EE1080/AI1110/EE2120 Probability, Quiz 1

20th Jan, 2025

Max. Marks: 16. **Time:** 1 hour.

Instructions

- Please **write your roll number** and **course id** prominently in the first page of the answer sheet.
- No laptops, mobile devices etc. allowed.
- please write supporting arguments for any of the statements you make. any result proved in the lectures can be used by stating it clearly, no need to prove them in your answer.

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1. (3) Identify the following sets as finite, countably infinite or uncountable as you seem appropriate. Given a set A, the notation A^n is used to indicate vectors of length n with elements taking values from A.
   (a) S^∞ where $S = \{1, 2, \cdots, 6\}$.
   (b) S^{1000}
   (c) Q^2 where Q is the set of rational numbers.
2. ($3 = 1.5 + 1.5$) Given an experiment where you roll a six faced die with sample space $\Omega = \{1, 2, \cdots, 6\}$.
   (a) Which of the following can be an event space $\mathcal{F}$:
      i. $\{\phi, \Omega\}$
      ii. $\{\phi, \Omega, \{1, 6\}, \{2, 3\}, \{4, 5\}\}$
      iii. $\{\phi, \Omega, \{1, 2, 3\}, \{4, 5, 6\}\}$
   (b) Assume $\mathcal{F} = 2^\Omega$. Consider probability function $P(\{i\}) = i * x$ for all $i \in \Omega$. Identify the value of x.
3. (1) Suppose A and B are events with very high probability: say $P(A) = 0.95$ and $P(B) = 0.85$. Then which of the following are true (check all that apply)
   (a) Either A or B occur with probability at most 0.85
   (b) Both A and B occur with probability at least 0.8
4. (2) Identify if the following statements are true or false
   (a) mutual independence of events $A, B, C \implies A, B$ are independent conditioned over C
   (b) A, B are independent conditioned over C implies A, B are independent
5. (2) An online store offers three types of products: books, electronics, and clothing. The probability that a book is defective is 0.02, the probability that an electronic item is defective is 0.05, and the probability that a piece of clothing is defective is 0.03. A customer orders one book, one electronic item, and one piece of clothing. What is the probability that at least one of the items is defective?
6. (2) We know that a treasure is located in one of two places, with probabilities β and $1 - \beta$, respectively, where $0 < \beta < 1$. We search the first place and if the treasure is there, we find it with probability $p > 0$. Given the treasure is not found in the first place, what is the probability of treasure being in the second place ?
7. (3) Alice and Bob want to choose between the opera and the movies.
   (a) Unfortunately, the only available coin is biased (and the bias is not known to Alice or Bob). How can they use the biased coin to make a decision so that either option (opera/movies) is equally likely to be chosen?
   (b) Now, instead, suppose that Alice and Bob have a fair coin. Alice and Bob want to toss a coin that is predisposed towards the movies - say they would want to pick the opera with probability $p = 1/8$. How can they use the fair coin to make a decision?

   (*Hint: Toss the coin multiple times*)