

EE2100: Matrix Theory
Midterm Exam

Name: _____

Roll/Reg. No.: _____

Instructions

1. The exam is for 80 minutes. It will commence at 16:05. You must hand over the question paper and the answer book before 17:25. Please note **any delay** in submission may attract **negative marking**.
2. Go through all the questions carefully. Any misinterpretations will **not** be considered.
3. The paper has three sections. Section I contains simple questions for which you should write answers in the space provided adjacent to the question. Note that answers written elsewhere will NOT be considered for evaluation.
4. The solutions to questions in Sections II and III should be written in the answer book.
5. For True or False questions, clearly write "True/False." Any other form of answer, such as (T/F, etc.) won't be considered.

I hereby declare that I have read all the instructions carefully and will adhere to them.

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Signature: _____

Section I (5 Points)

1. (5 points) State if the following statements are True/False.
 - (a) (1 point) Consider all possible square matrices (say $\mathbf{A} \in \mathcal{R}^{n \times n}$) whose rank is n . It is possible to express $\mathbf{A} = \mathbf{LU}$ where $\mathbf{L} \in \mathcal{R}^{n \times n}$ is a lower triangular matrix and $\mathbf{U} \in \mathcal{R}^{n \times n}$ is an upper triangular matrix. _____
 - (b) (1 point) It is possible to form a matrix $\mathbf{Q} \in \mathcal{R}^{m \times n}$ with orthogonal column vectors when $m < n$. _____
 - (c) (1 point) Consider a matrix $\mathbf{A} \in \mathcal{R}^{n \times n}$ whose rank is n . Then, there exists two distinct vectors (say $\mathbf{x}, \mathbf{y} \in \mathcal{R}^n$) such that $\mathbf{Ax} = \mathbf{Ay}$. _____
 - (d) (1 point) The column space of a matrix $\mathbf{A} \in \mathcal{R}^{m \times n}$ is a subspace of the vector space \mathbf{R}^m . _____
 - (e) (1 point) The null space of a symmetric matrix is perpendicular to its column space. _____

Section II (10 Points)

1. (3 points) Using the idea of a linear transformation, compute the inverse of the matrix $\mathbf{A} \in \mathcal{R}^{3 \times 3}$ given by (1).

$$\mathbf{A} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \quad (1)$$

2. (3 points) Let a, b and c denote positive real numbers i.e., $a, b, c \in \mathbb{R}^+$. Show that

$$\frac{a^2}{b+c} + \frac{b^2}{c+a} + \frac{c^2}{a+b} \geq \frac{a+b+c}{2} \quad (5)$$

3. (4 points) Express the matrix \mathbf{A} given by (9) as a product of a Lower triangular and Upper Triangular matrix.

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \quad (9)$$

Section III (5 Points) [You need to solve question 1 and one out of questions 2 and 3]

1. (3 points) Consider a matrix $\mathbf{A} \in \mathbb{R}^{n \times n}$, such that $\mathbf{A}^2 = \mathbf{0} \in \mathbb{R}^{n \times n}$. Show that $\text{Rank}(\mathbf{A}) \leq \frac{n}{2}$.
2. (2 points) **Similarity of signals and Cauchy-Schwarz inequality:** It is possible to define similarity index between two signals based on inner product. The similarity index between two signals $s_1(t)$ and $s_2(t)$ is given by the inner product between $\frac{s_1(t)}{\|s_1(t)\|}$ and $\frac{s_2(t)}{\|s_2(t)\|}$. According to Cauchy Schwarz inequality, the similarity index will be in the range $[0, 1]$. Two signals are dissimilar if the similarity index is 0. On the other hand, a higher value of similarity index indicates that the signals are similar. Consider a signal $s(t)$ shown in Fig. (1). Among the signals

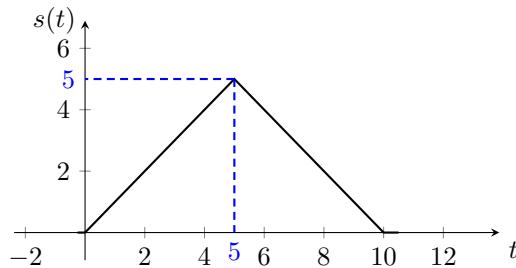


Fig. 1: Signal $s(t)$

$s_1(t)$, $s_2(t)$ and $s_3(t)$ shown in Fig. 2, identify the signal that is similar (based on the similarity index) to $s(t)$.

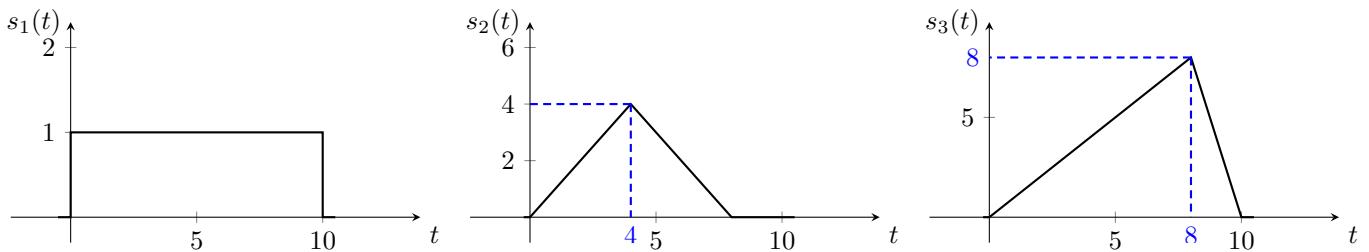


Fig. 2: Signals $s_1(t)$, $s_2(t)$, $s_3(t)$

3. (2 points) Consider two vectors $\mathbf{x} \in \mathbb{R}^N$ and $\mathbf{y} \in \mathbb{R}^N$. Simplify (a) $\|\mathbf{x}+\mathbf{y}\|_2^2 + \|\mathbf{x}-\mathbf{y}\|_2^2$ and (b) $\|\mathbf{x}+\mathbf{y}\|_2^2 - \|\mathbf{x}-\mathbf{y}\|_2^2$?