# 2004 TASI Lectures on Supersymmetry Breaking

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**Abstract**

These lectures give an introduction to the problem of finding a realistic and natural extension of the standard model based on spontaneously broken supersymmetry. Topics discussed at some length include the effective field theory paradigm, coupling constants as superfield spurions, gauge mediated supersymmetry breaking, and anomaly mediated supersymmetry breaking, including an extensive introduction to supergravity relevant for phenomenology.

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## 1 Introduction

Our present understanding of particle physics is based on effective quantum field theory. Quantum field theory is the inevitable result of combining quantum mechanics and special relativity, the two great scientific revolutions of the early twentieth century. An *effective* quantum field theory is one that includes only the degrees of freedom that are kinematically accessible in a particular class of experiments. Presently, the highest energies probed in accelerator experiments are in the 100 GeV range, and the standard model is an effective quantum field theory that describes all physical phenomena at energies of order 100 GeV and below.

The standard model contains a Higgs scalar that has not been observed as of this writing. Therefore, the minimal effective theory that describes the present data does not contain a Higgs scalar. This effective theory allows for the possibility that the dynamics that breaks electroweak symmetry does not involve elementary scalars (as in 'technicolor' theories, for example). This effective theory necessarily breaks down at energies of order a TeV, and therefore new physics must appear below a TeV. This is precisely the energy range that will be explored by the LHC starting in 2007-2008, which is therefore all but guaranteed to discover the interactions that give rise to electroweak symmetry breaking, whether or not it involves a Higgs boson.

If the physics that breaks electroweak symmetry does not involve particles with masses below a TeV, then it must be strongly coupled at a TeV. Contrary to what is sometimes stated, precision electroweak experiments have not ruled out theories of this kind. For example, if we estimate the size of the $S$ and $T$ parameters assuming that the electroweak symmetry breaking sector is strongly coupled at a TeV with no large or small parameters, we obtain

$$\Delta S \sim \frac{1}{\pi}, \qquad \Delta T \sim \frac{1}{4\pi}, \tag{1.1}$$

which are near the current experimental limits. The strongly-coupled models that *are* ruled out are those that contain $N \gg 1$ degrees of freedom at the TeV scale; in these models the estimates for $S$ and $T$ above are multiplied by $N$. Another difficulty with building models of strongly-coupled electroweak symmetry breaking is incorporating quark mixing without large flavor-changing neutral current effects. However, given our profound ignorance of strongly-coupled quantum field theory, it may be prudent to keep an open mind. Fortunately, the LHC will tell us the answer soon.

The subject of these lectures is weakly-coupled supersymmetry ('SUSY'). As you have heard in previous lectures, there are several hints that SUSY is correct. First, the simplest supersymmetric grand unified models predict a precise relation among the three gauge couplings of the standard model that is in excellent agreement with observation. Second, the best fit to precision electroweak data is obtained with a Higgs boson with a mass close to the experimental lower limit of 114 GeV from LEP. Such a light Higgs boson arises automatically in SUSY. Finally, SUSY naturally contains a viable cold dark matter candidate.

If SUSY is in fact discovered at the LHC, it will be the culmination of decades of work by many hands, starting with general theoretical investigations of spacetime symmetries and the construction of supersymmetric quantum field theories, to the realization that SUSY can solve the hierarchy problem and the construction of realistic models of broken supersymmetry. It will be an intellectual triumph comparable to general relativity, another physical theory that fundamentally changed our view of space and time, and which was also proposed based on very general theoretical considerations and later spectacularly verified by experiment.

If SUSY is realized in nature, it must be broken. In this case, the pattern of SUSY breaking can give us a great deal of information about physics at much higher energy scales. There are a number of theoretically well-motivated mechanisms for SUSY breaking, each of which give distinct patterns of SUSY breaking that can be experimentally probed at the weak scale. If nature is supersymmetric at the weak scale, then the experimental program in particle physics after the LHC turns on will be largely the study of superpartners.

These are exciting prospects, but we are not there yet. There is at present no direct experimental evidence for superpartners, or a light Higgs boson. Indirect experimental constraints place strong constraints on the simplest SUSY models, requiring either accidental cancelations (fine tuning) or additional non-minimal structure in the theory. It is premature to say that these constraints rule out the idea of SUSY, but they must be addressed by any serious proposal for weak scale SUSY.

These lectures will review both the progress that has been made in constructing realistic models of SUSY breaking, and the problems faced by SUSY in general, and these models in particular. It is my hope that these lectures will challenge and inspire—rather than discourage—the next generation of particle physicists.

## 2 Effective Field Theory and Naturalness

If we take seriously the idea that the standard model is an effective field theory, then the coupling constants of the standard model are not to be viewed as fundamental parameters. Rather they are to be thought of as effective couplings determined by a more fundamental theory. How do we know there is a more fundamental theory?

For one thing, we hope that there is a more fundamental theory that explains the $\sim 20$ free parameters of the standard model (mostly masses and mixings). One piece of evidence for a simpler fundamental theory comes from the fact that the standard model gauge couplings approximately unify at a scale $M_{\rm GUT} \sim 10^{16}$ GeV. This is evidence that $M_{\rm GUT}$ is a scale of new physics described by a more fundamental theory. Another evidence for a new scale is the fact that gravity becomes strongly interacting at the scale $M_{\rm P} \sim 10^{19}$ GeV, and we expect new physics at that scale. Finally, as we review below, the recent observation of neutrino mixing suggests the existence of another scale in physics of order $10^{15}$ GeV. These scales are so large that we cannot ever hope to probe them directly in accelerator experiments. The best that we can do is to understand how the effective couplings that we can measure are determined by the more fundamental theory.

### *2.1 Matching in a Toy Model*

Let us consider a simple example that shows how effective couplings are determined from an underlying theory. Consider a renormalizable theory consisting of a real scalar $h$ coupled to a Dirac fermion field $\psi$:

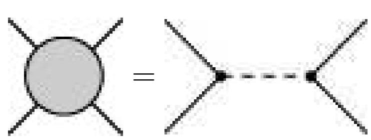
$$\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi + \tfrac{1}{2}(\partial h)^2 - \tfrac{1}{2} M^2 h^2 - \frac{\lambda}{4!} h^4 + y h \bar{\psi}\psi. \tag{2.1}$$

This theory has a discrete chiral symmetry

$$\psi \mapsto \gamma_5 \psi, \qquad h \mapsto -h, \tag{2.2}$$

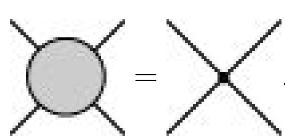
that forbids a fermion mass term, since $\bar{\psi}\psi \mapsto -\bar{\psi}\psi$. For processes with energy $E \ll M$, the scalar is too heavy to be produced. We should therefore be able to describe these processes using an effective theory containing only the fermions.

We determine the effective theory by matching to the fundamental theory. Let us consider fermion scattering. In the fundamental theory, we have at tree level



+ crossed, (2.3)

while in the effective theory the scattering comes from an effective 4-fermion coupling:



. (2.4)

Matching is simply demanding that these two expressions agree order by order in an expansion in $1/M$. This corresponds to expanding the scalar propagator in inverse powers of the large mass:

$$\frac{1}{p^2 - M^2} = -\frac{1}{M^2} - \frac{p^2}{M^4} + \mathcal{O}(p^4/M^6). \tag{2.5}$$

Equivalently, we can solve the classical equations of motion for $h$ in the fundamental theory order by order in $1/M^2$:

$$h = \frac{1}{M^2}\left[y\bar{\psi}\psi - \Box h - \frac{\lambda}{3!}h^3\right] \tag{2.6}$$

$$= \frac{y}{M^2}\bar{\psi}\psi - \frac{y}{M^4}\Box(\bar{\psi}\psi) + \mathcal{O}(1/M^6). \tag{2.7}$$

Substituting this into the lagrangain, we obtain the effective Lagrangian at tree level

$$\mathcal{L}_{\text{eff}} = \bar{\psi}i\partial\!\!\!/\psi + \frac{y^2}{2M^2}(\bar{\psi}\psi)^2 - \frac{y^2}{2M^4}\bar{\psi}\psi\Box(\bar{\psi}\psi) + \mathcal{O}(1/M^6). \tag{2.8}$$

This effective Lagrangian will give the same results as the Lagrangian Eq. (2.1) for all tree level processes, up to corrections of order $1/M^6$.

We can continue the matching procedure to include loop corrections, as shown in Fig. 2. The new feature that arises here is the presence of UV divergences in both the fundamental and effective theories. However, the matching ensures that the results in the effective theory are finite, and this determines the counterterms that are required in the effective theory. The result is that the renormalized couplings in the effective theory are a power series in $1/M$ even at loop level.

**Exercise 1:** Carry out the one-loop matching of the four fermion vertex using a momentum space cutoff regulator in both the fundamental and the effective theory. If we write the effective Lagrangian as

$$\mathcal{L}_{\rm eff} = \bar{\psi} i\partial\!\!\!/ \psi + G(\bar{\psi}\psi)^2 + \mathcal{O}(1/M^4), \tag{2.9}$$

show that

$$G = \frac{y^2}{2M^2}\left[1 + \frac{c_1 y^2 \Lambda^2}{16\pi^2 M^2} + \frac{y^2}{16\pi^2}\left(c_2 \ln\frac{\Lambda}{M} + c_3\right)\right], \tag{2.10}$$

where $\Lambda$ is the momentum space cutoff and $c_{1,2,3}$ are numbers of order 1. Note that all the non-analytic behavior of the loop amplitudes cancels in the matching computation. Compute the fermion-fermion scattering amplitude at one loop in the effective theory and show that it is independent of $\Lambda$ at one-loop order.

What is the value of $\Lambda$ that is appropriate for a matching calculation? Renormalization theory tells us that the physics is insensitive to the value of the cutoff, and so it does not matter. To get complete cutoff sensitivity one should take the cutoff to infinity, in which case the matching gives a relation among renormalized couplings in the fundamental and effective theory.

Note that the couplings in the effective theory are determined by dimensional analysis in the scale $M$, provided that the dimensionless couplings $y$ and $\lambda$ in the fundamental theory are order 1. If the scale $M$ is much larger than the energy scale $E$ that is being probed, then the effects of the higher-order effects in the $1/M^2$ expansion are very small.

### *2.2 Relevant, Irrelevant, and Marginal Operators*

The features we have seen in the example above are very general. A general effective Lagrangian is defined by the particle (field) content and the symmetries. An effective Lagrangian in principle contains an infinite number of operators, but only a finite number of them are important at a given order in the expansion in $1/M^2$, where $M$ is the scale of new physics. It is therefore useful to classify the terms in the effective Lagrangian according to their dimension. An operator of dimension $d$ will have coefficient

$$\Delta\mathcal{L}_{\rm eff} \sim \frac{1}{M^{d-4}}\mathcal{O}_d. \tag{2.11}$$

Inserting this term as a first-order perturbation to a given amplitude, we obtain an expansion of the form

$$\mathcal{A} \sim \mathcal{A}_0 \left[ 1 + \frac{E^{d-4}}{M^{d-4}} + \cdots \right], \tag{2.12}$$

where $E$ is the kinematic scale of the physical process. We see that the effects of operators with dimension $d > 4$ decrease at low energies, and these are called 'irrelevant' operators. The effects of operators with $d = 4$ are independent of energy, and these operators are called 'marginal.' Finally, the effects of operators with $d < 4$ increase with energy, and these operators are called 'relevant.' Loop effects change the scaling behavior of couplings. For weakly coupled theories, the most important change is that dimensionless couplings run logarithmically. The effects of quantum corrections on scaling behavior can be much more dramatic in strongly-coupled conformal field theories. See the lectures by Ann Nelson at this school.

For irrelevant and marginal operators, it makes sense for their coefficient to be fixed by dimensional analysis at the scale $M$. But if a relevant operator has a coefficient fixed by dimensional analysis at the scale $M$, then its effects are not small below the scale $M$. In this case, the simple picture based on dimensional analysis cannot be correct.

### *2.3 Naturally Small Parameters and Spurions*

In the toy model considered above, there is a relevant operator that could be added to the effective Lagrangian, namely a fermion mass term

$$\Delta\mathcal{L} = m\bar{\psi}\psi. \tag{2.13}$$

However, this was forbidden by the chiral symmetry Eq. (2.2), and so it is natural to omit this term. As we now discuss, this symmetry also means that it is natural for the mass to be nonzero but very small, *i.e.* $m \ll M$.

Suppose that we modify the theory by adding the fermion mass term Eq. (2.13). Now the chiral symmetry Eq. (2.2) is broken, but the effects of this breaking come only from the parameter $m$. Therefore, if we compute loop corrections to the fermion mass, we find that they are proportional to the scale $m$ itself. For example, computing the effective fermion mass by matching at one loop gives

$$m_{\text{eff}} = m + \frac{y^2}{16\pi^2} cm \ln \frac{\Lambda}{M}, \tag{2.14}$$

where $c \sim 1$. The size of the loop correction is controlled by $m$ rather than $M$ because an insertion of $m$ is required to break the symmetry.

This can be formalized in the following way. We can say that the mass parameter $m$ transforms under the discrete symmetry Eq. (2.2) as

$$m \mapsto -m. \tag{2.15}$$

What this means is that if we view $m$ as a parameter, all expressions must depend on $m$ in such a way that the chiral symmetry including the transformation Eq. (2.15) is a good symmetry. We can think of $m$ as a field, and the numerical value of $m$ as a vacuum expectation value for the field. We say that $m$ is a 'spurion field.' This spurion analysis immediately tells us that quantities that are even under the chiral symmetry will depend only on even powers of $m$, while quantities that are odd will depend on odd powers of $m$. This kind of spurion analysis will be very useful when we consider SUSY breaking.

### *2.4 Fine Tuning in a Toy Model*

To illustrate the problem with relevant operators that are not forbidden by any symmetry, let us consider another example of a light real scalar field $\phi$ coupled to a heavy Dirac fermion $\Psi$:

$$\mathcal{L} = \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 + \bar{\Psi}i\partial\!\!\!/\Psi - M\bar{\Psi}\Psi + y\phi\bar{\Psi}\Psi. \tag{2.16}$$

Note that the mass operator $\phi^2$ has $d = 2$, and is therefore relevant. The mass of the scalar cannot be forbidden by any obvious symmetry, but we simply assume that $m^2$ is chosen to make the scalar lighter than the fermion. We can describe processes with energies $E \ll M$ using an effective theory where the heavy fermion has been integrated out. Carrying out the one-loop matching calculation, we find that the scalar mass in the effective theory has the form

$$m_{\text{eff}}^2 = m^2 + \frac{y^2}{16\pi^2}\left[c_1\Lambda^2 + c_2 m^2 \ln\frac{\Lambda}{\mu} + c_3 M^2 + \mathcal{O}(M^4/\Lambda^2)\right]. \tag{2.17}$$

where $\Lambda$ is the cutoff used. If we use dimensional regularization in $4 - \epsilon$ dimensions and minimal subtraction, then we obtain

$$m_{\text{eff}}^2 = m^2 + \frac{y^2}{16\pi^2}\left[\frac{c_2}{\epsilon}m^2 + c_3 M^2 + \mathcal{O}(\epsilon)\right]. \tag{2.18}$$

In either case, we can write this in terms of the renormalized mass $m^2(\mu = M)$:

$$m_{\text{eff}}^2(\mu = M) = m^2(\mu = M) + \frac{c_3 y^2}{16\pi^2}M^2. \tag{2.19}$$

In this expression, the cutoff dependence has disappeared, but the dependence on the (renormalized) mass $M$ remains. This shows that if we want to make the scalar light compared to the scale $M$, we must tune the renormalized couplings in the fundamental theory so that there is a cancellation between the terms on the right-hand side of Eq. (2.19). The accuracy of this fine tuning is of order $y^2 m^2/(16\pi^2 M^2)$. There is no obvious symmetry or principle that makes the scalar naturally light in this model.

### *2.5 Fine Tuning Versus Quadratic Divergences*

We see that we must fine-tune parameters whenever the low-energy effective Lagrangian contains a relevant operator that cannot be forbidden by symmetries. The naturalness problem for scalar mass parameters is often said to be a consequence of the fact that scalar mass parameters are quadratically divergent in the UV. We have emphasized above that the fine-tuning can be formulated in terms of renormalized quantities, and has nothing to do with the regulator used. (In particular, we have seen in the example above that fine tuning can be present in dimensional regularization, where there are no quadratic divergences.) The naturalness problem is simply the fact that relevant operators that are not forbidden by symmetries are generally sensitive to heavy physical thresholds in the theory.

Although fine-tuning can be formulated without reference to UV divergences, there is a close connection that is worth commenting on. We can view a regulator for UV divergences as a UV modification of the theory that makes it finite. The dependence on the cutoff $\Lambda$ can therefore be viewed as dependence on a new heavy threshold.

### *2.6 Fine Tuning Versus Small Parameters*

Not all small parameters are finely tuned. We have seen in the first toy model above that a small fermion mass is not fine tuned, because there is an additional symmetry that results when the mass goes to zero. This automatically ensures that the radiative corrections to the fermion mass are small. In this case, we say that the small parameter is 'protected by a symmetry.'

There is another general mechanism by which a parameter in an effective Lagrangian can be naturally small, and that is if two sectors of the theory completely decouple as the parameter is taken to zero. As an example, consider a theory of two real scalars with Lagrangian

$$\mathcal{L} = \tfrac{1}{2}(\partial\phi_1)^2 - \tfrac{1}{2}m_1^2\phi_1^2 + \tfrac{1}{2}(\partial\phi_2)^2 - \tfrac{1}{2}m_2^2\phi_1^2 - \frac{\lambda_1}{4!}\phi_1^4 - \frac{\lambda_2}{4!}\phi_2^4 - \frac{\kappa}{4}\phi_1^2\phi_2^2. \qquad (2.20)$$

We have forbidden terms that with odd powers of the scalar fields with discrete symmetries

$$\phi_1 \mapsto -\phi_1, \qquad \phi_2 \mapsto \phi_2, \tag{2.21}$$

and

$$\phi_1 \mapsto \phi_1, \qquad \phi_2 \mapsto -\phi_2. \tag{2.22}$$

This symmetry also forbids mixing terms of the form $\phi_1\phi_2$. If we take $\kappa \to 0$, the theory becomes the sum of two 'superselection sectors,' *i.e.* two theories that are not coupled to each other. It is therefore natural to take $\kappa \ll \lambda_1, \lambda_2$. It is also easy to see that any radiative correction to the coupling $\kappa$ is proportional to $\kappa$ itself. In this case, we say that the parameter $\kappa$ is small because of an 'approximate superselection rule.'

Approximate superselection rules explain why it is natural for the electromagnetic coupling to be weaker than the strong coupling. If we take the electromagnetic coupling to zero, the theory splits into superselection sectors, consisting of QCD and a free photon. Approximate superselection rules also explain why it is natural for gravity to be much weaker than the standard model gauge interactions.[1]

### *2.7 To Tune or Not to Tune?*

Is fine tuning really a problem? If we want to explain the effective couplings of the standard model in terms of a more fundamental underlying theory, then it is at least disturbing that the underlying couplings must be adjusted to fantastic accuracy in order to reproduce even the qualitative features of the low-energy theory. A fine tuned theory is like finding a pencil balancing on its tip: it is possible that it arises by accident, but one suspects that there is a stabilizing force.

Recently, the possibility that the standard model may be fine tuned has received renewed attention, motivated in part by the fact that there is another grave naturalness problem: the cosmological constant problem. In general relativity, there is an additional relevant operator that must be added to the Lagrangian, namely the unit operator:

$$\Delta\mathcal{L}_{\rm cc} = -\Lambda^4. \tag{2.23}$$

[1] Gravity couples to all forms of matter with universal strength, and therefore sets the ultimate limit on how decoupled two approximate superselection sectors can be.

This can be thought of as a constant vacuum energy, which is not observable in the absence of gravity. In the presence of gravity, Eq. (2.23) is covariantized to

$$\Delta\mathcal{L}_{\rm cc} = -\sqrt{\det(g_{\mu\nu})}\,\Lambda^4, \tag{2.24}$$

where $g_{\mu\nu}$ is the metric field. This coupling means that vacuum energy couples to gravity. The term Eq. (2.23) gives rise to the infamous cosmological constant term in Einstein's equations, which gives rise to a nonzero spacetime curvature at a length scale

$$L \sim \frac{M_{\rm P}}{\Lambda^2}. \tag{2.25}$$

In order to explain the present universe, $L$ must be at least of order the size of the present Hubble horizon, $L_{\rm Hubble} \sim 10^{32}$ cm $\sim 10^{-42}$ GeV$^{-1}$. This requires $\Lambda \lesssim 10^{-3}$ eV.

This is an enormous problem, because loops of particles with mass $M$ give rise to a correction to the vacuum energy of order

$$\Delta\mathcal{L} \sim \frac{1}{16\pi^2} M^4. \tag{2.26}$$

For $M \sim M_Z \sim 100$ GeV, this is too large by 54 orders of magnitude! No one has ever found a symmetry that can cancel this contribution to the required accuracy.

This problem has prompted some physicists to consider the possibility that there could be a kind of anthropic selection process at work in nature. The idea is that there are in some sense many universes with different values of the effective couplings, and we live in one of the few that are compatible with our existence. If the cosmological constant were much larger than $\Lambda \sim 10^{-3}$ eV, then structure could not form in the universe [1]. The fact that cosmological observations favor a value of the cosmological constant in this range has given added impetus to this line of thinking. Also, string theory appears to have a large number of possible ground states, as required for anthropic considerations to operate. For recent discussions, see *e.g.* Refs. [2].

There is another possibility to save naturalness as we know it, advocated in Ref. [3]. The point is that although we have tested the standard model to energies of order 100 GeV, we have only tested gravity to a much lower energy scale, or much longer distances. The shortest distances probed in present-day gravitational force experiments are presently of order 0.1 mm, corresponding to an energy scale of $10^{-3}$ eV. If we assume that new gravitational physics comes in at the 0.1 mm scale, the small value of the cosmological constant may be natural. Note that this approach also predicts that the cosmological constant should be nonzero and close to its experimental value. The difficulty with this approach is that it is not known how to modify Einstein gravity in a consistent way to cut off the contributions to the cosmological constant. However, given our ignorance of UV completions of gravity, we should perhaps keep an open mind. See Ref. [4] for a recent idea along these lines.

These are interesting ideas, and worth pursuing. But in these lectures I will assume that naturalness is a good guide to non-gravitational physics at least.

## 3 Model-building Boot Camp

We are now ready to start building effective field theory models. If we believe in the naturalness principle articulated in the previous section, then the models should be defined by specifying the particle content and the symmetries of the theory. Then we should write down all possible couplings consistent with the symmetries.

### *3.1 The Standard Model*

Let us apply these ideas to the standard model. The standard model is defined to be a theory with gauge group

$$SU(3)_C \times SU(2)_W \times U(1)_Y. \tag{3.1}$$

The fermions of the standard model can be written in terms of 2-component Weyl spinor fields as[2]

$$\begin{aligned} Q^i &\sim (\mathbf{3}, \mathbf{2})_{+\frac{1}{6}}, \\ (u^{\rm c})^i &\sim (\bar{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}}, \\ (d^{\rm c})^i &\sim (\bar{\mathbf{3}}, \mathbf{1})_{+\frac{1}{3}}, \\ L^i &\sim (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}, \\ (e^{\rm c})^i &\sim (\mathbf{1}, \mathbf{1})_{+1}, \end{aligned} \tag{3.2}$$

where $i = 1, 2, 3$ is a generation index. In addition, the model contains a single scalar multiplet

$$H \sim (1, \mathbf{2})_{+\frac{1}{2}}. \tag{3.3}$$

[2]We use the spinor conventions of Wess and Bagger [5], which have become conventional in the SUSY literature. See *e.g.* Ref. [6] for a pedagogical introduction.

According to the ideas above, we must now write the most general interactions allowed by the symmetries. The most important interactions are the marginal and relevant ones. The marginal interactions include kinetic terms for the Higgs field, the fermion fields, and the gauge fields:

$$\mathcal{L}_{\text{kinetic}} = (D^\mu H)^\dagger D_\mu H + Q_i^\dagger i\tilde{\sigma}^\mu D_\mu Q_i + \cdots - \tfrac{1}{4} B^{\mu\nu} B_{\mu\nu} + \cdots \tag{3.4}$$

Note that these include the gauge self interactions. Also marginal is the quartic interaction for the Higgs

$$\Delta\mathcal{L}_{\text{quartic}} = -\frac{\lambda}{4}(H^\dagger H)^2 \tag{3.5}$$

and Yukawa interactions:

$$\Delta\mathcal{L}_{\text{Yukawa}} = (y_u)_{ij} Q^i H (u^{\text{c}})^j + (y_d)_{ij} Q^i H^\dagger (d^{\text{c}})^j + (y_e)_{ij} L^i H^\dagger (e^{\text{c}})^j. \tag{3.6}$$

Note that the Yukawa interactions are the only interactions that break a $SU(3)^5$ global symmetry that would otherwise act on the generation indices of the fermion fields. This means that the Yukawa interactions can be naturally small without any fine tuning. This is reassuring, since it means that the small electron Yukawa coupling $y_e \sim 10^{-5}$ is perfectly natural.

Finally, the marginal interactions include 'vacuum angle' terms for each of the gauge groups:

$$\mathcal{L}_{\text{vacuum angle}} = \frac{g_1^2 \Theta_1}{16\pi^2} \tilde{B}^{\mu\nu} B_{\mu\nu} + \frac{g_2^2 \Theta_2}{8\pi^2} \operatorname{tr}(\tilde{W}^{\mu\nu} W_{\mu\nu}) + \frac{g_3^2 \Theta_3}{8\pi^2} \operatorname{tr}(\tilde{G}^{\mu\nu} G_{\mu\nu}), \tag{3.7}$$

where $\tilde{B}^{\mu\nu} = \frac{1}{4}\epsilon^{\mu\nu\rho\sigma} B_{\rho\sigma}$, *etc.* These terms break $CP$, and are therefore very interesting. These terms are total derivatives, *e.g.*

$$\tilde{B}^{\mu\nu} B_{\mu\nu} = \partial^\mu K_\mu, \qquad K^\mu = \tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma} A_\nu F_{\rho\sigma}. \tag{3.8}$$

This is enough to ensure that they do not give physical effects to all orders in perturbation theory. They can give non-perturbative effects with parametric dependence $\sim e^{1/g^2}$, but these are completely negligible for the $SU(2)_W \times U(1)_Y$ terms, since these gauge couplings are never strong. The strong vacuum angle gives rise to $CP$-violating non-perturbative effects in QCD, most importantly the electric dipole moment of the neutron. Experimental bounds on the neutron electric dipole moment require $\Theta_3 \lesssim 10^{10}$. Explaining this small number is the 'strong $CP$ problem.' There are a number of proposals to solve the strong $CP$ problem. For example, there may be a spontaneously broken Peccei-Quinn symmetry [7] leading to an axion [8], or there may be special flavor structure at high scales that ensures that the determinant of the quark masses is real [9].

There is one relevant interaction that is allowed, namely a mass term for the Higgs field:

$$\mathcal{L}_{\text{relevant}} = -m_H^2 H^\dagger H. \tag{3.9}$$

Note that mass terms for the fermions such as $Le^c$ are not gauge singlets, and therefore forbidden by gauge symmetry. The Higgs mass parameter cannot be forbidden by any obvious symmetry, and therefore must be fine tuned in order to be light compared to heavy thresholds such as the GUT scale. For example, in GUT models there are massive gauge bosons with masses of order $M_{\text{GUT}}$ that couple to the Higgs with strength $g$, where $g$ is the unified gauge coupling. These will contribute to the effective Higgs mass below the GUT scale

$$\Delta m_H^2 \sim \frac{g^2 M_{\text{GUT}}^2}{16\pi^2} \sim 10^{30}\ \text{GeV}^2 \tag{3.10}$$

for $M_{\text{GUT}} \sim 10^{16}$ GeV. In order to get a Higgs mass of order 100 GeV we must fine tune to one part in $10^{26}$!

We can turn this around and ask what is the largest mass threshold that is naturally compatible with the existence of a light Higgs boson. The top quark couples to the Higgs with coupling strength $y_t \sim 1$, and top quark loops give a quadratically divergent contribution to the Higgs mass. Assuming that this is cut off by a new threshold at the scale $M$, we find a contribution to the Higgs mass of order

$$\Delta m_H^2 \sim \frac{y_t^2 M^2}{16\pi^2}, \tag{3.11}$$

which is naturally small for $M \lesssim 1$ TeV. We get a similar estimate for $M$ from loops involving $SU(2) \times U(1)$ gauge bosons. So the standard model is natural as an effective field theory only if there is new physics at or below a TeV. This is the principal motivation for the Large Hadron Collider (LHC) at CERN, which will start operation in 2007-2008 with a center of mass energy of 14 TeV. It is expected that the LHC will discover the mechanism of electroweak symmetry breaking and the new physics that makes it natural.

### *3.2 The GIM Mechanism*

One very important feature of the standard model is that it violates flavor in just the right way. The quark mass matrices are proportional to the up-type and down-type

Yukawa couplings. Diagonalizing the quark mass matrices requires that we perform independent unitary transformations on the two components of the quark doublet $Q_i$. This gives rise to the CKM mixing matrix, which appears in the interactions of the mass eigenstate quarks with the $W^{\pm}$ ('charged currents'). Crucially, the interactions with the photon and the $Z$ ('neutral currents') are automatically diagonal in the mass basis. This naturally explains the phenomenology of flavor-changing decays observed in nature, including the 'GIM suppression' of flavor changing neutral current processes such as $K^0$–$\bar{K}^0$ mixing.

For our purposes, what is important is that this comes about because the quark Yukawa couplings are the only source of flavor violation in the standard model. If there were other couplings that violated quark flavor, these would not naturally be diagonal in the same basis that diagonalized the quark masses, and would in general lead to additional flavor violation. A simple example of this is a general model with 2 Higgs doublets, in which there are twice as many Yukawa coupling matrices.

### *3.3 Accidental Symmetries*

It is noteworthy that the standard model was completely defined by its particle content gauge symmetries. In particular, we did not have to impose any additional symmetries to suppress unwanted interactions. If we look back at the terms we wrote down, we see that all of the relevant and marginal interactions are actually invariant under some additional global symmetries. One of these is baryon number, a $U(1)$ symmetry with charges

$$B(Q) = \tfrac{1}{3}, \quad B(u^{\rm c}) = B(d^{\rm c}) = -\tfrac{1}{3}, \quad B(L) = B(e^{\rm c}) = B(H) = 0. \tag{3.12}$$

Another symmetry is lepton number, another $U(1)$ symmetry with charges

$$L(Q) = L(u^{\rm c}) = L(d^{\rm c}) = 0, \quad L(L) = +1, \quad L(e^{\rm c}) = -1, \quad L(H) = 0. \tag{3.13}$$

These symmetries can be broken by higher-dimension operators. For example, the lowest-dimension operators that violate baryon number are dimension 6:

$$\Delta\mathcal{L} \sim \frac{1}{M^2} QQQL + \frac{1}{M^2} u^{\rm c} u^{\rm c} d^{\rm c} e^{\rm c}, \tag{3.14}$$

where the color indices are contracted using the $SU(3)_C$ invariant antisymmetric tensor. Consistency with the experimental limit on the proton lifetime of $10^{33}$ yr gives a bound $M \gtrsim 10^{22}$ GeV. Although this is larger than the Planck mass, these couplings also violate flavor symmetries, and it seems reasonable that whatever explains the small values of the light Yukawa couplings can suppresses these operators.

A very appealing consequence of this is that if the standard model is valid up to a high scale $M$, then the proton is automatically long-lived, without having to assume that baryon number is an exact or approximate symmetry of the fundamental theory. Baryon number emerges as an 'accidental symmetry' in the sense that the other symmetries of the model (in this case gauge symmetries) do not allow any relevant or marginal interactions that violate the symmetry.

### *3.4 Neutrino Masses*

Lepton number can be violated by the dimension 5 operator

$$\Delta\mathcal{L} \sim \frac{1}{M}(LH)(LH). \tag{3.15}$$

When the Higgs gets a VEV, these gives rise to Majorana masses for the neutrinos of order

$$m_\nu \sim \frac{v^2}{M}. \tag{3.16}$$

In order to get neutrino masses in the interesting range $m_\nu \sim 10^{-2}$ eV for solar and atmospheric neutrino mixing, we require $M \sim 10^{15}$ GeV, remarkably close to the GUT scale. The interaction Eq. (3.15) also has a nontrivial flavor structure, so the actual scale of new physics depends on the nature of flavor violation in the fundamental theory, like the baryon number violating interactions considered above.

The experimental discovery of neutrino masses has been heralded as the discovery of physics beyond the standard model, but it can also be viewed as a triumph of the standard model. The standard model *predicts* that neutrino masses (if present) are naturally small, since they can only arise from an irrelevant operator. We can view the discovery of neutrino masses as evidence for the existence of a new scale in physics. This is analogous to the discovery of weak $\beta$ decay, which can be described by an effective 4-fermion interaction with coupling strength $G_\mathrm{F} \sim 1/(100\ \mathrm{GeV})^2$. (Therefore, Fermi was doing effective quantum field theory in the 1930's!)

### *3.5 Extending the Standard Model*

The steps in constructing an extension of the standard model are the same ones we followed in constructing the standard model above. The model should be defined by its particle content and symmetries. We then write down all couplings allowed by these principles. The goal is to find an extension of the standard model that cures the naturalness problem, but preserves the successes of the standard model described above.

# 4 The Minimal Supersymmetric Standard Model

We now apply the ideas of the previous section to constructing a supersymmetric extension of the standard model. The motivation for this is that supersymmetry can naturally explain why a scalar is light. This because unbroken SUSY fixes scalar and fermion masses to be the same. Since fermion masses can be protected by chiral symmetries, the same chiral symmetries will also protect the masses of the scalar superpartners.

## *4.1 Superfields and Couplings*

To construct a supersymmetric extension of the standard model, we simply embed all fermions of the standard model into chiral superfields, and all gauge fields into vector superfields. The chiral superfields are therefore

$$
\begin{aligned}
Q^i &\sim (\mathbf{3}, \mathbf{2})_{+\frac{1}{6}}, \\
(U^c)^i &\sim (\bar{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}}, \\
(D^c)^i &\sim (\bar{\mathbf{3}}, \mathbf{1})_{+\frac{1}{3}}, \\
L^i &\sim (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}, \\
(E^c)^i &\sim (\mathbf{1}, \mathbf{1})_{+1},
\end{aligned}
\tag{4.1}
$$

where $i = 1, 2, 3$ is a generation index. The Higgs scalar fields are also in chiral superfields. If there is a single Higgs multiplet, the fermionic partners of the Higgs scalars will give rise to gauge anomalies. The minimual model is therefore one with two Higgs chiral superfields with conjugate quantum numbers

$$
\begin{aligned}
H_u &\sim (\mathbf{1}, \mathbf{2})_{+\frac{1}{2}}, \\
H_d &\sim (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}.
\end{aligned}
\tag{4.2}
$$

The next step is to write the most general allowed couplings between these fields. Let us begin with the relevant interactions. These are

$$
\Delta\mathcal{L}_{\text{relevant}} = \int d^2\theta \Big[\mu H_u H_d + \kappa_i L^i H_u\Big] + \text{h.c.} \tag{4.3}
$$

where $\mu$ and $\kappa_i$ have dimensions of mass. Right away, we have some explaining to do. We see that SUSY allows us to write a supersymmetric mass for the Higgs, as well as a term that mixes the Higgs with the lepton doublets. (Note that $L$ and $H_d$ have the same gauge quantum numbers, so the distinction between them is only a naming convention up to now.) The terms $\kappa_i$ can be forbidden by lepton number symmetry, defined by

$$\begin{gathered} L(Q) = L(U^c) = L(D^c) = 0, \quad L(L) = +1, \quad L(E^c) = -1, \\ L(H_u) = L(H_d) = 0. \end{gathered} \tag{4.4}$$

The '$\mu$ term' can be forbidden by a $U(1)$ 'Peccei-Quinn'[3] symmetry with charges

$$\begin{gathered} P(H_u) = P(H_d) = +1, \\ P(Q) = P(U^c) = P(D^c) = P(L) = P(E^c) = -\tfrac{1}{2}. \end{gathered} \tag{4.5}$$

There are many other symmetries that we could invent to control these terms. The motivation for the particular symmetries given here is that they are not violated by Yukawa interactions (see below). The important point is that the relevant terms in Eq. (4.3) can be naturally zero or small due to additional symmetries.

The marginal interactions include kinetic terms for all the gauge and chiral superfields, which we write schematically as

$$\mathcal{L}_{\text{kinetic}} \sim \int d^4\theta \left[ Q_i^\dagger e^V Q^i + \cdots \right] + \left( \int d^2\theta \, W^\alpha W_\alpha + \cdots + \text{h.c.} \right). \tag{4.6}$$

Note that the kinetic terms have been chosen to be diagonal in the flavor indices. It also includes the Yukawa couplings

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}} = \int d^2\theta \Big[ & (y_U)_{ij} Q^i i H_u (U^c)^j + (y_D)_{ij} Q^i H_d (D^c)^j \\ & + (y_E)_{ij} L^i H_d (E^c)^j \Big] + \text{h.c.} \end{aligned} \tag{4.7}$$

Note that the Yukawa couplings are invariant under both the lepton number symmetry Eq. (4.4) and the Peccei-Quinn symmetry Eq. (4.5). There are also additional Yukawa-like interactions

$$\begin{aligned} \mathcal{L}_{\text{dangerous}} = \int d^2\theta \Big[ & (\lambda_{LQD})_{ijk} L^i Q^j (D^c)^k + (\lambda_{LLE})_{ijk} L^i L^j (E^c)^k \\ & + (\lambda_{UDD})_{ijk} (U^c)^i (D^c)^j (D^c)^k \Big] + \text{h.c.} \end{aligned} \tag{4.8}$$

Once again, we have some explaining to do. These couplings violate lepton and baryon number symmetries, and therefore give rise to proton decay and other processes that

[3] A similar symmetry plays a role in the solution of the strong $CP$ problem by axions, as first discussed by Peccei and Quinn.

are not observed unless these couplings are small. The couplings $\lambda_{LQD}$ and $\lambda_{LLE}$ violate lepton number, and $\lambda_{LQD}$ and $\lambda_{UDD}$ violate baryon number, defined by

$$\begin{gathered} B(Q) = +\tfrac{1}{3}, \quad B(U^{\mathrm{c}}) = B(D^{\mathrm{c}}) = -\tfrac{1}{3}, \\ B(L) = B(E^{\mathrm{c}}) = B(H_u) = B(H_d) = 0. \end{gathered} \tag{4.9}$$

Imposing these symmetries therefore suppresses these terms.

### *4.2 R Parity*

Another type of possible symmetry in SUSY theories acts differently on different components of the same supermultiplet. For example, we can define a $U(1)_R$ symmetry acting on chiral and vector superfields as

$$\Phi(\theta) \mapsto e^{iR_\Phi \alpha}\Phi(\theta e^{-i\alpha}), \qquad V(\theta) \mapsto V(\theta e^{-i\alpha}), \tag{4.10}$$

where $R_\Phi$ is the '$R$ charge' of the chiral superfield $\Phi$. From this definition, we see that the $R$ charges of the scalar and fermion fields in $\Phi$ are

$$R(\phi) = R_\Phi, \qquad R(\psi) = R_\Phi - 1, \tag{4.11}$$

while the $R$ charge of a gaugino field is $+1$. In order for a supersymmetric Lagrangian

$$\mathcal{L} = \int d^4\theta \, K + \left( \int d^2\theta \, W + \text{h.c.} \right) \tag{4.12}$$

to be invariant, we require $R(K) = 0$ and $R(W) = +2$.

Note that if we define a $U(1)_R$ transformation in the MSSM where all chiral superfields have $R = \frac{1}{3}$, this is automatically preserved by all renormalizable couplings except the $\mu$ term.

Another $R$ symmetry in the MSSM is a discrete symmetry called '$R$ parity.' It can be defined by

$$\Phi(\theta) \mapsto \pm\Phi(-\theta), \tag{4.13}$$

where the sign is $-1$ for $Q$, $U^{\mathrm{c}}$, $D^{\mathrm{c}}$, $L$, and $E^{\mathrm{c}}$, and $+1$ for $H_u$, $H_d$. The idea is that the observed matter fermions have $R$ parity $+1$, while their scalar partners have $R$ parity $-1$. Note that the gauginos also have $R$ parity $-1$, so all superpartners have odd $R$ parity.

$R$ parity ensures that superpartners are produced in pairs, and that the lightest $R$ parity odd particle is absolutely stable. $R$ parity is sufficient to forbid all of the dangerous relevant and marginal interactions in Eqs. (4.3) and (4.8). (In fact, the couplings in Eq. (4.8) are often called '$R$ parity violating operators.') Conversely, if these operators are forbidden by another symmetry, such as $B$ and $L$ conservation, then $R$ parity emerges as an accidental symmetry. Unbroken $R$ parity is often taken as part of the definition of the MSSM, but it is worth keeping in mind that $R$ parity may only be an accidental and/or approximate symmetry.

Note that unlike the standard model, the MSSM requires that we impose certain exact or approximate global symmetries *in addition* to the gauge symmetries and particle content. In this sense, we have given up the attractive automatic explanation of the suppression of baryon and lepton number violation in the standard model.

## 5 Soft SUSY Breaking

We now begin our discussion of supersymmetry breaking. It is obvious that the world is not exactly supersymmetric, since SUSY predicts the existence of superpartners with the same mass and quantum numbers as existing particles. Once SUSY is broken the masses of the superpartners can be different from the observed particles, and must be larger than 100 GeV or so to have avoided detection in accelerator experiments performed so far. As we have already seen above, new physics at or below TeV is required in any solution of the naturalness problem. In SUSY the new physics is superpartners, and therefore these must be at or below the TeV scale, and can be discovered at LHC. If superpartners are discovered, the most important question in particle physics will be to understand the pattern of SUSY breaking. It is no exaggeration to say that SUSY phenomenology is SUSY breaking phenomenology.

A simple way to break SUSY is to break it explicitly in the effective Lagrangian. If we do this, we would like to ensure that the breaking terms do not introduce power-law sensitivity to heavy thresholds (*i.e.* 'quadratic divergences'), which must be canceled by fine-tuning. SUSY breaking terms with this feature are called 'soft breaking terms.' This way of breaking SUSY may be *ad hoc*, but it does realize the goal of constructing a natural extension of the standard model. Also, we will see below that if SUSY is spontaneously broken at high scales, the effective theory below the SUSY breaking scale is a softly broken SUSY theory.

### *5.1 Coupling Constants as Superfields*

To discuss soft breaking we will use a tool that will be very useful to us throughout these lectures. This is the idea of coupling constants as superfields. Note that if a superfield $\Phi$ has a nonzero value $\langle\Phi\rangle$, it does not break SUSY as long as

$$Q_\alpha\langle\Phi\rangle = \bar{Q}_{\dot\alpha}\langle\Phi\rangle = 0, \tag{5.1}$$

where

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu \tag{5.2}$$

is the SUSY generator. In particular, a constant nonzero value of the lowest component of a superfield does not break SUSY. We can therefore view the coupling constants that appear in a SUSY theory as superfields with only their lowest components nonzero. For example, in the Wess-Zumino model

$$\mathcal{L} = \int d^4\theta\, Z\Phi^\dagger\Phi + \left[\int d^2\theta \left(\tfrac{1}{2}M\Phi^2 + \tfrac{1}{6}\lambda\Phi^3\right) + \text{h.c.}\right], \tag{5.3}$$

we can view $Z = Z^\dagger$ as a real superfield and $M$ and $\lambda$ as chiral superfields.

### *5.2 Superfield Couplings in Perturbation Theory*

This simple idea is very useful for understanding the structure of loop corrections. For example, in the model defined by Eq. (5.3) the 1PI effective action at one loop contains the divergent term

$$\Delta\Gamma_{\text{1PI}} = \int d^4\theta\, Z\left[1 + \frac{\hat\lambda^2}{26\pi^2}\ln\frac{\Lambda}{\mu} + \text{finite}\right], \tag{5.4}$$

where

$$\hat\lambda = \frac{\lambda}{Z^{3/2}} \tag{5.5}$$

is the physical Yukawa coupling. By direct calculation, we find that there are no further divergences at one loop. In particular there are no corrections to the superpotential, even though these are allowed by dimensional analysis. The absence of corrections to the superpotential holds to all orders in perturbation theory. We now show that this can be understood very easily if we view the couplings as superfields.

Note that a one-loop correction the superpotential in the 1PI effective action would have the form

$$\Delta\Gamma_{\text{1PI}} = \int d^2\theta\, \frac{c\lambda^3}{16\pi^2}\ln\frac{\Lambda}{\mu} + \text{h.c.} \tag{5.6}$$

Note that we treat $\lambda$ as a chiral superfield, and therefore $\lambda^\dagger$ cannot appear in the 1PI superpotential. That is, the superpotential must be a holomorphic function of the couplings as well as the chiral superfields. For this argument, it is important that the couplings can be treated as superfields even in the fully regulated theory. In this theory, this can be easily done by using a higher derivative regulator:

$$\mathcal{L} = \int d^4\theta \, Z\Phi^\dagger \left(1 + \frac{\Box}{\Lambda^2}\right) \Phi + \cdots, \tag{5.7}$$

where the cutoff $\Lambda$ is a real superfield. For example, the scalar propagator is modified

$$\frac{i}{p^2} \to \frac{i}{p^2 - p^4/\Lambda^2}. \tag{5.8}$$

This makes loops of $\hat{Q}$ fields UV convergent.[4] Because $\Lambda$ is a real superfield, it cannot appear in the superpotential, immediately ruling out divergent corrections like Eq. (5.6).

What about finite contributions? This holomorphy of the superpotential allows us to easily show that these are also absent to all orders in perturbation theory. We consider a $U(1) \times U(1)_R$ symmetry with charges given below:

| | $U(1)$ | $U(1)_R$ |
|---|---|---|
| $\Phi$ | $+1$ | $0$ |
| $M$ | $-2$ | $2$ |
| $\lambda$ | $-3$ | $2$ |

(5.9)

Note that we are treating the couplings as spurions that transform nontrivially under these symmetries. The most general 1PI superpotential is therefore a function of the neutral (and dimensionless) ratio $\lambda\Phi^3/M\Phi^2$:

$$\Delta\Gamma_{\rm 1PI} = \int d^2\theta \, M\Phi^2 f\left(\frac{\lambda\Phi}{M}\right) + \text{h.c.} \tag{5.10}$$

Expanding this in powers of $\lambda$ we obtain

$$\Delta\Gamma_{\rm 1PI} \sim \int d^2\theta \left[\frac{M^2}{\lambda}\Phi + M\Phi^2 + \lambda\Phi^3 + \frac{\lambda^2}{M}\Phi^4 + \cdots\right] + \text{h.c.} \tag{5.11}$$

Only the $\Phi^2$ and $\Phi^3$ terms can be present, since the higher order terms are singular in the limit $\lambda \to 0$ or $M \to 0$. The conclusion is that the superpotential is not corrected in this theory.

[4]This regulator also introduces a ghost, *i.e.* a state with wrong-sign kinetic term at $p^2 = \Lambda^2$. However, this decouples when we take the limit $\Lambda \to \infty$ and does not cause any difficulties.

We can use the non-renormalization of the superpotential to understand the structure of the renormalization group (RG) equations for this theory. Since there is only wavefunction renormalization, the physical couplings

$$\hat{\lambda} = \frac{\lambda}{Z^{3/2}}, \qquad \hat{M} = \frac{M}{Z}, \tag{5.12}$$

run only because of the running of $Z$. We therefore have (exactly)

$$\mu \frac{d\hat{\lambda}}{d\mu} = -\tfrac{3}{2}\gamma\lambda_{\text{phys}}, \qquad \mu \frac{d\hat{M}}{d\mu} = -\gamma \hat{M}. \tag{5.13}$$

Demanding that the one loop 1PI effective action Eq. (5.4) is independent of $\mu$, we obtain the anomalous dimension

$$\gamma = \mu \frac{d \ln Z}{d\mu} = -\frac{|\hat{\lambda}|^2}{16\pi^2} \tag{5.14}$$

which summarizes the renormalization of the theory.

### *5.3 Soft SUSY Breaking from Superfield Couplings*

We can include SUSY breaking terms in the Lagrangian by allowing the superfield couplings to have nonzero higher components. For example, the Wess-Zumino model above, we can write

$$\begin{aligned} Z &\to 1 + (\theta^2 B + \text{h.c.}) + \theta^2\bar{\theta}^2 C, \\ M &\to M + \theta^2 F_M, \\ \lambda &\to \lambda + \theta^2 F_\lambda. \end{aligned} \tag{5.15}$$

Working out the potential by integrating out the auxiliary fields, we find

$$V = V_{\text{SUSY}} + m^2 \phi^\dagger \phi + \left[ \tfrac{1}{2} A_M \phi^2 + \tfrac{1}{6} A_\lambda \phi^3 + \text{h.c.} \right], \tag{5.16}$$

where

$$V_{\text{SUSY}} = \left| M\phi + \tfrac{1}{2}\lambda\phi^2 \right|^2 \tag{5.17}$$

is the supersymmetric potential, and

$$\begin{aligned} m^2 &= -C + |B|^2 = -[\ln Z]_{\theta^2\bar{\theta}^2}, \\ A_M &= -2(F_M - BM) = -2[\hat{M}]_{\theta^2}, \\ A_\lambda &= -2(F_\lambda - \tfrac{3}{2}B\lambda) = -2[\hat{\lambda}]_{\theta^2}, \end{aligned} \tag{5.18}$$

where $\hat{M} = M/Z$ is the physical mass.

Now let us consider the divergence structure of this theory including the SUSY breaking terms. The analysis in the previous subsection showed that in the supersymmetric case, the only divergence is in the wavefunction renormalization, given to one loop by Eq. (5.4). When we turn on higher components of the superfield couplings, this divergent contribution is given by the same expression, but now it contains SUSY breaking from the superfield couplings. We see that the renormalization of SUSY breaking terms that can be written as higher components of superfield couplings is completely fixed by the renormalization of the couplings in the SUSY limit. We will explore the consequences of this in the following subsections.

Are there any additional divergences in the presence of SUSY breaking that are not present in the SUSY limit? These can arise from couplings that vanish identically in the SUSY limit. We can get such couplings by taking the total superspace integral of a chiral quantity:

$$\Delta\Gamma_{\rm 1PI} = \int d^4\theta \left[\alpha_1 \Phi + \tfrac{1}{2}\alpha_2 \Phi^2\right] + \text{h.c.} \tag{5.19}$$

The couplings $\alpha_1$ and $\alpha_2$ are renormalizable by power counting, but are total derivatives if $\alpha_1$ and $\alpha_2$ have only the lowest component nonvanishing (which is why we did not include them in the original Lagrangian). However, if $\alpha_1$ and $\alpha_2$ depend on superfield couplings with higher components, they can have nontrivial effects. For example,

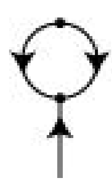
$$\int d^4\theta \, \alpha_1 \Phi = [\alpha_1]_{\theta^2\bar{\theta}^2} \phi + \cdots . \tag{5.20}$$

As above, we can understand the possible counterterms by considering the $U(1)$ and $U(1)_R$ symmetries defined in Eq. (5.9). Under these, the couplings $\alpha_1$ and $\alpha_2$ transform as

| | $U(1)$ | $U(1)_R$ |
|---|---|---|
| $\alpha_1$ | $-1$ | $0$ |
| $\alpha_2$ | $-2$ | $0$ |

(5.21)

From these symmetries, we can see that the diagram



allows a logarithmically divergent contribution

$$\alpha_1 \sim \frac{\lambda M^\dagger}{16\pi^2} \ln \frac{\Lambda}{\mu}. \tag{5.22}$$

Note that a counterterm of the form $\alpha_1$ is allowed only if the field $\Phi$ is a singlet. The following exercise illustrates the danger of this kind of divergence.

**Exercise 2:** Consider a model of two superfields $S$ and $X$ with Lagrangian

$$\begin{aligned} \mathcal{L} = &\int d^4\theta \left[ Z_S S^\dagger S + Z_X X^\dagger X \right] \\ &+ \int d^2\theta \left( \tfrac{1}{2}\lambda S X^2 + \tfrac{1}{2} M X^2 \right) + \text{h.c.} \end{aligned} \tag{5.23}$$

Show that in the SUSY limit $\langle X \rangle = 0$ while $\langle S \rangle$ is undetermined. The theory therefore has a space of vacua parameterized by $S$. Now break SUSY by turning on soft masses $m_S^2, m_X^2 \ll |M|^2$. Show that the loop corrections destabilize the vacuum at $\langle S \rangle = 0$ and force

$$\langle S \rangle \sim -\frac{M}{\lambda}. \tag{5.24}$$

Similar reasoning shows that

$$\alpha_2 \sim \frac{(\lambda M^\dagger)^2}{16\pi^2 M^\dagger M} = \text{finite} \tag{5.25}$$

because there cannot be a UV divergence that is singular in the limit $M \to 0$.

Another kind of term that can appear in the effective Lagrangian once SUSY is broken involves higher SUSY derivatives of the superfield couplings, such as

$$D^2\lambda = -4F_\lambda, \qquad \bar{D}^2 D^2 Z = 16C. \tag{5.26}$$

However, it is not hard to check that all such terms have positive mass dimension, and therefore cannot be UV divergent by simple power counting.

**Exercise 3:** Carry out an operator analysis to show that there are no additional divergent counterterms involving SUSY derivatives of the couplings in the Wess-Zumino model above. Note that $\bar{D}_{\dot\alpha}$ vanishes on chiral superfields, and that $\bar{D}^2 X$ is chiral for any superfield $X$.

We conclude that in the model Eq. (5.3) all SUSY breaking terms that can be parameterized by a nonzero higher component of coupling constant superfields are soft, in the sense that they do not lead to any quadratic divergences. Our argument has been rather abstract, and it is worth pointing out how the the absence of quadratic divergences comes about in explicit calculations. In component calculations, the quadratic divergences cancel between graphs involving loops of fermions and bosons. For example, the contributions to the counterterm for the scalar mass term in the model defined by Eq. (5.3) are given in Fig. 1. We obtain (after analytic continuation to Euclidean momenta)

$$\begin{aligned} \Delta m^2_{\text{boson loop}} &= +|\lambda|^2 \int \frac{d^4 k_E}{(2\pi)^4} \frac{1}{k_E^2 + (|M|^2 + m^2)}, \\ \Delta m^2_{\text{fermion loop}} &= -\tfrac{1}{2}|\lambda|^2 \int \frac{d^4 k_E}{(2\pi)^4} \frac{\operatorname{tr} \mathbf{1}}{k_E^2 + |M|^2}, \end{aligned} \tag{5.27}$$

where $\operatorname{tr} \mathbf{1} = 2$ is the trace over the Weyl fermion indices. We see that the quadratically divergent part cancels.

**Exercise 4:** Check the signs and combinatoric factors in Eq. (5.27).

The superfield analysis can be easily extended to include gauge fields. To fix our superfield conventions, we write a $U(1)$ gauge superfield as

$$V = V^\dagger = \cdots - 2\theta\sigma^\mu\bar{\theta}A_\mu + \cdots + \theta^2\bar{\theta}^2 D \tag{5.28}$$

and write the field strength as

$$W_\alpha = -\tfrac{1}{4}\bar{D}^2 D_\alpha V. \tag{5.29}$$

We then have

$$\int d^2\theta\, W^\alpha W_\alpha = -2F^{\mu\nu}F_{\mu\nu} + \cdots + 4D^2 \tag{5.30}$$

and

$$\int d^4\theta\, \Phi^\dagger e^V \Phi = (D^\mu\phi)^\dagger D_\mu\phi + \phi^\dagger D\phi + \cdots, \tag{5.31}$$

where $\phi$ is the scalar component of the chiral superfield $\Phi$, and

$$D_\mu = \partial_\mu - iA_\mu. \tag{5.32}$$

We therefore write the action of scalar QED as

$$\begin{aligned}\mathcal{L} = \int d^4\theta\, Z\Big[\Phi^\dagger e^V \Phi + \tilde{\Phi}^\dagger e^{-V} \tilde{\Phi}\Big] \\ + \int d^2\theta\, \frac{\tau}{8} W^\alpha W_\alpha + \text{h.c.},\end{aligned} \tag{5.33}$$

where

$$\tau = \frac{1}{2g^2} - \frac{i\Theta}{16\pi^2} - \theta^2 \frac{m_\lambda}{g^2} \tag{5.34}$$

is a chiral superfield that contains the gauge coupling $g$, vacuum angle $\Theta$, and gaugino mass $m_\lambda$. The gauge kinetic term is

$$\int d^2\theta\, \frac{\tau}{8} W^\alpha W_\alpha + \text{h.c.} = -\frac{1}{4g^2} F^{\mu\nu} F_{\mu\nu} + \cdots, \tag{5.35}$$

so the canonically normalized gauge field is

$$\hat{A}_\mu = g A_\mu. \tag{5.36}$$

A general renormalizable SUSY theory can be written in superfields as

$$\begin{aligned}\mathcal{L} = \int d^4\theta\, Q_a^\dagger \left(Z e^{V_A T_A}\right)^a{}_b Q^b \\ + \int d^2\theta\, \frac{\tau_A}{8}\, W_A^\alpha W_{\alpha A} + \text{h.c.} \\ + \int d^2\theta\, W(Q) + \text{h.c.},\end{aligned} \tag{5.37}$$

where $a, b, \ldots$ are field indices (including both gauge and flavor indices), $A, B, \ldots$ are gauge generator indices, and the superpotential $W$ is a cubic function of the fields.

$$W = \kappa_a Q^a + \tfrac{1}{2} M_{ab} Q^a Q^b + \tfrac{1}{6} \lambda_{abc} Q^a Q^b Q^c. \tag{5.38}$$

**Exercise 5:** Check that the $D$ term potential in the general theory above is given by (for $Z = 1$)

$$V_D = \sum_A \frac{g_A^2}{2} (Q^\dagger T_A Q)^2 \tag{5.39}$$

Using the same arguments as above, we can see that turning on nonzero higher components of these fields breaks SUSY softly, in the sense that there are no quadratic divergences in the theory. If there are singlets, then there may be logarithmic divergences of the type found in Eq. (5.20) that may destabilize the desired vacuum.

Is this the most general soft SUSY breaking? We can answer this question by again using higher components of superfield couplings to break SUSY. Any term that breaks SUSY can be written in this way. Consider for example the term

$$\Delta\mathcal{L} = \int d^4\theta\, \zeta D^\alpha \Phi i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu \bar{D}^{\dot\alpha}\Phi = 2i[\zeta]_{\theta^2\bar\theta^2}\psi^\alpha i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu\bar\psi^{\dot\alpha} + \cdots \tag{5.40}$$

changes the coefficient of the fermion kinetic term relative to the scalar kinetic term, and therefore gives rise to SUSY breaking perturbations in the physical couplings of the canonically normalized fermion relative to the scalar. The superfield coupling $\zeta$ has mass dimension $-2$, and at one loop we find divergent contributions of the form

$$\Delta\Gamma_{\rm 1PI} \sim \int d^4\theta\, \frac{\zeta|\lambda|^2}{16\pi^2}\Lambda^2\Phi^\dagger\Phi \sim \frac{|\lambda|^2[\zeta]_{\theta^2\bar\theta^2}}{16\pi^2}\Lambda^2\phi^\dagger\phi + \cdots. \tag{5.41}$$

We see that this gives rise to a quadratically divergent contribution to the scalar mass. In terms of diagrams, the couplings of the scalar and fermion in Eq. (5.27) are no longer equal, and the quadratic divergences no longer cancel.

A more subtle example is

$$\Delta\mathcal{L} = \int d^4\theta\, \tfrac{1}{2}C\Phi^\dagger\Phi^2 + \text{h.c.} = \tfrac{1}{2}[C]_{\theta^2\bar\theta^2}\phi^\dagger\phi^2 + \text{h.c.} + \cdots. \tag{5.42}$$

The superfield coupling $C$ has mass dimension $-1$, and at one loop we find divergent contributions of the form

$$\Delta\Gamma_{\rm 1PI} \sim \int d^4\theta\, \frac{C}{16\pi^2}\Lambda^2\Phi + \text{h.c.} \sim \frac{[C]_{\theta^2\bar\theta^2}}{16\pi^2}\Lambda^2\phi + \text{h.c.} + \cdots. \tag{5.43}$$

We see that this term is not soft in general. However, the quadratic divergence Eq. (5.43) is absent if there are no singlets in the theory, so that the tadpole is not allowed. For example, a term of the form

$$\Delta\mathcal{L} = \int d^4\theta\, \tfrac{1}{2}C\Phi_1^\dagger\Phi_2^2 + \text{h.c.} \tag{5.44}$$

is soft if there is a $U(1)$ symmetry with charges $\mathcal{Q}(\Phi_1) = 2\mathcal{Q}(\Phi_2)$. Although SUSY breaking terms of the form Eq. (5.42) are soft, they are usually neglected. We will see that they are not naturally generated in the more fundamental theories of SUSY breaking that we consider.

The result is therefore that the most general soft SUSY breaking terms are precisely those that can be written as higher components of superfield couplings in the Lagrangian, plus possible '$C$' terms. of the form Eq. (5.42).

### 5.4 Renormalization Group Equations for Soft SUSY Breaking

The fact that the divergences in this theory are controlled completely by the divergences in the SUSY limit gives rise to nontrivial relations between the renormalization of SUSY and SUSY breaking couplings. For example, the soft scalar mass is given by

$$m^2 = -[\ln Z]_{\theta^2\bar{\theta}^2}. \tag{5.45}$$

**Exercise 6:** Check that this formula is correct, even for the case where the lowest component of $Z$ is nonzero.

Therefore,

$$\mu \frac{dm^2}{d\mu} = -[\gamma]_{\theta^2\bar{\theta}^2} \tag{5.46}$$

$$= \frac{1}{16\pi^2}\left[\frac{|\lambda|^2}{Z^3}\right]_{\theta^2\bar{\theta}^2} + \cdots \tag{5.47}$$

$$= \frac{1}{16\pi^2}\left(|A_\lambda|^2 + 3m^2\right) + \cdots \tag{5.48}$$

where

$$A_\lambda = -\frac{F_\lambda + 3B\lambda}{Z^{3/2}}. \tag{5.49}$$

We see that the RG equations for the SUSY breaking parameters are completely determined in terms of the supersymmetric ones.

We can easily extend these results to gauge theories. The one loop renormalization group equation for a $U(1)$ gauge theory is

$$\mu \frac{d\tau}{d\mu} = -\frac{\mathrm{tr}\,\mathcal{Q}^2}{16\pi^2}, \tag{5.50}$$

where $Q$ is the $U(1)$ charge matrix of chiral superfields. For a non-abelian gauge theory with gauge group $SU(N)$ and $F$ fundamentals, the one-loop beta function is

$$\mu \frac{d\tau}{d\mu} = \frac{3N - F}{16\pi^2}, \tag{5.51}$$

Note that this immediately implies the RG equation for the gaugino mass

$$\mu \frac{d}{d\mu}\left(\frac{m_\lambda}{g^2}\right) = 0. \tag{5.52}$$

There is a subtlety in the gauge coupling superfield beyond one loop. It is not hard to see that the RG equation Eq. (5.51) for the gauge coupling superfield has no corrections to all orders in perturbation theory. This follows from the fact that $\tau$ is a chiral superfield, and therefore the beta function for $\tau$ must be a holomorphic function of $\tau$:

$$\mu \frac{d\tau}{d\mu} = \beta(\tau). \tag{5.53}$$

The RG equation for the real part of $\tau$ must be independent of the vacuum angle $\Theta$, which is a total derivative and therefore irrelevant in perturbation theory:

$$\frac{\partial \beta}{\partial \operatorname{Im} \tau} = 0. \tag{5.54}$$

But because $\beta$ is holomorphic, this implies that

$$\frac{\partial \beta}{\partial \tau} = 0, \tag{5.55}$$

*i.e.* $\beta$ is a constant. (It can be similarly shown that $\beta$ is independent of Yukawa couplings by considering $U(1)$ charges under which the Yukawa couplings are charged spurions.) We conclude that holomorphy implies that the one-loop RG equation Eq. (5.51) is in fact valid to all orders in perturbation theory.

This does not contradict the fact that the physical gauge coupling does run at two loops and beyond because the physical gauge coupling differs from the gauge coupling defined by $\operatorname{Re}(\tau)$ beyond one loop. The gauge coupling defined by the lowest component of $\tau$ is often called the holomorphic gauge coupling to distinguish it from the physical gauge coupling. The physical gauge coupling is the lowest component of a real superfield defined by

$$R = \tau + \tau^\dagger + \frac{N}{8\pi^2} \ln R - \frac{F}{8\pi^2} \ln Z + \mathcal{O}(1/R) \tag{5.56}$$

where the terms of order $1/R$ and higher are scheme dependent and

$$R = \frac{1}{g^2_{\text{phys}}} + \left( \theta^2 \frac{m_{\lambda,\text{phys}}}{g^2_{\text{phys}}} + \text{h.c.} \right) + \cdots. \tag{5.57}$$

Differentiating this expression, we obtain the famous expression for the beta function first written down in Ref. [11]

$$\mu \frac{d}{d\mu} \left( \frac{1}{g^2_{\text{phys}}} \right) = \frac{1}{8\pi^2} \frac{3N - F - F\gamma}{1 - \frac{N}{8\pi^2} g^2_{\text{phys}} + \mathcal{O}(g^4_{\text{phys}})}, \tag{5.58}$$

where

$$\gamma = \mu \frac{d \ln Z}{d\mu}. \tag{5.59}$$

For a complete discussion with many applications, see Ref. [10].

### *5.5 Soft SUSY Breaking in the MSSM*

We now apply these results to the MSSM. We assume $R$ parity (or equivalent symmetry) so that the '$R$ parity violating' terms in Eq. (4.8) and the $H_d$–$L$ mixing terms in Eq. (4.3) are absent. The most general soft terms are as follows. There are gaugino masses for the $SU(3)_C \times SU(2)_W \times U(1)_Y$ gauginos:

$$\Delta\mathcal{L}_{\text{gaugino}} = -M_1 \lambda_1 \lambda_1 - M_2 \lambda_2 \lambda_2 - M_3 \lambda_3 \lambda_3 + \text{h.c.} \tag{5.60}$$

These can be thought of as arising from the $\theta^2$ component of the gauge coupling superfield as in Eq. (5.34).[5] There are scalar masses for all scalars that can be thought of as arising from the $\theta^2\bar{\theta}^2$ component of the kinetic coefficients:

$$\begin{aligned}\Delta\mathcal{L}_{\text{scalar}} = -m_{Hu}^2 H_u^\dagger H_u - m_{Hd}^2 H_d^\dagger H_d \\ - (m_{\tilde{Q}}^2)^i{}_j \tilde{Q}_i^\dagger \tilde{Q}^j - (m_{\tilde{U}}^2)^i{}_j \tilde{U}_i^{\text{c}\dagger} \tilde{U}^{\text{c}j} + \cdots .\end{aligned} \tag{5.61}$$

Here we are using standard notation where the scalar components of matter fields (those with even $R$ parity) are denoted by a tilde, while the scalar components of the Higgs superfields are given the same name as the superfield itself. There are also '$A$' and '$B$' terms that can be thought of as arising from higher components of superfield couplings, or $\theta^2$ components of kinetic coefficients:

$$\Delta\mathcal{L}_B = -B\mu H_u H_d + \text{h.c.} \tag{5.62}$$

$$\Delta\mathcal{L}_A = -(A_U)_{ij} \tilde{Q}^i H_u \tilde{U}^{\text{c}j} + \cdots + \text{h.c.} \tag{5.63}$$

The names of these couplings have become traditional. Finally, there are cubic interactions arising from the '$C$' terms of the form Eq. (5.42):

$$\Delta\mathcal{L}_\beta = (C_U)_{ij} \tilde{Q}^i H_d^\dagger \tilde{U}^{\text{c}\dagger} + \cdots + \text{h.c.} \tag{5.64}$$

These are soft because they do not involve singlet fields. These are usually neglected, and we will see that they do not arise in any of the models of SUSY breaking that we consider. For an interesting possible application of these terms, see Ref. [12].

[5] Beyond one loop, the physical gaugino mass is defined by the $\theta^2$ component of the real gauge superfield Eq. (5.56). See Ref. [10].

What is the status of the MSSM with general soft breaking terms? First of all, we should note that there are enough terms allowed to give masses to all of the unobserved superpartners. This is obvious for the gauginos and squarks and sleptons. For the Higgs sector this requires some work. See *e.g.* the lectures by H. Haber at this school. This must be counted as a success.

On the other hand, there are an enormous number of parameters in the theory once SUSY is broken, about 100 even if we use our freedom to make field redefinitions to reduce the number of independent parameters.

### *5.6 The SUSY Flavor Problem*

To make matters worse, many of the soft SUSY breaking parameters have nontrivial flavor structure. This means that they will in general give an additional source of flavor mixing that is not diagonal in the basis where the quark masses are diagonal. This is the SUSY flavor problem.

Because the new flavor violation must be small, the scalar masses must be dominantly flavor-independent, *e.g.*

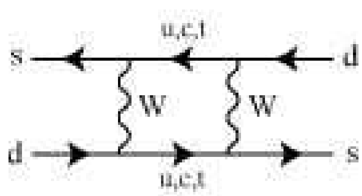
$$(m^2_{\tilde{Q}})^i{}_j = m^2_{\tilde{Q}}\delta^i{}_j + (\Delta m^2_{\tilde{Q}})^i{}_j, \tag{5.65}$$

with $\Delta m^2 \ll m^2$. Also, the $A$ terms must be dominantly proportional to the corresponding Yukawa couplings, *e.g.*

$$(A_U)_{ij} = a_U (y_U)_{ij} + (\Delta A_U)_{ij}, \tag{5.66}$$

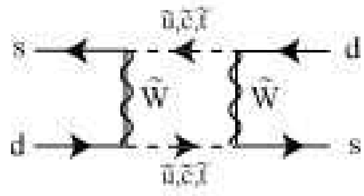
with $\Delta A \ll A$.

The flavor-violating parts of the scalar masses and $A$ terms are constrained to be small by observational constraints on flavor-changing neutral current processes. The most constraining process is $K^0$–$\bar{K}^0$ mixing. In the standard model, this arises from the famous box diagram




$$\sim \frac{g^4}{16\pi^2}(V_{ts}V_{td})^2\frac{m_t^2}{M_W^4}. \tag{5.67}$$

In the MSSM, there are additional diagrams from squark loops. We can treat the flavor-violating soft masses $\Delta m^2$ as insertions. This gives a new contribution from box diagrams involving superpartners, *e.g.*




$$\sim \frac{g^4}{16\pi^2} \frac{(\Delta m^2_{\tilde{s}\tilde{d}})^2}{m^6_{\tilde{Q}}}. \tag{5.68}$$

Because the standard model contribution does a good job in accounting for the observed rate, we must demand that the SUSY contribution is not larger. This gives the bound

$$\frac{\Delta m^2_{\tilde{s}\tilde{d}}}{m^2_{\tilde{Q}}} \lesssim m_{\tilde{Q}} V_{ts} V_{td} \frac{1}{M_W} \sim 10^{-3} \left( \frac{m_{\tilde{Q}}}{500 \text{ GeV}} \right). \tag{5.69}$$

We see that the squark masses have to be very nearly flavor diagonal in order to avoid flavor-changing processes that are much larger than what is observed. There are many similar constraints coming from various processes. See Ref. [13] for a comprehensive discussion.

Because flavor symmetry is broken in the Yukawa couplings, it cannot explain why the squark masses are nearly diagonal. This is the SUSY flavor problem.

### *5.7 The $\mu$ Problem*

In order to give the Higgsino a mass in the MSSM, we need a term of the form

$$\Delta \mathcal{L} = \int d^2\theta \, \mu H_u H_d + \text{h.c.} \tag{5.70}$$

If $\mu \gg 100$ GeV, the Higgs multiplet has a large supersymmetric mass and electroweak symmetry cannot be broken. If $\mu \ll 100$ GeV the Higgsino is lighter than $M_W^2/M_2$. The parameter $\mu$ must therefore be of order 100 GeV, that is, the same size as the other SUSY breaking parameters. Explaining why the $\mu$ term is the same size as the soft SUSY breaking parameters is the so-called '$\mu$ problem.' Since $\mu$ contributes to the mass term of the Higgs scalars, we cannot explain the weak scale without explaining the origin of the $\mu$ term.

It is not easy to understand why $\mu$ is the same size as the soft SUSY breaking parameters discussed above because it breaks a different set of symmetries. For example, it does not break SUSY, but it does break the Peccei-Quinn symmetry defined in Eq. (4.5).

## 5.8 The SUSY CP Problem

Since the MSSM with SUSY breaking has many new parameters, it is not surprising that there are new $CP$ violating phases in the soft SUSY breaking parameters that cannot be rotated away.

For example, there is a contribution to the strong CP phase from the phase of the gluino mass:

$$\Theta_{\rm QCD} = \Theta_3 - \arg\det(m_u) + \arg\det(m_d) - 3\arg(M_3). \tag{5.71}$$

Here $\Theta_3$ is the coefficient of $\mathrm{tr}(F^{\mu\nu}\tilde{F}_{\mu\nu})$ in the QCD Lagrangian, and the other terms are the phases in the masses of strongly coupled fermions. Only the combination $\Theta_{\rm QCD}$ is physically observable. Bounds on the neutron electric dipole moment require

$$\Theta_{\rm QCD} \lesssim 10^{-10}. \tag{5.72}$$

Explaining this small number is the 'strong CP problem.'

The strong CP problem must be solved somehow, whether or not nature is supersymmetric. For example, there may be a spontaneously broken Peccei-Quinn symmetry [7] leading to an axion [8], or there may be special flavor structure at high scales that ensures that the determinant of the quark masses is real [9]. These mechanisms work just as well with or without SUSY, although in the latter case, we must independently insure that the gluino mass does not have a phase that upsets the mechanism. There is also an interesting proposal for solving the strong CP problem that works only in SUSY models [15].

Phases in the squark masses can also give rise to electric dipole moments for quarks and leptons. Demanding that the quark eletric dipole moments do not give rise to a too-large neutron electric dipole gives the constraint

$$\frac{\mathrm{Im}\,\Delta m^2_{\tilde{Q}}}{m^2_{\tilde{Q}}} \lesssim 0.1 \left(\frac{m_{\tilde{Q}}}{500\ \mathrm{GeV}}\right)^2. \tag{5.73}$$

Even if the SUSY breaking violates CP, this can be satisfied if the phases are accidentally somewhat smaller than unity. There are several constraints that are similarly strong. See Ref. [13] for a review.

The 'SUSY CP problem' is the problem of explaining why these phases are small. It does not involve very large or small numbers, and is therefore not clearly a serious problem.

In the MSSM the Higgs potential is constrained by softly broken SUSY to have the form

$$\begin{aligned} V_{\text{Higgs}} &= (m_{Hu}^2 + |\mu|^2)|H_u|^2(m_{Hd}^2 + |\mu|^2)|H_d|^2 - B\mu(H_u H_d + \text{h.c.}) \\ &\quad + \tfrac{1}{8}(g_1^2 + g_2^2)(|H_u|^2 - |H_d|^2)^2 + \tfrac{1}{2}g_2^2|H_u^\dagger H_d|^2. \end{aligned} \tag{5.74}$$

In particular, the quartic terms are completely determined, since they arise entirely from the $D$ term potential from the $SU(2)_W \times U(1)_Y$ gauge multiplet.

Even though the quadratic terms are free parameters, we obtain an upper bound on the physical Higgs masses. The basic reason is that the Higgs potential has the form

$$V_{\text{Higgs}} \sim m_H^2 H^2 + g^2 H^4, \tag{5.75}$$

so the Higgs VEV is determined to be

$$v \sim \langle H \rangle \sim \frac{m_H}{g}. \tag{5.76}$$

This implies that we need $m_H \sim gv \sim M_Z$, and so the physical Higgs mass is also of order $M_Z$. This can be viewed as a consequence of eliminating the mass term $m_H$ in favor of the VEV. We might imagine that we can get larger physical higgs masses because there are several independent quadratic terms, but famously this is not the case for the neutral $CP$-even Higgs boson $h^0$. Computing the physical mass of $h^0$ from Eq. (5.74) we obtain a bound[6]

$$m_{h^0} \le M_Z |\cos 2\beta|. \tag{5.77}$$

Here

$$\tan\beta = \frac{v_u}{v_d}, \tag{5.78}$$

where

$$\langle H_u \rangle = \begin{pmatrix} v_u \\ 0 \end{pmatrix}, \qquad \langle H_d \rangle = \begin{pmatrix} 0 \\ v_d \end{pmatrix}, \tag{5.79}$$

with $v = \sqrt{v_u^2 + v_d^2} = 174$ GeV. A Higgs lighter than $M_Z$ was ruled out by LEP I, and the current limit from LEP II is $m_{h^0} \ge 114$ GeV. (It is ironic that in the standard model, the Higgs mass is in a sense too large, while in SUSY it is too small!)

[6] This assumes that the CP-odd neutral scalar $A^0$ is heavier than $h^0$. If this is not the case, then the bound is even stronger.

Because this bound holds independently of the Higgs quadratic terms, we need additional contributions to the Higgs quartic couplings in order to raise the mass of the $h^0$. In the MSSM, these corrections can come only from loop effects. We can compute the loop contribution to the Higgs potential systematically by integrating out the particles heavier than the $h^0$. Since the heavy particles have masses and couplings that break SUSY, the effective field theory below their masses is non-supersymmetric. It therefore contains non-supersymmetric quartic couplings such as

$$\Delta V_{\text{Higgs}} = \tfrac{1}{4}\Delta\lambda_u (H_u^\dagger H_u)^2 + \tfrac{1}{4}\Delta\lambda_d (H_d^\dagger H_d)^2 + \cdots. \tag{5.80}$$

(Another way to say this is that SUSY predicts relations among the most general allowed quartic couplings, which are violated at loop level.)

The largest loop contribution comes from the particles with the largest couplings to the Higgs, which are the gauge particles and the top and stop quarks.[7] Numerically, the top contribution dominates and gives

$$\Delta\lambda_u = +\frac{3y_t^4}{4\pi^2} \ln \frac{m_{\tilde{t}1} m_{\tilde{t}2}}{m_t^2}. \tag{5.81}$$

The log can be thought of as the result of RG running between the stop and the top mass scale. The precise argument of the logarithm is chosen to include the finite 1-loop matching corrections. The large size of the coefficient can be understood from the fact that it is enchanced by a color factor. This contributes to the physical mass of the $h^0$

$$\Delta m_{h^0}^2 = \frac{3y_t^4}{4\pi^2} v_u^2 \sin^4\beta \ln \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}. \tag{5.82}$$

For numerical estimates, an important correction to the value of $y_t$ comes from the QCD corrections to the physical top quark mass:

$$m_t = y_t v_u \left[ 1 + \frac{g_3^2}{3\pi^2} + \cdots \right]. \tag{5.83}$$

Again, the large coefficient of the 1-loop correction can be understood as color enhancement. For a top quark mass of 175 GeV, we have $y_t v_u \simeq 165$ GeV. The correction Eq. (5.81) is often written with the substitution $y_t \to m_t/v_u$, but it is the Yukawa coupling and not the mass that enters directly into the diagram.

The loop contribution to the Higgs quartic coupling Eq. (5.81) grows logaritmically with the stop mass, so we can try to get a large Higgs mass by increasing the stop

[7] If $\tan\beta \gtrsim 50$, then $y_b \sim 1$ in order to explain the observed bottom quark mass. In this case, the sbottom can also give a significant contribution.

mass. However, there is a heavy price to pay for this: the Higgs mass parameter $m_{Hu}^2$ also gets loop contributions that grow *quadratically* with the stop mass. In fact, this contribution is also logarithmically divergent, and therefore has a logarithmic sensitivity to the scale $\Lambda_{\text{SUSY}}$. These logs can be summed using the RG equations

$$\mu \frac{dm_{Hu}^2}{d\mu} = -\frac{3y_t^2}{8\pi^2}(m_{\tilde{Q}3}^2 + m_{\tilde{T}^c}^2) + \cdots. \tag{5.84}$$

If the logarithm is not large, we have

$$\Delta m_{Hu}^2 \simeq -\frac{3y_t^2}{4\pi^2} m_{\tilde{t}}^2 \ln \frac{\Lambda}{m_{\tilde{t}}} \tag{5.85}$$

where we have assumed a common stop mass for simplicity.

It should not be surprising that the Higgs mass is quadratically sensitive to SUSY breaking mass scales. This is just a particular manifestation of the fine-tuning problem for light scalars. We argued on very general grounds that naturalness of a light Higgs requires new physics below a TeV. Now that we have a specific model with a specific type of new physics, we can put in the numbers and make this more precise. To satisfy the current experimental bounds on the Higgs mass, we require

$$\Delta m_{h^0}^2 \geq (114\ \text{GeV})^2 - M_Z^2 = (69\ \text{GeV})^2. \tag{5.86}$$

We are assuming $\tan\beta$, which enhances the Higgs mass. We therefore require the stop mass to be

$$m_{\tilde{t}} \gtrsim m_t \exp\left\{\frac{2\pi^2 \Delta m_{h^0}^2}{3y_t^4 v^2}\right\} \simeq 620\ \text{GeV}. \tag{5.87}$$

To quantify how much fine tuning is involved, we note that the general scaling of Eq. (**??**) tells us that the natural size of the Higgs mass parameter is of order $M_Z$. An approximate measure of the fine tuning in the Lagrangian is therefore

$$\text{tuning} = \frac{\Delta m_{Hu}^2}{M_Z^2} \simeq \frac{3y_t^2 m_{\tilde{t}}^2}{4\pi^2 M_Z^2} \ln \frac{\Lambda}{m_{\tilde{t}}}. \tag{5.88}$$

This is approximately 16 for $m_{\tilde{t}} \simeq 620$ GeV and $\Lambda \simeq 100$ TeV. This means that the positive contribution to the quadratic Higgs mass terms must cancel the large negative contribution from the loop correction to an accuracy of at least 1 part in 16 to get acceptable electroweak symmetry breaking (see Eq. (5.74)). The full 1-loop corrections and the largest two loop corrections to the quartic of order $y_t^6$ and $y_t^4 g_3^2$ increase the required stop mass above 1 TeV for small $A_t$, so this analysis actually underestimates the amound of fine tuning.

This is not very much fine tuning if the parameters are at their current experimental limits, but note that the amount of fine tuning grows exponentially with the experimental bound on the Higgs mass (see Eq. (5.87)). This sensitivity also means that the precise amount of fine tuning is sensitive to other corrections. At present, we cannot say that there is anything clearly wrong with the MSSM, but this may be taken as a hint that the MSSM is not a completely natural solution to the fine-tuning problem of the standard model.

### *5.10 The Next-to-Minimal Supersymmetric Standard Model*

We can take the SUSY fine tuning problem as a motivation to go beyond the MSSM. A simple and well-motivated extension of the MSSM is obtained by adding a singlet chiral superfield $S$ to the theory.

This is relevant for the SUSY fine tuning problem because a superpotential term

$$\Delta W = \lambda S H_u H_d \tag{5.89}$$

gives a new contribution to the potential that is quartic in the Higgs fields:

$$\Delta V = |F_S|^2 = |\lambda|^2 |H_u H_d|^2. \tag{5.90}$$

This can raise the mass of the lightest neutral Higgs scalar at tree level. Another motivation for this model is that we can naturally obtain nonzero values for $\langle S \rangle$ and $\langle F_S \rangle$ of order the other SUSY breaking parameters. This gives weak scale $\mu$ and $B\mu$ terms, solving the $\mu$ problem.

The superpotential Eq. (5.89) preserves a Peccei-Quinn symmetry with $P(S) = -2$ (see Eq. (4.5)). If this is broken spontaneously, it gives rise to a weak-scale axion, which is ruled out. This problem is easily solved by adding the superpotential

$$\Delta W_{\text{NMSSM}} = \lambda S H_u H_d + \tfrac{1}{6} k S^3 \tag{5.91}$$

to the MSSM. This model is called the 'next-to-minimal supersymmetric standard model,' or NMSSM.

In principle, the lightest Higgs mass can be arbitrarily large by choosing the couplings $\lambda$ and $k$ large, but there is a constraint if we impose the condition that the coupling $\lambda$ remains perturbative up to the GUT scale. This is because the RG evolution

$$\mu \frac{d\lambda}{d\mu} = +\frac{\lambda^3}{8\pi^2} + \cdots \tag{5.92}$$

drives $\lambda$ larger in the UV, and $\lambda$ will diverge at a finite scale if its value at the weak scale is too large. This gives a bound of approximately 150 GeV on the lightest Higgs mass in the NMSSM if we require that the theory be perturbative up to the GUT scale $M_{\mathrm{GUT}} \simeq 2 \times 10^{16}$ GeV, as suggested by the success of gauge coupling unification. (Even in the standard model, requiring perturbativity of the quartic Higgs coupling up to the GUT scale gives an upper bound of 170 GeV on the Higgs mass.) Extending the model further does not relax this bound.

There are several ways to avoid these bounds. One possibility is that the coupling $\lambda$ indeed becomes strongly coupled, so that some or all of $S$, $H_u$, or $H_d$ are composite above the strong interaction scale. This need not interfere with perturbative gauge coupling unification, since the gauge couplings naturally remain weak going through a strong threshold. (For example, the electromagnetic coupling gets only a small renormalization going through the QCD threshold.) For examples of this kind of model, see Refs. [16].

The NMSSM phenomenology differs from the MSSM phenomenology mainly in that scalar component of $S$ mixes with the neutral Higgs, and can therefore change the signals of the lightest scalar. Also, the new singlet fermion mixes with the neutralinos.

## 6 Spontaneous SUSY Breaking

There are good reasons to be dissatisfied with the softly broken MSSM. It has $\mathcal{O}(100)$ unexplained parameters, and correspondingly no understanding of flavor conservation, one of the great successes of the standard model. To address this question, it is natural to consider models in which SUSY is broken spontaneously, with the hope that the many SUSY breaking parameters will be naturally explained in terms of a simpler underlying theory. That is, we look for models with a SUSY invariant Lagrangian in which the ground state breaks SUSY.

A famous condition for SUSY breaking comes from the fact that the Hamiltonian can be written in terms of the supercharges as

$$H = \tfrac{1}{4} \left[ \bar{Q}_1 Q_1 + Q_1 \bar{Q}_1 + \bar{Q}_2 Q_2 + Q_2 \bar{Q}_2 \right] \geq 0. \tag{6.1}$$

This shows that the energy of any state is positive or zero, and also that a state is SUSY invariant

$$Q_\alpha |\Omega\rangle = 0 \tag{6.2}$$

if and only if its energy is exactly zero

$$H|\Omega\rangle = 0. \tag{6.3}$$

The vacuum energy is therefore an order parameter for SUSY breaking. A VEV for a superfield $S$ can also act as an order parameter for breaking SUSY. SUSY is broken if

$$Q_\alpha \langle S \rangle = 0 \qquad \text{or} \bar{Q}_{\dot{\alpha}} \langle S \rangle = 0. \tag{6.4}$$

This occurs if higher $\theta$ components of $\langle S \rangle$ are nonzero, for example

$$\langle \Phi \rangle = \theta^2 F, \qquad \langle V \rangle = \theta^2 \bar{\theta}^2 D. \tag{6.5}$$

### *6.1 F-Type Breaking of SUSY*

The simplest examples of spontaneous breaking of SUSY are O'Raifeartaigh models. The idea behind this class of models is very simple. Consider a theory of chiral superfields with Lagrangian of the form

$$\mathcal{L}_{\mathrm{O'R}} = \int d^4\theta \, Q_a^\dagger Q^a + \left( \int d^2\theta \, W(Q) + \text{h.c.} \right). \tag{6.6}$$

In order for there to be a vacuum with unbroken SUSY, we must have

$$0 = \langle F_a^\dagger \rangle = \left\langle \frac{\partial W}{\partial Q^a} \right\rangle. \tag{6.7}$$

If there are $N$ fields $Q^a$, this gives $N$ conditions, which generically have a solution. However, for special choices of $W$, there may be no solution, and SUSY is spontaneously broken. The order parameter is the $F$ component of a chiral superfield, and this type of SUSY breaking is called '$F$-type SUSY breaking.'

The simplest example of this mechanism is a model with a single superfield $S$ with superpotential

$$W = \kappa S. \tag{6.8}$$

However, this theory is trivial, as is easily seen in components. It consists of a free chiral multiplet with a constant poential

$$V = |\kappa|^2. \tag{6.9}$$

This formally breaks SUSY, but there are no boson fermion splittings in the model.

Suppose however that we add higher-dimension operators in the $\int d^4\theta$ terms:

$$\mathcal{L} = \int d^4\theta \, f(S^\dagger, S) + \left( \int d^2\theta \, \kappa S + \text{h.c.} \right). \tag{6.10}$$

This model is often called the 'Polonyi model.' The higher order terms in $f$ may arise from integrating out heavy particles at scale $M$, in which case we expect

$$f = S^\dagger S + \frac{c}{4M^2}(S^\dagger S)^2 + \mathcal{O}(S^6/M^4), \tag{6.11}$$

where $c \sim 1$. The potential terms can be computed from

$$\mathcal{L} = f_{S^\dagger S} F^\dagger F + (\kappa F + \text{h.c.}) + \cdots, \tag{6.12}$$

where we use the abbreviation

$$f_{S^\dagger S} = \frac{\partial^2 f}{\partial S^\dagger \partial S}. \tag{6.13}$$

This gives

$$V = \frac{|\kappa|^2}{f_{S^\dagger S}}. \tag{6.14}$$

We see that this has a nontrivial minimum if $f_{S^\dagger S}$ has a maximum. For a potential of the form Eq. (6.11), we obtain $\langle S \rangle = 0$ for negative $c$. Expanding about this minimum, we find a scalar mass

$$m_S^2 = \frac{|c||\kappa|^2}{M^2}. \tag{6.15}$$

SUSY is broken because the fermion component of $S$ remains massless.

We can write a renormalizable model of $F$-type SUSY breaking following the original idea of O'Raifeartaigh. One simple model contains 3 chiral superfields $S_1$, $S_2$, and $X$, with superpotential

$$W = \tfrac{1}{2}\lambda_1 S_1 X^2 + \tfrac{1}{2}\lambda_2 S_2 (X^2 - v^2). \tag{6.16}$$

Note that

$$\frac{\partial W}{\partial S_1} = \tfrac{1}{2}\lambda_1 X^2, \qquad \frac{\partial W}{\partial S_2} = \tfrac{1}{2}\lambda_2 (X^2 - v^2). \tag{6.17}$$

Since these cannot both vanish, SUSY is spontaneously broken. Since

$$\frac{\partial W}{\partial X} = (\lambda_1 S_1 + \lambda_2 S_2) X, \tag{6.18}$$

the scalar potential is

$$V = \tfrac{1}{4}|\lambda_1|^2 |X|^4 + \tfrac{1}{4}|\lambda_2|^2 |X^2 - v^2|^2 + |\lambda_1 S_1 + \lambda_2 S_2|^2 |X|^2. \tag{6.19}$$

Extremizing this potential with respect to $S_1$ and $S_2$, it is easy to see that

$$\langle \lambda_1 S_1 + \lambda_2 S_2 \rangle = 0 \quad \text{or} \quad \langle X \rangle = 0. \tag{6.20}$$

Extremizing with respect to $X$ gives

$$\langle X^2 \rangle = \frac{|\lambda_2|^2}{|\lambda_1|^2 + |\lambda_2|^2} v^2 \quad \text{or} \quad \langle X \rangle = 0. \tag{6.21}$$

The value of the vacuum energy is

$$\langle V \rangle = \begin{cases} \dfrac{|\lambda_2|^2}{4} |v|^4 & \text{for } \langle X \rangle = 0, \\[2ex] \dfrac{|\lambda_1|^2 |\lambda_2|^2}{2(|\lambda_1|^2 + |\lambda_2|^2)} |v|^4 & \text{for } \langle X \rangle \neq 0. \end{cases} \tag{6.22}$$

It is easy to see that the vacuum energy is minimized at $\langle X \rangle \neq 0$ provided that $|\lambda_2| > |\lambda_1|$. In this case $\langle \lambda_1 S_1 + \lambda_2 S_2 \rangle = 0$, but one linear combination of $S_1$ and $S_2$ is completely unconstrained.

**Exercise 7:** Show that in an arbitrary O'Raifeartaigh model there is always one linear combination of the superfields that is unconstrained by minimizing the potential at tree level.

Working out the scalar and fermion masses at the minimum of the potential, we find that all scalar and fermion masses are of order $\lambda v$ except for the scalar and fermion components of the superfield that is orthogonal to the linear combination $\lambda_1 S_1 + \lambda_2 S_2$. This suggests that we can integrate out all the fields except one chiral superfield, and write the effective Lagrangian below the scale $\lambda v$ as an effective theory of a single chiral superfield. This can be formally justified by noting that

$$\langle F_X \rangle = 0, \tag{6.23}$$

$$\lambda_1 \langle F_{S1} \rangle + \lambda_2 \langle F_{S2} \rangle = \tfrac{1}{2} |\lambda_1|^2 X^{\dagger 2} + \tfrac{1}{2} |\lambda_2|^2 (X^{\dagger 2} - v^{\dagger 2}) = 0, \tag{6.24}$$

so that

$$X = 0, \qquad \lambda_1 S_1 + \lambda_2 S_2 = 0 \tag{6.25}$$

are superfield constraints that can be used to define the light degrees of freedom in the effective theory. The massless chiral multiplet can therefore be parameterized by $S_2$ (for example), which gives the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \int d^4\theta \left[1 + \frac{|\lambda_2|^2}{|\lambda_1|^2}\right] S_2^\dagger S_2 + \left(\int d^2\theta \left[-\tfrac{1}{2}\lambda_2 v^2\right] S_2 + \text{h.c.}\right). \tag{6.26}$$

This has precisely the form of the Polonyi model considered above! In this sense, O'Raifeartaigh models reduce to Polonyi models at low energies.

Note that in the O'Raifeartaigh model, the corrections to the effective kinetic function $f$ come from integrating out massive fields at the scale $\lambda v$, and is therefore fully calculable. Computing the Coleman-Weinberg potential in this model, one finds that the minimum is at $\langle S_2 \rangle = 0$.

## *6.2 D-Type Breaking of SUSY*

We can also break SUSY with an order parameter that is the highest component of a gauge superfield:

$$\langle V \rangle = \theta^2 \bar{\theta}^2 \langle D \rangle. \tag{6.27}$$

The simplest way works for only for a $U(1)$ gauge field, and consists of adding a 'Fayet-Iliopoulos term'

$$\Delta\mathcal{L}_{\text{FI}} = \int d^4\theta\, \xi V, \tag{6.28}$$

where $\xi$ is a coupling with mass dimension $+2$. This is is gauge invariant because under a gauge transformation $\delta V = \Omega + \Omega^\dagger$, where $\Omega$ is a chiral superfield, so under gauge transformations

$$\delta(\Delta\mathcal{L}_{\text{FI}}) = \int d^4\theta\, \xi(\Omega + \Omega^\dagger) = \text{total derivative}. \tag{6.29}$$

However, notice that this term is not gauge invariant if the coupling is a superfield. This means that this term cannot be generated by a more fundamental theory in which the couplings are superfields, such as a SUSY gauge theory. As we will discuss below, this also means that this term is not allowed by supergravity. We will therefore not discuss these terms further.

It is possible to get $\langle D \rangle \neq 0$ if SUSY and gauge symmetry are broken at the same time. For example, consider a theory with superfields $Q$, $\tilde{Q}$, $X$, and $\tilde{X}$, with $U(1)$ gauge charges $\pm 1$ and $\pm 2$ respectively, and one singlet $S$. The superpotential

$$W = \tfrac{1}{2}\lambda(\tilde{X}Q^2 + X\tilde{Q}^2) + \lambda' S(Q\tilde{Q} - v^2) \tag{6.30}$$

breaks SUSY and gauge symmetry at the same time, and gives rise to a nonzero value of $\langle D\rangle$. As this example illustrates, models of this type are necessarily somewhat complicated. An interesting open question is whether one can naturally get $\langle D\rangle \gg \langle F\rangle$ without Fayet-Iliopoulos terms.

### *6.3 Generalities of Tree-Level SUSY Breaking*

Let us consider a tree-level SUSY Lagrangian of the form given in Eq. (5.37). We are interested in tree-level SUSY breaking, so we take $Z = 1$ and all couplings to be SUSY preserving. The potential can be written as

$$V = F^a F^\dagger_a + \tfrac{1}{2} D_A D_A \tag{6.31}$$

where

$$F^a = W^{\dagger a} = \frac{\partial W^\dagger}{\partial Q^\dagger_a}, \qquad F^\dagger_a = \frac{\partial W}{\partial Q^a}, \qquad D_A = g_A Q^\dagger_a (T_A)^a{}_b Q^b. \tag{6.32}$$

(Note that we have absorbed a factor of the gauge coupling into the definition of $D_A$.) SUSY is spontaneously broken if $\langle F_a\rangle$ and/or $\langle D_A\rangle$ are nonzero.

We first show that if SUSY is spontaneously broken, there is a massless fermion in the spectrum, called the 'Goldstino.' The fermion mass terms in this notation are

$$\mathcal{L}_{\text{fermion mass}} = -\tfrac{1}{2}\psi^a W_{ab}\psi^b + \sqrt{2}\lambda_A D_{Aa}\psi^a + \text{h.c.}, \tag{6.33}$$

where

$$D_{Aa} = \frac{\partial D_A}{\partial Q^a}, \tag{6.34}$$

*etc.*, and we use the notation $W_{ab} = \langle W_{ab}\rangle$ in the remainder of this subsection. We can write the fermion masses in a matrix notation as

$$\mathcal{L}_{\text{fermion mass}} = \Psi M_{1/2}\Psi + \text{h.c.}, \tag{6.35}$$

where

$$\Psi = \begin{pmatrix} \psi^a \\ \lambda_A \end{pmatrix}, \tag{6.36}$$

and

$$M_{1/2} = \begin{pmatrix} W_{ab} & -\sqrt{2} g_B D_{Ba} \\ -\sqrt{2} D_{Ab} & 0 \end{pmatrix}. \tag{6.37}$$

We now claim that the fermion

$$\Psi_{\rm Goldstino} = \begin{pmatrix} W^{\dagger b} \\ -D_B/\sqrt{2} \end{pmatrix} \tag{6.38}$$

is massless. (Note that if SUSY is unbroken, $\Psi_{\rm Goldstino}$ is trivial.) This follows from

$$M_{1/2}\Psi_{\rm Goldstino} = \begin{pmatrix} W_{ab}W^{\dagger b} + D_B D_{Ba} \\ -\sqrt{2}D_{Ab}W^{\dagger b} \end{pmatrix}. \tag{6.39}$$

The upper component vanishes because

$$0 = \frac{\partial V}{\partial Q^a} = W_{ab}W^{\dagger b} + D_B D_{Ba}. \tag{6.40}$$

The lower component vanishes due to the gauge invariance of the superpotential:

$$0 = \delta W = W_a \delta Q^a = W_a (T_A)^a{}_b Q_b = \frac{1}{g_A} W_a D_A^a. \tag{6.41}$$

The existence of the Goldstino when SUSY is spontaneously broken is analogous to the existence of a Goldstone boson when a global symmetry is spontaneously broken. Its existence can also be established on general grounds, without referring to any Lagrangian. For a very clear discussion, see Ref. [18].

Let us continue our discussion of the masses with the scalar masses. Combining the scalar fields into a vector

$$\Phi = \begin{pmatrix} Q^a \\ Q_a^\dagger \end{pmatrix}, \tag{6.42}$$

we can write the scalar masses as

$$\mathcal{L}_{\rm scalar\,mass} = -\Phi^\dagger M_0^2 \Phi, \tag{6.43}$$

where

$$M_0^2 = \begin{pmatrix} W^{\dagger ac}W_{cb} + D_A^a D_{Ab} + D_{Ab}^a D_A & W^{\dagger abc}W_c + D_A^a D_A^b \\ W_{abc}W^{\dagger c} + D_{Aa}D_{Ab} & W_{ac}W^{\dagger cb} + D_{Aa}D_A^b + D_{Aa}^b D_A \end{pmatrix}. \tag{6.44}$$

Finally, the gauge boson masses are

$$\mathcal{L}_{\rm gauge\,mass} = \tfrac{1}{2} A^\mu M_1^2 A_\mu, \tag{6.45}$$

with

$$(M_1^2)_{AB} = 2D_A^a D_{Ba}. \tag{6.46}$$

From these formulas, we can read off the fact that the 'supertrace' of the mass matrix vanishes:

$$\mathrm{str}(M^2) \equiv \mathrm{tr}(M_0^2) - 2\,\mathrm{tr}(M_{1/2}^\dagger M_{1/2}) + 3\,\mathrm{tr}(M_1^2) = 0. \tag{6.47}$$

The supertrace is the sum of the squared masses of the particles, counting spin multiplicities, with fermions contributing with opposite sign as bosons. The vanishing of the supertrace puts strong constraints on how SUSY is broken, as we will see.

### *6.4 SUSY Breaking in the Observable Sector*

We have seen if SUSY makes electroweak symmetry breaking natural, superpartner masses must be below $\sim 1$ TeV, and to explain their non-observation they must have masses greater than $\sim 100$ GeV. The superpartner masses must therefore be at the weak scale, and a good model of SUSY breaking should explain why this is so.

An obvious thing to try is to break SUSY and electroweak symmetry at tree level by some extended Higgs sector. That is, we imagine a renormalizable extension of the MSSM to include extra fields and interactions that break SUSY and electroweak symmetry at tree level. In such a model, superpartner masses are nonzero because of direct couplings to the fields that break SUSY.

We can see that this is difficult from the supertrace constraint discussed in the previous subsection. First of all, note that the mass matrix has a block-diagonal form, with each block corresponding to states that do not mix with the states in the other blocks. (For example, colored particles to do not mix with color singlets.) Unless there are heavy fermions in every block containing observed quarks and leptons, the supertrace constraint immediately implies that the scalar superpartners must be lighter than the heaviest observed fermion, which is a phenomenological disaster.

It can be shown that even if we allow for the possibility of heavy particles, there are always light scalar color triplets (squarks) lighter than either $m_u$ or $m_d$, the lightest quark masses. Let us therefore restrict attention to the color triplet part of the mass matrix Eq. (6.44), which does not mix with anything else. Because color is unbroken we have

$$\langle W_a \rangle = \langle W^{\dagger a} \rangle = \langle D_{Aa} \rangle = \langle D_A^a \rangle = 0 \tag{6.48}$$

when $a$ is a color triplet index. The color triplet part of the scalar mass matrix is therefore

$$M_0^2 = \begin{pmatrix} M_{1/2}^\dagger M_{1/2} + \langle D_{Ab}^a D_A \rangle & \Delta \\ \Delta^\dagger & M_{1/2} M_{1/2}^\dagger + \langle D_{Aa}^b D_A \rangle \end{pmatrix}. \tag{6.49}$$

where

$$\Delta^{ab} = \langle W^{\dagger abc} W_c \rangle. \tag{6.50}$$

The idea is to think of $M_0^2$ as a Hamiltonian, with the lightest mass eigenvalue equal to the ground state energy. We can use the standard variational method from quantum mechanics to estimate the ground state energy. For any 'state vector' $\Phi_0$, we have

$$\frac{\Phi_0^\dagger M_0^2 \Phi_0}{\Phi_0^\dagger \Phi_0} \geq \text{smallest eigenvalue}. \tag{6.51}$$

To choose $\Phi_0$, note that the $D$ term contribution $\langle D^a_{Ab} D_A \rangle$ is proportional to charges, and is therefore negative at least one of the fields $U$, $U^{\mathrm{c}}$, $D$, and $D^{\mathrm{c}}$ in each generation (in a mass basis). Suppose for concreteness it is $U$ that gives a negative result. We then define

$$\Phi_0 = \begin{pmatrix} \phi \\ \phi^\dagger \end{pmatrix}, \tag{6.52}$$

where

$$\phi = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{6.53}$$

is a unit vector in the direction of the first generation $U$ field, with

$$\langle D^a_{Ab} D_A \rangle \phi^b = -|\lambda| \phi^a. \tag{6.54}$$

Then

$$\Phi_0^\dagger M_0^2 \Phi_0 = \tfrac{1}{2} \phi^\dagger (M_{1/2}^\dagger M_{1/2} + \langle D_A g_A T_A \rangle \phi + \text{h.c.}, \tag{6.55}$$

where we have used the fact that $\phi \Delta \phi$ vanishes, since there are no mass terms of this form $UU$ allowed. Therefore

$$\frac{\Phi_0^\dagger M_0^2 \Phi_0}{\Phi_0^\dagger \Phi_0} = m_u^2 - |\lambda|. \tag{6.56}$$

This implies that the matrix $M_0^2$ has at least one eigenvalue lighter than the quark mass $m_u^2$. This is clearly ruled out. If the quark with the negative eigenvalue is down-type, we have an eigenvalue less than $m_d^2$, which is also ruled out.

The basic problem is that $D$-type masses are proportional to charges, and therefore have both positive and negative signs, while the '$B$-type' masses parameterized by the off-diagonal $\Delta$ terms tend to split the eigenvalues, and therefore cannot raise the lowest eigenvalue.

### *6.5 The Messenger Paradigm*

To make a viable model of SUSY breaking, we need either large loop corrections, or non-renormalizable terms in the Kähler potential. SUSY breaking from either of these sources is suppressed, by loop factors and/or by high mass scales. This means that the theory must contain a sector in which SUSY is broken at a scale much larger than the weak scale. This large primordial SUSY breaking will then communicated to the standard model fields through 'messenger' interactions.

This gives us a way of thinking about the SUSY flavor problem. Since the interactions of the messengers with the standard model fields determine the pattern of SUSY breaking in the visible sector, a natural way to avoid additional flavor violation in the MSSM is if the messenger interactions do not violate flavor symmetries, *i.e.* are 'flavor blind.' We will see that this paradigm gives rise to successful models of SUSY breaking.

## 7 Hidden Sector SUSY Breaking

An obvious candidate for the messenger of SUSY breaking is gravity. From a particle physics perspective, the unique low-energy effective theory of gravity is general relativity.[8] Its consistency requires that gravity couples to matter through the stress-energy tensor, which is the origin of the equivalence principle. Because gravity couples to all forms of energy, it necessarily couples the SUSY breaking sector with the visible sector, even if there are no other interactions between the two sectors. All that is required for gravity to be the messenger of SUSY breaking is that there are no other stronger interactions between the two sectors. In this case, we refer to the SUSY breaking sector as the 'hidden sector.'

The fact that gravity couples to the stress-energy tensor also means that general relativity is flavor-blind. (Different flavors have different masses, and in this sense couple differently to gravity. But what we want is that there be no *additional* flavor violation beyond that of the Yukawa couplings.) The difficulty with this in practice is the fact that general relativity is only an effective theory, and requires UV completion above the Planck scale (if not at lower scales). It is far from clear that the fundamental theory of gravity can have flavor symmetries that guarantee that the UV couplings of gravity are flavor-blind. In fact, there are strong hints from what little is known

[8] This assumes that gravity is mediated by a spin-2 boson, and also assumes locality and Lorentz invariance. The cosmological constant problem has motivated attempts to relax the assumption of locality [20] or Lorentz invariance [21].

about the UV theory of gravity to suggest that the fundamental theory of gravity is unlikely to respect global symmetries such as flavor. One of these hints comes from studies of black holes, where Hawking radiation appears to be incapable of radiating away any global charge that was thrown into the black hole when it was formed. The final stages of black hole evaporation occur when the mass of the black hole becomes of order the Planck mass, and what happens there is not understood. However, requiring the conservation of global quantum numbers appears to require a large number of charged states at the Planck scale (corresponding to all the different possible charges of the initial black hole) which seems unlikely. Another hint comes from string theory, the only known candidate for a fundamental theory of gravity. String theory does not appear to allow exact global symmetries, although the full space of string theory vacua is still poorly understood.

From a low-energy point of view, we can parameterize the most general effects of the unknown physics at the Planck scale by higher dimension operators suppressed by powers of the Planck scale $M_{\rm P}$. Of particular interest to us are operators that connect the fields in the hidden sector with those in the observable sector. We assume that SUSY is broken in the hidden sector by the $F$ component of a field $X$, and without loss of generality we shift the field $X$ so that

$$\langle F_X \rangle \neq 0, \qquad \langle X \rangle = 0. \tag{7.1}$$

We can then write the most general interactions between $X$ and the visible sector fields:

$$\begin{aligned} \Delta\mathcal{L} = \int d^4\theta \bigg\{ & \frac{(z_Q)^i{}_j}{M_{\rm P}^2} X^\dagger X Q^\dagger Q + \cdots \\ & + \frac{b}{M_{\rm P}} X H_u H_d + \frac{b'}{M_{\rm P}} X^\dagger X H_u H_d + \text{h.c.} \bigg\} \\ & + \int d^2\theta \left[ \frac{s_1}{M_{\rm P}} X W_1^\alpha W_{1\alpha} + \cdots \right] + \text{h.c.} \\ & + \int d^2\theta \left[ \frac{a_{ij}}{M_{\rm P}} X Q^i H_u (U^{\rm c})^j + \cdots \right] \end{aligned} \tag{7.2}$$

When we substitute the SUSY breaking VEV $\langle F_X \rangle$, we find that this generates all the soft SUSY breaking terms of the MSSM (other than the '$C$' terms of Eq. (5.42)) with the size of all SUSY breaking masses of order

$$M_{\rm SUSY} \sim \frac{\langle F_X \rangle}{M_{\rm P}}. \tag{7.3}$$

Taking $M_{\text{SUSY}} \sim$ TeV gives

$$\langle F_X \rangle \sim M_{\text{P}} M_{\text{SUSY}} \sim (10^{11}\ \text{GeV})^2. \tag{7.4}$$

(The scale $10^{11}$ GeV is often called the 'intermediate scale.') Note that the $\mu$ and $B\mu$ terms are generated by the terms with coefficients $b$ and $b'$ in Eq. (7.2). This gives a very simple solution to the '$\mu$ problem', as first pointed out by Ref. [22]. Note these terms are only allowed if the SUSY breaking field $X$ is a singlet, as are the terms with couplings $s_1, \ldots$ that give rise to gaugino masses.

It is easy to see why SUSY breaking terms are suppressed in this approach. For example, '$C$ terms' are generated by operators of the form

$$\Delta\mathcal{L}_{\text{eff}} \sim \int d^4\theta \, \frac{X^\dagger X}{M_{\text{P}}^3} \, Q H_d^\dagger U^c + \text{h.c.}, \tag{7.5}$$

which give rise to SUSY breaking trilinear couplings of order $M_{\text{SUSY}}^2 / M_{\text{P}} \ll M_{\text{SUSY}}$. 'Hard' SUSY breaking is also small. For example, a fermion kinetic term arises from

$$\Delta\mathcal{L}_{\text{eff}} \sim \int d^4\theta \, \frac{X^\dagger X}{M_{\text{P}}^4} \, D^\alpha Q \sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu \bar{D}^{\dot{\alpha}} Q^\dagger, \tag{7.6}$$

which gives $\Delta Z \sim M_{\text{SUSY}}^2 / M_{\text{P}}^2 \ll 1$. It is striking that simply writing all possible terms connecting the hidden sector to the visible sector suppressed by powers of a single large scale gives all required SUSY breaking terms (including $\mu$ and $B\mu$ terms), all of the same order.

**Exercise 8:** Write the leading additional SUSY breaking allowed in the NMSSM coupled to a hidden sector. Does this automatically give rise to all allowed SUSY breaking of order $M_{\text{SUSY}}$, as in the MSSM? Can we impose symmetries so that all required SUSY breaking is generated with size $M_{\text{SUSY}}$?

The difficulty with this approach is that the soft masses and $A$ terms can violate flavor. The $A$ terms arise from the terms with coefficients $a_{ij}$, and we can imagine forbidding these by symmetries acting on the field $X$. However, the soft scalar masses are generated by the operators with coefficients $z^i{}_j$, which are invariant under all symmetries. Unless there are flavor symmetries at the Planck scale, there appears to be no reason for these coefficients to be flavor-diagonal. This is the flavor problem of hidden sector models of SUSY breaking.

One way to avoid the flavor problem is to assume that there is a gauged flavor symmetry at the Planck scale. The existence of gauge symmetries (as opposed to global symmetries) is compatible with what is known about string theory and black hole physics. A gauge symmetry must be free of anomalies, but extra fermions can always be added to cancel the anomalies, and once the flavor symmetries are broken, all these extra fermions can in principle become massive. The flavor gauge symmetry must be broken at a high scale to avoid dangerous flavor-changing neutral currents. The flavor symmetry may also be discrete. For an example of this kind of model, see Ref. [23].

**Exercise 9:** To illustrate some of the issues involved in models with flavor symmetries, consider the following model. Assume that the model at the Planck scale preserves an $SU(3)^5$ flavor symmetry that acts separately on the generation indices of the five multiplets $Q$, $U^{\rm c}$, $D^{\rm c}$, $L$, and $E^{\rm c}$. Find additional particle content that can make the $SU(3)^5$ flavor symmetry free of gauge anomalies, such that the additional particles can get masses below the $SU(3)^5$ that do not violate standard model gauge symmetry.

Suppose further that these symmetries are spontaneously broken by the VEVs of scalar fields $Y$ that have the same quantum numbers as the Yukawa couplings. (For example, there are one or more 'up-type' fields $(Y_U)_{ij}$, where the $i$ is a $SU(3)_Q$ and $j$ is a $SU(3)_U$ index.) The Yukawa couplings are of order

$$y \sim \frac{\langle Y \rangle}{M_{\rm P}}. \tag{7.7}$$

The hierarchy of VEV's for different components of $Y$ gives rise to the hierarchy for Yukawa couplings. Show that in this model, the off-diagonal squark masses have size

$$\frac{\Delta m^2_{\tilde{Q}}}{m^2_{\tilde{Q}}} \sim y^2, \tag{7.8}$$

where $y$ is an appropriate Yukawa coupling. Show that this is sufficient to suppress dangerous flavor-changing neutral currents.

The model in the example above is complicated, and contains many particles. The top quark Yukawa coupling is $y_t \sim 1$, which means that the expansion in powers of $\langle Y \rangle / M_{\rm P}$ is marginal. (Since loop effects are suppressed by powers of $1/(16\pi^2)$, the expansion really only breaks down for $y_t \sim 4\pi$, so this is not fatal.) However, all these features arise because the model is an ambitious attempt to explain the origin of the Yukawa couplings. Since the Yukawa couplings presumably have *some* explanation, and no simple one is known, this may not be a drawback of this approach.

We should note that this is not the way flavor arises in conventional string compactifications, in which there are no flavor symmetries at the string scale (see Ref. [24] for a review). On the other hand, string theory has not had any real success so far in explaining the observed features of our world. Maybe string theory is right, but we have not found the appropriate string vacuum. Given the richness of string theory, it might be right and we might never know. In this situation it is worth keeping an open mind about mechanisms that do not fit into string theory in an obvious way.

### *7.1 The 'Minimal SUGRA' Ansatz*

An *Ansatz* that has been extensively analyzed in the literature is to assume that the couplings that give rise to scalar masses are equal to a universal value at $\mu = M_{\rm P}$:

$$(z_Q)^i{}_j = (z_L)^i{}_j = \cdots = z_0 \delta^i{}_j, \quad z_{H_u} = z_{H_d} = z_0 \tag{7.9}$$

This is called 'minimal SUGRA' for historical reasons. One feature of this *Ansatz* is that the up-type Higgs mass runs negative because of the large top Yukawa coupling. This is called 'radiative symmetry breaking.' We have argued above that if there is no flavor symmetry at the Planck scale, this *Ansatz* is not natural. Nonetheless, there is an extensive literature on this, so you should at least know what it is.

**Exercise 10:** In this Ansatz, we must run the $z$ couplings down from the scale $M_{\rm P}$ down to the mass of the field $X$. Below this scale, we match onto a theory where $X$ is replaced by its SUSY breaking VEV. Show that if we include only standard model fields in the loops, this procedure is equivalent to running universal scalar masses down from the Planck scale.

## 8 Gauge Mediated SUSY Breaking

Following the messenger paradigm for SUSY breaking, another natural flavor-blind messenger to consider are the standard model gauge interactions themselves. We have seen that tree-level SUSY breaking in the visible sector has severe difficulties, so we look at loop effects.

## *8.1 Gauge Messengers*

A simple and predictive framework is to assume that SUSY breaking is communicated to the standard model via heavy chiral supermultiplets that are charged under the standard model gauge symmetries. If the masses of these messenger fields are not exactly supersymmetric, integrating them out will give rise to SUSY breaking in the visible sector.

A simple example that illustrates how this kind of SUSY breaking can arise is an O'Raifeartaigh-type model with superpotential

$$\begin{aligned} W = \tfrac{1}{2}\lambda_1 S_1 X^2 + \tfrac{1}{2}\lambda_2 S_2 (X^2 - v^2) + \lambda_3 S_3 \tilde{\Phi}\Phi \\ + (M + \lambda S_2)\tilde{\Phi}\Phi. \end{aligned} \tag{8.1}$$

Here $S_i$ $(i = 1, 2, 3)$ and $X$ are singlet fields, and $\Phi$ and $\tilde{\Phi}$ are charged under the standard model. This model breaks SUSY, and for appropriate choices of the parameters, the minimum occurs for

$$\langle F_{S_2} \rangle \sim \lambda v^2 \tag{8.2}$$

and $\langle S_1 \rangle, \langle S_3 \rangle, \langle \Phi \rangle, \langle \tilde{\Phi} \rangle = 0$. This means that $\Phi$ and $\tilde{\Phi}$ effectively have a SUSY breaking mass term, with a superfield mass parameter

$$\mathcal{M} = M + \lambda \langle S_1 \rangle + \theta^2 \langle F_{S_2} \rangle. \tag{8.3}$$

As long as the fields in this sector do not have any direct couplings to the MSSM fields, the leading effects on the standard model will come from loop graphs involving the fields $\Phi$ and $\tilde{\Phi}$, which depend on their mass Eq. (8.3). These fields are therefore the messengers of SUSY breaking. The messenger mass parameters are the only parameters in this model that will have an observable effect on physics in the visible sector.

The only unsatisfactory feature of this model is that the mass terms $M$ and $v^2$ are put in by hand. There is a large literature on models that break SUSY dynamically, in which the SUSY breaking messenger mass arises from dimensional transmutation (see Ref. [25] for a review). However, since the messenger mass parameters are the only parameters in the SUSY breaking sector that have observable effects in the visible sector, we will be content to assume that a fully satisfactory model exists and work out the consequences.

We therefore simply assume that there are charged messengers with SUSY breaking mass

$$\Delta \mathcal{L}_{\text{mess}} = \int d^2\theta \, \mathcal{M} \tilde{\Phi}\Phi + \text{h.c.} \tag{8.4}$$

with chiral superfield mass parameter

$$\mathcal{M} = M + \theta^2 F. \tag{8.5}$$

The messengers must be in a vector-like representation of the standard model gauge group to allow this mass. In order to keep the successful unification of couplings in the MSSM, we can take the messengers in complete $SU(5)$ multiplets, such as $\mathbf{5} \oplus \bar{\mathbf{5}}$. After integrating out the auxiliary fields, the scalar masses are

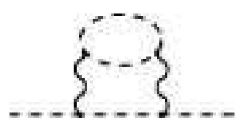
$$\Delta\mathcal{L}_{\text{mess}} \to -|M|^2(\phi^\dagger\phi + \tilde{\phi}^\dagger\tilde{\phi}) + (F\tilde{\phi}\phi + \text{h.c.}). \tag{8.6}$$

We can rephase $\phi$ and $\tilde{\phi}$ to make $F$ real, in which case the scalar mass eigenstates are $(\phi \pm \tilde{\phi})/\sqrt{2}$, with masses $|M|^2 \pm F$. We see that stability of the vacuum $\langle\phi\rangle = \langle\tilde{\phi}\rangle = 0$ requires $F \leq |M|^2$. The fermion masses are unaffected by $F$, and so the fermion mass is $|M|$.

When we integrate out the messengers at loop level, the resulting low-energy effective theory breaks SUSY. At one loop, we get a gaugino mass from the diagram

$m_{\text{gaugino}} \sim$  $\sim \dfrac{g^2}{16\pi^2}\dfrac{F}{M}.$ (8.7)

and at two loops, we get a scalar mass from diagrams like

$m^2_{\text{scalar}} \sim$  $+ \cdots \sim \left(\dfrac{g^2}{16\pi^2}\right)^2 \left|\dfrac{F}{M}\right|^2.$ (8.8)

These are the leading terms in an expansion in powers of $F$. (The corrections are suppressed by powers of $F/M^2$.) The estimates give the right order of magnitude even if $F \sim M^2$. One immediate consequence of this is that the masses of the scalars and gauginos are of the same order, which is important for getting a realistic and natural model of SUSY breaking. Also, note that the scalar masses depend only on the gauge quantum numbers of the scalars, and are therefore flavor-blind. This gives a natural solution to the SUSY flavor problem. Furthermore, the spectrum is determined by just a few parameters, so this is a highly predictive framework for SUSY breaking. This mechanism is called 'gauge mediation' of SUSY breaking, for obvious reasons.

**Exercise 11:** Show that the spurion $F/M$ has the right $U(1)_R$ charge to give rise to a gaugino mass. Use symmetry properties and dimensional analysis to show that the leading contribution to the scalar mass for small $F$ must be proportional to $|F/M|^2$. Also, show that the subleading terms for small $F$ give rise to fractional corrections of order $|F/M^2|^2$ to Eqs. (8.7) and (8.8).

If we want the superpartner masses to of order 100 GeV or more, we need to have $F/M \sim 5$–$50$ TeV. However, the mass scale $M$ can be quite large, keeping this ratio fixed. An upper bound on $M$ is obtained by requiring that the gauge mediated contributions to scalar masses to be sufficiently larger than the contributions from Planck suppressed higher-dimension operators. If SUSY is broken primordially by the VEV of a chiral superfield

$$\langle X \rangle = \theta^2 F_0, \tag{8.9}$$

then no symmetry can forbid operators of the form

$$\Delta \mathcal{L} \sim \int d^4\theta \, \frac{1}{M_{\rm P}^2} X^\dagger X Q^\dagger Q. \tag{8.10}$$

These will in general give rise to flavor-violating scalar masses of order

$$\Delta m_{\tilde{Q}}^2 \sim \frac{F_0^2}{M_{\rm P}^2}. \tag{8.11}$$

Demanding that $\Delta m_{\tilde{Q}}^2$ is small enough to avoid FCNC's gives the bound

$$\sqrt{F_0} \lesssim 10^{10} \text{ GeV} \left( \frac{m_{\tilde{Q}}}{500 \text{ GeV}} \right)^{3/2}. \tag{8.12}$$

Note that the primodial SUSY breaking scale $F_0$ need not be the same as the scale $F$ in the messenger mass. In particular, it is possible to have $F_0 \gg F$ if SUSY breaking is communicated weakly to the messengers. However, we must have $F_0 \gtrsim F$, so the largest possible value of $M$ is obtained when $F_0 \sim F$:

$$M \lesssim 10^{15} \text{ GeV} \left( \frac{m_{\tilde{Q}}}{500 \text{ GeV}} \right)^{3}. \tag{8.13}$$

### *8.2 The Gauge Mediated Spectrum*

We now turn to the calculation of the gauge mediated spectrum. Even at the qualitative level, it is crucial to know the signs of the squark and slepton mass-squared terms. If any of these are negative, the theory does not have a minimum that preserves color and electomagetism, which is certainly ruled out!

We will compute the induced SUSY breaking masses using an elegant method due to Giudice and Rattazzi [26] that makes essential use of superfield couplings. We treat the messenger mass $M$ as a chiral superfield

$$\mathcal{M} = M + \theta^2 F. \tag{8.14}$$

This reduces the problem to how the superfield couplings in the effective theory below the scale $M$ depend on $\mathcal{M}$. The leading dependence for large $M$ is given by the RG, making the calculation of the loop diagrams very simple. For example, the value standard-model gauge coupling at a scale $\mu < M$ can be obtained from the one-loop RG equation:

$$\frac{1}{g^2(\mu)} = \frac{1}{g'^2(\Lambda)} + \frac{b'}{8\pi^2} \ln \frac{M}{\Lambda} + \frac{b}{8\pi^2} \ln \frac{\mu}{M}. \tag{8.15}$$

Here $g'$ is the gauge coupling in the theory above the scale $M$, and $b'$ is the beta function coefficient in this theory, while $g$ and $b$ are the corresponding quantities in the effective theory below the scale $M$. We started the running at an arbitrary scale $\Lambda > M$. For a non-abelian group,

$$b - b' = N, \tag{8.16}$$

where $N$ is the number of messengers that get mass at the scale $M$ if the messengers are in the fundamental representation. Note that $N$ is always positive.

The idea of Giudice and Rattazzi is to extend the formula Eq. (8.15) to superfield couplings. We therefore have

$$\tau(\mu) = \tau'(\Lambda) + \frac{b'}{16\pi^2} \ln \frac{\mathcal{M}}{\Lambda} + \frac{b}{16\pi^2} \ln \frac{\mu}{\mathcal{M}}. \tag{8.17}$$

Here $\tau$ is the chiral superfield containing the (holomorphic) gauge coupling. Both sides of this equation are now well-defined chiral superfields. We can then compute the gaugino mass just by taking the higher $\theta$ components of both sides:

$$\begin{aligned} \frac{m_\lambda(\mu)}{g^2(\mu)} &= -[\tau(\mu)]_{\theta^2} = \frac{b - b'}{16\pi^2} [\ln \mathcal{M}]_{\theta^2} \\ &= \frac{N}{16\pi^2} \frac{F}{M}. \end{aligned} \tag{8.18}$$

Note that we have assumed that the only SUSY breaking is contained in $\mathcal{M}$. In particular, the couplings at the cutoff $\Lambda$ have no higher $\theta$ components, which means that SUSY is unbroken in the fundamental theory above the scale $M$. In components, this would have been a finite one-loop computation, but this method reduces it to a simple RG calculation.

Note that the result Eq. (8.18) includes the running from the matching scale $M$ down to scales $\mu < M$. We can find the value of the gaugino mass at the matching scale $M$ by expanding about $\mu = \mathcal{M}$. We illustrate this method here because it is very useful for the scalar masses to be discussed below. For the gaugino mass, we write

$$\tau(\mu) = \tau(\mathcal{M}) + \left.\frac{d\tau}{d\ln\mu}\right|_{\mu=\mathcal{M}} \ln\frac{\mu}{\mathcal{M}} + \mathcal{O}\left(\ln^2\frac{\mu}{\mathcal{M}}\right). \tag{8.19}$$

When we take the $\theta^2$ component of both sides, the terms of order $\ln^2(\mu/\mathcal{M})$ do not contribute in the limit $\mu \to M$. We then have

$$\lim_{\mu\to M}[\tau(\mu)]_{\theta^2} = [\tau(\mathcal{M})]_{\theta^2} + \left.\frac{d\tau}{d\ln\mu}\right|_{\mu=M} \left[\ln\frac{\mu}{\mathcal{M}}\right]_{\theta^2}. \tag{8.20}$$

We then compute

$$\begin{aligned} [\tau(\mathcal{M})]_{\theta^2} &= \frac{F}{M}\frac{\partial\tau(M)}{\partial\ln M} = \frac{F}{M}\frac{\partial\tau'(M)}{\partial\ln M} \\ &= \frac{F}{M}\frac{b'}{8\pi^2}. \end{aligned} \tag{8.21}$$

Note that in our expansion the UV couplings are held fixed, which is why the result is proportional to the beta function in the theory above the scale $M$. Putting this together, we obtain

$$\lim_{\mu\to M}[\tau(\mu)]_{\theta^2} = \frac{b'-b}{8\pi^2}\frac{F}{M}, \tag{8.22}$$

in agreement with Eq. (8.18).

This method is even more powerful when used to compute scalar masses. These are extracted from the wavefunction coefficient $Z$ via

$$m^2 = -[\ln Z]_{\theta^2\bar\theta^2}. \tag{8.23}$$

Here $Z$ is a real superfield, so it depends on $\mathcal{M}$ via the real superfield

$$\ln M \to \ln|\mathcal{M}| = \ln|M| + \tfrac{1}{2}\left(\theta^2\frac{F}{M} + \text{h.c.}\right). \tag{8.24}$$

Expanding about $\mu = \mathcal{M}$, we have

$$\begin{aligned} \lim_{\mu\to M}[\ln Z(\mu)]_{\theta^2\bar\theta^2} = [\ln Z(\mathcal{M})]_{\theta^2\bar\theta^2} &+ \left([\gamma(\mathcal{M})]_{\theta^2}\left[\ln\frac{\mu}{\mathcal{M}}\right]_{\bar\theta^2} + \text{h.c.}\right) \\ &+ \frac{1}{2}\frac{d\gamma}{d\ln\mu}(M)\left[\ln^2\frac{\mu}{\mathcal{M}}\right]_{\theta^2\bar\theta^2}, \end{aligned} \tag{8.25}$$

where

$$\gamma(\mu) = \frac{d \ln Z}{d \ln \mu} \tag{8.26}$$

is the anomalous dimension in the effective theory below the scale $M$. As in the calculation of the gaugino mass, we must perform the expansion keeping the UV cutoff fixed, which means that we must expand in $M$ in the fundamental theory. We therefore have

$$\begin{aligned} [\ln Z(\mathcal{M})]_{\theta^2\bar{\theta}^2} &= \frac{1}{4}\left|\frac{F}{M}\right|^2 \left(\frac{\partial}{\partial \ln M}\right)^2 \ln Z'(M) \\ &= \frac{1}{4}\left|\frac{F}{M}\right|^2 \frac{d\gamma'}{d\ln\mu}(M), \end{aligned} \tag{8.27}$$

where

$$\gamma'(\mu) = \frac{d \ln Z'}{d \ln \mu} \tag{8.28}$$

is the anomalous dimension in the theory above the scale $M$. Similarly,

$$\begin{aligned} [\gamma(M)]_{\theta^2} &= \frac{1}{2}\frac{F}{M}\frac{\partial}{\partial \ln M}\gamma(g'(M)) \\ &= \frac{1}{2}\frac{F}{M}\frac{\partial\gamma}{dg_i}(M)\beta_i'(M), \end{aligned} \tag{8.29}$$

where $g_i$ ($g_i'$) denotes the dimensionless couplings of the theory below (above) the scale $M$, and $\beta_i$ ($\beta_i'$) are the corresponding beta functions, *e.g.*

$$\beta_i = \frac{dg_i}{d\ln\mu}. \tag{8.30}$$

Putting it all together, we obtain

$$\begin{aligned} m^2(M) &= -\lim_{\mu\to M}[\ln Z(\mu)]_{\theta^2\bar{\theta}^2} \\ &= \frac{1}{4}\left|\frac{F}{M}\right|^2 \left[-\frac{\partial\gamma'}{\partial g_i'}\beta_i' + 2\frac{\partial\gamma}{dg_i}\beta_i' - \frac{\partial\gamma}{\partial g_i}\beta_i\right]. \end{aligned} \tag{8.31}$$

Here all anomalous dimensions are evaluated at $\mu = M$. This shows that the gauge mediated scalar mass at the threshold is a simple function of the anomalous dimensions of the theory. From this formula, we see that the scalar masses arise at two loops, since both $\gamma$ and $\beta$ start at one loop.

Note that we have performed a two-loop finite matching calculation using only the RG equations. We see that the threshold corrections are determined completely by the anomalous dimensions and beta functions of the theory. This is another illustration of the power of superfield couplings.

Squarks and sleptons do not couple directly to the messengers, so they have $\gamma' = \gamma$ at one loop. (This means that $\gamma'$ is the same function of the couplings $g'$ as $\gamma$ is of the couplings $g$.) In this case, the expression for the scalar mass simplifies further:

$$m^2(M) = \frac{1}{4}\left|\frac{F}{M}\right|^2 \frac{\partial\gamma}{\partial g_i}(\beta_i' - \beta_i). \tag{8.32}$$

The one-loop RG for a kinetic coefficient (of a quark field, say) from a gauge loop is

$$\mu\frac{d\ln Z}{d\mu} = \frac{c}{4\pi^2}g^2, \tag{8.33}$$

where $c$ is the quadratic Casimir of the field. For a fundamental representation of an $SU(N)$ gauge group, $c = (N^2 - 1)/(2N)$. Putting this in, we obtain

$$m^2(\mu = M) = \frac{g^4}{(16\pi^2)^2}2cN. \tag{8.34}$$

Note that the scalar masses are positive at the matching scale $\mu = |M|$, which is certainly a good starting point for a realistic model. RG evolution down to the weak scale can make the up-type Higgs mass run negative (due to the large top Yukawa coupling), triggering electroweak symmetry breaking.

Using the same techniques, we can see that

$$\lim_{\mu\to M}[\ln Z(\mu)]_{\theta^2} = \frac{1}{2}\frac{F}{M}(\gamma' - \gamma). \tag{8.35}$$

Again, for particles that do not couple directly to the messengers $\gamma' = \gamma$ at one loop, and so we do not get $A$ terms at one loop. (This is also obvious from the fact that there are no one-loop diagrams that could give an $A$ term.) Direct couplings of the quarks and leptons to the messengers violate flavor symmetries, but the Higgs can have nontrivial couplings to the messengers. Some of the consequences of this are explored in Refs. [27].

It is important to remember that the results above are only the leading result in an expansion in powers of $F/M^2$. In the effective theory below the messenger scale $M$, these additional terms are parameterized by terms with additional SUSY covariant derivatives, such as

$$\Delta\mathcal{L}_{\text{eff}} \sim \int d^4\theta \left|\frac{D^2 M}{M^2}\right|^2 Q^\dagger Q \sim \left|\frac{F^2}{M^3}\right|^2 \tilde{Q}^\dagger\tilde{Q} + \cdots \tag{8.36}$$

Unlike the leading terms computed above, these terms are not related to the dimensionless couplings of the low-energy theory, and therefore require an independent calculation. This calculation has been performed in Refs. [28]. The result is that the scalar mass in particular is very insensitive to corrections unless $F$ is very near $|M|^2$.

### *8.3 Phenomenology of Gauge Mediation*

We now mention briefly some highlights of the phenomenology of gauge-mediated SUSY breaking. For more detail, see Ref. [29] and references therein.

First, note that because superpartner masses are controlled by gauge couplings, colored states will be much heavier than uncolored states. In particular, the ratio of stop masses to right-handed slepton masses is of order

$$\frac{m_{\tilde{t}}}{m_{\tilde{e}_R}} \sim \sqrt{3}\,\frac{g_3^2}{g_1^2} \sim 10. \tag{8.37}$$

(The factor of $\sqrt{3}$ comes from the color factor $c \sim 3$ in Eq. (8.34).) The experimental bound $m_{\tilde{e}_R} \geq 99$ GeV therefore implies $m_{\tilde{t}} \gtrsim 980$ GeV, which implies a sizable fine-tuning for electroweak symmetry breaking. The actual value of $m_{\tilde{t}}$ can be smaller, but fine-tuning is a concern in gauge mediated SUSY breaking. As always, this fine-tuning is more severe if SUSY is broken at high scales.

Another important feature of gauge-mediated SUSY breaking is that the gravitino is generally the LSP. In standard scenarios for SUSY breaking, the gravitino gets a mass

$$m_{3/2} = \frac{F_0}{\sqrt{3}\,M_{\rm P}} \sim 100 \text{ GeV} \left( \frac{\sqrt{F_0}}{10^{10} \text{ GeV}} \right)^2, \tag{8.38}$$

where $F_0$ is the primordial scale of SUSY breaking. (We will review the origin of this formula below when we discuss supergravity.) The bound Eq. (8.13) implies that $m_{3/2} \ll 100$ GeV as long as $F_0$ is well below its maximum natural value, as suggested by fine-tuning considerations.

If the gravitino is the LSP, then all SUSY particles eventually decay to gravitinos and ordinary particles. To understand these decays, we use the fact that the gravitino mass can be thought of as arising from a 'super Higgs mechanism', in which a massless spin $\frac{3}{2}$ field (2 degrees of freedom) 'eats' the massless spin $\frac{1}{2}$ Goldstino field (2 degrees of freedom) to make a massive spin $\frac{3}{2}$ field. This is in direct analogy to a massive spin 1 particle, which can be thought of as a massless spin 1 particle (2 degrees of freedom) and an 'eaten' massless spin 0 Nambu-Goldstone field (1 degrees of freedom). We will not give the details here, but the important point is that the massless spin $\frac{3}{2}$ field is part of the supergravity multiplet, and therefore couples to matter with strength suppressed by powers of $1/M_{\rm P}$. On the other hand, the 'eaten' Goldstino couples to matter with strength determined by the priomordial SUSY breaking scale $F_0$. Since $F_0 \ll M_{\rm P}^2$, matter couples dominantly to the Goldstino field, and we can ignore the spin $\frac{3}{2}$ gravitino field.

Low energy theorems analogous to those for ordinary broken symmetries tell us that the coupling of the Goldstino field $\tilde{G}_\alpha$ couples to matter via the supercurrent $J^{\mu\alpha}$:

$$\mathcal{L}_{\rm int} = -\frac{1}{F_0} J^{\mu\alpha} \partial_\mu \tilde{G}_\alpha + \text{h.c.} + \mathcal{O}(\tilde{G}^2), \tag{8.39}$$

where the supercurrent is

$$J^\mu_\alpha = (\bar{\psi}_a \tilde{\sigma}^\mu \sigma^\nu)_\alpha \partial_\nu \phi^a - \frac{i}{4\sqrt{2}} (\lambda_A \tilde{\sigma}^\mu \sigma^\nu)_\alpha F_{\mu\nu A}. \tag{8.40}$$

This can be used to compute the decays of the other superpartners into Goldstinos.

Since $\sqrt{F_0}$ is much larger that superpartner masses, the heavy superpartners will decay rapidly to the next-to-lightest superpartner (NLSP), which will then decay more slowly into Goldstinos. (We are assuming that $R$ parity or a similar symmetry prevents rapid decays of the NLSP.) The phenomenology therefore depends on the value of $F_0$ and the identity of the NLSP. If the NLSP is Bino, its dominant decay is

$$\Gamma(\chi_1^0 \to \gamma \tilde{G}) \sim 10^{-3} \text{ eV} \left( \frac{m_{\chi_1^0}}{100 \text{ GeV}} \right)^5 \left( \frac{\sqrt{F_0}}{100 \text{ TeV}} \right)^{-4}. \tag{8.41}$$

If the NLSP is the right-handed stau (the lightest of the right-handed sleptons because of mixing effects), its dominant decay is

$$\Gamma(\tilde{\tau}_R \to \tau \tilde{G}) \sim 10^{-3} \text{ eV} \left( \frac{m_{\tilde{\tau}_R}}{100 \text{ GeV}} \right)^5 \left( \frac{\sqrt{F_0}}{100 \text{ TeV}} \right)^{-4}. \tag{8.42}$$

Depending on the value of $F_0$, the NLSP can have a visible decay length:

$$L = \frac{c\gamma}{\Gamma} \sim 10^{-2} \text{cm} \left( \frac{m}{100 \text{ GeV}} \right)^{-5} \left( \frac{\sqrt{F_0}}{100 \text{ TeV}} \right)^4 \times \sqrt{E^2/m^2 - 1}. \tag{8.43}$$

For $\sqrt{F_0} \lesssim 10^{-6}$ GeV this decay is inside the detector. Even for $\sqrt{F_0} \sim 100$ TeV (the smallest allowed value) this gives a displaced vertex small enough to be seen in a silicon vertex detector. Measurement of the decay length of the NLSP therefore gives direct information about the scale of primordial SUSY breaking!

### *8.4 Gravitino Cosmology*

We now make some brief remarks on gravitino cosmology in gauge-mediated models with $R$ parity. The gravitino is stable in these models, and can therefore contribute to the energy density of the universe today. If the gravitino has a thermal abundance early in the universe, it freezes out at temperatures of order its mass. The relic abundance today is of order

$$\Omega_{3/2} \sim \left( \frac{\sqrt{F_0}}{10^6 \text{ GeV}} \right)^{-1} \tag{8.44}$$

where $\Omega$ is the fraction of critical density contributed by the gravitino. (See Ref. [30] for a discussion of this standard calculation.) In order to avoid overclosing the universe we need $\Omega_{3/2} \lesssim 1$, or $\sqrt{F_0} \lesssim 10^6$ GeV. Note that this implies that the gravitino decays inside the detector in collider experiments! It is still possible to have $\sqrt{F_0} > 10^6$ GeV if the primordial gravitino abundance is diluted by inflation with a low reheat temperature or by significant late-time entropy production.

## 9 'Need-to-know' Supergravity

We now switch gears to a more formal subject: supergravity (SUGRA). SUGRA is the supersymmetric generalization of Einstein gravity, and as such unquestionably has a fundamental place in a supersymmetric world. However, the gravitational force is so weak that it is generally unimportant for particle physics experiments, so we start by explaining the motivation for a particle phenomenologist to learn about SUGRA.

One reason has already emerged in our discussion of gauge-mediated SUSY breaking. Namely, the superpartner of the spin-2 graviton is a the spin-$\frac{3}{2}$ gravitino. Its interactions with ordinary matter are suppressed by powers of the Planck scale, but it can have interesting (or dangerous) cosmological effects.

Another motivation is the cosmological constant problem, which is clearly a gravitational effect. The cosmological constant can be naturally zero in the limit of unbroken SUSY, but the cosmological constant problem comes back when SUSY is broken. There is at present no convincing solution to the cosmological constant problem, but SUSY is the only known symmetry that can explain why the cosmological constant smaller than the Planck scale, and may therefore play a role in the eventual solution.

However, the primary motivation for studying SUGRA in these lectures is that SUGRA can be the messenger of SUSY breaking. In section 7 we already considered gravity as the messenger of SUSY breaking. However, we really only considered the effect of integrating out heavy physics at the scale $M_{\rm P}$. We did not include the effects of the SUGRA fields, which are light! In particular, we want to explore the idea that SUSY breaking can be communicated to the visible sector via the VEV of an auxiliary field in the SUGRA multiplet. This auxiliary field is in some ways analogous to a $D$ field in SUSY gauge theory: if $\langle D \rangle \neq 0$, it will give rise to SUSY breaking for fields that are charged under the gauge group.[9] Since gravity couples universally, all fields are 'charged' under SUGRA, and we might expect that this gives rise to flavor-blind SUSY breaking. This idea can be made to work, but it turns out to be rather subtle and we will have to develop some formal machinery before we can get to the physics. Let us begin.

There are several different formalisms for SUGRA, all of which are related by field redefinitions and give equivalent physical results. The simplest formulation for our purposes is the tensor calculus approach. The basic idea of this approach is to write off-shell supermultiplets as a collection of component fields, and to define the usual superfield operations directly on this collection of components. For example, a chiral multiplet is written as

$$\Phi = (\phi,\ \psi_\alpha,\ F), \tag{9.1}$$

and products of chiral superfields are defined by

$$\Phi_1 \Phi_2 = (\phi_1 \phi_2,\ \phi_1 \psi_{2\alpha} + \phi_2 \psi_{1\alpha},\ \phi_1 F_2 + \phi_2 F_1 + \psi_1 \psi_2). \tag{9.2}$$

SUSY invariants are defined by taking the highest components of superfields, *e.g.*

$$\int d^2\theta\, \Phi = F. \tag{9.3}$$

For chiral and real superfields without SUGRA, this is just a rewriting of the usual rules for combining superfields. In the tensor calculus approach, matter and gauge supermultiplets are coupled to SUGRA by 'covariantizing' the rules for combining superfields and forming SUSY invariants. The minimal off-shell SUGRA multiplet is

$$({e_\mu}^a,\ \psi_{\mu\alpha},\ B_\mu,\ F_\phi), \tag{9.4}$$

where ${e_\mu}^a$ is the 4-bein, $\psi_{\mu\alpha}$ is the gravitino field, and $B_\mu$ and $F_\phi$ are vector and scalar auxiliary fields, respectively. In the tensor calculus approach, the SUGRA fields are

[9] One might wonder why we don't try to couple an additional $U(1)$ to the MSSM and break SUSY by $\langle D \rangle \neq 0$. One obstacle to building a model of this kind is that the scalar masses are proportional to $U(1)$ charges, which must occur with both signs in order to cancel gauge anomalies. Nonetheless, this approach may be viable [31].

included by suitably covariantizing the usual rules for multiplying supermultiplets and taking their highest components to define SUSY invariants. In particular, the 4-bein $e_\mu{}^a$ is coupled according to the standard rules from general relativity.[10]

For SUSY breaking, interested in $\langle F_\phi \rangle \neq 0$, since a VEV for $B_\mu$ would break Lorentz invariance. The dependence on $F_\phi$ is governed by supercovariance, and is closely related to a local (gauged) conformal invariance of the theory.

To understand this local conformal invariance, let us see how it can be introduced in Einstein gravity without SUSY. There we can write a theory in a way that is invariant under local scale transformations by introducing an additional real scalar. The additional gauge symmetry can be used to gauge away the scalar, so this theory is equivalent to ordinary Einstein gravity. We first introduce local scale transformations acting on the metric as

$$g_{\mu\nu}(x) \mapsto \Omega^2(x) g_{\mu\nu}(x). \tag{9.5}$$

It is easy to see that the usual Einstein kinetic term is not invariant:

$$\Gamma^\rho_{\mu\nu} \sim g^{-1}\partial g \quad \Rightarrow \quad \Gamma^\rho_{\mu\nu} \mapsto \Gamma^\rho_{\mu\nu} + \mathcal{O}(\partial\Omega), \tag{9.6}$$

$$R_{\mu\nu} \sim \partial\Gamma + \Gamma^2 \quad \Rightarrow \quad R_{\mu\nu} \mapsto R_{\mu\nu} + \mathcal{O}(\partial\Omega), \tag{9.7}$$

where $\mathcal{O}(\partial\Omega)$ denotes terms with derivatives acting on $\Omega$. Thererfore,

$$R = g^{\mu\nu} R_{\mu\nu} \mapsto \Omega^{-2} R + \mathcal{O}(\partial\Omega), \tag{9.8}$$

$$\sqrt{-g} \mapsto \Omega^4 \sqrt{-g}, \tag{9.9}$$

and we see that the Einstein Lagrangian $\sqrt{-g}\, R$ is not invariant. (Alternatively, it is clear that the Einstein Lagrangian is not conformally invariant because it has a dimensionful coefficient proportional to $M_{\rm P}^2$.) There is a 4-derivative action that is invariant under local scale transformations, but there is no obvious way to make sense out of theories whose leading kinetic term has 4 derivatives.

To make the Lagrangian invariant, we introduce a real scalar $\eta$ transforming under local scale transformations as

$$\eta(x) \mapsto \Omega^{-2}(x) \eta(x). \tag{9.10}$$

We can then write an invariant action in terms of the invariant 'metric'

$$\tilde{g}_{\mu\nu} = \eta\, g_{\mu\nu} : \tag{9.11}$$

[10] For a clear introduction to the 4-bein formalism of general relativity, see Ref. [32] or Ref. [24].

$$\int d^4x \sqrt{-\tilde{g}}\, \tilde{R} = \int d^4x \sqrt{-g} \left[ \eta^2 R - 6(\partial\eta)^2 \right]. \tag{9.12}$$

The signs are such that if the kinetic term for gravity has the right sign for positive energy, the kinetic term for the scalar has the 'wrong' sign (negative energy). Usually, a 'wrong' sign kinetic term means that the theory has a catastrophic instability to creation of negative energy modes. However, this is not a disaster in this case, because $\eta$ is not a physical degree of freedom: it can be gauged away. In fact, this theory is equivalent to Einstein gravity, as we can easily see by chosing the gauge

$$\eta(x) \to M_{\rm P}. \tag{9.13}$$

Eq. (9.13) is a good gauge choice as long as $\eta$ is everywhere nonzero, which is good enough for perturbative expansions.[11]

We have seen that we can rewrite Einstein gravity as a theory with an extra gauge symmetry (local scale invariance) and an extra scalar field. In fact, scale invariance implies invariance under an extended set of symmetries, the so-called conformal symmetries.[12] The scalar field $\eta$ is called the 'conformal compensator.'

The same trick is useful in writing the SUGRA Lagrangian. This approach is called the superconformal tensor calculus. The full group of symmetries is the superconformal tranformations, which includes scale transformations and a $U(1)_R$ symmetry.[13] One writes a theory that is invariant under local superconformal transformations, based on the superconformal supergravity multiplet

$$(e_\mu{}^a,\ \psi_{\mu a},\ B_\mu,\ R_\mu). \tag{9.14}$$

Here $B_\mu$ and $R_\mu$ are vector auxiliary fields. To break the superconformal symmetry dowm to super-Poincaré symmetry, one introduces a superconformal compensator supermultiplet, which is a chiral multiplet

$$\phi = (\eta,\ \chi,\ F_\phi). \tag{9.15}$$

The real part of the scalar complex field $\eta$ plays the same role as the real scalar field called $\eta$ above. The dimension and $R$ charge of are

$$d(\phi) = 1, \qquad R(\phi) = \tfrac{2}{3}. \tag{9.16}$$

[11]It may be disturbing that the strength of the gravitational coupling is apparently determined by a gauge choice. However, one must remember that only ratios of scales have physical significance. An operator of dimension $d$ in the lagragian will be proportional to $\eta^{4-d}$ by scale invariance, so the value of $\eta$ just sets the overall scale.

[12]For an introduction, see Ref. [33].

[13]The appearance of local scale symmetry can be understood in a deductive way in the superfield formulation of SUGRA. For a discussion of superfield SUGRA at the level of these lectures, see Refs. [34].

Note the dimension is the same as the real scalar $\phi$ of Eq. (9.12), and the $R$ charge is such that the superpotential $\int d^2\theta\phi^3$ is $U(1)_R$ invariant. To see that the theory with the compensator is equivalent to ordinary non-conformal SUGRA, one makes the gauge choice

$$\phi \to (1,\ 0,\ F_\phi), \qquad R_\mu \to 0. \tag{9.17}$$

This discussion has been very sketchy. For more details, see Ref. [35].

The utility of all this formalism is that the couplings of the superfield $\phi$, and hence $F_\phi$, are completely fixed by superconformal invariance. To determine the couplings of $\phi$ it is sufficient to keep track of scale transformations and $U(1)_R$ transformations, which are determined by the dimension $d$ and the $R$ charge. The basic rule is that the Lagrangian $\mathcal{L}$ has $d(\mathcal{L}) = 4$ and $R(\mathcal{L}) = 0$. For a SUSY Lagrangian of the form

$$\mathcal{L} = \int d^4\theta\, f + \left( \int d^2\theta\, W + \text{h.c.} \right) \tag{9.18}$$

this means that

$$d(f) = 2, \qquad R(f) = 0, \tag{9.19}$$

$$d(W) = 3, \qquad R(W) = +2. \tag{9.20}$$

It is convenient to choose all chiral and vector matter multiplets to have $d = 0$ and $R = 0$. (It may appear strange to choose $d = 0$ for matter fields, but we will see that we can make field redefinitions so that $d$ coincides with the usual mass dimension.) For a Lagrangian of the form Eq. (9.18), this implies in particular $d(f) = 0$, so it is not superconformally invariant. To make it invaraint, we use the conformal compensator. To convariantized kinetic and superpotential terms for a Lagrangian of the form Eq. (9.18) are then

$$\int d^4\theta\, \phi^\dagger \phi\, f = e \Big[ f|_{\theta^4} + (f|_{\theta^2} \cdot F_\phi^\dagger + \text{h.c.}) + f| \cdot (F_\phi^\dagger F_\phi + 6R(g) + \bar{\psi} i \not{\partial} \psi) + \text{fermions} \Big], \tag{9.21}$$

$$\int d^2\theta\, \phi^3\, W = e \Big[ W|_{\theta^2} + W| \cdot (3F_\phi + \psi\psi) + \text{fermions} \Big], \tag{9.22}$$

where $e = \det(e_\mu{}^a)$ is the determinant of the 4-bein and the terms involving the gravitino $\psi$ have been written only schematically. Note that the constant term in $f$ contains a kinetic term for the 4-bein and gravitino field, and a constant term in $W$ contains a mass term for the gravitino. We have not written terms involving the matter fermions, since we are interested in SUSY breaking.

For a gauge field with field strength $W_\alpha$, note that $d(W^\alpha W_\alpha) = 3$ and $R(W^\alpha W_\alpha) = 2$, so there is no $\phi$ dependence in the standard gauge kinetic terms:

$$\Delta\mathcal{L} = \int d^2\theta \, S(Q) W^\alpha W_\alpha + \text{h.c.} \tag{9.23}$$

The sum of Eqs. (9.21), (9.22), and (9.23) is the most general Lagrangian terms with 2 or fewer derivatives, coupled to supergravity.

Now we are (finally) ready to make our first main point. Consider a theory with no dimensionful couplings. The Lagrangian can be written schematically as

$$\mathcal{L} = \int d^4\theta \, \phi^\dagger \phi \, Q^\dagger e^V Q + \left[ \int d^2\theta \left( \frac{1}{g^2} W^\alpha W_\alpha \; + \phi^3 Q^3 \right) + \text{h.c.} \right]. \tag{9.24}$$

It appears that this has nontrivial couplings between $\phi$ and the matter fields, the field redefinition

$$\hat{Q} = \phi \, Q \tag{9.25}$$

removes the $\phi$ dependence completely:

$$\mathcal{L} = \int d^4\theta \, \hat{Q}^\dagger e^V \hat{Q} + \left[ \int d^2\theta \left( \frac{1}{g^2} W^\alpha W_\alpha \; + \hat{Q}^3 \right) + \text{h.c.} \right]. \tag{9.26}$$

Note that $\phi$ and $Q$ are both chiral superfields, so $\hat{Q}$ is also chiral. Note also that this redefinition implies that the superconformal dimension of $\hat{Q}$ coincides with its canonical dimension, *i.e.* $d(\hat{Q}) = 1$.

In the case of the MSSM, the only dimensionful parameter is the $\mu$ term. Coupling this to SUGRA and using the canonical 'hatted' fields defined in Eq. (9.25), the SUSY breaking part of the Lagrangian would be (assuming $\langle F_\phi \rangle \neq 0$)

$$\Delta\mathcal{L}_{\text{SUSY break}} = \mu \langle F_\phi \rangle H_u H_d + \text{h.c.} \tag{9.27}$$

That is, SUSY is broken only by a $B\mu$ term. This does not give rise to a realistic model (*e.g.* the squarks and sleptons are much lighter than the Higgs). However, we will see that there are important loop effects that can make this form of SUSY breaking realistic.

The fact that $F_\phi$ decouples from a conformally invariant Lagrangian at tree level would seem to imply that there are no supergravity corrections to the potential at tree level. However, this is not quite correct, because the kinetic term for scalars includes a non-minimal coupling to gravity:

$$\int d^4\theta \, \phi^\dagger \phi \, Q^\dagger Q = e \left[ |\partial Q|^2 + \tfrac{1}{6} |Q|^2 R(g) + \cdots \right]. \tag{9.28}$$

This coupling means that the scalar fields in general mix with gravity. We can eliminate the non-standard scalar couplings by a field redefinition. For a component Lagrangian of the form

$$\mathcal{L} = \sqrt{-g}\left[(M_{\rm P}^2 + f(Q))R(g) - V(Q)\right] \tag{9.29}$$

we can redefine the metric

$$g_{\mu\nu} = \Omega^2 \hat{g}_{\mu\nu}, \tag{9.30}$$

to obtain

$$\mathcal{L} = \sqrt{-\hat{g}}\,\Omega^4\left[\Omega^{-2}(M_{\rm P}^2 + f(Q))R(\hat{g}) - V(Q) + \mathcal{O}(\partial f/M_{\rm P}^2)\right], \tag{9.31}$$

where the omitted terms involve derivatives acting on $f$. Choosing

$$\Omega^2 = \frac{M_{\rm P}^2}{M_{\rm P}^2 + f(Q)}, \tag{9.32}$$

we obtain

$$\mathcal{L} = \sqrt{-\hat{g}}\left[M_{\rm P}^2 R(\hat{g}) - \hat{V}(Q) + \mathcal{O}(\partial f/M_{\rm P}^2)\right], \tag{9.33}$$

where

$$\hat{V}(Q) = \frac{V(Q)}{[1 + f(Q)/M_{\rm P}^2]^2}. \tag{9.34}$$

We have eliminated the non-standard couplings to gravity at the price of multiplicatively changing the scalar potential. This choice of definition of metric is often called 'Einstein frame.' The additional terms involving derivatives acting on $f$ mean that the new Lagrangian does not contain canonically normalized scalar fields in general. Further field redefinitions of the scalar fields can make these canonical. When all this is done, the expression for the potential for the canonically normalized scalar fields with no non-standard couplings to gravity is more complicated.

If we go to Einstein frame in supergravity, the connection to scale invariance is obscured. For purposes of understanding SUSY breaking, it is often better not to go to Einstein frame, as we will see.

### *9.1 SUSY Breaking in SUGRA: Polonyi Model*

In this subsection and the following two, we consider SUSY breaking in the presence of SUGRA. Readers who are willing to take it for granted that $\langle F_\phi\rangle$ will be nonzero in the presence of SUSY breaking can skip to section 10 below.

The Polonyi model is the simplest model of SUSY breaking, and we now consider what happens when it is coupled to SUGRA. The Lagrangian is

$$\begin{aligned}\mathcal{L} = &\int d^4\theta\, \phi^\dagger \phi[-3M_{\rm P}^2 + f(X, X^\dagger)] \\ &+ \left(\int d^2\theta\, \phi^3 [c + \kappa X] + \text{h.c.}\right).\end{aligned} \tag{9.35}$$

If the dimensionful parameters $c$ and $\kappa$ are small in units of $M_{\rm P}$ we expect the SUGRA corrections to the scalar potential to be small perturbations. In the absence of gravity, the scalar potential is

$$V = \frac{|\kappa|^2}{f_{X^\dagger X}}, \tag{9.36}$$

where $f_{X^\dagger X} = \partial^2 f / \partial X^\dagger \partial X$. We assume that the Kähler function $f$ is nontrivial so that this potential has a nontrivial minimum in which SUSY is broken. Since the superpotential is the most general linear function of $X$, we can shift the field so the minimum occurs at $\langle X \rangle = 0$.

To find the SUGRA corrections, we write out the terms without derivatives in the Lagrangian

$$\begin{aligned}\mathcal{L} = &F_\phi^\dagger F_\phi (-3M_{\rm P}^2 + f) + (F_X F_\phi^\dagger f_X + \text{h.c.}) && (9.37)\\ &+ f_{X^\dagger X} F_X^\dagger F_X \\ &+ 3F_\phi (c + \kappa X) + \text{h.c.} \\ &+ \kappa F_X + \text{h.c.} + \cdots && (9.38)\end{aligned}$$

Integrating out $F_X$, we obtain

$$\begin{aligned}\mathcal{L} \to &-\frac{|\kappa|^2}{f_{X^\dagger X}} + \left[-3M_{\rm P}^2 + f - \frac{|f_X|^2}{f_{X^\dagger X}}\right] F_\phi^\dagger F_\phi \\ &+ \left[3(c + \kappa X) - \frac{\kappa f_{X^\dagger}}{f_{X^\dagger X}}\right] F_\phi + \text{h.c.}\end{aligned} \tag{9.39}$$

If the dimensionful couplings in the Polonyi sector are small compared to $M_{\rm P}$, we can approximate the coefficient of $F_\phi^\dagger F_\phi$ by $-3M_{\rm P}^2$ and write

$$V = \frac{|\kappa|^2}{f_{X^\dagger X}} - \frac{3|c|^2}{M_{\rm P}^2} \left| 1 + \frac{\kappa}{c} \left( X - \frac{f_{X^\dagger}}{3 f_{X^\dagger X}} \right) \right|^2. \tag{9.40}$$

Note that the SUGRA corrections to the potential are negative definite. (This is related to the 'wrong-sign' kinetic term for the conformal compensator.) This is a crucial property that allows us to cancel the positive vacuum energy due to SUSY breaking and obtain a ground state with vanishing cosmological constant. This requires that we tune

$$|c|^2 \simeq \frac{M_{\rm P}^2 |\kappa|^2}{3\langle f_{X^\dagger X}\rangle}. \tag{9.41}$$

(Note that since $c \propto M_{\rm P}$, it is a good approximation to neglect the terms proportional to $\kappa/c$ in Eq. (9.40).) Because the Kähler function $f$ is renormalized, this is not stable under radiative corrections. Note that if the fluctuations of $X$ about $\langle X\rangle = 0$ are canonically normalized, then $\langle f_{X^\dagger X}\rangle \sim 1$ and $c \sim \kappa M_{\rm P}$.

In this vacuum, SUSY is broken by the auxiliary fields

$$\langle F_X\rangle = -\frac{\kappa^\dagger}{\langle f_{X^\dagger X}\rangle}, \tag{9.42}$$

$$\langle F_\phi\rangle = \frac{c^\dagger}{M_{\rm P}^2} \sim \frac{\langle F_X\rangle}{M_{\rm P}} \tag{9.43}$$

up to corrections suppressed by powers of $1/M_{\rm P}$, and where we have assumed $\langle f_{X^\dagger X}\rangle \sim 1$ in the last relation. If we include the gravitino couplings, we find that there is also a gravitino mass

$$m_{3/2} = \frac{\langle W\rangle}{M_{\rm P}} = \frac{c}{M_{\rm P}} = \langle F_\phi^\dagger\rangle. \tag{9.44}$$

This is the origin of the formulas for the gravitino mass used in the section on gauge mediated SUSY breaking.

We see that if SUSY is broken below the Planck scale by the usual O'Raifeartaigh (or Polonyi) mechanism, the only effect of SUGRA is to allow the fine-tuning of the cosmological constant, and to generate a nonzero VEV for the auxiliary field $F_\phi$.

### *9.2 'No Scale' SUSY Breaking*

The above analysis assumed that SUSY is broken in the absence of SUGRA. There is another possibility for SUSY breaking that can only occur in the presence of SUGRA: 'no scale' SUSY breaking. To see how this works, consider a model of a single chiral superfield $T$ with Lagrangian

$$\mathcal{L} = \int d^4\theta\, \phi^\dagger \phi (T + T^\dagger) + \left( \int d^2\theta\, \phi^3 c + \text{h.c.} \right), \tag{9.45}$$

where $c$ is a constant. Note that $T$ has been chosen to have dimensions of mass-squared. This choice of Lagrangian is rather arbitrary, and in fact it is not radiatively stable. We will address this below, but let us start by understanding this simple Lagrangian.

First, note that in the absence of gravity, the Kähler term would be a total derivative. The field $T$ acquires a kinetic term only by mixing with gravity, so this is only a healthy theory in the presence of gravity. The terms with no derivatives are

$$\begin{aligned}\mathcal{L} = F_\phi^\dagger F_\phi (T + T^\dagger) + (F_\phi^\dagger F_T + \text{h.c.}) \\ + 3F_\phi c + \text{h.c.} + \cdots \end{aligned}\tag{9.46}$$

Note that we have not included the Kinetic term for gravity, since this can be absorbed into a shift of $T$. In order to get the right strength for gravity, we need

$$\langle T \rangle = -\tfrac{3}{2} M_\text{P}^2. \tag{9.47}$$

Varying with respect to $F_T$ tells us that $F_\phi = 0$, and hence the potential vanishes identically. In particular, this means that the cosmological constant vanishes. On the other hand, the gravitino mass is nonzero:

$$m_{3/2} = \frac{\langle W \rangle}{M_\text{P}} = \frac{c}{M_\text{P}}. \tag{9.48}$$

Thus, SUSY is broken with vanishing cosmological constant! This kind of SUSY breaking is called 'no scale' SUGRA for historical reasons. However, the fact that the potential vanishes identically also means that the scalar field $T$ is completely undetermined. Also, the form of the Lagrangian Eq. (9.45) is not preserved by radiative corrections.

Suppose therefore that we add a nontrivial Kähler corrections to the Lagrangian above:

$$\Delta\mathcal{L} = \int d^4\theta\, \phi^\dagger \phi\, \Delta f(T, T^\dagger). \tag{9.49}$$

Such corrections will in any case be induced radiatively, and may play a role in the stabilization of $T$. Let us treat $\Delta f$ perturbatively, and ask what are the conditions that we get a vacuum that is 'close' to the one found above. It is easy to work out that the scalar potential to first order in $\Delta f$:

$$\Delta V = -|c|^2 \Delta f_{T^\dagger T}. \tag{9.50}$$

Therefore, if $f_{T^\dagger T}$ has a local maximum, the theory will have a stable minimum where $\langle T \rangle$ is given by Eq. (9.47), as required. In this vacuum, we have (to first order in $\Delta f$)

$$\langle F_T \rangle = -3c^\dagger, \tag{9.51}$$

$$\langle F_\phi \rangle = -\left\langle \frac{\Delta f_{T^\dagger T} F_T}{T + T^\dagger} \right\rangle = -\frac{c^\dagger \langle \Delta f_{T^\dagger T} \rangle}{3M_\text{P}^2} \tag{9.52}$$

Note if $\Delta f$ is small, we can make $\langle F_\phi \rangle$ as small as we want. This kind of vacuum can be thought of as 'almost no scale' SUGRA. For a more complete discussion see Ref. [36].

### *9.3 The SUGRA Potential*

We now consider the SUGRA corrections to the scalar potential. This is important to make contact between the approach to SUGRA taken here, which emphasizes the auxiliary fields, and more conventional treatments which use the SUGRA corrections to the potential as a starting point.

We consider a 2-derivative Lagrangian of the form

$$\mathcal{L} = \int d^4\theta\, \phi^\dagger \phi f + \left( \int d^2\theta\, \phi^3\, W + \text{h.c.} \right), \tag{9.53}$$

where $f$ and $W$ are functions of some matter fields $Q^a$. In order to get the right kinetic term for gravity, we require

$$\langle f \rangle = -3 M_{\text{P}}^2. \tag{9.54}$$

The terms in the Lagrangian with no derivatives are

$$\begin{aligned} \mathcal{L} = F_\phi^\dagger F_\phi f + (F_\phi^\dagger f_a F^a + \text{h.c.}) + f_a{}^b F_b^\dagger F_a \\ + 3F_\phi W + W_a F^a + \text{h.c.}, \end{aligned} \tag{9.55}$$

where $f_a = \partial f / \partial Q^a$, $f^a = \partial f / \partial Q_a^\dagger$, *etc.* Solving for the auxiliary fields, we find

$$F_a^\dagger = -(\tilde{f}^{-1})_a{}^b \left[ W_b - \frac{3 f_b}{f} W \right], \tag{9.56}$$

$$F_\phi^\dagger = -\frac{1}{f} (3W + f^a F_a^\dagger), \tag{9.57}$$

where $(\tilde{f}^{-1})_a{}^b$ is the matrix inverse of

$$\tilde{f}_a{}^b = f_a{}^b - \frac{1}{f} f_a f^b. \tag{9.58}$$

Integrating out the auxiliary fields, we find after some algebra

$$V = (\tilde{f})_b{}^a \left( W_a - \frac{3W}{f} f_a \right) \left( W^{\dagger b} - \frac{3W^\dagger}{f} f^b \right) - \frac{3|W|^2}{f}. \tag{9.59}$$

As discussed above, this is not the potential in Einstein frame. To make the gravity kinetic term canoncial, we define the Einstein frame metric

$$\hat{g}_{\mu\nu} = -\frac{f}{3M_{\mathrm{P}}^2} g_{\mu\nu}. \tag{9.60}$$

The potential in Einstein frame is then

$$\hat{V} = \left( \frac{3M_{\mathrm{P}}^2}{f} \right)^2 V. \tag{9.61}$$

Note also that the function $f$ is not what is called the Kähler potential in the SUGRA literature. The Kähler potnetial is related to $f$ by

$$f = -3M_{\mathrm{P}}^2 e^{-K/3M_{\mathrm{P}}^2}. \tag{9.62}$$

With these relations, Eq. (9.61) reduces to the standard expression for the SUGRA potential (see *e.g.* Ref. [5]).

Let us apply these results to find the conditions for a vacuum that *preserves* SUSY and has a vanishing cosmological constant. To preserve SUSY it is sufficient for all auxiliary fields to vanish in the vacuum. The condition $\langle F_a^\dagger \rangle = 0$ is equivalent to $W_a - 3W f_a/f = 0$ provided that $\langle \tilde{f}_a{}^b \rangle$ is a non-singular matrix. This in turn is equivalent to the condition that $W/f^3$ is stationary. The condition $\langle F_\phi^\dagger \rangle = 0$ then imposes the additional requirement that $\langle W \rangle = 0$. Combining these, we see that the conditions for a SUSY vacuum with vanishing cosmological constant are

$$W \text{ stationary}, \quad \langle W \rangle = 0. \tag{9.63}$$

Note that we can always impose $\langle W \rangle = 0$ by adding a constant to the superpotential. This shows that the SUSY-preserving vacua in the presence of SUGRA with vanishing cosmological constant are in one-to-one correspondence with the SUSY vacua in the absence of SUGRA. In particular, the SUGRA corrections to the potential cannot turn a SUSY preserving vacuum into a SUSY breaking one.

## 10 Anomaly Mediated SUSY Breaking

Now we finally have enough machinery to discuss SUGRA as the messenger of SUSY breaking. We therefore assume that the only source of SUSY breaking comes from a non-vanishing value of $\langle F_\phi \rangle$. This can be viewed as a SUGRA background in which we are calculating. Let us consider a SUSY model with no dimensionful couplings in the SUSY limit. (The NMSSM is such a model, as is the MSSM if we omit the $\mu$

term. We will see that the $\mu$ term in the MSSM will have to be treated differently.) As we have seen in the previous section, the fact that the there are no dimensionful couplings means that at tree level there is no SUSY breaking felt by the matter fields. However, at loop level scale invariance is broken by the running of the couplings. We therefore expect SUSY breaking related to the conformal anomaly. This is called 'anomaly mediated SUSY breaking' (AMSB). The original papers are Refs. [37, 38].

To see how this works in a very concrete way, note that the presence of UV divergences requires us to regulate the theory, and the regulator necessarily introduces a mass scale, and therefore breaks conformal symmetry. For example, in a Wess-Zumino model, we can use higher-derivative terms to regulate the theory:

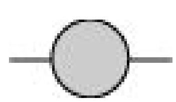
$$\mathcal{L} = \int d^4\theta \, Z_0 \hat{Q}^\dagger \left(1 + \frac{\Box}{\Lambda^2 \phi^\dagger \phi}\right) \hat{Q} + \left(\int d^2\theta \, \frac{\lambda}{6} \, \hat{Q}^3 + \text{h.c.}\right), \tag{10.1}$$

where $\Box$ is the covariant second derivative operator. We have directly written the Lagrangian in terms of the canonical 'hatted' fields defined by Eq. (9.25). The factors of $\phi$ are required because the operator $\Box$ has $d = 2$ and $R = 0$. The $\Box/\Lambda$ term modifies the propagator of the component fields in $\hat{Q}$. For example, setting $\langle F_\phi \rangle = 0$ for the moment, the scalar propagator is modified

$$\frac{i}{p^2} \to \frac{i}{p^2 - p^4/\Lambda^2}. \tag{10.2}$$

This makes loops of $\hat{Q}$ fields UV convergent.[14] Reintroducing $\langle F_\phi \rangle \neq 0$ introduces a small splitting between the regulated scalars and fermions required by the coupling to the SUGRA background, but the loop diagrams are still finite. Note that we are only regulating loops of $\hat{Q}$ fields, not SUGRA loops. SUGRA loops are suppressed by additional powers of $M_{\text{P}}$, and are therefore much smaller than loop effects from dimensionless couplings.

We now compute standard model loop corrections in the SUGRA background. We do this (once again) by treating the couplings as superfields. At one loop, the $\hat{Q}^\dagger \hat{Q}$ term in the 1PI effective action is logathimically divergent:



$$= Z_0 \left[1 + \frac{\lambda^2}{26\pi^2} \ln \frac{\Lambda |\phi|}{\mu} + \text{finite}\right], \tag{10.3}$$

where $\lambda$ is the physical Yukawa coupling. Note the $\phi$ dependence in Eq. (10.3) is required by conformal invariance. There is no $\phi$ dependence in the 'finite' part of the

[14] This regulator also introduces a ghost, *i.e.* a state with wrong-sign kinetic term at $p^2 = \Lambda^2$. However, this decouples when we take the limit $\Lambda \to \infty$ and does not cause any difficulties.

amplitude because it is by definition independent of $\Lambda$ as $\Lambda \to \infty$. The divergence ($\Lambda$ dependence) must be cancelled by absorbing it into the 'bare' coupling $Z_0$, but we cannot absorb the $\phi$ dependence into $Z_0$ because we want all SUSY breaking to be due to the nontrivial SUGRA background. (If the superfield $Z_0$ had some nonzero SUSY breaking components, this would be a theory with SUSY broken in the fundamental theory.) This means that there is some SUSY breaking left over in the finite part after we subtract the divergence. We can view this as the replacement

$$\ln \mu \to \ln \frac{\mu}{|\phi|} = \ln \mu - \tfrac{1}{2}(\theta^2 F_\phi + \text{h.c.}). \tag{10.4}$$

The fact that this substitution parameterizes all the SUSY breaking is clearly more general than than this example. The cutoff $\Lambda$ and the renormalization scale $\mu$ always appear in the combination $\Lambda/\mu$, and for any real superfield coupling the correct substitution is $\Lambda \to \Lambda|\phi|$, implying Eq. (10.4). Note also that Eq. (10.4) holds to all orders in perturbation theory.

We can therefore compute SUSY breaking in real superfield couplings by

$$\frac{\partial}{\partial \theta^2} = -\frac{1}{2} F_\phi \frac{\partial}{\partial \ln \mu}. \tag{10.5}$$

For example, the gaugino mass computed from the real superfield gauge coupling is

$$m_\lambda = -g^2 [R]_{\theta^2} = \frac{\beta(g)}{2g} F_\phi. \tag{10.6}$$

We also have

$$[\ln Z]_{\theta^2} = -\tfrac{1}{2} \gamma F_\phi, \tag{10.7}$$

which gives rise to nonzero $A$ terms. Finally, we have soft masses

$$m^2 = -[\ln Z]_{\theta^2 \bar\theta^2} = -\tfrac{1}{4} |F_\phi|^2 \frac{d\gamma}{d \ln \mu}. \tag{10.8}$$

Eqs. (10.6), (10.7), and (10.8) are exact in the sense that they hold to all orders in perturbation theory. They hold at each renormalization scale, and therefore define the 'AMSB renormalization group trajectory.' As in gauge mediation, the dominant source of SUSY breaking comes from gauge loops, and therefore the scalar masses are flavor-blind and the model solves the SUSY flavor problem. Also as in gauge mediation, gaugino masses arise at one loop and scalar mass-squared parameters at two loops, so all SUSY breaking masses are of the same order:

$$m_\lambda \sim \frac{g^2}{16\pi^2} \langle F_\phi \rangle, \qquad m_0^2 \sim \frac{g^4}{(16\pi^2)^2} \langle F_\phi \rangle^2. \tag{10.9}$$

To get SUSY breaking masses of order 100 GeV, we need $\langle F_\phi \rangle \sim 10$ TeV.

It is very interesting that the model gives the entire superpartner spectrum in terms of a single new parameter $\langle F_\phi \rangle$, which just sets the overall scale of the superpartner masses. Let us check the crucial sign of the scalar masses. For fields with only gauge interactions, we have $\gamma \sim +g^2$ and therefore

$$m^2 \sim -g\beta_g |\langle F_\phi \rangle|^2. \tag{10.10}$$

We see that if the gauge group is asymptotically free ($\beta_g < 0$) the scalar mass-squared parameter is positive. Unfortunately in the MSSM, the $SU(2)_W$ and $U(1)_Y$ gauge groups are not asymptotically free, so sleptons have negative mass-squared. Adding more fields can only make this worse. We cannot live on the AMSB renormalization trajectory.

Nonetheless, it is possible to have realistic SUSY breaking from AMSB. To understand this, let us consider the effect of massive thresholds in AMSB. Note that the formulas Eqs. (10.6), (10.7), and (10.8) are claimed to hold independently of the details of the high energy theory, in particular the nature of ultrahigh energy thresholds (*e.g.* at the GUT scale). Let us see how this works.

Consider some new chiral superfields $P$ and $\tilde{P}$ that transform as a vectorlike representation of the standard model gauge group, and which have a large supersymmetric mass term

$$\Delta\mathcal{L} = \int d^2\theta \, M\phi P\tilde{P} + \text{h.c.} \tag{10.11}$$

Note that we have included the superconformal compensator in a normalization where the fields $P$ and $\tilde{P}$ have canonical kinetic terms. Because the fields $P$ and $\tilde{P}$ are charged, the gauge beta functions will have different values above and below the scale $M$, so the SUSY breaking masses above and below the scale $M$ are different. To understand this, note that the $P$ threshold is not supersymmetric because of the $\phi$ dependence. Because of this, there is a *gauge*-mediated threshold correction at the scale $M$, with

$$\frac{F}{M} = F_\phi. \tag{10.12}$$

For example, for gaugino masses, the threshold correction is (see Eq. (8.18))

$$\Delta m_\lambda = \frac{\Delta\beta_g}{2g} F_\phi. \tag{10.13}$$

Adding this to the AMSB value above the threshold, we find that the gaugino mass below the threshold is precisely on the AMSB trajectory below the threshold.

Another way to understand this point is to consider again the superfield couplings. The holomorphic gauge coupling superfield $\tau$ below the threshold $M$ is given by

$$\begin{aligned}\tau(\mu) &= \tau_0 + \frac{b'}{16\pi^2}\ln\frac{M}{\Lambda} + \frac{b}{16\pi^2}\ln\frac{\mu}{M} \\ &\to \tau_0 + \frac{b'}{16\pi^2}\ln\frac{M\phi}{\Lambda\phi} + \frac{b}{16\pi^2}\ln\frac{\mu}{M\phi}.\end{aligned} \tag{10.14}$$

where $b$ and $b'$ are the beta function coefficients below and above the scale $M$, respectively. (The notation is the same as in section 8.) We see that the $\phi$ dependence induced by $\Lambda$ and $M$ exactly cancel in the contribution from above the scale $M$. The gaugino mass below the scale $M$ is correctly given by the substitution $\mu \to \mu/\phi$, just as if the threshold did not exist. We can think of $M$ as the new cutoff.

The fact that AMSB is independent of thresholds is very striking, and makes the theory very predictive. Unfortunately, we have seen that it is *too* predictive, and is ruled out by negative slepton mass-squared parameters.

This discussion however suggests a way out. If a heavy threshold is not supersymmetric, the cancelation discussed above no longer occurs, and we can be on a different RG trajectory below the threshold. If the $F/M$ of the threshold is much larger than $F_\phi$, we have gauge mediation, and if it is much smaller it is irrelevant. The only interesting case is where $F/M \sim F_\phi$, and we would like this to occur naturally.

A very simple class of models where this occurs was first discussed in Ref. [39]. Consider a theory with a singlet $S$ in addition to the vectorlike fields $P$ and $\tilde{P}$, with superpotential terms

$$\Delta\mathcal{L} = \int d^2\theta \left[\lambda S P \tilde{P} + \frac{S^n}{(M\phi)^{n-3}}\right] + \text{h.c.} \tag{10.15}$$

The potential for $S$ is

$$V = M^4 \left\{ n^2 \left|\frac{S}{M}\right|^{2(n-1)} + \left[(n-3)\left(\frac{S}{M}\right)^n \frac{F_\phi}{M} + \text{h.c.}\right]\right\}. \tag{10.16}$$

Minimizing the potential we find

$$\left(\frac{\langle S\rangle}{M}\right)^{n-2} = \frac{n-3}{n(n-1)}\frac{\langle F_\phi\rangle}{M}. \tag{10.17}$$

From this we see that

$$\frac{\langle F_S\rangle}{\langle S\rangle} = \frac{n-3}{n-1}\langle F_\phi\rangle. \tag{10.18}$$

For $n > 3$ and $M \gg \langle F_\phi \rangle$, Eq. (10.17) implies that $\langle F_\phi \rangle \ll \langle S \rangle \ll M$, while $\langle F_S \rangle / \langle S \rangle \sim \langle F_\phi \rangle$. Because the coefficient in Eq. (10.18) is different from unity, the theory will not be on the AMSB trajectory below the threshold. These theories can be viewed as a combination of gauge and anomaly mediation. We can obtain a realistic SUSY breaking spectrum in this way (in particular, the slepton masses can be positive). See Ref. [39] for more details.

There are also other ways proposed in the literature to make AMSB realistic. One class of models is similar to the proposal discussed above, in that they have an additional 'gauge mediated' contribution to SUSY breaking that is naturally the same size as the AMSB contribution [40]. For a very different approach, see Ref. [41].

### *10.1 The $\mu$ Problem in Anomaly Mediation*

The $\mu$ problem is more severe in AMSB because we cannot simply add a conventional $\mu$ term of the form

$$\Delta\mathcal{L} = \int d^2\theta \, \mu\phi H_u H_d + \text{h.c.} \tag{10.19}$$

The reason is that the explicit breaking of conformal invariance means that

$$B\mu = \langle F_\phi \rangle \mu, \tag{10.20}$$

which is far too large for $\langle F_\phi \rangle \sim 10$ TeV.

One possiblility is the NMSSM, discussed in subsection 5.10. This has no dimensionful couplings, and therefore this problem is absent. The effective $\mu$ term arises from a VEV for the singlet $\langle S \rangle$, which is ultimately triggered by AMSB itself. Given the fact that this

Another possibility was pointed out in Ref. [37]. If there is a chiral superfield $X$ with a shift symmetry $X \mapsto X +$constant, then the $\mu$ term can arise from an operator of the form

$$\Delta\mathcal{L} = \int d^4\theta \, \frac{\phi^\dagger}{\phi} \frac{1}{M} (X + X^\dagger) H_u H_d + \text{h.c.} \tag{10.21}$$

Assuming $\langle X \rangle = 0$, we have

$$\Delta\mathcal{L} = \int d^2\theta \, \frac{F_X^\dagger}{M} H_u H_d + \text{h.c.} \tag{10.22}$$

*i.e.* we generate a $\mu$ term with no $B\mu$ term. The $B\mu$ term can come from AMSB in this model.

Yet another possibility to generate the $\mu$ term from the VEV of a singlet is described in Ref. [39].

### *10.2 Anomaly-Mediated Phenomenology*

Since the theory cannot be on the AMSB trajectory at low energies, the low-energy phenomenology depends on how these problems are resolved. Discussions can be found in the original papers, quoted above.

We do want to point out, however, that these theories share the fine-tuning problem of gauge mediated SUSY breaking, since scalar masses arise from 2-loop gauge diagrams, and therefore

$$\frac{m_{\tilde{q}}^2}{m_{\tilde{e}_R}^2} \sim \frac{N_{\rm c} g_3^4}{g_1^4}. \tag{10.23}$$

### *10.3 Naturalness of Anomaly Mediation*

So far we have not addressed the question of whether it is natural for the theory to be on the AMSB trajectory. What we would like is to have a theory that breaks SUSY spontaneously in a hidden sector in such a way that the breaking is communicated to the observable sector dominantly through the SUGRA conformal compensator.

As we have seen in subsections 9.1 and 9.2, spontaneous SUSY breaking in SUGRA generally gives

$$\langle F_\phi \rangle \lesssim \frac{F_0}{M_{\rm P}}, \tag{10.24}$$

where $F_0$ is the primordial SUSY breaking scale. Generally, $F_0 = \langle F_X \rangle$, where $X$ is some chiral superfield. In this case, we expect the effective theory to contain operators of the form

$$\Delta\mathcal{L} \sim \int d^4\theta \, \frac{1}{M_{\rm P}^2} X^\dagger X Q^\dagger Q \tag{10.25}$$

where $Q$ are standard model fields. As already discussed in section 7, these couplings have the quantum numbers of a product of kinetic terms, and cannot be forbidden by any symmetries. We therefore expect them to be present in any UV completion of the theory at the Planck scale. This gives rise to scalar masses of order

$$m^2 \sim \frac{\langle F_X \rangle^2}{M_{\rm P}^2}, \tag{10.26}$$

which is much larger than the AMSB value. (Furthermore, there is no reason for the term Eq. (10.25) to conserve flavor, so we expect the masses to give rise to FCNC's.) If we consider all the other possible terms coupling the visible and the hidden sector suppressed by powers of $M_\mathrm{P}$ (see Eq. (7.2)), we find that they can all naturally be absent due to symmetries. Therefore, the viability of anomaly mediation depends on whether there are sensible models where the couplings Eq. (10.25) are naturally absent.

A simple rationale for this was given in Ref. [37]. The idea is that the hidden and visible sectors are localized on 'branes' in extra dimensions.[15] That is, the standard model matter and gauge interactions are localized on the visible brane, and the SUSY breaking sector is localized on the hidden brane. In fact, this type of setup naturally occurs in string theory, *e.g.* in the setup of Ref. [42]. In the higher-dimensional theory, interactions like Eq. (10.25) are fobidden because $X$ and $Q$ are localized on different branes, so the interaction is not local. Fields that propagate in the bulk can give rise to interactions between $X$ and $Q$, so we must check whether interactions like Eq. (10.25) are generated in the $3+1$ dimensional theory below the compactification scale $R^{-1}$, where $R$ is the distance between the visible and hidden branes. If the scale of new physics is $M$ (*e.g.* the string scale), then for $R \gg M^{-1}$ the propagator of a massive field (*e.g.* and excited string state) connecting the visible and hidden branes is suppressed by the Yukawa factor $e^{-MR} \ll 1$. (Since the suppression factor is exponential, $R \gtrsim$ few $\times\, M^{-1}$ is sufficient in practice.) Therefore, operators like Eq. (10.25) are not generated by the exchange of massive states. This leaves only the effect of fields that are light compared to the compactification scale. Only supergravity *must* propagate in the bulk, so the minimalthe minimal light fields in the bulk are the minimal 5D SUGRA multiplet. It was shown in Ref. [43] that this does not generate terms of the form Eq. (10.25). For details, see Refs. [37, 43]. In this setup, the SUSY breaking sector is 'more hidden' than in conventional hidden sector models, and is sometimes referred to as a 'sequestered sector.'

Another way to make this natural is to replace the extra dimension in the setup above with a conformal field theory via the AdS/CFT correspondence. For a discussion of this 'conformal sequestering,' see Ref. [44].

## 11 Gaugino Mediation

The final model we will mention (very briefly) is gaugino mediation. Like anomaly mediation, this can be motivated by an extra-dimensional setup. This time we assume that the standard-model gauge fields propagate in the bulk, while the matter fields

[15]We will not go into details here, but will just state the main ideas. For an introduction into many of the technical and conceptual issues in theories with extra dimensions and branes, see the lectures by Raman Sundrum at this school.

are localized on the visible brane. In this case, the gauginos can get a mass from contact terms on the hidden brane of the form

$$\Delta\mathcal{L} \sim \int d^2\theta \, \frac{X}{M} W^\alpha W_\alpha + \text{h.c.}, \tag{11.1}$$

where $X$ is the hidden sector field that breaks SUSY. In this type of model, the gaugino gets a mass at tree level, while the visible matter fields get a mass only at one-loop order. This means that the gaugino masses are much larger than scalar masses at the compactification scale, but the RG between this scale and the weak scale generates scalar masses of order the gaugino masses. This scenario is called 'gaugino mediated SUSY breaking' for obvious reasons. The original papers are Refs. [45].

Note that gaugino mediation shares the fine-tuning problem of gauge- and anomaly-mediation. In a GUT model, we expect that the gaugino masses are unified at the GUT scale:

$$M_1(M_{\rm GUT}) \simeq M_2(M_{\rm GUT}) \simeq M_3(M_{\rm GUT}). \tag{11.2}$$

(Even if there are GUT breaking splittings, we expect $M_1 \sim M_2 \sim M_3$, which is sufficient for our argument.) Since the quantity $M_i/g_i^2$ is RG invariant at one loop, at the weak scale we have

$$\frac{M_1}{g_1^2} \simeq \frac{M_2}{g_2^2} \simeq \frac{M_3}{g_3^2}. \tag{11.3}$$

The scalar masses are generated from the RG equation

$$\frac{dm^2}{dt} = -\frac{c}{2\pi^2} g^2 m_\lambda^2. \tag{11.4}$$

Using the fact that $m_\lambda^2/g^2$ is RG invariant, we have the solution

$$m^2(\mu) = \frac{2c}{b}\left[g^4(\mu) - g^4(M_{\rm GUT})\right]\left(\frac{m_\lambda}{g^2}\right)^2, \tag{11.5}$$

where we have assumed that $m^2(M_{\rm GUT}) \ll m^2(\mu)$. We therefore have

$$\frac{m_{\tilde{q}}^2}{m_{\tilde{e}_R}^2} \sim \frac{N_{\rm c} g_3^4}{g_1^4} \tag{11.6}$$

just as in gauge mediation and anomaly mediation.

## 12 No Conclusion

There is much more to say, but I will stop here. I hope that I have introduced some of the problems and issues with SUSY breaking, as well as introducing some ideas that may point in the right direction. I hope that some of the readers will be inspired by these lectures to go beyond them.

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