.

EE1101: Circuits and Network Analysis

Lectures 20 & 21: Inductance

September 15, 2025 | Sep 16, 2025

Topics :

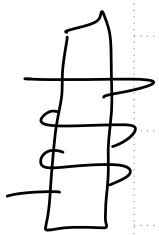
1. Response of inductors for Sinusoidal Signals

2. Power and Energy Associated with Inductors

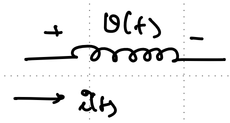
Inductors - Sinusoidal Response

Recall : Inductor $\longrightarrow$ 2 terminal ckt elem that satisfies $v_L(t) = \frac{d\lambda}{dt}$ $e(t) = d\lambda/dt$

where $\lambda = Li$
$\downarrow$
inductance (determined by the geometry & mat. prop)



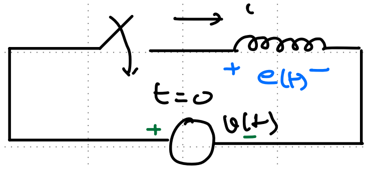
ckt symbol $\longrightarrow$



passive sign convention is used to ref v/i.

Typically 'L' is assumed constant $\Rightarrow$ $v(t) = L\frac{di}{dt}$

Example:



$\longrightarrow$ Time varying current $\Rightarrow$ time varying mag field
$\downarrow$
2 loops of interest

$e = v_L = L\frac{di}{dt} \Leftarrow \frac{d\lambda}{dt} \neq 0 \Leftarrow$ with an inductor $\longleftarrow$ 2 loops of interest

KVL. $\Leftarrow \frac{d\lambda}{dt} \approx 0$ { Loop formed by the ckt $\longleftarrow$
$\Downarrow$

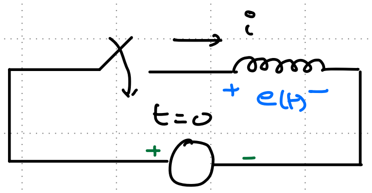
$$v(t) = e(t) = L\frac{di}{dt}$$

Example: $v(t)$ is applied to an inductor & goal is to compute $i(t)$. $\Rightarrow$ $i(t) = \frac{1}{L}\int v(t)\,dt$

when $v(t) = \delta(t) \Rightarrow i(t) = \frac{1}{L}u(t)$

when $v(t) = u(t) \Rightarrow i(t) = \frac{1}{L}\int u(t)\,dt \Rightarrow \frac{1}{L}$ 't' (for $t \geq 0$) (ramp signal)

Inductors - Impedance and Frequency Response



$V_m \cos(\omega t + \phi_V)$

flux linkage associated with an inductor cannot change instantaneously

↳ results in an impulse voltage (×)

$\Downarrow$

Current through an inductor cannot change instantaneously.

Example: $v(t) = V_m \cos \omega t$; $\frac{di}{dt} = \frac{1}{L} \underbrace{V_m \cos\omega t \, u(t)}_{v_L}$; $i(0) = 0$.

$$\Rightarrow di = \frac{V_m}{L} \cos\omega t \, u(t) \, dt$$

$$\int_{i(0)}^{i(t)} di = \int_0^t \frac{V_m}{L} \cos\omega t \, dt$$

$$\Rightarrow i = \frac{V_m}{\omega L} \sin\omega t$$

Example: $v(t) = V_m \cos(\omega t + \phi_V)$; $\frac{di}{dt} = \frac{1}{L} V_m \cos(\omega t + \phi_V) u(t)$; $i(0) = 0$

$$di = \frac{V_m}{L} \cos(\omega t + \phi_V) \, u(t) \, dt$$

$$i(t) = \frac{V_m}{\omega L} \left[\sin(\omega t + \phi_V) - \sin \phi_V \right]$$

Inductors - Impedance and Frequency Response

Example: $e(t) = V_m \cos(\omega t + \phi_V)$ (During sinusoidal steady state)

$$i(t) = \frac{1}{L}\int e(t)\,dt = \frac{V_m}{\omega L}\sin(\omega t + \phi_V)$$

$$= \frac{V_m}{\omega L}\cos(\omega t + \phi_V - \pi/2)$$

In sinusoidal steady state (use phasors to relate voltage & current associated with an inductor)

$$\vec{V}_L = \frac{V_m}{\sqrt{2}} \angle \phi_V$$

$$\vec{I}_L = \frac{V_m}{\sqrt{2}\,\omega L} e^{j(\phi_V - \pi/2)} = \frac{V_m}{\sqrt{2}\omega L} \angle \phi_i \text{ where } \phi_i = \phi_V - \pi/2.$$

Phase of current $(\phi_i) = \phi_V - \pi/2 \Rightarrow$ Current lags voltage by $\pi/2$

$$\vec{I}_L = \frac{1}{\omega L}\vec{V} e^{-j\pi/2} = \frac{1}{j\omega L}\vec{V}$$

def: only in sinusoidal steady state Impedance of an elem $(\vec{Z})$ $= \frac{\vec{V}}{\vec{I}} \propto f(\omega)$

for an inductor $= j\omega L$.

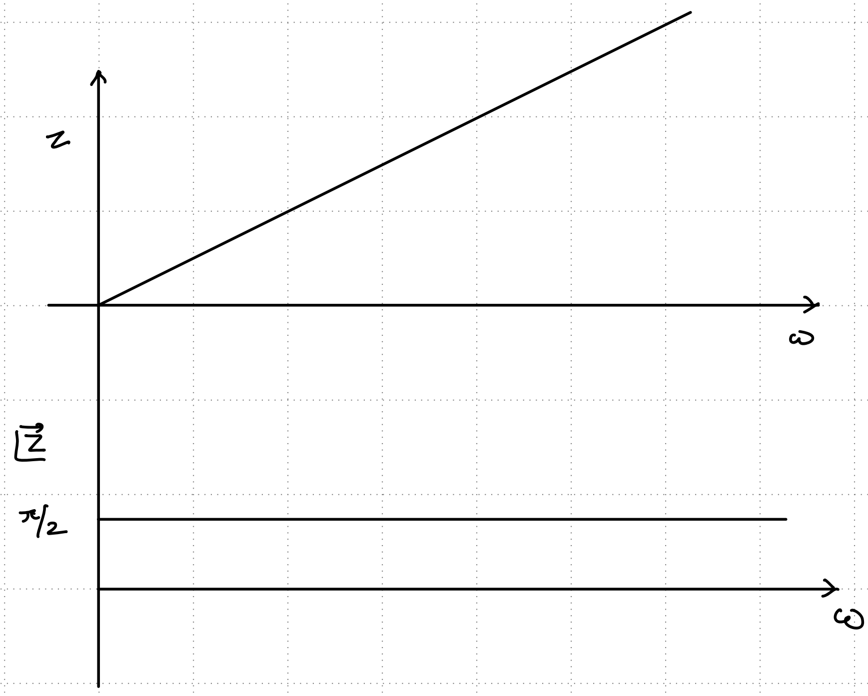
Inductors - Impedance and Frequency Response

$\vec{Z} = j\omega L \propto f(\omega)$

and $\vec{Z} \uparrow$ as $\omega \uparrow$

→ magnitude of impedance ($\propto \omega$)

→ Phase ($\pi/2$)

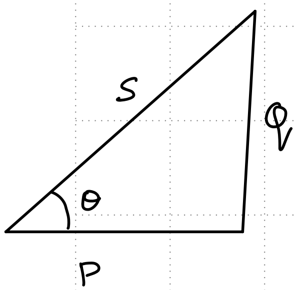


Complex Power Associated with Inductors

In steady state $\vec{V} = V\angle\phi_V$ and $\vec{I} = I\angle\phi_i = \frac{V}{Z}\angle\phi_V - \pi/2 = \frac{\vec{V}}{\vec{Z}}$

Complex power $\vec{S} = \vec{V}\vec{I}^* = (V\angle\phi_V)\left(\frac{\vec{V}}{\vec{Z}}\right)^* = \frac{V\angle\phi_V \; V\angle -\phi_V}{\vec{Z}^*}$

$$= j\frac{V^2}{Z} = j\left(\frac{V^2}{\omega L}\right) = P + jQ$$



$\Rightarrow$ $P = 0$

$Q = V^2/\omega L$ (>0)

Power factor $= \cos\theta = P/S = 0 \Rightarrow \theta = \pi/2$

(Lagging) or (Lag)

instantaneous Reactive power $= Q \sin 2\omega t = \frac{V^2}{\omega L}\sin 2\omega t.$

instantaneous power $= \frac{V^2}{\omega L}\sin 2\omega t$

Energy associated with Inductors

energy = $\int$ sin dt

Inductor $\longrightarrow$ Steady state & t_{ref} : $e(0) = 0$.

$$E(t) = \int_0^t \frac{V^2}{\omega L} \sin 2\omega t \, dt$$

$$E(t) = \frac{V^2}{2\omega^2 L} - \cos 2\omega t \Big|_0^t = \frac{V^2}{2\omega^2 L}(1 - \cos 2\omega t)$$

$$= \frac{V^2}{\omega^2 L} \sin^2 \omega t$$

$$= \left(\frac{V}{\omega L}\right)^2 L \sin^2 \omega t.$$

$$= L I^2 \sin^2 \omega t$$

$$= L \frac{I_m^2}{2} \sin^2 \omega t$$

$$= \frac{1}{2} L \underbrace{I_m^2 \sin^2 \omega t}_{i(t)}$$

$$= \frac{1}{2} L i^2$$