Classification of Dimension 5 Lorentz Violating Interactions in the Standard Model

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We give a complete classification of mass dimension five Lorentz-noninvariant interactions composed from the Standard Model fields, using the effective field theory approach. We identify different classes of Lorentz violating operators, some of which are protected against transmutation to lower dimensions due to the quantum loop effects. Within each class of operators we determine a typical experimental sensitivity to the size of Lorentz violation.

I. INTRODUCTION

Lorentz symmetry is one of the most important ingredients of the Standard Model (SM) of particles and fields, as well as its extensions at the electroweak scale. Even though there is a robust evidence that Lorentz symmetry is maintained to a high degree of accuracy, the searches of the so-called "Lorentz Violation" (LV) are still well-motivated. The reason is that an a priori unknown physics at high energy scales could lead to a spontaneous breaking of Lorentz invariance by giving an expectation value to certain non-SM fields that carry Lorentz indices, such as vectors, tensors, and gradients of scalar fields. The interaction of these fields with operators composed from the SM fields, fully Lorentz-symmetric before the spontaneous breaking, below the scale of the condesation will manifest itself as effective LV terms. Schematically, one has

$$O^{\rm SM}_{\mu\nu\dots}C^{\mu\nu\dots} \to O^{\rm SM}_{\mu\nu\dots}\langle C^{\mu\nu\dots}\rangle, \tag{1}$$

where $C^{\mu\nu\cdots}$ is an external field that undergoes condensation, and $O^{\rm SM}_{\mu\nu\dots}$ is the SM operator that transforms under the Lorentz group. It would be fair to say that the dynamical breaking of Lorentz invariance is difficult to achieve in a consistent UV-complete theory, and most of the papers studying LV shy away from this issue. Nevertheless, the spurion or the effective field theory approach has been instrumental in comparing the sensitivty of various LV tests.

If the generation of interaction (1) is a true UV phenomenon, it proves extremely useful to classify all operators of lowest dimensions coupled to a given spurion field $C^{\mu\nu\cdots}$. At dimension three and four levels, this was done nearly a decade ago by Kostelecky and co-workers. Dimension three Lorentz violating interactions, *e.g.* $b^\mu\bar{\psi}i\gamma_\mu\gamma_5\psi$ and $a^\mu\bar{\psi}i\gamma_\mu\psi$, must defy the naive dimensional counting, according to which a^μ and b^μ scale linearly with the UV energy scale responsible for LV, $a^\mu, b^\mu \sim \Lambda_{UV}$. This is, of course, in a stark contradiction with reality, where typical values for *e.g.* b_μ of light quarks are known to be smaller than 10^{-31} GeV. It can be hypothesized that spurions at dimension three and four levels are in fact not fundamental, but indeed effective, implying a different type of scaling, *e.g.* $a^\mu, b^\mu \sim \Lambda^2_{\rm IR}/\Lambda_{\rm UV}$, where Λ_{IR} is the energy scale associated with the SM. The appearence of Λ_{UV} in the denominator implies that above the scale $\Lambda_{\rm IR}$ one should be able to formulate all LV interactions in terms of the *higher-dimensional* LV operators.

The purpose of this paper is to give a complete account of LV operators of dimension five in QED and in the SM. Previously, only certain sublcasses of higher-dimensional operators have been considered. Notably, dimension six LV operators and their phenomenological consequences have been studied at length in the context of the canonical non-commutative field theories (refs). The operators that introduce the UV modifications of the dispersion relations for particles have been a subject of intense investigations over a number of years (refs). Finally, the supersymmetrized versions of the dimension five LV operators have been studied in detail in (refs). Our paper includes all previously known cases of LV dimenion five interactions, but also extends LV to operators that have never been discussed in the literature.

Our intention is to write down a generic LV theory with mass dimension five interactions, compatible with the Standard Model and to classify the inherent LV operators. We count the dimension of the SM operators $O^{\rm SM}_{\mu\nu\ldots}$, and thus dimension five corresponds to all background LV spurions to be of the inverse energy scale. It is easy to see that a theory with LV interactions of mass dimension five admits a more diverse set of operators and as a consequence more LV backgrounds $C^{\mu\nu\cdots}$ than one has at dimension three and four levels. Since there is a significant freedom in the choice of LV spurions, we take an approach that each of them represents an irreducible Lorentz tensor structure, which leads to a significant facilitation of the analysis of loop effects, and often protects dimension five operators from transmuting into lower dimensions when the quantum effects are taken into account. Thus, we augment the requirements specified in [1] by demandig hat LV spurions be irreducible tensors under the Lorentz group transformations. In total, these conditions look as follows: an LV operator of specific dimension must be

- gauge invariant
- Lorentz invariant, after contraction with a background tensor
- not reducible to lower dimension operators by the equations of motion
- not reducible to a total derivative
- coupled to an irreducible background tensor.

We find that operators built in this manner can be subdivided into three main groups. The first group includes the "unprotected operators", *i.e.* those which can generate lower-dimensional interactions by developing quadratic divergencies. Such operators are therefore dangerous, and as a rule, severely constrained by strong limits on lower dimensional operators multiplied by the square of the UV scale. The second group is the UV-enhanced operators, which induce modifications of the dispersion relations that grow with particle's energy. These operators induce new testable LV signatures in the laboratory experiments and in astrophysics, and are severly constrained by both. The last group is formed by "soft LV interactions" which are protected from developing quadratic divergencies at loop level and do not significantly modify the propagation of energetic particles in the UV. Typically, such operators are constrained by the laboratory searches of spatial anisotropy.

The structure of this paper is as follows. In Section II we analyze the case of Quantum Electrodynamics (QED), extended by all possible LV interactions of dimension five. QED is is one of the most popular testing grounds for LV [1–5], and the detailed study of LV QED facilitates a smooth transition to the Standard Model. Within QED, we develop one-loop renormalization group equations for the LV interactions. Going over to the Standard Model in Section III, we observe that the chirality of matter fields imposes further restrictions on the type of admissible LV interactions. However, the abundance of field content makes possible for more diverse structures and links between them. A complete RG analysis in the Standard Model may be highly desirable for refining the phenomenological constraints on LV operators. However, due to the excessively complicated structure of interactions, we only elaborate on the example of operators which modify dispersion relations, which are of the most phenomenological interest. In Section IV, we subdivide operators into the major groups according to their phenomenological consequences and give a brief acount of typical limits one can expect from the currently available tests of Lorentz violation.

II. DIMENSION 5 OPERATORS IN QED

in order to build a Lorentz-violating extension of QED, we tak an approach similar to Myers-Pospelov electrodynamics [1]. We modify the Lagrangian of QED by adding a number of LV operators which are generated by an absolutely symmetric 3-rank irreducible tensor background. Originally, the choice of symmetric tensors was motivated by the fact that LV operators can modify the dispersion relations, and also that they do not induce dangerous quadratic divergencies. Our intention is to classify *all* dimension five operators in Quantum Electrodynamics, and thus the list of the external LV tensors will necessarily be expanded.

Generic operators will produce new non-minimal interactions between the electron and the photon. The LV extension of the photon sector of QED appears to be the most rather simple, whereas the matter sector shows a rich structure of LV terms.

A. Purely Gauge Operators in QED

Dimension five LV interactions can admit LV backgrounds up to rank five. Higher ranks can appear only in combination with operators of dimension six or higher. There are 26 numbered Young tableaux to consider, which in fact lead to only one LV operator.

It can be shown that a generic content of a gauge invariant tensor has to be bilinear in the field strength $F_{\mu\nu}$ and contain one extra derivative, which must be a covariant derivative in the case of a non-abelian field.

The only non-vanishing terms that satisfy these properties are

$$F_{\mu\nu}\partial^{\nu}\widetilde{F}^{\mu\rho}\ , \qquad F_{\mu\nu}\partial^{\nu}F^{\rho\sigma}\ , \qquad F_{\mu\lambda}\partial_{\nu}\widetilde{F}^{\rho\lambda} \qquad \text{and} \qquad F^{\mu\nu}\partial^{\lambda}F^{\rho\sigma}\ . \tag{2}$$

It can be easily seen that the first two terms are *reducible* on the equations of motion, and, in accord with our requirements should be ignored. Amongst the two structures left, $F_{\mu\lambda}\partial_{\nu}\widetilde{F}_{\rho}{}^{\lambda}$ and $F^{\mu\nu}\partial^{\lambda}F^{\rho\sigma}$, the first has been studied in [1] and shown to modify the dispersion relations of the photon. It was shown in particular that this operator has to be contracted with an irreducible absolutely symmetric tensor,

$$C^{\mu\nu\rho}\, F_{\mu\lambda}\partial_{\nu}\widetilde{F}_{\rho}{}^{\lambda}\ , \qquad C^{\mu}{}_{\mu}{}^{\rho} = 0\ . \tag{3}$$

Conditions of absolute symmetry and irreducibility of the tensor $C^{\mu\nu\rho}$ follow from the requirement of independence of this operator of the lower-rank operators of (2), which is also a way of protection against the mixing with such operators at the loop level.

The last structure in (2), the five-index object $F^{\mu\nu}\partial^{\lambda}F^{\rho\sigma}$, upon a naive substitution into the equations of motion, seems to modify the dispersion relations in a manner similar to (3). However, that would be a misleading conclusion. As in the case of the 3-rd rank operator just discussed, one needs to separate it from all lower-rank interactions. In other words, one needs to subtract all possible $g^{\mu\nu}$ and $\epsilon^{\mu\nu\rho\sigma}$ traces of this term, and *then* substitute it into the equations of motion. Upon such reduction, this operator is completely expressible in terms of its $\epsilon^{\mu\nu\rho\sigma}$-trace, which coincides with the operator $C^{\mu\nu\rho}$:

$$
\begin{aligned}
&F_{\mu\nu}\partial_\lambda F_{\rho\sigma} = \\
&-\ \frac{1}{5}\,\epsilon_{\mu\nu\rho\chi}\,\widetilde{F}^{\zeta\chi}\partial_\lambda F_{\zeta\sigma} \ +\ \frac{1}{5}\,\epsilon_{\mu\nu\sigma\chi}\,\widetilde{F}^{\zeta\chi}\partial_\lambda F_{\zeta\rho} \ +\ \frac{1}{5}\,\epsilon_{\rho\sigma\mu\chi}\,\widetilde{F}^{\zeta\chi}\partial_\lambda F_{\zeta\nu} \ -\ \frac{1}{5}\,\epsilon_{\rho\sigma\nu\chi}\,\widetilde{F}^{\zeta\chi}\partial_\lambda F_{\zeta\mu} \\
&-\ \frac{1}{10}\,\epsilon_{\mu\lambda\rho\chi}\widetilde{F}^{\zeta\chi}\partial_\nu F_{\zeta\sigma} \ +\ \frac{1}{10}\,\epsilon_{\nu\lambda\rho\chi}\widetilde{F}^{\zeta\chi}\partial_\mu F_{\zeta\sigma} \ + \\
&+\ \frac{1}{10}\,\epsilon_{\mu\lambda\sigma\chi}\widetilde{F}^{\zeta\chi}\partial_\nu F_{\zeta\rho} \ -\ \frac{1}{10}\,\epsilon_{\nu\lambda\sigma\chi}\widetilde{F}^{\zeta\chi}\partial_\mu F_{\zeta\rho}\ .
\end{aligned}
$$

This relation shows that it is not possible to bring the rank five operator to an irreducible form, and consequently there is no dimension 5 LV interaction contracted with irreducible rank five tensor. We conclude that the only possible LV operator in QED is $C^{\mu\nu\rho}$. All these arguments trivially extend to a non-abelian gauge field.

B. Matter Sector of QED

In contrast to what we have seen in the gauge sector, the LV terms in the matter sector of QED have much wider variety. The reason for that is that the operators can be formed both by using covariant derivatives $\mathcal{D}_\mu$ and by inserting gamma matrices.

In order to make the enumeration of operators more systematic, we use Young tableaux. Omitting the details, we show the result for the LV operators in the matter sector,

$$
\begin{aligned}
&\mathcal{L}^{\rm matter}_{\rm QED} = \\
&\quad [c_1^\mu\cdot\overline{\psi}\,\gamma^\lambda F_{\mu\lambda}\psi^+] \ +\ [c_2^\mu\cdot\overline{\psi}\,\gamma^\lambda\gamma^5 F_{\mu\lambda}\psi^-] \ +\ \widetilde{c}_1^\mu\cdot\overline{\psi}\,\gamma^\lambda\widetilde{F}_{\mu\lambda}\psi^+ \ +\ \widetilde{c}_2^\mu\cdot\overline{\psi}\,\gamma^\lambda\gamma^5\widetilde{F}_{\mu\lambda}\psi^- \\
&+\ f_1^{\mu\nu}\cdot\overline{\psi}\,F_{\mu\nu}\psi^- \ +\ f_2^{\mu\nu}\cdot\overline{\psi}\,F_{\mu\nu}\gamma^5\psi^- \ +\ h_1^{\mu\nu}\cdot\overline{\psi}\,\mathcal{D}_{(\mu}\mathcal{D}_{\nu)}\psi^+ \ +\ h_2^{\mu\nu}\cdot\overline{\psi}\,\mathcal{D}_{(\mu}\mathcal{D}_{\nu)}\gamma^5\psi^+ \\
&+\ C_1^{\mu\nu\rho}\cdot\overline{\psi}\,\gamma_{(\mu}\mathcal{D}_\nu\mathcal{D}_{\rho)}\psi^- \ +\ C_2^{\mu\nu\rho}\cdot\overline{\psi}\,\gamma_{(\mu}\gamma^5\mathcal{D}_\nu\mathcal{D}_{\rho)}\psi^+ \\
&+\ D_1^{\mu\nu\rho}\cdot\overline{\psi}\,\gamma_{(\mu}F_{\rho)\nu}\psi^+ \ +\ D_2^{\mu\nu\rho}\cdot\overline{\psi}\,\gamma_{(\mu}F_{\rho)\nu}\gamma^5\psi^- \\
&+\ E_1^{\mu\nu\rho\lambda}\cdot\overline{\psi}\,\sigma_{\mu)\nu}\mathcal{D}_{(\rho}\mathcal{D}_\lambda\psi^- \ +\ E_2^{\mu\nu\rho\lambda}\cdot\overline{\psi}\,\sigma_{\mu)(\lambda}F_{\rho)(\nu}\psi^+ \ +\ E_3^{\mu\nu\rho\lambda}\cdot\overline{\psi}\,\sigma_{\mu)[\nu}F_{\rho](\lambda}\psi^+ \\
&+\ E_4^{\mu\nu\rho\lambda}\cdot\overline{\psi}\,\left(\sigma_{\mu)[\nu}\mathcal{D}_{\rho]}\mathcal{D}_{(\lambda} \ -\ \sigma_{\nu](\mu}\mathcal{D}_{\lambda)}\mathcal{D}_{[\rho} \ +\ 2\,\sigma_{\nu\rho}\mathcal{D}_{(\mu}\mathcal{D}_{\lambda)}\right)\psi^-\ .
\end{aligned}
\tag{4}
$$

In this formula, + and − superscripts reflect the parity of the corresponding LV term under the charge conjugation. We stress again that all structures shown here assume their coefficients to be irreducible tensors of the corresponding rank. Square brackets over the first two operators, c_1^μ and c_2^μ, indicate that these two terms vanish upon the use of EOM, but we list them for the reason they become nontrivial in the non-abelian case.

We would like to make a side note on the symmetrizations in the interactions in (4) and in all following formulae. We take the field operators to have certain symmetries (dictated by the corresponding Young tableaux), while their Wilson coefficients to be just traceless tensors. Equivalently, one could have cast all symmetrizations onto the Wilson coefficients, *e.g.* $E_1^{\mu\nu\rho\lambda}\overline{\psi}\,\sigma_{\mu)\nu}\mathcal{D}_{(\rho}\mathcal{D}_{\lambda}\psi \;\rightarrow\; E_1^{\mu)\nu(\rho\lambda}\overline{\psi}\,\sigma_{\mu\nu}\mathcal{D}_{\rho}\mathcal{D}_{\lambda}\psi$, or just imply $E_1^{\mu\nu\rho\lambda}$ to obey the corresponding symmetries: $E_1^{\mu\nu\rho\lambda}\cdot\overline{\psi}\,\sigma_{\mu\nu}\mathcal{D}_{\rho}\mathcal{D}_{\lambda}\psi$. We emphasize that this is only a matter of notation, and choose to expose the symmetry properties of tensors via explicit symmetrizations on the Lorentz indices of the field operators.

C. 1-loop RG coefficients

If the violation of Lorentz invariance is a true UV phenomenon, one has to evaluate the operators down to the IR scale, where the majority of tests is performed. For this purpose, we study the renormalization group (RG) equations for operators (3) and (4). The RG runninng brings about the change in the magnitude of Wilson coefficients and mxing of different operators.

Due to a rather large number of LV operators, one might expect that this mixings can be rather complicated. However, the two reasons, namely the discrete symmetries and irreducibility of the Lorentz tensors, reduce this mixing to a minimum. The charge conjugation symmetry, which is an exact symmetry in QED, prevents the mxing of C-odd and C-even operators. The irreducibility of the background tensors dictates that any tensor of higher rank will not mix with a tensor of a lower rank. Thus, only operators of the same rank can admix to each other.

A brief examination of (4) reveals that $\widetilde{c}_1^{\mu}$ cannot mix with $\widetilde{c}_2^{\mu}$ due to C-parity. Similarly, $f_{1,2}^{\mu\nu}$ cannot admix to $h_{1,2}^{\mu\nu}$, but they can mix within each other. At the level of rank three tensors, the photon operator (3) is even under charge conjugation and therefore it can mix only with the $C_2^{\mu\nu\rho}$ operator.

As we have admitted generic tensor structures to the theory we need to ensure that the latter is free of quadratic divergencies. It is obvious that quadratically divergent operators must necessarily couple to a vector background, as there are no dimension three structures which would be CPT-odd and contracted with a tensor background simultaneously. In our list (4), only the $\widetilde{c}_1^{\mu}$ term generates quadratically divergent corrections to LV dimension three operators. The result of explicit computation gives the following set of RG equations:

$$
\begin{aligned}
\frac{d}{dt}\,\widetilde{c}_1^{\mu} &\overset{\text{formally}}{=} -\frac{13e^2}{96\pi^2}\,\widetilde{c}_1^{\mu} & \frac{d}{dt}\,\widetilde{c}_2^{\mu} &= \frac{e^2}{32\pi^2}\,\widetilde{c}_2^{\mu} \\
\frac{d}{dt}\,f_{1,2}^{\mu\nu} &= -\frac{7e^2}{24\pi^2}\,f_{1,2}^{\mu\nu} + i\,\frac{7e^2}{48\pi^2}\,\widetilde{f}_{2,1}^{\mu\nu} & \frac{d}{dt}\,h_{1,2}^{\mu\nu} &= \frac{e^2}{6\pi^2}\,h_{1,2}^{\mu\nu} \\
\frac{d}{dt}\,C_1^{\mu\nu\rho} &= \frac{25e^2}{48\pi^2}\,C_1^{\mu\nu\rho} & \frac{d}{dt}\,C_2^{\mu\nu\rho} &= \frac{25e^2}{48\pi^2}\,C_2^{\mu\nu\rho} - \frac{5e^2}{48\pi^2}\,C^{\mu\nu\rho} \\
& & \frac{d}{dt}\,C^{\mu\nu\rho} &= \frac{e}{48\pi^2}\,C_2^{\mu\nu\rho} - \frac{e^2}{6\pi^2}\,C^{\mu\nu\rho} \\
\frac{d}{dt}\,D_1^{\mu\nu\rho} &= -\frac{e^2}{16\pi^2}\,D_1^{\mu\nu\rho} & \frac{d}{dt}\,D_2^{\mu\nu\rho} &= \frac{5e^2}{48\pi^2}\,D_2^{\mu\nu\rho} \\
\frac{d}{dt}\,E_1^{\kappa\mu\nu\rho} &= \frac{13e^2}{24\pi^2}\,E_1^{\kappa\mu\nu\rho} & \frac{d}{dt}\,E_3^{\kappa\mu\nu\rho} &= \frac{e^2}{3\pi^2}\,E_3^{\kappa\mu\nu\rho} \\
\frac{d}{dt}\,E_2^{\kappa\mu\nu\rho} &= \frac{e^2}{12\pi^2}\,E_2^{\kappa\mu\nu\rho} + & \frac{d}{dt}\,E_4^{\kappa\mu\nu\rho} &= \frac{e^2}{8\pi^2}\,E_4^{\kappa\mu\nu\rho}\,.
\end{aligned}
$$

Here we have introduced $\widetilde{f}_{1,2}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}(f_{1,2})_{\rho\sigma}$. As anticipated, most operators renormalize independently. It is also clear that one can easily form the linear combinations of LV interactions that are eigenvectors of one-loop RG equations.

III. CLASSIFICATION OF OPERATORS OF DIMENSION V IN THE STANDARD MODEL

In the Standard Model, the set of LV operators is more complicate, due to the wider gauge group. Since the LV physics in our approach is associated with the UV scale, the LV operators must respect all the symmetries which are present at that scale. Although the UV physics and its symmetries are not known, it is quite natural to require that LV interactions be invariant under $\mathrm{SU_C}(3) \otimes \mathrm{SU_L}(2) \otimes \mathrm{U}(1)$. Clearly the existence of families causes coefficients of all LV interactions in the matter sector (4) to be matrices in the flavor space [6]. Furthermore, the presence of the Higgs sector creates new possibilities for LV interactions. However, there is one simplification arising from intrinsic chirality of SM spinors, which together with gauge invariance would essentially prohibit all $E^{\kappa\mu\nu\rho}$ operators at dimension five level. In the rest of this section, we present our resuts for LV operators in different sectors of the SM.

A. Operators in the Gauge Sector of the Standard Model

As in the QED case, the gauge sector is the simplest since we already know that the only possible LV gauge structure is (3). Thus we replicate this structure for the three gauge groups of the SM:

$$\mathcal{L}_{\rm SM}^{\rm gauge} = C^{\mu\nu\rho}_{\rm U(1)} \cdot F_{\mu\lambda}\, \partial_\nu\, \widetilde{F}_\rho{}^\lambda \;\; + \;\; C^{\mu\nu\rho}_{\rm SU_L(2)} \cdot {\rm tr}\, W_{\mu\lambda}\, \mathcal{D}_\nu\, \widetilde{W}_\rho{}^\lambda \;\; + \;\; C^{\mu\nu\rho}_{\rm SU_C(3)} \cdot {\rm tr}\, G_{\mu\lambda}\, \mathcal{D}_\nu\, \widetilde{G}_\rho{}^\lambda\ . \quad (5)$$

B. Matter Sector of the Standard Model

Although the matter sector of the Standard Model is more diverse than that of QED, the number of "types" of operators is smaller. Due to chirality of both leptons and quarks, the structures with an even number of γ-matrices in (4) are not $\mathrm{SU_L}(2)$-gauge invariant. That greatly simplifies the structure of the LV lagrangian, as one can only have operators with an odd number of gamma matrices. In the resulting Lagrangian we have to abandon the C-parity eigenstates, (4) and list operators using $V - A$ and $V + A$ combinations of Dirac matrices.

Since in QED

$$\mathcal{D}_{[\mu}\mathcal{D}_{\nu]} = ieF_{\mu\nu}\ , \quad (6)$$

the first two terms in (4) actually vanish on the equations of motion. However, with the exception of right-handed leptons, the covariant derivatives for SM field contain different gauge potentials. For example, for quarks one has

$$\mathcal{D}_{[\mu}\mathcal{D}_{\nu]} = i\,Y g'\,F_{\mu\nu} \; + \; i\,g\,W_{\mu\nu} \; + \; i\,g_3\,G_{\mu\nu}\ , \quad (7)$$

where Y is the hypercharge of the quark. The use of equatios of motino allows then to express one of the operators $\overline{Q}\,\gamma^\lambda F_{\mu\lambda} Q$, $\overline{Q}\,\gamma^\lambda W_{\mu\lambda} Q$ and $\overline{Q}\,\gamma^\lambda G_{\mu\lambda} Q$ in terms of the other two but cannot eliminate such operators completely. Taking this into account, in the quark sector one obtains the following LV interactions:

$$
\begin{aligned}
\mathcal{L}_{\rm SM}^{\rm quark} = & \\
& c^\mu_{Q,1} \cdot \overline{Q}\,\gamma^\lambda F_{\mu\lambda}\,Q \;+\; c^\mu_{Q,3} \cdot \overline{Q}\,\gamma^\lambda W_{\mu\lambda}\,Q \;+\; c^\mu_{u} \cdot \overline{u}\,\gamma^\lambda F_{\mu\lambda}\,u \;+\; c^\mu_{d} \cdot \overline{d}\,\gamma^\lambda F_{\mu\lambda}\,d \;+ \\
+\;& \widetilde{c}^\mu_{Q,1} \cdot \overline{Q}\,\gamma^\lambda \widetilde{F}_{\mu\lambda}\,Q \;+\; \widetilde{c}^\mu_{Q,2} \cdot \overline{Q}\,\gamma^\lambda \widetilde{W}_{\mu\lambda}\,Q \;+\; \widetilde{c}^\mu_{Q,3} \cdot \overline{Q}\,\gamma^\lambda \widetilde{G}_{\mu\lambda}\,Q \;+ \qquad (8) \\
+\;& \widetilde{c}^\mu_{u,1} \cdot \overline{u}\,\gamma^\lambda \widetilde{F}_{\mu\lambda}\,u \;+\; \widetilde{c}^\mu_{u,3} \cdot \overline{u}\,\gamma^\lambda \widetilde{G}_{\mu\lambda}\,u \;+\; \widetilde{c}^\mu_{d,1} \cdot \overline{d}\,\gamma^\lambda \widetilde{F}_{\mu\lambda}\,d \;+\; \widetilde{c}^\mu_{d,3} \cdot \overline{d}\,\gamma^\lambda \widetilde{G}_{\mu\lambda}\,d \;+ \\
+\;& C^{\mu\nu\rho}_{Q} \cdot \overline{Q}\,\gamma_{(\mu}\mathcal{D}_\nu\mathcal{D}_{\rho)}\,Q \;+\; C^{\mu\nu\rho}_{u} \cdot \overline{u}\,\gamma_{(\mu}\mathcal{D}_\nu\mathcal{D}_{\rho)}\,u \;+\; C^{\mu\nu\rho}_{d} \cdot \overline{d}\,\gamma_{(\mu}\mathcal{D}_\nu\mathcal{D}_{\rho)}\,d \;+ \\
+\;& D^{\mu\nu\rho}_{Q,1} \cdot \overline{Q}\,\gamma_{(\mu}F_{\rho)\nu}\,Q \;+\; D^{\mu\nu\rho}_{Q,2} \cdot \overline{Q}\,\gamma_{(\mu}W_{\rho)\nu}\,Q \;+\; D^{\mu\nu\rho}_{Q,3} \cdot \overline{Q}\,\gamma_{(\mu}G_{\rho)\nu}\,Q \;+ \\
+\;& D^{\mu\nu\rho}_{u,1} \cdot \overline{u}\,\gamma_{(\mu}F_{\rho)\nu}\,u \;+\; D^{\mu\nu\rho}_{u,3} \cdot \overline{u}\,\gamma_{(\mu}G_{\rho)\nu}\,u \;+ \\
+\;& D^{\mu\nu\rho}_{d,1} \cdot \overline{d}\,\gamma_{(\mu}F_{\rho)\nu}\,d \;+\; D^{\mu\nu\rho}_{d,3} \cdot \overline{d}\,\gamma_{(\mu}G_{\rho)\nu}\,d \;.
\end{aligned}
$$

Here all coefficients are assumed to be Hermitian matrices in the flavor space, *e.g.*

$$
c^\mu_{Q,1} \cdot \overline{Q}\,\gamma^\lambda F_{\mu\lambda}\,Q \equiv \left(c^\mu_{Q,1}\right)_{ik} \cdot \overline{Q}_i\,\gamma^\lambda F_{\mu\lambda}\,Q_k \;.
$$

Similarly, LV interactions in the lepton sector of the Standard Model take the form:

$$
\begin{aligned}
\mathcal{L}_{\rm SM}^{\rm lepton} = & \\
& c^\mu_{L} \cdot \overline{L}\,\gamma^\lambda F_{\mu\lambda}\,L \;+\; \widetilde{c}^\mu_{L,1} \cdot \overline{L}\,\gamma^\lambda \widetilde{F}_{\mu\lambda}\,L \;+\; \widetilde{c}^\mu_{L,2} \cdot \overline{L}\,\gamma^\lambda \widetilde{W}_{\mu\lambda}\,L + \\
+\;& \widetilde{c}^\mu_{\nu} \cdot \overline{\psi}_\nu\,\gamma^\lambda \widetilde{F}_{\mu\lambda}\,\psi_\nu \;+\; \widetilde{c}^\mu_{e} \cdot \overline{\psi}_e\,\gamma^\lambda \widetilde{F}_{\mu\lambda}\,\psi_e \;+ \\
+\;& C^{\mu\nu\rho}_{L} \cdot \overline{L}\,\gamma_{(\mu}\mathcal{D}_\nu\mathcal{D}_{\rho)}\,L \;+\; C^{\mu\nu\rho}_{\nu} \cdot \overline{\psi}_\nu\,\gamma_{(\mu}\mathcal{D}_\nu\mathcal{D}_{\rho)}\,\psi_\nu \;+\; C^{\mu\nu\rho}_{e} \cdot \overline{\psi}_e\,\gamma_{(\mu}\mathcal{D}_\nu\mathcal{D}_{\rho)}\,\psi_e \;+ \\
+\;& D^{\mu\nu\rho}_{L,1} \cdot \overline{L}\,\gamma_{(\mu}F_{\rho)\nu}\,L \;+\; D^{\mu\nu\rho}_{L,2} \cdot \overline{L}\,\gamma_{(\mu}W_{\rho)\nu}\,L \;+ \qquad (9) \\
+\;& D^{\mu\nu\rho}_{\nu} \cdot \overline{\psi}_\nu\,\gamma_{(\mu}F_{\rho)\nu}\,\psi_\nu \;+\; D^{\mu\nu\rho}_{e} \cdot \overline{\psi}_e\,\gamma_{(\mu}F_{\rho)\nu}\,\psi_e \;.
\end{aligned}
$$

As one can see, the absence of strong interactions for leptons makes (9) more compact compared to (8).

C. Higgs sector

The scalar sector of the SM in its minimal form contains one electroweak doublet, which also admits LV extensions. All LV operators with the use of the Higgs field can be further subdivided into two groups. The first are interactions built of the Higgs field and derivatives:

$$
\begin{aligned}
&\mathcal{L}_{\mathrm{SM}}^{\mathrm{Higgs-gauge}} = \\
&\quad l^{\mu} \cdot i\, H^{\dagger}H \cdot H^{\dagger}\mathcal{D}_{\mu}H \quad + \quad \kappa^{\mu\nu\rho} \cdot i\, H^{\dagger}\mathcal{D}_{(\mu}\mathcal{D}_{\nu}\mathcal{D}_{\rho)}H \quad + \\
&+ \quad m_1^{\mu} \cdot i\, H^{\dagger}F_{\mu\lambda}\mathcal{D}^{\lambda}H \quad + \quad m_2^{\mu} \cdot i\, H^{\dagger}W_{\mu\lambda}\mathcal{D}^{\lambda}H \quad + \quad \text{h.c.} \quad + \\
&+ \quad \widetilde{m}_1^{\mu} \cdot i\, H^{\dagger}\widetilde{F}_{\mu\lambda}\mathcal{D}^{\lambda}H \quad + \quad \widetilde{m}_2^{\mu} \cdot i\, H^{\dagger}\widetilde{W}_{\mu\lambda}\mathcal{D}^{\lambda}H \quad + \\
&+ \quad n_1^{\mu\nu\rho} \cdot i\, H^{\dagger}F_{\nu(\mu}\mathcal{D}_{\rho)}H \quad + \quad n_2^{\mu\nu\rho} \cdot i\, H^{\dagger}W_{\nu(\mu}\mathcal{D}_{\rho)}H \quad + \quad \text{h.c.}
\end{aligned}
\tag{10}
$$

The second group contains all possible LV extensions of interaction of Higgs field and fermions. This group is somewhat larger and includes higher-rank structures. The following are the operators involving quarks:

$$
\begin{aligned}
\mathcal{L}_{\mathrm{SM}}^{\mathrm{Higgs-quark}} \quad = \quad & h_{QQ}^{\mu} \cdot \overline{Q}H\,\gamma_{\mu}\,H^{\dagger}Q \quad + \\
+ \quad & p_{QQ}^{\mu} \cdot \overline{Q}\,\gamma_{\mu}Q \cdot H^{\dagger}H \quad + \quad p_{uu}^{\mu} \cdot \overline{u}\,\gamma_{\mu}u \cdot H^{\dagger}H \quad + \quad p_{dd}^{\mu} \cdot \overline{d}\,\gamma_{\mu}d \cdot H^{\dagger}H \\
+ \quad & q_{Qd}^{(1)\mu} \cdot \overline{Q}\,d\,\mathcal{D}_{\mu}H \quad + \quad q_{Qu}^{(1)\mu} \cdot \overline{Q}\,u\,\mathcal{D}_{\mu}\epsilon H^{*} \quad + \quad \text{h.c.} \\
+ \quad & q_{Qd}^{(2)\nu} \cdot \overline{Q}\,\sigma^{\mu\nu}d\,\mathcal{D}_{\nu}H \quad + \quad q_{Qu}^{(2)\nu} \cdot \overline{Q}\,\sigma^{\mu\nu}u\,\mathcal{D}_{\nu}\epsilon H^{*} \quad + \quad \text{h.c.} \\
+ \quad & r_{Qd}^{(1)\mu\nu\rho} \cdot \overline{Q}\,\mathcal{D}_{(\mu}\sigma_{\nu)\rho}d \cdot H \quad + \quad r_{Qd}^{(2)\mu\nu\rho} \cdot \overline{Q}\,\sigma_{\nu)\rho}d\,\mathcal{D}_{(\mu}H \quad + \quad \text{h.c.} \\
+ \quad & r_{Qu}^{(1)\mu\nu\rho} \cdot \overline{Q}\,\mathcal{D}_{(\mu}\sigma_{\nu)\rho}u \cdot \epsilon H^{*} \quad + \quad r_{Qu}^{(2)\mu\nu\rho} \cdot \overline{Q}\,\sigma_{\nu)\rho}u\,\mathcal{D}_{(\mu}\epsilon H^{*} \quad + \quad \text{h.c.}
\end{aligned}
\tag{11}
$$

where ϵH^{*} is the charge conjugate of the Higgs field. One also has the similar set of operators for interaction of the Higgs with leptons:

$$
\begin{aligned}
\mathcal{L}_{\mathrm{SM}}^{\mathrm{Higgs-lepton}} \quad = \quad & h_{LL}^{\mu} \cdot \overline{L}H\,\gamma_{\mu}\,H^{\dagger}L \quad + \quad p_{LL}^{\mu} \cdot \overline{L}\,\gamma_{\mu}L \cdot H^{\dagger}H \quad + \quad p_{ee}^{\mu} \cdot \overline{e}\,\gamma_{\mu}e \cdot H^{\dagger}H \\
+ \quad & q_{Le}^{(1)\mu} \cdot \overline{L}\,e\,\mathcal{D}_{\mu}H \quad + \quad q_{Le}^{(2)\nu} \cdot \overline{L}\,\sigma^{\mu\nu}e\,\mathcal{D}_{\nu}H \quad + \quad \text{h.c.} \\
+ \quad & r_{Le}^{(1)\mu\nu\rho} \cdot \overline{L}\,\mathcal{D}_{(\mu}\sigma_{\nu)\rho}e \cdot H \quad + \quad r_{Le}^{(2)\mu\nu\rho} \cdot \overline{L}\,\sigma_{\nu)\rho}e\,\mathcal{D}_{(\mu}H \quad + \quad \text{h.c.} \\
+ \quad & \varsigma^{\mu\nu} \cdot \left(H^{\dagger}L\right)^{T}\sigma_{\mu\nu}\left(H^{\dagger}L\right) \quad + \quad \text{h.c.}
\end{aligned}
\tag{12}
$$

The last term in the Higgs-lepton sector, which couples to matrix $\varsigma^{\mu\nu}$ antisymmetric in the flavor space, is unusual. It is special in that it does not have analogs neither in other sectors, nor in lower dimensions — it violates the lepton number by $\Delta L = 2$.

This completes the list of the dimension V Lorentz-violating operators in the Standard Model. The set of operators in the Standard Model appears to be much wider than that in QED due to the diversity of fields and interactions. Loop corrections are expected to intermix the operators in an even more complicated way. Clearly, there are many operators that give rise to dimension 3 LV interaction with quadratically divergent coefficients. Although, as indicated earlier, studying renormalization of interactions is beneficial for refining costraints on LV, we are not setting the goal to derive all one-loop RG equations similarly to what we have done in QED.

On the other hand, of particular interest are the rank three absolutely symmetric operators $C_X^{\mu\nu\rho}$ and $\kappa^{\mu\nu\rho}$ that modify the dispersion relations for the SM particles. For these operators, we calculate the one-loop RG equations and present the results in Appendix A.

IV. PHENOMENOLOGICAL DISCUSSION

We now discuss typical limits on LV dimension 5 operators, which can be inferred from experimental tests of Lorentz symmetry in laboratory, astrophysical observations and data on neutrino oscillations. Given the abundance of non-minimal interactions we have derived in the last section, it would be useful to separate them in several classes and deduce a typical experimental sensitivity within each class.

Many of the constraints result from laboratory experiments or astrophysical observations at energies much lower than the weak scale. The Higgs boson, W and Z bosons and heavy SM fermions do not propagate at these energies and can be integrated out. Such integration at tree level provides new operators of higher mass dimensions which we are not considering here. One should keep in mind, however, that loop effects admix LV operators with heavy particles to the light quark and lepton LV operators of the same dimension. Such (typically one-loop) corrections include the logarithmic mixing under the RG running, as well as the finite threshold corrections. Therefore, the bounds discussed below contain an intrinsic sensitivity to LV interactions involving Higgs and weak bosons.

For most phenomenological applications it is useful to rewrite the LV Lagrangian at the normalization scale of around 1 GeV, the borderline of applicability for the quark-gluon description. At this scale it is useful to abandon chiral fermions and combine the left- and right-handed fields into full Dirac spinors as well as split the $\mathrm{SU_L}(2)$ doublets.

For practical reasons, one can also pass to the mass basis of the flavor matrices, as it facilitates the decoupling of heavy quarks:

$$
\begin{aligned}
c_Q^\mu,\ c_u^\mu \quad &\to \quad c_u^\dagger \Big|_{\text{below EW}} \;=\; \frac{1}{2}\left(W_u^\dagger\, c_u^\mu\, W_u \;+\; U_u^\dagger\, c_Q^\mu\, U_u\right) \\
&\to \quad c_{u,5}^\mu\Big|_{\text{below EW}} \;=\; \frac{1}{2}\left(W_u^\mu\, c_u^\mu\, W_u \;-\; U_u^\dagger\, c_Q^\mu\, U_u\right) \qquad (13) \\
&\ldots\ldots \\
u_L \;\to\; U_u\, u_L\,, \qquad u_R \;\to\; W_u\, u_R\,, &\qquad d_L \;\to\; U_d\, d_L\,, \qquad d_R \;\to\; W_d\, d_R\,.
\end{aligned}
$$

For consistency of the effective theory, we need to ensure that the operators that we have introduced do not transmute into lower dimensions, and thereby not develop quadratic divergencies. We can formulate certain criteria to ensure that operators cannot induce lower dimensional interactions:

- *Tensor structure.* Since in the Standard Model there are no CPT-odd dimension three operators of rank higher than one, any LV structure that is coupled to an irreducible tensor (which is not a vector) is unconditionally protected from developing quadratic divergencies.

- *Supersymmetry.* In the supersymmetric Standard Model, dimension three LV operators do not exist at all. Therefore, as long as the theory is considered above the supersymmetry breaking scale, those operators which fall into supermultiplets of the LV MSSM, are protected [5]. By cancellation of loop contributions due to superpartners, the quadratic divergencies turn into logarithmic ones if supersymmetry is exact. It turns out that there is only one type of such operators

$$
\mathcal{L}_{\text{SUSY}} \;=\; \tilde{c}^\mu_{\text{SUSY,Q}} \cdot \left(Y_Q g'\, \overline{Q}\gamma^\lambda \widetilde{F}_{\mu\lambda} Q \;+\; g\, \overline{Q}\gamma^\lambda \widetilde{W}_{\mu\lambda} Q \;+\; g_3\, \overline{Q}\gamma^\lambda \widetilde{G}_{\mu\lambda} Q\right) \;+\; \ldots\,, \qquad (14)
$$

  (here, Y_Q refers to the hypercharge of the left quark doublet) which, in the case of quarks, must form a certain linear combination to be part of a supermultiplet. Linear combinations orthogonal to the above one are not supersymmetric and therefore not protected. When the supersymmetry is broken, the above operators are allowed to induce quadratic divergencies, which will be stabilized at the supersymmetry breaking scale.

- *T-invariance.* Since in the Standard Model one needs multiple loops to flip T-parity of interactions, we assume that the operators which do not have dimension three counterpartners with the same T-parity, are protected.

- *Lepton-number violation.* There are no dimension three LV operators compatible with the Standard Model which would violate the lepton number. We know that there is only one $\Delta L = 2$ operator of dimension five — $\varsigma^{\mu\nu}$, which therefore is protected against developing quadratic divergencies.

The operators for which the above criteria do not apply, have no reason to be protected, and therefore will intermix with lower-dimensional interactions. We call such operators "unprotected". Such interactions are dangerous and we will exclude them from our low energy effective theory. Using T-parity it is easy to show that the dangerous operators in the quark and lepton sectors (Eqs. (8) and (9)) are the ones coupled to the dual field strengths

$$\begin{aligned}\mathcal{L}^{\rm divgt}_{\rm SM} \;&=\; \widetilde{c}^{\mu}_{Q,1}\cdot\overline{Q}\,\gamma^{\lambda}\widetilde{F}_{\mu\lambda}\,Q \;+\; \widetilde{c}^{\mu}_{Q,3}\cdot\overline{Q}\,\gamma^{\lambda}\widetilde{G}_{\mu\lambda}\,Q \;+\; \widetilde{c}^{\mu}_{q,1}\cdot\overline{q}\,\gamma^{\lambda}\widetilde{F}_{\mu\lambda}\,q \;+\; \widetilde{c}^{\mu}_{q,3}\cdot\overline{q}\,\gamma^{\lambda}\widetilde{G}_{\mu\lambda}\,q \\ &+\; \widetilde{c}^{\mu}_{L,1}\cdot\overline{L}\,\gamma^{\lambda}\widetilde{F}_{\mu\lambda}\,L \;+\; \widetilde{c}^{\mu}_{e}\cdot\overline{e}\,\gamma^{\lambda}\widetilde{F}_{\mu\lambda}\,e \;+\; \mathcal{L}^{\rm divgt}_{\rm Higgs}\;, \end{aligned} \tag{15}$$

where we have abbreviated $q = u, d$. In the Higgs sector, the following operators in Eqs. (10)-(12) are unprotected from transmuting into lower dimensional terms:

$$\begin{aligned}\mathcal{L}^{\rm divgt}_{\rm Higgs} \;&=\; l^{\mu}\cdot i\,H^{\dagger}H\cdot H^{\dagger}\mathcal{D}_{\mu}H \;+\; \widetilde{m}^{\mu}_{1}\cdot i\,H^{\dagger}\widetilde{F}_{\mu\lambda}\mathcal{D}^{\lambda}H \;+\; \widetilde{m}^{\mu}_{2}\cdot i\,H^{\dagger}\widetilde{W}_{\mu\lambda}\mathcal{D}^{\lambda}H \;+ \\ &+\; h^{\mu}_{QQ}\cdot\overline{Q}H\,\gamma_{\mu}\,H^{\dagger}Q \;+\; h^{\mu}_{LL}\cdot\overline{L}H\,\gamma_{\mu}\,H^{\dagger}L \;+ \\ &+\; p^{\mu}_{QQ}\cdot\overline{Q}\,\gamma_{\mu}Q\cdot H^{\dagger}H \;+\; p^{\mu}_{uu}\cdot\overline{u}\,\gamma_{\mu}u\cdot H^{\dagger}H \;+\; p^{\mu}_{dd}\cdot\overline{d}\,\gamma_{\mu}d\cdot H^{\dagger}H \;+ \\ &+\; p^{\mu}_{LL}\cdot\overline{L}\,\gamma_{\mu}L\cdot H^{\dagger}H \;+\; p^{\mu}_{ee}\cdot\overline{e}\,\gamma_{\mu}e\cdot H^{\dagger}H \\ &+\; q^{(2)\nu}_{Qd}\cdot\overline{Q}\,\sigma^{\mu\nu}d\,\mathcal{D}_{\nu}H \;+\; q^{(2)\nu}_{Qu}\cdot\overline{Q}\,\sigma^{\mu\nu}u\,\mathcal{D}_{\nu}\epsilon H^{*} \;+\; q^{(2)\nu}_{Le}\cdot\overline{L}\,\sigma^{\mu\nu}e\,\mathcal{D}_{\nu}H \;+\; \text{h.c.}\end{aligned} \tag{16}$$

Using the quadratic divergence of the loop corrections generated by operators (15), one can estimate the strength of the naturalness constraints resulting from experimental limits on dimension 3 LV terms [7]:

$$|b^{\mu}| \;=\; (\text{loop factor})\,\Lambda^{2}|\widetilde{c}^{\mu}| \;\lesssim\; 10^{-29}\ \text{GeV}\;. \tag{17}$$

Even in the very conservative assumption about the UV cutoff, *e.g.* $\Lambda = \Lambda_{\rm weak}$, this limit would make futile any efforts of detecting interactions (15) directly. We again note that even though certain linear combinations (14) might be protected by supersymmetry, below the supersymmetry breaking scale they are unprotected and therefore subject to constraints (17).

Leaving ourselves only with UV-safe operators, at the scale of 1 GeV we have the following effective interactions in the quark sector:

$$
\begin{aligned}
\mathcal{L}^{\rm quark}_{\rm SM\ 1\ GeV} = \\
& c^{\mu}_{q}\cdot\overline{q}\,\gamma^{\lambda}F_{\mu\lambda}\,q \;+\; c^{\mu}_{q,5}\cdot\overline{q}\,\gamma^{\lambda}\gamma^{5}F_{\mu\lambda}\,q \;+\; C^{\mu\nu\rho}_{q}\cdot\overline{q}\,\gamma_{(\mu}\mathcal{D}_{\nu}\mathcal{D}_{\rho)}\,q \;+\; C^{\mu\nu\rho}_{q,5}\cdot\overline{q}\,\gamma_{(\mu}\mathcal{D}_{\nu}\mathcal{D}_{\rho)}\gamma^{5}\,q \\
+ & \; D^{\mu\nu\rho}_{q}\cdot\overline{q}\,\gamma_{(\mu}F_{\rho)\nu}\,q \;+\; D^{\mu\nu\rho}_{q,5}\cdot\overline{q}\,\gamma_{(\mu}F_{\rho)\nu}\gamma^{5}\,q \\
+ & \; D^{\mu\nu\rho}_{qg}\cdot\overline{q}\,\gamma_{(\mu}G_{\rho)\nu}\,q \;+\; D^{\mu\nu\rho}_{qg,5}\cdot\overline{q}\,\gamma_{(\mu}G_{\rho)\nu}\gamma^{5}\,q\;,
\end{aligned}
\tag{18}
$$

and similarly in the lepton sector:

$$
\begin{aligned}
\mathcal{L}^{\rm lepton}_{\rm SM\ 1\ GeV} \;=\;& C^{\mu\nu\rho}_{l}\cdot\overline{\psi}\,\gamma_{(\mu}\mathcal{D}_{\nu}\mathcal{D}_{\rho)}\,\psi \;+\; C^{\mu\nu\rho}_{l,5}\cdot\overline{\psi}\,\gamma_{(\mu}\mathcal{D}_{\nu}\mathcal{D}_{\rho)}\gamma^{5}\,\psi \\
&+\; D^{\mu\nu\rho}_{l}\cdot\overline{\psi}\,\gamma_{(\mu}F_{\rho)\nu}\,\psi \;+\; D^{\mu\nu\rho}_{l,5}\cdot\overline{\psi}\,\gamma_{(\mu}F_{\rho)\nu}\gamma^{5}\,\psi\;.
\end{aligned}
$$

Passing to the gauge sector, we refer to Eq. (5). At low energies the LV lagrangian in the gauge sector takes the form

$$
\mathcal{L}^{\rm gauge}_{\rm SM\ 1\ GeV} \;=\; C^{\mu\nu\rho}_{\rm EM}\cdot F_{\mu\lambda}\,\partial_{\nu}\,\widetilde{F}_{\rho}^{\ \lambda} \;+\; C^{\mu\nu\rho}_{\rm SU_C(3)}\cdot{\rm tr}\,G_{\mu\lambda}\,\mathcal{D}_{\nu}\,\widetilde{G}_{\rho}^{\ \lambda}\;. \tag{19}
$$

Here, the electromagnetic operator $C^{\mu\nu\rho}_{\rm EM}$ emerges as a linear combination of the LV tensors from the U(1) and $\mathrm{SU_L}(2)$ sectors:

$$
C^{\mu\nu\rho}_{\rm EM}\Big|_{M_W} \;=\; C^{\mu\nu\rho}_{\rm U(1)}\Big|_{M_W}\cos^{2}\theta_W \;+\; C^{\mu\nu\rho}_{\rm SU_L(2)}\Big|_{M_W}\sin^{2}\theta_W\;.
$$

In the Higgs sector, in Eqs. (10)-(12) many “protected” terms involve a space-time derivative acting on the Higgs field. Below the EW scale, where Higgs does not propagate, such terms do not contribute. One is left with the following low-energy interactions:

$$
\begin{aligned}
\mathcal{L}^{\rm Higgs-induced}_{\rm SM\ 1\ GeV} \;=\;& \frac{v}{\sqrt{2}}\,r^{\mu\nu\rho}_{q}\cdot\overline{q}\,\mathcal{D}_{(\mu}\sigma_{\nu)\rho}q \;+\; \frac{v}{\sqrt{2}}\,r^{\mu\nu\rho}_{\psi}\cdot\overline{\psi}\,\mathcal{D}_{(\mu}\sigma_{\nu)\rho}\psi \;+\; {\rm h.c.} \\
&+\; \frac{v^{2}}{2}\,\varsigma^{\mu\nu}_{\nu}\cdot\nu^{T}\sigma_{\mu\nu}\nu \;+\; {\rm h.c.}\;,
\end{aligned}
\tag{20}
$$

where $q = u, d$ and $\psi = e, \nu$. The last operator in Eq. (20) violates the lepton number by two, and in the low-energy theory can only exist for neutrinos.

Table I: Typical constraints for dimension five operators

Operators	Typical constraints	Source of constraints
Unprotected operators		
$\tilde{c}^{\mu}_{Q,1}$ $\tilde{c}^{\mu}_{Q,3}$ $\tilde{c}^{\mu}_{q,1}$ $\tilde{c}^{\mu}_{q,3}$ $\tilde{c}^{\mu}_{L,1}$ $\tilde{c}^{\mu}_{\psi}$	$\ll 10^{-31}$ GeV^{-1}	constraints on dim 3 operators
Operators growing with energy (UV-enhanced operators)		
$C^{\mu\nu\rho}_{q}$ $C^{\mu\nu\rho}_{q,5}$ $C^{\mu\nu\rho}_{l}$ $C^{\mu\nu\rho}_{l,5}$ $C^{\mu\nu\rho}_{\mathrm{EM}}$	$\lesssim 10^{-33-34}$ GeV^{-1}	high energy cosmic rays
Soft LV interactions		
$c^{\mu}_{q,5}$ $D^{\mu\nu\rho}_{q,5}$ $D^{\mu\nu\rho}_{qg}$ $D^{\mu\nu\rho}_{q}$ $r^{\mu\nu\rho}_{q}$	$\lesssim 10^{-28-30}$ GeV^{-1}	nuclear spin precession
$c^{\mu}_{q,5}$ $D^{\mu\nu\rho}_{q,5}$ $D^{\mu\nu\rho}_{qg,5}$ $c^{\mu}_{e,5}$ $D^{\mu\nu\rho}_{e,5}$	$\lesssim 10^{-25}$ $e$cm	atomic and nuclear EDMs
$\Delta L \ = \ 2$ interaction		
$\varsigma^{\mu\nu}_{\nu}$	$\lesssim 10^{-23-24}$ GeV^{-1}	data on neutrino oscillations

The interactions in (18)-(19) can be divided into two groups. The first group is formed by the operators which modify dispersion relations and grow with energy [1], which we term the *UV-enhanced operators*. The second group, correspondingly, hosts all other structures, which we designate as "soft" LV interactions (see Table IV). We now outline the main sources of constraints applicable to these groups of low-energy LV interactions.

Ultra-high energy cosmic rays. The existence of high-energy cosmic rays of energies $E_{max} \sim 10^{12}$ GeV puts stringent bounds on UV-enhanced operators in certain sectors of the Standard Model, depending on relative magnitude of LV sources in these sectors [7]. Renormalization group equations (see Appendix A) then spread these limits on the other sectors. The UV-enhanced operators modify the dispersion relations of the particles, and this would allow the nucleons in the cosmic rays to emit photons or light leptons, and therefore efficiently lose all their energy before reaching the Earth. The fact of observation of high-energy cosmic rays sets typical constraints on UV-enhanced LV of the order of 10^{-33-34} GeV. Not all "corners" of the parameter space of UV-enhanced operators are covered by these limits. Sufficiently strong LV in the up quark sector would allow protons in the cosmic rays to decay into Δ^{++} which would be stable at high energies [7].

Precision experiments. Astrophysical constraints are not applicable to soft LV interactions, since the latter do not modify propagation of particles. For this type of interactions the bounds from low-energy precision experiments are in order. The strongest limits occur when an operator induces the interaction of nuclear spin with the nuclear electric or chromomagnetic field. One finds that the operators $D_q^{\mu\nu\rho}$, $D_{qg}^{\mu\nu\rho}$ and $r_q^{\mu\nu\rho}$, when averaged over the nucleus give an effective interaction

$$\mathcal{L}_{\rm eff} \quad \propto \quad \overline{N}\partial_{(\mu}\sigma_{\nu)\rho}N \ ,$$

multiplied by a coefficient $\sim \Lambda_{\rm QCD}$ which can be estimated by a naive dimensional counting of the nucleon matrix element [8]. For a non-relativistic nucleus this induces the interaction of the nuclear spin with the external preferred directions. Known limits [9] on interaction of nuclear spin with external directions allow one to estimate typical constraints on operators

$$|D_{qg}^{0ik}|, \ \ |D_{qg,5}^{ijk}| \ \ < \ \ 10^{-30} \ {\rm GeV}^{-1}, \qquad |r_q^{0ik}| \ \ < \ \ 10^{-31} \ {\rm GeV}^{-1} \ .$$

The limits on the interactions $D_q^{\mu\nu\rho}$, $D_{q,5}^{\mu\nu\rho}$ and c_q^μ are less strong by a factor of α due to the suppression of the nuclear electric field relative to the chromomagnetic fieldstrength. In constraining the leptonic operators $D_l^{\mu\nu\rho}$ and $r_e^{\mu\nu\rho}$ one loses the advantage of using the strong internal nuclear fields, and the corresponding bounds are weakened by the ratio of the characteristic atomic energy scale to the nuclear energy scale $p_{at}/p_{nucl} \sim \alpha m_e/\Lambda_{\rm QCD}$.

Electric dipole moments. The operators $D_{q,5}^{\mu\nu\rho}$, $D_{qg,5}^{\mu\nu\rho}$ and $D_{l,5}^{\mu\nu\rho}$, written in terms of low-energy effective Hamiltonian possess the signature of Electric Dipole Moment interactions. Averaged over the nucleus, the first two induce nuclear EDMs, for which the existing limits can be used to constrain the amount of Lorentz violation [10]:

$$|c_{q,5}^0|, \ \ |D_{q,5}^{i0k}| \ \ \lesssim \ \ 10^{-12} \ {\rm GeV}^{-1} \ .$$

The electron operator $D_{l,5}^{\mu\nu\rho}$ applied to paramagnetic atoms induces the electric dipole moment of the atom, and this way a bound of the similar strength is obtained [10].

Neutrino phenomenology. The operator $\varsigma_\nu^{\mu\nu} \cdot \nu^T \sigma_{\mu\nu} \nu$ is capable of changing the patterns of neutrino oscillations. Constraints from reactor and atmospheric neutrino oscillation data can be used [11] to limit various flavor components of $\varsigma_\nu^{\mu\nu}$:

$$|\varsigma_\nu^{\mu\nu}| \ \ \lesssim \ \ 10^{-23-24} \ {\rm GeV}^{-1} \ .$$

We comment that the constraints on LV operators displayed in this section do not generically restrict all the components of corresponding LV tensors. However, a customary argument applies, that the “unobservable” parts of the tensors could induce the detectable effects due to the Lorentz boost caused by motion of the Earth relative to the Galaxy, and so cannot exceed much more the observable components.

V. CONCLUSION

We have presented a generic Lorentz-violating extension of Quantum Electrodynamics and the Standard Model with dimension five operators. It has been shown that QED admits a plenty of interactions, parametrized by background vectors and tensors. Exploration of some of the operators was done earlier [1, 7]. The gauge sector of QED appears to only possess a single LV operator $C^{\mu\nu\rho}$, which is known to generate cubic modifications of dispersion relations. Other interactions are examples of non-minimal couplings which can be searched for in experiments. Obtaining the law of RG evolution of these operators is a useful step in a phenomenological investigation. We show that the method of irreducible tensors, which itself is a systematic tool for building a theory, appears useful for these purposes too.

QED left as a toy model, the accent of this paper was mainly put on the Standard Model domain. The wider gauge group, and the diversity of field content lead to a sufficiently broader set of LV structures, although the gauge sector still keeps its simplicity. Phenomenologically important is the effective theory below the EW scale, where the spectrum of LV operators significantly reduces. In the effective theory, interactions which induce quadratic divergencies are severely constrainted, besides of just introducing a naturalness problem. Imposing a requirement of absence of uncontrollable divergencies, one obtains a consistent effective theory with a narrower set of LV terms, which can be bounded by observations. We divide the interactions of the effective low-energy theory into two groups, the first one composed of the operators which grow with the energy (UV-enhanced operators), and the second group including other, soft interactions. We show typical limits that can be placed on the magnitude of these interactions. The bounds on UV-enhanced operators are the strongest as they come from astrophysical observations. Soft interactions can only be bounded by laboratory experiments (measurements of spin precession, electric dipole moments), which still impose very strong limits on such operators. Finally, the dimension five LV Standard Model admits a lepton-number violating operator, the bounds on which can be obtained from neutrino physics. Although some operators seem to escape direct constraints, the RG flow is expected to spread the limits over all the interactions. We can hardly see it possible for some interactions to outstand in magnitude with respect to other and yet not to induce observable effects at low energy. Given the strength of typical constraints [7], we pick up the natural conclusion that Lorentz violation at dimension five level in the Standard

Model is excluded by observations.

Due to the rich field content of the Standard Model, the spectrum of LV interactions is one of the widest. At the same time, due to experimental accessibility of the former, analysis of the LV operators in the Standard Model is directly related to observational constraints. In more abstract (and thus less accessible) theories with Lorentz violation (*e.g.* in those arising from non-commutative models or from quantum gravity), if they appeal for phenomenological viability, we can expect that the LV behavior in the low energy regime will always be described by some of the operators we have built.

Appendix A: RG EQUATIONS FOR DIMENSION FIVE OPERATORS WHICH MODIFY DISPERSION RELATIONS

We list one-loop RG equations for LV operators which modify propagation of particles, *i.e.* those which couple to absolutely symmetric tensors. Althought the set of such interactions in the Standard Model is not diverse, the equations appear to be complicated. The notations for the operators are introduced in section III B. All operators bear three indices μ, ν and ρ which we omit for brevity.

In what follows, Y_X are the hypercharges of the corresponding particles; λ_X are Yukawa coupling matrices for species X; g', g and g_3 are correspondingly the U(1), $\mathrm{SU_L}(2)$ and $\mathrm{SU_C}(3)$ gauge coupling constants; we introduce

$$\begin{aligned}\alpha_1 &= \frac{5/3\,N_g \;+\; 1/8}{6\pi^2}\\ \alpha_2 &= -\frac{19 \;-\; 8N_g}{48\pi^2}\\ \alpha_3 &= -\frac{5 \;-\; 4/3\,N_g}{8\pi^2}\;,\end{aligned}$$

which are are the gauge wavefunction renormalization coefficients for the Standard Model, where $N_g = 3$ is the number of generations; we also use the following notations:

$$\begin{aligned}N_W &= \;2 \quad (\text{dim fund SU(2)}) \qquad\qquad & T_W &= \frac{N_W^2 \;-\; 1}{2N_W}\\ N_S &= \;3 \quad (\text{dim fund SU(3)}) \qquad\qquad & T_S &= \frac{N_S^2 \;-\; 1}{2N_S}\;.\end{aligned}$$

Below we present the RG equations for UV-enhanced LV operators above the EW symmetry breaking scale. The Wilson coefficients are assumed to be flavor matrices given in the gauge basis.

$$
\begin{aligned}
\frac{d}{dt}\,C_L \;&=\; \frac{25}{48\pi^2}\left(g'^2 Y_L^2 \;+\; g^2 T_W\right)\,C_L \;\;+\;\; \frac{1}{32\pi^2}\left\{\lambda_e\lambda_e^\dagger\,,\,C_L\right\} \\
&+\;\; \frac{5g'^2}{48\pi^2} Y_L^2\,C_{\mathrm{U}(1)}\cdot \mathbf{1}_{\mathrm{flavor}} \;\;+\;\; \frac{5g^2}{48\pi^2} T_W\,C_{\mathrm{SU_L}(2)}\cdot \mathbf{1}_{\mathrm{flavor}} \\
&-\;\; \frac{1}{96\pi^2}\lambda_e C_e \lambda_e^\dagger \;\;-\;\; \frac{1}{64\pi^2}\lambda_e\lambda_e^\dagger\cdot\kappa \\
\frac{d}{dt}\,C_Q \;&=\; \frac{25}{48\pi^2}\left(g'^2 Y_Q^2 \;+\; g^2 T_W \;+\; g_3^2 T_S\right)\,C_Q \;\;+\;\; \frac{1}{32\pi^2}\left\{\lambda_d\lambda_d^\dagger \;+\; \lambda_u\lambda_u^\dagger\,,\,C_Q\right\} \\
&+\;\; \frac{5g'^2}{48\pi^2} Y_Q^2\,C_{\mathrm{U}(1)}\cdot \mathbf{1}_{\mathrm{flavor}} \;\;+\;\; \frac{5g^2}{48\pi^2} T_W\,C_{\mathrm{SU_L}(2)}\cdot \mathbf{1}_{\mathrm{flavor}} \;\;+\;\; \frac{5g_3^2}{48\pi^2} T_S\,C_{\mathrm{SU_C}(3)}\cdot \mathbf{1}_{\mathrm{flavor}} \\
&-\;\; \frac{1}{96\pi^2}\left(\lambda_d C_d \lambda_d^\dagger \;+\; \lambda_u C_u \lambda_u^\dagger\right) \;\;-\;\; \frac{1}{64\pi^2}\left(\lambda_d\lambda_d^\dagger \;-\; \lambda_u\lambda_u^\dagger\right)\cdot\kappa \\
\frac{d}{dt}\,C_e \;&=\; \frac{25g'^2}{48\pi^2} Y_e^2\,C_e \;\;+\;\; \frac{1}{16\pi^2}\left\{\lambda_e^\dagger\lambda_e\,,\,C_e\right\} \\
&-\;\; \frac{5g'^2}{48\pi^2} Y_e^2\,C_{\mathrm{U}(1)}\cdot \mathbf{1}_{\mathrm{flavor}} \;\;-\;\; \frac{1}{48\pi^2}\lambda_e^\dagger C_L \lambda_e \;\;+\;\; \frac{1}{32\pi^2}\lambda_e^\dagger\lambda_e\cdot\kappa \\
\frac{d}{dt}\,C_u \;&=\; \frac{25}{48\pi^2}\left(g'^2 Y_u^2 \;+\; g_3^2 T_S\right)\,C_u \;\;+\;\; \frac{1}{16\pi^2}\left\{\lambda_u^\dagger\lambda_u\,,\,C_u\right\} \\
&-\;\; \frac{5g'^2}{48\pi^2} Y_u^2\,C_{\mathrm{U}(1)}\cdot \mathbf{1}_{\mathrm{flavor}} \;\;-\;\; \frac{5g_3^2}{48\pi^2} T_S\,C_{\mathrm{SU_C}(3)}\cdot \mathbf{1}_{\mathrm{flavor}} \\
&-\;\; \frac{1}{48\pi^2}\lambda_u^\dagger C_Q \lambda_u \;\;-\;\; \frac{1}{32\pi^2}\lambda_u^\dagger\lambda_u\cdot\kappa \\
\frac{d}{dt}\,C_d \;&=\; \frac{25}{48\pi^2}\left(g'^2 Y_d^2 \;+\; g_3^2 T_S\right)\,C_d \;\;+\;\; \frac{1}{16\pi^2}\left\{\lambda_d^\dagger\lambda_d\,,\,C_d\right\} \\
&-\;\; \frac{5g'^2}{48\pi^2} Y_d^2\,C_{\mathrm{U}(1)}\cdot \mathbf{1}_{\mathrm{flavor}} \;\;-\;\; \frac{5g_3^2}{48\pi^2} T_S\,C_{\mathrm{SU_C}(3)}\cdot \mathbf{1}_{\mathrm{flavor}} \\
&-\;\; \frac{1}{48\pi^2}\lambda_d^\dagger C_Q \lambda_d \;\;+\;\; \frac{1}{32\pi^2}\lambda_d^\dagger\lambda_d\cdot\kappa
\end{aligned}
\tag{A1}
$$

The analogous RG equations for the gauge LV operators take the form:

$$
\begin{aligned}
\frac{d}{dt}\,C_{\mathrm{U(1)}} \;&=\; -\frac{g'^2}{48\pi^2}\,\mathrm{tr}\,\Big(Y_L^2\,C_L \;+\; N_S Y_Q^2\,C_Q \;-\; Y_e^2\,C_e \;-\; N_S Y_u^2\,C_u \;-\; N_S Y_d^2\,C_d\Big) \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad +\;\; \alpha_1\, g'^2\, C_{\mathrm{U(1)}} \\
\frac{d}{dt}\,C_{\mathrm{SU_L(2)}} \;&=\; -\frac{g^2}{192\pi^2}\mathrm{tr}\,\Big(C_L \;+\; N_S\,C_Q\Big) \;\;+\;\; \Big(\alpha_2 g^2 \;+\; \frac{7}{12\pi^2}N_W g^2\Big)\cdot C_{\mathrm{SU_L(2)}} \\
\frac{d}{dt}\,C_{\mathrm{SU_C(3)}} \;&=\; -\frac{g_3^2}{192\pi^2}\mathrm{tr}\,\Big(2C_Q \;-\; C_u \;-\; C_d\Big) \;\;+\;\; \Big(\alpha_3 g_3^2 \;+\; \frac{7}{12\pi^2}N_S g_3^2\Big)\cdot C_{\mathrm{SU_C(3)}}
\end{aligned}
\tag{A2}
$$

$$
\begin{aligned}
\frac{d}{dt}\,\kappa \;&=\; \frac{5}{12\pi^2}\left[(g')^2\,Y_H^2 \;+\; g^2\,T_W\right]\cdot\kappa \\
&+\; \frac{1}{8\pi^2}\mathrm{tr}\,\Big(\lambda_e\,\lambda_e^\dagger \;+\; N_S\,\lambda_u\,\lambda_u^\dagger \;+\; N_S\,\lambda_d\,\lambda_d^\dagger\Big)\cdot\kappa \\
&-\; \frac{1}{12\pi^2}\mathrm{tr}\,\Big(N_S\,\lambda_d^\dagger\,C_Q\,\lambda_d \;+\; \lambda_e^\dagger\,C_L\,\lambda_e \;-\; N_S\,\lambda_u^\dagger\,C_Q\,\lambda_u \;- \\
&\qquad -\; N_S\,\lambda_d\,C_d\,\lambda_d^\dagger \;-\; \lambda_e\,C_e\,\lambda_e^\dagger \;+\; N_S\,\lambda_u\,C_u\,\lambda_u^\dagger\Big) \;\;.
\end{aligned}
$$

We observe that mixing of RG operators is quite noticeable between all sectors of the Standard Model.

The RG equations for UV-enhanced LV interactions below the EW scale read as:

$$
\begin{aligned}
\frac{d}{dt}\,C_\nu \;&=\; 0 \\
\frac{d}{dt}\,C_{\nu,5} \;&=\; 0 \\
\frac{d}{dt}\,C_e \;&=\; \frac{25e^2}{48\pi^2}\,C_e \\
\frac{d}{dt}\,C_{e,5} \;&=\; \frac{25e^2}{48\pi^2}\,C_{e,5} \;\;+\;\; \frac{5e^2}{48\pi^2}\,C_{\mathrm{EM}}\cdot\mathbf{1}_{\mathrm{flavor}} \\
\frac{d}{dt}\,C_u \;&=\; \frac{25}{48\pi^2}\,\big(q_u^2 e^2 \;+\; T_S g_3^2\big)\;C_u \\
\frac{d}{dt}\,C_{u,5} \;&=\; \frac{25}{48\pi^2}\,\big(q_u^2 e^2 \;+\; T_S g_3^2\big)\;C_{u,5} \;\;+\;\; \frac{5}{48\pi^2}\,\big(q_u^2 e^2\,C_{\mathrm{EM}} \;+\; g_3^2 T_S\,C_{\mathrm{SU_C(3)}}\big)\cdot\mathbf{1}_{\mathrm{flavor}} \\
\frac{d}{dt}\,C_d \;&=\; \frac{25}{48\pi^2}\,\big(q_d^2 e^2 \;+\; T_S g_3^2\big)\;C_d \\
\frac{d}{dt}\,C_{d,5} \;&=\; \frac{25}{48\pi^2}\,\big(q_d^2 e^2 \;+\; T_S g_3^2\big)\;C_{d,5} \;\;+\;\; \frac{5}{48\pi^2}\,\big(q_d^2 e^2\,C_{\mathrm{EM}} \;+\; g_3^2 T_S\,C_{\mathrm{SU_C(3)}}\big)\cdot\mathbf{1}_{\mathrm{flavor}}
\end{aligned}
\tag{A3}
$$

for matter operators, and

$$
\begin{aligned}
\frac{d}{dt}\,C_{\mathrm{EM}} \;&=\; \frac{e^2}{48\pi^2}\,\mathrm{tr}\,\Big(C_{e,5} \;+\; N_S\,q_u^2\,C_{u,5} \;+\; N_S\,q_d^2\,C_{d,5}\Big) \;\;+\;\; e^2\,\alpha_{\mathrm{EM}}\cdot C_{\mathrm{EM}} \\
\frac{d}{dt}\,C_{\mathrm{SU_C(3)}} \;&=\; \frac{g_3^2}{96\pi^2}\,\mathrm{tr}\,\Big(C_{u,5} \;+\; C_{d,5}\Big) \;\;+\;\; \Big(\alpha_3 \;+\; \frac{7}{12\pi^2}N_S\Big)\,g_3^2\cdot C_{\mathrm{SU_C(3)}}
\end{aligned}
\tag{A4}
$$

for gauge LV interactions. Here the flavor matrices of the Wilson coefficients are given in the mass basis. Below the EW scale we have done an obvious transition to

$$\frac{C_L \quad + \quad C_e}{2} \quad \equiv \quad C_e\Big|_{\rm EW} \,, \qquad \frac{C_L \quad - \quad C_e}{2} \quad \equiv \quad C_{e,5}\Big|_{\rm EW} \,, \qquad \text{etc} \;.$$

We have denoted the electric charges by q_X, and introduced

$$\alpha_{\rm EM} \quad = \quad \frac{1}{6\pi^2} \times \sum_{\rm species} q_i^2 \;,$$

which is the wavefunction renormalization coefficient for the electromagnetic field. Here the sum runs over all species existing at the given scale μ.

Although the RG mixing in Eqs. (A3), (A4) is not very considerable, we again emphasize that the operators effectively mix above the EW scale, see Eqs. (A1), (A2).

Appendix B: YOUNG TABLEAUX AND IRREDUCIBLE TENSORS OF THE LORENTZ GROUP

To build irreducible tensors of an arbitrary rank one can use the Young tableaux. We describe here a recipe how to expand a tensor of a specific rank into its irreducible components*. In the text we most extensively exploit rank three tensors, which we here now use as a non-trivial but rather simple example.

For a tensor of rank r one builds all possible numbered Young tableaux consisting of r boxes. For each tableau one builds an irreducible component by (anti)symmetrizing its indices as described below. After that, to make a component truly irreducible, one has to subtract from it all its $g^{\mu\nu}$-traces.

For each numbered diagram, one builds a tensor such that each number corresponds to an index (*e.g.* for $T^{\mu\nu\rho}$, one could identify $1 \to \mu$, $2 \to \nu$, $3 \to \rho$). Indices whose numbers form horizontal rows in the diagram are symmetrized. Indices which form vertical columns are antisymmetrized. Symmetrization always occurs with respect to the name of the index. Antisymmetrization is always done with respect to the position of the index (in this case the number not always corresponds to one and the same index).

As an illustration to what have been said, we build the diagrams for a tensor $T^{\mu\nu\rho}$. One finds four different Young diagrams which can be built out of three boxes:

$$\begin{array}{|c|c|c|}\hline 1&2&3\\\hline\end{array} \qquad \begin{array}{|c|}\hline 1\\\hline 2\\\hline 3\\\hline\end{array} \qquad \begin{array}{|c|c|}\hline 1&2\\\hline 3\\\cline{1-1}\end{array} \qquad \begin{array}{|c|c|}\hline 1&3\\\hline 2\\\cline{1-1}\end{array} \tag{B1}$$

The first diagram corresponds to an absolutely symmetric component of the tensor:

$$\begin{array}{|c|c|c|}\hline 1&2&3\\\hline\end{array} \longrightarrow S^{\mu\nu\rho} = T^{(\mu\nu\rho)} = T^{\mu\nu\rho} + T^{\nu\rho\mu} + T^{\rho\mu\nu} + T^{\mu\rho\nu} + T^{\rho\nu\mu} + T^{\nu\mu\rho} .$$

The second diagram is the absolutely antisymmetric component:

$$\begin{array}{|c|}\hline 1\\\hline 2\\\hline 3\\\hline\end{array} \longrightarrow A^{\mu\nu\rho} = T^{[\mu\nu\rho]} = T^{\mu\nu\rho} + T^{\nu\rho\mu} + T^{\rho\mu\nu} - T^{\mu\rho\nu} - T^{\rho\nu\mu} - T^{\nu\mu\rho} .$$

The two "corner" diagrams generate, correspondingly,

$$\begin{array}{|c|c|}\hline 1&2\\\hline 3\\\cline{1-1}\end{array} \longrightarrow T_1^{\mu\nu\rho} = T^{\mu\nu\rho} - T^{\rho\nu\mu} + T^{\nu\mu\rho} - T^{\rho\mu\nu} ,$$

* For more details, the reader is referred to [12].

and

$$\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & \multicolumn{1}{c}{} \\ \cline{1-1}\end{array} \quad \longrightarrow \quad T_2^{\mu\nu\rho} \;=\; T^{\mu\nu\rho} \;-\; T^{\nu\mu\rho} \;+\; T^{\rho\nu\mu} \;-\; T^{\nu\rho\mu} \;.$$

All four components of (B1) (weighed by appropriate coefficients) sum into the original tensor $T^{\mu\nu\rho}$:

$$T^{\mu\nu\rho} \;=\; \frac{1}{3!}\;\Big(S^{\mu\nu\rho} \;+\; A^{\mu\nu\rho} \;+\; 2T_1^{\mu\nu\rho} \;+ 2T_2^{\mu\nu\rho}\Big)\;\;. \tag{B2}$$

The last step to perform is subtract from each component all traces obtained by contraction of any two indices which are not antisymmetrized (contraction of antisymmetrized indices is trivial). The solution can be sought by means of a tensor of a rank less by two:

$$T_{i\,(\mathrm{irr})}^{\mu\nu\rho} \;=\; T_i^{\mu\nu\rho} \;-\; a_i^\rho g^{\mu\nu} \;+\; a_i^\mu g^{\rho\nu} \;+\; \ldots\;, \tag{B3}$$

where $T_i^{\mu\nu\rho}$ is the i-the component obtained from the corresponding Young tableau. The trace part in the r.h.s. of (B3) should possess the same symmetries as $T_i^{\mu\nu\rho}$ so as to promote these symmetries to the l.h.s. Contracting any two indices in equation (B3) and requiring the result to vanish one can obtain the explicit expression for the trace a_i^ρ. For the tensors listed in Eq. (B2) one obtains:

$$\begin{aligned}
S_{(\mathrm{irr})}^{\mu\nu\rho} \quad &= \quad S^{\mu\nu\rho} \;-\; \frac{1}{6}\,\Big(b^\mu g^{\nu\rho} \;+\; b^\nu g^{\rho\mu} \;+\; b^\rho g^{\mu\nu}\Big)\;\;, \qquad b^\mu \;=\; S^{\mu\lambda\lambda}\;, \\
A_{(\mathrm{irr})}^{\mu\nu\rho} \quad &= \quad A^{\mu\nu\rho}\;, \\
T_{1\,(\mathrm{irr})}^{\mu\nu\rho} \quad &= \quad T_1^{\mu\nu\rho} \;-\; \frac{1}{3}\,\Big(a_{(1)}^{(\mu} g^{\nu)\rho} \;-\; 2\,a_{(1)}^{\rho} g^{\mu\nu}\Big)\;\;, \qquad a_{(1)}^\mu \;=\; T^{\mu\lambda\lambda} \;-\; T^{\lambda\lambda\mu}\;, \\
T_{2\,(\mathrm{irr})}^{\mu\nu\rho} \quad &= \quad T_2^{\mu\nu\rho} \;-\; \frac{1}{3}\,\Big(a_{(2)}^{(\mu} g^{\rho)\nu} \;-\; 2\,a_{(2)}^{\nu} g^{\mu\rho}\Big)\;\;, \qquad a_{(2)}^\mu \;=\; T^{\mu\lambda\lambda} \;-\; T^{\lambda\mu\lambda}\;.
\end{aligned} \tag{B4}$$

These arguments are easily generalized to tensors of arbitrary ranks.

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