BOISE STATE UNIVERSITY

Thermoelectric System Modelling and Design

Pasindu Gamarachchi
Department of Mechanical & Biomedical Engineering

Committee Members

Dr. Yanliang Zhang (Chair)
Dr. John Gardner
Dr. Inanc Senocak

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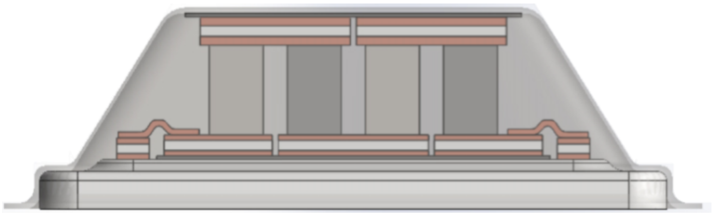
Introduction

Why Thermoelectrics?

- Solid-state heat to electricity conversion or thermoelectric cooling with same device
- Environmentally friendly
- Easy implementation with other systems
- Compact design



Thermoelectric Module

Thermoelectric Effect

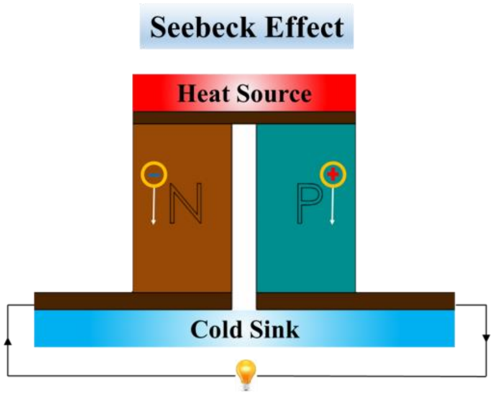
- Seebeck Effect

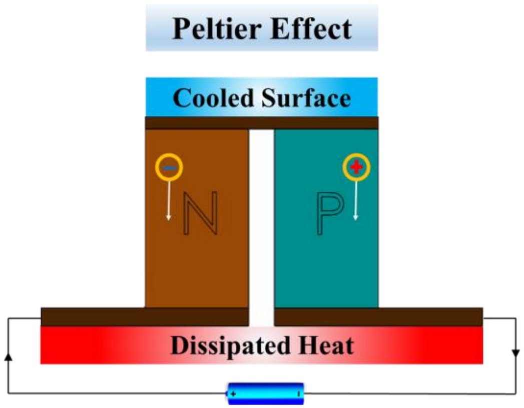
$$-\ \alpha = -\frac{\Delta V}{\Delta T}$$

- Peltier Effect

$$-\ \pi = \frac{I}{q}$$

V: Voltage
T: Temperature
I: Electric Current
q: Heat Dissipated





Thermoelectric Figure of Merit (FOM)

$$Z = \frac{\sigma\alpha^2}{\kappa}$$

σ : Electrical Conductivity
κ : Thermal Conductivity
α : Seebeck Coefficient
T : Absolute Temperature

$$ZT = \frac{\sigma\alpha^2 T}{\kappa}$$

Dimensionless Figure of Merit

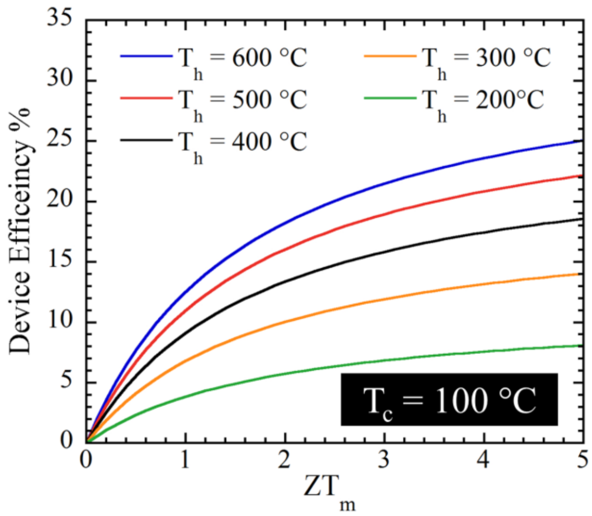
ZT – Thermoelectric Generator (TEG) Efficiency

Max Efficiency Condition

$$\eta_{max(E)} = \frac{\Delta T}{T_h} \frac{\sqrt{1 + Z \cdot T_m} - 1}{\sqrt{1 + Z \cdot T_m} + \frac{T_c}{T_h}}$$

Max Power Condition

$$\eta_{max\,(P)} = \frac{\Delta T}{T_h} \frac{ZT_h}{ZT_m + ZT_h + 4}$$



ΔT : Temperature Difference Between Hot and Cold Side

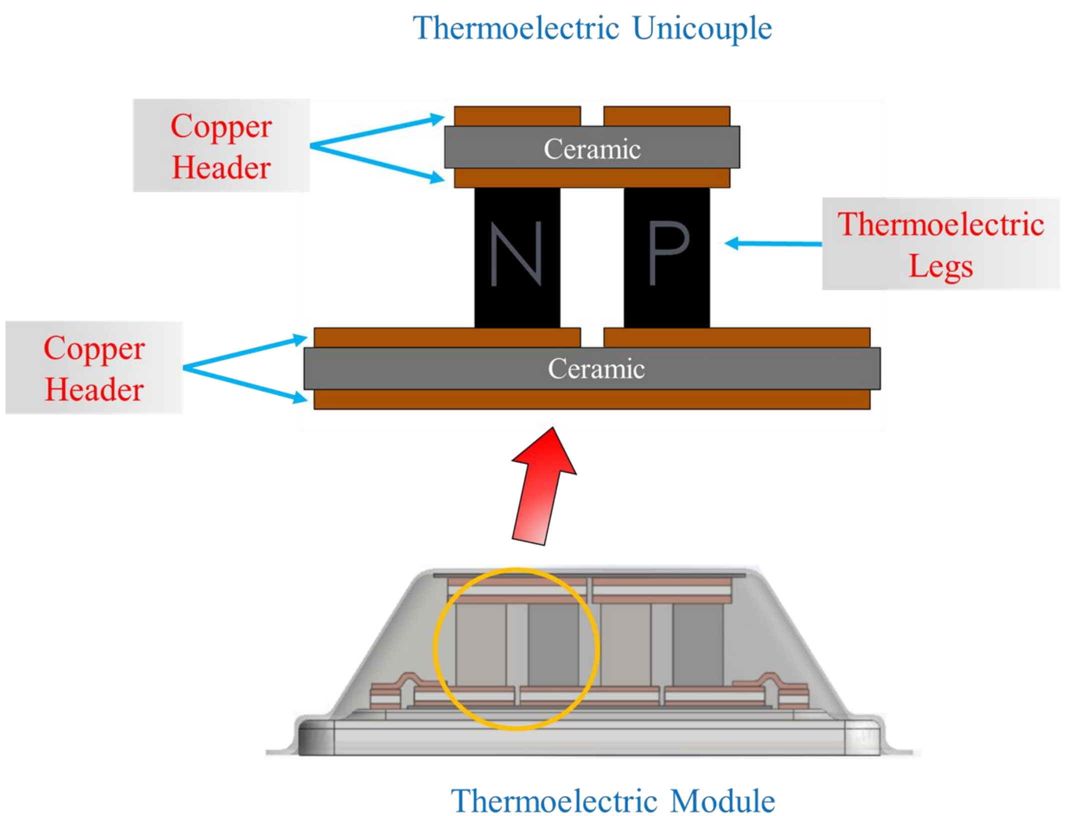
Z : Material FOM

T_m : Average of Hot and Cold Side Temperatures

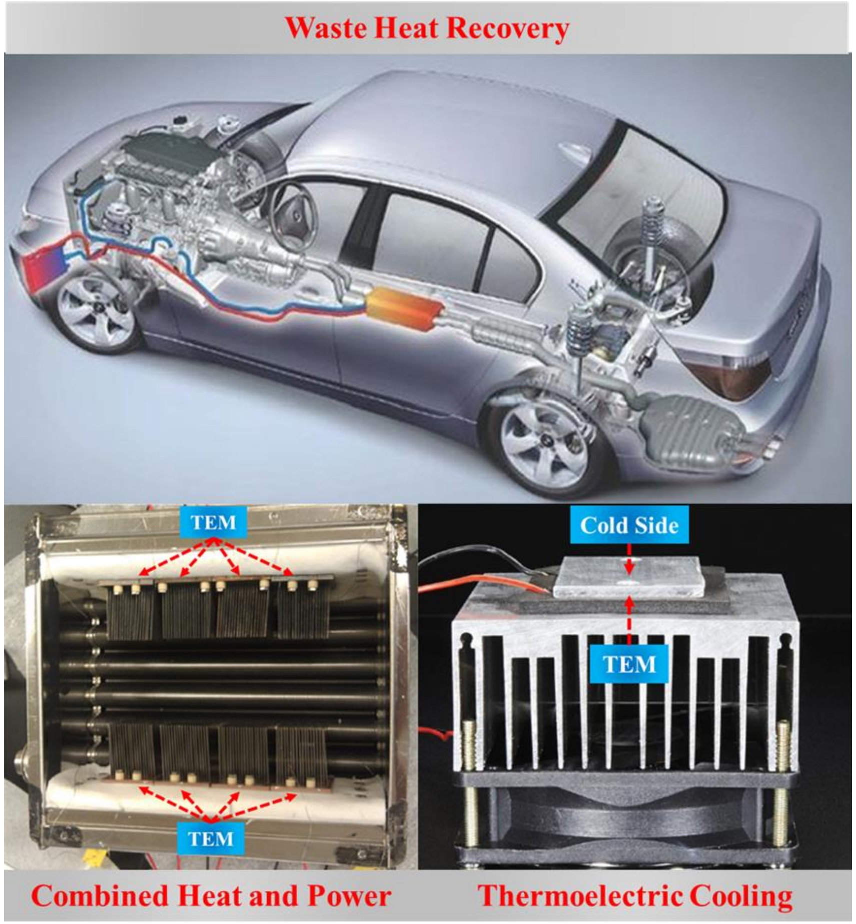
T_h : Hot Side Temperature

T_c : Cold Side Temperature

Thermoelectric Module Components



Thermoelectric Applications



[1]G. Snyder, "Small Thermoelectric Generators", *The Electrochemical Society Interface*, pp. 54-56, 2008.
[2]Y. L. Zhang, X. W. Wang, M. Cleary, L. Schoensee, N. Kempf, and J. Richardson, "High-performance nanostructured thermoelectric generators for micro combined heat and power systems," *Applied Thermal Engineering*, vol. 96, pp. 83-87, Mar 5 2016.

Organization of This Work

1. Temperature Dependent Finite Element Model for a Thermoelectric Generator (TEG)
   I. Model Introduction
   II. Model Validation
   III. Improvements to a Thermoelectric Generator
      a) Study of Ceramic Material
      b) Segmented TEG
2. TEG – Heat Exchanger Model
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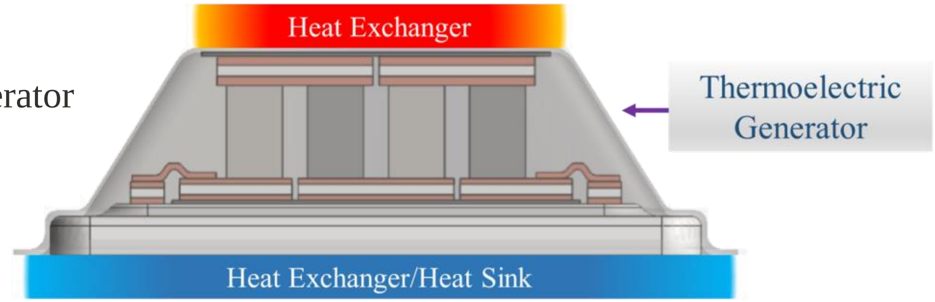
3. TEGs combined with Natural Convection Heat Sink
   I. Heat Sink Model Introduction
   II. Applications
      a) Wireless Sensor Node powered by a TEG
      b) Recovering Waste Heat from the Human Body
4. TEGs combined with Natural Convection Microwire Heat Sinks
   1. Microwire Heat Sink Model
   2. Recovering Waste Heat from the Human Body



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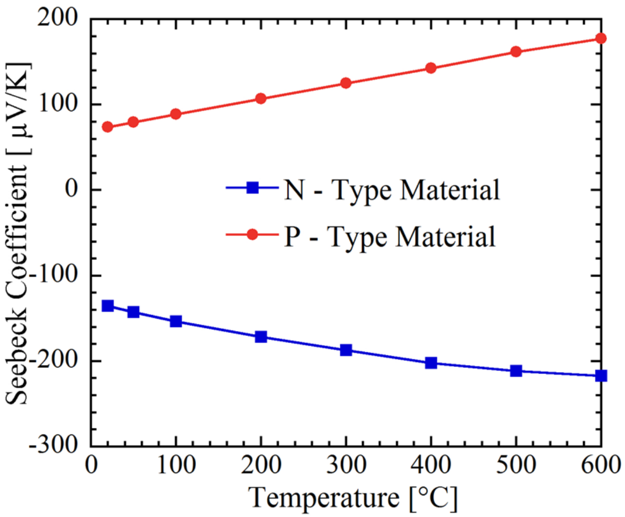
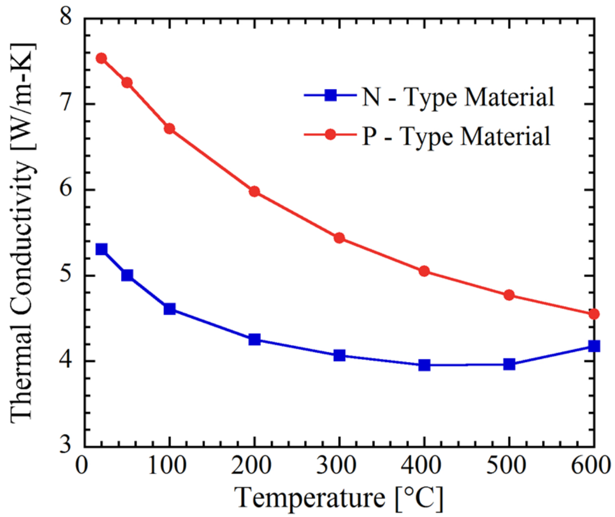
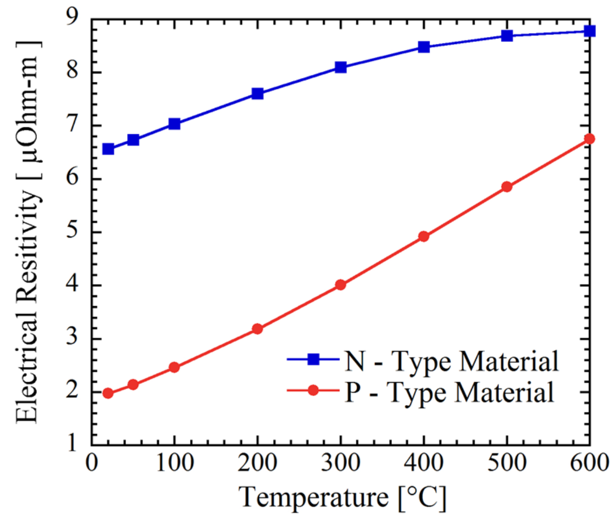
Why a Temperature Dependent Model is Needed?

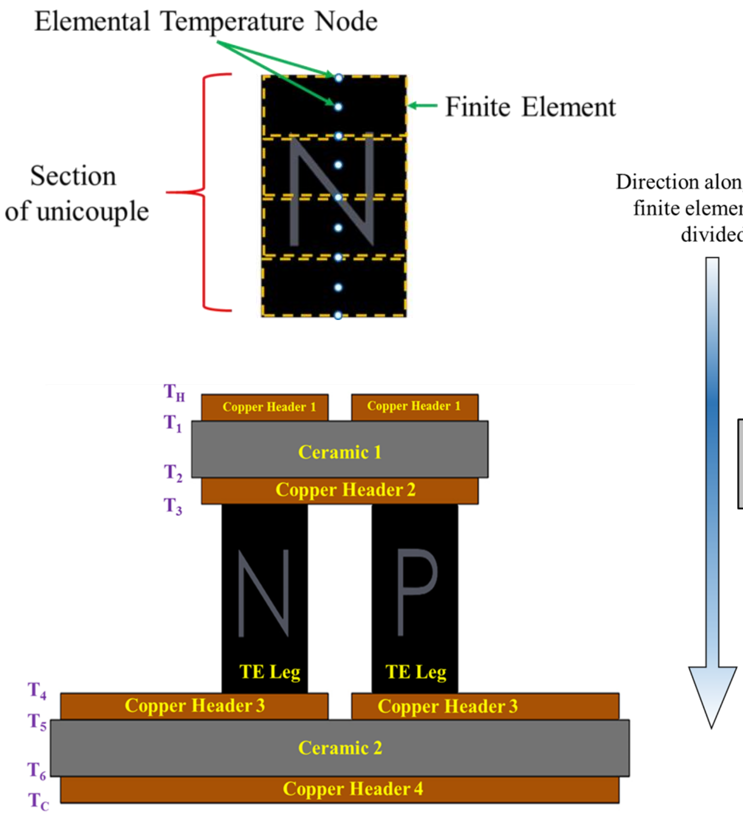


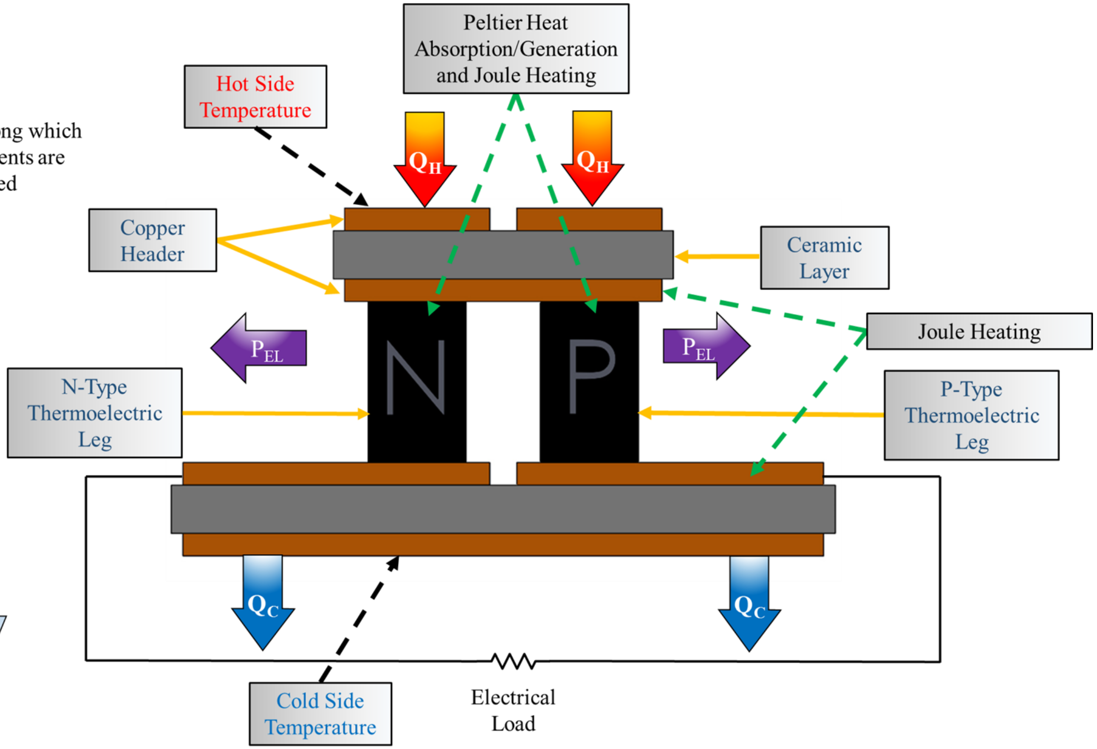




High Temperature Thermoelectric Material : Half – Heusler Alloy

Model Overview





Thermoelectric Power Generation

Heat Flow into Element	$Q_{h,p,n} = abs(\alpha_{p,n}(T)) \cdot (T_{no,p,n}) \cdot I + K_{p,n}(T_{no,p,n} - T_{no+2,p,n}) - \frac{1}{2} I^2 \cdot R_{el,p,n}$
Heat Flow leaving Element	$Q_{c,p,n} = abs(\alpha_{p,n}(T)) \cdot (T_{no+2,p,n}) \cdot I + K_{p,n}(T_{no,p,n} - T_{no+2,p,n}) + \frac{1}{2} I^2 \cdot R_{el,p,n}$
Power generated	$P_{p,n} = Q_{h,p,n} - Q_{c,p,n}$
Thermal Conductance	$K_{p,n} = \frac{\kappa_{p,n}(T) \cdot A_{p,n}}{l_{p,n}}$
Electrical Resistance	$R_{el,p,n} = \frac{\rho_{p,n}(T) \cdot l_{p,n}}{A_{p,n}}$
Open Circuit Voltage	$V_{oc} = \sum_{i=1}^{N} (\alpha_p - \alpha_n)(T_{no} - T_{no+2})$
Electric Current	$I = \frac{V_{oc}}{R_{el,TEC} + R_{el,L}}$

α : Seebeck Coefficient
κ : Thermal Conductivity
ρ : Electrical Resistivity
A : Element Area
l : Element length
$R_{el,L}$: Load Resistance
T : Temperature
no : Node
p : P - Leg
n : N - Leg
N : Number of elements
$R_{el, TEC}$: Electrical Resistance of Unicouple

Temperature Profile Along Unicouple

Temperature Profile

$$T = K^{-1}F$$

Elemental Stiffness Matrix

$$K_e = \int_l [B]^T[D][B]A_e dx$$

Elemental Stiffness Matrix Simplified for a quadratic element

$$K_e = \frac{A_e \kappa_e(T)}{l_e}\begin{bmatrix} 14 & -16 & 2 \\ -16 & 32 & -16 \\ 2 & -16 & 14 \end{bmatrix}$$

Elemental Loading Vector

$$F_e = \int_l G[N]^T A dx$$

Elemental loading vector simplified for a quadratic element

$$F_e = \frac{G_e A_e l_e}{6}\begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$$

T : Temperature Profile
K : Global Stiffness Matrix
F : Global Loading Vector
B : Strain Displacement Matrix
D : Thermal Conductivity Terms
N : Shape Function
$\mathbf{A_e}$: Elemental Area
$\mathbf{l_e}$: Elemental Length
$\boldsymbol{\kappa_e}$: Element Thermal Conductivity
$\mathbf{G_e}$: Volumetric Energy Generation

[3] P. Nithiarasu, R. Lewis and K. Seetharamu, *Fundamentals of the finite element method for heat and mass transfer*, 1st ed. Chichester, West Sussex: Wiley, 2016.

Model Assumptions

1. Temperature variation is assumed 1-D.
2. Constant energy generation or absorption throughout element
3. Convection and radiation from external surfaces ignored
4. The complete top and bottom surfaces are assumed to have a constant temperature
5. Material properties within an element is constant
6. Module power obtained by the product of power output per unicouple and number of unicouples. Similarly, for device voltage.

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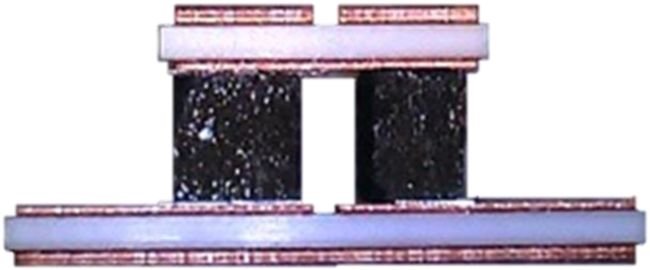

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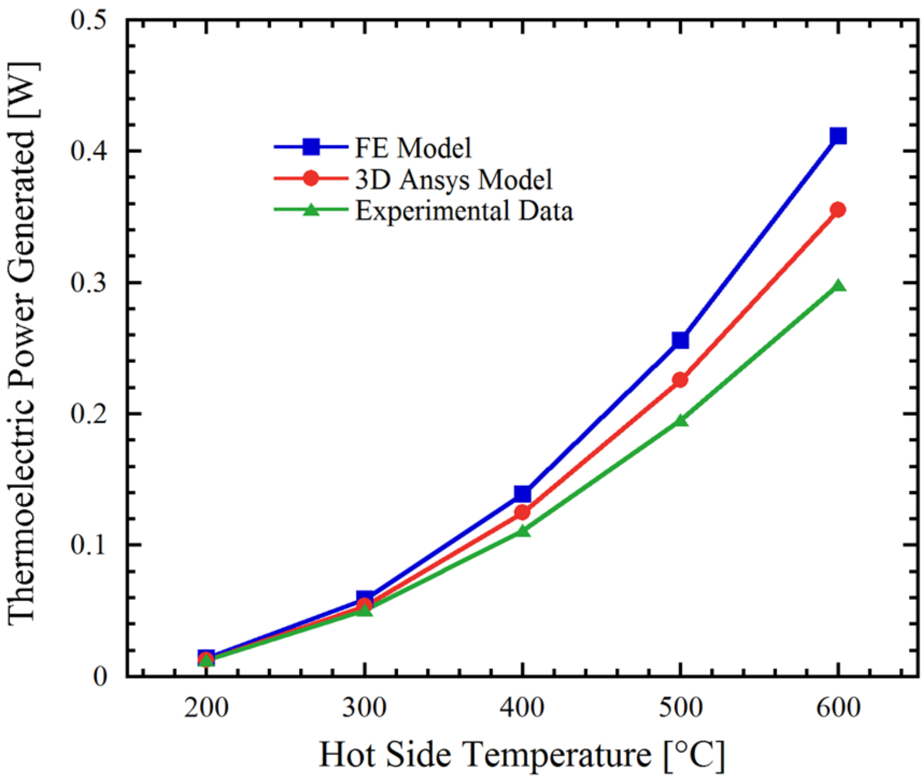
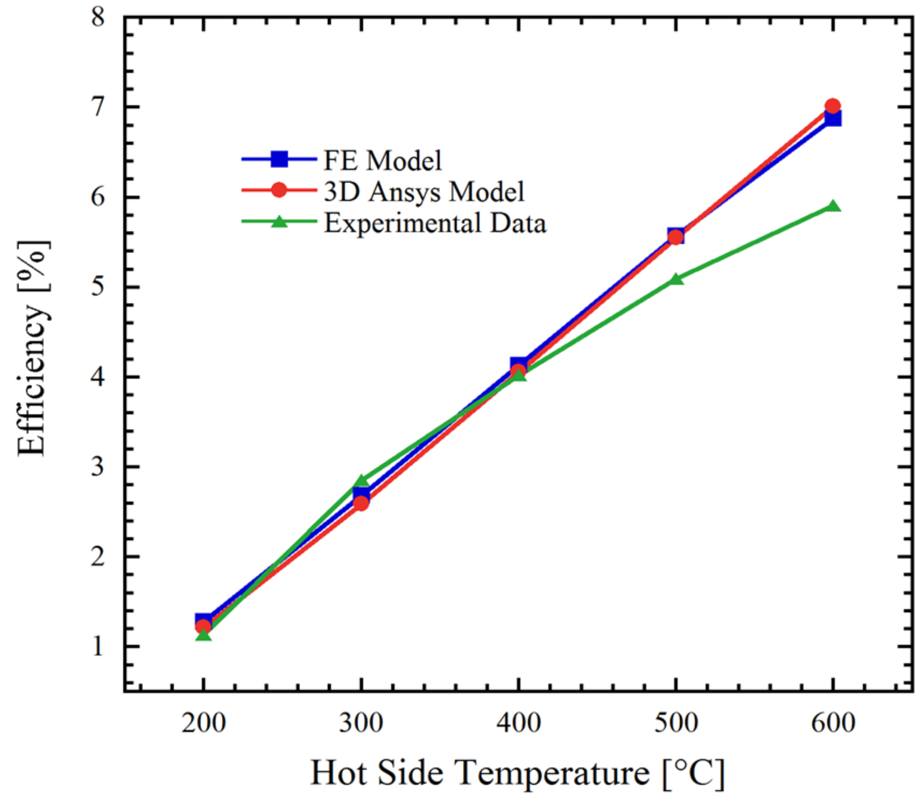
Geometric Input

Component	Thickness/Height [mm]	Area [mm x mm]
Copper Header 1	0.203	[1.93 x 1.96]*2
Ceramic 1	0.635	2.26 x 4.51
Copper Header 2	0.203	1.96 x 4.21
P-Leg	2.400	1.50 x 1.50
N-Leg	2.400	1.50 x 1.50
Copper Header 3	0.203	[1.96 x 4.07]*2
Ceramic 2	0.635	2.26 x 8.81
Copper Header 4	0.203	[1.96 x 8.50]*2

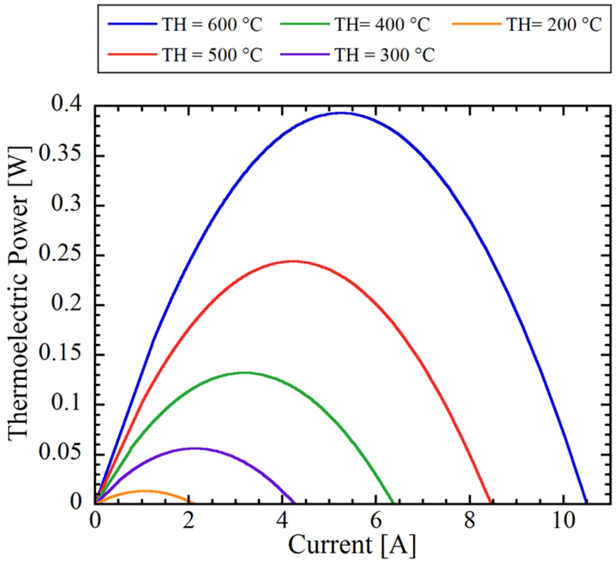
Half-Heusler Alloy Unicouple TEG Power and Efficiency

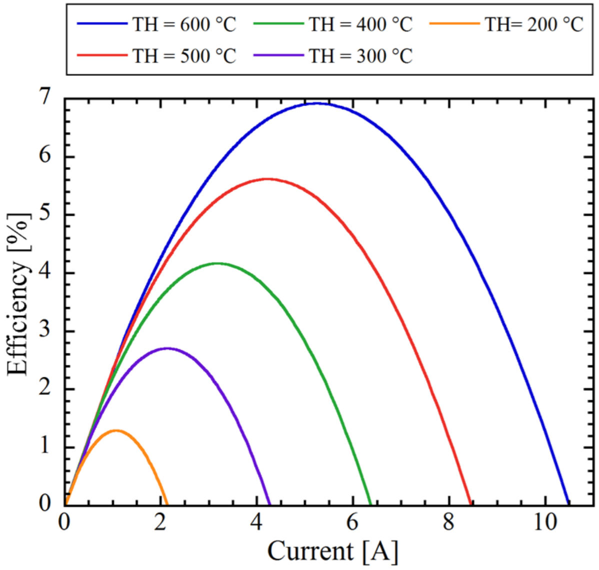


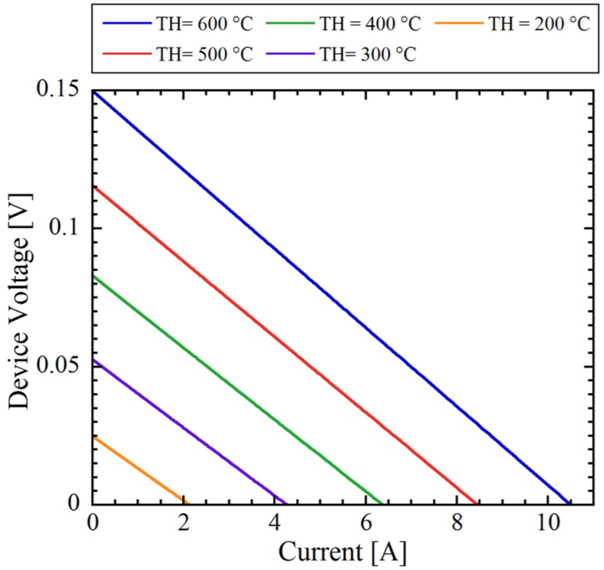


T_c = 100 °C

Half-Heusler Alloy Unicouple Power-Current, Efficiency-Current and Voltage-Current Curves







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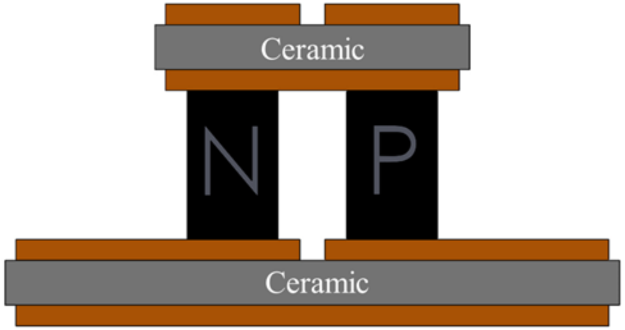
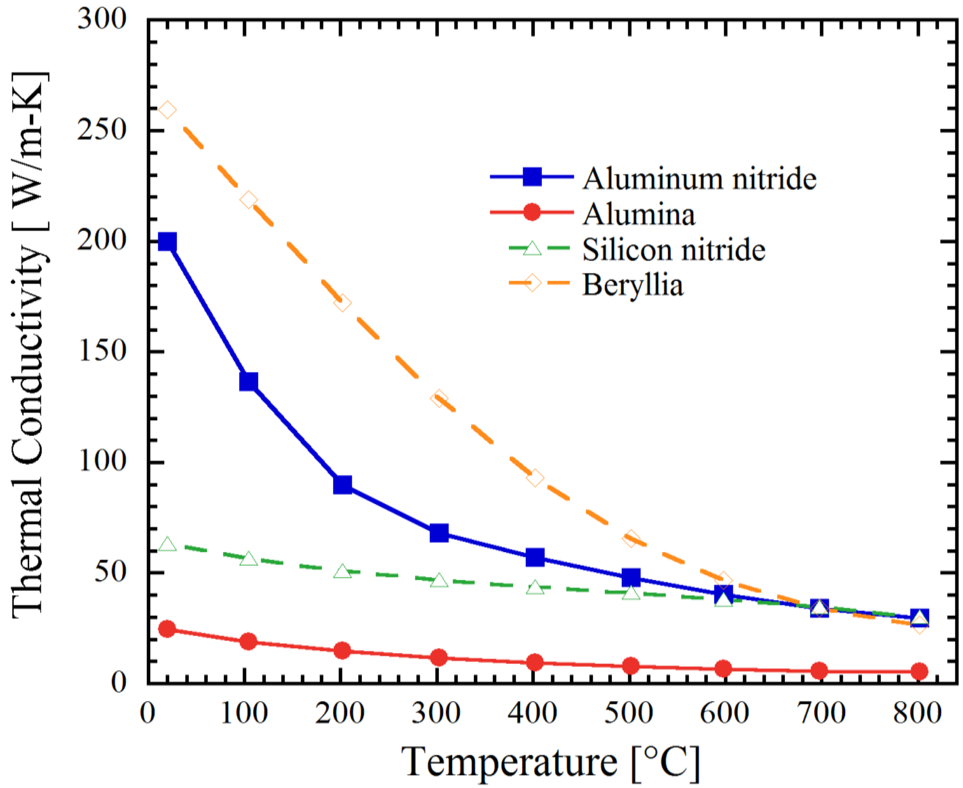
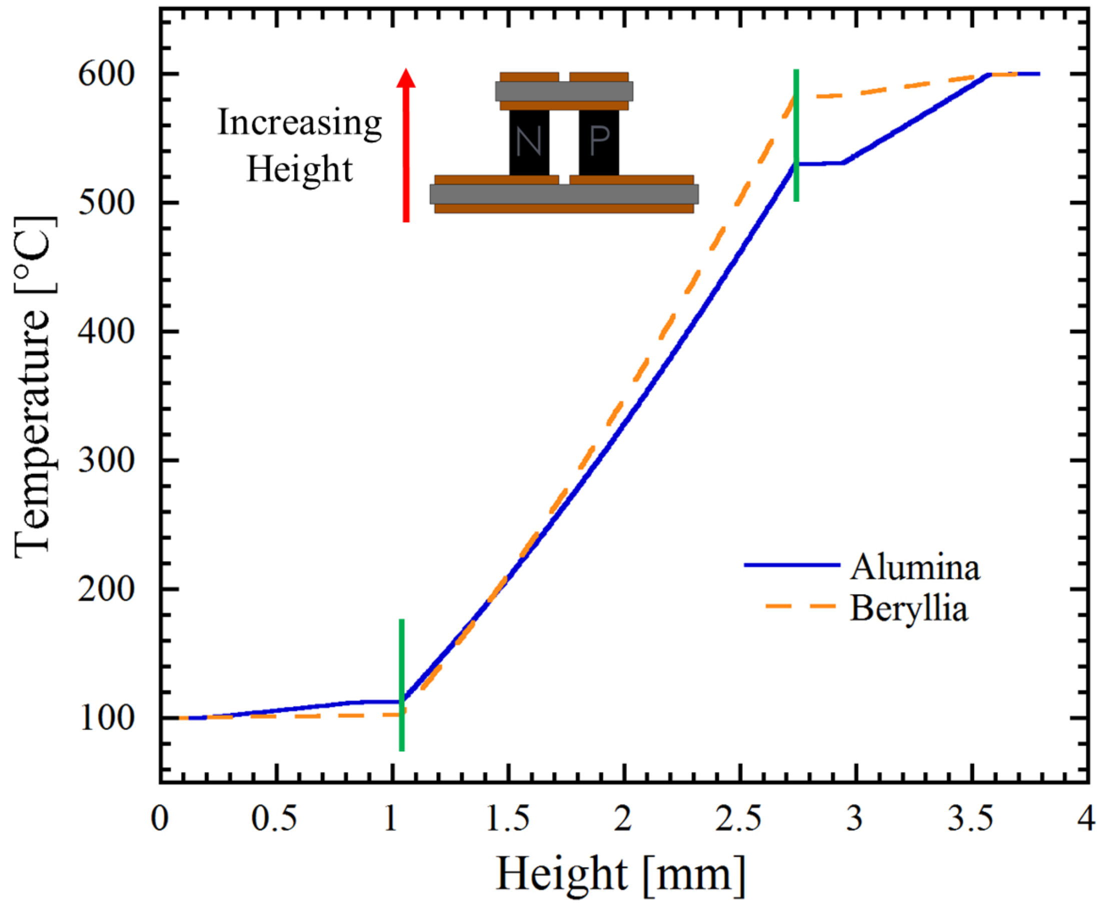


Thermal Conductivity of Various Ceramic Materials with Varied Temperature



[4] D. de Faoite, D. J. Browne, F. R. Chang-Diaz, and K. T. Stanton, "A review of the processing, composition, and temperature-dependent mechanical and thermal properties of dielectric technical ceramics," *Journal of Materials Science*, vol. 47, pp. 4211-4235, May 2012.

Temperature Profile of Unicouple with Two Different Ceramics



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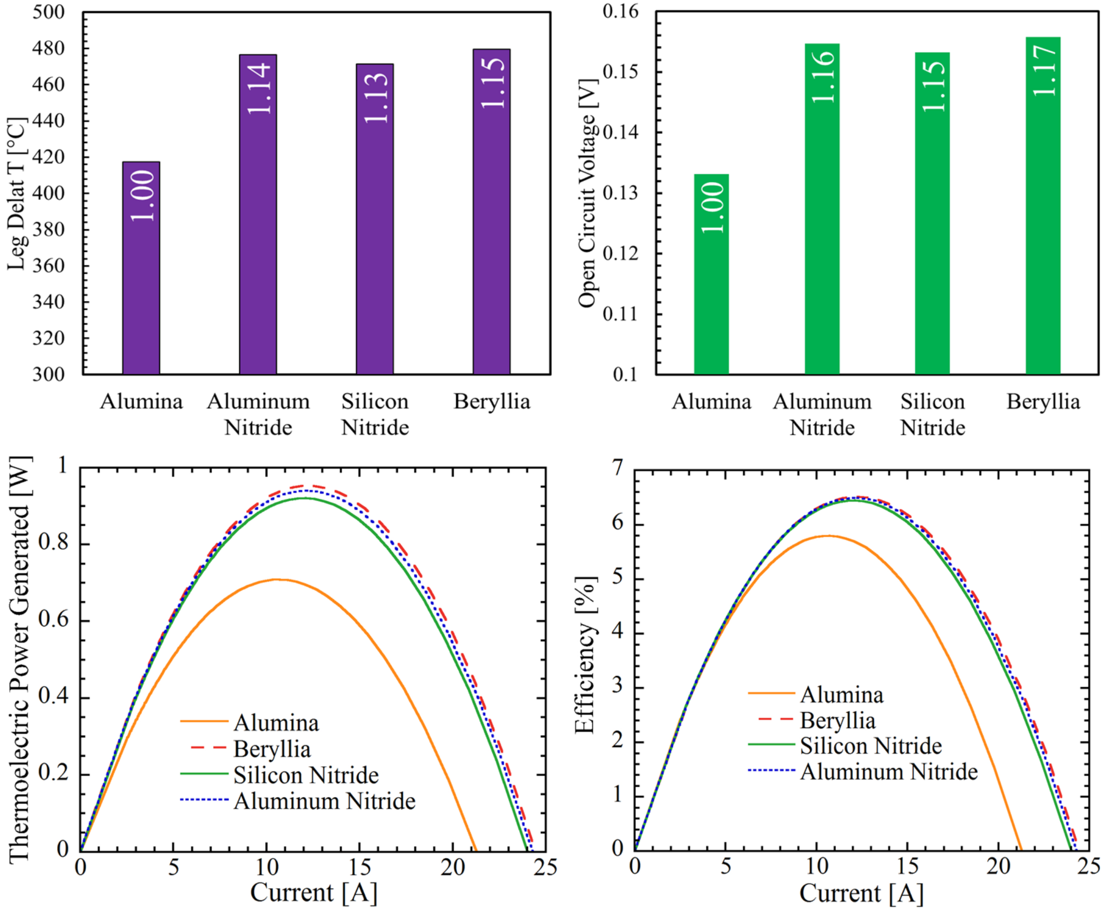
Updated Unicouple Dimensions

Performance Comparison with Different Ceramic Materials



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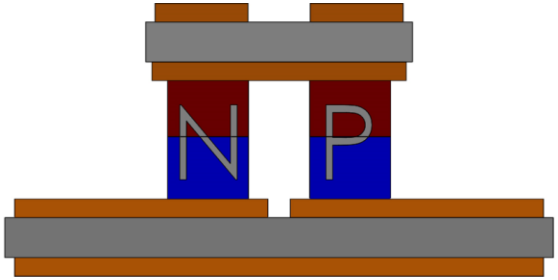
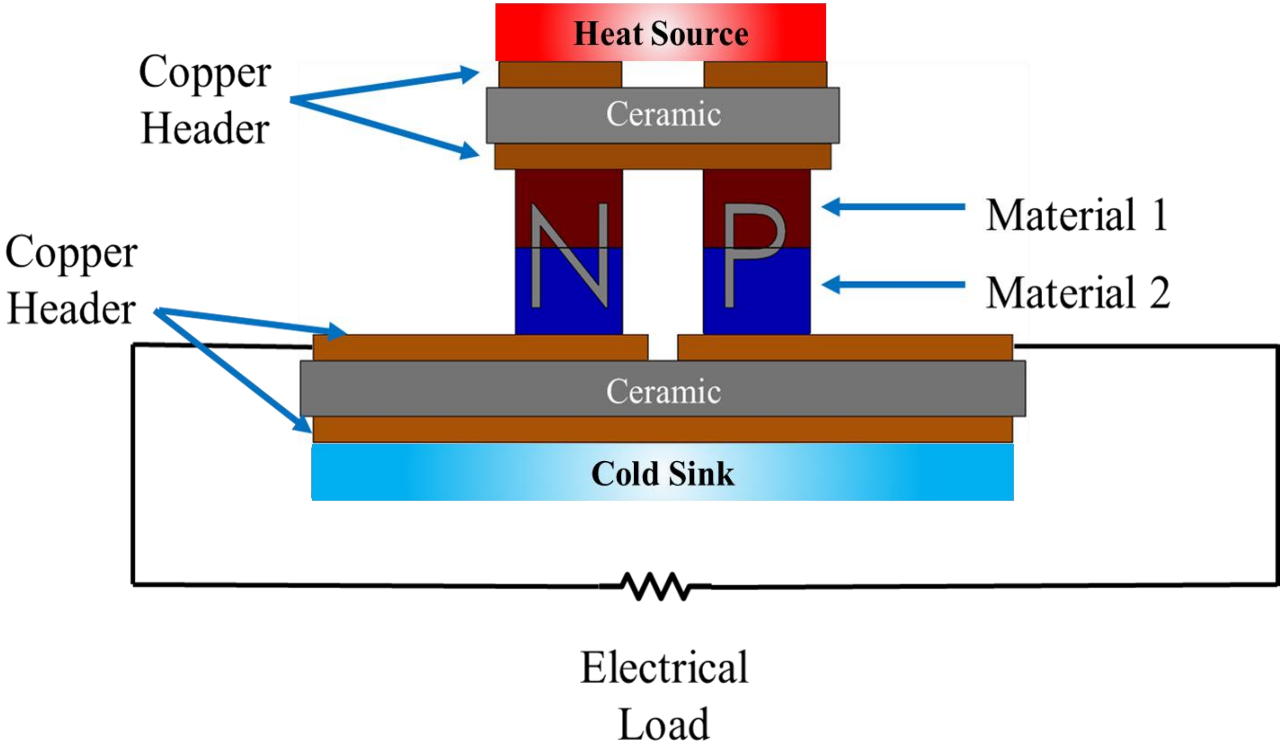


Segmented TEGs



Thermoelectric Compatibility

Optimum relative current density

$$u = \frac{J}{\kappa \nabla T}$$

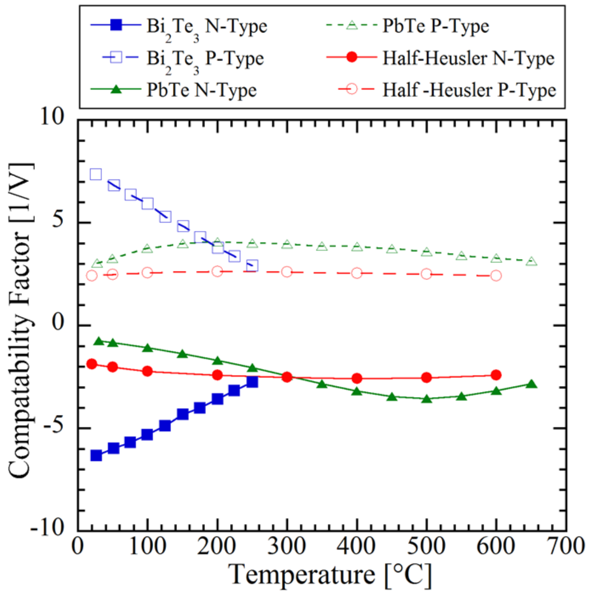
Compatibility Factor

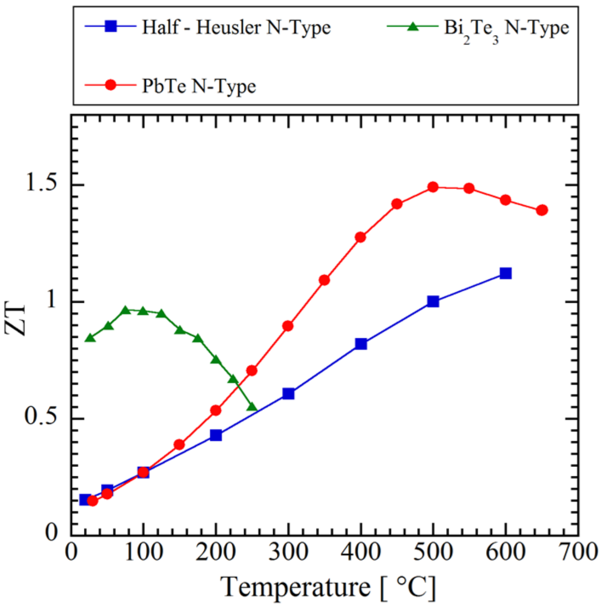
$$s = \frac{\sqrt{1 + zT} - 1}{\alpha T}$$

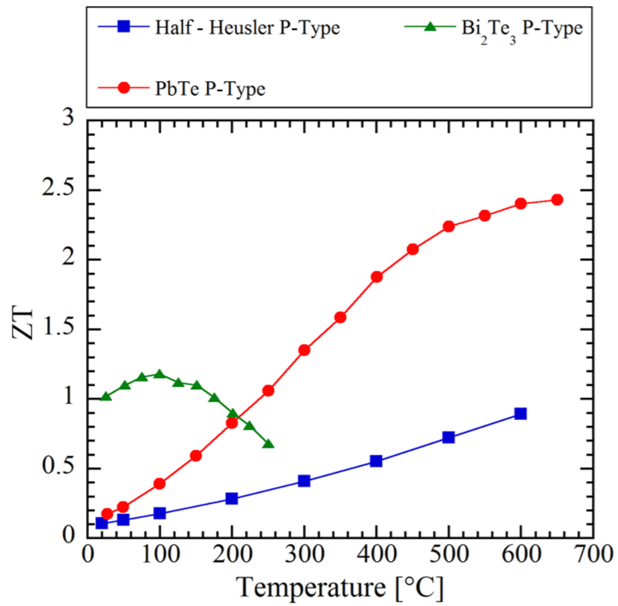
J : Current Density
κ : Thermal Conductivity
∇T : Temperature Gradient
zT : Dimensionless FOM
α : Seebeck Coefficient
T : Absolute Temperature

[5] G. Snyder and T. Ursell, "Thermoelectric efficiency and compatability", *Physics Review Letters*, vol. 91, no. 14, 2003.

Material Compatibility and ZT







Leg Height Ratios

P – Leg Ratio

$$\frac{l_{p1}}{l_{p2}} = \frac{\int_{T_{int}}^{T_h} \kappa_{p1}(T)dT}{\int_{T_c}^{T_{int}} \kappa_{p2}(T)dT}$$

N – Leg Ratio

$$\frac{l_{n1}}{l_{n2}} = \frac{\int_{T_{int}}^{T_h} \kappa_{n1}(T)dT}{\int_{T_c}^{T_{int}} \kappa_{n2}(T)dT}$$

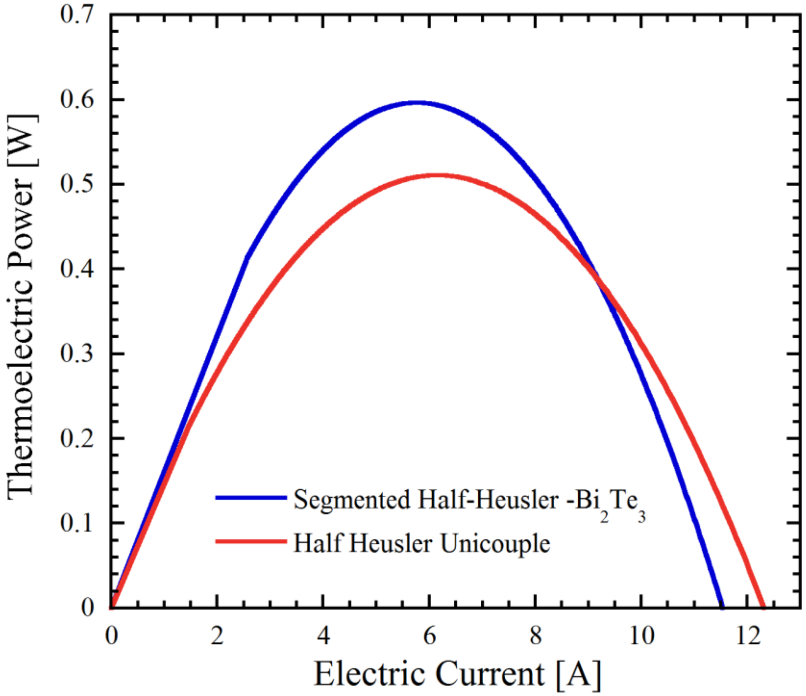
l : Leg Height
p : p-leg
n : n-leg
$\mathbf{T_h}$: Temperature at Intercept of Top Copper Header and Top Material
$\mathbf{T_{int}}$: Temperature at Interface Between the Two Materials
$\mathbf{T_c}$: Temperature at Interface Between Bottom Copper Header and Bottom Material
κ : Thermal Conductivity

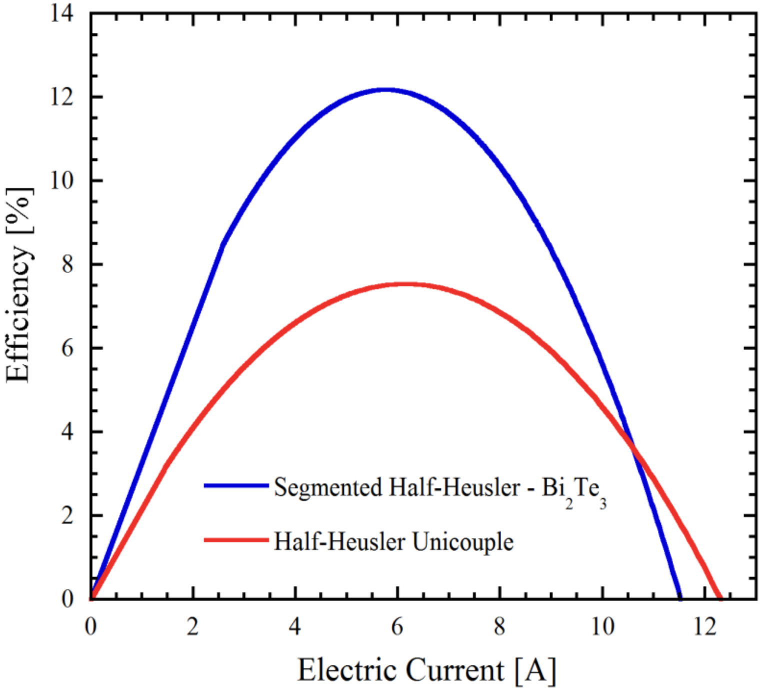
Segment	**Unicouple A**	**Unicouple B**
Half-Heusler alloy segment	2.106 mm	-
PbTe segment	-	1.618 mm
Bi_2Te_3 segment	0.294 mm	0.782 mm
Total Height	2.4 mm	2.4 mm

Leg Dimensions

[6] G. B. Zhang, L. H. Fan, Z. Q. Niu, K. Jiao, H. Diao, Q. Du, *et al.*, "A comprehensive design method for segmented thermoelectric generator," *Energy Conversion and Management*, vol. 106, pp. 510-519, Dec 2015.

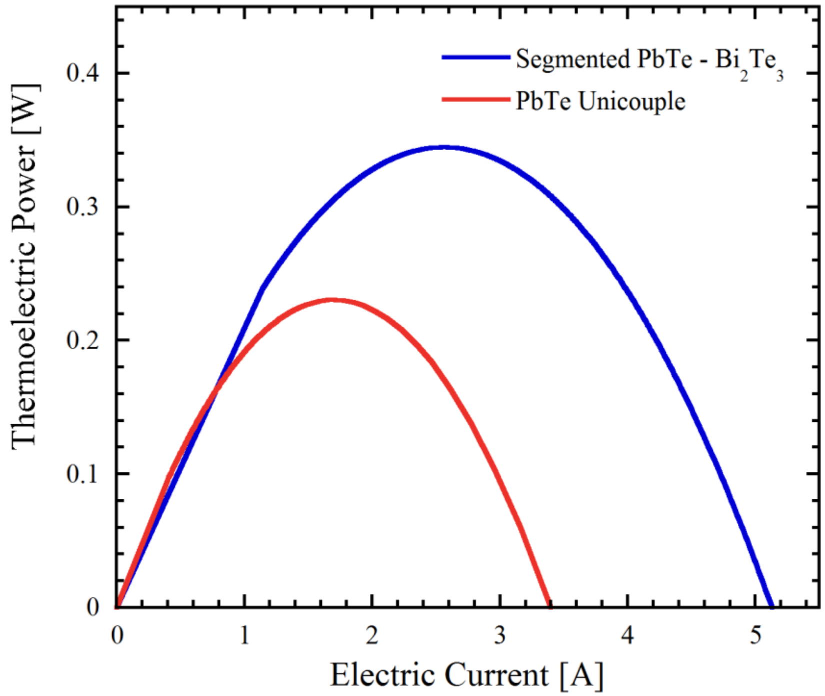
Segmented HH – Bi_2Te_3 Unicouple

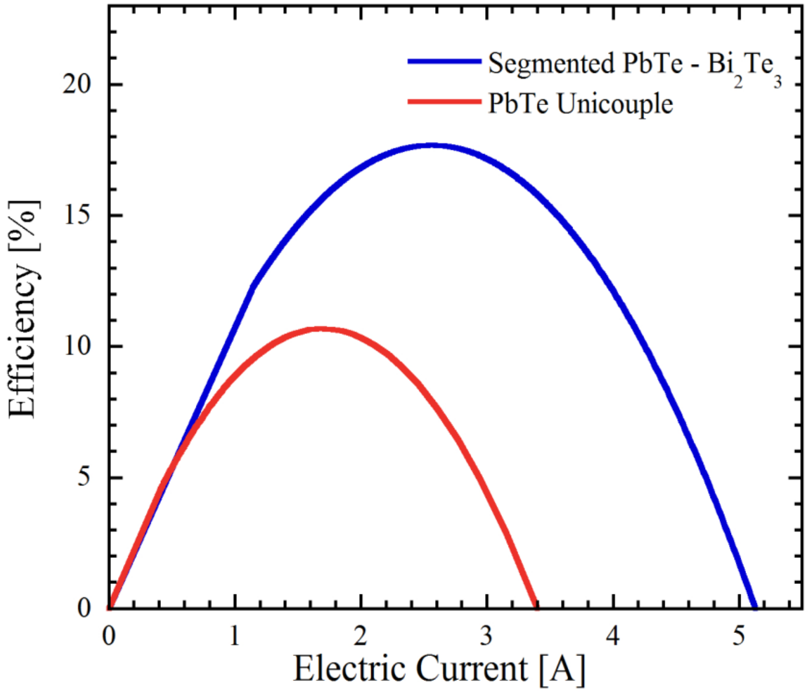




Delta T = 580 °C

Segmented PbTe – Bi_2Te_3 Unicouple





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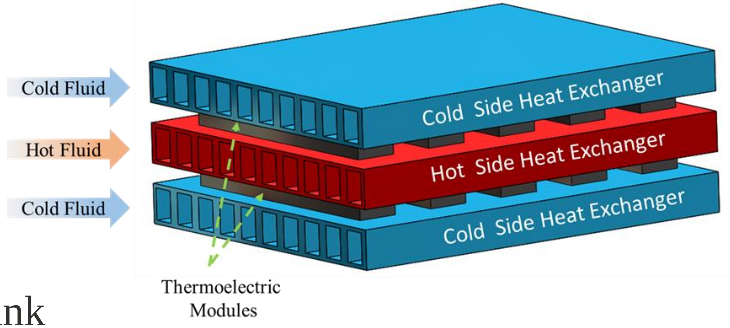
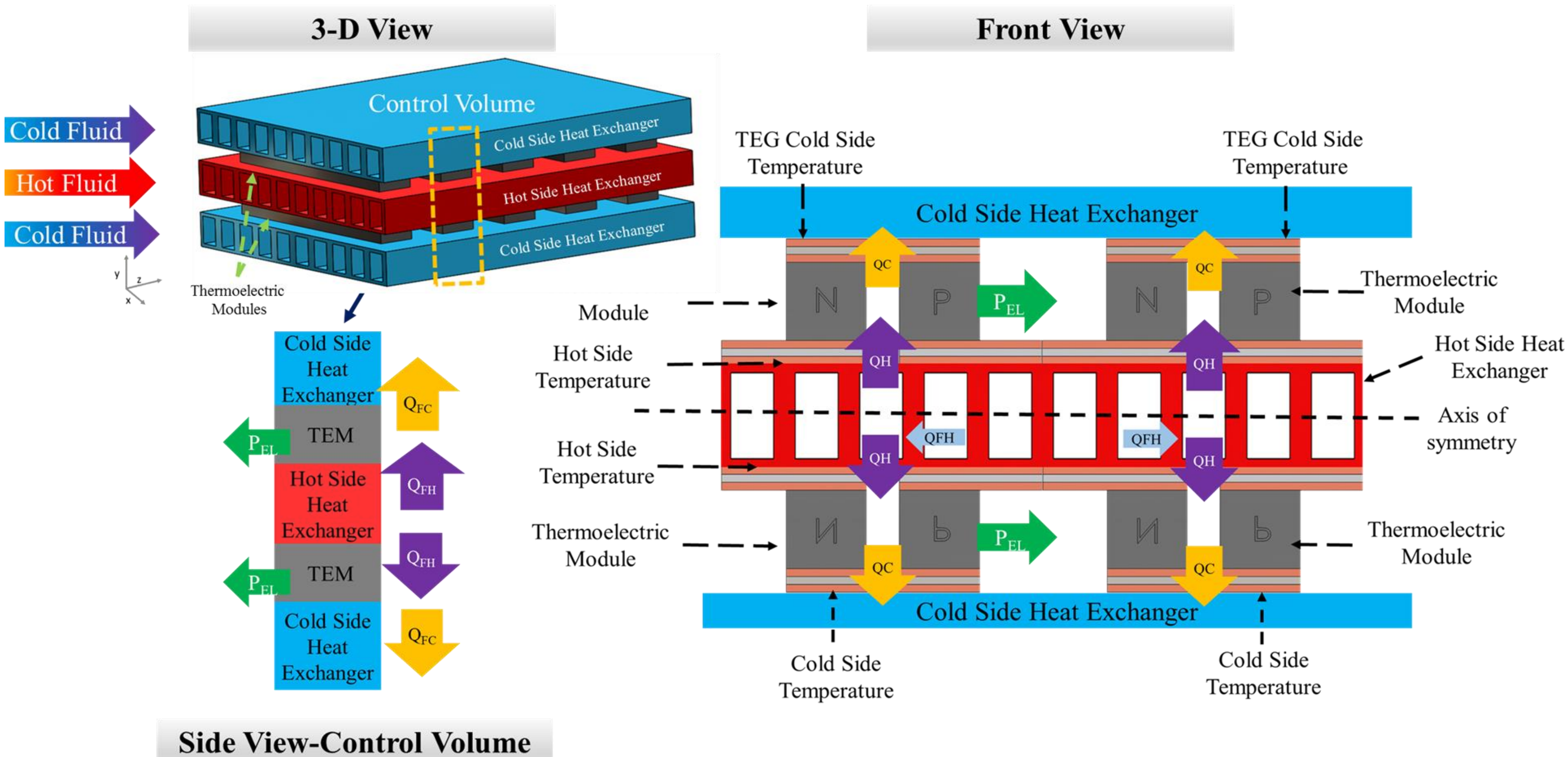


Model Overview



Model Assumptions

1. Heat exchanger divided into number of individual channels
   I. Mass flowrate is identical in each channel
   II. Heat transfer coefficient constant for each section of channel
2. Lateral variations in heat exchanger ignored
3. Assumes fully developed flow enters heat exchanger
4. Constant base temperature in each control volume
5. Lateral sides are assumed perfectly insulated

Primary Equations

Heat transferred through HX in Control Volume

$$Q_f = M \cdot \tanh(m \cdot \frac{fh}{2}) \cdot (N_f - 1) + A_{uf} \cdot (h) \cdot (T_b - T_f)$$

Fin Parameter

$$M = \sqrt{h \cdot P \cdot k \cdot A_c} \cdot (T_b - T_f)$$

Fin Parameter

$$m = \sqrt{\frac{h \cdot P}{k \cdot A_C}}$$

Fluid Temperature in control volume

$$T_{f,out} = T_{f,in} - \frac{Q_f}{\dot{m} \cdot c_p}$$

f_h : Fin Height
N_f : Number of Fins
h: Convection Coefficient
T_b : Base Temperature
T_f : Fluid Temperature
A_{uf} : Unfinned Area
k : Fin Thermal Conductivity
P : Fin Perimeter
A_c : Fin Cross-sectional Area
$T_{f,in,out}$: Inlet and Outlet Temperature of the Fluid in Control Volume
C_p : Specific Heat Capacity of Fluid

Channel Convection Coefficient - Duct

Reynolds Number

$$Re_{dh} = \frac{v_{ch} \cdot d_h}{\nu_f}$$

Hydraulic Diameter

$$d_h = \frac{4 \cdot A}{P}$$

Spacing Ratio

$$\alpha = \frac{s}{fh}$$

v_{ch} : Channel Velocity
ν_f : Fluid Kinematic Viscosity
A : Channel Cross Sectional Area
P: Wetting Perimeter
Pr : Prandtl Number
k_f : Fluid Thermal Conductivity

Nusselt Number

$$Nu = 0.026 \cdot Re_{dh}^{0.8} Pr^{0.3} \qquad Re_{dh} \geq 2500 \quad (T_s > T_f)$$

$$Nu = 0.024 \cdot Re_{dh}^{0.8} Pr^{0.4} \qquad Re_{dh} \geq 2500 \quad (T_f > T_s)$$

$$Nu = 7.541 \cdot (1 - 2.61\alpha + 4.970\alpha^2 - 5.199\alpha^3 + 2.702\alpha^4 - 0.548\alpha^5) \qquad Re_{dh} \leq 2500$$

Convection Coefficient

$$h = \frac{Nu \cdot k_f}{d_h}$$

[7] R. Shah and D. Sekulic, *Fundamental of Heat Exchanger Design*, 1st ed. Hoboken, NJ: John Wiley & Sons, Inc, 2003.

Channel Convection Coefficient – Compact HX

Colburn Factor

$$j = 0.483 \cdot \left(\frac{hxl}{d_h}\right)^{-0.162} \cdot \alpha^{-0.184} \cdot Re_{dh}^{-0.536} \qquad Re \leq 1000$$

$$j = 0.242 \cdot \left(\frac{hxl}{d_h}\right)^{-0.322} \cdot \left(\frac{t_f}{d_h}\right)^{-0.089} \cdot Re_{dh}^{-0.368} \qquad Re > 1000$$

Convection Coefficient

$$h = j \cdot Re_{dh} \cdot Pr^{1/3} \cdot \frac{k_f}{d_h}$$

Re_{dh} : Reynolds Number
α : Spacing Ratio
hxl : Heat Exchanger Length
d_h : Hydraulic Diameter
t_f : Fin Thickness
Pr : Prandtl Number
k_f : Fluid Thermal Conductivity

[8] A. R. Wieting, "Empirical Correlations for Heat-Transfer and Flow Friction Characteristics of Rectangular Offset-Fin Plate-Fin Heat-Exchangers," *Journal of Heat Transfer-Transactions of the Asme*, vol. 97, pp. 488-490, 1975.

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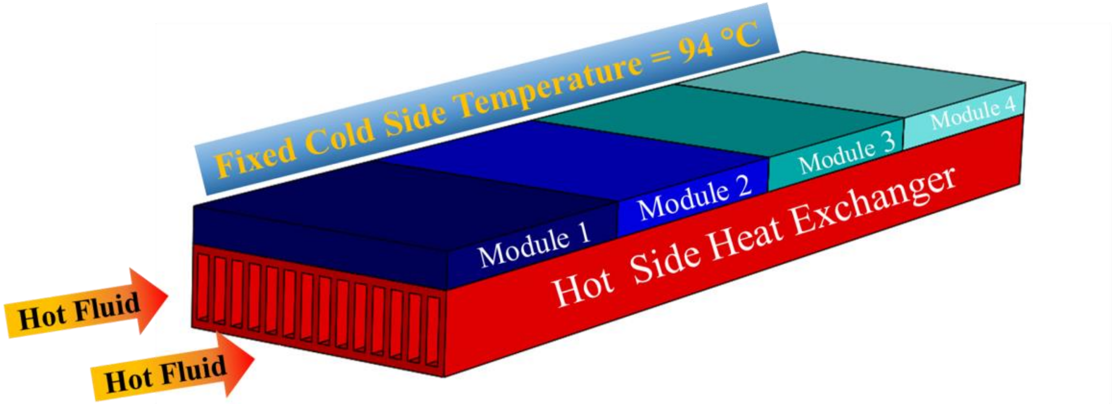
Model Input

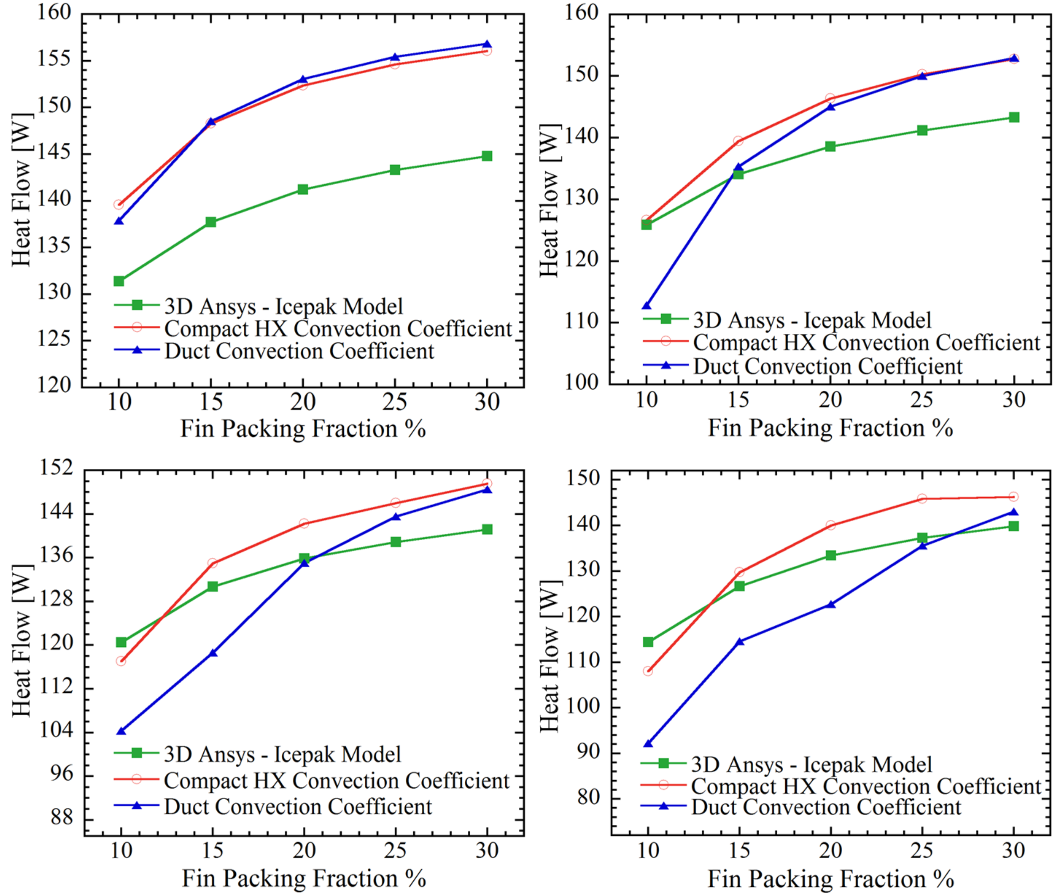
Input	Value
Exhaust Inlet Temperature	558 °C
Hot Side Fluid	Exhaust gas (Air)
Number of Modules	4
Module Size	40mm x 40 mm x 4.9 mm
TEM Cold Side Temperature	94 °C
Heat Exchanger Width	40 mm
Heat Exchanger Length	160 mm
Heat Exchanger Height	20 mm
Heat Exchanger Material	Nickel



Average Heat Flow Through TEG Comparison

Fin Thickness of 0.1mm

Fin Thickness of 0.2mm



Fin Thickness of 0.3mm

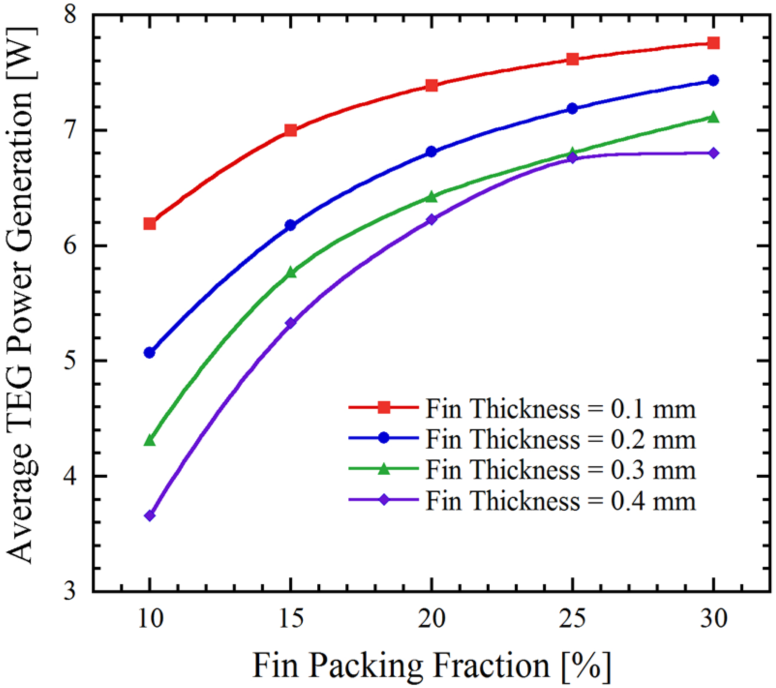
Fin Thickness of 0.4mm

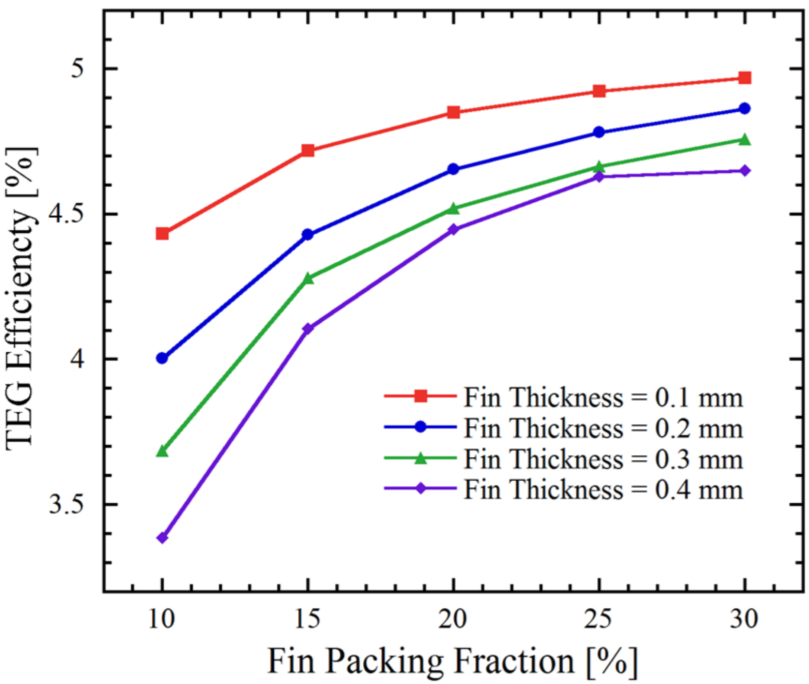
Average Percent Comparison

Fin Thickness	Duct Convection Coefficient Model	Compact Heat-Exchanger Convection Coefficient Model
0.1 mm	7.60 %	7.50 %
0.2 mm	5.79 %	4.63 %
0.3 mm	6.37 %	4.36 %
0.4 mm	8.13 %	4.76 %

- Compact HX convection coefficient more accurate
  - Geometry of heat exchanger used
- Produces less erratic results

TEG Power and Efficiency





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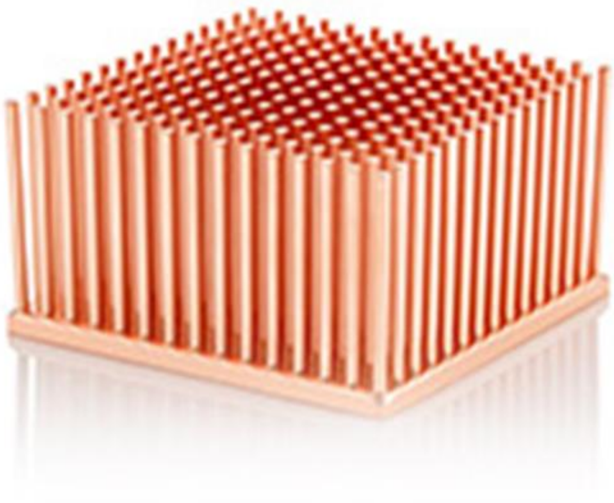
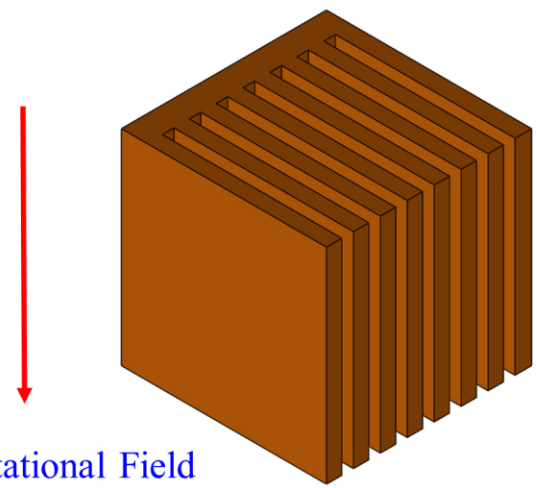


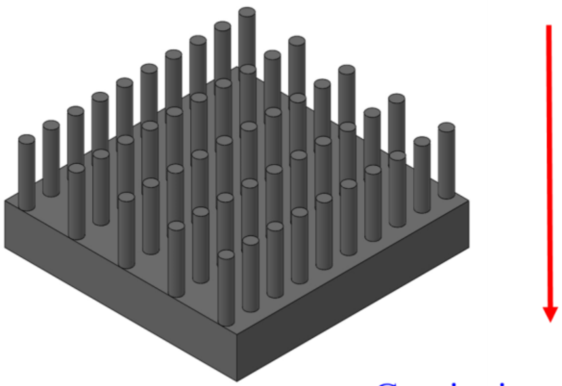
Heat Sinks Examined

Vertical Flat Plate
Heat Sink



Gravitational Field

Horizontal Base
Pin Fin Heat Sink



Gravitational Field

Heat Sink Model

Heat Flow from Heat Sink

$$Q_{HS} = n_{fin} \cdot q_{fin} + h_{base} \cdot A_{uf} \cdot (T_b - T_{amb})$$

Heat Flow from Fin

$$q_{fin} = M \cdot \tanh(m \cdot H)$$

Fin Parameter

$$M = \sqrt{h \cdot P \cdot k \cdot A_c} \cdot (T_b - T_{amb})$$

Fin Parameter

$$m = \sqrt{(h \cdot P)/(k \cdot A_c)}$$

n_{fin} : Number of Fins
h_{base} : Base Convection Coefficient
T_b : Base Temperature
T_{amb} : Fluid Ambient Temperature
A_{uf} : Unfinned Area
H : Fin Height
k : Fin Thermal Conductivity
P : Fin Perimeter
A_c : Fin Cross-sectional Area
h : Fin Convection Coefficient

Vertical Flat Plate Heat Sink Convection Coefficient

Base Convection Coefficient

$$h_{base} = 0.59 \cdot Ra_L^{1/4} \cdot k_f / L$$

Rayleigh Number

$$Ra_L = g \cdot \beta \cdot \theta_b \cdot Pr \cdot L^3 / \nu^2$$

Fin Convection Coefficient

$$h_{fin} = \frac{k_f}{s} \cdot \left(\frac{576}{(\eta_{fin} \cdot El)^2} + \frac{2.873}{(\eta_{fin} \cdot El)^{\frac{1}{2}}} \right)^{1/2}$$

Elenbaas Number

$$El = (g \cdot \beta \cdot \theta_b \cdot Pr \cdot s^4) / L\upsilon^2$$

L: Heat Sink Length Parallel to Gravity
d_h : Hydraulic Diameter
t_f : Fin Thickness
Pr : Prandtl Number
k_f : Fluid Thermal Conductivity
g : Gravity
β : Coefficient of Thermal Expansion
θ_b : Temperature Difference Between Base and Ambient
ν : Kinematic Viscosity
s : Fin Spacing
η_{fin} : Fin Efficiency

[9] A. Bar-Cohen, M. Iyengar, and A. D. Kraus, "Design of optimum plate-fin natural convective heat sinks," *Journal of Electronic Packaging*, vol. 125, pp. 208-216, Jun 2003.

Horizontal Base Vertical Square Pins

Fin Spacing Dependent Nusselt Number

$$\mathrm{Nu_s} = 0.0285\left\{1 - \exp\left(-\frac{H}{W}\right)\right\}\ldots$$

$$\ldots\left\{1 + 1.50\exp\left(-0.07\frac{L}{H}\right)\right\}\mathrm{Ra_s^{1/2}}\left\{1 - \exp\left[-\frac{7000}{\mathrm{Ra_s}}\right]\right\}^{1/3}$$

Fin Spacing Dependent Rayleigh Number

$$\mathrm{Ra_s} = \frac{g\beta\theta_b S^3 \mathrm{Pr}}{\nu^2}$$

Fin Spacing

$$S = P - W$$

Mean Convection Coefficient

$$h_c = \frac{k_f \cdot \mathrm{Nu_s}}{S}$$

H : Pin Height
W : Pin Width
L: Heat Sink Length
Pr : Prandtl Number
g : Gravity
β : Coefficient of Thermal Expansion
θ_b : Temperature Difference Between Base and Ambient
ν : Kinematic Viscosity

[10] D. Sahray, G. Ziskind, and R. Letan, "Scale-Up and Generalization of Horizontal-Base Pin-Fin Heat Sinks in Natural Convection and Radiation," *Journal of Heat Transfer-Transactions of the Asme*, vol. 132, Nov 2010.

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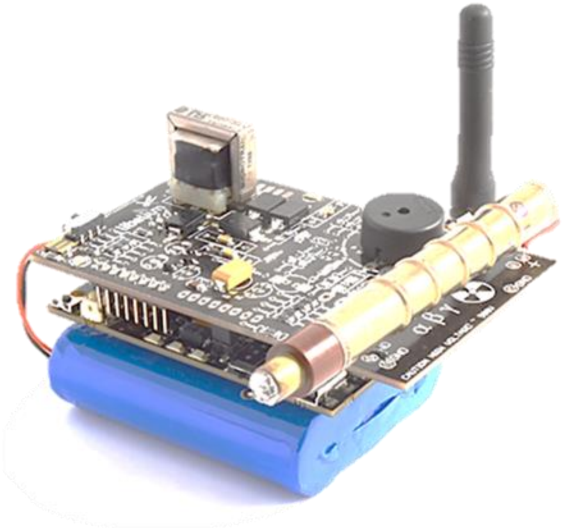
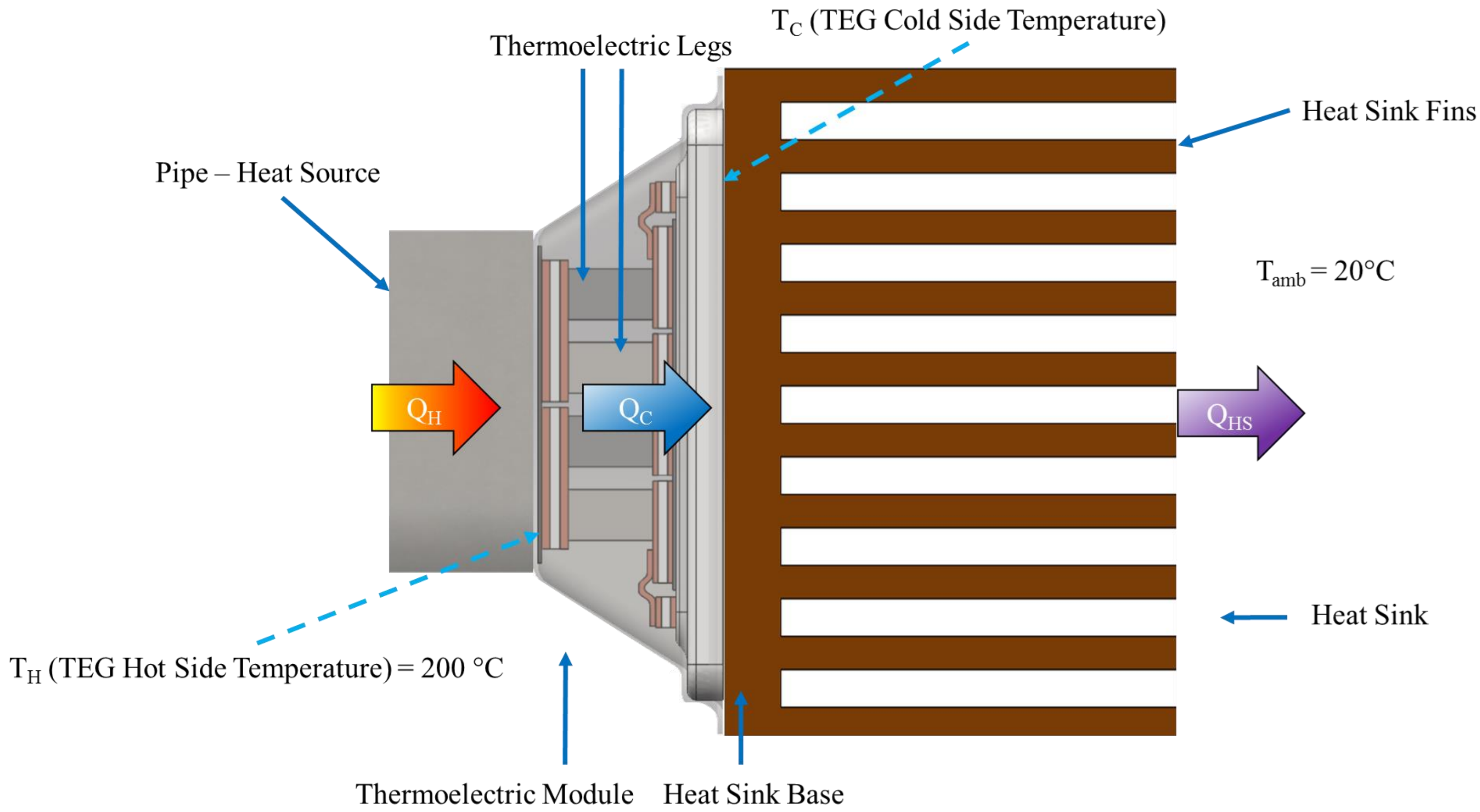


Model Overview

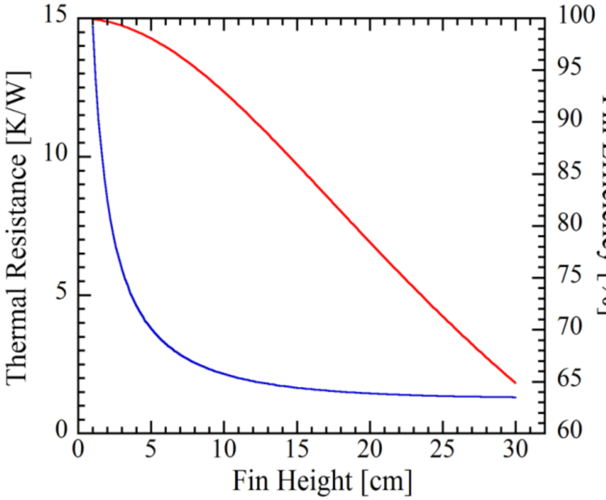


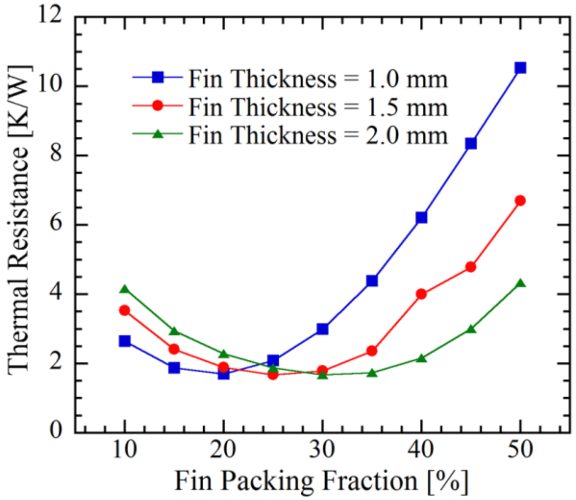
Heat Sink Optimization

Optimization Parameters

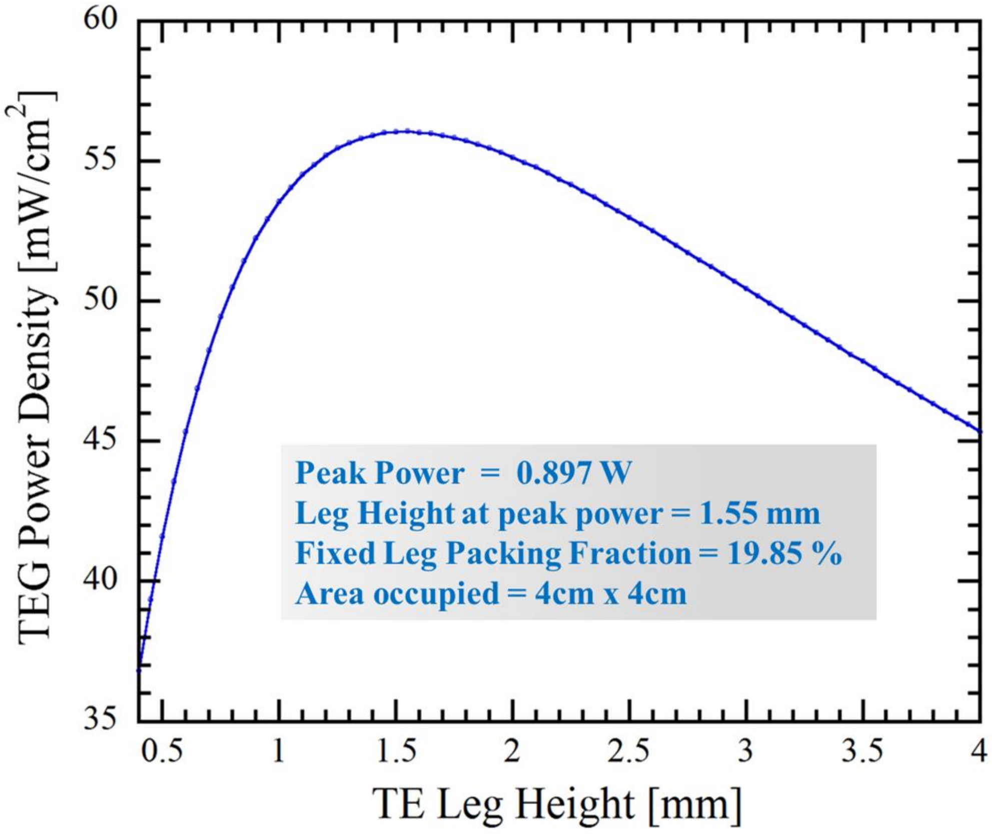
Parameter	Value
Fin Thickness	1mm – 10 mm with 0.5 mm increments
Fin Height	1cm – 30 cm with 1 cm increments
Fin Packing Fraction	10% - 50 %

Optimization Results





TEG Optimization



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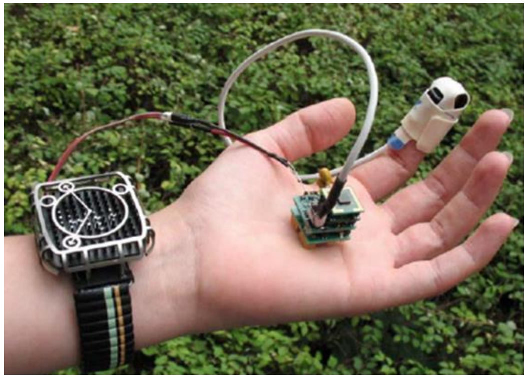
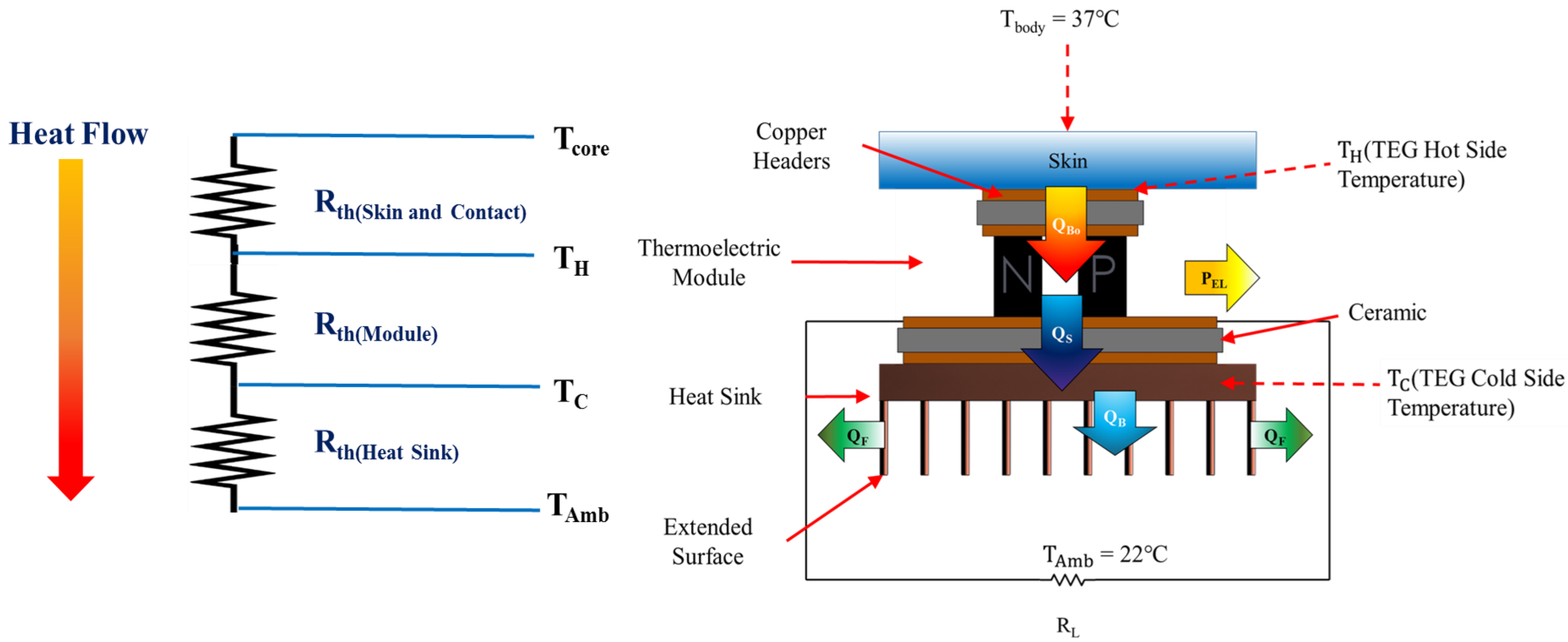
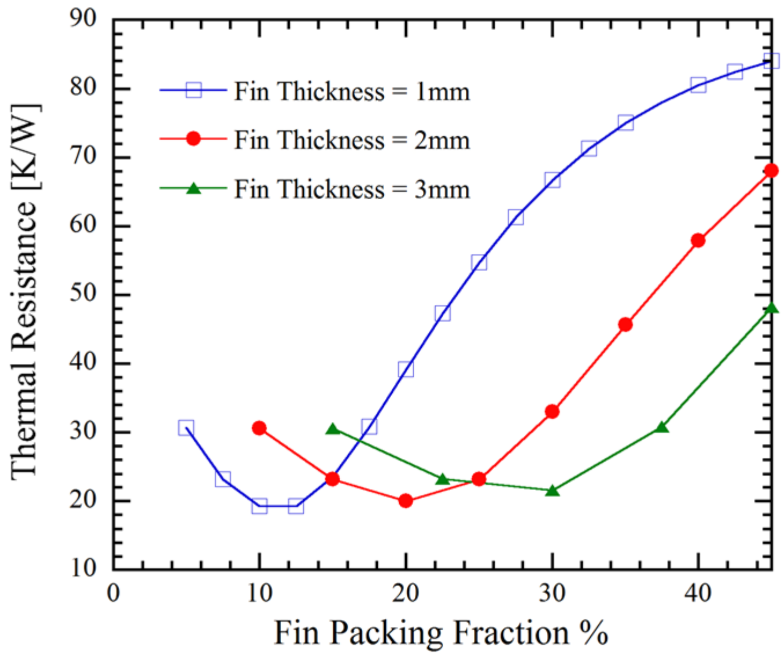


[11] T. Torfs, V. Leonov and R. Vullers, "Pulse Oximeter Fully Powered by Human Body Heat", *Sensors Transducers*, vol. 80, no. 6, pp. 1230-1238, 2007.

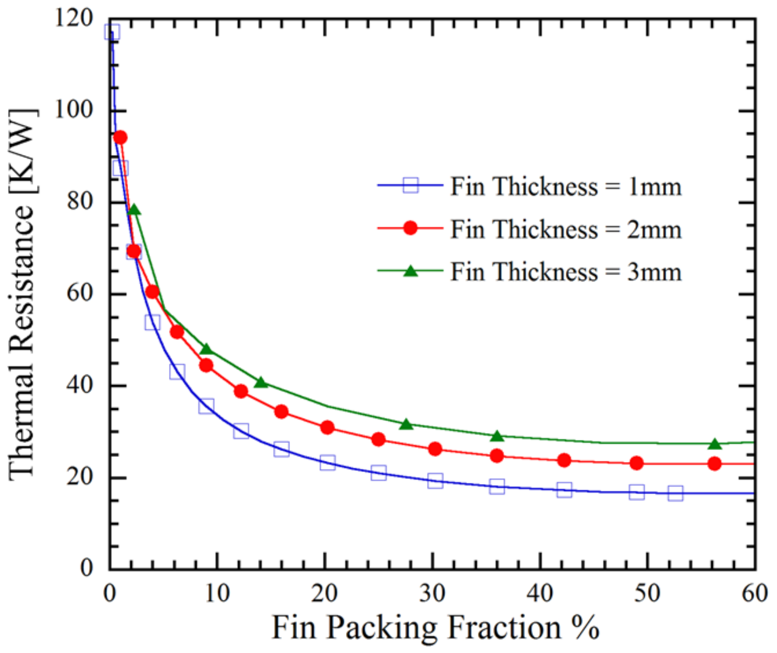
Model Overview



Heat Sink Optimization for a Fin Height of 3 cm

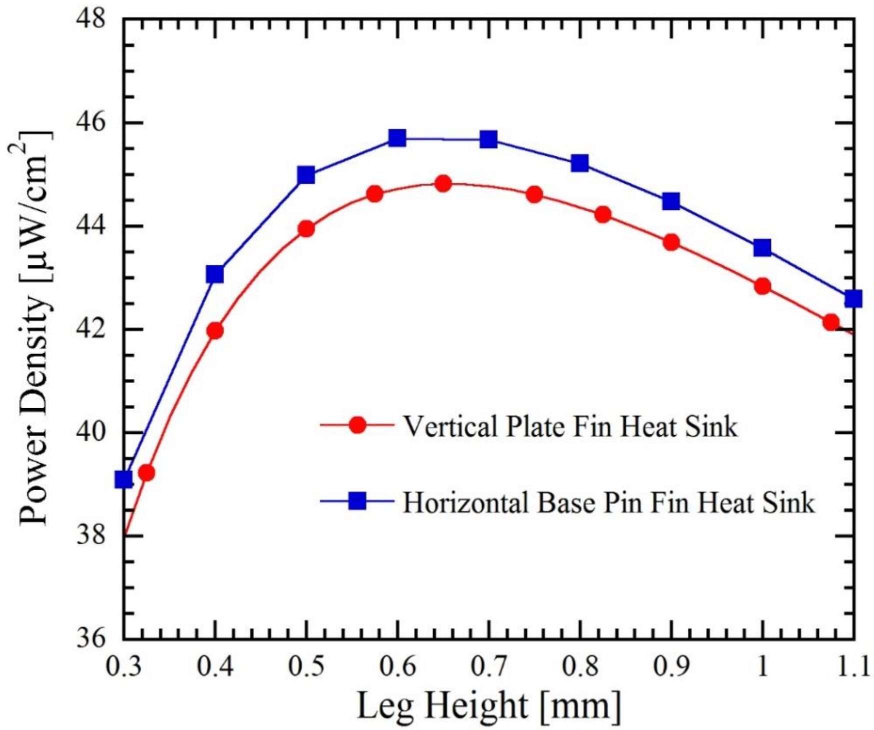


Flat Plate Natural Convection Heat Sink



Horizontal Base Square Pin Fin Natural Convection Heat Sink

TEG Optimization Results



Leg Packing Fraction = 0.638 %

Parameter	Flat Plate HS	Horizontal Base HS
Max Power	0.717 mW	0.731 mW
Max Heat Flow	8.074 mW/cm²	8.231 mW/cm²
TE Leg Height	0.65 mm	0.60 mm

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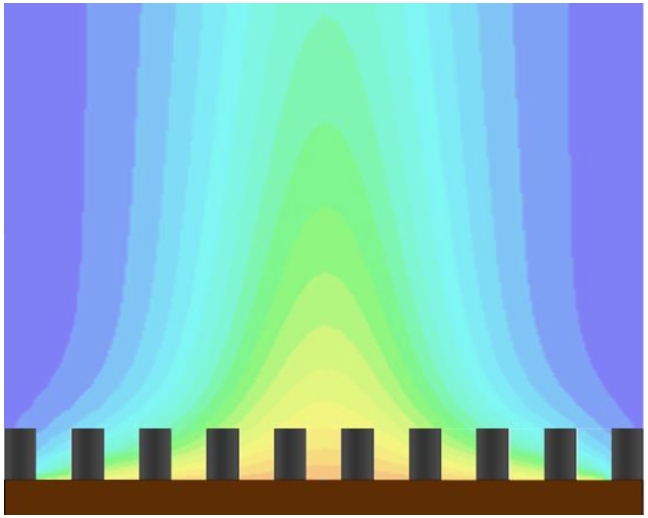


Microwire Convection Coefficient

Empirical Convection Coefficient From Ref [12]

$$h = \frac{k_a}{1 + K_n}\left(\frac{1}{d}\right)\left[\frac{1}{16}\ln^2\left(\frac{\alpha_a}{d^2}\right) - 0.292\ln\left(\frac{\alpha_a}{d^2}\right) + 0.958\right]^{-1/2}$$

Nusselt Number From Ref [13]

$$Nu = 1.03 \cdot (Gr \cdot Pr)^{0.035}$$

Grashof Number From Ref [13]

$$Gr = \frac{g \cdot \beta \cdot (T_s - T_\infty)d^3}{\upsilon^2}$$

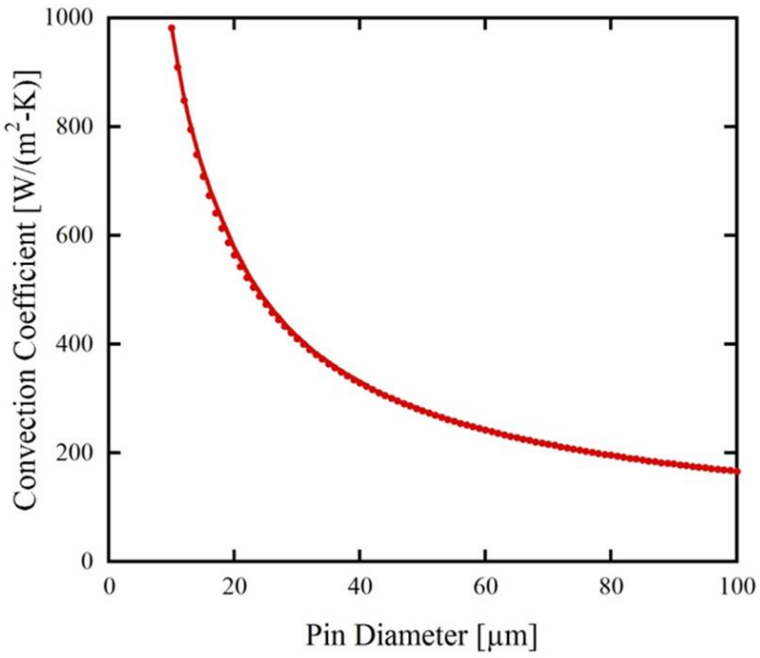
Empirical Convection Coefficient From Ref [13]

$$h = \frac{k_a \cdot Nu}{d}$$

k_a : Fluid Thermal Conductivity
α_a : Fluid Thermal Diffusivity
d : Fin Diameter
Pr : Prandtl Number
g : Gravity
β : Coefficient of Thermal Expansion
T_s : Temperature of Microwire Surface
T_∞ : Ambient Temperature
ν : Kinematic Viscosity of Air

[12] Z. L. Wang and D. W. Tang, "Investigation of heat transfer around microwire in air environment using 3 omega method," *International Journal of Thermal Sciences*, vol. 64, pp. 145-151, Feb 2013.
[13] N. Guan, Z. G. Liu, C. W. Zhang, and G. L. Jiang, "Natural convection heat transfer on surfaces of copper micro-wires," *Heat and Mass Transfer*, vol. 50, pp. 275-284, Feb 2014.

Microwire Convection Coefficient



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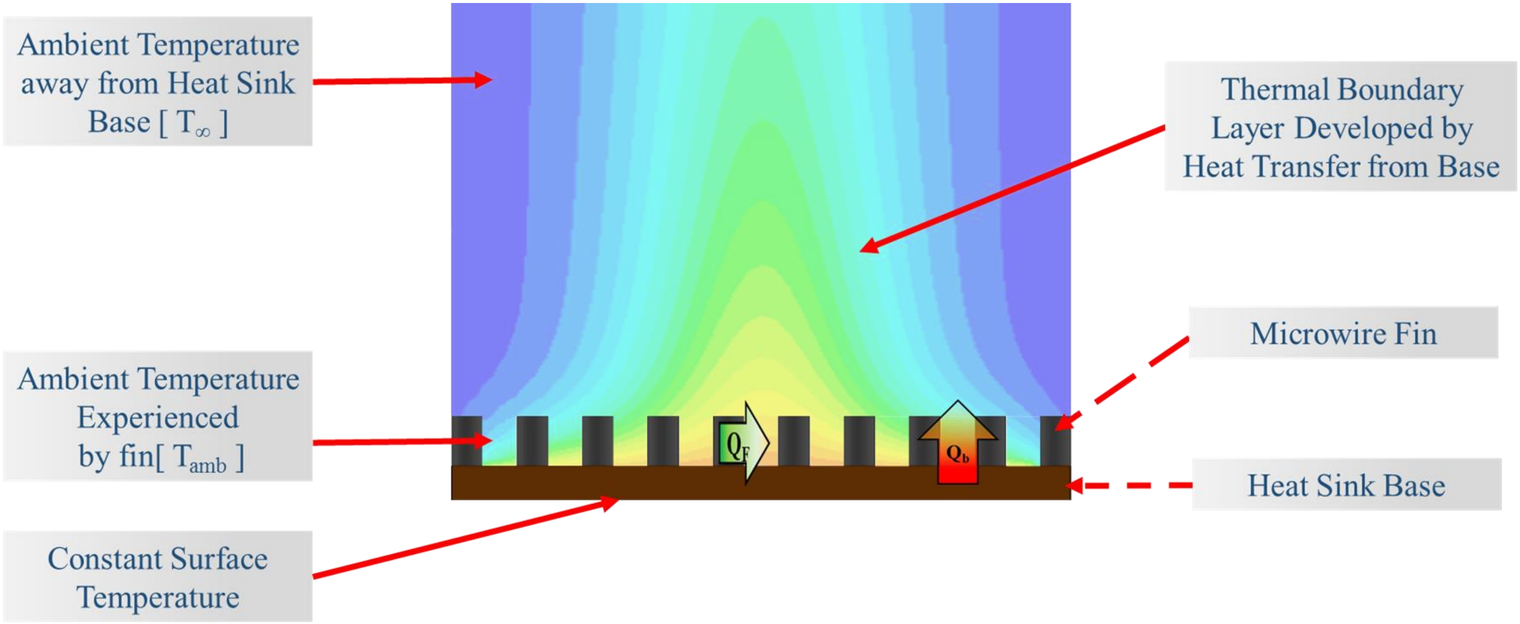
Model Overview



Model Assumptions

1. Heat Source is a thermal reservoir
2. Microwire boundary layer thickness has little influence on heat transfer from adjacent fin
3. Each fin treated as an individual fin based on spacing limitation
4. Microwire convection coefficient used as fin convection coefficient

Fin Heat Transfer

Fin Temperature Profile

$$T = K^{-1}F$$

Elemental Stiffness Matrix

$$K_e = \int_l [B]^T[D][B]A\,dx + \int_l hP[N]^T[N]\,dx$$

Elemental Stiffness Matrix Simplified for a linear element

$$K_e = \frac{A_e k_e}{l_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{hP}{l_e}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Elemental Loading Vector

$$F_e = \int_S hT_{amb}[N]^T dS = \frac{hPT_{amb}l_e}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Heat Transferred from an individual fin

$$Q_{T\_Fin} = \sum_{i=1}^{elems} h \cdot A_{e\_s} \cdot \left(\frac{T_n + T_{n+1}}{2} - T_{amb(n)}\right)$$

Total Heat Transferred from Heat Sink

$$Q_{HeatSink} = \sum_{i=1}^{Nf} Q_{T\_Fin} + h_{hp} \cdot A_{uf} \cdot (T_b - T_\infty)$$

T : Temperature Profile
K : Global Stiffness Matrix
F : Global Loading Vector
B : Strain Displacement Matrix
D : Thermal Conductivity Terms
N : Shape Function
h : Convection Coefficient
P : Fin Perimeter
A_e : Elemental Cross-sectional Area
l_e : Element Length
T_{amb} : Ambient Temperature
elems : Number of elements
A_{e_s} : Elemental Fin Surface Area
T : Temperature
n : Node
N : Number of Elements
h_{hp} : Convection Coefficient of a Hot Plate
A_{uf} : Unfinned Area
T_∞ : Ambient Temperature Away from Base
Nf : Number of fins

Ambient Temperature Above Base

Temperature Profile

$$\frac{T_{x,y} - T_\infty}{T_s - T_\infty} = (1 - y/\delta)^2$$

Boundary Layer Thickness

$$\delta = 4.317x\left[\frac{Pr + (16/21)}{Pr \cdot Gr_x}\right]^{1/5}$$

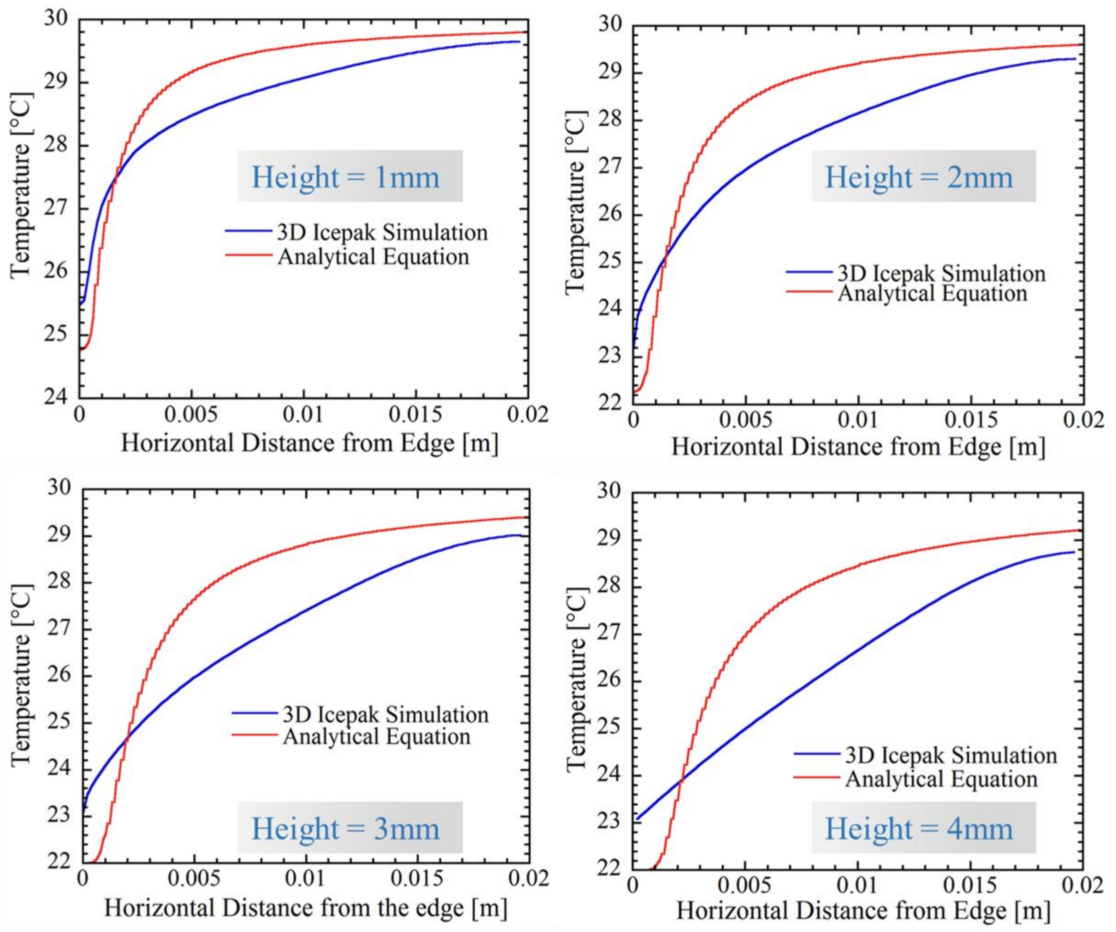
Local Grashof Number

$$Gr_x = \frac{g\beta x^3 (T_s - T_\infty)}{\upsilon^2}$$

T_∞ : Ambient Temperature
T_s : Surface Temperature
y: Vertical Distance
x : Horizontal Distance
Pr : Prandtl Number
g : Gravity
β : Coefficient of Thermal Expansion
ν : Kinematic Viscosity of Air

[14] A. Guha and S. Samanta, "Closed-Form Analytical Solutions for Laminar Natural Convection on Horizontal Plates," *Journal of Heat Transfer-Transactions of the Asme*, vol. 135, Oct 2013.

Temperature Profile Comparison



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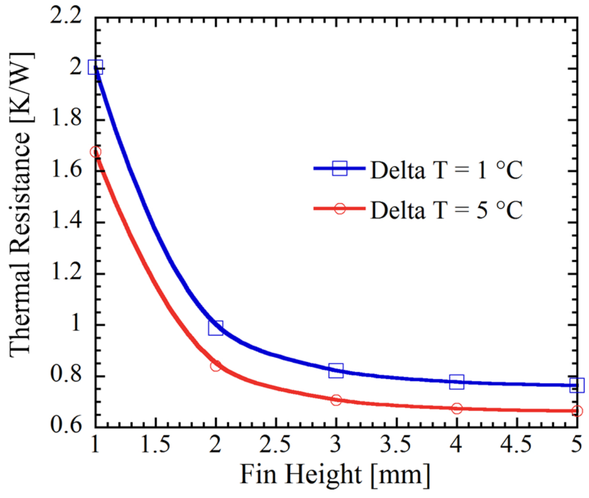
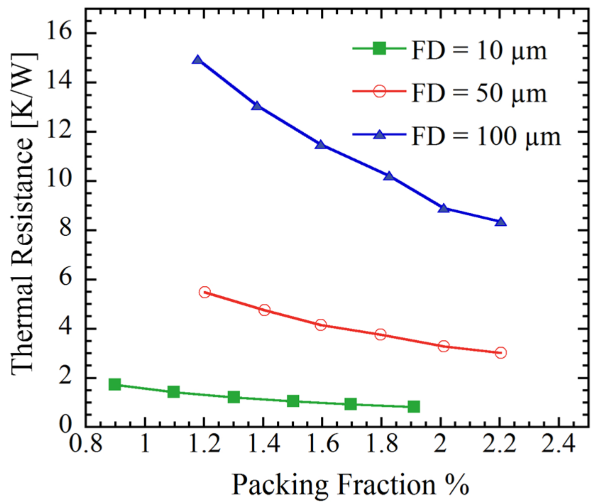
Heat Sink Optimization Input

Parameter	Value
Fin Diameter	10 µm, 50µm, 100 µm
Fin Height	1 mm , 2 mm, 3 mm, 4 mm, 5 mm
Fin Packing Fraction [10 µm, 50 µm and 100 µm]	0.9 % - 1.9 %, 1.2%, 2.2%

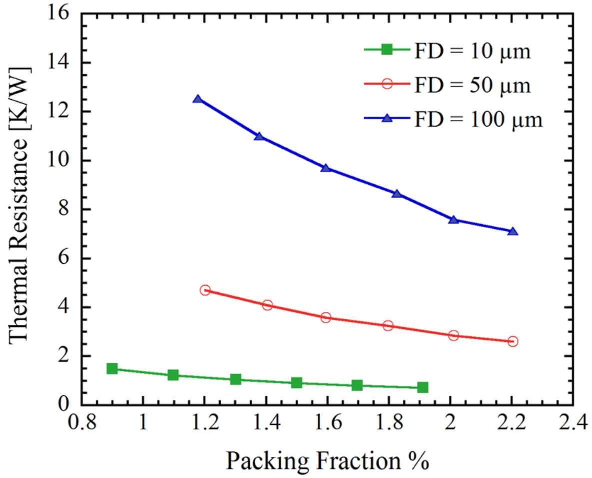
Heat Sink Optimization Results



Fin Height Optimization



Delta T = 1°C



Delta T = 5°C

Optimized Results and Comparison

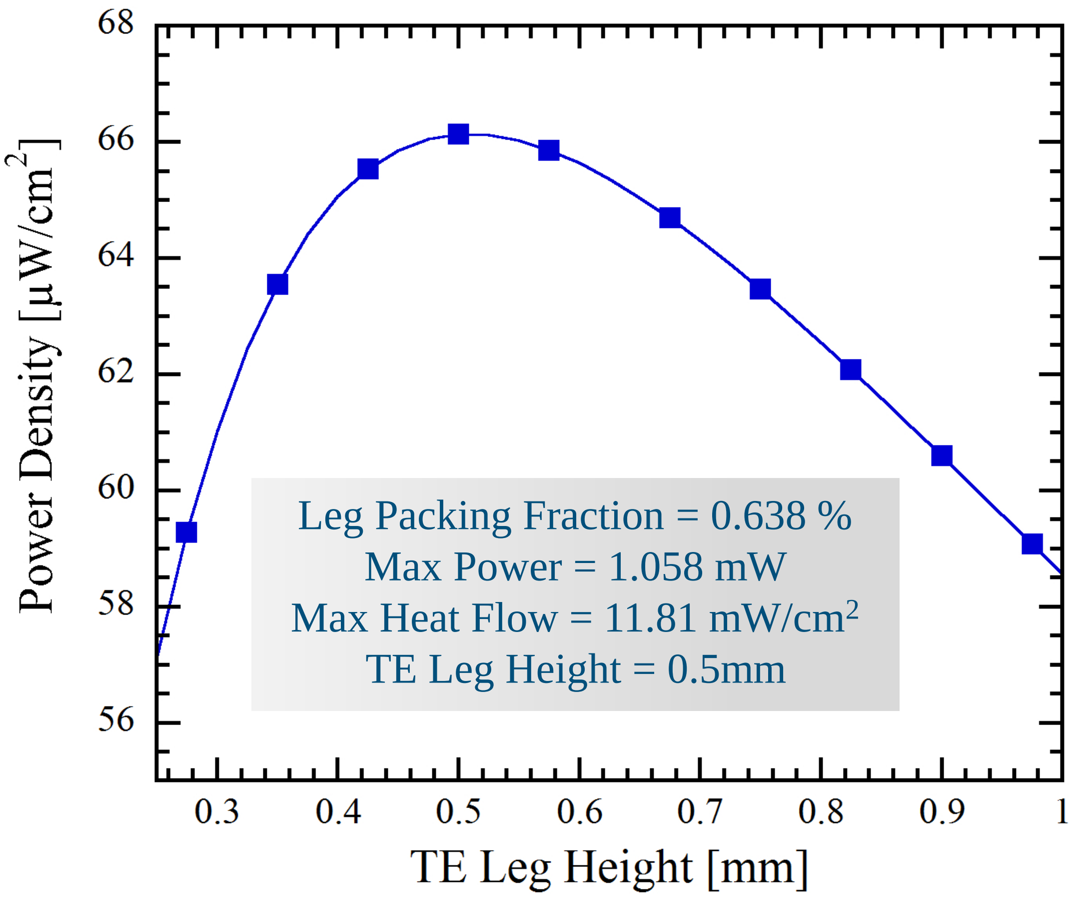
Optimized Heat Sink Parameter

Parameter	Value
Heat Exchanger Base Area	40 mm x 40 mm
Heat Exchanger Base Thickness	0.5 mm
Fin Diameter	10 µm
Fin Height	3 mm
Fin Packing Fraction	1.9 %

Comparison with a Macro-scale Heat Sink with height 3 cm

Heat Sink Type	Thermal Resistance $\Delta T = 1°C$	Adjusted Thermal Resistance $\Delta T = 1°C$	Thermal Resistance $\Delta T = 5°C$	Adjusted Thermal Resistance $\Delta T = 5°C$
Macro-scale	28.68 K/W	77.05 K-cm/W	12.90 K/W	38.70 K-cm/W
Micro-scale	0.820 K/W	0.246 K-cm/W	0.706 K/W	0.212 K-cm/W

TEG Optimization



Conclusion and Future Work

Conclusion

1. TEG Model
   - Can be used as initial step in TEG design
   - Validated with 3-D Model
   - Improvement to TEG design suggested
     - Ceramic layer
     - Segmented TEGs
2. TEG- HX Model
   - Compact HX model developed
   - Two convection coefficients implemented
   - Validated with 3-D Model

3. TEG – Heat Sink Model

- Heat sink model developed for two different heat sink types
- Optimized heat sink applied with TEG to power wireless sensor node and harvest waste heat from the human body.

4. TEG- Microwire Heat Sink Model

- Microwire heat sink model developed
- Optimized design used with TEG to harvest waste heat from the human body.

Improvements to Models

1. TEG Model
   I. Capture heat spreading effects in TEG model
   II. Incorporate radiation losses from outer surfaces of unicouple
2. Heat Exchanger Model
   1. Obtain pressure drop values for heat exchanger
   2. Obtain net power using pressure drop values

3. Natural Convection microwire heat sink

I. Model for varied base temperature

II. Consider vertical base for heat sink and related heat transfer

III. Obtain a global convection coefficient correlation for the heat sink based on fin height and fin diameter using data for various heat sink designs.

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[5] G. Snyder and T. Ursell, "Thermoelectric efficiency and compatability", *Physics Review Letters*, vol. 91, no. 14, 2003.

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Pasindu Gamarachchi

Email buddhimagamarachchi@u.boisestate.edu

THANK YOU