EE1205: Signals and Systems - TA

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.1 The Z-transform of $p(n)$ is defined as

$$P(z) = \sum_{n=-\infty}^{\infty} p(n)z^{-n} \tag{.1.1}$$

.2 If

$$p(n) = p_1(n) * p_2(n), \tag{.2.1}$$
$$P(z) = P_1(z)P_2(z) \tag{.2.2}$$

The above property follows from Fourier analysis and is fundamental to signal processing.

.3 For a Geometric progression

$$x(n) = x(0)\, r^n u(n)\,, \tag{.3.1}$$
$$\implies X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n} = \sum_{n=0}^{\infty} x(0)\, r^n z^{-n} \tag{.3.2}$$
$$= \sum_{n=0}^{\infty} x(0)\left(rz^{-1}\right)^n \tag{.3.3}$$
$$= \frac{x(0)}{1 - rz^{-1}}, \quad |z| > |r| \tag{.3.4}$$

.4 Let

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{.4.1}$$

Substituting $r = 1$ in (.3.4),

$$u(n) \overset{\mathcal{Z}}{\longleftrightarrow} U(z) = \frac{1}{1 - z^{-1}}, \quad |z| > 1 \tag{.4.2}$$

.5 From (.1.1) and (.4.2),

$$U(z) = \sum_{n=-\infty}^{\infty} u(n)z^{-n} \tag{.5.1}$$
$$\implies \frac{dU(z)}{dz} = -z^{-1} \sum_{n=-\infty}^{\infty} nu(n)z^{-n} \tag{.5.2}$$
$$\therefore nu(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}}{(1 - z^{-1})^2}, \quad |z| > 1 \tag{.5.3}$$

.6 For an AP,

$$x(n) = [x(0) + nd]\, u(n) = x(0)u(n) + dnu(n) \tag{.6.1}$$
$$\implies X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2}, \quad |z| > 1 \tag{.6.2}$$

upon substituting from (.4.2) and (.5.3).

.7 For the AP $x(n)$, the sum of first $n+1$ terms can be expressed as

$$y(n) = \sum_{k=0}^{n} x(k) \tag{.7.1}$$

$$\implies y(n) = \sum_{k=-\infty}^{\infty} x(k)u(n-k) \tag{.7.2}$$

$$= x(n) * u(n) \tag{.7.3}$$

Taking the Z-transform on both sides, and substituting (.6.2) and (.4.2) (making use of the convolution property of Z-transform),

$$Y(z) = X(z)U(z) \tag{.7.4}$$

$$\implies Y(z) = \left(\frac{x(0)}{1-z^{-1}} + \frac{dz^{-1}}{(1-z^{-1})^2}\right)\frac{1}{1-z^{-1}} \quad |z| > 1 \tag{.7.5}$$

$$= \frac{x(0)}{(1-z^{-1})^2} + \frac{dz^{-1}}{(1-z^{-1})^3}, \quad |z| > 1 \tag{.7.6}$$

$$= x(0)\frac{z^2}{(z-1)^2} + \frac{dz^2}{(z-1)^3}, \quad |z| > 1 \tag{.7.7}$$

$$= x(0)\left(\frac{z}{(z-1)^2} + \frac{1}{z-1}\right) + \frac{d}{2}\left(\frac{z^2+z}{(z-1)^3} + \frac{z}{(z-1)^2}\right), \quad |z| > 1 \tag{.7.8}$$

$$\tag{.7.9}$$

Taking the inverse Z-transform,

$$y(n) = \left(x(0)(n+1) + \frac{d}{2}\left(n^2+n\right)\right)u(n) \tag{.7.10}$$

$$= \frac{n+1}{2}\left(2x(0) + nd\right)u(n) \tag{.7.11}$$

$$\implies \text{Sum } S(n) = y(n-1) = \frac{n}{2}\left(2x(0) + (n-1)d\right), \quad n \geq 0 \tag{.7.12}$$