.

EE1101: Circuits and Network Analysis

Lecture 18: Power in AC Circuits

September 9, 2025

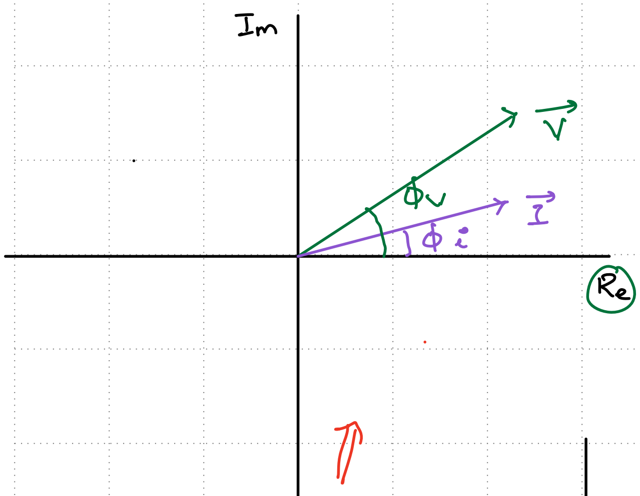
Topics :

1. Instantaneous Power, Active and Reactive Power
2. Complex Power and Power Factor

Phasor Diagram - Lag and Lead

time-domain $\begin{cases} v(t) = V_m \cos(\omega t + \phi_v) \Rightarrow \vec{V} = \frac{V_m}{\sqrt{2}} \angle \phi_v \\ i(t) = I_m \cos(\omega t + \phi_i) \Rightarrow \vec{I} = \frac{I_m}{\sqrt{2}} \angle \phi_i \end{cases}$ rep of signal in Phasor domain (or) freq. domain

Phasor diagram (scenario 1)



time to nearest peak

$\because \phi_v, \phi_i > 0 \Rightarrow t_{pv}, t_{pi} < 0$

$\phi_v > \phi_i \Rightarrow t_{pv} < t_{pi} \Rightarrow |t_{pv}| > |t_{pi}|$

(or) peak of $i(t)$ is closer to $t=0$ than peak of $v(t)$ (not efficient)

Instead def lagging phasor & leading phasor.

Phase (for this course) is always def in the range $-\pi$ to π.

$\Downarrow$ using this cond

$\vec{a}$ is leading $\vec{b}$ if $\phi_a > \phi_b$. where ϕ_a & ϕ_b phase angles of $\vec{a}$ & $\vec{b}$ [Phase ang. diff $= \phi_a - \phi_b$]

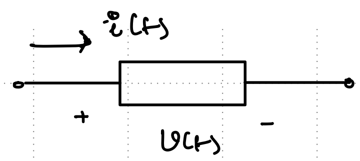
$\vec{a}$ is lagging $\vec{b}$ if $\phi_a < \phi_b$. where ϕ_a & ϕ_b phase angles of $\vec{a}$ & $\vec{b}$ [$\vec{a}$ lags $\vec{b}$ by $\vec{\phi}_b - \vec{\phi}_a$]

(or) Phase ang diff b/w $\vec{a}$ & $\vec{b}$ is $\vec{\phi}_a - \vec{\phi}_b$

Multiple statements

a) $\vec{V}$ leads $\vec{I}$ by $\phi_v - \phi_i$
(or)
b) $\vec{I}$ lags $\vec{V}$ by $\phi_v - \phi_i$
(or)
c) $\vec{V}$ and $\vec{I}$ have a phase difference of $\phi_v - \phi_i$

Instantaneous Power in AC Circuits



$v(t) = V_m \cos(\omega t + \phi_v)$

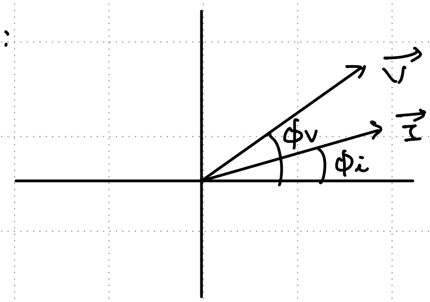
$i(t) = I_m \cos(\omega t + \phi_i)$

Phasor rep: $\vec{V} = \frac{V_m}{\sqrt{2}} \angle \phi_v$ and $\vec{I} = \frac{I_m}{\sqrt{2}} \angle \phi_i$

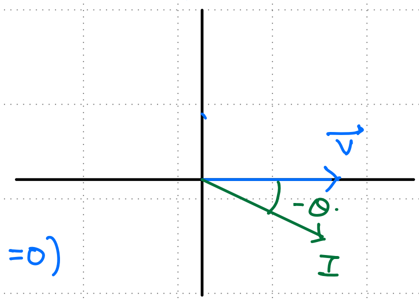
def θ as phase ang diff b/w $\vec{V}$ & $\vec{I}$

$\theta = \phi_v - \phi_i$

Phasor diagram:



$\Longrightarrow$ with a shifted time ref (s.t. $\phi_v = 0$)



Phasor domain

$\Downarrow$

$v(t) = V_m \cos(\omega t)$ time domain

$i(t) = I_m \cos(\omega t - \theta)$

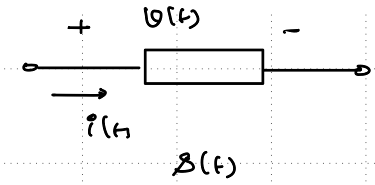
def $s(t)$: instantaneous power $= v(t)\, i(t)$

$= V_m \cos(\omega t)\, I_m \cos(\omega t - \theta)$

$= \frac{V_m I_m}{2} \left[\underbrace{\cos(2\omega t - \theta)}_{\text{double freq Comp}} + \underbrace{\cos\theta}_{\text{DC Component.}} \right]$

$s(t)$ is periodic with period $T/2$, freq $= 2\omega$ & Avg value $= \frac{V_m I_m}{2} \cos\theta$.

Active and Reactive Power



$$s(t) = \frac{V_m I_m}{2}\left[\cos(2\omega t - \theta) + \cos\theta\right]$$

(VA)

$$= \frac{V_m I_m}{2}\left[\cos 2\omega t \cos\theta + \sin 2\omega t \sin\theta + \cos\theta\right]$$

$$= \underbrace{\frac{V_m I_m}{2}\cos\theta\,(1+\cos 2\omega t)} + \underbrace{\frac{V_m I_m}{2}\sin\theta\,\sin 2\omega t}$$

def as instantaneous active Power $p(t)$ [W]

Avg value of $p(t) = \frac{V_m I_m}{2}\cos\theta$

def (Active power P)

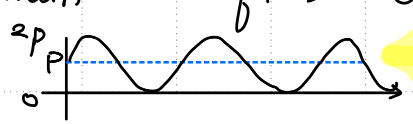
def of Active power (P) = Avg of $p(t)$ or $s(t)$

$= \underset{\sim}{VI}\cos\theta$

RMS values.

maximum value of $p(t) = V_m I_m \cos\theta$ (2P)

minimum value of $p(t) = 0$

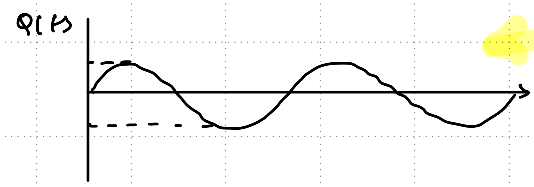


Instantaneous Reactive power (VAR) $q(t)$

Avg value of $q(t) = 0$

maximum value $= \frac{V_m I_m}{2}\sin\theta$

minimum value $= -\frac{V_m I_m}{2}\sin\theta$



Avg = 0

def : Reactive power $(Q) = \frac{V_m I_m}{2}\sin\theta$

$= VI\sin\theta$

$$s(t) = \underbrace{P(1+\cos 2\omega t)}_{p(t)} + \underbrace{Q\,\sin(2\omega t)}_{q(t)}$$

Complex Power

define Complex power $\vec{S} = VI\cos\theta + jVI\sin\theta = \underbrace{VI}_{\vec{V}}\, e^{j(\theta)} = VI\, e^{j(\phi_v - \phi_i)}$

$= \underbrace{Ve^{j\phi_v}}_{\vec{V}}\ \underbrace{Ie^{-j\phi_i}}_{(\vec{I})^*}$

$\vec{S} = \vec{V}\vec{I}^*$ [VA]

$= P + jQ$

def Apparent Power $S = |\vec{S}| = \sqrt{P^2 + Q^2} = VI$

def Power factor angle $\theta = \phi_v - \phi_i = \tan^{-1}(Q/P) = \cos^{-1}(P/S) = \sin^{-1}(Q/S)$

def Power factor $= \cos\theta = P/S$

Power Factor