

Задание 1.

Задание 1.1

$$f_1(x) = e^x, f_2(x) = 1, f_3(x) = x+1, f_4(x) = x - e^x.$$

$$f_3(x) = f_4(x) + f_1(x) + f_2(x) = x - e^x + e^x + 1 = x + 1$$

Ответ: лин. зависимое

Задание 1.2

$$f_1(x) = 2, f_2(x) = x, f_3(x) = x^2, f_4(x) = (x+1)^2$$

$$f_4(x) = f_3(x) + 2f_2(x) + 0.5f_1(x) = x^2 + 2x + 1 = (x+1)^2$$

Ответ: лин. зависимое

Задание 1.3

$$x = (2; 3; 5) \in \mathbb{R}^3; b_1 = (0; 0; 10); b_2 = (2; 0; 0); b_3 = (0; 1; 0).$$

$$x = 0.5b_1 + b_2 + 3b_3$$

Ответ: (0,5; 1; 3).

Задание 1.4

$$3x^2 - 2x + 2 \in \mathbb{R}^3[x]$$

$$\text{a)} 1; x; x^2;$$

$$3x^2 - 2x + 2 = 2 \cdot 1 + (-2) \cdot x + 3 \cdot x^2 \Rightarrow (2; -2; 3)$$

$$\text{б)} x^2; x-1; 1;$$

$$3x^2 - 2x + 2 = 3 \cdot x^2 + (-2) \cdot (x-1) + 0 \cdot 1 \Rightarrow (3; -2; 0)$$

Ответ: a) (2; -2; 3); б) (3; -2; 0).

Задание 1.5

$$\text{а)} (0; a; b) + (0; c; d) = (0; a+c; b+d)$$

$$(0; a; b) + (0; 0; c) = (0; a; b+c)$$

$$(0; a; b) + (0; 0; d) = (c; a; b+d) \quad \begin{matrix} \text{не определено} \\ \text{не лин. независимо} \end{matrix}$$

$$(0; 0; a) + (0; 0; b) = (0; 0; a+b)$$

$$(0; 0; a) + (b; 0; c) = (b; 0; a+c)$$

$$(a; 0; b) + (c; 0; d) = (a+b; 0; c+d)$$

\Rightarrow не лин. независимо

$$\text{в)} \bar{a} = d_1 \bar{u}_1 + d_2 \bar{u}_2 + \dots + d_n \bar{u}_n$$

$$\bar{B} = \mu_1 \bar{u}_1 + \mu_2 \bar{u}_2 + \dots + \mu_n \bar{u}_n$$

$$\bar{a} + \bar{B} = (d_1 + \mu_1) \bar{u}_1 + (d_2 + \mu_2) \bar{u}_2 + \dots + (d_n + \mu_n) \bar{u}_n$$

$$\bar{a} \cdot \bar{\mu} = \mu d_1 \bar{u}_1 + \mu d_2 \bar{u}_2 + \dots + \mu d_n \bar{u}_n$$

\Rightarrow обл. лин. независимы.

Ответ: а) не лин. независимо; в) лин. независимо.

Задание 2.1

$$\text{а)} x = (0; -3; 6); \bar{y} = (-4; 7; 9);$$

$$(x, y) = 0 \cdot (-4) + (-3) \cdot 7 + 6 \cdot 9 = 0 - 21 + 54 = 33.$$

$$\text{б)} \bar{x} = (7; -4; 0; 1); \bar{y} = (-3; 1; 11; 2)$$

$$(x, y) = 7 \cdot (-3) + (-4) \cdot 1 + 0 \cdot 11 + 1 \cdot 2 = -21 - 4 + 0 + 2 = -23.$$

Ответ: а) 33; б) -23.

Задание 2.2

$$a = (4; 2; 4); B = (12; 3; 4)$$

$$\text{L1: } \|a\|_1 = |4| + |2| + |4| = 10;$$

$$\|B\|_1 = |12| + |3| + |4| = 19;$$

$$\text{L2: } |a| = \sqrt{4^2 + 2^2 + 4^2} = \sqrt{36} = 6;$$

$$|B| = \sqrt{12^2 + 3^2 + 4^2} = \sqrt{169} = 13;$$

$$(a, B) = 4 \cdot 12 + 2 \cdot 3 + 4 \cdot 4 = 48 + 6 + 16 = 70;$$

$$\cos(\bar{a} \wedge \bar{B}) = \frac{(a, B)}{|a| \cdot |B|} = \frac{70}{6 \cdot 13} = \frac{70}{78} \approx 0,90;$$

$$(\bar{a} \wedge \bar{B}) = \arccos 0,9 \approx 0,46.$$

Ответ: L1: 10, 19; L2: 6, 13, $\approx 0,46$.

Задание 2.3

$$\text{в)} \bar{x} = (2; 0); \bar{y} = (5; 0); x = 3; \bar{x}_1 = (2; 0); \bar{x}_2 = (0; 4);$$

$$|\bar{x}| = 2; |\bar{y}| = 5; |\bar{x}_1| = 2; |\bar{x}_2| = 4; (x, y) = 2 \cdot 5 + 0 \cdot 0 = 10;$$

$$\bar{x} \cdot \bar{y} = (6; 0); |\bar{x}| = 6; (\bar{x}_1 \wedge \bar{x}_2) = (2; 4); (x, y) = 6 \cdot 5 + 0 \cdot 0 = 30;$$

$$|\bar{x}_1 + \bar{x}_2| = \sqrt{2^2 + 4^2} = \sqrt{20}; (x_1 + x_2, y) = 2 \cdot 5 + 0 \cdot 4 = 10; (x_1, y) = 5 \cdot 0 + 4 \cdot 0 = 0$$