EE2100: Matrix Theory

Assignment - 4

Handed out on : 01 - Sep - 2023 **Due on : 11 - Sep - 2023 (before 5 PM)**

Instructions :

1. Please submit the solutions to the assignment problems to the course page (on the canvas platform). Solutions submitted to the course page will only be evaluated. Refer to the assignment guidelines mentioned on the course page.
2. Submissions received after the deadline will attract negative marks.
3. The problems in this assignment are from the domains of Graphs, Networks, Signals and Systems, and Sparse Linear Algebra. A brief overview of these topics (sufficient enough to attempt the assignment is provided in Tutorial 4.
4. It is suggested that you attempt all the questions (preferably the ones indicated using *). However, submitting solutions for problems totalling at least 10 points is sufficient.

---

1. * (10 Points) Using a programming language of your choice, develop a code that

   (a) stores a given sparse matrix in a compressed sparse row (CSR) format and

   (b) computes the matrix-vector product.

   The input to the program/function will be a sparse matrix (you can define a random sparse matrix in the form of a 2-dimensional array) and the vector. It is suggested that the program be developed in such a way that (a) the storing of a sparse matrix in CSR format and (b) computing that matrix-vector product are two distinct functions/methods/modules that can be used in future applications. The input to the function that stores a spare matrix in CSR format must only be the 2-dimensional array, and the input to the function that computes the matrix-vector product must be the matrix (in CSR) format and the vector.

   Note: To appreciate the gain in computational time (attained by using sparse techniques), it is required to consider matrices whose size is much larger. However, for this assignment, you can choose a random matrix of any size or generate a random matrix using any inbuilt function.

2. * (10 Points) An Introduction to Graphs: A Graph is a useful mathematical tool to analyze large amounts of data and the relationship between various data points. The following question introduces you to the relationship between graphs and matrices and how we can use matrices to model specific graphs and perform useful operations on them. Graphs are generally denoted by $\mathbb{G}$ and their sets of edges and vertices by $\mathbb{E}$ and $\mathbb{V}$ respectively.

We can represent a graph in the form of an adjacency matrix. Let there be a Graph $\mathbb{G}$ with m nodes, then its adjacency matrix is defined as:

$$A_{ij} = \begin{cases} 1 & (v_i, v_j) \in \mathbb{E} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

i.e. if there exists an edge between the i^{th} and j^{th} node of the graph then the matrix has an entry 1, else, the entry is 0.

(a) (1 Point) What is the field of A?

(b) (1 Point) What property does the trace (Sum of diagonal entries of a matrix) hold if there are no loops in the graph? Under what conditions is the property violated?

(c) (1 Point) Mention the primary difference between the adjacency matrix of a directed and an undirected graph.

(d) (2 Points) Come up with an expression for the degree of a given vertex from the set $\mathbb{V}$ of a graph $\mathbb{G}$, with an adjacency matrix A.

(e) (2 Points) If N($\mathbb{E}$)=m, what is the trace of A^2, given that there are no self-loops in the graph?

(f) (3 Points) A walk of a graph is defined as a finite length alternating sequence of vertices and edges. The length of a walk is the number of edges encountered during its traversal. Find the total number of walks from vertex v_i to v_j of graph $\mathbb{G}$, of length k, as a function of the adjacency matrix A and parameter k. (Hint: Use mathematical induction)

Note: For attempting this assignment, you can use the following definitions for matrix multiplication and trace (which will be defined at a later point in the course).

- For $\mathbf{A} \in \mathcal{R}^{n\times n}$, the entries of $\mathbf{C} = \mathbf{A}^2$ are given by $C_{ij} = \mathbf{b}_i \cdot \mathbf{a}_j$ where b_i is the vector whose entries correspond to row i of the $\mathbf{A}$ and a_j is the column vector corresponding to column j of $\mathbf{A}$.
- The trace of a diagonal matrix (say $\mathbf{A} \in \mathcal{R}^{n\times n}$) denoted by $\mathbf{Tr}(\mathbf{A})$ is the sum of its diagonal entries i.e., $\mathbf{Tr}(\mathbf{A}) = \sum_{i=1}^{n} A_{ii}$

3. (5 points) Write down the incidence and adjacency matrix corresponding to the undirected graph shown in Fig. 1. While forming the matrices, order the links/nodes as per the sequence (i.e., from 1 to 8 for links and A to F for the nodes) indicated in the graph.

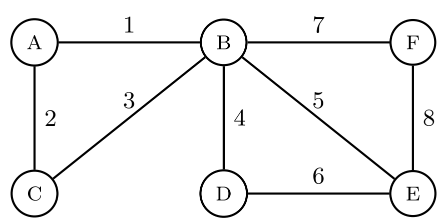

Fig. 1: Undirected graph

4. (5 Points) Consider a discrete-time system whose impulse response is $h[n] = [1, 1, 0, 0, 1, 0, 1, 0]^T$. Compute the response of the system to a finite length input $x[n] = [1, 2, 3, 4]^T$. Use the idea of computing the result of convolution as a matrix-vector product.

5. (5 Points) Represent the node equations for the circuits shown in Fig. 2 using matrices (or, more precisely, matrix-vector product).

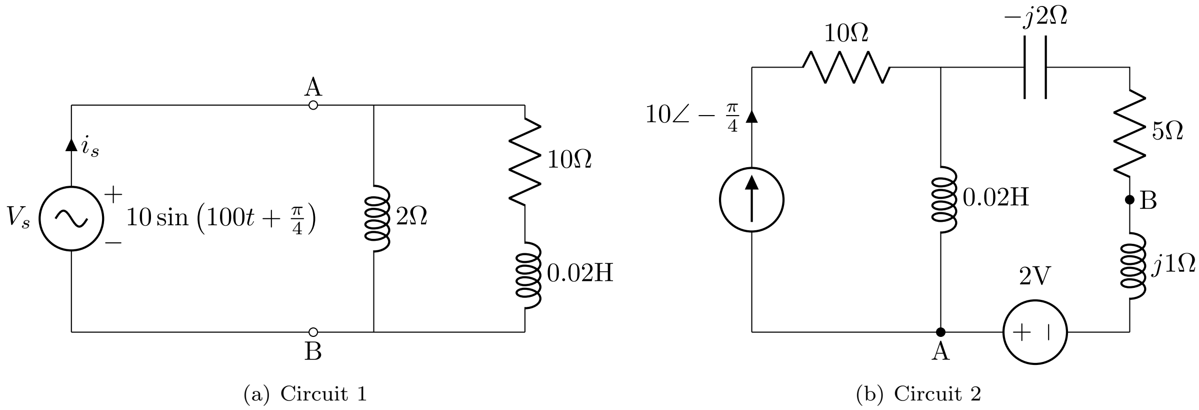

Fig. 2: Simple AC Circuits