.

EE1101: Circuits and Network Analysis

Lecture 28: Second-Order Circuits

October 10, 2025

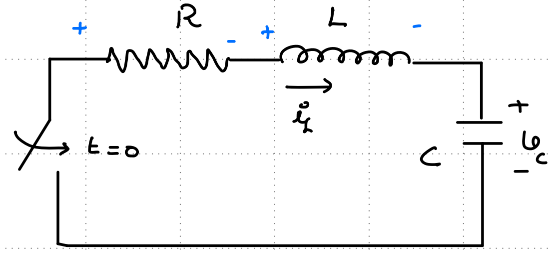
Topics :

1. Forced Response of Second-Order Circuits

Example (contd.)

$$R\,i(t) + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0 \longrightarrow (1)$$

Step (1) $\longrightarrow$ diff (1) w.r.t t $\Rightarrow$ $\dfrac{d^2 i}{dt^2} + \dfrac{R}{L}\dfrac{di}{dt} + \dfrac{1}{LC}\,i(t) = 0 \longrightarrow (2)$



Step (2): Initial Cond: $i_L(0) = 10\text{ A}$

$$\left.\frac{di_L}{dt}\right|_{t=0} = \frac{v_L(t=0)}{L} = \frac{-v_R(t=0) - v_c(0)}{L}$$

$$= -\frac{10R}{L}.$$

$i_L(0-) = 10\text{ A}$

$v_c(0-) = 0\text{ V}.$

Step (3): Nature of the Complementary Sol: Compare with standard 2^{nd} order ODE

$$\omega_n^2 = 1/LC \Rightarrow \omega_n = \sqrt{\frac{1}{LC}}$$

$$2\xi\omega_n = R/L \Rightarrow \xi = \frac{R}{2}\sqrt{\frac{C}{L}}.$$

if $\xi > 1$: $x(t) = c_1 e^{s_1 t} + c_2 e^{s_2 t}$ where s_1 & s_2 are roots of $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

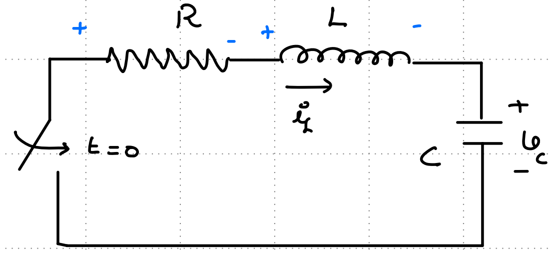
$\xi = 1$: $x(t) = c_1 e^{s_1 t} + c_2 t e^{s_1 t}$ where s_1 is the repeated root of $s^2 + 2\xi\omega_n s + \omega_n^2 = 0.$

$0 \le \xi < 1$: $x(t) = 2r\,e^{-\xi\omega_n t}\cos(\omega_d t + \theta)$ $\quad s = -\xi\omega_n \pm j\omega_d$

$\xi = 0$: $x(t) = 2r\cos(\omega_n t + \theta)$

Example (contd.)

Case A: $R = 30\Omega$, $L = 10H$, $C = 0.01F$



$\omega_n = 1\,rad/s$ $\quad \xi = 30/20 = 1.5 > 1$

characteristic Eqn : $s^2 + 3s + 1 = 0$.

roots are $s_1 = \frac{-3+\sqrt{5}}{2}$, $s_2 = \frac{-3-\sqrt{5}}{2}$

≈ -0.4, ≈ -2.6

$i_L(0-) = 10\,A$

$v_c(0-) = 0\,V$.

gen sol $i(t) = c_1 e^{-0.4t} + c_2 e^{-2.6t}$

Step ④: apply I.C. to get final sol $\quad i(0) = 10A = c_1 + c_2$.

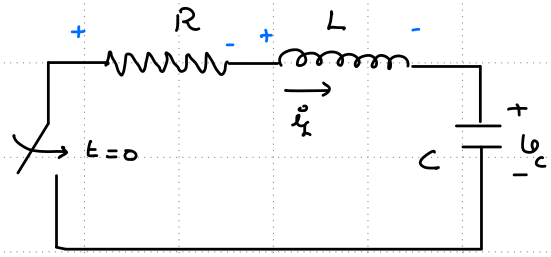
$$\left.\frac{di}{dt}\right|_{t=0} = -0.4c_1 - 2.6c_2 = -10(R/L) = -30.$$

$$c_2 = 130/11 \text{ and } c_1 = -20/11$$

$$i(t) = -20/11\, e^{-0.4t} + 130/11\, e^{-2.6t}.$$

Example (contd.)

Case B : $R = 10\,\Omega$, $L = 10\,H$, $C = 0.01\,F$.



$\omega_n = 1$ rad/s $\quad \xi = 10/20 = 0.5 \; (<1)$

characteristic Eqn : $s^2 + s + 1 = 0$.

roots are $= -\xi\omega_n \pm j\,\omega_d$.

$= -1/2 \pm j\frac{\sqrt{3}}{2} = \underbrace{\frac{-1}{2}} \pm j\frac{\sqrt{3}}{2}$.

$i_L(0-) = 10$ A

$v_c(0-) = 0$ V.

gen sol $i(t) = 2r\, e^{-0.5t} \cos\left(\frac{\sqrt{3}}{2}t + \theta\right)$

Step 4 : apply I.C. to get final sol $\quad i(0) = 10A = 2r\cos\theta \longrightarrow$ (I)

$$\left.\frac{di}{dt}\right|_{t=0} = -r\cos\theta - \sqrt{3}\,r\sin\theta = -10R/L = -10 \longrightarrow \text{(II)}$$

Solve (I) & (II) to get r and θ. & then the final sol.

Solution of Second-Order Differential Equations

Solve $\frac{d^2x}{dt^2} + 2\xi\omega_n \frac{dx}{dt} + \omega_n^2 x = f(t) \longrightarrow (1)$

$\longrightarrow A\cos(\omega t+\phi)$ or $A\sin(\omega t+\phi)$

$\hat{x}: \frac{d^2\hat{x}}{dt^2} + 2\xi\omega_n \frac{d\hat{x}}{dt} + \omega_n^2 \hat{x} = Ae^{j(\omega t+\phi)} \longrightarrow (2)$

Sol of (1) $x(t) = \mathrm{Re}\{\hat{x}(t)\}$ if $f(t)$ is Cos.

$= \mathrm{Im}\{\hat{x}(t)\}$ if $f(t)$ is Sin

Complementary Sol $\longrightarrow$ $\longleftarrow$ Particular Sol

$\hat{x} = \hat{x}_c(t) + \hat{x}_p(t)$

$\longrightarrow$ Sol to $\frac{d^2\hat{x}}{dt^2} + 2\xi\omega_n \frac{d\hat{x}}{dt} + \omega_n^2 \hat{x} = 0. \longrightarrow$ can be computed by solving CE & adopting the process discussed

as $t \to \infty$, $x_c(t) \longrightarrow 0$ unless $\xi = 0$.

$\hat{x}_p(t) \longrightarrow$ 2 ways
- $\longrightarrow$ guess the functional form (✓) for this Course
- $\longrightarrow$ Systematic approach. (in Course on D.E and Transform Techniques)

Method of Undetermined Coefficients $\longrightarrow$ guess $\hat{x}_p(t)$

$$\hat{x}_p(t) = r\, e^{j(\omega t + \phi - \theta)}$$

r and θ are not known

(for Complex exponential $f(t) = A e^{j(\omega t + \phi)}$)