MATHEMATICS

| Paper 9709/11 |
| Paper 11 |

Key messages

In order that candidates receive credit where possible for incorrect answers, they should remember to show full working and reasoning. Incorrect answers without working cannot be given any credit, whereas partial credit can be awarded if a correct method is shown. Where candidates are asked to 'show' or 'prove' a result, marks will be lost if any essential steps in the argument are omitted.

General comments

The paper allowed the majority of candidates to show what they had learned and understood. Candidates did not however gather as many marks as expected on the early shorter questions. These were intended to be reasonably 'routine' and candidates in the future could benefit by ensuring that they are confident on 'routine' work. Conversely, many candidates were able to boost unimpressive performances on the first half of the paper by strong performances in the last two questions.

The majority of scripts were well-presented but some candidates need to be reminded that they should work straight down the page rather than try to fit their answer to one question beside their answer to another question on the page and that the pages of their scripts need to be tied together securely. Some candidates also need to be warned of the danger of premature approximation. In particular, if a final result is required to be given correct to 3 significant figures, for example, it is necessary to carry at least 4 figures in intermediate results to ensure final accuracy.

The need to factorise quadratic expressions will always occur several times and it was noticeable that many candidates who employed decomposition into 4 terms before factorising 2 pairs of terms often did not complete the factorisation. For example, in **Question 10(ii)** candidates were faced with solving the equation $2x^2 - 5x - 3 = 0$ and many first wrote this as $2x^2 + x - 6x - 3 = 0$ followed by $x(2x+1) - 3(2x+1) = 0$. A significant proportion of candidates did not complete the factorisation but attempted to pick out the solutions from this position, making errors in doing so.

Comments on specific questions

Question 1

Many candidates attempted to write down and then simplify *all* the terms in the expansion – some never identifying the required term for the answer. A more efficient approach is to deduce that in order to obtain a term in x it is necessary to multiply together x^5 and $\left(\dfrac{2}{x^2}\right)^2$, and then to multiply finally by the coefficient $^7C_2$.

A very common error was to not include the 2 inside the bracket and thus to obtain 42 as the final answer.

Answer: 84.

Question 2

Of all the questions on the paper, candidates had the least success with this question. Many did not appreciate that differentiation was required, followed by use of the chain rule or similar. Of those that did start correctly many then confused the three variables involved, v, r and t, with the result that the chain rule was not implemented correctly.

Answer: $\dfrac{1}{8\pi}$ or 0.0398.

Question 3

For part **(i)** the majority of candidates produced an acceptable graph – but many of the graphs were drawn on graph paper following a time-consuming tabulation of values. It was expected that candidates would recognise the form of the equation and realise that it represented a parabola touching the *x*-axis at *x* = 2 and then to merely 'sketch' the curve as requested.

In part **(ii)** the most common error was to integrate $\pi(x-2)^2$ (assuming this was already πy^2), and a variety of incorrect limits were also seen. Those candidates who attempted to integrate the correct function often employed a very inefficient way of doing so – expanding the bracket (often with errors) before attempting to integrate rather than integrating directly to obtain $\dfrac{1}{5(x-2)^5}$.

Answer: **(ii)** $\dfrac{32\pi}{5}$.

Question 4

This was a good source of marks for many candidates with most being able to use the scalar product to find the angle between two vectors.

Answers: **(i)** −6**i** + 6**j** − 2**k**, −6**i** + 6**j** + 3**k**; **(ii)** 32.7°.

Question 5

On the whole, part **(i)** was generally well answered. In part **(ii)** most candidates successfully factorised the quadratic equation in $\sin^2\theta$ and also rejected the negative root to leave $\sin^2\theta = \dfrac{1}{2}$. After this point, however, candidates very often did not insert ± on square rooting, with the result that they obtained only two of the four solutions of θ .

Answer: **(ii)** 45°, 135°, 225°, 315°.

Question 6

Most candidates scored well on this question, although the majority of candidates did not write down the stationary value of *z*, thinking, no doubt, that having found that the stationary value occurred when *x* = 20 that they had answered the question.

Answers: **(ii)** 120, Minimum.

Question 7

In part **(i)**, while some candidates found the value of the gradient at (1, $\dfrac{1}{2}$) and then applied *y* = *mx* + *c*, most attempted the correct method of integrating the given function. However, a very significant number of candidates did not divide by 2 (the derivative of the contents of the bracket). At this stage also, some candidates did not add a constant of integration.

In part **(ii)** candidates were often successful, using a variety of methods, in finding the critical values but relatively few were able to apply the correct inequalities to these values.

Answers: **(i)** $y = \dfrac{-3}{2(1+2x)} + 1$; **(ii)** $x < -2$, $x > 1$.

Question 8

Marks obtained from this question were, on the whole, lower than expected. Candidates generally chose the correct progression for each part but in many cases candidates used the formula for the nth term rather than the sum to n terms and there was also confusion about how to apply the 5% to obtain the amount donated to charity. In part **(ii)** a significant proportion of candidates used $r = 0.1$ instead of 1.1.

Answers: **(i)** $41 000; **(ii)** $22 100.

Question 9

In part **(i)** there seemed to be a certain amount of confusion in the minds of candidates as to whether to use 2θ and subtract the area of sector *OPST* from the area of triangle *OAB* or to work with θ and double the answer at the end. In many cases some of the areas found were doubled and some were not, leading to incorrect answers.

Candidates seemed to be more comfortable in part **(ii)** where θ and r were given particular values, and in general candidates were more successful than in part **(i)**. Some candidates apparently assumed, without justification, that *P* was the mid-point of *OA*.

Answers: **(i)** $r^2(\tan\theta - \theta)$; **(ii)** $12 + 12\sqrt{3} + 4\pi$.

Question 10

This question was done well and even weaker candidates were able to obtain good marks. Candidates did particularly well in the first two parts, but in part **(iii)** failure to find the mid-point of *AP* accurately often cost candidates the last 3 marks.

Answers: **(i)** $2(x-1)^2 - 1$, $(1, -1)$; **(ii)** $(-0.5, 3.5)$; **(iii)** $5y + x = 17$.

Question 11

From the point of view of candidates this question was the most successful in the paper and even low-scoring candidates were able to score good marks. Very few candidates transposed the two answers to part **(i)**, although lack of simplification was reasonably common as were sign errors. Candidates often gained a high proportion of the marks in parts **(ii)** and **(iii)**, and in parts **(iv)** and **(v)** candidates generally showed good ability in finding inverses of functions, although relatively few obtained the correct sign in part **(v)**.

Answers: **(i)** $2x^2 - 3$, $4x^2 + 4x - 1$; **(ii)** -1; **(iii)** 2; **(iv)** $\frac{1}{2}(x^2 - 3)$; **(v)** $-\sqrt{x+2}$.

MATHEMATICS

> **Paper 9709/12**
> **Paper 12**

Key messages

In order that candidates receive credit where possible for incorrect answers, they should remember to show full working and reasoning. Incorrect answers without working cannot be given any credit, whereas partial credit can be awarded if a correct method is shown. Where candidates are asked to 'show' or 'prove' a result, marks will be lost if any essential steps in the argument are omitted.

General comments

While some parts of questions were found to be more demanding, there were many parts of questions that proved to be accessible to all but the very weakest of candidates. It was noticeable that on two questions (**Questions 7** and **8**), where a diagram was not provided on the question paper, many candidates did not sketch their own diagram and this often led to serious misunderstandings of the question.

Comments on specific questions

Question 1

This proved to be a straightforward starting question and most candidates integrated correctly. Unfortunately, a sizeable minority candidates omitted the constant of integration and many replaced $\dfrac{x^{-2}}{-2}$ by $\dfrac{-2}{x^2}$.

Answer: $\dfrac{x^4}{4} - \dfrac{x^{-2}}{2} + c$.

Question 2

There were many good responses to this question.

(i) The vast majority of candidates showed confidence in the way they used the binomial expansion. Binomial coefficients were nearly always correctly used and it was rare to see incorrect use of brackets with $\left(\dfrac{-3x}{2}\right)^2$ or $\left(\dfrac{-3x}{2}\right)^3$, though omission of the 'minus' sign was the most common error.

(ii) Many candidates did not realise that two terms were needed from the expansion of $(k+2x)\left(1-\tfrac{3}{2}x\right)^6$ and that the sum of these needed to be equated to 0.

Answers: **(i)** $\dfrac{135x^2}{4}$, $-\dfrac{540x^3}{8}$; **(ii)** 1.

Question 3

This question was poorly answered. Some candidates either did not attempt the question or made no progress.

(i) Most candidates did not realise that since the roots of the quadratic equation are −3 and 5, the equation can be expressed as $(x + 3)(x − 5)$, thereby leading directly to the values of p and q. The majority obtained two simultaneous equations from substituting $x = −3$ and $x = 5$ and generally were correct. Many others spent considerable time in attempting to use the quadratic formula and made little progress, and many others automatically associated 'roots' of a quadratic with '$b^2 − 4ac$'.

(ii) Of those candidates successfully coping with part **(i)**, the majority recognised the need to set '$b^2 − 4ac$' to 0, though many did not recognise that 'c' was $q + r$, whilst others took c to be $r + 15$ rather than $r − 15$.

Answers: **(i)** − 2, −15; **(ii)** 16.

Question 4

This question was very well answered.

(i) The differentiation of $4(3x − 4)^{−1}$ was usually correct and the majority of candidates realised the need to include the '×3'. Most then correctly found the numerical value of the gradient at $P(2, 2)$ and deduced the equation of the tangent. It is, however, still common to see some candidates expressing the gradient as a function of x, thereby obtaining the equation of a curve and it was surprising to see a large number of candidates assuming that the gradient of the tangent is $−1 \div \dfrac{dy}{dx}$.

(ii) Attempts at this part of the question were much improved on recent years with the majority of candidates realising that the angle made by the tangent with the x-axis could be obtained directly from the gradient $\dfrac{dy}{dx}$. Other attempts were seen in which candidates attempted to find two points on the line and use trigonometry. These too were often successful.

Answers: **(i)** $y + 3x = 8$; **(ii)** 108.4° or 71.6°.

Question 5

(i) Those candidates making progress with this identity generally started by replacing $\tan \theta$ by $\dfrac{\sin \theta}{\cos \theta}$, though some then had problems with the $\cos \theta$ in the denominator. Most however finished with $\cos^2 \theta$ in the numerator and this was automatically replaced by $1 − \sin^2 \theta$. At this point, most 'fiddled' the answer to obtain the correct expression on the right-hand side. Many candidates did not realise the need to factorise the numerator (difference of two squares) and then to cancel $(1 − \sin\theta)$.

(ii) This was very well answered and many candidates were not perturbed by their inability to cope with part **(i)**. Correct answers were common, though often four answers were given (one in each quadrant), and it was common to see the obtuse angle not given correct to 1 decimal place as required in the rubric.

Answer: **(ii)** 19.5°, 160.5°.

Question 6

(i)   This caused many problems, mainly because of some candidates' inability to cope with the algebraic manipulation of fractions. Recognition of the meaning of ff(x) was very good and it was rare to see the original expression incorrect. However, inability to cope with $(2x - 1)$ in the denominator of the fraction caused problems. Common errors were to express $2\left(\dfrac{x+3}{2x-1}\right)$ as $\dfrac{2x+6}{4x-2}$, or to cancel $(2x - 1)$ without adjusting the '+3' or '–1', or to take the numerator of the denominator as $(2x + 6 - 2x - 1)$ instead of $(2x + 6 - 2x + 1)$.

(ii)   Although only a few candidates recognised that because, from part **(i)**, ff(x) = x, then $f^{-1}(x) = f(x)$, this part was very well answered by the standard method of making x the subject of the equation. Solutions were nearly always correct and the only errors were to leave the answer in terms of y or to make simple sign errors in the manipulation.

Answer: **(ii)** $f^{-1}(x) = \dfrac{x+3}{2x-1}$.

Question 7

(i)   The fact that the question did not have a diagram led to a considerable number of misinterpretations. Confusion over the lines L_1, L_2 and *AC* were common and many candidates finished with the coordinates of points *A* and *C* being the same. This led to a nonsensical answer for part **(ii)**. The majority however correctly found the gradient of *AB* and realised that the gradient of L_2 was the same. Many candidates wasted time in finding the equation of *AB*, though most of these correctly ignored this and proceeded to L_2. Use of perpendicular gradients was good and the equation of *AC* presented few problems.

(ii)   Use of the distance formula was generally correct, though many candidates incorrectly gave the answer as 3.6 or 3.57.

Answers: **(i)** (3.6, 1.8); **(ii)** 3.58 or $\dfrac{8\sqrt{5}}{5}$.

Question 8

(i)   The candidate's ability to calculate an angle from a scalar product and to find vector $\overrightarrow{AB}$ from $\overrightarrow{OA}$ and $\overrightarrow{OB}$ or $\overrightarrow{BC}$ from $\overrightarrow{OB}$ and $\overrightarrow{OC}$ was excellent. Unfortunately many attempts used the scalar product of vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$ instead of $\overrightarrow{BA}$ and $\overrightarrow{BC}$ but this was an error that lost accuracy marks only.

(ii)   This caused most candidates difficulty and there were only a small number of correct solutions. Most candidates did not realise that vector $\overrightarrow{OD}$ could be calculated directly from $\overrightarrow{OA} + \overrightarrow{AD}$ and that $\overrightarrow{AD} = \overrightarrow{BC}$ since *ABCD* is a parallelogram. Confusion often resulted from the fact that candidates were not prepared to sketch a diagram.

Answers: **(i)** 112.4°; **(ii)** $\begin{pmatrix} 8 \\ 1 \\ 8 \end{pmatrix}$.

Question 9

The question as a whole caused many candidates difficulty.

(i) (a)   Correct solutions to this part of the question were rare for, whilst most candidates recognised that the limits for cos x are -1 to 1, it was rare for them to realise that the limits for cos² x are 0 to 1, leading to a range of -1 to 3, not -1 to 7.

(b)   This part of the question was answered more successfully with most candidates able to deduce that $\cos^2 x = \dfrac{1}{2}$. Very few realised however firstly that the term 'exact' means that decimal answers are not acceptable and secondly that the solution to the equation is found from $\cos x = \pm \dfrac{1}{\sqrt{2}}$ and not $\cos x = \dfrac{1}{\sqrt{2}}$.

(ii) (a)   The graph of $y = 3 - 4\cos x$ was reasonably drawn, though many candidates failed to realise that, for $0 < x < \pi$, the curve only has a half cycle, starting at $y = -1$ and finishing at $y = 7$. Many sketches were seen that either did not 'flatten out' at $x = 0$ and $x = \pi$ or did not have a point of inflexion at $x = \dfrac{1}{2}\pi$.

(b)   Candidates needed to read the question carefully. Answers were seen that just stated 'yes' and often any kind of reason was missing. Any of the following were acceptable: the function is one-one, the function is increasing or there are no turning points in the domain.

Answers: **(a)(i)** $-1 < f(x) < 3$; **(ii)** $\dfrac{1}{4}\pi$, $\dfrac{3}{4}\pi$; **(b)(ii)** f has an inverse, f is one-one or increasing in the domain.

Question 10

This question was poorly answered.

(i)   Many candidates failed to attempt this part of the question. Others recognised that '$a + 5d = 4a$' but only a small minority realised that because the six sectors came from a circle, that the sum of the six angles was $360°$ or 2π radians. Correct answers to the length of the perimeter were rare. Even when the angle of the smallest sector was correctly found as $24°$, candidates failed to realise the need to convert this to radians in order to find the arc length from '$s = r\theta$'.

(ii)   Many candidates did not realise that in a geometric progression the ratio of the first to second terms is the same as the ratio of the second to third terms. Others expressed $2k + 3$, $k + 6$, k as a, ar and ar^2 respectively and eliminated a and r. In many solutions, failure to cancel '$2k + 3$' led to a cubic rather than a quadratic equation, and this usually led to no further working. Many attempts however obtained a correct value for k and proceeded to find the sum to infinity.

Answers: **(a)** 12.1 cm; **(b)(i)** 12, **(ii)** 81.

Question 11

There were many pleasing responses to this question and it proved to be a source of high marks for many candidates. One serious error, however, affected many candidates when they misread the equation of the curve as $y = 4\sqrt{x-x}$ instead of $y = 4\sqrt{x} - x$. This error made the question nonsensical and meant that very little credit could be given.

(i) Most candidates realised that, at the point *A*, *y* = 0 and that at *M*, $\dfrac{dy}{dx} = 0$. Solving *y* = 0 led to the equation $\sqrt{x} = 4$, and this was often incorrectly solved as *x* = 2 instead of *x* = 16. The differentiation of *y* was generally accurate and most candidates were able to deduce that *x* = 4. Common errors were to make simple errors in calculating the value of the *y*-coordinate or to omit it altogether.

(ii) Despite the complicated arithmetic in evaluating the volume, there were many correct answers. Weaker candidates used the formula for area instead of volume and others assumed the volume to be found by integrating πy instead of πy^2. The most common error was to square $4\sqrt{x} - x$ incorrectly, with often only two terms being given and often with '$-x^2$' instead of '$+x^2$'. Use of limits was good.

Answers: **(i)** (16, 0), (4, 4); **(ii)** 137π.

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> **Paper 9709/13**
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> **Paper 13**

Key messages

In order that candidates receive credit where possible for incorrect answers, they should remember to show full working and reasoning. Incorrect answers without working cannot be given any credit, whereas partial credit can be awarded if a correct method is shown. Where candidates are asked to 'show' or 'prove' a result, marks will be lost if any essential steps in the argument are omitted.

General Comments

Many excellent scripts were seen. Candidates' work was usually neatly presented but some of the better candidates did not always show clearly the steps in their working.

Most candidates did give non-exact answers correct to 3 significant figures but many did not give non-exact angles in degrees correct to 1 decimal place. Some candidates need to read questions more carefully and ensure that they use the method implied by use of a word such as 'Hence'. A formula or idea must be applied to the specific question in order to earn method marks.

Comments on specific questions

Question 1

Most candidates found this fairly straightforward but the second part of the expansion caused problems for some with the minus sign and/or the 2 not being dealt with correctly. Some candidates were able to write down the relevant terms immediately and thus saved time.

Answer: 5.

Question 2

The most common way of tackling this question was to equate the two expressions for y and to collect terms, giving a quadratic in x. The use of '$b^2 - 4ac$' then led to the values 2 and -10 with the candidate now having to consider values of m that would give two distinct roots. Consequently the use of $\leqslant$ and $\geqslant$ in the answer was not worthy of the final accuracy mark. The main source of error was with the signs and/or the squaring of the x-coefficient.

A minority of candidates differentiated for the curve and used the result as m in the line equation. This led to a quadratic in x with no m in it and some stopped once they had found $x = 1$ or $x = -1$ instead of returning to find m values.

Answer: $m > 2$, $m < -10$.

Question 3

Candidates who immediately identified the points as *P*(*a*, 0) and *Q*(0, *b*) found this fairly straightforward; the most common mistake involved omitting the sign when using the fact that the gradient of *PQ* is $-\dfrac{b}{a}$. Use of the two facts given (gradient and length of *PQ*) gave simultaneous equations to solve. Some candidates offered both positive and negative solutions for *a* and *b* in spite of being told that they are positive constants. The majority of candidates worked through with *x* and *y*, sometimes specific relating to *P* and *Q*, sometimes with general *x* and *y*. There was a full range of achievement for these candidates from no marks for those who made no (or poor) attempts to use gradient and length, to full marks for those who drew a diagram to show to Examiners and themselves what they were doing and brought in *a* and *b* at the end. 'Trial and improvement' was sometimes offered with *a* = 2*b* or *b* = 2*a*. The use of √(45) led to an adjustment but often the gradient was not carefully considered.

Answers: 6, 3.

Question 4

(a) Only the best candidates took the simple step of dividing *x* into each of the terms in the numerator and differentiating each term. The majority of the rest used either product rule or quotient rule with varying degrees of success. A common error was to try to differentiate as if the function was all one term or as if it was a function of a function.

(b) The 3 was often ignored or misused but the majority made a reasonable attempt at the integration and the use of limits. Some candidates ignored the lower limit of zero, assuming it would give zero. Some candidates gave a final answer of 3.5 rather than −3.5, as if the question was about finding an area.

Answers: **(a)** $4x - 5x^{-2}$; **(b)** −3.5.

Question 5

Candidates seemed better prepared for this vectors question than has sometimes been the case with vectors questions in the past. Most made creditworthy attempts at vectors $\overrightarrow{PQ}$ and $\overrightarrow{RQ}$ and then used these (or $\overrightarrow{QP}$ and $\overrightarrow{QR}$) with a scalar product as required. Some candidates attempted to use the cosine rule. A significant number, however, incorrectly used $\overrightarrow{PQ}$ and $\overrightarrow{QR}$.

Sign errors (especially with **j**-components) were common among weaker candidates and some were inconsistent with the order of components.

Answers: **(i)** 3**i** + 6**j** − 3**k**, −3**i** + 8**j** + 3**k**; **(ii)** 63.2°.

Question 6

(a) The use of the sum to infinity led immediately to a value for the common ratio. Used in conjunction with the information about the third term, the first term could easily be found. Many candidates took a more lengthy route by forming two simple equations and solving, not noticing that cancelling was possible and often working with quadratic, or even cubic, equations.

(b) This was well done with candidates using a variety of methods. Some candidates used the information about the third and eighth terms to obtain a relationship between the first term and the common difference which they then used in expressions for the sums of eight and of four terms to prove the relationship. Others assumed the relationship and thus obtained a result verifying the earlier one. The main variation involved the use of 'sum = $\dfrac{n}{2} \times$ (first term + last term)', where in one sum the eighth term itself was used and in the other the fourth term as it related to the third term. Using the information regarding those terms an identity was reached. Candidates often omitted to state a conclusion.

Answer: **(a)** 45.

Question 7

(i) Many candidates did not show that the result was true. $\tan\left(\dfrac{\pi}{3}\right) = \dfrac{AX}{6}$ leads immediately to the required result but (for example) use of the sine rule requires use of the exact values (in surds as necessary) of $\sin\left(\dfrac{\pi}{3}\right)$ and $\sin\left(\dfrac{\pi}{6}\right)$.

(ii) This caused few problems for candidates.

(iii) Most candidates used '$s = r\theta$' correctly and added it to AX. However some then added 6 (or 12) with no explanation at all. Others said they were adding BX but seemed to assume, rather than evaluate, its length. Some candidates drew the standard 1, $\sqrt{3}$, 2 right-angled triangle and scaled up from this throughout for all linear measurements - a neat method. Some candidates inappropriately used their calculators at various stages of their working which did not yield an exact answer.

Answers: **(ii)** $18\sqrt{3} - 6\pi$ cm^2; **(iii)** $6\sqrt{3} + 2\pi + 6$ cm.

Question 8

(i) Many candidates used $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\sin^2\theta = 1 - \cos^2\theta$ but not all could cope with the algebra and the factorising needed (especially with regard to the signs). Some worked from the right-hand side, multiplying top and bottom by $1 - \cos\theta$ and proceeded on from there. Some worked from both ends at once resulting (if correct) in an identity, while others worked separately from both ends getting two identical expressions. The common problem to all methods was poor algebra (e.g. squaring the original left-hand side often led to no middle term(s) and an incorrect sign for $\dfrac{1}{\tan^2\theta}$).

(ii) 'Hence solve...' was the instruction in the question but some candidates failed to heed this and gained no credit. Most candidates found 64.6° as the solution to $\cos\theta = \dfrac{3}{7}$. However some candidates offered no second solution, or an incorrect one (or more).

Answer: **(ii)** 64.6°, 295.4°.

Question 9

(i) Working 'backwards' from a differential to a curve equation seemed to cause fewer problems than the surd on the denominator, but this was well done although quite a few candidates found the value of $\dfrac{dy}{dx}$ at P and attempted to use a line equation formula. A few of those integrating gave no thought to a constant of integration.

(ii) Some candidates used $y = 0$ rather than $\dfrac{dy}{dx} = 0$ but otherwise this was well done.

(iii) The better candidates had no problems; weaker candidates often struggled with the differentiation but still knew the significance of the sign of the second derivative.

(iv) Only a small number of candidates realised the direct link between the gradient of the normal and k. The majority found the gradient and the equation of the normal and went on from there, not always correctly or successfully.

Answers: **(i)** $y = 4\sqrt{x} - x + 2$; **(ii)** (4, 6); **(iii)** $-x^{-3/2}$, Maximum; **(iv)** 3.

Question 10

(i)   Finding g(2) first and then using this result as x in f(x) was required and was well done on the whole. A few candidates found gf(x) and some found f(x)g(x). Often the 4 or the 8 got lost in the calculation. Whilst most candidates worked numerically from the start there were those who worked generally to obtain an algebraic expression for fg(x) and then substituted for $x = 2$. This latter approach often produced errors.

(ii)   Some candidates worked out the expression for f^{-1}(x) in order to sketch the graph while others used the idea of reflection in $y = x$. Some failed to consider negative axes and others did not take their two graphs far enough for them to intersect. Having done the hard work and having produced correct lines, some candidates failed to label them or labelled them incorrectly.

(iii)   Many candidates obtained an expression for g^{-1}(x) rather than g$'$(x). For those that did differentiate, the statement 'it is one-one therefore an inverse exists' was often stated without an explanation using the expression for g$'$(x).

(iv)   The expression for f^{-1}(x) was almost always correct. More problems arose with g^{-1}(x). Some candidates expanded to form a cubic and made no progress. Amongst the rest some had sign errors with the 8 and the 1 whilst others divided by 2 at the wrong stage. Some candidates had a final answer in which the 2 was not cube-rooted. Sometimes this was because of poor presentation of their answer but sometimes because of dividing by 2 at the wrong stage.

Answers: **(i)** 26; **(iii)** $6(x-1)^2$; **(iv)** $\dfrac{x+4}{3}$, $\sqrt[3]{\dfrac{x-8}{2}}+1$.

MATHEMATICS

Key messages

In order that candidates receive credit where possible for incorrect answers, they should remember to show full working and reasoning. Incorrect answers without working cannot be given any credit, whereas partial credit can be awarded if a correct method is shown. Where candidates are asked to 'show' or 'prove' a result, marks will be lost if any essential steps in the argument are omitted.

General comments

There was a wide spread of marks on this paper. All parts of the paper, except **Question 8(iii)**, proved accessible to at least a reasonable number of candidates, though some questions were generally less well answered than others. Candidates found **Questions 1, 4, 5(ii)** and **8(i)** to be relatively straightforward and **Questions 3, 6** and **8(iii)** proved to be more difficult. Arithmetic and sign errors spoiled some otherwise good work, but this was not common problem. The standard of presentation was usually good and there was little to suggest that lack of time was an issue for candidates.

Comments on specific questions.

Question 1

This first question proved to be very accessible and the overwhelming majority of candidates scored full marks. Even the weaker solutions were awarded a mark by solving the equation $3x + 4 = 2x + 5$. Squaring both sides was a popular approach and was invariably a successful method, with only a few candidates not being able to produce a 3-term quadratic expression when squaring. Graphical solutions were very rare but usually led to correct results if graph paper was used.

Answer: $-\dfrac{9}{5}$, 1.

Question 2

Most candidates demonstrated the right approach but were thwarted by being unable to find the correct t-derivatives of the parametric equation(s). Some candidates started well but produced incorrect or approximate values of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ at $t = \dfrac{1}{6}\pi$. Weaker candidates began by substituting for $x(t)$ and $y(t)$ at $t = \dfrac{1}{6}\pi$ and attempting to use 'change in $y \div$ change in x'. A few stronger candidates used double-angle formulae to produce a correct result.

Answer: $-\dfrac{1}{2}\sqrt{3}$.

Question 3

There were many fully correct solutions, but a majority found the problem intractable, using the two given points as being on a graph of y against x. This is a feasible method only if the points are said to correspond to $x = 1$, $y = e^2$ and $x = e^6$, $y = e^{10.2}$ respectively. The most productive method was to note that $\ln y = \ln K + m \ln x$ and to use a simultaneous equation approach. Many candidates calculated the gradient of

the line but failed to connect it to 'm; others found the gradient to equal $\dfrac{41}{30}$ or 1.37 but associated it with 'K',

having looked at $y = Kx^m$ and assumed that the coefficient K must be the gradient. Some candidates failed to take logarithms correctly. The use of base 10 for logarithms occasionally spoiled solutions.

Answers: 7.39, 1.37.

Question 4

This question was well answered. Occasionally a numerical or sign slip produced an incorrect value in part **(i)**, but a correct $3x^2 + (a - 6)x$ was produced in part **(ii)**. Synthetic division, usually correct, was used by candidates in some Centres.

Answers: **(i)** 8; **(ii)** $3x^2 + 2x + 4$.

Question 5

This was often well answered. Stronger candidates had no problems with either part, though those using the product rule rather than the quotient rule were less successful in part **(ii)**. In part **(i)**, some candidates used $x\ln$ as their 'u' in the product rule.

Answers: **(i)** 4; **(ii)** 0.08.

Question 6

(a)   Only strong candidates coped well with the integration. Weaker candidates often did not attempt it or tried unsuccessfully to multiply out the integrand correctly.

(b)   Some good, fully correct, solutions were seen but there were a significant number involving integrating an unsimplified (though correct) expression which made the subsequent substitution of the limits rather longer than necessary. It was quite common to see mixed variables (x and θ) in integrated expressions. Many incorrect identities, such as $\tan^2\theta \equiv \sec^2\theta + 1$, were seen, and many believed they could integrate $\tan^2\theta$ directly.

Answer: **(a)** $12e^x + \dfrac{4}{3}e^{3x} + c$.

Question 7

(i)   Many candidates did not attempt the sketches and good quality sketches were rare. Straight lines were often seen as was the graph of $y = x^2 - 14$ plotted against $y = e^{2x}$. Some included sketches for only first quadrant or for an even more limited domain.

(ii)   Many solutions were fully correct, though weaker candidates were unsure here; many substitutions without rearrangement were seen, and many candidates did not define a function to be evaluated at $x = 1.2$ and $x = 1.3$. Some candidates evaluated f(1.2) and/or f(1.3) incorrectly.

(iii)   Some weak approaches attempted numerical solutions. Many candidates used $\ln(14 - x^2) = \ln 14 - \ln x^2$.

(iv)   This was usually well done. Sometimes an insufficient number of correct iterations were performed or candidates did not state the correct value of the root correct to 2 decimal places. A few candidates selected *x*-values from 1.2 to 1.3 and tried to use a 'trial and improvement' approach. Some candidates started outside the range [1.2, 1.3], *x* = 1 being common, or even began at an initial value of *x* so large as to make $(14 - x^2)$ negative.

Answer: **(iv)** 1.26.

Question 8

(i)   This was often very well answered, though some candidates got $R\cos\alpha$ and $R\sin\alpha$ the wrong way around or made a sign mistake. Others found *R* to be 7.21 instead of its exact value.

(ii)   Candidates who lost marks in part **(i)** were mainly restricted to method marks only. Those who had scored highly in part **(i)** did equally well here, but some did not spot the link with part **(i)** and tried to solve the original expression set equal to 3.

(iii)   This last part was found to be difficult. Invariably no link was seen with previous parts of the question and candidates attempted a differentiation approach which led to no viable solutions.

Answers: **(i)** $\sqrt{52}\,\sin(\theta - 56.31°)$; **(ii)** 80.9°, 211.7°; **(iii)** 60, 8.

MATHEMATICS

| Paper 9709/22 |
| Paper 22 |

Key messages

In order that candidates receive credit where possible for incorrect answers, they should remember to show full working and reasoning. Incorrect answers without working cannot be given any credit, whereas partial credit can be awarded if a correct method is shown. Where candidates are asked to 'show' or 'prove' a result, marks will be lost if any essential steps in the argument are omitted.

General comments

There was a wide spread of marks for this paper. The majority were well prepared and showed considerable skill in their working. However, there were also some weaker candidates who were less well prepared. **Questions 1**, **3** and **7** were particularly well done. Most candidates found **Questions 2(ii)**, **5** and **8(iii)** to be more difficult.

Presentation was generally excellent and there seemed no sign that a lack of time hindered candidates.

Comments on specific questions

Question 1

Candidates invariably noted that $x\log 3 = (x + 2)\log 2$, with e or 10 as a base for logarithms. Most followed this by solving for x, but the final decimal answer was often incorrect due to premature rounding of $\ln 2$ and $\ln 3$.

Answer: 3.42.

Question 2

(i) Those who used the trapezium rule generally did well, though some candidates believed that they could integrate $\sqrt{(1 + x^3)}$ 'on sight', and wrote, for example, $\frac{2}{3}(1 + x^3)^{\frac{3}{2}}$.

(ii) Explanations of why the approximation in part **(i)** underestimated the area generally were in terms of convexity or cancavity, or the curve being 'always on the increase'. Few candidates mentioned the two trapezia and their areas compared to that of the exact area of B.

Answers: **(i)** 3.41; **(ii)** 2.59.

Question 3

(i) This was generally very well done. A small minority worked to less than 5 decimal places in their interations or failed to round the final answer to 3 decimal places.

(ii) This was fairly well done, though some candidates were puzzled by the request and brought in values from part **(i)**.

Answers: **(i)** 0.952; **(ii)** $8x^3 - x^2 - 6 = 0$.

Question 4

(a) Many integrated to get $k\cos\left(\dfrac{1}{2}x\right)$, with $k = \pm1, 2$, and $\pm\dfrac{1}{2}$ being common. Others believed that a

form $\lambda\cos\left(\dfrac{1}{4}x^2\right)$ or $\mu\sin\left(\dfrac{1}{2}x\right)$ or $x\sin\left(\dfrac{1}{2}x\right)$ was involved. Many candidates quoted a result,

$2\cos\left(\dfrac{1}{2}x\right)$, and then substituted limits correctly.

(b) It was necessary to simplify the integrand to the form $e^{-x} + 1$ before integrating. Those who did

this made good progress, but those who did not quoted forms such as $\dfrac{e^{-x}\left(1+e^x\right)}{e^x}$ and made no

progress.

Answers: **(a)** 1; **(b)** $-e^{-x} + x\ (+\ c)$.

Question 5

A sizable number of candidates did no differentiation and proceeded with inappropriate methods. Others did not differentiate $2y^2$ using the chain rule, or kept the 10 in the right-hand side after differentiating. Many candidates obtained an incorrect value for $\dfrac{dy}{dx}$ at $(2, -1)$ after finding $\dfrac{dy}{dx}$ correctly.

Answer: $9x + 2y - 16 = 0$.

Question 6

(i) Weaker candidates had only one term in $\dfrac{dy}{dx}$, e.g. $8x.\dfrac{1}{x}$, instead of using the product rule. Even some very good candidates did not find the *y*-coordinate of the stationary point, settling for a correct *x*-value.

(ii) Most candidates knew how to differentiate between a maximum and a minimum point, but their second derivatives of *y* were often incorrect.

Answers: **(i)** $(0.607, -0.736)$; **(ii)** Minimum.

Question 7

(i) This part was very well done; only a few candidates had $p(-1) = 0$ and not 24.

(ii) Those with correct solutions to part **(i)** usually did well here though a few candidates did not factorise their correct quadratic factor $(6x^2 - 13x + 5)$. Candidates with incorrect answers to part **(i)** still earned method marks for intelligently seeking an appropriate quadratic factor.

Answers: **(i)** $-1, -21$; **(ii)** $(x + 2)(2x - 1)(3x - 5)$.

Question 8

(i) Candidates split into those trying to convert the left-hand side into the right-hand side and *vice-versa*. Many candidates showed correctly that $(\operatorname{cosec}^2 \theta - \sec^2 \theta) \equiv (\cot^2 \theta - \tan^2 \theta)$ but did not use $\sin^2 2\theta \equiv 4\sin^2 \theta \cos^2 \theta$. Those beginning with $4\cos 2\theta \equiv 4(\cos^2 \theta - \sin^2 \theta)$ had more success by beginning with the right-hand side.

(ii) (a) Most candidates used part **(i)** to convert the equation into the form $\cos 2\theta = \dfrac{3}{4}$, but some found only the first quadrant value for θ and omitted the solution in the second quadrant, or added spurious solutions.

(b) From part **(i)**, the expression $(\operatorname{cosec}^2 15° - \sec^2 150°)$ equals $\dfrac{4\cos 30°}{\sin^2 30°}$. However, many candidates used much longer methods. Some candidates only proceeded as far as an trigonometric expression involving 15° and 75°.

Answers: **(ii)(a)** 20.7°, 159.3°, **(b)** $8\sqrt{3}$.

MATHEMATICS

> **Paper 9709/23**
> **Paper 23**

Key messages

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General comments

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Presentation was generally excellent and there seemed no sign that a lack of time hindered candidates.

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this made good progress, but those who did not quoted forms such as $\dfrac{e^{-x}\left(1+e^x\right)}{e^x}$ and made no

progress.

Answers: **(a)** 1; **(b)** $-e^{-x} + x\ (+ c)$.

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A sizable number of candidates did no differentiation and proceeded with inappropriate methods. Others did not differentiate $2y^2$ using the chain rule, or kept the 10 in the right-hand side after differentiating. Many candidates obtained an incorrect value for $\dfrac{dy}{dx}$ at $(2, -1)$ after finding $\dfrac{dy}{dx}$ correctly.

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MATHEMATICS

Key messages

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General comments

The questions or parts of questions that were generally done well were **Question 1** (binomial expansion), **Question 2** (differentiation), **Question 4(i)** (factors of a polynomial), **Question 5(i)** (laws of logarithms), **Question 6(ii)** (iteration), **Question 7(i)** (change of variable in integral) and **Question 8(i)** (complex numbers). Those that were done least well were **Question 3** (vector geometry), **Question 4** (solving $a^x = b$), **Question 5(ii)** (implicit differentiation), **Question 8(ii)** (geometrical aspects of complex numbers), **Question 9** (trigonometry) and **Question 10** (differential equation).

Comments on specific questions

Question 1

When trying to accommodate the $-6x$ into their expansion, candidates often included simply $6x$, or even just x, throughout.

Answer: $1 - 2x - 4x^2 - \dfrac{40}{3}x^3$.

Question 2

Some good work was seen in both parts **(i)** and **(ii)**. However, in part **(i)** the derivative $1 + \sin 2x$ sometimes appeared as $1 + 2\cos 2x$.

Answers: **(i)** $\dfrac{2\cos 2x}{1 + \sin 2x}$; **(ii)** $\dfrac{x\sec^2 x - \tan x}{x^2}$.

Question 3

(i)   This proved a difficult question as many candidates did not realise that the vector $\overrightarrow{AB}$ was the normal to the plane and point B was in the plane. Instead they usually opted for vector $\overrightarrow{OB}$ being the normal to the plane and the point A, or even vector $\overrightarrow{AB}$, as the point in the plane. Even when the correct choice of the normal and the point were made there were often arithmetical errors in the numerical scalar product.

(ii)   Few candidates could give a correct vector representing the y-axis and many made errors in the evaluation of the scalar product of the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

Answers: **(i)** $3x - 4y + 6z = 80$; **(ii)** 30.8° or 0.538 radians.

Question 4

(i)   Showing that f(–2) = 0 required the actual evaluation of each term in the expression and not just leaving these as $12(-2)^3$, etc. Whilst the quadratic term was successfully produced the complete factorisation often had $(2x + \frac{3}{2})$ with (3x – 2), or other such terms halved with no compensation from elsewhere.

(ii)   Few candidates were able to spot the relationship between the roots of f(x) = 0 and the equation in this section. Hence the necessity to choose the positive root of $x = \frac{3}{2}$ rarely occurred; however when it did those few candidates usually made the appropriate choice.

Answers: **(i)** (x + 2)(4x + 3)(3x – 2); **(ii)** –0.369.

Question 5

Most candidates could use the laws of logarithms to evaluate $e^{2\ln 3}$, etc. However, the idea of differentiating the expression, even though the gradient was clearly mentioned, did not occur to many. To make matters worse, those that did realise to follow this correct approach often did not notice that implicit differentiation was also needed.

Answers: **(ii)** 5, 68.

Question 6

(i)   Many candidates simply ignored this part, but those that did attempt it were usually successful in establishing the given equation.

(ii)   The iteration work was usually sound, but many candidates failed to give the final answer to the requested number of decimal places once convergence had been established. The length of *AB* was successful obtained either using the half angle, or the sine or cosine rule.

Answers: **(ii)** 2.11, 17.4

Question 7

(i)   The details required for the change of variable were usually presented in sufficient detail, although occasionally limits were simply changed to their prescribed values without any working being shown.

(ii)   Most candidates knew that integration by parts was required but they had little idea as to where the various parts requiring differentiation and integration actually occurred within this formula. Those that did were usually successful as far as the second attempt at integration where they often did not deal with the signs correctly in the simplification of $-\left(\frac{x^2}{2} - 2x \right)$.

Answer: **(ii)** 15ln5 – 4.

Question 8

(i)   Whilst candidates knew to multiply the numerator and denominator by (1 – 2i) their actual evaluation often produced a large variety of errors. However, those that found –3i usually found its modulus and showed why the argument was $-\frac{\pi}{2}$.

(ii)   In order to understand what was required in this problem it was necessary to draw a line at $\frac{\pi}{4}$ above the horizontal from the point –3i, and realise that the closest distance that this locus passes to the origin is the required distance. Few candidates could produce such a sketch.

(iii) This given equation required the sketch of a circle of unit radius and centre $(1 + i)u = 3 - 3i$. The required distance was that from the origin through the centre of the circle to its circumference. With this part there appeared to be a better understanding than in part **(ii)** and some correct answers were observed.

Answers: **(i)** 3; **(ii)** $\frac{3}{2}\sqrt{2}$ or 2.12; **(iii)** $3\sqrt{2} + 1$ or 5.24.

Question 9

(i) The attempts at this part usually were unsuccessful from the start as few candidates could apply the double angle formula correctly or state $\cos 4\theta = 2\cos^2 2\theta - 1$, or its equivalent.

(ii) Whilst many candidates knew to use their result from part **(i)** in part **(ii)** they were often unable to successfully convert $8\cos^4 \theta - 3 = 1$ to $\cos^4 \theta = \frac{1}{2}$. Even those that reached this result found obtaining the correct θ values problematic.

(iii) The linkage between parts **(i)** and **(iii)** was only observed by few candidates, so most made no sensible progress with this part.

Answers: **(ii)(a)** ± 0.572, **(b)** $\frac{3}{32}\pi + \frac{1}{4}$.

Question 10

(i) The attempts at this question were either virtually all correct, except for an odd sign error, or were almost totally unsuccessful. This difference arose primarily from whether the candidate realised or not that to undertake the integration it was first necessary to split the expression into its partial fractions.

(ii) Some candidates who were unable to make any progress with part **(i)** were able to state correctly the number of birds after a long time.

Answers: **(i)** $N = \dfrac{1800e^{\frac{1}{2}t}}{5 + e^{\frac{1}{2}t}}$; **(ii)** Approaches 1800.

MATHEMATICS

> **Paper 9709/32**
>
> **Paper 32**

Key messages

In order that candidates receive credit where possible for incorrect answers, they should remember to show full working and reasoning. Incorrect answers without working cannot be given any credit, whereas partial credit can be awarded if a correct method is shown. Where candidates are asked to 'show' or 'prove' a result, marks will be lost if any essential steps in the argument are omitted.

General comments

The standard of work on this paper varied considerably and resulted in a wide spread of marks. No question or part of a question seemed to be of undue difficulty, and most questions discriminated well. The questions that were generally easily done were **Question 3** (trigonometry), **Question 6** (differential equation) and **Question 8** (partial fractions). Those that were done least well were **Question 2** (logarithms), **Question 5** (parametric differentiation), and **Question 7** (complex numbers).

In general the presentation of work was good and candidates appeared to have sufficient time to attempt all the questions. However, as referred to above, when attempting a question, candidates need to be aware that it is essential that sufficient working is shown to indicate how they arrive at their answers, whether they working towards a given answer, for example as in **Question 10(i)**, or an answer that is not given, as in **Question 10(ii)** and **Question 10(iii)**.

Where numerical and other answers are given after the comments on specific questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

Comments on specific questions

Question 1

This was fairly well answered. The most popular method involved working with a non-modular quadratic inequality. Whatever method was used the majority of candidates found the correct critical values -5 and $-\frac{5}{3}$. However the final answers to the question were frequently wrong. The commonest incorrect answers were *(a)* $x > -5$, $x > -\frac{5}{3}$, *(b)* $-5 < x < -\frac{5}{3}$, and *(c)* $-\frac{5}{3} < x < -5$; in the case *(c)*, it should be noted that since 'less than' is a transitive relation this answer implies that $-\frac{5}{3} < -5$ which is of course untrue.

Answer: $x < -5$, $x > -\frac{5}{3}$.

Question 2

Some candidates dealt with this easily. However there were many faulty attempts. Candidates need to be secure in their understanding of both the idea of a base for logarithms and the manipulation of logarithms (and exponentials). A common error was to start by replacing $\log_2 (x + 5)$ with $\log_2 x + \log_2 5$; another was to regard the symbol $\log_2$ as a quantity. When logarithms were removed, 5^2 occurred almost as often as 2^5. In the work that followed, those candidates who had obtained the correct quadratic $x^2 + 5x - 32 = 0$ quite often failed to reject the negative root -8.68.

Answer: **(ii)** 3.68.

Question 3

This question was answered well. Most candidates handled $4\cos 2\theta$ correctly and went on to reduce the equation to the quadratic $8\cos^2 \theta + \cos \theta - 7 = 0$. Most of the errors arose when solving for θ. The solution to $\cos \theta = \dfrac{7}{8}$ is inexact and was often not given correctly to 1 decimal place. The solution to $\cos \theta = -1$ frequently included the additional wrong answer $0°$.

Answer: 29.0°, 180°.

Question 4

In part **(i)** those who found $CT = r \tan x$ or $OT = r \sec x$ and knew the relevant area formulae did well. Generally sufficient working was shown to derive the given answer. However some candidates did not make any progress in this part.

Before starting an iteration involving a trigonometric function candidates need to check whether their calculator should be set in degree or radian mode. For part **(ii)** of this question such a check was particularly important since the previous question involved calculating angles in degrees. Those candidates who had a correct understanding of the iteration process and had their calculator in radian mode answered this part of the question very well.

Answer: **(ii)** 1.35.

Question 5

Part **(i)** discriminated well. The chain rule was needed in both the differentiations with respect to t and there was plenty of scope for errors in the division of the derivatives and any simplifications that were made.

In part **(ii)** those who used the given information to deduce $t = \dfrac{1}{4}\pi$ and found numerical values for y and the gradient usually formed the equation of the tangent in a correct manner. However the wrong answer $t = 0$ was common, and some candidates worked throughout with a gradient that was a function of t.

Answer: **(i)** $2 \sin^2 t \cos^2 t$; **(ii)** $y = \dfrac{1}{2}x + \dfrac{1}{2}$.

Question 6

Nearly all candidates interpreted the proportionality of the gradient to xy correctly and went on to answer part **(i)** very well. In part **(ii)** the request to find the gradient at $(-1, 2)$ was included in the question to assist candidates in making a sketch of the curve. The correct answer -4 for the gradient was almost universal and there were some excellent sketches. However many sketches resembled that of the standard exponential function and had a low positive gradient at $x = -1$ rather than a negative gradient as found previously.

Answer: **(ii)** -4.

Question 7

This question discriminated well throughout.

(a) Candidates usually started part **(i)** correctly. However, many of them, having obtained a real denominator embarked on false simplifications which led to answers with $a + 4$ as denominator, or even the answer $\dfrac{5}{a} - \dfrac{10\,i}{4}$.

In part **(ii)** the conjugate of the answer obtained in part **(i)** was usually formed correctly, but forming an equation to find a proved challenging. The most common errors were to equate the ratio of the imaginary and real parts of the conjugate to either $\dfrac{3}{4}\pi$ or to $\tan^{-1}\left(\dfrac{3}{4}\pi\right)$.

(b) Candidates need to set up their Argand diagrams with equal scales on the real and imaginary axes. Failure to do so leads to grave difficulties in questions such as this. Most sketches earned a mark for a correct circle. A mark was reserved for a correct plot of the point representing $2 + 2i$ or the equation $y = 2 - x$, or equivalent, but relatively few earned it. Those with a correct line and circle did not always shade the correct region. For example, some shaded the wrong segment of the circle and some only shaded the part of the correct region where x and y were positive.

Answers: **(a)(i)** $\dfrac{5a}{a^2 + 4} - \dfrac{10\,i}{a^2 + 4}$, **(ii)** -2.

Question 8

Part **(i)** was well answered. Nearly all candidates stated a correct general form such as $\dfrac{A}{1 + x} + \dfrac{Bx + C}{2 + x^2}$ and carried out the evaluation of the constants clearly and accurately. A common source of error was the failure to place $Bx + C$ in brackets in the basic identity $5x - x^2 \equiv A\,(2 + x^2) + (Bx + C)\,(1 + x)$.

Part **(ii)** was also well answered. However, before attempting to expand each partial fraction, candidates need to check that the partial fractions correctly record the outcome of part **(i)**. As well as errors in transcribing the values of A, B, and C, the miscopying of $(2 + x^2)$ as $(2 + x)^2$ was also seen. The final mark here is for a correct sum of terms in ascending powers of x. A few candidates incorrectly multiplied through by 4 to convert the coefficients to integers.

Answers: **(i)** $-\dfrac{2}{1 + x} + \dfrac{x + 4}{2 + x^2}$; **(ii)** $\dfrac{5}{2}x - 3x^2 + \dfrac{7}{4}x^3$.

Question 9

Part **(i)** was quite well answered. Most candidates formed a fraction consisting of a correct scalar product of normal vectors divided by the product of the moduli. But some then used the inverse sine to calculate the angle while others, using the inverse cosine, did not follow the answer 100.3° with its supplement but gave 10.3° instead.

In part **(ii)** five different methods of solution were seen. The error of multiplying a position vector such as

$$\begin{pmatrix} 0 \\ \dfrac{31}{8} \\ \dfrac{3}{8} \end{pmatrix}$$

by 8 to clear the components of fractions was not uncommon. The cartesian approach in which two expressions are obtained with one variable, say x, expressed in terms of a second variable y and also in terms of the third variable z, was used effectively. So was the closely related approach in which each of two variables, say y and z, is expressed in terms x. Candidates using these cartesian methods need to be particularly careful to avoid errors when forming a vector equation of the line.

Answers: **(i)** $79.7°$ or 1.39 radians; **(ii)** $\mathbf{r} = \mathbf{i} + 3\mathbf{j} + \lambda(8\mathbf{i} - 7\mathbf{j} - 3\mathbf{k})$.

Question 10

Most candidates realised that part **(i)** involved integrating by parts twice and many started well. But the indefinite integral $-x^2 e^{-x} - 2x e^{-x} - 2e^{-x}$ can only be reached if brackets and minus signs are handled accurately. For example, the minus sign in the third term of the integral is the product of five minus signs in the integration. Those that removed products of minus signs as soon as they arose seemed to have a better chance of reaching the correct indefinite integral than those who delayed till the end. Part **(ii)** was easily done by almost all. The answer $x = 2$ with no relevant working earned nothing. Part **(iii)** proved testing; only a minority had a sound method such as equating the gradient to $\dfrac{y}{x}$ and solving for x. The answer $x = 1$ with no relevant working scored nothing.

Answers: **(ii)** 2; **(iii)** 1.

MATHEMATICS

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General comments

The questions, or parts of questions, that candidates performed well on were **Question 2** (differentiation), **Question 4(i)** (trigonometry), **Question 5(i)** (remainder theorem), **Question 6** (numerical iteration), **Question 9** (differential equation) and **Question 10** (vector geometry), the latter question being well attempted by a large number of candidates. Those questions that candidates found difficult were **Question 1** (laws of logarithms), **Question 4(ii)** (solving trigonometric equation), **Question 5(ii)** (division of polynomials), **Question 7(ii)** (argument of complex numbers), **Question 7(iii)** (verification of complex equation), **Question 8(i)** (product and chain rules for differentiation) and **8(ii)** (integration involving change of variable).

Some candidates chose inappropriate time-consuming methods for some questions. For example: in **Question 1** it was common to see logarithms to the base 2, 3 and 5; in **Question 4(ii)**, instead of staying with an equation in $\tan^2 \theta$ it was common to see it manipulated into sine and cosine, and sometimes even a double angle. This poor choice of method often continued into **Question 7(iii)**, with candidates not heeding the guidance in part **(ii)** suggesting the use of the polar form, instead they expanded their roots to the power 6, once for each root in turn, and arithmetic errors usually prevented any successful conclusion. In **Question 8(i)** it was common to see candidates try, usually unsuccessfully, to convert the trigonometric function into one involving cos 4x, and even if they were successful there was little chance of their returning to an equation that they could solve when the derivative was set to zero.

Comments on specific questions

Question 1

Whilst it is possible to use logarithms to any base, many candidates had a mixture which proved difficult to handle. It was very common to see 2(3^x) taken as 6^x, or 2log6^x following the taking of logarithms of both sides of the equation. It should be stressed that it was necessary for candidates to reduce their equation at least as far as a linear equation in x before they resorted to their calculator in order to gain any marks.

Answer: 1.09.

Question 2

This question usually produced some sound mathematics. However, a few candidates muddled the product and the quotient formulae in that they had $v = x^{-3}$ in the quotient formula. Another common error was to see powers of x being incorrectly cancelled in only one of the numerator terms, or a similar error, resulting in an insoluble equation involving ln x, powers of x and a constant.

Answer: $e^{\frac{1}{3}}$ or 1.40.

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Question 3

Most candidates made a good attempt at the initial integration by parts. Some candidates differentiated the $e^{-\frac{x}{2}}$ within this formula whilst trying to integrate it. However, many candidates did not show any working when obtaining the given answer. Such candidates just *assumed* that if they substituted the given limits then this would produce the given answer and this resulted in the loss of both the method and the accuracy marks.

Question 4

(i)   The given result was usually well established, although some candidates omitted much of the required algebra. This evaluation required the multiplying out of a couple of brackets (twice) and the cancelling of the common terms. With a given answer it is essential that this detail is clearly displayed.

(ii)   It was expected that candidates would solve the new form of the given equation with the prescribed value of k. As this equation involved only terms in $\tan^2 \theta$ and constants the result $\tan^2 \theta = \frac{1}{11}$ should have been established immediately. However, many found the mental calculation difficult or finished with a quadratic equation in $\tan \theta$. Even those who correctly produced $\tan^2 \theta = \frac{1}{11}$ often did not manage to square root this value and obtain one of the two answers from their calculator. The negative square root was rarely in evidence.

Answer: **(ii)** 16.8°, 163.2°.

Question 5

(i)   Many candidates were able to obtain a and b correctly. However, the use of long division to establish the two equations involving a and b is not the ideal approach since errors are very likely and it is also time consuming as compared with substituting $x = \frac{1}{2}$ and $x = 2$ and equating to 0 and 12, respectively. Common errors included the use of $x = -\frac{1}{2}$, and equating the expression to -12.

(ii)   In obtaining the quadratic factor many candidates became confused and believed it was associated with $(x - 2)$ rather than $(2x - 1)$. Some very able candidates used the method of synthetic division but used it incorrectly due to there being a coefficient 2 as opposed to unity in the x-term. Those using long division were usually far more successful.

Answers: **(i)** 2, −3; **(ii)** $x^2 - x + 2$.

Question 6

(i)   Very weak attempts were seen at both graphs. The graph of $1 + x^2$ was often a straight line or did not cover the whole interval, something essential if one is to determine the number of roots, whilst the graph of $\cot x$ nearly always did not pass through $\frac{\pi}{2}$.

(ii)   This was usually well done. However, the question does say 'by calculation' so actual numerical values were expected, together with a clear statement mentioning change of sign, something that a few candidates did not include in their work.

(iii)   Some very sound numerical work was seen. However, candidates need to take care that they have the correct iterative expression in their calculator, as it was common to see convergence to some completely different value. Very few candidates did not set their calculator to radian mode in such calculations.

Answer: **(iii)** 0.62.

Question 7

(i)   This was usually correctly answered, but an answer of √3 ± 2i due to poor cancelling was common, as was the complete omission of i.

(ii)   This part was not well done for a variety of reasons. Firstly, many candidates failed to clearly state the modulus of *both* complex numbers. Secondly, some candidates simply evaluated $\pm\dfrac{\pi}{6}$ and then assumed these were the arguments, paying no heed to where the points were on the Argand diagram. In fact, sometimes each complex number was associated with two arguments.

(iii)   Very few candidates used their information from part **(ii)**, instead opting to multiply 6 brackets out. Most made little progress with this, but a few who spotted that one could use 3 or even 2 brackets were more successful. Some of the more able candidates who resorted to writing the complex numbers as $2e^{\pm\frac{5\pi}{6}i}$ consistently omitted the i here and in their cos θ + i sin θ expression.

Answers: **(i)** $-\sqrt{3}+\mathrm{i}$, $-\sqrt{3}-\mathrm{i}$; **(ii)** 2, $\dfrac{5}{6}\pi$; 2, $-\dfrac{5}{6}\pi$.

Question 8

(i)   Many candidates found this question difficult because they either tried to convert using some double angle formula or did not realise that the chain rule was required to differentiate both $\cos^2 x$ and $\sin^3 x$. Again candidates who actually reached $\tan^2 x = \dfrac{3}{2}$ or an equivalent expression often had difficulty in proceeding to the correct values. Many candidates gave an answer in degrees.

(ii)   This was another part that candidates found difficult. Candidates usually evaluated $\dfrac{\mathrm{d}u}{\mathrm{d}x}$ but became confused when substituting into the integrand, either because the sin x arising from their $\dfrac{\mathrm{d}u}{\mathrm{d}x}$ incorrectly appeared in their numerator, or because they did not realise how to write $\cos^2 x$ in terms of u. Many candidates proceeded to write it the given answer, omitting all the essential work on the substitution of limits.

Answers: **(i)** 0.886; **(ii)** $\dfrac{2}{3}$.

Question 9

(i)   Often candidates simply verified the value 0.01 instead of deriving this value by using $\dfrac{\mathrm{d}x}{\mathrm{d}t} = k(10-x)(20-x)$.

(ii)   There was some good partial fraction work, although occasional sign errors arose when producing the ln terms. Using the laws of logarithms to remove the logarithm terms was much improved compared with previous years.

(iii)   Most candidates believed the answer was 10, instead of 'approaches 10' or 'increases towards 10' or 'tends to 10'.

Answers: **(ii)** $x = \dfrac{20\left(e^{0.1t}-1\right)}{2e^{0.1t}-1}$; **(iii)** *x* approaches 10.

Question 10

(i)   There were many misreads taking 2**j** to be 2**i**. There were also many errors in trying to solve the 3 linear equations for λ and μ. Some candidates believed that there was no need to solve because λ could be expressed as two different expressions involving μ.

(ii)   Candidates usually obtained the correct answer. However, it was necessary to identify the very basic arithmetic steps when taking the scalar product, otherwise the method mark might not be earned when the answer was incorrect.

(iii)   This was approached by a variety of methods. The vector product method was popular and usually correctly applied to obtain a normal vector, but sometimes with one (or more) incorrect components. Often the point chosen to calculate the equation of the plane was not a point in the required plane.

Answers: **(ii)** 47.1°; **(iii)** $-2x + 4y + 3z = 26$.

MATHEMATICS

Key messages

In order that candidates receive credit where possible for incorrect answers, they should remember to show full working and reasoning. Incorrect answers without working cannot be given any credit, whereas partial credit can be awarded if a correct method is shown. Where candidates are asked to 'show' or 'prove' a result, marks will be lost if any essential steps in the argument are omitted.

General comments

The paper allowed candidates of all abilities to demonstrate their worth. There were candidates who could cope successfully with all, or nearly all, of the demands of the paper. Others found some questions, particularly **Questions 3** and **4** and the latter algebraic parts of **Question 6**, very demanding. Other candidates need to ensure that they have understood and learned the basic facts and techniques required by the syllabus.

Standards of presentation also varied. There were many well-presented scripts. However some candidates do need to heed the instructions on the front of the question paper and remember 'the need for clear presentation'.

Very few candidates had problems distinguishing between units of mass and force.

Comments on specific questions

Question 1

Most candidates scored well. They identified the need to use Newton's second law in part **(i)** and knew the relationship between power, force and speed required for part **(ii)**.

(i)   Most answers were fully correct. The commonest error was to subtract 600 from 1400 rather than to add.

(ii)   The vast majority of candidates correctly used the relationship $P = Fv$. Others attempted to use the equations of motion, not realising that the acceleration is not constant in such situations.

Answers: **(i)** 2000 N; **(ii)** 30 000 W (or 30 kW).

Question 2

This was another question that most candidates scored heavily on.

(i)   Most candidates realised that the work done was in two parts – that against the resistance to motion and that providing an increase in potential energy to the load.

(ii)   Most candidates knew that the power was equal to the work done in part **(i)** divided by the time they were seeking and capably dealt with the algebraic manipulation involved.

Answers: **(i)** 25 000 J (or 25 kJ); **(ii)** 20 s.

Question 3

This question proved difficult for the majority of candidates. Candidates need to know how to analyse equilibrium situations where all of the forces are not simply vertical or horizontal. Most knew the directions of the tension forces acting on the ring *R* and that they needed to resolve these and the other forces involved in the horizontal and vertical directions. But most could not do this correctly. Clear diagrams illustrating the situation were not always present. Candidates need to understand that such diagrams will help them in determining the correct equations and assist Examiners in assessing their work. Some candidates need also to understand that the process involved is essentially an application of Newton's second law with acceleration equal to zero and should yield equations, not simply a sum of components.

Candidates who could not show that $T \sin \theta = 12$ and $T \cos \theta = 3.5$ did not always realise that they could still find the value of θ by eliminating T from these 2 equations by division.

Answer: 73.7°.

Question 4

This was another question requiring resolution of forces in an equilibrium situation and this proved as unrewarding for most as the preceding question. Most candidates understood the need to resolve parallel and perpendicular to the inclined plane. They also understood the relationship between the coefficient of friction, the force of limiting friction (and its presence here) and the normal reaction.

(i)   The major difficulty was to obtain the correct equations from resolving the forces in the direction parallel to the inclined plane. Successful candidates drew a separate diagram for each situation and clearly marked the forces, including their directions, on them. Candidates should understand the great help that these diagrams provide during the resolving process. As in **Question 3** not all candidates understood that this is another example of the application of Newton's second law with zero acceleration.

(ii)   Most candidates knew that the normal reaction was given by $11g \cos \theta$. They also knew how to use this and their value of the force of limiting friction to evaluate a coefficient of friction.

Answers: **(i)** 10; **(ii)** 0.367.

Question 5

With two exceptions the various parts of this question proved to be a reliable source of marks for most candidates.

(i)   Almost all candidates realised that they needed the equations of motion for this question and confidently used the data for the first part of the motion to find the constant speed in the second part. This was then used to find the time involved in the third part. The calculations were usually correctly performed.

(ii)   Almost all candidates understood the correct form of the required graph. Unfortunately many failed to notice that the question also asked them to find the distance *AB*. It may have been because a separate sheet of graph paper was sometimes used for the graph (unnecessarily as only a 'sketch' was required). Candidates should understand that it always makes sense to check that all parts of a question have been answered.

(iii)   The time in the first section, 300 s, was always found correctly. Unfortunately many did not notice that a second time, in the third section, was also needed. Those that did almost always found this accurately.

Answers: **(i)** 3600 s; **(ii)** 46 500 m (or 46.5 km); **(iii)** 300, 3400.

Question 6

For most candidates both parts of this question proved to be of two distinct halves. They correctly understood that the velocity clearly did not vary linearly and thus that the equations of motion would be useless and thus integration and differentiation would be required. These two operations caused few problems but the ensuing algebra that was necessary did.

(i)   A very few candidates differentiated but most understood that velocity must be integrated to find distance. This was accurately done. The resultant quartic equation then defeated the majority. Those who realised that this equation was, in fact, a quadratic in t^2 usually solved this correctly and then remembered to take the square root to obtain the required answer.

A number of candidates correctly reached the stage: $2t^2 - \dfrac{t^4}{64} = 64$ then proceeded to factorise the left-hand side of this equation and then equate each of the two factors, t^2 and $2 - \dfrac{t^2}{64}$ to 64 – going on to obtain the 'correct' answer.

(ii)   The required differentiation was usually correct as was the initial inequality. Most candidates then obtained the required numerical value, 4.62 (8/√3 was also accepted) but very few successfully moved on to obtain the correct range of values..

Answers: **(i)** 8 s; **(ii)** 0 < t < 4.62.

Question 7

This question was generally answered quite well, at least in parts.

(i)   Many Centres have clearly prepared their candidates well in the requirements of this part and there were few problems in applying Newton's second law to the individual masses. The only problem for these candidates was to notice that the tension was required as well as the acceleration. Less successful candidates need to understand the value of a diagram with clearly marked forces. This helps the correct application of the required methods.

(ii)   Part **(a)** was almost always correct and so was part **(b)** – if solved by simply multiplying the value of the tension by the distance moved. Less successful were those candidates who tried to involve changes in both the kinetic and potential energies of *A*. For the former, part **(c)** usually followed naturally. Those following less direct methods often had difficulty in maintaining numerical accuracy.

(iii)   Most candidates correctly obtained the speed of *A* when the string becomes slack. Successful candidates then realised that the ensuing acceleration was now that due to gravity and not that found in part **(i)**. Of course, the really successful candidates also realised that *A* must come down as well as go up and offered the correct answer instead of the more common 0.4 s.

Answers: **(i)** 2.5 ms⁻²; 15 N; **(ii)(a)** 18 J, **(b)** 22.5 J, **(c)** 4.5 J; **(iii)** 0.8 s.

MATHEMATICS

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Paper 42
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Key messages

In order that candidates receive credit where possible for incorrect answers, they should remember to show full working and reasoning. Incorrect answers without working cannot be given any credit, whereas partial credit can be awarded if a correct method is shown. Where candidates are asked to 'show' or 'prove' a result, marks will be lost if any essential steps in the argument are omitted.

General comments

The work of candidates was generally well presented. Most candidates found **Questions 4** and **7** very straightforward although many candidates found difficulty with **Questions 5** and **6**.

Candidates for examination should be aware of the terminology specific to the syllabus. For example, in **Question 6** candidates should understand the significance of the word 'inextensible' which precludes the use of the formula for elastic strings which is not included in the M1 syllabus.

A misunderstanding that arose much more frequently in the same question is that acceleration is relevant to the question. Some candidates believed the question to be one of connected particles and wrote equations such as $0.6g - T = 0.6a$ and $T - 0.4g = 0.4a$. Candidates should have appreciated the meaning of the phrase 'the system is in equilibrium', which precludes acceleration from the question.

In addition to 'inextensible' and 'equilibrium' candidates should have a precise understanding of other syllabus terms.

Comments on specific questions

Question 1

Many candidates scored full marks for this question. Part **(i)** in particular was understood by candidates as a straightforward application of the formula 'work done = $Fd\cos\alpha$'.

Answers: **(i)** 4920 J; **(ii)** 97.1 W.

Question 2

A large proportion of candidates scored full marks for this question. Those who failed to do so usually scored two marks, obtaining the answer 1.25 m after subtracting 120 J from the kinetic energy gain instead of adding. Others scored no marks by using the principle of conservation of energy and ignoring the work done against resistance. These candidates should have realised that the mention of work done against the resistance to motion in the question, should preclude the use of the principle of conservation of energy.

Answer: 4.25 m.

Question 3

(i) Most candidates used the idea that the area under the graph represents the distance fallen and applied the principle accurately.

(ii) Most candidates were aware that the gradient of the graph represents the acceleration of the parachutist. However mistakes such as $a = \dfrac{50-8}{12-5} = 6$, and $d = \dfrac{8-50}{12-5}$ $= -6$ arose. Most candidates recognised the need to use Newton's second law, and wrote a 3-term equation with weight, upward force and 'ma' represented. However sign errors were common and 340 N was a common wrong answer.

Answer: **(ii)** 1360 N.

Question 4

(i) The word 'equilibrium' was more widely understood in this part of the question than in **Question 6**. Many candidates scored full marks for this part. However a very common answer for θ was 49.1. The likely explanation for this wrong answer is that in keying figures into the calculator to find θ from $\tan^{-1}\left(\dfrac{4}{6\times3^{\frac{1}{2}}}\right)$, the candidates used $\tan^{-1}\left(\dfrac{4}{6}\times3^{\frac{1}{2}}\right)$.

(ii) Most candidates attempted this part by finding the components in the 'x' and 'y' directions of the resultant of the two remaining forces. However very few candidates found both $X = 6 \times 3^{\frac{1}{2}}$ and $Y = 4 - 10$, despite the fact that the constituent figures are readily available following the work in part **(i)**.

Some candidates who used this method had unsatisfactory descriptions of the direction of the required resultant. The clearest answers seen were 'on a bearing of 120°', '30° below the x-axis', '30° clockwise from the positive x-axis' and '60° anticlockwise from the 10 N force', or in a sketch with the correct answer clearly shown.

Many candidates demonstrated a clear understanding of 'equilibrium' and 'resultant' by appreciating that the removal of one force destroys the equilibrium and has the effect of making the resultant of the remaining forces equal in magnitude and opposite in direction to the force removed.

Answers: **(i)** 11.1, 21.1; **(ii)** 12 N, 30° clockwise from the positive x-axis.

Question 5

(i) This was well attempted and most candidates recognised the significance of the times taken by P and Q to reach the highest point during their motion. However some candidates thought 'opposite directions' would require separate formulae for P and Q, one being $v = u - gt$ and the other $v = u + gt$. Such candidates made no progress.

Although most candidates found $t = 0.7$ and $t = 1.2$, many gave the duration of 0.5 s as the answer instead of the range. Others gave the range as $0.7 < t < 1.2$, instead of $0.7 < t < 1.2$, overlooking the fact that one particle is at rest instantaneously when $t = 0.7$ and when $t = 1.2$.

(ii) This part was poorly attempted by many candidates and the question as a whole was found to be the most difficult of the whole paper.

A very common error was to set up an equation for v, using $3h_P = 8h_Q$, as $\dfrac{3(v^2 - 144)}{-20} = \dfrac{8(v^2 - 49)}{-20}$ without distinguishing v_P and v_Q from each other.

Another common error was to substitute $s = 3h_P$ and $s = 8h_Q$ into equations for P and Q respectively, derived from $v^2 = u^2 + 2as$, $s = \dfrac{1}{2}(u+v)t$ or $s = ut + \dfrac{1}{2}at^2$.

Answers: **(i)** $0.7 < t < 1.2$; **(ii)** 4 ms^{-1}, -1 ms^{-1}.

Question 6

Angles of significance in this question are *ABR* and *MRB* (where *M* is the mid-point of *AB*). Incorrect values for these angles that were commonly used were 45° and 45°, 30° and 60°, 60° and 30°, 51.3° and 38.7° and 38.7° ($\sin^{-1}\left(\dfrac{50}{80}\right)$) and 51.3°.

(i) The most common error was to ignore one part of the string, obtaining *T*cos*α* = 0.6*g* instead of 2*T*cos*α* = 0.6*g*. Other errors were to include the weight of *B* as well as the weight of *R* in resolving forces on *R*, and resolving forces in the direction of one part of the string and ignoring the tension in the other part of the string.

(ii)/(iii) Most candidates who answered part **(i)** correctly found correct answers in these two parts.

Others who brought forward incorrect values of *T* often scored all three method marks and a follow through accuracy mark in these parts. This was especially so in cases for which *T* = 10 N was obtained in part **(i)** by ignoring one part of the string.

However new errors arose in these parts, including *F* = *W*sin*α* instead of *F* = *T*sin*α* and *R* = 0.4*g*.

Answers: **(i)** 5 N; **(ii)** 4 N, 7 N; **(iii)** 0.571.

Question 7

This was the best attempted question of the paper, which is unusual for a final question.

Almost all candidates scored all three marks in part **(i)** and those who realised the need to invoke the calculus usually proceeded to score full marks for the question.

Answers: **(i)** 100 s, 110 m; **(iii)** 0.132 ms^{-2}.

MATHEMATICS

> **Paper 9709/43**
> **Paper 43**

Key messages

In order that candidates receive credit where possible for incorrect answers, they should remember to show full working and reasoning. Incorrect answers without working cannot be given any credit, whereas partial credit can be awarded if a correct method is shown. Where candidates are asked to 'show' or 'prove' a result, marks will be lost if any essential steps in the argument are omitted.

General comments

Almost all candidates attempted all questions and were able to show what they knew. Questions requiring straightforward answers were generally done well for all parts of the syllabus.

Candidates need to ensure that they consider all forces when applying Newton's second law or when using a work/energy equation. A force diagram may help to ensure that no forces are omitted (e.g. **Questions 5(iii)** and **6**).

When candidates are asked to 'show that ...' they need to make sure that their answer is complete using the information given in the question (e.g. **Question 5(i)** and **Question 7(i)**).

Many well presented scripts showing work of a high standard were seen. **Questions 1**, **2** and **4** were found to be the best answered questions whilst **Question 5** was the least well answered.

Comments on specific questions

Question 1

The majority of candidates achieved full marks for this question. The occasional errors seen were usually either the use of sine instead of cosine or early approximations such as $\frac{8200}{9000}$ = 0.911 leading to a slight inaccuracy in the final answer.

Answer: 24.3.

Question 2

Most candidates attempted to use $P = Fv$ and $F = ma$ to solve the problem. The most usual methods of solution were to set up simultaneous equations from either $\frac{P}{v} - R = ma$ or $P = (ma + R)v$. Occasionally a sign error suggested that the driving force and the resistance were acting in the same direction, but many clear and accurate solutions were seen. The power was given in the question as P W and so the value $P = 28\,500$ rather than $P = 28.5$ was expected. Some candidates erroneously used $P = mav$ leading to two different values of the constant P.

Answers: 28500; 750.

Question 3

This question proved to be more challenging. The candidates who used constant acceleration formulae and recognised that $a = g\sin 30°$ for particle P often found both u and the speed at the bottom of the ramp correctly. Unfortunately, some candidates believed that the acceleration of both P and Q was g ms^{-2}. Other candidates formed energy equations for P and Q, sometimes assuming in part **(i)** that the speed of both particles at the bottom of the ramp was the same. In part **(i)**, a fully correct solution using an energy equation for particle P was rare but candidates could still gain some credit for 'correctly' forming an energy equation in part **(ii)** using an incorrect value of u found in part **(i)**.

Answers: **(i)** 6; **(ii)** 10 ms^{-1}.

Question 4

This question was often answered well.

(i)   The shading usually showed the correct region. Incorrect versions sometimes left the required region unshaded.

(ii)   Candidates frequently formed and correctly solved an equation in T. '$T = 5$' was a common incorrect answer from $20 \times 2.5 + 4T = 70$. In both part **(ii)** and part **(iii)** some candidates attempted to use $s = \dfrac{1}{2}(u + v)t$, even though the situation was one of constant speed rather than constant acceleration.

(iiI)   In finding the difference between distances travelled by P and by Q, candidates sometimes calculated the distance travelled by P as $70 + 20 \times 1.5$ or $70 + 20 \times 2.5$ instead of $70 + 20 \times 4$.

(iv)   There was considerable variation in the graphs drawn. Many correctly showed parallel line segments on a single diagram with appropriate values shown on the t-axis. The second line segment for each particle needed to be steeper than the first to indicate greater speed. Some graphs included curves, some included horizontal or vertical lines, some omitted to label values on the t-axes, some started Q from the wrong value of t and some showed only the first part of the graph for particle Q.

Answers: **(ii)** 25; **(iii)** 100 m.

Question 5

This was one of the less well answered questions.

(i)   The question asked candidates to show that the magnitude of the frictional force was 7.5 N and having resolved horizontally and achieved 7.5 N, many showed no further justification. For a complete solution resolving in two perpendicular directions was required.

(ii)   Candidates frequently assumed that the reaction was in the same horizontal plane as the other forces, stating $R = mg - 4.8 + 5\sin\theta$. Thus, although the value of 0.6 was found for μ it often followed from incorrect working.

(iii)   Some candidates omitted one of the forces when using Newton's second law leading to, for example, $a = 0.88$ from $8.6 - 7.5 = 1.25a$. Some found the acceleration but did not give the direction while others found the resultant force and direction but did not calculate the magnitude of the acceleration.

Answers: **(i)** Opposite in direction to the force of magnitude 6.1 N; **(ii)** 0.6;
        (iii) 2 ms^{-2} in the direction of the force of magnitude 6.1 N.

Question 6

Although this was a more demanding question it was often answered encouragingly well. The two approaches (work/energy and resolving forces/Newton's laws) were both regularly used.

(i)   Candidates needed to recognise that there were two elements to consider. Errors were usually due to the omission of either the weight or the resistance, or else from combining the two elements of work done by subtraction rather than by addition. Candidates need to make sure that all terms in an equation are in the same units. Equations were sometimes seen combining, for example, force with potential energy.

(ii)   Sign errors and missing terms were the main reasons for gaining an incorrect speed for the lorry at the bottom of the hill. For example, when using a work/energy equation the work done by the driving force (2000×5000) or the work done against the resistance (800×500) was sometimes omitted. Alternatively, when applying Newton's second law, a common error was to omit $15\,000g\sin 2.5°$ from the equation, stating $2000 - 800 = 15\,000a$ and leading to $v = 21.9$ ms^{-1}.

Answers: **(i)** 3 670 000 J; **(ii)** 30.3 ms^{-1}.

Question 7

The majority of candidates understood that integration was required for this question and they frequently scored well. Nevertheless, a few attempts to use constant acceleration formulae were seen and also a few attempts which confused differentiation with integration.

(i)   Candidates were required to show that the initial speed of the particle was zero. They were expected to use the information that, at B, $v = 0$ when $t = 20$, to find the constant of integration. Many solutions were incomplete with no consideration of a constant of integration.

(ii)   The maximum speed was sometimes rounded 5.3 ms^{-1}. This was usually the only place on the paper where insufficient accuracy was given.

(iii)   The distance was frequently calculated accurately with only occasional integration errors or attempts to use a constant acceleration formula.

Answers: **(ii)** 5.27 ms^{-1}; **(iii)** 50 m.

MATHEMATICS

| Paper 9709/51 |
| Paper 51 |

Key messages

In order that candidates receive credit where possible for incorrect answers, they should remember to show full working and reasoning. Incorrect answers without working cannot be given any credit, whereas partial credit can be awarded if a correct method is shown. Where candidates are asked to 'show' or 'prove' a result, marks will be lost if any essential steps in the argument are omitted.

General comments

g = 10 was used by most candidates with only a few candidates using g = 9.8 or 9.81.

Premature approximation and rounding to the wrong number of significant figures were rarely seen.

Question 1 was found to be the easiest question on the paper and **Question 7** proved to be the most difficult.

Comments on specific questions

Question 1

This proved to be a good starter question for candidates.

Two different approaches were taken with both leading to the correct result. Either $s = ut + \frac{1}{2}at^2$ for vertical motion was used or time of flight $t = \frac{2V \sin\theta}{g}$. Some candidates only found half the time of flight.

Answer: 1.93 s.

Question 2

(i)   The centre of mass of *G* from *O* was often found. Sometimes the wrong formula was used even though reference could have been made to the formula list.

Many candidates had difficulty with finding the distance of *G* from *A* which could have been calculated by using the cosine rule on triangle *AOG*.

(ii)   A solution to this part of the question was not often seen. The distance *AG* found in part **(i)** could be used in the sine rule for the triangle *AOG*.

Answers: **(i)** 0.429 m; **(ii)** 36.4°.

Question 3

(i)   Quite often an attempt to find the tension in the string was seen which did not help the candidate to solve the problem. By using KE loss = EE gain the result could be derived.

(ii)   In order to find the greatest acceleration it was essential to realise that this occurred when the particle had zero velocity. Newton's second law could be used to find the acceleration.

Answer: **(ii)** 1.92 ms^{-1}.

Question 4

(i) It was necessary to use the idea that the maximum velocity occurred when the acceleration was zero. The differential equation $\dfrac{dv}{dt} = v\dfrac{dv}{dx} = 15 - 6x$ needed to be solved using the correct limits.

(ii) By solving the differential equation in part **(i)** the candidate should arrive at $\dfrac{v^2}{2} = 15x - 3x^2$. If $v = 0$ was then used $15x - 3x^2 = 0$ follows and hence $x = 5$. Substituting this in the acceleration expression gives $a = -15$.

Answers: **(i)** 2.5, 6.12 ms^{-1}; **(ii)** -15 ms^{-2}.

Question 5

(i) Candidates were required to take moments about A for the forces acting on the lamina. These forces were T horizontally at B, T vertically at C and the weight of 19 N.

(ii) This part of the question required candidates to use the formula $T = \dfrac{\lambda x}{l}$.

(iii) The forces at A were the horizontal friction force F and vertical normal reaction R. These are calculated by resolving and the resultant is found by using $\sqrt{F^2 + R^2}$. The direction is $\tan^{-1}\left(\dfrac{R}{F}\right)$ to the horizontal.

Answers: **(ii)** 700N; **(iii)** 13.5 N, 42.0° with the horizontal.

Question 6

(i) Candidates attempting to use horizontal and vertical motion arrived at $V\cos\alpha = 12$ and $V\sin\alpha = 14.5$ with V equal to the initial velocity and α the angle of projection with the horizontal.

(ii) In order to find the direction it was necessary to find the horizontal and vertical velocities at time $t = 0.4$. Having done this the required angle was $\tan^{-1}\left(\dfrac{v_y}{v_x}\right)$ to the horizontal.

Answers: **(i)** 33.3 ms^{-1}, 25.8°; **(ii)** 19.3° to the horizontal.

Question 7

This question proved to be the most difficult question on the paper.

(i) Candidates needed to use Newton's second law for both particles, thus setting up two equations in T (the tension) and ω (the angular velocity). When solved the required values of T and ω were found.

(ii)(a) If r is the radius of the circle for P then $(1 - r)$ is the radius for Q and so $T = 0.2\omega^2 r = 0.3\omega^2(1 - r)$.

(b) Newton's second law could now be used radially for both particles.

Answers: **(i)** 6, 4.32 N; **(ii)(a)** 0.6 m, 0.4 m; **(b)** 1.2 ms^{-1}, 0.8 ms^{-1}.

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Paper 52

Key messages

In order that candidates receive credit where possible for incorrect answers, they should remember to show full working and reasoning. Incorrect answers without working cannot be given any credit, whereas partial credit can be awarded if a correct method is shown. Where candidates are asked to 'show' or 'prove' a result, marks will be lost if any essential steps in the argument are omitted.

General comments

Candidates found this to be a challenging paper.

The work of the candidates was generally well presented. g = 10 was used by most candidates with only a few applying g = 9.8 or 9.81. Premature approximations and rounding to less than 3 figures were seen only infrequently.

Candidates found **Questions 3(ii)** and **5** to be the most straightforward on the paper while **Questions 2(ii)** and **4** proved to be the most challenging. **Question 4** was found to be the most difficult question on the paper.

Comments on specific questions

Question 1

(i)   The basic principle of taking moments was recognised. Often candidates found the angle with the vertical and then failed to subtract their result from 90°. On occasions candidates simply resolved vertically, saying 16 = 4cos θ , and so ignored the forces at A.

(ii)   In this part of the question it was essential to resolve in two directions. The candidates who resolved horizontally and vertically often went on to score full marks. The candidates who resolved parallel and perpendicular to the rod often scored the first 3 marks. The direction was almost never clearly stated.

Answers: **(i)** 60°; **(ii)** 14.4 N, 76.1° to the horizontal.

Question 2

(i)   The centre of mass of the semicircle was usually correctly found. Some candidates found the centre of mass for a semicircular arc. Many errors occurred when candidates tried to set up a moment equation.

(ii)   A correct answer to this part of the question was very rarely seen. One way to to solve this part of the question was to find angle ADO by saying $\tan ADO = \dfrac{h}{0.2}$, and so $\cos ADO = \dfrac{XD}{0.2}$. This then results in the value of XD.

Answers: **(i)** 0.283; **(ii)** 0.115 m.

Question 3

(i)   Many candidates understood that it was necessary to equate vertical forces and to use Newton's second law radially. Errors occurred when resolving with candidates using the incorrect trigonometric functions.

(ii)   Most candidates realised that the greatest angular speed would be when the contact force was zero. This part of the question was generally well done.

Answers: **(i)** 0.518 ms^{-1}; **(ii)** 13.2 rad s^{-1}.

Question 4

This question proved to be the most difficult question on the paper.

(i)   The question could be solved by using an energy equation. The equilibrium position needs to be found first by using $T = \dfrac{\lambda x}{l}$, $T = 0.24g$ and so $0.24g = \dfrac{12x}{0.5}$ leading to $x = 0.1$ The energy equation would need 5 terms and quite often terms were missing or were incorrect. This equation should be initial(KE + EE) = final(KE + EE) + PE when the acceleration was zero.

(ii)   This part of the question can be done by considering an energy equation with initial(EE + KE) = final PE. Note that the final KE was zero.

Answer: **(i)** 3.61 ms^{-1}.

Question 5

(i)   Once the correct expression for $\dfrac{\mathrm{d}v}{\mathrm{d}t}$ has been found it is necessary to solve the equation and use the correct limits. Then by a process of manipulation the given answer can be found. The correct differential equation should be $\dfrac{\mathrm{d}v}{\mathrm{d}t} = -2.5k\sqrt{v}$ not $\dfrac{\mathrm{d}v}{\mathrm{d}t} = 2.5k\sqrt{v}$. Quite a number of candidates arrived at the correct result from incorrect working.

(ii)   This part of the question was generally well done. Sometimes 'c' was assumed to be zero.

Answers: **(i)** $\dfrac{\mathrm{d}v}{\mathrm{d}t} = -2.5k\sqrt{v}$; **(ii)** 15.75 m.

Question 6

This question caused many problems for candidates because they assumed that the angle of projection was above the horizontal when it should be below it.

(i)   Many candidates found the horizontal and vertical distances and then did not go on to find the actual distance from *O*.

(ii)   Sign errors often occurred when setting up the quadratic equation in *t*. Some candidates assumed that the time of flight was equal to $\dfrac{2V\sin\alpha}{g}$. These candidates assumed that the take off and landing were at the same level.

(iii)   The method required for this part of the question was clearly understood. Unfortunately because of earlier errors often only the method mark was scored.

Answers: **(i)** 76.5 m; **(ii)** 65.4 m; **(iii)** 47.7 ms^{-1}, 61.8° below the horizontal.

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Key messages

In order that candidates receive credit where possible for incorrect answers, they should remember to show full working and reasoning. Incorrect answers without working cannot be given any credit, whereas partial credit can be awarded if a correct method is shown. Where candidates are asked to 'show' or 'prove' a result, marks will be lost if any essential steps in the argument are omitted.

General comments

Most candidates used g = 10 with only a few candidates using g = 9.8 or 9.81.

Few candidates used premature approximation and rounded to less than 3 significant figures.

Questions 1, **2** and **6(i)** proved to be the easier questions, while **Questions 4** and **7** were found to be the most difficult questions.

Comments on specific questions

Question 1

(i)   Most candidates resolved vertically to find the correct value of the tension.

(ii)   Newton's second law was used radially and candidates usually arrived at the correct answer.

Answer: **(ii)** 1.86 ms^{-1}.

Question 2

(i)   Vertical motion was used to find the time the particle to hit the sea and this time was applied to the horizontal motion to find the required distance.

(ii)   Many candidates answered this correctly.

Answers: **(i)** 30 m; **(ii)** 25 ms^{-1}.

Question 3

(i)   Sometimes the wrong formula was used for the position of the centre of mass of the hemispherical shell even though reference could have been made to the formula list. By taking moments about O the correct answer for F was often found.

(ii)   Many candidates wrote down R = −16.48 + 12 + W and used R = 0 to find the least value of W.

Answer: **(ii)** 4.48.

Question 4

(i)   In this part of the question it was necessary to apply the principle of conservation of energy using PE loss = EE gain. Some candidates found the tension in the string and could not proceed.

(ii)   Again conservation of energy should have been used. The result could have been derived using PE loss = KE gain + EE gain.

Answers: **(i)** 10; **(ii)** 1.12 ms^{-1}.

Question 5

(i)   By using the formula $T = \dfrac{\lambda x}{l}$ and Newton's second law radially the extension could be found.

(ii)   The equation $0.2\,\omega^2(0.3 + x) = \dfrac{6x}{0.3}$ could be found by applying Newton's Second Law radially. From this the required result could be deduced.

Answers: **(i)** 0.1 m; **(ii)** $\omega < 10$.

Question 6

(i)   This part of the question was well done.

(ii)   The principle of separating the variables and integrating was clearly understood. Unfortunately some candidates obtained $v = -5x + \dfrac{2}{x}$ instead of $v^2 = -5x + \dfrac{2}{x}$ after the integration. Many candidates could not explain why the particle did not move. It was necessary to compare the force towards O with the friction force.

Answer: **(ii)** 0.5 m.

Question 7

This question was found to be the most difficult question on the paper.

(i)   The centre of mass of the quadrant from O was often found correctly to be $\dfrac{8}{3\pi\sqrt{2}}$. The next step was to take moments about O. This gave $a^2 \times \dfrac{a\sqrt{2}}{2} = \dfrac{\pi}{4} \times \dfrac{8}{3\pi\sqrt{2}} + \left(a^2 - \dfrac{\pi}{4}\right)OG$. Attempts were made but many errors occurred.

(ii)   It was not possible to arrive at the answer given if the expression in part **(i)** was incorrect. Most candidates verified the inequality.

Answer: **(i)** $\dfrac{2\sqrt{2}(3a^3 - 2)}{12a^2 - 3\pi}$.

MATHEMATICS

Key messages

In order that candidates receive credit where possible for incorrect answers, they should remember to show full working and reasoning. Incorrect answers without working cannot be given any credit, whereas partial credit can be awarded if a correct method is shown. Where candidates are asked to 'show' or 'prove' a result, marks will be lost if any essential steps in the argument are omitted.

General comments

There was no evidence of insufficient time being allowed and the general presentation of work was satisfactory. Marks were lost by approximating prematurely, truncating values and not giving answers to the required accuracy as detailed in the comments below. Questions involving probabilities caused particular problems to candidates in differentiating between distributions and whether to add or multiply probabilities.

Comments on specific questions

Question 1

Many candidates who correctly used the mean of the binomial distribution to find the probability of a broken biscuit then proceeded to use the inappropriate normal distribution. Truncating the three separate probabilities or correcting them to 3 significant figures prior to adding resulted in the loss of the final mark.

Answer: 0.655.

Question 2

Most candidates recognised the need for conditional probability and a quotient. However, many were unable to determine the required probabilities; a simple tree-diagram could have shown the three outcomes. Some candidates lost the final mark through giving the answer to 2 significant figures.

Answer: 0.921.

Question 3

(i) $\dfrac{9}{70}$ was often determined successfully. Candidates needed to be aware that $P(X < 2)$ in this instance meant adding $P(X = -2)$, $P(X = 0)$, $P(X = 1)$ and $P(X = -2)$ together.

(ii) The common errors were not subtracting $E^2(X)$ and the omission of the corresponding probabilities. Some candidates then divided by either 7 or 8.

(iii) The inequality signs, as in part **(i)**, were understood by only a few candidates.

Answers: **(i)** 0.486 or $\dfrac{17}{35}$; **(ii)** 5.33; **(iii)** 1.

Question 4

The candidates who made a serious attempt at this question recognised the need for the use of combinations in parts **(i)** and **(iii).**

(i)   The three possible team options were often overlooked and some candidates gave 1 option of 3 combinations either correctly multiplied or erroneously added.

(ii)   Candidates identified what was required here but some candidates miscalculated $\dfrac{11!}{5! \times 4! \times 2!}$ on a calculator as $\dfrac{11!}{5!} \times 4! \times 2!$.

(iii)   Few candidates realised that three different options needed to be summed.

Answers: **(i)** 148 176; **(ii)** 6930; **(iii)** 6930.

Question 5

The normal distribution is a topic that requires greater attention by candidates. Diagrams are always helpful.

(i)   This question was straightforward if a diagram was drawn initially and the basic concepts were applied: if $P(X < 2\mu) < (1 - 0.1016)$, then $\dfrac{2\mu - \mu}{\sigma} = 1.272$. Some candidates did not realise that $2\mu - \mu = \mu$ and consequently made mistakes when squaring. Some candidates quoted 1.27 either through truncation or the wrong tables.

(ii)   Again a diagram assists candidates to realise that the upper limit of the central 50% is 0.75 and the '$(x - \mu)$' is 'a'. Many candidates wrongly quoted the standard deviation as 21 and, as in part **(i)**, 0.674 was truncated to 0.67.

Answers: **(a)** 0.693, 0.545; **(b)** 3.09.

Question 6

This question was answered well by many candidates.

(i)   A mark was awarded for attempting to plot the cumulative frequencies against their respective upper class boundaries and joining them either by a curve or a series of straight lines. Unfortunately many candidates presented a series of rectangular blocks. A lot of candidates read off the value at 300 pupils.

(ii)   Most candidates ignored the 'less than' and therefore used the 4000 value instead of the 1000.

(iii)   Candidates need to realise that 'between 201 and 250 inclusive' meant a frequency obtained from $(2100 - 1600)$, i.e. 500.

(iv)   Several marks could be gained here by attempting to use mid-class values and class frequencies. However class widths, upper class boundaries and cumulative frequencies were often used and often candidates then divided by 7.

Answers: **(i)** 270; **(ii)** 160; **(iii)** 500; **(iv)** 268.

Question 7

(a) (i)   Few candidates attempted this part and those that did rarely used the concept of '1 – (none)'. The binomial distribution was often quoted.

  (ii)   Successful solutions were rarely seen. Those who did produce one showed a good understanding of solving an inequality involving indices, with most finding *n* and then choosing the correct integer.

(b)   Those candidates who drew a tree diagram were generally successful in obtaining the required solution. Others failed to realise that there were three possible successful outcomes and that there was no replacement.

Answers: **(a)(i)** 0.806, **(ii)** 13; **(b)** $\dfrac{11}{28}$ or 0.393.

MATHEMATICS

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Key messages

In order that candidates receive credit where possible for incorrect answers, they should remember to show full working and reasoning. Incorrect answers without working cannot be given any credit, whereas partial credit can be awarded if a correct method is shown. Where candidates are asked to 'show' or 'prove' a result, marks will be lost if any essential steps in the argument are omitted.

General comments

This paper proved accessible to the many candidates who had covered the syllabus and knew basic work. Most candidates knew when to apply the binomial distribution but surprisingly few appreciated what the conditions are for a normal approximation to the binomial distribution to be used. Candidates are reminded that they should label graphs and axes, show dotted lines for finding the median or quartiles from a graph, and choose sensible scales. Candidates need to work to 4 significant figure or greater accuracy in order to give answers correct to 3 significant figures.

Comments on specific questions

Question 1

Nearly all candidates recognised that this situation could be modelled by the binomial distribution. Many candidates however, did not appreciate that the 'average number was 5' meant that $np = 5$, and hence p could be found and used to find $P(X < 3)$ in a binomial situation where n is 20. A method mark was given for summing binomial probabilities using any probability in any binomial expression providing n was 5 or 20 and this mark was scored by nearly everybody. Some candidates used the normal approximation to the binomial, for which some credit was given as, although nq is considerably greater than 5, np is 4.8 which is close to 5. Premature approximation of the individual binomial terms gave an answer of 0.108 which lost the final accuracy mark. It is advised that candidates work with all the figures in their calculator.

Answer: 0.109.

Question 2

Candidates were told to use the normal approximation to the binomial and almost all did so successfully. There were some, however, who used a binomial situation, despite being told to use the normal approximation, and these candidates gained no marks. Only a small percentage of candidates realised that both np and nq had to be greater than 5 for the normal approximation to be justified.

Answer: **(i)** 0.590.

Question 3

This question proved to be the most difficult on the paper. Those candidates who knew that, when using summary statistics, the mean and standard deviation can be found either from using $\sum(x - a)$ everywhere or $\sum x$ everywhere (and this question used $\sum(x - a)$ everywhere) had no trouble and gained the correct answer immediately. There were many candidates who, when adding 29, failed to put it in coded form and hence were unable to find the new standard deviation, although they found the new mean by common sense.

A few centres expanded $\sum(x - a)^2$ but these were in the minority.

Answers: **(i)** 40.9, 8.30; **(ii)** 8.41.

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Question 4

This question was well done and candidates generally knew when to use permutations and when to use combinations. Even candidates who could not complete part **(ii)** successfully, managed to gain a method mark for writing 3! and another for writing 7! or 6! and another for dividing by their part **(i)** answer or using the 1 − probability idea. Part **(iii)** was the least well attempted but here too, candidates could pick up method marks for a sensible approach using combinations.

Answers: **(i)** 90 720; **(ii)** 0.917; **(iii)** 20.

Question 5

Many marks were lost in this question by candidates not labelling axes, not plotting points accurately, and not drawing lines across and up from their axes to read the graph. Another common mistake in part **(ii)** was for candidates to read the graph up from 30 and give their answer. This is the number of days when fewer than 30 rooms are occupied, not more than 30 rooms are occupied. Candidates who did not subtract from 200 and hence gave the wrong answer of 24 but drew a line up from 30 on their graph, gained a method mark, showing they knew that a line needed to be drawn up from the horizontal axis. Candidates who wrote down 24 with no line shown, gained no credit. Linear interpolation was acceptable.

Answers: **(ii)** 174 – 180; **(iii)** 59 or 60.

Question 6

Candidates are expected to use the normal tables in the formulae list and particularly the critical values at the foot of the page. This gives a z-value of 1.282. Values of z between 2.181 and 1.281 were accepted but not for instance a value of 1.28, as candidates have not shown evidence of being able to use the normal tables correctly. The answer was the same in this case and so no further penalties were applied, but it is worth noting that in the normal distribution questions any premature approximations or rounding can result in an answer that is not correct to the 3 significant figures required by this examination. Part **(ii)** involved some thought and was a good discriminator between candidates who could reason through a question, and those who could only do routine examples.

Answers: **(i)** 21.0; **(ii)** 0.746.

Question 7

Almost all candidates were able to make substantial efforts at the first three parts. Some candidates omitted the zero in the table for the score. For the final two parts, candidates needed to appreciate that play stopped as soon as one of the players won, and so it was not possible for Steve to win first and then for Judy to win.

Answers: **(ii)** 0, 0.2; 1, 0.24; 2, 0.08; 3, 0.08; 4, 0.16; 5, 0.16; 6, 0.08; **(iii)** 2.56; **(iv)** 0.08; **(v)** $(0.2)^{n-1} \times 0.4$

MATHEMATICS

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Paper 63
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Key messages

In order that candidates receive credit where possible for incorrect answers, they should remember to show full working and reasoning. Incorrect answers without working cannot be given any credit, whereas partial credit can be awarded if a correct method is shown. Where candidates are asked to 'show' or 'prove' a result, marks will be lost if any essential steps in the argument are omitted.

General comments

Candidates usually gained most of the available marks in **Questions 5** and **4(i)**, whilst **Questions 6** and **4(ii)** proved more challenging. An appreciable number of candidates failed to gain marks through not working to at least 4 significant figures in order to calculate an answer which is accurate to 3 significant figures. This was especially true in **Question 1**, but also applied to **Questions 5** and **6**. A few candidates gave their answers to 2, rather than 3, significant figures. Candidates need to understand the different circumstances under which the binomial and normal distributions should be applied.

There were no apparent problems with the time allowed to complete the paper.

Comments on specific questions

Question 1

(i)   This was usually answered well. Some of the candidates who calculated the mean age of the cars in Fairwheel Garage obtained the correct answer by using a weighted mean involving this and the given mean. The majority successfully found the combined total and divided by 24.

(ii)   Most candidates obtained the correct answer. A surprisingly high proportion of candidates gave the answer 2.18 after using the value 4.02 instead of 4.017 for the mean. A few answers terminated with the variance rather than the standard deviation. Common errors when using the variance formula to calculate Σx^2 for Red Street Garage were not squaring 1.925 and using 15 instead of 9.

Answers: **(i)** 4.02 years; **(ii)** 2.19 years.

Question 2

Parts **(i)** and **(iii)** were usually correctly answered. A few candidates were uncertain as to when factorials or permutations or combinations were appropriate.

(i)   A small number of candidates calculated either the sum of 3, 4 and 7 or the product of 3!, 4! and 7!.

(ii)   The most popular approach was to consider the number of arrangements of 10 items in a row when the items are all different and subtract the number of these arrangements in which the two blue items are next to each other. Nearly all of the errors when using this approach stemmed from ignoring the arrangements of the two blue items. The other frequently seen method consisted of arranging the 8 different items in a row and then inserting the two blue items into different gaps in that row. Often accompanied by an appropriate diagram, this method was nearly always successful.

(iii)   The method involving the sum of 9 combinations was used much more often than the 'take it' or 'leave it' approach leading to $2^9 - 1$. Both methods frequently resulted in the correct answer.

Answers: **(i)** 84; **(ii)** 2 903 040; **(iii)** 511.

Question 3

Attempts at this question usually gained most, if not all, of the marks. Some candidates had difficulty identifying the boundaries of the different classes.

(i)   Considering the class in which the 150 /150.5th candidate mark for each of the countries *A* and *B* occurred was the method used in most answers. Consideration of the number of candidates with marks below 50 in country *A* and 50 or more in country *B* was less popular with calculation of the median using linear interpolation rarely attempted. Sometimes two correct statements were made without a conclusion. A few candidates did draw a graph.

(ii)   The majority of candidates subtracted the appropriate cumulative frequencies and obtained the correct answer. Sometimes 159 or 134 was given as the answer.

(iii)   The values 5, 15, 27.5, etc., rather than 4.5, 14.5, 27, etc., were often used as the mid-interval values, leading to the answer 38.1. Several solutions involved the presentation of work in a table. This reduced the chance of omitting one of the terms from the calculation in the numerator. Several candidates used class widths or upper class boundaries. A smaller proportion used cumulative frequencies instead of frequencies. A few calculated the median instead of the mean.

Answers: **(ii)** 91; **(iii)** 37.6.

Question 4

The use of possibility spaces or tree diagrams appeared to assist in all parts of this question. In several of these a key would have helped clarify the outcomes under consideration.

(i) (a)   There were only a few incorrect answers. These often followed the use of 2 in the numerator or 12 or 25 or 30 in the denominator.

(b)   There were slightly fewer correct answers than in part **(a)**. Working, or a diagram, often indicated that the candidate had considered either the product of 1 and 5, where 5 was the second number thrown, or the four combinations which resulted in a sum of 5, or both. A wrong final answer in these cases meant that one or both method marks could be awarded. Answers of $\frac{6}{36}$ or $\frac{1}{6}$ without any working could not be awarded any marks.

(ii)   There were some very good answers to this part of the question. The majority of candidates had difficulty recalling and applying the condition under which two events are independent. Whilst several candidates gained one or both marks for the probabilities of the events *A*, *B* and *C* separately, only a small proportion were able to calculate the probability of, for example, the events *A* and *B* and use this in an appropriate equation. The value for P(*A*) was correct more often than the values for P(*B*) and P(*C*). The most frequent incorrect probabilities were $\frac{4}{36}$ and $\frac{1}{2}$ for P(*B*) and P(*C*) respectively. It was quite common for P(*A* and *B*) to be calculated from P(*A*) × P(*B*). Some candidates compared P(*A*) × P(*B*) with 0.

Answers: **(i)(a)** $\frac{1}{36}$, **(b)** $\frac{5}{36}$; **(ii)** None of the pairs are independent.

Question 5

An appreciable number of candidates gained full marks for this question. The use of normal tables was good, often accompanied by small sketches to help process the probabilities.

(i)   This was well answered with most candidates demonstrating a good grasp of the normal distribution. A few candidates equated their $\dfrac{20-\mu}{\mu/4}$ to a probability whilst others made a sign error or included a continuity correction.

(ii)   Nearly all candidates were able to use their answer from part **(i)** to standardise the value of 10. Several candidates did not appreciate that P(X > 20) had already been given and proceeded to standardise using their value for μ. Not using enough significant figures often resulted in a probability of 0.9602 or 0.9603.

(iii)   Most candidates had no problems using the normal approximation to the binomial distribution. Sometimes an incorrect answer was obtained because the continuity correction was overlooked, or a probability other than 0.96 was used. Since the question did not stipulate that an approximation had to be used, the binomial distribution could be applied. This involved summing 16 terms. Only a few of the candidates who attempted this method obtained the correct answer of 0.955.

Answers: **(i)** 13.9; **(ii)** 0.829 or 0.830; **(iii)** 0.962.

Question 6

This proved to be the most difficult question for many candidates. Some candidates who were unable to attempt one or two parts of the question realised that this did not prevent them from attempting the remaining part(s). Answers quite often erroneously involved the normal distribution.

(i)   Those candidates who realised that $(0.75)^n$ had to be less than 0.06 usually went on to calculate the value of 10 for n. Using logarithms was more popular than trial and error. Sometimes the former led to n < 9.78 and n = 9. Several attempts to produce an expression involving n did not include a power.

(ii)   Many candidates were unable to take advantage of the assumption. Several of those who did calculate 10.5 made no further progress. Only a minority were able to use this mean and hence calculate the binomial probabilities of completing 10 and 11 Sudoku puzzles successfully. Some of those candidates who started from a mean of 7, or used trial and error methodically, did gain all 3 marks.

(iii)   Stronger candidates identified the need to use the binomial distribution twice and proceeded to the correct answer. A few misunderstood the range and calculated the probability of '11 or more' rather than 'more than 11'. Many candidates did gain the final method mark for using the binomial distribution, with any probability, in an attempt to calculate the probability that Sue completes more than 11 Sudoku puzzles correctly in exactly 3 of the next 5 months.

Answers: **(i)** 10; **(ii)** 11; **(iii)** 0.115.

MATHEMATICS

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Paper 71
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Key messages

In order that candidates receive credit where possible for incorrect answers, they should remember to show full working and reasoning. Incorrect answers without working cannot be given any credit, whereas partial credit can be awarded if a correct method is shown. Where candidates are asked to 'show' or 'prove' a result, marks will be lost if any essential steps in the argument are omitted.

General comments

On this paper, candidates were largely able to demonstrate and apply their knowledge in the situations presented. Whilst there were some good scripts, there were also candidates who appeared unprepared for the paper. In general, candidates scored well on **Questions 2(a) and 4**, whilst **Questions 2(b) and 6** proved particularly demanding.

Some candidates gave answers to an inappropriate level of accuracy. It is important for candidates to realise that for a final answer to be accurate to 3 significant figures, then all working out up to the final answer needs to be accurate to at least 4 significant figures.

Lack of time did not appear to be a problem for candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also some good and complete answers.

Comments on specific questions

Question 1

In this question a Poisson distribution was the appropriate approximating distribution. Many candidates used this distribution with the correct parameter of 1.2. Successful candidates then went on to find the probability $1 - P(0, 1, 2)$. Some candidates incorrectly omitted 0 or included 3 from the event (0, 1, 2), illustrating the need to read the question carefully.

Some candidates chose an invalid distribution, or used the exact distribution rather than an approximating one.

Answer: 0.121.

Question 2

Part **(a)** of this question was well attempted, Successful candidates were able to quote a correct formula and substitute in the given values correctly. A 95% symmetrical confidence interval requires the *z*-value to be 1.96, and some candidates used an incorrect value. The final answer should have been given as an interval.

Part **(b)** was poorly answered, with many candidates unable to understand what was required.

Answers: **(a)** [39.6, 42.8]; **(b)** 87.5.

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Question 3

Successful candidates standardised using the correct normal distribution and equated it to 1.786. Common errors were in algebraic manipulation when rearranging the equation to find 'n'. Some candidates did not give the value of n as a whole number.

It was important in part **(ii)** that a clear comparison was made between 1.786 and 1.645; this could be as an inequality statement or on a diagram. Valid area comparisons were also accepted, but to compare an area with a z-value is not valid. The final conclusion should be related to the question and not be a definite statement. For example, 'There is evidence that the mean has increased' is better than the definite statement 'The mean has increased'.

Answers: **(i)** 150; **(ii)** Evidence that the mean has increased.

Question 4

Part **(a)** was well answered, with the majority of candidates understanding what was required for a probability density function. Part **b(i)** was also reasonably well attempted, though it was important, as is always the case in 'show that' questions, that all steps in the working out were clearly shown. Some candidates were unable to correctly reach the required answer due to poor understanding of the logarithmic function.

Many candidates successfully found the median of X, and many knew the method for finding the required probability. However, it was important that candidates kept to the required accuracy when finding this probability; those who rounded 30ln1.5 too early (i.e. used 12.2) were unable to obtain the final answer to the required 3 significant figures.

Answers: **(a)** Area under graph is greater than 1; probability density function cannot take negative values; **(b)(ii)** 12, 0.0337

Question 5

Use of N(2240, 848) was required here, which was successfully obtained by some candidates with others only obtaining one parameter correctly. Finding the probability between 2200 and 2300 was reasonably well attempted.

Part **(ii)** caused problems for some candidates. In particular, many candidates found P($X_1 - X_2 > 20$), but then left this as their final answer rather than doubling it to include P($X_2 - X_1 > 20$). This illustrates the importance of careful interpretation of the question.

Answers: **(i)** 0.896; **(ii)** 0.312.

Question 6

It was particularly important in part **(i)** of this question that all relevant working was clearly shown when finding the critical region. Some candidates found the correct region, but without full justification.

Successful candidates realised that once the critical region had been found, carrying out the test in part **(iii)** required a statement that 3 was not in the critical region and then a suitable conclusion. Other, slightly more lengthy, methods were also used.

In part **(iv)** some candidates correctly used N(25.2, 25.2), but often no continuity correction (or an incorrect one) was used when standardising.

This question was not well attempted.

Answers: **(i)** $X < 1$; **(ii)** 0.0134; **(iii)** No evidence that mean number of injuries has decreased; **(iv)** 0.128.

MATHEMATICS

Key messages

In order that candidates receive credit where possible for incorrect answers, they should remember to show full working and reasoning. Incorrect answers without working cannot be given any credit, whereas partial credit can be awarded if a correct method is shown. Where candidates are asked to 'show' or 'prove' a result, marks will be lost if any essential steps in the argument are omitted.

General comments

On this paper, candidates were able to demonstrate and apply their knowledge in the situations presented. There was a full range of marks with some very good scripts. In general, candidates scored well on **Questions 2, 4 and 7(i)**, **(ii) and (iii)**, whilst **Question 6** proved particularly demanding.

Questions requiring an explanation 'in context' continue to challenge candidates, explanations are not acceptable without some specific relation to the question. For example, in **Question 3** it was necessary to mention that the *goals* were scored at random. Just to state 'random' was not sufficient. It is also important for candidates to note that concluding statements made in hypothesis testing questions should be related to the question and should also not be 'definite' statements.

It is important for candidates to adhere to the 3 significant figure accuracy required on final answers. This means that greater accuracy (at least 4 significant figures) should be kept throughout the working in order for the final answer to be correct to 3 significant figures (as noted for **Question 3(iii)** below). On the whole, presentation was good and an adequate amount of working was shown by candidates; without adequate working marks can sometimes be withheld. This was of particularly importance in **Question 7(i)** where it was required to 'show that' k is $\frac{1}{2}$.

Lack of time did not appear to be an issue for candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also many very good and complete answers.

Comments on specific questions

Question 1

The responses given on this question illustrate the need for candidates to ensure that they read the question carefully. The question required the mean and standard deviation of the total weight of a random sample of 3 bags. Many candidates gave an answer that suggested they were considering samples of size 3 and finding the mean and the standard deviation of the distribution of sample means. Many candidates left their final answer as a variance, again illustrating the need to read the question carefully. Candidates must also understand the difference between $3X$ and $X_1 + X_2 + X_3$. This is a frequent source of confusion for candidates on this type of question, and meant that many candidates calculated the variance as $3^2 \times 0.0016$ rather than 3×0.0016.

Answers: 9.6 kg; 0.0693 kg.

Question 2

This question was reasonably well attempted. Many candidates correctly appreciated that the distribution of the sample means was normal, though not always with the correct parameters. Correct and incorrect continuity corrections were applied in some cases. Other methods were sometimes used which could have led to a correct answer, however candidates often became confused and mixed the different methods.

Answer: 0.151.

Question 3

It was important in part **(i)** that the conditions given were related to the question. In particular the condition '*X* occurs singly' was inherent in the context of the question, as goals do occur singly in a football match. Hence this was not a condition required. The conditions that were given by the candidate should have mentioned 'goals' in order to be in the context of the question.

In part **(ii)**, candidates were required to find the probability of 3, 4 or 5 goals. This was successfully calculated by many candidates, though a few common mistakes were evident which stemmed from confusion with inequality signs. The most common one was to include 2 or 6 or both, another was to calculate $P(X = 6) - P(X = 2)$, or $P(X < 6) - P(X > 2)$.

Part **(iii)** required two steps to obtain the final answer, firstly to find the probability of scoring at least one goal, then to use this to find the probability of at least one goal in each of the next 10 matches. Some candidates successfully attempted one of the two stages, but a completely correct answer proved to be challenging.

It was important here that full accuracy was adhered to; accuracy after the first step of the calculation should have been kept to at least 4 significant figures in order to keep the final answer to 3 significant figures.

Answers: **(i)** Two of: constant average rate of goals scored, goals occur at random, goals independent; **(ii)** 0.259; **(iii)** 0.164.

Question 4

The sample mean of 8.4 was usually correctly found from the data given, and candidates that were successful on this question quoted, and were able to correctly apply, the appropriate formula. The value of *z* that should have been used in the formula was 2.576; an incorrect value here was the most common error seen.

Consideration of the underlying statistical theorem (the Central Limit Theorem) in part **(ii)** showed a lack of understanding on the part of some candidates. Candidates needed to realise that the distribution of the population (the mass of fat in low-fat burgers) was known to be normal, so that the distribution of the sample means was therefore normally distributed. The Central Limit Theorem was therefore not required.

Part **(iii)** was generally well attempted with candidates realising that they needed to check if 8 g was within the confidence interval they found in part **(i)**.

On the whole candidates scored well on this question.

Answers: **(i)** [7.54, 9.26]; **(ii)** No, because the population is normally distributed, so sample means are normally distributed; **(iii)** Claim is justified.

Question 5

A Poisson distribution with $\lambda = 3.3$ was required in part **(i)** and many candidates successfully used this. Candidates who considered the adults and children separately and attempted a combination method were generally unsuccessful.

Part **(ii)** was quite well attempted, though it was important for candidates to show a clear comparison (either using an inequality statement or a diagram) with 1.96 (or equivalent area comparison) in order to justify their conclusion. The conclusion should be in context and should not be a definite statement.

Answers: **(i)** 0.359; **(ii)** There is evidence to support the claim.

Question 6

The null and alternative hypotheses in part **(i)** were not always correctly stated. Candidates must read the question carefully to determine the type of test, in this case a one-tail test.

Questions involving Type I and Type II errors often cause problems for candidates and this question was no exception. Candidates are often good at quoting the text book definitions, but have difficulty relating these definitions to the context of the question. Part **(iii)** was therefore not well answered.

The calculations in part **(ii)** and **(iv)** produced a variety of responses, including use of an incorrect value for p, calculation of the wrong tail or use of an incorrect distribution.

Answers: **(i)** $H_0 : P(6) = \dfrac{1}{6}$, $H_1 : P(6) > \dfrac{1}{6}$; **(ii)** 0.0697; **(iii)** Die is biased but die shows a six on less than 4 throws; **(iv)** 0.172.

Question 7

This question was well attempted. Most candidates successfully showed that k was $\dfrac{1}{2}$ (though, as this answer was given, all necessary steps in the working needed to be shown). Part **(ii)** and **(iii)** were similarly well attempted, though some candidates successful found −0.333 in part **(iii)**, only to then decide that the modulus of this value was required, suggesting that they thought that the mean could only be positive.

Part **(iv)** proved to be more challenging with candidates often using the wrong limits in their integration attempts. The most successful candidates found the correct quadratic equation, solved it and then considered whether both solutions were valid. In this case one of the solutions (2.732) needed to be rejected because of the context of the question.

Answers: **(ii)** 0.0625; **(iii)** −0.333; **(iv)** −0.732.

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Paper 73

Key messages

In order that candidates receive credit where possible for incorrect answers, they should remember to show full working and reasoning. Incorrect answers without working cannot be given any credit, whereas partial credit can be awarded if a correct method is shown. Where candidates are asked to 'show' or 'prove' a result, marks will be lost if any essential steps in the argument are omitted.

General comments

This paper enabled candidates to show their understanding of statistical methods. The vast majority of solutions were well presented, with some excellent work applying the normal distribution, and some elegant work on continuous random variables. Candidates' work on applying statistical theory in context is the area where the greatest improvement is possible. Candidates appeared to have sufficient time to finish the paper.

Comments on specific questions

Question 1

There were a large proportion of fully correct solutions. Most candidates combined the Poisson means for the two machines and multiplied by two to get 3.4, the mean number of breakdowns. They then used this figure to sum the probability of two or fewer breakdowns. Some candidates did not recognise that two years were needed and worked with $\lambda = 1.7$, the total for one year. A few candidates took less than 3 to include 3.

Answer: 0.340.

Question 2

(i) The correct solution required the application of the distribution of the sample proportion which is distributed $N(p, \frac{pq}{n})$. The majority of candidates used this distribution, together with the correct value of z (1.645) to obtain the correct interval. An incorrect z-value sometimes used was 1.282, indicating that a few candidates had not considered the symmetry of a confidence interval carefully enough. Candidates who did not score full marks were those who used 18 and not $\frac{18}{70}$ for the mean, and those who produced an interval for the number of undersized soap bars, using the normal approximation to the binomial distribution

(ii) Good responses were seen from some candidates, but the requirement to justify that the interval was an approximation was not well addressed by a significant number of candidates. Candidates tended to focus on the fact that the question involved a sample, but this is true of all confidence intervals.

Answers: **(i)** [0.171, 0.343]; **(ii)** Either: the variance (or standard deviation) is estimated; or that a normal approximation to a binomial distribution was being used.

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Question 3

(i) This question examined significance testing for a discrete distribution. Questions involving significance testing can usually be tackled in two stages. The first stage is the calculation of the test statistic, in this case the probability that 2 or fewer voted for the renewal party. The second stage is the comparison of this probability with 0.04, followed by a conclusion. The majority of candidates using this method applied both stages correctly. Some candidates only calculated the probability for less than 2 voters as their test statistic. Other candidates found only the probability that there were two voters. Performing the comparison was usually correct. When it was not, it was because of the lack of any comparison, while some other candidates compared with 0.15. Some candidates' conclusions contained contradictions meaning that the final mark could not be given. Candidates must avoid mixing distributions in significance testing and take particular care to avoid using the normal distribution when testing a discrete distribution. The other method to perform the test is to identify the critical region ($X = 0$), and to then indicate that 2 does not lie in this region. With this approach the method to identify a critical region needs to be shown, including as a minimum the summation of appropriate probabilities and comparison with 0.04. While many candidates did this well, there were some who did not show all the key stages.

(ii)(a) A high proportion of candidates identified that a Type I error was not possible as the conclusion had been to accept the null hypothesis. Responses sometimes confused type I and Type II errors, or merely defined a Type I error.

(b) The required response was that the probability of a voter voting for the renewal party was actually less than 0.15. Some candidates who did not respond correctly did not appreciate that the test was one-tailed and talked about a change in proportion. Candidates need to ensure that they can apply their knowledge to the given context.

Answers: **(i)** Accept H_0; **(ii)(a)** H_0 has been accepted; **(ii) (b)** If $p < 0.15$.

Question 4

(i) This part required the identification that the Poisson distribution was the appropriate approximation and then to use it to find $P(X > 3)$. Large numbers of candidates performed the two stages correctly. Those who did not get full marks tended to do one of the following. The first involved no approximation being used and calculation with a binomial. This, if done correctly, resulted in the same numerical answer. Solutions involving using the normal approximation were sometimes attempted, and if fully applied, including the appropriate continuity correction, were awarded one mark as a special case. The value of the parameters in this question meant that the normal approximation was not appropriate. Some candidates stated a correct method, but actually found $P(X < 3)$ or $P(X > 3)$

(ii) This part required using a correct probability involving n for $P(X = 0)$ and solving an inequality. Both binomial and Poisson expressions were allowed. Most candidates used the Poisson, and the majority were able to formulate and solve the inequality. A few candidates rounded down rather than up. Other candidates, when working with the Poisson distribution, inserted 2500 at a late stage into their solutions, but sometimes rounded their λ to the nearest whole number.

Answers: **(i)** 0.567; **(ii)** 11513 (Poisson) or 11511 (binomial).

Question 5

(i) The vast majority of candidates scored well on this question. They correctly found the distribution of $X + Y$ and used it to find $P(X + Y > 200)$. Most then correctly changed their answer into a percentage in order to score full marks.

(ii) As in part **(i)**, a high proportion of candidates scored very well. They found the distribution of $X - 4Y$ and calculated $P(X - 4Y > 0)$. Candidates should take particular care with finding the variance and not treat $4Y$ as a sum rather than a product, and they also need to ensure that the variance was for a combination of milk and coffee.

Answers: **(i)** 2.28%; **(ii)** 0.325.

Question 6

(i) Candidates often showed excellent integration skills throughout the question. In this part integrating between 2 and 3 and equating the integral to 1 led to the verification that $k = 6$. To verify that k is exactly 6 it was essential to avoid the use of decimals for $\frac{2}{3}$ or $\frac{1}{6}$.

(ii) While most candidates showed that E(X) = 2.5, most used integration rather than using the symmetry of the quadratic graph. Candidates often then scored all the remaining marks for the variance. Some candidates used integration to find E(X^2) and went no further. Those who found an incorrect value for the expectation still were credited with method marks for an attempt at $E(X^2) - [E(X)]^2$.

(iii) This part proved more demanding than the previous parts. There were two parts to the calculation. First, candidates needed to find a probability by integrating the given probability density function with limits 2 and 2.2. They then had to find the probability that in 4 brake pads at least one had to be replaced. This was normally done by finding 1 – P(none need replacing). Partial credit was awarded to those who could find the probability that a brake pad needed replacement (0.104), or to those candidates who had incorrectly found this probability and then used it to find P(one or more replacement needed). These incorrect probabilities sometimes came by using the normal distribution with their values found in part **(ii)**. This challenging part was not answered by a significant number of the candidates, who perhaps did not fully recognise how the given distribution could be used to find the initial probability.

Answers: **(ii)** 2.5, 0.05; **(iii)** 0.355 or 0.356.

Question 7

(i) A standardisation using the distribution of the sample mean was successfully performed by a good proportion of candidates. A few candidates standardised without using $\sqrt{n}$, suggesting that they had not identified that they were working with the mean of a sample.

(ii) This part expected candidates to recognise that, since they were working with the mean of a large sample, normality was not required for the mean number of cars per day since the central limit theorem was applicable. Candidates were not always able to explain this clearly. It was also clear from their responses that many candidates thought that this was an application of a normal approximation to a discrete distribution. Some responses were fully correct except that they concluded that normality for the underlying variable was required when it was not.

(iii) This final part of this question was a significance test applied to a sample mean. The hypotheses were correctly stated in the majority of candidates' responses, with just a few incorrectly applying a one-tailed test. Candidates need to ensure that their hypotheses must include a parameter. Using the correct figures gives a test statistic of $z = 2.571$, which should be compared with 1.96 and then be followed by the rejection of the null hypothesis. A follow through of a one-tailed test was allowed. Candidates must ensure that their significance tests include a clear comparison. The comparison can be done as an inequality or diagrammatically, but must explicitly compare the two values. Equivalent methods could also score full marks. Continuity corrections must be avoided.

Answers: **(i)** 0.0051; **(ii)** No, since n is large and CLT applies, or sample mean approximately normally distributed; **(iii)** Evidence that the population mean has changed.