# MATH 213 - Tutorial 10: Fourier series Round 2

1. Determine the pointwise limit of $f_n(x) = \frac{\sin(nx)}{n}$. Does the derivative of $f_n$ converge pointwise?

2. Use the Weierstrass $M$ test (introduced in A6 Q4) to prove that

$$\sum_{n=0}^{\infty} \sin^{2n}(x)$$

   converges uniformly on $[-a, a]$ for $a$ satisfying $0 < a < \pi/2$

3. Suppose you know that

$$x^3 - \pi^2 x = \sum_{n=1}^{\infty} \frac{12(-1)^n}{n^3} \sin(nx).$$

   Use Parseval's Theorem to compute $\sum_{n=1}^{\infty} \frac{1}{n^6}$.

4. Consider $f = x^5 - 80x$ on $[-\pi, \pi]$

   a) Compute $\|f\|_\infty$

   b) Compute $\|f\|_2$

   c) Confirm that $\|f\|_2 \leq \sqrt{b-a}\|f\|_\infty$ holds for this function.

5. For the following functions, determine if the Fourier series will converge pointwise. If it does converge pointwise, draw the periodic extension it converges to for at least 2 periods.

   a)

$$\begin{cases} x & -1 < x < 0 \\ \sqrt{x+1} & 0 < x < 1 \end{cases}$$

   b)

$$\begin{cases} \sin(\pi x) & -1 < x < 0 \\ \frac{x}{x+1} & 0 < x < 1 \end{cases}$$