Due: February 21st (Thursday), 6pm, submit to courseworks.

You may use "standard" arguments without a formal proof—either cite from class or add one sentence showing the main idea.

## Problem 1: Working Set implies Entropy

Let $T$ be a binary search tree with a Working Set/Move to Front property, as defined in the second lecture: if $t(x)$ distinct elements were accessed since the last time element $x$ was accessed, then the number of operations performed to access element $x$ will be $O(\log t(x))$ amortized. Prove that this implies that $T$ has entropy property: if access sequence is generated from i.i.d. samples from distribution on $\{1, \ldots, n\}$ with probabilities $(p_1, \ldots, p_n)$, then on average $T$ makes $O(\sum_i -p_i \log p_i)$ operations per access.

## Problem 2: Splay trees have Working Set property

Prove that splay trees have the Working Set/Move to Front property. Use the Access Lemma proved in class, which states that the amortized search time for key $x$, with respect to any non-negative weight function $w(x) > 0$, is at most $3(\text{rank}_w(root) - \text{rank}_w(x)) + 1$. Use the weight function $w(x) = 1/(t(x)+1)^2$ and the fact that the series $\sum_i 1/i^2$ converges.

## Problem 3: Splay trees insertion analysis

Complete analysis of splay trees: prove that amortized insertion time for splay trees is $O(\log n)$. To do that, look at the inserted node $y_0$, let $y_i := \text{parent}(y_{i-1})$, and upper bound the change in potential by $O(\log n)$ when all weights are $w(x) = 1$.