CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

MATHEMATICS 9709/22

Paper 2 Pure Mathematics 2 **(P2)** February/March 2018

**1 hour 15 minutes**

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

## READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.

**[Turn over**

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**1** Solve the inequality $|5x + 2| > |4x + 3|$. [4]

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**2** A curve has equation $y = 4x \sin \frac{1}{2}x$. Find the equation of the tangent to the curve at the point for which $x = \pi$. [4]

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**3** **(i)** Use the trapezium rule with four intervals to find an approximation to

$$\int_0^8 \ln(x+2)\,\mathrm{d}x,$$

giving your answer correct to 3 significant figures. [3]

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**(ii)** Hence find an approximation to

$$\int_0^8 3\ln(x^2+4x+4)\,\mathrm{d}x.$$ [2]

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**4** The polynomial p($x$) is defined by

$$p(x) = 4x^3 + 4x^2 - 29x - 15.$$

**(i)** Use the factor theorem to show that $(x + 3)$ is a factor of p($x$). [2]

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**(ii)** Factorise p($x$) completely. [3]

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**(iii)** Hence, given that

$$2^{3u+2} + 4^{u+1} = 29 \times 2^u + 15,$$

find the value of $2^u$ and, using logarithms, find the value of $u$ correct to 3 significant figures. [3]

**5** It is given that $\displaystyle\int_{-a}^{2a} 4e^{-2x}\, dx = 25$, where $a$ is a positive constant.

**(i)** Show that $a = \frac{1}{2}\ln(12.5 + e^{-4a})$. [4]

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**(ii)** Use the equation in part **(i)** to show by calculation that $1.0 < a < 1.5$. [2]

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**(iii)** Use an iterative formula based on the equation in part **(i)** to find the value of $a$ correct to 4 significant figures. Show the result of each iteration to 6 significant figures. [3]

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**6** **(i)** Show that $\operatorname{cosec} 2x + \cot 2x \equiv \cot x$. [4]

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**(ii)** Hence find the exact value of $\cot \frac{1}{12}\pi$. [2]

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**(iii)** Find $\displaystyle\int \sin 2x(\operatorname{cosec} 4x + \cot 4x)\,\mathrm{d}x$. [3]

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The diagram shows part of the curve defined by the parametric equations

$$x = t^2 + 4t, \qquad y = t^3 - 3t^2.$$

The curve has a minimum point at $M$ and crosses the $x$-axis at the point $P$.

**(i)** Find the gradient of the curve at $P$. [4]

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**(ii)** Find the coordinates of the point $M$. [3]

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**(iii)** The value of the gradient of the curve at the point with parameter $t$ is denoted by $m$. Show that

$$3t^2 - (2m + 6)t - 4m = 0$$

and hence find the set of possible values of $m$ for points on the curve. [4]

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## Additional Page

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