

Домашнее задание 4.

Задание №1 (пространение)

$$\det A_3 = \begin{vmatrix} 2 & -1 & 10 \\ 1 & 1 & -2 \\ 2 & 4 & 1 \end{vmatrix} = 2 \cdot \begin{vmatrix} 1 & -2 \\ 4 & 1 \end{vmatrix} + 5 + 10 \cdot 2 = 43;$$

$$x_1 = \frac{86}{43} = 2; x_2 = -\frac{43}{43} = -1; x_3 = \frac{43}{43} = 1;$$

Ответ: $x_1 = 2, x_2 = -1, x_3 = 1$.

Задание №2

$$a) \begin{pmatrix} 1 & 2 & 4 \\ 2 & 9 & 12 \\ 3 & 26 & 80 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 4 \\ 0 & 5 & 4 \\ 0 & 20 & 18 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 4 \\ 0 & 5 & 4 \\ 0 & 0 & 2 \end{pmatrix} = U; L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{pmatrix}$$

$$b) \begin{pmatrix} 1 & 1 & 2 & 4 \\ 2 & 5 & 8 & 9 \\ 3 & 18 & 29 & 18 \\ 4 & 22 & 53 & 33 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 2 & 4 \\ 0 & 3 & 4 & 1 \\ 0 & 15 & 23 & 6 \\ 0 & 18 & 45 & 17 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 2 & 4 \\ 0 & 3 & 4 & 1 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 21 & 11 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 1 & 2 & 4 \\ 0 & 3 & 4 & 1 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 4 \end{pmatrix} = U; L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 5 & 1 & 0 \\ 4 & 6 & 7 & 1 \end{pmatrix}.$$

Задание №3

$$\begin{cases} 2x_1 + x_2 + 3x_3 = 1; \\ 11x_1 + 7x_2 + 5x_3 = -6; \\ 9x_1 + 8x_2 + 4x_3 = -5; \end{cases}$$

$$\begin{pmatrix} 2 & 1 & 3 \\ 11 & 7 & 5 \\ 9 & 8 & 4 \end{pmatrix} \sim \begin{pmatrix} 2 & 1 & 3 \\ 0 & 1,5 & -11,5 \\ 0 & 3,5 & -9,5 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 2 & 1 & 3 \\ 0 & 1,5 & -11,5 \\ 0 & 0 & \frac{52}{3} \end{pmatrix} = U; L = \begin{pmatrix} 1 & 0 & 0 \\ 5,5 & 1 & 0 \\ 4,5 & \frac{7}{3} & 0 \end{pmatrix};$$

$$Ly = B;$$

$$\begin{cases} y_1 = 1; \\ 5y_1 + y_2 = -6; \\ 4,5y_1 + \frac{7}{3}y_2 + y_3 = -5; \end{cases} \Rightarrow \begin{cases} y_1 = 1; \\ y_2 = -11,5; \\ y_3 = -5 - 4,5 + \frac{7}{3} \cdot \frac{23}{2}; \end{cases} \Rightarrow \begin{cases} y_1 = 1; \\ y_2 = -11,5; \\ y_3 = \frac{52}{3}; \end{cases}$$

$$Lx = B;$$

$$\begin{cases} 2x_1 + x_2 + 3x_3 = 1; \\ 1,5x_2 - 11,5x_3 = -11,5; \\ \frac{52}{3}x_3 = \frac{52}{3}; \end{cases} \Rightarrow \begin{cases} x_3 = 1; \\ x_2 = 0; \\ x_1 = -1; \end{cases}$$

Ответ: $x_1 = -1, x_2 = 0, x_3 = 1$.

Задание №4.

$$\begin{cases} 81x_1 - 45x_2 + 45x_3 = 531; \\ -45x_1 + 50x_2 - 15x_3 = -460; \\ 45x_1 - 15x_2 + 38x_3 = 193; \end{cases}$$

$$l_{11} = \sqrt{a_{11}} = \sqrt{81} = 9;$$

$$l_{21} = \frac{a_{21}}{l_{11}} = -\frac{45}{9} = -5;$$

$$l_{31} = \frac{a_{31}}{l_{11}} = \frac{45}{9} = 5;$$

$$l_{22} = \sqrt{a_{22} - l_{21}^2} = \sqrt{50 - 25} = 5;$$

$$l_{32} = \frac{1}{l_{22}} (a_{32} - l_{21}l_{31}) = \frac{1}{5} \cdot (-15 \cdot 5 \cdot 5) = 2;$$

$$l_{33} = \sqrt{a_{33} - l_{31}^2 - l_{32}^2} = \sqrt{38 - 25 - 4} = 3;$$

$$L = \begin{pmatrix} 9 & 0 & 0 \\ -5 & 5 & 0 \\ 5 & 2 & 3 \end{pmatrix}, L^T = \begin{pmatrix} 9 & -5 & 5 \\ 0 & 5 & 2 \\ 0 & 0 & 3 \end{pmatrix};$$

$$Ly = B;$$

$$\begin{cases} 9y_1 = 531; \\ -5y_1 + 5y_2 = -460; \\ 5y_1 + 2y_2 + 3y_3 = 193; \end{cases} \Rightarrow \begin{cases} y_1 = 59; \\ y_2 = -33; \\ y_3 = -12; \end{cases}$$

$$Lx = Y;$$

$$\begin{cases} 9x_1 - 5x_2 + 5x_3 = 59; \\ 5x_2 + 2x_3 = -33; \\ 3x_3 = -12; \end{cases} \Rightarrow \begin{cases} x_3 = -4; \\ x_2 = -5; \\ x_1 = 6; \end{cases}$$

Ответ: $x_1 = 6, x_2 = -5, x_3 = -4$.