EE2100: Matrix Theory

Assignment - 13

Handed out on 10 - Nov - 2023 **Due on 20 - Nov - 2023 (before 5 PM)**

Instructions :

1. Please submit the solutions to the assignment problems to the course page (on the canvas platform). Solutions submitted to the course page will only be evaluated. Refer to the assignment guidelines mentioned on the course page.
2. Submissions received after the deadline will attract negative marks.
3. It is suggested that you attempt all the questions (preferably the ones indicated using *). However, submitting solutions for problems totaling at least 10 points is sufficient.

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1. (10 Points) The induced norm of order p of $\mathbf{A} \in \mathcal{R}^{n\times n}$ is defined as

$$\|\mathbf{A}\|_p = \max_{\mathbf{x}\neq\mathbf{0}} \frac{\|\mathbf{Ax}\|_p}{\|\mathbf{x}\|_p} \tag{1}$$

   (a) (8 Points) Compute $\|\mathbf{A}\|_1$ and $\|\mathbf{A}\|_\infty$.

   (b) (2 Points) Let $\|\mathbf{A}\| = \sigma_1$. Is it possible to compute $\|\mathbf{Ab}\|$ if $\|\mathbf{b}\| = c$. If so, compute $\|\mathbf{Ab}\|$.

2. (5 Points) Prove that $\|\mathbf{AB}\|_F \leq \|\mathbf{A}\|_F\|\mathbf{B}\|_F$.

3. (5 Points) Show that $\|\mathbf{AB}\|_F \leq \|\mathbf{A}\|_2\|\mathbf{B}\|_F$.

4. (10 points) The condition number of a matrix $\kappa(\mathbf{A})$, is an indicator of how susceptible the solution $\mathbf{x}$ due to errors. The condition number of matrix $(\kappa(\mathbf{A}))$ is defined as $\|A\|\|A^{-1}\|_n$. n indicates that the order of the norm is not fixed.

   Consider n as ∞ for now.

   (a) (2 Points) Find the condition number of a general 2×2 matrix A with entries a_{ij} and comment on when the condition number will be large (i.e. when the matrix will be ill-conditioned).

   (b) (2 Points) What is the condition number of a singular matrix?

   (c) (2 Points) Find the condition number for

$$\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 3.3 & 5 \end{bmatrix} \tag{2}$$

   (d) (4 Points) Solve $\mathbf{Ax} = \mathbf{b}$, when $\mathbf{b} = [5, 8.3]^T$. Solve it again for $\mathbf{b'} = [5.1, 8.4]^T$.