

# The Effects of Competition and Regulation on Error Inequality in Data-Driven Markets

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## ABSTRACT

Recent work has documented instances of *unfairness* in deployed machine learning models, and significant researcher effort has been dedicated to creating algorithms that intrinsically consider fairness. In this work, we highlight another source of unfairness: market forces that drive differential investment in the data pipeline for differing groups. We develop a high-level model to study this question. First, we show that our model predicts unfairness in a monopoly setting. Then, we show that under all but the most extreme models, competition does not eliminate this tendency, and may even exacerbate it. Finally, we consider two avenues for regulating a machine-learning driven monopolist - relative error inequality and absolute error-bounds - and quantify the price of fairness (and who pays it). These models imply that mitigating fairness concerns may require *policy-driven* solutions, not only technological ones.

## CCS CONCEPTS

• **Theory of computation** → **Market equilibria**; *Machine learning theory*; *Sample complexity and generalization bounds*; *Quality of equilibria*; • **Applied computing** → **Economics**.

## KEYWORDS

learning theory, algorithmic fairness, data markets, game theory, industrial organization, economics

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## 1 INTRODUCTION

As machine learning has become more integrated into products, markets, and decision-making throughout society, researchers, practitioners, and activists have identified many instances of *unfairness* in predictions or decisions made by machine-learned models (or by humans influenced by said models). A large and developing body

of work, which we briefly survey in Section 2, has empirically documented unfairness in practical machine learning settings, identified many theoretical sources and mechanisms of unfairness, and constructed innovative fairness-aware algorithms. Researchers have developed many innovative technical solutions to these problems, yet the issue in practice remains far from solved. This paper highlights a simple and important point: while technical solutions to unfairness are certainly important, mitigating unfairness in practice may require tackling *economic* incentives promoting unfairness.

Most of the existing literature assumes that a fixed dataset, possibly biased, arrives in the hands of a data scientist, and solutions often revolve around clever ways to mitigate this bias. In practice, however, economic incentives may create disparities well before the data scientist enters the picture. Consider, for example, the task of speech recognition: producing accurate models may require a large amount of data, and data from speakers with accented or rarer dialects may be more costly to collect. If the market size of a minority group is small relative to the costs a firm would expend in developing accurate speech recognition software, it is likely that the group will be served with lower quality products.

In this paper, we model the unfairness that arises when data-driven, profit-maximizing firms choose to differentially invest in data collection across groups, creating unequal error rates. In order to focus on this specific source of unfairness, we use a simple framework that elides the many other sources of bias that can seep into the machine learning pipeline. For instance, we assume that firms have unlimited budgets to purchase data at a cost from group-specific data sources of potentially infinite quantity. We also assume that both firms and users benefit from more accurate models, so that incentives are aligned. Furthermore, we assume that firms must build separate models for each group, to avoid unfairness that may come from fitting to the majority.

In order to construct our models, we borrow from the tools of learning theory and microeconomics to build simple, stylized models with crisp predictions of quantifiable unfairness. We assume each profit-maximizing firm faces a known demand curve as a function of the *worst-case* error rates for each group. Standard results from learning theory allow us to model worst-case error rates as a function of the amount of data the firm buys. We investigate three models of demand: linear demand, demand proportional to error rates, and (approximately) rational demand. For the precise description of our models and these assumptions, see Section 3.

We show in Section 4 that a profit-maximizing monopolist will choose to serve minorities (as defined by their market power) with lower quality models. Assuming linear demand, an oft-used benchmark in the economics literature, we quantify the difference in relative model quality between groups as a function of their market size, elasticity, and cost of data.

\*This work was completed while the author was an intern at Microsoft Research.

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We then consider two classical remedies to the ills of monopolies: competition and regulation. Under two natural models of competition – multilinear demand (Section 5.1) and proportional demand (Section 5.2) – introducing competition does not mitigate inequality, and proportional demand even exacerbates it. Only a model in which all consumers choose the firm with (even infinitesimally) smaller error until firms reach sufficient accuracy suggests that competition will mitigate inequality (Section 5.3); to do so, however, this model assumes a stringent notion of rationality that may not be reflective of consumer behavior in the real world.

Given that our most plausible models suggest that competition does improve the situation, we ask whether regulation could be used to mitigate error inequality by design. In particular, in Section 6 we examine two simple kinds of constraints: a ‘relative equality’ constraint where error rates across groups must be multiplicatively close to each other, and an ‘absolute equality’ constraint where error rates across all groups must be sufficiently small, but may be far apart from each other. We then formally quantify the costs to profits (and when relevant, to the majority group’s error rate) as a function of the threshold chosen. Finally, we conclude with takeaways, limitations, and directions for future work in Section 7.

## 2 RELATED WORK

Motivation for our work comes from the many documented instances of disparity in learned model performance between groups. The existing literature has demonstrated troubling disparities in a number of domains, including incentive-aligned domains (where both the firms and users receive benefit from more accurate models) that are the focus of this work. Wilson et al. [34] studies the performance of state-of-the-art object recognition systems, intended for applications like autonomous vehicles, and find that systems fail to recognize darker-skinned persons at much higher rates than lighter ones. Sweeney shows that search engine queries of black-associated names generated about four times the likelihood of ads for arrest records [30]. Blodgett and O’Connor show that on both complicated tasks like parsing and simple tasks like language identification, texts from speakers of African American English see vastly higher error rates [8]. Buolamwini and Gebru show that commercial facial recognition software systems misclassify race and gender among dark-skinned females at orders of magnitude higher rates than light-skinned males [9]. Mehrotra et al. [26] and Ekstrand et al. [13] identify differing satisfaction levels by age and gender in recommendation systems. The list goes on.

Researchers have engaged in many empirical and theoretical investigations to understand why these instances of unfairness occur, with the hope of developing solutions to mitigate them. Much of this work focuses on the learning algorithm itself as the source of unfairness, and attempts to incorporate fairness notions into the algorithm [21]; see e.g. Verma and Rubin [33] for a survey of fairness definitions. Training data has also been identified as source of unfairness; for example, Chen et al. identify sample size differences as a crucial source of unfairness, and decomposes induced unfairness into bias, variance, and noise [10]. Various feedback loops stemming from historical bias have also been identified as sources of unfairness [14, 15, 24]. There are a few others, including selection bias [20], using the wrong metric [27], or using a single

model across multiple underlying data generating processes [22]. However, to the best of our knowledge, market forces in data investment have seen little attention as a source of unfairness. See the survey of Cowgill and Tucker [11] for an in-depth survey of perspectives on the sources of unfairness from computer science and economics.

Our models are built on insights from two extensive, and historically separate, literatures: the formalization of learning from data embodied in computational and statistical learning theory, and models of strategic interactions from the theory of industrial organization (see e.g. Tirole [31]). From learning theory, we apply fundamental bounds on sample complexity derived from the *Probably Approximately Correct* (PAC) framework (see e.g. Kearns and Vazirani [23]) to relate firms’ costs to worst-case error rates; from industrial organization, we modify widely used models of demand (such as linear demand, multilinear models of imperfect substitutes [6], the Tullock contest [32], and Bertrand competition [29]) to link firms’ choices to consumer behavior.

Recently, these two fields have drawn closer, as both computer scientists and economists have begun to model markets for information and data. For example, Aridor et al. [1] and Mansour et al. [25] consider the exploration-exploitation tradeoffs faced by firms competing to win users in a bandits setting, while Ben-Porat and Tennenholtz formalize competition in the *prediction space* that can lead to models very different than those produced by empirical risk minimization algorithms [3, 4]. To the best of our knowledge, however, this is the first work to apply learning theory and industrial organization to explore differing incentives in the context of fairness. The work of Dong et al. [12] is the closest in form to ours, and uses a similar high-level abstraction of learning theory, as well as a proportional-error split in market share, but primarily explores questions of market concentration.

## 3 CONSUMER BEHAVIOR AND LEARNING THEORY

We begin by describing our framework at a high level. In our models, firms use data to create a classifier (or other machine learning model) that is then used to serve consumers. Consumers are split into non-overlapping groups, and choose a firm based on how well the firm’s model is performing for their group. Firms receive revenue based on how many consumers they attract, but must pay for the amount of the data they buy. The more data, the better their model. The firms aim to maximize their profits. In the case where there are multiple firms, the goal of each firm is to maximize their profit at equilibrium, as other firm’s choices affect the number of consumers that they get, and hence their choices. Here, the firm’s only (strategically relevant) choice is how much data to buy.

We start with the monopoly case, where there is only one firm. The firm chooses a number of data points  $M_g$  to buy for each group, where we write  $M$  for the vector of these choices; we write  $\epsilon_g(M_g)$  for the worst-case error the firm can guarantee for group  $g$ , and assume this error is known to consumers. The groups then respond by entering the market according to a demand function  $D_g(\epsilon_g)$ , where  $D_g(\epsilon_g)$  represents the proportion of  $g$  that uses the firm’s model. Each group has  $\mu_g$  total people, so the firm’s revenue is

$\sum_{g \in \mathcal{G}} \mu_g D_g(\varepsilon_g(M_g))$ . The firm also pays for the data, represented by a cost function  $C(M)$ .

We will discuss our choices for  $\varepsilon$ ,  $D$ , and  $C$  in Sections 3.1 and 3.2. But for now, the firm's profit is just the revenue the firm makes minus the cost it spends to acquire that data, leading to the following optimization problem:

**DEFINITION 3.1 (THE MONOPOLIST'S PROBLEM).** *The firm chooses  $M$  to maximize its profit  $\pi(M)$ , i.e.*

$$\max_M \pi(M) = \max_M \sum_{g \in \mathcal{G}} \mu_g D_g(\varepsilon_g(M_g)) - C(M).$$

Because we will assume in Section 3.1 that  $\varepsilon_g$  is a deterministic function of  $M_g$ , we can also rewrite this optimization problem as

$$\max_{\varepsilon} \pi(\varepsilon) = \max_{\varepsilon} \sum_g \mu_g D_g(\varepsilon_g) - C(\varepsilon),$$

where  $\varepsilon$  is the vector of  $\varepsilon_g$ . We define  $\pi(\varepsilon) = \sum_g \mu_g D_g(\varepsilon_g) - C(\varepsilon)$  as the total profit the monopolist makes. We will have an additively separable cost function in  $g$ , i.e.  $C(\varepsilon) = \sum_{g \in \mathcal{G}} C_g(\varepsilon_g)$ , which will allow us to also refer to the per-group profit:  $\pi_g(\varepsilon_g) = \sum_{g \in \mathcal{G}} \mu_g D_g(\varepsilon_g) - C_g(\varepsilon)$ .

On the other hand, when there are multiple firms  $\mathcal{F}$ , maximizing profit is not longer just an optimization problem, because each firm's optimal choice will depend on its opponent's choice. So instead, we search for a Nash equilibrium, which is the workhorse solution concept in classical game theory. Under such a Nash equilibrium, each firm plays their best response, given all the choices of the other firms. For a more thorough background, see [17].

Extending our notation, we have the same components as in the monopolist case, except now we write  $M_{gi}$  for the number of data points the  $i$ th firm buys for group  $g$ ,  $\varepsilon_{gi}$  is the error rate of the  $i$ th firm on group  $g$ , and  $D_{gi}(\varepsilon_g) = D_{gi}(\varepsilon_{gi}, \varepsilon_{g,-i})$  is the demand for the  $i$ th firm from group  $g$ , given the vector  $\varepsilon_g = (\varepsilon_{gi})_{i \in \mathcal{F}}$  of error rates.

**DEFINITION 3.2 (THE COMPETITOR'S PROBLEM).** *Firms simultaneously announce their choices, resulting in a matrix  $M = (M_{gi})$  of data points purchased. Each group in the market responds according to  $\varepsilon_g(M_g)$ .*

*Then a (pure) equilibrium under profit-maximizing firms is a set of vectors  $M_i^*$  chosen with a best response:*

*For all  $i$ ,*

$$M_i^* = \operatorname{argmax}_{M_i} \pi_i(M_i, M_{-i}^*)$$

*where for any  $M$ ,*

$$\pi_i(M_i, M_{-i}) = \sum_g \mu_g D_{gi}(\varepsilon_{gi}(M_{gi}), \varepsilon_{g,-i}(M_{g,-i})) - C(M_i).$$

We will only consider pure strategy Nash equilibria in this work.

Again, we can write an equivalent definition of the competitor's problem in terms of error:

$$\max_{\varepsilon_i} \pi_i(\varepsilon_i, \varepsilon_{-i}^*) = \max_{\varepsilon_i} \sum_{g \in \mathcal{G}} \mu_g D_{gi}(\varepsilon_{gi}, \varepsilon_{g,-i}^*) - C(\varepsilon_i),$$

where  $\varepsilon_i^*$  is the vector of error rates given by the associated equilibrium choice  $M_i^*$ . We also use  $\pi_{gi}(\varepsilon_{gi}, \varepsilon_{g,-i}^*) = \mu_g D_{gi}(\varepsilon_{gi}, \varepsilon_{g,-i}^*) - C_g(\varepsilon_i)$  to refer to the profit  $i$  makes on group  $g$ .

Note that a firm  $i$  only enters a market in the first place if  $\pi_i(\varepsilon^*) > 0$ . In this work, we do not consider the case when  $\pi_i(\varepsilon^*) \leq 0$ , as our goal is to show that even when firms *do* enter the market for each group, market forces may *still* create a disparity between groups.

Finally, in Section 6, we discuss imposing regulation on a monopolist to ensure some kind of 'fairness' across groups. We consider two different kinds of constraints a regulator could impose on a firm. The first is what we refer to as *relative error equality*, which roughly corresponds to group fairness in binary classification [5]. For all  $g, g' \in \mathcal{G}$ , we require

$$\frac{\varepsilon_g}{\varepsilon_{g'}} \leq (1 + \chi),$$

for parameter  $\chi \geq 0$ . On the other hand, we could ask for an *absolute error guarantee*, requiring that the error rates for both firms are low, regardless of how close to each other they are: For all  $g \in \mathcal{G}$ , we require instead

$$\varepsilon_g \leq \chi.$$

This roughly corresponds with maximin notions of fairness, e.g. [5, 7, 16].

We investigate what happens when a monopolist satisfies one of these two constraints. Because error is the relevant quantity from the regulator's perspective, and error and data investment are so tightly linked, we write the regulated monopolist's problem in terms of the choice of error:

**DEFINITION 3.3 (REGULATED MONOPOLIST'S PROBLEM).** *The firm chooses  $M$  to maximize its profit  $\pi(M)$  subject to a constraint:*

$$\max_{\varepsilon} \pi(\varepsilon) = \max_{\varepsilon} \sum_g \mu_g D_g(\varepsilon_g) - C(\varepsilon) \text{ s.t. } f_r(\varepsilon) \leq 0 \ \forall r \in R,$$

*where either  $R = \mathcal{G} \times \mathcal{G}$  and  $f_{g,g'}(\varepsilon) = \varepsilon_g - (1 + \chi)\varepsilon_{g'}$ , or  $R = \mathcal{G}$  and  $f_g(\varepsilon) = \varepsilon_g - \chi$ .*

Just as is the case in binary classification, where different settings may call for different notions of fairness, which version of fairness regulator should impose will depend on the context and the ethical assumptions she maintains.

### 3.1 Data, Costs, and Learning Theory

A key component to our model is how choices in data investment drive error rates. We assume that firms build a model to provide a product to consumers, and that this model is learned from data. The firms have access to independent and identically distributed data from fixed data sources that reflect the same distribution that consumers care about.

In the PAC model of learning [23], there is a class of hypotheses  $H$ , and each hypothesis  $h \in H$  has an associated risk  $R(h)$ , typically representing the error rate of  $h$ . For example, in binary classification,  $R(h) = E_{x,y \sim \mathcal{D}}[h(x) \neq y]$ , though our model will be applicable to other settings as well. With only access to data drawn from  $\mathcal{D}$ , rather than  $\mathcal{D}$  itself, the learner cannot guarantee its risk, but *can* achieve high probability upper bounds on its risk. In the agnostic PAC setting, there is a learning algorithm that upon seeing a sample of size  $M$ , except with probability  $\delta$ , returns a hypothesis  $h$  such that

$$R(h) - \min_{h' \in H} R(h') \leq K \sqrt{\frac{d_H + \log(1/\delta)}{M}},$$

where  $\min_{h' \in H} R(h')$  is the Bayes error,  $d_H$  is the VC dimension of  $H$ , and  $K$  is a universal constant. (See [28] for more on PAC learning, VC dimension, and the various kinds of PAC learning.)

To model the fact that getting appropriate data can have group-dependent sources and thus costs, we assume data for each group is drawn separately from distributions  $\mathcal{D}_g$ . The firms choose  $M_g$ , the number of data points to draw, and will use a learning algorithm with a PAC guarantee for each data set and give to a consumer of group  $g$  the output of the corresponding hypothesis.

Achieving such bounds would not be useful to the firm unless consumers make decisions based on these bounds. Here, we assume that the consumers have no more access than the firms: they do not have access to the distribution, so they cannot make decisions based on the true group-level error rates. Given this, we assume that the consumers use firms' bound on the excess error  $R(h) - \min_{h' \in H} R(h')$ , which we refer to as the *worst-case excess error* rate. Of course, in reality, consumer decisions are not necessarily based on the worst-case error rate. However, given that consumers often do in practice have to make choices using relatively little information about firms, and have trouble predicting how well exactly a firm will treat them, we believe this is a natural place to start. In particular, bounds on the excess error rates represent the minimal amount of information consumers need to make informed decisions.

Thus, we set

$$\varepsilon_{gi}(M_{gi}) = \frac{\gamma_g}{M_{gi}^{1/q}},$$

for constants  $\gamma_g > 0$  and  $q > 0$ . For example, in agnostic PAC learning,  $q = 2$  and  $\gamma_g = \sqrt{K(d_H + \log(1/\delta))}$ . Note that we are assuming  $\delta$  is fixed ahead of time, but we allow in general for  $\gamma_g$  to be group-dependent. Agnostic PAC learning is far from the only type of learning to have this form; the realizable PAC setting, the multi-class setting, and many regression settings all have this form [28].

This set-up does ignore the possibility of transfer learning, i.e. using the data from  $D_g$  to help with learning for another group  $g'$ . We avoid this scenario so as to concentrate on the 'unfairness' generated via the market incentives instead of the unfairness generated, for example, when an assumption about the similarity between  $D_g$  and  $D_{g'}$  fails to hold.

The choice of  $M_g$  determines not only the worst-case error rates, but the cost to the firm of generating that data, either by collecting it in the wild or buying it from another source. As mentioned above, we permit the costs to be group-dependent. We assume the cost is additively separable and linear in  $M_g$ :

$$C_{gi}(M_{gi}) = \phi_{gi} + c_{gi}M_{gi} \text{ and } C_i(M_i) = \sum_{g \in \mathcal{G}} C_{gi}(M_{gi}),$$

for constants  $\phi_{gi}, c_{gi}$ , where  $\phi_{gi}$  represents the fixed cost of entering the market.

Since we can rewrite  $M_{gi} = (\gamma_g / \varepsilon_{gi})^q$ , this model is equivalent to first choosing a worst-case error rate  $\varepsilon_{gi}$  and then paying a cost

$$C_{gi}(\varepsilon_{gi}) = \phi_{gi} + \frac{\gamma_g}{\varepsilon_{gi}^q}$$

for each group  $g$ , where  $\gamma_g$  is redefined to minimize the number of constants we employ.

So now

$$M_{gi} = \frac{\gamma_g}{c_{gi}\varepsilon_{gi}^q}.$$

This version is the one we will use for the remainder of the paper. Note that the cost function is convex, which means that whenever the demand is concave, so is the profit function.

### 3.2 Models of Consumer Choice

The firm's revenue is driven by how *demand* for its product reacts to its choice of worst-case error guarantees. We consider several models of this demand, each inspired by well-studied models in microeconomics. While firms are primarily concerned only with aggregate changes in demand, rather than the decisions of individual consumers, each of our models can be founded on natural models of individual consumer behavior, and we provide such models in several cases.

In the monopoly case, we use a simple model of *linear* demand; while an idealization, linear demand is often used even in econometric estimation (see e.g. [18]). In the competitive case, there are a variety of natural demand models, each embedding different assumptions about how consumers choose between firms and how stringently they react to differing error levels. We study three models along a spectrum of rationality: a *multilinear* demand, generalizing the monopoly case; a parameterized *proportional* demand; and an *approximately* rational demand, where consumers exclusively use the firm with lowest error (up to some tolerance).

We give the details of these models of demand in each appropriate subsection in Sections 4 and 5. Under each model, there are parameter regimes where firms choose not to invest in data collection for some groups at all; while this may reflect some real-world scenarios, the purpose of this paper is to highlight economic incentives that create inequality *even aside from such extreme scenarios*. As such, we will focus on *interior* optima or equilibria. In an interior optimum, the monopolist must make positive profit (so that it enters the market) and choose error rates strictly smaller than 1 for each group (so that it is investing in data collection for each group). Similarly, interior equilibria require that profits for both firms must be positive and each error rate strictly smaller than 1. Our theorems statements will highlight this focus.

## 4 MONOPOLY

We start with the case where there is one firm in the market and demand is linear:

**DEFINITION 4.1 (LINEAR DEMAND).** A linear demand function for each group  $g$  is given by:

$$D_g(\varepsilon_g) = \alpha_g - \beta_g \varepsilon_g,$$

where  $0 < \beta \leq \alpha \leq 1$ .

A linear demand curve can arise from a simple model of consumer behavior: suppose utility-maximizing consumers consider whether or not to use the product, and only use it if it is above some threshold (equivalent to being better than some 'outside option'). If these thresholds are uniformly distributed over some interval, then demand will be linearly decreasing over an interval. Strictly

speaking, this is a *piecewise* linear demand, but this does not greatly affect optimal behavior of the firm - it merely means that it will never choose outside the linearly decreasing range unless they are choosing not to invest in providing quality products at all. For simplicity of our theorem statements, we will assume that parameters are such that the firm's achievable errors are a subset of the linear portion of the demand curve, here  $0 < \beta \leq \alpha \leq 1$ , but in Appendix A, we generalize to arbitrarily large  $\alpha, \beta$  to ensure that our results still qualitatively hold.

Our main result here is the following:

**THEOREM 4.2 (MONOPOLY INEQUALITY).** *Suppose a monopolist with learning rate  $q$  faces linear demand. Then in any interior optimum, for every pair of groups  $g$  and  $g'$ , the error inequality is given by:*

$$\frac{\varepsilon_g^*}{\varepsilon_{g'}^*} = \left( \frac{\mu_{g'} \beta_{g'} \gamma_g}{\mu_g \beta_g \gamma_{g'}} \right)^{\frac{1}{q+1}}.$$

Again, we focus on an interior optimum. Three factors affect the error gap between the minority and the majority: the size of the minority as a share of the total market; the marginal cost of gathering data on the minority vs. on the majority; and the elasticities of the populations with respect to the error. It is also worth noting that the fundamental nature of the learning problem, via the learning rate  $q$ , affects the magnitude of error inequality.

Theorem 4.2 is a consequence of the following lemma:

**LEMMA 4.3.** *Suppose a monopolist with learning rate  $q$  faces linear demand. Then in any interior optimum, error levels set by the monopolist are given by:*

$$\varepsilon_g^* = \left( \frac{q \gamma_g}{\mu_g \beta_g} \right)^{\frac{1}{q+1}}.$$

**PROOF.** Recall that

$$\pi(\varepsilon) = \sum_{g \in \mathcal{G}} \mu_g (\alpha_g - \beta_g \varepsilon_g) - \sum_{g \in \mathcal{G}} \left( \phi_g + \frac{\gamma_g}{\varepsilon_g^q} \right).$$

Now, we notice that this profit function is separable into the sum of profits from each market. Differentiating with respect to  $\varepsilon_g$  separately and setting to zero, we arrive at the first-order conditions:

$$\frac{\partial \pi}{\partial \varepsilon_g} = -\mu_g \beta_g + \frac{q \gamma_g}{\varepsilon_g^{q+1}} = 0.$$

Solving this equation yields  $\varepsilon_g^* = \left( \frac{q \gamma_g}{\mu_g \beta_g} \right)^{\frac{1}{q+1}}$ . This is indeed a maximum because profit is concave, so the only alternative is an exterior optimum.  $\square$

Notice that if, for all  $g$ ,  $\pi_g(\varepsilon_g^*) > 0$ ,  $\pi_g(\varepsilon_g^*) > \pi_g(1)$ , and  $\varepsilon_g^* < 1$ , then the interior optimum exists and is unique.

## 5 COMPETITION

In this section, we show that under most reasonable models of competition, the introduction of competition does not mitigate error-inequality compared to the monopoly equilibrium, and may, in fact, increase it. Only under Bertrand-like competition, which assumes consumers are strictly rational, is inequality significantly

mitigated. In particular, we show that under both the Tullock and the multi-linear models of demand, the inequality between groups as measured by the error rates does not improve relative to the monopoly case. In the case of the Tullock model, as a function of the relative size of the groups, inequality is actually worse.

### 5.1 Multilinear Demand

Next, we consider a simple generalization of linear demand to the two-firm case. This model can be interpreted as a model of competition in identical products with differing quality levels as in [2], but can also be interpreted (as well as microfounded, and used to estimate structural parameters, as in [6]) as markets for imperfect substitutes.

**DEFINITION 5.1 (MULTILINEAR DEMAND).** *The multilinear linear demand function is, for firms  $i$  and  $j$ , and for each group  $g$ ,*

$$D_{gi}(\varepsilon_{gi}, \varepsilon_{gj}) = \alpha_{gi} - \beta_g \varepsilon_{gi} + \lambda_g \varepsilon_{gj},$$

where  $0 < \lambda_g < \beta_g \leq \alpha_{gi}$  and  $\alpha_{gi} + \lambda_g \leq 1$ .

We require  $\beta_g > \lambda_g$  so that demand reacts more strongly to a firm's own error rates than its opponents - this ensures that if both firms increase error, total demand decreases. The other conditions on the parameters are to ensure that the demand is truly (multi)-linear, as opposed to piece-wise linear.

Note it is not the case that all consumers choose the firm with lower error, as one might expect if the products of the firms were perfect substitutes. Instead, users switch between firms depending on their error rates, and even if firms achieved perfect accuracy, the split of the total market might not be even, as captured by differing  $\alpha_{gi}$ . This could represent brand loyalty, for example, or perhaps that firms' products are not perfectly identical.

Our main result for this case states that the gap between error rates is the same as in the monopoly case:

**THEOREM 5.2.** *Suppose that two firms with learning rate  $q$  compete under multilinear demand. Then in any interior equilibrium, for every pair of groups  $g$  and  $g'$ , error inequality is given by*

$$\frac{\varepsilon_{gi}^*}{\varepsilon_{g'i}^*} = \left( \frac{\mu_{g'} \beta_{g'} \gamma_{gi}}{\mu_g \beta_g \gamma_{g'i}} \right)^{\frac{1}{q+1}}.$$

Theorem 5.2 is a consequence of the fact that the firm's optimal choice does not depend on its opponent's error rates; that is they have a dominant strategy. This is formalized by the following lemma, which is enough to prove Theorem 5.2:

**LEMMA 5.3.** *Suppose that two firms with learning rate  $q$  compete under multilinear demand. Then in any interior equilibrium, error levels are given by*

$$\varepsilon_{gi}^* = \left( \frac{q \gamma_{gi}}{\mu_g \beta_g} \right)^{\frac{1}{q+1}}.$$

**PROOF.** This proof will be very similar to that of Lemma 4.3, only now, the behavior of the other firm will affect profit. Recall

$$\pi_i(\varepsilon_{gi}, \varepsilon_{gj}) = \sum_{g \in \mathcal{G}} \mu_g (\alpha_{gi} - \beta_g \varepsilon_{gi} + \lambda_g \varepsilon_{gj}) - \sum_{g \in \mathcal{G}} \frac{\gamma_{gi}}{\varepsilon_{gi}^q}.$$

We can see that even though the behavior of the other firm will affect profit, firm  $i$  still has a dominant strategy. This is because the first-order conditions do not depend on the other firm:

$$\frac{\partial \pi_i}{\partial \varepsilon_{gi}} = -\mu_g \beta_g + \frac{q \gamma_{gi}}{\varepsilon_{gi}^{q+1}}.$$

This is the same first order condition as in the monopolist's case, with the same implication:  $\varepsilon_{gi}^* = \left( \frac{q \gamma_{gi}}{\mu_g \beta_g} \right)^{\frac{1}{q+1}}$ .  $\square$

Similar to the case of the monopolist's, notice that if, for all  $g, i$ ,  $\pi_{gi}(\varepsilon_{gi}^*, \varepsilon_{gj}^*) > 0$ ,  $\pi_{gi}(\varepsilon_{gi}^*, \varepsilon_{gj}^*) > \pi_{gi}(1, \varepsilon_{gj}^*)$ , and  $\varepsilon_{gi}^* < 1$ , then an interior equilibrium exists.

## 5.2 Proportional Demand

In this section, we consider a model inspired by [12], and thus, indirectly, by the Tullock contest [32]. In particular, firms split the market proportionally to the other firms' error:

**DEFINITION 5.4 (PROPORTIONAL DEMAND).** *In a multi-firm market, we say that demand is proportionally split with competition exponent  $\rho$  if*

$$D_{gi}(\varepsilon) = 1 - \frac{\varepsilon_{gi}^\rho}{\sum_j \varepsilon_{gj}^\rho} = \frac{\sum_{j \neq i} \varepsilon_{gj}^\rho}{\sum_j \varepsilon_{gj}^\rho}.$$

Here, we focus on the two-firm case, in which case we can write

$$D_{gi}(\varepsilon_{gi}, \varepsilon_{gj}) = 1 - \frac{\varepsilon_{gi}^\rho}{\varepsilon_{gi}^\rho + \varepsilon_{gj}^\rho} = \frac{\varepsilon_{gj}^\rho}{\varepsilon_{gi}^\rho + \varepsilon_{gj}^\rho}.$$

Now we can write our inequality theorem:

**THEOREM 5.5 (INEQUALITY UNDER PROPORTIONAL DEMAND).** *Suppose two firms with learning rate  $q$  compete under proportional demand. Then in any interior equilibrium error inequality is given by*

$$\frac{\varepsilon_{gi}^*}{\varepsilon_{gi}^*} = \left( \frac{\rho g' \mu_{g'}}{\rho g \mu_g} \right)^{\frac{1}{q}} \cdot \frac{f(\gamma_{gi}, \gamma_{gj}, q)}{f(\gamma_{g'i}, \gamma_{g'j}, q)},$$

where

$$f(\gamma_{gi}, \gamma_{gj}, q) = \frac{(\gamma_{gi}^q + \gamma_{gj}^q)^{\frac{2}{q}}}{\gamma_{gi}^{1-\frac{1}{q}} \gamma_{gj}^q}.$$

Recall that in the monopoly case, the exponent was  $1/(q+1)$  instead of  $1/q$ , meaning that introducing competition under this model has actually *exacerbated* the effect of minority status on inequality. Note also that the relative inequality between two groups based on the results from a particular firm depends not only on that firm's cost structure for the two groups, but also on the opposing firm's cost structure for the two groups.

Proving Theorem 5.5 requires finding the equilibrium:

**LEMMA 5.6.** *Suppose two firms with learning rate  $q$  face proportional demand with competition exponent  $\rho$ . In any interior equilibrium, error rates are given by:*

$$\varepsilon_{gi}^* = \left( \frac{q \gamma_{gi}}{\rho g \mu_g} \right)^{\frac{1}{q}} \frac{(\gamma_{gi}^q + \gamma_{gj}^q)^{\frac{2}{q}}}{\gamma_{gi}^q \gamma_{gj}^q} = \left( \frac{q}{\rho g \mu_g} \right)^{\frac{1}{q}} \frac{(\gamma_{gi}^q + \gamma_{gj}^q)^{\frac{2}{q}}}{\gamma_{gi}^{1-\frac{1}{q}} \gamma_{gj}^q}.$$

If  $\varepsilon_{gi}^* < 1$  for all  $g$  and  $i$  then there exists a setting of parameters for which  $(\varepsilon_{gi}^*, \varepsilon_{gj}^*)$  is the unique equilibrium.

For brevity, we relegate the full proof and the characterization of when these conditions hold, to Section C. Below, we detail the instructive portion of the proof for the special case in which  $q = \rho = 1$ .

**LEMMA 5.7.** *Suppose two firms with learning rate 1 face proportional demand with competition exponent 1. In any interior equilibrium, error levels are given by:*

$$\varepsilon_{gi}^* = \frac{1}{\mu_g} \cdot \frac{(\gamma_{gi} + \gamma_{gj})^2}{\gamma_{gj}}.$$

Sufficient conditions for existence are that for all  $g$  and  $i$ :  $\phi_{gi} \leq \frac{\mu_g \gamma_{gj}^2}{(\gamma_{gi} + \gamma_{gj})^2}$ ,  $\varepsilon_{gi}^* < 1$ ,  $\frac{(\gamma_{gi} + \gamma_{gj})^2}{\mu_g \gamma_{gj}} < 1$ . If, moreover,  $\min_k \gamma_{gk} \geq (\max_k \gamma_{gk})^2$ , then  $(\varepsilon_{gi}^*, \varepsilon_{gj}^*)$  is the unique equilibrium.

The conditions of Lemma 5.7 are stated in terms of  $\gamma_{gi}$ ; recall, though, that  $\gamma_{gi}$  is not a primitive of our model, but rather the product of the per-datapoint cost and learning theory constants. These conditions thus imply conditions on these underlying constants. In the symmetric case, this asks that the per-datapoint cost  $c_g$  satisfies:

$$c_g \leq \frac{d_H + \log \frac{1}{\delta}}{12^q}.$$

which merely requires that the per-datapoint cost is not too large relative to the desired hypothesis class and success probability. In the asymmetric case, we require that firms do not face ratios of data cost to learning constant that are *too different* from each other. If either of these conditions is violated, then one or both firms may have an incentive to stop investing completely in data acquired for a group. Such non-interior equilibria can obviously lead to severe error inequality, but again, Theorem 5.5 demonstrates the existence of incentives to unfairness *even ruling out these extreme cases*.

**PROOF OF LEMMA C.1.** We can write Firm  $i$ 's profit from each group as:

$$\pi_{gi}(\varepsilon_{gi}, \varepsilon_{gj}) = \mu_g \frac{\varepsilon_{gj}}{\varepsilon_{gi} + \varepsilon_{gj}} - \frac{\gamma_{gi}}{\varepsilon_{gi}} - \phi_{gj}.$$

The strategy space of the firm is to select an  $\varepsilon_{gi}$  for each group in  $(0, 1]$ ; we search for a pure strategy Nash equilibrium. At a high level, our strategy to do so is as follows: first, we fix the opposing firm's action  $\varepsilon_{gj}$ . Optimizing Firm  $i$ 's profit gives a best-response to the fixed action  $\varepsilon_{gj}$ . An equilibrium pair must simultaneously satisfy *both* firms' first order conditions, given the other, so we obtain two simultaneous equations that yield the equilibrium relationship between the two firms' actions. Solving this yields a candidate solution. Then, we can show that the candidate solution is indeed a maximum via the concavity of the profit function. Finally, we

need but check that there are no solutions at the endpoints, and we provide conditions when this is ruled out.

Now, fixing Firm  $j$ 's choice  $\varepsilon_{gj}$ , the profit function  $\pi_g$  is just a function of  $\varepsilon_{gi}$ . Differentiating this gives:

$$\frac{\partial \pi_{gi}}{\partial \varepsilon_{gi}} = -\varepsilon_{gj} \mu_g (\varepsilon_{gi} + \varepsilon_{gj})^{-2} + \gamma_{gi} \varepsilon_{gi}^{-2}.$$

We set this equal to zero. Since satisfying this condition is required for  $\varepsilon_{gi}$  to be a best-response we can plug in  $\varepsilon_{gj}^*$ , whatever that may be, requiring:

$$\frac{\varepsilon_{gi}^2 \varepsilon_{gj}^*}{\mu_g \gamma_{gi}} = (\varepsilon_{gi} + \varepsilon_{gj}^*)^2, \quad (1)$$

and in particular, this must apply to the best-response  $\varepsilon_{gi}^*$ . We can apply similar logic to Firm  $j$ . Hence, for  $(\varepsilon_{gi}^*, \varepsilon_{gj}^*)$  to be best-responses to each other – that is, to be in (interior) equilibrium – we must have that

$$\frac{\varepsilon_{gi}^{*2} \varepsilon_{gj}^*}{\mu_g \gamma_{gi}} = (\varepsilon_{gi}^* + \varepsilon_{gj}^*)^2 = \frac{\varepsilon_{gj}^{*2} \varepsilon_{gi}^*}{\mu_g \gamma_{gj}}. \quad (2)$$

This implies that

$$\varepsilon_{gj}^* = \varepsilon_{gi}^* \frac{\gamma_{gj}}{\gamma_{gi}}.$$

Substituting this condition back into Equation 2, we obtain that

$$\frac{\varepsilon_{gi}^{*3} \gamma_{gj}}{\mu_g \gamma_{gi}} = (\varepsilon_{gi}^* + \varepsilon_{gi}^* \frac{\gamma_{gj}}{\gamma_{gi}})^2 \implies \varepsilon_{gi}^* = \frac{(\gamma_{gi} + \gamma_{gj})^2}{\mu_g \gamma_{gj}}.$$

Symmetric logic yields  $\varepsilon_{gj}^* = \frac{(\gamma_{gi} + \gamma_{gj})^2}{\mu_g \gamma_{gi}}$ .

Now, to show that this candidate solution is indeed an equilibrium, we must show that these actions are best-responses to each other. Fix  $\varepsilon_{gj}^* = \frac{(\gamma_{gi} + \gamma_{gj})^2}{\mu_g \gamma_{gi}}$ . Then we can view  $\pi_{gi}(\varepsilon_{gi}, \varepsilon_{gj}^*)$  as a continuous function on  $(0, 1]$ . By construction, evaluating  $\frac{\partial}{\partial \varepsilon_{gi}} \pi_{gi}(\varepsilon_{gi}, \varepsilon_{gj}^*)$  at  $\varepsilon_{gi}^*$  must give zero. (It is also easy to verify that this is indeed the case.) If  $\pi_{i, \varepsilon_{gj}^*}(\varepsilon_{gi})$  is concave at  $\varepsilon_{gi}^*$ , then that is a local maximum of the profit function (given  $\varepsilon_{gj}^*$ ).

To see that it is concave, note that

$$\frac{\partial^2}{\partial \varepsilon_{gj}^2} \pi_{gi}(\varepsilon_{gi}, \varepsilon_{gj}) = 2\varepsilon_{gj} \mu_g (\varepsilon_{gj} + \varepsilon_{gi})^{-3} - 2 \frac{\gamma_{gi}}{\varepsilon_{gi}^3}.$$

Evaluating this quantity at  $\varepsilon_{gi}^*$  gives:

$$\begin{aligned} & \frac{\partial^2}{\partial \varepsilon_{gj}^2} \pi_{gi}(\varepsilon_{gi}, \varepsilon_{gj}^*) \Big|_{\varepsilon_{gi} = \varepsilon_{gi}^*} \\ &= 2\mu_g \frac{(\gamma_{gi} + \gamma_{gj})^2}{\mu_g \gamma_{gi}} \left( \frac{(\gamma_{gi} + \gamma_{gj})^2}{\mu_g \gamma_{gj}} + \frac{(\gamma_{gi} + \gamma_{gj})^2}{\mu_g \gamma_{gi}} \right)^{-3} \\ & \quad - 2 \frac{\gamma_{gi}}{((\gamma_{gi} + \gamma_{gj})^2 / (\mu_g \gamma_{gi}))^3}. \end{aligned}$$

Straightforward, if tedious, algebra lets us rewrite the right hand side and conclude that

$$\frac{\partial^2}{\partial \varepsilon_{gj}^2} \pi_{gi}(\varepsilon_{gi}, \varepsilon_{gj}^*) \Big|_{\varepsilon_{gi} = \varepsilon_{gi}^*} = \frac{2\mu_g^3 \gamma_{gj}^3 \gamma_{gi}}{(\gamma_{gi} + \gamma_{gj})^6} \left[ \frac{\gamma_{gi}}{\gamma_{gi} + \gamma_{gj}} - 1 \right].$$

But notice that this quantity is always negative if costs are positive; hence,  $\varepsilon_{gi}^*$  is indeed a local maximum of  $\pi_{i, \varepsilon_{gj}^*}$ .

To ensure that this point is a global maximum, we must compare it with the profit at the endpoint. For brevity, we defer this calculation to the Appendix in Section C

Finally, note that equilibrium profits are positive if  $\pi_{gi}(\varepsilon_{gi}^*, \varepsilon_{gj}^*) \geq 0$ ; this is true whenever

$$\frac{\mu_g \gamma_{gj}^2}{(\gamma_{gi} + \gamma_{gj})^2} \geq \phi_{gi}, \quad (3)$$

i.e. fixed costs are not extremely large. Positive profits and the fact that  $\varepsilon_{gi}^*$  globally maximizes profit given  $\varepsilon_{gj}^*$  ensures that the putative equilibrium pair forms an equilibrium.

To identify conditions in which this equilibrium is unique, we need to eliminate the only other possible equilibrium (both firms choosing  $\varepsilon = 1$ ). Again, for brevity, we defer this calculation to the appendix.  $\square$

Again, we pause to highlight several intuitive properties of the equilibrium. First, Firm  $i$ 's choice of error for group  $g$  is *decreasing* with the market size of Group  $g$  as well as the *ferocity* of competition in Group  $g$ . These results are similar to those of Lemma 4.3, with a different functional form and the competition exponent of the Tullock game replacing the error elasticity of demand. It is also, intuitively, increasing in  $\gamma_{gi}$  and decreasing in  $\gamma_{gj}$ , though this is harder to see due to the functional form of  $f$ .

### 5.3 Approximately Rational Demand

Now we consider markets where consumers behave rationally. If we allow consumers to behave *fully* rationally, in the sense of always picking the firm with (even infinitesimally) smaller error, we obtain a model similar to the Bertrand model of competition [19]; accordingly, no equilibrium exists, as we show in Section B.3. Hence, we instead consider a slight relaxation of the fully rational model: Suppose consumers behave rationally, *except* that they do not care about excess error up to  $\zeta_g$  over the optimal error. That is, the lower firm will capture the whole market for errors that are not too small, but for  $\varepsilon_{gi}, \varepsilon_{gj} \in [0, \zeta_g]$ , firms again split the market.

We formally define this demand function below:

**DEFINITION 5.8 (BERTRAND-LIKE TOLERANT DEMAND).** *In a multi-firm market, we say demand is  $\zeta$ -tolerant rational with  $\zeta > 0$  if*

$$D_{gi}(\varepsilon) = \begin{cases} 1 & \min_k \varepsilon_{gk} > \zeta_g \text{ and } \varepsilon_{gi} < \min_{j \neq i} \varepsilon_{gj} \\ \frac{1}{\sum_j 1[\varepsilon_{gj} = \min_k \varepsilon_{gk}]} & \min_k \varepsilon_{gk} > \zeta_g \text{ and } \varepsilon_{gi} = \min_{j \neq i} \varepsilon_{gj} \\ 0 & \min_k \varepsilon_{gk} > \zeta_g \text{ and } \varepsilon_{gi} > \min_{j \neq i} \varepsilon_{gj} \\ \frac{1[\varepsilon_{gi} \leq \zeta_g]}{\sum_j 1[\varepsilon_{gj} \leq \zeta_g]} & \min_k \varepsilon_{gk} \leq \zeta_g \end{cases}.$$

We will show that there exists a unique equilibrium here (for appropriate parameters) in which groups' error levels are determined not by their sizes, but by their optimal errors and their tolerances.

**THEOREM 5.9 (APPROXIMATELY RATIONAL INEQUALITY).** *Suppose that two firms compete under  $\zeta$ -tolerant demand. Then in any interior*

equilibrium, error inequality is given by

$$\frac{\varepsilon_g}{\varepsilon_{g'}} = \frac{\zeta_g}{\zeta_{g'}}$$

where  $\zeta_g, \zeta_{g'}$  is users' tolerance threshold (assumed to be strictly positive). Moreover, if  $\gamma_{gi} < \frac{\zeta_g \mu_g}{2}$  for all  $g, i$ , the unique equilibrium is interior.

In particular, Theorem 5.9 shows that under this approximate Bertrand-like model of competition, the dependence on group size in the error inequality is eliminated. Instead, inequality depends merely on the optimal error achievable under the hypothesis class used by firms and groups' tolerances.

Note that the conditions of Theorem 5.9 is just asking that

$$c_{gi} \leq \left( \frac{\mu_g \zeta_g}{2} \right)^q \frac{1}{d_{H_i} + \log \frac{1}{\delta}}.$$

As before, we can interpret this as a condition that the per-datapoint cost is not too large relative to the total market size and the learning theory constants.

Theorem 5.9 follows from the following lemma:

**LEMMA 5.10 (APPROXIMATE RATIONAL EQUILIBRIUM).** *Suppose that two firms compete under  $\zeta$ -tolerant demand, and  $\gamma_{gi} < \frac{\zeta_g \mu_g}{2}$  for all  $g, i$ . Then an interior pure strategy equilibrium exists in which*

$$\varepsilon_{gi}^* = \zeta_g,$$

and this equilibrium is unique.

**PROOF.** We posit that the profile  $(\zeta_g, \zeta_g)$  is an equilibrium. To see this, note that a firm deviating to some  $\varepsilon > \zeta_g$  would lose its entire market share, and so would end up with negative profit. Under the conditions of the theorem, though,

$$\pi_i(\zeta_g, \zeta_{g'}) = \frac{\mu_g}{2} - \frac{\gamma_i}{\zeta_g} > 0$$

so deviating to a higher error, with negative profit, cannot be a profitable deviation. On the other hand, deviating to  $\varepsilon \in [0, \zeta_g)$  would result in the same market share, but with increased costs. Hence, deviating to decreased error is also not a profitable deviation.

To see that there can be no other equilibria, notice that if both firms were setting error in  $\varepsilon \in [0, \zeta_g)$ , they would have an incentive to deviate to  $\zeta_g$ ; if one firm's error were in that range and the other's were above, then the firm with higher error would have an incentive to deviate to  $\zeta_g$ ; and finally, if both firms were above  $\zeta_g$ , either firm could profitably deviate to slightly lower error.  $\square$

Unfortunately, even this relaxation of full rationality may not be a realistic model of competition in many cases; it still requires that outside of the range of  $[0, \zeta_g]$ , all consumers are perfectly discerning. This is unlikely to be true in practice. Without such an assumption, the conclusions of this model do not hold. Models like the proportional split and multilinear demand are more likely to capture salient market features in practice.

## 6 REGULATION

In this section, we consider the perspective of a regulator with the power to require one of two kinds of error equality, and analyze the response of the monopolistic firm to each. These constraints that the regulator may impose are *relative error equality* and *absolute error equality*. We quantify the direct cost associated with imposing these constraints, in terms of increased error to the majority group under the first kind and lost profit to the monopolist in both. This serves to give a sense of the direct tradeoffs involved in regulating machine-learning driven markets. We highlight, though, that there may be non-quantifiable benefits to equity across groups, and only societal deliberation can evaluate these tradeoffs.

Which of these two types of regulation is preferred will depend on the context. Requiring errors across groups to all be similar – relative error equality – may not be sufficiently strong if large error is harmful regardless of another group's error rate, but also may be too strict when small absolute errors are perceived as approximately equivalent. On the other hand, absolute error equality – where we require all errors to be below a threshold – treats all small absolute errors as equivalent, but still allows a large relative gap in error rates across groups. An absolute error bound shifts the 'burden' of fairness entirely to the firm, which may be preferable from a consumer standpoint; at the same time, decreasing profits for monopolies may reduce the incentive to innovate, which may also be undesirable.

We make the following assumption for the rest of the section for ease of exposition:

**ASSUMPTION 6.1.** *There are two groups  $\mathcal{G} = \{A, B\}$ , there is an interior optimum  $\varepsilon_A^M, \varepsilon_B^M < 1$  (i.e. the unconstrained monopoly enters the market), and B has lesser market power and higher data costs, i.e.*

$$\mu_B \beta_B \leq \mu_A \beta_A \quad \text{and} \quad \gamma_B \geq \gamma_A.$$

We refer to group A as the *majority* group and B as the *minority* group. We also define  $(\varepsilon_A^M, \varepsilon_B^M)$  and  $(\varepsilon_A^R, \varepsilon_B^R)$  to be the monopolist's and regulated monopolist's optimal choices, respectively.

Note that an immediate consequence of Assumption 6.1 and Theorem 4.3 is that  $\varepsilon_B^M \geq \varepsilon_A^M$ . Finally, we defer omitted proofs from this section to Sections D and E.

### 6.1 Relative Error Equality

In this section, we imagine that a regulator requires the monopolist to achieve error rates within a bounded ratio. We will show that a monopoly responds by investing less in majority data collection and more in minority data collection than it otherwise would, resulting in worse error rates for the majority, better error rates for the minority, approximate equality between groups, and lower profits for the firm. In particular we quantify by how much error rates worsen for the majority and by how much profits are lowered for the monopolist, which we refer to as the 'price' of fairness.

We formalize the regulator's constraint as follows:

**DEFINITION 6.2 (RELATIVE ERROR EQUALITY).** *The regulator forces the firm to achieve error guarantees of bounded ratio:*

$$\frac{\varepsilon_A}{\varepsilon_B} \leq 1 + \chi \quad \text{and} \quad \frac{\varepsilon_B}{\varepsilon_A} \leq 1 + \chi$$

where  $\chi$  is a positive constant.



As in Section 4, we consider a profit-maximizing monopolist. As before, each group has linear demand with market sizes  $\mu_A$  and  $\mu_B$ .

Now, if the regulation has ‘bite’ – that is, if it changes the outcome – the regulated monopolist does the minimum it can to satisfy the constraint; that is, it sets  $\varepsilon_B^R = \varepsilon_A^R(1 + \chi)$ . Formally:

LEMMA 6.3 (SATURATION). *Suppose that the unregulated monopoly sets  $\varepsilon_B^M > \varepsilon_A^M(1 + \chi)$ . Then the profit-maximizing monopoly facing the relative error constraint sets*

$$\varepsilon_B^R = \varepsilon_A^R(1 + \chi).$$

The proof follows from concavity and Jensen’s inequality; we provide details in D.

Lemma 6.3 allows us to characterize the regulated monopolist’s optimal choice of errors under this regulation:

THEOREM 6.4. *Suppose that the unregulated monopoly sets error  $\varepsilon_B^M > (1 + \chi)\varepsilon_A^M$ . Then in any interior optimum, the regulated monopoly sets the errors as*

$$\varepsilon_A^R = \left( q \cdot \frac{\gamma_A + \gamma_B/(1 + \chi)^q}{\mu_A\beta_A + \mu_B\beta_B(1 + \chi)} \right)^{\frac{1}{q+1}}$$

$$\text{and } \varepsilon_B^R = (1 + \chi)\varepsilon_A^R.$$

PROOF. By Lemma 6.3,  $\varepsilon_B^R = (1 + \chi)\varepsilon_A^R$ . Thus, the profit maximization problem can be written solely as a function of  $\varepsilon_A$ :

$$\begin{aligned} \pi(\varepsilon_A) &= \mu_A(\alpha_A - \beta_A\varepsilon_A) + \mu_B(\alpha_B - \beta_B\varepsilon_A(1 + \chi)) \\ &\quad - \left( \phi_A + \frac{\gamma_A}{\varepsilon_A^q} \right) - \left( \phi_B + \frac{\gamma_B}{\varepsilon_A^q(1 + \chi)^q} \right). \end{aligned}$$

Then, the first order condition is

$$\mu_A\beta_A + \mu_B\beta_B(1 + \chi) = \frac{q(\gamma_A + \gamma_B/(1 + \chi)^q)}{\varepsilon_A^{q+1}},$$

and hence we must have that

$$\varepsilon_A^R = \left( \frac{q(\gamma_A + \gamma_B/(1 + \chi)^q)}{\mu_A\beta_A + \mu_B\beta_B(1 + \chi)} \right)^{\frac{1}{q+1}}.$$

Concavity guarantees that this is a global optimum.  $\square$

These together provide insight into to what the regulation is doing. The monopolist’s problem can be written as:

$$\begin{aligned} \max_{\varepsilon} \pi(\varepsilon) &= \max_{\varepsilon} \mu_A\alpha_A + \mu_B\alpha_B - (\mu_A\beta_A + \mu_B\beta_B(1 + \chi))\varepsilon \\ &\quad - (\phi_A + \phi_B) - \frac{1}{\varepsilon^q}(\gamma_A + \gamma_B/(1 + \chi)^q). \end{aligned}$$

This is equivalent to facing a *single* population of with demand function  $\mu_A\alpha_A + \mu_B\alpha_B - (\mu_A\beta_A + \mu_B\beta_B(1 + \chi))\varepsilon$ , fixed cost  $\phi_A + \phi_B$ , and marginal cost  $(\gamma_A + \gamma_B/(1 + \chi)^q)$ . We later use this interpretation to quickly calculate the constrained monopolist’s profits.

One might worry that imposing fairness requires making both groups worse off in an absolute sense. It turns out that this is not the case; if the regulation has bite, then it necessarily increases the error of the majority group, and necessarily decreases the error of the minority group. That is, equality comes at a price for the majority group, but *does not* require a Pareto deterioration.

Our first result is that the monopolist will respond to regulation by increasing majority error rates.

COROLLARY 6.5.

$$\varepsilon_A^R \geq \varepsilon_A^M \quad \text{and} \quad \varepsilon_B^R \leq \varepsilon_B^M.$$

At this point, members of the majority group may be concerned because their error rate increases. We refer to the gap between their error rates under the constrained and unconstrained monopolies as a *price of fairness* for this reason, even though imposing this constraint may be on the whole desirable from a societal perspective:

$$\text{PoF}_{1+\chi} = \frac{\varepsilon_A^R}{\varepsilon_A^M}.$$

Fortunately, we can show this price is relatively small:

COROLLARY 6.6 (PRICE OF FAIRNESS UPPER BOUND).

$$\text{PoF}_{1+\chi} \leq \left( 1 + \frac{\gamma_B}{\gamma_A} \cdot \frac{1}{(1 + \chi)^q} \right)^{\frac{1}{q+1}}.$$

Unsurprisingly, this bound is increasing in the ratio of minority cost to majority cost and decreasing in the leniency of the regulator. Also unsurprisingly, decreasing the ratio  $\frac{\mu_B}{\mu_A}$  or  $\frac{\beta_B}{\beta_A}$  and increasing the ratio  $\frac{\gamma_B}{\gamma_A}$  all increase the price of fairness for the majority.

If regulation changes the monopolist’s behavior, it must weakly decrease profits. This loss is quantifiable as another price of fairness:

DEFINITION 6.7 (MONOPOLIST PRICE OF FAIRNESS, RELATIVE ERROR). *We define the price of fairness as the ratio between the unconstrained monopoly profit and constrained monopoly profit under the relative error constraint, i.e.*

$$\text{MonPoF}_{1+\chi} = \frac{\pi(\varepsilon_A^M, \varepsilon_B^M)}{\pi(\varepsilon_A^R, \varepsilon_B^R)} = \frac{\pi(\varepsilon_A^M, \varepsilon_B^M)}{\max_{\varepsilon_A, \varepsilon_B: \frac{\varepsilon_A}{\varepsilon_B} \leq 1+\chi, \frac{\varepsilon_B}{\varepsilon_A} \leq 1+\chi} \pi(\varepsilon_A, \varepsilon_B)}.$$

We can write down this price of fairness as a function of the parameters of the model:

THEOREM 6.8. *The Monopolist’s price of fairness is given by*

$$\begin{aligned} \text{MonPoF}_{1+\chi} &= \frac{\mu_A\alpha_A + \mu_B\alpha_B - Q(\mu_A\beta_A)^{\frac{q}{q+1}}\gamma_A^{\frac{1}{q+1}} - Q(\mu_B\beta_B)^{\frac{q}{q+1}}\gamma_B^{\frac{1}{q+1}}}{\mu_A\alpha_A + \mu_B\alpha_B - Q(\mu_A\beta_A + \mu_B\beta_B(1 + \chi))^{\frac{q}{q+1}}(\gamma_A + \gamma_B\frac{1}{(1+\chi)^q})^{\frac{1}{q+1}}}, \\ \text{where } Q &= q^{\frac{1}{q+1}} + \frac{1}{q^{\frac{q}{q+1}}}. \end{aligned}$$

PROOF. The optimal solution to the monopolist’s problem with parameters  $\mu_g, \alpha_g, \beta_g, \gamma_g$  for  $g \in \mathcal{G} = \{A, B\}$  is the following:

$$\pi^*(\varepsilon^*) = \sum_{g \in \{A, B\}} \mu_g\alpha_g - (\mu_g\beta_g)^{\frac{q}{q+1}}\gamma_g^{\frac{1}{q+1}}Q.$$

(See Appendix D.) Using this form and plugging in the market parameters, we obtain the optimal profit of the unconstrained monopolist for the numerator. The denominator is derived using the interpretation of the constrained monopolist’s problem as optimizing its profits against a single market with parameters modified by regulation, and plugging these parameters into the same form.  $\square$

Theorem 6.8 provides a quantitative *price of fairness* in terms of monopoly profits. However, it is somewhat unwieldy; Proposition 6.9 provides some clarity on the limiting behavior of this price of fairness as a function of the minority group’s size in absolute terms.

PROPOSITION 6.9 (MONPOF LIMIT - RELATIVE ERROR INEQUALITY). *Let  $\mu_B/\mu_A = r$  for constant ratio  $r$ . Then*

$$\lim_{\mu_B \rightarrow \infty} \text{MonPoF}_{1+\chi} = 1.$$

*On the other hand, for constant  $\mu_A$ ,*

$$\lim_{\mu_B \rightarrow 0} \text{MonPoF}_{1+\chi} = \frac{1 - (Q/q)\varepsilon_A^M}{1 - (Q/q)\varepsilon_A^M \left[ 1 + \frac{\gamma_B}{\gamma_A} \frac{1}{(1+\chi)^{\frac{1}{q+1}}} \right]^{\frac{1}{q+1}}}$$

where  $Q$  is as above.

## 6.2 Absolute Error Equality

In this section, we suppose instead that the regulator imposes an absolute upper bound on error rates for each group. We show that the monopolist responds by purchasing just enough data to meet the constraint using the profits from the majority to subsidize the minority. In this case, minority error rates can be improved without increasing error for the majority; the regulator can even improve error rates for the majority as well, up to a point. We characterize the price of fairness for the monopolist and the minimum error the regulator can guarantee. We formalize this constraint as follows:

DEFINITION 6.10 (ABSOLUTE ERROR EQUALITY). *For  $\chi < 1$ , the regulator forces the firm to achieve error of:*

$$\varepsilon_A \leq \chi \quad \text{and} \quad \varepsilon_B \leq \chi.$$

We have another saturation lemma for this kind of constraint too: either the unconstrained error was already less than  $\chi$ , or the profit maximizing error subject to regulation is exactly  $\chi$ . Formally:

LEMMA 6.11 (SATURATION).  $\forall g \in \{A, B\}$ , if  $\varepsilon_g^R \neq \varepsilon_g^M$  then  $\varepsilon_g^R = \chi$ .

Lemma 6.11 lets us reason very simply about the behavior of the regulated monopolist: for any group in which imposing regulation requires the firm to improve error rates, the firm will use up the entirety of this ‘error budget.’ Profit will decrease, of course, because imposing constraints can only decrease its objective. In this scenario, if the firm enters the market at all, it must enter the market for both groups so as to achieve the constrained error rates. A regulator then has to choose  $\chi$  so as to still induce the firm to enter the market at all if they want to ensure the constrained error rate for the minority group. Of course, a regulator may also wish to choose the smallest such error rate, which we refer to as the *minimum achievable error*. Lemma 6.11 let us characterize the minimum achievable error:

PROPOSITION 6.12. *Let  $\chi_0$  be the smallest  $\chi \in [0, 1]$  which solves*

$$K_1\chi^{q+1} + K_2\chi^q - K_3 = 0, \quad (4)$$

where  $K_1 = -(\mu_A\beta_A + \mu_B\beta_B)$ ,  $K_2 = \mu_A\alpha_A + \mu_B\alpha_B - \phi_A - \phi_B$ , and  $K_3 = \gamma_A + \gamma_B$ .  $\chi_0$  exists and is the minimum achievable error, i.e. the minimum  $\chi \in [0, 1]$  for which the monopolist still enters the market.

Equation 4 can be solved via the quadratic or cubic formulae in the realizable and agnostic cases, respectively, and learning rates in between can be accommodated numerically. This leads us to the monopoly’s optimal error rates as a function of  $\chi$ :

THEOREM 6.13 (ABSOLUTE OUTCOMES). *Outcomes fall into one of the following possibilities:*

- (1) If  $\chi \geq \varepsilon_B^R$  then  $(\varepsilon_B^R, \varepsilon_B^R) = (\varepsilon_A^M, \varepsilon_B^M)$ .

- (2) If  $\varepsilon_A^M \leq \chi < \varepsilon_B^M$  then  $(\varepsilon_B^R, \varepsilon_B^R) = (\varepsilon_A^M, \chi)$ .
- (3) If  $\chi_0 < \chi < \varepsilon_A^M$  then  $(\varepsilon_B^R, \varepsilon_B^R) = (\chi, \chi)$ .
- (4) If  $\chi < \chi_0$  then the firm exits the market.

PROOF. Case 1 is trivial. Case 2 and 3 follow from Lemma 6.11. Case 4 follows by the definition of  $\chi_0$ .  $\square$

Theorem 6.13 contrasts starkly with Theorem 6.5: as long as the constraint is not so strict the monopolist exits the market, outcomes either improve for the minority and remain just as good for the majority, or improve for *both* groups. In other words, this style of regulation *does not impose a price of fairness on the majority*. Note that unless  $\varepsilon_0 < \chi < \varepsilon_A^M$ , the regulator is *not* guaranteeing relative equality. Which type of equality is preferable will depend on the context. Of course, this regulation *does* still impact profit:

DEFINITION 6.14 (MONOPOLIST PRICE OF FAIRNESS). *We define the monopolist’s price of fairness under absolute error constraints as:*

$$\text{MonPoF}_\chi = \frac{\pi(\varepsilon_A^M, \varepsilon_B^M)}{\pi(\varepsilon_A^R, \varepsilon_B^R)} = \frac{\pi(\varepsilon_A^M, \varepsilon_B^M)}{\max_{\varepsilon_A, \varepsilon_B: \varepsilon_A \leq \chi, \varepsilon_B \leq \chi} \pi(\varepsilon_A, \varepsilon_B)}.$$

Notice that given the market parameters, Theorem 6.13 allows the regulator to evaluate the monopolist’s price of fairness for each potential choice of error threshold via straightforward calculation. Proposition 6.15 characterizes the limiting behavior of the monopolist’s price of fairness as a function of absolute size of the minority group under absolute error guarantees, and these are qualitatively similar to limiting behavior under relative error guarantees.

PROPOSITION 6.15 (MONPOF LIMIT - ABSOLUTE ERROR GUARANTEES). *For fixed  $\chi$ , and for  $\mu_B \rightarrow \infty$  at a constant ratio  $\mu_A/\mu_B = r$ :*

$$\lim_{\mu_B \rightarrow \infty} \text{MonPoF}_\chi = 1.$$

*On the other hand, let  $\chi_0$  be the minimal achievable error when  $\mu_B = 0$  (i.e. when the firm faces group A alone). Then if  $\chi > \chi_0$ , then  $\text{MonPoF}_\chi$  converges to a parameter-specific constant as  $\mu_B \rightarrow 0$ .*

## 7 DISCUSSION

In this work, we identify economic incentives leading to unfairness in data-driven markets. At a high level, we show that monopolists are incentivized to invest less in minority groups (as measured by market size, elasticity, and data costs) because they are less profitable; that competition does not mitigate this incentive towards inequality, under reasonable models; and that judicious regulation *can* improve outcomes, potentially at a cost in terms of profits or, depending on the regulation, error rates for the majority group.

We view this paper as highlighting an important and understudied point of view, but certainly not as the last word. We made many choices that situate our models in particular contexts; for example, the assumption that firms and users benefit from improved accuracy does not capture many settings that currently are or will soon be urgent domains of adjudicating fairness concerns - machine learning in loans, insurance, and facial recognition systems are obvious cases, but the potential, and consequent scope for unfairness, is vast. We hope that future work will further clarify the possibility - and perhaps necessity - of leveraging policy tools in addition to algorithmic solutions to combat unfairness in machine learning.

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