MATH 213 - Assignment 2

Submit to Crowdmark by 9:00pm EST on Friday, February 2.

Instructions:

1. Answer each question in the space provided or on a separate piece of paper. You may also use typetting software (e.g., Word, TeX) or a writing app (e.g., Notability).
2. All homework problems must be solved independently.
3. For full credit make sure you show **all** intermediate steps. If you have questions regarding showing intermediate steps, feel free to ask me.
4. Scan or photograph your answers.
5. Upload and submit your answers by following the instructions provided in an e-mail sent from Crowdmark to your uWaterloo e-mail address. Make sure to upload each problem in the correct submission area and only upload the relevant work for that problem in the submission area. Failure to do this **will** result in your work not being marked.
6. Close the Crowdmark browser window. Follow your personalized Crowdmark link again to carefully view your submission and ensure it will be accepted for credit. Any pages that are uploaded improperly (sideways, upside down, too dark/light, text cut off, out of order, in the wrong location, etc.) will be given a score of **zero**.

Questions:

1. (5 marks) Use the definition to compute the two-sided Laplace transform of $e^{-\alpha|t|}$ where α is a positive real number (i.e. $\alpha \in \mathbb{R}^+$). Simplify your result as much as possible and explicitly find the ROC with justification.

   For this problem you must show **all** of your work and must justify how you found the ROC.

2. (5 marks) Use the definition to compute $(f * g)(t)$ where

$$f(t) = \begin{cases} \sin(t) & 0 < t < \pi \\ 0 & else \end{cases} \qquad \text{and} \qquad g(t) = \begin{cases} 1 & 0 < t < T \\ 0 & else \end{cases}$$

   where $T > \pi$. For this problem you must show **all** of your work.

   Hint: You may find that thinking about the convolution geometrically simplifies the process of determining any cases that need to be considered.

3. (5 marks) Use the one-sided Laplace transform table and partial fractions to compute

$$\mathscr{L}^{-1}\left\{\frac{s^4}{s(2s^2+4)((s+1)^2+9)}\right\}.$$

   For this problem you must show **all** of your work and explicitly mention what properties of the Laplace transform you use.

   Pro tip: Before you blindly try to find a PF decomposition, look at the Laplace table and adjust the form of the PF decomposition to make your life nicer when computing the inverse transforms later.

4. (4 marks) Use the convolution theorem to compute

$$\mathscr{L}^{-1}\left\{\frac{s}{(s-1)(s^2+4)}\right\}$$

   where we mean the one sided Laplace Transform. For this problem you must show **all** of your work and explicitly mention what properties of the Laplace transform you use.

   Hint: For this question you are allowed to freely use any results from Assignment 1 as long as you cite them. If you do this, then make sure you look at the solutions in case you made errors you may have made in your A1.

5. (5 marks) Use the definition of the one-sided Laplace transform to compute

$$\mathscr{L}\{\sinh(4t)\}$$

   where $\sinh(at) = \frac{e^{at}-e^{-at}}{2}$. You **must** explicitly find and state the ROC with justification.

   For this problem you must show **all** of your work.

   Note: The final solution including ROC is explicitly listed in our Laplace table. Hence all marks for this question are given for the intermediate computations.

   Fun facts (optional): Students are often curious about what the hyperbolic trig functions "do" and/or why we care about them so I will elaborate here. In class we showed that $\sin(ax) = \frac{e^{ajx}-e^{-ajx}}{2j}$

and a similar result holds for cos. sinh and cosh typically defined by simply removing the js from these formulas. Geometrically, removing the js changes the underlying geometric curve from the complex unit circle (e^{jt} defines the "unit circle" in $\mathbb{C}$) to a hyperbola. One simple explanation for this is the fact $y = \sinh(t)$ and $x = \cosh(t)$ describe the x and y components on the unit hyperbola ($x^2 - y^2 = 1$). In short sin/cos are to the unit circle what sinh/cosh are to the unit hyperbola. Finally, for a real world application, all cables that are supported by their ends form a shape called a "Catenary". This shape is described by $\cosh(x)$. For details of the DE that gives this result see the "Analysis" section of this wiki page.