EE1205: Signals and Systems - TA

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.1 The Z-transform of $p(n)$ is defined as

$$P(z) = \sum_{n=-\infty}^{\infty} p(n)z^{-n} \tag{.1.1}$$

.2 If

$$p(n) = p_1(n) * p_2(n), \tag{.2.1}$$
$$P(z) = P_1(z)P_2(z) \tag{.2.2}$$

.3 For a Geometric progression

$$x(n) = x(0)\, r^n u(n), \tag{.3.1}$$
$$\implies X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n} = \sum_{n=0}^{\infty} x(0)\, r^n z^{-n} \tag{.3.2}$$
$$= \sum_{n=0}^{\infty} x(0) \left(rz^{-1}\right)^n \tag{.3.3}$$
$$= \frac{x(0)}{1 - rz^{-1}}, \quad |z| > |r| \tag{.3.4}$$

.4 Substituting $r = 1$ in (.3.4),

$$u(n) Z U(z) = \frac{1}{1 - z^{-1}}, \quad |z| > 1 \tag{.4.1}$$

.5 From (.1.1) and (.4.1),

$$U(z) = \sum_{n=-\infty}^{\infty} u(n)z^{-n} \tag{.5.1}$$
$$\implies \frac{dU(z)}{dz} = -z^{-1} \sum_{n=-\infty}^{\infty} nu(n)z^{-n} \tag{.5.2}$$
$$\therefore nu(n) Z \frac{z^{-1}}{(1 - z^{-1})^2}, \quad |z| > 1 \tag{.5.3}$$

.6 For an AP,

$$x(n) = [x(0) + nd]\, u(n) = x(0)u(n) + dnu(n) \tag{.6.1}$$
$$\implies X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2}, \quad |z| > 1 \tag{.6.2}$$

upon substituting from (.4.1) and (.5.3).

.7 From (**??**), the sum to n terms of a GP can be expressed as

$$y(n) = x(n) * u(n) \tag{.7.1}$$

where $x(n)$ is defined in (.3.1). From (.2.2), (.3.4) and (.4.1),

$$Y(z) = X(z)\,U(z) \tag{.7.2}$$

$$= \left(\frac{x(0)}{1 - rz^{-1}}\right)\left(\frac{1}{1 - z^{-1}}\right) \quad |z| > |r| \cap |z| > |1| \tag{.7.3}$$

$$= \frac{x(0)}{(1 - rz^{-1})(1 - z^{-1})} \quad |z| > |r| \tag{.7.4}$$

which can be expressed as

$$Y(z) = \frac{x(0)}{r - 1}\left(\frac{r}{1 - rz^{-1}} - \frac{1}{1 - z^{-1}}\right) \tag{.7.5}$$

using partial fractions. Again, from (.3.4) and (.4.1), the inverse of the above can be expressed as

$$y(n) = x(0)\left(\frac{r^{n+1} - 1}{r - 1}\right)u(n) \tag{.7.6}$$

.8 For the AP $x(n)$, the sum of first $n + 1$ terms can be expressed as

$$y(n) = \sum_{k=0}^{n} x(k) \tag{.8.1}$$

$$\implies y(n) = \sum_{k=-\infty}^{\infty} x(k)u(n - k) \tag{.8.2}$$

$$= x(n) * u(n) \tag{.8.3}$$

Taking the Z-transform on both sides, and substituting (.6.2) and (.4.1) (making use of the convolution property of Z-transform),

$$Y(z) = X(z)U(z) \tag{.8.4}$$

$$\implies Y(z) = \left(\frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2}\right)\frac{1}{1 - z^{-1}} \quad |z| > 1 \tag{.8.5}$$

$$= \frac{x(0)}{(1 - z^{-1})^2} + \frac{dz^{-1}}{(1 - z^{-1})^3}, \quad |z| > 1 \tag{.8.6}$$

Separating the terms,

$$Y(z) = x(0)\left[\frac{z^{-1}}{(1 - z^{-1})^2} + \frac{1}{1 - z^{-1}}\right] + \frac{d}{2}\left[-z\frac{d}{dz}\left(\frac{1}{(1 - z^{-1})^2}\right)\right], \quad |z| > 1 \tag{.8.7}$$

$$= x(0)\left[\frac{z^{-1}}{(1 - z^{-1})^2} + \frac{1}{1 - z^{-1}}\right] + \frac{d}{2}\left[-z\frac{d}{dz}\left(\frac{z^{-1}}{(1 - z^{-1})^2} + \frac{1}{1 - z^{-1}}\right)\right], \quad |z| > 1 \tag{.8.8}$$

Taking the inverse Z-transform using (.5.3), (.4.1) and applying the derivative property:

$$y(n) = \left(x(0)(n + 1) + \frac{d}{2}n\,(n + 1)\right)u(n) \tag{.8.9}$$

$$= \frac{n + 1}{2}\,(2x(0) + nd)\,u(n) \tag{.8.10}$$