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EE1101: Circuits and Network Analysis

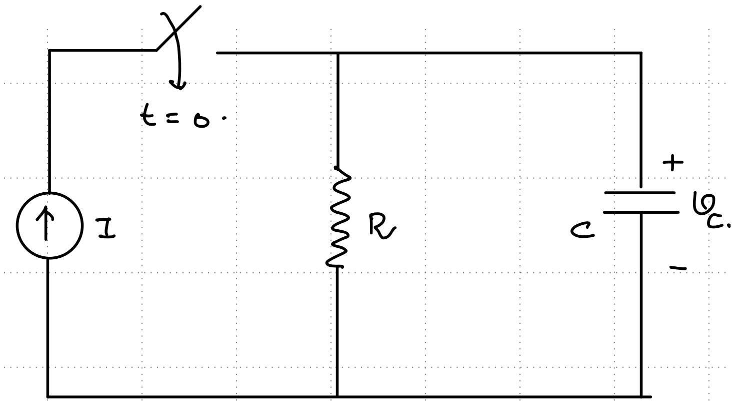
Lecture 24: First-Order Circuits

September 22, 2025

Topics :

1. Quantifying Transient Response
2. Examples of First-order Circuits

Examples of First-order Circuits



by KCL, $I_s = i_r + \underbrace{i_c}$

$$I_s = \frac{v_c}{R} + C\frac{dv_c}{dt}$$

$$\frac{dv_c}{dt} + \frac{1}{RC}v_c = \frac{I_s}{C}. \longrightarrow (1)$$

Integrating factor $= e^{\int \frac{1}{RC}dt} = e^{\frac{1}{RC}t}$

mul (1) on both sides with IF $\Rightarrow$

$$\frac{d}{dt}\left(e^{\frac{1}{RC}t} v_c(t)\right) = \frac{I_s}{C} e^{t/RC}$$

Integrate on both sides.

$$e^{t/RC} v_c = \frac{I_s}{C} \times RC\, e^{t/RC} + c$$

$$v_c = RI_s + ce^{-t/RC}$$

c: $v_c(0) = 0 \Rightarrow c = -RI_s$ & $v_c(t) = RI_s(1 - e^{-t/RC})$

as $t \to \infty$ $v_c = RI_s$ (steady state response) $\leftarrow$

Transient Response:

$$v_c(t) - \underbrace{v_c(\infty)}_{\text{Steady state Value.}} = -RI_s \underbrace{e^{-t/RC}}$$

determines the variation of transient response

Quantifying Transient Response

for first order circuits: Nature of transient response is exponentially decaying

one way to Quantify the transient response is to indicate the rate of decay.

time Constant : (τ)

$e^{-t/\tau}$.

for Series RL ckt

$\tau = L/R$

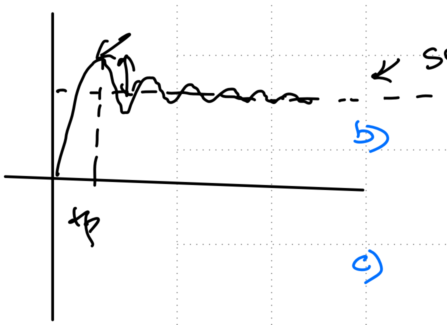
for parallel RC ckt

$\tau = RC$.

for higher order ckts (ckts having L, C & multiple energy storage elem)

a) rise time (t_r) : time taken from 2% of the final value to 98% of final value

(5% – 95%) (or) (10 – 90%)

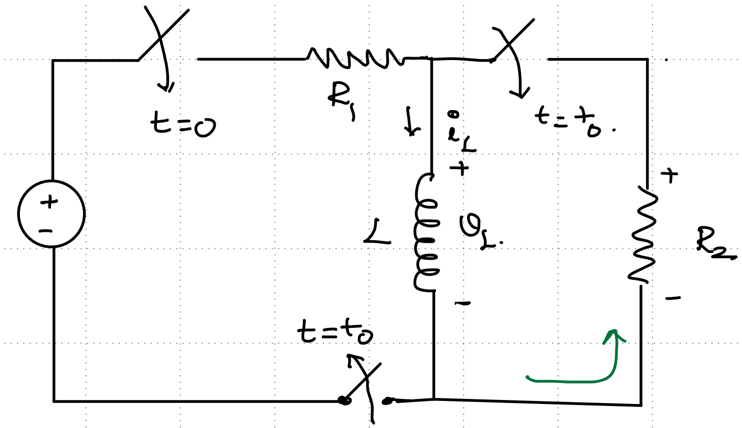


b) Settling time (t_s) : time to reach 98% (or) 95% of the final value. ($\pm$2% of final value)

c) Peak overshoot (M_p) : Peak value as a fraction of final value (%)

d) time to peak (t_p) : time to reach peak value

Examples of First-order Circuits



for $0 < t < t_0$: RL ckt driven by $V_s u(t)$

$$i(t) = \frac{V_s}{R_1}\left(1 - e^{-t/\tau}\right)$$

where $\tau = L/R_1$

at t_0 when switch transition happens

$$i_L(t_0-) = i_L(t_0+)$$

t_0: large enough for the ckt to operate in steady-state

$\Rightarrow$ t_0 is large enough for $i_L(t_0)$ = steady state current

$v_C(t_0)$ = steady state voltage

for $t > t_0$: $v_{R_2} = v_L$.

$$L\frac{di_L}{dt} = v_{R_2} = -i_L R_2.$$

$$\frac{di_L}{dt} + \frac{R_2}{L} i_L = 0. \longrightarrow (3)$$

$IF = e^{t/\tau_2}$ where $\tau_2 = L/R_2$

mul (3) by IF on both sides

$$\frac{d}{dt}\left(e^{t/\tau_2} i_L\right) = 0$$

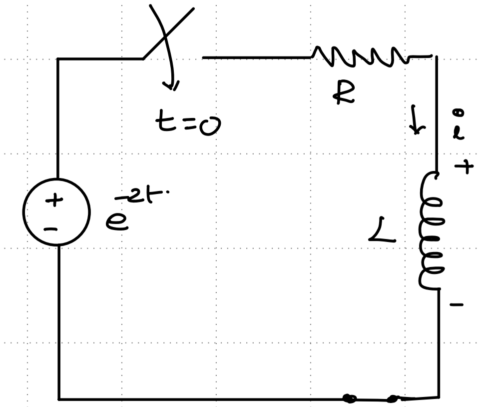
$$i_L = c e^{-t/\tau_2}.$$

at t_0: $i_L(t_0) = c e^{-t_0/\tau_2}$

$$c = i_L(t_0) e^{t_0/\tau_2}$$

$$\Rightarrow i_L(t) = i_L(t_0) e^{-(t-t_0)/\tau_2}$$

Examples of First-order Circuits (Response to e^{-2t})



by KVL (for $t > 0$):

$$\frac{di}{dt} + \frac{R}{L} i = \frac{1}{L} e^{-2t} \longrightarrow (1)$$

Integrating factor = $e^{R/L\, t}$

upon mul (1) by IF & rearranging

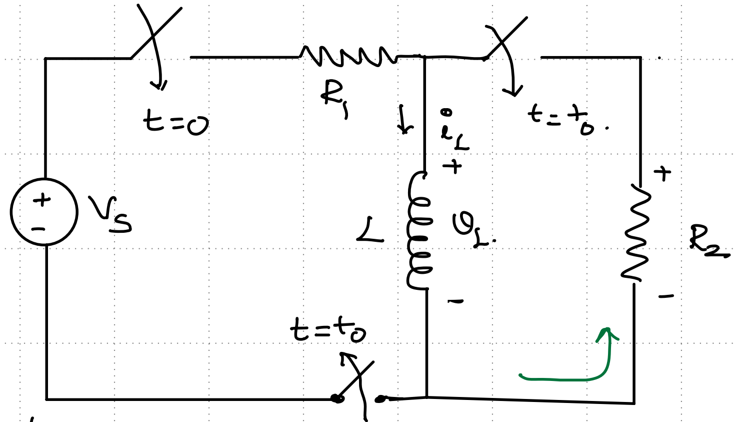
$$\frac{d}{dt}\left(e^{R/L\, t}\, i(t)\right) = \frac{1}{L} e^{(R/L - 2)t}$$

$$i(t) = \frac{e^{-R/L\, t}}{L} \cdot \frac{1}{(R/L - 2)} e^{R/L\, t} e^{-2t} + C e^{-R/L\, t}$$

$$i(t) = \frac{1}{(R - 2L)} e^{-2t} + C e^{-R/L\, t}$$

C: based on Init Cond.

Zero-input and Zero-state Response



for $0<t<t_0$: response is due to the source

$t>t_0$: response is due to init current through L.

Zero-state response
⇓
Complete response is due to source alone

Zero-input response
⇓
response is due to initial value of state variables
($i \to L$
$v \to C$)