Subarray - Contiguous part of array

Subsequences - Sequence generated by deleting 0 or more elements in array.

order matters

-2	-3	6	2	4	-1
×	✓	✓	×	×	✓

$\rightarrow \{-3, 6, -1\}$

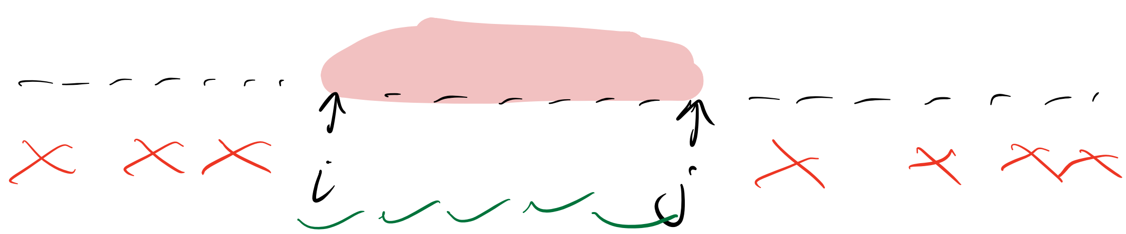
$\{-2, 2\}$ ✓

$\{-3, 6, 4\}$

-2	-3	6	2	4	-1
×			×		×

$\{2, -2\}$ ×

Are subarrays also subsequences?



1) All subarrays are subseq. but all subseq. are not subarrays.

2) Empty subseq is VALID subseq
(decided to delete everything)

3) Whole array is VALID subseq

{1, 2, 3} → { }
→ {1, 2, 3}
{1, 3}

Subset → Same as subseq BUT
1) order does NOT matter
2) no duplicates.

{1, 3, 1} → {1, 1}
valid subseq
invalid subset

{3, 1}   {1, 3, 1}
✗ ✓ ✓

{1, 2, 3}
→ {3, 2}
valid subset

{2, 3}   {3, 2}

in context of subset, both are same.

$\{1, 2, 3\}$ 1, 3 → valid subset
not subarray

Count of subarrays → $\frac{n(n+1)}{2}$

Count of subseq →

0	1	2	3	4
2	2	2	2	2

$$\underbrace{2 \times 2 \times 2 \times 2 \ldots \times 2}_{n \text{ times}} = 2^n$$

$n = 1 \longrightarrow \{100\} \rightarrow \{\ \},\ \{100\}$

$n = 2 \longrightarrow \{5, 9\} \rightarrow \{\ \},\ \{5\},\ \{9\}$

↳ {5, 9}

Q Given an array of N distinct elem check if there is a subset with sum = K

{3, -1, 0, 6, 2, -3, 5}   K = 10

{3, 2, 5}
{-1, 6, 5}   true
{3, -1, 6, 2}

{3, -1, 0, 6, 2, -3, 5}   K = 20

false.

Generate all subsets.

each subset ⟷ subsequence.

{1, 2, 3}   { }
{1}
{2}

{3}
{1, 2}
{2, 3}
{1, 3}
{1, 2, 3}

{3, 2}

Note: If distinct elem, no of subsets = no of subseq = 2^n

- Generate all subseq

BITMASK

-2	-3	6	2	4	-1	
✗	✓	✓	✗	✗	✓	
0	1	1	0	0	1	= 25

25 → 0 1 1 0 0 1

16 → 0 1 0 0 0 0

→ {-3}

{1, 2, 3}

0	0 0 0	$\longrightarrow$	$\{\ \}$
1	0 0 1	$\longrightarrow$	$\{3\}$
2	0 1 0	$\longrightarrow$	$\{2\}$
3	0 1 1	$\longrightarrow$	$\{2, 3\}$
4	1 0 0	$\longrightarrow$	$\{1\}$
5	1 0 1	$\longrightarrow$	$\{1, 3\}$
6	1 1 0	$\longrightarrow$	$\{1, 2\}$
7	1 1 1	$\longrightarrow$	$\{1, 2, 3\}$

N sized array $0 \longrightarrow 2^n - 1$

$0, 1, 2, 3, 4, \ldots\ldots\ldots 2^n - 1$

$n = 4$ $\quad 0 - 15$

$11 \longrightarrow 1011$

$\begin{array}{cccc} 1 & 0 & 1 & 1 \\ 4 & 3 & 2 & 1 \end{array}$

$\{a_0, a_2, a_3\}$

11 find if 1^{st} bit is ON

2^{nd}

3^{rd}

-

$>> 2$ $\qquad$ $1 << n \rightarrow 2^n$

$>> n \rightarrow$ division by 2^n

```
for(i=0 ; i < 2^n ; i++)
{          sum=0;
  // check which bits are on
  for(j=0 ; j<n ; j++)
  {     if (is_on(i,j))
            sum+= a_j
  }
  if (sum==k)
      return true.
}

  return false.
```

TC: $n 2^n \xrightarrow{DP} nk$

$n=3$ $\qquad$ $\{1, 3, 7\}$

i		
0	0 0 0	<3
1	0 0 1	
2	0 1 0	
3	⋮	
4		
5		
6		
7		

$n = 4$ 3

0 0 1 1

```
int is_on ( int n, int i)
{ return (n>>i & 1) }
```

$n = 5$ $n = 2$ $n = 1$

101 10 1

Break 10:30

$n=3$ $\qquad \underline{0 - 7}$

$7 \rightarrow \underline{1}\,\underline{1}\,\underline{1}$

$n = 500 \qquad 0 \qquad 2^{500}-1$

$1011011100\,1$

$1011011\,00$

$1011011\,0$

bitmask = 001001

$6 \qquad -1$

$\rightarrow 9$

Q. Given an array of N distinct elem, find sum of subset sums.

$\{-2, 6, 4\}$

$\{\} \rightarrow 0$

$\{-2\} \rightarrow -2$

$\{6\} \rightarrow 6$

$\{4\} \rightarrow 4$

$\{-2\ 6\} \rightarrow 4$

$\{-2\ 4\} \rightarrow 2$

$\{6\ 4\} \rightarrow 10$

$\{-2\ 6\ 4\} \rightarrow 8$

32

ans = 32

Brute force: Find all subseq & find their sums.

Contribution Technique.

$$\sum_{i=0}^{n-1} a_i * \text{contribution}.$$

contribution = no of subseq which have a_i.

$$\sum_{i=0}^{n-1} a_i * (\text{subseq with elem } a_i)$$

$a_0 \quad a_1 \quad a_2 \quad a_3 \quad a_4$

$1 \quad 2 \quad 2 \quad 2 \quad 2$

$= 2^4$

$a_0 \quad a_1 \quad a_2 \quad a_3 \quad a_4$

$2 \quad 1 \quad 2 \quad 2 \quad 2$

$= 2^4$

$5 \rightarrow 2^4$ for each elem.

N elements $\longrightarrow 2^{n-1}$

$$\sum_{i=0}^{n-1} a_i \, 2^{n-1}$$

$$2^{n-1} \times \sum_{i=0}^{n-1} a_i$$

sum of array.

$\boxed{2^{n-1} \times \text{sum of array}}$

TC: $O(n)$ (for sum)

```
sum = 0
for (i=0 ; i<n ; i++)
    sum += a_i

return ((1<<n-1) * sum)
```

---

zeta

Q Given an array, find sum of all subsets divided by 2^n (distinct elem)

$$\frac{2^{n-1} \times sum}{2^n} = \frac{sum}{2}$$

```
int sum = 0
for (i=0 ; i<n ; i++)
    sum += a_i

return sum/2.0
```

assume sum = 5 → ~~2~~

$5/2 = 2$

<u>Q</u> Given array of size = N, distinct elem.
find sum of ~~MAX~~ greatest elem. of all <u>subseq</u>.

$\{-2, 6, 4\}$

$\{\}$	→	0
$\{-2\}$	→	-2
$\{6\}$	→	6
$\{4\}$	→	4
$\{-2\ 6\}$	→	6
$\{-2\ 4\}$	→	4
$\{6\ 4\}$	→	6
$\{-2\ 6\ 4\}$	→	6
		30

<u>Brute force</u>: Same as Q2

Contribution technique.

$$\sum_{i=0}^{n-1} a_i * \text{contribution.}$$

contribution = no of subseq where a_i is MAX

a_0 a_1 a_2 a_3 a_4

1

6	10	4	2	12
1	1	2	2	1

2^2

everything greater → CANNOT take
everything smaller → your wish

$$\text{cont} = 2^{\text{smaller elem}}$$

6 10 4 2 12

2 1 2 2 1

2^3

$$\underbrace{\sum_{i=0}^{n-1} a_i \, \underline{2^{\text{smaller elem.}}}}$$

sort(a)

	3	5	7	10	12
idx	0	1	2	3	4
sm	0	1	2	3	4

$$\sum_{i=0}^{n-1} a_i 2^i$$

```
sum = 0
sort(a)

for(i=0; i<n; i++)
{
    sum += a_i * (1<<i)
}
return sum.
```

$$\underbrace{1<<i}_{2^i}$$

TC: $n\log n + n = n\log n$

$\log n < N < n\log n < n^2$

$\{-2, 6, 4\}$

$\{-2, 4, 6\}$

$-2 \; 2^0 + 4 \; 2^1 + 6 \; 2^2$

$-2 + 4 \times 2 + 6 \times 4$

$$-2+8+24 = 30$$