There Is No Largest Prime Number

Euclid of Alexandria[1]

[ISPN '80] 27th International Symposium of Prime Numbers

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Outline

Motivation

Results

Discussion

What Are Prime Numbers?

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A prime number is a number that has exactly two divisors.

Example

- 2 is prime (two divisors: 1 and 2).

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- 3 is prime (two divisors: 1 and 3).

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Example

- 2 is prime (two divisors: 1 and 2).
- 3 is prime (two divisors: 1 and 3).
- 4 is not prime (three divisors: 1, 2, and 4).

There Is No Largest Prime Number

The proof uses *reductio ad absurdum*.

Theorem

There is no largest prime number.

Proof.

1. Suppose p were the largest prime number.

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2. Let q be the product of the first p numbers.

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2. Let q be the product of the first p numbers.
3. Then $q + 1$ is not divisible by any of them.

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Theorem

There is no largest prime number.

Proof.

1. Suppose p were the largest prime number.
2. Let q be the product of the first p numbers.
3. Then $q + 1$ is not divisible by any of them.
4. Thus $q + 1$ is also prime and greater than p. □

The proof used *reductio ad absurdum*.

What's Still To Do?

Answered Questions

How many primes are there?

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Open Questions

Is every even number the sum of two primes?