# MATH 213 - Lecture 1: Introduction to Differential Equations (DEs)

Lecture goals: To understand how some DEs are derived, the importance of boundary conditions/initial conditions and how to solve some simple DEs.

### What are DEs and where do they come from?:

In 1687 Isaac Newton published the now famous equation $F = ma$ in his book *Philosophiae Naturalis Principia Mathematica* now commonly known as the *Principia Mathematica*. In $F = ma$, $F$ denotes the net force being applied to an object, $m$ denoted the mass of the object and $a$ is the acceleration of the object. This simple principle gives rise to many DEs.

> **Example 1: DE for the motion of the height of a ball**
> Consider a ball of mass $m$ being influenced by only the force of gravity. Use $F = ma$ to find equations for the vertical height of the ball.

> **Example 2: Solving a simple DE**
> Solve the DE you found in Example 1 to find an expression for the height of the ball as a function of time.

**Example 3: Initial Conditions (ICs)**
The solution in Example 2 is not unique. Determine the extra information you need to find the exact height of the ball as a function of time.
This information is known as the **initial conditions** or more generally (and depending on context) as the **boundary conditions**.

**Example 4: Constant Growth**
Suppose you have 1 E. coli bacteria (in Minecraft) at time $t = 0$ and it is known that each E. coli continuously splits to produce 5 new bacterium (we allow for fractional numbers of E. coli).

a. Find a DE for the number of E. coli as a function of time, $f(t)$.

b. State the initial condition(s).

c. Solve the **Initial Value Boundary Problem** (IVBP) found in parts a-b.

There is a slight problem with the previous model: In the real world, growth is not limitless!

But the number of E. coli in our previous model imply that there is a case of...

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which does not exist!!
To correct for this we need to include factors that limit the population growth so that the population remains bounded over time!

**Example 5: Limited growth**
Consider the new model for our E. coli population:

$$\frac{df}{dt} = af - bf^2$$

where $a, b \in \mathbb{R}^+$ are constants.

a. Suppose that $0 < f(0) \ll \frac{a}{b}$.
   **Without** solving the above DE, find the "maximum" value for $f(t)$.

b. Solve the DE for $f(t)$.

Sometimes things change over space and time leading to a **Partial Differential Equation** (PDE).

**Example 6: Linear Wave Equation**

Consider a string of length $L$, constant density $\rho$ and under uniform tension $T$.

a. Assuming the motion of the string is "small", find an equation for the perturbations, $y(x,t)$ of the string.

b. Show that $C_1 \sin\left(k\left(x - t\sqrt{\frac{T}{\rho}}\right)\right) + C_2 \sin\left(k\left(x + t\sqrt{\frac{T}{\rho}}\right)\right)$ solves the equation from part a.

c. Suppose our string is clamped at $x = 0, L$ so that $y(0,t) = y(L,t) = 0$. What limits do these boundary conditions impose of the solution given in part b?