

Our potential is

$$V = \frac{N}{4\pi} \left\{ -iD \ln |\sqrt{2}\sigma|^2 + \sum_{k \geq 1} \frac{(-1)^k}{k(k+1)} iD \left( \frac{iD}{|\sqrt{2}\sigma|^2} \right)^k \right\}. \quad (0.1)$$

The first term comes from Witten's superpotential. The first term of the series (with  $k = 1$ ) is taken into account by the kinetic term. Thus we can write our effective action as

$$\begin{aligned} S = & -\frac{N}{4\pi} \int d^4x \left\{ \int d^4\theta \frac{1}{2} \left| \ln \sqrt{2}\Sigma \right|^2 + \right. \\ & + i \int d^2\theta \left( \sqrt{2}\Sigma \ln \sqrt{2}\Sigma - \sqrt{2}\Sigma + \right. \\ & \left. \left. + \frac{1}{2} \sqrt{2}\Sigma \sum_{k \geq 2} \frac{(-1)^k}{(k-1)k(k+1)} \left( \frac{S}{\sqrt{2}\Sigma} \right)^k + \text{h.c.} \right) \right\}, \end{aligned} \quad (0.2)$$

where

$$S = \frac{i}{2} \overline{D}_R D_L \ln \sqrt{2}\overline{\Sigma}, \quad \overline{S} = \frac{i}{2} \overline{D}_L D_R \ln \sqrt{2}\Sigma. \quad (0.3)$$

The series in the third line can be written as a  $D$ -term. We get

$$\begin{aligned} S = & -\frac{N}{4\pi} \int d^4x \left\{ \int d^4\theta \frac{1}{2} \left| \ln \sqrt{2}\Sigma \right|^2 + \right. \\ & + i \int d^2\theta \left( \sqrt{2}\Sigma \ln \sqrt{2}\Sigma - \sqrt{2}\Sigma \right) + \\ & \left. + \frac{1}{2} \int d^4\theta \ln \sqrt{2}\overline{\Sigma} \sum_{k \geq 1} \frac{(-1)^{k+1}}{k(k+1)(k+2)} \left( \frac{S}{\sqrt{2}\Sigma} \right)^k \right\} + \text{h.c.} \end{aligned} \quad (0.4)$$

Here we can see explicitly that the only  $F$ -term is the Witten's superpotential, while all other terms are  $D$ -terms.

The last term can be re-written as

$$\begin{aligned} S = & -\frac{N}{4\pi} \int d^4x \left\{ \int d^4\theta \frac{1}{2} \left| \ln \sqrt{2}\Sigma \right|^2 + i \int d^2\theta \left( \sqrt{2}\Sigma \ln \sqrt{2}\Sigma - \sqrt{2}\Sigma \right) + \right. \\ & \left. + \frac{1}{4} \int d^4\theta \ln \sqrt{2}\overline{\Sigma} \left( \left( 1 + \frac{\sqrt{2}\Sigma}{S} \right)^2 \ln \left( 1 + \frac{S}{\sqrt{2}\Sigma} \right) - \frac{\sqrt{2}\Sigma}{S} \right) + \text{h.c.} \right\}. \end{aligned} \quad (0.5)$$

The kinetic (first) term can be combined with the last  $D$ -term, to write

$$\begin{aligned}
S = & -\frac{N}{4\pi} \int d^4x \left\{ i \int d^2\theta \left( \sqrt{2}\Sigma \ln \sqrt{2}\Sigma - \sqrt{2}\Sigma \right) + \right. \\
& + \frac{1}{4} \int d^4\theta \ln \sqrt{2}\Sigma \left[ \ln (S + \sqrt{2}\Sigma) + \right. \\
& \left. \left. + \frac{\sqrt{2}\Sigma}{S} \left( \left( 2 + \frac{\sqrt{2}\Sigma}{S} \right) \ln \left( 1 + \frac{S}{\sqrt{2}\Sigma} \right) - 1 \right) \right] \right\} + \text{h.c.}
\end{aligned} \tag{0.6}$$

Expansion in the components up to two derivatives in bosons and up to one derivative in fermions, and no further than  $1/|\sqrt{2}\sigma|^2$  gives,

$$\begin{aligned}
\frac{4\pi}{N} \mathcal{L} = & -iD \log |\sqrt{2}\sigma|^2 - \frac{1}{2} \frac{(iD)^2}{|\sqrt{2}\sigma|^2} - F_{03} \log \frac{\sqrt{2}\sigma}{\sqrt{2}\bar{\sigma}} + \\
& + \frac{\frac{1}{2} |\partial_\mu \sqrt{2}\sigma|^2 + \frac{1}{2} F_{03}^2 + \frac{1}{2} \left( \bar{\lambda}_R i \overleftrightarrow{\mathcal{D}}_L \lambda_R + \bar{\lambda}_L i \overleftrightarrow{\mathcal{D}}_R \lambda_L \right)}{|\sqrt{2}\sigma|^2} - \\
& - 2 \frac{i\sqrt{2}\sigma \bar{\lambda}_R \lambda_L + i\sqrt{2}\bar{\sigma} \bar{\lambda}_L \lambda_R}{|\sqrt{2}\sigma|^2} + \\
& + \frac{(iD + F_{03}) i\sqrt{2}\sigma \bar{\lambda}_R \lambda_L + (iD - F_{03}) i\sqrt{2}\bar{\sigma} \bar{\lambda}_L \lambda_R}{|\sqrt{2}\sigma|^4} + \\
& + 2 \frac{\bar{\lambda}_R \lambda_R \bar{\lambda}_L \lambda_L}{|\sqrt{2}\sigma|^4} + O\left(\frac{1}{|\sqrt{2}\sigma|^4}\right).
\end{aligned} \tag{0.7}$$

The long derivatives are

$$\begin{aligned}
\mathcal{D}_L &= \partial_L - \frac{1}{2} \partial_L \ln \sqrt{2}\sigma \\
\mathcal{D}_R &= \partial_R + \frac{1}{2} \partial_R \ln \sqrt{2}\sigma,
\end{aligned} \tag{0.8}$$

with conjugate expressions for the left-ward derivatives  $\overleftarrow{\mathcal{D}}_L$  and  $\overleftarrow{\mathcal{D}}_R$ . All the terms above actually come from the Witten's superpotential and from the “kinetic” term  $|\ln \sqrt{2}\Sigma|^2$ . The rest gives  $O(1/|\sqrt{2}\sigma|^4)$  corrections. The Witten's superpotential

generates the term  $iD \log |\sqrt{2}\sigma|^2$  of the potential, the anomaly and the Yukawa terms. The quartic fermionic term and all the kinetic terms (including the one for  $D$ ) come from the “kinetic” term  $|\ln \sqrt{2}\Sigma|^2$ .

As for the geometric description, I’m uncertain. Clearly, the Kahler potential is  $|\log \Sigma|^2$  — that’s what’s generated the  $1/|\sigma|^2$  metric. And Christoffel symbol too, as Misha explained. But the Riemann tensor vanishes for this metric. Normally that would cause the quartic fermionic term to vanish, and yet it does not vanish in our case. That’s because it only vanishes after elimination of the auxiliary fields. If we “eliminate” the complex combination  $iD + F_{03}$ , then indeed the quartic term vanishes. But the problem is that we can’t eliminate  $F_{03}$  this way because it’s not a variable itself but rather a fieldstrength (i.e. a derivative of a variable). Furthermore, we don’t actually want to eliminate either of them. So the quartic term stays.

Normally when the Riemann tensor vanishes, it perhaps means that the space is flat. Indeed, if we switch to the variables  $\log \Sigma$  instead of  $\Sigma$ , the metric disappears. As I wrote earlier, the action takes the normal form for the fact that  $\log \Sigma$  consists of

$$\ln \Sigma(\tilde{y}) = \ln \sigma - \sqrt{2} \theta_R \bar{\lambda}_L / \sigma + \sqrt{2} \bar{\theta}_L \lambda_R / \sigma + \theta_R \bar{\theta}_L \left( \frac{\sqrt{2} (D - iF_{03})}{\sigma} - 2 \frac{\bar{\lambda}_L}{\sigma} \frac{\lambda_R}{\sigma} \right). \quad (0.9)$$

That is, in terms of the variables  $\ln \sigma$ ,  $\lambda/\sigma$  and  $iD/\sigma$  the action looks just flat. This is because we are able to switch to such variables. Unlike in the Fubini-Study case, where it is not possible to switch to linear variables. Do you agree?