March 21, 2016

Conditions on the Effective D-term $G(u,v)$

V_{eff} and $F_{\mu\nu}^2$

$$\int d^4\theta\, G(u,v) = \int d^4\theta\, G(x,y) = -\tfrac{1}{4}\bar{D}_R D_L \bar{D}_L D_R\, G(u,v)\Big|$$

$$u = S/\sqrt{2}\Sigma \qquad x = \frac{u+v}{2} \qquad x\Big| \supset \frac{iD}{|\sqrt{2}\sigma|^2}$$

$$v = \bar{S}/\sqrt{2}\bar{\Sigma} \qquad y = \frac{v-u}{2} \qquad y\Big| \supset \frac{f_{03}}{|\sqrt{2}\sigma|^2}$$

= keeping only bosons, allowing D's only act on Σ's

and only in a chiral way

(otherwise spatial deriv of σ will be produced) $\supset$

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$$= -\tfrac{1}{4}\,\bar{D}_R D_L\, G_u\, (-1)\,\frac{S}{(\sqrt{\varepsilon}\Sigma)^2}\cdot \bar{D}_L D_R \sqrt{\varepsilon}\Sigma \Big| =$$

$$= -\tfrac{1}{4}\, G_{uv}\, (-1)\,\frac{S}{(\sqrt{\varepsilon}\Sigma)^2}\,\bar{D}_L D_R \sqrt{\varepsilon}\Sigma\,(-1)\,\frac{\bar{S}}{(\sqrt{\varepsilon}\bar{\Sigma})^2}\,\bar{D}_R D_L \sqrt{\varepsilon}\bar{\Sigma} \Big| =$$

$$= -\tfrac{1}{4}\, G_{uv}\, \frac{S\bar{S}}{|\sqrt{\varepsilon}\Sigma|^4}\,\bar{D}_R D_L \sqrt{\varepsilon}\bar{\Sigma}\cdot \bar{D}_L D_R \sqrt{\varepsilon}\Sigma \Big| =$$

$$= -\tfrac{1}{4}\, G_{\mu\nu}\, \frac{(iD)^2 - f_{03}^2}{|\Sigma 6|^6} \cdot (-2i)(iD + f_{03}) \cdot (-2i)(iD - f_{03}) =$$

$$= G_{\mu\nu}\, \frac{\left((iD)^2 - f_{03}^2\right)^2}{|\Sigma 6|^6} =$$

Note that $\partial_u = \frac{\partial_x - \partial_y}{2}$ $\quad \partial_v = \frac{\partial_x + \partial_y}{2}$

$$\Rightarrow \partial_u \partial_v = \frac{\partial_x^2 - \partial_y^2}{4}$$

$$= G_{uv} \frac{\left((iD)^2 - f_{03}^2\right)^2}{|\sqrt{2}b|^6} = \frac{\partial_x^2 - \partial_y^2}{4} G(x,y) \cdot \frac{\left((iD)^2 - f_{03}^2\right)^2}{|\sqrt{2}b|^6}$$

$$= G_{uv} |\sqrt{2}b|^2 \left\{ \left(\frac{iD}{|\sqrt{2}b|^2}\right)^2 - \left(\frac{f_{03}}{|\sqrt{2}b|^2}\right)^2 \right\}^2 = G_{uv} (x^2 - y^2)^2 |\sqrt{2}b|^2$$

here x, y are

the lowest components

of the superfields

- Potential condition – omitting $\frac{N}{4\pi}$:

$r_{03} \to 0$,

$$G_{uv}\Big|_{y=0}(x^2-y^2)^2\Big|_{y=0}|\tilde{\kappa}b|^2 = G_{uv}\Big|_{y=0} \cdot x^4\,|\tilde{\kappa}b|^2 =$$

$$= -|\tilde{\kappa}b|^2 \int_0^x \ln(1+x)\,dx$$

$$\rightarrow \quad G_{uv}\Big|_{y=0} = -\frac{1}{x^4}\int_0^x \ln(1+x)\,dx$$

· Gauge kinetic term condition — omitting $\frac{N}{4\pi}$:

Need $\frac{1}{4} F_{\mu\nu}^2 = \frac{1}{2} f_{03}^2$ times $\qquad |\ln \Sigma|^2$

$$\frac{1}{3}\left\{ \frac{1}{iD + |Gb|^2} - \frac{1}{|Gb|^2} \right\} + \frac{1}{|Gb|^2}$$

$$\Downarrow$$

$$- \frac{1}{3} \, \frac{iD / |Gb|^2}{iD + |Gb|^2} =$$

$$= - \frac{1}{3} \, \frac{x}{1+x} \, \frac{1}{|Gb|^2}$$

So we need

$$- \frac{1}{6} \, \frac{x}{1+x} \, \frac{f_{03}^2}{|Gb|^2} = - \frac{1}{6} \, \frac{x y^2}{1+x} |Gb|^2$$

Extracting y^2 from $G_{uv} \cdot (x^2 - y^2)^2 \, |\xi b|^2$:

$$\tfrac{1}{2} \partial_y^2 G_{uv} \Big|_{y=0} \cdot y^2 \cdot x^4 |\xi b|^2 - 2 \, G_{uv} \Big|_{y=0} x^2 y^2 \, |\xi b|^2 =$$

$$= - \tfrac{1}{6} \, \frac{x y^2}{1+x} \, |\xi b|^2$$

$$\rightarrow \quad \partial_y^2 G_{uv} \Big|_{y=0} x^4 \cdot \tfrac{1}{2} y^2 - 2 \, G_{uv} \Big|_{y=0} x^2 y^2 = - \tfrac{1}{6} \, \frac{x y^2}{1+x}$$

Let's gather the conditions:

$$G_{uv}\Big|_{y=0} = -\frac{1}{x^4}\int_0^x \ln(1+x)\,dx$$

$$\frac{1}{2}\partial_y^2 G_{uv}\Big|_{y=0} = +\frac{2}{x^2} G_{uv}\Big|_{y=0} - \frac{1}{6}\frac{1}{(1+x)x^3}$$

We can now introduce $H(x,y) = \partial_u \partial_v G(u,v)$

$$H\Big|_{y=0} = -\frac{1}{x^4}\int_0^x dx \ln(1+x)$$

$$\frac{1}{2}\partial_y^2 H\Big|_{y=0} = \frac{2}{x^2} H\Big|_{y=0} - \frac{1}{6}\,\frac{1}{(1+x)x^3}$$

Once again,

$$H(x,y)\Big|_{y=0} = -\frac{1}{x^4}\int_0^x \ln(1+x)\,dx$$

$$\frac{1}{2}\partial_y^2 H(x,y)\Big|_{y=0} = \frac{2}{x^2}H(x,y)\Big|_{y=0} - \frac{1}{6}\frac{1}{x^3(1+x)}$$

The conditions of symmetry in $u \longleftrightarrow v$ of d'Adda et al. then demands $H = H(y^2)$ — no odd powers of y

So we have now the first three terms

of $\underline{H}(x,y)$:

$$H(x,y) = -\frac{1}{x^4}\int_0^x \ln(1+x)\,dx + \frac{2}{x^2}y^2\frac{(-1)}{x^4}\int_0^x \ln(1+x)\,dx -$$

$$-\frac{1}{6}\frac{y^2}{x^3(1+x)} =$$

$$= -\frac{1}{x^4}\left\{1 + 2\frac{y^2}{x^2}\right\}\int_0^x \ln(1+x)\,dx - \frac{1}{6}\frac{y^2}{x^3(1+x)}$$

$$H(x,y) = G_{uv}(u,v) =$$

$$= -\frac{1}{x^4}\left[1 + 2\frac{y^2}{x^2}\right] \int_0^x \ln(1+x)\,dx - \frac{1}{6}\frac{y^2}{x^3(1+x)}$$

Now, $G(u,v) = \int du\, dv\, G_{uv} =$

$$= \int dv\, dx \left| \frac{\partial(u,v)}{\partial(x,v)} \right| \cdot G_{uv} =$$

$$= 2 \int dv\, dx\, G_{uv}$$

Need to express G_{uv} in terms of x, v: $\quad u = 2x - v$

$y = v - x$

$G_{uv}(x, v) =$

$$= -\left\{\frac{1}{x^4} + 2\frac{(2x-v)^2}{x^6}\right\} \cdot \int_0^x \ln(1+x)\,dx - \frac{1}{6}\frac{(2x-v)^2}{x^3(1+x)}$$

$2\int dx\, G_{uv}(x, v) =$

$$= -2\int dx \left\{\frac{1}{x^4} + 2\frac{(2x-v)^2}{x^6}\right\} \int_0^x \ln(1+x)\,dx \quad -$$

$$- \frac{1}{3}\int dx \frac{(2x-v)^2}{x^3(1+x)}$$

Therefore,

$$2\int dx\, G(x, v) =$$

$$= -2\int dx \left\{ \frac{1}{x^4} + 2\frac{(x-v)^2}{x^6} \right\} \int_0^x \ln(1+x)\, dx \, -$$

$$- \frac{1}{3}\int dx\, \frac{(x-v)^2}{x^3(1+x)}$$

$$\int dx \left[\frac{1}{x^4} + 2\frac{(x-v)^2}{x^6}\right] \int_0^x \ln(1+x)\, dx =$$

$\ast$ first $\displaystyle\int dx \left[\frac{1}{x^4} + \frac{2(x-v)^2}{x^6}\right] =$

$$= \int dx \left[\frac{1}{x^4} + 2\,\frac{x^2 - 2vx + v^2}{x^6}\right] =$$

$$= \int dx \left[\frac{3}{x^4} - 4\frac{v}{x^5} + 2\frac{v^2}{x^6}\right] =$$

$$= -\frac{1}{x^3} + \frac{v}{x^4} - \frac{2}{5}\frac{v^2}{x^5} =$$

$$= -\left[\frac{1}{x^3} - \frac{v}{x^4} + \frac{2}{5}\frac{v^2}{x^5}\right]$$

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$$= -\left[\frac{1}{x^3} - \frac{v}{x^4} + \frac{2}{5}\frac{v^2}{x^5}\right] \int_0^x dx \ln(1+x) +$$

$$+ \int \left[\frac{1}{x^3} - \frac{v}{x^4} + \frac{2}{5}\frac{v^2}{x^5}\right] \ln(1+x) =$$

$$= -\left[\frac{1}{x^3} - \frac{v}{x^4} + \frac{2}{5}\frac{v^2}{x^5}\right] \int_0^x dx \ln(1+x) +$$

$$+ \left[\frac{1}{2x^2} - \frac{v}{3x^3} + \frac{1}{10}\frac{v^2}{x^4}\right] \ln(1+x) +$$

$$+ \int \left[\frac{1}{2x^2} - \frac{v}{3x^3} + \frac{1}{10}\frac{v^2}{x^4}\right] \cdot \frac{1}{1+x} dx =$$

$$\int \left[\frac{1}{2x^2} - \frac{v}{3x^3} + \frac{1}{10} \frac{v^2}{x^4} \right] \frac{dx}{1+x} =$$

$$\frac{1}{x^2} \frac{1}{1+x} = \frac{1}{1+x} - \frac{1}{x} + \frac{1}{x^2}$$

$$\frac{1}{x^3} \frac{1}{1+x} = -\frac{1}{1+x} + \frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^3}$$

$$\frac{1}{x^4} \frac{1}{1+x} = \frac{1}{1+x} - \frac{1}{x} + \frac{1}{x^2} - \frac{1}{x^3} + \frac{1}{x^4}$$

$$= \frac{1}{2} \left[\ln \frac{1+x}{x} - \frac{1}{x} \right] +$$

$$+ \frac{v}{3} \left[\ln \frac{1+x}{x} - \frac{1}{x} + \frac{1}{2x^2} \right] +$$

$$+ \frac{v^2}{10} \left[\ln \frac{1+x}{x} - \frac{1}{x} + \frac{1}{2x^2} - \frac{1}{3x^3} \right]$$

$$= -\left[\frac{1}{x^3} - \frac{v}{x^4} + \frac{2}{5}\frac{v^2}{x^5}\right] \int_0^x dx \ln(1+x) -$$

$$-\left[\frac{1}{2x^2} - \frac{v}{3x^3} + \frac{1}{10}\frac{v^2}{x^4}\right] \ln(1+x) +$$

$$+\frac{1}{2}\left[\ln\frac{1+x}{x} - \frac{1}{x}\right] +$$

$$+\frac{v}{3}\left[\ln\frac{1+x}{x} - \frac{1}{x} + \frac{1}{2x^2}\right] +$$

$$+\frac{v^2}{10}\left[\ln\frac{1+x}{x} - \frac{1}{x} + \frac{1}{2x^2} - \frac{1}{3x^3}\right]$$

(began on p. 16)
(part of p. 14)

$$\cdot \int dx \, \frac{(x-v)^2}{x^3(1+x)} =$$

$$= \int dx \, (-1) \left\{ \frac{(v+1)^2}{1+x} - \frac{(v+1)^2}{x} + \frac{v^2+2v}{x^2} - \frac{v^2}{x^3} \right\} =$$

$=$

$$= -(v+1)^2 \ln \frac{1+x}{x} + \frac{v^2+2v}{x} - \frac{v^2}{2x^2}$$

begin on p. 20 (part of p. 14)

Now we combine $(-2)\times(p.19) - (1/3)\times(p.21)$

$$\int dx\, G_{wv}(x,v) =$$

$$= \left[\frac{2}{x^3} - \frac{2v}{x^4} + \frac{4}{5}\frac{v^2}{x^5}\right] \int_0^x dx \ln(1+x) +$$

$$+ \left[\frac{1}{x^2} - \frac{2v}{3x^3} + \frac{1}{5}\frac{v^2}{x^4}\right] \ln(1+x) -$$

$$- \left(\frac{v^2}{5} + \frac{2v}{3} + 1\right)\left[\ln\frac{1+x}{x} - \frac{1}{x}\right] -$$

$$- \left(\frac{v}{3} + \frac{v^2}{10}\right)\frac{1}{x^2} + \frac{v^2}{15x^3} +$$

$$+ \frac{(1+v)^2}{3}\ln\frac{1+x}{x} - \frac{v^2+2v}{3x} + \frac{v^2}{6x^2} =$$

$$= \left[\frac{2}{x^3} - \frac{2v}{x^4} + \frac{4}{5}\frac{v^2}{x^5} \right] \int_0^x dx \; \ln(1+x) \; +$$

$$+ \left[\frac{1}{x^2} - \frac{2}{3}\frac{v}{x^3} + \frac{1}{5}\frac{v^2}{x^4} \right] \cdot \ln(1+x) \; -$$

$$+ \left(\frac{2v^2}{15} - \frac{2}{3} \right) \ln \frac{1+x}{x} - \left(\frac{2v^2}{15} - 1 \right) \frac{1}{x} \; +$$

$$+ \left(\frac{v^2}{15} - \frac{v}{3} \right) \frac{1}{x^2} \; = \; \text{begin on p.22} \; =$$

$$= \int dx \; G_{uv}(x, v) =$$

$$= 2\left\{\frac{1}{x^3} - \frac{v}{x^4} + \frac{4}{5}\frac{v^2}{x^5}\right\}\int_0^x dx \, \ln(1+x) +$$

$$+ \left\{\frac{1}{x^2} - \frac{2}{3}\frac{v}{x^3} + \frac{1}{5}\frac{v^2}{x^4}\right\} \cdot \ln(1+x) +$$

$$+ \frac{2}{3}\left(\frac{v^2}{5} - 1\right)\ln\frac{1+x}{x} - \left(\frac{2v^2}{15} - 1\right)\frac{1}{x} +$$

$$+ \frac{1}{3}\left(\frac{v^2}{5} - v\right)\frac{1}{x^2} =$$

$$= 2\left[\frac{1}{x^3} - \frac{v}{x^4} + \frac{4}{5}\frac{v^2}{x^5}\right]\left[(1+x)\ln(1+x) - \underline{x}\right] +$$

$$+ \left[\frac{1}{x^2} - \frac{2}{3}\frac{v}{x^3} + \frac{1}{5}\frac{v^2}{x^4}\right] \ln(1+x) +$$

$$+ \frac{2}{3}\left(\frac{v^2}{5} - 1\right)\ln\frac{1+x}{x} - \underline{\left(\frac{2v^2}{15} - 1\right)\frac{1}{x}} +$$

$$+ \underline{\frac{1}{3}\left(\frac{v^2}{5} - v\right)\frac{1}{x^2}}$$

$$= \ln(1+x)\Big[\frac{2}{x^2} - \frac{2v}{x^3} + \frac{8}{5}\frac{v^2}{x^4} +$$

$$+ \frac{2}{x^3} - \frac{2v}{x^4} + \frac{8}{5}\frac{v^2}{x^5} +$$

$$+ \frac{1}{x^2} - \frac{2}{3}\frac{v}{x^3} + \frac{1}{5}\frac{v^2}{x^4} +$$

$$+ \frac{2}{3}\left(\frac{v^2}{5} - 1\right) \Big] -$$

$$- \frac{2}{3}\left(\frac{v^2}{5} - 1\right)\ln x -$$

$$- \frac{2}{x^2} + \frac{2v}{x^3} - \frac{8}{5}\frac{v^2}{x^4} -$$

$$- \left(\frac{2v^2}{15} - 1\right)\frac{1}{x} +$$

$$+ \frac{1}{3}\left(\frac{v^2}{5} - v\right)\frac{1}{x^2} =$$

$$= \left[\frac{3}{x^2} + \left(2 - \frac{8}{3}v\right)\frac{1}{x^3} + \left(\frac{2}{5}v^2 - 2v\right)\frac{1}{x^4} + \frac{8}{5}\frac{v^2}{x^5} + \right.$$

$$\left. + \frac{2}{3}\left(\frac{v^2}{5} - 1\right) \right] \ln(1+x) -$$

$$- \frac{2}{3}\left(\frac{v^2}{5} - 1\right) \ln x + \left(\frac{2v^2}{15} - 1\right)\frac{1}{x} +$$

$$+ \left[\frac{v^2}{15} - \frac{v}{3} - 2 \right] \frac{1}{x^2} +$$

$$+ \frac{2v}{x^3} - \frac{8}{5}\frac{v^2}{x^4} =$$

$$= \left[\frac{2}{3}\left(\frac{v^2}{5} - 1\right) + \frac{3}{x^2} - \left(\frac{8}{3}v - 2\right)\frac{1}{x^3} + \left(\frac{9}{5}v^2 - 2v\right)\frac{1}{x^4} + \frac{8}{5}\frac{v^2}{x^5} \right] \cdot$$

$$\cdot \ln(1+x) -$$

$$- \frac{2}{3}\left(\frac{v^2}{5} - 1\right)\ln x -$$

$$- \left(\frac{2v^2}{15} - 1\right)\frac{1}{x} + \left[\frac{v^2}{15} - \frac{v}{3} - 2\right]\frac{1}{x^2} + \frac{2v}{x^3} - \frac{8}{5}\frac{v^2}{x^4} =$$

$$= \int dx \; G_{ws}(x, v) =$$

$$= \int dx \; G_{uv}(x, v) =$$

$$= \left[-\frac{2}{3} + \frac{3}{x^2} + \frac{2}{x^3} \right] \ln(1+x) + \frac{2}{3} \ln x + \frac{1}{x} - \frac{2}{x^2} +$$

$$+ v \cdot \left[-\left(\frac{8}{3x^3} + \frac{2}{x^4} \right) \ln(1+x) - \frac{1}{3x^2} + \frac{2}{x^3} \right] +$$

$$+ v^2 \left[\frac{1}{5} \left(\frac{2}{3} + \frac{9}{x^4} + \frac{8}{x^5} \right) \ln(1+x) - \frac{2}{15} \ln x -\right.$$

$$\left. - \frac{2}{15} \frac{1}{x} + \frac{1}{15} \frac{1}{x^2} - \frac{8}{5} \frac{1}{x^4} \right] =$$

$$= \int dx \, G_{uv}(x, v) =$$

$$= \left[-\frac{2}{3} + \frac{3}{x^2} + \frac{2}{x^3} \right] \ln(1+x) + \frac{2}{3} \ln x + \frac{1}{x} - \frac{2}{x^2} +$$

$$+ \left[\left(\frac{8}{3x^3} + \frac{2}{x^4} \right) \ln(1+x) + \frac{1}{3x^2} - \frac{2}{x^3} \right] v +$$

$$+ \frac{1}{5} \left[\left(\frac{2}{3} + \frac{9}{x^4} + \frac{8}{x^5} \right) \ln(1+x) - \frac{2}{3} \ln x -\right.$$

$$\left. - \frac{2}{3} \frac{1}{x} + \frac{1}{3} \frac{1}{x^2} - 8 \frac{1}{x^4} \right] v^2$$

Then $G(u,v) = \int dx\,du\, G_{uv}$ + arbitrary $f(u)$ + arbitrary $f(v)$ =

$$= \left\{ \left\{ -\frac{2}{3} + \frac{3}{x^2} + \frac{2}{x^3} \right\} \ln(1+x) + \frac{2}{3}\ln x + \frac{1}{x} - \frac{2}{x^2} \right\} v -$$

$$- \left\{ \left(\frac{4}{3x^3} + \frac{1}{x^4} \right) \ln(1+x) + \frac{1}{6x^2} - \frac{1}{x^3} \right\} v^2 +$$

$$+ \frac{1}{15} \left\{ \left(\frac{2}{3} + \frac{9}{x^4} + \frac{8}{x^5} \right) \ln(1+x) - \frac{2}{3}\ln x -\right.$$

$$\left. - \frac{2}{3}\frac{1}{x} + \frac{1}{3}\frac{1}{x^2} - 8\frac{1}{x^4} \right\} v^3 +$$

$$+ \underset{\text{arbitrary}}{f(u)} + \underset{\text{arbitrary}}{f(v)}$$