Two-Port Networks

- When the primary focus of solving a circuit is to determine the circuit quantities associated with an element connected between a pair of terminals (or one-port), analysis techniques associated with one-port networks are appropriate. Two useful tools for dealing with scenarios involving one-ports are Thevenin's and Norton's theorems. The techniques that are used to analyze one-port networks can be extended to analyze circuits that contain elements connected between two pairs of terminals (or two-ports).

- **Two-port networks**: Before analyzing two-port networks, it is important to recall the following points about one-port networks:
  - It is important to determine whether the problem under consideration requires use of techniques related to one-port networks.
  - For cases where one-port network analysis is appropriate, it is necessary to establish efficient methods for determining the circuit response. This involves formulating the equivalent circuit of the one-port network and calculating its parameters.
  - The method used to establish the equivalent circuit of a one-port network is not necessarily the same as the method used to determine its parameters. For example, in proving Thevenin's theorem, we used a combination of the replacement theorem and the superposition theorem to establish the Thevenin equivalent circuit. However, the procedures used to compute the Thevenin equivalent voltage and the Thevenin equivalent resistance are different.

The techniques used to analyze two-port networks are useful when the primary interest is in determining the circuit quantities associated with elements connected between two pairs of terminals. These two pairs of terminals are typically referred to as an input port (labelled 1–1') and an output port (labelled 2–2'). The two-port network itself is the portion of the circuit that connectes these two ports. The two-port network can contain independent and dependent sources as well as resistors, capacitors, and inductors. A typical representation of a two-port network is shown in Figure 1. It is a common practice to choose the reference directions for voltages and currents in the manner shown in Fig. 1. The voltage v_1 is the voltage across the input port (1–1'), and the voltage v_2 is the voltage across the output port (2–2'). The current i_1 is the current entering the input port, and the current i_2 is the current entering the output port.

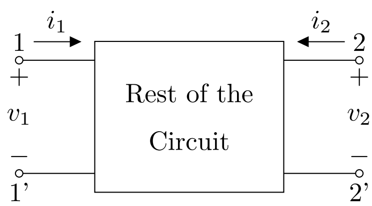

Figure 1: Two-Port Network Representation

Although the mathematical models of two-port networks to be developed in this discussion can, in principle, be formulated for any circuit consisting of linear elements, these models are meaningful only when the two-port network does not contain (a) any independent sources whose values may vary, and (b) any dependent sources whose controlling variables are external to the two-port network[1].

[1] A more general requirement for two-port modeling is that the network be bilinear, ensuring that the superposition and proportionality principles apply to both the excitation and response variables.

The models used to characterize two-port networks are derived in a manner similar to the one used to characterize one-port networks. Further, the models are developed for sinusoidal steady-state operation, and as a result, all circuit quantities are represented by complex phasors. The goal of the models used to characterize two-port networks is to establish relationships between the input and output port voltages and currents.

The way the models are introduced in this course assumes that the two-port networks are characterized by considering particular type of source at the input and output ports. Such an approach is useful because it leads to models that are easy to use in practical applications. More importantly, it helps in easily identifying the relation between various two-port parameters and the physical characteristics of the two-port network. The four types of two-port models that will be discussed are:

- **Admittance (Y) model**: In this model, the network is characterized by considering a voltage source at both the input and output ports and deriving the expression for currents at these ports (in terms of these currents).
- **Impedance (Z) model**: In this model, the network is characterized by considering a current source at both the input and output ports and derving the expression for voltages at these ports (in terms of these voltages).
- **Hybrid (H) model**: In this model, the network is characterized by considering a current source at the input port and a voltage source at the output port and deriving the expression for the input port voltage and output port current (in terms of the input port current and output port voltage).
- **Inverse Hybrid (G) model**: In this model, the network is characterized by considering a voltage source at the input port and a current source at the output port and deriving the expression for the input port current and output port voltage (in terms of the input port voltage and output port current).

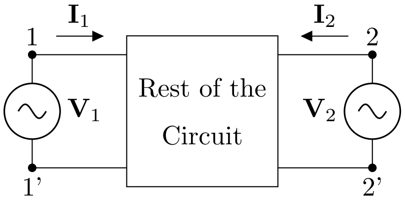

Figure 2: Two-Port Network characterization with Voltage Source Excitations

- **Admittance (Y) Model**: Conisder a two-port network as shown in Fig. 1. For the purpose of developing the admittance parameter model, consider that the input and output ports of the two-port network are excited by independent voltage sources, which in phasor form are denoted by $\mathbf{V}_1$ and $\mathbf{V}_2$, respectively (as shown in Fig. 2). The goal is to determine the expressions for the input and output port currents, $\mathbf{I}_1$ and $\mathbf{I}_2$, in terms of the input and output port voltages, $\mathbf{V}_1$ and $\mathbf{V}_2$. Since the two-port network is assumed to be linear, the current at both the ports can be computed as a linear combination of the two voltages (using superposition principle). The current at port 1-1$'$ can be computed as the sum of two components: (a) the first component $\mathbf{I}_{1a}$ is the current at $1-1'$ when the voltage source at port 2-2$'$ is nulled (i.e., $\mathbf{V}_2 = 0$), and (b) the second component $\mathbf{I}_{1b}$ is the current at 1-1$'$ when the voltage source at port 1-1$'$ is nulled (i.e., $\mathbf{V}_1 = 0$).

Thus,

$$\mathbf{I}_1 = \mathbf{I}_{1a} + \mathbf{I}_{1b} \tag{1}$$

The current $\mathbf{I}_{1a}$ can be expressed as

$$\mathbf{I}_{1a} = \frac{\mathbf{V}_1}{\mathbf{z}_{eq,1}} = \mathbf{y}_{11}\mathbf{V}_1 \tag{2}$$

where $\mathbf{z}_{eq,1}$ is the equivalent impedance looking into port 1-1′ with port 2-2′ short-circuited. The current $\mathbf{I}_{1b}$ represents the negative of the short-circuit current at port 1-1′ when the voltage source at port 2-2′ is active. Since the network is linear, $\mathbf{I}_{1b}$ can be expressed as

$$\mathbf{I}_{1b} = \mathbf{y}_{12}\mathbf{V}_2 \tag{3}$$

where $\mathbf{y}_{12}$ is a proportionality constant that relates the current at port 1-1′ to the voltage at port 2-2′. Thus, the total current at port 1-1′ can be expressed as

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2 \tag{4}$$

Similarly, the current at port 2-2′ can be computed as the sum of two components: (a) the first component $\mathbf{I}_{2a}$ is the current at 2-2′ when the voltage source at port 1-1′ is nulled (i.e., $\mathbf{V}_1 = 0$), and (b) the second component $\mathbf{I}_{2b}$ is the current at 2-2′ when the voltage source at port 2-2′ is nulled (i.e., $\mathbf{V}_2 = 0$). Thus,

$$\mathbf{I}_2 = \mathbf{I}_{2a} + \mathbf{I}_{2b} \tag{5}$$

The current $\mathbf{I}_{2a}$ can be expressed as

$$\mathbf{I}_{2a} = \frac{\mathbf{V}_2}{\mathbf{z}_{eq,2}} = \mathbf{y}_{22}\mathbf{V}_2 \tag{6}$$

where $\mathbf{z}_{eq,2}$ is the equivalent impedance looking into port 2-2′ with port 1-1′ short-circuited. The current $\mathbf{I}_{2b}$ represents the negative of the short-circuit current at port 2-2′ when the voltage source at port 1-1′ is active. Since the network is linear, $\mathbf{I}_{2b}$ can be expressed as

$$\mathbf{I}_{2b} = \mathbf{y}_{21}\mathbf{V}_1 \tag{7}$$

where $\mathbf{y}_{21}$ is a proportionality constant that relates the current at port 2-2′ to the voltage at port 1-1′. Thus, the total current at port 2-2′ can be expressed as

$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2 \tag{8}$$

The currents at the two ports can be expressed in the matrix form as

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} \tag{9}$$

where the parameters $\mathbf{y}_{11}$, $\mathbf{y}_{12}$, $\mathbf{y}_{21}$, and $\mathbf{y}_{22}$ are referred to as the **admittance parameters** (or **Y-parameters**) of the two-port network.

Computing the admittance parameters: For computing the admittance parameters of the two-port network it is not necessary to adopt the approach adopted in deriving the model. Observation into 9 indicates that the admittance parameters are defined as

$$\mathbf{y}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}, \quad \mathbf{y}_{12} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}, \quad \mathbf{y}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}, \quad \mathbf{y}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0} \tag{10}$$

Thus, the admittance parameters can be computed by performing the following steps:

– To compute $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, short-circuit the output port (i.e., set $\mathbf{V}_2 = 0$) and apply a voltage source $\mathbf{V}_1$ at the input port. Calculate the resulting currents $\mathbf{I}_1$ and $\mathbf{I}_2$. Then,

$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1}, \quad \mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} \tag{11}$$

– To compute $\mathbf{y}_{12}$ and $\mathbf{y}_{22}$, short-circuit the input port (i.e., set $\mathbf{V}_1 = 0$) and apply a voltage source $\mathbf{V}_2$ at the output port. Calculate the resulting currents $\mathbf{I}_1$ and $\mathbf{I}_2$. Then,

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2}, \quad \mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} \tag{12}$$

Since the admittance parameters are defined in terms of short-circuit conditions, they are also referred to as **short-circuit parameters**.

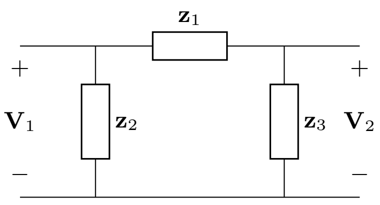

Figure 3: Two-Port Network for Example 1

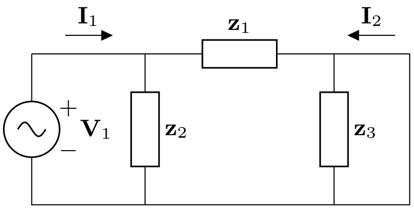

Figure 4: Circuit for computing $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$

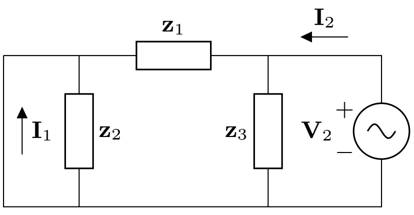

Figure 5: Circuit for computing $\mathbf{y}_{12}$ and $\mathbf{y}_{22}$

- Example 1: Determine the admittance parameters of the two-port network shown in Fig. 3.

Solution: The circuit is commonly referred to as a π-network. To compute the admittance parameters $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, we short-circuit the output port and apply a voltage source $\mathbf{V}_1$ at the input port as shown in Fig. 4. The parameter $\mathbf{y}_{11}$ is the equivalent admittance looking into port 1-1′ with port 2-2′ short-circuited. Thus,

$$\mathbf{y}_{11} = \frac{1}{\mathbf{z}_1||\mathbf{z}_2} = \frac{\mathbf{z}_1 + \mathbf{z}_2}{\mathbf{z}_1\mathbf{z}_2} = \mathbf{y}_1 + \mathbf{y}_2$$

The parameter $\mathbf{y}_{21}$ is the negative of the short-circuit current at port 2-2′ when the voltage source at port 1-1′ is active. Using current division,

$$\mathbf{I}_2 = -\frac{\mathbf{z}_2}{\mathbf{z}_1 + \mathbf{z}_2}\mathbf{I}_1 = -\frac{\mathbf{z}_2}{\mathbf{z}_1 + \mathbf{z}_2}\left(\frac{\mathbf{V}_1}{\mathbf{z}_1||\mathbf{z}_2}\right) = -\frac{\mathbf{V}_1}{\mathbf{z}_1} = -\mathbf{y}_1\mathbf{V}_1$$

To compute the admittance parameters $\mathbf{y}_{12}$ and $\mathbf{y}_{22}$, we short-circuit the input port and apply a voltage source $\mathbf{V}_2$ at the output port as shown in Fig. 5. The parameter $\mathbf{y}_{22}$ is the equivalent admittance looking into port 2-2′ with port 1-1′

short-circuited. Thus,

$$\mathbf{y}_{22} = \frac{1}{\mathbf{z}_1||\mathbf{z}_3} = \frac{\mathbf{z}_1 + \mathbf{z}_3}{\mathbf{z}_1\mathbf{z}_3} = \mathbf{y}_1 + \mathbf{y}_3$$

The parameter $\mathbf{y}_{12}$ is the negative of the short-circuit current at port 1-1′ when the voltage source at port 2-2′ is active. Using current division,

$$\mathbf{I}_1 = -\frac{\mathbf{z}_3}{\mathbf{z}_1 + \mathbf{z}_3}\mathbf{I}_2 = -\frac{\mathbf{z}_3}{\mathbf{z}_1 + \mathbf{z}_3}\left(\frac{\mathbf{V}_2}{\mathbf{z}_1||\mathbf{z}_3}\right) = -\frac{\mathbf{V}_2}{\mathbf{z}_1} = -\mathbf{y}_1\mathbf{V}_2$$

Thus, the admittance parameters of the two-port network are

$$\mathbf{y}_{11} = \mathbf{y}_1 + \mathbf{y}_2, \quad \mathbf{y}_{12} = -\mathbf{y}_1, \quad \mathbf{y}_{21} = -\mathbf{y}_1, \quad \mathbf{y}_{22} = \mathbf{y}_1 + \mathbf{y}_3$$

It is interesting to note that, if the admittance parameters are known (say computed experimentally), it is possible to construct an equivalent π-network whose admittance parameters are the same as those of the original two-port network provided $\mathbf{y}_{12} = \mathbf{y}_{21}$ (i.e., the network is reciprocal). The element values of such an equivalent π-network can be determined as

$$\mathbf{y}_1 = -\mathbf{y}_{12} = -\mathbf{y}_{21}, \quad \mathbf{y}_2 = \mathbf{y}_{11} + \mathbf{y}_{12}, \quad \mathbf{y}_3 = \mathbf{y}_{22} + \mathbf{y}_{12} \tag{13}$$

- **Impedance (Z) Model**: Conisder a two-port network as shown in Fig. 1. For the purpose of developing the impedance parameter model, consider that the input and output ports of the two-port network are excited by independent current sources, which in phasor form are denoted by $\mathbf{I}_1$ and $\mathbf{I}_2$, respectively (as shown in Fig. 6).

  The goal is to determine the expressions for the input and output port voltages, $\mathbf{V}_1$ and $\mathbf{V}_2$, in terms of the input and output port currents, $\mathbf{I}_1$ and $\mathbf{I}_2$. Since the two-port network is assumed to be linear, the voltage at both the ports can be computed as a linear combination of the two currents (using superposition principle). The voltage at port 1-1′ can be computed as the sum of two components: (a) the first component $\mathbf{V}_{1a}$ is the voltage at 1-1′ when the current source at port 2-2′ is nulled (i.e., $\mathbf{I}_2 = 0$), and (b) the second component $\mathbf{V}_{1b}$ is the voltage at 1-1′ when the current source at port 1-1′ is nulled (i.e., $\mathbf{I}_1 = 0$). Thus,

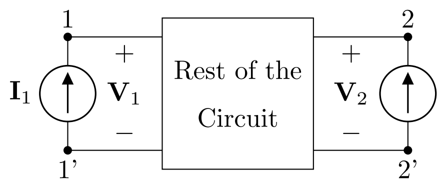

Figure 6: Two-Port Network characterization with Current Source Excitations

$$\mathbf{V}_1 = \mathbf{V}_{1a} + \mathbf{V}_{1b} \tag{14}$$

The voltage $\mathbf{V}_{1a}$ is the voltage across the input port when the current source at port 2-2′ is nulled. If $z_{eq,1}$ denotes the equivalent impedance looking into port 1-1′ with port 2-2′ open-circuited, then $\mathbf{V}_{1a}$ can be expressed as

$$\mathbf{V}_{1a} = \mathbf{z}_{eq,1}\mathbf{I}_1 = \mathbf{z}_{11}\mathbf{I}_1 \tag{15}$$

The voltage $\mathbf{V}_{1b}$ represents the open-circuit voltage at port 1-1′ when the source at port 2-2′ is active. Since the network is linear, $\mathbf{V}_{1b}$ can be expressed as

$$\mathbf{V}_{1b} = \mathbf{z}_{12}\mathbf{I}_2 \tag{16}$$

where $\mathbf{z}_{12}$ is a proportionality constant that relates the voltage at port 1-1′ to the current at port 2-2′. Thus, the total voltage at port 1-1′ can be expressed as

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \tag{17}$$

Similarly, the voltage at port 2-2′ can be computed as the sum of two components: (a) the first component $\mathbf{V}_{2a}$ is the voltage at 2-2′ when the current source at port 1-1′ is nulled (i.e., $\mathbf{I}_1 = 0$), and (b) the second component $\mathbf{V}_{2b}$ is the voltage at 2-2′ when the current source at port 2-2′ is nulled (i.e., $\mathbf{I}_2 = 0$). Thus,

$$\mathbf{V}_2 = \mathbf{V}_{2a} + \mathbf{V}_{2b} \tag{18}$$

The voltage $\mathbf{V}_{2a}$ is the voltage across the output port when the current source at port 1-1′ is nulled. If $z_{eq,2}$ denotes the equivalent impedance looking into port 2-2′ with port 1-1′ open-circuited, then $\mathbf{V}_{2a}$ can be expressed as

$$\mathbf{V}_{2a} = \mathbf{z}_{eq,2}\mathbf{I}_2 = \mathbf{z}_{22}\mathbf{I}_2 \tag{19}$$

The voltage $\mathbf{V}_{2b}$ represents the open-circuit voltage at port 2-2′ when the source at port 1-1′ is active. Since the network is linear, $\mathbf{V}_{2b}$ can be expressed as

$$\mathbf{V}_{2b} = \mathbf{z}_{21}\mathbf{I}_1 \tag{20}$$

where $\mathbf{z}_{21}$ is a proportionality constant that relates the voltage at port 2-2′ to the current at port 1-1′. Thus, the total voltage at port 2-2′ can be expressed as

$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \tag{21}$$

The voltages at the two ports can be expressed in the matrix form as

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} \tag{22}$$

where the parameters $\mathbf{z}_{11}$, $\mathbf{z}_{12}$, $\mathbf{z}_{21}$, and $\mathbf{z}_{22}$ are referred to as the **impedance parameters** (or **Z-parameters**) of the two-port network.

Computing the impedance parameters: Observation into 22 indicates that the impedance parameters are defined as

$$\mathbf{z}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}, \quad \mathbf{z}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0}, \quad \mathbf{z}_{21} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}, \quad \mathbf{z}_{22} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} \tag{23}$$

Thus, the impedance parameters can be computed by performing the following steps:

- To compute $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, open-circuit the output port (i.e., set $\mathbf{I}_2 = 0$) and apply a current source $\mathbf{I}_1$ at the input port. Calculate the resulting voltages $\mathbf{V}_1$ and $\mathbf{V}_2$. Then,
$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1}, \quad \mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} \tag{24}$$
- To compute $\mathbf{z}_{12}$ and $\mathbf{z}_{22}$, open-circuit the input port (i.e., set $\mathbf{I}_1 = 0$) and apply a current source $\mathbf{I}_2$ at the output port. Calculate the resulting voltages $\mathbf{V}_1$ and $\mathbf{V}_2$. Then,
$$\mathbf{z}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_2}, \quad \mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} \tag{25}$$

Since the impedance parameters are defined in terms of open-circuit conditions, they are also referred to as **open-circuit parameters**.

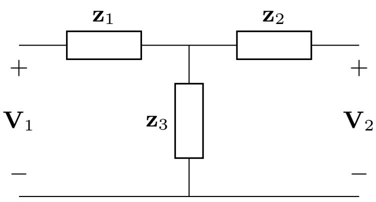

Figure 7: Two-Port Network for Example 2

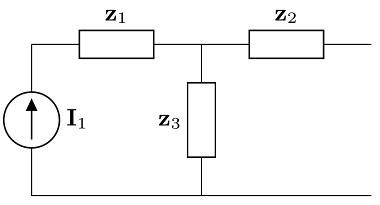

Figure 8: Circuit for computing $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$

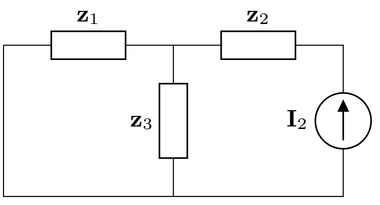

Figure 9: Circuit for computing $\mathbf{z}_{12}$ and $\mathbf{z}_{22}$

- Example 2: Determine the impedance parameters of the two-port network shown in Fig. 7.

  Solution: The circuit is commonly referred to as a T-network. To compute the impedance parameters $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, we open-circuit the output port and apply a current source $\mathbf{I}_1$ at the input port as shown in Fig. 8. The parameter $\mathbf{z}_{11}$ is the equivalent impedance looking into port 1-1′ with port 2-2′ open-circuited. Thus,
$$\mathbf{z}_{11} = \mathbf{z}_1 + \mathbf{z}_3$$
  The parameter $\mathbf{z}_{21}$ is the open-circuit voltage at port 2-2′ when the current source at port 1-1′ is active. Using voltage division,
$$\mathbf{V}_2 = \mathbf{I}_1\mathbf{z}_3 \quad \Rightarrow \quad \mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \mathbf{z}_3$$
  To compute the impedance parameters $\mathbf{z}_{12}$ and $\mathbf{z}_{22}$, we open-circuit the input port and apply a current source $\mathbf{I}_2$ at the output port as shown in Fig. 9. The parameter $\mathbf{z}_{22}$ is the equivalent impedance looking into port 2-2′ with port 1-1′ open-circuited. Thus,
$$\mathbf{z}_{22} = \mathbf{z}_2 + \mathbf{z}_3$$
  The parameter $\mathbf{z}_{12}$ is the open-circuit voltage at port 1-1′ when the current source at port 2-2′ is active. Using voltage division,
$$\mathbf{V}_1 = \mathbf{I}_2\mathbf{z}_3 \quad \Rightarrow \quad \mathbf{z}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_2} = \mathbf{z}_3$$
  Thus, the impedance parameters of the two-port network are
$$\mathbf{z}_{11} = \mathbf{z}_1 + \mathbf{z}_3, \quad \mathbf{z}_{12} = \mathbf{z}_3, \quad \mathbf{z}_{21} = \mathbf{z}_3, \quad \mathbf{z}_{22} = \mathbf{z}_2 + \mathbf{z}_3$$

It is interesting to note that, if the impedance parameters are known (say computed experimentally), it is possible to construct an equivalent T-network whose impedance parameters are the same as those of the original two-port network provided $\mathbf{z}_{12} = \mathbf{z}_{21}$ (i.e., the network is reciprocal). The element values of such an equivalent T-network can be determined as

$$\mathbf{z}_1 = \mathbf{z}_{11} - \mathbf{z}_{21}, \quad \mathbf{z}_2 = \mathbf{z}_{22} - \mathbf{z}_{21}, \quad \mathbf{z}_3 = \mathbf{z}_{12} = \mathbf{z}_{21} \tag{26}$$

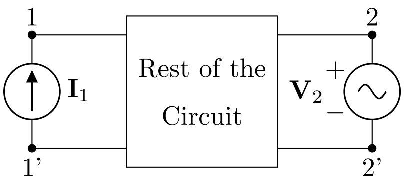

Figure 10: Two-Port Network characterization with Mixed Source Excitations

- **Hybrid parameters or h-parameters**: Consider a two-port network as shown in Fig. 1. For the purpose of developing the hybrid parameter model, consider that the input port of the two-port network is excited by a current source, which in phasor form is denoted by $\mathbf{I}_1$, and the output port is excited by a voltage source, which in phasor form is denoted by $\mathbf{V}_2$ (as shown in Fig. 10). In case of hybrid parameters, the goal is to determine the expressions for the input port voltage, $\mathbf{V}_1$, and the output port current, $\mathbf{I}_2$, in terms of the input port current, $\mathbf{I}_1$, and the output port voltage, $\mathbf{V}_2$. Since the two-port network is assumed to be linear, the voltage at port 1-1′ can be computed as the sum of two components: (a) the first component $\mathbf{V}_{1a}$ is the voltage at 1-1′ when the voltage source at port 2-2′ is nulled (i.e., $\mathbf{V}_2 = 0$), and (b) the second component $\mathbf{V}_{1b}$ is the voltage at 1-1′ when the current source at port 1-1′ is nulled (i.e., $\mathbf{I}_1 = 0$). Thus,

$$\mathbf{V}_1 = \mathbf{V}_{1a} + \mathbf{V}_{1b} \tag{27}$$

The voltage $\mathbf{V}_{1a}$ is the voltage across the input port when the voltage source at port 2-2′ is nulled (i.e., port 2-2′ is short-circuited). If $\mathbf{z}_{eq1}$ denotes the equivalent impedance looking into port 1-1′ with port 2-2′ short-circuited, then $\mathbf{V}_{1a}$ can be expressed as

$$\mathbf{V}_{1a} = \mathbf{z}_{eq1}\mathbf{I}_1 = \mathbf{h}_{11}\mathbf{I}_1 \tag{28}$$

It is interesting to recall that $\mathbf{z}_{eq1}$ (i.e., equivalent impedance looking into port 1-1′ with port 2-2′ short-circuited) is the reciprocal of $\mathbf{y}_{11}$. Thus,

$$\mathbf{h}_{11} = \frac{1}{\mathbf{y}_{11}} \tag{29}$$

The voltage $\mathbf{V}_{1b}$ represents the voltage at port 1-1′ when the source at port 1-1′ is nulled. Since the network is linear, $\mathbf{V}_{1b}$ can be expressed as

$$\mathbf{V}_{1b} = \mathbf{h}_{12}\mathbf{V}_2 \tag{30}$$

where $\mathbf{h}_{12}$ is a proportionality constant that relates the voltage at port 1-1′ to the voltage at port 2-2′. Thus, the total voltage at port 1-1′ can be expressed as

$$\mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2 \tag{31}$$

Similarly, the current at port 2-2′ can be computed as the sum of two components: (a) the first component $\mathbf{I}_{2a}$ is the current at 2-2′ when the voltage source at port 2-2′ is nulled (i.e., $\mathbf{V}_2 = 0$), and (b) the second component $\mathbf{I}_{2b}$ is the current at 2-2′ when the current source at port 1-1′ is nulled (i.e., $\mathbf{I}_1 = 0$). Thus,

$$\mathbf{I}_2 = \mathbf{I}_{2a} + \mathbf{I}_{2b} \tag{32}$$

The current $\mathbf{I}_{2a}$ is the current through the output port when the voltage source at port 2-2′ is nulled (i.e., port 2-2′ is short-circuited). Since the network is linear, $\mathbf{I}_{2a}$ can be expressed as

$$\mathbf{I}_{2a} = \mathbf{h}_{21}\mathbf{I}_1 \tag{33}$$

where $\mathbf{h}_{21}$ is a proportionality constant that relates the current at port 2-2′ to the current at port 1-1′. The term $\mathbf{h}_{21}\mathbf{I}_1$ represents the negative of the short-circuit current at port 2-2′ when the current source at port 1-1′ is active. The current $\mathbf{I}_{2b}$ represents the current through port 2-2′ when the source at port 1-1′ is nulled. If $\mathbf{z}_{eq2}$ denotes the equivalent impedance looking into port 2-2′ with port 1-1′ open, then $\mathbf{I}_{2b}$ can be expressed as

$$\mathbf{I}_{2b} = \frac{\mathbf{V}_2}{\mathbf{z}_{eq2}} = \mathbf{h}_{22}\mathbf{V}_2 \tag{34}$$

It is interesting to recall that $\mathbf{z}_{eq2}$ (i.e., equivalent impedance looking into port 2-2′ with port 1-1′ open) is same as $\mathbf{z}_{22}$. Thus,

$$\mathbf{h}_{22} = \frac{1}{\mathbf{z}_{22}} \tag{35}$$

Thus, the total current at port 2-2′ can be expressed as

$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2 \tag{36}$$

The voltages and currents at the two ports can be expressed in the matrix form as

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix} \tag{37}$$

where the parameters $\mathbf{h}_{11}$, $\mathbf{h}_{12}$, $\mathbf{h}_{21}$, and $\mathbf{h}_{22}$ are referred to as the **hybrid parameters** (or **h-parameters**) of the two-port network.

Computing the hybrid parameters: Observation into (37) indicates that the hybrid parameters are defined as

$$\mathbf{h}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0}, \quad \mathbf{h}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0}, \quad \mathbf{h}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0}, \quad \mathbf{h}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0} \tag{38}$$

Thus, the hybrid parameters can be computed by performing the following steps:

- To compute $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$, short-circuit the output port (i.e., set $\mathbf{V}_2 = 0$) and apply a current source $\mathbf{I}_1$ at the input port. Calculate the resulting voltage $\mathbf{V}_1$ and current $\mathbf{I}_2$. Then,

$$\mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1}, \quad \mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} \tag{39}$$

– To compute $\mathbf{h}_{12}$ and $\mathbf{h}_{22}$, open-circuit the input port (i.e., set $\mathbf{I}_1 = 0$) and apply a voltage source $\mathbf{V}_2$ at the output port. Calculate the resulting voltage $\mathbf{V}_1$ and current $\mathbf{I}_2$. Then,

$$\mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2}, \quad \mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} \tag{40}$$

- **Inverse Hybrid parameters or g-parameters**: Consider a two-port network as shown in Fig. 1. For the purpose of developing the inverse hybrid parameter model, consider that the input port of the two-port network is excited by a voltage source, which in phasor form is denoted by $\mathbf{V}_1$, and the output port is excited by a current source, which in phasor form is denoted by $\mathbf{I}_2$ (as shown in Fig. 11).

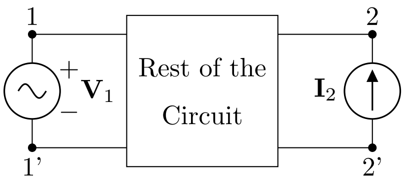

Figure 11: Two-Port Network characterization with Mixed Source Excitations

In case of inverse hybrid parameters, the goal is to determine the expressions for the input port current, $\mathbf{I}_1$, and the output port voltage, $\mathbf{V}_2$, in terms of the input port voltage, $\mathbf{V}_1$, and the output port current, $\mathbf{I}_2$. Since the two-port network is assumed to be linear, the current at port 1-1′ can be computed as the sum of two components: (a) the first component $\mathbf{I}_{1a}$ is the current at 1-1′ when the current source at port 2-2′ is nulled (i.e., $\mathbf{I}_2 = 0$), and (b) the second component $\mathbf{I}_{1b}$ is the current at 1-1′ when the voltage source at port 1-1′ is nulled (i.e., $\mathbf{V}_1 = 0$). Thus,

$$\mathbf{I}_1 = \mathbf{I}_{1a} + \mathbf{I}_{1b} \tag{41}$$

The current $\mathbf{I}_{1a}$ is the current through the input port when the current source at port 2-2′ is nulled (i.e., port 2-2′ is open-circuited). If $\mathbf{z}_{eq1}$ denotes the equivalent impedance looking into port 1-1′ with port 2-2′ open-circuited, then $\mathbf{I}_{1a}$ can be expressed as

$$\mathbf{I}_{1a} = \frac{\mathbf{V}_1}{\mathbf{z}_{eq1}} = \mathbf{g}_{11}\mathbf{V}_1 \tag{42}$$

It is interesting to recall that $\mathbf{z}_{eq1}$ (i.e., equivalent impedance looking into port 1-1′ with port 2-2′ open-circuited) is the same as $\mathbf{z}_{11}$. Thus,

$$\mathbf{g}_{11} = \frac{1}{\mathbf{z}_{11}} \tag{43}$$

The current $\mathbf{I}_{1b}$ represents the current at port 1-1′ when the source at port 1-1′ is nulled. Since the network is linear, $\mathbf{I}_{1b}$ can be expressed as

$$\mathbf{I}_{1b} = \mathbf{g}_{12}\mathbf{I}_2 \tag{44}$$

where $\mathbf{g}_{12}$ is a proportionality constant that relates the current at port 1-1′ to the current at port 2-2′. Thus, the total current at port 1-1′ can be expressed as

$$\mathbf{I}_1 = \mathbf{g}_{11}\mathbf{V}_1 + \mathbf{g}_{12}\mathbf{I}_2 \tag{45}$$

Similarly, the voltage at port 2-2′ can be computed as the sum of two components: (a) the first component $\mathbf{V}_{2a}$ is the voltage at 2-2′ when the current source at port 2-2′ is nulled (i.e., $\mathbf{I}_2 = 0$), and (b) the second component $\mathbf{V}_{2b}$ is the voltage at 2-2′ when the voltage source at port 1-1′ is nulled (i.e., $\mathbf{V}_1 = 0$). Thus,

$$\mathbf{V}_2 = \mathbf{V}_{2a} + \mathbf{V}_{2b} \tag{46}$$

The voltage $\mathbf{V}_{2a}$ is the voltage across the output port when the current source at port 2-2′ is nulled (i.e., port 2-2′ is open-circuited). Since the network is linear, $\mathbf{V}_{2a}$ can be expressed as

$$\mathbf{V}_{2a} = \mathbf{g}_{21}\mathbf{V}_1 \tag{47}$$

where $\mathbf{g}_{21}$ is a proportionality constant that relates the voltage at port 2-2′ to the voltage at port 1-1′. The voltage $\mathbf{V}_{2b}$ represents the voltage across port 2-2′ when the source at port 1-1′ is nulled. If $\mathbf{z}_{eq2}$ denotes the equivalent impedance looking into port 2-2′ with port 1-1′ short-circuited, then $\mathbf{V}_{2b}$ can be expressed as

$$\mathbf{V}_{2b} = \mathbf{z}_{eq2}\mathbf{I}_2 = \mathbf{g}_{22}\mathbf{I}_2 \tag{48}$$

It is interesting to recall that $\mathbf{z}_{eq2}$ (i.e., equivalent impedance looking into port 2-2′ with port 1-1′ short-circuited) is the reciprocal of $\mathbf{y}_{22}$. Thus,

$$\mathbf{g}_{22} = \frac{1}{\mathbf{y}_{22}} \tag{49}$$

Thus, the total voltage at port 2-2′ can be expressed as

$$\mathbf{V}_2 = \mathbf{g}_{21}\mathbf{V}_1 + \mathbf{g}_{22}\mathbf{I}_2 \tag{50}$$

The currents and voltages at the two ports can be expressed in the matrix form as

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{g}_{11} & \mathbf{g}_{12} \\ \mathbf{g}_{21} & \mathbf{g}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} \tag{51}$$

where the parameters $\mathbf{g}_{11}$, $\mathbf{g}_{12}$, $\mathbf{g}_{21}$, and $\mathbf{g}_{22}$ are referred to as the **inverse hybrid parameters** (or **g-parameters**) of the two-port network.

Computing the inverse hybrid parameters: Observation into (51) indicates that the inverse hybrid parameters are defined as

$$\mathbf{g}_{11} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{I}_2=0}, \quad \mathbf{g}_{12} = \left.\frac{\mathbf{I}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_1=0}, \quad \mathbf{g}_{21} = \left.\frac{\mathbf{V}_2}{\mathbf{V}_1}\right|_{\mathbf{I}_2=0}, \quad \mathbf{g}_{22} = \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{V}_1=0} \tag{52}$$

Thus, the inverse hybrid parameters can be computed by performing the following steps:

- To compute $\mathbf{g}_{11}$ and $\mathbf{g}_{21}$, open-circuit the output port (i.e., set $\mathbf{I}_2 = 0$) and apply a voltage source $\mathbf{V}_1$ at the input port. Calculate the resulting current $\mathbf{I}_1$ and voltage $\mathbf{V}_2$. Then,

$$\mathbf{g}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1}, \quad \mathbf{g}_{21} = \frac{\mathbf{V}_2}{\mathbf{V}_1} \tag{53}$$

- To compute $\mathbf{g}_{12}$ and $\mathbf{g}_{22}$, short-circuit the input port (i.e., set $\mathbf{V}_1 = 0$) and apply a current source $\mathbf{I}_2$ at the output port. Calculate the resulting current $\mathbf{I}_1$ and voltage $\mathbf{V}_2$. Then,

$$\mathbf{g}_{12} = \frac{\mathbf{I}_1}{\mathbf{I}_2}, \quad \mathbf{g}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} \tag{54}$$

- **Transmission parameters or ABCD-parameters**: Consider a two-port network as shown in Fig. 1. For the purpose of developing the transmission parameter model, consider the following scenario: The voltage and current at the output port of the two-port network (denoted by $\mathbf{V}_2$ and $\mathbf{I}_2$, respectively) are known, and the goal is to determine the voltage and current at the input port of the two-port network (denoted by $\mathbf{V}_1$ and $\mathbf{I}_1$, respectively) in terms of the output port voltage and current.

  It is important to keep in mind that, the current reference choosen for the output port, when computing the transmission parameters, is from 2 to 2′ (i.e., the current $\mathbf{I}_2$ is considered to be leaving the two-port network at port 2-2′). This is in contrast to the reference direction of current at port 2-2′ used while computing the impedance, admittance, hybrid, and inverse hybrid parameters, where the current $\mathbf{I}_2$ is considered to be entering the two-port network at port 2-2′. This change in current reference direction is essential to ensure that the transmission parameters of cascaded two-port networks can be computed by simply multiplying the individual transmission parameter matrices of the constituent two-port networks.

  Since the two-port network is assumed to be linear, the voltage and current at the input port vary linearly with the voltage and current at the output port. Thus, the voltage and current at the input port can be expressed as

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{V}_2 \\ \mathbf{I}_2 \end{bmatrix} \tag{55}$$

  where the parameters $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ are referred to as the **transmission parameters** (or **ABCD-parameters**) of the two-port network. The transmission parameters can be computed by performing the following steps:

  - To compute $\mathbf{A}$ and $\mathbf{C}$, open-circuit the output port (i.e., set $\mathbf{I}_2 = 0$) and apply a suitable source at the input port. Calculate the resulting voltage $\mathbf{V}_1$ and current $\mathbf{I}_1$. Then,

$$\mathbf{A} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0}, \quad \mathbf{C} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0} \tag{56}$$

– To compute $\mathbf{B}$ and $\mathbf{D}$, short-circuit the output port (i.e., set $\mathbf{V}_2 = 0$) and apply a suitable source at the input port. Calculate the resulting voltage $\mathbf{V}_1$ and current $\mathbf{I}_1$. Then,

$$\mathbf{B} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_2=0}, \quad \mathbf{D} = \left.\frac{\mathbf{I}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_2=0} \tag{57}$$

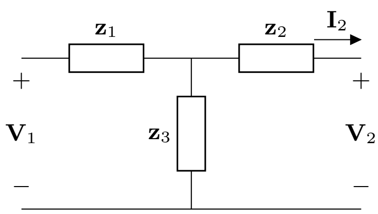

Figure 12: Two-Port Network for Example 3

- Example 3: Determine the transmission parameters of the two-port network shown in Fig. 12.

Solution: To compute the parameters $\mathbf{A}$ and $\mathbf{C}$, we proceed by open-circuiting the output port and applying a voltage source at the input port (as shown in Fig. 13). Since the output port is open-circuited, the current $\mathbf{I}_1$ is given by

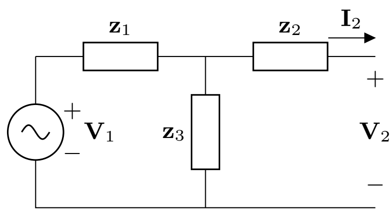

Figure 13: Circuit for computing $\mathbf{A}$ and $\mathbf{C}$

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{\mathbf{z}_1 + \mathbf{z}_3}$$

The voltage at output port is given by

$$\mathbf{V}_2 = \mathbf{z}_2\mathbf{I}_1 \quad \Rightarrow \mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{1}{\mathbf{z}_2}$$

Further, the voltage across the input and output port are related as

$$\mathbf{V}_2 = \frac{\mathbf{z}_3}{\mathbf{z}_1 + \mathbf{z}_3}\mathbf{V}_1 \quad \Rightarrow \quad \mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{\mathbf{z}_1 + \mathbf{z}_3}{\mathbf{z}_3}$$

Next, to compute the transmission parameters $\mathbf{B}$ and $\mathbf{D}$, we short-circuit the output port and apply a current source at the input port (as shown in Fig. 13). Since the output port is short-circuited, the current $\mathbf{I}_2$ is given by

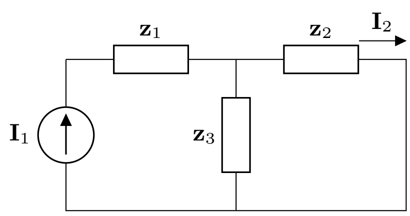

Figure 14: Circuit for computing $\mathbf{B}$ and $\mathbf{D}$

$$\mathbf{I}_2 = \frac{\mathbf{z}_3}{\mathbf{z}_2 + \mathbf{z}_3}\mathbf{I}_1 \Rightarrow \mathbf{D} = \frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{\mathbf{z}_2 + \mathbf{z}_3}{\mathbf{z}_3}$$

The voltage at input port is given by

$$\mathbf{V}_1 = \mathbf{z}_1\mathbf{I}_1 + \mathbf{z}_2\mathbf{I}_2 \quad \Rightarrow \quad \mathbf{B} = \frac{\mathbf{V}_1}{\mathbf{I}_2} = \mathbf{z}_2 + \mathbf{z}_1\frac{\mathbf{I}_1}{\mathbf{I}_2} = \mathbf{z}_2 + \mathbf{z}_1\frac{\mathbf{z}_2 + \mathbf{z}_3}{\mathbf{z}_3}$$

Thus, the transmission parameters of the two-port network are given by

$$\mathbf{A} = \frac{\mathbf{z}_1 + \mathbf{z}_3}{\mathbf{z}_3}, \quad \mathbf{B} = \mathbf{z}_2 + \mathbf{z}_1\frac{\mathbf{z}_2 + \mathbf{z}_3}{\mathbf{z}_3}, \quad \mathbf{C} = \frac{1}{\mathbf{z}_2}, \quad \mathbf{D} = \frac{\mathbf{z}_2 + \mathbf{z}_3}{\mathbf{z}_3}$$

Interconnection of Two-Port Networks: One of the significant advantages of using two-port network models is the ease with which multiple two-port networks can be interconnected to form more complex networks. The overall two-port network parameters of the interconnected network can be determined from the individual two-port network parameters of the constituent networks. The following types of interconnections are commonly encountered:

– **Series-Series Connection**: In a series-series connection, the input ports of the two two-port networks are connected in series, and the output ports of the two two-port networks are also connected in series (as shown in Fig. 15). Let

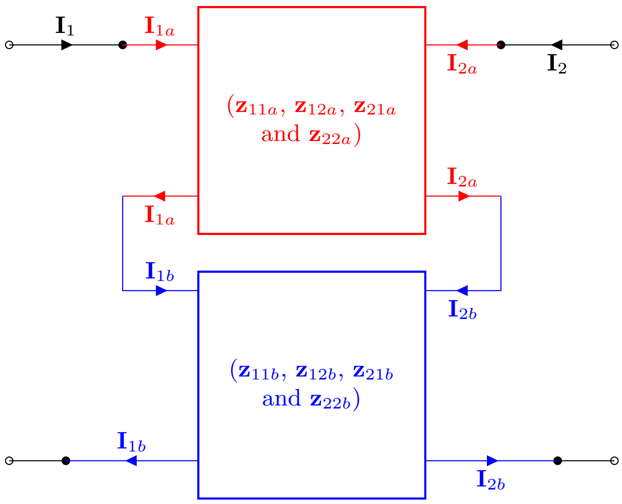

Figure 15: Series-Series Connection of Two-Port Networks

$\mathbf{z}_{11a}$, $\mathbf{z}_{12a}$, $\mathbf{z}_{21a}$, and $\mathbf{z}_{22a}$ be the impedance parameters of the first two-port network, and let $\mathbf{z}_{11b}$, $\mathbf{z}_{12b}$, $\mathbf{z}_{21b}$, and $\mathbf{z}_{22b}$ be the impedance parameters of the second two-port network. In terms of the impedance parameters of the individual two-port networks, the voltage at port 1-1′ can be expressed as

$$\begin{aligned}\mathbf{V}_1 &= \mathbf{V}_{1a} + \mathbf{V}_{1b} = \mathbf{z}_{11a}\mathbf{I}_{1a} + \mathbf{z}_{11b}\mathbf{I}_{1b} + \mathbf{z}_{12a}\mathbf{I}_{2a} + \mathbf{z}_{12b}\mathbf{I}_{2b} \\ &= \mathbf{z}_{11a}\mathbf{I}_1 + \mathbf{z}_{11b}\mathbf{I}_1 + \mathbf{z}_{12a}\mathbf{I}_2 + \mathbf{z}_{12b}\mathbf{I}_2 \text{ (since } \mathbf{I}_{1a} = \mathbf{I}_{1b} = \mathbf{I}_1 \ \& \ \mathbf{I}_{2a} = \mathbf{I}_{2b} = \mathbf{I}_2) \\ &= (\mathbf{z}_{11a} + \mathbf{z}_{11b})\mathbf{I}_1 + (\mathbf{z}_{12a} + \mathbf{z}_{12b})\mathbf{I}_2\end{aligned} \tag{58}$$

Similarly, the voltage at port 2-2′ can be expressed as

$$\begin{aligned}\mathbf{V}_2 &= \mathbf{V}_{2a} + \mathbf{V}_{2b} = \mathbf{z}_{21a}\mathbf{I}_{1a} + \mathbf{z}_{21b}\mathbf{I}_{1b} + \mathbf{z}_{22a}\mathbf{I}_{2a} + \mathbf{z}_{22b}\mathbf{I}_{2b} \\ &= \mathbf{z}_{21a}\mathbf{I}_1 + \mathbf{z}_{21b}\mathbf{I}_1 + \mathbf{z}_{22a}\mathbf{I}_2 + \mathbf{z}_{22b}\mathbf{I}_2 \text{ (since } \mathbf{I}_{1a} = \mathbf{I}_{1b} = \mathbf{I}_1 \ \& \ \mathbf{I}_{2a} = \mathbf{I}_{2b} = \mathbf{I}_2) \\ &= (\mathbf{z}_{21a} + \mathbf{z}_{21b})\mathbf{I}_1 + (\mathbf{z}_{22a} + \mathbf{z}_{22b})\mathbf{I}_2\end{aligned} \tag{59}$$

Thus, the overall impedance parameters of the series-series connected two-port networks are given by

$$\mathbf{z}_{11} = \mathbf{z}_{11a} + \mathbf{z}_{11b}, \quad \mathbf{z}_{12} = \mathbf{z}_{12a} + \mathbf{z}_{12b}, \quad \mathbf{z}_{21} = \mathbf{z}_{21a} + \mathbf{z}_{21b}, \quad \mathbf{z}_{22} = \mathbf{z}_{22a} + \mathbf{z}_{22b} \tag{60}$$

– **Parallel-Parallel Connection**: In a parallel-parallel connection, the input ports of the two two-port networks are connected in parallel, and the output ports of the two two-port networks are also connected in parallel (as shown in Fig. 16). Let $\mathbf{y}_{11a}$, $\mathbf{y}_{12a}$, $\mathbf{y}_{21a}$, and $\mathbf{y}_{22a}$ be the admittance parameters of the

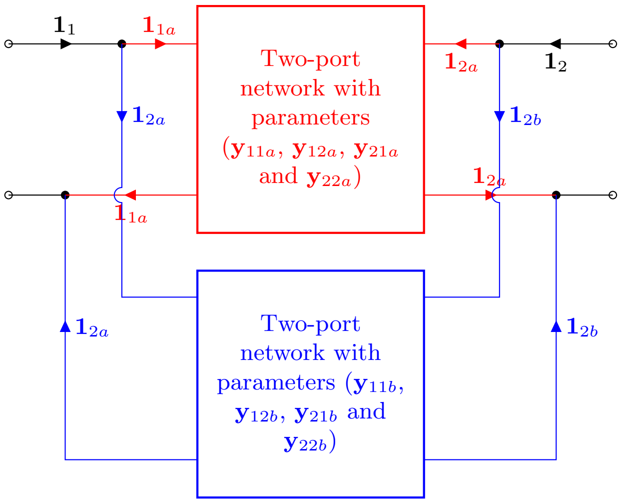

Figure 16: Parallel-Parallel Connection of Two-Port Networks

first two-port network, and let $\mathbf{y}_{11b}$, $\mathbf{y}_{12b}$, $\mathbf{y}_{21b}$, and $\mathbf{y}_{22b}$ be the admittance parameters of the second two-port network. In terms of the admittance parameters of the individual two-port networks, the current at port 1-1′ can be expressed as

$$\begin{aligned}\mathbf{I}_1 &= \mathbf{I}_{1a} + \mathbf{I}_{1b} = \mathbf{y}_{11a}\mathbf{V}_{1a} + \mathbf{y}_{11b}\mathbf{V}_{1b} + \mathbf{y}_{12a}\mathbf{V}_{2a} + \mathbf{y}_{12b}\mathbf{V}_{2b} \\ &= \mathbf{y}_{11a}\mathbf{V}_1 + \mathbf{y}_{11b}\mathbf{V}_1 + \mathbf{y}_{12a}\mathbf{V}_2 + \mathbf{y}_{12b}\mathbf{V}_2 \text{ (since } \mathbf{V}_{1a} = \mathbf{V}_{1b} = \mathbf{V}_1 \text{ \& } \mathbf{V}_{2a} = \mathbf{V}_{2b} = \mathbf{V}_2) \\ &= (\mathbf{y}_{11a} + \mathbf{y}_{11b})\mathbf{V}_1 + (\mathbf{y}_{12a} + \mathbf{y}_{12b})\mathbf{V}_2\end{aligned} \tag{61}$$

Similarly, the current at port 2-2′ can be expressed as

$$\begin{aligned}\mathbf{I}_2 &= \mathbf{I}_{2a} + \mathbf{I}_{2b} = \mathbf{y}_{21a}\mathbf{V}_{1a} + \mathbf{y}_{21b}\mathbf{V}_{1b} + \mathbf{y}_{22a}\mathbf{V}_{2a} + \mathbf{y}_{22b}\mathbf{V}_{2b} \\ &= \mathbf{y}_{21a}\mathbf{V}_1 + \mathbf{y}_{21b}\mathbf{V}_1 + \mathbf{y}_{22a}\mathbf{V}_2 + \mathbf{y}_{22b}\mathbf{V}_2 \text{ (since } \mathbf{V}_{1a} = \mathbf{V}_{1b} = \mathbf{V}_1 \text{ \& } \mathbf{V}_{2a} = \mathbf{V}_{2b} = \mathbf{V}_2) \\ &= (\mathbf{y}_{21a} + \mathbf{y}_{21b})\mathbf{V}_1 + (\mathbf{y}_{22a} + \mathbf{y}_{22b})\mathbf{V}_2\end{aligned} \tag{62}$$

Thus, the overall admittance parameters of the parallel-parallel connected two-port networks are given by

$$\mathbf{y}_{11} = \mathbf{y}_{11a} + \mathbf{y}_{11b}, \quad \mathbf{y}_{12} = \mathbf{y}_{12a} + \mathbf{y}_{12b}, \quad \mathbf{y}_{21} = \mathbf{y}_{21a} + \mathbf{y}_{21b}, \quad \mathbf{y}_{22} = \mathbf{y}_{22a} + \mathbf{y}_{22b} \tag{63}$$

– **Cascaded Connection**: In a cascaded connection, the output port of the first two-port network is connected to the input port of the second two-port network (as shown in Fig. 17). Let $\mathbf{A}_a$, $\mathbf{B}_a$, $\mathbf{C}_a$, and $\mathbf{D}_a$ be the transmission

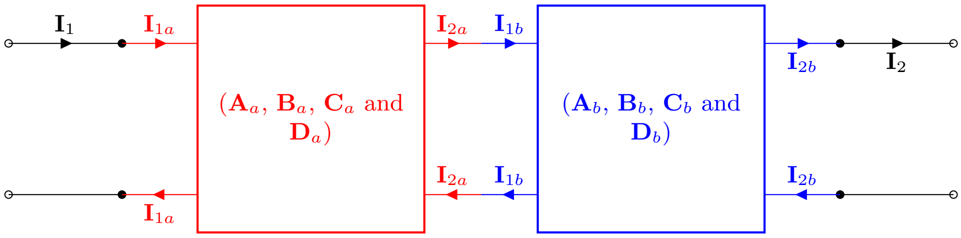

Figure 17: Cascaded Connection of Two-Port Networks

parameters of the first two-port network, and let $\mathbf{A}_b$, $\mathbf{B}_b$, $\mathbf{C}_b$, and $\mathbf{D}_b$ be the transmission parameters of the second two-port network. In terms of the transmission parameters of the individual two-port networks, the voltage and current at port 1-1$'$ can be expressed as

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_a & \mathbf{B}_a \\ \mathbf{C}_a & \mathbf{D}_a \end{bmatrix} \begin{bmatrix} \mathbf{V}_{2a} \\ \mathbf{I}_{2a} \end{bmatrix} \tag{64}$$

Also, the voltage and current at port 2-2$'$ can be expressed as

$$\begin{bmatrix} \mathbf{V}_{2a} \\ \mathbf{I}_{2a} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_b & \mathbf{B}_b \\ \mathbf{C}_b & \mathbf{D}_b \end{bmatrix} \begin{bmatrix} \mathbf{V}_2 \\ \mathbf{I}_2 \end{bmatrix} \tag{65}$$

Combining the above two equations, we obtain

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_a & \mathbf{B}_a \\ \mathbf{C}_a & \mathbf{D}_a \end{bmatrix} \begin{bmatrix} \mathbf{A}_b & \mathbf{B}_b \\ \mathbf{C}_b & \mathbf{D}_b \end{bmatrix} \begin{bmatrix} \mathbf{V}_2 \\ \mathbf{I}_2 \end{bmatrix} \tag{66}$$

Thus, the overall transmission parameters of the cascaded connected two-port networks are given by

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_a & \mathbf{B}_a \\ \mathbf{C}_a & \mathbf{D}_a \end{bmatrix} \begin{bmatrix} \mathbf{A}_b & \mathbf{B}_b \\ \mathbf{C}_b & \mathbf{D}_b \end{bmatrix} \tag{67}$$

- **Computing Single-Port Equivalent Circuit Parameters from Two-Port Network Parameters**: In some scenarios wherein the source at one of the ports is fixed, it is beneficial to represent the two-port network as a single-port equivalent circuit at the other port[2]. The parameters of the single-port equivalent circuit can be computed by careful manipulation of the two-port network equations. To illustrate the typical steps involved, consider a two-port network represented in terms of its hybrid ($\mathbf{H}$) parameters. Further, let a voltage source $\mathbf{V}_s$ be connected to port 1 of the two-port network (as shown in Fig. 18). The goal is to compute the Thevenin equivalent circuit associated with the output port (port 2). The hybrid parameter equations for the two-port network with $\mathbf{V}_s$ connected at port

[2] It is assumed that the two-port network does not comprise of any independent sources except the ones connected at the ports

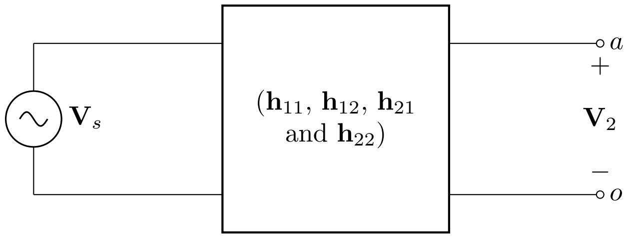

Figure 18: Two-Port Network with Voltage Source at Port 1

1 are given by

$$\begin{aligned} \mathbf{V}_s &= \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2 \\ \mathbf{I}_2 &= \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2 \end{aligned} \tag{68}$$

To compute the Thevenin equivalent voltage $\mathbf{V}_{th}$, we set the output port to be open-circuited (i.e., $\mathbf{I}_2 = 0$). Accordingly, from the second equation in 68, we have

$$0 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2 \quad \Rightarrow \quad \mathbf{I}_1 = -\frac{\mathbf{h}_{22}}{\mathbf{h}_{21}}\mathbf{V}_2$$

Substituting this expression for $\mathbf{I}_1$ into the first equation in 68, we obtain

$$\mathbf{V}_s = \mathbf{h}_{11}\left(-\frac{\mathbf{h}_{22}}{\mathbf{h}_{21}}\mathbf{V}_2\right) + \mathbf{h}_{12}\mathbf{V}_2 = \left(\mathbf{h}_{12} - \frac{\mathbf{h}_{11}\mathbf{h}_{22}}{\mathbf{h}_{21}}\right)\mathbf{V}_2 = -\frac{\Delta}{\mathbf{h}_{21}}\mathbf{V}_2$$

where Δ is the determinant of the hybrid parameter matrix. Thus, the Thevenin equivalent voltage at the output port is given by

$$\mathbf{V}_{th} = \mathbf{V}_2 = -\frac{\mathbf{h}_{21}}{\Delta}\mathbf{V}_s$$

Next, to compute the Thevenin equivalent impedance $\mathbf{z}_{th}$, we short-circuit the source at the input port (i.e., set $\mathbf{V}_s = 0$). Accordingly, from the first equation in 68, we have

$$0 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2 \quad \Rightarrow \quad \mathbf{I}_1 = -\frac{\mathbf{h}_{12}}{\mathbf{h}_{11}}\mathbf{V}_2$$

Substituting this expression for $\mathbf{I}_1$ into the second equation in 68, we obtain

$$\mathbf{I}_2 = \mathbf{h}_{21}\left(-\frac{\mathbf{h}_{12}}{\mathbf{h}_{11}}\mathbf{V}_2\right) + \mathbf{h}_{22}\mathbf{V}_2 = \left(\mathbf{h}_{22} - \frac{\mathbf{h}_{12}\mathbf{h}_{21}}{\mathbf{h}_{11}}\right)\mathbf{V}_2 = \frac{\Delta}{\mathbf{h}_{11}}\mathbf{V}_2$$

Thus, the Thevenin equivalent impedance at the output port is given by

$$\mathbf{z}_{th} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{\mathbf{h}_{11}}{\Delta}$$

In summary, the Thevenin equivalent circuit parameters at the output port of the two-port network are given by

– Thevenin Equivalent Voltage: $\mathbf{V}_{th} = -\dfrac{\mathbf{h}_{21}}{\Delta}\mathbf{V}_s$
– Thevenin Equivalent Impedance: $\mathbf{z}_{th} = \dfrac{\mathbf{h}_{11}}{\Delta}$