.

EE1101: Circuits and Network Analysis

Lecture 35: Two-Port Networks

October 28, 2025

Topics :

1. Admittance and Impedance Parameters

2. Hybrid and Inverse Hybrid Parameters

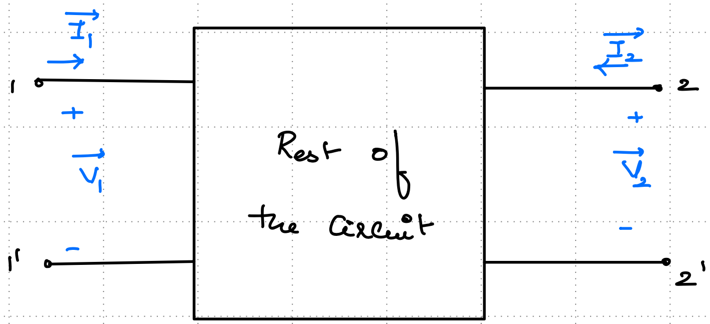
3 important aspects:

① Deriving the model

② Computing the parameters of the model

③ use case. → when Roc is linear & has no varying independent sources.

Admittance Parameters (Deriving the model → by Connecting a Voltage Source at both Ports)

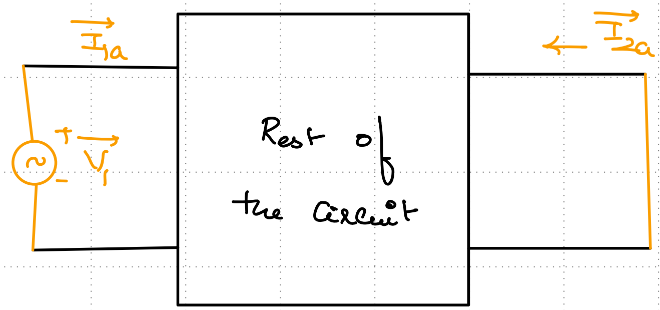


Goal: to derive expression for Currents $\vec{I}_1$ and $\vec{I}_2$

Sub ckt-a :- Null the Source at port 2-2' $(\vec{I}_{1a}, \vec{I}_{2a})$ (Short Circuit of Port 2-2')

Sub ckt-b :- Null the Source at Port 1-1' $(\vec{I}_{1b}, \vec{I}_{2b})$ (Short ckt of Port 1-1')

$\vec{I}_1 = \vec{I}_{1a} + \vec{I}_{1b}$ and $\vec{I}_2 = \vec{I}_{2a} + \vec{I}_{2b}$



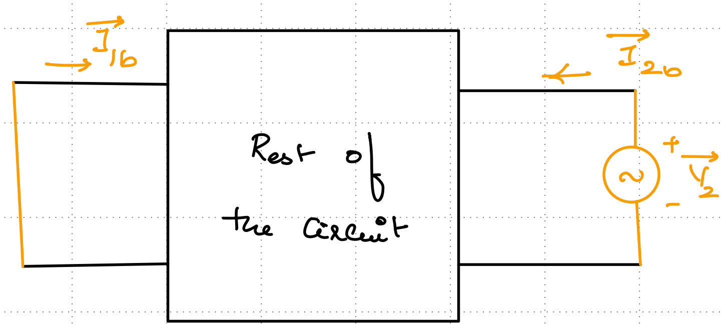
Sub ckt-a.

for Sub ckt-a :-

$\vec{I}_{1a} = \dfrac{\vec{V}_1}{\vec{Z}_{eq,1}}$ $\qquad \vec{Z}_{eq,1}$ → Equivalent imp of the ROC with Port 2-2' Shorted

$\vec{I}_{2a} = -$ (scc associated with Port 2-2')

$\vec{I}_{1a} = \vec{y}_{11} \vec{V}_1$ and $\vec{I}_{2a} = \vec{y}_{21} \vec{V}_1$ → Trans Conductance



Sub ckt-b.

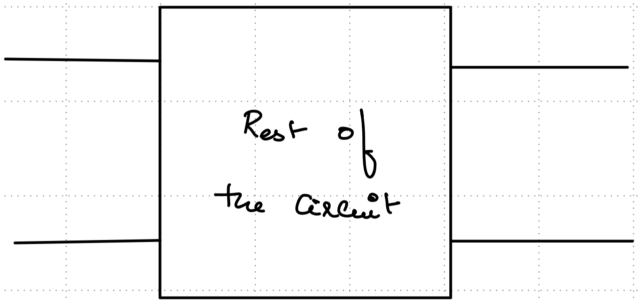
for Sub ckt-b :

$\vec{I}_{2b} = \dfrac{\vec{V}_2}{\vec{Z}_{eq,2}}$, $\vec{Z}_{eq,2}$ → Eq. imp of the Roc with Port 1-1' Shorted

$\vec{I}_{1b} = -$ (scc associated with Port 1-1')

$\vec{I}_{2b} = \vec{y}_{22} \vec{V}_2$ and $\vec{I}_{1b} = \vec{y}_{12} \vec{V}_2$

Admittance Parameters



final mathematical model

$$\begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \end{bmatrix} = \begin{bmatrix} \vec{y}_{11} & \vec{y}_{12} \\ \vec{y}_{21} & \vec{y}_{22} \end{bmatrix} \begin{bmatrix} \vec{V}_1 \\ \vec{V}_2 \end{bmatrix} \longrightarrow$$ Admittance parameter model of a 2-Port N/w.

Approach for computing the parameters:

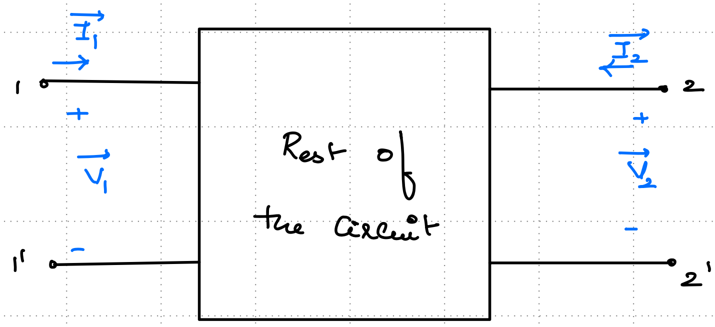
$$\vec{y}_{11} = \left.\frac{\vec{I}_1}{\vec{V}_1}\right|_{\vec{V}_2 = 0}$$ (Short Circuit of Port 2-2')

$$\vec{y}_{21} = \left.\frac{\vec{I}_2}{\vec{V}_1}\right|_{\vec{V}_2 = 0}$$ (Short Circuit of Port 2-2')

$$\vec{y}_{22} = \left.\frac{\vec{I}_2}{\vec{V}_2}\right|_{\vec{V}_1 = 0}$$ (Short Circuit of Port 1-1')

$$\vec{y}_{12} = \left.\frac{\vec{I}_1}{\vec{V}_2}\right|_{\vec{V}_1 = 0}$$ (Short Circuit of Port 1-1')

Impedance Parameters (Deriving the model → by Connecting a Current Source at both Ports)



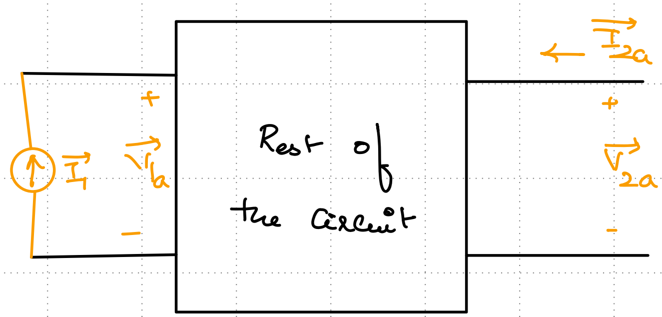
goal: to derive expression for voltages $\vec{V}_1$ and $\vec{V}_2$

Sub-ckt a: Null the Source at 2-2' (open circuit of Port 2-2')

$\vec{V}_{1a}$ and $\vec{V}_{2a}$

Sub-ckt b: Null the Source at 1-1' (open circuit of Port 1-1')

$\vec{V}_{1b}$ and $\vec{V}_{2b}$



Sub ckt-a.

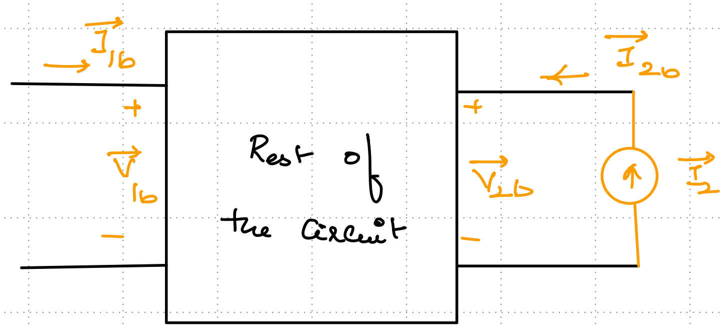
Sub-ckt a: (open circuit Port 2-2')

$\vec{V}_{1a} = \vec{I}_1 \vec{Z}_{eq,1}$ → Eq. imp of Port 1-1' with 2-2' open.

$= \vec{I}_1 \vec{Z}_{11}$

$\vec{V}_{2a}$ = open ckt vol of Port 2-2'

$= \vec{Z}_{21} \vec{I}_1$



Sub ckt-b.

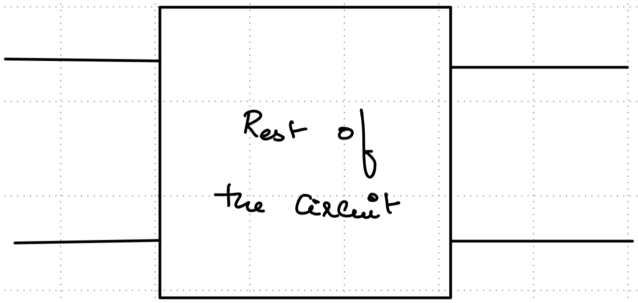
Sub-ckt b: (open circuit of Port 1-1')

$\vec{V}_{2b} = \vec{I}_2 \vec{Z}_{eq,2}$ → Eq. imp of Port 2-2' with 1-1' open

$\vec{V}_{1b}$ = open ckt vol of Port 1-1'

$= \vec{Z}_{12} \vec{I}_2$

Impedance Parameters



final mathematical model

$$\begin{bmatrix} \vec{V_1} \\ \vec{V_2} \end{bmatrix} = \begin{bmatrix} \vec{Z}_{11} & \vec{Z}_{12} \\ \vec{Z}_{21} & \vec{Z}_{22} \end{bmatrix} \begin{bmatrix} \vec{I_1} \\ \vec{I_2} \end{bmatrix}$$ ← impedance parameter model of a 2 port N/w

Approach for computing Parameters:

open circuit Parameters.

$$\vec{Z}_{11} = \left.\frac{\vec{V_1}}{\vec{I_1}}\right|_{\vec{I_2}=0} \quad \text{(open circuit of port 2-2')}$$

$$\vec{Z}_{21} = \left.\frac{\vec{V_2}}{\vec{I_1}}\right|_{\vec{I_2}=0} \quad \text{(open circuit of port 2-2')}$$

$$\vec{Z}_{22} = \left.\frac{\vec{V_2}}{\vec{I_2}}\right|_{\vec{I_1}=0} \quad \text{(open circuit of port 1-1')}$$

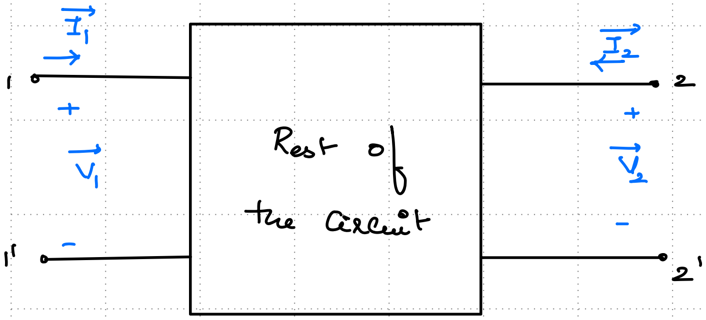
$$\vec{Z}_{12} = \left.\frac{\vec{V_1}}{\vec{I_2}}\right|_{\vec{I_1}=0} \quad \text{(open circuit of port 1-1')}$$

$$[Y_{param}] = [Z_{param}]^{-1}$$

$$\vec{y}_{11} = \frac{\vec{Z}_{22}}{\Delta} \quad \text{where } \Delta = \det[Z_{param}]$$

$$\vec{y}_{22} = \frac{\vec{Z}_{11}}{\Delta}$$

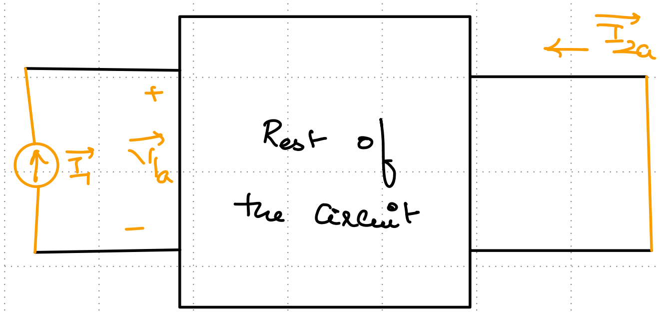
Hybrid Parameters (Deriving the model → by Connecting 1-1' → Current Source. 2-2' → voltage source.



goal: to derive expression $\vec{V}_1$ and $\vec{I}_2$

Sub ckt a: Null at 2-2' (short circuit of 2-2')

Sub ckt b: Null at 1-1' (open circuit of 1-1')



Sub ckt-a.

Sub ckt a: (Null the source at 2-2')

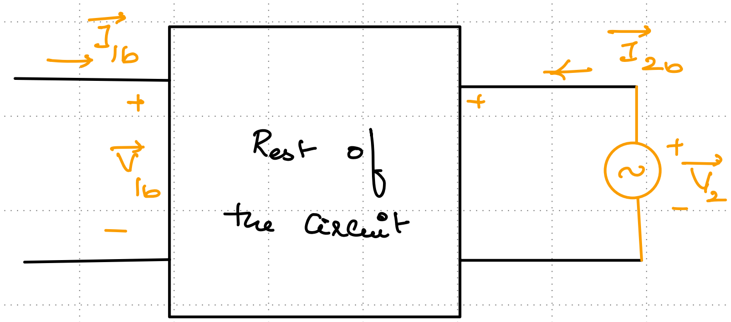
(Short circuit of 2-2')

$\vec{V}_{1a} = \vec{Z}_{eq,1} \vec{I}_1$ → EQ. imp of Port 1-1' w/n 2-2' Shorted

$= \vec{h}_{11} \vec{I}_1$ → $(= 1/\vec{y}_{11})$

$\vec{I}_{2a} = -(\text{Scc of } 2\text{-}2')$ ← in terms of y-Para $\vec{y}_{21}\vec{V}_1$

$= \vec{h}_{21} \vec{I}_1$



Sub ckt-b.

Sub ckt b: (Null the source at 1-1')

(open ckt of 1-1')

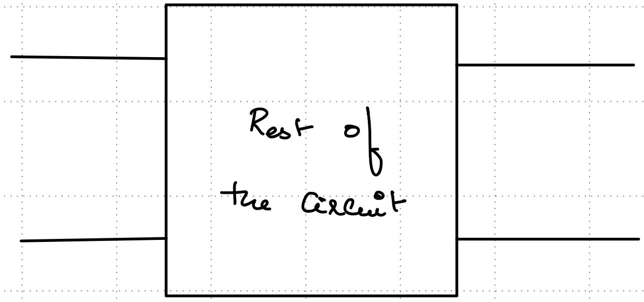
$\vec{V}_{1b} = \text{open ckt Vol of port 1-1'}$ ← in terms of z-param $\vec{Z}_{12}\vec{I}_2$

$= \vec{h}_{12} \vec{V}_2$

$\vec{I}_{2b} = \dfrac{\vec{V}_2}{\vec{Z}_{eq,2}} = \vec{h}_{22} \vec{V}_2 \quad (\vec{h}_{22} = 1/\vec{Z}_{22})$

$\vec{Z}_{eq,2}$ → Eq. imp of 2-2' w/n 1-1' open.

Hybrid Parameters



Final mathematical model:

$$\begin{bmatrix} \vec{V}_1 \\ \vec{I}_2 \end{bmatrix} = \begin{bmatrix} \vec{h}_{11} & \vec{h}_{12} \\ \vec{h}_{21} & \vec{h}_{22} \end{bmatrix} \begin{bmatrix} \vec{I}_1 \\ \vec{V}_2 \end{bmatrix} \leftarrow$$ hybrid parameter model of a 2-port n/w.

Computing the parameters

$$\vec{h}_{11} = \left.\frac{\vec{V}_1}{\vec{I}_1}\right|_{\vec{V}_2 = 0}$$ (2-2' shorted)

$$\vec{h}_{21} = \left.\frac{\vec{I}_2}{\vec{I}_1}\right|_{\vec{V}_2 = 0}$$ (2-2' shorted)

$$\vec{h}_{12} = \left.\frac{\vec{V}_1}{\vec{V}_2}\right|_{\vec{I}_1 = 0}$$ (1-1' open)

$$\vec{h}_{22} = \left.\frac{\vec{I}_2}{\vec{V}_2}\right|_{\vec{I}_1 = 0}$$ (1-1' open)

$$\vec{h}_{11} = 1/\vec{y}_{11}$$

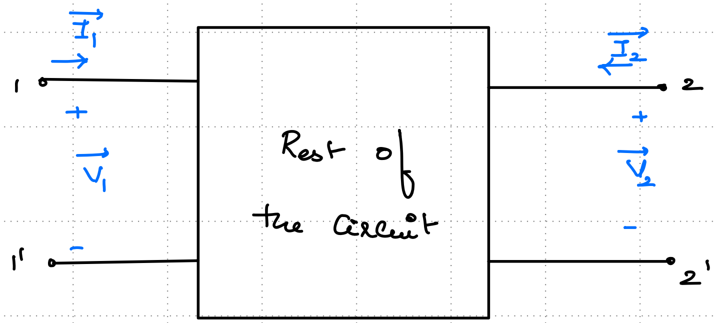
$$\vec{h}_{22} = 1/\vec{z}_{22}$$

$$\vec{h}_{21}\vec{I}_1 = \vec{y}_{21}\vec{V}_1$$

and $\vec{h}_{12}\vec{V}_2 = \vec{z}_{12}\vec{I}_2$

Inverse Hybrid Parameters

(Deriving the model → by Connecting $1-1' \rightarrow$ Voltage Source. $2-2' \rightarrow$ Current Source.



Goal: to derive expression $\vec{I}_1$ and $\vec{V}_2$

Sub Ckt a: Null the Source at 2-2' (open the Port)

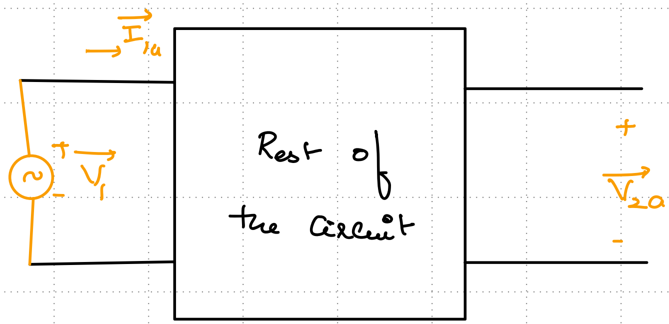
$$\vec{I}_{1a} = \frac{1}{\vec{Z}_{eq_1}} \vec{V}_1$$

$\vec{Z}_{eq_1} \rightarrow$ Eq. imp of 1-1' colts 2-2' open

$$\vec{g}_{11} = \frac{1}{\vec{Z}_{11}}$$

$\vec{V}_{2a}$ = (open ckt Vol of 2-2') = in terms of z-param $\vec{Z}_{21}\vec{I}_1$

$$= \vec{g}_{21}\vec{V}_1$$



Sub ckt - a.

Sub Ckt b: Null the Source at 1-1 (Shorting Port)

$\vec{I}_{1b}$ = -(scc associated colts 1-1') = in terms of y-Param $= \vec{y}_{12}\vec{V}_2$

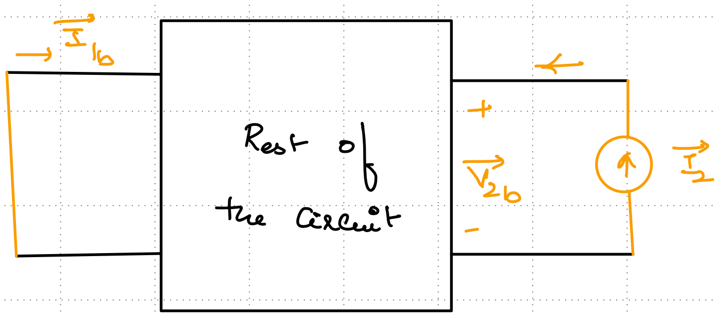
$$= \vec{g}_{12}\vec{I}_2$$

$$\vec{V}_{2b} = \vec{Z}_{eq_{12}} \vec{I}_2$$

$\vec{Z}_{eq_{12}} \rightarrow$ Eq. imp of 2-2' colts 1-1' Shorted

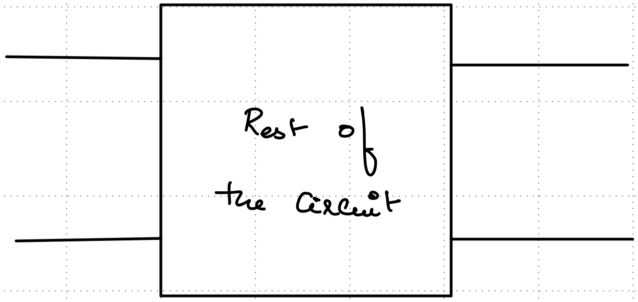
$$\vec{g}_{22} \vec{I}_2$$

$$\hookrightarrow = 1/\vec{y}_{22}$$



Sub ckt - b.

Inverse Hybrid Parameters



final model

$$\begin{bmatrix}\vec{I}_1\\ \vec{V}_2\end{bmatrix} = \begin{bmatrix}\vec{g}_{11} & \vec{g}_{12}\\ \vec{g}_{21} & \vec{g}_{22}\end{bmatrix}\begin{bmatrix}\vec{V}_1\\ \vec{I}_2\end{bmatrix}$$

← Inverse hybrid Param of a 2-port n/w.

$$\vec{g}_{11} = \frac{1}{\vec{Z}_{11}}$$

$$\vec{g}_{22} = \frac{1}{\vec{y}_{22}}$$

Computing the param

$$\vec{g}_{11} = \left.\frac{\vec{I}_1}{\vec{V}_1}\right|_{\vec{I}_2 = 0} \Rightarrow \text{open } 2\text{-}2'$$

$$\vec{g}_{21} = \left.\frac{\vec{V}_2}{\vec{V}_1}\right|_{\vec{I}_2 = 0} \Rightarrow \text{''}$$

$$\vec{g}_{12} = \left.\frac{\vec{I}_1}{\vec{I}_2}\right|_{\vec{V}_1 = 0} \Rightarrow \text{Short } 1\text{-}1'$$

$$\vec{g}_{22} = \left.\frac{\vec{V}_2}{\vec{I}_2}\right|_{\vec{V}_1 = 0} \Rightarrow \text{Short } 1\text{-}1$$

$$
\begin{array}{cc}
\begin{bmatrix} a & b \\ c & d \end{bmatrix} \\
\uparrow \\
\vec{a}_1 \quad \vec{a}_2
\end{array}
= \det(\vec{a}_1, \vec{a}_2) = \det(a\vec{e}_1 + c\vec{e}_2, \vec{a}_2)
$$

$$
= a \det(\vec{e}_1, \vec{a}_2) + c \det(\vec{e}_2, \vec{a}_2)
$$

$$
= a \det(\vec{e}_1, b\vec{e}_1 + d\vec{e}_2) + c \det(\vec{e}_2, b\vec{e}_1 + d\vec{e}_2)
$$

$$
= ab \underbrace{\det(\vec{e}_1, \vec{e}_1)}_{0} + ad \det(\vec{e}_1, \vec{e}_2) + bc \det(\vec{e}_2, \vec{e}_1) + cd \det(\vec{e}_2, \vec{e}_2)
$$

$$
= 0 + ad - bc + 0
$$