EE6327: Statistical Learning Theory **2025**

Exam 3: October 2025

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Instructions: *This is a closed-book exam. You are not permitted to refer to any material or discuss the problem with anyone. Malpractice will be severely punished. Please mention your ROLL Number and name clearly in the answer sheet.*

Justify all your statements clearly. You may use any result proved in class (but clearly state which results you are using), but everything else needs to be proved. Highlight your final answers. Clearly indicate which parts of your answer scripts are rough work.

Problem 3.1. For each of the following hypothesis classes (for binary classification with $0-1$ loss), state whether this is (a) agnostic PAC learnable (b) nonuniformly learnable, or (c) neither PAC nor nonuniformly learnable. For each statement, give a one-line justification (what property you use to conclude this, and why the property holds). Also give (nontrivial) upper bounds on the VC dimension for each (you do not have to provide any derivation/justification) $(2+2+2+1)\times 5$ points

1. $\mathcal{X} = \{0,1\}^{100}$, and $\mathcal{H}$ is the set of all functions from $\mathcal{X}$ to $\{0,1\}$
2. $\mathcal{X} = [0,1]$ and
$$\mathcal{H} = \left\{h(x) = 1_{\{\sum_{i=0}^{k} a_i x^i \geqslant 0\}} : k \in \mathbb{N}; a_1, \ldots, a_k \in \mathbb{R}\right\}$$
3. $\mathcal{X} = \mathbb{R}^d$ and
$$\mathcal{H} = \left\{h(x) = 1_{\{\|x\|^2 \leqslant r^2\}} : r > 0\right\}$$
4. $\mathcal{X} = \mathbb{R}$ and
$$\mathcal{H} = \left\{h(x) = 1_{\{\sin(\pi(x+\tau)/100) \geqslant 0\}} : \tau \in \mathbb{Z}\right\}$$
5. $\mathcal{X} = \mathbb{R}^2$ and
$$\mathcal{H} = \left\{h(x) = 1_{\{x \in \mathcal{A}\}} : \mathcal{A} \text{ is a convex polygon}\right\}$$

Problem 3.2. Consider the following hypothesis class on $\mathcal{X} = \mathbb{R}$
$$\mathcal{H} = \left\{h(x) = 1_{\{a \leqslant x \leqslant b\}} : a, b \in \mathbb{R}\right\}$$

1. Consider PAC learning the concept class $\mathcal{H}$. For a given $\mathcal{S} = \{(X_1, Y_1), \ldots, (X_N, Y_N)\}$, describe the structure of all possible empirical risk minimizers in explicit form (in terms of $\mathcal{S}$) (3pts)
2. From first principles (i.e., without invoking VC dimension/Rademacher complexity/growth function), prove that the concept class is PAC learnable by deriving an upper bound on the sample complexity (6pts)
3. Find the VC dimension of $\mathcal{H}$, and provide justification (4pts)
4. Find the growth function of $\mathcal{H}$ (5pts)

Problem 3.3. Consider a hypothesis class $\mathcal{H}$ with $|\mathcal{H}| = 1$.

1. Find the VC dimension of $\mathcal{H}$ (2pts)
2. Find the Rademacher complexity of $\mathcal{H}$ (4pts)