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Handout for Circuits and Network Analysis

Fall 2025

These handouts serve as **compact, informal summaries** of the grouped lecture topics covering the course. Rather than providing a full set of lecture-by-lecture notes, this material presents clusters of related topics in an approachable format. It is meant as a handout, not a replacement for textbooks. For in-depth explanations, students should consult the standard references listed below.

Please note: these notes are prepared in an informal style and may contain typos or minor mistakes. If you spot any errors, kindly bring them to my attention so they can be corrected in future versions.

References:

1. M. E. Van Valkenburg, *Network Analysis*, PHI.
2. C. A. Desoer and E. S. Kuh, *Basic Circuit Theory*, Tata McGraw-Hill.
3. W. H. Hayt Jr., J. E. Kemmerly, and S. M. Durbin, *Engineering Circuit Analysis*, McGraw-Hill.
4. C. K. Alexander and M. N. O. Sadiku, *Fundamentals of Electric Circuits*, McGraw-Hill.

Potential, Current, Power and Energy

(In the context of DC circuits)

- **Potential and Potential Difference**: Recall from electromagnetics that Potential at point P (v_p) in an electrical field is defined as the amount of work done per unit charge in bringing a positive test charge from infinity to point P. Mathematically, it is given by

$$v_p = -\int_{\infty}^{P} \mathbf{E} \cdot \mathbf{dl} \tag{1}$$

where $\mathbf{E}$ is the electric field vector and $\mathbf{dl}$ is the differential path element along the path from infinity to point P. There are two important points to note here:

  - What path should be taken to calculate the potential at point P?
  - Will the result depend on the path taken?

If the answer to the second question is yes, then the potential at point P is not well defined. Fortunately when dealing with DC citcuits, the $\mathbf{E}$ is conservative, i.e.,

$$\oint \mathbf{E} \cdot d\vec{l} = -\frac{d\lambda}{dt} = 0 \implies \nabla \times \mathbf{E} = 0 \tag{2}$$

Thus the result is independent of the path taken and as a result, the potential at point P is uniquely defined. Note the use of upper case V to denote the potential at point P for DC circuits. We will talk about the potential for AC circuits (for which we use the symbol v) later. For DC circuits, the electrical field at a point P is given by

$$\mathbf{E} = -\nabla V \tag{3}$$

Coming back to the potential, let A be a point in the circuit whose potential we aim to compute. Let B be some point on the path from A to infinity. The potential at point A is given by

$$V_a = \underbrace{-\int_{\infty}^{B} \mathbf{E} \cdot \mathbf{dl}}_{\text{Term I}} \underbrace{-\int_{B}^{A} \mathbf{E} \cdot \mathbf{dl}}_{\text{Term II}} \tag{4}$$

The first term (Term I) indicates the potential at point B (V_b) and the second term (Term II) indicates the external work done in bringing a positive test charge from point B to point A. This is defined as the potential difference between points A and B and is denoted by V_{ab}. Mathematically, it is given by

$$V_{ab} = V_a - V_b = -\int_{B}^{A} \mathbf{E} \cdot \mathbf{dl} \tag{5}$$

Note that potential difference V_{ab} denotes the external work done per unit charge in moving a positive test charge from point B to point A.

- **Current**: A precise definition of current from the point of view of electromagnetics requires understanding of the concept of current density (which can be one of three types: volume, surface, or line current density). If the volume current density $\mathbf{J}$ is specified, the current I through a differential surface $\mathbf{ds}$ is given by

$$I = \int_s \mathbf{J} \cdot \mathbf{ds} \tag{6}$$

However, in the context of circuits, we are primarily concerned with the net current flowing through a circuit element. The net current I through a circuit element is defined as amount of charge that passes through the cross section of the conductor/circuit element per unit time (in a given direction), i.e.,

$$I = \frac{dQ}{dt} \tag{7}$$

For a typical course in circuits, we will assume that the current is same through a circuit element (closely related to lumped circuit elements; more on that later). Also note that the definition of current requires the direction of flow of charge to be specified.

- **Notion of Reference and Polarity**: Consider a two-terminal circuit element (as shown in Fig. 1) with terminals (also referred to as nodes) a and b. The potential difference between nodes a and b (denoted by V_{ab}) can be uniquely

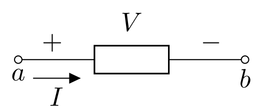


Fig. 1: A two-terminal circuit element with terminals a and b.

specfied in one of the two ways:

  - Specify V_{ab} as the potential at node a with respect to node b, i.e., $V_{ab} = V_a - V_b$ or V_{ba} as the potential at node b with respect to node a, i.e., $V_{ba} = V_b - V_a$.

    For example, assume that node a in Fig. 1 is at a potential that is 5 V higher than node b. It can be represented as $V_{ab} = 5V$. Note that the same potential difference can also be expressed as V_{ba}, in which case the value is $V_{ba} = -5$ V.

  - Another way to define the potential difference is to first define the polarity of the circuit element and then specify the potential difference with respect to the polarity. The polarity is specified by assigning one node as positive (denoted by $+$) and the other node as negative(denoted by $-$). Once the polarity is defined, the value of potential difference is the external energy required to move a positive test charge from the negative node to the positive node.

    For example, again assume that node a in Fig. 1 is at a potential 5 V higher than node b. If the polarity is defined as a being positive and b being negative, then we can write $V = +5$ V. On the other hand if we reverse the polarity, i.e., define b as positive and a as negative, then we can write $V = -5$ V.

Similarly, the current through the circuit element can be specified in one of the two ways:

  - Specify I_{ab} as the current flowing into node a and out of node b, or I_{ba} as the current flowing into node b and out of node a.

For example, if the current flowing into node a and out of node b is $2\,\mathrm{A}$, then we can write $I_{ab} = 2\,\mathrm{A}$ or alternatively, $I_{ba} = -2\,\mathrm{A}$.

- Another way to define the current is to first define the reference direction and then specify the current with respect to the reference direction. The reference direction is specified by drawing an arrow on the circuit element. The current is then defined as the amount of charge flowing in the direction of the arrow per unit time.

  For example, again assume that, in Fig. 1, the current flowing into node a and out of node b is $2\,\mathrm{A}$, then we can write $I = +2\,\mathrm{A}$ if reference is chosen from a to b (as shown in Fig. 1). If the reference is chosen from b to a, then we can write $I = -2\,\mathrm{A}$.

- **Energy**: We know from the potential that the external energy required to move a positive test charge from node b to node a is given by the potential difference V_{ab}. If we move a charge Q from node b to node a, then the total energy (external) required is given by

$$W = Q \cdot V_{ab} \tag{8}$$

If $V_a > V_b$, then the energy is positive, indicating that external energy is required to move the charge from b to a. If $V_a < V_b$, then the external energy is negative, which indicates that the energy stored in the electrical form (or the system) is being dissipated (or released). When referring to energy in circuits, we are concerned about the energy in the system or the electrical energy. Thus,

  - moving a positive test charge from a lower potential to a higher potential requires external energy (which can be considered as electrical energy being pumped into the system), and
  - moving a positive test charge from a higher potential to a lower potential releases energy (which can be considered as electrical energy being dissipated by the system). Note that this can also be interpreted as energy being extracted from the system. Since in circuits we are primarily concerned with the energy in the system (or energy in electrical form), we will refer to this as dissipation of energy.

- **Power**: The power P associated with a circuit element is defined as the rate at which energy is transferred or converted by the circuit element. Mathematically, it is given by

$$P = \frac{dW}{dt} \tag{9}$$

Substituting the expression for W, we get

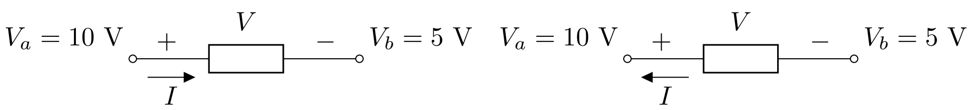
$$P = \frac{d}{dt}(QV) = Q\frac{dV}{dt} + V\frac{dQ}{dt} \tag{10}$$

For DC circuits, the potential V is constant over time, i.e., $\frac{dV}{dt} = 0$ and $\frac{dQ}{dt} = I$. Thus, we can simplify the expression for power as

$$P = V\frac{dQ}{dt} = VI \tag{11}$$

It is very hard to infer whether an element is disspiating/absorbing power from the sign of P, unless a systematic way of defining the polarity and reference direction is followed (commonly referred to as convention). We will discuss a couple of examples to illustrate this point.

Example 1: Consider a circuit element shown in Fig. 2(a). Let a current of 2 A flow from node a to node b and the potential at node a is 10 V and at node b is 5 V. For the chosen polarity and reference direction, $V = 5\,\text{V}$ and



(a) For Examples 1 and 2

(b) For Example 3

Fig. 2: A two-terminal circuit element.

$I = 2\,\text{A}$. Thus, the power is given by $P = VI = 10\,\text{W}$. Note that the changes are moving from a higher potential to a lower potential, which indicates that the circuit element is dissipating power. Note that the sign of power for the chosen polarity and reference direction is positive.

Example 2: Now consider the same circuit element shown in Fig. 2(a) but with the polarity reversed, i.e., a is negative and b is positive. The current is still 2 A flowing from node a to node b. The potential at node a is 10 V and at node b is 5 V. Now, for the chosen polarity and reference direction, $V = -5\,\text{V}$ and $I = 2\,\text{A}$. Thus, the power is given by $P = VI = -10\,\text{W}$. Note that the changes are still moving from a higher potential to a lower potential, which indicates that the circuit element is dissipating power. However, note that the sign of power for the chosen polarity and reference direction is negative.

Example 3: As a final example, consider the two-terminal circuit element shown in Fig. 2(b). Let a current of 2 A flow from node b to node a and the potential at node a is 10 V and at node b is 5 V. For the chosen polarity and reference direction, $V = 5\,\text{V}$ and $I = -2\,\text{A}$. Thus, the power is given by $P = VI = -10\,\text{W}$. Note that, in this case, the changes are moving from a lower potential to a higher potential, which indicates that the circuit element is generating power. Note that the sign of power for the chosen polarity and reference direction is positive.

- The set of examples above illustrates that the sign of power depends on the polarity and reference direction chosen. Thus, it is important to follow a systematic way of defining the polarity and reference direction in order to avoid confusion. This leads to the concept of convention in circuits, which is a systematic way of defining the polarity and reference direction for circuit elements. The most commonly used conventions are:
  - Passive Sign Convention: In this convention, the reference direction for current is chosen to be flowing into the positive terminal. This convention is used for passive circuit elements such as resistors, capacitors, and inductors. A positive sign of power indicates that the circuit element is absorbing power (i.e., dissipating energy), while a negative sign indicates that the circuit element is generating power (i.e., releasing energy) (try possible scenarios to convince yourself).
  - Active Sign Convention: In this convention, the reference direction for current is chosen to be flowing out of the positive terminal. This convention is used for active circuit elements such as voltage sources and current sources. In addition, it is a common practice to choose the polarity in such a way that the node at a higher potential is chosen as the positive terminal. Note that it is not always posible to determine from the circuit

diagram which node is at a higher potential, except in case of very simple circuits or circuit elements such as batteries. A positive sign of power indicates that the circuit element is generating power (i.e., releasing energy), while a negative sign indicates that the circuit element is absorbing power (i.e., dissipating energy) (try possible scenarios to convince yourself).

- **Sources and Sinks**: In the context of circuits, we often encounter two types of circuit elements: sources and sinks/loads. Sources are circuit elements that provide energy to the circuit, while sinks are circuit elements that dissipate energy from the circuit.