

- 1. Higher Dimensional Analysis:** Using the concepts you have learnt regarding planes in three dimensions, answer the following questions based on the information provided for a 4 dimensional space.

Consider a plane $P_1 : \vec{n}_1 \cdot (\vec{r} - \vec{r}_0) = \vec{m}_1 \cdot (\vec{r} - \vec{r}_0) = 0$

Consider another plane $P_2 : \vec{n}_2 \cdot (\vec{r} - \vec{r}_0) = \vec{m}_2 \cdot (\vec{r} - \vec{r}_0) = 0$

- (a) Will they intersect if they are not parallel?
 - (b) What will be the condition for the two planes to be parallel?
 - (c) What will be the condition for them to coincide?
 - (d) When will they intersect at a single point?
 - (e) When will they intersect in a line?
 - (f) Are there any other forms of intersection possible, that haven't been enumerated above?
2. If three vectors $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^3$, show that, $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$
- Try to explain geometrically what each term means and its orthogonality or parallel nature with respect to each of the other terms.

3. We know that every vector in \mathbb{R}^3 can be written as a scalar combination of the vectors $\hat{i}, \hat{j}, \hat{k}$. Can every vector in \mathbb{R}^3 be written as a scalar combination of just \hat{i}, \hat{j} , i.e. for any vector $\vec{v} \in \mathbb{R}^3$, are there scalars m, n such that $\vec{v} = m\hat{i} + n\hat{j}$? Justify your answer.

4. Find the points of intersection of the line $x = 3 + 2t, y = 7 + 8t, z = 2 + t$, that is, $l(t) = (3 + 2t, 7 + 8t, 2 + t)$, with the coordinate planes.
5. Do the lines $(x, y, z) = (t + 4, 4t + 5, t2)$ and $(x, y, z) = (2s + 3, s + 1, 2s - 3)$ intersect?
6. Find the line through $(3, 1, 2)$ that intersects and is perpendicular to the line $x = -1 + t, y = -2 + t, z = -1 + t$. [HINT: If (x_0, y_0, z_0) is the point of intersection, find its coordinates.]

7. **An elementary example of computing flux of a Vector field:** Let $\vec{F}(x, y, z)$ (or) $\vec{F}(r)$ denote a vector field whose value at point $(1, 1, 1)$ on a particular plane is $2\hat{i} + 3\hat{j} - \hat{k}$. The chosen plane for analysis has two other points $(3, 2, 1)$ and $(4, 1, 2)$.

- (a) Compute the equation of the plane chosen for analysis.
 - (b) Compute a unit vector (say \hat{n}) that is normal to the plane chosen for analysis.
 - (c) Compute $\vec{F}(r) \cdot \hat{n}$ at $(1, 1, 1)$ (Later in the course, you will see that the scalar product of the form plays a crucial role in computing flux of a vector field through a given surface).
8. **Representation of a Vector Fields:** Represent the following (3D) vector fields

(a) $\vec{F}(r) = -2 \frac{\vec{r}}{|\vec{r}|}$

(b) $\vec{F}(r) = 2 \frac{\hat{j} \times \vec{r}}{|\vec{r}|^2}$

It is recommended to represent the vector fields (without using any open-source tools) in an appropriate manner for your practice.