Q. Given 3 questions, each question has to be answered either True or False. In how many ways can we answer all the questions?

~~F F~~

F F F
F F T
F T F
F T T
T F F
T F T
T T F
T T T

$\underset{T\ F}{2}\ \overset{*}{\&}\ \underset{T\ F}{2}\ \overset{*}{\&}\ \underset{T\ F}{2} = 8$

$2 + 2 + 2$

$2 * 2 * 2$ multiply

Ex1:- Given 10 plates and 7 spoons. How many different pairs are possible?

pairs → 1 plate 1 spoon.

10 Plates	7 Spoons
P1	S1
P2	S2
P3	S3
P4	S4
P5	S5
P6	S6
P7	S7
P8	
P9	
P10	

P1 — 7

P2 — 7

P3 — 7

⋮

7

$7 * x$

$7 * 10$

P S

10 & 7

$10 * 7 = 70$

$\underbrace{7 + 7 + 7 + 7 \ldots}_{10}$

70

Ex :-

ways to reach Hyderabad to Delhi via Mumbai?

1st view

H1 → 4
H2 → 4
H3 → 4

12

2nd view

$\underbrace{\text{Hyd} \to \text{Mumbai}}_{3}$ AND $\underbrace{\text{Mumbai} \to \text{Delhi}}_{4}$

↓
×

= 12

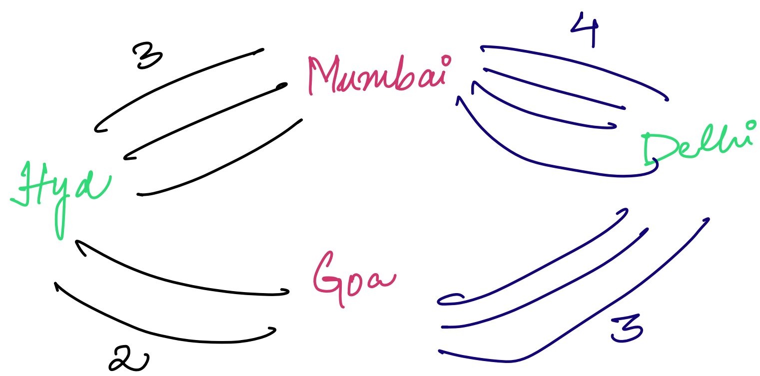
Ex:-

ways to reach Hyd to Delhi via Goa?

Ex:

5 * 7

= 35.



ways to reach from Hyd to Delhi?

(Hyd → Mum. & ~~Delhi~~ Mumbai to Delhi

3 * 4) — OR — (Hyd to Goa AND Goa to Delhi

2 * 3)

OR

$\boxed{12}$ ~~OR~~ $\boxed{6}$

Addition

Ex Say we need to buy a gift.

$$(\underbrace{\text{pen}}_{3} \;\&\; \underbrace{\text{book}}_{5}) \text{ or } (\underbrace{\text{flowers}}_{7} \;\&\; \underbrace{\text{chocolate}}_{3}) \text{ or } (\underbrace{\text{ring}}_{3})$$

$$3*5 + 7*3 + 5$$

$$15+21+3 \rightarrow \underline{\underline{39}}.$$

Permutations : Arrangement of objects

In general :- ORder matters.

$(i,j) \neq (j,i)$

Q. Given 3 distinct characters.
How many ways can you arrange them?

s = 'acd'

1st view.

a c d
a d c
c a d
c d a
d a c
d c a

$\left.\right\}$ 6 arrangements.

2nd view.

3 & 2 & 1 → 6 3!

a — c — d
a — d — c

c — a — d
c — d — a

d — a — c
d — c — a

Q. How many ways can you arrange 4 distinct characters? 'a b c d'

$$\underline{4} * \underline{3} * \underline{2} * \underline{1} = \underline{\underline{24}}. \rightarrow \underline{\underline{4!}}.$$

a $\{b, c, d\}$ $\{c, d\}$ $\{b, d\}$

b $\{a, c, d\}$

c $\{a, b, d\}$

d $\{a, b, c\}$

Q. How many ways to arrange n distinct elements?

$$\underline{N}\ \underline{N-1}\ \underline{N-2}\ \underline{N-3}\ \_\ \_\ \_\ \_\ \_\ \underline{1}$$

$$N * (N-1) * (N-2) * (N-3) * \ldots 1 = \underline{\underline{N!}}$$

Q. Given 5 distinct characters, in how many ways can we arrange them in 2 places?

N=5
R=2

{ a b c d e }

(1, 2) (2, 1)

Ayush Anshuman

$^5P_2 = \frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{5\times4\times\cancel{3\times2\times1}}{\cancel{3\times2\times1}} = 5\times4.$

5 * 4 = 20.

a	{b c d e}
b	{a c d e}
c	{a b d e}
d	{a b c e}
e	{a b c d}

a c

c a

a b

a c

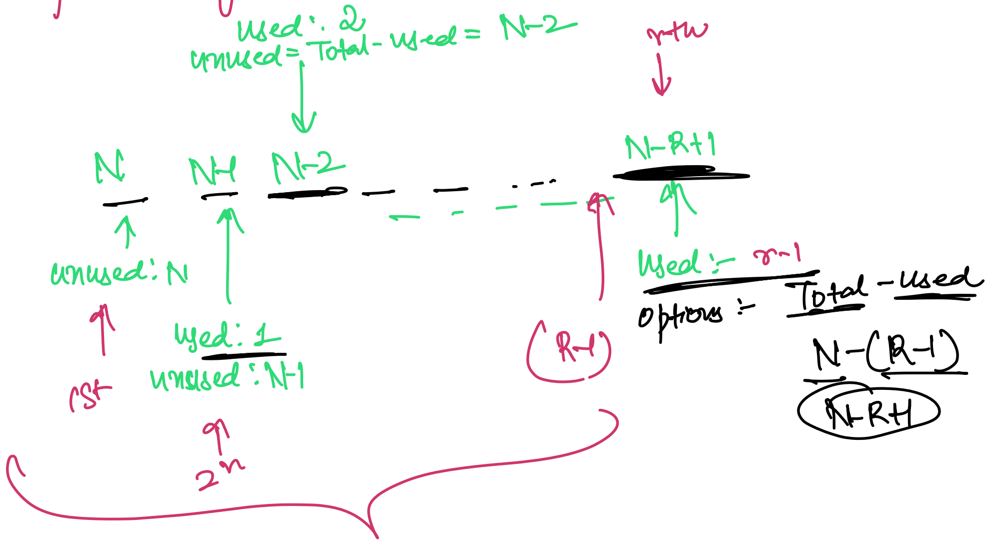
Q. N distinct characters, need to arrange 3 characters.

$\underline{(N)}\ \underline{(N-1)}\ \underline{(N-2)} \rightarrow N * (N-1) * (N-2)$

Q. N distinct characters, need to arrange 4 characters

$\underline{(N)} * \underline{(N-1)} * \underline{(N-2)} * \underline{(N-3)}$

Q. N distinct characters, need to arrange r characters.
# of arrangements?



ways to arrange R items in R places =

$$\underbrace{(N)(N-1)(N-2)(N-3)\cdots(N-R+1)}_{X}$$

$$\frac{(N)(N-1)(N-2)\cdots(N-R+1) * \underbrace{(N-R)(N-R-1)(N-R-2)\cdots(1)}}{(N-R)(N-R-1)(N-R-2)\cdots(1)}$$

$$\frac{N!}{(N-R)!} = {}^{N}P_{R}$$

Combinations (no. of ways to select)

Q. Given 4 cricketers, count ways of selecting 3 players.

$^4C_3$

4 P_1 P_2 P_3 P_4 } arrangement does not matter.

$$\frac{4!}{(4-3)!\,3!}$$

$P_1\ P_2\ P_3$

$P_1\ P_2\ P_4$

$P_1\ P_3\ P_4$

$P_2\ P_3\ P_4$

$P_1\ P_4\ P_2$

$$\frac{4!}{1!\,3!}$$

$$\frac{4 \times \cancel{3 \times 2 \times 1}}{\cancel{3 \times 2 \times 1}} = 4$$

$$^4P_3 = \frac{4!}{(4-3)!} = \frac{4!}{1!} = 24$$

Q. number of ways to Arrange 4 players in 3 slots.?

$P_1\ P_2\ P_3$	$P_1\ P_2\ P_4$	$P_1\ P_3\ P_4$	$P_2\ P_3\ P_4$
$P_1\ P_3\ P_2$	$P_1\ P_4\ P_2$	$P_1\ P_4\ P_3$	$P_2\ P_4\ P_3$
$P_2\ P_1\ P_3$	$P_2\ P_1\ P_4$	$P_3\ P_1\ P_4$	$P_3\ P_2\ P_4$
$P_2\ P_3\ P_1$	$P_2\ P_4\ P_1$	$P_3\ P_4\ P_1$	$P_3\ P_4\ P_2$
$P_3\ P_1\ P_2$	$P_4\ P_1\ P_2$	$P_4\ P_1\ P_3$	$P_4\ P_2\ P_3$
$P_3\ P_2\ P_1$	$P_4\ P_2\ P_1$	$P_4\ P_3\ P_1$	$P_4\ P_3\ P_2$
$\{P_1\ P_2\ P_3\}$	$\{P_1\ P_2\ P_4\}$	$\{P_1\ P_3\ P_4\}$	$\{P_2\ P_3\ P_4\}$

6 arrangements $\equiv$ 1 selection

24 arrangements $\equiv$ $\frac{24}{6} \times 1 = 4$ selections

N items.

Total arrangements $= {}^nP_r$.

↳ r items:

$\{a_1, a_2, a_3, \ldots a_r\}$

$${}^rP_r = \frac{r!}{(r-r)!} = \frac{r!}{0!} = \frac{r!}{1} = r!$$

↳ $r!$ factorial arrangements possible for this selection.

$r!$ factorial arrangements $\equiv$ 1 selection.

${}^nP_r$ arrangements $\equiv \frac{{}^nP_r}{r!} \times 1$ selections.

$$= \frac{n!}{(n-r)!\,r!}$$

1 arrangement → $\frac{1}{r!}$ selections

${}^nC_r$

${}^nP_r$ arrangements $\equiv {}^nP_r \times \frac{1}{r!}$ selections.

$$\equiv \frac{{}^nP_r}{r!} = \frac{n!}{(n-r)!\,r!} \to {}^nC_r$$

$$^{N}C_0 = \frac{N!}{(N-0)!\,0!} = \frac{N!}{N!} = 1$$

Subsets.

$\{a\,b\,c\}$ → Total subsets = 2^3

$^3C_0 \rightarrow \{\ \}$

$^3C_1 \rightarrow \{a\}, \{b\}, \{c\}$

$^3C_2 \rightarrow \{ab\}\ \{ac\}\ \{bc\}$

$^3C_3 \rightarrow \{a,b,c\}$

8

$$^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3 = 2^3$$

$$\boxed{^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \dots {}^nC_n = 2^n}$$

$2^4 = 16$

$\{a\,b\,c\,d\}$

$^4C_0 \rightarrow 1$

$^4C_1 \rightarrow 4$

$^4C_2 \rightarrow 6$

4

$1+4+6+4+1 = 16$

$$\frac{4!}{(4-2)!\,2!} = \frac{4!}{2!\,2!} = \frac{24}{4} = 6$$

$^4C_2 \to 4$
$^4C_4 \to 1$

Properties

Property 1

N distinct items.
Select r items.

Total remaining = N-1
selected remaining = R-1

Nth

pick
$^{N-1}C_{R-1}$

don't pick

start picking from here.

items to select => R

1st 2nd ....

Nth item

pick

Total items = N

remaining total -> N-1 (because we've made a decision regarding Nth item)

items remaining to be picked? R-1

OR

not pick

Total items = N

remaining total = N-1

items remaining to be picked = R

$^{N-1}C_R$

$^{N-1}C_{R-1}$

$$^nC_R = {}^{n-1}C_{R-1} + {}^{n-1}C_R$$

$$R! = (R)(R-1)!$$

$$(n-R)! = (n-R)*(n-R-1)!$$

$$\frac{(n-1)!}{(n-R)!\,(R-1)!} + \frac{(n-1)!}{(n-1-R)!\,R!}$$

$\cancel{n}-R\cancel{+1}$

$$(n-1)!\left[\frac{1}{(n-R)!\,(R-1)!} + \frac{1}{(n-R-1)!\,R!}\right]$$

$$(n-1)!\left[\frac{1}{(n-R)(n-R-1)!\,(R-1)!} + \frac{1}{(n-R-1)!\,(R)(R-1)!}\right]$$

$$\frac{(n-1)!}{(n-R-1)!\,(R-1)!}\left[\frac{1}{(n-R)} + \frac{1}{R}\right]$$

$$\frac{(n-1)!}{(n-R-1)!\,(R-1)!}\left[\frac{\cancel{R}+N-\cancel{R}}{(n-R)(R)}\right]$$

$$\frac{n!}{(n-R)!\,(R)!} \rightarrow {}^{n}C_{R}$$

Property 2

ex:- 5 boys :- {B1 , B2 , B3 , B4 , B5}

select 2 boys

$$5_{C_2} = \frac{5!}{3!\,2!} = \frac{\cancel{4}^{2} \times 5}{\cancel{2}} = 10$$

B1 B2 : B3 B4 B5

B1 B3

B1 B4

B1 B5

B2 B3

B2 B4

B2 B5

B3 B4

B3 B5

B4 B5

2 3

$${}^{n}C_{R} = {}^{n}C_{n-R}$$

$$\frac{n!}{(n-R)!\,R!}$$

$$\frac{n!}{(\cancel{n}-(\cancel{n}-R))!\,(n-R)!}$$

$$\frac{n!}{R!\,(n-R)!}$$

Ques. Given N, R and P. Calculate $({}^{N}C_{R})\,\%\,P$

Information : p is prime

$N, R < P$

constraints :

$1 <= (N, R) <= 10^5$

$R < N < P$

$$\left(\frac{N!}{(N-R)!\,R!}\right) \% p$$

a

$$\left(\frac{N!}{a}\right) \% p$$

$$\left(N! * a^{-1}\right) \% p$$

$$\left((N! \% p) * (a^{-1} \% p)\right) \% p$$

$R!\,(N-R)!$

$\gcd(a, p) = 1$

if p is prime ?

$\gcd(a, p) = 1$

p is prime

$a^{-1} \% p$

$$\boxed{\left(a^{p-2}\right) \% p}$$

$N! \% p$ * { $((N-R)!)^{-1} \% p$ * $(R!)^{-1} \% p$ }

$O(N)$ + $O(N-R) + \log p$ + $O(R) + \log p$

func { }

$((N-R)!)^{-1} \% p$

$\longrightarrow a^{-1} \% p \xrightarrow{gcd(a,p)=1}$

fun

$gcd((N-R)!, p)$

(p is prime)

$gcd(14, 7)$?

$(N-R)!$ $\quad p$

p → prime

$(N-R)! = 1 \times 2 \times 3 \times 4 \cdots \times (N-R-1) \times (N-R)$

$((N-R)!)^{-1} \% p$ exists.

$\longrightarrow (a^{p-2}) \% p$

x

$(N-R)!\ \%\ p$

$O(N-R)$

$O(N-R) + \log p$

$a^x \% p$

$((a\%p)^x)\%p$

$((N-R)!^{\,p-2})\ \%\ p$

$((N-R)!\ \%\ p)^{p-2}\ \%\ p$

$(N-R)!\ \%\ p$

x

$(x^{p-2})\ \%\ p$

$\log p$

```
fun Solve() {
    val x = [Calculate (N-R)! % p]          ← O(N-R)
    [Calculate (x^(p-2)) % p by fast exp.]  ← O(logp)
}
```

for( )
for( )

$$\left(\frac{N!}{(N-R)!\ R!}\right)\%\ p$$

$$\left(N! \times ((N-R)!)^{-1} \times (R!)^{-1}\right) \% p$$

$(a \cdot b \cdot c) \% p$

$(a \% p * b \% p * c \% p) \% p$

$$\left(\underbrace{(N! \% p)}_{\mathrm{I}} * \underbrace{((N-R)!)^{-1} \% p}_{\mathrm{II}} * \underbrace{((R!)^{-1} \% p)}_{\mathrm{III}} \right) \% p$$

T.C.

```
fun main() {
    A = calcI        // O(N)
    B = calcII       // O(N-R) + O(log p)
    C = calcIII      // O(R) + O(log p)
    return (A*B*C) % p
}
```

100

200

50

log p

O(N)

(100 * 200 * 50)

N!

N % p → N-1 % p → N-2 % p —

N

S.C = O(N)

$a^{-1} \% p$ — gcd(a, p) = 1, p is prime

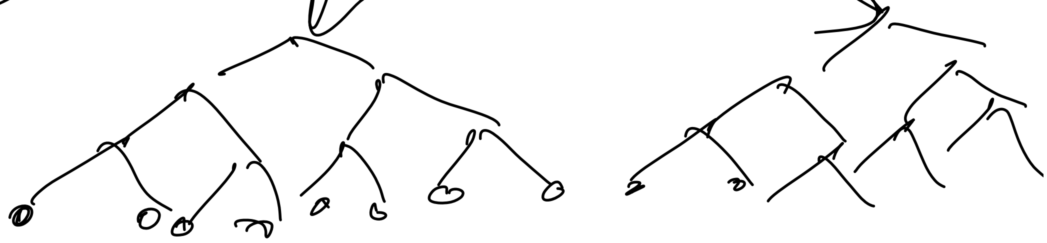
$(NCR) \% p$ when p is not prime

$\frac{n!}{(n-R)!\,R!}$

$(a+b)\,\%\,p = (a\,\%\,p + b\,\%\,p)\,\%\,p$

$({}^{n}C_{R})\,\%\,p = ({}^{n-1}C_{R-1}\,\%\,p + {}^{n-1}C_{R}\,\%\,p)\,\%\,p$

Q $\xrightarrow{\text{path 1}}$ sol'n



Permutation with Repetition.

3!

2!

BE₁E₂

$E_1 = E_2 = E$

E E

E_1 E_2

$E_1\,B\,E_2$ $E_2\,B\,E_1$

$E_1\,E_2\,B$ $E_2\,E_1\,B$

$B\,E_1\,E_2$ $B\,E_2\,E_1$

r! arrangements ≡ 1 selection.

2! arrangements ≡ 1 unique arrangement

argument

combination

for ...

argument for Permutation with repetition.

$^nP_r = \frac{^nP_r}{2!}$

BEEGG

$\frac{^5P_R}{2!\,2!}$

$\frac{^5P_R}{4!}$

N → N-1 → N-2 → N-3 ... 1

$N!$

O(N+1) → O(N) ← S.C. to calculate factorial

```
fun fact(int N){
  base condition
  return N*fact(N-1)
}
```

Recursion:

S.C = O(maximum fn calls active at any given time * space by each fn call)

T.C = O(Total fn calls * Time by each fn call)

A (B)(B-1) (B-2) . . . (1)

break this up & apply modulo individually

3hrs

8 hrs + ~~8 hr~~

16 18–20 hrs → 4 hrs.

12 hrs assignment.

~~56 hrs~~ 5 hrs → 2 days.

24 hrs

8 hr.

301

36–38 hrs

h.w/assignments.

12 hrs

9

3 × 3 hrs → ass

24 hrs / 3 = 8 hrs / hw class