

### Exact equation for $|u|^2$

(1)

We have,

$$[1 + u + \ln \mu^2] S' = \mu [1 + \ln(\mu S)]$$

in the Higgs phase.

$$\Rightarrow \lambda \equiv \frac{8\pi}{N} |u|^2 \approx \ln(\mu S); \Rightarrow S = e^\lambda / \mu$$

$$\text{then, } [1 + u + \ln \mu^2] e^\lambda = \mu^2 [1 + \lambda]$$

take  $\log$ ,

$$\log [1 + u + \ln \mu^2] + \lambda = \log \mu^2 + \log(1 + \lambda)$$

$$\Rightarrow \lambda - \log(1 + \lambda) = \log \frac{\mu^2}{1 + u + \ln \mu^2}$$

or

$$\lambda - \log(1 + \lambda) = \log \frac{\mu^2}{1 + u + \ln \mu^2}$$

$$\left\{ \frac{8\pi}{N} \rho_{\text{ren}} - \ln \left( \frac{8\pi}{N} \rho_{\text{ren}} + 1 \right) = \ln \frac{\mu^2}{1 + u + \ln \mu^2} \right\}$$