Electrical Engineering Department
IIT Hyderabad
EE2000 - Signal Processing
Homework-2

Note

* Plagiarism is strictly prohibited
* Deadline will not be extended under any circumstances.

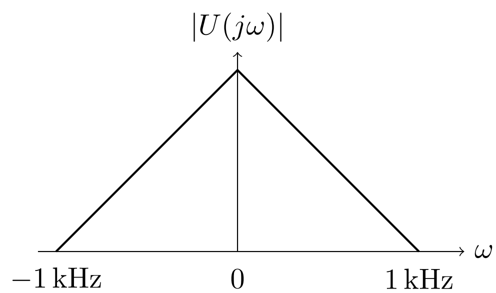
1. Let us consider a continuous time signal $x(t)$ and a periodic impulse train $p(t)$ with period $T_s = 0.3$ mSec. Determine the nyquist frequency of input signal $x(t)$ to avoid aliasing of the following signals when sampled with $p(t)$
   (a) $x(2t - 5)$
   (b) $x(t)cos(\Omega_0 t)$; For what values of Ω_0, the signal can be reconstructed
   (c) $x(t) + x(-t)$
2. Consider a continuous time signal
$$x(t) = 1 + \frac{1}{2}cos(2\pi t) + cos(10\pi t + \frac{\pi}{2}) + \frac{2}{3}cos(20\pi t - \frac{\pi}{3})$$
   The signal is passed through an ideal low pass filter with cutoff frequency of 10Hz. The signal is then sampled at 10Hz resulting in a sampled signal $x_p(t)$. Then
   (a) Determine the sampled signal $x_p(t)$?
   (b) Is the sampled signal $x_p(t)$ periodic ? If so, compute the fundamental frequency of $x_p(t)$?
3. Consider a continuous time periodic signal with fundamental frequency of 20Hz whose fourier series coefficients are a_k for $-\infty < k < \infty$. Let us suppose that this signal is sampled at $f_s = 100Hz$.
   (a) Does aliasing happen with sampling in this case ?
   (b) If aliasing happens, design an anti-aliasing filter and compute its highest cutoff frequency such that the nyquist theorem is satisfied. Determine time-domain expression for the reconstructed signal at the output after sampling ? Is the output signal periodic ? If so, compute its fundamental frequency ?
4. Consider a continuous time signal $x(t)$ bandlimited to Ω_N. The signal is sampled at $\Omega_s > 2\Omega_N$ to obtain a sampled signal $x_p(t)$. The discrete time signal $x[n]$ is obtained by collecting the strengths of the impulses from the sampled signal. Suppose $x(t)$ is an energy signal with energy E_c, compute the energy of the discrete time signal ?
5. Let us consider a continuous time signal bandlimited to Ω_N i.e., $X(j\Omega) = 0$ for $|\Omega| > \Omega_N$. Consider the sampling function
$$p(t) = \sum_n (-1)^{n+1}\delta(t - nT_s)$$
   (a) Compute the fourier transform of $p(t)$?

(b) Give the frequency domain interpretation of the sampling using $p(t)$?

(c) How is this sampling different from the one discussed in class ?

(d) How can you reconstruct the continuous time signal when sampled using the given $p(t)$ when you have only the ideal low pass filters at your disposal ?

6. The frequency spectrum of a signal is shown in the figure. If this is ideally sampled at intervals of 1 ms, then find the frequency spectrum of the sampled signal.



7. A sinusoid $x(t)$ of unknown frequency is sampled by an impulse train of period 20 ms. The resulting sample train is next applied to an ideal low-pass filter with a cut-off at 25 Hz. The filter output is seen to be a sinusoid of frequency 20 Hz. Find the component of frequency of $x(t)$ which is less than 100 Hz.