ABSENCE OF THE ANOMALOUS MAGNETIC MOMENT IN A SUPERSYMMETRIC ABELIAN GAUGE THEORY

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Received 2 October 1974

We give a general argument for the vanishing of the anomalous magnetic moment in a supersymmetric Abelian gauge theory recently constructed by Wess and Zumino, provided in such a model the appropriate Ward identities are consistent with renormalized perturbation theory. Our conclusion is in fact confirmed by explicit one-loop calculation, thus providing a new example of cancellation of Feynman graphs in supersymmetric theories.

Recently many renormalizable field theories, which have the property of being invariant under Fermi–Bose supersymmetry [1], have been constructed and investigated. In the simplest of these models [2–4], the super Φ^3 theory, renormalizable perturbation theory has been shown to preserve the symmetry at all orders and moreover only one infinite (wave function) multiplicative constant has been shown to be sufficient to renormalize the theory [2]. The absence of independent mass and coupling constant counterterms is merely due to the fact that the usual power counting argument for ultra-violet divergences is too pessimistic in this theory, and indeed remarkable cancellations of Feynman graphs seem to lower the over-all degree of divergence. These kinds of cancellations can be nicely understood in terms of supergraph techniques [5]. However, the situation is not completely clear when one deals with supersymmetric gauge theories, which have also been introduced [3,4], even if one restricts oneself to the Abelian case. The renormalization of such theories, consistent with supersymmetry, is an open question. On the other hand Wess and Zumino [3] showed that, at least at the one-loop level, the Abelian gauge model can be consistently renormalized preserving the symmetry. One hopes therefore that this trend is confirmed in higher-order calculations. In the present note, we point out that, as a consequence of supersymmetry, the magnetic moment of the charged fermion vanishes in the Abelian gauge model constructed by Wess and Zumino. This can be understood using superfield techniques, but in terms of component fields it is an example of cancellations among different graphs with different boson and fermion lines.

Let us recall the Lagrangian [3] which describes the Abelian supersymmetric theory

$$\begin{aligned}\mathcal{L} = &-\tfrac{1}{2}[(\partial A_1)^2 + (\partial A_2)^2 + (\partial B_1)^2 + (\partial B_2)^2 - F_1^2 - F_2^2 - G_1^2 - G_2^2 + \mathrm{i}\bar{\psi}_1\gamma\cdot\partial\psi_1 + \mathrm{i}\bar{\psi}_2\gamma\cdot\partial\psi_2] \\ &+ m(F_1A_1 + F_2A_2 + G_1B_1 + G_2B_2 - \tfrac{1}{2}\mathrm{i}\bar{\psi}_1\psi_1 - \tfrac{1}{2}\mathrm{i}\bar{\psi}_2\psi_2) - \tfrac{1}{4}F_{\mu\nu}^2 - \tfrac{1}{2}\mathrm{i}\bar{\lambda}\gamma\cdot\partial\lambda + \tfrac{1}{2}D^2 \\ &+ g_0[D(A_1B_2 - A_2B_1) - \mathcal{V}_\mu(A_1\partial_\mu A_2 - A_2\partial_\mu A_1 + B_1\partial_\mu B_2 - B_2\partial_\mu A_1 - \mathrm{i}\bar{\psi}_1\gamma_\mu\psi_2) \\ &- \mathrm{i}\bar{\lambda}\{(A_1 + \gamma_5B_1)\psi_2 - (A_2 + \gamma_5B_2)\psi_1\}] - \tfrac{1}{2}g_0^2\mathcal{V}_\mu^2(A_1^2 + A_2^2 + B_1^2 + B_2^2)\,.\end{aligned} \tag{1}$$

Eq. (1) rewritten in terms of superfields reads [6]

$$\mathcal{L} = W^\alpha W_\alpha + m(S_1^2 + S_2^2) + S_1\bar{S}_1 + S_2\bar{S}_2 + \mathrm{i}g_0(S_1\bar{S}_2 - S_2\bar{S}_1)V + \tfrac{1}{2}g_0^2(S_1\bar{S}_1 + S_2\bar{S}_2)V^2 + \text{h.c.}\,, \tag{2}$$

where $W_\alpha = \bar{D}\bar{D}D_\alpha V$, D_α, $\bar{D}_\alpha$ are the covariant derivatives [6] and $S = S_1 + iS_2$ is the complex chiral field ($\bar{D}_\alpha S = 0$) which interacts with the (real) vector supermultiplet V. The scalar multiplet describes the physical charged particles A, B, ψ with the same mass m interacting with the massless particles v_μ (photon) and λ (neutrino). As a consequence of (1) and (2), via elimination of the auxiliary fields F_1, F_2, G_1, G_2 and D, one finds that the effect of supersymmetry is to add to the usual minimal interaction of spinor and scalar QED, the new couplings

$$-ig_0\bar{\lambda}\{(A_1+\gamma_5 B_1)\psi_2 - (A_2+\gamma_5 B_2)\psi_1\} - \tfrac{1}{2}g_0^2(A_1B_2 - B_2B_1)^2 , \tag{3}$$

i.e., Yukawa and quartic couplings expressed in terms of the same gauge coupling constant g_0.

Let us consider now the anomalous magnetic moment of the charged spinor field $\psi = (1/\sqrt{2})(\psi_1 + i\psi_2)$ (ψ_1, ψ_2 being Majorana spinors) in such a model. It corresponds to an effective interaction term of the form

$$(g-2)\frac{1}{4m}\bar{\psi}\sigma_{\mu\nu}\psi F^{\mu\nu} ,$$

so the problem arises whether such a term can be written in a supersymmetric invariant way. We know, from the tensor calculus of superfields, that in order to be so, it must be contained in component of some multiplet of highest degree in $\theta, \bar{\theta}$, the anticommuting spinor coordinates of the superfield. It is a simple task to derive that the non-minimal coupling is contained in the M component of the superfield

$$L = S_1 \overleftrightarrow{D}_\alpha S_2 W^\alpha , \tag{4}$$

which is a linear multiplet, according to the definitions given in ref. [6], and has the general structure

$$L(x - i\theta\sigma\bar{\theta}, \theta, \bar{\theta}) = E + \chi\theta + \bar{\psi}\bar{\theta} + M\theta\theta + J_\mu\theta\sigma^\mu\bar{\theta} + \bar{\xi}\bar{\theta}\theta\theta . \tag{5}$$

The previous remark is sufficient to conclude that M does not transform as a total derivative under the supersymmetry transformation, this fact implying that the point-like anomalous magnetic term must vanish in a symmetry preserving interaction. In order to understand this result in terms of usual Feynman graphs, we computed the vertex in the one-loop approximation. We know, in fact, that the usual QED minimal coupling does actually generate on $O(g_0^2)$ point-like coupling of the type $\bar{\psi}\sigma_{\mu\nu}\psi F^{\mu\nu}$. What happens is that, due to the presence of the additional Yukawa couplings contained in eq. (3), the graphs contributing to the anomalous magnetic moment are those listed in fig. 1. and the two (actually equal) new graphs of fig. 1b, 1c, just cancel the conventional QED graph fig. 1a.

In order to show this, let us recall that the contribution to the onmass-shell Pauli form factor $F_2(t)$ from a fully off-mass-shell fermion-photon vertex amplitude $M_\mu(p_1, p_2, \Delta)$ is [7]

$$F_2(t) = \frac{m}{t(t-4m^2)}\,\mathrm{Tr}\left[(-i\not{p}_1 + m)\left(-m\gamma_\mu + i\frac{t+2m^2}{t-4m^2}P_\mu\right)(-i\not{p}_2 + m)M_\mu(p_1, p_2, \Delta)\right] , \tag{6}$$

to be evaluated at $p_1^2 = p_2^2 = -m^2$. Here p_1, p_2 are the four-momenta of the in and out fermions, $P = p_1 + p_2$ and

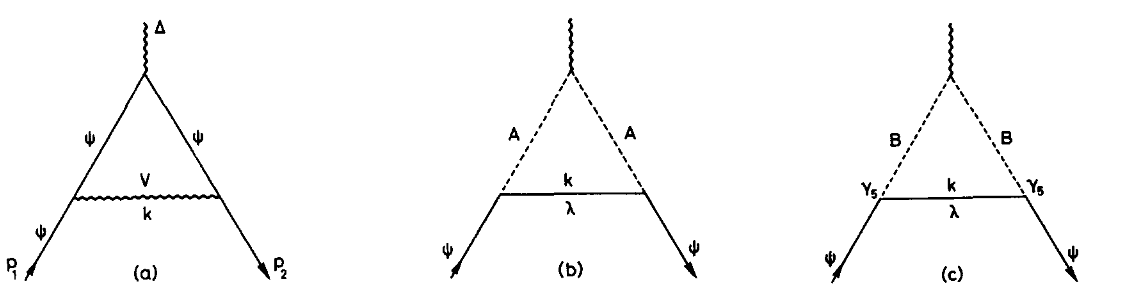

Fig. 1. The vertex graphs.

$\Delta = p_1 - p_2$ with $t = -\Delta^2$. The amplitude of the graph of fig. 1a, the only one present in conventional QED, can be written, dropping a $(-g_0)(g_0/2\pi)^2$ factor, as

$$M_\mu^{(a)} = \frac{i}{(2\pi)^2}\int \frac{d^4k}{D}\gamma_\nu(-i\not{p}_2 + i\not{k} + m)\gamma_\mu(-i\not{p}_1 + i\not{k} + m)\gamma_\nu\,, \tag{7}$$

with $D = k^2[(p_1-k)^2+m^2][(p_2-k)^2+m^2]$, and on account of eq. (4), the corresponding Pauli form factor is

$$F_2(t) = \frac{i}{(2\pi)^2}\int \frac{d^4k}{D} N^{(a)}\,,$$

with

$$N^{(a)} = \frac{8m^2}{t(t-4m^2)}\left[\frac{3t}{t-4m^2}(Pk)^2 - t(Pk) - tk^2 - (\Delta k)^2\right].$$

For the second graph, fig. 1b, the amplitude is

$$M_\mu^{(b)} = \frac{i}{(2\pi)^2}\int \frac{d^4k}{D}\, k(P-2k)_\mu\,, \tag{8}$$

where, as a consequence of supersymmetry, the denominator is exactly the same as for $M_\mu^{(a)}$. Eq. (6) gives corre spondingly

$$F_2^{(b)}(t) = \frac{i}{(2\pi)^2}\int \frac{d^4k}{D} N^{(b)}\,, \tag{9}$$

with $N^{(b)} = -\frac{1}{2}N^{(a)}$. Summing the contributions of the three graphs, as obviously $N^{(c)} = N^{(b)}$ one obtains

$$F_2(t) = F_2^{(a)}(t) + F_2^{(b)}(t) + F_2^{(c)}(t) = 0\,.$$

Note that the cancellation occurs already at the integrand level, prior to any loop integration. As a consequence, at least in the one-loop approximation, we not only obtain $F_2(0) = 0$, but the stronger result $F_2(t) = 0$ for the on-shell Pauli form factor. However, we have not been able to find a simple argument for its vanishing using symmetry arguments, even though it is plausible that this is so. Our result, at the one-loop level, is also in agreement with the fact that supersymmetry is consistent, in this approximation, with renormalization and gauge invariance.

As a concluding remark we would like just to comment on the rather negative results we have obtained. The vanishing of $g-2$ in this theory should not in fact astonish very much, and should not be taken as an argument against supersymmetry. Rather it should be attributed to the unphysical nature of the particular model considered. This model in fact requires the existence of a scalar and of a pseudoscalar particle with the same mass as that of the "electron". Such particles do not exist in Nature. One can visage a symmetry breaking [8] which makes the scalar and pseudoscalar masses very large, in such a way as to obtain a very small contribution from the additional graphs. Another possibility is to give a non-trivial mixing of the supersymmetry with some internal symmetry [9], in order to intrinsically change the algebraic structure of the supermultiplets.

Nevertheless, the present calculation provides and example of cancellation of graphs of the kind typical of supersymmetric theories, and one giving rise to possible observable effects.

The authors would like to thank Prof. Bruno Zumino for many helpful discussions.

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