Exemplar - 12.13.3.102

EE22BTECH11039 - Pandrangi Aditya Sriram*

Question: If A and B are two events such that $\Pr(A) > 0$ and $\Pr(A) + \Pr(B) > 1$, then

$$\Pr(B|A) \geq 1 - \frac{\Pr(B')}{\Pr(A)}$$

Solution:

$$\Pr(B|A) = \frac{\Pr(AB)}{\Pr(A)} \tag{1}$$

$$= \frac{\Pr(A) + \Pr(B) - \Pr(A + B)}{\Pr(A)} \tag{2}$$

For any event E, $\Pr(E) \leq 1$

$$\therefore \Pr(A + B) \leq 1 \tag{3}$$

$$-\Pr(A + B) \geq -1 \tag{4}$$

$$\frac{\Pr(A) + \Pr(B) - \Pr(A + B)}{\Pr(A)} \geq \frac{\Pr(A) + \Pr(B) - 1}{\Pr(A)} \tag{5}$$

$$\Pr(B|A) \geq \frac{\Pr(A) - (1 - \Pr(B))}{\Pr(A)} \tag{6}$$

$$\Pr(B|A) \geq \frac{\Pr(A) - \Pr(B')}{\Pr(A)} \tag{7}$$

$$\Pr(B|A) \geq 1 - \frac{\Pr(B')}{\Pr(A)} \tag{8}$$

$\therefore$ Proved.