Problem 1.4.3

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Verify that **O** satisfies (1.4.1.1). **O** is known as the circumcentre.

Solution: Let **A**, **B**, and **C** be the vectors representing the vertices of triangle ABC, and let **O** be the circumcentre of the triangle. We want to verify the condition (1.4.1.1) of the question:

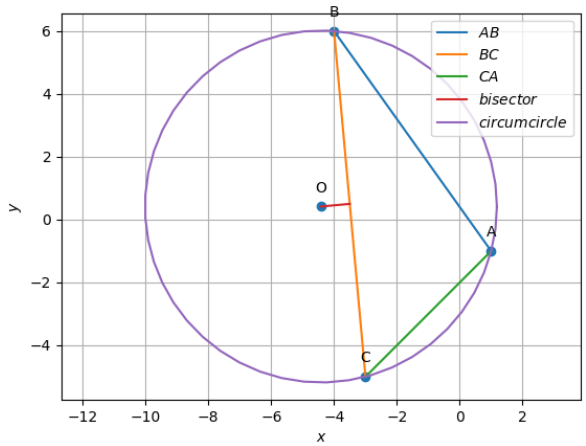

Fig. 0. Plot

$$\left(\mathbf{x} - \frac{\mathbf{B}+\mathbf{C}}{2}\right)^\top (\mathbf{B}-\mathbf{C}) = 0 \tag{1}$$

Substituting the values obtained in the previous problem:

$$\mathbf{x} = \mathbf{O} = \begin{pmatrix} -\frac{53}{12} \\ \frac{5}{12} \end{pmatrix} \tag{2}$$

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{3}$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{4}$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{5}$$

Calculating the required known vectors:

$$\frac{\mathbf{B}+\mathbf{C}}{2} = \begin{pmatrix} -\frac{7}{2} \\ \frac{1}{2} \end{pmatrix} \tag{6}$$

$$\mathbf{B}-\mathbf{C} = \begin{pmatrix} -1 \\ 11 \end{pmatrix} \tag{7}$$

Putting (2), (6) and (7) in condition (1):

$$\left(\begin{pmatrix} -\frac{53}{12} \\ \frac{5}{12} \end{pmatrix} - \begin{pmatrix} -\frac{7}{2} \\ \frac{1}{2} \end{pmatrix}\right)^\top \begin{pmatrix} -1 \\ 11 \end{pmatrix} = 0 \tag{8}$$

$$\implies \begin{pmatrix} -\frac{11}{12} & -\frac{1}{12} \end{pmatrix} \begin{pmatrix} -1 \\ 11 \end{pmatrix} = 0 \tag{9}$$

$$\implies \frac{11}{12} - \frac{11}{12} = 0 \tag{10}$$

Thus, we showed that **O** satisfies the required condition (1).