Summary as of May 16, 2016

We assume that the following contributions comprise the effective action:

$$\frac{4\pi}{N}\,\mathcal{L}_{\rm eff} \;=\; -\int\;\Bigg(d^4\theta\,\frac{1}{2}\left|\ln\sqrt{2}\Sigma\right|^2 \;+\; G(u,\overline{u}) \;+ \tag{1}$$

$$+\; i\,d^2\tilde{\theta}\,\Big(\sqrt{2}\Sigma\,\ln\sqrt{2}\Sigma \;-\; \sqrt{2}\Sigma\Big) \;+\; i\,d^2\bar{\tilde{\theta}}\,\Big(\sqrt{2}\overline{\Sigma}\,\ln\sqrt{2}\overline{\Sigma} \;-\; \sqrt{2}\overline{\Sigma}\Big)\Bigg)\,.$$

Matching the effective potential and the kinetic term $F_{\mu\nu}^2$ from arXiv:0803.0698 gives

$$G_{u\overline{u}} \;=\; -\,\frac{1}{x^4}\,\left(1 \;+\; 2\,\frac{y^2}{x^2}\right)\int_0^x \ln(1\,+\,x)\,dx \;-\; \frac{1}{6}\,\frac{y^2}{x^3\,(1\,+\,x)}\,. \tag{2}$$

Here

$$u \;=\; \frac{S}{\sqrt{2}\Sigma} \;\propto\; \frac{iD\;-\;F_{03}}{\phi} \qquad\qquad \overline{u} \;=\; \frac{\overline{S}}{\sqrt{2}\overline{\Sigma}} \;\propto\; \frac{iD\;+\;F_{03}}{\phi}$$

$$x \;=\; \frac{u\;+\;\overline{u}}{2} \;\propto\; \frac{iD}{\phi} \qquad\qquad y \;=\; \frac{\overline{u}\;-\;u}{2} \;\propto\; \frac{F_{03}}{\phi}\,,$$

neglecting fermions.

The effective action (1) produces the following kinetic term for λ:

$$-\,\frac{\overline{\lambda}_R\,i\overleftrightarrow{\partial}\lambda_R \;+\; \overline{\lambda}_L\,i\overleftrightarrow{\partial}\lambda_L}{\phi}\,x^2\,\Big(3\,G_{u\bar{u}} \;+\; x\,\big(G_{uu\bar{u}} \;+\; G_{\bar{u}\bar{u}u}\big)\Big)\,. \tag{3}$$

Substituting (2) gives,

$$-\,\frac{\overline{\lambda}_R\,i\overleftrightarrow{\partial}\lambda_R \;+\; \overline{\lambda}_L\,i\overleftrightarrow{\partial}\lambda_L}{\phi}\,\frac{\ln(1+x)\;-\;x}{x^2}\,, \tag{4}$$

which is the same coefficient that we obtained a year ago using our previous approach. This also is the *correct* answer for that diagram.

As for the kinetic term for σ, the correct expression is, as we know,

$$\frac{4\pi}{N}\,\mathcal{L}_{\rm eff} \;\supset\; \frac{1}{12}\,\frac{(\partial_\mu\,\phi)^2}{(\,\phi\;+\;iD\,)^2}\,. \tag{5}$$

The supersymmetric Lagrangian (1) produces the following expression,

$$\frac{4\pi}{N}\,\mathcal{L}_{\rm eff} \;\supset\; -\,\frac{(\partial_\mu\,\phi)^2}{\phi^2}\,x^2\,\Big(3\,G_{u\bar{u}} \;+\; x\,\big(G_{uu\bar{u}} \;+\; G_{\bar{u}\bar{u}u}\big)\Big)\,. \tag{6}$$

Notice that the bracket is the same as in (3), and so gives a logarithm.