Problem 1.4.2

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Find the intersection $\mathbf{O}$ of the perpendicular bisectors of AB and AC.

Solution: From (1.4.1), the normal equation of perpendicular bisectors of AB and AC as, respectively:

$$\begin{pmatrix} -5 & 7 \end{pmatrix} \mathbf{x} = 25 \tag{1}$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = -4 \tag{2}$$

The pair of linear equations can be solved using the augmented matrix $\begin{pmatrix} \mathbf{P} \mid \mathbf{Q} \end{pmatrix}$ as follows:

$$\mathbf{P} = \begin{pmatrix} -5 & 7 \\ 1 & 1 \end{pmatrix} \tag{3}$$

$$\mathbf{Q} = \begin{pmatrix} 25 \\ -4 \end{pmatrix} \tag{4}$$

$$\begin{pmatrix} \mathbf{P} \mid \mathbf{Q} \end{pmatrix} = \left(\begin{array}{cc|c} -5 & 7 & 25 \\ 1 & 1 & -4 \end{array}\right) \tag{5}$$

It can then can be solved using row reduction as follows:

$$\left(\begin{array}{cc|c} -5 & 7 & 25 \\ 1 & 1 & -4 \end{array}\right) \xleftrightarrow{R_2 \leftarrow R_1 + 5R_2} \left(\begin{array}{cc|c} -5 & 7 & 25 \\ 0 & 12 & 5 \end{array}\right) \tag{6}$$

$$\xleftrightarrow{R_1 \leftarrow 12R_1 - 7R_2} \left(\begin{array}{cc|c} -60 & 0 & 265 \\ 0 & 12 & 5 \end{array}\right) \tag{7}$$

$$\xleftrightarrow{R_1 \leftarrow \frac{R_1}{-60}} \left(\begin{array}{cc|c} 1 & 0 & -\frac{53}{12} \\ 0 & 12 & 5 \end{array}\right) \tag{8}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{R_2}{12}} \left(\begin{array}{cc|c} 1 & 0 & -\frac{53}{12} \\ 0 & 1 & \frac{5}{12} \end{array}\right) \tag{9}$$

We obtain:

$$\mathbf{O} = \begin{pmatrix} -\frac{53}{12} \\ \frac{5}{12} \end{pmatrix} \tag{10}$$

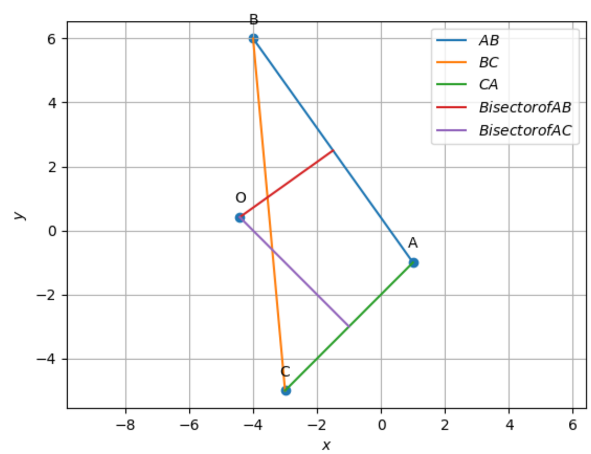

Fig. 0. Intersection point $\mathbf{O}$ of perpendicular bisectors of AB and AC