PRINCIPLE OF EQUIVALENCE FOR ALL STRONGLY INTERACTING PARTICLES WITHIN THE S-MATRIX FRAMEWORK*

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(Received October 30, 1961)

The notion, inherent in Lagrangian field theory, that certain particles are fundamental while others are complex, is becoming less and less palatable for baryons and mesons as the number of candidates for elementary status continues to increase. Sakata has proposed that only the neutron, proton, and Λ are elementary,[1] but this choice is rather arbitrary, and strong-interaction consequences of the Sakata model merely reflect the established symmetries. Heisenberg some years ago proposed an underlying spinor field that corresponds to no particular particle but which is supposed to generate all the observed particles on an equivalent basis.[2] The spirit of this approach satisfies Feynman's criterion that the correct theory should not allow a decision as to which particles are elementary,[3] but it has proved difficult to find a convincing mathematical framework in which to fit the fundamental spinor field. On the other hand, the analytically continued S matrix—with only those singularities required by unitarity[4]—has progressively, over the past half decade, appeared more and more promising as a basis for describing the strongly interacting particles. Our purpose here is to propose a formulation of the Feynman principle within the S-matrix framework.

Particles appear as energy poles of the S matrix—on the physical sheet if stable, and if unstable on an unphysical sheet. When one analyzes partial-wave elastic scattering amplitudes with neglect of competing processes, it is possible to distinguish between poles that correspond to bound states or dynamical resonances of the two-body system in question and Castillejo-Dalitz-Dyson (CDD) poles that correspond to particles "independent" of the two-body system.[5,6] However, some such CDD poles may not correspond to elementary particles when a more careful investigation includes competing channels. For example, the Dalitz-Tuan model of the Y^* describes this particle as a K-N bound state,[7] but with respect to the partial-wave elastic π-Λ amplitude such a model would normally be considered a CDD pole. Evidently one needs a criterion that emphasizes no particular configuration of particles; the criterion we propose below rests on the analytic structure of the S matrix regarded as a simultaneous function of angular momentum and energy, quantities which are meaningful for arbitrary particle combinations.

Regge has shown for elastic potential scattering[8] and Froissart for any amplitude satisfying the Mandelstam representation[9] that the S matrix can be simultaneously continued into the complex energy and angular-momentum planes. For scattering by a superposition of Yukawa potentials, all poles are associated with bound states and resonances and may be viewed either in the E plane for fixed J or in the J plane for fixed E. A corollary for the latter viewpoint is that the position, α_i, of a particular pole in the J plane is an analytic function of E, and $\alpha_i(E) = \text{constant}$ turns out not to be allowed. If at some energy the value of $\mathrm{Re}\,\alpha_i(E)$ passes through a positive integer or zero (with $d\,\mathrm{Re}\,\alpha_i/dE > 0$), one has here a physical resonance or bound state for J equal to this integer, so in general the trajectory of a single pole in the J plane as E changes corresponds to a family of "particles"—some stable and some unstable—of different J and different mass. It is possible for the trajectory of a particular pole to cross only the integer 0, but the failure to reach higher physical J values would in such a case be a dynamical circumstance and would not reflect a special role for $J=0$. It seems intuitively clear, therefore, that any such pole appearing in the union of the complex J and E planes of the full (relativistic) S matrix cannot be associated with the usual notion of an elementary particle—which emphasizes a particular value of J. We may satisfy Feynman's principle therefore by postulating that all poles of the S matrix are of this type (Regge poles).

The reader may wonder how anything but Regge poles can occur if simultaneous continuation to complex J and E is possible. The point is that for certain internal quantum numbers, the relativistic continuation in J may be restricted to a region somewhat smaller than that for the nonrelativistic case (where the limitation in the absence of spin is $\mathrm{Re}\,J > -\frac{1}{2}$). In particular, the region $\mathrm{Re}\,J < J_{\min}(E)$, $J_{\min}(E) > 0$, might be excluded. Following arguments given by Froissart on the basis of unitarity and analyticity in linear momenta,[10] one can show that there are some E for which

$J_{min}<1$, but "elementary particle" energy poles, admitting no continuation in J, could be associated with the unique angular momenta $J=0$ or $J=\frac{1}{2}$. Such a conclusion coincides with renormalizability requirements of conventional field theory, so if $J=\frac{1}{2}$ and $J=0$ elementary particle poles actually occur in nature, it may be argued that working directly with the S matrix is simply a technique for evaluating conventional field theory.[11] On the other hand, if all baryon and meson poles admit continuation in the J plane, then conventional field theory for strong interactions is not only unnecessary but grossly misleading and perhaps even wrong.

One may regard the principle that all strongly interacting particles are associated with Regge poles as a natural extension to angular momentum of the maximal analyticity principle,[12] which heretofore has been applied only to linear momenta. Maximal analyticity in linear momenta fails to specify precisely the asymptotic behavior in momentum transfer, which is the controlling factor in determining the region of analyticity in the J plane.[9]

How is one to distinguish experimentally between Regge poles and elementary particle poles? An essential characteristic of a Regge pole is that it moves in the J plane as a function of E, the trajectory being the same—regardless of multiplicity—for all S-matrix elements having the internal quantum numbers of the pole.[13] Experiments to establish this trajectory will be of two types, depending on the value of $s=E^2$. For $s>0$ one will seek to identify the existence of families of particles. Blankenbecler and Goldberger, for example, have mentioned the possibility that the nucleon is only the $J=\frac{1}{2}$ member of a family that may have unstable higher J members ($J=\frac{5}{2}$, $\frac{9}{2}$, etc.) to be found among the resonances of multiparticle systems with the same baryon number, isotopic spin, etc., as the nucleon.[14] One will seek to show that the angular momentum of such particles is a monotonic function of their masses, but since only discrete J values can be observed and the total number of family members is not necessarily large (there may be only one physical value crossed by the trajectory) one may confidently anticipate situations where the Regge character of a pole is not convincingly established by experiments with $s>0$.

For $s<0$, on the other hand, if the qualitative arguments about "strips" in the Mandelstam diagram presented by the authors can be taken seriously,[15] then one should be able to study the trajectory $\alpha_i(s)$ in a continuous sense within the strip $s_{min}^i<s<0$, where s_{min}^i is defined by $\alpha_i(s_{min}^i)=J_{min}(s_{min}^i)$. Here one is working experimentally in a "crossed" reaction where s is the negative square of a momentum transfer. It may turn out for some poles that s_{min}^i is greater than zero and the strip in question does not exist, but the essence of our earlier argument is that there should be important situations where the trajectory of the Regge pole is still inside the region of analyticity $\mathrm{Re}J>J_{min}$ for a range of negative s. Consider, for instance, the possibility that the recently discovered ρ meson is associated with a Regge pole whose internal quantum numbers are those of an $I=1$ two-pion configuration.[16] Then we know experimentally one point on the curve $\mathrm{Re}\alpha_\rho(s)$, namely, $\mathrm{Re}\alpha_\rho(28\,m_\pi^2)=1$, since the spin of the ρ meson is 1 and its mass $5.3\,m_\pi$. By analogy with potential theory, $d\,\mathrm{Re}\alpha_\rho/ds$ is probably positive for energies below this resonance.[8] However, it seems likely for these quantum numbers that we have $J_{min}\leqslant 0$,[16] so there is a chance that at zero or even slightly negative s, the value of α_ρ still is larger than J_{min}. [$\alpha(s)$ is real for real s below the lowest two-particle threshold, which here occurs at $s=4m_\pi^2$.] In such a circumstance the Regge pole should dominate the high-energy behavior of the crossed channel near the forward (or backward) direction where $|s|$ is small. In particular, in a two-body reaction, such as neutron-proton exchange scattering with center-of-mass energy $t^{1/2}$, the amplitude will have an energy dependence $\propto t^{\alpha(s)}$. Thus it may be possible, by studying the asymptotic energy variation of the backward peak in n-p scattering, to trace out a portion of the trajectory of the Regge pole associated with the ρ meson.[17]

In general, when forward or backward high-energy scattering is controlled by a Regge pole, one finds by an elementary calculation that

$$d\sigma/d\Delta^2 \sim F(\Delta^2)E^{2[\alpha(\Delta^2)-1]}, \tag{1}$$

where $\Delta=(-s)^{1/2}$ is the momentum transfer, and the lab energy E is proportional to t. Now α is expected to be a monotonically decreasing function of Δ^2, so two characteristic predictions are made: (a) The width of the peak should decrease logarithmically as E increases. (b) The tail of the peak should fall off exponentially with Δ^2 [the function $F(\Delta^2)$, which is related to the residue of the Regge pole, is expected to be an analytic function of Δ^2]. Both these effects are seen more clearly in a linear approximation for $\alpha(s)$:

$$\alpha(s)\approx\alpha(0)+s\alpha'(0), \tag{2}$$

from which follows

$$d\sigma/d\Delta^2 \sim F(\Delta^2)E^{-2[1-\alpha(0)]}\exp\{-[2\alpha'(0)\ln E]\Delta^2\}. \tag{3}$$

A third important prediction is that the same constants, $\alpha(0)$, $\alpha'(0)$, etc., should control all forward or backward peaks that relate to the same set of quantum numbers. Thus the trajectory of α_ρ, for example, should control backward $\pi^+\pi^0$ and K^+K^0 scattering if it controls backward np scattering.

In contrast to the above, an elementary particle of spin S would give a forward or backward peak characterized by replacing $\alpha(\Delta^2)$ by S, a constant, in formula (1). Thus the magnitude should fall off less rapidly and the shape of the peak should not change with increasing energy; also the tail should not be exponential. Of course, we already know from its quantum numbers that the ρ meson cannot be an elementary particle in the conventional sense, and that it almost certainly is associated with a Regge pole. The interesting cases will be those where the spin is equal to 0 or $\frac{1}{2}$. Here the effect on backward peaks of crossed reactions may be difficult to find experimentally (because J is smaller) but a detailed theoretical analysis of the various possibilities seems well worthwhile.[18] For example, it appears that to test whether the neutron is elementary one need only determine whether in backward π^+p scattering $d\sigma/d\Delta^2$ decreases like $1/E$ at high energy, or at a faster rate corresponding to a Regge pole with $\alpha(0) < \frac{1}{2}$.

It is a fascinating possibility that all forward (diffraction) peaks may be controlled by a Regge pole having the same internal quantum numbers as the vacuum and with a trajectory such that $\alpha_{\text{vac}}(s=0)=1$. Constant high-energy total cross sections satisfying the Pomeranchuk conditions are then guaranteed. Because of the even parity of the vacuum, odd J values here have no physical significance, but if at some positive energy $\text{Re}\alpha_{\text{vac}}$ crosses the value 2, there would be a spin-two particle associated with this pole. We apparently must exclude $J=0$ from the region of analyticity if the trajectory of this Regge pole is "normal," because then we would expect $\alpha_{\text{vac}}(s)$ to vanish at a negative value of s, corresponding to a ghost $J=0$ particle. An exceptional status for the quantum numbers of the vacuum appears unavoidable whether or not diffraction scattering is related to a Regge pole. The existence of constant high-energy cross sections will inevitably put special requirements on analyticity properties in J for these particular quantum numbers. Our principle of maximal analyticity in J, therefore, must be amended with the phrase, "consistent with the principle of maximum strength for strong interactions."[15]

The association of high-energy forward and backward peaks with Regge poles, together with the principle of maximal analyticity in J, provides a natural explanation of why low-mass particles tend to have low internal quantum numbers (isotopic spin, strangeness, and baryon number). High-energy peaks evidently are a result of coherence in scattering. Maximum coherence—i.e., the maximum value of $\alpha(0)$—occurs for exchange of the quantum numbers of the vacuum; the degree of coherence progressively decreases with an increase in the quantum numbers of the exchanged system—with a consequent decrease in the value of $\alpha(0)$. (This effect is explicitly illustrated for $\pi\pi$ scattering in reference 16.) By analytic continuation, such a correlation between $\alpha(0)$ and internal quantum numbers is likely to be maintained for $\alpha(s)$ in the region of positive s, with the consequence that low-energy bound states and resonances are most likely to occur for low isotopic spin, low strangeness, and low baryon number.

We note, in conclusion, two experimentally attractive features of the Regge-pole hypothesis for diffraction peaks: (a) Gribov's argument that total cross sections must decrease is circumvented.[19] He assumed a factorable high-energy dependence on Δ^2 and E, which is not true for formula (1). (b) As noted by Lovelace,[20] the observed exponential behavior in Δ^2 of the diffraction "tail" is difficult to understand on a classical basis but, as we have seen above, it follows immediately from a Regge pole. The disagreeable feature is the predicted logarithmic decrease of the elastic cross section while the total cross section remains constant. However, if one admits that, after all, this may not be a classical situation, no clear argument can be made that such behavior is inadmissible. We must wait for experiment to decide. A related experimental test that may be immediately possible is to see if the exponential occurring in the tail of pp diffraction scattering is the same as that occurring in πp and Kp scattering.

We should like to acknowledge enormous benefit received from conversations with M. Froissart, M. Gell-Mann, and S. Mandelstam, without implying that they agree with all the arguments given here.

*This work was supported by the U. S. Atomic Energy Commission.

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[11]S. Mandelstam, Report to the 1961 Solvay Conference on Field Theory, Brussels, 1961 (to be published); see also the last two reviews listed in reference 4.

[12]G. F. Chew, in Dispersion Relations, edited by G. R. Screaton (Oliver and Boyd, Ltd., Edinburgh, 1960), p. 167; and the reviews listed in reference 4. Also see H. P. Stapp, Lawrence Radiation Laboratory Report UCRL-9804, 1961 (unpublished).

[13]Unitarity couples all S-matrix elements of the same quantum numbers, so that a pole in one must appear in all. The possibility of continuation in the J plane has so far been demonstrated only for two-body elements, but we assume for the discussion here that it has a meaning in general. Note that a particular Regge pole is characterized among other things by its parity, so that only alternate J values have a physical meaning. This somewhat confusing point is discussed in reference 14 and will be elaborated in a forthcoming paper by M. Gell-Mann, S. C. Frautschi, and F. Zachariasen.

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[16]This possibility is discussed in detail in a forthcoming paper by G. F. Chew, S. C. Frautschi, and S. Mandelstam, Lawrence Radiation Laboratory Report UCRL-9925 (unpublished).

[17]One might expect the pole associated with the pion, whether Regge-like or elementary, to contribute to the backward n-p peak. Since the spin of the pion is zero, however, such a pole would probably lead to a lower power of t than that associated with the ρ. The ω has spin one, but because its isotopic spin is zero, it will not appear in backward n-p scattering.

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