Electrical Engineering Department
IIT Hyderabad
EE2000 - Signal Processing
Homework-3

Note

* Plagiarism is strictly prohibited
* Deadline will not be extended under any circumstances.

(a) Compute and roughly sketch the Fourier transform(FT) of the following sequences

i. $x[n] = x'[n-3]$ where $x'[n] = u[n+5] - u[n-5]$

ii. $x[n] = (n-1)^2 \left(\frac{1}{3}\right)^{n+3} u[n-3]$

iii. $x[n] = \left(\frac{1}{2}\right)^{|n|}$

iv. $x[n] = 3(0.8)^{|n|} cos(0.1\pi n)$

v. $x[n] = \begin{cases} 1 - \frac{|n|}{M+1} & -M \leq n \leq M \\ 0 & \text{else} \end{cases}$

vi. $x[n] = \begin{cases} \frac{1}{2}\left(1 + cos(\frac{\pi n}{M})\right) & -M \leq n \leq M \\ 0 & \text{else} \end{cases}$

(b) Let us consider a sequence $x_1[n]$ with FT $X_1(j\omega)$. Now we have a sequence $x_2[n] = (-1)^n x_1[n]$. Then

i. Represent the FT of $x_2[n]$ in terms of $X_1(j\omega)$

ii. Let's say $x_1[n]$ is an ideal low pass filter with cut-off frequency $\omega_c(< \pi)$. Then comment on the frequency domain characteristics of $x_2[n]$.

iii. For what $x_1[n]$, $x_1[n] = x_2[n]$.

(c) Let $X(j\omega)$ be the FT of the sequence $x[n] = [1, \underline{0}, 3, 2, 5, 2, 3, 0, 1]$ (Underline indicates the zeroth index). Then without explicitly computing $X(j\omega)$

i. Compute $X(j0), X(j\pi)$

ii. Find $\int_{-\pi}^{\pi} X(j\omega) d\omega$

iii. Evaluate $\int_{-\pi}^{\pi} |X(j\omega)|^2 d\omega$

iv. Evaluate $\int_{-\pi}^{\pi} |\frac{dX(j\omega)}{d\omega}|^2 d\omega$

v. Determine and sketch the signal $x'[n]$ whose FT is the real part of $X(j\omega)$ i.e., $X'(j\omega) = X_R(j\omega)$

(d) Compute the inverse Fourier transform of

$$\frac{1}{(1 - \alpha e^{-j\omega})^4} \qquad |\alpha| < 1$$

(e) Let's suppose someone is generating a sinusoidal signal $x[n] = cos\left(\frac{\pi}{4}n\right)$ but we do not have access to the signal for all n but only for $0 \leq n \leq 10$ (for other n we assume $x[n] = 0$). This is mathematically formulated as

$$x'[n] = x[n]w[n]$$

i. Sketch $x[n], w[n]$ and $x'[n]$

ii. Compute and roughly sketch $X(j\omega), W(j\omega)$ and $X'(j\omega)$

iii. Comment on the comparison between $X(j\omega)$ and $X'(j\omega)$

(f) A sequence has the DTFT

$$X(j\omega) = \frac{1-a^2}{(1-ae^{-j\omega})(1-ae^{j\omega})} \qquad |a| < 1$$

Find the sequence x[n]. Calculate $\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})cos(\omega)d\omega$.

(g) The linear constant-coefficient differential equation(LCCDE) representation of a discrete time system is given by

$$y[n] = x[n] - \alpha x[n-1] + x[n-2]$$

Determine the impulse response $h[n]$ of the system. For what values of α the system will be stable. Determine the frequency response($H(j\omega)$) and roughly sketch the magnitude and phase responses.

(h) For a system with input $x[n]$ and output $y[n]$ relation given by

$$y[n] - ny[n-1] = x[n]$$

Let's suppose the system is causal, then

i. Compute the output for the unit impulse input.

ii. Comment on the linearity, time-invariance and stability of the system with appropriate reasoning.

(i) For the impulse response

$$h[n] = \left(\frac{1}{2}\right)^n u[n] - \left(\frac{1}{2}\right)^{n+1} u[n-1]$$

compute the FT representation $(H(j\omega))$. Represent the input and output relation in LCCDE.