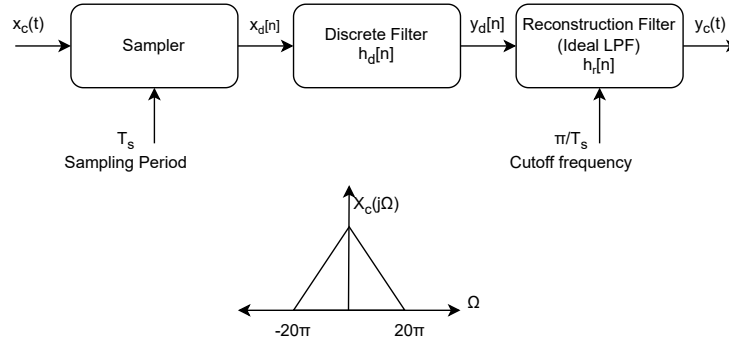


Homework-4

Note

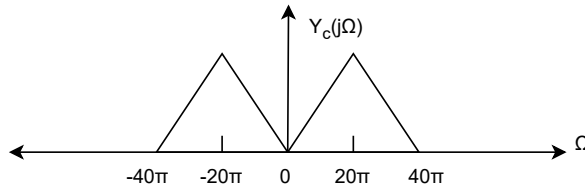
- * Plagiarism is strictly prohibited
- * Deadline will not be extended under any circumstances

1. Consider a system corresponding to discrete time(DT) processing of continuous time(CT) signal as shown below. Let us consider the sampling period $T_s = \frac{1}{20}s$ and the discrete



filter $h_d[n]$ is an ideal LPF with cutoff frequency $\frac{\pi}{4}$. Then, sketch $X_d(j\omega)$, $Y_d(j\omega)$, and $Y_c(j\Omega)$ for the input $X_c(j\Omega)$ shown in the figure.

2. Let us consider the input signal $x_c(t)$ is multiplied by a cosinusoid $\cos(30\pi t)$ in the above problem. Also consider that the discrete system in between is an ideal distortionless channel i.e., $H_d(j\omega) = 1 \quad \forall \omega$.
 - (a) Determine and sketch the output $Y_c(j\Omega)$ (with same T_s as in the above problem)
 - (b) Is it possible to reconstruct the signal $x_a(t)$ with any choice of T_s ? If yes, give one choice of T_s to reconstruct the signal $x_a(t)$
 - (c) Is it possible to have the output as



if yes, give a possible choice of T_s to have the above output.

3. Let us consider the block diagram from Problem.1, and in this case, the discrete filter's input-output relation is given by

$$y_d[n] = \frac{1}{2}y_d[n-1] + x_d[n]$$

then, determine $H_d(j\omega)$ and the effective continuous time frequency response $H_c(j\Omega)$

4. In some of the scenarios, an echo is recorded along with the signal. It is defined as the superimposition of delayed and attenuated replica of the signal $s(t)$. In the presence of an echo, the observed signal is represented as

$$x(t) = s(t) + \alpha s(t - T_0) \quad |\alpha| < 1$$

Now, we wish to process the signal $x(t)$ in discrete domain to obtain $s(t)$. The signal is sampled with a sampling period T . Determine the frequency response of the digital filter such that the reconstruction with ideal LPF with cut-off frequency $\frac{\pi}{T}$ is equal to $s(t)$? (Assume that the signal $s(t)$ is bandlimited with maximum frequency $\frac{\pi}{T}$)

5. If $X[K]$ is the DFT of the sequence $x[n]$, determine the N-point DFTs of the sequences

$$x_1[n] = x[n] \cos\left(\frac{2\pi k_0 n}{N}\right) \quad 0 \leq n \leq N-1$$

$$x_2[n] = x[n] \sin\left(\frac{2\pi k_0 n}{N}\right) \quad 0 \leq n \leq N-1$$

in terms of $X[K]$

6. Let $x[n]$ be a periodic signal with fundamental period N. Now consider $X_1[K]$ is N-point DFT and $X_2[K]$ is 3N-point DFT.

- (a) What is the relationship between $X_1[K]$ and $X_2[K]$?
(b) Verify the above result using the sequence

$$x[n] = [\dots 1, 2, 1, 2, 1, 2, 1, 2, \dots]$$

7. (a) Compute an 8-point DFT of the sequence $x[n] = [0, a, b, c, 0, -a, -b, -c]$. What do you observe from the $X[K]$?
(b) In general, comment on the N-point DFT of the sequences with the following symmetry

$$x\left[n + \frac{N}{2}\right] = -x[n] \quad n = 0, \dots, \frac{N}{2} - 1$$