

## Домашнее задание 8.

### Задание 8.1

$$z = \sqrt{1-x^3} + \ln(y^2-1)$$

$$\begin{cases} 1-x^3 \geq 0; \\ y^2-1 > 0; \end{cases} \Rightarrow \begin{cases} x \leq 1; \\ |y| > 1; \end{cases} \Rightarrow D(x,y) : x \in (-\infty; 1], y \in (-\infty; -1) \cup (1; +\infty)$$

### Задание 8.2

$$z = \left(1 + \frac{\ln x}{\ln y}\right)^3;$$

$$z'_x = \left(1 + \frac{\ln x}{\ln y}\right)^2 \cdot \frac{3}{x \ln y};$$

$$z'_y = 3 \cdot \left(1 + \frac{\ln x}{\ln y}\right)^2 \cdot \frac{\ln x}{(\ln y)^2} \cdot \frac{1}{y} \cdot (-1) = -\frac{3 \ln x}{y \ln^2 y} \cdot \left(1 + \frac{\ln x}{\ln y}\right)^2.$$

### Задание 8.3

$$z = \sqrt{2xy + \cos \frac{x}{y}}, z'(1; 1) - ?$$

$$z'_x = \frac{1}{2} \cdot \frac{1}{\sqrt{2xy + \cos \frac{x}{y}}} \cdot \left(2y - \sin \frac{x}{y} \cdot \frac{1}{y}\right) = \frac{2y - \frac{1}{y} \sin \frac{x}{y}}{2\sqrt{2xy + \cos \frac{x}{y}}};$$

$$z'_y = \frac{1}{2\sqrt{2xy + \cos \frac{x}{y}}} \cdot \left(2x - \sin \frac{x}{y} \cdot \frac{x}{y^2} \cdot (-1)\right) = \frac{2x + \frac{x}{y^2} \sin \frac{x}{y}}{2\sqrt{2xy + \cos \frac{x}{y}}};$$

$$z'_x(1; 1) = \frac{2 - \sin 1}{2\sqrt{2 + \cos 1}}, z'_y(1; 1) = \frac{2 + \sin 1}{2\sqrt{2 + \cos 1}};$$

$$dz(1; 1) = \frac{2 - \sin 1}{2\sqrt{2 + \cos 1}} dx + \frac{2 + \sin 1}{2\sqrt{2 + \cos 1}} dy;$$

### Задание 8.4

$$z = x^2 + xy + y^2 - 6x - 9y;$$

$$\begin{cases} \frac{\partial z}{\partial x} = 2x + y - 6 = 0; \\ \frac{\partial z}{\partial y} = x + 2y - 9 = 0; \end{cases} \Rightarrow \begin{cases} y = 6 - 2x; \\ x + 12 - 4x - 9 = 0; \end{cases} \Rightarrow \begin{cases} y = 6 - 2x; \\ 3x = 3; \end{cases} \Rightarrow \begin{cases} y = 4; \\ x = 1; \end{cases}$$

т. (1; 4) — критическийpunkt  $z(x; y)$ .