EE2100: Matrix Theory

Assignment - 6

Handed out on 15 - Sep - 2023 **Due on 26 - Sep - 2023 (before 5 PM)**

Instructions :

1. Please submit the solutions to the assignment problems to the course page (on the canvas platform). Solutions submitted to the course page will only be evaluated. Refer to the assignment guidelines mentioned on the course page.
2. Submissions received after the deadline will attract negative marks.
3. It is suggested that you attempt all the questions (preferably the ones indicated using *). However, submitting solutions for problems totaling at least 10 points is sufficient.

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1. *(10 Points) Using a programming language of your choice, implement the Gaussian Elimination technique to solve a system of linear equations of the form $\mathbf{Ax} = \mathbf{b}$. The developed code must be capable of

   (a) detecting the presence of no-solution,

   (b) detecting the presence of an infinite number of solutions and,

   (c) computing the solution (if the solution is unique)

   It is suggested that the program be developed in such a way that the core algorithm is implemented as an independent function that can be used in other codes (if necessary). The function must take the following inputs: (a) a matrix (as a two-dimensional array) and (b) the vector $\mathbf{b}$. To test the algorithm, you can use a randomly chosen $\mathbf{A}$ and $\mathbf{b}$.

   Note: The developed code must not use any built-in libraries available in the programming language (except for defining the random matrix and the random vector).

2. *(5 Points) Show that the column space of a matrix is orthogonal to its left null space.

3. (15 Points) Consider the system of linear equations given by

$$\begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{1}$$

   (a) (4 Points) Compute the solution to the system of equations using Gaussian Elimination. If the system of equations has no solution, prove that the system of equations has no solution; if the system of equations has infinitely many solutions, derive the parametric solution.

(b) (5 Points) For 3(a), compute the matrix corresponding to elementary row operations carried out during the Forward Elimination phase.

(c) (2 Points) Compute the Null space of the matrix.

(d) (4 Points) Let $\mathbf{z} = [1, 2, 3, 4]^T$. Compute $\mathbf{x}$ such that $\mathbf{A}\mathbf{x} = \mathbf{z}$.

4. (5 Points) Consider a matrix $\mathbf{A} \in \mathcal{R}^{m \times n}$. Show that

$$\text{rank}(\mathbf{A}) = \text{rank}(\mathbf{A}^T) = \text{rank}(\mathbf{A}\mathbf{A}^T) = \text{rank}(\mathbf{A}^T\mathbf{A}) \tag{2}$$

Please note: Although we have not started analyzing rectangular matrices, the concepts covered so far in this course are sufficient to prove the result.