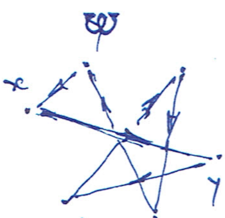
April 7, 2016

Kinetic Term for b in the Effective Action

$$Z = \int e^{-\bar n M n - |n|^2 \phi}\, dn\, d\bar n = e^{-\mathcal{L}_{int}\left(\delta/\delta v\right)} \int e^{-\bar n M n + v n + \bar n \bar v}\, dn\, d\bar n \Big|_{v,\bar v = 0} =$$

$$= e^{-\phi \frac{\delta}{\delta v}\frac{\delta}{\delta \bar v}} \cdot Z_0 \Big|_{v,\bar v=0} = \qquad \text{where } \phi = |\partial b|^2$$

$$= \sum_{N\geq 0} \frac{(-1)^N}{N!} \phi(x)\dots\phi(\varpi) \frac{\delta}{\delta v_x}\frac{\delta}{\delta \bar v_x}\cdot \dots \cdot \frac{\delta}{\delta v_\varpi}\frac{\delta}{\delta \bar v_\varpi} \cdot e^{+v M^{-1} \bar v} \Big| =$$



$(N-1)!$ ways

$$= \sum_{N\geq 0} \frac{(-1)^N}{N!} \phi(x)\phi(y)\dots\phi(\varpi) \cdot \frac{\delta}{\delta \bar v_x} \cdot \dots \frac{\delta}{\delta \bar v_\varpi} (M^{-1}\bar v)_x \dots (M^{-1}\bar v)_\varpi \cdot e^{v M^{-1}\bar v} \Big| \supset$$

$$\supset \sum_{N\geq 0} (-1)^N \frac{(N-1)!}{N!} \int \phi(x)\phi(y)\cdot\dots\phi(\varpi)\, D(x,y)\cdot \dots \cdot D(\varpi,x)\, dx\, dy \dots d\varpi \supset$$

$$\supset \sum_{N\geq 0} \frac{(-1)^N}{N} \int \frac{dp\, dq \dots ds}{(2\pi)^d \dots (2\pi)^d}\, dx \dots d\varpi \cdot$$

$$\cdot\, \phi(x) \cdot$$

$$\left\{\phi(x) + \partial_\mu \phi\, (y-x)^\mu + \tfrac{1}{2}\partial_\mu\partial_\nu \phi(x)\cdot (y-x)^\mu (y-x)^\nu\right\} \cdot$$

$$\cdot \dots \cdot$$

$$\cdot \left\{\phi(x) + \partial_\mu\phi\, (\varpi - x)^\mu + \tfrac{1}{2}\partial_\mu\partial_\nu\phi(x)(\varpi-x)^\mu(\varpi-x)^\nu\right\} \cdot$$

$m^2 \equiv iD$

$$\cdot \frac{e^{ip(y-x)}}{(p^2+m^2)} \cdot \dots \cdot \frac{e^{is(x-\varpi)}}{(s^2+m^2)} =$$

$$= \sum_{N\geq 0} \frac{(-1)^N}{N} \int \frac{dp\,dq \dots ds}{(2\pi)^d \dots (2\pi)^d}\, dx \dots d\omega \,.$$

$$\cdot\, f(x) \cdot$$

$$\left[f(x) + (y-x)^\mu \partial_\mu f + \tfrac{1}{2}(y-x)^\mu (y-x)^\nu \partial_\mu \partial_\nu f \right]$$

$$\dots$$

$$\left[f(x) + (\omega - x)^\mu \partial_\mu f + \tfrac{1}{2}(\omega-x)^\mu(\omega-x)^\nu \partial_\mu \partial_\nu f \right] \cdot$$

$$\cdot\, \frac{e^{ip(y-x)}}{(p^2+m^2)} \dots \frac{e^{is(x-\omega)}}{(s^2+m^2)} \Rightarrow$$

$$\boxed{\begin{aligned} x &= x_0 \\ y &= x_1 \\ z &= x_2 \\ &\vdots \\ \omega &= x_{N-1} \end{aligned}}$$

$$\Rightarrow \sum_{N\geq 0} \frac{(-1)^N}{N} \int \frac{dp \dots ds}{(2\pi)^d \dots (2\pi)^d} \,.\, dx \dots d\omega \,\cdot$$

$$\cdot\, f(x) \left[\sum_{\substack{k,l=1 \\ k\neq l}}^{N-1} f^{N-3}\, \tfrac{1}{2}(x_k - x)^\mu \partial_\mu f\, (x_l - x)^\nu \partial_\nu f \,.\, \Theta(N-3) + \right.$$

$$\left. + f^{N-2} \sum_{k=1}^{N-1} \tfrac{1}{2}(x_k - x)^\mu (x_k - x)^\nu \partial_\mu \partial_\nu f \,.\, \Theta(N-2) \right] .$$

$$\cdot\, \frac{e^{ip(y-x)}}{(p^2+m^2)} \dots \frac{e^{is(\tau-\omega)}}{(s^2+m^2)} =$$

$$= \sum_{N\geq 0} \frac{(-1)^N}{N!} \int \frac{dp \ldots ds}{(2\pi)^d \ldots (2\pi)^d}\, dx \ldots dw \,\cdot$$

$$\cdot\, \phi(x)\,\cdot$$

$$\Big\{ \frac{1}{2}\phi^{N-3}(x)\cdot\Theta(N-3)\sum_{\substack{k,l=1\\ k\neq l}}^{N-1}(x_k-x)^\mu(x_l-x)^\nu\,\partial_\mu\phi\,\partial_\nu\phi \;+$$

$$+\frac{1}{2}\phi^{N-2}(x)\cdot\Theta(N-2)\sum_{k=1}^{N-1}(x_k-x)^\mu(x_k-x)^\nu\,\partial_\mu\partial_\nu\phi\Big\}\,\cdot$$

$$\cdot\,\frac{e^{ip(y-x)}}{(p^2+m^2)}\cdots\frac{e^{is(x-w)}}{(s^2+m^2)} =$$

$$x_k - x = \sum_{m=0}^{k-1}(x_{m+1}-x_m) \equiv \sum_{m=0}^{k-1}\Delta x_m$$

$$\Delta x_j \equiv x_{j+1} - x_j$$

$$= \sum_{N\geq 0} \frac{(-1)^N}{N} \int \frac{dp \dots ds}{(2\pi)^d \dots (2\pi)^d}\, dx \dots d\omega \,\cdot$$

$$\cdot\, f(x)\,\cdot$$

$$\Big[\tfrac{1}{2}\, f^{N-3} \cdot \theta(N-3) \sum_{\substack{k,l=1\\k\neq l}}^{N-1} \sum_{m=0}^{k-1} \Delta x_m^\mu \sum_{n=0}^{l-1} \Delta x_n^\nu \cdot \partial_\mu f\, \partial_\nu f \,+$$

$$+\, \tfrac{1}{2}\, f^{N-2} \cdot \theta(N-2) \sum_{k=1}^{N-1} \sum_{m,n=0}^{k-1} \Delta x_m^\mu\, \Delta x_n^\nu \cdot \partial_\mu \partial_\nu f \Big] \,\cdot$$

$$\cdot\, \frac{e^{ip\Delta x_0}}{(p^2+m^2)} \cdots \frac{e^{is\Delta x_{N-1}}}{(s^2+m^2)} =$$

$$\boxed{\begin{aligned} p &= p_0 \\ q &= p_1 \\ &\vdots \\ s &= p_{N-1} \end{aligned}}$$

$$= \sum_{N\geq 0} \frac{(-1)^N}{N} \int \frac{dp \dots ds}{(2\pi)^d \dots (2\pi)^d}\, dx \dots d\omega \,\cdot$$

$$\cdot\, f(x)$$

$$\Big[\tfrac{1}{2}\, \theta(N-3)\, f^{N-3} \sum_{\substack{k,l=1\\k\neq l}}^{N-1} \sum_{m=0}^{k-1} \cancel{\Delta x_m} \sum_{n=0}^{l-1} \left(-i \frac{\partial}{\partial p_m^\mu}\right)\left(-i\frac{\partial}{\partial p_n^\nu}\right) e^{ip_0\Delta x_0 + \dots ip_{N-1}\Delta x_{N-1}} \,\cdot$$

$$\cdot\, \partial_\mu f\, \partial_\nu f \,+$$

$$+\, \tfrac{1}{2}\, \theta(N-2)\, f^{N-2} \sum_{k=1}^{N-1} \sum_{m,n=0}^{k-1} \left(-i \frac{\partial}{\partial p_m^\mu}\right)\left(-i\frac{\partial}{\partial p_n^\nu}\right) e^{ip_0\Delta x_0 + \dots ip_{N-1}\Delta x_{N-1}} \,\cdot$$

$$\cdot\, \partial_\mu \partial_\nu f \Big] \cdot \frac{1}{(p_0^2+m^2) \dots (p_{N-1}^2+m^2)}$$

by parts in p_m^μ, p_n^ν

$$= \sum_{N\geq 0} \frac{(-1)^N}{N} \int \frac{dp_0}{(2\pi)^d} \cdots \frac{dp_{N-1}}{(2\pi)^d}\, dx_0 \dots dx_{N-1}\, e^{ip_0\Delta x_0 + \dots + ip_{N-1}\Delta x_{N-1}} \cdot$$

$$\cdot\, \phi(x) \left\{ \frac{1}{2}\theta(N-3)\phi^{N-3}\partial_\mu\phi\,\partial_\nu\phi \sum_{\substack{k,l=1\\ k\neq l}}^{N-1}\sum_{m=0}^{k-1}\sum_{n=0}^{l-1} (-1)\frac{\partial}{\partial p_m^\mu}\frac{\partial}{\partial p_n^\nu} + \right.$$

$$\left. + \frac{1}{2}\theta(N-2)\phi^{N-2}\partial_\mu\partial_\nu\phi \sum_{k=1}^{N-1}\sum_{m,n=0}^{k-1} (-1)\frac{\partial}{\partial p_m^\mu}\frac{\partial}{\partial p_n^\nu} \right\} \frac{1}{(p_0^2+m^2)\dots(p_{N-1}^2+m^2)} =$$

$=$ see p. 5a $=$

$$= -\sum_{N\geq 0} \frac{(-1)^N}{N} \int \frac{dp_0}{(2\pi)^d} \cdots \frac{dp_{N-1}}{(2\pi)^d}\, dx_0 \dots dx_{N-1}\, e^{ip_0\Delta x_0 + \dots + ip_{N-1}\Delta x_{N-1}} \cdot$$

$$\cdot\, \phi(x) \cdot \left\{ \frac{1}{2}\theta(N-3)\phi^{N-3}\partial_\mu\phi\,\partial_\nu\phi \sum_{\substack{k,l=1\\ k\neq l}}^{N-1}\sum_{m=0}^{k-1}\sum_{n=0}^{l-1} \longrightarrow + \right.$$

$$\left. + \frac{1}{2}\theta(N-2)\phi^{N-2}\partial_\mu\partial_\nu\phi \sum_{k=1}^{N-1}\sum_{m,n=0}^{k-1} \longrightarrow \right\} \cdot$$

$$\cdot\, 4\left\{ \frac{(1+\delta_{mn})\, p_m^\mu p_n^\nu}{(p_m^2+m^2)(p_n^2+m^2)} - \frac{1}{2}\frac{\delta_{mn} g^{\mu\nu}}{p_n^2+m^2} \right\} \frac{1}{(p_0^2+m^2)\dots(p_{N-1}^2+m^2)}$$

$$\frac{\partial}{\partial p_m^\mu}\frac{\partial}{\partial p_n^\nu}\frac{1}{(p_0^2+m^2)\dots(p_{N-1}^2+m^2)} =$$

$$= \frac{\partial}{\partial p_m^\mu}\frac{-2p_n^\nu/(p_n^2+m^2)}{(p_0^2+m^2)\dots(p_{N-1}^2+m^2)} =$$

$$= \frac{1}{(p_0^2+m^2)\dots(p_{N-1}^2+m^2)}\left\{\frac{4\,p_m^\mu p_n^\nu}{(p_m^2+m^2)(p_n^2+m^2)} + \right.$$

$$\left. + \delta_{mn}\left[\frac{(-2p_m^\mu)(-2p_n^\nu)}{(p_n^2+m^2)^2} - \frac{2g_{\mu\nu}}{p_n^2+m^2}\right]\right\} =$$

$$= \frac{4}{(p_0^2+m^2)\dots(p_{N-1}^2+m^2)}\left\{\frac{p_m^\mu p_n^\nu(1+\delta_{mn})}{(p_m^2+m^2)(p_n^2+m^2)} - \frac{1}{2}\frac{\delta_{mn}g^{\mu\nu}}{p_n^2+m^2}\right\} =$$

$$= \frac{4}{(p_0^2+m^2)\dots(p_{N-1}^2+m^2)}\left\{\frac{(1+\delta_{mn})p_m^\mu p_n^\nu}{(p_m^2+m^2)(p_n^2+m^2)} - \frac{1}{2}\frac{\delta_{mn}g^{\mu\nu}}{p_n^2+m^2}\right\}$$

= integrating over $x_1, \dots x_{N-1}$, then $p_1 \dots p_{N-1}$ and averaging over $p^\mu p^\nu$ =

$$= -\sum_{N\geq 0} \frac{(-1)^N}{N} \int \frac{dp\,dx}{(2\pi)^d} e^{-ipx+ipx} \cdot$$

$$\cdot\, \phi(x) \left\{ \frac{1}{2}\Theta(N-3)\,\phi^{N-3}\,\partial_\mu\phi\,\partial_\nu\phi \sum_{\substack{k,l=1\\k\neq l}}^{N-1} \sum_{m=0}^{k-1} \sum_{n=0}^{l-1} + \right.$$

$$\left. +\, \frac{1}{2}\Theta(N-2)\,\phi^{N-2}\,\partial_\mu\partial_\nu\phi \sum_{k=1}^{N-1} \sum_{m,n=0}^{k-1} \right\} \cdot$$

$$\cdot\, 2g^{\mu\nu} \frac{p^2 - \delta_{mn} m^2}{(p^2+m^2)^2} \cdot \frac{1}{(p^2+m^2)^N} =$$

$$= -\sum_{N\geq 0} \frac{(-1)^N}{N} \int \frac{dx\,dp}{(2\pi)^d} \frac{\phi(x)}{(p^2+m^2)^{N+2}}$$

$$\left\{ \partial_\mu\phi\,\partial_\mu\phi \cdot \Theta(N-3)\,\phi^{N-3} \sum_{\substack{k,l=1\\k\neq l}}^{N-1} \sum_{m=0}^{k-1} \sum_{n=0}^{l-1} (p^2 - \delta_{mn} m^2) + \right.$$

$$\left. +\, \Box\phi \cdot \Theta(N-2)\,\phi^{N-2} \sum_{k=1}^{N-1} \sum_{m,n=0}^{k-1} (p^2 - \delta_{mn} m^2) \right\} =$$

= see p. 6a, 6b, 6c, 6d =

(6a)

$$\cdot \sum_{\substack{k,l=1 \\ k\neq l}}^{N-1} \sum_{m=0}^{k-1} \sum_{n=0}^{l-1} 1 = \sum_{\substack{k\neq l \\ k,l=1}}^{N-1} k\cdot l =$$

$$= \sum_{k,l=1}^{N-1} k\cdot l - \sum_{\substack{k,l=1 \\ k=l}}^{N-1} k\cdot l = \left[\frac{N(N-1)}{2}\right]^2 - \sum_{k=1}^{N-1} k^2 =$$

$$\left| \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \right.$$

$$\Rightarrow \sum_{k=1}^{N-1} k^2 = \frac{N(N-1)(2N-1)}{6}$$

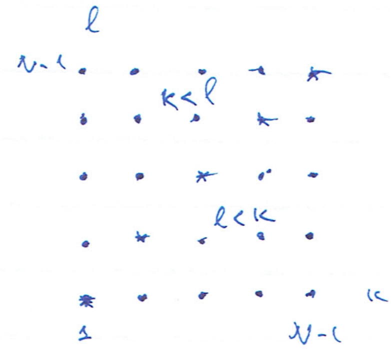
$$\frac{N(N-1)(N-2)(3N-1)}{12}$$

also

$$= \frac{N^2(N-1)^2}{4} - \frac{N(N-1)(2N-1)}{6} = \frac{N}{12}\left[3N^3 - 10N^2 + 9N - 2\right]$$

$$\cdot \sum_{k=1}^{N-1} \sum_{m,n=0}^{k-1} 1 = \sum_{k=1}^{N-1} k^2 = \frac{N(N-1)(2N-1)}{6}$$

$$\cdot \sum_{\substack{k,l=1\\k\neq l}}^{N-1} \sum_{m=0}^{k-1} \sum_{n=0}^{l-1} \delta_{mn} = \sum_{\substack{k,l=1\\k\neq l}}^{N-1} \sum_{m=0}^{\min(k,l)-1} 1 =$$



$$= \sum_{\substack{k<l\\k=1}}^{N-1} \sum_{m=0}^{k-1} 1 + \sum_{\substack{k>l\\k=1\\l=1}}^{N-1} \sum_{m=0}^{l-1} 1 =$$

$$= 2 \sum_{\substack{k<l\\k=1}}^{N-1} \sum_{m=0}^{k-1} 1 = 2 \sum_{\substack{k=1\\k<l}}^{N-1} k =$$

$$= 2 \sum_{k=1}^{N-2} \sum_{l=k+1}^{N-1} k = 2 \sum_{k=1}^{N-2} k\left(N-1-(k+1)+1\right) =$$

$$= 2 \sum_{k=1}^{N-2} k(N-k-1) = 2 \sum_{k=1}^{N-2} \left(-k^2 + kN - k\right) =$$

$$= 2\left[- \frac{(N-2)(N-1)(2(N-2)+1)}{6} + \frac{(N-1)^2(N-2)}{2} \cancel{\text{[illegible]}} \right] =$$

6c

$$= +2(N-1)(N-2)\left\{-\frac{2N-3}{6}+\frac{N-1}{2}\right\} =$$

$$= \frac{(N-1)(N-2)}{3}\left\{-2N+3+3N-3\right\} = \frac{N(N-1)(N-2)}{3}$$

$$\Rightarrow \sum_{\substack{k,l=1\\k\neq l}}^{N-1} \sum_{m=0}^{k-1} \sum_{n=0}^{l-1} \delta_{mn} = \frac{N(N-1)(N-2)}{3}$$

(6d)

$$\cdot \sum_{k=1}^{N-1} \sum_{m,n=0}^{k-1} 1 = \sum_{k=1}^{N-1} k^2 = \frac{(N-1)\,N\,.(2N-1)}{6} =$$

$$= \frac{N(N-1)(2N-1)}{6}$$ (done on p. 6a)

$$\cdot \sum_{k=1}^{N-1} \sum_{m,n=0}^{k-1} \delta_{mn} = \sum_{k=1}^{N-1} \sum_{m=0}^{k-1} = \sum_{k=1}^{N-1} k = \frac{N(N-1)}{2}$$

upper part already done on p. 6a

$$= -\sum_{N\geq 0} \frac{(-1)^N}{N} \int \frac{dx\,dp}{(2\pi)^d} \frac{\phi(x)}{(p^2+m^2)^{N+2}} \cdot$$

$$\Big\{ \partial_\mu \phi\, \partial_\mu \phi \cdot \Theta(N-3)\, \phi^{N-3} \cdot$$

$$\Big\{ p^2 \frac{N(N-1)(N-2)(3N-1)}{12} - m^2 \frac{N(N-1)(N-2)}{3} \Big\} +$$

$$+ \Box\phi \cdot \Theta(N-2) \cdot \phi^{N-2} \Big\{ p^2 \frac{N(N-1)(2N-1)}{6} - m^2 \frac{N(N-1)}{2} \Big\} \Big\} =$$

$$= -\int \frac{dx\,dp}{(2\pi)^d} \frac{\phi(x)}{(p^2+m^2)^2} \cdot$$

$$\Big\{ \partial_\mu \phi\, \partial_\mu \phi \sum_{N\geq 3} \frac{\phi^{N-3}(-1)^N}{(p^2+m^2)^N} (N-1)(N-2) \Big\{ \frac{3N-1}{12} p^2 - \frac{m^2}{3} \Big\} +$$

$$+ \Box\phi \sum_{N\geq 2} \frac{\phi^{N-2}(-1)^N}{(p^2+m^2)^N} (N-1) \Big\{ \frac{2N-1}{6} p^2 - \frac{m^2}{2} \Big\} \Big\} =$$

$$\sum_{N\geq 3} \frac{\phi^{N-3}}{(p^2+m^2)^N} (-1)^N \frac{(N-1)(N-2)(3N-1)}{12} =$$

$$= \frac{1}{\phi^3} \sum_{N\geq 3} \left(- \frac{\phi}{p^2+m^2} \right)^N \frac{3N^3 - 10N^2 + 9N - 2}{12} =$$

$$= \frac{1}{12\phi^3} \sum_{N\geq 3} q^N \{3N^3 - 10N^2 + 9N - 2\} =$$

$$q = - \frac{\phi}{p^2+m^2}$$

$$= \frac{1}{12\phi^3} \left\{ \sum_{N\geq 0} q^N \{3N^3 - 10N^2 + 9N - 2\} + 2 \right\} =$$

= see p. 8a =

$$\sum_{k\geq 0} q^k = \frac{1}{1-q} \qquad \sum_{k\geq 0} k q^k = \frac{q}{(1-q)^2}$$

$$\sum_{k\geq 0} k^2 q^k = q\,\frac{1+q}{(1-q)^3} \qquad \sum_{k\geq 0} k^3 q^k = q\,\frac{1+4q+q^2}{(1-q)^4}$$

$$= \frac{1}{12\phi^3}\left\{3q \cdot \frac{1+4q+q^2}{(1-q)^4} - 10q\,\frac{1+q}{(1-q)^3} + 9\,\frac{q}{(1-q)^2} - \frac{2}{1-q} + 2\right\} =$$

$$= \frac{q}{12\phi^3}\left\{3 \cdot \frac{1+4q+q^2}{(1-q)^4} - 10\,\frac{1+q}{(1-q)^3} + 9\,\frac{1}{(1-q)^2} - 2\,\frac{1}{1-q}\right\} =$$

$$= \frac{1}{12\phi^3}\,\frac{q}{1-q} \cdot \left\{3\,\frac{1+4q+q^2}{(1-q)^3} - 10\,\frac{1+q}{(1-q)^2} + \frac{9}{1-q} - 2\right\} =$$

$$= \frac{1}{12\phi^3} \cdot \frac{q}{(1-q)^4}\Big\{3 + \underline{12q} + \underline{\underline{q^2}} -$$

$$- 10(1 - \underline{\underline{q^2}}) +$$

$$+ 9(1 - \underline{2q} + \underline{\underline{q^2}}) -$$

$$- 2(1 - \underline{3q} + \underline{\underline{3q^2}} - \underline{\underline{\underline{q^3}}})\Big\} =$$

$$= \frac{1}{12\phi^3} \cdot \frac{q}{(1-q)^4}\Big\{2q^3 + (3+10+9-6)q^2 + (12-18+6)q +$$

$$+ 3 - 10 + 9 - 2\Big\} =$$

$$= \frac{1}{12\phi^3} \cdot \frac{q^3 \cdot (2q + 16)}{(1-q)^4} = \frac{1}{6}\,\frac{q^3}{\phi^3}\,\frac{q+8}{(1-q)^4}$$

$$\sum_{N\geq 3} \frac{\phi^{N-3}}{(p^2+m^2)^N} (-1)^N \frac{(N-1)(N-2)}{3} =$$

$$= \frac{1}{\phi^3} \sum_{N\geq 3} \left(-\frac{\phi}{p^2+m^2}\right)^N \frac{(N-1)(N-2)}{3} =$$

$$= \frac{1}{\phi^3}\left\{\sum_{N\geq 0} \left(-\frac{\phi}{p^2+m^2}\right)^N \frac{(N-1)(N-2)}{3} - \frac{2}{3}\right\} =$$

$$= \frac{1}{3\phi^3}\left\{\sum_{N\geq 0} q^N (N-1)(N-2) - 2\right\} =$$

$$= \frac{1}{3\phi^3}\left\{\sum_{N\geq 0} q^N (N^2-3N+2) - 2\right\} =$$

= see p. 8a =

$$= \frac{1}{3\phi^3}\left\{ q \cdot \frac{1+q}{(1-q)^3} - 3\frac{q}{(1-q)^2} + \frac{2}{1-q} - 2 \right\} =$$

$$= \frac{1}{3\phi^3}\frac{q}{(1-q)}\cdot\left\{ \frac{1+q}{(1-q)^2} - \frac{3}{1-q} + 2 \right\} =$$

$$= \frac{1}{3\phi^3}\frac{q}{(1-q)^3}\left\{ 1+q - 3(1-q) + 2(1-2q+q^2) \right\} =$$

$$= \frac{1}{3\phi^3}\frac{q}{(1-q)^3}\left\{ 2q^2 + (1+3-4)q + 1-3+2 \right\} =$$

$$= \frac{2}{3}\,\frac{q^3}{\phi^3}\,\frac{1}{(1-q)^3}$$

p. 9, 11 $\Longrightarrow$

$$\Rightarrow \sum_{n\geq 3} \frac{\phi^{n-3}}{(p^2+m^2)^n}(-1)^n (n-1)(n-2)\left[\frac{3n-1}{12}p^2 - \frac{m^2}{3}\right] =$$

$$= (p.9)\cdot p^2 + (p.11)\cdot(-m^2) =$$

$$= \frac{1}{6}\frac{q^3}{\phi^3}\frac{q+8}{(1-q)^4}\cdot p^2 - m^2\cdot\frac{2}{3}\frac{q^3}{\phi^3}\frac{1}{(1-q)^3} =$$

$$= \frac{1}{6}\frac{(-1)}{(p^2+m^2)^3}\left[\frac{q+8}{(1-q)^4}p^2 - 4\frac{1}{(1-q)^3}m^2\right] =$$

$$= -\frac{1}{6}\frac{1}{(p^2+m^2)^3}\frac{1}{(1-q)^4}\left[p^2q + 8p^2 - 4(1-q)m^2\right] =$$

$$=$$

$$\frac{1}{(1-q)^4} = \frac{1}{\left(1+\frac{\phi}{p^2+m^2}\right)^4} = \frac{(p^2+m^2)^4}{(p^2+m^2+\phi)^4}$$

$$= -\frac{1}{6}\,\frac{p^2+m^2}{(p^2+m^2+\phi)^4}\,\cdot$$

$$\cdot\left\{ \frac{-p^2\phi}{p^2+m^2} + 8p^2 - 4m^2\,\frac{p^2+m^2+\phi}{p^2+m^2} \right\} =$$

$$= -\frac{1}{6}\,\frac{1}{(p^2+m^2+\phi)^4}\left\{ -(p^2+m^2)\phi + m^2\phi + 8(p^2+m^2)^2 - 8m^2(p^2+m^2) - 4m^2(p^2+m^2) - 4m^2\phi \right\} =$$

$$= -\frac{1}{6}\,\frac{1}{(p^2+m^2+\phi)^4}\left\{ -3m^2\phi - (p^2+m^2)(\phi+12m^2) + 8(p^2+m^2)^2 \right\}$$

$$\bullet \quad \sum_{N\geq 2} \frac{\phi^{N-2}}{(p^2+m^2)^N} (-1)^N \frac{(N-1)(2N-1)}{6} =$$

$$= \frac{1}{6\phi^2} \sum_{N\geq 2} \left(\frac{-\phi}{p^2+m^2}\right)^N (N-1)(2N-1) =$$

$$= \frac{1}{6\phi^2}\left\{ \sum_{N\geq 0} q^N (N-1)(2N-1) - 1 \right\} =$$

$$= \frac{1}{6\phi^2}\left\{ \sum_{N\geq 0} q^N (2N^2-3N+1) - 1 \right\} =$$

$$= \frac{1}{6\phi^2}\left\{ 2q\frac{1+q}{(1-q)^3} - \frac{3q}{(1-q)^2} + \frac{1}{1-q} - 1 \right\} =$$

$$= \frac{1}{6\phi^2}\frac{q}{1-q}\left\{ \frac{2(1+q)}{(1-q)^2} - \frac{3}{1-q} + 1 \right\} =$$

$$= \frac{1}{6\phi^2}\frac{q}{(1-q)^3}\left\{ 2+2q-3(1-q)+1-2q+q^2 \right\} =$$

$$= \frac{1}{6\phi^2}\frac{\left[q^2+3q \right]}{(1-q)^3} = \frac{q(q+3)}{6\phi^2(1-q)^3} = \frac{1}{6\phi^2}\frac{q(q+3)}{(1-q)^3}$$

$$\bullet \quad \sum_{N\geq 2} \frac{\phi^{N-2}}{(p^2+m^2)^N} (-1)^N \frac{N-1}{2} =$$

$$= + \frac{1}{\phi^2} \sum_{N\geq 2} \left(-\frac{\phi}{p^2+m^2}\right)^N \cdot \frac{N-1}{2} = \frac{1}{2\phi^2} \sum_{N\geq 2} q^N \cdot (N-1) =$$

$$= \frac{1}{2\phi^2} \left\{ \sum_{N\geq 0} q^N (N-1) + 1 \right\} =$$

$$= \frac{1}{2\phi^2} \cdot \left\{ \frac{q}{(1-q)^2} - \frac{1}{1-q} + 1 \right\} = \frac{1}{2\phi^2} \left\{ \frac{q}{(1-q)^2} - \frac{q}{1-q} \right\} =$$

$$= \frac{1}{2\phi^2} \frac{q}{1-q} \left\{ \frac{1}{1-q} - 1 \right\} = \frac{1}{2\phi^2} \frac{q^2}{(1-q)^2} = \frac{1}{2} \frac{q^2}{\phi^2} \frac{1}{(1-q)^2}$$

p. 14, 15 $\Longrightarrow$

$$\Longrightarrow \sum_{N\geq 2} \frac{\phi^{N-2}(-1)^N}{(p^2+m^2)^N}(N-1)\left[\frac{2N-1}{6}p^2 - \frac{m^2}{2}\right] =$$

$$= (p.14)\cdot p^2 + (p.15)\cdot(-m^2) =$$

$$= \frac{1}{6\phi^2}\frac{q(q+3)}{(1-q)^3}p^2 - m^2\frac{q^2}{2\phi^2}\frac{1}{(1-q)^2} =$$

$$= \frac{1}{2\phi^2}\cdot\left\{\frac{1}{3}p^2\frac{(p^2+m^2)^3}{(p^2+m^2+\phi^2)^3}\left[\frac{\phi^2}{(p^2+m^2)^2} - \frac{3\phi}{p^2+m^2}\right] -\right.$$

$$\left. - m^2\frac{\phi^2}{\cancel{(p^2+m^2)^2}}\cdot\frac{\cancel{(p^2+m^2)^2}}{(p^2+m^2+\phi)^2}\right\} =$$

$$= \frac{1}{6}p^2\left[\frac{p^2+m^2}{(p^2+m^2+\phi^2)^3} - \frac{3\,1/\cancel{\phi}\,(p^2+m^2)^2}{(p^2+m^2+\phi^2)^3}\right] - \frac{m^2}{2}\frac{1}{(p^2+m^2+\phi)^2} =$$

$$= \frac{1}{6}p^2\frac{p^2+m^2}{(p^2+m^2+\phi^2)^3}\cdot\left[1 - 3\frac{p^2+m^2}{\phi}\right] - \frac{m^2}{2}\frac{1}{(p^2+m^2+\phi)^2} =$$

$$= \frac{1}{6}\frac{(p^2+m^2)^2}{(p^2+m^2+\phi)^3} - \frac{1}{6}m^2\frac{p^2+m^2}{(p^2+m^2+\phi)^3} -$$

$$- \frac{1}{2}\frac{(p^2+m^2)^2}{\phi}\frac{1}{(p^2+m^2+\phi)^2} + \frac{1}{2}\frac{(p^2+m^2)^2}{\phi}\frac{m^2+\phi}{(p^2+m^2+\phi)^3} -$$

$$- \frac{m^2}{2}\frac{1}{(p^2+m^2+\phi)^2}$$

So $\Sigma \supset$ p. 7 with subst. p. 13, 17 $\supset$

$$\supset \sim \int \frac{dx\, dp}{(2\pi)^d} \frac{\phi(x)}{(p^2+m^2)^2} \times$$

$$\Big\{ \times (\partial_\mu \phi)^2 \left(-\frac{1}{6}\right) \frac{-3m^2\phi - (p^2+m^2)(\phi+12m^2) + 8(p^2+m^2)^2}{(p^2+m^2+\phi)^4} +$$

$$+ n\phi \Big\{ \frac{1}{6} \frac{(p^2+m^2)^2}{(p^2+m^2+\phi)^3} - \frac{1}{6} m^2 \frac{p^2+m^2}{(p^2+m^2+\phi)^3} -$$

$$- \frac{1}{2} \frac{(p^2+m^2)^2}{\phi} \frac{1}{(p^2+m^2+\phi)^2} + \frac{1}{2} \frac{(p^2+m^2)^2}{\phi} \frac{m^2+\phi}{(p^2+m^2+\phi)^3} -$$

$$- \frac{m^2}{2} \frac{1}{(p^2+m^2+\phi)^2} \Big\} \Big\}$$

- terms at $(\partial_\mu \phi)^2$ (start on p. 18)

$$\int \frac{dx\, dp}{(2\pi)^d} \frac{\phi(x)}{(p^2+m^2)^2} \cdot (\partial_\mu \phi)^2 \cdot \frac{1}{6} \frac{-3m^2\phi - (p^2+m^2)(\phi+12m^2) + 8(p^2+m^2)^2}{(p^2+m^2+\phi)^4}$$

$$\cdot\cdot \int \frac{dx\, dp}{(2\pi)^d} (\partial_\mu \phi)^2 \cdot \phi^2 \cdot \left(-\tfrac{1}{2}\right) m^2 \frac{1}{(p^2+m^2)^2 (p^2+m^2+\phi)^4} =$$

$$\frac{d^d p}{(2\pi)^2} = \frac{2\pi p\, dp}{(2\pi)^2} = \frac{\pi\, dp^2}{4\pi^2} = \frac{dp^2}{4\pi} = \frac{d(p^2+m^2)}{4\pi} = \frac{ds}{4\pi}, \quad \boxed{s = p^2+m^2}$$

$$\int \frac{dx}{x^2(a+bx)^4} = -\left(\frac{1}{ax} + \frac{22b}{3a^2} + \frac{10b^2x}{a^3} + \frac{4b^3x^2}{a^4}\right)\frac{1}{(a+bx)^3} + \frac{4b}{a^5} \ln \frac{a+bx}{x}$$

$$= \int dx \left(-\tfrac{1}{2}\right) m^2 \phi^2 (\partial_\mu \phi)^2 \frac{ds}{4\pi} \frac{1}{s^2 (s+\phi)^4} =$$

$a = \phi, \quad b = 1, \quad x = s = p^2+m^2$

$$= + \int dx \left(-\frac{1}{8\pi}\right) m^2 \phi^2 (\partial_\mu \phi)^2$$

$$\cdot \left\{ - \left[\frac{1}{\phi (p^2+m^2)} + \frac{22}{3\phi^2} + \frac{10(p^2+m^2)}{\phi^3} + \frac{4(p^2+m^2)^2}{\phi^4} \right] \frac{1}{(p^2+m^2+\phi)^3} + \right.$$

$$\left. + \frac{4}{\phi^5} \ln \frac{p^2+m^2+\phi}{p^2+m^2} \right\}_0^\infty =$$

$$= \int dx \left(-\frac{1}{8\pi}\right) m^2 (\partial_\mu \phi)^2 \cdot$$

$$\cdot \left\{ \left[\frac{\phi}{m^2} + \frac{22}{3} + \frac{10m^2}{\phi} + \frac{4m^4}{\phi^2} \right] \frac{1}{(m^2+\phi)^3} - \frac{4}{\phi^3} \ln \frac{m^2+\phi}{m^2} \right\} =$$

$$= \int dx (\partial_\mu \phi)^2 \cdot \left(-\frac{1}{8\pi}\right) \left\{ m^2 \left[\frac{\phi}{m^2} + \frac{22}{3} + \frac{10m^2}{\phi} + \frac{4m^4}{\phi^2} \right] \frac{1}{(m^2+\phi)^3} - \right.$$

$$\left. - \frac{4m^2}{\phi^3} \ln \frac{m^2+\phi}{m^2} \right\}$$

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$$\therefore \int \frac{dx\,dp}{(2\pi)^{d}} \frac{\phi(x)}{(p^2+m^2)^2} (\partial_\mu \phi)^2 \frac{1}{6} \frac{-(p^2+m^2)(\phi+12m^2)}{(p^2+m^2+\phi)^4} =$$

$$= \int dx \frac{dp}{(2\pi)^d} \left(-\tfrac{1}{6}\right)(\partial_\mu \phi)^2 (\phi + 12m^2)\,\phi \frac{1}{(p^2+m^2)(p^2+m^2+\phi)^4} =$$

$$= \int dx \left(-\frac{1}{24\pi}\right) ds.\, (\partial_\mu \phi)^2 (\phi + 12m^2)\,\phi \cdot \frac{1}{s\,(s+\phi)^4} =$$

$$\int \frac{dx}{x(a+bx)^4} = \left\{\frac{11}{6a} + \frac{5bx}{2a^2} + \frac{b^2x^2}{a^3}\right\} \frac{1}{(a+bx)^3} - \frac{1}{a^4} \ln \frac{a+bx}{x}$$

$a=\phi,\ b=1,\ s=p^2+m^2 = x$

$$= \int dx \left(-\frac{1}{24\pi}\right)(\partial_\mu \phi)^2 \phi\,(\phi + 12m^2)\,\cdot$$

$$\left[\left\{\frac{11}{6\phi} + \frac{5(p^2+m^2)}{2\phi^2} + \frac{(p^2+m^2)^2}{\phi^3}\right\} \cdot \frac{1}{(p^2+m^2+\phi)^3} - \frac{1}{\phi^4} \ln \frac{p^2+m^2+\phi}{p^2+m^2}\right]_0^\infty =$$

$=$

$$= \int dx\, (\partial_\mu \phi)^2 \left(\frac{\phi + 12 m^2}{24\pi} \right) \times$$

$$\Rightarrow \quad \times \left\{ \left[\frac{11}{6} + \frac{5 m^2}{2\phi} + \frac{m^4}{\phi^2} \right] \frac{1}{(m^2+\phi)^3} - \frac{1}{\phi^3} \ln \frac{m^2+\phi}{m^2} \right\}$$

$$\int \frac{dx\, dp}{(2\pi)^d} (\partial_\mu \phi)^2 \frac{4}{3} \phi \cdot \frac{1}{(p^2 + m^2 + \phi)^4} =$$

$$= \int dx\, (\partial_\mu \phi)^2 \cdot \frac{4}{3} \phi \cdot \frac{ds}{4\pi} \frac{1}{(s + \phi)^4} =$$

$$= \int dx\, (\partial_\mu \phi)^2 \frac{4}{3} \phi \cdot \frac{1}{4\pi} \cdot \left(-\frac{1}{3}\right) \cdot \frac{1}{(p^2 + m^2 + \phi)^3} \Bigg|_0^\infty =$$

$$= \int dx\, (\partial_\mu \phi)^2 \frac{1}{9\pi} \phi \cdot \frac{1}{(m^2 + \phi)^3} =$$

$$\Rightarrow \quad = \int dx (\partial_\mu \phi)^2 \frac{1}{9\pi} \frac{\phi}{(m^2 + \phi)^3}$$

- terms at $\Box\phi$ (start on p. 18)

$$\int \frac{dx\,dp}{(2\pi)^d} \frac{\phi(x)}{(p^2+m^2)^2} \Box\phi \,\cdot$$

$$\Bigg\{ -\frac{1}{6} \frac{(p^2+m^2)^2}{(p^2+m^2+\phi)^3} + \frac{1}{6} m^2 \frac{p^2+m^2}{(p^2+m^2+\phi)^3} +$$

$$+ \frac{1}{2} \frac{(p^2+m^2)^2}{\phi} \frac{1}{(p^2+m^2+\phi)^2} - \frac{1}{2} \frac{(p^2+m^2)^2}{\phi} \frac{m^2+\phi}{(p^2+m^2+\phi)^3} +$$

$$+ \frac{m^2}{2} \frac{1}{(p^2+m^2+\phi)^2} \Bigg\} =$$

$$\therefore \int \frac{dx\,dp}{(2\pi)^d} \frac{\phi(x)}{(p^2+m^2)^2} \,\Box\phi \cdot \left(-\tfrac{1}{6}\right) \frac{(p^2+m^2)^2}{(p^2+m^2+\phi)^3} =$$

$$= \int \frac{dx\,dp}{(2\pi)^d} \,\Box\phi \cdot \left(-\tfrac{1}{6}\right) \phi \frac{1}{(p^2+m^2+\phi)^3} =$$

$$= \int dx \,\Box\phi \left(-\tfrac{1}{6}\right) \phi \frac{ds}{4\pi} \frac{1}{(s+\phi)^3} =$$

$$= \int dx \,\Box\phi \left(-\tfrac{1}{6}\right) \phi \frac{1}{4\pi} \left(-\tfrac{1}{2}\right) \frac{1}{(s+\phi)^2}\Bigg|_{p^2=0}^{\infty} =$$

$$= \int dx \,\Box\phi \cdot \frac{(-1)}{48\pi} \frac{\phi}{(m^2+\phi)^2} =$$

$$\Rightarrow \; = \int dx \,\Box\phi \cdot \left(-\frac{1}{48\pi}\right) \frac{\phi}{(m^2+\phi)^2}$$

$$\cdot\cdot \int \frac{dx\,dp}{(2\pi)^d} \frac{\phi}{(p^2+m^2)^2} \Box\phi \ast \frac{1}{6} m^2 \frac{p^2+m^2}{(p^2+m^2+\phi)^3} =$$

$$= \int \frac{dx\,dp}{(2\pi)^d} \Box\phi \cdot \frac{1}{6} m^2\phi \cdot \frac{1}{s\,(s+\phi)^3} =$$

$$= \int dx\, \Box\phi\, \frac{1}{24\pi} m^2\phi \int \frac{ds}{s\,(s+\phi)^3} =$$

$$\int \frac{dx}{x(a+bx)^3} = \left[\frac{3}{2a} + \frac{bx}{a^2}\right] \frac{1}{(a+bx)^2} - \frac{1}{a^3} \ln \frac{a+bx}{x}$$

$a = \phi, \quad b = 1, \quad x = s = p^2+m^2$

$$= \int dx\, \Box\phi\, \frac{1}{24\pi} m^2\phi \cdot$$

$$\left\{ \left[\frac{3}{2\phi} + \frac{p^2+m^2}{\phi^2}\right] \frac{1}{(p^2+m^2+\phi)^2} - \frac{1}{\phi^3} \ln \frac{p^2+m^2+\phi}{p^2+m^2} \right\}_0^\infty =$$

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$$= \int dx \; d\phi \, \cdot$$

$$\cdot \left\{ \left(-\frac{1}{24\pi}\right) m^2 \left\{ \left[\frac{3}{2} + \frac{m^2}{\phi} \right] \frac{1}{(m^2+\phi)^2} - \frac{1}{\phi^2} \ln \frac{m^2+\phi}{m^2} \right\} \right\} =$$

$$= \int dx \; d\phi \, \cdot$$

$$\Rightarrow \quad \cdot \left(-\frac{1}{24\pi}\right) m^2 \left\{ \left[\frac{3}{2} + \frac{m^2}{\phi} \right] \frac{1}{(m^2+\phi)^2} - \frac{1}{\phi^2} \ln \frac{m^2+\phi}{m^2} \right\}$$

$$\therefore \int \frac{dx\,dp}{(2\pi)^{d}} \frac{\phi}{(p^2+m^2)^2} \ln\phi \cdot \frac{1}{2} \frac{(p^2+m^2)^2}{\phi} \frac{1}{(p^2+m^2+\phi)^2} =$$

$$= \int \frac{dx\,dp}{(2\pi)^d} \ln\phi \cdot \frac{1}{2} \frac{1}{(p^2+m^2+\phi)^2} =$$

$$= \int dx \ln\phi \frac{ds}{8\pi} \frac{1}{(s+\phi)^2} = \int dx \ln\phi \left(-\frac{1}{8\pi}\right) \frac{1}{s+\phi}\Bigg|_{p^2=0}^{\infty} =$$

$$\Rightarrow \quad = \int dx \ln\phi \frac{1}{8\pi} \frac{1}{m^2+\phi}$$

$$\therefore \int \frac{dx\,dp}{(2\pi)^d} \frac{\phi}{(p^2+m^2)^2} \Box\phi \cdot \left(-\tfrac{1}{2}\right) \frac{(p^2+m^2)^2}{\phi} \frac{m^2+\phi}{(p^2+m^2+\phi)^3} =$$

$$= \int \frac{dx\,dp}{(2\pi)^d} \Box\phi \left(-\tfrac{1}{2}\right) \frac{m^2+\phi}{(p^2+m^2+\phi)^3} =$$

$$= \int dx \,\Box\phi \left(-\frac{1}{8\pi}\right) ds \frac{m^2+\phi}{(s+\phi)^3} =$$

$$= \int dx \,\Box\phi \,\frac{1}{16\pi} \frac{m^2+\phi}{(p^2+m^2+\phi)^2}\Bigg|_0^{\infty} =$$

$$\Rightarrow \quad = \int dx \,\Box\phi \left(-\frac{1}{16\pi}\right) \frac{1}{m^2+\phi}$$

$$\int \frac{dx\,dp}{(2\pi)^d} \frac{\phi}{(p^2+m^2)^2} \, a\phi \times \frac{m^2}{2} \frac{1}{(p^2+m^2+\phi)^2} =$$

$$= \int \frac{dx\,dp}{(2\pi)^d} \, a\phi \times \frac{m^2\phi}{2} \frac{1}{s^2(s+\phi)^2} =$$

$$= \int dx \; a\phi \, \frac{m^2\phi}{8\pi} \frac{ds}{s^2(s+\phi)^2} =$$

$$\int \frac{dx}{x^2(a+bx)^2} = -\left\{\frac{1}{ax} + \frac{2b}{a^2}\right\}\frac{1}{a+bx} + \frac{2b}{a^3}\ln\frac{a+bx}{x}$$

$$a=\phi, \quad b=1, \quad x=s=p^2+m^2$$

$$= \int dx \; a\phi \, \frac{m^2\phi}{8\pi} \times$$

$$\times \left\{ -\left\{\frac{1}{\phi(p^2+m^2)} + \frac{2}{\phi^2}\right\}\frac{1}{p^2+m^2+\phi} + \frac{2}{\phi^3}\ln\frac{p^2+m^2+\phi}{p^2+m^2} \right\}_0^\infty =$$

$$= \int dx \quad ut \times$$

$$\times \frac{1}{8\pi} m^2 \left\{ \left[\frac{1}{m^2} + \frac{2}{t} \right] \frac{1}{m^2 + t} - \frac{2}{t^2} \ln \frac{m^2 + t}{m^2} \right\} =$$

$$= \int dx \quad ut \times$$

$$\Rightarrow \times \frac{1}{8\pi} m^2 \left\{ \left[\frac{1}{m^2} + \frac{2}{t} \right] \frac{1}{m^2 + t} - \frac{2}{t^2} \ln \frac{m^2 + t}{m^2} \right\}$$

Altogether, gathering results on p. 18-31, dropping $\int dx$,

$$\bar{X} \supset$$

$$\supset (\partial_\mu \phi)^2 \cdot$$

$$\cdot \left\{ -\frac{1}{8\pi} \left\{ \left[\frac{\phi}{m^2} + \frac{22}{3} + \frac{10 m^2}{\phi} + \frac{4 m^4}{\phi^2} \right] \frac{m^2}{(m^2+\phi)^3} - \frac{4 m^2}{\phi^3} \ln \frac{m^2+\phi}{m^2} \right\} + \right.$$

$$+ \frac{1}{24\pi} (\phi + 12 m^2) \left\{ \left[\frac{11}{6} + \frac{5 m^2}{2\phi} + \frac{m^4}{\phi^2} \right] \frac{1}{(m^2+\phi)^3} - \frac{1}{\phi^3} \ln \frac{m^2+\phi}{m^2} \right\} +$$

$$\left. + \frac{1}{8\pi} \frac{\phi}{(m^2+\phi)^3} \right\} +$$

$$+ \square\phi \times \left\{ -\frac{1}{48\pi} \frac{\phi}{(m^2+\phi)^2} - \right.$$

$$- \frac{1}{24\pi} m^2 \left\{ \left[\frac{3}{2} + \frac{m^2}{\phi} \right] \frac{1}{(m^2+\phi)^2} - \frac{1}{\phi^2} \ln \frac{m^2+\phi}{m^2} \right\} +$$

$$+ \frac{1}{8\pi} \frac{1}{m^2+\phi} - \frac{1}{16\pi} \frac{1}{m^2+\phi} +$$

$$\left. + \frac{1}{8\pi} m^2 \left\{ \left[\frac{1}{m^2} + \frac{2}{\phi} \right] \frac{1}{m^2+\phi} - \frac{2}{\phi^2} \ln \frac{m^2+\phi}{m^2} \right\} \right\} =$$

$$= (\phi_a \phi)^2 \cdot$$

$$\cdot \left\{ \frac{1}{(m^2+\phi)^3} \left\{ -\frac{1}{8\pi}\phi - \cancel{\frac{11}{12\pi} m^2} - \cancel{\frac{5}{4\pi}\frac{m^4}{\phi}} - \cancel{\frac{1}{2\pi}\frac{m^6}{\phi^2}} + \right.\right.$$

$$+ \frac{11}{144\pi}\phi + \frac{5}{48\pi}m^2 + \frac{1}{24\pi}\frac{m^4}{\phi} +$$

$$\left. + \cancel{\frac{11}{12\pi}m^2} + \cancel{\frac{5}{4\pi}\frac{m^4}{\phi}} + \cancel{\frac{1}{2\pi}\frac{m^6}{\phi^2}} + \frac{1}{9\pi}\phi \right\} +$$

$$\left. + \left(\cancel{\frac{1}{2\pi}\frac{m^2}{\phi^3}} - \frac{1}{24\pi}\frac{1}{\phi^2} - \cancel{\frac{1}{2\pi}\frac{m^2}{\phi^3}} \right) \ln \frac{m^2+\phi}{m^2} \right\} +$$

$$-\frac{1}{8\pi}\phi + \frac{11}{144\pi}\phi + \frac{1}{9\pi}\phi = \frac{-18+11+16}{144\pi}\phi = \frac{9}{144\pi}\phi = \frac{1}{16\pi}\phi$$

$$\frac{5}{48\pi}m^2 + \frac{1}{16\pi}\phi = \frac{2}{48\pi}m^2 + \frac{1}{16\pi}(m^2+\phi) = \frac{1}{24\pi}m^2 + \frac{1}{16\pi}(m^2+\phi)$$

$$\frac{1}{24\pi}\frac{m^4}{\phi} + \frac{1}{24\pi}m^2 + \frac{1}{16\pi}(m^2+\phi) = \frac{1}{24\pi}\frac{m^2}{\phi}(m^2+\phi) + \frac{1}{16\pi}(m^2+\phi) =$$

$$= (m^2+\phi)\left\{\frac{1}{16\pi} + \frac{1}{24\pi}\frac{m^2}{\phi}\right\}$$

$+ \square \phi \cdot$

$$\left\{ \frac{1}{m^2+\phi} \cdot \left[-\frac{1}{24\pi}\frac{m^2}{\phi} + \frac{1}{16\pi} + \frac{1}{8\pi} + \frac{1}{4\pi}\frac{m^2}{\phi} \right] + \right.$$

$$+ \frac{1}{(m^2+\phi)^2}\left[-\frac{1}{48\pi}\phi - \frac{1}{24\pi}\frac{m^2}{2} \right] +$$

$$\left. + \left[\frac{1}{24\pi}\frac{m^2}{\phi^2} - \frac{1}{4\pi}\frac{m^2}{\phi^2} \right] \ln\frac{m^2+\phi}{m^2} \right\} =$$

$$\frac{1}{4\pi}\frac{m^2}{\phi} - \frac{1}{24\pi}\frac{m^2}{\phi} = \frac{5}{24\pi}\frac{m^2}{\phi}$$

$$-\frac{1}{48\pi}\phi - \frac{1}{48\pi}m^2 = -\frac{1}{48\pi}(m^2+\phi)$$

$$\frac{1}{16\pi} + \frac{1}{8\pi} = \frac{3}{16\pi}$$

$$\frac{3}{16\pi} - \frac{1}{48\pi} = \frac{8}{48\pi} = \frac{1}{6\pi}$$

$$\frac{1}{6\pi} + \frac{5}{24\pi}\frac{m^2}{\phi} = \frac{5}{24\pi} - \frac{1}{24\pi} + \frac{5}{24\pi}\frac{m^2}{\phi} =$$

$$= \frac{5}{24\pi}\frac{m^2+\phi}{\phi} - \frac{1}{24\pi}$$

$$\frac{1}{24\pi}\frac{m^2}{\phi^2} - \frac{1}{4\pi}\frac{m^2}{\phi^2} = -\frac{5}{24\pi}\frac{m^2}{\phi^2}$$

$$= (\partial_\mu \phi)^2 \left\{ \frac{1}{(m^2+\phi)^2}\left[\frac{1}{16\pi} + \frac{1}{24\pi}\frac{m^2}{\phi}\right] - \frac{1}{24\pi}\frac{1}{\phi^2}\ln\frac{m^2+\phi}{m^2}\right\} +$$

$$+ \Box\phi \left\{ \frac{5}{24\pi}\frac{1}{\phi} - \frac{1}{24\pi}\frac{1}{m^2+\phi} - \frac{5}{24\pi}\frac{m^2}{\phi^2}\ln\frac{m^2+\phi}{m^2}\right\} =$$

$$\frac{1}{16\pi} + \frac{1}{24\pi}\frac{m^2}{\phi} = \frac{1}{48\pi} + \frac{1}{24\pi} + \frac{1}{24\pi}\frac{m^2}{\phi} = \underline{\frac{1}{48\pi} + \frac{1}{24\pi}\frac{m^2+\phi}{\phi}}$$

$$\Box\phi\, \frac{5}{24\pi}\frac{1}{\phi} = (\partial_\mu\phi)^2 \frac{5}{24\pi}\frac{1}{\phi^2}$$

$$\Box\phi\left(-\frac{1}{24\pi}\right)\frac{1}{m^2+\phi} = (\partial_\mu\phi)^2\frac{(-1)}{24\pi}\frac{1}{(m^2+\phi)^2}$$

$$\Box\phi\left(-\frac{5}{24\pi}\right)\frac{m^2}{\phi^2}\ln\frac{m^2+\phi}{m^2} = \partial_\mu\phi\cdot\frac{5}{24\pi}\partial_\mu\left(\frac{m^2}{\phi^2}\ln\frac{m^2+\phi}{m^2}\right) =$$

$$= (\partial_\mu\phi)^2\left(-\frac{5}{12\pi}\right)\frac{m^2}{\phi^3}\ln\frac{m^2+\phi}{m^2} + (\partial_\mu\phi)^2\frac{5}{24\pi}\frac{m^2}{\phi^2}\frac{1}{m^2+\phi}$$

$$= (\tfrac{1}{2}\phi)^2 \cdot$$

$$\cdot \Big\{ \frac{1}{48\pi} \frac{1}{(m^2+\phi)^2} + \frac{1}{24\pi} \frac{1}{\phi} \frac{1}{m^2+\phi} +$$

$$+ \frac{5}{24\pi} \frac{1}{\phi^2} - \frac{1}{24\pi} \frac{1}{(m^2+\phi)^2} + \frac{5}{24\pi} \frac{m^2}{\phi^2} \frac{1}{m^2+\phi} -$$

$$- \frac{1}{24\pi} \frac{1}{\phi^2} \ln \frac{m^2+\phi}{m^2} - \frac{5}{12\pi} \frac{m^2}{\phi^3} \ln \frac{m^2+\phi}{m^2} \Big\} =$$

$$\frac{1}{48\pi} \frac{1}{(m^2+\phi)^2} - \frac{1}{24\pi} \frac{1}{(m^2+\phi)^2} = - \frac{1}{48\pi} \frac{1}{(m^2+\phi)^2}$$

$$\frac{5}{24\pi} \frac{m^2}{\phi^2} \frac{1}{m^2+\phi} = \frac{5}{24\pi} \frac{m^2+\phi-\phi}{\phi^2} \frac{1}{m^2+\phi} = \frac{5}{24\pi} \frac{1}{\phi^2} - \frac{5}{24\pi} \frac{1}{\phi} \frac{1}{m^2+\phi}$$

$$\frac{1}{24\pi} \frac{1}{\phi} \frac{1}{m^2+\phi} + \frac{5}{24\pi} \frac{1}{\phi^2} + \frac{5}{24} \frac{1}{\phi^2} - \frac{5}{24\pi} \frac{1}{\phi} \frac{1}{m^2+\phi} =$$

$$= \frac{5}{12\pi} \frac{1}{\phi^2} - \frac{1}{6\pi} \frac{1}{\phi} \frac{1}{m^2+\phi}$$

$$= (\phi f)^2$$

$$\left\{ \frac{5}{12\pi} \frac{1}{\phi^2} - \frac{1}{6\pi} \frac{1}{\phi} \frac{1}{m^2+\phi} - \frac{1}{48\pi} \frac{1}{(m^2+\phi)^2} - \right.$$

$$\left. - \frac{1}{24\pi} \frac{\phi + 10m^2}{\phi^3} \ln \frac{m^2+\phi}{m^2} \right\}$$