Домашнее задание 5.

Задание 5.4

c) $\lim\limits_{x\to\infty}\left(\frac{x+3}{x}\right)^{4x+1} = \lim\limits_{x\to\infty}\left(1+\frac{3}{x}\right)^{4x+1} = \left\{\begin{matrix} U(x) = 1+\frac{3}{x}; \\ V(x) = 4x+1; \end{matrix}\right\} =$

$= e^{\lim\limits_{x\to\infty}(1+\frac{3}{x}-1)(4x+1)} = e^{\lim\limits_{x\to\infty}(12+\frac{3}{x})} = e^{12}.$

Задание 5.5

d) $\lim\limits_{x\to 0}\left(\frac{4x+3}{4x-3}\right)^{6x} = (1^{\infty}) = \lim\limits_{x\to\infty}\left(1+\frac{6}{4x-3}\right)^{6x} = \left\{\begin{matrix} U(x) = 1+\frac{6}{4x-3} \\ V(x) = 6x \end{matrix}\right\} =$

$= e^{\lim\limits_{x\to\infty}(\cancel{1}+\frac{6}{4x-3}-\cancel{1})\cdot 6x} = e^{\lim\limits_{x\to\infty}\frac{36x}{4x-3}} = e^{\lim\limits_{x\to\infty}\frac{36}{4-\frac{3}{x}}} = e^{9};$

e) $\lim\limits_{x\to\infty}\frac{\sin x + \ln x}{x} = \lim\limits_{x\to\infty}\left(\frac{\sin x}{x} + \frac{\ln x}{x}\right) =$

$= \left\{\lim\limits_{n\to\infty}\frac{\log_a n}{n} = 0 \,\middle|\, a>1\right\} = \lim\limits_{x\to\infty}\frac{\sin x}{x} = 0.$

f) $\lim\limits_{x\to 0}\frac{\sin x + \ln x}{x} = \lim\limits_{x\to 0}\left(\frac{\sin x}{x}^{\nearrow 1} + \frac{\ln x}{x}^{\nearrow\infty}\right) = \infty.$