

ID2230 2025, Exam 1

September 25, 2025

Maximum Marks: 55. Time: 2 hours. Closed Books, No electronic devices including smart watches. All the notations and conventions are as discussed in the class.

1. Recall the insertion algorithm in a binary search tree of n nodes. Is it correct to say that its worst-case running time is $\Theta(n)$? Justify your answer (2 marks)
2. Write the recurrence relation for the least number of nodes in a modified-AVL tree of height h as discussed in the problem set 3. Recall that in a modified-AVL tree, the difference in the heights can be 2. (3 marks)
3. Convert the following infix expression to postfix using a stack: $(3 + 5) * 10^6 + 7 * 5 + 4$. Show the intermediate states of the stack. (5 marks)
4. What is the maximum number of keys in a B-Tree of height h and minimum degree t ? Explain your answer. (2+3 marks)
5. Design an algorithm to find the second largest element in a binary search tree. What is the worst-case running time of your algorithm? (5+3 marks)
6. Given a positive rational x , an algorithm to calculate e^x using the Taylor series, up to a given precision ϵ —the n th term should have a value at most ϵ . Prove its correctness using the invariants method discussed in the class. (4+6 marks)
7. Consider a static hash table where the keys are stored permanently—the table is used only for *find* operations. A hash function is said to be *perfect* if *finds* in such a situation only take $O(1)$ time.
 - (a) If a hash function h is chosen u.a.r from a universal family of hash functions (as discussed in the class), then show that, when $m \geq n^2$, the probability of h being perfect is at least $1/2$. You may use the Markov inequality:
$$\Pr(X \geq a) \leq \frac{E[X]}{a},$$
for nonnegative random variable X and constant $a > 0$. (10 marks)
 - (b) As you may have observed, this will need n^2 space (a hash table with n^2 slots). Can you do this in $O(n)$ space? Justify your answer. [Hint: you may resolve collisions through another hash table of appropriate size.] (6+6 marks)