Final Guidelines - MATH 213 Winter 2024

General Infromation

1. The final will cover content from all lectures of MATH 213 with a focus on materials covered after the midterm. A suggested list of topics you should study is given below but all course materials are fair game.
2. The test will consist of a single written exam. More information about the written component is given below.

Written Exam details

1. The exam will have 14 questions similar to what you would see on a traditional midterm or assignment. One problem will be a T/F problem similar to the midterm. The problems will be tailored to not require as much algebra (if done in the intended way) as the assignment questions and to have reasonable numbers (i.e. calculators are not needed or allowed) but you will have to do some standard calculations (integration, simplify sums, differentiation, partial fractions, take limits, simplify expressions, factor etc.).
2. The focus is on the topics covered after the midterm but you are responsible for pre-midterm content. In particular a lot of the post midterm content relies strongly on pre-midterm concepts.
3. The final will be in-person in PAC Upper on **April, 16 starting at 19:30 (7:30 PM)** and will be 2 hour 30 minutes long.

Authorized Aids:

- No aids are permitted (including but not limited to people, cats [this excludes Marmie], notes, calculators, rulers, slide rules, abaci, external books or websites). With the exception of the MATH 213 instructor and proctoring TAs, you are not to communicate with *anyone* about the test during the exam.

Review for final

Disclaimer: This is a guide and may not be complete (in particular there might be some prerequisite materials missing). Further some topics may not be directly tested on or even tested on at all but all topics are fair game and are things you should know when leaving this course. Please make sure you study all the material we have covered in lectures and tutorials and that you are able to do all of the assignment/tutorial problems.

DE Basics

- Dependent variables
- Independent variables
- Parameters
- Linear DEs
- Non-linear DEs
- Order of a DE
- ODE vs PDE
- Homogeneous vs Non-homogeneous
- Differential operators (i.e. $D(y(x)) = g(x)$ notation).
- Characteristic Polynomial
- Resonance

Laplace Transform and Convolutions

- Computing one-sided (unilateral) and two sided Laplace transforms by definition (i.e. integrating)
- Unit step function/Heaviside function/$u(t)$.
- Converting one-sided Laplace transforms to two sided transforms using the Heaviside function.
- Using Laplace transform tables and the algebraic formulas (linearity, time-scaling, shifting, $\mathcal{L}(f^{(n)})$, etc) to take Laplace transforms.
- Computing Laplace transforms of piecewise continuous functions
- Convolution definition and computing convolutions by definition.
- Convolution theorem including how to use it to avoid computing convolutions by definition.
- Convolution algebraic properties (commutative, linear)
- Using Laplace transforms to solve IVPs with continuous and/or discontinuous forcing terms
- $\delta(x)$ function/impulse function
- Solving constant coefficient IVPs with Impulses (i.e. $D(y) = \delta(x-1) + xu(x-5)$ etc.)
- Partial fractions and the Heaviside method.
- Rational functions
- Regularization
- Finite poles
- Proper and Strictly proper Rational functions
- Initial and Final value theorems
- Piecewise continuous functions
- How the poles of $F(s)$ determine the behaviour of its inverse Laplace transform $f(t)$.

Systems

- Zero-input and zero-state response
- Zero-input and zero state solutions
- Transfer function
- Analysis and Synthesis problems
- Continuous-time vs discrete time vs hybrid systems
- Memoryless vs Dynamic systems
- Causal systems
- Multivariable vs Scalar Systems
- Linearity for systems (remember this is different from the definition forDEs)
- Time-invariance
- LTIs
- LTI response to complex exponentials
- Impulse and step responses and how to compute them
- Using the impulse response to find the general solution for any input function $f(t)$
- Standard first order system: DC gain and time constant
- Standard second order system: Where the poles can be and how their position effects the system impulse and step responses.
- Standard second order system: undamped, under damped, critically damped, over damped.
- More complex stable systems and their decomposition to a linear combination of the standard first and second order systems.
- Linear stability: stable, unstable, marginally stable and BIBO stable.
- Stabilizing an unstable system with a P controller.
- Bode plots: How to draw them like in L19 and how to approximate the transfer function from the Bode plot
- What decibels are

Basic Control Theory

- Closed loop controllers and finding the transfer function
- Proportional controller
- Integral controller
- PI controllers

Fourier Series

- Inner product for $L^2([a, b])$
- Fourier series in complex form including how to find the coefficients by definition
- τ periodic functions
- even and odd functions
- Fourier sin series including how to find the coefficients by definition
- Fourier cos series including how to find the coefficients by definition
- Pointwise convergence of sequences of functions
- L^2 convergence (convergence in the mean) of sequences of functions
- Uniform convergence of sequences of functions
- Weierstrass M-test (A6 and Tut 10)
- Piecewise C^1 functions
- Periodic extension of a function
- Convergence in the mean for Fourier series
- Criteria for PW convergence of Fourier series
- Criteria for uniform convergence of Fourier series
- Gibbs phenomenon: when it happens
- Term-wise integration and differentiation of Fourier series to find a new series (with justification for applicability)
- Dirichlet's theorem
- Parseval's Theorem.
- Using known PW convergent Fourier series to evaluate sums.

Fourier Transform

- Definition
- Using the Laplace Transform algebraic properties to compute Fourier series (we did this with Laplace and nothing changes with Fourier so this is fair game)


CONVOLUTION
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FOURIER TRANSFORM