AI4010: Online Learning
First Midterm Exam
Aug 2025

Instructions:

- *The total number of marks is 20.*
- *The total duration of the exam is 90 minutes. No electronic aids are allowed. You can keep a maximum of one sheet of paper with formulas/notes.*
- *All questions are mandatory. A yes/no answer without proper proof or justification will be given zero marks even if it is correct.*
- *Any plagiarism case, if detected, will attract F grade in the course irrespective of overall performance.*

Problem 0.1 (3 marks). *Show that a straightforward extension of the* MAJORITY *algorithm makes* $O(m\log(N))$ *mistakes when the best expert makes* $m > 0$ *mistakes.*

Problem 0.2 (3 marks). *Let* $f(x) = e^{-\eta\ell(x,y)}$ *and consider the loss function* $\ell(x,y) = ||x - y||_2^2$ *over the closed ball of radius* $r > 0$ *and centered at origin i.e.* $\mathcal{B}_r(0) \subseteq \mathbb{R}$. *Find the range of values of* η *for which* f *is concave.*

Problem 0.3 (6 marks). *Prove or disprove the below statements.*

1. *LogSumExp (LSE) function is convex. [2 marks]*[1]
2. *Let the function* f *be* α_1*-strongly convex and* g *be* α_2*-strongly convex then* $f+g$ *is* $\alpha_1+\alpha_2$*-strongly convex. [2 marks]*
3. *If the function is* α*-exp-concave for some* $\alpha > 0$ *then it is also* α'*-exp-concave for any* $\alpha' \in (0, \alpha]$. *[2 marks]*

Problem 0.4 (8 marks). *We developed two important tools in this course; randomization and doubling trick. In this question, we will show the effectiveness of these tools with an example.*

Consider that there is a Cow located at $s = 0$ *on one side of an infinite fence (infinite towards both left and right ends). There is a hole on the fence at some location* $t \in \mathbb{R}$ *and the Cow wants to*

[1] Log-sum-exp is defined as

$$LSE(a_1, a_2, \cdots a_n) = \log(\sum_{i=1}^{n} e^{a_i})$$

go to the other side of the fence as the Cow believes that the grass is greener on the other side (as always!!). Cow's goal is to find the hole by travelling least possible distance. Had the Cow known t in advance, she would have found the hole by travelling $|t|$ distance in the direction of the hole. Let OPT $:= |t|$. *We are interested in minimizing the competitive ratio defined as* $CR_{Alg} := \frac{\text{ALG}}{\text{OPT}}$ *where* ALG *is the distance travelled by the Cow with algorithm **Alg** in the absence of knowledge about location of the hole.*

*To begin, consider the following **Bad** algorithm. Cow turns towards right (left) and keeps walking in hope of finding the hole. It is straightforward to see that the worst case competitive ratio of this strategy/algorithm is ∞ as the cow will never find the hole if it is on the left (right). It is clear that if the Cow has to find the hole it must explore to both right and left sides. Suppose now Cow decides to do the following;*

d_1 `distance right` $\rightarrow$ `origin` $\rightarrow$ d_2 `distance left` $\rightarrow$ `origin` $\rightarrow$ d_3 `distance right` $\rightarrow$ `...`

Answer the following questions,

1. *(2 marks) Let $d_k = k$ for all $k \geq 1$. Call this algorithm **Linear**. Show that the worst case competitive ratio $CR_{\textbf{Linear}}$ can be arbitrarily large; i.e., for any finite $m > 0$, there exists an instance (location t) such that $CR_{\textbf{Linear}} > m$.*

2. *(3 marks) Let $d_1 = 1$ and $d_k = 2d_{k-1}$ for all $k > 1$. Call this algorithm **Doubling**. What is worst case $CR_{\textbf{Doubling}}$?*

3. *(3 marks) Cow tosses a fair Coin to decide between right-first doubling strategy (as in above question) and left-first doubling strategy (complementary strategy where cow starts with moving towards left instead of right). Call this algorithm **R + D**. What is worst case expected $CR_{\textbf{R+D}}$?*

   Observe that both doubling trick and randomization help in improving the expected distance traveled to find the hole.