FTPI-MINN-XX/XX, UMN-TH-XXXX/XX August 18/2014/DRAFT

Appendix only

A Notations

{app:notations

A.1 Twisted superfields

Twisted superfields exist in two dimensions and are defined by a "twisted" chirality condition

$$D_L \Sigma = \overline{D}_R \Sigma = 0.$$
 (A.1) {gentwist}

Analogously, for twisted anti-chiral superfields,

$$D_R \overline{\Sigma} = \overline{D}_L \overline{\Sigma} = 0.$$
 (A.2) {genantitwist}

The reason these conditions look so similar to those for chiral superfields is that there is no ideomatic difference between chiral and twisted-chiral superfields. To say more, they are interchanged by the action of mirror symmetry — the transposition of supercharges turns condition (A.1) into the one for a chiral superfield. This way, as in the case with the chiral superfields, the constraints (A.1) and (A.2) are solved by letting the superfields be arbitrary functions of "chiral" variables \tilde{y}^{μ} ,

$$\Sigma = \Sigma(\widetilde{y}^{\mu}) \qquad \overline{\Sigma} = \overline{\Sigma}(\overline{\widetilde{y}}^{\mu})$$

$$\widetilde{y}^{0} = x^{0} + i(\overline{\theta}_{R}\theta_{R} - \overline{\theta}_{L}\theta_{L}) \qquad \overline{\widetilde{y}}^{0} = x^{0} - i(\overline{\theta}_{R}\theta_{R} - \overline{\theta}_{L}\theta_{L})$$

$$\widetilde{y}^{3} = x^{3} + (\overline{\theta}_{R}\theta_{R} + \overline{\theta}_{L}\theta_{L}) \qquad \overline{\widetilde{y}}^{3} = x^{3} - (\overline{\theta}_{R}\theta_{R} + \overline{\theta}_{L}\theta_{L}),$$

after which they will have the usual "chiral" component expansion

$$\Sigma(\widetilde{y}) = \sigma(\widetilde{y}) - \sqrt{2}\,\theta_R\overline{\lambda}_L + \sqrt{2}\,\overline{\theta}_L\lambda_R + \sqrt{2}\,\theta_R\overline{\theta}_L\,\widetilde{F}$$

$$\overline{\Sigma}(\overline{\widetilde{y}}) = \overline{\sigma}(\overline{\widetilde{y}}) - \sqrt{2}\,\theta_L\overline{\lambda}_R + \sqrt{2}\,\overline{\theta}_R\lambda_L + \sqrt{2}\,\theta_L\overline{\theta}_R\,\overline{\widetilde{F}}.$$

Here we understand that each function on the right hand side depends on \widetilde{y}^{μ} and $\overline{\widetilde{y}}^{\mu}$, correspondingly.

Exactly the same way as with chiral superfields, one constructs twisted superpotentials $\widetilde{\mathcal{W}}(\Sigma)$, just as functions that depend on Σ holomorphically. One then performs the twisted $d^2\widetilde{\theta}$ integration as,

$$\int d^2 \tilde{\theta} \, \widetilde{\mathcal{W}}(\Sigma) = \frac{1}{2} \, \overline{D}_L \, D_R \, \widetilde{\mathcal{W}}(\Sigma) \, \Big| \,, \qquad \int d^2 \overline{\tilde{\theta}} \, \overline{\widetilde{\mathcal{W}}}(\overline{\Sigma}) = \frac{1}{2} \, \overline{D}_R \, D_L \, \overline{\widetilde{\mathcal{W}}}(\overline{\Sigma}) \, \Big| \,. \quad (A.3)$$

And, of course, one can perform the full superspace integration of twisted superfields, provided that this holomorphicity is broken (e.g. by putting both chiral and anti-chiral factors),

$$\int d^4\theta \, \overline{\Sigma} \, \Sigma \,, \tag{A.4}$$

or the result will obviously be a total derivative.

One famous example of a twisted superfield is the *fieldstrength* of a $\mathcal{N}=(2,2)$ gauge supermultiplet V,

$$\Sigma = \frac{i}{\sqrt{2}} D_L \overline{D}_R V, \qquad \overline{\Sigma} = \frac{i}{\sqrt{2}} D_R \overline{D}_L V. \qquad (A.5)$$

In components it takes the form

$$\Sigma(\widetilde{y}) = \sigma(\widetilde{y}) - \sqrt{2} \theta_R \overline{\lambda}_L + \sqrt{2} \overline{\theta}_L \lambda_R + \sqrt{2} \theta_R \overline{\theta}_L \left(D - i F_{03} \right)$$

$$\overline{\Sigma}(\overline{\widetilde{y}}) \ = \ \overline{\sigma}(\overline{\widetilde{y}}) \ - \ \sqrt{2}\,\theta_L \overline{\lambda}_R \ + \ \sqrt{2}\,\overline{\theta}_R \lambda_L \ + \ \sqrt{2}\,\theta_L \overline{\theta}_R \left(D \ + \ i\,F_{03}\right) \,.$$

A.2 $\mathcal{N} = (0, 2)$ superfields

We define $\mathcal{N}=(0,2)$ superspace via reduction of $\mathcal{N}=(2,2)$ superspace by putting

$$\theta_L = \overline{\theta}_L = 0. \tag{A.6}$$

Each chiral and twisted-chiral $\mathcal{N}=(2,2)$ superfield this way splits into two $\mathcal{N}=(0,2)$ superfields. They still, however, retain their property of holomorphicity. While chiral superfields are usually described as being dependent on a "holomorphic" variable y^{μ} ,

$$y^{\mu} = x^{\mu} + i \overline{\theta \sigma_{\mu}} \theta, \qquad (A.7)$$

and twisted-chiral as dependent on a twisted variable \tilde{y}^{μ} ,

$$\widetilde{y}^{\mu} = x^{\mu} + i \overline{\theta \widetilde{\sigma}}_{\mu} \theta,$$
 (A.8)

the distinction between the two variables vanishes upon reduction to $\mathcal{N}=(0,2)$ superspace. As a result, $\mathcal{N}=(0,2)$ superspace defines only one kind of holomorphic

superfields — chiral $\mathcal{N}=(0,2)$ superfields, which depend upon the reduced variables v^{μ}

$$v^{0} = x^{0} + i \overline{\theta}_{R} \theta_{R}$$

$$v^{3} = x^{3} + i \overline{\theta}_{R} \theta_{R}.$$
(A.9)

To make a distinction with $\mathcal{N}=(2,2)$ superfields, we denote the $\mathcal{N}=(0,2)$ superfields by symbols with a hat on top $-\hat{\sigma}$, $\hat{\xi}$, etc.

Chiral superfields $\Phi(y)$ split into $\mathcal{N} = (0,2)$ superfields $\hat{\phi}(v)$ and $\hat{\xi}(v)$,

$$\begin{split} & \Phi(y) & \longrightarrow & \hat{\phi}(v) + \sqrt{2} \, \theta_L \, \hat{\xi}(v) \,, \\ & \overline{\Phi}(\overline{y}) & \longrightarrow & \hat{\overline{\phi}}(\overline{v}) - \sqrt{2} \, \overline{\theta}_L \, \hat{\overline{\xi}}(\overline{v}) \,, \end{split} \tag{A.10} \quad \{\text{Phisplit}\} \end{split}$$

while twisted-chiral superfields $\Sigma(\widetilde{y})$ split into $\hat{\sigma}(v)$ and $\hat{\lambda}(v)$,

$$\Sigma(\widetilde{y}) \longrightarrow \hat{\sigma}(v) + \sqrt{2}\,\overline{\theta}_L\,\hat{\lambda}(v)\,,$$

$$\overline{\Sigma}(\overline{\widetilde{y}}) \longrightarrow \hat{\sigma}(\overline{v}) - \sqrt{2}\,\theta_L\,\hat{\overline{\lambda}}(v)\,. \tag{A.11} {Sigmasplit}$$

We alert that relations (A.10) and (A.11) have only symbolical meaning demonstrating the effect of splitting, while there is no equality: the right-hand sides have incomplete dependence on θ_L , $\overline{\theta}_L$.

The individual $\mathcal{N} = (0, 2)$ superfields have a quite straightforward component expansion,

$$\hat{\phi}(v) = \phi - \sqrt{2} \theta_R \psi_L, \qquad \hat{\xi}(v) = \psi_R + \sqrt{2} \theta_R F,
\hat{\overline{\phi}}(\overline{v}) = \overline{\phi} + \sqrt{2} \overline{\theta_L \psi_L}, \qquad \hat{\overline{\xi}}(\overline{v}) = \overline{\psi}_R + \sqrt{2} \overline{\theta_R F}, \qquad (A.12)$$

and similarly do the ones that arise from splitting of the twisted chiral superfield $\Sigma(\tilde{y})$,

$$\hat{\sigma}(v) = \sigma - \sqrt{2} \theta_R \overline{\lambda}_L, \qquad \hat{\lambda}(v) = \lambda_R - \theta_R \widetilde{F},
\hat{\overline{\sigma}}(\overline{v}) = \overline{\sigma} + \sqrt{2} \overline{\theta}_R \lambda_L, \qquad \hat{\overline{\lambda}}(\overline{v}) = \overline{\lambda}_R - \overline{\theta}_R \overline{\widetilde{F}}. \qquad (A.13)$$

It is interesting to note that the simple structure of splitting shown in (A.10) and (A.11) makes fermionic superfields in $\mathcal{N} = (0, 2)$ superspace much more ubiqutious

than in $\mathcal{N} = (2,2)$ superspace. And the first example to this is the $\mathcal{N} = (0,2)$ superpotential. It has to be fermionic because the integration over *half* of the $\mathcal{N} = (0,2)$ superspace $d\theta_R$ is such. A superpotential can be constructed using an arbitrary holomorphic function — say $J(\hat{\sigma})$, and by multiplying it by an arbitrary (but still chiral) fermionic multiplet — say $\hat{\rho}$,

$$\int d\theta_R \,\hat{\rho} \, J(\hat{\sigma}) \,. \tag{A.14}$$

"Full" superspace integrals can conventionally be built using both chiral and antichiral fields,

$$\int d^2\theta_R \,\hat{\xi} \,\hat{\overline{\xi}} = \overline{\psi}_R \, i\partial_L \psi_R + \overline{F} \, F \,. \tag{A.15}$$

B Component Expansion of the Effective Action

{app:expansion

Here we give the complete component expansion of expression (??). Typically, one would be interested in a specific limit of this expression, such as the bosonic part of it, or the constant bosonic part, or an approximation in the certain number of space-time derivatives (some approximations, however, are easier to derive from the series representation (??)).

We make a remark that, according to [2], the whole fermionic part of the below effective Lagrangian can be obtained from its bosonic part simply via a replacement

$$\sqrt{2}\sigma \longrightarrow \sqrt{2}\sigma + 2\frac{i\overline{\lambda}_L\lambda_R}{iD + F_{03}}.$$
 (B.16)

It is useful to know the lowest component of the superfield S,

$$s = S \Big| = \frac{\sqrt{2}\overline{\sigma}(iD - F_{03}) - 2i\overline{\lambda}_R\lambda_L}{(\sqrt{2}\overline{\sigma})^2},$$

$$\overline{s} = \overline{S} = \overline{S} = \frac{\sqrt{2}\sigma (iD + F_{03}) - 2i\overline{\lambda}_L \lambda_R}{(\sqrt{2}\sigma)^2}.$$

In the expression below, however, we extensively make use of the lowest component

of the ratio $S/(\sqrt{2}\Sigma)$, which we denote as p,

$$p = \frac{S}{\sqrt{2}\Sigma} \Big| = \frac{1}{|\sqrt{2}\sigma|^2} \left(iD - F_{03} - \frac{2i\sqrt{2}\sigma\overline{\lambda}_R\lambda_L}{|\sqrt{2}\sigma|^2} \right),$$

$$\overline{p} = \frac{\overline{S}}{\sqrt{2\Sigma}} \Big| = \frac{1}{|\sqrt{2}\sigma|^2} \left(iD + F_{03} - \frac{2i\sqrt{2}\sigma\lambda_L\lambda_R}{|\sqrt{2}\sigma|^2} \right).$$

We have,

$$\frac{4\pi}{N}\mathcal{L} = iD - F_{03} \log \frac{\sqrt{2}\sigma}{\sqrt{2}\overline{\sigma}} - i \frac{\sqrt{2}\sigma\overline{\lambda}_{R}\lambda_{L} + \sqrt{2}\sigma\overline{\lambda}_{L}\lambda_{R}}{|\sqrt{2}\sigma|^{2}} - \frac{1}{4} \ln \sqrt{2}\sigma \square \ln \sqrt{2}\sigma - \frac{1}{4} \ln \sqrt{2}\overline{\sigma} \square \ln \sqrt{2}\overline{\sigma} - \frac{1}{2} \left(iD + F_{03} + \frac{1}{2}\square \ln \sqrt{2}\overline{\sigma}\right) \ln \left(|\sqrt{2}\sigma|^{2} + iD - F_{03} - 2i\frac{\overline{\lambda}_{R}\lambda_{L}}{\sqrt{2}\overline{\sigma}}\right) - \frac{1}{2} \left(iD - F_{03} + \frac{1}{2}\square \ln \sqrt{2}\sigma\right) \ln \left(|\sqrt{2}\sigma|^{2} + iD + F_{03} - 2i\frac{\overline{\lambda}_{L}\lambda_{R}}{\sqrt{2}\overline{\sigma}}\right) + \frac{-\overline{\lambda}_{R}i\overleftarrow{\mathcal{D}}_{L}\lambda_{R} + \overline{\lambda}_{L}i\mathcal{D}_{R}\lambda_{L} - \frac{1}{2}i\sqrt{2}\overline{\sigma}\lambda_{L}\lambda_{R} + \frac{1}{2}\frac{1}{\sqrt{2}\overline{\sigma}}i\overline{\lambda}_{R}\overleftarrow{\mathcal{D}}_{L}\mathcal{D}_{R}\lambda_{L}}{|\sqrt{2}\sigma|^{2} + iD - F_{03} - 2i\frac{\overline{\lambda}_{R}\lambda_{L}}{\sqrt{2}\overline{\sigma}}} + \frac{\overline{\lambda}_{R}i\mathcal{D}_{L}\lambda_{R} - \overline{\lambda}_{L}i\overleftarrow{\mathcal{D}}_{R}\lambda_{L} - \frac{1}{2}i\sqrt{2}\sigma\overline{\lambda}_{R}\lambda_{L} + \frac{1}{2}\frac{1}{\sqrt{2}\sigma}i\overline{\lambda}_{L}\overleftarrow{\mathcal{D}}_{R}\mathcal{D}_{L}\lambda_{R}}{|\sqrt{2}\sigma|^{2} + iD + F_{03} - 2i\frac{\overline{\lambda}_{L}\lambda_{R}}{\sqrt{2}\overline{\sigma}}} - \frac{1}{4}\frac{|\sqrt{2}\sigma|^{2}\left(iD - F_{03} + \square\ln\sqrt{2}\sigma\right)}{|\sqrt{2}\sigma|^{2} + iD + F_{03} - 2i\frac{\overline{\lambda}_{L}\lambda_{R}}{\sqrt{2}\sigma}} - \frac{1}{2}|\sqrt{2}\sigma|^{2}\left(iD - F_{03} + \square\ln\sqrt{2}\sigma\right)}{|\sqrt{2}\sigma|^{2} + iD + F_{03} - 2i\frac{\overline{\lambda}_{L}\lambda_{R}}{\sqrt{2}\sigma}} - \frac{1}{2}|\sqrt{2}\sigma|^{2}\left(iD - F_{03} + \square\ln\sqrt{2}\sigma\right)}{|\sqrt{2}\sigma|^{2} + iD + F_{03} - 2i\frac{\overline{\lambda}_{L}\lambda_{R}}{\sqrt{2}\sigma}}\right)^{2}$$

$$-\frac{1}{2}|\sqrt{2}\sigma|^{2}\left(1+\frac{1}{\overline{p}}\right)\frac{\overline{\lambda}_{R}iD_{L}\lambda_{R}-\overline{\lambda}_{L}i\overleftarrow{D}_{R}\lambda_{L}+i\sqrt{2}\sigma\overline{\lambda}_{R}\lambda_{L}+\frac{1}{\sqrt{2}\sigma}i\overline{\lambda}_{L}\overleftarrow{D}_{R}D_{L}\lambda_{R}}{\left(|\sqrt{2}\sigma|^{2}+iD+F_{03}-2i\frac{\overline{\lambda}_{L}\lambda_{R}}{\sqrt{2}\sigma}\right)^{2}}$$

$$+\frac{1}{2p^{3}}\left(-p^{2}\left(iD+F_{03}\right)+\frac{1}{2}p\Box\ln\sqrt{2}\overline{\sigma}-2p^{2}i\frac{\overline{\lambda}_{L}\lambda_{R}}{\sqrt{2}\overline{\sigma}}+\right)$$

$$+2i\frac{1}{\sqrt{2}\sigma}\frac{\overline{\lambda}_{R}\overleftarrow{D}_{L}D_{R}\lambda_{L}}{\left(\sqrt{2}\overline{\sigma}\right)^{2}}-2p\frac{\overline{\lambda}_{L}iD_{R}\lambda_{L}-\overline{\lambda}_{R}i\overleftarrow{D}_{L}\lambda_{R}}{\left|\sqrt{2}\sigma\right|^{2}}\right)\cdot\ln\left(1+p\right)$$

$$+\frac{1}{2\overline{p}^{3}}\left(-\overline{p}^{2}\left(iD-F_{03}\right)+\frac{1}{2}\overline{p}\Box\ln\sqrt{2}\sigma-2\overline{p}^{2}i\frac{\overline{\lambda}_{R}\lambda_{L}}{\sqrt{2}\overline{\sigma}}+\right)$$

$$+2i\frac{1}{\sqrt{2}\overline{\sigma}}\frac{\overline{\lambda}_{L}\overleftarrow{D}_{R}D_{L}\lambda_{L}}{\left(\sqrt{2}\overline{\sigma}\right)^{2}}-2\overline{p}\frac{\overline{\lambda}_{R}iD_{L}\lambda_{R}-\overline{\lambda}_{L}i\overleftarrow{D}_{R}\lambda_{L}}{\left|\sqrt{2}\overline{\sigma}\right|^{2}}\right)\cdot\ln\left(1+\overline{p}\right)$$

$$+\frac{1}{2p^{2}}i\left(\frac{-\sqrt{2}\sigma\overline{\lambda}_{L}-\overline{\lambda}_{R}\overleftarrow{D}_{L}}{\left|\sqrt{2}\overline{\sigma}\right|^{2}+iD-F_{03}-2i\frac{\overline{\lambda}_{L}\lambda_{R}}{\sqrt{2}\overline{\sigma}}}+\frac{\overline{\lambda}_{L}}{\sqrt{2}\overline{\sigma}}\right)\left(2p\lambda_{R}+\frac{D_{R}\lambda_{L}}{\sqrt{2}\overline{\sigma}}\right)$$

$$+\frac{1}{2p^{2}}i\left(-2p\overline{\lambda}_{L}+\frac{\overline{\lambda}_{R}\overleftarrow{D}_{L}}{\sqrt{2}\overline{\sigma}}\right)\left(\frac{\sqrt{2}\overline{\sigma}\lambda_{R}-D_{R}\lambda_{L}}{\left|\sqrt{2}\overline{\sigma}\right|^{2}+iD-F_{03}-2i\frac{\overline{\lambda}_{L}\lambda_{R}}{\sqrt{2}\overline{\sigma}}}-\frac{\lambda_{R}}{\sqrt{2}\overline{\sigma}}\right)$$

$$+\frac{1}{2p^{2}}i\left(-2p\overline{\lambda}_{L}+\frac{\overline{\lambda}_{L}\overleftarrow{D}_{R}}{\sqrt{2}\overline{\sigma}}\right)\left(\frac{\sqrt{2}\overline{\sigma}\lambda_{L}-D_{L}\lambda_{R}}{\left|\sqrt{2}\overline{\sigma}\right|^{2}+iD-F_{03}-2i\frac{\overline{\lambda}_{L}\lambda_{R}}{\sqrt{2}\overline{\sigma}}}-\frac{\lambda_{L}}{\sqrt{2}\overline{\sigma}}\right)$$

$$-\frac{1}{4p}|\sqrt{2}\sigma|^{2}\frac{iD+F_{03}+\Box\ln\sqrt{2}\overline{\sigma}}{\left|\sqrt{2}\sigma|^{2}+iD-F_{03}-2i\frac{\overline{\lambda}_{R}\lambda_{L}}{\sqrt{2}\overline{\sigma}}}+\frac{1}{4}|\sqrt{2}\sigma|^{2}\frac{\overline{p}}{p}$$

$$- \frac{1}{4\overline{p}} \left| \sqrt{2}\sigma \right|^2 \frac{iD - F_{03} + \Box \ln \sqrt{2}\sigma}{\left| \sqrt{2}\sigma \right|^2 + iD + F_{03} - 2i \frac{\overline{\lambda}_L \lambda_R}{\sqrt{2}\sigma}} + \frac{1}{4} \left| \sqrt{2}\sigma \right|^2 \frac{p}{\overline{p}}.$$

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