Lorentz Violating Supersymmetric Quantum Electrodynamics

Pavel A. Bolokhov

Introduction: Why Lorentz Violation?

There are many known examples in the history of physics when a symmetry of nature, which was assumed to be exact, has fallen under experimental scrutiny.
Lorentz symmetry is used as a crucial ingredient in the construction of fundamental theories of nature.
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One of the solutions is if such quadratic divergencies are suppressed by a symmetry.

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$$[LV]_{\mathrm{dim}\ 3} \;\sim\; (\text{loop factor})\; m_s^2 \;\times\; [LV]_{\mathrm{dim}\ 5}\,.$$

$\Rightarrow$ This might lead to a solution of the naturalness problem: why the lower dimensional LV operators are so much suppressed as compared to their natural scale.

Questions we ask

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- study the phenomenological consequences of the soft SUSY breaking; put constraints on the LV parameters

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This is enough to rule out the dimension three and four interactions, in particular the *Chern-Simons operator* — $k_\mu\, \epsilon^{\mu\nu\kappa\lambda} A_\nu \partial_\kappa A_\lambda$.

CPT-violating dimension five LV operators

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$$\mathcal{L}^{\text{matter}}_{\text{LV}} = \frac{1}{M} \int d^4\theta \Big\{ N^{\mu}_{+} \, \overline{\Phi}_{+} e^{2eV} i\nabla_{\mu}\Phi_{+} + N^{\mu}_{-} \, \overline{\Phi}_{-} e^{-2eV} i\nabla_{\mu}\Phi_{-} \Big\} \, ,$$

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In a non-abelian theory the last operator does not exist.

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We will be interested in dimension 5 operators only.

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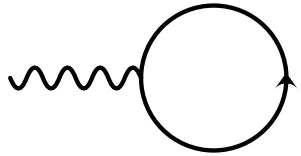

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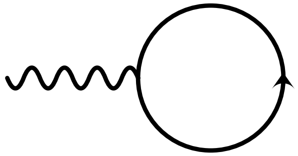

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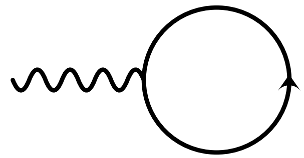

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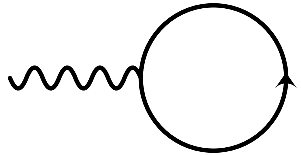

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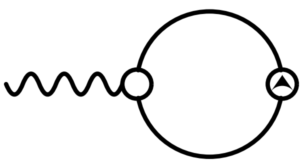

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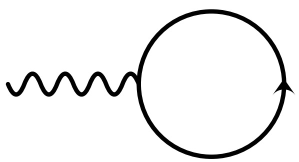

$$=\;0\,,$$

which vanishes for non-anomalous theories, where the sum of charges is zero.

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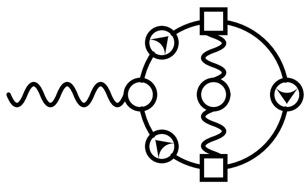
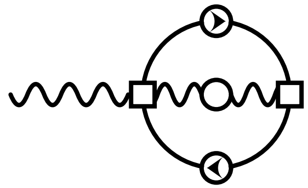
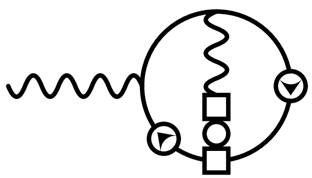
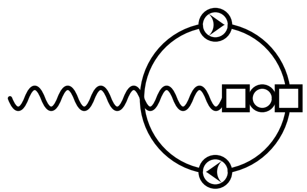
 +  ,

which by some standard superFeynman technique can be transformed into

 + 

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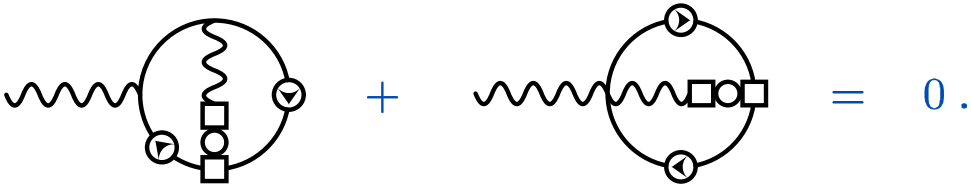

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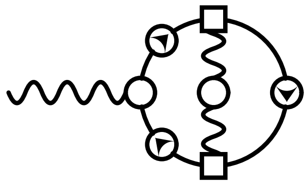
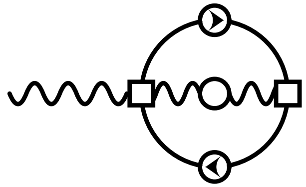
 $+$  ,

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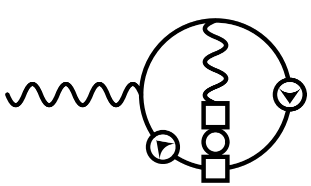
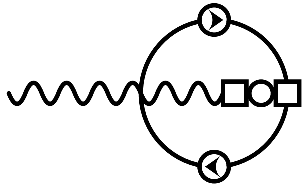
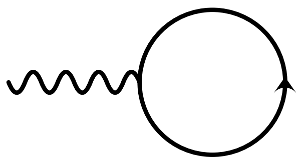
 $+$  $= \quad 0\,.$

In general, if one uses the *background superfield formalism*, any tadpole diagram will be an identical zero:

 $\equiv 0\,.$

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One can use the technique developed by Fujikawa and Konishi.

Our classical LV action is

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One considers the variation of the effective action obtained by integrating out the chiral superfield Φ_+, under a chiral gauge transformation $\delta\Lambda$:

$$\delta_\Lambda \Gamma(V) = \langle \delta_\Lambda S \rangle = \Big\langle \int d^8 z\, \overline{\Phi} e^V \Big(1 + iN^\mu \nabla_\mu\Big)(\delta\Lambda\,\Phi)\Big\rangle\,.$$

Plugging in an effective propagator for Φ_+ in the presence of the background field V one then notes that the LV part completely cancels out leaving the gauge variation the same as in the Lorentz invariant theory.

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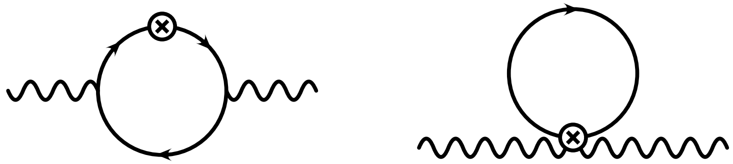

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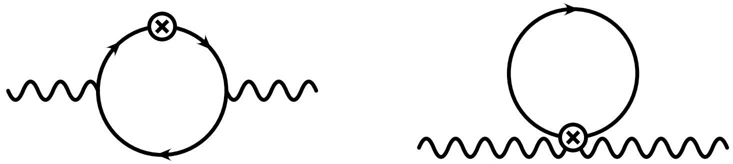

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One then obtains and solves the following RG equation:

$$\mu \frac{\partial}{\partial \mu} \begin{pmatrix} N^\nu \\ N_+^\nu \\ N_-^\nu \\ T^{\mu\nu\rho} \end{pmatrix} = \frac{\alpha}{2\pi} \begin{pmatrix} 2 & -1 & -1 & 0 \\ -6 & 3 & 0 & 0 \\ -6 & 0 & 3 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} N^\nu \\ N_+^\nu \\ N_-^\nu \\ T^{\mu\nu\rho} \end{pmatrix} .$$

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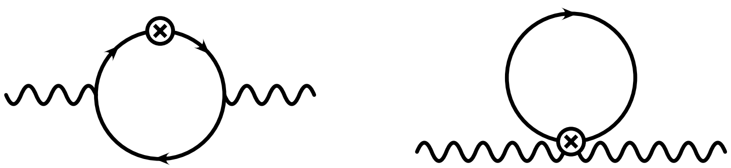

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For RG evolving the parameters from $M = M_{\rm Pl} \approx 10^{19}$ GeV to $\mu = m_s \approx 1$ TeV one obtains a change in the LV parameters of only about 10%.

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in the gauge sector, including the Chern-Simons term.

Dimension 3 Operators in the Matter Sector

There are two types of reduction of operators dimension 3 $\rightarrow$ dimension 5:

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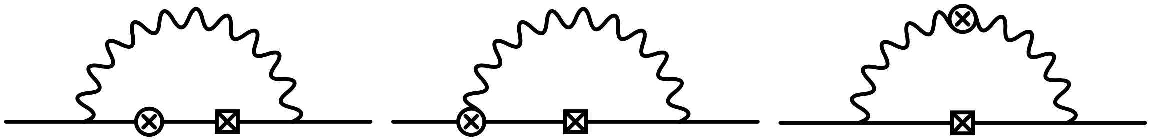
On the equations of motion, dimension 5 operators induce

$$\widetilde{A}^{\mu}_{\pm} \;=\; \pm\,2\,\frac{N^{\mu}_{V}}{M}\,(m_e^2 \,+\, m_s^2)\;.$$

1-loop Corrections to Matter Dimension 3 Operators

Including 1-loop corrections induced by dimension 5 operators



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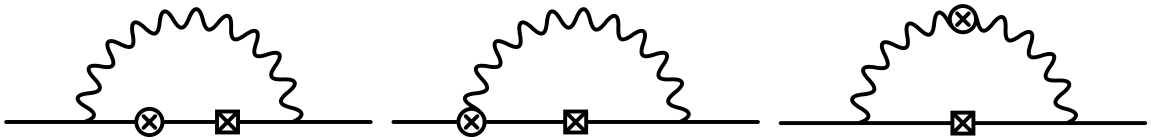

one arrives at the RG equations

$$\mu \frac{d\widetilde{A}^{\nu}_{+}}{d\mu} = \frac{\alpha}{\pi} \left(\widetilde{A}^{\nu}_{+} - \widetilde{B}^{\nu}_{+} \right) ,$$

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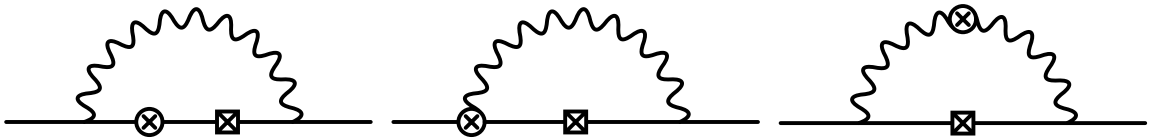
Including 1-loop corrections induced by dimension 5 operators



one arrives at the RG equations

$$\mu \frac{d\widetilde{A}^{\nu}_{+}}{d\mu} = \frac{\alpha}{\pi} \left(\widetilde{A}^{\nu}_{+} - \widetilde{B}^{\nu}_{+} \right) ,$$

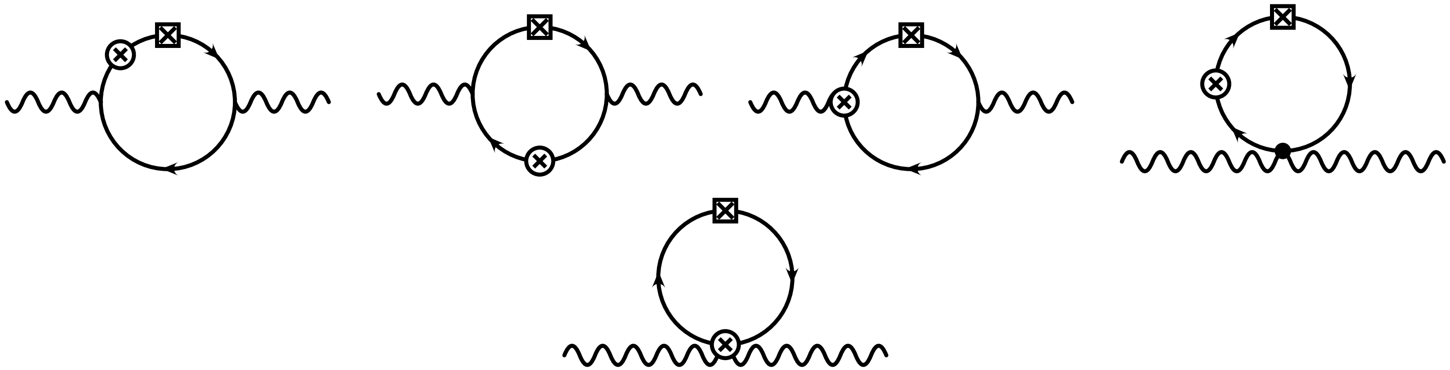
$$\mu \frac{d\widetilde{B}^{\nu}_{+}}{d\mu} = \frac{\alpha}{2\pi} \left(\widetilde{B}^{\nu}_{+} - \widetilde{A}^{\nu}_{+} + 3 \frac{(m_s^{+})^2}{M} N^{\nu} - 2 \frac{(m_s^{+})^2}{M} N^{\nu}_{+} \right) ,$$

with a leading α log solution

$$\widetilde{B}^{\pm\nu}(m_s) = \frac{\alpha}{\pi} \log(M/m_s) \frac{(m_s^{\pm})^2}{M} \left\{ \frac{3}{2} N^{\nu}(M) - N^{\nu}_{\pm}(M) \right\} .$$

1-loop Induced Gauge Dimension 3 Operators

The answer for gauge operators is given by



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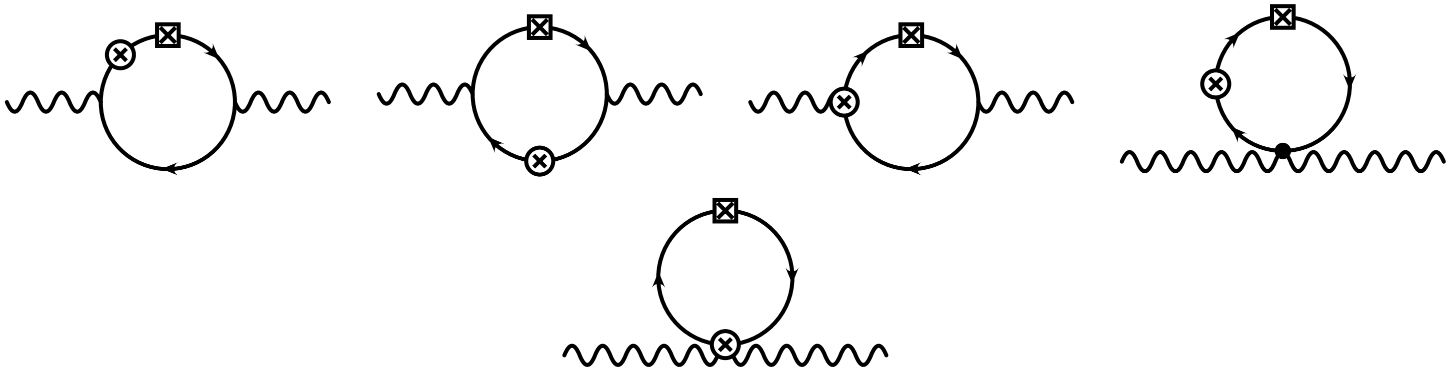

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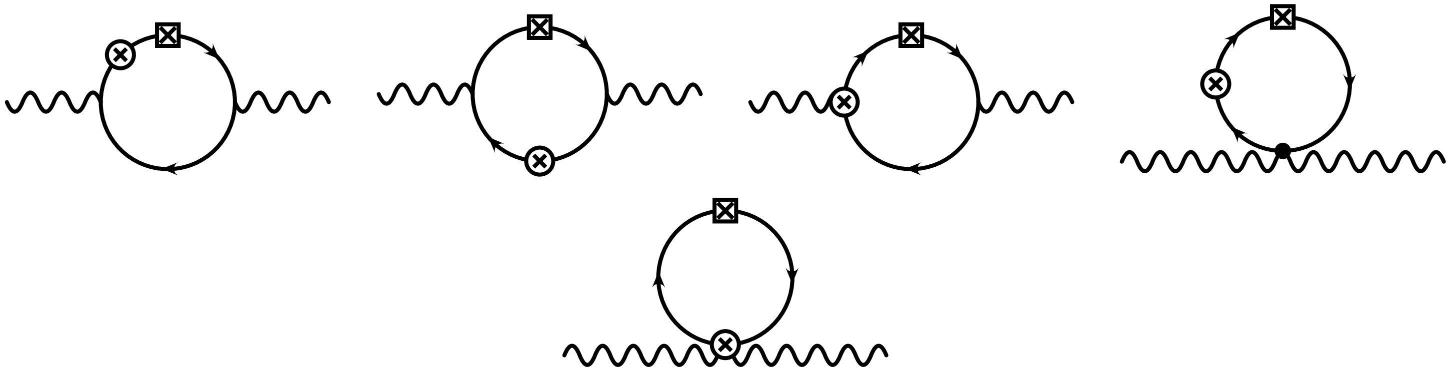

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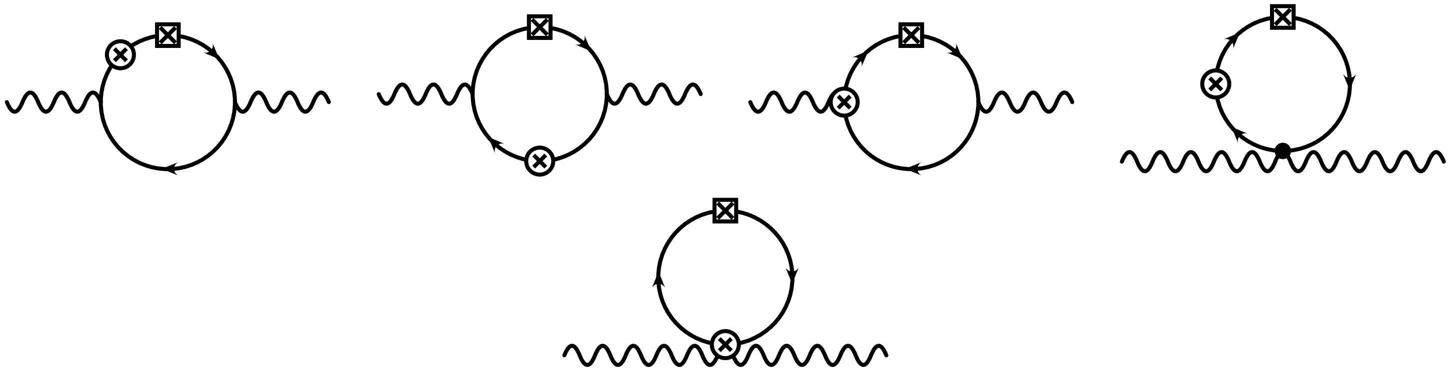

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The *Chern-Simons term is not generated* whether in a SUSY or in a SUSY-breaking theory. This is also confirmed by the Coleman-Glashow theorem.

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\mathcal{L}^{\text{matter}\,(+)}_{\text{LV dim 5}} \;=\; \frac{N_+^\mu}{M}\Big[\, & i\bar{F}_+ \mathcal{D}_\mu F_+ \;+\; ie\bar{z}_+ D\mathcal{D}_\mu z_+ \;-\; ie\mathcal{D}_\mu(\bar{z}_+)Dz_+ \;+\; \frac{1}{2}\bar{\psi}_+\mathcal{D}_{(\mu}\mathcal{D}_{\nu)}\bar{\sigma}^\nu\psi_+ \\
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$$

where the definite parity operators are

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N_V^\mu \;=\; \frac{N_+^\mu \;-\; N_-^\mu}{2}\,, \qquad N_A^\mu \;=\; \frac{N_+^\mu \;+\; N_-^\mu}{2}\,.
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At tree level this induces

$$\mathcal{L}_{\text{gauge (T)}}^{\text{EOM}} = 2\,e\,T_{\mu\nu\rho}\,\overline{\Psi}\gamma^\mu F^{\nu\rho}\Psi$$

in the observable sector.

Effective Lorentz Violating Lagrangian

We can now bring all terms together and write out the effective lagrangian

$$-\mathcal{L}_{\text{eff LV}} = \overline{\Psi}\,\gamma^{\mu}\Big(a_{\mu} + b_{\mu}\gamma^{5} + e\,c_{\nu}\,\widetilde{F}^{\nu\mu} + e\,d_{\nu}\,\widetilde{F}^{\nu\mu}\gamma^{5} + e\,f_{\mu\rho\sigma}\,F^{\rho\sigma}\Big)\,\Psi\,,$$

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In QED, the operator a^μ can be totally excluded via a phase redefinition $\Psi(x) \rightarrow e^{i a^\mu x_\mu}\,\Psi(x)$.

Effective Hamiltonian

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- and the energy scale of the external electromagnetic field.

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Dimension six operators were not subject of detailed examination. Rough estimates suggest only $M \sim 10^{14}$ GeV, which is lower than the Planck scale. More detailed study is desired.

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At the next order, $1/M^2_{\rm Pl}$, CPT-conserving operators are not excluded
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Estimates come close to experimental sensitivity, and therefore deserve further study in the framework of LV MSSM

Collaboration

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