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Breakdown of Lorentz Invariance

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The special theory of relativity results from the postulation of invariance under coordinate transformation of the hyperbolic wave equation

$$\Box\psi \equiv \nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = 0,$$

and it is required that all laws of physics (except perhaps general theory of relativity) be invariant under Lorentz transformations. Divergencies in present relativistic field equations may be removed by considering more general wave equations, for example,

$$-l_0^2\nabla^4\psi + \nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = 0.$$

This equation introduces a universal length $l_0 \approx 10^{-13}$ cm as a second invariant and destroys Lorentz invariance except as an approximate invariance. Some theoretical and experimental consequences of this four-dimensional wave equation are discussed.

I. INTRODUCTION

BESIDES the speed of light c and Planck's constant h, a third constant, a universal length $l_0 \approx 10^{-13}$ cm, seems to play an important role in physics. The distance of strong interactions is about 10^{-13} cm. The cross section of neutron-proton scattering is approximately 10^{-25} cm^2, which sets the diameter of the proton in the range of $\sim 10^{-13}$ cm. The "classical radius" of electrons is of the same magnitude.

A theory that predicts the masses of elementary particles must also contain a constant with the dimension of length for simple dimensional reasons. Using de Broglie's relation, we have in dimensions

$$\text{mass (gram)} \times \text{speed (cm sec}^{-1}) = \frac{h \text{ (erg cm)}}{\text{wavelength (cm)}}.$$

However, the special theory of relativity (STR) is hostile to the introduction of a universal length as a constant of nature, since, for two different frames of reference, the length is not invariant under Lorentz transformations.

Many attempts have been made to introduce a universal length into physics. In 1933, Pauli,[1] after discussing the divergencies in quantum electrodynamics and the infinite self-energy of electrons, closes with the following remark: "Wir möchten hierin einen Hinweis dafür erblicken, dass nicht nur der Feldbegriff, sondern auch der Raum-Zeit Begriff im kleinen einer grund- a universal length in physics. In a series of papers[2] he tried to modify the geometry of small lengths while maintaining relativistic invariance. With his method he hoped to avoid the divergencies in quantum electrodynamics. However, March was unable to connect his conceptions with the experimental facts of nuclear physics.

Heisenberg[3] in 1938 wrote a review article on the significance of a universal length in physics. He also discusses the divergencies in relativistic field theories and the unsatisfactory method of cutting off undesired divergent integrals at a length of magnitude l_0 which is done in a manner that destroys relativistic invariance.

In 1943, de Broglie in his book on elementary particles[4] devotes a chapter to the relation between the theory of relativity and quantum mechanics. From the quantum-mechanical point of view, the STR is a classical theory since the history of a point (worldline) similar to classical mechanics is described by its four coordinates x, y, z, and t, and the speed v. The concept of worldline excludes any Heisenberg uncertainty relation.

In 1950 Heisenberg pointed out[5] that future renormalized commutation relations which contain no singularities and describe the mass spectra of elementary particles should contain as a fundamental constant a length of the magnitude 10^{-13} cm.

Heisenberg *et al.* in 1955 developed a nonlinear theory of quantum mechanics which also contains a universal length.[6]

gate faster than the speed of light over distances of the order of 10^{-13} cm, which means that the time-direction of causality is not reserved at this small distance.

The foundation of the present theory of special relativity is briefly discussed here as a preliminary in order to provide the proper background for presenting an extension of the STR wherein previous assumptions are somewhat generalized.

because

$$\frac{\partial\psi}{\partial x_1}=\frac{\partial\psi}{\partial x_2}\frac{\partial x_2}{\partial x_1}+\frac{\partial\psi}{\partial y_2}\frac{\partial y_2}{\partial x_1}+\frac{\partial\psi}{\partial z_2}\frac{\partial z_2}{\partial x_1}+\frac{\partial\psi}{\partial t_2}\frac{\partial t_2}{\partial x_1}=\frac{\partial\psi}{\partial x_2},$$

$$\frac{\partial^2\psi}{\partial x_1{}^2}=\frac{\partial^2\psi}{\partial x_2{}^2},\ \text{etc.},$$

It can be seen that condition (1) requires the introduction of higher-order derivatives. For dimensional reasons, this would be the only way to introduce l_0. As a simple example, the following fourth-order partial differential equation may be considered as a model for an ET which fulfills conditions (1), (2), and (3):

$$-l_0^2\nabla^4\psi+\nabla^2\psi-\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}=0. \tag{2}$$

$\psi=e^{i(\omega t+\mathbf{k}\cdot\mathbf{x})}$ is a solution, and we obtain the dispersion relation

$$\omega^2=c^2k^2(1+l_0^2k^2). \tag{3}$$

The first minus sign in Eq. (2) prevents ω from becoming imaginary for $1<l_0^2k^2$, and Eq. (2) reduces to Eq. (1) for $l_0\to 0$.

Equation (2), which recently attracted attention, arises in the linearized dynamic theory of elastic solids if couple stresses are taken into account.[9] Unfortunately, no detailed study of this particular equation exists in the mathematical literature. This is also true for most of the other fourth-order partial differential equations.

Like Eq. (1), Eq. (2) is also invariant under translation in space and time.

For $1\ll l_0^2k^2$ (energy of the γ quantum above the rest energy of an elementary particle), the dispersion relation (3) reduces to

$$\omega^2=c^2l_0^2k^4, \tag{4}$$

to which the fourth-order partial differential equation,

$$l_0^2\nabla^4\psi+\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}=0, \tag{5}$$

corresponds. Surprisingly, this is a Schrödinger-type equation since

$$l_0^2\nabla^4\psi+\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}=\left(l_0\nabla^2+\frac{i}{c}\frac{\partial}{\partial t}\right)\left(l_0\nabla^2-\frac{i}{c}\frac{\partial}{\partial t}\right)\psi.$$

The static case,

$$-l_0^2\nabla^4\psi+\nabla^2\psi=0, \tag{6}$$

appears also in Bopp's treatise on field equations of higher order in electrodynamics,[10] but where the basic equation is of the type

$$\left(1-\frac{1}{k_0^2}\Box\right)\Box\psi=0.$$

The spherical symmetrical case of a point charge has the solution

$$\psi=\frac{\epsilon}{r}(1-e^{-r/l_0}). \tag{7}$$

[9] R. D. Mindlin and H. F. Tiersten, Arch. Ratl. Mech. Anal. **11**, 415 (1962).

[10] F. Bopp, Ann. Physik **38**, 345 (1940).

For $r\gg 0$, Eq. (7) reduces to the well-known electrostatic potential

$$\psi=\epsilon/r \tag{8}$$

and remains finite for $r\to 0$

$$\psi_0=\epsilon/l_0, \tag{9}$$

i.e., not singular at the origin. Therefore l_0 plays the role of an effective electron radius, and a pointlike electron is stable in this ET.

Equation (2) can be reduced to a second-order partial differential equation with the aid of Pauli's spin matrices

$$\sigma_1=\begin{pmatrix}0&1\\1&0\end{pmatrix},\quad \sigma_2=\begin{pmatrix}0&-i\\i&0\end{pmatrix},\quad \sigma_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix} \tag{10}$$

to the form

$$\left(l_0\sigma_1\nabla^2+i\sigma_2\nabla+\frac{\sigma_3}{c}\frac{\partial}{\partial t}\right)^2\psi=0, \tag{11}$$

where

$$\psi=\begin{pmatrix}\psi_+\\\psi_-\end{pmatrix}$$

is now a spinor. Equation (2) is not invariant under Lorentz transformation. But the transformations which leave Eq. (2) invariant should reduce to the Lorentz transformations for $l_0\to 0$.

However, the following should be considered: Contrary to the STR, for very short wavelength, the ET allows signals (group velocities) to propagate at speeds different from c. From Eq. (3) in particular it follows that

$$v\equiv\frac{d\omega}{dk}=c\frac{1+2l_0^2k^2}{(1+l_0^2k^2)^{1/2}} \tag{12}$$

for

$$1\gg l_0^2k^2,\quad v\approx c(1+\tfrac{3}{2}l_0^2k^2).$$

For visible light, the additional term would be of the order 10^{-16}, which is negligible.

IV. THE PROBLEM OF THE EXISTENCE OF SPEEDS WHICH EXCEED THE SPEED OF LIGHT AND THE STR

The legitimate question may now be asked: If the ET allows signals to propagate at speeds faster than c, could it then be possible to find a privileged coordinate system in nature? On a macro-scale (ether drift), the answer is no. A *Gedankenexperiment* may illustrate this.

We assume that Eq. (2) holds, i.e., we have signals at our disposal which propagate faster than c, e.g., $2c$. A Michelson-Morley experiment is carried out with these signals, but it is found that in different frames the speed of the signal is always exactly $2c$. No conclusion about the existence of a preferred coordinate system (ether) in nature can then be drawn. Therefore to conform to the special principle of relativity on a macro-scale, it is required only that signals propagate with the *same* constant velocity v (not necessarily equal to c) in different frames.

It should be mentioned that a Michelson-Morley experiment allows the measurement of differences only in speeds of signals in different frames. The absolute speed of the signals used is irrelevant.

The STR does not prove that c is the highest possible speed in nature. Postulation of invariance under coordinate transformation of Eq. (1) or postulate (b) *implies* that c is the highest speed in nature, since Eq. (1) allows signals to propagate only with the speed c (as shown). The Lorentz transformations then explicitly express this fact by becoming imaginary for $v>c$.

On a micro-scale, however, there seems to be a breakdown of the special principle of relativity. This will be discussed at the end of this paper.

Nevertheless, up to now the existence of an ET has been rather speculative. There are also other wave equations, which could replace Eq. (1) and lead to other ET different from that resulting from Eq. (2).[11] If Eq. (1) is considered as an approximation, one could formally write as a "dispersion relation" of an ET

$$\omega^2=c^2k^2(1+[f(lk)^2]) \tag{13}$$

with $f(lk)\ll 1$ for visible light, or respectively,

$$\omega^2=c^2k^2(1+l_0^2k^2+l_1^4k^4+\cdots).$$

The ET wave equation is then obtained by replacing ω and k by the operators $\omega\rightarrow i\partial/\partial t$, $k\rightarrow i\nabla$. However, the values of l_0, l_1, etc., can only be obtained by experiment. Therefore before going into any lengthy calculation, the speed of very short γ rays of wavelength $\lambda\approx 10^{-13}$ cm (and shorter) should be measured. If there is indeed an effect, one could immediately determine the proper form of the wave equation, leading to the correct ET, by measuring the speed v (group velocity) as a function of wavelength.

The dispersion relation

$$\omega=\int f(k)dk$$

would follow from $v\equiv d\omega/dk=f(k)$ by integration. This would also allow the determination of the magnitude of l_0 (and l_1, etc.).

A rather easy experiment to perform may be the following: One produces a very short burst of γ rays (containing wavelengths $\lambda\approx 10^{-13}$ cm, and also shorter and longer ones). One can follow now the shape of the burst as a function of time (distance) traveled. If indeed the different waves travel at different speeds, the length (shape) of the burst should also change.

[11] $\left(\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)(1-l_0^2\nabla^2)\psi=0$, for example.

V. CLOSING REMARKS

The constant l_0 has been introduced in a rather formal way. One may now be interested in a simple physical model that explains the significance of l_0. To construct this, we start with an ordinary elastic medium, pictured as an aggregate of points, where each point has coordinates x, y, z, and t assigned. In this medium, wave propagations are expressed by Eq. (1), where c is determined by the elastic constants of the medium. We note that the "electromagnetic medium" has also an elastic medium as a physical model.

The deformations of an ordinary elastic medium are expressed with the usual tensor formalism, which operates in an Riemannian space. However, there are more general elastic media, for example so-called "oriented media," where there are not only points, but also "directors" (or vectors) attached to each point. If the medium is now deformed, the directors may also undergo deformations (couple stresses) independent of the points. A standard of measure of length is supplied by the director frame. If the *length* of the directors is reduced to zero, one returns to the ordinary elastic case.

The concept of oriented media was first introduced by Duhem.[12] In 1909 the Cosserat brothers published their important work on a general theory of oriented bodies with three directors at each point.[13] Ericksen and Truesdell[14] generalized the Cosserats' work and presented it in modern mathematical language. The subject was further advanced by Toupin,[15] Mindlin,[16] and others.[17] Mindlin and Tiersten[9] derived Eq. (2), which describes wave propagation in oriented media, the wave propagation now being dispersive. The geometry, describing deformations in oriented media, is non-Riemannian (e.g. Finsler geometry[18,19]).

In relativistic mechanics, the motion of a point (particle) is expressed in a four-dimensional space with the metric

$$ds^2=dx^2+dy^2+dz^2-c^2dt^2. \tag{14}$$

If one now wants to extend relativistic kinematics to an ET kinematic and find the corresponding coordinate transformations, the following is indicated.

The "electromagnetic medium," in which wave propagation now obeys Eq. (2), does not have an ordinary elastic medium as a physical model, but an oriented medium. This imposes a necessity to express

[12] P. Duhem, Ecole Norm. **10**, 187 (1900).
[13] E. Cosserat and F. Cosserat, *Théorie Deformables* (Hermann & Cie., Paris, 1909).
[14] J. L. Ericksen and C. Truesdell, Arch. Ratl. Mech. Anal. **1**, 296 (1958).
[15] R. A. Toupin, Arch. Ratl. Mech. Anal. **11**, 385 (1962); **17**, 85 (1964).
[16] R. D. Mindlin, Arch. Ratl. Mech. Anal. **16**, 51 (1964).
[17] C. Truesdell and W. Noll, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1965), p. 389.
[18] P. Finsler, *Über Kurven und Flächen in Algemenien Raümen* (Birkhäuser, Basel, Switzerland, 1951). This is an unchanged reprint of Dissertation, University of Göttingen, 1918.
[19] H. Rund, *The Differential Geometry of Finsler Spaces* (Springer-Verlag, Berlin, 1959).

physical laws by more general geometrical concepts. A point (particle) is not only described by its four space-time coordinates, but director(s) have to be attached to each point. Therefore coordinate transformation would not only have to take the points into account, but also the director(s). These director(s) contain additional physical information. They constitute a preferred direction (or directions), and so are violating the special principle or relativity on a micro-scale. Symmetry operations in these more general spaces become also more complex. For example, in this model one would not generally have invariance under reflection.

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Multiple Meson Production by Heavy Primary Nuclei of Cosmic Origin and their Fragmentation Products at Energies above 10^{12} eV*

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Studies were made of the interactions initiated by fragments, including nucleons, from the gradual breakup of ultra high-energy heavy nuclei in a large block of nuclear emulsion. The results obtained by this approach are free from the detection biases that arise in scanning for, e.g., high-energy electromagnetic cascades. For one family of genetically related interactions the primary per-nucleon energy could be reliably established as ~1.3 TeV. The sample of nucleon-induced interactions with average multiplicity $n_s<25$, with primary energies in the region of 1 TeV, show strong bimodality in the angular distribution of the created particles. An upper limit of 1.5 BeV/c is found for the average transverse momentum of the possible fireballs that could have given rise to this bimodality. The average inelasticity for the same sample of collisions is ~0.6. The average multiplicity n_s for the nucleon-induced interactions with $N_h\leq 5$ is ~11.5. For the interactions initiated by heavy nuclei the lower limit to the average per-nucleon multiplicity, $n_s/\Delta A$, in the energy interval 1–20 TeV is consistent with the average multiplicity for the nucleon-induced interactions at about 1 TeV. A linear superposition, in nucleus-nucleus collisions, of elementary nucleon-nucleon acts of meson production is suggested.

INTRODUCTION

THE purpose of the present investigation is the study of multiple meson production in the TeV region using the detailed information obtained from the collisions of heavy primary nuclei of the cosmic radiation in a large nuclear emulsion block. In the collision with an emulsion nucleus, a primary heavy nucleus can dissociate into fragments and/or single nucleons. The nuclear interactions of the latter can be observed provided the dimensions of the detector are large compared with the collision mean free path in nuclear emulsion (~37 cm). A primary heavy nucleus thus initates a family or cascade of interactions, and it is plausible to expect that those caused by multiply-charged fragments are due to incident nuclei carrying the same per-nucleon energy as the parent primary nucleus. Some of the nucleon fragments as well may emerge from the original fragmentation with energy close to the per-nucleon energy of the heavy primary. However, the remainder of the fragmentation products will be represented by nucleons which have lost energy after having been involved in the production of mesons in the first collision. As a result, the breakup of a heavy primary nucleus is expected to give rise to a beam of fragments and nucleons, with some spread in energy but with a well-defined "hard edge" of the per-nucleon energy which will correspond to that of the primary heavy nucleus.

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