Q N points on a 2D plane

find max no. of colinear points

⇓

points which lie on the same line.

Point ⇒ (x, y)

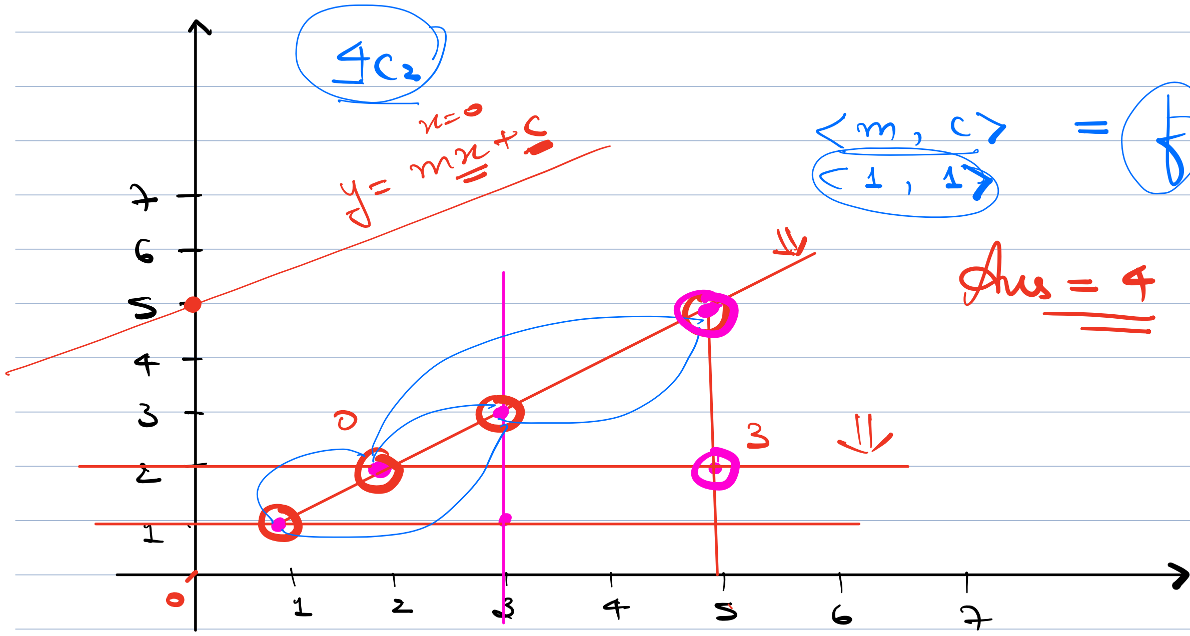
$X[N]$ ⇒ $X[i]$ denotes the x co-ord. of i

$Y[N]$ ⇒ $Y[i]$ " " y " " i

	0	1	2	3	4	5
X ⇒	1	2	3	5	3	5
Y ⇒	1	2	3	5	1	2

$N = 6$

$^4C_2$



$\langle m, c \rangle = f$

$\langle 1, 1 \rangle$

Ans = 4

① Equation of a line

(i) $y = mx + c$

slope $\quad$ y intercept

(ii) $(y_2 - y_1) = m(x_2 - x_1)$

2 points $\Rightarrow (x_1, y_1)$ & (x_2, y_2)

Brute force

$\Rightarrow$ for every pair of points, calculate the eqⁿ of the line.

(x_1, y_1) & (x_2, y_2)

Eqⁿ $\Rightarrow$ $y = mx + c$ —— (1)

$$\boxed{m = \frac{(y_2 - y_1)}{(x_2 - x_1)}} \quad —— (2)$$

$$y = \frac{(y_2 - y_1)}{(x_2 - x_1)} x + c \quad —— (3)$$

$$y_1 = \frac{(y_2 - y_1)}{(x_2 - x_1)} x_1 + c$$

$$c = y_1 - \frac{(y_2 - y_1)}{(x_2 - x_1)} x_1$$

$$c = \frac{y_1(x_2 - x_1) - (y_2 - y_1)x_1}{(x_2 - x_1)}$$

$$= \frac{y_1 x_2 - \cancel{y_1 x_1} - y_2 x_1 + \cancel{y_1 x_1}}{x_2 - x_1}$$

$$\boxed{c = \frac{y_1 x_2 - y_2 x_1}{x_2 - x_1}}$$

$$T.C. = O(\underline{\underline{N^2}} \times 1 \times N)$$

$$= O(N^3)$$

② Using $\underline{\underline{m}}$ & $\underline{\underline{c}}$

for every pair of points.

Calculate m & c $\rightarrow$ pair $\langle m, c \rangle$

store the freq of this $\langle m, c \rangle$ in a hashmap.

$\langle m, c \rangle \Rightarrow f$

$$x = {}^{x}C_2 = f$$

$ax^2 + bx + c = 0$

$$\frac{(x)(x-1)}{2} = f$$

$a = 1 \quad c = -2f$

$b = (-1)$

$$\underset{a}{x^2} \overset{(-1)}{\underset{b}{-x}} \overset{(-2)}{\underset{c}{-2f}} = 0$$

$$\Downarrow$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-1) \pm \sqrt{(1) - 4(1)(-2f)}}{2}$$

$$x = \frac{1 \pm \sqrt{1 + 8f}}{2}$$

$\sqrt{1}$

$$\frac{1 + (>1)}{2} \qquad \cancel{\frac{1 - (>1)}{2}}$$

$(\sqrt{1 + \dots}) > 1$

ans = 2

$$x = \frac{1 + \sqrt{1 + 8f}}{2}$$

```
for (i = 0; i < N; i++) {
    for (j = i+1; j < N; j++) {
        - P1 => X[i], Y[i]
        - P2 => X[j], Y[j].
        Generate the eqn of lin
        => count = 0;
        for (K = 0; K < N; K++) {
            P3 => X[K], Y[K]
            if (P3 satisfies eqn) {
                count++;
```

```
        }
    }
    ans = max(ans, count);
  }
}
```

$$T.C. = O(N^3)$$

Pair

⇓

pair< int, int >

x

$$^{x}C_2 = \frac{(x)(x-1)}{2}$$

0.333337

$$\frac{(x)(x-1)}{2} = f$$

$1/3 = 0.33|33......$

0.33

$\frac{2.5}{2}$ $\frac{2.7}{2}$

m c

float double

Code

Hashmap < Pair <

float $\Rightarrow$ approximation / precision

Error.

$$\uparrow m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow x_1 = x_2$$

$\underline{\underline{\infty}}$ $\qquad$ 0

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Pair $\langle \underset{\text{Int}}{(y_2 - y_1)}, \underset{\text{Int}}{(x_2 - x_1)} \rangle$

$\overset{x_1\ y_1}{(1, 1)}$ & $\overset{x_2\ y_2}{(2, 2)}$ $\qquad 1/1 = 1$

$$m = \langle 1, 1 \rangle$$

$(1, 1) \qquad (5, 5) \qquad 4/4 = 1$

$$m = \left\langle \frac{4}{gcd}, \frac{4}{gcd} \right\rangle$$

$$= \langle 1, 1 \rangle$$

HashMap $\qquad$ Key $\langle m, c \rangle$ $\qquad$ Value Freq.

$$m = \left\langle \frac{y_2 - y_1}{gcd}, \frac{x_2 - x_1}{gcd} \right\rangle$$

gcd of $(y_2 - y_1, x_2 - x_1)$

$$c = \left\langle \frac{y_1 x - y_2 x_1}{gcd}, \frac{x_2 - x_1}{gcd} \right\rangle$$

$gcd = (y_1 x_2 - y_2 x_1, \; x_2 - x_1)$

$$\frac{1}{2} = \frac{-1}{-2} \qquad \langle 1, 2 \rangle = \langle |-1|, +2 \rangle$$

i   j

(1, 2)	(2, 3)	(3, 4)	(4, 5) ✓
(1, 3) j	(2, 4)	(3, 5)	
(1, 4)	(2, 5)		
(1, 3)			

$$\langle \langle 1, 1 \rangle, \langle 1, 1 \rangle \rangle$$

$\langle m, c \rangle$ freq

$\langle \langle 1, 1 \rangle, \langle 1, 1 \rangle \rangle$ ~~1~~ ~~2~~ 3

$\langle \langle \; \rangle \; \langle \; \rangle \rangle$ 1

$$\frac{y_2 - y_1}{x_2 - x_1}$$

num	den	fer
+ve	+ve	+ve
−ve	−ve	+ve
+ve	−ve	−ve
−ve	+ve	−ve

$\frac{-1}{2}$ $\frac{1}{-2}$

$\langle -1, 2 \rangle$ $\langle 1, -2 \rangle$

H.W: Give 1 hr 30 min to code this approach.

$T.C. = O(N^2) * (T.C \text{ of } GCD)$

$S.C. = O(N^2)$

Q Given a string S of size N & an integer K.

Find whether it is possible to represent S as a concatenation of K similar strings.

S = " abbb b a bb a " K = 3

abb abb abb

bab bab bab

bba bba bba

True

abbabbabb

$K = 3$ $\quad$ $l = 20$

1) Size of string should be a multiple of K.

| 2a | | 2a | | 2a | . . . | 2a |

aab

count of a = 2a + 2a + 2a . . . . K times

= K × (2a)

freq of each char should also be a multiple of K.

1) Create a freq map

2) Check if all freq % K == 0

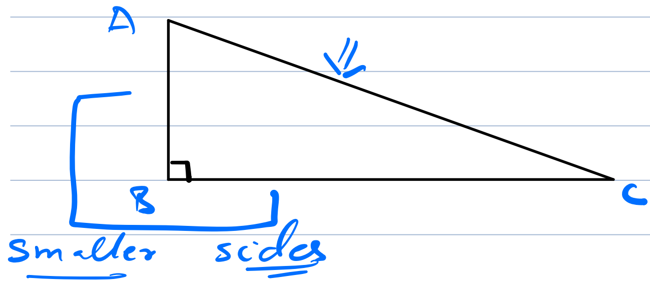
T.C. = O(N)

S.C. =

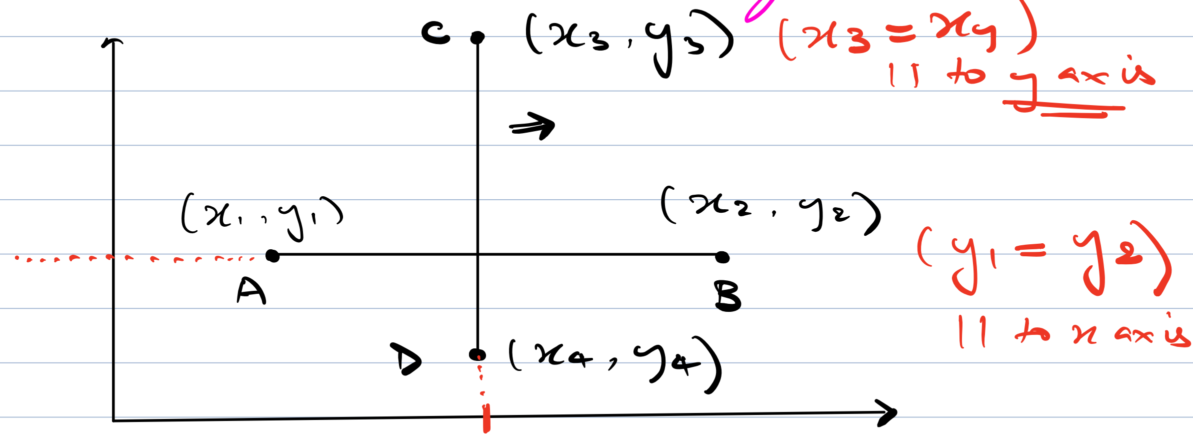
LC & UC = 52

Constant

Right angled triangle.



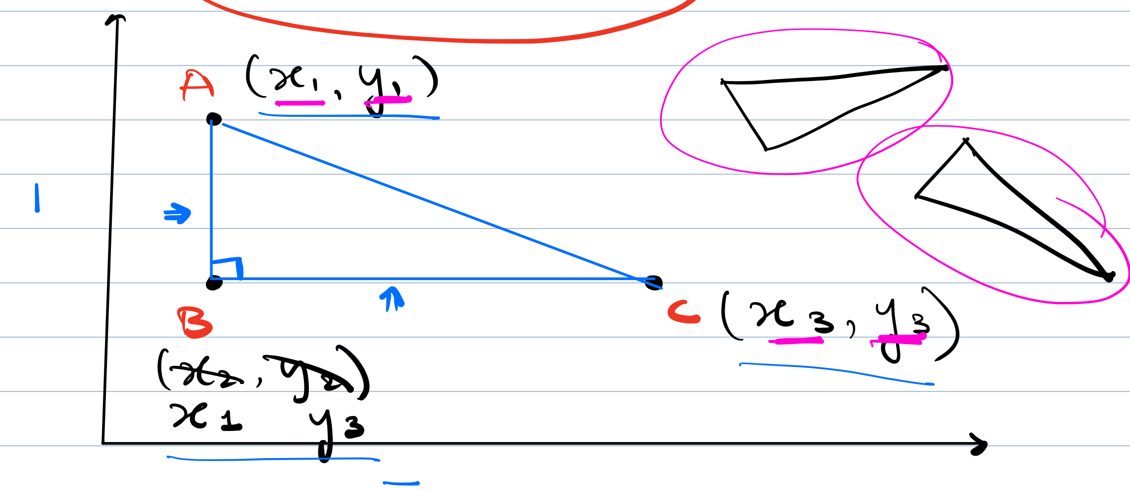
2D plane $\Rightarrow$ 3 points to represent a triangle.



Q = Given 3 distinct points in a 2D plane

Check if they form a right angled $\triangle$ with smaller sides || to x & y axis

A (1,8)   B (1,4)   C (5,4)

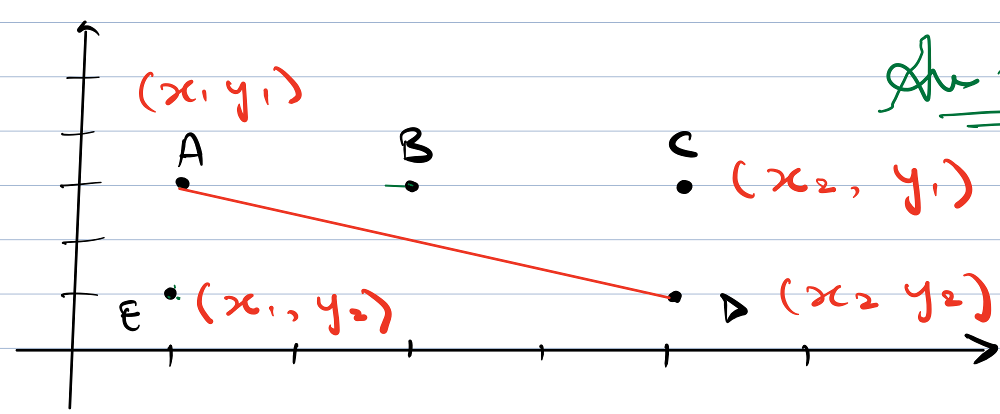


Q Given N distinct points on a 2D plane.

Calculate the total no. of right angled triangle whose smaller sides are || to x axis & y axis.

X :	1	1	3	5	5
Y :	1	3	3	1	3

Ans = 6



1) **Brute force**

for all distinct triplets, check if they form a right angled triangle.

```
count = 0;
for (i = 0; i < N; i++) {
    for (j = i+1; j < N; j++) {
        for (k = j+1; k < N; k++) {
            if (points i, j & k form a right angled triangle) {
                count++;
            }
        }
    }
}
```

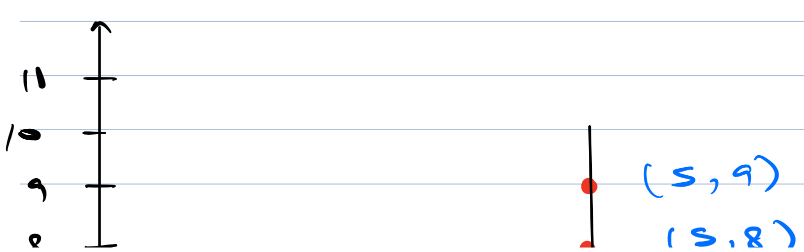
O(1)

$$T.C. = O(N^3)$$

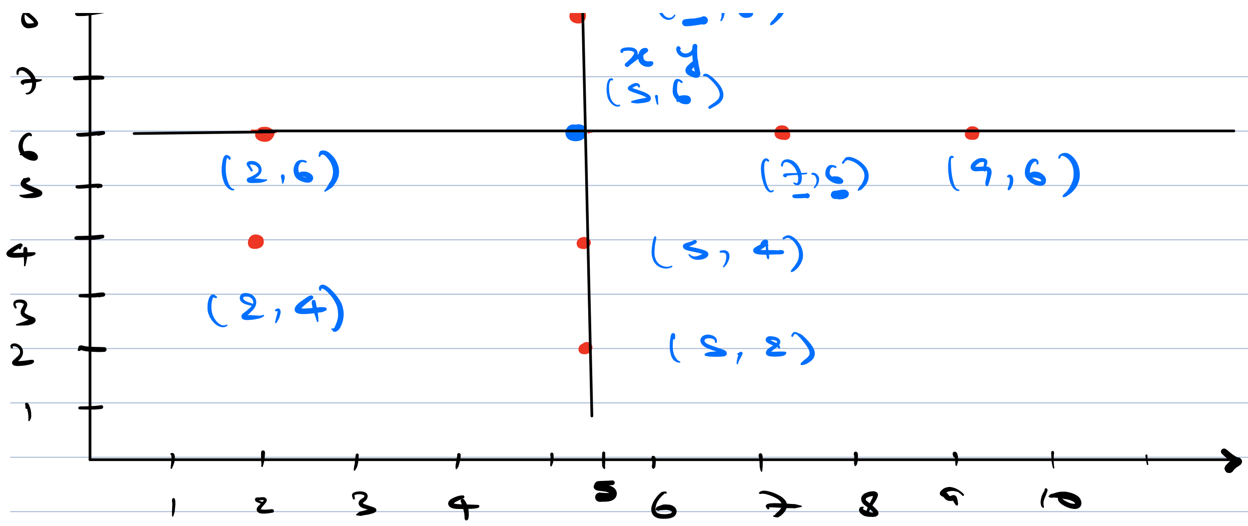
2) HashSet (Pair <x, y>)

```
for (i=0; i<N; i++) {
    for (j=i+1; j<N; j++) {
        if ((x[i] != x[j]) &&
                (y[i] != y[j])) {
            P1 = <x[i], y[j]>
            P2 = <x[j], y[i]>
            if (hs.contains(P1)) {
                count++;
            }
            if (hs.contains(P2)) {
                count++;
            }
        }
    }
}
```

T.C. = $\underline{O(N^2)}$



4×3



x   y

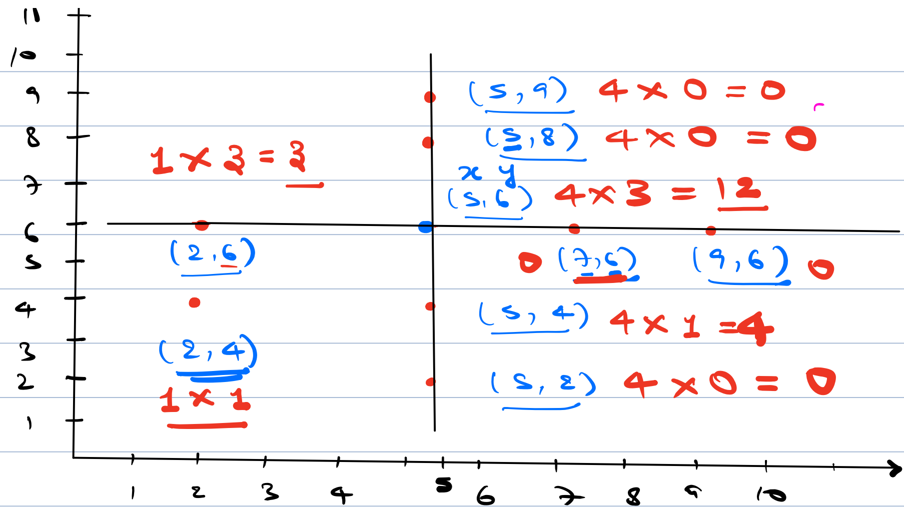
(freq of points with x − 1) × (freq of points with y − 1)

HM for x co-ordinate   hmx

HM for y co-ordinate   hmy

```
ans = 0
for (i = 0; i < N; i++) {
        ans += (hmx.get(x[i]) - 1) x
               (hmy.get(y[i]) - 1)
}
```

20



HMx

x	freq
2	~~1~~ 2
5	~~1~~ ~~2~~ ~~3~~ ~~4~~ 5
7	1
9	1

N

HMy

y	freq
6	~~1~~ ~~2~~ ~~3~~ 4
4	~~1~~ 2
9	1
8	1
2	1

N

T.C. = $O(N)$

S.C. = $O(2N) = O(N)$

2, 3, 4, 5, 6, 2, 6, 5, 1