CS1010: Discrete Mathematics for Computer Science

(Exam-3. Total: 30 marks.)
(Duration: 45 (you may take 90 minutes). Date: 23 Nov 2024)

Instructions:
A. If your mobile phone is found with you during the exam, you will lose one grade.
B. Anybody found copying will get an F grade for the course straight away.
C. It is a no-break exam. You cannot take a break in between. The exam is designed for 45 minutes. You may take up to 90 minutes. If you want to go out of the exam hall, you will have to submit your answer paper permanently.
D. You should sit far apart from each other. The halls are big enough. If we see two students sitting close to each other, both the students will lose one grade.
E. Unless mentioned explicitly, all graphs considered here are simple, finite, and undirected.

Questions

1. **Definition:** The minimum length of a cycle (contained) in a graph G is the *girth* of G.
   Question: Let k be any positive integer. Let G be a graph with girth 4 in which every vertex has degree at least k. Prove that G has at least $2k$ vertices. 10 marks.

   need not be connected

2. Use the principle of inclusion and exclusion to determine the number of ways in which three women and their spouses (, that means a total of 6 people,) may be seated around a round table such that no woman sits beside her spouse. 10 marks.

3. (a) A set of blocks contains blocks of heights 1, 2, and 4 centimeters. Imagine constructing towers by piling blocks of different heights directly on top of one another. (A tower of height 6 cm could be obtained using six 1-cm blocks, three 2-cm blocks, one 2-cm block with one 4-cm block on top, one 4-cm block with one 2-cm block on top, and so forth.) Let t_n be the number of ways to construct a tower of height n cm using blocks from the set. (Assume an unlimited supply of blocks of each size.) Find the values of $t_1, t_2, t_3,$ and t_4. Further, for any $n > 4$, write a recursive formula for t_n.

   (b) Let s_n denote the number of bit strings of length n that do not contain three consecutive 0s. Find the values of $s_1, s_2, s_3,$ and s_4. Further, for any $n > 4$, write a recursive formula for s_n. $5 + 5 = 10$ marks.

——————— ALL THE BEST ———————

1