

Our potential is

$$V = \frac{N}{4\pi} \left\{ -iD \ln |\sqrt{2}\sigma|^2 + \sum_{k \geq 1} \frac{(-1)^k}{k(k+1)} iD \left( \frac{iD}{|\sqrt{2}\sigma|^2} \right)^k \right\}. \quad (0.1)$$

The first term comes from Witten's superpotential. The first term of the series (with  $k = 1$ ) is taken into account by the kinetic term. Thus we can write our effective action as

$$\begin{aligned} S = & -\frac{N}{4\pi} \int d^4x \left\{ \int d^4\theta \frac{1}{2} \left| \ln \sqrt{2}\Sigma \right|^2 + \right. \\ & + i \int d^2\theta \left( \sqrt{2}\Sigma \ln \sqrt{2}\Sigma - \sqrt{2}\Sigma + \right. \\ & \left. \left. + \frac{1}{2} \sqrt{2}\Sigma \sum_{k \geq 2} \frac{(-1)^k}{(k-1)k(k+1)} \left( \frac{S}{\sqrt{2}\Sigma} \right)^k + \text{h.c.} \right) \right\}, \end{aligned} \quad (0.2)$$

where

$$S = \frac{i}{2} \overline{D}_R D_L \ln \sqrt{2}\overline{\Sigma}, \quad \overline{S} = \frac{i}{2} \overline{D}_L D_R \ln \sqrt{2}\Sigma. \quad (0.3)$$

The series in the third line can be written as a  $D$ -term. We get

$$\begin{aligned} S = & -\frac{N}{4\pi} \int d^4x \left\{ \int d^4\theta \frac{1}{2} \left| \ln \sqrt{2}\Sigma \right|^2 + \right. \\ & + i \int d^2\theta \left( \sqrt{2}\Sigma \ln \sqrt{2}\Sigma - \sqrt{2}\Sigma \right) + \\ & \left. + \frac{1}{2} \int d^4\theta \ln \sqrt{2}\overline{\Sigma} \sum_{k \geq 1} \frac{(-1)^{k+1}}{k(k+1)(k+2)} \left( \frac{S}{\sqrt{2}\Sigma} \right)^k \right\} + \text{h.c.} \end{aligned} \quad (0.4)$$

Here we can see explicitly that the only  $F$ -term is the Witten's superpotential, while all other terms are  $D$ -terms.

The last term can be re-written as

$$\begin{aligned} S = & -\frac{N}{4\pi} \int d^4x \left\{ \int d^4\theta \frac{1}{2} \left| \ln \sqrt{2}\Sigma \right|^2 + i \int d^2\theta \left( \sqrt{2}\Sigma \ln \sqrt{2}\Sigma - \sqrt{2}\Sigma \right) + \right. \\ & \left. + \frac{1}{4} \int d^4\theta \ln \sqrt{2}\overline{\Sigma} \left( \left( 1 + \frac{\sqrt{2}\Sigma}{S} \right)^2 \ln \left( 1 + \frac{S}{\sqrt{2}\Sigma} \right) - \frac{\sqrt{2}\Sigma}{S} \right) + \text{h.c.} \right\}. \end{aligned} \quad (0.5)$$

The kinetic (first) term can be combined with the last  $D$ -term, to write

$$\begin{aligned}
S = & - \frac{N}{4\pi} \int d^4x \left\{ i \int d^2\theta \left( \sqrt{2}\Sigma \ln \sqrt{2}\Sigma - \sqrt{2}\Sigma \right) + \right. \\
& + \frac{1}{4} \int d^4\theta \ln \sqrt{2\Sigma} \left[ \ln (S + \sqrt{2}\Sigma) + \right. \\
& \left. \left. + \frac{\sqrt{2}\Sigma}{S} \left( \left( 2 + \frac{\sqrt{2}\Sigma}{S} \right) \ln \left( 1 + \frac{S}{\sqrt{2}\Sigma} \right) - 1 \right) \right] \right\} + \text{h.c.}
\end{aligned} \tag{0.6}$$