CS1010: Discrete Mathematics for Computer Science

(Exam-1. Total: 30 marks.)
(Duration: 45 minutes. Date: 08 Sep 2025)

Instructions: ★ You may not get time to answer all the questions unless you have prepared really well. The exam is designed like that. ★ If your mobile phone is found with you during the exam, you will lose one grade. ★ Anybody found copying will get an F grade for the course straight away. ★ It is a no-break exam. You cannot take a break in between. The exam is only for 45 minutes. If you want to go out of the exam hall, you will have to submit your answer paper. ★ You should sit far apart from each other. The halls are big enough. If we see two students sitting close to each other, both the students will lose one grade.

Questions

1. Let p: 'I am awake', q: 'I work hard' and r: 'I dream of home'. Write each of the following in terms of p, q, r, and logical connectives.

   (a) Being awake is necessary for me not to dream of home.

   (b) Being awake implies that I work hard.

   (c) Working hard is sufficient for me to be awake. (1 x 3 = 3 marks)

2. Let $F(x, y)$ be 'x can fool y', where the domain consists of all people in the world. Use quantifiers and logical connectives to express the following statements:

   (a) There is someone whom everybody else can fool.

   (b) No one can fool himself or herself.

   (c) There is no one who can fool everybody else. (1 x 3 = 3 marks)

3. Use a diagonalization argument to prove that the set of all functions from $\mathbb{N}$ to $\mathbb{N}$ is uncountable. Here, $\mathbb{N}$ denotes the set of natural numbers. No marks will be given to proofs that are not based on diagonalization arguments. (5 marks)

4. Show that the set of real numbers that are solutions of quadratic equations of the form $ax^2 + bx + c = 0$, where a, b, and c are integers, is countable. One may use the following fact: Given a countable set S and a positive integer k, the Cartesian product $S^k := \underbrace{S \times S \times \cdots \times S}_{k \text{ times}}$ is countable. (6 marks)

5. A guest G at a party is a **celebrity** if G is known by every other guest, but G knows none of them. There is at most one celebrity at a party, for if there were two, they would know each other. A party may not necessarily have a celebrity. Further, note that in a party with only one guest, that guest is a celebrity.

   Consider a particular party. You are an outsider to this party. Your assignment is to find the celebrity, if one exists, at this party, by asking only one type of question - asking a guest whether they know a second guest. Everyone must answer your question truthfully. That is, if Alice and Bob are two people at the party, you can ask Alice whether she knows Bob; she must answer correctly. Use mathematical induction to show that if there are n people at the party, then you can find the celebrity, if there is one, with $3(n-1)$ questions. (6 marks)

6. Your task is to (logically) solve a murder mystery on behalf of Sherlock Holmes, which appeared in the novel "A Study in Scarlett", by Sir Arthur Conan Doyle. The arguments (simplified from the novel) go as follows:

   (i) There was a murder. If it was not done for robbery, then either it was a political assassination, or it might be for a woman.

   (ii) In case of robbery, usually something is taken.

   (iii) However, nothing was taken from the place of murder.

   (iv) Political assassins leave the place immediately after the assassination work gets completed.

   (v) On the contrary, the assassin left his/her tracks all over the place of murder.

   (vi) For an assassin, to leave tracks all over the place of murder indicates that (s)he was there all the time (for long duration).

   Please logically frame and derive the solution. Present your answer as asked in the following parts.

   (a) Write all propositions with English meaning (statements) that you have used.

   (b) Build suitable propositional logic formula to encode each of the six statements above.

   (c) Show all deduction steps (with the name of the rules you apply) to derive the goal (solve the mystery).

   (d) Conclude what the reason for the murder was. (1+2+3+1 = 7 marks)

——————— ALL THE BEST ———————