Agenda

1) More problems on Recursion
2) TC / SC analysis.

Q Given a number. Write a recursive code to calculate the sum of digits of given number.

Soln

Total sum

1 2 3 4 → 1+2+3+4 = 10

(123)

N % 10 = last digit

+

Sum(N / 10) = no. formed by remaining digits

552 = 55

10

Code

int calculateSum(N){

// Ass: calculateSum(N) returns the correct digit sum of the no. N.

// Base Case
if (N < 10) { return N; }

return (N % 10) + calculateSum(N/10);

}

calculate(1234) {

return $\overset{(N\%10)}{4}$ + calculate(123)

}

N digit ⇒ (N−1) digits ⇒ (N−2)

⋮

1 digit no. 7 → N

Q-2 Implement pow function

pow(a, n) ⇒ a^n

pow(2,3) = 8

$a^n = \underbrace{a \times a \times a \times a \cdots\cdots a}$ (N times)

$a \times a^{n-1}$

$2^5 = 2 \times 2^4$

$n = 0$

```
int pow(a, n) {
    // Base Case
    if (n==0) { return 1; }
    if (n==1) { return a; }

    return a * pow(a, n-1);
}
```

$2^5 \rightarrow 2^4 \rightarrow 2^3 \rightarrow 2^2 \rightarrow 2^1 \rightarrow 2^0$

$2^1 \Rightarrow 2$, $2^0 \Rightarrow 1$

$N \Rightarrow (N-1)$

a^5

$\hookrightarrow a \times a^4$

$\hookrightarrow a \times a^3$

$\hookrightarrow a \times a^2$

$\hookrightarrow a \times a^1$ a

$a^N \simeq O(N) \simeq T.C = O(N)$

Eg $\Rightarrow$

$$a^{10} \to a^1 \times a^9$$

$$a^{10} \to a^5 \times a^5 \quad (1\ RC)$$

$$a^{15} = a \times a^{14}$$

$$= a \times a^7 \times a^7 \quad (1\ RC)$$

N is even: $a^{N/2} \times a^{N/2}$

$pow(a, N)$

$a \times a^{N/2} \times a^{N/2}$

$$a^{15} = a \times a^7 \times a^7$$

$$\frac{N-1}{2} \qquad \frac{15-1}{2} \qquad \frac{15}{2} = 7$$

Code

```
int pow(a,n){
  if(n==0){ return 1; }
  if (n&1) {
    x = pow(a, (n-1)/2);
```

fast power method

```
int pow(a, N){
  if (n==0){ return 1; }
  x = pow(a, n/2);
  xsquare = x*x;
  if (n&1) {
    return a * xsquare;
```

```
        return a×x×x;
    }
    else {
        x = pow(a, n/2);
        return x²;
    }
}
```

```
}
else {
    return xsquare;
}
```

N

$a^{32} \rightarrow a^{16} \times a^{16}$

$a^{8} \times a^{8}$

$a^{16} \rightarrow a^{8} \times a^{8} \rightarrow a^{4} \times a^{4} \rightarrow a^{2} \times a^{2} \rightarrow a^{1} \times a^{1}$ $(N = 1)$

$N \Rightarrow N/2$ $\log N$ steps

$$T.C. = O(\log_2 N)$$

$a^n \uparrow$ $\quad 2^{32} - 1$ (Unsigned int)

2^{40} $\quad (100)^{250}$

$\Rightarrow$ mod

Google

Q

$pow(a, n, m) \Rightarrow a^N \,\%\, m$

Eg $\rightarrow$ $a = 5$, $N = 3$, $M = 20$

5^3

$(125) \% 20 = 5$ Ans

Code

$T(N)$ $\qquad 10^9+7$

```
int pow (a, n, m) {
    if (n==0) { return 1; }              // O(1)
                 T(N/2)
    x = pow (a, n/2, m);                 // O(1)
//O(1) xsquare = ((x % m) × (x % m)) % m;
O(1)   if (n&1) {
O(1)        return ((a % m) × (xsquare)) % m;
       }
       else
O(1)        return xsquare;
}
```

}

$$a \% m = [0, 10^9+7-1]$$

$$10^5 \times 10^5 = 10^{10} \% (10^9+7)$$

pow(5, 3, 20) {

$x = pow(5, \cancel{1}, 20);$ 5 $[0, m-1]$

$xsquare = ((x \% 20) \times (x \times 20)) \% 20$

$(5 \times 5) \% 20$

$(25) \% 20$

} return $(5 \times \overset{25}{5}) \% 20$ (5)

pow(5, 1, 20) {

$x = pow(5, \overset{1}{\cancel{0}}, 20);$

$xsquare = ((1 \% 20) \times (1 \% 20)) \% 20$

return $(5 \times 1) \% 20$ (5)

}

Calculate x (very sig)

$[0 - M-1]$

$(2M+4) \% M = 4$

$(3M+4) \% M = 4$

$(2M+8) \% M = 8$

TC of Recursion

1) $sum(N) = N + sum(N-1)$ (Main Logic)

Assume: $T(N)$ $\quad\quad$ $T(N-1)$

$$T(N) = T(N-1) + 1 \quad \text{--- } (1)$$

$T(N) \Rightarrow$ TC of $sum(N)$

$T(N-1) \Rightarrow$ TC of $sum(N-1)$

In eqn(1) put $N = N-1$

$$T(N-1) = (T(N-2) + 1)$$

put this in eqn ①

$$\Rightarrow T(N) = (T(N-2)+1) + 1$$

$$T(N) = T(N-2) + 2 \quad — ②$$

$$T(N-2) = T(N-3) + 1$$

$$T(N) = (T(N-3) + 1) + 2$$

$$T(N) = T(N-3) + 3 \quad — ③$$

⋮ K times (

$$T(N) = T(N-K) + K \quad —$$

Base Case

$$N - K = 0 \quad (K = N)$$

After N steps

$$T(N) = T(\underset{O(1)}{0}) + N$$

$$T(N) = 1 + N$$

$$T.C. = O(N)$$

Ex

$$\overset{TC}{T(N)} = 2\,T(N-1) + 1 \qquad T(0) = 1$$

$N = N-2$

$N = N-1$

$$T(N-1) = 2T(N-2) + 1$$

$$T(N) = 2\big(2T(N-2) + 1\big) + 1$$

$$T(N) = 4\big[T(N-2)\big] + 2 + 1$$

$$T(N) = 4\big[2T(N-3) + 1\big] + 2 + 1$$

$$T(N) = \underset{2^3}{\underline{8}}\big[T(N-3)\big] + \underbrace{4 + 2 + 1}_{2^3 - 1}$$

$$T(N) = 8\big[2T(N-4) + 1\big] + 4 + 2 + 1$$

$$T(N) = \underset{2^4}{\underline{16}}\,T(N-4) + \underbrace{8 + 4 + 2 + 1}_{(2^4 - 1)}$$

$$T(N) = 2^4(T(N-4)) + 2^4 - 1$$

$\vdots$

K steps

$$T(N) = 2^K(T(\underbrace{N-K}_{=0})) + 2^K - 1$$

$K = N$

$$T(N) = 2^N(\cancel{T(0)}^{1}) + 2^N - 1$$

$$T(N) = 2^N + 2^N - 1$$

$$T(N) = 2^{N+1} - 1$$

T.C. $= O(2^N)$

H.W.

$$T(N) = T(N-1) + T(N-2) + 1$$

$T(0) = 1$ $T(2) = 1$

$T(1) = 1$

$$T(N) = T(N/2) + 1 \qquad (=1)$$

power func

$$N = N/2$$

$$T(N/2) = T(N/4) + 1$$

$$T(N) = T(N/4) + 2$$

$$T(N) = \left(T\left(N/8\right) + 1\right) + 2$$

$$= T(N/8) + 3$$

$$T(N) = T\left(N/2^3\right) + 3$$

$$\vdots$$

$$T(N) = \underbrace{T\left(\cancel{\frac{N}{2^k}}\right)}_{\cancel{T(1)}\ 1} + k$$

$$\frac{N}{2^k} = 1 \qquad \frac{N}{2^k} = 0$$

$$N = 2^k$$

$$\log_2 N = \log_2 2^k$$

$$K = \log_2 N$$

$$T(N) = 1 + \log_2 N$$

$$T.C. = O(\log N)$$

Q pow $\Rightarrow$ return $a \times pow(a, n/2) \times pow(a, n/2)$

$$T(N) = 2 \times T(N/2) + 1$$

$N = N/2$

$$T(N/2) = 2 \times \left(T(N/4)\right) + 1$$

$$T(N) = 2\left(2T\left(\frac{N}{4}\right) + 1\right) + 1$$

$$= 4T\left(\frac{N}{4}\right) + 2 + 1$$

$$= 8T\left(\frac{N}{8}\right) + 4 + 2 + 1$$

$$= 16T\left(\frac{N}{}\right) + 8 + \overset{4+}{2} + 1$$

$$= 2^4 \, T\left(\frac{N}{2^4}\right) + 2^4 - 1$$

$\vdots$

k steps

$$\Rightarrow T(N) = 2^k \, T\left(\frac{N}{2^k}\right) + 2^k - 1$$

$$\frac{N}{2^k} = 1 \Rightarrow k = \log_2 N$$

$$T(N) = \cancel{2^{\log_2 N}}^{\,N} \, T(1) + \cancel{2^{\log_2 N}}^{\,N} - 1$$

$$T(N) = N + N - 1$$

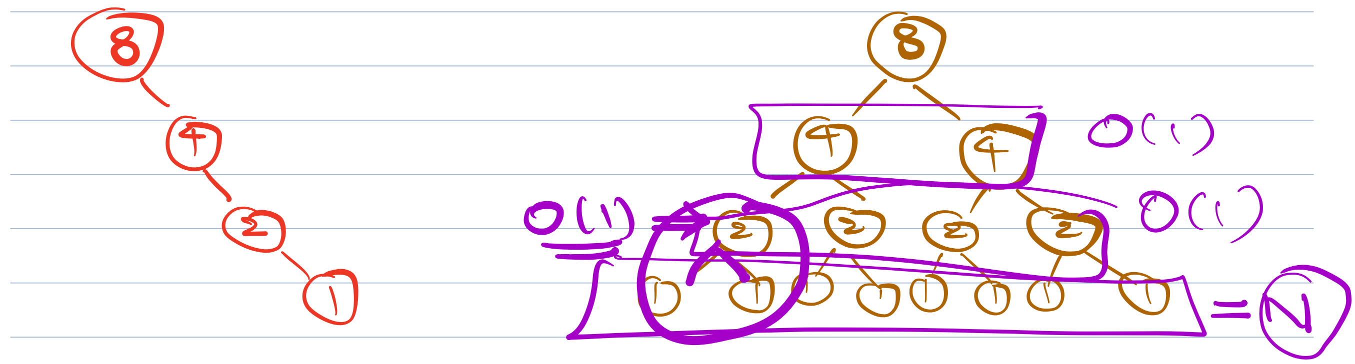
$$T.C. = O(N)$$

$$\log_2 8 = 3$$

$$(\log_2 N) \quad x$$

$$2^x = N$$

$2^{?} = N$



a^2 , a a

H.W

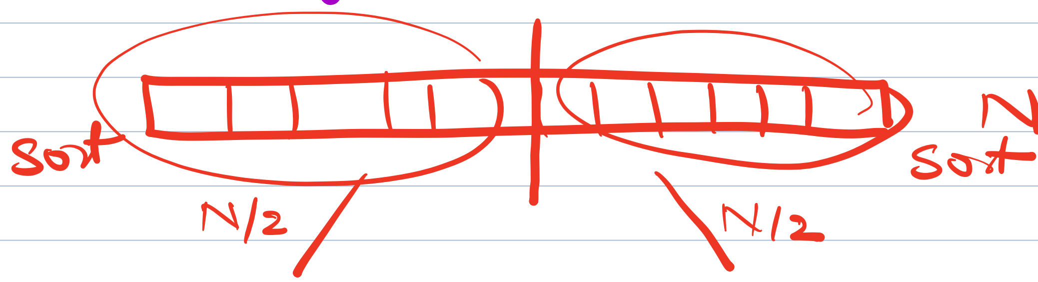
(1) $T(N) = 2T(N/2) + O(N) \Rightarrow$

$T(1) = 1$

$T(0) = 1$

$\Rightarrow N \log N$

$\Rightarrow$ Merge Sort (Recursion)



(N/2) (N/2)

O(N)

O(N)

Doubt

$x = 5;$

$y = x{+}{+};$ $\Rightarrow$ print(y) — 5 ✓ ~~6~~

postfix

$y = {+}{+}x;$ print(y) $\Rightarrow$ 6 ✓

prefix

$x+1$

swap

naman

~~ayush~~

~~bhaiya~~

Sum $\Rightarrow$ dp

O(N)

Rec

O(N)

O(N) ...

&&

if ((B&1) == 1)

$\rightarrow$ if ((B&1) == 1)

~~if ((B&1))~~