# Kinetic Term for $\sigma$ in the Effective Action

## Operator Method

in Extra sign

$$\Gamma = N \,\mathrm{Tr} \ln \{ -\Box + m^2 + \phi \} =$$

$$= N \int dx \, \langle x | \ln ( -\Box + \phi + m^2 ) | x \rangle =$$

$$= N \int dx \, dm^2 \, \langle x | \frac{1}{-\Box + \phi + m^2} | x \rangle =$$

$$= N \int dx \, dm^2 \left\langle x \right| \frac{1}{(\hat{p} + q)^2 + \phi + m^2} \left| x \right\rangle$$

$$a = \phi + m^2$$

(2)

$$-\frac{1}{(\hat{p}+q)^2+a} = \frac{1}{\hat{p}^2+2q\hat{p}+q^2+a} =$$

$$= \frac{1}{(p^2+a)+2pq+q^2} =$$

$$= \left[1 - \frac{1}{p^2+a}(2pq+q^2) + \frac{1}{p^2+a}(2pq+q^2)\frac{1}{p^2+a}(2pq+q^2)\cancel{\frac{1}{p^2+a}} + \dots\right]\frac{1}{p^2+a}$$

$$\supset -\frac{1}{p^2+a}\,q^2\frac{1}{p^2+a} + \frac{1}{p^2+a}2pq\frac{1}{p^2+a}2pq\frac{1}{p^2+a} + \dots =$$

$$= -\frac{1}{p^2+a}q^2\frac{1}{p^2+a} + \frac{1}{p^2+a}2pq\frac{1}{p^2+a}2pq\frac{1}{p^2+a}$$

$$\langle x | - \frac{1}{p^2+a} q^2 \frac{1}{p^2+a} + \frac{1}{p^2+a} 2p_q \frac{1}{p^2+a} 2p_q \frac{1}{p^2+a} | x \rangle = 0$$

averaging over the angles, and differentiating w.r.t. $q^2$,

$$\langle x | \frac{1}{(p^2+a)^2} | x \rangle = 2 \langle x | \frac{1}{p^2+a} p_\mu \frac{1}{p^2+a} p_\mu \frac{1}{p^2+a} | x \rangle =$$

$$\left[ p_\mu , \frac{1}{p^2+a} \right] = - \frac{1}{p^2+a} [p_\mu a] \frac{1}{p^2+a} = \frac{1}{p^2+a} (-i\partial_\mu a) \frac{1}{p^2+a}$$

$$= 2 \langle x | p_\mu \frac{1}{(p^2+a)^2} p_\mu \frac{1}{p^2+a} + \frac{1}{p^2+a} (i\partial_\mu a) \frac{1}{p^2+a} \frac{1}{p^2+a} p_\mu \frac{1}{p^2+a} | x \rangle =$$

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$$= 2 \langle x | p_\mu \frac{1}{(p^2+a)^3} p_\mu + p_\mu \frac{1}{(p^2+a)^2} (-i) \frac{1}{p^2+a}\, i\partial_\mu a \frac{1}{p^2+a} +$$

$$+ \frac{1}{p^2+a} (i\partial_\mu a) \frac{1}{(p^2+a)^3} p_\mu + \frac{1}{p^2+a} (i\partial_\mu a) \frac{1}{(p^2+a)^2} (-i) \frac{1}{p^2+a}\, i\partial_\mu a \frac{1}{p^2+a} | x \rangle$$

$$\to 2 \langle x | \frac{1}{p^2+a} \partial_\mu a \frac{1}{(p^2+a)^3} \partial_\mu a \frac{1}{p^2+a} | x \rangle =$$

$$= 2 \langle x | (\partial_\mu a)^2 \frac{1}{(p^2+a)^5} | x \rangle = \qquad |p\rangle\langle p| \frac{dp}{(2\pi)^2}$$

$$= 2 \int \frac{dp}{(2\pi)^2} (\partial_\mu a)^2 \frac{1}{(p^2+a)^5} =$$

$$= \frac{1}{2\pi} \int d(p^2+a) (\partial_\mu a)^2 \frac{1}{(p^2+a)^5} =$$

$$= -\frac{1}{8\pi} (\partial_\mu a)^2 \int \frac{1}{(p^2+a)^4} \Big|_0^\infty = \frac{1}{8\pi} \frac{(\partial_\mu a)^2}{a^4}$$

$$\langle x | \frac{1}{(p^2+a)^2} | x \rangle \supset \frac{1}{8\pi} \frac{(\partial_\mu a)^2}{a^4} =$$

$$= \frac{1}{8\pi} \frac{(\partial_\mu \phi)^2}{(\phi + m^2)^4}$$

$$\langle x | \ln(p^2 + a) | x \rangle = \langle x | \ln(p^2 + \phi + m^2) | x \rangle =$$

$$= -\left(\int\right)^2 dm^2 \, \langle x | \frac{1}{(p^2 + \phi + m^2)^2} | x \rangle =$$

$$= -\left(\int\right)^2 dm^2 \, \frac{1}{8\pi} \frac{(\partial_\mu \phi)^2}{(\phi + m^2)^4} =$$

$$= -\frac{1}{8\pi} (\partial_\mu \phi)^2 \left(-\frac{1}{3}\right)\left(-\frac{1}{2}\right) \frac{\cancel{(\partial_\mu \phi)} 1}{(\phi + m^2)^2} = -\frac{1}{48\pi} \frac{(\partial_\mu \phi)^2}{(\phi + m^2)^2}$$

$$\Gamma \supset -\int dx \, \frac{N}{48\pi} \frac{(\partial_\mu \phi)^2}{(\phi + m^2)^2}$$

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