

Danejene zadanie 6.

Zadanie 6.1.

a) $(\sin x \cdot \cos x)' = \cos x \cdot \cos x - \sin x \cdot \sin x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1;$

b) $(\ln(2x+1)^3)' = (3 \ln(2x+1))' = \frac{3 \cdot 2}{2x+1} = \frac{6}{2x+1};$

c) $(\sqrt{\sin^2(\ln(x^3))})' = (\left| \sin(\ln(x^3)) \right|)' =$
 $= \frac{\sin(\ln(x^3))}{\left| \sin(\ln(x^3)) \right|} \cdot \cos(\ln(x^3)) \cdot \frac{3}{x} =$
 $= \frac{3 \sin(\ln(x^3)) \cdot \cos(\ln(x^3))}{x \cdot \left| \sin(\ln(x^3)) \right|}; \quad x \neq 0, 1.$

d) $\left(\frac{x^4}{\ln x}\right)' = \frac{4x^3 \ln x - \frac{x^4}{x}}{(\ln x)^2} = \frac{x^3(4 \ln x - 1)}{(\ln x)^2};$

Zadanie 6.2.

$f(x) = \cos(x^2 + 3x)$, $x_0 = \sqrt{\pi}$; $f'(x_0) = ?$

$f'(x) = (\cos(x^2 + 3x))' = -\sin(x^2 + 3x) \cdot (2x + 3);$

$f'(x_0) = -\sin(\pi + 3\sqrt{\pi}) \cdot (2\sqrt{\pi} + 3) = \sin(3\sqrt{\pi}) \cdot (2\sqrt{\pi} + 3).$

$\approx -5,38.$

Odmien: $-5,38.$

Zadanie 6.3.

$f(x) = \frac{x^3 - x^2 - x - 1}{1 + 2x + 3x^2 - 4x^3}$; $x_0 = 0$; $f'(x_0) = ?$

$f'(x) = \left(\frac{x^3 - x^2 - x - 1}{1 + 2x + 3x^2 - 4x^3} \right)' = \frac{(3x^2 - 2x - 1)(1 + 2x + 3x^2 - 4x^3) - (x^3 - x^2 - x - 1)(2 + 6x - 12x^2)}{(1 + 2x + 3x^2 - 4x^3)^2}$

$$= \frac{(x^3 - x^2 - x - 1)(2 + 6x - 12x^2)}{(1 + 2x + 3x^2 - 4x^3)^2}.$$

$f'(0) = \frac{(-1) \cdot 4 - (-1) \cdot 2}{1^2} = \frac{-4 + 2}{1} = 1.$

Odmien: 1.

Zadanie 6.4.

$f(x) = \sqrt{3x} \cdot \ln x$; $x_0 = 1$; $\alpha = ?$

$\tan \alpha = f'(x_0) \Rightarrow \alpha = \arctg(f'(x_0)).$

$$f'(x) = (\sqrt{3x} \ln x)' = \frac{\overset{3x}{\cancel{3}} \ln x}{2\sqrt{3x}} + \frac{\sqrt{3x}}{x} =$$

$$= \frac{3\sqrt{3x} \ln x + 6\sqrt{3x}}{6x} = \frac{3\sqrt{3x}(\ln x + 2)}{6x} = \frac{\sqrt{3x}(\ln x + 2)}{2x};$$

$$f'(x_0) = f'(1) = \frac{\sqrt{3 \cdot 1}(\ln 1 + 2)}{2 \cdot 1} = \frac{\sqrt{3} \cdot 2}{2} = \sqrt{3};$$

$\alpha = \arctg \sqrt{3} = 60^\circ.$

Odmien: $60^\circ.$