Exemplar - 10.13.2.7

EE22BTECH11039 - Pandrangi Aditya Sriram*

Apoorv throws two dice once and computes the product of the numbers appearing on the dice. Peehu throws one die and squares the number that appears on it. Who has the better chance of getting the number 36? Why?

Solution: Let the random variables be defined as:

Random Variable	Values	Description
X	$1 \le X \le 6$	Apoorv's First Dice Roll
Y	$1 \le Y \le 6$	Apoorv's Second Dice Roll
E	$1 \le E \le 36$	Square of Peehu's Dice Roll

1) **Product:** Assuming all dice rolls and equally likely,:

$$p_X(k) = \begin{cases} \frac{1}{6} & \text{if } k \in \{1,2,3,4,5,6\} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$p_Y(k) = \begin{cases} \frac{1}{6} & \text{if } k \in \{1,2,3,4,5,6\} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

The probability mass function for Apoorv is:

$$p_{XY}(k) = \Pr(XY = k) \tag{3}$$
$$= \Pr\left(X = \frac{k}{Y}\right) \tag{4}$$
$$= E\left(p_X\left(\frac{k}{Y}\right)\right) \tag{5}$$
$$= \sum_{i=1}^{6} p_X\left(\frac{k}{i}\right) \cdot p_Y(i) \tag{6}$$
$$= \frac{1}{6}\sum_{i=1}^{6} p_X\left(\frac{k}{i}\right) \tag{7}$$
$$= \frac{1}{6}\sum_{i=1}^{6} \frac{[k \bmod i = 0]}{6} \tag{8}$$
$$= \frac{1}{36}\sum_{i=1}^{6} [k \bmod i = 0] \tag{9}$$

Thus, the probability of Apoorv rolling a 36 is:

$$p_{XY}(36) = \frac{1}{6}\sum_{i=1}^{6} p_X\left(\frac{36}{i}\right) \tag{10}$$
$$= \frac{1}{6}\left(0+0+0+0+0+\frac{1}{6}\right) \tag{11}$$
$$= \frac{1}{36} \tag{12}$$

The cumulative distribution function for Apoorv is:

$$F_{XY}(k) = \Pr(XY \le k) \tag{13}$$
$$= \frac{1}{6}\sum_{j=1}^{k}\sum_{i=1}^{6} p_X\left(\frac{j}{i}\right) \tag{14}$$

2) **Square:** The probability mass function for Peehu is:

$$p_E(k) = \begin{cases} \frac{1}{6} & \text{if } k \in \{1,4,9,16,25,36\} \\ 0 & otherwise \end{cases} \tag{15}$$

Thus, the probability of Peehu rolling a 36 is $p_E(36) = \frac{1}{6}$. The cumulative distribution

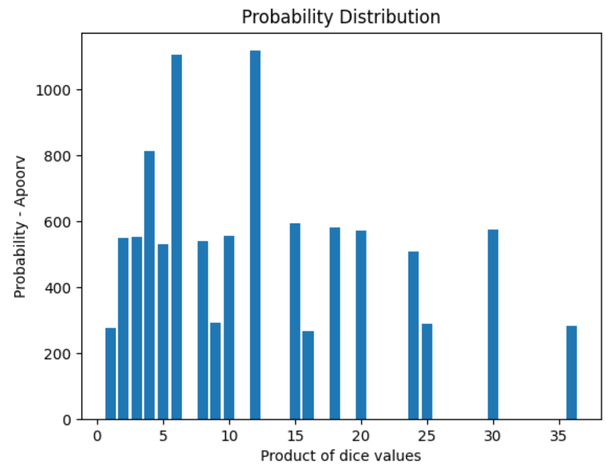
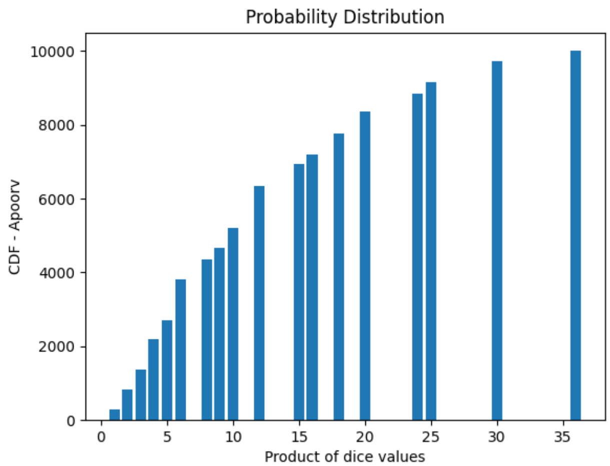


Fig. 1. Sketch of Probability Mass Function for Product obtained by taking a sample of random variables



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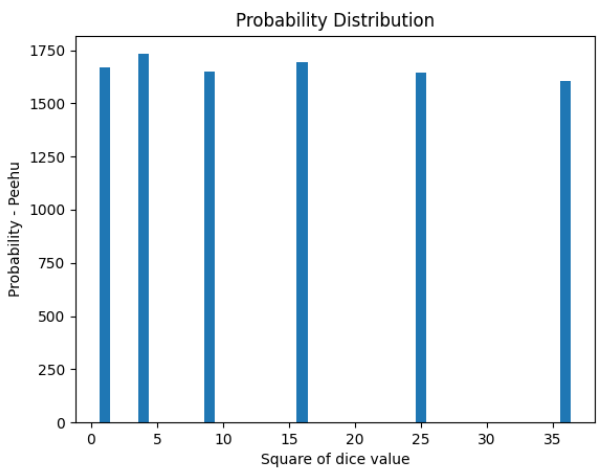

Fig. 2. Sketch of Probability Mass Function for square obtained by taking a sample of random variables

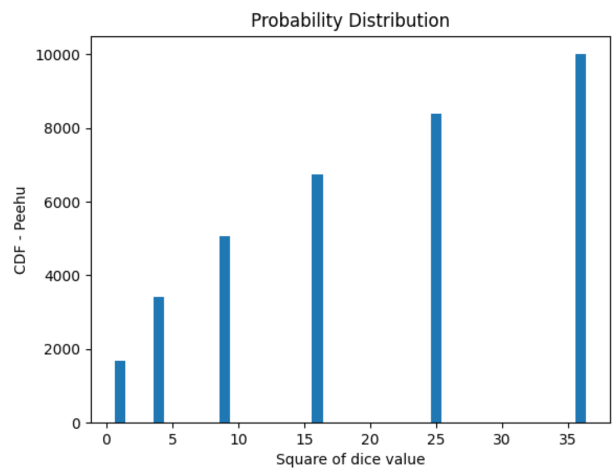

Fig. 2. Sketch of Cumulative Distribution Function for square obtained by taking a sample of random variables

function for Peehu is:

$$F_E(k) = \Pr(E \leq k) \tag{16}$$

$$= \begin{cases} 0 & \text{if } k \leq 0 \\ \frac{\lfloor \sqrt{k} \rfloor}{6} & \text{if } k \in \{1, 2, ..., 35\} \\ 1 & \text{if } k \geq 36 \end{cases} \tag{17}$$

As $p_E(36) > p_{XY}(36)$, Peehu has a better chance of getting the number 36 than Apoorv.