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LINEARIZED $N=2$ SUPERFIELD SUPERGRAVITY

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We present the formulation of linearized SU(2) supergravity and U(2) conformal supergravity in terms of unconstrained $N=2$ superfields: the gauge superfield, compensating superfields, gauge transformations, supertensors, actions, and covariant derivatives.

1. Introduction

Superfields are useful in classical supersymmetry and almost indispensable in quantum supersymmetry [1, 2]. In order to quantize a supersymmetric theory in a manifestly globally supersymmetric way, it is necessary to have a formulation in terms of *unconstrained* superfields. These superfields contain not only the physical and auxiliary component fields, but also additional gauge degrees of freedom which necessarily appear in globally supersymmetric gauge-fixing terms.

The program for constructing all supersymmetric theories in terms of unconstrained superfields is complete for simple supersymmetry, but little progress has been made for extended supersymmetry. So far, the only complete results have been for the $N=2$ tensor [3] and linearized $N=2$ vector [4, 5] multiplets. There have also been results for linearized multiplets which do not generalize to the interacting case [6], due to their containing variant field representations [7] and/or off-shell central charges [8]. For the case of $N=2$ supergravity the status is the following: The complete non-linear set of constraints on the covariant derivatives has been given [9]; they have been partially solved in fully non-linear form in terms of constrained $N=2$ superfields $H^{\alpha\dot{\beta}}$ (vector, dimension $=-1$) and $H^{a\alpha}$ (isospinor Lorentz spinor, dimension $=-\frac{1}{2}$) [10], analogous to the unconstrained superfields $H^{\alpha\dot{\beta}}$ and H^{α} of non-minimal $N=1$ supergravity [11]; the non-linear supergravity action has been expressed in terms of the supervierbein [10]. (Also, we have learned that Brink and Howe have obtained preliminary results for a complete linearized solution in terms of an unconstrained isospinor Lorentz spinor with dimension $=-\frac{5}{2}$ [12]. A similar solution has also been given for a proposed set of constraints for 6D $N=1$

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supergravity [13].) We will here complete the program for linearized SU(2) supergravity (and linearized U(2) conformal supergravity).

The outline of the paper is as follows: In sect. 2 we will discuss the $N=2$ tensor and linearized vector multiplets. These will be the compensating multiplets for $N=2$ supergravity. The unconstrained superfields for $N=2$ supergravity will be introduced in sect. 3. We will give their linearized gauge transformations, the supertensors invariant under these transformations, and the linearized actions for supergravity and conformal supergravity. In sect. 4 we will express the linearized covariant derivatives in terms of the unconstrained superfields. There is also an appendix of useful identities.

Our conventions are those of ref. [14]: small Latin letters denote isospinor indices (unless otherwise indicated); small Greek, Weyl spinor; underlined small Greek, combined isospinor + Weyl spinor; capital Latin, supervector. Vector indices are totally avoided: vectors are labeled by pairs (dotted + undotted) of Weyl spinor indices. For example, we have identities such as $\partial_{\underline{\alpha}}\vartheta^{\underline{\beta}} = \partial_{a\alpha}\vartheta^{b\beta} = \delta^{\underline{\beta}}{}_{\underline{\alpha}} = \delta_a^b\delta_\alpha^\beta$, $\partial_{\alpha\dot{\alpha}}x^{\beta\dot{\beta}} = \delta^{\beta\dot{\beta}}_{\alpha\dot{\alpha}} = \delta_\alpha^\beta\delta_{\dot{\alpha}}^{\dot{\beta}}$, $H^A D_A = H^{\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}} + H^{\underline{\alpha}}D_{\underline{\alpha}} + H^{\underline{\dot{\alpha}}}\bar{D}_{\underline{\dot{\alpha}}}$. We also find it convenient to introduce the totally antisymmetric symbol of SL(4, C):

$$C_{\underline{\alpha\beta\gamma\delta}}C^{\underline{\varepsilon\zeta\eta\vartheta}} = \delta^{\underline{\varepsilon}}_{[\underline{\alpha}}\delta^{\underline{\zeta}}_{\underline{\beta}}\delta^{\underline{\eta}}_{\underline{\gamma}}\delta^{\underline{\vartheta}}_{\underline{\delta}]}, \qquad \overline{C_{\underline{\alpha\beta\gamma\delta}}} = C_{\underline{\dot{\delta}\dot{\gamma}\dot{\beta}\dot{\alpha}}},$$

$$C_{\underline{\alpha\beta\gamma\delta}} = C_{ab}C_{cd}C_{\alpha\delta}C_{\beta\gamma} - C_{ad}C_{cb}C_{\alpha\beta}C_{\delta\gamma}, \tag{1.1a}$$

where C_{ab} is the antisymmetric symbol of SU(2):

$$C_{ab}C^{cd} = \delta^c_{[a}\delta^d_{b]}, \qquad \overline{C_{ab}} = C^{ab}. \tag{1.1b}$$

Due to ambiguities in sign conventions for raising and lowering SU(2) indices (e.g., $\psi_a^\alpha = C_{ba}\psi^{b\alpha} \to \bar{\psi}_a^{\dot{\alpha}} = -C_{ba}\bar{\psi}^{b\dot{\alpha}}$), and in order to make U(1) properties manifest, all C_{ab}'s will be written explicitly. Using $C_{\underline{\alpha\beta\gamma\delta}}$ we have the following useful definitions and identities for products of the spinor derivative $D_{\underline{\alpha}}$:

$$D^{(n)}_{\underline{\alpha}_1\cdots\underline{\alpha}_n} = D_{\underline{\alpha}_1}\cdots D_{\underline{\alpha}_n},$$

$$D^{(n)\underline{\alpha}_{n+1}\cdots\underline{\alpha}_4} = \frac{1}{n!}C^{\underline{\alpha}_4\cdots\underline{\alpha}_1}D^{(n)}_{\underline{\alpha}_1\cdots\underline{\alpha}_n},$$

$$D^{(n)}_{\underline{\alpha}_1\cdots\underline{\alpha}_n} = \frac{1}{(4-n)!}C_{\underline{\alpha}_4\cdots\underline{\alpha}_1}D^{(n)\underline{\alpha}_{n+1}\cdots\underline{\alpha}_4},$$

$$D^{(4-n)\underline{\alpha}_1\cdots\underline{\alpha}_n}D^{(n)}_{\underline{\beta}_1\cdots\underline{\beta}_n} = \delta^{\underline{\alpha}_1}{}_{[\underline{\beta}_1}\cdots\delta^{\underline{\alpha}_n}{}_{\underline{\beta}_n]}D^{(4)},$$

$$D^{(2)}_{\underline{\alpha\beta}} = C_{\beta\alpha}D^{(2)}_{ab} + C_{ba}D^{(2)}_{\alpha\beta}, \qquad D^{(2)\underline{\alpha\beta}} = C^{\beta\alpha}C^{ac}C^{db}D^{(2)}_{cd} + C^{ba}D^{(2)\alpha\beta}, \tag{1.2}$$

and similar identities for $\bar{D}$'s. Derivatives have the normalizations

$$\{D_{\underline{\alpha}}, \bar{D}_{\underline{\dot\beta}}\} = i\partial_{\underline{\alpha\dot\beta}} = i\delta_a^b\partial_{\alpha\dot\beta}, \qquad \{D_{\underline{\alpha}}, D_{\underline{\beta}}\} = 0,$$

$$\partial^{\underline{\alpha\dot\beta}} = \delta_b^a\partial^{\alpha\dot\beta}, \qquad \partial^{\underline{\alpha\dot\gamma}}\partial_{\underline{\beta\dot\gamma}} = \delta_{\underline{\beta}}^{\underline{\alpha}}\Box,$$

$$D^{(4)}\vartheta^{(4)} = 1, \qquad \bar{D}^{(4)}D^{(4)}\bar{D}^{(4)} = \Box^2\bar{D}^{(4)}, \qquad \int d^4x d^4\vartheta = \int d^4x D^{(4)}. \quad (1.3)$$

Hermitian supervectors satisfy $\bar{A}^A = A^A$, $\bar{B}_A = (-1)^A B_A$, $\overline{A^A B_A} = A^A B_A$ (where $(-1)^A = 1$ for $A = \alpha\dot\alpha$, -1 for $A = \underline{\alpha}$ or $\underline{\dot\alpha}$), and iD_A is hermitian. In order to make U(1) transformation properties manifest, we will sometimes add antisymmetrized pairs of isovector indices: e.g., $D^{(4)} \to D^{(4)}_{[ab][cd]} = C_{ab}C_{cd}D^{(4)}$, $\bar{D}^{(2)}_{\dot\alpha\dot\beta} \to \bar{D}^{(2)[ab]}_{\dot\alpha\dot\beta} = C^{ab}\bar{D}^{(2)}_{\dot\alpha\dot\beta}$.

2. Matter multiplets

In this section we discuss the superfield formulation of the $N=2$ tensor and linearized vector multiplets. These two massless multiplets occur as submultiplets in $N=2$ supergravity [15]. They are therefore the compensating multiplets for supergravity, and they occur in the linearized supergravity action as in (2.1.1) and (2.2.1) below (with $m=0$), but also in crossterms with the supergravity gauge superfield, and with a different gauge invariance which allows them to be gauged away completely. Alternatively, a gauge can be chosen where the supergravity gauge superfield consists entirely of its irreducible superconformal part. (For the analogy in $N=1$ supergravity, see ref. [11].) This will be shown explicitly in sect. 3. (Note that the "vector multiplet" of ref. [15] is our *tensor* multiplet, and vice versa: This is because the superfield $\Phi^{[ab]}$ acts as a *vector* multiplet in a *higher-derivative* action, and similarly $V_a{}^b$ as a tensor multiplet.)

2.1. LINEARIZED VECTOR MULTIPLET

The linearized $N=2$ vector multiplet is described by a real isovector pseudoscalar superfield $V_a{}^b$ (where $V_a{}^b = \overline{(V_b{}^a)}$, $V_a{}^a = 0$) with action and invariance [5]

$$S_V = \tfrac{1}{6}\int d^4x d^4\vartheta \left(\tfrac{1}{2}C_{ab}W^{[ab]}\right)^2 = \frac{1}{24}\int d^4x\, D^{(4)}_{[ab][cd]}W^{[ab]}W^{[cd]} = \bar{S}_V,$$

$$W^{[ab]} = C^{ab}\left(\bar{D}^{(4)}D^{(2)}_{cd}C^{ce}V_e{}^d\right) = \bar{D}^{(4)[ab][ce]}D^{(2)}_{cd}V_e{}^d,$$

$$\delta V_a{}^b = D_{c\alpha}\chi^\alpha{}_a{}^{(bc)} + \bar{D}^c_{\dot\alpha}\bar\chi^{\dot\alpha b}{}_{(ac)}, \qquad \delta W^{[ab]} = 0, \qquad \chi^\alpha{}_b{}^{(ab)} = 0. \qquad (2.1.1)$$

The second forms of S and $W^{[ab]}(\equiv C^{ab}W)$ make manifest their chiral U(1) invariance. (We could also write $\chi^{\alpha}{}_{c}{}^{(ab)} = C_{dc}\chi^{\alpha(abd)}$ to make manifest that χ is pure isospin $\frac{3}{2}$.) The Bianchi identity and field equation are:

$$\text{Bianchi identity:} \qquad D^{(2)}_{ac}W^{[bc]} = \bar{D}^{(2)bc}\bar{W}_{[ac]};$$

$$\text{field equation:} \qquad D^{(2)}_{ac}W^{[bc]} = 0. \tag{2.1.2}$$

[Many of the D-identities from the appendix have been used to derive (2.1.1) and (2.1.2).]

This multiplet can also be described by covariant derivatives $\nabla_{A} = D_{A} - i\Gamma_{A}$ $(D_{A} \equiv (D_{\underline{\alpha}}, \bar{D}_{\underline{\dot{\alpha}}}, \partial_{\alpha\dot{\alpha}}))$ satisfying the constraints [16]:

$$\left\{\nabla_{\underline{\alpha}}, \nabla_{\underline{\beta}}\right\} = 2C_{ab}C_{\alpha\beta}\bar{W}, \qquad \left\{\nabla_{\underline{\alpha}}, \bar{\nabla}_{\underline{\dot{\beta}}}\right\} = i\nabla_{\underline{\alpha\dot{\beta}}} = i\delta_a^{\ b}\nabla_{\alpha\dot{\beta}}. \tag{2.1.3}$$

As in the $N=1$ supersymmetric Yang-Mills theory, in the manifestly hermitian vector representation where $(i\nabla_A)^{\dagger} = (-1)^A i\nabla_A$, the transformation law of the covariant derivative is

$$\nabla'_A = e^{iK}\nabla_A e^{-iK}, \qquad K = \bar{K}. \tag{2.1.4}$$

In the following we will give a derivation of the solution of the constraints in (2.1.3) for the linearized theory. To begin we note from the Bianchi identity in (2.1.2) and a D-identity that $D^{(4)}W = \Box\bar{W}$. This means that the first constraint in (2.1.3) can be solved by writing $\Gamma_{a\alpha}$ as

$$\Gamma_{a\alpha} = iC_{ab}\Box^{-1}D^{(3)b}_{\ \ \alpha}W + D_{a\alpha}f, \qquad f = \bar{f}, \tag{2.1.5}$$

where $D_{a\alpha}f$ is a gauge transformation term chosen to make $\Gamma_{a\alpha}$ a local function of $V_a^{\ b}$. ($f=0$ is a supersymmetric Landau gauge.) We can take this solution for $\Gamma_{a\alpha}$ and calculate the $\{\nabla_{\underline{\alpha}}, \bar{\nabla}_{\underline{\dot{\beta}}}\}$ anticommutator. We find (after performing more D-algebra)

$$\left\{\nabla_{\underline{\alpha}}, \bar{\nabla}_{\underline{\dot{\beta}}}\right\} = i\delta_a^{\ b}\left[\partial_{\alpha\dot{\beta}} + \Box^{-1}\left(\partial^{\gamma}{}_{\dot{\beta}}D^{(2)}_{\alpha\gamma}W - \partial_{\alpha}{}^{\dot{\gamma}}\bar{D}^{(2)}_{\dot{\beta}\dot{\gamma}}\bar{W}\right) - i(\partial_{\alpha\dot{\beta}}f)\right]$$
$$+ i\Box^{-1}\partial_{\alpha\dot{\beta}}\left(D^{(2)}_{ac}C^{bc}W - \bar{D}^{(2)bc}C_{ac}\bar{W}\right). \tag{2.1.6}$$

The last term is seen to vanish due to the Bianchi identity in (2.1.2). The other terms are proportional to $\delta_a^{\ b}$ and therefore define $\nabla_{\alpha\dot{\beta}}$. (It is interesting to note that $\partial_{\alpha}{}^{\dot{\gamma}}\bar{D}^{(2)}_{\dot{\beta}\dot{\gamma}}\bar{W} = -\partial^{\gamma}{}_{\dot{\beta}}D^{(2)}_{\alpha\gamma}W$ due to the fact that $D^{(2)}_{\alpha\beta}W = i\partial_{(\alpha}{}^{\dot{\gamma}}\Gamma_{\beta)\dot{\gamma}}$.) So (2.1.5) solves both kinematic constraints of (2.1.3). However, we have yet to determine an explicit form for f. To fix this function we substitute W from (2.1.1) into $\Gamma_{a\alpha}$ in (2.1.5) yielding

$$\Gamma_{a\alpha} = iC_{ab}\Box^{-1}D^{(3)b}_{\ \ \alpha}\left(\bar{D}^{(4)}D^{(2)}_{cd}C^{ce}V_e^{\ d}\right) + D_{a\alpha}f. \tag{2.1.7}$$

Now we commute $D^{(2)}_{cd}$ through $\bar{D}^{(4)}$ to find

$$\Gamma_{a\alpha} = iC_{ab}D^{(3)b}{}_{\alpha}\bar{D}^{(2)cd}C_{de}V_c^{\ e} + \tfrac{1}{2}D_{a\alpha}\left(\Box^{-1}\partial^{\beta\dot{\gamma}}\left\{\bar{D}^{(3)}_{c\dot{\gamma}}, D^{(3)b}{}_{\beta}\right\}V_b^{\ c}\right)$$
$$+ D_{a\alpha}\left(f - \tfrac{1}{2}\Box^{-1}\partial^{\beta\dot{\gamma}}\left[\bar{D}^{(3)}_{c\dot{\gamma}}, D^{(3)b}{}_{\beta}\right]V_b^{\ c}\right). \tag{2.1.8}$$

Finally we define a new real superfield U by

$$f = U + \tfrac{1}{2}\Box^{-1}\partial^{\beta\dot{\gamma}}\left[\bar{D}^{(3)}_{c\dot{\gamma}}, D^{(3)b}{}_{\beta}\right]V_b^{\ c}. \tag{2.1.9}$$

Evaluating the anticommutator in (2.1.8), we find that $\Gamma_{a\alpha}$ is a local function of $V_a^{\ b}$ and U:

$$\Gamma_{a\alpha} = iC_{ab}D^{(3)b}{}_{\alpha}\left(C_{de}\bar{D}^{(2)cd}V_c^{\ e}\right) - i\tfrac{1}{2}D_{a\alpha}\left(\left\{D^{(2)}_{be}, \bar{D}^{(2)bc}\right\}V_c^{\ e}\right) + D_{a\alpha}U. \tag{2.1.10}$$

[The first term in $\Gamma_{a\alpha}$ was obtained in ref. [5], but, as our argument above has shown, the second term is required to solve the second constraint in (2.1.6).] The variation of the compensating superfield U is given by

$$\delta U = K - i\tfrac{1}{4}\left(D^{(2)}_{ab}\bar{D}^{(3)\dot{\alpha}}_{c}\bar{\chi}^{(abc)}_{\dot{\alpha}} - \bar{D}^{(2)ab}D^{(3)c\alpha}\chi_{\alpha(abc)}\right)$$
$$- i\tfrac{1}{2}\left(\bar{D}^{(3)\dot{\alpha}}_{c}D^{(2)}_{ab}\bar{\chi}^{(abc)}_{\dot{\alpha}} - D^{(3)c\alpha}\bar{D}^{(2)ab}\chi_{\alpha(abc)}\right),$$

where

$$\chi_{\alpha(abc)} = C_{ad}C_{eb}\chi_{\alpha c}{}^{(de)}. \tag{2.1.11}$$

The K gauge parameter can be used to gauge U to zero. However, in order to remain in such a gauge we must require $\delta U = 0$ as a consistency condition. Thus, when we set U equal to zero, the remaining gauge invariance is such that K becomes a function of χ.

The form of $\Gamma_{a\alpha}$ in (2.1.10) implies that the $N=2$ superfield formulation of Yang-Mills theories is different from its $N=1$ counterpart in one respect. In the latter case it was always possible to choose a representation (a chiral representation) such that $\Gamma_\alpha \neq 0$ and $\bar{\Gamma}_{\dot{\alpha}} = 0$ or vice versa. This is no longer possible in the $N=2$ theory. The first term in (2.1.10) is not of the form of a *complex* gauge transformation of $\Gamma_{a\alpha}$. So while it is possible to eliminate the latter two terms in $\Gamma_{a\alpha}$ by such a transformation, the first term remains. However, a new feature appears in the $N=2$ theory: we can use the gauge invariance of the theory to change the form of $\Gamma_{a\alpha}$ while remaining in the vector representation. Such a change of representation is equivalent to a redefinition of U. For instance, we may make the following redefinition on the connection in (2.1.10):

$$\tilde{U} = U - i\tfrac{1}{2}a\left[D^{(2)}_{bd}, \bar{D}^{(2)bc}\right]V_c^{\ d}$$

$$\rightarrow \Gamma_{a\alpha} = iC_{ab}D^{(3)b}_{\ \alpha}\left(C_{de}\bar{D}^{(2)cd}V_c^{\ e}\right) + D_{a\alpha}\tilde{U}$$

$$- i\tfrac{1}{2}D_{a\alpha}\left[(1+a)\bar{D}^{(2)bc}D^{(2)}_{bd} + (1-a)D^{(2)}_{bd}\bar{D}^{(2)bc}\right]V_c^{\ d}. \tag{2.1.12}$$

The "new" connection thus obtained still satisfies the constraints of (2.1.3). But the transformation law of $\tilde{U}$ is

$$\delta\tilde{U} = K - i\tfrac{1}{4}(1+a)\left(D^{(2)}_{ab}\bar{D}^{(3)\dot{\alpha}}_{c}\bar{\chi}^{(abc)}_{\dot{\alpha}} - \bar{D}^{(2)ab}D^{(3)c\alpha}\chi_{\alpha(abc)}\right)$$

$$- i\tfrac{1}{2}(1-a)\left(\bar{D}^{(3)\dot{\alpha}}_{c}D^{(2)}_{ab}\bar{\chi}^{(abc)}_{\dot{\alpha}} - D^{(3)c\alpha}\bar{D}^{(2)ab}\chi_{\alpha(abc)}\right). \tag{2.1.13}$$

For the special case of $a=3$ in the K-gauge $\tilde{U}=0$ we find the interesting but confusing result that $\Gamma_{a\alpha}$ transforms under χ as a spacetime derivative:

$$K(\chi) = i\left(\left[D^{(2)}_{ab}, \bar{D}^{(3)\dot{\alpha}}_{c}\right]\bar{\chi}^{(abc)}_{\dot{\alpha}} - \left[\bar{D}^{(2)ab}, D^{(3)c\alpha}\right]\chi_{\alpha(abc)}\right)$$

$$= -\partial_{\alpha\dot{\gamma}}\left(\bar{D}^{(2)ef}D^{\alpha}_{b}\bar{\chi}^{\dot{\gamma}b}{}_{(ef)} + D^{(2)}_{ef}\bar{D}^{b\dot{\gamma}}\chi^{\alpha}{}_{b}{}^{(ef)}\right). \tag{2.1.14}$$

Since then every *component* of $\Gamma_{a\alpha}$ transforms under χ as a spacetime derivative, it would seem impossible to obtain a Wess-Zumino gauge. However, in this K-gauge every component of $\Gamma_{a\alpha}$ contains a *term* which is a spacetime derivative: e.g., a component of the equation $\delta\Gamma_{a\alpha} = D_{a\alpha}K(\chi)$ may take the form $\delta(A + \partial_{\alpha\dot{\beta}}B^{\alpha\dot{\beta}}) = \partial_{\alpha\dot{\beta}}\lambda^{\alpha\dot{\beta}}$, indicating that A is not a gauge field while $B^{\alpha\dot{\beta}}$ can be gauged to 0 by $\lambda^{\alpha\dot{\beta}}$. Thus, to get a clear interpretation of gauge transformations one should in general examine directly the transformation law of the unconstrained superfields (in this case, $V_a^{\ b}$) rather than constrained superfields (such as $\Gamma_{a\alpha}$).

2.2. TENSOR MULTIPLET

The $N=2$ tensor multiplet is described by a chiral superfield $\Phi^{[ab]}(\equiv C^{ab}\Phi)$, with action [3]

$$S_\Phi = \int d^4x d^8\vartheta\left(\tfrac{1}{2}\bar{\Phi}_{[ab]}\Phi^{[ab]}\right)$$

$$+ \left[\tfrac{1}{2}\int d^4x d^4\vartheta\left(\tfrac{1}{2}C_{ab}\Phi^{[ab]}\right)(2m^2 - \Box)\left(\tfrac{1}{2}C_{cd}\Phi^{[cd]}\right) + \text{h.c.}\right]. \tag{2.2.1}$$

(Compare (2.1.1) for chiral U(1); note $d^8\vartheta = d^4\vartheta d^4\bar{\vartheta}$ is U(1) invariant.) In the massless case ($m=0$), this action has the invariance [8] and field strength

$$\delta\Phi^{[ab]}=C^{ab}\bar{D}^{(4)}D^{(2)}_{cd}C^{ce}K_e{}^d, \qquad K_a{}^b=\overline{(K_b{}^a)}, \qquad K_a{}^a=0,$$

$$F_a{}^b=i\left(D^{(2)}_{ac}\Phi^{[bc]}-\bar{D}^{(2)bc}\bar{\Phi}_{[ac]}\right), \qquad F_a{}^b=\overline{(F_b{}^a)}, \qquad F_a{}^a=0. \tag{2.2.2}$$

The $m=0$ Bianchi identity and field equation are

Bianchi identity: $\qquad C_{d(a}D^{\alpha}_bF_{c)}{}^d=0,$

field equation: $\qquad i\tfrac{2}{3}\bar{D}^{(2)ac}F_c{}^b=C^{ab}(\Box\Phi-\bar{D}^{(4)}\bar{\Phi})=0. \qquad (2.2.3)$

Note the correspondence between some quantities associated with the vector multiplet and similar quantities associated with the tensor multiplet: We can make the exchanges

$$\left(V_a{}^b, W^{[ab]}, (\text{Bianchi identity})_a{}^b\right)\leftrightarrow\left(K_a{}^b, \Phi^{[ab]}, F_a{}^b\right). \tag{2.2.4}$$

The $m=0$ action can be re-expressed in terms of the field strength, but in a form where locality is not manifest:

$$S_\Phi=\tfrac{1}{6}\int d^4x\,d^8\vartheta\,F_a{}^b\Box^{-1}F_b{}^a. \tag{2.2.5}$$

We also note that the $m=0$ action can be expressed in terms of a superprojector [14] acting on a general complex scalar superfield ψ. We relate ψ to $\Phi_{[ab]}$ by the definition $\Phi=\bar{D}^{(4)}\psi$, and the action in terms of ψ becomes

$$S_\psi=\int d^4x\,d^8\vartheta\left(\bar{\psi}\Box^2\Pi_{0,0-}\psi+\text{h.c.}\right), \tag{2.2.6}$$

where $\Pi_{0,0-}$ is a projection operator for one of the irreducible representations contained in ψ (see eq. (4.2.3) in ref. [14]).

We can also obtain the $N=2$ tensor multiplet from a "geometrical" formulation based on a super two-form gauge superfield Γ_{AB} as in the $N=1$ case [7]. The gauge transformation of Γ_{AB} is parametrized by a real supervector K_A, and the appropriate field strength is a three-form F_{ABC}. The gauge variations explicitly are

$$\delta\Gamma_{\underline{\alpha}\underline{\beta}}=D_{\underline{\alpha}}K_{\underline{\beta}}+D_{\underline{\beta}}K_{\underline{\alpha}}, \qquad \delta\Gamma_{\underline{\alpha}\beta\dot{\beta}}=D_{\underline{\alpha}}K_{\beta\dot{\beta}}-\partial_{\beta\dot{\beta}}K_{\underline{\alpha}},$$

$$\delta\Gamma_{\underline{\alpha}\underline{\dot{\beta}}}=D_{\underline{\alpha}}\bar{K}_{\underline{\dot{\beta}}}+\bar{D}_{\underline{\dot{\beta}}}K_{\underline{\alpha}}-i\delta_a^bK_{\alpha\dot{\beta}}, \qquad \delta\Gamma_{\alpha\dot{\gamma}\beta}{}^{\dot{\gamma}}=\partial_{(\alpha\dot{\gamma}}K_{\beta)}{}^{\dot{\gamma}}, \tag{2.2.7}$$

with the gauge-invariant field strengths which follow:

$$
\begin{aligned}
&F_{\underline{\alpha\beta\gamma}}=\tfrac{1}{2}D_{(\underline{\alpha}}\Gamma_{\underline{\beta\gamma})},\\
&F_{\underline{\alpha\beta\dot\gamma}}=D_{(\underline{\alpha}}\Gamma_{\underline{\beta})\dot\gamma}}+\bar{D}_{\underline{\dot\gamma}}\Gamma_{\underline{\alpha\beta}}+i\delta_a^c\Gamma_{\underline{\beta}\alpha\dot\gamma}+i\delta_b^c\Gamma_{\underline{\alpha}\beta\dot\gamma},\\
&F_{\underline{\alpha\beta}\gamma\dot\gamma}=D_{(\underline{\alpha}}\Gamma_{\underline{\beta})\gamma\dot\gamma}+\partial_{\gamma\dot\gamma}\Gamma_{\underline{\alpha\beta}},\\
&F_{\underline{\alpha\dot\beta}\gamma\dot\gamma}=D_{\underline{\alpha}}\bar\Gamma_{\underline{\dot\beta}\gamma\dot\gamma}+\bar{D}_{\underline{\dot\beta}}\Gamma_{\underline{\alpha}\gamma\dot\gamma}+\partial_{\gamma\dot\gamma}\Gamma_{\underline{\alpha\dot\beta}}-i\tfrac{1}{2}\delta_a^b\left(C_{\dot\beta\dot\gamma}\Gamma_{\alpha\gamma}+C_{\alpha\gamma}\bar\Gamma_{\dot\beta\dot\gamma}\right),\\
&F_{\underline{\alpha}\beta\dot\delta\gamma}{}^{\dot\delta}=D_{\underline{\alpha}}\Gamma_{\beta\dot\delta\gamma}{}^{\dot\delta}-\partial_{(\beta\dot\delta}\Gamma_{\underline{\alpha}\gamma)}{}^{\dot\delta},\\
&F_{\underline{\alpha}\delta\dot\beta}{}^{\delta}{}_{\dot\gamma}=D_{\underline{\alpha}}\Gamma_{\delta\dot\beta}{}^{\delta}{}_{\dot\gamma}-\partial_{\delta(\dot\beta}\Gamma_{\underline{\alpha}}{}^{\delta}{}_{\dot\gamma)},\\
&F_{\alpha\dot\alpha\gamma\dot\beta\delta}{}^{\dot\beta}=\tfrac{1}{2}C_{\alpha(\gamma|}\left(\partial_{\varepsilon\dot\alpha}\Gamma_{|\delta)\dot\beta}{}^{\varepsilon\dot\beta}+\partial_{|\delta)}{}^{\dot\varepsilon}\Gamma_{\beta\dot\varepsilon}{}^{\beta}{}_{\dot\alpha}\right).
\end{aligned}
\tag{2.2.8}
$$

As usual in a supersymmetric gauge theory, a set of kinematic constraints must be imposed on some components of the field strength supertensor. Here we impose

$$
F_{\underline{\alpha\beta\gamma}}=F_{\underline{\alpha\beta\dot\gamma}}=F_{\underline{\alpha\beta}\gamma\dot\gamma}=0,\qquad F_{\underline{\alpha\dot\beta}\gamma\dot\gamma}=C_{\alpha\gamma}C_{\dot\beta\dot\gamma}F_a{}^b,
\tag{2.2.9}
$$

where $F_a{}^b$ satisfies the algebraic restrictions given in (2.2.2). A solution to these constraints is:

$$
\begin{aligned}
&\Gamma_{\underline{\alpha\beta}}=C_{\alpha\beta}\bar\Phi_{[ab]},\qquad \Gamma_{\underline{\alpha\dot\beta}}=0,\\
&\Gamma_{\underline{\alpha}\beta\dot\beta}=iC_{\alpha\beta}\bar{D}^c_{\dot\beta}\bar\Phi_{[ac]},\qquad \Gamma_{\alpha\dot\gamma\beta}{}^{\dot\gamma}=C_{ab}D^{(2)}_{\alpha\beta}\Phi^{[ab]}.
\end{aligned}
\tag{2.2.10}
$$

The general solution is obtained by first performing a gauge transformation [see (2.2.7)] on this particular solution and then replacing the gauge parameter K_A by a compensating supervector U_A in the resulting expressions. However, if we remain in the gauge above, then under the variation $\delta\bar\Phi_{[ab]}$ in (2.2.2) the super 2-form Γ_{AB} transforms according to (2.2.7) with K_A given by

$$
\begin{aligned}
&K_{\underline{\alpha}}=\tfrac{1}{2}C_{ab}D^{(3)b}{}_{\alpha}\bar{D}^{(2)cd}\left(C_{ce}K_d{}^e\right),\qquad \bar K_{\underline{\dot\beta}}=-\overline{\left(K_{\underline{\beta}}\right)},\\
&K_{\alpha\dot\beta}=-i\tfrac{1}{4}\left(C_{ab}C_{cd}\bar{D}^a_{\dot\beta}D^{(3)b}{}_{\alpha}\bar{D}^{(2)ce}K_e{}^d+C^{ab}C^{cd}D_{a\alpha}\bar{D}^{(3)}_{b\dot\beta}D^{(2)}_{ce}K_d{}^e\right).
\end{aligned}
\tag{2.2.11}
$$

In general gauges (where U_A is restored to Γ_{AB}) the variation $\delta\Phi^{[ab]}$ is an invariance of Γ_{AB} due to the fact that the variation δU_A exactly cancels the term above. Finally, it can be shown that the Bianchi identity of (2.2.2) is implied by the constraints in (2.2.9) and the geometric Bianchi identities on F_{ABC}.

Now having obtained the complete (linearized) formulation of the $N=2$ vector and tensor gauge multiplets we can explain our parenthetical remark at the beginning of this section. Note that in (2.1.1) we can replace the vector multiplet gauge superfield $V_a{}^b$ by the field strength for the tensor multiplet $F_a{}^b$. The component field content, which can be found by acting on $W(F_a{}^b)$ with spinorial derivatives, is the same as a vector multiplet. But all of these component fields except $W(F_a{}^b)|_{\vartheta=0}$ have extra space-time derivatives so that the action in (2.1.1) now leads to a higher derivative action in terms of the fields of $\Phi^{[ab]}$. Remarkably enough we can also reverse this procedure and replace the tensor multiplet gauge superfield $\Phi^{[ab]}$ by the field strength for the vector multiplet $W^{[ab]}$ in (2.2.2). The argument above applies again and (2.2.1) leads to a higher derivative action in terms of the fields of $V_a{}^b$. Thus we see how the vector and tensor multiplets can be switched in considering higher derivative actions for component fields.

We end by noting that the unconstrained $N=2$ vector and tensor multiplets also fit into the hierarchical structure noted for $N=1$ super p-form gauge superfields [7].

3. Supergravity

An unconstrained superfield appearing in an action has many properties in common with the tensor superfield which results from varying the action with respect to it. The gauge invariance of the former is directly related to the Bianchi identities of the latter: For an infinitesimal gauge parameter K of the gauge field ψ, $S(\psi+OK)=S(\psi)\leftrightarrow O^t(\delta S/\delta\psi)=0$ (where transposition of an operator is defined by $\int AOB=\pm\int(O^tA)B$, with $-$ for O and A anticommuting, $+$ otherwise). Likewise, just as the vanishing of the $\vartheta=0$ component of a tensor implies the vanishing of the entire tensor, the ability to gauge away the highest ϑ-component of a gauge superfield would imply that the entire superfield could be gauged away. Thus, the highest ϑ-component of a superfield appearing in an action also appears in the action. Conversely, the highest-dimension component field appearing in an action is the highest ϑ-component of a superfield appearing in the action. In $N=2$ supergravity this component field is an isospinor Weyl spinor of dimension $=\frac{3}{2}$ [15, 17]. (We use gravity dimensions, with all terms in the supergravity action requiring a factor of $1/\kappa^2$.) The gauge superfield is thus also an isospinor Weyl spinor $\psi^{a\alpha}$, with dimension $=-\frac{5}{2}$. This superfield, and the chiral scalar $\Phi^{[ab]}$ of dimension $=-1$ and the real isovector $V_a{}^b$ of dimension $=-3$ described in sect. 2, give the complete superfield structure of $N=2$ supergravity. The gauge transformation of $\psi^{a\alpha}$ takes the form of a general local superconformal transformation. All locally supersymmetric actions are invariant under these transformations when the compensating fields $\Phi^{[ab]}$ and $V_a{}^b$ are included, and the truly locally superconformal actions are furthermore independent of the compensating fields. The local superconformal transformations are simpler than the local supersymmetry transformations which result from gauging the compensating fields to zero. Also, the compensating fields are necessary for quantization [2].

3.1. CONFORMAL SUPERGRAVITY

As explained in ref. [14], the "reality" of the chiral superconformal field strength $W_{(\alpha\beta)}^{[ab]}$ [18] (ie., the fact that there is a Bianchi identity [19] relating W and $\bar{W}$) implies that it takes the following (linearized) form:

$$W_{(\alpha\beta)}^{[ab]} = C^{ab} W_{(\alpha\beta)} = \tfrac{1}{2} \bar{D}^{(4)[ab][cd]} D^{(2)}_{[cd]\alpha\beta} V, \qquad V = D_{\underline{\alpha}} \psi^{\underline{\alpha}} + \bar{D}_{\underline{\dot\alpha}} \bar{\psi}^{\underline{\dot\alpha}}. \quad (3.1.1)$$

The fact that $V(=\bar{V})$ is expressed in terms of a field $\psi^{\underline{\alpha}}$ does not (and cannot) follow from a linearized Bianchi identity. However, $\psi^{\underline{\alpha}}$ is necessary for Poincaré supergravity with a compensating tensor multiplet $\Phi^{[ab]}$, and probably also for the fully non-linear conformal supergravity. In terms of components, $V = D_{\underline{\alpha}} \psi^{\underline{\alpha}} + \bar{D}_{\underline{\dot\alpha}} \bar{\psi}^{\underline{\dot\alpha}}$ merely says that the highest ϑ-component of V is a divergence. $W_{(\alpha\beta)}^{[ab]}$ of (3.1.1) is the chiral field strength which appears in the projection $\Pi_{3,1,0+} \psi^{\underline{\alpha}}$ [see the appendix, eq. (A.6)]:

$$\begin{aligned} \Pi_{3,1,0+} \psi^{a}_{\alpha} &= \tfrac{1}{4} \Box^{-2} C^{ab} D^{\beta}_{b} \bar{D}^{(4)} D^{(2)}_{\alpha\beta} \left(D_{\underline{\gamma}} \psi^{\underline{\gamma}} + \bar{D}_{\underline{\dot\gamma}} \bar{\psi}^{\underline{\dot\gamma}} \right) \\ &= \tfrac{1}{4} \Box^{-2} D^{\beta}_{b} W^{[ab]}_{(\alpha\beta)}. \end{aligned} \quad (3.1.2)$$

($\Pi_{3,1,0-}$ could also be used, simply making the redefinitions $\psi^{\underline{\alpha}} \to i\psi^{\underline{\alpha}}$.)

The superconformal action is [19, 20]

$$S = \int \mathrm{d}^4x \, \mathrm{d}^4\vartheta \left(\tfrac{1}{2} C_{ab} W^{[ab](\alpha\beta)} \right) \left(\tfrac{1}{2} C_{cd} W^{[cd]}_{(\alpha\beta)} \right). \quad (3.1.3)$$

It (and W itself) can easily be shown to be invariant under the transformations

$$\begin{aligned} &\delta \psi^{a\alpha} = D_{b\beta} X^{(ab)(\alpha\beta)} + C^{ab} D_{b}{}^{\alpha} X + D_{b}{}^{\alpha} X^{(ab)} + i \bar{D}^{b}_{\dot\beta} X_{b}{}^{a\alpha\dot\beta}; \\ &X_{a}{}^{a\alpha\dot\beta} = 0, \qquad X_{a}{}^{b\alpha\dot\beta} = \overline{X_{b}{}^{a\beta\dot\alpha}}. \end{aligned} \quad (3.1.4)$$

This will also be the transformation law of $\psi^{\underline{\alpha}}$ in the non-superconformal case. (3.1.4) can also be obtained by requiring that the Wess-Zumino gauge for the next-to-highest ϑ-components of $\psi^{\underline{\alpha}}$ give the usual superconformal component fields [15] of that dimension.

3.2. POINCARE SUPERGRAVITY

According to the representation analysis of ref. [14], vector and tensor multiplets are projected from $\psi^{\underline{\alpha}}$ only by the projectors $\Pi_{1,0,0+}$ and $\Pi_{3,0,0+}$, respectively, of eq.

(A.6). (Both vector and tensor multiplets have superspin and supersisospin 0, but the former corresponds to a chiral superfield of even dimension, like $W^{[ab]}$, whereas the latter corresponds to one of odd dimension, like $\Phi^{[ab]}$). By requiring that a linear combination of $\Pi_{3,1,0+}$ [eq. (3.1.2)], describing the conformal part of the Poincaré supergravity multiplet, with a projection operator each for the vector and tensor multiplets appear in the Poincaré supergravity action $\frac{1}{2}\int d^4x d^8\vartheta \bar{\psi} i\partial\!\!\!/ \Box(\Sigma\Pi)\psi + \text{h.c.}$, we find the following unique local result:

$$S(\psi)=\tfrac{1}{2}\int d^4x d^8\vartheta \bar{\psi}^{\underline{\dot{\beta}}} i\partial_{\underline{\alpha\dot{\beta}}}\Box(\Pi_{3,1,0+}-\Pi_{3,0,0-}-2\Pi_{1,0,0-})\psi^{\underline{\alpha}}+\text{h.c.}$$

$$=\int d^4x d^8\vartheta\Big\{-\tfrac{1}{4}\Big[\big(\bar{D}_b^{(3)\dot{\alpha}}\bar{\psi}_{a\dot{\alpha}}\big)\big(D^{(3)b\beta}\psi^a_\beta\big)+\big(\bar{D}_{\underline{\dot{\alpha}}}\bar{\psi}^{\underline{\dot{\alpha}}}\big)\big(\Box+i\partial^{\underline{\beta\dot{\gamma}}}\bar{D}_{\underline{\dot{\gamma}}}D_{\underline{\beta}}\big)\big(D_{\underline{\delta}}\psi^{\underline{\delta}}\big)\Big]$$

$$+\tfrac{1}{8}\Big[\big(C_{ba}\bar{D}^{a\dot{\beta}}\psi^{ba}\big)D^{(4)}\big(C_{dc}\bar{D}^c_{\dot{\beta}}\psi^d_\alpha\big)+\big(D_{\underline{\alpha}}\psi^{\underline{\alpha}}\big)\bar{D}^{(4)}\big(D_{\underline{\beta}}\psi^{\underline{\beta}}\big)+\text{h.c.}\Big]\Big\}. \tag{3.2.1}$$

The answer is unique up to overall magnitude (but the sign is unique) and the choice between $\Pi_{1,0,0+}$ and $\Pi_{1,0,0-}$. (We have performed the algebra using identities from the appendix.) The relative coefficients of the Π's can quickly be seen to be correct by noting that the $D^4\bar{D}^4$ terms cancel as $1\cdot 3-1\cdot 1-2\cdot 1=0$ ($\Pi_{3,1,0+}$ has weight 3 because it contains the chiral field strength $W_{(\alpha\beta)}$ with 3 components; the others have chiral scalars), and the $(\Pi_{3,1,0+}-\Pi_{3,0,0-})\psi^{\underline{\alpha}}$ combination is necessary for their $\bar{\psi}$ terms to combine in the form $\partial^{\underline{\alpha\dot{\gamma}}}f_{\underline{\dot{\gamma}}}$ and so produce a $\Box$ when multiplied by the $\partial_{\underline{\alpha\dot{\beta}}}$. The compensating fields can be introduced by the (non-local) field redefinition

$$\psi^{\underline{\alpha}}\to\psi^{\underline{\alpha}}+\tfrac{1}{2}iC^{bc}D_b{}^\alpha V_c{}^a+\Box^{-1}D^\alpha_b\Phi^{[ab]}$$

or

$$\psi^{\underline{\alpha}}+\tfrac{1}{2}i\Box^{-2}D^{(3)\underline{\alpha}}W+\tfrac{1}{3}i\Box^{-2}D^\alpha_b\bar{D}^{(2)ac}F_c{}^b, \tag{3.2.2}$$

resulting in

$$S(\psi)\to S(\psi)+S(V,\psi)+S(\Phi,\psi),$$

$$S(V,\psi)=-12S_V-\int d^4x d^8\vartheta\big(iWD_{\underline{\alpha}}\psi^{\underline{\alpha}}+\text{h.c.}\big),$$

$$S(\Phi,\psi)=-2S_\Phi+\tfrac{1}{3}\int d^4x d^8\vartheta F_b{}^a i\big(D_{a\alpha}\psi^{b\alpha}-\bar{D}^b_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}}_a\big), \tag{3.2.3}$$

with W, $F_a{}^b$, S_V, and S_Φ as defined in sect. 2.

The action (3.2.3) is now invariant under local superconformal transformations, with ψ transforming as in (3.1.4) and the compensating fields transforming as

$$\delta V_a{}^b = -i\left(C_{ac}X^{(bc)} - C^{bc}\bar{X}_{(ac)}\right), \qquad \delta\Phi = \bar{D}^{(4)}\bar{X}. \tag{3.2.4}$$

These are the unique (local) transformations of V and Φ which allow some cancellation of parameters in the transformation of the shifted ψ of (3.2.2).

As explained at the beginning of this section, there is a direct correspondence between transformation laws and Bianchi identities. We thus have, from (3.1.4) and (3.2.4),

$$\begin{aligned}
&D_{\underline{\alpha}}T_{\underline{\beta}} = C_{\beta\alpha}\left(\tfrac{1}{4}C_{ba}\bar{M} + \tfrac{1}{2}iC_{ac}P_b{}^c\right) + t_{[ab](\alpha\beta)},\\
&\bar{D}_{\underline{\dot\alpha}}T_{\underline{\beta}} = \delta_b^a A_{\beta\dot\alpha} + V_b{}^a{}_{\beta\dot\alpha},\\
&\bar{D}_{\underline{\dot\alpha}}M = 0, \qquad P_a{}^a = 0, \qquad P_a{}^b = \overline{P_b{}^a}, \qquad V_a{}^a{}_{\alpha\dot\beta} = 0, \qquad V_a{}^b{}_{\alpha\dot\beta} = \overline{V_b{}^a{}_{\beta\dot\alpha}},
\end{aligned} \tag{3.2.5a}$$

where

$$\delta S \equiv \int d^4x\, d^8\vartheta\left[\left(\delta\psi^{\underline{\alpha}}\right)T_{\underline{\alpha}} + \text{h.c.} + \left(\delta V_a{}^b\right)P_b{}^a\right] + \left[\int d^4x\, d^4\vartheta\,(\delta\Phi)M + \text{h.c.}\right]. \tag{3.2.5b}$$

$T_{\underline{\alpha}}$ is the dimension $=\frac{1}{2}$ tensor of the theory; M, $P_a{}^b$, $t_{[ab](\alpha\beta)}$, $A_{\alpha\dot\beta}$, and $V_a{}^b{}_{\alpha\dot\beta}$ are the dimension $=1$ tensors, along with $W_{(\alpha\beta)}^{[ab]}$ of (3.1.1). All tensors can be expressed as derivatives of $T_{\underline{\alpha}}$ and $W_{(\alpha\beta)}^{[ab]}$ [18].

4. Covariant derivatives

Before we give the explicit linearized solution for the covariant derivatives in terms of unconstrained superfields, we will first review previous results. The covariant derivatives take the form

$$\begin{aligned}
&\nabla_A = \hat{E}_A + \left(\varphi_{A\beta}{}^{\gamma}M_\gamma{}^\beta + \varphi_{A\dot\beta}{}^{\dot\gamma}\bar{M}_{\dot\gamma}{}^{\dot\beta}\right), \qquad \hat{E}_A = E_A{}^B\hat{D}_B + \Gamma_A Z,\\
&\hat{D}_A = \left(D_{\underline{\alpha}} + \tfrac{1}{2}iC_{ab}\vartheta_\alpha^b Z,\ \bar{D}_{\underline{\dot\alpha}} - \tfrac{1}{2}iC^{ab}\bar\vartheta_{b\dot\alpha}Z,\ \partial_{\alpha\dot\alpha}\right),\\
&\left\{\hat{D}_{\underline{\alpha}}, \hat{D}_{\underline{\beta}}\right\} = C_{ab}C_{\alpha\beta}Z, \qquad \left\{\hat{D}_{\underline{\alpha}}, \hat{\bar{D}}_{\underline{\dot\beta}}\right\} = i\partial_{\underline{\alpha\dot\beta}}, \qquad \left[\hat{E}_A, \hat{E}_B\right\} = C_{AB}{}^C\hat{E}_C + F_{AB}Z,\\
&\left[\nabla_A, \nabla_B\right\} = T_{AB}{}^C\nabla_C + F_{AB}Z + \left(R_{AB\gamma}{}^\delta M_\delta{}^\gamma + R_{AB\dot\gamma}{}^{\dot\delta}\bar{M}_{\dot\delta}{}^{\dot\gamma}\right),
\end{aligned} \tag{4.1}$$

where $Z(=-Z^{\dagger})$ is the central charge, which annihilates all fields. (It is closely analogous to $\partial/\partial x^5$, with all fields x^5 independent.) The constraints are [9] (up to arbitrary redefinitions)

$$T_{\underline{\alpha}\underline{\beta}}{}^{C}=0, \qquad F_{\underline{\alpha}\underline{\beta}}=C_{ab}C_{\alpha\beta}; \tag{4.2a}$$

$$T_{\underline{\alpha}\underline{\beta}}{}^{\gamma\dot{\gamma}}=i\delta_a^b\delta_\alpha^\gamma\delta_{\dot{\beta}}^{\dot{\gamma}}, \qquad T_{\underline{\alpha}\underline{\beta}}{}^{\dot{\gamma}}=-\tfrac{1}{4}\delta_{\dot{\beta}}^{\dot{\gamma}}\delta_c^{(b}T_{a)\alpha}, \qquad R_{a\alpha\dot{\beta}\gamma}^{\;\;\;a}{}^{\delta}=F_{\underline{\alpha}\underline{\dot{\beta}}}=0. \tag{4.2b}$$

Since (4.2a) and the Bianchi identities imply $R_{a\alpha b\beta\gamma}{}^{\delta}=0$ [18], we can choose a Lorentz gauge where $\varphi_{a\alpha\beta}{}^{\gamma}=0$. In this gauge:

$$\left\{\hat{E}_{\underline{\alpha}},\hat{E}_{\underline{\beta}}\right\}=C_{ab}C_{\alpha\beta}Z \tag{4.3a}$$

$$\to \hat{E}_{\underline{\alpha}}=\mathrm{e}^{-\Omega}\hat{D}_{\underline{\alpha}}\mathrm{e}^{\Omega}, \qquad \Omega=\Omega^{A}i\hat{D}_{A}+\hat{\Omega}iZ. \tag{4.3b}$$

The spinor covariant derivative thus takes the form

$$\nabla_{\underline{\alpha}}=\mathrm{e}^{-\Omega}\hat{D}_{\underline{\alpha}}\mathrm{e}^{\Omega}+\varphi_{\underline{\alpha}\dot{\beta}}{}^{\dot{\gamma}}\overline{M}_{\dot{\gamma}}{}^{\dot{\beta}}. \tag{4.4}$$

From (4.2b) we also have $\{\nabla_{a\alpha},\overline{\nabla}{}^{a}_{\dot{\beta}}\}=2i\nabla_{\alpha\dot{\beta}}$, so that

$$\nabla_{\alpha\dot{\beta}}=-\tfrac{1}{2}i\left\{\nabla_{a\alpha},\overline{\nabla}{}^{a}_{\dot{\beta}}\right\}. \tag{4.5}$$

We can also apply (4.2b) to determine $\varphi^{a}_{\dot{\alpha}\beta}{}^{\gamma}$ (and thus $\varphi_{a\alpha\dot{\beta}}{}^{\dot{\gamma}}$):

$$0=T^{(a}_{\dot{\alpha}}{}_{b)(\beta}{}^{b\gamma)}=C^{(a}_{\dot{\alpha}}{}_{b)(\beta}{}^{b\gamma)}+6\varphi^{a}_{\dot{\alpha}\beta}{}^{\gamma}\to\varphi^{a}_{\dot{\alpha}\beta}{}^{\gamma}=-\tfrac{1}{6}C^{(a}_{\dot{\alpha}}{}_{b)(\beta}{}^{b\gamma)}, \tag{4.6a}$$

$$T_{(a\alpha\dot{\beta}}{}^{b)\gamma\dot{\gamma}}=F_{(a\alpha\dot{\beta}}{}^{b)}=0\to\left\{\hat{E}_{(a\alpha},\hat{E}^{b)}_{\dot{\beta}}\right\}=C_{(a\alpha\dot{\beta}}{}^{b)c\gamma}\hat{E}_{c\gamma}+C_{(a\alpha\dot{\beta}c}{}^{b)\dot{\gamma}}\hat{E}^{c}_{\dot{\gamma}}. \tag{4.6b}$$

It is now convenient to go to the chiral representation, where $\Omega\to H(=\overline{H})$, $\overline{\Omega}\to 0$, so $\hat{E}_{\underline{\dot{\alpha}}}(=\mathrm{e}^{\overline{\Omega}}\hat{\overline{D}}_{\underline{\dot{\alpha}}}\mathrm{e}^{-\overline{\Omega}})\to\hat{\overline{D}}_{\underline{\dot{\alpha}}}$. Then

$$\left\{\hat{\overline{D}}{}^{(a}_{\dot{\alpha}},\hat{E}_{b)\beta}\right\}=\begin{cases}\left(\overline{D}{}^{(a}_{\dot{\alpha}}E_{b)\beta}{}^{C}\right)\hat{D}_{C}+\left(\overline{D}{}^{(a}_{\dot{\alpha}}\Gamma_{b)\beta}\right)Z+iE_{(b\beta}{}^{a)\gamma}\partial_{\gamma\dot{\alpha}}+C^{c(a}E_{b)\beta c\dot{\alpha}}Z\\ C^{(a}_{\dot{\alpha}}{}_{b)\beta}{}^{c\gamma}\left(E_{c\gamma}{}^{D}\hat{D}_{D}+\Gamma_{c\gamma}Z\right)+C^{(a}_{\dot{\alpha}}{}_{b)\beta c}{}^{\dot{\gamma}}\hat{\overline{D}}{}^{c}_{\dot{\gamma}}\end{cases}$$

$$\to C^{(a}_{\dot{\alpha}}{}_{b)\beta}{}^{c\gamma}E_{c\gamma}{}^{d\delta}=\overline{D}{}^{(a}_{\dot{\alpha}}E_{b)\beta}{}^{d\delta}$$

$$\to\varphi^{a}_{\dot{\alpha}\beta}{}^{\gamma}=-\tfrac{1}{6}\left(\overline{D}{}^{(a}_{\dot{\alpha}}E_{b)(\beta|}{}^{d\delta}\right)E_{d\delta}^{-1\,b|\gamma)}. \tag{4.6c}$$

($E_{a\alpha}^{-1\,b\beta}$ here refers to the inverse of the matrix $E_{a\alpha}{}^{b\beta}$, *not* the corresponding component of the inverse supervierbein E_A^{-1B}.) At this point we have ∇_A completely in terms of H, from (4.4), (4.5), and (4.6c). These results are equivalent to those of ref. [10]. (Although Sokatchev doesn't introduce Z, and instead of using the chiral representation he uses the gauge $e^{\bar{\Omega}}x^{\alpha\dot{\alpha}} = x^{\alpha\dot{\alpha}} + iH^{\alpha\dot{\alpha}}$, $H^{\alpha\dot{\alpha}} = \bar{H}^{\alpha\dot{\alpha}}$, $\Omega_{\dot{\alpha}}^{a} = 0$.) However, our exponential notation is simpler: although (4.5) and (4.6c) are the same as in ref. [10], the equation corresponding to the relatively complicated set of eqs. (21), (24)–(26) of that paper is our much simpler eq. (4.3b), which is also in a more general gauge and includes the central-charge potential.

From the Bianchi identities we also find $(-1)^B T_{AB}{}^B = 0$, from which follows the interesting fact [10]:

$$0 = -(-1)^B T_{AB}{}^B E^{-1} = \left(E^{-1}E_A{}^B\right)\overleftarrow{D}_B = E^{-1}\overleftarrow{\nabla}_A$$

$$\to 0 = E^{-1}\overleftarrow{\nabla}_{\underline{\dot{\alpha}}} = \bar{D}_{\underline{\dot{\alpha}}}E^{-1}. \tag{4.7}$$

($E = \det E_A{}^B$, $\overleftarrow{\nabla}_A = E_A{}^B\overleftarrow{D}_B + \dots$. Note that the vector-derivative case follows from the spinor-derivative case, since $\overleftarrow{\nabla}_{\alpha\dot{\beta}} = +\frac{1}{2}i\{\overleftarrow{\nabla}_{a\alpha}, \overleftarrow{\underline{\underline{\nabla}}}{}^{a}_{\dot{\beta}}\}$ from (4.5).) Thus E^{-1} is chiral in the chiral representation *and* antichiral in the antichiral representation. It was conjectured in ref. [10], and has recently been shown [21], that the O(2) supergravity action is, in an arbitrary representation,

$$S = \int d^4x\, d^4\vartheta\, E^{-1} e^{-\overleftarrow{\bar{\Omega}}}. \tag{4.8}$$

The transformation laws of the covariant derivatives of (4.3)–(4.6) are

$$e^{\Omega'} = e^{i\bar{\Lambda}}e^{\Omega}e^{-iK} \qquad (\text{or}\, e^{H'} = e^{i\bar{\Lambda}}e^{H}e^{-i\Lambda}, \text{with}\, e^{H} \equiv e^{\Omega}e^{\bar{\Omega}}),$$

$$K = K^A i\hat{D}_A + \hat{K}iZ = \bar{K}, \qquad \Lambda = \Lambda^A i\hat{D}_A + \hat{\Lambda}iZ, \tag{4.9a}$$

$$\left[\hat{\bar{D}}{}^{a}_{\dot{\alpha}}, \Lambda^{\beta\dot{\beta}}i\partial_{\beta\dot{\beta}} + \Lambda^{b\beta}i\hat{D}_{b\beta} + \Lambda^{\dot{\beta}}_{b}i\hat{\bar{D}}{}^{b}_{\dot{\beta}} + \hat{\Lambda}iZ\right] = \bar{A}_{(\dot{\alpha}}{}^{\dot{\beta})}i\hat{\bar{D}}{}^{a}_{\dot{\beta}}. \tag{4.9b}$$

In a more general Lorentz gauge, there would also be a vector-representation Lorentz invariance; the gauge condition $\varphi_{a\alpha\beta}{}^{\gamma} = 0$ is preserved only by antichiral Lorentz transformations, which are represented by $\Lambda^{\dot{\alpha}}_{a}$ through $A_{(\alpha}{}^{\beta)}$ ((4.9b) implies $D_{a\alpha}A_{(\beta}{}^{\gamma)} = 0$). In ref. [10], the constraints (4.9b) appeared with $\hat{D}_A$ replaced by ∂_A (and without $\hat{\Lambda}$): the solution there (as for $N = 1$ [11]) for $\Lambda^{\alpha\dot{\alpha}}$, $\Lambda^{a\alpha}$ (and $\hat{\Lambda}$) was simply that they were chiral. Here we have instead the solution (analogous to the manifestly globally supersymmetric $N = 1$ form [11])

$$\Lambda^{\alpha\dot{\alpha}}i\partial_{\alpha\dot{\alpha}} + \Lambda^{\underline{\alpha}}i\hat{D}_{\underline{\alpha}} = \left(-i\bar{D}^{(3)\dot{\alpha}}_{a}L^{a\alpha}\right)i\partial_{\alpha\dot{\alpha}} + \left(\bar{D}^{(4)}L^{a\alpha}\right)i\hat{D}_{a\alpha},$$

$$\hat{\Lambda}iZ + \Lambda^{\underline{\dot{\alpha}}}i\hat{\bar{D}}_{\underline{\dot{\alpha}}} = \left(-\bar{D}^{(2)}_{\dot{\alpha}\dot{\beta}}\bar{L}^{(\dot{\alpha}\dot{\beta})}\right)iZ + \left(\bar{D}^{(3)}_{a\dot{\beta}}\bar{L}^{(\dot{\alpha}\dot{\beta})}\right)i\hat{\bar{D}}{}^{a}_{\dot{\alpha}},$$

$$A_{(\alpha\beta)} = D^{(4)}L_{(\alpha\beta)}, \tag{4.10}$$

where $L^{a\alpha}$ and $L^{(\alpha\beta)}$ are new, unconstrained gauge parameters. When we solve the remaining linearized constraints below, we will find that the L's simply gauge away the fields additional to $\psi^{a\alpha}$, $V_a{}^b$, and $\Phi^{[ab]}$, and the X transformations of (3.1.4) and (3.2.4) emerge upon solving the constraints, just as the K transformations gauge away fields additional to H (including the imaginary part of Ω), while the L (or Λ) transformations emerged upon solving the constraints in (4.3).

The remaining unsolved parts of the constraints (4.2) are

$$T_{(a\alpha\dot{\beta}}{}^{b)\gamma\dot{\gamma}} = F_{(a\alpha\dot{\beta}}{}^{b)} = T_{(a\alpha\dot{\beta}b}{}^{d\dot{\gamma}}C_{c)d} = 0\,. \tag{4.11}$$

(The last of these equations was neglected in ref. [10], and the second appears as a higher-dimensional, nonlinear torsion constraint when Z is not introduced.) Also, (4.2b) and (4.6c) define $T_{a\alpha}$:

$$\overline{T}^a_{\dot{\alpha}} = -\tfrac{1}{3}\left(\overline{D}^{(a}_{\dot{\alpha}}E_{b)\beta}{}^{c\gamma}\right)E^{-1b\beta}_{c\gamma}\,. \tag{4.12}$$

The non-linear forms of (4.11) in terms of $\hat{E}_{a\alpha}$ are [from the coefficients of $\hat{D}_A$ and Z in (4.6c)]

$$\overline{D}^{(a}_{\dot{\alpha}}E_{b)\beta}{}^{\varepsilon\dot{\varepsilon}} + i\delta^{\dot{\varepsilon}}_{\dot{\alpha}}E_{(b\beta}{}^{a)\varepsilon} - \left(\overline{D}^{(a}_{\dot{\alpha}}E_{b)\alpha}{}^{d\delta}\right)E^{-1c\gamma}_{d\delta}E_{c\gamma}{}^{\varepsilon\dot{\varepsilon}} = 0\,,$$

$$\overline{D}^{(a}_{\dot{\alpha}}\Gamma_{b)\beta} + C^{c(a}E_{b)\beta c\dot{\alpha}} - \left(\overline{D}^{(a}_{\dot{\alpha}}E_{b)\alpha}{}^{d\delta}\right)E^{-1c\gamma}_{d\delta}\Gamma_{c\gamma} = 0\,,$$

$$C^{e(a}\left(\overline{D}^{b|}_{\dot{\alpha}}E_{e\beta}{}^{d\delta}\right)E^{-1|c)\gamma}_{d\delta} = 0\,. \tag{4.13}$$

Using the linearized $\hat{E}_{a\alpha}$,

$$\begin{aligned}\hat{E}_{a\alpha} &\approx \hat{D}_{a\alpha} + \left[\hat{D}_{a\alpha}, H\right]\\ &= \hat{D}_{a\alpha} + i\left(D_{a\alpha}H^B\right)\hat{D}_B + i\left(D_{a\alpha}\hat{H}\right)Z + \overline{H}^{\dot{\beta}}_a\partial_{\alpha\dot{\beta}} + iC_{ab}H^b_\alpha Z\,,\end{aligned} \tag{4.14}$$

we find the linearized form of (4.13):

$$\overline{D}^{(a}_{\dot{\alpha}}D_{b)\beta}H^{\gamma\dot{\gamma}} = i\left(\delta^{\gamma}_{\beta}\overline{D}^{(a}_{\dot{\alpha}}\overline{H}^{\dot{\gamma}}_{b)} - \delta^{\dot{\gamma}}_{\dot{\alpha}}D_{(b\beta}H^{a)\gamma}\right), \tag{4.15a}$$

$$\overline{D}^{(a}_{\dot{\alpha}}D_{b)\beta}\hat{H} = C_{cb}\overline{D}^{(a}_{\dot{\alpha}}H^{c)}_{\beta} - C^{ca}D_{(b\beta}\overline{H}_{c)\dot{\alpha}}\,, \tag{4.15b}$$

$$C^{d(a}\overline{D}^b_{\dot{\alpha}}D_{d\beta}H^{c)\gamma} = 0\,. \tag{4.15c}$$

Also, the linearized $\overline{T}^a_{\dot{\alpha}}$ takes the form

$$\overline{T}^a_{\dot{\alpha}} = -\tfrac{1}{3}i\overline{D}^{(a}_{\dot{\alpha}}D_{b)\beta}H^{b\beta}\,. \tag{4.16}$$

We will now give the solution for the linearized unsolved constraints (4.15) on $H^{\underline{\alpha}}$, $H^{\alpha\dot{\alpha}}$, and $\hat{H}$ in terms of $\psi^{\underline{\alpha}}$. This is most easily done by analogy to our method of subsect. 2.1 for solving the constraints on the covariant derivatives of the vector multiplet: $(T_{\underline{\alpha}}, H^{\underline{\alpha}}, \psi^{\underline{\alpha}})$ correspond to the vector multiplet's $(W, \Gamma_{\underline{\alpha}}, V_a{}^b)$. Therefore, we will first use (4.16) to express $H^{\underline{\alpha}}$ in terms of $\psi^{\underline{\alpha}}$ in its non-local Landau gauge form, plus gauge terms ($\Lambda^{\underline{\alpha}}$ and $\overline{\Lambda}^{\underline{\alpha}}$). We will then determine the non-local gauge transformation which makes $H^{\underline{\alpha}}$ a local function of $\psi^{\underline{\alpha}}$. With this solution for $H^{\underline{\alpha}}$, we can then use (4.15) to verify that our original constraints were correct and solve for $H^{\alpha\dot{\alpha}}$ and $\hat{H}$.

By using the following identity:

$$-\tfrac{1}{3}i\overline{D}_{\dot{\alpha}}^{(a}D_{b)\beta}(\Pi_{3,1,0\pm}, \Pi_{3,0,0\pm}, \Pi_{1,0,0\pm})\psi^{b\beta}$$
$$= \partial_{\beta\dot{\alpha}}(\Pi_{3,1,0\pm}, -\Pi_{3,0,0\pm}, 2\Pi_{1,0,0\pm})\psi^{a\beta} \tag{4.17}$$

and the expression for $\overline{T}_{\underline{\dot{\alpha}}}$ in terms of $\psi^{\underline{\alpha}}$ [from (3.2.1) and (3.2.5b)],

$$\overline{T}_{\underline{\dot{\alpha}}} = i\partial_{\underline{\beta\dot{\alpha}}}\Box(\Pi_{3,1,0+} - \Pi_{3,0,0-} - 2\Pi_{1,0,0-})\psi^{\underline{\beta}}, \tag{4.18}$$

we easily find the following solution for $H^{\underline{\alpha}}$ from (4.16):

$$H^{\underline{\alpha}} = i\Box(\Pi_{3,1,0+} + \Pi_{3,0,0-} - \Pi_{1,0,0-})\psi^{\underline{\alpha}} - i\overline{D}^{(4)}L^{\underline{\alpha}} + iD^{(3)}{}_{\beta}{}^{a}L^{(\alpha\beta)}, \tag{4.19}$$

where the gauge terms $\delta H^{\underline{\alpha}} = -i\Lambda^{\underline{\alpha}}(L^{\underline{\alpha}}) + i\overline{\Lambda}^{\underline{\alpha}}(L^{(\alpha\beta)})$ come from (4.9a) and (4.10). After plugging in the expressions for the Π's and performing some algebra, we find

$$H^{\underline{\alpha}} = \tfrac{1}{4}i\Big(C^{ab}C^{cd}\overline{D}^{(3)}_{b\dot{\gamma}}D^{\alpha}_{c}\overline{\psi}^{\dot{\gamma}}_{d} - \tfrac{1}{2}D^{(3)\underline{\alpha}}\overline{D}_{\underline{\dot{\beta}}}\overline{\psi}^{\underline{\dot{\beta}}} - \overline{D}^{(2)c(a}D^{(2)}{}_{b)c}\psi^{b\alpha}$$
$$+ C^{ab}C_{cd}i\partial^{\beta\dot{\gamma}}\overline{D}^{c}_{\dot{\gamma}}D_{b\beta}\psi^{d\alpha} - \Box\psi^{\underline{\alpha}}\Big) - i\overline{D}^{(4)}\tilde{L}^{\underline{\alpha}} + iD^{(3)a}_{\beta}\tilde{L}^{(\alpha\beta)}, \tag{4.20a}$$

$$\tilde{L}^{\underline{\alpha}} = L^{\underline{\alpha}} + \tfrac{1}{4}\Box^{-1}\Big(C^{ab}C^{cd}i\partial_{\beta\dot{\gamma}}D^{(2)\beta\alpha}_{bc}\overline{\psi}^{\dot{\gamma}}_{d} + D^{(4)}\psi^{\underline{\alpha}}\Big),$$

$$\tilde{L}^{(\alpha\beta)} = L^{(\alpha\beta)} + \tfrac{1}{8}\Box^{-1}\Big(\tfrac{1}{3}D^{(\alpha}_{c}i\partial^{\beta)\dot{\gamma}}\overline{D}^{(2)}{}_{\underline{\dot{\gamma}\dot{\delta}}}\overline{\psi}^{\underline{\dot{\delta}}} + \tfrac{1}{3}D^{(\alpha}_{b}\overline{D}^{(4)}\psi^{b\beta)} - i\partial^{(\alpha\dot{\gamma}}\overline{D}^{(3)}_{b\dot{\gamma}}\psi^{b\beta)}\Big).$$

$$\tag{4.20b}$$

To solve the constraints (4.15) it is most convenient to use the Landau gauge ($L^{\underline{\alpha}} = L^{(\alpha\beta)} = 0$) form of $H^{\underline{\alpha}}$ from (4.19). (4.15c) is then satisfied directly. Plugging (4.19) into the right-hand side of (4.15a) and (4.15b), we then find the interesting result that $\hat{H}$ is *pure gauge* (i.e., 0 in Landau gauge), and $H^{\alpha\dot{\alpha}}$ *contains only the Weyl* *multiplet* (a $\Pi_{3,1,0+}\psi^{\underline{\alpha}}$ term) plus gauge. Thus, in a general gauge [again using (4.9a) and (4.10)],

$$\hat{H} = -i\left(D^{(2)}{}_{\alpha\beta} L^{(\alpha\beta)} - \bar{D}^{(2)}{}_{\dot{\alpha}\dot{\beta}} \bar{L}^{(\dot{\alpha}\dot{\beta})} \right) = \hat{\bar{H}},$$

$$H^{\alpha\dot{\alpha}} = -\tfrac{1}{2} i \partial_\beta{}^{\dot{\alpha}} D_a^{(\alpha} \Pi_{3,1,0+} \psi^{a\beta)} + \left(D^{(3)a\alpha} \bar{L}_a^{\dot{\alpha}} - \bar{D}^{(3)}{}_a{}^{\dot{\alpha}} L^{a\alpha} \right) = \bar{H}^{\alpha\dot{\alpha}}. \qquad (4.21)$$

The normalization and $(\alpha\beta)$ symmetry of $H^{\alpha\dot{\alpha}}$ were determined by substitution into (4.15a). Note that (4.15b) can be satisfied only if $\Pi_{1,0,0-}$, not $\Pi_{1,0,0+}$, is used. For $\Pi_{1,0,0+}$ to appear in (3.2.1) and consequent equations, (4.2a) would have to be modified to $F_{\alpha\beta} = iC_{ab}C_{\alpha\beta}$. (Remember that either $\Pi_{1,0,0-}$ or $\Pi_{1,0,0+}$ can give a local action.) To find local expressions for $\hat{H}$ and $H^{\alpha\dot{\alpha}}$ without affecting our expression for $H^{\underline{\alpha}}$, we modify our expressions for $\tilde{L}^{\alpha}$ and $\tilde{L}^{(\alpha\beta)}$ in (4.20b) by terms which won't contribute to $H^{\underline{\alpha}}$ (i.e., that satisfy $\bar{D}^{(4)}\Delta \tilde{L}^{\underline{\alpha}} = D^{(3)}{}_{\underline{\beta}}{}^{a}\Delta \tilde{L}^{(\alpha\beta)} = 0$). Such terms are simply arbitrary chiral (+antichiral) contributions to $\hat{H}$ and $H^{\alpha\dot{\alpha}}$. Our final result for H is thus, including (4.20a), but replacing (4.20b),

$$\hat{H} = -\frac{i}{12} C_{ab}\left(2 D_c^{\alpha} \bar{D}^{(2)ac} + \bar{D}^{(2)ac} D_c^{\alpha}\right)\psi_\alpha^b - i D^{(2)}_{\alpha\beta} \tilde{L}^{(\alpha\beta)} + \text{h.c.},$$

$$H^{\alpha\dot{\alpha}} = -\tfrac{1}{4}\left(\bar{D}^{a\dot{\alpha}} D^{(2)\alpha\beta}_{ab} \psi_\beta^b + i\partial^{\beta\dot{\alpha}} D_{a\beta}\psi^{a\alpha} \right) - \bar{D}^{(3)\dot{\alpha}}_{a} \tilde{L}^{a\alpha} + \text{h.c.}; \qquad (4.22a)$$

$$\tilde{L}^{\underline{\alpha}} = L^{\underline{\alpha}} + \tfrac{1}{4}\Box^{-1}\left(C^{ab}C^{cd} i\partial_{\beta\dot{\gamma}} D^{(2)\beta\alpha}_{bc} \bar{\psi}_d^{\dot{\gamma}} + D^{(4)}\psi^{\underline{\alpha}} \right) - \tfrac{1}{4}\Box^{-2}\partial^{\alpha\dot{\beta}}\partial^{\gamma\dot{\delta}} \bar{D}_{\dot{\delta}}^{b} D^{(3)a}{}_{\gamma} \bar{\psi}_{b\dot{\beta}},$$

$$\tilde{L}^{(\alpha\beta)} = L^{(\alpha\beta)} + \tfrac{1}{8}\Box^{-1}\Big(\tfrac{1}{3} D_c^{(\alpha} i\partial^{\beta)\dot{\gamma}} \bar{D}^{(2)}_{\underline{\dot{\gamma}\dot{\delta}}} \bar{\psi}^{\underline{\dot{\delta}}} + \tfrac{1}{3} D_b^{(\alpha} \bar{D}^{(4)} \psi^{b\beta)}$$

$$- i\partial^{(\alpha\dot{\gamma}} \bar{D}^{(3)}_{b\dot{\gamma}} \psi^{b\beta)} + \tfrac{1}{3} D^{(2)\alpha\beta} C^{ab} \bar{D}^{(3)}_{a\dot{\alpha}} \bar{\psi}_b^{\dot{\alpha}} \Big). \qquad (4.22b)$$

$\tilde{L}^{\underline{\alpha}}$ and $\tilde{L}^{(\alpha\beta)}$ are now regarded as compensating superfields, analogous to $\tilde{U}$ for the vector multiplet in subsect. 2.1. The compensating superfields Φ and $V_a{}^b$ are included into the formalism by making the shift (3.2.2), *not* on H, but on the covariant derivatives themselves [which have been expressed in terms of $\psi^{\underline{\alpha}}$ using (4.20a) and (4.22)]. Since all of ∇_A can be expressed in terms of $\hat{E}_{\underline{\alpha}}$ and $\hat{E}_{\underline{\dot{\alpha}}}$, we can express the net result as

$$\hat{E}_{\underline{\alpha}} \to \hat{E}_{\underline{\alpha}} - i\bar{W}\hat{D}_{\underline{\alpha}} - iF_a{}^b \hat{D}_{b\alpha} + \Gamma_{\underline{\alpha}}(V)Z,$$

$$\hat{E}_{\underline{\dot{\alpha}}} \to \hat{E}_{\underline{\dot{\alpha}}} + iW\hat{\bar{D}}_{\underline{\dot{\alpha}}} + iF_b{}^a \hat{\bar{D}}_{\dot{\alpha}}^b + \bar{\Gamma}_{\underline{\dot{\alpha}}}(V)Z; \qquad (4.23)$$

where $\Gamma_\alpha(V)$ is the expression given by (2.1.12) (with $\tilde{U} = 0$). We have performed the shift on the Landau gauge form of H, and then made a non-local K transformation [using $K^{\underline{\alpha}}$ and $\hat{K}$ of (4.9a) in addition to the Lorentz transformation $K_\alpha{}^\beta$] to make the shifts in (4.23) local. This K transformation causes $\hat{E}_{\underline{\dot\alpha}}$ to be shifted in spite of the fact that it is $\psi^{\underline{\alpha}}$ independent (because it is H independent), and maintains the chiral-representation hermiticity condition $(i\nabla_A)^\dagger = e^H(-1)^A(i\nabla_A)e^{-H}$. (4.23) also expresses the linearized transformation properties under superscale ($V_a{}^b \to V_a{}^b + \Delta V_a{}^b$) and superisospin ($\Phi \to \Phi + \Delta\Phi$) transformations: W is the constrained superscale parameter and $F_a{}^b$ the constrained superisospin parameter [18]. As for $K(\chi)$ in subsect. 2.1, one can also easily calculate $L^{\underline{\alpha}}(X)$ and $L^{(\alpha\beta)}(X)$ in terms of the X's of (3.1.4) and (3.2.4).

Appendix

D-ALGEBRA

We here list all (graded) commutators $[D^{(m)}, \bar{D}^{(n)}\}$ and the superprojectors for $\psi^{\underline{\alpha}}$. Algebra is greatly simplified by the use of SL(4,C) notation. In addition, we use the representation

$$C_{\underline{\alpha\beta\gamma\delta}} = C^{\underline{\dot\delta\dot\gamma\dot\beta\dot\alpha}}, \qquad \left(C_{\alpha\beta} = C^{\dot\beta\dot\alpha}\right), \tag{A.1}$$

which normalizes the absolute values of the components to 0 or 1. By working in a Lorentz frame where $i\partial_{\underline{\alpha\dot\beta}}\Box^{-1/2} = \delta^{\underline{\beta}}{}_{\underline{\alpha}} = -i\partial^{\underline{\beta\dot\alpha}}\Box^{-1/2}$ and using (A.1), we can then derive the useful identities

$$C_{\underline{\alpha\beta\gamma\delta}} = \Box^{-2}C^{\underline{\dot\alpha\dot\beta\dot\gamma\dot\delta}}\partial_{\underline{\alpha\dot\alpha}}\partial_{\underline{\beta\dot\beta}}\partial_{\underline{\gamma\dot\gamma}}\partial_{\underline{\delta\dot\delta}},$$

$$C^{\underline{\alpha\beta\gamma\delta}} = \Box^{-2}C_{\underline{\dot\alpha\dot\beta\dot\gamma\dot\delta}}\partial^{\underline{\alpha\dot\alpha}}\partial^{\underline{\beta\dot\beta}}\partial^{\underline{\gamma\dot\gamma}}\partial^{\underline{\delta\dot\delta}}, \tag{A.2}$$

and similar identities for $C_{\underline{\dot\alpha\dot\beta\dot\gamma\dot\delta}}$ and $C^{\underline{\dot\alpha\dot\beta\dot\gamma\dot\delta}}$. Using (A.2) we can easily evaluate all commutators:

$$\left[D^{(2)}{}_{\underline{\alpha\beta}}, \bar{D}_{\underline{\dot\gamma}}\right] = -i\partial_{[\underline{\alpha\dot\gamma}}D_{\underline{\beta}]},$$

$$\left\{D^{(3)\underline{\alpha}}, \bar{D}_{\underline{\dot\gamma}}\right\} = -i\partial_{\underline{\beta\dot\gamma}}D^{(2)\underline{\alpha\beta}},$$

$$\left[D^{(4)}, \bar{D}_{\underline{\dot\gamma}}\right] = i\partial_{\underline{\beta\dot\gamma}}D^{(3)\underline{\beta}},$$

$$\left[D^{(2)}{}_{\underline{\alpha\beta}}, \bar{D}^{(2)}{}_{\underline{\dot\gamma\dot\delta}}\right] = -i\partial_{[\underline{\alpha}[\underline{\dot\gamma}}D_{\underline{\beta}]}\bar{D}_{\underline{\dot\delta}]} - \partial_{[\underline{\alpha\dot\gamma}}\partial_{\underline{\beta}]\underline{\dot\delta}},$$

$$\left[D^{(3)\underline{\alpha}}, \bar{D}^{(2)}{}_{\underline{\dot\gamma\dot\delta}}\right] = -i\partial_{\underline{\beta}[\underline{\dot\gamma}} D^{(2)\underline{\alpha\beta}} \bar{D}_{\underline{\dot\delta}]} + C^{\underline{\alpha\beta\gamma\delta}} \partial_{\underline{\beta\dot\gamma}} \partial_{\underline{\gamma\dot\delta}} D_{\underline{\delta}},$$

$$\left[D^{(4)}, \bar{D}^{(2)}{}_{\underline{\dot\gamma\dot\delta}}\right] = i\partial_{\underline{\alpha}[\underline{\dot\gamma}} D^{(3)\underline{\alpha}} \bar{D}_{\underline{\dot\delta}]} - \partial_{\underline{\alpha\dot\gamma}} \partial_{\underline{\beta\dot\delta}} D^{(2)\underline{\alpha\beta}},$$

$$\left\{D^{(3)\underline{\alpha}}, \bar{D}^{(3)\underline{\dot\beta}}\right\} = i\partial_{\underline{\beta\dot\gamma}} D^{(2)\underline{\alpha\beta}} \bar{D}^{(2)\underline{\dot\beta\dot\gamma}} - \partial^{[\underline{\alpha\dot\beta}} \partial^{\underline{\delta}]\underline{\dot\gamma}} D_{\underline{\delta}} \bar{D}_{\underline{\dot\gamma}} + i\Box\partial^{\underline{\alpha\dot\beta}},$$

$$\left[D^{(4)}, \bar{D}^{(3)\underline{\dot\beta}}\right] = -i\partial_{\underline{\beta\dot\delta}} D^{(3)\underline{\beta}} \bar{D}^{(2)\underline{\dot\beta\dot\delta}} + \partial^{\underline{\alpha\dot\beta}} \partial^{\underline{\gamma\dot\delta}} D^{(2)}{}_{\underline{\alpha\gamma}} \bar{D}_{\underline{\dot\delta}} - i\Box\partial^{\underline{\alpha\dot\beta}} D_{\underline{\alpha}},$$

$$\left[D^{(4)}, \bar{D}^{(4)}\right] = -i\partial_{\underline{\alpha\dot\beta}} D^{(3)\underline{\alpha}} \bar{D}^{(3)\underline{\dot\beta}} - \tfrac{1}{2}\partial_{\underline{\alpha\dot\gamma}} \partial_{\underline{\beta\dot\delta}} D^{(2)\underline{\alpha\beta}} \bar{D}^{(2)\underline{\dot\gamma\dot\delta}} - i\Box\partial^{\underline{\alpha\dot\beta}} D_{\underline{\alpha}} \bar{D}_{\underline{\dot\beta}} - \Box^2. \tag{A.3}$$

The only reduction from SL(4,C) to SU(2) ⊗ SL(2,C) is the reduction of $D^{(2)}{}_{\underline{\alpha\beta}}$ to $D^{(2)}{}_{\alpha\beta}$ and $D^{(2)}{}_{ab}$ as in (1.2) (and similarly for $\bar{D}^{(2)}{}_{\underline{\dot\alpha\dot\beta}}$); (A.3) then becomes

$$\left[D^{(2)}{}_{ab}, \bar{D}^{c}{}_{\dot\gamma}\right] = i\tfrac{1}{2}\delta^{c}_{(a}\partial_{\beta\dot\gamma} D_{b)}{}^{\beta},$$

$$\left[D^{(2)}{}_{\alpha\beta}, \bar{D}^{c}{}_{\dot\gamma}\right] = i\tfrac{1}{2}C^{cb}\partial_{(\alpha\dot\gamma} D_{b\beta)},$$

$$\left\{D^{(3)a}{}_{\alpha}, \bar{D}^{b}{}_{\dot\beta}\right\} = iC^{ac}C^{db}\partial_{\alpha\dot\beta} D^{(2)}{}_{cd} - iC^{ab}\partial^{\gamma}{}_{\dot\beta} D^{(2)}{}_{\alpha\gamma},$$

$$\left[D^{(4)}, \bar{D}^{b}{}_{\dot\beta}\right] = i\partial_{\gamma\dot\beta} D^{(3)b\gamma},$$

$$\left[D^{(2)}{}_{ab}, \bar{D}^{(2)cd}\right] = -\tfrac{1}{4}i\delta^{(c}_{(a}\partial_{\alpha\dot\beta} D_{b)}{}^{\alpha} \bar{D}^{d)\dot\beta} - \tfrac{1}{2}\delta^{c}_{(a}\delta^{d}_{b)}\Box,$$

$$\left[D^{(2)}{}_{\alpha\beta}, \bar{D}^{(2)cd}\right] = i\tfrac{1}{4}C^{e(c}\partial_{(\alpha\dot\gamma} D_{e\beta)} \bar{D}^{d)\dot\gamma},$$

$$\begin{aligned}\left[D^{(3)a}{}_{\alpha}, \bar{D}^{(2)cd}\right] = &-\tfrac{1}{2}iC^{ae}C^{f(c}\partial_{\alpha\dot\gamma} D^{(2)}{}_{ef} \bar{D}^{d)\dot\gamma} - \tfrac{1}{2}iC^{a(c}\partial^{\delta\dot\gamma} D^{(2)}{}_{\alpha\delta} \bar{D}^{d)}{}_{\dot\gamma} \\ &- \tfrac{1}{2}C^{a(c}C^{d)e}\Box D_{e\alpha},\end{aligned}$$

$$\left[D^{(4)}, \bar{D}^{(2)cd}\right] = -\tfrac{1}{2}i\partial_{\alpha\dot\beta} D^{(3)(c\alpha} \bar{D}^{d)\dot\beta} - \Box C^{ca}C^{bd} D^{(2)}{}_{ab},$$

$$\left[D^{(2)}{}_{\alpha\beta}, \bar{D}^{(2)}{}_{\dot\gamma\dot\delta}\right] = -\tfrac{1}{4}i\partial_{(\alpha(\dot\gamma} D_{d\beta)} \bar{D}^{d}{}_{\dot\delta)} - \tfrac{1}{2}\partial_{(\alpha\dot\delta}\partial_{\beta)\dot\gamma},$$

$$\left[D^{(3)a}{}_{\alpha}, \bar{D}^{(2)}{}_{\dot\gamma\dot\delta}\right] = \tfrac{1}{2}iC^{ab}\partial_{\alpha(\dot\gamma} D^{(2)}{}_{bc} \bar{D}^{c}{}_{\dot\delta)} - \tfrac{1}{2}i\partial^{\beta}{}_{(\dot\gamma} D^{(2)}{}_{\alpha\beta} \bar{D}^{a}{}_{\dot\delta)} + \tfrac{1}{2}C^{ab}\partial_{\alpha(\dot\gamma}\partial_{\beta\dot\delta)} D_{b}{}^{\beta},$$

$$\left[D^{(4)}, \bar{D}^{(2)}{}_{\dot\gamma\dot\delta}\right] = -\tfrac{1}{2}iC_{cd}\partial_{\beta(\dot\gamma} D^{(3)c\beta} \bar{D}^{d}{}_{\dot\delta)} - \partial_{\alpha\dot\gamma}\partial_{\beta\dot\delta} D^{(2)\alpha\beta},$$

$$\left\{D^{(3)a}_{\ \ \alpha}, \bar{D}^{(3)}{}_{b\dot{\beta}}\right\} = i\delta_b^{[a}\delta_d^{c]}\partial_{\alpha\dot{\beta}}D^{(2)}{}_{ce}\bar{D}^{(2)de} - iC_{bc}\partial^{\gamma}{}_{\dot{\beta}}D^{(2)}{}_{\alpha\gamma}\bar{D}^{(2)ac}$$

$$-iC^{ac}\partial_{\alpha}{}^{\dot{\gamma}}D^{(2)}{}_{cb}\bar{D}^{(2)}{}_{\dot{\beta}\dot{\gamma}} + i\delta_b^a\partial^{\gamma\dot{\delta}}D^{(2)}{}_{\alpha\gamma}\bar{D}^{(2)}{}_{\dot{\beta}\dot{\delta}}$$

$$-\tfrac{1}{2}\delta_b^{(a}\delta_d^{c)}\Box D_{c\alpha}\bar{D}^{d}{}_{\dot{\beta}} - \tfrac{1}{2}\delta_b^{[a}\delta_d^{c]}\partial_{(\alpha\dot{\beta}}\partial_{\gamma)\dot{\delta}}D_c{}^{\gamma}\bar{D}^{d\dot{\delta}} + i\delta_b^a\partial_{\alpha\dot{\beta}}\Box\,,$$

$$\left[D^{(4)}, \bar{D}^{(3)}{}_{b\dot{\beta}}\right] = -iC_{ac}C_{bd}\partial_{\gamma\dot{\beta}}D^{(3)a\gamma}\bar{D}^{(2)cd} - iC_{bc}\partial_{\alpha}{}^{\dot{\gamma}}D^{(3)c\alpha}\bar{D}^{(2)}{}_{\dot{\beta}\dot{\gamma}}$$

$$-\Box D^{(2)}{}_{bc}\bar{D}^{c}{}_{\dot{\beta}} - C_{bc}\partial^{\gamma\dot{\delta}}\partial^{\varepsilon}{}_{\dot{\beta}}D^{(2)}{}_{\gamma\varepsilon}\bar{D}^{c}{}_{\dot{\delta}} + i\Box\partial_{\gamma\dot{\beta}}D_b{}^{\gamma},$$

$$\left[D^{(4)}, \bar{D}^{(4)}\right] = -i\partial^{\alpha\dot{\beta}}D^{(3)a}_{\ \ \alpha}\bar{D}^{(3)}{}_{a\dot{\beta}} - \Box D^{(2)}{}_{ab}\bar{D}^{(2)ab}$$

$$-\partial^{\alpha\dot{\beta}}\partial^{\gamma\dot{\delta}}D^{(2)}{}_{\alpha\gamma}\bar{D}^{(2)}{}_{\dot{\beta}\dot{\delta}} - i\Box\partial_{\alpha\dot{\beta}}D_a^{\alpha}\bar{D}^{a\dot{\beta}} - \Box^2. \tag{A.4}$$

Some other useful identities which follow from (1.2) and (A.4) are:

$$D_{\underline{\alpha}}D_{\underline{\beta}}D_{\underline{\gamma}} = D_{\underline{\alpha}}D^{(2)}{}_{\underline{\beta\gamma}} = C_{\underline{\delta\gamma\beta\alpha}}D^{(3)\underline{\delta}},$$

$$D_{\underline{\alpha}}D_{\underline{\beta}}D_{\underline{\gamma}}D_{\underline{\delta}} = D^{(2)}{}_{\underline{\alpha\beta}}D^{(2)}{}_{\underline{\gamma\delta}} = D^{(3)}{}_{\underline{\alpha\beta\gamma}}D_{\underline{\delta}} = C_{\underline{\delta\gamma\beta\alpha}}D^{(4)}$$

$$\to D_{a\alpha}D^{(2)}_{bc} = \tfrac{1}{2}C_{a(b}C_{c)d}D^{(3)d}{}_{\alpha},$$

$$D_a{}^{\alpha}D^{(2)}{}_{\beta\gamma} = \tfrac{1}{2}C_{ab}\delta^{\alpha}_{(\beta}D^{(3)b}{}_{\gamma)},$$

$$D^{(2)}{}_{ab}D^{(2)}{}_{cd} = \tfrac{1}{2}C_{c(a}C_{b)d}D^{(4)},$$

$$D^{(2)}{}_{\alpha\beta}D^{(2)}{}_{\gamma\delta} = -\tfrac{1}{2}C_{\gamma(\alpha}C_{\beta)\delta}D^{(4)},$$

$$D^{(3)a\alpha}D_{b\beta} = \delta_b^a\delta_\beta^\alpha D^{(4)}. \tag{A.5}$$

Finally, we list the projection operators for $\psi^{\underline{\alpha}}$ [14]:

$$\Pi_{0,1/2,1/2}\psi^{\underline{\alpha}} = \Box^{-2}D^{(4)}\bar{D}^{(4)}\psi^{\underline{\alpha}},$$

$$\Pi_{4,1/2,1/2}\psi^{\underline{\alpha}} = \Box^{-2}\bar{D}^{(4)}D^{(4)}\psi^{\underline{\alpha}},$$

$$\Pi_{2,1/2,3/2}\psi^{\underline{\alpha}} = \tfrac{1}{6}\Box^{-2}D^{(2)}{}_{de}\bar{D}^{(4)}C^{b(d}C^{e|c}D^{(2)}{}_{bc}\psi^{|a)\alpha},$$

$$\Pi_{2,1/2,1/2}\psi^{\underline{\alpha}} = \tfrac{2}{3}\Box^{-2}C^{ad}C^{ec}D^{(2)}{}_{de}\bar{D}^{(4)}D^{(2)}{}_{cb}\psi^{b\alpha},$$

$$\Pi_{2,3/2,1/2}\psi^{\underline{\alpha}} = \tfrac{1}{6}\Box^{-2}D^{(2)}{}_{\beta\gamma}\bar{D}^{(4)}D^{(2)(\alpha\beta}\psi^{a\gamma)},$$

$$\Pi'_{2,1/2,1/2}\psi^{\underline{\alpha}} = \tfrac{2}{3}\Box^{-2}D^{(2)\alpha\beta}\bar{D}^{(4)}D^{(2)}{}_{\beta\gamma}\psi^{a\gamma},$$

$$\Pi_{1,1,1\pm}\psi^{\underline{\alpha}} = \tfrac{1}{8}\Box^{-2}\Big(D^{(3)}{}_{\beta}{}^{b}\bar{D}^{(4)}D^{(\alpha}_{(b}\psi^{a)\beta)} \mp i\partial^{\alpha\dot{\beta}}\bar{D}^{b\dot{\gamma}}D^{(4)}\bar{D}^{(a}_{(\dot{\beta}}\bar{\psi}_{b)\dot{\gamma})}\Big),$$

$$\Pi_{1,1,0\pm}\psi^{\underline{\alpha}} = \tfrac{1}{8}\Box^{-2}\Big(D^{(3)a}_{\beta}\bar{D}^{(4)}D^{(\alpha}_{b}\psi^{b\beta)} \mp i\partial^{\alpha\dot{\beta}}\bar{D}^{a\dot{\gamma}}D^{(4)}\bar{D}^{b}_{(\dot{\beta}}\bar{\psi}_{b\dot{\gamma})}\Big),$$

$$\Pi_{1,0,1\pm}\psi^{\underline{\alpha}} = \tfrac{1}{8}\Box^{-2}\Big(-D^{(3)b\alpha}\bar{D}^{(4)}D_{(b\beta}\psi^{a)\beta} \mp i\partial^{\alpha\dot{\beta}}\bar{D}^{b}_{\dot{\beta}}D^{(4)}\bar{D}^{(a}_{\dot{\gamma}}\bar{\psi}_{b)}{}^{\dot{\gamma}}\Big),$$

$$\Pi_{1,0,0\pm}\psi^{\underline{\alpha}} = \tfrac{1}{8}\Box^{-2}\Big(-D^{(3)\underline{\alpha}}\bar{D}^{(4)}D_{\underline{\beta}}\psi^{\underline{\beta}} \mp i\partial^{\underline{\alpha\dot{\beta}}}\bar{D}_{\underline{\dot{\beta}}}D^{(4)}\bar{D}_{\underline{\dot{\gamma}}}\bar{\psi}^{\underline{\dot{\gamma}}}\Big),$$

$$\Pi_{3,1,1\pm}\psi^{\underline{\alpha}} = \tfrac{1}{8}\Box^{-2}D_{b\beta}\bar{D}^{4}\Big(D^{(3)(a(\alpha}\psi^{b)\beta)} \pm C^{bc}C^{da}D^{(\alpha}_{(c}i\partial^{\beta)\dot{\gamma}}\bar{\psi}_{d)\dot{\gamma}}\Big),$$

$$\Pi_{3,1,0\pm}\psi^{\underline{\alpha}} = \tfrac{1}{8}\Box^{-2}C^{ab}D_{b\beta}\bar{D}^{(4)}\Big(-C_{cd}D^{(3)c(\alpha}\psi^{d\beta)} \mp C^{cd}D^{(\alpha}_{c}i\partial^{\beta)\dot{\gamma}}\bar{\psi}_{d\dot{\gamma}}\Big),$$

$$\Pi_{3,0,1\pm}\psi^{\underline{\alpha}} = \tfrac{1}{8}\Box^{-2}D^{\alpha}_{b}\bar{D}^{(4)}\Big(-D^{(3)(a}_{\beta}\psi^{b)\beta} \mp C^{bc}C^{da}D^{\beta}_{(c}i\partial_{\beta\dot{\gamma}}\bar{\psi}^{\dot{\gamma}}_{d)}\Big),$$

$$\Pi_{3,0,0\pm}\psi^{\underline{\alpha}} = \tfrac{1}{8}\Box^{-2}C^{ab}D^{\alpha}_{b}\bar{D}^{(4)}\Big(C_{cd}D^{(3)c}_{\beta}\psi^{d\beta} \pm C^{cd}D^{\beta}_{c}i\partial_{\beta\dot{\gamma}}\bar{\psi}^{\dot{\gamma}}_{d}\Big). \tag{A.6}$$

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