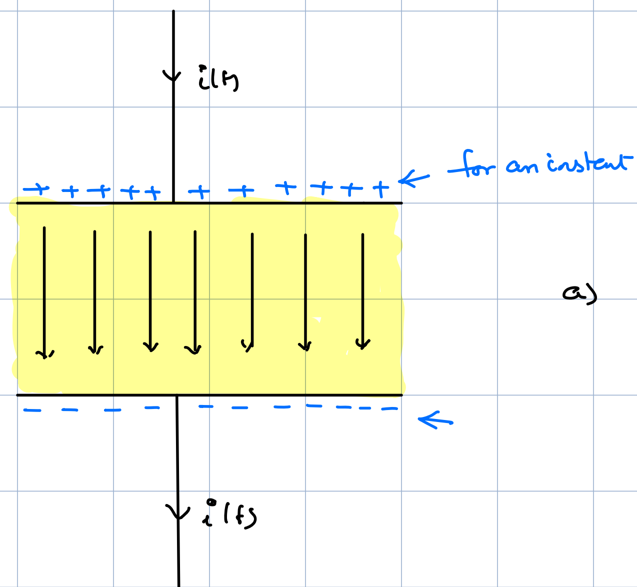
Capacitors (from a ckt point of view)

from a ckt point of view : 2 terminal elem

$$i(t) = \frac{dQ}{dt} = C\frac{dv}{dt} \quad \text{where} \quad Q = CV.$$

Capacitance [F] $\longrightarrow$ determined by geometry and Prop of dielectric medium.



a) when Current changes w.r.t time

$\Downarrow$

$Q(t)$ changes w.r.t time.

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$\vec{E}$ & $\vec{D}$ within a Cap changes w.r.t. time

$\Downarrow$

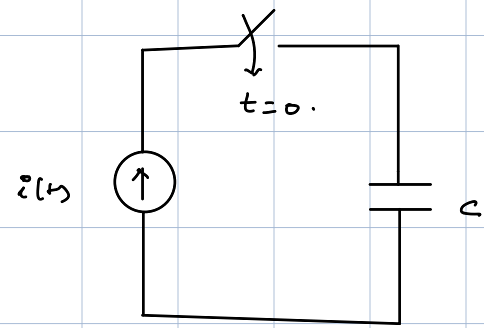
$$i_d \propto \frac{d\vec{D}}{dt} = i(t)$$

i_d: displacement Current

let $i_c(t)$ be Known $\Rightarrow$ $\frac{dv_c}{dt} = \frac{1}{C} i_c(t) \Rightarrow dv_c = \frac{1}{C} i_c(t)\,dt$

$$\int_{v_c(t_0)}^{v_c(t)} dv_c = \frac{1}{C}\int_{t_0}^{t} i_c(t)\,dt$$

$$\underbrace{v_c(t) = v_c(t_0) + \frac{1}{C}\int_{t_0}^{t} i_c(t)\,dt}$$



Example 1:- $i(t) = I$ (DC signal) ; $\underbrace{v_c(0) = 0}$.

$i_c(t) = I\,u(t)$

$$v_c(t) = \underbrace{v_c(0)}_{0} + \frac{1}{C}\int_{0}^{t} I\,u(t)\,dt$$

$v_c(t) = \frac{I}{C}t.$ for $t > 0$.

Example: $i(t) = I_m \cos(\omega t + \phi_i)$; $\phi_i = 0$ (evaluate it using indefinite integral)

$i_c(t) = i(t)\,u(t) = I_m \cos(\omega t + \phi_i)\,u(t)$

$$v_c(t) = \frac{1}{C}\int I_m \cos(\omega t)\,u(t)\,dt = \frac{I_m}{\omega C}\sin\omega t + \underline{\underline{C}}$$

C : some Cond $\rightarrow$??

Voltage across a capacitor cannot change instantly.

$$i_c = C \frac{dv}{dt}$$

if $v_c(t)$ changes instantly $\longrightarrow$ impulse current

(or)

to make $v_c(t)$ change instantly $\longrightarrow$ impulse current

$\Downarrow$

Not possible in a real world scenario.

$\Downarrow$

Voltage across a cap cannot change instantly ($\phi(t)$ as well)

Example: $i(t) = I_m \cos(\omega t + \phi_i)$; $\phi_i = 0$ (evaluate it using indefinite integral)

$$i_c(t) = i(t)\,u(t) = I_m \cos(\omega t + \phi_i)\,u(t)$$

$$v_c(t) = \frac{1}{C}\int I_m \cos(\omega t)\,u(t)\,dt = \frac{I_m}{\omega C}\sin\omega t + \underline{\underline{C}}$$

C : Voltage continuity $\Rightarrow v_c(0) = 0 \Rightarrow C = 0$

$$v_c(t) = \frac{I_m}{\omega C}\sin\omega t$$

Sinusoidal steady-state behavior of a capacitor:-

$$i_c(t) = I_m \cos(\omega t + \phi_i) \Rightarrow \vec{I_c} = \frac{I_m}{\sqrt{2}} \angle \phi_i$$

in sinusoidal steady state: $v_c(t) = \frac{1}{C}\int i_c(t)dt = \frac{I_m}{\omega C} \sin(\omega t + \phi_i) \Rightarrow \vec{V_c} = \frac{I_m}{\sqrt{2}\omega C} \angle \phi_i - \pi/2$

$$= \underline{\underline{V \angle \phi_v}}$$

$$V = \frac{I_m}{\sqrt{2}\omega C} = \frac{I}{\omega C}$$

$$\phi_v = \phi_i - \pi/2$$

In steady state, the current phasor leads the voltage phasor by $\pi/2$

$$\theta = \phi_v - \phi_i = -\pi/2.$$

Impedance of a cap $\vec{Z} = \frac{\vec{V}}{\vec{I}} = \underbrace{\frac{I}{\omega C} \angle \phi_v}_{\vec{V}} \frac{1}{I \angle \phi_i} = \frac{I \cancel{\angle \phi_i} \cdot e^{-j\pi/2}}{\omega C} \cdot \frac{1}{\cancel{I \angle \phi_i}}$

$$\vec{Z} = \frac{1}{j\omega C}.$$

mag of imp z of a cap $\propto 1/\omega \Rightarrow$ impedance is large at low freq

low at high freq.

Phase of imp of a cap : $\angle \vec{Z} = -\pi/2$

Power associated with a Capacitor: $\vec{I} = I\angle\phi_i$
(Sinusoidal steady-state)

$$\vec{V} = V\angle\phi_V = \frac{I}{\omega C}\angle\phi_i - \pi/2$$

Complex power $\vec{S} = \vec{V}\vec{I}^* = \frac{I}{\omega C}\angle\phi_i - \pi/2 \cdot \underbrace{I\angle -\phi_i}_{\vec{I}^*}$

$$= -j\frac{I^2}{\omega C} = P + jQ$$

Active power $= 0$

Reactive power $(Q) < 0 :=$ $|Q| = I^2/\omega C$; $Q = -I^2/\omega C$.

Power factor $= \cos\theta = 0$ (lead)

Power factor angle $\theta = -\pi/2$

Instantaneous Power $s(t) = Q \sin 2\omega t$ $(\because P = 0)$ $\Rightarrow$ $-I^2/\omega C \; \sin 2\omega t$.

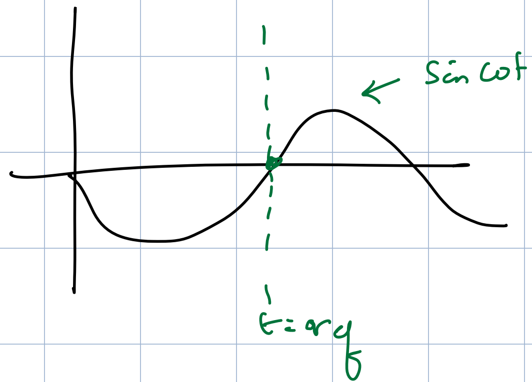
energy associated with a Cap:

$$E(t) = \int_0^t s(t)\,dt = \int_0^t \frac{I^2}{\omega C} \sin 2\omega t \quad \leftarrow \text{Shifting the time ref.}$$

$$= \frac{I^2}{\omega C} \left. \frac{\cos 2\omega t}{2\omega} \right|_0^t = \frac{I^2}{2\omega^2 C}(1 - \cos 2\omega t) = \frac{I^2}{2\omega^2 C}(2\sin^2\omega t)$$

$$= \frac{I^2}{\omega^2 C}\sin^2\omega t$$

$$= CV^2 \sin^2\omega t$$



$E(t) = -CV^2 \sin^2 \omega t \Rightarrow \frac{1}{2} C \underbrace{V_m^2 \sin^2 \omega t}_{v(t)}$

$E(t) = \frac{1}{2} C v^2$