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Final Project

1 Numerical Simplification

We have some errors because of numerical issues; the likelihoods are not scaled, so they cause numerical overflow and underflow. So, instead of computing likelihoods directly, we compute them as ratios

$$\frac{L_{new}}{L_{old}}.$$

For the likelihood of the noise model, we can simplify it in the following way

$$\begin{aligned}
\frac{L_{new}}{L_{old}} &= \frac{\sigma_c^{old}(2\pi(\sigma_c^{old})^2 T_{t-1})^{n_c} \exp\left\{\frac{-1}{2T_t}\sum_{s:x_s=c}\left(\frac{y_s-\mu_c}{\sigma_c}\right)^2\right\}}{\sigma_c(2\pi(\sigma_c)^2 T_t)^{n_c} \exp\left\{\frac{-1}{2T_{t-1}}\sum_{s:x_s=c}\left(\frac{y_s-\mu_c^{old}}{\sigma_c^{old}}\right)^2\right\}} \\
&= \left(\frac{\sigma_c^{old}}{\sigma_c}\right)^{3+n_c}\left(\frac{T_{t-1}}{T_t}\right)^{n_c}\exp\left\{\frac{-1}{2T_t}\sum_{s:x_s=c}\left(\frac{y_s-\mu_c}{\sigma_c}\right)^2+\frac{1}{2T_{t-1}}\sum_{s:x_s=c}\left(\frac{y_s-\mu_c^{old}}{\sigma_c^{old}}\right)^2\right\} \\
&= \left(\frac{\sigma_c^{old}}{\sigma_c}\right)^{3+n_c}\left(\frac{T_{t-1}}{T_t}\right)^{n_c}\exp\left\{\sum_{s:x_s=c}\frac{-1}{2T_t}\left(\frac{y_s-\mu_c}{\sigma_c}\right)^2+\frac{1}{2T_{t-1}}\left(\frac{y_s-\mu_c^{old}}{\sigma_c^{old}}\right)^2\right\} \\
&= \prod_{s:x_s=c}\left(\frac{\sigma_c^{old}}{\sigma_c}\right)^{3}\left(\frac{T_{t-1}}{T_t}\right)\exp\left\{\frac{-1}{2T_t}\left(\frac{y_s-\mu_c}{\sigma_c}\right)^2+\frac{1}{2T_{t-1}}\left(\frac{y_s-\mu_c^{old}}{\sigma_c^{old}}\right)^2\right\}
\end{aligned}$$