INFLATION

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Contents:

Abstract:

The inflationary Universe scenario is reviewed. The development of this cosmological scenario began with investigations of (primarily) first order phase transitions. In the context of grand unified theories, the dilemma of an overproduction of magnetic monopoles was resolved by a rapid expansion of the Universe. The original solution to the magnetic monopole problem quickly led to solutions to a host of other cosmological problems such as the horizon-isotropy and flatness-entropy problems. In this review I will take a detailed look at old and new inflation and its many variants including primordial, supersymmetric, chaotic, axionic, R^2, stochastic, eternal, superstring, power-law, multiple inflation and others. Special emphasis will also be given to consequences of inflation such as the generation of energy density perturbations and scalar field fluctuations. Implications for the cosmological baryon asymmetry, cosmic strings and dark matter will also be discussed.

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1. Introduction

1.1. The standard big bang model

The most successful cosmological model to date is undeniably the standard hot big bang model. It is based on the assumption of the cosmological principle [1], which states in effect that the Universe is both homogeneous and isotropic. Two immediate consequences of this assumption are: that there exists a direction independent measure of distance (which will yield the Friedmann–Robertson–Walker metric) and that the only true velocity fields are expansion or contraction. The latter implies the Hubble expansion of the Universe, where the relative velocities of two observers depend only on separation

$$v_{12} = Hr_{12} , \tag{1.1}$$

where H is a universal spatial constant.

Claims for the success of the hot big bang model are based primarily on the observation [2] of the 3 K microwave background radiation and the agreement with observations on the primordial abundances of the light elements [3]. The temperature of the microwave background radiation is known today to relatively high precision [4],

$$T_0 = 2.73 \pm 0.05\ \mathrm{K} . \tag{1.2}$$

The interpretation of this background is that it is a remnant of photons at their last scattering when the temperature was ~4000 K and the age of the Universe was about 10^5 years. Measurements of the microwave background also indicate a high degree of isotropy. After the dipole anisotropy of magnitude $\Delta T/T \sim 10^{-3}$, due to our motion with respect to the background, is subtracted out, no definite observable anisotropies have been discovered. On intermediate to large angular separations of 3°–180° limits are $\Delta T/T \leq (4\text{–}10) \times 10^{-5}$ [5, 6]. Similar limits on smaller scales of 10′–1° also indicate that $\Delta T/T \leq (6\text{–}10) \times 10^{-5}$ [7, 6] while at the very small angular scale of 4.5′ the limit is [8, 6]

$$\Delta T/T \leq 2 \times 10^{-5} . \tag{1.3}$$

Clearly, anisotropies in the microwave radiation appear to be perturbative.

Big bang nucleosynthesis is the model for the production of the light elements, primarily D, ^{3}He, ^{4}He and ^{7}Li [3]. Using measured nuclear cross sections and folding a nuclear network into the background of an expanding Universe it is possible to calculate the expected abundance of these light nuclei [9, 10]. Nucleosynthesis takes place when $T \sim 10^9$ K or when the age of the Universe is about 2 minutes. It is remarkable that using only standard model inputs such as three massless neutrino flavors etc., each of the calculated abundances (which ranges from 0.25 for ^{4}He and $\sim 10^{-9}$ for ^{7}Li) is within the uncertainties of the observational determinations. This is true, however, only if the net baryon number to photon ratio is $3\text{–}4 \times 10^{-10}$ [10]. Clearly nucleosynthesis must be considered a major success of the standard big bang model.

In the following section, we will look at some shortcomings of the standard model. Inflation is a proposed remedy to these problems. To discuss inflation and the cosmological problems it will be useful to first briefly review the standard big bang model. Assuming homogeneity and isotropy, space–time can be described by the spatially maximally symmetric Friedmann–Robertson–Walker metric in co-moving coordinates,

$$ds^2 = -dt^2 + R^2(t)[dr^2/(1-kr^2) + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)]\,, \tag{1.4}$$

where $R(t)$ is the cosmological scale factor and k is the three-space curvature constant ($k = 0, +1, -1$ for a spatially flat, closed or open Universe). I also assume the perfect fluid form for the energy–momentum tensor,

$$T_{\mu\nu} = pg_{\mu\nu} + (p+\rho)u_\mu u_\nu\,, \tag{1.5}$$

where $g_{\mu\nu}$ is the space–time metric described by (1.4), p is the isotropic pressure, ρ is the energy density and $u_\mu = (1, 0, 0, 0)$ is the velocity vector for the isotropic fluid. Einstein's equation (using the convention $R_{\mu\nu} = R^\sigma_{\mu\sigma\nu} = \Gamma^\sigma_{\mu\nu,\sigma} - \Gamma^\sigma_{\sigma\mu,\nu} + \Gamma^\sigma_{\mu\nu}\Gamma^\rho_{\sigma\rho} - \Gamma^\sigma_{\rho\mu}\Gamma^\rho_{\sigma\nu}$),

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R_c = \kappa^2 T_{\mu\nu} - \Lambda g_{\mu\nu}\,, \tag{1.6}$$

with space–time curvature $R_c = R^\mu_\mu$, $\kappa^2 = 8\pi G_N = 8\pi/M_P^2$ ($M_P = 1.22\times10^{19}$ GeV is the Planck mass), and Λ is the cosmological constant, yields the Friedmann equation

$$H^2 \equiv (\dot R/R)^2 = \tfrac{1}{3}\kappa^2\rho - k/R^2 + \tfrac{1}{3}\Lambda\,, \tag{1.7}$$

from the 0–0 component of (1.6), and

$$(\ddot R/R) = \tfrac{1}{3}\Lambda - \tfrac{1}{6}\kappa^2(\rho + 3p) \tag{1.8}$$

from the i–i components of (1.6) with a substitution from (1.7). Using either (1.7) and (1.8) or $T^{\mu\nu}{}_{;\nu} = 0$ we can write

$$\dot\rho = -3H(\rho + p)\,. \tag{1.9}$$

These equations form the basis of the standard big bang model.

At early times ($t \lesssim 10^5$ yr) the Universe is thought to have been dominated by radiation so that the equation of state can be given by $p = \rho/3$. If we neglect the contributions to H from k and Λ (this is always a good approximation for small enough R) then we find that

$$R(t) \sim t^{1/2} \text{ (radiation dominated)}\,, \tag{1.10}$$

and $\rho \sim R^{-4}$ so that $t \sim (3/4\kappa^2\rho)^{1/2}$. Similarly for a matter or dust dominated Universe with $p = 0$,

$$R(t) \sim t^{2/3} \text{ (matter dominated)}\,, \tag{1.11}$$

and $\rho \sim R^{-3}$. The Universe makes the transition between radiation and matter domination when $\rho_{\rm rad} = \rho_{\rm matter}$ or when $T \simeq$ few $\times 10^3$ K. In general, for $p = (\gamma - 1)\rho$ with $1 \le \gamma \le 2$, $R(t) \sim t^{2/3\gamma}$ and $\rho \sim R^{-3\gamma}$.

For $\Lambda = 0$, it is possible to define a critical energy density ρ_c such that $\rho = \rho_c$ for $k = 0$,

$$\rho_c = 3H^2/\kappa^2 . \tag{1.12}$$

In terms of the present value of the Hubble parameter this is

$$\rho_c = 1.88 \times 10^{-29} h_0^2 \text{ g cm}^{-3} , \tag{1.13}$$

$$h_0 = H_0/(100 \text{ km Mpc}^{-1} \text{ s}^{-1}) . \tag{1.14}$$

The cosmological density parameter is then defined by

$$\Omega \equiv \rho/\rho_c . \tag{1.15}$$

Equation (1.7) can be rewritten as

$$(\Omega - 1)H^2 = k/R^2 , \tag{1.16}$$

so that $k = 0$, $+1$, -1 corresponds to $\Omega = 1$, $\Omega > 1$ and $\Omega < 1$, respectively. Equation (1.8) can be rewritten in terms of the deceleration parameter q,

$$q \equiv -(\ddot{R}/R)/H^2 = \tfrac{1}{2}\Omega(1 + 3p/\rho) = \tfrac{1}{2}\Omega(3\gamma - 2) . \tag{1.17}$$

Observational limits on h_0 and Ω_0 (subscripts refer to present-day values) are [11]

$$0.4\text{–}0.5 \leq h_0 \leq 1, \qquad 0.1 \lesssim \Omega_0 \leq 4 . \tag{1.18a, b}$$

However, the age of the Universe, determined by

$$H_0 t_u = \int_0^1 (1 - \Omega_0 + \Omega_0 x^{-3\gamma+2})^{-1/2} \, dx \tag{1.19}$$

and the constraint $t_u \geq 13 \times 10^9$ yr requires the combination $\Omega_0 h_0^2 \leq 0.25$ for $h_0 \geq 0.5$, or $\Omega_0 h_0^2 \lesssim 0.45$ for $h_0 \geq 0.4$, while the constraint $t_u \geq 10 \times 10^9$ yr requires $\Omega_0 h_0^2 \leq 0.8$ for $h_0 \geq 0.5$ and $\Omega_0 h_0^2 \leq 1.1$ for $h_0 \geq 0.4$.

The energy density in the microwave background radiation is simply that of a black body, $\rho_\gamma = (\pi^2/15)T^4$ (in units where $\hbar = c = k_B = 1$). In the early radiation dominated era, the total density in radiation must include contributions from all relativistic particle species,

$$\rho = \tfrac{1}{30}\pi^2 N(T) T^4 , \tag{1.20}$$

$$N(T) = \sum_B g_B(T_B/T)^4 + \tfrac{7}{8} \sum_F g_F(T_F/T)^4 . \tag{1.21}$$

The sums are over all relativistic boson (fermion) degrees of freedom $g_{B(F)}$ with temperature $T_{B(F)}$

relative to the photon temperature T. (Often, as in the case of neutrinos or other particles which drop out of equilibrium [12], $T_{B(F)} < T$). At early times, when the Universe is radiation dominated, we can relate the age t of the Universe to the temperature T by

$$t = [45/2\pi^2\kappa^2 N(T)]^{1/2} T^{-2}, \tag{1.22}$$

or

$$tT^2 = 2.4[N(T)]^{-1/2}, \tag{1.23}$$

where t is measured in seconds and T in units of MeV.

As was noted above, nucleosynthesis takes place at temperatures $T \sim 1$ MeV. At higher temperatures ($T \gg 1$ MeV) the weak interaction rates for the processes

$$\mathrm{n} + \nu_e \leftrightarrow \mathrm{p} + \mathrm{e}^-, \qquad \mathrm{n} + \mathrm{e}^+ \leftrightarrow \mathrm{p} + \bar{\nu}_e, \qquad \mathrm{n} \leftrightarrow \mathrm{p} + \mathrm{e}^- + \bar{\nu}_e, \tag{1.24}$$

were all in equilibrium, i.e., $\Gamma_{wk} > H$. Thus we would expect that initially $(\mathrm{n}/\mathrm{p}) = 1$. Actually, in equilibrium the ratio is essentially controlled by the Boltzmann factor so that

$$(\mathrm{n}/\mathrm{p}) = \exp(-\Delta m/T), \tag{1.25}$$

where $\Delta m = m_n - m_p$ is the neutron–proton mass difference. For $T \gg \Delta m$, $(\mathrm{n}/\mathrm{p}) \simeq 1$.

At temperatures $T \gg 1$ MeV, nucleosynthesis cannot begin to occur even though the rate for forming the first isotope, deuterium, is sufficiently rapid. To begin with, at $T \gtrsim 1$ MeV, deuterium is photo-dissociated because $E_\gamma > 2.2$ MeV (the binding energy of deuterium; $\bar{E}_\gamma = 2.7T$ for a black body). Furthermore, the density of photons is very high, $n_\gamma/n_B \sim 10^{10}$. Thus the onset of nucleosynthesis will depend on the quantity

$$\eta^{-1} \exp[-2.2\ \mathrm{MeV}/T], \tag{1.26}$$

where $\eta = n_B/n_\gamma$. When this quantity (1.26) becomes $\lesssim O(1)$, the rate for $\mathrm{p} + \mathrm{n} \to \mathrm{D} + \gamma$ finally becomes greater than the rate for dissociation $\mathrm{D} + \gamma \to \mathrm{p} + \mathrm{n}$ and nucleosynthesis can begin. This occurs when $T \sim 0.1$ MeV.

Because the rates for processes (1.24) freeze out at $T \sim 1$ MeV, the neutron to proton ratio must be adjusted from its equilibrium value at freeze out to its value when nucleosynthesis begins. When freeze out occurs, the ratio (n/p) is relatively fixed at $(\mathrm{n}/\mathrm{p}) \sim \frac{1}{6}$.

This equilibrium value is adjusted by taking into account the free neutron decays up until the time at which nucleosynthesis begins. This reduces the ratio $(\mathrm{n}/\mathrm{p}) \sim \frac{1}{7}$.

Since virtually all the neutrons available end up in deuterium which gets quickly converted to ^{4}He, we can estimate the ratio of the ^{4}He nuclei formed compared with the number of protons left over,

$$X_4 \equiv N_{^4\mathrm{He}}/N_\mathrm{H} = \tfrac{1}{2}(\mathrm{n}/\mathrm{p})/[1 - (\mathrm{n}/\mathrm{p})], \tag{1.27}$$

or, more importantly, the ^{4}He mass fraction

$$Y_4 = 4X_4(1 + 4X_4) = 2(\mathrm{n}/\mathrm{p})/[1 + (\mathrm{n}/\mathrm{p})]. \tag{1.28}$$

For $(\mathrm{n}/\mathrm{p}) \simeq \frac{1}{7}$, we estimate that $Y_4 = 0.25$ which is very close to the observed value.

The actual calculated value of Y_4 will depend on a numerical calculation which runs through the complete sequence of nuclear reactions [9, 10]. The nuclear chain is temporarily halted because there are gaps at masses $A = 5$ and $A = 8$, i.e., there are no stable nuclei with those masses. There is some further production, however, which accounts for the abundances of ^{6}Li and ^{7}Li. Once again because of the gap at $A = 8$ there is very little subsequent nucleosynthesis in the big bang. A second chief factor in the ending of nucleosynthesis is that during this whole process the Universe continues to expand and cool. At lower temperatures it becomes exponentially difficult to overcome the Coulomb barriers in nuclear collisions. In spite of these effects, numerical calculations of the elemental abundance continue the chain up until Al.

In fig. 1, the predicted abundances of D, ^{3}He, ^{4}He and ^{7}Li are plotted as a function of η. The ^{4}He abundance is shown for $N_\nu = 2$, 3, and 4, where N_ν is the number of neutrino flavors. Also shown in the figure [10] are the observational ranges for these abundances. From deuterium alone, $\mathrm{D/H} \gtrsim 1\text{–}2 \times 10^{-5}$, we see that $\eta \lesssim 7\text{–}10 \times 10^{-10}$ while the combination of $(\mathrm{D} + {}^3\mathrm{He})/\mathrm{H} \lesssim 10^{-4}$ indicates that $3 \times 10^{-10} \lesssim \eta$. The constraint from ^{7}Li is consistent, $({}^7\mathrm{Li/H}) \lesssim 2 \times 10^{-10}$ implies $10^{-10} \lesssim \eta \lesssim 4 \times 10^{-10}$. (For the new upper limit on η from ^{7}Li (quoted) see the second and third references in [10].) Most importantly the range $0.22 \lesssim Y_4 \lesssim 0.25$ or at most $Y_4 \lesssim 0.26$ is again consistent with this same range for η.

Figure 1 actually contains significantly more information than just a limit on η. We can set a limit [10, 13, 14] on N_ν provided that we have a lower limit to η. Using $\eta \geq 3 \times 10^{-10}$ and $Y_4 \leq 0.25$ (0.26), we find that $N_\nu \leq 4$ (4.6) with the equality being at best marginal. This implies that at most one more generation is allowed, assuming that the neutrinos associated with each generation are light and stable.

The value of η cannot be determined directly from observations. Looking at the baryon and photon

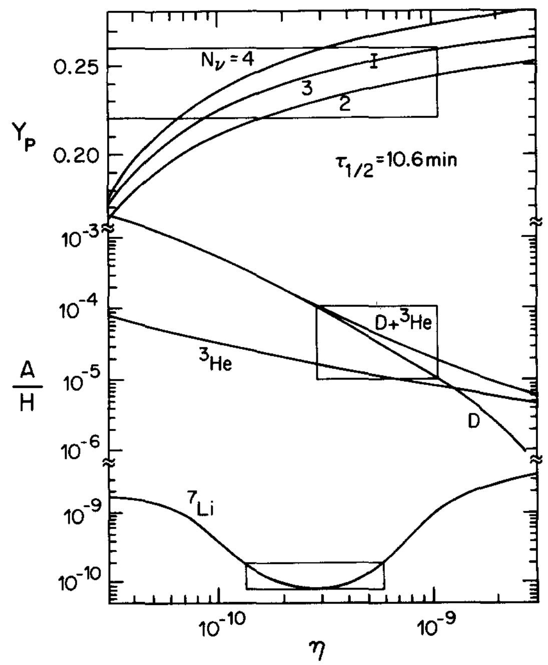

Fig. 1. The calculated abundances of the light elements: ^{4}He (Y_p, by mass) and D, ^{3}He and ^{7}Li (A/H, by number) produced in big bang nucleosynthesis as a function of the baryon-to-photon ratio η. The "error" bar indicates different neutron half-lives: $10.4 \leq \tau_{1/2} \leq 10.8$ min.

number densities separately,

$$n_{\mathrm{B}} = \rho_{\mathrm{B}}/m_{\mathrm{B}} = \Omega_{\mathrm{B}}\rho_{\mathrm{c}}/m_{\mathrm{B}} = 1.13\times 10^{-5}\Omega_{\mathrm{B}}h_0^2\ \mathrm{cm}^{-3}\,, \tag{1.29}$$

where ρ_{B} is the energy density in baryons, m_{B} is the nucleon mass, and Ω_{B} is that part of Ω which is in the form of baryons. The number density of photons is just

$$n_\gamma = [2\zeta(3)/\pi^2]T^3 \simeq 400(T_0/2.7)^3\ \mathrm{cm}^{-3}\,. \tag{1.30}$$

Taking the ratio of (1.29) and (1.30), we have

$$\eta = 2.81\times 10^{-8}\Omega_{\mathrm{B}}h_0^2(2.7/T_0)^3\,. \tag{1.31}$$

Thus we could determine η if we knew Ω_{B}, h_0, and T_0.

There is another important consequence of the nucleosynthesis limits and that is the limit on η [10]:

$$3\times 10^{-10} \le \eta \le 4\times 10^{-10}\,. \tag{1.32}$$

This can be converted to a limit on the baryon density and Ω_{B}. If we turn around eq. (1.31) we have

$$\Omega_{\mathrm{B}} = 3.56\times 10^{7}\eta h_0^{-2}(T_0/2.7)^3\,, \tag{1.33}$$

and using the limits on η (eq. 1.32), h_0 (eq. 1.18a), and T_0 [2.7–2.8 K], we find a range for Ω_{B}:

$$0.01 \le \Omega_{\mathrm{B}} \le 0.10\,. \tag{1.34}$$

Recall that for a closed Universe $\Omega > 1$, thus from eq. (1.34) we can conclude that the Universe is not closed by baryons. This has important consequences on the nature of dark matter [15] in the Universe when combined with the predicted value of Ω from inflation (see section 6).

The success of big bang nucleosynthesis allows us to have confidence in our description of the early Universe up to temperatures $T\sim 1$ MeV. To discuss inflation, we will have to feel comfortable with extrapolations to much higher temperatures. The possible explanation of the small non-zero value for η by grand unified theories (GUTs) is suggestive of the fact that such an extrapolation is reasonable.

The fact that $\eta = 3–4\times 10^{-10}$ is so small, may be viewed as a problem for the (pre-GUT) standard big bang model. We can define η as the net baryon to photon ratio, $\eta = (n_{\mathrm{B}} - n_{\bar{\mathrm{B}}})/n_\gamma$. In the standard model, the entropy density today is related to n_γ by

$$s = \tfrac{4}{3}(\pi^2/30)[g_\gamma + \tfrac{7}{8}N_\nu g_\nu(T_\nu/T_\gamma)^3]T^3 \simeq 7n_\gamma\,, \tag{1.35}$$

where $(T_\nu/T_\gamma)^3 = \frac{4}{11}$ due to the effect of $\mathrm{e}^\pm$ annihilation after neutrino decoupling. Then the nucleosynthesis bound on η (1.32) implies $n_{\mathrm{B}}/s \simeq (4–6)\times 10^{-11}$. If baryon number is conserved this ratio is also conserved and represents an undesirable initial condition.

Let us, for the moment, assume that in fact $\eta = 0$. We can compute the final number density of nucleons left over after annihilations of $\mathrm{B}\bar{\mathrm{B}}$ have frozen out. At very high temperatures (neglecting a quark–hadron phase transition) $T > 1$ GeV, nucleons were in thermal equilibrium with the photon background and $n_N = n_{\bar{N}} = \frac{3}{2} n_\gamma$ (a factor of 2 accounts for neutrons and protons and the factor $\frac{3}{4}$ for the difference between fermi and bose statistics). As the temperature fell below m_N, annihilations kept the nucleon density at its equilibrium value $(n_N/n_\gamma) = (m_N/T)^{3/2} \exp(-m_N/T)$ until the annihilation rate $\Gamma_A \simeq n_N m_\pi^{-2}$ fell below the expansion rate. This occurred at $T \simeq 20$ MeV. However, at that time the nucleon number density had already dropped to

$$n_N/n_\gamma = n_{\bar{N}}/n_\gamma \simeq 10^{-18} , \tag{1.36}$$

which is eight orders of magnitude too small [16]. Aside from this, there is the problem of having to separate the baryons from the antibaryons. If any separation did occur at higher temperatures (so that annihilations were as yet incomplete) the maximum distance scale on which separation could occur is the causal scale related to the age of the Universe at that time. At $T = 20$ MeV, the age of the Universe was only $t = 2 \times 10^{-3}$ s. At that time, a causal region (with distance scale defined by $2ct$) could only have contained $10^{-5}\, M_\odot$ which is very far from the galactic mass scales which we are asking for separations to occur, viz. $10^{12}\, M_\odot$.

Thus we are left with the problem as to the origin of a small non-zero value for η. We can assume that it was an initial condition to start off with and in a baryon number conserving theory it would remain nearly constant. (The production of entropy (photons) could cause it to fall.) In this case, however, we must still ask ourselves, why is it so small? A more attractive possibility, however, is to suppose that the baryon asymmetry was in some way generated by the microphysics. Indeed, if one can show that a small non-zero value for η developed from $\eta = 0$ (or any other value) as an initial condition, we could consider the question solved.

There are three basic ingredients necessary [17] to generate a non-zero η. They are: (1) baryon number violating interactions, (2) C and CP violation and (3) a departure from thermal equilibrium. The first two of these ingredients are contained in GUTs, the third in an expanding Universe where it is not uncommon that interactions come in and out of equilibrium. In SU(5), the fact that quarks and leptons are in the same multiplets allows for baryon non-conserving interactions such as $e^- + d \leftrightarrow \bar{u} + \bar{u}$, etc., or decays of the supermassive gauge bosons X and Y such as $X \to e^- + d$, $\bar{u} + \bar{u}$. Although today these interactions are very ineffective because of the masses of the X and Y bosons, in the early Universe, when $T \sim M_X \sim 10^{15}$ GeV, these types of interactions should have been very important. C and CP violation is very model dependent. In the minimal SU(5) model, the magnitude of C and CP violation is too small to yield a useful value of η. The C and CP violation in general comes from the interference between tree level and first loop corrections.

The departure from equilibrium is very common in the early Universe when interaction rates cannot keep up with the expansion rate. In fact, the simplest (and most useful) scenario for baryon production makes use of the fact that a single decay rate goes out of equilibrium. It is commonly referred to as the out of equilibrium decay scenario [18]. The basic idea is that the gauge bosons X and Y (or Higgs bosons) may have a lifetime long enough to insure that the inverse decays have already ceased so that the baryon number is produced by their free decays.

More specifically, let us call X either the gauge boson or Higgs boson which produces the baryon asymmetry through decays. Let α be its coupling to fermions. For X a gauge boson, α will be the GUT fine structure constant, while for X a Higgs boson, $(4\pi\alpha)^{1/2}$ will be the Yukawa coupling to fermions. The decay rate for X will be

$$\Gamma_D \sim \alpha M_X . \tag{1.37}$$

However, decays can only begin occurring when the age of the Universe is longer than the X lifetime Γ_D^{-1}, i.e., when $\Gamma_D > H$,

$$\alpha M_X \gtrsim N(T)^{1/2} T^2 / M_P \,, \tag{1.38}$$

or at a temperature

$$T^2 \lesssim \alpha M_X M_P N(T)^{-1/2} \,. \tag{1.39}$$

Scatterings on the other hand proceed at a rate

$$\Gamma_S \sim \alpha^2 T^3 / M_X^2 \,, \tag{1.40}$$

and hence are not effective at lower temperatures. To maintain equilibrium at temperatures $T < M_X$, decays must be occurring to lower the number density of X's and $\bar{X}$'s. Thus the condition that decays occur out-of-equilibrium is that at $T = M_X$, $\Gamma_D < H$, or

$$M_X \gtrsim \alpha M_P [N(M_X)]^{-1/2} \sim 10^{18} \alpha \text{ GeV} \,. \tag{1.41}$$

In this case, we can expect a net baryon asymmetry to be produced.

The total baryon asymmetry produced by X, $\bar{X}$ decay is then

$$n_B \sim (\Delta B) n_X \sim (\Delta B) n_\gamma \,, \tag{1.42}$$

where ΔB is the net baryon number produced by an $X\bar{X}$ pair since $n_X = n_{\bar{X}} = n_\gamma$. Although the net baryon number is conserved during the subsequent evolution of the Universe, the photon number density is not. As noted earlier, a more useful quantity is the baryon to entropy ratio n_B/s. At $T \lesssim M_X \sim 10^{15}$ GeV, we expect $N(T) \gtrsim O(100)$ so that $s \sim O(100)\, n_\gamma$. Thus the baryon-to-entropy ratio we would expect to be produced in the out-of-equilibrium decay scenario would be

$$n_B/s \sim 10^{-2} (\Delta B) \,. \tag{1.43}$$

The value of n_B/s that we are looking for must be related to the limits on η from nucleosynthesis. Depending on the model dependent value of ΔB, GUTs have a good chance in being able to explain the small value of η required by nucleosynthesis [19]. In the hope that this explanation might be correct, we extrapolate to temperatures approaching the Planck scale, $M_P \simeq 1.22 \times 10^{19}$ GeV.

1.2. Problems with the standard model

Despite the successes of the standard big bang model, there are a number of unanswered questions that appear difficult to explain without imposing unnatural initial conditions. As we have seen, before the application of GUTs to cosmology, the baryon to photon ratio seemed embarrassingly small. In this section, I will review many of the other problems (most of which were known before GUTs were applied to the standard big bang model) which can for the most part be solved by inflation.

(*a*) *The curvature problem*. The bound in eq. (1.18b) is curious in the fact that at the present time we do not know even the sign of the curvature term in the Friedmann equation (1.7), i.e., we do not know if the Universe is open, closed or spatially flat. Unless $k=0$ exactly and $\Omega=1$, the spatially flat Universe is unstable [20]. The reason being is that Ω evolves with time. If we rewrite eq. (1.16) we have (when $\Lambda=0$)

$$\Omega=(k/R^2H^2)+1\,. \tag{1.44}$$

We can also write $\rho=AR^{-3\gamma}$ so that $R^2H^2=(\frac{1}{3}\kappa^2A)R^{2-3\gamma}-k$ and

$$\Omega=k/[(\tfrac{1}{3}\kappa^2A)R^{2-3\gamma}-k]+1\,. \tag{1.45}$$

For small R, it is clear that $\Omega\rightarrow 1$ $(2-3\gamma<0)$. As R increases, if $k=-1$, $\Omega\rightarrow 0$ and $R\rightarrow\infty$, while for $k=+1$, $\Omega\rightarrow\infty$ as $R\rightarrow R_{\max}=(3/\kappa^2A)^{1/(2-3\gamma)}$. We can trace this behavior of Ω to the fact that $\Omega=1$ is an instability point of (1.44) whenever $\gamma>\frac{2}{3}$.

The curvature problem (or flatness problem) can manifest itself in several ways. First note that we can rewrite eq. (1.9) as

$$R^3\,\mathrm{d}p/\mathrm{d}t=(\mathrm{d}/\mathrm{d}t)[R^3(\rho+p)]\,, \tag{1.46}$$

or

$$(\mathrm{d}/\mathrm{d}t)(R^3s)=0\,, \tag{1.47}$$

where $s=(\rho+p)/T=\mathrm{d}p/\mathrm{d}T$ is the entropy density. For a radiation dominated gas $s\sim T^3$ and $R\sim T^{-1}$. Thus assuming an adiabatically expanding Universe, the quantity $\hat{k}=k/R^2T^2$ is a dimensionless constant. If we now apply the limit in eq. (1.18) to eq. (1.16) we find

$$\hat{k}=k/R^2T^2=(\Omega_0-1)H_0^2/T_0^2<2\times10^{-58}\,. \tag{1.48}$$

This limit on $\hat{k}$ represents an initial condition on the cosmological model. The problem then becomes what physical processes in the early Universe produced a value of $\hat{k}$ so extraordinarily close to zero (or Ω close to one).

A more natural initial condition might have been $\hat{k}\sim\mathrm{O}(1)$. In this case the Universe would have become curvature dominated at $T\sim10^{-1}M_{\mathrm{P}}$. For $k=+1$, this would signify the onset of recollapse. Even for $\hat{k}$ as small as $\mathrm{O}(10^{-40})$ the Universe would have become curvature dominated when $T\sim10\,\mathrm{MeV}$ or when the age of the Universe was only $\mathrm{O}(10^{-2})$ s. Thus not only is (1.48) a very tight constraint, it must also be strictly obeyed. Of course, it is also possible that $k=0$ and the Universe is actually spatially flat.

This problem can also be equivalently expressed as an initial condition of the proximity of Ω to 1 at a particular time. Noting that the dominant contribution to H at early times is the thermal energy density, we can write $H^2\simeq\frac{1}{3}\kappa^2\rho=\frac{1}{90}\kappa^2\pi^2\mathrm{N}\mathrm{T}^4$. Then we can write

$$|\Omega-1|\simeq(90/8\pi^3N)(M_{\mathrm{P}}^2/T^2)\hat{k}\,, \tag{1.49}$$

so that when $T \simeq 1\,\mathrm{MeV}$, $|\Omega - 1| \lesssim 10^{-15}$ and at the GUT scale $T \simeq 10^{15}\,\mathrm{GeV}$, $|\Omega - 1| \lesssim 10^{-52}$. Again we must demand for very special conditions at the epochs we discuss in our cosmological model.

Similarly, one can also express these problems in terms of the age of the Universe or the amount of total entropy in the Universe. The fact that Ω is still close to one is only a problem because the Universe is so old (i.e., H_0 is so small). One can justifiably ask how the Universe lasted for 10–20 billion years with Ω still close to unity. The total entropy in the Universe can be written as $S = R^3 s = (k/\hat{k}T^2)^{3/2} s > 10^{87}$. Again in an adiabatically expanding Universe S is constant and seems arbitrarily high.

(*b*) *The horizon problem*. Because of the cosmological principle, all physical length scales grew as the scale factor $R(t) \sim t^{2/3\gamma}$. However, causality implies the existence of a particle (or causal) horizon $d_{\mathrm{H}} \sim t$, $d_{\mathrm{H}}(t)$ (defined more precisely in section 2) is the maximal physical distance light can travel from the co-moving position of an observer at some initial time to time t. Because γ is greater than $\frac{2}{3}$, scales originating outside of the horizon will eventually become part of our observable Universe. Hence we would expect to see anisotropies on large scales [21].

In particular, let us consider the microwave background today. The photons we observe have been decoupled since recombination at $T_{\mathrm{d}} \sim 4000\,\mathrm{K}$. At that time, the horizon volume was simply $V_{\mathrm{d}} \propto t_{\mathrm{d}}^3$, where t_{d} is the age of the Universe at $T = T_{\mathrm{d}}$. If we take the present age of the Universe to be $t_0 = 1.5 \times 10^{10}\,\mathrm{yr}$ then $t_{\mathrm{d}} = t_0 (T_0/T_{\mathrm{d}})^{3/2} \sim 3 \times 10^5\,\mathrm{yr}$. Our present horizon volume $V_0 \propto t_0^3$ can be scaled back to t_{d} (corresponding to that part of the Universe which expanded to our present visible Universe), $V_0(t_{\mathrm{d}}) \propto V_0 (T_0/T_{\mathrm{d}})^3$. We can now compare $V_0(t_{\mathrm{d}})$ and V_{d}. The ratio

$$V_0(t_{\mathrm{d}})/V_{\mathrm{d}} \propto (V_0/V_{\mathrm{d}})(T_0/T_{\mathrm{d}})^3 \propto (t_0/t_{\mathrm{d}})^3 (T_0/T_{\mathrm{d}})^3 \sim 4 \times 10^4 \tag{1.50}$$

corresponds to the number of horizon volumes or causally distinct regions at decoupling which are encompassed in our present visible horizon.

As we discussed earlier, the microwave background appears highly isotropic on large scales with $\Delta T/T \lesssim (4\text{–}10) \times 10^{-5}$ [5, 6]. The horizon problem, therefore, is the lack of an explanation as to why nearly 10^5 causally disconnected regions at recombination all had the same temperature to within one part in 10^4.

(*c*) *Density perturbations*. Although it appears that the Universe is extremely isotropic and homogeneous (in fact the standard model assumes complete isotropy and homogeneity) it is very inhomogeneous on small scales. In other words, there are planets, stars, galaxies, clusters, etc. On small scales there are large density perturbations. The problem remains to understand the origin and development of such density perturbations, recalling that on large scales $\delta\rho/\rho \sim \delta T/T \lesssim 10^{-4}$ and these perturbations must have grown to $\delta\rho/\rho \sim 1$ on smaller scales.

The evolution of density perturbations in the Robertson–Walker Universe has been studied extensively [22–24]. Assuming small perturbations, it is possible to linearize the general relativistic equations of motion for $(\delta\rho/\rho)(x, t)$. The evolution of Fourier transformed quantity $(\delta\rho/\rho)(k, t)$ depends on the relative size of the wavelength $\lambda \sim k^{-1}$ and the horizon scale H^{-1}. For $k \ll H$ (always true at sufficiently early times), $\delta\rho/\rho \propto t$ while for $k \gg H$, $\delta\rho/\rho$ is constant (assuming a radiation dominated Universe). Because of the growth in $\delta\rho/\rho$, the microwave background limits force $\delta\rho/\rho$ to be extremely small at early times.

Consider a perturbation with wavelength on the order of a galactic scale. Between the Planck time and recombination, such a perturbation would have grown by a factor of $O(10^{57})$ and the anisotropy limit of $\delta\rho/\rho \lesssim 10^{-4}$ implies that $\delta\rho/\rho \lesssim 10^{-61}$ on the scale of a galaxy at the Planck time. One should compare this value with that predicted from purely random (or Poisson) fluctuations of $\delta\rho/\rho \sim 10^{-40}$ (assuming 10^{80} particles (photons) in a galaxy) [25]. The extent of this limit is of course related to the fact that the present age of the Universe is so great.

An additional problem is related to the formation time of the perturbations. A perturbation with a wavelength large enough to correspond to a galaxy today must have formed with wavelength modes much greater than the horizon size if the perturbations are primordial as is generally assumed. This is again due to the fact that the wavelengths red shift as $\lambda \sim R \sim t^{1/2}$ while the horizon size grows linearly. This is very reminiscent of the horizon problem discussed above. A solution would require perturbations with acausal wavelengths.

(*d*) *The magnetic monopole problem*. In addition to the much desired baryon asymmetry produced by grand unified theories, a less favorable aspect is also present. GUTs predict the existence of magnetic monopoles. The monopoles will be produced [26] whenever any simple group [such as SU(5)] is broken down to a gauge group which contains a U(1) factor [such as SU(3) × SU(2) × U(1)]. The mass of such a monopole would be

$$M_{\mathrm{m}} \sim M_{\mathrm{X}}/\alpha_{\mathrm{G}} \sim 10^{16}\ \mathrm{GeV}. \tag{1.51}$$

The basic reason monopoles are produced is that in the breaking of SU(5) the adjoint cannot align itself over all space [27]. On scales larger than the horizon, for example, there is no reason to expect the direction of the Higgs field to be aligned. Because of this randomness, topological knots are expected to occur and these are the magnetic monopoles. We can then estimate that the minimum number of monopoles produced [28] would be roughly one per horizon volume or causally connected region at the time of the SU(5) phase transition t_{c},

$$n_{\mathrm{m}} \sim (2t_{\mathrm{c}})^{-3}. \tag{1.52}$$

The time t_{c} can be expressed in terms of T_{c} through (1.23) as

$$t_{\mathrm{c}} \simeq 0.3 M_{\mathrm{P}} N(T_{\mathrm{c}})^{-1/2} T_{\mathrm{c}}^{-2}, \tag{1.53}$$

so that the monopole-to-photon ratio is

$$n_{\mathrm{m}}/n_{\gamma} \sim (10 T_{\mathrm{c}}/M_{\mathrm{P}})^3. \tag{1.54}$$

The overall mass density of the Universe can be used to place a constraint on the density of monopoles. The mass density of monopoles will be

$$\rho_{\mathrm{m}} = M_{\mathrm{m}} n_{\mathrm{m}}, \tag{1.55}$$

and the fraction of critical density in monopoles can be expressed as

$$\Omega_m h_0^2 = 9.5\times 10^4 M_m(\mathrm{GeV}) n_m(\mathrm{cm}^{-3})\,. \tag{1.56}$$

Thus for $M_m \sim 10^{16}$ GeV and $\Omega h_0^2 < 1$ we have that

$$n_m/n_\gamma < \mathrm{O}(10^{-25})\,. \tag{1.57}$$

The predicted density, however, from (1.54) for $T_c \sim M_X \sim 10^{15}$ GeV yields

$$(n_m/n_\gamma) \sim 10^{-9}\,. \tag{1.58}$$

Hence, we see that standard GUTs and cosmology have a monopole problem.

(*e*) *Examples of other problems.* To conclude this section I will just briefly mention two other problems which arise: one in the standard model and the other when supergravity is included.

The first problem is the rotation problem [29], which is similar to the curvature problem. The rotation of the Universe can be expressed as an anisotropy to which is associated a preferred direction and angular momentum. The strongest limits on the angular velocity are due to the microwave background radiation and yield $\omega < 10^{-21}\ \mathrm{s}^{-1}$ [30]. ω will scale with the expansion of the Universe and the scaling depends on the equation of state, $\omega \sim R^{3\gamma-5}$. At the Planck time, the limit on ω/T (a dimensionless constant) becomes $\omega/T < 2\times 10^{-29}$. Because a rotation term would enter into the field equations as ω^2, this limit is very similar to the limit in eq. (1.48). One would expect $\omega/T \sim \mathrm{O}(1)$.

The last problem I will discuss appears in the context of supergravity and is called the gravitino problem [31]. In some ways this problem is similar to the monopole problem. If gravitinos were initially in equilibrium and decoupled at $T \sim M_P$, the mass density of gravitinos today would be

$$\rho_{3/2} \simeq m_{3/2} n_{3/2} \simeq m_{3/2}[3\zeta(3)/\pi^2] T_{3/2}^3\,, \tag{1.59}$$

where $T_{3/2} < T_\gamma$ is the present temperature of gravitinos assuming they are stable. Taking $T_{3/2}^3 \sim 10^{-2} T_\gamma^3$ [12] and $\rho_{3/2} < \rho_c$, one would find $m_{3/2} < \mathrm{O}(1)$ keV [32]. If gravitinos were more massive they might be unstable, but one would expect their decay rate to be roughly $\Gamma_{3/2} \sim m_{3/2}^3/M_P^2$. To avoid problems with nucleosynthesis [31], one must require that the reheat temperature after decay must be $T_R \gtrsim 1$ MeV. The decay occurs when $H(T_d) \sim m_{3/2}^{1/2} T_d^{3/2}/M_P \sim m_{3/2}^3/M_P^2 \sim \Gamma_{3/2}$ or at $T_d \sim m_{3/2}^{5/3}/M_P^{2/3}$. After decay, $T_R^4 \sim m_{3/2} T_d^3$ or $T_R \sim m_{3/2}^{3/2}/M_P^{1/2}$, implying that $m_{3/2} > 20$ TeV. Unfortunately one expects $m_{3/2} \sim 10$–10^3 GeV if supersymmetry is to be consistent with low energy phenomenology and explain the gauge hierarchy problem.

1.3. *Inflation before inflation*

In this section I would like to briefly review the important work that went into the inflationary Universe scenario prior to the world-view changing work of Guth [33]. The earliest work that I am aware of is a paper by Gliner [34] in 1966. In that paper, Gliner looks at the various possible forms for the eigenvalues of the energy–momentum tensor and their description as different types of matter. He concludes that the case when the magnitudes of all four eigenvalues are equal (as in the case of a cosmological constant and no ordinary matter) corresponds to "matter" with the properties of a vacuum. Hence he concludes that a vacuum dominated Universe with positive energy density must correspond to a De Sitter model. This idea was further developed by Zeldovich [35].

In a later paper, Gliner and Dymnikova [36] came very close to what is the present theory of inflation. In that paper, they assume a transition from a vacuum dominated state to a radiation dominated one. Their idea was actually to remove the initial singularity with De Sitter space. Their model is then restricted by ensuring that the total entropy of the Universe agrees with the observed entropy. They also choose two possible values for the energy density of the vacuum: (1) the scale set by weak interactions $\rho \sim (10^2\ \mathrm{GeV})^4$, and (2) the Planck scale $\rho \sim (10^{19}\ \mathrm{GeV})^4$. Although grand unification was introduced [37] a year earlier, they can hardly be faulted for not discussing a GUT transition. Their results show that the transition produces an enormous growth of the scale factor and indeed for the Planck-scale vacuum, there is a change in the scale factor by about 30 orders of magnitude, remarkably similar to the present goals of inflationary models. Further work on utilizing a transition from a De Sitter expansion to a Friedmann Universe with the absence of singularities and horizons was done in ref. [38] and a connection to GUTs was made in ref. [39]. Quantum gravitational effects and the use of higher derivative curvature terms were studied in ref. [40] with a similar goal in mind.

Alternative mechanisms for the production of entropy in the Universe dissipation due to bulk viscosity surfaced briefly [41]. Attempts at using such a mechanism during the GUT [42] and electroweak [43] phase transitions did not succeed in producing enough entropy. Though these were failed attempts, they kept in focus the magnitude of the entropy problems.

Close to the eventual inflationary scenario, was the detailed examination of cosmological phase transitions in gauge theories [44]. Kolb and Wolfram [45] studied the cosmological consequences of the $SU(2) \times U(1)$ phase transition and showed in detail that for a first order transition, the Universe could have been dominated by the vacuum energy density of the symmetric phase and that acting like a cosmological constant, the expansion rate of the Universe was exponential rather than a simple power law. In addition, they noted that if strong enough, the phase transition could produce a great deal of entropy and perhaps even density inhomogeneities. Unfortunately they did not carry this thought further. The dramatic effects of the $SU(2) \times U(1)$ phase transition on the cosmological expansion were also discussed by Sher [46]. For the rest of this paper, a working definition of a first (second) order phase transition is a discontinuous (continuous) change in some order parameter (in most cases the vacuum expectation value of some scalar field) as the temperature of the Universe changes.

Sato [47] also studied the effects of a first order phase transition in the early Universe. Looking at a GUT phase transition, he showed that the horizon could be stretched exponentially large. Sato [47] was mainly concerned with preserving a baryon symmetric Universe and the domain walls produced by theories with spontaneously broken *CP*. Such a scenario, however, has little hope in deriving a baryon-to-photon ratio of the right order of magnitude, although such avenues have been pursued in ref. [48]. In a second paper, Sato [49] looked carefully at the mechanism in which the phase transition proceeds, i.e., through the nucleation of bubbles. He realized that, unless the nucleation rate was fairly large, such a phase transition might never be completed, a preview to the fate of the original model of inflation [50].

Independently, Kazanas [51] also showed that the effects of a first order transition could have greatly changed the expansion laws of the early Universe. More importantly, Kazanas had asked whether or not the exponential expansion could have lasted long enough to account for the observed isotropy of the Universe today, i.e. one of the key problems which inflation sets out to solve.

At about the same time, Lapchinsky, Rubakov and Veryaskin [52] also tried to account for the large amount of entropy in the Universe by means of a supercooled GUT and/or electroweak phase transition, using the renormalization group equations for the running scalar self-coupling and gauge coupling constants. They found, however, an insufficient increase in the entropy from either transition because of the absence of extreme supercooling. A similar result (in another context) was found by Sher [53]. It was speculated [52] that perhaps an enlarged [above SU(5)] gauge sector and a series of phase transitions could provide the desired entropy increase.

In addition to the awareness of the entropy and horizon problems, the excitement of the possible explanation of the baryon asymmetry by GUTs kept the monopole problem [28] in focus. A possible solution to the monopole problem was proposed by Guth and Tye [54] by considering a weakly first order GUT phase transition, where $T_c \lesssim 10^{10}$ GeV. With this value of T_c, the monopole abundance in eq. (1.54) is in agreement with the bound in eq. (1.57). Attempts to reduce the monopole abundance by enhanced annihilation due to the gravitational clustering of monopoles was discussed in ref. [55], and by means of a second order GUT transition by Bais and Rudaz [56]. For further discussion on the pre-inflationary suppression of monopoles by supercooled GUT phase transitions see ref. [57].

2. The inflationary Universe scenario

In the previous discussion of pre-inflationary solutions to cosmological problems, I have referred extensively to the effects of cosmological phase transitions in gauge theories and in particular to GUTs. Before reviewing Guth's original inflationary scenario [33], I will discuss in some detail the nature of phase transitions and symmetry restoration at high temperatures [44] in the early Universe.

Let us consider the simplest example of single (real) massive scalar field ϕ with ϕ^4 self-interactions described by the Lagrangian

$$\mathscr{L} = -\tfrac{1}{2}(\partial_\mu \phi)^2 - V(\phi)\,, \tag{2.1}$$

where the tree-level scalar potential is given by

$$V_0(\phi) = \tfrac{1}{4}\lambda\phi^4 - \tfrac{1}{2}\mu^2\phi^2 + \hat{V}\,, \tag{2.2}$$

and λ is the scalar self-coupling, $-\mu^2$ its mass and $\hat{V}$ is a constant. The potential is shown in fig. 2 ($T=0$). As is well known, such a potential leads to spontaneous symmetry breaking and the scalar field picks up a vacuum expectation value (vev) at its minimum value $\langle 0|\phi|0\rangle = v = \pm\sqrt{\mu^2/\lambda}$. If $\hat{V}$ is chosen to be $\mu^4/4\lambda$, the potential vanishes at the minima. By shifting the field to its vacuum value, $\hat{\phi} = \phi - v$, the potential becomes

$$V_0(\hat{\phi}) = \tfrac{1}{4}\lambda\hat{\phi}^4 + \lambda v\hat{\phi}^3 + \mu^2\hat{\phi}^2\,, \tag{2.3}$$

so that the physical scalar field, $\hat{\phi}$, has a mass $m_{\hat{\phi}}^2 = 2\mu^2$.

One-loop corrections to the potential (2.2) (see below) give rise to finite temperature corrections [58],

$$V(\phi) = V_0(\phi) + \frac{T}{2\pi^2}\int_0^\infty k^2\,\mathrm{d}k\,\ln\{1 - \exp[(k^2 + \partial^2 V_0/\partial\phi^2)/T^2]\}\,. \tag{2.4}$$

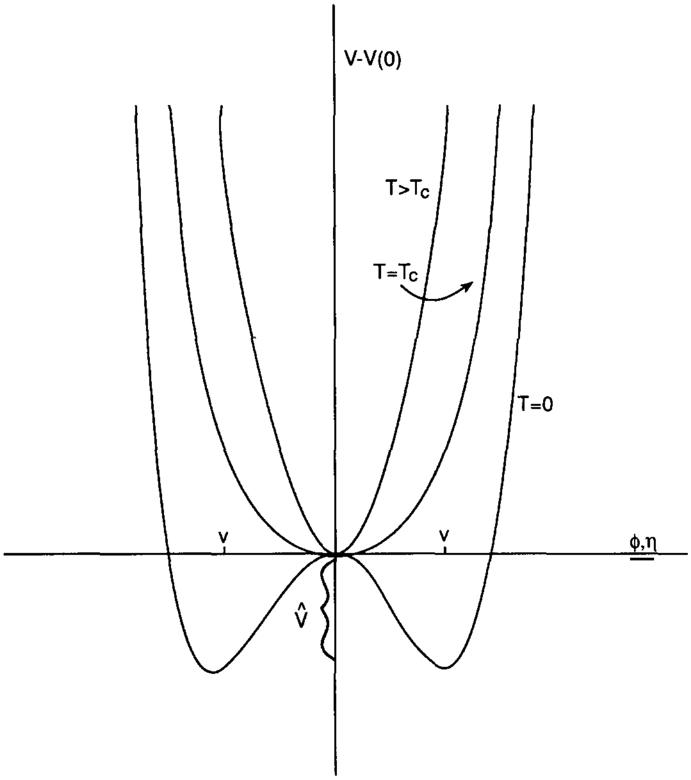

Fig. 2. Schematic drawing of the scalar potential $V - V(0)$, for either global symmetry breaking or local symmetry breaking when $\lambda \gg e^4$. In both cases there is a second order phase transition and the scalar field shifts continuously from the origin to $v = (\mu^2/\lambda)^{1/2}$.

In the range $T \gg m_\phi$,

$$V(\phi) = V_0(\phi) + \tfrac{1}{24} \frac{\partial^2 V_0}{\partial \phi^2} T^2 - \tfrac{1}{90}\pi^2 T^4 = \tfrac{1}{4}\lambda\phi^4 - \tfrac{1}{2}(\mu^2 - \tfrac{1}{4}\lambda T^2)\phi^2 - \tfrac{1}{24}\mu^2 T^2 - \tfrac{1}{90}\pi^2 T^4 + \hat{V}\,, \tag{2.5}$$

so that the effective mass is $m_\phi^2 = \frac{1}{4}\lambda T^2 - \mu^2$. At $T > T_c = (4\mu^2/\lambda)^{1/2}$, the minima at $\langle\phi\rangle = \pm v$ disappear and the symmetry is restored and $\langle 0|\phi|0\rangle = 0$.

Similarily, if we were to consider a complex scalar field, ϕ, which is not a singlet under some local U(1) gauge symmetry, the Lagrangian becomes

$$\mathscr{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - (\partial_\mu + \mathrm{i}eA_\mu)\phi^*(\partial^\mu - \mathrm{i}eA^\mu)\phi - V(\phi)\,, \tag{2.6}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and

$$V(\phi) = \lambda(\phi\phi^*)^2 - \mu^2\phi\phi^* + \hat{V}\,. \tag{2.7}$$

If we write $\phi = \eta\, \mathrm{e}^{\mathrm{i}\xi}/\sqrt{2}$, it is only the real field η which picks up a vev. After a gauge transformation

$\phi \to e^{-i\xi}\phi$ and $A_\mu \to A'_\mu = A_\mu - (1/e)\partial_\mu \xi$ the ξ field is eaten by A^μ and we are left with [after shifting the field η to $\hat{\eta} = \eta - v$ with $v = (\mu^2/\lambda)^{1/2}$]

$$\mathscr{L} = -\tfrac{1}{4}F'_{\mu\nu}F'^{\mu\nu} - \tfrac{1}{2}\partial_\mu \hat{\eta}\partial^\mu \hat{\eta} - \tfrac{1}{2}e^2v^2A'_\mu A'^\mu - \tfrac{1}{2}e^2A'_\mu A'^\mu \hat{\eta}(2v + \hat{\eta})$$
$$- \tfrac{1}{2}\hat{\eta}^2(3\lambda v^2 - \mu^2) - \lambda v \hat{\eta}^3 - \tfrac{1}{4}\lambda\hat{\eta}^4 . \tag{2.8}$$

At finite temperatures, one-loop corrections to the potential yield

$$V(\eta) = \tfrac{1}{4}\lambda\eta^4 - \tfrac{1}{2}[\mu^2 - \tfrac{1}{12}(4\lambda + 3e^2)T^2]\eta^2 + \hat{V}' , \tag{2.9}$$

where the additional constant terms have been absorbed into $\hat{V}'$. In this case there is again a second order phase transition with $T_c^2 = 12\mu^2/(4\lambda + 3e^2)$. See fig. 2. When $\lambda \lesssim e^4$, the approximation $T \gg m_\phi$, m_A is no longer valid near T_c, e.g. when $\lambda \ll e^4$, $m_A^2 \gtrsim T_c$. In this case there is a first order transition. There is a temperature $T_{c'} \sim ev$, where a second minimum appears. (Recall the previous second order transitions, there is always a single minimum to the potential which changes continuously from $|\phi| = 0$ to $|\phi| = v$). At a temperature between T_c and $T_{c'}$, the global minimum shifts from $|\phi| = 0$ to $|\phi| = v$, although there is still a barrier separating the two phases. Finally at $T = T_c$, the barrier too disappears and the sole minimum continues to evolve towards $\phi = v$. See fig. 3. For smaller values of λ, $\lambda < 3e^4/16\pi^2$, one-loop corrections at $T = 0$ also become important. For the simple model above and neglecting the scalar loops [59, 60], the scalar potential can be expressed as

$$V(\eta) = V_0(\eta) + \frac{3e^4\mu^2}{32\lambda\pi^2}\,\eta^2 + \frac{3e^4}{64\pi^2}\,\eta^4 \ln(\eta^2/v^2 - \tfrac{3}{2}) . \tag{2.10}$$

If we can neglect both μ and λ, we can approximate the potential and write it in general as

$$V(\eta) = A\eta^4(\ln \eta^2/v^2 - \tfrac{1}{2}) , \tag{2.11}$$

$$A = \frac{1}{64\pi^2v^4}\Big(\sum_{\mathrm{B}} g_{\mathrm{B}}m_{\mathrm{B}}^4 - \sum_{\mathrm{F}} g_{\mathrm{F}}m_{\mathrm{F}}^4\Big) , \tag{2.12}$$

and $g_{\mathrm{B(F)}}$ is the number of degrees of freedom for the bosons (fermions) of mass $m_{\mathrm{B(F)}}$ entering into the loop. For a more complete discussion of the one-loop effective potential see section 3. For our example, $\Sigma_{\mathrm{B}}\, g_{\mathrm{B}}m_{\mathrm{B}}^4 = 3(ev)^4$ where we neglect the scalar contribution to the loop, i.e. when $e^2 \gg \lambda$. At finite temperature, one adds an expression similar to eq. (2.4), which can be expanded for $\eta \ll T$ as

$$V_T(\eta) = -\tfrac{1}{90}\pi^2\Big(\sum_{\mathrm{B}} g_{\mathrm{B}} + \tfrac{7}{8}\sum_{\mathrm{F}} g_{\mathrm{F}}\Big)T^4 + \tfrac{1}{2}\Big(\sum_{\mathrm{B}} \tfrac{1}{12}g_{\mathrm{B}}m_{\mathrm{B}}^2 + \sum_{\mathrm{F}} \tfrac{1}{24}g_{\mathrm{F}}m_{\mathrm{F}}^2\Big)T^2 , \tag{2.13}$$

where the masses in eq. (2.13) may be dependent on η.

When $\lambda < 3e^4/16\pi^2$, the phase transition is still first order, but the barrier remains present even at very low temperatures. This possibility is displayed in fig. 4. Furthermore when $\lambda < 3e^4/32\pi^2$, the global minimum stays at $|\phi| = 0$.

Symmetry restoration at high temperatures for more complicated gauge theories follows a similar pattern. For the case of SU(5), for example, we can write the potential for the adjoint, Σ, as [61]

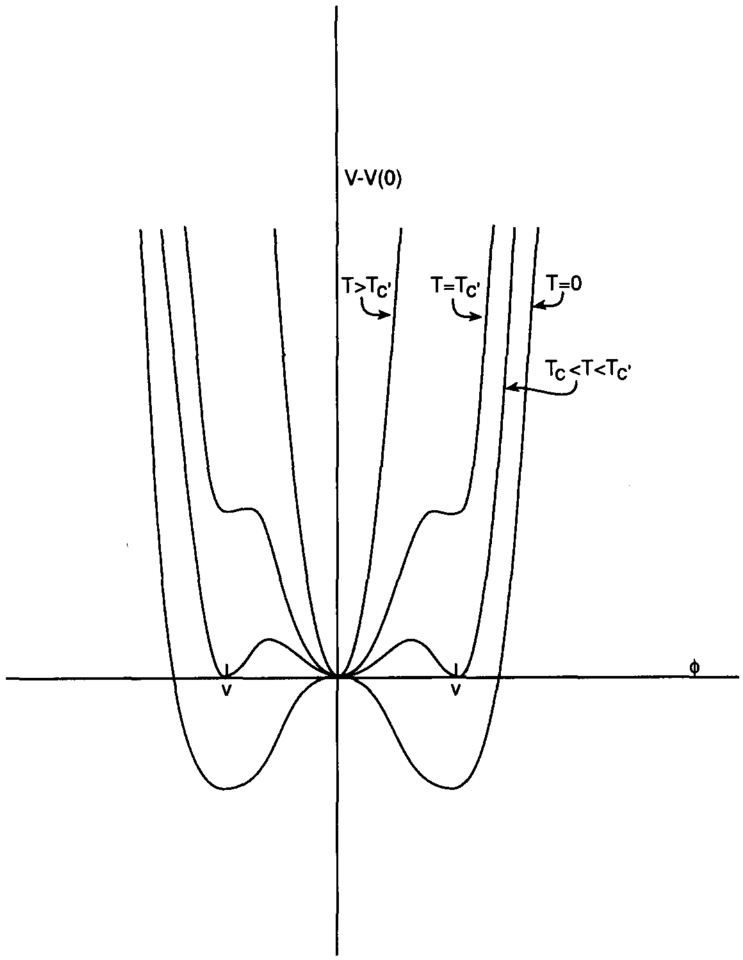

Fig. 3. Schematic drawing of the scalar potential $V - V(0)$ for local symmetry breaking with $3e^4/16\pi^2 < \lambda < e^4$. There is a weak first order phase transition.

$$V(\Sigma) = -\tfrac{1}{2}\mu^2\,\mathrm{Tr}(\Sigma^2) + \tfrac{1}{4}a[\mathrm{Tr}(\Sigma^2)]^2 + \tfrac{1}{2}b\,\mathrm{Tr}(\Sigma^4) + \tfrac{1}{3}c\,\mathrm{Tr}(\Sigma^3)\,. \tag{2.14}$$

For simplicity, consider first the case with $c = 0$. Then for $a > -\frac{7}{15}b$ and $b > 0$ [62], the ground state of the theory is the $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ vacuum with a Higgs expectation value

$$\langle 0|\Sigma|0\rangle = \mathrm{diag}(v, v, v, -\tfrac{3}{2}v, -\tfrac{3}{2}v)\,, \tag{2.15}$$

and $v^2 = 2\mu^2/(15a + 7b)$.

As in the previous examples, the SU(5) gauge symmetry is restored at high temperatures. For sufficiently small values of a and b, the Σ-dependent temperature corrections from eq. (2.13) are obtained by taking the masses of superheavy gauge bosons $m_X^2 = m_Y^2 = \frac{25}{8}g_5^2v^2$. g_5 is the SU(5) gauge coupling ($g_5^2/4\pi \sim \frac{1}{41}$) and $g_X + g_Y = 36$, so that

$$V_T(\Sigma) \simeq \tfrac{75}{16}g^2T^2\,\mathrm{Tr}(\Sigma^2)\,. \tag{2.16}$$

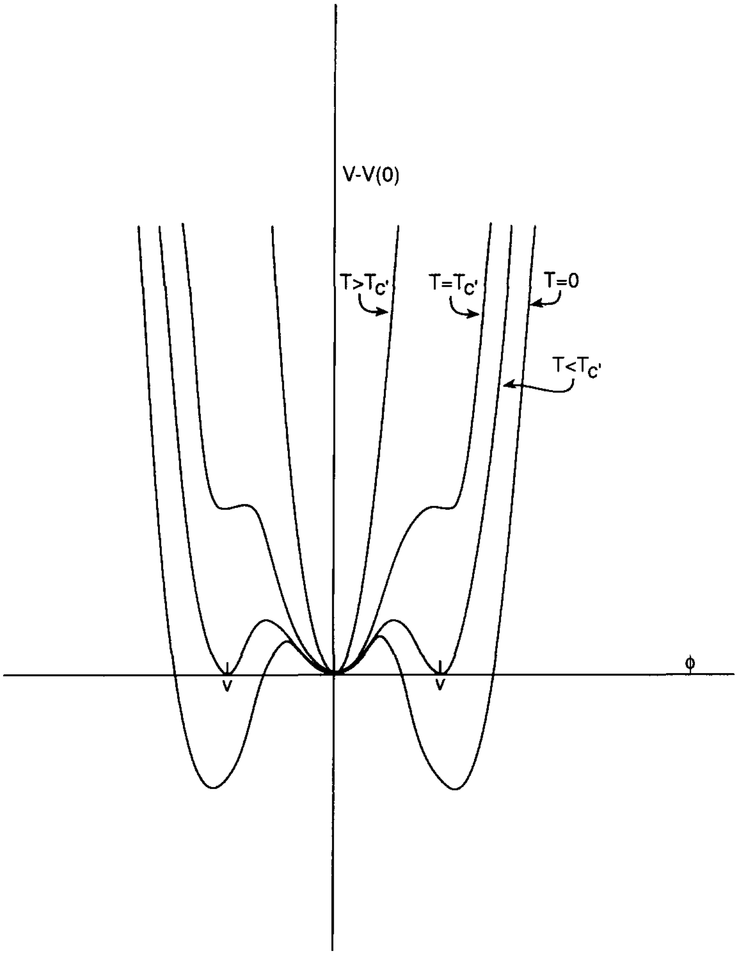

Fig. 4. Schematic drawing of the scalar potential $V - V(0)$ for local symmetry breaking with $3e^4/32\pi^2 < \lambda < 3e^4/16\pi^2$. There is a first order phase transition.

For small values of a and b, radiative corrections to eq. (2.14) may be important. The contribution from eq. (2.11) gives [63] $A = (5625/1024\pi^2)g_5^4$ and

$$V_1(\sigma) = A\sigma^4(\ln \sigma^2/v^2 - \tfrac{1}{2}) , \qquad (2.17)$$

where σ is the scalar field related to $\mathrm{Tr}\,\Sigma$ by an SU(5) transformation. The importance of radiative SU(5) symmetry breaking will be discussed in more detail in section 3.

In the absence of the one-loop corrections or the cubic terms in eq. (2.14), the SU(5) phase transition is a second order transition. For $c \neq 0$, there are additional possibilities [64, 54, 65, 66]. For example, the global minimum may be $\mathrm{SU}(4) \times \mathrm{U}(1)$ rather than $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$, and may involve more than a single phase transition as discussed in detail in refs. [64–66]. At high temperatures, the Universe begins in the SU(5) symmetric state. Depending on the couplings a and b, the transition may take one of the following possibilities: (1) for $a/b > -\frac{6}{15}$, the $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ phase is always preceded by an $SU(4) \times U(1)$ phase. (2) for $a/b < -\frac{6}{15}$ the transition proceeds directly to $SU(3) \times SU(2) \times U(1)$. The efficiency of the transition also depends somewhat sensitively on a and b. In an investigation of the gauge hierarchy problem it was concluded [63] that $a/b \gtrsim -\frac{7}{15}$ with $a \simeq -0.028$ and $b \simeq 0.06$. The smallness of these couplings implies the necessity of including one-loop corrections to the potential.

In a first order phase transition the Universe will generally supercool before the transition is completed. The presence of a barrier between the true and false vacuum means that, provided the cooling is fast enough, the Universe can get "hung up" in the metastable false vacuum, thus supercooling. At zero temperature, the transition is a quantum mechanical process corresponding to the tunneling of the scalar field (σ) through the potential barrier into the true vacuum [67]. The tunnelling rate or the probability of forming a bubble of the broken phase larger than a critical size per unit time per unit volume, was found to correspond to the Euclidean bounce action

$$P \sim M^4 \mathrm{e}^{-B}, \tag{2.18}$$

where M is some mass scale associated with the potential (e.g. the height of the barrier) and B is the bounce action. Depending on values of the parameters a, b, c and μ^2, the action may be quite large, thus producing a strong first order transition with a great deal of supercooling. At finite temperature [68–70], the action takes the form $B \sim E(T)/T$, where $E(T)$ is the bubble energy. It is generally smaller than the zero-temperature action and for the $SU(4) \times U(1) \rightarrow SU(3) \times SU(2) \times U(1)$ transition, it is roughly an order of magnitude smaller [65].

In the early Universe, the phase transition would proceed by the nucleation of bubbles at a rate given by eq. (2.18). Bubbles larger than a critical size would expand. The fraction of space remaining in the old phase is given by [54, 65]

$$f(t) = \exp\left(-\int_{t_0}^{t} \mathrm{d}t_1\, P(t_1) R^3(t_1) V(t_1, t)\right), \tag{2.19}$$

where

$$V(t_1, t) = \frac{4\pi}{3} \left(\int_{t_1}^{t} \mathrm{d}t_2\, R^{-1}(t_2)\right)^3 \tag{2.20}$$

is the coordinate volume of a bubble formed at time t_1 and t_0 is the onset of nucleation. If T^* corresponds to the minimal action, it was found [49, 65] that there is a critical value of $V(T^*)$ such that for smaller actions, the nucleation rate is rapid and the $f(t)$ decreases to zero (thus completing the phase transition). For larger actions, $P(t)$ is not driven to zero by the production of bubbles due to thermal fluctuations. Instead one must rely on the zero-temperature action and the quantum mechanical tunnelling. In this case supercooling may occur. As we shall see shortly, this might not be sufficient for completing the phase transition at all [50].

The utilization of cosmological phase transitions as described above, is the basis of the inflationary Universe scenario [33]. In a first order phase transition, the Universe supercools past T_c until bubbles of the low temperature phase nucleate, grow and percolate. If the nucleation rate is sufficiently slow, the

Universe will continue to supercool. If T_S is the temperature at which the transition is completed (assuming that it does complete), and T_R is the temperature to which the Universe reheats due to the release of the latent heat of transition, then the entropy released by the phase transition will be simply related to $(T_R/T_S)^3$. During this time, however, the scale factor will have increased by $(R_S/R_c) \sim T_R/T_S$ if $T_R \lesssim T_c$, and $R_s \sim R_R$.

The events leading to the release of entropy are a consequence of some dramatic changes in the evolution of the Universe. Suppose that prior to the phase transition, the Universe is in the symmetric false vacuum state. [For simplicity let us consider only a two-phase model with the symmetric state corresponding to $\langle \sigma \rangle = 0$, σ may be related to the Higgs adjoint of SU(5)]. The two vacua are separated by a barrier of sufficient height so as to produce supercooling. The energy density is then given by

$$\rho = \tfrac{1}{30}\pi^2 N(T) T^4 + V(0)\,, \tag{2.21}$$

where $V(0)$ is the vacuum energy density in the broken phase. At sufficiently high temperatures, the Universe is radiation dominated and the scale factor grows as $R(t) \sim t^{1/2}$ as is described in section 1 [cf. eq. (1.10)]. However, when $T < [30V(0)/\pi^2 N(T)]^{1/4}$, the Universe becomes dominated by the vacuum energy density $V(0)$. For a constant energy density ($p = -\rho$), eq. (1.7) becomes (neglecting k)

$$(\dot{R}/R)^2 = [\tfrac{1}{3}\kappa^2 V(0)]^{1/2}\,, \tag{2.22}$$

and one can associate $\kappa^2 V(0)$ with a cosmological constant Λ. The solution to eq. (2.22) is

$$R(t) \sim \exp[(\Lambda/3)^{1/2} t] = \exp(Ht)\,. \tag{2.23}$$

The Universe in this state asymptotically approaches a De Sitter Universe. The scale factor expands exponentially (inflates) and the temperature falls exponentially. As we shall see the cosmological problems of the previous section would be solved if $H\tau > 65$ where τ is the duration of the exponential expansion. When the transition is completed the collision of bubbles of the broken true vacuum reheat the Universe, thus restoring a radiation dominated Friedmann Universe. See fig. 5 for a typical potential in the old inflationary scenario. The energy density stored in the bubble walls is $\sim V(0)$ and the reheat temperature is approximately

$$\tfrac{1}{30}\pi^2 N(T_R) T_R^4 \simeq V(0)\,. \tag{2.24}$$

It is straightforward to check that this scenario indeed solves the main cosmological problems discussed in the previous section. First, recall the parameter $\hat{k} = k/R^2T^2$. During an *adiabatic* expansion this quantity remains constant, but during the inflationary phase transition $\hat{k}_f/\hat{k}_i = (R^2T^2)_i/(R^2T^2)_f \simeq (R_S^2 T_S^2)/(R_R^2 T_R^2) \simeq (T_S/T_R)^2$. Thus if $T_S/T_R \lesssim 10^{-29}$, even a value of $\hat{k} \sim O(1)$*) initially could be driven small enough to come into agreement with the bound in eq. (1.48).

Further consequences of such extreme supercooling (by at least 29 orders of magnitude) can be seen from eq. (1.49). After the phase transition, it would appear that because of the small values of $\hat{k}$, we

*) A large value of $\hat{k}$ may still be problematic if $k = +1$. The Universe may recollapse before it has a chance to inflate. This has been called the pre-inflation problem [71] but is certainly less severe than the original curvature problem.

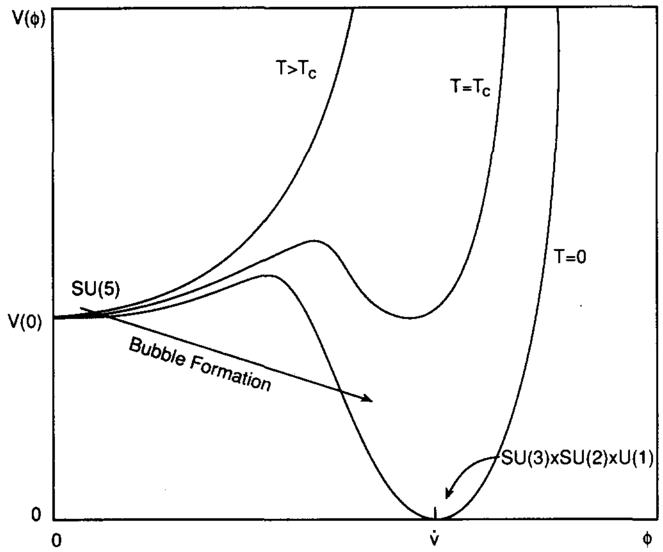

Fig. 5. Schematic drawing of a scalar potential in the old inflationary scenario. The scalar field ϕ can be associated with σ of the SU(5) adjoint. This first order phase transitions occurs via tunnelling and bubble formation.

would naturally expect Ω to be close to unity. Indeed, because of the exponential growth in the scale factor, we might expect values of $\hat{k}$ much smaller than its limiting value. In this case, Ω might be indistinguishable from unity. Consequently, the total entropy with our horizon may well exceed its minimal value of 10^{87}.

In addition to the curvature problem, inflation also solves the horizon problem. The horizon problem, recall, originates because the distance to the particle horizon

$$d_{\mathrm{H}}(t) = R(t) \int_0^t \mathrm{d}t'\, R^{-1}(t') \tag{2.25}$$

grows linearly with time during a radiation or matter dominated era,

$$d_{\mathrm{H}}(t) = 2t, \qquad \text{radiation dominated,} \qquad d_{\mathrm{H}}(t) = 3t, \qquad \text{matter dominated}\,, \tag{2.26}$$

and thus grows faster than co-moving separations given by the growth in the scale factor R, [cf. eqs. (1.10), (1.11)]. During the inflationary epoch d_{H} grows exponentially with time,

$$d_{\mathrm{H}}(t) = (1/H)(\mathrm{e}^{Ht} - 1)\,. \tag{2.27}$$

The problem of causality and the microwave background radiation at recombination now disappears. Whereas in the standard model, the horizon volume today contains $\sim 10^5$ disconnected causal regions scaled back to re-combination, in the inflationary model, because of the exponential expansion, the particle horizon has been pushed out to exponentially large distance scales and in fact the 10^5 volumes have been in causal contact since inflation (though it may happen that particles were separated to distances so large that the time it takes for them to interact again is much longer than the age of the Universe). In fig. 6, the particle horizon, and co-moving distance scales are shown schematically as a function of time. I have chosen the co-moving scale to correspond to one which is equal to the particle horizon today.

The problem of density perturbations is likewise easy to resolve in an inflationary Universe model. Any perturbation with a wavelength corresponding to a galaxy today, necessarily had at its origin, wavelengths outside the horizon at an early epoch. Though microphysical processes would be incapable of producing such "acausal" perturbations, inflation does. A given wavelength of a perturbation gets pushed ($\lambda \sim R(t) \sim e^{Ht}$) outside the scale we would have associated with the horizon in the absence of inflation, i.e., λ grows faster than t (of course nothing actually gets pushed outside the true particle horizon). After inflation, it seems that perturbations are falling within our horizon (cf. fig. 6). I will defer the resolution of the problem regarding the amplitude of the perturbation until section 4.

At this point, it should come as no surprise that inflation easily solves the monopole problem. Even before inflation, it was recognized [54] that a supercooled first order phase transition could dilute the monopole abundance. In an inflationary Universe the monopole abundance gets driven to exponentially small values $n_m \sim e^{-3Ht}$. Even the minimal amount of inflation required to resolve the standard cosmological problems would drop the monopole density by 87 orders of magnitude. In this case the observation of a single monopole would either be truely remarkable or invalidate the inflationary scenario. So long as the GUT symmetry was not restored after inflation, there would be no further production of monopoles. For other attempts at using GUTs to solve the monopole problem, see ref. [72]. I will return to the subject of monopoles and inflation in section 6.

Other problems discussed in the previous section are also easily solved. Rotation [29] would be reduced so that for minimal inflation, rotation might even be at an observable level [73]. Gravitinos as in the case of monopoles are inflated away [74] to acceptable levels, though there is a potential problem with the regeneration of gravitinos during reheating [31, 74–80]. I will return to gravitinos and other miscellaneous effects of inflation in section 6.

Despite the number of problems inflation appears to resolve, the escape from the De Sitter-like expansion or the so-called "graceful exit" to a Friedmann Universe must be addressed. As mentioned

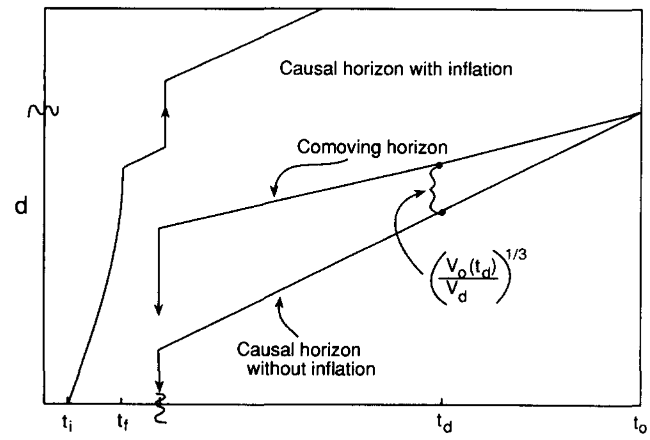

Fig. 6. The co-moving and causal horizons as a function of time with and without inflation. Without inflation the co-moving horizon volume is much greater than the particle horizon volume at the time of decoupling. With inflation the true particle horizon is pushed out to exponentially large distance scales.

previously, there seemed to be a delicate balance between sufficient inflation and the completion of the phase transition [49, 50]. It was in fact shown in detail by Guth and Weinberg [50] that the two are in fact incompatible. Their argument can be summarized fairly simply (see aso ref. [25]). Consider the formation of a bubble at time t_B. After its formation, it grows at the speed of light. From eq. (2.20) we would find that the volume of the bubble goes asymptotically to

$$V_B = \frac{4}{3}\pi H^{-3}\, e^{-3Ht_B}\,, \tag{2.28}$$

even though it continues to grow, as does the coordinate system. This means that two bubbles formed simultaneously with a separation greater than $\sim 2H^{-1}$ would never collide even though their surfaces are moving at the speed of light. This is just the realization of the existence of event horizons in De Sitter space. The integral in eq. (2.19) can easily be done assuming a constant bubble formation rate [eq. (2.18)],

$$f(t) = \exp\{-\frac{4}{3}\pi P H^{-4}[H(t-t_B) - \frac{11}{6} + O(e^{-H(t-t_0)})]\}\,. \tag{2.29}$$

Even though the fraction of space in the broken phase goes to zero exponentially, for $PH^{-4} < 9/4\pi$, the volume of space in the old phase *increases* with time.

Sufficient inflation would require $Ht \gtrsim 65$ before $f(t)$ was too small. Thus we must demand $PH^{-4} < 3/4\pi(65) \sim 10^{-2}$. Completion of the transition, i.e., the release of the latent heat through bubble collisions requires percolation. Without percolation, the Universe consists of largely isolated empty bubbles, hardly the picture of a homogeneous and isotropic hot Universe. Guth and Weinberg [50] showed that for $PH^{-4} < O(10^{-6})$, the Universe only consists of isolated finite sized clusters of bubbles. Complete percolation takes place if $PH^{-4} \gtrsim 0.24$. Because P depends exponentially on the calculated value of the action, the small range in which inflation takes place and percolates is extremely finetuned, e.g., values of P as small as 10^{-10^3} were found.

Attempts [81] at forming galaxies due to the collisions of large bubbles also failed due to low temperatures, $O(10^6)$ K, attained after thermalization. It would appear that the success of the (old) inflationary model requires a change in the tunnelling rate. A low value first to allow for inflation, then a large value to complete the transition. As is well known, this was not the doom of the inflationary Universe but a logical step towards new inflation [82, 83].

3. The new inflationary Universe scenario

As it will be of some importance (at least historically), I will begin this section with a more detailed look at radiative symmetry breaking.

In the absence of a bare Higgs mass, the spontaneous breakdown of a gauge symmetry such as $SU(2) \times U(1)$ would occur radiatively [59]. It was argued [84] that radiative symmetry breaking could be useful in generating large gauge hierarchies. Depending on the Higgs mass, there may be profound phenomenological and cosmological consequences.

We can begin by writing the general expression for the scalar potential through one loop [59],

$$V(\eta) = \frac{1}{4}\lambda\eta^4 - \frac{1}{2}\mu^2\eta^2 + \frac{1}{2}B\eta^2 + \frac{1}{4}C\eta^4 + \frac{1}{2}\int \frac{d^4k}{(2\pi)^4} \ln(1 + m^2/k^2)^{1/2} + \hat{V}\,, \tag{3.1}$$

where m^2 is the bare mass of the particle in the loop and B and C are the mass and quartic counterterms which are determined by a set of renormalization conditions. By choosing a cut-off Λ, the divergent integral in (3.1) can be done,

$$V(\eta) = \tfrac{1}{4}\lambda\eta^4 - \tfrac{1}{2}\mu^2\eta^2 + \tfrac{1}{2}B\eta^2 + \tfrac{1}{4}C\eta^4 + \frac{m^2}{32\pi^2}\Lambda^2 + \frac{m^4}{64\pi^2}(\ln m^2/\Lambda^2 - \tfrac{1}{2}) + \hat{V}\,. \tag{3.2}$$

One can fix B and C by demanding the following renormalization conditions to hold:

$$\partial^2 V/\partial\eta^2|_{\eta=0} = 0\,, \qquad \partial^4 V/\partial\eta^4|_{\eta=M} = 6\lambda\,, \tag{3.3a, b}$$

which fixes B and C,

$$B = \mu^2 - \frac{\partial^2 m^2}{\partial\eta^2}\frac{\Lambda^2}{32\pi^2}\,, \tag{3.4a}$$

$$6C = -\frac{1}{64\pi^2}\left[6\left(\frac{\partial^2 m^2}{\partial\eta^2}\right)^2(\ln M^2/\Lambda^2 - \tfrac{1}{2}) + 12\left(\frac{\partial m^2}{\partial\eta}\right)^2\frac{\partial^2 m^2}{\partial\eta^2}m^{-2} - 2m^{-4}\left(\frac{\partial m^2}{\partial\eta}\right)^4 + 9\left(\frac{\partial^2 m^2}{\partial\eta^2}\right)^2\right], \tag{3.4b}$$

where it has been assumed that $m^2 \propto \eta^2$ and M^2 is the renormalization scale at which eq. (3.3b) is fixed. The resultant potential becomes

$$V(\eta) = \tfrac{1}{4}\lambda\eta^4 + \frac{m^4}{64\pi^2}(\ln m^2/M^2 - \tfrac{25}{6}) + \hat{V}\,. \tag{3.5}$$

For a pure scalar theory, taking $m^2 = 3\lambda\eta^2$ we get the standard one-loop potential derived by Coleman and Weinberg. For a U(1) theory, if we can neglect the scalar masses relative to the gauge boson masses, we get $m^2 = 3e^2\eta^2$. In the scalar theory however, the minimum of (3.5) lies outside the range of validity of the one-loop approximation.

Instead of the renormalization condition fixed by (3.3b) we could equally well have chosen the condition that the minimum of the potential lie at some fixed value of η, say $\langle\eta\rangle = v = M$. And to maintain generality we can modify (3.3a) so that

$$\partial^2 V/\partial\eta^2|_{\eta=0} = D\,. \tag{3.3a$'$}$$

In this case our U(1) theory (neglecting scalar loops) is described by the potential

$$V(\eta) = \tfrac{1}{2}D\eta^2 + A\eta^4(\ln\eta^2/v^2 - \tfrac{1}{2} - D/4Av^2) + \hat{V}\,, \tag{3.6}$$

where A is given by eq. (2.12), and $\hat{V} = \tfrac{1}{2}Av^4 - \tfrac{1}{4}Dv^2$ is chosen to make $V(v) = 0$. Notice that for $D = 0$ ("true" Coleman–Weinberg), the potential (3.6) is identical to eq. (2.11).

In addition to eq. (3.3a′), the renormalization condition used to obtain eq. (3.6) amounts to

$$\lambda_{\rm eff} = \partial^4 V/\partial\eta^4|_{\eta=v} = 88A - 6D/v^2\,. \tag{3.3b$'$}$$

Finally, if we want to put the potential in the form $V(\eta) = V_0(\eta) + V_1(\eta)$, for the U(1) theory we can let

$$D = -\mu^2(1 - 3e^4/16\pi^2\lambda)\,, \tag{3.7}$$

and we recover the form of the potential given in eq. (2.10). Furthermore we see that $(D/Av^2) > 2$ corresponds to $\lambda < 3e^4/32\pi^2$ and the global minimum remains at $\eta = 0$.

Radiative symmetry breaking was first employed in a phenomenological context for the breaking of $SU(2) \times U(1)$. The above requirement for a global minimum at $\langle\eta\rangle = v$ becomes a constraint on the mass of the physical Higgs boson [60, 85, 86]:

$$m_{\rm ph}^2 = 8Av^2 - 2D > 4Av^2\,, \tag{3.8}$$

and the coefficient A is given by

$$A = \frac{1}{64\pi^2 v^4}\,(6M_{\rm W}^4 + 3M_{\rm Z}^4 - 12m_{\rm t}^4) = \frac{3e^4}{1024\pi^2}\,\frac{2 + \sec^4\theta_{\rm w}}{\sin^4\theta_{\rm w}} - \frac{3}{16\pi^2}\,h_{\rm t}^4\,, \tag{3.9}$$

where I have used $M_{\rm W} = \frac{1}{2}g_2 v$, $M_{\rm Z} = M_{\rm W}/\cos\theta_{\rm w}$, and have left only the contribution of the top quark in the loop with $m_{\rm t} = h_{\rm t} v$.

The cosmological consequences of radiative electroweak symmetry breaking have been examined in great detail [87, 46, 88–90]. For $D \gtrsim 0$, the $SU(2) \times U(1) \to U(1)$ phase transition is first order and the barrier separating the two phases is present at $T = 0$. For $D < 0$, the barrier disappears. The temperature dependent part of the potential can be obtained from eq. (2.13) as

$$V_T(\eta, T) = -\frac{\pi^2}{90}\,NT^4 + \left(\frac{e^2}{32\sin^2\theta_{\rm w}}\,(2 + \sec^2\theta_{\rm w}) + \tfrac{1}{4}h_{\rm t}^2\right)T^2\eta^2\,. \tag{3.10}$$

Because of the potential barrier, supercooling occurs in general. At the completion of the phase transition, depending on the value of D, an entropy excess may have been generated, possibly diluting the net baryon asymmetry [46, 88].

To determine the critical temperature, we can use a trick proposed by Witten [87]. In principle, eq. (3.10) is only valid when $e\eta \ll T$. In a supercooled transition, this is probably no longer valid. However, we can rewrite the logarithm in eq. (3.6) as $\ln\eta/v = \ln(e\eta/2T\sin\theta_{\rm w}) + \ln(2T\sin\theta_{\rm w}/ev) = \ln(e\eta/2T\sin\theta_{\rm w}) + \ln(T/M_{\rm W})$. At very low T, $\ln T/M_{\rm W}$ becomes very large whereas $e\eta/T$ remains 0(1). Therefore, we can approximate the potential by

$$\begin{aligned} V(\eta, T) = {}& -\frac{\pi^2}{90}\,NT^4 + \left(\frac{e^2}{32\sin^2\theta_{\rm w}}\,(2 + \sec^2\theta_{\rm w}) + \tfrac{1}{4}h_{\rm t}^2\right)T^2\eta^2 \\ & + \tfrac{1}{2}D\eta^2 - A\eta^4[\ln M_{\rm W}^2/T^2 + \tfrac{1}{2} + D/4Av^2] + \hat{V}\,. \end{aligned} \tag{3.11}$$

The tunnelling probability for the phase transition described by the potential (3.11) is again of the form (2.18) with $M^4 \sim T^4$. The potential (3.11) is effectively a quartic potential with a negative self-coupling. For the three-dimensional action,

$$I = \int d^3x\,[-\tfrac{1}{2}(\partial_\mu\phi)^2 + \tfrac{1}{2}m^2\phi^2 - \tfrac{1}{4}\lambda\phi^4]\,, \tag{3.12}$$

the bounce action has been computed [91] to be $B \simeq 18.897m/\lambda$. For $D \simeq 0$, and $\sin^2\theta_w = 0.23$ and $m_t = 60$ GeV, one finds $B \simeq 4000/\ln(M_W/T)$.

The phase transition will proceed when $P/H^4 \sim 1$. If we take $H^2 = 8\pi V(0)/3M_P^2$, then

$$P/H^4 \sim \frac{9M_P^4 T^4}{16\pi^2 A^2 v^8}\, e^{-B}\,, \tag{3.13}$$

and one would expect the transition to occur when $B \lesssim 150$, which results in supercooling down to $T \lesssim 1$ eV. Thus unless $D \lesssim 0$, extreme supercooling would occur. However, Witten [87] also pointed out that when the temperature has fallen to $T \sim$ few $\times 10^2$ MeV, chiral symmetry breaking and the formation of quark condensates may drive the transition.

By keeping the Yukawa term for quark masses in the potential,

$$V_Y = -\frac{\eta}{v}\sum_i m_i \bar{q}_i q_i\,, \tag{3.14}$$

a nonzero expectation value for $\langle \bar{q}q \rangle$ induces a linear term in the potential $V(\eta)$ and may destabilize the vacuum. Indeed, below the chiral symmetry restoration temperature such a condensate is expected to occur [92]. Nevertheless large values of $D > 0$ would not be cosmologically allowed [87, 46, 88–89].

The question of interest to inflation, is whether or not the Coleman–Weinberg potential for a GUT such as SU(5) would be suitable for inflation. The Coleman–Weinberg potential for SU(5) has also been studied in detail [63, 93–98]. For SU(5), the $T = 0$ scalar potential is of the form (3.6) and

$$A = (5625/1024\pi^2)\, g_5^4\,. \tag{3.15}$$

The finite temperature part of the potential is given by eq. (2.16) (neglecting the constant term). Thus we can write the dominant contribution as

$$V(\sigma, T) = -2A\sigma^4 \ln M_X/T + \tfrac{75}{16} g_5^2\sigma^2 T^2\,, \tag{3.16}$$

again having set $D = 0$.

As in the case of the electroweak phase transition a Coleman–Weinberg GUT phase transition also has a strong possibility of excessive entropy production [93]. A similar exercise in computing the action for the SU(5)$\rightarrow$SU(3) $\times$ SU(2) $\times$ U(1) transition gives

$$I = \tfrac{15}{2T}\int d^3x\,[-\tfrac{1}{2}(\partial_\mu\sigma)^2 + \tfrac{1}{2}m^2\sigma^2 - \tfrac{1}{4}\lambda\sigma^4]\,, \tag{3.17}$$

where now $m^2 = \frac{5}{4}g_5^2T^2$ and $\lambda = (375/64\pi^2)g_5^4 \ln M_X/T$ (again setting $D = 0$). From this one finds [95, 53] $B \simeq 6\alpha_5^{-3/2}/\ln(M_X/T)$, where $\alpha_5 = g_5^2/4\pi \sim \frac{1}{41}$. In this case, tunnelling becomes important when $B \lesssim 50$ (having set the determinental factor $M^4 \sim v^4$) or when $T \sim 1$ GeV.

The above simple analysis breaks down however, when one includes the running of the gauge coupling [53],

$$\alpha_5 = 3\pi/(10 \ln T^2/\Lambda^2)\,, \tag{3.18}$$

with

$$\Lambda_5 \simeq M_X e^{-6\pi/40\alpha_5}\,, \tag{3.19}$$

which for minimal SU(5) is $O(10^6)$ GeV. In this case $B \lesssim 50$ when $T \sim 10^{10}$ GeV. If $M^4 \sim T^4$, the transition takes place at even lower temperatures. Condensation phenomena are expected to again drive the phase transition when $T \sim \Lambda$ [95, 97]. Finally in the presence of a coupling of the Higgs field to gravity (by including a term in the potential of the form $\xi R\sigma^2$) the transition probability may be enhanced or suppressed depending on the sign of ξ [94, 99]. The scalar potential in non-flat spacetimes has been considered in ref. [100].

The new inflationary Universe scenario was based originally on the Coleman–Weinberg mechanism for symmetry breaking in a GUT [82, 83]. Assuming that the Universe supercools, it is necessary to consider the evolution of the scalar field *after* tunnelling (or the onset of the transition via strong coupling). Consider first the position of the maximum of the potential barrier,

$$\sigma_{\max} \simeq 4\pi T/[5\sqrt{3} g_5 (\ln M_X/T)^{1/2}]\,. \tag{3.20}$$

Inside a bubble (or a fluctuation region), the field will be roughly uniform and the magnitude of σ will be $\lesssim 3\sigma_{\max}$ (if the thin-wall approximation is to remain satisfied). The remaining evolution of the "bubble" becomes the inflationary epoch. The "bubble" begins to grow (at the speed of light) and the scalar field evolves toward the global minimum at $\sigma = v$. During the first part of its evolution, σ moves very slowly during which time, $V(\sigma)$ is roughly constant and an approximate De Sitter phase begins.

Let us look a little more closely at this evolution. The equation of motion for a scalar field ϕ can be derived from the energy–momentum tensor

$$T_{\mu\nu} = -\left(\frac{\partial \mathcal{L}}{\partial(\partial^\mu \phi)}\, \partial_\nu \phi - g_{\mu\nu}\mathcal{L}\right) = \partial_\mu \phi \partial_\nu \phi - \tfrac{1}{2} g_{\mu\nu}(\partial_\rho \phi\, \partial^\rho \phi) - g_{\mu\nu} V(\phi)\,. \tag{3.21}$$

By associating $\rho = T_{00}$ and $p = R^{-2}(t) T_{ii}$ we have

$$\rho = \tfrac{1}{2}\dot{\phi}^2 + \tfrac{1}{2} R^{-2}(t)(\nabla\phi)^2 + V(\phi), \qquad p = \tfrac{1}{2}\dot{\phi}^2 - \tfrac{1}{6} R^{-2}(t)(\nabla\phi)^2 - V(\phi)\,, \tag{3.22a, b}$$

and from eq. (1.9) we can write the equation of motion (by considering a homogeneous region, we can ignore the gradient terms)

$$\ddot{\phi} + 3H\dot{\phi} = -\partial V/\partial\phi\,. \tag{3.23}$$

Consider the approximation $\partial V/\partial\phi \simeq (\partial^2 V/\partial\phi^2)\phi$ (though this is not really true for a Coleman–Weinberg potential), the equation of motion becomes [101]

$$\ddot{\phi} + 3H\dot{\phi} + m^2(\phi)\phi = 0\,, \tag{3.24}$$

and $m^2(\phi) = \partial^2 V/\partial\phi^2 < 0$. The solution when $|m^2| \gg H^2$ grows exponentially as $\phi \sim \exp(|m|t)$, while for $|m^2| \ll H^2$ the scalar field grows as $\phi \sim \exp(|m|^2 t/3H)$. In the latter case the field moves very slowly during a time period $\tau \sim 3H/|m|^2$. During this period the Universe expands exponentially with $R(t) \sim \exp(H\tau)$, thus for

$$H\tau \sim 3H^2/|m|^2 > 65 \tag{3.25}$$

we will have sufficient inflation necessary to solve the cosmological problems discussed in section 1.

For the potential given by eq. (3.16), we can write

$$\partial V/\partial\sigma = \tfrac{75}{8} g_5^2 T_c^2 [1 - (\sigma/\sigma_{\max})^2]\sigma \,, \tag{3.26}$$

in which case we can identify $|m|^2 \sim \frac{2}{15} \cdot \frac{75}{8} g_5^2 T_c^2 (\sigma/\sigma_{\max})^2$ (where the factor $\frac{2}{15}$ is due to the normalization of σ). The condition (3.25) is easily satisfied,

$$H\tau \sim \frac{1125}{64\pi}\, g_5^2 \left(\frac{\sigma_{\max}}{\sigma}\right)^2 \frac{v^4}{M_P^2 T_c^2} \sim 10^4 \,, \tag{3.27}$$

if we take $T_c \sim 10^8$ as claimed in ref. [83]. One should be aware, however, that T_c should be computed using a full renormalization group analysis and in general depends on the particle content of the theory [102]. Clearly there is the possibility of abundant inflation. In this case the entire visible Universe was contained in a single bubble or fluctuation region.

In the old inflationary scenario, the main problem was not the number of e-foldings during the exponential expansion, but rather, the completion of the transition required the collision of bubble walls that in turn reheated and thermalized the Universe. In the new inflationary scenario, no such difficulty arises since the entire Universe is within a bubble. As the scalar field evolves towards its expectation value, it becomes a (nearly) homogeneous field oscillating about its global minimum.

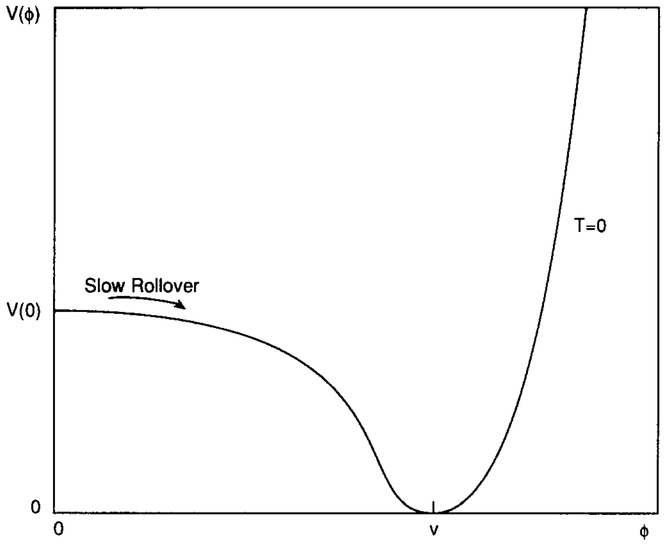

Fig. 7. Schematic drawing of a scalar potential in the new inflationary scenario. Inflation occurs during the roll-over of the scalar field ϕ from $\phi \simeq 0$ to $\phi = v$.

Interactions damp these oscillations and the Universe is reheated to a temperature $T_R \sim \hat{V}^{1/4} \sim (HM_P)^{1/2} \sim v$ [82, 103]. The problem of reheating and the baryon asymmetry will be treated in more detail in the next section.

Up until now, the potential mass term, $\frac{1}{2}D\phi^2$ has always been ignored. It is however possible to realize the new inflationary scenario even if $D \neq 0$ (so long as it is not too large) [104]. In this case gravity plays an important role [105] and there is a finite probability for the entire De Sitter volume [106] to make a quantum jump from $\sigma = 0$ to $\sigma = \sigma_{max}$, i.e., to the top of the barrier. The action for this solution was computed to be [104]

$$B = \tfrac{1}{8}M_P^4[\hat{V}^{-1} - V(\sigma_{max})^{-1}] . \tag{3.28}$$

So long as D is not too large, the subsequent evolution of the scalar field is as described above. For further discussion on the effect of gravity and the Hawking–Moss solution see ref. [107]. As we will see, in much of the discussion that follows, implementation of the new inflationary scenario does not depend on tunnelling phenomena [108, 109]. Figure 7 shows a typical scalar potential for new inflation.

4. More on the new inflationary scenario

Despite its marked improvement over the original inflationary scenario, new inflation, described by the radiative symmetry breaking of SU(5), was far from perfect. The major problems had to do with quantum fluctuations, fine-tuning, the correct low energy vacuum and above all the problem of density perturbations. In this section, I will first review these problems and discuss how one constructs generic models for new inflation. In addition, I will focus attention on reheating and the baryon asymmetry, scalar field fluctuations, and initial conditions for inflation.

4.1. Problems for inflation based on radiative SU(5) symmetry breaking

As was discussed in section 2, the spontaneous symmetry breaking patterns of SU(5) depend on the choice of the Higgs self-couplings in eq. (2.14). In the previous discussion of the new inflationary scenario, it was assumed that symmetry breaking occurs along the 3–2–1 direction. In some generality, we can write

$$V(\Sigma) = \frac{3g_5^4}{256\pi^2}\{b'[\mathrm{Tr}\,\Sigma^4 - \tfrac{7}{30}(\mathrm{Tr}\,\Sigma^2)^2] + \mathrm{Tr}\,M^4[\ln(4M^2/25v^2) - \tfrac{1}{2}]\} , \tag{4.1}$$

where M is the gauge boson mass matrix. (In terms of eq. (2.14) this corresponds to $a/b = -\frac{7}{15}$). In the 3–2–1 direction eq. (4.1) becomes eq. (2.17). For this potential, when $b' < 15$, there is a local minimum in the 4–1 direction and when $b' < -15 \ln \frac{3}{2}$, the global minimum is in the 4–1 direction [95, 110].

By solving the combined, cosmological equations of motion together with the equations of motion for σ [cf. eq. (3.23)], it was concluded [110] that for reasonable values of the coupling b', evolution of σ was always toward the local 4–1 minimum. In addition the subsequent phase transition to the global 3–2–1 minimum was found to be strongly first order, and thus one does not escape the problems encountered in old inflation. A similar conclusion was reached in ref. [111]. The tendency to pass through an intermediate phase was also noted in ref. [112] and in ref. [113] in the context of calculating the production of magnetic monopoles in the final phase transition. I will return to this in section 6. A detailed analysis of the phase transition and the passage through an intermediate phase was made in ref. [114], and confirmed the seriousness of this problem. Attempts to cure this problem using other Higgs representations [114] or an extremely weakly coupled adjoint [115] are not very realistic.

Quantum fluctuations of scalar fields, as we will see, play an important role in the theory of inflation. For $\sigma \simeq 0$, we can write the scalar potential as

$$V(\sigma) = \tfrac{1}{2}m^2\sigma^2 - \tfrac{1}{4}\lambda\sigma^4 + V(0)\,, \tag{4.2}$$

where $m^2 = D + cT^2 + \cdots$ (terms such as ξR could be included in $\cdots$) and $\lambda \sim 8A \ln M_X/T$ [cf. eq. (3.16)]. Classically, the initial condition with $\ddot{\sigma} = \dot{\sigma} = \sigma = 0$ is stable even with $m^2 < 0$. Also included in the total m^2, however, are terms proportional to the quantum fluctuations $\langle\sigma^2\rangle$ [116, 108, 109], and are of the form $-3\lambda\langle\sigma^2\rangle$. For large enough fluctuations, a destabilization of the false vacuum occurs.

For values of m^2 of interest for inflation, $m^2 \ll H^2$, the asymptotic value of $\langle\sigma^2\rangle$ can be derived [108] from the results of Bunch and Davies [117], who calculate $\langle\sigma^2\rangle$ in a De Sitter space,

$$\langle\sigma^2\rangle = 3H^4/8\pi^2 m_0^2\,. \tag{4.3}$$

However, because the inflationary Universe is not a true De Sitter space, care must be taken in computing $\langle\sigma^2\rangle$. (I will return to the details of how $\langle\sigma^2\rangle$ is calculated later in this section.) During inflation and in the massless limit $m_0^2 = 0$, one finds that $\langle\sigma^2\rangle$ grows linearly with time $\langle\sigma^2\rangle = H^3t/4\pi^2$. This can be compared with fluctuations in Minkowski space at finite temperature, $\langle\sigma^2\rangle \simeq T^2$ [44]. The reason for the anomalously large growth of $\langle\sigma^2\rangle$ in a De Sitter space, as we shall see, has to do with the accumulation of a large density of long wavelength modes of the scalar field. Clearly one sees that given enough time, $m^2 = m_0^2 - 3\lambda\langle\sigma^2\rangle$ will go negative and destabilize the vacuum.

Destabilization of the vacuum and the requirement for sufficient inflation will place a constraint on the value of λ. Consider for simplicity that $m_0^2 = 0$, and $\langle\sigma^2\rangle = H^3t/4\pi^2$. Then inflation will terminate when $|m^2| \sim 3\lambda\langle\sigma^2\rangle$ approaches H^2. To be consistent with the requirement in eq. (3.25), we must require that $|m^2| \simeq 3\lambda\langle\sigma^2\rangle \simeq 3\lambda H^3t/4\pi^2 < 3H^2/65$ with $Ht > 65$, or that [108]

$$\lambda \lesssim 9\times 10^{-3}\,. \tag{4.4}$$

This result is particularly troublesome [118, 119] since λ is fixed in the SU(5) theory,

$$\lambda \simeq 8A \ln M_X/T \gtrsim \mathrm{O}(1)\,. \tag{4.5}$$

It is also worth noting at this point that in addition to the above problems, the theory we have been considering is fine-tuned [118, 119]. Recall that throughout, we have assumed that bare mass contributions (D) and possible curvature contributions are all negligible. Indeed with $v \sim 5\times 10^{14}$ GeV [82, 83], inflation occurs with a Hubble parameter $H \lesssim 10^{10}$ GeV. The condition that $m^2 \ll H^2$ now appears particularly unnatural since typical values for D are $D^{1/2} \sim M_X$ or $\alpha_5 M_X$ (which are due to radiative corrections). Curvature terms would also be expected to be at least $\mathrm{O}(H)$. This however is only a technical problem as radiative corrections can be controlled by imposing supersymmetry. (I will return to this in the next section.)

The biggest blow to new inflation based on SU(5) radiative symmetry breaking came with the calculation of density perturbations produced during the roll-over transition. New inflation, it turns out, is not only capable of explaining the large scale homogeneity and isotropy, it can also explain the origin of primordial density perturbations. Due to quantum fluctuations, the scalar field will not be uniform over all space. The duration of inflation is therefore not uniform and this gives rise to adiabatic density perturbations [120, 121, 109, 122, 123].

The isotropy of the microwave background radiation indicates that any perturbations produced on large scales must have $\delta\rho/\rho \leq O(10^{-4})$. Ideally, what one wants from inflation are the scale-independent perturbations described by the Harrison–Zeldovich spectrum [124, 125]. Preferably, their magnitude must be $O(10^{-4})$. Any perturbations stronger then this would produce visible anisotropies in the microwave background radiation, while weaker perturbations would not have had enough time to grow during the present epoch of matter domination (since decoupling). The initial spectrum of perturbations can be classified by their magnitude on a given length scale,

$$(\delta\rho/\rho)_l \sim 1/l^{3n} \sim 1/M^n \,, \tag{4.6}$$

where M is the mass contained within the volume l^3. Perturbations on scales larger than the horizon grow in magnitude with $\delta\rho/\rho \sim t$. Inside the horizon, they act as sound waves and oscillate until further growth is possible when the Universe becomes matter dominated (when the Jeans mass falls below the mass within a perturbation). Perturbations on larger scales begin smaller and have a longer time to grow until $l \sim t$. For $n = \frac{2}{3}$, the magnitudes of the perturbations are equal as each scale l enters the horizon (recall l grows as $t^{1/2}$ after inflation). This is what is meant by scale-independent perturbations. See fig. 8. In addition, for $n > \frac{2}{3}$ perturbations are too strong and tend to close up and form island Universes, and for $n < \frac{2}{3}$ they are weak to form galaxies. Thus the motivation for an $n = \frac{2}{3}$ spectrum [124, 125].

As we will see, new inflation predicts a nearly (up to a logarithmic dependence) $n = \frac{2}{3}$ spectrum. Here I will follow the approach of Guth and Pi [122, 25] to calculate the magnitude of $\delta\rho/\rho$ in the case of radiative SU(5) symmetry breaking. Very similar results were obtained in refs. [121, 109, 123]. I

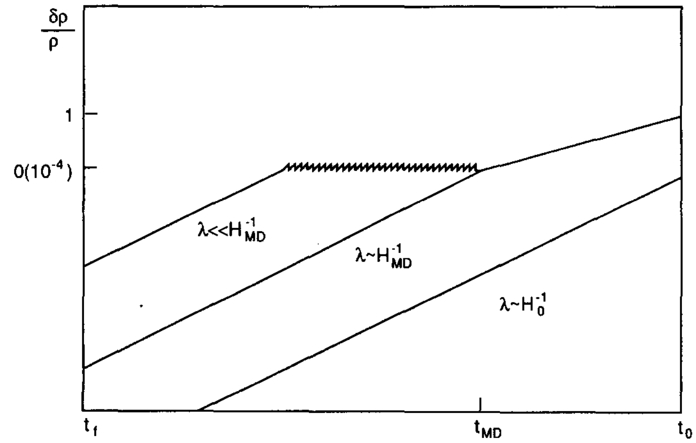

Fig. 8. The growth of density perturbations, $\delta\rho/\rho$, as a function of time for the $n = \frac{2}{3}$ spectrum. Shown is the magnitude of $\delta\rho/\rho$ for three wavelengths at horizon crossing: large wavelengths λ just entering the horizon today, $\lambda \sim H_0^{-1}$; wavelengths which enter the horizon at the time of matter domination, $\lambda \sim H_{\rm MD}^{-1}$; and much smaller wavelengths, $\lambda \ll H_{\rm MD}^{-1}$. Notice that at horizon crossing $\delta\rho/\rho \sim O(10^{-4})$ independent of λ.

choose this method only for simplicity; a more rigorous treatment is the gauge invariant formulation of Bardeen [23, 123]. Let us denote the scalar field by $\phi(x, t)$ and write

$$\phi(x, t) = \phi_0(t) + \delta\phi(x, t)\,, \tag{4.7}$$

where $\phi_0(t)$ is the homogeneous classical field which satisfies the equation of motion (3.23) (for the SU(5) case, we can set $\phi = \sigma$ at the end of the calculation). The perturbations $\delta\phi$ satisfy the following linearized equation of motion:

$$\delta\ddot{\phi} + 3H\,\delta\dot{\phi} = -\frac{\partial^2 V}{\partial\phi^2}(\phi_0)\,\delta\phi + e^{-2Ht}\partial_i^2\delta\phi\,. \tag{4.8}$$

If $\tilde{\phi}(k, t)$ is the Fourier transform of $\phi(x, t)$,

$$\phi(x, t) = \int d^3k\, e^{ikx}\tilde{\phi}(k, t)\,, \tag{4.9}$$

we can write

$$\delta\ddot{\tilde{\phi}} + 3H\,\delta\dot{\tilde{\phi}} = -\frac{\partial^2 V}{\partial\phi^2}(\phi_0)\,\delta\tilde{\phi} - e^{-2Ht}k^2\,\delta\tilde{\phi}\,. \tag{4.10}$$

Because we expect $\partial^2 V/\partial\phi^2 < 0$ for $\phi \simeq 0$, the two terms on the right-hand side of eq. (4.10) act to destabilize and stabilize the perturbations, respectively. However, because the second term is only important at early times, growth of $\delta\tilde{\phi}$ begins at a time $t^*(k)$ given by

$$e^{-2Ht^*}k^2 = \left|\frac{\partial^2 V}{\partial\phi^2}[\phi_0(t^*)]\right|\,. \tag{4.11}$$

In fig. 9, the time development of a given wavelength λ of the perturbation along with the particle horizon ($\equiv H^{-1}$) is plotted schematically. Although λ appears to go out of the (particle) horizon the proper distance to the causal horizon [cf. eq. (2.27) and fig. 6] always remains larger than λ. (One should not be misled to thinking that a perturbation ever leaves the causal horizon.) For late times, $Ht \gg Ht^*$, eq. (4.10) for $\delta\tilde{\phi}$ is identical to eq. (3.23) for $\dot{\phi}_0$ and we can write

$$\delta\tilde{\phi}(k, t) = -\delta\tilde{\tau}(k)\dot{\phi}_0(t)\,, \tag{4.12}$$

or, to first order in $\delta\tau$,

$$\phi(x, t) = \phi_0(t - \delta\tau(x))\,; \tag{4.13}$$

$\delta\tau$ represents the time delay in the evolution of ϕ. At early times, $Ht \ll Ht^*$, we can neglect the potential term and [117]

$$\langle\delta\tilde{\phi}(k, t)\rangle = (H/4\pi^{3/2})[1 + (k^2/H^2)\,e^{-2Ht}]^{1/2}\,. \tag{4.14}$$

The origin of eq. (4.14) will be made clear below.

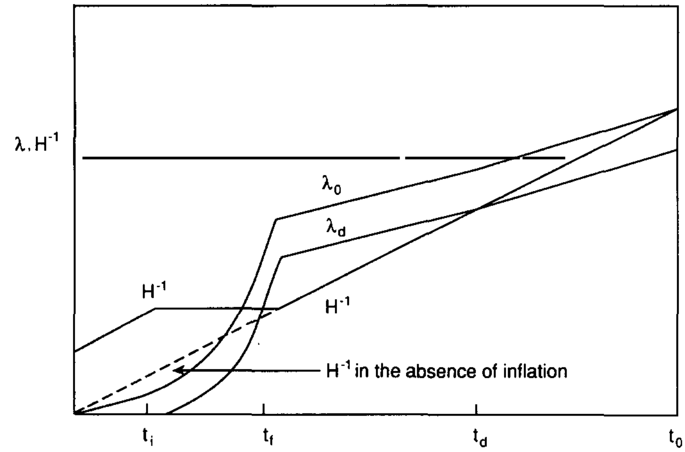

Fig. 9. Behavior of the wavelength of a density perturbation and the inverse Hubble parameter (effective particle horizon) as a function of time. Recall that during inflation the true (causal) horizon grows exponentially (cf. fig. 5). The dashed line corresponds to the particle (=causal) horizon in the absence of inflation. Two wavelengths are shown: λ_0, which reenters the horizon today, and λ_d, which reenters the horizon at the time of decoupling. In all cases $\lambda(t) \sim R(t)$.

Using the formalism of Olson [22], it was shown [122] that at horizon crossing

$$\delta\rho/\rho = 4H\,\delta\tau(k)\,, \tag{4.15}$$

and combining (4.12) and (4.14),

$$\delta\tau(k) = (H/4\pi^{3/2}\dot{\phi})[1 + (k^2/H^2)\,\mathrm{e}^{-2Ht}]^{1/2}\,, \tag{4.16}$$

or using eqs. (4.11) and (4.15),

$$\frac{\delta\rho}{\rho} = \frac{H^2}{\pi^{3/2}\dot{\phi}}\left(1 + \frac{\partial^2 V}{\partial\phi^2}(\phi_0)/H^2\right)^{1/2}. \tag{4.17}$$

For the cases of interest when $(\partial^2 V/\partial\phi^2)(\phi_0) \ll H^2$, we obtain the simple result

$$\delta\rho/\rho = H^2/\pi^{3/2}\dot{\phi}\,. \tag{4.18}$$

For the specific case of SU(5) inflation, writing the potential in the form given by eq. (4.2), and neglecting the m^2 contribution we have

$$\sigma_0^2(t) \simeq -3H/2\lambda t \tag{4.19}$$

as a solution for the homogeneous field (neglecting the $\ddot{\sigma}$ contribution) and from (4.11),

$$Ht^* \simeq \ln kH^{-1} + \tfrac{1}{2}\ln(-\tfrac{2}{9}Ht^*) \simeq \ln kH^{-1}\,. \tag{4.20}$$

Finally,

$$\dot{\sigma} \simeq (3/8\lambda)^{1/2} H^2/\ln^{3/2}(Hk^{-1}) , \tag{4.21}$$

$$\delta\rho/\rho = (8\lambda/3\pi^3)^{1/2} \ln^{3/2}(Hk^{-1}) . \tag{4.22}$$

The good news from eq. (4.22) is that density perturbations from new inflation are described by the desired $n = \frac{2}{3}$ spectrum (up to the log term). The bad news is that for galaxy sized perturbations $\ln Hk^{-1} \simeq 10^2$ and $\lambda \sim 1$ so that

$$\delta\rho/\rho \sim \mathrm{O}(10^2) \tag{4.23}$$

or about six orders of magnitude too large (this is not even a perturbation). Anisotropies in the microwave background radiation induced by density perturbations produced by inflation require $\lambda \lesssim 10^{-12}$ thus killing the original new inflationary scenario. The original scenario has of course been supplanted by many other variants which will be discussed in some detail in the next section. In the remainder of this section, I plan to discuss general features of a generic inflationary model.

4.2. *Reheating and the baryon asymmetry*

Unlike the old inflationary scenario, in the new inflationary scenario, there is a straightforward mechanism for reheating after inflation [82, 103, 126, 127, 75]. Because of the problems discussed above, the scalar field responsible for inflation should not be associated with the adjoint of SU(5). Instead we can consider an arbitrary gauge singlet scalar field, the inflaton [75], which drives inflation. As was stated in the previous section, the inflaton must satisfy the equation of motion (3.23), which after a time period $\tau \sim 3H/|m|^2$, grows exponentially to its vacuum expectation value v. At late times, the inflaton oscillates about the global minimum

$$\phi(t) \sim (v/mt) \sin mt \sim (H/m) v \sin mt . \tag{4.24}$$

In the absence of any couplings to matter the Universe would remain permanently in a state with a single oscillating field. The presence of interactions between the inflaton and ordinary matter allows for the conversion [127, 128] of the vacuum energy density and the reheating of the Universe.

For a non-interacting massive scalar field we can write the energy density in the oscillations as

$$\rho_\phi = \tfrac{1}{2} m^2 \phi^2(t) = \tfrac{1}{2} m^2 v^2 (R_i/R)^3 , \tag{4.25}$$

where R_i is the value of the scale factor when the oscillations begin. The Universe expands as if it were matter dominated, $\rho \sim R^{-3}$ (cf. ref. [129]). If the scalar field is interacting, we can assume for simplicity that it decays to ordinary matter with a rate Γ_D. If Γ_D is large ($\Gamma_D > H$) then the oscillations never really begin and the vacuum energy density is converted to radiation immediately, and the reheating temperature is determined by

$$\tfrac{1}{30} \pi^2 N(T_R) T_R^4 = \rho_\phi , \tag{4.26}$$

or

$$T_R \simeq [45/4\pi^3 N(T_R)]^{1/4} (HM_P)^{1/2} . \tag{4.27}$$

In contrast, if the field is only weakly coupled (as it must be if $\delta\rho/\rho \lesssim 10^{-4}$) then $\Gamma_D < H_I$, where H_I represents the value of H during inflation (exponential expansion). In this case, oscillations continue until the field decays (when $t \sim \Gamma_D^{-1}$),

$$\Gamma_D \simeq \tfrac{3}{2} H \simeq \sqrt{6\pi\rho}/M_P , \tag{4.28}$$

and from eq. (4.26),

$$T_R \simeq [5/\pi^3 N(T_R)]^{1/4} (\Gamma_D M_P)^{1/2} . \tag{4.29}$$

It is obvious, that in order to recover the standard results of the post-inflationary hot big bang model, T_R must be suitably large so that a mechanism for generating the baryon asymmetry is still operative. In the case of strong reheating (eq. 4.27) [103, 126, 130], for $H \simeq 10^{10}$ GeV, $T_R \simeq O(10^{14})$ GeV and it is possible that the standard out-of-equilibrium decay scenario follows. In the weak reheating case (eq. 4.29) [126, 75, 131], $T_R \ll v \sim M_X$. A sufficient baryon asymmetry will still be produced provided the inflaton is capable of decaying into Higgs fields which mediate baryon number violating interactions. From eq. (4.28), one finds that decays occur when

$$R_D/R_i \sim (mv/\Gamma_D M_P)^{2/3} . \tag{4.30}$$

For simplicity, if the inflaton decays exclusively to the desired Higggs boson then the number of Higgses $n_H \simeq n_\phi$ at a temperature $T_R < m_H < m$ so the Higgses will subsequently decay out of equilibrium. The final baryon asymmetry will be

$$n_B/s \simeq \varepsilon n_H/T_R^3 \simeq \varepsilon n_\phi/T_R^3 , \tag{4.31}$$

where $n_\phi = \rho_\phi/m$ and ε is the net baryon asymmetry produced by a pair of Higgses. At $R = R_D$, $n_\phi \simeq (\Gamma_D M_P)^2/m$ and

$$n_B/s \simeq \varepsilon T_R/m . \tag{4.32}$$

Thus the constraint on reheating becomes $\varepsilon T_R/m \gtrsim O(10^{-11})$.

An alternative to the more-or-less standard mechanism for producing a baryon asymmetry was originally proposed in the context of a supersymmetric GUT [132] and was shown to be easily applicable to inflationary scenarios [133, 134]. I will discuss this mechanism in detail in the next section when supersymmetric models of inflation are introduced.

4.3. *Scalar field fluctuations and inflation*

Previously, we have seen that quantum fluctuations may be responsible for symmetry breaking [116, 108, 109] and for the generation of energy density perturbations [109, 121–123]. In this subsection, I will discuss in more detail the arguments which lead to eqs. (4.3) and (4.14). An arbitrary scalar field $\phi(x, t)$, in a spatially flat Robertson–Walker background, can be expressed in terms of its plane wave modes as

$$\phi = \frac{1}{(2\pi)^{3/2}} \int d^3k \, [\Psi_k(t)\, e^{ik\cdot x} + \text{h.c.}] , \tag{4.33}$$

where the mode functions $\Psi_k(t)$ are found to be given by [117, 116, 135, 108]

$$\Psi_k(\eta) = (\pi/4)^{1/2} H|\eta|^{3/2}[c_1 H_\nu^{(1)}(k\eta) + c_2 H_\nu^{(2)}(k\eta)]\,, \tag{4.34}$$

$$\eta = -H^{-1}\,\mathrm{e}^{-Ht}, \qquad \nu^2 = \tfrac{9}{4} - m^2/H^2\,, \tag{4.35}$$

for a stable field with mass m (for the more general case of an unstable scalar field see [136]). $H_\nu^{(1)}$ and $H_\nu^{(2)}$ are Hankel functions and c_1 and c_2 are functions of (k/H) and are subject to the normalization conditions [136]

$$-|c_1|^2 - (1 + \mathrm{e}^{2\nu\pi\mathrm{i}} + \mathrm{e}^{-2\nu\pi\mathrm{i}})|c_2|^2 + (1 + \mathrm{e}^{-2\nu\pi\mathrm{i}})c_1 c_2^* + (1 + \mathrm{e}^{2\nu\pi\mathrm{i}})c_1^* c_2 = 1, \quad \nu \text{ real}\,, \tag{4.36a}$$

$$-|c_1|^2 - (1 + \mathrm{e}^{-2\mu\pi} + \mathrm{e}^{-4\mu\pi})|c_2|^2 + (1 + \mathrm{e}^{-2\mu\pi})(c_1 c_2^* + c_2 c_1^*) = \mathrm{e}^{-\mu\pi}, \quad \nu = \mathrm{i}\mu\ (\mu > 0) \text{ imaginary}\,. \tag{4.36b}$$

It is assumed that $\eta < 0$. Only when ν is half-integer do the conditions (4.36a, b) reduce to $|c_2|^2 - |c_1|^2 = 1$, which is the correct normalization condition when $\eta > 0$. ($\eta < 0$ must be chosen, however, in order to match the De Sitter solution with Minkowski space; η is actually the conformal time so that the metric can be written $\mathrm{d}s^2 = a^2(\eta)(\mathrm{d}\eta^2 - \mathrm{d}x^2)$. The approximate De Sitter solution in the inflationary Universe represents only half of the full De Sitter solution which allows all values $-\infty < \eta < \infty$.)

The general formalism for calculating quantum fluctuation $\langle\phi^2\rangle$ has been developed by Bunch and Davies [117]. Using a point splitting regularization scheme, they calculate a two point function

$$\langle\phi(x'')\phi(x')\rangle = \frac{2}{(2\pi)^3} \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}k\cdot(x''-x')}\Psi_k(\eta'')\Psi_k^*(\eta')\,\mathrm{d}^3k\,. \tag{4.37}$$

The Bunch–Davies vacuum was defined by $c_1 = 0$ and $c_2 = 1$. A less rigorous but physically motivated renormalization and regularization scheme was proposed by Vilenkin [137]. He argued that in the inflationary Universe, the only modes of interest are those with wavelengths greater than the horizon as these are the ones responsible for the growth of $\langle\phi^2\rangle$. Thus the important contributions must come when $k\eta \ll 1$ or $k \ll H\,\mathrm{e}^{Ht}$. In addition, when $k \lesssim H$, the magnitude of the fluctuations depends on the initial conditions of the Universe, but with a power law expansion rate. There is no reason to believe that $\langle\phi^2\rangle$ exceeds the anomalous De Sitter fluctuations before the onset of the inflationary era. (The Bunch–Davies fluctuations are infrared divergent.) Thus one can write [137]

$$\langle\phi^2\rangle \simeq \frac{1}{(2\pi)^3} \int_H^{H\,\mathrm{e}^{Ht}} \mathrm{d}^3k\,|\Psi_k|^2\,. \tag{4.38}$$

In the massless limit, for $k \gg H$, one recovers the Bunch–Davies vacuum and

$$\Psi_k \simeq (\tfrac{1}{4}\pi)^{1/2} H\eta^{3/2} H_{3/2}^{(2)}(k\eta) = -(2k)^{-1/2} H\eta(1 - \mathrm{i}/k\eta)\,\mathrm{e}^{-\mathrm{i}k\eta}\,. \tag{4.39}$$

The $\langle\phi^2\rangle$ fluctuations are easily computed:

$$\langle \phi^2 \rangle = \frac{1}{(2\pi)^3} \int\limits_H^{H e^{Ht}} d^3k \, \frac{H^2}{2k^3} \left(1 + \frac{k^2}{H^2} e^{-2Ht}\right). \tag{4.40}$$

The quantity $\langle \delta\tilde{\phi} \rangle$ in eq. (4.14) is a RMS fluctuation given by $[k^3/(2\pi)^3]|\Psi_k|^2$. Integrating eq. (4.40) one finds the standard result [108, 137, 136]

$$\langle \phi^2 \rangle \simeq (H^3/4\pi^2)t . \tag{4.41}$$

For the massive case when $\nu \neq \frac{3}{2}$, but with $m \ll H$ one finds [108, 135–137]

$$\langle \phi^2 \rangle = 2^{2\nu} \frac{H^{5-2\nu}}{8\pi^3} \Gamma^2(\nu) \left(\frac{e^{-Ht}}{H}\right)^{3-2\nu} \int\limits_1^{e^{Ht}} dz \, z^{2-2\nu} F(z) , \tag{4.42a}$$

and for $m^2 \ll H^2$, $F(z) \simeq 1$, we arrive at

$$\langle \phi^2 \rangle = (3H^4/8\pi^2 m^2)(1 - e^{-2m^2 t/3H}) , \tag{4.42b}$$

which has as its limiting value the result in eq. (4.3).

For more massive scalar fields the results are more complex [136]. When $m \sim H$, one expects $\langle \phi^2 \rangle \simeq H^2$. For the specific value of $m = \sqrt{2}\, H$ ($\nu = \frac{1}{2}$) one finds $\langle \phi^2 \rangle \to (H^2/8\pi^2)(1 + 2e^{-1})$. In the very massive case $m \gg H$ ($\nu = i\mu = im/H$) one finds $\langle \phi^2 \rangle \to H^3/12\pi^2 m$. Of course only for very light scalar fields can the long wavelength modes of $\langle \phi^2 \rangle$ be interpreted as a homogeneous field (see below).

4.4. *Density perturbations and the isotropy of the microwave background radiation*

Since the original work [109, 121–123], there have been numerous detailed and technical studies on the origin of density perturbations in the inflationary scenario. Most of this work agrees quite well with the original estimates of $\delta\rho/\rho$ [138–144] while some others do not [106, 145] (the latter have all been criticized in some way in refs. [138–144]). The relationship of density perturbations and Hawking radiation is discussed in ref. [146]. There have also been numerous studies of density perturbations in specific models of inflation. I will defer any discussion of this work until the next section. In this subsection, I will look at the constraints from $\delta\rho/\rho$ on inflationary models and related constraints coming from the generation of gravitational radiation.

Earlier in this section, it was shown that in new inflation, nearly scale-free density perturbations are produced. Because quantum fluctuations in the very light scalar field (light compared with the Hubble scale) causes a time delay in the rollover, inflation is completed at different times in different places. Among the main results in refs. [109, 121–123] was that

$$\delta\rho/\rho = H^2/\pi^{3/2}\dot{\phi} ; \tag{4.18}$$

when the scalar potential is dominated by a quartic term as in eq. (4.2), the resultant magnitude of $\delta\rho/\rho$ was found to be

$$\delta\rho/\rho = (8\lambda/3\pi^3)^{1/2} \ln^{3/2}(Hk^{-1}) , \tag{4.22}$$

where λ is the quartic self-coupling. It is straightforward to obtain a similar result when the potential is dominated by a cubic term [147],

$$\delta\rho/\rho = (\beta/\pi^{3/2}H)\ln^2(Hk^{-1}), \tag{4.43}$$

where β is the cubic self-coupling.

Clearly the magnitude of $\delta\rho/\rho$ is limited by the constraints on the microwave background anisotropy. Recall that on scales larger than the horizon ($k < H$) $\delta\rho/\rho|_k \sim k^{3/2}|\delta_k| \sim k^2 \sim 1/\ell^2$, where $|\delta_k|^2 \sim k$ for the Harrison–Zeldovich spectrum [124, 125, 148]. Sachs and Wolfe [149] showed that one can directly relate energy density perturbations $\delta\rho/\rho$ to fluctuations in the background temperature $\delta T/T$ [148–150],

$$\frac{\delta T}{T} = -\tfrac{1}{2}H^2 \int \frac{\mathrm{d}^3k}{(4\pi)} (\delta_k/k^2)\,\mathrm{e}^{\mathrm{i}k\cdot x}, \tag{4.44}$$

which can be expanded in spherical harmonics as

$$\frac{\delta T}{T} = \sum_{l,m} a_{lm} Y_{lm}, \tag{4.45}$$

$$a_{lm} = -2\pi \mathrm{i}^l H^2 \int \frac{\mathrm{d}^3k}{(4\pi)}\, j_l(2kH^{-1})(\delta_k/k^2)Y_{lm}(\Omega_k), \tag{4.46}$$

where j_l is a spherical Bessel function. One then obtains [150, 151]

$$\langle a_l^2\rangle = \left\langle \sum_{m=-l}^{l} |a_{lm}|^2 \right\rangle = A\pi H^4 \frac{2l+1}{2l(l+1)} = 2\pi^2(\delta\rho/\rho)^2 \frac{2l+1}{l(l+1)}. \tag{4.47}$$

From observed limits on the quadrupole anisotropy [5],

$$a_2^2 < 5.2\times 10^{-8}, \tag{4.48}$$

implying a limit on $\delta\rho/\rho$ [151]:

$$\delta\rho/\rho < 1.0\times 10^{-4}. \tag{4.49}$$

A stronger constraint was derived from the limit on the dipole anisotropy [152],

$$\delta\rho/\rho < 3\text{–}7\times 10^{-6}. \tag{4.50}$$

These bounds on $\delta\rho/\rho$ can in turn place limits on the value of the Hubble parameter during inflation [153–155]. Late in inflation, it is possible to write $\dot\phi \simeq -(\partial V/\partial\phi)/3H$. Integrating this equation from the time, t_{B}, the length scale corresponding to the observational limits on the microwave background which left the horizon during inflation until the end of inflation, t_{e}, we have

$$\int_{\phi_{\mathrm{B}}}^{\phi_{\mathrm{e}}} \frac{\mathrm{d}\phi}{\partial V/\partial\phi} = -\int_{t_{\mathrm{B}}}^{t_{\mathrm{e}}} \mathrm{d}t/3H. \tag{4.51}$$

Under the general conditions, $\partial V/\partial\phi > \partial V/\partial\phi|_{t_B}$ for $t > t_B$ and

$$\partial V/\partial\phi|_{t_B} < 3\sigma H^2/(H\tau')\,, \tag{4.52}$$

where $\sigma = \phi_e - \phi_B$ and $\tau' = \tau_e - \tau_B$. From eq. (4.18) we have $\delta\rho/\rho \simeq 3H^3/\pi^{3/2}(\partial V/\partial\phi)$ and

$$H^3 < \pi^{3/2}(\delta\rho/\rho)\sigma H^2/H\tau'\,, \tag{4.53}$$

or

$$H < (\pi^{3/2}/H\tau')\sigma\,\delta\rho/\rho\,. \tag{4.54}$$

Taking $H\tau' \simeq 50$ and $\sigma \lesssim M_P$ and from eqs. (4.49), (4.50) $\delta\rho/\rho < O(10^{-5})$, we find

$$H < O(10^{-6})M_P \sim 10^{13}\ \mathrm{GeV}\,. \tag{4.55}$$

A weaker bound cited in ref. [155] assumes that the perturbations re-enter the horizon during matter domination and the factor of $\pi^{-3/2}$ is replaced by [123, 143] $0.1\pi^{-3/2}$. Nevertheless, a factor of ten is easily made up by considering semi-realistic models and performing the correct integration over eq. (4.51) [154]. Thus the bound [153–155] in eq. (4.55) is believable.

It is also possible to derive a bound on the Hubble parameter due to the creation of gravitons during inflation [156–159]. Using the techniques developed by Grishchuk [160], it is possible to calculate the spectrum of gravitational radiation in a De Sitter space. The presence of relic gravitons tends to distort the microwave background (as do density perturbations) [149], and leads to an anisotropy [157–159]

$$(\delta T/T)^2 \sim O(10^{-1})H^2/M_P^2 \lesssim 5\times 10^{-8} \tag{4.56}$$

from eq. (4.48), and one finds a limit

$$H \lesssim 7\times 10^{-4}M_p\,. \tag{4.57}$$

4.5. Generic models of new inflation

Despite the problems associated with inflationary models based on SU(5) radiative symmetry breaking, it is straightforward to put together a workable model of inflation (as will become readily apparent in the next section) [161]. In the simplest possible terms, it usually suffices to construct the model such that it satisfies the following three constraints:

1. An inflationary time-scale long enough to produce sufficient expansion to solve the cosmological problems, i.e., from eq. (3.25),

$$(\partial^2 V/\partial\phi^2)|_{\phi\sim H} < 3H^2/65 = 8\pi V(0)/65M_P^2\,. \tag{4.58}$$

Thus we require a flat potential. As we will see in the next section, many models are constructed such that the initial condition is not $\phi = 0$, in this case we more generally require

$$(\partial^2 V/\partial\phi^2)|_{\phi_i} < 8\pi V(\phi_i)/65M_P^2\,. \tag{4.58'}$$

2. The magnitude of density perturbations at horizon crossing is consistent with observed limits due to the microwave background anisotropy. In what follows, I shall use the quadrupole limit, eq. (4.49). Depending on the form of the scalar potential, this places limits on the self-couplings of the inflaton.

3. The inflaton must be minimally coupled so as to reheat the Universe and allow for the subsequent generation of the baryon asymmetry. Only in a few cases, as we shall see, is this difficult to accomplish.

In addition to these constraints, the inflationary model builder must consider a choice of initial conditions. Up until now, we have assumed typically that at high temperature, the symmetry is restored and initially $\langle \phi \rangle = 0$. There are several potential problems with this approach. The first problem [162] is that even with symmetry restoration driving $\langle \phi \rangle \to 0$, this refers only to the average value of ϕ and that a typical field configuration is not homogeneous but contains domains with $|\phi|$ in the range 0 to v. In addition, in regions with $\phi \simeq 0$, kinetic terms in the Lagrangian would prevent the onset on inflation. Though this surely is a problem for certain inflationary models (such as the original new inflationary model) it is not generic [163–166]. In the cases of two types of models, Coleman–Weinberg type and a simple quartic double well potential, it was found that [164] provided that the inflation's self couplings are sufficiently small, or (in the models considered) equivalently, if v is sufficiently large as in models of primordial inflation [119] (see section 5), inflation indeed does occur and the problems of ref. [162] are avoided. These results were generalized to arbitrary scalar potentials (including possibly non-renormalizable interactions) in ref. [165]. An analysis [163] considering a distribution of initial values for ϕ and requiring a minimal volume in the false vacuum state also concluded the necessity for primordial inflation [119] with $v \gg H$. An additional possibility for avoiding the problem [162] is based on the presence of nonminimal kinetic terms for fields coupling to the dilaton [163, 166].

The second potential problem with the new inflationary paradigm has to do with the use of the finite temperature corrections to the scalar potential [167, 168]. Despite the possibility that gravitational interactions may have kept the inflaton in thermal equilibrium, below M_P an inflaton with only gravitational couplings will have decoupled and the use of the finite temperature potential might be invalidated. Simply put, an interaction rate $\Gamma \sim \alpha^2 T$, where $\alpha \sim m/M_P$ and $m \ll M_P$ is the inflaton mass, is only larger than the expansion rate when

$$\alpha^2 M_P > T . \tag{4.59}$$

After decoupling, it has been argued that temperature corrections should be ignored and only initial field configurations with $\langle \phi \rangle > M_P$ are justified [167] or any other configuration is equally plausible so that inflation occurring where $\phi \simeq 0$ would eventually dominate over those regions with $\phi \neq 0$ [168]. On the contrary however, it has been argued [169] that despite decoupling, the prior establishment of thermal equilibrium necessitates the use of the finite temperature potential. The reason is that the phase space distribution of ϕ appears as if it were in thermal equilibrium (as does the microwave background radiation today) and the one-loop calculation is in fact justified. Thus it has been argued [169, 170] that a finite temperature minimum with $\phi \simeq 0$ is necessary.

There has also been a good deal of numerical work regarding the onset of inflation. The conclusions of ref. [164] were confirmed for models assuming thermal initial condition in ref. [171]. Similar results were also found with random initial conditions [172] with the conclusion that field excitations, thermal or otherwise, contribute effective terms which can drive ϕ towards the origin, termed dynamical relaxation. In the models considered, the finite temperature minimum was at $\phi = 0$. This substantiates the claims of Binetruy and Gaillard [169], who argued that despite decoupling prior thermal excitations remain thermal in character. Further investigations including the back-reaction of gravity [173], or the coupling to a radiation bath [174], or the effect of gravitation perturbations [175] do not alter these conclusions.

5. Models of inflation

In the previous section, we have seen many of the pitfalls that arise in constructing models for inflation. Using the original SU(5) model for new inflation [82, 83] as an example, the requirements for successful models are now understood. As we will see, there is no intrinsic difficulty in constructing a successful model. We will see below that most of the models presented have features which are attractive while at the same time possessing some undesirable characteristics. In what follows, I will review most of the different classes of inflationary models that have been suggested to date.

5.1. Primordial, supersymmetric inflation

In this subsection, I will look at some of the original models of primordial inflation (i.e., inflaton models where the symmetry breaking scale $v \gg M_{\mathrm{GUT}}$). Many later models also involve the Planck scale for inflation as well. I will also look at the broad class of models based on supersymmetry (global) and supergravity.

We have already seen that the resolution of problems regarding the onset of inflation required primordial inflation [163–165], i.e., the scale associated with symmetry breaking v must be greater than the scale associated with the vacuum energy density driving inflation, H. Using the very simple example of the double well potential, it will be easy to show that even the requirement of a long roll-over time scale requires $v > M_{\mathrm{P}}$. Consider the potential

$$V(\phi) = \lambda(\phi^2 - v^2)^2 . \tag{5.1}$$

The Hubble parameter is $H^2 = 8\pi\lambda v^4/3M_{\mathrm{P}}^2$ and the mass^2 at the origin is $|m^2| \simeq 4\lambda v^2$ and the roll-over condition (3.25) yields

$$(v/M_{\mathrm{P}})^2 > 65/2\pi , \tag{5.2}$$

or $v > M_{\mathrm{P}}$. Typically one would not normally associate the potential (5.1) as one suitable for inflation, but the condition (5.2) together with $\lambda \simeq 10^{-12}$ (to make $\delta\rho/\rho \sim 10^{-4}$) satisfies at least the two basic requirements for inflation. (Such a model would of course be instantly plagued with large radiative corrections.) Primordial inflation was introduced [119] to alleviate some of the fine-tuning problems associated with satisfying the inflationary constraints. A more detailed example of a non supersymmetric model of primordial inflation can be found in refs. [176, 177].

In addition to separating the symmetry breaking scale from GUTs, it was recognized that supersymmetry would also be a powerful tool in alleviating fine-tuning problems encountered in inflationary models [118]. Supersymmetry, as is well known by now, was incorporated into GUTs because of its ability to resolve the gauge hierarchy problems [178]. There are two aspects to this problem: (1) there is a separation in physical mass scales, $M_{\mathrm{W}} \ll M_{\mathrm{X}} \ll M_{\mathrm{P}}$; (2) this separation must be stable with respect to radiative corrections. The first problem has to do with perhaps a tree-level choice of mass parameters. A single fine-tuning. The second problem requires fine-tuning at many orders in perturbation theory. As we have seen in section 3, radiative corrections to scalar masses are quadratically divergent,

$$\delta m_0^2 \sim g^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4}\, \frac{1}{k^2} \sim \mathrm{O}(\alpha/\pi)\Lambda^2 , \tag{5.3}$$

where Λ is some cut-off scale. In the low energy electroweak theory, the smallness of M_W requires the mass of the physical Higgs boson to be $m_H \lesssim O(1)$ TeV. Requiring $\delta m_H^2 < O(m_H^2)$ implies that $\Lambda \lesssim O(1)$ TeV as well. The trouble comes when we move to a GUT where the natural cut-off is M_X (or even M_P) rather than $O(M_W)$ and we expect $\delta m_H^2 > O(10^{15})$ GeV [84, 61]. A cancellation may be imposed by hand, but this must be done to each order in perturbation theory. A solution to this difficulty would be to cancel the radiative corrections by including fermion loops which have the opposite sign [cf. eq. (2.12)]. Then provided $|m_B^2 - m_F^2| < O(1)$ TeV, the stability of the mass scales would be guaranteed. Such a cancellation occurs automatically in a supersymmetric theory (in the limit of exact supersymmetry, these radiative corrections are absent entirely). In addition, although gauge couplings still get renormalized, the Yukawa couplings of theory, which are parameters of a superpotential, do not get renormalized [179]. For the purposes of inflation, this is a strong indication that models should be constructed directly from a superpotential [118, 119].

The scalar potential in a globally supersymmetric theory can be written as [180]

$$V(\phi^i, \phi_i^*) = \sum_i |F_i|^2 + \tfrac{1}{2} \sum_a g_a^2 |D^a|^2 \,. \tag{5.4}$$

Because scalars always appear in chiral multiplets (in $N = 1$ supersymmetry), all scalars in supersymmetric theories are complex. In eq. (5.4),

$$F_i \equiv \partial F / \partial \phi^i \,, \tag{5.5}$$

where F is the superpotential and

$$|D^a|^2 = \Big(\sum_{i,j} \phi_i^* T^{a\,i}_{\;j} \phi^j\Big)^2 \,, \tag{5.6}$$

for generators $T^{a\,i}_{\;j}$ of a gauge group with gauge coupling g_a. In most of what we will be considering for inflationary models, we will restrict our attention to the F-terms of eq. (5.4).

The incorporation of supersymmetry into a GUT can be described by a superpotential of the form

$$F_1 = \lambda(\tfrac{1}{3}\,\mathrm{Tr}\,\Sigma^3 + \tfrac{1}{2} M\,\mathrm{Tr}\,\Sigma^2) \,, \tag{5.7}$$

yielding

$$V = \lambda^2\,\mathrm{Tr}|M\Sigma + \Sigma^2 - \tfrac{1}{5}\,\mathrm{Tr}\,\Sigma^2|^2 + \tfrac{1}{2} g^2\,\mathrm{Tr}([\Sigma, \Sigma^+])^2 \,. \tag{5.8}$$

This potential has three *degenerate* minima with $V = 0$ corresponding to SU(5), SU(4) × U(1) and SU(3) × SU(2) × U(1). There are large barriers separating these minima, and when finite temperature corrections are included, the SU(5) minimum is the sole minimum at high temperatures. At lower temperatures the contribution of terms such as $-\frac{1}{90}\pi^2 N T^4$ make SU(5) the global minimum of theory. The resolution [181] of this difficulty requires the consideration of strong coupling [181, 182] phenomena to break SU(5).

Additional superpotential terms are needed to complete the theory. For example terms such as

$$F_2 = \lambda_d T \bar{F} \bar{H} + \lambda_u T T H \tag{5.9}$$

are required for fermion masses. In eq. (5.9), T, $\bar{F}$, H and $\bar{H}$ are chiral supermultiplets for the **10** and $\bar{\mathbf{5}}$ plets of SU(5) matter fields and the Higgs **5** and $\bar{\mathbf{5}}$ multiplets. The pure scalar interactions are derived from eqs. (5.4) and (5.5). In addition there are Yukawa terms of the form

$$V_{\mathrm{Y}} = (\partial^2 F/\partial\phi^i\,\partial\phi^j)\Psi^i\Psi^j\,, \tag{5.10}$$

where Ψ^i is the fermion component ϕ^i. Furthermore, Higgs interactions of the form

$$F_3 = \lambda'\bar{H}(\Sigma + 3M')H \tag{5.11}$$

must also be included. When $M' = M$ [from eq. (5.7)] there is the desired splitting of the Higgs doublets and triplets, though this itself is an extreme fine-tuning. For other solutions to the doublet–triplet problem see refs. [183–185].

Supersymmetry of course must be broken at very low energies, i.e., we know that $m_{\tilde{\gamma}} \neq m_\gamma$, $m_{\tilde{e}} \neq m_e$ etc. When global supersymmetry is spontaneously broken, there is the appearance of a goldstone fermion (goldstino) and the vacuum energy density $V > 0$. Thus supersymmetry will be broken when either or both of F_i, $D^a \neq 0$, corresponding to F-type or D-type supersymmetry breaking. Problems surrounding both D-type [186] and F-type [187] breaking has led to the widespread consideration of local supersymmetry (i.e., supergravity) [188].

Returning now to the subject at hand, how does supersymmetry help inflation? For one thing, it is clear that the scenario using radiative symmetry breaking must be greatly modified. The simple inclusion of massive fermions would lead to $A = 0$ in eq. (2.12) in the context of exact supersymmetry. When supersymmetry breaking effects are taken into account we can write [118, 119]

$$A = \frac{1}{32\pi^2 v^4}\sum_{\mathrm{B}} g_{\mathrm{B}} m_{\mathrm{s}}^2 M_{\mathrm{B}}^2\,, \tag{5.12}$$

where $m_{\mathrm{s}}^2 = M_{\mathrm{B}}^2 - M_{\mathrm{F}}^2 \ll M_{\mathrm{B}}^2 \simeq M_{\mathrm{F}}^2 \simeq M_{\mathrm{X}}^2$. Thus

$$A = \frac{75}{32\pi^2 v^2}\, g^2 m_{\mathrm{s}}^2\,. \tag{5.13}$$

For $gm_{\mathrm{s}} < 2\times 10^{-6} v$, $A < 10^{-12}$ and compatibility with $\delta\rho/\rho \lesssim 10^{-4}$ is achieved. For $v \sim M_{\mathrm{X}}$, this simply requires $gm_{\mathrm{s}} < 2\times 10^9$ GeV whereas we expect $m_{\mathrm{s}} \sim \mathrm{O}(1)$ TeV.

The first attempt to construct an inflationary model based on global supersymmetry [119] utilized an inflaton and two additional chiral supermultiplets and was based on the Hawking and Moss version [104] of the new inflationary scenario. The superpotential in this model was

$$F(\phi, X, Y) = aX\phi(\phi - \mu) + bY(\phi^2 - \mu^2)\,, \tag{5.14}$$

based on the superpotential in ref. [189]. High temperature corrections prefer $X = Y = 0$, after which one finds a scalar potential of the form

$$V(\phi) = (a^2 + b^2)\phi^4 - 2a^2\mu\phi^3 + (a^2 - 2b^2)\mu^2\phi^2 + b^2\mu^4 + cT^2\phi^2\,, \tag{5.15}$$

with $c = (2a^4 + a^2b^2 + 2b^4)/(2b^2 - a^2)$. The inflationary requirements lead to $a, b \sim \mathrm{O}(10^{-6})$ and

$(a^2/b^2-2) \sim O(10^{-7})$ if $\langle\phi\rangle = \mu \sim O(10^{16})$ GeV and a, $b \sim 10^{-3}$ and $(a^2/b^2-2) \sim O(10^{-1})$ with $\mu \sim 10^{19}$ GeV. Another indication towards primordial inflation. Density perturbations place a stronger constraint on these parameters [147]. To insure $\delta\rho/\rho \lesssim 10^{-4}$ one requires $a \lesssim O(10^{-6})\mu/M_P$. Though this is not a terribly compelling model it does satisfy the inflationary constraints and avoids the problems discussed in section 4 [119, 158]. It is interesting to note that the above model with $\mu \sim M_P$ may also resolve the degeneracy problem in supersymmetric GUTs [190]. During inflation with $M_\Sigma < H$, scalar field fluctuations might be capable of driving the SU(5) phase transition.

There was also an attempt [191] to construct an inflationary model based on the inverse hierarchy model [183, 192, 193] in which the GUT scale is determined from M_W. This model has been largely abandoned because of numerous difficulties [193] in constructing phenomenologically viable models. These problems include the generation of sufficiently large hierarchy, strong coupling above the unification scale, and proton decay. The inflationary model suffered a similar fate. The model has severe problems with reheating [191] and density perturbations [194].

Finally a supersymmetric-like but non-supersymmetric model was proposed in ref. [195] with the inclusion of heavy fermions. This is basically a Coleman–Weinberg type model. Renormalization group effects in all models of this type were discussed in ref. [196].

The next logical step in constructing supersymmetric inflationary models is to include supergravity [197]. The couplings in an $N=1$ supergravity theory are determined [198] by a real function of the chiral fields called the Kähler potential $G(\phi^i, \phi_i^*)$ and an analytic function $f_{\alpha\beta}(\phi)$ for the gauge fields. The function $f_{\alpha\beta}$ determines the kinetic terms as

$$\mathscr{L} \ni f_{\alpha\beta}(\phi) F_{\mu\nu}{}^{\alpha} F^{\mu\nu\beta} . \tag{5.16}$$

The role of $f_{\alpha\beta}$ will not be considered any further. The Kähler potential determines the scalar kinetic terms

$$\mathscr{L} \ni G_i^j \partial^\mu \phi^i \, \partial_\mu \phi_j^* \tag{5.17}$$

and the scalar potential

$$V = e^G (G_i (G^{-1})_j^i G^j - 3] + D \text{ terms} , \tag{5.18}$$

where

$$G_i \equiv \partial G/\partial \phi^i, \qquad G^j \equiv \partial G/\partial \phi_j^*, \qquad G_i^j \equiv \partial^2 G/\partial \phi^i \, \partial \phi_j^*, \qquad \text{etc} . \tag{5.19}$$

Minimal $N=1$ supergravity refers to those theories in which $G_j^i = \delta_j^i$ and G can be written as

$$G = \phi^i \phi_i^* + \ln|F|^2 , \tag{5.20}$$

where $F(\phi)$ is again the superpotential. (I will be using for convenience units such that $\kappa = 1$.) In minimal $N=1$ supergravity the scalar potential then takes the form

$$V = e^{\phi_i^* \phi^i} \left(\sum_i \left| \frac{\partial F}{\partial \phi^i} + \phi_i^* F \right|^2 - 3|F|^2 \right) + D \text{ terms} . \tag{5.21}$$

In the limit that $M_P \to \infty$, eq. (5.21) reduces to the global form given in Eq. (5.4). Note that eq. (5.21) is not positive semi-definite as was eq. (5.4), this will enable us to find minima in which supergravity is broken and $V = 0$.

The spontaneous breakdown of local supersymmetry is accompanied by the appearance of a massive spin $\frac{3}{2}$ state, namely the gravitino. The goldstino has now been eaten by the gravitino in the super-Higgs effect [199, 200, 198]. The condition for breaking supergravity is

$$e^{G/2} G_i \neq 0 . \tag{5.22}$$

The simplest model for breaking supergravity introduces a single chiral superfield z, called the Polonyi field [200]. The fermion component is the goldstino and gets eaten by the gravitino. The superpotential can be written as [200, 198, 201]

$$F(z) = \mu^2 (z + \Delta) , \tag{5.23}$$

so that

$$V(z, z^*) = \mu^4 e^{|z|^2} [(1 - 3\Delta^2) - 2(z + z^*)\Delta + |z|^2(-1 + \Delta^2 + (z + z^*)\Delta + |z|^2)] . \tag{5.24}$$

Along the direction $z = z^*$, V has a minimum at $\langle z \rangle = v = (\sqrt{3} - 1)$ and at the minimum $V(v) = 0$ if $\Delta = (2 - \sqrt{3})$. The two scalar fields have masses $m_A^2 = 2\sqrt{3}\, m_{3/2}^2$ and $m_B^2 = 2(2 - \sqrt{3}) m_{3/2}^2$ where the gravitino mass is

$$m_{3/2}^2 \equiv e^G = \mu^4 e^{(\sqrt{3}-1)^2} . \tag{5.25}$$

With $\mu \sim 10^{10}$ GeV, $m_{3/2} \sim 100$ GeV. This potential is very flat since $V(0)^{1/4} \sim \mu \ll v$.

Cosmologically, the breaking of $N = 1$ supergravity has been shown to be problematic [202]. In the early Universe the evolution of z is determined by the equation of motion [cf. eq. (3.23)]

$$\ddot{z} + 3H\dot{z} = -\partial V / \partial z . \tag{5.26}$$

When $H > \mu^2/M_P \sim m_{3/2}$, z is constant and when $H < \mu^2/M_P$, z begins to oscillate about the minimum at v. The initial value of $\langle z \rangle$ might be determined by thermal effects or by fluctuations during inflation. In the former case, the minimum at finite temperature is at $\langle z \rangle_T \neq v$. In the latter, because the mass of the Polonyi field z is so small, fluctuations in $\langle z^2 \rangle$ are induced during inflation. When $m_z^2 \ll H^2$, $\langle z^2 \rangle \sim H^3 t$ up to a limiting value $\langle z^2 \rangle = 3H^4/8\pi^2 m_z^2$ (in our case it would be much smaller since m_z^2 is very large when $\langle z \rangle \gtrsim v$). In either case we expect that $\langle z \rangle_i \simeq \langle z^2 \rangle^{1/2} \sim M_P$ but not at v so that $v - \langle z \rangle_i \sim M_P$ as well.

When z begins oscillating at $T \sim \mu$ the energy density is $\rho \sim m_z^2 (\Delta z)^2 \sim \mu^4$ and subsequently $\rho \sim \mu T^3$. As with the inflaton, oscillations continue until z decays at T_{dz} when $H \sim \mu^{1/2} T^{3/2}/M_P \sim \Gamma_z \simeq m_z^3/M_P^2 \sim \mu^6/M_P^5$, or at $T_{dz} \sim \mu^{11/3}/M_P^{8/3}$. For $\mu \sim 10^{10}$ GeV, $T_{dz} \sim 10^{-2}$ eV ($T_0 = 2.7\,\mathrm{K} \sim 2 \times 10^{-4}$ eV) and the Universe "reheats" to $T_R \sim \rho_{osc}^{1/4}(T_d) \sim \mu^3/M_P^2 \sim 1$ keV. The problem with that is that nucleosynthesis would have taken place during the oscillations which is characteristic of a matter dominated expansion rather than a radiation dominated one. Because of the difference in the expansion rate the abundances of the light elements would be greatly altered [203]. Even more problematic is the entropy release due to the decay of these oscillations. The entropy increase is [202]

$$S_f/S_i \simeq (T_R/T_{dz})^3 \sim (M_P/\mu)^2 \sim 10^{16}\,, \tag{5.27}$$

far too much to understand the present value of η in light of nucleosynthesis and baryosynthesis.

The presence of a flat direction for the breaking of supergravity clearly has an important impact, albeit a negative one, on the early Universe. The simplest solution, to raise the scale of μ, is not acceptable in this context because it would destroy the mass hierarchy which requires the mass splitting. The mass splitting is determined by the super-trace formula [198]

$$\mathrm{STr}\, M^2 = \sum_{J=0}^{3/2} (-1)^{2J}(2J+1)m_J^2 = 2(N-1)m_{3/2}^2 \tag{5.28}$$

(neglecting D-term contributions), where N is the number of chiral supermultiplets, and yields

$$\Delta m^2 \sim m_{3/2}^2 \sim \mu^4/M_P^2 \lesssim \mathrm{O}(1)\,\mathrm{TeV}^2\,. \tag{5.29}$$

Additional soft supergravity breaking terms may be present in the presence of non-minimal kinetic terms such as [198, 201, 204]

$$m_0^2\phi^i\phi_i^*\,, \qquad m_0 A F(\phi) + \mathrm{h.c.}\,, \qquad m_{1/2}\Psi^a\Psi^a\,, \tag{5.30}$$

where m_0, $m_{1/2}$ are soft supersymmetry breaking scalar and gaugino masses. Unlike the case for global supersymmetry, locally supergravity GUTs have led to interesting and viable low energy theories [205]. The problems regarding the breakdown of SU(5) in supergravity was discussed in ref. [206].

The first attempt to construct an inflationary model in the context of $N = 1$ supergravity was in ref. [207]. This was again a primordial inflation model with a single chiral superfield, the inflaton, with a general superpotential

$$F(\phi) = m^2 \sum_n \lambda_n \phi^n\,. \tag{5.31}$$

It was hoped that the couplings λ_n could be O(1) and be adjusted so that the scalar potential given by eq. (5.21) would satisfy the inflationary constraints. In addition, because of the difference in the mass scales involved in inflation and supersymmetry breaking the two were generally kept separate, i.e., $F(\phi)$ was chosen so that supersymmetry was unbroken at the global minimum.

Let us for the moment disregard the questions of initial conditions discussed in the previous section. The simplest example of an inflationary model of the type (5.31) is given by [168]

$$F(\phi) = m^2(1-\phi)^2\,, \tag{5.32}$$

when m is chosen to be $\mathrm{O}(10^{-4})$ to satisfy the constraint due to $\delta\rho/\rho$. To see this, we can write the scalar potential as

$$\begin{aligned} V = m^4\, \mathrm{e}^{|\phi|^2}[&1 + |\phi|^2 - (\phi^2+\phi^{*2}) - 2|\phi|^2(\phi+\phi^*) \\ &+ 5|\phi^2|^2 + |\phi|^2(\phi^2+\phi^{*2}) - 2|\phi^2|^2(\phi+\phi^*) + |\phi^3|^2] \\ \simeq{}& m^4(1 - 4\phi^3 + \tfrac{13}{2}\phi^4 + \cdots)\,, \end{aligned} \tag{5.33}$$

along the real direction (the imaginary direction is stable for $\phi_I = 0$ and $\phi_R \neq 0$ in this case). This potential is plotted in fig. 10. For this model,

$$H^2 = 8\pi m^4/3M_P^2\,, \tag{5.34}$$

and the cubic term is just

$$\beta = (4m^4/M_P^3)(8\pi)^{3/2}\,. \tag{5.35}$$

The inflationary time scale is

$$H\tau \sim 3H^2/|m|^2 \sim [H^2/(8Hm^4)](M_P/\sqrt{8\pi})^3 \sim (1/64\sqrt{3}\pi)(M_P^2/m^2)\,, \tag{5.36}$$

where $|m|^2$ is found from (5.33) at $\phi \simeq H$. For $m \sim O(10^{-4})M_P/\sqrt{8\pi}$, $H\tau \sim 10^6$, clearly enough inflation. From eq. (4.43),

$$\delta\rho/\rho = 32\sqrt{3/\pi}(m^2/M_P^2)\ln^2(Hk^{-1})\,, \tag{5.37}$$

so that for the same choice of m, $\delta\rho/\rho \sim 10^{-4}$. This result has been verified in detail in refs. [138, 208]. Thus there is only a single parameter which must be slightly tuned (to the level of 10^{-4}) to make an acceptable model of inflation. In addition because $F(1) = F_\phi(1) = 0$ ($v = 1$ is the global minimum in this model) supersymmetry is unbroken.

Let us now return to the question of initial conditions and the thermal constraint. The importance of this constraint with regard to supergravity models was stressed in refs. [170, 209]. The finite temperature potential in supergravity theories [210] has been calculated [211, 169, 212, 213]. Similar to eq. (2.13), we can write

$$V_T = -\tfrac{1}{48}\pi^2 N_B T^4 + \tfrac{1}{24}\mathrm{Tr}(m_B^2 + \tfrac{1}{2}m_F^2)T^2\,, \tag{5.38}$$

where N_B is the number of boson degrees of freedom and m_B^2 (m_F^2) is the boson (fermion) mass^2 matrix.

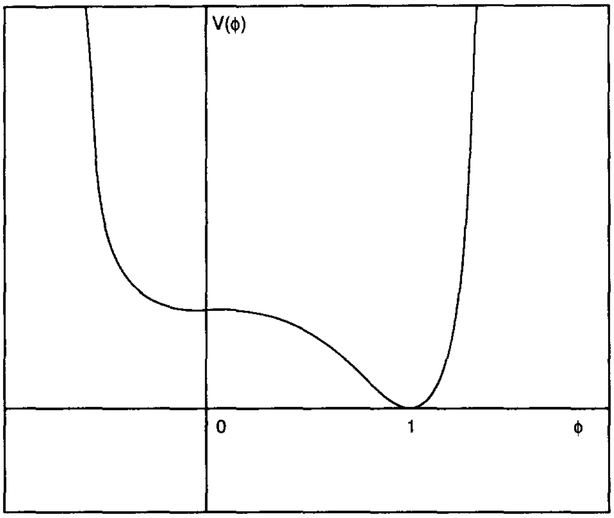

Fig. 10. The scalar potential V in eq. (5.33) derived from the superpotential in eq. (5.32). The potential given by eq. (5.56) is almost identical.

If we neglect D term contributions (these are included in ref. [213]), we find [211, 169, 212, 213]

$$\operatorname{Tr} m_{\mathrm{B}}^2 = 2(G^{-1})_j^i V_i^j \,, \tag{5.39a}$$

$$\operatorname{Tr} m_{\mathrm{F}}^2 = 2\mathrm{e}^G (G^{-1})_k^j Y_{ij} (G^{-1})_l^i Y^{lk} - 4\mathrm{e}^G \,, \tag{5.39b}$$

$$Y_{ij} = G_{ij} + G_i G_j - G_k (G^{-1})_p^k G_{ij}^p \,. \tag{5.39c}$$

In a minimal theory, this non-constant piece becomes [211, 169]

$$V_T = \tfrac{1}{12} T^2 \mathrm{e}^G [\tfrac{3}{2}(G^i G_{ij} G^j + G_i G^{ij} G_j + \tfrac{3}{2} G_{ij} G^{ij} + (G_i G^i + N)(G_i G^i - 2) + G_i G^i - 1 + \tfrac{1}{2}(G_i G^i)^2] \,. \tag{5.40}$$

The important point here is that in the limit of a large number N of chiral superfields the finite temperature potential becomes

$$V_T = \tfrac{1}{12} N T^2 \mathrm{e}^G (G_i G^i - 2) \,, \tag{5.41}$$

which is to be compared with the scalar potential

$$V = \mathrm{e}^G (G_i G^i - 3) \,. \tag{5.42}$$

There are two conclusions that one can draw at this point: (1) It is straightforward to show that in order to satisfy the thermal constraint one needs $(\partial V/\partial\phi)(0) = 0$ and $(\partial V_T/\partial\phi)(0) = 0$, which implies necessarily that $\mathrm{e}^G G_\phi = 0$ and in turn that $V(0) \le 0$, making a transition to a state with $V(\phi \ne 0) = 0$ impossible. (2) Equivalently, one can show [209] that any potential with $V(0) > 0$ and a supersymmetry preserving minimum with $V(\phi \ne 0) = 0$ has an additional minimum closer to the origin with $V < 0$. Thus one can conclude [169] that at the global minimum supersymmetry must be broken. It should be clear therefore that the model [168] described by eq. (5.32) does not satisfy the thermal constraint. A possible way around this could be to introduce a second field [168, 214] though one would require large [O(N)] couplings [214] making this scheme rather implausible.

The models considered up to now were constrained to preserve supersymmetry at the global minimum. Several models have been proposed [215, 214, 216, 217] which break supersymmetry at the global minimum. One approach [214] has been to start with a supersymmetry preserving minimum and perturb the theory to ensure that the gravitino mass remains small, $m_{3/2} \sim \varepsilon m^2/M_{\mathrm{P}}$. The thermal constraint in these models is also violated. Another approach [216] using two chiral superfields proposes to break supersymmetry at a low scale and has strong reheating producing a hierarchy of scales [218]. These models, however, required superpotential couplings with a ratio of $O(10^{-10})$ and were later found to be unstable with respect to perturbations in the imaginary direction [219, 217]. The use of gravitational radiative corrections [220] was suggested to cure this difficulty [217]. A two-component supergravity inflationary model was also considered in ref. [221].

Reheating in most of the above models takes place in a similar fashion as described in section 4 [75, 222, 223]. Inflaton oscillations about the global minimum and their subsequent decay reheat the Universe (provided the inflaton is gravitationally coupled to matter). Assuming a gravitationally coupled inflaton, one expects that the inflaton decay rate is

$$\Gamma_D \sim m_\phi^3/M_P^2 \sim m^6/M_P^5\,, \tag{5.43}$$

since $m_\phi \sim m^2/M_P$ [recall that m^2 is the parameter in the superpotential (5.31)]. The Universe reheats to

$$T_R \sim (\Gamma M_P)^{1/2} \sim m_\phi^{3/2}/M_P^{1/2} \sim m^3/M_P^2 \sim 10^7\text{–}10^{10}\ \text{GeV} \tag{5.44}$$

for $(m/M_P) \sim 10^{-3}$–10^{-4}. The expected baryon asymmetry is then

$$n_B/s = \varepsilon T_R/m_\phi = \varepsilon m_\phi^{1/2}/M_P^{1/2} \sim \varepsilon m/M_P\,. \tag{5.45}$$

This may be somewhat lower if subsequent particle decays produce additional entropy [222]. This requires, however, the existence of a Higgs boson mediating baryon number violating interactions with a mass $m_H < m_\phi \sim 10^{12}$ GeV. When combined with constraints on the reheating temperature from gravitino production [31, 74–80] (see section 6) these models may be in conflict with limits from proton decay [224]. The baryon asymmetry in models of the type in ref. [214] was examined in ref. [225].

As mentioned in the previous section, there is an alternative mechanism [132] for generating the baryon asymmetry in supersymmetric theories. In addition to the flat direction associated with the small scale of supersymmetry breaking, the standard supersymmetric model also contains numerous other flat directions as well. Along these directions squarks and sleptons have in general non-zero vacuum expectation values. Supersymmetry breaking lifts the degeneracy and the "symmetry restoration" can lead to a sizeable baryon asymmetry in the context of a GUT.

Let us consider the superpotential for the standard model,

$$F = \lambda_1 \bar{H} Q d^c + \lambda_2 H Q u^c + \lambda_e \bar{H} L e^c + \lambda_4 m H \bar{H}\,, \tag{5.46}$$

where Q and L represent $SU(2)_L$ doublets of quarks and leptons, u^c, d^c, e^c are the SU(2) singlets and H and $\bar{H}$ are doublet Higgses [cf. eq. (5.9)]. These are just the Yukawa mass terms for quarks and charged leptons plus a mixing term for H and $\bar{H}$. The scalar potential is given by eq. (5.4). A flat direction can be constructed by taking some linear combination of squark and slepton fields such that $V = 0$. For example, taking [132] $u_3^c = a$, $s_2^c = a$, $-u_1 = v$, $b_1^c = e^{i\alpha}(v^2 + a^2)^{1/2}$ and $\mu^- = v$ with $H = \bar{H} = 0$, both F and D are flat under $SU(3)_C \times SU(2)_L \times U(1)_Y$. Another simple example is [226] $d_1^c = s_2^c = t_3^c = v$. In both cases v and a are arbitrary. Supersymmetry breaking destroys the degeneracy so that scalars typically pick up masses of the order $\tilde{m}^2 \le (1\ \text{TeV})^2$.

If we let η represent one of these flat directions, then when $\langle\eta\rangle \ne 0$, in a GUT such as SU(5), there exists operators V, of the form $qqql$, such that $\langle V\rangle \ne 0$. We can then define a baryon number per particle as [132]

$$B = \mathrm{Im}\langle V\rangle/\tilde{m}^2\langle\eta\rangle_0^2 \sim \varepsilon\langle\eta\rangle_0^2/M_G^2\,, \tag{5.47}$$

where ε is a measure of CP violation. When the expansion rate of the Universe, H, falls below $\tilde{m}$, η begins oscillations about the origin so that the baryon density is

$$n_B \simeq B n_\eta \simeq B\tilde{m}\langle\eta\rangle^2 \simeq B\tilde{m}\langle\eta\rangle_0^2 (R_\eta/R)^3\,, \tag{5.48}$$

where $R = R_\eta$ when oscillations begin. As before we can compute the final asymmetry when the oscillations decay when $\Gamma_\eta \sim \tilde{m}^3/\langle\eta\rangle^2 \sim H \sim \tilde{m}\langle\eta\rangle/M_P$. The final baryon-to-photon ratio is [132]

$$n_B/n_\gamma \sim \varepsilon(\langle\eta\rangle_0^2/M_G^2)(M_P/\tilde{m})^{1/6}\,, \tag{5.49}$$

which as one can see could be O(1)!

Because of the fact that this asymmetry is produced late, $T_R \sim 10^4$ GeV, there are very few dissipative processes to damp this asymmetry before nucleosynthesis [227]. Is this asymmetry embarrassingly large?

Normally we consider the origin of $\langle\eta\rangle_0 \neq 0$ to be due to inflation, via the scalar field fluctuations discussed previously. That is, $\langle\eta\rangle_0^2 \simeq \langle\eta^2\rangle$, which grows as $H^3 t$ during inflation. We can therefore ask the questions: what is $\langle\eta\rangle_0$ in a typical inflation model of the type just discussed and can any of the entropy produced by inflation be used to damp the baryon asymmetry? For our inflaton the potential is $V(\phi) = m^4 P(\phi)$, where $P(\phi)$ is a polynomial [134] in ϕ as in eq. (5.33). Recall that the duration of inflation was given by (5.36) so that $\langle\eta^2\rangle \sim H^3\tau \sim m^2$ and we should take $\langle\eta\rangle_0^2 \sim \langle\eta^2\rangle \sim 10^{-7}\,M_P^2$. Furthermore, after inflation, the expansion rate is dominated by inflaton oscillations so that in reality fermion decay is determined by $\Gamma_\eta \sim \tilde{m}^3/\langle\eta\rangle^2 \sim H \sim m\langle\phi\rangle/M_P$ with $\langle\phi\rangle_0 \sim M_P$. Then the baryon asymmetry becomes [134]

$$n_B/n_\gamma \sim \varepsilon\langle\eta\rangle_0^4 m_\phi^{3/2}/M_G^2\tilde{m}M_P^{5/2}\,. \tag{5.50}$$

For $\varepsilon \sim 10^{-3}$, $\langle\eta\rangle_0^2 \sim 10^{-7}M_P^2$, $M_G \sim 10^{-1}M_P$, $m_\phi \sim 10^{-7}M_P$ and $\tilde{m} \sim 10^{-16}M_P$ we find $n_B/n_\gamma \sim 3\times10^{-10}$, in remarkable agreement with the desired value from nucleosynthesis. A general consideration of these effects including massive scalars was given in ref. [228].

Before moving on to no-scale supergravity it is worth noting that there have been attempts to cure the Polonyi problem of excess entropy generation. One attempt [229] uses a modified Polonyi superpotential of the O'Raifeartaigh [187] type. The other [230] assumes that during inflation, the Polonyi fields can settle at its minimum. This was correctly criticized, however, in ref. [231] noting that $\langle z^2\rangle$ fluctuations would be large during inflation.

Although it is not possible to construct an inflationary model in minimal $N = 1$ supergravity which satisfies all constraints including the thermal constraint, it is possible in the context of non-minimal supergravity theories [232]. By considering a general Kähler potential of the form

$$G = a_1(\phi + \phi^*) + a_2|\phi|^2 + a_3(\phi^2 + \phi^{*2}) + \cdots + \ln|F|^2\,, \tag{5.51}$$

it is possible to choose all $a_i \sim$ O(1) such that the only small scale m is again an overall constant in F and satisfy all constraints. Despite the fact that such a model is not very appealing it does suggest that non-minimal supergravity models may be important for inflation.

There exist several models of inflation [233, 234, 235] based on no-scale supergravity [236]. The two-component models in ref. [221] are also based on no-scale supergravity. The Kähler potential for no-scale supergravity is given by [237]

$$G = -3\ln(z + z^*) + \phi^i\phi_i^* + \ln|F(\phi)|^2 \tag{5.52}$$

for theories based on Kähler manifolds possessing non-compact SU(1, 1)/U(1) symmetry or may be generalized to

$$G = -3\ln[z + z^* + g(\phi^i, \phi_i^*)] + \ln|F(\phi)|^2\,, \tag{5.53}$$

based on an $SU(N, 1)/SU(N) \times U(1)$ symmetry. In the absence of fields ϕ^i and the superpotential F, the scalar potential vanishes identically. Taking $g(\phi^i, \phi_i^*) = -\frac{1}{3}\phi^i\phi_i^*$ leads to a very simple form for the scalar potential,

$$V = e^{2\bar{G}/3}|F_i|^2 = e^{\bar{G}}|F_{i'}|^2\,, \tag{5.54}$$

where $\bar{G} = G - \ln|F|^2$ and $F_{i'} = \partial F/\partial\phi^{i'}$. In this theory the fields z and ϕ no longer have properly normalized kinetic terms. The physical fields become $\xi_R = \bar{G}/\sqrt{6}$, $\xi_I = e^{\bar{G}/3}\sqrt{\frac{3}{2}}(z - z^*)$ and $\phi' = e^{\bar{G}/6}\,\phi$. Note that the potential closely resembles that of global supersymmetry. At the tree level, there is no potential for z, hence the gravitino mass is undetermined and can take values $m_{3/2} \sim M_W$ [238], $m_{3/2} \sim M_P$ [239] or $m_{3/2} \ll M_W$ [240].

The simplest model for inflation of this type is described by the superpotential [234]

$$F(\phi) = m^2(\phi - \phi^4/4)\,, \tag{5.55}$$

which effectively gives a scalar potential

$$V(\phi) = m^4|1 - \phi^3|^2\,. \tag{5.56}$$

Again the choice of $m \sim 10^{-3}$–$10^{-4}(M_P/\sqrt{8\pi})$ yields sufficient inflation and density perturbations with $\delta\rho/\rho \sim 10^{-4}$. This scalar potential at $T = 0$, is very similar to that shown in fig. 10. An explicit calculation of the high temperature potential [234, 213] shows that this model does satisfy the thermal constraint.

Despite the fact that it is clear that one can construct working and viable models of inflation, none of these nor any of the following models are too compelling. They all lack an integration with the rest of particle theory.

5.2. *Chaotic inflation*

Most of the models discussed in the previous subsection assumed a set of thermal initial conditions and made use of finite temperature corrections to the potential. If it is assumed that the initial state was not thermal, then at scales $T < M_P$, weakly coupled fields do not have time to attain thermal equilibrium until $T \ll M_P$ [167, 168]. In the supergravity model of ref. [168] where the thermal constraint was not satisfied, it was assumed that those regions with $\phi \simeq 0$ inflate while others do not. Arguably, the effective theory is well understood for $\phi \simeq 0$. Another approach to inflation which also assumes the lack of an initial thermal state, is chaotic inflation [167, 241]. In these models, it is assumed that as part of an initially chaotic state, $\phi \gtrsim M_P$ or even $\phi \gg M_P$. It is argued that so long as $V(\phi) \leq M_P^4$, these initial conditions are justified. Once these assumptions are made, these models of inflation are by far the simplest.

Typical models for chaotic inflation in terms of a single scalar field are described by the following scalar potentials [167, 241, 242]:

$$V=\tfrac{1}{4}\lambda\phi^4\,, \tag{5.57}$$

or

$$V=\tfrac{1}{2}m^2\phi^2\,. \tag{5.58}$$

That is all! Nothing more complicated is necessary. See fig. 11. It is assumed that at the Planck time, all fields $\phi(x)$ satisfy $V(\phi)\lesssim M_{\mathrm{P}}^4$ and $(\partial_\mu\phi)^2\lesssim M_{\mathrm{P}}^4$. It will also be assumed that there exist domains with sufficiently large $l\gtrsim H^{-1}$ with ϕ homogeneous and $\phi\gtrsim M_{\mathrm{P}}$.

For suitably large initial values of the scalar field ϕ, the Universe expands quasi-exponentially with

$$R=R_0\exp\left(\int_0^t H(t)\,\mathrm{d}t\right), \tag{5.59}$$

and the Hubble parameter is given by

$$H=(\tfrac{2}{3}\pi\lambda)^{1/2}\phi^2/M_{\mathrm{P}}\,, \tag{5.60}$$

for the potential given by eq. (5.57). By solving the equations of motion [cf. eq. (3.23)] [167, 243, 244], one finds that for $\phi\gtrsim M_{\mathrm{P}}$, the evolution of ϕ depends very weakly on the initial value of ϕ and neglecting the $\ddot{\phi}$ term in eq. (3.23) we have

$$\phi=\phi_0\exp(-\sqrt{\lambda}M_{\mathrm{P}}t/\sqrt{6\pi})\,, \tag{5.61}$$

where ϕ_0 is the initial value of ϕ. Inserting (5.61) into (5.59) one finds

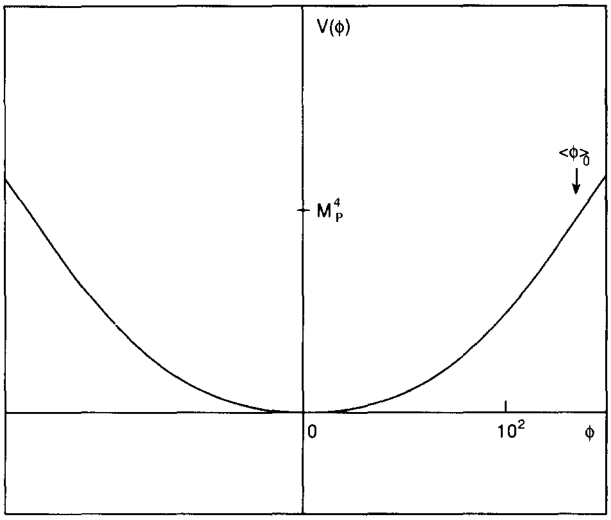

Fig. 11. A typical $m^2\phi^2$ potential for chaotic inflation. The point $\langle\phi\rangle_0$ corresponds to $V(\phi)\simeq M_{\mathrm{P}}^4$.

$$R = R_0 \exp\{(\pi\phi_0^2/M_P^2)[1 - \exp(-2\sqrt{\lambda}M_P t/\sqrt{6\pi})]\} , \tag{5.62}$$

and at small times

$$R(t) \sim \exp[(\tfrac{2}{3}\pi\lambda)^{1/2}\phi_0^2 t/M_P] . \tag{5.63}$$

The time-scale for inflation is

$$H\tau \simeq \pi\phi_0^2/M_P^2 \tag{5.64}$$

and is independent of λ [from eq. (3.25) one would find $H\tau \simeq \frac{2}{3}\pi\phi_0^2/M_P^2$]. Thus $H\tau > 65$ implies $\phi_0 \gtrsim 4.5 M_P$ and $V(\phi_0) \lesssim M_P^4$ only requires $\lambda < O(10^{-2})$.

Analogous results can be obtained [242, 244] for the case described by eq. (5.58). In this case the Hubble parameter is simply

$$H = (\tfrac{4}{3}\pi)^{1/2} m\phi/M_P , \tag{5.65}$$

and the equations of motion yield

$$\phi(t) = \phi_0 - mM_P t/\sqrt{12\pi} , \tag{5.66}$$

$$R(t) = R_0 \exp\{(2\pi/M_P^2)[\phi_0^2 - \phi^2(t)]\} . \tag{5.67}$$

The time-scale for inflation is now

$$H\tau \simeq 2\pi\phi_0^2/M_P^2 . \tag{5.68}$$

Now for $\phi_0 \gtrsim 3M_P$, there is a suitable amount of inflation. Models with a single non-interacting scalar field have been examined in detail in ref. [245].

Although the inflationary time-scale did not put a strong constraint on chaotic models, a stronger constraint is derivable from the consideration of density perturbations. Using eq. (4.22), one finds that $\delta\rho/\rho \lesssim 10^{-4}$ for $\lambda \lesssim 10^{-13}$ [246, 208, 243]. The condition that $V(\phi_0) \lesssim M_P^4$ now implies that $\phi_0 \simeq 10^3 M_P$. For the massive scalar field, a similar analysis for the calculation of $\delta\rho/\rho$ yields [242, 247]

$$\frac{\delta\rho}{\rho} = \left(\frac{4}{3\pi^2}\right)^{1/2} \frac{m}{M_P} \ln(Hk^{-1}) , \tag{5.69}$$

and $\delta\rho/\rho \lesssim 10^{-4}$ requires $m < O(10^{-5}) M_P$. These are stronger constraints, but especially in the latter case, they are certainly not too unnatural.

Because of the simplicity of the chaotic models, reheating and the generation of a baryon asymmetry is somewhat ambiguous, in that very few models have been discussed in detail (for an example see ref. [248]). Nevertheless, reheating should not be problematic. For example, if the field is only gravitationally coupled and $\Gamma_\phi \sim m_\phi^3/M_P^2$ then from eq. (4.29), $T_R \sim (\Gamma M_P)^{1/2} \sim m_\phi^{3/2}/M_P^{1/2} \sim 10^{11}$–$10^{12}$ GeV for the $m^2\phi^2$ version. If ϕ decays as do the squarks in the Affleck–Dine mechanism, then $\Gamma_\phi \sim m^3/\phi^2$ and

$T_R \sim m^{5/6} M_P^{1/6} \sim 10^{16}$ GeV. Without a definite model, it is difficult to say more than that reheating should not be a problem in these models.

Chaotic inflation has also been implemented in the context of $N=1$ supergravity [249] and in no-scale supergravity [250]. In simple supergravity a somewhat complicated model was necessary. The superpotential was defined as [249]

$$F(\phi) = \mu^3 e^{-\phi^2/4} \Psi(\phi), \quad \Psi(\phi) = \tanh\zeta \sinh\zeta, \quad \zeta = \sqrt{\tfrac{3}{2}}(\phi - \phi_0). \tag{5.70a, b, c}$$

The resulting potential along the real direction is

$$V(\xi) = 9\mu^6\left(1 - \frac{2}{3\cosh^2\xi} - \frac{1}{3\cosh^4\xi}\right), \tag{5.71}$$

where $\xi = \mathrm{Re}\,\zeta$. For $\phi \gg \phi_0$ and $\mu^3 \sim 3\times 10^{-6}$, this model satisfies the inflationary constraints. A much simpler model based on no-scale supergravity can be found. Choosing simply [250]

$$F(\phi) = \mu^3\phi^3, \tag{5.72}$$

one has

$$V(\phi) = 9\mu^6\phi^4, \tag{5.73}$$

or a standard chaotic model with $\lambda = 9\mu^6$. Chaotic inflation using the non-linear sigma model in a superconformal theory was looked at in ref. [251].

There have been some arguments against chaotic inflation. An early objection [246, 232] was based on the role of kinetic terms and initial conditions. Inflation requires $(\partial_\mu\phi)^2 < V(\phi) \lesssim M_P^4$. In particular, $\dot\phi^2$ and $(\partial_i\phi)^2 < V(\phi)$. Thus one seems to require (1) $\delta\phi \gtrsim M_P$ or even $\delta\phi \gg M_P$ since $\phi_0 \gtrsim M_P$ or $\phi_0 \gg M_P$ when $V(\phi) \simeq M_P^4$ while $\delta t \sim M_P^{-1}$ and $\dot\phi \sim (\delta\phi/\delta t) < M_P^2$; (2) $\delta\phi \gtrsim M_P$ and thus $\delta l > M_P^{-1} \sim H^{-1}$ over which the field is homogeneous so that $\partial_i\phi \le M_P^2$. The first of these difficulties has been shown explicitly not to be a problem for chaotic inflation [252, 244]. Even if one assumes that initially $\dot\phi^2 \gg V(\phi)$, using the Friedmann equation

$$H \simeq (\tfrac{4}{3}\pi)^{1/2}\dot\phi/M_P, \tag{5.74}$$

and the equation of motion and neglecting the potential, one has

$$\ddot\phi = \sqrt{12\pi}\,\dot\phi^2/M_P. \tag{5.75}$$

The solution to this equation is [252]

$$\dot\phi = -|\dot\phi_0|(1 + \sqrt{12\pi}|\dot\phi_0|t/M_P)^{-1}, \tag{5.76a}$$

$$\phi = \phi_0 - (M_P/\sqrt{12\pi})\ln(1 + \sqrt{12\pi}|\dot\phi_0|t/M_P). \tag{5.76b}$$

Thus ϕ and hence $V(\phi)$ decrease very slowly while $\dot\phi$ falls as t^{-1}. Such a situation corresponds to the Universe filled with matter obeying a stiff equation of state ($p = \rho$). The energy density of such matter redshifts and soon becomes dominated by the potential. The spatial gradients of the scalar field can be more problematic. It was aruged [253] that for $\phi \simeq \text{few} \times M_P$ (i.e., minimal chaotic inflation) extreme fine-tuning of spatial inhomogeneities is necessary. This fine-tuning disappears when $\phi \gg M_P$. If one assumes that initially the Universe is closed with size M_P^{-1} or the size of a topologically connected domain is only M_P^{-1}, then it is not unreasonable that since $(\partial_i\phi)^2$, $V(\phi) \lesssim M_P^4$, that $(\partial_i\phi)^2 < V(\phi)$ in some domains. Indeed Linde has argued [252] that it is more unreasonable to expect topologically connected domains with size $l \gg M_P^{-1}$ to exist. If the condition $(\partial_i\phi)^2 < V(\phi)$ is satisfied, inflation will most probably occur. Our lack of knowledge about the true initial conditions makes it difficult to be very quantitative. (Recently, a study [254] of various initial conditions on a double well potential was performed to determine the conditions in which chaotic inflation or new inflation via dynamical relaxation [171, 172] occurs.)

A more serious objection to the chaotic inflationary scenario has to do with our understanding of the scalar potential when $\phi > M_P$ [255]. In standard new inflation the potential is important when $\phi \simeq 0$. There, non-renormalizable gravitational terms of the form ϕ^{n+4}/M_P^n are sure to be small. Clearly when $\phi \geq M_P$, and we use a $\lambda\phi^4$ potential we have tacitly assumed that *all* higher order non-renormalizable terms are absent. In supergravity theories where $V \sim e^{\phi^2}$ this is a bad assumption. If we write the potential in general as

$$V(\phi) = (\lambda/n)\phi^n , \tag{5.77}$$

then the growth of the scale factor will be

$$H\tau \sim 4\pi\phi_0^2/nM_P^2 . \tag{5.78}$$

For $H\tau \gtrsim 65$ this implies

$$\phi_0 \gtrsim 2\sqrt{n}M_P . \tag{5.79}$$

Density perturbations are given by (for $n > 2$)

$$\frac{\delta\rho}{\rho} \simeq \frac{n-2}{\pi^{3/2}} H^{(n-4)/(n-2)}[\tfrac{1}{3}(n-2)\lambda]^{1/n-2} \ln^{(n-1)/(n-2)}(Hk^{-1}) , \tag{5.80}$$

or

$$\frac{\delta\rho}{\rho} \sim \frac{n-2}{\pi^{3/2}} \left(\frac{8\pi}{3n}\right)^{(n-4)/(2n-4)} [\tfrac{1}{3}(n-2)]^{1/n-2}\lambda^{1/2}\phi^{n(n-4)/(2n-4)} \ln^{(n-1)/(n-2)}(Hk^{-1}) . \tag{5.81}$$

For large n this becomes

$$\frac{\delta\rho}{\rho} \sim 0(50)\sqrt{n}\, n^{1/n}\lambda^{1/2}\phi_0^{n/2} , \tag{5.82}$$

and the constraint on λ becomes

$$\lambda_n \lesssim 4 \times 10^{-12}/2^n n^{n/2} . \tag{5.83}$$

For chaotic inflation, density perturbations depend on the largest value of n in the potential. The production of gravitinos [157–159] also puts limits [255] on λ for large values of n:

$$\lambda_n < 6\times 10^{-8} n/2^n n^{n/2} . \tag{5.84}$$

It was assumed [255] that $\lambda > 0(10^{-22})$ was necessary for reheating and producing a baryon asymmetry which meant that $n \gtrsim 12$ is forbidden. This is a strong constraint on chaotic models.

It was shown by Linde [256] however, that even when fields are constrained to be $\phi_0 \lesssim$ few M_P, chaotic inflation is still possible. If there is more than one field, say ϕ and Φ, such that $V = \lambda_\phi \phi^4 + \lambda_\Phi \Phi^4$ with $\lambda_\phi \sim 10^{-12}$ and $\lambda_\Phi \sim 10^{-2}$, one expects inflation to begin with Φ where $\Phi \sim 4M_P$ and $V(\Phi) \sim M_P^4$ even if $\phi \lesssim$ few M_P and $V(\phi) \ll M_P^4$. During Φ-inflation, fluctuations would drive $\phi \sim$ few M_P and eventually the Universe would inflate off of ϕ. With $\lambda_\phi \sim 10^{-12}$, $\delta\rho/\rho \sim 10^{-4}$. Thus with several (at least two) fields it is possible that chaotic inflation occurs with constrained fields. It was also pointed out [247] that when the fields are not constrained, because the "lightest" field inflates last, flat directions in supersymmetric theories would almost always yield the final period of inflation driven by the squark fields η. In this case, sufficient inflation occurs but $\delta\rho/\rho \sim 10^{-16}$ and another source of density perturbations must be found.

We have now seen models with complementary assumptions regarding initial conditions. Assuming an initially thermal state, new inflation is still a viable scenario [169, 171]. It has the added advantage that the theory can be well understood at $\phi \simeq 0$. Without the assumption of an initial thermal state, new inflation may work well via dynamical relaxation [171] or assuming some regions are present with $\phi \simeq 0$ as in ref. [168]. Chaotic inflation explicitly does not assume a thermal state. What it loses due to a potentially ill-defined theory with $\phi \gtrsim M_P$, it gains in its simplicity. All of the models which follow are in some way related to these two approaches.

5.3. *Inflation and axions*

Axions are pseudo-Goldstone bosons which arise in solving the strong *CP* problem [257, 258] via a global U(1) Peccei–Quinn symmetry. The invisible axion [158] is associated with the flat direction of the spontaneously broken PQ symmetry. Because the PQ symmetry is also explicitly broken (the *CP* violating $\theta F\tilde{F}$ (where F is the field strength tensor and $\tilde{F}$ its dual) coupling is not *PQ* invariant) the axion picks up a small mass similar to the pion picking up a mass when chiral symmetry is broken. We can expect that $m_a \sim m_\pi f_\pi / f_a$ where f_a, the axion decay constant, is the vacuum expectation value of the PQ current. If we write the axion field as $a = f_a \theta$, near the minimum, the potential produced by QCD instanton effects looks like

$$V(\theta) = \tfrac{1}{2} m_a^2 \theta^2 f_a^2 . \tag{5.85}$$

In the absence of any *CP* violating effects $\theta_0 = 0$.

The equation of motion for θ can be written as

$$\ddot{\theta} + 3H\dot{\theta} = -m_a^2 \theta . \tag{5.86}$$

For $H \gg m_a$, $\theta \simeq$ constant, while for $H < m_a$, θ begins to oscillate about θ_0. The energy density in these oscillations may be the dominant contribution to the mass density of the Universe today [259].

Depending somewhat on the value of f_a, oscillations begin when $T \sim T_i \sim 1$ GeV. The energy density is $\rho_a = V(\theta)$ where it understood that m_a is also a temperature dependent quantity. Entropy conservation tells us that $m_a\theta^2$ is constant, so that we can write

$$\rho_a = \tfrac{1}{2} m_a m_a(T_i)\theta_i^2 (R_i/R)^3 f_a^2 , \tag{5.87}$$

where $\theta_i \sim O(1)$ is the initial value of θ and R_i is the value of the scale factor when oscillations begin. Taking $m_a(T_i) \sim H(T_i) \sim T_i^2/M_P$, $R_i/R \sim T_0/T_i$ where $T_0 = 2.7$ K, and $m_a \sim 7\times 10^7$ eV(1 GeV/f_a) [260], we find [259]

$$\rho_a \sim m_a f_a^2 \theta_i^2 T_0^3/M_P T_i \sim 10^{-17}(f_a/1\ \mathrm{GeV})\ \mathrm{GeV/cm^3} , \tag{5.88}$$

so that $\Omega \leq 1$ implies that [259, 261]

$$f_a \lesssim 10^{12}\ \mathrm{GeV} . \tag{5.89}$$

There are numerous [262] limits of f_a from astrophysics; however, it is interesting to note that the recent supernova SN1987*a* places the strongest constraint from below on f_a [263]:

$$f_a \gtrsim 10^{11}\ \mathrm{GeV} . \tag{5.90}$$

The cosmological bound (5.89) has come under scrutiny recently [264] where it has been claimed that the production of axion strings and their subsequent production of axions leads to a stronger constraint, which may be in conflict with the astrophysical bound (5.90). Inflation would dilute the string density and the bound (5.89) would survive. The original bound is supported in ref. [265] including the presence of strings. More recent works, both numerical [266] and analytical [267] support Davis' [264] claims.

The cosmological bound (5.89) can be relaxed if the "bubble" that grew into our observable Universe had $\theta_i \ll 1$ [268, 269]. Indeed, taking $\theta_i \sim (10^{-1})$ allows $f_a \lesssim 10^{14}$ GeV and $\theta_i \sim (10^{-3})$ allows $f_a \lesssim 10^{18}$ GeV. Entropy production could also dilute the axion abundance relaxing the cosmological bound [270]. Inflation with a low value of the Hubble parameter would accomplish the same thing [271].

In addition to the above interplay between axions and inflation, there have been two more important roles played by the axion in inflationary scenarios. The first has to do with specific models for inflation, while the second has to do with fluctuations in the axion background caused by inflation.

The first step towards an axion-type model for inflation was based on non-supersymmetric SU(5) with an additional singlet field [272]. This model is based on a Coleman–Weinberg potential for the singlet due to its weak coupling with the SU(5) adjoint Σ and Higgs **5**-plet H_5. The potential (2.14) with $\mu = c = 0$ is augmented with standard couplings between Σ and H,

$$V_H = \alpha H_5^\dagger H_5 \,\mathrm{Tr}\, \Sigma^2 + \tfrac{1}{4}\lambda (H_5^\dagger H_5)^2 + \beta H_5^\dagger \Sigma^2 H_5 , \tag{5.91}$$

and couplings of the singlet ϕ,

$$V_\phi = \tfrac{1}{4}\lambda_1 \phi^4 - \tfrac{1}{2}\lambda \phi^2 \,\mathrm{Tr}\, \Sigma^2 + \tfrac{1}{2}\lambda_3 \phi^2 H_5^\dagger H_5 . \tag{5.92}$$

The coupling α, a, b and λ are assumed to be $\sim g^2$ so that radiative corrections can be neglected (recall, however, arguments in section 2 based on the gauge hierarchy [63] that in fact a, b are small). In contrast the couplings λ_i are assumed to be small so that the one-loop correction to (5.92) is given by

$$V = A\phi^4(\ln \phi^2/M^2 + C), \quad A = \frac{1}{64\pi^2}(24\lambda_2^2 + 10\lambda_3^2). \tag{5.93a, b}$$

C is a constant which depends on the choice of the minimum of the potential $\langle\phi\rangle = M$. To ensure $\delta\rho/\rho \lesssim 10^{-4}$, $A \lesssim 10^{-12}$ and λ_2, λ_3 are taken to be $O(10^{-5})$. The GUT scale is determined from $M_X \sim \lambda_2^{1/2} M$ so that one requires $M \sim 10^{18}$ GeV which corresponds to the minimum of the potential. (Another example of primordial inflation.)

In this model [272], SU(5) symmetry breaking occurs when $m_\Sigma \sim \lambda_2^{1/2}\phi \sim H$. Initially with $\Sigma = \phi = 0$ (due to high temperature effects) inflation begins and fluctuations drive Σ away from the origin. $m_\Sigma \sim H$ at about the same time inflation is completed and $H\tau \sim O(1)\,\lambda_2^{-1}$ and is certainly sufficient for inflation. Reheating is somewhat problematic in this model. There is a doublet–triplet separation problem which causes fine-tuning of the parameters α and β. Reheating by decays through the Higgs triplet, necessary for baryon number generation, requires $\beta \lesssim 10^{-6}$, though such a small value is not stable with respect to radiative corrections (recall the motivation for supersymmetry).

In a similar model [268], the identification of ϕ with the axion is made. This model contains an additional Higgs **5**-plet and the singlet is taken to be complex. The potential can be constructed so that it possess a U(1) Peccei–Quinn symmetry. Again the couplings a, b, α and λ are of order g^2 (there are now several couplings of the form α and λ due to the presence of the second Higgs **5**-plet). The axion in this model is $a \simeq \mathrm{Im}\,\phi$ and its mass is $m_a \sim m_\pi f_\pi/M$ corresponding to $f_a \sim M \sim 10^{18}$ GeV. Re ϕ is the inflaton responsible for inflation. In other respects this model is similar to that in ref. [272]. It has the additional burden of accounting for $f_a > 10^{12}$ as well as radiative corrections to β.

Axions are also important in cosmology as they act as dark matter with $\Omega \simeq 1$ when $f_a \simeq 10^{12}$ GeV. Perturbations in the axion background may then play a dominant role in the formation of galaxies [273–276]. Originally it was thought that the dominant perturbations were related to adiabatic perturbations produced during inflation [275, 276]. Later, it was realized that inflation is capable of producing substantial isothermal perturbations in the axion field [277, 278].

To calculate the spectrum of isothermal perturbations, recall the quantum fluctuation for a very light scalar field given by eq. (4.42). Long wavelength fluctuations at a scale k are then [243]

$$\delta\phi(k) \sim (H/\sqrt{2}\pi)(k/H)^{m^2/3H^2}. \tag{5.94}$$

This in turn leads to fluctuations in the scalar's energy density $\delta\rho_\phi(k) \sim (\partial V/\partial\phi)\delta\phi(k)$ so that

$$\delta\rho_\phi/\rho_\phi \sim \delta\phi(k)/\phi \sim (H/\sqrt{2}\pi\phi)(k/H)^{m^2/3H^2}. \tag{5.95}$$

When $m^2 \ll H^2$ this is almost exactly a scale free spectrum as we would expect. If these were fluctuations of the inflaton then $\delta\rho_\phi/\rho_\phi \sim \delta\rho_{\mathrm{tot}}/\rho_{\mathrm{tot}}$. These would become an adiabatic perturbation during reheating and $\delta\rho_\phi/\rho_\phi \sim \delta T/T$. If the scalar is largely non-interacting, yet comes to dominate the energy density after inflation, these perturbations appear isothermal. Such is the case with the axion (or any other decoupled field such as the Polonyi field). Fluctuations in the axion field are then given by [277, 278]

$$\delta\rho_a/\rho_{tot} \sim (\rho_a/\rho_{tot})(H/f_a)(k/H)^{m^2/3H^2} \sim (H/f_a)\,, \tag{5.96}$$

where it has been assumed that $a \sim f_a$ ($\theta \sim 1$) due to quantum fluctuations and that $m^2 \ll H^2$. Despite the fact that ρ_a/ρ_{tot} may initially be quite small, as we have seen in the case of axions, $\rho_a/\rho_{tot} \simeq 1$ today for $f_a \sim 10^{12}$ GeV.

Because of the periodicity in the axion potential, eq. (5.96) is somewhat oversimplified [277–279]. A detailed study of isothermal perturbations produced during inflation can be found in ref. [279]. A more accurate calculation of $\delta\rho/\rho$ yields

$$\delta\rho_a/\rho_{tot} \sim (H/2\sqrt{2}\pi f_a)(k/H)^{H^2/8\pi^2 f_a^2}\,. \tag{5.97}$$

When $f_a \sim 10^{12}$ GeV, $\delta\rho_a/\rho \sim 10^{-4}$ requires $H \simeq 10^8$ GeV and $f_a \gg H$ implies a very nearly flat spectrum. Isothermal fluctuations in the axion-inflationary models were discussed in ref. [278]. Isothermal fluctuations in the Affleck–Dine mechanism were found to be unimportant [134]. An axion model with non-Gaussian perturbations was discussed in ref. [280].

5.4. R^2-inflation

As was noted in section 1, a very early ("pre-inflationary") attempt at solving the singularity problem was the model of Starobinsky [40]. The model is based on obtaining a self-consistent solution of Einstein's equations when they are modified to include one-loop quantum corrections to $T_{\mu\nu}$. Such a solution is unstable, however, and evolves towards a Friedmann Universe. As we will see, though this model in fact does not solve the singularity problem, it is in fact equivalent to the inflationary model. It solves all cosmological problems solved by inflation including the generation of density perturbations.

The model is based on the Friedmann–Robertson–Walker metric [eq. (1.4)]. Quantum corrections to the right hand side of eq. (1.6) in the absence of matter can be written as [281]

$$\begin{aligned}\langle T_{\mu\nu}\rangle = {} & \frac{k_2}{2880\pi^2}\left(R_\mu{}^\rho R_{\nu\rho} - \tfrac{2}{3}RR_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R_{\rho\sigma}R^{\rho\sigma} + \tfrac{1}{4}g_{\mu\nu}R^2\right) \\ & + \frac{1}{6}\,\frac{k_3}{2880\pi^2}\left(2R_{;\mu;\nu} - 2g_{\mu\nu}R^{;\rho}{}_{;\rho} - 2RR_{\mu\nu} + \tfrac{1}{2}g_{\mu\nu}R^2\right).\end{aligned} \tag{5.98}$$

These terms appear in the process of regularization and are similar to the effects of vacuum polarization in QED. k_2 and k_3 are constants. k_2 is related to the number spin states while k_3 is arbitrary. It was shown [282] that such a theory admits a De Sitter solution. The 0–0 component analog of eq. (1.7) becomes [40] (with $\rho = 0$)

$$\left(\frac{\dot R}{R}\right)^2 = \frac{-k}{R^2} + \frac{1}{H'^2}\left(\frac{\dot R^2 + k}{R^2}\right)^2 - \frac{1}{M^2}\left[2\frac{\dot R\dddot R}{R^2} - \frac{\ddot R^2}{R^2} + 2\frac{\ddot R\dot R^2}{R^3} - 3\left(\frac{\dot R}{R}\right)^4 - 2k\frac{\dot R^2}{R^4} + \frac{k^2}{R^4}\right], \tag{5.99}$$

where $H'^2 = 2880\pi^2/\kappa k_2$ and $M^2 = 2880\pi^2/\kappa k_3$ with $k_{2,3} > 0$. The one-loop approximation requires H'/M_P, $M/M_P \ll 1$. Because of the exponential expansion in the De Sitter state, $R(t) \sim \exp(H't)$, the curvature terms in eq. (5.99) quickly become negligible. With $k = 0$, and $H = \dot R/R$, we can rewrite 5.99 as [283]

$$H^2(H^2 - H'^2) = (H'^2/M^2)(2\ddot H H + 2H^2\dot H - \dot H^2)\,. \tag{5.100}$$

The De Sitter solution corresponds to $H = H'$. By considering deviations from this solution, it is straightforward to show [40, 283] that it is in fact unstable. Consider the case where initially $H \lesssim H'$ and that $\dot{H} \ll H'^2$, $\ddot{H} \ll H\dot{H}$. In this approximation (5.100) becomes

$$H^2 - H'^2 = 6(H'^2/M^2)\dot{H} . \tag{5.101}$$

The solution of this equation is [283]

$$(H' - H)/(H' + H) = \tfrac{1}{2}\delta_0\, e^{M^2 t/3H'} , \tag{5.102a}$$

or

$$H = H' \tanh(\gamma - M^2 t/6H') , \tag{5.102b}$$

where $\gamma = \frac{1}{2}\ln(2/\delta_0)$ and $\delta_0 = |H' - H|/H'$ initially. As one can see from this equation, the "inflationary" time scale is

$$H\tau \sim 6H'^2/M^2 , \tag{5.103}$$

and sufficient inflation occurs when $M^2 \ll H'^2$.

At late times, $H \ll H'$, we can write eq. (5.100) as

$$2H\ddot{H} + 6H^2\dot{H} - \dot{H}^2 + M^2H^2 = 0 . \tag{5.104}$$

One finds an approximate damped oscillatory solution for H,

$$H \simeq \frac{4}{3t}\cos^2(Mt/2)\left(1 - \frac{\sin Mt}{Mt}\right) , \tag{5.105}$$

$$R(t) \propto t^{2/3}\left(1 + \frac{2}{3Mt}\sin Mt\right) . \tag{5.106}$$

Averaged over the oscillation period, $H \sim 2/3t$ and $R \sim t^{2/3}$, i.e., equivalent to a matter dominated Universe. Thus the transition to a Friedmann Universe.

An interpretation [40] of this late phase, is the production of zero modes of mass M (scalarons) which are expected to be unstable with lifetime $\tau \sim 48\pi M/\kappa\mu^4$ for decays into scalar particles of mass μ. In addition, there is a lifetime $\tau \sim 48\pi/\kappa M\mu^2$ for decays into fermions of mass μ with $\mu \ll M$. In this way the Universe is reheated and the standard cosmological model takes over. A detailed discussion of particle production is found in ref. [284]. In general the reheat temperature can be quite large.

Similar to constraints on "normal" inflationary models, the production of gravitons [157–159] also puts constraints on this model. In ref. [158], it was claimed that the requirement of $V < 3 \times 10^{-8} M_P^4$ corresponds to the limit on k_2, $k_2 \gtrsim 10^{10}$, implying the need for billions of spin degrees of freedom to be present. However, in contrast, it was argued in ref. [283] that the limit from gravitational waves depends on M rather than H' and requires $M \lesssim 10^{16}$ GeV or $k_3 \gtrsim 10^9$, which although might be construed as a strange choice, k_3 is arbitrary.

There has also been a considerable amount of work regarding the production of density perturbations in this model. In fact, the first calculation [285] by Mukhanov and Chibisov of the resulting perturbation spectrum found a nearly *flat, scale independent* spectrum which depends only logarithmically on the scale. In retrospect this should not be a surprise since the R^2-model is in fact an inflationary model. It should be stressed that this work was complete even before the inception of the new inflationary scenario. The evolution of perturbations was studied in ref. [286]. The magnitude of $\delta\rho/\rho$ places further constraints on M. M less than 10^{14} GeV [287] requires k_3 to be larger still. In this case, the reheat temperature is somewhat lower and the monopole problem can be solved. (If T_R is too large, R^2-inflation may be faced with a recurrent monopole problem.) Further work on the generation of density perturbations found [288]

$$\delta\rho/\rho \approx 0.3(\tfrac{8}{3}\pi)^{1/2}(M/M_P)\ln(Hk^{-1}), \tag{5.107}$$

which requires $M \lesssim 10^{13}$ GeV and corresponds to a reheat temperature $T_R \sim 10^8$–10^{10} GeV, comparable to the previous standard inflationary models. A similar result was found in ref. [289]. An explicit comparison to standard inflation was made in ref. [290]. A model with both chaotic and R^2-inflation was discussed in ref. [291].

It was first noted in ref. [287] that one can simplify the analyses of these models by considering a gravitational action of the form

$$I = \frac{1}{2\kappa^2}\int d^4x\,\sqrt{-g}(R + R^2/6M^2). \tag{5.108}$$

Indeed, as we shall soon see, such an action has motivation from unification theories involving gravity. By using the variational principle one can determine the gravitational field equations,

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R + \frac{1}{6M^2}(-2R_{;\mu;\nu} + 2RR_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R^2 + 2g_{\mu\nu}R^{;\mu}{}_{;\mu}) = \kappa^2 T_{\mu\nu}. \tag{5.109}$$

(For discussions on pure R^2-gravity theories see ref. [292].) One can see that with $M^2 \to \infty$ one recovers eq. (1.6) with $\Lambda = 0$. It was shown [293] that this theory is conformally equivalent to a theory with normal gravity (i.e., minimal kinetic terms for the graviton) plus a scalar field. By writing

$$\tilde{g}_{\mu\nu} = \left(1 + \frac{1}{3M^2}\phi\right)g_{\mu\nu}, \tag{5.110}$$

one has

$$\tilde{R}_{\mu\nu} = R_{\mu\nu} - \frac{1}{3M^2}\phi_{;\mu;\nu}\left(1 + \frac{1}{3M^2}\phi\right)^{-1} + \frac{1}{6M^4}\partial_\mu\phi\,\partial_\nu\phi\left(1 + \frac{1}{3M^2}\phi\right)^{-2}$$
$$- \frac{1}{6M^2}\phi^{;\mu}{}_{;\mu}g_{\mu\nu}\left(1 + \frac{1}{3M^2}\phi\right)^{-1}, \tag{5.111}$$

and the field equations become

$$\tilde{R}_{\mu\nu} - \tfrac{1}{2}\tilde{g}_{\mu\nu}\tilde{R} = \frac{1}{6M^4}\left(1 + \frac{1}{3M^2}\phi\right)^{-2}[\partial_\mu\phi\,\partial_\nu\phi - \tilde{g}_{\mu\nu}(\tfrac{1}{2}\partial^\rho\phi\,\partial_\rho\phi + \tfrac{1}{2}M^2\phi^2)], \tag{5.112}$$

which, as one can see, looks like a nearly ordinary theory of gravity with a massive scalar field. ϕ still does not have minimal kinetic terms but by a suitable field redefinition of ϕ it can be brought into canonical form. In particular, letting

$$\kappa\phi' = \sqrt{\tfrac{3}{2}}\ln\left(1 + \frac{1}{3M^2}\,\phi\right), \tag{5.113}$$

one finds

$$I = \int \mathrm{d}^4x\,\sqrt{-g}\left[\frac{1}{2\kappa^2}\,\tilde{R} - \tfrac{1}{2}\partial^\mu\phi'\,\partial_\mu\phi' - \frac{3}{4\kappa^2}\,M^2(1 - \mathrm{e}^{-\kappa\phi'\sqrt{2/3}})^2\right], \tag{5.114}$$

which corresponds to a real scalar field with potential (in units of $\kappa = 1$)

$$V = \tfrac{3}{4}M^2(1 - \mathrm{e}^{-\phi'\sqrt{2/3}})^2\,. \tag{5.115}$$

This potential is extremely flat for $\phi' \gg M_\mathrm{P}$ and has a minimum at $\phi' = 0$ with $V(\phi' = 0) = 0$. From our preceding discussions on chaotic inflation, we would expect this to be an ideal model of inflation provided initially ϕ' was very large. For further discussions of the conformal properties of the R^2-theory see refs. [294–297]. The cosmology of the R^2-theory is discussed in more detail in ref. [296, 298, 299] with special attention to reheating in ref. [300].

5.5. Stochastic inflation

Up until now, we have seen that quantum fluctuations play an integral role in the theory of inflation. The $\langle\phi^2\rangle$ fluctuations are responsible for density perturbations and can upset the notion of a slow roll-over phase transition. It was realized independently in refs. [301, 302] that effects such as those leading to large-scale perturbations ($l \gg H^{-1}$) which arise from quantum processes are stochastic in nature. In addition to problems regarding the slow roll-over discussed in section 4, the presence of large fluctuations would render the use of the simple equation of motion for the inflaton [eq. (3.23)] invalid [303]. In the stochastic approach, the formation of domains at which the inflaton is at its global minimum is examined by using a Langevin equation of motion. In analogy to the requirement of a flat potential it will be sufficient to have flatness along a $\langle\phi^2\rangle \neq \langle\phi\rangle^2$ direction.

The growth of fluctuations has been discussed in detail in section 4. Recall that for a massless scalar field, these fluctuations grow as a random walk $\langle\phi^2\rangle = H^3t/4\pi^2$. As the long wavelength modes grow beyond the horizon, extra-horizon interactions produce a stochastic force (or noise) term. The development of a fluctuation region (where the field can be treated as homogeneous) is then described by the Langevin equation

$$\ddot{\phi} + 3H\dot{\phi} + \partial V/\partial\phi = \Gamma(t)\,, \tag{5.116}$$

where $\Gamma(t)$ represents the stochastic force. $\Gamma(t)$ was assumed [301] to have the properties of white-noise appropriate for a De Sitter space, namely

$$\langle\Gamma(t)\rangle = 0, \qquad \langle\Gamma(t)\Gamma(t')\rangle = 2D\delta(t - t')\,, \tag{5.117}$$

and the diffusion coefficient is given by

$$D = 9H^5/8\pi^2 \,. \tag{5.118}$$

This was shown to be the case in ref. [302].

Equivalently, the system can be described by probability distributions of its stochastic variables using a Fokker–Planck (or Einstein–Smoluchowski) equation (to first order in $\hbar$),

$$\frac{\partial W}{\partial t} = -\frac{\partial}{\partial \phi}(\pi W) + 3H\frac{\partial}{\partial \pi}(\pi W) + D\frac{\partial^2 W}{\partial \pi^2} + \frac{\partial V}{\partial \phi}\frac{\partial W}{\partial \pi}\,, \tag{5.119}$$

where $\pi \equiv \dot{\phi}$ and $W(\pi, \phi; t)$ is the probability distribution function satisfying

$$\int \mathrm{d}\phi \,\mathrm{d}\pi \, W = 1 \,. \tag{5.120}$$

The ensemble average of a function of π and ϕ is determined by

$$\langle F(t)\rangle = \int \mathrm{d}\pi \,\mathrm{d}\phi \, F(\pi, \phi) W(\pi, \phi; t)\,. \tag{5.121}$$

In particular, the averaged tree-level scalar potential will determine the evolution of the system. In ref. [301] a Gaussian probability distribution was assumed. The slow rolling approximation simplifies the Langevin and Fokker–Planck equation so that $\ddot{\phi}$ in eq. (5.116) can be neglected and (5.119) becomes the Smoluchowski equation:

$$\frac{\partial W}{\partial t} = \frac{1}{3H}\frac{\partial}{\partial \phi}\left(\frac{\partial V}{\partial \phi} + \frac{3H^4}{8\pi^2}\frac{\partial}{\partial \phi}\right) W\,, \tag{5.122}$$

which was derived first via a coarse-graining technique in ref. [302]. The use of the probability distribution for inflation was also discussed in ref. [304].

There have been several applications of the stochastic approach to inflation. Using a simple double well potential, which as we saw earlier would require $v > M_\mathrm{P}$ for sufficient inflation, it was found [301] that sufficient inflation occurs when the effects of fluctuations are included. For the potential in eq. (5.1), and choosing the Gaussian probability distribution

$$W(\phi, \langle\phi\rangle, \sigma) = \frac{1}{\sqrt{2\pi\sigma}} \exp[-(\phi - \langle\phi\rangle)^2/2v^2\sigma]\,, \tag{5.123}$$

which yields the averaged potential

$$U(\langle\phi\rangle, \sigma) = \frac{1}{4\lambda v}\int \mathrm{d}\phi \, WV = -\tfrac{1}{2}\langle\phi\rangle^2 v^2 + \tfrac{1}{4}\langle\phi\rangle^4 + \tfrac{1}{4}\sigma(3\sigma v^4 - 2v^4 + 6\langle\phi\rangle^2 v^2)\,. \tag{5.124}$$

For $\sigma = 0$, the path for $\langle\phi\rangle$ is identical to the standard new inflationary scenario. However, there are paths with $\sigma \neq 0$ in which U is flat and leads to inflation. (The fluctuations σ can be defined by $\sigma v^2 = \langle\phi^2\rangle - \langle\phi\rangle^2$.)

The stochastic approach to inflation was also discussed in some detail in refs. [305–307]. The use of a Gaussian distribution was also used in ref. [308]. Its future evolution was determined, however, using the Smoluchowski equation, in the context of chaotic inflation (which was also considered in ref. [302] and will be discussed in more detail in section 6), and Coleman–Weinberg symmetry breaking. A simplified analytic solution for the probability distribution was found in ref. [309]. A non-Gaussian probability distribution was examined in refs. [305, 307]. Stochastic inflation in the context of new inflation was discussed in detail in ref. [310]. Stochastic inflation without the slow roll-over approximation was looked at in ref. [311]. Numerical methods were applied to stochastic inflation in ref. [312]. There has also been a considerable amount of work discussing general probability distributions [313–316].

5.6. *Eternal inflation*

Eternal inflation is not a specific model of inflation per se but rather a consequence of quantum fluctuations and stochastic processes. Indeed it provides one with a somewhat different view of the inflationary scenario. Instead of a Universe beginning with a singularity, a hot thermal state or an act of quantum creation (see section 6 for more on this), eternal inflation [317–320] postulates that the Universe is in an ever-present De Sitter-like state in which large fluctuation regions are formed and develop into hot dense "island universes". In this picture on small (now small refers to the size of the observable Universe or even the size of the region produced by inflation $l_1 \gg H_0^{-1}$) scales the Universe is homogeneous and isotropic up to $\delta\rho/\rho$ perturbations forming galaxies. Whereas on large scales, i.e., scales much larger than a single fluctuation region, quantum fluctuations, during inflation, produce a very inhomogeneous Universe. Recall that in the De Sitter phase, regions which are separated by a distance $l \gtrsim 2H^{-1}$ never meet despite the fact their volumes are growing exponentially. I am not aware of any observational consequences of eternal inflation.

The notion of a self-regenerating Universe was first discussed in ref. [317] in the context of new inflation. Once inflation begins, exponential inflation can continue indefinitely in regions where $\phi \simeq 0$. In ref. [318], it was suggested that such a scenario might solve the singularity problem, i.e., inflation might be eternal in the past as well as the future.

During inflation, the random walk of the quantum fluctuations, $\langle \phi^2 \rangle = H^3 t/4\pi^2$, implies that for points separated by a distance l and such that $H^{-1} \ll l \ll H^{-1}\exp(Ht)$, $\langle \phi^2 \rangle$ fluctuations are displaced from each other by an amount [319]

$$\delta\langle \phi^2 \rangle \sim (H^2/4\pi^2)\ln(Hl)\,. \tag{5.125}$$

Because of the random nature of the fluctuations, there will be regions where $\langle \phi^2 \rangle$ is very small. The probability of finding $\langle \phi^2 \rangle \lesssim H^2$ at late times was estimated by Vilenkin [319]:

$$P \sim (2\pi/\sqrt{3})(|m|/H)\exp(-\pi^2|m|^2 t/24H)\,, \tag{5.126}$$

and the number of horizon-size regions with $\langle \phi^2 \rangle \lesssim H^2$ is

$$N \sim P\,\mathrm{e}^{3Ht}\,, \tag{5.127}$$

where m is the inflaton mass. In this case when $|m^2| \ll H^2$ (as it must for inflation) the number of such regions is large and actually grows in time. Thus the Universe remains in an eternally inflating state.

The possibility of an eternal period of inflation in chaotic scenarios has been discussed in detail by Linde [320]. Density perturbations on small scales were discussed earlier. On large scales we can estimate $\delta\rho/\rho$ simply by [243]

$$\delta\rho/\rho \sim H^2/\dot{\phi} \sim \phi\sqrt{V(\phi)}/M_P^3\,, \tag{5.128}$$

in the slow-rolling approximation. Equation (5.128) reduces to earlier estimates when the value of $\phi \sim M_P$ is used to form perturbations on galactic scales. In chaotic inflation, perturbations on large scales are produced when $\phi \gg M_P$ and hence $\delta\rho/\rho \gtrsim 1$. As is the case for new inflation, the random walk of $\langle\phi^2\rangle$ creates an increasing number of domains with $\langle\phi^2\rangle$ *increasing* eventually up to the maximal value allowed by $V(\phi) \lesssim M_P^4$. For $V = (\lambda/n)\phi^n$, $\langle\phi^2\rangle$ will fluctuate to larger values when $\phi > (\lambda)^{-1/(n+2)}M_P$ [321].

An alternative way of looking at this picture is to consider the fluctuations as a stochastic process. By solving the Smoluchowski equation (5.122), one can determine the probability distribution function $P(\phi)$. (This was done in ref. [319] with the assumption that $\partial V/\partial\phi = 0$.) The use of stochastic processes for eternal inflation was discussed in detail in refs. [321, 322]. I will return to the connection between eternal inflation and the probability distribution determined by a stochastic analysis in the next section.

Further work on an eternal inflation by Aryal and Vilenkin showed [323] that on large scales, the Universe appears as a fractal. For a potential of the form $V = V_0 - \frac{1}{4}\lambda\phi^4$, they obtained a fractal dimension $d = 3 - (\lambda/3)^{1/2}(6.2/4\pi)$. Finally, the stochastic approach to eternal inflation in R^2 models was discussed in ref. [324].

5.7. *Inflation and superstrings*

The most promising candidate for a complete unification theory including gravity is superstring theory [325]. It is only natural, therefore, to search for a model of inflation in this context. Much of the work on inflation in superstring theories is based on models attempting to yield effective low energy theories compatible with phenomenological constraints. The starting point for these theories is the heterotic superstring [326] in a ten-dimensional space–time with gauge group $E_8 \times E_{8'}$. Compactification on a Calabi–Yau manifold (six-dimensional) yields a four-dimensional space–time with $N = 1$ supersymmetry [327]. If the six-dimensional manifold preserves SU(3) holonomy, the resulting low energy gauge group is a rank five or six subgroup of $E_6 \subset E_8$ [327, 328]. Thus one expects an extension to the low energy standard model. Furthermore it was found that the resulting supergravity displays a no-scale structure [329]. The breaking of supersymmetry in these theories is still somewhat uncertain, but most models assume that the breakdown is due to the formation of gaugino condensates in the hidden sector ($E_{8'}$) [330].

Early work on superstring inspired models leads almost uniquely to a rank five model based on $SU(3) \times SU(2) \times U(1) \times U(1)$ [331]. The matter content of these theories is found in a non-gauge singlet representation, the **27**, of E_6. The **27** has the following decomposition under SU(10) and SU(5):

$$\mathbf{27} = (\mathbf{16} + \mathbf{10} + \mathbf{1}) = (\mathbf{10} + \bar{\mathbf{5}} + \mathbf{1}) + (\mathbf{5} + \bar{\mathbf{5}}) + \mathbf{1}\,. \tag{5.129}$$

The number of fermion generations or, more precisely, the asymmetry between the number of **27**s and $\overline{\mathbf{27}}$s, is given by [327] half the Euler number of the compactification manifold. The number of $\overline{\mathbf{27}}$s (there is at least one) is also determined topologically by the Betti number [327] $b_{1,1}$. However, the Hosotani mechanism [332] breaking E_6 during compactification may leave [328] only part or none of the $(\mathbf{27}+\overline{\mathbf{27}})$ light. The most general form of the superpotential for the matter fields is

$$F = \lambda_1 HQu^c + \lambda_2 \bar{H}Qd^c + \lambda_3 \bar{H}Le^c + \lambda_4 H\bar{H}N + \lambda_5 NDD^c + \lambda_6 DQQ + \lambda_7 D^c u^c d^c + \lambda_8 D^c QL + \lambda_9 Du^c e^c + \lambda_{10} Dd^c \nu^c + \lambda_{11} HL\nu^c . \quad (5.130)$$

The first four terms in this superpotential are just standard model couplings, though in the fourth term the Higgs mixing mass is replaced by the SO(10) singlet field N. The rest are new terms. If all of the above terms were present, they would result in serious phenomenological problems [331, 333] having to do with neutrino masses and/or baryon number violating interactions.

In a rank-five model [331], only one of the two standard model singlets ν^c or N need pick up a vacuum expectation value. As noted above, a vev for N produces $H\bar{H}$ mixing and D masses. This phase transition has been shown [227] to be cosmologically safe from the point of view of entropy production. In a rank-six model, both ν^c *and* N need to pick up vacuum expectation values. It has been shown [334, 335] that without an intermediate scale (e.g., $\langle \nu^c \rangle \simeq O(10^{10}$ GeV) the addition of a ν^c vev produces problems for the down quark mass matrix, flavor charging neutral currents and lepton number violation.

There are, however, severe problems which can arise in models with intermediate scales [336]. Intermediate scales arise along flat directions of a scalar potential of the form

$$V(\phi, T) = (-m_\phi^2 + T^2)\phi^2 + \phi^{4+n}/M_c^n , \quad (5.131)$$

where ϕ might be ν^c, for example, and $m_\phi^2 \simeq \tilde{m}^2$ and $M_c \lesssim M_P$ is the compactification scale. The zero temperature potential has a minimum at

$$\langle \phi \rangle = M_I \simeq (\tilde{m}^2 M_c^n)^{1/(2+n)} , \quad (5.132)$$

so that for $n = 2$, $M_I \simeq 10^{10}$ GeV. Larger intermediate scales are possible along flatter directions (i.e., with $n > 2$). Despite other phenomenological problems, the cosmological phase transition generating M_I produces an entropy problem [337] similar to the Polonyi problem discussed earlier. Unless $M_I < 10^7$ GeV, the entropy increase in this phase transition is $\gtrsim O(10^6)$. The bound can be relaxed [336] somewhat in the presence of an inflationary period, or if the baryon asymmetry is generated à la Affleck and Dine (the Affleck–Dine mechanism in the context of superstring theory was considered in ref. [338]). Unfortunately, contrary to claims in ref. [339], just as inflation alone does not [231] solve [230] the Polonyi problem [202], inflation cannot do away with the bounds on intermediate scale models.

There has also been a considerable amount of work on "true" superstring models based on specific Calabi–Yau manifolds. The most studied models are based on $CP^3 \times CP^3$ [340] which is a rank-six model with a gauge group $SU(3)^3$ and a rank-five model based on CP^7 [341]. Unfortunately none of these models are problem free, phenomenologically.

Inflation in string theories as in GUTs is not expected to take place in the matter sector. There are numerous gauge singlets present in addition to the matter fields [342]. Given the no-scale structure present in superstring theories [329], one might wonder whether or not inflation might occur via a superpotential of the form in eq. (5.55) [234]. The problem is that string amplitudes are only consistent with superpotential terms of dimension three or higher [325]. A simple modification [343] would be to introduce additional singlet fields so that

$$F(\phi, X, Y) = m^2(\phi XY - \tfrac{3}{4}\phi^4) , \tag{5.133}$$

with $\langle X \rangle = \langle Y \rangle \simeq M_P$. However, the potential now becomes

$$V(\phi) = m^4(|1 - 3\phi^3|^2 + 2|\phi|^2) , \tag{5.134}$$

as in eq. (5.56) but with the addition of a mass term. Classically such a potential would not inflate but it was shown [343] that via stochastic processes, (5.134) can lead to inflation if initial fluctuations are large, $\langle \phi^2 \rangle \gtrsim 0.1\, M_P^2$. Outside of the chaotic scenario, such large fluctuations would not be expected to arise.

In a string theory however, one is not at liberty to simply introduce an inflaton. Inflation, if it occurs, must be due to one of the gauge singlet fields present in the theory to begin with. There are four types of singlets in superstring theories (for a discussion of some cosmological consequences of these singlets see ref. [343]): (1) $\tilde{\sigma}$, which is part of the gravitational multiplet, (2), (3) fields X^i and C^i, the so-called moduli of the Calabi–Yau space and (4) the fields S^j which are the descendents of the 10-dimensional gauge fields transforming as $(1, 8)$ under $E_6 \times SU(3)$. The number of $(1, 8)$ singlets is given topologically by H_1 (End T) for tangent space T. The numbers of X^i and C^i are given by $b_{1,1}$ and $b_{2,1}$, respectively. These fields are all complex. The real part of the scalar $\tilde{\sigma}$ is the dilaton ϕ, while the imaginary part is related to the scalar h, associated with the field strength H_{mnp} of the antisymmetric tensor B_{mn}.

It has become customary to define two chiral superfields S, T in terms of ϕ, h and one of the X^i (recall that there is always at least one)

$$S = \phi^{-3/4}\, e^{3\sigma} + 3i\sqrt{2}\, h, \qquad T = \phi^{3/4}\, e^{\sigma} - i\sqrt{2}\, \beta , \tag{5.135}$$

where σ and β are related to the real and imaginary parts of one of the X^i. The effective theory is then described by an $N = 1$ supergravity theory with Kähler potential G and gauge kinetic terms $f_{\alpha\beta}$, where

$$G = -\ln(S + S^*) - 3\ln(T + T^* - y^i y_i^*) + \ln|F(y_i)|^2), \qquad f_{\alpha\beta} = \delta_{\alpha\beta} S . \tag{5.136}$$

The scalar potential is then derived from eq. (5.136).

Superstring breaking may come from two sources [330]: (1) a non-zero vev for the field strength H_{mnp} (where mnp are indices on the compact manifold) or (2) through gaugino condensation in the hidden sector. The resultant superpotential is given by

$$F(S) = c + h\, e^{-3S/2b_0} , \tag{5.137}$$

where c receives a contribution from the vev of H_{mnp} and/or the non-zero vev of $F(y)$ [345], h is a constant (not to be confused with the scalar in the gravitational multiplet) and b_0 is the coefficient of the one-loop β function for the gauge group of the hidden sector. The potential can be written as [346]

$$V(s) = \frac{s_0}{s}\, \mu^4 \left| 1 - \frac{1+s}{1+s_0}\, e^{(s_0-s)/2} \right|^2 , \tag{5.138}$$

where $s = 3\kappa S_R/b_0$, $S_I = 0(S_R = \mathrm{Re}\, S,\ S_I = \mathrm{Im}\, S)$, $-c/h = (1+s_0)\exp(-s_0/2)$, where s_0 is the position of the minimum of V with $V(s_0) = 0$ for $s_0 > 1$ and

$$\kappa^4 \mu^4 = \frac{3h^2}{2b_0} \frac{(1+s_0)^2}{s_0} \mathrm{e}^{-s_0} . \tag{5.139}$$

The potential for the T-field is highly uncertain and I will not discuss it here. I note only that a proposal for inflation using the T-field can be found in ref. [347].

Much of the work on inflation in superstring theories is based on the potential for the S-field [348–351, 346, 352, 353]. The problem with this approach is that the potential (5.138) is not really suitable for inflation [349, 350, 346]. In ref. [349], inflation via a Hawking–Moss [104] transition was discussed. In that model inflation required a specific value of c/h with $\delta(c/h)/(c/h) \lesssim 2\times 10^{-5}$. A similar constraint under more general circumstances was found in ref. [346], $-c/h = 0.9374 + \delta(-c/h)$ and $\delta(-c/h) \lesssim 3\times 10^{-4}$. This type of fine-tuning hardly makes these models of inflation attractive.

The possibility of R^2-inflation in string theories is also probably non-existent [354, 350]. Although R^2-terms arise in string theories as I will discuss below, they come perturbatively, i.e., with $M > M_P$ in the notation of eq. (5.108). In this case no inflation would occur. Also string theories predict a specific combination which in four dimensions does not have an effect on the equations of motion.

On a more stringy level, there have been some arguments [355] that at times near the Planck time, string creation could drive inflation. Unfortunately, very special initial conditions were required. Furthermore at the Planck time, the standard cosmological equations of motion are expected to be altered due to string corrections. To order R^2, one can write the string generated Lagrangian in D space–time dimensions as [356, 357]

$$\mathscr{L} = \frac{1}{2\kappa^2} R - \mathrm{e}^{-4\kappa\phi/\sqrt{D-2}} H^2 - \tfrac{1}{2}\partial_\mu\phi\, \partial^\mu\phi - \frac{\alpha'}{16\kappa^2}(F_{ab}F^{ab} - \tilde{R}^2) + \mathscr{L}_m , \tag{5.140}$$

where $H^2 = H_{mnp}H^{mnp}$, ϕ is the dilaton, F_{ab} is the gauge field strength tensor and g is the gauge coupling. In four dimensions the scalar potential in $\mathscr{L}_m$ is of the form $\exp(\sqrt{2}\kappa\phi)V(y)$. The Gauss–Bonnet combination $\tilde{R}$ is defined by

$$\tilde{R}^2 = R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2 . \tag{5.141}$$

The string tension is defined by

$$\alpha' = (2\kappa^2/g^2)\,\mathrm{e}^{-2\kappa\phi/\sqrt{D-2}} . \tag{5.142}$$

Neglecting the field strengths F and H the equations of motion become [358]

$$\begin{aligned} R_{ab} - \tfrac{1}{2}g_{ab}R &= \partial_a\phi\,\partial_b\phi - \tfrac{1}{2}g_{ab}\,\partial^\mu\phi\,\partial_\mu\phi \\ &\quad + \tfrac{1}{8}\alpha'(\tfrac{1}{2}g_{ab}\tilde{R}^2 + 4R_{ac}R^c{}_b + 4R_{acbd}R^{cd} - 2R_{ab}R - 2R_a{}^{cde}R_{bcde}) \\ &\quad + \tfrac{1}{4}\frac{\partial\alpha'}{\partial\phi}(R\phi_{;a;b} - Rg_{ab}\phi^{;c}{}_{;c} - 2R_{ac}\phi_{;b}{}^{;c} - 2R_{cb}\phi_{;a}{}^{;c} + 2g_{ab}R^{cd}\phi_{;c;d} + 2R_{ab}\phi^{;c}{}_{;c} - 2R_{acbd}\phi^{;c;d}) \\ &\quad + \tfrac{1}{4}\frac{\partial^2\alpha'}{\partial\phi^2}(R\partial_a\phi\,\partial_b\phi - Rg_{ab}\partial^c\phi\,\partial_c\phi - 2R_{ac}\partial_b\phi\,\partial^c\phi - 2R_{cb}\partial_a\phi\,\partial^c\phi + 2g_{ab}R^{cd}\partial_c\phi\,\partial_d\phi \\ &\quad + 2R_{ab}\partial^c\phi\,\partial_c\phi - 2R_{acbd}\partial^c\phi\,\partial^d\phi) + \kappa^2 T_{ab} , \end{aligned} \tag{5.143a}$$

$$\phi^{;c}{}_{;c} = (\alpha'/8\kappa\sqrt{D-2})\tilde{R}^2 - \partial\mathcal{L}_m/\partial\phi \,. \tag{5.143b}$$

Cosmological solutions using the Gauss–Bonnet term were examined in ref. [359]. In four dimensions, the Gauss–Bonnet term alone has no effect on the equations of motion. The coupled set of equations for the dilaton plus gravity lead to strong constraints [360–362]. When the dilaton is taken to be constant, one finds from eq. (5.143b)

$$\alpha'\tilde{R}^2 = 8\kappa\sqrt{(D-2)}\partial\mathcal{L}_m/\partial\phi \,, \tag{5.144}$$

so that in the absence of matter, eq. (5.144) implies $\tilde{R}^2 = 0$ [356]. For $D = 4$ one finds [361] that the only perturbative solution is $p = \rho = 0$. Thus we expect that non-trivial cosmological solutions are incompatible with $\phi =$ constant. In particular, we expect that inflation must incorporate the dynamics of the dilaton. Attempts at explicitly constructing De Sitter solutions [360, 362] lead to a similar conclusion. Self-interactions of the dilaton as described by the potential in eq. (5.138) can lead to an approximate De Sitter solution [362] when $\alpha' R \ll 1$ so that the R^2-term can be neglected. The role of the dilaton in inflation was also considered in ref. [363].

Attempts at cosmological solutions directly at the string level [364] have found only two possible non-trivial solutions: (1) an Einstein static Universe and (2) a Universe with $R(t) \sim t$, i.e., linear expansion. The linear expansion was also found [362] as a solution to the equations of motion when dilaton self interactions and R^2-terms are neglected.

String theories can also be expressed as true four-dimensional theories [365]. An example of a promising theory of this sort is based on the gauge group $SU(5) \times U(1)$ [185, 366]. The model is attractive from a model-building point of view in that it does not require an adjoint to break SU(5), it is done with a **10** plet. There is also a very natural doublet–triplet separation mechanism and a see-saw for neutrino masses. Although $\Delta B \neq 0$ interactions require a scale $M_I \simeq 10^{16}$ GeV, strong coupling effects become important at $T \simeq \Lambda_5 \simeq 10^{10}$ GeV and can avoid the entropy problem normally associated with intermediate scale models [367]. Models with smaller gauge groups such as $SU(3)^3$ as described earlier can not exploit this solution [181]. A model of chaotic inflation based on $SU(5) \times U(1)$ was described in ref. [368]. Inflation based on the breaking of an anomalous Fayet–Iliopoulos U(1) in string theories was considered in ref. [369].

Unfortunately, I do not believe that a truely integrated model of inflation in a string theory has yet been constructed.

5.8. Power-law inflation

Up to this point, all models of inflation have been based on a period of exponential expansion. The solution of the cosmological problems, however, does not necessarily require this. Power-law expansion of the form $R(t) \sim t^p$ is sufficient given a large enough value of p. From the requirement that $(R_f T_R / R_i T_i) \gtrsim 10^{29}$ (assuming sufficient reheating) our previous condition $H\tau \gtrsim 65$ now becomes $(t_f/t_i)^p \gtrsim 10^{29}$ (T_i/T_R), where t_f and t_i are the final and initial times where the $R \sim t^p$ expansion law holds and T_i and T_R are the initial and reheat temperature, respectively. Constraints on these models have been studied in some detail [370–373].

Reheating in this case is never as efficient as in exponential inflation because H is not constant. Strong reheating implies that $T_R \sim (HM_P)^{1/2} \sim (pM_P/t_f)^{1/2}$. If we take $T_i = M_P$, then we can write [371]

$$(t_f/t_i)^{p-1} \gtrsim 10^{29}(T_R/M_P)\,. \tag{5.145}$$

Power-law inflation would arise if the equation of state $p/\rho < -\frac{1}{3}$. Thus, in all of these cases $\ddot{R} > 0$. To see this one can simply solve the cosmological equations (1.7) and (1.9). With $\gamma - 1 = p/\rho$ one arrives at the general solution $R \sim t^p$ with $p = 2/3\gamma$. If the power-law expansion is due to the evolution of a scalar field, it is relatively straightforward to see that the scalar potential will be an exponential in ϕ [371, 374–379]. From eq. (3.22), we can write the Friedmann equation as

$$(\dot{R}/R)^2 = \tfrac{1}{6}\kappa^2\dot{\phi}^2 + \tfrac{1}{3}\kappa^2 V(\phi)\,. \tag{5.146}$$

If we assume the following:

$$R(t) \sim t^p, \qquad \phi(t) \sim c\ln(t/t_0), \qquad V(\phi) \sim V_0\,e^{\lambda\phi}\,. \tag{5.147a, b, c}$$

Then we find that from eq. (3.23) and (5.146) $c\lambda = -2$ and

$$p^2 = \tfrac{1}{6}\kappa^2 c^2 + \tfrac{1}{3}\kappa^2 V_0 t_0^2, \qquad -c + 3pc + \lambda V_0 t_0^2 = 0\,. \tag{5.148a, b}$$

The solution to these equations is

$$c^2 = 2p/\kappa^2, \qquad \kappa^2 V_0 t_0^2 = p(3p-1)\,. \tag{5.149a, b}$$

Thus there is a general class of potentials admitting a power-law expansion.

It is interesting to note that in a string theory exponential potentials occur. However the couplings are not adjustable. If V_0 is a contribution from $\mathscr{L}_m$, then the coupling λ is uniquely determined, $\lambda = 2\kappa/\sqrt{D-2}$, or $\lambda = \sqrt{2}\kappa$ in four dimensions. In that case $c^2 = 4\lambda^{-2} = 2/\kappa^2$, implying that $p = 1$. Recall that the linearly expanding solution in string theories was discussed earlier [364, 362]. Such a result actually holds in D space–time dimensions as well [remember that eqs. (3.23) and (1.7) are modified when $D \neq 4$]. Thus power-law inflation is not expected in a string theory.

The spectrum of density fluctuations is also different in power-law inflation [370–373, 380, 381]. In a relatively simple calculation, it was found [370] that the analogue of eq. (4.14) is

$$\langle \delta\tilde{\phi}(k,t)\rangle \sim \Gamma^2(\nu)2^{2\nu-5}/k^{2\nu-3}|1-p|^{1-2\nu}\pi^4\,, \tag{5.150}$$

where $\nu = (3p-1)/2(p-1)$. In the limit that $p \to \infty$ (exponential inflation), $\nu \to \frac{3}{2}$ and $\langle\delta\tilde{\phi}\rangle$ is scale independent. For other values of p, however, one predicts a k dependence for $\delta\rho/\rho$,

$$\delta\rho/\rho \sim k^{-1/p-1}\,. \tag{5.151}$$

Equation (5.151) can easily be derived from eqs. (4.11), (4.18), (5.147b, c) with the replacement of $\exp(-2Ht)$ by t^{-2p}. For large values of p, there is only a weak dependence on k. It was also found [371] that from the anisotropy of microwave background, $\delta\rho/\rho \lesssim 10^{-4}$ requires $p > 1.9$ and $(t_f/t_i) < p^{(3p-1)/(2p-1)}10^{4(8p+33)/3(2p-1)}$. For $p = 2$ this corresponds to $t_f/t_i \lesssim 10^{22}$ and the cosmological problems are easily solved. The reheat temperature in this case is $T_R \sim 10^8$ GeV. T_R increases slowly with p so that $T_R \gtrsim 10^{13}$ GeV corresponds to $p \gtrsim 10$.

The effects of damping were considered in refs. [374, 378]. The behavior of the $\langle\phi^2\rangle$ fluctuations was discussed in detail in ref. [380]. Stochastic methods were applied to power-law inflation in refs. [381, 311, 315].

Although power-law inflation does not arise in string theories, it was shown [382] that power-law expansion occurs if the Universe becomes dominated by domain walls.

Theories of induced gravity [383] based on an action of the form

$$I = \int \mathrm{d}^4x \sqrt{-g}\left[\tfrac{1}{2}\varepsilon\phi^2 R - \tfrac{1}{2}\partial_\mu\phi\, \partial^\mu\phi - V(\phi)\right] \tag{5.152}$$

lead to power-law inflation [384–387]. In ref. [385] it was shown that for small ε,

$$R(t) \sim t^{1/4\varepsilon}\,. \tag{5.153}$$

Density perturbations in this model were considered in detail in ref. [386]. When $V(\phi)$ can be neglected (and R^2-terms as well), it was shown [387] that the expansion parameter is given by $p = -[4\varepsilon - 1 \pm \sqrt{\varepsilon}(\varepsilon - \frac{1}{6})/(3 - 16\varepsilon)]$ admitting both power-law expansion ($p > 0$) and super-exponential expansion ($p < 0$) in which $\dot{H} > 0$. It should be noted that a solution (non-inflationary) to the horizon problem in induced gravity models was suggested in ref. [388].

Power-law inflation also arises in the context of Brans–Dicke gravitation [389]. An action of the form

$$I = \frac{1}{2\kappa^2}\int \mathrm{d}^4x \sqrt{-g}\left(\phi R - \frac{\omega}{\phi}\,\partial^\mu\phi\,\partial_\mu\phi + \mathscr{L}_\mathrm{m}\right) \tag{5.154}$$

is equivalent to the action

$$I = \int \mathrm{d}^4x \sqrt{-g}\left(\frac{R}{2\kappa^2} - \tfrac{1}{2}\partial_\mu\phi'\,\partial^\mu\phi' + \mathrm{e}^{-2\sqrt{2}\kappa\phi'/\sqrt{3+2\omega}}\mathscr{L}_\mathrm{m}\right) \tag{5.155}$$

after a conformal transformation, and a field redefinition of the dilaton ϕ [390]. If there is a non-zero vacuum energy in $\mathscr{L}_\mathrm{m}$, i.e. $\mathscr{L}_\mathrm{m} = -V_0 + \cdots$, one has an exponential potential as in eq. (5.147c) with $\lambda = -2\sqrt{2}\kappa/\sqrt{3+2\omega}$. Thus $c = \sqrt{3+2\omega}/\sqrt{2}\kappa$ and from eq. (5.149a), $p = \frac{1}{2}\omega + \frac{3}{4}$. When converted back to physical coordinates by $\mathrm{d}t' = \Omega\,\mathrm{d}t$ for a conformal transformation $\Omega^2 = \phi \sim t'$ one finds the expected expansion rate $R(t) \sim t^{\omega+1/2}$, i.e., a power-law expansion [295]. (A similar conformal transformation can be used to show $R(t) \sim t^{(1+2\varepsilon)/4\varepsilon}$ in the induced gravity model [295].) In ref. [389] the effect of Brans–Dicke gravity on new inflation was discussed. It was later noted in ref. [391] that using $V(y)$ from a first-order transition as in old inflation, allows the bubbles to merge because of the slower rate of expansion. However for pure Brans–Dicke, bubble production puts a constraint on ω which is incompatible with observations $\omega \lesssim 25$ [392]. This idea was applied to an induced gravity model in ref. [393].

5.9. Kaluza–Klein inflation

Theories utilizing $D > 4$ space–time dimensions are useful for unification theories involving gravity. A case in point is the ten-dimensional heterotic superstring theory. This approach, however, dates back to the 1920s to Kaluza's and Klein's [394] work on the unification of gravity with electromagnetism using a theory in five dimensions. For unification with larger gauge groups, more dimensions are required [395]. Symmetries of the internal compactified space appear as gauge symmetries in an effective four-dimensional theory. It is unlikely, however, that realistic models of low energy physics can be constructed in this way. It has been shown [396] that these theories contain only real field representations for particles as opposed to chiral representations needed in the standard model. Nevertheless, models of inflation specifically utilizing the presence of extra (to be compactified) dimensions have been discussed in the literature. Cosmologies based on Kaluza–Klein theories have been discussed in ref. [397].

There have been two basically different approaches to inflation in Kaluza–Klein theories. One approach [398–401] has been to consider the solution of Einstein's equation (with normal gravity) with three expanding spatial dimensions and D-4 contracting dimensions. Entropy (in the four-dimensional space–time) can be generated when the number of dimensions is suitably large. In ref. [399], it was suggested that $D \gtrsim 40$ would be sufficient to solve the cosmological problems. Numerical work [400] verified this. In ref. [401], it was noted that entropy production depends on assumptions made regarding the time scales for compactification and the decoupling of the extra dimensions. Kaluza–Klein inflation (with normal gravity) was also considered in ref. [402].

Another approach to inflation in higher dimensions utilizes higher derivative corrections to Einstein's equations [403]. The appearance of these corrections was argued to be necessary for compactification [404]. The gravitational Lagrangian in these models contains ordinary gravity plus terms such as R^2, $R_{ab}R^{ab}$ and $R_{abcd}R^{abcd}$ but not necessarily in the Gauss–Bonnet combination. The evolution of the extra dimensions can be cast in the form of the evolution of a scalar field with an exponential potential. This model, it turns out, leads to a power-law expansion or superexponential expansion as $\gamma - 1 = p/\rho < -1$ leading to $\dot{H} > 0$ which is quite different from all other models of inflation [405]. The perturbation spectrum is still given by eq. (5.151) [405, 406]. Reheating in this model was considered in ref. [407]. Power-law expansion in higher dimensional theories was also considered in ref. [374] and in models of induced gravity in ref. [408]. Theories with additional $\log R$ corrections were considered in ref. [409].

5.10. Multiple inflation

Multiple inflation is again not really a specific model of inflation but is rather a combination of previous models. The effects of a series of inflationary or near inflationary phase transitions is examined.

A series of phase transitions is always possible if one goes beyond the minimal SU(5) theory by either including gauge singlets or even extended gauge groups. Even in minimal SU(5), we saw that an intermediate stage of SU(4) × U(1) could lead to two-inflationary epochs (though the second might be an old inflationary epoch). In axionic models [272, 268] the inflationary transition induced the breaking of SU(5). The appearance of a linear term due to chiral symmetry was proposed [410] to break SU(5) with inflation. A series of (near inflationary) transitions with only a partial suppression of magnetic monopoles was suggested in ref. [411]. The supersymmetric inflationary models in ref. [216, 221] are also two component inflationary models.

In the context of R^2-inflationary models, two periods of inflation, one by R^2-terms, the other by the evolution of a scalar field was considered in ref. [291]. A generalization of this picture was studied as multi-component inflation in ref. [412] using several scalar fields and an R^2-correction. The production of density fluctuations was also considered in this context.

It was also suggested [413] that by designing the inflationary periods appropriately, one might be able to produce density fluctuations with different amplitudes on different scales. In double inflation [413], one tries to arrange the first period of inflation so that $\delta\rho/\rho \sim 10^{-5}$–$10^{-4}$ on large scales and the latter period of inflation with $\delta\rho/\rho \sim 10^{-2}$ (followed by a late period where the microwave background is reionized) on small scales. The effects of double inflation on galaxy formation are studied in more detail in ref. [414]. Typically one would expect that because the Hubble parameter for the first period of inflation is larger, $\delta\rho/\rho$ on large scales would be larger than $\delta\rho/\rho$ on small scales [415]. However, by carefully fine-tuning the inflationary potentials, this can be avoided [413]. A detailed study of multiple inflation is found in ref. [415]. The designing of density fluctuations was looked at in detail in ref. [416]. A combination of adiabatic and isothermal perturbations from inflation was examined in ref. [417].

6. More on inflation and related topics

6.1. Quantum mechanics of inflation and the quantum creation of the Universe

We have seen that quantum physics plays an important role in the global effects due to inflation. Concrete examples of this were the role of quantum fluctuations on symmetry breaking and energy density fluctuations. While a classical description of inflation usually leads to the correct picture for the evolution of the Universe, the correct resolution to ambiguities such as those described in ref. [162] require a quantum analysis of inflation. These investigations eventually led to models of stochastic inflation described in the previous section.

Early investigations [303, 304, 418,–420] used quantum techniques to determine the viability of the classical description of a slow roll-over phase transition. A toy model consisting of an upside-down harmonic oscillator [304] in a De Sitter space was used to determine the late time classical behavior of a roll-over transition. It was concluded [304, 418] in a complementary fashion to the method used in ref. [164], that given a small enough coupling, a slow roll-over transition leads to inflation. Another toy model [419] utilized an N-vector model and both single and double well potentials. The need for using non-equilibrium dynamics was stressed. Inflation was found to occur under a wide class of initial conditions. The use of non-equilibrium dynamics is investigated in detail in ref. [421].

Closely related to inflation is the idea of the creation of the Universe from nothing. The idea that the Universe may have been a vacuum fluctuation has been around long before the inflationary Universe scenario [422]. The possibility that the Universe emerged as a quantum tunnelling effect was discussed in refs. [423, 38]. By making an analogy with the process of pair production, Vilenkin [424] was able to take a more quantitative approach to this problem. It was suggested that the probability for producing a Universe from "nothing" could be expressed as

$$P \sim e^{-S_E}\,, \tag{6.1}$$

where S_E corresponds to the Euclidean action for the De Sitter instanton [425, 104],

$$S_E = -3M_P^4/8V\,, \tag{6.2}$$

where V is the energy density of the vacuum. It was further argued [319] that tunnelling should occur to a local maximum of V, the scalar potential. This is in close parallel to the Hawking and Moss version

[104] of the new inflationary scenario where tunnelling occurs from a state $\phi = 0$ to a local maximum. Inflation follows the subsequent evolution of the scalar field to its global minimum. Quantum tunnelling was also discussed in ref. [426].

The probability (6.1) and (6.2) was criticized by Linde [427] as having the wrong sign on physical grounds. He argued [427] that quantum gravitational effects should only be important for $V \sim M_P^4$ and length scales should be confined to $l \lesssim M_P^{-1}$. Therefore, he concluded that the probability for the quantum creation of the Universe should be

$$P \sim \exp(-3M_P^4/8V)\,. \tag{6.3}$$

Such a probability function strongly suppresses $V \ll M_P^4$ and would clearly favor a chaotic approach to the inflationary Universe. Using a minisuperspace model based on the Wheeler–DeWitt equation [428], Vilenkin [429] later agreed that

$$P \sim e^{-|S_E|}\,. \tag{6.4}$$

Vilenkin stressed that such an expression only makes sense for $S_E \gg 1$ and concluded that tunnelling would occur to the highest local maximum. See ref. [430] for further discussion. The possibility of quantum creation has been discussed in a variety of contexts: in extended $N = 8$ supergravity [431]; in R^2-models of inflation [283]; in higher dimensional theories of gravity and inflation [432]; and in string theories [433]. Particle production in the quantum creation scenario was examined in ref. [434].

Another approach to the question of quantum cosmology is the wave function of the Universe proposed by Hartle and Hawking [435]. Using a path integral approach summing over all compact metrics, they proposed a wave function given by

$$\Psi(h_{\mu\nu}, \phi) = \int \mathrm{d}[g_{\mu\nu}]\,\mathrm{d}[\phi]\,\exp -S_E(g_{\mu\nu}, \phi) \tag{6.5}$$

for matter fields ϕ and total Euclidean action S_E. $h_{\mu\nu}$ is the spatial part of the metric $g_{\mu\nu}$. Note the resemblance of eq. (6.5) to (6.1) and (6.2). The wave function was also applied to the inflationary Universe in a number of contexts: a single massive scalar field [436] (this leads to a scenario very reminiscent of chaotic inflation); a conformally coupled scalar field with a Coleman–Weinberg potential [437]; a minimally coupled scalar field in a double well potential [438] and a $\lambda\phi^n$ potential [439]; in a Kaluza–Klein theory [440]; and in an R^2-cosmology [441].

The question of the sign in the action for the wave function of the Universe was also criticized by Linde [427, 242]. The Euclidean action is found by making a rotation $t \to -i\tau$ and $P \sim e^{iS} \to e^{-S_E}$. Linde argued that for the gravitational action the rotation $t \to i\tau$ should be made, leading to a wavefunction with the opposite sign in eq. (6.5). Without a more complete theory of gravity (i.e., quantum gravity), it is difficult to make any strong claims with regard to such a probability distribution.

It is interesting to note, however, that the stochastic approach also yields a similar probability distribution [302]. Indeed, the probability distribution

$$W \sim P \sim \exp[3M_P^4/8V(\phi)] \tag{6.6}$$

is a stationary solution ($\partial P/\partial t = 0$) to the Smoluchowski equation (5.122). Again we see the positive sign in the exponential. The interpretation of this distribution is difficult because it is not renormalizable. It has been argued [321, 442] that eq. (6.6) is not applicable because the stationary solution must correspond to the evolution of scalar field downward (for $V = m^2\phi^2$ or $\lambda\phi^4$) compensated by the diffusion of the scalar field upward. When P is large, $\phi \ll M_P$, diffusion does not occur. The derivation of P with either sign was criticized in ref. [306].

A non-stationary solution to eq. (5.122) was proposed [321] for the massive scalar theory. With an initial distribution $P(\phi, t=0) = \delta(\phi - \phi_0)$, it was found that

$$P(\phi, t) = \exp\left(- \frac{3[\phi_0 - \phi(t)]^2 M_P^4}{2m^2[\phi_0^4 - \phi^4(t)]} \right), \tag{6.7}$$

leading to an exponential expansion of the Universe. The stochastic approach to various inflationary models was used in refs. [306, 443].

6.2. *Inflation and anisotropic models*

It may seem strange to the alert reader, that up until now, nearly our entire analysis of inflation assumed a priori that the Universe is homogeneous and isotropic. However, the homogeneity and isotropy is what inflation is supposed to explain. The main reason this is done is out of convenience. The Friedmann–Robertson–Walker metric is much simpler to deal with than the metric (if it can be found) in some arbitrary inhomogeneous spacetime. There have been numerous studies on anisotropic spacetimes and the question of inflation. It is very widely believed that given a theory which inflates in a FRW spacetime, it will also inflate under more general circumstances.

Homogeneous but anisotropic spacetimes in which the metric is of the form [444]

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \gamma_{\alpha\beta}(t) e_\mu{}^\alpha e_\nu{}^\beta \,\mathrm{d}x^\mu \,\mathrm{d}x^\nu \tag{6.8}$$

(in a triad notation), where $\gamma_{\alpha\beta}$ is the generalization of the scale factor and $e_\mu{}^\alpha e_\beta{}^\mu = \delta^\alpha_\beta$, are classified into equivalence classes known as the Bianchi types. The commutator

$$e_\alpha{}^\mu e_\beta{}^\nu (e_\mu{}^\rho{}_{;\nu} - e_\nu{}^\rho{}_{;\mu}) = C_{\alpha\beta}{}^\rho \tag{6.9}$$

determines the structure constants $C_{\alpha\beta}{}^\rho$, which classify the Bianchi types. Writing $C_{\alpha\beta}{}^\rho = \varepsilon_{\alpha\beta\sigma} C^{\sigma\rho}$, where $\varepsilon_{\alpha\beta\sigma}$ is the unit antisymmetric symbol, and $C^{\alpha\beta} = n^{\alpha\beta} + \varepsilon^{\alpha\beta\sigma} a_\sigma$ with $n^{\alpha\beta} a_\beta = 0$ and the symmetric tensor $n^{ab} = \mathrm{diag}(n_1, n_2, n_3)$, one can classify the Bianchi types as shown in table 1.

The simplest case is Bianchi type I,

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \gamma_i(t)\,\mathrm{d}x^{i2} , \tag{6.10}$$

or in its Kasner [445] form

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + t^{2p_1}\,\mathrm{d}x^2 + t^{2p_2}\,\mathrm{d}y^2 + t^{2p_3}\,\mathrm{d}z^2 , \tag{6.11}$$

where p_1, p_2, p_3 are any numbers satisfying $p_1^2 + p_2^2 + p_3^2 = 1$.

Table 1
Classification of Bianchi types

	a	n_1	n_2	n_3
I	0	0	0	0
II	0	1	0	0
III	1	0	1	−1
IV	1	0	0	1
V	1	0	0	0
VI	0	1	−1	0
VI_h	a	0	1	−1
VII	0	1	1	0
VII_h	a	0	1	1
VIII	0	1	1	−1
IX	0	1	1	1

Types I, V, VII with matter were shown [446] to have late time behavior evolving towards a FRW solution. It had been conjectured [146, 104] that any cosmology with a positive cosmological constant would asymptotically (in time) approach a De Sitter spacetime. The Bianchi types, with the possible exception of type IX, have been shown [447] to obey the "no hair" conjecture. For a large enough cosmological constant type IX also tends to De Sitter. Other general solutions also followed suit [448]. A general proof was attempted in ref. [449].

In an inflationary model, the cosmological constant is only temporary, and it is a natural question to ask whether or not inflation occurs in an anisotropic model. The issue was first raised in the context of old inflation [450]. In simple cases, the Friedmann equation becomes

$$(\dot{R}/R)^2 = \tfrac{1}{3}\kappa\rho - k/R^2 + \Sigma^2/R^6\,, \tag{6.12}$$

where Σ represents the effect of the anisotropy. If the energy density associated with the anisotropy (shear, for example) is still dominant at the time of the GUT phase transition ($\Sigma^2/R^6 > \rho_v$), then the transition would proceed without an exponential expansion. Later, the anisotropy would redshift away. Thus the presence of a large anisotropy could forego an inflationary phase. This interpretation, however, has been recognized to be erroneous [451–454]. It was argued that [451] the presence of large anisotropies tends to restore gauge symmetries much like high temperature effects. Thus symmetry breaking would not occur until the anisotropies diminish. Also it was shown [452–454] that the time scale needed to become vacuum dominated is independent of Σ. In addition, this time-scale is always shorter than the time-scale associated with symmetry breaking if sufficient inflation is to occur without the anisotropy [454].

In the context of new inflation, it was shown that under a wide variety of circumstances, inflation occurs in anisotropic models. Initial investigations [455, 456, 452, 457] showed the absence of a problem for simple models such as Bianchi type I. Indeed, new inflation with anisotropies inflates more than without [455, 457] these anisotropies. Problems regarding a classical treatment of inflation in type I models were examined in ref. [457]. In Bianchi type IX, it was noted [458] that even new inflation would not occur if the anisotropies were too large (recall that type IX with a cosmological constant was the only example which did not always tend to De Sitter space [447]). An application of Wald's results [447] to the inflationary scenario was made in ref. [459] where it was also argued that the presence of anisotropies would be felt again, albeit in the distant future. The methods used to reach these conclusions were criticized in ref. [460] though these authors also believe inflation will begin in the Bianchi models.

Chaotic inflation in Bianchi types I and V were studied in refs [461, 453] and with large metric and matter perturbations in ref. [462]. The effects of spacetime topology on chaotic inflation were considered in ref. [463] with the conclusions that though chaotic inflation may not be realized under certain circumstances, the non-trivial topologies may favor R^2-type inflation. Inflation in models with rotation was considered in refs. [29, 460, 464], in the Einstein–Cartan Bianchi I gravitational model with non-zero torsion in ref. [465] and in the mixmaster model in ref. [466]. Inflation in the Kantowski–Sacks class of anisotropic models was considered in ref. [467].

The no-hair conjecture has been examined in extended gravitational models, such as the R^2-model [294, 470], the induced gravity model [469] and Brans–Dicke gravity [470]. A discussion of the no-hair conjecture in the power-law expanding models can be found in ref. [375].

The behavior of $\langle \phi^2 \rangle$ in an anisotropic higher dimensional model was computed in ref. [471].

Finally, for discussions of inflation in inhomogeneous space–times see ref. [472].

6.3. Monopoles, cosmic strings and gravitinos

The magnetic monopole problem [28] was one of the original motivations for the inflationary Universe scenario. In nearly all cases, sufficient inflation reduces the monopole abundance exponentially. In other words, we would not expect more than one monopole in our horizon if the SU(5) Higgs field was coherent over an exponentially large volume.

As discussed in section 4, the original models of new inflation based on radiative SU(5) symmetry breaking were problematic in part because of the tendency to pass through an intermediate SU(4) × U(1) phase [110–114]. The subsequent breaking of the intermediate phase could lead to an over production of monopoles [113]. A detailed calculation of the monopole production in this case was made in ref. [139]. An attempt to resolve this problem using higher Higgs representations was made in ref. [473].

It was also feared that in models of primordial inflation [119], that because the inflationary scale $v \sim M_P \gg M_{GUT}$, it was possible that the GUT symmetry breaking would occur after inflation, thereby not necessarily resolving the monopole problem. Conditions under which the breaking of SU(5) by strong coupling effects occurs before the end of inflation were addressed in ref. [474]. It was argued however [475], that in almost all circumstances, GUT symmetry breaking would occur during inflation, and unless reheating were so efficient so as to restore the GUT, the monopole problem does not in fact exist in primordial inflationary models.

It was also argued that monopoles might be thermally produced during reheating even if the GUT symmetry was not restored [476]. These arguments are based on detailed balance and are not appropriate for magnetic monopoles.

The production of magnetic fields whose long wavelength fluctuations lead to the appearance of monopoles was discussed in ref. [477]. Assuming relatively light monopoles $M_m \ll H$ and $T_R \ll M_m$ it was argued that $\Omega_m h^2 \sim 10^{27}(H/M_P)^{4/3}(T_R/M_P)^{7/3}$, and a magnetic field strength $B_0 \simeq 2\times10^{-5} \times (H/M)(HT_R/M_P^2)^{2/3}G$. The production of magnetic fields was also discussed in ref. [478].

Like magnetic monopoles, it is expected that one-dimensional topological defects called cosmic strings are expected to be formed during a cosmological phase transition [479, 27]. By a process as simple as the breaking of a U(1) symmetry, cosmic strings would form. The evolution of a network of cosmic strings [480–482] has been studied with the hope of producing seeds for galaxy formation (for a comparison between cosmic strings and inflation for producing large scale structure see refs. [141, 483]). It has been argued [481] that the resulting structure from cosmic strings is in very good agreement with observed large scale structure though this has been criticized in ref. [482].

In order for cosmic strings to be relevant for galaxy formation, they must be produced after (or at the very end of) inflation. This may be quite problematic [154, 484], since strings relevant for galaxy formation must have a string tension $\mu \sim 1\text{–}4 \times 10^{-6} M_P^2$ [481] and μ is typically related to the vev of the "Higgs" breaking the U(1) (i.e., $\mu \simeq v^2$). For $T_c \sim \nu \sim \mu^{1/2} \sim 10^{16}$ GeV, producing strings after inflation requires $T_R > T_c$. In section 4, I have already discussed limits on the Hubble parameter (and hence T_R) from limits due to the microwave background anisotropy [153–155]. Assuming strong reheating, from eqs. (4.27) and (4.55) one finds a limit [154]

$$T_R \lesssim 3 \times 10^{15}\ \text{GeV} \tag{6.13}$$

for $N(T_R) \sim O(100)$. In any "realistic" model of inflation based on a slow roll-over, the inflaton decay rate $\Gamma \ll H$ and the limit on T_R via eq. (4.29) becomes much stronger.

There have been attempts to reconcile this incompatibility. One possibility is that strings form at the end of inflation [485, 154]. This can be done by coupling the inflaton to the string Higgs so that the U(1) factor is only broken when the inflaton approaches its global minimum. In a model related to the axion-inflation model [485], additional couplings between a GUT singlet, S, breaking the U(1) and the SU(5) adjoints and five-plets as well as a coupling between the S and the inflaton, are needed. These couplings vary from 10^{-16}–10^{-20}. Another example is a supersymmetric construction [154] requiring two additional singlets, η_1 and η_2, and a superpotential of the form

$$F = \mu^2[\eta_1(\phi^3 - 1 + S^4) + \eta_2(S^2 - a\phi^2)]\,, \tag{6.14}$$

where ϕ is the inflaton and S breaks the U(1). One can choose $m^2 \simeq 10^{-7} M_P^2$ for $\delta\rho/\rho \simeq 10^{-4}$ and $a \sim (2\text{–}3) \times 10^{-5}$ to form strings at the end of inflation. Unfortunately, neither of these models appear very attractive. In the event that strings do form late, their role for galaxy formation may be quite different from the standard scenario [154].

Other possibilities have also been discussed. The breaking of a U(1) at an intermediate-scale phase transition occurring late may provide strings after inflation [486]. (Recall, however, problems associated with intermediate scale models [336, 337].) In the higher dimensional models using higher derivative terms in the gravitational Lagrangian [403], the reheat temperature might be large enough to overcome these problems [484, 486]. Another possibility is to couple S to the Ricci scalar so that [487]

$$V(S) = \tfrac{1}{4}\lambda(|S|^2 - v^2)^2 + \tfrac{1}{2}\xi R|S|^2\,. \tag{6.15}$$

In a chaotic model with $V(\phi) = \frac{1}{2}m^2\phi^2$ strings form late when $\xi R < \lambda v^2$.

Finally in this subsection, I would like to comment on the relationship between inflation and gravitinos. In section 1, I described the gravitino problem [31]. It was quickly noticed that inflation could dilute the gravitino abundance to safe levels [74]. For unstable gravitinos, the most restrictive bound on their number density comes from the photo-destruction of the light elements produced during nucleosynthesis [79, 80],

$$n_{3/2}/n_\gamma \lesssim 10^{-13}(100\ \text{GeV}/m_{3/2})\,, \tag{6.16}$$

for lifetimes $\tau > 10^4$ s. Gravitinos are regenerated after inflation and one can estimate [74, 77, 79]

$$n_{3/2}/n_\gamma \sim (\Gamma/H)(T_{3/2}/T_\gamma)^3 \sim \alpha N(T_R)^{1/2}(T_R/M_P)(T_{3/2}/T_\gamma)^3 , \tag{6.17}$$

where $\Gamma \sim \alpha N(T_R)T_R^3/M_P^2$ is the production rate of gravitinos. Combining eqs. (6.16) and (6.17) one can derive bounds on T_R:

$$T_R \lesssim 5 \times 10^8\ \mathrm{GeV}\,(100\ \mathrm{GeV}/m_{3/2}) . \tag{6.18}$$

This bound can be avoided when $m_{3/2}$ is so large that $\tau \sim M_P^2/m_{3/2}^3 \times 10^4$ s. Similar problems regarding gauge singlets arise in superstring theories [344]. The role played by gravitinos in inflationary models was discussed in refs. [75, 78, 492].

6.4. Dark matter

There are not very many predictions of the inflationary Universe scenario. The homogeneity and isotropy must be considered a postdiction. A flat density perturbation spectrum is one prediction, although as we have seen, power-law and super-exponential inflation can lead to other spectra. Another prediction is the value of Ω. Recall from eq. (1.44) that the present value of Ω can be expressed as

$$\Omega_0 = 1 + (k/R_0^2H_0^2) = 1 + \hat{k}T_0^2/H_0^2 . \tag{6.19}$$

In the inflationary model, $\hat{k}$ is no longer constant ($\hat{k} = k/R^2T^2$). Sufficient inflation i.e., $H\tau \simeq 65$, necessary to solve the cosmological conundrums, produces only $\Omega \sim \mathrm{O}(1)$ [489]. However, as we have seen, nearly all of the inflationary models considered in the previous section have $H\tau \gg 65$ with $H\tau$ as much as $\mathrm{O}(10^7)$ for simple polynomial models with $V(\phi) = m^4P(\phi)$ and m adjusted for $\delta\rho/\rho \simeq 10^{-4}$. In these models Ω_0 becomes indistinguishable from unity. This is indeed an important prediction.

One problem for this prediction is that nearly all observations are consistent with $\Omega \sim 0.1$–0.3 [11, 490]. An alternative to fine-tuning initial conditions or the inflationary parameters, one can fine-tune the present value of the cosmological constant [491, 492]. Depending on the form of the matter comprising $\Omega = 1$ and the scenario for galaxy formation, $\Omega = 1$ is not necessarily inconsistent with observation. For discussions see refs. [491, 493].

Given $\Omega = 1$, we can use the constraint (eq. 1.34) $\Omega_B < 0.1$ to further predict the presence of non-baryonic dark matter in great abundance. There is of course additional evidence for dark matter (DM) [15] beyond inflation. On galactic scales, there is good evidence from rotation curves of spiral galaxies for the presence of dark matter and a galactic halo [494]. The rotation curve is a measure of the velocity as a function of distance from the center of the galaxy of a star as it revolves around the galaxy. If there were no DM, one would expect that at distances beyond the bulk of the luminous matter that $v^2 \sim 1/r$. Instead one finds flat rotation curves ($v^2 \sim$ constant) out to very large distances (> 50 kpc). This implies that the mass of the galaxy must continue to increase $M \sim r$ beyond the luminous region.

Another piece of (theoretical) evidence for dark matter is the evolution of density perturbations. The key feature of the scale independent perturbations produced by inflation is that the magnitude of $\delta\rho/\rho$ on each scale is the same as that scale enters the horizon. Once within the horizon, these modes cannot really begin to grow further as long as the Universe is radiation dominated. At a temperature of a few thousand degrees, the Universe becomes matter dominated and density perturbations begin growing as

$$\delta\rho/\rho \sim R(t) \sim 1/T\ . \tag{6.20}$$

Now to reach non-linear growth we must have had $\delta\rho/\rho \lesssim 1$ at the time when the oldest galaxies and quasars were forming or at $T \sim 4T_0 \sim 10\,\mathrm{K}$. This means that at the time of matter dominance $\delta\rho/\rho \sim 10^{-3}$. However, we know from limits on the anisotropy of the microwave background that $\delta\rho/\rho < 10^{-4}$. If there exists some form of non-baryonic DM, the Universe may have become matter dominated earlier. For example [495] in the case of massive neutrinos $T_{\mathrm{MD}} \sim m_\nu/10$. Density perturbations could then begin to grow earlier at say $T \sim 10^4$–$10^5\ T_0$ while baryonic perturbations could not, until decoupling at $T \sim 10^3 T_0$. After decoupling, the baryons would fall into the perturbations already formed by the neutrinos. Hence the existence of DM could help enormously in the growth of perturbations for galaxy formation. For a complete review see ref. [496].

Inflation and nucleosynthesis imply that about 90% of the Universe is dark and non-baryonic. (There are other arguments against baryonic dark matter in galactic halos [497].) There are of course many non-baryonic candidates for dark matter. It is well beyond the scope of this review to be more complete on the question of dark matter. For further details see ref. [15].

6.5. *Left-over topics*

As is evident from the diversity of models described in section 5, inflation has developed a long way from the original ideas based on strong first-order phase transitions [45–52, 33]. The great majority of the inflationary models do not even require the tunnelling through a barrier.

The study of strong first-order phase transitions was pursued in great depth. The production of black holes and wormholes was considered in ref. [498]. The treatment of percolation in this analyses was criticized in ref. [50]. (However, see also ref. [499].) A preview to eternal inflation was discussed in ref. [500], though this was again in the context of an old inflationary scenario. The formation of density perturbations in first-order transitions was considered in [507] and due to the production of wormholes in [502].

The formation and evolution of bubbles in cosmological phase transitions have also been studied in great detail [67–70, 81, 105, 503]. In the new inflationary scenario, the production of bubbles of horizon size becomes possible [104, 106, 107, 504]. The interesting possibility that bubbles would form in the absence of a barrier in new inflation was raised in ref. [505] and further elaborated upon in ref. [506].

The role of particle creation [507] has also been studied in the context of inflation [508, 509]. Particle creation in anisotropic models was considered in ref. [510]. Particle creation in the inflationary Universe has been argued to serve as an alternative approach to reheating [508]. The production of massive scalars was examined in ref. [509].

Finally I would like to note that the role of bulk viscosity on inflation has been considered in a number of papers [511, 512]. Bulk viscosity can serve to lower the pressure and it had been suggested that if these effects are strong enough, $p < 0$ would result inducing an inflationary expansion. I believe these models have been correctly criticized in ref. [512].

6.6. *Concluding remarks*

Have existing models of inflation accomplished their set goal? Well, yes and no. The standard big bang model (without inflation) is plagued with several problems. Many of these have to do with small numbers and our feeling that it should be improbable that our Universe, with small number initial conditions, exists. There is an anthropic [513] answer: If it were not so, we would not be around to worry about. Another answer is that actually the Universe can begin with arbitrary initial conditions and micro/macro physical processes always lead to a similar set of final states. A similar situation existed with the problem regarding the net baryon asymmetry, η. If η were very different from $O(10^{-10})$, life as we know it would not be possible. But one of the most important contributions of grand unified theories to cosmology was the explanation of a small value of η. (The exact number of course depends on a knowledge of the specific GUT, which we do not have yet.) Any initial value of η will be altered to yield the same final value. For universes with $\eta \ll O(10^{-10})$ a baryon asymmetry will be generated. For $\eta \gg O(10^{-10})$, early scatterings can wipe out the pre-existing asymmetry and regenerate the same final value as an $\eta = 0$ initial condition. To say the least, this is a satisfying explanation of η.

Inflation has a much larger set of goals to accomplish. There is the overall isotropy, homogeneity, entropy, the absences of nearby horizons, flatness, density perturbations, and perhaps the absence of a past singularity. In addition, there are additional problems due to GUTs and supergravity such as a possible overabundance of monopoles, and gravitinos. Unfortunately, I do not think anyone can claim complete success at this time, though there is clearly the potential (hopefully a flat one) for such an all-encompassing solution.

Obviously, we do not know the likely initial conditions for a Universe. Although most models of inflation assume homogeneity and isotropy, it appears that many anisotropic [446–448, 459, 460] and perhaps inhomogeneous [472] models would end up in an inflating state. The sign of the curvature is unknown, and if inflation occurred, it will perhaps never be known for certain. But what about the magnitude of $\hat{k}$? Recall that for $\hat{k} \sim O(1)$ and $k = +1$, the Universe may recollapse before inflation begins [71, 372, 514]. For further discussion on this problem and the avoidance of recollapse see ref. [515]. After inflation begins, provided the model allows for reheating, most of the cosmological problems are resolved. Perhaps eternal inflation can resolve the question of a past singularity. Unless anisotropies are definitively discovered soon, density perturbations from inflation would have to be too small to act as seeds for galaxy formation.

A glaring problem, in my opinion, is our lack of being able to fully integrate inflation into a unification scheme or any scheme having to do with our fundamental understanding of particle physics and gravity. Old inflation and new inflation with radiative GUT symmetry breaking was simple. SU(5) symmetry breaking occurs and with sufficient supercooling, the Universe inflates. The inflaton is the SU(5) Higgs adjoint. An inflaton as an inflaton and nothing else can only be viewed as a toy, not a theory. The axionic models [272, 268] were a step in the right direction. R^2-models and higher dimensional models are on the same level as inflaton models until a more complete understanding of gravitation is available. As superstring theory remains the most promising hope of unification of gravity and the other fundamental interactions, perhaps an inflaton or the role of R^2 and/or extra dimensions is yet to be uncovered. Inflation is, at this time, our only solution to the many problems in the big bang model. Whether or not it is the only solution, time will tell.

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