Agenda

1) Negative numbers

2) Range

3) Overflow.

4) TLE (Time Limit Exceed)

8 bit no.

$$\underline{\ \ }\,2^7 \quad \underline{\ \ }\,2^6 \quad \underline{\ \ }\,2^5 \quad \underline{\ \ }\,2^4 \quad \underline{\ \ }\,2^3 \quad \underline{\ \ }\,2^2 \quad \underline{\ \ }\,2^1 \quad \underline{\ \ }\,2^0$$

MSB
Most significant Bit

LSB
Least significant bit

10000000 > 01111111

I II

2^7 $2^0 + 2^1 + 2^2 \ldots\ldots 2^6$

$$\underline{\underline{2^7 - 1}}$$

N Bit no.

$$\begin{array}{cccccccccc} 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 2^{N-1} & 2^{N-2} & 2^{n-3} & & & & 2^2 & 2^1 & 2^0 \end{array} > \begin{array}{ccccccc} 0 & 1 & 1 & \cdots & 1 & 1 & 1 \\ 2^{N-1} & 2^{N-2} & 2^{N-3} & & 2^2 & 2^1 & 2^0 \end{array}$$

$$2^{N-1} > 2^0 + 2^1 + 2^2 \cdots 2^{N-2}$$

$$= 2^{N-1} - 1$$

MSB overpowers

Integer $\Rightarrow$ 32

$$\begin{array}{ccccccccc} \_ & \_ & \_ & \_ & \_ & \cdots & \_ & \_ & \_ & \_ \\ 2^{N-1} & 2^{N-2} & 2^{N-3} & \cdots & & & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$$

Min value $\Rightarrow$ 0 (When all bits are 0)

Max Value $\Rightarrow 2^0 + 2^1 + 2^2 \cdots 2^{31}$

$$= 2^{32} - 1$$

Unsigned Integer (All +ve)

s e

Range $\Rightarrow [0, 2^{32} - 1]$

unsigned integers = 2^{32}

bits reserved for an int = 32

2^{32}

2^{31} (Negative)

$2^{31} - 1$ (positive) (1 ⇒ 0)

(0) 32 -ve +ve (1)

Father (Property) 1 S / 1 D

2^{30} 2^{29} 2^{28} 2 1 0

MSB (Sign bit) 2^0

⇓

31 bits

MSB = 0 ⇒ +ve

MSB = 1 ⇒ -ve

Assume 8 bit numbers

10 : 0 0 0 0 1 0 1 0

Sign bit

7 Bit

-10 : $\underline{1}$ $\underline{0}$ $\underline{0}$ $\underline{0}$ $\underline{1}$ $\underline{0}$ $\underline{1}$ $\underline{0}$

$-$ve no.

$\Rightarrow$

int x = 3

Signed Integer

32 bits

MSB — Sign

31 bits (Value)

unsigned int x = 3

32 bits

value

8 bits

1

-3 : $\underline{1}$ $\underline{0}$ $\underline{0}$ $\underline{0}$ $\underline{0}$ $\underline{0}$ $\underline{1}$ $\underline{1}$

-4 : $\underline{1}$ $\underline{0}$ $\underline{0}$ $\underline{0}$ $\underline{0}$ $\underline{1}$ $\underline{0}$ $\underline{0}$

-7 : 0 0 0 0 0 1 1 1

6 : 0 0 0 0 0 1 1 0

−2 : 1 0 0 0 0 0 1 0

4 : 1 0 0 0 1 0 0 0 ✗

0 : 0 0 0 0 0 0 0 0

−0 : 1 0 0 0 0 0 0 0

Two represatations for 0

How to get Negative Representation of N in Binary

$N \Longrightarrow -N$

2's compliment

Steps

1) Write Binary representation of abs(N)

2) Toggle all the bits of N ⇒ 1's compliment
(1's to 0's
0's to 1's)

3) Add 1 to N → 2's Compliment.

Assume 8 bit no.

N: 10

10 :	0	0	0	0	1	0	1	0
	7	6	5	4	3	2	1	0
1st Compliment: (−10)	1	1	1	1	0	1	0	1
	2^7						1	
Add (1)	0	0	0	0	0	0	0	1
2's compliment (−10)	1	1	1	1	0	1	1	0
	-2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

$$-2^7\times 1 + 2^6\times 1 + 2^5\times 1 + 2^4\times 1 + 2^2\times 1 + 2^1\times 1$$

$$= -128 + 64 + 32 + 16 + 4 + 2$$

$$-64 + 32$$

$$-32 \quad -16 = -10$$

(2^7)

MSB $= 1$

$$\begin{array}{c|ccccccc} - & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ \Downarrow & & & & & & & \\ 0 & \multicolumn{7}{c}{\underbrace{\hspace{8em}}_{7\text{ bits}}} \end{array}$$

$2^8 \qquad (2^7 - 1)$

2^7

+ve no. $\Rightarrow$ $[0, \; 2^7 - 1]$

+ve range

2^7 (−ve no.)

−ve no $\Rightarrow$ $[-2^7, \; -1]$ $\qquad (2^7)$

$$10 : \quad \underbrace{\overset{1}{2^1} + \overset{1}{2^3}}$$

$$-2^7 \Rightarrow -\left(\underbrace{(2^7 - 1)}_{"} + 1\right)$$

$\Downarrow$
max +ve value

1st Complimet $\Rightarrow$

$$-2^7 + \underbrace{2^6 + 2^5 + 2^4 + 2^2 + 2^0}_{\underbrace{((2^7 - 1) - 10)}} \Rightarrow$$

2st Complimet $\Rightarrow$ $+1$

$$2^7 \cancel{-1} - 10 + \cancel{1}$$

$$= -2^7 + 2^7 - 10 = -10$$

Hashing $\Rightarrow$ Negative refers to hashing

N bit no.

MSB for -ve contribution

(N-1) bits for +ve contri.

4 Bit no.

Max $\Rightarrow$ 7

=

$$\underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}$$
$$-2^3 \quad 2^2 \quad 2^1 \quad 2^0$$

0 1 1 1

1 0 0 0 Min ⇒ $-\underline{\underline{8}}$

$-2^3 + 0 + 0 + 0$

Bits

		Min	Max
1)	2	$\underset{-2^1}{\underline{1}}\ \underset{2^0}{\underline{0}} \Rightarrow -2$	$\underline{0}\ \underline{1} \Rightarrow 1$
2)	5	$\underset{-2^4}{\underline{1}}\ \underset{2^3}{\underline{0}}\ \underset{2^2}{\underline{0}}\ \underset{2^1}{\underline{0}}\ \underset{2^0}{\underline{0}} \Rightarrow -16$	$01111 \Rightarrow 15$
⋮			
3)	N	$\underset{-2^{N-1}}{\underline{1}}\ \underset{2^{N-2}}{\underline{0}}\ \underset{2^{N-3}}{\underline{0}} \cdots \underset{2^2}{\underline{0}}\ \underset{2^1}{\underline{0}}\ \underset{2^0}{\underline{0}}$ ⇒ -2^{N-1}	$0111\ldots\ldots11$ $2^0+2^1+2^2\ldots\ 2^{N-2}$ $(2^{N-1}-1)$

Signed N Bit no.

$$\text{Range} \Rightarrow \{\overset{s}{-2^{N-1}},\ \overset{e}{2^{(N-1)}-1}\}$$

$$(2^{(N-1)} - \cancel{1}) - (-2^{(N-1)}) + \cancel{1}$$

$$= 2 \times 2^{N-1} = 2^N$$

Signed

1 Byte $\Rightarrow$ 8 bits $\{-2^7, 2^7-1\}$

Short Int
2 Bytes $\Rightarrow$ 16 Bits $\{-2^{15}, 2^{15}-1\}$

$\{-32768, 32767\}$

Int
4 Bytes $\Rightarrow$ 32 bits $\Rightarrow \{-\underset{2 \times 2^{30}}{\underline{2^{31}}}, 2^{31}-1\}$

$\{-2 \times 10^9, 2 \times 10^9 - 1\}$

Long

8 Bytes $\Rightarrow$ 64 bits $\{-2^{63}, \; 2^{63}-1\}$

$\{-8\times10^{18}, \; 8\times10^{18}-1\}$

Approximation

$2^{10} = 1024 \simeq 10^3$

1) $(2^{10})^3 \simeq (10^3)^3$

$2^{30} \simeq 10^9$

2) $(2^{10})^6 \simeq (10^3)^6$

$2^{60} \simeq 10^{18}$

$8\times 2^{60} \simeq 8\times 10^{18}$

$2^{63} \simeq 8\times10^{18}$

Overflow

$\text{int } a = 10^5, \quad b = 10^6$

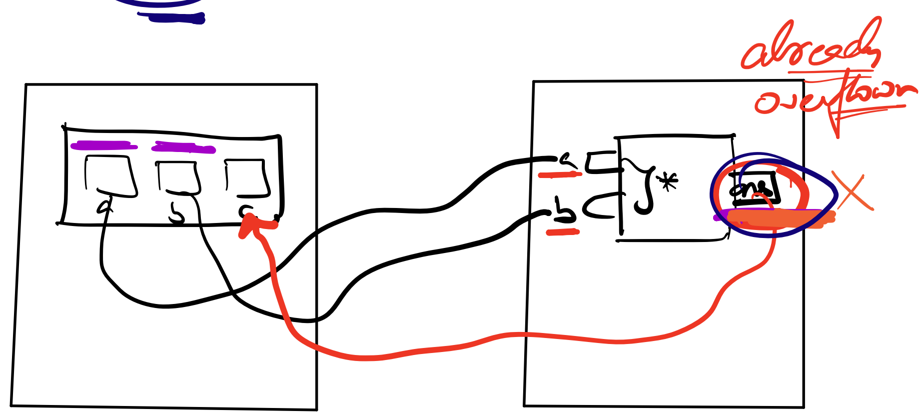
$\text{int } c = \underline{a \times b}$ × — ①

$10^5 \times 10^6 = \underline{10^{11}}$

$\text{long } c = a \times b$ × — ②

$\text{long } c = \text{long}(a \times b)$ × — ③

OS ⇒



$\text{long } c = (\text{long})a * b$

$\text{long } c = a * \text{long}(b)$

long c = long(a) * long(b)

Q Given N array elements. cal sum of all elements.

```
~~int~~ long sum = 0
for (i=0; i<N; i++) {
        sum += A[i];
}
return sum.
```

Constraints

$1 <= N <= 10^5$

$1 \leq arr[i] \leq 10^6$

$$\{ \underbrace{10^6, 10^6, 10^6, \ldots\ 10^6}_{N = 10^5} \}$$

$$10^6 \times 10^5 = \underline{10^{11}}$$

TLE

1 GHz $\rightarrow$ 1 B instructions / sec

10^9

+1
only once

$i = i + 1$ (2)

```
for (i=1; i<=100; i++) {
    print(i);   1
}
```

$4 \times 100 \Rightarrow 400$ instructions $+ 1$

Assume

1 ins $= \frac{1}{10}$

1) 1 iteration = 10 instruction

10^9 instruction $=$ 10^8 iterations.

$10^9 \rightarrow \frac{1}{100} \times 10^9 = 10^7$

2) 1 iteration = 100 instructions

Total no. of iterations

10^9 instruction $= \frac{1}{100} \times 10^9$ iterations

$= \underline{\underline{10^7}}$ ==

1 iteration $\Rightarrow$ $[10, \quad 100]$ # instruction

$\Rightarrow$ $[10^7$ to $10^8]$ iterations

Constraints

1) $1 \leq N \leq 10^5$

$1 \leq A[i] \leq 10^9$

$O(N^2)$
$\Downarrow$
$O(10^{10})$ ✗

$O(N)$
$\Downarrow$
$O(10^5)$ ✓

$O(N\sqrt{N})$
$\Downarrow$
$O(10^5 \times \underbrace{\sqrt{10^5}}_{< 10^3})$ ✓

$> O(10^8)$

$N^2 \simeq 10^8$

Q $O(N^2) \Rightarrow$ Best approach

$1 \le N \le 10^4$

Q

count =

1 1 0 1 1 0 1 1 1 1 1 1 0 1 1

l 2 r 2 6

Convert one 0 to 1 ??

CL0's

CR0's

$l+r+1$ max

swap $0 \Rightarrow 1$

1 0 0 1

For every 0 $\Rightarrow$ max consecutive 1's in left

" " " in right.

Carry forward technique

if # 1's > $l+r$

ret = $l+r+1$

$O(N)$

if # 1's = $l+r$

$ret = (\underline{l+r})$ ✓

Today

next 3 days

after 1 week

15 days

1 month

2/3 months

Q

$(\#1 \Rightarrow \underline{0})$

$A = \underline{000000000000}\,1$

$\Rightarrow 1 \Rightarrow$ take help $(\underline{+1})$

$\times 2$

$0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,1\,0$

$A = 0010100010|0|0111011$

$\#1 \Rightarrow \cancel{x}$ $\times$ helps

$A = 9$

$A: \quad 0\underline{1}001$

$A': \quad 00000$
$+1$ (Help)

$00001 \quad \underline{\underline{<<3}}$

01000
$+1$ (Help)

$\underline{\underline{01001}}$

$N \Rightarrow \quad \_\ \underline{1}\ \_\ \underline{1}\ \_\ \_\ \underline{0}\ \underline{1}$
$\underline{\underline{2^1}} \quad 2^0$

$\underline{\underline{A:}} \quad \underline{\underline{2^0}} + \cancel{2^1}$