CS1010: Discrete Mathematics for Computer Science

(Exam-2. Total: 30 marks.)
(Duration: 45 minutes. Date: 30 Sep 2024)

Instructions:

- You may not get time to answer all the questions unless you have prepared really well. The exam is designed like that.
- If your mobile phone is found with you during the exam, you will lose one grade.
- Anybody found copying will get an F grade for the course straight away.
- It is a no-break exam. You cannot take a break in between. The exam is only for 45 minutes. If you want to go out of the exam hall, you will have to submit your answer paper.
- You should sit far apart from each other. The halls are big enough. If we see two students sitting close to each other, both the students will lose one grade.

Questions

1. Let n be a positive integer. You are given a set A of n integers. Prove that there exists a set B such that $B \subseteq A$ and the sum of the integers in B is divisible by n. 8 marks.

2. Let $0 \leq \ell \leq k \leq n$, where ℓ, k, n are integers. Using double counting, show that

$$\binom{n}{k}\binom{k}{\ell} = \binom{n}{\ell}\binom{n-\ell}{k-\ell}.$$

7 marks.

3. Prove that, for all integers $n \geq 0$, $2^{2n} - 1$ is divisible by 3. 8 marks.

4. Let a, b be two integers. Let $d := gcd(a, b)$. It is known that d can be expressed as an integer linear combination of a and b (Note: no need to prove this).

   Prove that d is the smallest positive integer which can be expressed as an integer linear combination of a and b. 7 marks.

——————— ALL THE BEST ———————