.

EE1101: Circuits and Network Analysis

Lecture 36: Examples of Two-Port Networks

~~October 28, 2025~~

October 31, 2025

Topics :

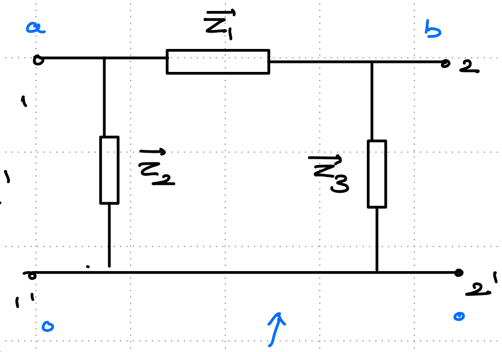
1. Examples of Two-Port Networks

Example 1: Pi Network (Y- Parameters)

$$\begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \end{bmatrix} = \begin{bmatrix} \vec{y}_{11} & \vec{y}_{12} \\ \vec{y}_{21} & \vec{y}_{22} \end{bmatrix} \begin{bmatrix} \vec{V}_1 \\ \vec{V}_2 \end{bmatrix}$$

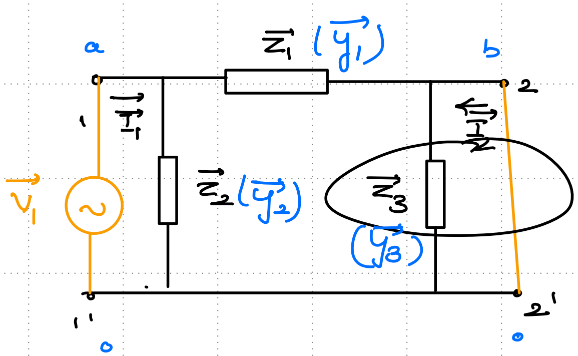
$$\left.\begin{aligned} \vec{y}_{11} &= \frac{\vec{I}_1}{\vec{V}_1}\Bigg|_{\vec{V}_2 = 0} \\ \vec{y}_{21} &= \frac{\vec{I}_2}{\vec{V}_1}\Bigg|_{\vec{V}_2 = 0} \end{aligned}\right\}$$ Solving a ckt with 2-2' Shorted

$$\left.\begin{aligned} \vec{y}_{22} &= \frac{\vec{I}_2}{\vec{V}_2}\Bigg|_{\vec{V}_1 = 0} \\ \vec{y}_{12} &= \frac{\vec{I}_1}{\vec{V}_2}\Bigg|_{\vec{V}_1 = 0} \end{aligned}\right\}$$ Solving a Ckt with 1-1' Shorted



Two port n/w with 3-terminals

$$\vec{y} = \frac{1}{\vec{Z}}$$



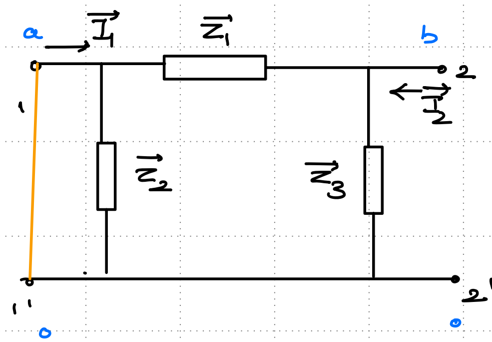
= Short Circuited branch

$$\vec{I}_1 = \frac{\vec{V}_1}{(\vec{Z}_1 \| \vec{Z}_2)} = \frac{\vec{V}_1}{\vec{Z}_1\vec{Z}_2}(\vec{Z}_1 + \vec{Z}_2)$$

$$= \vec{V}_1(\vec{y}_1 + \vec{y}_2)$$

$\vec{I}_2$ = -ve of $\vec{I}$ through $\vec{Z}_1$

$$= -\frac{\vec{V}_1}{\vec{Z}_1} = -\vec{y}_1\vec{V}_1$$



$$\vec{I}_1 = -\vec{y}_1\vec{V}_2 \quad (\text{or}) \quad -\frac{\vec{V}_2}{\vec{Z}_1}$$

$$\vec{I}_2 = \frac{\vec{V}_2}{\vec{Z}_3 \| \vec{Z}_1} \quad (\text{or}) \quad \vec{V}_2(\vec{y}_3 + \vec{y}_1)$$

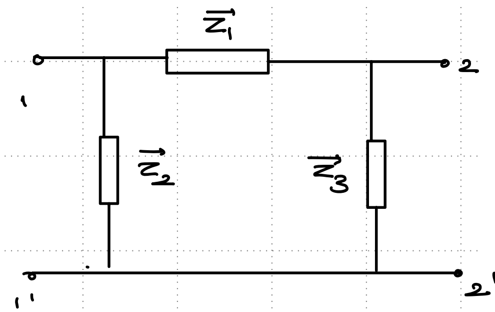
$$\begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \end{bmatrix} = \begin{bmatrix} \vec{y}_1 + \vec{y}_2 & -\vec{y}_1 \\ -\vec{y}_1 & \vec{y}_1 + \vec{y}_3 \end{bmatrix} \begin{bmatrix} \vec{V}_1 \\ \vec{V}_2 \end{bmatrix}$$

Example 1: Pi Network (z - Parameters)

$$\begin{bmatrix} \vec{V_1} \\ \vec{V_2} \end{bmatrix} = \begin{bmatrix} \vec{Z_{11}} & \vec{Z_{12}} \\ \vec{Z_{21}} & \vec{Z_{22}} \end{bmatrix} \begin{bmatrix} \vec{I_1} \\ \vec{I_2} \end{bmatrix}$$

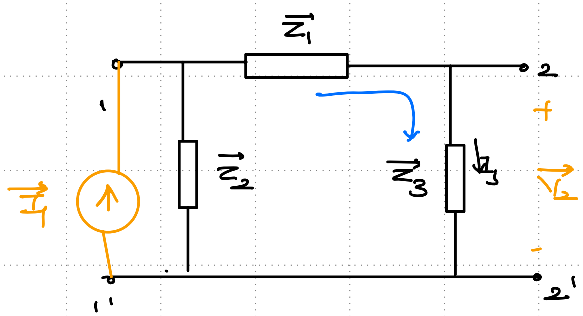
$$\vec{Z_{11}} = \left.\frac{\vec{V_1}}{\vec{I_1}}\right|_{\vec{I_2}=0} \qquad \vec{Z_{21}} = \left.\frac{\vec{V_2}}{\vec{I_1}}\right|_{\vec{I_2}=0}$$

Solving a ckt with 2-2' open



$$\vec{Z_{12}} = \left.\frac{\vec{V_1}}{\vec{I_2}}\right|_{\vec{I_1}=0} \qquad \vec{Z_{22}} = \left.\frac{\vec{V_2}}{\vec{I_2}}\right|_{\vec{I_1}=0}$$

Solving a ckt with 1-1' open.

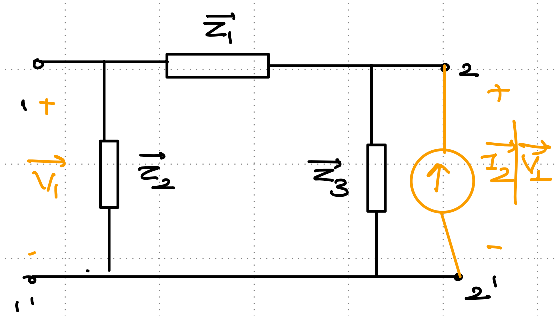


$$\vec{V_1} = \left[(\vec{Z_1}+\vec{Z_3}) \| \vec{Z_2}\right] \vec{I_1}$$

$$\Rightarrow \vec{Z_{11}} = \vec{Z_2} \,\|\, (\vec{Z_1}+\vec{Z_3})$$

$$\vec{V_2} = \vec{Z_3}\,(\vec{I_3}) = \vec{Z_3}\left(\frac{\vec{Z_2}}{\vec{Z_1}+\vec{Z_2}+\vec{Z_3}}\right)\vec{I_1}$$

$$\Rightarrow \vec{Z_{21}} = \frac{\vec{Z_2}\vec{Z_3}}{\vec{Z_1}+\vec{Z_2}+\vec{Z_3}}$$



$$\vec{V_2} = \left[\vec{Z_3} \,\|\, (\vec{Z_1}+\vec{Z_2})\right] \vec{I_2}$$

$$\Rightarrow \vec{Z_{22}} = \vec{Z_3} \,\|\, (\vec{Z_1}+\vec{Z_2})$$

$$\vec{V_1} = \frac{\vec{Z_3}\vec{Z_2}}{\vec{Z_1}+\vec{Z_2}+\vec{Z_3}}\,\vec{I_2}$$

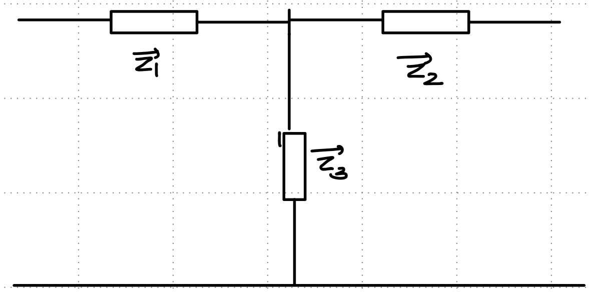
$$\vec{Z_{12}} = \frac{\vec{Z_3}\vec{Z_2}}{\vec{Z_1}+\vec{Z_2}+\vec{Z_3}}$$

Example 2: T Network (z-parameters)

$$\begin{bmatrix} \vec{V_1} \\ \vec{V_2} \end{bmatrix} = \begin{bmatrix} \vec{Z}_{11} & \vec{Z}_{12} \\ \vec{Z}_{21} & \vec{Z}_{22} \end{bmatrix} \begin{bmatrix} \vec{I_1} \\ \vec{I_2} \end{bmatrix}$$

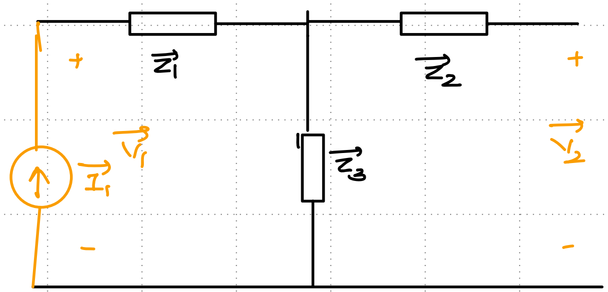
$$\left.\begin{aligned} \vec{Z}_{11} &= \frac{\vec{V_1}}{\vec{I_1}}\Bigg|_{\vec{I_2}=0} \\ \vec{Z}_{21} &= \frac{\vec{V_2}}{\vec{I_1}}\Bigg|_{\vec{I_2}=0} \end{aligned}\right\}$$

Solving a ckt with 2-2' open



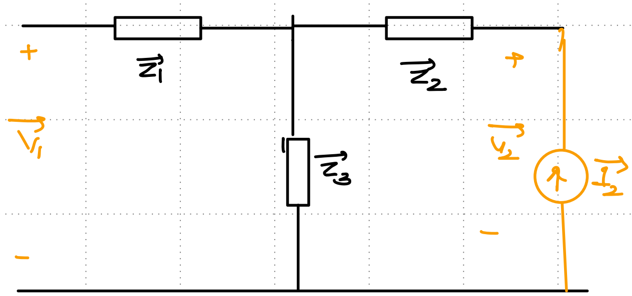
$$\left.\begin{aligned} \vec{Z}_{12} &= \frac{\vec{V_1}}{\vec{I_2}}\Bigg|_{\vec{I_1}=0} \\ \vec{Z}_{22} &= \frac{\vec{V_2}}{\vec{I_2}}\Bigg|_{\vec{I_1}=0} \end{aligned}\right\}$$

Solving a ckt with 1-1' open.



$$\vec{V_1} = (\vec{Z_1} + \vec{Z_3})\vec{I_1} \Rightarrow \vec{Z}_{11} = \vec{Z_1} + \vec{Z_3}$$

$$\vec{V_2} = \vec{Z_3}\vec{I_1} \Rightarrow \vec{Z}_{21} = \vec{Z_3}$$

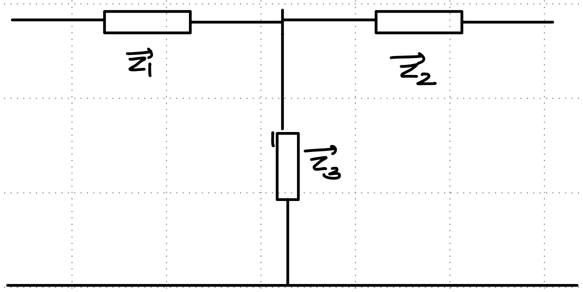


$$\vec{V_1} = \vec{Z_3}\vec{I_2} \Rightarrow \vec{Z}_{12} = \vec{Z_3}$$

$$\vec{V_2} = (\vec{Z_2} + \vec{Z_3})\vec{I_2} \Rightarrow \vec{Z}_{22} = \vec{Z_2} + \vec{Z_3}$$

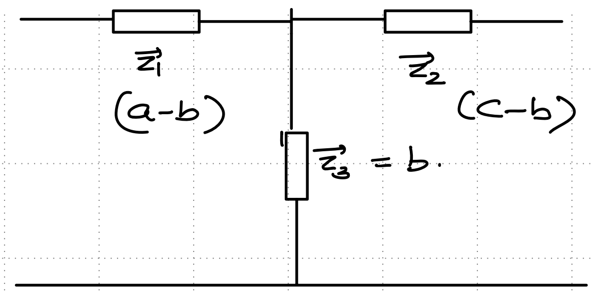
$$\begin{bmatrix} \vec{V_1} \\ \vec{V_2} \end{bmatrix} = \begin{bmatrix} \vec{Z_1} + \vec{Z_3} & \vec{Z_3} \\ \vec{Z_3} & \vec{Z_2} + \vec{Z_3} \end{bmatrix} \begin{bmatrix} \vec{I_1} \\ \vec{I_2} \end{bmatrix}$$

Example 2: T Network (Y-parameters)



Example 3: Equivalent Circuit for a Given Z-Parameter Matrix

given $\vec{Z} = \begin{bmatrix} a & b \\ b & c \end{bmatrix} \Rightarrow$



$$\vec{Z_3} = \vec{b}$$

$$\vec{Z_1} + \vec{Z_3} = a$$

$$\vec{Z_1} = a-b$$

given $\vec{h} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \Rightarrow \begin{bmatrix} \vec{V_1} \\ \vec{I_2} \end{bmatrix} = \begin{bmatrix} \vec{h_{11}} & \vec{h_{12}} \\ \vec{h_{21}} & \vec{h_{22}} \end{bmatrix} \begin{bmatrix} \vec{I_1} \\ \vec{V_2} \end{bmatrix}$