

MATHEMATICS

Paper 9709/11

Paper 11

General comments

Most candidates found the paper accessible. The questions that proved to be more challenging (**Questions 1, 4(i), 7, 9(iii) and 11**) were still accessible for the more able candidates. **Questions 1** and **11** required different approaches for the integration (and in **Question 11**, also for the differentiation) of a function of a function and candidates' performance may be improved by learning to recognise the situations that require these different approaches.

Comments on specific questions

Question 1

Many candidates did not appreciate that the function could not be integrated as it stood but the bracket needed to be expanded first. Hence there were many incorrect answers seen such as $\frac{1}{3} \left(x + \frac{1}{x} \right)^3$. Some of the candidates who did attempt to expand the bracket unfortunately lost the 'product' term arriving at $\int (x^2 + \frac{1}{x^2}) dx$.

Answer. $\frac{x^3}{3} - \frac{1}{x} + 2x + c$.

Question 2

The majority of candidates wrote down at least the first four terms of the expansion of $(1 + ax)^6$ and on the whole this was accomplished accurately. Most candidates were able to deduce that the value of a is -5 and also that the required term is $\binom{6}{3}(ax)^3$. There were a few incorrect evaluations of $\binom{6}{3}$ but the most common mistake was to forget to raise a to the power of 3 which gave answers of 20×-5 instead of $20 \times (-5)^3$.

Answer. -2500 .

Question 3

The vast majority of candidates attempted the composition of the two functions in the correct order and very few candidates found $fg(x)$. A fairly common mistake, however, was to see $(2x + 3)^2 - 2x$ instead of $(2x + 3)^2 - 2(2x + 3)$; algebraic mistakes were also made in the simplification of this expression. Expressing a quadratic expression (the correct expression is $4x^2 + 8x + 3$) in the form $a(x + b)^2 + c$ needs a lot of care and a significant number of candidates made algebraic errors. The most successful approach was to expand $a(x + b)^2 + c$ to reach $ax^2 + 2abx + ab^2 + c$ and then to equate coefficients.

Answer. $4(x + 1)^2 - 1$.



Question 4

Part (i) proved to be difficult for candidates and there were relatively few candidates who scored full marks for this part. A substantial number of candidates wrote down the full identity to be proved and proceeded to treat this as though it were an equation to be solved – multiplying, for example, both sides by $\cos x(1 - \cos x)$. This is not an approach to be recommended since it will lead to a truism such as $1 = 1$ from which presumably one is meant to deduce that the original statement is true. The standard approach is to start with the left-hand side of the identity to be proved and manipulate it until the right-hand side is reached.

In this case it requires three steps: replacing $\tan x$ by $\frac{\sin x}{\cos x}$; replacing $\sin^2 x$ by $1 - \cos^2 x$; replacing $1 - \cos^2 x$ by $(1 - \cos x)(1 + \cos x)$.

Providing candidates used the result from part (i), part (ii) was found to be more straightforward.

Answer: (ii) $109.5^\circ, 250.5^\circ$.

Question 5

This question was found more difficult than usual for questions of this type for two reasons. Candidates first had to identify which two vectors enclosed the required angle. A significant number found \vec{AC} and \vec{CB} which gives the obtuse angle. Secondly, the two vectors each contained only 2 components instead of the expected 3 components and this appeared to confuse some candidates who need to understand that in these circumstances the third component has to be assigned a coefficient of zero.

Answer: 48.0° .

Question 6

On the whole this question was done very well. In part (a) weaker candidates showed a certain amount of confusion in knowing which formula to apply for the fifth term and which to apply for the sum to five terms.

Providing the value of the common ratio was found to be $\frac{3}{4}$, part (b) was found to be straightforward by most candidates.

Answers: (a) 12, 1.5; (b) 64.

Question 7

Part (i) was found difficult. Candidates needed to realise that, for $0 < x < \pi$, $0 < \frac{1}{2}x < \frac{1}{2}\pi$ and therefore that

$\tan\left(\frac{1}{2}x\right)$ is always positive and hence that $f(x) < 3$. In part (ii) many candidates did not appreciate the

significance of the word ‘exact’ in the question and gave the answer as a decimal number. Candidates seemed to find part (iii) particularly difficult. Very often questions are constructed so that early parts of the question give assistance in later parts. In this question, the first two parts are quite helpful in sketching the graph in part (iii) but many candidates did not seem to appreciate this. Part (iv), on the other hand, was done well and many candidates demonstrated that they understood how to find the inverse of the given function.

Answers: (i) $f(x) < 3$; (ii) $3 - 2\sqrt{3}$; (iv) $2\tan^{-1}\left(\frac{3-x}{2}\right)$.



Question 8

In part (i) some candidates expressed the curved part of the perimeter incorrectly as $90x$ (using degrees instead of radians) or more generally θx . In part (ii) the same types of mistakes occurred in expressing the area of the quarter-circle. Candidates who avoided making these errors were often quite successful in the first two parts. Part (iii) was done very well. In part (iv) most candidates stated correctly that the stationary value is a maximum but many candidates omitted to say what the stationary value of A is.

Answers: (i) $y = 30 - x - \frac{\pi x}{4}$; (iii) 15; (iv) 225, Maximum.

Question 9

A significant number of candidates did not recognise that part (i) required an application of Pythagoras' Theorem. The majority of candidates used the correct method for obtaining the required angle in part (ii), although there were occasional cosine/sine misunderstandings seen. Part (ii) specifically required answers to be given in radians correct to 4 significant figures – an instruction which was ignored by some candidates. In addition, the request for 4 significant figures should have been helpful to candidates as a hint in part (iv) that they needed to keep (at least) 4 significant figures in their calculations in order to be able to give the final answer accurately correct to 3 significant figures.

Answers: (ii) 0.9273; (iii) 5.90 cm².

Question 10

The whole of this question was generally done very well with a substantial proportion of candidates achieving full credit.

Answers: (ii) $(-4.5, 2.25)$; (iii) $(0.5, 4.75)$.

Question 11

In part (i), since $9(2-x)^{-1}$ is a function of a linear function, in addition to multiplying by the power -1 it is necessary to also multiply by -1 (the derivative of $2-x$), and some candidates omitted to do this. Candidates also needed to recognise that there are no values of x for which the derivative has a value of zero and hence the curve has no stationary points. In part (ii) the function to be integrated is $81\pi(2-x)^{-2}$, which is again a function of a linear function. This time it is necessary to divide by -1 (the derivative of $2-x$) and again some candidates omitted to do this. Part (iii) required considerable care, particularly in algebraic simplification, and although many candidates used the right method only a minority were able to complete this part without error.

Answers: (i) $\frac{9}{(2-x)^2}$; (ii) $\frac{81\pi}{2}$; (iii) $k < -8, k > 4$.



MATHEMATICS

Paper 9709/12

Paper 12

General comments

Candidates generally found this paper to be accessible and there were many excellent scripts. Some parts of questions caused most candidates some difficulty, particularly **Questions 7(iii), 9(i),(ii) and 10(i)**. Candidates had been well prepared for the paper and there were relatively few very poor scripts. The standards of algebra and numerical manipulation were good and, in general, presentation was satisfactory. One noticeable point was the lack of detail and explanation in questions in which candidates had to show or prove a particular result and marks were unnecessarily lost, particularly in **Questions 2, 9 and 10**.

Comments on specific questions

Question 1

- (i) Although this question was generally accurately answered, a significant number of candidates misread $(1-2x)^2$ as $(1-2x)$. The most common errors occurred in finding the third term and $(-2x^2)^2$ was often replaced by $-4x^4$ or by $\pm 2x^4$.
- (ii) The vast majority of candidates realised the need to consider two products in finding the coefficient of x^4 .

Answers: (i) $1-16x^2+112x^4$; (ii) 240.

Question 2

Although most candidates started from the left-hand side, those preferring to start from the right-hand side were equally, if not more, successful. Knowledge of the basic trigonometric functions was good; most marks were lost in the failure to provide full and detailed explanations. In a question of this nature, it is not sufficient to state, without any evidence, that, for example, $\frac{\sin^2 x(1-\cos^2 x)}{\cos^2 x}$ is equal to $\tan^2 x \sin^2 x$.

Question 3

- (i) This was well answered and was generally correct, though a surprising number of weaker candidates expressed $\sqrt{2t-1}$ as $(2t-1)^{-\frac{1}{2}}$ or as $(2t-1)^{-1}$.
- (ii) This part proved to be more of a problem with about a third of all candidates failing to realise that the rate of growth of a 5-year old snake referred to the value of $\frac{dx}{dt}$ when $x = 5$. It was common to see the rate of growth given as the value of x when $t = 5$.

Answers: (i) $\frac{0.7}{\sqrt{2t-1}}$; (ii) 0.233 metres/year.



Question 4

- (i) Many candidates failed to realise that angle AOB equalled $2\pi - 2.3 - 2.3$, and many others disregarded the instruction to provide an answer correct to 4 significant figures.
- (ii) A large number of candidates incorrectly assumed that the shaded region was itself a sector of a circle with centre C. Another common error was to find the non-shaded region within the circle. The majority of candidates however, found the sum of the areas of two triangles and added this to the area of a sector. Candidates made competent use of radian measure throughout the question.

Answers: (i) 1.683; (ii) 14.3 cm².

Question 5

- (i) This was well answered with most candidates finding the common difference as -7 (though $+7$ was a common error) and then using the correct formula for the sum of m terms. Many candidates made hard work of the solution of the quadratic equation by failing to cancel m in the equation.
- (ii) This presented more difficulty and the algebraic nature of the question led to many candidates omitting it altogether. On the other hand, many candidates realised that $S_n < 0.9S_\infty$ and completed the solution accordingly, though failure to cancel ' a ' and ' $(1 - r)$ ' caused many to give up on the resulting algebra.

Answer: (i) 47.

Question 6

- (i) Most candidates realised the need to eliminate y from the two equations and to use ' $b^2 - 4ac$ ' on the resulting quadratic equation. Although the values $k = 0$ and $k = 4$ were usually obtained, a minority of candidates were able to solve the inequality correctly, with $k < 0$ and $k < 4$ being the usual answer.
- (ii) A few candidates used calculus to deduce that $x = \frac{1}{2}$ and proceeded to deduce that $k = 4$ and $y = 2$. The majority however realised from part (i) that $k = 4$ and substituted into the equation in part (i) to find $x = \frac{1}{2}$. Surprisingly, many solutions omitted to give the y -value.

Answers: (i) $0 < k < 4$; (ii) $4, (\frac{1}{2}, 2)$.

Question 7

- (i) The use of 'completing the square' was very good, though at least a half of all candidates failed to realise that the range of $(x - a)^2 + b$ is $f(x) > b$.
- (ii) Although a few weaker candidates attempted to make x the subject of $x^2 - 4x + 7$ without using the answer to part (i), the majority performed the operation correctly, though many incorrectly left ' \pm ' in the final answer. Most candidates realised that the domain of f^{-1} was the same as the range of f .
- (iii) Only a few candidates realised that the answer could be deduced directly from part (i) as $x^2 + 3$.

Answers: (i) $(x - 2)^2 + 3$, $f(x) > 3$; (ii) $2 + \sqrt{x - 3}$, $x > 3$; (iii) $x^2 + 3$.



Question 8

This question proved to be a source of high marks.

- (i) Virtually all candidates realised the need to equate the equations of the curve and the line and to solve the resulting quadratic equation. Apart from a few careless algebraic errors, most solutions were correct.
- (ii) A few candidates lost marks through incorrectly assuming that the mid-point is $\left(\frac{x_1 - x_2}{2}, \frac{y_1 - y_2}{2}\right)$ or that the distance is $\sqrt{(x_1 + x_2)^2 + (y_1 + y_2)^2}$.

Answers: (i) $(-1, 1)$, $(\frac{2}{3}, 6)$; (ii) 5.27 , $(-\frac{1}{6}, \frac{7}{2})$.

Question 9

- (i) This part caused many problems with most candidates hopefully writing $a = 10 - 6$ with no explanation whatsoever. Successful efforts came from using similar triangles PDE and POA or from deducing that triangle POA was isosceles implying an angle of 45° and that $PD = 6$ cm.
- (ii) Less than a half of all solutions correctly found vector \vec{BG} , with the coefficients of \mathbf{j} and \mathbf{k} causing most difficulty.
- (iii) Knowledge and manipulation of scalar product was good, though many candidates used the vectors $\pm \vec{AG}$, $\pm \vec{BG}$ instead of the vectors \vec{BG} and \vec{BA} to find angle GBA . The use of $\vec{BG} \cdot \vec{AB}$ instead of $\vec{BG} \cdot \vec{BA}$, thereby giving the obtuse angle, was another common error.

Answers: (ii) $-10\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$; (iii) 69.6° .

Question 10

- (i) This presented many candidates with difficulty. The majority realised that the volume in m^3 (4) could be equated to the product of x , $\frac{1}{2}x$ and h and that this led to an expression for h in terms of x . Expressions for A however were often incorrect, with many candidates considering 7, or even 8, faces instead of 6 (base, 4 sides and the lid).
- (ii) Most candidates realised the need to set the differential, $\frac{dA}{dx}$, to zero and solutions were generally accurate, though $x^3 = 8$ was often solved as $x = \pm 2$. Use of the second differential to deduce that A was a minimum was also well carried out.

Answers: (i) $h = \frac{8}{x^2}$; (ii) 2.



Question 11

- (i) A sizeable minority of candidates started by differentiating the equation of the curve, thereby failing to realise that this was a question on coordinate geometry and not on calculus. Those deducing that A was $(0, 1)$ and B was $(5, \frac{1}{2})$ had little difficulty in finding the equation of the line AB .

- (ii) A significant number of candidates misinterpreted ‘volume’ for ‘area’ and many others, whilst quoting $V = \int y^2 dx$, did not square either the equation of the line or the equation of the curve.

The standard of integration was good, though omission of ‘÷3’ in the integration of $(3x+1)^{-\frac{1}{2}}$ was common. Errors in squaring were common, with the term in x often missing from $(-\frac{1}{10}x+1)^2$, and $\left((3x+1)^{-\frac{1}{4}}\right)^2$ often expressed as $(3x+1)^{\frac{1}{16}}$. The use of limits was generally accurate, though automatic omission of a value at $x = 0$ remains a common error.

Answer: (ii) $\frac{11}{12}\pi$.



MATHEMATICS

Paper 9709/13
Paper 13

General comments

Much of the work examined was of a high standard, being detailed, well presented and showing a thorough understanding of the subject matter. Candidates mostly made good use of their time. A small number should have tried to leave more time to answer the later, longer questions which carry larger numbers of marks.

Most of the instructions on the front of the question paper were widely observed. An exception was the one relating to the accuracy required for non-exact numerical answers (see comments below on **Question 8**).

Comments on specific questions

Question 1

Many candidates answered this question correctly. Some saw that the term independent of x is ${}^9C_3 x^6 \left(-\frac{1}{x^2}\right)^3$. Some used the general term ${}^9C_r x^r \left(-\frac{1}{x^2}\right)^{a-r}$ and wrote $r = 18 - 2r$, giving $r = 6$. Others wrote out the terms of the expansion in descending powers of x until they arrived at -84 . Closer attention should have been paid to the signs, as many answers of 84 were seen.

Answer: -84 .

Question 2

- (i) This part was answered correctly by nearly all candidates.
- (ii) This part was also well answered. Most candidates found the gradient of AB to be -2 , and used $m_1 m_2 = -1$ to find the gradient of the perpendicular ($= \frac{1}{2}$), and the equation of the line through $(8, 6)$. Some chose the direct method by writing $y - 6 = \frac{1}{2}(x - 8)$. Others used $y = mx + c$ and found c . A more careful reading of the question would have led to fewer candidates finding the equation of the perpendicular bisector of AB .

Answers: (i) $(3.5, 2)$; (ii) $x - 2y + 4 = 0$.

Question 3

This question was well answered. Most candidates correctly used the identity $\cos^2 x + \sin^2 x \equiv 1$ to form a quadratic in $\cos x$, solved it to find $\cos x = \frac{1}{3}, -\frac{2}{5}$ and hence $x = 70.5^\circ, 113.6^\circ$. Some gave values outside the required range $0 < x < 180$. Closer attention in some cases might have been paid to the instruction to give angles correct to 1 decimal place, as some answers of 113° were seen, and a few were given in radians (1.23, 1.95)

Answer: $70.5^\circ, 113.6^\circ$.



Question 4

Many candidates correctly sketched the curve $y = 2 \sin x$ for $0 < x < 2\pi$. Fewer found the equation of the line required to determine the number of roots of the equation $2\pi \sin x = \pi - x$ to be $y = 1 - \frac{x}{\pi}$. Most of those who did, drew the line correctly and so determined the number of roots to be 3.

Answers: (ii) 3, $y = 1 - \frac{x}{\pi}$.

Question 5

- (i) This part was generally well answered. The commonest omission was the differentiation of the second term of $\frac{1}{x-3} + x$, leaving the answer $\frac{-1}{(x-3)^2}$.
- (ii) The omission referred to above led to difficulties in this part of the question, as equating $\frac{dy}{dx}$ to 0 gave an equation which could not be solved. Starting by equating the correct $\frac{dy}{dx}$ to 0, most candidates found the values of x at the turning points and the corresponding values of y . Many used $\frac{d^2y}{dx^2}$ correctly to identify the maximum and minimum as required. Some candidates assumed incorrectly that the larger value of x gave the maximum and the smaller value gave the minimum.

Answers: (i) $-\frac{1}{(x-3)^2} + 1, \frac{2}{(x-3)^3}$; (ii) (2, 1), (4, 5).

Question 6

- (i) Many candidates correctly found the values of x , 1 and $-\frac{5}{3}$, for which $f'(x) = 0$. Some needed to pay more attention to the inequality $f'(x) > 0$. Those who illustrated the method they were using, for example on a number line, nearly always found the correct answer.
- (ii) This part was very well answered. Nearly all candidates correctly integrated to find $f(x) = x^3 + x^2 - 5x + c$, and used the fact that the curve passes through (1, 3) to find $c = 6$.

Answers: (i) $x < -\frac{5}{3}, x > 1$; (ii) $x^3 + x^2 - 5x + 6$.

Question 7

- (i) Many candidates showed a good understanding of range.
- (ii) It was understood by many candidates that the graph of $f^{-1}(x)$ is the reflection of $f(x)$ in the line $y = x$. Many of these would have made greater progress had they identified the key coordinates (2, 2) and (6, 4) in the given diagram. Without them, it is hard to locate the line $y = x$ and the graph of $y = f^{-1}(x)$ correctly.
- (iii) This part was well answered. The point needing more attention was the sign in the inverse of $x \mapsto \frac{1}{2}x^2$. Candidates obtaining $y = \pm\sqrt{2x}$ should have rejected the minus sign.

Answers: (i) $0 < f(x) < 4$; (iii) $x \mapsto \sqrt{2x}$ for $0 < x < 2$, $x \mapsto 2x - 2$ for $2 < x < 4$.

Question 8

- (i) Many candidates correctly found the area of a sector from $\frac{1}{2} \times 5^2 \times 1.2$. Fewer found the area of a triangle from $\frac{1}{2} \times 5^2 \sin 1.2$, using instead base $2 \times 5 \sin 0.6$ and height $5 \cos 0.6$. Closer attention should have been paid to the instruction to give answers correct to 3 significant figures, as 6.70 was frequently written as 6.7. A common misconception was to state the area of the rhombus as 5×5 .
- (ii) Closer attention to accuracy would have helped the many candidates who knew how to calculate the required length, but worked with too few significant figures to ensure a correct final answer. There was a need for more explanation in some answers in which candidates should have made clear which lengths they were trying to find.

Answers: (i) 6.70 cm²; (ii) 1.75 cm.

Question 9

- (a) This part was well answered, with candidates showing a sound understanding of a geometric progression and its sum to infinity.
- (b) (i),(ii) Equally good understanding of an arithmetic progression was shown. In part (ii) most candidates equated the expressions for the sum of m and $m + 1$ terms to find m and many were successful with the quite difficult algebra to obtain $m = 20$. Hardly any candidates saw the implication that if $S_m = S_{m+1}$, the $(m + 1)$ th term must be 0, and so $100 + (-5)m = 0$, leading to $m = 20$.
- (iii) This part was well answered. Candidates were able to solve the quadratic $\frac{n}{2}(200 + (n - 1)(-5)) = 0$, to give $n = 41, 0$ and most rejected the 0.

Answers: (a) 95; (b)(i) 100, -5; (ii) 20; (iii) 41.

Question 10

- (i),(ii) These parts were very well answered.

- (iii) Some candidates could have taken more care in arranging $3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k} + p(2\mathbf{i} + \mathbf{j} - 3\mathbf{k})$ as $\mathbf{i}(3 + 2p) + \mathbf{j}(-2 + p) + \mathbf{k}(4 - 3p)$ in order to write $\overrightarrow{BC} \cdot \overrightarrow{OA} = 0$. Nevertheless a lot of accurate work was seen. There was a misunderstanding in the work of a few candidates who used $\overrightarrow{BC} \cdot \overrightarrow{OA} = -1$ instead of $\overrightarrow{BC} \cdot \overrightarrow{OA} = 0$.

Answers: (i) 66.6°; (ii) $3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k} + p(2\mathbf{i} + \mathbf{j} - 3\mathbf{k})$; (iii) $\frac{4}{7}$.

Question 11

- (i) Many candidates solved the equation $x^6 - 9x^3 + 8 = 0$, giving $x = 1, 8$ and hence $a = 1, b = 2$. Fewer appreciated the need to equate $9 - x^3$ and $\frac{8}{x^3}$ in order to obtain the given equation.
- (ii) This part was well answered. Most candidates chose to find $\int_1^2 (9 - x^3) dx$ and $\int_1^2 \frac{8}{x^3} dx$ separately, rather than finding $\int_1^2 \left(9 - x^3 - \frac{8}{x^3}\right) dx$.



- (iii) Most candidates understood the need to find the gradients of the two curves used to equate them, obtaining $-3x^2 = -\frac{24}{x^4}$ correctly. The next step, to give $x^6 = 8$, was less well understood, as $x^2 \times x^4$ was sometimes wrongly written as x^8 . Nevertheless, many completely correct answers were seen.

Answers: (i) 1, 2; (ii) 2.25; (iii) $\sqrt{2}$.



MATHEMATICS

Paper 9709/21

Paper 21

General comments

Questions 1, 4 and 7 were very well attempted. Many candidates however struggled with Questions 2, 3, 5 and 8. Sign and arithmetical errors caused marks to be lost for some candidates.

Work was clearly and neatly presented and the Examiners had no difficulty understanding candidates' thought processes and reasoning.

There was no evidence of a lack of time producing curtailed work. The value of carefully working through past examination papers and an understanding of the basic techniques and results of calculus remain as important as ever.

Comments on specific questions

Question 1

This question was well attempted in general. Sometimes candidates obtained the inequality sign in the final answer the wrong way round (note that the value $x = 0$ does not satisfy the original inequality and so cannot belong to the solution set), and sometimes candidates made sign or arithmetic errors after successfully squaring on both sides.

Answer: $x > \frac{3}{2}$.

Question 2

Nearly everyone successfully took logarithms to obtain $x \ln 5 = (2x + 1)\ln 2$. However, isolating x in terms of $\ln 5$ and $\ln 2$ proved beyond many candidates who made serious algebraic errors.

Answer: 3.11.

Question 3

The correct method involves expanding the given integrand into the form $(e^{2x} + 2e^x + 1)$ and then integrating these three terms. Unfortunately, the majority of candidates were under the impression that integration yielded $\frac{1}{3}(e^x + 1)^3$ or another function of a similar form.

Question 4

- (i) Candidates proved very competent in obtaining $\frac{dy}{dx}$ as the quotient of $\frac{dy}{dt}$ over $\frac{dx}{dt}$ or via multiplying $\frac{dy}{dt}$ by $\frac{dt}{dx}$. Few errors were seen.



- (ii) Many candidates expanded out $(t^2 - 9)(t - 2)$ and sought fruitlessly to solve the resulting cubic equation in t . It was expected that candidates would use $\frac{dy}{dx} = 0$ and the factorised form to find the values $t = \pm 3, 2$. Of these three values, only one is greater than 2, namely $t = 3$.

Answer: (ii) (1, 6).

Question 5

Those who began well, by noting that $\operatorname{cosec}^2 \theta \equiv 1 + \cot^2 \theta$, for example, usually went on to produce excellent solutions. Some candidates did not produce a quadratic equation in $\cot \theta$ or $\tan \theta$, and many incorrect equations in $\sin \theta$ or $\cos \theta$ (or both) were obtained by using incorrect trigonometric identities.

Answer: $26.6^\circ, 146.3^\circ, 206.6^\circ, 326.3^\circ$.

Question 6

- (i) Most candidates formed a function $f(x) = \pm(6x^{-2} - x - 1)$ or $f(x) = \pm(x^3 + x - 6)$ and calculated $f(1.4)$ and $f(1.6)$, noting that these were different in sign. A minority argued, incorrectly, from the values of $6x^{-2}$ or thought that it was sufficient to show that $6x^{-2} = x + 1$ is equivalent to the form $x^3 + x - 6 = 0$.
- (ii) Most obtained the iteration formula from the fact that $6x^{-2} = x + 1$. Some candidates attempted to iterate here instead of in part (iii).
- (iii) Usually the iteration was methodically correct. However many candidates worked to only 2 or 3 decimal places in the iterative process or did not round the final answer to 2 decimal places or rounded incorrectly.

Answer: (iii) 1.54.

Question 7

- (i) This part was generally well attempted. Virtually everyone set $p(1) = 0$ and $p(2) = 10$, though a few set $p(2) = 0$. Having obtained two simultaneous linear equations in a and b , some candidates could not correctly solve the equations $a + b = -5, 2a + b = -22$.
- (ii) The use of long division to divide $p(x)$ by $(x - 1)$ was generally excellent, but some candidates did not note that $x = 1$ was a solution to $p(x) = 0$. A minority lost marks by correctly factorising $p(x)$ but not solving $p(x) = 0$.

Answers: (i) $-17, 12$; (ii) $-3, 1, \frac{4}{3}$.

Question 8

- (i) Many candidates wrongly believed that y' had only a single term and did not use $(uv)' = uv' + u'v$. Others, having correctly shown that the gradient of the normal at Q is -1 , then failed to obtain the equation of the normal at that point.
- (ii) Many solutions featured only a single term (usually $\pm x \sin x$) as the derivative of $x \cos x$.
- (iii) Those who correctly obtained $x \sin x$ as the solution to part (ii) proceeded successfully in part (iii). However, most solutions featured false integrals such as $-x \cos x$.

Answers: (ii) $x \sin x$; (iii) 1.



MATHEMATICS

Paper 9709/22

Paper 22

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Answer: $x > \frac{3}{2}$.

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- (i) Candidates proved very competent in obtaining $\frac{dy}{dx}$ as the quotient of $\frac{dy}{dt}$ over $\frac{dx}{dt}$ or via multiplying $\frac{dy}{dt}$ by $\frac{dt}{dx}$. Few errors were seen.



- (ii) Many candidates expanded out $(t^2 - 9)(t - 2)$ and sought fruitlessly to solve the resulting cubic equation in t . It was expected that candidates would use $\frac{dy}{dx} = 0$ and the factorised form to find the values $t = \pm 3, 2$. Of these three values, only one is greater than 2, namely $t = 3$.

Answer: (ii) (1, 6).

Question 5

Those who began well, by noting that $\operatorname{cosec}^2 \theta \equiv 1 + \cot^2 \theta$, for example, usually went on to produce excellent solutions. Some candidates did not produce a quadratic equation in $\cot \theta$ or $\tan \theta$, and many incorrect equations in $\sin \theta$ or $\cos \theta$ (or both) were obtained by using incorrect trigonometric identities.

Answer: $26.6^\circ, 146.3^\circ, 206.6^\circ, 326.3^\circ$.

Question 6

- (i) Most candidates formed a function $f(x) = \pm(6x^{-2} - x - 1)$ or $f(x) = \pm(x^3 + x - 6)$ and calculated $f(1.4)$ and $f(1.6)$, noting that these were different in sign. A minority argued, incorrectly, from the values of $6x^{-2}$ or thought that it was sufficient to show that $6x^{-2} = x + 1$ is equivalent to the form $x^3 + x - 6 = 0$.
- (ii) Most obtained the iteration formula from the fact that $6x^{-2} = x + 1$. Some candidates attempted to iterate here instead of in part (iii).
- (iii) Usually the iteration was methodically correct. However many candidates worked to only 2 or 3 decimal places in the iterative process or did not round the final answer to 2 decimal places or rounded incorrectly.

Answer: (iii) 1.54.

Question 7

- (i) This part was generally well attempted. Virtually everyone set $p(1) = 0$ and $p(2) = 10$, though a few set $p(2) = 0$. Having obtained two simultaneous linear equations in a and b , some candidates could not correctly solve the equations $a + b = -5$, $2a + b = -22$.
- (ii) The use of long division to divide $p(x)$ by $(x - 1)$ was generally excellent, but some candidates did not note that $x = 1$ was a solution to $p(x) = 0$. A minority lost marks by correctly factorising $p(x)$ but not solving $p(x) = 0$.

Answers: (i) $-17, 12$; (ii) $-3, 1, \frac{4}{3}$.

Question 8

- (i) Many candidates wrongly believed that y' had only a single term and did not use $(uv)' = uv' + u'v$. Others, having correctly shown that the gradient of the normal at Q is -1 , then failed to obtain the equation of the normal at that point.
- (ii) Many solutions featured only a single term (usually $\pm x \sin x$) as the derivative of $x \cos x$.
- (iii) Those who correctly obtained $x \sin x$ as the solution to part (ii) proceeded successfully in part (iii). However, most solutions featured false integrals such as $-x \cos x$.

Answers: (ii) $x \sin x$; (iii) 1.



MATHEMATICS

Paper 9709/23

Paper 23

General comments

There were insufficient attempts to make a valid comment on many of the questions.

Comments on specific questions

Question 1

Some candidates solved $3x + 1 = 8$ only. Others were unable to solve the equation $3x^2 + 2x - 21 = 0$ resulting from squaring each side of the given modular inequality.

Answer: $x < -3$, $x > \frac{7}{3}$.

Question 2

- (i) Some candidates worked to less than 4 decimal places or did not round the final iterate.
- (ii) There were insufficient attempts to make a valid comment.

Answers: (i) 1.82; (ii) $x = \frac{7x}{8} + \frac{5}{2x^4}$.

Question 3

There were insufficient attempts to make a valid comment.

Answers: (i) -7; (ii) $x^2 + 5x - 2$.

Question 4

There were insufficient attempts to make a valid comment.

Answers: (a) $-\frac{1}{2}e^{1-2x}$; (b) $\frac{1}{2} - \frac{1}{2}\cos 6x$, $\frac{1}{2}x - \frac{1}{12}\sin 6x$.

Question 5

There were insufficient attempts to make a valid comment.

Answers: 1.11; 1.65.

Question 6

There were insufficient attempts to make a valid comment.

Answers: (i) $\sqrt{5} \sin(\theta - 26.57^\circ)$; (ii) 16.3° , 216.9° .



Question 7

There were insufficient attempts to make a valid comment.

Answers (i) $\left(\sqrt{e}, \frac{1}{2e}\right)$; (ii) 0.34.

Question 8

There were insufficient attempts to make a valid comment.

Answer: (ii) $y = 2x - 2$.



MATHEMATICS

Paper 9709/31

Paper 31

General comments

There was considerable variation in the standard of work on the paper which resulted in a broad spread of marks. No question or part of a question seemed to be of undue difficulty, and most questions discriminated well. The questions that were generally done well were **Question 3** (trigonometry), **Question 5** (integration) and **Question 8** (partial fractions). Those that were done least well were **Question 4(i)** (graph sketching), **Question 7** (vector geometry) and **Question 10** (differential equation).

In general the presentation of work was good and candidates appeared to have sufficient time to attempt all the questions. In two questions, **Question 5** and **Question 9**, a request was made for the 'exact value' of an answer. Candidates need to be aware of the meaning of such requests, for some either gave an approximate answer, or else, having found an exact answer, spent time on the fruitless calculation of an approximate value. Similarly they need to know that if they are expected to reach an exact answer, such as $\frac{1}{6}\pi$ in **Question 6(iii)**, then only work with exact values can earn full marks. Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

Comments on specific questions

Question 1

This was fairly well answered. The most common method involved working with a non-modular quadratic inequality and the main error was the failure to square the factor of 2.

Answer: $-7 < x < 1$.

Question 2

Some candidates dispatched this swiftly and accurately. However there were many faulty attempts indicating the need for candidates to be more secure in their manipulation of logarithms (and exponentials). A common error was to start by replacing $\ln(1 + x^2)$ with $\ln 1 + \ln x^2$, and sometimes a further error led to a quadratic in $\ln x$. Other errors occurred after logarithms had been removed and the correct equation

$x^2 = \frac{1}{e-1}$ had been reached. In the work that followed some candidates failed to take the square root and gave the answer $x = 0.582$, whilst others did not reject the negative root and gave the answer $x = \pm 0.763$.

Answer: 0.763.

Question 3

Most candidates expanded $\cos(\theta + 60^\circ)$ correctly and went on to reduce the equation to one in $\tan \theta$. The most common errors arose in the algebraic work leading to the value of $\tan \theta$. Occasionally $\cos 60^\circ$ and $\sin 60^\circ$ were given incorrect values and a small number of candidates took $\cos(\theta + 60^\circ)$ to be $\cos \theta + \cos 60^\circ$, but generally this question was answered well.

Answer: $9.9^\circ, 189.9^\circ$.



Question 4

In part (i) correct sketches of two relevant graphs were rare. Sketches of $y = 4x^2 - 1$ were often straight lines and those of $y = \cot x$ that bore some resemblance to the correct graph often failed to maintain the correct concavity or reach the x -axis at $x = \frac{1}{2}\pi$. The sketching of simple graphs is an area where many candidates appear to be weak.

Part (ii) was fairly well answered, with most candidates stating an appropriate function and looking for a change of sign in calculated values.

Before starting an iteration involving a trigonometric function candidates need to check whether their calculator should be set in degree or radian mode. For part (iii) of this question such a check was particularly important since the previous question involved calculating angles in degrees. Those candidates who had a correct understanding of the cotangent function and had their calculator in radian mode answered this part of the question very well.

Answer: (iii) 0.73.

Question 5

In part (i) there were many excellent answers, for most candidates realised that a full justification of the given answer, including work supporting the change of limits was required. The most common errors in faulty attempts were (a) replacing $\sqrt{(4 - 4 \sin^2 \theta)}$ with $2 - 2 \sin \theta$, (b) replacing dx by $d\theta$, and (c) omitting a justification of the change in limits.

In part (ii) those who knew to replace $4 \sin^2 \theta$ by an expression involving $\cos 2\theta$ were generally successful, though quite a few made trivial errors in the handling of the factor of 4. However there were a number of candidates who appeared unaware of this strategy and went directly to incorrect integrals such as $\frac{4 \sin^3 \theta}{3 \cos \theta}$.

As mentioned earlier in the general comments, some candidates failed to give an exact answer, or having obtained one went on to calculate an approximation of it.

Answer: (ii) $\frac{1}{3}\pi - \frac{\sqrt{3}}{2}$.

Question 6

Part (i) was usually done correctly. The answers to the pair of problems in part (ii) suggest that some candidates need to be more familiar with the notation for the conjugate of a complex number. For there were a number who used the conjugate of z in part (b), but failed to state the correct conjugate of z when asked for it explicitly in part (a). However, on the whole, this part was fairly well answered. The main errors arose when candidates were collecting terms $\sqrt{3}$ and in i , with $2i - i$ becoming $-i$ quite regularly.

There were many sound attempts at part (iii). Since the answer $\frac{1}{6}\pi$ was given, it was important to work with exact values rather than approximate ones. A variety of successful methods were seen, including the use of a scalar product.

Answers: (i) 2 , $\frac{1}{6}\pi$; (ii)(a) $3\sqrt{3} + i$, (b) $\frac{1}{2}\sqrt{3} + \frac{1}{2}i$.



Question 7

Part (i) was generally answered correctly, though some candidates omitted ' $\mathbf{r} =$ '. A minority incorrectly gave $\overrightarrow{AB} = 2\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ as the equation of the line.

Most candidates commenced part (ii) by equating an appropriate scalar product to zero. The frequency with which damaging slips were made in solving for the parameter emphasised the need for candidates to check their work as they go along. In this passage of work common errors included ' $-2(2 - 2\lambda) = -4 - 4\lambda$ ' and ' $12\lambda + 2 = 0, \therefore \lambda = \frac{1}{6}$ '. At the end some of those who had obtained the correct position vector for \overrightarrow{OP} , went on to present another answer $2\mathbf{i} + 5\mathbf{j} + 7\mathbf{k}$. Tripling the correct components in this way falsifies the answer and loses the mark.

Very few candidates started part (iii) by stating or implying that \overrightarrow{OP} , or a non-zero multiple of it, was normal to the required plane. Those that did solved the problem quickly. Most candidates found and presented as final answer the equation of a plane which was in fact the plane OAB rather than the one requested. The majority of those who made a successful attempt at the problem worked with two pairs of simultaneous equations, or used a pair of vector products to arrive at a multiple of \overrightarrow{OP} before going on to form the equation of the required plane.

Answers: (i) $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k} + \lambda(2\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$; (ii) $\frac{2}{3}\mathbf{i} + \frac{5}{3}\mathbf{j} + \frac{7}{3}\mathbf{k}$; (iii) $2x + 5y + 7z = 26$.

Question 8

Part (i) was well answered. Nearly all candidates stated a correct general form such as $\frac{A}{1+x} + \frac{Bx+C}{1+2x^2}$ and carried out the evaluation of the constants clearly and accurately. A common source of error was the failure to place $Bx + C$ in brackets in the basic identity $3x \equiv A(1+2x^2) + (Bx+C)(1+x)$.

Part (ii) was also well answered. However before attempting to expand each partial fraction, candidates need to check that the partial fractions correctly record the outcome of part (i). As well as errors in transcribing the values of A , B , and C , the miscopying of $(1+2x^2)$ as $(1+2x)^2$ was also seen. The final mark here is for a correct sum of terms in ascending powers of x . Some candidates unnecessarily went on to factorise the expansion and some incorrectly discarded the factor of 3.

Answers: (i) $-\frac{1}{1+x} + \frac{2x+1}{1+2x^2}$; (ii) $3x - 3x^2 - 3x^3$.

Question 9

Part (i) was quite well answered. Most candidates differentiated correctly and reached at least $3\ln x = -1$. Many continued by finding the exact value of x , but then usually converted it to a decimal and calculated the corresponding value of y as a decimal.

Some omitted the y -value altogether or miscalculated its exact value as $\frac{1}{e} - \frac{1}{3}$.

In part (ii) the integration was generally done well with most candidates finding and using the correct lower limit $x = 1$. Some gave an approximate answer rather than an exact one, or followed an exact answer with an unnecessary decimal approximation.

Answers: (i) $(e^{-\frac{1}{3}}, -\frac{1}{3e})$; (ii) $4\ln 2 - \frac{15}{16}$.



Question 10

Nearly all candidates answered part (i) correctly, stating a general equation and using the given values to find the constant of proportionality. The few who simply checked that the given values satisfy the given equation scored nothing.

There were good solutions to parts (ii) and (iii). Variables were usually separated correctly but candidates need to be careful to introduce a minus sign into the integral when integrating $\frac{1}{20-x}$ and also be sure to introduce a constant of integration. Here, as in **Question 2**, errors arose from incorrect manipulation of logarithms (and exponentials), the main errors being to take $\ln(20-x)$ to be $\ln 20 - \ln x$ and $\exp(0.5 - \ln 20)$ to be $\exp(0.5) - \exp(\ln 20)$. Only a few answered part (iv) correctly. Some did recognise 20 was a limiting value but spoiled their answer by saying 'x = 20' or 'x becomes 20', rather than 'x tends to 20' or some equivalent.

Answers: (ii) $-\ln(20-x) = 0.05t - \ln 20$; (iii) 7.9; (iv) x approaches 20.



MATHEMATICS

Paper 9709/32

Paper 32

General comments

There was considerable variation in the standard of work on the paper which resulted in a broad spread of marks. No question or part of a question seemed to be of undue difficulty, and most questions discriminated well. The questions that were generally done well were **Question 3** (trigonometry), **Question 5** (integration) and **Question 8** (partial fractions). Those that were done least well were **Question 4(i)** (graph sketching), **Question 7** (vector geometry) and **Question 10** (differential equation).

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Answers: (ii) $-\ln(20-x) = 0.05t - \ln 20$; (iii) 7.9; (iv) x approaches 20.



MATHEMATICS

Paper 9709/33

Paper 33

General comments

This paper proved to be accessible to most candidates, the majority offering attempts at solutions to all questions, and a full range of marks was seen. Much of the work was of a very good standard, demonstrating a strong understanding of the syllabus content. The best work was clearly set out, with methods clearly explained and implemented with care. Candidates gave confident responses to familiar questions. More unusual questions, such as the end of **Question 10**, provided a challenge for even the best candidates. Candidates should be reminded of the benefits of making their method clear to the Examiner. Where an answer is not correct it is not possible to give credit for a correct method unless this is shown. Where the value of an integral is given, as in **Question 5**, candidates need to be particularly rigorous in their working to demonstrate that they have achieved the given result. They should also be reminded that it is not appropriate to rely on a calculator method to verify the result. A few candidates persist in working in double columns. This often makes their work difficult to follow, and this practice should not be encouraged.

Comments on specific questions

Question 1

Most candidates were familiar with the binomial expansion, and many correct answers were seen. Candidates dealt well with the negative power. Most errors involved arithmetic slips or not dealing with the 2 correctly. Some candidates simply ignored the 2, and some tried to take it out as a factor. Many obtained $+12x^2$ with no working shown. It was also common to see a correct unsimplified expansion involving $\frac{(-3)(-4)}{2!}(2x)^2$ leading to $+12x^2$. When the correct unsimplified expression was seen, candidates were given credit for the correct initial method. A few candidates were able to write down an expression involving unsimplified binomial coefficients $\binom{-3}{1}$ and $\binom{-3}{2}$ but they did not gain any credit until they expanded these coefficients.

A very small number of candidates opted to use the McLaurin expansion, with varied success.

Answer: $1 - 6x + 24x^2$.



Question 2

Most candidates made correct use of $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ with their expressions for $\frac{dy}{dt}$ and $\frac{dx}{dt}$. The expression for $\frac{dy}{dt}$

$\frac{dy}{dt}$ was often correct, but the factor of -2 was sometimes missing. The use of product or quotient rule to

obtain $\frac{dx}{dt}$ or $\frac{dy}{dx}$ proved to be more difficult. Although many candidates did not state the rule that they were using, it was often clear that the rule was being misused, with confusion between terms and sign errors. Starting with a statement of the rule might assist candidates in making sure that they are using a correct rule and they are substituting terms correctly. Many candidates did not recognise the expression as a quotient at all – they treated the numerator as a constant and obtained $\frac{dx}{dt} = \frac{-2t}{(2t+3)^2}$ for their derivative. Some

candidates with a correct expression for $\frac{dx}{dt}$ were not able to use it correctly. This was a particular problem for candidates who had used the product rule, for whom the incorrect statement $\frac{1}{1} = \frac{2t+3}{2t} - \frac{(2t+3)^2}{2t}$ was a common error. Candidates who preferred not to use the

$\frac{1}{2t+3} - \frac{1}{(2t+3)^2}$ parametric form were often successful in obtaining $y = e^{\frac{-6x}{1-2x}}$ but rarely succeeded in differentiating this correctly.

Algebraic and arithmetic slips were common in this question.

Answer: -6 .

Question 3

Part (i) proved to be straightforward for most candidates. The small number of incorrect answers usually involved errors in expanding $(2+i)^2$, but there were also some candidates apparently unaware of the value of i^2 .

In part (ii) many candidates used graph paper for their Argand diagram. Although this was not expected, candidates working on graph paper did often achieve a better result. Candidates should be familiar with the common loci, and should have recognised the basic circular form in this expression. Diagrams drawn freehand often lacked some of the key features expected, particularly that the circle should pass through the origin. Candidate using different scales on the two axes should obtain an ellipse rather than a circle. Some candidates with a correct circle drawn also had a straight line drawn through their circle and shaded only part of the circle for their answer. The source of the misunderstanding that led to the inclusion of the line was not obvious.

Answers: (i) $3 + 4i$, 5.

Question 4

- (i) Many candidates differentiated the function correctly as a ‘function of a function’ and went on to substitute correct exact values correctly. The most common errors by this method were to omit a factor of 3 in the differentiation or to lose it from the trigonometric term resulting in an expression of the form $k \cos 3x \sin x$. The other common correct method was to start by rewriting $4 \cos^2 3x$ as $2 + 2 \cos 6x$ and then differentiate. Invalid attempts to differentiate often resulted in answers of the form $k \cos 3x$ or $k \sin^2 3x$.



- (ii) Many candidates recognised this standard integral and completed it by correct use of the double angle formula. Several candidates struggled to make any progress with the integral, including some who had started part (i) by using the correct double angle form. Incorrect answers commonly involved $\cos^3 3x$ or $\sin^3 3x$.

Answers: (i) $-6\sqrt{3}$; (ii) $2x + \frac{1}{3} \sin 6x + c$.

Question 5

This integral required the use of partial fractions. Many candidates took advantage of a fraction that is simpler than some they have had to deal with in recent papers, and they dealt confidently with this stage in the process. Most went on to integrate to obtain terms involving logarithms. There were some errors in coefficients, most commonly involving $4\ln|2x+1|$, but this did not prevent progress with substitution of the limits and an attempt to simplify to the given answer. Because the answer was given, candidates were expected to give full working to explain how their answer simplified to the given form. A calculator method demonstrating approximate decimal equivalence was not acceptable. There were some examples of poor work with logarithms, including incorrect statements such as ' $\ln 9 - \ln 2 = \ln 7$ '.

Candidates who did not recognise the need to use partial fractions offered a number of incorrect approaches to the integral but they were not able to make any valid progress with this question.

Question 6

The quality of the vector work was generally very good.

- (i) Most candidates started correctly by finding the direction of the line l and attempting a vector equation of the line. A few candidates stopped at this point, but most went on to substitute the components of a general point on l into the equation for the plane. Many correct solutions were seen, although most candidates expressed their answer as a position vector of the point, rather than stating the coordinates of the point. Most errors were due to algebra and arithmetic slips in the substitution of the point into the plane equation and solution of the resulting equation.
- (ii) Most candidates identified a correct normal vector for the plane and went on to use scalar product correctly to find the angle between the normal and the line. The question asks for the angle between the line and the plane. Some candidates found this correctly, but many stopped when they had found the angle between the line and the normal to the plane. Most errors in the working were due to starting with the wrong vectors, often involving the position vectors of one or both of the two points given at the start of the question.

Answers: (i) $(-1, 5, 4)$; (ii) 5.1° .

Question 7

- (i) Most candidates recognised the need to use integration by parts, and usually completed this correctly. The fact that the answer is given meant that full and clear working was expected. Errors in the integration were usually due to errors in integrating the term x^{-2} . Some candidates appeared to be differentiating the function rather than integrating.
- (ii) The majority of candidates converted the given equation to the expected iteration formula $a_{n+1} = \frac{5}{3}(1 + \ln a_n)$. They used this formula correctly and clearly understood the process and accuracy required to determine the value of the root. The question recommends an initial value, but some candidates preferred to use an alternative. Where the process was not properly understood, the most common error was to see calculation of values for $a_n = \frac{5}{3}(1 + \ln n)$ for integer values of n . Some candidates appeared to be confused between 'decimal places' and 'significant figures'.

Answer: (ii) 3.96.



Question 8

- (i) The majority of candidates gave correct answers to the first part of this question, many clearly remembering a standard form for the answers. The most common error was to see $\alpha = 52.2^\circ$ rather than the 2 decimal places asked for in the question. Some candidates reached the correct expression $\alpha = \tan^{-1}\left(\frac{\sqrt{10}}{\sqrt{6}}\right)$ but did not evaluate this correctly. The question asks for the form $R \cos(\theta - \alpha)$. Some candidates misquoted the expansion for this, and some clearly attempted to use $R \sin(\theta + \alpha)$ instead. Candidates should always be careful to avoid nonsense in their working – there were a few who wrote $\sin \alpha = \sqrt{10}$ and $\cos \alpha = \sqrt{6}$ in their attempt to find the value of α .
- (ii) (a) Many candidates recognised the need to find $\cos^{-1}(-1)$ and arrived at the correct answer with no difficulty. Large numbers of candidates showed no working and there was no evidence of the use of 180° . It was quite common to see $\cos^{-1}(1)$. Where the candidate went on to take account of the negative value and use this correctly they obtained the correct answer, but candidates who ignored the minus sign gained no credit.
- (b) It was unusual to see a correct answer to this part. Many candidates started by finding a value of $\cos^{-1}\left(\frac{3}{4}\right)$, but most used 41.4° and did not consider possible alternative values. Candidates made a variety of errors in attempting to deal with the $\frac{1}{2}\theta$, often treating this part as a completely separate question. There were also many algebraic slips in attempting to find θ from $\frac{1}{2}\theta - 52.24^\circ =$ their angle. Here again, candidates are advised to show their working. There were many incorrect answers following no working.

Answers: (i) $4 \cos(\theta - 52.24^\circ)$; (ii)(a) 232.2° , (b) 21.7° .

Question 9

- (i) Many candidates were successful in expressing the given information as a differential equation. Some were not confident with the use of proportionality in this context and set about trying to find a linear relationship between A and t . Others understood that rate of change involved a derivative, but then omitted the constant of proportionality. There was also some confusion between $\frac{dA}{dt}$ and $\frac{dt}{dA}$.
- (ii) Candidates with a differential equation were usually successful in separating the variables and making a start on the integration. Having integrated, the majority of candidates used the given values to evaluate a constant of integration and the constant of proportionality, and then went on to evaluate the required area. The alternative method of using the given values to form a definite integral was equally successful. The most common error in the integration was the mistaken use of logarithms to obtain a term in $\ln(\sqrt{2A - 5})$. Candidates with the correct form for the integral often made a mistake with the coefficients, usually by a factor of 2, and some forgot to include the constant of integration. Some candidates mistakenly thought that they could use the given values in the differential equation to evaluate the constant of proportionality before integrating.

Answers: (i) $\frac{dA}{dt} = k\sqrt{2A - 5}$; (ii) 63 m^2 .



Question 10

- (i) The majority of candidates applied the factor theorem correctly and deduced the correct value for m . Although there are only 2 marks available, some candidates preferred the longer method of working through the long division and equating a constant remainder to zero. This was often completed correctly, but there were also many arithmetic and algebraic errors in the working and some candidates did not complete the division.
- (ii) (a) Most candidates who had completed the long division in part (i) were able to use their quotient here. The alternatives of creating an identity or factorising by inspection were also widely used. Most candidates equated their quadratic factor to zero and then applied the quadratic formula to solve for values of z . Although the question uses the variable z , commonly used for complex values, many candidates were clearly expecting real roots from this equation. Some candidates said that the quadratic had no roots, and some stopped as soon as they realised that the value of the discriminant was negative, but made no comment. Some, either by accident or by design, obtained a positive value for the discriminant and real roots. Many candidates do not understand the difference between factors of an expression and roots of an equation. Consequently, the 'root' $z + 2$ was often offered alongside the two roots of the quadratic equation. Candidates were often so involved in their work on the quadratic that they forgot all about the root $z = -2$.
- (b) Many candidates misunderstood the notation $p(z^2)$ and did a lot of work squaring the original cubic expression and then attempting to factorise it. Some even started with a factorised form with three pairs of repeated roots and attempted to multiply out and simplify their expression. A minority of candidates recognised that the roots of the new equation would be the square roots of their original values. Most of these gave correct simplifications of $\sqrt{-2}$. Some left the remaining four roots in square root form, but others did go on to complete a correct process for finding the square root of a complex number. Of these candidates, most assumed that the roots would be of the form $a + ib$ and set up and solved equations to find the values of a and b . The modulus argument method also works well for this particular pair of numbers, giving exact values relatively easily.

Answers: (i) 6; (ii)(a) $-2, -2 \pm 2\sqrt{3}i$, (b) $\pm i\sqrt{2}, \pm(1+i\sqrt{3}), \pm(1-i\sqrt{3})$.



MATHEMATICS

Paper 9709/41

Paper 41

General comments

Candidates generally answered **Questions 4(i)** and **5(i)** with confidence. Each of these questions required an ability to use formulae for motion with constant acceleration in a straight line. Unfortunately candidates generally demonstrated only a limited understanding of any other topic examined.

Comments on specific questions

Question 1

Almost all candidates scored either full marks or no marks. Candidates with an understanding of the subject matter dealt with the question with apparent confidence and generally scored full marks.

Question 2

There was no apparent difficulty in this question for candidates who recognised the need to apply Newton's second law in conjunction with the formula connecting power, force and velocity for a force acting in the direction of motion. This was required firstly to find an acceleration in straightforward circumstances, and secondly to find the velocity when the acceleration is halved in otherwise unchanged circumstances.

Question 3

This was the worst attempted question with the majority of candidates scoring no marks. A considerable proportion of candidates introduced acceleration into the question, notwithstanding the clear reference in the question to 'hanging freely in equilibrium'.

Few candidates recognised that the tensions in the strings meeting at X are 5.5 N, 7.3 N and W N, which would have revealed that the question is one on the equilibrium of three forces acting at a point.

Many candidates assumed without justification that each of the angles AP_1X and CP_2X is 45° .

Question 4

Part **(i)** was well attempted and most candidates scored both marks. In part **(ii)** candidates who recognised that the acceleration is a function of time found this part of the question straightforward and usually scored full marks. However many candidates did not recognise this and did not make use of calculus.

Question 5

Part **(i)** was well attempted and most candidates scored both marks. Thereafter further scoring was usually limited to single marks for finding the kinetic energy gain or the frictional force in part **(ii)**, and the distance AX in part **(iii)**.

Question 6

Parts **(i)(a)** and **(i)(b)** were reasonably well attempted, although few candidates made progress in part **(c)** or part **(ii)**.

Part **(i)(c)** was better attempted than part **(ii)**, perhaps because the actual values of v_B and v_G are more accessible in part **(i)(c)** (4 and 7.75) than in part **(ii)** (2.83 and 7.21).



Question 7

A significant proportion of candidates introduced acceleration into part (i) of the question, notwithstanding the clear reference in the question to ‘the system is in limited equilibrium’.

Candidates should be aware that in most questions of this type, the normal reaction is found by resolving forces in the direction of the normal to the surface. Some candidates made the incorrect statement $R = W$.

Although candidates were more comfortable with part (ii), few candidates applied Newton’s second law to Q correctly. One common mistake made was that the force of magnitude 3.2 N was not removed. Another was that the same frictional force was used as in part (i); in some cases the second mistake arose as a consequence of the first.



MATHEMATICS

Paper 9709/42

Paper 42

General comments

Among the early questions candidates found **Question 2** the easiest and **Question 3** the most difficult. Among the later questions, **Question 6** was found to be the easiest and **Question 5** the most difficult.

Misconceptions among weaker candidates included: $\sin(A + B) = \sin A + \sin B$; PE gain (always) equals KE loss; if an initial value is given in a question it must be equal to some constant of integration.

Comments on specific questions

Question 1

In the first sentence of the question it is stated that a block rests on *horizontal* ground. Some candidates nevertheless had their block resting on a plane inclined at 15° to the horizontal. The information in the second sentence of the question was not applied correctly by the many candidates who had the force acting at 15° above the horizontal, and not at 15° to the upwards *vertical* as stated in the question.

Candidates should be aware that in most questions of this type, the normal reaction is found by resolving forces in the direction of the normal to the surface. Some candidates made the incorrect statement ' $R = W$ '.

Question 2

Almost all candidates recognised the need to apply Newton's second law in this question. Some candidates omitted the component of the cyclist's weight and some did not deal correctly with the information that the rate of working is 400 W and the speed is 4 ms^{-1} .

Question 3

Most candidates used the strategy of resolving forces in the 'x-direction' and the 'y-direction' and equating the two resulting expressions for F . In proceeding further many candidates used the incorrect ' $\cos(90^\circ - \alpha^\circ) = 0 - \cos \alpha^\circ$ ' and the incorrect ' $\sin(90^\circ - \alpha^\circ) = 1 - \sin \alpha^\circ$ ' and thus met with the difficulty of trying to solve $11\sin \alpha^\circ - \cos \alpha^\circ = 1$.

Some candidates recognised that the resultant of the two forces of magnitude $F\text{ N}$ must be equal and opposite to the resultant of the two forces of magnitudes 5 N and 6 N . This was occasionally exploited to obtain the value of F from $2F^2 = 5^2 + 6^2$, but few candidates used the fact that the direction of the resultant of the two forces of magnitudes 5 N and 6 N is at 45° anticlockwise from the dotted line in the diagram on the question paper.

Question 4

- (i) This part was generally well attempted. Some candidates gave 40 J as the answer for both KE loss and PE gain.
- (ii) Most candidates had the relevant three terms in linear combination, but not always with correct signs.
- (iii) This part was well attempted, many candidates benefitting from the 'follow through' attached to the accuracy mark.



Question 5

- (i) This part of the question was much better attempted than the second part. Some candidates used the 25 ms^{-1} and the 0.4 s , which have no relevance in this part. Other mistakes arose from the inequality. For example $t > 1$ and $t > 3$ was the answer given by several candidates, and many abandoned their attempt after writing $20 - 5t^2 > 15$.

Many candidates obtained the values $t = 1$ and $t = 3$ but did not proceed to an acceptable answer (2 s or $1 < t < 3$).

- (ii) This part of the question was poorly attempted. Many candidates tried to proceed directly to the required velocities, without realising the need to find the time when P and Q are at the same height. The work of such candidates was varied and often muddled.

Among the candidates who equated expressions for height gained by P and by Q at time t , most made one of two common errors. One such error was to use t as the time taken for both P and Q , which was very quickly found to be inappropriate and the attempt was usually abandoned. The other error was to use the elapsed time of Q 's motion to be $t + 0.4$ instead of $t - 0.4$ at the instant when the elapsed time of P 's motion is t .

In using $v = u - gt$ to find the required velocities, many candidates who reached this stage used the same value of t for finding both v_P and v_Q .

Question 6

- (i) Almost all candidates who used the idea of area representing distance were successful in scoring both marks. The few candidates who used $\frac{1}{2}(0 + v)t + \frac{1}{2}(v + 0)(2.5 - t) = 4$ were almost all successful. However candidates who used either $\frac{1}{2}(0 + v) \times 2.5 = 4$ or $\frac{1}{2}(v + 0) \times 2.5 = 4$ had implicitly assumed that the P 's acceleration is the same throughout the 2.5 s of motion. A look at the given graph shows the assumption to be unjustifiable and the 'correct' answer does not therefore arise from correct work.

Candidates who used $2 \times \frac{1}{2} \times 1.25v = 4$ made the implicit assumption that the maximum speed occurs when $t = 1.25$. Although the assumption is unjustifiable, it is nevertheless plausible, and such candidates were eligible for some credit.

- (ii) This part of the question was very well attempted. Some candidates used the gradient property for acceleration and some used $V = 0 + at$.
- (iii) This part of the question was well attempted. The use of the idea of area representing distance was almost universally understood. Sometimes two triangles and a rectangle were used, sometimes one triangle and a trapezium, and sometimes one trapezium.
- (iv) Almost all candidates used the gradient property for acceleration and many correctly obtained $a = -1$. However many such candidates did not follow this with $d = 1$. Some candidates found 'd = -1'.



Question 7

- (i) This part was well attempted by candidates who recognised the need to use calculus. A significant minority of candidates used $a = \frac{v - u}{t}$ leading to $a = 0.002t^2 - 0.12t + 1.8$ and hence, in some cases, to $a = 0 \Rightarrow t = 30, 30$.

Some candidates who did not differentiate to find $a(t)$ nevertheless used integration to find $s(t)$. Thus a large majority of candidates scored the method mark for integration and many of them also scored the accuracy mark. The use of limits or its equivalent was not so well attempted. Some candidates just used the initial value (of velocity) as the constant of integration and a significant number of candidates used limits from 10 to 30.

- (ii) Almost all candidates who scored the accuracy marks for the values of T_1 and T_2 also scored the mark for finding the required velocity in this part of the question.

In many cases the curves sketched by candidates had adjacent segments meeting at cusps instead of a smooth maximum and a smooth minimum. Many candidates' graphs did not consist of a curve or curves, but consisted of 3 line segments, one from (0, 5) to (10, 13), one from (10, 13) to (30, 5) and one upwards from (30, 5) with positive gradient.



MATHEMATICS

Paper 9709/43

Paper 43

General comments

This paper was generally well attempted and many good scripts were seen. Most candidates were able to answer at least part of each question although some struggled on the last question.

The presentation of work for many candidates was of a high standard and most answers were given to the required accuracy. Exceptions to this were the final answers in **Questions 3(ii)** and **7(ii)**.

When a question begins ‘Show that’, as in **Questions 3(i), 6(i) and 7(iii)**, candidates need to check that they have fully answered the question and that they have reached the given statement accurately.

Comments on specific questions

Question 1

The majority of candidates answered the first part of this question correctly. Those who calculated $a = g \sin \theta$ were able to use constant acceleration formulae correctly to answer both parts of the question. However, some candidates assumed that the acceleration was $g \text{ ms}^{-2}$. Many candidates used the energy principle to answer part (i) and some were then unsure how to proceed in part (ii). A few candidates thought that in part (ii) the distance travelled was still 0.9 m.

Answers: (i) 3 ms^{-1} ; (ii) 4 ms^{-1} .

Question 2

- (i) Most candidates equated kinetic and potential energies appropriately to obtain the speed of the particle at *B*. However, some candidates misinterpreted the situation using the constant acceleration formula $v^2 = u^2 + 2as$ and obtaining 6 ms^{-1} from incorrect work.
- (ii) Some candidates formed a work / energy equation for the motion from *A* to *C* whilst others formed the equation for motion from *B* to *C*. A common error was to include an additional term using the potential energy at *A* and the kinetic energies at both *B* and *C*. A sign error for the work done against the resistance was sometimes seen. Occasionally candidates appeared to be finding the resistance rather than the work done even though this was equated to the loss in kinetic energy.

Answers: (i) 6 ms^{-1} ; (ii) 2.75 J.

Question 3

This question was found to be one of the more challenging questions.

- (i) A variety of suitable methods were used to show that the tension in the string was 3 N. The most common methods seen were to resolve in the direction of the resultant, to resolve horizontally or vertically or to use the sine rule or the cosine rule usually on a 120° triangle. Some candidates appeared to look for a calculation that would provide the given value of 3, such as $3\sqrt{3}\tan 30^\circ$ or even $3\sqrt{3}\sin 30^\circ$ which to the nearest whole number rounds to 3.



- (ii) The resolution of the forces on Q should have led to $T = F + mgsin30^\circ$ in order to find the mass of Q. Common errors were to omit the weight component leading to a mass of 0.462 kg or to use an incorrect frictional force such as $F = \mu mg$ or $F = \mu mgsin30^\circ$. Some solutions assumed acceleration of the system rather than equilibrium. Others gave the two significant figure answer 0.26 kg rather than three significant figures as required.

Answer: (ii) 0.261 kg.

Question 4

This question was well answered with many candidates scoring full marks. A common error in part (i) was to miscalculate the velocity when $t = 60$ usually obtaining either -30 from -0.5×60 , -27 from $3 + -0.5 \times 60$, 3 from 0.5×6 or -3 from -0.5×6 . Such calculation errors often resulted in a similarly incorrect sketch graph. Those candidates who obtained a negative value for v when $t = 60$ in part (i) and then attempted to calculate areas to find the distance XY in part (ii) often treated the third stage of the journey as the area of one triangle rather than two triangles (above and below the time axis). An alternative method for calculating the distance using constant acceleration formulae was less frequently seen. When this method was used some candidates obtained 900 m from applying $s = ut + \frac{1}{2}at^2$ with $u = 0$, $a = 0.5$ and $t = 60$ rather than considering the three stages of the motion.

Answers: (i) 3 ms^{-1} , 0 ms^{-1} ; (ii) 165 m.

Question 5

- (i) Whilst many candidates found the correct values of F and α , these did not always follow fully correct solutions. Negative signs in working sometimes disappeared in answers leaving inconsistent work e.g. $F\cos\alpha = 27.5$ and $F\sin\alpha = -24$ leading to $\alpha = 41.1^\circ$.
- (ii) Some candidates worked with the second force acting at 90° clockwise rather than anticlockwise from the force of magnitude F N whilst others confused the directions of the x - and y -axes. Amongst those who had a correct diagram, the angles 67.4° and 48.9° were the most common incorrect values given for angle θ .

Answers: (i) 36.5, 41.1; (ii) 94.9, 26.3.

Question 6

- (i) Although many candidates used $a = \frac{dv}{dt}$ for the first stage of motion and obtained $a = 0$ when $t = 5$, only some continued to consider the acceleration in the second stage of motion. Candidates were expected to show that the acceleration when $t = 5$ was the same for both functions of t rather than that it was zero for one (or both) of the functions. Some candidates showed no change in velocity when $t = 5$ rather than no change in acceleration.
- (ii) Candidates, on the whole, knew that integration and the use of limits was needed to find the distance AB. Some used the substitution of $t = 5$ but not $t = 10$ for the second integral obtaining the distance $AB = 35$ m but from incorrect working. Others who used a constant of integration and thus obtained $s_2(10) = 35$ did not always realise that this was the distance AB, adding $s_1(5) = 15$ to give 50 m as their answer.

Answer: (ii) 35 m.

Question 7

The first and last parts of this question were the best attempted with many fully correct answers seen for part (i). In part (ii) those who omitted to consider the resistance found $a = 0.642 \text{ ms}^{-2}$, whilst some of those who calculated correctly gave their answer correct to 3 significant figures rather than 3 decimal places. Many candidates seemed to be unsure what was required in parts (iii), (iv) and (v). In part (iii) the best solutions considered the implication of increasing speed rather than showing the solution of a velocity equation formed using $a = 0$ which suggested constant speed. In contrast, the final part of the question, part (vi), required the assumption that speed was constant although a few candidates attempted to use constant acceleration formulae with the acceleration found earlier in the question. Candidates should check the units used to prevent errors such as $t = \frac{1200}{24000}$ instead of $t = \frac{1200}{24}$.

Answers: (i) 800; (ii) 0.002 ms^{-2} ; (v) 30 ms^{-1} , 0.1 ms^{-1} ; (vi)(a) 50 s, (b) 1500 m.



MATHEMATICS

Paper 9709/51

Paper 51

General comments

Compared to previous papers, more candidates drew clear diagrams to help with their solutions and aid the clear presentation of their work.

Not many candidates used premature approximations and most candidates gave their answers to the required accuracy. On this paper $g = 10$ should be used and few candidates used $g = 9.8$ or 9.81 . The formulae list can be used by candidates to check that they are using the correct formula.

Comments on specific questions

Question 1

The use of $F = mr\omega^2$ was readily used by candidates in order to find the horizontal component of the force. Unfortunately some candidates thought that this was the required answer. The final answer which comes from $\sqrt{1.62^2 + 0.5^2}$ was not often seen. A minority of the candidates misread the question and used a speed of 9 ms^{-1} rather than the angular speed of 9 rad s^{-1} given.

Answer: 1.7 N.

Question 2

The correct formula for the centre of mass of a circular arc was used by most of the candidates. This question was generally well done.

Answer: 0.0649.

Question 3

- (i) A common mistake was to use 0.2 m for the radius instead of $0.2\cos 30^\circ$ m. Newton's second law could be used with the horizontal component of the tension in order to find the tension. Having found the tension an equation in 3 terms could be set up by equating vertical forces to find the required force.
- (ii) Most candidates realised that the contact force would become zero at the greatest possible speed.

Answers: (i) 1.5 N; (ii) 1.73.



Question 4

- (i) Most candidates attempted to take moments about A. Sometimes in doing this mistakes were made by mixing up sines and cosines.
- (ii) The principle of resolving horizontally and vertically was generally used and then the angle was calculated by using the two components.

Answers: (i) 562.5 N; (ii) 52.1° to the downward vertical.

Question 5

- (i) A common mistake was to say $T\cos\theta = 0.28g$ instead of $2T\cos\theta = 0.28g$ where θ is the angle the two parts of the string make with the vertical. Having found T most candidates went onto use $T = \frac{\lambda x}{l}$.
- (ii) Most candidates realised that they needed to set up an energy equation but unfortunately sometimes only 3 terms were present instead of 4 terms. The elastic energy at the start was usually omitted.

Answer: (ii) 2.75 ms^{-1} .

Question 6

- (i) The correct differential equation was usually set up and both marks often scored.
- (ii) Many candidates scored well on this part of the question. When integrating $\frac{1}{15-v}$ candidates who made mistakes often obtained $\ln(15 - v)$ and not $-\ln(15 - v)$. Sometimes the constant of integration was omitted or the limits of the integration were not used and so no further progress was made.
- (iii) Candidates often made mistakes when integrating $e^{\frac{t}{9}}$ and again the constant of integration was not seen.

Answer: (iii) 49.7 m.

Question 7

- (i) Sometimes candidates just quoted the trajectory equation as given in the formula booklet and then substituted $u = 10$ and $\theta = 45^\circ$. This did not answer the question asked and obtained no credit.
- (ii) Some candidates used $\tan 30^\circ = \frac{v_y}{v_x}$ instead of $\tan 30^\circ = \frac{y}{x}$.
- (iii) Few candidates realised that if the trajectory equation was differentiated and $\frac{dy}{dx}$ found and equated to $\tan\theta$ then by substituting the value of x found in part (ii) the required angle could be found. Most candidates tried to find the time taken to reach A and then used $\tan\theta = \frac{v_y \text{ at } A}{v_x \text{ at } A}$.

Answers: (i) $x = (10 \cos 45^\circ)t$, $y = (10 \sin 45^\circ)t - \frac{1}{2}gt^2$, $y = x - \frac{1}{10}x^2$; (ii) 4.23; (iii) 8.79° .



MATHEMATICS

Paper 9709/52

Paper 52

General comments

Compared to previous papers, more candidates drew clear diagrams to help with their solutions and aid the clear presentation of their work.

Most candidates gave their answers to the required accuracy and few candidates used premature approximations. On this paper $g = 10$ should be used and few candidates used $g = 9.8$ or 9.81 . The formulae list can be used by candidates to check that they are using the correct formula.

Comments on specific questions

Question 1

The use of $F = mr\omega^2$ was readily used by candidates in order to find the horizontal component of the force. Unfortunately some candidates thought that this was the required answer. The final answer, which comes from $\sqrt{1.62^2 + 0.5^2}$, was not often seen. A minority of the candidates misread the question and used a speed of 9 ms^{-1} rather than the angular speed of 9 rad s^{-1} given.

Answer: 1.7 N.

Question 2

The correct formula for the centre of mass of a circular arc was used by most of the candidates. This question was generally well done.

Answer: 0.0649.

Question 3

- (i) A common mistake was to use 0.2 m for the radius instead of $0.2\cos 30^\circ$ m. Newton's second law could be used with the horizontal component of the tension in order to find the tension. Having found the tension an equation in 3 terms could be set up by equating vertical forces to find the required force.
- (ii) Most candidates realised that the contact force would become zero at the greatest possible speed.

Answers: (i) 1.5 N; (ii) 1.73.

Question 4

- (i) Most candidates attempted to take moments about A. Sometimes in doing this mistakes were made by mixing up sines and cosines.
- (ii) The principle of resolving horizontally and vertically was generally used and then the angle was calculated by using the two components.

Answers: (i) 562.5 N; (ii) 52.1° to the downward vertical.



Question 5

- (i) A common mistake was to say $T\cos\theta = 0.28g$ instead of $2T\cos\theta = 0.28g$ where θ is the angle the two parts of the string make with the vertical. Having found T most candidates went onto use $T = \frac{\lambda x}{l}$.
- (ii) Most candidates realised that they needed to set up an energy equation but unfortunately sometimes only 3 terms were present instead of 4 terms. The elastic energy at the start was usually omitted.

Answer: (ii) 2.75 ms^{-1} .

Question 6

- (i) The correct differential equation was usually set up and both marks often scored.
- (ii) Many candidates scored well on this part of the question. When integrating $\frac{1}{15-v}$ candidates who made mistakes often obtained $\ln(15 - v)$ and not $-\ln(15 - v)$. Sometimes the constant of integration was omitted or the limits of the integration were not used and so no further progress was made.
- (iii) Candidates often made mistakes when integrating $e^{\frac{t}{9}}$ and again the constant of integration was not seen.

Answer: (iii) 49.7 m.

Question 7

- (i) Sometimes candidates just quoted the trajectory equation as given in the formula booklet and then substituted $u = 10$ and $\theta = 45^\circ$. This did not answer the question asked and obtained no credit.
- (ii) Some candidates used $\tan 30^\circ = \frac{v_y}{v_x}$ instead of $\tan 30^\circ = \frac{y}{x}$.
- (iii) Few candidates realised that if the trajectory equation was differentiated and $\frac{dy}{dx}$ found and equated to $\tan\theta$ then by substituting the value of x found in part (ii) the required angle could be found. Most candidates tried to find the time taken to reach A and then used $\tan\theta = \frac{v_y \text{ at } A}{v_x \text{ at } A}$.

Answers: (i) $x = (10 \cos 45^\circ)t$, $y = (10 \sin 45^\circ)t - \frac{1}{2}gt^2$, $y = x - \frac{1}{10}x^2$; (ii) 4.23; (iii) 8.79° .



MATHEMATICS

Paper 9709/53

Paper 53

General comments

Compared to previous papers, more candidates drew clear diagrams to help with their solutions and aid the clear presentation of their work.

Not many candidates used premature approximations and most candidates gave their answers to the required accuracy. On this paper $g = 10$ should be used and few candidates used $g = 9.8$ or 9.81 . The formulae list can be used by candidates to check that they are using the correct formula.

Comments on specific questions

Question 1

This question was generally well answered by the candidates. Errors when they occurred were usually made in finding the centre of mass of the triangle. Occasionally the angle found was with the horizontal and not the vertical.

Answers: (i) 0.4 m, 0.7 m; (ii) 29.7° .

Question 2

- (i) A common error made in this question was to substitute $v = 5$ and not $v = -5$ resulting in $t = 0.8$ and not $t = 1.8$ when using $v = u + at$. This does give $s = 7.2$ but this is the height when the particle is on the way up.
- (ii) Some candidates worked out the time to the highest point and used this time to find OA. This time should have been doubled.

Answers: (i) 12.5° below the horizontal, 7.2 m; (ii) 2.6 s, 58.5 m.

Question 3

- (i) This part of the question was generally answered well.
- (ii) When resolving vertically the tension in PQ was often omitted, resulting in $T \cos \alpha = 0.8g$. The horizontal resolution was usually carried out correctly.

Answers: (i) 4 N, 3 N; (ii) 13.4 N, 26.6° .

Question 4

- (i) When moments were taken about A for the rod the value of the applied force at B was immediately found. Candidates who took moments about the centre of the rod and then resolved parallel to the plane often made sign errors or a calculation error when solving the simultaneous equations.
- (ii) A number of approaches can be used in this part of the question in order to find the magnitude of the frictional force. Most candidates answered this part well.
- (iii) Occasionally when resolving perpendicular to the plane candidates omitted the force applied at B.

Answers: (ii) 3.75 N; (iii) 0.247.



Question 5

- (i) This part of the question was well answered.
- (ii) When setting up the energy equation some candidates did not included the elastic energy at the start.

Answer: (ii) 2.42 ms^{-1} .

Question 6

- (i) Most candidates used Newton's second law with three force terms to set up the required differential equation.
- (ii) This part of the question was well done by the majority of the candidates.
- (iii) Very few candidates managed to answer this part of the question correctly. Often the original differential equation was used. A new differential equation was required as the particle has now started to move down the plane.

Answers: (ii) 0.204 ; (iii) 0.4 ms^{-1} .



MATHEMATICS

Paper 9709/61

Paper 61

General comments

This paper proved accessible to the majority of candidates and there was a wide range of marks. Candidates generally expressed themselves clearly and concisely. There was no evidence that lack of time was a problem for candidates.

Comments on specific questions

Question 1

The majority of candidates found this a straightforward first question. There were some who tried to expand $\sum(t - \bar{t})^2$, not realising that this formula for the variance is given in the tables, so all they needed to do was look it up.

Answers: 18.2, 4.19.

Question 2

Many candidates recognised that this question used a normal approximation to the binomial distribution. They found the mean and variance and the majority used a continuity correction and used the correct probability area. A few candidates used the binomial distribution, which was not penalised since the question did not stipulate that a normal approximation should have been used. However, it took a long time and accuracy was invariably lost.

Answer: 0.807.

Question 3

The first part of this question concerned a normal distribution. Again, those candidates who were familiar with the normal distribution had no problems in standardising and finding the correct probability area. The question then continued by asking candidates how many of 350 candidates met the required conditions. Candidates had to multiply their probability by 350 but many omitted to do this and thus lost the final mark. In part (ii) ‘very slow’ students were defined to be those who took a longer time than 1.645 standard deviations above the mean. Candidates had to find the probability associated with this z-value, which meant looking up 1.645 backwards in the normal distribution tables, to get 0.95. Thus the required probability was 0.05. Many candidates did not appreciate that the z-value is the number of standard deviations from the mean and added on 1.645 standard deviations to the mean, and then standardised to end up with $z = 1.645$. This wasted a lot of time and again many lost accuracy. The binomial was recognised and used by the majority of candidates.

Answers: (i) 335 or 336; (ii) 0.994.



Question 4

Most candidates knew that frequency density = frequency / class width. A common mistake was to say the class width was 9 instead of 10 ($10.5 - 0.5$) in the first interval. Candidates then found the new height, and were awarded method marks for knowing what to do. To find an estimate of the mean meant that candidates had to use the mid-intervals multiplied by the frequencies. Some candidates used class widths, some used lower or upper class boundaries and both these scored no marks. Some used the correct frequency ratios but not actual frequencies, and as the x's cancelled out this worked satisfactorily.

Answers: (i) 15, 0.75 cm; (ii) 26.6 grams

Question 5

This turned out to be a straightforward question with many candidates evaluating the associated probabilities of Rick choosing entrance *B* etc., correctly. It was good to note that nearly every candidate multiplied their probabilities when finding the probabilities of independent events happening, and then added these to find the total probability of all the options. The most common mistake was for candidates, when finding the probability of 2 out of 3 friends choosing entrance *B*, to multiply the 2 probabilities together for those who chose entrance *B*, and to forget to multiply this by the probability of the third friend *not* choosing *B*. Another common slip was to forget to add on the probability of all 3 friends choosing entrance *B*. The second part of this question was well done with many candidates scoring full marks.

Answers: (i) $\frac{24}{315}$ or 0.0762; (ii) $\frac{2}{35}$ or 0.0571.

Question 6

The first three parts of this question were well attempted. The remaining two parts were found more difficult and discriminated between candidates. Part (iv) was a generalisation of part (ii) and those candidates who spotted this managed successfully to multiply their answer to part (ii) by 3 and then by 6C_3 . Part (v) was an extension of the other parts. Candidates had to realise that 'any of the 12 pegs' had been covered by 4 different colours, 3 different colours, and only needed 2 different colours to complete the question, and then to add the three options together.

Answers: (i) 360; (ii) 12; (iii) 360; (iv) 720; (v) 1170.

Question 7

Some candidates did not subtract $[E(X)]^2$ when calculating the variance. In part (iv) candidates had to add the probabilities associated with first and second throw scores of (6, 10) and (10, 6) and (8, 8). Some candidates omitted the (8, 8) possibility and some did not appreciate that (6, 10) and (10, 6) are two possibilities. The last part was a conditional probability and 'many candidates recognised this and applied the correct method.'

Answers: (i) $\frac{1}{3}$; (iii) 4.02; (iv) $\frac{6}{81}$ or 0.0741; (v) $\frac{1}{6}$.



MATHEMATICS

Paper 9709/62

Paper 62

General comments

In general, candidates expressed their working clearly and were competent with questions across a range of topics. A few questions asked for an answer that involved candidates having to think carefully about what was happening, and not rushing into the first option that presented itself.

Comments on specific questions

Question 1

Most candidates used the fact that the sum of the probabilities should be equal to 1 to establish a quadratic equation in p . Many candidates had difficulty after that, however, because they could not solve the quadratic equation. Of those who solved it correctly many candidates gave both solutions, not realising that a probability cannot be negative, so $p = -2$ should have been rejected. There were a small number who mistakenly used the formula for the expectation ($\sum xp(x)$) and assumed that would be 1 or 0.

Answer: 0.1.

Question 2

The idea of ‘coding’ the data caused problems and it was apparent that many candidates needed to improve their coded data practice to be confident in answering this question well. Candidates were expected to know that $\sum(x - 50) = \sum x - \sum 50 = \sum x - 16 \times 50$. To evaluate $\sum(x - 50)^2$ they were expected either to use everything in coded form, which comes out in one line, or expand it which takes a lot more time and effort but is a valid method and was successfully used by the majority of those who got the correct answer. In the second part, many candidates found the new mean correctly and the new variance either by using $\sum x^2$ together with $\sum x$, or $\sum(x - 50)^2$ together with $\sum(x - 50)$. The problems arose when candidates tried to mix the two.

Answers: (i) 24, 712; (ii) 52.7, 7.94.

Question 3

This question needed to be read carefully. It consists of two operations, namely throwing a 5-sided spinner and then throwing 2 dice. Many candidates correctly found the numbers on the dice which would generate 12 by multiplying, to be (4, 3) or (3, 4) or (2, 6) or (6, 2), and to generate 12 by adding to be (6, 6). They needed to understand that this gives the probability of obtaining a score of 12, i.e. $P(12) = \frac{4}{36}$ for multiplying

or $P(12) = \frac{1}{36}$ for adding. The question was formulated with the word ‘Given’ which tells candidates that this

is a conditional probability question, requiring two probabilities to be divided. Candidates were given credit for finding $P(\text{even number on spinner and } 12)$ and a further credit for finding $P(12)$. Credit was then given for dividing one by the other. A few candidates used an alternative ‘intuitive’ solution which was given full marks.

Answer: $\frac{8}{11}$.



Question 4

A back-to-back stem-and-leaf diagram is routine work but candidates need to understand that ‘leaves’ can only consist of single digits and the ‘stem’ cannot have decimals. These factors are accounted for in the key, which needs to state what the units are (kg) and also what the substances are (sugar, flour). Clearly if these are missing the stem-and-leaf diagram cannot be expected to gain full marks. The interquartile range was well done and almost all candidates subtracted their lower quartile from their upper quartile.

Answers: (ii) 1.989 kg, 0.034 kg.

Question 5

Both parts of this question were well done with many candidates scoring full marks in part (ii) by looking up $1 - 0.409 (= 0.591)$ backwards in the normal distribution tables, obtaining $z = 0.23$ and solving the standardised equation for d .

Answers: (i) 0.985, 0.988; (ii) 4.97.

Question 6

Those candidates who knew the conditions for a binomial distribution gained full marks for part (i). The remaining two parts were recognised by almost all the candidates, part (ii) as being the binomial distribution and part (iii) as being the normal approximation to the binomial distribution. Candidates should be aware that being requested to find the probability that Julie’s train is late on more than 7 days or fewer than 2 days means finding both probabilities and adding them together. Many candidates found both probabilities separately and did not add them together, thus losing marks in this part and also in the third part, where a similar situation was considered.

Answers: (ii) 0.196; (iii) 0.480.

Question 7

Many correct solutions were obtained for part (i) with candidates recognising that they had to find the probability of each required option, (4M 2W or 5M 1W), and add the probabilities together. The second part used a similar principle. Albert and Tracey were fixed on the committee, leaving 4 places which could be filled by either 3M and 1W or 4M. Candidates need to remember that since Albert and Tracey have been taken out, only 9 men and 8 women remain. The 3M 1W option could be chosen in ${}^9C_3 \times {}^8C_1 (= 672)$ ways and the 4M chosen in ${}^9C_4 (= 126)$ ways. Part (iii) could be solved in a similar manner by considering Albert and not Tracey on the committee, and Tracey but not Albert. Many candidates attempted this but forgot to reduce the number of possible men from 10 to 9. The last part was finding a probability, not the number of ways, and so one mark was given for dividing by 6! to get a fraction.

Answers: (i) 9828; (ii) $\frac{798}{9828} (0.0812)$; (iii) 4494; (iv) $\frac{2}{3} (0.667)$.



MATHEMATICS

Paper 9709/63

Paper 63

General comments

Nearly all candidates were able to show that they understood some of the topics within the syllabus. **Questions 2** and **3** were found to be accessible for most candidates, while **Questions 4** and **6** proved to be more challenging. There was no evidence of problems with the time allowed to complete the paper, except possibly for those who used longer methods than necessary.

Candidates should be aware that probabilities must lie between 0 and 1, so an answer outside this range ought to be checked for errors if time allows.

Comments on specific questions

Question 1

The majority of responses correctly identified the normal distribution as being appropriate in these circumstances. Many of these stated that the mean and variance were the relevant parameters, without appreciating the need to give appropriate numerical values for them.

Answer: normal distribution, mean 60 kg and standard deviation 9 kg (for example).

Question 2

- (i) Nearly all of the candidates produced a table containing x -values 1, 2, 3, 4 and 5. The correct probabilities k , $2k$, $3k$, $4k$ and $5k$ were usually given. The most frequent incorrect probabilities were k , k , k , k and k . Often these were followed through correctly to obtain the answer 3 in part (ii), thereby gaining all 3 method marks. A few candidates overlooked the requirement to write their probabilities in terms of k .
- (ii) The method for calculating $E(X)$ from a probability distribution was understood well, with very few candidates dividing their $\sum px$ by 5.

Answers: (i) $1, k; 2, 2k; 3, 3k; 4, 4k; 5, 5k; \frac{1}{15}$; (ii) $3\frac{2}{3}$ or 3.67.

Question 3

- (i) Candidates demonstrated a sound grasp of tree diagrams. Labelling of the diagram was very well done. Sometimes the branch corresponding to those between 30 and 65 years old who did not use a cell phone during their train journey was omitted, or the probability was given as zero. Very few candidates did not give their probabilities as decimals.
- (ii) There were many correct answers, but conditional probability proved a difficult topic for some candidates. The most frequent errors were to either give 0.68×0.25 only or to calculate the probability of a passenger being between 30 and 65 years by summing three products instead of two, often resulting in a denominator of 0.68 and a final answer of 0.25.

Answer: (ii) 0.842.



Question 4

- (i) This was usually correctly answered. Some candidates did not add 60, leaving their answer as 3.5. A few added 60 to 245 before dividing by 70.
- (ii) There were many correct answers. Several candidates did not multiply the difference between $(x - 60)$ and $(x - 50)$ by 70 before adding to 245.
- (iii) Attempts usually involved the appropriate formula for the standard deviation from the formulae list, occasionally with a mistake in one of the powers. Often 3.5 was used instead of 13.5. Sometimes a candidate used their answer to part (ii) without dividing this by 70. A few candidates used the expansion of $\sum(x - 50)^2$. This was less successful than using the standard deviation formula.

Answers: (i) 63.5; (ii) 945; (iii) 20 600.

Question 5

- (i) Most candidates gave a correct frequency table. Some gave the frequency density as their frequency. A few calculated frequency density divided by class width.
- (ii) This was usually answered correctly, the most frequent error being to use class width or class upper boundary instead of the class mid-point. Several candidates omitted the second frequency of 30 from their calculation.
- (iii) A frequent mistake was to assume that the candidates were selected with replacement, leading to denominators of 194 and 194 instead of 194 and 193. This error was less frequent when candidates used a tree diagram. Attempts involving ${}^{60}C_1 \times {}^{134}C_1 \div {}^{194}C_2$ usually resulted in the correct answer. The normal distribution, involving $\sum fx^2$ to calculate a variance, was not appropriate.

Answers: (i) 44, 34, 30, 30, 36; (ii) 7.55; (iii) 0.427.

Question 6

Candidates often had more difficulty with parts (ii) and (iii) of this question than any other parts of the paper. Weaker candidates often added their different ways instead of multiplying them. Some candidates benefitted from either drawing a sketch or examining some of the possibilities.

- (i) This was answered correctly by most candidates. A few overlooked the instruction to give their answer to three significant figures. Incorrect answers included ${}^{14}C_{12}$, 12! and 14!.
- (ii) Candidates usually identified that the business people could be arranged in $3!$ different ways. Considering the candidates next should have resulted in $5!$. This appeared less frequently than $3!$, with 7P_5 or 6P_5 featuring instead. Several different approaches were used to calculate the 24 different ways of arranging the two couples and candidate in the three sets of two seats which each had a window seat.
- (iii) Attempts involving the number of arrangements divided by the answer to part (i) often overlooked the fact that, since Mrs Brown must sit in the front row only 10, not 11, seats were available to Mrs Lin in order that she might sit behind a candidate. Similarly, when using probabilities, $\frac{11}{14}$ was used instead of $\frac{10}{13}$.

Answers: (i) 43 600 000 000; (ii) 17280; (iii) 0.0687.



Question 7

Some candidates used the normal distribution to calculate the probability that a visit to the certain dentist lasts less than 10 minutes in part (iii) and/or less than 8.2 minutes in part (iv). Continuity corrections were usually correct and applied when relevant. Many candidates realised that parts (iii) and (iv) could be answered independently of parts (i) and (ii).

- (i) This was usually well answered. Occasionally a probability was used instead of a z-value. The most frequent incorrect z-values were 0.803 and 0.81.
- (ii) Many candidates calculated the probability that the time spent visiting this dentist was more than 9.2 minutes and not either more than 9.2 minutes or less than 7.2 minutes. Sketches assisted in the calculation of probabilities.
- (iii) This was usually answered well. A few candidates interchanged 0.21 and 0.79. Sometimes $P(2)$ only was calculated and in other answers either $P(0)$ or $P(2)$ was omitted.
- (iv) The majority of candidates realised that a normal approximation to the binomial should be used. Those candidates who used an incorrect value for the probability often gained all three method marks. The use of an approximation was not stipulated, so candidates could use the binomial distribution. On this occasion, with probabilities of 0.5 and 0.5, the terms all involved 0.5^{34} and addition of the coefficients using a calculator could be completed in a relative short space of time.

Answers: (i) 2.23; (ii) 0.654; (iii) 0.112; (iv) 0.250.



MATHEMATICS

Paper 9709/71

Paper 71

General comments

On this paper, candidates were largely able to demonstrate and apply their knowledge in the situations presented. There was a wide range of scores with some very good scripts. In general, candidates scored well on **Questions 2, 7(b)(i),(ii)**, whilst **Question 6** proved particularly demanding.

Questions requiring an explanation ‘in context’ continue to challenge candidates. It is important that concluding statements made in hypothesis testing questions are related to the question and are not ‘definite’ statements. For example in **Question 5**, the conclusion was to reject H_0 ; a statement such as ‘There is evidence that the Grinford mean is lower’ would be an appropriate statement, whereas a definite statement that ‘The Grinford mean is lower’ would not be an appropriate statistical claim. The null hypothesis has not been ‘disproved’, it has only had contradictory evidence provided by the sampling information.

It is important for candidates to adhere to the 3 significant figure accuracy required on final answers. This means that greater accuracy (at least 4 significant figures) should be kept throughout the working in order for the final answer to be correct to 3 significant figures. On the whole, presentation was good and an adequate amount of working was shown by candidates; without adequate working marks are sometimes not earned. Lack of time did not appear to be a problem for candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also many very good and complete answers.

Comments on specific questions

Question 1

Many candidates answered this question well. The z-value of 1.645 was usually correctly found. It is important that candidates do not confuse the confidence interval for a population proportion (as required here) with the confidence interval for the population mean. Candidates that were successful on this question quoted, and were able to correctly apply, the appropriate formula.

Answer: [0.580, 0.630].

Question 2

A Poisson distribution was required for this question. This was recognised by the majority of candidates, though the value of λ was not always correctly found.

In part (i), the value required for λ was $\frac{10}{3}$, and it was important here that this value was not prematurely approximated; if a value of 3.3 were to be used in the expression for $P(4)$, an accuracy error would occur in the final answer. In part (ii), again λ was not always correctly found, but on the whole a correct method for finding ‘fewer than three people’ was used.

Answers: (i) 0.184; (ii) 0.125.



Question 3

This question tested the approximation of a binomial distribution to a Poisson distribution. It is important that candidates understand what is meant by the ‘exact distribution’ and ‘approximate distribution’ and know, and can apply, the appropriate conditions for an approximation to be valid. Many candidates used an invalid approximation to a normal distribution.

Answers: (i) $B(40\ 000, 0.0001)$; (ii) $Po(4)$; (iii) 0.567.

Question 4

Some candidates approached part (i) by using the diagram to find the area between 1 and 1.5. On the whole this was a successful method. Other candidates attempted to find the equation of the straight line, and then integrate. This was often successfully done and the equation of the line was then used for the rest of the question. The equation of the line was $y = \frac{1}{2}x$; errors included finding an incorrect gradient and using a non-zero intercept. Many candidates used a correct method in parts (ii) and (iii). On the whole once an expression for $f(x)$ had been found, this was a well attempted question.

Answers: (i) $\frac{5}{16}$; (ii) $\sqrt{2}$; (iii) $\frac{4}{3}$.

Question 5

The calculation of $E(F)$ and $\text{Var}(F)$ in part (i) was generally done well. Many candidates realised that the variance was found by calculating $5.6^2 + \frac{1}{4} \times 12.4^2$; some made the common error of calculating $5.6^2 + \frac{1}{2} \times 12.4^2$. In part (ii) it was necessary to state the null and alternative hypotheses, which should have been in terms of the ‘population mean’ μ (or in this case the ‘Grinford mean’), which was 54 and not 49 as commonly stated. The z-value (-1.893) was usually correctly calculated, and a comparison with -1.645 was then required. It is important that this comparison is clearly shown to justify the conclusion made. An alternative clear comparison between the probability ($z < -1.893$) and 0.05 was also acceptable. The final conclusion ('there is evidence that the Grinford mean is lower') needed to be clearly stated with no contradictory statements, and, as mentioned above, it is preferable not to give a definite conclusion.

Answers: (i) 54, 69.8.

Question 6

This question proved to be particularly challenging for a large number of candidates. The null and alternative hypotheses needed to be clearly stated, and many errors were made here. H_0 should have been $P(6) = \frac{1}{6}$, some candidates said $\mu = \frac{1}{6}$, but the main error was to say $P(6) = \frac{3}{10}$ which then led to incorrect values for p in the subsequent binomial expressions. Some candidates calculated $P(3)$ or $P(<3)$ rather than $P(>3)$ and in some cases the wrong distribution was used. A clear comparison with 0.1 was required to justify the conclusion. As in Question 5, the conclusion should not be definite. In part (ii), many candidates were unable to find the probability of a Type I error. Part (iii) needed to be in the context of the question and not merely a text book definition of a Type II error. Again this was not answered well.

Answers: (ii) 0.0697; (iii) Concluding that the die is fair when it is biased.



Question 7

Parts **(b)(i)** and **(ii)** were well attempted by many candidates. In part **(a)** it was important for candidates to realise why sampling was necessary; some candidates realised that the population in part **(b)(i)** would be large, but few realised that testing involved destruction in part **(b)(ii)**. In part **(b)** unbiased estimates were required; a number of candidates calculated a biased estimate for the variance, or confused the two formulae obtaining a totally incorrect answer. Errors in part **(b)(ii)** included standard deviation and variance mixes, and some candidates thought the sample size was 560, or even still 500. Consideration of the underlying statistical theorem (the Central Limit Theorem) showed a lack of understanding on the part of some candidates. Candidates needed to realise that the distribution of the population (X) was not known, so the Central Limit Theorem was required.

Answers: **(a)(i)** Population too large, **(ii)** Testing involves destruction; **(b)(i)** 19.7, 0.160, **(ii)** 0.281,
(iii) Yes; X is not necessarily normal, and the sample is large.



MATHEMATICS

Paper 9709/72

Paper 72

General comments

On this paper, candidates were largely able to demonstrate and apply their knowledge in the situations presented. There was a wide range of scores with some very good scripts. In general, candidates scored well on **Questions 2, 7(b)(i),(ii)**, whilst **Question 6** proved particularly demanding.

Questions requiring an explanation ‘in context’ continue to challenge candidates. It is important that concluding statements made in hypothesis testing questions are related to the question and are not ‘definite’ statements. For example in **Question 5**, the conclusion was to reject H_0 ; a statement such as ‘There is evidence that the Grinford mean is lower’ would be an appropriate statement, whereas a definite statement that ‘The Grinford mean is lower’ would not be an appropriate statistical claim. The null hypothesis has not been ‘disproved’, it has only had contradictory evidence provided by the sampling information.

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Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also many very good and complete answers.

Comments on specific questions

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Many candidates answered this question well. The z-value of 1.645 was usually correctly found. It is important that candidates do not confuse the confidence interval for a population proportion (as required here) with the confidence interval for the population mean. Candidates that were successful on this question quoted, and were able to correctly apply, the appropriate formula.

Answer: [0.580, 0.630].

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A Poisson distribution was required for this question. This was recognised by the majority of candidates, though the value of λ was not always correctly found.

In part (i), the value required for λ was $\frac{10}{3}$, and it was important here that this value was not prematurely approximated; if a value of 3.3 were to be used in the expression for $P(4)$, an accuracy error would occur in the final answer. In part (ii), again λ was not always correctly found, but on the whole a correct method for finding ‘fewer than three people’ was used.

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Question 4

Some candidates approached part (i) by using the diagram to find the area between 1 and 1.5. On the whole this was a successful method. Other candidates attempted to find the equation of the straight line, and then integrate. This was often successfully done and the equation of the line was then used for the rest of the question. The equation of the line was $y = \frac{1}{2}x$; errors included finding an incorrect gradient and using a non-zero intercept. Many candidates used a correct method in parts (ii) and (iii). On the whole once an expression for $f(x)$ had been found, this was a well attempted question.

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Answers: (ii) 0.0697; (iii) Concluding that the die is fair when it is biased.



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Answers: **(a)(i)** Population too large, **(ii)** Testing involves destruction; **(b)(i)** 19.7, 0.160, **(ii)** 0.281,
(iii) Yes; X is not necessarily normal, and the sample is large.



MATHEMATICS

Paper 9709/73

Paper 73

General comments

Many good scripts were seen, with many candidates showing a useful understanding of the topics in the syllabus. Much of the work was well presented and the methods chosen by the candidates were clearly presented. The required 3 significant figure accuracy was usually in evidence. In general candidates scored well on **Questions 1, 3(i) and 4** whilst **Questions 2, 3(iii), 6(ii)** and **7** proved more demanding. Most candidates attempted all of the questions in the available time.

In the significance testing questions candidates should not express their conclusions as 'definite' statements. The null hypothesis has not been 'proved' or 'disproved', but has only had supporting or contradictory evidence provided by the sampling information. Examples appear in **Questions 6(i) and 7(i)**, and in similarly in **Question 3(ii)**.

Comments on specific questions

Question 1

Most candidates realised that the normal distribution was the appropriate approximating distribution and stated that the mean was 31. It was also necessary to state that the standard deviation was $\sqrt{31}$ (or 5.57). Stating that the variance was 31 was not sufficient.

Answers: normal; 31, $\sqrt{31}$ (5.57).

Question 2

- (i) Two distinct reasons why the readers who returned the questionnaire would not form a random sample were required. One reason involved the idea that only readers who were not too busy would return the questionnaire. Candidates who stated this idea in reverse form as 'those readers who were too busy would not return it' were also correct. Some candidates correctly identified possible groups of people such as retired people, housewives or students who would meet this requirement. The second reason required candidates to notice that the questionnaire was available only to readers who bought that particular issue of the magazine.
- (ii) As the regular readers were numbered from 1 to 7302 three randomly generated 4-digit numbers were required. The random numbers given in the question were presented in blocks of 5 digits as is commonly found in tables. Some candidates correctly found the required set of numbers by treating the given numbers as a continuous list of 20 individual random numbers and using successive sets of 4-digit numbers, giving 4975, 7802, 3952, 0386, from which 7802 is discarded as being too large. Alternative consistent methods that produced a set of 3 randomly generated 4-digit numbers as required were accepted. For example, candidates who took the first 4 digits from each given block and discarded any that were too large, giving 4975, 5203, 6088 scored both marks. Many attempts such as dividing the given blocks by 50 were not valid.

Answer: (ii) 4975, 3952, (0)386 (for example).



Question 3

- (i) Many candidates correctly found the 99% confidence interval. Some candidates incorrectly used 2.326 instead of 2.576 for the critical z-value.
- (ii) A good number of candidates correctly stated that the claim was not supported. The reason given for this was often not clear or incorrect. Some candidates thought that 29.9 was close to 30 and suggested that the claim was supported, which was incorrect.
- (iii) Some candidates realised that the different conclusion indicated a different confidence interval, which in turn indicated a different sample mean for the new sample of 65 sweets, even though the population mean was unchanged. Many candidates incorrectly stated that the variance had changed.

Answers: (i) (29.3, 29.9); (ii) The claim is not supported.

Question 4

A good number of candidates found the probability correctly. Some candidates incorrectly used 2×10^2 instead of 4×10^2 when calculating the variance. Most candidates standardised with their mean and variance. A few candidates incorrectly tried to apply a continuity correction. The appropriate area was used in many answers. Some candidates incorrectly used '1 – the area'.

Answer: 0.941.

Question 5

The first two parts were well answered by many candidates. Just a few candidates were expecting to be asked to find a constant for $f(x)$ (given in this case) and included the integration of $f(x)$ between the limits 2 and 4.

- (i) Many candidates integrated $xf(x)$ between the limits 2 and 4 correctly to find $E(X)$.
- (ii) Many candidates knew that they needed to integrate $f(x)$ from 2 to the median M or from M to 4 and to equate to 0.5. Errors included using $xf(x)$ or using the incorrect limits. Also, after commencing with a correct method, some calculation errors were seen.
- (iii) Fewer candidates completed this part successfully. Many candidates found the probability for one value of the variable (usually $\frac{7}{12}$), but then did not continue to find the probability for the two values of X . Errors included doubling their probability (instead of squaring) or applying an incorrect binomial distribution. Many candidates expected to have to find the variance at this stage and proceeded to do so and then applied a normal distribution to try to find the probability.

Answers: (i) $\frac{28}{9}$ (3.11); (ii) $\sqrt{10}$ (3.16); (iii) $\frac{49}{144}$ (0.340).



Question 6

- (i) To commence this significance test a null hypothesis and an alternative hypothesis were required. The null hypothesis should be written as ‘population mean = 0.336’, though many candidates wrote ‘ $\mu = 0.336$ ’ and this was accepted. The unbiased estimate of the variance (0.128) was required. Many candidates found this correctly. Some candidates found the biased variance (0.126) and proceeded with this, gaining some of the following marks. Many candidates standardised correctly, using $\frac{\sigma^2}{100}$, with their value of σ^2 . Having found z (2.77), candidates used one of three possible processes to test the statistic. The most popular (and possibly the most successful) way was to compare 2.77 with the critical z -value 2.576 for the two-tailed test. ‘ $2.77 > 2.576$ ’ (or equivalent) should be written down, as a good number of candidates did. The second process involved the comparison of the tail probability (0.0028 for one of the tails) with the critical probability (0.005) for that tail. Some candidates incorrectly compared 0.0028 with 0.01. The third process involved finding the critical values of X (0.244 and 0.428) and comparing the test statistic (0.435) with 0.428. This way was the least successful of the three ways. The conclusion was that there is evidence that B mean amount differs from the A mean amount. Some candidates gave a conclusion that contradicted their comparison of their statistical values.
- (ii) Many candidates incorrectly thought that because they had used the normal distribution in their test calculations in part (i) then the distribution of B itself had to be normal, or thought that as n was large the data was normal. Some candidates did appreciate that as n was large then the Central Limit Theorem applied and the sample mean was approximately normally distributed. Some candidates incorrectly suggested that the normal distribution must be used as the binomial and Poisson distributions were not applicable.

Answer: (ii) No, n is large and Central Limit Theorem applies.

Question 7

- (i) Some candidates correctly stated that H_0 : mean number of sales = 2.4 and H_1 : mean number of sales > 2.4 , clearly showing that they were dealing with the mean sales in the 3 week period. But others just used 0.8 or used μ . Some correct responses were seen when the tail probability (> 5) was found (0.0959) and then compared to 0.05. A common error was to calculate only the probability of 5 sales and compare this value to 0.05, instead of using the probability of the whole tail. Summation of the relevant Poisson terms was essential. More errors were seen when the probability of 0, 1, 2, 3, 4 sales was compared with 0.95, instead of the probability of 5 or more sales with 0.05. Although this was an equivalent process, it caused more difficulties for candidates. Often 0, 1, 2, 3, 4, 5 sales were found incorrectly. Most candidates did use the correct type of distribution, the Poisson. Very few instances of a fallacious normal were seen. However, some responses that began with Poisson terms were later incorrectly concluded with a comparison with a normal critical value such as 1.645.
- (ii) A good understanding of a Type I error was shown in many responses, and the correct Poisson terms calculated. A few candidates incorrectly suggested that this was 0.05.
- (iii) Many sensible clear explanations were seen, in context as required. Some candidates merely quoted a ‘bookwork’ definition, instead of expressing this in the context of the mean sales.
- (iv) Many correct statements were seen, showing a sound understanding of a Type II error.

Answers: (ii) 0.0357; (iii) Concluding that the mean sales have increased when this is not true;
(iv) The new value of the mean.

