

Our potential is

$$V = \frac{N}{4\pi} \left\{ -iD \ln \frac{2|\sigma|}{\Lambda^2} + iD \sum_{k=1}^{\infty} \frac{(-1)^k}{k(k+1)} \left(\frac{iD}{2|\sigma|^2} \right)^k \right\} \quad (0.1)$$

The first term comes from Witten's superpotential. The first term of the series (with $k=1$) is taken into account by kinetic term. Thus we can write our effective action as

$$\begin{aligned} S = & \frac{N}{4\pi} \int d^4x \left\{ \int d^4\theta \frac{1}{2} |\ln \Sigma|^2 \right. \\ & -i \int d^2\theta \left[\sqrt{2}\Sigma \ln \sqrt{2}\Sigma - \sqrt{2}\Sigma \right. \\ & \left. \left. - \frac{1}{2} \sqrt{2}\Sigma \sum_{k=2}^{\infty} \frac{(-1)^k}{k(k+1)(k-1)} \left(\frac{S}{\sqrt{2}\Sigma} \right)^k \right] \right\} + c.c. \end{aligned} \quad (0.2)$$

Is this expression correct? Here I use your definition of S .

The series in the third line can be written as a D -term. We get

$$\begin{aligned} S = & \frac{N}{4\pi} \int d^4x \left\{ \int d^4\theta \frac{1}{2} |\ln \Sigma|^2 \right. \\ & -i \int d^2\theta \left[\sqrt{2}\Sigma \ln \sqrt{2}\Sigma - \sqrt{2}\Sigma \right] \\ & \left. - \int d^4\theta \frac{1}{2} \ln \bar{\Sigma} \sum_{k=1}^{\infty} \frac{(-1)^{(k+1)}}{k(k+1)(k+2)} \left(\frac{S}{\sqrt{2}\Sigma} \right)^k + c.c. \right\} \end{aligned} \quad (0.3)$$

Here we see explicitly that the only F -term is the Witten's superpotential while all other terms are D -terms.

The last term can be rewritten as

$$\begin{aligned} S = & \frac{N}{4\pi} \int d^4x \left\{ \int d^4\theta \frac{1}{2} |\ln \Sigma|^2 - i \int d^2\theta \left[\sqrt{2}\Sigma \ln \sqrt{2}\Sigma - \sqrt{2}\Sigma \right] \right. \\ & - \int d^4\theta \frac{1}{2} \ln \bar{\Sigma} \left[\frac{1}{2} \left(1 + \left(\frac{\sqrt{2}\Sigma}{S} \right) \right)^2 \ln \left(1 + \left(\frac{S}{\sqrt{2}\Sigma} \right) \right) - \frac{3}{4} - \frac{1}{2} \frac{\sqrt{2}\Sigma}{S} \right] \\ & \left. + c.c. \right\} \end{aligned} \quad (0.4)$$