Exemplar - 12.13.3.50

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Question: The probability distribution of a random variable X is given as under:

$$p_X(x) = \begin{cases} kx^2 & \text{for } x = 1, 2, 3 \\ 2kx & \text{for } x = 4, 5, 6 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant. Calculate:

(i) $E(X)$
(ii) $E\left(3X^2\right)$
(iii) $\Pr(X \geq 4)$

Solution: From the axiom of total probability,

$$\sum_{i=1}^{6} p_X(i) = 1 \tag{1}$$

$$\implies \sum_{i=1}^{3} ki^2 + \sum_{i=4}^{6} 2ki = 1 \tag{2}$$

$$\implies k + 4k + 9k + 8k + 10k + 12k = 1 \tag{3}$$

$$\implies k = \frac{1}{44} \tag{4}$$

Thus, the probability distribution of X is

$$p_X(x) = \begin{cases} \frac{x^2}{44} & \text{for } x = 1, 2, 3 \\ \frac{x}{22} & \text{for } x = 4, 5, 6 \\ 0 & \text{otherwise} \end{cases}$$

(i) Calculating $E(X)$:

$$E(X) = \sum_{i=1}^{6} i p_X(i) \tag{5}$$

$$= 1\left(\frac{1}{44}\right) + 2\left(\frac{4}{44}\right) + 3\left(\frac{9}{44}\right) + 4\left(\frac{4}{22}\right) + 5\left(\frac{5}{22}\right) + 6\left(\frac{6}{22}\right) \tag{6}$$

$$= \frac{95}{22} \tag{7}$$

(ii) Calculating $E\left(3X^2\right)$:

$$E\left(3X^2\right) = 3E\left(X^2\right) \tag{8}$$

$$= 3\sum_{i=1}^{36} i p_{X^2}(i) \tag{9}$$

$$= 3\left(1\left(\frac{1}{44}\right) + 4\left(\frac{4}{44}\right) + 9\left(\frac{9}{44}\right) + 16\left(\frac{4}{22}\right) + 25\left(\frac{5}{22}\right) + 36\left(\frac{6}{22}\right)\right) \tag{10}$$

$$= \frac{2724}{44} \tag{11}$$

(iii) Calculating $\Pr(X \geq 4)$:

$$\Pr(X \geq 4) = 1 - \Pr(X \leq 3) \tag{12}$$

$$= 1 - F_X(3) \tag{13}$$

$$= 1 - \sum_{i=1}^{3} p_X(i) \tag{14}$$

$$= 1 - \left(\frac{1}{44} + \frac{4}{44} + \frac{9}{44}\right) \tag{15}$$

$$= \frac{15}{22} \tag{16}$$