.

EE1101: Circuits and Network Analysis

Lecture 17: Sinusoidal Signals and Phasors

September 8, 2025

Topics :

1. Sinusoidal Signals
2. Phasor Representation

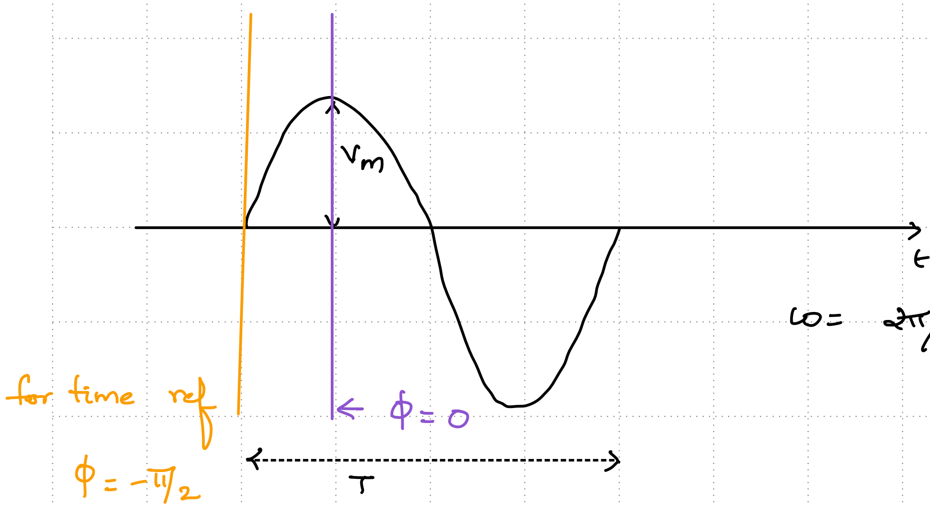
Sinusoidal Signals - Significance of time reference

general form : $V_m \cos(\omega t + \phi)$

if the given signal is $V_m \sin(\omega t + \gamma) \Rightarrow V_m \cos(\omega t + \phi)$ where $\phi = \gamma - \pi/2$.

3 parameters in a sinusoidal signal :
a) $V_m \longrightarrow$ Peak value.
b) $\omega \longrightarrow$ freq. (rad/s)
c) $\phi \longrightarrow$ Phase.

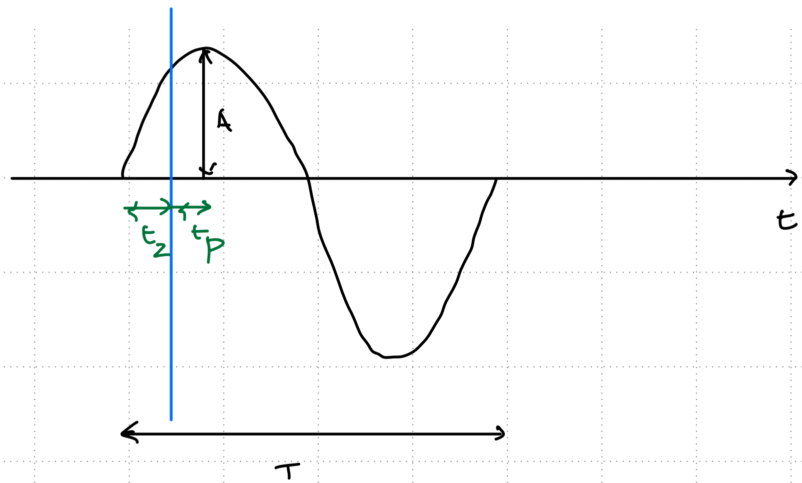
Ideal Scenario (No Noise, $\omega \rightarrow$ fixed & $V_m \rightarrow$ fixed)



$\phi \leftarrow$ dependent on time ref.

$\omega = 2\pi/T$

Phase of a Sinusoidal Signal



Peak Value & freq in an ideal Scenario can be computed from the waveform.

$V_m = A$

$\omega = 2\pi/T$.

How to Compute ϕ ?

a) look at Nearest Zero Crossing (t_z)

(or) time to nearest positive peak (t_p)

$v(t_p) = V_m = V_m \cos(\omega t_p + \phi)$

$\Rightarrow \cos(\omega t_p + \phi) = 1$

$\Rightarrow \phi = -\omega t_p$.

if $\phi < 0 \Rightarrow$ Nearest Positive peak occurs after the time ref $(t=0)$

if $\phi > 0 \Rightarrow$ Nearest Positive peak is bef time ref.

(x) b) if $v(0)$ can be measured accurately

$v(0) = V_m \cos(\omega t + \phi) = V_m \cos\phi$

$\Rightarrow \phi = \cos^{-1}\left(\frac{v(0)}{V_m}\right)$

Computing Phase for a real-world signal is challenging.

Phasor Representation

Consider a signal $v(t) = V_m \cos(\omega t + \phi)$

When dealing with a ckt where all sources are at same freq

$\Downarrow$ (when ckt is a linear ckt)

all voltages & currents in the ckt are also of the same freq.

$\Downarrow$ under this scenario

all signals can be characterized by

a) Peak value

b) Phase

$\Downarrow$ leads to the def of a Phasor.

$$\vec{V} = V_m \angle\phi \quad (\text{or}) \quad \boxed{\vec{V} = \underbrace{\frac{V_m}{\sqrt{2}}}_{\text{RMS value}} \angle\phi}$$

def for this course.

mag of Phasor (or) Sinusoidal signal $\longrightarrow \dfrac{V_m}{\sqrt{2}}$

Phase $\longrightarrow \phi$

$$v(t) = \mathrm{Re}\left\{V_m e^{j(\omega t + \phi)}\right\}$$

$$= \mathrm{Re}\left\{V_m e^{j\phi} \underbrace{e^{j\omega t}}_{\text{redundant}}\right\}$$

$\vec{V} = V_m \angle\phi$ ← Polar form

or $V_m e^{j\phi}$ ← Complex exponential form

$= V_m(\cos\phi + j\sin\phi)$

↑ Rectangular form

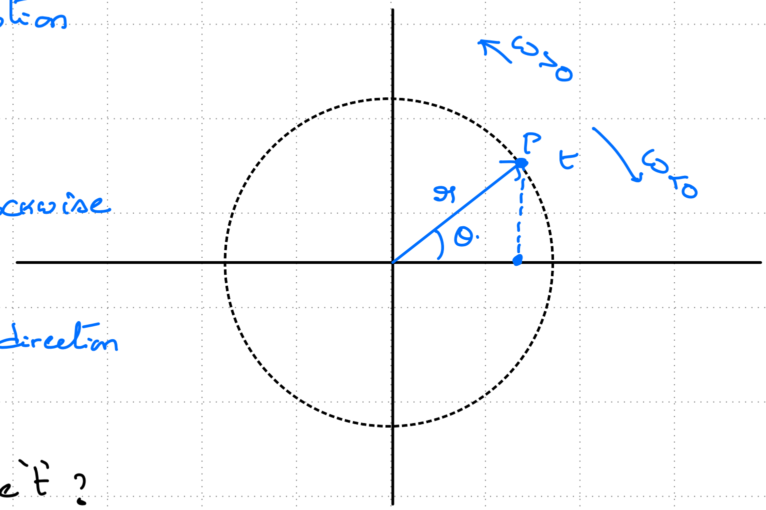
Phasors and Uniform Circuit Motion

object exhibits uniform circular motion

ω = fixed

$\omega > 0 \longrightarrow$ Counter Clockwise direction

$\omega < 0 \longrightarrow$ Clockwise direction



Position of obj in 2-D plane at time 't' ?

a) radius & angle (r and θ)

if $(r, \omega, \theta_0) \longrightarrow$ specified. $[\theta = \theta_0 + \omega t]$ for uniform circular motion

θ_0: angle w.r.t x-axis at $t=0$

$$P(t) = r e^{j\theta} = r e^{j(\omega t + \theta_0)}$$

Proj. of $P(t)$ onto x-axis (x-component) $= r \cos(\omega t + \theta_0)$

When a signal of form $V_m \cos(\omega t + \phi) \longrightarrow$ uniform circular motion.

Phasors and Uniform Circuit Motion

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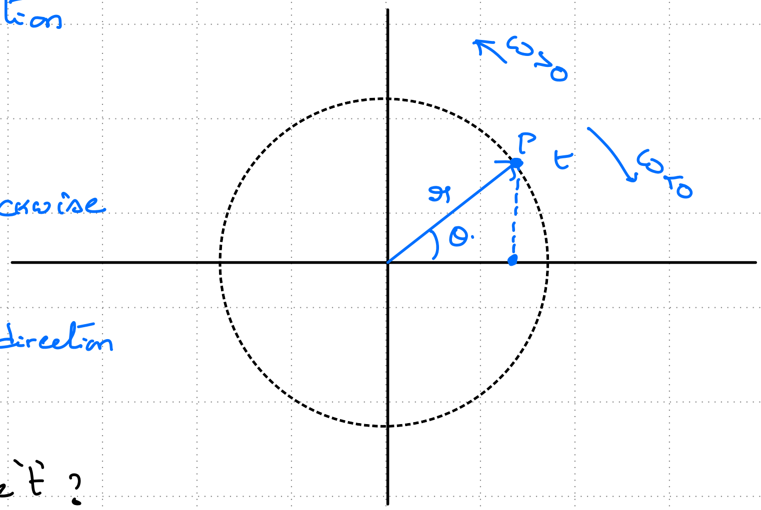

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$[\theta = \theta_0 + \omega t]$ for uniform circular motion

$$P(t) = re^{j\theta} = re^{j(\omega t + \theta_0)}$$

b) rotating position vector $P(t)$

Position vector to $P_0 \longrightarrow re^{j\theta_0} \longrightarrow$ related to Phasor of the signal

Phasor Representation

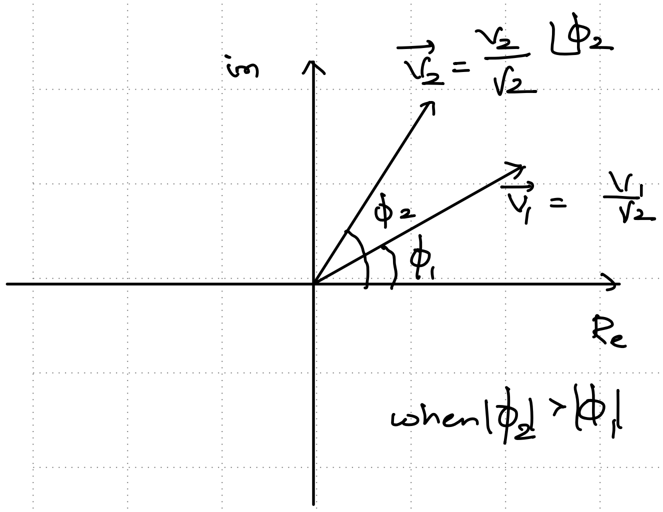
Scenario: a) Signals are of same freq (v, i)

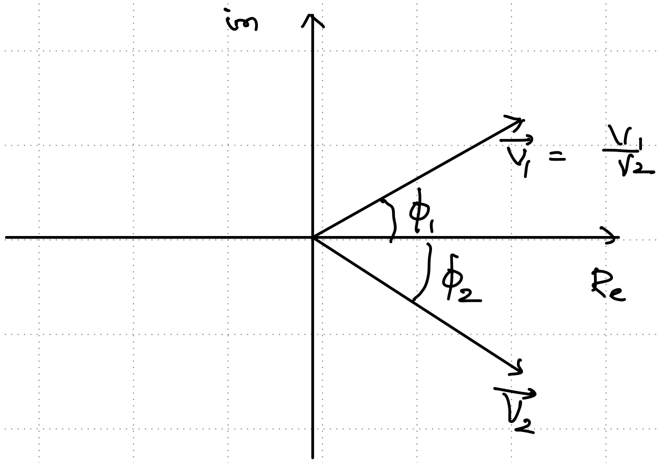
b) Perfectly sinusoidal steady state

→ rep (v, i) signals as vectors in 2D plane.

Ex: $v_1 = V_1 \cos(\omega t + \phi_1)$

$v_2 = V_2 \cos(\omega t + \phi_2)$





Relative sense of peaks of the signal ⟹ Lag & Lead.