

Question No.	1	2	3	4	5	6	7	8	9	10	Total
MARKS	8	7	8	7							

1. Given set A contains n integers

let, the elements in set A are

$s_1, s_2, \dots, s_k, \dots, s_n$ (elements of A)

Let, $T_k = s_1 + s_2 + \dots + s_k$ (sum of elements (k elements))

and, let R_k is remainder when T_k is divided with n

so, $T_k \equiv R_k \pmod{n}$

The most possible values of R_k are $0, 1, 2, 3, \dots, n-1$

these are n values.

if, $R_k=0$ then T_k (is subset elements sum)
elements are s_1, s_2, \dots, s_k .
subset is $\{s_1, s_2, \dots, s_k\}$

else if, $R_k \neq 0$
then R_k may be $1, 2, 3, \dots, n-1$

let these R_k values be pigeonholes $(n-1)$ but we want

to get to choose n R_k values so for some $i > j$
 \uparrow
(Reminder)

$$R_i = R_j$$

\uparrow
(by Pigeon
hole principle)

$$T_i \equiv R_i \pmod{n}$$

$$T_j \equiv R_j \pmod{n}$$

$$T_i - T_j \equiv 0 \pmod{n} (\because R_i = R_j)$$

So, the difference of $T_i - T_j$ will remain with some elements. Therefore the subset of these elements divisible by n . It will remain with some elements because $i > j$. Hence Proved by using Pigeonhole Principle.

- 3 Given, $2^{2n}-1$ is divisible by 3

$4^n - 1$ ✓

$$\text{we can write } 4^n - 1 = (3+1)^n - 1$$

from binomial theorem we know that

$$(a+b)^n = n_0 a^n + n_1 a^{n-1} b + \dots + n_n b^n$$

$$(3+1)^n = n_0 3^n + n_1 3^{n-1} + \dots + n_n (1)$$

$$= 3^n + n_1 3^{n-1} + \dots + 1$$

$$4^n - 1 = 3^n + n_1 3^{n-1} + \dots + n_{n-1} 3 + 0$$

we know n_1 is choosing 1 element from n elements

n_2 is choosing 2 elements from n elements

n_{n-1} is choosing $n-1$ elements from n elements

These all are integers. So, in every term of $4^n - 1$ we have 3 in every term we can write $4^n - 1 = 3(k)$ let where k be some integer.

Hence Proved. ✓

2.

Given,

$$(n \choose k) (k \choose l) = (n \choose l) (n-l \choose k-l)$$

If means we are choosing k elements from n elements and we are choosing l elements from the chosen k elements.

1st:- we can choose in two different ways

let, we have a set of n elements

we want to choose k elements so it is $n \choose k$

Now we want to choose l elements from the k elements

it is, $(n \choose k) (k \choose l)$

2nd:- By another way

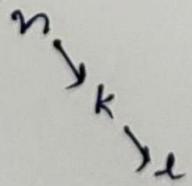
we can directly choose l elements from n elements which can be, $n \choose l$ ways

Now we will choose $k-l$ elements from $n-l$ elements

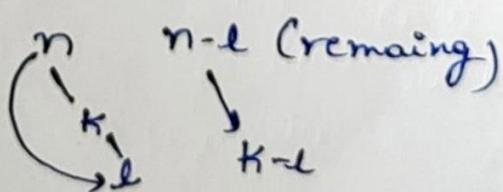
We are doing same with one also we are end up with l elements (with k elements chosen)

In 2nd we are choosing l elements from n and taking $k-l$ from $n-l$ where we are getting k elements also.

1st



2nd



4. Given $\gcd(a, b) = d$

so, $d = Sa + tb$ (we use contradiction to prove)
 by using ~~well ordering principle~~ let us take some
 linear combination which is less than $Sa + tb$ let it
 be $s'a + t'b < d$
 \downarrow (two integers)

but $d/a, d/b$ then $d/s'a + t'b$

but $s'a + t'b < d$
 \downarrow (two integers)

So our assumption that there will be a number

$s'a + t'b < d$ is wrong

(two integers) d is smallest such positive integer which can
 be written as linear combination of a, b .

Hence by, contradiction we have

Proved.