Gaussian - 9.3.19

EE22BTECH11039 - Pandrangi Aditya Sriram*

Question: Suppose X is a binomial distribution $B\left(6, \frac{1}{2}\right)$. Show that $X = 3$ is the most likely outcome. (Hint : $P(X = 3)$ is the maximum among all $P(x_i), x_i = 0, 1, 2, 3, 4, 5, 6$)

Solution:

$$X \sim B\left(6, \frac{1}{2}\right) \tag{1}$$

This implies $n = 6,\ p = \frac{1}{2}$ and $q = 1 - p = \frac{1}{2}$.

$$p_X(x) = {}^nC_x p^x q^{n-x} \tag{2}$$

$$= {}^6C_x \left(\frac{1}{2}\right)^6 \tag{3}$$

Thus, evaluating:

$$p_X(0) = \left(\frac{1}{2}\right)^6 \tag{4}$$

$$p_X(1) = 6\left(\frac{1}{2}\right)^6 \tag{5}$$

$$p_X(2) = 15\left(\frac{1}{2}\right)^6 \tag{6}$$

$$p_X(3) = 20\left(\frac{1}{2}\right)^6 \tag{7}$$

$$p_X(4) = 15\left(\frac{1}{2}\right)^6 \tag{8}$$

$$p_X(5) = 6\left(\frac{1}{2}\right)^6 \tag{9}$$

$$p_X(6) = \left(\frac{1}{2}\right)^6 \tag{10}$$

Thus, $X = 3$ is the most likely outcome.