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EE1101: Circuits and Network Analysis

Lecture 31: Network Theorems

October 15, 2025

Topics :

1. Linearity and Superposition
2. Thevenin's Theorem

Linearity and Superposition

A typical time domain description of a ckt: (encountered so far) n^{th} order ordinary DE with constant coefficients

$$a_n \frac{d^n x}{dt} + a_{n-1}\frac{d^{n-1}x}{dt} + \dots + a_1 x = f(t) \longrightarrow (1)$$

$a_n \dots a_1 \longrightarrow$ Constants

from the context of DEs: (1) as a linear non-homogeneous ODE if $f(t) \not\equiv 0$

(2) homogeneous ODE if $f(t) \equiv 0$

if $x(t)$ is a sol to DE then $\alpha x(t)$ must also be a solution $\forall \alpha$.

if x_1 is a sol of DE to $f_1(t)$

and x_2 is a sol of DE to $f_2(t)$

then $\alpha x_1 + \beta x_2$ is also a sol to $\alpha f_1(t) + \beta f_2(t)$

$\alpha, \beta \in \mathbb{C}$

for this Course: Any circuit whose describing equation is a linear ODE is considered as a linear circuit.

Linearity and Superposition

for a linear ckt, $\Rightarrow$ $a_n \frac{d^n x}{dt} + a_{n-1} \frac{d^{n-1} x}{dt} + \dots + a_1 x = f(t)$

$\Downarrow$ given linearity

$f(t)$ as a sum of much simpler functions

$\downarrow$ (if complex)

$\downarrow$ Compute the response of DE to the simpler functions

$\Downarrow$

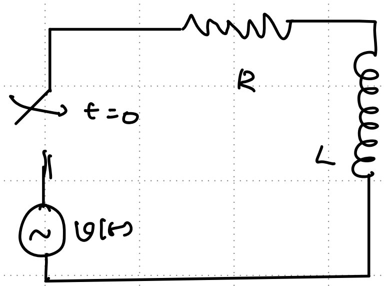
Sum of all the responses

① $f(t) = \sum_i f_i(t)$

② Solve the DE with $f_i(t)$ on the RHS

$x_i(t)$: $a_n \frac{d^n x_i}{dt} + a_{n-1} \frac{d^{n-1} x_i}{dt} + \dots + a_1 x_i = f_i(t)$

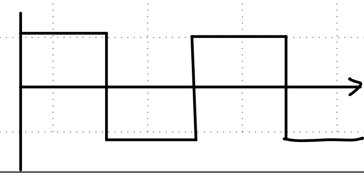
③ $x = \sum_i x_i(t)$



if $v(t) = V$, $i(t) = \frac{V}{R}(1 - e^{-t/\tau})$

if $v(t) = V_m \cos\omega t$ $\quad i(t) = \frac{V}{\sqrt{\omega^2 L^2 + R^2}} \cos(\omega t - \theta) + \underbrace{\sim}_{\text{Transient Part.}}$

if $v(t)$ is as shown below



$\Rightarrow \sum_n \sin(n\omega t + \phi_n) A_n \longrightarrow x_n$

$\downarrow$

$\sum x_n$

Linearity and Superposition (Steady State Analysis).

for a given ckt (R, L, C, sources) $\longrightarrow$ Convert into Phasor domain

$\downarrow$ apply Node analysis

$[Y][V] = [I]$

assumption: is full rank

$[V] = [Y]^{-1}[I]$

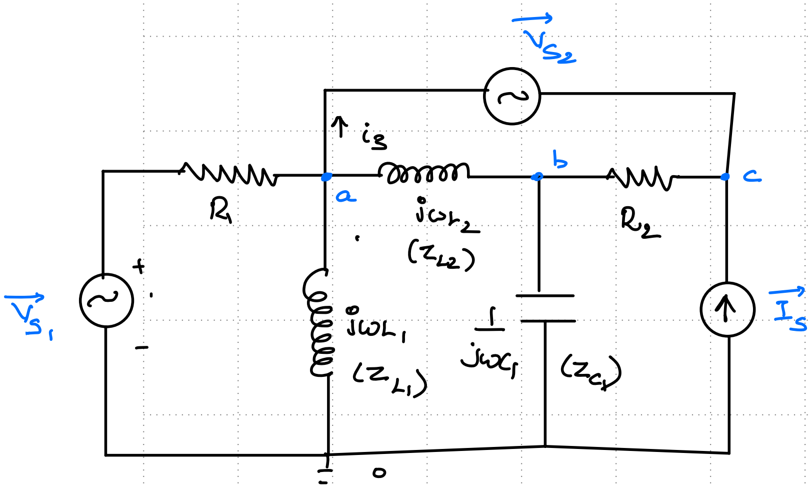
if $[I] = [I_1] + [I_2]$, then

$[V] = [V_1] + [V_2]$ where $[V_1] = [Y]^{-1}[I_1]$ and

$[V_2] = [Y]^{-1}[I_2]$

if $[I] = \sum I_i [e_i]$, then $[V] = \sum [V_i]$ where

$[V_i] = I_i (Y^{-1}) [e_i]$



$$\begin{bmatrix} -1/Z_{L2} & \frac{1}{Z_{L_2}} + \frac{1}{Z_{C_1}} + \frac{1}{R_2} & -\frac{1}{R_2} \\ \left(\frac{1}{R_1} + \frac{1}{Z_{L_1}} + \frac{1}{Z_2}\right) & -\frac{1}{Z_{L_2}} - \frac{1}{R_2} & \frac{1}{R_2} \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} \vec{V}_a \\ \vec{V}_b \\ \vec{V}_c \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ \frac{\vec{V}_{S_1}}{R_1} + \vec{I}_S \\ \vec{V}_{S_2} \end{bmatrix}}$$

$$\longrightarrow \begin{bmatrix} 0 \\ \vec{I}_S \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\vec{V}_{S_1}}{R_1} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vec{V}_{S_2} \end{bmatrix}$$

$Y V_1 = I_1$ $\quad$ $Y V_2 = I_2$ $\quad$ $Y V_3 = I_3$

$V = V_1 + V_2 + V_3$

Linearity and Superposition

The overall response of any circuit is the sum of responses of sub circuits when one independent source is present at a time

(or)

sub ckts might contain combination of sources present at a time.

Typical Practice :-

Consider sub ckts when only one independent source is present (Null all other sources)

↳ if I source → oc
V " → sc.

⇓

response of orig ckt = resp of all sub ckts

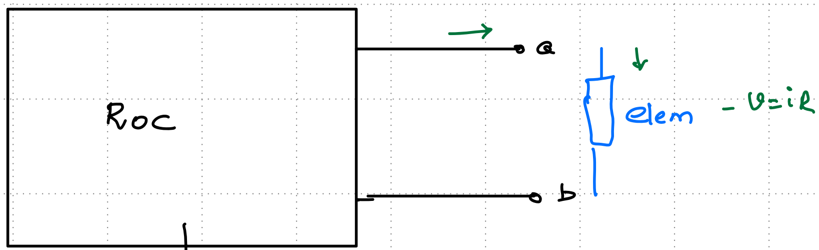
↓

make sure that all sources are spanned

$V = V_1 + V_2 + V_3$

Substitution Theorem

aim: to explore if its possible to replace a ckt elem with an other elem such that response does not change.

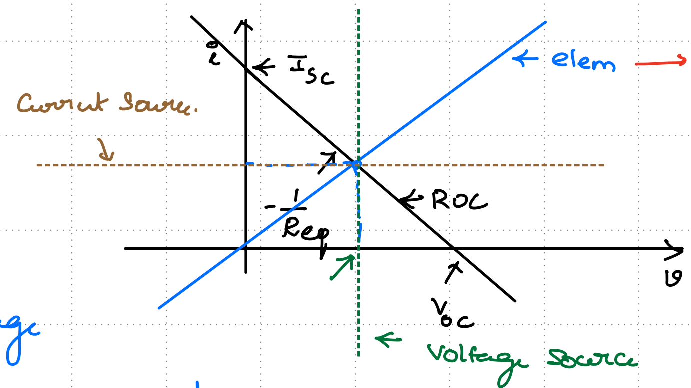


characterized by V_{oc}, I_{sc} and R_{eq}

output / input resistance.

one way to find the sol at nodes a & b: graphical route (load line analysis)

formal statement: if $\vec{V_x}$ and $\vec{I_x}$ denote the voltage & current associated with a ckt elem, replacing it with a voltage source of $\vec{V_x}$ or a current source of I_x does not effect the ckt response.



→ change the characteristic in any manner as long as the point of intersection does not change
⇓ linearity
entire ckt response does not change