CS1010: Discrete Mathematics for Computer Science

(Exam-3. Total: 30 marks.)
(Duration: 45 minutes. Date: 17 Nov 2025)

Instructions: ★ You may not get time to answer all the questions unless you have prepared really well. The exam is designed like that. ★ If your mobile phone is found with you during the exam, you will lose one grade. ★ Anybody found copying will get an F grade for the course straight away. ★ It is a no-break exam. You cannot take a break in between. The exam is only for 45 minutes. If you want to go out of the exam hall, you will have to submit your answer paper. ★ You should sit far apart from each other. The halls are big enough. If we see two students sitting close to each other, both the students will lose one grade.

Questions

1. Let n be a positive integer. Let $\mathcal{M}$ denote the set of all $n \times n$ matrices with real entries. Let $A, B \in \mathcal{M}$. We say A is *similar* to B if $A = PBP^{-1}$, where $P \in \mathcal{M}$ is some invertible matrix. Show that the similarity relation is an equivalence relation.
   5 marks [1 page]

2. Let $n \geq 1$ be an integer. Find recurrene relations (along with the requisite number of initial conditions) for s_n and t_n in the questions below. Give a brief explanation of your answer.

   (a) Let s_n denote the number of **binary strings** of length n that **do not contain three consective** 0's.

   (b) Let t_n denote the number of **strings over** $\{0, 1, 2\}$ that **neither contain two consecutive** 1's **nor contain two consecutive** 2's.

   (3+1) + (5+1) = 10 marks. [1 page]

3. A test has four sections. Section A contains many questions of 2 marks each. Section B has many questions of 5 marks each. Section C has a single question of 4 marks. Section D has a single question of 1 mark. Assume that the questions in Sections A, B and D are of objective numerical type. You either get full marks or zero. The Section C question is essay-type, and you can get an integer mark in the range [0,4]. We are interested to know in how many ways can you get a total of n marks.

   Model this question using a generating function in x such that the coefficient of x^n would be the quantity we are intereseted in. 5 marks [$\frac{1}{2}$ page].

4. Let T be a tree that has no vertex of degree exactly 2. **Without** using mathematical induction, prove that the number of leaf vertices in T is more than the number of non-leaf vertices. (Recall: In a tree, (i) a vertex of degree equal to one is called a *leaf vertex*, and (ii) if a vertex is not a leaf vertex, then it is called an *non-leaf vertex*.) 10 marks [$\frac{1}{2}$ page].

——— ALL THE BEST ———