Question No.	1	2	3	4	5	6	7	8	9	10	Total
MARKS	8	7	8	7							

1. Given set A contains n integers

let, the elements in set A are

$S_1, S_2, \ldots, S_K, \ldots S_n$ (elements of A)

let, $T_K = S_1 + S_2 + \ldots + S_K$ (sum of elements (K elements))

and, let R_K is remainder when T_K is divided with n

so, $T_K \equiv R_K \pmod{n}$

The most possible values of R_K are $0, 1, 2, 3 \ldots, n-1$

these are n values.

if, $R_K = 0$ then T_K (is subset elements sum) elements are $S_1, S_2 \ldots S_K$. subset is $\{S_1, S_2 \ldots S_K\}$

else if, $R_K \neq 0$

then R_K may $1, 2, 3 \ldots n-1$

let these R_K values be pegion holes $(n-1)$ but we want to get to choose n R_K values (Reminder) so for some $i > j$ (by pegion hole principle)

$R_i = R_j$

$$T_i \equiv R_i \pmod{n}$$
$$T_j \equiv R_j \pmod{n}$$
$$T_i - T_j \equiv 0 \pmod{n} \quad (\because R_i = R_j)$$

So, the difference of $T_i - T_j$ will remain with so (Some) elements. Therefore the subset of these elements i divisible by n. It will remain will some elements Becau $i>j$. Hence Proved by using Pegion hole Princi

3. Given, $2^{2n}-1$ is divisible by 3
$4^n - 1$

we can write $4^n - 1 = (3+1)^n - 1$

from binomial theorem we know that

$$(a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1} b + \cdots\cdots + {}^nC_n b^n$$

$$(3+1)^n = {}^nC_0 3^n + {}^nC_1 3^{n-1} + \cdots\cdots + {}^nC_n (1)$$

$$= 3^n + {}^nC_1 3^{n-1} + \cdots\cdots + 1$$

$$4^n - 1 = 3^n + {}^nC_1 3^{n-1} + \cdots\cdots + {}^nC_{n-1} 3 + 0$$

we know ${}^nC_1$ is chosing 1 element from n elements

${}^nC_2$ is choosing 2 elements from n elements

⋮

${}^nC_{n-1}$ is choosing $n-1$ elements from n elements

These all are integers. So, in every term of $4^n - 1$ we have 3 in every term we can write $4^n - 1 = 3(k)$

Let where k be some integer.

Hence Proved.

2. Given,
$$\binom{n}{k}\binom{k}{l} = \binom{n}{l}\binom{n-l}{k-l}$$
$$({}^{n}C_{k})({}^{k}C_{l}) = ({}^{n}C_{l})({}^{n-l}C_{k-l})$$

It means we are choosing k elements from n elements and we are choosing l elements from the choosen k elements.

<u>1st</u>:- we can choose in two different ways

let, we have a set of n elements

we want to choose k elements so it is ${}^{n}C_{k}$

Now we want to choose l elements from the k elements choose

it is, $({}^{n}C_{k})({}^{k}C_{l})$ $\quad n \to k \to l$

7

<u>2nd</u>:- By another way

we can directly choose l elements from n elements which can be, ${}^{n}C_{l}$ ways

Now we will choose $k-l$ elements from $n-l$ elements ${}^{n-l}C_{k-l}$ $\quad n \to l,\ k \to l,\ n-l \to k-l$

We are doing same with one also we are end up with l elements (with k elements choosen)

In 2nd we are choosing l elements from n and taking $k-l$ from $n-l$ where we are getting k elements also.

1st $\quad n \to k \to l$

2nd $\quad n \to l,\ k \to l \quad n-l$ (remaing) $\to k-l$

4. Given $gcd(a,b) = d$

So, $d = sa + tb$ (we use contradiction to prove)

by using ~~well ordering principle~~. let us take some linear combination which is less than $sa+tb$ let it be $s'a + t'b < d$
(+ve integer)

but d/a, d/b then $d/s'a+t'b$

but $s'a+t'b < d$
(+ve integer)

So our assumption that there will be a number $s'a + t'b < d$ is wrong
(+ve integer) d is smallest such positive integer which can be written as linear combination of a, b.

Hence by, contradiction we have Proved.

7