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EE1101: Circuits and Network Analysis

Lecture 34: Two-Port Networks

October 27, 2025

Topics :

1. Introduction to Two-Port Networks
2. Admittance and Impedance Parameters

Introduction to Two-Port Networks

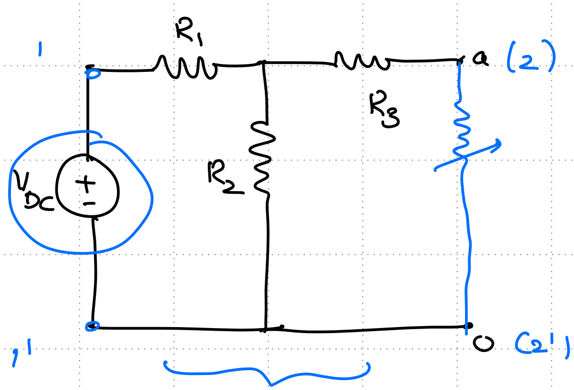
Analysis of single-port Networks

a) Port properties (OCV, SCC, Eq. impedance)

b) Thevenin & Norton Equivalent
(OCV + Eq imp) (SCC || Eq. imp)

→ Deriving the Equivalent

→ Computing the Parameters of the Eq. ckt.

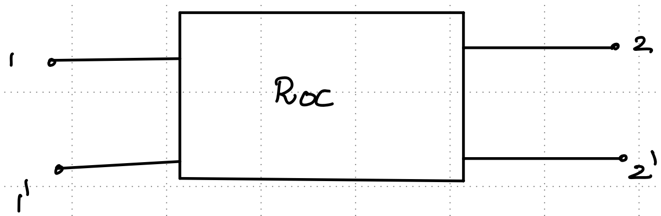


$$V_{Th} = \frac{R_2}{R_1+R_2} V_{DC}$$

$$R_{Th} = (R_1 || R_2) + R_3$$

when computing parameters, the commonly employed route does not involve Substitution / Superposition Theorem.

Two-port Network :-



Use case :

a) Useful when the goal is to compute response associated with elements connected across two ports

→ can vary.

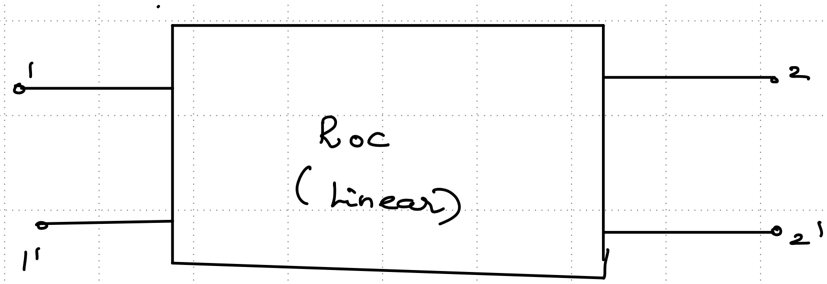
Goal: Come with a mathematical rep of the Roc

so that computing the resp associated with the two ports is efficient

Can determine as along as ckt is linear

→ useful: if Roc: does not contain indep sources.

Introduction to Two-Port Networks



⟹ Come up with a simpler mathematical rep

↓

Can be achieved in multiple ways

↓

each way has a unique use case.

4 Possibilities

① − ①' → V_1 , ② − ② → V_2

① − ①' → I_1 , 2 − 2' → I_2

① − ①' → I_1 , 2 − 2' → V_2

1 − 1' → V_1 , 2 − 2' → I_2

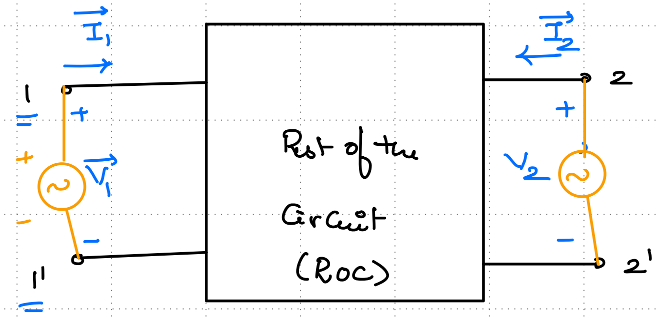
① Connect a Vol Source & Compute the Current associated with it

② Connect a Curr Source & Compute the Voltage associated with it.

Transmission & inverse Transmission ← ③ other ways.

Admittance Parameters (deriving the model)

→ involves using independent Vol. sources at both ports & trying to express the associated currents



$$\vec{I_1} = \vec{I_{1a}} + \vec{I_{1b}}$$

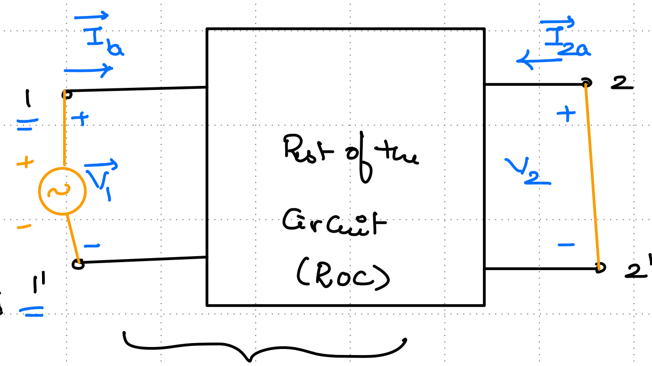
$$\vec{I_2} = \vec{I_{2a}} + \vec{I_{2b}}$$

Sub-ckt (a): Source at Port 2-2' is Null. (Short Circuit of Port 2-2') ($\vec{V_2} = 0$)

$$\vec{I_{1a}} = \left(\frac{1}{\vec{Z_{11}}}\right)\vec{V_1}$$

$\vec{Z_{11}}$ → Eq. imp of Port 1-1' with 2-2' Short Circuited

$\vec{y_{11}}$ → admittance.



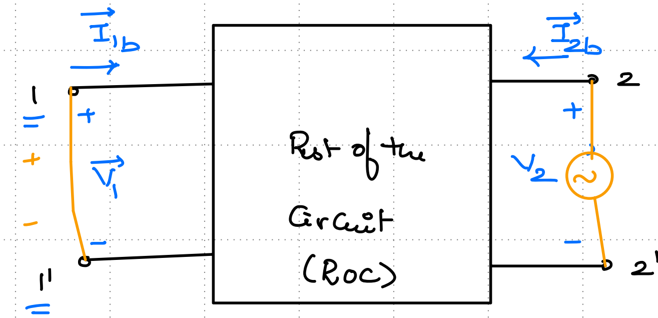
$$\vec{I_{2a}} = -\vec{I_{sc}} \text{ of Port } 2\text{-}2'$$
$$= \vec{y_{21}}\,\vec{V_1}$$

$\vec{y_{21}}$ → admittance

Admittance Parameters (short circuit parameters)

Sub-ckt (b): Source at Port 1-1' & Null. (Short circuit of Port 1-1')

$(\vec{V}_1 = 0)$



$\vec{I}_{1b} = -($ sc current of Port 1-1'$)$

$= \vec{y}_{12} \vec{V}_2$

$\vec{I}_{2b} = \dfrac{1}{\vec{Z}_{22}} \vec{V}_2 = \vec{y}_{22} \vec{V}_2$

$\vec{Z}_{22} \rightarrow$ Eq. imp of Port 2-2' when 1-1' is short circuited

$\vec{y}_{22}$

Overall model:

$\vec{I}_1 = \vec{I}_{1a} + \vec{I}_{1b} = \vec{y}_{11}\vec{V}_1 + \underbrace{\vec{y}_{12}\vec{V}_2}$ → −(scc associated with 1-1')

$\vec{I}_2 = \vec{I}_{2a} + \vec{I}_{2b} = \underbrace{\vec{y}_{21}\vec{V}_1} + \vec{y}_{22}\vec{V}_2$

−(scc associated with 2-2')

$\vec{y}_{11} \rightarrow$ Eq. admittance with 2-2' shorted

$\vec{y}_{22} \rightarrow$ Eq. admittance with 1-1' shorted

$$\begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \end{bmatrix} = \begin{bmatrix} \vec{y}_{11} & \vec{y}_{12} \\ \vec{y}_{21} & \vec{y}_{22} \end{bmatrix} \begin{bmatrix} \vec{V}_1 \\ \vec{V}_2 \end{bmatrix}$$

Methodology to derive the mathematical model of a 2-Port n/w.

Step 1: Connect a Source at each port

(if $\vec{V}$ source is connected, Derive an expression for Current)

(if $\vec{I}$ " " " , Derive " " " Voltage)

function of Port Prop.

(or)

response of 2 Sub ckts

Step 2: Compute the desired responses as a Sum of resp. obtained from 2 Sub Ckts

Sub Ckt ⓐ & ⓑ

Step 3: Put them together & build matrix up of the 2- port n/w.

for admittance parameters

$$\begin{bmatrix} \vec{I_1} \\ \vec{I_2} \end{bmatrix} = \begin{bmatrix} \vec{y_{11}} & \vec{y_{12}} \\ \vec{y_{21}} & \vec{y_{22}} \end{bmatrix} \begin{bmatrix} \vec{V_1} \\ \vec{V_2} \end{bmatrix}$$

Computing

$$\vec{y_{11}} = \left.\frac{\vec{I_1}}{\vec{V_1}}\right|_{\vec{V_2}=0} \qquad \vec{y_{21}} = \left.\frac{\vec{I_2}}{\vec{V_1}}\right|_{\vec{V_2}=0}$$

$$\vec{y_{12}} = \left.\frac{\vec{I_1}}{\vec{V_2}}\right|_{\vec{V_1}=0} \qquad \vec{y_{22}} = \left.\frac{\vec{I_2}}{\vec{V_2}}\right|_{\vec{V_1}=0}$$