# Exemplar - 12.13.3.50

EE22BTECH11039 - Pandrangi Aditya Sriram*

**Question:** The probability distribution of a random variable $X$ is given as under:

$$p_X(x) = \begin{cases} kx^2 & \text{for } x = 1, 2, 3 \\ 2kx & \text{for } x = 4, 5, 6 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant. Calculate:

1) $E(X)$
2) $E\left(3X^2\right)$
3) $\Pr(X \geq 4)$

**Solution:** From the axiom of total probability,

$$\sum_{i=1}^{6} p_X(i) = 1 \tag{1}$$

$$\implies \sum_{i=1}^{3} ki^2 + \sum_{i=4}^{6} 2ki = 1 \tag{2}$$

$$\implies k + 4k + 9k + 8k + 10k + 12k = 1 \tag{3}$$

$$\implies k = \frac{1}{44} \tag{4}$$

Thus, the probability distribution of $X$ is

$$p_X(x) = \begin{cases} \frac{x^2}{44} & \text{for } x = 1, 2, 3 \\ \frac{2x}{44} & \text{for } x = 4, 5, 6 \\ 0 & \text{otherwise} \end{cases}$$

1) Calculating $E(X)$:

$$E(X) = \sum_{i=1}^{6} i p_X(i) \tag{5}$$

$$= 1\left(\frac{1}{44}\right) + 2\left(\frac{4}{44}\right) + 3\left(\frac{9}{44}\right) + 4\left(\frac{8}{44}\right) + 5\left(\frac{10}{44}\right) + 6\left(\frac{12}{44}\right) \tag{6}$$

$$= \frac{95}{22} \tag{7}$$

$$= 4.32 \tag{8}$$

2) Calculating $E\left(3X^2\right)$:

$$E\left(3X^2\right) = 3E\left(X^2\right) \tag{9}$$

$$= 3\sum_{i=1}^{6} i^2 p_X(i) \tag{10}$$

$$= 3\left(1\left(\frac{1}{44}\right) + 4\left(\frac{4}{44}\right) + 9\left(\frac{9}{44}\right) + 16\left(\frac{8}{44}\right) + 25\left(\frac{10}{44}\right) + 36\left(\frac{12}{44}\right)\right) \tag{11}$$

$$= \frac{2724}{44} \tag{12}$$

$$= 61.91 \tag{13}$$

3) Firstly, calculating the cumulative function:

$$F_X(x) = \sum_{i=1}^{x} p_X(i) \tag{14}$$

$$= \begin{cases} \sum_{i=1}^{x} \frac{i^2}{44} & \text{if } x \leq 3 \\ \sum_{i=1}^{3} \frac{i^2}{44} + \sum_{i=4}^{x} \frac{2i}{44} & \text{if } x \geq 4 \end{cases} \tag{15}$$

$$= \begin{cases} \frac{x(x+1)(2x+1)}{6\times 44} & \text{if } x \leq 3 \\ \frac{14}{44} + \frac{x(x+1)}{44} - \frac{3\times 4}{44} & \text{if } x \geq 4 \end{cases} \tag{16}$$

$$= \begin{cases} \frac{x(x+1)(2x+1)}{264} & \text{if } x \leq 3 \\ \frac{x(x+1)+2}{44} & \text{if } x \geq 4 \end{cases} \tag{17}$$

Calculating $\Pr(X \geq 4)$:

$$\Pr(X \geq 4) = 1 - \Pr(X \leq 3) \tag{18}$$
$$= 1 - F_X(3) \tag{19}$$
$$= 1 - \frac{3 \times 4 \times 7}{264} \tag{20}$$
$$= \frac{15}{22} \tag{21}$$
$$= 0.68 \tag{22}$$