MATH 213 - Tutorial 9: Fourier series

1. a) Compute the Fourier series of $f(x) = x^2$ on $-\pi < x < \pi$.
   b) Draw a picture of the periodic continuation of f on the interval from -3π to 3π.
   c) Plot the truncated Fourier series to $N = 8$ (Using some software). Do you see Gibbs phenomena in this case?

2. Recall that a function is C^1 if it is **differentiable and its derivative is continuous**.

   Marmie found that
   $$f(x) = \begin{cases} x^3 \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$$
   is C^1 on $\mathbb{R}$ but wants you yo double check her work. Show that $f(x)$ is C^1 on all of $\mathbb{R}$.

   Hint 1: Consider two cases, $x \neq 0$ and $x = 0$.

   Hint 2: At $x = 0$ you must use the definition of the derivative as the limit of the difference quotient (from calc 1) in order to compute the derivative.

   Hint 3: The squeeze theorem is a thing.

3. Consider the function $g(t) = |\sin(t)|$ on the interval $t \in (0, \pi)$.

   a) Find the complex Fourier series of $g(t)$. Hint: To evaluate the integral, it may be helpful to rewrite $\sin(t)$ in terms of exponential functions by using Euler's formula ($e^{i\theta} = \cos(\theta) + i\sin(\theta)$).
   b) Use this Fourier series along with the assumption that the series converges to $g(t)$ (it does and we will be able to prove it later) to show that
   $$\sum_{n=1}^{\infty} \frac{4}{\pi(4n^2 - 1)} = \frac{2}{\pi}$$