Q Party

N people attending a party.
Every person can enjoying the party
2 ways
1) Enjoy the party alone
2) Enjoy the party in a pair

Find the total no. of ways in which N people can enjoy the party.

$N = 3$ (A, B, C) = 4

(A) (B) (C) , (AB) (C)
(AC) (B) , (A) (BC)

Sol^n Recursion

1) Elements Of Choice → Party Alone / Party in pairs.

2) How to represent state Ways(i)

No. of ways in which i people can party

3) Recurrance Relation

Party Alone

$$\text{Ways}(i) \begin{cases} \longrightarrow 1 \times \text{Ways}(i-1) \\ \xrightarrow{\text{Party in Pair}} (i-1) \times \text{Ways}(i-2) \end{cases}$$

N = 1   A   = 1

N = 2   A   B

$$\begin{cases} \xrightarrow{\text{Alone}} 1 \times \cancel{\text{Way}(1)}\ 2 \\ \xrightarrow{\text{Pair}} 1 \times 1 \end{cases}$$

= 2

N = 3   A   B   C

$$\begin{cases} \xrightarrow{\text{Alone}} 1 \times \cancel{\text{Ways}(2)}\ 2 \\ + \\ \xrightarrow{\text{Pair}} 2 \times \cancel{\text{Ways}(1)}\ 1 \end{cases}$$

= 4

$$\text{Ways}(i) = \text{ways}(i-1) + (i-1) \times \text{Ways}(i-2)$$

4) Which state is my answer ??

$\text{Ways}(N)$

Q Given a dice (6 faced) & a no. N

Count the no. of ways to get a sum

N if you can 'roll' the 'dice' as many times as you want.

Eg: $N = 4$

1	2	3	4	...	~~5, 6~~
↓	↓	↓	↓		
3	2	1	0		

1	2	3
↓	↓	↓
2	1	0

0 → Base Case

Soln Recursion (Optimal Substructure)

1) Element Of Choice ⇒ 6 [1, 6]

2) What is a state representing

Ways(i) ⇒ Ways to make the total sum i by rolling dice

3) Recurrance Relation

$$\text{Ways}(i) \xrightarrow{1} \text{ways}(i-1)$$
$$\text{Ways}(i) \xrightarrow{2} \text{ways}(i-2)$$
$$\text{Ways}(i) \xrightarrow{3} \text{ways}(i-3)$$
$$\text{Ways}(i) \xrightarrow{4} \text{ways}(i-4)$$
$$\text{Ways}(i) \xrightarrow{5}$$

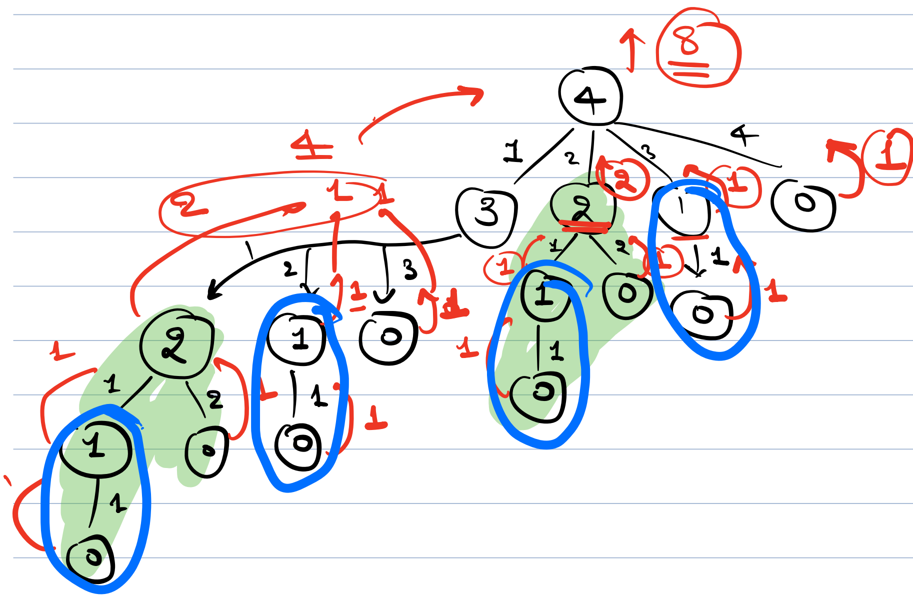
Add $\xrightarrow{6}$ ways (i−5)

ways (i−6)

Base Case

if (N==0) { return 1; }

if (N<0) { return 0; }



Code

DP[N+1] = {−1} ⇒ Memoization array [1, 6]

remaining sum

```
int Ways(N) {
    if (N<0) { return 0; }
    if (N==0) { return 1; }
    if (DP[N] != -1) { return DP[N]; }
    ans = 0;
    for (i=1; i<=6; i++) {
```

Outcomes of dice

```
        ans = ans + Ways(N-i);
    }
    DP[N] = ans;
    return ans;
}
```

0	1	2	3	4	5	6	7	8	9
1									

$$DP[i] = DP[i-1] + DP[i-2] + DP[i-3] + DP[i-4] + DP[i-5] + DP[i-6]$$

```
DP[N+1]
DP[0] = 1;
for (i=1; i<=N; i++) {
    ans = 0;
    for (j=1; j<=6; j++) {
        if (i >= j) {
            ans = ans + DP[i-j]
```

}
DP[i] = ans;
}

4) Final Ans

DP[N]

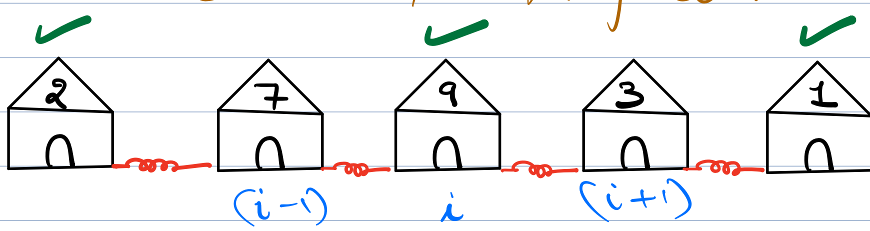
T.C. = O(N)

S.C. = O(N) → O(1)

House Robber

Max Sum w/o Adjacent Elements

Amazon
Flipkart
Wallmart
DE Shaw



2+9+1 = 12

Alerted

Max loot possible

Approach :

Max (~~Sum of all even indexed houses,~~
~~" " odd " "~~)

wrong!

Eg

0	1	2	3	4	5
1,	1,	100,	2,	1,	101

202 Ans

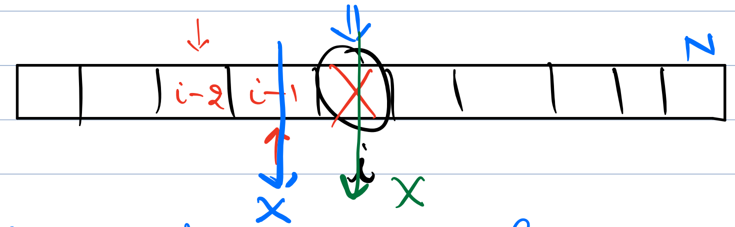
Even: 1 + 100 + 1 = 102

Odd: 1 + 2 + 101 = 104 ✓

Soln Recursion

1) Elements of choices. (rob or Not rob)

2) What does a state represents.



Max loot from the house 1 to ith house.

Loot(i)

3) Recurrance Relation

Loot(i) —rob→ Loot(i-2) + A[i]

Loot(i) —Not rob→

Max

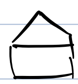
$\searrow$ Loot $(i-1)$

$$\text{Loot}(i) = \max(\text{Loot}(i-2) + A[i], \; \text{Loot}(i-1))$$

Base Case

Transition $(i) \longrightarrow (i-1), \; (i-2)$

$\Rightarrow$ For what value of i, transition goes out of bounds

$i = 1$  0 (return (A[0]))

$DP[1] = A[0]$

$i = 0$

return 0;

$DP[0] = 0$;

Code

$T.C. = O(N)$

$S.C. = O(1)$