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May 5, 2016

Kinetic Term for ϕ in the Effective ActionProper Time Method

$$Z = \int d\mu d\bar{\mu} e^{-\int d\mu u - \int \bar{\mu} \phi} = \int d\mu d\bar{\mu} e^{-\int (-\nabla + m^2 + \phi) u}$$

$$\text{where } m^2 = iD, \quad \phi = kE\beta l^2$$

$$e^{-T} = Z \propto \frac{1}{\det^N(-\nabla + m^2 + \phi)}$$

$$T = \ln \det^N(-\nabla + m^2 + \phi) = N T_r \ln [-\nabla + m^2 + \phi] =$$

$$= N T_r \int dm^2 \frac{1}{-\nabla + m^2 + \phi} = N T_r \int dm^2 \left[\int_0^\infty e^{-(\nabla + m^2 + \phi)s} ds \right] =$$

$$= -N T_r \int_0^\infty \frac{ds}{s} e^{-(\nabla + m^2 + \phi)s} =$$

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$$= -N \int \langle \alpha | \frac{ds}{s} e^{-(m^2 + \phi - \omega)s} \langle \alpha | dx =$$

$$1 = \int \frac{dp}{(2\pi)^d} \langle p | p \rangle$$

$$= -N \int \langle \alpha | p \rangle \frac{ds}{s} \frac{dp}{(2\pi)^d} dx e^{-(m^2 + \phi - \omega)s} \langle p | \alpha \rangle =$$

$$\langle p | \alpha \rangle = e^{ipx}$$

$$= -N \int \frac{dx dp}{(2\pi)^d} \langle \alpha | p \rangle \frac{ds}{s} e^{-(m^2 + \phi - \omega)s} \langle p | \alpha \rangle =$$

$$= -N \int \frac{dx dp}{(2\pi)^d} \frac{ds}{s} e^{-ipx} e^{-(m^2 + \phi - \omega)s} e^{ipx} =$$

$$= N \int \frac{dx dp}{(2\pi)^d} \frac{ds}{s} e^{-ms} e^{-ipx} e^{(\phi - \phi)s} e^{ipx}$$

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$$e^{(a-f)s} =$$

$$\cdot [as, -fs] = -s^2(a(f) + 2\partial_p f \partial_p)$$

need only $\cdot [as [as, -fs]] =$

two

derivatives

$$= -s^3 [a, a(f) + 2\partial_p f \partial_p] =$$

$$\approx -s^3 [a, a\partial_p f \partial_p] \approx -4s^3 \partial_p a(f) \cdot \partial_p \partial_p$$

$$\cdot [-d = [as, -fs]] = s^3 [f, a(f) + 2\partial_p f \partial_p] =$$

$$= s^3 2\partial_p f \cdot [f \partial_p] = -2s^3 \partial_p f \cdot \partial_p f$$

- all single commutators will have too many derivatives

- or will vanish

$$e^{x+y} = \left(e^{\frac{x}{N}} e^{\frac{y}{N}} \right)^N$$

$N \rightarrow \infty$

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Zusammenfassung:

$$e^{x+y} = e^x e^y e^{-\frac{1}{2}(xy)} e^{\frac{1}{3}[y(xy)] + \frac{1}{6}[x(xy)]}$$

.....

In our case,

$$= e^{(a-t)s} = e^{-ts} e^{as} e^{-\frac{1}{2}(-ts, as)} e^{\frac{1}{3}[as(-ts, as)] + \frac{1}{6}[-ts(-ts, as)]} =$$

$$= e^{-ts} e^{as} e^{\frac{1}{2}[as, -ts]} e^{-\frac{1}{3}[as(as, -ts)] - \frac{1}{6}[-ts(as, -ts)]} =$$

$$= e^{-ts} e^{as} e^{-\frac{1}{2}(at + 2\delta_p t + \delta_p^2)}$$

$$\times e^{-\frac{1}{3}(-4) \stackrel{3}{\delta_p} \delta_p \delta_p (\phi) \delta_p \delta_p} e^{-\frac{1}{6}(-2) \stackrel{3}{\delta_p} \delta_p \delta_p \phi} =$$

$$= e^{-ts} e^{as} e^{-\frac{1}{2}(at + 2\delta_p t + \delta_p^2)}$$

$$\times e^{\frac{4}{3} \stackrel{3}{\delta_p} \delta_p \delta_p (\phi) \delta_p \delta_p} e^{\frac{1}{3} \stackrel{3}{\delta_p} \delta_p \delta_p \phi} =$$

$$\begin{aligned}
 &= e^{-\frac{1}{2}s + \frac{\gamma_3}{3}s^3 \partial_\mu \phi \partial_\mu \phi} \\
 &\times e^{\frac{\alpha s}{2} - \frac{\alpha^2}{2}(\partial_\mu \phi + 2\partial_\mu \phi \partial_\mu \phi) - \frac{\gamma_3}{3}s^3 \partial_\mu \phi \partial_\mu \phi (\partial_\mu \phi)^2} \\
 &= e^{-\frac{\alpha^2}{2}\partial_\mu \phi} \\
 &= e^{-\frac{\alpha^2}{2}s + \frac{\gamma_3}{3}s^3 \partial_\mu \phi \partial_\mu \phi} \cdot e^{-\frac{\alpha^2}{2}\partial_\mu \phi}
 \end{aligned}$$

$$x e^{\frac{as}{e} - \frac{s^2}{2} \delta_{\mu}^2 t} e^{y_3 \frac{s^3}{3} \delta_{\mu}^3 t} e^{\frac{y_4 s^4}{4} \delta_{\mu}^4 t}$$

need only up to 2nd power

$$a - \frac{d^3}{2} \Delta^2 + \frac{1}{3} d^3 \Delta^2 \Delta^2$$

$$\therefore e^{As} \left[1 - \frac{3}{2} \partial_p \partial_q + \frac{1}{2} \partial_p \partial_q \partial_r \partial_s \right].$$

$$4/3 \times^3 \partial_\mu \bar{b}_\nu \partial^\mu b_\nu$$

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only 1st power

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$$\approx e^{-ts} \left(1 - \frac{s^2}{2} \partial_t + \gamma_3 s^3 (\partial_\mu t)^2 \right) \cdot$$

$$e^{ts} \left\{ 1 - s^2 \partial_\mu t \partial_\mu + \frac{s^4}{2} \partial_\mu t \partial_\nu t \partial_\mu \partial_\nu \right\} \cdot$$

$$\times \left[1 + \frac{s^3}{3} \partial_\mu \partial_\nu t \partial_\mu \partial_\nu \right] =$$

$$\approx e^{-ts} \left[1 - \frac{s^2}{2} \partial_t + \frac{1}{3} s^3 (\partial_\mu t)^2 \right] \cdot$$

$$e^{ts} \left\{ 1 - s^2 \partial_\mu t \partial_\mu + \frac{s^4}{2} \partial_\mu t \partial_\nu t \partial_\mu \partial_\nu + \right.$$

$$\left. + \frac{s^3}{3} \partial_\mu \partial_\nu t \partial_\mu \partial_\nu + \dots \right] =$$

(6a)

$$+ e^{\frac{is}{2}} (-s^2) \partial_r \phi \partial_r =$$

$$= \left\{ 1 + ns + \frac{1}{2} (ns)^2 + \dots \right\} (-s^2) \partial_r \phi \partial_r =$$

$$= (-s^2) \left\{ 1 + ns + \frac{1}{2} (ns)^2 + \dots \right\} \partial_r \phi \partial_r =$$

$$= -s^2 \sum_{j=0}^{\infty} \frac{s^j}{j!} \square^j \cdot \partial_r \phi \partial_r =$$

$$= -s^2 \sum_{j=0}^{\infty} \frac{s^j}{j!} \left\{ \partial_r \phi \square^j \partial_r + 2 \partial_r \phi \partial_r \square^{j-1} \right\} =$$

$$= -s^2 \sum_{j=0}^{\infty} \left\{ \partial_r \phi \partial_r \cdot \frac{(s\square)^j}{j!} + 2 \frac{s^j}{(j-1)!} \partial_r \phi \partial_r \cdot \frac{\partial_r \square^{j-1}}{\square^{j-1}} \right\} =$$

$$= -s^2 \left[\partial_r \phi \partial_r \cdot e^{\frac{s\square}{2}} + 2 \sum_{j \geq 1} \partial_r \phi \partial_r \cdot \frac{\square^{j-1}}{(j-1)!} s^j \right] =$$

(6b)

$$= -s^2 \left[\partial_\mu \partial_\nu e^{s\Omega} + 2s \partial_\mu \partial_\nu \partial_\rho e^{s\Omega} \right]$$

$$e^{Us} (-s) \partial_\mu \partial_\nu \approx$$

$$= -s^2 \cdot \partial_\mu \partial_\nu e^{s\Omega} + 2s^3 \partial_\mu \partial_\nu \partial_\rho e^{s\Omega} =$$

$$= -s^2 \partial_\mu \partial_\nu e^{s\Omega} - 2s^3 \partial_\mu \partial_\nu \partial_\rho e^{s\Omega}$$

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$$\approx e^{-\frac{1}{2}s^2} \left(1 - \frac{s^2}{2} \partial_u \partial_v + \frac{1}{3} s^3 (\partial_u \partial_v)^2 \right) \approx$$

$$\approx \left\{ 1 - s^2 \partial_u \partial_v - \frac{2s^3}{3} \partial_u \partial_v \partial_u \partial_v + \frac{s^4}{2} \partial_u \partial_v \partial_u \partial_v + \right. \\ \left. + \frac{1}{3} s^3 \partial_u \partial_v \partial_u \partial_v + \dots \right\} e^{\frac{sU}{2}} \approx$$

$$\approx e^{-\frac{1}{2}s^2} \left(1 - \frac{s^2}{2} \partial_u \partial_v + \frac{1}{3} s^3 (\partial_u \partial_v)^2 \right) \approx$$

$$\approx \left\{ 1 - s^2 \partial_u \partial_v - \frac{2s^3}{3} \partial_u \partial_v \partial_u \partial_v + \frac{s^4}{2} \partial_u \partial_v \partial_u \partial_v \right\} e^{\frac{sU}{2}} \approx$$

$$\approx e^{-\frac{1}{2}s^2} \left[1 - s^2 \partial_u \partial_v - \frac{s^2}{2} \partial_u \partial_v + \right.$$

$$\left. + \frac{2s^3}{3} (\partial_u \partial_v)^2 - \frac{s^5}{3} s^3 \partial_u \partial_v \partial_u \partial_v + \frac{s^6}{2} \partial_u \partial_v \partial_u \partial_v \right] e^{\frac{sU}{2}} \approx$$

$$\approx e^{(\frac{1}{2} - \frac{1}{2})s}$$

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$$\int_{-\infty}^{\infty} \frac{dp}{(2\pi)^d} e^{-ipx} e^{(k-p)^2 s} e^{ipx} =$$

$$= \int_{-\infty}^{\infty} \frac{dp}{(2\pi)^d} e^{-ipx}.$$

$$= e^{-ds} \left\{ 1 - \frac{s^2}{2} \partial_k \partial_p - \frac{s^2}{2} \partial_k + \right. \\ \left. + \frac{s^3}{3} (\partial_k +)^2 - \frac{1}{3} s^3 \partial_k \partial_p + \partial_k \partial_p + \frac{s^4}{2} \partial_k^2 + \partial_k \partial_p \partial_k \partial_p \right\} e^{-ipx}$$

$\times e^{ipx} = \text{averaging} =$

$$= \int_{-\infty}^{\infty} \frac{dp}{(2\pi)^d} e^{-ds} \left\{ 1 + \frac{s^2}{2} \partial_k^2 + \right. \\ \left. + \frac{s^3}{3} (\partial_k +)^2 + \frac{1}{3} s^3 \partial_k \cdot \partial_p - \frac{s^4}{4} (\partial_k +)^2 \partial_p \right\} e^{-sp^2} =$$

$$= e^{-ds} \int_{-\infty}^{\infty} \frac{dp}{(2\pi)^d} \left\{ e^{-sp^2} \cdot \left\{ 1 - \frac{s^2}{2} \partial_k + \frac{s^3}{3} (\partial_k +)^2 \right\} + \right. \\ \left. + p^2 e^{-sp^2} \left[\frac{s^3}{3} \partial_k - \frac{s^4}{4} (\partial_k +)^2 \right] \right\} =$$

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$$= \frac{e^{-ts}}{4\pi} \left\{ \int dp^2 e^{-sp^2} \left[s - \frac{s^2}{2} \alpha d + \frac{s^3}{3} (\alpha d)^2 \right] + \right.$$

$$\left. + \int dp^2 p^2 e^{-sp^2} \left[\frac{s^3}{3} \alpha d - \frac{s^4}{4} (\alpha d)^2 \right] \right\} =$$

$$\int dp^2 e^{-sp^2} = -\frac{1}{s} e^{-sp^2} \Big|_{\infty}^{\infty} = \frac{1}{s}$$

$$\int dp^2 p^2 e^{-sp^2} = -\frac{d}{ds} \int dp^2 e^{-sp^2} = \frac{1}{s^2}$$

$$= \frac{e^{-ts}}{4\pi} \left\{ \frac{1}{s} - \frac{s}{2} \alpha d + \frac{s^2}{3} (\alpha d)^2 + \frac{s}{3} \alpha d - \frac{s^3}{4} (\alpha d)^2 \right\} =$$

$$= \frac{e^{-ts}}{4\pi} \left\{ \frac{1}{s} - \frac{s}{6} \alpha d + \frac{s^2}{12} (\alpha d)^2 \right\} =$$

$$= \int \frac{dp}{(2\pi)^d} e^{-ipx} e^{(\alpha - d)s} e^{ipx}$$

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$$r = -N \int dx \frac{ds}{s} e^{-as^2} \times \frac{e^{-sd}}{4\pi} \left\{ \frac{1}{s} - \frac{s}{6} ad + \frac{1}{12} (ad)^2 \right\} =$$

$$= -\frac{N}{4\pi} \int dx e^{-(m^2 + d^2)s^2} \left\{ \frac{1}{s^2} - \frac{1}{6} ad + \frac{1}{12} (ad)^2 \right\} ds =$$

$$\int \frac{e^{-as}}{s^2} ds = -\frac{e^{-as}}{s} \Big|_0^\infty + \int_a e^{-as} \frac{-a}{s} ds =$$

$$= -\frac{e^{-as}}{s} + a \int da e^{-as} ds =$$

$$= -\frac{e^{-as}}{s} + a \int da \frac{1}{a} e^{-as} =$$

$$= \frac{e^{-as}}{s_0} + a \ln a$$

$$\int e^{-as} ds = -\frac{1}{a} e^{-as} \Big|_0^\infty = \frac{1}{a}$$

$$\int_0^\infty s e^{-as} ds = -\int_{a\infty}^0 e^{-as} ds = \frac{1}{a^2}$$

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$$= -\frac{N}{4\pi} \int dx \left\{ \frac{e^{-s_0(\phi+m^2)}}{s_0} + (\phi+m^2) \ln(\phi+m^2) - \right.$$

$$\left. - \frac{1}{6} \square \phi \cdot \frac{1}{\phi+m^2} + \frac{1}{12} (\partial_\mu \phi)^2 \frac{1}{(\phi+m^2)^2} \right\} =$$

$$\supset -\frac{N}{4\pi} \int dx \left\{ \frac{1}{6} \frac{(-\square \phi)}{\phi+m^2} + \frac{1}{12} \frac{(\partial_\mu \phi)^2}{(\phi+m^2)^2} \right\} =$$



$$= \int dx \left(+ \frac{N}{48\pi} \right) \frac{(\partial_\mu \phi)^2}{(\phi+m^2)^2} < \Gamma$$