EE1080/AI1110/EE2120 Probability, Quiz 7

April 28, 2025

Max. Marks: 14. **Time:** 1 hour.

Instructions

- Please **write your roll number**, **serial number** (used for attendance) and **course id** prominently in the first page of the answer sheet.
- No laptops, mobile devices etc. allowed.
- please write supporting arguments for any of the statements you make. any result proved in the lectures can be used by stating it clearly, no need to prove them in your answer.

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1. (4 marks) Identify all the covariance matrices from the below list:

   $$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$$

   Please indicate the reasoning for your answer.

2. (3+3+3+1) Given a 2-Gaussian zero-mean random vector, X with covariance matrix

   $$K_X = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

   (a) Find orthogonal matrix Q, and diagonal matrix D such that $K = QDQ^T$

   (b) identify the points that have pdf evaluating to 90 percent of the maximum value that the pdf can take and plot them

   (c) Find the covariance matrix of random vector Y defined as $Y = AX$ where $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$

   (d) Find the pdf of Y. Simplify as much as possible.

   Note that the pdf of a n-Gaussian random vector is given by:

   $$f_X(x) = \frac{1}{(2\pi)^{\frac{n}{2}}(\det(K))^{\frac{1}{2}}} e^{-\frac{1}{2}x^T K^{-1} x}, \quad x \in \mathbb{R}^n$$