Dependent Sources and Three-Terminal Elements

- The sources considered for analysis so far namely voltage source and the current source are commonly referred to as independent sources. The voltage across an independent voltage source is not dependent on the external circuit. Similarly, the current through a current source is not dependent on the external circuit. The other class of sources are the dependent sources, whose voltage/current is dependent/controlled by some other voltage or current in the circuit.

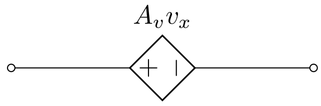


Figure 1: Representation of Voltage controlled voltage source.

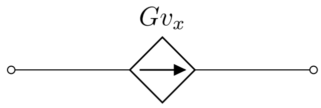


Figure 2: Representation of Voltage controlled current source.

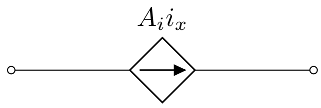


Figure 3: Representation of Current controlled current source.

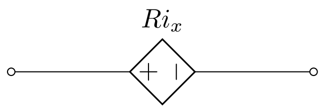


Figure 4: Representation of Current controlled voltage source.

  Dependent voltage sources are often used for modeling three terminal circuit elements like operational amplifiers, transistors, etc. There are four types of dependent sources:

  - Voltage-controlled voltage source (VCVS): The voltage across the source is proportional to a voltage elsewhere in the circuit. The VCVS are commonly used to model operational amplifiers. The circuit symbol of a VCVS is shown in Figure 1. The voltage across the source is given by $A_v v_x$, where A_v is the gain (in V/V) of the VCVS and v_x is the controlling voltage.
  - Voltage-controlled current source (VCCS): The current through the source is proportional to a voltage elsewhere in the circuit. The VCCS are commonly used to model Field Effect Transistors (FETs). The circuit symbol of a VCCS is shown in Figure 2. The current through the source is given by Gv_x, where G is the transconductance (in S) of the VCCS and v_x is the controlling voltage.
  - Current-controlled current source (CCCS): The current through the source is proportional to a current elsewhere in the circuit. The CCCS are also used in the modeling of BJTs. The circuit symbol of a CCCS is shown in Figure 3. The current through the source is given by $A_i i_x$, where A_i is the gain (in A/A) of the CCCS and i_x is the controlling current.
  - Current-controlled voltage source (CCVS): The voltage across the source is proportional to a current elsewhere in the circuit. The circuit symbol of a CCVS is shown in Figure 4. The voltage across the source is given by Ri_x, where R is the transresistance (in V/A) of the CCVS and i_x is the controlling current.

  The circuit techniques such as node and mesh analysis can also be used for analyzing circuits with dependent sources. However, some circuit analysis techniques to follow such as superposition and Thevenin/Norton equivalents may require special consideration when dependent sources are present.

- Instead of simply analyzing circuits with dependent sources using node and mesh analysis, this course focuses on making the study more meaningful by examining circuits that incorporate semiconductor devices, which can be modeled with dependent sources. We will introduce semiconductor devices briefly, focusing on how

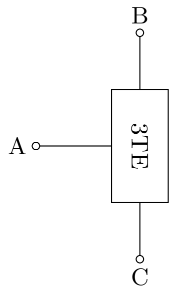


Figure 5: Generic representation of a three-terminal element.

they work from a circuit point of view. We'll look at the conditions that cause these devices to operate in different modes and how to model them using circuit elements like dependent sources. The goal is to apply circuit analysis principles and use the established equivalent circuit models to solve circuits involving these devices.

- **Circuits with three terminal elements**: The devices covered in this course broadly fall under the category of three-terminal devices. A key feature of these devices is that the voltage or current at one terminal (called the control port/terminal) controls the voltage at or current through the other two terminals (called the main port/circuit). The main port is where the external circuit is connected. Unlike two-terminal devices, three-terminal devices can operate in different modes, namely, as a switch or as an amplifier. The mode of operation is determined by the voltage or current at the control port. In switch mode, the elements exhibits very low resistance between the terminals of the main/circuit (ON state) or very high resistance (OFF state). In amplifier mode, the device behavior can be modeled using dependent sources.

  To begin with, circuits comprising of generic three terminal elements will be analyzed. When analyzing generic circuits, the definition of a three terminal element includes a clear description of its control and main ports, conditions under which it behaves as a switch and the equivalent circuit model when operating as an amplifier.

  Example Definition: The three-terminal element shown in Figure 5 is defined as follows:

  – Control Port: Terminals A and C
  – Main Port: Terminals B and C
  – Switch Mode: When the voltage between A and C is below a certain threshold (say V_t), the element behaves as a switch in OFF state and when the voltage between A and C is above the threshold(say V_{on}), the element behaves as a switch in ON state.
  – Amplifier Mode: When the voltage between A and C satisfies $V_t < V < V_{on}$, the element behaves as an amplifier. The equivalent circuit model in this mode is a VCCS as shown in Fig. 6.

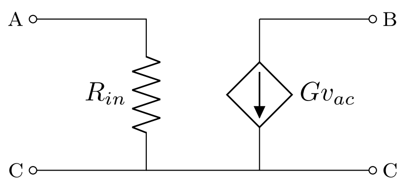


Figure 6: Example of an equivalent circuit model of a three-terminal element in amplifier mode.

  Note that under ideal device modeling, it is a common practice to consider a device operating as a switch to be considered as a perfect short circuit (i.e., zero resistance) in ON state and as a perfect open circuit (i.e., infinite resistance) in OFF state. However, there are practical limitations to this idealization. For instance, a device in ON state will have a small but non-zero resistance and a device in OFF state will have a very high but finite resistance. These non-idealities can be incorporated into the device model if required.

Also, the actual circuit models of the three terminal devices operating in amplifier mode can be more complex and non-linear. The models considered in this course are simplified linear models (or small signal models, in some cases) that capture the essential behavior of the device for the purpose of circuit analysis.

To get ourselves familiar with solving circuits with three terminal elements, we will consider an example. For this example, the three terminal element is defined as per the definition provided above i.e., the device is in OFF state, when $V_{ac} < V_t$, in ON state when $V_{ac} > V_{on}$ and in amplifier mode when $V_t < V_{ac} < V_{on}$. The equivalent circuit model in amplifier mode is a VCCS as shown in Fig. 6.

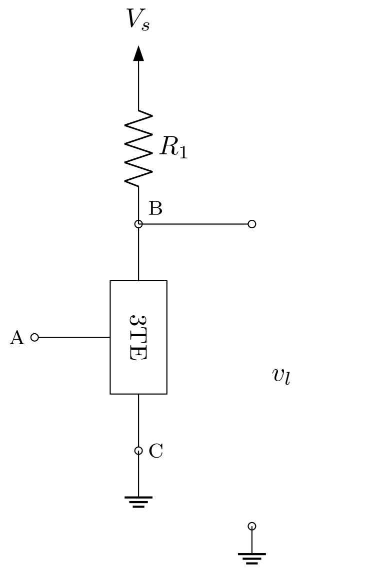


Figure 7: Circuit for Example 1

Example 1: Consider the circuit shown in Figure 7. The values of V_t and V_{on} for the three terminal element are given as follows: $V_t = 0.5V$ and $V_{on} = 5V$. Analyze the circuit for the following input conditions:

- Case 1: $V_{AC} = 0.5V$
- Case 2: $V_{AC} = 1.0V$
- Case 3: $V_{AC} = 5.0V$

For the circuit being analyzed, the voltage v_l is given by $v_l = V_s - I_1R_1$, where I_1 is the current flowing through the resistor R_1.

- When $V_{AC} = 0.5V$, the three terminal element is in OFF state i.e., the device acts as an open circuit. Thus no current flows through the resistor R_1 and the output voltage V_l is equal to the supply voltage V_s.
- When $V_{AC} = 5.0V$, the three terminal element is in ON state i.e., the device acts as a short circuit. The voltage across the terminals B and C is zero. Thus node B is also at the ground potential. Hence, the output voltage V_l is zero. The current flowing through the resistor R_1 is given by $I_1 = V_s/R_1$.
- When $V_{AC} = 1.0V$, the three terminal element is in amplifier mode. The circuit to be analyzed in this mode (obtained by replacing the three terminal element with its equivalent circuit model) is shown in Figure 8. The current through the resistor R_1 is given by $I_1 = G \cdot V_{AC}$. Thus, the output voltage V_l is given by

$$V_l = V_s - I_1R_1 = V_s - GV_{AC}R_1.$$

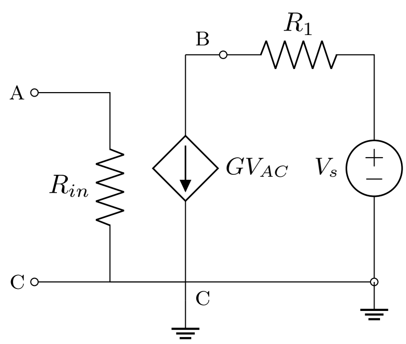


Figure 8: Circuit in Example 1 when the three terminal element is in amplifier mode.

It is interesting to note that the output voltage v_l equals V_s when $V_{AC} < V_t$ and is 0 when $V_{AC} > V_{on}$. Thus, by varying the input voltage frequently between V_t and V_{on}, the output voltage toggles between 0 and V_s. This binary switching behavior forms the fundamental principle behind digital logic circuits. Additionally, it illustrates how a three-terminal device can be used to generate a AC (not perfectly sinusoidal) signal from a DC voltage (in this case V_s) [1].

[1] Practical circuits are more complex than this basic model!

- **Bipolar Junction Transistor (BJT) - from a circuit point of view**: The Bipolar Junction Transistor (BJT) is a three-terminal semiconductor device that

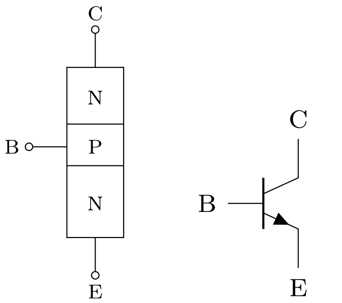


Figure 9: Structural representation (left) and circuit symbol (right) of an NPN BJT.

can be used as a switch or an amplifier. The three terminals of a BJT are called the Emitter (E), Base (B) and Collector (C). The BJT comes in two types: NPN and PNP, which differ in the arrangement of the semiconductor materials. In this course, we will focus on the NPN type of BJT. The structural representation and circuit symbol of an NPN BJT are shown in Fig. 9. The BJT comprises of two PN junctions: the emitter-base junction and the collector-base junction.

From a circuit point of view, the BJT can be defined as follows:

- Control Port: Terminals B and E
- Main Port: Terminals C and E
- Switch Mode (OFF state): When the voltage between B and E (V_{BE}) is below a certain threshold (say $V_t \approx 0.7V$ for silicon BJTs), the BJT behaves as a switch in OFF state.
- Switch Mode (ON state): When the voltage between B and E (V_{BE}) is above $V_t(\approx 0.7V)$ and the voltage between C and E is below $V_{CE,sat}$ (typically 0.2 V).
- Amplifier Mode: When the voltage between B and E (V_{BE}) is above $V_t(\approx 0.7V)$ and the voltage between C and E (V_{CE}) is above $V_{CE,sat}$ (typically 0.2 V), the BJT behaves as an amplifier. The equivalent circuit model in this mode is a CCCS as shown in Fig. 10.

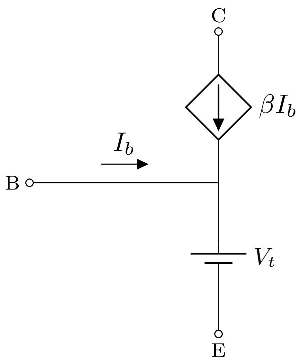


Figure 10: Equivalent circuit model of a BJT in amplifier mode.

It is interesting to note that when the V_{BE} is above the threshold voltage V_t, the BJT can operate either as a switch (in ON state) or as an amplifier. The factor that determines the mode of operation is the voltage V_{CE}. While this might give the impression that the controlling quantity is not associated with the control port, this is not True. The V_{CE} voltage is influenced by the base-emitter voltage V_{BE}, and thus the control port does play a role in determining the mode of operation. To understand this better, the operation of BJT as a switch is considered next.

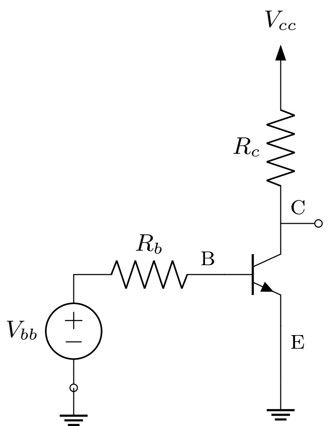


Figure 11: Circuit to operate a BJT as a switch.

- **BJT as a Switch**[1]: Consider the circuit shown in Fig. 11, where it is desired to operate the BJT as a switch. When V_{bb} is such that $V_{BE} < V_t$, the BJT is in the OFF state and no current flows through the collector resistor R_c. When V_{bb} is such that $V_{BE} > V_t$, the BJT can operate either in the ON state or in amplifier mode. To ensure that the BJT operates in the ON state, the collector-to-emitter voltage must be less than $V_{ce,\text{sat}}$. For the given circuit, the collector-emitter voltage V_{CE} is given by

$$V_{CE} = V_{cc} - I_c R_c,$$

where I_c is the collector current. To ensure that the BJT is in the ON state, the

[1]Suggested reference: A. S. Sedra and K. C. Smith, *Microelectronic Circuits*, Oxford University Press

following condition must be satisfied:

$$V_{CE} < V_{ce,\text{sat}} \implies V_{cc} - I_c R_c < V_{ce,\text{sat}} \implies I_c > \frac{V_{cc} - V_{ce,\text{sat}}}{R_c}.$$

As long as $I_c < \frac{V_{cc}-V_{ce,\text{sat}}}{R_c}$, the BJT will operate in amplifier mode. The critical value of I_c at which the BJT transitions from amplifier mode to the ON state is called the Edge-of-Saturation ($I_{c,\text{eos}}$) and is given by

$$I_{c,\text{eos}} = \frac{V_{cc} - V_{ce,\text{sat}}}{R_c}.$$

Since the collector current (in amplifier mode) is a function of base current, increasing the base current (achieved by increasing the voltage V_{bb}) results in an increase in collector current. The critical value of base current at which the BJT transitions from amplifier mode to the ON state is given by

$$I_{b,\text{eos}} = \frac{I_{c,\text{eos}}}{\beta} = \frac{V_{cc} - V_{ce,\text{sat}}}{\beta R_c}.$$

One way to drive the base current above $I_{b,\text{eos}}$ is to choose the voltage V_{bb} such that the base current I_b is much higher[2] than $I_{b,\text{eos}}$, i.e.,

$$V_{bb} > I_{b,\text{eos}} R_b + V_t \quad \text{(by KVL to base-emitter loop)}.$$

[2] To account for any variations in the circuit parameters or operating conditions, the circuit is typically designed such that I_b is much higher than $I_{b,\text{eos}}$

Note that increasing the base current beyond $I_{b,\text{eos}}$ does not increase the collector current beyond $I_{c,\text{eos}}$. The collector current remains approximately constant at $I_{c,\text{eos}}$.

It is, however, **important** to note that a varying V_{bb} is not always possible. Circuits are often driven at fixed voltage levels (say, 5 V or 3.3 V). In such cases, to ensure that the BJT operates in the ON state for the available voltage levels, it is necessary to choose the circuit elements (such as resistors) such that $V_{CE} < V_{ce,\text{sat}}$.

- **Analyzing BJT circuits at DC**: The analysis of BJT circuits at DC involves determining the operating mode of the BJT (i.e., OFF, ON, or amplifier mode) and calculating the relevant voltages and currents in the circuit. To begin with, the voltage between the base-emitter junction is first observed. If $V_{BE} > V_t$, the BJT operates either in the ON state or in amplifier mode. At this point, it is common practice to assume that the BJT is operating in amplifier mode and analyze the circuit accordingly.

  If the V_{CE} obtained based on the computed values is greater than $V_{CE,sat}$, the device is in amplifier mode and the analysis is complete. On the other hand, if the computed V_{CE} is less than $V_{CE,sat}$, the assumption of amplifier mode is incorrect, and the BJT is actually in the ON state. The circuit must then be re-analyzed with the BJT replaced by a short circuit between the collector and emitter.

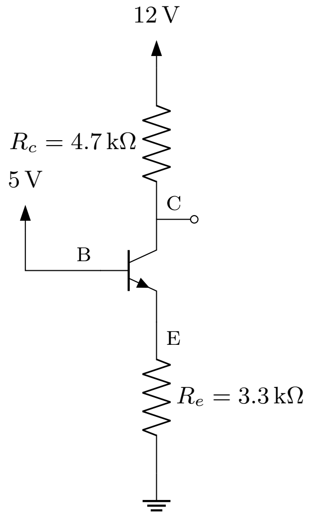


Figure 12: Circuit for Example 2.

Example 2: Consider the circuit shown in Fig. 12. Analyze the circuit to determine the operating mode and the voltages and currents associated with various elements of the circuit[2]. The parameters of the BJT are given as follows: $\beta = 100$, $V_t = 0.7V$ and $V_{CE,sat} = 0.2V$. Since $V_b = 5\,\mathrm{V}$, we begin tha anslysis by assuming that the BJT is operating in amplifier mode. Applying KVL to the base-emitter loop results in

$$-5 + V_t + (3.3\,\mathrm{k})I_e = 0 \implies I_e = 1.30\,\mathrm{mA}.$$

The collector current is given by

$$I_c = \frac{\beta}{\beta+1} I_e = 1.29\,\mathrm{mA}.$$

Applying KVL to the collector-emitter loop results in

$$-12 + (4.7\,\mathrm{k})I_c + V_{CE} + (3.3\,\mathrm{k})I_e = 0 \implies V_{CE} \approx 4.0\,\mathrm{V}.$$

Since $V_{CE} > V_{CE,sat}$, the assumption of amplifier mode is correct. Thus, the BJT is operating in amplifier mode with $I_b = 13\,\mu\mathrm{A}$, $I_c = 1.29\,\mathrm{mA}$, $I_e = 1.30\,\mathrm{mA}$, $V_{BE} = 0.7\,\mathrm{V}$ and $V_{CE} \approx 4.0\,\mathrm{V}$.

Example 3: Determine the operating mode of the BJT in the circuit shown in Fig. 12 when V_b is increased to 6 V and R_e is chosen to be $2\,\mathrm{k}\Omega$. Assume that the other parameters remain the same.

Assuming that the BJT is in amplifier mode, applying KVL to the base-emitter loop results in

$$-6 + V_t + (2\,\mathrm{k})I_e = 0 \implies I_e = 2.65\,\mathrm{mA}.$$

The collector current is given by

$$I_c = \frac{\beta}{\beta+1} I_e = 2.62\,\mathrm{mA}.$$

Applying KVL to the collector-emitter loop results in

$$-12 + (4.7\,\mathrm{k})I_c + V_{CE} + (2\,\mathrm{k})I_e = 0 \implies V_{CE} \approx -5.63\,\mathrm{V}.$$

Since $V_{CE} < V_{CE,sat}$, the assumption of amplifier mode is incorrect and the BJT is actually in the ON state. The circuit must be re-analyzed with the BJT replaced by a short circuit between the collector and emitter[3]. Applying KVL to the base-emitter loop results in

$$I_e = 2.65\,\mathrm{mA}.$$

[3] For this course. Although, it should be noted that it is a common practice to consider the ON-state drop i.e., $V_{CE,sat}$ between the collector and emiiter terminals during saturation mode/ON-state.

[2] Values taken from A. S. Sedra and K. C. Smith, *Microelectronic Circuits*, Oxford University Press

The voltage at the emitter and the colllector terminals of the the BJT is given by

$$V_e = R_E I_E = 5.3\,\text{V}, \quad V_c = V_E = \ 5.3\,\text{V (Neglecting ON-state drop)}$$
$$= V_E + V_{CE,sat} = \ 5.5\,\text{V (considering ON-state drop).}$$

The collector current (neglecting ON-state drop) is given by

$$I_c = \frac{V_{cc} - V_c}{R_c} = \frac{12 - 5.3}{2\,\text{k}} = 3.35\,\text{mA}.$$

Thus, the BJT is operating in the ON state with $I_c = 3.35\,\text{mA}$, $I_e = 2.65\,\text{mA}$, $V_{BE} = 0.7\,\text{V}$ and $V_{CE} = 0\,\text{V}$.

- **MOSFET - from a circuit point of view**: The Metal-Oxide-Semiconductor Field-Effect Transistor (MOSFET) is another three-terminal semiconductor device that can be used as a switch or an amplifier. The three terminals of a MOSFET are called the Source (S), Gate (G) and Drain (D). The MOSFET comes in two types: N-channel and P-channel, which differ in the arrangement of the semiconductor materials. In this course, we will focus on the N-channel type of MOSFET. The circuit symbol of an N-channel MOSFET are shown in Fig. 13. The MOSFET comprises of a channel region between the source and drain terminals, which can be modulated by the voltage applied at the gate terminal. From a circuit point of view, the MOSFET can be defined as follows:

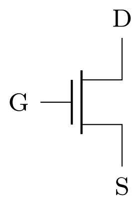


Figure 13: Structural representation (left) and circuit symbol (right) of an N-channel MOSFET.

  - Control Port: Terminals G and S
  - Main Port: Terminals D and S
  - Switch Mode (OFF state): When the voltage between G and S (V_{GS}) is below a certain threshold (say $V_t \approx 2V$ for typical N-channel MOSFETs), the MOSFET behaves as a switch in OFF state.
  - Switch Mode (ON state): When the voltage between G and S (V_{GS}) is above V_t and the voltage between D and S (V_{DS}) is below $V_{GS} - V_t$.
  - Amplifier Mode: When the voltage between G and S (V_{GS}) is above V_t and the voltage between D and S (V_{DS}) is above $V_{GS} - V_t$, the MOSFET behaves as an amplifier. The equivalent circuit model (for Large Signl and DC Analysis) in this mode is a VCCS as shown in Fig. 14. The large signal equivalent model is modlled with a dependent current source whose value is given by $K(V_{GS} - V_t)^2$, where K is a constant that depends on the physical and electrical characteristics of the MOSFET. The model is nonlinear due to the square-law relationship between the gate-source voltage and the drain current. For small signal analysis, the MOSFET can be linearized around a specific operating point, resulting in a small signal equivalent model that includes a transconductance parameter (g_m) and an output resistance (r_d). The small signal model of the MOSFET is shown in Fig 15.

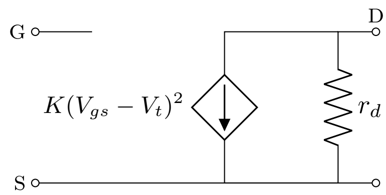


Figure 14: Equivalent circuit model of a MOSFET (for DC/Large Signal Analysis) in amplifier mode.

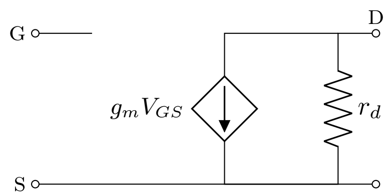


Figure 15: Small signal equivalent circuit model of a MOSFET in amplifier mode.

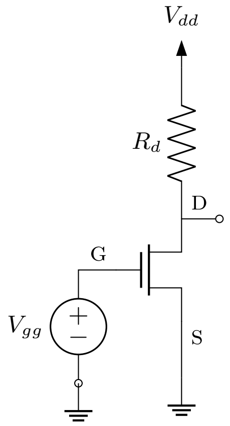


Figure 16: Circuit to operate a MOSFET as a switch.

- **MOSFET as a Switch**: Consider the circuit shown in Fig. 16, where it is desired to operate the MOSFET as a switch. When V_{gg} is such that $V_{GS} < V_t$, the MOSFET is in the OFF state and no current flows through the drain resistor R_d. When V_{gg} is such that $V_{GS} > V_t$, the MOSFET can operate either in the ON state or in amplifier mode. To ensure that the MOSFET operates in the ON state, the drain-to-source voltage must be less than $V_{GS} - V_t$. For the given circuit, the drain-source voltage V_{DS} is given by

$$V_{DS} = V_{dd} - I_d R_d,$$

where I_d is the drain current. To ensure that the MOSFET is in the ON state, the following condition must be satisfied:

$$V_{DS} < V_{GS} - V_t \implies V_{dd} - I_d R_d < V_{GS} - V_t \implies I_d > \frac{V_{dd} - (V_{GS} - V_t)}{R_d}.$$

As long as $I_d < \frac{V_{dd}-(V_{GS}-V_t)}{R_d}$, the MOSFET will operate in amplifier mode. The critical value of I_d (denoted by I_d^*) at which the MOSFET transitions from amplifier mode to the ON state is given by

$$I_d^* = \frac{V_{dd} - (V_{GS} - V_t)}{R_d}.$$

The drain current is a nonlinear function of gate-source voltage. However, for a given V_{GS}, the drain current can be determined using the large signal equivalent model shown in Fig. 14. The critical value of gate-source voltage at which the MOSFET transitions from amplifier mode to the ON state (say V_{GS}^*) can be determined by solving the following equation:

$$K(V_{GS}^* - V_t)^2 = I_d^* = \frac{V_{dd} - (V_{GS}^* - V_t)}{R_d}.$$

The resulting quadratic equation that needs to be solved is

$$KR_d(V_{GS}^* - V_t)^2 + V_{GS}^* - V_t - V_{dd} = 0 \implies V_{GS}^* - V_t = \frac{-1 \pm \sqrt{1 + 4KR_d(V_{dd} + V_t)}}{2KR_d}.$$

Similar to BJTs, MOSFETs are often driven at fixed voltage levels (say, 5 V or 3.3 V). In such cases, to ensure that the MOSFET operates in the ON state for the available voltage levels, it is necessary to choose the circuit elements (in this case, R_d) such that $V_{DS} < V_{GS} - V_t$.

- **Operational Amplifier (Op-Amp) - From a circuit point of view**: The Operational Amplifier (Op-Amp) is a three-terminal device with two input terminals and one output terminal. The two input terminals are the non-inverting terminal (marked +) and the inverting terminal (marked −). Op-Amps are designed to operate exclusively as a high-gain amplifier, and unlike devices such as BJT or

MOSFET, it is not typically used as a switch. Instead, Op-Amps are intended for analog signal processing applications where linear behavior is desired.

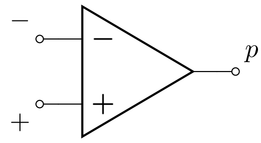


Figure 17: Circuit symbol of an Op-Amp. Note: Power supply terminals are not shown.

The circuit symbol of an Op-Amp is shown in Fig. 17. The voltage at the output terminal of an ideal Op-Amp (when no feedback is present) is given by

$$v_{out} = A\,(v_+ - v_-) \tag{1}$$

where v_+ and v_- are the voltages at non-inverting and inverting terminals of the Op-Amp respectively and A is the gain of the Op-Amp. The gain of the Op-Amp is very high and thus equation 1 tends to suggest that a very large voltage can be generated at the output. However, this is not the case. Op-Amps are powered by two additional supply terminals namely, a positive supply ($+V_{s1}$) and a negative supply ($-V_{S2}$), which are usually not shown in circuit diagrams for simplicity. These supply voltages define the maximum and minimum bounds of the output voltage. When the Op-Amp operates in **open-loop differential mode** (i.e., $v_+ \neq v_-$ and no feedback is present), the output rapidly saturates to one of the supply rails depending on the polarity of the input difference. The transfer characteristic of the Op-Amp (i.e., v_{out} vs $v_+ - v_-$) is shown in Fig. 18. The equivalent circuit of an Op-Amp considering non-idealities is shown in Fig. 19. The open-loop gain A of Op-Amp is very large, the input resistance R_i is very high and the output resistance R_o is very low.

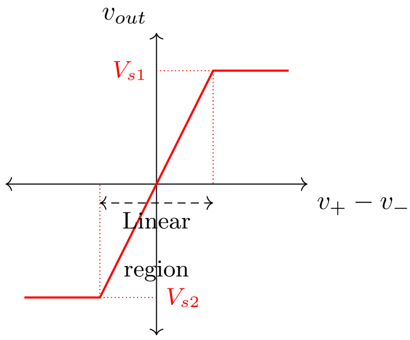


Figure 18: Transfer characteristic of an Ideal Op-Amp

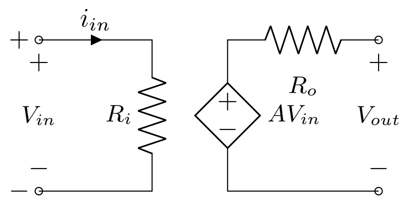


Figure 19: Equivalent circuit of an Op-Amp

A high voltage gain implies that the range over which the Op-Amp operates linearly is very narrow (typically in the order of millivolts or less). However, practical Op-Amp circuits are designed with negative feedback that keeps the input terminals nearly equal in voltage. Under this scenario, to keep the output within the linear region (i.e., avoid saturation), the voltage difference between the inputs must be very close to zero. This condition is referred to as a virtual short i.e.,

$$v_+ \approx v_- \tag{2}$$

Because the input resistance R_i is extremely high, the current flowing into the input terminals of the Op-Amp is negligibly small. For the purpose of analysis, the input current is typically assumed to be zero. These properties, along with the virtual short concept, significantly simplify circuit analysis and will be used extensively in this course. However, it is important to note that not all Op-Amp circuits can be analyzed using the virtual short approximation.

Virtual Short - Definition and Validity: The term "virtual short" refers to the condition where the voltage difference between the two input terminals of an Op-Amp is negligibly small, even though they are not physically connected. This condition is valid only when:

– The Op-Amp operates in the linear region (i.e., the output is not saturated),

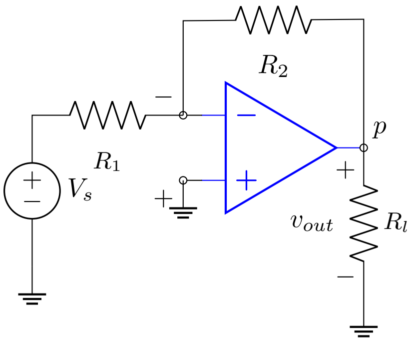


Figure 20: Inverting amplifier using Op-Amp

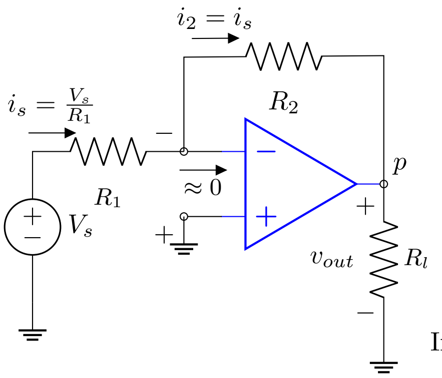


Figure 21: Analysis of Inverting amplifier using virtual short concept

- Negative feedback is present,
- The open-loop gain A_v is sufficiently large.

A useful graphical method to justify the virtual short assumption involves analyzing the intersection of terminal characteristics:

- Consider one input terminal (say v_-) as part of a feedback network connected to the output v_{out}.
- Treat the other terminal (e.g., v_+) as driven by a known source.
- Plot the v_{out} vs v_- characteristic imposed by the feedback network.
- Simultaneously, use the Op-Amp's characteristic $v_{out} = A(v_+ - v_-)$.
- The intersection of these two plots gives the operating point. If it lies within the linear region of the Op-Amp (i.e., within the supply rails), then $v_+ \approx v_-$ and the virtual short condition is valid.

In this course, we will simplify our analysis by considering only those circuits where the virtual short assumption holds. You do not need to explicitly validate it in each case. However, in practice, it is good engineering practice to verify whether the conditions for a virtual short are satisfied before applying this assumption.

- Example 4: Inverting Amplifier: The circuit shown in Fig. 20 is a classic inverting amplifier configuration implemented using an Op-Amp. Assuming the Op-Amp is ideal, we can apply the virtual short concept[4] to simplify the analysis significantly.

[4] Use the graphical approach to justify its use.

  - Since the gain $A_v \to \infty$, $v_- \approx v+$ and equal to 0 (since v_+ is conencted to ground). This the current through R_1 (see Fig. 21 for reference direction) is given by
$$i_s = \frac{V_s}{R_1}$$
  - Further, since $R_i \to \infty$, no current flows into the input terminals of the Op-Amp (i.e., $i_{in} \approx 0$). As a result, the current through R_2 is given by
$$i_2 = i_s = \frac{V_s}{R_1}$$
  - The potential at node p (the output voltage) is given by
$$v_p = v_{out} = \underbrace{v_-}_{0} - i_s R_2 = -\left(\frac{R_2}{R_1}\right) V_s$$

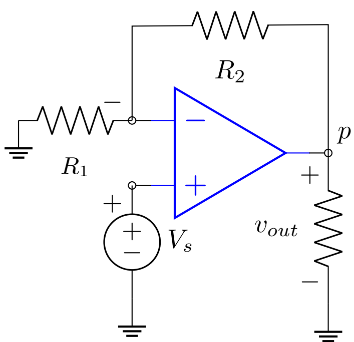


Figure 22: Non-inverting amplifier using Op-Amp

Example 5: Non-inverting Amplifier: The circuit shown in Fig. 22 is the non-inverting amplifier designed using Op-Amp. Assuming the Op-Amp is ideal, we can apply the virtual short concept to simplify the analysis significantly.

- Since the gain $A_v \to \infty$, $v_- \approx v+(=0)$. This the current through R_1 (see Fig. 4(b) for reference direction) is given by

$$i_s = \frac{V_s}{R_1}$$

- Further, since $R_i \to \infty$, no current flows into the input terminals of the Op-Amp (i.e., $i_{in} \approx 0$). As a result, the current through R_2 is given by

$$i_2 = -i_s = -\frac{V_s}{R_1}$$

- The potential at node p (the output voltage) is given by

$$v_p = v_{out} = \underbrace{v_-}_{V_s} - i_2 R_2 = \left(1 + \frac{R_2}{R_1}\right) V_s$$

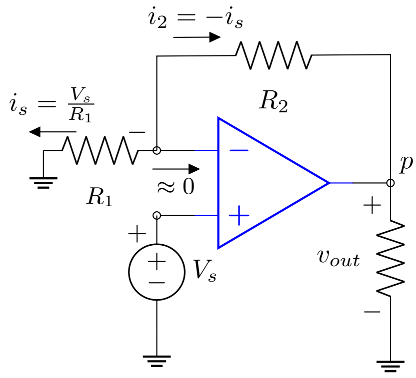


Figure 23: Anslysis of Non-inverting amplifier using Virtual Short Concept

Example 6: Summing amplifier designed using Op-Amp: The circuit shown in Fig. 24(a) is the summing amplifier designed using Op-Amp. Using virtual short,

- Since the gain $A_v \to \infty$, $v_- \approx v_+(=0)$. This the current through R_1 and R_2 (see Fig. 24(b) for reference direction) is given by

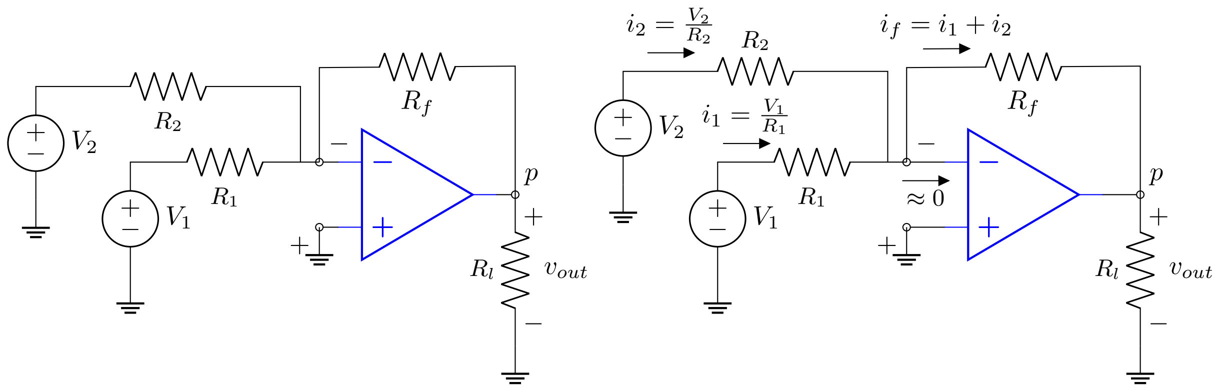
$$i_1 = \frac{V_1}{R_1} \text{ and } i_2 = \frac{V_2}{R_2}$$

- Further, since $R_i \to \infty$, no current flows into the input terminals of the Op-Amp (i.e., $i_{in} \approx 0$). As a result, the current through R_f is given by

$$i_f = i_1 + i_2 = \frac{V_1}{R_1} + \frac{V_2}{R_2}$$

- The potential at node p (the output voltage) is given by

$$v_p = v_{out} = \underbrace{v_-}_{0} - i_f R_f = -\left[\left(\frac{R_f}{R_1}\right) V_1 + \left(\frac{R_f}{R_2}\right) V_2\right]$$



(a) Circuit

(b) Analysis using virtual short

Figure 24: Summing amplifier using Op-Amp