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# Unified Interactions of Leptons and Hadrons*

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It is suggested that a unifying description of leptons and hadrons can be obtained within a nonabelian gauge theory where the gauge group is a symmetry group of a set of massless elementary fermions (leptons, quarks). We investigate the consequences of such an approach for the strong, electromagnetic, and weak interactions. We study both gauge theories with and without fermion number conservation, e.g., theories based on the groups $SU_n \times SU_n$ ($n = 8, 12, 16$) and $SO_n$ ($n = 10, 14$).

## 1. INTRODUCTION

In this paper, we show how several hypotheses proposed during the last few years about nonabelian gauge theories for the weak and electromagnetic interactions [1], permanently confined colored quarks [2, 3] and color octet vector gluons [4, 5] can be combined to give a unified picture of the strong, electromagnetic, weak, and other ineractions. Among the ideas used to construct interacting field theories the nonabelian (Yang–Mills [6]) gauge principle seems to be singled out by nature. The reason for this preference is not yet well understood, but it is presumably related to the fact that the minimal coupling required by gauge invariance is the only way to formulate renormalizable vector and axial vector interactions.

There are significant indications that the electromagnetic and weak interactions can be described within a theory containing nonabelian gauge fields. Furthermore, there is reason to believe that the strong interactions are caused by nonabelian vector fields (gluons) coupled to the color degree-of-freedom of the quarks [5, 7, 8].

At present, it seems convincing to suppose that the conventional weak and electromagnetic interactions on the one hand and the strong interactions on the other hand are due to the existence of two commuting gauge groups, e.g., $SU_2 \times U_1$ for the weak and electromagnetic interactions and $SU_3$ (color) for the strong interactions, with the weak and electromagnetic currents as color singlets. Only

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in this case is it natural to understand why the strong interactions differ in their properties (e.g. their conserved quantum numbers) from the nonstrong interactions [9].

However, from a fundamental point of view, the existence of several different theories unrelated to each other is disturbing. In this paper, we discuss the logical possibility that both the strong gauge group and the gauge group for the weak and electromagnetic interactions are embedded as commuting subgroups into a larger group and both the strong and nonstrong interactions are different manifestations of the same underlying basic theory.[1]

This idea presupposes some kind of universality not only between the electromagnetic and weak interactions but also between the electromagnetic or weak and the strong interactions. Although it is rather unclear at present at which energies such a universality sets in, the general idea of a universality between the strong and nonstrong interactions is not so unnatural today as it would have been some years ago. During the last few years it has been recognized that after the translation of statements about strong interaction phenomena into statements about the behavior or quarks and gluons, bound together to form hadrons, there is no need for the existence of a strong coupling between the quarks and gluons provided that a mechnism is found to explain the absence of states with quark and gluon quantum numbers in the particle spectrum. Furthermore, it seems that the naive free quark model, which neglects the binding between quarks, works very well for many purposes.

The phenomenon of Bjorken scaling and, in particular, the experimental facts that the scaling behavior sets in so rapidly, that the symmetry and sum rules deduced from the quark-parton model or the quark-type light-cone algebra are fulfilled at rather low energies, and that the nucleon wavefunction shows only very small contributions from quark-antiquark pairs can be understood within the asymptotically free quark–gluon field theory if one assumes that the quark–gluon coupling constant is small when renormalized at typical hadronic mass parameters. In this case, perturbation theory for the current commutators can be applied, with the free quark commutator as the dominant term [12, 13]. The picture that emerges from this assumption is as follows. At distances between $\exp(-4\pi/g^2)\,10^{-14}$ cm and $\exp(4\pi/g^2)\,10^{-14}$ cm, the quarks behave essentially as free spin $\frac{1}{2}$ objects, with relatively small effective mass. If one probes distances smaller than $\exp(-4\pi/g^2)\,10^{-14}$ cm, the fine structure of the quarks due to the strong interactions shows up and generates logarithmic violations of Bjorken scaling, which may be interpreted as "radiative corrections" due to the strong interactions. At distances larger than $\exp(4\pi/g^2)\,10^{-14}$ cm the quark–gluon coupling

[1] Unifications of the weak, electromagnetic and strong interactions have been previously proposed by Pati and Salam [10], based on Han–Nambu quarks and the gauge group $SU_4 \times SU_4$, and by Georgi and Glashow [11], based on the gauge group $SU_5$.

constant reaches its critical value of the order of one and is supposed to approach infinity very rapidly from there. If this unproven hypothesis is true, the quarks will be confined at the critical distance and the quark–gluon coupling constant cannot be extremely small, but $g^2/4\pi$ should be of the order of $\frac{1}{5}$ at $10^{-14}$ cm [12]. In this case, the quarks will behave as free particles roughly in the range between $10^{-13}$ cm and $10^{-15}$ cm.

The smallness of the strong interaction coupling constant suggests the outstanding possibility that strong, electromagnetic, weak, and possibly other interactions can be described by one unified theory based on a gauge group $G$ that does not factorize in the form $G_s \times G_w$.

In this paper, we propose general principles in constructing a unified theory and consider special models that may be serious candidates for such a theory [14]. In Section 2, we discuss the different aspects of the strong, weak, and electromagnetic interactions related to our work. We start by assuming that the strong interactions are caused by the interaction of colored quarks with nonabelian color octat gluons. Color is taken to be an exact symmetry, and we assume that the hypotheses about the perfect confinement of all colored states, and in particular, the quarks and gluons, is correct. The strong interactions are supposed to be caused by the infrared instability of the theory, which leads to the confinement of color and is the origin of the strong phenomenological coupling constant between color singlet hadrons (e.g., the pion–nucleon coupling constant).

It is assumed that the quark–gluon coupling constant $g^2/4\pi$ is appreciably smaller than unity when renormalized at distances of the order of the nucleon Compton wavelength. This gives the possibility of understanding why the free quark model is so successful in many applications.

We continue with comments about the electromagnetic and weak interactions of leptons. In particular we briefly describe the model of Weinberg, based on the group $SU_3^L \times SU_3^R$ [15].

In Section 2(c), we discuss the weak interactions involving hadrons. It is pointed out that the arguments in favor of the so-called universality (i.e., the Cabibbo structure of the charged weak hadronic current) may be incorrect.

In Section 3, we propose the general principles that we adopt in constructing unified theories. In particular, we take it as a basic principle that the fundamental Lagrangian should be completely symmetric under all conceivable symmetries and that all asymmetries observed in the real world are due to the asymmetries of the vacuum state.[2] For example, the preference of the observed weak interactions to left-handed fermions or the qualitative difference between the weak and strong interactions should be due entirely to the vacuum asymmetry and should not

[2] To our knowledge this principle in a general context was first proposed by Heisenberg (see [16]).

reflect themselves in the basic field equations. Furthermore, it is assumed that the basic elementary degrees-of-freedom are provided by a set of massless spin $\frac{1}{2}$ fields that are later identified with leptons and quarks.

The interaction is supposed to arise through a nonabelian gauge invariant theory. The gauge group must be either simple or semisimple such that the different factor groups are related to each other by a discrete symmetry. It is only then that there is one coupling constant for all interactions in the symmetry limit and the theory can be considered as a unified theory.

The principle of maximal symmetry requires the maximal gauge group one can construct. The natural gauge group that follows from the basic principles under the additional assumption that there exists a fermion number operator that commutes with the gauge group is the chiral group of the elementary fermions $SU_f{}^L \times SU_f{}^R$, where $f$ is the number of elementary fermions in the scheme. The number of gauge bosons is, of course, $2(f^2 - 1)$, i.e., in the symmetry limit there exists for each fermion–antifermion pair a gauge boson that is coupled exclusively to it.[3]

In Section 4, we describe the exactly conserved quantum numbers of the scheme. It is assumed that one $SU_f$-vector generator, identified with the electric charge, and a vectorial subgroup $SU_3^{L+R}$ (color) that commutes with the electric charge remain unbroken. We suppose that the infrared instability of the quark–gluon theory does not allow the existence of colored states in the particle spectrum.

The exactly conserved color quantum numbers divide the elementary fermions into two groups: (i) color triplets, denoted as quarks; and (ii) color singlets, denoted as leptons. Since the electric charge must be an $SU_f$-generator, the sum of the lepton charges and the sum of the quark charges must be equal but of opposite sign.

Besides color and electric charge the fermion number operator $F$, which commutes with the gauge group, is supposed to be exactly conserved.

At the end of Section 4, we classify the gauge bosons according to their transformation properties under color. It is characteristic for our scheme as well as for any scheme that unifies strong and nonstrong interactions that it generates new kinds of colored gauge bosons besides the neutral vector gluons, in particular, colored and charged gauge bosons.

In Sections 5 and 6, we discuss the spontaneous symmetry breaking in general. Formally, the symmetry breaking is described by the coupling of the gauge fields and fermions to an, in general, reducible representation of scalar fields, which

[3] Such a theory with $f > 2$ is not free of anomalies that spoil the renormalizability. However, the anomalies become relevant only at a relatively high order and we do not regard the cancellation of anomalies as a constraint on model building with respect to the electromagnetic, weak, and associated interactions. We require only that the strong interactions be anomaly free, since there all orders of perturbation theory are relevant for the confinement mechanism to work.

develop nonvanishing vacuum expectation values [17]. Since the gauge group in our approach is typically very large, involving at least a few hundred generators, the number of scalar fields needed to generate the symmetry breaking becomes correspondingly large. We feel that this proliferation of scalar fields only can be tolerated if one interprets the usual symmetry breaking mechanism as preliminary as far as the treatment of the scalar fields as elementary fields is concerned. It may be replaced by a dynamical symmetry breaking [18], in which case the scalar fields appear as bound states of the basic fields and one may tolerate a very large number of them; perhaps even infinitely many. Invoking a dynamical symmetry breaking of this kind, one must face one important consequence: There is no reason to attach a special meanining to algebraic mass relations that follow from the choice of a specific representation of scalar fields. If one allows for an unlimited number of scalar fields, one is able to generate any mass matrix for the gauge bosons and fermions.

One consequence of the large symmetry of the basic Lagrangian of leptons and quarks is that one predicts many new kinds of interactions unobserved thus far. The only way to tolerate these is to generate the symmetry breaking in such a way that the associated gauge bosons become much heavier than, for example, the conventional weak bosons, i.e., at least several hundred GeV [15]. How heavy these bosons are is a question that may be answered by experiment. The existence of the superheavy bosons amounts to small violations of the usual low energy weak interaction theory, e.g., to reactions with parity properties opposite to those of the conventional weak interactions. It would be very interesting to check by refined experiments whether such violations are indeed present.

We assume, in accordance with the observed interactions, that the symmetry breaking generates a complicated hierarchy of different levels in the gauge boson mass spectrum. At the first stage of the symmetry breaking, the hadron and lepton worlds become separated and the gauge bosons, which couple to the lepton–quark pairs (leptoquark bosons) formally acquire a mass that is assumed to be the largest in the scheme (e.g., $10^3$–$10^5$ GeV). The next stages of the symmetry breaking involve the generation of masses for the color singlet bosons, responsible for the weak and superweak interactions, and their color octet partners (diotons) [19].

In Section 7, we describe Model A, based on the "observed" 12 fermions (nine colored quarks, three leptons), with the gauge group $SU_{12}^L \times SU_{12}^R$. In this model the subgroup of $SU_{12}^L \times SU_{12}^R$ involving the color singlet gauges is $(SU_3{}^L \times SU_3{}^R)_{\mathrm{hadr}} \times (SU_3{}^L \times SU_3{}^R)_{\mathrm{lept}} \times (U_1)^2_{\mathrm{lept\&hadr}}$, i.e., one is dealing with a set of color singlet bosons coupled exclusively to leptons and a set of bosons coupled exclusively to quarks, besides two bosons coupled to both. The symmetry breaking is arranged as follows. First, one considers the ideal case where the quark masses vanish and the Cabibbo angle can be set to zero. In this limit, one supposes that the minimal model of the weak and electromagnetic interactions based on the group $SU_2 \times U_1$

is a very good approximation, and all color singlet bosons that are singlets under this group are supposed to become superheavy. The basic gauge group $SU_{12}^L \times SU_{12}^R$ fixes the $SU_2 \times U_1$-mixing angle such that $\sin^2 \theta_W = \frac{5}{16} \approx 0.31$.

We introduce the strangeness violating weak interactions by enlarging the minimal gauge group $SU_2 \times U_1$ to the group $(SU_3{}^L)_{\text{hadr}} \times (SU_2)_{\text{lept}} \times (U_1)_{\text{lept\&hadr}}$. The breaking of this group is constructed such that the usual charged weak current results, but there are no $|\Delta S| = 1$ – neutral currents coupled to leptons and no $|\Delta S| = 2$ – transitions (e.g., $K_0 \leftrightarrow \bar{K}_0$) up to order $G_F \cdot \alpha$ ($G_F$ : Fermi coupling constant). In Section 7, one particular example of such a symmetry breaking is given which, in addition, has the property that not only the charged weak current has the usual form, but the neutral current has the same algebraic structure as in the limit $\theta_C = 0$.

In Section 8, we describe Model B, which is based on 16 fermions. It differs from Model A by the introduction of a charmed quark triplet and of the right-handed counterparts of the usual neutrinos. The introduction of charm allows, of course, a more economical description of the weak and electromagnetic interactions. If we suppose that the latter can be described by an $SU_2 \times U_1$ subtheory, the phenomenological consequences of this model for the weak interactions are the same as for the $SU_2 \times U_1$ model in the limit where the superheavy boson effects are neglected, with the restriction that the $SU_2 \times U_1$ mixing angle $\theta_W$ is determined by the basic symmetry such that $\sin^2 \theta_W = \frac{3}{8}$.

The main phenomenological consequences of Models A and B for high energy neutrino hadron scattering are discussed in Section 9.

Section 10 is devoted to the study of the fermion mass spectrum in both models. In accordance with the observed lepton masses and the quark mass values estimated from PCAC, it is reasonable to assume that the spontaneous symmetry breaking generates only a mass for the muon and $s$, or perhaps the $s$ and $c$-quarks, while the other lepton and quark masses arise by second-order effects due to the emission and reabsorption of gauge bosons.

In addition to a qualitative study of the quark masses, we discuss the generation of the electron mass in both models. The latter is supposed to be generated by the vector and axial vector gauge bosons coupling to the $(e, \mu)$ system. In Model A, these bosons have charges $\pm 2$, while in Model B they are neutral. Assuming that the mixing angle of these bosons is canonical such that the eigenstates of the mass matrix are vector and axialvectors one finds in Model A:

$$m_e/m_\mu = 3\alpha/\pi \log(m_A{}^2/m_V{}^2) + 0(\alpha \cdot (m_\mu{}^2/m_V{}^2), \alpha^2) \tag{1.1}$$

and in Model B:

$$m_e/m_\mu = 4\alpha/\pi \log(m_A{}^2/m_V{}^2) + \cdots. \tag{1.2}$$

The observed mass ratio agrees very well with the formula[4]

$$m_e/m_\mu \simeq 3\alpha/\pi \log 2, \tag{1.3}$$

hence, in the case of Model A, one finds the very simple ratio $m_A{}^2/m_V{}^2 = 2$, which reminds us of the familiar formula $m_{A_1}^2/m_\rho{}^2 = 2$ derived from PCAC and the saturation of the spectral function sum rules for the vector and axialvector currents [20]. In Model B, one finds the ratio $m_A{}^2/m_V{}^2 = 8^{1/4}$.

In Section 11, we describe the interpretation of lepton number as fourth color.

In Section 12, we discuss possible phenomenological implications of the existence of colored gauge bosons (leptoquark bosons, diotons) besides the gluons. Those bosons are confined like the quarks and gluons inside color singlet hadrons, i.e., serve besides the quarks and gluons as hadronic constituents. The ideal experiment to detect those new constituents is $e^+e^-$-annihilation in the one-photon channel. In particular, the dioton part of the electromagnetic current contributes to the hadronic cross section such that the cross section ratio

$$R = \frac{\sigma_{e^+e^-\to\text{hadrons}}}{\sigma_{e^+e^-\to\mu^+\mu^-}} \tag{1.4}$$

becomes flat at rather large values (18 in Model A, $35\frac{1}{3}$ in Model B) [19]. Possible effects due to the leptoquark bosons are also discussed.

In Section 13, we outline other possibilities to construct unified theories. In particular we concentrate on the fermion set of Scheme B and show that a variety of other models are possible; in particular we discuss the ones based on the gauge groups $SU_8{}^2$, $SU_4{}^4$, $SO_{10}$, and $SO_{14}$. All models have the property that either fermion number is conserved, but there are anomalies, or the theory is free of anomalies, but fermion number is not conserved and the proton decays into leptons in second order of the gauge coupling. It is shown that there exists no unified theory based on the 16 fermions in which fermion number is conserved and which is anomaly-free. General consequences for the construction of unified theories are discussed.

## 2. Gauge Theory Aspects of the Basic Interactions

### (a) *The Strong Interactions*

If the strong interactions are described by a gauge field theory, the strong interaction gauges must commute with the weak and electromagnetic gauges. Otherwise, the strong interactions would mix with the weak and electromagnetic interactions

[4] To our knowledge, this formula has been first suggested by Bjorken (private communication), who found a certain justification for it in an $SU_6$ gauge model.

in an uncontrolled way and there would be no reason why the strong interactions preserve quantum numbers that are broken by the weak interactions like strangeness or parity.

The structure of the baryon spectrum and the magnitude of the dacay $\pi^\circ \to 2\gamma$ when interpreted within the quark model indicate that there is a hidden degree-of-freedom (color) present in the strong interactions, which can be described by colored quarks [2, 3]. One represents the quarks by the matrix

$$\begin{pmatrix} u_r & u_g & u_b \\ d_r & d_g & d_b \\ s_r & s_g & s_b \\ \vdots & \vdots & \vdots \end{pmatrix}$$

where the horizontal index ($r$, $g$, $b$: red, green, blue)[5] is the color index and the ordinary hadronic symmetries (isospin, $SU_3$ ...) act vertically. To be general, we allow the possibility of a larger hadronic symmetry than $SU_3$, indicated by the dots (for example, the introduction of charm) [21].

The structure of the baryon spectrum requires the number of colors to be three, i.e., the color group is $SU_3$. The hadron spectrum can be described by adopting the superselection rule that only color singlets are realized as hadronic states and color nonsinglets (e.g., the quarks) are fictitious. This idea requires color to be an exact symmetry and the usual hadronic symmetry generators to commute with the color generators. In particular, the physical currents (electromagnetic, weak) must be color singlets.

With these requirements in mind, it is most natural to construct a gauge model for the strong interactions that is based on $SU_3$ (color) as an unbroken gauge group [5]. Only then both the quarks (color triplets) and the gluons (color octet) are colored, i.e., fictitious. The color singlet nature of the physical currents ensures automatically that the color gauges commute with the gauges used in the gauge theory for the electromagnetic, weak, and possibly other interactions.

The Lagrangian for the strong interactions is given by

$$\mathscr{L} = -\frac{1}{4} \sum_{A=1}^{8} G_{\mu\nu}^A G_A^{\mu\nu} + \bar{q} \left[ i\gamma^\mu \left( \partial_\mu + i g_{st} \frac{\chi^A}{2} B_{A\mu} \right) - \mathscr{M} \right] q. \tag{2.1}$$

The color matrices are denoted by $\chi_A$ ($A = 1...8$), they are defined such that

$$\left[ \frac{\chi_A}{2}, \frac{\chi_B}{2} \right] = i f_{ABC} \frac{\chi_C}{2} \quad \text{and} \quad \mathrm{tr}(\chi_A, \chi_B) = 2\delta_{AB}. \tag{2.2}$$

[5] We use red, green, and blue, since they average to white. Color singlet states may be regarded as white states.

Furthermore, we have

$$G_{\mu\nu}^{A} = \partial_\nu B_\mu{}^{A} - \partial_\mu B_\nu{}^{A} - g_{st} f^{ABC} B_\nu{}^{B} B_\mu{}^{C}. \tag{2.3}$$

The quarks transform as three different color triplets, or perhaps as four, if one adds a charmed quark triplet. We also introduce the strong interaction analogue of $\alpha$: $\kappa = g_{st}^2/(4\pi)$. The quark mass matrix is denoted by $\mathscr{M}$.

The theory is asymptotically free, provided there are not more than 16 quark triplets present, i.e., the renormalized quark–gluon coupling constant approaches zero logarithmically in the short distance limit [7]. Furthermore, due to the infrared divergencies caused by the massless gluons, the theory is unstable in the infrared region and perturbation theory breaks down at large distances. Hence, it is conceivable that the Fock space, used in perturbation theory, is not useful to describe the asymptotic states of the theory.

It has been suggested that the infrared instability leads to a shielding of the colored states from the physical world. The color singlet part of the $S$-matrix is alleged to factorize, and in a scattering process, involving color singlet states as in-states, only color singlet states are produced. The colored states, in particular, the quarks and gluons, are relevant only as field theoretic coordinates but not as asymptotic states.

In this paper, we assume that the hypotheses about the perfect confinement of color nonsinglets is true. Below, we indicate the possibility that the confinement is not perfect ("partial confinement"), i.e., that color could be excited at some presumably rather high energy $M_c$ ("color threshold"). In this case, one may differentiate between the following three possibilities:

(1) Color is still a perfect symmetry. A colored state, eventually produced in a reaction, cannot decay into low-lying color singlet hadrons, but only into stable colored states with mass $> M_c$. Such a scheme is likely to contain heavy stable quarks.

(2) Color is broken by the strong interactions, but not by the electromagnetic and weak interactions. Colored particles can decay strongly into colorless ones. It is conceivable that in this scheme there exist quark states with nonintegral electric charges, unless all triality nonzero states are still confined by some unknown mechanism.

(3) Color is broken or unbroken by the strong interactions, but broken by the electromagnetic and weak interactions. The electric and weak currents have colored components, while the usual expressions for these currents are obtained after averaging over color. Quarks can be real and can have conventional quantum numbers in this scheme, i.e., they can decay into ordinary hadrons, as, for example, in a version of the Han–Namby quark model [2, 23].

Within our approach to the strong interactions (description of the strong interactions by the canonical quark–gluon gauge theory with infrared instability) only perfect color confinement makes sense. Possibility (1) cannot occur in the canonical unmodified gauge theory, since the exact color symmetry requires the gluons to be massless. In (2) or (3), color is a broken symmetry, and the theory is infrared stable. In Possibility (3) the photon and the weak intermediate bosons are themselves colored, i.e., the strong interaction gauges mix with the gauges for the weak and electromagnetic interactions, and there is no reason why the strong interactions conserve quantum numbers that are typically broken by the weak interactions (e.g., parity).

The infrared instability generates the essential dynamics of the quark–gluon theory. It may produce the strong force between color singlet hadrons even if the basic quark–gluon coupling constant is relatively small when renormalized at usual spacelike hadronic distances, e.g., $\kappa = g_{st}^2/(4\pi) \lesssim 1/5$. It has been emphasized recently that a small quark–gluon coupling constant could be the origin of Bjorken scaling, and in particular of the fact that the symmetry relations and sum rules derived from the free quark model light-cone algebra or the quark parton model work at surprisingly low energies [12, 13]. The reason is that because of the small coupling constant a perturbation expansion for current commutators is valid with the free quark commutator as the lowest order term. Thus far, only the free quark singularities are seen in the experiments, while the interaction dependent logarithmic corrections to the free quark commutator are negligible at the energies explored. They become relevant only at energies of the order of $\exp 1/\kappa \cdot$ typical hadronic energy.

Due to the infrared instability, perturbation theory can only be applied to current commutators at relatively small distances, but not for the relevant hadron matrix elements of bilocal operators, which determine the scaling functions. For the latter, a perturbation expansion is inappropriate.

The permanent confinement of colored states is supposed to be a consequence of the singular behavior of the renormalized quark–gluon coupling constant at large distances. The renormalized quark–gluon coupling constant, although supposed to be relatively small at distances where Bjorken scaling sets in, reaches a critical value where perturbation theory breaks down at distances of the order of the pion Compton wavelength and then either approaches infinity very rapidly, giving rise to the permanent confinement of colored states, or the canonical quark–gluon field theory fails to be realistic.[6]

One important aspect of the quark–gluon theory must be that it generates its

[6] It has been emphasized by Wilson [22], that in the lattice gauge field theory the confinement of color occurs if the quark–gluon coupling becomes strong. This may serve as an indication, but by no means proof, that the color confinement hypotheses may be a consequence of the field equations of the quark–gluon field theory.

own scale spontaneously. Scale invariance is broken formally by the bare quark mass term. However, PCAC suggests that the bare quark masses are very unsymmetrical (the strange quark mass is about 20 times larger than the nonstrange quark masses). Thus, there must be another symmetric scale parameter, giving rise to the typical strong interaction mass scale of 1 GeV. The latter would be responsible for the $SU_3$-symmetric part of the baryon masses, the meson decay constants, the universal slope of the Regge trajectories, etc.

We assume that this scale is generated spontaneously by the choice of the subtraction point and the specification of the renormalized coupling constant at this point. How this works in detail is just as unresolved as the confinement property itself. There is, for example, the possibility that the strong interaction scale is related to the distance at which the quark–gluon coupling constant reaches its critical value.

Finally, we should like to add a comment about the role of the bare quark masses. They arise in the same way as the lepton masses and serve to parameterize the divergencies of the hadronic currents, i.e., the hadronic symmetry breaking. In the limit $\mathscr{M} \to 0$ the strong interactions are symmetric. The lowest lying pseudoscalar mesons become massless and serve as Nambu–Goldstone particles.

The real world is supposed to be not very far from such a situation and not at all far from the case where $SU_2 \times SU_2$ becomes exact and the nonstrange quark masses vanish. Thus, the introduction of the bare quark masses causes only minor disturbances of the strong interactions and perhaps also other interactions. Later, we make use of the possibility to turn off the quark masses and to study particular questions in this limit.

### (b) *Leptonic Weak and Electromagnetic Interactions*

The most economical scheme to describe the weak and electromagnetic interactions of leptons is the model based on the gauge group $SU_2 \times U_1$ [1]. However, this model cannot be considered as a unification of weak and electromagnetic interactions since two commuting gauge groups, i.e., two independent coupling constants, are involved. A unified theory can be obtained if one embeds the $SU_2 \times U_1$-group into a larger group that has enough symmetry to allow only one independent coupling constant for the associated gauge theory.

Some time ago, such a model was proposed by Weinberg [15], based on the gauge group $SU_3^L \times SU_3^R$ ($L$, $R$ denotes left-handed or right-handed, respectively). In this model the leptons transform as the triplet [24]:

$$l = \begin{pmatrix} \nu \\ e^- \\ \mu^+ \end{pmatrix} \tag{2.4}$$

where the four-component neutrino is formed by the left-handed electron neutrino and the right-handed muon antineutrino

$$\nu = \begin{pmatrix} \nu^L \\ \nu^R \end{pmatrix} = \begin{pmatrix} \nu_e \\ \bar{\nu}_\mu \end{pmatrix}. \tag{2.5}$$

The interaction Lagrangian is defined to conserve parity; hence, there is only one independent coupling constant. The model contains 16 gauge bosons. Many of them give rise to interactions not observed, for example, to interactions that violate the conservation of muon number. It is however, remarkable that the reaction $\mu \to e\gamma$ is forbidden by lepton number conservation. This reaction has not been observed; its branching ratio is less than $10^{-8}$ [25].

Obviously, the only way to interpret the observed weak and electromagnetic interactions is to assume that those "unwanted" interactions actually exist, but are suppressed by the spontaneous symmetry breaking, i.e., by a relatively high mass of the associated gauge bosons.

The spontaneous symmetry breaking is assumed to occur in two steps described by the following diagram:

$$\begin{array}{cc} & SU_3{}^L \times SU_3{}^R \\ \mathrm{I} & \downarrow \\ & SU_2 \times U_1 \\ \mathrm{II} & \downarrow \\ & U_1{}^e \end{array} \tag{2.6}$$

During the first step, all gauge bosons except the four needed to describe the usual weak and electromagnetic interactions in the form of an $SU_2 \times U_1$-theory acquire a mass of the order of at least a few hundred GeV (superheavy level of the symmetry breaking). During the second step, the $SU_2 \times U_1$ subtheory is broken such that the photon remains massless and the remaining three gauge bosons acquire a mass.

One essential virtue of the basic gauge group $SU_3{}^L \times SU_3{}^R$ is that it fixes the ratio of the two coupling constants of the $SU_2 \times U_1$-subtheory parameterized usually by the mixing angle $\theta_w$ :

$$\tan \theta_w = \frac{g(U_1)}{g(SU_2)} = \frac{1}{3^{1/2}}, \qquad \sin^2 \theta_w = \frac{1}{4}. \tag{2.7}$$

As emphasized by Weinberg, there could be another virtue due to the existence of the basic gauge group. In the $SU_2 \times U_1$ subtheory, the electron and muon belong to different representations; however, with respect to $SU_3{}^L \times SU_3{}^R$ they belong to the same representation. Thus, in principle, there is the possibility of calculating the electron mass in terms of the muon mass. If only the muon acquires a mass by its coupling to the calar "Higgs" fields, a mass term for the electron can be generated by the emission and absorption of the doubly charged gauge bosons, which couple to the $(\mu e)$-system. In this case, the electron–muon mass ratio may be calculable in terms of the basic coupling constant, i.e., in terms of $\alpha$, and the mass matrix of the doubly charged superheavy gauge bosons.

(c) *The Weak Interactions of Hadrons*

The phenomenological properties of the weak interactions involving hadrons are compatible with the following statements:

(1) The $W$-bosons mediating the weak interactions of hadrons couple to vector or axialvector currents whose charges generate the group $SU_3^L \times SU_3^R$, i.e., the symmetry group of three massless spin $\frac{1}{2}$ quark fields.

(2) The charged $W$-bosons mediating the semileptonic interactions couple only to the left-handed hadronic currents. This is well established for the $\Delta S = 0$ reactions, but less well for the $\Delta S = 1$ reactions.

(3) There must be intermediate bosons that couple both to $\Delta S = 0$ and $\Delta S = 1$ currents. Otherwise, there would be no strangeness violating nonleptonic decays.

(4) The experimental values for the decay constants $G_\mu$ ($\mu$-decay), $G_\beta$ ($\beta$-decay) and $G_\Lambda$ ($\Delta S = 1$ decay) are subject to radiative corrections. In case of $G_\beta$ and $G_\Lambda$, these corrections are logarithmically divergent within the phenomenological current–current theory of the weak interactions. It can be argued that a cut-off of the order of 100 GeV reduces the value of $G_\beta$ such that the bare constants $G_0$ agree with the so-called universality principle [26]:

$$G_\Lambda{}^0/G_\mu{}^0 = \sin\theta_c\,, \qquad G_\beta{}^0/G_\mu{}^0 = \cos\theta_c \tag{2.8}$$

($\theta_c$ : Cabibbo angle).

If one introduces charm as a new hadronic degree-of-freeom and constructs a $SU_2 \times U_1$-theory of the weak interactions, the logarithmically diverging radiative correction of $G_\beta$ due to the charged intermediate bosons is cut off at the mass of the neutral intermediate boson $Z$. If the latter has a mass of the order of 100 GeV the situation is the same as above, where a cut-off was introduced by hand [27].

Despite the fact that the radiative corrections modify the $\beta$-decay constant such that it may agree with the universality relation [Eq. (2.8)], we stress the fact that the treatment of the radiative corrections is still unsatisfactory because of the following argument.

The radiative corrections, which must be finite in a renormalizable theory, depend to a large extent on the theory one is using. They are not model independent. For example, within the $SU_2 \times U_1$-theory, one can calculate the radiative corrections to $\beta$-decay, and the numerical result is later used to justify the universality, which, on the other hand, was the basis of the model. Of course, in a different theory the situation will be somewhat different. For example, it could be that a theory of the weak interactions involves not only one, but several neutral intermediate bosons, which all give rise to second-order corrections to $G_\beta$. Furthermore, there may be not only one charged weak boson, but several. In such models, the higher order corrections to the various decay constants are different. Nevertheless, it may be that the net result for the bare coupling constants $G^0$ comes close to the universality relation [Eq. (2.8)], while universality in its strict sense is not a deep underlying principle of the weak interactions.

## 3. Unification of the Weak, Electromagnetic, and Strong Interactions

As already remarked in Section 2, Bjorken scaling can be understood within the framework of the canonical quark–gluon theory, provided that the quark–gluon coupling constant is small enough when renormalized at usual hadronic distances. This suggests the outstanding possibility that perhaps a unification of the weak and electromagnetic interactions can only be achieved if one introduces the strong interactions into the scheme as well, and that all known interactions (besides the gravitational interactions) can be described by one unified theory based on one independent coupling constant (see Footnote 1).

In this section we propose the following general principles in constructing a unified theory:

I. *Principle of maximal symmetry.* The basic field equations (i.e., the Lagrangian) are completely symmetric under all conceibable symmetries, including the discrete symmetries $P$, $C$, and $T$. The apparent asymmetries of the real world (e.g., the qualitative difference between weak and strong interactions, the violations of scale and conformal symmetry, parity or PC-invariance) arise spontaneously by the asymmetry of the vacuum state (see Footnote 2).

II. *Principle of elementarity.* The basic elementary degrees-of-freedom are provided by a set of elementary spin $\frac{1}{2}$ fields. Leptons are supposed to be elementary, while hadrons are considered as bound states of elementary spin $\frac{1}{2}$ fields (quarks).

III. *Gauge principle.* The interaction arises within a nonabelian gauge invariant theory. In order to be a unified theory, its gauge group $G$ must be either simple or semisimple such that the different factor groups are related to each other by a discrete symmetry. Furthermore, we require the group $G$ to consist only of local transformations that commute with the Lorentz transformations.

According to Principle I we would like to have the maximal gauge group. The largest group one can construct is the symmetry group of the kinetic energy of the elementary spin $\frac{1}{2}$ fields. Parity invariance requires that the kinetic energy consists of the energies of $f$ left-handed and $f$ right-handed spin $\frac{1}{2}$ fields, where $f$ is an arbitrary number. Thus, the maximal symmetry group is the semisimple group $U_{2f}$. Note that the $U_1$-charge that commutes with $SU_{2f}$ does not commute with parity.

For reasons related to the stability of the proton and which will be described in more detail in Section 6, we find it natural to suppose that one of the generators of $SU_{2f}$ can be defined as the fermion number operator $F$, which is exactly conserved and has $f$ eigenvalues plus one and $f$ eigenvalues minus one, and that $F$ is not coupled to a gauge boson. We are left with the subgroup of $SU_{2f}$, which commutes with $F$ and does not contain $F$. This is the group $SU_f^L \times SU_f^R$, i.e., the chiral symmetry group of $f$ massless fermion fields. The basic Lagrangian is given by (see Footnote 3):

$$\mathscr{L} = -\frac{1}{4}\operatorname{tr} G_{\mu\nu}^{L} G_{L}^{\mu\nu} - \frac{1}{4}\operatorname{tr} G_{\mu\nu}^{R} G_{R}^{\mu\nu} + \bar{f}\left[\gamma^{\mu}\frac{1+\gamma_5}{2}(i\partial_\mu - gB_\mu{}^{L}) + \gamma^{\mu}\frac{1-\gamma_5}{2}(i\partial_\mu - gB_\mu{}^{R})\right] f \tag{3.1}$$

where $L(R)$ refers to the bosons coupled to left-handed (right-handed) fermions and the fermion row $f$ transforms as $(f, 1) + (1, f)$ under $SU_f^L \times SU_f^R$. The field strength operator is given by

$$G_{\mu\nu}^{h} = \partial_\nu B_\mu{}^{h} - \partial_\mu B_\nu{}^{h} + ig[B_\nu{}^{h}, B_\mu{}^{h}] \tag{3.2}$$

($h = L, R$ respectively).

For the self-representation of the group $U_f$ we use the $E_m{}^n$-matrices introduced by Weyl [28]. They are defined as the $f \times f$ matrices

$$E_m{}^n = \begin{array}{cc} \begin{pmatrix} 0 & \cdots & 0 & \cdots \\ \vdots & & \vdots & \\ 0 & \cdots & 1 & \cdots \\ \vdots & & \vdots & \end{pmatrix} & \begin{matrix} 1 \\ \vdots \\ m \\ \vdots \end{matrix} \\ \begin{matrix} 1 & \cdots & n & \cdots \end{matrix} & \end{array} \tag{3.3}$$

They fulfill the algebra

$$[E_m{}^n, E_q{}^p] = \delta_q{}^n E_m{}^p - \delta_m{}^p E_q{}^n. \tag{3.4}$$

The self-representation of the group $SU_f$ is then given by the matrices $M_m{}^n = E_m{}^n - (1/f)\,\delta_m{}^n \cdot 1$, which satisfies the same algebra as the $E_m{}^n$. The vectorial $U_1$-charge of the group $U_f^L \times U_f^R$ that commutes with $SU_f \times SU_f^R$ defines the fermion number operator $F$.

The number of gauge bosons is $2(f^2 - 1)$. Note that for each fermion–antifermion pair (left-handed or right-handed), there exists a gauge boson coupled to it.

Our choice of the gauge group is different from conventional model building. Usually one chooses the gauge group first and subsequently describes the fermion by certain representations of the group, sometimes rather complicated ones. We find this procedure rather ad hoc since it assigns a major role to the gauge group. We take instead the attitude that the elementary fermions of the theory play the guiding role and that they determine the structure of the gauge group.

In the basic Lagrangian [Eq. (3.1)], all elementary fermions are treated on the same level, i.e., there is no distinction between "weakly" interacting ones (leptons) and "strongly" interacting ones (quarks). All specifications of the elementary fermions have to be provided by the spontaneous symmetry breaking, which we discuss in the next sections.

## 4. Conserved Quantum Numbers

The basic $SU_f^L \times SU_f^R$ symmetry is broken by the asymmetries of the vacuum state. The specific form of the spontaneous symmetry breaking and, of course, the number of elementary fermions $f$, are not determined by the theory but have to be made compatible with observation.

Before we describe the symmetry breaking in detail we discuss the exactly conserved quantum numbers of the scheme. We require the spontaneous symmetry to have the following properties:

(a) We suppose that one $U_1$-subgroup remains unbroken. Its generator defines the electric charge and the associated gauge boson (photon) stays massless. The eigenvalues of the electric charge matrix define the electric charges of the elementary fermions. Since the electric charge is an $SU_f^{L+R}$-generator the sum of all fermion charges must be zero:

$$\sum_{\text{fermions}} Q_i = 0. \tag{4.1}$$

If we renormalize at sufficiently small distances where the symmetry breaking is negligible, we can determine $g$ in terms of the electromagnetic coupling $e$, defined at the same renormalization point. One finds:

$$g^2/4\pi = 2 \cdot \left( \sum_{\text{fermions}} Q_i^2 \right) \cdot e^2/4\pi. \tag{4.2}$$

(b) A subgroup $SU_3^{L+R}$ of $SU_f^L \times SU_f^R$ remains unbroken, and it defines the color group $SU_3$(color). The color generators are constructed such that the $f$ elementary fermions can be decomposed into $l$ color singlets, denoted as leptons, and $q/3$ color triplets, denoted as quarks. Note $f = l + q$. Since color is exactly conserved, there is an exact distinction between quarks and leptons, i.e., between the hadronic and leptonic world. According to [Eq. (4.1)] one has the following relation between the lepton and quark charges:

$$\sum_{\text{leptons}} Q_i = - \sum_{\text{quarks}} Q_i . \tag{4.3}$$

There is no fundamental reason why the color group is $SU_3$. Any nonabelian subgroup of $SU_f^L \times SU_f^R$ could serve as the color group. We restrict ourselves to $SU_3$ (color), since the structure of the baryon spectrum and also the strength of the decay $\pi^\circ \to 2\gamma$ suggest strongly that each quark has three color degrees-of-freedom.

The eight massless vector gluons are assumed to be responsible for the strength of the strong interactions. The associated infrared divergences are supposed to confine all color nonsinglets inside color singlets.

According to the confinement assumption, the electric charge has to commute with the color generators, i.e., the quark charges for each color triplet must be degenerate.

By the basic symmetry group the strong coupling constant $g_{st}$ is related to the basic coupling constant $g$ and therefore also by Eq. (4.2) to the electric coupling constant $e$ at sufficiently small distances where the symmetry breaking plays no role. One finds

$$\kappa = \frac{g_{st}^2}{4\pi} = \frac{g^2}{4\pi} \cdot \frac{3}{q} = \frac{6 \sum_{\text{fermions}} Q_i^2}{q} \cdot \frac{e^2}{4\pi} . \tag{4.4}$$

Thus, the quark–gluon coupling constant $g_{st}$ is a definite multiple of the electromagnetic coupling constant $e$.

It is a matter of speculation at which distances Eq. (4.4) starts to hold. This depends on details of the symmetry breaking. Note that Eq. (4.4) certainly cannot hold at large distances, since $\kappa$ is supposed to approach infinity in the infrared limit, while $e^2/(4\pi)$ approaches the fine structure constant $\alpha$.

(c) In the $SU_f^L \times SU_f^R$-scheme there are $2f - 2$ diagonal generators. One has $2q - 2$ within the quark sector and $2l - 2$ within the lepton sector. In addition, there are two charges $D^{L\pm R}$ that are proportional to the unit matrices both in the quark space and in the lepton space. They are represented by the following matrix $D$:

$$D = \begin{array}{c} \begin{bmatrix} 1/q & & & & & 0 \\ & \ddots & & & & \\ & & 1/q & & & \\ & & & -1/l & & \\ & & & & \ddots & \\ 0 & & & & & -1/l \end{bmatrix} \\ 1 \quad \cdots \quad q \quad \cdots \quad q+l. \end{array}$$

The $U_1$-degrees-of-freedom corresponding to the $f \times f$ unit matrix ($L$ and $R$, respectively) are not associated to gauge bosons. The unit vector charge defines the fermion number $F$. Using the $D$-charges above we can construct the baryonic charge $B$:

$$B = \begin{array}{c} \begin{bmatrix} \frac{1}{3} & & & & & \\ & \ddots & & & & \\ & & \frac{1}{3} & & & \\ & & & 0 & & \\ & & & & \ddots & \\ & & & & & 0 \end{bmatrix} \\ 1 \quad \cdots \quad q \quad \cdots \quad q+l \end{array}$$

and the leptonic charge $L$:

$$L = \begin{array}{c} \begin{bmatrix} 0 & & & & & \\ & \ddots & & & & \\ & & 0 & & & \\ & & & 1 & & \\ & & & & \ddots & \\ & & & & & 1 \end{bmatrix} \\ 1 \quad \cdots \quad q \quad \cdots \quad q+l \end{array}$$

They are related to $F$ and $D$ as follows:

$$B = \frac{q}{3(l+q)}(F + l \cdot D), \qquad L = F - 3B = \frac{l}{l+q}(F - qD). \tag{4.5}$$

Note that $D$ is meant to be the vectorial $D$-charge.

We assume that the fermion number $F$ is exactly conserved. Since $F$ is not coupled to a gauge boson, its conservation does not imply the existence of a massless, color singlet gauge boson besides the photon.

(d) Finally, we classify the different gauge bosons according to their conserved quantum numbers. The $f$ fermions are decomposed into leptons (color singlets) and $q$ quarks ($q/3$-color triplets). Hence, we can write:

$$2(f^2 - 1) = 2[(q^2 - 1) + (l^2 - 1) + 2ql + 1)]. \tag{4.6}$$

There are 2 $(q^2 - 1)$ bosons whose generators couple to quarks only, 2 $(l^2 - 1)$ bosons whose generators couple to leptons only, and 2 $ql$ bosons that transform quarks into leptons and vice versa, before the spontaneous symmetry breaking mixes the different gauge bosons. In addition one has two neutral bosons associated with the $D$-matrix.

Classifying the gauge bosons according to color we find:

(1) $2\ q \cdot l$ bosons that couple to lepton–quark (leptoquark bosons), which are color triplets and carry baryon number $\pm\frac{1}{3}$ and lepton number $\mp 1$.

(2) $2 \cdot 8\ (q/3)^2$ bosons that couple to quarks and are color octets, among them the eight vector gluons, which stay massless.

(3) $2[(q/3)^2 - 1) + (l^2 - 1) + 1]$ color singlet bosons.

All colored gauge bosons are fictitious, like the quarks and gluons, according to the confinement mechanism, i.e., they are bound inside color singlet hadrons. The physical implications of the presence of colored gauge bosons in addition to to the eight massless gluons will be discussed in Section 11. The confinement mechanism allows only relatively few, namely the color singlet gauge bosons, to appear as real spin-1 bosons in the physical spectrum.

## 5. Symmetry Breaking I: General Remarks

The spontaneous symmetry breaking that generates the asymmetries of the real world and, in particular, the different gauge boson masses, must be constructed such that it respects the conserved quantities mentioned in the last section (fermion number, color, electric charge). Formally, we describe the spontaneous symmetry breaking by the gauge invariant coupling of the gauge fields to scalar fields which develop nonzero vacuum expectation values [17]. Introducing scalar fields the Lagrangian [Eq. (3.1)] takes the form

$$\begin{aligned}\mathscr{L} = &- \frac{1}{4} G^L_{\mu\nu} G^{\mu\nu}_L - \frac{1}{4} G^R_{\mu\nu} G^{\mu\nu}_R \\ &+ \bar{f} \left[\gamma^\mu \frac{1+\gamma_5}{2} (i\partial_\mu - gB_\mu{}^L) + \gamma^\mu \frac{1-\gamma_5}{2} (i\partial_\mu - gB_\mu{}^R)\right] f \\ &+ \frac{1}{2} |(\partial_\mu + ig \operatorname{tr} B_\mu{}^L M^L + ig \operatorname{tr} B_\mu{}^R M^R)\, \phi\, |^2 - V(\phi) \\ &- H \cdot \bar{f} T \frac{1+\gamma_5}{2} f \cdot \phi - H \cdot \bar{f} T \frac{1-\gamma_5}{2} f \cdot \phi \\ &+ \text{gauge defining terms.}\end{aligned} \tag{5.1}$$

The scalar field $\phi$ transformas as a real, in general, complicated reducible representation of the gauge group. The matrices $M^h$ ($h = L, R$, respectively) satisfy the algebra [Eq. (3.4)] for $L$ and $R$ separately, while $[M^L, M^R] = 0$ and one has the reality condition

$$(M_k{}^i)^\dagger = M_i{}^k = -(M_i{}^k)^T.$$

The fermion masses are generated by their Yukawa couplings to the scalar fields; the coupling constant is denoted by $H$. The matrices $T$ are constructed such that the projected field $T\phi$ transform as an $(\bar{f}, f)$-representation under $SU_f{}^L \times SU_f{}^R$.

The scalar potential $V(\phi)$ can be written as

$$V(\phi) = -\frac{1}{2} \lambda_{\alpha\beta} \phi^\alpha \phi^\beta + \frac{1}{3!} \mu_{\alpha\beta\gamma} \phi^\alpha \phi^\beta \phi^\gamma + \frac{1}{4!} f_{\alpha\rho\gamma\delta} \phi^\alpha \phi^\rho \phi^\gamma \phi^\delta \tag{5.2}$$

and is invariant under $SU_f{}^L \times SU_f{}^R$:

$$(\delta V / \delta\phi)\, M^{ih}_k \cdot \phi = 0. \tag{5.3}$$

After rewriting $\phi = \phi' + \Phi$, where $\Phi$ is the vacuum expectation value of $\phi$: $\langle 0 \mid \phi' \mid 0 \rangle = 0$, we find the Lagrangian

$$\begin{aligned}\mathscr{L} = &- \frac{1}{4} G^L_{\mu\nu} G^{\mu\nu}_L - \frac{1}{4} G^R_{\mu\nu} G^{\mu\nu}_R + \frac{1}{2} B_\mu{}^h m_B^{2hh'} B^{h'\mu} \\ &+ \bar{f} \left[\gamma^\mu \frac{1+\gamma_5}{2} (i\partial_\mu - gB^L) + \gamma^\mu \frac{1-\gamma_5}{2} (i\partial_\mu - gB_\mu{}^R)\right] f - \bar{f} m_f f \\ &+ \frac{1}{2} |(\partial_\mu + ig \operatorname{tr} B_\mu{}^h M^h) \cdot \phi'\, |^2 - \tilde{V}(\phi', \xi) \\ &+ g^2 \Phi^T (M^h B_\mu{}^h)(M^{h'} B^{\mu h'}) \phi' - \frac{1}{2} \xi^2 \operatorname{tr}(\partial_\mu B_h^\mu)(\partial_\nu B_h^\nu) + L_{\text{ghost}}(\xi) + L_{\text{yukawa}}(\phi')\end{aligned}$$

where $\xi$ is the gauge defining parameter.

Because of the large gauge group, naturally a very large number of scalar fields is needed to break the $SU_f^L \times SU_f^R$-symmetry such that only color and the electric charge remain conserved. With respect to this proliferation of "Higgs" fields, we take the attitude that the "Higgs" mechanism of the soft symmetry breaking is only a preliminary formalism as far as the treatment of the scalar fields is concerned. In particular, it could be that a formulation of the symmetry breaking in purely dynamical terms is possible following the original proposal of Nambu and Jona–Lasinio [18]. In such a scheme the scalars may be introduced as bound states of the basic fields (fermions, gauge bosons). With this possible modification of the theory in mind, we see no reason not to tolerate a large number of scalar fields.

Another feature of the models considered by us is the large number of gauge bosons (typically several hundred). Such a large number of bosons is naturally introduced by our principle of maximal symmetry, for which a gauge group with many generators is very welcome. One also should keep in mind that most of the bosons are colored and therefore they do not exist in the particle spectrum, but exist only as hadronic degrees-of-freeom. Only relatively few bosons are color singlets and exist as actual spin-1 bosons. The latter must be made responsible for the usual weak and electromagnetic interactions and the associated superweak interactions.

We add the following remarks about the role of the Higgs mechanism for a colored field. In our scheme, there are several kinds of colored fields (quarks, colored gauge bosons, and colored scalars). By the spontaneous symmetry breaking, these fields acquire a mass, except the eight massless gluons. According to the confinement hypotheses, there are no colored states in the physical Hilbert space, i.e., the physical mass of a colored state may be interpreted as being infinite. Thus, the mass parameters arising by the spontaneous symmetry breaking cannot be the physical masses of actual particles. What else are they? It is obvious that this question cannot be answered without solving the confinement problem itself, and we cannot provide a definite answer to it. For our discussions, we treat the propagators of all colored fields as if they had a pole at the mass value given by the spontaneous symmetry breaking. Higher order gluon corrections must remove this pole; otherwise, the corresponding colored state would be present in the spectrum.

We conclude this section with some remarks about the onset of the universality of the strong and nonstrong interactions. Spontaneously broken gauge theories are not asymptotically free, if the number of scalar fields is sufficiently high. However, if the scalar fields are composite fields, as we suppose, they do not contribute to the renormalization group equations in the ultraviolet region. Consequently, the basic theory based on the gauge group $SU_f^L \times SU_f^R$ is asumptotically free, even if the symmetry is broken, and the basic coupling constant approaches zero at small distances, provided a dynamical symmetry breaking can be shown to be a consistent framework [18, 29].

Qualitatively, we can describe the behavior of the electromagnetic and strong coupling constants as follows. At infinite distance the electromagnetic coupling constant is given by $\alpha$, while the strong coupling constant is infinite. As the distance becomes smaller, $e^2/4\pi$ increases according to the vacuum polarization contributions of the charged fermions. But as soon as the weak intermediate bosons contribute to the vacuum polarization (at a distance of $10^{-16}$ cm) and the electromagnetic interactions are built into a nonabelian theory, it starts to decrease. The strong interaction coupling constant decreases all the way. At a certain distance, which is determined by the $SU_f^L \times SU_f^R$-breaking, the electromagnetic and strong coupling constants start to fulfill the universality relation [Eq. (4.4)]. From this point they move to zero in proportion to decreasing distance. Further details on the universality between strong and nonstrong interactions will be discussed within a special model (see the end of Section 8).

## 6. Symmetry Breaking II

In this section, we describe the symmetry breaking in general terms, without committing ourselves yet to specific models for the fermions. More details with respect to the symmetry breaking will be described within special models in the following two sections.

It is obvious from the beginning that the spontaneous symmetry breaking has to produce several qualitatively different levels in the boson mass spectrum, i.e., there must exist a complicated hierarchy of the symmetry breaking interactions. The basic Lagrangian [Eq. (3.1)] contains many interactions unobserved thus far. Therefore, the associated gauge bosons must be very heavy, i.e., heavier than a few hundred GeV. Furthermore, the Lagrangian must describe the usual weak interactions, mediated by gauge bosons with masses of the order of 100 GeV. It also contains the massless gluon fields, which produce the typical 1 GeV mass scale of the strong interactions by their infrared instability.

It is instructive to illustrate the symmetry breaking by several different stages. We describe each stage by the corresponding subgroup that is left unbroken at the stage in question. We do not indicate the different irreducible representations of the scalar fields that generate those stages. The typical number of scalars needed is of the order of $f^n$, where $n$ is the average number of fermion representations composing the scalar representations. In the models discussed in Sections 7 and 8, $n$ is at least three. The main stages of the symmetry breaking are given by the following diagram:

$$\begin{array}{ll}
\text{Stage} & SU_f{}^L \times SU_f{}^R \\
\text{I} & \downarrow \\
& (SU_q{}^L \times SU_q{}^R) \times (SU_l{}^L \times SU_l{}^R) \times U_1^2 \\
\text{II} & \downarrow \\
& (SU_3{}^L \times SU_3{}^R)_{\text{color}} \times (SU_m{}^L \times SU_m{}^R) \times (SU_l{}^L \times SU_l{}^R) \times U_1^2 \quad m = q/3 \\
\text{III} & \downarrow \\
& (SU_3^{L+R})_{\text{color}} \times (SU_m{}^L \times SU_m{}^R)_{\text{hadr}} \times (SU_l{}^L \times SU_l{}^R) \times U_1^2 \\
\text{IV} & \downarrow \\
& (SU_3^{L+R})_{\text{color}} \times U_1{}^e.
\end{array}$$

Below we describe the different stages of the symmetry breaking in more detail.

*Stage I*

At this stage the lepton and hadron worlds become separated. The $4 \cdot q \cdot l$ leptoquark bosons acquire a mass. The chiral subgroups of the quarks and leptons as well as the left-handed and right-handed $D$-generators remain unbroken.

The simplest breaking of the leptoquark boson generators will not mix the different leptoquark bosons and will generate a common mass for the latter. Consequently, the leptoquark bosons cause an effective interaction between the quarks and leptons as follows:

$$\mathscr{L}_{\text{eff}} \cong \frac{g^2}{m_{LQ}^2} \left[\bar{q}\gamma_\mu \frac{1+\gamma_5}{2} l \,\bar{l}\gamma^\mu \frac{1+\gamma_5}{2} q + (\gamma_5 \to -\gamma_5)\right] \tag{6.1}$$

where $m_{LQ}$ denotes the common leptoquark boson mass. After Fierz reordering, one obtains

$$\mathscr{L}_{\text{eff}} \cong \frac{g^2}{m_{LQ}^2} \left[\bar{q}\gamma_\mu \frac{1+\gamma_5}{2} q \,\bar{l}\gamma^\mu \frac{1+\gamma_5}{2} l + (\gamma_5 \to -\gamma_5)\right]. \tag{6.2}$$

The net effect of the leptoquark bosons is to contribute a semileptonic neutral current interaction where the corresponding neutral currents are proportional to the baryon and lepton currents (left-handed and right-handed, respectively). We want to avoid any interference of this neutral current interaction with the usual weak interactions. This can be achieved only by requiring that the leptoquark boson mass be superheavy, i.e., much higher than 100 GeV (a mass of the order of 500 GeV or higher would be sufficient).

Nevertheless, it is interesting to consider the possibility that $m_{LQ}$ is not superheavy. In this case, the leptoquark bosons will be affected substantially by the symmetry breaking that generates the mass of the usual weak bosons and mixes strongly the different quantum numbers. The reason for this is that the leptoquark bosons couple to the same scalar multiplets as the weak bosons, and it is impossible to avoid a mixing of the different leptoquark bosons. Consequently, the effective interaction caused by the leptoquark bosons will not be the neutral current interaction [Eq. (6.2)], but it will also contain charged semileptonic interactions. However, such interactions would spoil the usual interpretation of the weak interactions, for example, there would be no reason why purely leptonic and semileptonic interactions are of similar strength (e.g., $G_\mu \approx G_\beta$). We conclude that the masses of the leptoquark bosons must be superheavy.

*Stage II*

The gauge bosons belonging to the chiral group of the quarks $SU_q{}^L \times SU_q{}^R$ can be classified according to the subgroups $SU_3^{L+R}$ (color and the group $SU_m^{L+R}$ generated by the color singlet gauges. We denote the corresponding matrices by $\chi_A$ ($A = 1...8$), $\lambda_i$ ($i = 1...[(q/3)^2 - 1]$). One has:

$2 \cdot 8$ color octet, $SU_m$-singlet bosons, transforming like $\chi \times 1$ under $SU_3$(color) $\times$ $SU_m$ (vector gluons and the associated axial vector bosons).

$2[(q/3)^2 - 1]$ color singlet bosons transforming like $1 \times \lambda$ under $SU_3$(color) $\times$ $SU_m$.

$2 \cdot 8 \cdot [(q/3)^2 - 1]$ color octet bosons transforming like $\chi \cdot \lambda$ under $SU_3$(color) $\times$ $SU_m$.

The last group of gauge bosons is particularly interesting. They carry two different types of conserved charges. Their group generators transform both the different colors and the different color singlet charges among each other. Since the electric charge is among the color singlet charges, some of these bosons carry both color and electric charge. For this reason, the last group of gauge bosons has been denoted previously as diotons [19]. Just like the quarks, gluons, and leptoquark bosons, they are fictitious, i.e., confined inside color singlet hadrons. The possible importance of these bosons for hadron spectroscopy and $e^+e^-$-annihilation will be discussed in Section 12.

In Stage II, the diotons become massive, while all other bosons except the diotons and leptoquark bosons remain massless. The diotons transform as a color octet; consequently, the gauge boson mass matrix cannot mix them with either the leptoquark bosons (color triplets) or the color singlet bosons. Therefore, the diotons cause only interactions among the quarks. We suppose that the masses of the diotons are at least of the same order as the color singlet bosons to be discussed below. In this case, the dioton contributions to the hadronic weak interactions are of the same order or weaker than the contributions of the usual weak bosons.

*Stage III*

During Stage III, the group $(SU_3^L \times SU_3^R)_{\text{color}}$ breaks down to the vectorial group $SU_{3\,\text{color}}^{L+R}$. The axial vector counterparts of the vector gluons acquire a mass. In principle, it is possible to leave out this stage since the group $(SU_3^L \times SU_3^R)_{\text{color}}$ is broken in any case by the introduction of the quark masses. However, we introduce an extra breaking of the axial color generators independently of the quark masses, in order to allow for the possibility that some of the quarks remain massless. Massless axial vector gluons are unwanted because of the anomalies caused by them for the strong interactions (see also Footnote 3).

*Stage IV*

Thus far, the $(l^2 - 1) + [(q/3)^2 - 1]$ color singlet bosons and the $D$-mesons have remained massless. During Stage IV, all these bosons, with the exception of the photon, acquire a mass. This cannot be discussed in detail without commitment to a particular model for the fermions; thus, we restrict ourselves to a few comments. The subtheory of the color singlet gauges is based on the gauge group $U_1^2 \times (SU_m^L \times SU_m^R)_q \times (SU_l^L \times SU_l^R)$, i.e., before the symmetry breaks down there is, in particular, a set of gauge bosons coupled exclusively to quarks and a set of gauge bosons coupled exclusively to leptons. The boson mass matrix generated by the symmetry breaking must mix these two sets of bosons in order to reproduce the photon and the usual weak intermediate bosons, which are known to couple both to quarks and leptons.[7]

Besides the known electromagnetic and weak interactions, many more interactions are introduced by the group $(SU_m^L \times SU_m^R) \times (SU_l^L \times SU_l^R) \times U_1^2$. Therefore, the associated gauge bosons must acquire superheavy masses $\gtrsim$500 GeV) and the corresponding interactions become superweak. Note that the PC-violation may well be an effect due to those superweak interactions.

At Stage IV of the symmetry breaking the $D$-mesons also acquire a mass. In particular, the vectorial $D$-charge is no longer conserved. However, note that this does not necessarily imply by Eq. (4.5) a violation of baryon and lepton number conservation.[8] If baryon number is not conserved, the exact conservation of fermion number, combined with the exact conservation of color, requires that the decay of the proton occur only in a rather high order of the gauge coupling (at least sixth order), and crudely speaking one may view the proton decay as a triple

[7] The separate existence of hadronic and leptonic bosons has also been discussed in [30], although within a completely different approach.

[8] We emphasize that it is possible to define an exactly conserved baryon and lepton number operator, even if there is no massless gauge boson coupled to the $D$-charge. This can be arranged by using the general mechanism proposed by t' Hooft [31]. In this case the Lagrangian of the scalar fields possesses a larger symmetry than the gauge group (see for example, [39]), and one can define a $U_1$ symmetry, whose generator commutes with the gauge group.

$\beta$-decay with a Fermi constant $G \sim \alpha/m_{LQ}^2$, where $m_{LQ}$ is the mass of the lepto-quark bosons. A rough calculation shows that masses of at least $10^3$ GeV are needed to ensure the lifetime of the proton of more than $10^{33}$ sec.

This mechanism, which suppresses the decay of the proton, is similar to the one proposed by Pati and Salam (see [10]). However, note that we cannot view the proton decay as the separate decay of each quark, since in our case the quarks have nonintegral electric charges and color is exactly conserved.

The lower limit for the masses of the leptoquark bosons denoted above can be viewed as the lower limit in the energy scale at which the unification of strong and nonstrong interactions might set in. Of course, we have no information about the different mass scales that arise at very high energies. But it seems natural to interpret the mass of those gauge bosons that are responsible for the proton decay as the highest mass parameter in physics in addition to the mass parameters given by the gravitational interaction. If this is true and the leptoquark bosons really have a mass of the order of $10^3$–$10^5$ GeV, the unification of the strong and nonstrong interactions can set in at distances that are not too much smaller than the typical weak interaction distance. One might already have universality at $10^{-18}$ cm!

Finally, we discuss the possibility that the ordinary weak interactions together with the electromagnetic interactions, belong to an $SU_2 \times U_1$-theory in the limit where the superweak interactions are neglected, i.e., there is an intermediate stage in IV as follows:

$$\begin{array}{ll} & (SU_3^{L+R})_{\text{color}} \times (SU_m{}^L \times SU_m{}^R)_{\text{hadr}} \times (SU_l{}^L \times SU_l{}^R)_{\text{lept}} \times U_1{}^2 \\ \text{IV(a)} & \downarrow \\ & (SU_3^{L+R})_{\text{color}} \times SU_2 \times U_1 \\ \text{IV(b)} & \downarrow \\ & (SU_3^{L+R})_{\text{color}} \times U_1{}^e. \end{array} \tag{6.3}$$

In this case, the basic symmetry $SU_f{}^L \times SU_f{}^R$ determines the $SU_2 \times U_1$-mixing angle by the requirement that the photon and the charged intermediate weak bosons are eigenstates of the gauge boson mass matrix. If we denote the generators of the $SU_2 \times U_1$-group by $T_i$, $i = 1, 2, 3$, and $Y$, the electric generator must be a linear combination of the neutral generators $T_3$ and $Y$:

$$Q^l = \alpha \cdot T_3 + \beta \cdot Y. \tag{6.4}$$

Since any scheme of the elementary fermions must involve at least the observed lepton doublet $(\nu_e, e^-)$, the coefficient of $T_3$ in Eq. (6.4) is determined by the charges of the $(\nu_e, e^-)$-doublet:

$$Q^e = T_3 + \beta \cdot Y. \tag{6.5}$$

On the other hand, the $SU_2 \times U_1$-mixing angle $\theta_w$ is given by

$$\frac{T_3}{(\mathrm{tr}\ T_3^2)^{1/2}} = \cos\theta_w \cdot \frac{Z}{(\mathrm{tr}\ Z^2)^{1/2}} + \sin\theta_w \cdot \frac{Q^e}{(\mathrm{tr}\ Q^{e^2})^{1/2}} \tag{6.6}$$

where $Z$ denotes the neutral $SU_2 \times U_1$-generator orthogonal to the electric charge. One finds:

$$\sin\theta_w = \frac{\mathrm{tr}(T_3 \cdot Q^e)}{(\mathrm{tr}\ T_3^2 \cdot \mathrm{tr}\ Q_e^2)^{1/2}} \tag{6.7}$$

and using Eq. (6.5):

$$\sin^2\theta_w = \frac{\mathrm{tr}\ T_3^2}{\mathrm{tr}\ Q_e^2}. \tag{6.8}$$

If the elementary fermions are either $SU_2$-singlets or $SU_2$-doublets, one can rewire Eq. (6.8) in the form:

$$\sin^2\theta_w = \frac{N_d}{4 \cdot \sum_{\text{fermions}} Q_i^2} \tag{6.9}$$

where $N_d$ is the number of all $SU_2$-doublets (left-handed and right-handed).

## 7. Model A: Twelve Elementary Fermions

In this section, we describe a particular model that is minimal in the sense that that it is based on the observed leptons and the known hadronic degrees-of-freedom including color, represented by nine colored quarks. The fermions are denoted by the matrix

$$\begin{pmatrix} u_r & u_g & u_b & \nu \\ d_r & d_g & d_b & e^- \\ s_r & s_g & s_b & \mu^+ \end{pmatrix} \tag{7.1}$$

where the leptons are represented by the triplet $(\nu, e^-, \mu^+)$ (see Section 2). The gauge group is $SU_{12}^L \times SU_{12}^R$. The electric charge operator is required to be a color singlet; consequently, the electric charges are given by the matrix

$$\begin{pmatrix} \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & 0 \\ -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & -1 \\ -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & 1 \end{pmatrix} \tag{7.2}$$

Note that in this scheme the electric charges for the quarks and the leptons add to zero separately, while the squares of the charges add to two, both for the quarks and the leptons. According to Eq. (4.2), the basic coupling constant is related to the electromagnetic coupling constant by

$$g^2/(4\pi) = 8 \cdot e^2/(4\pi). \tag{7.3}$$

In the region where the universality between strong and electromagnetic interactions holds, the strong and electromagnetic couplings are related by

$$\kappa = g_{st}^2/(4\pi) = 8/3\ e^2/(4\pi). \tag{7.4}$$

The $D$-operator in Scheme A is given by the matrix

$$D = \begin{bmatrix} \frac{1}{9} & & & & & \\ & \ddots & & & & \\ & & \frac{1}{9} & & & \\ & & & -\frac{1}{3} & & \\ & & & & -\frac{1}{3} & \\ & & & & & -\frac{1}{3} \end{bmatrix} \tag{7.5}$$

and the baryon and lepton number can be expressed in terms of $D$ and the fermion number $F$:

$$B = \tfrac{1}{4}(F + 3D), \qquad L = F - 3B. \tag{7.6}$$

In this scheme $e^-$ and $\mu^+$ have the same lepton number assignment; consequently, the decay $\mu \to e\gamma$ is forbidden by lepton number conservation. This is well in agreement with observation.

(a) *Spontaneous Symmetry Breaking I*

The different stages of the symmetry breaking are described by the diagram

$$\begin{array}{ll} \text{Stage} & SU_{12}^L \times SU_{12}^R \\ \text{I} & \downarrow \\ & (SU_9{}^L \times SU_9{}^R)_{\text{quarks}} \times (SU_3{}^L \times SU_3{}^R)_{\text{leptons}} \\ \text{II} & \downarrow \\ & (SU_3{}^L \times SU_3{}^R)_{\text{color}} \times (SU_3{}^L \times SU_3{}^R)_{\text{hadr}} \times (SU_3{}^L \times SU_3{}^R)_{\text{leptons}} \\ \text{III} & \downarrow \\ & (SU_3^{L+R})_{\text{color}} \times (SU_3{}^L \times SU_3{}^R)_{\text{hadr}} \times (SU_3{}^L \times SU_3{}^R)_{\text{leptons}} \end{array} \tag{7.7}$$

During Stage I, the 108 leptoquark bosons acquire a mass, together with the two $D$-mesons. The smallest "Higgs" representation needed for this symmetry breaking has the dimension 220. It is assumed that the leptoquark bosons and the $D$-mesons are superheavy (at least $10^3$ GeV), such that they do not interfere substantially with the usual weak interactions.

At Stage II, the 128 diotons acquire a mass. There is no reason to suppose that the dioton masses are superheavy as well. They cause only interactions among the quarks and cannot mix with the usual weak intermediate bosons because of their color. We assume that the dioton masses are of the same order as the masses for the color singlet weak bosons. At this stage the chiral group of the nine colored quarks $SU_9{}^L \times SU_9{}^R$ splits into the product $(SU_3{}^L \times SU_3{}^R)_{\text{color}} \times (SU_3{}^L \times SU_3{}^R)_{\text{hadr}}$ while the second factor is the usual chiral group for the hadrons generated by the 16 hadronic color singlet generators.

The diotons transform nontrivially both under color isospin and color hypercharge and usual isospin and hypercharge. The squares of all electric dioton charges can be calculated easily since there are $2 \cdot 8\ SU_3$-octets of diotons, and each octet contributes two to the sum. One finds

$$\sum_{\text{diotons}} Q^2 = 32. \tag{7.8}$$

(Note that antiparticles are not counted separately).

During Stage IV of the symmetry breaking, the subgroup $(SU_3{}^L \times SU_3{}^R)_{\text{color}}$ breaks down to the vectorial group( $SU_3^{L+R}$; this leaves us with the yet unbroken group $(SU_3^{L+R})_{\text{color}} \times (SU_3{}^L \times SU_3{}^R)_{\text{hadr}} \times (SU_3{}^L \times SU_3{}^R)_{\text{lept}}$. The further decomposition of this group will be discussed in the following subsections.

(b) *Weak and Superweak Interactions: General Remarks*

In this and the following two subsections, we indicate how the spontaneous symmetry breaking of the group $(SU_3{}^L \times SU_3{}^R)_{\text{hadr}} \times (SU_3{}^L \times SU_3{}^R)_{\text{lept}}$ generated by the 32 color singlet gauges could be arranged such that the resulting interactions have the strength and symmetry properties of the observed weak interactions. Before the symmetry of the color singlet gauges is broken, there is no connection between the hadronic and leptonic gauge bosons: The hadronic bosons cause only diagonal interactions among the quarks. the leptonic ones only diagonal interactions among the leptons (see Footnote 7). The boson-mass matrix generated by the symmetry breaking has to mix the hadronic and leptonic bosons such that only one combination, namely the one given by the matrix [Eq. (7.1)] stays massless and can be identified with the photon, while all the other ones acquire a mass of the order of 100 GeV (the usual weak intermediate bosons) or even higher masses (describing the strength of the superweak interactions).

Due to the relatively large number of color singlet bosons, there is, of course, a considerable amount of freedom in arranging the scheme in agreement with observation. We select one particular path among the open possibilities, which to us seems to be the most simple one. We assume that the group $SU_2$, generated by the usual weak charges, plays an exceptional role. This, of course, confronts us immediately with the problem of the $\Delta S = 1$ neutral currents, since the commutator $[W^+, W^-]$ has a $\Delta S = 1$-piece, proportional to $\sin \theta_c$ giving rise to the decay $K_0^L \to \mu^+\mu^-$. As far as the relevance of the absence of $\Delta S = 1$ neutral currents for model building is concerned, we accept the following point of view. Obviously, there is no problem in the $SU_3$-limit, where the bare quark masses are degenerate. Furthermore it is possible, as conjectured by several authors [32], that the Cabibbo angle $\theta_c$ vanishes in the limit, where the nonstrange bare quark masses vanish (hadronic $SU_2 \times SU_2$-limit, $m_\pi^2 = 0$). Also taking into account the observed smallness of $\theta_c$, we find it a reasonable approximation to neglect the Cavibbo angle and the difficulties due to $\Delta S = 1$ neutral currents for the moment and to work in the $SU_3$-limit (or perhaps $SU_2 \times SU_2$ or $SU_3 \times SU_3$-limit). The real world, i.e., a world in which the hadronic symmetry is broken by the quark masses, is supposed to be a relatively good approximation to a fictitious one in which the bare quark masses are degenerate or vanish and the Cabibbo angle is zero. The latter arises together with the hadronic symmetry breaking by a disturbance of the ideal case $\theta_c = 0$.

One can easily see that there is no problem with $\Delta S = 1$ neutral currents. Although the group $(SU_3^L \times SU_3^R)_{\text{hadr}} \times (SU_3^L \times SU_3^R)_{\text{leptons}}$ contains two neutral $|\Delta S| = 1$ gauge bosons, it is possible to arrange the boson mass spectrum such that the latter do not mix with the neutral leptonic bosons, and therefore, the decay $K_0^L \to \mu^+\mu^-$ does not occur. This will be described in detail in Section 7(d).

### (c) *The Ideal Case*

In this section, we discuss the symmetry breaking for Model A in the ideal case $\theta_c = 0$. We assume the following symmetry breaking scheme as a continuation of the scheme [Eq. (7.7)]:

$$
\begin{array}{ll}
\text{Stage} & (SU_3^{L+R})_{\text{color}} \times (SU_3^L \times SU_3^R)_{\text{hadr}} \times (SU_3^L \times SU_3^R)_{\text{leptons}} \\
\text{IV} & \qquad\downarrow \\
 & (SU_3^{L+R})_{\text{color}} \times (SU_3^L \times SU_3^R)_{\text{hadr}} \times (SU_2 \times U_1^3)_{\text{leptons}} \\
\text{V} & \qquad\downarrow \\
 & (SU_3^{L+R})_{\text{color}} \times (SU_2^L \times U_1^3)_{\text{hadr}} \times (SU_2 \times U_1^3)_{\text{leptons}} \\
\text{VI} & \qquad\downarrow \\
 & (SU_3^{L+R})_{\text{color}} \times (SU_2)_{\text{hadr+leptons}} \times U_1^6 \\
\text{VII} & \qquad\downarrow \\
 & (SU_3^{L+R})_{\text{color}} \times (SU\ )_{\text{hadr+leptons}} \times U_1 \\
\text{VIII} & \qquad\downarrow \\
 & (SU_3^{L+R})_{\text{color}} \times U_1^e
\end{array}
\tag{7.9}
$$

In Stage IV, only the $SU_2$-subgroup relevant for the ordinary leptonic weak interactions remains unbroken, together with three neutral generators. This breaking is closely related to the one discussed by Weinberg (see Section 2).

During Stage V, the hadronic group $SU_3^L \times SU_3^R$ breaks down to $SU_2^L \times U_1^L \times (U_1^R)^2$, i.e., the right-handed $I_3$ and $Y$-generator, and the left-handed $SU_3$ is broken down to $SU_2 \times U_1$ (isospin and hypercharge).

At Stage VI, both isospin groups mix with each other such that the difference $(SU_2)_{\text{hadr-lept}}$ is broken while the bosons belonging to $(SU_2)_{\text{hadr+lept}}$ remain massless. Besides this group there are still six $U_1$ groups left; one is dealing with the group $SU_2 \times U_1^6$. This group may be the gauge group for the conventional weak and electromagnetic interactions, together with some neutral current interactions, in the ideal limit $\theta_c = 0$. All bosons that do not belong to this group are supposed to acquire superheavy masses, of the order of 500 GeV or higher. There is no particular reason for the assumption that one or several neutral $U_1$ generators are broken at the superheavy level. If, however, we suppose that the most econonomical case, "all neutral $U_1$ generators are broken at the superheavy level, except the particular linear combination needed to generate the photon" is preferred by nature, we arrive at the group $SU_2 \times U_1$. This is indicated by Stage VII.

At the last stage, VIII, the $SU_2 \times U_1$ subtheory is broken such that the photon remains massless, while the other three bosons acquire masses.

The ideal case $\theta_c = 0$ is supposed to be a relatively good approximation to the real world, and a very good one for reactions, in which the $\Delta S = 1$ processes are of second order in $\theta_c$ (note that $\sin^2 \theta_c \approx 0.04$), e.g., neutrino–hadron total cross sections. Therefore it seems to be worthwhile to analyze this case in more detail and, in particular, to study the implications of the underlying basic symmetry group for the ordinary weak interactions.

If we neglect the Cabibbo angle, the weak generators are given by the following matrices, which act on the fermion vector $(u, d, s, \nu, e^-, \mu^+)$:

$$T_+ = \frac{1}{5^{1/2}} \begin{pmatrix} 0 & 1_L & 0 & & & \\ 0 & 0 & 0 & & 0 & \\ 0 & 0 & 0 & & & \\ & & & 0 & 0 & -1_R \\ & 0 & & 1_L & 0 & 0 \\ & & & 0 & 0 & 0 \end{pmatrix} \tag{7.10}$$

These matrices should be understood as $12 \times 12$ matrices; each quark is to be counted three times because of the color indix, which is not denoted explicitly. The matrices are normalized such that $\sum_{i,k=1}^{12} a_{ik,L}^2 + a_{ik,R}^2 = 1$.

The neutral generator that forms together with the charged weak generators an isospin triplet is given by

$$T_3 = \frac{1}{10^{1/2}} \begin{pmatrix} \begin{matrix} 1_L & 0 & 0 \\ 0 & -1_L & 0 \\ 0 & 0 & 0 \end{matrix} & \\ & \begin{matrix} 1_R & -1_L & 0 & 0 \\ & 0 & 1_L & 0 \\ & 0 & 0 & -1_R \end{matrix} \end{pmatrix} \tag{7.11}$$

The electric charge and the neutral generator $T_3$, of course, are not orthogonal with respect to each other. The linear combination $Z$, which is orthogonal to $Q^e$, couples to the following neutral current:

$$J_\mu{}^n = \frac{4}{110^{1/2}} \Big( \frac{1}{12} \bar{u}\gamma_\mu u + \frac{1}{2} \bar{u}\gamma_\mu\gamma_5 u - \frac{7}{24} \bar{d}\gamma_\mu d - \frac{1}{2} \bar{d}\gamma_\mu\gamma_5 d + \frac{5}{24} \bar{s}\gamma_\mu s$$

$$+ \bar{\nu}\gamma_\mu\gamma_5\nu + \frac{1}{8} \bar{e}^-\gamma_\mu e^- - \frac{1}{2} \bar{e}^-\gamma_\mu\gamma_5 e^- - \frac{1}{8} \bar{\mu}^+\gamma_\mu\mu^+ - \frac{1}{2} \bar{\mu}^+\gamma_\mu\gamma_5\mu^+ \Big). \tag{7.12}$$

The neutral current can be parameterized in terms of the $SU_2 \times U_1$-mixing angle $\theta_w$. Since the number of $SU_2$ doublets is five (three quark doublets, two lepton doublets) we find from Eq. (6.9):

$$\sin^2 \theta_w = \tfrac{5}{16} \approx 0.31. \tag{7.13}$$

We reach the conclusion:

In the ideal case $\theta_c = 0$ the weak and electromagnetic interactions of leptons and hadrons can be described by an $SU_2 \times U_1$-gauge theory in which the mixing angle is fixed by $\sin^2 \theta_w = \frac{5}{16}$ ($\theta_w \approx 34°$).

We would like to add some remarks about the breaking of $SU_2 \times U_1$. The simplest way to break $SU_2 \times U_1$ is to couple the gauge bosons to an $SU_2$ doublet of scalar fields and let the neutral member of the doublet develop a nonzero vacuum expectation value [1]. In this case, one finds a specific prediction for the $W$-boson masses:

$$\begin{aligned} m_{W_\pm} &= \frac{37.3}{|\sin \theta_w|} \text{ GeV} \cong 67 \text{ GeV} \\ m_Z &= \frac{74.6}{|\sin^2 \theta|} \text{ GeV} \cong 80 \text{ GeV}. \end{aligned} \tag{7.14}$$

Not only the algebraic form of the neutral current [Eq. (7.12)], but also its coupling strength is therefore determined by $\theta_w$. However, it is important to note that the prediction [Eq. (7.14)] about the $Z$-mass is merely a consequence of the scalar fields being an $SU_2$ doublet. Within our approach this seems a very special assumption, which may not be relevant. For this reason we should like to keep the mass of the $Z$-boson as a free parameter, to be determined by experiment. In general, we find the following phenomenological Lagrangian for the weak interactions in the case $\theta_c = 0$, neglecting the superweak interactions:

$$\mathscr{L}_{\text{weak}} = \frac{G_F}{2^{1/2}} (J_\mu{}^+)^* J^{+\mu} + \frac{G_F{}'}{2^{1/2}} (J_\mu{}^n)^* J^{n\mu}. \tag{7.15}$$

where $G_F$ is the usual Fermi constant, $J_\mu{}^+$ is the charged current, and $G_F{}'$ is a new Fermi constant describing the coupling strength of the neutral current [Eq. (7.12)]. The phenomenological consequences of the Lagrangian [Eq. (7.15)] will be described in Section 9.

(d) *Strangeness Violating Weak Interactions*

In Section 2, we described the basic features of the weak interactions that we want to incorporate into the scheme. It was pointed out that the arguments in favor of the so-called Cabibbo universality are still not satisfying, and it is quite possible that the relation $G_\beta = \cos\theta_c G_\mu$ is not correct. Furthermore, we should like to stress that little is known about the purely hadronic weak interactions. Thus far, there is no concrete evidence for the assumption that the usual hadronic weak current is solely responsible for the hadronic weak interactions. In fact, it is known that the $\Delta I = \frac{1}{2}$ rule for the hadronic weak interactions leads to difficulties with respect to that assumption.

We find it quite possible that the conventional scheme of the weak interactions and its interpretation within an $SU_2 \times U_1$ gauge theory is too narrow a scheme and we propose to enlarge the set of gauge bosons for the weak interactions, i.e., to enlarge the corresponding gauge group. This does not exclude the possibility that an $SU_2 \times U_1$ theory may describe the purely leptonic and the semileptonic weak interactions rather well, in particular, in the limit where one neglects the strangeness violating weak interactions (i.e., the Cabibbo angle).

The largest possible subgroup of $SU_{12}^L \times SU_{12}^R$ for the weak and electromagnetic interactions would be the group of the color singlet gauges $(SU_3{}^L \times SU_3{}^R)_{\text{hadr}} \times (SU_3{}^L \times SU_3{}^R)_{\text{lept}}$. However, as far as the leptonic weak interactions are concerned, there is no reason for the assumption that more currents than the usual leptonic weak currents are relevant. Furthermore, there is no reason to believe that right-handed hadronic currents play a substantial role. Thus, we take the subgroup

$$(SU_3{}^L)_{\text{hadr}} \times (SU_2)_{\text{lept}} \times (U_1)_{\text{hadr+lept}} \tag{7.16}$$

as the gauge group for the description of the usual weak and electromagnetic interactions where the $U_1$ subgroup is constructed such that the photon belongs to the group. The other color singlet bosons are assumed to be superheavy.

We have the following symmetry breaking scheme as the continutation of scheme [Eq. (7.7)]:

$$\begin{array}{ll}
\text{Stage} & (SU_3^{L+R})_{\text{color}} \times (SU_3{}^L \times SU_3{}^R)_{\text{hadr}} \times (SU_3{}^L \times SU_3{}^R)_{\text{lept}} \\
\text{V} & \downarrow \\
 & (SU_3^{L+R})_{\text{color}} \times (SU_3{}^L \times SU_3{}^R)_{\text{hadr}} \times (SU_2 \times U_1)_{\text{lept}} \\
\text{VI} & \downarrow \\
 & (SU_3^{L+R})_{\text{color}} \times (SU_3{}^L)_{\text{hadr}} \times (SU_2)_{\text{lept}} \times (U_1)_{\text{lept+hadr}} \\
\text{VII} & \downarrow \\
 & (SU_3^{L+R})_{\text{color}} \times (U_1{}^e).
\end{array} \tag{7.17}$$

At Stage IV, we are left with the group [Eq. (7.16)]. The advantage of using this group is that one is able to avoid the decay $K_0{}^L \to \mu^+\mu^-$ in lowest order by requiring that the $\Delta S = 1$ neutral current does not couple to the leptonic current, i.e., the strangeness carrying neutral hadronic gauge bosons do not mix with the neutral leptonic bosons. Before the symmetry is broken, one is dealing with a set of eight bosons coupling to the eight left-handed hadronic currents, a set of three bosons coupling to the leptonic currents, and one neutral boson which couples to both leptons and hadrons (see Footnote 7). There are many possible ways to arrange the symmetry breaking by the choice of the vacuum expectation values of the "Higgs" fields such that the usual weak interactions arise, but $\Delta S = 1$ neutral currents coupled to leptons are avoided. We impose the following requirements on the symmetry breaking:

(1) The photon described by the generator

$$\begin{pmatrix} \frac{2}{3} & & \\ & -\frac{1}{3} & \\ & & -\frac{1}{3} \end{pmatrix} + \begin{pmatrix} 0 & & \\ & -1 & \\ & & 1 \end{pmatrix}$$

remains massless.

(2) The $SU_2 \times U_1$ subtheory discussed in the previous section is a good approximation in the limit where the Cabibbo angle is neglected.

(3) The quarks acquire masses by the symmetry breaking and define strangeness and isospin in the hadronic $SU_3$-space.

(4) The charged $W$-bosons couple both to strangeness conserving and strangeness changing currents and one has approximately $G_\mu{}^2 \approx G_\beta{}^2 + G_\Lambda{}^2$.

(5) The neutral bosons that couple to leptons, do not carry strangeness.

The generators of the hadronic group $SU_3^L$ are denoted by $H_i$ ($i = 1...8$), the leptonic isospin generators are denoted by $L_\alpha$ ($\alpha = 1, 2, 3$), while the remaining $U_1$ generator is denoted by $Y'$.

We single out two noncommuting $SU_2$ subgroups of $SU_3^L$, namely, the isospin acting in the $(u, d)$ space and the $V$-spin, acting in the $(u, s)$ space. Our general procedure to avoid $\Delta S = 1$ neutral currents is as follows. Suppose we arrange to break the group $SU_3^L \times SU_2 \times U_1$ according to

$$\begin{array}{c} (SU_3^L)_{\text{hadr}} \times (SU_2)_{\text{lept}} \times U_1 \\ \downarrow \\ (SU_2)_{(u,d)+\text{lept}} \times U_1 \\ \downarrow \\ U_1^e \end{array} \tag{7.18}$$

In this case the resulting weak interactions conserve strangeness, i.e., $\theta = 0$. We arrange the analogous breaking

$$\begin{array}{c} (SU_3^L)_{\text{hadr}} \times (SU_2)_{\text{lept}} \times U_1 \\ \downarrow \\ (SU_2)_{(u,s)+\text{lept}} \times U_1 \\ \downarrow \\ U_1^e \end{array} \tag{7.19}$$

In that case the weak interactions conserve $I_3$ ($\theta = \pi/2$). In both cases the weak neutral current has no $\Delta S = 1$ component.

It is assumed that the real breakdown of $SU_3^L \times SU_2 \times U_1$ arises by a superposition of both kinds of symmetry breaking. In other words, it is assumed that an interference between the two cases $\theta = 0$, $\theta = \pi/2$ takes place, and the observed strangeness violation of the weak interactions is due to this interference. This mechanism ensures that there are no strangeness violating effects caused by neutral bosons that couple to leptons.

The boson mass matrix depends, of course, on the "Higgs" representation breaking the symmetry. In the appendix, a typical example of the kind of symmetry breaking outlined above is studied. The mass matrix for the three charged $W$-bosons $\hat{H}_{1+i2}$, $\hat{H}_{4+i5}$, $\hat{L}_{1+i2}$, ($\hat{G}$ denotes the boson belonging to the generator $G$) depends on three parameters, $F_c$, $\epsilon$, and $\mu$, where $F_c$ is fixed by the "Higgs" representation, $\mu$ is the overall scale of the vacuum expectation values of the "Higgs" fields, and $\epsilon$ describes the relative weight of the $\theta = 0$ case compared to the case $\theta = \pi/2$.

Using these three parameters, one can calculate the decay constants as follows:

$$\frac{G_\beta}{G_\mu} = \frac{1}{1 + F_c\epsilon}, \qquad \frac{G_\Lambda}{G_\mu} = \frac{1}{1 + F_c/\epsilon}. \tag{7.20}$$

If we require the Cabibbo form of the weak current one can express $F_c$ and $\epsilon$ in the form of the Cabibbo angle:

$$F_c^2 = (\sec\theta_c - 1)(\operatorname{cosec}\theta_c - 1)$$
$$\epsilon^2 = \frac{\sec\theta_c - 1}{\operatorname{cosec}\theta_c - 1}. \tag{7.21}$$

Using the value $\sin\theta_c = 0.2$ as an example, one finds: $F_c = 0.29$, $\epsilon = 0.07$. The eigenvalues of the mass matrix for the charged bosons are:

$$m_{W_1^+} \approx 67 \text{ GeV}, \qquad m_{W_2^+} \approx 189 \text{ GeV}, \qquad m_{W_3^+} \approx 502 \text{ GeV}. \tag{7.22}$$

All three charged $W$-bosons contribute to the charged leptonic and semileptonic currents. We have arranged the parameters $F_c$ and $\epsilon$ such that the resulting weak current is the usual weak current rotated in $SU_3$ space by the Cabibbo angle $\theta_c$. This is put in by hand and seems within our approach not a very natural constraint. Because of the smallness of $\epsilon$, the relative amount of strangeness violation by the weak current is small, and $G_\beta$ is naturally close to $G_\mu$, but there is no reason for the exact relation $G_\beta = \cos\theta_c G_\mu$. It would be very interesting to find a breakdown of this relation. This would indicate strongly that the gauge group for the weak interactions is indeed larger than $SU_2$.

In our scheme, one is dealing with four neutral generators $H_3$, $H_8$, $L_3$, and $Y'$. One linear combination of these is identified with the photon and remains unbroken. In the appendix, the symmetry breaking is constructed such that the lowest-lying neutral boson dominates the neutral current and is the boson that is obtained in the $SU_2 \times U_1$ theory with vanishing strangeness violation, i.e., the $SU_2 \times U_1$ type coupling for the neutral current with $\sin^2\theta_w = 5/16$ is preserved. However, because of the more complicated symmetry breaking, the mass of the lowest neutral boson does not follow Eq. (7.14). It depends on a representation parameter $F_n$, and reasonable choices for this parameter give values between 40 and 200 GeV for the dominating neutral boson (see the appendix).

By construction, the $\Delta S = 1$ neutral bosons, which couple to $H_{6\pm i7}$, do not mix with the leptonic bosons, nor do they mix with other neutral hadronic bosons. Their mass is given by

$$M_{6\pm i7} = (\tfrac{2}{5})^{1/2}\mu[(1+\epsilon)F_c]^{1/2} \approx 190 \text{ GeV}, \tag{7.23}$$

i.e., they are almost degenerate with $W_2^\pm$, which couple predominantly to $H_{4\pm i5}$. Since the $\Delta S = 1$ neutral bosons do not mix with each other, there are no $K_0 - \bar{K}_0$ transitions of order $G_F$.

Finally, we mention the corrections to the decay $K_0^L \to \mu^+\mu^-$ and the $K_1 - K_2$ mass difference of order $G_F \cdot \alpha$. Since we are dealing with altogether 12 intermediate bosons for the weak and electromagnetic interactions, the higher order corrections to the decay $K_0{}^L \to \mu^+\mu^-$ and the $K_0 - \overline{K}_0$ transitions receive contributions from many diagrams. However, it is easy to see that one can arrange the mass matrix of the gauge bosons such that all terms of order $G_F \cdot \alpha$ cancel. Just as we were able to suppress the $K_0{}^L \to \mu^+\mu^-$ decay and the $K_0 - \overline{K}_0$ transitions of order $G_F$ by imposing constraints on the boson mass matrix, one can require the same reactions to be suppressed to order $G_F \cdot \alpha$ by imposing this as a constraint on the mass matrix.

Our approach to avoid the $\Delta S = 1$ neutral current problem is certainly less economical and predictive than the usual approach of introducing new hadronic degrees-of-freedom (e.g., charm). However, if new hadronic degrees-of-freedom should not be found, an approach to the weak interactions like ours or similar to ours, which involves a larger group than $SU_2 \times U_1$, seems to be the only one possible.

## 8. Model B: Sixteen Elementary Fermions

In this model, we allow for the possibility of introducing charm in the form of a new charmed triplet of quarks [21]. The number of elementary fermions is 16, and they are represented by the matrix

$$\begin{pmatrix} c_r & c_g & c_b & \nu_e \\ u_r & u_g & u_b & \nu_\mu \\ d_r & d_g & d_b & e^- \\ s_r & s_g & s_b & \mu^- \end{pmatrix} \tag{8.1}$$

where $(c_r, c_g, c_b)$ denotes the charmed quark triplet [33].

The essential differences between this scheme and the previous one are as follows:

(1) Both the electron and muon-type neutrinos are full four-component Dirac fields, i.e., there are four leptons instead of three as in the $SU_{12}^L \times SU_{12}^R$ scheme.

(2) The electric charges of the leptons do not add to zero. Therefore, the electric charge of the charmed quark has to be $\frac{2}{3}$ in order for the electric charges to be a $SU_{16}$ generator. It is given by the matrix

$$Q^e = \begin{pmatrix} \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & 0 \\ \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & 0 \\ -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & -1 \\ -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & -1 \end{pmatrix} \tag{8.2}$$

Using Eq. (4.2) the basic coupling constant can be calculated in terms of the electromagnetic one:

$$g^2/4\pi = 32/3 \cdot e^2/4\pi \tag{8.3}$$

while the strong and electromagnetic coupling constants are related to each other in the symmetry limit by

$$\kappa = g_{st}^2/4\pi = 8/3 \cdot e^2/4\pi. \tag{8.4}$$

i.e., one obtains the same result as in the $SU_{12} \times SU_{12}$ model.

(3) The number of gauge bosons in Scheme B is 510.

(4) The $D$-charge is represented by the $16 \times 16$ mattix

$$D = \begin{bmatrix} \frac{1}{12} & & & & & \\ & \ddots & & & & \\ & & \frac{1}{12} & & & \\ & & & -\frac{1}{4} & & \\ & & & & \ddots & \\ & & & & & -\frac{1}{4} \end{bmatrix} \tag{8.5}$$

and the baryon and lepton numbers are given by

$$B = \tfrac{1}{4}(F + 4D), \qquad L = F - 3B. \tag{8.6}$$

Note that in Scheme B the decay $\mu^+ \to e^+\gamma$ is allowed by fermion number conservation. It must be suppressed by the spontaneous symmetry breaking.

The symmetry breaking is described by the following diagram:

$$\begin{array}{ll}
\text{Stage} & SU_{16}^L \times SU_{16}^R \\
\text{I} & \downarrow \\
& (SU_{12}^L \times SU_{12}^R)_{\text{quarks}} \times (SU_4{}^L \times SU_4{}^R)_{\text{leptons}} \times (U_1{}^L \times U_1{}^R)_D \\
\text{II} & \downarrow \\
& (SU_3{}^L \times SU_3{}^R)_{\text{color}} \times (SU_4{}^L \times SU_4{}^R)_{\text{hadr}} \times (SU_4{}^L \times SU_4{}^R)_{\text{lept}} \times (U_1{}^L \times U_1{}^R)_D \\
\text{III} & \downarrow \\
& (SU_3^{L+R})_{\text{color}} \times (SU_4{}^L \times SU_4{}^R)_{\text{hadr}} \times (SU_4{}^L \times SU_4{}^R)_{\text{lept}} \times (U_1{}^L \times U_1{}^R)_D \\
\text{IV} & \downarrow \\
& (SU_3^{L+R})_{\text{color}} \times (SU_4{}^L \times SU_4{}^R)_{\text{hadr}} \times (SU_2{}^L \times U_1{}^4)_{\text{lept}} \times (U_1{}^L \times U_1{}^R)_D \\
\text{V} & \downarrow \\
& (SU_3^{L+R})_{\text{color}} \times (SU_2{}^L \times SU_1{}^R)_{\text{hadr}} \times (SU_2{}^L \times U_1{}^4)_{\text{lept}} \times (U_1{}^L \times U_1{}^R)_D \\
\text{VI} & \downarrow \\
& (SU_3^{L+R})_{\text{color}} \times (SU_2{}^L)_{\text{hadr+lept}} \times U_1^{10} \\
\text{VII} & \downarrow \\
& (SU_3^{L+R})_{\text{color}} \times (SU_2{}^L)_{\text{hadr+lept}} \times U_1 \\
\text{VIII} & \downarrow \\
& (SU^{L+R})_{\text{color}} \times U_1{}^e
\end{array} \tag{8.7}$$

The eight different stages of the symmetry breaking are analogous to those in Model A for $\theta_c = 0$ and are not described in detail. The main differences between the two symmetry breaking schemes are:

(a) The number of leptoquark bosons is 192, the number of diotons 240. The sum of the squares of all dioton charges is 64.

(b) The hadronic symmetry group is $SU_4^L \times SU_4^R$ instead of $SU_3^L \times SU_3^R$. This symmetry has to be broken very strongly, since thus far no charmed hadrons were observed.

(c) The photon has a nontrivial $D$ content, since the electric charges do not add to zero in the lepton and hadron space separately. Consequently, the $D$-generator must be broken at the same stage where the hadronic and leptonic $(SU_4^L \times SU_4^R)$-groups are broken. During Stage I only the leptoquark bosons acquire a mass, while the $D$-generators remain unbroken.

(d) The introduction of charm allows the addition of a charmed piece to the usual weak hadronic current such that the $\Delta S = 1$ operators in the neutral current cancel:

$$J_\mu^+ = \bar{u}\gamma_\mu(1 + \gamma_5)\, d' + \bar{c}\gamma_\mu(1 + \gamma_5)\, s'$$

where

$$\begin{aligned} d' &= d \cos \theta_c + s \sin \theta_c \\ s' &= -d \sin \theta_c + s \cos \theta_c\,. \end{aligned} \tag{8.8}$$

This allows us to arrange the symmetry breaking in a simpler way as in Model A and to assume that the $SU_2 \times U_1$ subtheory is a perfect description of the weak and electromagnetic interactions insofar as the superheavy boson effects can be neglected.

The charged weak generator is given by the following matrix, which acts on the fermion vector $(cud's'\nu_\mu\nu_e e^-\mu^-)$:

$$T_+ = \frac{1}{8^{1/2}} \begin{bmatrix} \begin{matrix} 0 & 0 & 0 & 1_L \\ 0 & 0 & 1_L & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} & 0 \\ 0 & \begin{matrix} 0 & 0 & 0 & 1_L \\ 0 & 0 & 1_L & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} \end{bmatrix} \tag{8.9}$$

while the third component of the weak isospin is given by:

$$T_3 = \frac{1}{4}\begin{bmatrix} 1 & & & & & & & \\ & 1 & & & & 0 & & \\ & & -1 & & & & & \\ & & & -1 & & & & \\ & & & & 1 & & & \\ & 0 & & & & 1 & & \\ & & & & & & -1 & \\ & & & & & & & -1 \end{bmatrix} \tag{8.10}$$

After orthogonalization one finds that the neutral generator $Z$ couples to the neutral current

$$\begin{aligned} J_\mu{}^n = \frac{1}{2(10)^{1/2}} \Big(&\bar{c}\gamma_\mu\gamma_5 c + \bar{u}\gamma_\mu\gamma_5 u - \frac{1}{2}\bar{d}\gamma_\mu d - \bar{d}\gamma_\mu\gamma_5 d - \frac{1}{2}\bar{s}\gamma_\mu s - \bar{s}\gamma_\mu\gamma_5 s \\ &+ \bar{\nu}_\mu\gamma_\mu(1+\gamma_5)\,\nu_\mu + \bar{\nu}_e\gamma_\mu(1+\gamma_5)\,\nu_e \\ &+ \frac{1}{2}\,\bar{e}^-\gamma_\mu e^- - \bar{e}^-\gamma_\mu\gamma_5 e^- + \frac{1}{2}\,\bar{\mu}^-\gamma_\mu\mu^- - \bar{\mu}^-\gamma_\mu\gamma_5\mu^-\Big). \end{aligned} \tag{8.11}$$

For the $SU_2 \times U_1$ mixing angle one finds from (6.9):

$$\sin^2\theta_w = 3/8 \approx 0.38. \tag{8.12}$$

The mass of the charged $W$-bosons is then fixed to

$$m_{W^c} = 37.3/\sin\theta_w = 61 \text{ GeV}. \tag{8.13}$$

With respect to the mass of the neutral boson $Z$ the same remarks as in Section 7 are in order. Only if one makes the very special assumption that the $SU_2 \times U_1$ subtheory is broken by an $SU_2$ doublet of Higgs fields, the $Z$-mass is determined by

$$m_Z = 74.6/\sin 2\theta_w = 77 \text{ GeV}. \tag{8.14}$$

Again we want to keep the $Z$-mass, and consequently the coupling strength of the neutral current, as a free parameter of the scheme. The phenomenological implications of Eq. (8.11) will be discussed in the following section.

Finally, we discuss within Model B the question of universality between strong and nonstrong interactions. Analogous considerations can be carried out for Model A and for any model within the general approach discussed in Section 3. We assume that the scalar fields can be treated as bound states and therefore can be neglected if one considers the behavior of the Greens functions at small distances.

We concentrate on the subgroup $(SU_3)_{\text{color}} \times (U_1)_e \subset SU_{16}^L \times SU_{16}^R$. The strong interaction coupling constant $\kappa = g_{st}^2/4\pi$ is supposed to be of order 1/5 if one renormalizes at typical hadronic euclidean distances $l_0$, say $l_0 = 10^{-14}$ cm. If we apply perturbation theory to the renormalization group equations for the subtheory based on $(SU_3)_{\text{color}}$, the renormalized coupling constant as a function of the euclidean distance $l$ behaves as follows [7, 12, 13]:

$$\kappa(l) = \frac{\kappa_0}{1 + (25/6\pi)\,\kappa_0 t} \tag{8.15}$$

where $t = \log(l_0/l)$.

According to Eq. (8.4) the strong coupling constant is related to the electromagnetic coupling constant at distances where the universality can be applied: $\kappa = 8/3 \cdot e^2/4\pi$. Because of the smallness of the electromagnetic coupling at $l = l_0$, the renormalization of the electromagnetic coupling (and also of the weak couplings) is very small compared to the renormalization of the strong coupling constant at distances large relative to $\exp(-1/\alpha) \cdot l_0$. Thus, as a rough approximation, one may neglect the renormalization of the electromagnetic coupling and ask at which distances the strong coupling constant becomes equal to $8/3\alpha \approx 1/50$. Solving the equation

$$\frac{\kappa_0}{1 + (25/6\pi)\,\kappa_0 t} = \frac{8}{3}\,\alpha \tag{8.16}$$

with $\kappa_0 \approx 1/5$ one finds

$$t \approx 34, \qquad l^* \approx 10^{-29}\ \text{cm}. \tag{8.17}$$

One may expect therefore, that the universality between strong and nonstrong interactions does not set in before one reaches distances of the order of $l^* \approx 10^{-29}$ cm and all algebraic implications of the basic lepton–quark symmetry would be subject to sizable renormalization effects.[9] This conclusion, however, is not justified if one considers the effects due to the diotons, leptoquark bosons, and color singlet bosons setting in at much larger distances than $l^*$. The point is that once we reach distances of the order of $m_D^{-1}$ or $m_{LQ}^{-1}$, $10^{-17}$ cm say, the vacuum polarization effects of those bosons become important.

Considering the renormalization of $\kappa$ one finds that after reaching distances of the order of the dioton Compton wavelenth the whole chiral group of the 12 quarks $SU_{12}^L \times SU_{12}^R$ becomes relevant, and instead of Eq. (8.15) one has to use the equation appropriate to $SU_{12}^L \times SU_{12}^R$ [7]:

$$\kappa(t) = \frac{\kappa_0}{1 + (524/3\pi)\,\kappa_0 t} \tag{8.18}$$

[9] Within the $SU_5$ model of [11] those corrections were calculated recently in [34].

i.e., the factor $25/6\pi$ in Eq. (8.15) is enhanced to $524/3\pi$, about 42 times.[10] After reaching distances corresponding to the formal leptoquark boson mass, the full group $SU_{16}^L \times SU_{16}^R$ becomes relevant, and one has from there on:

$$\kappa(t) = \frac{\kappa_0}{1 + (700/3\pi)\,\kappa_0 t} \tag{8.19}$$

i.e., the factor $25/6\pi$ in Eq. (8.15) is enhanced 56 times by the group $SU_{16}^L \times SU_{16}^R$. Connecting the different regions, one finds the picture for the behavior of $\kappa$ and the electromagnetic coupling constant given in Fig. 1.

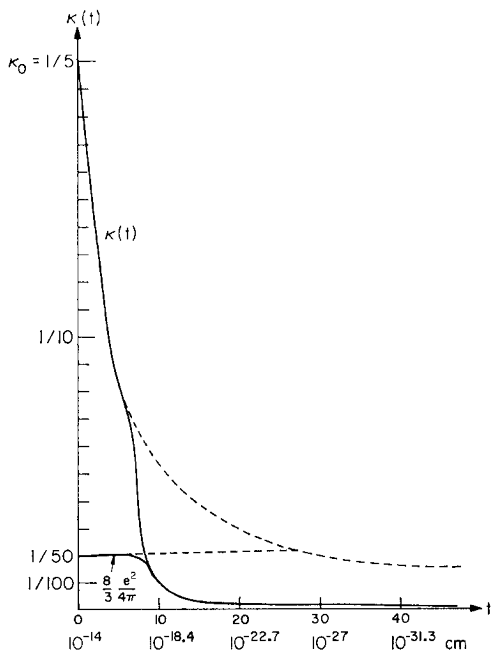



FIG. 1. The behavior of the strong and electromagnetic coupling constants as a function of $t = \log l_0/l$ ($l_0 = 10^{-14}$ cm).

[10] Of course, there is no discontinuity in the behavior of $\kappa(l)$; the transition between the $SU_3$ region and the $SU_{12}^L \times SU_{12}^R$ region is smoothed out by the mass insertion terms in the renormalization group equations. Note that the rate of decrease of the coupling constant is to a rough approximation proportional to the number of colored gauge bosons that can couple to the gluons. The group $SU_3$ has eight bosons, and the group $SU_{12} \times SU_{12}$ has 256 colored bosons, which explains the large factor, 42.

The strong coupling constant $\kappa$ reaches the value $8/3\alpha \approx 1/50$ rather quickly, at energies only slightly larger than the leptoquark boson mass. If the latter is about 500–1000 GeV, the universality between strong and nonstrong interactions is expected to be essentially valid already at distances of the order of $10^{-18}$ cm. Consequently, there are no sizable renormalization corrections to algebraic relations derived from the basic symmetry (they are of order $\alpha/2\pi \log(m_{LQ}/m_W) \sim \alpha/2\pi \log 10$). If Model B is correct, and the conventional weak and electromagnetic interactions are indeed described by an $SU_2 \times U_1$ subtheory, one should observe $\sin^2 \theta_w \simeq 3/8$ in the neutrino production experiments.

## 9. Phenomenological Applications

We concentrate on the total cross sections for (anti) neutrino hadron scattering and define the following ratios, which involve the total cross sections induced by the charged and neutral currents in case of $I = 0$ targets:

$$R^c = \frac{\sigma^+_{\bar{\nu}\mu}}{\sigma^-_{\nu\mu}}, \qquad R^\nu = \frac{\sigma_{\nu\nu}}{\sigma^-_{\nu\mu}}, \qquad R^{\bar{\nu}} = \frac{\sigma_{\bar{\nu}\bar{\nu}}}{\sigma^+_{\bar{\nu}\mu}}. \tag{9.1}$$

Assuming that the current commutator can be abstracted from the quark model (quark lightcone algebra, quark parton model) and neglecting the antiquark contributions to the nucleon wave functions, one finds in the scaling region

$$R^c \cong \tfrac{1}{3} \tag{9.2}$$

which is known to be in good agreement with experiment [35, 36]. For the neutral current we assume that the $SU_2 \times U_1$ subtheory is a good approximation to the conventional wek interactions if we neglect the Cabibbo angle. Then the same assumptions as above hold for the ratios involving neutral currents [37]:

$$\begin{aligned} R^\nu &\cong \left(\frac{1}{2} - \sin^2\theta_w + \frac{20}{27}\sin^4\theta_w\right)\cdot\frac{\rho^2}{\cos^4\theta_w} \\ R^{\bar{\nu}} &\cong \left(\frac{1}{2} - \sin^2\theta_w + \frac{20}{9}\sin^4\theta_w\right)\frac{\rho^2}{\cos^4\theta_w}. \end{aligned} \tag{9.3}$$

In Eq. (9.3), $\rho$ parameterizes the coupling strength of the neutral current compared to the coupling strength of the charged current, and is given by the mass ratio

$$\rho = m_W{}^2/m_Z{}^2. \tag{9.4}$$

Only if we assume that the $SU_2 \times U_1$ theory is broken by an $SU_2$ doublet of scalar fields can we express $\rho$ in terms of the $SU_2 \times U_1$-mixing angle:

$$\rho^2 = \cos^4\theta_w. \tag{9.5}$$

The ratio $R^{\bar{\nu}}/R^{\nu}$ is, of course, independent of $\rho$. Only the measurement of this ratio is therefore of relevance for testing our different predictions for the $SU_2 \times U_1$ mixing angle (see Table 1).

TABLE I

| Model | A: 12 fermions | B: 16 fermions |
|---|---|---|
| $\sin^2 \theta_w$ | $5/16$ | $3/8$ |
| $R^{\bar{\nu}}$ (general) | $\rho^2 \cdot 932/1089$ | $\rho^2 \cdot 7/10$ |
| $R^{\bar{\nu}}$ (doublet breaking) | $233/576 \approx 0.41$ | $7/16 \approx 0.44$ |
| $R^{\nu}$ (general) | $\rho^2 \cdot 1796/3267$ | $\rho^2 \cdot 11/30$ |
| $R^{\nu}$ (doublet breaking) | $449/1728 \approx 0.26$ | $11/48 = 0.230$ |
| $R^{\bar{\nu}}/R^{\nu}$ | $699/449 \approx 1.56$ | $21/11 = 1.91$ |

In Table I, we summarize the predictions for the neutrino production cross sections off isoscalar targets only (antiquark contributions to the target wave functions are neglected). We stress that the ratio $R^{\bar{\nu}}/R^{\nu}$ is very sensitive to $\sin^2 \theta_w$ if $0.3 < \sin^2 \theta_w < 0.4$. Thus, a relatively exact determination of this ratio allows us to determine $\sin^2 \theta_w$ very accurately, provided the $SU_2 \times U_1$ model applies.

## 10. Fermion Masses and the Electron–Muon Mass Ratio

The fermions acquire, in general, a mass by the spontaneous symmetry breaking. We assume that the quark masses generated by the symmetry breaking determine the $SU_3 \times SU_3$ breaking parameters and are therefore measurable quantities. The present available experimental information supports the strong PCAC picture in which the quark masses are very unsymmetrical. In particular, the nonstrange quark masses $m_u$, $m_d$ are small compared to $m_s$, and one has approximately the relation

$$2m_s/(m_u + m_d) \approx 2m_K{}^2/m_\pi{}^2 \approx 25. \tag{10.1}$$

It is possible to get information about the absolute magnetude of the quark masses by using the light cone commutators abstracted from the quark model, generalized to scalar and pseudoscalar densities. One finds typically a value between 100 and 200 MeV for $m_s$ [38].

The mass differences inside the different isotopic multiplets can be understood by assigning different values to $m_u$ and $m_d$. This splitting is presumably of non-electromagnetic origin, a crude estimate gives

$$0.01 \lesssim (m_d - m_u)/m_s \lesssim 0.02. \tag{10.2}$$

Taking for example, $m_s = 100$ MeV one finds

$$m_u \approx 3 \text{ MeV}, \qquad m_d \approx 5 \text{ MeV}. \tag{10.3}$$

If there is a charmed quark, its mass must be considerably larger than the $s$-quark mass, in order to account for the strong $SU_4$ symmetry breaking. It is presumably of the order of 1 GeV.

The lepton and quark masses are supposed to be generated by the invariant coupling of the fermion fields to scalar fields transforming like $(f, \bar{f}) + (\bar{f}, f)$ under $SU_f{}^L \times SU_f{}^R$. It is typical for the lepton and quark mass spectrum that one lepton (muon) and one or perhaps two quarks ($s$-quark, $c$-quark) are much heavier than the other leptons and quarks. This fact suggests the possibility that only one lepton and one or two quarks receive a mass through the scalar potential, but the other lepton and quark masses are generated by the emission and reabsorption of gauge bosons, etc., and are in principle calculable.

The fermion propagator in lowest order of the symmetry breaking is given by

$$S_0(p) = 1/(\not{p} - m_0). \tag{10.4}$$

We define the full propagator by

$$S(p) = 1/[\not{p} - m_0 - \Sigma(p)] \tag{10.5}$$

where $m_0$ and $\Sigma$ are matrices in the fermion space ($m_0^{kj}$, $\Sigma^{kj}$, $k, j = 1 \ldots f$).

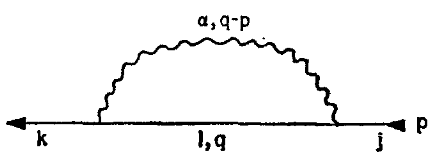



FIG. 2. Diagram contributing to the fermion self-energy. $p$, incoming fermion momentum; $j$, $l$, $k$, fermion indices; $\alpha$, gauge boson index.

Taking into account gauge boson effects to second order (see Fig. 2), the self-energy in the Landau gauge is given by

$$\Sigma^{kj}_{(p)} = ig^2 \sum_{\alpha, l, h, h'} \int \frac{d^4q}{(2\pi)^4} \frac{1}{[(q-p)^2 - m_\alpha{}^2]} \frac{1}{(q^2 - m_l{}^2)}$$
$$\times\, C^{\alpha}_{klh} \gamma_\nu \frac{1 + h\gamma_5}{2} (\not{q} + m_l)\, \rho^{\nu\mu} C^{\alpha}_{ljh'} \gamma_\nu \frac{1 + h'\gamma_5}{2} \tag{10.6}$$

where

$$\rho^{\nu\mu} = -g^{\nu\mu} + (l-p)^\nu (l-p)^\mu \frac{1}{(l-p)^2}$$

and we have used a basis in which the gauge boson mass matrix and $m_0$ is diagonalized. Note that $C^{\alpha}_{klh}$ is the coupling of the $\alpha$-gauge boson to

$$\bar{f}_k \gamma_\mu \frac{1 + h\gamma_5}{2} f_l$$

where $h = \pm 1$ and

$$\sum_{k,l,h} C^{\alpha}_{klh} C^{*\beta}_{klh} = \delta^{\alpha\beta} \sum_{\alpha} C^{\alpha}_{klh} C^{*\alpha}_{k'l'h'} = \delta_{kk'} \delta_{ll'} \delta_{hh'} .$$

As shown in [29, 39], it is convenient to use the Landau gauge since there the calculable self-energies are separated from the contributions that can be absorbed in a redefinition of the "Higgs" potential. Note that according to Eq. (10.6) a contribution to $\Sigma$ arises only if the corresponding gauge boson couples both to left-handed and right-handed fermions.

The situation is particularly simple if only one fermion acquires a mass through the scalar potential evaluated and renormalized to all orders, e.g., $m_l \neq 0$, and if the gauge bosons coupling to $\bar{f}_k \gamma_\mu [(1 \pm \gamma_5)/2] f_l$ do not mix with other bosons, but only with themselves. In this case, one can write:

$$\begin{aligned} C^{\alpha_1}_{kl+} &= \cos \phi_\alpha , \qquad & C^{\alpha_1}_{kl-} &= \sin \phi_\alpha \\ C^{\alpha_2}_{kl+} &= - \sin \phi_\alpha , \qquad & C^{\alpha_2}_{kl-} &= \cos \phi_\alpha . \end{aligned} \tag{10.7}$$

Provided the gauge boson masses are much heavier than the fermion masses, one finds for $k \neq l$:

$$m_k = m_l \cdot \frac{3g^2}{4\pi} \cdot \frac{1}{8\pi} \sum_{\alpha} \sin^2 \phi_\alpha \log \frac{m^2_{\alpha_2}}{m^2_{\alpha_1}} + O\left(\frac{g^2}{4\pi} \cdot \frac{m_l^2}{m_\alpha^2}\right) + O\left[\left(\frac{g^2}{4\pi}\right)^2\right]. \tag{10.8}$$

The assumption that certain fermion masses vanish if the gauge boson self-energy effects are neglected is not a zeroth order mass relation in the sense defined in [39]. The latter arise if the number of renormalization constants in the Lagrangian is not large enough due to special symmetries of the scalar potential to adjust all fermion masses, in which case deviations from the zeroth order mass relations are necessarily finite and of order $g^2/4\pi$. Such fermion mass relations do not arise if the gauge group is the chiral group of the fermions $SU_f^L \times SU_f^R$.

We propose to accept certain mass relations as exactly valid in the limit where the fermions decouple from the gauge bosons, e.g., $m_e = m_\nu = 0$. These relations are physical constraints and not consequences of certain symmetries of the scalar potential. The corrections to those constraints calculated in the Landau gauge are then necessarily of order $g^2/4\pi$ and arise only from gauge boson self-energy effects, while by definition, effects due to the renormalization of the scalar potentials (tadpoles) are not present.

*Model A*

The fermion mass spectrum of Model A is very symmetric between leptons and quarks. Note that the $s$-quark mass is of the same order as the muon mass, and it could well be that both are identical [38]. Furthermore, both the masses of the nonstrange quarks and the electron and neutrino masses are very small, and the mass spectrum suggests that the quarks and leptons are related to each other as follows: $u \leftrightarrow \nu$, $d \leftrightarrow e^-$, $s \leftrightarrow \mu^+$. This relationship will be discussed in more detail below. According to the fermion mass spectrum it seems very reasonable to assume that only one quark and one lepton acquire a mass by their coupling to the scalar fields. The masses of the nonstrange quarks and the electron mass would then arise by the emission and reabsorption of gauge bosons and are calculable in terms of the gauge boson mass matrix and, perhaps, in terms of the Cabibbo angle.

We concentrate first on the lepton mass spectrum. Both the electron and the neutrino acquire masses due to the emission and absorption of: (1) the color singlet leptonic gauge bosons, and (2) the colored leptoquark bosons.

According to the properties of the leptonic weak interactions, the gauge bosons that couple to $\bar{\nu}\gamma_\mu[(1 \pm \gamma_5)/2]\,\mu$ or $\bar{\mu}\gamma_\mu[(1 \pm \gamma_5)/2]\,\nu$ remain chiral, i.e., they do not generate a mass for the neutrino. The latter can acquire a mass only by the leptoquark bosons, which couple to $\bar{\nu}\gamma_\mu[(1 \pm \gamma_5)/2]\,s$ and $\bar{s}\gamma_\mu[(1 \pm \gamma_5)/2]\,\nu$. Consequently, the leptoquark bosons that couple to $(\bar{\nu}s)$ must be chiral in order to leave the neutrino massless. We assume in general that the leptoquark bosons, which are the heaviest bosons of the scheme, do not mix left-handed and right-handed fermions (see also Section 6).

In this case, only the color singlet leptonic gauge bosons are able to generate a mass for the electron. The latter may adquire a mass by emitting and absorbing the superheavy doubly-charged bosons that couple to $\bar{\mu}^+\gamma_\mu[(1 \pm \gamma_5)/2]\,e^-$ or $\bar{e}^-\gamma_\mu[(1 \pm \gamma_5)/2]\,\mu^+$. These bosons must be superheavy. There is no reason why these bosons should not mix between left and right. Suppose the eigenstates of the mass matrix couple as follows:

$$\begin{aligned} W_1^{++} &\sim \cos\phi\,\bar{\mu}^+\gamma_\mu\,\frac{1+\gamma_5}{2}\,e^- + \sin\phi\,\bar{\mu}^+\gamma_\mu\,\frac{1-\gamma_5}{2}\,e^- \\ W_2^{++} &\sim -\sin\phi\,\bar{\mu}^+\gamma_\mu\,\frac{1+\gamma_5}{2}\,e^- + \cos\phi\,\bar{\mu}^+\gamma_\mu\,\frac{1-\gamma_5}{2}\,e^-. \end{aligned} \tag{10.9}$$

This is the only type of mixing that can occur, since there are no other gauge bosons of charge 2. Using Eq. (10.8) one finds:

$$\frac{m_e}{m_\mu} = \frac{3}{8\pi} \cdot \left(\frac{g^2}{4\pi}\right) \cdot \sin 2\phi \log \frac{m_2{}^2}{m_1{}^2} + O\left(\frac{g^2}{4\pi} \cdot \frac{m_\mu{}^2}{m_{1,2}^2}\right) + O\left[\left(\frac{g^2}{4\pi}\right)^2\right]. \quad (10.10)$$

Inserting the universality relation [Eq. (7.3)], one obtains:

$$\frac{m_e}{m_\mu} = \frac{3\alpha}{\pi} \sin 2\phi \log \frac{m_2{}^2}{m_1{}^2} + \cdots . \quad (10.11)$$

The gauge bosons that couple to the $(\bar{\nu}\mu)$-system are chiral, i.e., $\phi = 0$. Perhaps the mixing angle for the doubly charged bosons is also a simple canonical value. Another simple situation besides $\phi = 0$ would be $\phi = \pi/4$ i.e., $W_1$ is a pure vector, $W_2$ is a pure axial vector. In this case the formula [Eq. (10.11)] reduces to

$$\frac{m_e}{m_\mu} = \frac{3\alpha}{\pi} \log \frac{m_A{}^2}{m_V{}^2} + \cdots . \quad (10.12)$$

It is remarkable that this formula agrees very well with experiment if we choose $m_A{}^2/m_V{}^2 = 2$ (see Footnote 4):

$$\frac{m_e}{m_\mu} = \frac{3\alpha}{\pi} \log 2 + O\left(\alpha^2, \alpha \cdot \frac{m_\mu{}^2}{m_W{}^2}\right). \quad (10.13)$$

(Note that $[3\alpha/\pi \log 2]^{-1} \cdot m_e = 105,\ 7943$ MeV, while $m_\mu^{\text{exp}} = 105,\ 6599 \pm 0.0014$ MeV).

Formula (10.13) relates the $m_e/m_\mu$ mass ratio to the simple algebraic mass ratio $m_A{}^2/m_V{}^2 = 2$ for the doubly charged gauge bosons. Alternatively it can be used to predict a mass ratio $m_A{}^2/m_V{}^2 = 2$ for these bosons on the basis of the observed $m_e/m_\mu$ mass ratio.

It is important to note the relevance of the $SU_{12}^L \times SU_{12}^R$ scheme to obtain Eq. (10.12). The main input was the relation $g^2/4\pi = 8\alpha$, which relates the coupling constant for the doubly charged bosons to the electromagnetic coupling and enabled us to obtain the factor $3\alpha/\pi$ in Eq. (10.11). It is also worthwhile to note that the factor 8 is equal to $2 \sum_{\text{fermions}} e_i{}^2$, i.e., the sum of the squares of the lepton *and* quark charges is important for the $m_e/m_\mu$ ratio.

Formula (10.13) could be considered as a strict calculation of the $m_e/m_\mu$ mass ratio if one finds a justification for $m_A{}^2/m_V{}^2 = 2$. It is possible to choose the representation of the Higgs fields such that $m_A{}^2/m_V{}^2 = 2$, but so far we have not been able to find a reason why these representations should be chosen by nature.

Note that this ratio may be related to a similar phenomenon in hadron physics. Assuming broken $SU_2 \times SU_2$ invariance, PCAC and the saturation of the spectral function sum rules for the vector and axial vector currents with the $\rho$ and $A_1$ mesons, one can derive the relation $m_{A_1}^2/m_\rho^2 = 2$ [20]. It is possible that a similar technique applied to the $SU_2 \times SU_2$ group generated by the doublet $\binom{e^-}{\mu^+}$ can be used to derive $m_A{}^2/m_V{}^2 = 2$.

We emphasize that formulas (10.10)–(10.13) are based on the additional assumption made above. They are not consequences of the spontaneously broken gauge field theory.

The generation of the quark masses in Model A is more complicated as the generation of the electron mass. According to the weak interactions of hadrons (see Section 2) the hadronic color singlet bosons coupling to $(\bar{u}s)$ and $(\bar{d}s)$ are purely chiral, i.e., they cannot generate a $u$ or $d$-quark mass of second order in the gauge coupling; this is similar to the case of the $(e\mu)$ system. Neglecting leptoquark boson effects again, the only source of the quark masses can be the diotons. Both the $u$ and the $d$-quark will receive masses by emitting and reabsorbing diotons (see Fig. 3). Precise knowledge of the dioton mass matrix is required to calculate these effects. The $d$ and $s$-quark have the same electric charge; therefore, it can happen that not only the $s$-quark receives a mass by the spontaneous symmetry breaking, but a certain mixture of them, whereby the mixing angle is related to the Cabibbo angle, e.g. [32],

$$\begin{aligned} m_d{}^0 &= m \cdot \sin^2 \theta_c \\ m_s{}^0 &= m \cdot \cos^2 \theta_c \end{aligned} \tag{10.14}$$

and the $u$-quark receives its mass solely by dioton self-energy effects. The addition of both dioton self-energy and mixing effects may fully account for the quark masses.

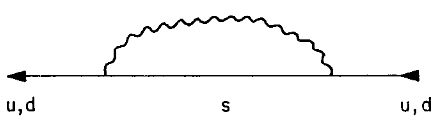



FIG. 3. Dioton contribution to the $u$ and $d$ quark masses.

*Model B*

The inclusion of the charmed quark triplet (Model B) generates an assymetry between leptons and quarks, as far as their mass spectra are concerned. The relationship,

$$c \leftrightarrow \nu_u, \qquad u \leftrightarrow \nu_e, \qquad d \leftrightarrow e^-, \qquad s \leftrightarrow \mu^-,$$

is spoiled by the high mass for the charmed quark.

We treat the lepton mass spectrum in a manner similar to that used in Model A. It is assumed that only the muon acquires a mass to lowest order in the symmetry breaking, while the other lepton masses are supposed to be generated by the emission and absorption of the color singlet leptonic gauge bosons. The muon neutrino cannot acquire a mass to second order, since the bosons coupling to $(\nu_\mu, \mu)$ are chiral. On the other hand the electron neutrino could acquire such a mass by second-order effects due to the bosons coupling to $(\nu_e, \mu)$.

The neutral bosons coupling to the $(e^-, \mu^-)$ pair, in general, will generate a mass for the electron. As opposed to Model A, these bosons have charge 0; therefore, they can mix in principle with any color singlet neutral boson. If we make the simplifying assumption that they mix only among themselves, i.e., between left and right with the canonical mixing angle $\pi/4$, formula (10.8) takes the form:

$$\frac{m_e}{m_\mu} = \frac{4\alpha}{\pi} \log \frac{m_A{}^2}{m_V{}^2} + \cdots . \tag{10.15}$$

Agreement with observation, i.e., with (10.13) is reached if one has the mass ratio

$$m_A{}^2/m_V{}^2 = 8^{1/4}. \tag{10.16}$$

The quark mass spectrum in Model B can be understood by assuming that both the $c$-quark and the $s$-quark acquire a mass to lowest order in the symmetry breaking, while the $u$ and $d$-quark masses are due to dioton self-energies. As before, it may happen in addition that parts of the $u$ and $d$-quark masses are generated by mixing, since both the $c$ and $u$-quark and the $d$ and $s$-quark have the same charge.

## 11. Lepton Number as Fourth Color

In both Schemes A and B it is possible to interpret the leptons as the fourth color row [40], i.e., one may think that the group $SU_3$(color) is a subgroup of a still larger group $SU_4$(color). It is obvious that this group must be strongly broken. The additional gauge degrees-of-freedom needed to enlarge $SU_3$(color) to $SU_4$ are leptoquark bosons, which have a high mass. Nevertheless, it seems that certain traces of this symmetry are left in both Models A and B. As pointed out in Section 10, the mass matrix for quarks and leptons in Model A is roughly symmetric under the exchange of leptons and quarks, i.e., is almost an $SU_4$ singlet. The $SU_4$ symmetry is broken by the electric charges of the quarks and leptons, but in a very specific way. Let us classify the electric charges using the group $SU_4(\text{color}) \times SU_3$ acting horizontally and vertically in the Scheme A:

$$\begin{pmatrix} u & u & u & \nu \\ d & d & d & e^- \\ s & s & s & \mu^+ \end{pmatrix}$$

Suppose we introduce both a hadronic photon

$$\gamma_h = \frac{1}{2^{1/2}} \begin{pmatrix} \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & 0 \\ -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & 0 \\ -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & 0 \end{pmatrix}$$

and a leptonic photon

$$\gamma_l = \frac{1}{2^{1/2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Note that the real photon $\gamma$ is just the sum

$$\gamma = \gamma_h + \gamma_l \tag{11.1}$$

due to the fact that the sum of the squares of the quark charges equals the sum of the squares of the lepton charges. Furthermore, as far as the $SU_3$ content of $\gamma_h$ and $\gamma_l$ is concerned, they are otrhogonal with respect to each other:

$$(\gamma_h \cdot \gamma_l)_{SU_3} = 0. \tag{11.2}$$

Thus, the $SU_4$ breaking affects the photon generator in such a way that its leptonic part $\gamma_l$ takes a direction orthogonal to its hadronic part $\gamma_h$ in $SU_3$ space, but both have the same normalization. This gives rise to the quantization of the electric charge, i.e., to the fact that the proton charge is exactly equal to the positron charge.

In Scheme B

$$\begin{pmatrix} c & c & c & \nu_\mu \\ u & u & u & \nu_e \\ d' & d' & d' & e^- \\ s' & s' & s' & \mu^- \end{pmatrix}$$

the situation is quite different. The fermion masses do not respect the $SU_4{}^c$ symmetry at all because of the large $c$-quark mass. Also, it is not possible to define separate hadronic and leptonic photon generators, since the quark and lepton charges do not add to zero separately. However, the weak generators are not only color singlet, but are also $SU_4{}^c$ singlet generators. Moreover the electric charges show the following simple pattern. Note that the leptonic charges can be obtained by subtracting $\frac{2}{3}$ from the quark charges, i.e., the photon generator is a sum of an $SU_4$ singlet and an $SU_4$ quindecimet operator, the latter being the $SU_4$ analogue of the hypercharge in $SU_3$ space.

Let us define the weak generators by $T_i^L$ ($i = 1, 2, 3$), the associated generators acting on the right-handed fermions by $T_i^R$, and the $SU_4^c$ generator $R$ by

$$R = \frac{1}{3}\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -3 \end{pmatrix}.$$

Then we find

$$Q^e = \begin{pmatrix} \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & 0 \\ \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & 0 \\ -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & -1 \\ -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 \end{pmatrix} + \frac{1}{6}\begin{pmatrix} 1 & 1 & 1 & -3 \\ 1 & 1 & 1 & -3 \\ 1 & 1 & 1 & -3 \\ 1 & 1 & 1 & -3 \end{pmatrix}$$

$$= T_3^{L+R} + \tfrac{1}{2}R. \tag{11.3}$$

This equation shows that the electric charge can be embedded in the Lie algebra of the semisimple group $SU_4^c \times SU_2^L \times SU_2^R$ [37]. It is interesting to see that within this algebra only the specification of one charge is necessary to obtain all fermion charges. Suppose we start with the group $SU_4 \times SU_2^L \times SU_2^R$ and describe the 16 fermions by the representation $(\mathbf{4}, \mathbf{2}, \mathbf{1}) + (\mathbf{4}, \mathbf{1}, \mathbf{2})$. If we require the electric charge to be a singlet under the subgroup $SU_3^c$, it must be a linear combination of $T_3^{L+R}$ and $R$. If we now require the neutrino to be neutral and if we normalize the electron charge to $-1$, one fixes all coefficients and finds combination (11.3). Hence, the quark charges are uniquely determined by the $SU_4^c \times SU_2^L \times SU_2^R$ algebra.

The fact that the $SU_4^c \times SU_2^L \times SU_2^R$ algebra fixes the quark charges to just the right values and explains within Scheme B why the electric charges in nature are quantized leads one to believe that the gauge group $SU_4^c \times SU_2^L \times SU_2^R$ plays an important role. However, since a theory based on this group depends on two coupling constants (provided parity is exact in the Lagrangian), it cannot be regarded as a unified theory. To obtain a unified theory, the group $SU_4 \times SU_2^L \times SU_2^R$ must be embedded in a larger group. The previously considered $SU_{16}^L \times SU_{16}^R$ theory is one way to do that. Other possibilities, e.g., the gauge group $SO_{10}$, will be considered in Section 13.

We conclude this section with the following remark. The idea of spontaneous symmetry breaking has been accompanied from the beginning by the idea that the symmetry breaking may be different in different space-time regions of the universe. Thus, one may entertain the possibility that the $SU_4$ symmetry discussed above is actually exact somewhere in the universe, or has been exact in the past, perhaps during the first stages of the universe.

In this case, the leptons would have the same charge and mass as the quarks and are confined inside $SU_4$ singlets. All physical states ($SU_4$ singlets) are bosons, no spin $\frac{1}{2}$ objects would exist as free particles. If in such a world the symmetry is broken down to $SU_3$(color), the $SU_4$ singlets break up into $SU_3$(color) singlets and leptons. However, this breakup is such that one has three times as many quarks as leptons, i.e., the baryon and lepton number of the universe would have to be equal.

## 12. Colored Massive Gauge Bosons: New Hadronic Constituents

In Sections 6 and 7, we demonstrated that, necessarily, within our framework, new kinds of gauge bosons arise that are colored and acquire a mass by the spontaneous symmetry breaking. Leaving aside the axial counterparts of the vector gluons, there are two qualitatively different groups of new colored gauge bosons: the leptoquark bosons and the diotons. Both types of bosons are supposed to be bound permanently inside color singlet hadron states like the quarks and gluons.

One special importance of these bosons arises from the fact that some of them carry electric charge. In this section, we discuss the possible implications caused by the existence of these bosons for hadron spectroscopy, deep inelastic scattering, and $e^+e^-$-annihilation.

We denote the leptoquark bosons and diotons by the product of fermion fields to which they couple, i.e., leptoquark bosons by $(\bar{q}l)$ or $(\bar{l}q)$ and diotons by $(\bar{q}q)$.

The leptoquark bosons were assumed to be much heavier than the conventional weak intermediate bosons so that they do not contribute to the usual weak interactions.

Since the diotons transform nontrivially under the hadronic symmetry group ($SU_3$ in Model A, $SU_4$ in Model B), they couple to the same set of "Higgs" fields that generates the masses of the color singlet $W$-bosons, ie., the dioton masses are of the same order as the color singlet $W$-boson masses. Thus, one may think that the leptoquark bosons and diotons do not play a role for strong interactions phenomena. However, this is not true. Suppose we construct a color singlet field as a product of leptoquark boson fields, dioton fields, and quarks and gluons (for example, the dioton pair $\sum_A (\bar{q}\lambda_i\chi_A q)(\bar{q}\lambda_j\chi_A q)$). Because of the strong interactions, such an operator causes transitions between the vacuum and any hadronic state with its quantum numbers. This implies that the wavefunction of all hadron states must have a certain content of dioton and leptoquark bosons besides the quarks and gluons. This does not contradict the results found in the leptoproduction experiments [36, 41]. The latter indicate that to a good approximation the nucleon can be considered as consisting of three "valence quarks" (plus gluons) while all other configurations, e.g., the quark–antiquark pairs, do contribute essentially only to small $\xi = -q^2/(2\nu)$. Diotons, which carry the quantum numbers of quark–antiquark pairs, or leptoquark bosons, which carry the quantum numbers of lepton–quark pairs, are assumed to contribute to the nucleon wavefunction not more than the quark–antiquark pairs. Using the present experimental data, it is impossible to distinguish them from the latter. Thus, the nucleon, or presumably any low-lying classified hadron state, is not a good target to be investigated to detect these new hadronic constituents: The low-lying hadrons are polarized by their dynamical preference to consist mainly of quarks and gluons. However, the vacuum is polarized by all constituents, and the ideal opportunity to detect any new colored and charged hadronic constituents besides the quarks is $e^+e^-$-annihilation into hadrons.

Because of their different properties, we discuss the diotons and leptoquark bosons separately in the subsections below. Their possible importance for $e^+e^-$-annihilation will be discussed in Section 12(c).

(a) *Diotons*

The simplest color singlet configuration involving diotons is the operator $\sum_A (\bar{q}\lambda_i\chi_A q)(\bar{q}\lambda_j\chi_A q)$. Within the quark classification, this operator is exotic i.e., of the type $\bar{q}q\,\bar{q}q$. Consequently, one expects that the low-lying classified mesons and baryons, which follow the simple quark pattern ($\bar{q}q$ and $qqq$, respectively) rather well, have only a very small dioton content. However, at sufficiently high energies, hadron states (color singlets that consist almost entirely of diotons and have only little quark content will exist. Such states necessarily belong to exotic representations within the quark classification, i.e., cannot be represented as $\bar{q}q$ or $qqq$.

It is, of course, not clear whether or not such states are realized in the hadronic spectrum as real and presumably relatively broad resonances, or simply as continuum states. Nevertheless, there will exist a threshold for states consisting almost entirely of diotons, which can be interpreted as a measure of the effective mass of a dioton $m_d^{\text{eff}}$ inside a color singlet hadron ("dioton threshold"). This threshold will provide a new mass scale for the strong interactions. As argued below, this mass may be relatively low (below 5 GeV), and it may coincide with the threshold at which exotic resonances in the particle spectrum arise.

Diotons carry both electric and weak charges. Consequently, they modify the algebraic properties of the electromagnetic and weak currents. For example, the electromagnetic current is given by

$$j_\mu = \bar{q}\gamma_\mu Q q - i\left\{\sum_{A=1}^{8}\sum_{h=L,R}(\partial_\mu\rho_\nu^{Ah} - \partial_\nu\rho_\mu^{Ah})\,\rho_A^{*\nu h}\right.$$

$$\left. + (\partial_\mu K_\nu^{Ah} - \partial_\nu K_\mu^{Ah})\,K_A^{*\nu h} + \text{h.c.} + O(e)\right\} \qquad (12.1)$$

where $Q$ is the quark charge matrix and $\rho_\nu^{hA}$, $K_\nu^{hA}$ denotes the dioton field transforming like $\rho$, $K^*$ under $SU_3$ and which couples to the left-handed ($h = L$) or right-handed ($h = R$) quark fields.

Let us consider the commutator of two electromagnetic currents at distances relatively small to hadronic distances but still large compared to typical weak interaction length parameters. There are reasons to believe that this commutator is particularly simple due to the smallness of the quark–gluon or dioton–gluon coupling constant (see Section 2), i.e., the leading part of the commutator is given by free field theory, while the higher order logarithmic corrections to the free field theory behavior are small. Neglecting the logarithmic terms, the leading lightcone singularity of the part of the commutator that is due to the quarks and the transverse dioton modes is scale invariant. The next to leading singularities are supposed to be greatly influenced by the strong interactions. Their scale could be interpreted as an effective quark or dioton mass, but they have little to do with the formal masses of these fields, acquired by the spontaneous symmetry breaking. Thus, despite the fact that the formal dioton masses are of the order of the $W$-boson masses, the transverse dioton modes are expected to contribute to the current commutators at distances much larger than the Compton wavelength of the $W$-bosons. This is not true for the longitudinal dioton modes. The part of the current commutator due to the latter is affected by the $k_\mu k_\nu/m_D{}^2$-part of the vector propagator and is therefore suppressed at distances large compared to $\lambda\!\!\!^{-}_D$. We conclude that at distances large compared to $\lambda\!\!\!^{-}_D$ but small compared to the length associated to the dioton threshold $\lambda\!\!\!^{-\,\mathrm{eff}}_D$, the commutators of color singlet dioton currents look like the corresponding commutators in free massless vector meson theory, if one neglects the logarithmic violations of the free field lightcone commutators.

One can easily see that in the presence of diotons, all sum rules and symmetry principles of the quark parton model or the quark lightcone algebra are modified. In particular, the leading lightcone singularity of a current commutator now contains exotic representations of $SU_3$. Since the three-quark part of the nucleon wavefunction dominates all other configurations, in particular, the ones including diotons, there is no contradiction with experiment. Diotons are supposed to contribute to the nucleon wavefunction in the same way as the quark–antiquark pairs, i.e., only to small $\xi$ values. The existence of diotons as charged hadronic constituents requires, of course, that the ratio $\sigma_l/\sigma_t$ in electroproduction remains different from zero near $\xi = 0$ as $-q^2 \to \infty$. This is compatible with present experimental information.

(b) *Leptoquark Bosons*

The leptoquark bosons are the most exotic new fields required by the unification of strong, electromagnetic, and weak interactions. They carry both baryon and lepton number as well as nonintegral electric charges. The maximal charge in both Models A and B is $\frac{5}{3}$.

The leptoquark bosons can form color singlet hadron states with quarks, gluons, and diotons; in particular, they form hadron states with nonvanishing lepton number (for example, consider the state $|(\bar{q}l), q\rangle$. There must be a threshold in the hadronic spectrum at which states with a substantial leptoquark content arise. This threshold can be considered as a measure for the effective mass of the leptoquark bosons inside a color singlet hadron. An essential characteristic of such states is that they decay partly into leptons. Therefore, it is expected that these resonances are relatively stable and narrow, and it may be possible to detect them in the spectrum. Thus far, there is no indication that such a threshold exists in the hadron spectrum at low energies. It is concluded that the effective leptoquark boson mass is rather high. This may be related to the possibility that the formal leptoquark boson mass generated by the spontaneous symmetry breaking is higher than all other mass parameters of the scheme, i.e., the masses of the weak intermediate bosons and the dioton masses, although we have no way to show how this mass is related to the leptoquark boson threshold.

It is crucial for the scheme discussed by us that at a certain threshold exotic hadrons, consisting of leptoquark bosons, are produced. The first indication that such a threshold is reached is a copious production of leptons in high-energy reactions. It would be very interesting to observe such an anomalous amount of lepton production in the experiments.

### (c) $e^+e^-$-*Annihilation into Hadrons*

The recent experiments on $e^+e^-$-annihilation into hadrons have yielded the surprising result that the total cross section ratio

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \tag{12.2}$$

increases to rather high values [42, 43]. This is to be compared with the prediction from quark current algebra, which predicts, when combined with the strength of the decay $\pi^\circ \to 2\gamma$, PCAC, and the assumption about scale invariance at small distances of current products in the hadronic vacuum, an asymptotic value two (or 10/3 in case of charm) for this ratio [3]. This is also the result obtained considering the behavior of current products at short timelike distances in the hadronic vacuum within a field theory model based on three (or perhaps four) color triplet quark fields or applying the quark parton model ideas [44].

One possible interpretation of the data is that the quark triplets are not the only charged hadronic fields, but there are new constituents that, for some dynamical reason, give only small contributions to the wavefunctions of the low-lying hadron states [19]. It is interesting to observe that any unification of the strong, weak, and electromagnetic interactions yields such new hadronic constituents in the form of colored gauge bosons that, unlike the gluons, acquire a mass by the symmetry breaking and in general have nontrivial charges; in our case the diotons and leptoquark bosons. Thus, one has, besides the usual one-photon annihilation diagram for $e^+e^-$-annihilation into hadrons (see Fig. 4), also the diagram for the virtual dioton production (Fig. 5), and for virtual leptoquark boson production (Fig. 6). The leptoquark bosons contribute to $e^+e^-$-annihilation also by the diagram (Fig. 7) [40]. These contributions are suppressed by the large leptoquark boson mass. Note that their presence would also spoil the usual interpretation of the electroproduction experiments.

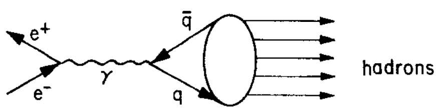



FIG. 4. $e^+e^-$-annihilation into hadrons via quark-antiquark intermediate states.

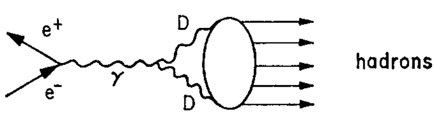



FIG. 5. $e^+e^-$-annihilation into hadrons via dioton intermediate states.

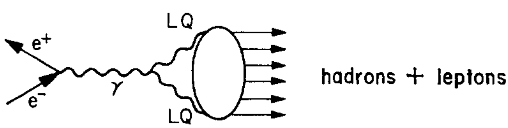



FIG. 6. $e^+e^-$-annihilation into hadrons and leptons via intermediate leptoquark bosons.

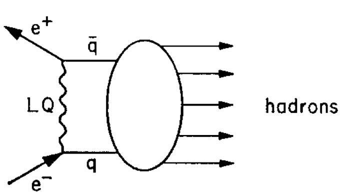



FIG. 7. Nonelectromagnetic contribution to $e^+e^-$-annihilation involving leptoquark boson exchange.

Once the energy of the virtual photon reaches the leptoquark boson threshold, the contributions from the diagram (Fig. 6) become relevant. The final state arising from the decay of the virtual leptoquark boson pair will consist not only of hadrons, but also of leptons. It would be interesting to investigate the final state in $e^+e^-$-annihilation in this respect. If there is an anomalously large lepton production it would indicate that virtual leptoquark production has become relevant and the leptoquark boson threshold has been passed.

In the following we concentrate on the hadronic final state and neglect leptoquark boson effects. The hadronic final state is supposed to arise by the decay of either a quark–antiquark pair or a dioton pair. One finds:

$$R \to \sum_{\text{quarks}} Q_i^2 + \tfrac{1}{2} \sum_{\text{diotons}} Q_j^2 = 2 + 16 = 18 \qquad \text{(Model A)}$$

$$= 3\tfrac{1}{3} + 32 = 35\tfrac{1}{3} \qquad \text{(Model B)}. \qquad (12.3)$$

This result is, of course, not an asymptotic result, but valid only for the intermediate region where the energy is larger than the dioton threshold, but smaller than the energy at which other contributions become relevant (weak interactions effects, neutral currents, longitudinal dioton modes, etc.).

We obtain the following two-component picture for $e^+e^-$-annihilation into hadrons. At energies below the "dioton threshold," only the quark part of the electromagnetic current is relevant, since one produces mainly resonances ($\rho$, $\rho'$,...) that are nonexotic in the quark classification, and therefore, the dioton part of the electromagnetic current has only a very small chance of producing such a state.

It is conceivable that in $e^+e^-$-annihilation the scaling limit for the quark part of the electromagnetic current is reached at rather low energies (below 2 GeV). This idea is compatible with the data and supported by simple duality considerations [45]. As soon as the energy reaches the threshold at which the dioton part of the electromagnetic current is activated, $R$ increases above two and approaches its asymptotic value 18 or $35\frac{1}{3}$.

Since low-lying hadrons consist almost entirely of quarks and gluons, the probability that a virtual dioton pair disintegrates into a final hadron state containing one fast moving particle (e.g., a pion or a nucleon), which carries a sizable amount of the available energy, should be very small.[11] It will rather disintegrate into a multihadron state with many particles at relatively low momenta. Hence, scaling in the inclusive reaction $e^+e^- \to p$ + anything ($p$: low-lying hadron state) is expected to work rather well away from the central region ($\xi = q^2/(2\nu)$, small) even at rather low energies (beyond the energy at which the scaling limit for the quark part of the electromagnetic current in the total cross section is reached),

[11] Generalizing an argument by Feynman, this probability is related to the hadron distribution function of a dioton, which is expected to be significantly different from zero only for large $\xi = q^2/2\nu$.

since it is due essentially to the quark part of the current. As the energy increases beyond the "dioton threshold," the central region blows up and gives the main contribution to the cross section.

The present experiments show that $R$ increases to values above five at about $\sqrt{q^2} = 5$ GeV. If we interpret the data in the sense described above, the dioton threshold must be between 2.5 and 4 GeV. This is about the energy one might expect for exotic resonances to arise. If the present rate of increase in $R$ continues the asymptotic values 18 or $35\frac{1}{3}$ may be reached at energies between 10 and 20 GeV.

Finally, we should like to mention the case where both diotons and leptoquark bosons contribute besides the quarks to the final state in $e^+e^-$-annihilation. Instead of Eq. (11.3) one finds the following rather large numbers:

$$\frac{\sigma_{e^+e^-\to\text{hadrons}\,\&\,\text{leptons}}}{[\sigma_{e^+e^-\to\mu^+\mu^-}]_{\text{QED}}} \to \sum_{\text{quarks}} Q_i^2 + \tfrac{1}{2}\sum_{\text{diotons}} Q_j^2 + \tfrac{1}{2}\sum_{\substack{\text{leptoquark}\\ \text{bosons}}} Q_i^2$$

$$\begin{aligned} &= 2 + 16 + 24 = 42 && \text{(Model A)} \\ &= 3\tfrac{1}{3} + 32 + 45\tfrac{1}{3} = 80\tfrac{2}{3} && \text{(Model B).} \end{aligned} \tag{12.4}$$

## 13. Other Possible Models

In this section, we discuss other possibilities to unify the interactions that may be considered within the general scheme of Section 3. One obvious possibility, which was already indicated in Section 3, is to include fermion number as a gauge degree-of-freedom and to consider the group $SU_{2f}$ or, in general, the gauge group $SU_n$ generated by all $n$ left-handed spin $\frac{1}{2}$ fields (fermions and antifermions together). Examples of such models are the ones based on the gauge groups $SU_{24}$ (Scheme A) and $SU_{32}$ (Scheme B).

In the $SU_{2f}$ scheme, the fermion number generator is represented by the matrix

$$\frac{1}{(2f)^{1/2}}\begin{bmatrix} 1_1 & & & & & \\ & \ddots & & & & \\ & & 1_f & & & \\ & & & -1_1 & & \\ & & & & \ddots & \\ & & & & & -1_f \end{bmatrix}$$

The gauge boson can be represented by $\bar{f}f$, $ff$, or $\bar{f}\bar{f}$, i.e., in such a scheme one has in addition to the previously considered gauge bosons also diquarks $(qq)$, dileptons $(ll)$, and new leptoquark bosons of the type $(ql)$.

In the $SU_{2f}$ scheme the fermion number need not be conserved, since the associated gauge boson must be massive. In particular the symmetry breaking may mix gauge bosons of different fermion numbers. For example, one can have transitions like $(qq) \leftrightarrow (\bar{q}\bar{l})$, which cause the decay of the proton to second order in the gauge coupling. The measured stability of the proton requires that these gauge bosons acquire an extraordinarily large mass by the symmetry breaking, at least of the order of $10^{14}$ GeV. Consequently, the $SU_{2f}$ scheme may be relevant only at very small distances (smaller than $10^{-28}$ cm), which are almost of the order of the gravitational length of $10^{-33}$ cm (Planck's elementary length).

One may interpret the $SU_{2f}$ scheme as follows. Only at distances less than say $10^{-28}$ cm are all $SU_{2f}$ gauge degrees-of-freedom excited. Howefer, at much larger distances, only those gauge degrees-of-freedom that commute with the fermion number are relevant, i.e., one is dealing with the subtheory based on the gauge group $SU_f \times SU_f$. Then the symmetry breaking could be as follows:

$$\begin{array}{cl} SU_{2f} & \\ \downarrow & 10^{-28}\ \text{cm} \\ SU_f \times SU_f & \\ \downarrow & 10^{-18}\ \text{cm} \\ SU_3{}^c \times U_1{}^e. & \end{array}$$

The use of the maximal gauge group $SU_n$ ($n$: number of all left-handed basic fermions and antifermions) is very convenient in calculating the $SU_2 \times U_1$ mixing angle, provided that an $SU_2 \times U_1$ subtheory applies for the description of the usual weak and electromagnetic interactions. The point is that our formulas (6.8) and (6.9) are also valid for the maximal gauge group. Moreover, they are valid for any subgroup of the maximal group that is either simple or semisimple such that only one independent coupling constant is allowed.

This observation makes formulas (6.8) and (6.9) generally true for unified theories, i.e., the $SU_2 \times U_1$ mixing angle, which is calculable in a unified theory, is not specific to the gauge group, but only specific to the maximal group and to the set of elementary fermions. For example, the gauge group $SU_5$ discussed in [11] is a subgroup of $SU_{32}$. Consequently, it is not astonishing that both the $SU_5$ model and the previously considered Model B give the same value for $\sin^2 \theta_w$ ($\sin^2 \theta_w = \frac{3}{8}$).

One question that arises naturally with regard to the principle of maximal symmetry is the following. Although it would be very symmetric, if nature used all possible degrees-of-freedom provided by the elementary fermions in its gauge group, it may well be that the gauge group is smaller than the maximal group. In particular one may ask: What is the smallest gauge group that one can construct with a realistic fermion context?

In case of the fermion arrangement A (12 fundamental fermions) there is no smaller group, i.e., one has either $SU_{12} \times SU_{12}$ or $SU_{24}$ as gauge group. However, in case of arrangement B, there are several possibilities to construct smaller groups. We discuss first the case where the fermion number commutes with the gauge group and the fermions and antifermions belong to different representations of the gauge group. Hence, the gauge group must be a subgroup of $SU_{16} \times SU_{16}$.

In Scheme B the 16 fermions can be arranged in two sets of eight fermions as follows:

$$\begin{pmatrix} u_r & u_g & u_b & \nu_e \\ d_r' & d_g' & d_b' & e^- \end{pmatrix}\begin{pmatrix} c_r & c_g & c_b & \nu_\mu \\ s_r' & s_g' & s_b' & \mu_- \end{pmatrix}$$

Both groups have the same physical properties, except the fermions in the second group have in general higher masses, and it may be useful to think of the fermions in the second group as excitations of the analogous ones in the first group. Thus, one may regard the fermions as (**8**) representations of the group $SU_8$ and start with the gauge group $SU_8{}^L \times SU_8{}^R$. The symmetry bteaking is as follows:

$$\begin{gathered} SU_8{}^L \times SU_8{}^R \\ \downarrow \\ (SU_4{}^L \times SU_2{}^L) \times (SU_4{}^L \times SU_2{}^R) \\ \downarrow \\ (SU_3{}^L \times SU_3{}^R)_c \times SU_2{}^L \times SU_2{}^R \times U_1{}^2 \\ \downarrow \\ SU_3{}^c \times SU_2 \times U_1 \\ \downarrow \\ SU_3{}^c \times U_1{}^e \end{gathered}$$

This model is analogous to the previously considered $SU_{16} \times SU_{16}$ model. However, note that here there is no way of getting information about the $m_e/m_\mu$ mass ratio since the electron and muon belong to different irreducible representations of $SU_8$.

Another interesting possibility is the gauge group $(SU_4)$ [40]. This group is obtained as follows. Besides the group $SU_4 \times SU_4$, which transforms the left-handed and right-handed fermions in the same row of Scheme B

$$\begin{pmatrix} c & c & c & \nu \\ u & u & u & \nu \\ d & d & d & e^- \\ s & s & s & \mu^- \end{pmatrix}$$

among each other, one considers also the group $(SU_4^L \times SU_4^R)$, acting on the fermions in the same column (see also [47]). Thus, one obtains the group $(SU_4^L \times SU_4^R)_{\text{hor}} \times (SU_4^L \times SU_4^R)_{\text{vert}}$. This theory is not yet a unified theory, since one is dealing with two commuting gauge groups $G_{\text{hor}}$ and $G_{\text{vert}}$. However, a unified theory can be obtained if we require that the Lagrangian be invariant under the exchange of columns and rows. This new kind of discrete symmetry would imply that the physical $SU_4$ group and the group $SU_4^{\text{color}}$ (extended from $SU_3^{\text{color}}$ by adding the leptons as fourth color column) can be interchanged freely. In particular, quarks and leptons can be interchanged in the symmetry limit.

The symmetry breaking can be arranged as follows:

$$\begin{gathered}(SU_4^L \times SU_4^R)_{\text{hor}} \times (SU_4^L \times SU_4^R)_{\text{vert}} \\ \downarrow \\ SU_4^{\text{color}} \times SU_2^L \times SU_2^R \\ \downarrow \\ SU_3^{\text{color}} \times SU_2^L \times U_1 \\ \downarrow \\ SU_3^{\text{color}} \times U_1^e\end{gathered}$$

The theory based on the gauge group $SU_4^4$ is a subtheory of the $SU_{16} \times SU_{16}$ theory; hence, it yields the same consequence for the $SU_2 \times U_1$ mixing angle: $\sin^2 \theta_w = \frac{3}{8}$.

The $SU_4^4$ model discussed above is the model with the smallest number of gauge bosons (60), which has the fermion content of Scheme B and has an exactly conserved fermion number.

Finally, we come to the discussion of models that break fermion number conservation. In these models, the irreducible fermion representations of the gauge group contain in general fermions and antifermions. The gauge group is a subgroup of $SU_n$ ($n$: number of all left-handed fermions and antifermions).

We concentrate on orthogonal subgroups of $SU_{32}$, where the 32 spin $\frac{1}{2}$ fields are identified with the fermions and antifermions of Scheme B. As is well-known, the orthogonal groups have the advantage of being free of anomalies.

If the idea of lepton number as fourth color makes sense, it is natural for the group $SU_4^{\text{color}} \times SU_2^L \times SU_2^R$ to be a subgroup of the gauge group (see Section 11.) This group is isomorphic to $SO_6 \times SO_4$, which is naturally embedded in $SO_{10}$.[12] Indeed, one finds that $SO_{10}$ has a 16-dimensional complex representation. We describe all 32 fermions and antifermions by the reducible representation

[12] After the completion of this article we were informed that the $SO_{10}$-model has also been considered by Georgi and Glashow (see [47]).

**(16)** + **(16)** of $SO_{10}$. The symmetry breaking can be constructed according to the following chain:

$$\begin{array}{l} SO_{10} \\ \downarrow \\ SO_6 \times SO_4 = SU_4{}^c \times SU_2{}^L \times SU_2{}^R \\ \qquad\qquad\qquad\quad \downarrow \\ \qquad\qquad\qquad SU_3{}^c \times SU_2{}^L \times SU_2{}^R \times U_1 \\ \qquad\qquad\qquad\quad \downarrow \\ \qquad\qquad\qquad SU_3{}^c \times SU_2{}^L \times U_1 \\ \qquad\qquad\qquad\quad \downarrow \\ \qquad\qquad\qquad SU_3{}^c \times U_1{}^e \end{array}$$

The **(16)** representation transforms under $SU_4 \times SU_2 \times SU_2$ as: **(16)** = (4, 2, 1) + ($\bar{4}$, 1, 2). Hence, we identify the fermions as follows:

$$(\mathbf{16})_1 = \begin{pmatrix} \bar{u} & \bar{u} & \bar{u} & \bar{\nu}_e & \vdots & u & u & u & \nu_e \\ \bar{d}' & \bar{d}' & \bar{d}' & e^+ & \vdots & d' & d' & d' & e^- \end{pmatrix}_L$$

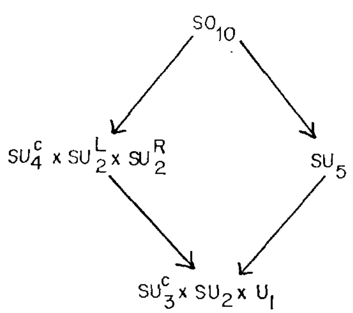
$$(\mathbf{16})_2 = \begin{pmatrix} \bar{c} & \bar{c} & \bar{c} & \bar{\nu}_\mu & \vdots & c & c & c & \nu_\mu \\ \bar{s}' & \bar{s}' & \bar{s}' & \mu^+ & \vdots & s' & s' & s' & \mu^- \end{pmatrix}_L$$

The $SO_{10}$ theory described above has the following features.

(a) Since $SO_{10}$ has the subgroup $SU_4{}^c \times SU_2 \times SU_2$, the parity transformation is an automorphism of the gauge group and the Lagrangian is parity invariant. Parity must be broken spontaneously.

(b) The $SU_5$ model of [11] is a subtheory of the $SO_{10}$ model, since $SO_{10}$ contains $SU_5$. The **(16)** representation breaks up under $SU_5$ as **(1)** + **($\bar{5}$)** + **(10)**.

The breakup of the $SO_{10}$ group can proceed in two different ways, given by the diagram

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The intermediate stage $SU_4^c \times SU_2^L \times SU_2^R$ seems to us more natural because of the remarks made in Section 11. A breaking of $SO_{10}$ using the intermediate stage $SU_5$ presumably has no physical meaning.

The 45 generators of $SO_{10}$ transform under $SU_4^c \times SU_2^L \times SU_2^R$ as follows:

$$\begin{aligned} (45) = &\ (15, 1, 1) && \text{[gluon, leptoquarks } (\bar{q}l), R] \\ &+ (1, 3, 1) && \text{[weak bosons]} \\ &+ (1, 1, 3) && \text{[superheavy weak bosons]} \\ &+ (6, 2, 2) && \text{[diquarks, leptoquarks } (ql)]. \end{aligned}$$

(c) Like the $SU_5$ theory, the $SO_{10}$ theory contains gauge bosons that couple to diquarks and antiquark–antilepton pairs. Consequently, the proton decays in second order, and the associated gauge boson has to have an extraordinarily high mass (larger than $10^{14}$ GeV). Hence, the unification of the strong and nonstrong interactions by the 24 $SO_{10}$ gauge degrees-of-freedom not contained in the subtheory $SU_4^c \times SU_2 \times SU_2$ can only occur at very small distances ($\sim 10^{-28}$ cm).

(d) The theory does not minimize the number of neutral currents, as does the $SU_2 \times U_1$ theory. Since the group $SU_4 \times SU_2 \times SU_2$ contains three $SU_3^c$ singlet neutral generators, one of which is identified with the photon, there are two massive netural bosons that may produce neutral current effects. However, if one requires that the third neutral boson be superheavy, the conventional weak and electromagnetic interactions are described by a $SU_2 \times U_1$ theory with $\sin^2 \theta_w = \frac{3}{8}$. However, note that this value is subject to sizable renormalization corrections, depending on the energies where the unification of the interactions sets in and where the $SU_4^c \times SU_2 \times SU_2$ theory becomes relevant. In general $\sin^2 \theta_w$ may be larger or smaller than $\frac{3}{8}$.

(e) The basic fermions are described by two irreducible (16) representations of $SO_{10}$. This implies that there is no way to obtain information about the $m_e/m_\mu$ ratio. Furthermore, one checks easily that there is no other group such that all 32 fermions and antifermions belong to an irreducible representation and the theory is anomaly-free. For example, the group $SO_{12}$ has a 32-dimensional representation, however this representation is real and therefore does not give the desired fermion representation under the subgroup $SU_3^c \times SU_2 \times U_1$.

It is interesting that the requirement that the theory be free of anomalies leads automatically to 16-dimensional irreducible representations, i.e., to representations involving eight fermions and antifermions, as the maximal irreducible fermion representations. Does this mean that the fermions in the second (16) representation are dynamical excitations of the fermions in the first one? If this is true, there is no reason to stop at the second level. It may well be that an infinite sequence of such excitation, i.e., of "elementary" fermions exist, that can be classified according to irreducible representations of $SO_{10}$.

If one is willing to introduce new fermions, one can still go a step further and ask the question: What is the smallest orthogonal group that contains all 32 (anti) fermions and possibly new fermions and antifermions in one irreducible representation?

This group is $SO_{14}$, and all fermions can be described by a complex (**64**)-dimensional representation, i.e., one must introduce 32 new (anti) fermions, which must be heavier than the 32 (anti) fermions of Scheme B.

If we characterize the group $SO_{14}$ by the subgroups $SO_{10}$ and $SU_4{}^c \times SU_2{}^L \times SU_2{}^R$, the (**64**)-representation decomposes as follows:

$$\begin{array}{ccc} SO_{14} & & (64) \\ \downarrow & & \downarrow \\ SO_{10} & & 2 \times \mathbf{16} + 2 \times \overline{\mathbf{16}} \\ \downarrow & & \downarrow \qquad\qquad \downarrow \\ SU_4{}^c \times SU_2{}^L \times SU_2{}^R & & 2(\mathbf{4}, \mathbf{2}, \mathbf{1}) \qquad 2(\mathbf{4}, \mathbf{1}, \mathbf{2}) \\ & & + 2(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \quad + 2(\bar{\mathbf{4}}, \mathbf{2}, \mathbf{1}). \end{array}$$

The 32 light (anti) fermions are identified with the representations $2(\mathbf{4}, \mathbf{2}, \mathbf{1}) + 2(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$, while the 32 new, heavy (anti) fermions transform under $SU_4{}^c \times SU_2{}^L \times SU_2{}^R$ as $2(\mathbf{4}, \mathbf{1}, \mathbf{2}) + 2(\bar{\mathbf{4}}, \mathbf{2}, \mathbf{1})$. Note that the 32 new (anti) fermions have the opposite transformation property under $SU_4{}^c$ as compared to the light (anti) fermions.

In the $SO_{14}$ scheme the proton decays into leptons as in the $SO_{10}$ scheme to second order in the gauge coupling, i.e., the unification of strong and nonstrong interactions can only set in at distances of the order of $10^{-30}$ cm.

The $SU_5$ and $SO_{10}$ models are anomaly-free, but they do not allow the definition of a conserved fermion number operator, whereas the other models discussed in this section allow fermion number to be conserved, but have anomalies. This is not an accident, but the consequence of the following theorem:

*There is no anomaly-free unified theory involving only the* 16 *fermions and antifermions of Scheme B such that fermion number is conserved and commutes with the gauge group.*

This theorem can be proven as follows. The gauge group $G$ has the subgroup $SU_3{}^c \times SU_2 \times U_1$. Under this group the 16 left-handed fermions transform as $f = 2 \times (\mathbf{3}, \mathbf{2}) + 2 \times (\mathbf{1}, \mathbf{2})$, and the 16 left-handed antifermions transform as $\bar{f} = 4 \times (\bar{\mathbf{3}}, \mathbf{1}) + 4 \times (\mathbf{1}, \mathbf{1})$. Since the fermion number $F$ does not belong to the gauge group, the 16 fermions $f$ and the 16 antifermions $\bar{f}$ belong to different, and in general reducible, representations of the gauge group. Note that the representations $f$ and $\bar{f}$ are complex, but are not related to each other by complex conjugation and are also not equivalent. Hence, it is sufficient to consider all compact simple Lie groups that have two inequivalent complex 16-dimensional representations and have the group $SU_3 \times SU_2 \times U_1$ as subgroups. The only groups that have complex representations are the unitary and $SO_{2n}$ groups (besides $E_6$, which, however, demands more than 16 fermions). One checks easily that the groups $SU_5 ... SU_{16}$ have no 16-dimensional representations with the desired fermion content under $SU_3 \times SU_2 \times U_1$. The only $SO_{2n}$ group with a 16-representation is the group $SO_{10}$, but this group was considered above. There we found that the 16-representation contains both $SU_3{}^c$ triplets and antitriplets, and hence, fermion number cannot be conserved.

It remains to check the case where the gauge group $G$ is not simple, but consists of the direct product $G = G_1{}^L \times G_2{}^R$, where both factors are related by parity. Here the left-handed fermions transform under $G^L$, and the right-handed fermions transform under $G^R$. However, this theory is certainly not anomaly-free, since one may restrict oneself to the subgroup $SU_3^{cL}$, which has anomalies (note that the representation $f$ contains only (3) representations of $SU_3{}^c$).

The theorem above implies that the problem of fermion number conservation (proton decay) and the problem of anomalies are complementary issues in constructing a unified theory of strong and nonstrong interactions. On the basis of the presently known or suspected fermions, one can construct an anomaly-free theory only by giving up fermion number conservation and running the risk of generating a sizable instability of the proton. To be compatible with the observed lifetime of the proton one must introduce gauge bosons of a mass $\sim 10^{14}$ GeV. Hence, the unification of strong and nonstrong interactions becomes some kind of an illusion; it takes place only at distances of the order of $10^{-30}$ cm, i.e., at distances where the gravitational interaction, which was left out, becomes relevant also.

On the contrary, one can construct a unified theory with a conserved fermion number only by allowing the presence of anomalies and admitting the possibility that nature has her own way to go around the infinities generated by the anomalies in high order, or by enlarging the set of elementary fermions. Of course, anomalies can be avoided by adding new elementary fermions such that the new fermions have conjugate transformation properties under the gauge group and the fermions and antifermions transform separately as real representations. This would mean in case of the 16-fermion Scheme B, that at sufficiently high energies 16 new fermions arise, doubling the number of leptons and the number of hadronic degrees-of-freedom. We conclude that if anomalies are taken seriously as a principle to construct a unified theory and if the unification of strong and nonstrong interactions is required to set in much before gravitational effects become important, one must include heavy quarks and heavy leptons with conjugate transformation properties under the gauge group.

## 14. Concluding Remarks

In this paper we have tried to show how a unification of the interactions can be achieved on the basis of the assumption that the unifying gauge group is given by a symmetry group of the kinetic energy in the fermion Lagrangian. We have demonstrated that such a unification leads to interesting algebraic consequences for the interactions between quarks and leptons, e.g., to the determination of the neutral current within an $SU_2 \times U_1$ theory for the weak and electromagnetic interactions or of the couplings of the superheavy bosons, which are supposed to be responsible for the generation of the electron mass. Furthermore, we have shown that the unification of the strong and nonstrong interactions gives rise to an extension of the list of hadronic constituents in the form of colored gauge bosons with different quantum numbers than the vector gluons. Possible implications of their existence for $e^+e^-$-annihilation have been discussed.

The hypotheses of a unified theory of the type we are proposing can only be turned into a real theory of leptons and hadrons if the following problems can be resolved.

(1) It must be shown that the idea of perfect confinement of color within the canonical Yang–Mills theory of colored quarks and gluons makes sense. Any unification of the leptons and quarks must necessarily fail if additional ingredients are needed for the confinement, e.g., a modification of the canonical field theory, the introduction of strings or the postulation of boundary conditions for the color charges, i.e., mechanisms that do not apply for the leptonic world and that would generate an intrinsic difference between leptons and quarks.

(2) The mechanism of dynamical symmetry breaking has to be understood. Does it make sense to treat the scalar fields as bound state fields and to neglect them for the study of the ultraviolet behavior of the theory? If so, what is the nature of the bound state fields? Are they bound states of colored objects only?

(3) Are the parameters of the spontaneous symmetry breaking universal, or are there domains in the universe that exhibit different properties? For example, are there domains where $(SU_4)_{\text{color}}$ is exact, as described in Section 11? Is it conceivable that the color symmetry is broken in certain domains of the universe and that the confinement mechanisms fail? In such a domain there would be no hadrons and no strong interactions, but instead there would exist quarks, which act like leptons and presumably have integral electric charges.

(4) What is the origin of the symmetric hadronic mass scale, i.e., the 1 GeV mass scale of the strong interactions? Does it arise spontaneously by the infrared instability of the quark–gluon theory, or is it due to the existence of new hadronic degrees-of-freedom, e.g., the formal mass of a charmed quark [48]?

(5) If the generators of the electric charge really belong to a simple algebra of gauge generators, as is the case in the unified models discussed here, and if the dynamical symmetry breaking makes sense, the electromagnetic coupling $e$ approaches zero at very small distances and there is no nontrivial fixed point to be approached by $e$ in the ultraviolet limit. Consequently, there is no hope of determining $\alpha$ as a function of such a fixed point, and the question arises: What physical parameters determine $\alpha$. Presumably, this problem is related to the unknown dynamical details of the symmetry breaking, in particular, to the spontaneous generation of the fermion masses.

(6) Both in Model A and in Model B it is apparent that the leptons and quarks come in families of four fermions, i.e., three colored quarks and one lepton. Is this $3 + 1$ structure simply an accident, or perhaps the consequence of an underlying $SU_4$-color group, or is there a deep underlying principle that requires such a structure? In this connection it is very interesting to note that the recent proposal to generalize the quantum mechanics to an octonion quantum mechanics automatically gives a $3 + 1$ structure for the fermions, with the quarks being fictitious fermions due to the nonassociativity of the Cayley algebra [49].

(7) As shown in Section 13, the problem of anomalies is a central issue in constructing a unified theory, since anomaly-free theories based on the known or suspected fermions cannot really achieve a unification (the latter would set in only at $\sim 10^{-30}$ cm). Therefore, we must ask: Is it possible that the "true" theory has anomalies, but nature has found a special way to cut off the infinities generated by the anomalies? Or, does nature not tolerate anomalies, and do there exist many new quarks and leptons contrary to common beliefs?

## APPENDIX: Strangeness Violations in Weak Interactions

It is assumed that the gauge group for the electromagnetic and weak interactions is $(SU_3{}^L)_{\text{hadr}} \times (SU_2)_{\text{lept}} \times (U_1)_{\text{hadr+lept}}$ where the $U_1$ subgroup is constructed such that the electric charge is within the group. The generators are denoted as follows:

$$\begin{aligned} (SU_3{}^L)_{\text{hadr}} &: H_i \qquad (i = 1...8) \\ (SU_2)_{\text{lept}} &: L_a \qquad (a = 1, 2, 3) \\ (U_1)_{\text{hadr+lept}} &: Y_1 \end{aligned}$$

First, consider the following breaking:

$$\begin{gathered} (SU_3{}^L)_{\text{hadr}} \times (SU_2)_{\text{lept}} \times (U_1)_{\text{hadr+lept}} \\ \downarrow \\ (SU_2)^{(ud)}_{\text{hadr+lept}} \times (U_1)_{\text{hadr+lept}} \end{gathered} \tag{A.1}$$

The group $(SU_2)^{(u,d)}_{\text{hadr+lept}}$ is constructed as the direct sum of $(SU_2)_{\text{lept}}$ and $(SU_2)^{(u,d)}_{\text{hadr}}$, where the latter is an $SU_2{}^L$ subgroup of $SU_3{}^L$ to be identified with the group generated by the left-handed $(u, d)$ quarks. The gauge group $(SU_2^{(u,d)})_{\text{lept+hadr}} \times U_1$ is the group studied in Section 7(c).

The symmetry breaking is supposed to be generated by the coupling of the gauge fields to a scalar field $\phi^{(u,d)}$ that transforms as a certain representation of $SU_3{}^L \times SU_2 \times U_1$. The vacuum expectation value is denoted by $\phi^{(u,d)}$. As an example, we take the following vacuum expectation value:

$$\phi^{(u,d)} = M_0 \cdot X^h(I_h, m_h, \eta) \cdot X^{*l}(I_l, m_l, y') \cdot 1/(2I+1)^{1/2} \tag{A.2}$$

where $(I, m)$ are the quantum numbers for $SU_2^{(u,d)}$ or $(SU_2)^l$, respectively, and $\eta, y'$ are the eigenvalues of $H_8$ and $Y'$. $M_0$ is an overall scale parameter of dimension $[l^{-1}]$.

Since the direct sum $(SU_2^{(u,d)})_{\text{lept+hadr}} \times U_1$ is supposed to be conserved, one has $I_h = I_l = I$, $m_h = m_l$, $\eta = y'$. We normalize (A.2) such that $|X^h|^2 = |X^l|^2 = 1$.

We also introduce a scalar field $\phi^{(u,s)}$, whose vacuum expectation value breaks the symmetry according to

$$\begin{array}{c} (SU_3{}^L)_{\text{hadr}} \times (SU_2)_{\text{lept}} \times (U_1)_{\text{lept+hadr}} \\ \downarrow \\ SU^{(u,s)}_{2\,\text{hadr+lept}} \times U_1 \end{array} \tag{A.3}$$

$\phi^{(u,s)}$ is constructed the same way as $\phi^{(u,d)}$, but $M_0$ is replaced by $M_0(\epsilon)^{1/2}$, where $\epsilon$ is a free parameter.

The breaking of the group $SU_3{}^L \times SU_2 \times U_1$ is supposed to arise by the coupling of the gauge bosons to both $\phi^{(u,d)}$ and $\phi^{(u,s)}$. The resulting mass matrix is determined by $\epsilon$, $M_0$, parameters $F_c$ and $F_n$, which depend on the representation, in particular on the eigenvalues $I$ and $y'$:

$$F_c = \frac{3}{4}\frac{\langle\phi \mid I + n' \mid \phi\rangle}{\langle\phi \mid I(I+1) \mid \phi\rangle}, \qquad F_n = \frac{11}{25}\frac{\langle\phi \mid (I+n')^2 \mid \phi\rangle}{\langle\phi \mid I(I+1) \mid \phi\rangle} \tag{A.4}$$

$n'$ is defined by

$$H_8 X^h(I, m, \eta) = \pm(\tfrac{2}{3})^{1/2}(I + n')\, X^h \tag{A.5}$$

and is a non-negative integer. Note that for each positive eigenvalue of $H_8$ there exists a corresponding value with negative sign in order to preserve CP.

Consider the mass-matrix for the positively charged bosons first. The eigenvalues of the mass matrix are linear combinations of the bosons coupled to the generators

$H_{1+i2}$, $H_{4+i5}$, $L_{1+i2}$. The associated $SU_{12}^L \times SU_{12}^R$ matrices can be represented by the following short notation

| | $H_{1+i2}$ | $H_{4+i5}$ | $L_{1+i2}$ | $(H+L)_{1+i2}$ |
|---|---|---|---|---|
| $(u, d)$ | $\frac{1}{3^{1/2}}$ | 0 | 0 | $\frac{1}{5^{1/2}}$ |
| $(u, s)$ | 0 | $\frac{1}{3^{1/2}}$ | 0 | 0 |
| $(\nu, e)$ | 0 | 0 | $\frac{1}{2^{1/2}}$ | $\frac{1}{5^{1/2}}$ |
| $(\mu, \nu)$ | 0 | 0 | $-\frac{1}{2^{1/2}}$ | $-\frac{1}{5^{1/2}}$ |

The generator $(H + L)_{1+i2}$ is defined such that in the limit $\theta_c = 0$, its boson describes the positively charged $W$-boson. Note:

$$(H + L)_{1+i2} = \cos \phi H_{1+i2} + \sin \phi L_{1+i2} \tag{A.6}$$

with $\sin \phi = (2/5)^{1/2}$). The mass-matrix takes the form

$$M^2 = \mu^2 \cdot \begin{pmatrix} (1 + F_c\epsilon) \sin^2 \phi & 0 & -\sin \phi \cos \phi \\ 0 & (\epsilon + F_c) \sin^2 \phi & -\epsilon \sin \phi \cos \phi \\ -\sin \phi \cos \phi & -\epsilon \sin \phi \cos \phi & (1 + \epsilon) \cos^2 \phi \end{pmatrix} \tag{A.7}$$

with

$$\mu^2 = 20/9\langle \phi \mid I(I + 1) \mid \phi\rangle\, M_0^2. \tag{A.8}$$

Using this mass-matrix one can easily calculate the weak current and, in particular, the different decay constants. One finds

$$\frac{G_\beta}{G_\mu} = \frac{1}{1 + F_c\epsilon}, \qquad \frac{G_A}{G_\mu} = \frac{1}{1 + (F_c/\epsilon)} \tag{A.9}$$

$$\frac{G_\mu}{2^{1/2}} = \frac{4\pi\alpha}{\mu^2 \det(M^2/\mu^2)} (1 + F_c\epsilon)(\epsilon + F_c) \sin^4 \phi. \tag{A.10}$$

If we require Cabibbo type universality $G_\beta = \cos \theta_c G_\mu$, $G_A = \sin \theta_c G_\mu$, we obtain:

$$\begin{aligned} F_c^2 &= (\sec \theta_c - 1)(\operatorname{cosec} \theta_c - 1) \\ \epsilon^2 &= (\sec \theta_c - 1)/(\operatorname{cosec} \theta_c - 1). \end{aligned} \tag{A.11}$$

Using the value $\sin \theta_c = 0.2$ one finds

$$F_c = 0.29, \qquad \epsilon = 0.07. \tag{A.12}$$

According to ($A$.8) isospin values of at least $I = 5/2$ are needed to generate such a small value for $F_c$. Using the experimental value for $G_\mu$, one finds $\mu = 530$ GeV and the following eigenvalues of the mass matrix:

$$m_{W_1^+} = 67 \text{ GeV}, \qquad m_{W_2^+} = 189 \text{ GeV}, \qquad m_{W_3^+} = 502 \text{ GeV}. \tag{A.13}$$

The lowest-lying boson couples to lowest order in $\theta_c$ in the form

$$W_1^+ \sim \cos\theta_c(\bar{u}d)_L + \sin\theta_c(\bar{u}s)_L + (\nu e)_L + (\mu\nu)_L\,, \tag{A.14}$$

the corrections to the pure Cabibbo case are of order $\theta^2$. $W_2^+$ couples predominantly to $(\bar{u}s)_L$.

For the description of the neutral bosons it is appropriate to introduce the following linear combinations:

$$\begin{aligned} \cos\phi H_3 + \sin\phi L_3 &= (H+L)_3 = W_0 \\ -\sin\phi H_3 + \cos\phi L_3 &= (-H+L)_3 = V_0 \\ \cos\psi H_8 + \sin\psi Y' &= Y_0 \\ -\sin\psi H_8 + \cos\phi Y' &= X_0\,. \end{aligned} \qquad \begin{aligned} \sin\phi &= (\tfrac{2}{5})^{1/2} \\ \sin\psi &= (\tfrac{10}{11})^{1/2} \end{aligned} \tag{A.15}$$

The electric charge is then a combination of $(H+L)_3$ and $Y_0$:

$$\gamma = \sin\theta_w W_0 + \cos\theta_w Y_0 \tag{A.16}$$

where $\theta_w$ is the $SU_2 \times U_1$ mixing angle: $\sin^2\theta_w = \frac{5}{16}$. The orthogonal combination is

$$Z_0 = -\cos\theta_w W_0 + \sin\theta_w Y_0'. \tag{A.17}$$

The mass-matrix in terms of $(Z_0, X_0, V_0)$ is then given in terms of $F_n$:

$$(M^2)^{\text{neutral}} = \begin{pmatrix} \left(\frac{72}{121}F_n + \frac{24}{275}\right)\epsilon & -\left(\frac{24}{121}F_n + \frac{6}{55}\right)2^{1/2}\epsilon & \left(\frac{18}{11}F_n - \frac{24}{25}\right)\left(\frac{2}{33}\right)^{1/2}\epsilon \\ \sim & \left(\frac{16}{121}F_n + \frac{3}{11}\right)\epsilon + F_n & \left(-\frac{12}{11}F_n + \frac{12}{5}\right)\frac{\epsilon}{(33)^{1/2}} \\ \sim & \sim & \left(\frac{3}{11}F_n + \frac{16}{25}\right)\epsilon + 1 \end{pmatrix} \tag{A.18}$$

In the limit $\epsilon = 0$ it takes the simple form

$$M^{2\,\text{neutral}}_{(\epsilon=0)} = \mu^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & F_n & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{A.19}$$

i.e., the lowest eigenvalue is zero, which has to be, since the $SU_2 \times U_1$ subtheory is exact in this limit.

For $\epsilon \neq 0$ the three bosons mix. If $\epsilon$ is small compared to $F_n$ and 1, the lowest lying neutral boson essentially does not mix with the other ones, i.e., the $SU_2 \times U_1$ coupling for the neutral current [Eq. (7.14)] is almost exactly maintained. However, the mass of the lowest lying boson does not follow Eq. (7.16), but depends on $F_n$.

For example one obtains:

$$\begin{aligned} F_n &= 0, \qquad M_Z = 42 \text{ GeV} \\ F_n &= \tfrac{1}{2}, \qquad M_Z = 70 \text{ GeV}. \end{aligned} \tag{A.20}$$

In the mass matrix (A.18) we have not included the $|\Delta S| = 1$ neutral bosons $\hat{H}_{6\pm i7}$. By construction these bosons do not mix at all, neither with the leptonic or hadronic $\Delta S = 0$ bosons nor with themselves. Their mass is given by

$$M_{6\pm i7}^2 = \tfrac{2}{5}\mu^2(1+\epsilon)\, F_c \cdot \approx [190 \text{ GeV}]^2. \tag{A.21}$$

The $|\Delta S| = 1$ neutral bosons are essentially degenerate with the $|\Delta S| = 1$ charged bosons $W_2{}^+$ (see A.13).

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