Pasqal challenge

QUBO constrains

2024

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Introduction

We aim to solve the problem of creating M bus lines each with P bus stops in a city. The formulation is based on the QUBO problem, i.e, minimizing

$$f_Q(X) = X^T Q X \tag{0.1}$$

where X is a n-dimensional column vector and Q an $n \times n$ matrix. In our context, X will represent the adjacency matrix and Q the cost function (distance). As we want to minimize the distance from all bus lines, we perform summation over the M lines

$$f_Q(X) = \sum_{l}^{M} (X^l)^T Q X^l \tag{0.2}$$

where l is a dummy superscript.

Conditions

We define first the type of path that we aim to achieve for every single line: a closed non-self-crossing path of length p. To achieve this, it suffices to impose:

(i) p edges per path $\sum_{ij} X^l_{ij} = p \; \forall l$

(ii) At most, one edge pointing inwards to each node$j \to i$: $\sum_j X^l_{ij} \leq 1 \; \forall l, i$

(iii) At most, one edge pointing outwards from each node $i \to j$: $\sum_i X^l_{ij} \leq 1 \; \forall l, j$

(iv) (Together with (i)) All paths are cyclic and length (order) $\leq p$: $\sum_i (X^l)^p_{ii} = p \forall l$

(v) There are no cycles of order $< p$: $\sum_{i,p} (X^l)^p_{ii} = 0 \; \forall l$

Sufficient condition proof: Clearly, (v) + (iv) $\implies$ (a) All paths are p-order cycles.
Now, (a) + (i) $\implies$ (b) There is only one p-order cyclic path with p edges.
Finally, (b) + (2) + (3) $\implies$ There is only one p-order cyclic path with p edges and p nodes.