Electrical Engineering Department
IIT Hyderabad
EE2000 - Signal Processing
Homework-4

Note

* Plagiarism is strictly prohibited
* Deadline will not be extended under any circumstances.

1. Show that the roots of a polynomial with real coefficients are real or form complex conjugate pairs.
2. Show that for a real and even sequence, if there is a pole or zero at $z = \rho e^{j\omega_0}$, then there is a pole or zero at $z = \frac{1}{\rho}e^{j\omega_0}$ and at $z = \frac{1}{\rho}e^{-j\omega_0}$
3. Compute the z-transform for the following signals. Sketch the pole-zero plot and indicate the region of convergence (ROC). State whether the Fourier transform exists or not for each of the signals.
   (a) $[9, 27, 81, 243, ...]$
   (b) $\frac{1}{(n-p)!}$
   (c) $\sum_{k=-\infty}^{\infty} \delta[n-kM]$ (M is a finite positive integer)
   (d) $(-1)^n 2^{-n} u[n]$
   (e) $n\left(\frac{1}{2}\right)^n (u[n+3] - u[n-6])$
   (f) $(n^2+n)\left(\frac{1}{3}\right)^{n-1} u[n-1]$
   (g) $4^n cos[\frac{2\pi}{6}n + \frac{\pi}{4}]u[n]$
4. Determine the inverse z-transform of the following
   (a) $X(z) = \frac{1-2z^{-1}}{1+2z^{-1}}$, $|z| < 2$
   (b) $X(z) = \frac{1+z^{-2}}{\left(1+\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)^2}$, $x[n]$ is causal
   (c) $X(z) = e^{z^{-1}}$
   (d) $X(z) = cos(z)$
   (e) $X(z) = log(1-z^{-1})$, $|z| > 1$
5. A causal sequence $g[n]$ has the z-transform

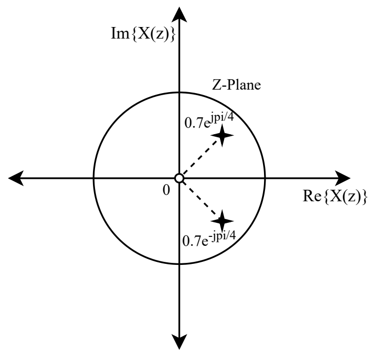
$$G(z) = cos(z^{-1})(2z^{-1} + 3z^{-3})$$

   Find the values of $g[n]$ for $0 \leq n \leq 10$
6. Let us suppose $X(z)$ is z-transform of the sequence $x[n]$, then express the z-transform of the following in terms of $X(z)$
   (a) $x_1[n] = \sum_{k=-\infty}^{n} x[k]$
   (b) $x_2[n] = \begin{cases} x\left[\frac{n}{2}\right] & \text{if } \frac{n}{2} \text{ is integer} \\ 0 & \text{otherwise} \end{cases}$
   (c) $x_3[n] = x[2n]$

(d) $x_4[n] = x[n]e^{j\omega_0 n}$

(e) $x_5[n] = \begin{cases} \frac{1}{n}x[n] & n > 0 \\ 0 & \text{otherwise} \end{cases}$

7. Let us suppose that the poles and zeros of $X(z)$ are given as shown in the figure below.



Sketch the pole-zero plots for all the sequences in problem-6.

8. Determine the output for all n, for the systems defined by following difference equations and the corresponding initial conditions

(a) $y[n] = \frac{1}{2}y[n-1] + x[n] - \frac{1}{2}x[n-1]$
input: $x[n] = u[n]$
Initial Conditions: $y[-1] = 1$

(b) $y[n] = \frac{1}{2}y[n-2] + x[n]$
input: $x[n] = \left(\frac{1}{3}\right)^n u[n]$
Initial Conditions: Rest (i.e., $y[-1] = 0,\ y[-2] = 0$)

(c) $y[n] - \frac{3}{2}y[n-1] + \frac{1}{2}y[n-2] = 0$
Initial Conditions: $y[-1] = 1$