Reduction of Vacuum Equations in the Higgs Phase

Vacuum Equations:

$$
\begin{cases}
|u|^2 = \dfrac{1}{4\pi} \sum\limits_k \ln \dfrac{iD + |\sqrt{2}\sigma - m^k|^2}{\Lambda^2} \\[2ex]
\left[iD + |\sqrt{2}\sigma - m|^2 \right] u = 0 \\[2ex]
(\sqrt{2}\sigma - m)|u|^2 - \dfrac{1}{4\pi} \sum\limits_k (\sqrt{2}\sigma - m^k) \ln \dfrac{iD + |\sqrt{2}\sigma - m^k|^2}{|\sqrt{2}\sigma - m^k|^2} + \dfrac{N}{4\pi} \cdot u \cdot \sqrt{2}\sigma = 0
\end{cases}
$$

Introducing $S \equiv \sqrt{2}\sigma/\Lambda$ and $\mu \equiv m/\Lambda$,

$$
\sum_{k=1}^{N-1} \left\{ (S-\mu) \ln\left[iD + |S-\mu^k|^2\right] - (S-\mu^k) \ln \frac{iD + |S-\mu^k|^2}{|S-\mu^k|^2} \right\} + N u \cdot S = 0
$$

or

$$
\sum_k \left\{ (\mu^k - \mu) \ln\left[|S-\mu^k|^2 - |S-\mu|^2 \right] + (S-\mu^k) \ln|S-\mu^k|^2 \right\} +
$$

$$
+ N u \cdot S = 0
$$

$$|S-\mu^k|^2 - |S-\mu|^2 = \text{as shown by Misha,}$$

$$= 4\mu S \cdot \sin^2\frac{\alpha_k}{2}, \quad \text{for real } S$$

$\Rightarrow$ the Equation is:

$$\sum\left\{(\mu^k-\mu)\ln 4\mu S\cdot\sin^2\frac{\alpha_k}{2} + (S-\mu^k)\ln|S-\mu^k|^2\right\} +$$

$$+ N u S = 0$$

or

$$\frac{1}{N}\sum\left\{(\mu^k-\mu)\ln\left[4\mu S\cdot\sin^2\frac{\alpha_k}{2}\right] + (S-\mu^k)\ln|S-\mu^k|^2\right\} + uS = 0$$

but
$$\frac{1}{N}\sum(\mu-\mu^k)\ln\left(4\mu^2\sin^2\frac{\alpha_k}{2}\right) = \mu(\ln\mu^2+1)$$
(again from Section 5.3)

$$\Rightarrow \frac{1}{N}\sum(\mu^k-\mu)\ln\left(4\mu S\cdot\sin^2\frac{\alpha_k}{2}\right) = -\mu(\ln\mu^2+1) + \mu\ln\mu/S =$$

$$= -\mu\left[1+\ln(\mu S)\right]$$

now, $\frac{1}{N}\sum (S - \mu^i)\ln|S-\mu^i|^2 = S\left[1+\ln\mu^2\right]$, for $|S|<\mu$,
again from Section 6

altogether, eqn:

$$-\mu\left[1+\ln(\mu S)\right] + S\left[1+\ln\mu^2\right] + uS = 0$$

or

$$\Rightarrow \quad \boxed{\left[1+u+\ln\mu^2\right]S = \mu\left[1+\ln(\mu S)\right]}$$

confirmed by numeric calculations