Lorentz Violating Supersymmetric Quantum Electrodynamics

Pavel Bolokhov, **Stefan Groot Nibbelink**[1] and **Maxim Pospelov**[1,3,4]

[1] *Department of Physics and Astronomy, University of Victoria, Victoria, BC, V8P 1A1, Canada*

[2] *William I. Fine Theoretical Physics Institute, University of Minnesota, Minneapolis, MN 55455, USA*

[3] *Perimeter Institute, 31 Caroline Street North, Waterloo, ON, N2J 2W9, Canada*

[4] *Department of Physics, University of Guelph, Guelph, ON, N1G 2W1, Canada*

Abstract

We extend the Supersymmetric QED by the interactions with external tensor backgrounds which are assumed to be generated by some Lorentz-noninvariant dynamics at an ultraviolet scale M. Exact supersymmetry is compatible with operators of dimension five and higher, solving the naturalness problem in the Lorentz-violating sector. The Lorentz-noninvariant extension at dimension five level is studied in detail, including the renormalization group properties of these operators and their phenomenological consequences. Once the supersymmetry is broken, dimension three operators are allowed and their size is controlled by the scale of the soft breaking masses. The low-energy precision measurments set the typical constraints on the size of the dimension five operators at $10-...$ level from the inverse Planck mass. Dimension 6 LV operators in SQED/SQCD are classified.

I. INTRODUCTION

Recent years have seen an increase in number of theoretical studies of Lorentz non-invariant physics as well as intensified efforts to detect a signature of the Lorentz symmetry breakdown in terrestrial, astrophysical and cosmological settings [? ? ? ?]. The interest to theories with Lorentz violation (LV) is stimulated by several seemingly unrelated motives. Firstly, a combination of different sets of cosmological data firmly indicate that the dominant component of the energy density in the Universe is the dark energy, which can be either a cosmological constant or some energy density associated with the new infrared degree of freedom such as *i.e.* an ultra-light scalar field (quintessence). The time evolution of quintessence creates a preferred frame which could in principle be detected as a Lorentz non-symmetric background if quintessence couples to the Standard Model fields. Secondly, string theory predicts a number of massless or nearly massless moduli fields with some of them carrying open Lorentz indices. A well studied example, a non-vanishing background of the antisymmetric field $B_{\mu\nu}$ (for a review see [?]) leads to the effects that are seen at low energies as an effective violation of Lorentz symmetry. Thirdly, there has been a number of conjectures that a quantum nature of gravity at distances $1/M_{\rm Pl}$ can manifest itself at low energies through the Lorentz breaking signatures that scale as $(E/M_{\rm Pl})^n$ (See, e.g. [?] and references therein), where E is the energy in the process and $n \geq 1$. Although such conjectures are undoubtedly very speculative, if true they would provide a very powerful tool of probing ultra-short distance scales via Lorentz-violating physics. Direct experimental constraints on modifications of dispersion relations come from astrophysical processes [? ? ? ? ? ?] and terrestrial clock comparison experiments [? ? ? ?]. In both cases the typical sensitivity to the size of the coefficients in front of these operators is at the level of $10^{-5}/M_{\rm Pl}$ creating a definite problem for those theories that predict $\sim 1/M_{\rm Pl}$ effects.

In effective field theory framework the breakdown of Lorentz symmetry can be described by the presence of external tensors fixed by some unspecified dynamics and coupled to the operators of the Standard Model. It is very useful to characterize the operators by the powers of increasing dimension as it gives a first guidance to the possible scaling of the LV effects with the ultraviolet scale M. In quantum electrodynamics the generic expansion in terms of the gauge invariant operators starts at dimension three (see *e.g.* [?]),

$$\mathcal{L}^{(3)}_{\rm QED} = -a_\mu \bar{\psi}\gamma_\mu\psi - b_\mu\bar{\psi}\gamma^\mu\gamma_5\psi - \frac{1}{2}H_{\mu\nu}\bar{\psi}\sigma^{\mu\nu}\psi - k_\mu\epsilon^{\mu\nu\alpha_\beta}A_\nu\frac{\partial}{\partial x^\alpha}A_\beta, \qquad (1)$$

where ψ is the electron Dirac spinor, A_μ is the electromagnetic vector-potential, a_μ, b_μ, k_μ and $H_{\mu\nu}$ are external vector and anti-symmetric tensor backgrounds that introduce the preferred frame and therefore break Lorentz invariance. Note that a possible coupling to the vector current, $a_\mu\bar{\psi}\gamma^\mu\psi$, can be removed by introducing a space-time dependent phase for a fermion field. The last term in the Lagrangian density is gauge invariant up to a total derivative that can be neglected.

Even at this level, there is a problem in ascribing the Lorentz breaking to the dynamics at some UV scale M. If Lorentz symmetry is broken by some dynamics at the high-energy scale, then one might expect that $a_\mu,\ b_\mu, ... \sim O(M)$ and therefore LV is very large and unadmissible. For example, a Higgs mechanism allowing for a condensation of the vector field $V_\mu \sim Mn_\mu$, where n_μ is a "unit" vector [?], can create disastrous consequences in the observable sector if V_μ is coupled to a non-conserved current, *i.e.* $\bar{\psi}\gamma_\mu\gamma_5\psi$. One can hope that operators of dimension three and four are forbidden by some symmetry arguments or tuned to be small so that the LV effects first appear at dimension five level [?] or higher. However, such hopes can be shattered by quatum loop effects that can dimensionally transmute higher-dimensional operators into lower dimensional operators with a square divergence,

$$[\text{dim } 3]_\mu \sim (\text{loop factor}) \times [\text{dim } 5]_\mu \frac{\Lambda_{UV}^2}{M}. \tag{2}$$

Here $[\text{dim } 3(5)]_\mu$ represent generic vector backgrounds parametrising LV at dimension 3 and 5 levels. If the scale of the ultraviolet cutoff Λ_{UV} is on the order M, then huge dimension three operators will be generated. In that case, all higher dimensional operators would also have to be tuned, leaving no room for LV physics. This is the problem of naturalness in the LV sector. It can be avoided if the quadratic divergencies are suppressed by certain symmetry arguments. In Ref. [?] it has been shown that the dimension five LV operators coupled to the three-index symmetric irresducible tensor are protected against developing quadratic divergences inside the loops. This solves the naturalness problem only partially, as there are no arguments why dimension 3 and 4 operators cannot be induced at the tree level, which have to be tuned to experimentally acceptable values "by hand".

In a recent paper [?] it has been shown that supersymmetry provides a powerful selection rule on admissible forms of the LV interactions. In particular, it has been shown that in the minimal supersymmetric Standard Model (MSSM) the requirements of supersymmetry and gauge invariance restrict the LV operators be dimension five or higher. In the case of exact supersymmetry, it leads to a solution of the hierarchy problem in the LV sector. Once the supersymmetry is softly broken, one can expect the stabilization of the UV divergencies by the scale of the soft-breaking masses.

Explicit example of how the supersymmetry (SUSY) restricts the form of possible Lorentz-noninvariant interactions and leads to a drammatic numerical change for the predicted observables can be seen on the example of the non-commutative field theories. At a fundamental level, the noncommmutative background $\theta_{\mu\nu}$ enters via the Moyal product. It has the canonical dimension -2, and the scale of the noncommutativity $\Lambda_{NC} \sim (\theta)^{-1/2}$ gives a natural UV scale. As a result, a linearized expansion in θ is justified as long as the momenta of fields are much smaller than Λ_{NC}. This expansion leads to a series of dimension 6 operators that at the tree level induce the interaction between the spins of particles and θ-background [?], $H_{\mu\nu} \sim \Lambda_{IR}^3 \theta_{\mu\nu}$, with Λ_{IR} being the relevant infrared scale, such as Λ_{QCD} in the case of hadrons. However, it can be shown that loop effects in the non-commutative field theories lead to a quadratically divergent integrals [?], $H_{\mu\nu} \sim \Lambda_{IR}\Lambda_{UV}^2\theta_{\mu\nu}$, essentially invalidating the expansion in terms of $\theta_{\mu\nu}$. If the scale of the cutoff is very high, e.g. comparable to Λ_{NC}, then the resulting spin anisotropy is very large and certainly excluded by experiment. However, this conclusion is premature as one can argue that the operator $\bar{q}\sigma_{\mu\nu}q$ is incompatible with SUSY [?] and thus should not be induced in the domain of the loop momenta higher than the SUSY breaking. This means that the cutoff is essentially coincides with the energy splitting between fermions and bosons, $\Lambda_{UV} \sim m_{\rm soft}$. This has been confirmed by an explicit two-loop calculation in non-commutative QED [?]. With the quadratic divergence stabilized at $m_{\rm soft} \sim 1\text{TeV}$, the Planck scale non-commutativity is safely within the experimental bounds. This example illustrates that the existence of SUSY is important in understanding the actual size of the expected LV effects.

The purpose of this work is to analyze in detail the LV operators in the supersymmetric quantum electrodynamics (SQED), as a miniature version of the LV MSSM, prove the absence of the naturalness problem in the LV sector, and derive phenomenological constraints on LV parameters in SQED. Following Ref. [?], we parametrize all dimension five operators in the SQED sector by three vectors (n^μ, n_e^μ and $n_{\bar{e}}^\mu$) that enter in the LV operators composed from vector superfield (photons and photinos) and chiral superfields corresponding to left- and right-handed (s)electrons. Besides these three vectors, there is one irreducible tensor of rank three that parametrizes additional LV effects in the vector multiplet sector. We introduce these operators in the superfield formalism, and then derive their component form. We observe that upon the use of the equations of motion some parts of dimension five operators can be reduced to dimension three LV operators, and the relation between them is controlled by the electron mass m_e, $[\dim 3]_\mu \sim m_e^2[\dim 5]_\mu$.

The main emphasis of this study is on quantum effects. We derive the renormalization group evolution for the LV operators, showing explicitly that only the logarithmic divergencies arise in the limit of exact supersymmetry. We solve the one-loop renormalization equations to obtain the low-energy values of LV parameters in terms of the original values formulated at the UV scale M. We notice that the photon LV operator mixes with one specific combination of chiral operators, symmetric under the charge conjugation.

We further break supersymmetry in the chiral sector by introducing the soft-breaking mass via the spurion superfield and study the consequences for the LV operators. As expected, dimension three LV operators can now be induced, and the relation between the parameters is now given by $[\dim 3]_\mu \sim m_{soft}^2[\dim 5]_\mu$. Although a loop effect, this constitutes a dramatic enhancement over case with unbroken SUSY, as $m_{soft}^2/m_e^2 > 10^4$. The study of dimension 3 operators also raises the question of possibility of inducing a Chern-Simons term from the radiative corrections. Our analysis shows that the Chern simons term is not generated by the radiative corrections in the spontaneously broken SUSY.

We investigate phenomenological consequences of LV in the framework of softly-broken SQED. The strongest constraints on the parameters of the model come from the (non)observation of the anomalous spin precession around the direction given by the linear combination of the spatial parts of n-vectors. We utilise other constraints as well, such as comparison of the anomalous magnetic moments of electrons and positrons. It is important to note that all constraints obtained in this work are the laboratory contstraints, as the astrophysical and cosmological searches of LV are not sensitive to the effects induced by LV in SUSY QED. Other questions considered in this work include the study of a possible D-term induced by the LV operators, and classification of the next order dimension six operators in SUSY QED.

We present our results in the following order. Section 2 introduces the LV operators and backgrounds. Section 3 addresses the running of LV dimension 5 operators in the exact SUSY limit. Section 4 studies the consequences of the soft SUSY breaking for the LV sector, and derives the RG equations for the induced dimension 3 LV operators. In section

5 we study the phenomenology of the model, and obtain the predictions for the relevant LV observables. In section 6, we generalize the discussion to the next, dimension 6 level of LV operators and make their classification. We reach our conclusions in section 7.

II. DIMENSION 5 LV OPERATORS IN SQED

Supersymmetric Quantum Electrodynamics (SQED) is described by two chiral superfields Φ_+ and Φ_-, which are oppositely charged under the U(1) supersymmetry gauge group, and a gauge superfield V:

$$\begin{aligned}\mathcal{L}_{SQED} &= \int d^4\theta\, \overline{\Phi}_+ e^{2eV} \Phi_+ \;+\; \int d^4\theta\, \Phi_- e^{-2eV} \overline{\Phi}_- \;+ \\ &+ \frac{1}{16e^2}\int d^2\theta\; WW \;+\; \frac{1}{16e^2}\int d^2\overline{\theta}\;\overline{WW} \;+ \\ &+ \int \{\, d^2\theta\; m\,\Phi_-\Phi_+ \;+\; d^2\overline{\theta}\; m\,\overline{\Phi}_+\overline{\Phi}_- \,\}\;,\end{aligned} \tag{3}$$

where $W_\alpha = -\frac{1}{16e^2}\overline{D}^2 e^{-2eV} D_\alpha\, e^{2eV}$ is a gauge-invariant expression for the field strength [1]. Here,

$$\begin{aligned} d^4\theta &= d^2\theta\, d^2\overline{\theta} \\ WW &= W^\alpha W_\alpha \\ \overline{WW} &= \overline{W}_{\dot\alpha}\overline{W}^{\dot\alpha} \end{aligned}$$

The fermion components of superfields Φ_+ and Φ_- correspond to the the left-handed electron and right-handed charge-conjugated electron fields. With a slight abuse of the language, we are going to call them electron and positron superfields or just the electron and the positron for brevity. We define the charge of electron as $+e = -|e|$ for the electron.

As was shown in [**?**], the LV extension of SQED can be constructed as a series of effective operators containing the superfields Φ_-, Φ_+ and V and *arbitrary* constant tensor coefficients that specify the breakdown of the Lorentz symmetry. The general rules according to which such operators can be constructed are listed in Ref. [**?**]. Here we add an additional requirement: the supersymmetry is preserved in the presence of LV operators. **Here we need more on SUSY algebra, and the subalgebra that remains untached in our approach. I leave it to Stefan**.

Supersymmetry and supersymmetric gauge invariance impose strong restrictions on the number of generic terms of specific mass dimension one can write. In the matter sector, there is only one LV operator for each chiral superfield [2]:

$$\mathcal{L}_{\rm LV}^{\rm matter} = \int d^4\theta \left\{ \frac{i}{M} n_e^\mu \, \overline{\Phi}_+ e^{2eV} \nabla_\mu^+ \Phi_+ \;-\; \frac{i}{M} n_{\bar e}^\mu \, \Phi_- e^{-2eV} \nabla_\mu^- \overline{\Phi}_- \right\} . \tag{4}$$

The covariant derivatives are defined as

$$\nabla_\mu^\pm = -\frac{i}{4} \bar\sigma_\mu^{\dot\alpha\alpha} \{\nabla_\alpha^\pm \overline{\nabla}_{\dot\alpha}^\pm\}$$

$$\nabla_\alpha^+ = e^{-2eV} D_\alpha e^{2eV} \quad , \quad \overline{\nabla}_{\dot\alpha}^+ = \overline{D}_{\dot\alpha}$$
$$\nabla_\alpha^- = D_\alpha \quad , \quad \overline{\nabla}_{\dot\alpha}^- = e^{2eV} \overline{D}_{\dot\alpha} e^{-2eV}$$

Defined this way, the operators (4) are completely invariant under the gauge tranformation, **Pasha, put your favorite form for g.t. here**. The operators (4) are parameterized by their external "preferred" frames, n_e^μ and $n_{\bar e}^\mu$. We chose the negative sign for the positron operator so that both operators transform into similar expressions:

$$\begin{aligned} \mathcal{L}_{\rm LV}^{\rm matter} &= \frac{n_e^\mu}{4M} \overline{\sigma}_\mu^{\dot\alpha\alpha} \, \overline{\Phi}_+ \, e^{2eV} \, \overline{D}_{\dot\alpha} \, e^{-2eV} D_\alpha e^{2eV} \Phi_+ \;\; + \\ &+ \; \frac{n_{\bar e}^\mu}{4M} \overline{\sigma}_\mu^{\dot\alpha\alpha} \, \overline{\Phi}_- e^{-2eV} \overline{D}_{\dot\alpha} \, e^{2eV} D_\alpha e^{-2eV} \Phi_- . \end{aligned}$$

Here D_α and $\overline{D}_{\dot\alpha}$ act all the way to the right. Clearly, these two termss produce almost identical LV vertices with a difference only in the sign of the charge.

For the gauge supermultiplet (or the photon, for brevity), we get two possible operators. One of them is a Kähler term

$$\mathcal{L}_{\rm LV}^{\rm gauge\ (K)} = \int d^4\theta \, \overline{W} \not{n} W \quad , \qquad \bar{\not{n}}^{\dot\alpha\alpha} \equiv n^\mu \overline{\sigma}_\mu^{\dot\alpha\alpha}, \tag{5}$$

parameterized by a background vector n^μ. It is the *only* term of dimension 5 within the gauge sector which can be parameterized by a vector background. There is an additional superpotential-type term that can be added in the gauge sector,

$$\mathcal{L}_{\rm LV}^{\rm gauge\ (T)} = \int d^2\theta \, T^{\mu\nu\rho} \, W \sigma_{\nu\rho} \partial_\mu W + h.c. \tag{6}$$

It is parameterized by a rank three tensor $T^{\mu\nu\rho}$, antisymmetric in $\nu\rho$. Without the loss of generality, it suffices to take an irreducible $T^{\mu\nu\rho}$,

$$\begin{aligned} T_\mu{}^{\mu\rho} &= 0 \\ \epsilon_{\mu\nu\rho\sigma} T^{\nu\rho\sigma} &= 0 . \end{aligned}$$

Combinations $\epsilon_{\mu\nu\rho\sigma}T^{\nu\rho\sigma}$ and $T_{\mu}{}^{\mu\rho}$ correspond to vector backgrounds. It can be shown that the use of these vector backgrounds inside Eq. (6) does not create new LV interactions, as both structures can be reduced down to the form of operator (5).

Note that an operator (6) does not allow for a generalization to a non-abelian theory. In the latter case, W_α's are not gauge invariant, but are gauge covariant. Thus, to maintain gauge invariance of the operator, the partial derivative ∂_μ in (6) would have to be replaced by a covariant derivative ∇^+_μ. Then, the chirality property of the integrand is lost, and one can not write it as a superpotential term.

The set of operators (4), (5) and (6) is a complete set of Lorentz violating operators of dimension 5 in SQED. Here we present them in the component form in the Wess-Zumino gauge, using the two-spinor notations for the fermion fields. The matter operator (4) for the electron takes the form

$$\begin{aligned}
&\frac{n_e^\mu}{M}\int d^4\theta\,\overline{\Phi}_+ e^{2eV}\nabla^+_\mu\Phi_+ = \frac{\overline{\not{n}}_e^{\dot\alpha\alpha}}{4M}\int d^4\theta\,\overline{\Phi}_+ e^{2eV}\{\overline{\nabla}^+_{\dot\alpha}\nabla^+_\alpha\}\Phi_+ = \\
&= \frac{n^\mu}{M}\Bigg[\; i\bar{F}_+\mathcal{D}_\mu F_+ \;+\; ie\bar{z}_+ D\mathcal{D}_\mu z_+ \;-\; ie\mathcal{D}_\mu(\bar{z}_+)Dz_+ \;+ \\
&+\; ie\frac{\sqrt{2}}{2}\left\{\overline{\psi_+\sigma^\mu}\lambda F_+ \;-\; \overline{F_+\lambda\sigma^\mu}\psi_+\right\} \;+\; \frac{1}{2}e\overline{\psi_+\sigma^\mu}D\psi_+ \;+ \\
&+\; e^2\bar{z}_+\left\{\lambda\sigma^\mu\bar\lambda \;-\; \overline{\lambda\sigma^\mu}\lambda\right\}z_+ \;- \\
&-\; \frac{\sqrt{2}}{2}e\left\{\overline{\psi_+\sigma^\nu}\sigma^\mu\bar\lambda\mathcal{D}_\nu z_+ + \mathcal{D}_\nu(\bar{z}_+)\lambda\sigma^\mu\bar\sigma^\nu\psi_+\right\} \;- \\
&-\; \sqrt{2}e\left\{\mathcal{D}_\mu(\overline{\psi_+})\lambda z_+ \;+\; \bar{z}_+\lambda\mathcal{D}_\mu\psi_+\right\} \;+\; \frac{1}{2}\bar\psi_+\mathcal{D}_{(\mu}\mathcal{D}_{\nu)}\bar\sigma^\nu\psi_+ \;- \\
&-\; \frac{1}{4}e\bar\psi_+\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}\bar\sigma_\nu\psi_+ \;+\; i\bar{z}_+\mathcal{D}^\nu\mathcal{D}_\mu\mathcal{D}_\nu z_+ \;+\; \frac{1}{2}ie\mathcal{D}_\nu(\bar{z}_+)\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}z_+\Bigg]\;,
\end{aligned} \tag{7}$$

where the gauge covariant derivative $\mathcal{D}_\mu$ is defined in a usual way

$$\mathcal{D}_\mu = \partial_\mu + i\,ev_\mu \quad .$$

The "positron" part of (4) is obtained from (7) by changing "+" subscripts to "–", and changing sign of all components of the vector superfield V. See Appendix C for more details.

The gauge Kähler LV operator (5) in components reduces to a simple expression,

$$\begin{aligned}
\mathcal{L}_{\rm LV}^{\rm gauge\ (K)} &= \int d^4\theta\,\overline{W}\not{n}W \;= \\
&= 2\,\overline{\lambda\not{n}}\,\Box\,\lambda \;+\; 2\,\lambda\, n^\mu\partial_\mu\bar{\not\partial}\,\overline{\lambda} \;-\; 2\,D\,n_\mu\partial_\nu F^{\mu\nu} \;+\; \partial_\lambda F^{\lambda\mu}\,\widetilde{F}_{\mu\nu}n^\nu\;.
\end{aligned} \tag{8}$$

Finally, the tensor operator (6) can be expanded in the following form,

$$\begin{aligned}\mathcal{L}_{\rm LV}^{\rm gauge\ (T)} &= \int d^2\theta\, T^{\mu\nu\rho}\, W\sigma_{\nu\rho}\partial_\mu W \;+\; h.c. \;= \\ &= \left\{T^{\mu\nu\rho} \;+\; i\,\frac{1}{2}\,\epsilon^{\nu\rho\sigma\tau}T^{\mu}{}_{\sigma\tau}\right\}\times \\ &\times\left(2i\,\overline{\lambda}\,\partial_\mu\partial_\nu\overline{\sigma}_\rho\,\lambda \;+\; \partial_\mu D\,\widetilde{F}_{\nu\rho} \;-\; \frac{1}{2}\left\{F_{\sigma\nu} \;+\; i\widetilde{F}_{\sigma\nu}\right\}\partial_\mu F_\rho{}^\sigma\right) \;+\; h.c.\end{aligned} \tag{9}$$

We notice here that this operator actually depends on a "self-dual" combination

$$T^{\mu\nu\rho} \;+\; \frac{1}{2}\,i\,\epsilon^{\nu\rho\sigma\tau}T^{\mu}{}_{\sigma\tau}$$

rather than just on $T^{\mu\nu\rho}$ itself. This combination is invariant under

$$T^{\mu\nu\rho} \;\to\; \frac{1}{2}\,i\,\epsilon^{\nu\rho\sigma\tau}T^{\mu}{}_{\sigma\tau}\ .$$

This is rather natural, since the expression in paranthesis of (9) obeys the same property upon the use of identity:

$$\frac{1}{2}\,i\,\epsilon^{\mu\nu\rho\sigma}\sigma_{\rho\sigma} = \sigma^{\nu\rho}\ .$$

Many terms in the component form of LV operators can be further reduced on the equations of motion. The use of the equations of motion is justifiable in the effective operators. We list the resulting expression in Appendix ???.

As mentioned in the Introduction, we assume that operators (4), (5) and (9) are generated at the UV scale M by some unspecified LV dynamics. However, all experimental limits are obtained at much lower energy scales. Therefore, in order to derive meaningful experimental constraints on parameters of LV SQED, we have to evolve the LV operator down to the low-energy scale. Furthermore, we know that supersymmetry is broken, and the operators of dimension 5 will source the dimension 3 LV operators via the supersymmetry breaking, leading to tight bounds on LV parameters of the model.

III. RG EVOLUTION OF THE LV OPERATORS

In this section we study the renormalization group evolution of LV operators introduced in section II. To do that we use the superfield technique which allows for significant simplifications over the calculations in the component form. In this section we find and solve the renormalization group equations for dimension 5 LV operators assuming unbroken SUSY.

We work in the linear approximation in LV parameters, and neglect all terms that involve higher powers of $1/M$. Since the size of LV terms is small, all diagrams can be expanded in LV parameters, which in effect enter as corrections to either superfield propagator or the superfield vertex.

The only place where we have to consider higher orders in LV parameters is the Fayet-Illiopoulos D-term, $\int d^4\theta V$, that a priori could be induced by the LV terms. In the absence of LV interactions the cancellation of the D-term in SQED is automatic, but the presence of LV could affect this cancellation. If the result of a tadpole calculation

A FIGURE OF A TADPOLE HERE (10)

is not zero, the D-term will be generated with a divergent coefficient presumably stabilized at the scale M. This could lead to disastrous phenomenological consequences, since a non-zero VEV for D will source the mass term for the selectrons:

$$\mathcal{L}_{SQED} \supset \frac{e}{2} D \{ \bar{z}_+ z_+ - \bar{z}_- z_- \} \ . \tag{11}$$

We are able to show that the cancellation of the anomalous D-term is not modified due to LV operators included in the tadpole calculation in all orders. See Appendix B for the details.

Since the dimension 3 LV operators are prohibited by exact supersymmetry, the evolution of operators over energy scales is logarithmic. Fig. 1 shows the diagrams contributing to the renormalization of the electron/positron operators at one loop level. The crossed circle in Fig. 1 is the LV vertex arising from (4), (5) or (6) where appropriate. It is important to mention here that it is sufficient to consider the case of massless SQED. Indeed, m_e is much smaller than the characteristic momenta inside the loops. Therefore the retention of m_e can create only convergent (and rather small) contributions which should be interpreted as part of the mass threshold corrections, which we can safely neglect. As a consequence, all diagrams in this section can be calculated for Φ_+ and Φ_- separately. Moreover, since all these diagrams are proportional to the *square* of the charge of the multiplet, it is sufficient to consider only one set of diagrams keeping in mind that the second set is obtained from the first one by a trivial replacement of tags: "$+$" $\to$ "$-$".

The operator (5) gets one loop corrections from the diagrams shown in Fig. 2. Note that since we work with an Abelian theory, the self-interaction in the vector superfield

Figure 1: One-loop corrections to the chiral operators (4). Solid line denotes the chiral field propagator, wiggled line represents the gauge superfield propagator, and the crossed circle represents an insertion of the LV operators (4), (5).

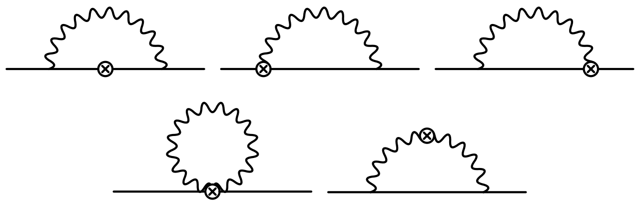
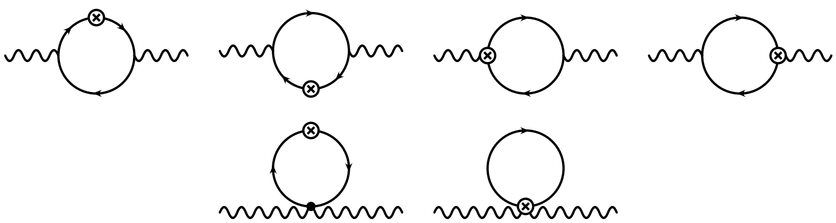
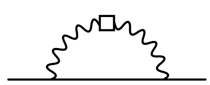


Figure 2: 1-loop corrections to the gauge LV operator $\overline{W}\not{\eta}W$.



sector does not exist, and the LV operators (5) (6) cannot generate any non-trivial running other than through interaction with matter fields. Since we work in the first order in LV, the $T^{\mu\nu\rho}$-proportional operator cannot receive any corrections from operators that depend on vector backgrounds. Moreover, the inverse is also true due to irreducible properties of $T^{\mu\nu\rho}$ background: the loop diagram (3) vanishes in the limit of exact SUSY. Thus, not

Figure 3: 1-loop corrections to the matter operators (4) due to the tensor operator (6). The box symbolizes the insertion of the operator (6).



surprisingly, operator (6) does not mix with other LV operators and runs only due to the renormalization of the wave function.

Finally, we observe that all quadratic divergencies in the diagrams of Figs. 1, 2 cancel identically. In particular, this proves that a Chern-Simons term does not get generated by Lorentz violation in massless SQED at one loop in the limit exact SUSY.

The renormalization group equation following from the loop diagrams in Figs. 1, 2 reads as:

$$\mu\frac{\partial}{\partial\mu}\begin{pmatrix} n^\mu \\ n^\mu_e \\ n^\mu_{\tilde{e}} \\ T^{\mu\nu\rho} \end{pmatrix} = \frac{e^2}{8\pi^2}\begin{pmatrix} 2 & -1 & -1 & 0 \\ -6 & 3 & 0 & 0 \\ -6 & 0 & 3 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} n^\mu \\ n^\mu_e \\ n^\mu_{\tilde{e}} \\ T^{\mu\nu\rho} \end{pmatrix} . \tag{12}$$

Here we notice again that the electron and the positron multiplet LV operators both give and receive equal contributions to/from the vector supermultiplet LV operator n^μ. The (1,1) and (4,4) elements of the matrix in (12) are equal and show the running of the corresponding coefficients due to the wave function renormalization. In particular, we could have defined the operator (6) with a coefficient $\frac{1}{e^2}T^{\mu\nu\rho}$ such that it would not run at all.

We can diagonalize the matrix of RG coefficients in Eq. (12) in order to determine the eigenvectors, *i.e.* such linear combinations of the LV operators (4–5) that are renormalized only due to themselves. At this point it is useful to introduce the combinations of LV parameters in the matter sector that have definite charge conjugation properties.

$$N^\mu_\pm \equiv \frac{n^\mu_e \pm n^\mu_{\tilde{e}}}{2} . \tag{13}$$

In terms of these quantities and of the photon operator n^μ, RG equations (12) can be written as an independent renormalization group flow of the following parameters:

$$\begin{aligned} \mu\frac{\partial}{\partial\mu}N^\mu_- &= \frac{3e^2}{8\pi^2}\,N^\mu_- \\ \mu\frac{\partial}{\partial\mu}\left(2n^\mu + N^\mu_+\right) &= -\frac{e^2}{8\pi^2}\left(2n^\mu + N^\mu_+\right) \\ \mu\frac{\partial}{\partial\mu}\left(3n^\mu - 2N^\mu_+\right) &= \frac{3e^2}{4\pi^2}\left(3n^\mu - 2N^\mu_+\right) \\ \mu\frac{\partial}{\partial\mu}T^{\mu\nu\rho} &= \frac{e^2}{4\pi^2}\,T^{\mu\nu\rho} . \end{aligned} \tag{14}$$

We can see that besides $T^{\mu\nu\rho}$, the operator N^μ_- also runs independently. This is because the operator coupled to N^μ_- is odd under the charge conjugation while n^μ and N^μ_+ are even, and the electromagnetic interactions preserve this discrete symmetry. Now we express the values of the LV parameters at the low-energy scale in terms of those at the UV scale M. For the low-energy scale we take the scale of the superpartner masses because below this scale the supersymmetry is broken completely,

$$n^\mu\Big|_{m_{soft}} = \frac{4\zeta_2 + 3\zeta_3}{7}\,n^\mu\Big|_M + \frac{2}{7}\left(\zeta_2 - \zeta_3\right)\,N^\mu_+\Big|_M$$

$$
\begin{aligned}
N_+^\mu\Big|_{m_{soft}} &= \frac{6}{7}\,(\zeta_2-\zeta_3)\;n^\mu\Big|_M \;\;+\;\; \frac{3\zeta_2+4\zeta_3}{7}\,N_+^\mu\Big|_M \\
N_-^\mu\Big|_{m_{soft}} &= \zeta_1\,N_-^\mu\Big|_M \\
T^{\mu\nu\rho}\Big|_{m_{soft}} &= \zeta_4\,T^{\mu\nu\rho}\Big|_M \,,
\end{aligned}
\tag{15}
$$

where

$$
\begin{aligned}
\zeta_i \;&=\; \left(\frac{\alpha_{\rm SQED}(m_s)}{\alpha_{\rm SQED}(M)}\right)^{\frac{\alpha_i}{2\beta_0}} \;=\; \left(\frac{1\;-\;2\,\beta_0\,e_0^2\log m_s/\mu_0}{1\;-\;2\,\beta_0\,e_0^2\log M/\mu_0}\right)^{-\frac{\alpha_i}{2\beta_0}} \\
(\alpha_1,\;\alpha_2,\;\alpha_3,\;\alpha_4) \;&=\; \left(\frac{3}{8\pi^2},\;-\frac{1}{8\pi^2},\;\frac{6}{8\pi^2},\;\frac{2}{8\pi^2}\right)\;,
\end{aligned}
$$

and β_0 is the 1-loop coefficient of the SQED β-function:

$$
\beta_0^{\rm SQED} = \frac{1}{8\pi^2}\;.
$$

Should one want to extend the evolution of these operators below the soft breaking scale m_{soft}, one must use the RG equations of ordinary, non-supersymmetric QED.

IV. INDUCED OPERATORS OF DIMENSION 3

Once the SUSY is broken, dimension 3 LV operators can be induced with coefficients controlled by the soft-breaking mass scale. Following a usual approach [*a reference here*], we introduce a spurion singlet chiral superfield S which interacts with the SQED multiplets and provides masses for the scalar particles in the matter sector. We could consider other soft-breaking terms including a gaugino mass, but in this paper we restrict ourselves to the limit $m_{selectron} \gg m_{gaugino}$ which often holds in benchmark MSSM scenarios [**?**]. Generically, we can assume that parity is broken so that the selectron and spositron have different masses. To do this we introduce two different spurion superfields S_+ and S_- for the selectron and spositron correspondingly. The spurions give masses to the electron and positron via the interaction

$$
\mathcal{L}_{SB} = -\frac{1}{M}\int d^4\theta\;\left[\overline{S}_+S_+\overline{\Phi}_+\Phi_+\;\;+\;\overline{S}_-S_-\overline{\Phi}_-\Phi_-\right]\;. \tag{16}
$$

Some hidden dynamical mechanisms are assumed to be responsible for $S_\pm$ developing a nonzero VEVs for their F-components:

$$
\left|\langle F_S^\pm\rangle\right| = M^2 {m_{soft}^\pm}^2\;, \qquad \langle\phi_S^\pm\rangle = \langle\psi_S^\pm\rangle = 0\;.
$$

Note that for expression (16) to be gauge-invariant one would have to introduce extra factors of e^{2eV}. However, provided that $S_\pm$ condenses to $\theta^2\langle F_S^\pm\rangle$, in the Wess-Zumino gauge we can take $e^{2eV}=1$. We assume ${m^+_{soft}}^2 \approx {m^-_{soft}}^2 \equiv m_s^2$. The operator (16), combined with a dimension 5 LV operator generate dimension 3 LV terms in the fermion matter sector at one loop level or in selectron sector upon the use of the equations of motion. Presence of such terms must be carefully investigated, because if they can easily dominate over dimension 5 operators in observable effects.

We start with some general remarks about dimension LV 3 operators that one can expect to appear. We can easily list all such operators in the component form. In the matter sector these are

$$\begin{aligned}
& i\,\widetilde{A}^\mu_\pm\,\overline{z}_\pm\mathcal{D}_\mu z_\pm \\
& \widetilde{B}^\mu_\pm\,\overline{\psi_\pm\sigma}_\mu\psi_\pm \\
& i\,\widetilde{C}^\mu\,z_-\mathcal{D}_\mu z_+ \\
& \widetilde{D}^{\mu\nu}\,\psi_-\sigma_{\mu\nu}\psi_+\ .
\end{aligned}\tag{17}$$

In superfield notation they can be re-written as:

$$\begin{aligned}
i\,d^4\theta\;\theta^4\,\widetilde{A}^\mu_+\,\overline{\Phi}_+\nabla^+_\mu\Phi_+ \quad - \quad & i\,d^4\theta\;\theta^4\,\widetilde{A}^\mu_-\,\Phi_-\nabla^-_\mu\overline{\Phi}_- \\
\frac{1}{2}\,d^4\theta\;\theta^4\,\widetilde{B}^\mu_\pm\,&\overline{\nabla\Phi_\pm\sigma_\mu}\nabla\Phi_\pm \\
d^4\theta\;\theta^4\,\widetilde{C}^\mu\,&\Phi_-\nabla^+_\mu\Phi_+ \\
d^4\theta\;\theta^4\,\widetilde{D}^{\mu\nu}\,&\nabla\Phi_-\sigma_{\mu\nu}\nabla\Phi_+\ ,
\end{aligned}\tag{18}$$

where

$$\theta^4=\theta^2\bar{\theta}^2\ .$$

It is quite obvious that the operators $\widetilde{C}^\mu$ and $\widetilde{D}^{\mu\nu}$ in (17) cannot be generated by inclusion of supersymmetry breaking (16). It turns out that at tree level there are no suitable dimension 5 operators that would generate them on the equations of motion (see Appendix C).It can be shown that an operator of at least dimension 6 is required in order to produce the $\widetilde{C}^\mu$ and $\widetilde{D}^{\mu\nu}$ dimension 3 operators.

Alternatively, we can obtain all SUSY breaking LV operators in the superfield form by inserting θ^2, $\bar{\theta}^2$, $\theta_\alpha\bar{\theta}_{\dot{\alpha}}$, … inside gauge-invariant supersymmetric LV operators. In particular, one way of doing this [?] is to introduce a spurion vector superfield

$$\widetilde{V} = -\, \widetilde{v}^{\mu} \cdot \theta\sigma_{\mu}\bar{\theta} \ ,$$

which is effectively an insertion of $\theta_\alpha\bar{\theta}_{\dot{\alpha}}$. But it turns out that upon the use of identities

$$\begin{aligned}
\theta^2 &= \overline{D}^2(\theta^2\bar{\theta}^2) \\
\theta_\alpha\bar{\theta}_{\dot{\alpha}} &= D^2\overline{D}^2(\theta^2\bar{\theta}^2) \\
& \cdots\cdots
\end{aligned}$$

and integration by part, possible operators with insertion of $\widetilde{V}$ can be re-written as a linear combination of operators (18).

In the gauge sector, in WZ gauge the only LV dimension 3 operators are:

$$\begin{gathered}
\widetilde{E}_\mu \, \epsilon^{\mu\nu\rho\sigma} A_\nu \partial_\rho A_\sigma \\
\widetilde{F}_\mu \, \lambda\sigma^\mu\overline{\lambda} \ ,
\end{gathered}$$

which have the following superfield expression:

$$\begin{aligned}
\int d^4\theta \ \widetilde{V}\Omega &= \frac{\widetilde{v}_\mu}{4}\left(\epsilon^{\mu\nu\rho\sigma} A_\nu \partial_\rho A_\sigma \ + \ \lambda\sigma^\mu\overline{\lambda}\right) \\
\widetilde{F}_\mu \int d^4\theta \ \theta^4 \, W\sigma^\mu\overline{W} &= \widetilde{F}_\mu \, \lambda\sigma^\mu\overline{\lambda} \, .
\end{aligned}$$

Here Ω is the Chern-Simons superfield [?]:

$$\Omega = -\frac{1}{4}\left\{ D^\alpha(V\, W_\alpha) \ + \ \overline{D}_{\dot{\alpha}} V\, \overline{W}^{\dot{\alpha}} \right\} \ .$$

We now return to the discussion of possible ways dimension 5 can transmute to dimension 3. We notice that there are two generic ways this may occur, at tree level and via loop effects.

$$\begin{aligned}
&O^{(5)} \ \xrightarrow{\text{EOM}} \ (m^2_{soft} + m^2_e)\, O^{(3)} \quad \text{for selectrons} \\
&O^{(5)} \ \xrightarrow{\text{1 loop}} \ m^2_{soft}\, O^{(3)} \quad \text{for fermions and bosons} \ .
\end{aligned}$$

The soft supersymmetry breaking in the form (16) will affect the LV interactions for selectron and spositron already at tree level. The masses of scalar particle are lifted with respect to the masses of the electron and positron. This alter selectrons' equations of motion, leading to the *enhancement* of certain dimension 3 operators. Ignoring the difference between the selectron and spositron masses, one cane easily show that combination of LV operators (4) and SUSY breaking (16) leads to the following dimension 3 LV operator,

$$\mathcal{L}^{\mathrm{EOM}}_{\mathrm{sparticle}} = \frac{N^{\mu}_{-}}{M}\, 2i\, \left(m_e^2 + m_s^2\right) \left\{ \overline{z}_{+} \mathcal{D}_{\mu} z_{+} \; - \; \overline{z}_{-} \mathcal{D}_{\mu} z_{-} \right\} \tag{19}$$

effectively generating the $\widetilde{A}^{\mu}_{\pm}$-terms in the list (17).

$$\widetilde{A}^{\mu}_{\pm} = \pm\, 2\, \frac{N^{\mu}_{-}}{M} \left\{ m_e^2 \;+\; m_s^2 \right\} \; . \tag{20}$$

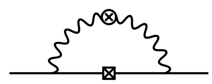
However, we will not be interested in these particular operators due to current impossibility to study experimentally the superpartner sector. In the matter sector only the operators invloving electrons and positrons are of particular value for phenomenology. For the same reason, in the gauge sector we will only be interested in the Chern-Simons term that might be induced for photons.

At one-loop level, the transmition of SUSY breaking to the LV sector of chiral fermions, gauge bosons may indeed be possible. We start with the 1-loop effects in the matter sector.

A. Operators in the matter sector

When SUSY is softly broken, all radiative corrections involving superpartners can be divided roughly in the two category. The first is the logarithmic running of the soft-breaking operators and Kahler terms for which the interval of the loop momenta is $m_{soft} \ll |p_{loop}| \ll M$. In this case, the soft breaking parameters inside the loops can be treated as perturbations, and inserted explicitly in the lines. The second category corresponds to $|p_{loop}| \sim m_{soft}$, where insertion approximation breaks down and the m_{soft} has to be taken into account exactly. These are corrections from the sparticle threshold. We concentrate on the first category, as it is enhanced relative to threshold corrections by a large logarithm, $\log(M/m_s oft)$. Thus, we have to consider all diagrams with two external chiral fields containing one LV insertion and one SUSY-breaking (SB) insertion in all possible ways. That is achieved by inserting a SUSY-breaking interaction in all diagrams of Figs 1, 2 with the exception of the tadpole diagrams in Fig. 1 that cannot be complemented by soft-breaking insertion so as to stay one-particle-irreducible. This way we find that the only diagram that trasmit LV from the gauge sector to dim 3 operators in the electron sector is the diagram shown in Fig. 4, The crossed box symbol stands for the SUSY-breaking operator

Figure 4: Diagram that generates dimension 3 LV operators for electrons and positrons due to soft supersymmetry breaking and dimension 5 LV operator in (5) in the gauge sector. Crossed box denotes the insertion of the SUSY breaking operator (16).



insertion. A straightforward calculation reveals the result

$$\mathcal{L}_{\rm ct}^{n^\mu} = \frac{3e^2}{16\pi^2} \log M/m_s \cdot \int d^4\theta \; \overline{\nabla^\pm \Phi_\pm \not n} \, \nabla^\pm \Phi_\pm \, \theta^4 \, {m_{soft}^\pm}^2 \tag{21}$$

which in components takes the form

$$\mathcal{L}_{\rm ct}^{n^\mu} = \frac{3e^2}{8\pi^2} {m_{soft}^\pm}^2 \log M/m_s \cdot \overline{\psi_\pm \not n} \psi_\pm \quad . \tag{22}$$

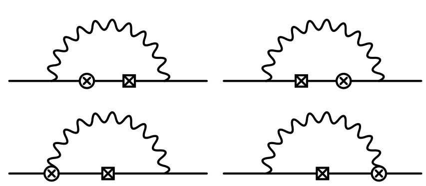
This can be rewritten in Dirac spinor notations (see Section V for details) as

$$\mathcal{L}_{\rm ct}^{n^\mu} = \frac{3e^2}{8\pi^2} \left(- {m_{soft}^+}^2 \, \overline{\Psi} \not n P_L \Psi \; + \; {m_{soft}^-}^2 \, \overline{\Psi} \not n P_R \Psi \right) \; , \tag{23}$$

where P_L, P_R are the left and right projectors. defined in Appendix A.

Now we pass on to the dimension 3 LV operators induced by dimension 5 operators (4) in the matter sector. Inserting soft-breaking into the first three graphs of Fig. 1, we arrive at the set of diagrams given in Fig. 5. As before, m_e can be nglected everywhere inside the

Figure 5: Dimension 3 LV operators induced by dimension 5 LV operators (4) in the matter sector and soft supersymmetry breaking (16).



diagrams and it is sufficient to consider only one set of diagrams of Fig.5, as calculations for Φ_+ and Φ_- yield identical results:

$$\mathcal{L}_{\rm ct}^{n_e^\mu} = -\frac{e^2}{8\pi^2} \log M/m_s \cdot \int d^4\theta \; \overline{\nabla^\pm \Phi_\pm \not n_{e,\bar e}} \, \nabla^\pm \Phi_\pm \; \theta^4 \, {m_{soft}^\pm}^2 \; , \tag{24}$$

(here n_e corresponds to the upper plus sign and $n_{\bar{e}}$ corresponds to the lower minus sign), which in components gives rise to

$$\mathcal{L}_{\rm ct}^{n_e^\mu} = -\frac{e^2}{4\pi^2} \log M/m_s \, {m_{soft}^{\pm}}^2 \cdot \overline{\psi_\pm \not{n}_{e,\bar{e}} \psi_\pm} \quad , \tag{25}$$

or in Dirac spinors,

$$\mathcal{L}_{\rm ct}^{n_e^\mu} = -\frac{e^2}{4\pi^2} \log M/m_s \, \left(-{m_{soft}^{+}}^2 \, \overline{\Psi} \not{n}_e P_L \Psi \; + \; {m_{soft}^{-}}^2 \, \overline{\Psi} \not{n}_{\bar{e}} P_R \Psi \right) \; . \tag{26}$$

Pasha, I actually think that all formulae here that use $\pm$ are quite cumbersome. A reader have to think what they actually are, decipher them. Why don't we split them into "e-quantities" + "ebar-quantities"? This is a minor point though. The main thing is that we are too naive here. This answer is correct only if $e^2/\pi \times \log(M/m)$ is a small number. But the RG equations should do better than that! What we should do is to form a new RG equations for dimension 3 operators. The result is going to be different from equation (24) and alike. The reason is because e, m_s^2 and n_μ all depend on the normalization scale as well. This will modify many formulae in this section, and in the phenomenology sections as well. In the end it might not be that significant change in terms of numbers, but since we solve the RG equation with dim 5 operators, we must do it in a similar way here. In any event, I expect serious revisions here. Alternatively, we can say that $e^2/\pi \times \log(M/m)$ is always small but then formulae in the previous section should also be simplified. Do you see my point?

Analyzing (21) and (24), we can see that out of the whole sequence of operators (17), only one got generated, with a coefficient:

$$\widetilde{B}_\mu^\pm = {m_{soft}^{\pm}}^2 \cdot \log M/m_s \left\{ \frac{3e^2}{8\pi^2} n^\mu \; - \; \frac{e^2}{4\pi^2} n_{e,\bar{e}}^\mu \right\} \; . \tag{27}$$

Other operators will presumably receive contributions at higher loop order and/or due to a different pattern of SUSY breaking (for example, upon the inclusion of gaugino masses). As we remarked earlier, the term with $\widetilde{A}_\mu^\pm$ can be generated already at tree level. Finally, we note that the tensor operator (**??**) does not mix with dimension 3 operators in any order in SUSY breaking, because there are no dimension 3 operators composed from the available fields that could couple to an irreducible three-index object.

As before, it proves advantageous to re-express the results (26) and (23) in terms of the combinations $N^\mu_\pm$ (see (13)). In the soft-breaking sector it is also convenient to adopt a definite parity basis,

$$\Delta m^2 = \frac{{m^+_{soft}}^2 - {m^-_{soft}}^2}{2} \quad ; \qquad {m^\pm_{soft}}^2 \equiv m_s^2 \pm \frac{\Delta m^2}{2} \; , \tag{28}$$

Furthemore, we assum that Δm^2 to be small compared to m_s^2. Taking $\Delta m^2 \to 0$ then restores the parity conservation in the SUSY breaking sector. Combining the results (**??**) and (**??**), we arrive at the total contribution to dimension 3 electron operators generated by the SUSY breaking and LV operators of dimension 5:

$$\begin{aligned} &\mathcal{L}^{\rm matter}_{\rm SB\ dim\ 3} = \\ &\overline{\Psi}\gamma^\mu\Psi \cdot \left\{ \frac{e^2}{4\pi^2}\, m_s^2\, N^\mu_- + \frac{e^2}{4\pi^2}\frac{\Delta m^2}{2}\, N^\mu_+ - \frac{3e^2}{8\pi^2}\frac{\Delta m^2}{2}\, n^\mu \right\} \log M/m_s \\ &\qquad\qquad\qquad\qquad\qquad + \\ &\overline{\Psi}\gamma^\mu\gamma^5\Psi \cdot \left\{ \frac{e^2}{4\pi^2}\, m_s^2\, N^\mu_+ + \frac{e^2}{4\pi^2}\frac{\Delta m^2}{2}\, N^\mu_- - \frac{3e^2}{8\pi^2}\, m_s^2\, n^\mu \right\} \log M/m_s \; . \end{aligned} \tag{29}$$

B. Operators in the gauge sector. Chern-Simons term.

The absence of optical activity effects caused by the Chern-Simons (CS) term has been checked over the cosmological distances. This creates an incredible sensitivity to k_μ in (1), at the level better than the value of the Hubble scale (see *e.g.* Ref. [**?**] and references therein). This is about ten orders of magnitude better than the level of sensitivity for the best terrestrial experiments searching for LV parameters in (1). Not surprisingly, the issue of CS term generated by the radiative corrections from other LV interactions has drawn a lot of interest [**? ? ? ?**], exhibiting the whole range of answers for k_μ being induced by b_μ (including zero). Although we expect a no-go theorem of Coleman and Glashow [**?**] to hold in our case **Pasha, please have a look at their proof**, we would like to check explicitly the cancellation of certain groups of diagrams potentially leading to a CS term.

We find that dimension 5 LV terms in SQED do not lead to the CS term at one loop level, and this statement holds even if the supersymmetry is softly broken. Note that this statement is only valid for the pure Chern-Simons term $\epsilon^{\mu\nu\rho\sigma} A_\nu \partial_\rho A_\sigma$, while there is no evidence against another possible operator in the photon sector, $\lambda\sigma^\mu\overline{\lambda}$, that corresponds to a gauge-invariant Lagrange density, rather than just a gauge-invariant action. The presence/absence of the latter term is not very relevant for phenomenological applications due to obvious reasons.

A generic argument against the possibility to induce the CS term is based on the gauge invariance of LV terms (4) versus only a "partial" gauge invariance, up to a total derivative, of the CS term in (19). This difference become very serious if in (4) we allow n_e^μ to represent a field or at least a slow-varying function of space-time. This will keep the gauge invariant property of operator (4) and hence of all diagrams with its insertion, while the n_e^μ-induced Chern-Simons will loose gauge invariance completely. Therefore, the most reasonable possibility is to have *no connection* between the CS term and operators (4). A usual worry is that the UV regularization may invalidate a no-go theorem [?], and thus generating a CS term at the UV scale. This possibility, however, is excluded by our assumption of the supersymmetric dynamics at the UV scale, and the explicit breaking of SUSY by the CS term [? ?].

First we checked the all possible diagrams with the chiral loop and two external gauge supefields in exact SUSY, retaining a non-zero value of m_e. It is quite easy to show that for such diagrams the only possible operator which may contain the CS term when expressed in terms of the vector superfield V is

$$\int d^4\theta \, V \overline{D \not{n}_e} D V \quad . \tag{30}$$

It is then very sufficient to check for presence/absence of (30)-proportional contributions in the diagrams of Fig. 6. As before, the diagrams with $n_{\bar{e}}^\mu$ are obtained by flipping all the

Figure 6:

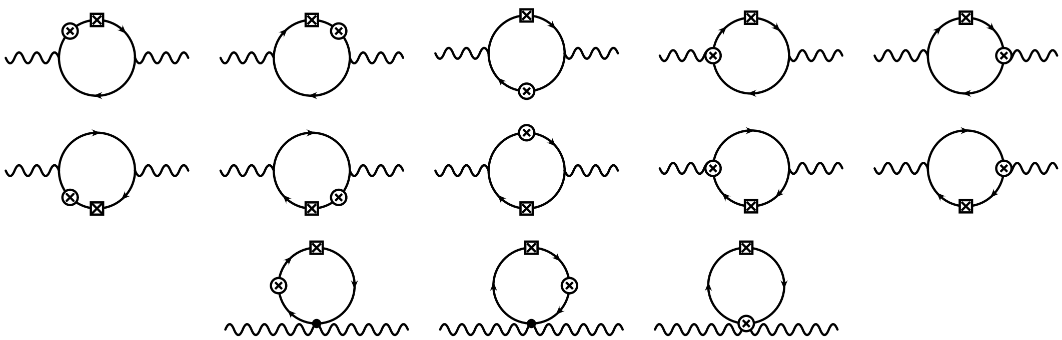
MASSIVE DIAGRAMS HERE

charges in diagrams for n_e^μ. An explicit calculation reveals that all contributions of the type (30) cancel for both n_e^μ and $n_{\bar{e}}^\mu$ backgrounds. We conclude now that Chern-Simons is not generated at one loop in *massive exact SQED.*

We expect that nothing will change even if the supersymmetry is broken, at least at one loop level. Indeed, the CS term can only be generated by a fermion running in the loop, as a bosonic looip cannot produce $\epsilon^{\mu\nu\rho\sigma}$ entering the expression for CS. However, the SUSY breaking terms (16) only provide a mass to the bosonic component of chiral superfields and thus only affect the parts of the diagrams that are not capable of inducing the CS in the first place.

This argument can be solidified by a direct calculation in the presence of the soft-breaking. The relevant diagrams are obtained by inserting the soft breaking vertex $\mathcal{L}_{\rm SB}$ (16) into the diagrams shown in Fig. 2. This yields the set of graphs represented in Fig. 7.

Figure 7: Dimension 3 1-loop contributions arising from the dimension 5 LV operators (4) and soft supersymmetry breaking.



Again, instead of calculating every possible term, including the threshold corrections to dimension 5 operators, we only concentrate on one structure

$$|F_S|^2 \int dx \; \mathrm{tr} \; \left\{ \not{v} \, \bar{\not{\partial}} \, \not{n} \, \bar{\not{v}} \right\} \quad , \tag{31}$$

where v_μ is the photon, and tr means taking the trace of the product of Pauli σ-matrices. Here, again, the vertex cancellation property can be used quite effectively to mutually cancel contributions of particular diagrams. A straightforward calculation shows that indeed, as anticipated, all (31)-proportional terms cancel.

We conclude that the Chern-Simons term is not induced by dimension 5 LV operators in SQED, which leaves us a task of using other constraints that [**?**] to limit the LV parameters of the model.

V. PHENOMENOLOGY OF LV SQED: LV OBSERVABLES AND EXPERIMENTAL LIMITS

This section has to be re-written, after we straighten out the flaw that I spotted in section IV. Moreover, certain important things (e.g. dimensions) are

wrong, and we need to correct all that. This is for Pasha and myself. I did not edit it, as I expect serious changes

As mentioned earlier, besides the supersymmetry breaking, dimension 3 operators can be induced by dimension 5 LV operators via the equations of motion. Dimension 5 operators will turn into dimension 3 operators with a factor of the mass dimension two. This factor depends on the particular operator of consideration. Some of them (e.g. those involving fermions) get multiplied by m_e^2 on the equations of motion. Those involving scalars get a factor of $m_e^2 + m_s^2$ due to the supersymmetry breaking (16). Others are multiplied by the electromagnetic fieldstrength.

First we consider the matter operators (4). The component form for the electron part is given by (7). The subject of the most interest are the operators which appear in the fermion matter sector — that is where we can directly impose constraints. We do not show the intermediate results, just sketch the main steps, and refer the reader to Appendix C for more details. First, we resolve the equations of motion of the auxiliary fields D, $F_\pm$. Then we resolve the unperturbed equations of motion for the fields: this allows us to replace the 2nd derivative of the scalar fields and the derivative of the fermion fields by the RHS of the corresponding equations of motion. For convenience of phenomenological studies we convert all Weyl spinors into Dirac/Majorana fermions:

$$\Psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \quad \text{and} \quad \lambda = \begin{pmatrix} \lambda \\ \overline{\lambda} \end{pmatrix} \quad . \tag{32}$$

The complete result of this is listed in the Appendix C. Here we only show the resulting operators in the fermionic matter sector:

$$\begin{aligned} \mathcal{L}_{\rm LV}^{\rm quark} \;&=\; \frac{N_+^\mu}{M}\,\frac{1}{2} e\,\overline{\Psi}\widetilde{F}_{\mu\nu}\gamma_\nu\Psi \;+\; \frac{N_-^\mu}{M}\,\frac{1}{2} e\,\overline{\Psi}\widetilde{F}_{\mu\nu}\gamma^\nu\gamma^5\Psi \;- \\ &-\; \frac{N_-^\mu}{M}\, m\overline{m}\,\overline{\Psi}\gamma_\mu\Psi \quad , \end{aligned} \tag{33}$$

which appear to depend on the combinations $N_\pm^\mu$ defined in (13) evaluated at the scale m_{soft}. Note that soft SUSY breaking does not affect these operators as it only adds m_{soft}^2 to the mass of the scalar field.

The next operator to consider is the photon operator (8):

$$\begin{aligned} \mathcal{L}_{\rm LV}^{\rm gauge\ (K)} &= \int d^4\theta\, \overline{W} \not{n} W \;= \\ &= 2\,\overline{\lambda}\not{n}\,\Box\,\lambda \;+\; 2\,\lambda\, n^\mu \partial_\mu \not{\partial}\,\overline{\lambda} \;-\; 2\,D\, n_\mu \partial_\nu F^{\mu\nu} \;+\; \partial_\lambda F^{\lambda\mu}\,\widetilde{F}_{\mu\nu}\cdot n^\nu \;. \end{aligned} \tag{34}$$

When reducing it on the equations of motion, it is not difficult to show that resolution of the unperturbed equations of motion for either of λ or D is not going to give a contribution in the observable sector. Only the last term does provide such a contribution, if we replace $\partial_\lambda F^{\lambda\mu}$ with the electromagnetic current $e\,\overline{\Psi}\,\gamma^\mu\Psi$:

$$\mathcal{L}^{\mathrm{EOM}}_{\mathrm{gauge\ (K)}} = -\,e\,\overline{\Psi}\,n^\mu\gamma^\nu\widetilde{F}_{\mu\nu}\,\Psi\ , \tag{35}$$

which contributes to the interaction of the electromagnetic current with the dual electromagnetic field.

The tensor operator (9),

$$\begin{aligned}\mathcal{L}^{\mathrm{gauge\ (T)}}_{\mathrm{LV}} &= \int d^2\theta\, T^{\mu\nu\rho}\, W\sigma_{\nu\rho}\partial_\mu W \ + \ h.c. \ = \\ &= \ 2\left[-D\,\partial_\mu\widetilde{F}_{\nu\rho} \ + \ \frac{1}{2}F_{\nu\lambda}\partial_\mu F_\rho{}^\lambda\right]\cdot\left\{T_{(r)}{}^{\mu\nu\rho} \ - \ \frac{1}{2}\epsilon^{\nu\rho\tau\varphi}T_{(i)}{}^\mu{}_{\tau\varphi}\right\} \\ &- \ 2\left[2\,\overline{\lambda}\,\partial_\mu\partial_\nu\overline{\sigma}_\rho\lambda \ + \ \frac{1}{2}F_{\nu\lambda}\partial_\mu\widetilde{F}_\rho{}^\lambda\right]\cdot\left\{T_{(i)}{}^{\mu\nu\rho} \ + \ \frac{1}{2}\epsilon^{\nu\rho\tau\varphi}T_{(r)}{}^\mu{}_{\tau\varphi}\right\}\ ,\end{aligned} \tag{36}$$

where we have defined

$$T_{(r)}{}^{\mu\nu\rho} = \mathrm{Re}\ T^{\mu\nu\rho}\ , \qquad T_{(i)}{}^{\mu\nu\rho} = \mathrm{Im}\ T^{\mu\nu\rho}\ , \tag{37}$$

also can be reduced on the equations of motion. Applying the unperturbed equations of motion to the electromagnetic terms in the square brackets of (36), we obtain in the observable sector

$$\begin{aligned}&\mathcal{L}^{\mathrm{EOM}}_{\mathrm{gauge\ (T)}} = \\ &= \ -\ \frac{1}{2}e\,\overline{\Psi}F_{\mu\rho}\gamma_\nu\Psi\left\{T_{(r)}{}^{(\mu\nu)\rho} \ - \ \frac{1}{2}\epsilon^{\nu)\rho\tau\varphi}T_{(i)}{}^{(\mu}{}_{\tau\varphi}\right\} \ + \\ &+ \ 2\,e\,\overline{\Psi}\widetilde{F}_{\mu\rho}\gamma_\nu\Psi\left\{T_{(i)}{}^{\mu\nu\rho} \ + \ \frac{1}{2}\epsilon^{\nu\rho\tau\varphi}T_{(r)}{}^\mu{}_{\tau\varphi}\right\}\ .\end{aligned} \tag{38}$$

It can be shown [**?**] that neither of the gauge operators (34) and (36) modify the dispersion relation for the photon, **"and equations (35) and (38) confirm this." ?**

Now we can gather all operators of phenomenological interest of dimensions 5 and 3 — (29), (33), (35) and (38):

$$\begin{aligned}\mathcal{L}_{\mathrm{eff}} \ &= \ a_\mu\,\overline{\Psi}\gamma^\mu\Psi \ + \ b_\mu\,\overline{\Psi}\gamma^\mu\gamma^5\Psi \ + \ c_\mu\,e\,\overline{\Psi}\widetilde{F}^{\mu\nu}\gamma_\nu\Psi \ + \\ &+ \ d_\mu\,e\,\overline{\Psi}\widetilde{F}^{\mu\nu}\gamma_\nu\gamma^5\Psi \ + \ f^{\mu\nu\rho}\,e\,\overline{\Psi}F_{\mu\rho}\gamma_\nu\Psi \ + \ \widetilde{f}^{\mu\nu\rho}\,e\,\overline{\Psi}\widetilde{F}_{\mu\rho}\gamma_\nu\Psi\ ,\end{aligned} \tag{39}$$

where we use the notations of [?] for the coefficients of the dimension three operators. The coefficients of (39) take the form:

$$
\begin{aligned}
a^\mu &= -\frac{1}{M}\, m_e^2 N_-^\mu\Big|_{m_s} \;+\; \frac{1}{M}\left\{ \frac{\alpha}{\pi}\, m_s^2\, N_-^\mu \;+\; \frac{\alpha}{\pi}\,\frac{\Delta m^2}{2}\, N_+^\mu \;- \right.\\
&\qquad\qquad\left. -\;\frac{3\alpha}{2\pi}\,\frac{\Delta m^2}{2}\, n^\mu \right\}\Big|_M \log M/m_s \\
b^\mu &= \frac{1}{M}\left\{ \frac{\alpha}{\pi}\, m_s^2\, N_+^\mu \;+\; \frac{\alpha}{\pi}\,\frac{\Delta m^2}{2}\, N_-^\mu \;-\; \frac{3\alpha}{2\pi}\, m_s^2\, n^\mu \right\}\Big|_M \log M/m_s \\
c^\mu &= \frac{1}{M}\left\{ \frac{1}{2} N_+^\mu \;-\; n^\mu \right\}\Big|_{m_s} \\
d^\mu &= \frac{1}{M}\,\frac{N_-^\mu}{2}\Big|_{m_s} \\
f^{\mu\nu\rho} &= -\,\frac{1}{2}\left\{ T_{(r)}{}^{(\mu\nu)\rho} \;-\; \frac{1}{2}\epsilon^{\nu)\rho\tau\varphi} T_{(i)}{}^{(\mu}{}_{\tau\varphi} \right\}\Big|_{m_s} \\
\widetilde{f}^{\mu\nu\rho} &= 2\left\{ T_{(i)}{}^{\mu\nu\rho} \;+\; \frac{1}{2}\epsilon^{\nu\rho\tau\varphi} T_{(r)}{}^{\mu}{}_{\tau\varphi} \right\}\Big|_{m_s}\,,
\end{aligned}
\tag{40}
$$

where $\alpha = e^2/4\pi$, and the operators at the scale m_s are expressed in terms of those at the UV scale M via (15). We now impose experimental constraints on the coefficients of the effective low-energy lagrangian (39).

The operator a^μ is believed not to lead to any physical effects as it can be totally absorbed into the kinetic term $i\,\overline{\Psi}\partial\!\!\!/\Psi$ via a plain gauge rotation [*reference*]: $\Psi(x) \to e^{ia^\mu x_\mu}\Psi(x)$.

As for the other operators, we first make estimates for the c^μ, d^μ, $f^{\mu\nu\rho}$ and $\widetilde{f}^{\mu\nu\rho}$ ones. The operator c^μ represents an interaction of the EM-current with the EM-field. When considered inside a nucleus, this interaction can be limited [?] by

$$
c^\mu \lesssim 10^{-5}\ .
$$

Operator d^μ induces the interaction of the CPT-odd electron spin with the electromagnetic field, and thus contributes to the anomalous magnetic moment of the electron. Existence of this operator itself is a remarkable property, since Ferrara-Remiddi theorem forbids emergence of anomalous magnetic moment of the electron in supersymmetric abelian gauge theories [?]. In our model, however, two very basic concepts of a generic field theory — Lorentz-invariance and CPT-invariance — are not fulfilled, thus admitting the existence of the anomalous magnetic moment. **Need to affirm this.**

The operator d^μ does only induce a small correction to the magnetic moment and is therefore very unlikely to be detectable. However, due to its CPT-oddness, it contributes the same amount of magnetic moment to the electron and positron:

$$H^e_{\rm eff} = e\,d^0\,\frac{\vec{B}\cdot\vec{S}}{S} - |\mu|\,\frac{\vec{B}\cdot\vec{S}}{S}$$
$$H^{\bar{e}}_{\rm eff} = e\,d^0\,\frac{\vec{B}\cdot\vec{S}}{S} + |\mu|\,\frac{\vec{B}\cdot\vec{S}}{S}\ .$$

Hence, using the estimate for the difference of magnetic moments of the electron and positron given in [?] we can put a limit on d^0 of:

$$d^0 \lesssim 10^{-12}\,\frac{m_e}{M}\ . \tag{41}$$

Operators $f^{\mu\nu\rho}$ and $\widetilde{f}^{\mu\nu\rho}$ are harder to constrain. They represent a modification of the interaction of the electromagnetic current with the electromagnetic field. Rather strong electromagnetic fields exist in nuclei. Effectively, these operators are an average of the interaction of the current with the EM field over the nucleus, which is of the same order of magnitude as the processes caused by c^μ. Thus, we can use the approximate limit which we have used to constrain c^μ:

$$|f|\,,\ |\widetilde{f}|\ \lesssim\ 10^{-5}\ . \tag{42}$$

A very speculative derivation, though.

The best known limits exist for the dimension 3 operator b^μ. It is severely constrained by the clock comparison experiments which measure the interaction of nuclei spin with the preferred direction:

$$|\mathbf{b}| \lesssim \frac{1}{M} 10^{-29}\ \mathrm{GeV}\ . \tag{43}$$

This is the strongest constraint among all of the operators we are considering in $\mathcal{L}_{\rm eff}$. In our model, this constraint appears to be advantageous also due to a different reason. As mentioned above, dimension 5 operators can be roughly thought of as dimension 3 operators interacting with the electromagnetic field in the nucleus, that is, multiplied by a characteristic scale of the electromagnetic field in the nucleus. At best, this scale will not exceed (200 MeV)^2 [*a reference*]. However, dimension 3 LV operators (40) generated at the soft supersymmetry breaking scale, are enhanced by the factor of this scale

$$b^\mu \sim m_s^2\ ,$$

which is many orders of magnitude greater than the characteristic scale of electromagnetic energy in the nucleus.

VI. CPT-CONSERVING DIMENSION 6 OPERATORS

In this section we list all possible Lorentz violating operators of dimension 6. They are most easily derived in the *vector* representation with covariantly chiral superfields (see [*reference to 1001 nights*]). We list here the operators arising in SQCD (and in SQED in particular). We get, for the Kahler term the following operators:

$$
\begin{aligned}
&\overline{\Phi}_+ e^{2eV}\, \nabla_{(\mu}\nabla_{\nu)}\, \Phi_+\ , \quad \text{its charge conj. ,}\\
&\overline{\Phi}_+ e^{2eV}\, \nabla\sigma_{\mu\nu} W\, \Phi_+ \quad + \quad \text{h.c.,} \quad \text{its charge conj. ,}\\
&\Phi_- \nabla\sigma_{\mu\nu} W\, \Phi_+ \quad + \quad \text{h.c.,}\\
&\mathrm{T}r\ W\sigma^{\mu\nu}\nabla^2 W \quad + \quad \text{h.c.,}\\
&\mathrm{T}r\ \overline{W\sigma}_{(\mu}\nabla_{\nu)} W \quad .
\end{aligned}
\tag{44}
$$

For the superpotential terms we have fewer possibilities:

$$
\begin{aligned}
&\Phi_-\ W\sigma^{\mu\nu} W\, \Phi_+ \quad + \quad \text{h.c.,} \qquad\qquad \text{(nonabelian theory)}\\
&\mathrm{T}r\ \partial_\mu W\, \partial_\nu W \quad + \quad \text{h.c.}
\end{aligned}
\tag{45}
$$

As $\sigma^{\mu\nu}$ is antisymmetric in $(\mu\nu)$, the first term in (45) vanishes in abelian theories. The operators listed in (44), (45) are evident to classify into symmetric and antisymmetric ones in $(\mu\nu)$.

From the results (44), (45) a conclusion can be made that in SQED/SQCD, no terms like $F_{\mu\rho}F_{\nu\sigma}F^{\rho\sigma}$ or $F_{\rho\sigma}F^{\rho\sigma}F_{\mu\nu}$ can arise. It is an important statement since these terms naturally arise in NC QED and Yang-Mills [*a reference here*]. However, as we find, there is no way to supersymmetrize them.

The reason is akin to the fact that Seiberg-Witten map cannot be defined for NC supersymmetric gauge theories in Minkowski space. As pointed out in (*a reference here*), an originally non-commuting supersymmetric Yang-Mills can be rewritten in usual commuting component fields using the *-product. Such a theory is invariant under the supersymmetric transformations which involve the *-product. But there is no warranty the theory is invariant under the ordinary SUSY transformations. In particular, the first term of $\Theta_{\mu\nu}$-expansion (i.e. a dimension 6 operator) will not be supersymmetric with respect to usual commuting SUSY. **Something more needs to be said here. Do those $(F_{\mu\nu})^3$ terms arise in the NC YM?**

VII. DISCUSSION AND CONCLUSIONS

We have constructed the LV extension of supersymmetric quantum electrodynamics, as a subset of the LV minimal supersymmetric standard model. The LV modification are power-like suppressed by the scale of the UV physics, and decouple in the limit of $M \to \infty$.

In the leading M^{-1} order, dimension 5 LV operators can by coupled to two types of the LV backgrounds. There are three four-vectors n^μ, n_e^μ and $n_{\bar{e}}^\mu$ that parametrize Lorentz violation in the Kähler terms for vector and chiral superfields, as well as the irreducible rank three tensor $T^{\mu\nu\lambda}$, antisymmetric in $\nu\lambda$. We have obtained the explicit expressions in component form for LV interactions generated by these backgrounds.

The renormalization group equations for LV operators are derived in exact and softly-broken SUSY. In case of the exact supersymmetry, the mixing of operators and their logaritmic evolution over the energy scales are found at one loop. Once the SUSY is broken the generation of dimension three operators become possible. The energy scale that controls the transmutation of dimension 5 into dimension 3 is the scale of the soft-breaking masses. In other words, quadratic divergencies that pose a naturalness problem for the SM extended by higher-edimensional LV operators are stabilized at the scale of the SUSY breaking, and thus the naturalness problem is alleviated. In order to obtain phenomenologically relevant formulae, we broke supersymmetry in the scalar electron sector, and calculated the resulting LV effective Lagrangian for the electrons. We also checked that a corresponding dimension 3 operator for photons, the Cern-Simons term, is not generated.

Barring accidental cancellation among different LV sources, we derive stringent limits on the linear combination of LV parameters in the SQED. The most stringent results come from the one-loop generated coupling between the electron axial vector current and the external 4-vector, given by the linear combination of n^μ, n_e^μ and $n_{\bar{e}}^\mu$, combined with the absence of the anomalous psin precession for electrons checked at better than 10^{-28} GeV level in the torsion balance experiment [**?**]. Allowing for order one values for vectors n^μ, we conclude that the scale of the LV must be *significantly* higher than the Planck scale, $M >$....GeV. **We need to get our act together here and get an actual number**. It is also remarkable that none of the operators considered lead to high-energy modifications of dispersion relations. Therefore, none of the stringent astrophysics-derived limits on LV parameters [**? ?**] apply to the case of SQED. This also refers to potentially very strong constraints that exist for the Chern-Simons term **??**. We have presented the arguments why the Chern-Simons term is not generated in the loop corrections even if the supersymmetry is broken.

The existence of such strong constraints at dimension 5 level (with or without supersymmetry), pose a very serious challenge for theories that predict LV at $1/M_{\rm Pl}$ level. Therefore such theories would necessarily abandon the effective field theory description, which does not look to us as a reasonable alternative. However, it might be that dimension five operators are forbidden by some additional symmetry reasons, such as *i.e.* CPT. Then in the next order, $O(M^{-2})$ Planck-scale-suppressed LV effects are not excluded. The best constraints come close [**?**], but applicable only to operators that modify high-energy dispersion relations. We classified dimension six LV operators in SQED and found that they couple to symmetric or antisymmetric two-index tensor backgrounds. At the next step one can study their transmutation to dimension four operators in the presence of the soft-breaking terms. Naturally, we expect the approximate relation [dim 4] $\sim m_s^2$ [dim 6] to hold, which gives the estimate for the size of the LV backgrounds at dimension four as $m_s^2/M^2 \sim 10^{-32}$. We notice that this prediction comes close to the experimental sensitivity to such operators [**?**], and therefore deserves further studies in the framework of LV MSSM.

anything else anyone wants to add?

Acknowledgements

The work of P.B. and M.P. is supported in part by the N.S.E.R.C. of Canada.

Appendix A: CONVENTIONS AND NOTATIONS

Our notations for the superfield formalism are based on Wess & Bagger [**?**]. For conversion between Weyl and Dirac spinors we use the notations of [**?**]. Covariant derivatives and hermitean conjugation are taken from [**?**] with a proper adaptation.

The signature of the metric is $(-+++)$. All spinor algebra definitions can be found in the canonical treatment [**?**], and we list here only some minor conventional departures.

Unlike in [**?**], we denote the space-time Lorentz indices with the letters from the middle of the *greek* alphabet: v_μ, σ_ν, N^ρ, etc, as it is normally accepted in QFT. Spinor indices are taken, also as commonly accepted, from the beginning of the greek alphabet: θ^α, $\epsilon_{\beta\gamma}$, $\overline{\psi}_{\dot\delta}$. Spinor derivatives are designated as

$$\partial_\alpha = \frac{\partial}{\partial\theta^\alpha}\,, \qquad \partial^\alpha \equiv \epsilon^{\alpha\beta}\partial_\beta\,.$$

We use a "slashed" vector notation in the case where a Lorentz vector is contracted with a σ-matrix, or a γ-matrix:

$$\not{v} = v^\mu\,\sigma_\mu\,, \qquad \overline{\not{A}} = A^\mu\,\overline{\sigma}_\mu\,, \qquad \not{n} = n^\mu\gamma_\mu \quad , \tag{A1}$$

where the case of σ-matrices is used with Weyl spinors, and the case of γ-matrices is only used with Dirac spinors. There should normally be no confusion as to which kind of spinor is meant in a particular expression. Note the appearance of the bar upon the slashed vector in (A1), when it is contracted with a $\overline{\sigma}_\mu$.

For aesthetical reasons, in cases when two or more close-standing factors bear such a bar, we unite them to have one single long bar:

$$\overline{WW} = \overline{W}_{\dot\alpha}\overline{W}^{\dot\alpha}\,, \qquad \overline{W\sigma_{\mu\nu}\not{n}}\,W = \overline{W}_{\dot\alpha}\;\left(\overline{\sigma}_{\mu\nu}\right)^{\dot\alpha}{}_{\dot\beta}\;\overline{\not{n}}^{\dot\beta\gamma}\;W_\gamma \quad .$$

Note, that such a long bar does not symbolize complex conjugation of the object beneath it, but instead, indicates that each particular factor should be understood as having a bar.

For switching from Weyl to Dirac spinors we followed the notations of [**?**]. Weyl basis for Dirac spinors is the most appropriate in this case, where two Weyl spinors combine into one Dirac spinor:

$$\Psi = \begin{pmatrix} \xi_\alpha \\ \overline{\chi}^{\dot\alpha} \end{pmatrix}, \qquad \overline{\Psi} = \begin{pmatrix} \chi^\alpha \\ \overline{\xi}_{\dot\alpha} \end{pmatrix},$$

and the γ-matrices take the form

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix} , \qquad \gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} .$$

A bunch of useful conversion formulae can be found [?] which allow for quick transfer from Weyl-fermionic bilinear terms to Dirac-fermionic bilinears:

$$\begin{matrix} \overline{\Psi}_1\, \gamma^\mu P_L \Psi_2 = \overline{\xi_1 \sigma_\mu} \xi_2 \\ \overline{\Psi}_1\, \gamma^\mu P_R \Psi_2 = \chi_1 \sigma_\mu \overline{\chi}_2 \end{matrix} \quad , \quad \Psi_1 = \begin{pmatrix} \xi_1 \\ \overline{\chi}_1 \end{pmatrix} , \quad \Psi_2 = \begin{pmatrix} \xi_2 \\ \overline{\chi}_2 \end{pmatrix} . \tag{A2}$$

The chirality projectors $P_{L,R}$ are defined as [?]:

$$P_L = \frac{1\ +\ \gamma_5}{2} , \qquad P_R = \frac{1\ -\ \gamma_5}{2} .$$

Gauge-covariant derivatives are denoted in our paper as

$$\begin{aligned} \nabla^+_\alpha &= e^{-2eV} D_\alpha e^{2eV} , & \overline{\nabla}^+_{\dot\alpha} &= \overline{D}_{\dot\alpha} \\ \nabla^-_\alpha &= D_\alpha \ , & \overline{\nabla}^-_{\dot\alpha} &= e^{2eV} \overline{D}_{\dot\alpha} e^{-2eV} , \end{aligned}$$

where the $+$ or $-$ superscript relates them to the *chiral* or *antichiral* representation [?] correspondingly (in our case, this corresponds to the electron or the positron). The symbol ∇^2 stands for $\nabla^\alpha \nabla_\alpha$, not for $\nabla^\mu \nabla_\mu$, and also should not be confused with the d'Alembertian, which we denote as:

$$\Box = \partial^\mu \partial_\mu \ .$$

For the conjugation, we use the notion of *hermitean conjugation* defined in [?]. When translated into the Wess & Bagger notations, it implies

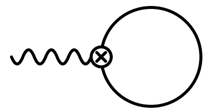
$$\begin{aligned} (\psi_\alpha)^\dagger &= \overline{\psi}_{\dot\alpha} \ , & (\psi^\alpha)^\dagger &= \overline{\psi}^{\dot\alpha} \\ \partial^\dagger_\alpha &= \overline{\partial}_{\dot\alpha} \ , & \partial^\dagger_\mu &= -\,\partial_\mu \\ D^\dagger_\alpha &= -\,\overline{D}_{\dot\alpha} \ , & (\nabla^\pm_\alpha)^\dagger &= -\,\overline{\nabla}^\mp_{\dot\alpha} \\ W^\dagger_\alpha &= \overline{W}_{\dot\alpha} \end{aligned}$$

Its basic principle is that in any expression being conjugated *all* objects must be put in the reverse order:

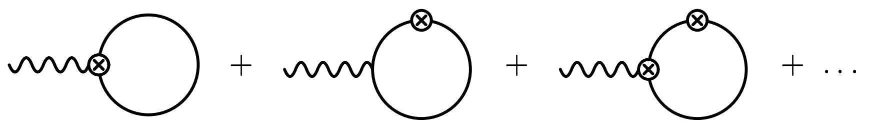
$$\begin{aligned} \left(\overline{\Phi}_1 \partial_\mu \Phi_2\right)^\dagger &= -\,\overline{\Phi}_2 \partial_\mu \Phi_1 \\ \left(\chi \sigma_\mu \overline{\psi}\right)^\dagger &= \quad \psi \sigma_\mu \overline{\chi} \\ (\chi \sigma_{\mu\nu} \psi)^\dagger &= -\,\overline{\psi \sigma_{\mu\nu} \chi} \ . \end{aligned}$$

Appendix B: CANCELLATION OF TADPOLES AND THE VERTEX CANCELLATION PROPERTY

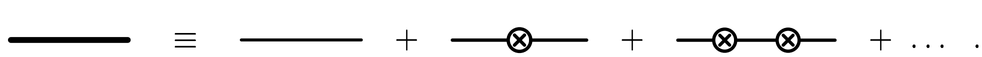
Cancellation of tadpoles



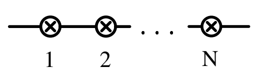
can be proved to all orders of Lorentz violation. This corresponds to summing all diagrams with arbitrary number of LV insertions (4)

 (B1)

In order to show this we evaluate the "exact" LV propagator, i.e. a propagator which contains all powers of LV:

 (B2)

It is straightforward to show that a propagator with N LV insertions is

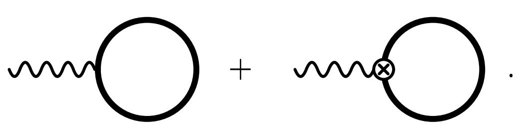


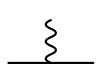
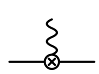
$$= -\frac{i}{p^2}\,\delta^4(\theta_1-\theta_2)\,(n_e^\mu p_\mu)^N \ . \qquad \text{(B3)}$$

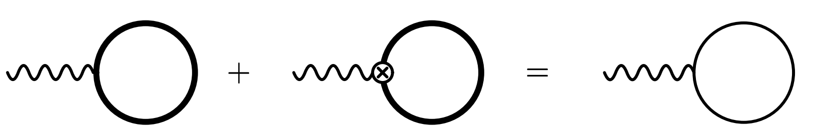
Then, the sum in (B2) is a geometric progression:

$$= -\frac{i}{p^2}\,\delta^4(\theta_1-\theta_2)\,\frac{1}{1-n^\mu p_\mu} \ . \qquad \text{(B4)}$$

The full tadpole (B1), in terms of the full propagator, is:

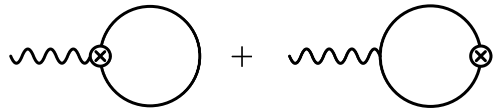


Here the vertices  and  exactly sum up into the structure $1 - n^\mu p_\mu$ and cancel the denominator in (B4), thus making it to be just a free propagator. We are then left with

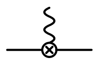
 (B5)

That means that Lorentz violation cancels *to all orders starting from the first.* Cancellation of the zeroth order is of course a matter of vanishing of the sum of charges in the theory.

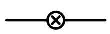
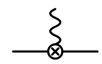
One useful property which is handy for calculations can be unscrambled by examining the process of the tadpole cancellation (B5) at the first order of LV:

 $= 0$. (B6)

This is easy to see from the first order propagator (B3), when N is put to be equal one.

In general,  corresponds to

$$\frac{1}{2}\overline{n}_e^{\alpha\dot{\alpha}} e\,\overline{\Phi}\left\{\overline{D}_{\dot{\alpha}} D_\alpha(V) - 2iV\partial\!\!\!/_{\alpha\dot{\alpha}}\right\}\Phi \ .$$

Thus,  cancels the second part (the term with $\partial\!\!\!/_{\alpha\dot{\alpha}}$) of  (in the case of the tadpole (B6) this lead to a total cancellation, because the $\overline{D}_{\dot{\alpha}} D_\alpha V$ part obviously turns into a total derivative after evaluating the first diagram in (B6)). This can significantly decrease the amount of calculations by cancelling some parts of (usually different) diagrams.

This "vertex cancellation" property can be extensively exploited in loop calculations.

Appendix C: REDUCTION OF CHIRAL LV OPERATORS ON EQUATIONS OF MOTION

Here we show the operators (4)

$$\mathcal{L}_{\rm LV}^{\rm matter} = \frac{i}{M} n_e^\mu\,\overline{\Phi}_+ e^{2eV}\nabla_\mu^+\Phi_+ \quad - \quad \frac{i}{M} n_{\bar{e}}^\mu\,\Phi_- e^{-2eV}\nabla_\mu^-\overline{\Phi}_-$$

in component form with Dirac fermions. The Weyl fermion form of the electron operator is given by (7). The analogous form for the positron operator can be easily obtained by replacing

$$\begin{aligned} \Phi_+ &\to \Phi_-^T \\ V &\to -V^T \end{aligned} \qquad \text{(C1)}$$

(and $n_e^\mu \to n_{\bar{e}}^\mu$, of course) for SQCD, and, correspondingly, by

$$\begin{aligned} \Phi_+ &\to \Phi_- \\ V &\to -V \end{aligned} \qquad \text{(C2)}$$

for SQED. This is in accord with our definition (4), in particular with the minus sign of the positron operator. We again note on this sign, that we could have chosen it differently. And that might have seemed more natural from the point of view of antichiral or vector representation, when drawing an analogy of the electron operator to all possible choices of the positron one (in fact, there are only two choices). But it is only our sign that leads to relations (C1) and (C2) without reverting the sign of the overall expression.

Our first step is to resolve for the auxiliary fields D, $F_\pm$ in these operators to the first order in Lorentz violation. This is done by considering the full lagrangian with all the kinetic terms and with the mass terms:

$$\begin{aligned}
\mathcal{L}_{\text{SQED}} + \mathcal{L}_{\text{LV}}^{\text{matter}} = & \\
= \; & \int d^4\theta\, \overline{\Phi}_+ e^{2eV}\Phi_+ \;+\; \int d^4\theta\, \Phi_- e^{-2eV}\overline{\Phi}_- \;+ \\
+ \; & \frac{1}{16e^2}\int d^2\theta\ \mathrm{Tr}\, WW \;+\; \frac{1}{16e^2}\int d^2\bar\theta\ \mathrm{Tr}\,\overline{WW} \;+ \\
+ \; & \int \{ d^2\theta\, m\,\Phi_-\Phi_+ \;+\; d^2\bar\theta\, \overline{m\,\Phi_+\Phi_-} \} \;+ \\
+ \; & \mathcal{L}_{LV} \;\; .
\end{aligned} \tag{C3}$$

Then we eliminate the auxiliary fields via their equations of motion. The obtained expression can still be reduced, however, on the unperturbed equations of motion of the dynamical fields. That is, derivatives of the fields can be expressed in terms of the fields themselves using the zero order equations of motion (we can use the unperturbed equations as we are only interested in the LV to the first order). Since we are looking for the phenomenological implications of the LV operators, i.e. the effects induced on real observable particles (rather than on their superpartners), we only resolve derivatives of $\psi_\pm$ and $z_\pm$. And, finally, we rewrite the result in terms of Dirac fermions by exploiting (32):

$$\Psi = \begin{pmatrix} \psi_+ \\ \\ \psi_- \end{pmatrix} \quad \text{and} \quad \lambda = \begin{pmatrix} \lambda \\ \\ \overline{\lambda} \end{pmatrix} \quad , \tag{C4}$$

(there should not normally be a confusion due to the same letter λ which designates both Weyl and Majorana spinors, because the two ones never meet). This leads to the following expression:

$$\mathcal{L}_{\text{LV}}^{\text{matter}} = \frac{\mathbf{N}_+^\mu}{\mathbf{M}}\frac{\mathbf{1}}{\mathbf{4}}\,\mathbf{e}\,\overline{\mathbf{\Psi}}\epsilon_{\mu\nu\rho\sigma}\mathbf{F}^{\rho\sigma}\gamma^\nu\mathbf{\Psi} \;+\; \frac{\mathbf{N}_-^\mu}{\mathbf{M}}\frac{\mathbf{1}}{\mathbf{4}}\,\mathbf{e}\,\overline{\mathbf{\Psi}}\epsilon_{\mu\nu\rho\sigma}\mathbf{F}^{\rho\sigma}\gamma^\nu\gamma^{\mathbf{5}}\mathbf{\Psi} \;+$$

$$
\begin{aligned}
&+ \frac{n_e^\mu}{M}\Big[\, \frac{1}{2} i\, e\, \mathcal{D}^\nu \overline{z}_+ \, \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} z_+ \;\; + \;\; \frac{1}{2}\, e \left(\overline{z}_+ F_{\mu\nu} \mathcal{D}^\nu z_+ \; + \; \mathcal{D}^\nu \overline{z}_+ \, F_{\mu\nu} z_+ \right) \\
&\qquad - \; \frac{i}{2}\, e^2 \left(\mathcal{D}_\mu \overline{z}_+ \, T^a z_+ \; - \; \overline{z}_+ T^a \mathcal{D}_\mu z_+ \right) \Big\{ z_- T^a \overline{z}_- \; - \; \overline{z}_+ T^a z_+ \Big\} \Big] \;\; + \\
&+ \frac{n_{\bar{e}}^\mu}{M}\Big[- \frac{1}{2} i\, e\, z_- \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} \mathcal{D}_\nu \overline{z}_- \; - \; \frac{1}{2}\, e \left(z_- F_{\mu\nu} \mathcal{D}^\nu \overline{z}_- \; + \; \mathcal{D}^\nu z_- \, F_{\mu\nu} \overline{z}_- \right) \\
&\qquad - \; \frac{i}{2}\, e^2 \left(\mathcal{D}_\mu z_- T^a \overline{z}_- \; - \; z_- T^a \mathcal{D}_\mu \overline{z}_- \right) \Big\{ z_- T^a \overline{z}_- \; - \; \overline{z}_+ T^a z_+ \Big\} \Big] \;\; + \\
&+ \frac{n_e^\mu}{M}\, e^2\, \overline{z_+ \lambda} \gamma^\mu \gamma^5 \lambda\, z_+ \;\; + \;\; \frac{n_{\bar{e}}^\mu}{M}\, e^2\, z_- \, \overline{\lambda} \gamma^\mu \gamma^5 \lambda\, \overline{z}_- \;\; + \\
&+ \frac{n_{e\mu}}{M} \frac{\sqrt{2}}{2}\, e \left(\overline{\Psi} \gamma^\nu \gamma^\mu P_R \lambda \cdot \mathcal{D}_\nu z_+ \; + \; \mathcal{D}_\nu \overline{z}_+ \cdot \overline{\lambda} \gamma^\mu \gamma^\nu P_L \Psi \right) \;\; - \\
&- \frac{n_{\bar{e}\mu}}{M} \frac{\sqrt{2}}{2}\, e \left(\mathcal{D}_\nu z_- \cdot \overline{\lambda} \gamma^\mu \gamma^\nu P_R \Psi \; + \; \overline{\Psi} \gamma^\nu \gamma^\mu P_L \lambda \cdot \mathcal{D}_\nu \overline{z}_- \right) \;\; - \\
&- \frac{n_e^\mu}{M} \frac{\sqrt{2}}{2}\, e \left(\overline{\Psi} P_R \, \mathcal{D}_\mu \lambda \cdot z_+ \; + \; \overline{z}_+ \, \mathcal{D}_\mu \overline{\lambda}\, P_L \Psi \right) \;\; + \\
&+ \frac{n_{\bar{e}}^\mu}{M} \frac{\sqrt{2}}{2}\, e \left(z_- \, \mathcal{D}_\mu \overline{\lambda}\, P_R \Psi \;\; + \;\; \Psi P_L \, \mathcal{D}_\mu \overline{\lambda}\, \overline{z}_- \right) \;\; - \\
&- \frac{N_{+\mu}}{M} \frac{1}{2}\, e^2\, \overline{\Psi} \gamma^\mu T^a \Psi \cdot \Big\{ z_- T^a \overline{z}_- \; - \; \overline{z}_+ T^a z_+ \Big\} \;\; - \\
&- \frac{N_{-\mu}}{M} \frac{1}{2}\, e^2\, \overline{\Psi} \gamma^\mu \gamma^5 T^a \Psi \cdot \Big\{ z_- T^a \overline{z}_- \; - \; \overline{z}_+ T^a z_+ \Big\} \;\; + \\
&+ \frac{N_+^\mu}{M} \frac{\sqrt{2}}{2}\, i\, e \left(\overline{m\, \Psi} \gamma^\mu P_L \lambda \overline{z}_- \; - \; m\, z_- \overline{\lambda} \gamma^\mu P_L \Psi \right) \;\; - \\
&- \frac{N_+^\mu}{M} \frac{\sqrt{2}}{2}\, i\, e \left(m\, \overline{\Psi} \gamma^\mu P_R \lambda\, z_+ \; - \; \overline{m\, z_+}\, \overline{\lambda} \gamma^\mu P_R \Psi \right) \;\; - \\
&- \frac{\mathbf{N}_{-\mu}}{\mathbf{M}}\, \mathbf{m\overline{m}}\, \overline{\boldsymbol{\Psi}} \gamma^\mu \boldsymbol{\Psi} \;\; + \;\; \frac{N_-^\mu}{M}\, 2i\, m\overline{m}\, \left(\overline{z}_+ \mathcal{D}_\mu z_+ \; + \; z_- \mathcal{D}_\mu \overline{z}_- \right) \quad ,
\end{aligned}
\tag{C5}
$$

($T^a = 1$ for SQED). We have highlighted the operators which fall into the Standard Model sector with the bold font.

---

[1] Throughout this paper, we use predominantly the notations of Wess and Bagger [**?**]. Appendix A summarizes these conventions as well as few cases where we deviate from them.

[2] Our notations in this paper for the background vectors deviate from those of [**?**]. We use n with subscripts for the backgrounds, and capital $N_\pm$ for the emerging linear combinations of them.