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EE1101: Circuits and Network Analysis

Lecture 39: Series and Parallel Resonance

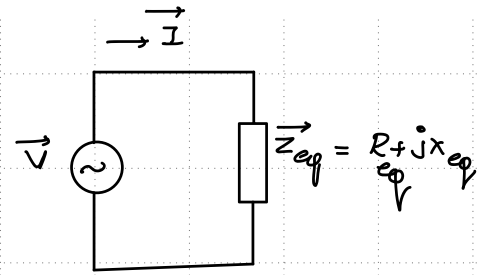
November 7, 2025

Topics :

1. Series and Parallel Resonance
2. Quality Factor and Bandwidth

Series Resonance

Question: Condition on freq for which $|\vec{I}|$ is maximum?



Sol: $\omega : X_{eq} = 0$.

Example 1

(Series RL Ckt)

for $\omega = 0$, $X_{eq} = 0$

↓

True for any ckt that comprises of R & L (along with sources)

(Series RC Ckt)

for $\omega = \infty$, $X_{eq} = 0$

↓

True for any ckt that comprises of R & C (along with sources)

Example 2

Series RLC ckt

for $\omega = \frac{1}{\sqrt{LC}}$, $X_{eq} = 0$. $[\omega_0]$

Prop 1: at resonance $\vec{V}$ and $\vec{I}$ are in phase

Prop 2: Net power drawn from the source is active power $P = |\vec{I}|^2 R$

$$P = \frac{|\vec{V}_s|^2}{R}$$

Prop 3: $e_L(t) = \frac{1}{2} L i^2$ & $e_C(t) = \frac{1}{2} C v_c^2$

$e_L(t) + e_C(t) = \frac{1}{2} L I_m^2$ (Constant)

Prop 4: $e_d \propto t$; E_d (per cycle) $= |\vec{I}|^2 R T = \frac{1}{2} I_m^2 R T$

Prop 5: energy from source $= e_d(t)$. [in steady state]

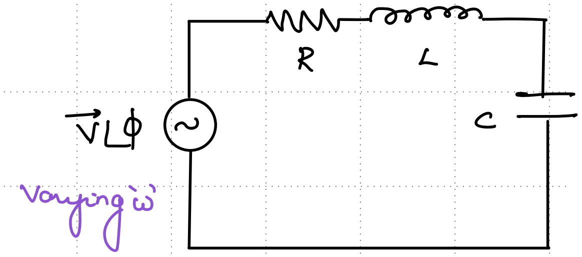
Series Resonance

def: for a ckt that comprises of L & C (in addition to other linear ckt elem)

the cond at which $X_{eq} = 0$ (→ if it exists) is called Resonance.

and the freq at which Resonance occurs → Resonant freq (ω_0)

Resonant freq for Series RLC ckt $\omega_0 = \frac{1}{\sqrt{LC}}$.



At frequencies other than ω_0 :-

(1) $\vec{Z}_{eq} = R + j\left(\omega L - \frac{1}{\omega C}\right)$

$|\vec{Z}_{eq}| =$

$\angle \vec{Z}_{eq} =$

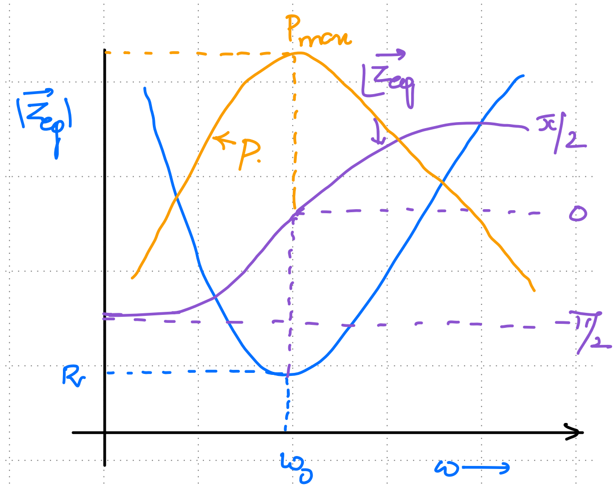
at low freq → capacitive in nature

at high freq → inductive in nature

(2) P, Q from source

$P = |\vec{I}|^2 R_{eq}$

$Q = |\vec{I}|^2 X_{eq}$.



Quality Factor and Bandwidth

$$Q = (2\pi)\ \underline{\text{energy stored in ckt at resonance}} \leftarrow$$

$E_d(t) \longrightarrow e_d(t)$ per cycle.

$$= 2\pi\ \frac{e_s(t)}{E_d(t)} = 2\pi\ \frac{\frac{1}{2} L I_m^2}{|\vec{I}|^2 R T}$$

$$= \underbrace{\frac{2\pi}{T}}\ \frac{L}{R}$$

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\sqrt{LC}}\ \frac{L}{R} = \underbrace{\sqrt{\frac{L}{C}}}\cdot\frac{1}{R}$$

$\hookrightarrow$ characteristic imp.

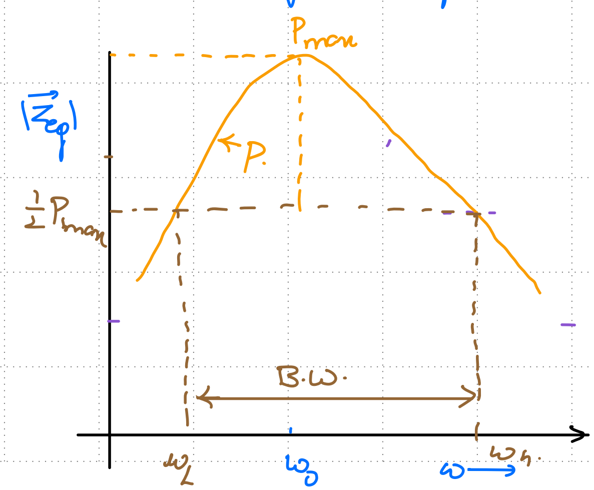
$Q = \dfrac{1}{\omega_0 R C}$ when energy stored is computed in terms of $v(t)$ across the cap.

Bandwidth: range of freq for which $P \geq \frac{1}{2} P_{max}$.

$$P = |\vec{I}|^2 R = \frac{|\vec{V}|^2}{|\vec{Z}_{eq}|^2} R = \frac{V^2}{R^2 + (\omega L - 1/\omega C)^2} R$$

$$\frac{1}{2} P_{max} = \frac{1}{2}\ \frac{|\vec{V}|^2}{R}$$

ω_L & ω_h are freq for which $\omega L - 1/\omega C = \pm R$



Quality Factor and Bandwidth

Compute ω : $\omega L - \frac{1}{\omega C} = \pm R$: if ω_a is sol to $\omega L - \frac{1}{\omega C} = \pm R$

then $-\omega_a$ is also a sol to $\omega L - \frac{1}{\omega C} = \pm R$

$$\frac{\omega L}{R} - \frac{1}{\omega RC} = -1$$

$$\frac{\omega Q}{\omega_0} - \frac{Q\omega_0}{\omega} = -1 \qquad Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$$

$$\Rightarrow \quad Q\omega^2 + \omega_0\omega - Q\omega_0^2 = 0.$$

$$\omega = \frac{-\omega_0 \pm \sqrt{\omega_0^2 + 4Q^2\omega_0^2}}{2Q} = \frac{\omega_0}{2Q}\left(-1 \pm \sqrt{1+4Q^2}\right)$$

$\sqrt{1+4Q^2} > 1$: roots $= \frac{\omega_0}{2Q}\left(-1 + \sqrt{1+4Q^2}\right) > 0 \longrightarrow \omega_L|\omega_h$ $\leftarrow \omega_L$

$$\frac{\omega_0}{2Q}\left(-1 - \sqrt{1+4Q^2}\right) < 0$$

other Positive root $\left(1 + \sqrt{1+4Q^2}\right)\frac{\omega_0}{2Q} \longrightarrow \omega_h$

$$B.\omega = \omega_h - \omega_L = \frac{\omega_0}{Q}.$$