December 11, 2013

Written Notes on Effective Potential in CP^{N-1} at large N

1. The kinetic term (which contains $\left|\frac{\partial_\mu \sigma}{\sigma}\right|^2$) is obtained as $-\frac{1}{2}\int d^4\theta \ \ln\Sigma \ \ln\bar{\Sigma}$

2. $iD \cdot \ln|\sigma|^2$ is found from

$$\int d^2\hat{\theta} \ \Sigma(\ln\Sigma - 1) + \text{h.c.}$$

* * *

These things we already know. Remind, we need to get

$$\frac{4\pi}{N}\mathcal{L}_{eff} = \frac{1}{|\sqrt{2}\sigma|^2}\Big\{ \frac{1}{4}F_{\mu\nu}^2 + |\partial_\mu\sigma|^2 + \bar{\lambda}_R i\partial_L\lambda_R + \bar{\lambda}_L i\partial_R\lambda_L +$$

$$+ \underbrace{4|\sigma|(\sigma-\bar{\sigma})\cdot F_{03}}_{\text{anomaly}} + 2i\left(\sqrt{2}\bar{\sigma}\bar{\lambda}_L\lambda_R + \sqrt{2}\sigma\bar{\lambda}_R\lambda_L\right) + \dots \Big\}$$

$$+ \left(iD + |\sqrt{2}\sigma|^2\right)\cdot\ln\left(iD + |\sqrt{2}\sigma|^2\right) + iD + |\sqrt{2}\sigma|^2\cdot\ln|\sqrt{2}\sigma|^2$$

I will omit various $\sqrt{2}$ and "i" here, but I do track all of them in calculations.

(2)

3. $\ln(1 + iD/|b|^2)$ is found as the lowest component of

$$\ln\left\{1 + \frac{\bar{D}_R D_L \ln \bar{\Sigma}}{\Sigma}\right\}$$ or its hermitean conjugate. — it's possible that both are needed in order to balance various right- and left-handed fermions; if we include both this log and its h.c., we'll get $\ln\left\{|b|^2 + iD + \frac{(iD)^2}{|b|^2}\right\}$ in the result — and that does not yet even include fermions and field derivatives, so I don't know whether this is bad or good.

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Although, if we include both $\ln\{..\}$ and its hermitean conjugate, then item "2." and "3." combine into

$$\int d^2\theta\, \Sigma\left\{\ln\left\{\Sigma + \bar{D}_R D_L \ln \bar{\Sigma}\right\} - 1\right\} + h.c.$$

This expression I need to fully expand.

4. $|b|^2 \cdot \ln\left(1 + \frac{iD}{|b|^2}\right)$ can be found as

$$\int d^2\hat{\theta} \; \frac{\Sigma^2}{\bar{D}_R D_L \ln \bar{\Sigma}} \cdot \ln\left(1 + \frac{\bar{D}_R D_L \bar{\Sigma}}{\Sigma}\right)$$

one Σ gives "iD" (we don't need it),

another $\Sigma \to b$, and logarithm $\to$ $iD/_b$ in denominator
in denom.

{ the big $\ln(\;)$, as in item "3.", gives $\ln(1 + {iD}/_{|b|^2})$ }.

cancellation of the un-needed "iD" happens only if

we ignore $F_{\mu\nu}$. In full expression, there will be

a factor $\frac{iD + F_{03}}{iD - F_{03}}$.

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Again, I don't know if this is good enough "cancellation"

5. The effective potential V_{eff} can be written as

a formal expression

$$-\int_0^{iD} d(iD)\,.\,\ln\left(|\phi|^2 + iD\right)$$

this can be expanded as

$$-\int_0^{iD} d(iD)\left\{\ln\phi + \ln\bar{\phi} + \ln\left\{1 + \frac{iD}{|\phi|^2}\right\}\right\}$$

and easily supersymmetrized using the log of item "3."

6. I have not obtained yet anything reasonable

from the supergraph approach, besides of "guessing"

$\ln\Sigma \cdot \ln\bar{\Sigma}$ which we know anyway

7. Expansion of the logarithms in series, supersymmetrization, and then combining back into logarithms, quite likely will give the same as items "2." and "3.".

This is not very difficult, and I have done this at the component level (without supersymmetrization).

By the way, the sign of $\frac{1}{2}\frac{D^2}{|\sigma|^2}$ from the kinetic term **does not** cancel the sign of $\frac{1}{2}\frac{(iD)^2}{|\sigma|^2}$ from the series expansion {again, I do track all factors of "i", and confirm that they don't cancel}. I don't know if it would have been useful if they did.

8. To compare the result with D'Adda et al. is still too early. It is possible (hopefully) to do this at the component level (as series expansions) — since they too, first obtained a component expression (and then supersymmetrized it). Unlike our Veff, however, their result is unreadable even at the component level.

One surely useful thing to borrow from their work is the chiral variable $\dfrac{\bar{D}_R D_L \ln \bar{\Sigma}}{\Sigma}$

[it's a bit hard to decrypt this, but this indeed is what they use]

$\dfrac{\bar{D}_R D_L \bar{\Sigma}}{|\Sigma|^2}$ is more straightforward in giving $\dfrac{iD}{|\sigma|^2}$, but is not chiral. Plus, they say a lot of good words about transformation properties of $\dfrac{\bar{D}_R D_L \ln \bar{\Sigma}}{\Sigma}$