Boltzmann equations for CPT-odd

Leptogenesis with sphaleron processes

The sphaleron rate we'll be using here is

$$\Gamma_{sph} = [25.4 \pm 2.0]\,\alpha_w^5 T^4 \simeq 10^{-6} T^4 \equiv \omega T^4 ; \quad \omega \simeq 10^{-6}$$

as by Bödeker et al PRD 61

This rate enters the Boltzmann eqns:

$$\frac{d}{dt} n_B, \quad \frac{d}{dt} n_L \supset - \frac{\Gamma_{sph}}{\tilde{V}^{-1}} \left\{ n_B - n_B^{eq} + n_L - n_L^{eq} \right\}$$

where $\tilde{V}^{-1} \equiv \phi T^3$;

Kuzmin, Rubakov, Shaposhnikov, Phys. Lett. 155B

References

[1] "General form of Boltzmann equations for CPT-odd Leptogenesis"

[2] "Derivation of rates for Boltzmann equations in a form suitable for CPT-odd leptogenesis"

let's take $\tau^{-1} \equiv \phi T^3$, $\left(\phi \equiv \frac{1}{\pi^2} \right)$; c.b. $\frac{2}{\pi^2}$ for n_γ

so that

$$\dot{n}_B, \dot{n}_L \supset -\frac{\omega T^4}{\phi T^3}\left\{ [n_B + n_L] - [n_B^{eq} + n_L^{eq}] \right\} =$$

$$= -\chi T\left[(n_B - n_B^{eq}) + (n_L - n_L^{eq}) \right], \quad \chi \equiv \omega/\phi$$

we then write

$$\begin{cases} \dot{n}_L \supset -\chi T[n_B - n_B^{eq} + n_L - n_L^{eq}]; & \dot{n}_B \supset -\chi T[n_B - n_B^{eq} + n_L - n_L^{eq}] \\ \dot{n}_{\bar{L}} \supset -\chi T\{n_{\bar{B}} - n_{\bar{B}}^{eq} + n_{\bar{L}} - n_{\bar{L}}^{eq}\}; & \dot{n}_{\bar{B}} \supset -\chi T\{n_{\bar{B}} - n_{\bar{B}}^{eq} + n_L - n_{\bar{L}}^{eq}\} \end{cases}$$

$$\Rightarrow \dot{[n_L - n_{\bar{L}}]} \supset \chi T\left\{ (n_B - n_B^{eq}) - (n_{\bar{B}} - n_{\bar{B}}^{eq}) + (n_L - n_L^{eq}) - (n_{\bar{L}} - n_{\bar{L}}^{eq}) \right\} \quad (2.5)$$

$$\dot{\{n_B - n_{\bar{B}}\}} \supset -\chi T[\quad \text{the same} \quad]$$

now, using the ansatz in [1], see p. 10-12

$$n_L - n_L^{eq} = n_L^{eq}\left\{ e^{\mu/T} - 1 \right\} \simeq n_L^{eq}\left[\mu/T \right] = n_L^{eq}\left[Y_L - \Theta_L T \right] \simeq$$

$$\simeq n_L^{0}\left\{ Y_L - \Theta_L T \right\},$$

$$n_{\bar{L}} - n_{\bar{L}}^{eq} \simeq \text{similarly} \quad n_L^{0}\left\{ -Y_L + \Theta_L T \right\}$$

therefore the RHS of (2.5) is

$$-\chi T \cdot 2 \left\{ n_L^0 \left[Y_L - \Theta_L T \right] + n_B^0 \left[Y_B - \Theta_B T \right] \right\}$$

and then the Boltzmann eqs are

$$\begin{cases} -\dfrac{T^3}{2\alpha M_{Pl}} n_L^0 Y_L' = -a \dfrac{T^6}{M_R^2} \left[Y_L - \Theta_L T \right] - \\ \qquad\qquad - \chi T \left[n_L^0 \left[Y_L - \Theta_L T \right] + n_B^0 \left[Y_B - \Theta_B T \right] \right] \\ -\dfrac{T^3}{2\alpha M_{Pl}} n_B^0 Y_B' = -\chi T \left[n_L^0 \left[Y_L - \Theta_L T \right] + n_B^0 \left[Y_B - \Theta_B T \right] \right] \end{cases}$$

$$n_L^0 \equiv \delta_L T^3; \quad n_B^0 \equiv \delta_B T^3, \qquad \delta_{L,B} = \frac{g_{L,B}}{\pi^2}$$

$$\begin{cases} -\dfrac{\delta_L}{2\alpha M_{Pl}} T^6 Y_L' = -\eta \dfrac{T^6}{M_R^2}\left[Y_L - \Theta_L T\right] - \\ \qquad \chi T^4 \left[\delta_L \left[Y_L - \Theta_L T\right] + \delta_B \left[Y_B - \Theta_B T\right]\right] \\ -\dfrac{\delta_B}{2\alpha M_{Pl}} T^6 Y_B' = -\chi T^4 \left[\delta_L \left[\qquad\right] + \delta_B \left[\qquad\right]\right] \end{cases}$$

$$\Rightarrow \begin{cases} Y_L' = \dfrac{2\alpha\eta}{\delta_L} \dfrac{M_{Pl}}{M_R^2}\left[Y_L - \Theta_L T\right] + \\ \qquad + \dfrac{2\alpha\chi}{\delta_L} M_{Pl} \cdot T^{-2}\left[\delta_L\left[Y_L - \Theta_L T\right] + \delta_B\left[Y_B - \Theta_B T\right]\right] \\ \Rightarrow Y_B' = \dfrac{2\alpha\chi}{\delta_B} M_{Pl} \cdot T^{-2}\left[\delta_L\left[Y_L - \Theta_L T\right] + \delta_B\left[Y_B - \Theta_B T\right]\right] \end{cases}$$

Now introduce $\Upsilon_L \equiv Y_L - \Theta_L T = \mu_L/T$; $\Upsilon_B = Y_B - \Theta_B T$;

$$\begin{cases} \delta_L\left[\Upsilon_L' + \Theta_L\right] = 2\alpha\eta \dfrac{M_{Pl}}{M_R^2} \cdot \Upsilon_L + \\ \qquad + 2\alpha\chi M_{Pl} \cdot T^{-2}\left[\delta_L \Upsilon_L + \delta_B \Upsilon_B\right] \\ \delta_B\left[\Upsilon_B' + \Theta_B\right] = 2\alpha\chi M_{Pl} \cdot T^{-2}\left[\delta_L \Upsilon_L + \delta_B \Upsilon_B\right] \end{cases} \qquad (4.5)$$

Let's now rewrite (4.5) in terms of

$$\hat{U}_+ = \delta_L \Upsilon_L + \delta_B \Upsilon_B, \qquad \hat{U}_- = \delta_L \Upsilon_L - \delta_B \Upsilon_B;$$

$$\begin{cases} \hat{U}_+' + \left[\delta_L \Theta_L + \delta_B \Theta_B\right]' = 2\alpha\gamma \dfrac{M_{Pl}}{M_R^2} \dfrac{\hat{U}_+ + \hat{U}_-}{2\delta_L} + 4\alpha\chi M_{Pl} T^{-2} \hat{U}_+, \\[2ex] \hat{U}_-' + \left[\delta_L \Theta_L - \delta_B \Theta_B\right]' = 2\alpha\gamma \dfrac{M_{Pl}}{M_R^2} \dfrac{\hat{U}_+ + \hat{U}_-}{2\delta_L} \end{cases}$$

reverse transformation: $\Upsilon_L = \dfrac{\hat{U}_+ + \hat{U}_-}{2\delta_L}$ $\qquad \Upsilon_B = \dfrac{\hat{U}_+ - \hat{U}_-}{2\delta_L}$ /

remind, $\Upsilon_{L,B} = \tilde{Y}_{L,B} - \Theta_{L,B} T = \mu_{L,B}/T$ /

$$\tilde{Y}_{L,B} = \frac{n_{L,B} - n_{\bar{L},\bar{B}}}{2 n^0_{L,B}}, \quad \text{where } n^0_{L,B} = \frac{g_{L,B}}{\pi^2} T^3 /$$