Задание 11.6

$f(x) = x^2, \ x \in [-\pi; \pi]$

1) $a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} x^2 dx = \frac{1}{2\pi}\cdot\frac{x^3}{3}\Big|_{-\pi}^{\pi} = \frac{\pi^3}{6\pi} + \frac{\pi^3}{6\pi} = \frac{\pi^2}{3};$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cos nx\, dx = \left(\frac{1}{\pi}\cdot\frac{x^2 \sin nx}{n} - \frac{1}{\pi}\int\frac{2x \sin nx\, dx}{n}\right)\Big|_{-\pi}^{\pi} =$

$u_1 = x^2; \quad du_1 = 2x\,dx; \quad \begin{cases} u_2 = x; & du_2 = dx; \\ dv_2 = \sin nx\,dx; & v_2 = -\frac{\cos nx}{n}; \end{cases}$

$dv_1 = \cos nx\,dx; \quad v_1 = \frac{\sin nx}{n};$

$= \left(\frac{x^2 \sin nx}{\pi n} - \frac{2}{\pi n}\int x \sin nx\,dx\right)\Big|_{-\pi}^{\pi} = \left(\frac{x^2 \sin nx}{\pi n} - \frac{2}{\pi n}\left(-\frac{x\cos nx}{n} + \right.\right.$

$\left.\left. + \int\frac{\cos nx\,dx}{n}\right)\right)\Big|_{-\pi}^{\pi} = \left(\frac{x^2 \sin nx}{\pi n} + \frac{2x\cos nx}{\pi n^2} - \frac{2\sin nx}{\pi n^3}\right)\Big|_{-\pi}^{\pi} =$

$= \frac{\pi^2 \sin \pi n}{\pi n} + \frac{2\pi \cos \pi n}{\pi n^2} - \frac{2\sin\pi n}{\pi n^3} - \frac{\pi^2 \sin(-\pi n)}{\pi n} + \frac{2\pi\cos(-\pi n)}{\pi n^2} +$

$+ \frac{2\sin(-\pi n)}{\pi n^3} = 0 + \frac{2\cos\pi n}{n^2} - 0 - 0 + \frac{2\cos\pi n}{n^2} + 0 = \frac{4\cos\pi n}{n^2};$

$f(x) = x^2$ – четная функция, т.к. $f(-x) = x^2$.

$\Rightarrow f(x) \sim \frac{\pi^2}{3} + \sum_{n=1}^{\infty}\frac{4\cos\pi n}{n^2}\cdot\cos nx = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} 4\cdot\frac{(-1)^n}{n^2}\cdot\cos nx;$

2) $S_1(x) = \frac{\pi^2}{3} + (-4\cos x) = \frac{\pi^2}{3} - 4\cos x;$

$S_2(x) = \frac{\pi^2}{3} - 4\cos x + \cos 2x;$

$S_3(x) = \frac{\pi^2}{3} - 4\cos x + \cos 2x - \frac{4}{9}\cos 3x.$