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EE1101: Circuits and Network Analysis

Lecture 37: Two Port Networks

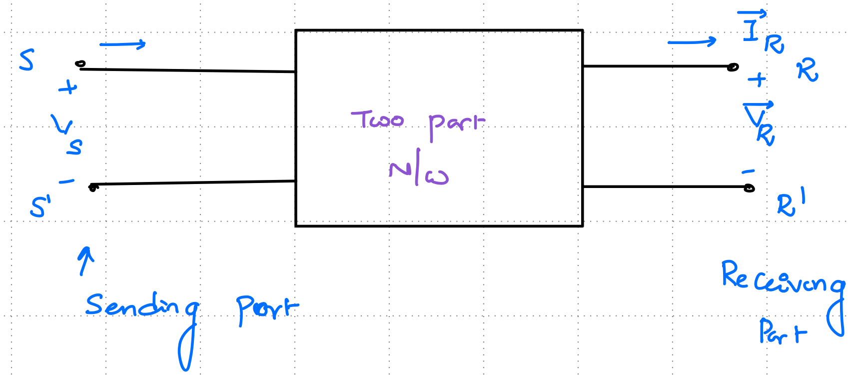
Topics :

1. Transmission Parameters
2. Series and Cascade Connection of Two-port networks

Transmission parameters

Model

$$\begin{bmatrix} \vec{V}_S \\ \vec{I}_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \vec{V}_R \\ \vec{I}_R \end{bmatrix}$$



Complex valued

$$A = \left.\frac{\vec{V}_S}{\vec{V}_R}\right|_{\vec{I}_R = 0} \qquad C = \left.\frac{\vec{I}_S}{\vec{V}_R}\right|_{\vec{I}_R = 0}$$

open of Port R−R'

$$B = \left.\frac{\vec{V}_S}{\vec{I}_R}\right|_{\vec{V}_R = 0} \qquad D = \left.\frac{\vec{I}_S}{\vec{I}_R}\right|_{\vec{V}_R = 0}$$

Short circuit of Port R−R'

2 diff :-

a) Ports are ref as Sending & Receiving Port

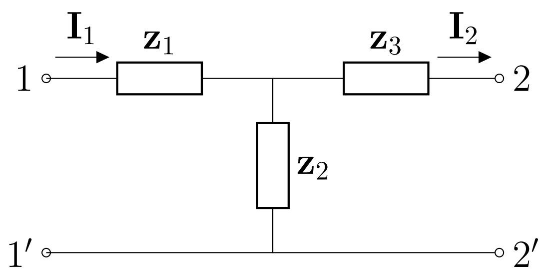
(b) Sending Port & Receiving Port adopt different sign convention.

Computing Parameters (A, B, C, D)

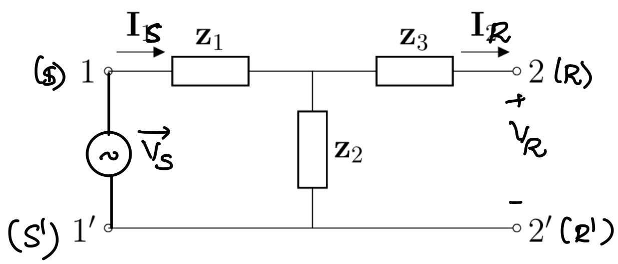
open Circuit of port R−R' (A, C)

Short Circuit of Port R−R' (B, D).

Transmission parameters - Example



$$\begin{bmatrix} \vec{V}_s \\ \vec{I}_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \vec{V}_R \\ \vec{I}_R \end{bmatrix}$$



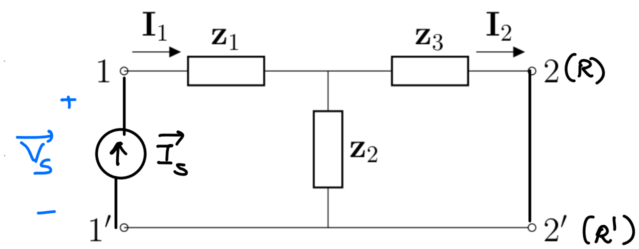
$$A = \left.\frac{\vec{V}_s}{\vec{V}_R}\right|_{\vec{I}_R = 0} =$$

$$\vec{V}_R = \frac{\vec{Z}_2}{\vec{Z}_2 + \vec{Z}_1}\vec{V}_s \Rightarrow A = \frac{\vec{Z}_2 + \vec{Z}_1}{\vec{Z}_2}$$

$$C = \left.\frac{\vec{I}_s}{\vec{V}_R}\right|_{\vec{I}_R = 0}$$

$$\vec{I}_s = \frac{\vec{V}_s}{\vec{Z}_1 + \vec{Z}_2} \Rightarrow \frac{A \cdot \vec{V}_R}{\vec{Z}_1 + \vec{Z}_2}$$

$$C = \frac{A}{\vec{Z}_1 + \vec{Z}_2} = \frac{1}{\vec{Z}_2}$$



$$D = \left.\frac{\vec{I}_s}{\vec{I}_R}\right|_{\vec{V}_R = 0}$$

$$\vec{I}_R = \frac{\vec{Z}_2}{\vec{Z}_3 + \vec{Z}_2}\vec{I}_s \Rightarrow D = \frac{\vec{Z}_3 + \vec{Z}_2}{\vec{Z}_2}$$

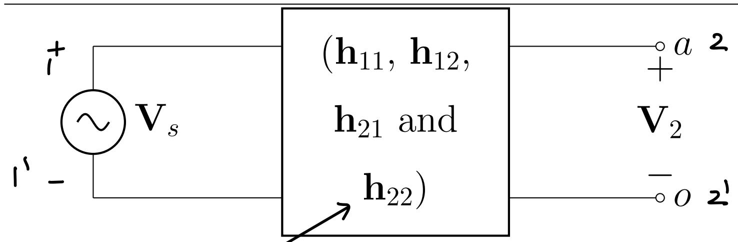
$$\vec{B} = \left.\frac{\vec{V}_s}{\vec{I}_R}\right|_{\vec{V}_R = 0} =$$

$$\vec{V}_s = \vec{V}_{z_1} + \vec{V}_{(\vec{Z}_2 \| \vec{Z}_3)} \quad \Big| \quad \vec{Z}_{eq} = \vec{Z}_1 + (\vec{Z}_2 \| \vec{Z}_3)$$

$$\vec{V}_s = \vec{I}_s \vec{Z}_{eq}$$

$$\vec{V}_s = D\vec{I}_R \vec{Z}_{eq}$$

Computing Thevenin Equivalent of a Port



given: a) a 2-port rep of the N/w & → h-parameters

b) Port 1-1' has a fixed source connected to it.

Compute: Thevenin EQ. as seen from 2-2'

$$\begin{bmatrix} \vec{V_1} \\ \vec{I_2} \end{bmatrix} = \begin{bmatrix} \vec{h_{11}} & \vec{h_{12}} \\ \vec{h_{21}} & \vec{h_{22}} \end{bmatrix} \begin{bmatrix} \vec{I_1} \\ \vec{V_2} \end{bmatrix}$$

$\vec{V_1} = \vec{h_{11}}\vec{I_1} + \vec{h_{12}}\vec{V_2}$ — ⓐ

$\vec{I_2} = \vec{h_{21}}\vec{I_1} + \vec{h_{22}}\vec{V_2}$ — ⓑ

$\vec{V_{Th}}$ = open ckt vol of 2-2' ($\vec{I_2} = 0$) & given $\vec{V_1} = \vec{V_S}$

$\because \vec{I_2} = 0 \Rightarrow \vec{V_2} = -\dfrac{\vec{h_{21}}}{\vec{h_{22}}}\vec{I_1} \rightarrow$ ① (from ⓑ)

from ⓐ : $\vec{I_1} = \dfrac{\vec{V_S} - \vec{h_{12}}\vec{V_2}}{\vec{h_{11}}} \rightarrow$ ②

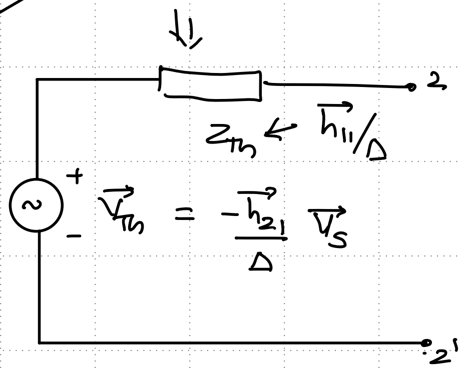
Plugin ② in ① $\Rightarrow \vec{V_2} = \dfrac{-\vec{h_{21}}}{\vec{h_{22}}}\left(\dfrac{\vec{V_S} - \vec{h_{12}}\vec{V_2}}{\vec{h_{11}}}\right)$

$\Rightarrow \vec{V_2} = -\dfrac{h_{21}}{\Delta}\vec{V_S}$

$\vec{V_{Th}}$

$\Delta \leftarrow$ det of h-param matrix.

When R_{oc} has no independent source.



$$\vec{Z_{Th}} = \frac{\vec{V_2}}{\vec{I_2}} \Bigg| \; \vec{V_S} = 0 \text{ (or) } \vec{V_1} = 0$$

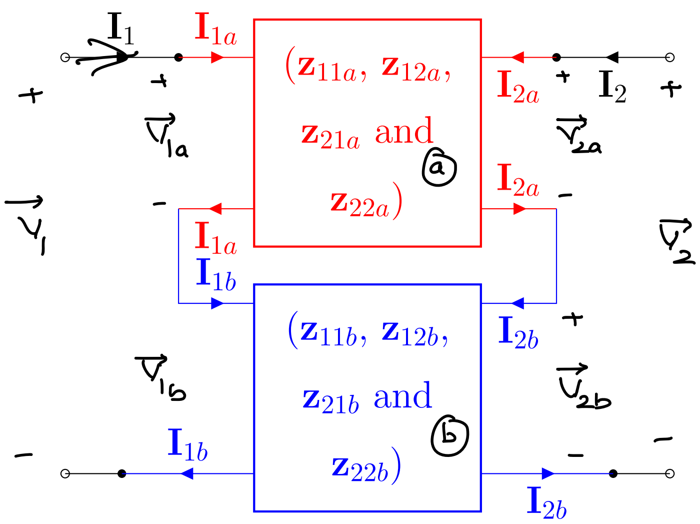
from ⓐ : $0 = \vec{h_{11}}\vec{I_1} + \vec{h_{12}}\vec{V_2} \rightarrow$ ③

from ⓑ : $\vec{I_2} = \vec{h_{21}}\vec{I_1} + \vec{h_{22}}\vec{V_2} \rightarrow$ ④

Compute $\vec{I_1}$ from ③ & plug in ④

$$\frac{\vec{V_2}}{\vec{I_2}} = \frac{\vec{h_{11}}}{\Delta}.$$

Series Connection of Two Port Networks



for 2-port N/w (a)

$$\begin{bmatrix} \vec{V}_{1a} \\ \vec{V}_{2a} \end{bmatrix} = \begin{bmatrix} \vec{Z}_{11a} & \vec{Z}_{12a} \\ \vec{Z}_{21a} & \vec{Z}_{22a} \end{bmatrix} \begin{bmatrix} \vec{I}_{1a} \\ \vec{I}_{2a} \end{bmatrix}$$

for 2-port N/w (b)

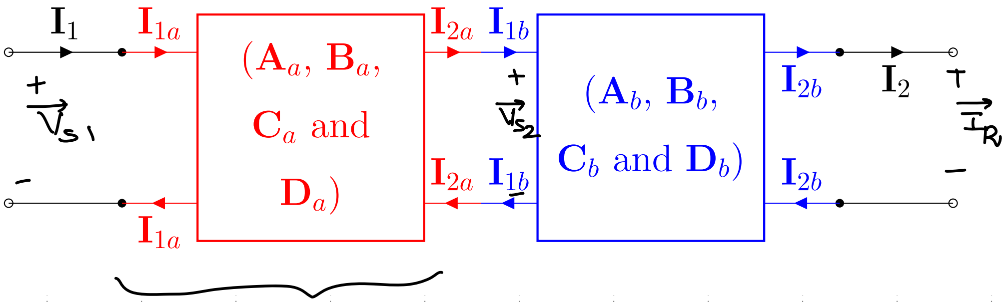
$$\begin{bmatrix} \vec{V}_{1b} \\ \vec{V}_{2b} \end{bmatrix} = \begin{bmatrix} \vec{Z}_{11b} & \vec{Z}_{12b} \\ \vec{Z}_{21b} & \vec{Z}_{22b} \end{bmatrix} \begin{bmatrix} \vec{I}_{1b} \\ \vec{I}_{2b} \end{bmatrix}$$

(a) & (b) are connected in series.

$$\Rightarrow \left.\begin{aligned} \vec{V}_1 &= \vec{V}_{1a} + \vec{V}_{1b} \\ \vec{V}_2 &= \vec{V}_{2a} + \vec{V}_{2b} \end{aligned}\right\} \quad \begin{aligned} \vec{I}_{1a} &= \vec{I}_{1b} = \vec{I}_1 \\ \vec{I}_{2a} &= \vec{I}_{2b} = \vec{I}_2 \end{aligned}$$

$$\begin{bmatrix} \vec{V}_1 \\ \vec{V}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \vec{Z}_{11a} + \vec{Z}_{11b} & \vec{Z}_{12a} + \vec{Z}_{12b} \\ \vec{Z}_{21a} + \vec{Z}_{21b} & \vec{Z}_{22a} + \vec{Z}_{22b} \end{bmatrix}}_{\text{Sum of imp. param matrices}} \begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \end{bmatrix}$$

Cascade Connection of Two Port Networks



for 2-port N/w (a)

$$\begin{bmatrix} \vec{V}_{sa} \\ \vec{I}_{sa} \end{bmatrix} = \underbrace{\begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix}}_{T_a} \begin{bmatrix} \vec{V}_{Ra} \\ \vec{I}_{Ra} \end{bmatrix}$$

for N/w (b)

$$\begin{bmatrix} \vec{V}_{sb} \\ \vec{I}_{sb} \end{bmatrix} = \underbrace{\begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}}_{T_b} \begin{bmatrix} \vec{V}_{Rb} \\ \vec{I}_{Rb} \end{bmatrix}$$

$$\begin{bmatrix} \vec{V}_{Ra} \\ \vec{I}_{Ra} \end{bmatrix} = \begin{bmatrix} \vec{V}_{sb} \\ \vec{I}_{sb} \end{bmatrix}$$

$$\begin{bmatrix} \vec{V}_{sa} \\ \vec{I}_{sa} \end{bmatrix} = [T_a][T_b] \begin{bmatrix} \vec{V}_{Rb} \\ \vec{I}_{Rb} \end{bmatrix}$$