QUIZ 1: EE2102

READ THESE INSTRUCTIONS CAREFULLY BEFORE ATTEMPTING TO ANSWER:

1. This paper has 3 questions with a max. total score of 50 marks. NO QUESTION IS OPTIONAL.
2. Your answers are expected to be LOGICAL, CONCISE and LEGIBLE. **You are to supply a proof for any result that was not covered in the class.** CALCULATORS ARE NOT NECESSARY to answer any of the questions.
3. **The use of any internet-connectable devices is strictly prohibited; their use will be deemed plagiarism. Institute Academic Plagiarism and Malpractice Policy (Section 3.5.3 Academic Handbook) that allows for a fail grade to be assigned by the instructor will be strictly enforced for any practice deemed plagiarism.**
4. **Your are required to return the question paper along with your answer sheets.** OTHERWISE YOUR ANSWERS WILL NOT BE GRADED.

Problem 1 (15 marks)

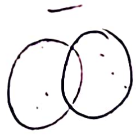
Suppose A_1 and A_2 are events defined on a probability space. Answer the following questions using the probability axioms discussed in the class:

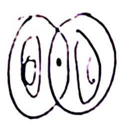
Hint: You may find the following result useful:

$$\mathbb{P}(A_1 \cup A_2) = \mathbb{P}(A_1) + \mathbb{P}(A_2) - \mathbb{P}(A_1 \cap A_2).$$

1. **(7 marks).** If A_1 and A_2 are mutually independent, then prove that $\bar{A}_1$ and A_2 are mutually independent.
2. **(8 marks).** If A_1 and A_2 are *mutually exclusive* with $\mathbb{P}(A_1) + \mathbb{P}(A_2) < 1$, prove that $\bar{A}_1$ and $\bar{A}_2$ are *not* mutually exclusive.

Note: Verifying the statement holds for a special case does NOT constitute a valid proof; such answers will be not be awarded any marks.





Problem 2 (20 marks)

A spectator uses the clock app on their phone to count the number of laps their favorite horse has run in a race. The plan is to tap the "lap" button on the phone screen, so that the phone records the current lap (and the lap time) and starts a timing the new lap.

The trouble is, in their excitement of watching the race, they don't check whether or not their tap landed on the button or not, leading to a potential *undercounting* of the total number of laps. In other words, the number of laps recorded on the app will be no larger than the actual number of laps completed. Suppose

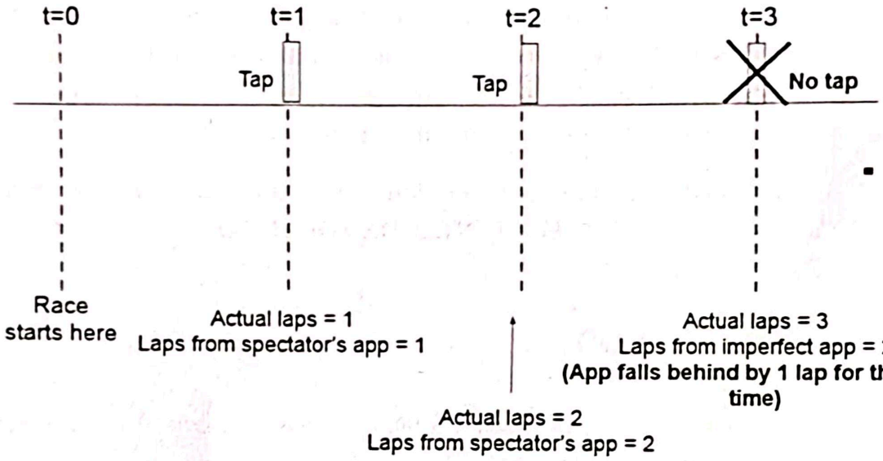


Illustration of a skipped lap. At the end of the third actual lap $t = 3$, the app falls 1 lap because only 2 laps were recorded.

that in each lap, the spectator does *not* press the lap button correctly with probability $p \in [0, 1)$, and that their pressing/not pressing events are independent across laps.

Based on the above, answer the following:

1. Let Y denote the number of laps fallen behind by the phone app after $n = 100$ actual laps.
   (a) **(6 marks)** Identify the sample space, event set, and the probability measure for this random experiment.
   (b) **(3 marks)** Evaluate $\mathbb{P}(Y < 1)$ when $p = 10^{-1}$.
2. Starting from the first lap, denote by X_i the minimum number of actual laps that have occured when the spectator's app falls behind by exactly i laps ($i = 1, 2, ...$).
   (a) **(4 marks)** Find the pmf of X_1 and its region of support.
   (b) **(7 marks)** Find the pmf of X_m and its region of support, $m \in \mathbb{N}$.

Problem 3 (15 marks)

Two players A and B each have their own (possibly unfair) coins. The game involves the players taking turns to toss their respective coins and noting whether it's a head or a tail. The coin tosses can be considered independent trials. The player who gets the *same* outcome as the previous player's toss wins, and the game is stopped. For example, if A gets H, and if B gets H, then B is the winner. If instead, A gets H, B gets T and A gets T in the next toss, A wins.

Assuming A tosses his/her coin first, answer the following questions:

> **Hint:** You may find it useful to recall how we analyzed the "toss-until-head" sample space in the class.

1. **(7 marks)** Derive the sample space Ω, the set $\mathcal{F}$ of events and the appropriate probability measure $\mathbb{P}$ for this random experiment. Identify which of these spaces are countable and uncountable, giving reasons for your answers.

2. **(4 marks)** Let the rv X denote the total number of die rolls from both players at which the game terminates. Find its pmf p_X and identify the possible values X can take (i.e., its region of support).

3. **(4 marks)** If both the coins are fair, does A or B have a greater chance of winning the game? Give reasons for your answer based on an analysis of the pmf of X.

H

H H

H T T

H T H T T