Before you read my remarks inserted between your formulas below, please consider the following.

[You don't have to perform the manipulations that I describe immediately, just give it a read, it may clarify. I also should ask you to use antisymmetrization on indices in $\partial_{[\mu} A_{\nu]}$ and $\partial_{[\mu} B_{\nu]}$ everywhere and in all occurences, or the coefficients will be wrong]

- There are two auxiliary fields — $F_{\mu\nu}$ and $G_{\mu\nu}$. We want to eliminate them (although we don't have to, as they are useful for Hamiltonian approach, but let us do)

- In my January note I chose to eliminate them both at once. There is no miscalculation in that note, and the result is a self-dual QED with two gauge fields constrained by duality

- We can choose to eliminate $F_{\mu\nu}$ and $G_{\mu\nu}$ one–by–one instead. Fine. Choose any of them, they are equivalent, say $G_{\mu\nu}$. If you eliminate it, you will find that the other field — $F_{\mu\nu}$ — enters only linearly. That is — it is a Lagrange multiplier of the duality constraint. And the rest of the action will again be a QED, therefore, forced to be self-dual

- Finally, as you have noticed, it may be convenient to introduce a *sum*

  $$H_{\mu\nu} \quad = \quad F_{\mu\nu} \quad + \quad \widetilde{G}_{\mu\nu} \, .$$

  I suggest to introduce a *difference* as well

  $$M_{\mu\nu} \quad = \quad F_{\mu\nu} \quad - \quad \widetilde{G}_{\mu\nu} \, .$$

  Now without any elimination you'll see that the difference $M_{\mu\nu}$ acts as a Lagrange multiplier for the same duality constraint. Consequent elimination of $H_{\mu\nu}$ again leads to the same result as above

- The approach of your email (copied on the next page), is a kind of mix, where you introduce the sum $H_{\mu\nu}$, while keeping $G_{\mu\nu}$, instead of trading it for the difference. Fine. I argue below that it still certainly leads to the same result

Dear Dr. Bolokhov

Sorry, I have misanderstood a statement of your previous mail; I thought that your claim was that the two big round brackets in eq. (6) vanish but now I see that you intended that say cancel the last (non big) brackets which is correct if one use the field eqs. (5).

In any case your approach remains inconsistent. Indeed your lagrangian (3) can be rewritten as

$$L = 1/4\partial_\mu A_\nu H^{\mu\nu} - (1/16)(H_{\mu\nu})^2 - 1/4(\partial_\mu A_\nu - 1/2\epsilon_{\mu\nu\rho\sigma}\partial^\rho B^\sigma)\tilde{G}^{\mu\nu} + k_\mu A^\mu + j_\mu B^\nu$$

[you seem to be implying antisymmetrization on $\partial_\mu A_\nu$ and $\partial_\rho B_\sigma$ here. Otherwise the coefficients would be incorrect — e.g. in the first term]

where $H = F + \tilde{G}$ or, integrating over H,

$$L = (1/16)(\partial_{[\mu}A_{\nu]})^2 - (1/4)(\partial_\mu A_\nu - 1/2\epsilon_{\mu\nu\rho\sigma}\partial^\rho B^\sigma)\tilde{G}^{\mu\nu} + k_\mu A^\mu + j_\mu B^\nu$$

[Now, (implied) antisymmetrization is still missing in the second term, while the first term has it, but its coefficient should be "1/4" as well]

and since

$$\int (1/2\epsilon_{\mu\nu\rho\sigma}\partial^\rho B^\sigma)\tilde{A}^{\mu\nu}) = 0$$

[Now this is crucial! While there is something wrong with notations in this formula, I certainly do understand what you mean:

$$\int \boldsymbol{1/2\,\epsilon_{\mu\nu\rho\sigma}\,\partial^\rho B^\sigma \partial^\mu A^\nu} \quad = \quad \mathbf{0}$$

and this is not right. The integrand is a total derivative, which cannot be discarded in the presence of monopoles]

your action is identical to the action

$$\int L = \int [-(1/4)(\partial_\mu A_\nu - 1/2\epsilon_{\mu\nu\rho\sigma}\partial^\rho B^\sigma)K^{\mu\nu} + k_\mu A^\mu + j_\mu B^\nu]$$

[the first term vanishes, I agree, but once the above "topological" integral is added back, everything comes into place]

where

$$K_{\mu\nu} = \tilde{G}_{\mu\nu} - 1/4(\partial_{[\mu}A_{\nu]})\,,$$ ***[in fact there is no "1/4"]***

which is clearly inconsistent.

With my best regards
Mario Tonin

*   *   *

You could ask how is this better to have $F_{\mu\nu}$ and $G_{\mu\nu}$ if this or another way we are just enforcing the duality constraint by a Lagrange multiplier, the thing I wanted to avoid from the start?

True, there is a Lagrange multiplier. But I stress again that it carries physical sense — first, being the fieldstrength tension — and second, being the conjugate momentum, which is useful for applying the Hamiltonian approach. But let's leave this discussion for later because we just can't break through the difficulties with the initial formulas