Requisites

Title: ... Lorentz Violation in SQED

Abstract

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1 Dimension 5 LV operators in SQED

Here we list Lorentz violating operators of dimension five arising in SQED. Massive SQED contains two multiplets with opposite charges which we'll call the electron and the positron multiplets, or just loosely electron and positron. We denote electron as Φ_+ (assuming charge $+e = -|e|$), and positron as Φ_-. We get one dimension 5 LV operator for electron and one for positron

$$\int d^4\theta \left\{ \frac{i}{M} n_e^\mu \, \overline{\Phi}_+ e^{2eV} \nabla^+_\mu \Phi_+ \; - \; \frac{i}{M} n_{\bar{e}}^\mu \, \Phi_- e^{-2eV} \nabla^-_\mu \overline{\Phi}_- \right\} \tag{1}$$

Both operators are parameterized by their external "preferred" directions n_e^μ and $n_{\bar{e}}^\mu$. We chose the negative sign for the positron operator so that both operators transform into similar expressions:

$$\begin{array}{ll} \dfrac{n_e^\mu}{4M} & \overline{\sigma}_\mu^{\dot{\alpha}\alpha} \, \overline{\Phi}_+ \, e^{2eV} \, \overline{D}_{\dot{\alpha}} \, e^{-2eV} D_\alpha e^{2eV} \Phi_+ \; , \\ \dfrac{n_{\bar{e}}^\mu}{4M} & \overline{\sigma}_\mu^{\dot{\alpha}\alpha} \, \overline{\Phi}_- e^{-2eV} \overline{D}_{\dot{\alpha}} \, e^{2eV} D_\alpha e^{-2eV} \Phi_- \end{array}$$

The covariant derivatives are defined as

$$\nabla^\pm_\mu = -\frac{i}{4} \bar{\sigma}_\mu^{\dot{\alpha}\alpha} \{ \nabla^\pm_\alpha \overline{\nabla}^\pm_{\dot{\alpha}} \}$$

$$\begin{array}{lll} \nabla^+_\alpha = e^{-2eV} D_\alpha e^{2eV} & , & \overline{\nabla}^+_{\dot{\alpha}} = \overline{D}_{\dot{\alpha}} \\ \nabla^-_\alpha = D_\alpha \;\; , & & \overline{\nabla}^-_{\dot{\alpha}} = e^{2eV} \overline{D}_{\dot{\alpha}} e^{-2eV} \end{array}$$

For photon (i.e. vector multiplet), we get two operators, one of which is the Kahler term

$$\int d^4\theta \, \overline{W \not{n}} W \quad , \qquad \bar{\not{n}}^{\dot{\alpha}\alpha} \equiv n^\mu \overline{\sigma}_\mu^{\dot{\alpha}\alpha} \tag{2}$$

parameterized by the vector n^μ, the other one is the superpotential kind

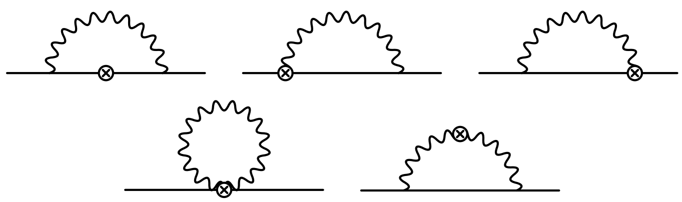
$$\int d^2\theta \, T^{\mu\nu\rho} \, W \sigma_{\nu\rho} \partial_\mu W + h.c. \tag{3}$$

parameterized by an irreducible tensor $T^{\mu\nu\rho}$, antisymmetric in $\nu\rho$. Note that such an operator cannot exist in a non-abelian theory due to W^α's being a non-invariant object, and thus requiring a covariant ∇_μ instead of ∂_μ, which would spoil the chirality property of the $d^2\theta$ integrand.

2 RG evolution of the LV operators

In this section we study how the LV operators introduced in the section 1 evolve under the RG equations. We find out which operators mix with each other and which operators run independently of each other. The following diagrams contribute to the renormalization of the electron and positron superfield operators (1):

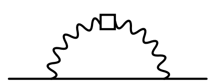
Figure 1:



where ????? is the LV vertex arising from (1) or from (2) where appropriate.

All diagrams are proportional to the square of the charge of the corresponding superfield, thus both electron and positron get equal one loop corrections.

The operator (3) gives a contribution via

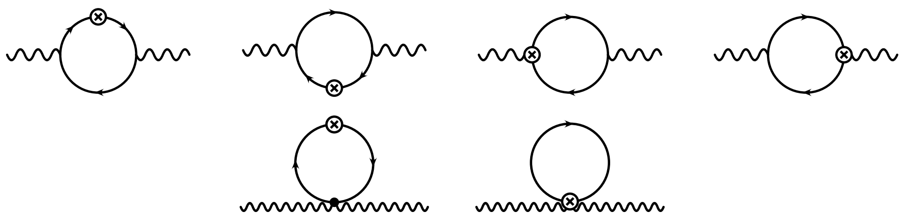


Here ????? is the insertion of the LV operator (3). The contribution of this diagram appears to vanish provided we take an irreducible tensor $T_{\mu\nu\rho}$. Thus, this operator can only run due to the wavefunction renormalization and does not mix with the other operators. *Place a few words about this operator in components and whether or not it has any implications on the propagation of photon due to its component form.* We shall not consider this operator anymore.

The operator (2) gets one loop contributions from the following diagrams:

(Note that in SQED, the gauge operator (2) does not renormalize due to itself due to the same reason why in ordinary QED the photon does not run due to itself). It appears that in the diagrams the Figs 1 and 2 all quardratic divergencies cancel. This particularly shows that Chern-Simons term does not get generated by Lorentz violation at one loop in *exact SUSY*. *Maybe*

Figure 2:



more intricate reason for that is that there is no supersymmetric form for Chern-Simons??

The renormalization group equation following from the loop diagrams on Figs. 1, 2 reads as

$$\mu\frac{\partial}{\partial\mu}\begin{pmatrix} n^\mu \\ n_e^\mu \\ n_{\bar{e}}^\mu \end{pmatrix} = \frac{e^2}{8\pi^2}\begin{bmatrix} 2 & -1 & -1 \\ -6 & 3 & 0 \\ -6 & 0 & 3 \end{bmatrix}\begin{pmatrix} n^\mu \\ n_e^\mu \\ n_{\bar{e}}^\mu \end{pmatrix} \tag{4}$$

Here we again note that the electron and the positron supermultiplet LV operators both give and receive equal contributions to/from the vector supermultiplet LV operator n^μ. We can diagonalize the matrix that emerged in (4) so as to find out which linear combinations of the LV operators (1,2) are renormalized only due to themselves. It will appear later to be useful to introduce

$$N_\pm^\mu \equiv \frac{n_e^\mu \pm n_{\bar{e}}^\mu}{2} \quad . \tag{5}$$

In terms of these quantities and of the photon operator n^μ the RG equations (4) can be rewritten as the renormgroup flow of the following operators:

$$\mu\frac{\partial}{\partial\mu}N_-^\mu = \frac{3e^2}{8\pi^2}\, N_-^\mu \tag{6}$$

$$\mu\frac{\partial}{\partial\mu}\left(2n^\mu + N_+^\mu\right) = -\frac{e^2}{8\pi^2}\left(2n^\mu + N_+^\mu\right) \tag{7}$$

$$\mu\frac{\partial}{\partial\mu}\left(3n^\mu - 2N_+^\mu\right) = \frac{3e^2}{4\pi^2}\left(3n^\mu - 2N_+^\mu\right) \quad . \tag{8}$$

We see, in particular, that N_-^μ does not mix with other operators via the renormalization group equations.

3 Induced operators of dimension 3

Dimension 3 LV operators can arise if we expand (1) and (2) in components and use the free equations of motions to solve for the derivatives of the fields to the first order in the Lorentz violation. But in fact, those dimension 3 operators are only enhanced by the mass of the quarks???. We can get a much stronger enhancement due to a logarithm at one loop. We will return to the component form of LV operators later in the section 4.

In this section, we will mostly concentrate on the dimension 3 operators arising from soft SUSY breaking mass terms added to an originally massless supersymmetric QED. We will also be interested in the issue of potential possibility of emergence of the Chern-Simons term. At the end of this section we will consider unbroken massive SQED.

Following a common method, we introduce a vertex

$$\mathcal{L}_{SB} = \int d^4\theta\, \overline{\Phi S} S \Phi \tag{9}$$

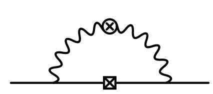
which generates a mass term for the scalar component of Φ if the spurious gauge-singlet superfield S develops a non-zero VEV for its F-component. (For this expression to be gauge-invariant one would have to introduce an extra factor of e^V. However, provided that S condenses to $\theta^2\langle F_S\rangle$, in the Wess-Zumino gauge, the only contribution of this exponent is 1).

Such an operator, combined with a dimension 5 LV operator can (and does) generate dimension 3 LV SUSY-breaking terms at one loop. We first study operators arising in the matter sector.

3.1 Operators in the matter sector

We have to consider all diagrams with 2 external chiral fields containing one LV insertion and one SUSY-breaking (SB) insertion in all possible ways. That is, we need to complement the diagrams on the Figs 1, 2 with a SUSY-breaking insertion.

Evidently, the tadpole diagram in the Fig. 1 can not be SB-complemented so as to stay 1PI. Then, the last diagram in the Fig. 1 generated by the gauge LV operator (2) can only be complemented to



which is a logarithmical diagram. Here ???? symbolizes the SUSY-breaking operator insertion. The straightforward calculation reveals the result

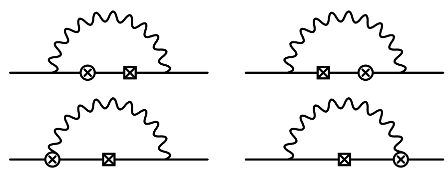
$$\frac{3ie^2}{16\pi^2}\log\Lambda/\mu\int d^4\theta\,\overline{D\Phi\eta\!\!\!/}D\Phi\cdot\overline{S}S \tag{10}$$

which in components takes the form

$$\frac{3ie^2}{8\pi^2}\log\Lambda/\mu\;|F_S|^2\cdot\overline{\psi\eta\!\!\!/}\psi\quad . \tag{11}$$

Now we pass on to the matter operators (1), the first three graphs on the Fig. 1, at the first order of SB generate

Figure 3:



In calculating them, one can use the vertex cancellation property to save some job. Notice, that because we are considering a massless theory, the electron and positron do not mix and thus it is again enough to consider only one set of diagrams of the Fig.3 and to assign corresponding charges to distinguish between electron and positron. Obviously, the graphs are proportional to e^2 and so both of them yield identical results:

$$-\frac{ie^2}{8\pi^2}\log\Lambda/\mu\int d^4\theta\,\overline{D\Phi\eta\!\!\!/_{e,\bar{e}}}D\Phi\cdot\overline{S}S \tag{12}$$

which in components gives rise to

$$-\frac{ie^2}{4\pi^2}\log\Lambda/\mu\;|F_S|^2\cdot\overline{\psi\eta\!\!\!/_{e,\bar{e}}}\psi\quad , \tag{13}$$

where $|F_S|^2$ bears the scale of mass difference of SUSY breaking.

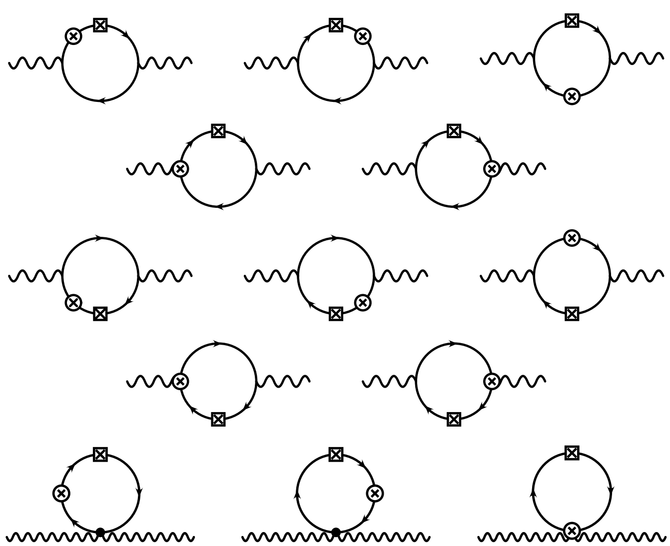
3.2 Operators in the gauge sector

Now we proceed with calculations of the dimension 3 operators generated at one loop by soft SUSY breaking. In these subsections we don't try solve the problem of RG coefficients, but rather, try to answer whether the Chern-Simons term, which is gauge-invariant up to a total derivative, can be generated by SUSY-breaking at one loop.

Partly, we can guess the answer from gauge invariance of the LV term (1). The idea is that the terms (1) are *exactly* gauge invariant, whereas any "supersymmetric" notation for Chern-Simons will be gauge invariant only up to a total derivative (just because Chern-Simons itself is such). Alternatively and more strongly, if in (1) we now declare n_e^μ a field or at least a function of space-time, it (and hence all diagrams with it) will pretty well stay gauge-invariant, whilest Chern-Simons will not any longer be such. All this hints that indeed Chern-Simons will not be generated by SUSY-breaking. However, here we justify this by a direct calculation.

All relevant SB diagrams are obtained by making all possible insertions of the SB vertex $\mathcal{L}_{SB}$ into the diagrams presented in the Fig.2. This yields the set of graphs represented on the Fig.4.

Figure 4:



The goal is to extract from each diagram all contributions containing

$$|F_S|^2 \int dx \; tr\,(\not\psi \,\bar{\not\partial}\, \not v \,\bar{\not\psi}) \quad ,$$

where v_μ is the photon, and tr means taking trace of the product of Pauli σ-matrices. Also, quite an effort can be saved if using the vertex cancellation property. A straightforward calculation shows that indeed, as it was expected, all terms of the kind (14) cancel.

We therefore conclude that in the *massless SQED with soft SUSY breaking* dimension 5 LV operators do not generate Chern-Simons at one loop.

3.3 Massive SUSY case

It is quite easy to show that the only possible operator which contains Chern-Simons term, when expressed in terms of the vector superfield V, takes the form

$$\int d^4\theta \, V \overline{D \not{n}_e} D V \cdot \bar{S} S \quad . \tag{14}$$

It will then save some job to discover only the (14)-like contributions in the diagrams in the Fig. 5.

Figure 5:

We have only illustrated diagrams containing the operator n_e^μ. However, the diagrams with $n_{\bar{e}}^\mu$ are obtained by flipping all the charges in the Fig. 5. An attentive look reveals that this won't in fact change anything, so, again, the result for $n_{\bar{e}}^\mu$ is the same as for n_e^μ (except that it's proportional to the positron background).

Like previously conjectured, all contributions of the type (14) cancel for both of the backgrounds. We conclude now that Chern-Simons is not generated at one loop in *massive SQED.*

4 Phenomenological aspects

In order to study the phenomenological consequences of introducing the operators (1) we first need to derive the component expression for them. In the case of electron, the component expression of the n_e^μ operator looks as

$$\begin{aligned}
&\frac{n_e^\mu}{M}\int d^4\theta \, \overline{\Phi} e^{2eV} \nabla_\mu \Phi = \frac{\overline{\not{n}}_e^{\dot\alpha\alpha}}{4M}\int d^4\theta \, \overline{\Phi} e^{2eV} \{\overline{\nabla}_{\dot\alpha} \nabla_\alpha\} \Phi = \\
&= \frac{n^\mu}{M}\Big[i\bar{F}\mathcal{D}_\mu F + ie\bar{z}D\mathcal{D}_\mu z - ie\mathcal{D}_\mu(\bar{z})Dz + ie\frac{\sqrt{2}}{2}\left[\overline{\psi\sigma^\mu}\lambda F - \overline{F\lambda\sigma^\mu}\psi\right] + \\
&+ \frac{1}{2}e\overline{\psi\sigma^\mu}D\psi + e^2\bar{z}\left\{\lambda\sigma^\mu\bar{\lambda} - \overline{\lambda\sigma^\mu}\lambda\right\}z \; - \\
&- \frac{\sqrt{2}}{2}e\left\{\overline{\psi\sigma^\nu}\sigma^\mu\bar{\lambda}\mathcal{D}_\nu z + \mathcal{D}_\nu(\bar{z})\lambda\sigma^\mu\bar{\sigma}^\nu\psi\right\} - \sqrt{2}e\left\{\mathcal{D}_\mu(\overline{\psi)\lambda}z + \bar{z}\lambda\mathcal{D}_\mu\psi\right\} + \\
&+ \frac{1}{2}\bar{\psi}\mathcal{D}_{(\mu}\mathcal{D}_{\nu)}\bar{\sigma}^\nu\psi - \frac{1}{4}e\bar{\psi}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}\bar{\sigma}_\nu\psi + i\bar{z}\mathcal{D}^\nu\mathcal{D}_\mu\mathcal{D}_\nu z \; + \\
&+ \frac{1}{2}ie\mathcal{D}_\nu(\bar{z})\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}z\Big] \quad ,
\end{aligned} \tag{15}$$

where

$$\mathcal{D}_\mu = \partial_\mu + i\, ev_\mu \quad .$$

The subject of the most interest are the operators which appear in the quark sector — that's where we can directly impose constraints. First, however, we need to resolve for the auxiliary fields. We also resolve the free equations of motion for the derivatives of the fields and rewrite all Weyl spinors into Dirac fermions:

$$\Psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \quad \text{and} \quad \lambda = \begin{pmatrix} \lambda \\ \overline{\lambda} \end{pmatrix} \quad . \tag{16}$$

The complete result of this is listed in the Appendix A. Here we only show the resulting operators in the quark sector:

$$\begin{aligned} \mathcal{L}^{\rm quark}_{\rm LV} &= \frac{N^\mu_+}{M}\frac{1}{4}e\,\overline{\Psi}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}\gamma_\nu\Psi + \frac{N^\mu_-}{M}\frac{1}{4}e\,\overline{\Psi}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}\gamma^\nu\gamma^5\Psi - \\ &- \frac{N^\mu_-}{M}m\overline{m}\,\overline{\Psi}\gamma_\mu\Psi \quad , \end{aligned} \tag{17}$$

which appear to depend on the combinations $N^\mu_\pm$ defined in (5).

5 Experimental limits

6 CPT-conserving dimension 6 operators

In this section we list all possible Lorentz violating operators of dimension 6. They are most easily derived in the *vector* representation with covariantly chiral superfields (see ???). We list here the operators arising in SQCD (and in SQED in particular). We get, for the D-term the following operators:

$$\begin{aligned} &\overline{\Phi}_+ e^{2eV}\,\nabla_{(\mu}\nabla_{\nu)}\,\Phi_+ \ , \quad \text{its charge conj. ,} \\ &\overline{\Phi}_+ e^{2eV}\,\nabla\sigma_{\mu\nu}W\,\Phi_+ \quad + \quad \text{h.c.,} \quad \text{its charge conj. ,} \\ &\overline{\Phi}_- e^{2eV}\,\nabla\sigma_{\mu\nu}W\,\Phi_+ \quad + \quad \text{h.c.,} \\ &Tr\ W\sigma^{\mu\nu}\nabla^2 W \quad + \quad \text{h.c.,} \\ &Tr\ \overline{W\sigma}_{(\mu}\nabla_{\nu)}W \quad . \end{aligned} \tag{18}$$

For the F-term we have fewer possibilities:

$$\begin{aligned} &\Phi_-\, W\sigma^{\mu\nu}W\,\Phi_+ \quad + \quad \text{h.c.,} \qquad \text{(SQCD only)} \\ &Tr\ \partial_\mu W\,\partial_\nu W \quad + \quad \text{h.c.} \end{aligned} \tag{19}$$

As $\sigma^{\mu\nu}$ is antisymmetric in $(\mu\nu)$, the first term in (19) vanishes in abelian theories. The operators listed in (18), (19) are evident to classify into symmetric and antisymmetric ones in $(\mu\nu)$.

From the result (18) a conclusion can be made that in SQED/SQCD, no terms like $F_{\mu\rho}F_{\nu\sigma}F^{\rho\sigma}$ or $F_{\rho\sigma}F^{\rho\sigma}F_{\mu\nu}$ can arise. It is an important statement because these terms naturally arise in NC QED and Yang-Mills *a reference here*. However, as we find, there is no way to supersymmetrize them.

The reason is akin to the fact that Seiberg-Witten map cannot be defined for NC supersymmetric gauge theories in Minkowski space. As pointed out in (*a reference here*), an originally noncommuting supersymmetric Yang-Mills can be rewritten in usual commuting component fields using the *-product. However, the first term of $\Theta_{\mu\nu}$-expansion (i.e. a dimension 6 operator) will not be supersymmetric with respect to usual commuting SUSY.

A Appendix A