#### **Notations**

$$\Phi = \phi + \sqrt{2}\lambda\theta + F\theta^2$$

$$V = -\overline{\theta}\overline{\sigma}_{\mu}\theta A_{\mu} + i\theta^{2}\overline{\theta}\overline{\lambda} + i\overline{\theta}^{2}\theta\lambda - \frac{i}{2}\theta^{2}\overline{\theta}^{2}D$$

$$W^{\alpha} = -\lambda^{\alpha} + \theta^{\alpha}D + \frac{1}{2} \left[\sigma_{\mu}\overline{\sigma}_{\nu}\theta\right]^{\alpha} F_{\mu\nu} - \theta^{2}i\mathcal{D}^{\alpha\dot{\alpha}}\overline{\lambda}_{\dot{\alpha}}$$

$$\sigma_{\mu}^{\alpha\dot{\alpha}} = \left(1, -i\tau^{a}\right), \quad \overline{\sigma}_{\mu\dot{\alpha}\alpha} = \left(1, i\tau^{a}\right)$$

$$D^{\alpha} = \partial^{\alpha} - i \partial^{\alpha \dot{\alpha}} \overline{\theta}_{\dot{\alpha}} \quad \epsilon_{\alpha\beta} = \epsilon_{\dot{\alpha}\dot{\beta}} = \begin{pmatrix} 0 & 1 \\ & & \\ -1 & 0 \end{pmatrix} \quad \psi_{\alpha} = \epsilon_{\alpha\beta} \psi^{\beta} \quad \overline{\psi}_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}} \overline{\psi}^{\dot{\beta}}$$

$$\overline{D}_{\dot{\alpha}} = \overline{\partial}_{\dot{\alpha}} - i \overline{\partial}_{\dot{\alpha}\alpha} \theta^{\alpha} \quad \epsilon^{\alpha\beta} = \epsilon^{\dot{\alpha}\dot{\beta}} = \begin{pmatrix} 0 & -1 \\ & \\ 1 & 0 \end{pmatrix} \quad \psi^{\alpha} = \epsilon^{\alpha\beta} \psi_{\beta} \quad \overline{\psi}^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}} \overline{\psi}_{\dot{\beta}}$$

$$W^2 = W_{\alpha}W^{\alpha}, \qquad \overline{W}^2 = \overline{W^{\dot{\alpha}}W_{\dot{\alpha}}}$$

$$\overline{W}_{\dot{\alpha}} = -\overline{\lambda}_{\dot{\alpha}} + \overline{\theta}_{\dot{\alpha}}D - \frac{1}{2} \left[ \overline{\sigma}_{\mu}\sigma_{\nu}\overline{\theta} \right]_{\dot{\alpha}} F_{\mu\nu} - \overline{\theta}^{2} (i\overline{\mathcal{D}}\lambda)_{\dot{\alpha}} \qquad \text{(unconfirmed)}$$

$$x_L \equiv y \equiv x_\mu + i \overline{\theta \sigma_\mu} \theta , \quad \Phi = \Phi(y) , \quad \overline{\Phi} = \overline{\Phi}(x_R)$$

$$\nabla_{\mu} = \partial_{\mu} - iA_{\mu}^{a} \frac{\tau^{2}}{2} , \text{ for SU(2)}$$

$$V(y) = -\overline{\theta}\overline{\sigma}_{\mu}\theta A_{\mu}(y) + i\theta^{2}\overline{\theta}\overline{\lambda}(y) + i\overline{\theta}^{2}\theta\lambda(y) - \frac{i}{2}\theta^{2}\overline{\theta}^{2}\left[D(y) - \partial_{\mu}A^{\mu}(y)\right]$$

$$W^{\alpha} = -\frac{i}{4}\overline{DD}D^{\alpha}V$$
 (has to be renormalized for  $e^{-2V}$  in the chiral representation)

Shift: 
$$x_L \rightarrow x$$
 is  $e^{-i\overline{\theta}\phi\theta}$ 

## Useful formulae

$$\begin{array}{llll} \theta^{\alpha}\theta^{\beta} & = & \frac{1}{2}\theta^{2}\epsilon^{\alpha\beta} & \theta^{1}\theta^{2} & = & -\frac{1}{2}\theta^{2} \\ \theta_{\alpha}\theta_{\beta} & = & -\frac{1}{2}\theta^{2}\epsilon_{\alpha\beta} & \theta_{1}\theta_{2} & = & -\frac{1}{2}\theta^{2} \\ \overline{\theta}\dot{\alpha}\theta^{\beta} & = & -\frac{1}{2}\overline{\theta}^{2}\epsilon^{\dot{\alpha}\beta} & \overline{\theta}^{1}\theta^{2} & = & \frac{1}{2}\overline{\theta}^{2} \\ \overline{\theta}\dot{\alpha}\theta_{\dot{\beta}} & = & \frac{1}{2}\overline{\theta}^{2}\epsilon^{\dot{\alpha}\beta} & \overline{\theta}^{1}\theta^{2} & = & \frac{1}{2}\overline{\theta}^{2} \\ \overline{\theta}\dot{\alpha}\theta_{\dot{\beta}} & = & \frac{1}{2}\overline{\theta}^{2}\lambda\psi & \overline{\theta}\lambda\overline{\theta}\psi & = & -\frac{1}{2}\overline{\theta}^{2}\lambda\overline{\psi} \\ \sigma_{\mu}\overline{\sigma}_{\nu}\sigma_{\rho} & = & \eta_{\mu\nu}\sigma_{\rho} + \eta_{\nu\rho}\sigma_{\mu} - \eta_{\mu\rho}\sigma_{\nu} - \epsilon_{\mu\nu\rho\lambda}\sigma_{\lambda} \\ \overline{\sigma}_{\mu}\sigma_{\nu}\overline{\sigma}_{\rho} & = & \eta_{\mu\nu}\overline{\sigma}_{\rho} + \eta_{\nu\rho}\overline{\sigma}_{\mu} - \eta_{\mu\rho}\overline{\sigma}_{\nu} + \epsilon_{\mu\nu\rho\lambda}\overline{\sigma}_{\lambda} \\ Tr(\overline{\sigma}^{\mu}\sigma^{\nu}) & = & 2\eta^{\mu\nu} & \overline{\theta}\overline{\sigma}^{\mu}\theta\overline{\theta}\overline{\sigma}^{\nu}\theta & = & \frac{1}{2}\theta^{2}\overline{\theta}^{2}\eta^{\mu\nu} \\ \partial_{\alpha}\theta_{\beta} & = & \epsilon_{\alpha\beta} & \overline{\theta}\overline{\theta}\overline{\theta}^{2} & = & 2\overline{\theta}\dot{\alpha} \\ \partial^{2}\theta^{2} & = & -4 & \overline{\theta}\overline{\theta}^{2} & = & 2\overline{\theta}\dot{\alpha} \\ \partial^{2}\theta^{2} & = & -4 & \overline{\theta}\overline{\theta}^{2} & = & 2\overline{\theta}\dot{\alpha} \\ \varepsilon^{\alpha\gamma}\epsilon_{\gamma\beta} & = & \delta^{\alpha}_{\beta} & \varepsilon^{\dot{\alpha}\dot{\gamma}}\epsilon_{\dot{\gamma}\dot{\beta}} & = & \delta^{\dot{\alpha}}_{\dot{\beta}} \\ \sigma^{(\mu}\overline{\sigma}^{\nu)} & = & 2\eta^{\mu\nu} & \overline{\theta}\overline{\theta}^{2} & = & \overline{\theta}\overline{\theta}^{2} \\ \overline{D}\dot{\alpha}D_{\dot{\alpha}} & = & \overline{\theta}^{2} + & 2i\overline{\theta}\dot{\alpha}\alpha\theta^{\alpha}\overline{\theta}^{\dot{\alpha}} + & \theta^{2}\Box \\ \{D^{\alpha}\overline{D}\dot{\alpha}^{\dot{\alpha}}\} & = & 2i\,\theta^{\alpha\dot{\alpha}} \\ \end{array}$$

# Chiral Superfields

$$\begin{array}{llll} \Phi(y) & = & \phi(x) \; + \; \sqrt{2} \, \theta \psi \; + \; \theta^2 F \; + \; i \, \overline{\theta} \overline{\theta} \, \theta \, \phi \; - \; i \, \frac{\sqrt{2}}{2} \, \theta^2 \, \overline{\theta} \overline{\theta} \, \psi \; - \; \frac{1}{4} \, \theta^4 \, \Box \phi \\ \\ \overline{\Phi}(\overline{y}) & = & \overline{\phi}(x) \; + \; \sqrt{2} \, \overline{\theta} \overline{\psi} \; + \; \overline{\theta}^2 \overline{F} \; + \; i \, \theta \overline{\theta} \, \overline{\theta} \, \overline{\phi} \; - \; i \, \frac{\sqrt{2}}{2} \, \overline{\theta}^2 \, \theta \overline{\theta} \, \overline{\psi} \; - \; \frac{1}{4} \, \theta^4 \, \Box \overline{\phi} \end{array}$$

$$\int d^4\theta = \frac{1}{16} D^2 \overline{D}^2 \Big| \qquad \int d^4\theta = \frac{1}{16} \overline{D}^2 D^2 \Big|$$

$$\int d^2\theta = -\frac{1}{4} D^2 \Big| \qquad \int d^2\overline{\theta} = -\frac{1}{4} \overline{D}^2 \Big|$$

$$\ln \Phi(y) = \ln \phi + \sqrt{2} \theta \psi / \phi + \theta^2 \left( F / \phi + \frac{1}{2} \left( \psi / \phi \right)^2 \right)$$

$$\ln \overline{\Phi}(\overline{y}) = \ln \overline{\phi} + \sqrt{2} \overline{\theta \psi} / \overline{\phi} + \overline{\theta}^2 \left( \overline{F} / \overline{\phi} + \frac{1}{2} \left( \overline{\psi} / \overline{\phi} \right)^2 \right)$$

# $\mathcal{N}=1$ Supersymmetry Transformations

$$Q^{\alpha} = \partial^{\alpha} + i \partial^{\alpha \dot{\alpha}} \overline{\theta}_{\dot{\alpha}}$$
$$\overline{Q}_{\dot{\alpha}} = \overline{\partial}_{\dot{\alpha}} + i \overline{\partial}_{\dot{\alpha}\alpha} \theta^{\alpha}$$

## Formulae for Two Dimensions

#### Chiral superfields

$$\Phi(y) = \phi(y) - \sqrt{2} \theta_R \psi_L + \sqrt{2} \theta_L \psi_R - 2 \theta_R \theta_L F$$

$$\Phi(\overline{y}) \ = \ \overline{\phi}(\overline{y}) \ + \ \sqrt{2}\,\overline{\theta_R\psi}_L \ - \ \sqrt{2}\,\overline{\theta_L\psi}_R \ + \ 2\,\overline{\theta_R\theta}_L\,\overline{F}$$

$$y^0 = x^0 + i(\overline{\theta}_R \theta_R + \overline{\theta}_L \theta_L) \qquad y^3 = x^3 + (\overline{\theta}_R \theta_R - \overline{\theta}_L \theta_L)$$

$$\overline{y}^0 = x^0 - i(\overline{\theta}_R \theta_R + \overline{\theta}_L \theta_L) \qquad \overline{y}^3 = x^3 - (\overline{\theta}_R \theta_R - \overline{\theta}_L \theta_L)$$

#### Twisted Chiral superfields

$$\Sigma(\widetilde{y}) = \sigma(\widetilde{y}) - \sqrt{2} \theta_R \overline{\lambda}_L + \sqrt{2} \overline{\theta}_L \lambda_R + \sqrt{2} \theta_R \overline{\theta}_L \left( D - i F_{03} \right)$$

$$\overline{\Sigma}(\overline{\widetilde{y}}) = \overline{\sigma}(\overline{\widetilde{y}}) - \sqrt{2}\theta_L \overline{\lambda}_R + \sqrt{2}\overline{\theta}_R \lambda_L + \sqrt{2}\theta_L \overline{\theta}_R \left(D + iF_{03}\right)$$

$$\widetilde{y}^{\mu} = x^{\mu} + i \overline{\theta \widetilde{\sigma}}_{\mu} \theta$$
  $\widetilde{\sigma}_{0} = i \sigma_{3}$   $\widetilde{\sigma}_{3} = -i \sigma_{0}$ 

$$\overline{\widetilde{y}}^{\mu} = x^{\mu} + i \theta \widetilde{\sigma}_{\mu} \overline{\theta} \qquad \overline{\widetilde{\sigma}}_{0} = i \overline{\sigma}_{3} \qquad \overline{\widetilde{\sigma}}_{3} = -i \overline{\sigma}_{0}$$

$$\widetilde{y}^0 = x^0 + i(\overline{\theta}_R \theta_R - \overline{\theta}_L \theta_L) \qquad \widetilde{y}^3 = x^3 + (\overline{\theta}_R \theta_R + \overline{\theta}_L \theta_L)$$

$$\overline{\widetilde{y}}^{0} = x^{0} - i \left( \overline{\theta}_{R} \theta_{R} - \overline{\theta}_{L} \theta_{L} \right) \qquad \overline{\widetilde{y}}^{3} = x^{3} - \left( \overline{\theta}_{R} \theta_{R} + \overline{\theta}_{L} \theta_{L} \right)$$

$$\widetilde{\partial} = \begin{pmatrix} \partial_L & \\ & -\partial_R \end{pmatrix}^{\alpha\dot{\alpha}} \qquad \widetilde{\overline{\partial}} = \begin{pmatrix} -\partial_R & \\ & \partial_L \end{pmatrix}_{\dot{\alpha}\dot{\alpha}}$$

$$D_R \ = \ \frac{\partial \! /}{\partial \theta_R} \ - \ 2i\overline{\theta}_R \partial_L \Big|_{\widetilde{y}} \ = \ \frac{\partial \! /}{\partial \theta_R} \Big|_{\overline{\widetilde{y}}} \ \overline{D}_R \ = \ \frac{\partial \! /}{\partial \overline{\theta}_R} \Big|_{\widetilde{y}} \ = \ \frac{\partial \! /}{\partial \overline{\theta}_R} \ - \ 2i\theta_R \partial_L \Big|_{\overline{\widetilde{y}}}$$

$$D_L \ = \ \frac{\partial\!\!/}{\partial\theta_L}\Big|_{\widetilde{y}} \ = \ \frac{\partial\!\!/}{\partial\theta_L} \ - \ 2i\overline{\theta}_L\partial_R\Big|_{\overline{\widetilde{y}}} \quad \overline{D}_L \ = \ \frac{\partial\!\!/}{\partial\overline{\theta}_L} \ - \ 2i\theta_L\partial_R\Big|_{\widetilde{y}} \ = \ \frac{\partial\!\!/}{\partial\overline{\theta}_L}\Big|_{\overline{\widetilde{y}}}$$

$$\int d^4\theta = \frac{1}{4} \overline{D}_R D_R \overline{D}_L D_L \Big| \qquad \int d^4\theta = -\frac{1}{4} \overline{D}_R D_L \overline{D}_L D_R \Big| 
\int d^2 \widetilde{\theta} = \frac{1}{2} \overline{D}_L D_R \Big| \qquad \int d^2 \overline{\widetilde{\theta}} = \frac{1}{2} \overline{D}_R D_L \Big| 
D_R \sqrt{2} \Sigma \Big| = -2 \overline{\lambda}_L \qquad \overline{D}_R \sqrt{2} \overline{\Sigma} \Big| = +2 \lambda_L 
\overline{D}_L \sqrt{2} \Sigma \Big| = +2 \lambda_R \qquad D_L \sqrt{2} \overline{\Sigma} \Big| = -2 \overline{\lambda}_R 
\frac{1}{2} \overline{D}_L D_R \sqrt{2} \Sigma \Big| = D - i F_{03} \qquad \frac{1}{2} \overline{D}_R D_L \sqrt{2} \overline{\Sigma} \Big| = D + i F_{03}$$

$$\ln \Sigma(\widetilde{y}) = \ln \sigma - \sqrt{2} \,\theta_R \overline{\lambda}_L / \sigma + \sqrt{2} \,\overline{\theta}_L \lambda_R / \sigma + \theta_R \overline{\theta}_L \left( \frac{\sqrt{2} \,(D - iF_{03})}{\sigma} - 2 \frac{\overline{\lambda}_L}{\sigma} \frac{\lambda_R}{\sigma} \right)$$

$$\ln \overline{\Sigma}(\widetilde{\widetilde{y}}) = \ln \overline{\sigma} - \sqrt{2} \,\theta_L \,\overline{\lambda}_R / \overline{\sigma} + \sqrt{2} \,\overline{\theta}_R \,\lambda_L / \overline{\sigma} + \theta_L \overline{\theta}_R \left( \frac{\sqrt{2} \,(D + iF_{03})}{\overline{\sigma}} - 2 \frac{\overline{\lambda}_R}{\overline{\sigma}} \frac{\lambda_L}{\overline{\sigma}} \right)$$

#### Twisted Chiral Superfield S

$$S = \frac{i}{2}\overline{D}_{R}D_{L} \ln \sqrt{2}\overline{\Sigma} \qquad \overline{S} = \frac{i}{2}\overline{D}_{L}D_{R} \ln \sqrt{2}\Sigma$$

$$S = \frac{i}{2}\overline{D}_{L}D_{R} \ln \sqrt{2}\Sigma$$

$$S = \frac{i}{2}\overline{D}_{L}D_{R} \ln \sqrt{2}\Sigma$$

$$S = \frac{\sqrt{2}\sigma (iD - F_{03}) - 2i\overline{\lambda}_{L}\lambda_{R}}{(\sqrt{2}\sigma)^{2}}$$

$$D_{R}S = \partial_{L}\frac{D_{L}\sqrt{2}\overline{\Sigma}}{\sqrt{2}\overline{\Sigma}} \qquad D_{R}S = -2\partial_{L}\frac{\overline{\lambda}_{R}}{\sqrt{2}\sigma}$$

$$\overline{D}_{R}S = -\partial_{L}\frac{\overline{D}_{L}\sqrt{2}\Sigma}{\sqrt{2}\Sigma} \qquad \overline{D}_{R}S = -2\partial_{L}\frac{\lambda_{R}}{\sqrt{2}\sigma}$$

$$\overline{D}_{L}S = -\partial_{R}\frac{\overline{D}_{R}\sqrt{2}\Sigma}{\sqrt{2}\overline{\Sigma}} \qquad \overline{D}_{L}S = -2\partial_{R}\frac{\lambda_{L}}{\sqrt{2}\sigma}$$

$$D_{L}\overline{S} = \partial_{R}\frac{D_{R}\sqrt{2}\Sigma}{\sqrt{2}\Sigma} \qquad D_{L}\overline{S} = -2\partial_{R}\frac{\overline{\lambda}_{L}}{\sqrt{2}\sigma}$$

$$\frac{1}{2}\overline{D}_{L}D_{R}S = -i\Box \ln \sqrt{2}\Sigma \qquad \frac{1}{2}\overline{D}_{L}D_{R}S = -i\Box \ln \sqrt{2}\sigma$$

$$\frac{1}{2}\overline{D}_{R}D_{L}\overline{S} = -i\Box \ln \sqrt{2}\Sigma \qquad \frac{1}{2}\overline{D}_{R}D_{L}\overline{S} = -i\Box \ln \sqrt{2}\sigma$$

#### Vector superfield

$$V(x) = -A_L \overline{\theta}_R \theta_R - A_R \overline{\theta}_L \theta_L - i\sqrt{2} \sigma \overline{\theta}_R \theta_L - i\sqrt{2} \overline{\sigma} \overline{\theta}_L \theta_R - 2i \overline{\theta}_R \theta_L \overline{\theta}_{[R} \lambda_{L]} - 2i \overline{\theta}_R \theta_L \overline{\theta}_{[R} \lambda_{L]} - 2i \overline{\theta}_R \theta_R \overline{\theta}_L \theta_L D(x)$$

$$\begin{array}{lclcrcl} V(x) & = & - & A_L \, \overline{\theta}_R \theta_R \, - \, A_R \, \overline{\theta}_L \theta_L & - & i \, \sqrt{2} \, \sigma \, \overline{\theta}_R \theta_L & - & i \, \sqrt{2} \, \overline{\sigma} \, \overline{\theta}_L \theta_R & - \\ \\ & & - & 2 \, i \, \theta_R \theta_L \, \overline{\theta}_{[R} \overline{\lambda}_{L]} & - & 2 \, i \, \overline{\theta}_R \overline{\theta}_L \, \theta_{[R} \lambda_{L]} & - & 2 \, i \, \overline{\theta}_R \theta_R \overline{\theta}_L \theta_L \, \left(D \, \mp \, i \, F_{03} \right)_{\widetilde{y}, \overline{\widetilde{y}}} \end{array}$$

$$A_R = A_0 + i A_3$$
  $\sigma = -\frac{A_1 + i A_2}{\sqrt{2}}$   $\partial_{[R} A_{L]} = -2i F_{03}$ 

$$\Sigma = \frac{i}{\sqrt{2}} D_L \overline{D}_R V \qquad \overline{\Sigma} = \frac{i}{\sqrt{2}} D_R \overline{D}_L V$$

# Vortices

$$e^{i\alpha n\overline{n}}\cdot \hat{M}\cdot e^{-i\alpha n\overline{n}} \ = \ \hat{M} \ + \ i\,\sin\alpha\left[\,n\overline{n},\,\hat{M}\,\right] \ - \ (1 \, - \, \cos\alpha)\left[\,n\overline{n}\left[\,n\overline{n},\,\hat{M}\,\right]\,\right]$$

## **Profiles**

$$A_{\pm}^{\rm U(1)} = \mp i \frac{x^{\pm}}{r^2} \frac{1}{N} f(r)$$

$$A_{\pm}^{\rm SU(N)} = \mp i \frac{x^{\pm}}{r^2} f_N(r) \left( n\overline{n} - 1/N \right)$$

$$\partial_r \left( \phi_1 \, \phi_2^{N-1} \right) = \frac{f}{r} \cdot \phi_1 \, \phi_2^{N-1}$$

$$\partial_r \, \frac{\phi_1}{\phi_2} = \frac{f_N}{r} \, \frac{\phi_1}{\phi_2}$$

## Gauge Field Normalization

$$\hat{\mu}_1 = \sqrt{\frac{2}{N}} \mu_1 \qquad \hat{A}^a = \frac{1}{\sqrt{2N}} A^a \qquad \hat{T}^a = \sqrt{2N} T^a$$