In this notebook I will explore the topic of Time-Series Forecasting as part a Project Work for my University, and eventually make submission(s) for the competition described below.

Contents

1. Defining the problem and the goal

The ***problem*** is a time-series prediction problem presented as a competition, more specifically the [Store Sales Time-Series Forecasting Competition](),

where the ***goal*** is to use time-series forecasting to forecast store sales on data from Corporación Favorita, a large Ecuadorian-based grocery retailer. Essencially, to use machine learning to predict grocery sales.

[- Dataset description]()

The training data includes dates, store and product information, whether that item was being promoted, as well as the sales numbers. Additional files include supplementary information that may be useful in building your models.

Training data: train.csv

- The training data, comprising time series of features **store_nbr**, **family**, and **onpromotion** as well as the target **sales**.
- **store_nbr** identifies the store at which the products are sold.
- **family** identifies the type of product sold.

- **sales** gives the total sales for a product family at a particular store at a given date. Fractional values are possible since products can be sold in fractional units (1.5 kg of cheese, for instance, as opposed to 1 bag of chips).
- **onpromotion** gives the total number of items in a product family that were being promoted at a store at a given date.

Test data: test.csv

- The test data, having the same features as the training data. You will predict the target **sales** for the dates in this file.
- The dates in the test data are for the 15 days after the last date in the training data.

Submission file: sample_submission.csv

A sample submission file in the correct format.

Additional information

1. Store metadata: stores.csv

- Store metadata, including **city**, **state**, **type**, and **cluster**.
- **cluster** is a grouping of similar stores.

2. Daily oil price: oil.csv

Includes values during both the train and test data timeframes. (Ecuador is an oil-dependent country and it's economical health is highly vulnerable to shocks in oil prices.)

3. Holidays and Events, with metadata: holidays_events.csv

- NOTE: Pay special attention to the **transferred** column. A holiday that is transferred officially falls on that calendar day, but was moved to another date by the government. A transferred day is more like a normal day than a holiday. To find the day that it was actually celebrated, look for the corresponding row where type is Transfer. For example, the holiday Independencia de Guayaquil was transferred from 2012-10-09 to 2012-10-12, which means it was celebrated on 2012-10-12. Days that are type Bridge are extra days that are added to a holiday (e.g., to extend the break across a long weekend). These are frequently made up by the type Work Day which is a day not normally scheduled for work (e.g., Saturday) that is meant to payback the Bridge.
- Additional holidays are days added a regular calendar holiday, for example, as typically happens around Christmas (making Christmas Eve a holiday).

Additional Notes

- Wages in the public sector are paid every two weeks on the 15 th and on the last day of the month. Supermarket sales could be affected by this.
- A magnitude 7.8 earthquake struck Ecuador on April 16, 2016. People rallied in relief efforts donating water and other first need products which greatly affected supermarket sales for several weeks after the earthquake.

- Evaluation

The evaluation metric for this competition is Root Mean Squared Logarithmic Error.

The RMSLE is calculated as: $[\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\log(1+\hat{y}_i) - \log(1+y_i)\right)^2}]$ where:

n is the total number of instances,
$\hat{y}\ i$ is the predicted value of the target for instance (i),

yi is the actual value of the target for instance (i), and,

log is the natural logarithm.

2. Importing the data

```
# This Python 3 environment comes with many helpful analytics libraries installed
# It is defined by the kaggle/python Docker image: https://github.com/kaggle/docker-python
# For example, here's several helpful packages to load
import os
for dirname, _, filenames in os.walk('/kaggle/input'):
    for filename in filenames:
        print(os.path.join(dirname, filename))

# You can write up to 20GB to the current directory (/kaggle/working/) that gets preserved as output when you create a version using "Save & Run All"
# You can also write temporary files to /kaggle/temp/, but they won't be saved outside of the current session

import datetime
from pathlib import Path
#from learntools.time_series.style import *  # plot style settings
#from learntools.time_series.utils import (seasonal_plot,
#                                           plot_periodogram,
#                                           make_lags,
#                                           make_leads,
#                                           plot_lags,
#                                           make_multistep_target,
#                                           plot_multistep)

import pandas as pd
import matplotlib.pyplot as plt
import numpy as np
import seaborn as sns
from sklearn.metrics import mean_squared_log_error
from statsmodels.graphics.tsaplots import plot_pacf
from statsmodels.tsa.deterministic import CalendarFourier, DeterministicProcess

from sklearn.model_selection import train_test_split
from sklearn.metrics import mean_squared_log_error
from sklearn.preprocessing import LabelEncoder

# Model 1 (trend)
from sklearn.linear_model import LinearRegression

# Model 2
from sklearn.neighbors import KNeighborsRegressor
```

```
/kaggle/input/store-sales-time-series-forecasting/oil.csv
/kaggle/input/store-sales-time-series-forecasting/sample_submission.csv
/kaggle/input/store-sales-time-series-forecasting/holidays_events.csv
/kaggle/input/store-sales-time-series-forecasting/stores.csv
/kaggle/input/store-sales-time-series-forecasting/train.csv
/kaggle/input/store-sales-time-series-forecasting/test.csv
/kaggle/input/store-sales-time-series-forecasting/transactions.csv
```

```
## Utils Plot tools
## from learntools.time_series
## reference: https://github.com/Kaggle/learntools/tree/master/learntools/time_series

def seasonal_plot(X, y, period, freq, ax=None):
    if ax is None:
        _, ax = plt.subplots()
    palette = sns.color_palette(
        "husl",
        n_colors=X[period].nunique(),
    )
    ax = sns.lineplot(
        x=freq,
        y=y,
        hue=period,
        data=X,
        ci=False,
        ax=ax,
        palette=palette,
        legend=False,
    )
    ax.set_title(f"Seasonal Plot ({period}/{freq})")
    for line, name in zip(ax.lines, X[period].unique()):
        y_ = line.get_ydata()[-1]
        ax.annotate(
            name,
            xy=(1, y_),
            xytext=(6, 0),
            color=line.get_color(),
            xycoords=ax.get_yaxis_transform(),
            textcoords="offset points",
            size=14,
            va="center",
        )
    return ax


def plot_periodogram(ts, detrend='linear', ax=None):
    from scipy.signal import periodogram
    fs = pd.Timedelta("1Y") / pd.Timedelta("1D")
    freqencies, spectrum = periodogram(
        ts,
        fs=fs,
        detrend=detrend,
        window="boxcar",
        scaling='spectrum',
    )
    if ax is None:
        _, ax = plt.subplots()
    ax.step(freqencies, spectrum, color="purple")
    ax.set_xscale("log")
    ax.set_xticks([1, 2, 4, 6, 12, 26, 52, 104])
    ax.set_xticklabels(
        [
            "Annual (1)",
            "Semiannual (2)",
            "Quarterly (4)",
```

```
            "Quarterly (4)",
            "Bimonthly (6)",
            "Monthly (12)",
            "Biweekly (26)",
            "Weekly (52)",
            "Semiweekly (104)",
        ],
        rotation=30,
    )
    ax.ticklabel_format(axis="y", style="sci", scilimits=(0, 0))
    ax.set_ylabel("Variance")
    ax.set_title("Periodogram")
    return ax


# From Lesson 4
def lagplot(x, y=None, shift=1, standardize=False, ax=None, **kwargs):
    from matplotlib.offsetbox import AnchoredText
    x_ = x.shift(shift)
    if standardize:
        x_ = (x_ - x_.mean()) / x_.std()
    if y is not None:
        y_ = (y - y.mean()) / y.std() if standardize else y
    else:
        y_ = x
    corr = y_.corr(x_)
    if ax is None:
        fig, ax = plt.subplots()
    scatter_kws = dict(
        alpha=0.75,
        s=3,
    )
    line_kws = dict(color='C3', )
    ax = sns.regplot(x=x_,
                     y=y_,
                     scatter_kws=scatter_kws,
                     line_kws=line_kws,
                     lowess=True,
                     ax=ax,
                     **kwargs)
    at = AnchoredText(
        f"{corr:.2f}",
        prop=dict(size="large"),
        frameon=True,
        loc="upper left",
    )
    at.patch.set_boxstyle("square, pad=0.0")
    ax.add_artist(at)
    title = f"Lag {shift}" if shift > 0 else f"Lead {shift}"
    ax.set(title=f"Lag {shift}", xlabel=x_.name, ylabel=y_.name)
    return ax


def plot_lags(x,
              y=None,
              lags=6,
              leads=None,
```

```
              nrows=1,
              lagplot_kwargs={},
              **kwargs):
    import math
    kwargs.setdefault('nrows', nrows)
    orig = leads is not None
    leads = leads or 0
    kwargs.setdefault('ncols', math.ceil((lags + orig + leads) / nrows))
    kwargs.setdefault('figsize', (kwargs['ncols'] * 2, nrows * 2 + 0.5))
    fig, axs = plt.subplots(sharex=True, sharey=True, squeeze=False, **kwargs)
    for ax, k in zip(fig.get_axes(), range(kwargs['nrows'] * kwargs['ncols'])):
        k -= leads + orig
        if k + 1 <= lags:
            ax = lagplot(x, y, shift=k + 1, ax=ax, **lagplot_kwargs)
            title = f"Lag {k + 1}" if k + 1 >= 0 else f"Lead {-k - 1}"
            ax.set_title(title, fontdict=dict(fontsize=14))
            ax.set(xlabel="", ylabel="")
        else:
            ax.axis('off')
    plt.setp(axs[-1, :], xlabel=x.name)
    plt.setp(axs[:, 0], ylabel=y.name if y is not None else x.name)
    fig.tight_layout(w_pad=0.1, h_pad=0.1)
    return fig


# def make_lag_features(y, lags):
#     name = 'lag' if lags > 0 else 'lead'
#     steps = range(1, lags + 1) if lags > 0 else range(-1, lags - 1, -1)
#     return pd.concat(
#         [y.shift(i, freq='infer') for i in steps],
#         axis=1,
#         join='outer',
#         keys=[f'{y.name}_{name}_{i if lags > 0 else -i}' for i in steps],
#     )

# From Lesson 5


class BoostedHybrid:
    def __init__(self, model_1, model_2):
        self.model_1 = model_1
        self.model_2 = model_2
        self.y_columns = None
        self.stack_cols = None

    def fit(self, X_1, X_2, y, stack_cols=None):
        # Train model_1
        self.model_1.fit(X_1, y)

        # Make predictions
        y_fit = pd.DataFrame(
            self.model_1.predict(X_1),
            index=X_1.index,
            columns=y.columns,
        )
        # Compute residuals
        y_resid = y - y_fit
```

```
        y_resid = y_resid.stack(stack_cols).squeeze()  # wide to long

        # Train model_2 on residuals
        self.model_2.fit(X_2, y_resid)

        # Save column names for predict method
        self.y_columns = y.columns
        self.stack_cols = stack_cols

    def predict(self, X_1, X_2):
        # Predict with model_1
        y_pred = pd.DataFrame(
            self.model_1.predict(X_1),
            index=X_1.index,
            columns=self.y_columns,
        )
        y_pred = y_pred.stack(self.stack_cols).squeeze()  # wide to long

        # Add model_2 predictions to model_1 predictions
        y_pred += self.model_2.predict(X_2)
        return y_pred.unstack(self.stack_cols)


# From Lesson 6
def make_lags(ts, lags, lead_time=1, name='y'):
    return pd.concat(
        {
            f'{name}_lag_{i}': ts.shift(i)
            for i in range(lead_time, lags + lead_time)
        },
        axis=1)


def make_leads(ts, leads, name='y'):
    return pd.concat(
        {f'{name}_lead_{i}': ts.shift(-i)
         for i in reversed(range(leads))},
        axis=1)


def make_multistep_target(ts, steps, reverse=False):
    shifts = reversed(range(steps)) if reverse else range(steps)
    return pd.concat({f'y_step_{i + 1}': ts.shift(-i) for i in shifts}, axis=1)


def create_multistep_example(n, steps, lags, lead_time=1):
    ts = pd.Series(
        np.arange(n),
        index=pd.period_range(start='2010', freq='A', periods=n, name='Year'),
        dtype=pd.Int8Dtype,
    )
    X = make_lags(ts, lags, lead_time)
    y = make_multistep_target(ts, steps, reverse=True)
    data = pd.concat({'Targets': y, 'Features': X}, axis=1)
    data = data.style.set_properties(['Targets'], **{'background-color': 'LavenderBlush'}) \
                   .set_properties(['Features'], **{'background-color': 'Lavender'})
```

```
    return data


def load_multistep_data():
    df1 = create_multistep_example(10, steps=1, lags=3, lead_time=1)
    df2 = create_multistep_example(10, steps=3, lags=4, lead_time=2)
    df3 = create_multistep_example(10, steps=3, lags=4, lead_time=1)
    return [df1, df2, df3]


def plot_multistep(y, every=1, ax=None, palette_kwargs=None):
    palette_kwargs_ = dict(palette='husl', n_colors=16, desat=None)
    if palette_kwargs is not None:
        palette_kwargs_.update(palette_kwargs)
    palette = sns.color_palette(**palette_kwargs_)
    if ax is None:
        fig, ax = plt.subplots()
    ax.set_prop_cycle(plt.cycler('color', palette))
    for date, preds in y[::every].iterrows():
        preds.index = pd.period_range(start=date, periods=len(preds))
        preds.plot(ax=ax)
    return ax

import warnings

import matplotlib.pyplot as plt
from IPython import get_ipython

warnings.simplefilter("ignore")

plt.style.use("seaborn-whitegrid")
plt.rc(
    "figure",
    autolayout=True,
    figsize=(11, 4),
    titlesize=18,
    titleweight='bold',
)
plt.rc(
    "axes",
    labelweight="bold",
    labelsize="large",
    titleweight="bold",
    titlesize=16,
    titlepad=10,
)
plot_params = dict(
    color="0.75",
    style=".-",
    markeredgecolor="0.25",
    markerfacecolor="0.25",
    legend=False,
)

get_ipython().config.InlineBackend.figure_format = 'retina'
```

```
input_dir = Path('../input/store-sales-time-series-forecasting')

# Training data: train.csv

# For the first part of the analysis (time-dependence), we are going to
# use a restricted training data, using information about the store number,
# family, date and the sales; we will use the rest of the training data
# as we expand the analysis.
store_sales = pd.read_csv(
    input_dir / 'train.csv',
    usecols=['store_nbr', 'family', 'date', 'sales'],
    dtype={
        'store_nbr': 'category',
        'family': 'category',
        'sales': 'float32',
    },
    parse_dates=['date'],
    infer_datetime_format=True,
)
store_sales['date'] = store_sales.date.dt.to_period('D')
store_sales = store_sales.set_index(['store_nbr', 'family', 'date']).sort_index()

# Holidays and Events, with metadata
holidays_events = pd.read_csv(
    input_dir / "holidays_events.csv",
    dtype={
        'type': 'category',
        'locale': 'category',
        'locale_name': 'category',
        'description': 'category',
        'transferred': 'bool',
    },
    parse_dates=['date'],
    infer_datetime_format=True,
)
holidays_events = holidays_events.set_index('date').to_period('D')

# Test data: test.csv
df_test = pd.read_csv(
    input_dir / 'test.csv',
    dtype={
        'store_nbr': 'category',
        'family': 'category',
        'onpromotion': 'uint32',
    },
    parse_dates=['date'],
    infer_datetime_format=True,
)
df_test['date'] = df_test.date.dt.to_period('D')
df_test = df_test.set_index(['store_nbr', 'family', 'date']).sort_index()
```

3. EDA: Exploratory Data Analysis

Let's take a look at the training data.

```
print("Training Data", "\n" + "-" * 14 + "\n", store_sales)
print("\n")
print("Test Data", "\n" + "-" * 10 + "\n", df_test)
print("\n")
print("Holidays events Data", "\n" + "-" * 14 + "\n", holidays_events)
```

```
Training Data
--------------
                                       sales
store_nbr family     date
1         AUTOMOTIVE 2013-01-01   0.000000
                     2013-01-02   2.000000
                     2013-01-03   3.000000
                     2013-01-04   3.000000
                     2013-01-05   5.000000
...                                    ...
9         SEAFOOD    2017-08-11  23.830999
                     2017-08-12  16.859001
                     2017-08-13  20.000000
                     2017-08-14  17.000000
                     2017-08-15  16.000000

[3000888 rows x 1 columns]


Test Data
----------
                                        id  onpromotion
store_nbr family     date
1         AUTOMOTIVE 2017-08-16  3000888            0
                     2017-08-17  3002670            0
                     2017-08-18  3004452            0
                     2017-08-19  3006234            0
                     2017-08-20  3008016            0
...                                    ...          ...
9         SEAFOOD    2017-08-27  3022271            0
                     2017-08-28  3024053            0
                     2017-08-29  3025835            0
                     2017-08-30  3027617            0
                     2017-08-31  3029399            0

[28512 rows x 2 columns]


Holidays events Data
--------------
                  type    locale locale_name                    description  \
date
2012-03-02     Holiday     Local       Manta             Fundacion de Manta
2012-04-01     Holiday  Regional    Cotopaxi  Provincializacion de Cotopaxi
2012-04-12     Holiday     Local      Cuenca            Fundacion de Cuenca
2012-04-14     Holiday     Local    Libertad      Cantonizacion de Libertad
2012-04-21     Holiday     Local    Riobamba      Cantonizacion de Riobamba
...                ...       ...         ...                            ...
2017-12-22  Additional  National     Ecuador                      Navidad-3
2017-12-23  Additional  National     Ecuador                      Navidad-2
2017-12-24  Additional  National     Ecuador                      Navidad-1
```

```
2017-12-25     Holiday  National     Ecuador                       Navidad
2017-12-26  Additional  National     Ecuador                     Navidad+1

            transferred
date
2012-03-02        False
2012-04-01        False
```

```
store_sales #train.csv
```

store_nbr	family	date	sales
1	AUTOMOTIVE	2013-01-01	0.000000
		2013-01-02	2.000000
		2013-01-03	3.000000
		2013-01-04	3.000000
		2013-01-05	5.000000
...	...	...	...
9	SEAFOOD	2017-08-11	23.830999
		2017-08-12	16.859001
		2017-08-13	20.000000
		2017-08-14	17.000000
		2017-08-15	16.000000

3000888 rows × 1 columns

```
df_test
```

			id	onpromotion
store_nbr	family	date		

holidays_events

date	type	locale	locale_name	description	transferred
2012-03-02	Holiday	Local	Manta	Fundacion de Manta	False
2012-04-01	Holiday	Regional	Cotopaxi	Provincializacion de Cotopaxi	False
2012-04-12	Holiday	Local	Cuenca	Fundacion de Cuenca	False
2012-04-14	Holiday	Local	Libertad	Cantonizacion de Libertad	False
2012-04-21	Holiday	Local	Riobamba	Cantonizacion de Riobamba	False
...	...	...	...	...	...
2017-12-22	Additional	National	Ecuador	Navidad-3	False
2017-12-23	Additional	National	Ecuador	Navidad-2	False
2017-12-24	Additional	National	Ecuador	Navidad-1	False
2017-12-25	Holiday	National	Ecuador	Navidad	False
2017-12-26	Additional	National	Ecuador	Navidad+1	False

350 rows × 5 columns

3.1 Indexes

Dates

```
print("Total duration of data:")
print(f"    Training data: {store_sales.index.get_level_values(2).min()} --> {store_sales.index.get_level_values(2).max()}")
print(f"    Test data: {df_test.index.get_level_values(2).min()} --> {df_test.index.get_level_values(2).max()}")
```

```
Total duration of data:
    Training data: 2013-01-01 --> 2017-08-15
    Test data: 2017-08-16 --> 2017-08-31
```

Family

```
print(f"Total number of families: {len(store_sales.index.unique(level=1))}")

print(f"First 5 families (in alphabetical order): {[x.capitalize() for x in store_sales.index.unique(level=1)[:5]]}")

print(f"Last 5 families (in alphabetical order): {[x.capitalize() for x in store_sales.index.unique(level=1)[-5:]]}")

print(f"List of families: {[x.capitalize() for x in store_sales.index.unique(level=1)]}")
```

```
Total number of families: 33
First 5 families (in alphabetical order): ['Automotive', 'Baby care', 'Beauty', 'Beverages', 'Books']
Last 5 families (in alphabetical order): ['Poultry', 'Prepared foods', 'Produce', 'School and office supplies', 'Seafood']
List of families: ['Automotive', 'Baby care', 'Beauty', 'Beverages', 'Books', 'Bread/bakery', 'Celebration', 'Cleaning', 'Dairy', 'Deli', 'Eggs', 'Frozen foods', 'Grocery i', 'Grocery ii', 'Hardw
```

Stores

```
print(f"Total number of stores: {len(store_sales.index.unique(level=0))}")
```

```
Total number of stores: 54
```

4. Modelling time dependency

Time dependece is one of the two essencial component of a time-series. Specifically, a series is time dependent if its values can be predicted from the time they occured.

In this part we will model the features of time series that are time dependent (trends and seasonality), and as such the time series based on those feature require linear algorithm (e.g. Linear Regression) to make a forecasting.

4.1 Trends

A trend represents a persistent, long-term change in the mean of the series. It is the slowest-moving part of a series, the part representing the largest time scale of importance.

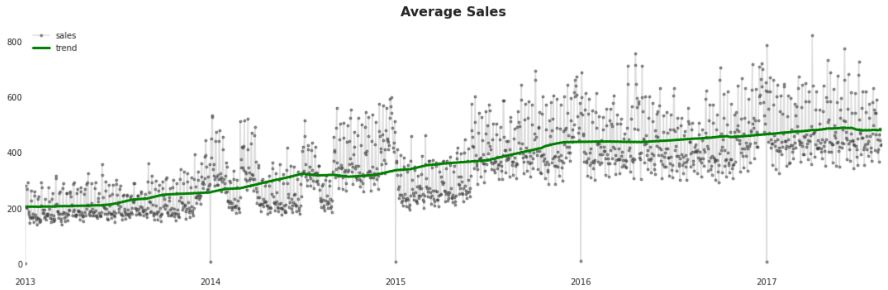
```
avg_sales = store_sales.groupby('date').mean()['sales']
avg_sales.head()
```

```
date
2013-01-01      1.409438
2013-01-02    278.390808
2013-01-03    202.840195
2013-01-04    198.911148
2013-01-05    267.873230
Freq: D, Name: sales, dtype: float32
```

Moving average plot

```
# moving average plot to estimate the trend

trend = avg_sales.rolling(
    window=365, #smooth over short-term changes within the year
    center=True,
    min_periods=183,
).mean()


fig, ax = plt.subplots(figsize=(15,5))
avg_sales.plot(**plot_params, alpha=0.5, title="Average Sales", ax=ax)
```

```
trend.plot(linewidth=3, label='trend', color='g', ax=ax)
ax.spines['top'].set_visible(False)
ax.spines['right'].set_visible(False)
ax.spines['bottom'].set_visible(False)
ax.spines['left'].set_visible(False)
ax.grid(False)
ax.legend();
ax.set_xlabel("")
plt.savefig('avg_sales_trend.png', dpi=300)
```



Time-series based on Trends

```
# the target

y = avg_sales.copy()
y.head()
```

```
date
2013-01-01      1.409438
2013-01-02    278.390808
2013-01-03    202.840195
2013-01-04    198.911148
2013-01-05    267.873230
Freq: D, Name: sales, dtype: float32
```

and evaluate perfomance on the validation set.

Polynomial order 1

```
dp = DeterministicProcess(
    index=y.index,          # dates from the training data
    constant=True,          # dummy feature for the bias (y_intercept)
```

```
    order=1,                # the time dummy (trend): linear trend
    drop=True,              # drop terms if necessary to avoid collinearity
)

X = dp.in_sample()
X.head()
```

date	const	trend
2013-01-01	1.0	1.0
2013-01-02	1.0	2.0
2013-01-03	1.0	3.0
2013-01-04	1.0	4.0
2013-01-05	1.0	5.0

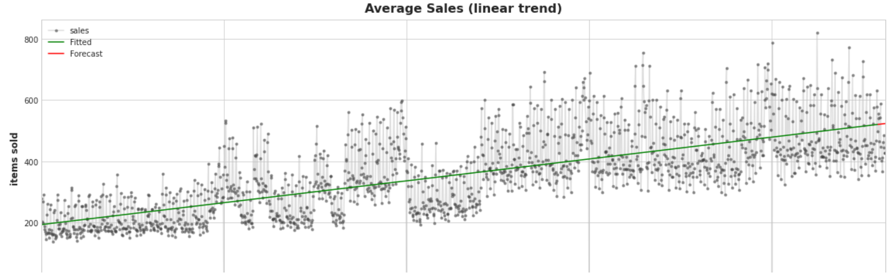
The validation set is choosen to have size 15 such as the total duration of the test set (15 days).

```
X_train, X_valid, y_train, y_valid = train_test_split(X, y, test_size=15, shuffle=False)

model = LinearRegression(fit_intercept=False).fit(X_train, y_train)
y_fit = pd.Series(model.predict(X_train), index=X_train.index).clip(0.0)
y_pred = pd.Series(model.predict(X_valid), index=X_valid.index).clip(0.0)
```

For conguence with the test set, we use the **Root Mean Squared Logarithmic Error** as the evaluation metric (described above in the problem definition) also for the validation set.

```
rmsle_train = mean_squared_log_error(y_train, y_fit) ** 0.5
rmsle_valid = mean_squared_log_error(y_valid, y_pred) ** 0.5
print(f'Training RMSLE: {rmsle_train:.5f}')
print(f'Validation RMSLE: {rmsle_valid:.5f}')

fig, ax = plt.subplots(figsize=(16,6))
ax = y.plot(**plot_params, alpha=0.5, title="Average Sales (linear trend)", ylabel="items sold")
ax = y_fit.plot(ax=ax, label="Fitted", color='g')
ax = y_pred.plot(ax=ax, label="Forecast", color='r')
ax.legend();
```

```
Training RMSLE: 0.32472
Validation RMSLE: 0.18144
```



Polynomial order 3

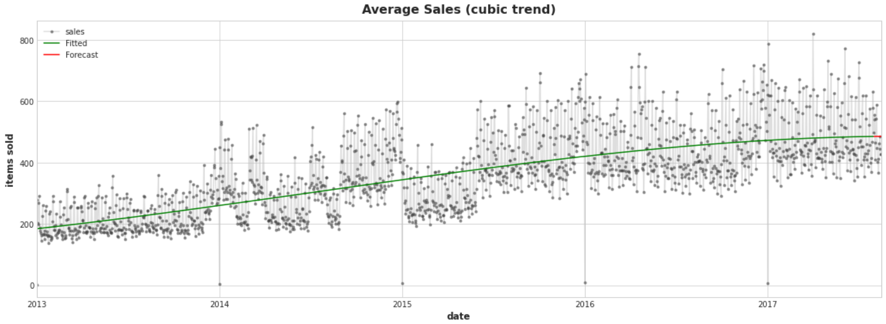
```
dp = DeterministicProcess(
    index=y.index,  # dates from the training data
    constant=True,       # dummy feature for the bias (y_intercept)
    order=3,             # the time dummy (trend): cubic trend
    drop=True,           # drop terms if necessary to avoid collinearity
)

X = dp.in_sample()
X.head()
```

	const	trend	trend_squared	trend_cubed
date				
2013-01-01	1.0	1.0	1.0	1.0
2013-01-02	1.0	2.0	4.0	8.0
2013-01-03	1.0	3.0	9.0	27.0
2013-01-04	1.0	4.0	16.0	64.0
2013-01-05	1.0	5.0	25.0	125.0

```
X_train, X_valid, y_train, y_valid = train_test_split(X, y, test_size=15, shuffle=False)

model = LinearRegression(fit_intercept=False).fit(X_train, y_train)
y_fit = pd.Series(model.predict(X_train), index=X_train.index).clip(0.0)
y_pred = pd.Series(model.predict(X_valid), index=X_valid.index).clip(0.0)


rmsle_train = mean_squared_log_error(y_train, y_fit) ** 0.5
rmsle_valid = mean_squared_log_error(y_valid, y_pred) ** 0.5
print(f'Training RMSLE: {rmsle_train:.5f}')
print(f'Validation RMSLE: {rmsle_valid:.5f}')

fig, ax = plt.subplots(figsize=(16,6))
ax = y.plot(**plot_params, alpha=0.5, title="Average Sales (cubic trend)", ylabel="items sold")
ax = y_fit.plot(ax=ax, label="Fitted", color='g')
```

```
ax = y_pred.plot(ax=ax, label="Forecast", color='r')
ax.legend();
```

```
Training RMSLE: 0.32328
Validation RMSLE: 0.14174
```



Interpretation: we see that the linear and cubic polynomial perform similarly on the training data but the cubic polynomial generalizes better on the validation set. For this reason from now on we will consider the cubic trend.

Trend for each family

Analysing the structure of the store sale data it is obvious that each family has their own trend. Let's consider the trend as combination of the trend of each family. This is possibile by redefining the target data as date and family.

```
y = store_sales.unstack(['store_nbr', 'family'])  # the target
y.head()
```

	sales							
store_nbr	1							
family	AUTOMOTIVE	BABY CARE	BEAUTY	BEVERAGES	BOOKS	BREAD/BAKERY	CELEBRATION	CLEAN

```
dp = DeterministicProcess(
    index=y.index,  # dates from the training data
    constant=True,       # dummy feature for the bias (y_intercept)
    order=3,             # the time dummy (trend): cubic trend
    drop=True,           # drop terms if necessary to avoid collinearity
)

X = dp.in_sample()
X.head()
```

date	const	trend	trend_squared	trend_cubed
2013-01-01	1.0	1.0	1.0	1.0
2013-01-02	1.0	2.0	4.0	8.0
2013-01-03	1.0	3.0	9.0	27.0
2013-01-04	1.0	4.0	16.0	64.0
2013-01-05	1.0	5.0	25.0	125.0

```
X_train, X_valid, y_train, y_valid = train_test_split(X, y, test_size=15, shuffle=False)

model = LinearRegression(fit_intercept=False).fit(X_train, y_train)

y_fit = pd.DataFrame(model.predict(X_train), index=X_train.index, columns=y_train.columns).clip(0.0)
y_pred = pd.DataFrame(model.predict(X_valid), index=X_valid.index, columns=y_valid.columns).clip(0.0)

rmsle_train = mean_squared_log_error(y_train, y_fit) ** 0.5
rmsle_valid = mean_squared_log_error(y_valid, y_pred) ** 0.5
print(f'Training RMSLE: {rmsle_train:.5f}')
print(f'Validation RMSLE: {rmsle_valid:.5f}')

y_pred.head()
```

```
Training RMSLE: 1.08942
Validation RMSLE: 0.59417
```

	sales						
store_nbr	1						
family	AUTOMOTIVE	BABY CARE	BEAUTY	BEVERAGES	BOOKS	BREAD/BAKERY	CELEBRATION
date							
2017-08-01	3.989250	0.0	3.531821	2196.628603	0.716912	343.640481	14.867376
2017-08-02	3.987273	0.0	3.534058	2196.550189	0.718472	343.374526	14.857873

➜ Forecasting based on target (first submission)

2017-08-	3.983288	0.0	3.538537	2196.385892	0.721597	342.840136	14.838750

```
# Create features for test set
X_test = dp.out_of_sample(steps=16)
X_test.index.name = 'date'

X_test.head()
```

	const	trend	trend_squared	trend_cubed
date				
2017-08-16	1.0	1685.0	2839225.0	4.784094e+09
2017-08-17	1.0	1686.0	2842596.0	4.792617e+09
2017-08-18	1.0	1687.0	2845969.0	4.801150e+09
2017-08-19	1.0	1688.0	2849344.0	4.809693e+09
2017-08-20	1.0	1689.0	2852721.0	4.818246e+09

```
y_submit = pd.DataFrame(model.predict(X_test), index=X_test.index, columns=y.columns)
y_submit = y_submit.stack(['store_nbr', 'family'])
y_submit = y_submit.join(df_test.id).reindex(columns=['id', 'sales'])
y_submit.to_csv('submission_trend.csv', index=False)

y_submit.head()
```

			id	sales
date	store_nbr	family		

> Kaggle Score: 0.63608

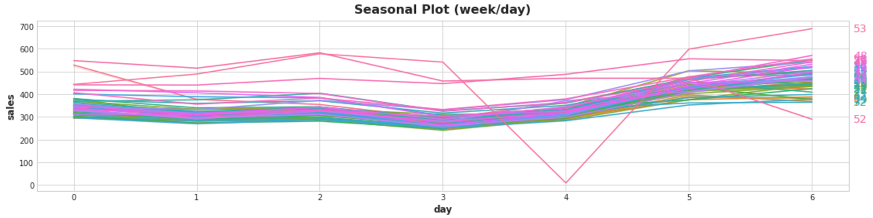
BEAUTY 3000890 3.565596

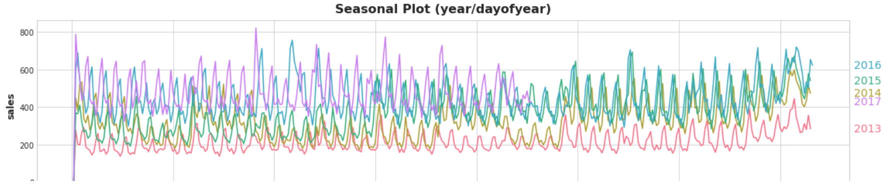
4.2 Seasonality

While trends represent long-term changes in the mean of the series, [seasonality](#) represents **regular, periodic changes in the mean of the series** caused by weekly, monthly, seasonal or yearly patterns of social behaviour. In this context a season can mean a week, month, year or an actual 'season' (e.g., Vivaldi's "The Four Seasons"). However, depending on the number of observations in a season, we might use **two distinct features** to model seasonality:

- [Seasonal indicators](#): For a season with few observations (eg, a weekly season of daily observations) seasonal differences in the level of the time series (eg, difference between daily observations in a week) can be represented through binary features, or more specifically, one-hot-encoded categorical features. These features are called **seasonal indicators** and can be represented through seasonal plots.
- [Fourier features](#): Seasonal indicators create a feature for every unit of the period of the season. Hence, they have the tendency to blow up for long seasons, e.g., daily observations over a year. For such cases, we use **Fourier features**, pairs of sine and cosine curves, one pair for each potential frequency in the season starting with the longest, to capture the overall shape of the seasonal curve with just a few features. We can choose these features using a **periodogram** which tells us the strength of the frequencies in a time series.

```
X = avg_sales.to_frame()

# days within a week
X["day"] = X.index.dayofweek  # the x-axis (freq)
X["week"] = X.index.week  # the seasonal period (period)

# days within a year
X["dayofyear"] = X.index.dayofyear
X["year"] = X.index.year
fig, (ax0, ax1) = plt.subplots(2, 1, figsize=(16, 8))
seasonal_plot(X, y="sales", period="week", freq="day", ax=ax0)
seasonal_plot(X, y="sales", period="year", freq="dayofyear", ax=ax1);
plt.savefig('seasonal plot all.png', dpi=300)
```





Interpretation: The two plot show an evident increase of sales during week-ends.

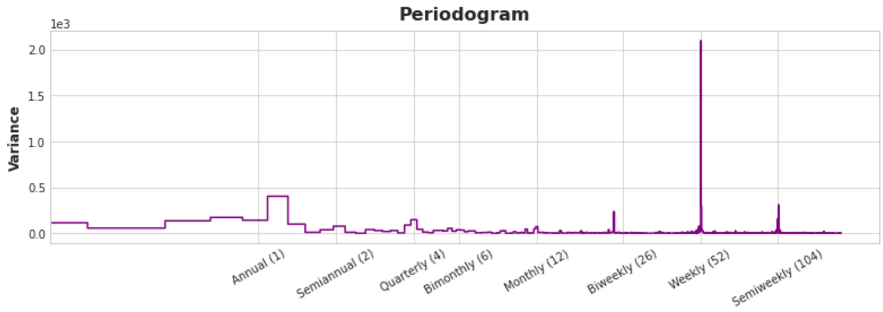
More specifically

- The *weekly seasonal plot* shows for each weeks within a year (1-53) the number of sales in days from Monday (0) to Sunday (6). On week 53, the last of the year, on Friday (4) there are very few sales. This is jusified by the fact that most years have 52 week, in the period of the traning data from 2013 to 2017 there is only one 53th week. On the graph, that friday corresponds to new year's eve, which usually has few sales, and the line representing the 53th week show exactly the sales of that week from 2015-12-28 to 2016-01-03. This analysis shows that it is important to consider the holidays in the modelling process.
- In fact in the *yearly seasonal plot* it is shown that sales are low on every 1st of Januarry. And also that the sales changes with a similar frequency, but they don't match with the exact day of year. It is worth noticing that the end of the year there is an increase of sales, probabily due to holidays.

```
# Show the sales of the 53th week
X[X['week']==53]
```

date	sales	day	week	dayofyear	year
2015-12-28	443.145294	0	53	362	2015
2015-12-29	488.643127	1	53	363	2015
2015-12-30	578.026794	2	53	364	2015
2015-12-31	541.325195	3	53	365	2015
2016-01-01	9.221882	4	53	1	2016
2016-01-02	598.584412	5	53	2	2016
2016-01-03	688.403870	6	53	3	2016

```
plot_periodogram(avg_sales);
```



Interpretation: as expected, there is a strong weekly seasonality. The periodogram shows also a week annually seasonality, to be modelled with Fourier features.

> **Choosing Fourier Features:** How many Fourier pairs should we actually include in our feature set? From right to left, the periodogram falls off between Bimonthly (6) and Monthly (12), so let's use 10 Fourier pairs.

Adding Seasonal features:

While creating the cubic trend, let's add the weekly seasonality (seasonal indicators) by setting seasonal parameter to True and adding the annual seasonality by adding Fourier features as additional terms.

```
y = store_sales.unstack(['store_nbr', 'family'])#.loc["2017"]

fourier = CalendarFourier(freq='A', order=10) ## 2 pairs of sine/cosine curves to model monthly/biweekly seasonality
dp = DeterministicProcess(
    index=y.index,
    constant=True,               # dummy feature for bias (y-intercept)
    order=1,                     # linear trend
    seasonal=True,               # weekly seasonality (indicators)
    additional_terms=[fourier],  # annual seasonality (fourier)
    drop=True,                   # drop terms to avoid collinearity
)

X = dp.in_sample()


X_train, X_valid, y_train, y_valid = train_test_split(X, y, test_size=15, shuffle=False)

model = LinearRegression(fit_intercept=False).fit(X_train, y_train)

y_fit = pd.DataFrame(model.predict(X_train), index=X_train.index, columns=y_train.columns).clip(0.0)
y_pred = pd.DataFrame(model.predict(X_valid), index=X_valid.index, columns=y_valid.columns).clip(0.0)

rmsle_train = mean_squared_log_error(y_train, y_fit) ** 0.5
rmsle_valid = mean_squared_log_error(y_valid, y_pred) ** 0.5
print(f'Training RMSLE: {rmsle_train:.5f}')
print(f'Validation RMSLE: {rmsle_valid:.5f}')
```

```
y_pred.head()
```

```
Training RMSLE: 1.16594
Validation RMSLE: 0.58326
```

	sales						
store_nbr	1						
family	AUTOMOTIVE	BABY CARE	BEAUTY	BEVERAGES	BOOKS	BREAD/BAKERY	CELEBRATION
date							
2017-08-01	4.953936	0.0	3.306737	2246.974965	0.373197	369.529873	15.482020
2017-08-02	4.882141	0.0	3.254167	2225.239544	0.345177	386.690007	16.517681
2017-08-03	5.170742	0.0	3.314313	2263.517438	0.308487	403.668162	24.454581
2017-08-04	5.383187	0.0	3.449976	2351.753812	0.288438	440.781395	19.391537
2017-08-05	5.079181	0.0	3.372870	2372.711992	0.289279	431.397997	19.581998

5 rows × 1782 columns

➜ Forecasting based on Trends and Seasonality (second submission)

```
# Create features for test set
X_test = dp.out_of_sample(steps=16)
X_test.index.name = 'date'

X_test.head()
```

	const	trend	s(2,7)	s(3,7)	s(4,7)	s(5,7)	s(6,7)	s(7,7)	sin(1,freq=A-DEC)	cos
date										

```
y_pred = pd.DataFrame(model.predict(X_test), index=X_test.index, columns=y.columns)
y_submit = y_pred.stack(['store_nbr', 'family'])
y_submit = y_submit.join(df_test.id).reindex(columns=['id', 'sales'])
y_submit.to_csv('submission_trend_seasonality.csv', index=False)

y_submit.head()
```

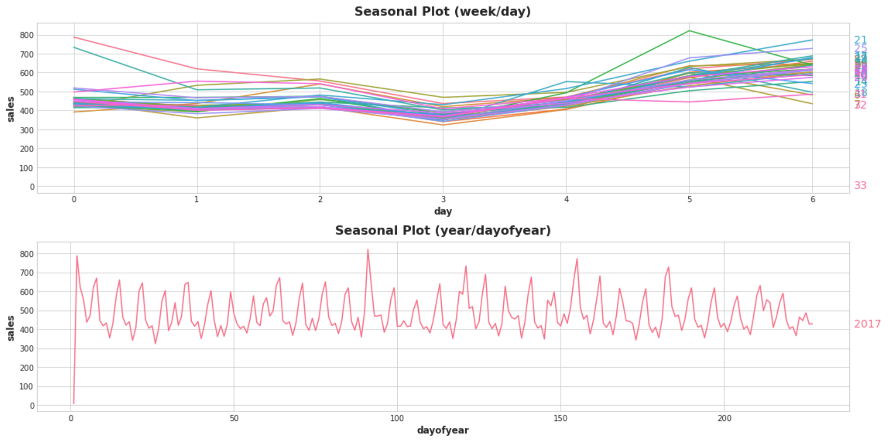
date	store_nbr	family	id	sales
2017-08-16	1	AUTOMOTIVE	3000888	4.299846
		BABY CARE	3000889	0.000000
		BEAUTY	3000890	3.182246
		BEVERAGES	3000891	2159.074062
		BOOKS	3000892	0.328001

Kaggle Score: 0.62179

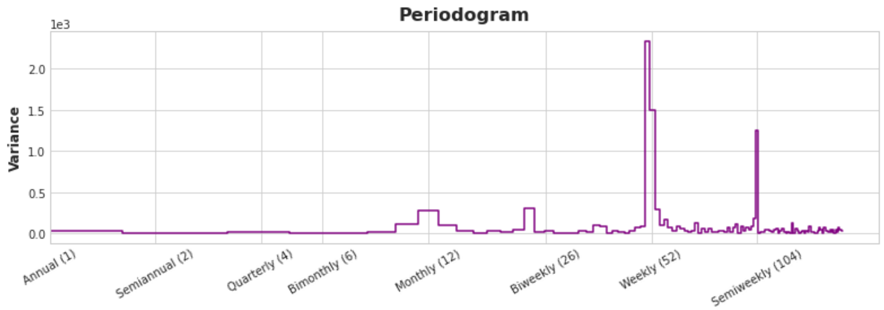
Restricting the period of store sales (train data)

Intuition from the above plots and results: Since the period of the training data is 4.5 years while the period of the test data is 15 days, we can restrict the period of the training data to be considered to make the trend of the past year (2017) generalize to the validation/test data.

```
X = avg_sales.loc['2017'].to_frame()

# days within a week
X["day"] = X.index.dayofweek  # the x-axis (freq)
X["week"] = X.index.week  # the seasonal period (period)

# days within a year
X["dayofyear"] = X.index.dayofyear
X["year"] = X.index.year
fig, (ax0, ax1) = plt.subplots(2, 1, figsize=(16, 8))
seasonal_plot(X, y="sales", period="week", freq="day", ax=ax0)
seasonal_plot(X, y="sales", period="year", freq="dayofyear", ax=ax1);

plt.savefig('seasonal plots 2017.png', dpi=300)
```




```
plot_periodogram(avg_sales.loc['2017']);
```



> Interpretation: now the plot of yearly seasonality and periodogram are clearer since we isolated the 2017 line. From the periodogram we conclude that Furier Features to model monthly/biweekly seasonality by using 2 pair of sine/cosine.

```
y = store_sales.unstack(['store_nbr', 'family']).loc["2017"]

fourier_M = CalendarFourier(freq='M', order=4) ## 2 pairs of sine/cosine curves to model monthly/biweekly seasonality
dp = DeterministicProcess(
```

```
    index=y.index,
    constant=True,               # dummy feature for bias (y-intercept)
    order=1,                     # linear trend
    seasonal=True,               # weekly seasonality (indicators)
    additional_terms=[fourier_M],  # annual seasonality (fourier)
    drop=True,                   # drop terms to avoid collinearity
)

X = dp.in_sample()

X_train, X_valid, y_train, y_valid = train_test_split(X, y, test_size=15, shuffle=False)

model = LinearRegression(fit_intercept=False).fit(X_train, y_train)

y_fit = pd.DataFrame(model.predict(X_train), index=X_train.index, columns=y_train.columns).clip(0.0)
y_pred = pd.DataFrame(model.predict(X_valid), index=X_valid.index, columns=y_valid.columns).clip(0.0)

rmsle_train = mean_squared_log_error(y_train, y_fit) ** 0.5
rmsle_valid = mean_squared_log_error(y_valid, y_pred) ** 0.5
print(f'Training RMSLE: {rmsle_train:.5f}')
print(f'Validation RMSLE: {rmsle_valid:.5f}')

y_pred.head()
```

```
Training RMSLE: 0.62390
Validation RMSLE: 0.57367
```

	sales						
store_nbr	1						
family	AUTOMOTIVE	BABY CARE	BEAUTY	BEVERAGES	BOOKS	BREAD/BAKERY	CELEBRATION
date							
2017-08-01	5.024247	0.0	3.961651	2244.459247	0.386078	328.939471	16.708611
2017-08-02	5.650650	0.0	4.007962	2516.636447	0.532333	414.733731	18.544272
2017-08-03	5.837047	0.0	3.110995	2238.215966	0.237725	372.601868	22.473153
2017-08-04	7.115996	0.0	2.883839	2529.708792	0.257185	390.066055	23.641936
2017-08-05	5.820920	0.0	3.745052	2684.328165	0.022511	414.932839	15.459501

5 rows × 1782 columns

➜ Forecasting based on Trends and Seasonality with period restriction of 2017 (third submission)

```
# Create features for test set
X_test = dp.out_of_sample(steps=16)
X_test.index.name = 'date'
y_pred = pd.DataFrame(model.predict(X_test), index=X_test.index, columns=y.columns)
y_submit = y_pred.stack(['store_nbr', 'family'])
y_submit = y_submit.join(df_test.id).reindex(columns=['id', 'sales'])
y_submit.to_csv('submission_trend_seasonality_2017.csv', index=False)
```

Kaggle Score: 0.58780

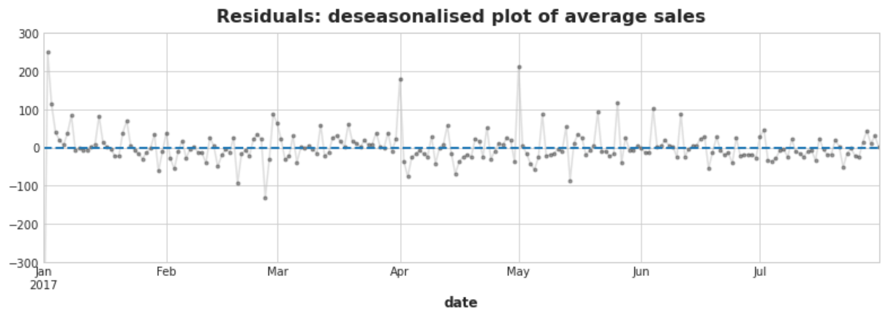
From now on we will consider time-series of 2017 as training set, since it gives better score: meaning that the choice to remove the past year for forecasting task is great. Especially forecasting in the same year of 2017 as our goal.

4.3 Residual Analysis: Deseasonalising

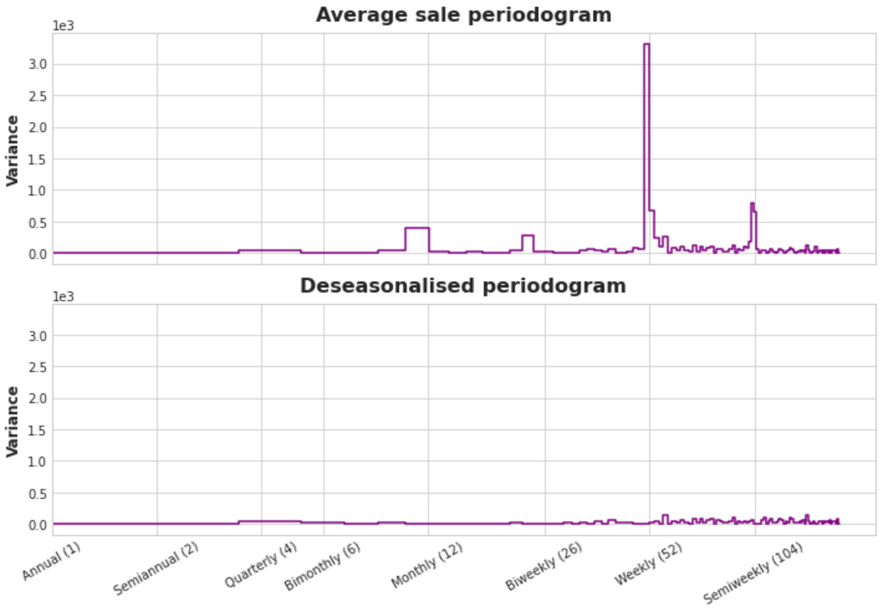
At this point, Residual Analysis is made to check the effectiveness of trend and seasonality modelling, but also to check what to model next: serial dependent features. A residual is what is left behind when we subtract from the data, predictions of our model. At the end of the analysis we are left with just random fluctuations (errors), if we had modelled time and serial dependence properly.

Below we will plot a visual representation of the residual, considering the average sale of the training set. Naturally we could easily visualize deseasonalised residuals of families of products and other features as well.

```
y_train_avg = y_train.stack(['store_nbr', 'family']).groupby('date').mean()['sales']
y_fit_avg = y_fit.stack(['store_nbr', 'family']).groupby('date').mean()['sales']
y_deseason_avg = y_train_avg - y_fit_avg

ax = y_deseason_avg.plot(**plot_params, alpha=0.5, ylim=[-300,300], title='Residuals: deseasonalised plot of average sales')
ax.axhline(y=0, ls='dashed', lw=2)

plt.savefig('deseasonalised average sales.png', dpi=300)
```



```
fig, (ax1, ax2) = plt.subplots(2, 1, sharex=True, sharey=True, figsize=(10, 7))
ax1 = plot_periodogram(y_train_avg, ax=ax1)
ax1.set_title("Average sale periodogram")
ax2 = plot_periodogram(y_deseason_avg, ax=ax2);
ax2.set_title("Deseasonalised periodogram");

plt.savefig('Deseasonlised Periodogram.png', dpi=300)
```



Interpretation: on the deseasonalised periodogram, weekly seasonality is lower, even thought it is not absent, probability due to its strong presence on the time series. Also, Bimonthly and Monthly seasonality are reduced to be almost absent.

4.4 Holidays Features

As anticipated, holidays need to be considered as a time dependent feature. This is what the dataset looks like:

```
holidays_events
```

	type	locale	locale_name	description	transferred
date					
2012-03-02	Holiday	Local	Manta	Fundacion de Manta	False
2012-04-01	Holiday	Regional	Cotopaxi	Provincializacion de Cotopaxi	False
2012-04-12	Holiday	Local	Cuenca	Fundacion de Cuenca	False
2012-04-14	Holiday	Local	Libertad	Cantonizacion de Libertad	False
2012-04-21	Holiday	Local	Riobamba	Cantonizacion de Riobamba	False
...	...	...	...	...	...
2017-12-22	Additional	National	Ecuador	Navidad-3	False

From the whole dataset we consider only:

- national and regional holidays, ignoring local holidays (assuming a local holiday will have minimum impact on the average national sales)
- holidays in 2017 that fall within our training (2017-01-01 : 2017-07-31) + validation (2017-08-01 : 2017-08-15) + test set (2017-08-16 : 2017-08-31)

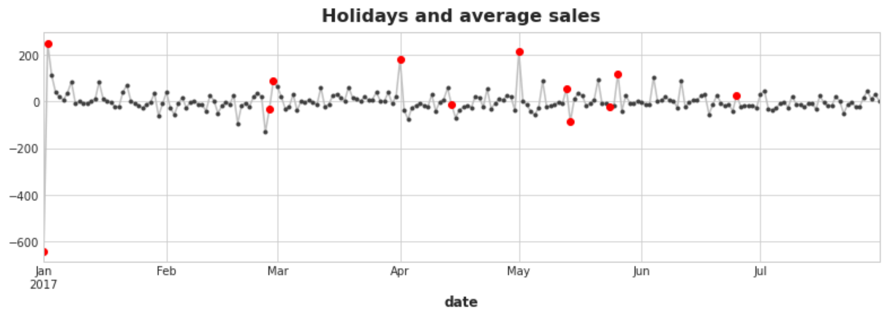
=> 14 holidays considered

```
holidays = (
    holidays_events
    .query("locale in ['National', 'Regional']")
    .loc['2017':'2017-08-31', ['description']] # restricting to the dates in the training set
    .assign(description=lambda x: x.description.cat.remove_unused_categories()) # remove categories which are not used
)

holidays
```

	description
date	
2017-01-01	Primer dia del ano

Holidays on deseasonalised plot of average sales

```
ax = y_deseason_avg.plot(**plot_params)
plt.plot_date(holidays.index[:-2], y_deseason_avg[holidays.index[:-2]], color='r') # the [:-2] is to remove the last 2 dates, 2017-08-10 and 2017-08-11 because they are in the validation set
ax.set_title('Holidays and average sales');
plt.savefig('Holidays and average sales.png', dpi=300)
```



Creating holiday features as seasonal indicators

These features modelling Holidays are created through one-hot-encoding of categorical feature. Then they are joint to the previously modelled feature, and the result is all the time dependent features (X_time).

```
X_holidays = pd.get_dummies(holidays)
X_holidays.head()
```

description_Batalla de Pichincha	description_Carnaval	description_Dia de la Madre	description_Dia de la Madre-1	description_Dia del Trabajo	description_Primer Grito de Independencia	description_Primer dia del ano	description_Provincializacion de Cotopaxi	description_Provincializ de Imb

```
X_time = X.join(X_holidays, on='date').fillna(0.0)
X_time_test = X_test.join(X_holidays, on='date').fillna(0.0)

X_time.head()
```

date	const	trend	s(2,7)	s(3,7)	s(4,7)	s(5,7)	s(6,7)	s(7,7)	sin(1,freq=M)	cos(
2017-01-01	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.000000	
2017-01-02	1.0	2.0	1.0	0.0	0.0	0.0	0.0	0.0	0.201299	
2017-01-03	1.0	3.0	0.0	1.0	0.0	0.0	0.0	0.0	0.394356	
2017-01-04	1.0	4.0	0.0	0.0	1.0	0.0	0.0	0.0	0.571268	
2017-01-05	1.0	5.0	0.0	0.0	0.0	1.0	0.0	0.0	0.724793	

5 rows × 29 columns

```
X_train, X_valid, y_train, y_valid = train_test_split(X_time, y, test_size=15, shuffle=False)

model = LinearRegression(fit_intercept=False).fit(X_train, y_train)

y_fit = pd.DataFrame(model.predict(X_train), index=X_train.index, columns=y_train.columns).clip(0.0)
y_pred = pd.DataFrame(model.predict(X_valid), index=X_valid.index, columns=y_valid.columns).clip(0.0)

rmsle_train = mean_squared_log_error(y_train, y_fit) ** 0.5
rmsle_valid = mean_squared_log_error(y_valid, y_pred) ** 0.5
print(f'Training RMSLE: {rmsle_train:.5f}')
print(f'Validation RMSLE: {rmsle_valid:.5f}')
```

```
Training RMSLE: 0.51651
Validation RMSLE: 0.56997
```

➜ Forecasting based on Trends, Seasonality and Holidays of 2017 (forth submission)

```
# Create features for test set
X_test = dp.out_of_sample(steps=16)
X_test.index.name = 'date'

y_submit = pd.DataFrame(model.predict(X_time_test), index=X_time_test.index, columns=y.columns)
y_submit = y_submit.stack(['store_nbr', 'family'])
```

```
y_submit = y_submit.join(df_test.id).reindex(columns=['id', 'sales'])
y_submit.to_csv('submission_trend_seasonality_holidays_2017.csv', index=False)
y_submit.head()
```

			id	sales
date	store_nbr	family		
2017-08-16	1	AUTOMOTIVE	3000888	4.434636
		BABY CARE	3000889	0.000000
		BEAUTY	3000890	3.425094
		BEVERAGES	3000891	2436.376406
		BOOKS	3000892	0.531519

> Kaggle Score: 0.58129

5. Modelling Serial Dependency

Serial dependence are behaviours in a time series that are *time-independent*, i.e., they have less to do with a particular time of occurance, but more to do with what happened in the **recent past**; thus bringing a sense of irregularity to the events. An example of serial dependent properties are cycles.

Types of serial dependency:

- **Linear**: where past and present observations are linearly related. Such linear serial dependence can be explored through lag series/plots where **the lag features are chosen by calculating (partial) autocorrelation**. They can also be anticipated through leading indicators, like online trends or promotions.
- **Non-linear**: where past and present observations can not be related by a simple linear relationship, hence **we can't calculate lag features through (partial) autocorrelations.** Non-linear relationships like these can either be transformed to be linear or else learned by an appropriate algorithm (e.g. XGBoost).

5.1 Modelling Serial Dependecy on School and Office Supply

If we look deeper into the train dataset, we can see that there is a clear evidence that in the family of School and Office Supplies shows a cyclic behaviour in 2017.

So, let's reload the dataset to include other information for serial dependency modelling. From the reloaded dataset that now includes "onpromotion" features, let's select the family School and Office Supplies.

Loading Supply Sales

```
store_sales = pd.read_csv(
    input_dir / 'train.csv',
    usecols=['store_nbr', 'family', 'date', 'sales', 'onpromotion'],
```

```
    dtype={
        'store_nbr': 'category',
        'family': 'category',
        'sales': 'float32',
        'onpromotion': 'uint32', ## NEW FEATURE: To be introduced later
    },
    parse_dates=['date'],
    infer_datetime_format=True,
)
store_sales['date'] = store_sales.date.dt.to_period('D')
store_sales = store_sales.set_index(['store_nbr', 'family', 'date']).sort_index()

family_sales = (
    store_sales
    .groupby(['family', 'date'])
    .mean()
    .unstack('family')
    .loc['2017']
)

supply_sales = family_sales.loc(axis=1)[:, 'SCHOOL AND OFFICE SUPPLIES']
y_supply_sales = supply_sales.loc[:, 'sales'].squeeze()
y_supply_sales
```

```
date
2017-01-01     0.000000
2017-01-02     2.925926
2017-01-03     2.018518
2017-01-04     1.722222
2017-01-05     1.425926
                 ...
2017-08-11    65.240738
2017-08-12    67.481483
2017-08-13    68.851852
2017-08-14    52.333332
2017-08-15    46.851852
Freq: D, Name: SCHOOL AND OFFICE SUPPLIES, Length: 227, dtype: float32
```

Deseasonalised Sales of "School and Office Supplies"

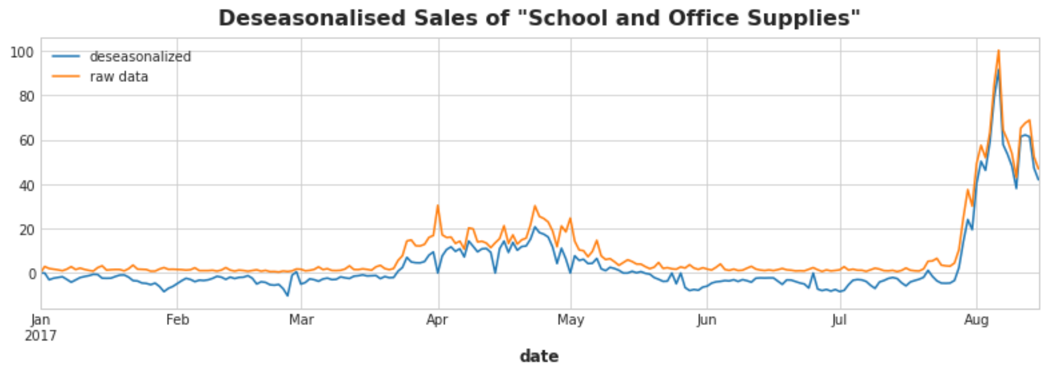
Obtained by residuals (the considered and complete residual includes both train and valid data) and plotted as such.

```
y_resid_train = y_train - y_fit
y_resid_valid = y_valid - y_pred

y_resid = pd.concat([y_resid_train, y_resid_valid])

y_resid_supply_sales = y_resid.stack(['store_nbr', 'family']).groupby(['family', 'date']).mean() .unstack('family').loc(axis=1)[:, 'SCHOOL AND OFFICE SUPPLIES'].loc[:, 'sales'].squeeze().rename('sales_
y_resid_supply_sales
```

```
date
2017-01-01   -9.436896e-15
2017-01-02   -1.257173e-14
2017-01-03   -3.050247e+00
2017-01-04   -2.342834e+00
2017-01-05   -1.975145e+00
```

```
                    ...
2017-08-11    6.142390e+01
2017-08-12    6.211941e+01
2017-08-13    6.135346e+01
2017-08-14    4.721377e+01
2017-08-15    4.197002e+01
Freq: D, Name: sales_deseason, Length: 227, dtype: float64
```

```
ax = y_resid_supply_sales.plot(label='deseasonalized')
y_supply_sales.plot(ax=ax, label='raw data')
ax.set_title("Deseasonalised Sales of \"School and Office Supplies\"");
ax.legend()
plt.savefig('Deseasonalised Supply Sales.png', dpi=300)
```

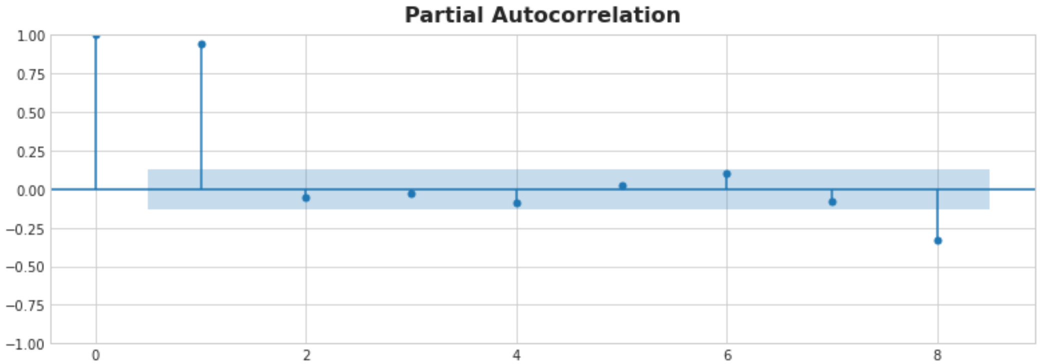


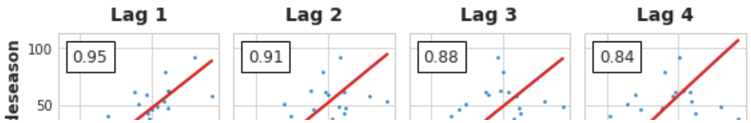
> Interpretation: the two curves are close to each other, showing a predominant cyclic behaviour of the raw data of supply sales (target) and the need to model a serial dependence on the residual curve.

(Partial) Autocorrelation plots

Usefull tool to identify the number of lag series and to determine the right one(s) to be a lag feature(s).

```
plot_pacf(y_resid_supply_sales, lags=8);
plot_lags(y_resid_supply_sales, lags=8, nrows=2);
```





> Interpretation: From the partial autocorrelation plot and lag plots it looks like the relevant lag to consider is Lag 1.

Lag Features

From the partial autocorrelation plot (correlogram) it is clear that we need only Lag 1 feature, created below.

```
X_lags = make_lags(y_resid_supply_sales, lags=1)
X_supply_sales = pd.concat([X_time, X_lags], axis=1).dropna()
print(f"Total features in our combined feature set: {len(X_supply_sales.columns)}")
```

```
Total features in our combined feature set: 30
```

```
X_lags
```

date	y_lag_1
2017-01-01	NaN

Below it is visualize the Lag series placed side by side with the corresponding target residual series.

```
pd.concat([y_resid_supply_sales,X_lags], axis=1)
```

date	sales_deseason	y_lag_1
2017-01-01	-9.436896e-15	NaN
2017-01-02	-1.257173e-14	-9.436896e-15
2017-01-03	-3.050247e+00	-1.257173e-14
2017-01-04	-2.342834e+00	-3.050247e+00
2017-01-05	-1.975145e+00	-2.342834e+00
...	...	...
2017-08-11	6.142390e+01	3.804515e+01
2017-08-12	6.211941e+01	6.142390e+01
2017-08-13	6.135346e+01	6.211941e+01
2017-08-14	4.721377e+01	6.135346e+01
2017-08-15	4.197002e+01	4.721377e+01

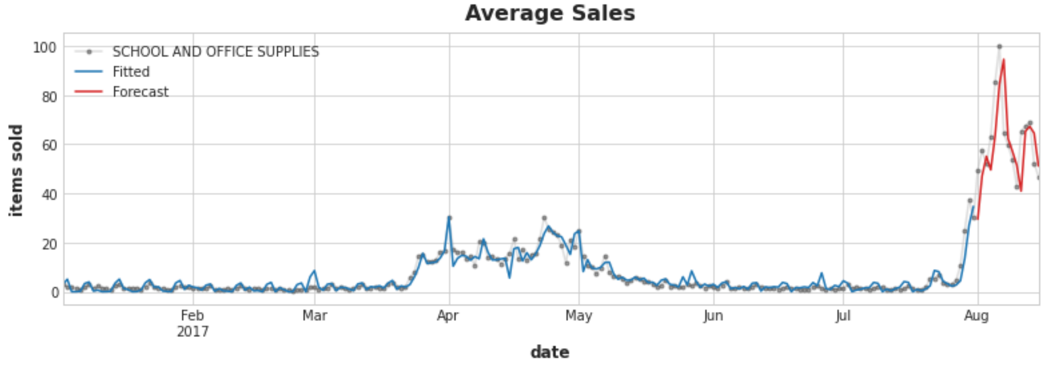
227 rows × 2 columns

Training and forecasting on Supply Sales

```
# Target considering Lag 1 feature and time dependence features
y_supply_sales, X_supply_sales = y_supply_sales.align(X_supply_sales, join='inner')


X_train_supply_sales, X_valid_supply_sales, y_train_supply_sales, y_valid_supply_sales = train_test_split(X_supply_sales, y_supply_sales, test_size=15, shuffle=False)

model = LinearRegression(fit_intercept=False).fit(X_train_supply_sales, y_train_supply_sales)
y_fit_supply_sales = pd.Series(model.predict(X_train_supply_sales), index=X_train_supply_sales.index).clip(0.0)
y_pred_supply_sales = pd.Series(model.predict(X_valid_supply_sales), index=X_valid_supply_sales.index).clip(0.0)

rmsle_train = mean_squared_log_error(y_train_supply_sales, y_fit_supply_sales) ** 0.5
rmsle_valid = mean_squared_log_error(y_valid_supply_sales, y_pred_supply_sales) ** 0.5
print(f'Training RMSLE: {rmsle_train:.5f}')
print(f'Validation RMSLE: {rmsle_valid:.5f}')

ax = y_supply_sales.plot(**plot_params, alpha=0.5, title="Average Sales", ylabel="items sold")
ax = y_fit_supply_sales.plot(ax=ax, label="Fitted", color='C0')
```

```
ax = y_pred_supply_sales.plot(ax=ax, label="Forecast", color='C3')
ax.legend();
```

```
Training RMSLE: 0.43198
Validation RMSLE: 0.24564
```



5.2 Leading Indicator Feature

A leading indicator provides "early notice" of changes in the target.

From our dataset we have identified an important leading indicator: **onpromotion**, which is a Series indicating whether a sale for a product (belonging to a family of product) was in promotion.

```
# onpromotion feature as a boolean variable
X_serial = store_sales.unstack(['store_nbr', 'family']).loc["2017"].loc[:, 'onpromotion']
```

```
X_serial.head()
```

store_nbr	1										...	9								
family	AUTOMOTIVE	BABY CARE	BEAUTY	BEVERAGES	BOOKS	BREAD/BAKERY	CELEBRATION	CLEANING	DAIRY	DELI	...	MAGAZINES	MEATS	PERSONAL CARE	PET SUPPLIES	PLAYERS AND ELECTRONICS	POULTRY	PREPARED FOODS	PRODUCE	SCHOOL AND OFFICE SUPPLIES
date																				
2017-01-01	0	0	0	0	0	0	0	0	0	0	...	0	0	0	0	0	0	0	0	0
2017-01-02	0	0	0	31	0	2	0	7	11	3	...	0	0	13	0	0	2	1	4	0
2017-01-03	0	0	1	42	0	2	0	18	14	5	...	0	0	11	0	0	1	2	150	0
2017-01-04	0	0	1	54	0	8	1	15	32	5	...	0	0	15	0	0	1	8	9	0
2017-01-05	0	0	2	32	0	7	0	10	24	1	...	0	21	8	0	0	1	1	5	0

X_serial stands for features from serial dependence modelling, used in the next step to train the non-linear part of the Hybrid model.

6. Hybrid Model

Linear regression excels at extrapolating trends, but can't learn interactions. XGBoost excels at learning interactions, but can't extrapolate trends. Let's create a "hybrid" forecasters that combine complementary learning algorithms with the following schema:

```
# 1. Train and predict with first model
model_1.fit(X_train_1, y_train)
y_pred_1 = model_1.predict(X_train)

# 2. Train and predict with second model on residuals
model_2.fit(X_train_2, y_train - y_pred_1)
y_pred_2 = model_2.predict(X_train_2)

# 3. Add to get overall predictions
y_pred = y_pred_1 + y_pred_2
```

Note: the number 1 in a variable name is refered to the part of modelling time dependency, while the number 2 is referred to modelling serial dependency.

6.1 BoostedHybrid Class

It is just an auxiliary class abstracting the workflow described above. We can pass different kind of models: I choose LinearRegression as model_1, and XGBRegressor as model_2.

```
class BoostedHybrid:

    def __init__(self, model_1, model_2):
        self.model_1 = model_1
        self.model_2 = model_2
        self.y_columns = None  # store column names from fit method

def fit(self, X_1, X_2, y):
    # Train model_1
    self.model_1.fit(X_1, y)

    # Make predictions
    y_fit = pd.DataFrame(
        self.model_1.predict(X_1),
        index=X_1.index, columns=y.columns,
    )

    # Compute residuals
    y_resid = y - y_fit
```

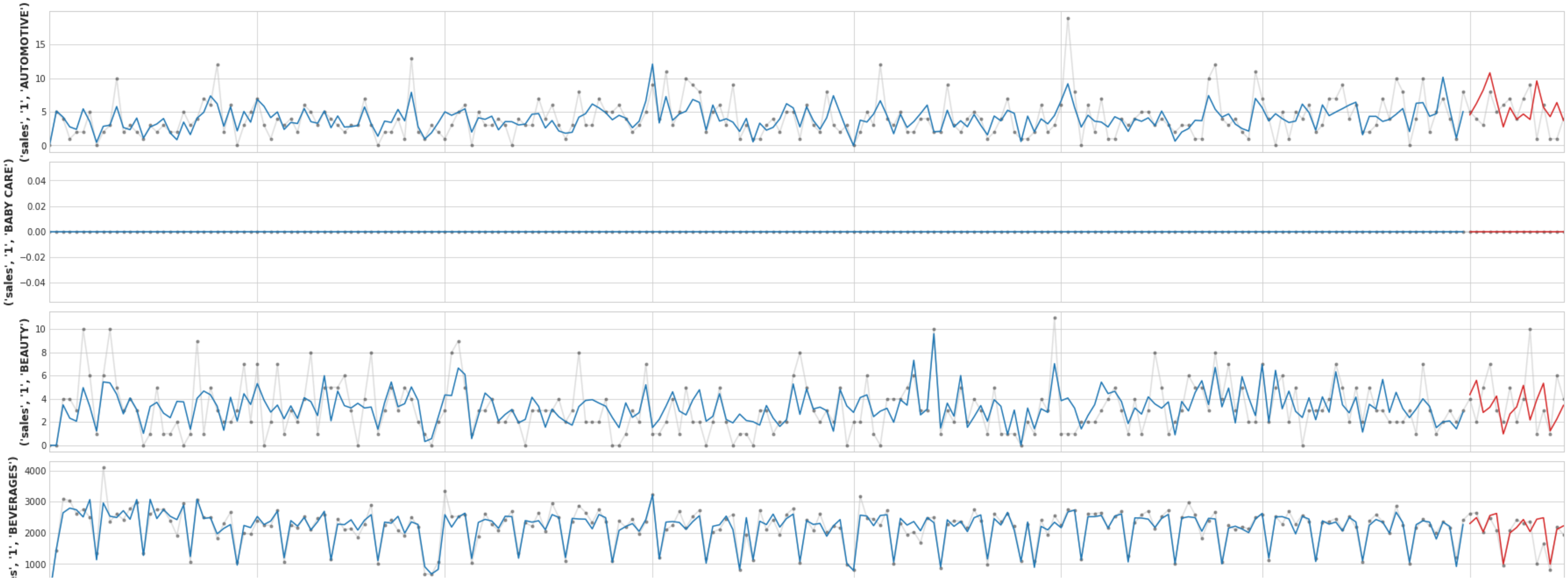
```
    #y_resid = y_resid.stack().squeeze()

    # Train model_2 on residuals
    self.model_2.fit(X_2, y_resid)


    # Save column names for predict method
    self.y_columns = y.columns
    # Save data for question checking
    self.y_fit = y_fit
    self.y_resid = y_resid


# Add method to class
BoostedHybrid.fit = fit

def predict(self, X_1, X_2):
    # Predict with model_1
    y_pred = pd.DataFrame(
        self.model_1.predict(X_1),
        index=X_1.index, columns=self.y_columns,
    )
    # Add model_2 predictions to model_1 predictions
    y_pred += self.model_2.predict(X_2)
    return y_pred


# Add method to class
BoostedHybrid.predict = predict

# X_1: Time-dependence features

X_1 = X_time # training set time-features
X_1_test = X_time_test # test set time-features

# Splitting between training and validation sets

X_1_train, X_1_valid, y_train, y_valid = train_test_split(X_1, y, test_size=15, shuffle=False)


# X_2: Features for serial dependence
# onpromotion feature as a boolean variable
X_2 = store_sales.unstack(['store_nbr', 'family']).loc["2017"].loc[:, 'onpromotion']

# Label encoding for seasonality
X_2["day"] = X_2.index.day  # values are day of the month

# Splitting between training and validation sets
X_2_train, X_2_valid, y_train, y_valid = train_test_split(X_2, y, test_size=15, shuffle=False)


mod_1 = LinearRegression(fit_intercept=False) # for time-dependence
mod_2 = KNeighborsRegressor() # for serial-dependence

model = BoostedHybrid(model_1=mod_1, model_2=mod_2)

model.fit(X_1_train, X_2_train, y_train)
```

```
y_fit = model.predict(X_1_train, X_2_train).clip(0.0)
y_pred = model.predict(X_1_valid, X_2_valid).clip(0.0)

rmsle_train = mean_squared_log_error(y_train, y_fit) ** 0.5
rmsle_valid = mean_squared_log_error(y_valid, y_pred) ** 0.5
print(f'Training RMSLE: {rmsle_train:.5f}')
print(f'Validation RMSLE: {rmsle_valid:.5f}')
```

```
Training RMSLE: 0.45829
Validation RMSLE: 0.62112
```

➜ Forecasting with BoostedHybrid model based on linear time-dependent and non-linear serial-dependence (fifth submission)

```
families = y.columns[0:6]
axs = y.loc(axis=1)[families].plot(
    subplots=True, sharex=True, figsize=(25, 15), **plot_params, alpha=0.5,
)
_ = y_fit.loc(axis=1)[families].plot(subplots=True, sharex=True, color='C0', ax=axs)
_ = y_pred.loc(axis=1)[families].plot(subplots=True, sharex=True, color='C3', ax=axs)
for ax, family in zip(axs, families):
    ax.legend([])
    ax.set_ylabel(family)

plt.savefig('forecasting_hybrid.png', dpi=300)
```



Interpretation: In some product family the model is fitting and predicting well (e.g. BABY CARE, BREAD/BAKERY, BEVERAGES). In others it does not perform as well as for other family of products. This might suggest that it is a good idea to implement more models, one for each product family and train them with their product family sales data, which means identifing different features for each model/family. (e.g. as we did with School and Office Supplies)

```
# X_1_test: Time-dependence features: test set

# X_2_test: Serial-dependence features: test set
X_2_test = df_test.unstack(['store_nbr', 'family']).loc["2017"].loc[:, 'onpromotion']

# Label encoding for seasonality
X_2_test["day"] = X_2_test.index.day  # values are day of the month

# making submission predictions
y_submit = model.predict(X_1_test, X_2_test).clip(0.0)
y_submit = pd.DataFrame(y_submit.stack(['store_nbr', 'family']))#.rename('sales'))
y_submit = y_submit.join(df_test.id).reindex(columns=['id', 'sales'])
y_submit.to_csv('submission_hybrid.csv', index=False)
y_submit
```

			id	sales
date	store_nbr	family		
2017-08-16	**1**	**AUTOMOTIVE**	3000888	4.549086
		BABY CARE	3000889	0.000000
		BEAUTY	3000890	4.467870
		BEVERAGES	3000891	2384.559095
		BOOKS	3000892	0.300994
...	**...**	**...**	...	...
2017-08-31	**9**	**POULTRY**	3029395	366.041333
		PREPARED FOODS	3029396	97.622797
		PRODUCE	3029397	1235.178864
		SCHOOL AND OFFICE SUPPLIES	3029398	5.120332
		SEAFOOD	3029399	17.265244

28512 rows × 2 columns

> Kaggle Score: 0.58717