Problem 1.2.4

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Verify that

$$\frac{BG}{GE} = \frac{CG}{GF} = \frac{AG}{GD} = 2 \tag{1}$$

Solution: Calculating the length BG:

$$\begin{aligned}
BG &= \|\mathbf{G} - \mathbf{B}\| && (2)\\
&= \sqrt{(\mathbf{G} - \mathbf{B})^\top (\mathbf{G} - \mathbf{B})} && (3)\\
&= \sqrt{\left(\begin{pmatrix}-2\\0\end{pmatrix} - \begin{pmatrix}-4\\6\end{pmatrix}\right)^\top \left(\begin{pmatrix}-2\\0\end{pmatrix} - \begin{pmatrix}-4\\6\end{pmatrix}\right)} && (4)\\
&= \sqrt{\begin{pmatrix}2 & -6\end{pmatrix}\begin{pmatrix}2\\-6\end{pmatrix}} && (5)\\
&= \sqrt{40} && (6)\\
&= 2\sqrt{10} && (7)
\end{aligned}$$

Calculating the length GE:

$$\begin{aligned}
GE &= \|\mathbf{E} - \mathbf{G}\| && (8)\\
&= \sqrt{(\mathbf{E} - \mathbf{G})^\top (\mathbf{E} - \mathbf{G})} && (9)\\
&= \sqrt{\left(\begin{pmatrix}-1\\-3\end{pmatrix} - \begin{pmatrix}-2\\0\end{pmatrix}\right)^\top \left(\begin{pmatrix}-1\\-3\end{pmatrix} - \begin{pmatrix}-2\\0\end{pmatrix}\right)} && (10)\\
&= \sqrt{\begin{pmatrix}1 & -3\end{pmatrix}\begin{pmatrix}1\\-3\end{pmatrix}} && (11)\\
&= \sqrt{10} && (12)
\end{aligned}$$

Calculating the length CG:

$$\begin{aligned}
CG &= \|\mathbf{G} - \mathbf{C}\| && (13)\\
&= \sqrt{(\mathbf{G} - \mathbf{C})^\top (\mathbf{G} - \mathbf{C})} && (14)\\
&= \sqrt{\left(\begin{pmatrix}-2\\0\end{pmatrix} - \begin{pmatrix}-3\\-5\end{pmatrix}\right)^\top \left(\begin{pmatrix}-2\\0\end{pmatrix} - \begin{pmatrix}-3\\-5\end{pmatrix}\right)} && (15)\\
&= \sqrt{\begin{pmatrix}1 & 5\end{pmatrix}\begin{pmatrix}1\\5\end{pmatrix}} && (16)\\
&= \sqrt{26} && (17)
\end{aligned}$$

Calculating the length GF:

$$\begin{aligned}
GF &= \|\mathbf{F} - \mathbf{G}\| && (18)\\
&= \sqrt{(\mathbf{F} - \mathbf{G})^\top (\mathbf{F} - \mathbf{G})} && (19)\\
&= \sqrt{\left(\frac{1}{2}\begin{pmatrix}-3\\5\end{pmatrix} - \begin{pmatrix}-2\\0\end{pmatrix}\right)^\top \left(\frac{1}{2}\begin{pmatrix}-3\\5\end{pmatrix} - \begin{pmatrix}-2\\0\end{pmatrix}\right)} && (20)\\
&= \sqrt{\frac{1}{4}\begin{pmatrix}1 & -5\end{pmatrix}\begin{pmatrix}1\\5\end{pmatrix}} && (21)\\
&= \frac{1}{2}\sqrt{26} && (22)
\end{aligned}$$

Calculating the length AG:

$$\begin{aligned}
AG &= \|\mathbf{G} - \mathbf{A}\| && (23)\\
&= \sqrt{(\mathbf{G} - \mathbf{A})^\top (\mathbf{G} - \mathbf{A})} && (24)\\
&= \sqrt{\left(\begin{pmatrix}-2\\0\end{pmatrix} - \begin{pmatrix}1\\-1\end{pmatrix}\right)^\top \left(\begin{pmatrix}-2\\0\end{pmatrix} - \begin{pmatrix}1\\-1\end{pmatrix}\right)} && (25)\\
&= \sqrt{\begin{pmatrix}-3 & 1\end{pmatrix}\begin{pmatrix}-3\\1\end{pmatrix}} && (26)\\
&= \sqrt{10} && (27)
\end{aligned}$$

Calculating the length GD:

$$\begin{aligned}
GD &= \|\mathbf{D} - \mathbf{G}\| && (28)\\
&= \sqrt{(\mathbf{D} - \mathbf{G})^\top (\mathbf{D} - \mathbf{G})} && (29)\\
&= \sqrt{\left(\frac{1}{2}\begin{pmatrix}-7\\1\end{pmatrix} - \begin{pmatrix}-2\\0\end{pmatrix}\right)^\top \left(\frac{1}{2}\begin{pmatrix}-7\\1\end{pmatrix} - \begin{pmatrix}-2\\0\end{pmatrix}\right)} && (30)\\
&= \sqrt{\frac{1}{4}\begin{pmatrix}-3 & 1\end{pmatrix}\begin{pmatrix}-3\\1\end{pmatrix}} && (31)\\
&= \frac{1}{2}\sqrt{10} && (32)
\end{aligned}$$

From these statements, we can conclude:

$$\frac{BG}{GE} = \frac{2\sqrt{10}}{\sqrt{10}} = 2 \tag{33}$$

$$\frac{CG}{GF} = \frac{\sqrt{26}}{\frac{1}{2}\sqrt{26}} = 2 \tag{34}$$

$$\frac{AG}{GD} = \frac{\sqrt{10}}{\frac{1}{2}\sqrt{10}} = 2 \tag{35}$$

The required condition is verified.