Homework 2

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1. **Question 1:** For the unity feedback system shown in Figure 1, where

$$G(s) = \frac{500}{s(s+28)(s^2+8s+12)}$$

Find the steady-state error for the following inputs.
a) $20u(t)$
b) $60tu(t)$
c) $81t^2u(t)$

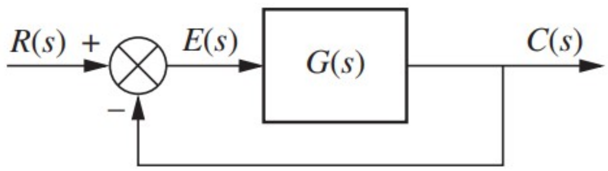

Figure 1: Control System

2. **Question 2:** For the system shown in Figure 2, What steady-state error can be expected for the following test inputs?

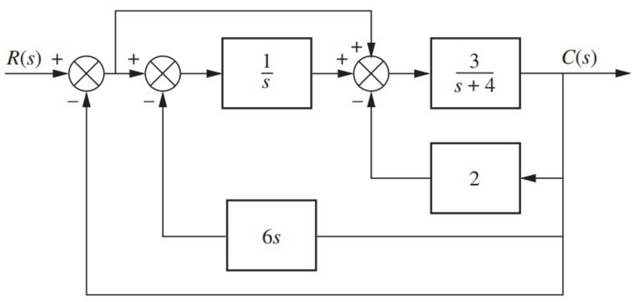

Figure 2: Control System

a) $10u(t)$
b) $10tu(t)$
c) $10t^2u(t)$

Answers:
$a)$ 0
$b)$ 93.3

$c) \infty$

3. **Question 3:** A unity feedback system has the following forward transfer function:

$$G(s) = \frac{K(s+12)}{(s+14)(s+18)}$$

Find the value of K to yield a 10% error in the steady state.

Answer: $K = 189$

4. **Question 4:** For the unity feedback system shown in Fig 3, where

$$G(s) = \frac{K(s^2-2s+2)}{(s+2)(s+4)(s+5)(s+6)}$$

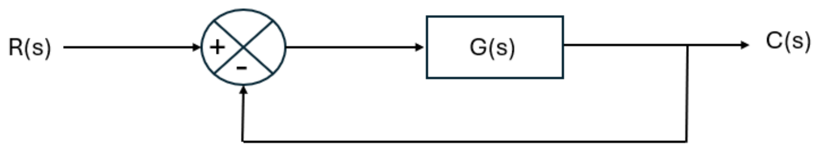

Figure 3: Control system

do the following:

(a) Sketch the root locus.
(b) Find the asymptotes.
(c) Find the range of gain K that makes the system stable.
(d) Find the breakaway points.
(e) Find the value of K that yields a closed-loop step response with 25% overshoot.

5. **Question 5:** For the unity feedback system shown in Figure 3, where

$$G(s) = \frac{K(s+10)(s+20)}{(s+30)(s^2-10s+100)}$$

do the following:

(a) Sketch the root locus.
(b) Find the range of gain K that makes the system stable.
(c) Find the value of K that yields a damping ratio of 0.707 for the system's closed-loop dominant poles.
(d) Find the value of K that yields closed-loop critically damped dominant poles.

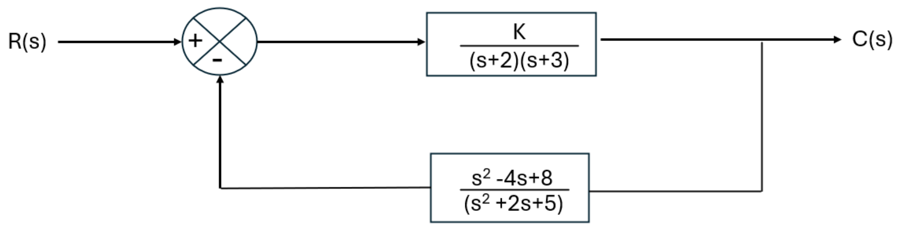

Figure 4: Control system

6. **Question 6:** For the system shown in Figure 4, do the following:
   (a) Sketch the root locus.
   (b) Find the $j\omega$-axis crossing and the gain K at the crossing.
   (c) Find the real-axis breakaway to two-decimal-place accuracy.
   (d) Find angles of arrival to the complex zeros.
   (e) Find the closed-loop zeros.
   (f) Find the gain K for a closed-loop step response with 30% overshoot.

7. **Question 7:** For the unity feedback system shown in Figure 3, where

$$G(s) = \frac{K(s+2)(s+3)}{(s^2+2s+2)(s+4)(s+5)(s+6)}$$

   do the following:
   (a) Sketch the root locus.
   (b) Find the $j\omega$-axis crossing and the gain K at the crossing.
   (c) Find all breakaway and break-in points.
   (d) Find angles of departure from the complex poles.
   (e) Find the gain K to yield a damping ratio of 0.3 for the closed-loop dominant poles.

8. **Question 8:** Design a PI controller to drive the step response error to zero for the unity feedback system shown in Figure 5, where

$$G(s) = \frac{K}{(s+1)^2(s+10)}$$

   The system operates with a damping ratio of 0.6. Compare the specifications of the uncompensated and compensated systems.

9. **Question 9:** Consider the unity feedback system shown in Figure 5, where

$$G(s) = \frac{K}{s(s+3)(s+6)}$$

   Design a PI controller to drive the ramp response error to zero for any k that yields stability

10. **Question 10:** The unity feedback system shown in Figure 5 with

$$G(s) = \frac{K}{(s+2)(s+3)(s+7)}$$

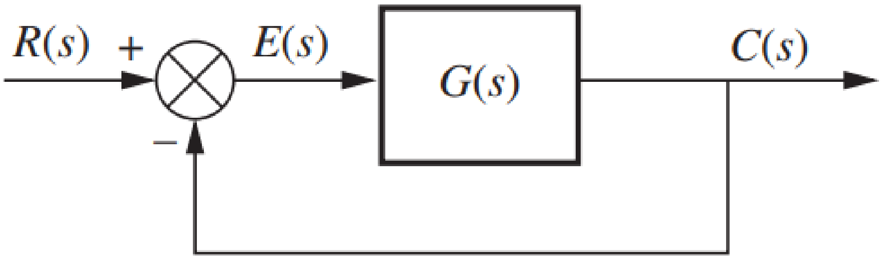

Figure 5: Control system

is operating with 10% overshoot.

a. What is the value of the appropriate static error constant?

b. Find the transfer function of a lag network so that the appropriate static error constant equals 4 without appreciably changing the dominant poles of the uncompensated system.

11. **Question 11:** Repeat Problem 10 for

$$G(s) = \frac{K}{s(s+3)(s+7)}$$