

Question No.	1	2	3	4	5	6	7	8	9	10	Total
MARKS	8	7	8	7							

1. Given Set A contains n integers

let, the elements in set A are

$S_1, S_2, \dots, S_k, \dots, S_n$ (elements of A)

let, $T_k = S_1 + S_2 + \dots + S_k$ (sum of elements (k elements))

and, let R_k is remainder when T_k is divided with n

$$\text{So, } T_k \equiv R_k \pmod{n}$$

The most possible values of R_k are $0, 1, 2, 3, \dots, n-1$
these are n values.

if, $R_k = 0$ then T_k (is subset elements sum)
elements are S_1, S_2, \dots, S_k .
subset is $\{S_1, S_2, \dots, S_k\}$

else if, $R_k \neq 0$

then R_k may $1, 2, 3, \dots, n-1$

let these R_k values be pigeonholes $(n-1)$ but we want
to get to choose n R_k values so for some $i > j$
 \downarrow
(Reminder)

$$R_i = R_j$$

(by pigeon hole principle)

$$T_i \equiv R_i \pmod{n}$$

$$T_j \equiv R_j \pmod{n}$$

$$T_i - T_j \equiv 0 \pmod{n} (\because R_i = R_j)$$

So, the difference of $T_i - T_j$ will remain with some (Some) elements. Therefore the subset of these elements divisible by n . It will remain with some elements because $i > j$. Hence Proved by using Pigeonhole Principle.

3. Given, $\frac{2^{2n}-1}{4^n-1}$ is divisible by 3 ✓

$$\text{we can write } 4^n - 1 = (3+1)^n - 1$$

from binomial theorem we know that

$$(a+b)^n = nC_0 a^n + nC_1 a^{n-1} b + \dots + nC_n b^n$$

$$(3+1)^n = nC_0 3^n + nC_1 3^{n-1} + \dots + nC_n (1)$$

$$= 3^n + nC_1 3^{n-1} + \dots + 1$$

$$4^n - 1 = 3^n + nC_1 3^{n-1} + \dots + nC_{n-1} 3 + 0 \quad \checkmark$$

we know nC_1 is choosing 1 element from n elements

nC_2 is choosing 2 elements from n elements

nC_{n-1} is choosing $n-1$ elements from n elements

These all are integers. So, in every term of $4^n - 1$ we have 3 in every term we can write $4^n - 1 = 3(k)$ let where k be some integer.

Hence Proved. ✓

2. Given,

$$({}^n C_k) ({}^k C_l) = ({}^n C_l) ({}^{n-l} C_{k-l})$$

If means we are choosing k elements from n elements and we are choosing l elements from the chosen k elements.

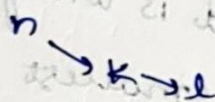
1st:- we can choose in two different ways

Let, we have a set of n elements

we want to choose k elements so it is ${}^n C_k$

Now we want to choose l elements from the k elements chosen

It is, $({}^n C_k) ({}^k C_l)$

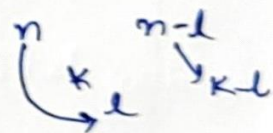


2nd:-

By another way

we can directly choose l elements from n elements which can be ${}^n C_l$ ways

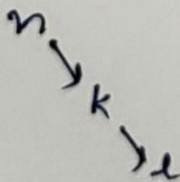
Now we will choose $k-l$ elements from $n-l$ elements ${}^{n-l} C_{k-l}$



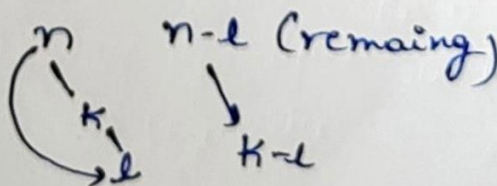
We are doing same with one also we are end up with l elements (with k elements chosen)

In 2nd we are choosing l elements from n and taking $k-l$ from $n-l$ where we are getting k elements also.

1st



2nd



4. Given $\gcd(a, b) = d$

so, $d = sa + tb$ (we use contradiction to prove)

by using ~~well ordering principle~~ • let us take some linear combination which is less than $sa + tb$ let

$$be \ s'a + t'b < d$$

↓ (+ve integer)

but $d/a, d/b$ then $d/s'a + t'b$

$$but \ s'a + t'b < d$$

↓ (+ve integer)

So our assumption that there will be a number

$s'a + t'b < d$ is wrong

(+ve integer) d is smallest such positive integer which can be written as linear combination of a, b .

Hence by, contradiction we have
Proved.