CBSE Class 12 65/2/S

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1 Matrices

1. If $A = \begin{pmatrix} 0 & 3 \\ 2 & -5 \end{pmatrix}$ and $kA = \begin{pmatrix} 0 & 4a \\ -8 & 5b \end{pmatrix}$ find the values of k and a.

2. If $A = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -4 \\ 3 & -2 \end{pmatrix}$ find $|AB|$.

3. If A is a square matrix such that $|A| = 5$, write the value of $\left|AA^T\right|$.

4. Ishan wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2. Using matrices, find the dimensions of the plot. Also give reason why he wants to donate the plot for a school.

5. Using properties of determinant, prove that: $\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$

6. Using elementary row operations, find the inverse of the following matrix: $\begin{pmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{pmatrix}$

2 Probability

7. There are two bags A and B. Bag A contains 3 white and 4 red balls whereas bag B contains 4 white and 3 red balls. Three balls are drawn at random (without replacement) from one of the bags and are found to be two white and one red. Find the probability that these were drawn from bag B.

8. Three numbers are selected at random (without replacement) from first six positive integers. If X denotes the smallest of the three numbers obtained, find the probability distribution of X. Also find the mean and variance of the distribution.

3 Algebra

9. Prove that $2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \frac{\pi}{4}$.

10. Solve the equation for x: $\cos\left(\tan^{-1} x\right) = \sin\left(\cot^{-1} \frac{3}{4}\right)$.

4 Vectors

11. Find the distance between the planes $\mathbf{r} \cdot (2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}) - 4 = 0$ and $\mathbf{r} \cdot (6\mathbf{i} - 9\mathbf{j} + 18\mathbf{k}) + 30 = 0$
12. If $\mathbf{a}$ and $\mathbf{b}$ are unit vectors, then what is the angle between $\mathbf{a}$ and $\mathbf{b}$ for $\mathbf{a} - \sqrt{2}\mathbf{b}$ to be a unit vector?
13. If vectors $\mathbf{a}$ and $\mathbf{b}$ are such that $|\mathbf{a}| = \frac{1}{2}, |\mathbf{b}| = \frac{4}{\sqrt{3}}$ and $|\mathbf{a} \times \mathbf{b}| = \frac{1}{\sqrt{3}}$, then find $|\mathbf{a} \cdot \mathbf{b}|$.
14. Find the equation of the plane passing through the points A $(3, 2, 1)$, B $(4, 2, -2)$ and C $(6, 5, -1)$ and hence find the value of λ for which A $(3, 2, 1)$, B $(4, 2, -2)$, C $(6, 5, -1)$ and D $(\lambda, 5, 5)$ are coplanar.
15. Find the co-ordinates of the point where the line $\mathbf{r} = (-\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}) + \lambda(3\mathbf{i} + 4\mathbf{j} + 3\mathbf{k})$ meets the plane which is perpendicular to the vector $\mathbf{n} = \mathbf{i} + \mathbf{j} + 3\mathbf{k}$ and at a distance of $\frac{4}{\sqrt{11}}$ from origin.
16. Given that vectors $\mathbf{a}$, $\mathbf{b}$. $\mathbf{c}$ form a triangle such that $\mathbf{a} = \mathbf{b} + \mathbf{c}$. Find p, q, r, s such that area of triangle is $5\sqrt{6}$ where $\mathbf{a} = p\mathbf{i} + q\mathbf{j} + r\mathbf{k}$, $\mathbf{b} = s\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ and $\mathbf{c} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$.
17. Find the equation of the plane containing two parallel lines $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{3}$ and $\frac{x}{4} = \frac{y-2}{-2} = \frac{z+1}{6}$. Also, find if the plane thus obtained contains the line $\frac{x-2}{3} = \frac{y-1}{1} = \frac{z-2}{5}$ or not.

5 Functions

18. Find k, if $f(x) = \begin{cases} k\sin\left(\frac{\pi}{2}(x+1)\right), & x \le 0 \\ \frac{\tan x - \sin x}{x^3}, & x > 0 \end{cases}$ is continuous at $x = 0$.
19. Let $f : \mathbb{N} \to \mathbb{N}$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f : \mathbb{N} \to \mathbb{S}$ is invertible (where $\mathbb{S}$ is range of f). Find the inverse of f and hence find $f^{-1}(31)$ and $f^{-1}(87)$

6 Differentiation

20. Differentiate $(\sin 2x)^x + \sin^{-1}\left(\sqrt{3x}\right)$ with respect to x.
21. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}\right)$ with respect to $\cos^{-1} x^2$
22. Find the equation of normal to the curve $ay^2 = x^3$ at the point whose x coordinate is am^2.
23. Determine the intervals in which the function $f(x) = x^4 - 8x^2 + 22x^2 - 24x + 21$ is strictly increasing or strictly decreasing.
24. Find the maximum and minimum values of $f(x) = \sec x + \log\left(\cos^2 x\right)$, $0 < x < 2\pi$.

7 Integration

25. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx$
26. Evaluate: $\int_0^1 \cot^{-1}\left(1 - x + x^2\right) dx$
27. Find: $\int [\log(\log x) + \frac{1}{(\log x)^2}] dx$
28. Find: $\int \frac{1 - \sin x}{\sin x(1 + \sin x)} dx$
29. Solve the differential equation: $2ye^{\frac{x}{y}} dx + \left(y - 2xe^{\frac{x}{y}}\right) dy = 0$.
30. Solve the differential equation: $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^3$
31. Using integration, find the area of the region $\left\{(x, y) : y^2 \le 6ax, x^2 + y^2 \le 16a^2\right\}$.

8 Optimization

32. A diet is to contain 80 units of Vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available, costing ₹5 per unit and ₹6 per unit, respectively. One unit of food F_1 contains 4 units of vitamin A and 3 units of minerals whereas one unit of food F_2 contains 3 units of vitamin A and 6 units of minerals. Formulate this as a linear programming problem. Find the minimum cost of diet that consists of mixture of these two foods and also meets minimum nutritional requirement.