EE2100: Matrix Theory

Assignment - 2

Handed out on : 18 - Aug - 2023 **Due on : 28 - Aug - 2023 (before 5 PM)**

Instructions :

1. Please submit the solutions to the assignment problems to the course page (on the canvas platform). Solutions submitted to the course page will only be evaluated. Refer to the assignment guidelines mentioned on the course page.

2. Submissions received after the deadline will attract negative marking.

3. There are two sections in the assignment.

4. It is suggested that you attempt all the questions (preferably the ones indicated using *). However, it is sufficient to submit solutions for problems that total to at least 10 points.

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1. * (8 Points) **Complex vectors as basis of $\mathcal{R}^N$:** While attempting this question, it is natural to think of complex exponentials as one choice of complex basis. The objective of this question is to explore if there are other possibilities.

   Consider the vector space $\mathcal{R}^2$.

   (a) (2 points) Can the vector space have a basis whose basis vectors are complex i.e., can $\mathcal{R}^2$ have a basis whose basis vectors $\mathbf{b}_1, \mathbf{b}_2 \in \mathcal{C}^2$?.

   (b) (2 points) What conditions do the elements of basis vectors $\mathbf{b}_1, \mathbf{b}_2 \in \mathcal{C}^2$ and the corresponding coefficients of linear combination (α_1 and α_2) need to satisfy in order to realize every vector in $\mathcal{R}^2$.

   (c) (2 points) Indicate one complex basis of $\mathcal{R}^2$ and derive an expression for computing the coordinate vector in the chosen complex basis.

   (d) (2 points) Does all complex basis of $\mathcal{R}^2$ constitute an orthogonal basis?

2. (8 Points) **Inner product of a functional space:** Consider a vector space $\mathbb{V}$ of real functions. The inner product between any two functions $f(x), g(x) \in \mathbb{V}$ (denoted by $< f(x), g(x) >$ or $f(x) \cdot g(x)$) is defined as

$$< f(x), g(x) > \;=\; \int_{-\infty}^{\infty} f(x)g(x)\, dx \tag{1}$$

   The norm of a function $f(x) \in \mathbb{V}$ is defined as

$$\|f(x)\|_2 \;=\; \sqrt{f(x) \cdot f(x)} \tag{2}$$

   (a) (4 Points) Show that (2) satisfies all the properties of norm.

(b) (2 Points) Consider two functions $\sin(mx)$ and $\cos(nx)$ (where $n \neq m$) defined in the range $[-2\pi, 2\pi]$. Are the two functions orthogonal?

(c) (2 Points) Let $f(x) = 1 \in [a, b]$. Is there a function/set of functions (say $g(x) \in [a, b]$) that is orthogonal to $f(x)$.

3. (2 Points) Let $\mathbf{u}_1, \mathbf{u}_2$ be two linearly independent vectors in $\mathcal{R}^2$. Further, let $\mathbf{b}_1 = 2\mathbf{u}_1 + 3\mathbf{u}_2$ and $\mathbf{b}_2 = 4\mathbf{u}_1 + 5\mathbf{u}_2$. Does $\mathbb{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ constitute the basis of $\mathcal{R}^2$. Prove or disprove with necessary arguments.

4. (10 Points) Develop a code (in a programming language of your choice) to compute a desired DFT coefficient for a given discrete time signal of Length N. Note: Later in the course, we'll explore more efficient ways to compute all DFT coefficients.