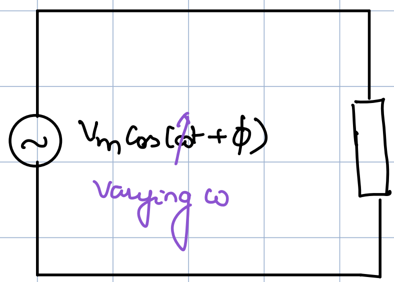
Resonance in AC circuits :- (Sinusoidal Steady state) Lec-38



AC Voltage Source (Sinusoidal & Variable freq)

$V_m \cos(\omega t + \phi)$

Varying ω

$\vec{Z}_{eq} = R + jX$

Reactance $\propto$ freq.

but $L, C \longrightarrow$ fixed.

Question : when is the mag. of Current drawn from the source is Max

RMS value of I

$\Downarrow$

Value of 'ω' for which I is Max $|\vec{I}|$

$$\vec{I} = \frac{\vec{V}}{\vec{Z}_{eq}} \Rightarrow |\vec{I}| = \frac{|\vec{V}|}{|\vec{Z}_{eq}|}$$

for $|\vec{I}|$ to maximum, $|\vec{Z}_{eq}|$ must be minimum.

$\Downarrow$

minimum possible value : $Z_{eq} = R$

$\Rightarrow X = 0$

if such a possibility exists, ckt is under resonance & freq at which $X = 0$ occurs is called the resonant freq.

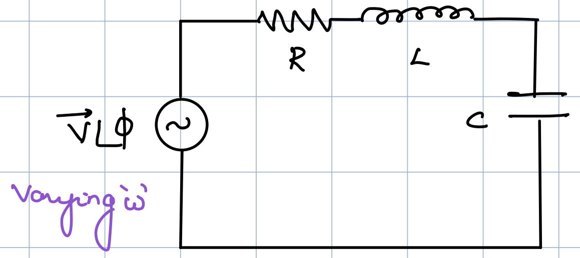
when applied to first-order ckts :

Resonance (??) $\begin{cases} \text{Series } RL \ (X = 0 \text{ when } \omega = 0) \\ RC \ (X = 0 \text{ when } \omega = \infty) \end{cases}$

$\Downarrow$

Some additional condition rather than $X = 0$.

Series RLC :-



At freq $\omega = \frac{1}{\sqrt{LC}}$

$\vec{Z}_{eq} = R$

$\vec{I} = \frac{\vec{V}}{R} \Rightarrow |\vec{I}| = \frac{V}{R}$

observation ①: $\vec{V}$ and $\vec{I}$ are in phase.

observation ②: $\vec{S} = \vec{V}\,\vec{I}^*$

Source.

$= \vec{V}\,\frac{\vec{V}^*}{R} = \frac{|\vec{V}|^2}{R}$

$= |\vec{I}|^2 R.$

Net power from source

= active

observation ③: $v(t) = R I_m \cos(\omega_0 t + \phi)$

$i(t) = I_m \cos(\omega_0 t + \phi)$

$e_L(t) = \frac{1}{2} L\, i^2(t) = \frac{1}{2} L I_m^2 \cos^2(\omega_0 t + \phi)$

$e_C(t) = \frac{1}{2} C\, v_C^2(t) =$

$$\vec{Z}_{eq} = R + j\underbrace{(\omega L - 1/\omega C)}_{X.}$$

ω for which $X = 0$ results in Max. mag of $|\vec{I}|$

$\omega_0 : \omega L - 1/\omega C = 0 \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$ ← resonant freq

Question: for a Series RLC $\omega_0 = \frac{1}{\sqrt{LC}}$ is the cond for $X = 0$

(commonly ref to as resonant freq)

while for a Series RL ckt $\omega = 0$ is the cond for $X = 0$

(Not considered as resonance)

What completes the def of resonance?

$$v_C(t) = \frac{1}{C}\int i\,dt = \frac{I_m}{\omega_0 C}\sin(\omega_0 t+\phi)$$

$$e_C(t) = \frac{1}{2} C v_C^2(t) = \frac{1}{2}\frac{I_m^2}{\omega_0^2 C}\sin^2(\omega_0 t+\phi)$$

$$e_s(t) = e_L(t) + e_C(t) = \frac{1}{2} L I_m^2 \cos^2(\omega_0 t+\phi) + \frac{1}{2}\frac{I_m^2}{\omega_0^2 C}\sin^2(\omega_0 t+\phi)$$

energy stored in the ckt

$$\omega_0^2 C = \frac{1}{LC}\cdot C = \frac{1}{L}.$$

$$e_s(t) = \frac{1}{2} L I_m^2.$$

(energy exchange b/w reactive elem in the ckt)

→ for a first order RL ckt $e_s(t)$ is not constant when $x = 0$.

observation ④: Voltage across L+C (x): when $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\vec{V}_L = (j\omega_0 L)\vec{I} = j\omega_0 L\frac{\vec{V}}{R} = j\sqrt{\frac{L}{C}}\frac{\vec{V}}{R}$$

$$\vec{V}_C = \frac{1}{j\omega_0 C}\vec{I} = \frac{-j}{\omega_0 C}\frac{\vec{V}}{R} = -j\sqrt{\frac{L}{C}}\frac{\vec{V}}{R}$$

$\vec{V}_L$ & $\vec{V}_C$ are out of phase (same mag)

$$\underline{\underline{\vec{V}_L + \vec{V}_C = 0}}$$

Voltage across individual LC is not zero. which is not the case with a first order RL ckt.

observation ⑤: $e_d(t)$ (or) $e_R(t)$

$$e_d(t) = \int s(t)\,dt = \int R I_m\cos(\omega_0 t+\phi)\, I_m\cos(\omega_0 t+\phi)\,dt$$

$$= I_m^2 R$$

$$E_d(t) = \int_0^T e_d(t) = \left(\frac{1}{2} I_m^2 R\right) T.$$

energy per cycle

Observation (6) : energy from Source per cycle = $E_d(t)$ + energy stored in CKt per cycle

is this true??

(True provided initially the CKt is at rest)

= .