EE5605: Kernel Methods **2024**

Exam 1: November 2024

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Instructions: *This is a closed-book exam. You are not permitted to refer to any material or discuss the problem with anyone. Malpractice will be severely punished. Please mention your ROLL Number and name clearly in the answer sheet.*

Justify all your statements clearly. You may use any result proved in class (but clearly state which results you are using), but everything else needs to be proved.

Problem 1.1 (5pts). Let $\underline{a}, \underline{x}$ be arbitrary vectors in $\mathbb{R}^d$ for some positive integer d, with $\|\underline{a}\|_2 = 1$ and $b \in \mathbb{R}$. Let

$$\mathcal{S} = \{\underline{y} \in \mathbb{R}^d : \underline{a}^T \underline{y} = b\}.$$

Compute

$$\min_{\underline{y} \in \mathcal{S}} \|\underline{x} - \underline{y}\|_2$$

Problem 1.2 (6pts). Let d be a positive integer. Define the hypothesis class of linear classifiers in $\mathbb{R}^d$:

$$\mathcal{H} = \{h(\underline{x}) = 1_{\{\underline{a}^T \underline{x} \geq b\}} : \underline{a} \in \mathbb{R}^d, b \in \mathbb{R}\}.$$

Compute the VC dimension of this hypothesis class.

Problem 1.3 (5+5+4pts). Find the differential sets for the following functions:

- $f : \mathbb{R} \to \mathbb{R}$, with $f(x) = |x|$
- $f : \mathbb{R} \to \mathbb{R}$, with

$$f(x) = \begin{cases} -x & \text{if } x < 0 \\ 0 & \text{if } 0 \leqslant x \leqslant 1 \\ x - 1 & \text{if } x > 1 \end{cases}$$

- $f : \mathbb{R}^d \to \mathbb{R}$, with $f(\underline{x}) = \exp\left(\|\underline{x} - \underline{a}\|_2^2\right)$, for some fixed $\underline{a} \in \mathbb{R}^d$

Problem 1.4 (6pts). Describe the hard-SVM algorithm. Set up the lagrangian, and use the KKT conditions to show that the optimizing $\underline{a}$ is a linear combination of support vectors (i.e., vectors that lie on the marginal hyperplane).