CS1010: Discrete Mathematics for Computer Science

(Exam-1. Total: 30 marks.)
(Duration: 45 minutes. Date: 26 Aug 2024)

Instructions:

- All the questions, except Q7, have short answers. Even the answer to Q7 can be written in 3-5 lines.
- You may not get time to answer all the questions unless you have prepared really well. The exam is designed like that.
- If your mobile phone is found with you during the exam, you will lose one grade.
- Anybody found copying will get an F grade for the course straight away.
- It is a no-break exam. You cannot take a break in between. The exam is only for 45 minutes. If you want to go out of the exam hall, you will have to submit your answer paper.
- You should sit far apart from each other. The halls are big enough. If we see two students sitting close to each other, both the students will lose one grade.

Questions

1. Let G represent the set of all T.V. game shows. Let P be the set of people in your neighbourhood. Let $C(p, g)$ be the predicate that the person p appeared on game show g and $D(p)$ be the predicate that p is a doctor. Using the above definitions,
   (a) Translate: "There is a person in your neighbourhood who has been a contestant on a game show but is not a doctor" into symbolic notation.
   (b) Negate the symbolic expression of (a)
   (c) Finally, translate the expression from (b) into English. (1 x 3 = 3 marks)
2. Using the algebraic rules for boolean functions, show that $\sim (\sim p \wedge q) \wedge (p \vee q) \equiv p$. At each step, identify the rule used. (2 marks)
3. Let $\mathbf{I} =$ Interest rates go down; $\mathbf{S} =$ Stock Market goes up. For the implications below, write the implication, its converse and its contrapositive in words.
   $I \Rightarrow S$ (1 x 3 = 3 marks)

4. Rewrite the following statements in if-then form:

   (a) Either you get to work on time or you are fired.

   (b) Kamala's attaining age 35 is a necessary condition for her being the president of India.

   (c) Pedro's birth on U.S. soil is a sufficient condition for him to be a U.S. citizen.

   (d) Arun will go to school unless it rains. (1 x 4 = 4 marks)

5. Suppose that p and q are statements so that $p \Rightarrow q$ is false. Find the truth values of each of the following:

   (a) $\sim p \Rightarrow q$.

   (b) $p \vee q$.

   (c) $q \Rightarrow p$. (1 x 3 = 3 marks)

6. Prove that if x is irrational, then $\frac{1}{x}$ is irrational. (3 marks)

7. Let T be the set of all infinite 0-1 bit sequences. We know that T is *uncountable*. For any non-negative integer k, the k-th element of a sequence $\alpha \in T$ is denoted by $\alpha(k)$. Prove the countability/uncountability of the subsets of T given in sub-questions (a) and (b). Solve the two sub-questions independently. That is, do not use the result of any sub-question in the other.

   (a) $T_1 = \{\alpha \in T \ : \ \alpha(k) = 1 \text{ and } \alpha(k+1) = 0 \text{ for } \underline{\text{some}} \ k \geq 0\}$

   (b) $T_2 = \{\alpha \in T \ : \ \alpha(k) = 1 \text{ and } \alpha(k+1) = 0 \text{ for } \underline{\text{no}} \ k \geq 0\}$

   (6+6 = 12 marks)

——————— ALL THE BEST ———————