

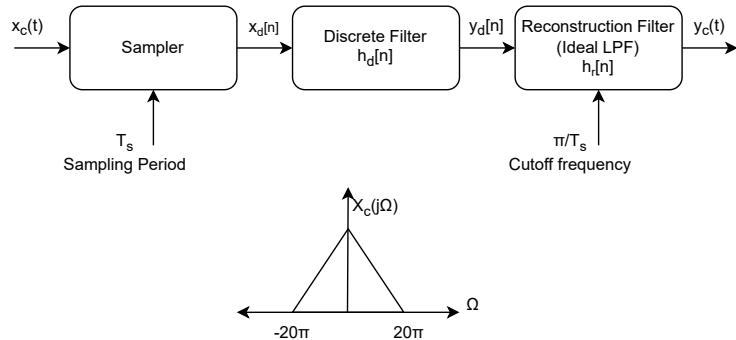


## Homework-4

### Note

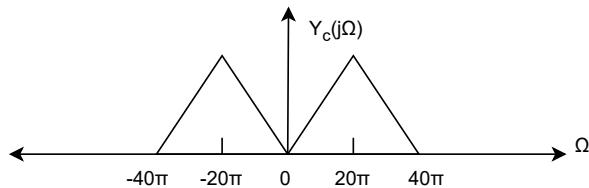
- \* Plagiarism is strictly prohibited
- \* Deadline will not be extended under any circumstances

1. Consider a system corresponding to discrete time(DT) processing of continuous time(CT) signal as shown below. Let us consider the sampling period  $T_s = \frac{1}{20}s$  and the discrete



filter  $h_d[n]$  is an ideal LPF with cutoff frequency  $\frac{\pi}{4}$ . Then, sketch  $X_d(j\omega)$ ,  $Y_d(j\omega)$ , and  $Y_c(j\Omega)$  for the input  $X_c(j\Omega)$  shown in the figure.

2. Let us consider the input signal  $x_c(t)$  is multiplied by a cosinusoid  $\cos(30\pi t)$  in the above problem. Also consider that the discrete system in between is an ideal distortionless channel i.e.,  $H_d(j\omega) = 1 \quad \forall \omega$ .
  - (a) Determine and sketch the output  $Y_c(j\Omega)$  (with same  $T_s$  as in the above problem)
  - (b) Is it possible to reconstruct the signal  $x_a(t)$  with any choice of  $T_s$ ? If yes, give one choice of  $T_s$  to reconstruct the signal  $x_a(t)$
  - (c) Is it possible to have the output as



if yes, give a possible choice of  $T_s$  to have the above output.

3. Let us consider the block diagram from Problem.1, and in this case, the discrete filter's input-output relation is given by

$$y_d[n] = \frac{1}{2}y_d[n-1] + x_d[n]$$

then, determine  $H_d(j\omega)$  and the effective continuous time frequency response  $H_c(j\Omega)$

4. In some of the scenarios, an echo is recorded along with the signal. It is defined as the superimposition of delayed and attenuated replica of the signal  $s(t)$ . In the presence of an echo, the observed signal is represented as

$$x(t) = s(t) + \alpha s(t - T_0) \quad |\alpha| < 1$$

Now, we wish to process the signal  $x(t)$  in discrete domain to obtain  $s(t)$ . The signal is sampled with a sampling period  $T$ . Determine the frequency response of the digital filter such that the reconstruction with ideal LPF with cut-off frequency  $\frac{\pi}{T}$  is equal to  $s(t)$ ? (Assume that the signal  $s(t)$  is bandlimited with maximum frequency  $\frac{\pi}{T}$ )

5. If  $X[K]$  is the DFT of the sequence  $x[n]$ , determine the N-point DFTs of the sequences

$$x_1[n] = x[n] \cos\left(\frac{2\pi k_0 n}{N}\right) \quad 0 \leq n \leq N-1$$

$$x_2[n] = x[n] \sin\left(\frac{2\pi k_0 n}{N}\right) \quad 0 \leq n \leq N-1$$

in terms of  $X[K]$

6. Let  $x[n]$  be a periodic signal with fundamental period  $N$ . Now consider  $X_1[K]$  is N-point DFT and  $X_2[K]$  is  $3N$ -point DFT.

- (a) What is the relationship between  $X_1[K]$  and  $X_2[K]$ ?
- (b) Verify the above result using the sequence

$$x[n] = [\dots, 1, 2, 1, \underline{2}, 1, 2, 1, 2, \dots]$$

7. (a) Compute an 8-point DFT of the sequence  $x[n] = [0, a, b, c, 0, -a, -b, -c]$ . What do you observe from the  $X[K]$ ?
- (b) In general, comment on the N-point DFT of the sequences with the following symmetry

$$x\left[n + \frac{N}{2}\right] = -x[n] \quad n = 0, \dots, \frac{N}{2} - 1$$