EE1203/MA1130: Vector Calculus

Practice problem Set-2: Jan-Feb 2025 22nd January 2025

1. Express (a) the surface $xz = 1$ and (b) the surface $x^2 + y^2 - z^2 = 1$ in spherical coordinates.
2. Consider a modified spherical coordinate system (purely hypothetical), where any point in the three-dimensional space is represented in the following form$P(\rho, \phi, \theta)$ where ρ is the magnitude of the position vector to the point, ϕ is angle with respect to yaxis and θ is angle that the position vector to P_1 (projection of P onto zx-plane) subtends with x-axis. Derive expressions for representing any point $P(x, y, z)$ in this hypothetical coordinate system, with supporting diagram.
3. If $z = e^x sin y$, where $x = st^2$, $y = s^2t$, find $\partial z/\partial s$, $\partial z/\partial t$ (Hint: Chain rule in partial derivatives)
4. If $g(s, t) = f(s^2 - t^2, t^2 - s^2)$ and f is differentiable, show that g satisfy the equation;

$$t\frac{\partial g}{\partial s} + s\frac{\partial g}{\partial t} = 0$$

5. Find all local-maxima and minima of $f(x, y) = (x - 2)^4 + (x - 2y)^2$
6. Find the equation of the tangent plane to the surface $x^2 + y^2 + z^2 = 9$ at the point $(2, 2, 1)$.
7. Wheat production W in a given year depends on the average temperature T and the annual rainfall R. Scientists estimate that the average temperature is rising at a rate of 0.15^0C/year and rainfall is decreasing at a rate of 0.1 cm/year. They also estimate that at current production levels, $\frac{\partial W}{\partial T} = -2$ and $\frac{\partial W}{\partial R} = -8$.
   (a) What is the significance of the signs of these partial derivatives?
   (b) Estimate the current rate of change of wheat production, $\frac{dW}{dt}$.
8. A boat is sailing northeast at 20 km/h. Assuming that the temperature drops at a rate of $0.2^0C/km$ in the northerly direction and $0.3^0C/km$ in the easterly direction, what is the time rate of change of temperature as observed on the boat?