① $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$\lambda_1 + \lambda_2 = 2$

$\lambda_1 \lambda_2 = 0. \quad \Rightarrow \lambda_1 = 2, \lambda_2 = 0.$

0.5 { Yes $\Rightarrow$ It is a covariance matrix } 0.5 for reasoning

$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ 0.5 { No as it is not symmetric } 0.5 for reasoning

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 0.5 { yes , This is covariance matrix of iid standard normal Gaussian } 0.5 ..

$\begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$ $\begin{vmatrix} 1-\lambda & 2 \\ 2 & 6-\lambda \end{vmatrix} = 0$

$\Rightarrow \quad 6 - 7\lambda + \lambda^2 - 4 = 0.$

$\lambda^2 - 7\lambda + 2 = 0.$

0.5 { yes

$\lambda_1 = \dfrac{7 + \sqrt{49-8}}{2}, \quad \lambda_2 = \dfrac{7 - \sqrt{41}}{2}.$

$\lambda_1, \lambda_2 \geq 0.$ } 0.5 for reasoning

② $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$(2-\lambda)(2-\lambda) - 1 = 0$

$4 - 4\lambda + \lambda^2 - 1 = 0$

$(\lambda - 3)(\lambda - 1) = 0$

$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 3 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$2x_1 + x_2 = 3x_1$ 2 for correct eigen vectors

$x_2 = x_1$

$\Rightarrow$ { $D = \begin{bmatrix} 3 & \\ & 1 \end{bmatrix}$, $Q = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

1 mark for correct eigen values

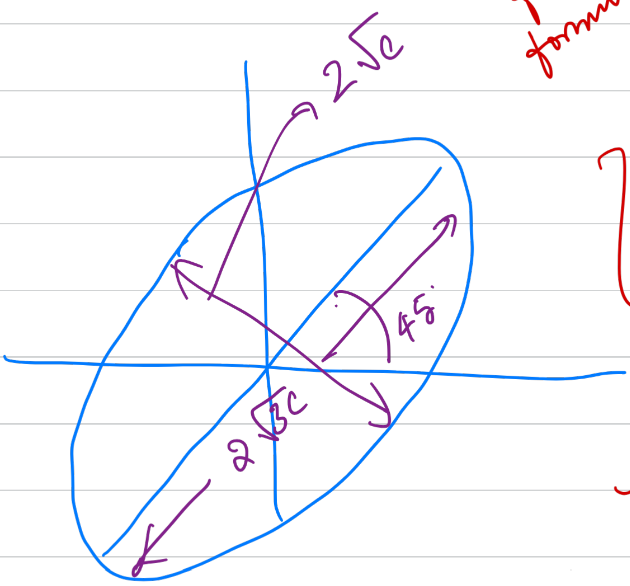
(b) $K^{-1} = Q D^{-1} Q^T$ $\qquad f_X(x) = \dfrac{e^{-\frac{x^T Q D^{-1} Q^T x}{2}}}{(\sqrt{2\pi})^2 (\det(K))^{1/2}}$

To find $x \in R^2$ st

$$\frac{e^{-\frac{x^T Q D^{-1} Q^T x}{2}}}{(\sqrt{2\pi})^2 (\det(K))^{1/2}} = \frac{(0.9)}{(\sqrt{2\pi})^2 (\det K)^{1/2}}$$

1.5 for formulation

$$x^T Q D^{-1} Q^T x = 2 \ln\left(\frac{1}{0.9}\right) = c$$



let $y = Q^T x$

$$y^T D^{-1} y = c.$$

1.5 for accurate tilt (45°) length descriptions.

$$\frac{y_1^2}{3} + \frac{y_2^2}{1} = c.$$

(c) $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ $\qquad Y = AX$

$$\underbrace{K_Y = E[YY^T] = E[AXX^TA^T] = A K_X A^T}_{2 \text{ for the formula}}$$

1 for correct calculation

$$= \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 5 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 14 & 9 \\ 9 & 6 \end{bmatrix}$$

(d) $f_Y(y) = \dfrac{e^{-\frac{y^T K_Y^{-1} y}{2}}}{(2\pi) \det(K_Y)}$ $\Big\}$ 0.5

$$\det(K_y) = 14 \times 6 - 81$$

$$= 84 - 81 = 3$$

$$K_y^{-1} = \begin{bmatrix} 6 & -9 \\ -9 & 14 \end{bmatrix} \frac{1}{3}$$

0.5 for this computation

$$= f_Y(y) = \frac{e^{-\frac{1}{6}(y_1, y_2)\begin{bmatrix} 6 & -9 \\ -9 & 14 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}}}{2\pi\sqrt{3}}$$

$$f_{Y_1,Y_2}(y_1, y_2) = \frac{e^{-\frac{1}{6}\left[6y_1^2 + 14y_2^2 - 18y_1y_2\right]}}{2\pi\sqrt{3}}$$