MATH 213 - Assignment 5

Submit to Crowdmark by 9:00pm EST on Friday, March 22.

Instructions:

1. Answer each question in the space provided or on a separate piece of paper. You may also use typetting software (e.g., Word, TeX) or a writing app (e.g., Notability).
2. All homework problems must be solved independently.
3. For full credit make sure you show **all** intermediate steps. If you have questions regarding showing intermediate steps, feel free to ask me.
4. Scan or photograph your answers.
5. Upload and submit your answers by following the instructions provided in an e-mail sent from Crowdmark to your uWaterloo e-mail address. Make sure to upload each problem in the correct submission area and only upload the relevant work for that problem in the submission area. Failure to do this **will** result in your work not being marked.
6. Close the Crowdmark browser window. Follow your personalized Crowdmark link again to carefully view your submission and ensure it will be accepted for credit. Any pages that are uploaded improperly (sideways, upside down, too dark/light, text cut off, out of order, in the wrong location, etc.) will be given a score of **zero**.

Read before starting the assignment: For this assignment you must do all your work independently and without the use of external aids.

Questions:

1. (5 marks) In tutorial we showed that a proportional controller could be used to stabilize a system. It is also the case that such a controller can cause a BIBO stable system to become unstable! Suppose we are applying a proportional controller to a BIBO stable system with transfer function

$$T(s) = \frac{s+a}{s^2+bs+c}, \qquad a, b, c \in \mathbb{R}.$$

   Find and analyze the poles of the transfer functions of the controlled and uncontrolled systems to find conditions on k_p and potentially the coefficients a, b and/or c such that the proportionally controlled system is **not** BIBO stable while uncontrolled system **is** BIBO stable.

2. (4 marks) Consider the system with transfer function

$$H(s) = \frac{(s+20)^6}{((s+1)((s+20)^2+9)(s+2)((s+21)^2+16)(s+46)^2}.$$

   Find all of the poles of $H(s)$. Identify with justification the dominate pole(s) and then use these poles to find $h_{approx}(t)$ such that $h_{approx}(t) \approx h(t)$.

3. (9 marks) Manually construct the Bode plot of

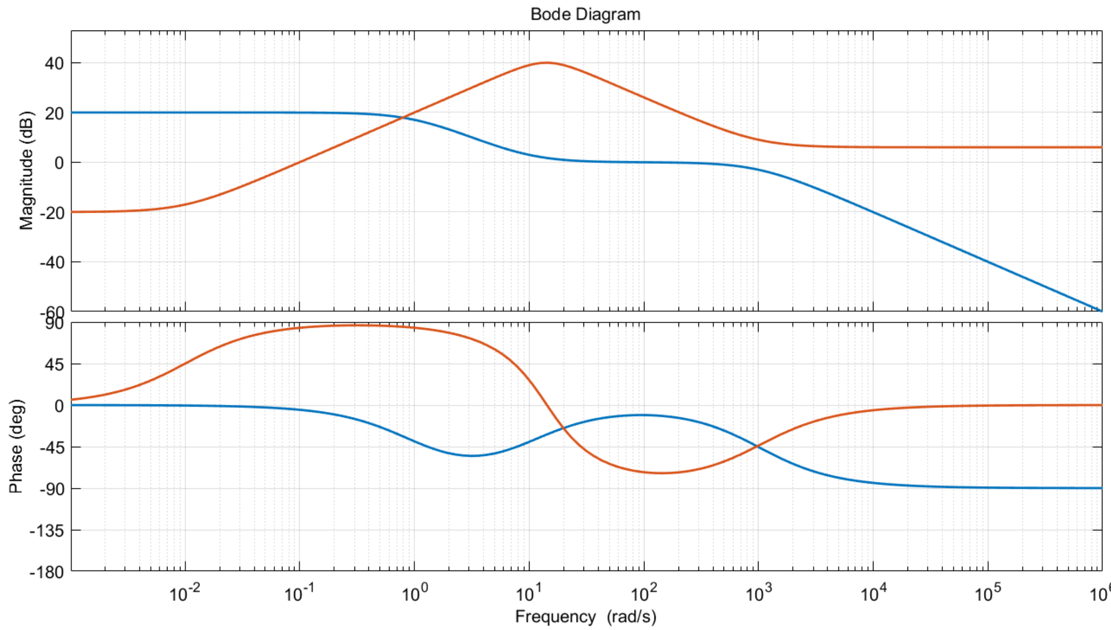
$$H(s) = (s+0.1) \cdot \frac{10^6}{((s+1)^2+100)} \cdot \frac{1}{s+1000}$$

   for $\log_{10}(\omega)$ ranging from -2 to 6.

   Note: I am **not** looking for an exact plot. You need to plot the asymptotic behaviours (blue/red lines I drew in class), approximately correct timescales for the adjustments to the asymptotics, and show how you use this to construct the approximate curves. You need to show all your work but can use the results from class where applicable.

   For ease of plotting, a blank Bode plot is included at the end of this assignment.

4. Marmie the engineering cat generated the following Bode plots:

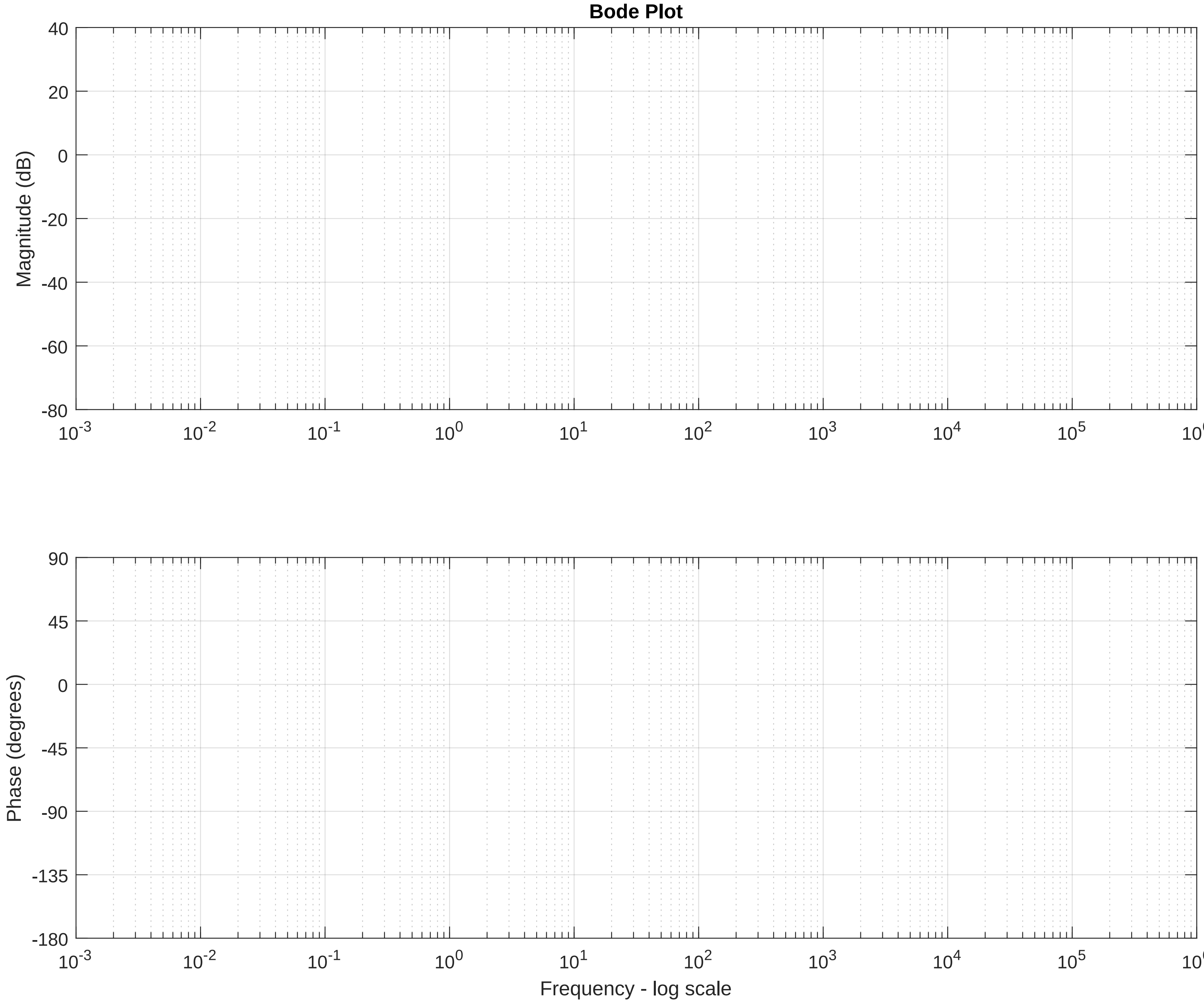


(a) (8 marks)Use the curves to approximate the transfer functions $H_{red}(s)$ and $H_{blue}(s)$ from the bode plots for the two curves Marmie gave you. Your transfer function is **not** expected to be perfect but should be good enough to be in the ballpark of being correct.

(b) (4 marks) Use either your transfer functions or the given plots to draw an approximate Bode plot for $H_{red}(s)H_{blue}(s)$. For ease of plotting, a blank Bode plot is included at the end of this assignment.

(c) (2 marks) Based on the Bode plot for the blue curve can you conclude that the system with transfer function $H_{blue}(s)$ is unstable?

(d) (2 marks) Suppose that $H_{red}(s)$ is the transfer function for a proposed controller of a system with transfer function $H_{blue}(S)$. Use your plot in part (b) determine if the closed loop controller is stable. Explain why or why not.



Bode Plot
Magnitude (dB)
40
20
0
-20
-40
-60
-80
Phase (degrees)
90
45
0
-45
-90
-135
-180
10^-3
10^-2
10^-1
10^0
10^1
10^2
10^3
10^4
10^5
10^6
Frequency - log scale