

1. Find the line integral of $f(x, y, z) = 2xy + \sqrt{z}$ over the helix

$$\mathbf{r}(t) = \cos t \mathbf{i} + \sin t \mathbf{j} + t \mathbf{k}, \quad 0 \leq t \leq \pi.$$

2. A fluid's velocity field is

$$\mathbf{F} = x \mathbf{i} + z \mathbf{j} + y \mathbf{k}.$$

Find the flow along the helix

$$\mathbf{r}(t) = (\cos t) \mathbf{i} + (\sin t) \mathbf{j} + t \mathbf{k}, \quad 0 \leq t \leq \frac{\pi}{2}.$$

3. Calculate the outward flux of the vector field

$$\mathbf{F}(x, y) = 2e^{xy} \mathbf{i} + y^3 \mathbf{j}$$

across the square bounded by the lines $x = \pm 1$ and $y = \pm 1$.

4. Find the circulation of the field $\mathbf{F} = (x^2 - y) \mathbf{i} + 4z \mathbf{j} + x^2 \mathbf{k}$ around the curve C in which the plane $z = 2$ meets the cone $z = \sqrt{x^2 + y^2}$, counterclockwise as viewed from above.
5. Use the surface integral in Stokes' Theorem to calculate the flux of the curl of the field \mathbf{F} across the surface S in the direction of the outward unit normal \mathbf{n} .

Given vector field:

$$\mathbf{F} = x^2 y \mathbf{i} + 2y^3 z \mathbf{j} + 3z \mathbf{k}$$

And the parameterized surface:

$$\mathbf{r}(r, \theta) = (r \cos \theta) \mathbf{i} + (r \sin \theta) \mathbf{j} + r \mathbf{k},$$

where $0 \leq r \leq 1$, $0 \leq \theta \leq 2\pi$.

6. (a) Calculate the flux of the vector field

$$\mathbf{F} = x \mathbf{i} + 4xyz \mathbf{j} + ze^x \mathbf{k}$$

out of the box-shaped region $D : 0 \leq x \leq 3, 0 \leq y \leq 2, 0 \leq z \leq 1$.

(b) Integrate $\nabla \cdot \mathbf{F}$ over this region and show that the result is the same value as in part (a), as asserted by the Divergence Theorem.

7. Find the net outward flux of the field

$$\mathbf{F} = \frac{x \mathbf{i} + y \mathbf{j} + z \mathbf{k}}{\rho^3}, \quad \rho = \sqrt{x^2 + y^2 + z^2}$$

across the boundary of the region $D : 0 < b^2 \leq x^2 + y^2 + z^2 \leq a^2$.