

Do We Understand the Sphaleron Rate?^a

Guy D. Moore

Department of Physics, University of Washington, Seattle, Washington 98195

I begin by answering a different question, “Do we know the sphaleron rate?” and conclude that we do. Then I discuss a crude but purely analytic picture which provides an estimate of the sphaleron rate within the context of Bödeker’s effective theory. The estimate, which comes surprisingly close to the numerically determined sphaleron rate, gives a physical picture of baryon number violation in the hot phase, and provides a conjecture of the N_c dependence of the sphaleron rate in $SU(N_c)$ gauge theory.

1 Introduction

Originally I intended to entitle this talk, “Do we Know the Sphaleron Rate?” Unfortunately the talk would then have been much too short:

Yes.

Speaking seriously, it is appropriate here to remind the audience of what the sphaleron rate is, why we are interested in it, and how we go about calculating it. As I will discuss, we know the correct effective theory for determining the sphaleron rate; it is called Bödeker’s effective theory. We also understand what the physics which goes into deriving that effective theory, is. Further, we can compute within the effective theory quite accurately. But exactly what is going on, *within* the effective theory, is more opaque. In the second half of the talk I will try to present my picture of what the physics is, which will lead to a purely analytic estimate for the sphaleron rate which is closer than you might think to the right answer.

Baryon number (number of baryons minus anti-baryons, essentially the net amount of matter) is known to be very nearly conserved, but *not* exactly conserved, within the standard model and under ordinary conditions, as shown long ago by t’Hooft¹. The reason is the anomaly, which shows that the baryon number current is coupled to the $SU(2)$ nonabelian field strength;

$$\partial_\mu J_B^\mu = N_F \frac{g^2 F^{\mu\nu} \tilde{F}_{\mu\nu}}{32\pi^2}, \quad (1)$$

where $F^{\mu\nu}$ is the $SU(2)$ weak field strength and $N_F = 3$ is the number of generations (and I have dropped an irrelevant term involving hypercharge field

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strength). Further, the total derivative on the right hand side, integrated over spacetime in a vacuum to vacuum process, need not be zero; the topology of the group SU(2) shows that it can be any integer, and equals the instanton number of the vacuum to vacuum process. Hence baryon number is violated, but in vacuum to vacuum processes its violation only occurs nonperturbatively. Since the weak coupling is actually small, $\alpha_w \simeq 1/30$, the instanton suppression factor $\exp(-4\pi/\alpha_w)$ is enormous, and baryon number violation is of no consequence under normal conditions.

However, the very hot conditions relevant in the very early universe did not constitute “normal conditions.” At temperatures in excess of the electroweak phase transition (or crossover) temperature $T \sim 80\text{GeV}$, the population functions for infrared electroweak gauge bosons are large, and nonperturbative physics can be unsuppressed^{2,3}. This means that baryon number violation becomes efficient; in fact it is only polynomially, not exponentially, suppressed by the size of the weak coupling⁴.

Since the universe has a small, nonzero, and very interesting (to us!) abundance of baryons, understanding any cosmological process which changes baryon number seems well motivated, especially when it is associated with physics which we all believe must be there (the standard model gauge group). So there has been significant interest in understanding the efficiency of baryon number violation in the early universe.

The sphaleron rate, the topic of this talk, is defined to be the diffusion constant for the quantity on the right hand side in Eq. (1),

$$\Gamma \equiv \lim_{V \rightarrow \infty} \lim_{t \rightarrow \infty} \frac{\left\langle \left(\int dt \int d^3 \mathbf{x} \frac{g^2 F^{\mu\nu} \tilde{F}_{\mu\nu}}{32\pi^2} \right)^2 \right\rangle}{Vt} = \frac{1}{9} \Gamma_{N_B}, \quad (2)$$

with Γ_{N_B} the diffusion constant for baryon number. This is related to how fast a baryon number excess will decay, by a fluctuation dissipation relation. There is a free energy cost associated with having a net baryon number;

$$F = \frac{13}{12} \frac{N_B^2}{VT^2}. \quad (3)$$

Averaging over possible N_B with $\exp(-F/T)$ weight gives

$$\langle N_B^2 \rangle = \frac{6VT^3}{13}, \quad (4)$$

which must be sustained by a balance between baryon number diffusion and baryon number decay, resulting in

$$\frac{1}{N_B} \frac{dN_B}{dt} = \frac{39}{4} \frac{\Gamma}{T^3}. \quad (5)$$

Hence, the sphaleron rate tells us how fast a baryon number abundance will decay. Understanding its size is the object of the rest of the talk.

2 Bödeker's effective theory

The context where we want to compute the sphaleron rate is the hot standard model above the temperature of “electroweak symmetry restoration,” which to good accuracy means⁵ that we may work in Yang-Mills theory. It has been known for some time that the infrared behavior of Yang-Mills theory, even at weak coupling $\alpha_w \ll 1$, is non-perturbative³. The easiest way to understand this is to consider the thermodynamics. It is well known⁶ that the full path integral for the thermodynamics of the standard model is approximated up to corrections suppressed by powers of α_w by a 3-dimensional path integral,

$$Z = \int \mathcal{D}A_i \mathcal{D}\Phi \exp(-H/T),$$

$$H = \int d^3x \left[\frac{1}{4} F_{ij}^a F_{ij}^a + (D_i \Phi)^\dagger D_i \Phi + m^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \right]. \quad (6)$$

It is also well known that this 3-dimensional theory is super-renormalizable. That means that it has a characteristic momentum scale, in this case $g^2 T$. On scales very large compared to this, it is weakly coupled; in fact it grows more weakly coupled as a *power* of scale (until the effective theory breaks down at $p \sim \pi T$, where it is as weakly coupled as the underlying theory); conversely, as we look at more infrared scales it becomes more strongly coupled, until at the scale $g^2 T$ its interactions are nonperturbative. Crudely speaking we could say that this occurs because the occupation number of a Bosonic state with momentum p is $\sim T/p$; and that the loop counting parameter is enhanced by one power of the occupation number. This also suggests a nice way of understanding the physics which occurs at strong coupling; large occupation number is the classical field correspondence limit, and Eq. (6) is the partition function of the classical field theory at finite temperature.

This correspondence led Grigoriev and Rubakov to conjecture⁷ that the right effective theory to determine the sphaleron rate is the classical field theory, regulated for instance on a lattice. We see that this cannot be true in Fig. 1; the dependence on the lattice spacing is nontrivial and does not go away at suitably small spacing. This physics was predicted, before the data for the figure were taken, by Arnold, Son, and Yaffe⁹. They argued correctly that “hard” (momentum scale $p \sim \pi T$) physics is important to the dynamics of the infrared gauge fields, which are responsible for Chern-Simons number diffusion. Though not originally presented this way, their argument boils down

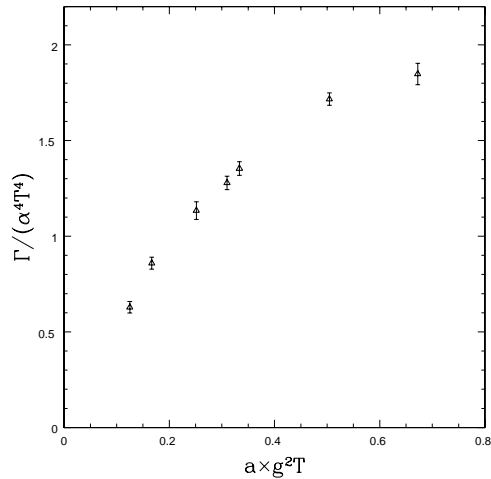


Figure 1: Γ for pure lattice classical Yang-Mills theory against lattice spacing. It does not show rapid convergence to a nonzero limit at zero spacing. Data from ⁸.

to fairly well known plasma physics. A plasma—abelian or nonabelian—is highly conducting. In a conducting medium, magnetic fields evolve slowly, because electric fields interact strongly with the plasma. In particular, the time evolution of a magnetic field is related to the magnitude of an electric field by (the nonabelian) Faraday’s law. But an electric field is quickly shorted out, because it generates a current. The nonabelian Ampere’s law reads

$$-\dot{\mathbf{E}} + \mathbf{D} \times \mathbf{B} = \mathbf{j}. \quad (7)$$

The greater the strength of screening in the medium, the greater the importance of the current term, which because it is a conducting medium is of form $\sim \int \mathbf{E} dt$, and causes the $\dot{\mathbf{E}}$ term to “undo” past electric fields. More accurately, the current is

$$\mathbf{j}^a(\mathbf{x}, t) = m_D^2 \int \frac{d\Omega_v}{4\pi} \mathbf{v} \int dy U^{ab}(\mathbf{x}, t; \mathbf{x} - \mathbf{v}y, t - y) \mathbf{v} \cdot \mathbf{E}^b(\mathbf{x} - \mathbf{v}y, t - y), \quad (8)$$

with \mathbf{v} a unit vector and its integral $d\Omega_v$, taken over the unit sphere. Here U^{ab} is the straight line, adjoint parallel transporter with initial and final points shown, which renders the equation gauge covariant. Together, Eq. (7) and Eq. (8) give classical Yang-Mills theory with added hard thermal loop effects^{10,11}. (One must also add a noise term, which has a form completely determined by equilibrium thermodynamics; the noise correlator is nonlocal

and we will not present it, see^{12,13}.) Arnold, Son, and Yaffe advocated considering this effective theory, and argued that the sphaleron rate should scale as $\Gamma \propto 1/m_D^2$.

Two numerical implementations of this effective theory exist^{14,15}. Both are quite complicated and I will not describe them here, though I will momentarily discuss their results for Γ .

Bödeker has argued that it is possible to integrate out the scale gT from the effective theory just presented to arrive at a new, “more effective” theory¹⁶. The physical content of his calculation (which is fairly complicated^{16,17,18,19}) is that scatterings between the excitations carrying the current j , exchange the nonabelian charges of the current carriers. The mean rate for nonabelian charge randomization turns out to be

$$\gamma = \frac{N_c g^2 T}{4\pi} \left[\log \frac{m_D}{g^2 T} + O(1) \right]. \quad (9)$$

This is parametrically larger than the scale where nonperturbative physics happens, $\sim g^2 T$, if we are willing to expand in $\log(m_D/g^2 T)$. Doing so, we conclude that a current carrier scatters frequently compared to the time it takes to traverse the length scale we are interested in, $\sim 1/g^2 T$. Therefore, to leading order the current can only depend on the local value of fields, and we may replace Eq. (8) with a local expression, which must be of form

$$\mathbf{j} = \sigma \mathbf{E}. \quad (10)$$

For obvious reasons the constant σ has been christened the nonabelian conductivity [or “color conductivity,” if we think about the $SU(N_c)$ theory rather than $SU(2)$]. It turns out one may determine σ to next to leading log order; at this order it is

$$\sigma^{-1} = \frac{3}{m_D^2} \gamma, \quad \gamma = \frac{N_c g^2 T}{4\pi} \left[\ln \frac{m_D}{\gamma} + 3.041 \right]. \quad (11)$$

The value for the constant 3.041 is from²⁰. Further, on the scales of interest the term $\dot{\mathbf{E}}$ in Eq. (7) can be dropped relative to the current term, yielding Bödeker’s effective theory,

$$\mathbf{j} = \sigma \mathbf{E} = \mathbf{D} \times \mathbf{B} + \xi, \quad \langle \xi_i^a(\mathbf{x}, t) \xi_j^b(\mathbf{y}, t') \rangle = 2\sigma T \delta^{ab} \delta_{ij} \delta(\mathbf{x} - \mathbf{y}) \delta(t - t'), \quad (12)$$

with the noise Gaussian and white, with the autocorrelator shown.

To determine Γ within this effective theory requires dealing with the non-linear form of $\mathbf{D} \times \mathbf{B}$, necessarily including scales where the appearance of the

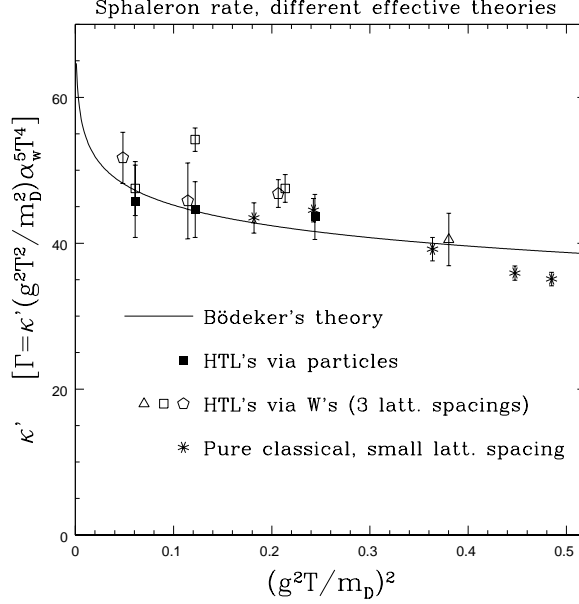


Figure 2: Sphaleron rate in Bödeker's effective theory, two lattice implementations of HTL effective theory^{14,15}, and pure lattice theory interpreted as HTL effective theory (see²⁴).

gauge field in \mathbf{D} is as important as the derivative term. It is a hard nonlinear problem and the only controllable techniques known are lattice techniques. However as a lattice problem it is particularly amenable to solution. In particular, the effective theory has a good, well defined limit as the UV regulator is taken to infinity¹⁸, and it is possible to compute analytically a matching between a lattice implementation and the continuum theory so the first errors are quadratic in lattice spacing²¹, and to define $\int F^{\mu\nu} \tilde{F}_{\mu\nu}$ topologically^{22,23}. One finds numerically that^{21,5}

$$\Gamma = \kappa' \left(\frac{g^2 T^2}{m_D^2} \right) \alpha_w^5 T^4, \quad \kappa' = (10.0 \pm 0.2) \left[\ln \frac{m_D}{\gamma} + 3.041 \right]. \quad (13)$$

This is also in good agreement with the results of the HTL effective theory, as summarized in Fig. 2.

3 Model for the nonperturbative behavior

I would argue that we understand pretty well *why* Bödeker's effective theory is the correct effective description of the the infrared physics behind baryon

number violation. We understand why the IR behavior is classical, and why the length scale $1/g^2T$ naturally sets the scale for topology change. We understand why IR gauge field evolution is over-damped, in terms of plasma physics. And we understand why, in a nonabelian theory, a nonabelian Ohm's law becomes valid on the scale $1/g^2T \log(1/g)$. But the number in front of Eq. (13) was computed via lattice Monte-Carlo means, which is sort of a “black box.” It would be nice to have some physical picture of what is going on *within* the effective theory. Here I provide a crude picture, which leads to a crude analytic estimate of the sphaleron rate in Bödeker's effective theory.

The point by point distribution of $F^{\mu\nu}\tilde{F}_{\mu\nu}$ is as ultraviolet sick as that of F^2 . So why is its diffusion constant finite and infrared dominated? The answer is that $F^{\mu\nu}\tilde{F}_{\mu\nu}$ has topological meaning. It is a total derivative and can be written as the spacetime gradient of a current, $(g^2/32\pi^2)F^{\mu\nu}\tilde{F}_{\mu\nu} = \partial^\mu K_\mu$. The associated charge is the Chern-Simons number,

$$\int d^3x K_0 \equiv N_{\text{CS}} = \frac{g^2}{32\pi^2} \int d^3x \epsilon_{ijk} \left(F_{ij}^a A_k^a - \frac{g}{3} f_{abc} A_i^a A_j^b A_k^c \right). \quad (14)$$

In vacuum the integral of the A^3 term is always an integer; so there must be nonvanishing fields for N_{CS} to take on a non-integral value. And in particular, since $F_{ij}A_k$ has one fewer derivative than F^2 , the more concentrated the region where N_{CS} is stored, the greater the energetic cost. Physically, this means that a fluctuation which creates a small amount of N_{CS} leaves an imprint or “strain” behind in the fields, so there is gradient energy present until either the fields relax in a way which undoes the N_{CS} generated, or enough more is added to get up to the next integer. This is what prevents there from being any UV divergences—in fact, any unsuppressed UV contribution at all—in Γ .

The other piece of physics we know about the symmetric electroweak phase, or pure 3-D Yang-Mills theory, is that there is magnetic screening. Colored degrees of freedom interact with each other exclusively through gradient interactions, so a colored field at \mathbf{x} feels the fields at \mathbf{y} only through a parallel transporter from \mathbf{y} to \mathbf{x} . In fact, the field equations mean that the fields at \mathbf{x} will generally evolve under the influence of \mathbf{y} through some average over paths for parallel transportation.

For \mathbf{x} and \mathbf{y} of small separation $\ll 1/g^2T$, two typical paths between them will have approximately the same parallel transporter. Therefore, the interactions via the different paths will add coherently. But for \mathbf{x} and \mathbf{y} separated by $\gtrsim 1/g^2T$, a generic pair of paths give completely different parallel transportations. (Equivalently, the mean trace of large Wilson loops goes to zero.) Roughly, there is a length scale l_{mag} , called the magnetic screening length, beyond which parallel transportation differs by $O(1)$ on change of path. This

means that the influence of fields at \mathbf{y} on fields at \mathbf{x} add incoherently over paths and will approximately cancel. A field cannot “see” colored objects more than l_{mag} away. This arises because of large nonabelian fields in between the two points, which are responsible for making the parallel transportations differ.

The physical picture I argue for is that magnetic screening renders the “gradient energy” argument for keeping N_{CS} from changing, ineffective for field fluctuations on scales $\gtrsim l_{\text{mag}}$. Hence, N_{CS} will diffuse due to $F^{\mu\nu}\tilde{F}_{\mu\nu}$ from fields at the scale l . These fields will evolve diffusively; but fields on less IR scales will not contribute to the diffusion of N_{CS} .

Within this picture, we can make a crude calculation of Γ , as follows. Consider some large volume V as a collection of boxes l_{mag} on a side. Within each box, the most IR gauge field degrees of freedom diffuse, and all higher wave number modes do not contribute; and the contribution of different boxes are independent. Writing $F^{\mu\nu}\tilde{F}_{\mu\nu} = 4E_i^a B_i^a$, Eq. (2) becomes

$$\Gamma = \frac{1}{Vt} \sum_{\text{boxes}} \left\langle \left(\int dt' d^3x \frac{g^2 E_i^a(x, t') B_{i, \text{IR}}^a(x, t')}{8\pi^2} \right)^2 \right\rangle, \quad (15)$$

where the subscript on the magnetic field means that only the most infrared component is kept. The sum over directions gives the two independent transverse polarizations, giving

$$\frac{2g^4}{64\pi^4 Vt} \sum_{\text{boxes}} \int dt_1 dt_2 d\mathbf{x}_1 d\mathbf{x}_2 \langle E^a(\mathbf{x}_1, t_1) E^b(\mathbf{x}_2, t_2) B_{\text{IR}}^a(\mathbf{x}_1, t_1) B_{\text{IR}}^b(\mathbf{x}_2, t_2) \rangle. \quad (16)$$

We determine the E field correlator from Eq. (12). Our argument about scales says that, if we consider only the most infrared component of the magnetic field, then we can neglect the gradient term, $\mathbf{D} \times \mathbf{B}$, in evaluating the E correlators. In this case E and B are uncorrelated, and the E correlators come only from the noise,

$$\langle E^a(x_1, t_1) E^b(x_2, t_2) \rangle = \frac{2T}{\sigma} \delta^{ab} \delta(x_1 - x_2) \delta(t_1 - t_2). \quad (17)$$

(For higher frequency excitations the $\mathbf{D} \times \mathbf{B}$ term is what will ensure no contribution.) This performs one space and one time integral:

$$\begin{aligned} & \int dx_1 dx_2 dt_1 dt_2 \langle E^a(x_1, t_1) E^b(x_2, t_2) B_{\text{IR}}^a(x_1, t_1) B_{\text{IR}}^b(x_2, t_2) \rangle \\ &= \frac{2T}{\sigma} \delta^{ab} \int dx_1 dx_2 dt_1 dt_2 \delta(x_1 - x_2) \delta(t_1 - t_2) \langle B_{\text{IR}}^a(x_1, t_1) B_{\text{IR}}^b(x_2, t_2) \rangle \\ &= \frac{2T}{\sigma} \delta^{ab} \int_0^t dt_1 \int dx \langle B_{\text{IR}}^a(x, t_1) B_{\text{IR}}^b(x, t_1) \rangle. \end{aligned} \quad (18)$$

This integral can be estimated by equipartition,

$$\int_0^t dt_1 \int d^3x \langle B_{\text{IR}}^a(x, t_1) B_{\text{IR}}^b(x, t_1) \rangle = T \delta^{ab} \int_0^t dt_1 = T t \delta^{ab}. \quad (19)$$

Probably there is more power in the infrared magnetic fields than equipartition would estimate; but this is only one point at which our analysis is crude.

The group indices have reduced to $\delta^{ab} \delta^{ab} = N_c^2 - 1$. The volume of a box is l_{mag}^3 , so in a total volume V there are V/l_{mag}^3 boxes. Putting everything together, we get

$$\Gamma = \frac{4(N_c^2 - 1)g^4 T^2}{64\pi^4 l_{\text{mag}}^3 \sigma}, \quad (20)$$

which, on substituting Eq. (11) and re-arranging, gives

$$\Gamma = \frac{48N_c^3(N_c^2 - 1)}{(N_c g^2 T l_{\text{mag}})^3} \left(\frac{N_c g^2 T^2}{m_D^2} \right) \alpha_w^5 T^4 \left[\log \frac{m_D}{\gamma} + 3.041 \right]. \quad (21)$$

It remains to estimate l_{mag} , which is a thermodynamic question and can be answered within the 3-D Euclidean theory, Eq. (6). The only natural length scale of hot Yang-Mills theory at weak coupling is $l_{\text{mag}} \sim 1/g^2 T$, and at large N_c the N_c dependence must be $l_{\text{mag}} \sim 1/N_c g^2 T$. The only question is what the coefficient is. I estimate it by trying to find on what length scale the perturbative expansion for 3-D Yang-Mills theory (describing the thermodynamics of the thermal theory) has completely broken down.

One estimate is that $l_{\text{mag}} \sim 1/p$ for the momentum p with $|\Pi_T(p)| = p^2$, that is, where the one loop transverse self-energy computed with tree propagators is as large as the tree inverse propagator. In Landau gauge (strict Coulomb gauge from the 3+1 dimensional perspective) this is

$$\Pi(p, \text{one loop, tree propagators}) = \frac{11N_c g^2 T}{64|p|} (\delta_{ij} p^2 - p_i p_j), \quad (22)$$

so $l_{\text{mag}} \sim 64/(11N_c g^2 T)$. This estimate is gauge dependent; in Feynman gauge the 11 becomes 14. However, since unequal separation correlators are maximal in Landau gauge, it seems the appropriate gauge for estimating where those correlators break down.

Alternately, we can take l_{mag} as the reciprocal of the “magnetic mass” found self-consistently by solving a gap equation²⁵. Without endorsing the efficacy of gap resummed perturbation theory to make useful predictions, one can nevertheless say that the mass found gives a characteristic scale where higher order terms in the perturbative expansion are of order the leading ones.

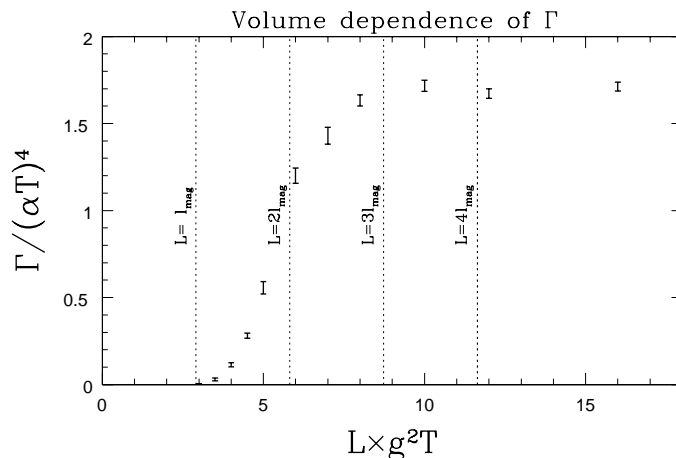


Figure 3: Volume dependence of Γ (pure lattice theory, $g^2 a T = 1/2$) illustrates that Γ turns on for a box about $2l_{\text{mag}}$ across.

A two loop gap equation calculation²⁶ gives $l_{\text{mag}} = 1/m_{\text{mag}} = 1/0.17N_c g^2 T$ which agrees with the self-energy estimate.^b

One check on this estimate of l_{mag} is to look at lattice data in finite volume, and see how large a lattice has to be before Γ “turns on.” We expect that it should have to be about $2l_{\text{mag}}$ across, because we use periodic boundary conditions on the lattice. Fig. 3 indicates that our estimate of l_{mag} is reasonable. In particular, the inverse glueball mass $< 1/g^2 T$ is *not* the appropriate scale.

Using the estimate for l_{mag} , we get

$$\Gamma \sim .24 N_c^3 (N_c^2 - 1) \left(\frac{N_c g^2 T^2}{m_D^2} \right) \alpha_w^5 T^4 \left[\log \frac{m_D}{\gamma} + 3.041 \right]. \quad (23)$$

Plugging in $N_c = 2$ gives $\Gamma = 11.5 (g^2 T^2 / m_D^2) \alpha_w^5 T^4 \log(\dots)$. The 11.5 should be compared with the actual number, 10.0, presented earlier. The quality of the agreement is probably fortuitous.

4 Conclusion

After reviewing the current status of what the sphaleron rate *is*, and what the right effective theory for its determination is (Bödeker’s effective theory),

^bThe computation²⁶ is in $SU(2)$, but since all graphs at one and two loops are planar, the N_c dependence must be as shown.

I have proposed a physical picture of what is going on in the strongly coupled IR physics which determines the rate. My picture is that, on scales longer than l_{mag} , magnetic screening prevents “any record of being kept” of what past changes have occurred to N_{CS} ; therefore, very IR gauge fields evolve diffusively. I have presented a crude estimate of this diffusive evolution and of l_{mag} , and find that the resulting sphaleron rate Γ is surprisingly close to the value determined in lattice simulations.

The model here should not be taken as an accurate quantitative predictor of Γ ; rather it should be viewed as a parametric estimate where an effort has been made to include factors of 2 or π where possible. It also gives an interesting physical picture of N_{CS} diffusion in Yang-Mills theory; rather than occasional, integer changes, N_{CS} diffuses from the incoherent accumulation of many contributions individually much smaller than an integer.

A more robust prediction of this analysis is for the dependence of Γ on N_c . Of course the scaling behavior $l \sim 1/N_c g^2 T$ has $O(1/N_c^2)$ corrections, so the N_c dependence in Eq. (23) cannot be relied on absolutely at small N_c . However, it would be interesting to numerically investigate Γ within Bödeker’s effective theory for a few larger N_c and see how well the prediction holds. In particular the case of SU(3) is physically interesting²⁷.

In conclusion: Do we understand the sphaleron rate? I think, perhaps, that we do.

Acknowledgments

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