Electrical Engineering Department

IIT Hyderabad

EE2000 - Signal Processing

Homework-1

Note

* Plagiarism is strictly prohibited
* Deadline will not be extended under any circumstances. Deadline: August 17, 2025.

# 1 Signal Transformations and Plotting

1. Let $x[n] = \sin(n\pi)$. Then plot $x[2n]$, $x[n/2]$, $x[n+1]$, $x[n-1]$.
2. Plot $2x[n-4]$, $3x[n-5]$, and $x[3-n]$ for the sequence

$$x[n] = \begin{cases} n & 0 \leq n \leq 10 \\ 0 & \text{otherwise} \end{cases}$$

# 2 Periodicity of Signals

1. Check whether the following discrete-time signals are periodic or not. If they are periodic, find their fundamental period.
   (a) $x[n] = \sin\left(\frac{n}{6}\right)$
   (b) $x[n] = \cos(0.7\pi n) + \sin(1.1\pi n)$
   (c) $x[n] = (-1)^n + e^{j\frac{n\pi}{2}}$
   (d) $x[n] = u[n] + u[-n]$
   (e) $x[n] = \cos\left(\frac{\pi n}{6}\right)$
   (f) $x[n] = \cos\left(\frac{n}{8} - \pi\right)$
   (g) $x[n] = \cos\left(\frac{\pi n^2}{4}\right)$
   (h) $x[n] = \cos\left(\frac{\pi n}{5}\right) \sin\left(\frac{\pi n}{3}\right)$
   (i) $x[n] = e^{j\frac{5\pi n}{7}}$
2. For the following continuous-time signals, find the fundamental period $T_0$ if periodic.
   (a) $x(t) = \cos(18t) + \sin(12\pi t)$
   (b) $x(t) = \sin\left(\frac{2t}{3}\right) \cos\left(\frac{4t}{5}\right)$
   (c) $x(t) = \cos(3t) + \sin(5\pi t)$
   (d) $x(t) = \cos(5t)u(t)$
   (e) $x(t) = \cos^2(2\pi t)$
   (f) $x(t) = \sum_{k=-\infty}^{\infty} (-1)^k h(t-2k)$
   (g) $x(t) = \cos(2\pi t) + \sin(3\pi t) + \cos\left(5\pi t - \frac{3\pi}{4}\right)$

# 3 System Properties

1. Give an example of a system that is:
   (a) Additive, but not homogeneous.
   (b) Homogeneous, but not additive.

2. Check whether the following systems are linear, time-invariant, causal, and stable. Justify your answers.
   (a) $y[n] = n \cos\left(\frac{n\pi}{4}\right) u[n]$
   (b) $y[n] = \max(x[n], x[n-1], x[n-2])$
   (c) $y[n] = (1 + (-1)^n)x[n]$
   (d) $y[n] = 2x[n+1] + x^2[n-1]$
   (e) $y[n] = x[2n]$. It is called a down-sampling system.
   (f) $y[n] = x[n] + 0.5x[n-1] + 0.25x[n-2]$
   (g) $y[n] = \begin{cases} 0 & \text{if } n \text{ is odd} \\ x[n/2] & \text{if } n \text{ is even} \end{cases}$
   (h) $y[n] = x_e[n-1]$ where $x_e[n]$ denotes the even part of the signal $x[n]$ and it is defined as $x_e[n] = \frac{x[n]+x[-n]}{2}$
   (i) $y[n] = \begin{cases} x[n] & n \geq 1 \\ 0 & n = 0 \\ x[n+1] & n \leq -1 \end{cases}$

3. Comment on the causality and stability of the systems with the following unit sample responses.
   (a) $h[n] = \delta[n-d]$ where $d$ is an integer.
   (b) $h[n] = u[n] - u[n-N]$
   (c) $h[n] = \frac{1}{2N+1} \sum_{k=-N}^{N} \delta[n-k]$
   (d) $h[n] = a^n u[-n]$. Derive condition on $a \in \mathbb{R}$ for stability.
   (e) $h[n] = \sum_{k=-\infty}^{n} \delta[k]$

4. Recall the condition on the unit sample response for stability

$$\sum_k |h[k]| < \infty.$$

   Show that the above is a sufficient and necessary condition for stability with appropriate examples.

# 4 System Design

1. Give examples of systems that satisfy the following properties. Provide justifications for your examples.
   (a) Linear, Time-Invariant, Unstable
   (b) Nonlinear, Time-Invariant, Stable
   (c) Linear, Time-Variant, Causal
   (d) Static, Nonlinear, Stable

# 5 LTI System Outputs

1. Given that the output of an LTI system to an input

$$x_0[n] = \delta[n+1] + 2\delta[n] + \delta[n-1]$$

is

$$y_0[n] = \delta[n+2] - 2\delta[n+1] + 2\delta[n-1] + \delta[n-2].$$

Find the output of the system to an input

$$x_1[n] = \delta[n-1] + 2\delta[n-2] + 3\delta[n-3] + 4\delta[n-4] + 3\delta[n-5] + 2\delta[n-6] + \delta[n-7].$$

2. Compute and sketch the LTI system output for the following input and unit sample response sequence pairs.
   (a) $x[n] = [4, 2, 2, 3, 1, 6]$, $h[n] = [1, 0, 3, 1, 5]$
   (b) $x[n] = [-1, -1, 0, 0, 1, 1, 0, 0, -1, -1]$, $h[n] = \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right]$

3. Compute the LTI system output ($y[n]$) for the following input and unit sample response pairs.
   (a) $x_1[n] = a^n u[n]$, $|a| < 1$; $h_1[n] = u[n] - u[n-M]$
   (b) $x_2[n] = a^n u[n]$, $|a| < 1$; $h_2[n] = u[n+M] - u[n]$
   (c) $x_3[n] = a^n u[n]$, $|a| < 1$; $h_3[n] = u[n+M] - u[n-M]$

   How do you relate the outputs $y_1[n]$, $y_2[n]$, and $y_3[n]$?

# 6 Frequency Response and Eigenfunctions

1. What is an eigenfunction? Check whether $e^{sn}$ is an eigenfunction of an LTI system with unit sample response $h[n]$. If so, derive the eigenvalues.

2. Compute and sketch the magnitude and phase response for the LTI systems with the following unit sample responses.
   (a) $h[n] = e^{j\omega_0 n}$
   (b) $h[n] = a^n u[n-10]$, $|a| < 1$

(c) $h[n] = \begin{cases} 1 & 0 \leq n \leq M \\ 0 & \text{otherwise} \end{cases}$

(d) $h[n] = \frac{\sin(\omega_c n)}{\pi n}$

3. For an LTI system with input and unit sample response

$$x[n] = -2 + 3\cos\left(\frac{\pi n}{4} + \frac{\pi}{3}\right) + 10\cos\left(\frac{3\pi n}{4} - \frac{\pi}{5}\right)$$

$$h[n] = 3\delta[n] + 2\delta[n-1] + \delta[n-3]$$

(a) Compute frequency response $H(e^{j\omega})$.

(b) Sketch the magnitude ($|H(e^{j\omega})|$) and phase ($\angle H(e^{j\omega})$) response.

(c) Compute the system output $y[n]$.