.

EE1101: Circuits and Network Analysis

Lecture 07: Node Analysis

August 11, 2025

Topics :

1. Node Analysis - Circuits with Voltage Sources

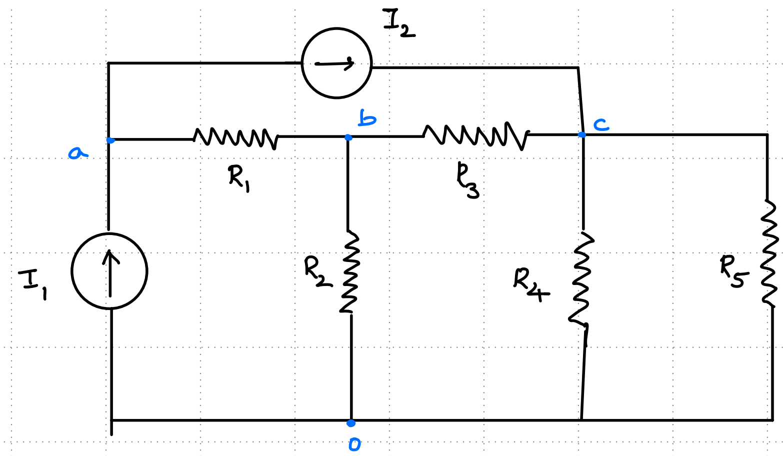
Example - Use of Matrices in Node Analysis

Recall :- by applying KCL at nodes of unknown potential → $\sum I_{leaving} = 0$

$\sum_{j \in i} I_{ij} = 0$

Current from node 'i' to node 'j'

$j \in i$ ↳ all nodes connected to node 'i'



for the Example :-

Node a: $\frac{1}{R_1} V_a - \frac{1}{R_1} V_b = I_1 - I_2 \longrightarrow (1)$

Node b: $-\frac{1}{R_1} V_a + \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) V_b - \frac{1}{R_3} V_c = 0 \longrightarrow (2)$

Node c: $-\frac{1}{R_3} V_b + \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right) V_c = I_2 \longrightarrow (3)$

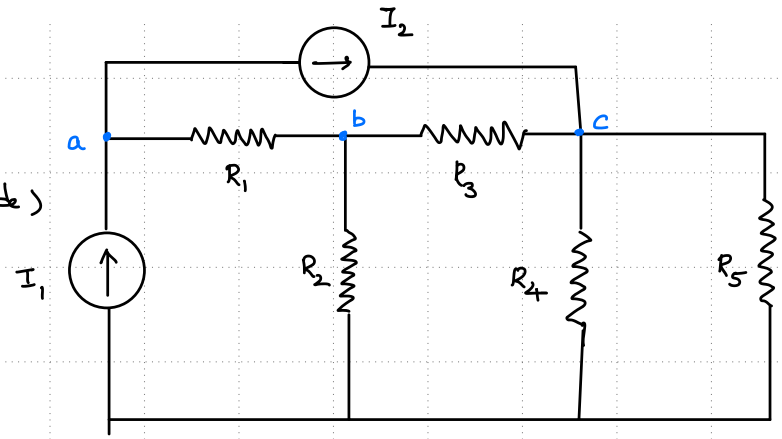
Example - Use of Matrices in Node Analysis

Develop an Equation of the form

$[G][V] = [I] \longrightarrow I_{Eq}$ (injected at each node)

$[G] \longrightarrow$ Conductance matrix

$[V] \longrightarrow$ Vector of unknown node voltages.



For the system of Eqn's to be consistent, we first need to define $[V] = [V_a, V_b, V_c]^t$

For Example:

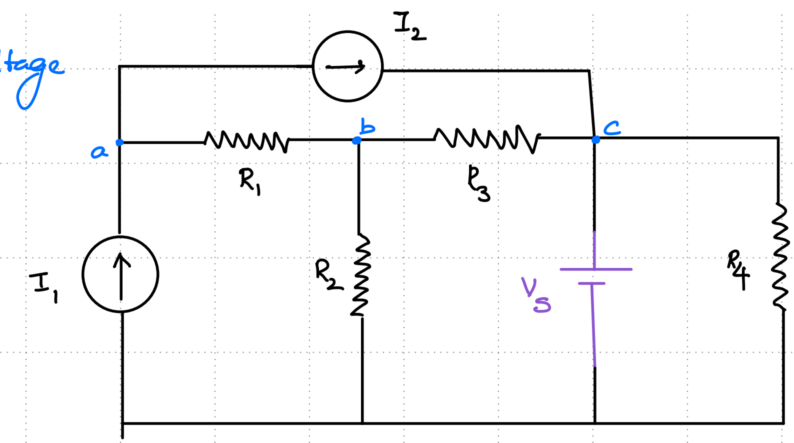
$$\begin{bmatrix} \frac{1}{R_1} & -\frac{1}{R_1} & 0 \\ -\frac{1}{R_1} & \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_3} \\ 0 & -1/R_3 & \sum_{i=3}^{5} \frac{1}{R_i} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ 0 \\ I_2 \end{bmatrix}$$

Circuits with Voltage Sources - Between Node and Reference Node

that particular node voltage is known

$\Downarrow$

apply KCL at nodes of unknown potential.



for example: $V_c = V_s$

Node a :- $\frac{1}{R_1} V_a - \frac{1}{R_1} V_b = I_1 - I_2$.

Node b : $-\frac{1}{R_1} V_a + \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) V_b = \underbrace{\frac{V_s}{R_3}}_{}$ Eq. Current.

In matrix form

$[V] = [V_a, V_b]$

$$\begin{bmatrix} \frac{1}{R_1} & -\frac{1}{R_1} \\ -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ \frac{V_s}{R_3} \end{bmatrix}$$

Circuits with Voltage Sources - Between two non-reference Nodes

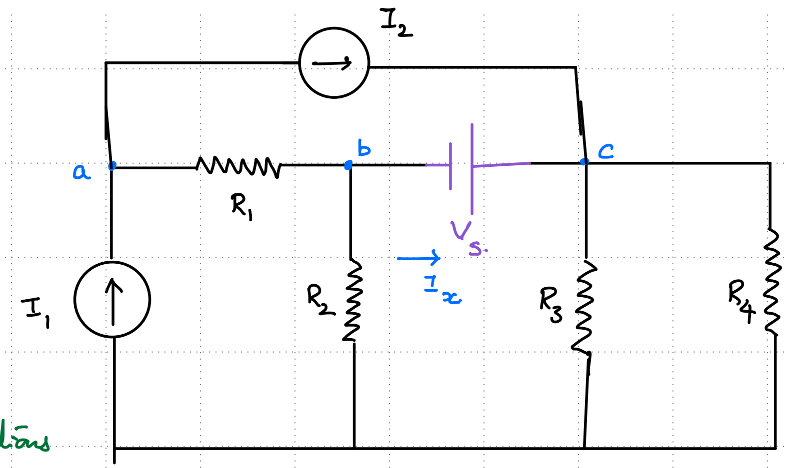
Approach 1:- Introduce a new variable (i.e, Current through the Voltage Source)

⇓

introduces new set of unknowns

⇓

as many no. of equations from the P.D from the the two nodes



3 unknown $\underbrace{\text{node voltages } (V_a, V_b, V_c)}_{\text{3 Eqn's by applying KCL}}$ & $\underbrace{\text{unknown current } I_x}_{V_c - V_b = V_s}$.

at Node a: $\frac{1}{R_1} V_a - \frac{1}{R_1} V_b = I_1 - I_2 \longrightarrow (1)$

at Node b: $-\frac{1}{R_1} V_a + \left(\frac{1}{R_2} + \frac{1}{R_1}\right) V_b + I_x = 0 \longrightarrow (2)$

at Node c: $\left(\frac{1}{R_3} + \frac{1}{R_4}\right) V_c - I_x = I_2 \longrightarrow (3)$

$V_c - V_b = V_s \longrightarrow (4)$

Solve (1) - (4) to find V_a, V_b, V_c, I_x

(or)

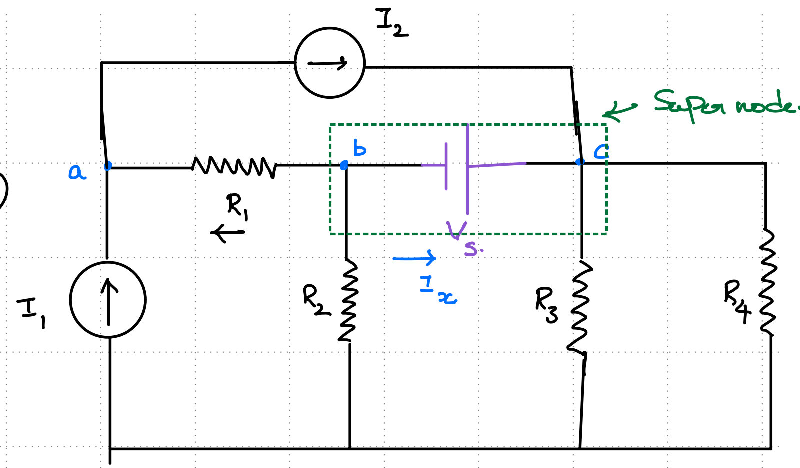
Solve (1), (5) & (4) to find V_a, V_b, V_c.

from (2) & (3) $\Rightarrow$ $-\frac{1}{R_1} V_a + \left(\frac{1}{R_2} + \frac{1}{R_1}\right) V_b + \left(\frac{1}{R_3} + \frac{1}{R_4}\right) V_c = I_2 . \longrightarrow (5)$

Circuits with Voltage Sources - Between two non-reference Nodes

focus on Eq (5)

$$\underbrace{-\frac{1}{R_1} V_a + \left(\frac{1}{R_2}+\frac{1}{R_1}\right) V_b}_{\text{KCL at Node } b} + \underbrace{\left(\frac{1}{R_3}+\frac{1}{R_4}\right) V_c}_{\text{KCL at Node } c} = I_2 . \longrightarrow (5)$$



Approach 2 :- def Super node

Apply KCL at $\underbrace{\text{Super node + PD Eqn}}_{\text{2 set of Eqn's for the 2 unknown node potentials.}}$

2 set of Eqn's for the 2 unknown node potentials.

for the example :

at Supernode bc :- $\dfrac{V_b - V_a}{R_1} + \dfrac{V_b}{R_2} + \dfrac{V_c}{R_3} + \dfrac{V_c}{R_4} = I_2 .$