

Домашнее задание 9.

Задание 9.1

$$U = 3 - 8x + 6y, \text{ если } x^2 + y^2 = 36.$$

$$L(\lambda; x; y) = 3 - 8x + 6y + \lambda(x^2 + y^2 - 36);$$

$$\begin{cases} L'_x = -8 + \lambda \cdot 2x = 0; \\ L'_y = 6 + \lambda \cdot 2y = 0; \\ L'_{\lambda} = x^2 + y^2 - 36 = 0; \end{cases} \Rightarrow \begin{cases} x = \frac{4}{\lambda}; \\ y = -\frac{3}{\lambda}; \\ 25/\lambda^2 = 36; \end{cases} \Rightarrow \begin{cases} x = \frac{4}{\lambda}; \\ y = -\frac{3}{\lambda}; \\ \lambda^2 = \frac{25}{36}; \end{cases}$$

$$\left(\frac{5}{6}; 4,8; -3,6 \right), \left(-\frac{5}{6}; -4,8; 3,6 \right).$$

$$L''_{xx} = 2\lambda; \quad L''_{yy} = 2\lambda; \quad L''_{x\lambda} = 0; \quad L''_{xy} = 0; \quad L''_{x\lambda} = 2x; \quad L''_{yx} = 2y;$$

$$\begin{vmatrix} 0 & 2x & 2y \\ 2x & 2\lambda & 0 \\ 2y & 0 & 2\lambda \end{vmatrix} = 0 + 0 - 8y^2\lambda - 8x^2\lambda - 0 = -8\lambda(x^2 + y^2);$$

$$\Rightarrow \left(\frac{5}{6}; 4,8; -3,6 \right) - \min; \left(-\frac{5}{6}; -4,8; 3,6 \right) - \max.$$

Задание 9.2

$$U = 2x^2 + 12xy + 32y^2 + 15, \text{ если } x^2 + 16y^2 = 64.$$

$$L(\lambda; x; y) = 2x^2 + 12xy + 32y^2 + 15 + \lambda(x^2 + 16y^2 - 64);$$

$$\begin{cases} L'_x = 4x + 12y + \lambda \cdot 2x = 0; \\ L'_y = 12x + 64y + \lambda \cdot 32y = 0; \\ L'_{\lambda} = x^2 + 16y^2 - 64; \end{cases} \Rightarrow \begin{cases} (2+\lambda)x = -6y; \\ (2+\lambda)y = -\frac{3x}{8}; \\ x^2 + 16y^2 - 64 = 0; \end{cases} \Rightarrow \begin{cases} \frac{-6y}{x} = -\frac{3x}{8y}; \\ \lambda = -\frac{64}{x} - 2; \\ x^2 + 16y^2 - 64 = 0; \end{cases}$$

$$\begin{cases} x^2 = 16y^2; \\ x = -\frac{64}{x} - 2; \\ x^2 + 16y^2 - 64 = 0; \end{cases} \Rightarrow \begin{cases} x_1 = 4y; \\ x_2 = -4y; \\ \lambda_1 = -3,5; \\ \lambda_2 = -0,5; \end{cases} \Rightarrow \begin{cases} x_1 = 4y; \\ x_2 = -4y; \\ \lambda_1 = -3,5; \\ \lambda_2 = -0,5; \end{cases} \Rightarrow \begin{cases} y^2 = 2; \\ 16y^2 + 16y^2 - 64 = 0; \end{cases}$$

$$\Rightarrow (-3,5; 4\sqrt{2}; \sqrt{2}), (-3,5; -4\sqrt{2}; -\sqrt{2}), (-0,5; -4\sqrt{2}; \sqrt{2}), (-0,5; 4\sqrt{2}; -\sqrt{2}).$$

$$L''_{xx} = 4 + 2\lambda; \quad L''_{yy} = 64 + 32\lambda; \quad L''_{x\lambda} = 0; \quad L''_{xy} = 12; \quad L''_{x\lambda} = 2x; \quad L''_{yx} = 32y;$$

$$\begin{vmatrix} 0 & 2x & 32y \\ 2x & 4+2\lambda & 12 \\ 32y & 12 & 64+32\lambda \end{vmatrix} = 0 + 768xy + 768xy - 2048y^2(2+\lambda) - 128x^2(2+\lambda) - 0 = 1536xy - 128(2+\lambda)(x^2 + 16y^2) = 1536xy - 8192(2+\lambda);$$

$$(-0,5; -4\sqrt{2}; \sqrt{2}), (-0,5; 4\sqrt{2}; -\sqrt{2}) - \min$$

$$(-3,5; 4\sqrt{2}; \sqrt{2}), (-3,5; -4\sqrt{2}; -\sqrt{2}) - \max$$

Задание 9.3

$$U = x^2 + y^2 + z^2, \quad \vec{C}(-9; 8; -12), \quad L(8; -12; 9).$$

$$U_x = 2x;$$

$$U_y = 2y; \quad \text{grad } U|_M = (16; -24; 18)$$

$$U_z = 2z;$$

$$|\vec{C}| = \sqrt{81 + 64 + 144} = 17;$$

$$\vec{c}_0 = \frac{\vec{C}}{|\vec{C}|} = \left(-\frac{9}{17}; \frac{8}{17}; -\frac{12}{17} \right);$$

$$\frac{\partial U}{\partial C}|_M = 16 \cdot \left(-\frac{9}{17} \right) - 24 \cdot \frac{8}{17} + 18 \cdot \left(-\frac{12}{17} \right) = \frac{-144 - 192 - 216}{17} = -\frac{552}{17}.$$

Задание 9.4

$$U = e^{x^2 + y^2 + z^2}, \quad J(4; -13; -16), \quad L(-16; 4; -13).$$

$$U_x = 2x \cdot e^{x^2 + y^2 + z^2},$$

$$U_y = 2y \cdot e^{x^2 + y^2 + z^2}; \quad \text{grad } U|_L = (-32e^{441}; 8e^{441}; -26e^{441})$$

$$U_z = 2z \cdot e^{x^2 + y^2 + z^2},$$

$$|\vec{C}| = \sqrt{16 + 169 + 256} = 21;$$

$$\vec{c}_0 = \frac{\vec{C}}{|\vec{C}|} = \left(\frac{4}{21}; -\frac{13}{21}; -\frac{16}{21} \right);$$

$$\frac{\partial U}{\partial d}|_L = -\frac{128}{21}e^{441} - \frac{104}{21}e^{441} + \frac{416}{21}e^{441} = \frac{184}{21}e^{441};$$