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EE1101: Circuits and Network Analysis

Lecture 23: First-Order Circuits

September 19, 2025

Topics :

1. Solving First-order Differential Equations
2. Examples of First-order Circuits

① for the ckts covered in this course both KCL & KVL are valid.

Solving First-order Differential Equations - Integrating Factor Method

given $\frac{dx}{dt} + P(t)\,x(t) = \phi(t)$

goal: compute $x(t) \longrightarrow$ parametric form (constants)

$\downarrow$

determined by using either

current cont. (inductors)

voltage cont. (capacitors)

when $P(t) \equiv 0$ $(=0\ \forall t)$: $\frac{dx}{dt} = \phi(t)$

$$\Rightarrow \quad x(t) = \int \phi(t)\,dt$$

when $P(t) \not\equiv 0$: $\frac{dx}{dt} + P(t)\,x(t) = \phi(t) \longrightarrow ①$

Integrating factor : mul ① on both sides with $u(t)$ s.t. it can be reduced into a form where integrating on both sides can be employed.

$① \times u(t)$: $u(t)\frac{dx}{dt} + u(t)\,P(t)\,x(t) = u(t)\,\phi(t) \longrightarrow ②$

LHS of ② can be taken to the form $\frac{d}{dt}\left(u(t)\,x(t)\right)$ if

$$\frac{du}{dt} = u(t)\,P(t) \quad \Rightarrow \quad u(t): \quad \frac{du}{u} = P(t)\,dt$$

$$\Rightarrow \quad \ln(u) = \int P(t)\,dt \quad \Rightarrow u = e^{\int P(t)\,dt}$$

Solving First-order Differential Equations - Integrating Factor Method (Contd.)

orig DE when mul $u(t) \Rightarrow \frac{d}{dt}\left(u(t)\,x(t)\right) = u(t)\,q(t)$

Integrate on both sides

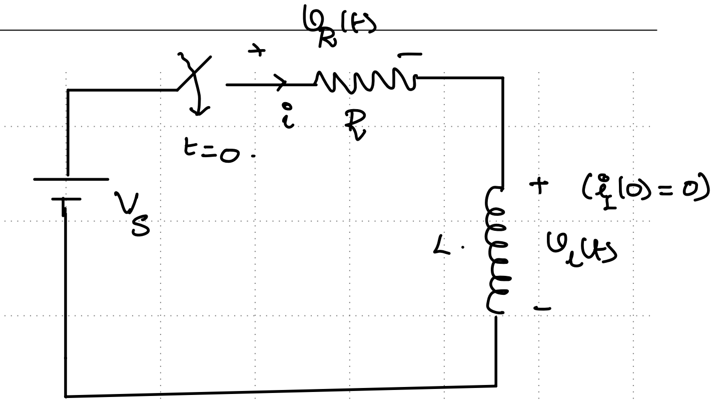
$$u(t)\,x(t) = \int u(t)\,q(t)\,dt + C$$

$$\Rightarrow x(t) = \frac{1}{u(t)}\left[\int u(t)\,q(t)\,dt + C\right]$$

Steps involved in solving $\frac{dx}{dt} + P(t)\,x(t) = q(t)$:-

1. Determine the integrating factor : $e^{\int P(t)\,dt}$
2. Multiply on both sides of DE with IF & simplify $\Rightarrow \frac{d}{dt}\left(u(t)\,x(t)\right) = u(t)\,q(t)$
3. Integrate on both sides to get $x(t)$
4. Determine the constant based on the appropriate principle.

Examples of First-order Circuits



Set 1: for $t>0$: $-V_s + v_R(t) + v_\ell(t) = 0$

$$\Rightarrow \quad L\frac{di}{dt} + Ri = V_s$$

for $t<0$: $i_L(t)=0$.

Alt : $L\frac{di}{dt} + Ri = V_s u(t)$ and $i(0)=0$.

In std form : $\frac{di}{dt} + \underbrace{\frac{R}{L}}_{P(t)} i(t) = \underbrace{\frac{V_s}{L}}_{Q(t)} u(t)$.

① Integrating factor $= e^{\int P(t)dt} = e^{\int R/L\, dt} = e^{R/L\, t}$

② mul on b.s. by IF $\Rightarrow \frac{d}{dt}\left(e^{R/L\, t} i(t)\right) = e^{R/L\, t}\frac{V_s}{L}u(t)$

$$\Rightarrow \quad e^{R/L\, t} i(t) = \frac{V_s}{L}\int e^{R/L\, t} u(t)\,dt$$

$$= \frac{V_s}{\cancel{L}} e^{R/L\, t} \cdot \frac{\cancel{L}}{R} + c.$$

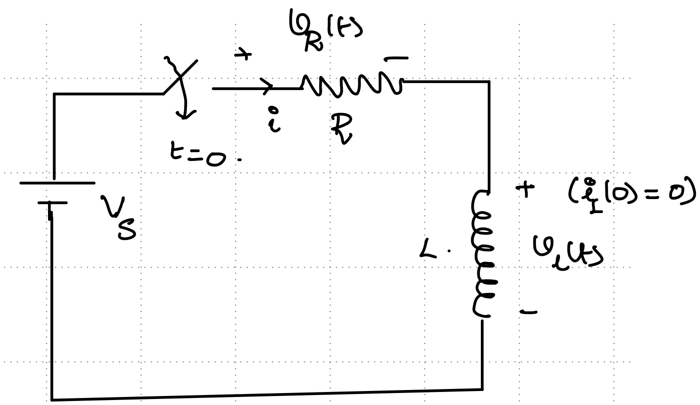
$$e^{R/L\, t} i(t) = \frac{V_s}{R} e^{R/L\, t} + c$$

$$\Rightarrow \quad i(t) = \frac{V_s}{R} + \underline{c}\, e^{-R/L\, t}.$$

c: $i(0)=0 \Rightarrow c = -V_s/R$ & $i(t) = \frac{V_s}{R}\left(1-e^{-R/L\, t}\right)$ $(t\geq 0)$

and $i(t)=0 \ \forall t<0$.

Examples of First-order Circuits (Contd.)



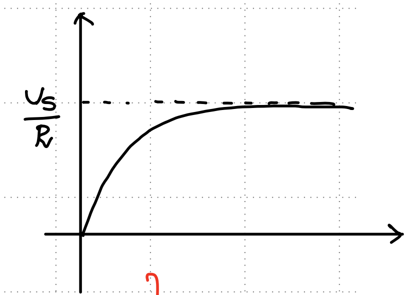
Response: $i(t) = \frac{V_s}{R}\left(1-e^{-R/L\, t}\right)\ (t \geq 0)$

and $i(t) = 0 \ \forall t < 0$.

① Check if resp of circuit is finite as $t \longrightarrow \infty$.

if true $\longrightarrow$ Steady-state exists.

when $t \longrightarrow \infty$: $i(t) = \frac{V_s}{R}$ (steady state current)



resp of the ckt till it reaches steady state ← Transient Part

↓ To characterize

- a) rise time
- b) settling time
- c) peak overshoot.

more on this in next lecture

time-constant ← In addition

↓

$\left(\tau = L/R\right)$