EE2100: Matrix Theory

Assignment - 7

Handed out on 22 - Sep - 2023 **Due on 03 - Oct - 2023 (before 5 PM)**

Instructions :

1. Please submit the solutions to the assignment problems to the course page (on the canvas platform). Solutions submitted to the course page will only be evaluated. Refer to the assignment guidelines mentioned on the course page.
2. Submissions received after the deadline will attract negative marks.
3. It is suggested that you attempt all the questions (preferably the ones indicated using *). However, submitting solutions for problems totaling at least 10 points is sufficient.

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1. *(10 Points) Using a programming language of your choice, factorize a given square matrix as a product of the lower and upper triangular matrices. For this assignment, you can assume that the matrix is of full rank.

   It is suggested that the program be developed in such a way that the core algorithm is implemented as an independent function that can be used in other codes (if necessary). The function must take the following inputs: (a) a matrix (as a two-dimensional array). You can use a randomly chosen $\mathbf{A}$ to test the algorithm.

   Note: The developed code must not use any built-in libraries available in the programming language (except for defining the random matrix).

2. *(5 Points) Consider a scenario where it is necessary to solve m systems of linear equations of the form $\mathbf{Ax} = \mathbf{b}_i$, where $\mathbf{b}_i$ varies in each set. Compare the computational complexity involved in solving these m sets of linear equations using (a) Gaussian Elimination and (b) $\mathbf{LU}$ decomposition

3. (15 Points) Consider the system of linear equations given by

$$\underbrace{\begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 \end{bmatrix}}_{\mathbf{A}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{1}$$

   (a) (8 Points) Factorize $\mathbf{A}$ as a product of a lower and upper triangular matrix.

   (b) (4 Points) Compute the solution to the system of linear equations (1)

   (c) (3 Points) Let $\mathbf{z} = [1, 2, 3, 4]^T$. Compute $\mathbf{x}$ such that $\mathbf{Ax} = \mathbf{z}$.

4. (10 points) Introduction to **Cholesky Decomposition**: Consider a matrix $\mathbf{M}$ given by 2. Factorize the matrix into a product of lower triangular matrix and an upper triangular matrix (i.e., $\mathbf{M} = \mathbf{LU}$).

$$\mathbf{M} = \begin{bmatrix} 6 & 15 & 55 \\ 15 & 55 & 225 \\ 55 & 225 & 979 \end{bmatrix} \tag{2}$$

Note: You will notice that, for the given matrix, $\mathbf{U} = \mathbf{L}^T$. Later in this course, we will show that every positive definite matrix with real entries can be decomposed as a product of L is a lower triangular matrix and its transpose. To be more precise, any positive definite matrix (with real or complex entries) can be expressed as $\mathbf{LL}^H$ (**Cholesky Decomposition**), where $\mathbf{L}^H$ denotes the Hermitian of a given matrix i.e., $(\mathbf{L}_{ij}^H = \mathbf{L}_{ji}^*)$).