

Consider just the electromagnetic potential of *one* of the U(1) fields, say A_μ . Then the long-wave solution will look like

$$A_\lambda \propto \tau^n, \quad n \neq 0 \quad (1)$$

with some n . Correspondingly,

$$A'_\lambda \propto \tau^{n-1}. \quad (2)$$

We have then,

$$\rho_E \propto \frac{\langle 0 | \vec{E}^2 | 0 \rangle}{2 a^4} \propto \int k^2 dk \tau^{2n-2}, \quad (3)$$

$$\rho_B \propto \frac{\langle 0 | \vec{B}^2 | 0 \rangle}{2 a^4} \propto \int k^4 dk \tau^{2n}, \quad (4)$$

where $k^2 dk$ is basically the remnant of the Fourier integrals, and the extra factor of k^2 in ρ_B comes from the spatial derivative in $\vec{B} = [\vec{\nabla} \vec{A}]$.

The integral over k goes over all the inflation,

$$\rho_E \propto \int_{Ha_i}^{Ha_0} k^2 dk \tau^{2n-2} \propto (Ha_0)^3 \cdot \tau_0^{2n-2} = \tau_0^{2n-5}, \quad (5)$$

$$\rho_B \propto \int_{Ha_i}^{Ha_0} k^4 dk \tau^{2n} \propto (Ha_0)^5 \cdot \tau_0^{2n} = \tau_0^{2n-5}. \quad (6)$$

In this case we have that $\rho_E \rightarrow 0$ at the same rate as ρ_B