MATH 213 - Tutorial 4: Using Laplace to solve DEs and analyzing poles

1. Compute the following limits without the use of L'Hospital's rule

   (a) $\lim_{x\to\infty} \frac{4x^4+1}{x^4-2x+1}$

   (b) $\lim_{x\to 1} \frac{x^2-1}{x-1}$

2. Johnathan computed that the transfer function for the system he is building is given by

$$T(s) = \frac{1}{s^3 + 2s^2 + 3s}.$$

   Suppose that Johnathan starts his system with the initial conditions $y(0) = 1$, $y'(0) = 1$ and $y''(0) = 1$.

   (a) What is the zero-input response of the system?

   (b) Without solving for $y(t)$, determine if Johnathan's system has a bounded solution. If the system has a bounded solution find $\lim_{t\to\infty} y(t)$ without computing $y(t)$.

3. Find the zeros and poles of

$$F(s) = \frac{s^2 + s}{s^6 + 2s^4 + s^2}$$

   and use the poles to write $f(t)$ as a linear combination of appropriate terms.

   Here you do not need to find the actual coefficients for the terms in $f(t)$.

4. Recall the Minecraft chicken problem from A1Q1 and lecture 6 example 6. In class we discussed some of the limitations of the model we considered in A1Q1 and replaced the model with the system of equations

$$\begin{aligned} E'(t) &= -\alpha E(t) + \beta C(t) \\ C'(t) &= \quad \alpha E(t) \end{aligned}$$

   or in matrix form

$$\begin{bmatrix} E' \\ C' \end{bmatrix} - \begin{bmatrix} -\alpha & \beta \\ \alpha & 0 \end{bmatrix} \begin{bmatrix} E \\ C \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

   where $\alpha, \beta > 0$ are the rates of egg hatching and egg laying.

   Use the Laplace transform to first find the Laplace transforms of E and/or C under the initial conditions $E(0) = 0$ and $C(0) = 2$ and then to find the solutions in the case where $\alpha = 1$ and $\beta = 2$.

   In this problem we will show two standard ways of using the Laplace transform to solve this linear system of ODEs with constant coefficients and one novel way that will not be covered in tutorial but will be uploaded to Learn.