POWER CORRECTIONS $1/Q^2$ TO PARTON SUM RULES FOR DEEP INELASTIC SCATTERING FROM POLARIZED NUCLEONS

I.I. BALITSKY, V.M. BRAUN and A.V. KOLESNICHENKO [1]

Leningrad Nuclear Physics Institute, 188 350 Gatchina, USSR

Received 19 August 1989; revised manuscript received 15 March 1990

Power corrections to $\int dx\, g_1(x)$ and the second moments $\int dx\, x^2 g_2(x)$ are calculated in the framework of QCD sum rules.

1. The data [1] on deep inelastic scattering from polarized protons have signalled the inadequacy of the conventional picture for the distribution of spin among the constituents of the proton. The "spin crisis" is not resolved yet, although some possibilities have already been outlined, see, e.g., ref. [2]. In particular, possible corrections to the parton sum rules are being discussed, they fall off as inverse powers of the large momentum Q^2. The experimental data on g_1 integrated over different regions of x do not show significant deviations from the scaling law. Regarding this evidence, most of the authors consider large power corrections to be unlikely. However, in ref. [3] arguments are given for the power corrections to g_1 accounting probably for half of the deviation of the experimental value from predictions of the standard model. The key observation of ref. [3] is that some nonperturbative mechanism is needed to modify crucially the slow logarithmic evolution of $\int dx\, g_1(x, Q^2)$ in order to satisfy the Gerasimov–Drell–Hearn sum rule [4] at $Q^2=0$. The tail of this effect could stretch up to rather high values of Q^2. Thus, the status of power corrections is not yet clear.

In this letter we attempt a direct calculation of the correction $1/Q^2$ to the value of $\int x\, g_1^{p\pm n}(x, Q^2)$ in the framework of the QCD sum rules. Our main result is that the power corrections are very small both in the singlet and non-singlet cases and for $Q^2>1$ GeV^2 are beyond the accuracy of the experiment. As a fraction of the total correction $1/Q^2$, we calculate the contributions of twist three to the second moment of the structure function $g_2(x, Q^2)$, which turn out to be of the same order of magnitude as the contributions of the leading twist. Our predictions for $\int dx\, x^2 g_2^{p\pm n}(x)$ can soon be checked experimentally by the European Muon Collaboration.

2. The starting point for our analysis are the results [5] for the contributions of twist four to the amplitudes of deep inelastic scattering from polarized nucleons that imply power corrections $1/Q^2$ to the classical parton sum rules [6,7] in the form

$$\int dx\, g_1^{p\pm n}(x, Q^2) = \begin{pmatrix} 5/18 \\ 1/6 \end{pmatrix} \{ g_A^{S(NS)} [1-\alpha_s(Q^2)/\pi] - (8/9Q^2) \langle\!\langle U^{S(NS)} \rangle\!\rangle \}$$

$$+ \frac{8}{3}\frac{m_N^2}{Q^2} \int dx\, x^2 [g_2^{p\pm n}(x) + \tfrac{5}{6} g_1^{p\pm n}(x)] + O(1/Q^4) \,. \qquad (1)$$

[1] Deceased.

The upper value in parentheses corresponds to the sum of structure functions for the proton and neutron, and vice versa. The same identification is used below. The axial constant g_A^{NS} is simply $|G_A/G_V| \simeq 1.25$, while a more unusual form for g_A^S is $g_A^S = (\sqrt{3}/5)(a_8 + 2\sqrt{2}a_0)$, $a_8 = (1/\sqrt{3})(3F-D)$, $S_z^q = \sqrt{3/8}\, a_0 = 0.014 \pm 0.056 \pm 0.121$ [1]. Using the standard values for F and D one obtains $g_A^S = 0.16 \pm 0.2$ [#1]. $\langle\!\langle U^S \rangle\!\rangle$ and $\langle\!\langle U^{NS} \rangle\!\rangle$ are the reduced matrix elements of the local operators of spin one and twist four [5]

$$U_\mu^S = \bar{u} g \tilde{G}_{\mu\nu}^a \gamma_\nu \cdot \tfrac{1}{2}\lambda^a u + (u \to d) + \tfrac{18}{5}(u \to s)\,, \quad U_\mu^{NS} = \bar{u} g \tilde{G}_{\mu\nu}^a \gamma_\nu \cdot \tfrac{1}{2}\lambda^a u - (u \to d)\,, \tag{2}$$

$$\langle \mathrm{N} | U_\mu | \mathrm{N} \rangle = s_\mu \langle\!\langle U \rangle\!\rangle\,, \quad s_\mu = \bar{N}\gamma_\mu\gamma_5 N\,. \tag{3}$$

$\bar{N}$, N are nucleon spinors. In what follows we neglect the contribution of strange quarks to U^S, this being presumably less by a factor 2–3 than that of u and d. The simplification seems to be justified since our accuracy a priori does not exceed 20%.

The term $\sim m_N^2$ gives the so-called kinematical power correction to the RHS of (1) and involves a contribution of the second structure function $g_2(x, Q^2)$. As it is well known the latter contains both the contributions of twist two which are similar to that in $g_1(x, Q^2)$, and of twist three. These two types of terms are easily separated in terms of moments [5]. In particular,

$$\int \mathrm{d}x\, x^2 g_2^{p\pm n}(x) = -\tfrac{2}{3} \int \mathrm{d}x\, x^2 g_1^{p\pm n}(x) - \frac{1}{6}\begin{pmatrix} 5/18 \\ 1/6 \end{pmatrix} \langle\!\langle V^{S(NS)} \rangle\!\rangle\,, \tag{4}$$

where

$$V_{\mu\nu,\sigma}^{S(NS)} = s_{\nu,\sigma} \bar{u} g \tilde{G}_{\mu\nu}^\alpha \gamma_\sigma \cdot \tfrac{1}{2}\lambda^a u \pm (u \to d) - \text{traces}\,, \quad \langle \mathrm{N} | V_{\mu\nu,\sigma} | \mathrm{N} \rangle = S_{\nu,\sigma} A_{\mu,\nu} s_\mu p_\nu p_\sigma \langle\!\langle V \rangle\!\rangle - \text{traces}\,, \tag{5}$$

$A_{\mu,\nu}$ and $S_{\nu,\sigma}$ stand for the (anti) symmetrization over the given subscripts. It seems to be more physical to rewrite (1) in order to include the terms $\sim \langle\!\langle V \rangle\!\rangle$ into the dynamical power correction in braces. Then the latter takes the form

$$-\frac{8}{9Q^2}\left(\langle\!\langle U \rangle\!\rangle + \tfrac{1}{2} m_N^2 \langle\!\langle V \rangle\!\rangle\right),$$

while the remaining kinematical correction contains $g_1(x)$ only. Our goal is the evaluation of the matrix elements $\langle\!\langle U \rangle\!\rangle$ and $\langle\!\langle V \rangle\!\rangle$.

3. To this end we consider the three-point correlation functions of the operators of interest with the proton current $\eta = \epsilon^{abc}(u^a C \gamma_\lambda u^b)\gamma_5 \gamma_\lambda d^c$ [8]

$$\Gamma_\mu(p) = \mathrm{i}^2 \int \mathrm{d}x \exp(\mathrm{i}px) \int \mathrm{d}y \langle \mathrm{T}[\eta(x) U_\mu(y) \bar{\eta}(0)] \rangle = -2 p_\mu \not{p} \gamma_5 \frac{\lambda_p^2 \langle\!\langle U \rangle\!\rangle}{(m_N^2 - p^2)^2} + \ldots\,, \tag{6}$$

$$\Gamma_{\mu\nu,\sigma}(p) = \mathrm{i}^2 \int \mathrm{d}x \exp(\mathrm{i}px) \int \mathrm{d}y \langle \mathrm{T}[\eta(x) V_{\mu\nu,\sigma}(y) \bar{\eta}(0)] \rangle = -2 S_{\nu,\sigma} A_{\mu,\nu} p_\sigma p_\mu \gamma_\nu \gamma_5 \frac{\lambda_p^2 \langle\!\langle V \rangle\!\rangle}{(m_N^2 - p^2)^2} + \ldots\,. \tag{7}$$

λ_p is the proton coupling to the current η. The suppressed contributions to the RHS of (6), (7) are less singular in the vicinity of $p^2 = m_N^2$.

We proceed further to the unusual program of QCD sum rules. The main work is the expansion of the correlation functions (6), (7) in the euclidean region $p^2 \to -\infty$ in inverse powers of p^2. Then we apply the Borel transformation over p^2 and match between the operator expansion series and the expansion over the hadron states, making use of the standard procedure to account for the hadronic continuum. This technique has become

[#1] The validity of the operator expansion (1) has been confirmed once more in ref. [2] by a direct calculation of the box graph (the leading behavior). In higher twists there is no anomaly.

a standard one, so we shall outline the main points only. A similar calculation has been described in detail in refs. [9,10].

A straightforward calculation yields (for the chosen Lorentz structures)

$$\Gamma_\mu^{\mathrm{S(NS)}}(p)=p_\mu \not{p}\gamma_5\Bigg[\begin{pmatrix}9\\5\end{pmatrix}\frac{\alpha_s}{360\pi^5}p^4\ln^2(\mu^2/-p^2)-\begin{pmatrix}0\\1\end{pmatrix}\frac{\langle(\alpha_s/\pi)G^2\rangle}{72\pi^2}\ln(\mu^2/-p^2)-\begin{pmatrix}1\\0\end{pmatrix}\frac{f_\pi^2\delta^2}{3\pi^2}\ln(\mu^2/-p^2)$$
$$+\begin{pmatrix}\ln(\mu^2/-p^2)-1/24\\ \ln(\mu^2/-p^2)+11/24\end{pmatrix}\frac{16\alpha_s}{27\pi}\frac{\langle\bar\psi\psi\rangle^2}{-p^2}+\frac{\Pi}{36\pi^2}\frac{1}{-p^2}-\begin{pmatrix}1\\3\end{pmatrix}\frac{m_0^2\langle\bar\psi\psi\rangle^2}{9p^4}\Bigg]+\dots, \tag{8}$$

$$\Gamma_{\mu\nu,\sigma}^{\mathrm{S(NS)}}(p)=S_{\nu,\sigma}A_{\mu,\nu}p_\sigma p_\mu\gamma_\nu\gamma_5\Bigg[\begin{pmatrix}7\\-13\end{pmatrix}\frac{\alpha_s}{2160\pi^5}p^4\ln^2(\mu^2/-p^2)-\begin{pmatrix}0\\1\end{pmatrix}\frac{\langle(\alpha_s/\pi)G^2\rangle}{36\pi^2}\ln(\mu^2/-p^2)$$
$$-\begin{pmatrix}\ln(\mu^2/-p^2)-71/12\\ 10\ln(\mu^2/-p^2)+157/12\end{pmatrix}\frac{4\alpha_s}{27\pi}\frac{\langle\bar\psi\psi\rangle^2}{-p^2}+\begin{pmatrix}-1\\2\end{pmatrix}\frac{R}{72\pi^2}\frac{1}{-p^2}+\begin{pmatrix}2\\-2\end{pmatrix}\frac{m_0^2\langle\bar\psi\psi\rangle^2}{9p^4}+\dots\Bigg], \tag{9}$$

where $f_\pi=133$ MeV, $\delta^2\simeq0.2\ \mathrm{GeV}^2$ [11], $m_0^2=\langle\bar\psi\sigma G\psi\rangle/\langle\bar\psi\psi\rangle$, and

$$\Pi=\mathrm{i}\int\mathrm{d}y\langle\mathrm{T}[U_\mu(y)U_\mu(0)]\rangle\simeq3\times10^{-3}\ \mathrm{GeV}^6,\quad R=\mathrm{i}\int\mathrm{d}y\langle\mathrm{T}[V_{\mu\nu,\sigma}(y)V_{\mu\nu,\sigma}(0)]\rangle\simeq1\times10^{-3}\ \mathrm{GeV}^6. \tag{10}$$

The terms proportional to Π, R and $f_\pi^2\delta^2$ correspond to the so-called bilocal power corrections to (6), (7) which come from the region of integration over parametrically large $y\gg x\gg 1/p$. In this case only the nucleon currents are contracted by the large momentum p, producing a sequence of local operators of increasing dimension. The lowest dimension operator turns out to be the axial current $J_{\mu5}$, cf. ref. [10]. The bilocal corrections are written as correlation functions at zero momentum of $U(y)$ or $V(y)$ and these operators, multiplied by certain functions of p^2. The leading contribution $\mathrm{i}\int\mathrm{d}y\langle\mathrm{T}[U_\mu(y)J_{\mu5}(0)]\rangle$ can be evaluated by the contribution of the intermediate pseudoscalar meson which is $4f_\pi^2\delta^2$ [#2], $\langle0|U_\mu|\Pi(q)\rangle=\mathrm{i}f_\pi\delta^2q_\mu$. Thus one arrives to the third term on the RHS of (8).

The evaluation of Π, R is more involved since the operators U_μ and $V_{\mu\nu,\sigma}$ are not conserved. This is done by a standard, although tedious procedure, considering additional sum rules for the correlation functions (10) at nonzero momentum q and taking special care to define properly the (finite!) part of Π, R coming from large y, which is relevant only for the calculation of bilocal corrections. We estimate the accuracy of (10) within $\pm30\%$ for Π and within a factor of two for R. Since the corresponding contribution to the matrix elements of interest turns out to be small, we shall not go into details here. An interested reader can find the detailed discussion of all principal points in an example of a similar contribution in refs. [9,12].

4. Next, we apply the Borel transformation to (8), (9), introduce the cutoff s_0 in the dispersion representation for the logarithmic terms, and equate the result to the contribution of the proton. Let A, B, C, D be the coefficients in (8), (9) in front of $p^4\ln^2(\mu^2/-p^2)$, $\ln(\mu^2/-p^2)$, $1/-p^2$, and $1/p^4$, respectively. Then the general form of the sum rule is $(O=U,\ V)$

$$\langle\!\langle O\rangle\!\rangle+c(\mu^2)M^2$$
$$=-\frac{\exp(m_\mathrm{N}^2/M^2)}{2\lambda_\mathrm{p}^2}\left(2AM^2\int_0^{s_0}\mathrm{d}s\exp(-s/M^2)s^2\ln(\mu^2/s)+BM^4[1-\exp(-s_0/M^2)]+CM^2+D\right). \tag{11}$$

An extra contribution $c(\mu^2)M^2$ to the LHS reminds of possible single-pole terms $\sim(m_\mathrm{N}^2-p^2)^{-1}$, e.g., produced by nondiagonal transitions "proton→continuum". We assume tacitly that all the vacuum expectation values and the coupling are taken at the normalization point 1 GeV (which is a typical matching point in the Borel

[#2] To our accuracy the η meson gives the same contribution as the pion, cf. ref. [10].

variable M^2), and disregard the weak dependence on M^2, produced by anomalous dimensions. In particular, we use $\langle(\alpha_s/\pi)G^2\rangle=0.012$ GeV^4, $\langle\bar\psi\psi\rangle=-0.017$ GeV^3, $m_0^2=0.65$ GeV, $\alpha_s(1\text{ GeV})=0.37$. Two more entries are $32\pi^4\lambda_p^2=2.5$ GeV^6 and $s_0=2.25$ GeV^2.

One more source of dependence of the sum rule (11) on the parameter of the ultraviolet cutoff μ^2 (that is explicit in (11)) is the mixing of the three-point correlation functions (6), (7) with the two-point correlation functions of the nucleon current $\eta(x)$ and complicated operators produced by the contraction of $O(y)$ and $\bar\eta(0)$ (and h.c.). This mixing is present in the perturbative contribution to (8), (9) as well as in the $\langle\bar\psi\psi\rangle^2$ correction. One can easily see that the contraction of internal loops in the corresponding diagrams does produce counterterms in the form of new operators of dimension $\frac{11}{2}$. The coefficient function in front of $(\bar\psi\psi)^2$ has been calculated to two-loop accuracy, i.e. with accounting of the constant under logs. The numbers in (8), (9) correspond to $\mu^2_{\overline{\rm MS}}$. With the natural choice $\mu^2_{\overline{\rm MS}}=-p^2\sim1$ GeV^2 the term $\sim\ln(\mu^2_{\overline{\rm MS}}/-p^2)/p^2$ vanishes. Our calculation of the perturbative graphs $\sim p^4\ln^2(\mu^2/-p^2)$ does not have this accuracy and greater freedom is left here in fixing the cutoff. Note that the Borel transformation does not eliminate the dependence on μ^2, see the first contribution to the RHS of (11). Respectively, the LHS of the sum rule depends explicitly on μ^2 via the coefficient $c(\mu^2)$. This dependence reflects uncertainties in the terms $\sim(m_N^2-p^2)^{-1}$ that are being due to the renormalization of the above mentioned two-point correlation functions which get admixed to (6), (7). However, since we separate the double-pole and single-pole contributions numerically by a different dependence on M^2 in a narrow region of stability, a certain artificial dependence on μ^2 will actually be present in $\langle\!\langle O\rangle\!\rangle$ as well. Fortunately, the effect is not large.

The numerical results are shown in fig. 1 (twist 3) and in fig. 2 (twist 4) for two different values of the cutoff in the perturbative graphs $\mu^2=1$ GeV^2 and $\mu^2=0.1$ GeV^2. In each figure the RHS of the sum rule (11) is shown by the dotted curve and gives the sum of double-pole and single-pole contributions that is supposed to be a linear function of M^2. To single out $\langle\!\langle U\rangle\!\rangle$ and $\langle\!\langle V\rangle\!\rangle$ we apply the differential operator $1-M^2(\mathrm{d}/\mathrm{d}M^2)$ to both sides of (11) and arrive at the solid curves in figs. 1, 2. In agreement with our expectations the dependence on μ^2 mainly affects the cM^2 term which is the difference between the solid and the dashed curves. Taking $\mu^2=1$ GeV^2, $M^2=m_N^2$ we obtain

$$\langle\!\langle U^{\rm NS}\rangle\!\rangle\simeq0.18\text{ GeV}^2,\quad \langle\!\langle U^{\rm S}\rangle\!\rangle\simeq-(0\text{–}0.1)\text{GeV}^2,\tag{12}$$

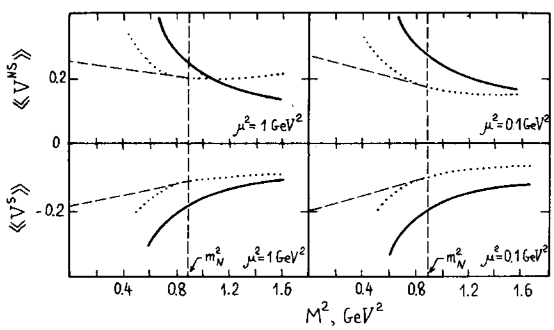

Fig. 1. Stability of the sum rules for matrix elements of the operators V of twist three. The values of $\langle\!\langle V\rangle\!\rangle$ depending on the matching point M^2 are shown by solid curves while the dotted curves present the RHSs of the corresponding sum rules that are supposed to be linear functions of M^2 in the region of stability.

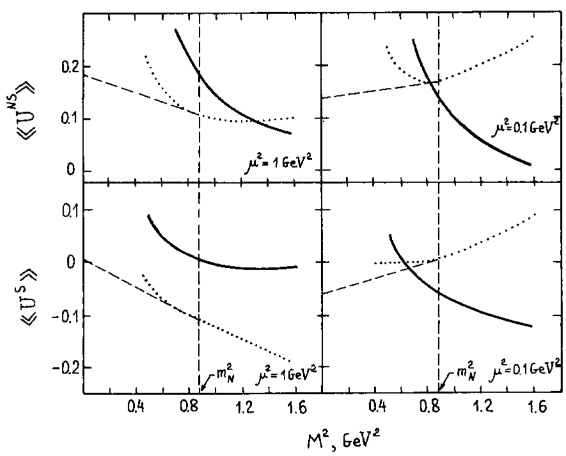

Fig. 2. Stability of the sum rules for the reduced matrix elements of the operators U of twist four. The notations are similar to those for fig. 1.

$$\langle\!\langle V^{\rm NS}\rangle\!\rangle \simeq 0.3\,, \quad \langle\!\langle V^{\rm S}\rangle\!\rangle \simeq -0.2\,, \tag{13}$$

with a possible error of the order of ± 0.1 owing to poor stability. Respectively, the sum rules (1) read

$$\int {\rm d}x\, g_1^{\rm p-n}(x, Q^2) = \tfrac{1}{6}\{g_{\rm A}[1-\alpha_s(Q^2)/\pi] - 0.3\ {\rm GeV}^2/Q^2\} + \frac{4}{9}\frac{m_{\rm N}^2}{Q^2}\int {\rm d}x\, x^2 g_1^{\rm p-n}(x)\,, \tag{14}$$

$$\int {\rm d}x\, g_1^{\rm p+n}(x, Q^2) = \tfrac{5}{18}\{g_{\rm A}^{\rm S}[1-\alpha_s(Q^2)/\pi] + 0.1\ {\rm GeV}^2/Q^2\} + \frac{4}{9}\frac{m_{\rm N}^2}{Q^2}\int {\rm d}x\, x^2 g_1^{\rm p+n}(x)\,. \tag{15}$$

We estimate the uncertainties in the power correction terms to be $\pm 0.15\ {\rm GeV}^2/Q^2$.

The kinematical correction $\sim m_{\rm N}^2/Q^2$ can directly be extracted from the data. Taking for orientation the simplest form $g_1 \sim (1-x)^3$ from which $\langle x^2\rangle = \frac{1}{15}$, we obtain the kinematical correction to the Bjorken sum rule (14) $+0.005 m_{\rm N}^2/Q^2$, which is absolutely negligible.

The total $1/Q^2$ correction to $g_1^{\rm p+n}$ is also very small. Finally, let us mention a general argument for the smallness of $1/Q^4$ corrections to deep inelastic scattering [5] which is due to the vanishing in this case of a set of diagrams with the emission of soft gluons from the external quark legs. We conclude that power corrections are not essential for the interpretation of modern data in the whole region of Q^2 where the expansion in $1/Q^2$ makes sense, i.e. for $Q^2 > 1\ {\rm GeV}^2$. Thus we expect that the Gerasimov–Drell–Hearn sum rule is saturated by the contributions that die off faster than $1/Q^4$.

Substituting (13) in (4) and using the $(1-x)^3$ dependence for $g_1(x)$ we obtain the second moments of $g_2(x, Q^2)$ in the scaling limit

$$\int {\rm d}x\, x^2 g_2^{\rm p+n}(x) \simeq -0.002 + 0.009\,, \quad \int {\rm d}x\, x^2 g_2^{\rm p+n}(x) \simeq -0.009 - 0.008\,. \tag{16}$$

(The two figures are the contributions of twist two and three, respectively). It is seen that the contributions of twist three are predicted to be small for the proton while being large and positive for the neutron. This result can be traced to a large correction $\sim m_0^2\langle\bar\psi\psi\rangle^2$ of dimension eight in eq. (9). Estimating the uncertainties involved in the calculation we arrive at the following final values for the contributions of twist three:

$$\int {\rm d}x\, x^2 g_{2,{\rm twist3}}^{\rm p}(x) = +0.001 \pm 0.002\,, \quad \int {\rm d}x\, x^2 g_{2,{\rm twist3}}^{\rm n}(x) = +0.009 \pm 0.004\,. \tag{17}$$

We hope that this prediction could be checked in the immediate future.

Thus we have calculated the matrix elements of operators of twist three and four of the lowest spin that enter in deep inelastic scattering from polarized targets. We have neglected the possible contributions of strange quarks which would inevitably acquire an extra factor $\alpha_s/\pi \sim 0.1$ within our technique. Although experimental data suggest rather a much more moderate damping $s/d \sim \frac{1}{2} - \frac{1}{3}$ (which signals a large contribution of gluons), still these contributions probably cannot alter the results of this paper.

References

[1] G. Baum et al., Phys. Rev. Lett. 51 (1983) 1135;
J. Ashman et al., Phys. Lett. B 206 (1988) 364.
[2] A.V. Efremov and O.V. Teryaev, Dubna preprint E2-88-287 (1988);
G. Altarelli and G.G. Ross, Phys. Lett. B 212 (1988) 391;
R.D. Carlitz, J.C. Collins and A.H. Mueller, Phys. Lett. B 214 (1988) 229.
[3] M. Anselmino, B.L. Ioffe and E. Leader, Yad. Fiz. 49 (1989) 214.
[4] S.B. Gerasimov, Yad. Fiz. 2 (1965) 598;
S.D. Drell and A.C. Hearn, Phys. Rev. Lett. 16 (1966) 908.
[5] E.V. Shuryak and A.I. Vainshtein, Nucl. Phys. B 201 (1982) 144.

[1] J.D. Bjorken, Phys. Rev. 48 (1966) 1467.
[7] J. Ellis and R.L. Jaffe, Phys. Rev. D 9 (1974) 1444; D 10 (1974) 1669 (E).
[8] B.L. Ioffe, Nucl. Phys. B 188 (1981) 317; B 191 (1981) 591 (E).
[9] V.M. Braun and A.V. Kolesnichenko, Nucl. Phys. B 283 (1987) 723.
[10] V.M. Belyaev, B.L. Ioffe and Ya. I. Kogan, Phys. Lett. B 151 (1985) 290.
[11] V.A. Novikov et al., Nucl. Phys. B 237 (1984) 525.
[12] V.M. Braun and A.V. Kolesnichenko, Yad. Fiz. 44 (1986) 756.