SLIATE

SRI LANKA INSTITUTE OF ADVANCED TECHNOLOGICAL EDUCATION

(Established in the Ministry of Higher Education, vide in Act No. 29 of 1995)

Higher National Diploma in Information Technology

First Year, First Semester Examination – 2017

HNDIT 1107 / HNDIT 11072: Mathematics for IT / Mathematics for Computing

Marking Scheme

Instructions:	No. of questions: 05
Answer four questions only.	No. of pages : 03
All questions carry equal marks.	Time :Two(02) hours

Question 01 **[Total Marks 25]**

(i) Define the following terms with examples: (08 marks)

a) Power Set

The power set is the set of all subsets that can be created from a given set
Eg: power set of {1,2} is {{1},{2},{1,2},{ }} **{2 marks}**

b) Disjoint Set

If A and B have no common elements they are said to be disjoint
Eg: A={1,2} B={3,4} **{ 2marks}**

c) Complement of a set

the set of elements which belongs to the universal set but which does not belong to itself **{2 marks}**
eg: U={1,2,3,4,5} A={3,4} A' = {1,2,5}

d) Sub Set

If every element of set A is also contained in set B then set A is a subset of set B
A={2,3} B={1,2,3,4} **{2 marks}**

(ii) If A and B are disjoint sets state whether following statements are true or false. (06 marks)

a) $A \subseteq B$ **false {1 mark}**
b) $A \cap B = \phi$ **true {1 mark}**
c) $A - B = A$ **true {1 mark}**
d) $B \subseteq A$ **false {1 mark}**
e) $n(A \cup B) = n(A) + n(B)$ **true {1 mark}**
f) $A^c \cap B^c = \phi$ **false {1 mark}**

i. Given U= {1,2,………,12}
A ={x: x is an odd integers}
B ={x: x is an odd integer, $3x - 5 \geq 10$}
C = {x: x is divisible by 3}

Find

a) $A \cup B \cup C$ (02 marks)

{1,3,5,7,9,11}

b) $A^c \cap (B \cup C)$ (02 marks)

{}or ϕ

c) $A \cap (B \setminus C)$ (02 marks)

{5,7,11}

d) $(A \oplus B) \oplus C$ (02 marks)

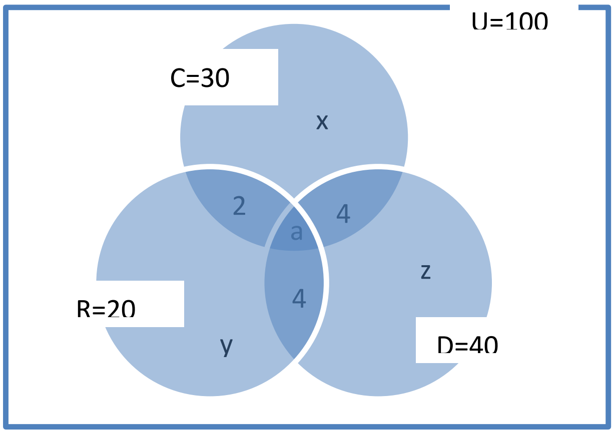
{1,6,9}

e) P(C) = **{{3},{6},{9},{3,6},{3,9},{6,9}{3,6,9},{}}** (03 marks)

Question 02 **[Total Marks 25]**

(i) In a city of 100 people, 60 people own cats, dogs, or rabbits. 30 people owned cats, 40 owned dogs, 20 owned rabbits.4 owned a cat and a dog only. 2 owned a cat and a rabbit only. 4 owned a dog and a rabbit only.

a) Draw a Venn diagram to illustrate the above information (05 marks)



{2 marks}

n(A∪B∪C)=n(A)+n(B)+n(C)-n(A∩B)- n(A∩C)- n(C∩B)+n(n(A∩B∩C)

60=30+20+40-(4+a)-(4+a)-(2+a)+a

60=90-10-2a

2a=20

a=10 **{02 marks}**

or

{02 Marks}

40+x+y+2 = 60

a+y+2+4=20

x+a+2+4=30

a+z+4+4=40

x=14 **{1 mark}**

y=4

a=10

z=22

How many people own

b) All three **=10** (02 marks)

c) Cats but neither dogs nor rabbits **=14** (02 marks)

d) Only dogs **=22** (02 marks)

e) Exactly two pets **=10** (02 marks)

f) How many did not own any pet? **=40** (02 marks

(ii) Show that $(A \cap B^C) \cup (A^C \cap B) \cup (A \cap B) = A \cup B$ using set theory laws (06 marks)

$(A \cap B^C) \cup B\cap(A \cup A^C)$ (Distributive law) {1 mark}
$(A \cap B^C) \cup (B\cap U)$ (Complement law) {1 mark}
$(A \cap B^C) \cup B$ (Identity law) {1 mark}
$(A\cup B) \cap (B^C \cup B)$ (Distributive law) {1 mark}
$(A\cup B) \cap U$ (Complement law) {1 mark}
$(A\cup B)$ (Identity law) {1 mark}
0r
$(A \cap B^C) \cup (A^C \cap B) \cup (A \cap B)$
$A\cap (B^C \cup B) \cup (A^C \cap B)$
$(A\cap U) \cup (A^C \cap B)$
$A\cup (A^C \cap B)$
$(A\cup A^C) \cap (A\cup B)$
$U\cap (A\cup B)$
$(A\cup B)$

(iii) Write the dual of the followings (02 marks)

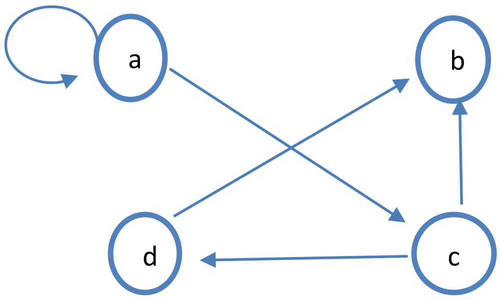
a) $A = (A \cup B) \cap (A \cup \phi)$
$A = (A \cap B) \cup (A \cap U)$

b) $(B^C \cup \phi) \cup (A^C \cap U) = (A\cap B)^C$
$(B^C\cap U) \cap (A^C \cup\phi) = (A\cup B)^C$ (02 Marks)

Question 03 **[Total Marks 25]**

(i). Let R be the following relations on B = {a, b, c, d}:
R = {(a, a), (a, c), (c, b), (c, d), (d, b)}

a) Draw a directed graph for relation R (02 marks)



b) Determine the matrix of the relation R (02 marks)

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

c) Using the above matrix find the composition relation RoR (03 marks)

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix}\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$ **{2 marks}**

RoR = {(a,a),(a,b),(a,c),(a,d)} **{1 marks}**

(ii). What are the type of relations? Give an example for each.
[04 Marks]

a) Reflexive relation
b) Symmetric relation
c) Antisymmetric relation
d) Transitive relation

A={1,2,3,4}

Reflexive relation

R1={(1,1),(2,2),(3,3),(4,4),(1,3)}

Symmetric relation

R2={(1,2),(2,1),(1,3)}

Antisymmetric relation

R3={(1,3),(2,1)}

Transitive relation

R4={((1,2),(2,3),(1,3)} (1/2*4=02 Marks)

(iii). Let X = {1, 2, 3, 4}. Determine whether each relation on X is a function from X into X giving reasons. (06 marks)

a) f = {(2, 3), (1, 4), (2, 1), (3.2), (4, 4)}
b) g = {(3, 1), (4, 2), (1, 1)}

c) h = {(2, 1), (3, 4), (1, 4), (2, 1), (4, 4)}

(a) No. Two different ordered pairs (2, 3) and (2, 1) in f have the same number 2 as their first coordinate. {2 marks}

(b) No. The element 2 ∈ X does not appear as the first coordinate in any ordered pair in g. {2 marks}

(c) Yes. {2 marks}

(iv). Let the function $f:R \rightarrow R$ be defined as follows

$$f(x) = \begin{cases} 2x + 1 & \text{if } x \leq -5 \\ 3x^2+2 & \text{if } -5<x<6 \\ 5x - 2 & \text{if } x \geq 6 \end{cases}$$

Find

a) f (2) **= 14 {2 marks}**
b) f (0) **= 2 {2 marks}**
c) f (6) = **28 {2 marks}**

(06 marks)

Question 04 **[Total Marks 25]**

(i) Define the following terms:

a) Vector
Matrices with one row or one column are called vectors {2 marks}
b) Singular matrix
A square matrix without inverse is called singular matrix {2 marks}
c) Lower triangular matrix
It is a square matrix where all the entries above the main diagonal are zero {2 marks} (06 marks)

(ii) State two applications of matrices (02 marks)
Solving liner application, graph theory, cryptography

(iii)Find the values of x and y that satisfy the matrix equation

$$\begin{bmatrix} 0 & y+x \\ 4 & 3x+4 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ 2+2x & 7y-14 \end{bmatrix}$$

2+2x=4
x+y=4 **{2 marks}**
x=1 , y=3 **{2 marks}** (04 marks)

(iv)Let $A = \begin{bmatrix} 1 & 2 \\ 5 & -2 \end{bmatrix}$, B=$\begin{bmatrix} 0 & 3 \\ 1 & 4 \end{bmatrix}$ and C=$\begin{bmatrix} -9 & 3 \\ 1 & -1 \end{bmatrix}$

Verify

a) A(B+C)=AB+AC

LHS

B+C = $\begin{bmatrix} -9 & 6 \\ 2 & 3 \end{bmatrix}$ **{1 mark}**

A(B+C) = $\begin{bmatrix} -5 & 12 \\ -49 & 24 \end{bmatrix}$ **------- 1** **{1 mark}**

RHS

AB = $\begin{bmatrix} 2 & 11 \\ -2 & 7 \end{bmatrix}$ **{1 mark}**

AC = $\begin{bmatrix} -7 & 1 \\ -47 & 17 \end{bmatrix}$ **{1 mark}**

AB+AC = $\begin{bmatrix} -5 & 12 \\ -49 & 24 \end{bmatrix}$**--------- 2**

1=2

So A(B+C)=AB+AC

b) $(AB)^T=B^TA^T$

AB = $\begin{bmatrix} 2 & 11 \\ -2 & 7 \end{bmatrix}$

$\mathbf{(AB)^T}$ **=** $\begin{bmatrix} 2 & -2 \\ 11 & 7 \end{bmatrix}$ **---------1**

$\mathbf{B^T}$ **=** $\begin{bmatrix} 0 & 1 \\ 3 & 4 \end{bmatrix}$

$\mathbf{A^T}$ **=** $\begin{bmatrix} 1 & 5 \\ 2 & -2 \end{bmatrix}$

$\mathbf{B^TA^T}$ **=** $\begin{bmatrix} 2 & -2 \\ 11 & 7 \end{bmatrix}$ **---------2**

1=2

So $\mathbf{(AB)^T=B^TA^T}$ (08 marks)

(v) Find the inverse of the matrix A=$\begin{bmatrix} 5 & 8 & 4 \\ 0 & -3 & -5 \\ 1 & 1 & 3 \end{bmatrix}$ (05 marks)

$$\mathbf{A^{-1}}=\begin{bmatrix} 1/12 & 5/12 & 7/12 \\ 5/48 & -11/48 & -25/48 \\ -1/16 & -1/16 & 5/16 \end{bmatrix}$$

Question 05 **[Total Marks 25]**

(i) State three properties of determinants (06 marks)

If two rows or columns are equal then detA =0
If a row(or column) of A consists entirely of 0 then det(A)=0
det(AB) = detA.detB = det(BA)

(ii) Find the determinant of $A=\begin{bmatrix}3 & 2 & 1\\ 4 & 1 & 3\\ 1 & 1 & 1\end{bmatrix}$ (06 marks)

$3\begin{vmatrix}1 & 3\\ 1 & 1\end{vmatrix}-2\begin{vmatrix}4 & 3\\ 1 & 1\end{vmatrix}+1\begin{vmatrix}4 & 1\\ 1 & 1\end{vmatrix}$ **{2 marks}**
3(1-3)-2(4-3)+(4-1) **{2 marks}**
3*-2 – 2*1+3
|A|= -5 **{2 marks}**

(iii) Souvenir pens, caps and mugs are sold by SLIATE for 25th anniversary .Three pens ,two caps and one mug cost Rs.140. two pens, two caps and two mugs cost Rs.170. One pen , three caps and two mugs cost Rs.180.

a) Write a system of linear equations for the above problem (03 marks)

No of pens – x
No of caps – y
No of mugs – z

3x+2y+z = 140
2x+2y+2z=170
X+3y+2z=180

b) Find the prices of individual items by solving the system of linear equations using Cramer's rule. (10 marks)

$\boldsymbol{D} = \begin{bmatrix}3 & 2 & 1\\ 2 & 2 & 2\\ 1 & 3 & 2\end{bmatrix}$

|D| = -6 ***{1 mark}***

$\boldsymbol{D_x} = \begin{bmatrix}140 & 2 & 1\\ 170 & 2 & 2\\ 180 & 3 & 2\end{bmatrix}$ ***{1 mark}***

| D_x | = -90 ***{1 mark}***

$$D_y = \begin{bmatrix} 3 & 140 & 1 \\ 2 & 170 & 2 \\ 1 & 180 & 2 \end{bmatrix}$$ ***{1 mark}***

| D_y | = -150 ***{1 mark}***

$$D_z = \begin{bmatrix} 3 & 2 & 140 \\ 2 & 2 & 170 \\ 1 & 3 & 180 \end{bmatrix}$$ ***{1 mark}***

| D_z| = -270 ***{1 mark}***

x = Δ1 / Δ = (-90) / (-6) = 15 ***{1 mark}***

y = Δ2 / Δ = (-150) / (-6) = 25 ***{1 mark}***

z = Δ3 / Δ = (-270) / (-6) = 45 ***{1 mark}***