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EE1101: Circuits and Network Analysis

Lecture 26: Sinusoidal Response of First-Order Circuits

October 6, 2025

Topics :

1. Sinusoidal Response using complex exponentials
2. Role of Phasors and Impedances

Sinusoidal Response of First-Order Circuits : Recap from Lecture 25

$$\frac{dx}{dt} + Px(t) = \underbrace{A\cos(\omega t + \phi)}_{} \longrightarrow (1)$$

use Complex exponentials : $\frac{d\hat{x}}{dt} + P\hat{x}(t) = A e^{j(\omega t+\phi)} \longrightarrow (2)$

Sol of (1) $x(t) = \mathrm{Re}\{\hat{x}(t)\}$

$$= \frac{A}{\sqrt{P^2+\omega^2}} \cos(\omega t + \phi - \theta) + Ce^{-Pt}.$$

where $\theta = \tan^{-1}(\omega/P)$

if $P>0$: then as $t \to \infty$, $x(t) = x_{ss}(t) = \underbrace{\frac{A}{\sqrt{P^2+\omega^2}} \cos(\omega t + \phi - \theta)}_{}$

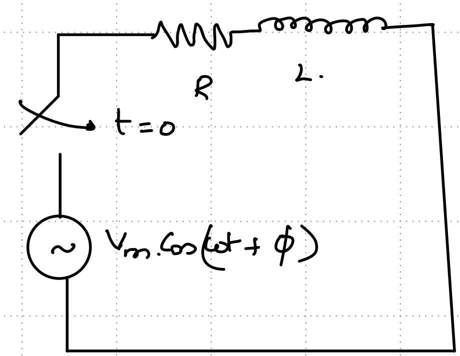
for Computing S.S response $\longrightarrow$ Phasor approach.

$$\vec{F} = \frac{A}{\sqrt{2}} \angle\phi$$

$$\vec{X}_{ss} = \frac{\vec{F}}{P+j\omega} C$$

Steady-State Response-Phasors and Impedances (Equivalent Impedances and Divider Circuits)

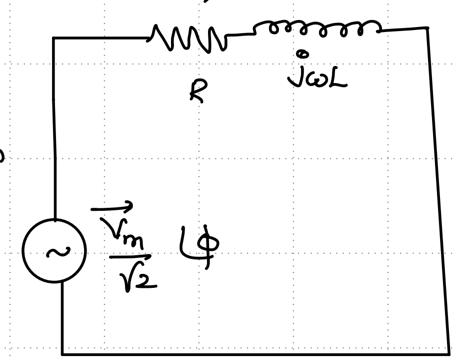
governing DE : $\frac{di}{dt} + \frac{R}{L} i = \frac{V_m}{L} \cos(\omega t + \phi)$



Complete response $\longrightarrow$ $i(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta) + c e^{-Rt/L}$

where $\theta = \tan^{-1}(\omega L / R)$

$\Downarrow$ Phasor domain Equivalent



Steady-state response $\longrightarrow$ Phasor domain approach.

a) Replace source with phasor representation

b) " ckt elem with impedances (switch with final pos)

c) apply ckt laws / Mesh or node analysis

ckt elem		Impedance
R	$\longrightarrow$	R
L	$\longrightarrow$	$j\omega L$
C	$\longrightarrow$	$1/j\omega C$

Series Connection $\vec{Z}_{eq} = \sum_i \vec{Z}_i$ $\longrightarrow$ Voltage divider

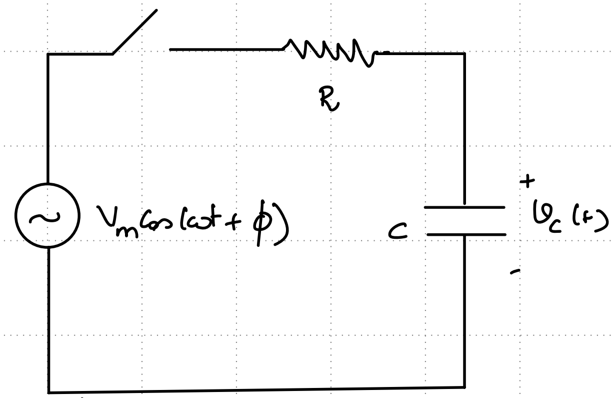
Parallel Connection $\frac{1}{\vec{Z}_{eq}} = \sum_i \frac{1}{\vec{Z}_i}$ $\longrightarrow$ Current divider.

Example - RC Circuit

governing DE :- $v(t) = R\underbrace{i(t)}_{} + v_c(t)$

$$v(t) = RC\frac{dv_c}{dt} + v_c(t)$$

$$\Rightarrow \frac{dv_c}{dt} + \underbrace{\frac{1}{RC}}_{P(t)} v_c(t) = \underbrace{\frac{1}{RC}}_{A} V_m \cos(\omega t + \phi)$$

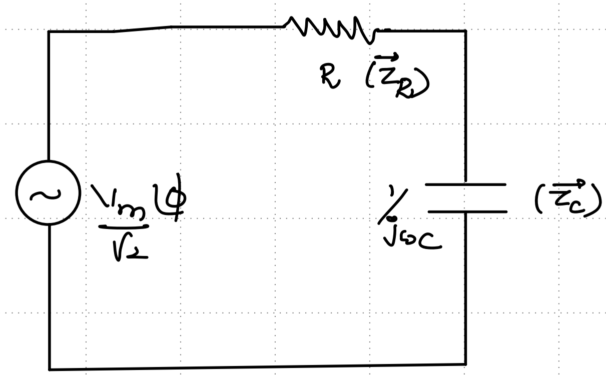


Complete response $= \dfrac{A}{\sqrt{P^2+\omega^2}} \cos(\omega t + \phi - \theta) + ce^{-Pt}$

$$= \frac{V_m}{RC\sqrt{\left(\frac{1}{RC}\right)^2+\omega^2}} \cos(\omega t + \phi - \theta) + ce^{-t/RC} \quad \text{when } \theta = \tan^{-1}(\omega RC)$$

Steady state response (using phasors)

$$\vec{V_c} = \frac{\vec{Z_c}}{\vec{Z_R}+\vec{Z_c}} \vec{V_s} = Be^{j\phi_B}$$



$$v_{c,ss}(t) = \sqrt{2}\, B \cos(\omega t + \phi_B)$$

Introduction to Response of Second-Order Circuits (2 energy storage elements)

Std form :- $\frac{d^2x}{dt^2} + 2\xi\omega_n \frac{dx}{dt} + \omega_n^2 x(t) = f(t) \leftarrow (1)$

$\xi \rightarrow$ damping factor

$\omega_n \rightarrow$ natural freq

(or)

$\frac{d^2x}{dt^2} + \frac{\omega_n}{Q}\frac{dx}{dt} + \omega_n^2 x(t) = f(t)$

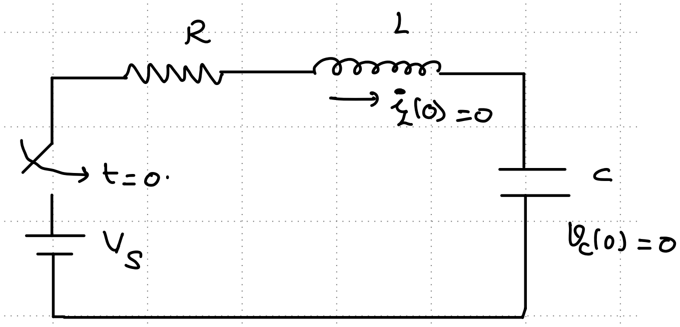
$Q \rightarrow$ Quality factor $= \frac{1}{2\xi}$

2 initial Cond : $x(t_0)$ & $\left.\frac{dx}{dt}\right|_{t_0}$.

Sol :- $x(t) = x_c(t) + x_p(t) \longrightarrow$ Particular Sol

$x_c(t) \downarrow$ Complementary Sol $\downarrow$

$x_c(t) = \left\{x : \frac{d^2x}{dt^2} + 2\xi\omega_n\frac{dx}{dt} + \omega_n^2 x(t) = 0\right\}$



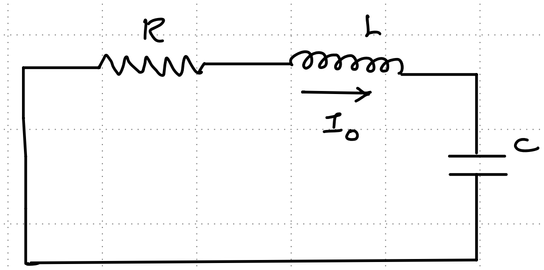
by KVL : $V_S = Ri(t) + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$ (for $t>0$)

diff w.r.t 't' $\quad 0 = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}i(t)$

$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i(t) = 0$

$i(0) = 0$

$\left.\frac{di}{dt}\right|_{t=0} = \frac{V_S(0)}{L}$ $\quad (\because v_c(0) = 0)$



by KVL $\quad \frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i(t) = 0$

$i(0) = I_0$ and $\left.\frac{di}{dt}\right|_{t=0} = -RI_0/L$.