EE2100: Matrix Theory

Assignment - 5

Handed out on : 08 - Sep - 2023 **Due on : 18 - Sep - 2023 (before 5 PM)**

Instructions :

1. Please submit the solutions to the assignment problems to the course page (on the canvas platform). Solutions submitted to the course page will only be evaluated. Refer to the assignment guidelines mentioned on the course page.
2. Submissions received after the deadline will attract negative marks.
3. It is suggested that you attempt all the questions (preferably the ones indicated using *). However, submitting solutions for problems totaling at least 10 points is sufficient.

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1. (5 Points) Prove that rotation as a linear transformation in $\mathcal{R}^2$ preserves distances between two vectors.
2. (5 Points) Design a step-by-step algorithm to find the matrix associated with a black box that performs a linear transformation from $\mathcal{R}^n \to \mathcal{R}^m$. One is allowed to give any input to it, and the black box will generate the corresponding output for that input.
3. *(10 Points) Let $\mathbb{B} = \{\mathbf{b}_1, \cdots, \mathbf{b}_n\} \in \mathcal{R}^n$ and $\mathbb{V} = \{\mathbf{v}_1, \cdots, \mathbf{v}_n\} \in \mathcal{R}^n$ denote two basis of the vector space $\mathcal{R}^n$. Let $\mathbf{y}$ and $\mathbf{z}$ denote the coordinate vectors of $\mathbf{x} \in \mathcal{R}^n$ in basis $\mathbb{B}$ and $\mathbb{V}$ respectively. Let T denote a transformation that transforms the coordinate vector of $\mathbf{x}$ in basis $\mathbb{B}$ to coordinate vector in basis $\mathbb{V}$.
   (a) (5 Points) Is T a linear transformation?
   (b) (5 Points) If so, can the transformation matrix be computed with T? (based on the concepts covered so far).
4. *(5 Points) Let $T : \mathbb{V} \to \mathbb{U}$, where $\mathbf{dim}(\mathbb{V}) = n$ and $\mathbf{dim}(\mathbb{U}) = m$ be a linear transformation. Let $\mathbb{S} = \{v_1, \cdots, v_n\} \subset \mathbb{V}$ and $\mathbb{W} = \{w_1, \cdots, w_m\} \subset \mathbb{U}$ be the bases for $\mathbb{V}$ and $\mathbb{U}$ respectively. Let $\mathbf{y}$ denote the coordinate vector of $\mathbf{x} \in \mathbb{V}$ in the basis $\mathbb{S}$. Prove that there exists a unique $m \times n$ matrix $\mathbf{A}$ such that for every $\mathbf{x} \in \mathbb{V}$, $(T(\mathbf{y}))_{basis\ W} = \mathbf{A}(\mathbf{y})$
5. *(5 Points) Using the idea of Linear transformation (and its inverse), compute the inverse of $\mathbf{A} \in \mathcal{R}^{n\times n}$ given by

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

6. (10 Points) Let T and T_1 denote a transformation that corresponds to rotating a vector by angle θ in counter-clockwise direction (about the $x-$axis) and reflection about a given vector $\mathbf{v} \in \mathcal{R}^2$. Consider a transformation T_2 defined by (2)

$$T_2(\mathbf{x}) := T\left(T_1(\mathbf{x})\right) \tag{2}$$

   (a) (5 Points) Show that T_2 is a linear transformation.

   (b) (5 Points) Derive the transformation matrix corresponding to T_2.