

# ID2230 2025, Exam 1

September 25, 2025

**Maximum Marks: 55. Time: 2 hours. Closed Books, No electronic devices including smart watches. All the notations and conventions are as discussed in the class.**

1. Recall the insertion algorithm in a binary search tree of  $n$  nodes. Is it correct to say that its worst-case running time is  $\Theta(n)$ ? Justify your answer (2 marks)
2. Write the recurrence relation for the least number of nodes in a modified-AVL tree of height  $h$  as discussed in the problem set 3. Recall that in a modified-AVL tree, the difference in the heights can be 2. (3 marks)
3. Convert the following infix expression to postfix using a stack:  $(3 + 5) * 10^6 + 7 * 5 + 4$ . Show the intermediate states of the stack. (5 marks)
4. What is the maximum number of keys in a B-Tree of height  $h$  and minimum degree  $t$ ? Explain your answer. (2+3 marks)
5. Design an algorithm to find the second largest element in a binary search tree. What is the worst-case running time of your algorithm? (5+3 marks)
6. Given a positive rational  $x$ , an algorithm to calculate  $e^x$  using the Taylor series, up to a given precision  $\epsilon$ —the  $n$ th term should have a value at most  $\epsilon$ . Prove its correctness using the invariants method discussed in the class. (4+6 marks)
7. Consider a static hash table where the keys are stored permanently—the table is used only for *find* operations. A hash function is said to be *perfect* if *finds* in such a situation only take  $O(1)$  time.
  - (a) If a hash function  $h$  is chosen u.a.r from a universal family of hash functions (as discussed in the class), then show that, when  $m \geq n^2$ , the probability of  $h$  being perfect is at least  $1/2$ . You may use the Markov inequality:
$$Pr(X \geq a) \leq \frac{E[X]}{a},$$
for nonnegative random variable  $X$  and constant  $a > 0$ . (10 marks)
  - (b) As you may have observed, this will need  $n^2$  space (a hash table with  $n^2$  slots). Can you do this in  $O(n)$  space? Justify your answer. [Hint: you may resolve collisions through another hash table of appropriate size.] (6+6 marks)