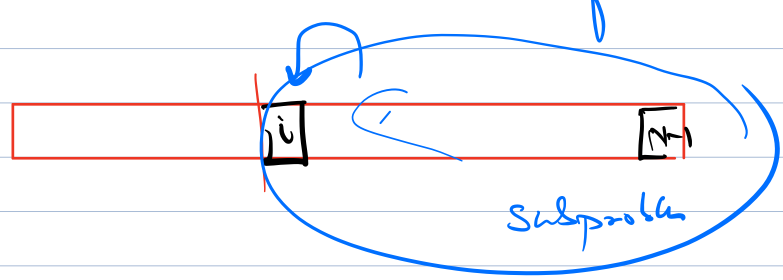
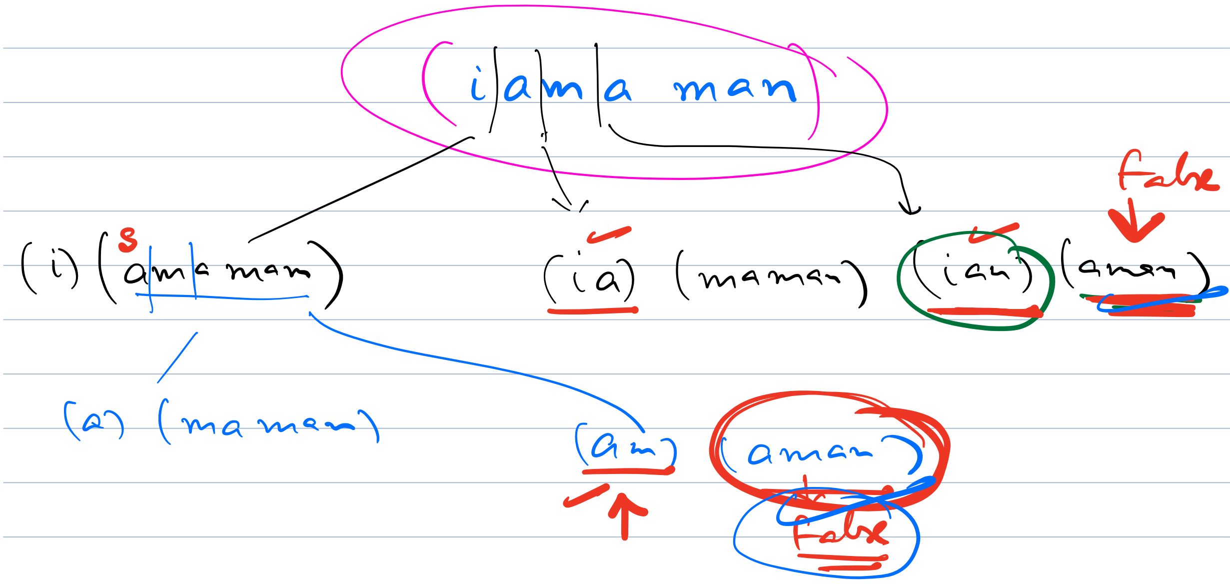
```
int DP[N+1] = {-1} 0
int wordBreak( s, str) {
    // if (s == str.length()) {
              return True;
       }
  if ( DP[s] != -1) {
              ret DP[s];
  }

  for ( i = s ; i <= (N-1); i++ ) {

        if ( isValidWord( s, i , str)
                 && WordBreak( i+1, str))
                 DP[s] = 1;
                 return DP[s]
        }
  }
  DP[s] = 0;
  return DP[s];
}
```

DP[i] ⇒ True if it is possible to break the string from (i to N-1) into valid words from the dictionary



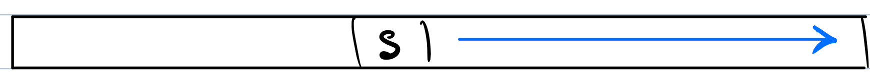


T.C.

w/o mem → 2^N

with mem. → $(1 \times N)\ (N)$

$= O(N^2)$



R → L

```
bool    DP[N+1];
```

```
DP[N] = True;

for (s = N-1; s >= 0; s--) {
    bool ans = false;
    for (i = s; i <= N-1; i++) {
        if (isValidWord(s, i, str) &&
                        DP[i+1]) {
            ans = True;
            break;
        }
    }
    DP[s] = ans;
}
```

Partition

H.W. (DP ?!?)

HM of valid words.

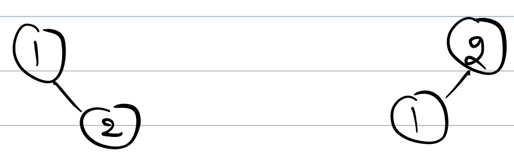
$T.C. = O(N^2)$

Q. Count the total no. of BST's possible with N distinct nodes.

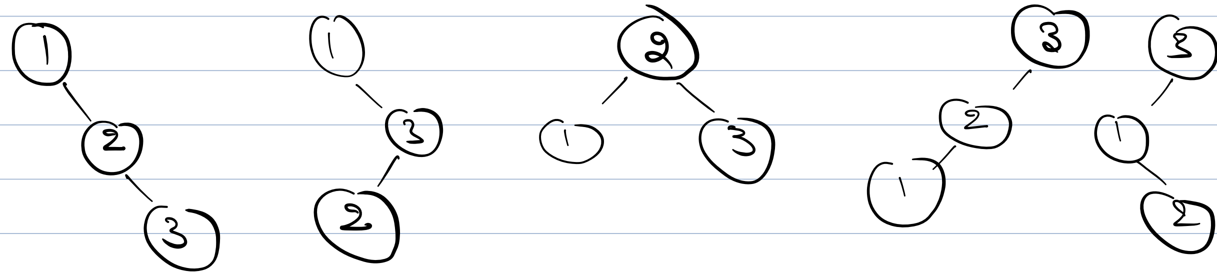
Amazon
flipkart

$N = 1$ ⓵ $= 1$

$N = 2$ [1, 2] $= 2$



$N = 3 \quad [1, 2, 3] \quad = 5$

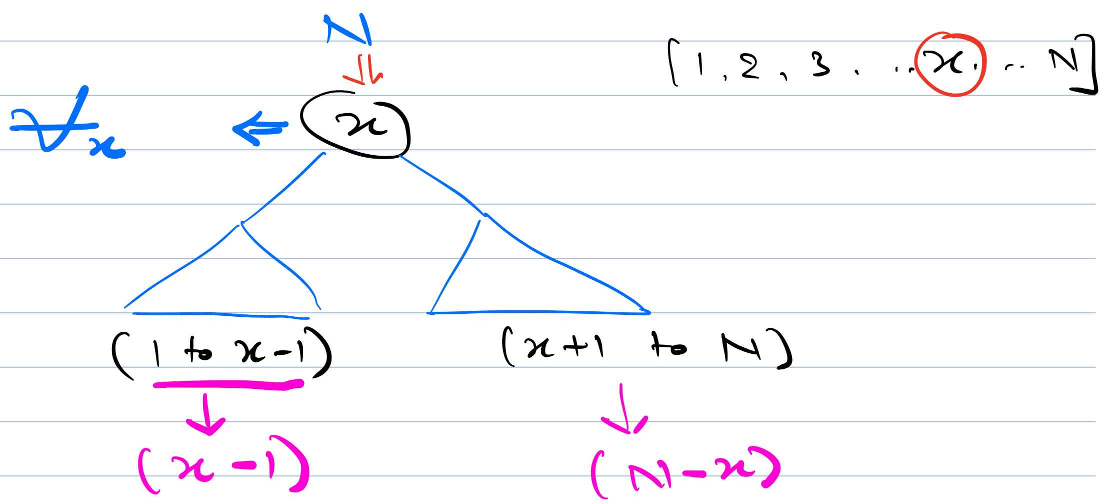


Observations

1) Every element can be the root node.

2) Distinct ele $\Rightarrow$ [1 to N]

3) x is the root node



1) Element of Choice (N)

↳ Which ele will be root

[1 to N]

2) State

$\text{count}(i)$ = No. of BSTs with i distinct nodes

3) Recurrance Relⁿ

Elements $= i$

possible choices for root $(x) = (1 \text{ to } i)$

$\forall x$ from 1 to i

x

$\text{Count}(x-1) \times \text{Count}(i-x)$

$$\boxed{\text{count}(i) = \sum_{x=1}^{i} \text{count}(x-1) \times \text{count}(i-x)}$$

$N = 5$ $(c(5))$

1: { } {2, 3, 4, 5} + 2: {1} {3, 4, 5} + 3: {1, 2} {4, 5}

$C(0) \times C(4)$ $\qquad$ $C(1) \times C(3)$ $\qquad$ $C(2) \times C(2)$

$+$ (4): $\{1,2,3\}$, $\{5\}$ $\qquad$ $+$ (5): $\{1,2,3,4\}$, $\{\}$

$C(3) \times C(1)$ $\qquad$ $C(4) \times C(0)$

$N = 4$

(1): $\{\}$, $\{2,3,4\}$

$C(0) \times C(3)$

$C(2)$ $\qquad$ $C(1)$

$$\boxed{C[N] = \sum_{i=0}^{N-1} c(i) \times (c(N-i-1))}$$

↳ Catalan numbers.

$N =$	0	1	2	3	4	5	6	7	8
	↓	↓	↓	↓	↓	↓	↓	↓	↓
$C(N) =$	1	1	2	5	14	42	132	424	1434

9
↓
4862

Code

```
DP[N+1] = {0}

DP[0] = 1
DP[1] = 1

for (i=2; i<=N; i++) {
        sum = 0;
        for (j=0; j<i; j++) {
              sum += DP[j] × DP[i-j-1];
        }
        DP[i] = sum;
}

return DP[N];
```

$$T.C. = O(N^2)$$

Pascal's triangle

$$C(N) = \frac{(2N)!}{(N+1)!\,(N)!} = \frac{{}^{2N}C_N}{(N+1)}$$

H.W.

T.C

Uses

1) No. of unique BST with N distinct nodes.

2) Count valid parenthesis with N pair of brackets (())

N = 1   () $\Rightarrow$ 1

N = 2   ()()   (())

N = 3 (5) $\Rightarrow$

((())), (()()), (())(), ()(()), ()()()

2 matrices

$$[\quad]_{a \times b} \times [\quad]_{c \times d} = [\quad]_{x \times y}$$

$b = c$ $a \times d$

$$[\quad]_{a \times b} \times [\quad]_{b \times c} = [\quad]_{x \times y}$$

$$x \times y \rightarrow a \times c$$

Cost of multiplying 2 matrices

$$\begin{bmatrix} \cancel{a} & \cancel{b} & \cancel{c} \\ d & e & f \end{bmatrix}_{\substack{a=2 \\ b=3}} \times \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix}_{\substack{b=3 \\ c=3}}$$

$$\begin{bmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \end{bmatrix}_{2 \times 3} \ (a \times c)$$

$$a * x_1 + b * y_1 + c * z_1 \Rightarrow b$$

1 cell = b multiplication

(a×c) cells = a×b×c multiplications

⇓

Cost of multiplying 2 matrices.

Q. Given N matrices that needs to be multiplied.

Find the min cost to multiply these N matrices.

M_1	M_2	M_3	M_4
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
5×4	4×6	6×2	2×7

1) $$\left(\left(\underbrace{M_1 \times M_2}_{5 \times 6}\right) \times M_3\right) \times M_4$$

5×6

5×2

5×7

$$5 \times 4 \times 6 \;+\; 5 \times 6 \times 2 \;+\; 5 \times 2 \times 7$$

$$= 250$$

2) $$\left(\underbrace{M_1 \times M_2}_{5 \times 6}\right) \times \left(\underbrace{M_3 \times M_4}_{6 \times 7}\right)$$

5×7

$5 \times 4 \times 6 + 6 \times 2 \times 7 + 5 \times 6 \times 7$

$= 414$

$$\begin{array}{cccc} 0 & 1 & 2 & 3 \\ M_1 & M_2 & M_3 & M_4 \\ i & & & j \end{array} \quad (0-3)$$

- $(M_1 M_2 M_3)(M_4)$ — $(0-2)$ $(3-3)$
  - $(M_1 M_2)(M_3)$ — $(0-1)$ $(2-2)$
  - $(M_1)(M_2 M_3)$ — $(0-0)$ $(1-2)$
- $(M_1 M_2)(M_3 M_4)$ — $(0-1)$ $(2-3)$
- $(M_1)(M_2 M_3 M_4)$ — $(0-0)$ $(1-3)$
  - $(M_2 M_3)(M_4)$ — $(1-2)$ $(3-3)$
  - $(M_2)(M_3 M_4)$ — $(1-1)$ $(2-3)$

$2 \rightarrow (s, e)$

$M_0 \times M_1 \times M_2 \times M_3 \times M_4 \ldots\ldots\ldots M_{N-1}$ (N)

$d_0 \quad d_1 \quad d_2 \quad d_3 \quad d_4 \quad d_5 \ldots\ldots \quad d_{N-1} \quad d_N$

min no. of variables. $= N+1$

$M_i \Rightarrow d_i \times d_{i+1}$

$(i \rightarrow j) \Rightarrow d_i \times d_{j+1}$

DP[i][j] ⇒ Min cost to multiply matrices from M_i to M_j

Dry Run

$$\begin{array}{ccccccccc} & & i & & & & j & & \\ & & M_0 & & M_1 & & M_2 & & M_3 \\ d_0 & & d_1 & & d_2 & & d_3 & & d_4 \\ 5 & & 4 & & 6 & & 2 & & 7 \end{array}$$

	0	1	2	3
0	0	120		
1	0	0	48	
2	0	0	0	84
3	0	0	0	0

1) $M[0-1] = d_0 \times d_1 \times d_2$
2) $M[1-2] = d_1 \times d_2 \times d_3$
3) $M[2-3] = d_2 \times d_3 \times d_4$

	0	1	2	3
0	0	120	88	
1	0	0	48	104
2	0	0	0	84
3	0	0	0	0

1) $M[0-2]$

$\rightarrow \underbrace{M[0][0]}_{d_0 \times d_1} + \underbrace{M[1-2]}_{d_1 \times d_3} + d_0 \times d_1 \times d_3$

$\rightarrow \overbrace{M[0][1]}^{d_0 \times d_2} + \overbrace{M[2][2]}^{d_2 \times d_3} + d_0 \times d_2 \times d_3$

2) $M[1-3]$

$\rightarrow \overbrace{M[1][1]}^{d_1 \times d_2} + \overbrace{M[2][3]}^{d_2 \times d_4} + d_1 \times d_2 \times d_4$

$\rightarrow \overbrace{M[1][2]}^{d_1 \times d_3} + \overbrace{M[3][3]}^{d_3 \times d_4} + d_1 \times d_3 \times d_4$

	0	1	2	3
0	0	120	88	
1	0	0	48	104
2	0	0	0	84
3	0	0	0	0

$M[0][3]$

$M[0][0] + M[1][3]$
$+$
$d_0 \times d_1 \times d_4$

$M[0][1] + M[2][3]$
$+$
$d_0 \times d_2 \times d_4$

$M[0][2] + M[3][3]$
$+$
$d_0 \times d_3 \times d_4$

$$M[i][j] = \min_{k \to i+1}^{j} \Big(M[i][k-1] + M[k][j] + d_i \times d_k \times d_{j+1} \Big)$$