EE2100: Matrit Theory

Assignment - 3

Handed out on : 25 - Aug - 2023 **Due on : 04 - Sep - 2023 (before 5 PM)**

Instructions :

1. Please submit the solutions to the assignment problems to the course page (on the canvas platform). Solutions submitted to the course page will only be evaluated. Refer to the assignment guidelines mentioned on the course page.
2. Submissions received after the deadline will attract negative marks.
3. There are two sections in the assignment.
4. It is suggested that you attempt all the questions (preferably the ones indicated using *). However, it is sufficient to submit solutions for problems that total to at least 10 points.

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1. (4 Points) Let $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\} \in \mathcal{R}^3$ denote a set of linearly independent vectors. Further, let $\{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\} \in \mathcal{R}^3$ denote the orthogonal basis obtained by applying Grahm Schmidt Algorithm to $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\} \in \mathcal{R}^3$.

   (a) (2 Points) Show that $\mathbf{b}_2 \perp \mathbf{b}_3$ and $\mathbf{b}_3 \perp \mathbf{b}_1$.

   (b) (2 Points) Consider the subspace $\mathbb{W} = \textbf{Span}\,\{\mathbf{b}_2, \mathbf{b}_3\}$. Compute $\textbf{Proj}_{\mathbb{W}}\ \mathbf{b}_1$

2. *(10 Points) Signal Space and Orthogonal Basis: A vector space comprising of a collection of functions constitutes a signal space (A brief overview of signal space, projection and inner product will be covered in Tutorial 3). Consider a signal space comprising of four real-valued functions i.e., $\{s_1(t), s_2(t), s_3(t), s_4(t)\}$ where

$$s_1(t) = \begin{cases} 0 & t \leq 0 \\ 1 & 0 \leq t \leq 2 \\ 0 & t \geq 2 \end{cases}, \; s_2(t) = \begin{cases} 0 & t \leq 0 \\ 1 & 0 \leq t \leq 1 \\ -1 & 1 \leq t \leq 2 \\ 0 & t \geq 2 \end{cases}, \; s_3(t) = \begin{cases} 0 & t \leq 0 \\ 1 & 0 \leq t \leq 2 \\ -1 & 2 \leq t \leq 3 \\ 0 & t \geq 3 \end{cases} \text{ and } s_4(t) = \begin{cases} 0 & t \leq 0 \\ -1 & 0 \leq t \leq 3 \\ 0 & t \geq 3 \end{cases} \tag{1}$$

   (a) (6 Points) Find an orthogonal basis to represent the above signals.

   (b) (4 Points) Is the orthogonal basis same if the Grahm Schmidt algorithm is applied to $\{s_1(t), s_2(t), s_3(t), s_4(t)\}$ and $\{s_2(t), s_3(t), s_1(t), s_4(t)\}$. [The intention of the question is to explore the possibility of reduction in computational complexity when the order of the signals/vectors are changed].

3. (10 Points) Consider finite length ($N = 8$) discrete time signals given by (2).

$$x_1[n] = \begin{cases} 1 & n = 0, \cdots, 7 \end{cases}, \; x_2[n] = \begin{cases} 1 & n = 0, \cdots, 3 \\ -1 & n = 4, \cdots, 7 \end{cases},$$
$$x_3[n] = \begin{cases} 0 & n = 0, 7 \\ 0.25 & n = 1, 6 \\ 0.5 & n = 2, 5 \\ 1.0 & n = 3, 4 \end{cases} \text{ and } x_4[n] = \begin{cases} -1 & n = 0, \cdots, 3 \\ 1 & n = 4, \cdots, 7 \end{cases} \tag{2}$$

(a) (1 Point) Represent the discrete time signals as vectors (say $\mathbf{x}_1, \cdots, \mathbf{x}_4$.

(b) (6 Points) Compute the orthogonal basis of the subspace spanned by $\mathbf{x}_1, \cdots, \mathbf{x}_4$.

(c) (1 Point) Represent (graphically) the orthogonal basis obtained in 3(b) as a discrete time signal.

(d) (2 Points) Will the orthogonal basis obtained by applying Grahm Schmidt in the following order $\mathbf{x}_2, \mathbf{x}_1, \mathbf{x}_4, \mathbf{x}_3$ be the same as the orthogonal basis obtained in 3(b).

4. * (10 Points) Using a Programming Language of your choice, develop a code to compute a set of orthogonal basis for the subspace spanned by the given vectors. Note that the input to the algorithm will be the set of vectors, some of which can be linearly dependent. In addition to computing the orthogonal basis of the subspace spanned by the given vectors, the program is also expected to detect the presence of linearly dependent vectors.