EE2100: Matrix Theory

Assignment - 1

Handed out on : 11 - Aug - 2023   **Due on : 21 - Aug - 2023 (before 5 PM)**

Instructions :

1. Please submit the solutions to the assignment problems to the course page (on the canvas platform). Solutions submitted to the course page will only be evaluated. Refer to the assignment guidelines mentioned on the course page.

2. Submissions received after the deadline will attract negative marking.

3. There are two sections in the assignments. It is suggested that you attempt all the questions. However, it is sufficient to submit solutions for problems that total to at least 10 points.

1. (5 Points) Let $\mathbf{x} \in \mathcal{R}^N$ denote the daily maximum price (within the day) of a particular stock. Assume that the data corresponding to n days ia available at the point when the analysis is carried out.

   (a) (1 Point) Derive an expression (in terms of Norm/inner product) to compute the average maximum price (over the n days) of the stock.

   (b) (1 Point) Derive an expression (in terms of Norm/inner product) to compute the variance of maximum price (over the n days) of the stock.

   Assume that the data corresponding to day $n + 1$ is now available i.e., $\mathbf{x} \in \mathcal{R}^{n+1}$. Derive an expression (preferably recursive) to compute

   (c) (1 Point) the updated average price and,

   (d) (2 Points) the updated variance.

2. (2 Points) Show that the magnitude of scalar projection of $\mathbf{a}$ onto $\mathbf{b}$ is less than $\|\mathbf{a}\|_2$.

3. (3 points) Consider a vector $\mathbf{a} \in \mathcal{R}^+$ i.e., a vector whose entries are real and positive. If $\|\mathbf{a}\|_1 = 1$, what is the minimum limit on $\|\mathbf{a}\|_2$.

4. (5 Points) Let $\mathbf{a}, \mathbf{b} \in \mathcal{R}^n$. Compute the value of α_1 and α_2 such that $\|\mathbf{a} + \alpha_1 \mathbf{b}\|_2$ is minimum and $\|\mathbf{a} - \alpha_2 \mathbf{b}\|_2$ is minimum.

5. (2 Points) Prove that the L_1 norm of any vector $\mathbf{x} \in \mathcal{R}^n$ is always greater than or equal its L_2 norm.

6. (10 Points) [Understanding the role of vectors/matrices in solving differential equations of order n]: Using a programming language of your choice, develop a code to implement the following numerical methods for solving a differential equation of the form (1).

$$a_n \frac{d^n x}{dt^n} + a_{n-1} \frac{d^{n-1} x}{dt^{n-1}} + \cdots + a_1 \frac{dx}{dt} = f(x, t) \tag{1}$$

(a) Forward Euler's approach

(b) Modified Euler's approach (or Runge-Kutta 2 method)

(c) Runge-Kutta 4 method

A short supplement emphasizing the key ideas in solving a differential equation of the form (1) is available here (Part A) and Part B. Please keep the following points in mind while submitting the code.

- The input to the code must be the a) order n, b) an array that contains the coefficients $[a_n, \cdots, a_1]$ and $f(x, t)$ (in the given order), c) the initial conditions and d) time step (Δt).

- It is preferred that you develop the code in Jupyter Notebook/equivalent with appropriate documentation. Also, develop the main part of the code as a function/module so that you can use it in future assignments.