Book and Course Evaluation

- Book:

  Control System Engineering

  Author: Norman Nise

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  Control System Engineering

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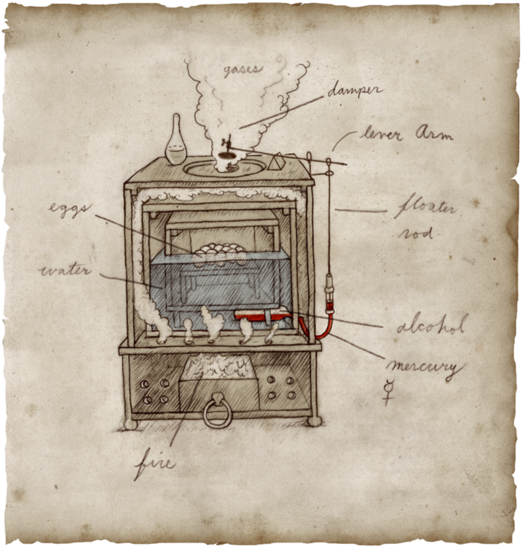
- Evaluation:

3 quizzes	$3 \times 15 = 45\%$
Matlab assignment	5%
End sem	50%

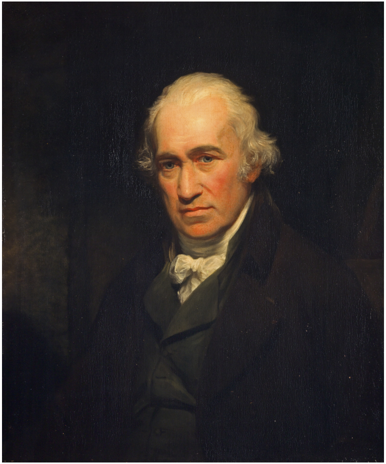
Cornelis Drebbel (1572-1633)

Drebbel's Constant Temprature Oven



Christiaan Huygens (1629-1695)

James Watt (1736-1819)



Centrifugal Governor in Steam Engine

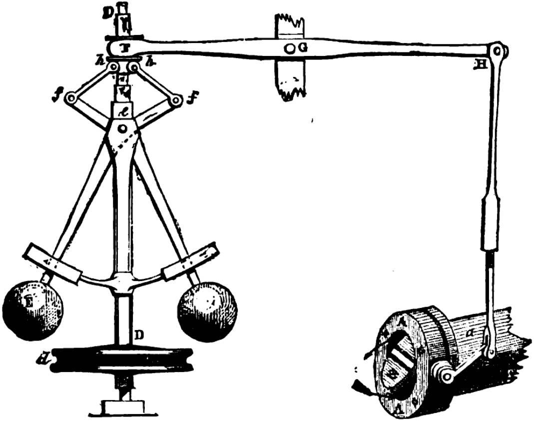


FIG. 4.—*Governor and Throttle-Valve.*

James Clerk Maxwell (1831-1879)

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- First paper in Control Theory

  On governors, *Proceedings of the Royal Society of London*, 1868.

Negative Feedback Amplifier

- Year of invention: 1927 Inventor: Harold Black (1898-1983)

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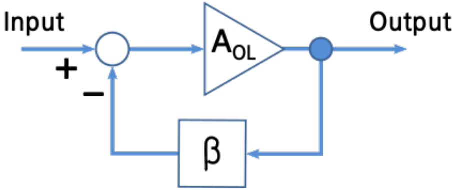
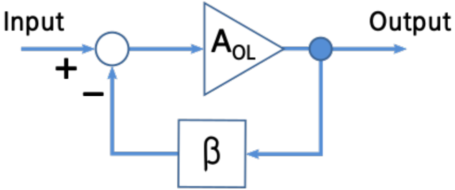


Negative Feedback Amplifier

- Year of invention: 1927 Inventor: Harold Black (1898-1983)



- Feedback gain:

$$A_{FL} = \frac{V_{out}}{V_{in}} = \frac{A_{OL}}{1 + \beta A_{OL}}$$

Negative Feedback Amplifier

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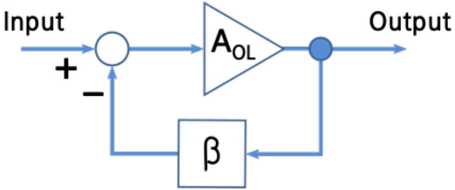

- Feedback gain:

$$A_{FL} = \frac{V_{out}}{V_{in}} = \frac{A_{OL}}{1 + \beta A_{OL}}$$

- If $A_{OL} >> 1$, then $A_{FL} \approx \dfrac{1}{\beta}$

Harry Nyquist (1889-1976)

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- Nyquist stability criteria
- Year: 1932

Hendrik Bode (1905-1982)

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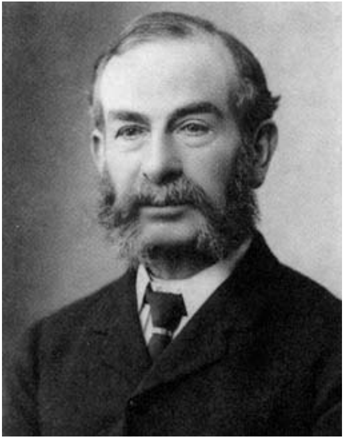
- Bode plot
- Year: 1930s

Routh-Hurwitz Stability Criterion

Routh-Hurwitz Stability Criterion



Edward Routh

1876

Adolf Hurwitz

1895

Aleksandr Lyapunov (1857-1918)

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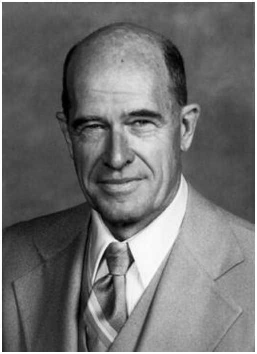
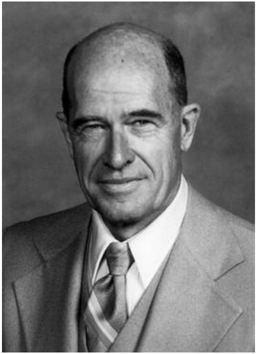
- Lyapunov stability
- Year: 1892

Walter Evans (1920-1999)



Walter Evans (1920-1999)



- Root locus
- Year: 1948

Rudolf Kalman (1930-2016)

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- Kalman filtering

Rudolf Kalman (1930-2016)

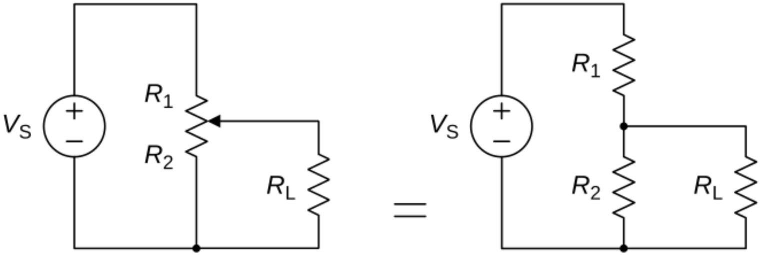
- Kalman filtering
- Controllability, Observability, Kalman decomposition
- Optimal Control

The images are sourced from the Wikipedia website and the book Control Systems Engineering by Norman Nise. The author extends gratitude to these sources.

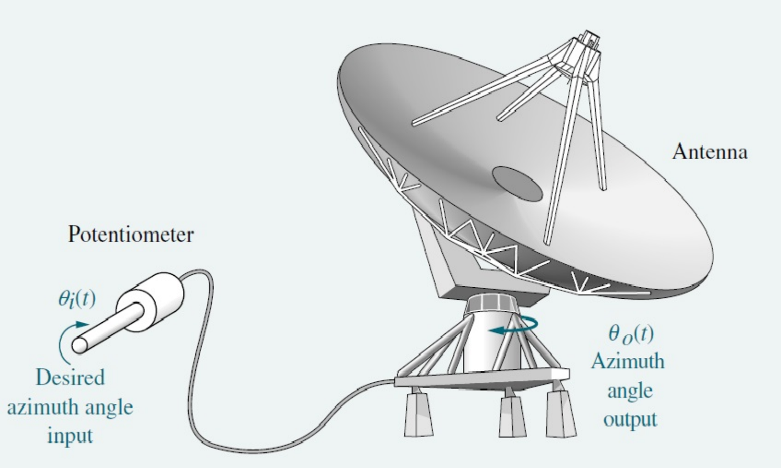
ISRO Antenna used in Chandrayan Mission

Potentiometer

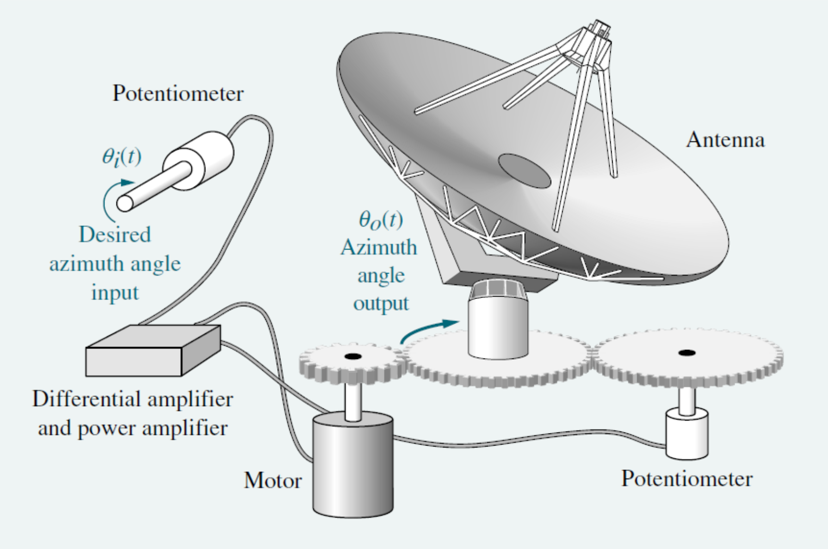
Potentiometer



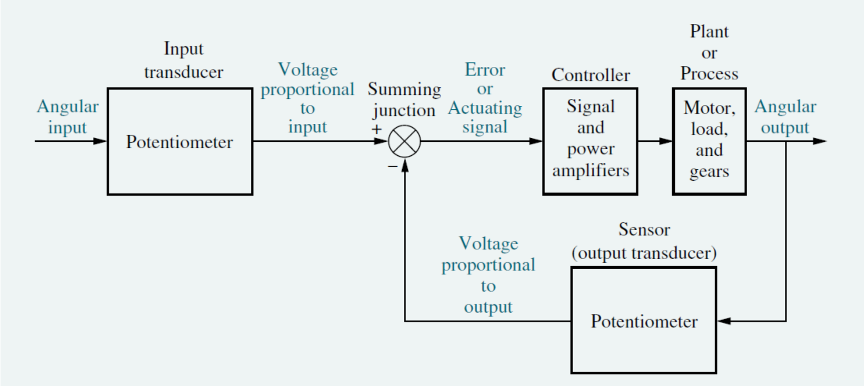
Potentiometer with Antenna



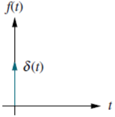
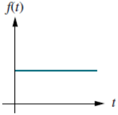
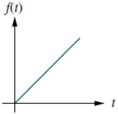
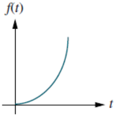
Antenna Position Control System



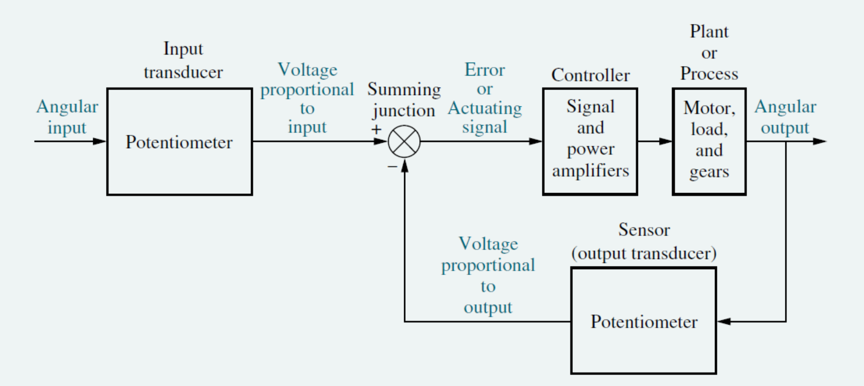
Block Diagram



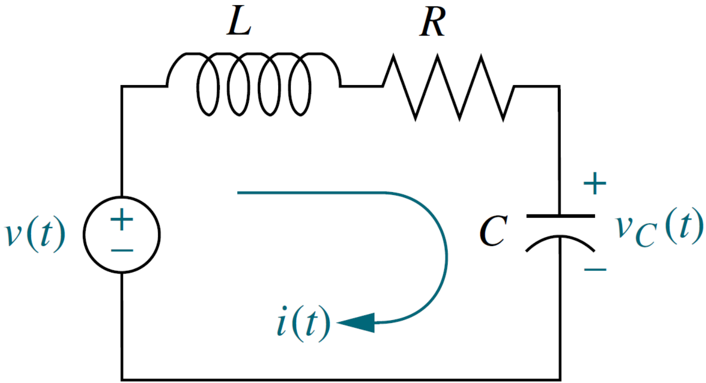
Test Signals

Input	Function	Description	Sketch
Impulse	$\delta(t)$	$\delta(t) = \infty$ for $0- < t < 0+$ $= 0$ elsewhere $\int_{0-}^{0+} \delta(t)dt = 1$	
Step	$u(t)$	$u(t) = 1$ for $t > 0$ $= 0$ for $t < 0$	
Ramp	$tu(t)$	$tu(t) = t$ for $t \geq 0$ $= 0$ elsewhere	
Parabola	$\frac{1}{2}t^2u(t)$	$\frac{1}{2}t^2u(t) = \frac{1}{2}t^2$ for $t \geq 0$ $= 0$ elsewhere	

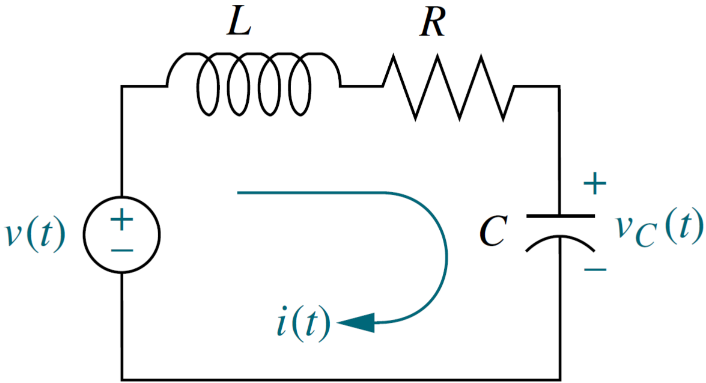
Block Diagram



RLC

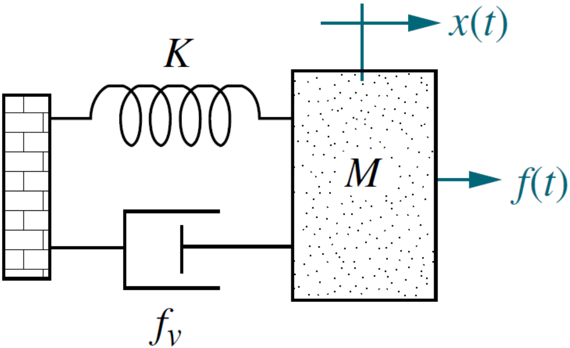


RLC

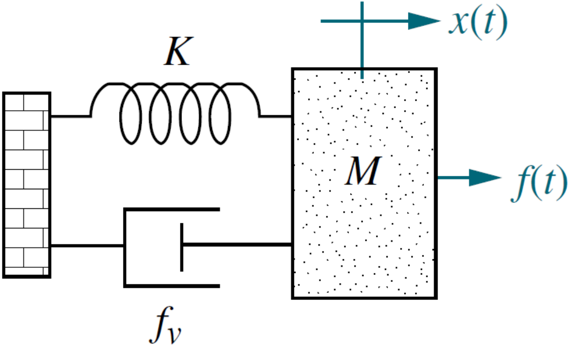


$$LC\frac{d^2v_c(t)}{dt^2} + RC\frac{dv_c(t)}{dt} + v_c(t) = v(t)$$

Mass Spring Damper System



Mass Spring Damper System



$$M\frac{d^2x(t)}{dt^2} + f_v\frac{dx(t)}{dt} + Kx(t) = f(t)$$

Linear Constant Coefficient Ordinary Differential Equation

$$a_n \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \ldots + a_1 \frac{dy(t)}{dt} + a_0 y(t)$$

$$=$$

$$b_n \frac{d^n r(t)}{dt^n} + b_{n-1} \frac{d^{n-1} r(t)}{dt^{n-1}} + \ldots + b_1 \frac{dr(t)}{dt} + b_0 r(t)$$

Laplace Transform

$$F(s) := \int_{0^-}^{\infty} f(t)\, e^{-st} dt$$

$s = \sigma + jw$: complex variable

$$f(t) = ae^{-bt}$$

$$f(t) = ae^{-bt}$$

$$F(s) = \frac{a}{s+b}$$

Item no.	***f*(*t*)**	***F*(*s*)**
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^n + 1}$
5.	$e^{-at}u(t)$	$\frac{1}{s + a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2 + \omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2 + \omega^2}$

Properties of Laplace Transform

Item no.	Theorem	Name
1.	$\mathscr{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}dt$	Definition
2.	$\mathscr{L}[kf(t)] = kF(s)$	Linearity theorem
3.	$\mathscr{L}[f_1(t) + f_2(t)] = F_1(s) + F_2(s)$	Linearity theorem
4.	$\mathscr{L}[e^{-at}f(t)] = F(s+a)$	Frequency shift theorem
5.	$\mathscr{L}[f(t-T)] = e^{-sT}F(s)$	Time shift theorem
6.	$\mathscr{L}[f(at)] = \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathscr{L}\left[\frac{df}{dt}\right] = sF(s) - f(0-)$	Differentiation theorem
8.	$\mathscr{L}\left[\frac{d^2f}{dt^2}\right] = s^2F(s) - sf(0-) - f'(0-)$	Differentiation theorem
9.	$\mathscr{L}\left[\frac{d^nf}{dt^n}\right] = s^nF(s) - \sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$	Differentiation theorem
10.	$\mathscr{L}\left[\int_{0-}^{t} f(\tau)d\tau\right] = \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty) = \lim_{s\to 0} sF(s)$	Final value theorem[1]
12.	$f(0+) = \lim_{s\to\infty} sF(s)$	Initial value theorem[2]

Transfer Function

$$a_n \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \ldots + a_1 \frac{dy(t)}{dt} + a_0 y(t)$$

$$=$$

$$b_n \frac{d^n r(t)}{dt^n} + b_{n-1} \frac{d^{n-1} r(t)}{dt^{n-1}} + \ldots + b_1 \frac{dr(t)}{dt} + b_0 r(t)$$

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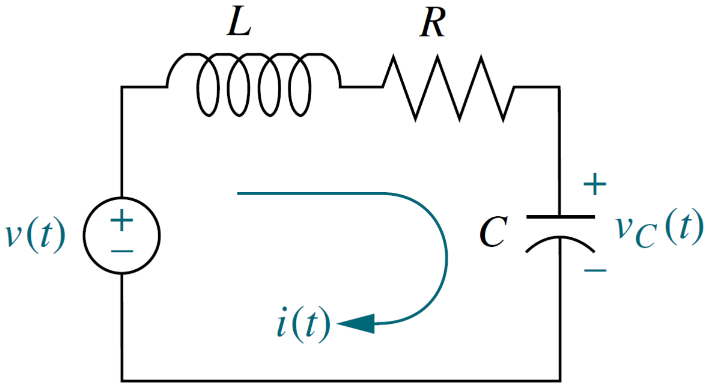
$$=$$

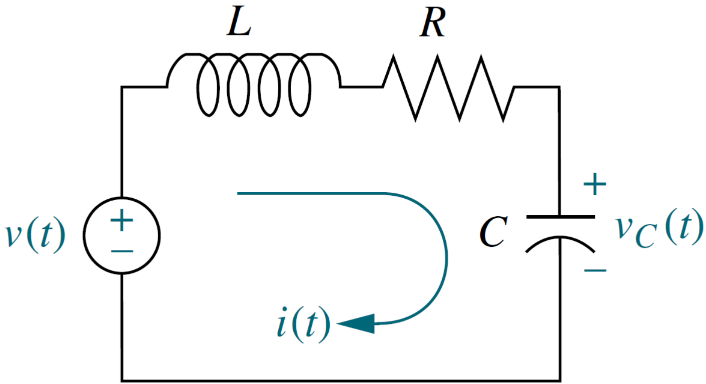
$$b_n \frac{d^n r(t)}{dt^n} + b_{n-1} \frac{d^{n-1} r(t)}{dt^{n-1}} + \ldots + b_1 \frac{dr(t)}{dt} + b_0 r(t)$$

$$G(s) = \frac{Y(s)}{R(s)} = \frac{b_n s^n + b_{n-1} s^{n-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$$

RLC: Transfer function

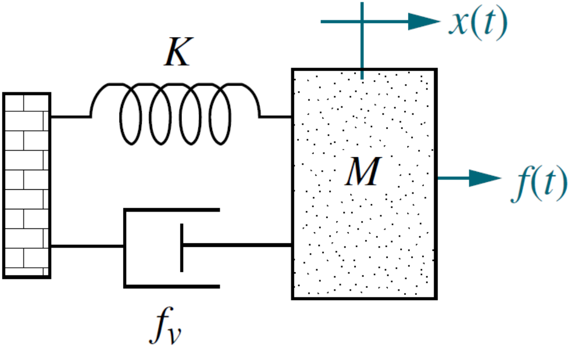


RLC: Transfer function

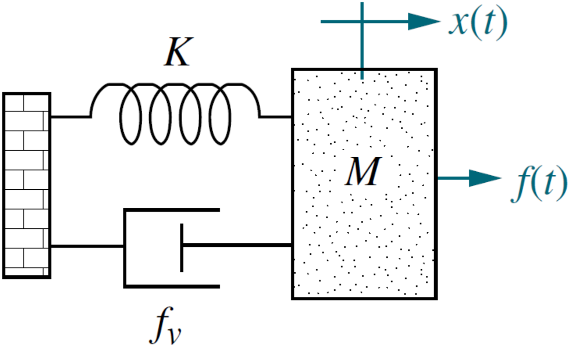


$$G(s) = \frac{V_c(s)}{V(s)} = \frac{1}{LCs^2 + RCs + 1}$$

Mass Spring Damper: Transfer Function



Mass Spring Damper: Transfer Function



$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + f_v s + K}$$

$$G(s) = \frac{V_c(s)}{V(s)} = \frac{1}{LCs^2 + RCs + 1}$$

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + f_v s + K}$$

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$$\boxed{\frac{d^2y(t)}{dt^2} + 12\frac{dy(t)}{dt} + 32y(t) = 32r(t)}$$

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$$G(s) := \frac{Y(s)}{R(s)} =$$

$$\boxed{\frac{d^2y(t)}{dt^2} + 12\frac{dy(t)}{dt} + 32y(t) = 32r(t)}$$

$$G(s) := \frac{Y(s)}{R(s)} = \frac{32}{s^2 + 12s + 32} =$$

$$\boxed{\frac{d^2y(t)}{dt^2} + 12\frac{dy(t)}{dt} + 32y(t) = 32r(t)}$$

$$G(s) := \frac{Y(s)}{R(s)} = \frac{32}{s^2 + 12s + 32} = = \frac{32}{(s+4)(s+8)}$$

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$$Y(s) = \frac{32}{(s+4)(s+8)}R(s)$$

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$$Y(s) = \frac{32}{(s+4)(s+8)}R(s)$$

Step Response:

$$\boxed{\frac{d^2y(t)}{dt^2} + 12\frac{dy(t)}{dt} + 32y(t) = 32r(t)}$$

$$G(s) := \frac{Y(s)}{R(s)} = \frac{32}{s^2 + 12s + 32} = = \frac{32}{(s+4)(s+8)}$$

$$Y(s) = \frac{32}{(s+4)(s+8)}R(s)$$

Step Response: $\quad R(s) = \frac{1}{s}$

$$\boxed{\frac{d^2y(t)}{dt^2} + 12\frac{dy(t)}{dt} + 32y(t) = 32r(t)}$$

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$$Y(s) = \frac{32}{(s+4)(s+8)}R(s)$$

Step Response: $R(s) = \frac{1}{s}$

$$Y(s) = \frac{32}{s(s+4)(s+8)}$$

$$Y(s) = \frac{1}{s(s+4)(s+8)} = \frac{A_1}{s} + \frac{A_2}{s+4} + \frac{A_3}{s+8}$$

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$$\boxed{y(t) = A_1 u(t) + A_2 e^{-4t} u(t) + A_3 e^{-8t} u(t)}$$

Computation of A_1, A_2 and A_3

$$\frac{32}{s(s+4)(s+8)} = \frac{A_1}{s} + \frac{A_2}{s+4} + \frac{A_3}{s+8} \tag{1}$$

Computation of A_1, A_2 and A_3

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1) Computation of A_1

i) Multiply Equation (1) with s

Computation of A_1, A_2 and A_3

$$\frac{32}{s(s+4)(s+8)} = \frac{A_1}{s} + \frac{A_2}{s+4} + \frac{A_3}{s+8} \tag{1}$$

1) Computation of A_1

i) Multiply Equation (1) with s

$$\frac{32}{(s+4)(s+7)} = A_1 + \frac{A_2 s}{s+4} + \frac{A_3 s}{s+8} \tag{2}$$

Computation of A_1, A_2 and A_3

$$\frac{32}{s(s+4)(s+8)} = \frac{A_1}{s} + \frac{A_2}{s+4} + \frac{A_3}{s+8} \tag{1}$$

1) Computation of A_1

i) Multiply Equation (1) with s

$$\frac{32}{(s+4)(s+7)} = A_1 + \frac{A_2 s}{s+4} + \frac{A_3 s}{s+8} \tag{2}$$

ii) Put $s = 0$ in Equation (2)

Computation of A_1, A_2 and A_3

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i) Multiply Equation (1) with s

$$\frac{32}{(s+4)(s+7)} = A_1 + \frac{A_2 s}{s+4} + \frac{A_3 s}{s+8} \tag{2}$$

ii) Put $s = 0$ in Equation (2)

$$\boxed{A_1 = 1}$$

Computation of A_1, A_2 and A_3

$$\frac{32}{s(s+4)(s+8)} = \frac{A_1}{s} + \frac{A_2}{s+4} + \frac{A_3}{s+8} \tag{3}$$

2) Computation of A_2

i) Multiply Equation (3) with $(s+4)$

Computation of A_1, A_2 and A_3

$$\frac{32}{s(s+4)(s+8)} = \frac{A_1}{s} + \frac{A_2}{s+4} + \frac{A_3}{s+8} \tag{3}$$

2) Computation of A_2

i) Multiply Equation (3) with $(s+4)$

$$\frac{32}{s(s+8)} = \frac{A_1(s+4)}{s} + A_2 + \frac{A_3(s+4)}{s+8} \tag{4}$$

Computation of A_1, A_2 and A_3

$$\frac{32}{s(s+4)(s+8)} = \frac{A_1}{s} + \frac{A_2}{s+4} + \frac{A_3}{s+8} \tag{3}$$

2) Computation of A_2

i) Multiply Equation (3) with $(s+4)$

$$\frac{32}{s(s+8)} = \frac{A_1(s+4)}{s} + A_2 + \frac{A_3(s+4)}{s+8} \tag{4}$$

ii) Put $s = -4$ in Equation (4)

Computation of A_1, A_2 and A_3

$$\frac{32}{s(s+4)(s+8)} = \frac{A_1}{s} + \frac{A_2}{s+4} + \frac{A_3}{s+8} \tag{3}$$

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ii) Put $s = -4$ in Equation (4)

$$\boxed{A_2 = -2}$$

$$A_1 = 1 \qquad A_2 = -2 \qquad A_3 = 1$$

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$$Y(s) = \frac{32}{s(s+4)(s+8)} = \frac{1}{s} - \frac{2}{s+4} + \frac{1}{s+8} \tag{5}$$

$$A_1 = 1 \qquad A_2 = -2 \qquad A_3 = 1$$

$$Y(s) = \frac{32}{s(s+4)(s+8)} = \frac{1}{s} - \frac{2}{s+4} + \frac{1}{s+8} \tag{5}$$

$$\boxed{y(t) = u(t) - 2e^{-4t}u(t) + e^{-8t}u(t)}$$

$$A_1 = 1 \qquad A_2 = -2 \qquad A_3 = 1$$

$$Y(s) = \frac{32}{s(s+4)(s+8)} = \frac{1}{s} - \frac{2}{s+4} + \frac{1}{s+8} \tag{5}$$

$$\boxed{y(t) = u(t) - 2e^{-4t}u(t) + e^{-8t}u(t)}$$

$$\frac{Y(s)}{R(s)} = \frac{32}{(s+4)(s+8)}$$

Denominator With Real Repeated Roots

$$G(s) = \frac{Y(s)}{R(s)} = \frac{2}{(s+1)(s+2)^2}$$

Denominator With Real Repeated Roots

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Denominator With Real Repeated Roots

$$G(s) = \frac{Y(s)}{R(s)} = \frac{2}{(s+1)(s+2)^2}$$

$$Y(s) = \frac{2}{s(s+1)(s+2)^2} = \frac{A_1}{s} + \frac{A_2}{s+1} + \frac{A_3}{s+2}$$

Denominator With Real Repeated Roots

$$G(s) = \frac{Y(s)}{R(s)} = \frac{2}{(s+1)(s+2)^2}$$

$$Y(s) = \frac{2}{s(s+1)(s+2)^2} = \frac{A_1}{s} + \frac{A_2}{s+1} + \frac{A_3}{s+2} + \frac{A_4}{(s+2)^2}$$

Denominator With Real Repeated Roots

$$G(s) = \frac{Y(s)}{R(s)} = \frac{2}{(s+1)(s+2)^2}$$

$$Y(s) = \frac{2}{s(s+1)(s+2)^2} = \frac{A_1}{s} + \frac{A_2}{s+1} + \frac{A_3}{s+2} + \frac{A_4}{(s+2)^2}$$

$$\boxed{A_1 = \frac{1}{2} \qquad A_2 = -2 \qquad A_4 = 1}$$

Computation of A_3

$$\frac{2}{s(s+1)(s+2)^2} = \frac{A_1}{s} + \frac{A_2}{s+1} + \frac{A_3}{s+2} + \frac{A_4}{(s+2)^2} \tag{6}$$

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Multiply Equation (6) with $(s+2)^2$

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Multiply Equation (6) with $(s+2)^2$

$$\frac{2}{s(s+1)} = \frac{A_1(s+2)^2}{s} + \frac{A_2(s+2)^2}{s+1} + A_3(s+2) + A_4 \tag{7}$$

Computation of A_3

$$\frac{2}{s(s+1)(s+2)^2} = \frac{A_1}{s} + \frac{A_2}{s+1} + \frac{A_3}{s+2} + \frac{A_4}{(s+2)^2} \tag{6}$$

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Differentiate Equation (7) and put $s = -2$

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Differentiate Equation (7) and put $s = -2$

$$\left.\frac{-4s-2}{s^2(s+1)^2}\right|_{s=-2} = A_3$$

Computation of A_3

$$\frac{2}{s(s+1)(s+2)^2} = \frac{A_1}{s} + \frac{A_2}{s+1} + \frac{A_3}{s+2} + \frac{A_4}{(s+2)^2} \tag{6}$$

Multiply Equation (6) with $(s+2)^2$

$$\frac{2}{s(s+1)} = \frac{A_1(s+2)^2}{s} + \frac{A_2(s+2)^2}{s+1} + A_3(s+2) + A_4 \tag{7}$$

Differentiate Equation (7) and put $s=-2$

$$\left.\frac{-4s-2}{s^2(s+1)^2}\right|_{s=-2} = A_3 = \frac{3}{2}$$

$$\boxed{A_1 = \frac{1}{2} \qquad A_2 = -2 \qquad A_3 = \frac{3}{2} \qquad A_4 = 1}$$

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$$Y(s) = \frac{2}{s(s+1)(s+2)^2} = \frac{\frac{1}{2}}{s} - \frac{2}{s+1} + \frac{\frac{3}{2}}{s+2} + \frac{1}{(s+2)^2}$$

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$$y(t) = \frac{1}{2}u(t) - 2e^{-t}u(t) + \frac{3}{2}e^{-2t}u(t) + te^{-2t}u(t)$$

Complex Roots in Denominator

$$G(s) = \frac{Y(s)}{R(s)} = \frac{3}{s^2 + 2s + 5}$$

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$$Y(s)$$

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Complex Roots in Denominator

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$$Y(s) = \frac{3}{s(s^2 + 2s + 5)} = \frac{A_1}{s} + \frac{A_2 s + A_3}{s^2 + 2s + 5}$$

$$Y(s) = \frac{3}{s(s^2 + 2s + 5)} = \frac{A_1 s^2 + 2A_1 s + 5A_1 + A_2 s^2 + A_3 s}{s(s^2 + 2s + 5)}$$

$$3 = (A_1 + A_2)s^2 + (2A_1 + A_3)s + 5A_1$$

$$3 \;=\; (A_1 + A_2)s^2 \;+\; (2A_1 + A_3)s \;+\; 5A_1$$

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$$\boxed{A_1 = \frac{3}{5} \qquad A_2 = -\frac{3}{5} \qquad A_3 = -\frac{6}{5}}$$

$$Y(s) \;=\; \frac{1}{s(s^2 + 2s + 5)} \;=\; \frac{3}{5}\frac{1}{s} - \frac{3}{5}\left(\frac{s+2}{s^2 + 2s + 5}\right)$$

Property of Laplace Transform

$$\begin{aligned} f(t) &\quad\Longleftrightarrow\quad F(s) \\ e^{-at}f(t) &\quad\Longleftrightarrow\quad F(s+a) \end{aligned}$$

Property of Laplace Transform

$$f(t) \iff F(s)$$

$$e^{-at}f(t) \iff F(s+a)$$

$$f(t) = cos(2t) \iff F(s) = \frac{s}{s^2+4}$$

$$e^{-t}cos(2t) \iff \frac{s+1}{(s+1)^2+4} = \frac{s+1}{s^2+2s+5}$$

Property of Laplace Transform

$$f(t) \iff F(s)$$

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$$f(t) = cos(2t) \iff F(s) = \frac{s}{s^2+4}$$

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$$f(t) = sin(2t) \iff F(s) = \frac{2}{s^2+4}$$

$$e^{-t}sin(2t) \iff \frac{2}{(s+1)^2+4} = \frac{2}{s^2+2s+5}$$

$$Y(s) = \frac{1}{s(s^2+2s+5)} = \frac{3}{5}\frac{1}{s} - \frac{3}{5}\left(\frac{s+2}{s^2+2s+5}\right)$$

$$Y(s) = \frac{1}{s(s^2+2s+5)} = \frac{3}{5}\frac{1}{s} - \frac{3}{5}\left(\frac{s+2}{s^2+2s+5}\right)$$

$$Y(s) = \frac{3}{5}\frac{1}{s} - \frac{3}{5}\left(\frac{(s+1)+\frac{1}{2}2}{s^2+2s+5}\right)$$

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$$Y(s) \;=\; \frac{3}{5}\frac{1}{s} - \frac{3}{5}\left(\frac{(s+1)+\frac{1}{2}2}{s^2+2s+5}\right)$$

$$\boxed{y(t) \;=\; \frac{3}{5}u(t) \;-\; \frac{3}{5}e^{-t}\left(cos2t + \frac{1}{2}sin2t\right)}$$

Transfer Function

$$G(s) = \frac{N(s)}{D(s)}$$

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- Pole of $G(s)$: Roots of denominator $D(s)$

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- Pole of $G(s)$: Roots of denominator $D(s)$
- Zero of $G(s)$: Roots of numerator $N(s)$
- Order of $G(s)$: Degree of denominator $D(s)$

$$G(s) = \frac{Y(s)}{R(s)} = \frac{s+2}{s+5}$$

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$$Y(s) = \frac{2}{5}\frac{1}{s} + \frac{3}{5}\frac{1}{s+5}$$

$$G(s) = \frac{Y(s)}{R(s)} = \frac{s+2}{s+5}$$

$$Y(s) = \frac{s+2}{s(s+5)}$$

$$Y(s) = \frac{2}{5}\frac{1}{s} + \frac{3}{5}\frac{1}{s+5}$$

$$y(t) = \frac{2}{5}u(t) + \frac{3}{5}e^{-5t}u(t)$$

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$\frac{2}{5}u(t)$: Forced response

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$\frac{2}{5}u(t)$: Forced response

$\frac{3}{5}e^{-5t}u(t)$: Natural response

Relation between Poles and Form of Response

$$G(s) = \frac{1}{(s+4)(s+8)}$$

Relation between Poles and Form of Response

$$G(s) = \frac{1}{(s+4)(s+8)}$$

$$Y(s) = \frac{1}{s(s+4)(s+8)} = \frac{A_1}{s} + \frac{A_2}{s+4} + \frac{A_3}{s+8}$$

Relation between Poles and Form of Response

$$G(s) = \frac{1}{(s+4)(s+8)}$$

$$Y(s) = \frac{1}{s(s+4)(s+8)} = \frac{A_1}{s} + \frac{A_2}{s+4} + \frac{A_3}{s+8}$$

$$\boxed{y(t) = A_1 u(t) + A_2 e^{-4t} u(t) + A_3 e^{-8t} u(t)}$$

First Order System

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$$G(s) = \frac{a}{s+a}$$

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$$G(s) = \frac{a}{s+a}$$

$$Y(s) = \frac{a}{s(s+a)}$$

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$$y(t) = \left(1 - e^{-at}\right) u(t)$$

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$$y(t)\Big|_{t=\frac{1}{a}}$$

First Order System

$$G(s) = \frac{a}{s+a}$$

$$Y(s) = \frac{a}{s(s+a)}$$

$$y(t) = \left(1 - e^{-at}\right)u(t)$$

$$y(t)\Big|_{t=\frac{1}{a}} = 1 - \frac{1}{e} = 0.63$$

Transient Specifications of First Order System

$$\boxed{\text{Time Constant} \;=\; \frac{1}{a}}$$

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$$\left.\frac{dy(t)}{dt}\right|_{t=0} \;=\; a$$

Transient Specifications of First Order System

Rise Time: Time taken by $y(t)$ to rise from $y(t) = 0.1$ to $y(t) = 0.9$

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$$y\left(t_1\right) = \left(1 - e^{-at_1}\right)u(t) = 0.1 \qquad t_1 = \frac{0.11}{a}$$

$$y\left(t_2\right) = \left(1 - e^{-at_2}\right)u(t) = 0.9$$

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Rise Time: Time taken by $y(t)$ to rise from $y(t) = 0.1$ to $y(t) = 0.9$

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$$y\left(t_2\right) = \left(1 - e^{-at_2}\right)u(t) = 0.9 \qquad t_2 = \frac{2.31}{a}$$

Transient Specifications of First Order System

Rise Time: Time taken by $y(t)$ to rise from $y(t) = 0.1$ to $y(t) = 0.9$

$$y(t_1) \; = \; \left(1 - e^{-at_1}\right) u(t) = 0.1 \qquad\qquad t_1 = \frac{0.11}{a}$$

$$y(t_2) \; = \; \left(1 - e^{-at_2}\right) u(t) = 0.9 \qquad\qquad t_2 = \frac{2.31}{a}$$

$$\boxed{\text{Rise Time} \; = \; T_r \; = \; \frac{2.31}{a} \; - \; \frac{0.11}{a} \; = \; \frac{2.2}{a}}$$

Transient Specifications of First Order System

Settling Time: Time taken by $y(t)$ to reach and remain within 2% of final value, which is 1

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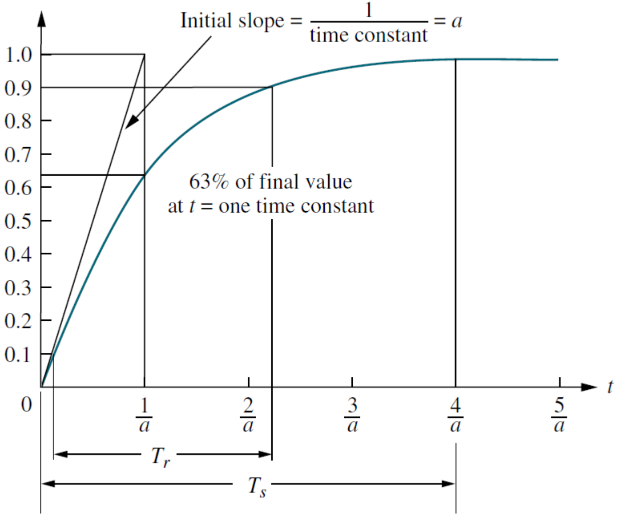
Transient Specifications of First Order System

Settling Time: Time taken by $y(t)$ to reach and remain within 2% of final value, which is 1

$$y(T_s) = \left(1 - e^{-aT_s}\right) = 0.98$$

$$\boxed{T_s = \frac{4}{a}}$$

Transient Specifications of First Order System



First Order System

$$G(s) = \frac{K}{s+a}$$

First Order System

$$G(s) = \frac{K}{s+a}$$

$$Y(s) = \frac{K}{s(s+a)}$$

First Order System

$$G(s) = \frac{K}{s+a}$$

$$Y(s) = \frac{K}{s(s+a)}$$

$$y(t) = \frac{K}{a}\left(1 - e^{-at}\right)u(t)$$

Second Order System

$$G(s) = \frac{b}{s^2 + as + b}$$

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$$\zeta := \frac{a}{2\sqrt{b}} \qquad w_n := \sqrt{b}$$

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Second Order System

$$G(s) = \frac{b}{s^2 + as + b}$$

$$\zeta := \frac{a}{2\sqrt{b}} \qquad w_n := \sqrt{b}$$

$$G(s) = \frac{w_n^2}{s^2 + 2\zeta w_n s + w_n^2}$$

Poles: $$\sigma_{1,2} = -\zeta w_n \pm w_n\sqrt{\zeta^2 - 1}$$

$\zeta > 1$

Poles are Real

$$\sigma_1 = -\zeta w_n - w_n\sqrt{\zeta^2 - 1}$$
$$\sigma_2 = -\zeta w_n + w_n\sqrt{\zeta^2 - 1}$$

$\zeta > 1$

Poles are Real

$$\sigma_1 = -\zeta w_n - w_n\sqrt{\zeta^2 - 1}$$
$$\sigma_2 = -\zeta w_n + w_n\sqrt{\zeta^2 - 1}$$

$$\zeta w_n > w_n\sqrt{\zeta^2 - 1}$$

$\zeta > 1$

Poles are Real

$$\sigma_1 = -\zeta w_n - w_n\sqrt{\zeta^2 - 1}$$
$$\sigma_2 = -\zeta w_n + w_n\sqrt{\zeta^2 - 1}$$

$$\zeta w_n > w_n\sqrt{\zeta^2 - 1}$$

$\sigma_1 < 0$ $\sigma_2 < 0$

$\zeta > 1$

$$G(s) = \frac{Y(s)}{R(s)} = \frac{w_n^2}{(s-\sigma_1)(s-\sigma_2)}$$

$\zeta > 1$

$$G(s) = \frac{Y(s)}{R(s)} = \frac{w_n^2}{(s-\sigma_1)(s-\sigma_2)}$$

$$Y(s) = \frac{w_n^2}{s(s-\sigma_1)(s-\sigma_2)} = \frac{A_1}{s} + \frac{A_2}{s-\sigma_1} + \frac{A_3}{s-\sigma_2}$$

$\zeta > 1$

$$G(s) = \frac{Y(s)}{R(s)} = \frac{w_n^2}{(s-\sigma_1)(s-\sigma_2)}$$

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$$y(t) = \left(A_1 + A_2 e^{-\sigma_1 t} + A_3 e^{-\sigma_2 t}\right) u(t)$$

$\zeta > 1$

$$G(s) = \frac{Y(s)}{R(s)} = \frac{w_n^2}{(s-\sigma_1)(s-\sigma_2)}$$

$$Y(s) = \frac{w_n^2}{s(s-\sigma_1)(s-\sigma_2)} = \frac{A_1}{s} + \frac{A_2}{s-\sigma_1} + \frac{A_3}{s-\sigma_2}$$

$$y(t) = \left(A_1 + A_2 e^{-\sigma_1 t} + A_3 e^{-\sigma_2 t}\right) u(t)$$

OVERDAMPED system

$\zeta = 1$

$\zeta = 1$

Poles are real and repeated

$$\sigma_1 \;=\; \sigma_2 \;=\; -w_n$$

$\zeta = 1$

Poles are real and repeated

$$\sigma_1 \;=\; \sigma_2 \;=\; -w_n$$

$$G(s) \;=\frac{Y(s)}{R(s)} = \;\frac{w_n^2}{(s+w_n)^2}$$

$\zeta = 1$

Poles are real and repeated

$$\sigma_1 = \sigma_2 = -w_n$$

$$G(s) = \frac{Y(s)}{R(s)} = \frac{w_n^2}{(s + w_n)^2}$$

$$Y(s) = \frac{w_n^2}{s(s + w_n)^2} = \frac{1}{s} + \frac{A_1}{s + w_n} + \frac{A_2}{(s + w_n)^2}$$

$\zeta = 1$

Poles are real and repeated

$$\sigma_1 = \sigma_2 = -w_n$$

$$G(s) = \frac{Y(s)}{R(s)} = \frac{w_n^2}{(s+w_n)^2}$$

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$$y(t) = \left(1 + A_1 e^{-w_n t} + A_2 t e^{-w_n t}\right) u(t)$$

$\zeta = 1$

Poles are real and repeated

$$\sigma_1 = \sigma_2 = -w_n$$

$$G(s) = \frac{Y(s)}{R(s)} = \frac{w_n^2}{(s+w_n)^2}$$

$$Y(s) = \frac{w_n^2}{s(s+w_n)^2} = \frac{1}{s} + \frac{A_1}{s+w_n} + \frac{A_2}{(s+w_n)^2}$$

$$y(t) = \left(1 + A_1 e^{-w_n t} + A_2 t e^{-w_n t}\right) u(t)$$

CRITICALLY DAMPED system

$\zeta = 0$

$\zeta = 0$

Poles are imaginary

$$\sigma_1 = jw_n \qquad \sigma_2 = -jw_n$$

$\zeta = 0$

Poles are imaginary

$$\sigma_1 = jw_n \qquad \sigma_2 = -jw_n$$

$$G(s) = \frac{Y(s)}{R(s)} = \frac{w_n^2}{s^2 + w_n^2}$$

$\zeta = 0$

Poles are imaginary

$$\sigma_1 = jw_n \qquad \sigma_2 = -jw_n$$

$$G(s) = \frac{Y(s)}{R(s)} = \frac{w_n^2}{s^2 + w_n^2}$$

$$Y(s) = \frac{w_n^2}{s(s^2 + w_n^2)} = \frac{1}{s} - \frac{s}{s^2 + w_n^2}$$

$\zeta = 0$

Poles are imaginary

$$\sigma_1 = jw_n \qquad \sigma_2 = -jw_n$$

$$G(s) = \frac{Y(s)}{R(s)} = \frac{w_n^2}{s^2 + w_n^2}$$

$$Y(s) = \frac{w_n^2}{s(s^2 + w_n^2)} = \frac{1}{s} - \frac{s}{s^2 + w_n^2}$$

$$y(t) = \Big(1 - cos(w_n t)\Big) u(t)$$

$\zeta = 0$

Poles are imaginary

$$\sigma_1 = jw_n \qquad \sigma_2 = -jw_n$$

$$G(s) = \frac{Y(s)}{R(s)} = \frac{w_n^2}{s^2 + w_n^2}$$

$$Y(s) = \frac{w_n^2}{s(s^2 + w_n^2)} = \frac{1}{s} - \frac{s}{s^2 + w_n^2}$$

$$y(t) = \Big(1 - cos(w_n t)\Big)u(t)$$

UNDAMPED system

$0 < \zeta < 1$

$0 < \zeta < 1$

Poles are complex

$$\begin{aligned} \sigma_1 &= -\zeta w_n - j w_n \sqrt{1-\zeta^2} \\ \sigma_2 &= -\zeta w_n + j w_n \sqrt{1-\zeta^2} \end{aligned}$$

$0 < \zeta < 1$

Poles are complex

$$\begin{aligned} \sigma_1 &= -\zeta w_n - j w_n \sqrt{1-\zeta^2} \\ \sigma_2 &= -\zeta w_n + j w_n \sqrt{1-\zeta^2} \end{aligned}$$

$$G(s) = \frac{Y(s)}{R(s)} = \frac{w_n^2}{s^2 + 2\zeta w_n s + w_n^2}$$

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$$Y(s) = \frac{w_n^2}{s(s^2 + 2\zeta w_n s + w_n^2)} = \frac{A_1}{s} + \frac{A_2 s + A_3}{s^2 + 2\zeta w_n s + w_n^2}$$

$$Y(s) = \frac{w_n^2}{s(s^2 + 2\zeta w_n s + w_n^2)} = \frac{A_1\left(s^2 + 2\zeta w_n s + w_n^2\right) + A_2 s^2 + A_3 s}{s\left(s^2 + 2\zeta w_n s + w_n^2\right)}$$

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$$Y(s) = \frac{1}{s} - \frac{s + 2\zeta w_n}{(s + \zeta w_n)^2 + w_n^2(1 - \zeta^2)}$$

$$Y(s) \;=\; \frac{1}{s} \;-\; \frac{s + 2\zeta w_n}{(s + \zeta w_n)^2 \;+\; w_n^2(1 - \zeta^2)}$$

$$Y(s) \;=\; \frac{1}{s} \;-\; \frac{(s \;+\; \zeta w_n) \;+\; \zeta w_n}{(s + \zeta w_n)^2 \;+\; w_n^2(1 - \zeta^2)}$$

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$$Y(s) = \frac{1}{s} - \frac{(s + \zeta w_n)}{(s+\zeta w_n)^2 + w_n^2(1-\zeta^2)} - \frac{\zeta w_n}{(s+\zeta w_n)^2 + w_n^2(1-\zeta^2)}$$

$$Y(s) = \frac{1}{s} - \frac{(s + \zeta w_n)}{(s+\zeta w_n)^2 + w_n^2(1-\zeta^2)} - \frac{\frac{\zeta}{\sqrt{1-\zeta^2}} w_n \sqrt{1-\zeta^2}}{(s+\zeta w_n)^2 + w_n^2(1-\zeta^2)}$$

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$$w_d \;:=\; w_n\sqrt{1-\zeta^2}$$

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$$y(t) \;=\; u(t) \;-\; e^{-\zeta w_n t} cos(w_d t) u(t) \;-\; \frac{\zeta}{\sqrt{1-\zeta^2}} e^{-\zeta w_n t} sin(w_d t) u(t)$$

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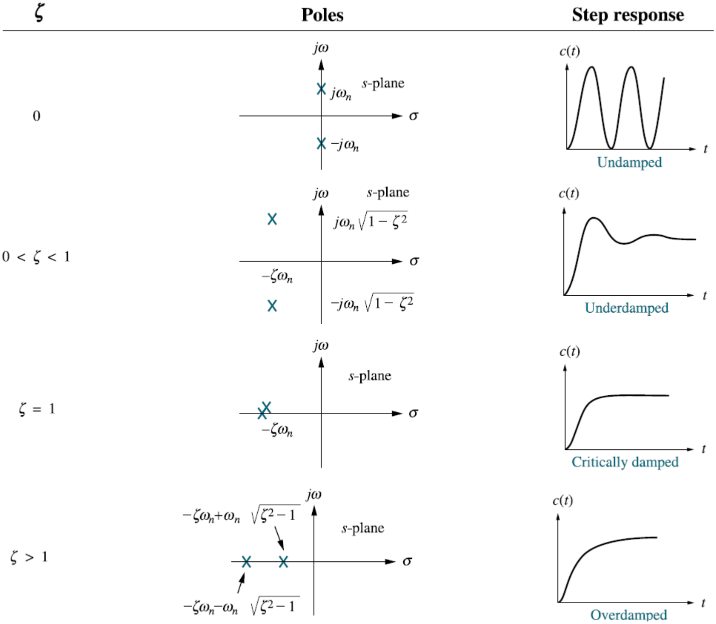
Step Response of Second Order System



Peak Time T_p

Time instant at which $y(t)$ reaches its first peak

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Time instant at which $y(t)$ reaches its first peak

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$$\frac{dy(t)}{dt} = -e^{-\zeta w_n t}\left(-w_d sin(w_d t) + \frac{\zeta w_d}{\sqrt{1-\zeta^2}} cos(w_d t)\right) + \zeta w_n e^{-\zeta w_n t}\left(cos(w_d t) + \frac{\zeta}{\sqrt{1-\zeta^2}} sin(w_d t)\right)$$

Peak Time

$$\frac{dy(t)}{dt} = -e^{-\zeta w_n t}\left(-w_d sin(w_d t) + \frac{\zeta w_n \sqrt{1-\zeta^2}}{\sqrt{1-\zeta^2}} cos(w_d t)\right) + \zeta w_n e^{-\zeta w_n t}\left(cos(w_d t) + \frac{\zeta}{\sqrt{1-\zeta^2}} sin(w_d t)\right)$$

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$$\frac{dy(t)}{dt} = e^{-\zeta w_n t} w_d sin(w_d t) - \zeta w_n e^{-\zeta w_n t} cos(w_d t) + \zeta w_n e^{-\zeta w_n t} cos(w_d t) + \frac{\zeta^2 w_n e^{-\zeta w_n t}}{\sqrt{1-\zeta^2}} sin(w_d t)$$

Peak Time

$$\frac{dy(t)}{dt} = e^{-\zeta w_n t} w_n \sqrt{1-\zeta^2} sin(w_d t) + \frac{\zeta^2 w_n e^{-\zeta w_n t}}{\sqrt{1-\zeta^2}} sin(w_d t)$$

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$$w_d t = k\pi$$

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$$w_d t = k\pi$$

$$T_p = \frac{\pi}{w_d}$$

$$\frac{d^2y(t)}{dt^2} = -\frac{\zeta w_n^2 e^{-\zeta w_n t}}{\sqrt{1-\zeta^2}} sin(w_d t) + \frac{w_n e^{-\zeta w_n t}}{\sqrt{1-\zeta^2}} w_d cos(w_d t)$$

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$$\text{At } t = T_p = \frac{\pi}{w_d} \qquad \frac{d^2y(t)}{dt^2} = -\frac{w_n e^{-\zeta w_n t}}{\sqrt{1-\zeta^2}} w_d < 0$$

Second Order System

$$G(s) = \frac{w_n^2}{s^2 + 2\zeta w_n s + w_n^2}$$

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Poles:

$$\sigma_1 = -\zeta w_n - j w_n \sqrt{1-\zeta^2}$$
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Transient Specifications

- Peak Time T_p:

$$T_p = \frac{\pi}{w_d} = \frac{\pi}{w_n\sqrt{1-\zeta^2}}$$

- % Overshoot:

$$\%\ \text{Overshoot} = e^{\dfrac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100$$

- Rise Time:

$$T_r = \frac{\pi-\theta}{w_d} = \frac{\pi-\theta}{w_n\sqrt{1-\zeta^2}} \qquad \theta = \arctan\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

$$y(t) = u(t) - e^{-\zeta w_n t}\left(cos(w_d t) + \frac{\zeta}{\sqrt{1-\zeta^2}} sin(w_d t)\right) u(t)$$

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$$\tan\left(w_d T_r\right) = \frac{-\sqrt{1-\zeta^2}}{\zeta}$$

$$y(t) \;=\; u(t) \;-\; e^{-\zeta w_n t}\left(cos(w_d t) \;+\; \frac{\zeta}{\sqrt{1-\zeta^2}} sin(w_d t)\right) u(t)$$

$$\tan\left(w_d T_r\right) \;=\; \frac{-\sqrt{1-\zeta^2}}{\zeta}$$

$$\tan\left(-w_d T_r\right) \;=\; \frac{\sqrt{1-\zeta^2}}{\zeta}$$

$$y(t) = u(t) - e^{-\zeta w_n t}\left(cos(w_d t) + \frac{\zeta}{\sqrt{1-\zeta^2}} sin(w_d t)\right)u(t)$$

$$\tan\left(w_d T_r\right) = \frac{-\sqrt{1-\zeta^2}}{\zeta}$$

$$\tan\left(-w_d T_r\right) = \frac{\sqrt{1-\zeta^2}}{\zeta}$$

$$\tan\left(k\pi - w_d T_r\right) = \frac{\sqrt{1-\zeta^2}}{\zeta}$$

$$y(t) = u(t) - e^{-\zeta w_n t}\left(cos(w_d t) + \frac{\zeta}{\sqrt{1-\zeta^2}} sin(w_d t)\right) u(t)$$

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$$\tan\left(k\pi - w_d T_r\right) = \frac{\sqrt{1-\zeta^2}}{\zeta}$$

$$T_r = \frac{\pi - \theta}{w_d} \qquad \theta = \arctan\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

Settling Time

Settling Time T_s: Time taken by $y(t)$ to reach and stay within 2% of final value, i.e., 1

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$$y(t) \,=\, u(t) \,-\, \frac{e^{-\zeta w_n t}}{\sqrt{1-\zeta^2}}\left(cos(w_d t)\sqrt{1-\zeta^2} \,+\, \zeta sin(w_d t)\right)u(t)$$

$$y(t) = u(t) - \frac{e^{-\zeta w_n t}}{\sqrt{1-\zeta^2}}\left(cos(w_d t)\sqrt{1-\zeta^2} + \zeta sin(w_d t)\right)u(t)$$

$$\tan\phi = \frac{\zeta}{\sqrt{1-\zeta^2}}$$

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$$\tan\phi = \frac{\zeta}{\sqrt{1-\zeta^2}} \qquad \cos\phi = \sqrt{1-\zeta^2} \qquad \sin\phi = \zeta$$

$$y(t) \;=\; u(t) \;-\; \frac{e^{-\zeta w_n t}}{\sqrt{1-\zeta^2}}\Big(cos(w_d t)\sqrt{1-\zeta^2} \;+\; \zeta sin(w_d t)\Big)u(t)$$

$$\tan\phi \;=\; \frac{\zeta}{\sqrt{1-\zeta^2}} \qquad \cos\phi \;=\; \sqrt{1-\zeta^2} \qquad \sin\phi \;=\; \zeta$$

$$y(t) \;=\; u(t) \;-\; \frac{e^{-\zeta w_n t}}{\sqrt{1-\zeta^2}} cos\big(w_d t \;-\; \phi\big)u(t)$$

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$$\frac{e^{-\zeta w_n t}}{\sqrt{1-\zeta^2}} \;=\; 0.02$$

$$T_s = \frac{-\ln\left(0.02\sqrt{1-\zeta^2}\right)}{\zeta w_n}$$

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$$T_s = \frac{4}{\zeta w_n}$$

Relation between Peak Time and Pole Location

Poles:

$$
\begin{aligned}
\sigma_1 &= -\zeta w_n - j w_n \sqrt{1-\zeta^2} \\
\sigma_2 &= -\zeta w_n + j w_n \sqrt{1-\zeta^2}
\end{aligned}
$$

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Poles:

$$\begin{aligned}\sigma_1 &= -\zeta w_n - j w_n \sqrt{1-\zeta^2} \\ \sigma_2 &= -\zeta w_n + j w_n \sqrt{1-\zeta^2}\end{aligned}$$

Peak Time:

$$T_p = \frac{\pi}{w_d} = \frac{\pi}{w_n\sqrt{1-\zeta^2}} = \frac{\pi}{|\text{ Imaginary part of pole}|}$$

Relation between Settling Time and Pole Location

Poles:

$$\begin{aligned}\sigma_1 &= -\zeta w_n - j w_n \sqrt{1-\zeta^2} \\ \sigma_2 &= -\zeta w_n + j w_n \sqrt{1-\zeta^2}\end{aligned}$$

Relation between Settling Time and Pole Location

Poles:

$$\sigma_1 = -\zeta w_n - j w_n \sqrt{1-\zeta^2}$$
$$\sigma_2 = -\zeta w_n + j w_n \sqrt{1-\zeta^2}$$

Settling Time:

$$T_s = \frac{4}{\zeta w_n} = \frac{4}{|\text{ Real part of pole}|}$$

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$$\%\text{ Overshoot } = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100$$

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Relation between % Overshoot and Pole Location

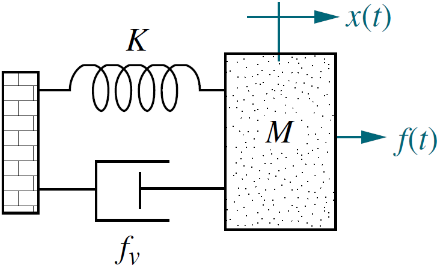
$$\% \text{ Overshoot } = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100$$

Poles:

$$\sigma_1 = -\zeta w_n - j w_n\sqrt{1-\zeta^2}$$
$$\sigma_2 = -\zeta w_n + j w_n\sqrt{1-\zeta^2}$$

$$\cos\theta = \zeta$$

Mass Spring Damper



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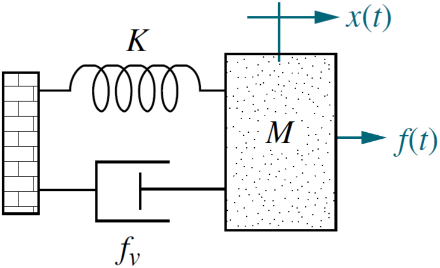

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + f_v s + K}$$

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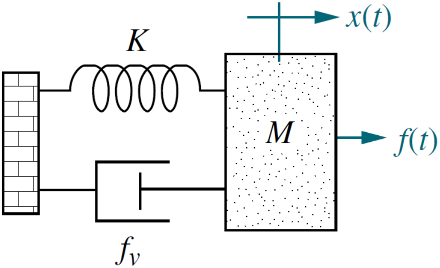
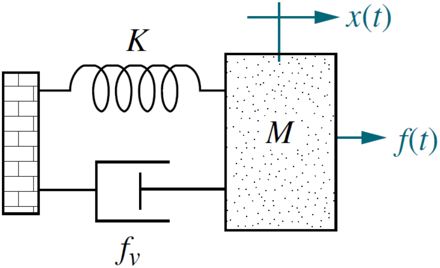


$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + f_v s + K}$$

Problem: Take $K = 5$. Find M and f_v which results in settling time 2 sec and overshoot 20%

Mass Spring Damper



$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + f_v s + K}$$

Problem: Take $K = 5$. Find M and f_v which results in settling time 2 sec and overshoot 20% $\quad M = 0.26$ and $f_v = 1.04$

Second Order System with Additional Pole

Second Order System with Additional Pole

$$G(s) = \frac{w_n^2}{\left(s^2 + 2\zeta w_n s + w_n^2\right)}$$

Second Order System with Additional Pole

$$G(s) = \frac{w_n^2}{\left(s^2 + 2\zeta w_n s + w_n^2\right)} \frac{\alpha_r}{\left(s + \alpha_r\right)}$$

Second Order System with Additional Pole

$$G(s) = \frac{w_n^2}{\left(s^2 + 2\zeta w_n s + w_n^2\right)} \frac{\alpha_r}{\left(s + \alpha_r\right)}$$

Poles: $-\zeta w_n \pm j w_n \sqrt{1-\zeta^2}$ $\quad -\alpha_r$

Second Order System with Additional Pole

$$G(s) = \frac{w_n^2}{\left(s^2 + 2\zeta w_n s + w_n^2\right)} \frac{\alpha_r}{\left(s + \alpha_r\right)}$$

Poles: $\quad -\zeta w_n \pm j w_n \sqrt{1-\zeta^2} \qquad -\alpha_r$

$$Y(s) = \frac{A_1}{s} + \frac{A_2(s+\zeta w_n) + A_3 w_d}{(s+\zeta w_n)^2 + w_d^2} + \frac{A_4}{s+\alpha_r}$$

Second Order System with Additional Pole

$$G(s) = \frac{w_n^2}{\left(s^2 + 2\zeta w_n s + w_n^2\right)} \frac{\alpha_r}{\left(s + \alpha_r\right)}$$

Poles: $-\zeta w_n \pm j w_n \sqrt{1 - \zeta^2}$ $\quad -\alpha_r$

$$Y(s) = \frac{A_1}{s} + \frac{A_2(s + \zeta w_n) + A_3 w_d}{(s + \zeta w_n)^2 + w_d^2} + \frac{A_4}{s + \alpha_r}$$

$$y(t) = A_1 u(t) + A_2 e^{-\zeta w_n t}\left(A_2 cos(w_d t) + A_3 sin(w_d t)\right) + A_4 e^{-\alpha_r t}$$

Residues

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$$G(s) = \frac{b}{(s^2 + as + b)} \frac{\alpha_r}{(s + \alpha_r)}$$

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$$G(s) = \frac{b}{(s^2 + as + b)} \frac{\alpha_r}{(s+\alpha_r)}$$

$$Y(s) = \frac{1}{s} \frac{b}{(s^2 + as + b)} \frac{\alpha_r}{(s+\alpha_r)} =$$

Residues

$$G(s) = \frac{b}{(s^2 + as + b)} \frac{\alpha_r}{(s + \alpha_r)}$$

$$Y(s) = \frac{1}{s} \frac{b}{(s^2 + as + b)} \frac{\alpha_r}{(s + \alpha_r)} = \frac{A_1}{s} + \frac{A_2 s + A_3}{(s^2 + as + b)} + \frac{A_4}{(s + \alpha_r)}$$

Residues

$$G(s) \;=\; \frac{b}{(s^2 \;+\; as \;+\; b)}\,\frac{\alpha_r}{(s+\alpha_r)}$$

$$Y(s) \;=\; \frac{1}{s}\,\frac{b}{(s^2 \;+\; as \;+\; b)}\,\frac{\alpha_r}{(s+\alpha_r)} \;=\; \frac{A_1}{s} \;+\; \frac{A_2 s + A_3}{(s^2 \;+\; as \;+\; b)} \;+\; \frac{A_4}{(s+\alpha_r)}$$

$$A_1 \;=\; 1 \qquad\qquad A_2 \;=\; \frac{\alpha_r a \;-\; \alpha_r^2}{\alpha_r^2 + b - \alpha_r a}$$

$$A_3 \;=\; \frac{\alpha_r a^2 - \alpha_r^2 a - b\alpha_r}{\alpha_r^2 + b - \alpha_r a} \qquad\qquad A_4 \;=\; \frac{-b^2}{\alpha_r^2 + b - \alpha_r a}$$

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$$G(s) = \frac{b}{(s^2 + as + b)} \frac{\alpha_r}{(s+\alpha_r)}$$

$$Y(s) = \frac{1}{s} \frac{b}{(s^2 + as + b)} \frac{\alpha_r}{(s+\alpha_r)} = \frac{A_1}{s} + \frac{A_2 s + A_3}{(s^2 + as + b)} + \frac{A_4}{(s+\alpha_r)}$$

$$A_1 = 1 \qquad A_2 = \frac{\alpha_r a - \alpha_r^2}{\alpha_r^2 + b - \alpha_r a}$$

$$A_3 = \frac{\alpha_r a^2 - \alpha_r^2 a - b\alpha_r}{\alpha_r^2 + b - \alpha_r a} \qquad A_4 = \frac{-b^2}{\alpha_r^2 + b - \alpha_r a}$$

$$A_1 = 1 \qquad A_2 = -1 \qquad A_3 = -a \qquad A_4 = 0$$

Second Order System With Additional Zero

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$$G(s) = \frac{1}{(s+b)(s+c)}$$

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$$G_1(s) = \frac{\left(\frac{-b+a}{-b+c}\right)}{s+b} + \frac{\left(\frac{-c+a}{-c+b}\right)}{s+c}$$

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$$G_1(s) = \frac{\left(\frac{-b+a}{-b+c}\right)}{s+b} + \frac{\left(\frac{-c+a}{-c+b}\right)}{s+c}$$

$$G_1(s) \approx \frac{\left(\frac{a}{-b+c}\right)}{s+b} + \frac{\left(\frac{a}{-c+b}\right)}{s+c}$$

Second Order System With Additional Zero

$$G(s) = \frac{1}{(s + b)(s + c)}$$

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$$G_1(s) \approx \frac{\left(\dfrac{a}{-b + c}\right)}{s + b} + \frac{\left(\dfrac{a}{-c + b}\right)}{s + c} = \frac{a}{(s+b)(s+c)}$$

$$G(s) = \frac{1}{(s+b)(s+c)}$$

$$G(s) = \frac{1}{(s+b)(s+c)}$$

$$G_1(s) = (s+a)G(s)$$

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$$y_1(t) = \frac{dy(t)}{dt} + ay(t)$$

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Pole Zero Cancellation

$$G(s) = \frac{k(s+z)}{(s+p)(s^2+as+b)}$$

Pole Zero Cancellation

$$G(s) = \frac{k(s + z)}{(s + p)(s^2 + as + b)}$$

$$G(s) = \frac{A_1}{s+p} + \frac{A_2 s + A_3}{s^2 + as + b}$$

Examples

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$$G_1(s) = \frac{26.25(s+4)}{(s+3.5)(s+5)(s+6)}$$

Examples

$$G_1(s) = \frac{26.25(s+4)}{(s+3.5)(s+5)(s+6)}$$

$$Y_1(s) = \frac{26.25(s+4)}{s(s+3.5)(s+5)(s+6)}$$

Examples

$$G_1(s) = \frac{26.25(s+4)}{(s+3.5)(s+5)(s+6)}$$

$$Y_1(s) = \frac{26.25(s+4)}{s(s+3.5)(s+5)(s+6)} = \frac{1}{s} - \frac{1}{s+3.5} - \frac{3.5}{s+5} + \frac{3.5}{s+6}$$

Examples

$$G_1(s) = \frac{26.25(s+4)}{(s+3.5)(s+5)(s+6)}$$

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$$G_2(s) = \frac{26.25(s+4)}{(s+4.01)(s+5)(s+6)}$$

Examples

$$G_1(s) = \frac{26.25(s+4)}{(s+3.5)(s+5)(s+6)}$$

$$Y_1(s) = \frac{26.25(s+4)}{s(s+3.5)(s+5)(s+6)} = \frac{1}{s} - \frac{1}{s+3.5} - \frac{3.5}{s+5} + \frac{3.5}{s+6}$$

$$G_2(s) = \frac{26.25(s+4)}{(s+4.01)(s+5)(s+6)}$$

$$Y_2(s) = \frac{26.25(s+4)}{s(s+4.01)(s+5)(s+6)} = \frac{0.87}{s} + \frac{0.033}{s+4.01} - \frac{5.3}{s+5} + \frac{4.4}{s+6}$$

Second Order System with Additional Pole

$$G(s) = \frac{w_n^2}{\left(s^2 + 2\zeta w_n s + w_n^2\right)} \frac{\alpha_r}{\left(s + \alpha_r\right)}$$

Second Order System with Additional Pole

$$G(s) \;=\; \frac{w_n^2}{\left(s^2 \;+\; 2\zeta w_n s \;+\; w_n^2\right)} \frac{\alpha_r}{\left(s + \alpha_r\right)}$$

$$G(s) \;=\; \frac{w_n^2}{s^2 \;+\; 2\zeta w_n s \;+\; w_n^2}$$

Pole Zero Cancellation

$$G(s) = \left(\frac{b}{s^2 + as + b}\right)\left(\frac{s + 2.99}{s + 3}\right)$$

Pole Zero Cancellation

$$G(s) = \left(\frac{b}{s^2 + as + b}\right)\left(\frac{s + 2.99}{s + 3}\right)$$

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Examples

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$$G_1(s) = \frac{26.25(s+4)}{(s+3.5)(s+5)(s+6)}$$

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Examples

$$G_1(s) = \frac{26.25(s+4)}{(s+3.5)(s+5)(s+6)}$$

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$$G_2(s) = \frac{26.25(s+4)}{(s+4.01)(s+5)(s+6)}$$

Examples

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$$G_2(s) = \frac{26.25(s+4)}{(s+4.01)(s+5)(s+6)}$$

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Feedback Loop

Feedback Loop

- Transfer function:

$$T(s) := \frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

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- Closed loop transfer function

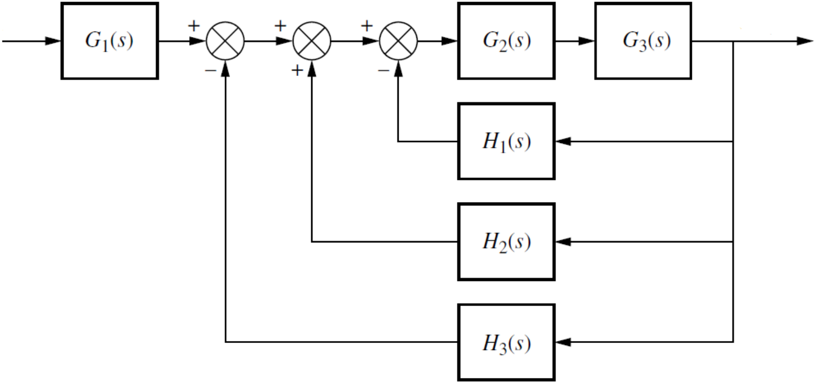
Feedback Loop

- Transfer function:

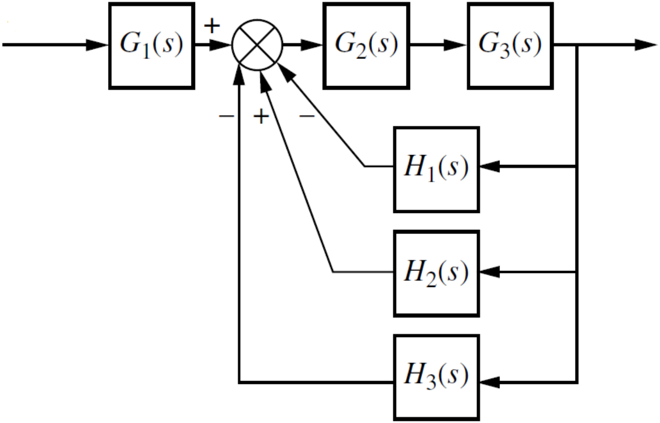
$$T(s) := \frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

- Closed loop transfer function

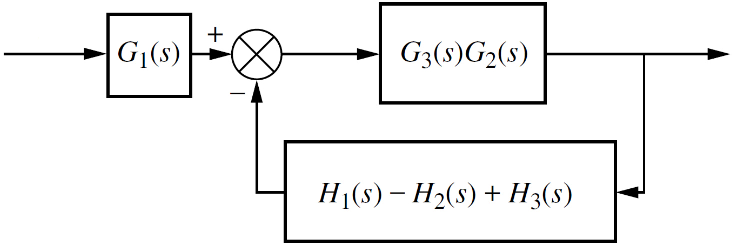
- $G(s)H(s)$: Loop Gain or Open-Loop transfer function

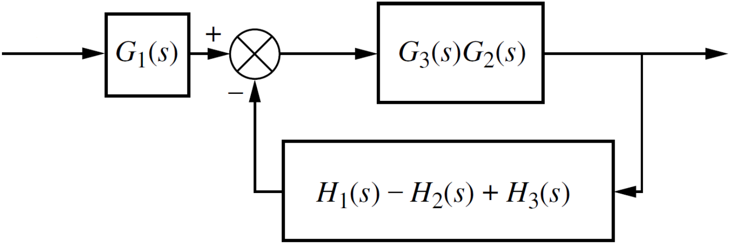


$G_1(s)$
$G_2(s)$
$G_3(s)$
$H_1(s)$
$H_2(s)$
$H_3(s)$
+
−
+
+
+
−



G1(s)
+
G2(s)
G3(s)
−
+
−
H1(s)
H2(s)
H3(s)

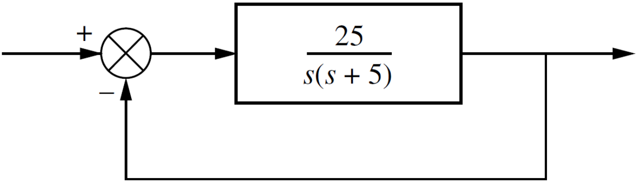

$G_1(s)$
+
−
$G_3(s)G_2(s)$
$H_1(s) - H_2(s) + H_3(s)$



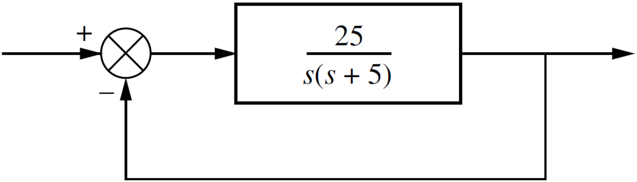
$$T(s) = \frac{G_3(s)G_2(s)G_1(s)}{1 + G_3(s)G_2(s)\Big(H_1(s) - H_2(s) + H_3(s)\Big)}$$

Unity Feedback

Unity Feedback

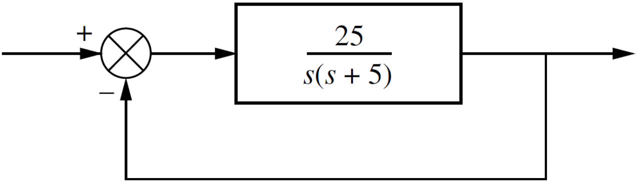


Unity Feedback



$$T(s) = \frac{25}{s^2 + 5s + 25}$$

Unity Feedback



$$T(s) = \frac{25}{s^2 + 5s + 25}$$

$\zeta = 0.5$ $\qquad$ $w_n = 5$

Specifications of Control Systems

- Transient response
- Stability
- Steady state response

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- Transient response
- Stability
- Steady state response

Bounded-Input Bounded-Output (BIBO) Stability

Bounded Signal

Bounded Signal

- Definition: A signal $x(t)$ is said to be a bounded signal if

$$\left|x(t)\right| \;\leq\; M_x \;<\; \infty, \qquad\qquad \forall t \geq 0$$

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- Examples: Unit Step Signal

  Sine / Cosine Signal

BIBO Stability

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- Definition: A system is said to be BIBO stable if every bounded input signal produces a bounded output signal

BIBO Stability

- Definition: A system is said to be BIBO stable if every bounded input signal produces a bounded output signal , i.e.,

$$
\begin{aligned}
\left|r(t)\right| \;\le\; M_r \;<\; \infty \qquad\qquad & \forall t \ge 0 \\
\Downarrow \qquad\qquad & \\
\left|y(t)\right| \;\le\; M_y \;<\; \infty \qquad\qquad & \forall t \ge 0
\end{aligned}
$$

Impulse Response

Impulse Response

$$G(s) = \frac{Y(s)}{R(s)}$$

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$$r(t) = \delta(t) \qquad\qquad R(s) = 1$$

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$$Y(s) = G(s)$$

Impulse Response

$$G(s) = \frac{Y(s)}{R(s)}$$

$$r(t) = \delta(t) \qquad\qquad R(s) = 1$$

$$Y(s) = G(s)$$

Impulse Response $\quad h(t) := \mathscr{L}^{-}1\Big(G(s)\Big)$

$$G(s) = \frac{Y(s)}{R(s)}$$

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$$Y(s) = G(s)R(s)$$

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$$Y(s) = G(s)R(s)$$

$$\mathscr{L}^{-}1\Big(Y(s)\Big) =$$

$$G(s) = \frac{Y(s)}{R(s)}$$

$$Y(s) = G(s)R(s)$$

$$\mathscr{L}^{-}1\Big(Y(s)\Big) = \mathscr{L}^{-}1\Big(G(s)\Big) * \mathscr{L}^{-}1\Big(R(s)\Big)$$

$$G(s) = \frac{Y(s)}{R(s)}$$

$$Y(s) = G(s)R(s)$$

$$\mathscr{L}^{-}1\Big(Y(s)\Big) = \mathscr{L}^{-}1\Big(G(s)\Big) * \mathscr{L}^{-}1\Big(R(s)\Big)$$

$*$ Convolution

$$G(s) = \frac{Y(s)}{R(s)}$$

$$Y(s) = G(s)R(s)$$

$$\mathscr{L}^{-}1\Big(Y(s)\Big) = \mathscr{L}^{-}1\Big(G(s)\Big) * \mathscr{L}^{-}1\Big(R(s)\Big)$$

$*$ Convolution

$$y(t) = h(t) * r(t)$$

Convolution Integral

$$y(t) = \int_0^\infty r(t - \tau)\, h(\tau)\, d\tau$$

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$$\left|y(t)\right| \leq \int_0^{\infty} \left| r(t - \tau)\, h(\tau)\, d\tau \right|$$

$$\left|y(t)\right| \;\leq\; \int_0^\infty \left|r\left(t \;-\; \tau\right)\right| \left|h(\tau)\right| d\tau$$

$$\left|y(t)\right| \;\leq\; \int_0^\infty \left|r\left(t - \tau\right)\right| \left|h\left(\tau\right)\right| d\tau$$

$$\left|y(t)\right| \;\leq\; \int_0^\infty M_r \left|h\left(\tau\right)\right| d\tau$$

$$\left|y(t)\right| \;\leq\; \int_0^\infty \left|r\left(t - \tau\right)\right| \left|h\left(\tau\right)\right| d\tau$$

$$\left|y(t)\right| \;\leq\; \int_0^\infty M_r \left|h\left(\tau\right)\right| d\tau \quad = \quad M_r \int_0^\infty \left|h\left(\tau\right)\right| d\tau$$

$$\left|y(t)\right| \ \leq \ \int_0^\infty \left|r\left(t - \tau\right)\right| \left|h(\tau)\right| d\tau$$

$$\left|y(t)\right| \ \leq \ \int_0^\infty M_r \left|h(\tau)\right| d\tau \quad = \quad M_r \int_0^\infty \left|h(\tau)\right| d\tau$$

If $\displaystyle\int_0^\infty \left|h(\tau)\right| d\tau \ \leq \ M_h \ < \ \infty$

$$\left|y(t)\right| \;\leq\; \int_0^{\infty} \left|r(t-\tau)\right| \left|h(\tau)\right| d\tau$$

$$\left|y(t)\right| \;\leq\; \int_0^{\infty} M_r \left|h(\tau)\right| d\tau \quad = \quad M_r \int_0^{\infty} \left|h(\tau)\right| d\tau$$

If $\displaystyle\int_0^{\infty} \left|h(\tau)\right| d\tau \;\leq\; M_h \;<\; \infty$

$$\left|y(t)\right| \;\leq\; M_r M_h \;<\; \infty$$

LTI System

$$\int_0^\infty |h(\tau)|\, d\tau \;\leq\; M_h \;<\; \infty$$

$h(t)$ is absolutely integrable

LTI System

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$$\Downarrow$$

BIBO Stable

LTI System

$$\int_0^{\infty} \left|h(\tau)\right| d\tau \;\leq\; M_h \;<\; \infty$$

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BIBO Stable

BIBO Stable $\Longrightarrow$ $h(t)$ is absolutely integrable ????

LTI System

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$h(t)$ is absolutely integrable

$$\Downarrow$$

BIBO Stable

BIBO Stable $\Longrightarrow$ $h(t)$ is absolutely integrable ????

BIBO Stable $\Longleftrightarrow$ $h(t)$ is absolutely integrable

BIBO Stability and Transfer Function

BIBO Stability and Transfer Function

$$G(s) = \frac{b - a}{(s + a)(s + b)} =$$

BIBO Stability and Transfer Function

$$G(s) = \frac{b - a}{(s + a)(s + b)} = \frac{1}{s + a} - \frac{1}{s + b}$$

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Poles: $\quad s = -a, \quad s = -b$

BIBO Stability and Transfer Function

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$$h(t) = e^{-at}\, u(t) - e^{-bt}\, u(t)$$

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BIBO Stability and Transfer Function

$$G(s) = \frac{b - a}{(s + a)(s + b)} = \frac{1}{s + a} - \frac{1}{s + b}$$

Poles: $\quad s = -a, \quad s = -b$

$$h(t) = e^{-at}\, u(t) - e^{-bt}\, u(t)$$

$$\int_0^\infty \left| h(\tau) \right| d\tau \leq M_h < \infty$$

$a > 0, \; b > 0$ Both Poles in LHP of complex plane

$$G(s) = \frac{1}{s}$$

$$G(s) = \frac{1}{s} \qquad\qquad h(t) = u(t)$$

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Take $r(t) = u(t)$ $\qquad\qquad R(s) = \frac{1}{s}$

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Take $r(t) = u(t)$ $\qquad R(s) = \frac{1}{s}$

$$Y(s) = G(s)R(s) = \frac{1}{s^2}$$

$$G(s) = \frac{1}{s} \qquad h(t) = u(t)$$

Take $r(t) = u(t)$ $\qquad R(s) = \frac{1}{s}$

$$Y(s) = G(s)R(s) = \frac{1}{s^2}$$

$$y(t) = tu(t)$$

$$G(s) = \frac{1}{s} \qquad h(t) = u(t)$$

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$$y(t) = tu(t)$$ Not Bounded

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$y(t) = tu(t)$ $\qquad$ Not Bounded

System is NOT BIBO stable

$$G(s) = \frac{1}{s^2 + 1}$$

$$G(s) = \frac{1}{s^2 + 1} \qquad\qquad h(t) = sin(t)u(t)$$

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$y(t) = tsint(t)u(t) +$ $\qquad$ Not Bounded

System is NOT BIBO stable

BIBO stability

$h(t)$ is absolutely integrable

$$\equiv$$

BIBO stability

$$\equiv$$

All Poles of $G(s)$ in LHP of complex plane

BIBO stability

$h(t)$ is absolutely integrable

$$\equiv$$

BIBO stability

$$\equiv$$

All Poles of $G(s)$ in LHP of complex plane

$$T(s) = \frac{N(s)}{D(s)}$$

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Definition: If all roots of polynomial $D(s)$ are in LHP, then $D(s)$ is called as the hurwitz polynomial

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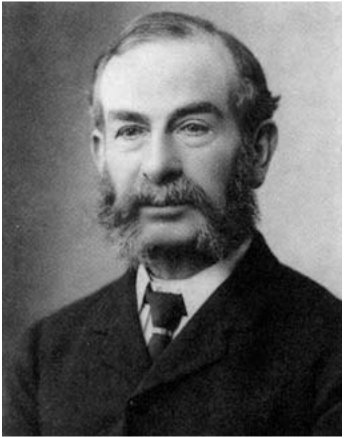
Definition: If all roots of polynomial $D(s)$ are in LHP, then $D(s)$ is called as the hurwitz polynomial

$$T(s) \text{ is BIBO stable} \iff D(s) \text{ is hurwitz}$$

Routh-Hurwitz Stability Criterion

Routh-Hurwitz Stability Criterion



Edward Routh

1876

Adolf Hurwitz

1895

Polynomial with Degree 1

Polynomial with Degree 1

$$D(s) = a_1 s + a_0$$

Polynomial with Degree 1

$$D(s) \;=\; a_1 s \,+\, a_0$$

$$D_1(s) \;=\; s \,+\, \frac{a_0}{a_1}$$

Polynomial with Degree 1

$$D(s) = a_1 s + a_0$$

$$D_1(s) = s + \frac{a_0}{a_1} = s + a$$

Polynomial with Degree 1

$$D(s) = a_1 s + a_0$$

$$D_1(s) = s + \frac{a_0}{a_1} = s + a$$

$D(s)$ is Hurwitz $\iff$ $a_1 \neq 0, \quad a_0 \neq 0, \quad$ and
a_1 and a_0 have same sign

Polynomial with Degree 2

$$D(s) = a_2 s^2 + a_1 s + a_0$$

Polynomial with Degree 2

$$D(s) = a_2 s^2 + a_1 s + a_0$$

$$\text{Roots of } D(s) = \frac{-a_1 \pm \sqrt{a_1^2 - 4a_2 a_0}}{2a_2}$$

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$$D(s) \;=\; a_2 s^2 \;+\; a_1 s \;+\; a_0$$

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1) $a_0 < 0, \quad a_1 > 0 \quad \text{and} \quad a_2 > 0$

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$\Longrightarrow$ One root of $D(s)$ lies in RHP

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Polynomial with Degree 2

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Polynomial with Degree 2

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Polynomial with Degree 2

3) $a_2 < 0, \quad a_1 > 0 \quad \text{and} \quad a_0 > 0$

$\implies$ One root of $D(s)$ lies in RHP

4) $a_2 > 0, \quad a_1 > 0 \quad \text{and} \quad a_0 > 0$

Polynomial with Degree 2

3) $a_2 < 0, \quad a_1 > 0 \quad \text{and} \quad a_0 > 0$

$\Longrightarrow$ One root of $D(s)$ lies in RHP

4) $a_2 > 0, \quad a_1 > 0 \quad \text{and} \quad a_0 > 0$

$\Longrightarrow$ Both roots of $D(s)$ lie in LHP

Polynomial with Degree 2

3) $a_2 < 0, \quad a_1 > 0$ and $a_0 > 0$

$\implies$ One root of $D(s)$ lies in RHP

4) $a_2 > 0, \quad a_1 > 0$ and $a_0 > 0$

$\implies$ Both roots of $D(s)$ lie in LHP

$D(s)$ is Hurwitz $\iff$ $a_2 \neq 0, \quad a_1 \neq 0, \quad a_0 \neq 0,$

$a_2, \; a_1$ and a_0 have same sign

Routh Table

Routh Table

$$D(s) = a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s_1 + a_0$$

Routh Table

$$D(s) = a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s_1 + a_0$$

s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2			
s^1			
s^0			

Routh Table

$$
\begin{array}{cccc}
s^4 & a_4 & a_2 & a_0 \\
s^3 & a_3 & a_1 & 0 \\
s^2 & \dfrac{-\begin{vmatrix} a_4 & a_2 \\ a_3 & a_1 \end{vmatrix}}{a_3} = b_1 & \dfrac{-\begin{vmatrix} a_4 & a_0 \\ a_3 & 0 \end{vmatrix}}{a_3} = b_2 & \dfrac{-\begin{vmatrix} a_4 & 0 \\ a_3 & 0 \end{vmatrix}}{a_3} = 0 \\
s^1 & \dfrac{-\begin{vmatrix} a_3 & a_1 \\ b_1 & b_2 \end{vmatrix}}{b_1} = c_1 & \dfrac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0 & \dfrac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0 \\
s^0 & \dfrac{-\begin{vmatrix} b_1 & b_2 \\ c_1 & 0 \end{vmatrix}}{c_1} = d_1 & \dfrac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0 & \dfrac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0
\end{array}
$$

Routh Stability Criteria

$D(s)$ is Hurwitz

if and only if

1) All coefficients of $D(s)$ are non-zero and of same sign
2) All elements in the first column of Routh table are non-zero
3) No sign changes in the first column of Routh table

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Number of sign changes in first column

$$=$$

Number of roots of $D(s)$ in RHP

Example

$$D(s) = s^3 + 10s^2 + 31s + 1030$$

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$$D(s) = s^3 + 10s^2 + 31s + 1030$$

$$\begin{array}{llll}
s^3 & 1 & 31 & 0 \\
s^2 & \cancel{10}\quad 1 & \cancel{1030}\quad 103 & 0 \\
s^1 & \dfrac{-\begin{vmatrix} 1 & 31 \\ 1 & 103 \end{vmatrix}}{1} = -72 & \dfrac{-\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}}{1} = 0 & \dfrac{-\begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix}}{1} = 0 \\
s^0 & \dfrac{-\begin{vmatrix} 1 & 103 \\ -72 & 0 \end{vmatrix}}{-72} = 103 & \dfrac{-\begin{vmatrix} 1 & 0 \\ -72 & 0 \end{vmatrix}}{-72} = 0 & \dfrac{-\begin{vmatrix} 1 & 0 \\ -72 & 0 \end{vmatrix}}{-72} = 0
\end{array}$$

Example

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s^3 & 1 & 31 & 0 \\
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s^1 & \dfrac{-\begin{vmatrix} 1 & 31 \\ 1 & 103 \end{vmatrix}}{1} = -72 & \dfrac{-\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}}{1} = 0 & \dfrac{-\begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix}}{1} = 0 \\
s^0 & \dfrac{-\begin{vmatrix} 1 & 103 \\ -72 & 0 \end{vmatrix}}{-72} = 103 & \dfrac{-\begin{vmatrix} 1 & 0 \\ -72 & 0 \end{vmatrix}}{-72} = 0 & \dfrac{-\begin{vmatrix} 1 & 0 \\ -72 & 0 \end{vmatrix}}{-72} = 0
\end{array}$$

Two roots in RHP

$$D(s) = s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3$$

$$D(s) = s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3$$

$$\begin{array}{cccc}
s^5 & 1 & 3 & 5 \\
s^4 & 2 & 6 & 3 \\
s^3 & \cancel{0} \quad \epsilon & \dfrac{7}{2} & 0 \\
s^2 & \dfrac{6\epsilon - 7}{\epsilon} & 3 & 0 \\
s^1 & \dfrac{42\epsilon - 49 - 6\epsilon^2}{12\epsilon - 14} & 0 & 0 \\
s^0 & 3 & 0 & 0
\end{array}$$

Label	**First column**	$\epsilon = +$	$\epsilon = -$
s^5	1	$+$	$+$
s^4	2	$+$	$+$
s^3	$\cancel{0}\quad \epsilon$	$+$	$-$
s^2	$\dfrac{6\epsilon - 7}{\epsilon}$	$-$	$+$
s^1	$\dfrac{42\epsilon - 49 - 6\epsilon^2}{12\epsilon - 14}$	$+$	$+$
s^0	3	$+$	$+$

Routh Stability Criteria

$D(s)$ is Hurwitz

if and only if

1) All coefficients of $D(s)$ are non-zero and of same sign
2) All elements in the first column of Routh table are non-zero
3) No sign changes in the first column of Routh table

Routh Stability Criteria

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1) All coefficients of $D(s)$ are non-zero and of same sign
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Number of sign changes in first column

$$=$$

Number of roots of $D(s)$ in RHP

Routh Stability Criteria

Special case *I*: 0 in first column of Routh table

Routh Stability Criteria

Special case *I*: 0 in first column of Routh table

Number of sign changes in first column

$$=$$

Number of roots of $D(s)$ in RHP

Example 1

$$D(s) = s^3 - 14s^2 + 49s - 36$$

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s^3	1	49	0
s^2	-14	-36	0
s	46.43	0	0
s^0	-36	0	0

Example 1

$$D(s) = s^3 - 14s^2 + 49s - 36$$

s^3	1	49	0
s^2	-14	-36	0
s	46.43	0	0
s^0	-36	0	0

Roots: 1, 4, 9

$$D(s) = a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0$$

$$D(s) = a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0$$

$$D(s) = a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0$$

Substitute: $$s = \frac{1}{r}$$

$$D(s) = a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0$$

Substitute: $$s = \frac{1}{r}$$

$$Q_1(r) = a_4 \left(\frac{1}{r}\right)^4 + a_3 \left(\frac{1}{r}\right)^3 + a_2 \left(\frac{1}{r}\right)^2 + a_1 \left(\frac{1}{r}\right) + a_0 = 0$$

$$D(s) \;=\; a_4 s^4 \;+\; a_3 s^3 \;+\; a_2 s^2 \;+\; a_1 s \;+\; a_0 \;=\; 0$$

Substitute: $\qquad s \;=\; \dfrac{1}{r}$

$$Q_1(r) \;=\; a_4\left(\frac{1}{r}\right)^4 \;+\; a_3\left(\frac{1}{r}\right)^3 \;+\; a_2\left(\frac{1}{r}\right)^2 \;+\; a_1\left(\frac{1}{r}\right) \;+\; a_0 \;=\; 0$$

$$Q_2(r) \;=\; a_0 r^4 \;+\; a_1 r^3 \;+\; a_2 r^2 \;+\; a_3 r \;+\; a_4 \;=\; 0$$

Example 2

$$D(s) = s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3$$

$$\begin{array}{llll}
s^5 & 1 & 3 & 5 \\
s^4 & 2 & 6 & 3 \\
s^3 & \cancel{0} \quad \epsilon & \dfrac{7}{2} & 0 \\
s^2 & \dfrac{6\epsilon - 7}{\epsilon} & 3 & 0 \\
s^1 & \dfrac{42\epsilon - 49 - 6\epsilon^2}{12\epsilon - 14} & 0 & 0 \\
s^0 & 3 & 0 & 0
\end{array}$$

$$D(s) = s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3$$

$$D(s) = s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3$$

$$Q_2(r) = 3r^5 + 5r^4 + 6r^3 + 3r^2 + 2r + 1$$

$$D(s) = s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3$$

$$Q_2(r) = 3r^5 + 5r^4 + 6r^3 + 3r^2 + 2r + 1$$

s^5	3	6	2
s^4	5	3	1
s^3	4.2	1.4	
s^2	1.33	1	
s^1	–1.75		
s^0	1		

Example 3

$$D(s) = s^5 + 7s^4 + 6s^3 + 42s^2 + 8s + 56$$

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$$D(s) = s^5 + 7s^4 + 6s^3 + 42s^2 + 8s + 56$$

s^5	1	6	6
s^4	7	42	56
s^3	0	0	0

Even/Odd Polynomial

Even Polynomial: $$D(-s) = D(s)$$

Even/Odd Polynomial

Even Polynomial: $$D(-s) = D(s)$$

$$D(s) = s^2 + 1$$

Even/Odd Polynomial

Even Polynomial: $D(-s) = D(s)$

$$D(s) = s^2 + 1$$

$$D(s) = s^8 + 2s^6 + 1$$

Even/Odd Polynomial

Even Polynomial: $D(-s) = D(s)$

$$D(s) = s^2 + 1$$

$$D(s) = s^8 + 2s^6 + 1$$

Odd Polynomial: $D(-s) = -D(s)$

Even/Odd Polynomial

Even Polynomial: $$D(-s) = D(s)$$

$$D(s) = s^2 + 1$$

$$D(s) = s^8 + 2s^6 + 1$$

Odd Polynomial: $$D(-s) = -D(s)$$

$$D(s) = s^5 + 2s^3 + s = s(s^4 + 2s^2 + 1)$$

$$D(s) = s^5 + 7s^4 + 6s^3 + 42s^2 + 8s + 56$$

$$D(s) = s^5 + 7s^4 + 6s^3 + 42s^2 + 8s + 56$$

s^5	1	6	6
s^4	7	42	56
s^3	0	0	0

$$D(s) = s^5 + 7s^4 + 6s^3 + 42s^2 + 8s + 56$$

s^5	1	6	6
s^4	7	42	56
s^3	0	0	0

$$D_1(s) = s^4 + 6s^2 + 8$$

$$D(s) = s^5 + 7s^4 + 6s^3 + 42s^2 + 8s + 56$$

s^5	1	6	6
s^4	7	42	56
s^3	0	0	0

$$D_1(s) = s^4 + 6s^2 + 8$$

$$\frac{dD_1(s)}{ds} = 4s^3 + 12s$$

$$
\begin{array}{lccccccccc}
s^5 & & & 1 & & & 6 & & & 8 \\
s^4 & & \cancel{7} & 1 & & \cancel{42} & 6 & & \cancel{56} & 8 \\
s^3 & \cancel{0} & \cancel{4} & 1 & \cancel{0} & \cancel{12} & 3 & \cancel{0} & \cancel{0} & 0 \\
s^2 & & & 3 & & & 8 & & & 0 \\
s^1 & & & \frac{1}{3} & & & 0 & & & 0 \\
s^0 & & & 8 & & & 0 & & & 0
\end{array}
$$

s^5			1			6			8
s^4		$\cancel{7}$	1		$\cancel{42}$	6		$\cancel{56}$	8
s^3	$\cancel{0}$	$\cancel{4}$	1	$\cancel{0}$	$\cancel{12}$	3	$\cancel{0}$	$\cancel{0}$	0
s^2			3			8			0
s^1			$\frac{1}{3}$			0			0
s^0			8			0			0

Roots: $-7, \quad \pm 2i, \quad \pm\sqrt{2}i$

Example 3

$$D(s) = s^8 + s^7 + 12s^6 + 22s^5 + 39s^4 + 59s^3 + 48s^2 + 38s + 20$$

Example 3

$$D(s) = s^8 + s^7 + 12s^6 + 22s^5 + 39s^4 + 59s^3 + 48s^2 + 38s + 20$$

s^8	1	12	39	48	20
s^7	1	22	59	38	0
s^6	$-\cancel{10}\ -1$	$-\cancel{20}\ -2$	$\cancel{10}\ \ 1$	$\cancel{20}\ \ 2$	0
s^5	$\cancel{20}\ \ 1$	$\cancel{60}\ \ 3$	$\cancel{40}\ \ 2$	0	0
s^4	1	3	2	0	0
s^3	$\cancel{0}\ \ \cancel{4}\ \ 2$	$\cancel{0}\ \ \cancel{6}\ \ 3$	$\cancel{0}\ \ \cancel{0}\ \ 0$	0	0
s^2	$\cancel{\frac{3}{2}}\ \ 3$	$\cancel{2}\ \ 4$	0	0	0
s^1	$\frac{1}{3}$	0	0	0	0
s^0	4	0	0	0	0

$$D_1(s) = s^4 + 3s^2 + 2$$

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All 4 roots of $D_1(s)$ are on the imaginary axis

$$D_1(s) = s^4 + 3s^2 + 2$$

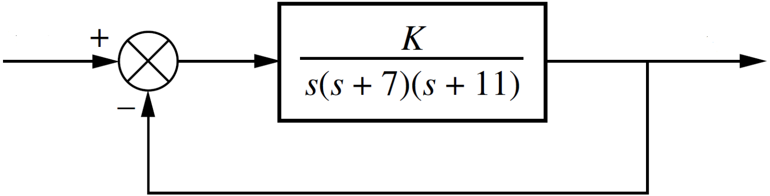
All 4 roots of $D_1(s)$ are on the imaginary axis

Out of remaining 4 roots of $D(s)$:

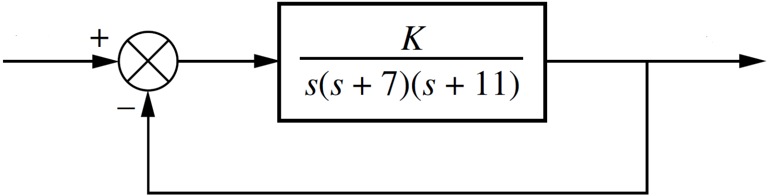
2 roots in RHP

2 roots in LHP

Design Using Routh Hurwitz



Design Using Routh Hurwitz



$$T(s) = \frac{K}{s^3 + 18s^2 + 77s + K}$$

s^3	1	77
s^2	18	K
s^1	$\dfrac{1386 - K}{18}$	
s^0	K	

s^3	1	77
s^2	18	K
s^1	$\dfrac{1386 - K}{18}$	
s^0	K	

$K < 1386$ All roots in LHP

s^3	1	77
s^2	18	K
s^1	$\dfrac{1386 - K}{18}$	
s^0	K	

$K < 1386$	All roots in LHP
$K > 1386$	1 roots in LHP and 2 roots in RHP

s^3	1	77
s^2	18	K
s^1	$\dfrac{1386 - K}{18}$	
s^0	K	

$K < 1386$ All roots in LHP

$K > 1386$ 1 roots in LHP and 2 roots in RHP

$K = 1386$ 2 roots on imaginary axis $\pm\sqrt{77}i$

1 roots in LHP