# EE1080/AI1110/EE2120 Probability, Quiz 3

17th Feb, 2025

**Max. Marks:** 16. **Time:** 1 hour.

## Instructions

- Please **write your roll number** and **course id** prominently in the first page of the answer sheet.
- No laptops, mobile devices etc. allowed.
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---

1. *Function of Uniform Distribution: (2)* A stick of length 1 is split at a point $U$ that is uniformly distributed over $(0, 1)$. Determine the expected length of the piece that contains the point $1/4$.

2. *Function of Uniform Distribution: (2)* A man aiming at a target receives 10 points if his shot is within 1 inch of the target, 5 points if it is between 1 and 3 inches of the target, and 3 points if it is between 3 and 5 inches of the target. Find the expected number of points scored if the distance from the shot to the target is uniformly distributed between 0 and 10.

3. *Exponential Random Variable: (1+2)* Suppose that the number of miles that a car can run before its battery wears out is exponentially distributed with an average value of 20,000km miles.

   (a) Find the CDF of exponentially distributed random variable $X$ with mean 20,000km.

   (b) If a person desires to take a 2000km trip, what is the probability that he or she will be able to complete the trip without having to replace the car battery? *(Can assume that the car has already travelled say $x$ kms. Hint: Use memory less property of exponential random variables)*

4. *Normal Random Variable: (1+2)* The annual rainfall (in inches) in a certain region is normally distributed with $\mu = 40$ and $\sigma^2 = 4$.

   (a) What is the probability that this years rainfall is over 50 inches?

   (b) What is the probability that starting with this year, it will take over 10 years before a year occurs having a rainfall of over 50 inches? What assumptions are you making?

5. (1+1+2)Let $X$ be a random variable with probability density function

$$f(x) = c(1 - x^2) \quad -1 < x < 1$$
$$0 \quad \text{otherwise}$$

   (a) What is the value of $c$?

   (b) What is the cumulative distribution function of $X$?

   (c) What is $E[X], Var(X)$.

6. *Joint PDF: (2)* Consider two continuous random variables $X$ and $Y$, the joint PDF is given by

$$f(x, y) = \begin{cases} cxy & 0 < x < 4, 1 < y < 5 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find the value of constant $c$.

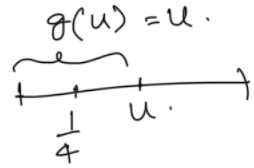
   (b) Find $P(X \geq 3, Y \geq 2)$.

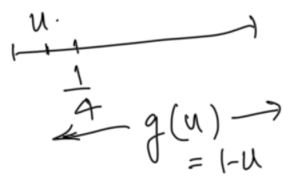
   (c) Find the marginal distributions of $X$ and $Y$.

   (d) Find $P(X + Y < 3)$

①

$$\underbrace{g(U)}_{\text{Length of piece that has } 1/4.} = \begin{cases} U & U \geq 1/4 \\ 1-U & U < 1/4 \end{cases}$$




$$E[g(U)] = \int_{1/4}^{1} u \, du + \int_{0}^{1/4} (1-u) \, du$$

$$= \frac{u^2}{2}\Big|_{1/4}^{1} + \left(u - \frac{u^2}{2}\right)\Big|_{0}^{1/4}$$

$$= \boxed{\frac{11}{16}}$$

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   (a) Find the CDF of exponentially distributed random variable $X$ with mean $20,000$km.
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5. (1+1+2)Let $X$ be a random variable with probability density function

$$f(x) = c(1 - x^2) \quad -1 < x < 1$$
$$0 \quad \text{otherwise}$$

   (a) What is the value of $c$?
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6. *Joint PDF: (2)* Consider two continuous random variables $X$ and $Y$, the joint PDF is given by

$$f(x, y) = \begin{cases} cxy & 0 < x < 4, 1 < y < 5 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find the value of constant $c$.
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(2) $g(u) = \begin{cases} 10 & u \leq 1. \\ 5 & 1 \leq u \leq 3. \\ 3 & 3 \leq u \leq 5 \end{cases}$

$U \sim \text{uniform}(0, 10)$

$E[g(U)] = \int_0^1 g(u) f_U(u)\, du.$

$= 10 \times \frac{1}{10} + 5 \times \frac{2}{10} + 3 \times \frac{2}{10}$

$= \boxed{2.6.}$

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3a

$$F_X(x) = \int_{\infty}^{x} f_X(t)\, dt$$

$$= \begin{cases} \int_0^x \lambda e^{-\lambda t}\, dt & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} 1 - e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Here $\lambda = \frac{1}{20 \times 10^3}$ as $E[X] = \frac{1}{\lambda}$.

3b

$$P(X > x + 2000 \mid X > x)$$

$$= P(X > 2000)$$

Memoryless property.

$$= 1 - F_X(2000)$$

$$= \boxed{1 - e^{-1/10}}$$

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4. *Normal Random Variable: (1+2)* The annual rainfall (in inches) in a certain region is normally distributed with $\mu = 40$ and $\sigma^2 = 4$.

   (a) What is the probability that this years rainfall is over ~~50~~ 35 inches?

   (b) What is the probability that starting with this year, it will take over 10 years before a year occurs having a rainfall of over ~~50~~ 35 inches? What assumptions are you making?

5. (1+1+2)Let $X$ be a random variable with probability density function

$$f(x) = \begin{matrix} c(1-x^2) & -1 < x < 1 \\ 0 & \text{otherwise} \end{matrix}$$

   (a) What is the value of $c$?

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4 $X \sim N(\mu, \sigma^2)$ $\mu = 40$ $\sigma^2 = 4$

a $P(X > 35) = P(X - 40 > -5)$

$= P\left(\frac{X-40}{2} > \frac{-5}{2}\right)$

$\frac{X-\mu}{\sigma}$ is a standard normal R.V

$= 1 - \Phi(-2.5)$

$= \Phi(2.5)$

$\Phi(\cdot)$ is cdf of standard normal RV

$= \boxed{0.99379}$

b Let $X_i$ be rainfall in year $i$

$E_i = P(X_i > 35)$.

$P(E_1^c \cap E_2^c \cap \cdots E_{10}^c) = \prod_{i=1}^{10} P(E_i^c)$

assumes independence of $X_1, \dots X_{10}$ indentical

$= (P(E_1^c))^{10}$

$= \boxed{(0.0062)^{10}}$

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5 (a) $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$

$$\int_{-1}^{1} c(1-x^2)\,dx = 2c\int_0^1 (1-x^2)\,dx$$

$$= 2c\left(x - \frac{x^3}{3}\right)\Big|_0^1$$

$$\Rightarrow 2c \times \frac{2}{3} = 1 \Rightarrow \boxed{c = 3/4}$$

(b) $F_X(x) = \int_{-\infty}^{x} f_X(t)\,dt$

$$= \boxed{\begin{cases} 0 & x < -1 \\ \frac{3}{4}\left(t - \frac{t^3}{3}\right)\Big|_{-1}^{x} = \frac{3}{4}\left(x - \frac{x^3}{3}\right) + \frac{1}{2} & -1 < x < 1 \\ 1 & x \geq 1 \end{cases}}$$

(c) $E[X] = \frac{3}{4}\int_{-1}^{1} \underbrace{(1-x^2)x}_{\text{odd fn.}}\,dx = \boxed{0}$

(d) $Var[X] = E[X^2] - (E[X])^2 = E[X^2]$

$$= \frac{3}{4}\int_{-1}^{1} x^2(1-x^2)\,dx = 2 \times 3/4\left(\frac{x^3}{3} - \frac{x^5}{5}\right)\Big|_0^1 = \frac{3}{2} \times \frac{2}{15} = \boxed{\frac{1}{5}}$$

6 a $f_{X,Y}(x, y) = c\,x\,y \qquad 0 \le x \le 4, \quad 1 \le y \le 5$

$$\int_1^5\int_0^4 f_{X,Y}(x,y)\,dx\,dy = \left[\int_0^4 c\,x\,dx\right]\left[\int_1^5 y\,dy\right] = 1.$$

$$= c\left(\frac{x^2}{2}\Big|_0^4\right)\left(\frac{y^2}{2}\Big|_1^5\right)$$

$$= c \times \frac{\cancel{16}^{\,4}}{\cancel{2}} \times \frac{24}{\cancel{2}} \Rightarrow \boxed{c = 1/96.}$$

b $$P(X \ge 3, Y \ge 2) = \frac{1}{96}\int_2^5\int_3^4 x\,y\;dx\,dy$$

$$= \frac{1}{96}\left(\frac{x^2}{2}\right)\Big|_3^4 \left(\frac{y^2}{2}\right)\Big|_2^5$$

$$= \frac{1}{\cancel{96}_{32}} \times \frac{7}{2} \times \frac{\cancel{21}^{\,7}}{2} = \boxed{\frac{49}{128}}$$

c $$f_X(x) = \int_y f_{X,Y}(x,y)\,dy = \frac{1}{96}\int_1^5 x\,y\,dy.$$

$$= \frac{1}{96}\,x\,\frac{y^2}{2}\Big|_1^5 = \boxed{\frac{x}{8}}$$

$$f_Y(y) = \frac{1}{96}\int_0^4 x\,y\,dx = \frac{x^2}{2}\Big|_0^4 \frac{y}{96}.$$

$$= \boxed{y/12}$$

$\left\{(x,y) \mid x+y<3,\ 0\le x\le 4,\ 1\le y\le 5\right\}$

$= \left\{(x,y) \mid 0\le x\le 2;\ 1\le y<3-x\right\}$.

(d) $P(X+Y<3)$

$= \frac{1}{96}\int_0^2\int_1^{3-x} x\,y\,dy\,dx$

$= \frac{1}{96}\int_0^2 x\left(\frac{y^2}{2}\right)\Big|_1^{3-x} dx$

$= \frac{1}{96}\int_0^2 x\left[\frac{(3-x)^2-1}{2}\right] dx$

$(9+x^2-6x)x$

$\boxed{\frac{4}{192}} = \frac{1}{192}\int_0^2 (x^3-6x^2+8x)\,dx = \frac{\frac{2^4}{4}-\frac{6\times 2^3}{3}+\frac{8\times 2^2}{2}}{192}$

| $z$ | + 0.00 | + 0.01 | + 0.02 | + 0.03 | + 0.04 | + 0.05 | + 0.06 | + 0.07 | + 0.08 | + 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| **0.0** | 0.50000 | 0.50399 | 0.50798 | 0.51197 | 0.51595 | 0.51994 | 0.52392 | 0.52790 | 0.53188 | 0.53586 |
| **0.1** | 0.53983 | 0.54380 | 0.54776 | 0.55172 | 0.55567 | 0.55962 | 0.56360 | 0.56749 | 0.57142 | 0.57535 |
| **0.2** | 0.57926 | 0.58317 | 0.58706 | 0.59095 | 0.59483 | 0.59871 | 0.60257 | 0.60642 | 0.61026 | 0.61409 |
| **0.3** | 0.61791 | 0.62172 | 0.62552 | 0.62930 | 0.63307 | 0.63683 | 0.64058 | 0.64431 | 0.64803 | 0.65173 |
| **0.4** | 0.65542 | 0.65910 | 0.66276 | 0.66640 | 0.67003 | 0.67364 | 0.67724 | 0.68082 | 0.68439 | 0.68793 |
| **0.5** | 0.69146 | 0.69497 | 0.69847 | 0.70194 | 0.70540 | 0.70884 | 0.71226 | 0.71566 | 0.71904 | 0.72240 |
| **0.6** | 0.72575 | 0.72907 | 0.73237 | 0.73565 | 0.73891 | 0.74215 | 0.74537 | 0.74857 | 0.75175 | 0.75490 |
| **0.7** | 0.75804 | 0.76115 | 0.76424 | 0.76730 | 0.77035 | 0.77337 | 0.77637 | 0.77935 | 0.78230 | 0.78524 |
| **0.8** | 0.78814 | 0.79103 | 0.79389 | 0.79673 | 0.79955 | 0.80234 | 0.80511 | 0.80785 | 0.81057 | 0.81327 |
| **0.9** | 0.81594 | 0.81859 | 0.82121 | 0.82381 | 0.82639 | 0.82894 | 0.83147 | 0.83398 | 0.83646 | 0.83891 |
| **1.0** | 0.84134 | 0.84375 | 0.84614 | 0.84849 | 0.85083 | 0.85314 | 0.85543 | 0.85769 | 0.85993 | 0.86214 |
| **1.1** | 0.86433 | 0.86650 | 0.86864 | 0.87076 | 0.87286 | 0.87493 | 0.87698 | 0.87900 | 0.88100 | 0.88298 |
| **1.2** | 0.88493 | 0.88686 | 0.88877 | 0.89065 | 0.89251 | 0.89435 | 0.89617 | 0.89796 | 0.89973 | 0.90147 |
| **1.3** | 0.90320 | 0.90490 | 0.90658 | 0.90824 | 0.90988 | 0.91149 | 0.91308 | 0.91466 | 0.91621 | 0.91774 |
| **1.4** | 0.91924 | 0.92073 | 0.92220 | 0.92364 | 0.92507 | 0.92647 | 0.92785 | 0.92922 | 0.93056 | 0.93189 |
| **1.5** | 0.93319 | 0.93448 | 0.93574 | 0.93699 | 0.93822 | 0.93943 | 0.94062 | 0.94179 | 0.94295 | 0.94408 |
| **1.6** | 0.94520 | 0.94630 | 0.94738 | 0.94845 | 0.94950 | 0.95053 | 0.95154 | 0.95254 | 0.95352 | 0.95449 |
| **1.7** | 0.95543 | 0.95637 | 0.95728 | 0.95818 | 0.95907 | 0.95994 | 0.96080 | 0.96164 | 0.96246 | 0.96327 |
| **1.8** | 0.96407 | 0.96485 | 0.96562 | 0.96638 | 0.96712 | 0.96784 | 0.96856 | 0.96926 | 0.96995 | 0.97062 |
| **1.9** | 0.97128 | 0.97193 | 0.97257 | 0.97320 | 0.97381 | 0.97441 | 0.97500 | 0.97558 | 0.97615 | 0.97670 |
| **2.0** | 0.97725 | 0.97778 | 0.97831 | 0.97882 | 0.97932 | 0.97982 | 0.98030 | 0.98077 | 0.98124 | 0.98169 |
| **2.1** | 0.98214 | 0.98257 | 0.98300 | 0.98341 | 0.98382 | 0.98422 | 0.98461 | 0.98500 | 0.98537 | 0.98574 |
| **2.2** | 0.98610 | 0.98645 | 0.98679 | 0.98713 | 0.98745 | 0.98778 | 0.98809 | 0.98840 | 0.98870 | 0.98899 |
| **2.3** | 0.98928 | 0.98956 | 0.98983 | 0.99010 | 0.99036 | 0.99061 | 0.99086 | 0.99111 | 0.99134 | 0.99158 |
| **2.4** | 0.99180 | 0.99202 | 0.99224 | 0.99245 | 0.99266 | 0.99286 | 0.99305 | 0.99324 | 0.99343 | 0.99361 |
| **2.5** | 0.99379 | 0.99396 | 0.99413 | 0.99430 | 0.99446 | 0.99461 | 0.99477 | 0.99492 | 0.99506 | 0.99520 |
| **2.6** | 0.99534 | 0.99547 | 0.99560 | 0.99573 | 0.99585 | 0.99598 | 0.99609 | 0.99621 | 0.99632 | 0.99643 |
| **2.7** | 0.99653 | 0.99664 | 0.99674 | 0.99683 | 0.99693 | 0.99702 | 0.99711 | 0.99720 | 0.99728 | 0.99736 |
| **2.8** | 0.99744 | 0.99752 | 0.99760 | 0.99767 | 0.99774 | 0.99781 | 0.99788 | 0.99795 | 0.99801 | 0.99807 |
| **2.9** | 0.99813 | 0.99819 | 0.99825 | 0.99831 | 0.99836 | 0.99841 | 0.99846 | 0.99851 | 0.99856 | 0.99861 |
| **3.0** | 0.99865 | 0.99869 | 0.99874 | 0.99878 | 0.99882 | 0.99886 | 0.99889 | 0.99893 | 0.99896 | 0.99900 |
| **3.1** | 0.99903 | 0.99906 | 0.99910 | 0.99913 | 0.99916 | 0.99918 | 0.99921 | 0.99924 | 0.99926 | 0.99929 |
| **3.2** | 0.99931 | 0.99934 | 0.99936 | 0.99938 | 0.99940 | 0.99942 | 0.99944 | 0.99946 | 0.99948 | 0.99950 |
| **3.3** | 0.99952 | 0.99953 | 0.99955 | 0.99957 | 0.99958 | 0.99960 | 0.99961 | 0.99962 | 0.99964 | 0.99965 |
| **3.4** | 0.99966 | 0.99968 | 0.99969 | 0.99970 | 0.99971 | 0.99972 | 0.99973 | 0.99974 | 0.99975 | 0.99976 |
| **3.5** | 0.99977 | 0.99978 | 0.99978 | 0.99979 | 0.99980 | 0.99981 | 0.99981 | 0.99982 | 0.99983 | 0.99983 |
| **3.6** | 0.99984 | 0.99985 | 0.99985 | 0.99986 | 0.99986 | 0.99987 | 0.99987 | 0.99988 | 0.99988 | 0.99989 |
| **3.7** | 0.99989 | 0.99990 | 0.99990 | 0.99990 | 0.99991 | 0.99991 | 0.99992 | 0.99992 | 0.99992 | 0.99992 |
| **3.8** | 0.99993 | 0.99993 | 0.99993 | 0.99994 | 0.99994 | 0.99994 | 0.99994 | 0.99995 | 0.99995 | 0.99995 |
| **3.9** | 0.99995 | 0.99995 | 0.99996 | 0.99996 | 0.99996 | 0.99996 | 0.99996 | 0.99996 | 0.99997 | 0.99997 |
| **4.0** | 0.99997 | 0.99997 | 0.99997 | 0.99997 | 0.99997 | 0.99997 | 0.99998 | 0.99998 | 0.99998 | 0.99998 |

Table 1: Shown below are the values of $\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$. First row implies $\phi(0) = 0.5$, $\phi(0.01) = 0.50399$, $\cdots$. Second row implies $\phi(0.1) = 0.53983$, $\phi(0.11) = 0.54380$, $\cdots$