

Two-dimensional — four-dimensional duality:
Towers of kinks \leftrightarrow towers of monopoles
in $\mathcal{N} = 2$ theories

Pavel A. Bolokhov

University of Minnesota · St.Petersburg State University

May 8, 2012

Two-dimensional models have interesting similarities to 4-d gauge theories

CP^{N-1} theory has been shown to have chiral symmetry breaking, mass gap, asymptotic freedom, *etc* and all these properties are much easier to show in two dimensions

In general, two-dimensional theories are simpler and two-dimensional methods are more powerful

There is also a two-dimensional — four-dimensional duality of their BPS spectra

$\mathcal{N} = 2$ $N_c = N_f$ SYM in four dimensions at the root of the first baryonic Higgs branch



$\mathcal{N} = (2, 2)$ CP^{N-1} theory in two dimensions



supports non-Abelian vortex strings



kinks interpolate between worldsheet vacua

monopoles



kinks

strings



vacua

At weak coupling the Seiberg-Witten theory contains Quarks, W-bosons, Monopoles, Dyons and bound states

At strong coupling — we consider CP^{N-1} theory

Strong coupling spectrum is accessible via the mirror theory

CP^{N-1} theory with *twisted* masses:

$$r \left(|\mathcal{D}_\mu n^l|^2 + |\sigma - m^l|^2 |n^l|^2 + i D (|n^l|^2 - 1) + \dots \right) \\ + \frac{1}{4e^2} F_{\mu\nu}^2 + \frac{1}{e^2} |\partial_\mu \sigma|^2 + \frac{1}{2e^2} D^2 + \dots,$$

in the $e^2 \rightarrow \infty$ limit

The theory has N vacua — both classically and exactly

CP^{N-1} theory with *twisted* masses:

$$r \left(|\mathcal{D}_\mu n^l|^2 + |\sigma - m^l|^2 |n^l|^2 + i D (|n^l|^2 - 1) + \dots \right) \\ + \frac{1}{4e^2} F_{\mu\nu}^2 + \frac{1}{e^2} |\partial_\mu \sigma|^2 + \frac{1}{2e^2} D^2 + \dots,$$

in the $e^2 \rightarrow \infty$ limit

CP^{N-1} model with \mathcal{Z}_N twisted masses

$$m_l = m_0 \cdot e^{2\pi i l/N}$$

in this case $\mathcal{Z}_N \subset U_R(1)$ remains unbroken

Exact superpotential

The theory possesses an “exact” superpotential of Veneziano-Yankielowicz type

$$\mathcal{W}_{\text{eff}} = -i\tau\hat{\sigma} + \frac{1}{2\pi} \sum_l (\hat{\sigma} - m_l) \left(\ln \frac{\hat{\sigma} - m_l}{\mu} - 1 \right)$$

μ = UV cut-off scale

Exact superpotential

Vacuum values:

$$\mathcal{W}_{\text{eff}} = -\frac{1}{2\pi} \left(N \sigma_p + \sum_l m_l \ln (\sigma_p - m_l) \right)$$

where

$$\sigma_p = \sigma_0 \cdot e^{2\pi i p/N}$$

$$\sigma_0 = \sqrt[N]{1 + m_0^N}$$

vacuum equation

$$\prod_l (\sigma - m_l) = 1.$$

Mirror dual of \mathbb{CP}^{N-1}

The mirror dual is the affine Toda theory

$$\mathcal{W}_{\text{mirror}}^{\mathbb{CP}^{N-1}} = -\frac{1}{2\pi} \left(x_1 + x_2 + \dots + x_n + \sum m_l \ln x_l \right),$$

$$x_1 x_2 \dots x_n = 1$$

Only the superpotential is known in that theory

The vacuum values coincide with $\mathcal{W}_{\text{eff}}(\sigma_p)$ upon identification

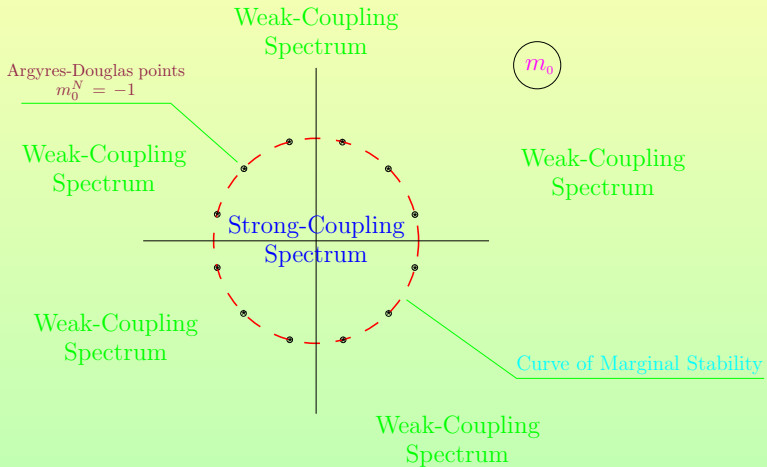
$$x_l^{(p)} = \sigma_p - m_l$$

Based on work of K.Hori and C.Vafa, N mirror kinks can be found at strong coupling

$$|m_k| \leq 1$$

Limiting to the sector interpolating between 0^{th} and 1^{st} vacua

$$\mathcal{Z} = \mathcal{W}(\sigma_1) - \mathcal{W}(\sigma_0) + i m_k, \quad k = 0, \dots, N-1,$$



The spectrum

$$\mathcal{Z} = \mathcal{W}(\sigma_1) - \mathcal{W}(\sigma_0) + i m_k, \quad k = 0, \dots, N-1,$$

describes N dyonic kinks (of either \mathbb{CP}^{N-1} or the mirror theory) in the fundamental of $\mathrm{SU}(N)$

The spectrum

$$\mathcal{Z} = \mathcal{W}(\sigma_1) - \mathcal{W}(\sigma_0) + i m_k, \quad k = 0, \dots, N-1,$$

describes N dyonic kinks (of either CP^{N-1} or the mirror theory) in the fundamental of $\mathrm{SU}(N)$

Question:

What happens to them at weak coupling?

The spectrum

$$\mathcal{Z} = \mathcal{W}(\sigma_1) - \mathcal{W}(\sigma_0) + i m_k, \quad k = 0, \dots, N-1,$$

describes N dyonic kinks (of either CP^{N-1} or the mirror theory) in the fundamental of $\mathrm{SU}(N)$

Question:

What happens to them at weak coupling?

What states do they correspond to?

Previously known picture

Weak coupling:

$$\mathcal{Q}_{ik}, \quad \mathcal{M}_{ik}, \quad \mathcal{D}_{ik}^n$$

$$\mathcal{D}_{ik}^n + Q \quad \text{---} \quad \text{bound states}$$

$$\mathcal{Q}_{ik} = i(m_i - m_k)$$

$$\mathcal{M}_{ik} \text{ --- purely topological kink interpolating from } (k) \rightarrow (i)$$

$$\mathcal{D}_{ik}^n = \mathcal{M}_{ik} + i n(m_i - m_k) \text{ --- tower of dyonic kinks upon quasiclassical quantization}$$

\mathbb{CP}^2 theory

We focus on \mathbb{CP}^2 model — the first non-trivial theory.

We find the curves of marginal stability (*c.m.s.*) — analogues of wall crossing — for this model, where the spectrum can change due to decays

\mathbb{CP}^2 theory

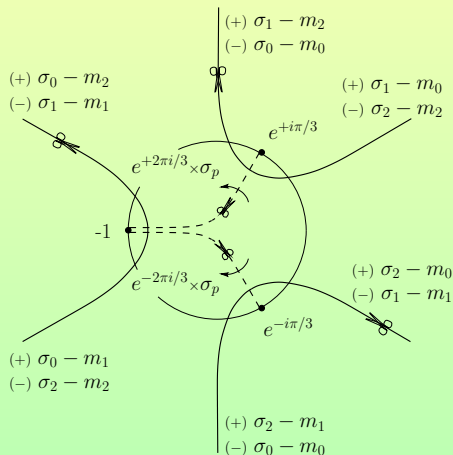
We focus on \mathbb{CP}^2 model — the first non-trivial theory.

We find the curves of marginal stability (*c.m.s.*) — analogues of wall crossing — for this model, where the spectrum can change due to decays

c.m.s. are a supersymmetric version of a “phase transition”

there are practically no phase transitions in supersymmetric theories except for, perhaps, when supersymmetry is broken, or for theories with $N_c \rightarrow \infty$

Moduli space of \mathbb{CP}^2 — plane of m_0

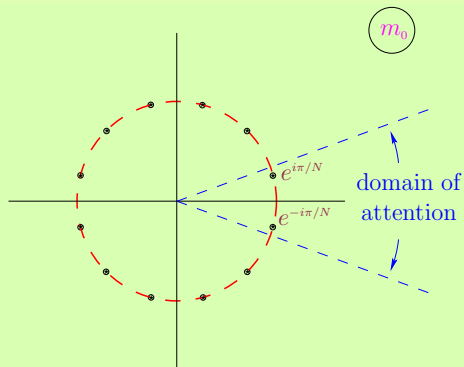


there are three \mathbb{Z}_3 -equivalent sectors

So we choose only one topological sector:

$$\text{kinks:} \quad (0) \longrightarrow (1)$$

and one sector in m_0 -plane:



the other two sectors are completely equivalent

other kinks have the same masses, just central charges shifted by a phase

Weak coupling spectrum

$$\mathcal{Q}_{ik}$$

$$\mathcal{M}_{10}$$

$$\mathcal{D}_{10}^{(n)}$$

$$\mathcal{D}_{10}^{(n)} \mathcal{Q}_{02}$$

Weak coupling spectrum

decays



\mathcal{Q}_{ik}

\mathcal{M}_{10}

$\mathcal{D}_{10}^{(n)}$

$\mathcal{D}_{10}^{(n)} \mathcal{Q}_{02}$

Weak coupling spectrum

decays



\mathcal{Q}_{ik}

\mathcal{M}_{10}

decay all
but one



$\mathcal{D}_{10}^{(n)}$

$\mathcal{D}_{10}^{(n)} \mathcal{Q}_{02}$

Weak coupling spectrum

decays



\mathcal{Q}_{ik}

\mathcal{M}_{10}

decay all
but one



$\mathcal{D}_{10}^{(n)}$

all decay



$\mathcal{D}_{10}^{(n)} \mathcal{Q}_{02}$

Strong coupling spectrum

$$\mathcal{W}_1 = \mathcal{W}_0 + i m_0$$

$$\mathcal{W}_1 = \mathcal{W}_0 + i m_1$$

$$\mathcal{W}_1 = \mathcal{W}_0 + i m_2$$

Strong coupling spectrum

$$\mathcal{W}_1 = \mathcal{W}_0 + i m_0$$

$$\mathcal{W}_1 = \mathcal{W}_0 + i m_1$$

$$\mathcal{W}_1 = \mathcal{W}_0 + i m_2 \quad \text{--- the lightest}$$

Strong coupling spectrum

$$\mathcal{W}_1 = \mathcal{W}_0 + i m_0$$

$$\mathcal{W}_1 = \mathcal{W}_0 + i m_1$$

decays \Leftarrow $\mathcal{W}_1 = \mathcal{W}_0 + i m_2$ — the lightest

Strong coupling spectrum

$$\mathcal{W}_1 = \mathcal{W}_0 + i m_0 \quad \text{--- next lightest}$$

$$\mathcal{W}_1 = \mathcal{W}_0 + i m_1$$

$$\text{decays} \iff \mathcal{W}_1 = \mathcal{W}_0 + i m_2 \quad \text{--- the lightest}$$

Strong coupling spectrum

$$\mathcal{M}_{10} \iff \mathcal{W}_1 - \mathcal{W}_0 + i m_0 \quad \text{--- next lightest}$$

$$\mathcal{W}_1 - \mathcal{W}_0 + i m_1$$

$$\text{decays} \iff \mathcal{W}_1 - \mathcal{W}_0 + i m_2 \quad \text{--- the lightest}$$

Strong coupling spectrum

$$\mathcal{M}_{10} \iff \mathcal{W}_1 - \mathcal{W}_0 + i m_0 \quad \text{--- next lightest}$$

$$\mathcal{W}_1 - \mathcal{W}_0 + i m_1 \quad \text{--- the heavier}$$

$$\text{decays} \iff \mathcal{W}_1 - \mathcal{W}_0 + i m_2 \quad \text{--- the lightest}$$

Strong coupling spectrum

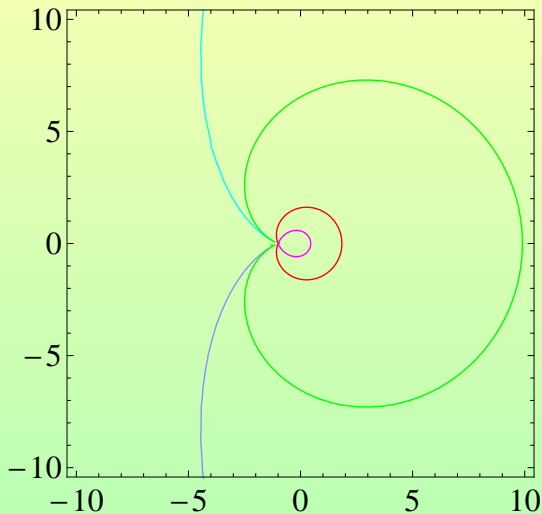
$$\mathcal{M}_{10} \iff \mathcal{W}_1 - \mathcal{W}_0 + i m_0 \quad \text{--- next lightest}$$

$$\mathcal{D}_{10} \iff \mathcal{W}_1 - \mathcal{W}_0 + i m_1 \quad \text{--- the heavier}$$

$$\text{decays} \iff \mathcal{W}_1 - \mathcal{W}_0 + i m_2 \quad \text{--- the lightest}$$

Strong coupling spectrum

$$\begin{array}{llllll}
 \mathcal{M}_{10} & \Leftarrow & \mathcal{W}_1 - \mathcal{W}_0 + i m_0 & \text{---} & \text{next lightest} & \leftarrow \text{build their own tower} \\
 \mathcal{D}_{10} & \Leftarrow & \mathcal{W}_1 - \mathcal{W}_0 + i m_1 & \text{---} & \text{the heavier} & \leftarrow \\
 \text{decays} & \Leftarrow & \mathcal{W}_1 - \mathcal{W}_0 + i m_2 & \text{---} & \text{the lightest} &
 \end{array}$$

$C.M.S.$ 

Decays

Simple decays:

$$\mathcal{D}^{(n)} Q \longrightarrow \mathcal{D}^{(n)} + Q \quad n < 1$$

$$\mathcal{D}^{(n)} Q \longrightarrow \mathcal{D}^{(n-1)} + \tilde{Q} \quad n > 1$$

Decays

Simple decays:

$$\mathcal{D}^{(n)} Q \longrightarrow \mathcal{D}^{(n)} + Q \quad n < 1$$

$$\mathcal{D}^{(n)} Q \longrightarrow \mathcal{D}^{(n-1)} + \tilde{Q} \quad n > 1$$

Topological decays:

$$\mathcal{D}_{10}^{(1)} Q \longrightarrow \overline{\mathcal{M}}_{02} + \overline{\mathcal{D}}_{21}^{(1)}$$

$$\mathcal{V}_{10}^2 \longrightarrow \overline{\mathcal{D}}_{02} + \overline{\mathcal{M}}_{21}$$

Generalizing to higher N

Two states become part of the “monopole” tower

Generalizing to higher N

Two states become part of the “monopole” tower

All extra states decay

Generalizing to higher N

Two states become part of the “monopole” tower

All extra states decay

(Indeed for even N there are *no bound states*)

Generalizing to higher N

Two states become part of the “monopole” tower

All extra states decay

(Indeed for even N there are *no bound states*)

We can infer that most likely they decay for all N

SQCD in four dimensions

there are also N states at strong coupling

SQCD in four dimensions

there are also N states at strong coupling

our results indicate that two of them are part of one tower of dyons

SQCD in four dimensions

there are also N states at strong coupling

our results indicate that two of them are part of one tower of dyons

other states decay before reaching the weak coupling

Thank you