

EE1080/AI1110/EE2120 Probability, Quiz 6

April 24, 2025

Max. Marks: 23. **Time:** 1 hour.

Instructions

- Please **write your roll number, serial number** (used for attendance) and **course id** prominently in the first page of the answer sheet.
 - No laptops, mobile devices etc. allowed.
 - please write supporting arguments for any of the statements you make. any result proved in the lectures can be used by stating it clearly, no need to prove them in your answer.
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1. Let X, Z be identical independent normal random variables with mean 2 and variance 4. Let $Y = X + Z$. PDF of a normal random variable with mean μ and variance σ^2 is given by
$$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$$
 - (a) Find $f_{Y|X}(y|x)$, $f_Y(y)$, $f_{X|Y}(x|y)$. (3 marks)
 - (b) Find the MMSE (Minimum Mean Square Error) estimator of X upon observing $Y = y$ (3 marks)
 - (c) Find the MAP (Maximum Aposteriori Probability) and ML (Maximum Likelihood) estimator of X upon observing $Y = y$? (3 marks)
 - (d) Find $\text{Var}(Y)$, $E[Y]$ and $\text{Cov}(X, Y)$ (4 marks)
 - (e) Find the Linear MMSE estimator of X upon observing $Y = y$ (2 marks)
 - (f) Find the mean square error for Linear MMSE estimator (2 marks)
 - (g) Find the mean square error for MMSE estimator (3 marks)
 - (h) Find the mean square error of the MAP estimator (3 marks)

You can use the following if needed:

$$\begin{aligned}\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} dx &= 1 \\ \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} dx &= \mu \\ \int_{-\infty}^{\infty} x^2 \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} dx &= \mu^2 + \sigma^2\end{aligned}$$