Two-dimensional — four-dimensional duality: Towers of kinks $\leftrightarrow$ towers of monopoles in $\mathcal{N} = 2$ theories

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Two-dimensional models have interesting similarities to 4-d gauge theories

CP^{N-1} theory has been shown to have chiral symmetry breaking, mass gap, asymptotic freedom, *etc* and all these properties are much easier to show in two dimensions

In general, two-dimensional theories are simpler and two-dimensional methods are more powerful

There is also a two-dimensional — four-dimensional duality of their BPS spectra

$\mathcal{N}=2$ $N_c = N_f$ SYM in four dimensions at the root of the first baryonic Higgs branch	$\longleftrightarrow$	$\mathcal{N}=(2,2)$ CP^{N-1} theory in two dimensions
$\downarrow$		$\downarrow$
supports non-Abelian vortex strings		kinks interpolate between worldsheet vacua
monopoles	$\longleftrightarrow$	kinks
strings	$\longleftrightarrow$	vacua

At weak coupling the Seiberg-Witten theory contains Quarks, W-bosons, Monopoles, Dyons and bound states

At strong coupling — we consider CP^{N-1} theory
Strong coupling spectrum is accessible via the mirror theory

CP^{N-1} theory with *twisted* masses:

$$r\,\Big(\left|\mathcal{D}_\mu\, n^l\right|^2 \;+\; \left|\sigma\,-\,m^l\right|^2 \left|n^l\right|^2 \;+\; i\,D\left(\left|n^l\right|^2 - 1\right) \;+\; \ldots\Big)$$
$$+\;\frac{1}{4e^2}\,F_{\mu\nu}^2 \;+\; \frac{1}{e^2}\left|\partial_\mu\sigma\right|^2 \;+\; \frac{1}{2e^2}\,D^2 \;+\; \ldots,$$

in the $e^2 \;\to\; \infty$ limit

The theory has N vacua — both classically and exactly

CP^{N-1} theory with *twisted* masses:

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CP^{N-1} model with $\mathcal{Z}_N$ twisted masses

$$m_l \;=\; m_0 \cdot e^{2\pi i l/N}$$

in this case $\mathcal{Z}_N \subset U_R(1)$ remains unbroken

Exact superpotential

The theory possesses an "exact" superpotential of Veneziano-Yankielowicz type

$$\mathcal{W}_{\text{eff}} \;=\; -\,i\,\tau\hat{\sigma} \;+\; \frac{1}{2\pi}\sum_l \left(\hat{\sigma} - m_l\right)\left(\ln\frac{\hat{\sigma} - m_l}{\mu} - 1\right)$$

μ = UV cut-off scale

Exact superpotential

Vacuum values:

$$\mathcal{W}_{\text{eff}} \;=\; -\,\frac{1}{2\pi}\,\left(N\,\sigma_p \,+\, \sum_l m_l \,\ln\left(\sigma_p - m_l\right) \right)$$

where

$$\sigma_p \;=\; \sigma_0 \cdot e^{2\pi i p/N}$$

$$\sigma_0 \;=\; \sqrt[N]{1 + m_0^N}$$

vacuum equation

$$\prod_l (\,\sigma - m_l\,) \;=\; 1\,.$$

Mirror dual of CP^{N-1}

The mirror dual is the affine Toda theory

$$\mathcal{W}_{\text{mirror}}^{\text{CP}^{N-1}} \;=\; -\,\frac{1}{2\pi}\left(x_1 \,+\, x_2 \,+\, \dots \,+\, x_n \,+\, \sum m_l \,\ln x_l \right),$$

$$x_1\, x_2 \dots x_n \;=\; 1$$

Only the superpotential is known in that theory

The vacuum values coincide with $\mathcal{W}_{\text{eff}}(\sigma_p)$ upon identification

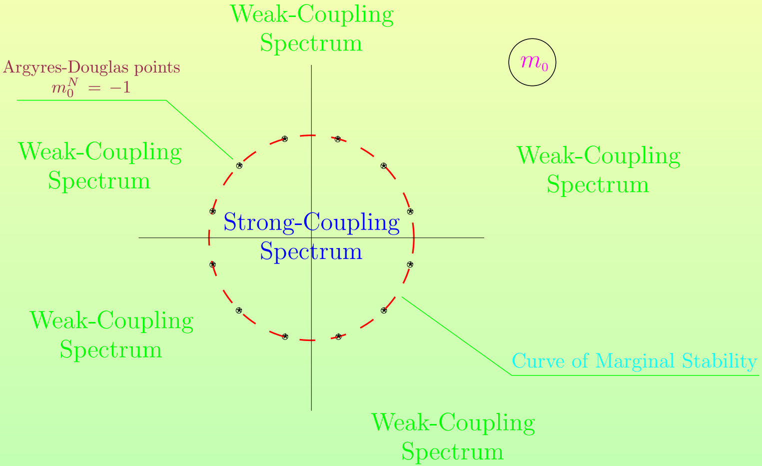
$$x_l^{(p)} \;=\; \sigma_p - m_l$$

Based on work of K.Hori and C.Vafa, N mirror kinks can be found at strong coupling

$$|m_k| \;\leq\; 1$$

Limiting to the sector interpolating between 0^{th} and 1^{st} vacua

$$\mathcal{Z} \;=\; \mathcal{W}(\sigma_1) \,-\, \mathcal{W}(\sigma_0) \,+\, i\,m_k\,, \qquad\qquad k \,=\, 0\,,...,N-1,$$


Weak-Coupling
Spectrum
m_0
Argyres-Douglas points
$m_0^N = -1$
Weak-Coupling
Spectrum
Weak-Coupling
Spectrum
Strong-Coupling
Spectrum
Weak-Coupling
Spectrum
Curve of Marginal Stability
Weak-Coupling
Spectrum

The spectrum

$$\mathcal{Z} \;=\; \mathcal{W}(\sigma_1) \;-\; \mathcal{W}(\sigma_0) \;+\; i\,m_k\,, \qquad\qquad k \,=\, 0\,,...,N-1,$$

describes N dyonic kinks (of either CP^{N-1} or the mirror theory) in the fundamental of $\mathrm{SU}(N)$

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Question:

What happens to them at weak coupling?

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Question:

What happens to them at weak coupling?

What states do they correspond to?

Previously known picture

Weak coupling:

$$\mathcal{Q}_{ik}, \quad \mathcal{M}_{ik}, \quad \mathcal{D}^n_{ik}$$

$$\mathcal{D}^n_{ik} \;+\; Q \quad \text{— bound states}$$

$\mathcal{Q}_{ik} \;=\; i\,(\,m_i - m_k\,)$

$\mathcal{M}_{ik}$ — purely topological kink interpolating from $(k) \;\to\; (i)$

$\mathcal{D}^n_{ik} \;=\; \mathcal{M}_{ik} \;+\; i\,n\,(\,m_i - m_k\,)$ — tower of dyonic kinks upon quasiclassical quantization

CP^2 theory

We focus on CP^2 model — the first non-trivial theory.

We find the curves of marginal stability (*c.m.s.*) — analogues of wall crossing — for this model, where the spectrum can change due to decays

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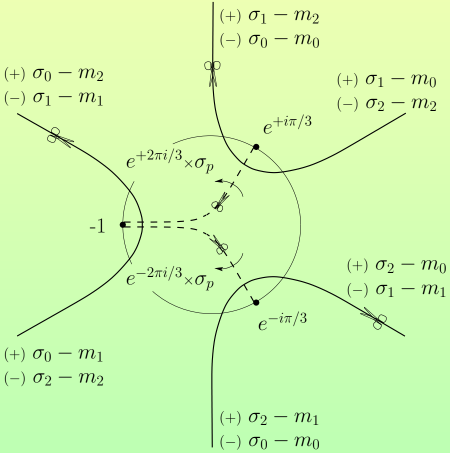
We find the curves of marginal stability (*c.m.s.*) — analogues of wall crossing — for this model, where the spectrum can change due to decays

c.m.s. are a supersymmetric version of a "phase transition"

there are practically no phase transitions in supersymmetric theories except for, perhaps, when supersymmetry is broken, or for theories with $N_c \to \infty$

Moduli space of CP^2 — plane of m_0

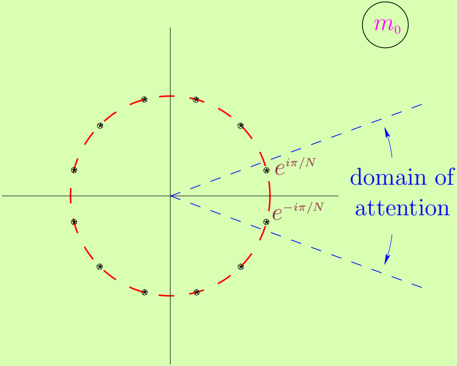


there are three $\mathcal{Z}_3$–equivalent sectors

So we choose only one topological sector:

kinks: $(0) \longrightarrow (1)$

and one sector in m_0-plane:



the other two sectors are completely equivalent

other kinks have the same masses, just central charges shifted by a phase

Weak coupling spectrum

$\mathcal{Q}_{ik}$

$\mathcal{M}_{10}$

$\mathcal{D}_{10}^{(n)}$

$\mathcal{D}_{10}^{(n)}\ \mathcal{Q}_{02}$

Weak coupling spectrum

decays $\Longleftarrow$ $\mathcal{Q}_{ik}$

$\mathcal{M}_{10}$

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Weak coupling spectrum

decays	$\Longleftarrow$	$\mathcal{Q}_{ik}$
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Weak coupling spectrum

decays	$\Longleftarrow$	$\mathcal{Q}_{ik}$
		$\mathcal{M}_{10}$
decay all but one	$\Longleftarrow$	$\mathcal{D}_{10}^{(n)}$
all decay	$\Longleftarrow$	$\mathcal{D}_{10}^{(n)}\ \mathcal{Q}_{02}$

Strong coupling spectrum

$$\mathcal{W}_1 \;-\; \mathcal{W}_0 \;+\; i\,m_0$$

$$\mathcal{W}_1 \;-\; \mathcal{W}_0 \;+\; i\,m_1$$

$$\mathcal{W}_1 \;-\; \mathcal{W}_0 \;+\; i\,m_2$$

Strong coupling spectrum

$$\mathcal{W}_1 - \mathcal{W}_0 + i\,m_0$$

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$$\mathcal{W}_1 - \mathcal{W}_0 + i\,m_2$$ — the lightest

Strong coupling spectrum

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decays $\Longleftarrow$ $\mathcal{W}_1 \; - \; \mathcal{W}_0 \; + \; i\,m_2$ — the lightest

Strong coupling spectrum

$\mathcal{W}_1 - \mathcal{W}_0 + i\,m_0$ — next lightest

$\mathcal{W}_1 - \mathcal{W}_0 + i\,m_1$

decays $\Longleftarrow$ $\mathcal{W}_1 - \mathcal{W}_0 + i\,m_2$ — the lightest

Strong coupling spectrum

$\mathcal{M}_{10} \quad \Longleftarrow \quad \mathcal{W}_1 - \mathcal{W}_0 + i\,m_0$ — next lightest

$\mathcal{W}_1 - \mathcal{W}_0 + i\,m_1$

decays $\Leftarrow= \quad \mathcal{W}_1 - \mathcal{W}_0 + i\,m_2$ — the lightest

Strong coupling spectrum

$\mathcal{M}_{10} \quad \Longleftarrow \quad \mathcal{W}_1 - \mathcal{W}_0 + i\,m_0$ — next lightest

$\mathcal{W}_1 - \mathcal{W}_0 + i\,m_1$ — the heavier

decays $\Longleftarrow \quad \mathcal{W}_1 - \mathcal{W}_0 + i\,m_2$ — the lightest

Strong coupling spectrum

$\mathcal{M}_{10} \quad \Longleftarrow \quad \mathcal{W}_1 - \mathcal{W}_0 + i\,m_0$ — next lightest

$\mathcal{D}_{10} \quad \Longleftarrow \quad \mathcal{W}_1 - \mathcal{W}_0 + i\,m_1$ — the heavier

decays $\Longleftarrow \quad \mathcal{W}_1 - \mathcal{W}_0 + i\,m_2$ — the lightest

Strong coupling spectrum

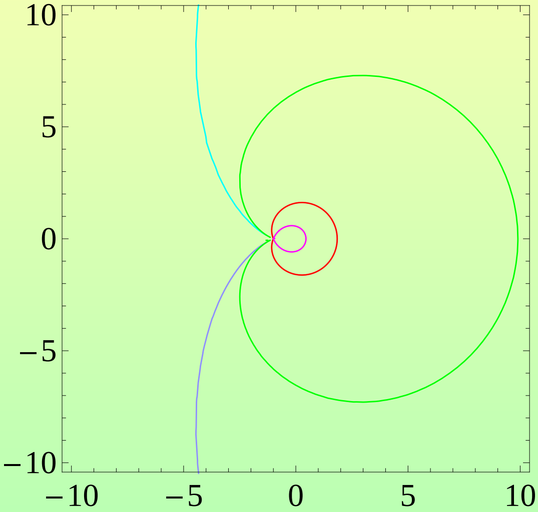
$\mathcal{M}_{10} \quad \Longleftarrow \quad \mathcal{W}_1 - \mathcal{W}_0 + i\,m_0$ — next lightest $\longleftarrow$

build their own tower

$\mathcal{D}_{10} \quad \Longleftarrow \quad \mathcal{W}_1 - \mathcal{W}_0 + i\,m_1$ — the heavier $\longleftarrow$

decays $\Longleftarrow \quad \mathcal{W}_1 - \mathcal{W}_0 + i\,m_2$ — the lightest

$c.m.s.$



Decays

Simple decays:

$$
\begin{array}{llllllll}
\mathcal{D}^{(n)}\,\mathcal{Q} & \longrightarrow & \mathcal{D}^{(n)} & + & \mathcal{Q} & & n & < & 1 \\
\mathcal{D}^{(n)}\,\mathcal{Q} & \longrightarrow & \mathcal{D}^{(n-1)} & + & \widetilde{\mathcal{Q}} & & n & > & 1
\end{array}
$$

Decays

Simple decays:

$$\mathcal{D}^{(n)}\,\mathcal{Q} \;\longrightarrow\; \mathcal{D}^{(n)} \;+\; \mathcal{Q} \qquad n < 1$$

$$\mathcal{D}^{(n)}\,\mathcal{Q} \;\longrightarrow\; \mathcal{D}^{(n-1)} \;+\; \widetilde{\mathcal{Q}} \qquad n > 1$$

Topological decays:

$$\mathcal{D}^{(1)}_{10}\,\mathcal{Q} \;\longrightarrow\; \overline{\mathcal{M}}_{02} \;+\; \overline{\mathcal{D}}^{(1)}_{21}$$

$$\mathcal{V}^{2}_{10} \;\longrightarrow\; \overline{\mathcal{D}}_{02} \;+\; \overline{\mathcal{M}}_{21}$$

Generalizing to higher N

Two states become part of the “monopole” tower

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All extra states decay

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(Indeed for even N there are *no bound states*)

Generalizing to higher N

Two states become part of the “monopole” tower

All extra states decay

(Indeed for even N there are *no bound states*)

We can infer that most likely they decay for *all N*

SQCD in four dimensions

there are also N states at strong coupling

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there are also N states at strong coupling

our results indicate that two of them are part of one tower of dyons

SQCD in four dimensions

there are also N states at strong coupling

our results indicate that two of them are part of one tower of dyons

other states decay before reaching the weak coupling

Thank you