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Compute $3G_{uv} + x(G_{uuv} + G_{uvv})$ at $t_{0s} = 0$

$$G_{uv} = -\frac{1}{x^4}\left\{1 + 2\frac{y^2}{x^2}\right\}\int_0^x \ln(1+x)\,dx - \frac{1}{6}\frac{y^2}{x^3(1+x)}$$

to differentiate w.r.t. u, v, need to

differentiate w.r.t. $x = \frac{u+v}{2}$ and $y = \frac{v-u}{2}$,

then put $y = 0$

So we can put $y^2 = 0$ in G_{uv}:

$$G_{uv} \simeq -\frac{1}{x^4}\int_0^x \ln(1+x)\,dx$$

$$G_{uuv}\Big|_{y=0} = G_{uvv}\Big|_{y=0} = \frac{\partial G_{uv}}{\partial x}\cdot \frac{1}{2} =$$

$$= \frac{1}{2}\frac{\partial}{\partial x}\left\{-\frac{1}{x^4}\int_0^x \ln(1+x)\,dx\right\} =$$

$$= \frac{1}{2}\frac{\partial}{\partial x}\left[-\frac{1}{x^4}\int_0^x \ln(1+x)\,dx\right] =$$

$$= \frac{1}{2}\left\{\frac{4}{x^5}\int_0^x \ln(1+x)\,dx - \frac{\ln(1+x)}{x^4}\right\} =$$

$$\int_0^x \ln(1+x)\,dx = (1+x)\ln(1+x) - (1+x) + 1 =$$

$$= (1+x)\ln(1+x) - x$$

$$= \frac{2(1+x)\ln(1+x) - 2x}{x^5} - \frac{1}{2}\frac{\ln(1+x)}{x^4} =$$

$$= \frac{1}{2}\,\frac{4\ln(1+x) + 4x\ln(1+x) - 4x - x\ln(1+x)}{x^5} =$$

$$= \frac{1}{2}\,\frac{4\ln(1+x) + 3x\ln(1+x) - 4x}{x^5}$$

$$3\, G_{uv} + x\left(G_{uuv} + G_{uvv}\right) =$$

$$= -\frac{3 \cdot (1+x)\ln(1+x) - 3x}{x^4} +$$

$$+ \frac{4\ln(1+x) + 3x\ln(1+x) - 4x}{x^4} =$$

$$= \frac{\ln(1+x) - x}{x^4}$$