

Q Given an array, find the  $K$  smaller elements.  
 (print)

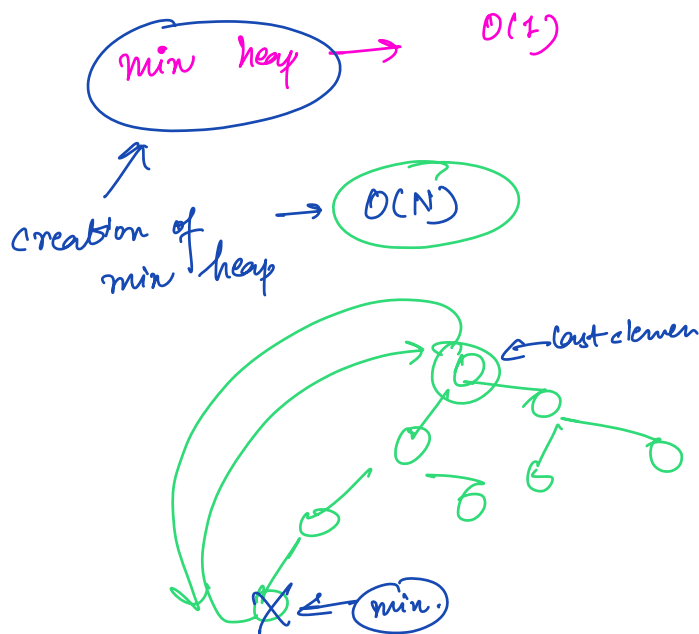
I/P:  $A: [8, 3, 10, 4, 11, 2, 7, 6, 5, 1], K=4$

O/P:  $1, 2, 3, 4$

Approach:

$$T.C = O(n \log n)$$

$$O(N) / O(1) \quad O(\log n)$$

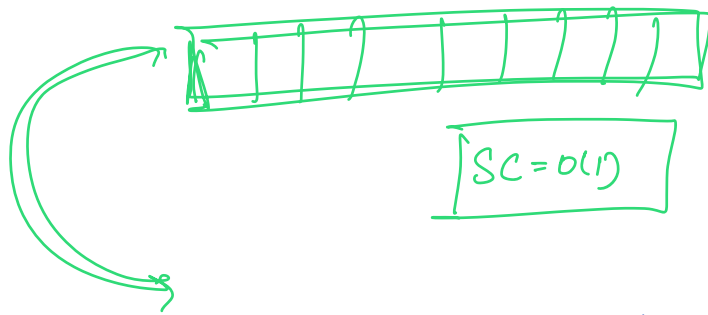


Min-Heap

$$K \rightarrow K \log N$$

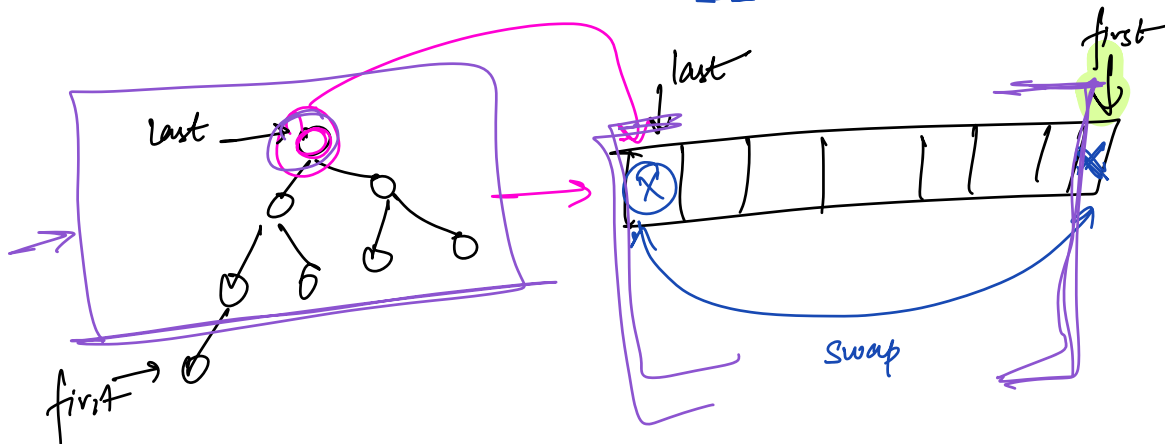
$$T.C = O(N) + O(K \log N)$$

$$N \log N \checkmark$$



Cannot modify the array

$SC = O(N)$



Approach :-

$A = [8, 5, 10, 4, 3, 2, 7, 6, 5, 1], k=4$

first 4 elements.

$[8, 5, 10, 4]$

~~8~~  
~~3~~  
~~10~~  
~~4~~  
 2  
 7  
 6  
 5  
 1

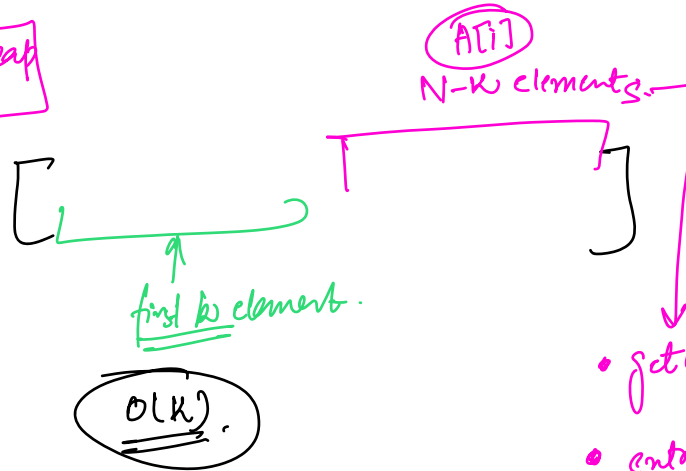
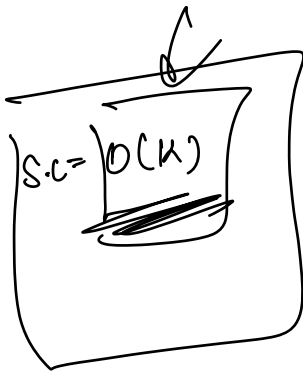
$[3, 4, 2, 1]$

2



$A[i] < \text{max of current group}$

if yes: remove (max of current group)  
insert  $A[i]$  into the group.



- $A[i]$   
N-k elements
- get Current Max  $\rightarrow O(1)$
  - extract Max  $\rightarrow \log k$
  - insert  $A[i]$   $\rightarrow \log k$

$$(N-k) * (2 \log k)$$

T.C =  $O(k) + (N-k) * \log k$

$\underline{m \log N}$

Q

$k$  sorted Array.

Given an array - It is nearly sorted.

↳  $k$  nearly sorted.

↓  
Every element is at most  $k$  positions away from its sorted position.

Goal:- Sort the array

→ A:  $\begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 3 & 2 & 8 & 10 & 9 \end{matrix}$  ,  $k=3$

Sorted A:-  $\begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 5 & 6 & 8 & 9 & 10 \end{matrix}$

$$| \text{Actual index} - \text{Sorted Index} | \leq k$$

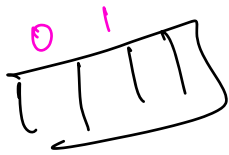
Possible indices for the min element

→  $[0, 1, 2, 3]$

(K-)

possible indices for min element

→  $(0 \rightarrow k)$



$0 \rightarrow 1 \div 2$   
 $0 \rightarrow 2 \div 3$   
 $0 \rightarrow 3 \div 4$

$0 \rightarrow k \div \boxed{(k+1) \text{ elements}}$

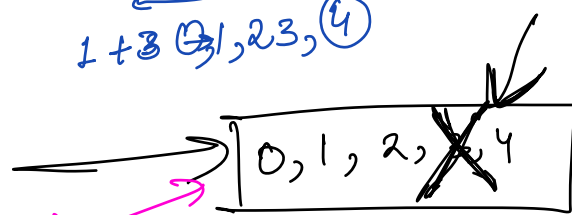
- pick  $(k+1)$  element.
- get min.
- place it at  $S[0]$  → new sorted array.

$2-3$  to  $3+3$   
 $0$  to  $6$

$A: \max(0, 4-3) \rightarrow 1$   
 $\min(4+3, 6) \rightarrow 7$   
 $[0, 1, 2, 3, 4, 5, 6]$ ,  $K=3$

$K+1 = 4$   
 $C: [2, 3, 5, 6, \dots]$   
 $(1, 6)$   
 $2+3 \rightarrow 5$   
 $\max(2-3, 0) \rightarrow 0$

$1+3 \rightarrow 4, 2+3 \rightarrow 5$



what is the min cleared?

$[0, 1, 2, 3, 4, 5]$  ← 2nd min. element (index 2)

group 3 elements

if no. of elements in the group becomes  $< k+1$

8

[10, 9]

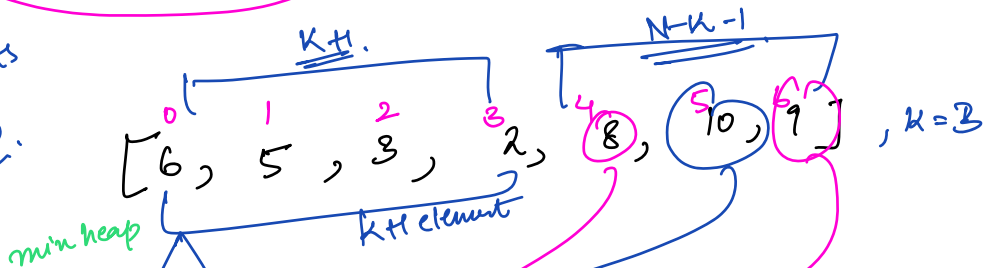
[9]

[10]

→ Created group of first  $k+1$  elements.

- what is the min element.
- remove the min element.
- add 1 element to the group.

heap of  $(k+1)$  elements  
→  $O(k)$ .



↓  
look to extract  
look to insert

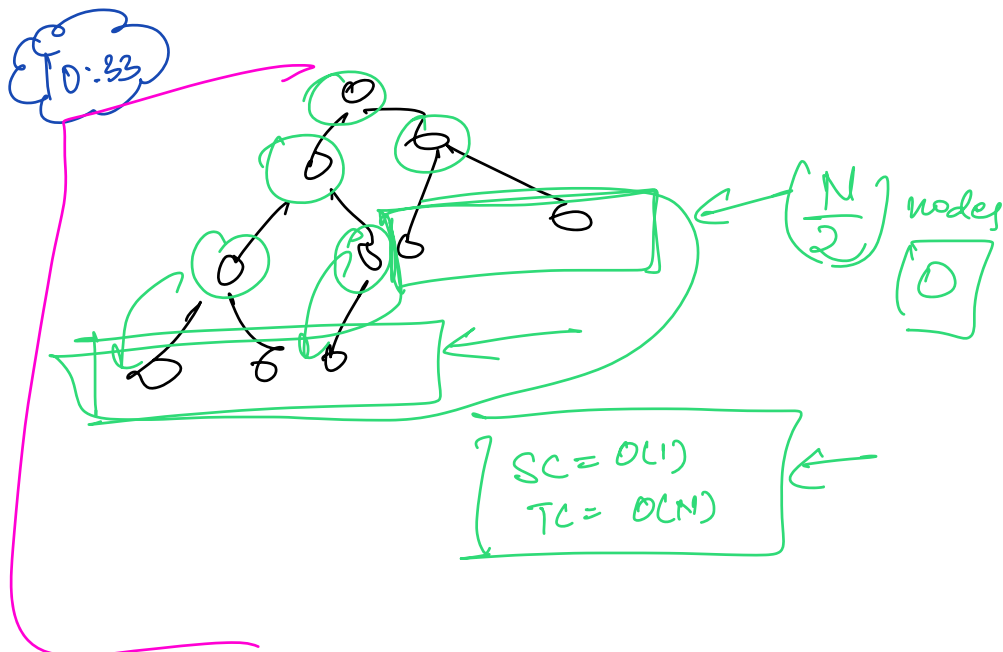
2 3 5 6 8 9 10

min heap of  $k+1$  elements  $\rightarrow O(k)$

$$(N-k-1) * \log k$$

$$TC = O(k) + \underline{(N-k-1) \log k}$$

$$SC = O(k) \rightarrow \text{size of the heap}$$



Q. Given an array, sort it in place.

$$S.C = O(1)$$

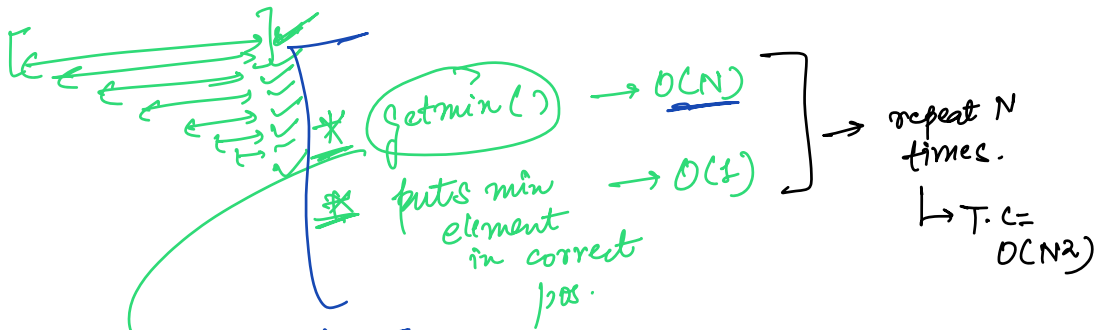
Insertion Sort.  
Bubble Sort  
Selection Sort

$$O(N^2) \leftarrow T.C$$

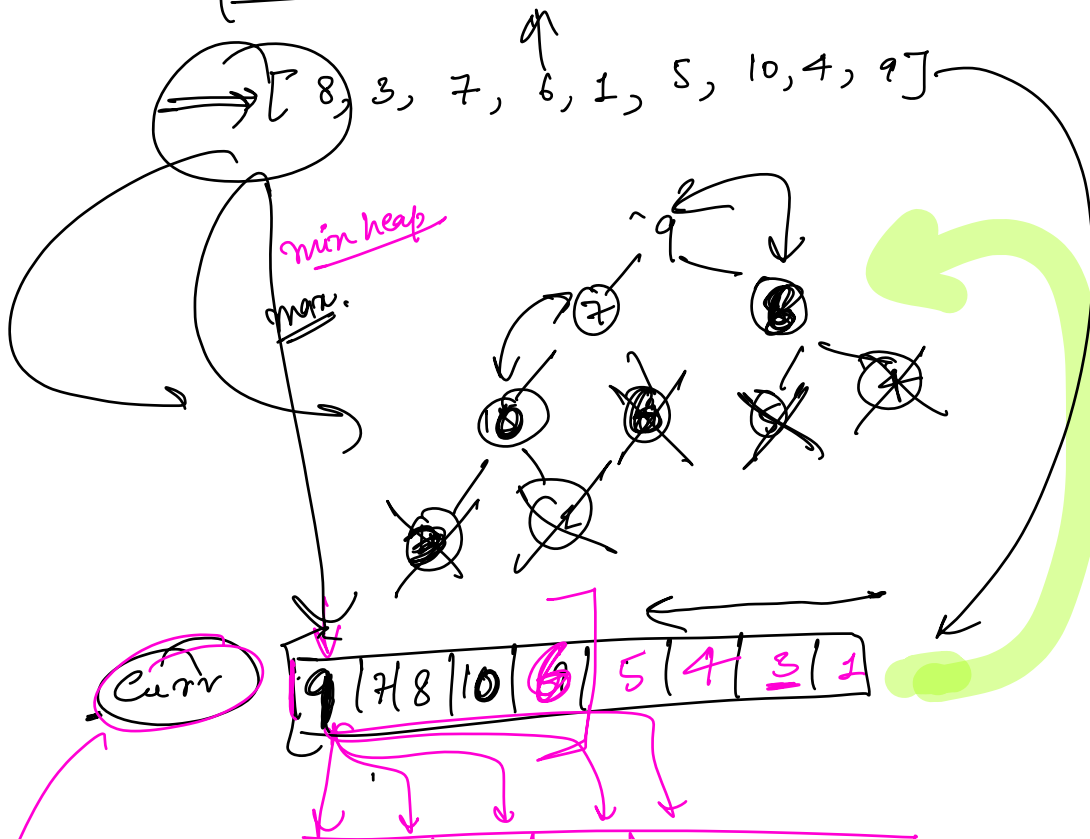
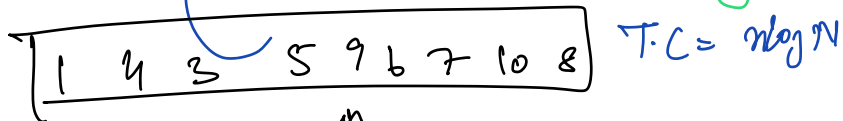
$$S.C = O(1)$$

S.C =  $O(1)$

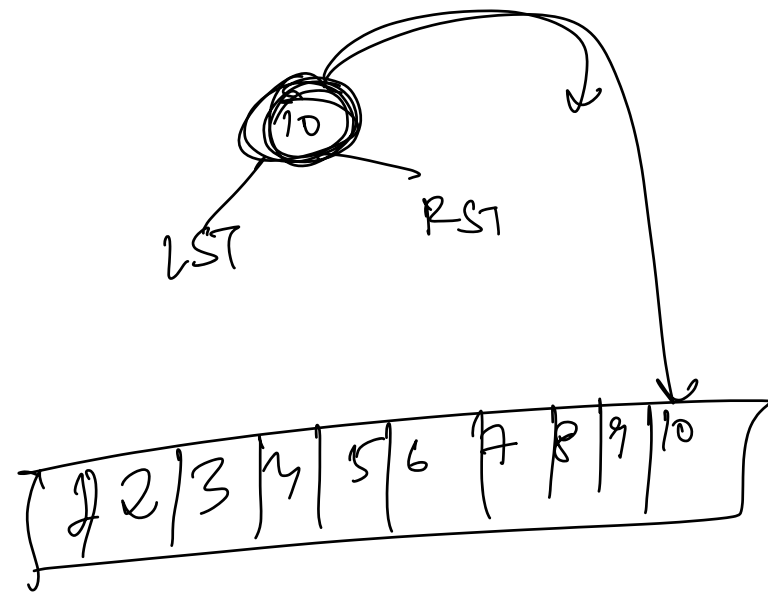
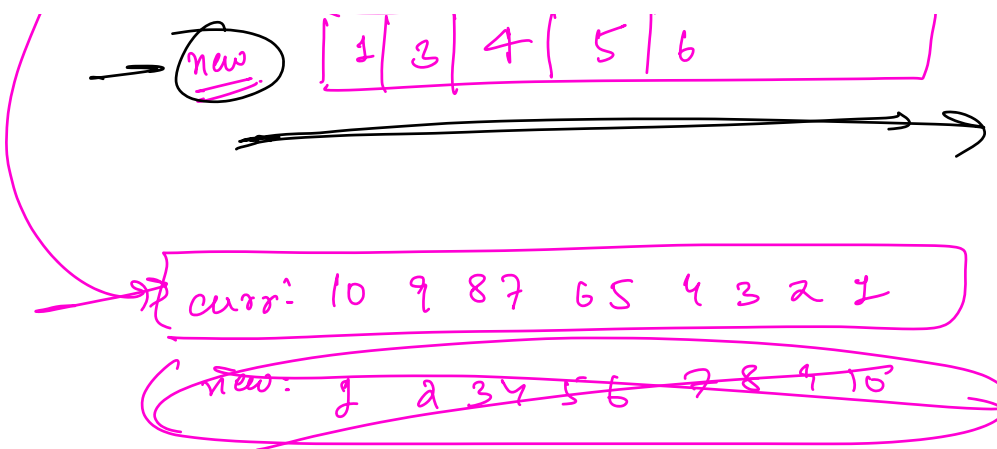
T.C =  $O(n \log n)$



I can use min heap.  $\rightarrow O(1)$   
 extract it as well  $\rightarrow (\log n)$   $\rightarrow N$  time







## Q Running median.

Given a stream of integers, find the median with every new insertion.

$q \rightarrow \text{median}$   
 $q, 6 \rightarrow 7$   
 $q, 6, 3 \rightarrow 6$   
 $q, 6, 3, 10 \rightarrow 7$   
 $[3, 6, 9, 10] \rightarrow \frac{6+9}{2} = \frac{15}{2} = 7.5$

$\frac{q+6}{2} = \frac{15}{2} = 7.5$   
 $\frac{q+6+11}{3} = \frac{16}{3} = 5$

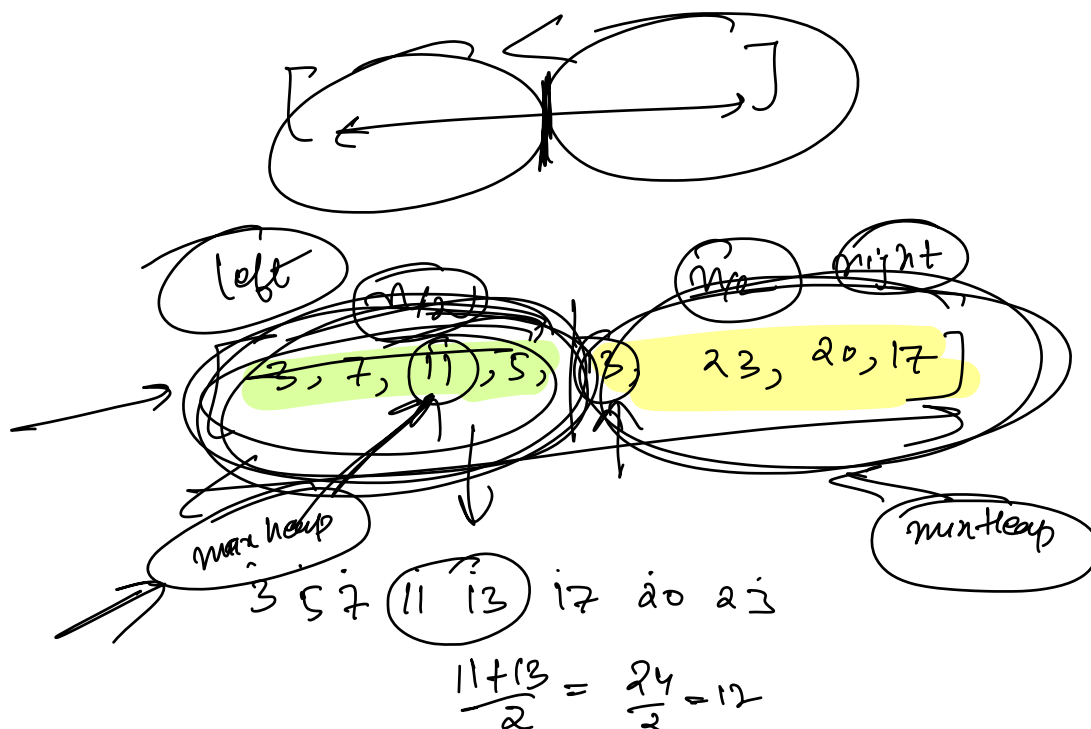
Sort

odd:  $\frac{n+1}{2}$  middle element  
 even:  $\frac{n}{2}$  two middle elements

$n$   $\rightarrow$   $n \log n$   
 $TC = n \log n$

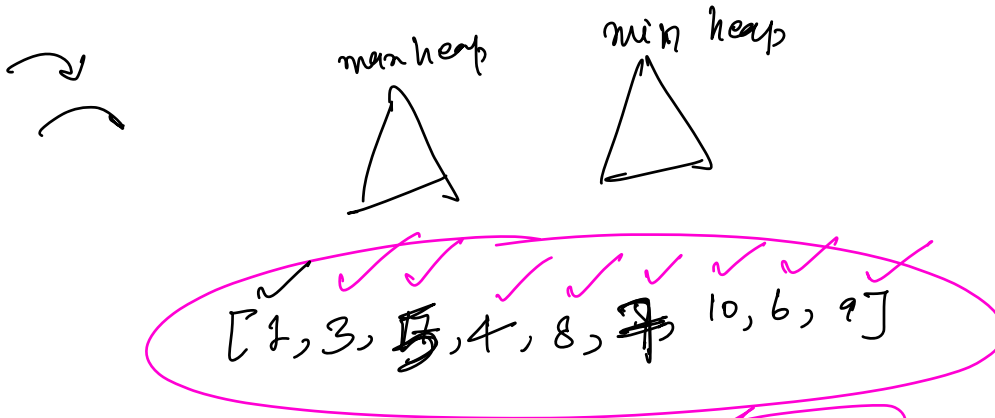
$9, 6$   $\rightarrow$   $T.C = \text{insertion sort} = O(n^2)$

Do we need to sort the whole array to figure out median?



left group: If the array was sorted,  
these are first half elements

right group: If the array was sorted,  
these are the second half  
elements



i → parent  
2i+1  
2i+2



index of the first non  
leaf node  
in a Complete  
Binary Tree →

$$\left\lfloor \frac{(N-1)}{2} \right\rfloor$$

```
for (i = (n-1) / 2 to 0) {  
    heapify (i, A)  
}  
  
heapify (i, A) {  
    left child → 2i + 1  
    right child → 2i + 2  
}
```