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Convert $\Box\phi$ to normal kinetic term

$$G_{uu} \frac{(iD)^2}{\phi^4} (\partial_\mu \phi)^2 +$$

$$+ G_u \, iD \cdot \left[\frac{2(\partial_\mu \phi)^2}{\phi^3} - \frac{\Box\phi}{\phi^2} \right] -$$

$$- G_{uv} \frac{(iD)^2}{\phi^2} \Box \ln\phi$$

$$\mathcal{L}_{eff} \supset - \frac{(\partial_\mu \phi)^2}{\phi^2} x^2 \left\{ 3 G_{uv} + (G_{uuv} + G_{uvv}) x \right\}$$

$$\cdot\quad G_u\left(-\frac{\Box\phi}{\phi^2}\right) = \partial_\mu\phi\cdot\partial_\mu\left(\frac{G_u}{\phi^2}\right) \supset$$

$$\supset \partial_\mu\phi\left(-\frac{2\partial_\mu\phi}{\phi^3}\right)G_u +$$

$$+ \frac{\partial_\mu\phi}{\phi^2}\left\{G_{uu}\frac{iD}{(-\phi^2)}\partial_\mu\phi + G_{uv}\frac{iD}{(-\phi^2)}\partial_\mu\phi\right\} =$$

$$= -(\partial_\mu\phi)^2\left\{\frac{2G_u}{\phi^3} + (G_{uu}+G_{uv})\frac{iD}{\phi^4}\right\} =$$

$$= -\frac{(\partial_\mu\phi)^2}{\phi^3}\left[2G_u + (G_{uu}+G_{uv})\frac{iD}{\phi}\right]$$ ← multiply by iD now

$$\Rightarrow G_u\, iD\left\{\frac{2(\partial_\mu\phi)^2}{\phi^3} - \frac{\Box\phi}{\phi^2}\right\} \supset$$

$$\supset -\frac{(\partial_\mu\phi)^2}{\phi^4}(iD)^2(G_{uu}+G_{uv})$$

$$\cdot \quad -G_{\mu\nu}\frac{(iD)^2}{\phi^2}\,\square\ln\phi =$$

$$= G_{\mu\nu}\frac{(\partial_\mu\phi)^2}{\phi^4}(iD)^2 - G_{\mu\nu}\frac{(iD)^2\,\square\phi}{\phi^3} =$$

$$= G_{\mu\nu}(iD)^2\left[\frac{(\partial_\mu\phi)^2}{\phi^4} - \frac{\square\phi}{\phi^3}\right] =$$

$$\cdot\cdot \quad -G_{\mu\nu}(iD)^2\frac{\square\phi}{\phi^3} = \partial_\mu\phi\,(iD)^2\,\partial_\mu\left(\frac{G_{\mu\nu}}{\phi^3}\right) \supset$$

$$\supset \partial_\mu\phi\,(iD)^2\left[-\frac{3\,\partial_\mu\phi\cdot G_{\mu\nu}}{\phi^4} + \frac{G_{\mu\mu\nu}}{\phi^3}\frac{iD}{(-\phi^2)}\partial_\mu\phi \quad \frac{G_{\mu\nu\nu}}{\phi^3}\frac{iD}{(-\phi^2)}\partial_\mu\phi\right]$$

$$= (\partial_\mu\phi)^2(iD)^2\left[-3\frac{G_{\mu\nu}}{\phi^4} - (G_{\mu\mu\nu}+G_{\mu\nu\nu})\frac{iD}{\phi^5}\right]$$

$$= -(\partial_\mu\phi)^2(iD)^2\left[2\frac{G_{\mu\nu}}{\phi^4} + (G_{\mu\mu\nu}+G_{\mu\nu\nu})\frac{iD}{\phi^5}\right] =$$

$$= -\frac{(\partial_\mu\phi)^2}{\phi^4}(iD)^2\left[2G_{\mu\nu} + (G_{\mu\mu\nu}+G_{\mu\nu\nu})\frac{iD}{\phi}\right]$$

Altogether, the effective action contains,

$$\cancel{G_{uu} \frac{(iD)^2}{\phi^4} (\partial_\mu \phi)^2} - \frac{(\partial_\mu \phi)^2}{\phi^4} (iD)^2 (\cancel{G_{uu}} + G_{uv}) -$$

$$- \frac{(\partial_\mu \phi)^2}{\phi^4} (iD)^2 \left\{ 2\, G_{uv} + (G_{uuv} + G_{uvv}) \frac{iD}{\phi} \right\} =$$

$$= - \frac{(\partial_\mu \phi)^2}{\phi^4} (iD)^2 \left\{ 3\, G_{uv} + (G_{uuv} + G_{uvv}) \frac{iD}{\phi} \right\}$$

$$\mathcal{L}_{eff} \supset - \frac{(\partial_\mu \phi)^2}{\phi^2} x^2 \left\{ 3\, G_{uv} + (G_{uuv} + G_{uvv})\, x \right\}$$