Fair Algorithms for Learning in Allocation Problems

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ABSTRACT

Settings such as lending and policing can be modeled by a centralized agent allocating a scarce resource (e.g. loans or police officers) amongst several groups, in order to maximize some objective (e.g. loans given that are repaid, or criminals that are apprehended). Often in such problems *fairness* is also a concern. One natural notion of fairness, based on general principles of *equality of opportunity*, asks that conditional on an individual being a candidate for the resource in question, the probability of actually receiving it is approximately independent of the individual's group. For example, in lending this would mean that equally creditworthy individuals in different racial groups have roughly equal chances of receiving a loan. In policing it would mean that two individuals committing the same crime in different districts would have roughly equal chances of being arrested.

In this paper, we formalize this general notion of fairness for allocation problems and investigate its algorithmic consequences. Our main technical results include an efficient learning algorithm that converges to an optimal fair allocation even when the allocator does not know the frequency of candidates (i.e. creditworthy individuals or criminals) in each group. This algorithm operates in a *censored* feedback model in which only the number of candidates who received the resource in a given allocation can be observed, rather than the true number of candidates in each group. This models the fact that we do not learn the creditworthiness of individuals we do not give loans to and do not learn about crimes committed if the police presence in a district is low.

As an application of our framework and algorithm, we consider the *predictive policing* problem, in which the resource being allocated to each group is the number of police officers assigned to each district. The learning algorithm is trained on arrest data gathered from its own deployments on previous days, resulting in a potential feedback loop that our algorithm provably overcomes. In this case, the fairness constraint asks that the probability that an individual who has committed a crime is arrested should be independent of the district in which they live. We investigate the performance of our learning algorithm on the *Philadelphia Crime Incidents* dataset.

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CCS CONCEPTS

• **Theory of computation** → *Online learning algorithms*; *Machine learning theory*; *Online learning theory*; • **Computing methodologies** → *Learning from implicit feedback*;

KEYWORDS

algorithmic fairness, resource allocation, censored feedback, online learning

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1 INTRODUCTION

The bulk of the literature on algorithmic fairness has focused on classification and regression problems (see e.g. [3, 4, 6–8, 10, 14, 16, 17, 19, 20, 24–26] for a collection of recent work), but fairness concerns also arise naturally in many resource allocation settings. Informally, a resource allocation problem is one in which there is a limited supply of some *resource* to be distributed across multiple groups with differing needs. Resource allocation problems arise in financial applications (e.g. allocating loans), disaster response (allocating aid), and many other domains — but the primary example that we will focus on in this paper is policing. In the predictive policing problem, the resource to be distributed is police officers, which can be dispatched to different districts. Each district has a different crime distribution, and the goal (absent additional fairness constraints) might be to maximize the number of crimes caught.[1]

Of course, fairness concerns abound in this setting, and recent work (see e.g. [11, 12, 21]) has highlighted the extent to which algorithmic allocation might exacerbate those concerns. For example, Lum and Isaac [21] show that if predictive policing algorithms such as PredPol are trained using past arrest data to predict future crime, then pernicious feedback loops can arise, which misestimate the true crime rates in certain districts, leading to an overallocation of police.[2] Since the communities that Lum and Isaac [21] showed to be overpoliced on a relative basis were primarily poor and minority, this is especially concerning from a fairness perspective. In this work, we study algorithms that avoid this kind of under-exploration and incorporate quantitative fairness constraints.

In the predictive policing setting, Ensign et al. [11] implicitly consider an allocation to be *fair* if police are allocated across districts in direct proportion to the district's crime rate; generally extended, this definition asks that units of a resource are allocated according to the group's share of the total candidates for that resource. In our work, we study a different notion of allocative fairness that has a similar motivation to the notion of *equality of opportunity* proposed by Hardt et al. [14] in classification settings. Informally speaking, it asks that the probability that a candidate for a resource be allocated a resource be independent of his group membership. In the predictive policing setting, it asks that conditional on committing a crime, the probability that someone is apprehended should not depend on the district in which they commit the crime.

To illustrate that our notions of fairness do not depend on whether individuals would prefer to receive or not receive the resource, we highlight another setting in which allocative fairness is a natural concern: hiring.[3] Suppose a company wishes to recruit machine learning programmers by advertising on a social media platform. Many such platforms offer the ability to advertise to different demographics of users and charge by the number of times the advertisement is shown to different users (i.e., the number of *impressions*); a fixed advertising budget can then be viewed as a number of impressions to allocate. Depending on how well the platform can identify programmers within each demographic, the ad may be shown to a higher or lower number of programmers. In this setting, our notion of allocative fairness asks that the probability a programmer is exposed to the hiring ad (and thus, receives the opportunity to apply for a job) does not depend on the programmer's demographic, and the allocation problem is to maximize the number of programmers reached via the choice of impressions across each demographic, subject to fairness constraints.

[1]We understand that policing has many goals besides simply apprehending criminals, including preventing crimes in the first place, fostering healthy community relations, and generally promoting public safety. But for concreteness and simplicity we consider the limited objective of apprehending criminals.

[2]Predictive policing algorithms are often proprietary, and it is not clear whether in deployed systems, arrest data (rather than 911 reported crime) is used to train the models.

[3]Dwork and Ilvento [9] consider such a setting under different fairness notions and with different research questions in mind.

1.1 Our Results

To define the extent to which an allocation satisfies our fairness constraint, we must model the specific mechanism by which resources deployed to a particular group reach their intended targets. We study two such *discovery models*, and we view the explicit framing of this modeling step as one of the contributions of our work; the implications of a fairness constraint depend strongly on the details of the discovery model, and specifying such a model is an important step in making one's assumptions transparent.

We study two discovery models, which capture two extremes of targeting ability. In the *random* discovery model, regardless of the number of units allocated to a given group, all individuals within that group are equally likely to be assigned a unit, regardless of whether they are a candidate for the resource or not. In other words, the probability that a candidate receives a resource is equal to the ratio of the number of units of the resource assigned to his group to the size of his group (*independent* of the number of candidates in the group).

At the other extreme, in the *precision* discovery model, units of the resource are given only to actual candidates within a group, as long as there is sufficient supply of the resource. That is, the probability that a candidate receives a resource is equal to the ratio of the number of units of the resource assigned to his group to the number of *candidates* in his group.

In the policing setting, these models can be viewed as two extremes of police targeting ability for an intervention like *stop-and-frisk*. In the random model, police are viewed as stopping people uniformly at random. In the precision model, police have the omniscient ability to identify individuals with contraband, and stop only them. Of course, reality lies somewhere in between.

These discovery models have different implications for fairness. In the random model, fairness constrains resources to be distributed in amounts proportional to group sizes, regardless of the distribution of candidates, and so is uninteresting from a learning perspective. On the other hand, the precision model yields an interesting fairness-constrained learning problem when the distribution of the number of candidates in each group must be learned via observation, and what counts as a 'fair' allocation depends greatly on these distributions.

We study learning in a censored feedback setting: each round, the algorithm can choose a feasible deployment of resources across groups. Then the number of candidates for the current round in each group is drawn from a fixed, but unknown group-dependent distribution (which might be not be independent from the distributions in other groups). The algorithm does not observe the number of candidates present in each group, but only the number of candidates that received the resource. In the policing setting, this corresponds to the algorithm being able to observe the number of arrests, but not the actual number of crimes in each of the districts. Thus, the extent to which the algorithm can learn about the distribution in a particular group is limited by the number of resources it deploys there. The goal of the algorithm is to converge to an optimal fairness-constrained allocation, where here both the objective value of the solution and the constraints imposed on it depend on the unknown distributions.

One trivial solution to the learning problem is to sequentially deploy *all* of one's resources to each group in turn for a sufficient amount of time to accurately learn the candidate distributions. This would reduce the learning problem to an offline constrained optimization problem, which we show can be efficiently solved by a greedy algorithm. But this algorithm is unreasonable: it has a large exploration phase in which it uses nonsensical deployments, vastly overallocating to some groups and underallocating to others. A much more realistic, natural approach is a greedy-style learning algorithm which at each round uses its current best-guess estimate for the distribution in each group and deploys an optimal fairness-constrained allocation according to these estimates. Unfortunately, as we show, if one makes no assumptions on the underlying distributions, any algorithm that has a guarantee of converging to a fair allocation must behave like the trivial one, deploying vast numbers of resources to each group in turn.

This impossibility result motivates us to consider the learning problem in which the unknown distributions are from a known parametric family. The natural greedy algorithm uses an optimal fair deployment at each round given the maximum likelihood estimates of candidate distributions given its (censored) observations so far; for concreteness, we consider the case of the Poisson distribution, and show that it converges to an optimal fair allocation, but our analysis generalizes for any single-parameter Lipschitz-continuous family of distributions.

Finally, we conduct an empirical evaluation of our algorithm on the *Philadelphia Crime Incidents* dataset, which records all crimes reported to the Philadelphia Police Department's INCT system between 2006 and 2016. We verify that the crime distributions in each district are in fact well-approximated by Poisson distributions, and that our algorithm converges quickly to an optimal fair allocation (as measured according to the empirical crime distributions in the dataset). We also systematically evaluate the *Price of Fairness*, and plot the Pareto curves that trade off the number of crimes caught versus the slack allowed in our fairness constraint, for different sizes of police force, on this dataset. For the random discovery model, we prove worst-case bounds on the Price of Fairness.

1.2 Further Related Work

Our precision discovery model is inspired by and has technical connections to Ganchev et al. [13], which models the *dark pool problem* from quantitative finance, in which a trader wishes to execute a specified number of trades across a set of exchanges of unknown but independently distributed liquidity. In Ganchev et al. [13], the authors design an optimal allocation algorithm under the censored feedback of the precision model. It is straightforward to map their setting onto ours, but they assume independence between different exchanges, while the candidate distributions in our setting need not be independent. Regardless, we show that their allocation algorithm can be used to compute an optimal allocation (ignoring fairness) even when the independence assumption is relaxed (see Remark 1). Later, Agarwal et al. [1] extend the dark pool problem to an adversarial (rather than distributional) setting. This is quite closely related to the work of Ensign et al. [12] who also consider the precision model (under a different name) in an adversarial predictive policing setting. They provide no-regret algorithms for this setting by reducing the problem to learning in a partial monitoring environment. Since their setting is equivalent to that of Agarwal et al. [1], the algorithms in Agarwal et al. [1] can be directly applied to the problem studied by Ensign et al. [12].

Our desire to study the natural greedy algorithm rather than an algorithm which uses "unreasonable" allocations during an exploration phase is an instance of a general concern about exploration in fairness-related problems [5]. Recent works have studied the performance of greedy algorithms in different settings for this reason [2, 18, 23].

Lastly, the term *fair allocation* appears in the *fair division* literature (see e.g. [22] for a survey), but that body of work is technically quite distinct from the problem we study here.

2 SETTING

We study an *allocator* who has $\mathcal{V}$ units of a resource and is tasked with distributing them across a population partitioned into $\mathcal{G}$ groups. Each group is divided into *candidates*, who are the individuals the allocator would like to receive the resource, and *non-candidates*, who are the remaining individuals. We let m_i denote the total number of individuals in group i. The number of candidates c_i in group i is a random variable drawn from a fixed but unknown distribution C_i called the *(marginal) candidate distribution*. We do not make any assumptions about the relationship between the candidate distributions across different groups and in particular these distributions need not be independent. We use M to denote the total size of all groups (i.e., $M = \Sigma_{i \in [\mathcal{G}]} m_i$). An allocation $\mathbf{v} = (v_1, \ldots, v_{\mathcal{G}})$ is a partitioning of these $\mathcal{V}$ units, where $v_i \in \{0, \ldots, \mathcal{V}\}$ denotes the units of resources allocated to group i. Every allocation is bound by a *feasibility* constraint which requires that $\Sigma_{i \in [\mathcal{G}]} v_i \leq \mathcal{V}$.

A *discovery model* $\mathrm{disc}(v_i, c_i)$ is a (possibly randomized) function mapping the number of units v_i allocated to group i and the number of candidates c_i in group i to the number of candidates discovered in group i. In the learning setting, upon fixing an allocation $\mathbf{v}$, the learner will get to observe (a realization of) $\mathrm{disc}(v_i, c_i)$ for the realized value of c_i for each group i. Fixing an allocation $\mathbf{v}$, a discovery model $\mathrm{disc}(\cdot)$ and candidate distributions for all groups $C = \{C_i : i \in [\mathcal{G}]\}$, we define the total expected number of discovered candidates, $\chi(\mathbf{v}, \mathrm{disc}(\cdot), C)$, as

$$\chi\left(\mathbf{v}, \mathrm{disc}(\cdot), C\right) = \sum_{i \in [\mathcal{G}]} \mathop{\mathbb{E}}_{c_i \sim C_i} \left[\mathrm{disc}(v_i, c_i) \right], \tag{1}$$

where the expectation is taken over C_i and any randomization in the discovery model $\mathrm{disc}(\cdot)$. When the discovery model and the candidate distributions are fixed, we will simply write $\chi(\mathbf{v})$ for brevity. We also use the total expected number of discovered candidates and *(expected) utility* exchangeably. We refer to an allocation that maximizes the expected number of discovered candidates over all feasible allocations as an *optimal allocation* and denote it by $\mathbf{w}^*$.

2.1 Allocative Fairness

For the purposes of this paper, we say that an allocation is *fair* if it satisfies *approximate equality of candidate discovery probability* across groups. We call this *discovery probability* for brevity. This formalizes the intuition that it is unfair if candidates in one group have an inherently higher probability of receiving the resource than candidates in another. Formally, we define our notion of *allocative fairness* as follows.

Definition 1. *Fix a discovery model* disc(·) *and the candidate distributions* C. *For an allocation* $\mathbf{v}$*, let*

$$f_i\left(v_i, \mathrm{disc}(\cdot), C_i\right) = \mathop{\mathbb{E}}_{c_i \sim C_i}\left[\frac{\mathrm{disc}\left(v_i, c_i\right)}{c_i}\right],$$

denote the expected probability that a random candidate from group i receives a unit of the resource at allocation $\mathbf{v}$ *(i.e. the discovery probability in group i). Then for any* $\alpha \in [0,1]$*,* $\mathbf{v}$ *is* α*-fair if*

$$\left|f_i\left(v_i, \mathrm{disc}(\cdot), C_i\right) - f_j\left(v_j, \mathrm{disc}(\cdot), C_j\right)\right| \leq \alpha,$$

for all pairs of groups i and j.

When it is clear from the context, for brevity, we write $f_i(v_i)$ for the discovery probability in group i. We emphasize that this definition (1) depends crucially on the chosen discovery model, and (2) requires nothing about the treatment of non-candidates. We think of this as a *minimal* definition of fairness, in that one might want to further constrain the treatment of non-candidates — but we do not consider that extension.

Since discovery probabilities $f_i(v_i)$ and $f_j(v_j)$ are in $[0,1]$, the absolute value of their difference is in $[0,1]$. By setting $\alpha = 1$ we impose no fairness constraints whatsoever on the allocations, and by setting $\alpha = 0$ we require *exact* fairness.

We refer to an allocation $\mathbf{v}$ that maximizes $\chi(\mathbf{v})$ subject to α-fairness and the feasibility constraint as an *optimal* α*-fair allocation* and denote it by $\mathbf{w}^{\alpha}$. In general, $\chi(\mathbf{w}^{\alpha})$ is a non-increasing quantity in α, since as α diminishes, the utility maximization problem becomes more constrained.

Remark 1. *We note that both the utility and discovery probabilities can be written solely in terms of the* marginal *candidate distributions in each of the groups, even when these distributions are not independent. This is because we have (implicitly) assumed that the number of candidates discovered in a group depends only on the number of candidates in the group and the allocation to that group, regardless of the allocations to and the number of candidates in other groups. This assumption together with the linearity of expectation allows us to write the expected utility as in the right hand side of Equation 1.*

3 THE PRECISION DISCOVERY MODEL

We begin by describing the *precision model* of discovery. Allocating v_i units to group i in the precision model results in the discovery of $\mathrm{disc}(c_i, v_i) \triangleq \min(c_i, v_i)$ candidates. This models the ability to perfectly discover and reach candidates in a group with resources deployed to that group, limited only by the number of deployed resources and the number of candidates present.

The precision model results in *censored* observations that have a particularly intuitive form. Recall that in general, a learning algorithm at each round gets to choose an allocation $\mathbf{v}$ and then observes $\mathrm{disc}(v_i, c_i)$ for each group i. In the precision model, this results in the following kind of observation: when v_i is larger than c_i, the allocator learns the number of candidates c_i present on that day exactly. We refer to this kind of feedback as an *uncensored observation.* When v_i is smaller than c_i, all the allocator learns is that the number of candidates is *at least* v_i. We call this a *censored observation.*

The rest of this section is organized as follows. In Sections 3.1 and 3.2 we characterize optimal and optimal fair allocations for the precision model when the candidate distributions are known. In Section 3.3 we focus on learning an optimal fair allocation when these distributions are unknown. We show that any learning algorithm that is guaranteed to find a fair allocation in the *worst case* over candidate distributions must have the undesirable property that at some point, it must allocate a vast number of its resources to each group individually. To bypass this hurdle, in Section 3.4 we show that when the candidate distributions have a parametric form, a natural greedy algorithm which always uses an optimal fair allocation for the current maximum likelihood estimates of the candidate distributions converges to an optimal fair allocation.

3.1 Optimal Allocation

We first describe how an optimal allocation (absent fairness constraints) can be computed efficiently when the candidate distributions C_i are known. In Ganchev et al. [13], the authors provide an algorithm for computing an optimal allocation when the distributions over the number of shares present in each dark pool are known and the trader wishes to maximize the expected number of traded shares. They assume that the distributions of shares across different dark pools are independent, but our formulation does not require this assumption of independence. Still, we can use the same algorithm as in Ganchev et al. [13] to compute an optimal allocation in our setting; this is because, as stated in Remark 1, the utility in both settings can be written solely in terms of the (marginal) candidate distributions even when the candidate distributions are not independent across groups. Here, we present the high level ideas of their algorithm in the language of our model.

Let $\mathcal{T}_i(c) = \Pr_{c_i \sim C_i}[c_i \geq c]$ denote the probability that there are at least c candidates in group i. We refer to $\mathcal{T}_i(c)$ as the *tail probability of* C_i *at* c. Recall that the value of the *cumulative distribution function* (CDF) of C_i at c is defined to be

$$\mathcal{F}_i(c) = \sum_{c' \leq c} \Pr_{c_i \sim C_i}\left[c_i = c'\right].$$

So $\mathcal{T}_i(c)$ can be written in terms of CDF values as $\mathcal{T}_i(c) = 1 - \mathcal{F}_i(c-1)$.

First, observe that the expected total number of candidates discovered by an allocation in the precision model can be written in terms of the tail probabilities of the candidate distributions i.e.

$$\chi(\mathbf{v}, \mathrm{disc}(\cdot), C) = \sum_{i \in [\mathcal{G}]} \mathop{\mathbb{E}}_{c_i \sim C_i}\left[\min\left(v_i, c_i\right)\right] = \sum_{i \in [\mathcal{G}]} \sum_{c=1}^{v_i} \mathcal{T}_i(c).$$

Since the objective function is concave (as $\mathcal{T}_i(c)$ is a non-increasing function in c for all i), a greedy algorithm which iteratively allocates the next unit of the resource to a group in

$$\arg\max_{i \in [\mathcal{G}]} \left(\mathcal{T}_i \left(v_i^t + 1 \right) - \mathcal{T}_i \left(v_i^t \right) \right),$$

where v_i^t is the current allocation to group i in the t^{th} round achieves an optimal allocation.

3.2 Optimal Fair Allocation

We next show how to compute an optimal α-fair allocation in the precision model when the candidate distributions are known and do not need to be learned.

To build intuition for how the algorithm works, imagine that the group i has the highest discovery probability in $\mathbf{w}^\alpha$, and the allocation w_i^α to that group is somehow known to the algorithm ahead of time. The constraint of α-fairness then implies that the discovery probability for each other group j in $\mathbf{w}^\alpha$ must satisfy $f_j \in [f_i - \alpha, f_i]$. This in turn implies upper and lower bounds on the feasible allocations w_j^α to group j. The algorithm is then simply a constrained greedy algorithm: subject to these implied constraints, it iteratively allocates units so as to maximize their marginal probability of reaching another candidate. Since the group i maximizing the discovery probability in $\mathbf{w}^\alpha$ and the corresponding allocation w_i^α are not known ahead of time, the algorithm simply iterates through each possible choice of i.

Algorithm 1 Computing an optimal fair allocation in the precision model

Input: α, C and $\mathcal{V}$.
Output: An optimal α-fair allocation $\mathbf{w}^\alpha$.

```
w^α ← 0⃗.
χ_max ← 0.
for i ∈ [G] do
    v ← 0⃗.
    for v_i ∈ {0, ... V} do
        Set v_i in v and compute f_i(v_i).
        ub_i ← v_i.
        lb_i ← v_i.
        for j ≠ i, j ∈ [G] do
            Update lb_j and ub_j using f_i(v_i), α and C_j.
            v_j ← lb_j.
        if Σ_{i∈[G]} v_i > V then
            continue.
        V_r = V − Σ_{i∈[G]} v_i
        for t = 1, ..., V_r do
            j* ∈ argmax_{j∈[G]} (T_j(v_j + 1) − T_j(v_j)) s.t. v_j < ub_j.
            v_{j*} ← v_{j*} + 1.
        χ(v) = Σ_{i∈[G]} Σ_{c=1}^{v_i} T_i(c).
        if χ(v) > χ_max then
            χ_max ← χ(v).
            w^α ← v.
return w^α.
```

Pseudocode is given in Algorithm 1. We prove that Algorithm 1 returns an optimal α-fair allocation in Theorem 1. We defer all the omitted proofs and details to the full version.

Theorem 1. *Algorithm 1 computes an optimal α-fair allocation for the precision model in time $O(\mathcal{G}\mathcal{V}(\mathcal{G}\mathcal{V} + M))$.*

3.3 Learning Fair Allocations Generally Requires Brute-Force Exploration

In Sections 3.1 and 3.2 we assumed the candidate distributions were known. When the candidate distributions are unknown, learning algorithms intending to converge to optimal α-fair allocations must learn a sufficient amount about the distributions in question to certify the fairness of the allocation they finally output. Because learners must deal with feedback in the censored observation model, this places constraints on how they can proceed. Unfortunately, as we show in this section, if candidate distributions are allowed to be worst-case, this will force a learner to engage in "brute-force exploration" — the iterative deployment of a large fraction of the resources to each subgroup in turn. This is formalized in Theorem 2.

Theorem 2. *Define $m^* = \max_{i \in [\mathcal{G}]} m_i$ to be the size of the largest group and assume $m_i > 6$ for all i and $\mathcal{G} \geq 2$. Let $\alpha \in [0, 1/(2m^*))$, $\delta \in (0, 1/2)$, and $\mathcal{A}$ be any learning algorithm for the precision model which runs for a finite number of rounds and outputs an allocation. Suppose that there is some group i for which $\mathcal{A}$ has not allocated at least $m_i/2$ units for at least $k \ln(1/\delta)/(\alpha m_i)$ rounds upon termination, where k is an absolute constant. Then there exists a candidate distribution such that, with probability at least δ, $\mathcal{A}$ outputs an allocation that is not α-fair.*

Sketch of the Proof. Let i denote a group in which $\mathcal{A}$ has not allocated at least $m_i/2$ units for at least $k \ln(1/\delta)/(\alpha m_i)$ rounds upon its termination and let $\mathbf{v}$ denote an arbitrary allocation. We will design two candidate distributions for group i which have true discovery probabilities that are at least 2α apart given v_i, but which are indistinguishable given the observations of the algorithm with probability at least δ. If the $\mathcal{A}$ cannot distinguish between C_i and C_i', it cannot distinguish between f_i and f_i', and thus cannot guarantee whether group i's discovery probability is indeed within α of every other group's discovery probability.

To design these candidate distributions, consider distributions C_i and C_i' which satisfy the following four conditions.

(1) C_i and C_i' agree on all values less than $m_i/2 - 2$.
(2) The total mass of both distributions below $m_i/2 - 2$ is $1 - 2\alpha m_i$.
(3) The remaining $2\alpha m_i$ mass of C_i is on the value $m_i/2 - 1$.
(4) The remaining $2\alpha m_i$ mass of C_i' is on the value m_i.

Distinguishing between C_i and C_i' requires at least one uncensored observation beyond $m_i/2 - 2$. However, conditioned on allocating at least $m_i/2$ units, the probability of observing an uncensored observation is at most $2\alpha m_i$. So to distinguish between C_i and C_i' with confidence $1-\delta$, and therefore to guarantee an α-fair allocation, a learning algorithm must allocate at least $m_i/2$ units to group i for $k \ln(1/\delta)/(\alpha m_i)$ rounds. □

Recall that we used m^* to denote the size of the largest group. When $m^* > 2\mathcal{V}$, then Theorem 2 implies that no algorithm can guarantee α-fairness for sufficiently small α. Moreover, even when $m^* \le 2\mathcal{V}$, Theorem 2 shows that in general, if we want algorithms that have provable guarantees for *arbitrary* candidate distributions, it is impossible to avoid something akin to brute-force search (recall that there is a trivial algorithm which simply allocates *all* resources to each group in turn, for sufficiently many rounds to approximately learn the CDF of the candidate distribution, and then solves the offline problem). In the next section, we circumvent this by giving an algorithm with provable guarantees, assuming that the candidate distributions have a known parametric form.

3.4 Poisson Distributions and Convergence of the MLE

In this section, we assume that all the candidate distributions have a particular and known *parametric form* but that the parameters of the these distributions are not known to the allocator. Concretely, we assume that the candidate distribution for each group is Poisson[4] (denoted by $C(\lambda)$) and write $\boldsymbol{\lambda}^* = (\lambda_1^*, \ldots, \lambda_{\mathcal{G}}^*)$ for the true underlying parameters of the candidate distributions; this choice appears justified, at least in the predictive policing application, as the candidate distributions in the Philadelphia Crime Incidents dataset are well-approximated by Poisson distributions (see Section 4 for further discussion). This assumption allows an algorithm to learn the tails of these distributions without needing to rely on brute-force search, thus circumventing the limitation given in Theorem 2. Indeed, we show that (a small variant of) the natural greedy algorithm incorporating these distributional assumptions converges to an optimal fair allocation.

For simplicity, we assume a parametric form on the marginal candidate distribution in each of the groups. We could have equivalently assumed that the candidates across groups are drawn from a multivariate Poisson distribution to highlight the (potential) correlation between candidates distributions. However, since for a given multivariate Poisson distribution the marginal distribution on each group is itself a Poisson distribution [15], we made our parametric assumption directly on these marginal distributions.

At a high level, in each round, our algorithm uses Algorithm 1 to calculate an optimal fair allocation with respect to the current maximum likelihood estimates of the group distributions; then, it uses the new observations it obtains from this allocation to refine these estimates for the next round. This is summarized in Algorithm 2. The algorithm differs from this pure greedy strategy in one respect, to overcome the following subtlety: there is a possibility that Algorithm 1, when operating on a preliminary estimate for the candidate distributions, will suggest sending zero units to some group, even when the optimal allocation for the true distributions sends some units to every group. Such a deployment would result in the algorithm receiving no feedback for the zero-allocated group that round. If this suggestion is followed and a lack of feedback is allowed to persist indefinitely, the algorithm's parameter estimate for the zero-allocated group will also stop updating — potentially at an incorrect value. In order to avoid this problem and continue making progress in learning, our algorithm chooses another allocation in this case. As we show, any allocation that allocates positive resources to all groups will suffice; in particular, our algorithm simply repeats the allocation from the previous round.

Algorithm 2 Learning an optimal fair allocation

Input: $\alpha, \mathcal{V}$ and T (total number of rounds).
Output: An allocation $\mathbf{v}^{T+1}$ and estimates to parameters $\{\lambda_i^T\}$.
  $\mathbf{v}^1 \leftarrow (\lfloor(\mathcal{V}/\mathcal{G})\rfloor, \ldots, \lfloor(\mathcal{V}/\mathcal{G})\rfloor)$.
  for rounds $t = 1, \ldots, T$ **do**
    if $\exists i$ such that $v_i^t == 0$ **then**
      $\mathbf{v}^t \leftarrow \mathbf{v}^{t-1}$.
    Observe $o_i^t = \min\{v_i^t, c_i^t\}$ for each group.
    for $i = 1, \ldots, \mathcal{G}$ **do**
      Update history $\mathbf{h}_i^{t+1}$ with o_i^t and v_i^t.
      $\hat{\lambda}_i^t \leftarrow \arg\max_{\lambda \in [\lambda_{\min}, \lambda_{\max}]} \hat{\mathcal{L}}(\mathbf{h}_i^{t+1}, \lambda)$.
    $\mathbf{v}^{t+1} \leftarrow$ Algorithm 1$(\alpha, \{C(\hat{\lambda}_i^t)\}, \mathcal{V})$.
  return $\mathbf{v}^{T+1}$ and $\{\lambda_i^T\}$.

Notice that Algorithm 2 chooses an allocation at every round which is fair with respect to its estimates of the parameters of the candidate distributions; hence, asymptotic convergence of its output to an *optimal α-fair* allocation follows directly from the convergence of the estimates to true parameters. However, we seek a stronger, *finite sample* guarantee, as stated in Theorem 3.

Theorem 3. *Let $\epsilon, \delta > 0$. Suppose that the candidate distributions are Poisson distributions with unknown parameters in the vector $\boldsymbol{\lambda}^*$, where $\boldsymbol{\lambda}^*$ lies in the known interval $[\lambda_{\min}, \lambda_{\max}]^{\mathcal{G}}$. Suppose we run Algorithm 2 for $t > \tilde{O}(\ln(\mathcal{G}/\delta)/(\eta(\epsilon))^2) \triangleq T_{\max}$ rounds, where $\eta(\cdot)$ is some distribution specific function to get an allocation $\hat{\boldsymbol{v}}$ and estimated parameters $\hat{\lambda}_i$ for all groups i. Then with probability at least $1 - \delta$*

1. *For all i in $[\mathcal{G}]$, $|\hat{\lambda}_i - \lambda_i^*| \le \epsilon$.*
2. *Let $D = \max_{i \in [\mathcal{G}]} D_{TV}(C(\lambda_i^*), C(\hat{\lambda}_i))$ where D_{TV} denotes the total variation distance between two distributions. Then $\hat{\boldsymbol{v}}$*
   - *is $(\alpha + 4D)$-fair.*
   - *has utility at most $4D\mathcal{G}\mathcal{V}$ smaller than the utility of an optimal $(\alpha - 4D)$-fair allocation i.e. $\chi(\hat{\boldsymbol{v}}) \ge \chi(\boldsymbol{w}^{\alpha-4D}) - 4D\mathcal{G}\mathcal{V}$.*

Remark 2. *Theorem 3 implies that in the limit, the allocation from Algorithm 2 will converge to an optimal α-fair allocation. As $t \to \infty$, $\hat{\lambda}_i \xrightarrow{p} \lambda_i^*$ for all i, meaning $D \to 0$ and more importantly, $\hat{\boldsymbol{v}}$ will be α-fair and optimal.*

To prove Theorem 3, we first show that *any* sequence of allocations selected by Algorithm 2 will eventually recover the true parameters. There are two conceptual difficulties here:

[4] To match our model, we would technically need to assume a *truncated* Poisson distribution to satisfy the bounded support condition. However, the distinction will not be important for the analysis, and so to minimize technical overhead, we perform the analysis assuming an untruncated Poisson.

the first is that standard convergence results typically leverage the assumption of *independence*, which does not hold in this case as Algorithm 2 computes *adaptive* allocations which depend on the allocations in previous rounds; the second is the censoring of the observations. Despite these difficulties, we give quantifiable rates with which the estimates converge to the true parameters. Next, we show that computing an optimal α-fair allocation using the estimated parameters will result in an allocation that is $(\alpha + 4D)$-fair with respect to the true candidate distributions where D denotes the maximum total variation distance between the true and estimated Poisson distributions across all groups. Finally, we show that this allocation also achieves a utility that is comparable to the utility of an optimal $(\alpha - 4D)$-fair allocation.

Remark 3. *Although we assumed Poisson distributions in this section, all our results hold for any single-parameter Lipschitz-continuous distribution whose parameter is drawn from a compact set. However, the convergence rate of Theorem 3 depends on the quantity $\eta(\epsilon)$ which* depends *on the family of distributions used to model the candidate distributions.*

4 EXPERIMENTS

In this section, we apply our allocation and learning algorithms for the precision model to the Philadelphia Crime Incidents dataset, and complement the theoretical convergence guarantee of Algorithm 2 to an optimal fair allocation with empirical evidence suggesting fast convergence in practice. We also study the trade-off between fairness and utility in the dataset.

4.1 Experimental Design

The Philadelphia Crime Incidents dataset[5] contains all the crimes reported to the Police Department's INCT system between 2006 and 2016. The crimes are divided into two types. Type I crimes include violent offenses such as aggravated assault, rape, and arson among others. Type II crimes include simple assault, prostitution, gambling and fraud. For simplicity, we aggregate all crime of both types, but in practice, a police department would of course treat different categories of crime differently. We note as a caveat that these crimes are *reported* and may not represent the entirety of *committed* crimes.

To create daily crime frequencies in Figure 1, we first calculate the daily counts of criminal incidents in each of the 21 geographical police districts in Philadelphia by grouping together all the crime reports with the same date; we then normalize these counts to get frequencies.[6] Each subfigure in Figure 1 represents a police district. The horizontal axis of the subfigure corresponds to the number of reported incidents in a day and the vertical axis represents the frequency of each number on the horizontal axis. These frequencies approximate the true (marginal) distributions of the number of reported crimes in each of the districts in Philadelphia. Therefore, throughout

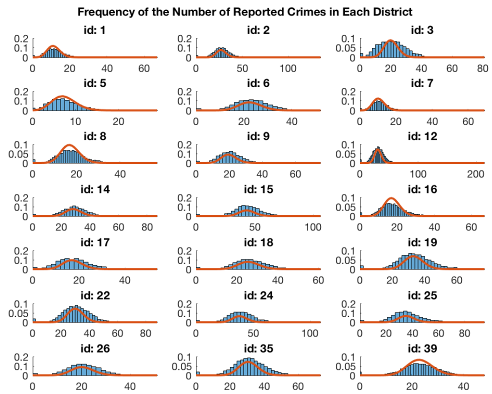

Figure 1: Frequencies of the number of reported crimes in each district in the Philadelphia Crime Incidents dataset. The red curves display the best Poisson fit to the data.

this section we take these frequencies as the *ground truth* candidate distributions for the number of reported incidents in each of the districts.

Figure 1 shows that crime distributions in different districts can be quite different; e.g., the average number of daily reported incidents in District 15 is 43.5, which is much higher than the average of 11.35 in District 1. Despite these differences, each of the crime distributions can be approximated well by a Poisson distribution. The red curves overlayed in each subfigure correspond to the Poisson distribution obtained via maximum likelihood estimation on data from that district. Throughout, we refer to such distributions as the *best Poisson fit* to the data. districts as the resource to be distributed, the ground truth crime frequencies as candidate distributions, and aim to maximize the sum of the number of crimes discovered under the precision model of discovery.

4.2 Results

We can quantify the extent to which fairness degrades utility in the dataset through a notion we call *Price of Fairness* (PoF henceforth). In particular, given the ground truth crime distributions and the precision model of discovery, for a fairness level α, we define $\mathrm{PoF}(\alpha) = \chi(\mathbf{w}^*)/\chi(\mathbf{w}^\alpha)$. The PoF is simply the ratio of the expected number of crimes discovered by an optimal allocation to the expected number of crimes discovered by an optimal α-fair allocation. Since $\chi(\mathbf{w}^*) \geq \chi(\mathbf{w}^\alpha)$ for all α, the PoF is at least one. Furthermore, the PoF is monotonically non-increasing in α. We can apply the algorithms given in Sections 3.1 and 3.2 respectively for computing optimal unconstrained, and optimal fair allocations with the with ground truth distributions as input and numerically compute the PoF. This is illustrated in Figure 2. The x axis corresponds to different α values and the y axis displays $1/\mathrm{PoF}(\alpha)$. Each curve corresponds to a different number of total police officers denoted by $\mathcal{V}$. Because feasible allocations must be integral, there can sometimes be no feasible α-fair allocation for small α. Since the PoF in these cases is infinite we instead opt to display the inverse, 1/PoF, which is always bounded in $[0, 1]$. Higher values of inverse PoF are more desirable.

[5]https://www.opendataphilly.org/dataset/crime-incidents accessed 2018-05-16.
[6]The current list of 21 districts can be found at https://www.phillypolice.com/districts-units/index.html. The dataset however contains 25 districts; we removed Districts 77 and 92, which correspond to the PHL airport and urban parks, as well as 4 and 23, which were dissolved in 2010.

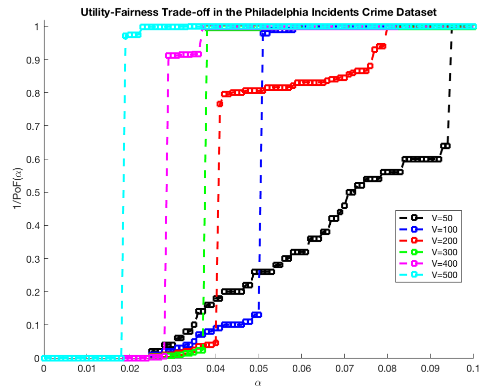

Figure 2: Inverse PoF plots for the Philadelphia Crime Incidents dataset. Smaller values indicate greater sacrifice in utility to meet the fairness constraint.

Figure 2 shows a diverse set of utility/fairness trade-offs depending on the number of available police officers. It also illustrates that the cost of fairness is rather low in most regimes. For example, in the worst case, with only 50 police officers (the black curve) (which is much smaller than the average number of daily reported crimes: 563.88), the inverse PoF is 1 for $\alpha \geq 0.1$, which corresponds to a 10% difference in the discovery probability across districts. When we increase the number of available police officers to 400 (the magenta curve), tolerating only a 4% difference in the discovery probability across districts is sufficient to guarantee no loss in the utility. Figure 2 also shows that for any fixed α, the inverse PoF(α) tends to increase as the number of police increases (i.e. the cost of fairness decreases).[7] This captures the intuition that fairness becomes a less costly constraint when resources are in greater supply. Finally, we observe a thresholding phenomenon in Figure 2; in each curve, increasing α beyond a threshold will significantly increase the inverse PoF. This is due to discretization effects, since only integral allocations are feasible.

We next turn into analyzing the performance of Algorithm 2 in practice. We run the algorithm instantiated to fit Poisson distributions, but use observations from the ground truth distribution at each round. As we have shown in Figure 1,the ground truth is well approximated by a Poisson distribution.

We measure the performance of Algorithm 2 as follows. First, we fix a police budget $\mathcal{V}$ and unfairness budget α and run Algorithm 2 for 2000 rounds using the dataset as the ground truth. That is, we simulate each round's crime count realizations in each of the districts as being sampled from the ground truth distributions, and return censored observations under the precision model to Algorithm 2 according to the algorithm's allocations and the drawn realizations. The algorithm returns an allocation after termination and we can measure the expected number of crimes discovered and fairness violation (the maximum difference in discovery probabilities over all pairs of districts) of the returned allocation using the ground truth distributions. Varying α while fixing $\mathcal{V}$ allows us to trace out the Pareto frontier of the utility/fairness trade-off for a fixed police budget. Similarly, for any fixed $\mathcal{V}$ and α, we can run Algorithm 1 (the offline algorithm for computing an optimal fair allocation) with the ground truth distributions as input and trace out a Pareto curve by varying α. We refer to these two Pareto curves by the *learned* and *optimal* Pareto curves, respectively.[8] To measure the performance of Algorithm 2, we can compare the learned and optimal Pareto curves.

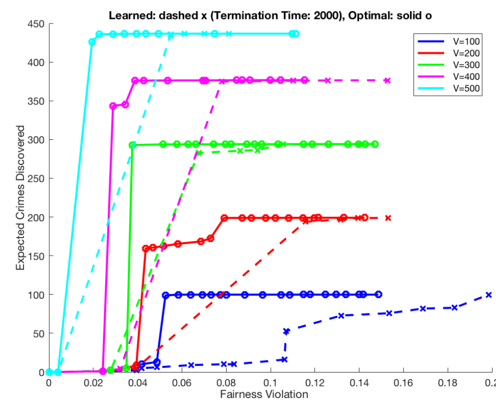

Figure 3: Pareto frontier of expected crimes discovered versus fairness violation.

In Figure 3, each curve corresponds to a police budget. The x and y axes represent the expected number of crimes discovered and fairness violation for allocations on the Pareto frontier, respectively. In our simulations we varied α between 0 and 0.15. For each police budget $\mathcal{V}$, the 'x' s connected by the dashed lines show the learning Pareto frontier. Similarly, the circles connected by solid lines show the optimal Pareto frontier. We point out that while it is possible for the fairness violations in the learned Pareto curves to be higher than the level of α set as an input to Algorithm 2, the fairness violations in the optimal Pareto curves are always bounded by α.

The disparity between the optimal and learned Pareto curves are due to the fact that the learning algorithm has not yet fully converged. This can be attributed to the large number of censored observations received by Algorithm 2, which are significantly less informative than uncensored observations. Censoring happens frequently because the number of police used in every case plotted is less than the daily average of 563.88 crimes across all the districts in the dataset — so it is unavoidable that in any allocation, there will be significant censoring in at least some districts.

Figure 3 shows that while the learning curves are dominated by the optimal curves, the performance of the learning

[7]There are exceptions to this observation – for example, in the regime when α is between 0.03 and 0.04, the inverse PoF decreases as $\mathcal{V}$ increases from 100 to 200. This occurs because only integral allocations are feasible, so achieving a particular fairness level may require leaving some resources unallocated until significantly more resources become available; increasing $\mathcal{V}$ in this regime improves the utility of an optimal allocation while leaving the utility of an optimal fair allocation unchanged.

[8]We can also generate *fitted* Pareto curves using best Poisson fit distributions instead of the ground truth distributions. These curves look very similar to the optimal Pareto curves.

algorithm approaches the performance of the offline optimal allocation as $\mathcal{V}$ increases. Again, this is because increasing $\mathcal{V}$ generally decreases the frequency of censoring. We study the $\mathcal{V} = 500$ regime in more detail, to explore the empirical rate of convergence. In Figure 4, we study the round by round performance of the allocation computed by Algorithm 2 in a single run with the choice of $\mathcal{V} = 500$ and $\alpha = 0.05$.

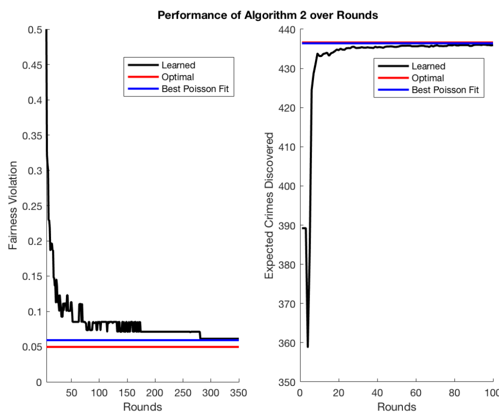

Figure 4: The per round expected number of crimes discovered and fairness violation of Algorithm 2. $\mathcal{V} = 500$ and $\alpha = 0.05$.

In Figure 4, the x axis labels progression of rounds of the algorithm. The y axis measures the fairness violation (left) and expected number of crimes discovered (right) of the allocation deployed by the algorithm, as measured with respect to the ground truth distributions. The black curves represent Algorithm 2. For comparison we also show the same quantities for an offline optimal fair allocation as computed with respect to the ground truth (red line), and an offline optimal fair allocation as computed with respect to the best Poisson fit to the ground truth (blue line). Note that in the limit, the allocations chosen by Algorithm 2 are guaranteed to converge to the blue baselines — but not the red baseline, because the algorithm is itself learning a Poisson approximation to the ground truth. The disparity between the red and blue lines quantifies the degradation in performance due to using Poisson approximations, rather than due to non-convergence of the learning process.

Figure 4 shows that Algorithm 2 converges to the Poisson approximation baseline well before the termination time of 2000, and substantially before the convergence bound guaranteed by our theory. Examining the estimated Poisson parameters used internally by Algorithm 2 reveals that although the allocation has converged to an optimal fair allocation, the estimated parameters have not yet converged to the parameters of the best Poisson fit in any of the districts. In particular, Algorithm 2 systematically underestimates the parameters in all of the districts: the correlation coefficient between the true and estimated parameters is 0.9975.

We see also in Figure 4 that convergence to the optimum expected number of discovered crimes occurs more quickly than convergence to the target fairness violation level. This is also apparent in Figure 3 where the learning and optimal Pareto curves are generally similar in terms of the maximum number of crimes discovered, while the fairness violations are higher in the learning curves.

5 THE RANDOM DISCOVERY MODEL

Finally, we consider the *random model* of discovery. In the random model, when v_i units are allocated to a group with c_i candidates, the number of discovered candidates is a random variable corresponding to the number of candidates that appear in a uniformly random sample of v_i individuals from a group of size m_i. Equivalently, when v_i units are allocated to a group of size m_i with c_i candidates, the number of candidates discovered by $\text{disc}(\cdot)$ is a random variable $\text{disc}(v_i, c_i) \triangleq o_i$, where o_i is drawn from the hypergeometric distribution with parameters m_i, c_i and v_i. Furthermore, the expected number of candidates discovered when allocating v_i units to group i is $\mathbb{E}[\text{disc}(v_i, c_i)] = v_i\, \mathbb{E}[c_i]/m_i$.

For simplicity, throughout this section, we assume $m_i \geq \mathcal{V}$ for all i. This assumption can be completely relaxed.Moreover, let $\mu_i = \mathbb{E}[c_i]/m_i$ denote the expected fraction of candidates in group i. Without loss of generality, for the rest of this section, we assume $\mu_1 \geq \mu_2 \geq \ldots \geq \mu_{\mathcal{G}}$.

5.1 Optimal Allocation

In this section, we characterize optimal allocations. Note that the expected number of candidates discovered by the allocation choice $v_i \leq m_i$ in group i is simply $v_i \mu_i$. This suggests a simple algorithm to compute $\mathbf{w}^*$: allocating every unit of the resource to group 1. More generally, let $\mathcal{G}^* = \{i \mid \mu_i = \mu_1\}$ denote the subset of groups with the highest expected number of candidates. An allocation is optimal if and only if it only allocates *all* resources to groups in $\mathcal{G}^*$.

5.2 Properties of Fair Allocations

We next discuss the properties of fair allocations in the random discovery model. First, we point out that the discovery probability can be simplified as

$$f_i(v_i) = \mathop{\mathbb{E}}_{c_i \sim C_i}\left[\frac{c_i v_i / m_i}{c_i}\right] = \frac{v_i}{m_i}.$$

So an allocation is α-fair in the random model if $|v_i/m_i - v_j/m_j| \leq \alpha$ for all groups i and j. Therefore, fair allocations (roughly) distribute resources in proportion to the size of the groups, essentially ignoring the candidate distributions within each group.

5.3 Price of Fairness

Recall that PoF quantifies the extent to which constraining the allocation to satisfy α-fairness degrades utility. While in Section 4 we study the PoF on the Philadelphia Crime Incidents dataset, we can define a worst-case variant as follows.

Definition 2. *Fix the random model of crime discovery and let $\alpha \in [0, 1]$. We define the PoF as*

$$PoF(\alpha) = \max_{C} \frac{\chi(\boldsymbol{w}^*, C)}{\chi(\boldsymbol{w}^{\alpha}, C)}.$$

where C ranges over all possible candidate distributions.

We can fully characterize this worst-case PoF in the random discovery model.

THEOREM 4. *The PoF in the random discovery model is*

$$PoF(\alpha) = \begin{cases} 1, & \frac{\mathcal{V}}{m_1} \leq \alpha, \\ \frac{M}{m_1+\alpha(M-m_1)}, & \frac{\mathcal{V}}{m_1} > \alpha. \end{cases}$$

The PoF in the random model can be as high as M/m_1 in the worst case. If all groups are identically sized, this grows linearly with the number of groups.

6 CONCLUSION AND FUTURE DIRECTIONS

Our presentation of allocative fairness provides a family of fairness definitions, modularly parameterized by a "discovery model". What counts as "fair" depends a great deal on the choice of discovery model, which makes explicit what would otherwise be unstated assumptions about the process of tasks like policing. The random and precision models of discovery studied in this paper represent two extreme points of a spectrum. In the predictive policing setting, the random model of discovery assumes that officers have no advantage over random guessing when stopping individuals for further inspection. The precision model assumes they can oracularly determine offenders, and stop only them. An interesting direction for future work is to study discovery models that lie in between these two.

We have also made a number of simplifying assumptions that could be relaxed. For example, we assumed the candidate distributions are *stationary* — fixed independently of the actions of the algorithm. Of course, the deployment of police officers can *change* crime distributions. Modeling this kind of dynamics, and designing learning algorithms that perform well in such dynamic settings would be interesting. Finally, we have assumed that the same discovery model applies to all groups. One friction to fairness that one might reasonably conjecture is that the discovery model may differ between groups — being closer to the precision model for one group, and closer to the random model for another. We leave the study of these extensions to future work.

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