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SUPERGRAPHITY
(II). Manifestly covariant rules and higher-loop finiteness

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We continue our discussion of the background field formalism in supersymmetric theories, deriving new covariant Feynman rules for chiral superfields. As a result, we obtain improved power-counting rules for both simple and extended supersymmetry which can be used to make the following statements: If the corresponding extended superfield formalisms exist, (a) $N=2$ supersymmetric Yang-Mills theory is finite beyond one loop, (b) $N=4$ Yang-Mills is finite at all loops, and (c) $N=8$ supergravity is finite through six loops. We also find that in simple super-Yang-Mills the radiative corrections to the Fayet-Iliopoulos ("D") term, which are known to vanish for higher loops, also vanish automatically at one loop for arbitrary couplings.

1. Introduction

In a previous paper [1] we introduced the background field formalism for superfield supergravity. By combining the advantages of supergravity supergraphs [2] and background superfields [3] we obtained a formulation for quantum supergravity which makes one-loop calculations no more difficult than (and in many respects quite similar to) corresponding calculations in QCD. In this sequel to [1] we present further developments of the method and some applications.

Our main goal in this paper is to make the covariance of the background field method more explicit when performing perturbation theory calculations. The usual Feynman rules still contain, in general, explicit reference to the unconstrained background fields, even though the method guarantees that at the end of the calculation the effective action depends only on covariant quantities. We present here a background-quantum splitting, equivalent to that described in [1], for both supersymmetric Yang-Mills and supergravity, which is completely expressible in terms of the (constrained) background covariant derivatives and the (unconstrained) quantum gauge fields. We also avoid the explicit reference to unconstrained background gauge fields of the previous Feynman rules for covariantly chiral superfields by deriving new covariant rules which again involve only the background covariant

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derivatives: no explicit exponentials of unconstrained background gauge-superfields occur in the vertices.

As a result of the new rules, gauge invariance is manifest and further algebraic simplifications are possible for chiral superfields. More important, perhaps, we can conclude that the effective action must be completely expressible in terms of objects which appear explicitly in the background covariant derivatives: superconnections and, in supergravity, the supervierbein, with integrals over full superspace. This result is true not only in $N=1$ theories; the method of derivation of the covariant Feynman rules could be translated immediately, without modifications, to N-extended theories if and when the corresponding actions are written in terms of extended superfields. As a consequence, we can derive improved power-counting rules for all such theories. These rules imply finiteness of $N=2$ Yang-Mills theory beyond one loop, of $N=4$ Yang-Mills theory for all loops, and of N-extended supergravity through $N-2$ loops (thus extending the usual statements of two-loop finiteness).

Our paper is organized as follows: In sect. 2 we describe the new background-quantum splitting method, and in sect. 3 we review the quantization procedure. In sect. 4 we derive the new covariant Feynman rules for chiral superfields. In sect. 5 we discuss the higher-loop divergences. Appendix A contains notational conventions and useful formulae. Appendix B contains an alternative derivation of the covariant Feynman rules for chiral superfields.

2. The background-quantum splitting

We have discussed in [1] and elsewhere [3] the superfield background-quantum splitting, both in supersymmetric Yang-Mills theory and in supergravity. The essential feature of this splitting, in contradistinction to the situation in ordinary field theory (Yang-Mills or gravity) where the splitting is linear, is that because of the correlation to the transformation properties of the gauge fields it involves the exponential of these fields. One has, respectively,

$$e^{V} \rightarrow e^{\bar{\boldsymbol{W}}} e^{V} e^{\boldsymbol{W}},$$

or

$$\exp\left(H^{A} i\boldsymbol{E}_{A}\right) \rightarrow \exp\left(\boldsymbol{W}^{M} iD_{M}\right) \exp\left(H^{A} i\boldsymbol{E}_{A}\right) \exp\left(\bar{\boldsymbol{W}}^{M} iD_{M}\right),$$

where bold letters denote background fields and $\boldsymbol{E}_{A}=\boldsymbol{E}_{A}{}^{M} D_{M}$ in terms of flat superspace derivatives. The splitting is correlated to the transformation properties in such a way that the quantum fields transform tensorially under background gauge transformations.

The main virtue of the background field method is that it leads to the evaluation of an effective action which is gauge invariant. This invariance manifests itself by having the effective action depend on the gauge fields through field strengths and covariant derivatives only. In this respect, a splitting which involves the unconstrained background gauge fields explicitly does not take full advantage of the method. We therefore present here a method, completely equivalent to that of [1], in which the splitting is directly expressed entirely in terms of unconstrained quantum fields and the constrained background covariant derivatives. As we showed in [1], the final expression in the action involves only these quantities. Here we proceed more directly to this final form.

2.1. SUPER-YANG-MILLS

The covariant formulation of the theory introduces connections defined in terms of covariant derivatives

$$\nabla_A = D_A + \Gamma_A \,, \tag{2.1}$$

with transformation laws

$$\nabla_A{}' = e^{iK} \nabla_A e^{-iK} \,, \qquad K = \bar{K} \,. \tag{2.2}$$

The covariant derivatives satisfy constraints which can be solved by expressing them in terms of flat superspace derivatives and the unconstrained gauge field V (in the chiral representation):

$$\begin{aligned} \bar{\nabla}_{\dot\alpha} &= \bar{D}_{\dot\alpha} \,, \qquad \nabla_\alpha = e^{-V} D_\alpha e^{V} \,, \\ \nabla_{\alpha\dot\alpha} &= -i\{\nabla_\alpha, \bar{\nabla}_{\dot\alpha}\} \,. \end{aligned} \tag{2.3}$$

In the background field formalism we have background covariant derivatives $\boldsymbol{\nabla}_A$ and full covariant derivatives ∇_A which satisfy the same constraints. We perform the background-quantum splitting by solving the constraints on the full derivatives, expressing them in terms of the background covariant derivatives and unconstrained quantum gauge superfields as follows:

$$\begin{aligned} \bar{\nabla}_{\dot\alpha} &= \bar{\boldsymbol{\nabla}}_{\dot\alpha} \,, \qquad \nabla_\alpha = e^{-V} \boldsymbol{\nabla}_\alpha e^{V} \,, \\ \nabla_{\alpha\dot\alpha} &= -i\{\nabla_\alpha, \bar{\nabla}_{\dot\alpha}\} \,. \end{aligned} \tag{2.4}$$

(This is the analog of the usual splitting in ordinary Yang-Mills theory, $\nabla_{\alpha\dot\alpha} = \boldsymbol{\nabla}_{\alpha\dot\alpha} + A_{\alpha\dot\alpha}$, where now $A_{\alpha\dot\alpha}$ is the quantum vector potential.)

The full covariant field strength is defined by

$$W_\alpha = \tfrac{1}{2}[\bar{\nabla}^{\dot\alpha}, \{\bar{\nabla}_{\dot\alpha}, \nabla_\alpha\}] \,, \tag{2.5}$$

and the action is given by

$$S_{\mathrm{C}} = -\frac{1}{g^2}\operatorname{tr}\int \mathrm{d}^4x\,\mathrm{d}^2\theta\,\tfrac{1}{2}W^{\alpha}W_{\alpha}. \tag{2.6}$$

If we substitute in it the expression for W_α given in (2.5) and use (2.4), we obtain immediately the action of ref. [3], expressed in terms of the quantum gauge fields and the background covariant derivatives and field strengths.

Fully covariantly chiral superfields φ ($\overline{\nabla}_{\dot\alpha}\varphi = 0$) can be expressed in terms of the quantum gauge fields and background covariantly chiral superfields:

$$\varphi \to \varphi, \qquad \bar{\varphi} \to \mathrm{e}^{-V}\bar{\varphi}, \qquad \overline{\nabla}_{\dot\alpha}\varphi = 0. \tag{2.7}$$

For quantum perturbation calculations one would have to reexpress this φ in terms of an ordinary chiral superfield at the expense of introducing explicit dependence on the unconstrained background gauge fields. As we'll see later on, our new method for handling chiral superfields avoids this.

2.2. SUPERGRAVITY

We recall that in supergravity the covariant derivatives are defined by

$$\nabla_A = E_A{}^M D_M + \left(\Phi_{A\beta}{}^{\gamma} M_{\gamma}{}^{\beta} + \Phi_{A\dot\beta}{}^{\dot\gamma}\overline{M}_{\dot\gamma}{}^{\dot\beta}\right),$$

$$[\nabla_A, \nabla_B\} = T_{AB}{}^C\nabla_C + \left(R_{AB\gamma}{}^{\delta}M_{\delta}{}^{\gamma} + R_{AB\dot\gamma}{}^{\dot\delta}\overline{M}_{\dot\delta}{}^{\dot\gamma}\right). \tag{2.8}$$

They are covariant under the following (vector-representation) transformations:

$$\nabla_A{}' = \mathrm{e}^{iK}\nabla_A \mathrm{e}^{-iK}, \qquad K = \overline{K},$$

$$K = K^M iD_M + \left(K_{\alpha}{}^{\beta} iM_{\beta}{}^{\alpha} + K_{\dot\alpha}{}^{\dot\beta} i\overline{M}_{\dot\beta}{}^{\dot\alpha}\right). \tag{2.9}$$

Here $E_A{}^M$ and $\Phi_{A\beta}{}^{\gamma}(\Phi_{A\dot\beta}{}^{\dot\gamma})$ are the supervierbein and Lorentz superconnection, $T_{AB}{}^C$ and $R_{AB\gamma}{}^{\delta}$ ($R_{AB\dot\gamma}{}^{\dot\delta}$) are the supertorsion and supercurvature, while the action of the Lorentz generators $M_{\alpha}{}^{\beta}$ ($\overline{M}_{\dot\alpha}{}^{\dot\beta}$) is given by

$$\left[X_{\beta}{}^{\gamma}M_{\gamma}{}^{\beta}, \psi_{\alpha}\right] = X_{\alpha}{}^{\beta}\psi_{\beta}, \qquad \left[X_{\beta}{}^{\gamma}M_{\gamma}{}^{\beta}, \bar{\psi}_{\dot\alpha}\right] = 0,$$

$$M_{\alpha}{}^{\alpha} = 0, \qquad \overline{M}_{\dot\alpha}{}^{\dot\beta} = -\left(M_{\alpha}{}^{\beta}\right)^{\dagger}. \tag{2.10}$$

The (modified) Wess-Zumino constraints ($n=-\frac{1}{3}$)

$$
\begin{aligned}
&T_{\alpha\beta}{}^{\gamma}=T_{\alpha\beta}{}^{\dot\gamma}=T_{\alpha\dot\beta}{}^{\dot\gamma}=T_{\alpha\beta}{}^{\gamma\dot\gamma}=T_{\alpha,\beta\dot\beta}{}^{\gamma\dot\gamma}=R_{\alpha\dot\beta\gamma}{}^{\delta}=0\,,\\
&T_{\alpha\dot\beta}{}^{\gamma\dot\gamma}=i\delta_{\alpha}{}^{\gamma}\delta_{\dot\beta}{}^{\dot\gamma}\,,
\end{aligned}
\tag{2.11}
$$

can be solved by expressing the covariant derivatives in terms of the flat superspace derivatives and the axial vector superfield $H^{\alpha\dot\alpha}$ (as well as the compensating chiral superfield φ) [4].

To obtain the background-quantum splitting we proceed just as in the Yang-Mills case. We introduce background covariant derivatives ∇ and full covariant derivatives ∇, both satisfying the Wess-Zumino constraints, and we solve the constraints on the full derivatives by expressing them in terms of the background covariant derivatives and unconstrained quantum gauge fields. This procedure exactly parallels the solution of the constraints in ordinary superfield supergravity [4]. In a quantum-chiral representation one finds (note the appearance of the background super-vierbein determinant in the formula for ψ)

$$
\begin{aligned}
&\overline{\nabla}_{\dot\alpha}=\bar\psi\left(\overline{\nabla}_{\dot\alpha}+\bar\omega_{\dot\alpha\beta}{}^{\gamma}M_{\gamma}{}^{\beta}+\bar\omega_{\dot\alpha\dot\beta}{}^{\dot\gamma}\overline{M}_{\dot\gamma}{}^{\dot\beta}\right),\\
&\nabla_{\alpha}=\mathrm{e}^{-H}\psi\left(\nabla_{\alpha}+\omega_{\alpha\beta}{}^{\gamma}M_{\gamma}{}^{\beta}+\omega_{\alpha\dot\beta}{}^{\dot\gamma}\overline{M}_{\dot\gamma}{}^{\dot\beta}\right)\mathrm{e}^{H},\\
&\nabla_{\alpha\dot\alpha}=-i\left\{\nabla_{\alpha},\overline{\nabla}_{\dot\alpha}\right\},\\
&\bar\psi=\varphi^{-1}(\mathrm{e}^{-H}\bar\varphi)^{1/2}(1\cdot\mathrm{e}^{-\overleftarrow{H}})^{1/6}\hat{E}^{-1/6}\boldsymbol{E}^{1/6}(\mathrm{e}^{-H}\boldsymbol{E})^{-1/6}\equiv\psi^{\dagger},\\
&\bar\omega_{\dot\alpha\dot\beta}{}^{\dot\gamma}=-\delta_{\dot\alpha}{}^{(\dot\gamma}\overline{\nabla}_{\dot\beta)}\ln\bar\psi\equiv-\left(\omega_{\alpha\beta}{}^{\gamma}\right)^{\dagger},\\
&\bar\omega_{\dot\alpha\beta}{}^{\gamma}=-\tfrac{1}{2}\hat{T}_{\dot\alpha,(\beta\dot\delta}{}^{\gamma)\dot\delta}\equiv-\left(\omega_{\alpha\dot\beta}{}^{\dot\gamma}\right)^{\dagger};
\end{aligned}
\tag{2.12}
$$

where $H=H^{A}i\nabla_{A}$, and the hatted objects are defined as in (2.8) but from $\hat\nabla_{A}$:

$$
\begin{aligned}
&\hat{\overline{\nabla}}_{\dot\alpha}=\overline{\nabla}_{\dot\alpha},\qquad \hat\nabla_{\alpha}=\mathrm{e}^{-H}\nabla_{\alpha}\mathrm{e}^{H},\qquad \hat\nabla_{\alpha\dot\alpha}=-i\left\{\hat\nabla_{\alpha},\hat{\overline{\nabla}}_{\dot\alpha}\right\},\\
&\hat\nabla_{A}=\hat{E}_{A}{}^{B}\nabla_{B}+\left(\hat\Phi_{A\beta}{}^{\gamma}M_{\gamma}{}^{\beta}+\hat\Phi_{A\dot\beta}{}^{\dot\gamma}\overline{M}_{\dot\gamma}{}^{\dot\beta}\right)=-(-1)^{A}\mathrm{e}^{-H}\overline{(\hat\nabla_{A})}\mathrm{e}^{H}.
\end{aligned}
\tag{2.13}
$$

We have also defined the usual superdeterminants, with their appropriate hermiticity properties (along with $\bar\varphi\equiv\varphi^{\dagger}$):

$$
\begin{aligned}
&E=\det E_{A}{}^{M}, &&\boldsymbol{E}=\det\boldsymbol{E}_{A}{}^{M}, &&\hat{E}=\det\hat{E}_{A}{}^{B},\\
&E^{-1}=(E^{-1})^{\dagger}\mathrm{e}^{-\overleftarrow{H}}, &&\boldsymbol{E}^{-1}=(\boldsymbol{E}^{-1})^{\dagger}, &&\hat{E}^{-1}=(\hat{E}^{-1})^{\dagger}\mathrm{e}^{-\overleftarrow{H}}.
\end{aligned}
\tag{2.14}
$$

In these formulae φ is background covariantly chiral: $\overline{\nabla}_{\dot{\alpha}}\varphi = \nabla_\alpha \bar{\varphi} = 0$. For quantum calculations it would seem necessary to express it in terms of an ordinary chiral superfield, thereby producing explicit background gauge field dependence. Our later modification of the Feynman rules will allow us to avoid this unsatisfactory feature.

The splitting of the supergravity action is simply

$$S_{\mathrm{C}} = -\frac{3}{\kappa^2}\int \mathrm{d}^4x\,\mathrm{d}^4\theta\, E^{-1},$$

$$E^{-1} = \boldsymbol{E}^{-1}\hat{E}^{-1/3}\left(1\cdot \mathrm{e}^{-\overleftarrow{H}}\right)^{1/3}\varphi \mathrm{e}^{-H}\bar{\varphi}, \tag{2.15}$$

which is obtained again by following the steps in ordinary superfield supergravity, ultimately expressing the full supervierbein determinant in terms of background covariants and the unconstrained quantum fields. To quadratic order in the quantum fields $H^{\alpha\dot{\alpha}}$ (in the gauge $H^\alpha = 0$) and $\chi(=\varphi - 1)$ we have

$$\begin{aligned}
\boldsymbol{E}E^{-1} = 1 &+ \left\{\chi + \bar{\chi} + \tfrac{1}{3}H^{\alpha\dot{\alpha}}\boldsymbol{G}_{\alpha\dot{\alpha}}\right\} \\
&+ \Big\{\chi\bar{\chi} + \tfrac{1}{3}i(\bar{\chi}-\chi)\nabla_{\alpha\dot{\alpha}}H^{\alpha\dot{\alpha}} + \tfrac{1}{6}H^{\alpha\dot{\alpha}}\Box H_{\alpha\dot{\alpha}} + \tfrac{1}{12}\left(\nabla_{\alpha\dot{\alpha}}H^{\alpha\dot{\alpha}}\right)^2 \\
&\qquad + \tfrac{1}{36}\left(\left[\overline{\nabla}_{\dot{\alpha}}, \nabla_\alpha\right]H^{\alpha\dot{\alpha}}\right)^2 - \tfrac{1}{3}\left[\left(\overline{\nabla}^2 + \tfrac{3}{2}\boldsymbol{R}\right)H^{\alpha\dot{\alpha}}\right]\left[\left(\nabla^2 + \tfrac{3}{2}\bar{\boldsymbol{R}}\right)H_{\alpha\dot{\alpha}}\right]\Big\} \\
&+ \Big\{\tfrac{1}{3}(\chi + \bar{\chi})H^{\alpha\dot{\alpha}}\boldsymbol{G}_{\alpha\dot{\alpha}} + \tfrac{1}{18}\left(H^{\alpha\dot{\alpha}}\boldsymbol{G}_{\alpha\dot{\alpha}}\right)^2 + \tfrac{2}{3}\boldsymbol{R}\bar{\boldsymbol{R}}H^{\alpha\dot{\alpha}}H_{\alpha\dot{\alpha}} \\
&\qquad + \tfrac{1}{4}\left(\nabla^2\boldsymbol{R} + \overline{\nabla}^2\bar{\boldsymbol{R}}\right)H^{\alpha\dot{\alpha}}H_{\alpha\dot{\alpha}} - \tfrac{1}{12}H^{\alpha\dot{\alpha}}\boldsymbol{G}^{\beta\dot{\beta}}\left[\overline{\nabla}_{\dot{\beta}}, \nabla_\beta\right]H_{\alpha\dot{\alpha}} \\
&\qquad + \tfrac{1}{12}H^{\alpha\dot{\alpha}}\left[\left(\nabla_{(\alpha}\boldsymbol{G}_{\beta)}{}^{\dot{\beta}}\right)\overline{\nabla}_{\dot{\beta}}H^{\beta}{}_{\dot{\alpha}} - \left(\overline{\nabla}_{(\dot{\alpha}}\boldsymbol{G}^{\beta}{}_{\dot{\beta})}\right)\nabla_\beta H_\alpha{}^{\dot{\beta}}\right] \\
&\qquad - \tfrac{1}{18}\left(H^{\alpha\dot{\alpha}}\boldsymbol{G}_{\alpha\dot{\alpha}}\right)\left[\overline{\nabla}_{\dot{\beta}}, \nabla_\beta\right]H^{\beta\dot{\beta}} + \tfrac{1}{6}H^{\alpha\dot{\alpha}}\left(\boldsymbol{W}_\alpha{}^{\beta\gamma}\nabla_\beta H_{\gamma\dot{\alpha}} + \bar{\boldsymbol{W}}_{\dot{\alpha}}{}^{\dot{\beta}\dot{\gamma}}\overline{\nabla}_{\dot{\beta}}H_{\alpha\dot{\gamma}}\right)\Big\}.
\end{aligned} \tag{2.16}$$

The first set of braces gives the terms linear in the quantum fields, cancelled by the source terms. ($\delta S_{\mathrm{C}}/\delta H^{\alpha\dot{\alpha}} = -\kappa^{-2}\boldsymbol{G}_{\alpha\dot{\alpha}}$, $\delta S_{\mathrm{C}}/\delta\varphi^3 = -\kappa^{-2}R$.) The second set of braces contains the covariantization of the flat space expression. $\boldsymbol{R}, \boldsymbol{G}_{\alpha\dot{\alpha}}, \boldsymbol{W}_{\alpha\beta\gamma}$ are the background field strengths and $\Box = \frac{1}{2}\nabla^{\alpha\dot{\alpha}}\nabla_{\alpha\dot{\alpha}}$.

The background field formalism has the following set of transformations which leave the action invariant:

Background transformations:

$$\nabla_{A'} = \mathrm{e}^{iK}\nabla_A \mathrm{e}^{-iK}, \qquad H' = \mathrm{e}^{iK}H\mathrm{e}^{-iK}, \qquad \varphi' = \mathrm{e}^{iK}\varphi,$$

$$K = \bar{K} = K^A i\nabla_A + \left(K_\alpha{}^\beta iM_\beta{}^\alpha + K_{\dot{\alpha}}{}^{\dot{\beta}} i\bar{M}_{\dot{\beta}}{}^{\dot{\alpha}}\right). \tag{2.17}$$

Quantum transformations:

$$\nabla_{A'} = \nabla_A\,, \qquad e^{H'} = \langle e^{i\bar{\Lambda}} e^{H} e^{-i\Lambda} \rangle\,,$$

$$\varphi' = \exp\left[i\Lambda - \tfrac{1}{3}\left(\nabla_{\alpha\dot{\alpha}} \Lambda^{\alpha\dot{\alpha}} - \nabla_\alpha \Lambda^\alpha - i G_{\alpha\dot{\alpha}} \Lambda^{\alpha\dot{\alpha}} \right) \right] \varphi\,,$$

$$\Lambda = \Lambda^A i \nabla_A \neq \bar{\Lambda}\,. \tag{2.18}$$

Here the operation $\langle\ \rangle$ is defined by:

$$\text{for any } A = A^A \nabla_A + \left(A_\alpha{}^\beta M_\beta{}^\alpha + A_{\dot{\alpha}}{}^{\dot{\beta}} \bar{M}_{\dot{\beta}}{}^{\dot{\alpha}} \right),$$

$$\langle A \rangle \equiv A^A \nabla_A\,, \qquad \langle e^A \rangle \equiv e^{\langle A \rangle}\,. \tag{2.19}$$

Due to the fact that the quantum transformations preserve the chirality of φ ($\bar{\nabla}_{\dot{\alpha}}\varphi = 0 \rightarrow [\bar{\nabla}_{\dot{\alpha}}, \Lambda]\varphi = 0$), Λ takes the following form in terms of the unconstrained supergravitational gauge parameter L^α:

$$\Lambda_{\alpha\dot{\alpha}} = -i \bar{\nabla}_{\dot{\alpha}} \varphi^{-3} L_\alpha\,, \qquad \Lambda_\alpha = \bar{\nabla}^2 \varphi^{-3} L_\alpha\,. \tag{2.20}$$

Furthermore, we choose the (quantum) $\Lambda_{\dot{\alpha}}$ gauge $H^\alpha = 0$ $(= H^{\dot{\alpha}})$, which determines $\Lambda_{\dot{\alpha}}$ in terms of L_α:

$$\Lambda_{\dot{\alpha}} = e^{-H} \nabla^2 \bar{\varphi}^{-3} \bar{L}_{\dot{\alpha}}\,. \tag{2.21}$$

3. Quantization and Feynman rules

The quantization procedure for the Yang-Mills case was discussed in ref. [3] and will not be repeated here. It leads to the quantum action

$$S = S_C - \int d^4x\, d^4\theta (\bar{\nabla}^2 V)(\nabla^2 V)$$

$$+ \int d^4x\, d^4\theta \left\{ (c' + \bar{c}') L_{V/2} \left[(c + \bar{c}) + (\coth L_{V/2})(c - \bar{c}) \right] + \bar{c}_3 c_3 \right\},$$

$$L_X Y \equiv [X, Y]\,, \tag{3.1}$$

where c, c', c_3 are background covariantly chiral Faddeev-Popov and Nielsen-Kallosh ghosts. If in addition fully covariant chiral matter superfields are present, they are to be expressed in terms of background chiral superfields η and have an action

$$S = \int d^4x\, d^4\theta\, \bar{\eta} e^V \eta\,. \tag{3.2}$$

We review now the quantization procedure for supergravity. Given the action of the previous section, we must fix the gauge for quantum transformations in a background covariant manner. We first of all write the quantum transformations of H in infinitesimal form:

$$\delta H = \langle - iL_{H/2}\left[(\Lambda + \bar{\Lambda}) + (\coth L_{H/2})(\Lambda - \bar{\Lambda})\right]\rangle . \tag{3.3}$$

In terms of the unconstrained spinor parameter L_α we have

$$\delta H_{\alpha\dot{\alpha}} = \nabla_\alpha \bar{L}_{\dot{\alpha}} - \bar{\nabla}_{\dot{\alpha}} L_\alpha + \text{higher-order terms},$$

$$\delta\varphi^3 = (\bar{\nabla}^2 + \boldsymbol{R})\nabla_\alpha L^\alpha . \tag{3.4}$$

For simplicity we give here the quantization procedure with the background fields on shell ($\boldsymbol{R} = \boldsymbol{G}_{\alpha\dot{\alpha}} = 0$). The gauge-fixing function is

$$F_\alpha = \bar{\nabla}^{\dot{\alpha}}\left(H_{\alpha\dot{\alpha}} + \tfrac{1}{3} i \nabla_{\alpha\dot{\alpha}} \Box^{-1} \bar{\varphi}^3\right). \tag{3.5}$$

This introduces Faddeev-Popov ghosts ψ_α, ψ'_α, occurring in the action as $\psi'^\alpha \delta F_\alpha(\psi_\beta)$ + h.c., where δF_α is the variation of the gauge-fixing function under the gauge transformations (3.4) with gauge parameter L_α replaced by ψ_α. It is important to observe that while both terms in (3.5) lead to interactions of the ghosts with the background fields, only the first term $\bar{\nabla}^{\dot{\alpha}} H_{\alpha\dot{\alpha}}$ leads to (local) interactions between the ghosts and the quantum fields and hence will contribute to higher loops. The second term gives non-local terms quadratic in the ghosts but independent of the quantum fields, and therefore contributes only at the one-loop level. The non-locality can be eliminated by the shifts

$$\psi_\alpha \to \psi_\alpha + \nabla_\alpha (V_1 + iV_2), \qquad \psi'_\alpha \to \psi'_\alpha + \nabla_\alpha (V'_1 + iV'_2), \tag{3.6}$$

which introduce the catalyst ghosts V_i, V'_i [1].

All the fields introduced so far contribute at all loops. The original quantum fields H, χ have self-interactions obtained by expanding the action in (2.15) to higher order, and interactions with the ghosts ψ, ψ', V_i, V'_i from the non-linear terms in the variation of $\bar{\nabla}^{\dot{\alpha}} H_{\alpha\dot{\alpha}}$. In addition there are a number of ghost fields which contribute only at the one-loop level. (They enter quadratically, and interact only with the background fields.) These are first of all the Nielsen-Kallosh ghost $\psi_{3\alpha}$ introduced by the usual averaging procedure in the presence of background fields, and then a variety of ghosts associated with the gauge transformations of the ghosts introduced so far. This has been discussed in [1], and we will only quote the result for the quadratic action (in units $\kappa = 1$ and, for simplicity, with the background fields on-shell):

$$S=\int d^4x\, d^4\theta\, E^{-1}\Big[-\tfrac{1}{2}H^{\alpha\dot\alpha}\hat\Box H_{\alpha\dot\alpha}-\tfrac{9}{5}\chi\bar\chi$$

$$+\left(\bar\psi'^{\dot\alpha}i\nabla^{\alpha}{}_{\dot\alpha}\psi_\alpha+\psi'^{\alpha}i\nabla_{\alpha}{}^{\dot\alpha}\bar\psi_{\dot\alpha}+\bar\psi_3{}^{\dot\alpha}i\nabla^{\alpha}{}_{\dot\alpha}\psi_{3\alpha}\right)+\left(3V'_1\Box V_1+V'_2\Box V_2\right)$$

$$+\sum_{i=1}^{4}\tfrac{1}{2}V''_i\Box V''_i+\sum_{i=1}^{2}\left(\tfrac{1}{2}\varphi_i{}^{\alpha}\nabla^2\varphi_{i\alpha}+\text{h.c.}\right)\Big], \tag{3.7}$$

$$\hat\Box=\Box+W^{\alpha}{}_{\beta}{}^{\gamma}\nabla_\alpha M_\gamma{}^\beta+\bar W^{\dot\alpha}{}_{\dot\beta}{}^{\dot\gamma}\bar\nabla_{\dot\alpha}\bar M_{\dot\gamma}{}^{\dot\beta}. \tag{3.8}$$

This action is sufficient for one-loop calculations with the background fields on shell. For higher-loop calculations we need terms of higher order in the quantum fields. These quantum interaction terms are given by *the higher-order expansion of*

$$S_C-\left\{\nabla^\alpha\left[\bar\psi'^{\dot\alpha}+\bar\nabla^{\dot\alpha}(V'_1-iV'_2)\right]-\bar\nabla^{\dot\alpha}\left[\psi'^{\alpha}+\nabla^\alpha(V'_1+iV'_2)\right]\right\}$$

$$\times\delta H_{\alpha\dot\alpha}\left[\psi_\beta+\nabla_\beta(V_1+iV_2)\right], \tag{3.9}$$

where $\delta H_{\alpha\dot\alpha}[L_\beta]$ is the expression obtained by substituting (2.20) and (2.21) into (3.3). In these formulae ψ^α, ψ'^α, $\psi_3{}^\alpha$, V_i, and V'_i have abnormal statistics. χ and $\varphi_{i\alpha}$ are the only chiral superfields. In addition, at the one-loop level, because of opposite statistics, the contribution of V''_i cancels that of V_i and V'_i. (In [1] ψ^α, ψ'^α, V_1+iV_2, $V'_1+iV'_2$, and V''_i were called $\psi_1{}^\alpha$, $\psi_2{}^\alpha$, ψ_1, ψ_2, and ψ'_i, respectively, up to numerical factors.)

As discussed in [1], an equivalent formulation (up to topological considerations) is given by the replacement (even off shell)

$$\varphi^3\to 1+(\bar\nabla^2+\boldsymbol{R})V, \qquad V=\bar V. \tag{3.10}$$

V has the following transformation laws:

Background transformation:

$$V'=e^{iK}V,$$

Quantum transformation:

$$V'=V+\left(\nabla_\alpha L^\alpha+\bar\nabla_{\dot\alpha}\bar L^{\dot\alpha}\right). \tag{3.11}$$

The gauge-fixing function is now

$$F_\alpha = \overline{\nabla}^{\dot\alpha} H_{\alpha\dot\alpha} - \tfrac{1}{5}\nabla_\alpha V. \tag{3.12}$$

The shifts (3.6) are again made, and the terms in the action of higher than quadratic order are the same as before, with the substitution (3.10) into (3.9), plus the higher-order terms resulting from making this nonlinear substitution in the quadratic term $-\frac{9}{5}\chi\bar\chi$ in (3.7). However, the quadratic terms are now

$$\begin{aligned} S = \int \mathrm{d}^4x\,\mathrm{d}^4\theta\, E^{-1} \Bigg[& -\tfrac{1}{2} H^{\alpha\dot\alpha} \hat\Box H_{\alpha\dot\alpha} - \tfrac{1}{10} V \Box V \\ & + \left(\bar\psi'^{\dot\alpha} i \nabla^\alpha{}_{\dot\alpha} \psi_\alpha + \psi'^\alpha i \nabla_\alpha{}^{\dot\alpha} \bar\psi_{\dot\alpha} + \bar\psi_3{}^{\dot\alpha} i \nabla^\alpha{}_{\dot\alpha} \psi_{3\alpha} \right) \\ & + \left(3V'_1 \Box V_1 + V'_2 \Box V_2\right) + \sum_{i=1}^{7} \tfrac{1}{2} V''_i \Box V''_i + \sum_{i=1}^{7} \chi_i \bar\chi_i \Bigg]. \end{aligned} \tag{3.13}$$

Here ψ^α, ψ'^α, ψ_3, V_i, V'_i, and χ_i have abnormal statistics. Also, the χ_i are chiral.

When performing one-loop calculations, we start with either (3.7) or (3.13). For convenience, we square the kinetic operator of ψ^α, ψ'^α, and $\psi_3{}^\alpha$, taking half the resulting contribution to the effective action. The necessary relation is:

$$\begin{aligned} \left(i\nabla_\alpha{}^{\dot\gamma}\right)\left(i\nabla^\beta{}_{\dot\gamma}\right) &= \tfrac{1}{2}\delta_\alpha{}^\beta \left(i\nabla_\gamma{}^{\dot\gamma}\right)\left(i\nabla^\gamma{}_{\dot\gamma}\right) + \tfrac{1}{2}\left[i\nabla_\alpha{}^{\dot\gamma}, i\nabla^\beta{}_{\dot\gamma}\right] \\ &= \delta_\alpha{}^\beta \Box + \left\{\nabla_\alpha, W^\beta{}_\gamma{}^\delta M_\delta{}^\gamma\right\}. \end{aligned} \tag{3.14}$$

[We have used (A.4) and (A.5) on shell.] The action for the ψ_α's thus becomes

$$\begin{aligned} & \int \mathrm{d}^4x\,\mathrm{d}^4\theta E^{-1} \sum_{i=1}^{3} \tfrac{1}{2}\psi_i{}^\alpha \left(\delta_\alpha{}^\beta \Box + \left\{\nabla^\alpha, W^\beta{}_\gamma{}^\delta M_\delta{}^\gamma\right\}\right)\psi_{i\beta} + \text{h.c.} \\ & = \int \mathrm{d}^4x\,\mathrm{d}^4\theta E^{-1} \sum \tfrac{1}{2}\psi_i{}^\alpha \hat\Box \psi_{i\alpha} + \text{h.c.}, \end{aligned} \tag{3.15}$$

with $\hat\Box$ as in (3.8).

For higher-loop calculations we return to the forms in (3.7) or (3.13) for the quadratic action, with higher-order terms obtained from the expansion of (3.9). To actually perform calculations we must express the background covariant derivatives in terms of flat superspace ones:

$$\nabla_A = E_A{}^M D_M + \cdots. \tag{3.16}$$

To read off propagators from (3.7) or (3.13) we must also split off the free part:

$$E_A{}^M = \delta_A{}^M + \Delta_A{}^M . \tag{3.17}$$

In one-loop calculations vertices involve interactions with only the background fields, which can be read off from the quadratic action after the splitting above. For higher-loop calculations vertices are obtained from the non-quadratic action as well and involve in general both quantum and background fields. For non-chiral superfields the procedure is straightforward. In particular, we observe that even after the splitting off of the flat superspace parts the background gauge fields do not appear explicitly but only through field strengths, connection terms, and the background supervierbein. However, for chiral superfields there are two complications: First, the standard Feynman rules [3] require factors of D^2, $\bar{D}^2$, acting on lines at each vertex, whose manipulation may lead to algebraically complicated expressions. Second, and more important, because the fields are background covariantly chiral, as mentioned earlier the splitting of a free part requires writing, for example, $\varphi = e^{\bar{W}}\hat{\varphi}$, and this introduces explicitly in the action the unconstrained background gauge fields. In the next section we shall discuss a method for getting around this problem. The resulting Feynman rules, especially for one-loop calculations, will be simpler and involve the background fields only through connections and supervierbeins.

4. Covariant rules for chiral superfields

In this section we shall consider for simplicity the case of scalar chiral superfields. (Generalization to arbitrary Lorentz representations is trivial.) They are assumed to be covariantly chiral with respect to the full gauge fields: $\bar{\nabla}_{\dot{\alpha}}\eta = 0$. Such superfields would appear, for example, if we considered in the background field method some matter multiplet described by η. After the background-quantum splitting it would interact with both the background and the quantum gauge fields in a manner which is determined by the above chirality condition. On the other hand, if the chiral field happens to be a supergravity field φ or χ_i, it would be just background chiral. For such a case, in all the formulae below we would replace the full derivatives, curvatures, supervierbeins, etc., with just the background ones. We can keep the background fields off shell.

We begin by defining covariant functional differentiation of quantum superfields. For a general superfield Ξ and a fully covariantly chiral superfield η we define

$$\delta\Xi(z)/\delta\Xi(z') \equiv E\delta^8(z-z') ,$$

$$\delta\eta(z)/\delta\eta(z') \equiv (\bar{\nabla}^2 + R)E\delta^8(z-z') = \varphi^{-3}\bar{D}^2\delta^8(z-z') . \tag{4.1}$$

(In the Yang-Mills case we set $E=\varphi=1$ and $R=0$.) If Ξ or η is a supergravity field (physical or ghost), one would replace $E, \overline{\nabla}^2, R$, by just the background $\boldsymbol{E}, \overline{\nabla}^2, \boldsymbol{R}$.

These forms are obtained by introducing in the definition of the functional differentiation suitable factors multiplying the variations of the fields. The second form of $\delta\eta/\delta\eta$ is obtained by using a particular chiral representation defined by

$$\Gamma_{\dot\alpha}=0\,, \qquad E_{\dot\alpha}{}^{\mu}=E_{\dot\alpha}{}^{\mu\dot\mu}=\Phi_{\dot\alpha\beta}{}^{\gamma}=0\,, \tag{4.2}$$

as well as the identity

$$(\overline{\nabla}^2+R)\boldsymbol{f}=\varphi^{-3}\bar{D}^2E^{-1}\boldsymbol{f}. \tag{4.3}$$

In this representation the covariantly chiral superfields are chiral in the usual sense: $\overline{\nabla}_{\dot\alpha}\eta_{\beta\cdots\gamma}=0$ implies $\bar{D}_{\dot\alpha}\eta_{\beta\cdots\gamma}=0$. (Chiral superfields can't have dotted indices, since $R_{\dot\alpha\beta\dot\gamma}{}^{\delta}\sim R\neq 0$.) It can be shown that this representation exists by actually solving the constraints in terms of the unconstrained gauge fields [4]. However, since we wish to avoid talking explicitly about these gauge fields, it is better to define the chiral representation as in (4.2), directly in terms of objects which appear in ∇_A. We emphasize again that in the above expressions the covariant derivatives and all other quantities are full, except when Ξ or η are the (physical or ghost) Yang-Mills or supergravity fields themselves. (For one-loop calculations the distinction is irrelevant.)

Manifestly covariant supergraph rules for chiral superfields can now be found by a direct covariantization of the usual method. In what follows we shall describe the situation in supergravity, but the Yang-Mills case can immediately be obtained by dropping factors of E, φ, R, etc. We observe first that the covariantization of the identity $\bar{D}^2D^2\eta=\Box\eta$ becomes

$$(\overline{\nabla}^2+R)(\nabla^2+\bar{R})\eta_{\alpha\cdots\beta}=\Box_+\eta_{\alpha\cdots\beta}\,,$$

$$\Box_+=\Box+W^{\alpha}{}_{\beta}{}^{\gamma}\nabla_{\alpha}M_{\gamma}{}^{\beta}-\tfrac{1}{2}i\left(\nabla^{\gamma\dot\alpha}G_{\beta\dot\alpha}\right)M_{\gamma}{}^{\beta}-\tfrac{1}{2}iG^{\alpha\dot\alpha}\nabla_{\alpha\dot\alpha}$$

$$-R\nabla^2-\tfrac{1}{2}(\nabla^{\alpha}R)\nabla_{\alpha}+R\bar{R}+\overline{\nabla}^2R \tag{4.4}$$

in the supergravity case, or

$$\Box_+=\Box+W^{\alpha}\nabla_{\alpha}-\tfrac{1}{2}(\nabla_{\alpha}W^{\alpha}) \tag{4.5}$$

in the Yang-Mills case, with the fully covariant $\Box=\frac{1}{2}\nabla^{\alpha\dot\alpha}\nabla_{\alpha\dot\alpha}$. Note that for one-loop, on-shell calculations only the first two terms in $\Box_+$ need be kept. Furthermore, comparing the on-shell form of $\Box_+$ in (4.4) with $\hat\Box$ of (3.8) (and noting that chiral superfields have only undotted indices), we find $\Box_+=\hat\Box$. Thus, we see that $\square$ universally appears as $\hat{\square}$ in our one-loop, on-shell calculations, a result which can be very useful [5].

We start with the action

$$S = S_0 + S_{\mathrm{INT}}(\eta, \bar{\eta}), \qquad S_0 = \int \mathrm{d}^4x\, \mathrm{d}^4\theta\, E^{-1}\bar{\eta}\eta \tag{4.6}$$

(full E^{-1} unless η is a supergravity field), where we have indicated explicitly the dependence on the particular chiral field η. S_{INT} also contains the other quantum fields. We concentrate on the functional integral over η which gives, using (4.1),

$$Z(J, \bar{J}) = \int D\eta D\bar{\eta} \exp\left[S + \left(\int \mathrm{d}^4x\, \mathrm{d}^2\theta\, \varphi^3 J\eta + \mathrm{h.c.}\right)\right]$$

$$= \Delta \cdot \left[\exp S_{\mathrm{INT}}(\delta/\delta J, \delta/\delta \bar{J})\right]\left[\exp\left(-\int \mathrm{d}^4x\, \mathrm{d}^4\theta\, E^{-1}\bar{J}\square_+{}^{-1}J\right)\right], \tag{4.7}$$

where Δ is the functional determinant

$$\Delta = \int D\eta D\bar{\eta}\, \mathrm{e}^{S_0}. \tag{4.8}$$

In general the above expression for Z depends on, and is to be integrated over, the remaining quantum fields.

We shall devote the remainder of this section mainly to the evaluation of the factor Δ, but let us first discuss the other factors (which enter only in higher-loop calculations of contributions from η). From S_{INT} and the definition of the covariant functional derivative in (4.1) we obtain vertices which have factors of $(\bar{\nabla}^2 + R)E$ or $(\nabla^2 + \bar{R})E$ for each chiral or antichiral line leaving the vertex. These factors act on the full propagator $\square_+{}^{-1}$, which in general must be expanded further. First, we separate the full covariant derivatives in terms of background covariant derivatives and quantum gauge fields. (If η is φ itself, $\square_+{}^{-1}$ contains only the background covariant derivatives and this step is unnecessary.) Second, we separate the background covariant derivatives in terms of a flat superspace part and background connections, field strengths, etc., and eventually expand $\square_+{}^{-1}$ in terms of the flat space propagator $1/p^2$ and additional vertices involving both quantum and background interactions. The procedure is straightforward although the algebra may become involved for complicated graphs. We will work this out explicitly when we perform higher-loop calculations.

We turn now to the evaluation of Δ. It gives the complete one-loop contribution of the chiral superfield and could be evaluated by using the standard superfield Feynman rules, but this we wish to avoid. There are many ways to evaluate it, but perhaps the most transparent way is to use the "doubling" trick [1] (analogous in

QED to evaluating det $\nabla\!\!\!/$ in terms of det($\square + \cdots$) for the spinor). The action S_0 leads to the equations of motion (in the presence of sources)

$$\boldsymbol{O}\begin{pmatrix} \eta \\ \bar{\eta} \end{pmatrix} + \begin{pmatrix} J \\ \bar{J} \end{pmatrix} = 0, \qquad \boldsymbol{O} = \begin{pmatrix} 0 & \bar{\nabla}^2 + R \\ \nabla^2 + \bar{R} & 0 \end{pmatrix}, \tag{4.9}$$

which define the operator $\boldsymbol{O}$. We define an action S'_0 whose equations of motion are

$$\boldsymbol{O}^2\begin{pmatrix} \eta \\ \bar{\eta} \end{pmatrix} + \begin{pmatrix} J \\ \bar{J} \end{pmatrix} = 0, \qquad \boldsymbol{O}^2 = \begin{pmatrix} (\bar{\nabla}^2 + R)(\nabla^2 + \bar{R}) & 0 \\ 0 & (\nabla^2 + \bar{R})(\bar{\nabla}^2 + R) \end{pmatrix} \tag{4.10}$$

in terms of the square of $\boldsymbol{O}$. This action is given by

$$S'_0 = \int d^4x\, d^2\theta\, \varphi^3 \eta \square_+ \eta = \int d^4x\, d^4\theta\, E^{-1} \eta(\nabla^2 + \bar{R})\eta, \tag{4.11}$$

and in terms of it we can write the functional integral

$$\Delta^2 = \int D\eta D\bar{\eta} \exp[S'_0(\eta) + \text{h.c.}] = \left| \int D\eta e^{S'_0} \right|^2. \tag{4.12}$$

We have used the fact that S'_0 and its hermitian conjugate contribute equally to Δ, as can be seen, for example, by examining the resulting Feynman rules below. [In QED the analogue of $\boldsymbol{O}$ is $\nabla\!\!\!/$, and of $\square_+$, $\square + F^{\alpha\beta}M_{\alpha\beta}$, with $F^{\alpha\beta}$ the electromagnetic field strength; cf. (4.4.).]

We now integrate S'_0 in (4.12) by separating out $\varphi^3 D^2$ from $E^{-1}(\nabla^2 + \bar{R})$, treating $\eta[E^{-1}(\nabla^2 + \bar{R}) - \varphi^3 D^2]\eta$ as an interaction term. The result is (with $\square_0$ the empty-space d'Alembertian)

$$\begin{aligned} \Delta &= \int D\eta e^{S'_0} \\ &= \left\{ \exp \int d^4x\, d^2\theta\, \varphi^3 \frac{\delta}{\delta J} [(\bar{\nabla}^2 + R)(\nabla^2 + \bar{R}) - \bar{D}^2 D^2] \frac{\delta}{\delta J} \right\} \\ &\quad \times \left[\exp - \int d^4x\, d^2\theta\, \varphi^3 J \square_0{}^{-1} J \right] \Bigg|_{J=0}. \end{aligned} \tag{4.13}$$

(Note that writing instead $(\bar{\nabla}^2 + R)(\nabla^2 + \bar{R}) \to \bar{D}^2 E^{-1} e^{-H} D^2$ would give the rules for the one-loop expression in the usual non-covariant formalism.) Therefore, a calculation of the one-loop contribution of η to the effective action (i.e., $\ln \Delta$)

consists in evaluating graphs with propagators $p^{-2}\delta^4(\theta-\theta')$ and vertices $[(\bar{\nabla}^2\cdots]$, giving rise to a string

$$\cdots\left[(\bar{\nabla}^2+R)(\nabla^2+\bar{R})-\bar{D}^2D^2\right]_i\delta^4(\theta_i-\theta_{i+1})$$
$$\times\left[(\bar{\nabla}^2+R)(\nabla^2+\bar{R})-\bar{D}^2D^2\right]_{i+1}\cdots, \tag{4.14}$$

with $\int d^4\theta_i$ integrals at each vertex and one loop-momentum integral. We concentrate on a given vertex and at the next vertex we rewrite $(\bar{\nabla}^2+R)=\bar{D}^2\varphi^{-3}E^{-1}$ (see (4.3) and the remark which follows it). We then temporarily transfer the $\bar{D}^2$ factor across the delta-function. We can now use the identity $[(\bar{\nabla}^2+R)(\nabla^2+R)-\bar{D}^2D^2]\bar{D}^2=(\Box_+-\Box_0)\bar{D}^2$ [see (4.4)]. We can further simplify this expression as follows: In $\Box_+$ we use the anticommutation relations to move each $\bar{D}_{\dot\alpha}$ over to the right until it is annihilated by the $\bar{D}^2$. The resulting equivalent expression, which we call $\hat{\Box}_+$, contains no $\bar{D}$'s. Having performed this maneuver we return the $\bar{D}^2$ to its original place, reexpress that vertex in its original form, and proceed to manipulate it the same way. Clearly this procedure can be carried out at all vertices in the loop but one, which retains its original form. The resulting rules for the one-loop vertices are thus (with propagators $p^{-2}\delta^4(\theta-\theta')$):

For Yang-Mills:

$$\text{one vertex:}\quad \bar{D}^2(\nabla^2-D^2),$$

$$\text{other vertices: } \Box_+-\Box_0 .$$

For supergravity:

$$\text{one vertex: } \bar{D}^2[\varphi^{-3}E^{-1}(\nabla^2+\bar{R})-D^2],$$

$$\text{other vertices: } \hat{\Box}_+-\Box_0 . \tag{4.15}$$

Now only one vertex in the loop contributes any $\bar{D}$'s. As a consequence, the explicit evaluation of one-loop contributions which with the previous rules required significant manipulation of D's and $\bar{D}$'s is now simplified. We have one $\bar{D}^2$ in the loop and this means that all D's but two must be integrated by parts outside the loop. Furthermore, it is clear now that the one-loop contribution of the chiral field η is completely expressible in terms of objects which appear in the covariant derivatives, with no explicit unconstrained gauge fields appearing anywhere. The higher-loop contributions obtained, as discussed earlier, from (4.7) also have this property.

An interesting independent way of understanding the introduction of the $(\bar{\nabla}^2+R)$ factors into (4.13) via (4.1) is by considering η as the field strength of a non-chiral, complex gauge superfield [6]. The interested reader may refer to appendix B for a discussion of this alternative procedure. Our methods can also be applied to massive chiral superfields. In that case $\bar{J}\Box_+{}^{-1}J$ of (4.7) is replaced with $\bar{J}\Box_+{}^{-1}J - \frac{1}{2}m[J(\nabla^2+\bar{R})\Box_+{}^{-1}(\Box_+ - m^2)^{-1}J + \text{h.c.}]$, a direct covariantization of the free-field result [3]. In performing the doubling trick, we first note that $\Delta(m)=\Delta(-m)$ (as seen by replacing $\eta \to i\eta$ in the mass term $\frac{1}{2}m\int d^2\theta\, \varphi^3\eta^2$), so we now replace the kinetic operator $\boldsymbol{O}(m)$ with $\boldsymbol{O}(-m)\boldsymbol{O}(m)$:

$$\boldsymbol{O}(m)=\begin{pmatrix} m & \bar{\nabla}^2+R \\ \nabla^2+\bar{R} & m \end{pmatrix},$$

$$\boldsymbol{O}(-m)\boldsymbol{O}(m)=\begin{pmatrix} (\bar{\nabla}+R)(\nabla^2+\bar{R})-m^2 & 0 \\ 0 & (\nabla^2+\bar{R})(\bar{\nabla}^2+R)-m^2 \end{pmatrix}. \tag{4.16}$$

The corresponding replacement is made in (4.13) and (4.15): the propagator is now $-(\Box_0 - m^2)^{-1}$. (The vertices are the same as before.)

5. Improved power counting and absence of divergences

We shall assume in this section that unconstrained superfield formalisms exist for all supersymmetric systems of interest. As a result of our fully covariant background field formalism, we have improved power-counting rules for discussing local divergences in simple and extended supersymmetric systems. These rules follow simply from the fact that *all quantum* terms in the effective action are automatically expressed directly in terms of the constrained covariant derivatives (and their field strengths), and an explicit expansion in terms of unconstrained background fields in unnecessary. (One might also expect to need the superspace generalization of antisymmetric tensor gauge fields, but the background-quantum split classical action can always be written in a form where such background fields appear only as their field strengths, and these field strengths already appear among the field strengths of the background covariant derivatives.) This implies that all divergent terms must be expressible as *local functions of the covariant derivatives*, and furthermore, because in the derivation of the Feynman rules vertices are always *integrated over full superspace*, they will carry a full $\int d^4x\, d^{4N}\theta$ for N-extended supersymmetry. Thus, for super-Yang-Mills all counterterms must be local functions of $\Gamma_{\underline{\alpha}}$, and for supergravity of $\Gamma_{\underline{\alpha}}$ and also $E_{\underline{\alpha}}{}^M$. (Conventional constraints determine all of ∇_A from $\Gamma_{\underline{\alpha}}$ and $E_{\underline{\alpha}}{}^M$. We are using a notation where $\underline{\alpha}$ stands for a combined Lorentz + internal symmetry spinor index.)

For the case of extended supersymmetry, treated with extended superfields, there would appear to be a difficulty with the background field method because of the existence of an infinite number (second, third, etc., generation) of ghost superfields with progressively increasing superspin (as in, e.g., $N=2$ Yang-Mills [7]). This is basically due to the fact that the gauge superfields unavoidably contain fields of spin higher than those (physical and auxiliary) occurring in the gauge-invariant action, so that the gauge superparameters (and therefore the corresponding ghosts) contain higher spins than the gauge superfields (to gauge away the higher-spin fields). However, only a finite number of ghosts (i.e., the usual Faddeev-Popov ghosts, plus perhaps certain catalyst ghosts [1]) contribute at more than one loop. Therefore, the higher-loop contributions to the effective action can be calculated in a manifestly background covariant form and obey our power-counting rules, whereas the one-loop contribution may have to be treated separately. (For example, we could choose background non-covariant gauges for some of the ghosts which contribute only at one loop in such a way that all but a finite number of these ghosts decouple. The effect of such a choice would be to produce a one-loop effective action which would be non-covariant, but this would have no effect on physical quantities.) We discuss now the implications of these remarks.

5.1. SUPER-YANG-MILLS

There has been some discussion in the literature about the generation of a "D" term (a term in the action of the form $\int d^4x\, d^4\theta\, V$ linear in a U(1) gauge superfield) in $N=1$ Yang-Mills by loop corrections. In particular, it has been shown [8] using the older form of chiral supergraph rules [3] that such a term cannot be generated at more than one loop. With our new rules it can be shown *without any supergraph calculations* that to *any* loop order (even at one loop) such a term cannot be generated, because the only quantity linear in the gauge field that could appear in the effective action is Γ_α, and it contains a derivative of the U(1) gauge field. This can be seen more explicitly at the one-loop level by examination of the relevant tadpole graph, where not enough D_α's appear in the loop to give rise to a non-vanishing contribution: From (4.15), the one vertex takes the form $\bar{D}^2(D_\alpha + \Gamma_\alpha)^2 - \bar{D}^2 D^2 = \bar{D}^2[\Gamma^\alpha D_\alpha + (\Gamma_\alpha)^2 + \frac{1}{2}(D^\alpha \Gamma_\alpha)]$, so at most one D_α (but two $\bar{D}_{\dot\alpha}$'s) can appear in the loop. However, even with the older rules the cancellation occurs at one loop (for arbitrary couplings of the gauge superfield) when the graph is regularized gauge covariantly and supersymmetrically. Massive fields do not contribute [8] (since they must occur in pairs of opposite charge, due to gauge invariance). The massless superfields give contributions $\sim \int d^4p/p^2$, which necessarily vanish in (supersymmetric) dimensional regularization ($\lim_{m\to 0}\int d^{4+\epsilon}p/(p^2+m^2) \sim \lim_{m\to 0} m^{2+\epsilon} = 0$). In (covariant) Pauli-Villars regularization, the regulators give no contribution (to the tadpole graph), since they are massive, so the massless superfields' contribution must be defined to vanish separately. In our new covariant rules for chiral superfields, the manifest covariance automatically causes the cancellation of the tadpole graph before explicit covariant regularization.

The new covariant Feynman rules can be used to discuss the higher-loop divergences. In supersymmetric Yang-Mills, which is renormalizable, the only allowed divergence in the background field method is proportional to the classical action. On the other hand, it must have the form at the linearized level $\int d^4x\, d^{4N}\theta\, \Gamma^{\underline{\alpha}}\Omega\Gamma_{\underline{\alpha}}$, because of the Feynman rules. Here $\Gamma_{\underline{\alpha}}$ has dimension $\frac{1}{2}$ and Ω is a local operator (non-negative dimension). Since the classical action is dimensionless, we obtain the inequality $-4+2N+\frac{1}{2}+\frac{1}{2}\leqslant 0$, which implies $N=0$ or 1. Thus, $N=2$ (as has been conjectured previously [9]) and $N=4$ super-Yang-Mills must be finite beyond one loop. Furthermore, we know from explicit one-loop calculations [10] (obtained most simply by merely counting $N=1$ chiral superfields in a background gauge [3]) that $N=4$ is one-loop finite as well. On the other hand, $N=2$ does have one-loop divergences. We emphasize again that we had to make a separate one-loop argument because of the difficulty with infinite numbers of ghosts for extended supersymmetry. The conclusion we reach is that $N=2$ has only one-loop divergences, while $N=4$ is completely finite.

5.2. SUPERGRAVITY

Beyond one loop we can apply similar arguments to N-extended supergravity. The local divergent part of the effective action consists of the integral of E^{-1} times a product of factors of supervierbein and connections. To insure covariance these factors must arrange themselves in a form which contains at most one noncovariant factor times a covariant object which satisfies a Bianchi identity. The lowest dimensional such term is $\kappa^{2(L-1)}\int d^4x\, d^{4N}\theta\, E^{-1}$ [11] (for L loops, multiplied perhaps by some function of the dimensionless scalar field strength for $N\geqslant 4$ [12], which may, however, be forbidden by global on-shell invariances). Requiring this action, with factors of non-negative dimension, to be dimensionless, we obtain the inequality $-2(L-1)-4+2N\leqslant 0$, which implies $L\geqslant N-1$. (Similar arguments for Yang-Mills give the improved $-4+2N+2\leqslant 0$ instead of the above $-4+2N+1\leqslant 0$.) Thus, from these arguments alone we find that N-extended supergravity only allows divergences at $N-1$ loops and greater. (This is so even though possible lower-loop counterterms have been proposed [13]. The important point is that our Feynman rules imply integration over the full superspace with integrands which involve covariant objects.) With appropriate choice of auxiliary fields, only $N=1$ and $N=2$ have one-loop topological divergences on shell [14, 5]. At two loops all the theories are finite by the usual supersymmetry arguments [15]. Note that the divergences excluded by our new rules are absent *both on and off shell*.

Our proof of the absence of divergences cannot be made rigorous until the corresponding supergraph rules are explicitly constructed. At the present time, the only foreseeable difficulties in such a program are the explicit construction of the classical action, and possible infrared problems due to large negative powers of momenta in the superfield propagators. In the $N=4$ Yang-Mills case, our finiteness argument should be compared to the one in terms of $N=1$ superfields [16] which follows from the impossibility of writing the chiral φ^3 self-interaction with a $\int d^4\theta$ integral. The absence of the $\int d^2\theta\,\varphi^3$ counterterm implies by $N=4$ supersymmetry the absence of any other counterterm. Here the finiteness follows directly from the impossibility of writing a counterterm in terms of $\Gamma_{\underline{\alpha}}$ with a $\int d^{16}\theta$ integral. What we need to make the proof rigorous in the $N=1$ formulation is an explicit proof of $N=4$ supersymmetry, and in the present formulation, an explicit construction of the $N=4$ superfield action and Feynman rules. However, we emphasize that both here and in supergravity, once the action has been actually written, the power counting rules and therefore our conclusions immediately follow.

A similar analysis in higher dimensions gives the result that higher-loop divergences are absent in super-Yang-Mills for $L<2(N-1)/(D-4)$, and in supergravity for $L<2(N-1)/(D-2)$ (for L loops in D dimensions, where N refers to the four-dimensional value: i.e., the number of anticommuting coordinates is $4N$). For lower dimensions, (super-)Yang-Mills is superrenormalizable anyway; for supergravity, the above inequality holds for $D=3$, and for $D=2$ we find higher-loop finiteness for $N>1$.

At the one-loop level, in $N=1$ language, the only contributions to the (on-shell, topological) divergences come from chiral superfields. Such divergences are already

TABLE 1
Absence of divergences in super-Yang-Mills and supergravity

		loops						
	N	1	2	3	4	5	6	$\geqslant 7$
Yang-Mills	0							
	1							
	2		C	C	C	C	C	C
	4	A	C	C	C	C	C	C
supergravity	0							
	1		B					
	2		B					
	3	A	B					
	4	A	B, C					
	5	A	B, C	C				
	6	A	B, C	C	C			
	8	A	B, C	C	C	C	C	

(A) One loop for $N\geqslant 3$, due to zero net $N=1$ chiral superfields; (B) two-loop supergravity, due to lack of invariant counterterms; (C) $\geqslant 2$ loops for $N\geqslant 2$ Yang-Mills and $\geqslant 2$ but $\leqslant N-2$ loops for supergravity, due to our new power counting.

present in the two-point function and can be obtained by evaluating self-energy graphs. However, in our gauge, only chiral-field vertices have enough D's and $\bar{D}$'s to give non-zero contributions. Therefore, in a theory with zero net number (physical minus ghost) of chiral superfields (*any* $N \geqslant 3$ theory) there are no topological one-loop divergences.

We summarize our results in table 1, which lists all known cases where divergences are absent in pure supersymmetric gauge theories. The results can be classified into three types: (A) due to one-loop cancellations in $N \geqslant 3$ supersymmetry [14] of contributions of $N=1$ chiral superfields [5]; (B) absence of two-loop supergravity counterterms by invariance arguments [15]; (C) finiteness at more than one loop for $N \geqslant 2$ Yang-Mills and at $L \leqslant N-2$ for $N \geqslant 4$ supergravity, which follows from our new background covariance arguments.

Appendix A

NOTATION AND USEFUL FORMULAE

We use the following index conventions: lower-case Greek letters for Weyl spinor indices, upper-case Roman letters for supervector indices; vector indices appear as pairs of dotted and undotted spinor indices. Letters from the beginning of the alphabets represent indices which transform under local Lorentz but not general supercoordinate transformations; the opposite is true for letters from the middle (except in the case of global supersymmetry or when a Lorentz gauge has been chosen, when we make no distinction). Spinor indices are raised and lowered by $C^{\alpha\beta}$ and $C_{\alpha\beta}$:

$$C_{\alpha\beta} = -C_{\beta\alpha}, \qquad \overline{C_{\alpha\beta}} = C_{\dot{\beta}\dot{\alpha}}, \qquad C_{\alpha\beta}C^{\gamma\delta} = \delta_{[\alpha}{}^{\gamma}\delta_{\beta]}{}^{\delta},$$
$$\psi_\alpha = \psi^\beta C_{\beta\alpha}, \qquad \psi^\alpha = C^{\alpha\beta}\psi_\beta, \qquad \psi^2 = \tfrac{1}{2}\psi^\alpha\psi_\alpha. \tag{A.1}$$

The empty-space covariant derivatives satisfy the following identities:

$$\{D_\alpha, D_\beta\} = 0, \qquad \{D_\alpha, \bar{D}_{\dot{\beta}}\} = i\partial_{\alpha\dot{\beta}}, \qquad D^\alpha D_\beta = \delta_\beta{}^\alpha D^2,$$
$$\partial^{\alpha\dot{\gamma}}\partial_{\beta\dot{\gamma}} = \delta_\beta{}^\alpha \Box_0, \qquad \bar{D}^2 D^2 \bar{D}^2 = \Box_0 \bar{D}^2, \qquad \int d^2\theta = D^2. \tag{A.2}$$

Hermitian conjugation is generally represented by bars or daggers. However, we define "$^{-}$" of an operator to also return the original ordering: $W = W^M iD_M \to \bar{W} \equiv \bar{W}^M iD_M \neq W^\dagger = (-1)^M iD_M \bar{W}^M$. Here $(-1)^M$ is $+1$ for $M = \mu\dot{\mu}$, -1 for $M = \mu$ or $\dot{\mu}$; and $D_M = (\partial_{\mu\dot{\mu}}, D_\mu, \bar{D}_{\dot{\mu}})$, $W^M D_M = W^{\mu\dot{\mu}}\partial_{\mu\dot{\mu}} + W^\mu D_\mu + W^{\dot{\mu}}\bar{D}_{\dot{\mu}}$. Also, we define for any spinor $(\psi^\alpha)^\dagger \equiv \bar{\psi}^{\dot{\alpha}}$, so that $(\psi_\alpha)^\dagger = -\bar{\psi}_{\dot{\alpha}}$. Thus, a hermitian supervector V^A

satisfies $V^A=(V^A)^\dagger$, while a hermitian supervector V_A satisfies $V_A=(-1)^A(V_A)^\dagger$. D_M is antihermitian ($D_M=-(-1)^M(D_M)^\dagger$), as usual for derivatives. Furthermore, *in the chiral representation*, which is obtained by a non-unitary transformation from the vector representation (where reality takes the usual form), covariant objects use a modified hermitian conjugation, represented by tildes:

$$\tilde{f}\equiv e^{-H}f^\dagger, \qquad (-1)^A i\nabla_A=\left(i\tilde{\nabla}_A\right)\equiv e^{-H}\overline{(i\nabla_A)}e^H . \tag{A.3}$$

However, we will replace tildes with bars in equations which are representation independent, and over individual symbols, such as $\overline{\nabla}_{\dot\alpha}\equiv(\tilde{\nabla}_\alpha)$. In general, if the ambiguity is not resolved by the context, we will make an explicit distinction.

For supergravity, we have defined the covariant derivatives in (2.8), and the constraints they satisfy in (2.11). These constraints, and the consequent Bianchi identities, imply the following (graded) commutation relations:

$$\begin{aligned}
&\left\{\nabla_\alpha,\overline{\nabla}_{\dot\beta}\right\}=i\nabla_{\alpha\dot\beta},\\
&\left\{\overline{\nabla}_{\dot\alpha},\overline{\nabla}_{\dot\beta}\right\}=-2R\overline{M}_{\dot\alpha\dot\beta},\\
&\left[\overline{\nabla}_{\dot\alpha},i\nabla_{\beta\dot\gamma}\right]=C_{\dot\gamma\dot\alpha}\left[-R\nabla_\beta-G_\beta{}^{\dot\delta}\overline{\nabla}_{\dot\delta}+\left(\overline{\nabla}^{\dot\delta}G_{\beta\dot\varepsilon}\right)\overline{M}_{\dot\delta}{}^{\dot\varepsilon}+W_{\beta\delta}{}^{\varepsilon}M_\varepsilon{}^{\delta}\right]-\left(\nabla_\beta R\right)\overline{M}_{\dot\alpha\dot\gamma}.
\end{aligned} \tag{A.4}$$

The remaining commutator $[\nabla_{\alpha\dot\beta},\nabla_{\gamma\dot\delta}]$ follows easily by substituting the others. The supergravitational field strengths $R, G_{\alpha\dot\alpha}, W_{\alpha\beta\gamma}$ satisfy the following Bianchi identities:

$$\begin{aligned}
&\overline{\nabla}_{\dot\alpha}R=\overline{\nabla}_{\dot\alpha}W_{\beta\gamma\delta}=0, \qquad G_{\alpha\dot\alpha}=\overline{G}_{\alpha\dot\alpha}, \qquad W_{\alpha\beta\gamma}\ \text{totally symmetric},\\
&\overline{\nabla}^{\dot\alpha}G_{\alpha\dot\alpha}=\nabla_\alpha R, \qquad \nabla^\alpha W_{\alpha\beta\gamma}=\tfrac{1}{2}i\nabla_{(\beta}{}^{\dot\alpha}G_{\gamma)\dot\alpha}.
\end{aligned} \tag{A.5}$$

In the chiral representation $\Phi_{\dot\alpha\beta}{}^{\gamma}=0$, we also have $W_{\alpha\beta\gamma}=(\overline{\nabla}^2+R)\Phi_{\alpha\beta\gamma}$. The field equations are $R=G_{\alpha\dot\alpha}=\nabla^\alpha W_{\alpha\beta\gamma}=0$.

Appendix B

AN ALTERNATIVE APPROACH TO THE COVARIANT FEYNMAN RULES

An interesting independent way of understanding the introduction of the $\overline{\nabla}^2+R$ factors into (4.13) via (4.1) is by considering η as the field strength of a non-chiral, complex gauge superfield [6]. In the non-covariantized case, we have:

$$\overline{D}_{\dot\alpha}\eta=0\to\eta=\overline{D}^2\Xi, \tag{B.1}$$

which introduces the gauge invariance

$$\delta\Xi = \bar{D}_{\dot\alpha}\bar{\omega}^{\dot\alpha}. \tag{B.2}$$

Unlike the variant representation where Ξ is real [17], the representation of a scalar multiplet by the complex Ξ is identical to the standard representation, and Ξ can belong to any (global or local) group representation that η can:

$$\eta' = e^{i\Lambda}\eta, \qquad [\bar{D}_{\dot\alpha}, \Lambda] = 0 \to \Xi' = e^{i\Lambda}\Xi. \tag{B.3}$$

Here Λ can represent a local Yang-Mills transformation and/or the supergravity transformation $\Lambda = \Lambda^{\mu\dot\mu} i\partial_{\mu\dot\mu} + \Lambda^{\mu} iD_{\mu} = (\bar{D}^{\dot\mu}L^{\mu})\partial_{\mu\dot\mu} + i(\bar{D}^2 L^{\mu})D_{\mu}$. The gauge-fixing function and term for the invariance of (B.2) can easily be found:

$$F_\alpha = D_\alpha \Xi,$$

$$S_{\rm GF} = \int d^4x\, d^4\theta\, \bar{F}^{\dot\alpha}\left(D_\alpha \bar{D}_{\dot\alpha} + \tfrac{3}{4}\bar{D}_{\dot\alpha}D_\alpha\right)F^\alpha = \int d^4x\, d^4\theta\, \bar{\Xi}(\Box - D^2\bar{D}^2)\Xi,$$

$$S_{\rm C} + S_{\rm GF} = \int d^4x\, d^4\theta\, \bar{\eta}\eta + S_{\rm GF} = \int d^4x\, d^4\theta\, \bar{\Xi}\Box\Xi. \tag{B.4}$$

F_α contains only the $\Pi_{1/2\pm}$ and Π_{0+} projections of Ξ, whereas η contains only the Π_{0-}. ($\Pi_{1/2\pm}\Xi = -\Box^{-1}D^\alpha\bar{D}^2 D_\alpha \frac{1}{2}(\Xi \pm \bar{\Xi})$, $\Pi_{0+}\Xi = \Box^{-1}\bar{D}^2 D^2 \Xi$, $\Pi_{0-}\Xi = \Box^{-1}D^2\bar{D}^2\Xi$.) It can easily be shown that quantization of (B.4) (plus interaction terms) leads to supergraph rules identical to the standard ones for η. This is due to the fact that Ξ appears only through its field strength η. Furthermore, due to the abelian nature of (B.2), no ghosts are needed.

In the interacting case one can make (Yang-Mills and supergravitational) gauge covariance manifest by working directly with covariantly chiral superfields:

$$\bar{\nabla}_{\dot\alpha}\eta = 0 \to \eta = (\bar{\nabla}^2 + R)\Xi, \qquad \delta\Xi = \bar{\nabla}_{\dot\alpha}\bar{\omega}^{\dot\alpha}. \tag{B.5}$$

If gauge fixing is then performed by addition of the term

$$S_{\rm GF} = \int d^4x\, d^4\theta\, E^{-1}\bar{\Xi}(\Box_0 - D^2\bar{D}^2)\Xi, \tag{B.6}$$

and the $\bar{D}^2$ manipulations of sect. 4 are again applied, the rules of (4.7) and (4.15) are immediately obtained. However, the Faddeev-Popov analysis of ghosts is complicated, although we see by comparison with the results of (4.7) and (4.15) that all of Ξ's ghosts (which would only contribute at the one-loop level in any case) must cancel in this gauge.

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Note added

The 3-loop β-function for $N=2$ supersymmetric Yang-Mills has been calculated by Avdeev and Tarasov [18], and found to be zero in agreement with our higher-loop finiteness predictions.

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