EE2100: Matrix Theory

Assignment - 11

Handed out on 20 - Oct - 2023 **Due on 03 - Nov - 2023 (before 5 PM)**

Instructions :

1. Please submit the solutions to the assignment problems to the course page (on the canvas platform). Solutions submitted to the course page will only be evaluated. Refer to the assignment guidelines mentioned on the course page.

2. Submissions received after the deadline will attract negative marks.

3. It is suggested that you attempt all the questions (preferably the ones indicated using *). However, submitting solutions for problems totaling at least 10 points is sufficient.

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1. *(10 Points) Using a programming language of your choice implement the Cholesky decomposition algorithm to factorize a positive semidefinite matrix as $\mathbf{L}\mathbf{L}^T$.

   Note: The developed program must not adopt any of the existing libraries. The input to the program should be a positive semidefinite matrix given as a two dimensional array.

2. (5 points) Let $\mathbf{A} \in \mathcal{R}^{n\times n}$ and $\mathbf{B} \in \mathcal{R}^{n\times n}$ be positive semidefinite. Prove that

   (a) $\mathbf{Tr}(\mathbf{A}^T\mathbf{B}) \geq 0$.

   (b) if $\mathbf{Tr}(\mathbf{A}^T\mathbf{B}) = 0$, then $\mathbf{AB} = 0$

3. (5 points) Prove that if a matrix $\mathbf{A}$ satisfies $\mathbf{A} = \mathbf{A}^T$, then it is positive semi-definite if $\mathbf{Tr}(\mathbf{A}^T\mathbf{B}) \geq 0\ \forall \mathbf{B}$, where $\mathbf{B}$ belongs to the set of positive semi-definite matrices.