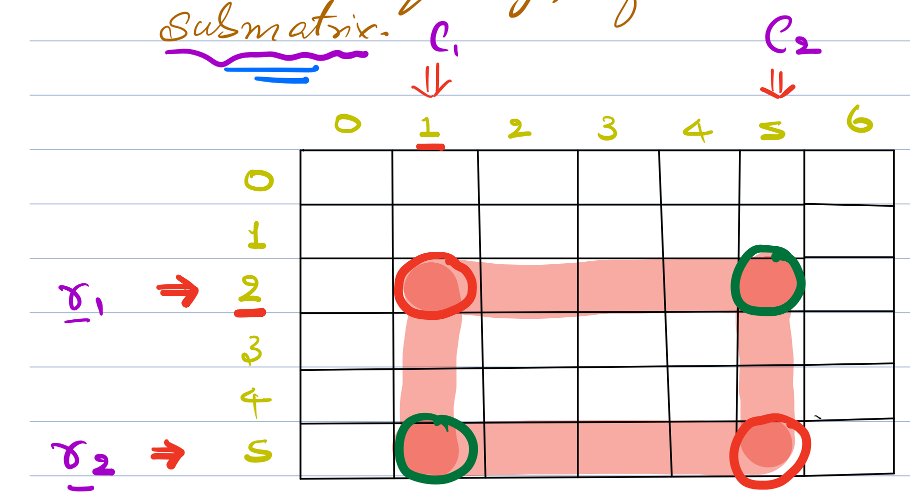
Agenda → 2D matrices

1) Submatrix sum Queries
2) Sum of all submatrices
3) Max Submatrix Sum.

Q Given a matrix of size N × M. & Q queries.
Amazon
for each query, find the sum of given submatrix.



6 × 7

To identify a submatrix.

(Top Left) (r_1, c_1) (Bottom Right) (r_2, c_2)

	0	1	2	3	4	5	6
0	7	1	-6	3	12	-2	4
1	10	5	-2	0	9	4	8
2	6	4	-3	8	11	3	2
3	13	-8	-5	12	4	6	7
4	3	2	1	9	3	9	6
5	4	3	-2	6	8	8	8

6 × 7

→ 0,0 to 5,6

Q	TL (r_1, c_1)	BR (r_2, c_2)	
Q1	(1, 2)	(4, 3)	⇒ 20
Q2	(1, 1)	(4, 5)	⇒ 72

Sol^n ① Brute force

```
sum = 0;
for(i = r1 ; i <= r2 ; i++) {   //All rows
    for (j = c1; j <= c2; j++) {
        sum += M[i][j];
    }
}
```

T.C. of 1 query ⇒ $O(N \times M)$ ⇒ $O(1)$

" Q queries ⇒ $O(Q \times N \times M)$

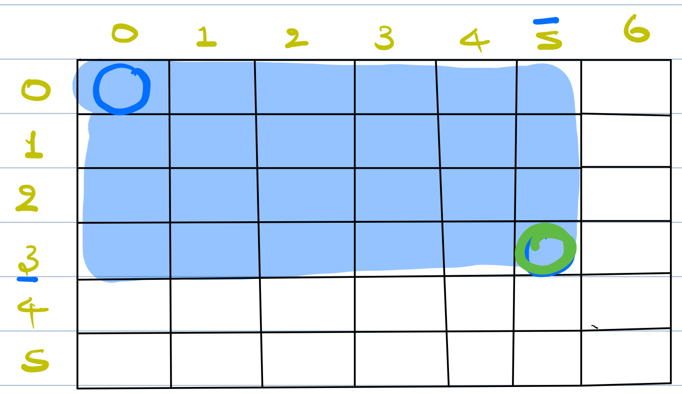
2) Can we optimise ??

1D Array ⇒ prefix sum Array.

PF[i] ⇒ sum of all elements from index 0 to i.

2D Matrix → Prefix Sum Matrix

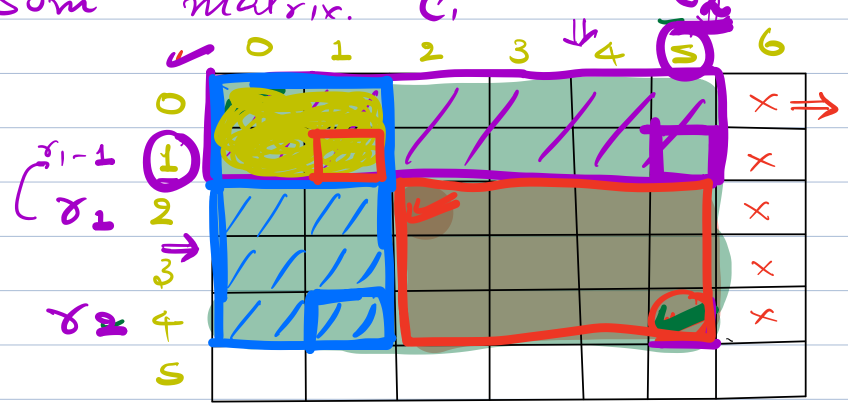
Pf[i][j] → sum of all elements in submatrix from (0, 0) to i, j



6 × 7

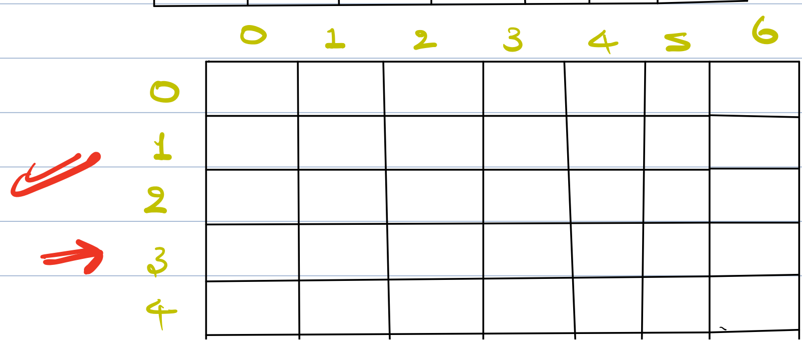
Pf [3, 5]

Assume we already have a prefix sum matrix. C_1 C_2



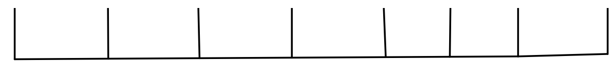
6 × 7

⇒ M

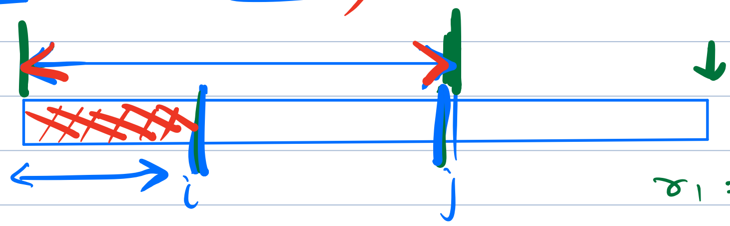


6 × 7

⇒ Pf

s 

Sum((2, 2) to (4, 5))



$r_1 = 2$

$r_2 = 4$

$c_1 = 2$

$c_2 = 5$

$ps[j] - ps[i-1]$

$\left(PF[4][5] - PF[1][5] - PF[4][1] + PF[1][1] \right)$

$r_2 \quad c_2$

M

	0	1	2
0	3	4	1
1	6	2	9
2	5	3	1

$\Rightarrow$

PF

	0	1	2
0	3	7	8
1	9	15	25
2	14	23	34

PF[0][0] → (0,0) to (0,0)
PF[0][1] ⇒ (0,0) to (0,1)
PF[0][2] ⇒ (0,0) to (0,2)

PF[1][0] → (0,0) to (1,0)
PF[1][1] ⇒ (0,0) to (1,1)
PF[1][2] ⇒ (0,0) to (1,2)

PF[2][0] ⇒ (0,0) to (2,0)
PF[2][1] ⇒ (0,0) to (2,1)
PF[2][2] ⇒ (0,0) to (2,2)

$O(N \times M)$

Using a prefix matrix, we can calculate

each "query" in $O(1)$ time

$$T.C. = O(1 \times Q) + \text{Time to calculate PF matrix}$$

Edge Cases → if $(r_1 == 0) \,||\, (c_1 == 0)$

$$Sum\left((r_1, c_1) \text{ to } (r_2, c_2)\right) = PF[r_2][c_2] - PF[r_1-1][c_2] - PF[r_2][c_1-1] + PF[r_1-1][c_1-1]$$

$Q \Rightarrow O(1)$

$r_1 = 0 \;||\; c_1 = 0$ H.W.

M

a_0	a_1	a_2
b_0	b_1	b_2
c_0	c_1	c_2

⇒ PS row wise

PF

a_0	a_0+a_1	$c_0+a_1+a_2$
b_0	b_0+b_1	$b_0+b_1+b_2$
c_0	c_0+c_1	$c_0+c_1+c_2$

$i=0$

a_0	a_0+a_1	$a_0+a_1+a_2$
a_0+b_0	$a_0+a_1+b_0+b_1$	$a_0+a_1+a_2+b_0+b_1+b_2$
$a_0+b_0+c_0$	$a_0+a_1+b_0+b_1+c_0+c_1$	$a_0+a_1+a_2+b_0+b_1+b_2+c_0+c_1+c_2$

Code

M ⇒ $N \times M$

PF[N][M] row wise

$O(2 \times N \times M)$

1

```
for (i = 0; i < N; i++) {
    PF[i][0] = M[i][0];    // Edge Case
    for (j = 1; j < M; j++) {
        PF[i][j] = PF[i][j-1] + M[i][j];
    }
}
```

3, 4, 7, 9, 2

P 3 7 14 23 25

Col wise

2

```
for (j = 0; j < M; j++) {
    for (i = 1; i < N; i++) {
        PF[i][j] = PF[i-1][j] + PF[i][j];
    }
}
```

T. C. = $O(N \times M)$

Overall T.C. = $O(0 + N \times M)$

PS in array S.C. = $O(N \times M)$

N C(A) (PS)

Q-2 Given a matrix of size N x M

Calculate the sum of all submatrix sum.

Eg

$$\begin{bmatrix} 4, & 9, & 6 \\ 5, & -1, & 2 \end{bmatrix}$$

[4] ⇒ 4
[9] → 9
[6] ⇒ 6
[5] ⇒ 5
[-1] ⇒ -1
[2] ⇒ 2

[4, 9] ⇒ 13
[9, 6] ⇒ 15
[5, -1] 4
[-1, 2] 1
[4, 9, 6] 19
[5, -1, 2] 6

$\begin{bmatrix} 4 \\ 5 \end{bmatrix}$ 9

$\begin{bmatrix} 9 \\ -1 \end{bmatrix}$ 8

$\begin{bmatrix} 6 \\ 2 \end{bmatrix}$ 8

$\begin{bmatrix} 4 & 9 \\ 5 & -1 \end{bmatrix}$ 17

$\begin{bmatrix} 9 & 6 \\ -1 & 2 \end{bmatrix}$ 16

$\begin{bmatrix} 4, & 9, & 6 \\ 5 & -1 & 2 \end{bmatrix}$ 25

Sum = 166 Ans

4 x 6 + 9 x 8 + 6 x 6 + 5 x 6 + (-1) x 8

+ 2 x 6

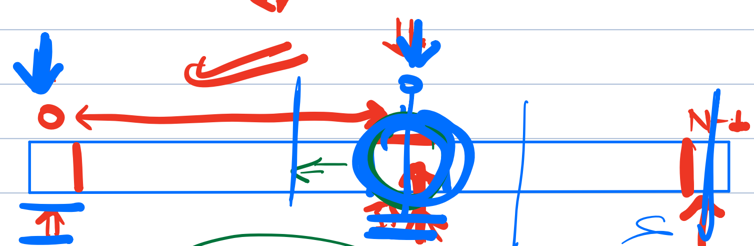
$= 24 + 72 + 36 + 30 - 8 + 12$

$= 166$

Sum of all submatrices

sum

$$= \sum_{\forall i,j} \left(\begin{array}{c} \text{No. of submatrices} \\ A[i][j] \text{ is a part of} \end{array} \right) \times A[i][j]$$



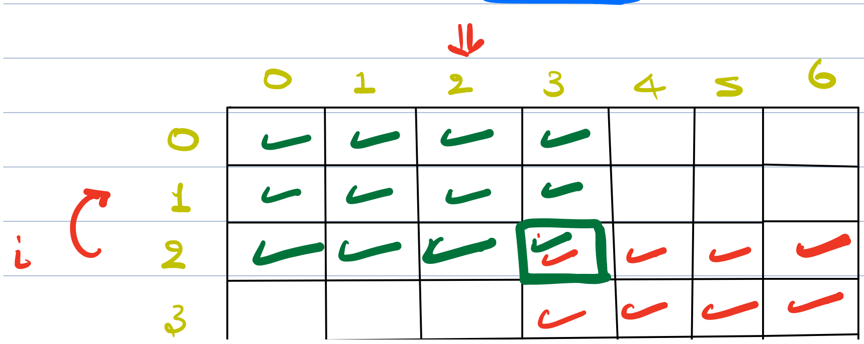
i to $N-1$

$(N-1) - i + 1 = N-i$

Subarray

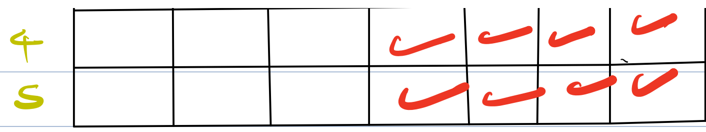
s e

$\Rightarrow (i+1) \times (N-i)$



6×7

(i, j)



Start ⇒ green
End ⇒ red

TL (r_1, c_1) $\quad$ $i - 0 + 1 = i$ $\quad$ BR (r_2, c_2)

$r_1 <= i \Rightarrow [0, i] \Rightarrow (i+1)$

$c_1 <= j \Rightarrow (j+1)$

$[i, N-1]$

$r_2 >= i \Rightarrow (N-i)$

$c_2 >= j \Rightarrow M-j$

$[j, M-1]$

submatrix (i,j) is a part of $= (i+1)(j+1) \times (N-i)(M-j)$

Code

```
Sum = 0;
for (i = 0; i < N; i++) {
    for (j = 0; j < M; j++) {
        Contri = M[i][j] × ((i+1)(j+1)(N-i)(M-j))
```

sum = sum + contri;

}

}

$T.C = O(N \times M)$

$S.C. = O(1)$

Q3 Given a matrix of size $N \times M$.

Find the max submatrix sum.

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-2	3	1
3	6	-3

$\Rightarrow \underline{\underline{10}}$

Soln 1) Brute force

→ Create all submatrices
for each submatrix, calc sum & take max

TL: r_1, c_1 $\Rightarrow$ BR: r_2, c_2

for ($r_1 = 0$; $r_1 < N$; r_1++) { // N

TL { 0 — for ($c_1 = 0$; $c_1 < M$; c_1++) { // M

BR { for ($r_2 = r_1$; $r_2 < N$; r_2++) { // N

for ($c_2 = c_1$; $c_2 < M$; c_2++) { // M

// r_1, c_1, r_2, c_2

$\Downarrow$

// Prefix Sum Matrix

}

}

}

}

$\cancel{N^4}$

$T.C. = O(N^2 M^2)$

Q How to optimise $\Rightarrow$ 2D Kadane's

1) Find the max submatrix sum starting at row 0 & ending at row N-1

2) // " ... starting at row 0 & ending at any row.

3) " " " starting at any row & ending at any row.

Doubt

0 , 0 , 0 , 0 , 0 , 0

+5 , +5 , +5

-5 , -5

Q sindex , eindex value

1 3 5 ⇒

e + 1

0, 5, 0, 0, -5, 0

+2 -2

2, 4, 2

Q times

LLD ⇒

Normalisation

1 + 1

1 + ~

1) Requirement gathering

[illegible]