Exemplar - 12.13.3.28

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Question: A die is thrown three times. Let X be 'the number of twos seen'. Find the expectation of X.

Solution: Let the random variables be:

RV	Values	Description
X	{0, 1, 2, 3}	The number of twos rolled in three dice rolls
Y	{0, 1}	The number of twos rolled in one die roll

TABLE 0

RANDOM VARIABLES

For a single die roll, the probability distribution function of Y is:

$$p_Y(k) = \begin{cases} \frac{5}{6} & \text{if } k = 0 \\ \frac{1}{6} & \text{if } k = 1 \end{cases} \tag{1}$$

Thus, the expectation of Y for a single roll of die is:

$$E(Y) = \sum_{k=0}^{1} k p_Y(k) \tag{2}$$

$$= \frac{5}{6}(0) + \frac{1}{6}(1) \tag{3}$$

$$= \frac{1}{6} \tag{4}$$

But, as all three dice rolls are independent, and expectation is linear:

$$X = 3Y \tag{5}$$

$$\therefore E(X) = E(3Y) \tag{6}$$

$$= 3E(Y) \tag{7}$$

$$= \frac{1}{2} \tag{8}$$