Module: 1
The Concept of Time value of Money

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Module-I

CO1: Understand the concept of economics, engineering economics and its application in engineering

- Engineering Economics-its meaning and importance
- Basic problems of an economy.
- The concept of time value of money, Concept of Interest.
- Time value of equivalence, Compound interest factors.
- Cash flow diagrams,
- Calculation of time value of equivalence,
- Present worth comparison,
- Future worth comparison,
- Pay-back period comparison

Time Value of Money

- Money has time value
- Money value differs from one point of time to another
- Money has time value because of two important reasons:

1. Earning power of money
2. Purchasing power of money

Earning power of Money

- Time value of money is the idea that money that is available at the present time is worth more than the same amount in the future, due to its potential earning capacity.
- In simpler terms, it would be safe to say that a dollar was worth more yesterday than today and a dollar today is worth more than a dollar tomorrow.
- Money earns money as time advances
- The amount of money increases

Purchasing Power

- Money has purchasing power
- As purchasing power of money changes due to change in price level, value of money changes
- At the time of inflation value of money falls and at the time of deflation value of money rises
- Suppose you an income of Rs. 100 and you want to purchase rice

Price	Qty
20	5kg
10	10 Kg
5	20 Kg.

Interest

What is interest?

- Interest is manifestation of time value of money.
- It is the amount we owe using some ones else's money.
- In other words it is the amount some one pays for using your money

Why interest are charged?

- Money has time value
- Interest is charged by the lender for risk of non payment
- is a price for sacrifice of present consumption and waiting.
- From the borrower's point of view interest is given because a loan is both an obligation and opportunity

Concept of Interest

- Interest can be calculated in two ways:

Simple Interest

Compound Interest

Simple Interest: In case of simple interest, interest earned is directly proportional to the capital involved in loan. Simple interest is earned on the beginning principal.

Simple Interest: $I = P\ i\ N$

P = Present amount or principal amount

i = Interest Rate

N = Number of interest period.

$F = P+I$

$F= P + PiN$

$F= P(1+iN)$

F= Future amount

Ex: P =100 i = 10% N=2

$F = 100[\ 1 + (0.10 \times 2)\ ]= 120$

- When N is not a full year, there are two ways to calculate simple interest earned during the period of the load
- When ordinary simple interest is used , the year is divided in to twelve 30 day periods
- In exact simple interest, a year has precisely the calendar number of days, and N is the fraction of the number of days the loan is in effect that year
- Example: P =100 i= 10% N= 2months (January, Feb)

Ordinary Simple interest :

$F = 100\ [1 + (0.10 \times 2/12)]$

Exact Simple interest: $F = 100\left[1+(0.10)\left(\frac{31+28}{365}\right)\right]$

Compound Interest

- It is good to receive compound interest, but not so good to pay.
- interest is calculated not only on the beginning principal, but on any interest accumulated in the mean time

Example : P= 100 i = 10% N = 2

	At the beginning of the year	At the end of year 1	At the end of year 2
Simple Interest	100	110	120
Compound interest	100	110	121

Compound interest Formula

$$Compound\ Amount\ due\ in\ two\ years =$$

$$amount\ borrowed + year1\text{int}\ erest +$$

$$\left(amount\ borrowed\ \ plus\ year1\ \text{int}\ erest\ due\right)\left(\text{int}\ erest\ rate\right)$$

$$F_2 = P + Pi + \left(P + Pi\right)i$$

$$= P\left(1 + i + i + i^2\right)$$

$$= P\left(1 + i\right)^2$$

$$F_N = P\left(1 + i\right)^N$$

Nominal Interest rate

- Generally interest rates are quoted on annual basis. But sometimes interest rates may be compounded number of times, like monthly, quarterly and semi-annually
- For an example if the amount is compounded semi-annually there will be two compounding period. With interest rate of 5% for each half is typically called 10% compounded semi-annually
- **The nominal interest rate is base rate without the compounding effect.**

Example: P =1000 i = 12 %

$$F_N = P(1+i)^N$$
$$= 1000(1+0.12)^1$$
$$= 1000(1+0.06)^2$$
$$= 1000(1+0.03)^4$$
$$= 1000(1+0.01)^{12}$$

Effective Interest Rate

- The effective annual interest rate is simply the ratio of the interest charged for 1 year to the principal.
- In other words effective interest rate is the rate of interest which include the compounding effect of interest on interest

$$Effective\text{int}\ erest\ rate = \frac{F - P}{P}$$

$$i_{eff} = \left(1 + \frac{r}{m}\right)^{m} - 1$$

$$Ex: P = 1000\ \ i = 18\%$$

$$Calculate\ \ effective\text{int}\ erest\ rate\ if\ \ the\ amount\ \ is\ \ compounded$$

$$Semi - annually$$

$$Monthly$$

$$F_N = 1000(1 + 0.09)^2 = 1188$$

$$F_N = 1000(1 + 0.015)^{12} = 1196$$

$$i_{eff} = \frac{1188 - 1000}{1000} = 18.8\%$$

$$i_{eff} = \frac{1196 - 1000}{1000} = 19.6\ \%$$

$$Effective annual\ \ \text{int}\ erest\ can\ be\ obtained\ \ without\ \ reference\ to\ the\ \ principal$$

$$i_{eff} = \left(1 + \frac{0.18}{2}\right)^{2} - 1 = 18.8\%$$

Continuous Compounding

- In case of continuous compounding there is no limit of compounding period. The interest may compounded continually moment by moment. The effective interest rate for continuous compounding for nominal interest rate r is developed as follows:

$$i_{eff} = \lim_{m \mapsto \infty} \left(1 + \frac{r}{m}\right)^{m} - 1$$

Time- Value Equivalence

End of Year	Receipt from A (Rs)	Receipt from B (Rs.)
0	12,500	
1		2000
2	-	2000
3		2000
4	-	2000
5		2000
6	-	2000
7	-	2000
8	-	2000
9	-	2000
10	-	2000
Total	12,500	20,000

Time value of Equivalence

- If two and more situations are to be compared their characteristics must be placed on an equivalent basis. Which is worth more, 4 kg of product A or 500 gms of product . To answer this question, it is necessary to place the two amounts on an equivalent basis by use of the proper conversion factor. After conversion of kg to grams, the question becomes, which is worth more, 4 kg of product A or 500 gms of product

Two things are said to be equivalent when they have the same effect.

- Three elements are involved in the equivalence of series of money. These are
- The amounts of the sums
- The times of occurrence of the sums and
- The interest rate

In the above example we can compare both the option by braining them to a common base.

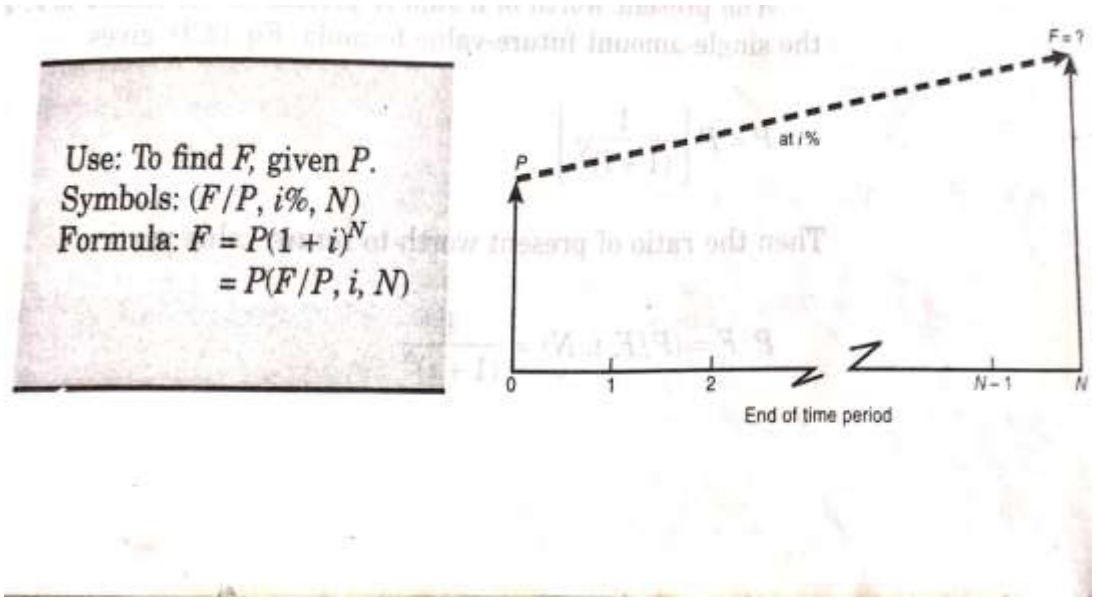
- Either we can convert the single value to series of values

  12500(A/P,10,10)=2034.75
- Or we can convert series of values to single value
- Series can be compared with series
- **2000 (P/A,10,10)= 12289.14**
- Single present value can be compared with single present value
- Single future values can be compared with single future values

Compound-Interest Factors

There are 8 conversion factors. They are called as compound-interest factors. Some conversion factors help us in converting single value to single value and some conversion factors help in converting single value to series of values.

Compound amount Factor (single value to single value):



Compound Amount Factor

Example: **If you deposit Rs. 100 in your bank account today at an interest rate of 10% , what amount you will withdraw from the bank after two years**

P= 100 i =10% N= 2 F = ?

F = P(F/P,10%,2)

= 100 (F/P, 10% ,2)

= 100 (1.2100)= 121

Discrete Interest Table

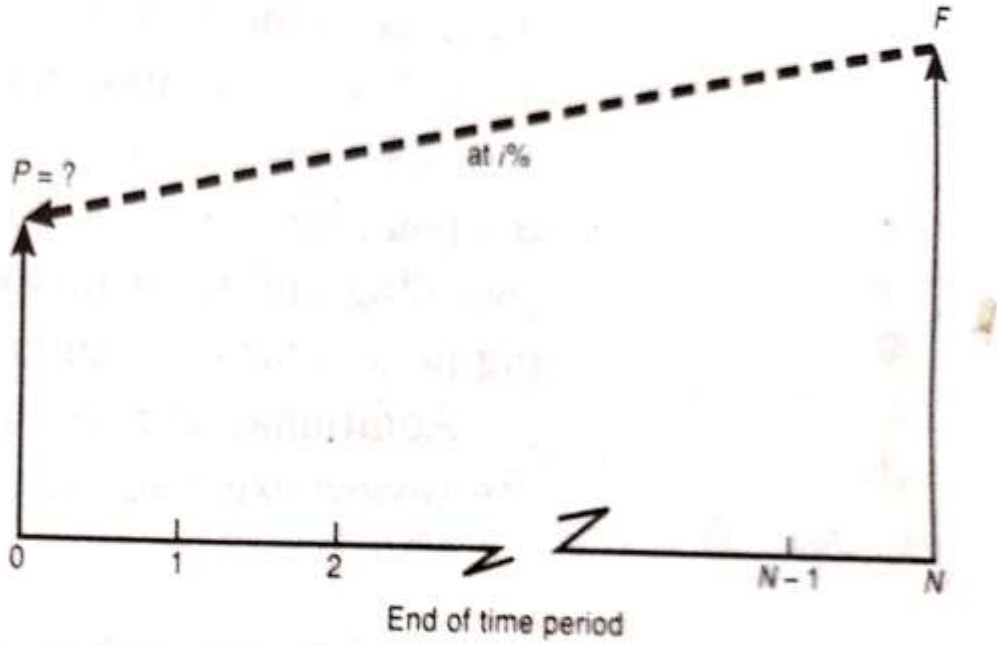
A-76

10% interest factors for discrete compounding periods

N	SINGLE PAYMENT		UNIFORM SERIES					N
	Compound Amount Factor	Present Worth Factor	Capital Recovery Factor	Present Worth Factor	Sinking Fund Factor	Compound Amount Factor	Gradient Factor	
	(F/P,10,N)	(P/F,10,N)	(A/P,10,N)	(P/A,10,N)	(A/F,10,N)	(F/A,10,N)	(A/G,10,N)	
1	1.10000	0.90909	1.10000	0.90909	1.00000	1.00000	0.00000	1
2	1.21000	0.82645	0.57619	1.73554	0.47619	2.10000	0.47619	2
3	1.33100	0.75131	0.40211	2.48685	0.30211	3.31000	0.93656	3
4	1.46410	0.68301	0.31547	3.16987	0.21547	4.64100	1.38117	4
5	1.61051	0.62092	0.26380	3.79079	0.16380	6.10510	1.81013	5
6	1.77156	0.56447	0.22961	4.35526	0.12961	7.71561	2.22356	6
7	1.94872	0.51316	0.20541	4.86842	0.10541	9.48717	2.62162	7
8	2.14359	0.46651	0.18744	5.33493	0.08744	11.43589	3.00448	8
9	2.35795	0.42410	0.17364	5.75902	0.07364	13.57948	3.37235	9
10	2.59374	0.38554	0.16275	6.14457	0.06275	15.93742	3.72546	10
11	2.85312	0.35049	0.15396	6.49506	0.05396	18.53117	4.06405	11
12	3.13843	0.31863	0.14676	6.81369	0.04676	21.38428	4.38840	12
13	3.45227	0.28966	0.14078	7.10336	0.04078	24.52271	4.69879	13
14	3.79750	0.26333	0.13575	7.36669	0.03575	27.97498	4.99553	14
15	4.17725	0.23939	0.13147	7.60608	0.03147	31.77248	5.27893	15
16	4.59497	0.21763	0.12782	7.82371	0.02782	35.94973	5.54934	16
17	5.05447	0.19784	0.12466	8.02155	0.02466	40.54470	5.80710	17
18	5.55992	0.17986	0.12193	8.20141	0.02193	45.59917	6.05256	18
19	6.11591	0.16351	0.11955	8.36492	0.01955	51.15909	6.28610	19
20	6.72750	0.14864	0.11746	8.51356	0.01746	57.27500	6.50808	20
21	7.40025	0.13513	0.11562	8.64869	0.01562	64.00250	6.71888	21
22	8.14027	0.12285	0.11401	8.77154	0.01401	71.40275	6.91889	22
23	8.95430	0.11168	0.11257	8.88322	0.01257	79.54302	7.10848	23
24	9.84973	0.10153	0.11130	8.98474	0.01130	88.49733	7.28805	24
25	10.83471	0.09230	0.11017	9.07704				

Present-worth Factor
(Single value to single value)

Use: To find P, given F.
Symbols: $(P/F, i\%, N)$
Formula: $P = F[1/(1+i)^N]$
$= F(P/F, i, N)$



Present-worth Factor

Ex: If you want to have Rs. 121 in your bank account after two years at 10 interest rate, what amount you will deposit today

F=121 , i = 10% , N=2 , P =?

P = F(P/F,10%, 2)

P= 121 (0.82645)

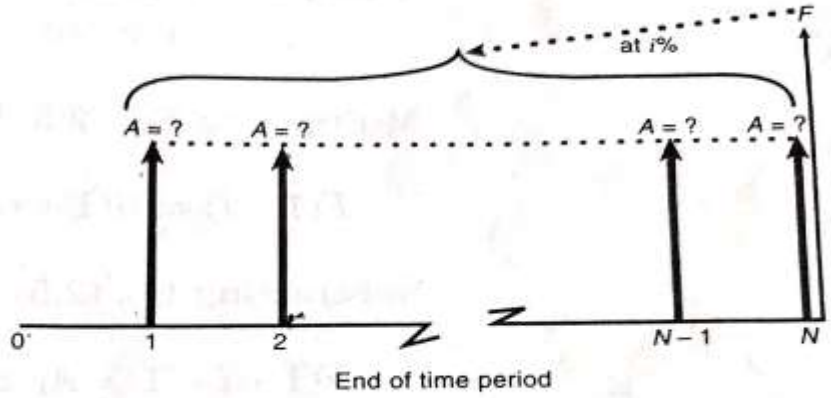
= 100

Sinking-Fund Factor (A/F , i, N)

(Single future to series)

A fund established to accumulate a given future amount through the collection of uniform series of payments is called a sinking fund. Each payment has a constant value A is made at the end of an interest period

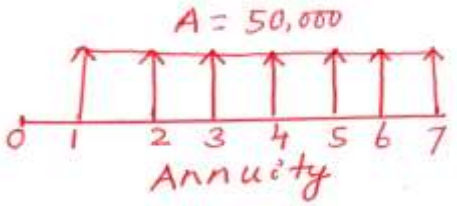
Use: To find A, given F.
Symbols: $(A/F, i\%, N)$
Formula: $A = F\{i/[1 + i)^N - 1]\}$
$= F(A/F, i, N)$



Annuity

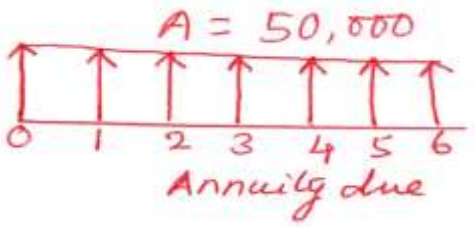
A is called Annuity

Annuity is a series of payments having three important characteristics



- **Series of uniform payments**
- **Uniform time gap**
- **The payments are made at the end of the period**

Annuity due is also a series of payments having uniform time gap and uniform payments , but the payments are made at the beginning of the period rather than the end of the period.



Sinking-fund factor

Ex:

You want to have Rs. 50,000 in your bank account after five years, If the prevailing interest rate is 10 percent compounded annually what amount you will deposit at the end of each year?

F = 50,000 , i = 10% , N=5 , A=

A= F(A/F, 10%, 5)

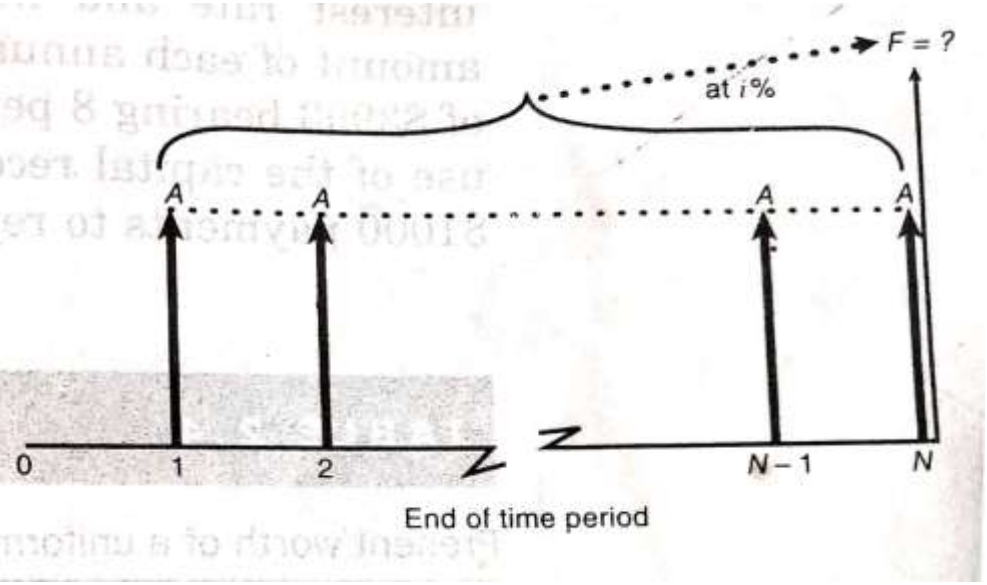
=50,000(0.16380)

= 8190

Series-compound amount factor(F/A ,i, N)
(Series to single Future)

Use: To find F, given A
Symbols: $(F/A, i\%, N)$
Formula:

$$F = A\frac{(1+i)^N - 1}{i}$$
$$= A(F/A, i, N)$$



Ex: You decided today to make a deposit of Rs. 8190 in a bank at the end of each year for coming 5 years. If the prevailing interest rate is 10 % compounded annually, what amount you will withdraw at the end of the 5^{th} Year?

A= 8190 , i = 10% , N= 5 , F = ?

F= 8190 (F/A,10, 5)

= 8190 (6.10610)

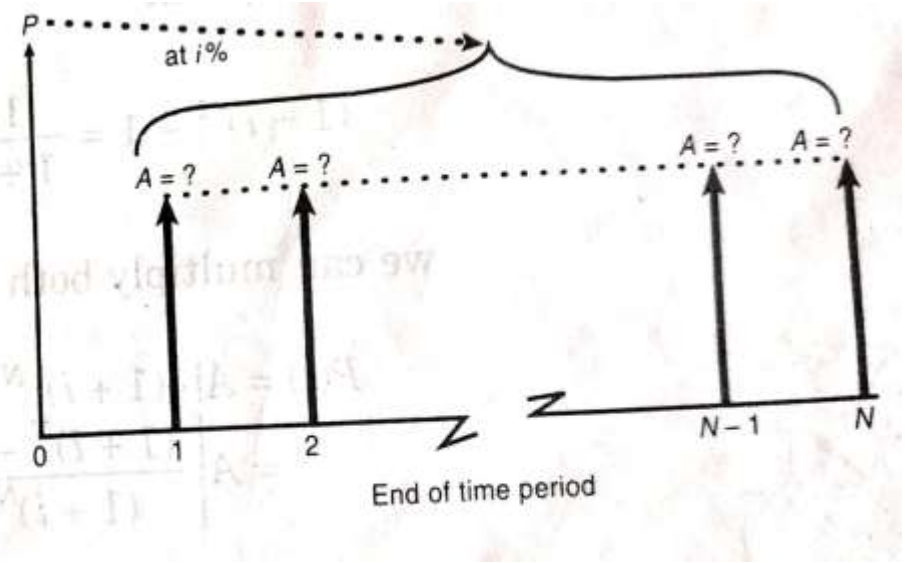
= 50,000

Capital-recovery factor (A/P,i, N) (Single present to series)

The capital-recovery factor is used to determine the amount of each future annuity payment required to dissipate a given present value when the interest rate and number of payments are known

Use: To find A, given P.
Symbols: $(A/P, i\%, N)$
Formula:

$$A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right]$$
$$= P(A/P, i, N)$$



Example: A company has borrowed Rs.200,000 to purchase equipment. The loan carries an interest rate of 5% per year and is to be repaid in equal installments over the next 7 years. What is the amount of the annual installment

Solution:

P= 200000 , i= 5% N=7 A=?

200000 (A/P,5,7)

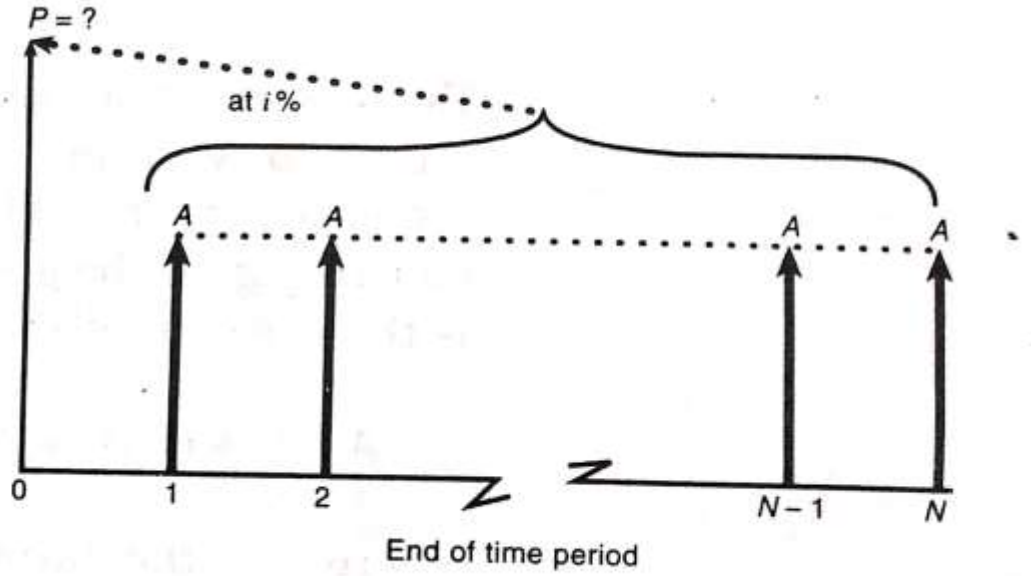
= 200000(0.1728)

- = 34560

Series present-worth Factor (P/A,i,N)

Use: To find P, given A.
Symbols: $(P/A, i\%, N)$
Formula:

$$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right]$$
$$= A(P/A, i, N)$$



A working woman is planning for her retired life. She has 20 more years of service. She would like to have an annual equivalent amount of Rs. 3, 00,000 starting from the end of the first year of her retirement. Find the single amount that should be deposited now so that she receives the above mentioned annual equivalent amount at the end of every year for 20 years after her retirement. Assume i = 15 %, compounded annually.

- A = Rs. 3,00,000, i= 15%, N= 20, Find P =?

P = A(P/A,i, N)

P = 3,00,000 (P/A,15,20)

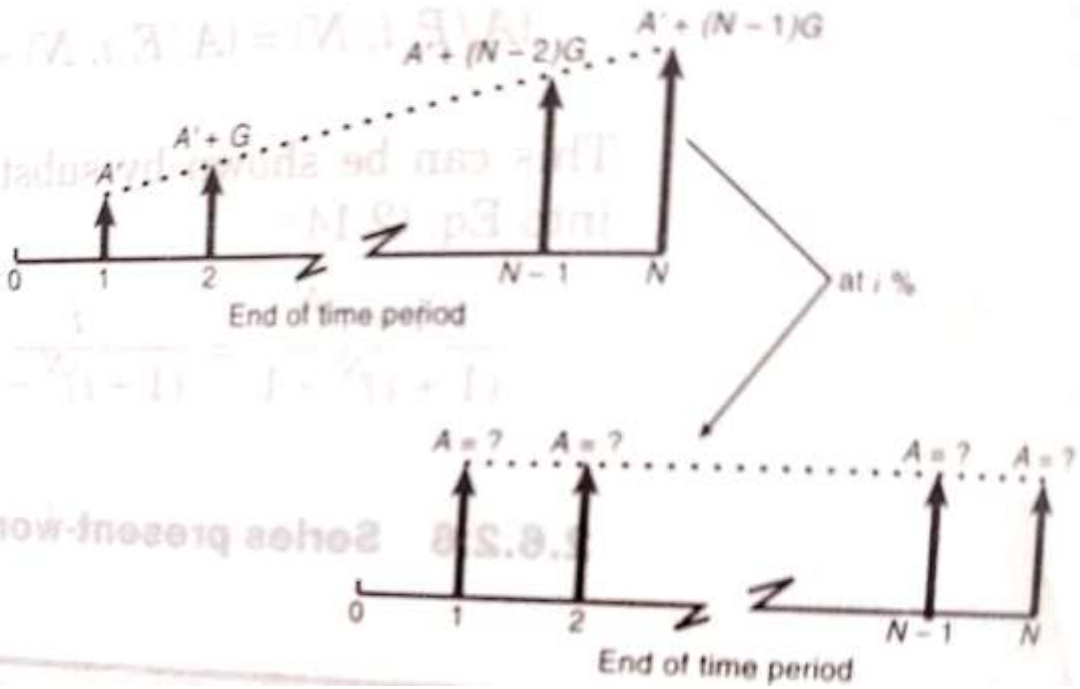
P = 3,00,000 (6.25933)

P = Rs. 1877799

Arithmetic gradient conversion factor (A/G,i,N)

Use: To find A, given G.
Symbols: $(A/G, i\%, N)$
Formula:

$$A = G\left[\frac{1}{i} - \frac{N}{(1+i)^N - 1}\right]$$
$$= G(A/G, i, N)$$



Arithmetic gradient series

The series may be Increasing gradient or Decreasing gradient. In both cases the series may be converted to annuities

$A = A' + G (A/G, i, N)$

$A = A' - G (A/G, i, N)$

A person is planning for his retired life. He has 10 more years of service. He would like to deposit 20% of his salary, which is Rs. 10,000 at the end of the first year and thereafter he wishes to deposit the same amount (Rs.10,000) with an annual increase of Rs. 2,000 for the next 9 years with an interest rate of 20%. Find what equivalent annual series of income it will create and also find the total amount at the end of the 10th year of the above series.

A' = 10,000, G = 2000, N=10, i = 20%, A =? F= ?

A = A' + G (A/G,20,10)

A = 10,000 + 2000 (4.4643)

=10,000 + 8928.6

= 18928.6

F = 18928.6(F/A,20,10)

= 1898.6(186.688)

= 3533742.47

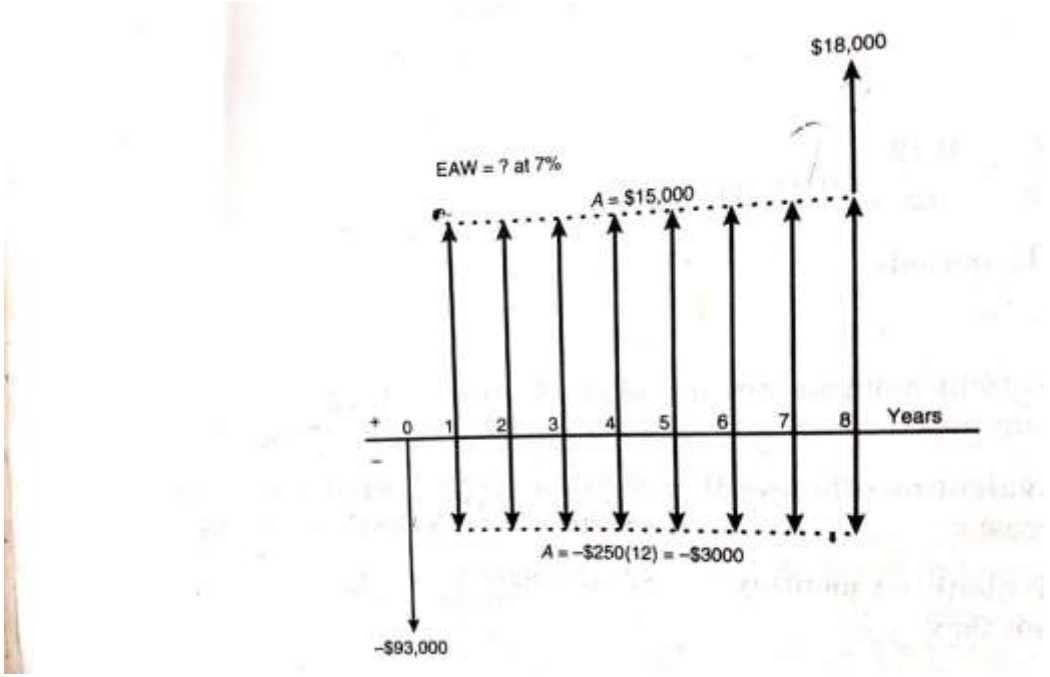
Geometric Gradient Series

A geometric gradient series is a nonuniform progression that grows or declines at a constant perc.entage rate per period

Cash –Flow Diagrams

A **cash flow diagram** allows you to graphically depict the timing of the cash flows as well as their nature as either inflows or out-flows. Such a diagram is very easy to construct. We start with a simple horizontal time line and then add arrows to represent the **inflows** (arrows pointing *from* the line) or **outflows** (arrows pointing *to* the line) of cash …

Particulars	$
Initial Cost	93,000
Annual Revenue	15,000
Annual expenses	3000
Salvage Value	18000



Calculations of Time Value of Equivalence

So far we have learned

Given Value			Find	Symbol
P	*i*	N	F	F/P
F	*i*	N	P	P/F
F	*i*	N	A	A/F
A	i	N	F	F/A
P	i	N	A	A/P
A	i	N	P	P/A
P	**F**	**N**	**i**	
P	**i**	**F**	**N**	

Unknown interest rate

Ex: At what interest rate Rs. 10,000 invested today will be equivalent to Rs. 30,000 after 10 years.

Given P= 10,000, F= 30,000 n= 10 years Find I

F= P(F/P, i , N)

F/P= F/P, i,N

30000/10000= F/P, i,N

3= F/P, i, N

Assuming i at 11% , (F/P,11,10) = 2.83942

Assuming i at 12% ,(F/P,12,10) = 3.10586

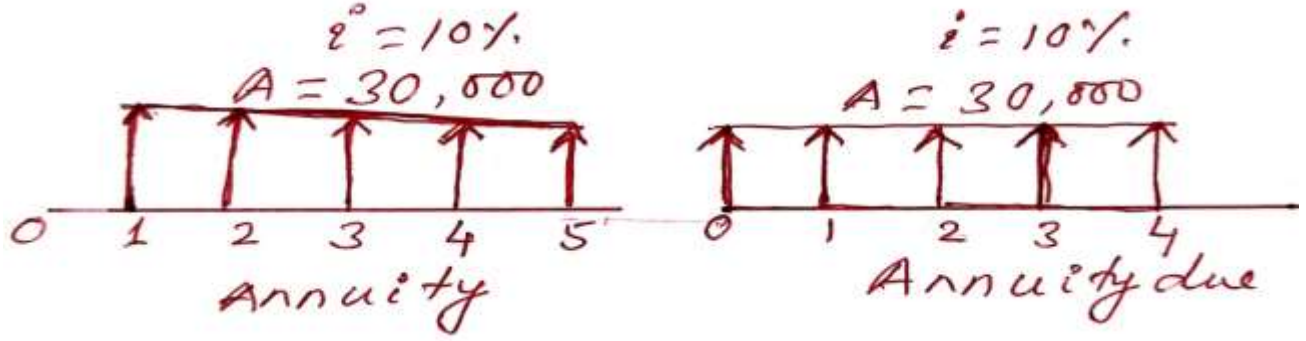
Then by interpolation.

$$i = 0.11 + 0.01\left[\frac{3 - 2.83942}{3.10586 - 2.83942}\right]$$

$$= 11.5\%$$

Present worth of Annuity Due

Find the present-worth of the given figures:



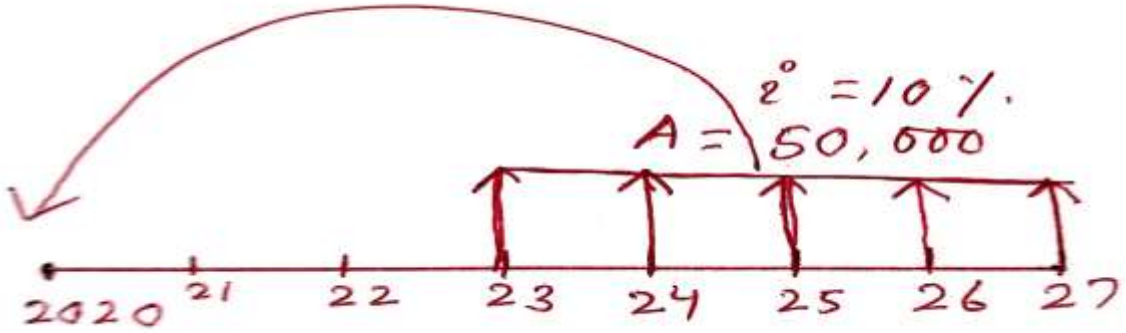
P = 30,000 (P/A,10, 5)

P= 30,000 + 30,000 (P/A,10,4)

Present-worth of Deferred Annuity

Another pattern for a series of payments, in which the first payment does not begin until some date later than the end of the first period , is called a deferred annuity.

Ex: **With interest rate of 10 percent, what is the worth on December 31,2020, of a series of year-end payments of Rs 50,000 made from the years 2023 through 2027.**

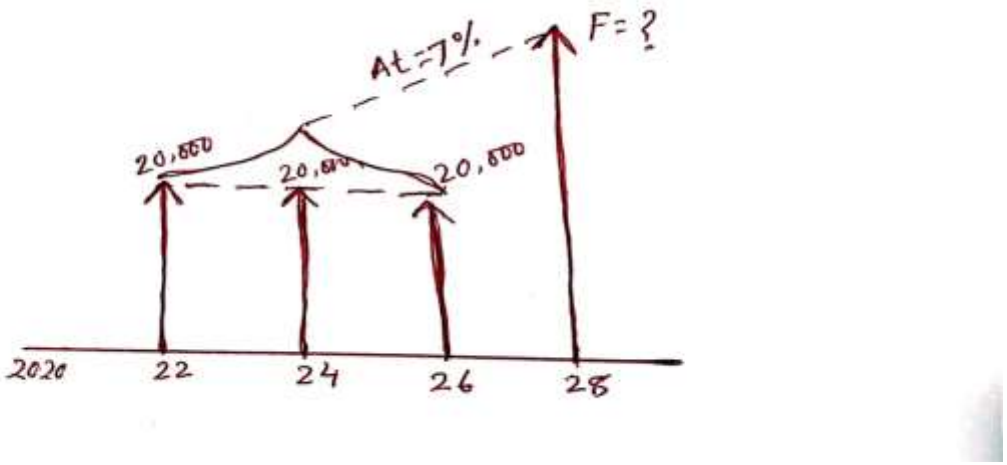


P = 50,000 (P/A,10,5) (P/F,10,2)

Multiple payment cash-flows

(More compounding period than Payments)

Today(2020) we decided to make three payments of Rs. 20,000 each in every two years. The prevailing interest rate is 7 percent per year. How large the will the bank account be in 2028.



FW= 20,000 (F/P,7,6) + 20,000 (F/P,7,4) + 20,000(F/P,7,2)

Income & outlay

Initial cost = 1,00,00

Annual revenue = 30,000

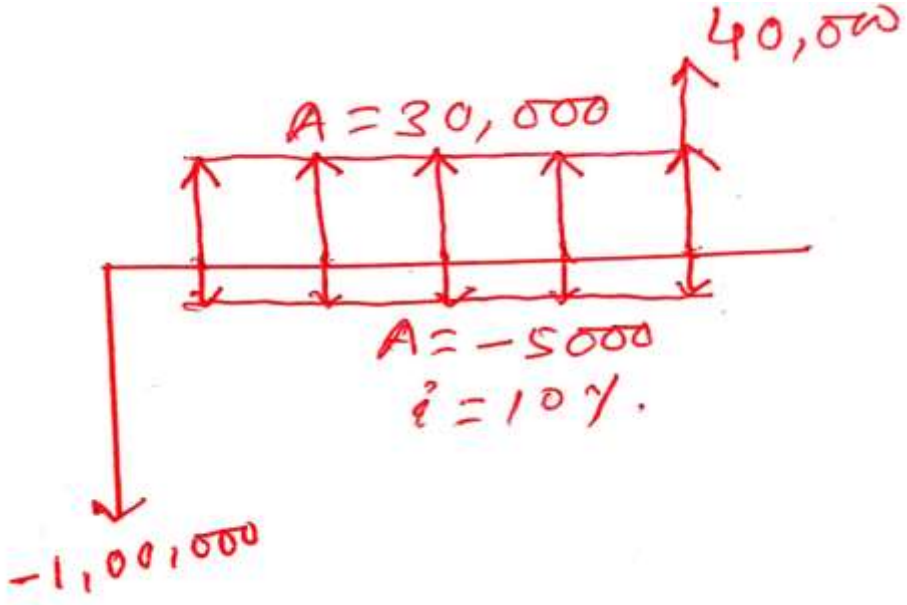
Annual expenses = 5000

Salvage value= 40,000

i = 10%

N= 5

Find present worth and future worth



PW = -1,00,000 + 30,000(P/A,i, N) – 5000 (P/A,i,N) + 40,000 (P/F,i,N)

PW = -1,00,000 + 30,000(P/A,10, 5) – 5000 (P/A,10,5) + 40,000 (P/F,10,5)

PW = -1,00,000 + 25000 (P/A,10,5) + 40,000 (P/F,10,5)

FW= -1,00,000(F/P,10,5) + 25000 (F/A,10,5) + 40,000

Problems

The net income from a newly purchased piece of construction equipment is expected to be Rs 12,000 in the first year and to decrease by Rs. 1500 each year as maintenance cost increase. The equipment will be used for four years. What annual annuity will produce an equivalent income , when the interest rate is 8 percent.

Answer

Given;

A’ =12,000

G=1500

i=8%

N=4

Find A

A= A’-1500 (A/G,i,4)

A= 12,000-1500(1.40396)

=12,000-2105.94= 9894.06

The rights to patent have been sold under an agreement in which annual year-end of payments of Rs. 10,000 are to be made for the next ten years. What is the current worth of the annuity at an interest rate of 7 percent.

Given:

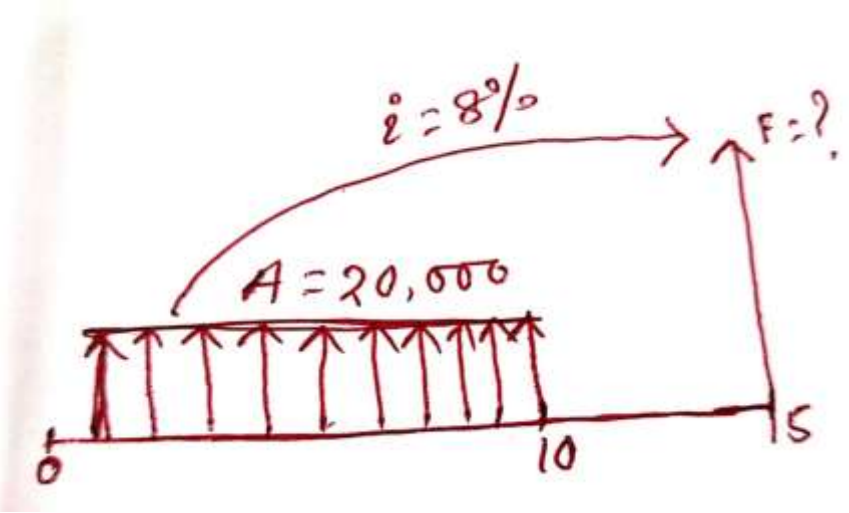
A=10,000 , i=7% N= 10, Find P

P= 10,000 (P/A,7,10)

P= 10,000(7.02358)

=70235.8

A company deposits Rs. 20,000 in a bank at the end of every year for 10 years. The company makes no deposit during the subsequent 5 years. If the bank pays 8 percent interest, how much would be in the account at the end of 15 years.



20,000(F/A,8,10) (F/P,10,5)

20,000(14.48656)(2.15892)

=625506.48

A company buys a machine for Rs. 1,20,000 which it agrees to pay for in 5 equal annual payments, beginning one year after the date of purchases, at an interest rate of 4 percent per annum. Immediately after the second payment, the terms of agreement are changed to allow the balance due to be paid off in a single payment the next year. What is the final single payment?

1,20,000 (A/P,4,5)=

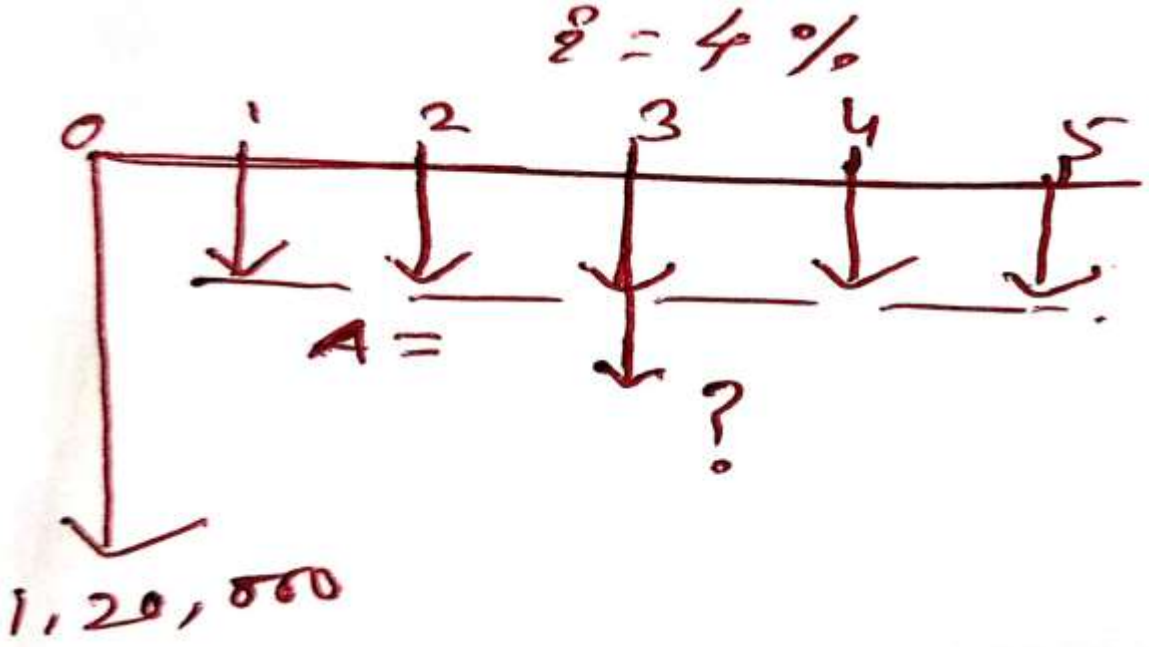
120000(0.22463)

=26955.6

P= 26955.6+ 26955 .6(P/A,4,2)

= 26955.6+ 26955 .6(1.88609)

=26955.6+50840.687=77796.287



You estimate you will need Rs. 1,00,000 per year for thirty years starting on your 65^{th} birthday to live on during your retirement. Today is your 50^{th} birthday and you want to make equal deposits to an account paying 7 percent per year, the first deposit today and the last deposit on your 64^{th} birthday. How much each deposit be?

1,00,00(P/A,7,30)

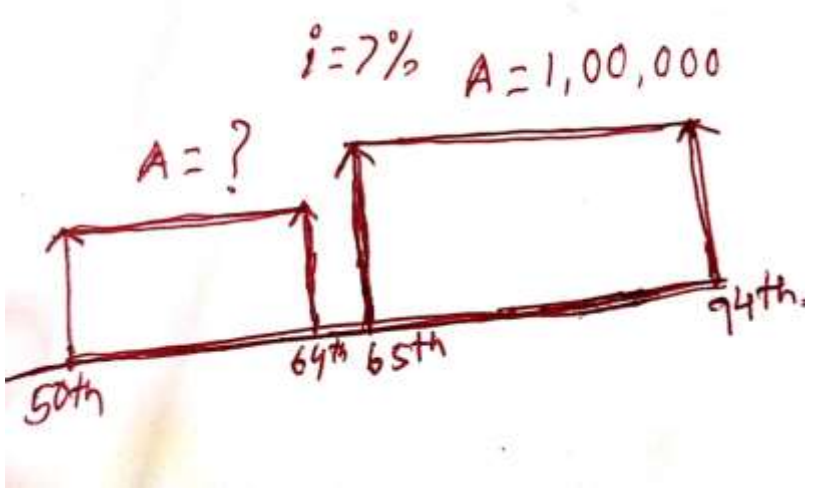
100000(12.40904)

=1240904

1240904(A/F,7,15)

1240904(0.04434)

=55021.683



An engineer borrowed Rs. 3000 from the bank , payable in 6 equal end-of year payments at 8 percent. The bank agreed to reduce the interest on the loan if interest rates declined before the loan was fully repaid. At the end of 3 years, at the time of third payment, the bank agreed to reduce the interest rate from 8 percent to 7 percent on the remaining debt. What was the amount of the equal annual end of year payments for each of the first 3 years? What was the amount of the equal annual end-of –year payments for each of the last 3 years?

3000 (A/P,8,6)

3000 (0.21632)

=648.96

648.96(P/A,8,3)

648.96(2.57710)=1672.434

1672.434(A/P,7,3)

1672.434(0.38105)

=637.21

A building site for new gasoline station was purchased 10 years ago for Rs. 50,000.The site has been recently sold for Rs.1,20,000. Disregarding any taxes , determine the rate of interest obtained on the initial investment.

A person needs a sum of Rs. 2,00,000 for his daughter's marriage which will take place 15 years from now. Find the amount of money that he should deposit now in a bank if the bank gives 18 percent interest, compounded annually.

F =2,00,000

i= 18%

N= 15

P=?

P = 2,00,000 (P/F,18,15)

200000(0.0835)

=16700

- A sinking fund is created for the redemption of a loan of Rs. 50,000 at the end of 10 years. How much money should be provided out of profits each year of the sinking fund, if the interest rate is 5% p.a.

F = 50,000

i= 5%

A= ?

N= 10

A= 50,000 (A/F,5,10)

=50,000 (0.07950)

= 3975

Present-worth Comparisons

Dr. Mahendra P. Agasty

Meaning of Present-worth Comparison

- There are different of comparing proposals like, present worth, Future-worth, equivalent annual worth, Rate of return
- present worth comparisons are the most widely used, utilized methods for making investment decisions
- Many economists prefer a present worth analysis because it reveals the sum in today's rupee that is equivalent to a future cash flow stream

Conditions

- Cash flows are given
- The interest rate is known.
- Purchasing power of money remain constant
- Comparisons should be made before taxes.

Selection Criteria

Single Proposal:

PW> 0 (Select the proposal)

PW < 0 (Reject the proposal)

PW = 0 (Indifferent)

Mutual Exclusive cases:

PW (A)= 5423

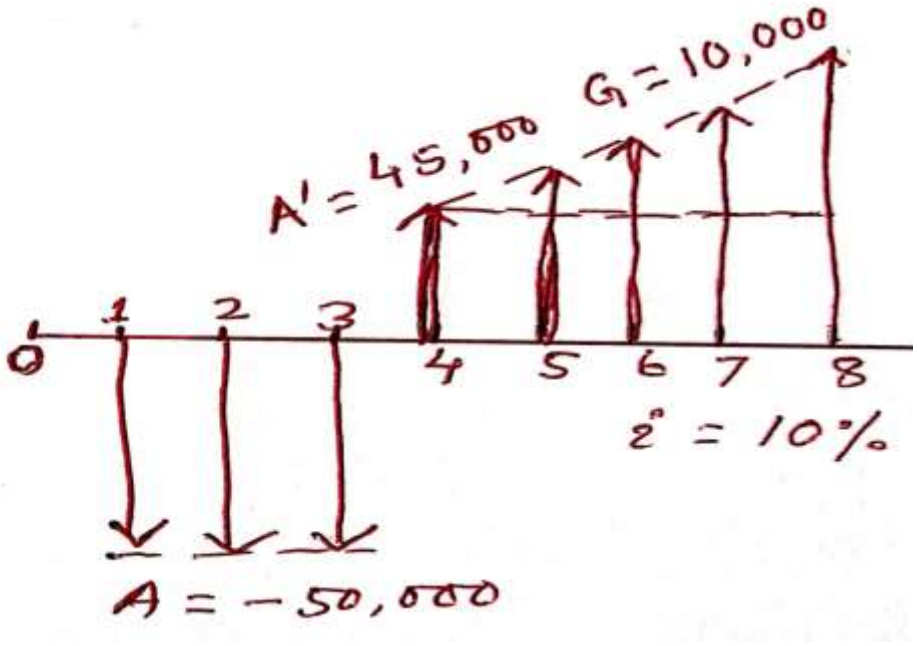
PW(B)= 5432

PW (Cost) = 5423

PW (Cost) = 5432

Single Proposal

An Investor can make three end-of-year payments of Rs. 50,000 , which are expected to generate receipts of Rs, 45,000 at the end of year 4 that will increase annually by Rs.10,000 for the following 4 years. If the investor can earn a rate of return of 10 percent on other 8 year investments ,is this alternative attractive



Solution

PW = -50,000 (P/A,10,3) + [45,000 + 10000 (A/G,10,5)](P/A,105)(P/F,10,3)

= -50,000 (2.48685) + [45,000 + 10000 (1.81013)](3.79079)(0.75131)

=-124342.5 + [45,000 + 18101.3](3.79079)(0.75131)

=-124342.5 + 63101.3 (3.79079)(0.75131)

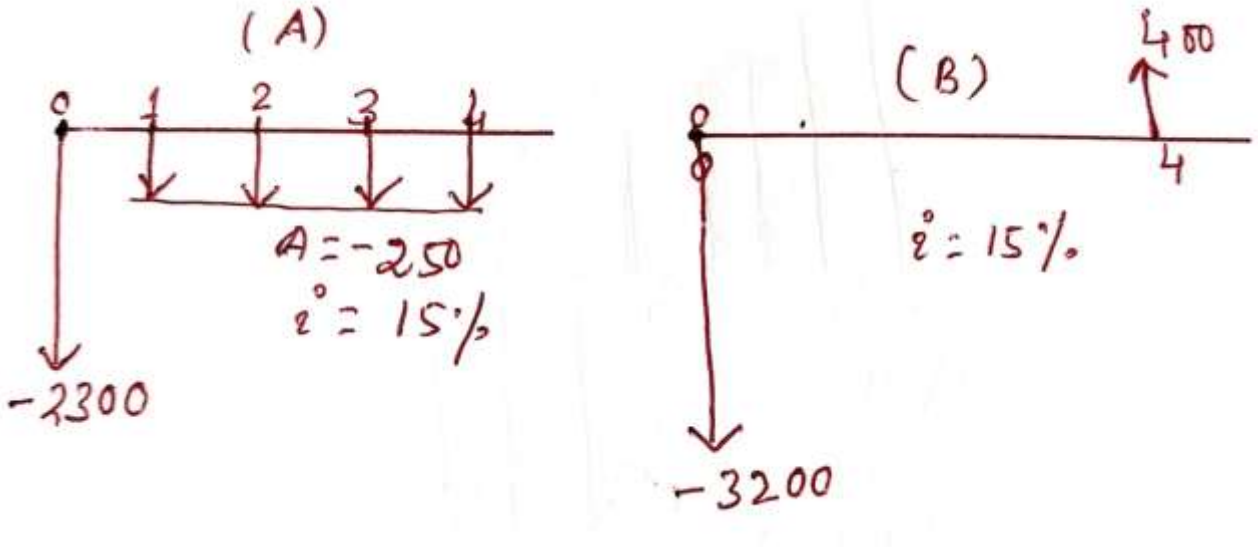
=-124342.5 + 179716.189

=55363.689

Mutual Exclusive case

(Comparison of assets having equal lives)

	A	B
Initial Cost	2300	3200
Annual Expenses	250	0
Salvage Value	0	400
Life (N)	4	4
i	15 %	



PW (A) = -2300 -250(P/A,15,4)

= -2300 – 250 (2.85498)

= -2300 -250 (2.85498)

= -2300 – 713.745

= **-3013.745**

PW (B)= -3200 +400 (P/F,15,4)

= -3200 +400 (0.49718)

=-3200 + 198.872

= **-3001.128**

Comparison Assets having unequal lives

There are two prominent methods to compare the assets having unequal lives.

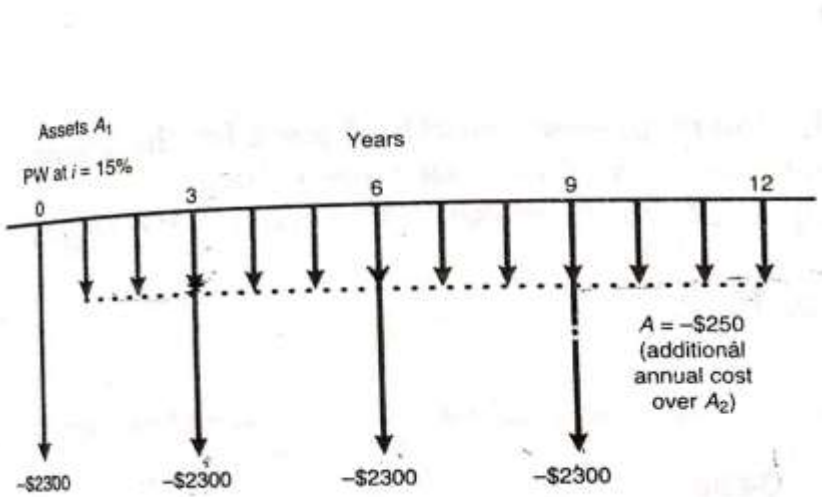
1. Common-multiple Method

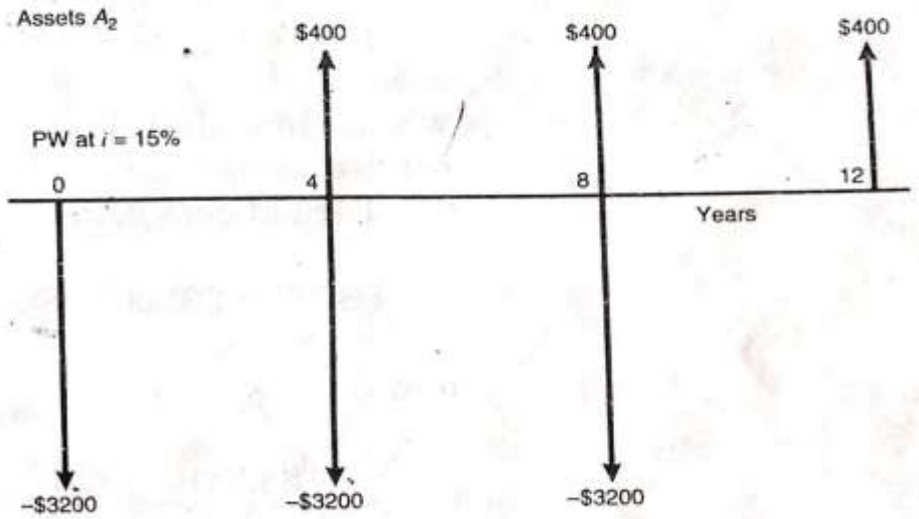
In common multiple method we have to select one analysis period for comparing the assets (Common multiple of the lives of the asset involved)

2. Study-period method (Shortest life span)

Comparison of Assets having unequal lives (Common-multiple method)

	A	B
Initial Cost	2300	3200
Annual expenses	250	0
Salvage Value	0	400
Life (N)	3	4
i	15	





PW (A)= -2300 -2300 (P/F,15,3) -2300 (P/F,15,6) -2300 (P/F,15,9) -250 (P/A,15,12)

= -2300 -2300 (0.65752)- 2300 (0.43233)-2300 (0.28426)- 250 (5.42062)

= -6816

PW (B) = -3200 -2800 (P/F,15,4)-2800 (P/F,15,8)+ 400 (P/F,15,12)

PW (B) = -3200 -2800 (P/F,15,4) -2800 (P/F,15,8) + 400 (P/F,15,12)

= -3200 -2800 (0.57175) -2800 (0.32691)+ 400 (0.18691)

=-5642

Study Period

Incase of study period method we have to select a common period from both the assets for comparison, we assume we sell the asset at the end of the study period, which results in salvage value of the asset. It may be very difficult to assume the salvage value at the end of the study period. Suppose in this example, we assume the study period is 2 years having no salvage value

PW (A) = -2300 -250(P/A,15,2)

= -2300 -250(1.62571)

=-2707

Pw (B) = **-3200**

In this case we select A, but the result contradict. The reason is salvage value that was present in the original question.

Suppose now we want to bring back salvage value to B after two years and what should be the salvage of B, if we want to select B again

2707= 3200 –S(P/F,15,2)

S = 3200-2707/P/F,15,2= 493/0.75614= 652

Comparison of Assets having infinite lives

The sum of the first cost and the present –worth of disbursements last forever is called a capitalized cost

$$Capitalised\ \ Cost = First\ \ Cost + A(P/A, i, \infty)$$

$$We\ know P/A, i, N = \frac{1}{i}\frac{(1+i)^N - 1}{(1+i)^N}$$

$$The \lim it\ of\ (P/A, i, N) as\ N\ approaches \inf inity\ is$$

$$(P/A, i, \infty) = \frac{1}{i}$$

$$So\ \ C.C = P + A(P/A, i, \infty)$$

$$= P + A\left(\frac{1}{i}\right)$$

$$= P + \frac{A}{i}$$

$$where\ A\ is\ the\ uniform\ \ difference\ between\ annual\ receipts\ \ and\ disbursements.$$

$$when\ \ there\ \ is\ no\ \ \ revenue, the\ \ formula\ \ becomes$$

$$C.C = P + \frac{disbursements}{i}$$

You are given Rs. 10,00,000 by your institute for the opening and upkeep of a music cell in your campus. Annual maintenance of the music cell is expected to be Rs. 25,000. You also need Rs. 1,00,00 in every 10 years for major equipments purchase. How much will be left for the initial opening costs ,after funds are allocated for perpetual upkeep? Deposited funds can earn 7 percent annual interest .

$$First\ Cost = capitalised\ \cos t - \frac{Annual\ disbursements}{i}$$

$$= 1000000 - \frac{25{,}000 + 1{,}00{,}000(A/F,7,10)}{0.07}$$

$$= 10{,}00{,}000 - \frac{25{,}000 + 1{,}00{,}000(0.07238)}{0.07}$$

$$= 10{,}00{,}000 - \frac{25000 + 7238}{0.07}$$

$$= 10{,}00{,}000 - \frac{32238}{0.07}$$

$$= 539457.1429$$

Future-worth Comparison

- Net present worth measures the surplus in an investment project at time 0
- Net future worth is a particularly useful investment situation where we need to compute the equivalent worth of a project at the end of its investment period, rather than it's beginning

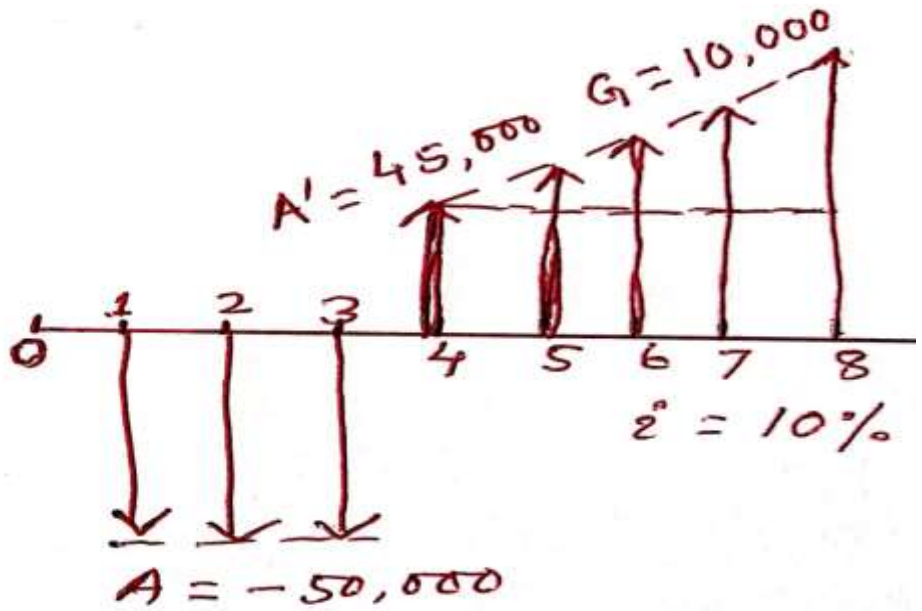
Single proposal

If FW (i)>0, accept the investment

If FW (i)=0 remain indifferent to investment

If FW (i)<0, reject the investment

Find the Future-worth of the cash-flow diagram given below.



FW=50,000(F/A,10,3)(F/P,10,5)+[45,000+10,000(A/G,10,5)(F/A,10,5)

Pay-back Period Comparison

- The pay-back method is otherwise known as the pay out method
- It is used to obtain a rough estimate of the time that an investment will take to pay for itself
- **Conventional payback period method:** The simplest method. Ignores the time value of money
- **Discounted payback period method:** It includes the time value of money; it is more complex to conduct that method

$$Pay-back\ period = \frac{\text{Required Investment}}{Annual\ \text{Receipts} - Annual\ disbursements}$$

$$= \frac{\text{Required Investment}}{Net\ Annual\ Savings}$$

End of year	A	B	C
0	-1000	-1000	-700
1	500	200	-300
2	300	300	500
3	200	500	500
4	200	1000	0
5	200	2000	0
6	200	4000	0

Problems

- If Utkal limited expects cash inflows from its investment proposal it has undertaken in time period zero, Rs 2,00,000 and Rs 1,50,000 for the first two years respectively and than expects annuity payment of Rs 1,00,000 for the next eight years, what would be the present value of cash inflows , assuming a ten percent rate of interest?

PW = 2,00,000 (P/F,10,1)
+150000(P/F,10,2)+100000(P/A,10,8)(P/F,10,2)

PW = 2,00,000 (0.90909)
+150000(0.82645)+100000(5.33493)(0.82645)

PW= 181818+123967.5+440905.289

PW=746690.789

- Mr. Patnaik has ten more years of service .He is planning for his retired life. He would like to deposit Rs 8500 in the first year and thereafter he wants deposit the amount with an annual decrease of Rs 500 for the next 9 years with an interest rate of 15%. Find the total amount at the end of 10^{th} year of the above series

A’= 8500, G=500, N=10, i=10% F=?

A= 8500-500 (A/G,10,10)

= 8500 -500(3.72546)

= 8500 -1862.73=6637.27

6637.27(F/A,10,10)

= 6637.27(15.93742)

= 105780.959

- An investor has an option to purchase a tract of land that will be worth Rs. 10,000 in six years. If the value of the land increases at 8% each year, how much should the investor be willing to pay now for this property?

F= 10000 , N=6 , i=8%, P= ?

P = 10000 (P/F,8,6)

= 10000 (0.63017)

=6301.7

A proposed improvement in an assembly line will have n initial purchase and installation cost of Rs. 1,75,000.The annual maintenance cost will be Rs 6000; periodic overhauls once every three years excluding the last year of use, will cost Rs 11,500 each. The improvement will have a useful life of 9 years at which time it will have no salvage value. What is the present worth of the 9 year cost of the improvement at i = 8 percent.

PW (Cost) = 175000 + 11,500(P/F,8,3) +11500 (P/F,8,6) + 6000 (P/A,8,9)

= 175000 + 11,500(0.79383) +11500 (0.63017) + 6000 (6.24689)

= 175000 + 9129.045+7246.955 +37481.34

=228857.34

A refinery company entered in to a contract for raw materials with an agreement to pay Rs 6,00,000 now and Rs 1,50,000 per year beginning at the end of the fifth year. The contract was made for 10 years. At the end of the third year, because of unexpected profits , the company requested that it will be allowed to make a lump sum payment in advance for the rest of the contract. Both parties agreed that 7 percent compounded annually was a fair interest rate. What was the amount of lump sum?

PW = 1,50,000 (P/A,7,6) (P/F,7,1)

= 150000(4.10020)(0.93458)

= 574794.737

A company wants to set up a reserve which will help the company to have an annual equivalent amount of Rs 10,00,000 for the next 20 years towards employees welfare measures. The reserves assumed to grow at the rate of 15 percent annually. Find the single payment that must be made now as the reserve amount.

PW = 1000000(P/A,15,20)

=1000000(6.25933)

=6259330

- Mrs. Sabitri Rao Invests a sum of Rs. 50,000 in a bank at a nominal rate of interest of 18% for 15 years. The compounding is done half yearly. Find the maturity amount.

P =50,000 i=9% N= 30

F = 50000 (F/P,9,30)

50000(66.21177)

=3310588.5

A company wants to buy a machine. It has received two offers. The details are as under

Initial Cost	Rs.6,00,000	Rs,8,00,000
Life Span	4 years	4 years
Salvage Value	Rs. 1,00,000	Rs. 1,00,000
Annual maintenance cost	Rs.50,000	Nil

At 10% interest rate, which offer should be selected. Use Future worth Method.

FW (A)=-600000(F/P,10,4) +100000-50000(F/A,10,4)

FW (A)=-600000(1.46410) +100000-50000(4.64100)

FW (A) = -878460+ 100000-232050

FW(A)= -1010510

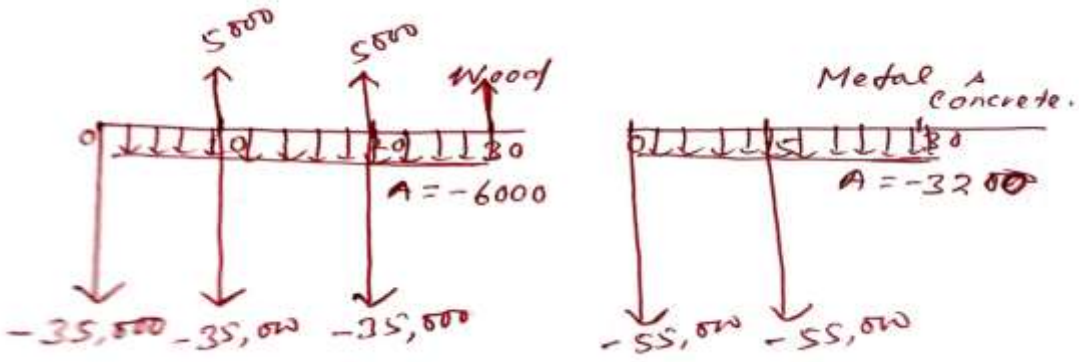
FW (B) = -800000 (F/P,10,4) +100000

FW (B) = -800000 (1.46410) +100000

FW (B) = -1071280

A marina has two alternative plan for constructing a small-boat landing on lake behind the sales building, one is a wooden dock and the other is a metal and concrete wharf. Data for the two plans are as shown. Using a minimum rate of return of 10 percent compare the present worth of the two plans.

	Wood	Metal and Concrete
First Cost	35,000	55,000
Period before Replacement	10 years	15years
Salvage value	5000	0
Annual maintenance	6000	3200



PW = -35,000 -30,000 (P/F,10,10) - 30,000(P/F,10,20) +5000(P/F,10,30)-6000(P/A,10,30)

PW(W) = -35,000 -30,000 (0.38554) - 30,000(0.14864) +5000(0.055731)-6000(9.42691)

PW(W) = -35,000 -11566.2 – 4459.3 +278.655-56561.46

PW= -107308.305

PW(M) = -55,000 -55,000(0.23939) -3200(9.42691)

PW (M) = -55,000 -13166.45 -30166.112

PW(M)= -98332.562

A finance company advertises two investment plans. In Plan-I, The Company pays Rs. 12,000 after 15 years for every Rs 1,000 invested now. In plan-II, for every Rs. 1000 invested the company pays Rs. 4000 at the end of 10^{th} year and Rs. 4000 at the end of 15^{th} year. Select the best alternative plan for the investors point of view at i = 12% compounded annually.

FW(I) = -1000 (F/P,15,12) + 12,000

FW(I) = -1000 (5.35025) + 12,000

FW(I) = -5350.25 +12000

= **6649.75**

FW(II) = -1000 (5.35025) + 4000 (2.01136) + 4000

= -5350.25 + 8045.44 +4000

= -5350.25+ 12045.44

=6695.19

= Rs. 47,293.30

The present worth of the small business is Rs. 47,293.30.

QUESTIONS

1. A project involves an initial outlay of Rs. 30,00,000 and with the following transactions for the next five years. The salvage value at the end of the life of the project after five years is Rs. 2,00,000. Draw a cash flow diagram of the project and find its present worth by assuming $i = 15\%$, compounded annually.

End of year	*Maintenance and operating expense* (Rs.)	*Revenue* (Rs.)
1	2,00,000	9,00,000
2	2,50,000	10,00,000
3	3,00,000	12,00,000
4	3,00,000	13,00,000
5	4,00,000	12,00,000

2. Find the present worth of the following cash flow series. Assume $i = 15\%$, compounded annually.

End of year	0	1	2	3	4	5
Cash flow (Rs.)	–10,000	30,000	30,000	30,000	30,000	30,000

3. Consider the following cash flow series over a 20-year period. Assuming the interest rate as 18% compounded annually, compute the present worth of the series; give your comments.

End of year	*Cash flow* (Rs.)
0	–50,00,000
1	6,00,000
2	6,00,000
.	.
.	.
.	.
20	.

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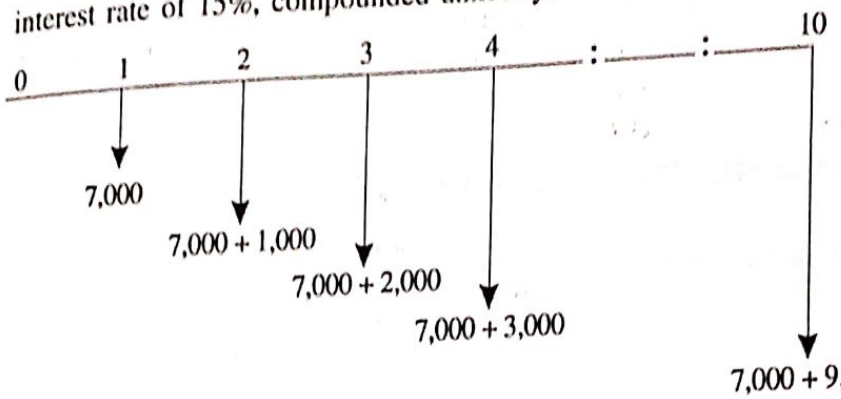
4. The cost of erecting an oil well is Rs. 1,50,00,000. The annual equivalent yield from the oil well is Rs. 30,00,000. The salvage value after its useful life of 10 years is Rs. 2,00,000. Assuming an interest rate of 18%, compounded annually, find out whether the erection of the oil well is financially feasible, based on the present worth method.

5. The details of the feasibility report of a project are as shown below. Check the feasibility of the project based on present worth method, using $i = 20\%$.
   Initial outlay = Rs. 50,00,000
   Life of the project = 20 years.
   Annual equivalent revenue = Rs. 15,00,000
   Modernizing cost at the end of the 10th year = Rs. 20,00,000
   Salvage value at the end of project life = Rs. 5,00,000.

6. Consider the following cash flow diagram. Find the present worth using an interest rate of 15%, compounded annually.



7. An automobile company recently advertised its car for a down payment of Rs. 1,50,000. Alternatively, the car can be taken home by customers without making any payment, but they have to pay an equal yearly amount of Rs. 25,000 for 15 years at an interest rate of 18%, compounded annually.
   You are asked to advise the best alternative for the customers based on the present worth method of comparison.

8. The cash flows of two project proposals are as given below. Each of the project has an expected life of 10 years. Select the best project based on present worth method of comparison using an interest rate of 18%, compounded annually.

	Initial outlay (Rs.)	*Annual equivalent revenue* (Rs.)	*Salvage value after 10 years* (Rs.)
Project 1	–7,50,000	2,00,000	50,000
Project 2	–9,50,000	2,25,000	1,00,000

9. A company has two alternatives for satisfying its daily travel requirements of its employees for the next five years:

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Alternative 1: Renting a vehicle at a cost of Rs. 10,00,000 per year.

Alternative 2: Buying a vehicle for Rs. 5,00,000 with an operating and maintenance cost of Rs. 3,50,000 per year. The salvage value of the vehicle after five years is Rs. 1,00,000.

Select the best alternative based on the present worth method of comparison using the interest rate of 20%, compounded annually.

10. A working woman is planning for her retired life. She has 20 more years of service. She would like to have an annual equivalent amount of Rs. 3,00,000, starting from the end of the first year of her retirement. Find the single amount that should be deposited now so that she receives the above mentioned annual equivalent amount at the end of every year for 20 years after her retirement. Assume $i = 15\%$, compounded annually.

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QUESTIONS

1. A suburban taxi company is considering buying taxis with diesel engines instead of petrol engines. The cars average 50,000 km a year, with a useful life of three years for the taxi with the petrol engine and four years for the diesel taxi. Other comparative information are as follows:

	Diesel	*Petrol*
Vehicle cost	Rs. 5,00,000	Rs. 4,00,000
Fuel cost per litre	Rs. 9.00	Rs. 24.00
Mileage, in km/litre	30	20
Annual insurance premium	Rs. 500	Rs. 500
Salvage value at the end of vehicle life	Rs. 70,000	Rs. 1,00,000

Determine the more economical choice based on the future worth method of comparison if the interest rate is 15%, compounded annually.

2. A motorcycle is sold for Rs. 50,000. The motorcycle dealer is willing to sell it on the following terms:

   (a) Make no down payment but pay Rs. 1,500 at the end of each of the first four months and Rs. 3,000 at the end of each month after that for 18 continuous months.

   (b) Make no down payment but pay a total amount of Rs. 90,000 at the end of the 22nd month; till that time the buyer should mortgage property worth of Rs. 50,000, at present.

   Based on these terms and a 12% annual interest rate compounded monthly, find the best alternative for the buyer based on the future worth method of comparison.

3. Consider the following two mutually exclusive alternatives.

	A	*B*
Cost	Rs. 4,000	Rs. 6,000
Uniform annual benefit	Rs. 640	Rs. 960
Useful life (years)	20	20

Using a 15% interest rate, determine which alternative should be selected based on the future worth method of comparison.

4. A company must decide whether to buy machine *A* or machine *B*:

	Machine A	*Machine B*
Initial cost	Rs. 4,00,000	Rs. 8,00,000
Useful life, (years)	5	5
Salvage value at the end of machine life	Rs. 2,00,000	Rs. 5,50,000
Annual maintenance cost	Rs. 40,000	0

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At 15% interest rate, which machine should be selected? (Use the future worth method of comparison.)

5. Due to increasing awareness of customers, two different television manufacturing companies started a marketing war. The details of advertisements of the companies are as follows:

	Brand X	*Brand Y*
Selling price of a TV set	Rs. 15,000	Rs. 10,000
Amount returned to buyer after 5 years	Rs. 8,000	–

Select the most economical brand from the customer's point of view using the future worth method of comparison, assuming an interest rate of 15%, compounded annually.

6. Alpha Finance Company is coming with an option of accepting Rs. 10,000 now and paying a sum of Rs. 1,60,000 after 20 years. Beta Finance Company is coming with a similar option of accepting Rs. 10,000 now and paying a sum of Rs. 3,00,000 after 25 years. Compare and select the best alternative based on the future worth method of comparison with 15% interest rate, compounded annually.

7. An insurance company gives an endowment policy for a person aged 30 years. The yearly premium for an insured sum of Rs. 1,00,000 is Rs. 4,000. The policy will mature after 25 years. Also, the person is entitled for a bonus of Rs. 75 per thousand per year at the end of the policy. If a person survives till the end of the 25th year:

   (a) What will be the total sum that he will get from the insurance company at that time?

   (b) Instead of paying the premiums for the insurance policy, if the person invests an equal sum of Rs. 4,000 at the end of each year for the next 25 years in some other scheme which is having similar tax benefit, find the future worth of the investment at 15% interest rate, compounded annually.

   (c) Rate the above alternatives assuming that the person is sure of living for the next 25 years.

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Brand D

Tyre warranty = 48 months

Price/tyre = Rs. 2,700

The cash flow diagram for brand D is shown in Fig. 6.25.

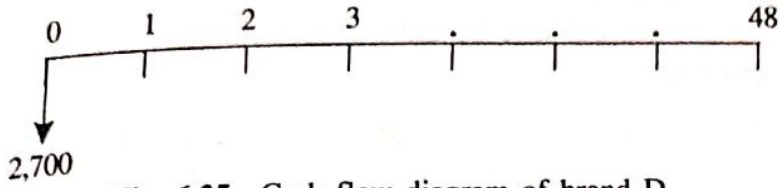

Fig. 6.25 Cash flow diagram of brand D.

The annual equivalent cost expression of the above cash flow diagram is

$$AE(1\%) = 2{,}700(A/P, 1\%, 48)$$
$$= 2{,}700(0.0263)$$
$$= \text{Rs. } 71.01$$

Here, minimum common multiple lives of tyres is considered. This is 144 months. Therefore, the comparison is made on 144 month's basis.

The annual equivalent cost of brand C is less than that of other brands. Hence, it should be used in the vehicles of the trucking company. It should be replaced four times during the 144-month period.

QUESTIONS

1. A company has three proposals for expanding its business operations. The details are as follows:

Alternative	*Initial cost* (Rs.)	*Annual revenue* (Rs.)	*Life* (years)
A1	25,00,000	8,00,000	10
A2	20,00,000	6,00,000	10
A3	30,00,000	10,00,000	10

Each alternative has insignificant salvage value at the end of its life. Assuming an interest rate of 15%, compounded annually, find the best alternative for expanding the business operations of the company using the annual equivalent method.

2. An automobile dealer has recently advertised for its new car. There are three alternatives of purchasing the car which are explained below.

Alternative 1 The customer can take delivery of a car after making a down payment of Rs. 25,000. The remaining money should be paid in 36 equal monthly installments of Rs. 10,000 each.

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Alternative 2 The customer can take delivery of the car after making a down payment of Rs. 1,00,000. The remaining money should be paid in 36 equal monthly installments of Rs. 7,000 each.

Alternative 3 The customer can take delivery of the car by making full payment of Rs. 3,00,000.

Suggest the best alternative of buying the cars for the customers by assuming an interest rate of 20% compounded annually. Use the annual equivalent method.

3. A small-scale industry is in the process of buying a milling machine. The purchase value of the milling machine is Rs. 60,000. It has identified two banks for loan to purchase the milling machine. The banks can give only 80% of the purchase value of the milling machine as loan. In Urban Bank, the loan is to be repaid in 60 equal monthly installments of Rs. 2,500 each. In State Bank, the loan is to be repaid in 40 equal monthly installments of Rs. 4,500 each. Suggest the most economical loan scheme for the company, based on the annual equivalent method of comparison. Assume a nominal rate of 24%, compounded monthly.

4. There are two alternatives of replacing a machine. The details of the alternatives are as follows:

Alternative 1

Purchase value of the new machine	: Rs. 2,00,000
Life of the machine	: 10 years
Salvage value of the new machine at the end of its life	: Rs. 20,000
Annual operation and maintenance cost	: Rs. 40,000
Buyback price of the existing machine	: Rs. 25,000

Alternative 2

Purchase value of the new machine	: Rs. 3,00,000
Life of the machine	: 10 years
Salvage value of the new machine at the end of its life	: Rs. 15,000
Annual operation and maintenance cost	: Rs. 35,000
Buyback price of the existing machine	: Rs. 5,000

Suggest the best replacement option for the company using the annual equivalent cost method of comparison by assuming 20% interest rate, compounded annually.

5. A company receives two options for purchasing a copier machine for its office.

Option 1 Make a down payment of Rs. 30,000 and take delivery of the copier machine. The remaining money is to be paid in 24 equal monthly installments of Rs. 4,500 each.

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Option 2 Make a full payment of Rs. 1,00,000 and take delivery of the copier machine.

Suggest the best option for the company to buy the copier machine based on the annual equivalent method of comparison by assuming 15% interest rate, compounded annually.

6. Find the best alternative using the annual equivalent method of comparison. Assume an interest rate of 15% compounded annually.

Alternative	*A*	*B*	*C*
Initial cost (Rs.)	5,00,000	8,00,000	6,00,000
Annual receipt (Rs.)	2,00,000	1,50,000	1,20,000
Life (years)	10	10	10
Salvage value (Rs.)	1,00,000	50,000	30,000

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$= -8,00,000 + 2,70,000(2.8036)$

$= \text{Rs. } -26,828$

Thus, the rate of return of alternative 2 is

$$i = 20\% + \frac{7{,}462 - 0}{7{,}462 - (-26{,}828)} \times 2\%$$

$$= 20.435\%$$

Since the rate of return of alternative 1 is greater than that of the alternative 2, select alternative 1.

QUESTIONS

1. Consider the following cash flow of a project:

Year	0	1	2	3	4	5
Cash flow	–10,000	4,000	4,500	5,000	5,500	6,000

Find the rate of return of the project.

2. A person invests a sum of Rs. 2,00,000 in a business and receives equal net revenue of Rs. 50,000 for the next 10 years. At the end of the 10th year, the salvage value of the business is Rs. 25,000. Find the rate of return of the business.

3. A company is in the process of selecting the best alternative among the following three mutually exclusive alternatives:

Alternative	*Initial investment*	*Annual revenue* (Rs.)	*Life* (years)
*A*1	Rs. 5,00,000	1,00,000	10
*A*2	Rs. 8,00,000	1,40,000	10
*A*3	Rs. 3,00,000	70,000	10

Find the best alternative based on the rate of return method of comparison.

4. A shipping firm is considering the purchase of a materials handling system for unloading ships at a dock. The firm has reduced their choice to three different systems, all of which are expected to provide the same unloading speed. The initial costs and the operating costs estimated for each system are now tabulated.

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System	Initial cost	Annual operating expenses
S1	Rs. 6,50,000	Rs. 91,810
S2	Rs. 7,80,000	Rs. 52,600
S3	Rs. 7,50,000	Rs. 68,417

The life of each system is estimated to be five years and the firm's minimum attractive rate of return is 15%. If the firm must select one of the materials handling systems, which one is the most desirable?.

5. A firm has identified three mutually exclusive alternatives. The life of all three alternatives is estimated to be five years. The minimum attractive rate of return is 12%. Find the best alternative based on the rate of return method.

Alternative	A1	A2	A3
Initial investment (Rs.)	2,00,000	2,80,000	3,60,000
Annual income (Rs.)	52,000	72,000	1,00,000

6. An automobile company is planning to buy a robot for its forging unit. It has identified two different companies for the supply of the robot. The details of cost and incremental revenue of using robots are summarized in the following table:

	Brand	
	Speedex	Giant
Initial cost (Rs.)	5,00,000	9,00,000
Annual incremental revenue (Rs.)	80,000	2,50,000
Life (years)	8	8
Life-end salvage value (Rs.)	40,000	60,000

The minimum attractive return for the company is 12%. Suggest the best brand of robot to the company based on the rate of return method.

7. A bank introduces two different investment schemes whose details are as follows: Find the best investment alternative from the investor's point of view.

	Alpha Bank	Beta Bank
Deposit amount (Rs.)	1,00,000	2,00,000
Period of deposit (years)	5 years	3 years
Maturity amount (Rs.)	3,00,000	4,50,000

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8. A company is planning for its expansion programme which will take place after five years. The expansion requires an equal sum of Rs. 5,00,000 for consecutive three years. Gamma Bank has recently introduced a scheme in this line. If the company invests Rs. 7,00,000 now with this bank, it will make equal repayments of Rs. 5,00,000 for three consecutive years starting from the end of the fifth year from now. The minimum attractive rate of return for the company is 12%. Suggest whether the company should invest with the Gamma Bank for its expansion programme.

9. Consider the following table which summarizes data of two alternatives.

	First cost	*Annual return*	*Life*
Alternative 1	Rs. 5,00,000	Rs. 1,50,000	10 yrs
Alternative 2	Rs. 8,00,000	Rs. 2,50,000	10 yrs

Find the best alternative based on the rate of return method of comparison.

10. A company is planning to expand its present business activity. It has two alternatives for the expansion programme and the corresponding cash flows are given in the following table. Each alternative has a life of five years and a negligible salvage value. The minimum attractive rate of return for the company is 15%. Suggest the best alternative to the company.

	Initial investment (Rs.)	*Yearly revenue* (Rs.)
Alternative 1	4,50,000	1,50,000
Alternative 2	7,50,000	2,50,000

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Equivalent Annual-worth Comparison

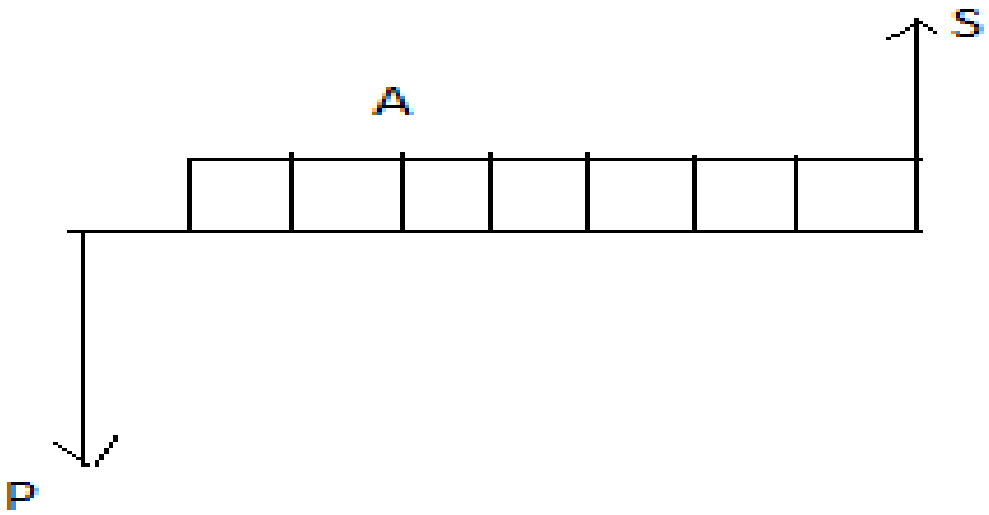
Dr. Mahendra P. Agasty

- Equivalent annual worth method is another important method for comparing engineering alternatives
- With an annual worth method, all the receipts and disbursements occurring over a period are converted to an equivalent uniform yearly payment
- Cost accounting procedures, depreciation ,profit and loss are calculated annually .
- These yearly cost tabulations generally make the annual worth method easier to apply and comprehend than the other comparison methods
- The annual worth calculation is derived from capital recovery factor as it converts a lump sum to an equivalent annuity

PW= **-P** +A(P/A,i,N)+ S(P/F,i,N)

FW= -P(F/P,i,N) + A(F/A,i,N)+**S**

EAW= -P(A/P,I,N)+ **A** +S(A/F,I,N)



- The capital recovery factor (A/P, i, N) accounts for both the repayment of invested capital and the interest earned on the unrecovered portion of the investment

End of Period	Capital not recovered by end of period	Interest due on Unrecovered capital	Amount of capital recovered	Period capital recovery charge
0	40,000.00			
1	31,381.17	4000	8618.83	12,618.83
2	21900.45	3138.12	9480.72	12,618.83
3	11,471.67	2190.05	10,428.79	12,618.83
4	0.00	1147.17	11471.67	12,618.83
		10,475.33	40,000.00	50,475.33

Capital Recovery Calculations

When only the amount of unrecovered capital at a certain time is sought, it can be determined directly from the present worth of the remaining payments:

Unrecovered capital (year 3)= A(P/A,10,1)

=12618.83(0.90909)= 11471.65

Unrecovered capital (year 2)= 12618.83(P/A,10,2)

=12618.83(1.73554)= 21,900.48

EAW & EAC

EAW = Equivalent Annual worth

Net Annual worth

It has both positive cash and Negative cash flow

EAC= Equivalent Annual worth Comparison

When only the cost of the projects are given and we convert all the cost to equivalent annual amount, it. is called equivalent annual cost.

EAC = P(A/P,i,N)- S(A/F,,i,N)

EAC = (P-S)(A/P,i, N) + S(i)

$$EAC = P(A/P,i,N) - S(A/F,i,N).....(i)$$

We knowthat

$$(A/P,i,N) = \frac{i(1+i)^N}{(1+i)^N - 1}.....(ii)$$

$$(A/F,i,N) = \frac{i}{(1+i)^N - 1}.....(iii)$$

Now substracting i from eq(ii), we get

$$\frac{i(1+i)^N}{(1+i)^N - 1} - i = \frac{i(1+i)^N - i(1+i)^N + i}{(1+i)^N - 1}$$

which reduces to

$$\frac{i}{(1+i)^N - 1}$$

and which is the right – hand side of equation (iii)

$$(A/P,i,N) - i = (A/F,i,N)$$

$$EAC = P(A/P,i,N) - S[(A/P,i,N) - i]$$

$$= (P - S)(A/P,i,N) + S(i)$$

Example:

Initial Cost 1,00,000

Salvage value= 20,000

i=10%

N=4

EAC= 1,00,000(A/P,10,4) -20,000 (A/F,10,4)

= 1,00,000(0.31547) -20,000 (0.21547)

=31547 -4309.4

=27237.6

EAC= (1,00,000-20,000)(A/P,10,4) +20,000(0.10)

EAC= (1,00,000-20,000)(0.31547) +20,000(0.10

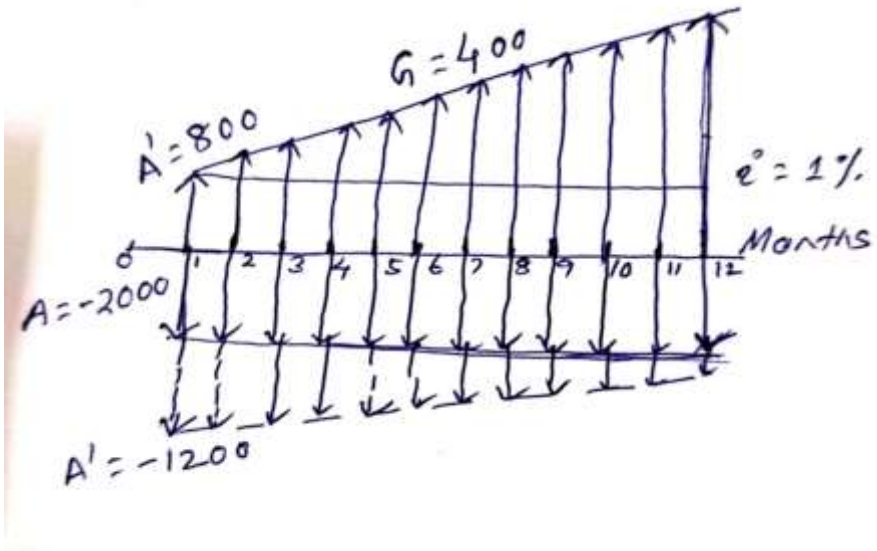
EAC= 25237.6+2000

=27237.6

Situations for equivalent Annual-worth Comparison

Consolidation of Cash-Flows:

The government of Odisha wanted to introduce a training programme to increase the efficiency of its employees. The programme last 1 year, costs $2000 per month. It is expected that in the first month the savings from the training programme will be $800 and should increase by $400 per month .However, operational confusion and work interferences are expected to boost clerical costs by $1200 the first month ,but this amount should subsequently decline in equal increments at the rate of $100 per month. If the required return on money is 12 percent compounded monthly and there is a stipulation that the programme must pay for itself within 1 year, should the training be introduced.



EAW (Savings)= 800 +400 (A/G,1,12)

= 800 +400 (5.36815)= 2947

EAW(Costs)= -2000-[1200-100(A/G,1,12)]

=-3200 + 100(5.36815)= -2663

Equivalent net monthly cash flow= 2947-2663= 284

Recovery of Invested Capital
(Will it pay off?)

Initial Cost : Rs. 1,00,000

Annual Revenue : 20,000

Annual Expenses: 5000

Salvage Value: 40,000

i= 8 percent

N= 5

EAW= -1,00,000 (A/P,8,5) + 15,000 + 40,000(A/F,8,5)

EAW = -1,00,000(0.25046) + 15,000 + 40,000(0.17046)

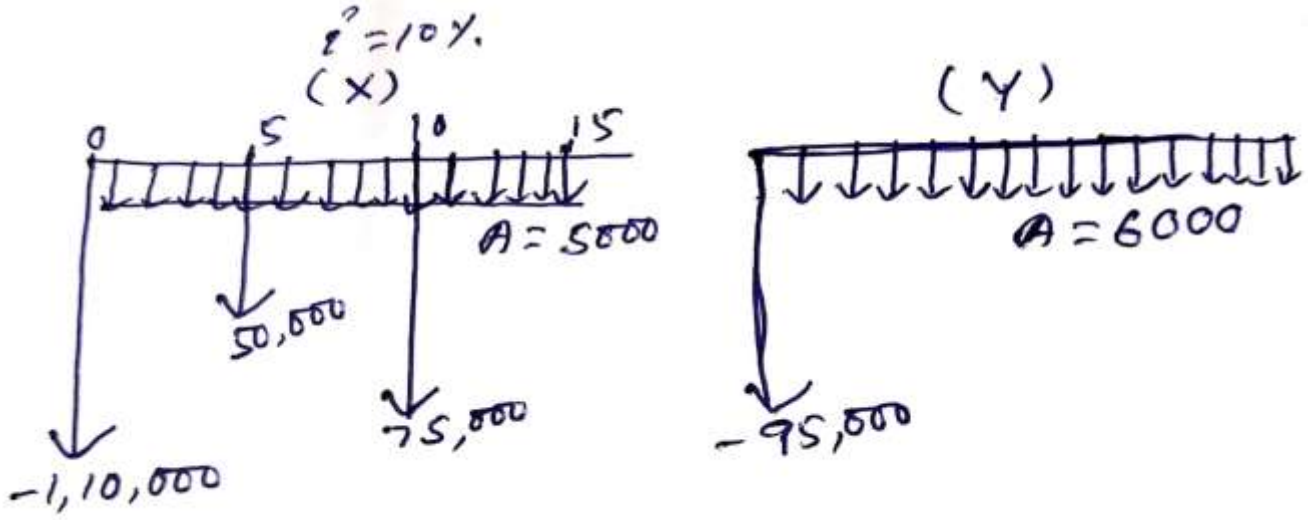
=-25046 + 15,000 + 6818.4

= -3227.6

EAW= - [(1,00,000-20,000)(A/P,8,5) +20,000(0.10)]+15,000

Comparison of Net Annual Costs

	X	Y
Initial Cost	1,10,000	95,000
Investment in 5th Year	50,000	-
Investment in 10th Year	75,000	-
Annual Cost	5000	6000
Life	15	15
i	10	10



EAC (X) = 1,10,00 (A/P,10,15) + 50,000(P/F,10,5)(A/P,10,15) + 75,000 (P/F,10,10)(A/P,10,15) + 5000

EAC (X) = 1,10,00 (0.13147) + 50,000(1.61051)(0.13147) + 75,000 (0.38554)(0.13147) + 5000

EAC(X) = 14461.7 + 10586.687 + 3801.520

EAC(X) = 14461.7 + 10586.687 + 3801.520

=28849.907

EAC (Y) = 95,000 (A/P,10,15) + 6000

= 95,000 (0.13147) + 6000

=12489.65 + 6000

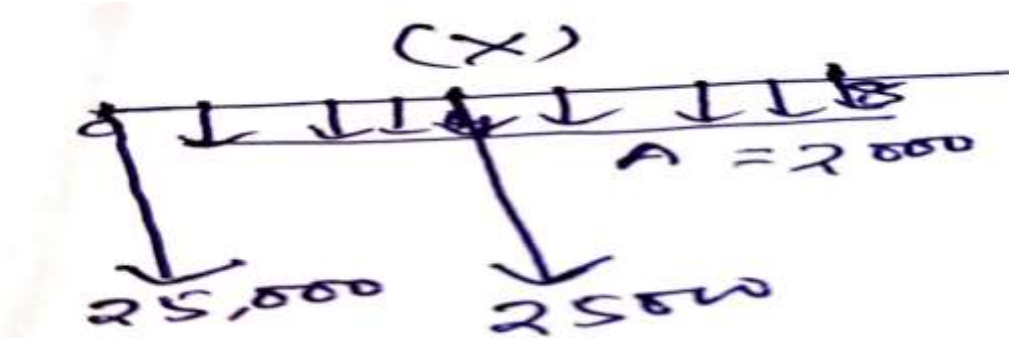
=18489.65

Consideration of Asset Life

- In time value mechanics N is simply the number of compounding period. N has special meaning when it represents life of the asset. The life of the asset can be defined as follows
- **Ownership life:** Service life (It is the period of time an asset kept in service). It is the period of useful service from the time of purchase until disposal.
- **Accounting life:** This is a life expectancy based primarily on book keeping and tax considerations.
- **Economic** life: It is the time period that minimises the asset's total equivalent annual cost and maximizes its equivalent net annual income.

Comparison of Assets having unequal lives

	X	Y
Initial Cost	25000	30,000
AMC	2000	1000
N	4	8
i	10	10



EAC (X)= 25,000(A/P,10,4) + 2000

EAC (X)= 25,000(0.31547) + 2000

EAC= 7886.75 + 2000

=9886.75

EAC(Y)= 30,000 (A/P,10,8) + 1000

=30,000 (0.18744) +1000

=5623.2+1000

=6623.2

Repeated Life Method

EAC(X) = 25000 (A/P,10,8) + 25000(P/F,10,4)(A/P,10,8)+ 2000

EAC(X) = 25000 (0.18744) + 25000(0.68301)(0.18744)+ 2000

EAC(X) = 4686 + 3200.584+ 2000

=9886.7

EAC(Y)= 30,000 (A/P,10,8)+ 1000

=6623.2

Problems

A company can purchase a piece of equipment for Rs 20,000 and sell it for Rs. 4000 at the end of 6 year service life, or it can lease the unit for the same period by making first-of-the year payments of Rs 3000. Compare the equivalnent annual cost of the alternatives, using an interest rate of 15 percent

EAC (Purchase)= (20,000-4000)(A/P,15,6)+ 4000(0.15)

= (20,000-4000)(0.26424)+ 4000(0.15)

=4227.84+600

=**4827.84**

EAC (Lease)= [3000 +3000(P/A,15,5)](A/P,15,6)

= [3000 +3000(3.35218)](0.26424)

= [3000 +3000(3.35218)](0.26424)

= [3000 +10056.54](0.26424)

=3450.060

- A stand by electric power generator was purchased 6 years ago for Rs 8000. At that time it was expected that the equipment would be used for 15 years and would have a salvage value of 10 percent of the first cost. The generator is no longer needed and is to be sold for Rs. 2500. Using an interest rate of 15 percent determine the difference between the anticipated and actual equivalent annual capital cost.

EAC(Anticipation)=(8000-800)(A/P,15,15)+ 800(0.15)

EAC(Anticipation)=(8000-800)(0.17102)+ 800(0.15)

EAC(Anticipation)= 1231.344+120

=1351.344

EAC(Actual)= (8000-2500)(A/P,15,6) + 2500(0.15)

EAC(Actual)-=(8000-2500)(0.26424) + 2500(0.15)

EAC(Actual)=1453.32+375

=1828.32

Laser beams are to be used on a major construction project to ensure the exact alignment of components. Two types of laser alignment systems, with the cost shown below, are suitable for the projects.

	IC System	UC System
First Cost	5000	3200
Salvage Value	1000	0
Annual operating cost	600	950
Annual taxes	180	0

If both system have a life of 4 years and the minimum rate of return is 15 percent, which offers the lower equivalent annual cost.

EAC(IC System)= [(5000-1000)(A/P,15,4) +1000(0.15)]+780

EAC(IC System)= [(5000-1000)(0.35017) +1000(0.15)]+780

EAC(IC System)= [(5000-1000)(0.35017) +1000(0.15)]+780

EAC(IC System)=1400.68 + 150+780

=**2330.68**

EAC(UC System)= 3200(0.35027) + 950

EAC(UC System)= 1120.864+ 950

=2070.864

A company receives two options for purchasing a copier machine for its office.

Option 1: Make a down payment of Rs. 30,000 and take delivery of the copier machine. The remaining money is to be paid in 24 equal monthly installments of Rs. 4500 each.

Option: 2 Make a full payment of Rs. 1,00,000 and take delivery of the copier machine.

Suggest the best option for the company to buy the copier machine based on the equivalent method of comparison by assuming 12% interest rate, compounded annually.

EAC (Option I) =30,000(A/P,1,24) + 4500

=30,000 (0.04707) + 4500

=1412.1+4500

=**5912.1**

EAC (Option II) = 1,00,000 (A/P,1,24)

1,00,000 (0.04707)

= 4707

PROBLEMS

3.1 The lease on a warehouse amounts to $5000 per month for 5 years. If payments are made on the first of each month, what is the present worth of the agreement at a nominal annual interest rate of 12 percent, compounded monthly?

3.2 Determine the present worth of the lease in Prob. 3.1 if the payments are to increase at the beginning of each year equal to $250 per month for each month of occupancy.

3.3 What is the future worth at the end of 5 years of the payments to be made in Prob. 3.1? Do *not* use your PW results from Prob. 3.1!

3.4 A company borrowed $100,000 to finance a new product. The loan was for 20 years at a nominal interest rate of 8 percent compounded semiannually. It was to be repaid in 40 equal payments. After one-half the payments were made, the company decided to pay the remaining balance in one final payment at the end of the 10th year. How much was owed?

3.5 A proposed improvement in an assembly line will have an initial purchase and installation cost of $175,000. The annual maintenance cost will be $6000; periodic overhauls once every 3 years, excluding the last year of use, will cost $11,500 each. The improvement will have a useful life of 9 years, at which time it will have no salvage value. What is the present worth of the 9-year costs of the improvement at $i = 8$ percent?

3.6 The assembly line in Prob. 3.5 will have potential income increases, due to higher production volumes, of $29,000 in the first year with $3000 per year additional increases in years 2 through 5. These increases can be expected to decrease from the year 5 value at $5000 per year until the increase in year 9 is $21,000. Are the future expected increases sufficient to justify the expenditures given in Prob. 3.5?

3.7 Amjay Company is currently renting a parking lot for employee and visitor use at an annual cost of $9000, payable on the first of each year. The company has an opportunity to buy the lot for $50,000. Maintenance and taxes on the property are expected to cost $2500 annually. Given that the property will be needed for 10 more years, determine what sales price must be obtained at the end of that period in order for Amjay to break even, when the interest rate is 12 percent.

3.8 A bakery is thinking of purchasing a small delivery truck that has a first cost of $18,000 and is to be kept in service for 6 years, at which time the salvage value is expected to be $2500. Maintenance and operating costs are estimated at $2500 the first year and will increase at a rate of $200 per year. Determine the present worth of this vehicle, using an interest rate of 12 percent.

3.9 A small dam and an irrigation system are expected to cost $300,000. Annual maintenance and operating costs are expected to be $40,000 the first year and will increase at a rate of 10 percent per year. Determine the equivalent present worth of building and operating the system with interest of 10 percent over a 30-year life.

3.10 It is estimated that additional reinforcement of the dam in Prob. 3.9 in the first year at a cost of $65,000 will reduce annual maintenance costs to $25,000 in

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Rate of Return

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Rate of Return

The rate of return is a percentage that indicates the relative yield on different uses of capital.

- **MARR** *(Minimum Acceptable Rate of Return):* It is the rate set by an organization to designate the lowest level of return that makes an investment acceptable.
- IRR *(Internal Rate of Return):* It is the rate on the unrecovered balance of the investment in a situation where the terminal balance is zero.
- ERR *(External rate of Return):* It is the rate of return that is possible to obtain for an investment under current economic conditions.

IRR

IRR is the interest rate at which present worth of the cash flow of a project is zero, or the rate which when employed in computing the present worth of all the costs and present worth of all returns will make them equal.

N	CASH FLOW
0	– 50,000
1	15000
2	15000
3	15000
4	15,000

IRR

PW = – 50,000 + 15,000(P/A, i, 4) = 0

$$P/A,i,4 = \frac{(1+i)^4 - 1}{i(1+i)^4}$$

Can we Solve for the value of i ?

Determining the type of investment

We can classify the investment project by counting the number of sign changes in its net cash flow sequence/ A change either "+" to "–" or "–" to "+"is counted as one sign change. (We ignore zero cash flow change). Then,

A simple (conventional) investment is one in which the initial cash flows are negative, and only one sign change occurs in the net cash flow series. If the initial flows are positive, and only one sign change occurs in the subsequent net cash flows, they are referred as the simple borrowing cash flows.

A non-simple (or non-conventional) investment is one in which more than one sign change occurs in the cash flow series.

Multiple i* occur only in non-simple investments

Selection Criteria:

For single investment projects, the decision rule is

If IRR>MARR, accept the project

If IRR=MARR, remain indifferent.

If IRR<MARR, reject the project

For Mutually exclusive cases:

Incremental IRR:

N	A	B	B –A
0	–1000	–5000	–4000
1	2000	7000	5000
IRR	100%	40%	125%
PW(10%)	818	1364	

MARR in the above case is 10%
The decision rule is
If IRR_{B-A}>MARR select B
If IRR_{B-A}=MARR, select either one
If IRR_{B-A}<MARR select A

IRR Calculation:

There are three practical solution methods that need to be discussed for calculation of the interest rate i*. These methods are;

Direct solution method

Trial and error method

Computer solution method (CHEER Method)

Trial and error method

Initial Cost =Rs. 80,000/-

Annual Revenue = Rs.1,500/-

Annual Expenses = Rs.850/-

Salvage value = Rs.1,50,000/-

N = 5 years

MARR = 13%

Find IRR.

Calculation of IRR:

PW = – 80,000 + 650(P/A, i, 5) + 150000(P/F,i,5) = 0

Assuming i as 14%

PW = 137

Assuming i as 15%

PW = – 3244 . 10

By linear interpolation

$$IRR = 14\% + \left[\frac{(15-14)\%\left(137-0\right)}{137.0-\left(-3244.10\right)}\right] = 14.04\%$$

Clues for IRR Calculation:

(i) To avoid unnecessary trial and error if we have a simple investment, it is suggested to sum the cash flows. If we have negative total we should not go for the calculation of IRR. If by putting PW(0) it came by negative the calculation should be avoided.

(ii) When the salvage value is close to 100% of the first cost, the net annuity divided the first cost give a close approximation to i that is $A/P = i$.

(iii) If most of the income is at year N and is about twice the initial outlay than $i = 72/N$.

(iv) If MARR is given that value can be assumed as i

Incremental IRR:

N	X	Y	X – Y
0	–1000	–1000	0
1	100	1000	–900
2	350	200	150
3	600	200	400
4	850	200	650
MARR = 10%			

Calculate :

PW =

FW =

EAW =

IRR(x) =

IRR(y) =

IRR(x-y) =

PW(x) = – 1000 + [100 + 250(A/G,10, 4)] (P/A, 10,4)

PW(Y) = – 1000 + [1000 + 200 (P/A, 10, 3)] *(P/F,* 10, 1)

PW(X)= 411.52

PW(Y)= 361.24

FW(X) =411.52(F/P,10,4)=602.57

FW(Y) =361.24(F/P,10,4)=528.89

AEW(X) = 602.57(A/F,10,4)=129.83

AEW(X) = 528.89(A/F,10,4)=113.95

IRR = PW(x) = – 1000 + [100 + 250(A/G,i, 4)] (P/A, i,4)= 0

By assuming i as 20 percent, PW = 83.53

ias 25 percent PW = –40.64

By interpolation=

$$IRR = 20\% + \left[\frac{(25-20)\%\left(83.53-0\right)}{83.53-\left(-40.64\right)}\right] = 23.4$$

IRR(Y) = – 1000 + 1000 *(P/F, i*, 1) + 200 (P/A, i, 3) *(P/F, i*, 1)=0

By assuming I as 30%

PW = 48.63

By assuming I as 40%

PW = –58.72

$$IRR = 30\% + \left[\frac{(40-30)\%\left(48.63-0\right)}{48.63-\left(-58.72\right)} \right] = 34.26$$

IRR(X – Y)

PW = – 900 + 150 (P/F, i, 1)+ 400(P/F, i, 2) + 650(P/F, i, 3) = 0

By assuming i as 10%

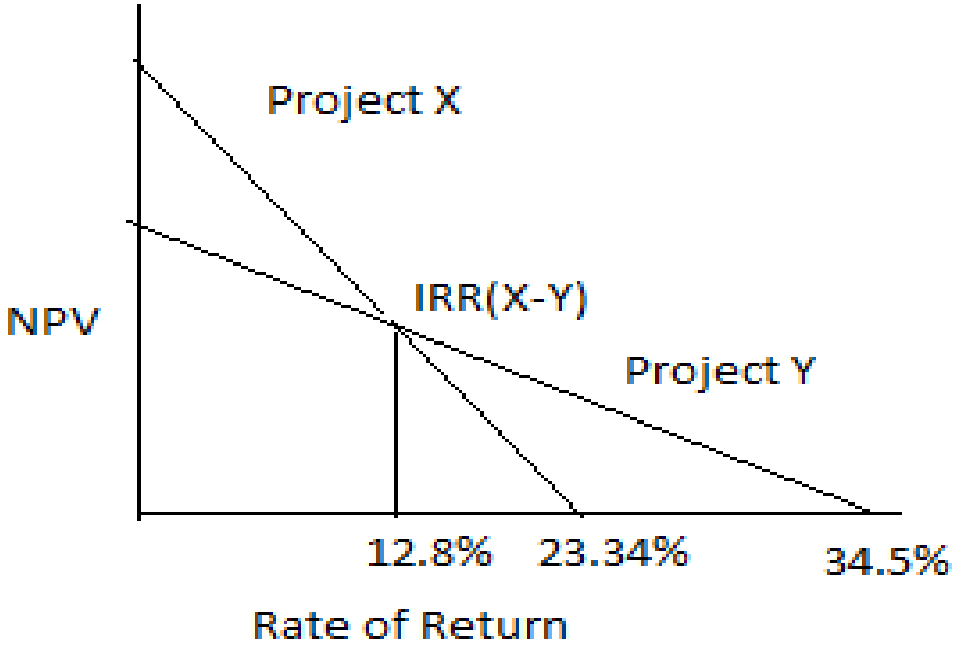
PW = 55.29

By assuming i as 15%

PW = – 33.22

$$IRR = 10\% + \left[\frac{(15-10)\%\left(55.29-0\right)}{55.29-\left(-33.22\right)}\right] = 12.8\%$$

NPV Vs. IRR



Unequal lives

N	A	B
0	-100	-200
1	60	120
2	50	270
3	170	
IRR	58.85 %	50 %

Unequal lives

N	A	B	B–A
0	–100	–200	–100
1	60	120	60
2	50	270–200=70	20
3	170–100=70	120	50
4	60	270–200=70	10
5	50	120	70
6	170	270	100

Problems

- *Project X involves an initial outlay of Rs 32,400. Its working life is expected to be three years. The cash stream generated by it are expected as follows. Calculate the Internal Rate of Return. Assuming the minimum required rate of return to be 14%. Would the project be selected?*

Year	Cash Flow
1	16,000
2	14,000
3	12,000

Solution

- PW = −32400 + 16000 (P/F,i, 1) + 14000 (P/F,i,2) + 12000 (P/F,i,3) = 0
- Assuming i as 14%
- PW = -32400 + 16000(P/F,14,1) + 14000 (P/F,14,2) + 12000 (P/F,12,3)
- PW = 508.2
- So we have to increase the value to 16%
- PW = - 32400 + 16000(P/F,16,1) + 14000 (P/F,16,2) + 12000 (P/F,16,3)
- PW = -513.2

$$IRR = 14\% + \left[\frac{(16-14)\%(508.2-0)}{508.2-(-513.2)} \right] = 14.99\%$$

Problem

- *A capital investment of Rs 10000 can be made in a project that will produce uniform annual revenue of Rs 5,310 for five years then have a salvage value of Rs 2,000, Annual expense will be Rs 3000. The company is willing to accept any project that will earn at least 10% per year, before income taxes on all invested capital. Determine whether it is acceptable by using the IRR method*

Solution

- PW = -10000 + 2310 (P/A,i,5) + 2000 (P/F,i,N) = 0
- Assuming i as 10%
- PW = - 1.452
- If we assume i as 9%
- PW =285.007

$$IRR = 9\% + \left[\frac{(10-9)\%\left(285.007-0\right)}{285.007-\left(-1.452\right)} \right] = 9.94\%$$

In 1994 *a small apartment building was purchased for Rs. 2,00,000. Receipt them sum rent have averaged Rs. 30,200 a year, taxes, maintenance and repair cost have totalled Rs. 8620 annually. The owner intends to held the property until she retires in 2004. If at that time the property sells for Rs. 2,00,000, what rate of return will obtained on the investment.*

Solution

- We can take A/P =2,00,000/21580 = 10.79

- PW = -2,00,000 + 21580 (P/A,i,10) + 200000 (P/F,i,10) = 0
- Assuming i as 10%
- PW = 9700
- If we assume i as 15%
- PW =-42254.296

$$IRR = 10\% + \left[\frac{(15-10)\%(9700-0)}{9700-(-42254.296)}\right] = 10.93\%$$

From the given table find IRR

Year	0	1	2	3	4	5
Cash-flow	-1000	-800	500	500	500	1200

Solution

PW=-1,000- 800 *(P/F, i, 1)* + 500 (P/A, i, 3) (P/F, i, 1) + 1200 *(P/F, i,* 5)=0

Assuming i as 10%

PW = 148.21

Assuming i as 15%

PW = -106.40

$$IRR = 10\% + \left[\frac{(15-10)\%(148.21-0)}{148.21-(-106.4)}\right] = 12.91\%$$

Two mutually exclusive programs are being considered for finding. Projected cash flow are as follows: Determine which program to recommend, assessing a MARR OF *12%.*

Year	A	B	B-A
0	-10,000	-15,000	-5000
1	3,000	5000	2000
2	5,000	5000	0
3	2,000	5000	3000
4	4,000	5000	1000