Multi-Category Fairness in Sponsored Search Auctions

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ABSTRACT

Fairness in advertising is a topic of particular concern motivated by theoretical and empirical observations in both the computer science and economics literature. We examine the problem of fairness in advertising for general purpose platforms that service advertisers from many different categories. First, we propose inter-category and intra-category fairness desiderata that take inspiration from individual fairness and envy-freeness. Second, we investigate the "platform utility" (a proxy for the quality of allocation) achievable by mechanisms satisfying these desiderata. More specifically, we compare the utility of fair mechanisms against the unfair optimum, and show by construction that our fairness desiderata are compatible with utility. Our mechanisms also enjoy nice implementation properties including metric-obliviousness, which allows the platform to produce fair allocations without needing to know the specifics of the fairness requirements.

CCS CONCEPTS

• **Theory of Computation** → Algorithmic Mechanism Design.

KEYWORDS

algorithmic fairness, advertisement auctions, utility, individual fairness, envy-freeness

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1 INTRODUCTION

In the ongoing discussion of what it means for automated decision-making systems to be fair, the topic of online advertising has merited particular interest. In the United States, segregated employment ads for men and women proved to be a flashpoint in the 1960s, and the introduction of ever-more finely-tuned advertising online has renewed concerns about discrimination in ads for critical categories such as employment, housing and credit. Although individual advertisers certainly have opportunities to use fine-grained targeting (in some cases targeting individual users) to implement biased advertising strategies, there is both empirical evidence and theoretical support for the idea that skewed advertisement between different demographic groups is not solely due to biased advertiser behavior.

Indeed, recent work has shown that **even when the advertisers all act fairly in isolation, revenue-optimized platform mechanisms can result in unfairness.** In particular, even when each advertiser intends to be fair in its delivery, competition between advertisers in first-price auctions, particularly across categories, can introduce a significant skew in the types of ads people see (e.g., [18]). For example, two software engineers, who are targeted equally by software engineering companies, may see wildly different numbers of employment ads depending on *competition* for their attention from other categories. Competition from lucrative categories like children's products can be difficult for an individual advertiser to correct for, particularly in categories like employment where parental status is considered very sensitive.

While the idea of parents being excluded from employment ads due to competition is troubling, blunt policies like equalizing the number of ads across users within *each* category can also result in poor outcomes for individuals. For example, suppose Bob is highly qualified for a new credit card, and he is also searching for daycare options in his area. Alice is equally qualified for the credit card, but she is not interested in daycare. Bob would not want to see fewer ads for daycare services to ensure that he sees as many ads for credit as Alice, and likewise Alice would not want good credit offers suppressed in order to ensure that she doesn't see more credit ads than Bob who doesn't care as much about credit cards anyway!

Competition between ads *within* a single category can also result in undesirable skew. Even if two equally qualified individuals see the same number of ads in a category, these ads may not be equally relevant or valuable to them. For example, these individuals could see the same total number of job ads, but with one receiving ads for jobs with significantly higher salaries than the other.[1]

How can we formalize these intuitive fairness desiderata? Are they compatible with consumer, platform, and advertiser utility?

Our Contributions

Model and Definitions. In this work, we examine the problem of fairness in advertising across multiple categories from a theoretical perspective. We propose a stylized model and fairness requirements that match the intuitive fairness desiderata of inter- and intra-category guarantees.[2] Moreover, our fairness requirements incorporate the qualifications of users as well as their preferences across different ad categories. Our definitions are inspired by and combine the complementary notions of **envy-freeness** and **individual fairness**. Envy-freeness [8] requires that every user should prefer their own allocation to that of everyone else; it ignores users'

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[1]Subtle issues, like willingness to relocate, company size and type, and other policies may also significantly influence the usefulness of the ads to each user.

[2]Our aim is to propose a model and examine its properties to gain insight into how certain notions of fairness interact with platform utility as well as our intuition about what is fair rather than to propose a specific notion of fairness for practical implementation.

qualifications and considers preferences as paramount. Individual or metric fairness [12], on the other hand, ignores preferences and essentially requires that similar users should be treated similarly. Its extension to multi-dimensional allocations as introduced in [13], **multiple task fairness**, requires that individual fairness is satisfied separately and simultaneously for all categories.

For inter-category competition, we note that multiple task fairness is at odds with consumer and platform utility. Consider the example of Alice and Bob discussed above. Taking this example to the extreme suppose that a user, whom we will call Jack, is a "jack of all trades," i.e., qualified or interested in many categories. Per multiple task fairness, another user who is as qualified as Jack in a single category but unqualified in all other categories, would have their allocation within the single category of interest limited to match Jack's allocation within that category. Multiple task fairness would then either (1) enforce a minimum amount of exposure for Jack to ads they may not care about or (2) dictate a maximum amount of exposure to relevant ads for other users who are qualified for a smaller number of categories. In effect, by ignoring users' own preferences, this stringent notion of fairness helps no one. We thus introduce **inter-category envy-freeness**, which allows users to specify a set of categories that they "care" about, and guarantees that they see at least as many ads that they care about as any other individual.[3]

For intra-category competition, we show that multiple-task fairness can be too weak to avoid discriminatory allocations and is therefore vulnerable to certain subversion attacks. For example, if Alice sees *every* high-paying job ad slightly less often than Bob and *every* low-paying job ad slightly more often than Bob, then for any single ad the two users obtain similar allocations, but across the set of all high-paying jobs, Alice may receive a far smaller share than Bob. We thus introduce **total variation fairness**, which offers protection against these subversion attacks and captures fairness over all possible sets of substitutes.[4]

Combining these fairness aims yields a definition giving fairness both across categories and within categories. More specifically, we demonstrate how to combine inter-category envy-freeness and total-variation fairness into a hybrid fairness notion that we call **compositional fairness**. For example, suppose two individuals, Alice and Bob, are equally qualified and interested in a particular type of job. Alice is also interested in the latest movie releases, while Bob is interested in buying a new car. We provide the guarantee that Alice and Bob see the same *mix* of ads for jobs, but are permitted to see job ads overall with different probabilities, so long as each sees their preferred categories in total at least as often as the other.

Mechanism design with fairness constraints. In practice, fairness is obviously not the only consideration in mechanism design. Platforms typically aim to optimize for (1) short-term revenue, i.e. the sum of payments made by advertisers for displaying their ads, or (2) allocative efficiency a.k.a. social welfare, i.e. the sum of the perceived values of the user-ad matches, or a combination of these objectives. As a proxy for these complex objectives, we use the sum of the bids of the ads displayed as a measure of the quality of an allocation. We call this objective **platform utility**. High platform utility correlates both with advertiser happiness (ads get matched with targeted "high bid" users) as well as with platform revenue (advertisers often pay their bid or an increasing function of their bid). While our goal is to design *fair* mechanisms that achieve high platform utility, we compare the performance of our mechanisms to the *unfair* optimum, following the intuition that platforms and advertisers are unlikely to adopt mechanisms which deviate wildly in utility from the status quo. In this respect, we show that our fairness desiderata are compatible with high platform utility.

Apart from these fairness and utility objectives, from a mechanism design perspective, it is desirable for our mechanisms to satisfy certain implementation properties. The first question that needs to be addressed is how to distribute the responsibility for ensuring fairness among the advertisers and the platform. In this work, we give a clear separation of responsibilities: the advertisers are required to bid "fairly", and the platform is required to fairly allocate advertisements assuming the bids are fair.[5] As a result, any perceived unfairness can be clearly tracked to the responsible party. Moreover, we propose that the determination of what constitutes fairness in each category and auditing of advertiser bids be outsourced to a neutral third-party or governing body, so that the platform can be oblivious to the particulars of fairness requirements.[6] Thus, the underlying fairness conditions are, in effect, encoded into the bids when the advertisers are required to satisfy fair bidding constraints.

Allocation algorithms achieving high utility. In this work, we take a first step towards designing fair auction mechanisms for ad platforms. We focus on constructing fair allocation algorithms, where the allocation is fair so long as the bids are fair and that achieve a high platform utility. In particular, we do not consider incentives. Nonetheless, our allocation algorithms already achieve some of the nice properties that are needed for incentive compatibility, like monotonicity and individual rationality, though a full consideration of incentives would require considering how advertisers behave when their true values are not fair.

Our main result is a family of allocation algorithms that achieve compositional fairness and utility close to optimal within the class of fair algorithms, while also being oblivious to fairness requirements. The nature of the fairness definitions enables us to compose an algorithm satisfying total-variation fairness within each category with an algorithm satisfying inter-category envy-freeness across categories to obtain a algorithm satisfying compositional fairness. Thus, we can separately design inter- and intra-category algorithms.

[3]Kim et al. [17] has similar motivation, but takes a different approach to blending user-preferences and individual fairness.

[4]Although "high-paying" or "low-paying" are well-defined attributes in the employment setting, there are many subtle properties which may be harder to articulate, e.g., "commute time." By protecting all sets of substitutes, we obviate the need to enumerate all possible attributes.

[5]An alternate option would be to allow advertisers to bid arbitrarily and force the platform to achieve fairness, potentially by changing advertiser bids. However, this would make it impossible for the platform to be oblivious to fairness requirements, and could also cause transparency issues in the bidding process. Moreover, the advertiser is generally in a better position than the platform to understand the nuances of fairness in their particular category. For example, a car insurance advertiser is more likely to be aware of the problems associated with using zip code to determine insurance rates than the advertising platform, which may have advertisers from thousands of different categories. For this reason, we argue for a separation where the advertisers are required to bid "fairly" and the platform is allowed to assume bids are fair.

[6]Particularly in cases in which determining fairness requires sensitive information (or sensitive information may be leaked by the determination procedure), it may be desirable to avoid sharing this information with the platform.

The inter-category algorithms that we propose are always envy-free regardless of the bids, and the intra-category algorithms that we propose are oblivious to fairness requirements: our intra-category algorithm implicitly uses fairness of advertiser bids to infer and provide the necessary fairness requirements. Unsurprisingly, if advertisers are allowed to bid very differently on similar individuals, it is impossible to simultaneously achieve fairness and high platform utility. As part of our study, we also explore upper and lower bounds on the strength of a bid fairness condition needed to achieve a high platform utility.

Outline for the rest of the paper

The remainder of this work is organized as follows: related work is discussed in Section 2; definitions and our formal model are introduced in Section 3; algorithms achieving intra-category fairness are discussed in Section 4, and algorithms achieving inter-category envy-freeness are discussed in Section 5; composing these algorithms is addressed in Section 6; finally future work is discussed in Section 7. We defer proofs and some discussion to the full version of the paper [11].

2 RELATED WORK

Fairness in advertising is a topic of particular interest in the popular press, as well as the empirical and theoretical computer science and economics literature. There are number of compelling empirical studies and popular press articles demonstrating "skew" in advertisement delivery between groups, either due to advertiser targeting choices, platform delivery issues, or competition between advertisers [2–4, 7, 18].

In the theoretical computer science literature, Dwork et al. [12] proposed the notion of individual fairness as a fairness concept for settings including advertising. Dwork and Ilvento [13] consider individual fairness (and group fairness) in advertising as a composition problem, and pose a limited set of fair composition mechanisms. Similar to this work, Kim et al. [17] propose "Preference Informed Individual Fairness" (PIIF) to capture the idea that deviation from individual fairness is acceptable as long as it is based on individuals' preferences. However, while PIIF is intended to expand the definition of fairness within the context of task-specific metrics, our preferences-based envy-freeness concept applies across multiple categories independent of the metric for each category. Furthermore, whereas Kim et al. focus primarily on optimizing user utility via offline optimization across all fair outcomes, we study online allocation algorithms and measure utility relative to the unfair optimum. We view our work as complementary to [17], and we anticipate that combining the insights from their perspective and ours will be useful for proposing alternative mechanisms and fairness relaxations.

Other works on fair advertising and related problems employ different notions of fairness. With respect to group or statistical notions of fairness, Celis et al. [9] propose a mechanism for ad delivery while maintaining certain group level statistics. A variety of fairness notions have also been considered in related problems such as ranking [10, 14], recommendation systems [5], and news search engines [16].

Finally, mostly disjoint from the work referenced above, there is an extensive literature in fair division concerning other notions of fairness [8]. The goal of one such notion, called "envy-freeness", is to partition a limited shared resource among multiple agents with heterogenous preferences. A major difference between this literature and the fairness literature referenced above is that envy-freeness is defined exclusively based on agents' preferences and not on their traits or qualifications. Two recent works, Balcan et al. [6] and Zafar et al. [20] seek to combine notions of envy-freeness with parity-based fairness in machine learning in the context of classification.

3 ADVERTISING AND FAIRNESS MODELS

We model the online advertising problem as follows. A universe U of users arrive in an online fashion. There are k advertisers indexed by $i \in [k]$. When a user u arrives, each advertiser i places a bid $b_u^i \geq 0$ on the user. The allocation algorithm then assigns allocation probabilities $p_u^i \in [0, 1]$ to the advertisers with $\sum_{i=1}^k p_u^i \leq 1$. The allocation algorithm is an online algorithm, i.e. it assigns allocation probabilities without observing bids on users that arrive in the future or the ordering of future user arrivals. Moreover, we assume for simplicity that each user arrives at most once. We use $\mathbf{p}$ to denote the allocation rule output by the allocation algorithm.

The goal of the allocation algorithm is to maximize the sum of the bids of the ads displayed. Formally, this is given by:

$$\text{Utility}(\mathbf{p}) = \sum_{u \in U} \sum_{i \in [k]} p_u^i b_u^i.$$

We measure the utility of our algorithms against the best achievable in the absence of any fairness constraints. The utility is easy to maximize in the absence of any constraints on how allocations vary across users: the algorithm can simply assign a probability mass of 1 to the highest bidder for every user.[7] We call the corresponding utility the unfair optimum:

$$\text{Unfair-OPT} = \sum_{u \in U} \max_{i \in [k]} b_u^i.$$

The **Fair Value** of an allocation algorithm is the ratio of its utility to the Unfair-OPT. Note that this ratio is always less than 1; the larger the fair value the better the utility of the allocation is.

Why compare with the *unfair* optimal utility instead of the fair optimal (as do other related works [9, 17])? First, the utility of the fair optimal algorithm can be difficult to analyze due to the fairness requirements being revealed in an online fashion. Second, if there is a large gap between the utility of the algorithms we propose and the utility of the unfair-optimal algorithm, the platform and advertisers may be unwilling to adopt the algorithm, so it is critical that we compare with the platform's status quo.

3.1 Fairness of allocation

Our fairness guarantees are based on the concept of individual fairness defined by [12] and the well-studied concept of envy-freeness [8]. At a high level, individual fairness guarantees that similar individuals are treated similarly. Similarity between individuals is captured through a fairness metric $\mathbf{d}$ over U and similarity

[7] This is equivalent to running a first-price auction.

between outcomes is captured by defining a metric D over distributions over outcomes.

Definition 3.1 (Individual Fairness [12]). A function $f : U \to \Delta(O)$ assigning users to distributions over outcomes is said to be **individually fair** with respect to distance metrics $\mathbf{d}$ over U and D over $\Delta(O)$, if for all $u, v \in U$ we have $D(f(u), f(v)) \leq \mathbf{d}(u, v)$.

Dwork and Ilvento [13] proposed extending the notion of individual fairness to settings involving multi-dimensional allocations by ensuring fairness separately within each dimension or "task". This gives rise to the notion of multiple-task fairness, which we now define in the context of online advertising. Let $\{C_1, \ldots, C_c\}$ denote a partition of the set $[k]$ of advertisers into c categories. For $1 \leq j \leq c$, let $\mathbf{d}^j$ denote a pseudometric over the users relevant to all advertisers in category j; $\mathbf{d}^j : U \times U \to [0, 1]$. In our setting, the outcome assigned to each user u corresponds to the advertiser who is assigned to the slot for user u. Our algorithm maps users to distributions over outcomes, i.e. to the allocation probabilities $\{p_u^i\}_{1 \leq i \leq k}$. We use the absolute difference between these probabilities to capture the similarity of allocations, and multiple-task fairness becomes the following condition:[8]

Definition 3.2 (Multiple-Task Fairness [13]). An allocation function $\mathbf{p}$ satisfies **multiple-task fairness** with respect to distance metrics $\{\mathbf{d}^j\}_{j \in [c]}$ if for all $u, v \in U$, $j \in [c]$, and $i \in C_j$, we have $|p_u^i - p_v^i| \leq \mathbf{d}^j(u, v)$.

We will demonstrate that multiple-task fairness is too weak for fairness within a single category, and it results in suboptimal allocations across categories from the perspective of envy-freeness. We propose two new multi-dimensional fairness notions, one applying across different categories and the other to multiple advertisers within the same category, and combine these into the notion of **compositional fairness** that overcomes the shortcomings of multiple-task fairness.

Intra-category fairness. First, we consider a setting in which all of the advertisers belong to a single category (i.e. $c = 1$) with a single metric that we denote by $\mathbf{d}$ (e.g., a tech job-search website containing advertisements only from tech employers). We observe that multiple-task fairness is insufficient to protect against two broad classes of problems.

(1) **Intentional unfairness:** It is vulnerable to subversion by malicious advertisers. In particular, consider an advertiser that submits multiple different ads (pretending to be distinct advertisers) for the same job and bids separately on each user for each of those ads. The advertiser effectively poses as multiple sub-advertisers; let S denote the set of these sub-advertisers. In this case, the multiple-task fairness constraint only ensures $|p_u^i - p_v^i| \leq \mathbf{d}(u, v)$ for each i, and so it is possible that $|\sum_{i \in S} p_u^i - \sum_{i \in S} p_v^i| = |S|\mathbf{d}(u, v)$. As a result, the advertisers may be able to amplify the difference in probabilities of allocation between two users to an arbitrarily large extent.
(2) **Unintentional unfairness:** Multiple-task fairness can also interact in undesirable ways with well-intentioned advertisers. Suppose that the set S consists of all high-paying job ads.[9] Suppose high-paying advertisers all bid higher on user u than on user v, and the algorithm sets $|p_u^i - p_v^i| = \mathbf{d}(u, v)$ for all $i \in S$ (to maximize utility). Then, it would again be the case that $|\sum_{i \in S} p_u^i - \sum_{i \in S} p_v^i| = |S|\mathbf{d}(u, v)$, so the total allocation on high-paying job ads (i.e. advertisers in S) can be vastly different for u and v.

To rectify these issues, we propose **total variation fairness** which requires that the allocation vectors p_u and p_v are not only close component-wise, but are also close in terms of ℓ_1 distance or total variation distance.

Definition 3.3 (Total Variation Fairness). An allocation function $\mathbf{p}$ satisfies **total variation fairness** with respect to a metric $\mathbf{d}$ if for all $u, v \in U$ and all $S \subseteq [k]$, we have $|\sum_{i \in S} p_u^i - \sum_{i \in S} p_v^i| \leq \mathbf{d}(u, v)$. Equivalently, for all $u, v \in U$, $\|p_u - p_v\|_1 \leq \mathbf{d}(u, v)$.

This stronger definition, which provides guarantees on all subsets of advertisers S, effectively mitigates the issues outlined above. First, it prevents the multiple bid attack by ensuring that $|\sum_{i \in S} p_u^i - \sum_{i \in S} p_v^i| \leq \mathbf{d}(u, v)$. Second, it provides nice guarantees over substitutes in the following sense. Consider a user u who regards some arbitrary subset S of advertisers to be substitutes. In that case, the probability that the user observes an ad from this subset is $\sum_{i \in S} p_u^i$ and total variation fairness ensures that this sum is close to the corresponding sum for similar users.[10]

Inter-category fairness. Next consider a setting where every advertiser belongs to a different category, i.e. where $c = k$. As the "jack of all trades" example in the introduction (formalized in Section 5) shows, multiple-task fairness can lead to outcomes that, although technically fair, are undesirable from every stakeholder's perspective – (1) users get low allocations in their desired categories, (2) advertisers reach far fewer qualified users and (3) the platform gets low utility owing to the poor quality of the matching produced. In this example, allocations available to the jack of all trades are constrained by the limited attention of this single user (a single ad slot in our model). Multiple-task fairness combined with this constraint limits the allocations of all of the other users, thereby hurting their utility, without in turn providing any benefit to the jack of all trades. Within this context, we view the multiple task fairness objective to be unduly skewed in favor of a single individual over the collective good.

Is it possible to achieve a better balance? It is if we slightly shift our viewpoint. Consider a platform where each user is allowed to select the category they are most interested in with the guarantee that they see at least as many ads in this category as *any other* user. In effect, no user is envious of other users given their own choice of preferred category.

From a practical perspective, the platform generally has a good deal of information about their users based on their previous web browsing behavior and may also have existing interfaces to allow users to view and modify these imputed preferences.[11] In practice, we anticipate platforms would include a fixed set of sensitive

[8]We can also view the assignment $\{p_u^i\}_{1 \leq i \leq k}$ as a fractional allocation. In this case, the distance corresponds to the difference in the portion of allocation for each advertiser.

[9]The set S can also be job ads from a certain geographical area.

[10]This definition can be viewed as combination of the multiple-task fairness and OR-fairness definitions put forth in [13]. The definition essentially provides OR-fairness over all possible subsets of advertisers.

[11]See Google's Ad options https://support.google.com/accounts/answer/2662856.

categories (e.g., housing, credit, employment) in every user's preferences by default and augment this list with additional interests or preferences learned implicitly (or explicitly specified by the user) over time. This approach accounts for the importance of ensuring inter-category fairness for certain sensitive categories, either due to legal constraints or other ethical concerns, and allowing a more flexible approach for less critical categories.

Now, we discuss the fairness implications of this definition. First, in the special case where every category has a single advertiser,[12] within the chosen category, the fairness guarantee provided to the user is much stronger than that guaranteed by individual fairness – the user does not just obtain an allocation close to that of other similar users, but rather obtains one that is as good or better than that of everyone else.[13] Second, this is balanced by the lack of any fairness guarantee on non-chosen categories. However, note that from the user's perspective those other categories are anyway not important. To take an example, suppose that Alice and Bob are identical in terms of their credit-worthiness as well as job qualifications. Suppose Alice is looking for a job and Bob for a credit card. Then a platform that shows a job ad to Alice and a credit card ad to Bob makes both users happy and envy-free. In contrast, a platform that shows each user one of the two ads uniformly at random is multiple-task fair but makes both users worse off. Third, while our definitions focus on a single arrival of each user, one may envision a system where a user interacting with the platform multiple times can change their preferred category at each interaction and thereby obtain a fair allocation within the chosen category at each individual interaction. Finally, observe that the envy-freeness guarantee is directional and entirely independent of distance metrics.

More generally, we allow users to select multiple preferred categories and provide an envy-freeness guarantee with respect to the total probability of seeing an ad within their set of preferred categories. Formally, each user u picks a preferred set $S_u \subseteq [c]$ of categories.[14] We then ensure that $\sum_{i \in S_u} p_u^i$ is at least as large as the corresponding sum for any other user v.

Definition 3.4 (Inter-Category Envy-Freeness). An allocation function $\mathbf{p}$ satisfies **inter-category envy-freeness** with respect to preferred sets $\{S_u\}_{u \in U}$ if for all $u, v \in U$, we have $\sum_{i \in S_u} p_v^i \leq \sum_{i \in S_u} p_u^i$.

Compositional fairness. Now we consider the general setting where there can be multiple categories and multiple advertisers in each category. We discuss how to combine the two definitions above to provide hybrid fairness guarantees. We have two goals: (1) each user should be envy-free with respect to the categories of ads they see and (2) within each category, the mix of ads presented to each user should satisfy our strengthened notion of individual fairness. For example, suppose that two similarly qualified users Alice and Bob both select jobs as their preferred ad category. Not only should the two users then see job ads with the same total likelihood, but they should also see a similar mix of high-paying and low-paying job ads.

This composition of definitions becomes subtle when users select *multiple* preferred categories. Suppose that Alice continues to choose jobs as her preferred category, but Bob chooses both jobs and household product ads. Suppose, further, that Alice is allocated a job ad with probability 1 and Bob sees an ad in each of the two categories with probability $1/2$ each. This allocation satisfies inter-category envy-freeness. However, within the job ads category there is no way to assign probabilities that satisfy unconditional total variation fairness simply because of the fact that we have different total probabilities to distribute. Intuitively, we want Bob to be able to see the same *mix* of ads as Alice even though Alice may see job ads more frequently overall. Accordingly, we enforce total variation fairness on the *conditional distribution* of allocation within each category.

Formally, we define compositional fairness as follows. We use $\mathbf{d}^j$ to denote the metric specific to category C_j and $q_u^j = \sum_{i \in C_j} p_u^i$ to denote the total allocation within category C_j for user u.

Definition 3.5 (Compositional Fairness). An allocation function $\mathbf{p}$ satisfies **compositional fairness** with respect to distance metrics $\{\mathbf{d}^j\}_{j \in [c]}$ if the assignments $\left\{q_u^j\right\}_{u \in U, j \in [c]}$ satisfy inter-category envy-freeness, and for each $j \in [c]$ such that $q_u^j > 0$, the conditional probabilities $\left\{\frac{p_u^i}{q_u^j}\right\}_{i \in C_j}$ satisfy total variation fairness with respect to $\mathbf{d}^j$.

Multiplicative Relaxations. We can further refine each of the above notions by defining multiplicative relaxations parameterized by $\beta \in [1, \infty)$:

- β **total variation fairness:** for all $u, v \in U$ we have $\|p_u - p_v\|_1 \leq \beta \mathbf{d}(u, v)$.
- β **inter-category envy-freeness:** for all $u, v \in U$ we have $\sum_{i \in S_u} p_v^i \leq \beta \sum_{i \in S_u} p_u^i$.
- β **compositional fairness:** $\left\{q_u^j\right\}_{u \in U, j \in [c]}$ satisfies β inter-category envy-freeness, and for each $j \in [c]$ such that $q_u^j > 0$, $\left\{\frac{p_u^i}{q_u^j}\right\}_{i \in C_j}$ satisfies β total variation fairness.

4 INTRA-CATEGORY FAIRNESS

In this section, we focus on the case where advertisers are in a single category ($c = 1$), i.e. where advertisers face the same metric over users, in particular, $\mathbf{d} = \mathbf{d}^1 = \mathbf{d}^2 = \ldots = \mathbf{d}^k$. Our main result in this section is an allocation algorithm that is fair when advertiser bids are "fair" and achieves a high fair value. Our allocation algorithm also enjoys the property of **metric-obliviousness**. That is, the platform does not require knowledge of the fairness metric $\mathbf{d}$.

In Section 4.1, we describe the need for a fairness condition on advertiser bids, that we call a bid ratio condition. In Section 4.2, we investigate the special case of uniform metrics and establish impossibility results, i.e. upper bounds on the fair value of any allocation algorithm that satisfies multiple-task fairness as a function of the bid ratio condition. In Section 4.3, we consider settings with

[12]We discuss the case of multiple advertisers per category and allocation between those advertisers in the subsection on compositional fairness.

[13]The reader may worry that a user who is "unqualified" in their chosen category could obtain a large allocation at the expense of advertisers that don't want to target such a user. Note that advertisers can bid 0 on users that are not targeted and thereby pay nothing for those users. Furthermore, it is rare for a user to be wholly unqualified for a category from the perspective of *advertising*. For example, an individual who is unqualified for a job is likely a good candidate for a job training ad.

[14]From an implementation perspective, we might imagine that users specify these categories on a user profile.

arbitrary distance metrics and exhibit an allocation algorithm that is metric-oblivious, history-oblivious, and achieves total variation fairness with respect to the given metric with an appropriate bid ratio condition. We then bound the fair value achieved by this algorithm as a function of k (the number of advertisers) and a parameter defining the bid ratio constraint. Moreover, we show that this algorithm achieves a near-optimal tradeoff between bid ratio condition and fair value within a restricted class of algorithms. In Section 4.4, we show that the fair value achieved by this algorithm is close to the upper bound established in Section 4.2 for allocation algorithms with a uniform metric. We emphasize that while our negative result in Section 4.2 applies to general online algorithms which satisfy multiple-task fairness with access to the underlying metric, our positive result applies to a algorithm that is metric-oblivious and satisfies the stronger notion of total variation fairness. All of the proofs can be found in the full version of the paper [11].

4.1 Fairness in bids

Our goal is to develop an allocation algorithm that simultaneously satisfies total variation fairness and achieves large fair value with respect to the Unfair-OPT. As one might expect, it is impossible to achieve this if advertisers are allowed to place arbitrary bids on users without regard to the relevant similarity metric over users.The question thus becomes: what kind of fairness constraint on bids enables a reasonable fair value? The following example illustrates the need for a fairness constraint that requires an advertiser's bids on pairs of similar users to be close in ratio, even when allocations are merely required to satisfy the weaker condition of multiple-task fairness. In particular, being close in terms of their absolute difference is not sufficient to achieve a good fair value.

Example 4.1. Suppose that there are k advertisers and exactly k users. Suppose that the metric is uniform: for some parameter $d \in [0, 1]$, every pair of users is a distance of d apart. For each $i \in [k]$, advertiser i bids b^{high} on user i and $b^{\text{low}} (< b^{\text{high}})$ on all other users $j \neq i$. Observe that Unfair-OPT = kb^{high}. On the other hand, due the symmetry of this instance and the fact that a fair allocation requires that $|p_i^i - p_i^j| \leq d$, for each user, it turns out the optimal fair allocation assigns an allocation probability of $(1-d)/k$ to all advertisers with the low bid and a probability of $(1-d)/k + d$ to the advertiser with the high bid. The fair value of this allocation turns out to be $d + \frac{1-d}{k} + (1-d)\frac{(k-1)}{k}\frac{b^{\text{low}}}{b^{\text{high}}}$. Observe that a fair value of $d + (1-d)/k$ is trivial to achieve via a fair allocation.[15] When d is very small, this trivial bound is tiny. If $b^{\text{high}} \gg b^{\text{low}}$, then, in this example, no fair allocation can perform much better than the trivial algorithm. In order to be able to do better, $b^{\text{high}}/b^{\text{low}}$ must be bounded.

Motivated by the example above, we require that for every advertiser and every pair of users, the ratio of the bids the advertiser places on the users is bounded by a function of the distance between the users according to $\mathbf{d}$. The closer the two users, the closer this ratio bound should be to 1; on the other hand, the ratio bound should be large between far apart users. We formally define this constraint as follows.

[15] In particular, for each user, assigning an allocation probability of $(1-d)/k + d$ to the highest bidder and $(1-d)/k$ to all other advertisers achieves this bound.

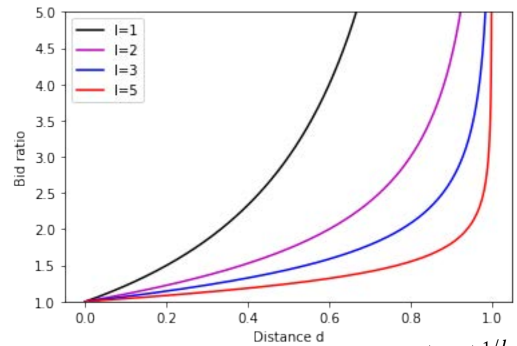


Figure 1: Bid ratio condition $f_l(d) = \left(\frac{1+d}{1-d}\right)^{1/l}$

Definition 4.2. A **bid ratio constraint** is a function $f : [0, 1] \to [1, \infty]$. We say that the bid function b^i of advertiser i satisfies the bid ratio constraint f with respect to metric $\mathbf{d}$ if we have for all $u, v \in U$: $\frac{1}{f(\mathbf{d}(u,v))} \leq \frac{b_u^i}{b_v^i} \leq f(\mathbf{d}(u, v))$.

How should we choose f? On the one hand, the bid ratio constraint needs to be sufficiently strict to provide meaningful fairness and utility guarantees and on the other hand it cannot be so overly restrictive as to prohibit reasonably expressive bidding strategies. We characterize f by boundary conditions for identical users and maximally distant users with the requirement that f is weakly increasing and $f > 1$ in the intermediate range. In the case of identical users, i.e. $\mathbf{d}(u, v) = 0$, the advertiser is required to place identical bids, i.e., $f(0) = 1$. For maximally distant users, i.e. $\mathbf{d}(u, v) = 1$, we choose $f(1) = \infty$, allowing advertisers to bid arbitrarily differently on this pair. For ease of analysis, we also make f continuous.

In this work, we show that a specific parameterized class of bid ratio constraints that satisfy the above properties performs well in terms of fair value. The family is parameterized by $l \geq 1$, and is defined as: $f_l(d) = \left(\frac{1+d}{1-d}\right)^l$. Figure 1 displays some functions in this family. Note that as the parameter l increases, the bid ratio condition becomes more and more strict. A further discussion of the structural properties of $f_l(d)$ can be found in the full version [11].

We emphasize that bid ratios are not required to satisfy the constraint exactly, but rather should lie *at or below* the imposed curve. From an algorithmic viewpoint, this means that if we design an algorithm based on the polynomial family described above but the actual bid ratio constraint imposed on bids, say g, does not belong to this family, it nevertheless suffices for the algorithm to find a value of the parameter l for which $f_l(d) \geq g(d)$ for all $d \in [0, 1]$ and use the function f_l in making allocations.

4.2 Upper bounds on fair value with uniform metrics

We prove upper bounds on the fair value of any allocation algorithm that satisfies multiple-task fairness. Observe that these upper bounds apply also to total variation fairness, which is a stronger requirement. We use uniform metrics, i.e. metrics of the form $\mathbf{d}(u, v) = d$ for all $u \neq v$. We use Example 4.1 to show an upper bound on the fair value as a function of the bid ratio constraint.

Lemma 4.3. *Given $d \in [0,1]$ and $\alpha = f(d)$, there exists an instance of the online advertising problem for which the fair value of every offline allocation algorithm satisfying multiple-task fairness with respect to the uniform metric $\mathbf{d}(u,v) = d$ is at most $d + \frac{1-d}{k} + \frac{(1-d)(k-1)}{k\alpha} \le \frac{1}{k} + \frac{1}{\alpha} + d$.*

Observe that as d increases, the upper bound on the fair value increases, due to a weaker fairness constraint on setting allocation probabilities. On the other hand, for any fixed d and k, as α increases, weakening the fairness constraint on bids, the algorithm's performance becomes worse.

In the online setting it is possible to prove even stronger bounds on the fair value.[16] More specifically, we can tighten the $1/\alpha$ term in the fair value to $1/\alpha^2$. In the full version of the paper, we develop online allocation algorithms that are tailored to uniform metrics and achieve a fair value that nearly matches these lower bounds, demonstrating that stronger lower bounds for general online algorithms cannot be obtained through uniform metrics.

Lemma 4.4. *Given $d \in [0, 1 - 1/k]$ and $\alpha = f(d)$, there exists an instance of the online advertising problem for which no online allocation algorithm satisfying multiple-task fairness with respect to the uniform metric $\mathbf{d}(u,v) = d$ can obtain fair value better than $\alpha^{-2}\left(1 - \frac{1}{k} - d\right) + \frac{1}{k} + d \le \frac{1}{k} + \frac{1}{\alpha^2} + d$.*

4.3 Algorithms for the general metric case

We construct an allocation algorithm for a general fairness metric $\mathbf{d}$ that achieves total variation fairness and a large fair value. From a algorithm design standpoint, it is desirable for our algorithm to satisfy certain other properties. Most important is *metric-obliviousness*, i.e. where the algorithm doesn't require direct knowledge of $\mathbf{d}$. In addition, the following other properties are desirable:

(1) *Identity-obliviousness*: that is, the algorithm treats advertisers in a symmetric manner in each individual iteration — the allocation to an advertiser does not depend on their identity.
(2) *Monotonicity*: that is, the allocation probabilities increase monotonically as functions of the advertisers' bids, which means that we can make this algorithm truthful by setting the payoffs appropriately by Myerson's lemma.
(3) *History-obliviousness*, which has the nice implication that the memory required by the algorithm is independent of the number of users and the solution is *independent* of the ordering of the users so we don't need to worry about impact of the order in which the users are given on advertiser strategies and/or utility.
(4) *Protecting against advertiser splitting*: We would like the algorithm to disincentivize an advertiser from splitting up into sub-advertisers (or submitting multiple ads) in an attempt to obtain a higher probability allocation.

We now describe a class of algorithms that satisfy these properties. The key intuition is to convert the bids on a user into probabilities using a function that places higher probabilities on higher bids. Note that the first three properties (identity-oblivious, monotonic, and history-oblivious) mean that the algorithm must be defined by a symmetric, coordinate-wise increasing function $G : \left(\mathbb{R}^{\ge 0}\right)^k \to \left\{(p^1, \ldots, p^k) \mid p^i \ge 0, \sum_{i=1}^k p^i \le 1\right\}$ that maps bids into probabilities. We observe that the overall optimal solution (Unfair-OPT) can be placed in this framework: the algorithm that distributes the probability mass equally among the highest bidders for each user corresponds to the function that places the full mass on the highest bids. This function can be viewed as assigning allocation probabilities in proportion to their contribution to the ℓ_∞-norm over the bids.

Proportional allocation algorithms. One rich class of algorithms of this form are algorithms where the probabilities are proportional to some deterministic function g of the bids, i.e. where $p_u^i \propto g(b_u^i)$, with appropriate normalization to make $\sum_{i=1}^k p_u^i = 1$. We call these algorithms proportional allocation algorithms, defined as follows:[17]

Algorithm 1. *Let $g : \mathbb{R}^{\ge 0} \to \mathbb{R}^{\ge 0}$ be a continuous, superadditive (i.e. $g(x) + g(y) \le g(x+y)$), increasing function. The* ***proportional allocation algorithm with parameter*** *g assigns $p_u^i = \frac{g(b_u^i)}{\sum_{j=1}^k g(b_u^i)}$ for every user $u \in U$ and advertiser $i \in [k]$.*

It is straightforward to verify that proportional allocation algorithms are identity-oblivious, monotonic, history-oblivious, and protect against submitting multiple ads (this last property follows from the super-additivity of g). These algorithms bear similarity to position auctions[18], e.g., the highest bidder is always assigned the highest probability regardless of their identity.

A proportional allocation algorithm with high fair value. We construct a family of functions g where the fair value of the proportional allocation algorithms is high. More specifically, we show that $g(x) = x^l$ for $l \ge 1$ can achieve fair value approaching 1 as $l \to \infty$. This algorithm can be viewed as assigning allocations in proportion to each bid's contribution to the ℓ_l-norm of the bid vector. We emphasize that our bound on the fair value does not require bids to satisfy the bid ratio constraint.

Theorem 4.5. *Let M be a proportional allocation algorithm with parameter $g(x) = x^l$ for a positive integer l. If $k \ge 9$ and $l \ge 1$, then the fair value of M is at least $(k-1)^{-1/l}\frac{k-1}{k} + \frac{1}{k}$.*

Observe that fair value is an increasing function of l, and as $l \to \infty$ (where M places the entire mass on the highest bid), the bound equals 1 as expected. This means that fair value can be made arbitrarily close to 1 by sufficiently strengthening the bid ratio constraint. To achieve a fair value of r, we can set $l \ge \frac{\log(k-1)}{\log(1/r)}$.

We show that with the bid ratio condition $f_l(d) = \left(\frac{1+d}{1-d}\right)^{1/l}$ shown in Figure 1, this algorithm satisfies total variation fairness.

Theorem 4.6. *Let M be a proportional allocation algorithm with parameter $g(x) = x^l$ for a positive integer l. If all advertisers in a category satisfy the bid ratio condition $f_l(d) = \left(\frac{1+d}{1-d}\right)^{1/l}$, M satisfies total variation fairness in that category.*

[16] To be explicit, the online lower bound applies in the limit as $|U| \to \infty$, where the adversary is given access to the probabilities output by the algorithm when designing the next set of bids.

[17] The special case of the proportional allocation algorithm with $g(x) = x$ was considered in a different context in [1].

[18] Roughly speaking, a position auction [15, 19] only uses the ordering of the bids (and not the bid values or advertiser identities) in determining the allocation. See the full version of the paper [11] for a discussion of why position auctions cannot achieve both fairness and a high fair value.

Observe that the bid ratio condition becomes stronger as l increases. Thus, for a fixed number of advertisers, a higher fair value is accompanied by a stronger bid ratio condition. Moreover, observe that to maintain a fair value of r, a stronger bid ratio condition is needed as the number of advertisers increases, since $l = \frac{\log(k-1)}{\log(1/r)}$ grows with k.

Near-optimality within proportional allocation algorithms. A natural question is to ask is whether Algorithm 1 with a different function g can achieve a a much better fair value, potentially using a differently shaped bid ratio condition. In fact, we show that Algorithm 1 is nearly optimal within the family of proportional allocation algorithms. We specifically prove that *any* proportional allocation algorithm achieving a certain fair value will have a corresponding bid ratio condition that is *point-wise* stronger than $f_l(d) = \left(\frac{1+d}{1-d}\right)^{1/l}$, where l within a constant factor of what is achieved by Algorithm 1 with $g(x) = x^l$. This result demonstrates that changing the *shape* of the bid ratio condition will not significantly improve the fair value. We show the following lower bound:

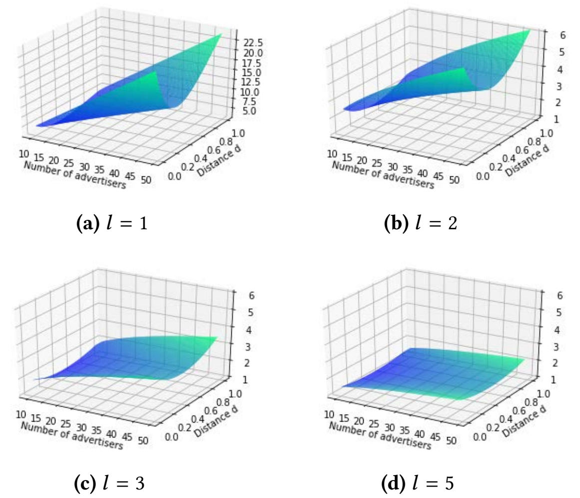
LEMMA 4.7. *Suppose that M is a proportional allocation algorithm achieves fair value r and achieves total variation fairness with a bid ratio condition of f. For $l = \frac{\log(k-1)}{2\log(1/r)} - 0.5$, we have that $f(d) \leq f_l(d) = \left(\frac{1+d}{1-d}\right)^{1/l}$ for infinitely many points on f (more specifically, for $d \in D := \left\{d \mid f(d) = \left(\frac{1}{r^2}\right)^m, m \in \left\{n, \frac{1}{n} \mid n \in \mathbb{N}\right\}\right\}$).*

How does the lower bound in Lemma 4.7 compare to the proportional allocation algorithms with parameter x^l? Lemma 4.7 shows that any proportional allocation algorithm will satisfy $f(d) \leq f_l(d)$ where $l = \frac{\log(k-1)}{2\log(1/r)} - 0.5$ for infinitely many points on the curve. Meanwhile, Theorem 4.5 and Theorem 4.6 show that to achieve a fair value r, it suffices to take $f_l(d)$ with $l = \frac{\log(k-1)}{\log(1/r)}$. Thus, there is essentially a constant factor difference in the lower and upper bounds on l.

A consequence of Lemma 4.7 is in the family of proportional allocation algorithms, the fair value must necessarily degrade with the number of advertisers k if the bid ratio condition is fixed. Or equivalently, to maintain a fair value of r, a stronger bid ratio condition is needed as the number of advertisers increases. In fact, Lemma 4.7 implies that that the proportional algorithm with parameter x^l achieves the optimal rate of change of the bid ratio condition as a function of k: within the family of proportional allocation algorithms, a better dependence on k is not possible.

4.4 Discussion

In Section 4.3, we showed that Algorithm 1 with $g(x) = x^l$ is nearly optimal in the class of proportional allocation algorithms. Now, we compare Algorithm 1 with $g(x) = x^l$ to general online algorithms, using our negative results in Section 4.2. It is a little tricky to directly compare the fair value lower bounds achieved by Algorithm 1 with $g(x) = x^l$ with the upper bounds in Section 4.2, because the bounds depend on different parameters. In particular, the lower bounds hold for arbitrary metrics whereas the upper bounds are designed only for the uniform metric. To perform an apples-to-apples comparison, we fix parameters k, l, and some number $d \in (0, 1)$, and set $\alpha = f_l(d)$.



(a) $l = 1$ **(b)** $l = 2$

(c) $l = 3$ **(d)** $l = 5$

Figure 2: Illustrations of the ratio of upper and lower bounds on fair values for different values of l. Note that the $z-$axis scale for $l = 1$ differs from the others.

Figure 2 displays the ratio of the upper bound and lower bound for various parameter settings. Observe that the ratio is bounded by a reasonably small constant except when l is very small. This indicates that Algorithm 1 with parameter $g(x) = x^l$ in general obtains a large fraction of the utility that can be obtained by any online algorithm satisfying multiple-task fairness, despite satisfying a stronger form of fairness and nice algorithm design properties.

5 INTER-CATEGORY FAIRNESS

We consider the setting where different product categories correspond to very different metrics. We specifically consider the setting where every category has exactly one advertiser (i.e. where $c = k$). We first show it is not possible to achieve the same tradeoff between multiple-task fairness and utility as in the case of identical metrics. We then show that with inter-category envy-freeness, significantly better tradeoffs are possible. We present tight upper and lower bounds on the fair value achievable as a function of upper bounds on the sizes of the preferred sets. In Section 5.1, we focus on the Unfair-OPT benchmark considered in the previous section. In Section 5.2, we argue in favor of a relaxed benchmark to evaluate the performance of fair allocation algorithms, and show that this benchmark can be exactly met by algorithms that satisfy inter-category envy-freeness. Proofs are in the full version of the paper [11].

5.1 Inter-category envy-freeness and fair value relative to Unfair-OPT

First, we show a upper bound that demonstrates that multiple-task fairness is in conflict with utility, even in the offline setting, if the metrics are permitted to be different. The result is based on the "jack of all trades" example in the introduction. Formally:

Example 5.1 (Jack-of-all-trades). Suppose that the universe has $c + 1$ users $u_1, \ldots, u_{c+1}$ and there are c categories (with one advertiser per category). The metric $\mathbf{d}^i$ is defined so that $\mathbf{d}^i(u_{c+1}, u_i) = 0$

and all other distances are 1. Suppose that advertiser i bids 1 on u_i and u_{c+1}, and 0 on everybody else. Observe that the bids are fair assuming $f(1) = \infty$.

Consider any allocation algorithm and suppose that this algorithm chooses an ad from distribution $(q_1, \cdots, q_c)$ to display for user u_{c+1}. Then, in order to respect multiple task fairness as defined above, the algorithm cannot allocate ad i to user u_i with probability greater than q_i. As a result, most of the "specialists" necessarily obtain a low allocation within their desired categories: since the "jack of all trades" can only be served a single ad, the "specialists" are penalized. Multiple-task fairness thus limits the allocation of almost all of the users. Moreover, the utility of any fair allocation is bounded by some constant, whereas an unfair allocation can achieve utility $c + 1$. We obtain the following bound on fair value:

Proposition 5.2. *Suppose that bids satisfy the bid ratio constraint f, then no offline algorithm that satisfies multiple-task fairness across c categories can obtain a fair value more than $\frac{2}{c+1}$.*

We now consider the weaker fairness notion of inter-category envy-freeness defined in Section 3 that limits the number of fairness constraints imposed by any given user. We assume that the platform obtains from each user $u \in U$ a preferred set $S_u \subseteq [c]$ of categories as the user arrives (i.e. the preferences are not known to the allocation algorithm in advance). Inter-category envy-freeness requires that the total allocation of ads in S_u to the user u should be at least as large as the total allocation of ads in S_u to any other user. Observe that our definition of fairness doesn't involve a metric over the users. Moreover, obtaining good utility requires that we place a reasonable amount of mass on the highest bid(s) for the user u.

We develop an allocation algorithm that reserves some allocation mass for the category with the highest bid on each user u, and distributes the remaining allocation probability across categories in S_u. In doing so, we must ensure that no subset of categories gets too much mass in total. This is because if such a set S exists, and a future user v sets $S_v = S$, then the algorithm is forced to give a high allocation to this set for user v. Unless S is small, this may then leave little probability mass for the highest bidder for v.

We now formally define our algorithm and bound its fair value.

Algorithm 2. *The* **equal-spread algorithm with parameters** β **and** C *is defined as follows. We assume that every user u specifies a subset $S_u \subset [c]$ with $|S_u| \leq C$. Let $C_u = \text{argmax}_{1 \leq j \leq c}\{b_u^j\}$ be a category with the highest bid. The algorithm assigns an allocation probability of p_{high} to C_u and $p_{fairness}$ to categories in $S_u \setminus \{C_u\}$, where $p_{high} = \frac{1}{1+\beta^{-1}+\ldots+\beta^{-C}}$ and $p_{fairness} = \frac{\beta^{-1}+\ldots+\beta^{-|S_u|}}{(|S_u|)(1+\beta^{-1}+\ldots+\beta^{-C})}$.*

The parameter $\beta \geq 1$ allows the algorithm to trade-off between fairness and utility. By allowing the algorithm to achieve β-inter-category envy-freeness for larger β values, we obtain a better approximation to the Unfair-OPT.

Theorem 5.3. *If every user's preferred set contains $\leq C$ categories, then for any $\beta \geq 1$, the equal-spread algorithm with parameters β and C (Algorithm 2) satisfies β-inter-category envy-freeness and achieves a fair value of $\geq \frac{1}{1+\beta^{-1}+\ldots+\beta^{-C}}$.*

We show a matching upper bound on the fair value, thus showing that Algorithm 2 is optimal. Our proof boils down to bounding the amount of mass that can be placed on the highest bid. We construct a sequence of users with the property that bids in S_u are always 0 and each user has a category $C_u \notin S_u$ that bids 1. We adaptively construct the sets S_u and C_u to minimize the fair value.

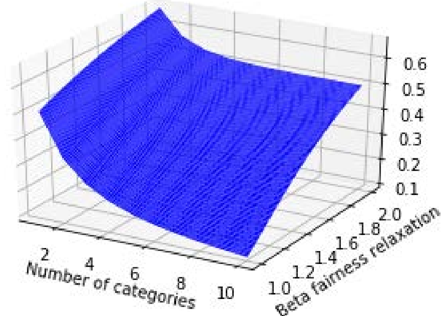


Figure 3: Fair value for inter-category envy-freeness

Lemma 5.4. *Suppose that $C < c$, and every user's preferred set can contain any number of categories $\leq C$. Then, regardless of the bid ratio constraint f imposed on the advertisers (but assuming $f(1) = \infty$), any online algorithm that satisfies β-inter-category envy-freeness obtains a fair value of at most $\frac{1}{1+\beta^{-1}+\ldots+\beta^{-C}}$.*

In the above construction, we consider a uniform metric with distance 1 (where the bid ratio conditions do not implicitly provide any guarantees). Nonetheless, even though the users are maximally distant, the inter-category envy-freeness still provides strong uni-dimensional guarantees between these users. A natural question to ask is: can we achieve a higher fair value by considering a relaxed version of inter-category envy-freeness that reintroduces the metric? We show in the full version of the paper [11] that it is not possible to do better.

We plot the tight bound on fair value obtained for inter-category envy-freeness as a function of β and C in Figure 3. Observe that as β increases, the weakened fairness guarantees cause the fair value to increase. For the dependence on C, observe that the algorithm must balance between allocating to a category with the highest bid to achieve high utility, and allocating to categories in S_u to achieve inter-category envy-freeness. As C increases, the algorithm has a greater number of categories to consider for each user (and the highest bid can still be outside of S_u), thus causing the optimal fair value to decrease.

5.2 Relaxing the utility benchmark

The results in the previous section, for the strongest form of fairness, place an upper bound of $1/2$ (or lower) on the fair value that can achieved by inter-category envy-freeness. In this section, we consider the setting where we restrict to algorithms that receive utility only for allocations in S_u: that is, the utility is $\sum_{u \in U} \sum_{i \in S_u} p_u^i b_u^i$. In this case, we assume that user-specified categories are aligned with interest, and in a click-through-rate based revenue/utility model, the platform only obtains benefit from allocations within the user-specified sets.

It is straightforward to see that the best possible utility achieved by any (potentially unfair) algorithm in this restricted class is $\sum_{u \in U} \max_{i \in S_u} b_u^i$, which may be much smaller than unrestricted optimal utility of $\sum_{u \in U} \max_{i \in [c]} b_u^i$. This motivates considering the relaxed fair value given by: $\frac{\sum_{u \in U} \sum_{i \in S_u} p_u^i b_u^i}{\sum_{u \in U} \max_{i \in S_u} b_u^i}$. When there can be more one advertiser per category, we define the relaxed fair value as $\frac{\sum_{u \in U} \sum_{i \in S_u} \sum_{j \in C_i} p_u^j b_u^j}{\sum_{u \in U} \max_{i \in S_u, j \in C_i} b_u^j}$.

Returning to the case where there is at most one advertiser per category, we show a strong positive result: the simple highest-bidder-wins algorithm achieves inter-category envy-freeness and a relaxed fair value of 1 even when the sets $|S_u|$ are large.

Algorithm 3. *For each $u \in U$, the algorithm allocates a probability of 1 to the category in S_u with the highest bid. If there are multiple categories tied for the highest bid, then the algorithm splits the probability equally between these categories.*

Theorem 5.5. *Algorithm 3 achieves inter-category envy-freeness with $\beta = 1$ and achieves a relaxed fair value of 1.*

Given the simplicity of Algorithm 3, a natural question is to ask is: can we obtain better fairness guarantees while still maintaining a high relaxed fair value? We show in the full version of the paper [11] that it is not possible to have multiple-task fairness guarantees and a high fair value even against the relaxed benchmark.

6 COMPOSITIONAL FAIRNESS

We now discuss how to combine our algorithms to handle the setting where there can be multiple categories and multiple advertisers in each category. Let $\{C_1, \ldots, C_c\}$ denote a partition of the set $[k]$ of advertisers into c categories. We can compose our algorithms from Section 4 and Section 5 by running algorithms from Section 5 to allocate probability between categories, and algorithms from Section 4 to distribute the probability assigned to a category between advertisers in that category.

The intuition for fairness is that compositional fairness is implied by inter-category envy-freeness and intra-category total variation fairness.[19] Moreover, these combined constructions achieve a high fair value, since the fair value of the composed algorithm is the product of the fair values of the individual algorithms. Finally, the composed algorithms are metric-oblivious, as a direct consequence of our intra-category and inter-category algorithms enjoying this property. Proofs are in the full version of the paper [11].

Note that the two choices of category selection algorithms in Section 5 result in combined constructions that differ in terms of both utility guarantees (i.e. choice of benchmark) and category distribution properties (i.e., concentrating or spreading probability between categories), which are important for practical considerations.

High fair value compared to relaxed benchmark

To achieve a high fair value compared to the relaxed utility benchmark (taking user preferences as an indicator of click probability discussed in Section 5.2), we can compose Algorithm 3 with Algorithm 1. The idea is that we run Algorithm 3 to identify the category in S_u with the highest bid, and then we run Algorithm 1 to divide the probability mass between advertisers in this category.

[19] In order to conclude that $\sum_{i \in C_j} p_u^i$ is actually the probability the first algorithm assigned to category C_l, we require that the second algorithm assigns the full 1 probability mass to each user. This is true of proportional allocation (Algorithm 1).

Algorithm 4. *For each user u, the algorithm runs Algorithm 3 to allocate probabilities between categories, where for $1 \le j \le c$, the bid B_u^j is taken to be $\max_{i \in C_j} b_u^i$. The algorithm then determines the conditional probabilities for advertisers within each category using Algorithm 1 with parameter $g(x) = x^l$.*

Theorem 6.1. *Let $k' = \max_{1 \le j \le c} |C_j|$ be the maximum number of advertisers in any category. If all advertisers $1 \le i \le k$ satisfy the bid ratio condition $f^i(d) \le \left(\frac{1+d}{1-d}\right)^{1/l}$, then Algorithm 4 achieves compositional fairness and a relaxed fair value of at least $(k'-1)^{-1/l} \frac{k'-1}{k'} + \frac{1}{k'}$.*

High fair value compared to Unfair-OPT

It is likewise possible to combine algorithms 1 and 2 to obtain bounds on the fair value against Unfair-OPT.

Algorithm 5. *For each user u, the algorithm runs Algorithm 2 with parameters β and C to allocate probabilities between categories, where for $1 \le j \le c$, the bid B_u^j is taken to be $\max_{i \in C_j} b_u^i$. The algorithm then determines the conditional probabilities for advertisers within each category using Algorithm 1 with parameter $g(x) = x^l$.*

Theorem 6.2. *Suppose that every user's preferred set contains at most C categories. Let $k' = \max_{1 \le j \le c} |C_j|$ be the maximum number of advertisers in any category. If all advertisers $1 \le i \le k$ satisfy the bid ratio condition $f^i(d) \le \left(\frac{1+d}{1-d}\right)^{1/l}$, then Algorithm 5 achieves compositional fairness and a fair value of at least*

$$\left(\frac{1}{1+\beta^{-1}+\ldots+\beta^{-C}}\right)\left((k'-1)^{-1/l}\frac{k'-1}{k'}+\frac{1}{k'}\right).$$

7 FUTURE WORK

In this work, we give an initial framework for understanding the utility and fairness properties of a combination of individual fairness and envy-freeness. We highlight areas for future work below.

- *Advertiser incentives.* In order to use implement our allocation algorithms as mechanisms, it is necessary to consider advertiser incentives. Our allocation algorithms satisfy nice properties such as monotonicity and individual rationality, which serves as a starting point. Nonetheless, a full consideration of incentives requires considering how the bid fairness constraints impact advertiser bidding strategies.
- *Closing the gaps and tighter upper bounds.* In Section 4, we showed general fair value upper bounds using uniform metrics, and showed stronger upper bounds for proportional allocation algorithms. An interesting direction for future work would be to show upper bounds for general metric oblivious algorithms and compare these bounds to Algorithm 1 with parameter $g(x) = x^l$.
- *Multiple slot auctions.* In this work, we have focused on the case of single slot auctions. However, in practice multiple slot auctions, or auctions where each user appears at several times, are more common. This leaves open several considerations in defining fairness, such as how to account for the ordering of advertisements.
- *Different combinations of inter- and intra-category fairness.* We give one combination of inter- and intra-category fairness, but it's likely that other combinations (e.g., intra-category total variation fairness and inter-category PIIF) may be of interest.

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