g_V vs g_A plane each of the above cross sections limits the physical region to an elliptical band. The physically allowed region is the intersection indicated by the two black domains. To resolve the two-valued ambiguity we discuss next the result from electron-positron ring experiments.

(δ) Electroweak Effects in e^+e^- Annihilation.

Electroweak effects arise from the fact that leptons couple not only to photons but also to the Z^0 boson.

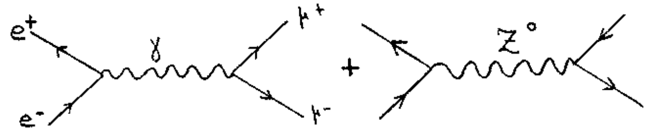


Fig. (10.3)

At the high energies they change the normalization of the cross section, giving

$$\Delta R = \frac{\sigma(\gamma + Z^0) - \sigma(\gamma)}{\sigma(\gamma)} = -\left(\frac{G}{2\sqrt{2}\,\pi\alpha}\right)\left\{\frac{2 s g_V^2}{s/M_Z^2 - 1} - \frac{s^2(g_V^2 + g_A^2)}{(s/M_Z^2 - 1)^2}\right\} \qquad (10.11)$$

They also produce a forward-backward asymmetry

$$A_\mu(\theta) = \frac{d\sigma/d\Omega(\theta) - d\sigma/d\Omega(\pi-\theta)}{d\sigma/d\Omega(\theta) + d\sigma/d\Omega(\pi+\theta)} = \frac{\sqrt{2}\,G s}{4\pi\alpha}\ \frac{\cos\theta}{1+\cos^2\theta}\ g_A^2 \qquad (10.12)$$

Both of the quantities increase with $S = (2E)^2$. They arise from the presence of neutral currents, but they are not manifestly parity violating. Therefore higher order QED diagrams produce similar effects. In the experiments in progress, the results are corrected for higher order QED effects. The size of the corrections depends on the c.m. energy and the experimental cuts which are used to select the data. At $\sqrt{s} = 35$ GeV the corrections are typically $\sim 1.5\%$. Values for the asymmetries extracted by the PETRA groups [12] are shown in Table 10.2

	JADE	MARK J	PLUTO	TASSO
measured	-11 ± 4	-3 ± 4	7 ± 10	-11.3 ± 5
predicted	- 7.8	- 7.1	- 5.8	- 8.7

Table 10.2 Charge asymmetry in muon production in percent.

We can combine these results and obtain an asymmetry $A_\mu = (-7.7 \pm 2.4)\%$ with a χ^2 of 3.6 for 3 degrees of freedom.

At PEP the muon asymmetry has been measured [13] by the MAC group $A_\mu = (-0.9 \pm 5.2 \pm 1.5)$ % and by the MARK II group $A_\mu = (-4.0 \pm 3.5)$ % at $\sqrt{s} = 29$ GeV.

Finally the Bhabha cross section

$$e^+ + e^- \rightarrow e^+ + e^-$$

is used to set limits on $\sin^2\theta_W$ by the method shown in Equation (10.11). The sum total of the measurements at PETRA [13] allow at the 68 % and 95 % confidence level the two shaded regions in the g_V vs g_A plane of fig. (10.4). These data resolve the two-ambiquity and select the back spot at the left as the correct

solution. This region is consistent with the GSW model. The value for $\sin^2\theta_W$ is approximately 1/4 with a 30 - 40 % error.

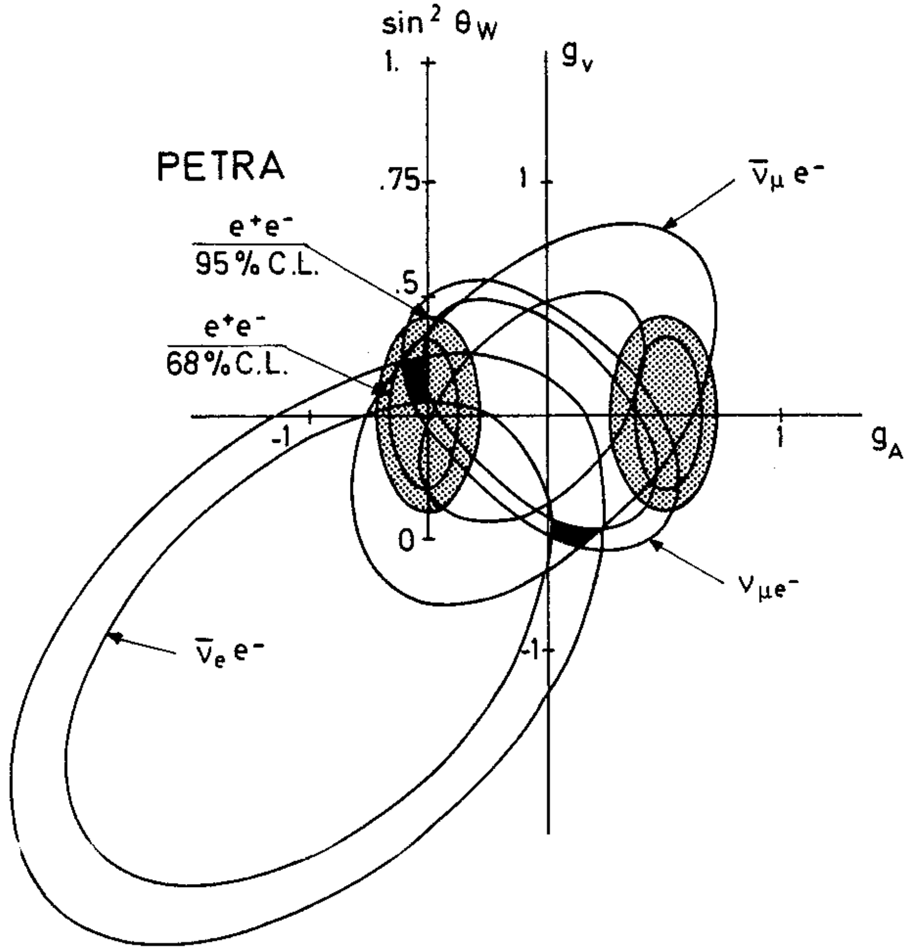


Fig. (10.4). Determination of neutral current couplings in leptonic reactions.

References

1. F. Reines, H.S. Gurr and H.W. Sobel, Phys. Rev. Lett. 37, 315 (1976).
2. J. Blietschau et al., Nucl. Phys. B114, 189 (1976); Phys. Lett. 73B, 232 (1978).
3. H. Faissner et al., Phys. Rev. Lett. 41, 213 (1978).
4. P. Alibran et al., High-energy elastic scattering off electrons in Gargamelle, preprint CERN-EP/79-38 (1979).
5. A.M. Cnops et al., Phys. Rev. Lett. 41, 357 (1978).
6. CHARM Collaboration.
7. R.H. Heisterberg et al., Phys. Rev. Lett. 44, 635 (1980).
8. P. Alibran et al., Phys. Lett. 74B, 422 (1978).
9. D. Bertrand et al., An upper limit to the cross-section for the reaction $\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-$ at SPS energies, preprint PITHA 79/07 (1979).
10. J.P. Berge et al., A search at high energies for antineutrino-electron elastic scattering, preprint Fermilab Pub 79/27-EXP (1979).
11. N. Armenise et al., Phys. Lett. 81B, 385 (1979).
12. J.G. Branson, 1981 Symposium on Lepton and Photon Interactions at High Energies, Bonn August 1981.
13. K.H. Mess and B.H. Wiik, Recent result in electron-positron and lepton-hadron interactions, DESY preprint 82-011/1982.

11. THE QUARK-PARTON MODEL

In the following applications of the electroweak theory it will be necessary to use the quark-parton model for the analysis of semileptonic reactions. We describe its salient features in this chapter.

The basic idea in the parton model[1] is to represent the deep inelastic scattering as quasifree scattering from pointlike constituents within the proton. This happens when the scattering is viewed from a frame in which the proton has infinite momentum. The neutrino-proton center-of-mass system is, at high energies, a good approximation of such a frame. In the infinite momentum frame, the proton is Lorentz-contracted into a thin pancake, and the lepton scatters instantaneously. Furthermore, the proper motion of the constituents within the proton is slowed down by time dilatation. We estimate the interaction time and lifetime of the virtual states within the proton. By using the notation in Fig.(11.1), we find the following .

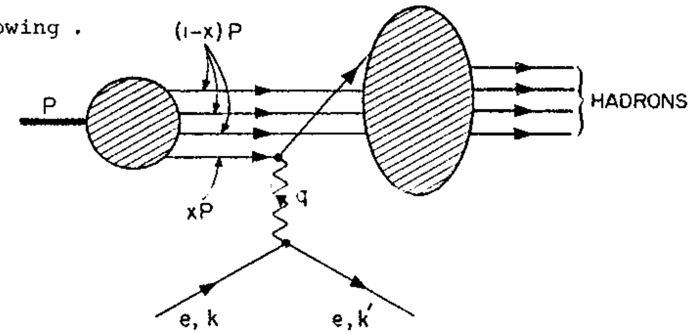


Fig. (11.1) Kinematics of lepton-nucleon scattering in parton model.

From the uncertainty principle the time of interaction is

$$\tau = \frac{1}{q_0} = \frac{4P}{4M\nu - Q^2} \qquad (11.1)$$

where q_o is the energy of the virtual current calculated in the lepton-proton center-of-mass frame. In this frame

$$p \cdot q = M\nu = p_o\left(q_o + q_z\right) \qquad (11.2)$$

$$k \cdot q = -\frac{Q^2}{2} = k_o\left(q_o - q_z\right) \qquad (11.3)$$

with $\vec{k} = -\vec{p}$, $k_o = p_o = P$

and

$$q_o = \frac{1}{2P}\left(M\nu - \frac{Q^2}{2}\right) \qquad (11.4)$$

We visualize the proton as composed of virtual states called partons. We denote by x the fraction of the proton's momentum carried by a constituent. The lifetime of the virtual states:

$$T = \frac{1}{E_x + E_{1-x} - E_p} = \frac{1}{\sqrt{(xP)^2 + \mu_1^2} + \sqrt{(1-x)^2P^2 + \mu_2^2} - \sqrt{P^2 + M_p^2}}$$

$$= \frac{2P}{\left(\mu_1^2 + p_{1\perp}^2\right)/x + \left(\mu_2^2 + p_{2\perp}^2\right)/(1-x) - M_p^2} \qquad (11.5)$$

If we now require that

$$\tau << T \qquad (11.6)$$

then we can consider the partons, contained in the proton, as free during the interaction. In this limit the current interacts with just one of the constituents leaving the rest undisturbed, thus making the impulse approximation valid. The above conditions appear to be satisfied in the high-energy, large momentum-transfer electron-nucleon scattering and in the high-energy neutrino-nucleon scattering.

Let us denote by $\frac{d\sigma_i(x)}{dQ^2 d\nu}$ the cross section between the lepton and a parton of type i, which carries a fraction x of the proton's momentum. The cross section from a proton is the incoherent sum of cross-sections from the individual constituents, one at a time,

$$\frac{d\sigma}{dQ^2 d\nu} = \sum_i \int_0^1 \frac{d\sigma_i(x)}{dQ^2 d\nu} f_i(x)\, dx \qquad (11.7)$$

The function $f_i(x)$ is the probability of finding an i^{th} constituent carrying fraction x of the proton's momentum. The summation is over all possible species within the proton and the integral is over the momentum distributions. The point cross sections are obtained from our previous results on neutrino-lepton scattering. For neutrino-parton scattering

$$\frac{d\sigma_i(x)}{dQ^2 d\nu} = \frac{G^2}{\pi}\,\delta\left(\nu - Q^2/2Mx\right) \qquad (11.8)$$

For neutrino-antiparton scattering

$$\frac{d\sigma_i(x)}{dQ^2 d\nu} = \frac{G^2}{\pi}\,\delta\left(\nu - Q^2/2Mx\right)\left(1 - \nu/E\right)^2 \qquad (11.9)$$

The species within the proton are quarks: Two up quarks and a down quark, which define the protons's quantum numbers. We denote the probability of finding an up-quark carrying a fraction x of the proton's momentum by u(x). Similarly we denote by d(x) the probability of finding a down quark carrying a fraction x of the proton's momentum. In addition to the up- and the down-quarks it is possible to find in the proton's cloud quark-antiquark pairs of any flavor. This necessitates the introduction of additional quark distribution functions. For instance, $\bar{u}(x)$ and $\bar{d}(x)$ correspond to up and down antiquarks. Similarly there are distributions s(x), $\bar{s}(x)$, c(x), $\bar{c}(x)$, ... for strange, charm and other flavors.

We consider in detail neutrino-proton scattering. The charged W^+ can scatter from d, s and b quarks or $\bar{u}$, $\bar{c}$ and $\bar{t}$ quarks. Since b and t quarks are very heavy the probability of finding them in the proton is very small and will be neglected. In fact in order to demonstrate the techniques used in the parton model, I will restrict the discussion of this section to the four-quark model in which case the K-M matrix reduces to the Glashow-Iliopoulos-Maiani form (Equ. 8.25). The scattering of a d-quark excites u and c quarks with the strengths $\cos^2\theta_c$ and $\sin^2\theta_c$, respectively. The total scattering on a d quark is proportional to the full strength of the Fermi coupling constant. The same holds for the other quarks. Introducing the variables $x = Q^2/_{2M\nu}$ and $y = \nu/_E$ and using (10.7) - (10.9) we obtain

$$\frac{d\sigma^{\nu p}}{dx\,dy} = \frac{G^2}{\pi}\, 2ME \times \left\{ d(x) + s(x) + \left[\bar{u}(x) + \bar{c}(x)\right](1-y)^2 \right\} \qquad (11.10)$$

For the scattering of neutrinos on neutrons it suffices to make an isospin rotation. The rotation which transforms protons into neutrons also transforms

$$u(x) \longrightarrow d(x)\ , \qquad \bar{u}(x) \longrightarrow \bar{d}(x)$$
$$d(x) \longrightarrow u(x)\ , \qquad \bar{d}(x) \longrightarrow \bar{u}(x)$$

but leaves the remaining quark distributions unchanged. Thus

$$\frac{d\sigma^{\nu n}}{dx\,dy} = \frac{G^2}{\pi}\, 2ME \times \left\{ u(x) + s(x) + \left[\bar{d}(x) + \bar{c}(x)\right](1-y)^2 \right\} \qquad (11.11)$$

The cross section on isoscalar target follows now trivially from (10.10) and (10.11).

Before discussing antineutrino reactions we simplify the notation by denoting

$$\sigma_o = \frac{G^2}{\pi}\, 2ME = \frac{G^2}{\pi}\, s \qquad (11.12)$$

The neutrino-proton cross section is the incoherent sum of the neutrino scattering on a d, s, $\bar{u}$ and $\bar{c}$ quarks. For the antineutrino-hadron scattering we obtain again an incoherent sum with lepton-quark cross sections related to each other by CP:

$$d\sigma_{\nu q} = d\sigma_{\bar{\nu}\bar{q}} \qquad (11.13)$$

Thus for antineutrino-proton scattering

$$\frac{d\sigma^{\bar{\nu} p}}{dx\,dy} = \sigma_o \times \left\{ \bar{d}(x) + \bar{s}(x) + \left[u(x) + c(x)\right](1-y)^2 \right\} \qquad (11.14)$$

and by an isospin rotation

$$\frac{d\sigma^{\bar{\nu} n}}{dx\,dy} = \sigma_o \times \left\{ \bar{u}(x) + \bar{s}(x) + \left[d(x) + c(x)\right](1-y)^2 \right\} \qquad (11.15)$$

Deep inelastic electron-proton scattering is also described in the model. The point cross section for electron-quark scattering is

$$\frac{d\sigma}{dQ^2 d\nu} = \frac{4\pi\alpha^2}{Q^4}\, \delta(\nu - Q^2/_{2Mx})\, Q_i^2 \qquad (11.16)$$

with Q_i the charge of the quark in units of e. The electron-proton cross section is

$$\frac{d\sigma}{dQ^2 d\nu} = \frac{4\pi\alpha^2}{Q^4}\, \frac{x}{\nu} \left\{ \frac{4}{9}\left[u(x) + c(x) + \bar{u}(x) + \bar{c}(x)\right] + \frac{1}{9}\left[d(x) + s(x) + \bar{d}(x) + \bar{s}(x)\right] \right\}, \qquad (11.17)$$

Other cross sections are easily obtained following the same methods.

The model deals with the inner structure of the hadrons. For instance $\int_0^1 s(x)\,dx$ is the probability of finding a strange quark within the proton and since the proton has zero strangeness

$$\int_0^1 \left[s(x) - \bar{s}(x) \right] dx = 0 \qquad (11.18)$$

Similarly we calculate the baryon number and isospin of the proton which give

$$\int_0^1 \left[u(x) - \bar{u}(x) \right] dx = 2 \qquad (11.19)$$

$$\int_0^1 \left[d(x) - \bar{d}(x) \right] dx = 1 \qquad (11.20)$$

Likewise, we find

$$\int_0^1 \left[c(x) - \bar{c}(x) \right] dx = 0 \qquad (11.21)$$

and similar relations for other heavy quarks. Similarly, the integral $\int_0^1 xq(x)\,dx$ gives the fraction of the proton's momentum carried by the q-quark. Thus

$$\Sigma = \int_0^1 x \left[u(x) + d(x) + s(x) + c(x) + \bar{u}(x) + \bar{d}(x) + \bar{s}(x) + \bar{c}(x) \right] dx$$

$$= 0.47 \pm 0.02 \qquad (11.22)$$

is the momentum carried by all the quarks [2] inside the proton. This integral was determined by combining the data from several processes. It is much less than one, indicating that the quarks carry only half of the proton's momentum. The remaining part is believed to be carried by gluons, the particles that mediate the strong interactions.

The quark structure functions were measured in numerous experiments. They are in general functions of two variables: (Q^2, ν) or (Q^2, x). But in the model, discussed here, they are functions of a single variable $x = Q^2/2M\nu$. This scaling phenomenon was introduced, on general grounds, by Bjorken just before the development of the parton model. Experimental results from electron and neutrino experiments show that it works remarkably well for q^2 in the range of a few $(\mathrm{GeV})^2$ to about 20 $(\mathrm{GeV})^2$; beyond that there are systematic departures. Such departures are expected in quantum chromodynamics (QCD) from higher-order corrections due to gluons. These extensions and other applications of the model are beyond the scope of these lectures. In later chapters we have a chance to use several applications of the model.

References Ch. 11

1. R.P. Feynman, Phys. Rev. Lett. 23, 1415 (1969); Proceedings of the Third Int. Conference of High Energy Collisions at Stony Brook (New York Gordon and Breach 1969);
   J.D. Bjorken and E.A. Paschos, Phys. Rev. 185, 1975,(1969).

2. These sum rules are the parton analogous of current algebra results
   S. Adler, Phys. Rev. 143, 144 (1966)
   J.D. Bjorken, Phys. Rev. 163, 1769 (1967)
   D.J. Gross and C.H. Llewellyn Smith, Nucl.Phys. B14, B37 (1969)

3. J.G.H. de Groot et al. Zeitschr. für Physik 1C, 143 (1979)

4. J.D. Bjorken, Phys. Rev. 179, 1547 (1969)

5. A. Buras, Rev. Mod. Phys. 52, 199 (1980);
   J. Ellis and C.T. Sachrajda, Quantum Chromodynamics and its Application, CERN-preprint Ref. Th. 2782 (1980);
   E. Reya, Perturbative Quantum Chromodynamic, Vol. 63, 3 (1981).

12. STRUCTURE OF THE CHARGED CURRENTS

The structure of the charged current involves three angles and the phase, which occur in the K-M matrix. It suffices to determine four elements of the matrix and a phase. The others follow from the unitary character of the matrix. Estimates of the V_{ij} elements draw from many experimental results.

Beta Decay. For superallowed $0^+ \to 0^+$ transitions only the vector current contributes. For nuclei belonging to an I = 1 isospin multiplet one can extract the Fermi coupling constants G_V. The matrix element V_{ud} is extracted from the ratio

$$V_{ud} = \frac{G_V}{G_\mu} \qquad (12.1)$$

with G_μ the Fermi coupling constant extracted from the muon lifetime. In actual computations the situation is more complicated because to the ratio (12.1) we must also include radiative corrections from the exchange of photons and/or intermediate gauge bosons. In fact, if one does not include radiative corrections the values of V_{ud} for different nuclei do not converge. It is necessary to include, for the nuclei, additional corrections.

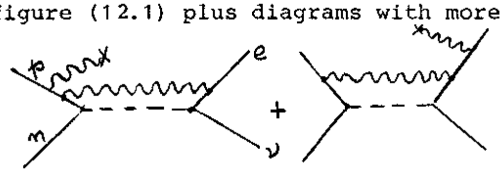
(i) Coulomb corrections to the nuclear wave functions are denoted by δ_c. For superallowed transitions between members of the same I = 1 isomultiplet, one encounters the matrix element

$$M_V = \langle I=1, I_3+1 | T^+ | I=1, I_3 \rangle \qquad (12.2)$$

This is a direct application of CVC. In nuclei however, there is isospin-mixing between neighboring states and the matrix element is modified to

$$M_V^2 \rightarrow |M_V'|^2 = 2(1-\delta_c) \qquad (12.3)$$

(ii) Outer radiative corrections are denoted by δ_R. They are characterized by the fact that in the decay of a neutron the resulting particles carry charge and feel the electromagnetic field of the nucleus. They are described by the diagrams in figure (12.1) plus diagrams with more photons.



The cross at the end of a photon denotes interactions with the static charge of the nucleus.

(iii) Inner radiative corrections are denoted by Δ_R. They are characterized by the exchange of photons and intermediate gauge bosons in the primitive Born-diagrams. Some of the contributions are shown in figures (12.2) - (12.3). They were calculated in the GSW model by Sirlin.[1]

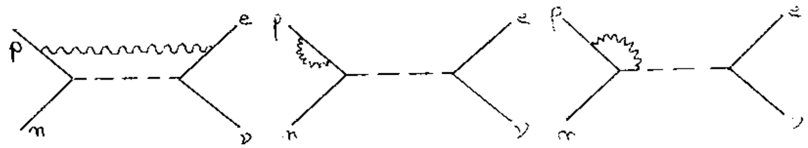


Fig. (12.2). Some inner photonic terms.

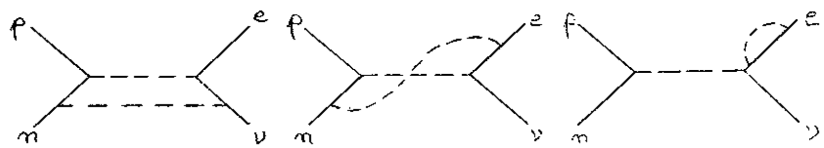


Fig. (12.3). Some inner contribution from charged and neutral gauge bosons.

It is standard in analyses of β-decay to introduce the ft-values, that is, the product of the Fermi integral times the half lifetime of the state. They depend on matrix element M_V and the Fermi coupling constant

$$ft = \frac{2\pi^3 \ln 2}{|M_V|^2 G_V^2 m_e^5} \qquad (12.4)$$

Radiatively corrected ft-values are defined as

$$\mathcal{F}t = ft(1-\delta_c)(1+\delta_R) = \frac{\pi^3 \ln 2}{G_V^2 (1+\Delta_R) m_e^5} \qquad (12.5)$$

and are the same for all nuclei. The ft-values and radiative corrections for several transitions are shown in Table (12.1).

Table (12.1). ft-values, corrections and corresponding results for the more accurate decays.[2]

nucleus	ft (s)	$\delta_R(\%)$	$\delta_c(\%)$	$\delta_w(\%)$ +	$\lvert V_{ud}\rvert$
^{14}O	3047.6±3.6	1.57	0.18	2.10	0.97223
^{26}Al^m	3037.9±2.9	1.61	0.24	2.10	0.97377
^{34}Cl	3052 ±12	1.68	0.51	2.10	0.97255
^{38}K^m	3063 ±10	1.74	0.44	2.10	0.97015
^{42}Sc	3052 ±13	1.81	0.44	2.10	0.97154
^{46}V	3039 ±16	1.87	0.40	2.10	0.97311
^{50}Mn	3038.1±7.1	1.95	0.47	2.10	0.97322
^{54}Co	3041.4±5.0	2.01	0.56	2.10	0.97289

Finally, the matrix element V_{ud} is given by

$$|V_{ud}| = \frac{G_V}{G'_\mu}$$

+ We denote by $\bar{\delta}_W = \Delta_R - \Delta_\mu$, the difference of weak corrections to the ft-value and the muon decay.

with G'_μ the Fermi coupling constant obtained from muon decay including one-loop radiative corrections. This means that we use the formula [1]

$$\frac{1}{\tau_\mu} = \frac{G_\mu'^2 m_\mu^5}{192\pi^3}\left[1-\frac{8m_e^2}{m_\mu^2}\right]\left\{1+\frac{3}{5}\frac{m_\mu^2}{M_W^2}+\frac{\alpha}{2\pi}\left[\frac{25}{4}-\pi^2-\frac{3}{2}\tan^2\theta_W\frac{R}{R-1}\ln R+\dots\right]\right\} \qquad (12.6)$$

with $R = m_W^2/m_Z^2$.

The resulting values for V_{ud} appear in the last column of the table. The weighted average is [2,3]

$$|V_{ud}| = 0.9730 \pm 0.0004 \pm 0.0020 \qquad (12.7)$$

The first error is statistical from the ft-values in table (10.1). However, there are additional theoretical uncertainties. Estimates of the isospin breaking correction δ_c vary in the published articles. The photonic corrections from the box diagrams are computed using quarks, but the low frequency part of the integration involves some uncertainties associated with the binding of quarks into hadrons. These uncertainties are hard to estimate. By studying the deviations in the corrections reported in articles we obtain the last error.

Strangeness Changing Decays. The element V_{us} is obtained from $K_{\ell 3}$ and hyperon decays. In $K_{\ell 3}$ decays only the vector current contributes through the matrix element

$$\langle \pi^0 | V_\mu | K^- \rangle = \frac{1}{\sqrt{2E_K E_\pi}}\left\{ f_+(q^2)(p_\mu + p'_\mu) + f_-(q^2)\, q_\mu \right\} \qquad (12.8)$$

The largest contribution comes from $f_+(q^2)$, whose q^2 dependence is studied by the distribution of events in the Dalitz plot. The value for $f_+(0)$ is taken from SU(3), allowing for SU(3) and

SU(3)xSU(3) breaking effects. The end result is [3]

$$|V_{us}| = 0.219 \pm 0.02 \pm 0.011 \tag{12.9}$$

with the first error statistical and the second error representing a theoretical uncertainty of 5 %.

An alternative determination of V_{us} is from hyperon decays. Here there are many more decays and the range of q^2 is smaller. The data for hyperon decays are constantly improving. Most of the decays are in very good agreement with the predictions of the SU(3) symmetry group. An exception is brought by recent results on the $\Sigma^- \to \Lambda e \nu$ decay, which conserves strangeness and is independent of V_{us}.

The hadronic matrix element for the decay $B_1 \to B_2 \ell \nu_\ell$ is written as

$$\langle B_2 | J_\mu^h | B_1 \rangle = \bar{u}_{B_2} \left\{ f_1 \gamma_\mu - i f_2 \frac{q^\nu}{m} \sigma_{\mu\nu} + f_3 \frac{q_\mu}{m} - \left[g_1 \gamma_\mu - i g_2 \frac{q^\nu}{m} \sigma_{\mu\nu} + g_3 \frac{q_\mu}{m} \right] \gamma_5 \right\} u_{B_1} \tag{12.10}$$

where q^2 is the momentum transfer $(p_{B_2} - p_{B_1})^2$, and f_1, f_2, f_3 are the vector form factors and g_1, g_2, g_3 the axial vector form factors. The form factors f_3 and g_3 are usually neglected, because their contribution to the matrix element is of the order m_{e/M_3}. We assume the absence of second class currents, therefore the form factor $g_2 = 0$. The momentum transfer in the decay is limited by the mass difference of the baryons, which allow one to expand the form factors

$$f_i(q^2) = f_i(0) \left\{ 1 + \lambda_{f_i} \frac{q^2}{m_\alpha^2} \right\} \tag{12.11}$$

$$g_i(q^2) = g_i(0) \left\{ 1 + \frac{2 q^2}{m_A^2} \right\} \tag{12.12}$$

For the masses m_α one uses the average mass of the baryons in the decay process. The form factors $f_i(o)$ are related to the electromagnetic form factors by SU(3). The axial form factors $g_i(o)$ are computed in terms of the antisymmetric and symmetric F and D couplings. Table 12.2 gives the form factors for the decays in terms of F, D and the magnetic moments of the proton and neutron.

Table 12.2. Results for the formfactors. [4]

	$f_1(0)$	$f_2(0)$	$g_1(0)$	$g_3(0)$	λ_{f_1}
$n \to p e \bar\nu$	V_{ud}	$\frac{1}{2} V_{ud} (\mu_p - 1 - \mu_n)$	$(D+F)\, V_{ud}$	$2(D+F) \frac{m_1^2}{m_\pi^2} V_{ud}$	$(1 - \mu_p + \frac{3}{2}\mu_n + 8 \frac{m_1^2}{m_V^2})/4$
$\Sigma^- \to \Lambda e \bar\nu$	0	$-\frac{1}{2}\sqrt{\frac{3}{2}} V_{ud}\, \mu_n$	$\sqrt{\frac{2}{3}} V_{ud} D$	$2\sqrt{\frac{2}{3}} V_{ud} D\, m_2^2/m_\pi^2$	0
$\Sigma^+ \to \Lambda e \nu$	0	$-\frac{1}{2}\sqrt{\frac{3}{2}} V_{ud}\, \mu_n$	$\sqrt{\frac{2}{3}} V_{ud} D$	$2\sqrt{\frac{2}{3}} V_{ud} D\, m_3^2/m_\pi^2$	0
$\Lambda \to p e \bar\nu$	$-\sqrt{\frac{3}{2}} V_{us}$	$-\frac{1}{2}\sqrt{\frac{3}{2}} V_{us} (\mu_p - 1)$	$-\frac{V_{us}}{\sqrt{6}} (3F+D)$	$-\frac{2V_{us}}{\sqrt{6}} (3F+D)\, m_4^2/m_K^2$	$(1 - \mu_p + 8 \frac{m_4^2}{m_V^2})/4$
$\Lambda \to p \mu \bar\nu$	$-\sqrt{\frac{3}{2}} V_{us}$	$-\frac{1}{2}\sqrt{\frac{3}{2}} V_{us} (\mu_p - 1)$	$-\frac{V_{us}}{\sqrt{6}} (3F+D)$	$-\frac{2V_{us}}{\sqrt{6}} (3F+D)\, m_5^2/m_K^2$	$(1 - \mu_p + 8 \frac{m_5^2}{m_V^2})/4$
$\Sigma^- \to n e \bar\nu$	$-V_{us}$	$-\frac{1}{2} V_{us} (\mu_p - 1 + 2\mu_n)$	$V_{us} (D-F)$	$V_{us} (D-F)\, 2 m_6^2/m_K^2$	$(1 - \mu_p - 2\mu_n + 8 \frac{m_6^2}{m_V^2})/4$
$\Sigma^- \to n \mu \bar\nu$	$-V_{us}$	$-\frac{1}{2} V_{us} (\mu_p - 1 + 2\mu_n)$	$V_{us} (D-F)$	$V_{us} (D-F)\, 2 m_7^2/m_K^2$	$(1 - \mu_p - 2\mu_n + 8 \frac{m_7^2}{m_V^2})/4$
$\Xi^- \to \Lambda e \bar\nu$	$\sqrt{\frac{3}{2}} V_{us}$	$\frac{1}{2}\sqrt{\frac{3}{2}} V_{us} (\mu_p - 1 + \mu_n)$	$\frac{V_{us}}{\sqrt{6}} (3F-D)$	$\frac{V_{us}}{\sqrt{6}} (3F-D)\, 2 m_8^2/m_K^2$	$(1 - \mu_p - \mu_n + 8 \frac{m_8^2}{m_V^2})/4$
$\Xi^- \to \Lambda \mu \bar\nu$	$\sqrt{\frac{3}{2}} V_{us}$	$\frac{1}{2}\sqrt{\frac{3}{2}} V_{us} (\mu_p - 1 + \mu_n)$	$\frac{V_{us}}{\sqrt{6}} (3F-D)$	$\frac{V_{us}}{\sqrt{6}} (3F-D)\, 2 m_9^2/m_K^2$	$(1 - \mu_p - \mu_n + 8 \frac{m_9^2}{m_V^2})/4$

It is now straightforward to calculate decay rates and polarization asymmetries using the form factors in Table 12.2. Several groups determined the parameters F, D and V_{us} using only strangeness-changing decays or both $\Delta S=0$ and 1 decays. Results from a recent fit [2] are shown in Table 12.3

Table 12.3. Input data and results of the fits for the hyperon decays. (The quoted errors in V_{us} , F and D are purely statistical.)

decay	experiment	ΔS=0,1 fit	ΔS=1 fit
$\tau(n\to pe\bar{\nu})$	(917±14) sec	938 sec	-
$br(\Sigma^-\to\Lambda e\bar{\nu})$	$(5.40\pm0.31)\,10^{-5}$	6.64×10^{-5}	-
$br(\Sigma^+\to\Lambda e^+\nu)$	$(2.02\pm0.47)\,10^{-5}$	2.15×10^{-5}	-
$br(\Lambda\to pe\bar{\nu})$	$(8.37\pm0.14)\,10^{-4}$	8.63×10^{-4}	8.47×10^{-4}
$br(\Lambda\to p\mu\bar{\nu})$	$(1.57\pm0.35)\,10^{-4}$	1.38×10^{-4}	1.35×10^{-4}
$br(\Sigma^-\to ne\bar{\nu})$	$(1.08\pm0.04)\,10^{-3}$	0.99×10^{-3}	1.06×10^{-3}
$br(\Sigma^-\to n\mu\bar{\nu})$	$(4.5\pm0.4)\,10^{-4}$	4.5×10^{-4}	4.8×10^{-4}
$br(\Xi^-\to\Lambda e\bar{\nu})$	$(5.57\pm0.37)\,10^{-4}$	4.91×10^{-4}	4.79×10^{-4}
$br(\Xi^-\to\Lambda\mu\bar{\nu})$	$(2.6\pm2.6)\,10^{-4}$	1.4×10^{-4}	1.3×10^{-4}
$\frac{g_1}{f_1}(n\to pe\bar{\nu})$	1.254±0.007	1.253	1.253
$\frac{g_1}{f_1}(\Lambda\to pe\bar{\nu})$	0.699±0.035	0.728	0.715
$\left\lvert\frac{g_1}{f_1}\right\rvert(\Sigma^-\to ne\bar{\nu})$	0.385±0.070	-0.320	-0.362
	$\lvert V_{us}\rvert$ =	0.226±0.003	0.227±0.003
	F =	0.466±0.006	0.446±0.006
	D =	0.786±0.006	0.807±0.007
	χ^2/D.F. =	24.6/9	5.6/6

Both fits give practically the same value for $|V_{us}| = 0.227 \pm 0.003$, because it occurs only in the ΔS = 1 transitions. The $^2/_{D.F.}$ for the combined ΔS = 0,1 fit is poor mainly because of the $\Sigma^- \to \Lambda e^- \bar{\nu}$ decay which carries a small error.[5] The discrepancy was interpreted[6] as a symmetry breaking effect. The ΔS = 1 fit is still very good and it by itself determined the V_{us} element. It was also checked that changes of the axial mass m_A in the range 0.90

to 1.10 GeV does not affect the V_{us} value. In order to check the theoretical uncertainty, associated with the discrepancy in the Σ-decay, a 10 % symmetry breaking effect was introduced in D. The resulting value is[2,3]

$$|V_{us}| = 0.227 \pm 0.003 \pm 0.013 \qquad (12.13)$$

with the first error being statistical and the second error representing the uncertainty associated with a 10 % symmetry breaking effect in D.

Dimuon production by neutrinos and antineutrinos. The following reactions have been observed at high energies

$$\nu + N \longrightarrow \mu^- + \mu^+ + \text{hadrons} \qquad (12.14)$$

$$\bar{\nu} + N \longrightarrow \mu^- + \mu^+ + \text{hadrons} \qquad (12.15)$$

The fast muon is associated with the incident neutrino or anti-neutrino and the other muon we believe to come from the production and subsequent decay of a charmed particle. The diagram in figure (12.4) shows a typical production of a dimuon pair.

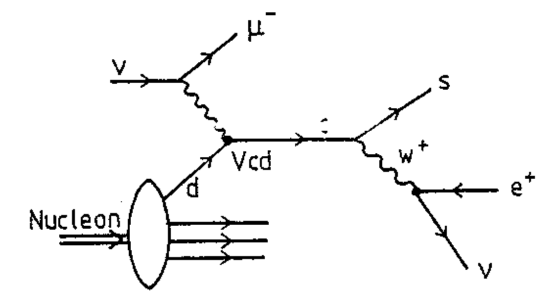


Fig. (12.4)

Opposite sign dilepton events were observed in several experiments. These processes are described in the quark-parton model and provide new information on the V_{cd} and V_{cs} matrix elements.

There is a slight complication in the model related to the excitation of heavy quarks and we must consider carefully the threshold effects. When a W-boson with momentum q scatters on a light quark with momentum ξp producing a charmed quark, it follows

$$m_c^2 = (q+\xi p)^2 \simeq q^2 + 2\xi\, q\cdot p$$

$$\Rightarrow \quad \xi = x + \frac{m_c^2}{2M\nu} \tag{12.16}$$

Thus the quark distribution functions depend on the new scaling variable ξ. The cross section for the excitation of charmed quarks on an isoscalar target is

$$\frac{d\sigma^\nu}{dx\,dy} = \frac{G^2ME}{\pi}\left(1-y+\frac{xy}{\xi}\right)\left\{|V_{cd}|^2\,\xi\left[u(\xi)+d(\xi)\right] + |V_{cs}|^2\,\xi\, 2s(\xi)\right\}\theta(1-\xi) \tag{12.17}$$

In this formula we use the general form of the Kobayashi-Maskawa matrix and the θ-function controls the beginning of charm excitation. The integrated form is much more compact

$$\sigma^{\nu N\to c} = \frac{G^2ME}{\pi}\left\{|V_{cd}|^2 r_d\,(U+D) + |V_{cs}|^2 r_s\, 2S\right\} \tag{12.18}$$

with the abbreviations

$$U = \int_0^1 u(x)\,x\,dx\ ,\quad D = \int_0^1 d(x)\,x\,dx \quad \text{and} \quad S = \int_0^1 s(x)\,x\,dx \ . \tag{12.19}$$

The factors r_s and r_d are defined as

$$r_s \equiv \iint \left(1-y+\frac{xy}{\xi}\right)\xi\, s(\xi)\,\theta(1-\xi)\,dx\,dy \Big/ S \tag{12.20}$$

$$r_d \equiv \iint \left(1-y+\frac{xy}{\xi}\right)\xi\, d(\xi)\,\theta(1-\xi)\,dx\,dy \Big/ D \tag{12.21}$$

The cross section for production of anticharm by antineutrinos is

$$\sigma^{\bar\nu N\to \bar c+\dots} = \frac{G^2ME}{\pi}\left\{|V_{cd}|^2 r_{\bar d}\,(\bar U+\bar D) + |V_{cs}|^2 r_{\bar s}\, 2S\right\} \tag{12.22}$$

with evident notation and the mild assumption $S = \bar S$.

The experiments measure dimuon pairs whose rate follows from the above formulas after multiplication by B_e, the average semileptonic branching ratio for charmed states; $B_e = (8.5 \pm 0.5)$ %. The reported values are [7]

$$R^\nu = \frac{\sigma^{\nu N\to\mu^-\ell^+\dots}}{\sigma^{\nu N\to\mu^-\dots}} = (0.77 \pm 0.08)\,\% \text{ at } E_\nu = 220 \text{ GeV}$$

and

$$R^{\bar\nu} = \frac{\sigma^{\bar\nu N\to\mu^+\ell^-\dots}}{\sigma^{\bar\nu N\to\mu^+\dots}} = (0.73 \pm 0.09)\,\% \text{ for } E_{\bar\nu} = 150 \text{ GeV}.$$

The solution of these equations determines the V_{cd} and V_{cs} elements as follows [2]

$$|V_{cd}| = \left\{\frac{R^\nu[U+D+2S+\frac{4}{3}(\bar U+\bar D)] - R^{\bar\nu}[\bar U+\bar D+2\bar S+\frac{4}{3}(U+D)]}{B_e\,[r_d(U+D) - r_{\bar d}(\bar U+\bar D)]}\right\}^{1/2} \tag{12.23}$$

$$|V_{cs}| = \left\{\frac{R^{\bar\nu}[\bar U+\bar D+2\bar S+\frac{4}{3}(U+D)]\,r_d(U+D) - R^\nu[U+D+2S+\frac{4}{3}(\bar U+\bar D)]\,r_{\bar d}(\bar U+\bar D)}{2B_e\,S\,r_s\,[r_d(U+D) - r_{\bar d}(\bar U+\bar D)]}\right\}^{1/2} \tag{12.24}$$

The V_{cd} element in equation (1) is independent of S, since instead of the quark distribution functions in the numerators we can introduce measurements of the absolute cross section.

As additional input we need

$$\begin{array}{lrl} (\mathrm{i}) & U &= 0.285 \pm 0.012\ , \\ & D &= 0.129 \pm 0.010\ , \\ & \overline{D}+\overline{S} &= 0.034 \pm 0.004\ , \\ \text{and} & \overline{U}+\overline{S} &= 0.021 \pm 0.003 \end{array} \qquad (12.25)$$

from the BEBC[8] collaboration.

(ii) The threshold suppression factors

$$\begin{array}{ll} & r_d(220\ \text{GeV}) = 0.91\ , \\ & r_s(220\ \text{GeV}) = 0.72\ , \\ \text{and} & r_{\bar{s}}(150\ \text{GeV}) = 0.66 \end{array} \qquad (12.26)$$

are taken from the work of Brock.[9]

$$r_{\bar{d}}(150\ \text{GeV}) = 0.70$$

was calculated using the Field-Feynman parametrization[10], with m_c = 1.5 GeV.

The value for $|V_{cd}|$, reads

$$|V_{cd}| = 0.25 \pm 0.04\ . \qquad (12.27)$$

If we bound $0<\bar{S}<\bar{U}$, we obtain a lower bound for

$$|V_{cs}| > 0.81\ . \qquad (12.28)$$

For decreasing values of S, the element $|V_{cs}|$ increases and becomes bigger than one. This gives us the lower bound

$$S = \int x s(x) dx \geq 0.008 \qquad (12.29)$$

Summarizing the results of this chapter we analysed data in β-decay, in strangeness changing decays and accelerator data on dileptons in order to determine four elements in the K-M matrix. The result in equations (12.7), (12.9), (12.13), (12.27) and (12.28) are very restrictive and reliable. Now using the unitarity of the matrix we can bound the other elements [2]

$$\begin{pmatrix} |V_{ud}| = 0.9730 \pm 0.0024 & |V_{us}| = 0.227 \pm 0.016 & |V_{ub}| < 0.12 \\ |V_{cd}| = 0.25 \pm 0.04 & |V_{cs}| > 0.81 & |V_{cb}| < 0.54 \\ |V_{td}| < 0.12 & |V_{ts}| < 0.55 & |V_{tb}| > 0.83 \end{pmatrix} \qquad (12.30)$$

Another determination [11] of the angles and the phase gives for $S_i = \sin\theta_i$ the curves shown in figure 12.5. The upper branch is for δ near π. For these curves we need in addition the constraint imposed by the K^0-$\bar{K}^0$ system, which the reader can find in the published articles.

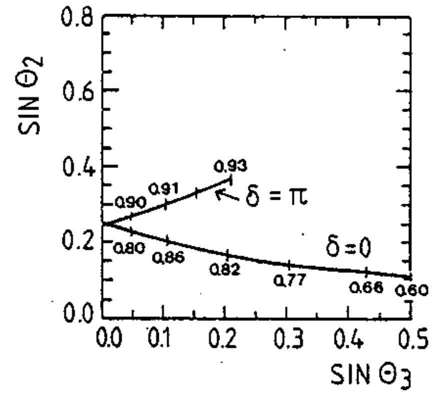


Fig. (12.5). The combined constraints for $\sin\theta_2$ vs $\sin\theta_3$ based on the K^0-$\bar{K}^0$ system. The value of $V_{cs}^{\ 2}$ is indicated along the curve.

A topic of central importance to the electroweak theory is the origin of CP-violation. There are several theoretical proposals for its origin and an extensive phenomenology. Unfortunately, this topic was not covered in my lectures at Maria Laach. The interested student will find a lucid introduction in the following articles. [12]

References Ch. 12

1. A. Sirlin, Rev. Mod. Phys. 50, 573 (1978).
2. For further details and references see
   E.A. Paschos and U. Türke, Dortmund preprint DO-TH/82-07 (1982), to be published.
3. It is consistent with previous determinations
   M. Roos, Nucl. Phys. B77, 420 (1974)
   R.E. Shrock and L.L. Wang, Phys. Rev. Lett. 41, 1692 (1978).
4. U. Türke, Diplomarbeit University of Dortmund (Feb. 1982).
5. WA2 Collaboration, LAL 81/18 June 1981
   M. Bourquin et al., Z. Phys. C12, 307 (1982)
6. A. Garcia and P. Kielanowski, Phys. Lett. 110B, 498 (1982)
   J. Donoghue and B. Holstein, Phys. Rev. D25, 2015 (1982)
7. J. Knobloch et al., (CDHS Collaboration), Proc. 1981 Int. Conf. on Neutrino Physics and Astrophysics P. 421; Maui, Hawaii Edited by R.J. Cence, E. Ma and A Roberts
8. D. Allasia et al., Paper submitted to the International Conference on Neutrino Physics and Astrophysics, Maui, Hawaii, U.S.A., July 1-8, 1981 and Bonn Conference on Photon hadron Interactions, pg. 464 (1981)
9. R. Brock, Phys. Rev. Lett. 14, 1027 (1980)
10. R.D. Field and R.P. Feynman, Phys. Rev. D15, 2590 (1977)
11. S. Pakvasa, S.F. Tuan and J.J. Sakurai, Phys. Rev. D23, 2799 (1981)

13. NEUTRAL CURRENTS: Semileptonic Reactions.

The subject of semileptonic neutrino interactions is now well-founded and the determination of their coupling constants is becoming very precise. The effective current-current interaction between neutrinos and quarks is given by

$$\mathcal{L} = \frac{G}{\sqrt{2}} \bar{\nu} \gamma^{\mu} (1-\gamma_5) \nu \, J^z_{\mu} \tag{13.1}$$

with the hadronic current given as

$$\begin{aligned} J^z_\mu = {} & \bar{u}\gamma_\mu \{u_L(1+\gamma_5) + u_R(1-\gamma_5)\} u \, + \\ & + \bar{d}\gamma_\mu \{d_L(1+\gamma_5) + d_R(1-\gamma_5)\} d \, + \\ & + \bar{s}\gamma_\mu \{s_L(1+\gamma_5) + s_R(1-\gamma_5)\} s \, + \\ & + \bar{c}c + \ldots \end{aligned}$$

or alternatively

$$\begin{aligned} J^z_\mu = \frac{1}{2} \Big\{ & \alpha \left[\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right] + \beta \left[\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d\right] \\ & + \gamma \left[\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d\right] + \delta \left[\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d\right] + \bar{s}\ldots s + \ldots \Big\} \end{aligned} \tag{13.3}$$

The constants α, β, γ and δ are the coupling constants of the vector-isovector, axial vector-isovector, vector-isoscalar and axial vector-isoscalar currents, respectively. Similarly, u_L, u_R, etc. are the coupling constants for the left-handed up-quark current, right-handed up-quark current, and so on. However, the use of this form does not require a quark picture sin e the occurrence of the quarks is merely a mnemonic for the transformation properties of various parts of the total hadronic neutral current. We can write the hadronic current as

$$J_\mu^{2} = \alpha V_\mu^3 + \beta A_\mu^3 + \gamma V_\mu^o + \delta A_\mu^o + \cdots \qquad (13.4)$$

with the vector and axial currents having specific transformation properties under strong isospin. For instance V_μ^3 is the third component of the strong isospin current, V_μ^o is the baryon current and the rest of the notation is evident.

In the electroweak theory there is only one parameter $\sin^2\theta_W$. The above couplings are then expressed in terms of the angle and the ρ-parameter defined in (7.13). We can read them off the table (8.1)

$$u_L = \frac{1}{2}\rho\left(g_V^u + g_A^u\right) = \rho\left(\frac{1}{2} - \frac{2}{3}s^2\right)$$

$$u_R = \frac{1}{2}\rho\left(g_V^u - g_A^u\right) = -\frac{2}{3}\rho s^2$$

$$d_L = \rho\left(-\frac{1}{2} + \frac{1}{3}s^2\right) \quad \text{and} \quad d_R = \frac{1}{3}\rho s^2 \qquad (13.5)$$

A good deal of effort [1,2] was devoted in order to establish that all semileptonic neutral current data are consistent with a single parameter $\sin^2\theta_W$. The effort was carried out as follows: the coupling u_L, ..., d_R, S_L, ... were considered as free parameters to be determined by experiment. The matrix elements of the operators V_μ^3, A_μ^3, ... are expressed in terms of known quantities measured in charged current reactions or in electroproduction. For instance, in the deep inelastic region the reactions are well understood in terms of the parton model. For elastic scattering, on the other hand, neutral current form factors are determined in terms of elastic electron and charged current form factors. Combining data from several reactions [3,4] the following values were determined [3,4]

$$u_L = 0.340 \pm 0.033, \qquad d_L = -0.424 \pm 0.026$$

$$u_R = -0.179 \pm 0.019, \qquad d_R = -0.017 \pm 0.058$$

An independent analysis [5], using different criteria for data selection, gave for the couplings the values [5] are shown in figures (13.1) and (13.2). In the same figures, shown the regions allowed by 1.0, 1.5 and 2.0 standard deviations. The results depend somewhat on M_A; the axial mass occurring in elastic scattering. Results for different values of M_A can be found in the published article.

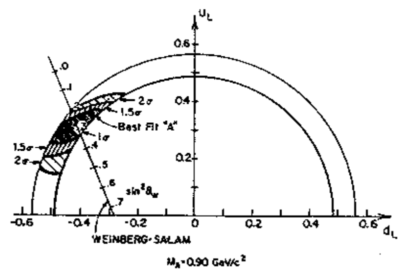


Fig. (13.1)

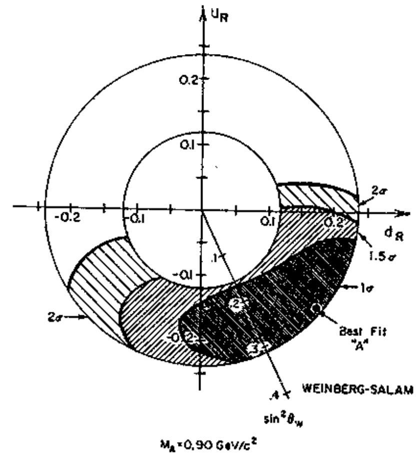


Fig. (13.2)

This type of analysis was carried out by many experimental and theoretical collaborations. [1,2,3] Their final conclusion was that, allowing the neutral current couplings in Equation (13.2) as independent parameters, there is only one solution; the solution is consistent with the standard model.

An alternative approach is to adopt the theory and require the best value for the mixing angle. Deep inelastic measurements are crucial in this approach and we discuss the relevant formulas. Neutral current cross sections are rather long and we give here the neutrino-proton result and simple rules to obtain the other reactions.

$$\frac{d\sigma^{\nu p}}{dx\,dy} = \frac{G^2 ME}{4\pi}\Big\{(u+c)\left[(1-y)^2 g_-^2 + g_+^2\right] + (d+s)\left[(1-y)^2 g_-'^2 + g_+'^2\right]$$
$$+ (\bar{u}+\bar{c})\left[(1-y)^2 g_+^2 + g_-^2\right] + (\bar{d}+\bar{s})\left[(1-y)^2 g_+'^2 + g_-'^2\right]\Big\} \qquad (13.6)$$

with

$$g_+ = -\frac{4}{3}s^2 \quad , \quad g_- = 1 - \frac{4}{3}s^2 \qquad (13.7)$$

the positive and negative helicity couplings for upper quarks and

$$g_+' = \frac{2}{3}s^2 \quad , \quad g_-' = -1 + \frac{2}{3}s^2 \qquad (13.8)$$

the corresponding couplings for down quarks. To obtain formulas for other reactions, follow the simple rules:

(i) for cross sections on neutrons exchange $u \leftrightarrow d$, and

(ii) for cross sections induced by antineutrinos exchange $g_+ \leftrightarrow g_-$ and $g_+' \leftrightarrow g_-'$.

As an example, we give the cross section on an isoscalar target $N = (p+n)/2$

$$\frac{d\sigma^{\nu N}}{dx\,dy} = \frac{G^2 ME}{4\pi}\, 2x \Big\{(u+d)\left[(1-y)^2 \frac{10}{9}s^4 + \left(1-2s^2+\frac{10}{9}s^4\right)\right]$$
$$+ (\bar{u}+\bar{d})\left[(1-y)^2\left(1-2s^2+\frac{10}{9}s^4\right) + \frac{10}{9}s^4\right]$$
$$+ \left[c\left(1-\frac{8}{3}s^2+\frac{32}{9}s^4\right) + s\left(1-\frac{4}{3}s^2+\frac{8}{9}s^4\right)\right]\left[1+(1-y)^2\right]\Big\} \qquad (13.9)$$

Keeping only the valence quarks, we obtain from (11.10), (11.11) and (13.9) the ratio

$$R_\nu \equiv \frac{\left(d\sigma^{\nu N}/dx\right)_{NC}}{\left(d\sigma^{\nu N}/dx\right)_{CC}} = \frac{1}{2} - s^2 + \frac{20}{27}s^4 \quad . \qquad (13.10)$$

Following similar steps we obtain the corresponding ratio for antineutrinos

$$R_{\bar\nu} \equiv \frac{\left(d\sigma^{\bar\nu N}/dx\right)_{NC}}{\left(d\sigma^{\bar\nu N}/dx\right)_{CC}} = \frac{1}{2} - s^2 + \frac{20}{9}s^4 \quad . \qquad (13.11)$$

The ratios R and $\bar{R}$ are used extensively in the determination of $s^2 = \sin^2\theta_W$. The results in (13.10) and (13.11) neglect the effects of other quark distributions and represent a first-order approximation. In data analysis one should use the complete expressions of (13.6) and (13.9).

The PW-relation [6] is frequently used to determine $\sin^2\theta_W$. It reads

$$D = \frac{\sigma_{NC} - \bar\sigma_{NC}}{\sigma_{CC} - \bar\sigma_{CC}} = \frac{1}{2} - \sin^2\theta_W \qquad (13.12)$$

where σ and $\bar\sigma$ are the cross sections for neutrinos and antineutrinos on isoscalar targets. This relation holds for the differential cross section as well, provided the same cuts in x and y are taken in the numerator and denominator. It is remarkably stable to corrections.

3. The hadronic model

Equ. (13.12) was originally derived [6] in the GIM-model with four quarks and we repeat its derivation with six quarks [7]. The charged current is obtained from the Kobayashi-Maskawa matrix

$$J_\mu^{CC} = J_{1\mu}^+ c_1 + J_{2\mu}^+ s_1 c_3 + J_{3\mu}^+ s_1 c_2 + \left(c_1 c_2 c_3 + s_2 s_3 e^{i\delta}\right) J_{4\mu}^+ , \tag{3.1}$$

with c_i (s_i) denoting $\cos\theta_i$ ($\sin\theta_i$), the angles appearing in the KM matrix. The quantum numbers and quark content of the current are shown in table 1. For isoscalar targets charged current transitions produce final states which differ by at least one quantum number and thus contribute incoherently.

The neutral current is of the form

$$J_\mu^{NC} = J_\mu^3 - 2\sin^2\theta_W J_\mu^{em} + \gamma J_\mu^s + \delta J_\mu^c + \cdots$$

$$= A_\mu^3 + \left(1 - 2\sin^2\theta_W\right) V_\mu^3 - 2\sin^2\theta_W V_\mu^0 + \cdots , \tag{3.2}$$

where $J^i = V^i - A^i$ is the ith component of the usual isospin current, J^{em} is the electromagnetic current and $J^s, J^c, \cdots$ are isoscalar currents consisting of strange, charm and heavier quarks. Their structure is shown in table 2. For isoscalar targets there is no isoscalar-isovector interference. In the deep inelastic region there is no interference with the J^s, J^c current as well, since they produce distinct final states.

TABLE 1
Structure of the charged currents

Current \ Property	ΔI_3	Δs	Δc	Quark transition
$J_{1\mu}^+$	1	0	0	d → u
$J_{2\mu}^+$	$\frac{1}{2}$	1	0	s → u
$J_{3\mu}^+$	$\frac{1}{2}$	0	1	d → c
$J_{4\mu}^+$	0	1	1	s → c

I wish to thank North-Holland Publishing Company
for using pages 194-197 from Nuclear Physics B194(1982).

TABLE 2
Structure of the neutral currents

Current \ Property	Transformation	Quark structure
$V_\mu^3 - A_\mu^3$	isovector, V − A	$\bar u \gamma_\mu (1-\gamma_5) u - \bar d \gamma_\mu (1-\gamma_5) d$
V_μ^0	isoscalar/baryon	$\bar u \gamma_\mu u + \bar d \gamma_\mu d$
J_μ^s	strange	$\bar s s$
J_μ^c	charm	$\bar c c$

The charged current cross section has the form

$$\sigma_{CC}^{(\nu,\bar\nu)} = \sum_i \left(V_i + A_i \pm I_i'\right) \tag{3.3}$$

with the V (A) denoting the contribution to the cross section from the vector (axial) current alone and I' is the interference term. The sum runs over the four currents in eq. (3.1) and the angles have been suppressed. The difference of neutrino- and antineutrino-induced reactions involves only $F_3(x, Q^2)$ structure functions. Its explicit form is

$$\sigma_{CC}^{\nu} - \sigma_{CC}^{\bar\nu} = 2\left[I_1 c_1^2 + I_2 s_1^2 c_3^2 + I_3 s_1^2 c_2^2 + I_4 |c_1 c_2 c_3 + s_2 s_3 e^{i\delta}|^2\right] \tag{3.4}$$

with the weak angles now explicitly exhibited. For kinematic regions that investigate short distances,

$$I_1 = I_3 , \tag{3.5}$$

since they both involve d-quark distribution functions.

Similarly the neutral current difference,

$$\sigma_{NC}^{\nu} - \sigma_{NC}^{\bar\nu} = \left(1 - 2\sin^2\theta_W\right) I_0^3 , \tag{3.6}$$

is the isovector vector-axial interference term. There is no contribution from V_μ^0, since there is no axial part term to interfere with. An isoscalar term could arise from the J_μ^s but its contribution vanishes for $s(x) = \bar s(x)$. This equality for the strange quark distributions is expected in QCD, where $s\bar s$ pairs are radiated by gluons. Later on, we estimate a very small correction that would arise from a difference between $s(x)$ and $\bar s(x)$.

From (3.4) and (3.6) we obtain

$$D = \frac{\sigma_{NC} - \bar\sigma_{NC}}{\sigma_{CC} - \bar\sigma_{CC}} = \left(\tfrac{1}{2} - \sin^2\theta_W\right) \frac{1}{s_1^2 c_2^2 \xi + c_1^2} + O(1\%), \tag{3.7}$$

where $\xi = I_3/I_1$. The parameter ξ measures the excitation of charm in charged current reactions. Far above the threshold for production of charm ξ is one; as indicated in eq. (3.5). For $\xi = 1$ and $c_2 = 1$ eq. (3.7) reduces to

$$D = \tfrac{1}{2} - \sin^2\theta_W \tag{3.8}$$

which is the relation derived [6] in the GIM model. At present energies the production of charm may not be fully developed. This dependence was studied by $\mu^- e^+$ and $\mu^-\mu^+$ events in neutrino and antineutrino charged current interactions and the excitation curve was understood by slow rescaling factors [8]. We folded the neutrino spectrum of the CDHS experiment with the charm excitation curves and obtained that more than 80% of the asymptotic charm production is already excited in the CDHS experiment [8]*. The correction arising from this source is positive (increases $\sin^2\theta_W$) by

$$+1.0\% \text{ for } \sin\theta_2 = 0.00,$$

$$+2.4\% \text{ for } \sin\theta_2 = 0.60.$$

This is the allowed range for $\sin\theta_2$ as obtained from other experiments [18]. The maximum correction to $\sin^2\theta_W$ from this source is $+0.005$, but it could be smaller. A smaller correction is obtained when $\sin\theta_2$ is small or when ξ is determined to be closer to unity. Both of these parameters can be better understood in future experiments.

Instead of the previous general method, the relation is also derived in the parton model. Restricting to the four quarks of the GIM model its differential form is

$$\left(\frac{d\sigma}{dx\,dy} - \frac{d\bar\sigma}{dx\,dy}\right)_{NC} \Bigg/ \left(\frac{d\sigma}{dx\,dy} - \frac{d\bar\sigma}{dx\,dy}\right)_{CC}$$

$$= \frac{\left[1-(1-y)^2\right]\left\{(u+d-\bar u-\bar d)(1-2\sin^2\theta_W) + (s-\bar s)\left(1-\tfrac{4}{3}\sin^2\theta_W\right) + (c-\bar c)\left(1-\tfrac{8}{3}\sin^2\theta_W\right)\right\}}{2\left\{\left[1-(1-y)^2\right](u+d-\bar u-\bar d) + 2(s-\bar s) + 2(\bar c-c)(1-y)^2\right\}}. \tag{3.9}$$

It is evident that for $s = \bar s$ and $c = \bar c$ it reduces to eq. (3.8).

The specific form of eq. (3.9) was derived in the parton model and in principle could be modified by scaling violations. The corrections from QCD leave it invariant, because it involves only non-singlet structure functions. There are multiplicative scaling violating corrections to both numerator and denominator, but they involve one anomalous dimension and cancel out in the ratio.

* Neutrino spectra for other experiments give similar results. Improved estimates, to be discussed in Chapter 12, reduce the correction to 2.5% .

TABLE 3
Uncertainties in the hadronic model

Origin of uncertainty	Correction to $\sin^2\theta_W$
Charm development	$\leq +0.005$
$s(x) \neq \bar s(x)$, $\bar c(x) \neq c(x)$	± 0.0005
Baryon axial current	-0.0008

Eq. (3.9) also provides an estimate for the correction arising from unequal strange quark and antiquark distributions. Conservation of flavor demands that

$$\int s(x)\,dx = \int \bar s(x)\,dx, \qquad \int c(x)\,dx = \int \bar c(x)\,dx, \qquad \ldots. \tag{3.10}$$

Furthermore QCD predicts that the above structure functions are equal for each point of x. A difference in $\int xs(x)\,dx$ and $\int x\bar s(x)\,dx$ by 15% will produce a change of $\Delta\sin^2\theta_W = \pm 0.0005$.

Finally, it has been argued that a baryon-axial term could arise from higher order gluonic corrections [19]. This term interferes with the V_μ^0 current. Its strength is*

$$A_\mu^B = \frac{1}{4}\left(\frac{\alpha_s}{\pi}\right)^2 \ln\frac{Q^2+m_+^2}{Q^2+m_-^2}\left[\bar u\gamma_\mu\gamma_5 u + \bar d\gamma_\mu\gamma_5 d\right], \tag{3.11}$$

with α_s the strong coupling constant and (m_+, m_-) the quark masses belonging to an isospin doublet. Its contribution to (3.9) is

$$-\frac{1}{6}\left(\frac{\alpha_s}{\pi}\right)^2 \ln\frac{Q^2+m_+^2}{Q^2+m_-}\sin^2\theta_W. \tag{3.12}$$

For $Q^2 = 20\ \text{GeV}^2$ and $\alpha_s = 0.30$ it produces a change $\Delta\sin^2\theta_W = -0.0008$.

A summary of hadronic corrections is shown in table 3. We note that the corrections are small. The largest uncertainty comes from the development of charmed final states in charged current reactions. This uncertainty will be reduced by future experiments. In summary, a determination of the angle by this method is very precise, and with a better understanding of charm development, it would carry a theoretical uncertainty of $\pm 2\%$.

<u>Electron-Nucleon Interactions.</u> Neutrino and antineutrino experiments alone cannot determine whether the neutral current interaction is parity violating. This limitation stems from the fact that neutrino and antineutrino interactions are related to each other by a CP-transformation as discussed in equ. (11.13). This motivated a new class of experiments whose purpose was to measure the electron-nucleon neutral-current interaction. The first experiments were designed to look for parity violating effects in atomic physics. One type of experiment measures the rotation of linearly polarized light as it passes trough Bismuth vapor. The effect is very small, but precise experiments reported the values in Table (13.1)

Atom	Measured Quantity	$R = \mathrm{Re}\ (\frac{E1}{M1})$	Ref.
Bi	ϕ_φ	$(-9.3 \pm 2.9) \times 10^{-8}$	10
"	"	$(-20.6 \pm 3.2) \times 10^{-8}$	11
"	"	$(-2.4 \pm 0.6) \times 10^{-8}$	10
"	"	$(-10.4 \pm 1.7) \times 10^{-8}$	12
Tl	$\frac{d\sigma_L - d\sigma_R}{d\sigma_L + d\sigma_R}$	$(+2.8 \pm 1.0) \times 10^{-3}$ 0.9	13

Table (13.1). Summary of neutral-current effects in Atoms.

Other experiments [13] in Thallium and Caesium measure optical dichroism and saw the effects included in the table. The articles also mention that these results are consistent with the GSW-model for $\sin^2\theta_W = 0.23$.

A manifestly parity-violating effect was observed in an experiment [14] with polarized electrons scattered off deuterons or protons, i.e.

$$e\ (\text{polarized}) + d \rightarrow e' + x$$

The experiment measures the asymmetry

$$A = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$$

where σ_R and σ_L are the cross sections for right-handed and left-handed electrons. The dependence of the asymmetry on the scaling variables is

$$\frac{A(x,y,q^2)}{q^2} = a_1(x) + a_2(x)\,\frac{1-(1-y)^2}{1+(1-y)^2}$$

The functions $a_1(x)$ and $a_2(x)$ depend on the couplings of the neutral current to electrons and nucleons and in addition on the structure functions of the nucleons. Several authors [15] studied the model dependence of the specific form for $a_1(x)$ and $a_2(x)$ and concluded that the dependence is minimal.

The experiment established the y-dependence of the asymmetry in the range $0.14 \leq y \leq 0.40$ and determined a_1 and a_2 separately

$$a_1 = (-9.7 \pm 2.6) \times 10^{-5}$$

$$a_2 = (\ 4.9 \pm 8.1) \times 10^{-5}$$

The results are consistent with the electroweak theory with a mixing angle

$$\sin^2\theta_W = 0.224 \pm 0.020$$

A investigation of the sensitivity of this value on reasonable changes of the model parameters varies the central value between 0.207 and 0.227. A very recent analysis [16] of radiative corrections reports an additional reduction of - 0.007.

References Ch. 13

1. The structure of the neutral currents was deciphered at stages with contributions from many authors. Here I list papers which made specific contributions to the final solution
   L.M. Sehgal, Phys. Lett. 71B, 99 (1977)
   P.Q. Hung, Phys. Lett. 69B, 216 (1977)
   P.Q. Hung and J.J. Sakurai, Phys. Lett. 72B, 208 (1977)
   G. Ecker, Phys. Lett. 72B, 450 (1978)
   L.F. Abbott and R.M. Barnett, Phys. Rev. Lett. 40, 1303 (1978)
   E.A. Paschos, Phys. Rev. D15, 83 (1979)
   M. Claudson, E.A.Paschos, L.R. Sulak and J.Straight, Phys. Rev.
2. For a recent review:
   P.Q. Hung and J.J. Sakurai, Annual Review of Nuclear Science (1981) and references therein.
3. J.E. Kim, P. Langacker, M. Levine and H.H. Williams, Rev. Mod. Phys. 53, 211 (1981)
4. I. Liede and M. Roos, Nucl. Phys. B167, 397 (1980)
5. M. Claudson et al. ibid 1
6. E.A. Paschos and L. Wolfenstein, Phys. Rev. D7, 91 (1973)
7. The following four pages are from a recent article by E.A. Paschos and M. Wirbel, Nucl. Phys. B134, 189 (1982)
8. J.G.H. de Groot et al., Z. Phys. C1, 143 (1979)
9. J. Collins, F. Wilcek and A. Zee, Phys. Rev. D18, 242 (1978)
10. J. Missimer, Habilitationsarbeit, Univ. of. Mainz (1982)
11. L.M. Barkov and M.S. Solotorev, Phys. Lett. 85B, 309 (1979)
12. J.H. Hollister et al., Phys. Rev. Lett. 46, 643 (1981)
13. P. Bucksmaun, E. Commins and L. Hunter, Phys. Rev. Lett. 46, 640 (1981)
14. C.Y. Prescott, et al., Phys. Lett. 84B, 347 and 525 (1978)
15. J.D. Bjorken, Phys. Rev. D18, 3239 (1978); L. Wolfenstein, Nucl. Phys. B146, 477 (1978)
16. C.H. Llewellyn Smith and J.F. Wheater, Phys. Lett. 105B, 486 (1981)

14. PREDICTIONS FOR HIGGSES.

An important, outstanding problem in the present theory is the origin of the masses. In chapters six and seven we described the origin for the gauge boson masses. They originate from the Higgs Lagrangian with one of the Higgs mesons acquiring a finite vacuum expectation value. It was also demonstrated that the gauge transformation (6.11) eliminated three fields ξ_1, ξ_2, ξ_3. At the end there was only one physical field . A related problem is the origin of the fermion masses. They originate, in principle, from the term $\mathcal{L}_Y$ in (6.5), when ϕ_o develops an expectation value. In practice, however, the overall constant G_e is arbitrary and the predictions for fermion masses are limited.

Masses for Higgs mesons are generated by substituting (6.16) into (6.4) and collecting the terms quadratic in η. After substitution we find that the terms linear in η vanish because of the condition

$$v^2 = \mu^2/2 \qquad (14.1)$$

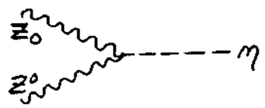
and the term quadratic in η is

$$\left(-\frac{\mu^2}{2} + \frac{3}{2}\lambda v^2\right)\eta\eta = \mu^2\eta\eta \qquad (14.2)$$

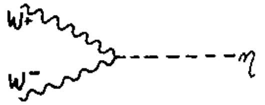
Thus the mass of the Higgs is $M_\eta = \sqrt{2}\,\mu = \sqrt{2\lambda}\,v$. The constant $v \approx 250$ GeV was determined in (7.18), but the Higgs coupling λ is unknown. Assuming that $\lambda \lesssim 1$, for the validity of perturbation theory, we conclude

$$M_\eta \lesssim 1\ \mathrm{TeV}. \qquad (14.3)$$

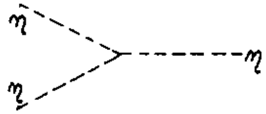
The coupling of Higgs particles are



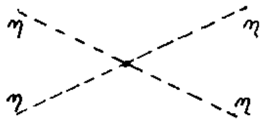
$$\frac{igM_W}{\cos^2\theta} g_{\mu\nu}$$



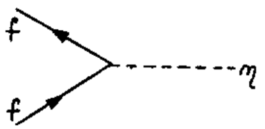
$$\frac{-i}{4} g^2 v g_{\mu\nu} = ig M_W g_{\mu\nu} = \sqrt{G} M_W^2 g_{\mu\nu}$$



$$-i\,3!\lambda v \sim M_\eta^2 \sqrt{G}$$



$$-i\,3!\lambda$$



$$\frac{m_f}{v} = \sqrt{2G}\, m_f$$

It is clear from the above that the Higgses couple more strongly to the heavier particles. Thus the production of Higgses by (e, μ, ν_e, ν_μ) and light quarks is miniscule. In the same spirit the <u>tree level</u> couplings to photons $g_{\gamma\gamma H}$ or gluons g_{GGH} are zero. The same observation suggest that Higgses will decay predominantly into heavy fermions and bosons.

<u>Masses.</u> We already remarked on the upper bound (14.3) that follows from validity of perturbation theory. This bound could perhaps be improved by considering partial wave unitary [2] with the result

$$M_\eta^2 \leq \frac{8\pi\sqrt{2}}{3G} = 1\ (\text{TeV})^2 \qquad (14.8)$$

Lower bounds follow from the stability of the vacuum. In this case, the requirement that radiative corrections to the potential should leave $\phi_{min} \neq 0$ translates [3] into the condition

$$M_\eta \geq 7.2 \text{ GeV}.$$

In the model of Coleman and Weinberg [4] the symmetry is dynamically broken. One begins in an theory with $\mu^2 = 0$, $\lambda > 0$ and generates minima through radiative corrections. This method fixes the mass of the Higgs meson

$$M_{H^0} = 10.4 \pm {}^{0.5}_{0.4} \text{ GeV} \quad \text{for} \quad \sin^2\theta_W = 0.20 \pm 0.01.$$

The value is just above the masses of the produced $(Q\bar{Q})$ bound states $\Upsilon, \Upsilon', \ldots,$ but below the $(t\bar{t})$-bound states.

<u>Decays.</u> Higgses can decay in any of the following channels, provided they are allowed by energy considerations

$$\begin{aligned} H^0 &\longrightarrow Z^0 Z^0 \\ &\longrightarrow W^+ W^- \\ &\longrightarrow l^+ l^- \\ &\longrightarrow q\bar{q} \\ &\longrightarrow GG \\ &\longrightarrow \gamma\gamma \end{aligned}$$

Realistic decays are into quarks and leptons. The widths are

$$\Gamma\left(H^0 \to \ell^+\ell^-\right) = \frac{G m_\ell^2}{4\sqrt{2}\,\pi} m_H \left(1 - \frac{4m_\ell^2}{m_H^2}\right) \qquad (14.9)$$

and

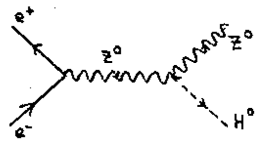
$$\Gamma\left(H^0 \to q\bar{q}\right) = 3\frac{G m_q^2}{4\sqrt{2}\,\pi} m_H \left(1 - \frac{4m_q^2}{m_H^2}\right) \qquad (14.10)$$

where the factor of 3 in (14.10) is due to color. The decays are stronger into the heaviest fermions. They are very fast and leave no detectable tracks.

Decays into W's and Z's require very high masses. The widths in this case are $\gtrsim 1$ GeV.

<u>Production.</u> Of practical importance is the production and detection of Higgses. We discuss four promising possibilities.

1) Electron-positron rings can directly produce Higgses through the reaction



$$e^+ + e^- \longrightarrow Z^0 + H^0$$

The ZZH coupling in (14.4) is large and one expects large cross sections. The signature is a Z^0 with its conventional decays and a recoiling H^0. For the cross section we define the ratio

$$R_{Z^0H^0} = \frac{\sigma(e^+e^- \to Z^0H^0)}{\sigma(e^+e^- \to \mu^+\mu^-)} = \frac{3}{64}\left(\frac{m_Z}{38\,\text{GeV}}\right)^4 \frac{m_Z}{2\sqrt{2}\,m_H}\left[1+(1-4\sin^2\theta_W)^2\right]$$

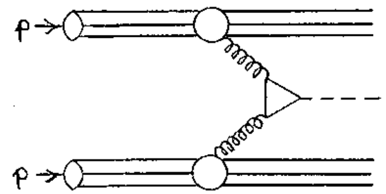
Values of the ratio at $\sqrt{S} = M_Z + \sqrt{2}\,M_H$ are given in table (14.1). Clearly for these mass ranges

M_H (GeV)	$\sqrt{S}$ GeV	R_{Z^0H}
10	104	4.7
30	132	1.56
60	175	0.78

Table (14.1)

the production rate is comparable to the point cross section.

2) In proton-proton reactions the largest contribution comes from the gluon-fusion mechanism. Estimates of the cross section are in the range $\sim 10^{-34}$ cm^2 for $\sqrt{s} = 800$ GeV and $M_H = 11$ GeV. This is comparable to Drell-Yan pairs of the same mass, which makes their identification difficult.



3) A very interesting and promising mechanism was proposed by Wilczek [5]. In this case Higgses are produced in the decays of heavy Quarkonia states

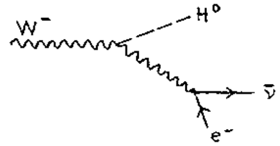
$$V_{Q\bar{Q}} \longrightarrow H^0 + \gamma$$

The signal is a characteristic two body decay with monochromatic gammas. Its width relative to the electromagnetic decay is

$$\frac{\Gamma(V_{Q\bar{Q}} \to H^0+\gamma)}{\Gamma(V_{Q\bar{Q}} \to \gamma \to e^+e^-)} = \frac{G}{4\sqrt{2}\,\pi}\,\frac{m_V^2}{\alpha}\left(1-\frac{m_H^2}{m_V^2}\right) \qquad (14.11)$$

$$\approx \quad 10\% \quad \text{for} \quad m_V = 36 \text{ GeV}$$

4) If the physical Higgs masses are lighter than W and Z bosons, they can decay through the second order diagrams (for $m_H \ll m_W$)

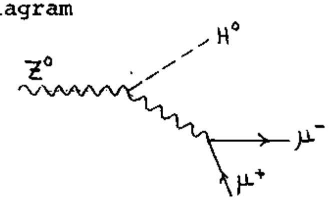


$$W^- \to H^0 + e^-\bar{\nu}$$

Compared to the leptonic decay

$$B_H^W = \frac{\Gamma(W^- \to H^0 e\bar{\nu})}{\Gamma(W^- \to e\bar{\nu})} = \frac{\alpha}{4\pi\sin^2\theta_W}\left[\ln\frac{2m_H}{m_W} + \frac{23}{24}\right.$$

This formula indicates that for light Higgses there is a logarithmic enhancement. For $m_H \approx 1$ GeV it gives $B_H \approx 1\%$. Similarly, Higgses will be produced in the Z^0 decays through the diagram



$$Z^0 \rightarrow H^0 + \mu^+\mu^-$$

with $$B_H^2 = \frac{\Gamma(Z \rightarrow H^0\mu^+\mu^-)}{\Gamma(Z \rightarrow \mu^+\mu^-)} = \frac{\alpha}{(\sin 2\theta_w)^2}\,\frac{1}{\pi}\left[\ln\frac{2m_H}{m_Z} + \frac{23}{24}\right]$$

The signature in this decay mode is a pair of muons with definite mass.

These are some of the methods for searches of fundamental Higgses. There are other possibilities discussed in the review articles. The discussion of the chapter was limited to the standard model with one physical particle. Other models with more Higgses including charged ones have being contemplated. There are also alternative models without Higgses. The subject of mass generation and the role of the Higgses is still an open and active topic of research.

References

(1) The subject of Higgses is reviewed by

J. Ellis, M.K. Gaillard and D.V. Nanopoulos, Nucl. Phys. B106, 292 (1976)

A. Ali, Proceedings of the 1981 Isabelle Summer Workshop pg. 192

L.-F. Li, Properties of Higgs Particles Carnegie-Mellon preprint COO-3066-145 (1980)

(2) M. Veltman, Phys. Lett. 70B, 253 (1977)

B.W. Lee, C. Quigg and H. Thacker, Phys. Rev. Lett. 38, 883 (1977)

(3) A. Linde, JETP Lett. 23, 64 (1976);
S. Weinberg, Phys. Rev. Lett. 36, 294 (1976)

(4) S. Coleman and E. Weinberg, Phys. Rev. D7, 1888 (1973)

(5) F. Wilczek, Phys. Rev. Lett. 39, 1304 (1977).

15. THE METHOD OF COUNTERTERMS.

The discussion of the previous sections and especially the applications were based on Born diagram computations. We had the opportunity to refer occasionally to higher-order corrections whose main discussion was postponed for this part. The fact that the new theory is renormalizable is a great advantage. It makes possible the computation of higher order corrections and the precise determination of physical parameters. This subject is extensive and difficult. Here we will develop it in several stages.

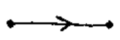
For reasons of clarity consider first the fermion part of the Lagrangian

$$\mathcal{L}_{1B} = \bar{\Psi}_{R0}\, i\gamma^\mu\left(\partial_\mu - ig_0'B_{0\mu}\right)\Psi_{R0} + \bar{\Psi}_{L0}\, i\gamma^\mu\left(\partial_\mu - \frac{i}{2}g_0'B_{0\mu} - i\frac{\tau^i}{2}g_0 A^i_{0\mu}\right)\Psi_{L0}$$

$$+ \text{ fermion mass terms} \qquad (15.1)$$

This we call the bare Lagrangian and all fields and coupling constants are bare quantities indicated by the subscript zero. If we take the above Lagrangian and calculate Born (tree) diagrams the results are finite, but in one-loop or many-loop diagrams we encounter infinities. The theory as stated is incomplete. The aim of renormalization is to give a method for dealing with the infinities by absorbing them into a few arbitrary constants.

Consider the fermion propagator. The term $\bar{\Psi}_0(i\gamma - m_0)\Psi_0$ in the Lagrangian gives the free propagator



$$= \frac{i}{\not{p} - m_0} \qquad (15.2)$$

which is then modified by higher-order interactions. In perturbation theory one can compute one or more loop diagrams, like those in figure (15.1). The first difficulty is how to handle infinite quantities. The method of handling infinities is known as regularization. There are several methods to regularize infinite integrals to be discussed at the end of this chapter. For the moment, we can think that we make the integrals finite by introducing a large momentum cut-off. Next we define the fermion self energy as the sum of the diagrams in figure (15.1)

$$-i\Sigma(\not{p}, m) =$$

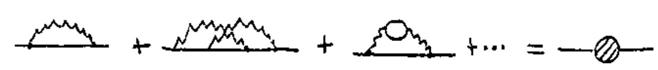


Fig. (15.1)

Each diagram has the special property that cutting an internal line does not produce two disconnected diagrams. Such diagrams are called one-particle-irreducible. A final piece of notation: When an external line ends in a dot, then the fermion propagator for this line must be included. In order to obtain the full fermion propagator we add all reducible diagrams in figure (15.2) obtaining

$$\frac{i}{\not{p} - m_0} + \frac{i}{\not{p} - m_0}\cdot -i\Sigma(\not{p}, m_0)\frac{i}{\not{p} - m_0} + \cdots = \frac{i}{\not{p} - m_0 - \Sigma(\not{p}, m_0)} \qquad (15.3)$$

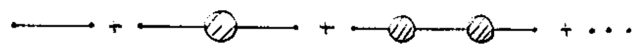


Fig. (15.2)

The new propagator does not have its pole at the position $\not{p} = m_0$ as in the free propagator but at a new position m_R. The position of the new pole depends on m_0 and the cut-off . In addition to the change of the position of the pole, the residue at the new pole is no longer one. We can define it formally by expanding around the point m_R.

$$\Sigma(\not{p}, m_0) = \Sigma(m_R) + (\not{p} - m_R)\left(\frac{\partial \Sigma}{\partial \not{p}}\right)_{m_R} + \cdots$$

$$= \Sigma_1 + (\not{p} - m_R)\Sigma_2 + \cdots \tag{15.4}$$

Upon substitution

$$\frac{i}{\not{p} - m_0 - \Sigma(\not{p}, m_0)} = \frac{i}{(\not{p} - m_R)(1 - \Sigma_2)} = \frac{iZ_2}{\not{p} - m_R} \tag{15.5}$$

with $m_R = m_0 + \Sigma_1$ and $Z_2 = \frac{1}{1 - \Sigma_2}$. We can make the residue at the pole equal to one, by absorbing Z_2 into a redefinition of the fermion wave function

$$\Psi_0 = Z_2^{1/2}\Psi_R \tag{15.6}$$

Ψ_R is now the renormalized fermion field.

We have discussed the fermion propagator in detail, since the same steps can be repeated for the propagators of gauge bosons. Again we begin with a free propagator which is modified by higher order interactions. The position of the new pole

$$\frac{-i}{p^2 - M_0^2 - \Pi(p^2, M_0, \Lambda)}$$

is denoted by M_R. We expand $\Pi(p^2, M_0, \Lambda)$ around the new pole

$$\Pi(p^2, M_0, \Lambda) = \Pi_1 + (p^2 - M_R^2)\Pi_2 + \cdots \tag{15.7}$$

and define the renormalized mass $M_R^2 = M_0^2 + \Pi_1$ and a wave function renormalization

$$Z_3 = \frac{1}{1 - \Pi_2} \tag{15.8}$$

We apply these steps to each of the gauge fields in (15.1) defining renormalized fields

$$B_{0\mu} = Z_B^{1/2} B_\mu \quad \text{and} \quad A^i_{0\mu} = Z_A^{1/2} A^i_\mu \tag{15.9}$$

Additional infinities occur in the vertices from the diagrams.

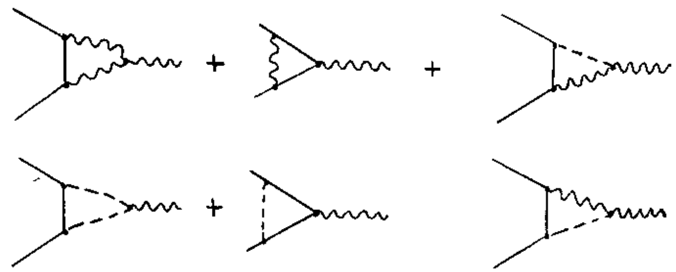


Fig. (15.3)

Here the wavy lines are gauge bosons and the dotted lines Higgses. We absorb these infinities into vertex renormalization functions and define renormalized coupling constants as

$$i g_0 \Gamma_\mu(p', p) = i g Z_g^{-1} \gamma_\mu \tag{15.10a}$$

$$i g'_0 \Gamma_\mu(p', p) = i g' Z_{g'}^{-1} \gamma_\mu \tag{15.10b}$$

The renormalized fields and coupling constants are finite quantities. In terms of them the Lagrangian reads

$$\mathcal{L}_{18} = Z_2 \bar{\Psi}_R i\gamma^\mu\left(\partial_\mu + i g' Z_{g'} Z_B^{1/2} B_\mu\right)\Psi_R$$
$$+ Z_2 \bar{\Psi}_L i\gamma^\mu\left(\partial_\mu + \frac{i}{2} g' Z_{g'} Z_B^{1/2} B_\mu + i\frac{\tau^i}{2} g Z_g Z_A^{1/2} A^i_\mu\right)\Psi_L \tag{15.11}$$

The infinities, of course, did not go away, but they are contained in the renormalization functions Z_i. In perturbation theory each renormalization function is a power series in the coupling constant

$$Z_i = 1 + \delta Z_i = 1 + g^2 f_1 + g^4 f_4 + \cdots \tag{15.12}$$

where the functions f_i are independent of the coupling constants. Finally introducing

$$\delta g' = g' \left(\delta Z_{g'} + \tfrac{1}{2} \delta Z_B \right) \qquad (15.13)$$

and
$$\delta g = g \left(\delta Z_g + \tfrac{1}{2} \delta Z_B \right) \qquad (15.14)$$

we obtain

$$\mathcal{L}_{1B} = \mathcal{L}_R + \mathcal{L}_{cT} \qquad (15.16)$$

with

$$\mathcal{L}_R = \overline{\Psi}_R i\gamma^\mu \left(\partial_\mu + i g' B_\mu \right) \Psi_R + \overline{\Psi}_L i\gamma^\mu \left(\partial_\mu + \tfrac{i}{2} g' B_\mu + i \tfrac{\tau^i}{2} g A^i_\mu \right) \Psi_L \qquad (15.17)$$

and

$$\mathcal{L}_{cT} = \overline{\Psi}_R i\gamma^\mu \left[\delta Z_2 \partial_\mu + i g' \left(\delta Z_2 + \frac{\delta g'}{g'} \right) B_\mu \right] \Psi_R$$

$$+ \overline{\Psi}_L i\gamma^\mu \left[\delta Z_2 \partial_\mu + i g' \left(\delta Z_2 + \frac{\delta g'}{g'} \right) B_\mu - i \frac{\tau^i}{2} \left(\delta Z_2 + \frac{\delta g}{g} \right) A^i_\mu \right] \Psi_L \,. \qquad (15.18)$$

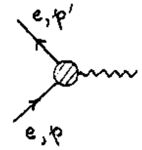
The renormalized Lagrangian L_R contains only renormalized quantities, which will be identified with physical observables. The second term L_{CT} contains the counter terms. Their role is to cancel the infinities generated by L_R.

Consider a realistic situation. At the tree level, amplitudes from L_R are finite. At the one loop level L_R generates infinities, but the counterterms have the same form to cancel out these infinities. The property that the counterterms have the same structure as the terms in L_{1B} is a characteristic of a renormalizable theory. Renormalization then amounts to a rescaling of the bare fields and the parameters of the theory and does not result in any physical effects.

QED as an example. The bare Lagrangian in quantum electrodynamics has the form

$$\mathcal{L}_o = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} - \tfrac{1}{2} \left(\partial_\mu A^\mu \right)^2 + \overline{\Psi} \left(i\gamma\cdot\partial - m \right) \Psi - e \overline{\Psi} \gamma_\mu \Psi A^\mu \qquad (15.19)$$

After correction the photon propagator has a pole at $p^2 = 0$ whose residue defines the photon wave - function renormalization Z_3. The electron has a pole at $m_R = m - \delta m$ and a residue Z_2, the electron wave function renormalization. The corrected electron-photon vertex



$$= ie\,\Gamma(p', p) = i \frac{e}{Z_1} \gamma_\mu \quad \text{at } p = p' \text{ and } p^2 = m_R^2 \,. \qquad (15.20)$$

In terms of them and renormalized fields the Lagrangian takes the form§§

$$\mathcal{L}_o = -\tfrac{1}{4} Z_3 \left(\partial_\mu A^R_\nu - \partial_\nu A^R_\mu \right)^2 - \tfrac{1}{2} Z_3 \left(\partial^\mu A^R_\mu \right)^2 + Z_2 \overline{\Psi}_R \left(i\gamma\cdot\partial - m_R \right) \Psi_R$$

$$- Z_2 \,\delta m\, \overline{\Psi}_R \Psi_R - Z_2 Z_3^{1/2} e\, \overline{\Psi}_R \gamma_\mu \Psi_R A^\mu_R \qquad (15.21)$$

We introduce a renormalized charge $e_R = Z_1^{-1} Z_2 Z_3^{1/2} e$ and the interaction term becomes

$$- Z_1 e_R \overline{\Psi}_R \gamma_\mu \Psi_R A^\mu_R$$

In QED the Ward identity requires

$$Z_1 = Z_2 \qquad (15.22)$$

to all orders and the formulas simplify somewhat. Finally we can use (15.12) in order to obtain the renormalized Lagrangian and the counter terms.

§§ In this example the index R denotes renormalized fields. Ψ_R is not a right-handed fermion field, but the wave-function of an electron.

<u>Regularization of Integrals.</u> Before we carry through the renormalization program, it is necessary to give meaning to divergent integrals. Several procedures have been devised. The most widely used method is dimensional regularization because it respects the gauge invariance of the theory.

In dimensional regularization the integrals are considered in n-dimensions. By this method a new parameter is introduced, the dimension of space. The physical situation obtains in the limit $n \to 4$. Consider the integral

$$I_n = \int \frac{d^n p}{(2\pi)^n} \frac{1}{(p^2 - \Delta + i\varepsilon)^\alpha} \qquad \text{with} \qquad \Delta > 0 \quad . \tag{15.23}$$

For $\alpha = 2$ the integral converges in one, two and three dimensions, but it diverges in 4-dimensions. We will treat n as a complex variable and consider the analytic continuation of I_n. With the help of a Wick's rotation

and
$$p_0 \to i p_0$$

$$I_n = (-1)^\alpha i \int \frac{d^n p}{(2\pi)^n} \frac{1}{(p^2 + \Delta)^\alpha} \tag{15.24}$$

with p now an n-dimensional Euclidean vector. The problem is reduced to an integration over a Euclidean space, where in spherical coordinates

$$d^n p = p^{n-1}\, dp\, d\Omega_n \quad .$$

The integrand is angle independent giving

$$I_n = \frac{i(-1)^\alpha}{(2\pi)^n} \frac{2\pi^{n/2}}{\Gamma(n/2)} \int_0^\infty dp \, \frac{p^{n-1}}{(p^2 + \Delta)^\alpha} \tag{15.25}$$

The last integral is now done with the help of Γ-functions (as in Jahnke + Emde pg. 20)

$$I_n = \int \frac{d^n p}{(2\pi)^n} \frac{1}{(p^2 - \Delta + i\varepsilon)^\alpha} = \frac{i(-1)^\alpha \pi^{n/2}}{(2\pi)^n} \frac{\Gamma(\alpha - n/2)}{\Gamma(\alpha)\, \Delta^{\alpha - n/2}} \tag{15.26}$$

For the case $\alpha = 2$ a pole in the Γ-function occurs when $n \to 4$. Extensions of the method and tables of integrals can be found in books [1,2].

<u>References Ch. 15</u>

1. G. 't Hooft and M. Veltman, Diagrammar CERN-preprint 73-9 (1973)
2. P. Ramond, Field Theory, A Modern Primer, Appendix B, The Benjamin/Cummins Publishing Comp. (1981).

16. RENORMALIZATION OF THE ELECTROWEAK THEORY [1]

In the electroweak theory the renormalized Lagrangian describing the vector boson-lepton interactions and their masses is

$$\mathcal{L}_R = \frac{g}{2\sqrt{2}} W_\mu^+ \left[\bar{\nu} \gamma^\mu (1-\gamma_5) L \right] + \frac{g}{2\sqrt{2}} W_\mu^- \left[\bar{L} \gamma^\mu (1-\gamma_5) \nu \right]$$
$$+ \frac{g}{4c} Z_\mu \left[\bar{\nu} \gamma^\mu (1-\gamma_5) \nu \right] + gs A_\mu \bar{L} \gamma_\mu L + \frac{g}{4c} Z_\mu \left[\bar{L} \gamma^\mu (4s^2 - 1 + \gamma_5) L \right]$$
$$+ M_W^2 W_\mu^+ W_\mu^- + \frac{1}{2} \frac{M_W^2}{c^2} Z_\mu Z_\mu + \text{fermion mass terms.} \qquad (16.1)$$

All fields and coupling constants are renormalized quantities with c and s being $\cos\theta_W$ and $\sin\theta_W$. We identify

g with the SU(2) coupling
gs with the electromagnetic charge
M_W with the mass of the $W^\pm$ bosons.

The masses of the fermions are, in any case, arbitrary (see pg.50) so that absolute predictions are impossible. Again computation of Born diagrams gives finite results. This is the term of the Lagrangian that is used in most applications. It is straight forward to apply the methods of the previous chapter and generate the counterterms, which are given in Appendix (16.A).

Using $\mathcal{L}_R$ we encounter infinities at the one loop level. The lowest order counterterms cancel out these infinities. The renormalized amplitudes are determined, except for arbitrary additive constants. The constants are fixed by the renormalization conditions. Here we select to renormalize on the mass-shell. For the gauge bosons

the self energies are defined as

$$q^2 \Pi^i_{\mu\nu}(q^2) = a_i(q^2)\left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right] + b_i(q^2) \frac{q_\mu q_\nu}{q^2} \quad . \qquad (16.2)$$

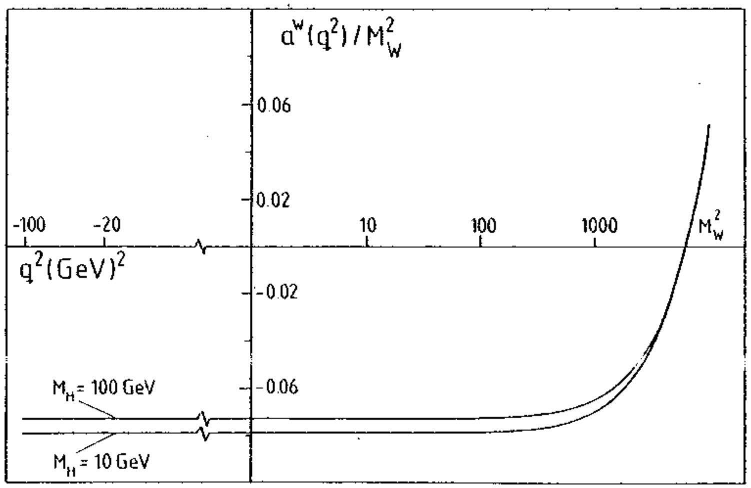
We define renormalized Green functions as

$$a^R(q^2) = a(q^2) + a_{CT} \qquad (16.3)$$

with $a(q^2)$ the contribution from Feynman diagrams dimensionally regularized and a_{CT} the contribution from the counterterms. We impose the conditions

$$a^R_W(q^2 = M_W^2) = a^R_Z(q^2 = M_Z^2) = a^R_\gamma(q^2 = 0)/q^2 = 0 \qquad (16.4)$$

They imply that the masses occurring in L_R are the physical masses. The renormalized self energy $a^R{}_W/M_W^2$ is shown [2] in figure (16.1)



The two curves depend on the masses of the Higgs particles which we have chosen as 10 and 100 GeV/c^2. They are very flat over extended regions of q^2 and vanish at the masses of the gauge bosons. The curves show that the electroweak corrections are -7.3%. The largest contribution comes from intermediate states [2] with light fermions.

The photon self energy also receives the largest contribution from the fermion intermediate states.

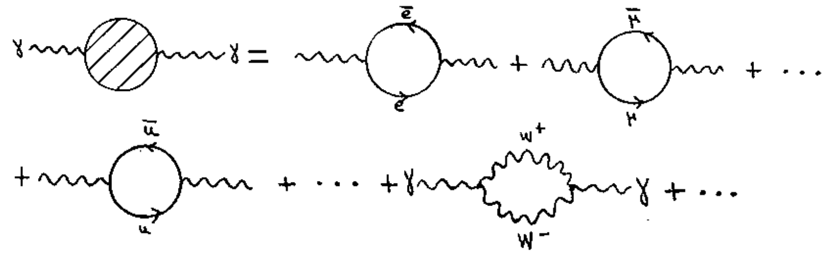


Fig. (16.2)

Diagrams with quark intermediate states are estimated using dispersion relations and include all hadronic interactions. The results are shown [3,4] in figure (16.3). The solid curve is the hadronic contribution and the broken line the sum of hadronic and leptonic terms.

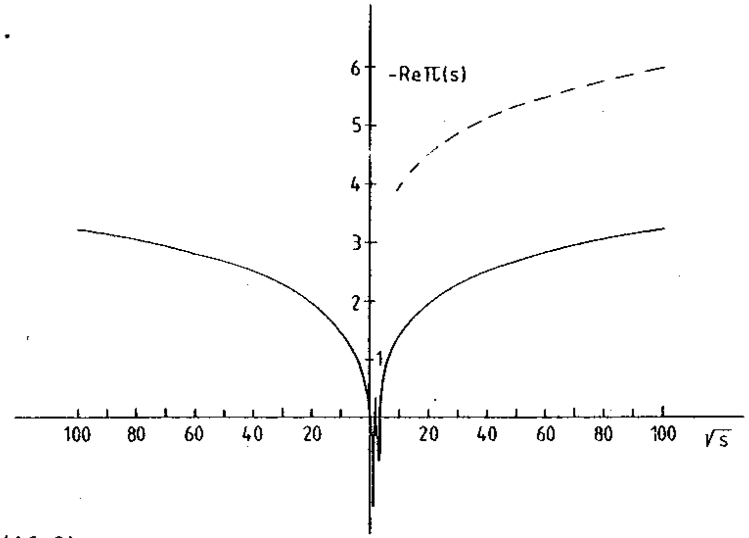


Fig. (16.3)

Even though the theory is now finite, the constants g, gs and M_W occurring in equ. (16.1) are still undetermined. They must be fixed by experiments. For this purpose we consider the following experimental quantities. [5]

(i) The electric charge is defined as the electromagnetic coupling constant measured at very low energies. As measurement [a] we can use the Thomson limit or the determination of $\alpha = \frac{e^2}{4\pi}$ at the Josephson junction.

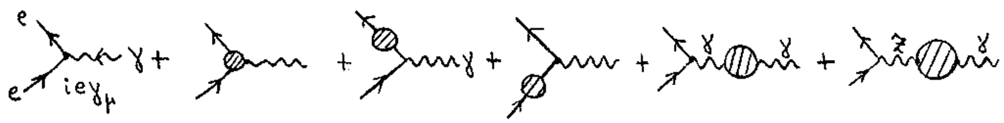
(ii) The muon decay measures G_F and also determines g^2/M_W^2 directly.

(iii) In order to determine $\sin^2\theta_W$ we use a low energy neutral current measurement. This could be the ratio of a neutral current to a charged current neutrino cross section. We wish to determine the parameters including renormalization corrections and we elaborate on each of them separately.

From the Thomson limit we extract

$$e^2 = \left[\frac{g^2 s^2}{1 - \Pi_\gamma(q^2)}\right]_{q^2=0} = g^2 s^2 \qquad (16.5)$$

The last equation follows from the conditions (16.4). The renormalized electron photon vertex is the sum of the diagrams in figure (16.4) plus their corresponding counter terms.



(a) (b) (c) (d) (e) (f)

Fig. (16.4)

In the definition of the electric charge through (16.5) there is the implicit condition that the sum of the terms (b-d) plus their counterterms vanish at $q^2 = 0$. We observe that the definition of electric charge dictates a second renormalization condition.

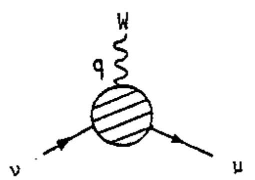
Any weak decay, which is well understood, can serve the purpose of determining g^2/M_W^2. The muon decay is free of hadronic model complications and has been studied extensively. The total decay was computed[6] as

$$\tau_\mu^{-1} = \left\{ \frac{g^2}{8} \left[\frac{1}{q^2 - M_W^2 - \alpha_W^R(q^2)} \right]_{q^2=0} \right\}^2 \frac{m_\mu^5}{192\pi^3} \left(1 - \frac{8m_e^2}{m_\mu^2} \right) \left[1 + \frac{\alpha}{2\pi} \left(\frac{25}{4} - \pi^2 \right) + \cdots \right] \quad (16.6)$$

The origin of the terms is as follows

(i) The Fermi coupling constant G_F is replaced by $\dfrac{g^2}{q^2 - M_W^2 - \alpha_W^R(q^2)}$

with the vacuum polarization correction explicitly in the denominator. This term was plotted in fig. (16.1) and produces a measurable effect. Other corrections from vertices and fermion self energies are also included, but they are much smaller. Fig. (16.5) shows the correction to the $\nu\mu W$-vertex as a function of q^2, for $0 \leq Q^2 \leq 15000$ GeV^2.



$$= \frac{ig}{2\sqrt{2}} \gamma_\mu \gamma_- \left\{ 1 + F_1(Q^2) \right\} \qquad (Q^2 = -q^2)$$

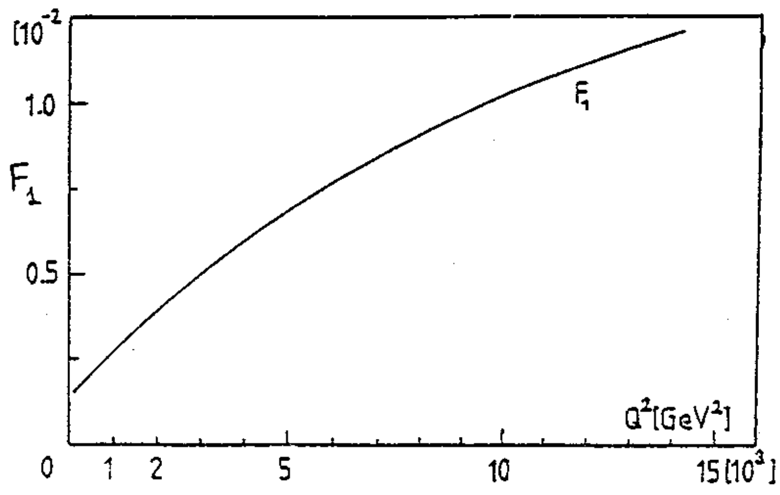


Fig. (16.5)

Vertex corrections for space-like momentum transfers.

ii) The photonic correction from bremsstrahlung and virtual photons have characteristic logs in the fermion masses, but in the total rate they only give the $\frac{\alpha}{2\pi}\left(\frac{25}{4} - \pi^2\right)$ term. A potentially large correction from the sum of diagrams in fig. (17.1) is known to vanish in the decay of muons.

The last experimental measurement could be taken as a leptonic neutral current reaction. Unfortunately, the experimental results for leptonic reactions

$\sin^2\theta_W = 0.22 \pm 0.08$ from $e^+e^- \to \mu^+\mu^-$

and

$$\sin^2\theta_W = 0.25 \pm {}^{0.07}_{0.05} \quad \text{from} \quad \nu_\mu e^- \rightarrow \nu_\mu e^-$$

are not very accurate.
The experimental situation is better in semileptonic reactions where the errors are between 5 - 10 %. The quantities which have been studied in greater detail are the ratios

$$D_- = \frac{\sigma(\nu N \rightarrow \nu X) - \sigma(\bar{\nu} N \rightarrow \bar{\nu} X)}{\sigma(\nu N \rightarrow \mu^- X) - \sigma(\bar{\nu} N \rightarrow \mu^+ X)} = \frac{1}{2} - \sin^2\theta_W \qquad (16.7)$$

and

$$R_\nu = \frac{\sigma(\nu N \rightarrow \nu X)}{\sigma(\nu N \rightarrow \mu^- X)} = \frac{1}{2} - \sin^2\theta_W + \frac{20}{27}\sin^4\theta_W \qquad (16.8)$$

on isoscalar targets, denoted by N. These ratios are computed in terms of the parameters occuring in L_R including the relevant $O(G\alpha)$ corrections.

When this program is carried through we obtain α and $\sin^2\theta_W$ directly. Then from the muon decay we determine M_W. Finally, the ρ-parameter, including radiative corrections, determines M_Z. An outline of the corrections and numerical values are given in the next chapter.

References Ch. 16

1. Specific renormalization schemes were developed by

   D.A. Ross and J.C. Taylor, Nucl. Phys. B51, 125 (1973)

   There are many articles by M. Veltman and collaborators; the following will serve as a guide to their previous work

   M. Green and M. Veltman, Nucl. Phys. B169, 137 (1980)
   G. Passarino and M. Veltman, Nucl. Phys. B160, 155 (1980)
   M. Veltman, Phys. Lett. 91B, 95 (1980)
   A. Sirlin, Phys. Rev. D22, 971 (1980)
   S. Sakakibara, Phys. Rev. D24, 1149 (1981)
   F. Antonelli, G. Corbò, M. Cousoli and O. Pellegrino, Nucl. Phys. B183, 195 (1981)
   T. Appelquist, Broken gauge theories and effective Lagrangians, lectures at the 21 Scottish Un. Summer School in Physics, St. Andrews (1980)

2. R. Murzik, Diplomarbeit University of Dortmund (Dec. 1981)

3. E.A. Paschos, Nucl. Phys. B159, 285 (1979)

4. F.A. Berends and G.J. Komen, Phys. Lett. 63B, 439 (1976)
   W. Wetzel, Z. Phys. C11, 117 (1981)

5. For a parallel discussion see the lectures by
   M. Veltman, Cargese Lectures (1980)

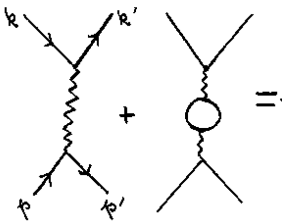
6. Compare with equ. (12.6)

7. H. Usler, Diplomarbeit University of Dortmund (Jan. 1982).

17. The Weak Mixing Angle and its Relation to M_Z and M_W.

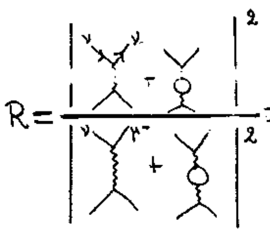
This chapter deals with the calculations of the ratios D- and R_ν including one-loop radiative corrections. In these computations one uses the Feynman rules for L_R and L_{CT} treating $\sin^2\theta_W$, g and M_W as renormalized parameters. They are constants with all radiative corrections explicitly included in the computation of the ratios. Some authors choose different definitions for the parameters by including in them part of the radiative corrections; thus introducing q^2 dependence in the parameters (running coupling constants).

Self energies of Gauge Bosons and the ρ-parameter. We consider a neutrino or antineutrino induced reaction. The sum of Born and gauge boson self energy diagrams is



$$= \frac{g^2}{8(q^2 - M_W^2 - \alpha_W^R(q^2))} \bar{u}(k')\gamma_\mu(1-\gamma_5)u(k)\,\bar{u}(p')\gamma^\mu(1-\gamma_5)u(p) \qquad (17.1)$$

with $a^R_W(q^2)$ defined in equ. (16.3). For a typical ratio



$$R = \frac{M_W^2}{M_Z^2 c^2}\left[1 + 2\frac{\alpha_Z^R(q^2)}{q^2 - M_Z^2} - 2\frac{\alpha_W^R(q^2)}{q^2 - M_W^2}\right]R_0 \qquad (17.2)$$

The renormalized self energies are evaluated at the q^2 of the experiment. They are obtained from explicit calculation of the relevant one loop diagrams plus the addition of the relevant counter-terms. Explicitly,

$$\alpha_W^R(q^2) = \alpha_W(q^2) + \delta Z_\phi M_W^2 - \delta Z_W q^2 + \delta M_W^2 \qquad (17.3)$$

$$\alpha_Z^R(q^2) = \alpha_Z(q^2) + \delta Z_\phi M_Z^2 - \delta Z_Z q^2 + \delta M_Z^2 \qquad (17.4)$$

$a_W(q^2)$, $a_Z(q^2)$ are the contributions from the Feynman diagrams and the terms δz_ϕ, δz_W, δz_Z, δM_W^2 and δM_Z^2 are wave function and mass counter terms. They are given in reference 1 and 2. The renormalized W-self energy was plotted in figure (16.1). A plot for a^R_Z is found in reference 3 . The only uncertainty in these curves is the treatment of intermediate quark states. The weak correction for each self energy is $\approx$ -7%. The quantity relevant to the ratio (17.2) involves a difference and is indeed much smaller $\approx$0.3%.

An additional correction comes from the ρ-parameter

$$\rho = \frac{M_W^2}{M_Z^2\cos^2\theta} = 1 + \sin^2\theta_W\frac{\delta g}{g} + \frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \qquad (17.5)$$

with $\delta g/g$ the coupling constant counter terms. The combined effect from the self energies and the ρ-parameter is a fraction of a percent. Other corrections from fermion self energies, vertices, and box diagrams are also available. They are included in the computations and their combined effect on $\sin^2\theta_W$ is less than 1%. A much larger correction comes from the exchange of photons.

Photonic Corrections.

Several Green's functions receive contributions from diagrams with internal $W^{\pm}$, Z^{0}s and in addition internal photons. Diagrams with internal photons are handled as follows: (i) the high frequency range of integrations, denoted by $\ln(\Lambda/m)$ is combined with other diagrams and counter terms to give finite Green's functions, (ii) the low frequency parts, with finite logarithms and all constant terms, are added to the bremsstrahlung terms. I will refer to them together as the photonic correction. A large photonic logarithm arises from the diagrams in figure (17.1).

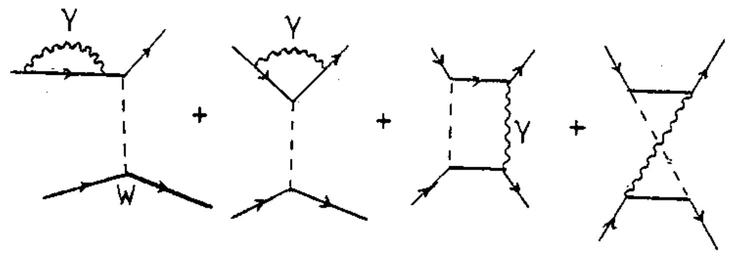


Figure 17.1

Their combined effect is

$$\Delta_M = \Delta_\Sigma + \Delta_V + \Delta_B$$
$$= \frac{\alpha}{\pi}\left\{-\frac{1}{4}\sum_i f_i^2 + \frac{1}{2}\sum_{i,j} f_i f_j + \frac{3}{2}\right\} \ln\left(\frac{M_W^2}{Q^2}\right)$$
$$= \frac{\alpha}{\pi}\ln\left(\frac{M_W^2}{Q^2}\right) \approx 1.3\ \% \text{ for } Q^2 = 20\ (\mathrm{GeV}/c)^2 \qquad (17.6)$$

This term occurs only in the charged currents. The summation i runs over charged fermions and ij over the vertices. The logs are again truncated at Q^2, with the rest combined in the bremsstrahlung terms. During the past year several groups[4-8] pointed out that there is a potentially large logarithm shown in equation (17.6). Its net effect is to lower the mixing angle.

The bremsstahlung terms are harder to handle since they must be computed explicitly in the kinematic regions of the experiments. In the experiments there is a hadronic energy cut which excludes the small region of y and a muon energy cut which excludes the high region of y. I describe the bremsstrahlung corrections as developed in references 4,9, 10. Bremsstrahlung corrections are larger for the charged currents, where the diagrams

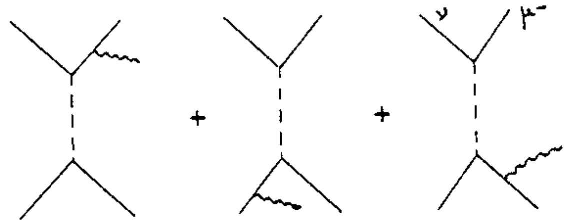


Figure (17.2)

in fig. (17.2) contribute together with diagrams containing virtual photons needed in order to cancel the infrared singularities. We describe the effects from bremsstrahlung by $\delta_{cc}(x,y)$ and $\delta_{cc}(y)$ defined as

$$\frac{d\sigma}{dxdy} = \frac{d\sigma_0}{dxdy}\,[1 + \delta_{cc}(x,y)] \qquad (17.7)$$

and

$$\delta_{cc}(y) = \int_0^1 \frac{d\sigma_0}{dxdy}\,\delta_{cc}(x,y)dx, \qquad (17.8)$$

with the subscript zero indicating the Born diagram cross section. Figures (17.3a,b) show corrections for two parametrizations of the quark distribution functions. We note that for some values of x and y the corrections are ± 10 %. After integration over x the corrections are reduced to those shown in fig. (17.4). The reduction is understood once we realize that the largest correction comes from terms of the form $\log(Q^2/m_f^2)$, where m_f is the mass of a fermion. Furthermore, it is evident that by appropriate choices of the region of integration, in fig. (17.4), we can make the bremsstrahlung correction zero. This is consistent with an old

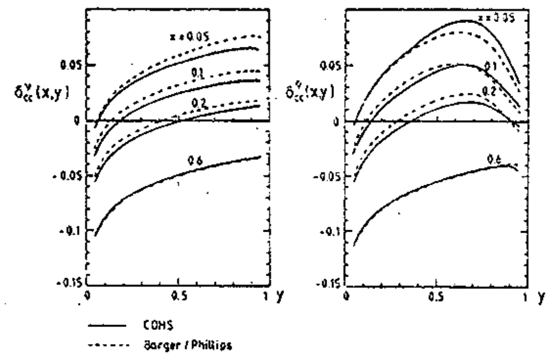


Fig. (17.3)

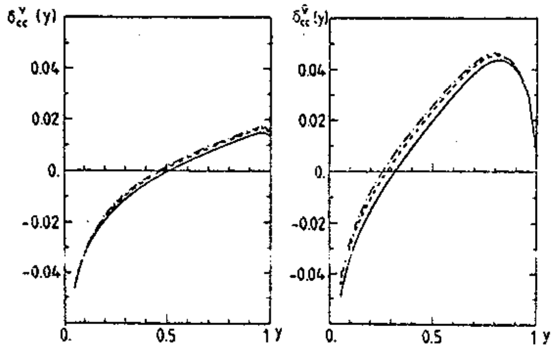


Fig. (17.4)

m_q = 0.003 GeV
m_q = 0.3 GeV
m_q = 3.0 GeV

theorem[11], which states that the integrated cross section cannot depend on log $m_{f'}$, where $m_{f'}$ is the mass of a fermion in the final state. Finally, fig.(17.4) also shows the dependence of $\delta_{cc}(y)$ on three quark masses: m_q = 3, 300, 3000 MeV/c^2. Evidently the dependence on quark masses is very small.

Numerial Results.

Before discussing specific results it is worthwhile to study the sensitivity of the different ratios. The sum total of the corrections can be combined into the terms δ_- and δ_ν defined by

$$D_- = (1 + \delta_-)\, D_-^0 \qquad (17.9)$$

$$R_\nu = (1 + \delta_\nu)\, R_\nu^0 \qquad (17.10)$$

with the quantities on the lefthand sides representing the experimental ratios and D_-^0, R_ν^0 denoting the ratios as functions of $\sin^2\theta_W$ computed in the Born approximation. The change of the angle due to the corrections δ_- and δ_ν is

$$\Delta\sin^2\theta_W \approx +\frac{1}{4}\,\delta_- \qquad (17.11)$$

and

$$\Delta\sin^2\theta_W \approx +\frac{27}{17}\, R_\nu\, \delta_\nu \approx +0.46\,\delta_\nu \qquad (17.12)$$

Both relations have the remarkable property that the change in $\Delta\sin^2\theta_W$ is a fraction of the δ's. In other words, the determination of the angle through D_- and R_ν is very stable. The stability of D_- is better than that of R_ν by a factor of 2. In the following I discusss each of the relations separately including values for the angle, ambiguities on the hadronic model and the experimental data.

Analysis of the PW relation: Relation (13.2) was studied extensively. In the limit where the s, c, b, t quarks are neglected and θ_c is set equal to zero, the relation follows from strong isospin invariance as demonstrated in the original paper[12]. In this limit, it is valid independent of any details of the structure functions; it is valid in the presence of arbitrary amounts of scaling violations, and in the presence of arbitrary higher twists, for example. In this limit there are corrections due to a violation of strong isospin invariance[13] as a result of the d-u mass difference. These might be of the order $[(m_d^2 - m_u^2)/q^2]$, which is very small. Thus one is left with corrections from the presence of the heavier quarks which are discussed in detail in a recent article[4] and are found to be small. The main correction arises from the charged currents involving s and c quarks. The total uncertainty from the hadronic model was estimated to bring in a correction for $\sin^2\theta_W$, equal to

$$\Delta\sin^2\theta_W = 0.0035 \text{ to } 0.0045.$$

Note that this correction is positive and increases the angle.

This relation is also attractive from the experimental point of view, because it is independent of the kinematic region so long as the same region is used for both charged and neutral currents. It gives experimentalists the freedom to employ kinematic cuts that reduce the charged current contamination of the neutral current sample. Three experimental groups analysed their data using the PW relation and obtained the values denoted by $\sin^2\theta_W^o$ in table 1.

Experiment	$\sin^2\theta_W^o$	$\sin^2\theta_W$	Remark
CDHS [14]	0.228 ± 0.018	0.225 ± 0.016	It is necessary to add a correction
CHARM [15]	0.230 ± 0.023	0.225 ± 0.023	$\Delta\sin^2\theta_W = +0.0035$
CFRR [16]	0.243 ± 0.016	0.231 ± 0.016	from the hadronic model uncertainty.
Weighted Average		0.227 ± 0.010	

Table 1

The third column gives values for $\sin^2\theta_W$ including $O(\alpha)$ weak and electromagnetic corrections. The electroweak corrections are from reference 4. However, if I adopt the corrections of

Marciano and Sirlin[5], their effect is to decrease $\sin^2\theta_W^o$ by 2 % and the above results remain practically the same. This is a consequence of the remarkable stability of D_-, as indicated by equ. (17.11). Finally the angle as obtained from the D_- ratio carries an experimental error of ± 0.010, which equals the experimental error on the angle as obtained from the R_ν ratio.

<u>Analysis of the R_ν ratio</u>. The data used in the analysis were compiled by Kim et al.[17] in their extensive review on weak neutral currents. Subsequently, several authors adopted their QCD model calculation and deduced values for the angle indicated as $\sin^2\theta_W^o$. In particular the CHARM collaboration repeated the analysis by taking into account the beam spectra and selection criteria of their experiments and obtained the value shown in table 2. The same collaboration also analysed[18] their data using the hadronic energy distributions and I shall return to their results later on.

Experiment	R_ν	$\sin^2\theta_W^o$
CDHS [14]	0.307 ± 0.008	0.230 ± 0.013
CHARM [15]	0.320 ± 0.010	0.220 ± 0.014
BEBC [19]	0.32 ± 0.03	0.217 ± 0.045
CITF [20]	0.28 ± 0.03	0.272 ± 0.055
HPWF [21]	0.30 ± 0.04	0.274 ± 0.075
Kafka et al.[22]	0.30 ± 0.03	

Table 2

In table 2 I show the values of R_ν reported by the experimental groups and values for $\sin^2\theta_W^o$ deduced in ref.5. The analysis by Kim et al.[17] <u>adopts</u> the quark-parton model and includes QCD corrections.

Three groups reported results on the $O(\alpha)$ radiative corrections to R_ν. Their values for the angle are shown in table 3. The errors in the third and fourth columns are the same as in column two. The values in column two and three almost agree with each other . The values by Wirbel are consistently larger by the small amount 0.005. Part of the

Experiment	$\sin^2\theta_W(-20\text{GeV}^2)$	$\sin^2\theta_W(M_W)$	$\sin^2\theta_W(-20\text{GeV}^2)$
Authors	Marciano-Sirlin[5]	Llewellyn-Smith-Wheater[7]	Wirbel[9]
CDHS	0.217 ± 0.013	0.219	0.222
CHARM	0.211 ± 0.015	0.210	0.215
BEBC	0.203 ± 0.045	0.206	0.208
CITF	0.259 ± 0.055	0.263	0.263
HPWF	0.264 ± 0.075	0.266	0.27
Weighted Average	0.216 ± 0.010	0.217±0.010	0.221 ± 0.010

difference comes from the computation of the bremsstrahlung terms. In their work, Marciano and Sirlin[5] computed the total-neutrino-nucleon cross section, including bremsstrahlung terms, by introducing average values of y. Then they corrected for the $0 \le y \le y_{min}$ region, which is not observed in the experiments. On the other hand, Wirbel[9] integrated the leading-log approximation of the bremsstrahlung terms over the region $y_{min} \le y \le y_{max}$ detected in the experiments. The small difference comes partly from the different methods of including the experimental cuts and partly from the leading-log approximation.

An independent determination of the angle is possible using a more recent analysis[18] of the CHARM collaboration. In this analysis they studied the differential cross sections $d\sigma/dy$ in neutrino and antineutrino reactions. They also included charged current radiative corrections [18] and obtained

$$\sin^2\theta_W^0 = 0.222 \pm 0.016.$$

Since the weak corrections are small and the bremsstrahlung terms were included, we must correct only for the term $\alpha/\pi \ln(M_W^2/Q^2)$ which reduces the angle by 0.008 to

$$\sin^2\theta_W = 0.214 \pm 0.016.$$

In summary the radiative corrections to R_ν are reliable and there is good agreement between the groups. Much more uncertain, in my opinion, are the hadronic model ambiguities associated with the quark-parton model and also with higher twists.

Conclusion.

1. The values for the mixing angle are summarized in figure 7. The results are consistent with each other, with the neutrino data giving more accurate results.

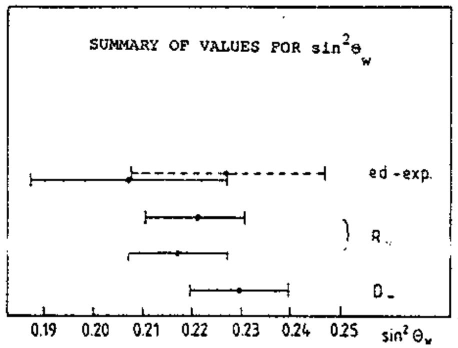


Fig. (17.5)

2. As described at the end of the previous chapter, the mass of the W boson is determined from the decay rate of the muon. Except for M_W and $\frac{\alpha_W^R}{M_W^2}$ all other terms in equ. (16.6) are known. Taking the $q^2 = 0$ value from figure (16.1), we obtain

$$M_W = \left(\frac{\pi\alpha}{\sqrt{2}\,G_F}\right)^{1/2} \frac{\left(1+\frac{\Delta r}{2}\right)}{\sin^2\theta_W}$$

with Δr being practically $\frac{\alpha_W^R(0)}{M_W}$ plus a few smaller terms. Numerically

$$M_W = 37.81\ \text{GeV}\ \frac{\left(1+\frac{\Delta r}{2}\right)}{\sin\theta_W} = \frac{38.5}{\sin\theta_W}\ \text{GeV}$$

with the angle as an input we calculate M_W including one loop radiative corrections. Taking the angle from the D_- ratio

$$M_W = 80.6\ {}^{+1.8}_{-1.7}\ \text{GeV}$$

or from the R_ν ratio

$$M_W = 82.6\ {}^{+2.0}_{-1.8}\ \text{GeV}.$$

The mass for the Z boson now follows from equ. (17.5) with

$$\rho = \begin{cases} 1.0014 & M_H = 10\ \text{GeV} \\ 1.0009 \quad \text{for} & M_H = 100\ \text{GeV} \end{cases}$$

Again taking the angle from D_-

$$M_Z = 91.7 \pm 1.4\ \text{GeV}$$

or from R_ν

$$M_Z = 93.3\ {}^{+1.6}_{-1,5}\ \text{GeV}.$$

3. Finally the values of the angle in fig. (17.5) are in good agreement with the prediction of the SU(5) grand unified theory

$$\sin^2\theta_W/_{SU(5)} = 0.209\ {}^{+0.003}_{-0.002}$$

They also restrict the lifetime of the proton below 10^{30} years, which is within the range of the contructed experiment.

References Ch. 17

1. For several different renormalization schemes see ref. 1 in chapter 16.
2. For the general structure of counter terms see Itzykson and Zuber pgs. 588-594. For the specific results in this chapter I use the renormalization scheme of the Dortmund group.
3. R. Murzik, Diplomarbeit University of Dortmund (Dec. 1981)
4. E.A. Paschos and M. Wirbel, Nucl. Phys. B197, 189 (1982)
5. R. Sirlin and W. Marciano, Nucl. Phys. B159, 442 (1981)
6. C.H. Llewellyn-Smith and J.F. Wheater , Phys. Lett. 105B, 486 (1981) and Univ. of Oxford preprint 5/82 (1982)
7. F. Antonelli and L. Maiani, Nucl. Phys. B186, 269 (1981)
8. S. Dawson, J.S. Hagelin and L. Hall, Harvard preprint HUTP-80/1090
9. M. Wirbel, Ph.D.Thesis, University of Dortmund (1981), and Heidelberg preprint HD-THEP 82-7
10. A.DeRújula, R. Petronzio and A. Savoy-Navarro, Nucl. Phys. B154, 394 (1979)
11. T. Kinoshita, J. Math. Phys. 3, 650 (1962)
T.D. Lee and M. Nauenberg, Phys. Rev. 133, 1549 (1964)
12. See chapter 13.
13. L. Wolfenstein, private communication (1982)

14. C. Geweniger, CDHS Collaboration, Proc.Int.Conf. Neutrino 79 (Bergen 1979) p. 392

15. M. Jonker, CHARM Collaboration, Phys. Lett. 99B, 265 (1981)

16. M. Shaevitz (CFRR Collaboration), Proceed. of 1981 Int. Conf. in Neutrino Physics und Astrophysics, Vol. I, p. 311

17. J.E. Kim, P. Langacker, M. Levine and H.H. Williams, Rev. Mod. Phys. 53, 211 (1981)

18. M. Jonker et al., (CHARM Collaboration), Phys.Lett. 102B, 67 (1981)

19. BEBC ABCLOS Collab., M. Deden et al., Nucl. Phys. B149, 1 (1979)

20. CITF Collab., F. Merrit et al., Phys. Rev. D17, 2199 (1979)

21. HPWF Collab., P. Wanderer et al., Phys. Rev. D17, 1679 (1978)

22. T. Kafka et al., Stony Brook - Illinois Inst. - Maryland - Tohoku-Tufts Collaboration; preprint (1982)

23. J. Ellis, M.K. Gaillard, D. Nanopoulos and S. Rudaz, Nucl. Phys. B176, 61 (1980)

    P. Langacker, Grand Unified Theories, "Int. Symp. on Lepton and Photon Int. at High Energies", Bonn (1981)

    I. Antoniadis, C. Kounnas and Roiessnel, A 466.1081 Ecole Polytechnique Preprint (1981)