Joe Tranquillo

An Introduction to Complex Systems

Making Sense of a Changing World

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Joe Tranquillo
Bucknell University
Lewisburg, PA, USA

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Preface

Every book emerges from an author's belief that something is missing. In the case of a scholarly work, it may be a new idea, critical connection, or a deeper exploration. In the case of an educational work, it is that some important topic has not been made sufficiently digestible to those not already in the know. In either case the publication of the work, and whether anyone reads it, is contingent on if something really is missing and if the book fills that hole. I cannot promise that this book fills a gap, but I can explain a bit of the backstory.

Over the course of working with undergraduate students on research projects, I found that I was constantly bouncing new students between literature in neuroscience, game theory, network theory, nonlinear dynamics, and systems theory. The existing texts and articles seemed to be either written as popularizations for the lay audience or were encyclopedias of the entire field (often filled with equations) written for those already in the field. To help my students, I began writing short introductions to each of the areas that surrounded my research area. My selfish intent was to quickly bring them up to speed so that they could read the literature, start forming and answering their own hypotheses, and contribute to my research program.

Around the same time, a few senior faculty sat in on my neural signals and systems course. Every Friday the class would discuss "big topics," such as free will, consciousness, neurolaw, or artificial intelligence. So as not to distract the students, the faculty formed their own small discussion group. That group continued on for the next 4 years, meeting every week, and eventually grew to 32 faculty, staff, and administrators. Complex systems readings and topics made many appearances at these informal meetings. After our discussions I found myself jotting down short snippets into a file on my computer.

My scattered notes remained in that file for at least a decade until I had the opportunity to teach a new elective on Complex Systems. Due to a fortunate glitch in scheduling, students majoring in engineering, finance, anthropology, economics, innovation and design, management for sustainability, physics, and biochemistry all signed up. With such a wide range of disciplinary expertise in the room, I made the decision to teach the course using principles from complex systems. For example, the interrelationships between students formed a network in which ideas could bubble up and be passed around. My role was not only to maintain this network as the instructor but also to participate as a student. Discussions encouraged all of us to connect together ideas and experiences from outside of class. Individual writings, class exercises, group meta-reflections, and a few guest discussion leaders made for a very interactive learner-centered environment.

Teaching in this way meant that class time could not be used for lecture. I found it helpful to introduce concepts through reading assignments. My scattered notes began to take some shape. As they did, and through the course discussions, three broad themes emerged, which I thought of as *sparks* I hoped I would ignite: Curiosity, Connections, and Citizenship. All three are expanded upon in the introduction and should be thought of as muscles that can be exercised while reading the main text. Intentional practice really does help one improve.

The text is not meant to be anywhere near a complete picture of Complex Systems. You will find many holes that will hopefully pique your interest to look at the sources, start your own a discussion group, or visit one of the many complex systems centers. As I often hear from past students, I would love to hear from you—whether it is to correct an issue with the text or to share a new insight or project you are working on. You can find me at jvt002@bucknell.edu.

I must give a heartfelt thank you to all of the research students I have had over the years. They were the first readers of this text and helped me more clearly define my audience. I must also thank my many colleagues, especially the members of the Brain, Mind and Culture discussion group who showed me just how far outside of my traditional scientific background complex systems could be applied. A special thanks goes out to George DeMartino, Jim Baish, Mark Haussmann, John Hunter, Tom Solomon, Kevin Meyers, Joe Meiser, Charles Kim, Margot Vigeant, Jason Leddington, Mathew Slater, and Doug Candland. Many of them have read drafts of this text and provided outstanding suggestions. I must also thank Martin Davalos, a graphic design student who made my awful hand-drawn images into works of art. My publisher, and more specifically Michael McCabe, Marta Moldvai, and Charles Glaser, helped give me some much-needed focus and advice throughout the project. I also need to reluctantly thank Wikipedia. As a faculty member, I have been trained to not respect or trust it as a source. But it is an outstanding example of emergence and proved useful in tracking down obscure sources. In fact, you might gain more by reading up on the index terms on Wikipedia than marching linearly through this book. My first class of Complex Systems students struggled through the earliest complete draft of this text and were so patient and forgiving. I know that they taught me more than I taught them. My second class of Complex Systems students kept me honest and made many excellent suggestions on how to improve the readability. I also must thank Green Day, Kate Rusby, Keith Urban, Bach, deadmau5, Dave Matthews Band, Rage Against the Machine, Taylor Swift, Iron and Wine, Clifford Brown, and countless other artists that I listened to while writing and editing. If you press your ears closely to these pages, you might even hear them. Lastly, I must thank my family for bearing with me early in the mornings, on weekends, in the car, on vacation, and at night, while I wrote, revised, and revised some more.

Lewisburg, PA, USA Joe Tranquillo

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Chapter 1
Introduction

Marco Polo describes a bridge, stone by stone.
"But which is the stone that supports the bridge?" Kublai Khan asks.

"The bridge is not supported by one stone or another," Marco answers, "but by the line of the arch that they form."

Kublai Khan remains silent, reflecting. Then he adds: "Why do you speak to me of the stones? It is only the arch that matters to me."

Polo answers: "Without stones there is no arch."

- Italo Calvino in La citta' invisibili (trans. William Weaver)

Complex systems are all around us. Some were created by nature, such as ant colonies, gene networks, ecosystems, and brains. Others were created by us, such as political systems, stock markets, language, technology, and cities. In this book we will explore the interdisciplinary field of complex systems theory that aims to gain deep insights into how real-world systems work. Along the way, we will study tools that include chaos and fractals, game theory, networks, agent-based models and information theory. These tools will help us approach some important questions in new ways and find common patterns that appear in many systems. How do complex systems emerge or self-organize? How are they more than the sum of their parts? How do they develop and adapt but eventually decay? What does it mean for a system to be healthy or sick? What does it mean for a system to be diverse? Is it possible to intentionally design a complex system? Are there general principles that might apply to all complex systems?

These are the kinds of questions a physicist would ask. You likely experienced physics through mind-numbing problem sets in which you calculated how long it takes for a ball to drop from the Leaning Tower of Pisa. You should throw away the misconception that physics is about solving math problems. Rather, image Jen the physicist. One morning Jen makes herself a cup of coffee, swirls in some sugar, and adds a few drops of milk. She notices a dynamic spiral of the light

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milk against the dark coffee. Later that day Jen visits her friend the biologist who works on cyclic AMP signaling in slime molds. Her friend explains that although a slime mold is a tiny organism, many thousands of them can cluster together and communicate using cyclic AMP. What is more, these communication patterns, under the right conditions, can form beautiful spirals. Jen digs deeper and finds even more examples. Spiral galaxies arise from the balance of competing forces. The circadian rhythm is a molecular spiral that develops over time. Spirals are the root cause of cardiac arrhythmias, the dangerous self-looping electrical patterns that can lead to cardiac fibrillation. It turns out that electrical spirals occur in every human heart but are only sustained in sick hearts. It seems that everywhere Jen looks she sees spirals.

Because Jen is a physicist, she does not care if the media is electrical, chemical, or mechanical or if the spiral occurs in space or time. Perhaps at some deeper level, a spiral is a spiral. Jen will ask herself some questions: What is it about these particular systems that allow them to support a spiral pattern? Why are some spirals stable while others are ephemeral? How do spirals form from non-spirals? What kinds of events can break up a spiral? Jen is an imaginary physicist, but real physicists have asked these questions. They have found that spirals can form in systems that have a balance between the two competing phenomena of reaction and diffusion. When the balance is right, spirals can form.

Spirals are just one pattern we will explore. In fact, you can think of complex systems as a sort of catalog of interesting real-world patterns that appear over and over again. By collecting universal patterns of complex systems and building up the tools needed to get to the heart of how they form and are sustained, a more holistic picture of all complex systems will emerge.

1.1 Why Complex Systems?

There is some reason you are reading this book. It may have been assigned. You may be looking for tools to apply to your own discipline. Perhaps it was general curiosity. Whether your reasons are intrinsic or extrinsic, my hope is that you will discover additional reasons to keep reading. Below I offer three reasons, what I call sparks, that I hope will inspire you to keep exploring.

The first spark is the ability to make *connections* across time, space, actors, ideas, feelings, and patterns. Forming connections is how networks form. Connections are also the basis for learning new knowledge and skills, making sense of the world around us through narratives and using life's experiences to build new things. All too often we think of learning as the acquisition of knowledge. But a mind can also expand and become better able to synthesize new information. Making new connections is at the heart of cognitive stretching—that feeling when we realize our capacity to juggle ideas has just expanded. An expanded mind can contain within it multiple, perhaps even conflicting, ideas. The ability to synthesize meaning out of the barrage of information coming from multiple sources, formats, and degrees of trustworthiness is something that will only become more important in the future.

The second spark I hope to ignite will be dispositional *curiosity*—a general interest in the world around you. This can be contrasted with situational curiosity which is focused much more narrowly on a specific topic that you may find immediately useful. Dispositional curiosity is what I hope you will display when you are not reading this text—to go out into the world and find your own examples. Not because you must but because it is intrinsically fun to learn how things work. It is unreasonable to think, however, that a specific topic will hold your interest for very long. A wide range of concepts, disciplines, and examples will be touched upon that deserve much deeper exploration and if followed will exercise your dispositional curiosity.

The third spark is to engage with the world as a *citizen* of multiple nested and intertwined disciplines and organizations. Being a careful observer is a first step. A good citizen does not ignore things they do not understand but instead tries to make sense of new information. Knowing a lot on its own, however, does not make one a good citizen. Taking action out in the world is critical. Throughout the text there are ideas on how to diagnose problems, strategically develop interventions as experiments, execute tactics, marshal resources and assess impacts. Regardless of what you might call this process (e.g., change management, science, design), complex systems thinking can serve as a grounding framework for change and action out in the world.

The world is becoming radically more interconnected and networked. As a result, almost all corners of the world and fields of knowledge are accelerating. In his book *The Seventh Sense*, Joshua Cooper Ramo (1968–) claims that the ability to see and leverage networks will define the next generation of leaders. The field of complex systems lives in the interstitial spaces between disciplines. This is where the action is—in finding ways to create new network connections between disciplines. For example, the interactions between biological and non-biological systems (e.g., neuro-marketing, humanlike "smart" robots, epidemiology based upon big data and algorithms, buildings that can "breathe") are all hot areas that are ripe for exploration.

If network thinking is a seventh sense, then perhaps complex systems thinking is an eighth sense. Seeing the evolution of patterns in the world will become even more critical in the future. Yet we are often pushed to become disciplinary experts as quickly as possible and therefore only recognize patterns in our own fields. In this regard studying complex systems will challenge you to look outward to other disciplines. The framework intersects the nature of free will and consciousness, the workings of various political theories, the origin of life, human behavior, moral systems, and other tantalizing philosophical topics. It can also have a fun side, revealing a bit about how ideas pop into your head, why jokes are funny, and how it is that traffic can back up and then paradoxically release even when there is no construction or accidents. No single book can dive deeply into these questions. We will only be able to scratch the surface. But it is hoped that you will begin to see complex systems concepts everywhere in the world.

1.2 About This Text

Your understanding of complex systems will not come all at once. Rather it will emerge throughout the book. Concepts will be introduced, often in a simplistic way first and then refined. For example, you likely have an intuitive idea of what it means for a system to be stable. Throughout the text, however, the idea of stability will occur again and again, each time enriching the concept of stability. This approach follows one of the tenets of complex systems theory—that meaning is achieved through connections. The more connected a concept is, the richer the meaning it achieves. The epigram that stated this chapter parallels this approach. We can try to understand an arch by studying the parts, but an arch can only truly be understood as a holistic pattern of connections between parts. The dilemma is that reading a book is inherently a linear process, showing one idea and then the next. I have laid out a particular narrative that I hope will help you form a larger whole. But it is just one possible story. For that reason, rereading sections in a different order may reveal new connections between concepts.

Each chapter will introduce a major tool or approach, introduce terminology, and give examples from a wide range of systems. Some will be quantitative and others qualitative. The qualitative parts are meant to be readable by anyone with an open mind. The equations within the text are at the level of undergraduate math and aim to demonstrate how some of the ethereal concepts discussed by philosophers can be approached (sometimes well and other times not so well) by scientists. Try not to skip over the text in the mathematical sections as there are important insights hidden within. Along the way several key players (both historical and current) will be introduced along with the titles of seminal works in the field. I would encourage you to exercise your dispositional curiosity by reading more about their lives and work.

Each chapter ends with prompts that are meant to serve two related purposes. First, additional concepts are embedded in the questions. Simply reading them over may supplement the ideas in the chapter. Second, you can pick two or three prompts that resonate with you and then wrestle with them. This will allow you to engage in a more active way with the concepts from that particular chapter. A format I have found helpful in my classes is to answer the prompts in the form of *meditations* (modeled after Descartes). These are musings, not essays, that encourage an active approach to searching and thinking. Making this practice a habit will be the key to making the concepts personal rather than theoretical. You might also use the prompts as a way to spark discussion among your friends, relatives, and colleagues. As my students did, you will begin seeing complex systems concepts waiting in line at Dunkin' Donuts, kayaking down a creek, or having a conversation with a friend.

The term "complex systems" will be used throughout the text in three different ways. First, we will assume that complex systems are coherent entities that exist in the world. Second, the field of complex systems is a human-created field or discipline, composed of people, interconnections of ideas, terminology, processes, and analysis methods. Third, complex systems thinking is an individual mindset and a way of learning about and acting upon the world. There is a tight relationship between these three different uses of "complex systems"—the discipline of complex systems uses complex systems thinking to try to uncover the underlying nature of the complex systems that are out in the world.

Furthermore, what it means for a system to be "complex" is not easy to define. To some, complexity is about order and disorder. To others, it is about degrees of symmetry or the distribution of network connections. Complexity is also sometimes defined as the richness, diversity, or unpredictability of the flow of information or materials. Still others measure complexity as the range of functions that can be expressed. As you will discover throughout the text, there is no unifying definition of complexity.

I have made every attempt to distinguish between well-established ideas and those that are more speculative. This presents a problem. Some of the ideas have been robustly proven and adopted by a wide range of disciplines. Others are only accepted within particular disciplines. Still others are intriguing and presented for illustrative purposes. A few are my own musings. In general, each chapter begins with the most established ideas, proceeds to the more speculative, and then concludes with the most controversial topics. Although some effort has been made to distinguish between these levels of rigor and certainty, complex systems theory is a dynamic field where ideas are coming and going all of the time. You are encouraged to question everything.

Every broad introductory text will be missing, or will underemphasize, important topics. I could blame this on the fact that complexity is a new and rapidly evolving discipline. In reality it is because of my limit background. It is not possible to be an expert in all of the fields we will touch upon. To help fill in some gaps, each chapter concludes with a list of resource where you can go to dig deeper.

1.3 Complex Systems Concepts

You already know a great deal about complex systems because you are immersed in them all of the time. The formal study of complex systems, however, can be disorienting. The field is inherently interdisciplinary, drawing from sociology, genetics, economics, ecology, immunology, city planning, mathematics, embryology, computer science, politics, and many other fields. Each field has their own tools, definitions, terminology, methods, and historical twists and turns, as well as particular ways of asking and answering questions. Sometimes these fields are well aligned, but more often than not, there are tensions, contradictions, and misunderstandings. As we explore complex systems, keep in mind that we are looking for patterns that cross and bind together disciplines.

To make sense of a new field, it can help to have an initial framework. New information can be hung on this framework, and in the process, the framework itself becomes richer. Daniel Dennett (1942–) provides a nice analogy for frameworks in his book *Intuition Pumps and Other Tools for Thinking*. If one is creating an original painting, every stroke is obsessed over, and particular brushes and mixes of colors are chosen for specific purposes. On the other hand, if a home painter is

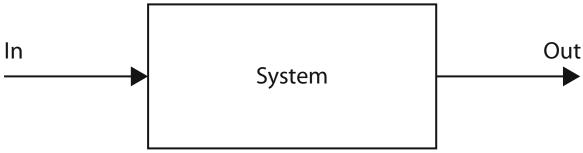


Fig. 1.1 Inputs to and outputs from a system. The definition of the inputs and outputs will define the system boundary

looking to patch up peeling paint on an archway, they might grab any old brush and get to work. If that same home painter is painting an entire room, perhaps needing to apply several coats, they might invest in rollers, painter's tape, and other tools. Industrial painters who paint the outsides of houses and building will set up scaffolding. Although setting up a scaffold system takes time, once up, the rest of the job goes much more quickly. Think of this section as setting up the intellectual scaffolding that we will use throughout the text.

A System Is Any Collection of Parts That Interact The parts can be physical or conceptual. A car is a system. So is a political system. A human body is a system and so is a cell. Spoken language and writing are systems created from words and phonemes. But language is itself a system, composed of rules that are used to construct new sentences. Religious systems are composed of historical events, the writings of important thinkers, moral principles, beliefs, physical structures, and policies. Each of these systems is formed from interlocking parts that work together to create a coherent whole.

A System Divides the World into Two Parts *Inside* the system is everything that is included, and *outside* is everything that is not included. A system definition or model draws the boundary between inside and outside. Depending on the parts, this boundary could be physical or conceptual, but it is a logical necessity. As inputs and outputs are considered to be outside the system, they can help define the system boundary. Figure 1.1 shows a graphical representation of a system (as a box) with an input (arrow pointing in) and an output (arrow pointing out). The arrows are sometimes called *signals* and may contain flows of information, matter, or energy.

Some systems are *closed*, meaning that they are entirely self-contained and do not interact with anything outside. Aside from a few special conceptual systems, no real physical system is truly closed. Open systems, on the other hand, accept inputs and send out outputs. In this case the system boundary is permeable to matter, energy, or information.

Structure Is How the Internal Parts of a System Are Related Relationships may be defined by simple rules, equations, or logical operations that connect variables or ideas. They may also be physical proximity or pathways such as tunnels, pipes, or channels that connect parts together. Structure is about relationships.

Networks Are a Useful Way to Represent Complicated Structures When a structure is composed of many parts or many relationships (or both), it is often called *complicated*. Representing such systems using boxes and arrows can become cumbersome. Networks are a more compact graphical way to show relationships in

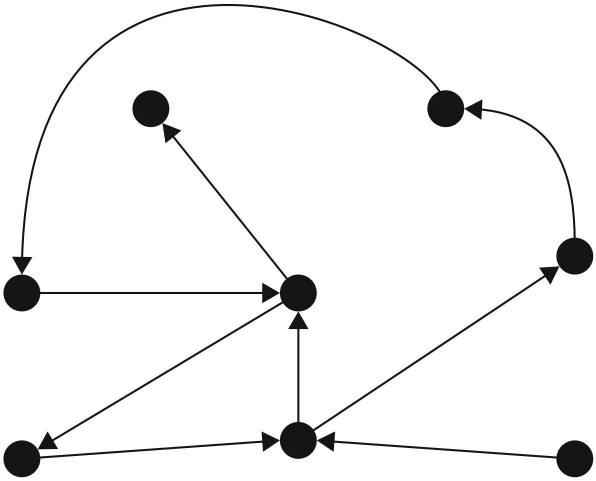


Fig. 1.2 A network as a graphical representation of a system

complicated structures. A relatively simple example is shown in Fig. 1.2. Chapter 5 will explore networks.

Flows on a Structure Determine Function Flows can take many forms. They might be logical or mathematical, such as A follows B or iterations of a rule over and over again in time. They might also be the physical flow of some material, energy, or information in space. Some systems, such as the agent-based models in Chaps. 2 and 4, will have flows in both time and space. It is the flows within a system that express functions or behaviors. As such, function and structure are inseparable. We will explore the flow of information in Chap. 6 and the flow of information on networks in Chaps. 7–9.

Structure Is Necessary But Not Sufficient for Functions to Be Expressed Structure does not dictate function but rather acts as a constraint for flows. Different structures can in fact produce identical functions. For example, a Rube Goldberg machine performs a very simple task in an overly complicated way. They are amusing because a much simpler structure could perform the same function. On the other hand, a system might contain within it several ways to perform the same function. Consider two cities that are connected by a highway and several backroads. On a normal day, most of the traffic would stay on the highway. But if there is an accident, traffic could be rerouted onto backroads. Structural redundancy in flow pathways allows multiple ways for a function to be achieved.

Systems Are Constrained by the Physical Laws of the Universe All of the complex systems we will study, even the most mathematical, will live in the real world. Without becoming immersed in a long philosophical debate, one can assume that an idea or equation still exists somewhere in the physical world, perhaps in a book, on the internet or the synaptic connections of your brain.

All functions, being based upon physical flows, will be constrained in some important ways. All will require time to complete. These *delays*, perhaps so short that they are imperceptible, can occur for a variety of reasons. At a minimum, nothing can travel faster than the speed of light. All flows also rely in one way or another on some *conservation law*. Some quantity within the system cannot be created or destroyed. For electrical systems this is a unit of charge. For chemical systems it is matter. It would be tempting to think of money as non-conserved in that it can be taken out of circulation, accrue interest, or become devalued due to inflation. When framed in terms of flows, however, these additions or subtractions, whether they are internal or external to the system, are treated as money sources or sinks. When accounting for all of these sources and sinks, the flow of money is assumed to be conserved. In fact, it is often a violation of conservation that tips off an auditor that some financial transactions are occurring "off the books."

Functional Adaptation Is a Way to Selectively Express Latent Functions Complex systems rarely, if ever, express all possible functions at the same time. For example, the blinkers on your car are always present, but they are only expressed when a user flips a switch. The blinkers on your car are a *latent function*. People are not always consumers in an economy. Predators are not hunting prey all the time. Ants are not always exploring. Rather, functions in a complex system are adaptively expressed, calling up latent functions by diverting flows away from other functions. As such, the same structure can flexibly express a range of functions in response to both internal and external events. As we will explore in Chaps. 3, 4, and 9, several simple functions might be turned on together to form a more sophisticated function.

Feedback Is a Mechanism for Controlling Function Expression Functions of a complex system are not only selectively turned on and off, but they may also contain features such as speed, strength, color, or other attributes that can be tuned. Something must be in charge of this tuning. Tuning can be achieved by an outside source in the form of an input. Such is the case of a user-controlled device such as a switch, knob, or slider. Another possibility is that the system could tune itself. Self-tuning is usually achieved through *feedback*. The most basic version of feedback is shown on the left side of Fig. 1.3, whereby an output is looped around and reenters the system as an input.

Negative Feedback Is a Mechanism for Error Correction and Control When an output is fed back in as an input, it can act to make the next output smaller. When repeated over and over again such a loop is known as *negative feedback*. In general, negative feedback prevents an output from becoming too large. It is associated with making a system more stable by converging to an equilibrium.

A common need of a system is to tune two variables until they match one another. Consider a variable that is desired to be at a particular value, often called a *set point*. The actual value, however, is not at this point. Negative feedback can be used to minimize the difference, sometimes called an error. Maintaining a variable (e.g., temperature, pressure, pH, wealth) at an ideal point is how systems

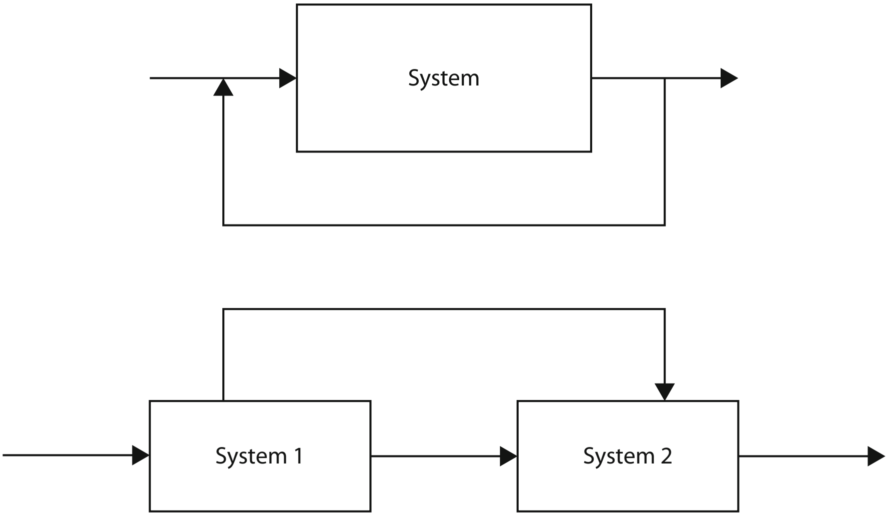


Fig. 1.3 Generic feedback (top) and feedforward (bottom) systems

can remain *dynamically stable*—deviating for a time from some desired point but always snapping back when the deviations become too great. This is the mechanism by which variables such as temperature can be controlled in your home and in your body.

Positive Feedback Can Lead to Vicious or Virtuous Cycles When an output is fed back in as an input, it may also result in a larger output. Positive feedback amplifies small flows and signals and is associated with growth and instability. As positive feedback will amplify, it can lead to *virtuous cycles* (e.g., the more you exercise, the better it feels and so the more you exercise) or *vicious cycles* (e.g., lower socioeconomic classes may lack adequate health care and education which causes the next generation to be even less well-off). This idea was highlighted in a psychological study where participants were asked to push one another. Participant 1 would give Participant 2 a slight push. Participant 2 would then push back with what they perceived to be the same force. This would continue on for several back-and-forth cycles. What scientists found was that the force of the pushes quickly escalated. A vicious cycle had been created through positive feedback.

Feedforward Can Coordinate Functions When There Are Delays Feedforward, shown on the right hand side of Fig. 1.3, is a way of alerting a system that information is coming. Feedforward can be positive (a sort of wake-up call for a function) or negative (inhibiting a function to stay in the background). While feedback is used to react to past events, feedforward is a means of anticipating future events. As such it forms the basis for planning and is important in systems that have significant delays.

Feedback and Feedforward Are the Basis of Many Simple Functions Above it was argued that they form the basis for control, amplification, and planning. Feedback and feedforward also help gate flows on simple structures that give rise to sensors, memory, clocks, and actuators. For example, when a system needs to react to a small signal (either internal or external), it can use positive feedback as an amplifier. On the other hand, if a function should be turned off, some combination of negative feedback and feedforward can be used to quiet that function so that another function can be expressed. Other simple functions can detect, store, time, and compare internal and external signals.

Recombination and Iteration Expand Possibilities A simple example can be illustrated with pennies. Given a single penny, there are only two possible configurations, heads or tails. When we add a second penny, there are four possibilities—both heads, both tails, and two ways to have one head and one tail. As more pennies are added, the number of possibilities explodes. This process becomes even more explosive if we consider playing cards where there are more possible states for each card. A similar argument can be applied to the four genetic bases, which code for 20 amino acids, which can be used to create thousands of different proteins. In the context of systems, the recombination of functions works in the same manner.

Recombined Functions Can Become Non-linear Flows between functions are the means by which functions can be recombined. Complex functions, however, require not only the right simpler functions but also the right interconnections between those functions. An arch is a nice structural example—it requires the individual stones but also the proper relationships, for an arch to be achieved. More complex flows can give rise to behaviors that are *more (or less) than the sum of their parts*. Such a phenomenon is known as *nonlinearity* and will be explored in Chap. 3.

Processes and Ideas Can Also Be Recombined Auguste Escoffier (1846–1935) revolutionized French cooking by recombining processes. Deglazing, whereby ingredients are purposely burned on the bottom of a pot and then reconstituted to form a savory sauce, does not use any new ingredients. It was the process that was new. Capitalism and Marxists are the recombination of economic ideas that aim to form sustainable structures. Evolutionary theory is also a series of interrelated ideas. Most fields of study, from geology to politics, contain a set of grounding ideas that are recombined in different ways to create subfields or camps. We will dive deeper into the origin and dynamics of disciplines in Chap. 10.

Iteration Is Also a Recombination but in Time Imagine having many copies of the same function. An input goes into the first instance of the function. The output of this first instance is then sent into the second instance as an input. Then the output of the second function becomes the input to a third copy of the function and so on. Feedback (both positive and negative) achieves iteration with only one copy of the function and will be explored in simple mathematical functions in Chap. 2. Just as new possibilities can arise when parts are recombined in space, iteration can lead to new possibilities in the evolution of a system.

Patterns Emerge in Space and Time in Both Structure and Function The iteration of simple rules in time and the recombination of parts in space can lead to patterns that are larger in space or longer in time than any individual part or function. As Fritjof Capra (1939–) describes in his book *The Systems View of Life*, "Emergent properties cannot be reduced to the properties of the parts." A classic example is of water (H_2O) molecules. One molecule on its own might be interesting, but something magical happens when we allow water molecules to interact. A fluid can display properties that a single molecule cannot. Another example is the description of a pin factory by Adam Smith (1723–1790) in *An Inquiry In the Nature and Causes of the Wealth of Nations*. No single worker knows how a pin is actually made because the labor and knowledge are divided among them. Somehow out of local communication between workers, pins are made. Smith famously dubbed this phenomena the *invisible hand* to describe how the macro-level patterns of an economy emerge from the micro-level decisions of people and organizations.

To return to the playing card analogy, there is an entirely different kind of possibility that emerges with two cards that is not possible with one card. They could be positioned such that they counterbalance one another and rise off of the table. The ability to move cards into three-dimensional space is fundamentally different than being trapped on a two-dimensional surface. Rising into a third dimension can only be achieved when there are multiple cards. Similar arguments can be made for atoms, people, ants, brush strokes, DNA base pairs, phonics, bricks, tones, and neurons. Complex systems generally cannot be designed top down by simply putting parts together. Instead, almost all complex systems *emerge* over time, with various components coadapting to form long-lasting and large-scale patterns.

Phase Transitions Radically Alter Structure and Function Under some conditions a system can fundamentally change, not in its parts but in the pattern of relationships between the parts. Such a system-wide structural rearrangement changes flow pathways and therefore the functions that can be expressed. In fact, the same system may behave so different on either side of a phase transition as to be unrecognizable. The canonical example is of H_2O molecules changing their state from solid ice to liquid water or from liquid water to vapor. As will be discussed in Chap. 7, there are different kinds of transitions and in particular one type of transition that seems to appear very often in complex systems.

Emergence and Non-linear Relationships Can Lead to Domino Effects As the parts or functions of a system become more intertwined and dependent upon one another, a small change or perturbation has the potential to have a system-level impact. In the playing card example, an individual card cannot lift off of the table because a single card is only stable when it lies flat. But two cards, which are both in individually unstable positions, can counterbalance one another. Stable systems can in fact be made out of unstable parts. By adding more counterbalanced cards, a House of Cards can be constructed. In such a structure, moving one card has the potential to disrupt the entire structure, potentially causing it to lose its emergent three-dimensional nature. In fact *House of Cards* is often used to describe any

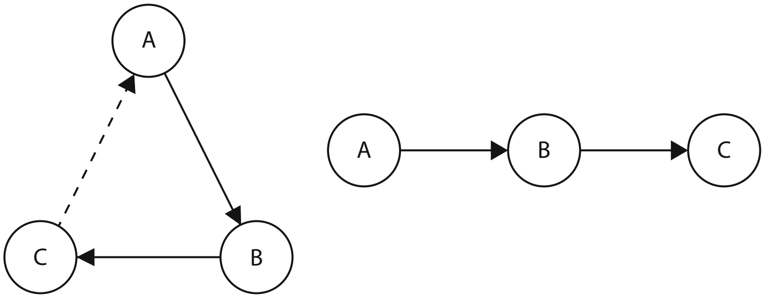


Fig. 1.4 The partially (looped) causal system (left) as in reality. A causal chain (right) as measured in an experiment. If the arrow between C and A is not measured by the experiment (dotted line), it would lead one to assume the linear causal chain in the right panel

system that is built upon a shaky foundation or has some critical vulnerability. Another phrase often used to describe a similar idea is the *domino effect*, whereby the falling of one domino can create a chain reaction that leads to all dominos falling. Most complex systems, at least the ones that last for any time in the real world, contain a degree of robustness to counteract the domino effect. Fail-safes, checks and balances, and copies of functions protect against perturbation that might compromise the integrity of the system.

Complex Systems Are Causal But Not in a Simplistic Sense The definition of causality is that inputs (causes) must precede outputs (effects). This will be assumed in discussing complex systems. Many fields, however, focus on causal chains—A causes B causes C—as on the left side of Fig. 1.4. Chains are often discovered when correlations are detected—when event A occurs, events B and C usually follow. When there is a drop in income, behaviors associated with poverty often follow. It is far from obvious, however, that the causes are direct or necessary. Some low-income areas do not exhibit behaviors associated with poverty.

Aristotle understood that causality comes in a variety of flavors: parallel causality (correlation without a causal relationship), acausality (causality does not exist—essentially random processes), and partial causality (loops in causality). It is partial causality, called circular causality by Hermann Haken (1927–), that plays an important role in complex systems. For example, in the left panel of Fig. 1.4, A causes B causes C causes A again. This kind of self-looping or self-reference will be a theme through the text.

Homeostasis Maintains Multiple Parameters of a System Within Acceptable Ranges If a particular variable of a system strays too far in any direction, there is a restorative feedback loop that will nudge the system back on track. When applied to many variables simultaneously, the system gains a sort of inertia that makes it resistant to change. Such an ecosystem of variables is known as homeostasis and allows a system to self-regulate and maintain its integrity against perturbations from the outside. A great example is the systems of checks and balances that are built into many political systems. In theory, these checks and balances form feedback loops that prevent one voice or ideology from becoming too dominate. This same set of checks and balances, however, can also lead to political impasses and resistance to new policies.

The flows and variables within a homeostatic system are almost always dependent upon one another. Corrective nudges applied to one variable will often come at the expense of moving another variable away from its own preferred range. For example, when blood sugar moves out of normal ranges, other body systems react. This is entirely normal and happens every time you eat a meal. It is in fact healthy for a system to be temporarily pushed outside of normal bounds for short periods of time (e.g., an acute stress). Exercising the adaptive capabilities of the system will prepare the system for future challenges.

Structural Adaptation Occurs When Physical Structures or Rules Change Whenever relationships between parts change, the flows and the functions that result will also change. These structural changes may be achieved through cutting, adding, strengthening, or weakening interconnections or parts. For example, the on-paper design of cars has evolved over the past 150 years. These designs have acted as a guide to the physical manufacturing of cars. Changes in the design will result in changes to the cars. There can also be changes in the factories that make the cars, optimizing flows of materials, energy, workers, and manufacturing processes. When a car requires maintenance, or is broken and requires fixing, a mechanic will make structural changes to that individual car. As these cases reveal, structural change is the basis for evolution, reproduction, and healing.

Cars, phones, and other designed systems do not change on their own. When a new road is proposed, and later built, the designer intended some change in traffic patterns. The road, however, does not make the change itself. That requires some outside arbiter to direct the change. Likewise, language, as a system of rules and words, requires human cultures to make changes.

Some Systems Can Turn an Internal Functional Adaptation into a Structural Adaptation Such systems can direct their own internal growth, development, healing, and learning. The relationship between structural and functional adaptation will be explored in the second half of Chap. 7 and then built upon in Chaps. 8 and 9. A nice treatment of these kinds of learning systems can be found in Peter Senge's (1947–) *The Fifth Discipline: The Art and Practice of The Learning Organization*. The view presented is of an organization that alters its own flows and processes by reorganizing internal structures and policies.

System Structures Can Become Modular Whether structural changes are internally or externally directed, or a combination, patterns often emerge. A *module* or *motif* is simply a collection of parts that often appear together in the same or similar relationship. Despite many possible recombinations, particular configurations of neurons, chemical reactions, and country alliances will appear much more often than others. Generally motifs are simultaneously useful and stable. Once established within a system, a module can become a basic building block itself and recombined with other modules. Complex structures emerge a hierarchical nesting of modules. As we will explore in Chap. 5, modularity enables a system to evolve and adapt more quickly than a non-modular system.

Growth and Disease Are Triggered by Chronic Disruptions of Homeostasis In an interconnected system, when one variable moves outside of its range, other variables will functionally adapt. Chronic deviations, however, may result in structural adaptations that are more permanent. For example, if glucose levels persist outside of normal ranges, as in diabetes, structural changes will impact the circulatory, nervous, and other organ systems. A similar argument can be made for the interconnections between socio-political-economic systems that can disrupt one another outside of their respective stable ranges. In these cases, corrective actions that are meant to be temporary become more permanent and lead to a domino effect that spreads the imbalance throughout the system. If not corrected this positive feedback cycle can become a vicious cycle with the end result being the inability to maintain homeostasis. In both organic and nonorganic systems when functions cease, the system dies. On the other hand, one way to view growth and development is when tensions within a system prompt that system to find a new homeostasis that is the next phase in development.

Open Systems Have Permeable Boundaries That Metabolize Energy, Information, and Materials Systems require a boundary to separate inside from outside. An entirely closed system, however, will eventually be unable to maintain the flows needed to function. Such systems reach an equilibrium where nothing happens. The physical interpretation will be explored in Chap. 7, but the practical meaning is that systems must be at least partially open to their environment. They must take in or feed upon external energy, information, or materials to keep going. Likewise they generally give off some kind of waste. The combination of taking in energy and giving off waste is the broad definition of a *metabolism* and can be applied to both natural and human-created systems.

Some Complex Adaptive Systems Can Change Their Own Environment Given that open systems can communicate with their environment, and structurally adaptive systems contain functions that can make structural changes, it should come as no surprise that some systems can in fact change the environment around them. This could be unintentional (e.g., the expulsion of waste) or directed (e.g., terraforming a landscape). In these situations a tight, and perhaps dependent, relationship may form between a system and its environment. Such a relationship means the system has become just one player in a more complex ecosystem.

Complex Systems Can Become More Complex over Time From a simple seed, recombination, iteration, modularity, hierarchies, and emergence, when combined with structural adaptation, can enable a system to add layers of complexity. The evolution of a system might be directed by a designer (e.g., a mechanic supercharging a car). As we will explore throughout the text, it is also possible for a system to direct its own development. For example, a sensor that measures concentration in the environment might be copied and turned inward to measure internal concentrations.

There Are More Complex Systems Principles to Be Discovered The concepts above were presented as a starting point. They will be refined and connected together throughout the text. It is suspected, however, that there are still grounding principles waiting to be discovered. In fact, this is one of the most exciting aspects of studying complex systems. A few guesses as to these new areas for exploration will be discussed in Chaps. 10 and 11.

1.4 Understanding the Real World

The intent of nearly every discipline is to deeply understand phenomena in the world. Complex systems are no different. As an interdisciplinary field, however, it has inherited tools, processes, ways of thinking, histories, tensions, and world views from other disciplines. It is not always clear where one discipline ends and the field of complex systems picks up. The lines are so blurry that some don't even consider the field of complex systems to be unique discipline. In an attempt to stake out the boundaries of complex systems theory, this section will bounce back and forth between various ways of knowing. It will at times have the feel of philosophical throat clearing. Do stick with it as knowing where complex systems theory fits into the overall intellectual landscape will help you become a more intentional explorer.

1.4.1 Ways of Knowing

There are three ways of knowing the world that directly intersect the study of complex systems. First, knowledge might be handed down from somewhere else. The core of this way of knowing is to have a trusted source. Examples include divine revelation, fortune-tellers, parents, muses, charismatic teachers, websites, and books such as this one. Knowledge often propagates throughout a culture using trusted sources. Second, logical arguments, both qualitative and quantitative, derive knowledge from a small set of first principles. There is a long tradition of "armchair" philosophers building *a priori* knowledge that is claimed to exist independent from worldly experience. Some knowledge derived in this way was well ahead of its time, for example, Democritus' (c.460–c.370 BC) theory of atoms. Others were wrong but insightful, for example, Aristotle's (384–322 BC) ideas of motion. Third, experimental approaches, ranging from "try it" or "tinker" to the more formal design of repeatable experiments and statistical analysis of data, can be employed. Such *a posteriori* knowledge relies upon, and gives weight to, experiences (both successes and failures), repeated exposure and practice, as well as histories and stories.

There are many hybrids of these ways of knowing that combine two or more of the above ways of knowing. For example, Leonardo da Vinci (1452–1518) and Galileo Galilei (1564–1642) combined theoretical and experimental approaches. Likewise, some philosophers such as Thomas Aquinas (1225–1274) combined logic and religion to arrive at a new way to view the world. When an engineer builds a working device, they are using both theory and experiments. Likewise much psychological research aims to rigorously test folk knowledge by combining theory and real data.

The brain is a nice model system to understand how different approaches ask and answer questions about phenomena in the world. First, you might be told about the brain by a parent, teacher, sage, mystic, book, or website. You take it on faith that what you hear or see about the brain is factual. Second, you might arrive at ideas about how the brain works through logical or mathematical derivations. The statement, “I think therefore I am” (*cognito ergo sum*) by Descartes is argued from first principles. Likewise, Yoshiki Kuramoto (1940–) derived how oscillators synchronize and desynchronize to demonstrate how idealized neurons might coordinate to perform actions. Third, experimental work on the brain ranges from the study of protein folding to entire brain imaging using functional magnetic resonance imaging (fMRI). These three ways of knowing about the brain are often used in concert to develop a deeper understanding.

Complex systems thinking does not present one unique way of knowing. We will mix and match tools, ways of asking questions, and interpretations of observations. We will borrow from philosophy, anthropology, mathematics, physics, sociology, and the arts. While complex systems theory is grounded in a scientific way of knowing, it does at times make bold claims that will seem unscientific. We will explore more of these deviations throughout the book and take a critical look at the field in the last chapter.

1.4.2 How Much of the Real World Can We Really Know?

The question posed in the title of this section is a source of tension within and across disciplines. On the one hand, perhaps the world is what we observe and measure. We should take at face value the patterns we see, hear, taste, feel, and smell. New tools and technologies often extend our senses to reveal even more of the world. That we currently have an incomplete picture may simply be that we don’t have the right tools or perhaps haven’t asked the right questions. In principle we might be able to understand and explain everything.

On the other hand, perhaps there are truths that will forever remain outside the province of human knowledge. Plato (428–348 BC) argued that what we experience in the world is merely the shadow of ideal forms that we can never access. David Hume (1711–1776) claimed that we would forever be limited by our senses and, in agreement with much of the spiritual tradition, the flawed nature of our human mind. There are also some logical arguments which we will touch upon in Chap. 6 that imply that all systems will have blind spots.

On the surface, how much we can know seems to be an Ivory Tower debate. Yet how much we can know is at the heart of our understanding of the origin of life and consciousness, what we mean when we say someone possess a mind or a self, the nature of language and morals, and what it means to be a human. The practical weight of these topics is exposed when questions arise about the beginning and end of life, the purposes of educational and economic policy, and the balance between technological advancement and ecological stewardship.

Fields often will disagree, each believing that their own tools and theories explain the true nature of things. One side may use the weaknesses of another field to highlight how their own is superior. Retaliation often only serves to sharpen the focus on differences and escalate tension. The ancient Indian parable of the blind men and an elephant sums up the problem nicely. A group of blind men encounter an elephant for the first time and begin to report back what they feel. Not surprisingly, they disagree on what they experience. The morale of the story is that each blind man thinks he has discovered an absolute truth and has data to support his position. The true nature of the elephant will always escape these blind men until they combine their collective experiences.

The field of complex systems provides a framework for integrating knowledge across fields with the hope of gaining new insights. Sometimes this approach confronts big questions by integrating information across disciplines, with the hope of gaining new insights. Sometimes this will validation the perspective of one discipline. Other times it will propose a compromise. In some cases it will propose a new way to understand a phenomenon. For example, in Chap. 10, a proposal will be made that perhaps free will should not be assigned to an individual but rather should be thought of as an emergent property that arises only when groups of people gather. This kind of arbitration between disciplines and perspectives can appear arrogant. In the interest of transparency, we will explore such criticisms of complex systems theory in the last chapter.

1.4.3 Models as Ways of Knowing

A unifying methodology across all ways of knowing is the creation of models. A historian constructs a model when they tell a story that weaves together various facts. A politician debates another politician by arguing that they have a better model of how the world works and therefore know what to do to solve its problems. The same goes for CEOs, religious leaders, scientists, and musicians. Mental models are the lens through which each of us makes sense of the world. We even create models to explain our thoughts to others.

Models are powerful for a variety of reasons. They generally strip away the messiness of the real world to bring into high definition a particular phenomenon. Irrelevant information and connections can be ignored. The focus is placed narrowly on the phenomenon, making the formulation of questions and the interpretation of answers, more clear. Furthermore, simplified models are easier to communicate to others. As such they will diffuse more easily within a field as well as perhaps outward to other fields. On the other hand, as the statistician George Box (1919–2013) said, “all models are wrong.” By this statement Box meant that all models are representations of a more complex reality. As such they must be recognized as only partially right and applicable under particular circumstances. It is sometimes easy to fall in love with the model one has constructed and become blinded by it. It is the art of modeling to balance the simplicity needed to understand a phenomenon at its most basic level and the true complexity of the world.

One tenant of complex systems theory is that simple models can reveal universal phenomenon. For example, the Ising model of ferromagnetism, introduced in Chap. 4, describes how magnetism varies with temperature. It would normally be of interest to only a small group of physicists. Yet in complex systems theory it is a simple illustration of a type of phase transition that shows up again and again in other systems.

A second tenant of complex systems theory is that the same system can often be better understood, not with one model but rather with many complimentary models. For example, aspects of the economy will be explored as an agent-based system (Chap. 2), a set of coupled non-linear differential equations (Chap. 3), a complex pattern generator (Chap. 4), a network of interactions (Chap. 5), and a balance between competition and cooperation (Chap. 8). No single model can represent the nature of an economy. But together these models can provide deep insight into how an economy works.

1.4.4 Analogies and Isomorphisms

Analogies and isomorphisms are two kinds of models that are used often in complex systems. Both compare two seemingly different systems or phenomena and point out the dimensions along which they are similar. An isomorphism makes the claim that at some deep level, the two systems are identical in their form and function. Despite perhaps operating on different time and space scales and being created out of different materials, there is a one-to-one mapping of everything from one system to the other. At a deep level, all spirals are the same. Analogies are a somewhat weaker version of system comparison, usually showing similarity along one or a few dimensions, but not all. An alarm clock and a rooster both might serve the same function, but in all other ways they are different. Whenever you catch yourself saying that something is *like* something else, you are generally using an analogy.

There is a fuzzy line between isomorphisms and analogies. For example, in Douglas Hofstadter’s *Godel, Escher, Bach*, he repeatedly makes connections between how anthills and human minds work. It is not entirely clear if Hofstadter is using the anthill as an analogy for explanatory purposes or is in fact claiming that anthills and brains operate using the same structures, flows, and functions. This will mirror many of the connections made through this text. It will not always be clear if a connection between two systems is an analogy or an isomorphism. For example, when traffic flow is compared to flows in the economy, it will not be clear if this is a helpful analogy or if both systems operate upon the same general principles. It is perhaps safest to assume any system comparison is an analogy that might point to some deeper principle that deserves further exploration.

1.4.5 Decomposability and Reductionism

Modeling relies on the idea that the world is decomposable. This concept has been around for a very long time, exemplified by Plato's assessment in *Phaedrus* that the way to study the world is to "carve Nature at its joints." The world seems to have presented us with logical ways to create categories. As such many disciplines attempt to reduce their field down to elementary ideas or parts. To a sociologist this is a person. To a neuroscientist it is a neuron. To a physicist it is particles and forces.

Decomposability means that we can make progress in a particular intellectual corner or level of a hierarchy without needing to know much about other disciplines or what happens above or below our chosen level. For example, a cell biologist likely doesn't need to understand nuclear physics to study biology. They also do not need to understand the social behavior of humans. It is helpful to understand basic concepts at the chemical level below and the formation of tissues above, but that understanding is for the purpose of having a context in which to propose and perform cellular experiments and interpret results. For the cell biologist, once chemical structures emerge at the level of large molecules, the focus can be on how large molecules interact within a cell. Likewise, a historian of post-1945 Italy can be an outstanding scholar without knowing anything about the Old Kingdom dynasties of Memphis in ancient Egypt. Very little information moves between levels in most systems or fields of study. Herbert Simon (1916–2001) called these types of systems *nearly decomposable*.

A pure reductionistic approach assumes that understanding parts will lead to an understanding of the whole. As long as the parts can be sealed off from everything else, essentially treating them as a closed system, this approach works remarkably well. The canonical example is mathematics, which would claim to stand apart from human or even natural laws. We will address this claim, at least in part, in Chap. 6.

1.4.6 Holism and the Relationships Between Parts

The reductionistic approach begins to break down when relationships between parts or levels of a hierarchy matter. Relationships form the basis for recombination and iteration that can lead to system-level non-linearity and emergence. Some systems, for example, cultures, politics, ethics, and economics, are so intertwined that studying one necessitates studying the others. For example, when studying taboos, as we will touch upon in Chap. 10, it is not easy to separate out legal, moral, and historical elements. Messy systems are messy because the relationships between parts matter at least as much as the parts themselves. They are called "complex" for precisely this reason. As complex systems theory is more about the study of relationships than parts, it is these kinds of messy systems where the field stands to provide the greatest insights.

Tension exists between the reductionistic and holistic approaches to understanding the world. One is not better than the other; they simply have different purposes. Both approaches come with epistemic risks and blindspots. Although this text will focus mainly on relationships, holism, and putting parts together, these concepts could not be meaningfully discussed without having parts in the first place.

1.4.7 Determinism and the World as a Machine

A powerful analogy has arisen from the use of models—the world is a machine governed by unchanging rules that we can discover. It can be taken apart and understood. We can make predictions. Interventions can be made that will have predictable outcomes. We can hack the natural laws of the world to build our own machines. Almost all disciplines, practical and theoretical, academic or professional, have been impacted by the idea of the world as one giant machine. Complex systems theory builds upon this mechanistic tradition. There will be several references to simple conceptual mechanisms throughout the text, and so it is important to understand the history and allure of thinking of the world as a machine.

Immutable laws that we can derive or discover go back to the ancient world. It was, however, Newton, Galileo, Lavoisier, Boyle, Laplace, Dalton, and others who revealed just how powerful this approach could be. These early physicists, chemists, and mathematicians initiated the "clockwork" world view—everything we experience is the complex operation of an elaborate machine. Every event is the one and only thing that could have come next and is therefore predictable given enough information and the right set of rules. The later successes of James Clerk Maxwell (1831–1879), Albert Einstein (1879–1955), and others helped reinforce this determinism view of the world.

Biologists also were seeking to show how life could be explained through chemical and physical principles. Early work by Galen of Pergamon (130–210), William Harvey (1578–1657), and others demonstrated that the body could be thought of as a machine. The use of tools such as the microscope revealed that we are composed of many submachines called cells. Erwin Schrodinger's (1887–1961) *What is Life* hypothesized the type of molecule that could act as the carrier for heredity information. It was less than a decade later that DNA was characterized and initiated a revolution in cell and molecular biology. The strong stance for much of the second half of the twentieth century was that all biological functions were determined by genes. Biological systems seemed to be machines at all levels of granularity. Likewise, fields that built upon biology, such as clinical medicine and psychology, followed along in the spirit of this mechanistic paradigm.

The success in the sciences inspired other fields to search for their own universal laws. If the natural world is governed by laws, and humans are a part of that world, then anything created by humans should also follow discoverable laws. Modern economics began with William Petty (1623–1687) and John Locke (1632–1704), who laid down the concept of supply and demand. Adam Smith based many of his ideas on the Newtonian concepts of motion and equilibria. David Ricardo (1772–1823) took another step by showing how economic models could make predictions. The mathematical view of economics continued to grow, advanced in large part by the theories of John Maynard Keynes (1883–1946). Keynes viewed the economy as a system that we could predictably fine-tune through interventions such as interest adjustments, tax rate modulation, and investment in large-scale projects.

Social-political thinkers also took up a mechanistic view. John Locke, based upon the work of Thomas Hobbes (1588–1679), described the human mind as a *tabula rasa* (blank slate) but one that is endowed with basic underlying rules. Everyone was equal at birth and should be granted certain unalienable rights. Auguste Comte (1798–1857) searched for general social-political rules, and Emile Durkheim (1858–1917), considered the founder of social science, claimed that societies were governed by generalizable laws that could be discovered.

The industrial revolution injected machines into everyday experiences. Businesses began to view their internal workings as machinelike, governed by rigid policies, hierarchies, and efficient assembly-line flows. For example, Frederick Taylor (1856–1915) in *Principles of Scientific Management* treated an organization as a machine to be designed. Many of these ideas have continued on to present-day business practices in the form of Agile, Lean, and Six Sigma methodologies that attempt to systematize production.

1.4.8 Organic Thinking

Complex systems thinking adopts much of the mechanistic way of thinking but layers on some other critical elements. For example, some systems are adaptive. They grow and evolve over time. New possibilities are continually being created and destroyed. These systems display path dependences and sensitivity to historical context. For example, Stephen Wolfram's (1959–) book *A New Kind of Science* advocates for a particular view of the world as unfolding by some underlying algorithm. But the algorithm itself is developing too. Perhaps rules exist but only as constraints that generate the next set of rules. Such thinking is most applicable to organic systems and will make many appearances throughout this text.

A less mechanistic view of the world has come in and out of favor several times over the centuries. Leonardo Da Vinci viewed nature as the best teacher of design, adopting many of the practices of modern-day biomimetics and ecological design 500 years before these became disciplines. Evolutionary thinking progressed from Georg Wilhelm Friedrich Hegel (1770–1831) and Immanuel Kant (1724–1804) to Jean-Baptiste Lamarck (1744–1829), to Friedrich Engels (1820–1895), and finally to Charles Darwin (1809–1882). Running in parallel, Rudolf Virchow (1821–1902) developed the cell theory of biology and Louis Pasteur (1822–1892) initiating microbiology and biochemistry. Both had a developmental view of life. The idea of a system that could self-regulate was first put forward by the French physiologist and Catholic priest Claude Bernard (1813–1878), saying that the *milieu interne* was not only balanced but self-correcting.

Early ecologists, such as Ernst Haeckel (1834–1919) described adaptive interrelationships between species as being critical to understanding the biological world. Likewise Charles Elton (1900–1991) advocated studying food chains, cycles, and flows of material and energy, while A.G. Tansley (1871–1955) coined the word ecosystem to describe a grouping of organisms, interacting and coevolving as a coherent whole. Vladimir Vernadsky (1863–1945) introduced the idea of the biosphere as the living skin that penetrates some depth into the earth and extends up in the atmosphere. A more modern version of this idea will be expanded upon in the final chapter when we discuss the Gaia theory of the earth.

Organic thinking was also applied to human-created systems. For example, John Stuart Mill (1806–1873) in his *Principles of Political Economy* (1848) made a distinction between the more mathematical parts of economic study that are based upon unchanging laws and the parts that were contingent on human-made policies, social forces, and environments. Karl Marx (1818–1883) developed this social thread into a new economic theory. Likewise, the leading artists, composers, philosophers, and novelists of the Romantic movement (1800–1850) rebelled against the mechanism of the industrial revolution by infusing their work with emotion and natural themes.

The view of systems as organic entities eventually became a field of its own. Lawrence Henderson (1878–1942) was the first to use the word "system" to describe both natural and human-made patterns. C.D. Broad (1887–1971) used the term "emergent properties" to describe how patterns might self-organize. Christian von Ehrenfels (1895–1932) used the word *gestalt*, which in German means "organic form," to express that in some systems "the whole is more than the sum of the parts." The implication is that taking a system apart to study it will in fact destroy the very subject of study. Complex systems thinking is at least in part the study of how interconnected parts can form an adaptive and coherent whole.

1.4.9 Repeatability, Refutability, and Predictability

The field of complex systems builds upon a scientific tradition. As such it inherits three pillars of what is considered to be good science—predictability, repeatability, and refutability. Although not mutually exclusive, we will discuss each of these supporting ideals below.

Predictability is linked strongly to determinism—a particular event is the one and only thing that could have possibly occurred. The ability to predict is a sign that the model accurately represents the real world. Predictability in science is therefore a sort of guiding light that helps choose between competing hypotheses and models. The better the ability to predict, the more accurate the model is assumed to be.

Repeatability is based upon patterns in time. It is often assumed that identical inputs should result in the same or similar output every time. The desire for repeatability sometimes twists scientists into knots as they attempt to design experiments that will "freeze" a naturally changing system. Likewise many analysis techniques assume the system to be linear and at an equilibrium. There are nuances, however, that allow science to be applied to the messy real world. For example, we do not expect a particular event or observation to be exactly the same every time. Some predictions are probabilistic, and the effects of context can sometimes be removed or minimized, through experimental procedures, post hoc statistics, or both.

Refutability was articulated most clearly by the philosopher of science Karl Popper (1902–1994) as creating questions (hypotheses) that can be disproven by real data. In the scientific tradition, if no experiment, observation, or data can answer the question, then there must be something wrong with the question. Refutability is a sort of litmus test of a good scientific question.

While complex systems theory is a science, it sometimes is aimed at systems that seem to defy rigorous scientific study. The next three sections will explore how the field of complex systems thinking sometimes conforms to and at other times breaks with scientific tradition.

1.4.10 The Relationship Between Randomness and Unpredictability

Some systems are easy to predict while others are not. The first total solar eclipse in the 2300 century can be predicted, whereas the location and size of the largest earthquake between now and then cannot. The inability to measure everything at once is often the source of unpredictability. For example, the paradigm shift of quantum mechanics was that all we can measure are probabilistic distributions on the micro-level that give rise to predictable macro-level events. Alternatively, even macro-level events might be unpredictable if there are many interacting parts. In the language of science, these systems are effectively of infinite dimensions and behave as a random process.

One of the interesting contributions of complex systems theory is to catalog the ways in which systems defy prediction. For example, some systems are very sensitive to perturbations, amplifying the impact of context rather than having context averaged out. Likewise, randomness may be viewed not as some separate process but rather as an emergent property of certain types of systems.

1.4.11 Repeatability in Adaptive and Non-linear Systems

Repeatability can become a problem when studying complex systems. In a constantly developing system, the same input will not always produce the same result.

To an experimental scientist, such systems are a moving target. It is considered good scientific methodology to freeze the system (not literally) so that it can be studied without the change. Such a technique essentially forces the system to be at or near an equilibrium. For example, when studying a living organism such as a rat, a researcher might only limit their study to subjects that are between 30 and 45 days old—the equivalent of a human teenager. This type of methodology means that hypotheses about development will not be formulated.

Adaptive and non-linear systems are in a constant state of becoming. They are often sensitive to onetime events that may forever change the trajectory of the system. How does one study events that will happen only once? A historical approach creates what are known as "just so" stories that attempt to retroactively connect together events. A story, however, is generally not repeatable and therefore not scientific. Complex systems theory does not entirely break with the scientific tradition but does attempt to uncover broad patterns within systems that lead to adaptation, sensitivity, and onetime events. As such it can sometimes provide insights into systems that are becoming or contextually dependent.

1.4.12 Refutability and Complex Causal Loops

Refutability is based upon the idea that logical chains of reasoning will eventually lead to a more accurate model of the world. A hypothesis is a guess that if proven by real data makes a new logical connection within that model of how the world works. The practical difficulty of this approach lies in the relationship between correlation and causation. Causation is about the true nature of a relationship. Correlation on the other hand is about what is measured or observed. The two do not always agree, as will be illustrated below.

When science encounters a causal loop, like the one in Fig. 1.4, a controlled experiment will often make a particular cut in the loop. Once the cut is made, the loop becomes a linear chain and correlations are much easier to detect. Cutting in a different location, however, can lead to different conclusions. If all of the researchers working on the system shown in Fig. 1.4 are publishing in similar journals, they would read each other's work, and a more complete picture would quickly emerge.

Consider the slightly more complex network in Fig. 1.5. Because the signals are carried by different molecules, multiple measurement techniques would be needed. If only $[Na^{+}]$ and a hormone were measured, the system would appear to be a pure causal chain, with A being the root cause. In reality we can see that there is feedback from two other molecules, but these were not measured. Furthermore, such a system may become even more difficult to untangle if the signals occur on different time or space scales or are intermittent. Findings from these kinds of studies, because they use different measurement techniques, might be scattered across a range of journals. Building a coherent model of such a system would need to wait until all of the key signals and relationships were known.

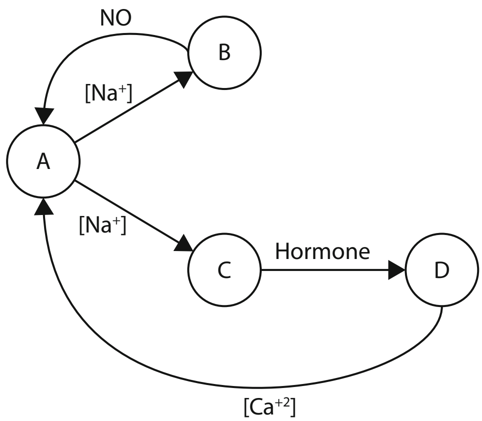


Fig. 1.5 A hypothetical system containing multiple causal loops carried by different signals

1.4.13 Tools and Literature That Span Disciplines

The tools and processes used to understand the real world matter a great deal. Many fields are defined by the systems they study and the tools they use, often siloing their findings in disciplinary journals. In this regard complex systems as a field is an anomaly. It is not tied to a particular system, tool, or process. Rather it is a motley collection of ideas, principles, and tools, both analytical and experimental, for studying a wide range of systems that often do not behave well when approached through the usual scientific lenses of predictability, repeatability, and refutability. As a result complex systems research is often scattered throughout the journals of other disciplines. It has only been recently that venues have been created specifically for complex systems research.

Depending upon your particular field, some of the ideas and tools contained in this text will seem familiar. For example, to an evolutionary biologist, much of game theory, with its focus on multiplayer competition and cooperation on a fitness landscape, will seem to be just a reformulation of evolutionary theory. Other tools will seem like an awkward fit for your discipline. You are invited to try them out anyway. It is often the application of a new tool that provides fresh insight.

1.4.14 Complex Systems and the Computer

The purpose of a model is not only to describe a phenomenon but to make predictions and direct future work. Analytical models have historically required assumptions to be made to achieve these goals. For example, when encountering a non-linear system, the equations are linearized. When encountering an adaptive system, the temptation is to freeze it. This generally means either ignoring some parameters, holding them constant in a cleverly designed experiment, averaging phenomenon, or exploring parameters over a small range.

Over the past several decades, a critical new tool has become available—computing. Computation endows the programmer with a sort of godlike control over rules and environments. Simulations can be repeated as many times as desired and can proceed when pencil and paper calculations fail. Parameters can be varied. The inputs from the environment can be systematically tuned. Unlike in the real world, just about anything can be measured at the same time. As such, simulations are not dependent upon particular experimental apparatus. Phenomenon that span space and time scales can be investigated using multi-scale simulations. It therefore becomes possible to study the kinds of systems that are less decomposable. Development, healing, learning, and reproduction can be studied by coding in the feedback loops between functional and structural adaptation. "What-if" questions can be entertained. In some cases a hypothesis can even be refuted by a computer model.

It is not a coincidence that the formal study of complex systems arose around the same time as electronic computing. For all of the reasons above, complex systems researchers often turn to computation. For example, the ability to numerically integrate non-linear equations led to the discovery and exploration of chaos. Likewise, new computer-aided visualization and analysis techniques sparked a renaissance in network science. Simulations have also helped refine hypotheses to be tested out in the real experiments. For example, without knowing the exact feedback signals in Fig. 1.5, a series of simulation could explore the type of signals that might be missing, thereby suggesting where experiments should look next.

1.5 Gatherings and Continued Work

Breakthroughs sometimes happen through individual insight. Upon deeper examination, however, these breakthroughs are often due to the efforts and influences of groups. Throughout history there have been concentrations of thinkers who have collectively made extraordinary jumps in thinking. The construction and warfare ideas of ancient China and Egypt, the political ideas of ancient Greece, the mathematical innovations of the Middle East, the artists and scientists in Florence who sparked the Renaissance, the Western European origins of the industrial revolution, the information revolution centered at Bell Labs, and Japan's industrial process innovations in the 1960s all come to mind. Below we highlight three interdisciplinary groups who have laid the groundwork for complex systems theory.

1.5.1 Vienna Circle

The Vienna Circle was a group of scientists, social scientists, philosophers, and mathematicians who met on and off between approximately 1921 and 1936 at the University of Vienna. We will encounter a number of their members through the text, including Ludwig Wittgenstein (1889–1951), Oskar Morgenstern (1902–

1977), Kurt Gödel (1906–1978), and Karl Popper (1902–1994). They documented their work in a series of publications, many of which have become classics of systems theory. The group dissolved after one of their members was murdered as Hitler rose to power. The spirit of the group, however, continued on as the members of the Vienna Circle were welcomed in to other universities.

As a group, the Vienna Circle advocated for the twin schools of empiricism and positivism, meaning that they believed that knowledge could only be gained through experience but that experience included the manipulation of symbols (e.g., math and logic). Their ultimate aim was to provide a comprehensive framework for understanding all phenomena in the world. That framework contains many of the grounding principles of complex systems theory.

1.5.2 The Macy Conferences

In 1942 a number of young and brilliant minds from a variety of fields met for several days for the first of what became known as the Macy Conferences. They were not really conferences but more like informal gatherings. The topics at that first meeting were focused on the general workings of the mind, but conversation ranged across many topics. The following year, there was another meeting, with additional topics. The meetings continued, with members rotating in and out for the next 19 years over approximately 160 meetings. These meetings were characterized by a wide range of backgrounds and skill sets, intense immersion, the movement of information across disciplinary boundaries, tension, and productive intellectual conflict.

Many of the participants went on to make enormous contributions to their respective fields and in several cases even created entirely new fields. Gregory Batson (1904–1980) pioneered ideas in anthropology, linguistics, and psychology, Claude Shannon (1916–2001) laid down the fundamentals of information theory which we will discuss in Chap. 6, John McCarthy (1927–2011) coined the word artificial intelligence (AI), and John von Neumann (1903–1957) experimented with cellular automaton that we will explore in Chaps. 2 and 6.

It was at the Macy Conferences that Norbert Wiener (1894–1964) introduced the idea of cybernetics. The name cybernetics has taken on a slightly different connotation in the modern press, being associated with cyborgs, cybersecurity, and other terms that blend the real and virtual worlds. Wiener's *Cybernetics* and Ross Ashby's (1903–1972) *An Introduction to Cybernetics*, however, were generally about feedback as a way for a system to monitor and control its internal variables and maintain homeostasis.

1.5.3 The Santa Fe Institute

The current epicenter of complexity is the Santa Fe Institute in Santa Fe, New Mexico. Members such as Stuart Kauffman (1939–), Geoffrey West (1940–), John

Holland (1929–2015), and David Axelrod (1955–) have invented new ways to think about and formally study complex systems. Agent-based systems, chaos theory, network theory, self-organized criticality, and a range of other topics are being actively explored at the Santa Fe Institute.

The institute started in 1984 as an outgrowth of the nearby government-funded Los Alamos Labs. Most government labs and university departments had become siloed as they became focused on the predictable sources of publications and funding that reductionistic disciplinary science could promise. As a nonprofit research and education center, the Santa Fe Institute has full control over the directions they pursue.

One of the original, and perhaps most unlikely, members of the Santa Fe Institute is Murray Gell-Mann (1929–). Gell-Mann won the Nobel Prize in physics for the discovery of the quark, the smallest known family of particles. As such he is the very definition of a reductionistic scientist. Yet he became a strong advocate for a more holistic study of the emergent properties of complex systems. Gell-Mann was in many ways the ideal scientist to bring legitimacy to the growing field of complex systems.

The organization of the institute is based upon many of the principles of complex systems. It has a very small number of semipermanent staff, with a large number of visiting and external faculty that have appointments elsewhere. This ensures a constant churn of new people and ideas. Small offices are shared, usually radically across disciplines. The architecture is dominated by large common areas, centered around a kitchen where members and guests often congregate. Walks around the trails surrounding the institute are a common activity. There is an excellent book, *Complexity: The Emerging Science at the Edge of Order and Chaos* by M. Mitchell Waldrop that details the history and ideas of the Institute. The website for the Santa Fe Institute also is a nice resource, containing white papers and published works of the various members.

1.5.4 Group You Could Join

As complex systems theory is still a growing field, there are many opportunities to get involved. Many of the people mentioned throughout the text are still working. Those who have passed on have left direct intellectual descendants who are carrying on their work. For example, the students of John Holland (1929–2015) are very active and continuing on with his work in genetic algorithms and complex adaptive systems.

There are also many conferences that focus on complex systems. Some are regular meetings such as the International Conference on Complex Systems. Others focus on the intersection between complex systems and a particular discipline. It is a fairly regular occurrence that someone in a field "discovers" the relationship between their field and complex systems and organizes a onetime conference to explore the intersection in more depth. If you keep your eyes open, you may find one nearby. These are wonderful opportunities to learn about the field from those who are at the center of the action. Do reach out, as complex systems researchers are often very supportive of those who are interested in the field.

1.6 Questions

- Find two different systems that contain positive feedback. Show that in one system it leads to a virtuous cycle and in the other it leads to a vicious cycle. What might be done to break the vicious cycle?
- Music is one of the most fundamental of human inventions, developed independently by nearly all cultures. Different musical traditions may use different frequency spacings between notes, giving Western music and Eastern music distinctly different sounds. But fundamentally, all tonal music is based upon musical notes. A note in isolation, however, does not on its own mean anything. It must either be sequenced in time with other notes (think of a kid's song such as Twinkle Twinkle Little Star) to create a melody, or it must be played synchronously with other notes to form a chord (think here of the strumming of multiple notes on a guitar). Furthermore, the way that notes are used in relation to other notes can even be used to identify different genres of music and in many cases individual composers. The Beatles, Tom Petty, Aaron Copland, and Igor Stravinsky all have a characteristic sound. Pick a song and take it apart, not musically but in terms of what units are being put together to form the song. What instruments, harmonies, lyrics, and other musical elements are being blended together? How do they all support one another?
- In 2010 Henrich, Heine, and Norenzayan published a very readable and fascinating meta-study "The weirdest people in the world?" that criticized much of what we think we know about the social and psychological behaviors of humans. Their argument was that most published experiments have been conducted on WEIRD populations, those that are Western, Educated, Industrialized, Rich, and Democratic. Furthermore the study goes on to report that two-thirds of psychological studies published in Western journals use college students as research subjects. In an unusual move, the article is accompanied nearly 30 one-page responses from anthropologists, evolutionary biologists, sociologists, philosophers, economists, linguists, and historians. Nearly all of the commentary agrees with and expands upon the caution that universals cannot be extracted from the WEIRD population. Find a behavior or phenomenon that was mentioned in this chapter or the Introduction, and be critical of it. Explain why that behavior, or the framing of the phenomenon, may not be universal across all human populations. What kind of studies might you conduct to demonstrate your point?
- It is rare for an intervention to have one impact on a complex system. There may have been an intended impact for the intervention, but then there are *cascade effects*, also sometimes called *secondary order effects* or *trickle down*. These effects are due to the interconnected nature of the system—it is almost impossible

to change one thing without impacting other parts of the system. A nice example is the first rule of pharmacology, which states that no drug will have one impact. This is the origin of side effects. Likewise, it is often the case a new law or product aims to solve one problem but inadvertently causes several more. Identify a historical intervention, exploring the intention and the side effects. Can you dissect how the interconnections within the wider system led to the side effects?
- Beer is one of the world's most popular beverages. In many countries it outranks water in gallons consumed. As the craft beer revolution in the United States of the 1980s and 1990s has shown, distinct flavors, colors, and aromas can be created using just four ingredients: water, yeast, barley, and hops. These were not always the four ingredients of beer, with the bittering of hops being achieved through other sources (e.g., soured beer, sweet gale). But the German purity law, the Reinheitsgebot passed in 1516, decreed that beer would only contain these four ingredients. Yet, with that constraint, Germanic people defined a huge number of distinct beer styles. In beer, these four ingredients do not take on any meaning of their own and in fact must be combined in the right way to become beer. In other words, there is a process for transforming the raw ingredient into beer. The Belgians, not being under the Reinheitsgebot, added some other units (e.g., spices, wild yeast) and different processing (e.g., warm fermentation, odd step infusions) that bring out flavors that are unattainable with the four basic ingredients and usual processes. What other examples might you find where some simple parts, combined together using relatively simple processes, lead to something entirely new? Have others innovated on adding in extra components to add to the diversity of the outcomes?
- Pick a series of events in your life that clearly form a causal chain. Draw this relationship out. Now compare this to a different series of events in your life that are more circularly causal, where it is not clear what caused what. What makes these two case studies different from one another?
- In this chapter a system that is robust against perturbations was framed largely as a healthy feature of a complex system. It was hinted, however, that sometimes system inertia can prevent new ideas or corrections from taking hold. Sclerosis is often associated with biological diseases when tissues harden and become very resistant to change. The term, however, has a system meaning—the generation of so many parts or functions that the system stops working. This is related to Ross Ashby's ideas about a system being too richly joined or connected. What is an example of a system that has undergone sclerosis and is nearing the point of locking up? What has been added to this system over time that is causing it to lock up? Speculate on what might destabilize the system, and open the door to a needed change.
- Scientists and engineers have often looked to the body and biology to help them develop new ideas. For example, neural networks are very similar to adaptive filters. The field of biomimetics steals form and function from nature to design artifacts. Scientists and engineers are not the only ones who steal from nature. Find some examples from the humanities, social sciences, legal, political, management, or education fields that have knowingly or unknowingly adopted

ideas from the natural world. Be clear about what they have adopted and how it works in the human-created system.

- *Laying Down A Path In The Walking*, inspired by the poem *Wander* by Antonio Machado, is a way in which many complex systems emerge over time. A similar idea appears in *The Lord of the Rings* in the phrase, "not all who wander are lost." Spend at least 15 minutes intentionally wandering, letting cues in the environment guide where you should go next. After your wandering, deconstruct your experience. Do not focus on where you wandered but rather the cues and thoughts that prompted a next move. How might your wandering have been different if the circumstances or environmental cues were different?
- Find a conference or institute of complex systems online. What is the makeup of their membership? What areas do they work in? Have they made any significant advances? How does their work intersect with the general systems ideas in this chapter?
- A foundational principle of group improvisation and brainstorming is to accept what another person is saying or doing and build upon it, summed up in the phrase "yes, and...." This is a form of positive feedback, whereby an initial creative idea is amplified. The opposite is the phrase "yes, but," which generally is a form of negative feedback—minimizing an idea. In improvisation it is assumed that one should always use "yes, and." However, "yes, but" can serve an important purpose in that it will quickly make room for another idea. Have a conversation with another person, and note the times that you both use language that says "yes, and" or "yes, but" (likely not in those exact words). How does a combination of both positive and negative feedback keep the conversation going?
- Jay Forrester (1918–2016), one of the founders of systems dynamics, made many contributions to systems thinking. One of those has become known as the *Forrester effect*, sometimes called the *bullwhip effect*. The original idea was outlined in 1961 in terms of how supply chains are impacted when there are fluctuations in customer demand. Consider that customers can change their opinions and desires very quickly, based upon fads and seasonal variations. But the development, manufacturing, and distribution of products takes considerably longer. When customer demand spikes, there is a delay before it can be satisfied. It is entirely possible that the supply chain and manufacturing could ramp up to meet the demand, but by the time the product is distributed, the demand could be back down again. Such dynamics occur when two systems are connected together but operate on different time scales. Describe two systems that communicate with one another but operate on different time scales. What mismatches arise? Does the bullwhip effect come into play?
- Throughout the text we will discuss how complex systems emerge as coherent patterns in both time and space. A hypothesis is that to survive such systems must somehow strike a balance being able to generate solutions to the immediate problems at hand but also somehow still be robust against changes. Fill in at least five entries in the table below:
- In 1953 Isaiah Berlin (1909–1997) wrote an essay that compared hedgehogs and foxes, based upon the Greek poet Archilochus who wrote, "A fox knows

System	How is the systems generative?	How is the system robust?

many things, but a hedgehog one important thing." Berlin explained that people can be divided into these two categories as well. Foxes draw on many ways of knowing and doing and adaptively change strategies when encountering a challenge. Hedgehogs have one way of knowing and doing and apply it, often to great effect, when encountering a challenge. When mapping this classification to systems thinking, it would seem that foxes would be drawn to environments that are rapidly changing, where multiple strategies can be mixed and matched. Hedgehogs on the other hand would be drawn to challenges where the rules are well defined and relatively constant. Which one do you feel you are? Can you make some guesses as to the hedgehog or fox nature of some of your friends?

- The hedgehog and fox concept can be extended beyond people. For example, cancer is notoriously hard to eliminate because it employs a wide range of strategies that include rewiring the vasculature, going dormant, banding together in the form of a tumor, or going mobile by entering blood or lymph flow. Cancer is essentially a fox. Can you identify other natural and human-made systems that behave either as a hedgehog or fox?
- The term checks and balances is often used in political systems where there is oversight of one body by another. In the political system in the United States, the executive, legislative, and judicial branches of government are often said to balance one another. Others would add the role of the media and journalists as a way to ensure that critical information reaches citizens. The education system can likewise be viewed as a part of this complex system as well. What other elements of a society act to balance the government? Are their weights that you could assign to their impact? Do these weights change over time?
- Many theories and styles of leadership have been classified, from the more authoritarian/autocratic to strategic to laissez-faire. One leadership philosophy that has received more recent attention is the servant leader, coined in the article "The Servant as Leader" in 1970 by Robert Greenleaf (1904–1990). Broadly, a servant leader will turn the traditional power pyramid upside down and use their own authority to support everyone with the resources they need to be productive, find their own purpose, and explore the many ways they can contribute to an overall goal. The idea has also been expanded to organizations and even countries that can act to lead by serving the greater good. In what roles or ways do you act as a servant leader?
- A popular learning theory advocates for *constructivism*. You learn, early in life or perhaps through some innate skills, basic motor movements and cognitive

abilities. We will in fact encounter a neurological basis for the repetitive motions of locomotion in Chap. 4. Throughout your life you recombine them to perform more and more complex functions. For example, consider how a baby learns to crawl, then walk, and later run. These are very basic functions that they will use later in a variety of situations. But when we combine running with kicking a ball, running to spaces, passing to other players, and overlapping with a defender, we arrive at the game of soccer. Just as we can create complex structures from simpler structures, we can also create complex functions from simpler functions. Deconstruct some simple physical or cognitive action that you now can do unconsciously. What are the parts that were recombined? When did you learn these various parts? When were you able to unconsciously recombine them together to generate the higher-level function?

1.7 Resources and Further Reading

There are a number of excellent introductions to complexity theory that range from non-mathematical and historical to highly technical. Although systems have been studied for as long as we have written records, and perhaps thinking in terms of systems is a universal trait of all humans, it was not until the last few centuries that the formal study of systems began. The credit for the more modern approach is generally given to Karl Ludwig von Bertalanffy (1901–1972) an Austrian biologist who laid out what he called general systems theory. His master works, *Modern Theories of Development: An Introduction to Theoretical Biology* (1928) and *Problems of Life: An Evaluation of Modern Biological and Scientific Thought* (1949), outlined a framework for exploring any kind of system.

For a nice technical treatment, you may wish to try James Sethna's *Statistical mechanics: entropy, order parameters, and complexity*. Similar to the end-of-chapter questions in this text, Sethna's end-of-chapter questions, although of a more technical nature, are meant to spark new ideas and future exploration. Stuart Kauffman's *The Origins of Order* is a tome of a book but contains some deep technical insights into self-organization in biology. John Holland's *Hidden Order*, although not mathematical, is very technical. A slightly less technical introduction is Ricard Solé (1962–) and Brian Goodwin's (1931–2009) *Signs of Life: How Complexity Pervades Biology*.

Several non-mathematical introductions dive deeply into the various theories and methods of complex systems. Fritjof Capra has long been an articulate advocate for complex systems. Of his many books two intersect complex systems most directly, *The Web of Life: A New Scientific Understanding of Living Systems* and *The Systems View of Life: A Unifying Vision*. A wonderful general book on systems aimed at the change agent is Donella Meadows (1941–2001) and Diana Wright's book *Thinking in Systems*. John Holland's *Emergence: From Chaos to Order* and *Signals and Boundaries* are nice summaries of complex systems principles.

Several disciplines have explored the intersection between their central ideas and complex systems theory. For example, Brian Arthur's (1946–) *Increasing Returns and Path Dependence in the Economy* and Philip Anderson (1923–), Kenneth Arrow (1921–2017), and David Pines' (1924–2018) *The Economy as an Evolving Complex System* apply systems concepts to the economy. *Understanding Complex Ecosystem Dynamics* by William Yackinous does the same for natural ecosystems. *Complexity and Postmodernism* by Paul Cilliers (1956–2011) focuses on the social and philosophical intersections with systems theory. *Systems Thinking for Social Change* by David Stroh (1950–) provides insights into how to use systems thinking to change policy and take action out in the world.

A number of books have been written for a broad audience. Some outstanding examples are Steven Johnson's (1968–) *Emergence: The Connected Lives of Ants, Brains, Cities, and Software*, Melanie Mitchell's *Complexity: A Guided Tour*, John Gribbin's (1946–) *Deep Simplicity*, and Jack Cohen (1933–) and Ian Stewart's (1945–) *The Collapse of Chaos: Discovering Simplicity in a Complex World*. Two works by Stuart Kauffman, *At Home in the Universe: The Search for the Laws of Self-Organization and Complexity* and *Investigations*, and Murray Gell-Mann's *The Quark and the Jaguar* are tours of much of the work at the Santa Fe Institute. Heinz Pagels' (1939–1988) book *The Dreams of Reason: The Computer and the Rise of the Sciences of Complexity* is a wonderful exploration of the impact of computation on the study of emergence. A fun book of games that teach systems concepts can be found in *The Systems Thinking Playbook* by Linda Booth Sweeney and Dennis Meadows (1941–).

Chapter 2
Simple Rules

The areas around Chicago were previously prairie land, populated by a unique mix of grasses, wildflowers, termites, beetles, worms, jack rabbits, prairie dogs, golden eagles, and coyotes. This ecosystem has been largely crowded out by the growth of the human population and buildings. But suppose that you were asked to turn a suburban area just outside of Chicago back to prairie. How would you go about such a task?

A first intuitive step would be to remove human fingerprints and then begin planting seeds that should be part of the ecosystem. With some careful monitoring, these plants would grow and establish the preconditions for the survival of prairie animals. Through a literature search, you might discover that it would be best to add in new species in some stepwise way: first the insects, then lower-level herbivores, and finally the dominate predators.

Such experiments have been tried and they failed. The flaw was not accounting for a wide variety of system interactions that are necessary for an ecosystem to develop and thrive. For example, urban weeds grew much more quickly than prairie plants and easily crowded out the early seedlings. Ecologists did succeed in establishing new prairie, but not through the forced means described above. Instead they allowed urban weeds to first colonize the area. Once these plants established a foothold, the ecological niche for the slow-growth prairie grass presented itself.

What was correct from the initial approach, however, was that the order of introducing new species did matter. For example, reversing the order of introduction of two predators (e.g., coyotes and eagles) could make or break the new ecosystem. In addition, adding or subtracting a species could fundamentally change how resilient the final ecosystem would be. One of the most significant findings was how important periodic fires were in establishing a robust and diverse ecosystem. Fire induces some prairie grass seeds to sprout while killing off urban weeds.

How might a system as complex as a prairie arise? A generalization is how the complexity we observe around us can arise from just a few laws of physics. In this chapter we will introduce the idea that complexity can arise from very simple

J. Tranquillo, *An Introduction to Complex Systems*,
https://doi.org/10.1007/978-3-030-02589-2_2

rules. It is a theme that we will revisit many times. In this chapter we will focus on two ways in which simple rules can lead to complexity. The first is iteration in time. In complex system the phrase *less is more* is sometimes used, meaning that iteration of simple rules can over time lead to complex behaviors. The second mechanism we will explore is the interactions between many-bodied systems. These interactions can lead to complexity through emergence of unexpected and unpredictable behaviors and often are summed up by the phrase *more really is more*.

2.1 Simple Rules: Complex Behavior

It is a long-standing goal of science to search for the simplest rules that will completely explain a particular phenomenon. This is known *Occam's razor*, after the Franciscan friar William of Ockham (1287–1347), and sometimes is called the *law of parsimony*. The idea has a much longer history but essentially advocates starting with the fewest axioms or *a priori* assumptions. Such simplicity also fits well into the reductionistic tradition and supports the falsifiability of hypothesis development. Occam's razor is also used as a heuristic to judge two more descriptions of a phenomenon, preferring the simplest. A number of findings from complexity theory in fact give credence to the idea that there exist a small set of discoverable rules.

2.1.1 Logistic Equation of Population Growth

One of the early models of population growth was published by Pierre Francois Verhulst (1804–1849) in 1835. He hypothesized the growth of a population as being a balance between two competing forces. Ignoring death (a big assumption), there is a force that is causing an exponential growth, represented by

$$\frac{dx}{dt} = ax$$

where x is the number of that species and a is a growth rate. The solution to this differential equation is

$$x(t) = e^{at}$$

which is exponential growth, as shown in Fig. 2.1.

Acting to counter exponential growth is the availability of a limited resource. When the population is small, growth is exponential. As numbers rise, the population consumes more natural resources, and the growth slows down. Eventually the population plateaus at some value, known as the *carrying capacity*. Putting these

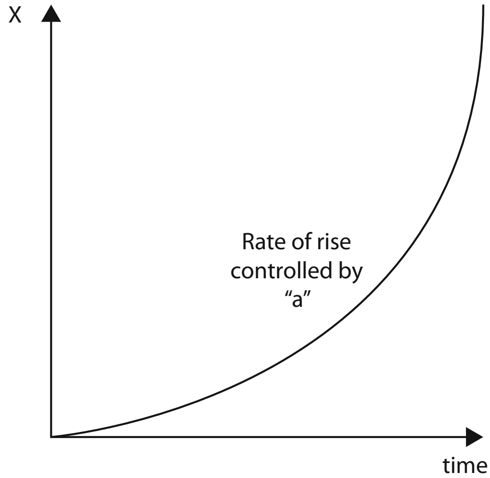


Fig. 2.1 Example of exponential growth. The parameter a controls the rate of growth such that a larger magnitude results in a faster rise

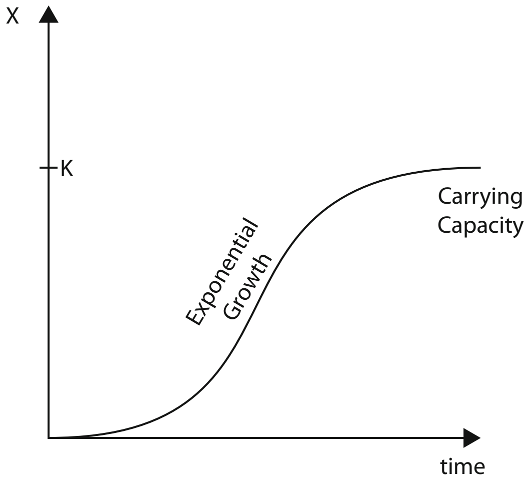


Fig. 2.2 Exponential growth limited by a carrying capacity, K. When the value of x is relatively small, growth is exponential. As x reaches the carrying capacity, growth slows down, eventually plateauing at K

two ideas together, and using K as the carrying capacity, the differential equation becomes

$$\frac{dx}{dt} = ax(1 - x/K)$$

Starting with a small initial population, the numbers will rise exponentially but then level off at the carrying capacity K, as shown in Fig. 2.2.

A similar idea was expressed by Thomas Malthus (1766–1834) in 1798 in *Essay on the Principle of Population*. Improved farming practices or farming on new land would increase the food supply, thus increasing the standard of living and growth rate. But as population growth outstripped the food supply, the standard of living would go back down, most especially for those at the bottom of the economic ladder.

The resulting famine, disease, and hardship would eventually lead to a decrease in the population. Although not framed mathematically, Malthus had understood the dynamics of resource-limited systems.

2.1.2 The Discrete Logistic Equation

A way to dissect the dynamics of a differential equation is to transform it into a discrete equation. In discrete form the behavior can be explored through a simple iteration over time. When the logistics equation above is transformed into a discrete equation, it becomes

$$x_{t+1} = rx_t(1 - x_t)$$

where r is a combination of a and K. The reason for this change is that the system is now *dimensionless*. This is a trick used by mathematicians and physicists so that they can study phenomena independent of the actual values. From a systems perspective, such a change allows for the behavior to be studied rather than being distracted by specific numbers.

To determine the behavior of the discrete logistic equation, an initial value of x is chosen, x_t. The next value of x is found by plugging into the equation to get x_{t+1}. This new value then becomes the current value. Then x_{t+1} can be reentered into the equation again to get x_{t+2}. In this way we can *iterate* on the equation to find an entire time course for x, denoted as

$$x = [x_1, x_2, x_3, \dots]$$

2.1.3 Dynamics of the Discrete Logistic Equation

We can explore the dynamics of this simple equation by varying the value of r and then, by iterating, build up the time course for x. You can explore this yourself with a calculator, pencil and paper, or a computer program. Below are some examples of the types of behavior that can come from this simple equation.

Let's pick $r = 2.2$ and start with $x_1 = 0.5$. If we continue to iterate on the logistics equation, we get the following sequence:

$$x = [0.5, 0.055, 0.5445, 0.5456, 0.5454, 0.5455, 0.5455, 0.5455, 0.5455, \dots]$$

Over time x is trending to 0.5455. There is an initial period of settling down, often called a *transient* in systems language, but eventually a static equilibrium is achieved. This is very similar to what we see in the continuous logistic equation.

If we turn up r to 3.1, and again start at $x_1 = 0.5$, we arrive at the following sequence:

$$x = [0.5, 0.775, 0.5406, 0.76, 0.55, 0.77, 0.55, 0.77, 0.55, \dots]$$

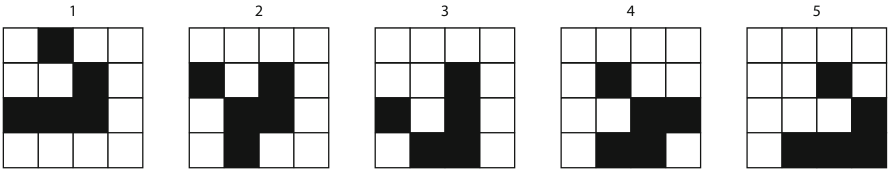


Fig. 2.3 Various behaviors of the discrete logistic equation. The left panel reaches a steady state. The middle panel shows oscillatory behavior. The right panel is statistically random

After a transient, x does something qualitatively different—it oscillates between two values, 0.55 and 0.77. This is a bit unexpected because the continuous logistic equation will never exhibit oscillations.

We can increase r to 3.6, and starting with $x_1 = 0.5$, the iterations yield

$$x = [0.5, 0.9, 0.324, 0.7885, 0.6, 0.8637, 0.4238, 0.8791, 0.3827, 0.8505, \ldots]$$

This is even more surprising. The trajectory never settles down but also does not seem to repeat itself. You may wonder if it ever settles down. As we will find in Chap. 3, it does not and acts as an infinitely long transient—the sequence of seemingly random numbers continues forever. What is more, this is the first time we have encountered a system that is governed by strict deterministic rules yet generates what appears to be random numbers. Figure 2.3 is a graphical summary of the three different behaviors of the discrete logistic equation.

The logistic equation is one of the classic examples of a very simple rule that, with a change in only one parameter, can exhibit a wide variety of behaviors. This is exactly the kind of system that complexity theory aims to understand. It also highlights a cornerstone of complexity theory—just because a system displays a wide range of behaviors (its function) does not mean it must necessarily be complicated (its structure). The opposite also turns out to be true as well—observing a simple behavior does not necessarily mean the system is not capable of generating complexity.

2.2 Cellular Automaton

Throughout the 1930s and 1940s, biology began to uncover many of the fundamental molecular mechanisms of life and reproduction. The various internal structures and functions of the cell had been cataloged. Proteins and enzymes had been isolated and experiments had narrowed down the possible mechanisms by which a molecule could pass along instructions for the self-assembly of cells and organisms. Around the same time, anthropologists were uncovering general principles of how a culture is propagated from one generation to the next. Likewise some business theorists recognized the interconnected networks of factories and the automation of the

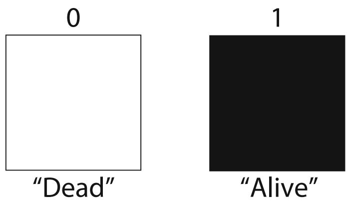


Fig. 2.4 The two most basic states of an individual cell in a cellular automaton. Cells may take on more than two states in more complex cellular automata

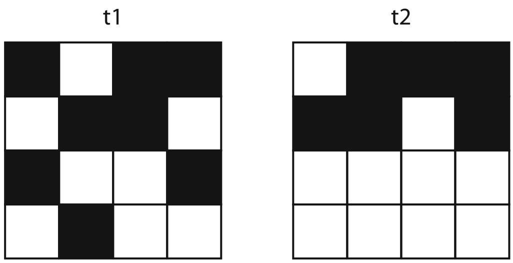


Fig. 2.5 Geometry of a two-dimensional cellular automaton at time = t_1 and time = t_2

assembly line as another form of self-replication. Theories of self-propagation were also being developed by the pioneers of the new field of automated computing.

The theoretical idea of self-replication was taken up by Stanislaw Ulam (1909–1984) and John von Neumann (1903–1957) in the form of cellular automata. In an automaton there are cells that can contain a *state*. A state in this context is simply a value that a cell contains. To simplify their argument, Ulam and von Neumann limited the state of a cell to either 1 or 0, as shown in Fig. 2.4, but allowed for this state to be able to change over time.

In space, cells can be placed next to one another. At any given snapshot in time, say t_1, the states at each cell are in some pattern of 0s and 1s. The way a cell changes state is then dependent upon the states of its neighbors. The rules of the automaton determine how the pattern of states at t_1 are transformed into a different pattern of states at t_2. Just as in the logistic equation, there are rules that can be iterated upon, transforming a previous state into the next state. The only difference is that now the rules can include what is happening in space.

To make the idea more concrete, imagine a two-dimensional grid of cells, as in Fig. 2.5, showing states t_1 and t_2. The rule that governs the change from t_1 to t_2 may be very simple. In Fig. 2.5, if 50% or less of the neighboring cells (up, down, left, right, and diagonal) are a 1, then that cell will be a 1 on the next step. If more than 50% are a 0, then that cell will be a 0 on the next step.

Of course there can be different types of neighborhoods, rules, and even spaces (e.g., one-dimensional, three-dimensional networks as explored in Chap. 5). Again the key is iteration, in both time and space, to uncover the dynamics of how patterns change over time. Below are two classic examples of the range of patterns that can be generated in relatively simple cellular automaton.

2.2.1 Conway's Game of Life

In 1970, the mathematician John Conway (1937–) created a set of rules that were published in Scientific American as part of Martin Gardner's column on mathematical games. The challenge put to the readers was to uncover the various dynamics that the rules could generate and then to report back what they found. The rules, which have become known as the *Game of Life*, are

- If you are alive and have fewer than two live neighbors, you die, due to underpopulation or loneliness
- If you are alive and have two or three live neighbors, you live.
- If you are alive and have more than three live neighbors, you die due to overpopulation
- If you are dead and have three live neighbors, you become alive due to reproduction.

What returned from the readers of Gardner's column, many of them working on graph paper, was an outpouring of different dynamical patterns. It is difficult to capture all the richness in a static book, but there are several excellent online examples that are animated if one searches under "Conway's Game of Life."

One of the most basic dynamic patterns is known as the glider gun, composed of just five alive cells. What is striking about the pattern is that over time it "crawls" diagonally downward. As the crawl occurs, there are four conformational changes, always returning back to the same shape, only shifted down and to the right. Note that, in Fig. 2.6, the first and fifth patterns are the same only shifted diagonally.

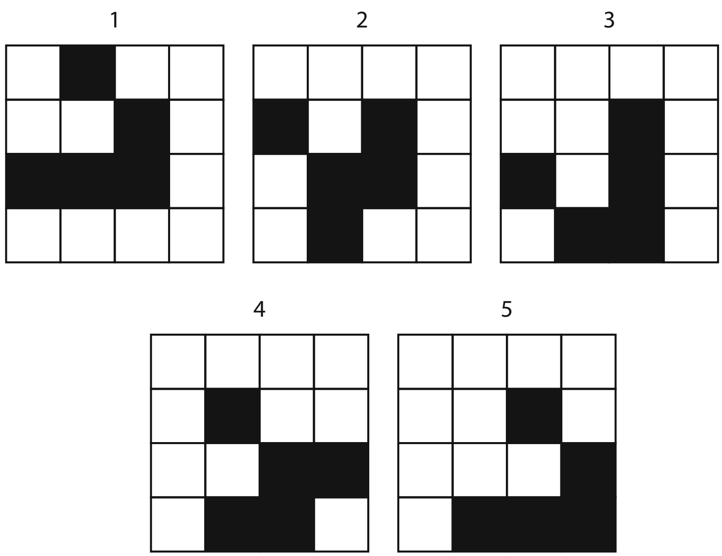


Fig. 2.6 Five-step repeating motion of a glider gun in Conway's Game of Life automaton

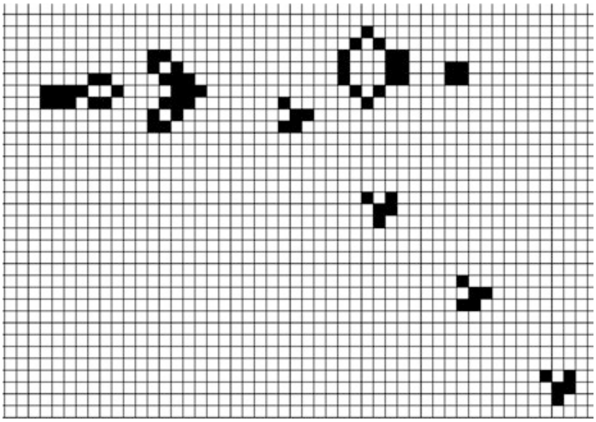


Fig. 2.7 Snapshot in time of the 30-step Gosper glider gun. Every 30 steps, one glider gun, shown in Fig. 2.6, is generated and sent diagonally downward and to the right

What is more, one of the early submissions to the challenge, by Bill Gosper, was a way to continuously create glider guns. It is now known as a Gosper glider gun and is composed of two "queen bee shuttles" that move back and forth. Each time they touch one another, a smaller glider gun is sent outward. The result is a 30-step cycle that generates a glider gun on each cycle. A snapshot of these dynamics is shown in Fig. 2.7.

Although the Gosper glider gun is not self-reproducing, it does maintain itself against many perturbations. Without going into details, it also can function as a logical NOT gate. Other types of dynamics can also be created that will form other logical gates. It was hypothesized that the rules of the Game of Life could in fact be used to create a fully functional general computer. A more formal definition of a general computer will be given in Chap. 6, but the possibility sent later researchers to explore how this might be possible. Over time it was in fact proven mathematically that the Game of Life could form a fully functional computer. What is more, in 2013 Dave Greene created the first fully self-reproducing cellular automata using the Game of Life rules, fulfilling the initial goal of Ulam and von Neumann.

2.2.2 *Wolfram's Automaton*

What happens if we slightly change the rules for the Game of Life? For example, what happens if rule four is changed such that the number of alive neighbors needed to resurrect a dead cell is increased to 4? It turns out that small changes in the rules can have a profound effect on the dynamics. Over the years, the "rule space" of cellular automaton was explored. Although the rules Conway developed are not entirely unique, they are certainly special when all possible rules are considered.

We will return to this idea in Chap. 10. The problem, however, is that, in a two-dimensional geometry, it is difficult to study all of the possible rules that could exist.

Stephen Wolfram (1959–), a mathematical prodigy who received a PhD from the California Institute of Technology at 20 and then founded the software company Mathematica, took up the challenge in the early 1980s. His simplification was to study cellular automaton on a one-dimensional line where iteration of each cell state would only depend on its current state and the current state of its two neighbors. To study how patterns evolve over time, Wolfram used a two-dimensional grid. On the top line would be the initial values (t_0) for each cell. The line underneath that line would be the next time step (t_1). And the line under that would be the time step after that (t_2) and so on. The result for one of Wolfram's rules (Rule 105) is shown in Fig. 2.8.

The rules determine whether a cell will be alive or dead on the next time step by taking into account the state of that cell and its two neighbors. Given that there are two states that a cell can be in, and three cells are part of the rule, there are $2^3 = 8$ different possible configurations of the state of three cells. The top row in Fig. 2.9 shows these eight possible states, labeled 7 through 0. The reason for labeling them in this manner is that the top row corresponds to binary numbers. In other words when all three cells are alive, it is a representation of the binary number 7. Likewise, the pattern 011 represents a binary 3. In this way, all possible current states of three neighboring cells are completely described.

The single cell below each line of three cells shows the next state of the middle cell. Wolfram kept the order of the top line the same and then explored what

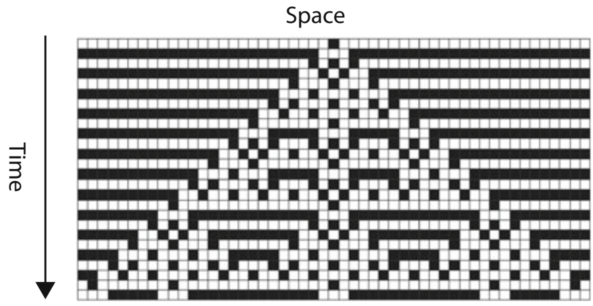


Fig. 2.8 Example of Wolfram's rule 105. The automaton geometry is a one-dimensional line across to top, with each row being the next iteration in time. The rule for the pattern is explained in Fig. 2.9

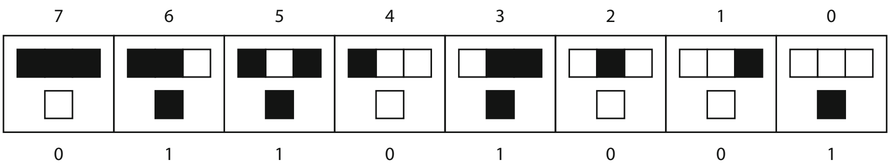


Fig. 2.9 Description of Wolfram rule 105. Each grouping of three cells in the top row represents the eight possible states for three consecutive cells. The single cell in the second row represents the state of the middle cell on the next time iteration

happened with different next states. A particular rule in his automaton is therefore the settings of each of the next steps for the eight possible current states. Because he always listed the current states (top row) in the same order (binary 7 through 0), all that needed to be coded were the next states (the bottom cells). In Fig. 2.9, the rule would be 01101001, which is a binary number 105. In this particular one-dimensional automaton, there are $2^8 = 256$ different possible rules.

In this narrowed rule space, Wolfram was able to exhaustively explore the dynamics of his simple automaton. He found different types of behavior and labeled them one (I) through four (IV). Class I rules resulted in a stable homogeneous state, generally all alive or all dead. Any randomness that might have been present in the initial conditions is quickly erased. Examples of this class are Rules 12, 21, and 96. Class II rules evolve to some sort to oscillation, perhaps alternating between alive and dead, but perhaps having oscillations of 5 or 100 cycles. In general, local patterns tended to stay local. Examples of this class are Rule 15.

Class III rules were chaotic. Stable structures would appear and disappear as they interact with other nearby patterns. The result was that an initial pattern might spread out to the entire structure. An example is shown in Fig. 2.10 of Rule 90, which generates a well-known mathematical structure known as the Sierpinski Gasket. It was first described mathematically in 1915 but has appeared in art as early as the thirteenth century. One reminder is that the iterations are in both time and space. To determine how a particular cell evolves in time is simply reading down a column.

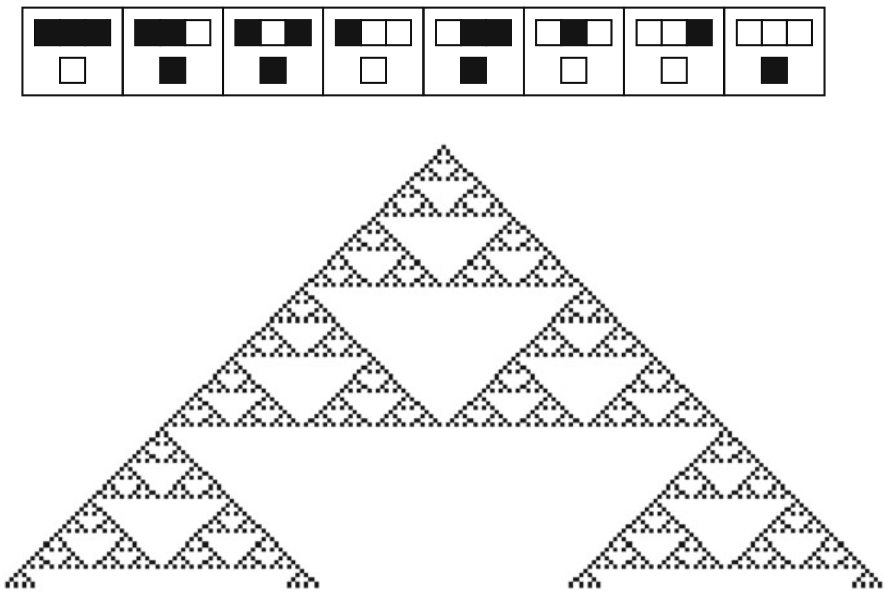


Fig. 2.10 Wolfram Rule 90 (top) generates the Sierpinski gasket (bottom), a well-known fractal

In general, local patterns do not stay local. For this reason, perturbations (e.g., changing one cell from alive to dead) can have a large impact on future iterations.

Many of the rules in Class III form fractals, as will be discussed in Chap. 4. What is special about Class III is that in general each cell undergoes a pseudorandom progression. It is called "pseudo" for a particular reason—it is not truly a random process because it is deterministic (e.g., governed by rules), but the statistics are the same as a random process. This is the second time that we have encountered a system that appears to be random yet is generated by rigid rules.

Class IV rules produce an even more surprising result, especially coming from such simple rules. In Class IV rules, all of the previous three classes of behavior can coexist. This means that one place in space might become very stable, while another part next to it may undergo some cyclic process, but nearby might be chaotic behavior. Over a long period of time, these regions can invade one another—a chaotic area may overtake an oscillator or vice versa. A nice example is Rule 30, shown in Fig. 2.11. Wolfram has conjectured that any Class IV rule can in principle form a general computer. It has already been proven that rule 110 can be used to form a computer, but a universal proof has not yet appeared.

As in the Game of Life, a slight change in the rules can lead to very different behavior. Due to the numbering scheme, similar rules are near one another. For example, there is only a small difference between Rule 29 and Rule 30. Yet, Rule 30 is of Class IV, while Rule 29 is of Class II. Furthermore there does not appear, at least on the surface, to be any order or pattern to how rules are distributed. For example, it is not as if we see Class IV rules surrounded by Class II rules.

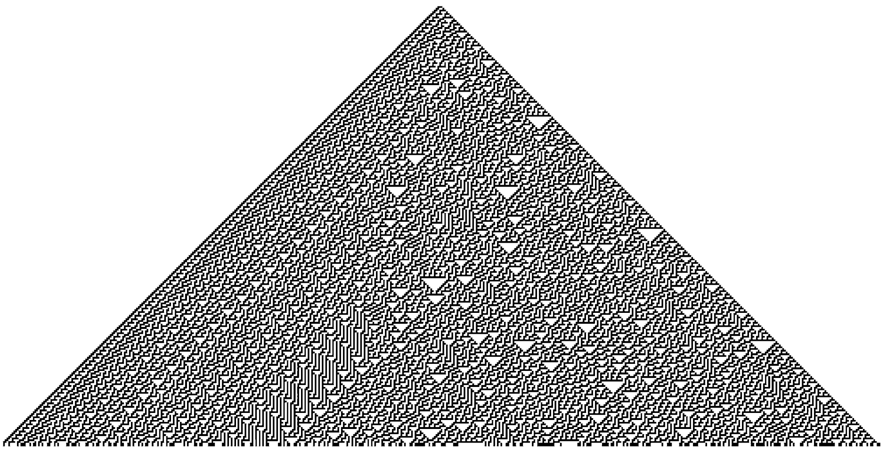


Fig. 2.11 Wolfram Rule 30 as an example of a Class IV rule that displays a mix of steady-state, oscillatory, and chaotic behaviors

2.2.3 The Scientific Process and Abduction

The scientific process is a way to learn about the world around us. It is generally taught as an iterative process that alternates between induction and deduction. Induction starts with many observed examples and then derives general theory. This is sometimes called bottom-up logic. Deduction starts with a general theory and then moves toward specifics, sometimes called top-down logic. The original version of the scientific method viewed science as progressing from induction to deduction. Many observations are made, from which a general theory is developed (induction). But then the general theory can be applied to explain and predict specific examples (deduction). Most of us are taught in school that science is a combination of induction and deduction.

Charles Sanders Peirce (1839–1914) was an American philosopher who created a new term that he felt better characterized how the scientific process works in practice. *Abduction* is a form of guessing about hypotheses that Peirce claimed was used early in the scientific process. The logical flow of abduction is:

The surprising fact C is observed
But if A were true, C would be a matter of course
Hence, there is reason to *suspect* that A is true.

Based upon an observation, C, we make a hypothesis about the cause, A, that is very likely. Rather than strict logic, abduction admits that A might not be the only cause and that more observations are needed. Until new observations are made, however, A will be assumed to be the logical cause of C. C.S. Peirce's claim is that this is the true nature of how new scientific hypotheses are created from observations. Deduction and induction only come into play later in the scientific process. Abduction is a sort of intuitive and probabilistic logic that is always running in the background, connecting deduction and induction.

An illustration of abduction is to start with the observation that the grass is wet in the morning (C above). The most probable logical reason for this observation is that it rained the night before (A above). This hypothesis is a guess because C may have been caused by other events (e.g., a sprinkler system, a flooded basement). The simple rules discussed in this chapter were likely derived through abduction—rules were guessed based upon observation, and then the consequences were explored to determine whether or not they agreed with the observations.

Abduction may also be related to partial causality. A might be necessary but not sufficient for C to be true. For example, there might be an unseen player, D, that works together with A to lead to C. No matter how hard we try, the partial cause, D, might be hidden from us. The only way for D to expose itself is when we have a surprising result that does not fit with our current hypothesis. This framing fits very well with the idea of emergence and the relationship between structure and function. Structure may constrain function, but it is not in and of itself a cause. Likewise, the observation of a complex phenomenon likely means that some form of emergence has occurred. But that may not be the only explanation.

2.3 Many Players

The takeaway from cellular automata is that simple rules can generate complex dynamic patterns in both time and space. A generalization is the study of *agent-based systems*. In these systems, the rules are embedded within an individual agent, and those rules tell that agent how to interact with other agents. In a cellular automaton, the neighbors are fixed. In agent-based models, however, the rules may dictate how an agent can move to a new neighborhood, thereby having new neighbors with which to interact.

A simple game can illustrate the idea of agent-based models. Imagine that a group of people are walking around a space randomly. They are then told that they are to keep track of where someone else is in the space. Next they are asked to keep track of where a second person is in the space. On a signal they are then told to make an equilateral triangle with these two people. The result is a sort of dynamic repositioning, as everyone is reacting to the movements of their two selected people. As in most agent-based systems, there are many agents all following the same (or very similar) rules. If one were to study the agents in isolation, they would appear to be very simplistic. This is in fact the case for quarks, atoms, ants, neurons, and, in some contexts, people. Of course it could be argued that an ant or a neuron or a person is far from simple. What is meant here is the *relative* simplicity of the agent when compared to the emergent patterns of many interacting agents.

In general agents have internal rules that govern both their internal states and also how they interact with other agents. This provides a very simple reason why a purely reductionistic approach can never completely describe a complex system. If one were to study an individual agent, changes in the internal states could be uncovered, but the rules governing interactions between agents would be missed. For example, studying a neuron in isolation will yield interesting information, but it will not expose any rules about how neuroelectrical impulses can propagate through a network of neurons. Likewise, if we study one H_2O molecule, the world of ionic bonds can be discovered. With three or four H_2O molecules, a new kind of bond is possible, namely, van der Pol bonds. These bonds exist in principle but only reveal themselves when several water molecules are near one another. Scaled up to millions or billions of water molecules, the dynamics of fluid flow can be discovered.

2.3.1 Mobs

The formation of mobs, also known as the "bandwagon effect," is a social form of emergence. Imagine a group of 100 people. Each of them has a personal threshold beyond which they will join the mob. The meaning of this threshold is how many other people will need to be part of the mob before that person will join. If someone has a threshold of 15, that means that once 15 people are part of the mob, they will join. With these rules, we can explore how a mob might get started.

If everyone has the same threshold, say 10, the mob will not get started. Alternatively, imagine a diversity of thresholds such that some people are ready to join (low threshold), while others will take a lot of convincing (high threshold). If, perhaps by chance, there are a few people with a low threshold, they can create a runaway process whereby more and more people will join the mob. If the distribution of thresholds is right, it could be that everyone joins the mob. Although this example is for mobs, it could apply to all sorts of ideas (sometimes called *crowdthink*), from fashions and fads to cultural practices and norms. The idea of mob formation is also an example of *sensitivity to initial conditions*, which will be explored more deeply in Chap. 3—in a large population, a few people can trigger the domino effect that becomes a mob.

2.3.2 Flocking and Schooling

In 1986 Craig Reynolds (1953–) created an agent-based model that simulated how flocking might arise in a population of birds. He called his agents Boids, each of which moved according to three basic rules:

Separation: Birds keep some distance from other birds.
Alignment: Birds move toward the average heading of the neighbors that they can see around them.
Cohesion: Birds move toward the average center of mass of the birds around them.

It is worth noting that embedded within the rules are counterbalancing forces. For example, Cohesion will tend to move birds together, but Separation keeps them from collapsing into one lump. This is similar to other agent-based models where rules that expand or add are counterbalanced by rules that contract or subtract. The balancing of rules will be explored in greater detail in Chap. 10.

When Reynolds ran his simulations with an initial random distribution, after a transient, the agents would settle into a dynamic that looked remarkably like flocking birds. The well-known "V" profile as shown in Fig. 2.12. What is mesmerizing is that they keep a semi-stable pattern, never truly repeating the same exact pathway. The "V" pattern does not appear anywhere in the rules; it is an emergent phenomenon.

Flocking behavior turns out to also be very robust. After Reynolds, others showed that the rules also naturally allow the flock to avoid obstacles, splitting up to go around obstacles but then rejoining again on the other side. In this way a group of Boids can explore new environments. Others have shown that the Boids rules can even evade predators in much the same way that a school of fish will scatter when attacked but then rejoin afterward. The US military has in fact funded the creation of small flying robots, programmed with a version of the Boids rules to scout out areas. The key is that coordination can occur without the need for any sort of global

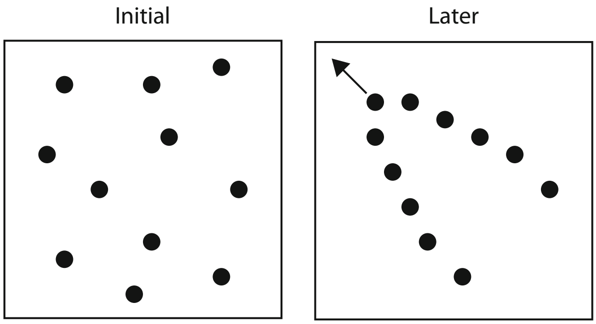


Fig. 2.12 The Boids model of flocking. The left panel shows an initial random distribution of simulated birds. The right panel shows a later time when the flock has formed a "V"

transmission signal, making the flock autonomous and difficult to trace back to a source. Likewise, the destruction of one or more Boids will not destroy the overall pattern of exploration.

The Boids model is a simple example of *artificial life*, simulating lifelike properties on a computer. In fact, the idea of Boids has been used in video games and movies. A company, MASSIVE, has created a software platform using agent-based models that have simulated the bats in the Batman movies and the Uruk-hai army in the Lord of the Rings.

2.3.3 Segregation Model

The goal of many agent-based models is to demonstrate how a more complex behavior might arise from simple rules, distilling a phenomenon down to its essence. As such simple models can serve as rich arenas for scientific study but also can be powerful as educational tools as well. One of the earliest agent-based models that served both purposes was created by the Nobel Prize winning economist Thomas Schelling (1921–2016) to describe the phenomenon of segregation. In his original version, he used pennies that can be head up or tail down. The pennies are placed randomly on a chessboard, but any 2D grid will work. It is important that not all positions are filled with pennies as in Fig. 2.13. A penny can be "happy" with the neighborhood or not. In general a penny is happy when it is surrounded by some percentage of like-state pennies. For example, if the percentage is 40%, that means a penny will be happy as long as 40% or more of the neighbors are in the same state. When a penny is happy, it stays where it is. When a penny is not happy, it moves to some vacant spot on the board at random. After iterating on this simple model, the heads and tails largely segregate into pockets, after which they are all (or mostly all) happy and no more changes take place. Schelling's model demonstrates how simple rules can lead to segregation.

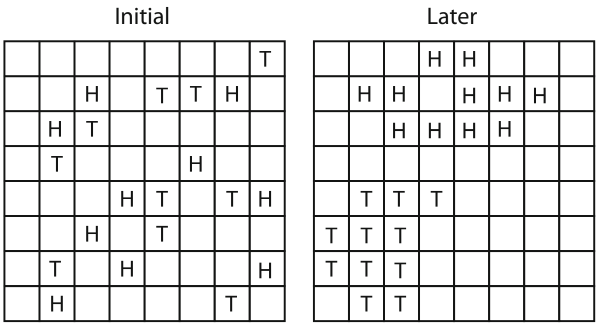


Fig. 2.13 Evolution of the Schelling segregation model. The left panel shows an initial random distribution of heads and tails. Sometime later, the heads and tails have segregated

A similar agent-based model created by Guillaume Deffuant in 2006 shows how extreme political positions can emerge. An individual is endowed with two properties, an initial thought on their political position (their position on the line) and an uncertainty (given by error bars). Two individuals are chosen at random to simulate a chance encounter. When these two individuals meet, they will check to see if their uncertainties overlap. If they do not, they simply go on their way and forget the interaction. If their uncertainties do overlap, they will exchange information through a conversation. At the end of the interaction, they will move closer together. But the distance they each move is proportional to their uncertainty. So, a person who is very certain will not move much, whereas a person who is very uncertain could move a great distance.

When these rules are iterated upon, a polarization occurs. What is striking, and perhaps disturbing, is that those with the narrowest uncertainty (or tolerance for the views of others) will capture others and draw them in more closely. They become attractors of those who are within range. What is more, they recruit others who have wider tolerances who can bring in even more people, much like the mob model above. Over time the population will split into factions, centered around those who are most certain of their positions.

To add to the segregation model, we can superimpose the *proximity principle*, first articulated in the 1960s by Theodore Newcomb (1903–1984). The general idea is that there is a tendency to form interpersonal relationships with those who are geographically close by. The combination of the proximity principle and the segregation model can lead to like-minded groups to clump together.

Many more examples can be found in *Agent-Based Models* by Nigel Gilbert. As the text is part of a series in the social sciences, most of the examples lean in the direction of human interactions. The book does, however, review how one builds an agent-based model. More information can also be found at the NetLogo website which has a free agent-based software along with many pre-programmed models.

2.3.4 Video Games and Leaking Buckets

In older computer games, humans would play against very simplistic agents that would only be able to attack and fight to the death. Over time the agents inside of computer games have become much more sophisticated. Complex agents were designed to have multiple actions, react to a wide range of situations in a contextual way, and in some cases learn from a past history. In many ways they contain within them a range of functions that they can express to achieve much more complex goals. In this section we will explore how these types of agents can be designed.

Agent design typically begins by identifying a series of goals that the agent is trying to achieve along with associated actions that may move in the direction of those goals. Because there are multiple goals, the drive to express a particular action may be in conflict with the drive to express another action. This necessitates some higher-level decision-making strategies, often based upon external context, to determine which action to enact. It is important to note that no one goal (or action) will necessarily have a higher priority than another goal. Such internal actions and decision-making strategies endow an agent with contextual flexibility.

Imagine an agent within a video game that has three buckets, representing three general goals: Survive, Resources, and Win. We can imagine that each of these three buckets can be filled anywhere between 0 and 100% at any given point in time. The agent will also have three actions it can take: flee (corresponding to the goal to survive), explore (corresponding to the goal to gather resources), and attack (corresponding to the goal to win). Each agent might also have particular internal parameters, such as health, amount of ammunition, kinds of weapons, and so on.

The way in which the buckets rise or fall depends upon actions taken as well as external events and context. Below is an example of the rules that might be contained within an agent.

Event	Bucket change
Seen enemy	+5% Survive, +10% Win
Low ammunition	+20% Resource
Low health	+20% Survive, +10% Resource
High ammunition and health	+50% Win, −20% Survive, −20% Resource
Lost 25% health in one hit	+50% Survive, +20% Resource, −50% Win
Been a long time since seeing an enemy	+50% Win
Gathering of more than 10 other agents	+80% Win

The second to the last rule is an example of how a bucket by default may fill (or empty) over time when no events happen. The implication of this specific rule is that an agent will become bored and will actively look for a fight. The last rule hints at ways in which agents might begin to collaborate with one another. There could be many more possible rules that could be added that might enable multifunctional teams of agents to form.

The exact numbers in the table above can also open up the possibility of individual agents having a variety of "personalities." Below are three personalities and the reaction to having seen an enemy (Rule 1 above).

Personality	Bucket change
Normal	+5% Survive, +10% Attack
Coward	+20% Survive, +5% Attack
Berserk	+80% Attack

The "personality" table above also opens up the possibility that an agent might change over time based upon experience. For example, if an agent with a Normal personality has recently been in many fights, that agent might over time have the percentage for attack rise, meaning they have become closer to the Berserk personality. Alternately an agent that begins with the Berserk personality may become tired of fighting. Agents might also die quickly or survive for a long time. Those that survive might gain "experience points" that might be included in the rules as well.

The idea of having a diversity of agents is also the premise behind a number of sport-inspired games that contain the likenesses of real sport figures. With some finite number of parameters, a game could mimic the range of abilities and playing styles of Michael Jordan, Dwyane Wade, Stephen Curry, Wilt Chamberlain, or Allen Iverson.

The decision to execute an action can be simply based upon which bucket is most filled. Of course some additional strategy would need to handle the case of two buckets having the same level. Decision-making rules can, however, become far more complex, adaptive, and based upon experience. Such agents may begin to resemble human players. Some thinkers even go so far as to claim that this same line of reasoning (e.g., buckets, actions, events, and personalities) explains how humans navigate the world, even though we may not be consciously aware of this process. We will explore how strategies can guide decisions in Chap. 8.

2.4 Implications of Simple Rules and Emergence

The idea that simple rules can generate complex behavior when repeated in time or when many agents follow similar rules strengthens the claims made in Chap. 1 about emergence and the ways in which complex systems can be studied. In this section we will explore more deeply the nature of emergence, effectual thinking, and the viewpoints from which complex systems might be studied.

2.4.1 Logical Types and Epiphenomenon

The emergence of new behaviors and phenomena that arise when agents interact has a convoluted history that weaves throughout many fields. From a philosophical perspective, the idea of *logical types* was introduced by the philosopher Bertrand Russell (1872–1970). Russell distinguished between two systems being different in *degree* or in *type*. Systems that are different in degree can be compared. Mathematically, if x and y are numbers, and $x \neq y$, then either $x > y$ or $y > x$. Russell extended this idea to all logical operations. On the other hand, when two systems are different in type, they cannot be compared logically or mathematically. If $x = 5$, and y is a cheeseburger, logically there are no operations to compare them. They are of different types. We might compare two dogs, or cars or people, but it does not make sense to compare a dog to the Constitution of the United States. While that may be true on the surface, there are still some dimensions that can be compared—we can compare the age of a particular dog to the age of the Constitution, thereby allowing some simple logical operations even between systems of different types.

Logical types can be helpful in detecting when a new phenomenon has emerged. For example, a water molecule is of a different logical type than a snowflake. Likewise a car is not the same as a traffic jam, an ant is not an anthill, an apple is of a different type than an apple tree, a CEO is not the company, the musical note B is not the same as Bach's B minor mass, and a neuron is fundamentally different than the neuroelectric flows that form a thought.

A closely related idea is the concept of an *epiphenomena*. In the context of complex systems, an epiphenomenon is essentially the same as emergent behavior. The fluid properties of water would be called an epiphenomenon of the properties of water molecules. In some arenas, however, the word epiphenomena is used disparagingly. For example, consciousness would be described as "simply" an epiphenomenon of the interactions of neurons. There is no mystery; it is simply what happens when you put a bunch of neurons together in the right way. Gregory Bateson explains in *Mind and Nature* how combining together two or more members of a particular logical type can lead to a new logical type—an example being how two images can be combined to generate depth perception. In reality, epiphenomena, emergence, and logical types are all related to one another, but, depending upon the field, they are used in slightly different ways.

2.4.2 Effectual Thinking

Deeply embedded in the traditional way of doing and thinking about science is the idea of causality. Time is presumed to flow in one direction, an idea we will explore more in Chap. 7, and therefore the inputs of a system necessarily must come before the outputs appear. Once we have a prescribed order of events in time, we can begin to make statements such as "this input *caused* that output." There is evidence of this kind of thinking almost everywhere in our society, from malpractice suits and playground disputes to our how children are educated to the ordering of morning routines.

The implication that we can separate causes (input) and effects (output) means that we can, at least in principle, predict ahead of time what the effect will be for a given action. Much of the scientific method relies on this fact, as summed up in the idea of repeatable experiments. There are phenomena in the world that are strongly repeatable, such as when we drop an object and gravity pulls it toward the ground. Other phenomena are more probabilistic, such as exactly what date a particular leaf will turn from green to yellow during the fall. In principle we can make hypotheses about what, when, how, and where an action will have an effect. The causal way of thinking also leads to Karl Popper's (1902–1994) claim that real science is driven by hypotheses that can be refuted by real data.

The study of complex system admits that there is strict causality but that, when new structures or behaviors come into being, it may become difficult to make simplistic causal statements. An alternative to causal thinking is *effectual thinking*. The idea of effectuation acknowledges that, as time moves forward, new possibilities emerge that did not exist before. New products and behaviors of a system arise from the recombination, reinterpretation, or modification of existing processes. New processes may also be generated by a reordering of how processes are turned on and off, whether they compete or cooperate, and how they interact with one another and the environment. These are emergent systems that unfold over time. As the system unfolds, causes and effects become entangled, making it impossible to predict what will happen next. Effectual thinking also admits that there can be agents in the system that might make their own decisions (a weak version of free will) that can impact everything around them.

The power of effectual thinking reveals itself when working with systems that are constantly changing. Because we experience time as moving in one direction, these systems cannot be studied in the usual scientific way of repeating experiments over and over again. After the event occurs, it is too late to go back and study a onetime phenomenon in detail. One cannot rewind the tape of one's life or the history of ancient Egypt and rerun it. The assumption is that your life would turn out differently in a rerun, because so many events in your life were contextual and due to the timing and magnitude of events not under your control. Evolutionary biologists and politicians often make similar claims.

To overcome this limitation, many who study emergent systems resort to the rhetorical style of knowing characterized by "just so" (used often as a derogatory term) stories. They are narratives that are not necessarily true but are a plausible way that a particular system may have unfolded. Historians fight with one another over their stories, as do evolutionary biologists. For example, one way to think about the origin of humans is that we rose upright, which caused a lengthening of the vocal track, that enabled speech, that enabled communication and the binding together of groups which led to social-political-economic systems that then caused the rapid increase in the size of the brain. We could just as easily argue that simple toolmaking required higher-order grammars that induced increases in brain size that then allowed for socio-political-economic systems, which enabled those who could walk upright an advantage in communication, because of the extended vocal track. In both stories there is a plausible causal story that connects events. The effectual way of thinking is to admit that it likely did happen one way but that it was not the only way that it could have happened, explaining why in retrospect multiple story lines could have led to the same (or similar) outcome. To add in a perspective of complex systems theory, it is also possible that many of these events happened in parallel, driving one another in a sort of push and pull. Just so stories are used to great effect by economists, political pundits, evolutionary biologists, psychologists, historians, and many others.

The difference between causal and effectual thinking can be summed up in a quote from Fritjof Capra's *The Systems View of Life*, "In the deterministic world of Newton, there is no history and no creativity. In the living work of self-organization and emergent structures, history plays an important role, the future is uncertain, and this uncertainty is at the heart of creativity." In general, causal thinking works well for systems that are not changing over time. Effectual thinking is better suited to those systems that are growing, developing, dying, and recovering. The study of complex systems, however, requires both points of view to be adopted.

2.4.3 Points of View

The various simple rules and emergent behaviors explored in this chapter also open up the possibility that fundamentally different viewpoints can be adopted. The predominant perspective of science is to view the system being studied from the outside and to remain *objective*. Because multiple observers can all view this same system, this view is called the *third-person perspective*. Scientists are even taught to write in the passive third-person voice. It is a goal of science, as well as other fields, to agree upon this viewpoint, making independent observations that agree with one another. As such, scientists aim to place themselves outside of the system being studied.

The predominate perspective of the humanities is the view from inside the system as a *first-person perspective*. In the gestalt tradition, this is known as an umwelt and roughly translates into an organism's perspective of their environment. Of course a *subjective* first-person perspective can go beyond the human-centric perspective. For example, one could ask what it is like to be a car, or your mother, the economy of Thailand, or a bat. In fact, this is what the philosopher Thomas Nagel (1937–) discussed in a now famous article *What Is it Like to Be a Bat?*. It is easy as Westerners to dismiss these kinds of thoughts—it isn't *like* anything to be an economy or a car. One of the most speculative ideas that will come out in this text is the possibility that complex systems may be able to bridge the first- and third-person perspectives.

2.5 Questions

- A favorite thought experiment of philosophers of mind is to imagine a brain in a vat of sustaining liquid. For example, in Daniel Dennett's essay "Where am I," he tells the story of his own brain being entirely disconnected from the outside world but still able to have thoughts. Do you think such a brain is of the same logical type as a fully embodied human brain (e.g., a brain connected to a body)? Neuroscientists are beginning to grow three-dimensional networks of neurons. Do you think these tissue-engineered neural networks are of the same type as a human brain?
- Autonomous drones are approaching the size of insects such that, if one were in your room right now, you would likely not even notice it. How might a swarm of drone insects be used? What virtuous things might come from such technology? What harm could result?
- Most scientific disciplines adopt what is known as the *Olympian stance*, meaning that the researcher imagines themselves as separate and, often above, the object of their study. This is a variation of the third-person perspective—anyone who stands where the researcher stands would make the same observations and come to the same conclusions. A few scientists, however, have rebelled and claimed that real scientists cannot truly adopt a pure third-person perspective. Scientists they claim are an active part of the research process, and their observations, methods, analyses, and subjective interpretation will always introduce a first-person perspective. The result is that science is a combination of a first- and third-person perspective. Are there other fields that also adopt an Olympian stance? Do you think they have more or less of a claim to this stance than the scientific disciplines?
- In the age of autonomous vehicles, how might simple agent-based rules be used to create efficient traffic flow through a city? Obviously there are many technical challenges. But what might some of the simple rules be that would enable safe and efficient travel?
- One of the impacts of the digital age has been to alter the proximity principle to go beyond geographic location. Faster communication between people has enabled non-geographic clumping and segregation to occur much more easily. How do you think the ability to interact with like-minded people online has changed politics, business, and friendships?
- Ants communicate using a relatively simple set of chemical messages that they can exchanged directly with another ant or indirectly by laying chemical along the path of other ants. The rules by which an individual ant sends and receives these messages are well mapped out and turn out to be fairly simple. Despite the simplicity of the rules, groups of ants can self-organize into teams, perform a job, and later disband. These same ants may then go off to join other teams. Within the colony, the jobs that are performed include cleaning tunnels, harvesting food, defending the hill, removing waste, and burying dead ants. There are in fact some researchers who have created agent-based ant models to study how these

functions emerge from ant interactions. How might the complex functionality of a company be like an ant colony? What is the equivalent of the chemical messages? Are there perhaps some simple "work rules" that might guide interactions? What higher-level functions do people band together in a company to accomplish?

- Can you give an example from your life where you have used abduction to come to a conclusion? Did you ever find out if your assumptions were right or not? Are there entire fields or disciplines that use abduction as a way to move forward?
- When was the last time you were sick? Do you know how you got sick? Do you know if it was bacterial or viral? How did your body adapt as a complex system?
- Cities are known to grow, usually from some central founding square or resource, building both outward and upward. Over time, however, forces act to cause certain areas to decay. In some cases, these decayed areas might come back in a new way. The classic game SimCity, created by Will Wright (1960–), allows a player to construct such a city from scratch. But underlying the game are some rules that govern how cities evolve over time. What kinds of simple rules might capture some aspect of these dynamics? What elements might be important in such rules?
- The adaptive immune system, which we will discuss further in Chap. 8, must be able to generate a way to create antigens that are a unique match to an invader. There are far too many possible shapes to code for every possible antigen. Instead the DNA codes for proteins that can take on a small set of basic shapes. Out of these shapes there are theoretically 10^14 possible combinations for B-cell receptors and 10^18 for T-cells. This is more than the number of neurons in the brain! And it is clearly more than the DNA could ever express directly. During the days that the adaptive immune system is mounting a response, various combinations are being tested. A similar type of combinatorial power comes from 7 to 8 simple glyphs (dots, lines, and curves) that can be used to form the entire Western alphabet. From that alphabet, a nearly infinite number of words can be generated. Choose a non-biological example where a system has a heterogeneous alphabet that is recombined to generate a massive number of possibilities. Why do you think this strategy has been taken by that particular system?
- Carrying capacity, as a limit to growth, follows from the limited supply of some resource. What are your limited resources? In what ways do they create limits to your multidimensional growth?
- The triangle game was introduced as a simple agent-based game that can be played with a group of people. When playing the game, it is often the case that the group will settle into an equilibrium where no one is moving anymore. Theoretically this should not happen, except in very specific initial conditions. The reason it happens is that each individual has an error in their perception of an equilateral triangle. Most people will get close to a solution and assume it is good enough. This perception error in fact stabilizes the system and helps it find a "solution." A similar concept is found in *stochastic resonance* where white noise is added to a signal. If the signal is relatively weak, the frequencies of the noise will selectively resonate with the signal, helping to amplify it relative to the noise. What are other examples where noise or errors can actually help a system find a direction or solution?

- The Game of Life, also more simply called Life, is known in pop culture as a board game created in 1860 by Milton Bradley. It was the first popular board game in the United States and launched the Milton Bradley Company that makes many other games. There are several modern versions of the game, but they all center around taking a journey through life, considering health, family, housing, finances, and career. In Bradley's version, the winner was the player with the most money (considering stocks, bank accounts, debts, and other elements of the game). As such, players will often play using particular strategies to maximize their potential for winning. What other metrics could be used at the end of the game to determine winning? How would it change the strategies and play of the game? Are there other games that you could imagine hacking by changing what it means to win? How might it change the strategies used by players?
- A variation of the triangle game endows each individual with one of two rules. For the first rule, the individual must position themselves exactly between their two randomly chosen partners. If everyone adopts this rule, the group will immediately clump together. For the second rule, the individual must position themselves so that one of their partners is exactly in the middle (i.e., they would move to be on an end). If everyone adopts this second rule, the group will disperse. Something interesting happens however, if we change the percentages of individuals following the first and second rules. Experimentally there appears to be a range over which the forces keeping the group together (from rule 1) and forces sending the group apart (from rule 2) result in a system which stays contained within a defined space but never settles to into a repeating pattern. This is similar to the triangle game but now with a diversity of rules. What other systems seem to have this sort of balance between agents following different rules? Can you identify how the diversity in rules is creating the dynamic functional balance in the system?
- Anthropologists have found that most cultures have *liminal* spaces and times, when the usual social norms are lifted or more fluid. They can be times of creativity, reinvention, and experimentation. They also play prominent roles in the hero's journey (worth looking up if you are unfamiliar with this term) and shamanistic rituals. New Year's celebrations, Mardi Gras, Carnivale, and other celebrations are some Western examples. What is a local or family liminal space or time that you have encountered? Explain how the rules are different in this space or during this time.
- A nice way to sum up the difference between causal and effectual thinking is how one goes about cooking a meal. The causal cook will read a recipe, shop for the ingredients, and then execute the procedures in the recipe. The effectual cook makes a meal from what is available, often inventing processes and adapting as they go. Give an example of when you have applied causal thinking and another when you have applied effectual thinking. Compare the two approaches and which felt more natural to you.

2.6 Resources and Further Reading

The idea of simple rules leading to complex behavior is a cornerstone of complex systems thinking. For that reason, almost all books on complex systems, including the general introductions mentioned in Chap. 1, will contain significant discussion of simple rules. Stephen Wolfram's *A New Kind of Science* covers a great deal of ground but through the lens of cellular automata. Nigel Gilbert's *Agent-Based Models* is an excellent short introduction to agents and models. The logistic equation is expertly covered in James Gleick's *Chaos*. A wonderful computational resource that is worth exploring is the open-source NetLogo simulation system that can be found online. NetLogo is designed for programming agent-based models, and the free download contains hundreds of pre-programmed models. Nearly all of the models mentioned in this chapter can be explored in more detail within NetLogo.

Chapter 3
Non-linear Systems

Citations have become an important element of most academic disciplines. When a work is published, the author acknowledges the sources and inspirations of past works through some form of citation. A citation system allows for ideas to be traced backward in time to their origins. In this way, ideas are modular—a new idea is a recombination of previous ideas, often accompanied by some new insight. The form of acknowledgment varies by field, but in most cases, there are social norms or strict rules on when a citation is necessary. Such is the case in science, history, patents, journalism, philosophy, law, and many other fields.

Some new ideas become a basic building block of a field. When that occurs, the work is cited many times by future works in a virtuous feedback cycle. Of course the opposite occurs too—many ideas never catch on, are not cited, and are relegated to obscurity. The number of citations to a given work is therefore a measure of the relative importance of that idea as judged by the field. A similar kind of scoring system exists for individuals within a field as well. We can total up the number of citations an author has accumulated for all of their publications. Some individuals have one seminal work that is cited many times but few other publications. Other individuals may have published a great deal, but their works are not well cited. A middle ground is found in what has become known as the h-index, suggested in 2005 by the physicist Jorge Hirsch (1953–). To illustrate the idea, imagine that an author has 35 total papers. Four of those papers have been cited four or more times. Despite this particular author having 35 papers, their h-index would be considered four. The only way they could achieve an h-index of five would be if five of their papers were cited five or more times. There are a select number of authors who have an h-index above 100, meaning that they have 100 papers that have been cited 100 or more times. A similar idea can be applied to other levels of granularity, for example, the h-index of an entire department, division, or university.

The h-index is an example of a non-linear system. Consider how an author increases their h-index. Each citation gained early in one's career is a milestone as it can quickly increase the h-index. For example, moving from a 2 to a 3 may

J. Tranquillo, *An Introduction to Complex Systems*,
https://doi.org/10.1007/978-3-030-02589-2_3

involve only a few additional citations. As the h-index grows, however, it becomes more and more difficult to increment. An author with an h-index of 50 will need many more citations to reach 51. The index is therefore *non-linear*, meaning that it does not scale—a doubling of the number of total citations does not double one's h-index.

In this chapter, we will explore non-linear systems through simple mathematical models, encoded in differential equations. In most courses on differential equations, the intent is to learn a variety of techniques for "solving" the equations. Here we are more interested in graphical techniques for taking apart non-linear differential equations that will enable us to understand the underlying system dynamics.

3.1 Differential Equations

Mathematics provides a powerful set of tools for modeling real-world systems. The ability to capture the essence of a system, whether it is functional, structural, or both, allows for a wide range of theoretical experiments to be conducted that may be impossible, or very costly, to carry out in the real world. The dilemma is that all models are wrong in some way, meaning that they are necessarily a simplification of the real world. Skilled modelers know how to make simplifications that enable a phenomenon to be studied without superfluous parts, variables, interactions, or outside influences. Model building is more art than science.

One of the more important tools of mathematical modeling is the differential equation. Originally developed by Isaac Newton (1643–1727) to describe how bodies interact with one another, differential equations can be applied to any system that changes, from cells crawling along a membrane, to money flowing through an economic system, to the neural firing that give rise to mental functions.

One of the most common ways to write down a quantitative rule is with a differential equation such as

$$\frac{dx}{dt} = f(x)$$

The term $\frac{dx}{dt}$ stands for how x changes over time. $f(x)$ is the rule by which that change takes place. Many systems thinkers reframe this as x being a *stock* of some quantity (like the amount of water in a reservoir) and $f(x)$ describing the *flows* that will change the amount of water. There are entire texts and college courses dedicated to tricks and techniques for solving these equations. In this context a "solution" means finding how x changes over time, or $x(t) = \ldots$, sometimes called a *signal* or *trajectory*.

In the world of system dynamics, a *state* is a snapshot of the system variables at a particular instant. In the example above, we would say that the snapshot of the system at some time (t_1) would be the value of x at that time (x_1). The rules, $f(x)$, then describe how to get from x_1 to x_2.

In some models, there are several interdependent, or coupled, differential equations:

$$\frac{dx}{dt} = f(x, y, z)$$
$$\frac{dy}{dt} = g(x, y, z)$$
$$\frac{dz}{dt} = h(x, y, c)$$

In this system, there are three variables that together form the snapshot of the system (e.g., x_1, y_1, and z_1 at t_1). The rules then tell how these three variables will change together in the future. The *order* or *dimension* of a system is simply the number of state variables in each snapshot. Because three variables are needed in the system above, this set of equation would be referred to as being a third-order system.

Some well-known differential equations that are relevant to nearly all physical systems are

$$F = M\frac{d^2x}{dt^2}\text{(Newton's Second Law of Motion)}$$
$$J = Da\frac{dC}{dx}\text{(Fick's Law of Diffusion)}$$
$$I = \frac{1}{R}\frac{d\phi}{dx}\text{(Ohm's Law of Current Flow)}$$

It is important to note that the rules can unfold in time or space, or both. When the differential equations are in space (as in the equation for J above), the rules describe what happens to state variable that might vary in space, often referred to as a *gradient*. In this chapter, we will only be concerned with rules that describe changes in time. Therefore, our differential equations will be the rules that tell us how state variables change from one time to the next.

3.1.1 Linear Systems

There are many ways to classify differential equations, but the one that will concern us is the difference between linear and non-linear equations. Although we will focus on non-linear systems for most of the chapter, it is important to first understand what it means for a system to be linear. The word *linear* does in fact imply a line in the following sense. All variables (or derivatives of variables) that make up the terms to the right of the differential (e.g., $f(x)$) appear by themselves (e.g., no multiplication of state variables such as xy) and are only raised to the first power (e.g., no x^2

terms), and any coefficients must be independent (e.g., $3x$ and bx are linear, but $b(y)x$ is not). Furthermore, no transcendental functions may appear (e.g., $sin(xt)$, $atan(xt)$).

Practically, these constraints lead to two important and powerful conclusions about linear systems. First, if one doubles the input to a linear system, the output is doubled. More formally, this property is known as *homogeneity*. Second, if one recombines one linear system with another linear system, the result is a larger system that is also linear. This property is known as *additivity*. The term *superposition* is used to describe systems that follow both homogeneity and additivity and means the system is linear.

Linear systems are relatively easy to analyze mathematically and generally can be solved. Much of our understanding of the world has been built up with the idea that we can take apart the world, study the parts in isolation, and then put them back together. This idea is summed up in the phrase "the whole *is* the sum of the parts" and has led to much of the success of reductionistic science. Linear systems are no more and no less than the sum of their parts. Despite the simplicity of linear systems, and the mathematical tools that have been created to study linear systems, most real-world systems are non-linear.

3.1.2 Non-linear Systems

A non-linear system is one in which at least one term to the right of the differential, $f(x)$, is not linear. As such, non-linear systems are defined as all systems that have at least one non-linear term or rule. It turns out that linear systems are a special (and small) subset of all possible system models.

Except in some very special cases, it is not possible to write down an analytical solution of the form $x(t) = \ldots$ for a non-linear system. There is, however, a great deal that can be learned about the dynamics of a non-linear system without finding an analytical solution. First, the solution to a differential equation can be transformed into numerical approximations that can be iterated upon on a computer. The next section will briefly cover this approach. Second, geometric methods may be used. Most of the chapter will focus on this second approach.

3.1.3 Numerical Simulations

Numerical simulations are a means of iteratively evolving a differential equation over time on a computer. Essentially, the rules are encoded in such a way that an algorithmic process can find an approximation to $x(t)$. This is similar in spirit to how we discretized the logistics equation in Chap. 2. Most weather and economic predictions are the result of a computer simulation.

Generally differential equations are written using time as a continuous variable, t. The implication is that there are infinitely many little time instants between any two moments. Computers are limited by the size of their internal memory and cannot handle an infinite number of data points. A trick, however, is to *discretize* the differential equation—to only compute the value of the state variables at particular time instants. If the state variable is not changing quickly with respect to how often it is updated, then the approximation will be good. Moving a differential equation forward in time is known as *time integration*.

To gain some insight into how these methods work, we can consider the non-linear equation $\frac{dx}{dt} = 3x^2 + 5$. Note that this system is non-linear because of the x^2 term. The discretized equivalent is

$$\begin{aligned} \frac{\Delta x}{\Delta t} &= 3x^2 + 5 \\ \Delta x &= \Delta t(3x^2 + 5) \end{aligned}$$

The interpretation of the second equation is that we can find how much x will change (Δx) if we know the current value of x, and how much farther forward in time we want to predict (Δt). With this information, we can use the current value of x_t to find the next value of $x_{t+\Delta t}$ as

$$\begin{aligned} x_{t+\Delta t} &= x_t + \Delta x \\ x_{t+\Delta t} &= x_t + \Delta t(3x_t^2 + 5) \\ x_2 &= x_1 + \Delta t(3x_1^2 + 5) \end{aligned}$$

Graphically, what we are doing is using the slope at the current point to predict where the future point will be. This method is the simplest example of a numerical method and is known as Euler iteration, after Leonhard Euler (1707–1783). This method can be generalized for any function, whether it is non-linear or of very high order:

$$\begin{aligned} \frac{dx}{dt} &= f(x) \\ x_{t+\Delta x} &= x_t + \Delta t f(x) \end{aligned}$$

There is an entire field of numerical methods that balance speed, complexity, number of computational operations performed, stability, and error. Numerical integration techniques are an enormous reason why computers have been so useful in studying complex systems. They also allow for systems to be as complex as the modeler desires, without the constrain of finding a closed form analytical solution.

3.2 The Geometry of One-Dimensional Systems

The remainder of the chapter will explore the geometric interpretation of differential equations. To gain some initial insights, first-order (one-dimensional) systems will be explored. To begin, consider a simple one-dimensional system of the form

$$\frac{dx}{dt} = f(x)$$

where $f(x)$ might take many different forms. For example,

$$f(x) = -x^3$$
$$f(x) = x^3$$
$$f(x) = x^2$$

3.2.1 Classification of Equilibrium Points

The first step in a geometric analysis is to find the *equilibrium* or *fixed* points. These are simply the roots of $f(x)$ (e.g., where $f(x) = 0$). The meaning of these points is that x is not changing because $\frac{dx}{dt} = 0$. In our example above, all equations have a single fixed point at $x = 0$. If these equations described a physical system (say movement of a ball in the x-direction), then placing the ball at $x = 0$ would mean that the ball would stay at that point forever. Note that finding this point does not require solving the differential equation, only finding roots. A graphical way to find the equilibrium point is to plot $\frac{dx}{dt}$ against x, which is the same as plotting $f(x)$. On this plot, called a *phase plot*, the equilibrium points are the locations where $f(x)$ crosses the x-axis, for example, in Fig. 3.1.

Using these diagrams, the behavior of the system near the equilibrium can also be described. Examining what happens in a small region around an equilibrium is a common theme in non-linear dynamics and is related to the idea of *perturbation theory*. Although we will not dive deeply into perturbation theory, the basic concept is very simple. A system, defined by $f(x)$, is perturbed slightly from an equilibrium by some external forcing function, I, for a single instant:

$$\frac{dx}{dt} = f(x) + I$$

This is like hitting a bell with a hammer. Then we can explore how the system (the bell) returns back to the equilibrium. For example, if we consider $f(x) = -x^3$ and start our system at its equilibrium value of $x = 0$, it will stay there. If we then perturb the system ($I \neq 0$ for a single instant), $\frac{dx}{dt} \neq 0$ and therefore x will change. Let us assume that I has forced x to be slightly positive—shown in Fig. 3.1 as 1. In the plot of $-x^3$, we see that when x is positive, the change in x (characterized

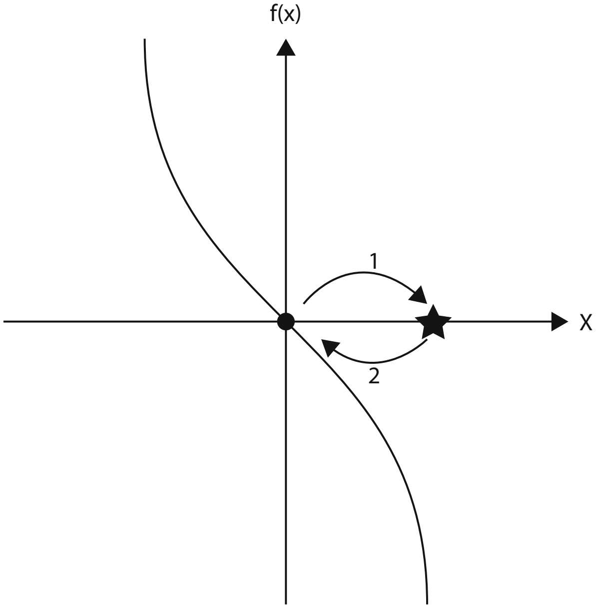


Fig. 3.1 Plot of $f(x) = -x^3$ showing the movement of x given a short perturbation, I

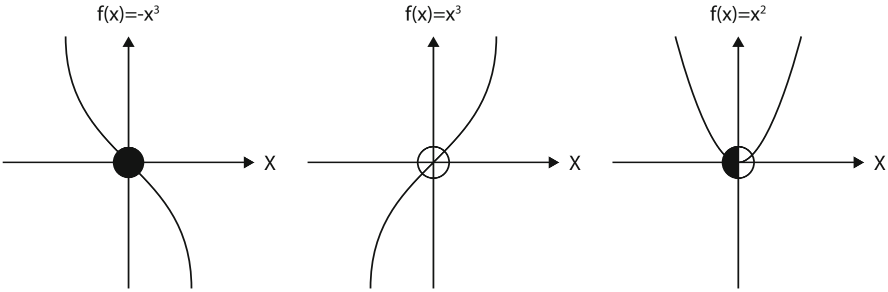


Fig. 3.2 Comparison of three simple functions and their equilibrium points. Solid circles represent stable equilibria. Open circles represent unstable equilibria. Half-filled circles represent saddle equilibria

by $f(x)$) will be negative and tend to *decrease* the value of x—indicated by 2. In other words, the dynamics will push the value of x back to 0. If the perturbation is such that x is negative (i.e., to the left of the equilibrium point), the change will be positive and tend to increase x back to zero. Therefore, any perturbation (positive or negative) will relax the system back to the equilibrium point. Points that behave in this way are referred to as *stable equilibria*, also known as the *attractors* of the system.

If the same logic is applied to $f(x) = x^3$ (shown in the middle panel of Fig. 3.2), the result is very different. Any perturbation (positive or negative) will tend to send x away from the equilibria. Points that behave in this way are referred to as *unstable equilibria* or *repellors* of a system.

The final example system, $f(x) = x^2$ (shown in the right panel of Fig. 3.2), can be explored in the same way with an unusual result. If the perturbation causes x to decrease, it will be drawn back in to the equilibrium point. However, if the perturbation causes x to increase, it will move away from the equilibrium. The meaning is that the system is stable in one direction (negative x) but unstable in another (positive x). The name for this type of equilibrium point is a *saddle*. This may be expected since the quadratic resembles the left-hand side of $f(x) = -x^3$ combined with the right-hand side of $f(x) = x^3$. Of course, we could design a function that would be a saddle in the opposite direction (e.g., $f(x) = -x^2$). It has become common practice to denote stable points with a filled circle, unstable points with open circles, and saddle points with half-filled circles.

It is also interesting to note that stable and unstable fixed points could easily have been created with linear equations (e.g., $f(x) = -x$ and $f(x) = x$). The saddle equilibrium, however, can only appear in a non-linear system. This result is known as the Hartman-Grobman theorem.

It is important to pause at this point to introduce the idea that attractors and repellors can serve as *decision boundaries* in a system. Different states will move toward or away from various equilibrium states of the system. We now have a way of determining these points. Being on one side of an equilibrium may result in a very different behavior than being on the other side. As we will explore later, this is a fundamental property of a pattern detector, or sorting device, and what some learning theorists believe is the essence of how we learn new information. It should also not be lost that initial conditions and the direction that a perturbation sends a system make a difference. We will encounter this idea again when we explore chaos theory.

3.2.2 Velocity Vectors

If we reexamine $f(x) = x^2$, we can also classify the type of equilibrium point. We can in fact quantify how fast x will decay to the equilibrium when it is negative and how fast it will escape from the equilibrium when it is positive. At each point, we can evaluate the function $f(x)$, to create a *velocity vector*, or how fast x is changing at that point. This vector indicates the strength of the pull toward (or push away from) an equilibrium point. Graphically, these vectors are indicated by using an arrow, where the longer the arrow, the stronger the force (either toward or away). In other words, the length of the arrow is proportional to the value of $f(x)$ evaluated at that point. Figure 3.3 shows a few velocity vectors for $f(x) = x^2$.

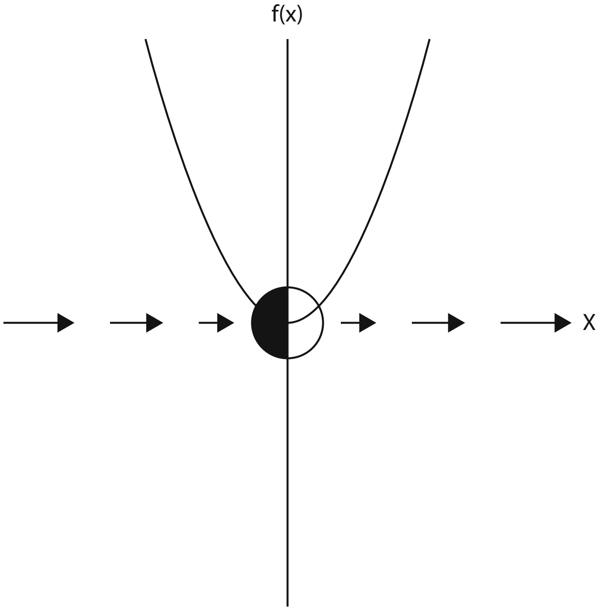


Fig. 3.3 Velocity vectors, represented by arrows for $f(x) = x^2$. The length of the arrows represents the strength of the force moving the trajectory of the system variable

3.2.3 Basins of Attractions

The function $f(x)$ can contain any number of equilibrium points, as well as a complex set of velocity vectors. Even a slightly more complex function can reveal new behaviors. Consider

$$\frac{dx}{dt} = 1 - x^2$$

The equilibrium points are the roots of $1 - x^2$, which are ± 1. The next step is to determine the stability of each of these points, which can be found by graphing x against $\frac{dx}{dt}$, as in Fig. 3.4.

On either side of these two points, we can explore the direction that velocity vectors will move the state variable x. To the left of -1, the system will move away from -1 because velocity vectors point to the left. Between -1 and 1, the system x will move positive toward 1. To the right of 1, however, x will decrease and move trajectories back to 1. Therefore 1 is a stable equilibrium, and -1 is an unstable equilibrium. These two points together define a *basin of attraction* that is everything from -1 to positive infinity. Any trajectory that finds itself in this basin will be attracted to the stable equilibrium at 1.

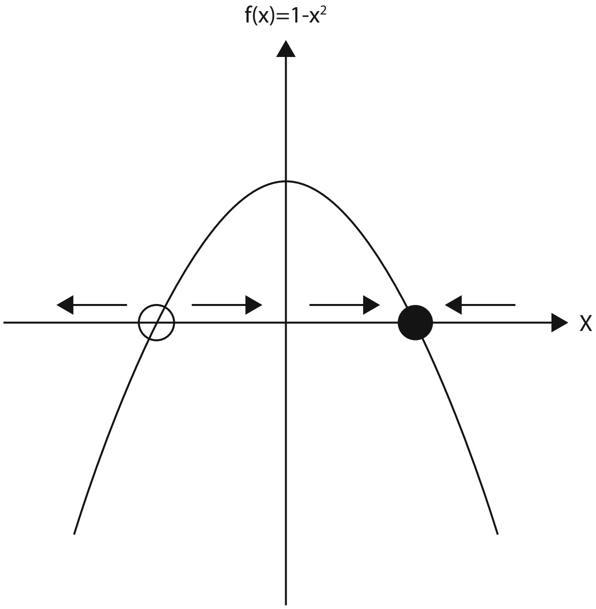


Fig. 3.4 Basin of attraction for $f(x) = 1 - x^2$, between -1 and positive infinity and attracted to $x = 1$

Basins of attraction are one of the key concepts that help move the analysis of differential equations into the realm of geometric analysis. Figure 3.5 is a graphical representation of a system, with no differential equation given. In this system, we can classify equilibrium points using ideas about velocity vectors. These points have been labeled 1 through 5. Based upon the stability of the points, we can then define basins of attraction. Any trajectory to the left of point 1 will move toward the stable equilibrium at 1. Likewise any time the system is between point 2 and 4, the trajectories will move toward the stable point at 3. And any point that is to the right of 4 will move toward point 5. The space is therefore carved up into three separate basins of attraction, each centered on one of the stable points and bounded by unstable points.

From the examples above, you may have noticed that the stable and unstable points alternate. This is a direct result of the Fundamental Theorem of Algebra, which states that a continuous function cannot cross the x-axis with the same signed slope twice. Therefore, stable points must be surrounded by unstable points (or the unstable side of a saddle). We can therefore define a basin of attraction as a stable attracting point surrounded by unstable points that determine the size of the basin.

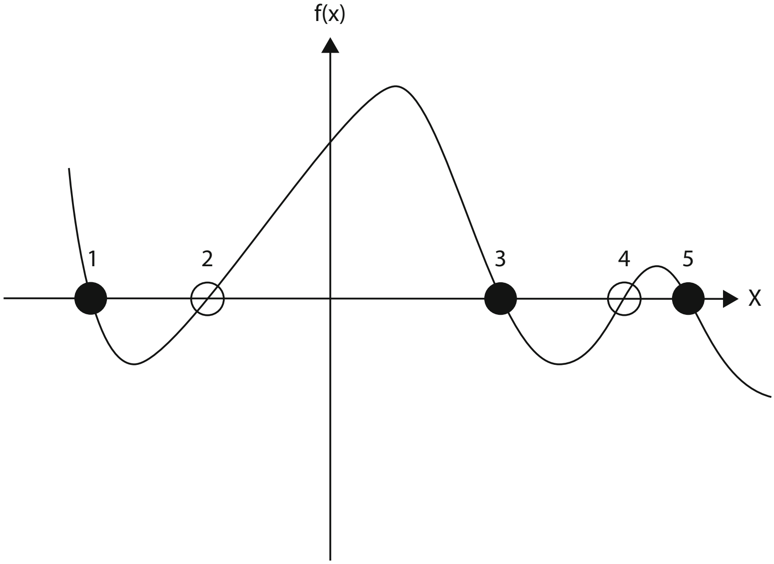


Fig. 3.5 Multiple basins of attraction and repulsion in one dimension

3.2.4 Bifurcations

The examples above can be generalized to include a parameter, r

$$\frac{dx}{dt} = x^2 + r$$

The effect of r is to move the parabola, formed by the x^2 term, up and down in phase space. First let us consider if $r < 0$ (left side of Fig. 3.6). From the graphical analysis, there are two equilibrium points with the left point being stable and the right point being unstable. If $r > 0$ (right side of Fig. 3.6), there are no equilibrium points, and any initial condition will eventually increase x to infinity. When $r = 0$, we are back to the system in equation $\frac{dx}{dt} = x^2$, with a single equilibrium that is a saddle point.

The parameter r is known as a *bifurcation parameter* because it controls how equilibrium points come in and out of existence or change type (from stable to unstable to saddle). It is important to point out that many real systems have parameters that when varied can alter equilibrium stability, thereby fundamentally changing the behavior of the system.

A valuable way to summarize the changes due to a bifurcation is to plot the locations of the stable, unstable, or saddle values of x_{eq}, as r is varied. Figure 3.7 shows a bifurcation diagram where solid lines represent stable equilibria and dashed lines represent unstable equilibria. As expected, when $r < 0$ there is one stable and

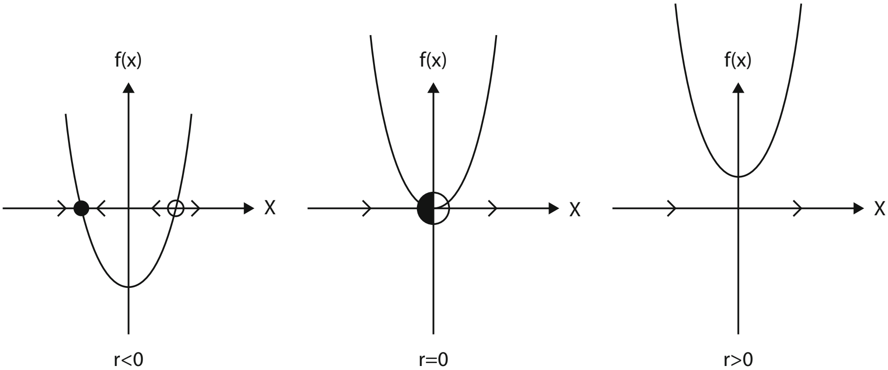
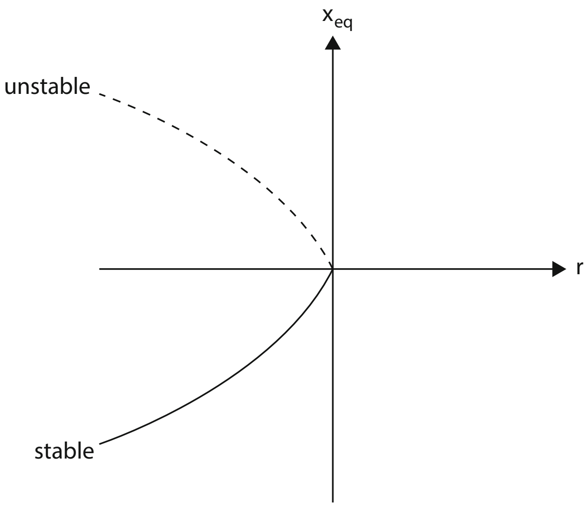


Fig. 3.6 Phase plots for $x^2 + r$ as r is varied

Fig. 3.7 Bifurcation diagram for $x^2 + r$. As r is varied an unstable and stable equilibrium point, x_{eq}, move closer together, eventually annihilating when $r = 0$



one unstable point. As r approaches zero, the stable and unstable point approach one another and eventually collide in a saddle (when $r = 0$). It is as if the stable and unstable points annihilate one another. Many bifurcations have names, and this one is called a *saddle-node* bifurcation.

Consider what the velocity vectors look like as r lifts the parabola off of the axis (Fig. 3.8), leaving the system with no equilibria. As determined above, the system will push any initial value of x to become more positive, eventually headed out to infinity. If we imagine an initial x at some larger negative number, however, the velocity vectors will show that x will increase quickly at first but then slow down when approaching $x = 0$, only to speed up again as x becomes positive. This is because the velocity vectors near $x = 0$ are very small. This is known as the "ghost" of the annihilation of the two previous equilibrium points. Another way to say this is that $x = 0$ is almost an equilibrium point, and around these points, the

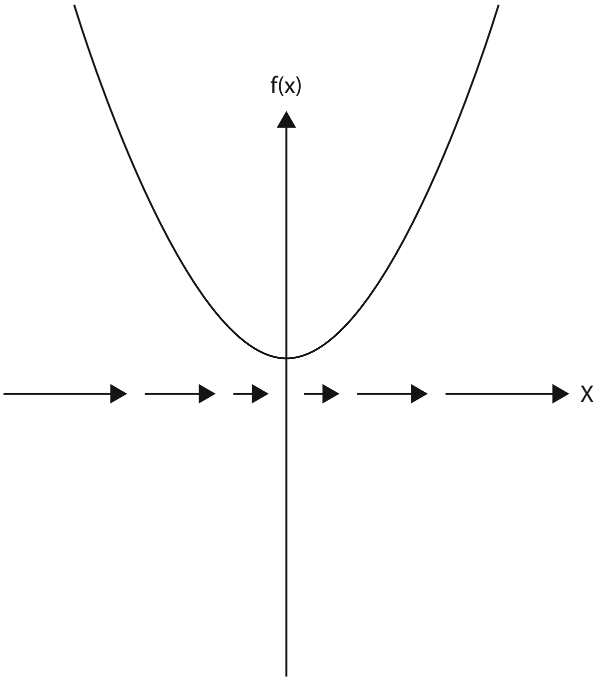


Fig. 3.8 Critical slow down as the velocity vectors become small near the "ghost" of an equilibrium point

velocity vectors become very small. The pause in fast dynamics near the ghost of an equilibrium point is known as *critical slow down* and is a signature of a system that is near a bifurcation.

A bifurcation diagram can also be created without even knowing the actual differential equation. For example, imagine moving the function in Fig. 3.5 up and down with a constant bifurcation parameter (like r in the example above). What we can imagine is that points will come into and out of existence. All of these changes can be captured in a bifurcation diagram.

The ability to characterize a system based upon a bifurcation diagram can also be a first step toward modeling a system. If one were to experimentally map out the basins of attraction and bifurcation diagram, it would be relatively simple to write down a plausible $f(x)$ that would fit these data. External perturbations and parameter changes can be used to map out the basins of attraction. By examining how basins of attraction grow (or shrink), and the types of bifurcations, $f(x)$ could be mapped out. Unfortunately, this approach becomes cumbersome in higher dimensional systems.

Although we have referred to r as a parameter of the system, it could itself be a constant perturbation or flow of energy, I. For example, in biological systems, this may be some sustained external chemical, electrical, or mechanical stimulus. We will explore this idea much more in Chap. 7 when we discuss systems that can be far from an equilibrium. The form of r may also be some other variable (say y) coming from another part of the system. We will explore the dynamics of these systems below when discussing two- and three-dimensional systems.

3.2.5 Classification of Bifurcations

The saddle-node bifurcation is only one type of bifurcation. There are many others that have been cataloged. Consider the following polynomials for $f(x)$, where r is a bifurcation parameter. It is left up to you to determine the nature of the bifurcation diagrams:

$$f(x) = r + x^2$$
$$f(x) = r - x^2$$
$$f(x) = rx - x^2$$
$$f(x) = rx - x^3$$
$$f(x) = rx + x^3$$
$$f(x) = rx + x^3 - x^5$$

3.2.6 Hysteresis

An interesting phenomenon occurs for the system $\frac{dx}{dt} = rx + x^3 - x^5$ above. The bifurcation diagram, shown in Fig. 3.9, is known as a subcritical pitchfork bifurcation. There is a range (between $-r_s$ and 0) where there are three stable points (one along the x-axis and then two branches). There are also two unstable equilibria

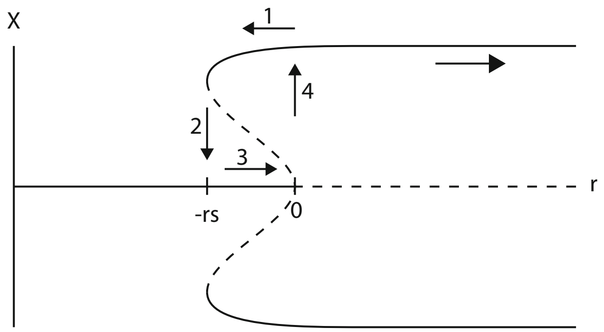


Fig. 3.9 Subcritical pitchfork bifurcation diagram for $\frac{dx}{dt} = rx + x^3 - x^5$, as r is varied

as well. It is of interest because of how equilibrium values of x will move as r is varied. Imagine starting with a high positive value for r and with x on the upper branch. Then slowly start to turn down r, and x will move to the left along the upper bifurcation branch (1). When r is decreased to $-r_s$, suddenly x will jump down to the origin (2). Then, if r is increased again (3) x will stay on the origin until $r = 0$, at which point x will jump back up to the upper branch (4). Continuing to increase r and x will stay on the upper branch (5). This same pattern can also occur on the lower branch and is a behavior that can only occur in a non-linear system.

There are two important system concepts embedded in this example. First is the concept of *hysteresis*, which has to do with the reversibility and memory of a system when there are multiple possible stable equilibria. The pathway by which a system arrives at an equilibrium point can determine how it will react in the future. It is similar in some ways to the inertia of a system that may make it easier or harder to change that system in the future. In this case, the system has settled into a particular basin of attraction, and it may take more to bump a system into a basin than out of it and into a different basin. Yet another way to think of this is that the trajectory that enters a basin of attraction does not need to be the trajectory that it takes out of that basin. Hysteresis is in fact a mechanism by which a system can retain a history of its past trajectories.

A fun way to think about hysteresis is the clothing that one wears during transitional seasons (e.g., fall and spring seasons). For example, when transitioning from summer to fall, one might switch to long sleeve shirts when the temperature reaches 15 °C. But when switching from winter to spring, one might switch from long sleeves to short sleeves at a lower temperature (perhaps 10 °C). So there is a range of temperatures where short sleeves and long sleeves are both permitted, but the choice might strongly depend on the past history. This is an illustration that the memory of a system (in this case your tolerance for cold weather) can have an effect on choosing between two possible equilibria.

The second important point about Fig. 3.9 is that sometimes a parameter is varied (r) and no result is apparent (x stays as $x = 0$), until suddenly there is a dramatic change (x moves back up to the upper branch). This idea is sometimes called a *tipping point* in the popular literature. If one were to understand the entirety of the system, it would not be a surprise (e.g., if one knew ahead of time the exact analytical expression and the location of all the stable and unstable points). But in real-world systems, we generally do not know all of the underlying dynamics, and so these points of radical change are not easy to predict and seem surprising when they occur.

3.3 Two-Dimensional Systems

In the past section, we studied the rich dynamics that can occur in a one-dimensional system. In this section, we will explore the additional dynamics that arise when we allow two variables to interact with one another. As is often the case in complex systems, adding a new parameter or dimension to a system can sometimes radically change the types of behaviors that are possible. For example, there are only so many ways that a one-dimensional system can be non-linear (e.g., usually by creating powers of the variable—x^2, $x^{-1/3}$). In two-dimensional systems, there are many more new ways for a system to be non-linear (through combinations of variables—x/y).

In general, two-dimensional systems are of the form

$$\frac{dx}{dt} = f(x, y)$$
$$\frac{dy}{dt} = g(x, y)$$

When the rules for x and y are dependent upon one another, they are said to form a *coupled system*. This simply means that changes in x depend in some way on what is happening to y and vice versa. Another way to think of coupled equations is that there is feedback between them—when one changes, the other will necessarily change too. This is fundamentally different than if they were independent of one another, where $\frac{dx}{dt}$ is only dependent on x and $\frac{dy}{dt}$ is only dependent on y. We would call these systems *decoupled*.

3.3.1 Phase Space and Trajectories

The evolution of $x(t)$ and $y(t)$ are often the variables that we can measure, such as pH, voltage, or bank account balance. It is typical that we would display these data in a plot of the variable versus time. An alternative graphical representation is to plot the variables against one another, for example, by plotting x versus y. Anytime variables other than time are plotted against one, the plot is referred to as a *phase space*. We already saw an example of this in Fig. 3.1 for one-dimensional systems, but the concept of a phase plot becomes much more powerful when studying higher dimensional systems. Like the one-dimensional system, because time is not represented, we do not know how long it will take x and y to move to a new point. What we can define, however, is a trajectory, showing how x and y vary together.

Figure 3.10 shows a phase trajectory that spirals down to an equilibrium point, indicted by a solid circle. This means that there is a push and pull of the variables, with x increasing while y decreases, and vice versa. On each turn around the spiral, the deviation from the equilibrium point gets smaller and smaller, as the trajectory returns to the equilibrium point.

In a real system, it is sometimes difficult to record both x and y. Luckily, there is a way to reconstruct a trajectory (x–y pairs) in a two-dimensional phase space with only one variable. The basic idea is to create a surrogate second (or higher-order) variable from $x(t)$, using Taken's method. $x(t)$ is plotted against a delayed version of itself, $x(t + \tau)$ (where τ is the delay). We will explore this method more in Chap. 4, in the context of higher-order systems.

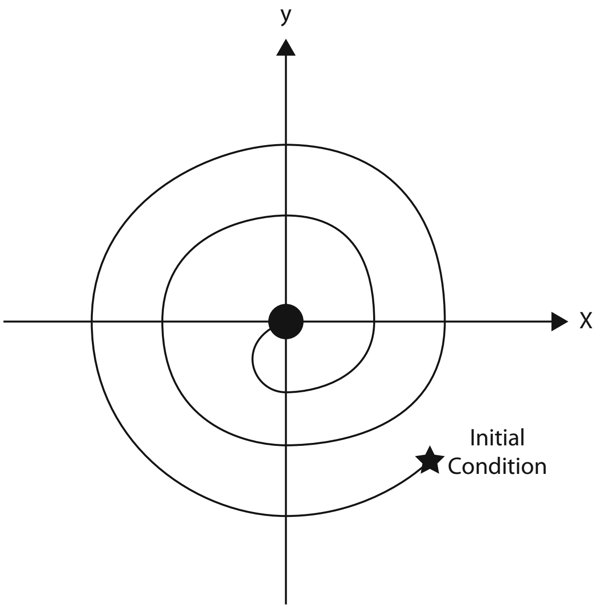


Fig. 3.10 A spiral trajectory, centered around a stable equilibrium, in a two-dimensional phase space

In any deterministic system, the trajectory cannot cross itself. The proof will not be given, but it can be understood intuitively by considering a proof by contradiction. Imagine that a trajectory did cross itself. If the system was traveling down one branch, it would eventually get to a crossing point. The question then would become which way it should go? In a deterministic system, the rules must be absolutely clear about which way to go—there can be no uncertainty about the next step. Therefore, trajectories cannot cross one another in a deterministic system.

3.3.2 Nullclines

A trajectory in phase space is deterministic but dependent upon initial conditions and any perturbations. Therefore examining one specific trajectory does not reveal all of the dynamics that might be possible. There are, however, some graphical ways to uncover the general behavior of a system. Consider the coupled differential equations:

$$\frac{dx}{dt} = f(x, y) = x^3 - y + r$$

$$\frac{dy}{dt} = g(x, y) = y^3 + x$$

For now, we will assume that $r = 3$. From these equations, we can find the equilibrium point where x and y do not change ($\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$):

$$0 = x^3 - y + 3$$
$$0 = y^3 + x$$

Although these equations can be solved analytically, there is a much easier graphical way. First the function $y = x^3 + 3$ is plotted in the x–y phase space. This is known as the *x-nullcline* and is shown as the solid line in Fig. 3.11. The x-nullcline shows all of the combinations of x and y where the system cannot move in the x direction ($\frac{dx}{dt} = 0$). Any trajectories must cross the x-nullcline in the y direction.

This same idea can apply to the y-nullcline, as a plot of the function $x = -y^3$. This function is shown as a dotted line in Fig. 3.11. Trajectories can only cross the y-nullcline in the x direction. By plotting the two nullclines against one another, the equilibrium points can be determined graphically as the locations where the nullclines intersect.

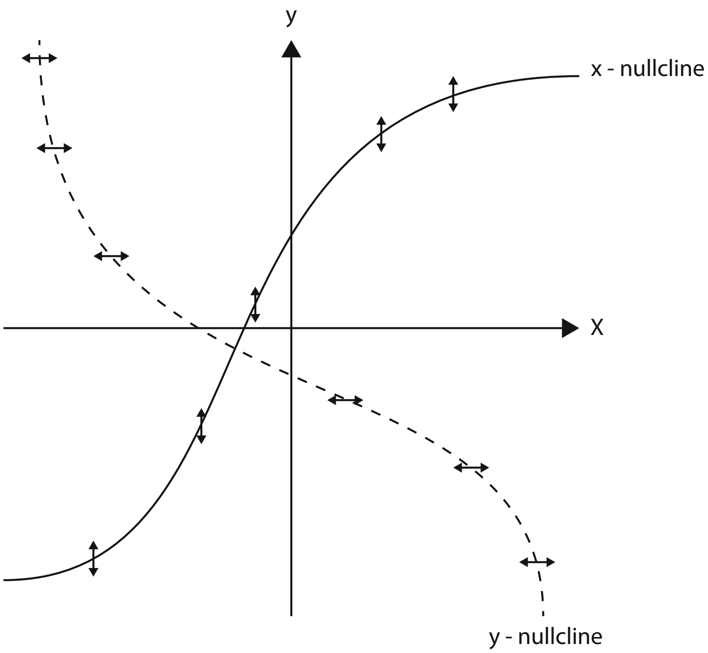


Fig. 3.11 x (solid) and y (dotted) nullclines in two dimensions. Arrows indicate the directions in which nullclines may be crossed. Note that magnitudes and directions are not given so they are not velocity vectors and are only meant to indicate the possible directions of crossing a nullcline

In Fig. 3.11 we assumed that $r = 3$. But consider what happens as r is varied. The impact is to move the x-nullcline up and down. A bit of thinking will show that doing so will change where the equilibrium point is. In some systems, such parameter variations could give rise to a whole new world of bifurcations, a few of which will be explored below.

3.3.3 2D Velocity Vectors

As in one dimension, we can gain more information about trajectories by plotting velocity vectors. At any point (x_o, y_o), we can define the velocity vector as $[f(x_o, y_o), g(x_o, y_o)]$. This is the evaluation of the differential equation at a particular point, as shown in Fig. 3.12. An intuitive way of thinking of the velocity vectors is as fluid flow lines. If you were to start at any initial x–y point, the trajectory of the system would "flow" along the velocity vector lines, being sure to cross nullclines only in one direction.

When nullclines and velocity vectors are plotted together, the result is known as a *phase portrait*, as shown in Fig. 3.13. Although the differential equations are not shown, we can gain a sense for how this system will behave. There is one stable equilibrium point where the two nullclines intersect. But, remember that trajectories can only cross the x-nullcline in the y direction, and the y-nullcline in the x direction. So, if the system is perturbed from the stable equilibrium (indicated by a star), it is usually assumed that it will simply decay back to the stable equilibrium. But if it is displaced from the equilibrium and cannot cross a nullcline in a particular direction, it may need to return to the equilibrium through a long trajectory, shown in the example as a solid line with arrows.

This particular phase portrait is what is known as a one-shot. It will "fire" when the perturbation is large enough, after which the system will settle back to a stable equilibrium. How hard the system needs to be perturbed to fire is known as a *threshold*. In this particular phase space, the threshold has been set by the x-

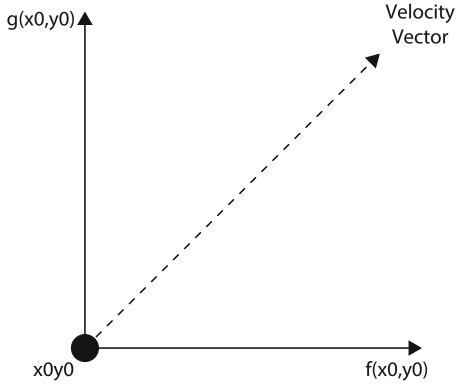


Fig. 3.12 Two-dimensional velocity vector. The vector is composed of components from $\frac{dx}{dt}$ and $\frac{dy}{dt}$ evaluated at x_o,y_o

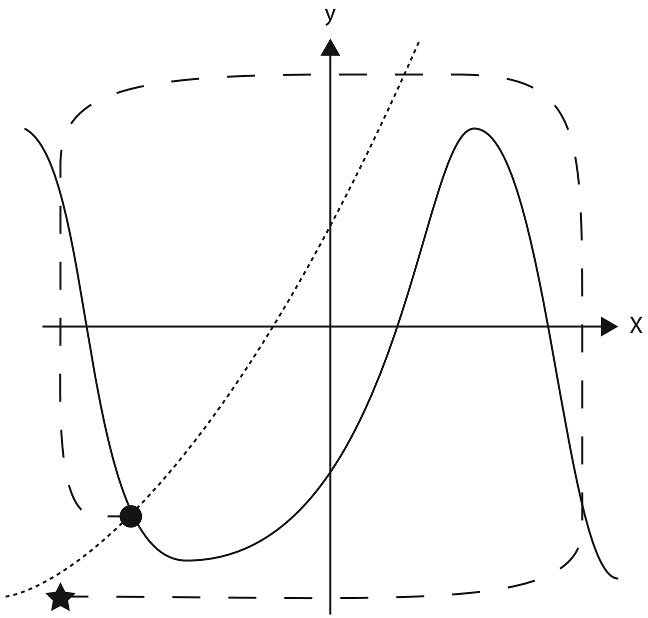


Fig. 3.13 Phase portrait of a two-dimensional system showing x (solid) and y (dotted) nullclines, an equilibrium point (large dot), initial condition (star) and a representative trajectory (dashed)

nullcline. An initial condition (or stimulus) must move the state of the system to the other side, forcing it to go on a long trajectory. In fact, the phase portrait shown is a simplified representation of the Hodgkin-Huxley neural model, which won the two mathematical biologists who created it the Nobel Prize in 1963.

3.3.4 Stability of Equilibrium Points

In two dimensions, classifying the stability of points is performed in much the same way as in one dimension—the velocity vectors near an equilibrium are assessed. If the vectors are pointing away from the equilibria in all directions, then it is unstable. If the vectors are pointing toward the point in all directions, then the equilibrium is stable. The more formal method of checking these slopes involves creating a matrix of the slopes.

$$\mathbf{J} = \begin{bmatrix} \dot{f}_x & \dot{f}_y \\ \dot{g}_x & \dot{g}_y \end{bmatrix}$$

This matrix, known as a Jacobian, assesses the velocity vectors in the x and y directions. The notation $\dot{f}_x$ is the derivative of $f(x, y)$ with respect to x, while $\dot{f}_y$

is the derivative of $f(x, y)$ with respect to y. The Jacobian can then be evaluated at the equilibrium point

$$\mathbf{J} = \begin{bmatrix} \dot{f}_x & \dot{f}_y \\ \dot{g}_x & \dot{g}_y \end{bmatrix}_{x_{eq}, y_{eq}}$$

The resulting matrix will be numbers rather than variables that indicate the direction and magnitude of the velocity vectors in the x and y directions. Just as in one dimension, knowing how trajectories move away from or toward an equilibrium point can classify the type of equilibrium. Unlike in one-dimensional systems, it is entirely possible to have stability in one direction but not in another.

3.3.5 Basins of Attraction and the Separatrix

As in one-dimensional systems, we can define basins of attraction. In two dimensions, the basins become areas of the phase portrait, as shown in Fig. 3.14. But unlike in one dimension, unstable and saddle points cannot completely separate the basins of attraction. Instead there is a special trajectory called a *separatrix* that separates basins of attraction. A separatrix is an unstable line, from which other nearby trajectories will diverge. The interpretation is that any initial condition will follow the velocity vectors toward the stable equilibrium within a particular basin of attraction. A trajectory cannot on its own cross a separatrix. Intuitively, a separatrix is like the edge of a sharp ridge—stray a bit to one side and you will tumble down into one valley. But stray just a little in the other direction and you will land in the next valley over. Such behavior is another example of sensitivity to initial conditions.

Of course there can be multiple separatrices in the same phase portrait, defining all manner of basins of attraction. In this way, a two-dimensional phase space can contain many separate behaviors. For example, we could imagine that in one basin, there is a one-shot (as in Fig. 3.13), but in the next basin over, there might be a spiral (as in Fig. 3.10).

Once a system is in a basin of attraction, it will by definition stay in that basin. The trajectory is expressing the behavior in that basin, with all other basins being latent behaviors. Some perturbation will be required to kick the dynamics across the separatrix into another basin. To express a latent behavior can therefore be thought of as moving the system across a separatrix, or in the language of chemistry to overcome some *activation energy*.

In theory just about any continuous shape or vector field that can be imagined could be created in a two-dimensional phase portrait. A helpful analogy is to imagine placing a large sheet over a series of objects. Where the sheet touches an object, the topography will rise. Where it is between objects, the sheet will fall. The result will be a series of peaks and valleys. Each x–y point on the sheet will have

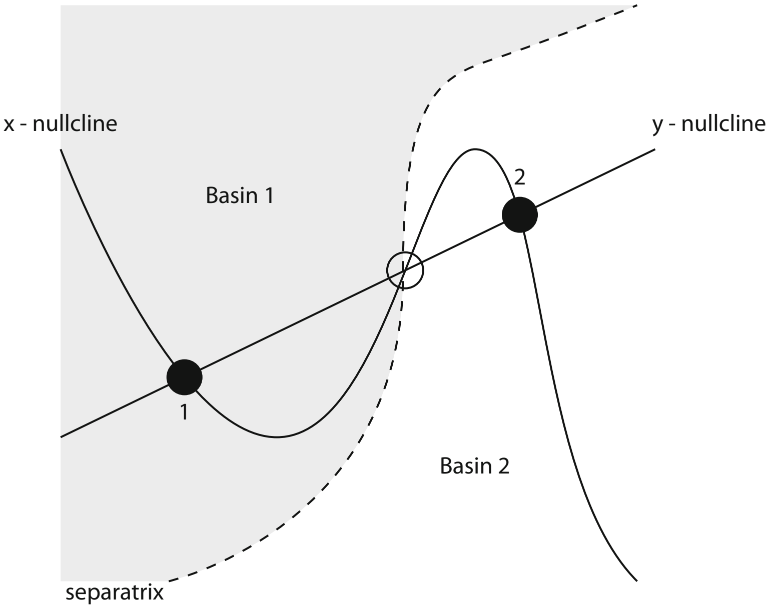


Fig. 3.14 Separatrix (dotted line) separating two basins of attraction

some elevation, such that if a ball were placed there as an initial condition and let go, the ball would move in a direction. That direction is analogous to a velocity vector. In general, the steeper the slope, the faster the ball will move, mimicking a larger velocity vector. Peaks are unstable points, ridges are separatrices, valleys are basins of attraction, and the lowest point of a valley is a stable equilibrium. Using this analogy, bifurcations can also be understood as moving the sheet (or objects below) as parameters of the system are altered such that the various equilibria, velocity vectors, and separatrices are altered. The sheet and ball model can be helpful, but as it is only an analogy, it should not be stretched too far.

3.3.6 Limit Cycles

A common theme through this text has been, and will continue to be, the idea of cycles and repeating rhythms. In phase space, this idea can be represented as a line that feeds back on itself to form a closed shape. If we take the stable line from the previous section and bend it to rejoin with itself, in phase space, the result is a *limit cycle*, as shown in the left side of Fig. 3.15. Although the simplest kind of closed shape is a circle, a limit cycle can form any closed shape. Intuitively, a stable limit cycle will attract all nearby trajectories, but once the trajectory is on the limit cycle, it will continue to circle forever. What this means is that the same combination of

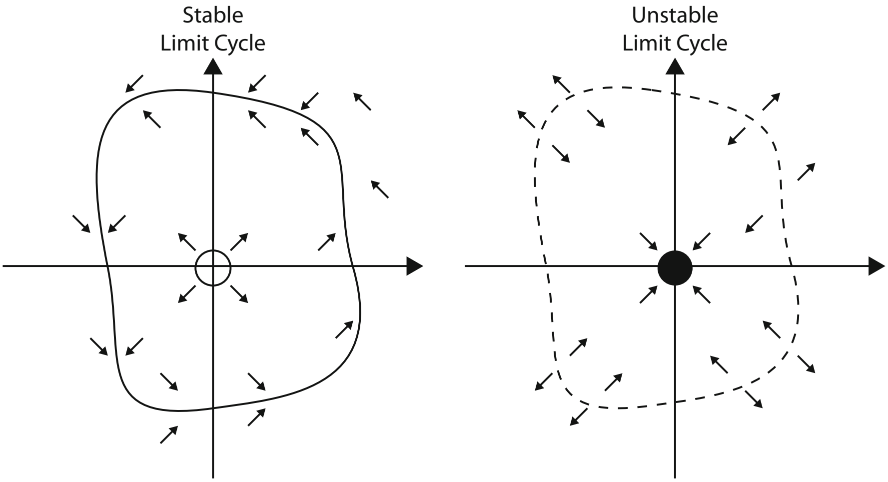


Fig. 3.15 Stable (left) and unstable (right) limit cycles with velocity vectors and equilibria

x and y values will continue to repeat over and over again in a kind of oscillation. In non-linear dynamics, stable limit cycles are drawn as solid lines. Likewise, a separatrix (an unstable line) can be joined end to end to form an unstable limit cycle. Here all points near the limit cycle will move away from the cycle, as shown in the right-hand side of Fig. 3.15. Typically an unstable limit cycle is represented by a dotted line.

There are a number of mathematical theorems about limit cycles in phase space. One of the most important is the Poincare-Bendixson theorem. The proof demonstrates that somewhere inside of a stable limit cycle, there must be an unstable point. To understand why, imagine a stable limit cycle and imagine an initial condition inside of the limit cycle. Something must push the velocity vectors so that they point outward. The geometric means of doing this is to place an unstable point somewhere inside the limit cycle. Likewise, somewhere inside of an unstable limit cycle must be a stable point.

3.3.7 Bifurcations in 2D Systems

In two dimensions, all of the bifurcations discussed above have analogs in two dimensions, but many of them take on new behaviors in a second dimension. Likewise, there are new bifurcations that can only occur in two dimensions. Many of these were classified in the 1960s and 1970s, with the star being Rene Thom (1923–2002) who used the term *catastrophe theory*. The mathematical theories Thom described were meant to catalog the varieties of ways that some parameter could be moved slowly, yet the behavior could jump to a new type suddenly. Below we will give one example, but the key is to understand how points (and now lines) that are stable or unstable can annihilate or recombine, when parameters of the system are changed. Geometrically, they can be understood as the movement of the system nullclines and their intersections.

A family of bifurcations, known as *Hopf* bifurcations after Eberhard Hopf (1902–1983), a pioneer of Ergodic theory and Bifurcation theory, become possible in two dimensions. This family describes the creation and destruction of limit cycles. Imagine a stable equilibrium point at the intersection of two nullclines, around which a trajectory will spiral inward (e.g., Fig. 3.10). A parameter change can make this point more or less attractive to nearby trajectories. As a result, the spiral may become more and more lazy, taking longer and longer to spiral down to the stable point. Further changes in the parameter could in fact cause the equilibrium to switch from stable to unstable. The slow spiraling inward now becomes a stable limit cycle around an unstable point. This change from a stable point to an unstable point surrounded by stable limit cycle is one type of Hopf bifurcation. Several other Hopf bifurcations have been cataloged and studied with all them involving the creation or destruction of limit cycles.

3.3.8 Building and Simplifying Systems

Moving points and lines and limit cycles around in phase space suggests that mathematical systems might be designed to have particular properties. The methods will not be covered here but imagine combining together various polynomials, $f(x, y)$ and $g(x, y)$, such that the nullclines can express particular behaviors and bifurcations. One of the earliest and most important of these designed mathematical models is the van der Pol oscillator, created by Balthasar van der Pol (1889–1959):

$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = \mu(1 - x^2)y - x$$

where μ is a parameter of the system. It is left up to you to determine how this system will behave. But given the tools built up in this chapter, you should be able to find out without needing to solve the equations.

A phase portrait can also be used to simplify the dynamics of a more complex system. An excellent example is the FitzHugh-Nagumo model derived in 1961 from the Hodgkin-Huxley model. The Hodgkin-Huxley model is a complex four-variable model, involving several exponential functions, that at least in the 1950s and 1960s required a great deal of computation. The FitzHugh-Nagumo model, on the other hand, is a two-variable model that only relies on polynomials and can reproduce a one-shot, as well as the neural oscillations that form a limit cycle in phase space. The model is of the following form

$$\frac{dx}{dt} = x - \frac{x^3}{3} - y + I_{ext}$$
$$\frac{dy}{dt} = \frac{1}{\tau}[x + a - by]$$

where a, b and τ are parameters that control various aspects of the phase portrait. I_{ext} is a stimulus from the outside or perhaps another neuron. Exploring models in phase space is a very rich area of research, summarized in an excellent book by Eugene Izhikevich, *Dynamical Systems in Neuroscience: The Geometry of Excitability and Bursting*.

3.4 Three-Dimensional Systems

The trend we have seen in going from one- to two-dimensional systems is that with each increase in dimension, there are new behaviors that become possible. This holds true in moving to three-dimensional systems. A classic example of a non-linear three-dimensional system is the Lorenz equations:

$$\frac{dx}{dt} = \sigma(y - x)$$
$$\frac{dy}{dt} = rx - y - xz$$
$$\frac{dz}{dt} = zy - bz$$

Edward Lorenz (1917–2008), a theoretical meteorologist, developed these equations in the early 1960s as a simple model of how air moves in the atmosphere. As expected, for particular parameters (r, σ, and b), the nullclines can intersect to yield stable and unstable points that can display all of the complexity seen in 2D systems (e.g., stable and unstable points, lines, and limit cycles). But Lorenz, through a series of computer simulations (remember this was the 1960s), found a new behavior that was very unexpected.

3.4.1 Chaos in Three-Dimensional Systems

The numerical simulations Lorenz was running at the time would often take a long time. For some simulations, he would save the state variables (x, y, and z) and then restart the simulation with those values as the initial conditions. In a deterministic system, there should be no problem with this approach.

Lorenz noticed something, however, that he first thought must be an error in his methodology. He would stop a simulation at t_1 (saving x_1,y_1 and z_1) and then restart to run until some later time (t_f). He also ran the same simulation but stopped at t_2 (saving x_2, y_2 and z_2), restarted from that point, and again ran the simulation until t_f. The error he saw was that the values of x_f, y_f, and z_f were radically different in the two cases. Even more mysterious was that this kind of anomaly only seemed to happen for certain parameters values.

What Lorenz had found was an example of the behavior of a *chaotic system*. A chaotic system is by definition deterministic (e.g., governed by precise rules) but is very sensitive to initial conditions and perturbations. Because numerical simulations on a computer have limited precision, when Lorenz was saving state variables, he necessarily needed to slightly round off numbers. For some parameters of his equations, these slight roundoff errors were causing nearby trajectories to radically diverge from one another over time, as shown in Fig. 3.16.

Even more strange was that the time course of an individual variable (e.g., $x(t)$) seemed to fit the statistical definition of randomness, despite being generated from a deterministic system. Lorenz's finding called into question recordings of signals from the real world that appeared random. Were they really random or the result of a deterministic but chaotic system? Throughout the 1970s and 1980s (and into the 1990s), several analysis tools were created that could distinguish between random and chaotic and as a result a number of systems that were previously thought of as random being reclassified as chaotic.

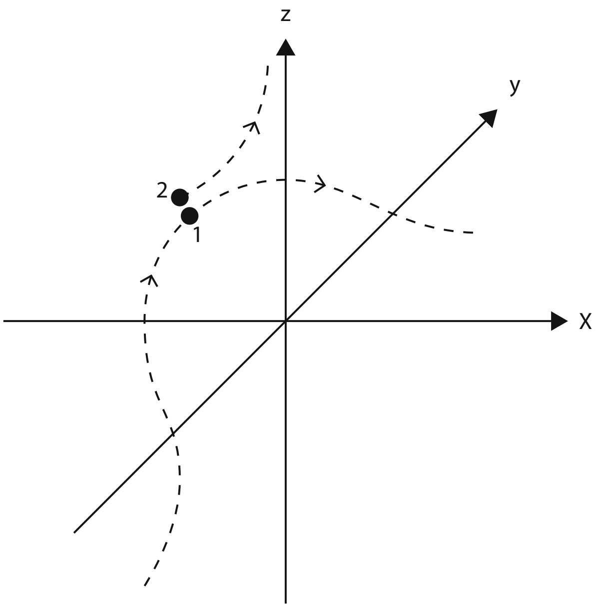


Fig. 3.16 The divergence of nearby trajectories. A trajectory is moving along to a point (1). At this time, the simulation state is saved and then restarted at a very nearby point (2). These two nearby trajectories can diverge in a chaotic system

3.4.2 Strange Attractors

When viewed in phase space, Lorenz found that the trajectories of chaotic systems never settled down to a particular value (e.g., stable point) but also did not repeat themselves (e.g., limit cycle). Furthermore trajectories remained within some finite volume of the phase space, never veering off to infinity. Because the equations were deterministic, the trajectories also could not cross. Step by step, Lorenz eliminated possibilities, showing that no combination of stable or unstable, limit cycles or points could describe this behavior. We will not go through each twist and turn, but the original paper is surprisingly readable with the background you have from this chapter.

Lorenz concluded that he had discovered some new object in phase space that would attract nearby trajectories. The shape he described, a representative of which is shown in Fig. 3.17, has become known as a *strange attractor*. An important consequence is that a strange attractor can only occur in three dimensions or higher. It was also shown that strange attractors must have at least one positive eigenvalue, meaning that there is some force causing trajectories to move outward. This eigenvalue is balanced by negative eigenvalues that pull the trajectories back in along some other direction. It is the tension between the push and pull that generates

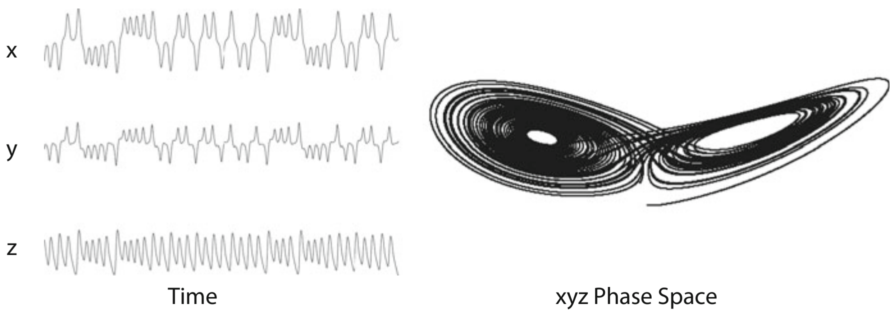


Fig. 3.17 The Lorenz attractor. On the left are the three time courses of the x, y, and z variables. On the right is the attractor shown in a 3D phase space

chaos. We will explore the unique geometry of strange attractors in more depth in Chap. 4 in the context of fractals.

3.4.3 Fast-Slow Systems and Bursting

Chaos is the poster child for the new behaviors that can occur in higher dimensional system. There are other important behaviors that can only emerge in three-dimensional phase space as well. One example is the phenomena of *bursting*. In a two-dimensional system, there might exist a parameter r that in three dimensions could become a variable of the system. The difference is that now the system itself can, using a bifurcation in two dimensions, turn on and off a limit cycle. A simplified bursting system is of the form

$$\frac{dx}{dt} = f(x, y) + r$$
$$\frac{dy}{dt} = g(x, y)$$
$$\frac{dr}{dt} = \frac{h(x, y)}{\tau_{slow}}$$

In this system, the interaction between x and y generates a limit cycle. The state variable r, however, controls the x-nullcline that can potentially push the system through a Hopf bifurcation (the creation and destruction of a limit cycle). It is the system itself that has control over the bifurcation through $\frac{dr}{dt}$, which is governed by the dynamics of x and y. It should be noted that the dynamics of r have been slowed down by including the time constant τ_{slow} such that changes in r are much slower than the oscillations of the limit cycle between x and y. As the limit cycle of x–y causes an oscillation, in the background r is slowly changing to make that limit cycle less and less stable. At some point, a Hopf bifurcation occurs and the x–y system spirals into a stable point. But, in the background, r then reverses direction, moving the x-nullcline such that the limit cycle reappears. The result is an oscillating burst in $x(t)$ and $y(t)$, followed by a period of quiet, and then a restart of the burst again.

The Hindmarsh-Rose model is a real example that describes bursting neurons:

$$\frac{dx}{dt} = y - ax^3 + bx^2 - z$$
$$\frac{dy}{dt} = c - dx^2 - y$$
$$\frac{dz}{dt} = e\left[s(x - x_r) - z\right]$$

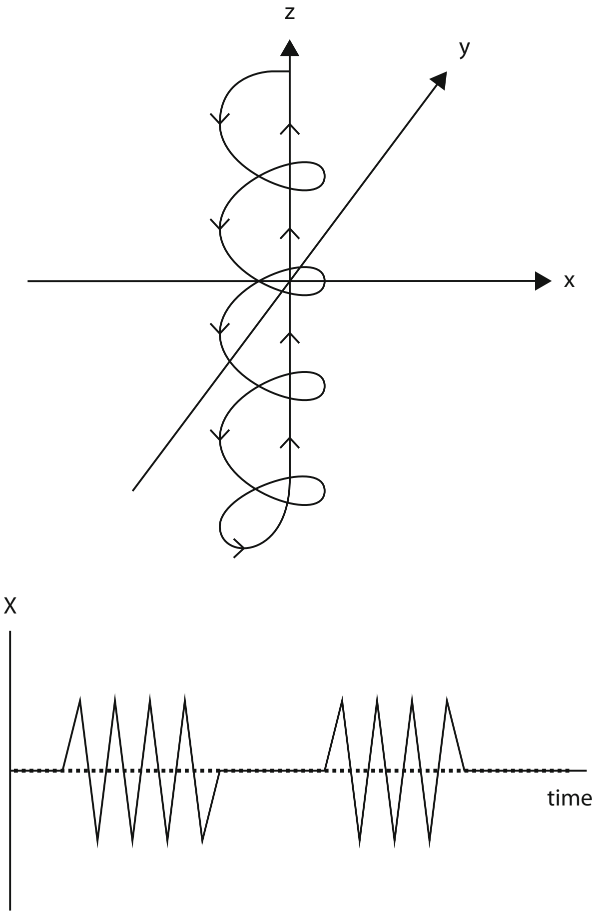


Fig. 3.18 Bursting behavior in a three-dimensional system. The top panel shows a trajectory that forms a downward spiraling helix. When the trajectory reaches the bottom, the system resets by moving back up the z-axis. The bottom panel shows the x time course that bursts four times, corresponding to the four loops of the helix. There is a period of *quiescence* as z resets the system, followed by another burst

where a–e, x_r and s are all variables that control the shape of the polynomials. Figure 3.18 shows an example of the phase space and dynamics for x over time. The limit cycle spirals around the z-axis, with fast oscillations in x and y. All the while z is slowly decreasing. The resulting shape is a spiral in three dimensions (a helix). When the spiral gets to the bottom, x and y stop changing, but the trajectory travels back up the z axis. Once it is at the top, the helix is reentered. The result is that a plot of x, or y, will oscillate for a time and then suddenly stop, only to start up again at some later time. Note that it takes two variables to create a limit cycle, but a third variable to turn the limit cycle on and off.

3.5 Implications of Chaos

When the foundations of chaos theory were being developed there were some strong underlying philosophical and practical questions in play. In this section we will explore a few of these ideas and questions.

3.5.1 *Predictability of Deterministic Systems*

In the clockwork universe of Newton and Laplace everything was determined and could in principle be predicted. That we could not exactly predict the future was due to having an incomplete set of rules and initial conditions. Determinism seemed to imply predictability. With chaos theory a new set of hurdles appeared in the quest for prefect prediction, namely sensitivity to initial conditions and the effect of small perturbations.

An early glimpse of the difference between determinism and predictability arose in the form of a prize to the first person to analytically solve what is known as the *three body problem*. The challenge was to derive, using Newton's Laws, the complete solution to the motion of three interacting planets. The solution to two interacting planets was fairly easy to derive but for some reason all attempts to derive a more general solution, even one that added one more planet, was strangely difficult. The person who eventually won the prize was the French mathematician Henri Poincare (1854–1912). Ironically what he provided in 1887 was evidence that an analytical solution to the three body problem was not possible, with the implication that predictions might not be possible. As part of his proof, he glimpsed the nature of a 3D strange attractor as a two-dimensional slice, shown in Fig. 3.19. The crossings never repeated but also didn't completely fill the two dimensional slice either. Given better three-dimensional graphical abilities, Poincare would likely have characterized the first strange attractor several decades before Lorenz.

3.5.2 *The Robustness of Non-linear Systems*

Stability is a key concept in complex systems. With non-linear dynamics, we find an example of how coupled systems can prop one another up in such a way that they are more stable. This is a mathematical version of the house of cards introduced in Chap. 1—each card on its own is unstable, but together they can form a stable structure. The stability of the cards, however, was static.

In this chapter, we explored how non-linearity can allow systems to be *dynamically stable*. To understand what this means, imagine the merry-go-round that is sometimes on playgrounds. As the children spin the merry-go-round, there is a repeating dynamic cycle. But the frequency of rotation is not set by the system

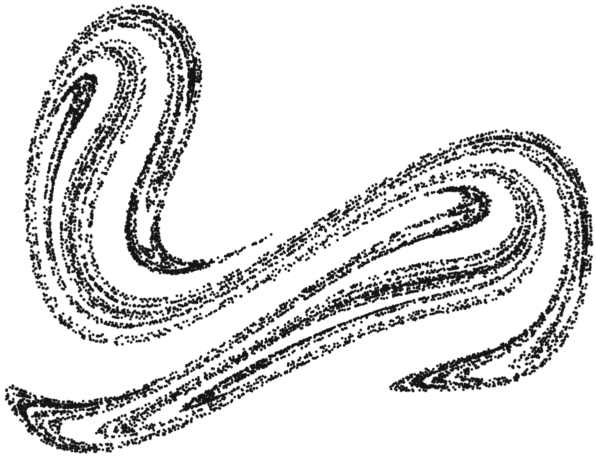


Fig. 3.19 Example of a Poincare map as a two-dimensional slice through a three-dimensional strange attractor

itself. It is set by how hard it is pushed by the children. Although a merry-go-round displays behavior like a limit cycle, it is not stable on its own.

A limit cycle, on the other hand, has a built-in frequency and shape in phase space. It will move toward this frequency and shape regardless of the initial condition and will be robust against perturbations. Even if another system tries to speed it up, stop it, or move it off course, it will return to its original cycle once the perturbation is gone. Such behavior is one of dynamic robustness and can only appear in non-linear systems.

3.5.3 The Arts and Non-linear Dynamics

It is sometimes said that the sciences are the home of those who crave linearly, while the arts and humanities attract those who think non-linearly. But this is not always the case. Schenkerian analysis of tonal music is very mathematical. Likewise, Laban movement analysis is a quantitative language for describing human movement and is used in dance, occupational theory, and anthropology. And there are sculptors who live in the realm of programming, material science, and high-end manufacturing technology. There are also some scientists who have embraced the idea of non-linear dynamics as a way to generate new art. For example, Diana Dabby explored the idea of mapping various kinds of music onto a strange attractor. A particular piece of music would be mapped to a strange attractor. By slightly changing the parameters of the attractor, a new piece of music could be generated that would have the same character as the original. It is as if the strange attractor is calibrated by an existing piece and then used to generate new music. There is in fact some evidence that music may have a strange attractor-like shape in a hypothetical tonal phase space. We will explore this idea more in Chap. 10. Elizabeth Bradley did something very similar with dance steps to create new choreography. Methods derived to study chaos have even been used to authenticate certain works of art. For example, the drip painting of Jackson Pollock have been shown to have a strange attractor-like pattern (a fractal, as will be discussed in Chap. 4). Chaotic patterns turn out to be very hard to fake and can be used to separate the authentic from the forgeries.

3.5.4 Metastable Systems

Figure 3.20 is a representation of multiple basins of attraction that may be present in the same phase space. One could imagine a strange attractor, limit cycle, and various combinations of stable and unstable points all coexisting, but only one being expressed at any given time. This is essentially the idea from Chap. 1 of latent or dormant functions. The general idea extends to higher dimensional system to separate different behaviors of a system.

There are at least a few ways that a trajectory can cross from one basin into another. First a constant stimulus may push the system through a bifurcation, creating a basin that impacts trajectories. When the stimulus is released, the system may move back to a default or innate basin. Second, a perturbation from the outside could jump a trajectory across a separatrix and into another basin. Third, systems

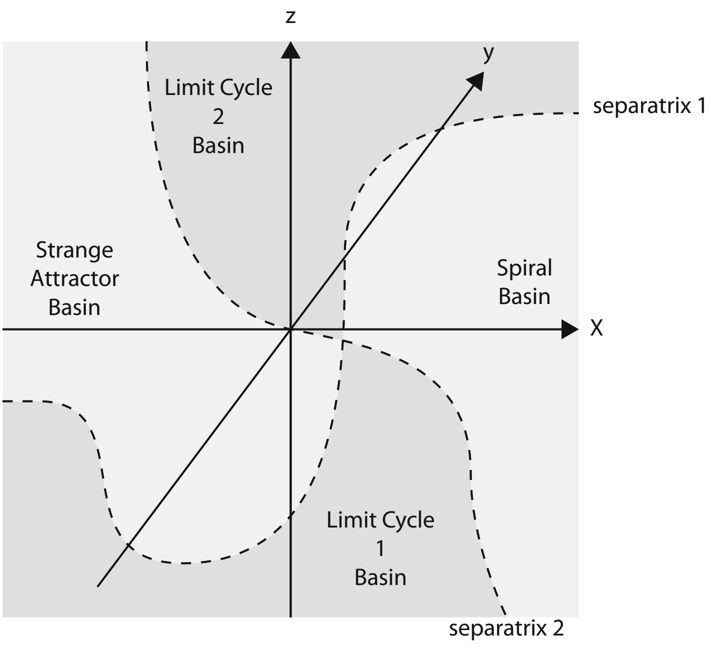


Fig. 3.20 Multiple basins of attraction in three dimensions bounded by separatices. Each basin might contain its own behavior

may have a higher dimensional version of the bursting behavior. Once a trajectory enters a basin, other parameters of the system start working in the background to make that basin progressively less and less stable. At some point, the basin becomes unstable, and the trajectory must enter some new basin of attraction.

These kinds of systems are what are called *meta-stable*, meaning that they are only temporarily stable. In fact, it is possible to have a system in which no particular basin of attraction is stable forever. The long-term trajectory jumps from one basin to another, displaying one behavior, then another, and then another, and so on. This picture becomes even more complex and interesting if the idea of hysteresis is included—it might be easier (or harder) to get into a basin than out of it. And it might be that it is easier (or harder) to get into a particular basin coming from some other basin of attraction. Again, a geometric intuition in phase space can help understand how this is possible.

All three mechanisms of moving from basin to basin may in fact work together. For example, it might be that when a trajectory enters a new basin of attraction, it is very stable. A perturbation from the outside might have little impact at this time. But, over time, as the basin becomes less stable, that same perturbation might be able to kick the trajectory to a new basin of attraction. What is more, this new basin of attraction might not be the same as the basin that would have eventually absorbed the trajectory without the perturbation. A critical aspect of metastable systems is therefore the importance of the timing (and direction) that outside perturbations can have on the future trajectory. Many complex systems display this type of sensitivity to the timing of events.

The idea of a metastable system might be useful in how to think about building a non-linear "machine." This would not be a machine in the traditional sense but rather one that can express various functions in a way that functionally adapts to outside influences and contexts. This view of the brain is expertly detailed in Scott Kelso's (1947–) *Dynamic Patterns*. For example, perhaps the formation and recall of memories operate on this kind of principle. Likewise, gene expression, economic patterns, and technology advancement might also be thought of as continually jumping from basin to basin. Metastable systems may also explain some aspects of learning systems as those that can structurally change the sequencing of how basins of attraction pass a trajectory between them, resulting in a coordinated ordering of different functions. Learning will be discussed in more depth in Chaps. 7 and 9.

3.5.5 Parallel Trajectories

The study of non-linear systems generally assumes that only one trajectory is being followed at a time. Is it possible that the same system might have multiple trajectories running at the same time? In some sense, each individual human is following their own trajectory through a larger sociocultural system. Likewise, we could imagine that neural firings in the brain are following multiple trajectories as they domino through the neural tissue. Mathematicians would argue that these kinds of systems can in fact be represented by a single much higher dimensional system that would have only one trajectory. It can often be helpful, however, to think of a single system that can multitask by allowing several trajectories to coexist. It is worth noting that the idea of multiple trajectories is similar to the agent-based models described in Chap. 2—the overall system provides the rules and container for interactions, and the trajectories represent individual agents.

The concept of multiple trajectories (although not framed in this way) is the centerpiece of *Modularity of Mind*, written by the philosopher Jerry Fodor (1935–), and *Society of Mind*, written by the cognitive scientist Marvin Minsky (1927–2016). Both introduced their ideas in the 1980s. Although their approaches are a bit different from one another, they described the mind as a system of simultaneously competing and cooperating functions. These functions are likely to be distributed throughout the brain, so they don't have a precise location. But they are all vying for expression in thought, attention, and behavior. Combining together metastability and multiple trajectories might explain some differences between our conscious and unconscious thoughts. The unconscious would be the sum total of all trajectories running in parallel. Consciousness would be the particular trajectory that is captured by our attention and followed through a metastable system. What is more, the spotlight of attention can jump when it is perturbed in some way to focus on some other unconscious trajectory, making it the subject of conscious thought. We will continue to map other complex systems concepts to the mind in later chapters.

3.5.6 Free Will as Chaos

Free will is often thought of as an agent's ability to create, evaluate, and execute on choices that are generated internally. This view seems to be in conflict with the idea that an agent must obey the deterministic laws of a universe. According to this reasoning, there is no room for free will because every decision would necessarily be the only decision that could be made. The experience of free will is an illusion created by our consciousness that doesn't enable us to see that the strings are being pulled by the laws of the universe. We mistakenly attribute causes to a "self." Chaos theory, at least to some, may offer a compromise. Perhaps everything is deterministic, but free will is instead the inability to predict decisions (your own and others) ahead of time. Free will then means something different—the inability to predict what will happen next. Chaos theory is a demonstration that deterministic but unpredictable system can in fact exist.

3.6 Questions

- Interview another person about some discipline with which they are more familiar than you. Ask them to explain their field. Keep listening until you find some concept that you can relate to the content of this chapter. To continue your conversation, share with them this connection. Keep the conversation going and see how deeply you can explore together. Write up what you discovered.

- In 1884, Edwin Abbott Abbott (1838–1926, yes his middle and last name are the same) published *Flatland: A Romance of Many Dimensions*. The book is meant to be a satire of Victorian English society but uses a variety of geometric shapes as characters. The main character, known as "Square," is a shape that lives in a two-dimensional world. In Flatland, the more sides you have, the more highly you are regarded. For example, priests have so many sides as to be nearly circular. On the other hand, women, satirically, are one-dimensional lines. Square is visited by a shape calling itself "Sphere," a three-dimensional object that is able to lift Square into the third dimension. Once this occurs, Square can see the world in a way that is fundamentally different than anyone else. There is much more to this short book than can be told here, and it is worth an evening to read it. In this chapter, we saw how adding in new dimensions, with new variables, added new possibilities. Where have you seen this type of phenomenon in the real world? When has viewing something from another "dimension" provide new insights?
- We discussed systems that might have a constant input that behave one way, but when the input is removed, the system suddenly changes its behavior. When a system develops with some persistent input, and then suddenly that input is taken away, the system will often radically change. What are some examples of systems that became dependent upon a constant input and then suddenly had that input taken away. What happened? You are encouraged to draw from historical examples.
- Speculate on the differential equations that would describe a parent-child relationship. What would the variable or variables be? What kinds of equilibria points (stable, unstable, limit cycle, strange attractors) might you need to design into the equations? What parameters might be adjusted over time to have various attractors and repellers come into and out of existence?
- The idea of critical slowdown was discussed as occurring when some event may be inevitable but proceeds slowly through some set of steps along the way. These cases are described colloquially as *bottlenecks* in a system. Identify a bottleneck in a system and discuss what you think the critical hold up might be.
- Many optical illusions are based upon a bifurcation in how we interpret a sensory input. The most famous is the Necker cube, a fancy term for the three-dimensional cube many of us draw on a flat sheet of paper. The image can flip back and forth between two equally possible cubes. What is important for our purposes is that you cannot see both at once. The cube is metastable, having two equally likely (stable) ways to be perceived. In fact, there have been many philosophical musings as to why this is the case, and even some claims that this phenomenon is the key to understanding a wide range of tricky topics, from the mind and consciousness to the "real" structure of the universe. What are some examples where two equally possible and consistent phenomena do not seem to be able to coexist? Speculate on why you think they cannot coexist.
- Consider a simple model of your bank account. Let the variable x represent how much money you have in your account. Can you draw a phase diagram that might

represent how you spend your money? Be sure to include values both above and below some "equilibrium" point. Explain this diagram along with your thought process.

- Describe an event in your life that seemed to be sensitive to context and perturbations from others. Use vocabulary from this chapter to discuss how the unfolding of this event in time was non-linear.
- Draw out a typical time period of your life (a day, week, month, year) and what a two-dimensional phase space might look like for you. This will be similar to Fig. 3.20 with multiple basins of attraction, representing actions or events, separated by separatrices. Annotate your phase space to explain your rationale.
- In 1985, Scott Kelso described how coordination between limbs can switch modes based upon frequency. He asked subjects to extend both index fingers and slowly begin wagging their fingers in opposite directions to the beat of a metronome. As the speed reaches some critical frequency, the subjects would switch to in-phase finger wagging. If they were then asked to slow down, the finger wagging remained in phase. Such a physical phenomenon is an example of a bifurcation (the switch from out of phase to in phase at a particular frequency) and hysteresis (the system remembers the direction by which it arrived at a particular frequency). What other examples of bifurcation and hysteresis can you identify in human-made or natural systems? Is there a single variable that acts as a bifurcation parameter?
- Natural habitats and ecosystems are dependent on the timing of a seasonal cycle. This cycle is not perfectly constant—the first spring thaw is at a different date each year. But ecosystems can bend within reason to accommodate this variability. If the seasonable variability oscillates too widely, or if it is skewed in a particular direction, however, that could have longer-term consequences. To make this argument more concrete, it has been observed that some species of bear are not hibernating as long or in some cases at all. As such they require more food intake over the course of a year are turning to new food sources. The result is a ripple effect throughout the ecosystem. What other non-environmental systems undergo slight variations in cycles? On what time scale is the cycle? What happens if this cycle is broken or disturbed in some way?
- It is often the case that individuals make decisions and take actions around some thematic attractor. Just as the dynamic attractors in this chapter captured nearby trajectories, a thematic attractor captures attention and is the organizational principle that grounds your world view. If you were to describe a thematic attractor in your life, what would it be? How do you think it impacts the way you operate in the world? Do you have more than one?
- Habits are patterns, generally in time, that form the cycles or rhythms of our lives. In *The Power of Habit*, Charles Duhigg (1974–) explains how habits are a feedback cycle between routines, rewards, and cues. Forming a habit is making this cycle tighter and less conscious, whereas breaking a habit is making the connection less tight. A related idea is expressed at a sociological level in the book *Nudge: Improving Decisions about Health, Wealth and Happiness* written by Richard Thaler (1945–, 2017 Nobel Prize Winner in Economics) and Cass

Sunstein (1954–) that advocates for governments and other powerful entities to think carefully about the processes, defaults, and policies that will encourage good habits to form. Pick one good and one bad habit that you have? How have they been encouraged (or discouraged) by some policy or process that is out in the world? What might you do to strengthen the feedback cycle of the good habit but weaken the cycle of the bad habit?

- Consider a long-running relationship that you have had in your life. Use ideas from this chapter to dissect the various phases of the relationship. What have been the cycles, stable points, bifurcations, hysteresis, chaotic, or burst dynamics?
- History is said to repeat itself. Do you believe this to be true or is history better described as being metastable or perhaps chaotic or even random? Are there examples of patterns that have emerged throughout history that were entirely new? Give real historical examples to back up your claims.
- Mariano Rivera is considered the greatest closing baseball pitcher of all time. He had a cut fastball that, even though everyone knew it was coming, would still fool them and result in broken bats and strikeouts, often late in important games. Rivera, however, was merely a good pitcher until a day in 1997. He claims, and there is evidence to support, that his cut fastball simply emerged one day. There was no change in grip or mechanics. Has there been some event in your life when all of a sudden something happened (a bifurcation) when you were able to surprisingly do something you couldn't beforehand?

3.7 Resources and Further Reading

A number of popular books have been written about non-linear dynamics, with the most read being James Gleick's *Chaos: Making a New Science*. *The Essence of Chaos* by Lorenz is also a nice summary. An excellent technical introduction can be found in Steven Strogatz's *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering*. Several of the texts mentioned within this chapter also are wonderful introductions.

There is much more to non-linear dynamics and chaos than was presented here. For example, an entire field of mathematics, called *ergodic theory*, deals with non-linear mapping of one space to another space. *Perturbation theory* and *bifurcation theory* are extensions of many of the ideas presented here. An online search for these terms will reveal many more focused books and articles.

Chapter 4
Pattern Formation

The human body develops over the course of a lifetime yet remains remarkably intact. In a series of experiments where atoms were radiolabeled, it was found that about 98% of atoms currently in your body will not be there in 1 year. Over the course of 5 years, nearly all atoms will be different. The pattern of your human form and function, however, remains largely coherent despite a turnover of the parts. Likewise the pattern of who you are changes but maintains consistent themes that span across long periods of time. A similar type of claim can be made more broadly about the stable patterns of cells, cities, and ecosystems.

Patterns are important in complex systems because they are based upon differences in time, space, or both. Differences in a complex systems are the foundation of how information and energy flows (as discussed in Chap. 6) based upon *gradients* that store energy and can perform work (as discussed in Chap. 7). In this chapter we will discuss how patterns form and the critical role they play in the development and maintenance of complex systems. Although it is easy to think of patterns as arising in space, such as spots or stripes, we will adopt a broad definition of a pattern as any coherent organization.

4.1 Symmetry Breaking

Defining what is meant by a pattern is not as simple a task as it may first appear. To help, mathematicians use the idea of symmetry. An object, whether physical or conceptual, is said to have symmetry if an operation can be performed on it that yields the same object back again. Such operations preserve the symmetry. Imagine a square (the object) and rotation (the operation). When the square is rotated by 90°, it will result in the same square. This rotation could be performed any number of times with the same result. The same cannot be said for angles of rotation other than 90°. A circle on the other hand can be rotated by any number of degrees and still be

J. Tranquillo, *An Introduction to Complex Systems*,
https://doi.org/10.1007/978-3-030-02589-2_4

indistinguishable from the original circle. To generalize, the object does not need to be a geometric image, and the operation does not need to take place in space. As such, the concept of symmetry can be applied to the functions and structures of a system.

The simplest way for a system to display symmetry is for it to be *homogeneous*, or the same everywhere. Imagine three identical bowls containing the same number of cashew nuts. If someone hid these bowls behind their back and presented one to you, you would not be able to tell which one you had received. A perfectly homogeneous object cannot contain a pattern.

Many systems contain different parts or functions, in which case they are *heterogeneous*. They may still have a high degree of symmetry by being *well mixed*, a kind of statistical state whereby all samples look the same. Such a state occurs when there is a random distribution. Imagine the same three bowls, but this time they are filled with a mix of three different types of nuts. Again you would not be able to tell the difference between the bowls because they are well mixed.

Patterns arise when a symmetry is broken. For our purposes this means that the statistics of a sample will vary. In the case of only cashew nuts, a simple way for symmetry to be broken would be if many more nuts were in one bowl, with less in the other two. An uneven distribution of the same parts will form the basis for gradients which will form the basis for much of Chap. 7. In the case of the mixed nuts, we could imagine the three types of nuts being separated into their own bowls. In both cases, a symmetry has been broken which enables you to discern a pattern.

In complex systems, patterns generally exist somewhere between perfect homogeneity and a well-mixed heterogeneity. As such, there are two pathways by which a pattern might form. A system could start perfectly well mixed and become more homogeneous or start homogeneous and become more mixed. In this section, we will explore both possibilities in the form of Benard cells (the breaking of homogeneity) and the Ising model (the breaking of a well-mixed heterogeneity). In both cases, breaking a symmetry is due to some form of tension that is built up and released within the system.

4.1.1 Benard Cells

Imagine slowly heating a thin layer of fluid from below. The liquid closest to the heat source will see a temperature rise, which will decrease the density of the fluid. This less dense fluid will be pushed upward toward the denser fluid at the surface. But, the cooler fluid on the surface is in the way, forming a sort of cap. Such a phenomenon happens in certain cities that are in a valley surrounded by mountains (e.g., Los Angeles, Salt Lake City) in what is called an inversion. Warm air (often polluted) is trapped under a cooler layer of air.

The heater at the bottom of the plate injects thermal energy into the system, while the air at the top of the fluid dispels heat energy. Each layer conducts heat energy upward to the next fluid layer, forming a gradient of temperature from top to bottom.

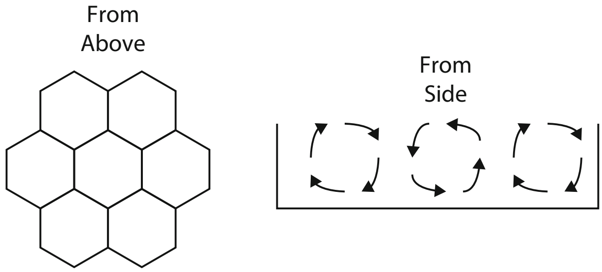


Fig. 4.1 Hexagonal Benard cells from above (left) and the side (right)

This is a break in symmetry, but each horizontal layer has the same temperature. When the heater is at a low setting, the heat energy will flow through the system, from one layer to the next, exiting the system at the top.

As the temperature of the heater is turned up, conduction will no longer be enough to expel all of the energy from the system. This excess energy is released through motion of the fluid, as less dense fluid at the bottom moves upward and the denser fluid at the top falls. This motion, known as convection (or more specifically Rayleigh-Benard convection), is seen in many other systems, including the atmospheric phenomena of wind. As channels for fluid flow open up, gradients are created within the horizontal layers. The hexagonal patterns that arise were first characterized by Henri Benard (1874–1939) and are called Benard cells. Benard cells, shown in Fig. 4.1, are the canonical example of a system where patterns emerge from an initial homogeneous state.

There are many factors that impact when cells will form, their shape, how big they will be, and how quickly fluid is recirculated. Even if the heat is turned up slowly, the patterns suddenly break symmetry—an example of emergence. Even more striking is that Benard cells can also display hysteresis—once the hexagonal patterns are formed, and the flow channels have opened up, the heat can be turned down, and the pattern will remain.

4.1.2 Ising Model

Patterns can also arise from an entirely heterogenous state, with the canonical example being the Ising model of ferromagnetism. In general charged particles form dipoles—little magnets that have a north and south orientation. In most objects the orientation of these north-south dipoles are randomly distributed (heterogeneous) and so cancel out. The result is an object with no apparent magnetism. In objects that are magnetic, the dipoles line up to form a more global pattern. The more aligned the dipoles, the stronger the magnetism.

The Ising model was first developed by Ernst Ising (1900–1998) in the 1920s as a simple way to describe how magnetism varies with temperature. In the simplified

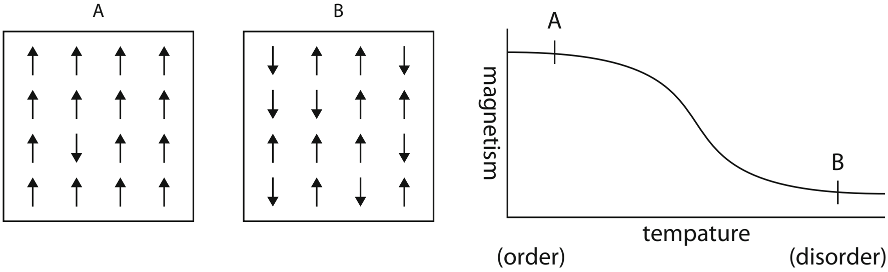


Fig. 4.2 Ising model of ferromagnetism at low (left, A) and high (middle, B) temperatures. At low temperatures, most dipoles align, and the material will be magnetic. At high temperatures the dipoles are not aligned, and the material loses magnetism. The phenomenon is summarized (right) in a plot of temperature versus magnetism

model, a two-dimensional grid of particles each having their own dipole orientation as shown in Fig. 4.2. To simplify further, the dipoles at a particular node can only be either spin up or spin down. The magnetic strength of the material is a global property that reflects how aligned the dipoles are.

Dipoles will interact with their neighbors, based upon the temperature of the material. At low temperatures, the interactions are strong, and therefore neighboring dipoles will tend to align. At higher temperatures the heat energy causes much jostling of the dipoles such that neighbors cannot communicate. As shown in Fig. 4.2, when temperature is high, there is no magnetism. When the temperature is lower, magnetism emerges.

In the Ising model, temperature acts as an *order parameter* in that it controls the patterns that emerge. We will explore order parameters more in Chap. 7, but in general they are some variable that controls whether a system will be more or less symmetric. The transition between magnetic and nonmagnetic behavior in the Ising model also serves as an example of a *second-order phase transition* which will be discussed more in Chap. 7.

4.1.3 Artists Breaking Symmetry

The idea of breaking symmetry is not new to science. Artists play with symmetry and the breaking of symmetry in their work, as shown in Fig. 4.3. Often times an artist will purposely break a symmetry to draw, or sometimes confuse, our attention. Many examples are embedded in a wonderful book by Margaret Livingston (1950–) called *Vision and Art: The Biology of Seeing*. One method that artists use is to play with the difference between figure (the attention draw) and ground (the background). Another approach is to have multiple possible stable

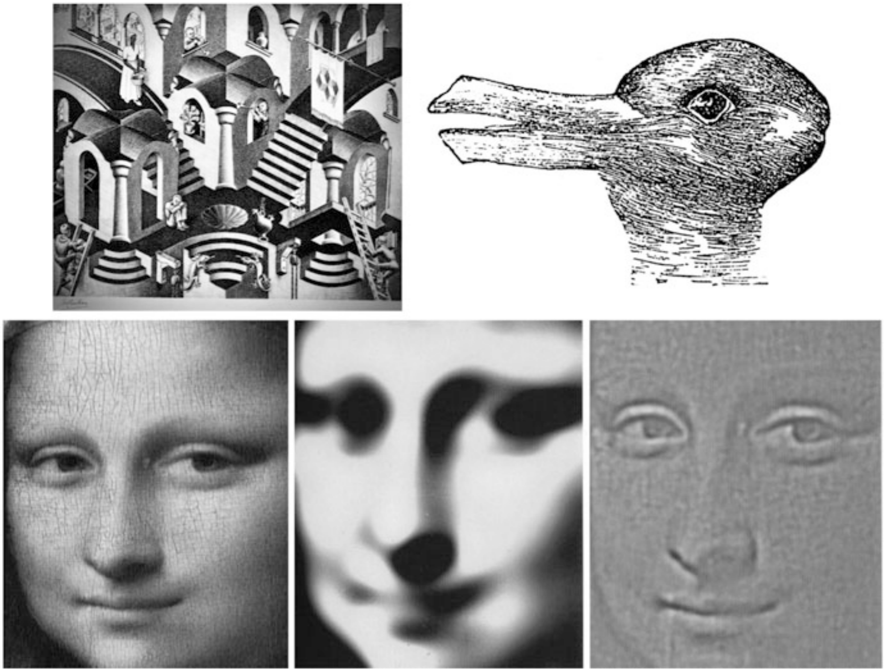


Fig. 4.3 Pattern breaking is used in artwork and illusions. The top figures illustrate visual contradictions. The bottom shows that the Mona Lisa smile only appears in the shadows that are seen holistically in peripheral vision

points of view, for example the rabbit-duck illusion (upper right panel of Fig. 4.3). Most optical illusions force your perception to choose between two metastable states. M.C. Escher (1898–1972) played with perception in nearly all of his work. Likewise, many impressionistic paintings have a contrast in scale, with the small scale being composed of what appear to be random colorful points, while at the larger scale meaningful patterns emerge when viewed from farther away. Even the Mona Lisa contains interesting contradictions in patterns that give rise to her enigmatic smile. For centuries there seemed to be something strange about the smile. When you look directly at her mouth it appears to be straight, but the minute you look at her eyes, you detect a slight smirk. The slight change in your visual attention from mouth to eyes somehow brings about that enigmatic smile. This is a classic example of a break in symmetry that occurs when viewing the same image through different perspectives. So as not to ruin the explanation, only a hint is contained in Fig. 4.3 with a more thorough discussion in Livingston's book.

4.2 Turing Pattern Formation

Alan Turing (1912–1954) is best known for his work on the theory of computing, which we will discuss in Chap. 6. Like many other brilliant thinkers, his mind wandered to many other topics. One of those was the idea of pattern generation. In 1952 he wrote a classic article titled, "The Chemical Basis of Morphogenesis." Turing's hypothesis was that the patterns that we see in living organisms, such as spots and stripes, arise during development as spatial gradients of some chemical that he called morphogens. The dynamics of such a system would also explain how a small mass of nondescript cells could differentiate into a variety of cell types that would group together into more complex patterns. In Turing's model, cellular differentiation is a break in symmetry that results in a pattern.

The morphogen theory is based upon the reaction-diffusion equation:

$$\frac{du}{dt} = D\frac{d^2u}{dx^2} + f(u)$$

where u is the concentration of the morphogen. As this equation is a partial differential equation in time and space, it is worth taking apart. Just as in Chap. 2, individual nodes change over time based upon internal rules and the state of their neighbors. The $D\frac{d^2u}{dx^2}$ term establishes how neighbors will interact through simple diffusion. The classic example of diffusion is if you squeeze a single drop of food coloring into still water. Over time that drop will spread out until the entire system become homogeneous. Diffusion is inherently a homogenizing force, tending to spread out differences over time. The value of D controls how much neighbors matter and therefore how quickly the process of diffusion occurs.

The function $f(u)$, sometimes called a forcing function, describes the internal rules of each point. When disconnected from neighbors, it is $f(u)$ that will determine how a particular node will behave. There are many possible forms for these rules, but $f(u)$ could be a spiral, limit cycle or any of the other behaviors explored in Chap. 3. $f(u)$ leads to variations of u in space, thereby counteracting the homogenization due to diffusion.

What Turing showed was that these two competing processes can result in instabilities, symmetry breaking, and the emergence of patterns. This is similar to the formation of Benard cells where two forces are in tension, with the resolution being a coherent pattern. What is more, additional forcing functions might come from outside the system adding another dimension to these dynamics. We will explore this idea more in Chap. 7 in discussing open systems that are far from equilibrium.

4.2.1 Growth and Differentiation

Turing's original motivation was to present a mechanism by which dividing cells could self-organize into patterns. It is important to point out that Turing's article came out in 1952, the year before Watson and Crick published their landmark paper on DNA as the molecular basis of heredity. Much of the genetic process at a molecular level was therefore not known, making Turing's guesses all that much more impressive. What is below is a modern version of Turning's reasoning.

Imagine a mass of undifferentiated stem cells that begin to multiply. Each cell contains all of the DNA needed to create a variety of different cell types, ranging from muscle to liver to bone. The only difference between these cells is which genes are being expressed. Differentiation is the turning on (excitation or promotion) or off (inhibition or repression) of particular genes that will express the proteins that will define a particular cell type. Turing's assumption was that as cells divide they produce chemical signals, what he called a morphogen, that would trigger the activation (or inhibition) of particular genes. A spatial distribution in these morphogens would then trigger different genes, and therefore different cell types, to be expressed in different locations.

To illustrate, imagine a small sphere of cells as in Fig. 4.4. Each cell, in the process of dividing, gives off some morphogen. But cells at the center (1) will be bathed in more morphogen than those cells near the boundary (3). The resulting gradient can be used to trigger a first change in genetic differentiation. The cells at the center would become differentiated and stop producing the morphogen. Cells at the boundary would continue to divide and grow outward, leading to a new gradient, again due to boundaries, that could drive another wave of differentiation (the region of Fig. 4.4 denoted as (2)). The end result would be a sphere with different cell types.

The example given was for a sphere because it is easier to imagine, but a similar kind of gradient in a chemical can drive differentiation in any geometry. When

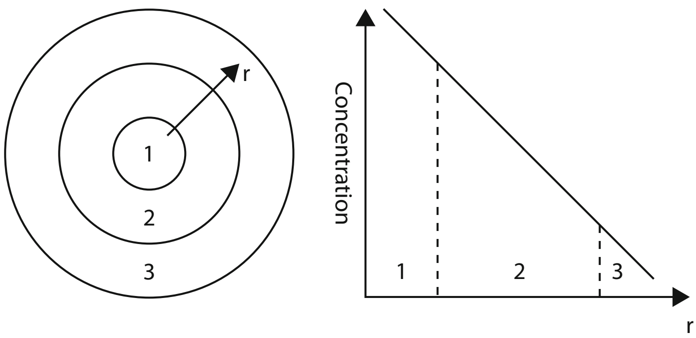


Fig. 4.4 Turning morphogenesis in a sphere (left panel). The morphogen falls in concentration the farther from the center (right panel). The cell regions, numbered 1, 2, and 3, are triggered to differentiate based upon the concentration

multiple morphogens, some excitatory and some inhibitory, are involved, they might work in concert. Likewise, once a cell has completed one round of differentiation, it may begin to produce a different morphogen that drives further specialization.

The most common example of this phenomenon comes from the development of a multicellular organism and the development of various tissue types (e.g., muscle, neuron, bone). All derive from the same genetic code. The roughly 12,000 species of ants provide yet another example. All ants within a particular hill have the same genetic makeup, yet they differentiate into queens, mates for the queen, and workers. Only a few types of ants, when numbering in the hundreds of thousands, give rise to the phenomena of an ant colony. Similar arguments could be made for workers in a company or people in an economy.

4.2.2 Spirals, Spot, and Stripes

The model Turing proposed has been used by others to explain how patterns of hair, feathers, ribs, limbs, and other features may emerge from a homogeneous mass. By tuning the balance between $f(u)$ and the rate of diffusion, a wide range of patterns can be generated, as shown in Fig. 4.5.

Turing's model can also explain the origin of spirals. Recall that spirals are a pattern that repeat over and over again in nature in different media, over an enormous range of time and space scales. Such a display is an example of what physicists would call universality—a pattern or phenomenon that occurs regardless of the materials, time, or spatial scales. It turns out that any system that can support reaction and diffusion has the potential to generate spirals.

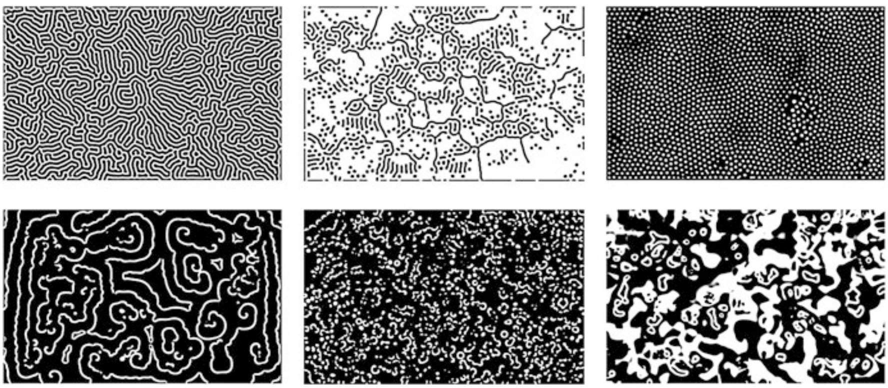


Fig. 4.5 Natural patterns are often created through the balance of reaction and diffusion. Changing the balance between the two will result in different patterns. Patterns created using the Gray-Scott system of equations implemented at https://pmneila.github.io/jsexp/grayscott/

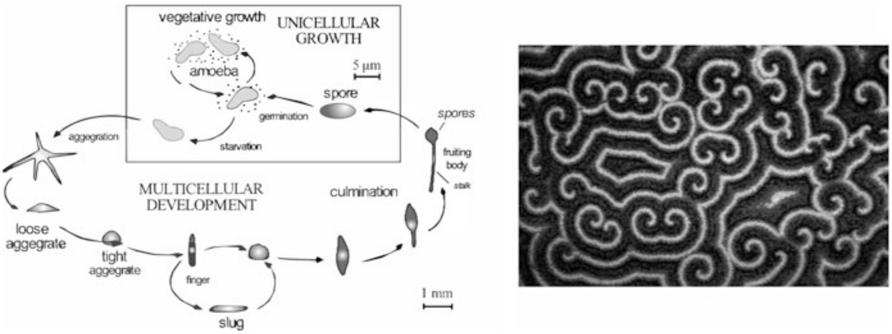


Fig. 4.6 Slime molds alternate between being independent agents and social collections of many agents. Under the right conditions, slime molds will naturally generate spirals. Underlying these patterns is a balance of reaction and diffusion. Figure generated by Florian Siegert

One of the most unusual and striking displays of differentiation and the generation of spirals is found in slime molds. The life cycle of a slime mold is shown on the left side of Fig. 4.6. The organism is a single cell and lives most of its life as an independent entity feeding mostly on dead organic matter. When food becomes scarce, however, an individual slime mold sends out a signal that helps it find and gather with other individuals. In close proximity, the cells can share various chemicals that act as morphogens that trigger a kind of differentiation of the individuals. Over the next several hours, the homogeneous mass, which can be nearly 30 cm across, will move much like a common garden slug. All the while the individuals are undergoing a differentiation whereby parts of the mass differentiation into an anchor, stalk, and kind of flower. This flower will eventually burst open, spreading some individuals off to new areas. From an evolutionary perspective, this cycle allows the slime mold to operate as individuals when there is food available but then redistribute widely when food is limited. Some evolutionary biologists claim the slime mold is an early "experiment" in how multicellular organisms might work, long before they became commonplace on our planet.

Slime molds in nature use cyclic AMP to communicate in three dimensions. In a lab, however, sheets of individual slime molds can be induced to display spiral patterns, as in the right-hand side of Fig. 4.6. This speaks to the idea of universality—any reaction-diffusion system, under the right conditions, can display spirals.

4.3 Limit Cycles as Patterns in Time

Patterns in time exist all around us but are sometimes difficult to detect because time flows from one moment to the next. In Chap. 3 the idea of a limit cycle was introduced as a pattern that repeats over time, with its own frequency and robustness in the face of perturbations. Limit cycles range from pacemaker cells in the heart to seasonal variations in shopping habits. In this section we will revisit limit cycles as patterns in time.

4.3.1 Pacemakers and Other Rhythms

Limit cycles govern many of the most basic biological rhythms. There are very short rhythms that we don't even notice, for example, the stroke and reset of the actin-myosin fibers that lead to muscle contraction. There are also rhythms that we do notice such as respiration and our heart rate. While the exact rate might vary, the pattern is roughly the same for each breath or beat. The circadian rhythm governs the cycle of protein expression throughout a day. Longer cycles may last months or even years. An individual organism can also be considered as one larger cycle that repeats itself over and over again in an evolutionary development cycle. One way to think about a life is as a series of multiple rhythms, all operating at different rates, superimposed on top of one another.

Limit cycles are also present in human-made systems. Some of them are linked to human rhythms, such as human sleep-wake cycles being intertwined with when businesses are typically open. Yearly cycles, such as shopping during holidays and the spike in bathing suit sales in late spring and early summer, are tuned to the change in seasons. Others, such as the boom-bust economic cycle or political orientations, may span generations or even centuries.

A disruption of a regular pattern can have major consequences, especially when it drives or interacts with other cycles. A cardiac arrhythmia is the loss of regular beating. The disruption of the usual cascade of molecular switches that are flipped on and off to control cell division can become a cancer. Such disruptions most often move a system out of homeostasis by placing one or more variables of the system outside of acceptable ranges. Likewise, major economic or political events can disrupt business cycles.

4.3.2 Sychronization

A striking phenomenon can occur when several limit cycles synchronize. A classic example is the bioluminescence of fireflies (Photinus carolinus) in the Great Smoky Mountains National Park and Allegheny National Forest during the summer months. Individual fireflies contain the compound luciferin, an organic molecule that, when combined with calcium and ATP, will send off a pulse of light. During summer nights, fireflies will suck air into their abdomens that will mix with the other compounds and lead to a single blink. What is special about the Great Smoky Mountains and Allegheny is that thousands of fireflies will synchronize their blinking to the same rhythm, leading to an eerie periodic illumination of the forest.

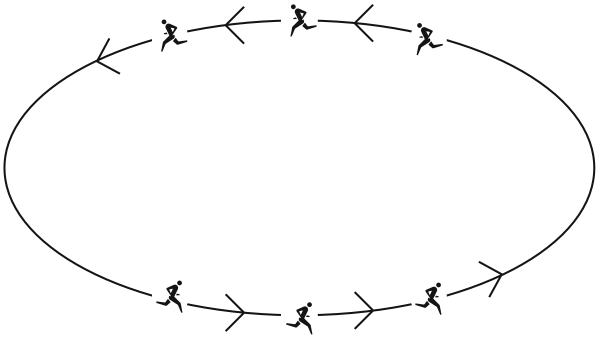


Fig. 4.7 Groups of runners synchronizing (out of phase in this case) on a track

Individual oscillators may influence one another through some form of coupling. A simple demonstration is through *phase coupling* of runners going around a track, as described by Steven Strogatz (1959–) in his book *Sync: How Order Emerges From Chaos in the Universe, Nature, and Daily Life*. As shown in Fig. 4.7, all runners are moving through all phases at the same rate, but two groups have formed. They will in effect synchronize such that the groups are exactly 180° out of phase.

A similar idea can be applied to cardiac arrhythmias. Individual cells in the heart have their own intrinsic rhythm at which they will beat. But, when placed together in a group, they will synchronize to some agreed-upon rate (usually whichever cells are beating fastest).

The original discovery of synchronization is traced back to 1665 as Christiaan Huygens (1629–1695), the inventor of the pendulum clock, was sick in bed. In his room he had several examples of his invention. He noticed over the extended time that he was in bed that eventually the pendulums on his clocks would all find themselves in the same phase. Huygens started to experiment with how this could be possible. He hypothesized that as long as two clocks were coupled, for example, being connected to the same wall, they would synchronize over time. This remained the dominate idea of how clocks would synchronize and was only mathematically demonstrated in 2016.

Consider two identical van der Pol oscillators. The first oscillator is

$$\frac{dx_1}{dt} = y_1$$
$$\frac{dy_1}{dt} = \mu_1\left(1 - x_1^2\right)y_1 - x_1$$

and the second is

$$\frac{dx_2}{dt} = y_2$$

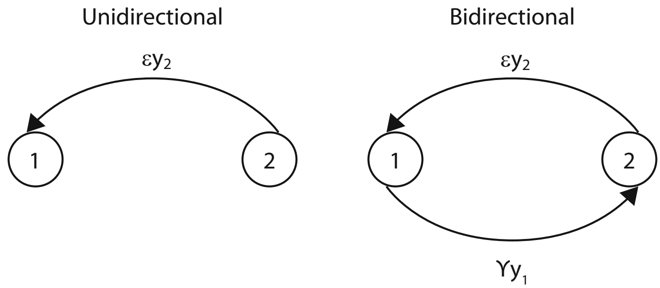


Fig. 4.8 Unidirectional (left) and bidirectional (right) coupling of oscillators

$$\frac{dy_2}{dt} = \mu_2\left(1 - x_2^2\right)y_2 - x_2$$

On their own, both will form a limit cycle in phase space. Because μ_1 and μ_2 are different, the oscillators will take on different shapes and rates in phase space.

Coupling the two oscillators means entangling the variables x and y in some way. A graphical example is shown on the left side of Fig. 4.8 and described by

$$\frac{dx_1}{dt} = y_1 - \epsilon y_2$$
$$\frac{dy_1}{dt} = \mu_1\left(1 - x_1^2\right)y_1 - x_1$$
$$\frac{dx_2}{dt} = y_2$$
$$\frac{dy_2}{dt} = \mu_2\left(1 - x_2^2\right)y_2 - x_2$$

The term $-\epsilon y_2$ has been added to the dynamics of x_1 to allow the second system to "talk" to the first system. This form of coupling is unidirectional, as system one does not talk back to system two. What is striking is that the addition of this term (if ϵ is large enough) will in fact cause the two oscillators to become *phase locked* together. Oscillator 2 still operates on its own and will drive oscillator 1 to be more like itself.

There are many ways coupling might occur. For example, the oscillators may be bidirectionally coupled, as in the right-hand side of Fig. 4.8, by adding a similar term (where γ is a constant) to the dynamics of x_2.

$$\frac{dx_1}{dt} = y_1 - \epsilon y_2$$
$$\frac{dy_1}{dt} = \mu_1\left(1 - x_1^2\right)y_1 - x_1$$

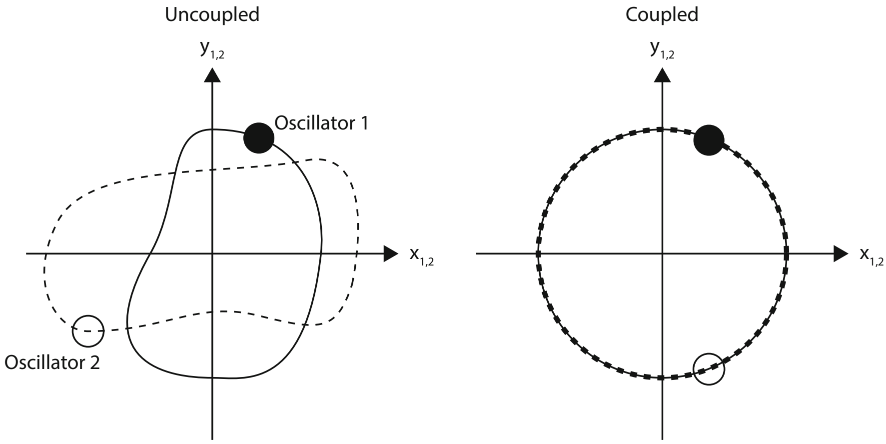


Fig. 4.9 Two oscillators before (left) and after (right) being coupled. Before coupling they operate as independent trajectories. After coupling they follow one another 180° out of phase

$$\frac{dx_2}{dt} = y_2 - \gamma y_1$$
$$\frac{dy_2}{dt} = \mu_2\left(1 - x_2^2\right)y_2 - x_2$$

Now the two oscillators are bidirectionally coupled together. What generally arises is that they settle upon a third limit cycle that is a compromise. This is shown on the right-hand side of Fig. 4.9. You may notice that in Figs. 4.7 and 4.9, the two trajectories are 180° out of phase (as indicated by the closed and open circles). This is because in both cases we have assumed inhibitory connections (note the "-" sign in the coupling). In the track example we could imagine that the two groups of runners do not like one another but still want to run at the same rate. A dynamically stable equilibrium is to move 180° out of phase. On the other hand, if everyone did like one another, it would be a stable solution for all runners to form one large pack.

The concept of coupling and synchronization can be applied to many aspects of our lives. For example two people may have their own habits that govern the cycles of their respective days. When they live together, there are many ways they can combine their habits. It could be that one person does not change their habit and it is up to the other person to conform. They might also recombine their various habits. Alternatively, they may create entirely new habits that emerge over time.

There are many more mathematical ways that two (or more) limit cycles can be coupled. In the example above, the two oscillators could have been coupled together in the y variable. Likewise, coupling could be through some non-linear term, such as xy. In some systems, the coupling might even be more complex than the oscillators. Such is the case of a synapse between neurons where the coupling is itself a complex function of the pre- and postsynaptic electrical potentials in both space and time.

4.3.3 Complex Synchronization

The oscillators do not need to be identical, perhaps varying in magnitude, natural oscillation frequency, or the shape of the limit cycle. To use the analogy of runners going around a track, what if the runners are not going at the same rate? In general slower runners will band together, and faster runners will band together. Over time the faster runners might lap the slower runners. For example, perhaps the faster runners complete ten laps for every nine that the slower runners complete.

Synchronization may become more complex than one to one when multiple frequencies are present, a phenomenon known as *beating*. Rather, there will be a lock in to some other ratio, perhaps 2:1, or 4:3 or as above 10:9. Many athletes will have encountered this phenomenon in the relationship between their heart rate, respiration, and gait (e.g., swimming stroke rate, running or cycling or rowing turnover rate). What emerges is that there may be six revolutions of the gait, for every four heart beats, for every one inspiration-expiration cycle. In fact, in kinesthesiology these ratios can be used to measure progress and efficiency of an athlete. What is more striking is that there is evidence that there are particular ratios that seem to never appear. Similar ideas can be applied to many other systems that have oscillators that have varying natural rates. Making these maps, called Arnold Tongues after Vladimir Arnold (1937–2010), experimentally is very challenging but can reveal important insights into the behavior of a complex system.

4.3.4 Swimming, Walking, and Flying

All limit cycles go through phases that can be mapped in some way to the hands of a clock. When at noon they are in one state. When at one they are in a different state and so on. What is important is that we can begin to think about how these states, various phases of a limit cycle, can be used to coordinate other events or functions. In this sense one limit cycle can be a master command signal that generates a repeatable sequence of other actions. In other words, when the cycle is at noon, it will trigger a particular series of events. When the cycle reaches one, it will trigger a different series of events and so on. In this way a limit cycle can serve as the dynamical glue that allows functions to recombine with one another to form a more complex process.

One of the classic models of limit cycles controlling functions is animal locomotion, namely, walking, swimming, and flying. Walking is driven by the precisely timed firing of particular muscles. Consider the motion of your right leg when you walk. You will notice that there is a cycle that begins when your heel hits the ground, proceeds to rolling your foot, to toe-off, and then to the swing of your leg. Each of these actions involves a complex series of muscle contractions throughout your entire body. This cycle can be thought of as being mapped to the hands of a clock, shown in Fig. 4.10, with each complete rotation being one

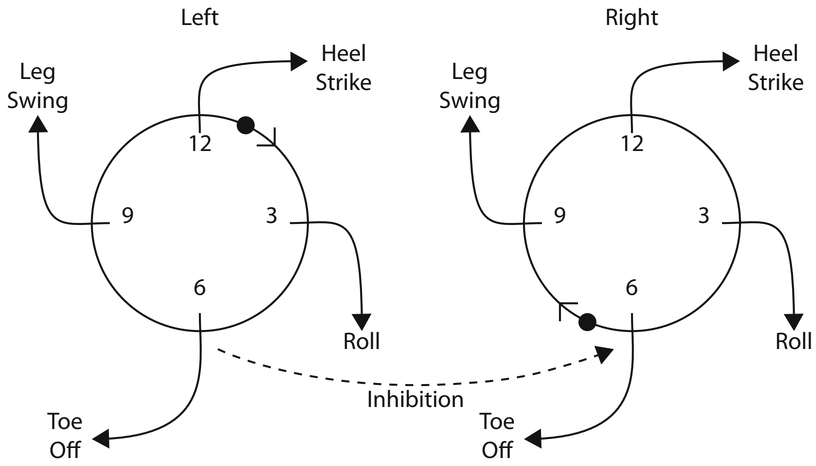


Fig. 4.10 Coupled oscillator model of locomotion. Each state along the limit cycle will trigger particular muscle contractions. The limit cycles on the left and right side are mirrors of one another, which due to inhibition at one point will be 180° out of phase. Such a situation enables bipedal walking

complete cycle of your gait. What is more, neuroscientists have found that small groups of neurons exist, called a *central pattern generator*, that act as a master controller. A central pattern generator can be perturbed by causing it to miss fire (e.g., if you trip), but it will eventually settle back to a regular pattern. Such neural control has been found in locomotion ranging from jellyfish and worms to wasps, snakes, and people. As stated by Hugh Wilson, the author of *Spikes, Decisions, and Actions*, “The brain does not micro-manage locomotion.”

What we described is for only one foot. The other foot goes through exactly the same progression, only 180° out of phase (or in the clock analogy, offset by 6 hours). In reality there are two neural circuits that control the left and right side of the body, both cycling through their limit cycles. To coordinate the two, all that needs to happen is for them to be exactly 180° out of phase, as shown in Fig. 4.10. This is in fact easy to accomplish by coupling the two oscillating circuits together through inhibition. The purpose of inhibition is to ensure that they (in the analogy of the two runner groups that do not like one another) will repel each other. A biped learning to walk (run, swim, or fly) is a tuning of the limit cycles that properly time muscle firings, combined with inhibition across the body. Of course this kind of synchronization becomes slightly more complex with four (or more) legs, but the principle remains the same.

In swimming a new complexity emerges. Butterfly and breast stroke are about synchronizing the motion of one’s arms with the same phase, while freestyle and backstroke are about having exactly the opposite phase. The circuits that control arm motions must therefore be flexible enough to be either in-phase or out-of-phase synchronized. Likewise, some animals hop, gallop, or travel using other gaits that are variations of the synchronization of neural-muscular limit cycles.

Central pattern generators are at the heart of some very basic functions such as the rhythmic contraction of intestinal smooth muscles that drives peristaltic digestion. Outside of the neural world, there are predictable annual changes in markets, the life and death cycle of an ant colony, and the molecular rhythms of the circadian rhythm, each of which displays robustness in the face of external forces.

A basic clock moves from state to state in a predictable and repeatable way. But it is possible that given some condition, it might reverse direction or perhaps skip a state when a signal is received. Alternatively, there might be a reset signal that moves the clock to a particular initial state. All of these are present in biological, economic, and ecological clocks. Clocks might also link to some sensor that can detect an externally repeating event (e.g., the sun rising). Likewise interacting clocks might be superimposed to create a more complex clock.

4.3.5 Delays

As systems must contain some physical basis, they also must follow the laws of the universe. As such, delays, even if they are hard to measure, are a part of all systems. For example, if it has rained very hard, a river will swell, and it will take time for the water level to come down. Likewise, you may need to wait for your hot water in the morning as the cool water in the pipes is flushed out. The volume of cold water in your pipes must be moved to make room for hot water. Delays generally mean that a system cannot react instantaneously. In some cases this can be very benefiting in preventing a system from simply reacting to an internal change or external perturbation. A delay enables planning, marshaling of resources, and coordination of the expression of functions. In the context of a central pattern generator, delays will naturally set the rate at which actions are generated, enabling a system to perform the same sequence of actions at a slower or faster rate. As such managing delays in a system can be the source of learning and optimization.

4.3.6 Bursts and Queuing

There are some phenomena that defy a kind of cyclic pattern and are seemingly unpredictable. Earthquakes, bridge failures, the deaths of many famous people in a short span of time, stock market crashes, and unexpected good fortune are all examples. This was a question that Albert Barabasi (1967–) explored in his book *Bursts*. We all experience the phenomena of "when it rains it pours," for example, when our work load seems to pile up beyond what we can handle. Then there are down times when we do our best to catch up. Although one might suspect that work should come in and get done at a statistically steady rate, it does not seem that way. The question is if there are perhaps underlying patterns that govern the pattern of bursts.

Consider the work you receive as being generated by a complex system. That complex system has bottlenecks that are inherent in the people and processes that make up the system. One person checking something off their list will often become an action item on someone else's list, as work on a larger project moves throughout a system. Imagine that there is a group that often passes you work and that they are swamped with some other project. For a while they will not be sending you work. Once they finish their larger project, their requests on the backlog will all of a sudden be passed to you. From your perspective it will appear as a "burst" in work. Of course this is a very simplistic view. In reality you might normally be passed work from six groups. If five of those groups were impacted by the same large project, they will end up all dumping work on you at the same time.

Bottlenecks (either functional or structural) collect future work to be done. The result is that when the bottleneck is released a burst can appear downstream. Consider what happens when you suddenly receive many requests all at once. You become the bottleneck in the system, and other groups will now wait for you.

Bursts are a different type of pattern than a limit cycle. In a limit cycle one state predictably follows another. In bursts there is unpredictability that can lead to delays and more unpredictability. This same idea has been applied to many other phenomena including earthquakes, asteroid strikes, technological breakthroughs, and stock market fluctuations. Statistically what seems to emerge from these systems is known as a *power law*, which means there is no characteristic delay between events. Power laws will receive a more formal treatment later in this chapter and will be one of core ideas of Chap. 10.

4.3.7 The Simple Machines of Complex Systems

The field of complex systems is a work in progress. A modest proposal is that there may be a series of basic phenomenon or "machines" that nearly all complex systems express in some way. The word machine is here used in a particular historical context. The Ancient Greeks identified six basic mechanical machines from which they believed other more complex machines would be derived. They were the lever, pulley, screw, wheel, inclined plane, and wedge. If you look around at the technology in our world, from elevators to cars to rockets, you will see these six basic machines almost everywhere. With the electronic revolution, some have proposed that there are analogous electronic machines. These would be oscillators, amplifiers, transistors, transformers, and so on. Most electrical machines can be constructed from these most basic components. One could in fact begin to think of simple machines (or more broadly phenomenon) in almost any field that are recombined to create the dynamics of that field. A particular machine is then the unique expression and recombination of more basic elements.

Due to the range of complex systems, these machines would necessarily be abstract and not dependent upon material, time, or spatial scales. If such an approach is helpful, it is almost certain that repeatable, stable rhythms that can be synchronized in some fashion would be one of those machines. What is more, there might be parallels between the machines from different domains. For example, a limit cycle is in reality the phase space version of a wheel as well as an electrical oscillator—a phenomenon that repeats over and over again. In this respect, there may be a much deeper underlying connection between complex system machines and the machines from other disciplines. We will explore this idea in a variety of ways throughout the remainder of the text.

4.4 Fractals as Patterns

We often think of patterns as being repeating in space (e.g., spots, stripes, and spirals) or in time (e.g., limit cycles, bursts). In this section, we will explore the idea of fractals as patterns.

4.4.1 The Logistic Equation Revisited

In Chap. 2, the discrete logistics equation was introduced:

$$x_{n+1} = rx_n(1 - x_n)$$

Depending upon the parameter r, very different dynamics could be expressed. These variations can be summed up in a single bifurcation map that plots r versus the stable values (after any transients have died out), of x. The bifurcation map is shown in Fig. 4.11. You may recall that when r is below 3.0, the system will move toward a single stable point. As r is increased, however, the system will begin to oscillate

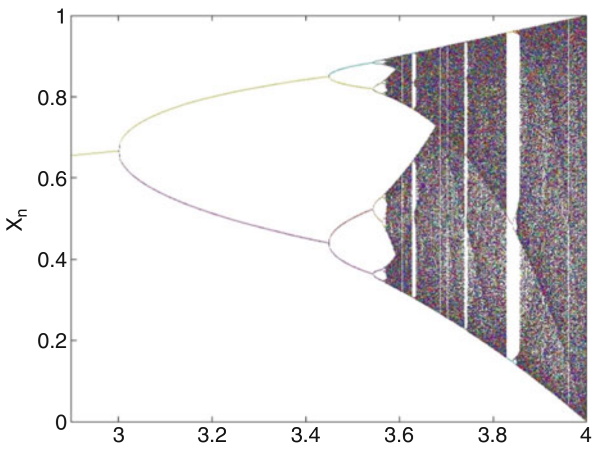


Fig. 4.11 The bifurcation map of the discrete logistic equation showing period doubling as a pathway to chaotic behavior. The map is also self-similar in that each small portion is a copy of the larger pattern

Table 4.1 Bifurcation table for logistics equation showing the convergence to the Feigenbaum constant

n	Period	Bifurcation parameter (r)	Ratio $\left(\frac{r_{n-1}-r_{n-2}}{r_n-r_{n-1}}\right)$
1	2	3.0	–
2	4	3.449	–
3	8	3.544	4.7514
4	16	3.564	4.6562
5	32	3.658	4.6683
6	64	3.569	4.6686
7	128	3.5696	4.6692
8	256	3.5699	4.6694

between two values, then four, then eight, and so on, in what is known as *period doubling*, until eventually it will achieve an infinite period oscillation, or chaos. When in the realm of chaos, the signal x appears completely random. What is more, as r is increased, small pockets appear where x returns to some finite period oscillation, only to move back into chaos.

Period doubling turns out to be a universal pathway to chaos. Mitchell Feigenbaum (1944–), while working at Los Alamos National Lab, would pass time during boring meetings by computing the logistics equation by hand. What he found was that the location of doublings always followed a predictable pattern as shown in Table 4.1. More specifically, successive doublings (calculated as $\frac{r_{n-1}-r_{n-2}}{r_n-r_{n-1}}$) move toward a particular constant—4.669201609. . . . What is even more surprising is that any system that undergoes period doublings on the way to chaos does so by this same ratio. So the constant, now known as Feigenbaum's constant, seems to be a universal number, similar to π.

The logistics equation is the most studied example of *period doublings* as a route to chaos. There are, however, many other mathematical and real systems that transition to chaos in the same way. For example, laminar fluid flow becomes turbulent through period doublings as the flow rate is increased. You may have observed this phenomenon when you turn on your faucet. When the flow is low, the stream of water is relatively smooth but becomes more and more turbulent as the flow increases.

4.4.2 *Fractals and Power Laws*

The logistic bifurcation map is one example of a *fractal*, an object that looks similar when different parts are explored. For example, each branch of the logistic bifurcation map looks "like" the overall logistic map. It is not exactly the same in every way, but there seems to be a pattern that is repeated over and over again at different scales. To characterize self-similarity, mathematicians created the concept of *fractal dimension*.

We tend to think of a point as having zero dimensions, a line as having one dimension, a plane having a dimension of two, and a solid object as having three dimensions. A fractal dimension describes objects that fall between dimensions. What does it mean for an object to have a dimension of 1.63?

The mathematical definition of dimension is when a variable of a system is represented as a power of another variable of that same system. More formally, x and y have a power law relationship if

$$y = y_o x^a$$

where a is the power and y_o is a constant. If we have actual values for x and y, then we can find the slope on a log-log plot as

$$\log(y) = a \log(x) + \log(y_o)$$

This is simply in the form of a line where x is the size of some measurement tool and y is the total measurement. Remember $y = mx + b$; only in this case m is the dimension a, and b is the intercept $\log(y_o)$.

To make this idea more concrete, consider the Koch curve, first described by Niels Fabian Helge von Koch (1870–1924). To generate a Koch curve, we start with a straight line. On each iteration, every straight line is transformed into four line segments, with the middle two forming an upside-down "V" shape. This process is then continued for some number of iterations, as shown in the left-hand side of Fig. 4.12. In fact, the Koch curve, when iterated upon an infinite number of times, is only edges, meaning that the derivative does not exist anywhere.

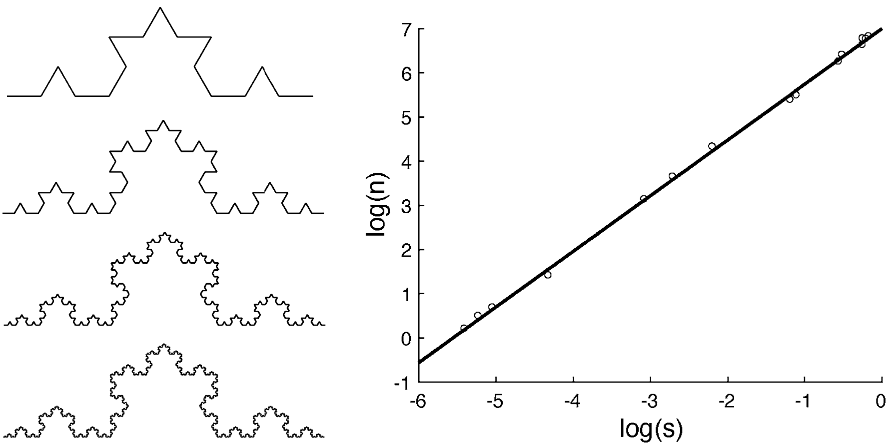


Fig. 4.12 The generation of a Koch curve (left) along with measurement of the fractal dimension. The middle panel demonstrates using smaller and smaller boxes to cover the curve. The right panel is a log-log plot of the box size plotted against the number of boxes needed to cover the curve

Imagine trying to cover the Koch curve with various sized boxes. One could start with moderately sized boxes (the measuring tool, or x above). Anytime a box contains some part of the curve, it is counted and added to the total area (y above). This process will yield an $x - y$ pair. Then the box size is decreased, and a new $x - y$ pair is generated. When plotted on a log-log plot, as shown in the right panel of Fig. 4.12 the slope can be found. For the Koch curve, $a = 1.26186\ldots$, where a will be called the fractal dimension of the curve.

For comparison, imagine this same procedure applied to a straight line. What we find is that for a line, the number of boxes needed scales linearly, meaning with a power of $a = 1$. A similar result would be found for a purely two-dimensional object in which case $a = 2$. The Koch curve, on the other hand, has a fractional dimension. Another way to see this is that the Koch curve moves into a second dimension—there would be no way to draw a Koch curve in one dimension. At the same time it does not fill the second dimension. That is the intuitive reason; it has a fractional dimension between 1 and 2.

Objects that obey power laws are sometimes called *scale invariant* or *self-similar*. In space, this means that if you compare images from a close and a far vantage point, it will be difficult to tell the difference. Practically what this means is that there are some phenomena, for example, the cratered landscape of the moon shown in Fig. 4.13 that when photographed from a few meters and again from several kilometers will be virtually indistinguishable. Yet another phrase used to describe a power law is that the object has *no characteristic scale*, meaning that there is no inherent scale bar that could be put on a diagram. In real systems, the power law eventually breaks down—if we kept zooming into the image of the moon, we would eventually begin seeing structures (e.g., individual grains of powder) that would give away the scale. Likewise, zooming out would eventually reveal the curvature of the moon, again giving away the scale.

When viewed in this way, many objects seem to obey power laws. There are certainly some obvious candidates, such as ferns, trees, lightening, broccoli, river networks, snowflakes, clouds, mountain ranges, the veins in leaves and animals, and

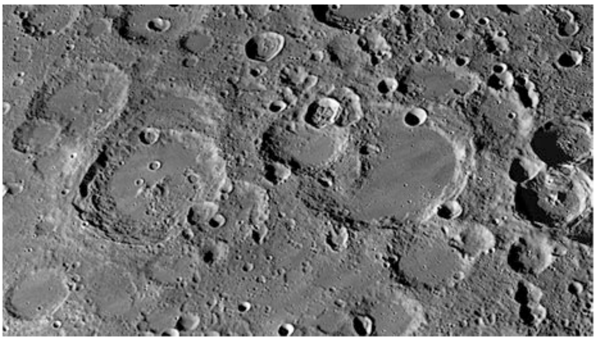


Fig. 4.13 Self-similarity of craters on the moon

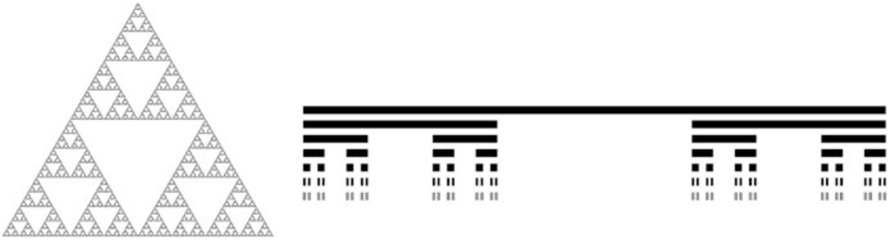


Fig. 4.14 Fractals can be generated through iteration of some operation. The Sierpinski gasket is shown on the left and is created by starting with a solid triangle and then iteratively removing and upside-down triangle from the center. The Cantor set is shown on the right and is created by starting with a line and iteratively removing the center third portion

stalagmites and stalactites in caves. But, there are some less obvious self-similarities that do not reveal themselves in space. For example, in Chaps. 5 and 6, we will explore the frequency of word usage as a power law. Likewise the rise and fall of companies, the volatility of the stock market, and citations in the scientific literature have all been shown to follow a power law.

Figure 4.14 shows two other well-studied fractal objects. The Sierpinski gasket was encountered in Chap. 2 as the result of Wolfram Rule 90. However, the shape in the figure can also be created by starting with a solid triangle and then removing an upside-down triangle from the center. The result will be three triangles that just touch one another. But then the center of those triangles is taken out too. On each iteration, smaller and smaller triangles are subtracted out. In theory, the final objective would have infinitely fine detail. When the box counting method is used, the fractal dimension is $a = 1.585\ldots$, meaning that it is about halfway between a one- and two-dimensional object.

The first person to mathematically study fractal objects was the mathematician Georg Cantor (1845–1918), best known as the creator of set theory. Cantor also explored in more detail than anyone before him the nature of infinity, work that some believe contributed to him entering a sanatorium several times during his lifetime. One of Cantor's explorations was to imagine a line, to which the middle third would be removed. This would leave two line segments, each of which would have their middle third removed as well. This process would continue, as shown on the right side of Fig. 4.14. The object that results would be somewhere between a zero-dimensional point and a one-dimensional line. It can be proven that the fractal dimension of this object is $a = 0.631\ldots$.

Making measurements in this way is not limited to mathematical objects. For example, one can find the fractional dimension of the coast of England using different-sized boxes to measure the coastline length. The slope of this line can be used to determine the fractal dimension, as in Fig. 4.15.

Geoffrey West (1940–) has catalogued power laws that describe the metabolism of organisms. The idea that metabolism follows a power law is known as *allometric scaling* and was first described in 1932 by Max Kleiber (1893–1976). When

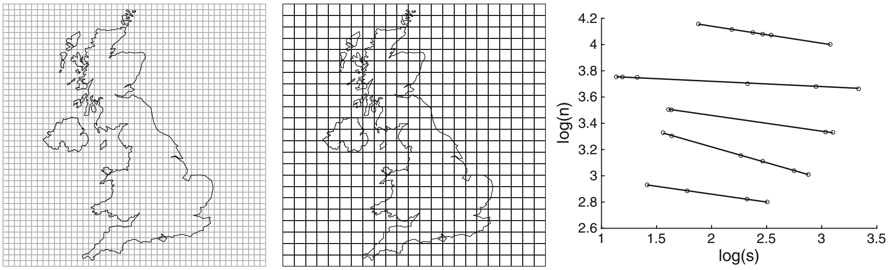


Fig. 4.15 The fractal dimension of any shape can be computed by changing the size of the measuring device. In a classic example the fractal dimension of the coastline of England is computed by changing the box size and counting the number of boxes that then intersect the outline (left panel). The fractal dimension of the border of other countries can be computed in the same manner (right panel)

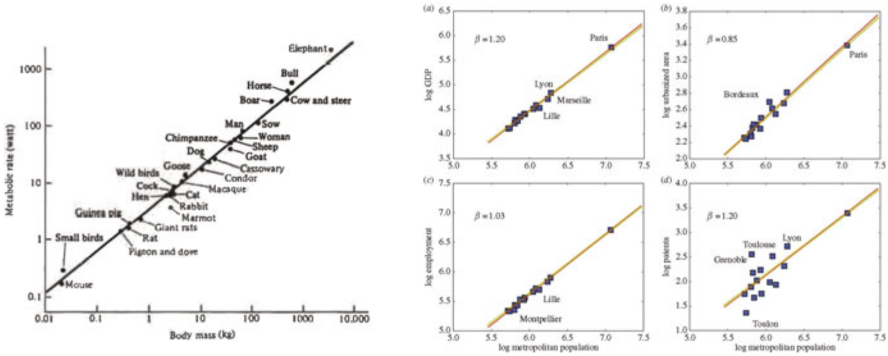


Fig. 4.16 Allometric scaling laws describe how various functional properties scale with body size or mass. A common example is to plot how metabolism scales with body mass (from *A Fractional Probability Calculus of Allometry* by West in Systems 2(2) 89–118, 2014). Such functional scaling can also be applied to nonorganic systems such as cities and economies (from *Urban Scaling in Europe* by Bettencourt and Lobo in *Journal of the Royal Society Interface* 13:20160005, 2016)

metabolism is measured, for example, as energy input required per day, and plotted against body mass, the power law has an exponent of 0.75. West was able to show that this relationship holds over 21 orders of magnitude, from 10^{-13} g bacteria to 10^8 g whales. Such a relationship is assumed to be due to a deep connection between how organisms use resources to grow and maintain their internal structures as well as expel waste. A summary plot is shown on the left-hand side of Fig. 4.16.

The researchers at Santa Fe Institute, however, did not stop with biological species. They continued on to explore the "metabolism" of groups of humans in the formation of cities, as described in West's book *Scale: The Universal Laws of Growth, Innovation, Sustainability, and the Pace of Life in Organisms, Cities, Economies, and Companies*. Their remarkable claim is that, when metabolism is defined the right way, groups of humans also seem to obey the 0.75 power law distribution, perhaps extending the curve in Fig. 4.16 even farther to the right. The right-hand side of Fig. 4.16 in fact shows the scaling against several measures of city output such as GDP, urbanized area, employment, and patents. Similar arguments were made (although not mathematically) by Michael Batty and Paul Longley in their 1994 book *Fractal Cities* and Jane Jacobs (1916–2006) in her 1961 book *The Death and Life of Great American Cities*.

4.4.3 Mandelbrot and Fractals in Nature

The Koch curve and other shapes become fractals through iterative bending, folding, or removal of some original shape. But the computer revolution enabled a whole new exploration of complex fractal shapes. In 1967 Benoit Mandelbrot (1924–2010) began to explore the idea of self-similarity and in fact performed the original fractal measurement of the coastline of England. In 1975, he continued to explore the idea of "rough" objects, more specifically Julia sets, a family of iterative mathematical equations that could be displayed graphically. In 1979 Mandelbrot developed a sort of master equation that contained all Julia sets in one equation. It was Mandelbrot's object, along with a beautifully illustrated book *The Fractal Geometry of Nature* that launched a worldwide fascination with fractals.

The Mandelbrot set, as it has come to be known, is shown in Fig. 4.17 and is a perfect mathematical fractal, meaning that it can be blown up as many times as desired and will remain scale invariant. The full set is shown in Fig. 4.17, along with two zoomed-in images, displaying the fractal nature of the object. To generate the Mandelbrot set, a point in complex z space is iterated using the equation

$$z_{n+1} = z_n^2 + c$$

In this equation, c is a complex number ($c = x + yi$) that can be represented on a complex plane (plot of x against y). To generate the image, start with a particular point, say $c = 1 + 2i$, and an initial condition for z_n. Then the equation is iterated upon, finding successive values for z_{n+1}. Depending upon the value of c, sometimes the iterations settle down (or converge), while other times the iterations never settle down (or diverge). If the value of c leads to convergence, that pixel is colored black. If the value of c leads to divergence, that particular pixel of the image is given a color related to how quickly the divergence happens. This iterative procedure cuts up the two-dimensional x–y space into two sets—values of c that stabilize and values of c that do not stabilize. More than almost any other single image, the Mandelbrot set has come to symbolize complexity to the general public, appearing in artwork and early computer screen savers.

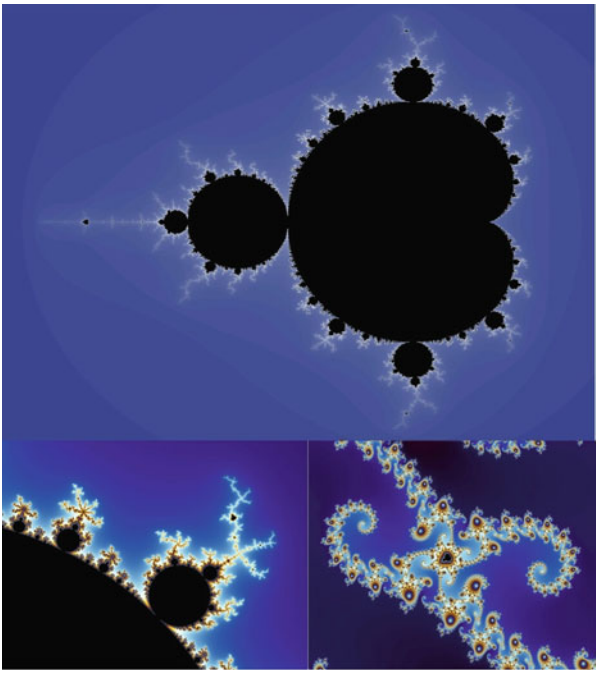


Fig. 4.17 The Mandelbrot set (top), along with a zoomed-in version (lower left) and an even further zoomed-in version (lower right)

4.4.4 Strange Attractors as Fractals in Phase Space

In Chap. 3 the concept of a strange attractor was introduced as a trajectory in phase space that remains within a given volume but never repeats. When strange attractors are viewed as objects, it turns out that they are fractals, being composed of trajectories (lines) that do not completely fill a space. A number of real-world phenomena have been found that are fractals in phase space. For example, one can plot economic measurements such as stock market fluctuations and then compute their fractal dimension.

Finding the fractal dimension of a real system is not always easy because it is not clear what variables should be plotted against one another. There is, however, an important theorem, known as the Takens sampling theorem, which proves that a strange attractor can be reconstructed from a long enough time course of a single variable. We will leave the proof of the theorem aside, but its practical meaning is relatively easy to understand. Given a time course of one variable of a system,

$$x = [x_1, x_2, x_3, \ldots, x_n]$$

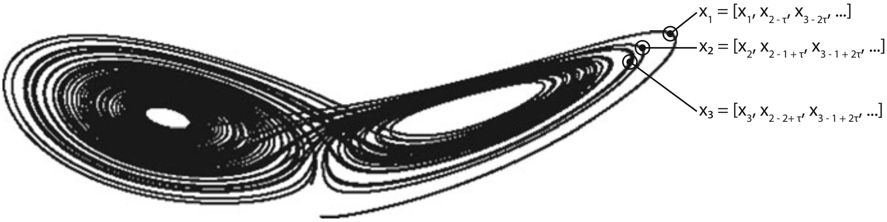


Fig. 4.18 The method of Takens can be used to reconstruct a strange attractor from a single signal

multiple surrogate variables (also called signals) are created that are all separated by some time delay (τ).

$$x[1] = [x_1, x_\tau, x_{2\tau}, \ldots]$$
$$x[2] = [x_2, x_{1+\tau}, x_{1+2\tau}, \ldots]$$
$$x[3] = [x_3, x_{2+\tau}, x_{2+2\tau}, \ldots]$$

The choice of τ is important in that neighboring values must have nothing to do with one another (e.g., x_1 and x_τ should be unrelated). The standard way of finding τ is to take the first zero crossing of the autocorrelation function (not to be discussed here).

Figure 4.18 shows a mathematical reconstruction of the Lorenz attractor using only the x signal. The same method, however, can be used to estimate the underlying strange attractors from economic, ecosystem, city, cellular, or other complex systems from real data. Once regenerated, the fractal dimension can be estimated using three-dimensional (or higher) boxes of different sizes to cover the attractor.

4.5 Autocatalytic Systems

In Chap. 1, the idea of partial causality was introduced simplistically as A causes B causes C causes A. This is a very basic *autocatalytic* system, sometimes called a hypercycle. The term autocatalysis comes from chemistry, where a family of reactions depend upon one another. We will first explore a classic chemical example and then move on to how logical relationships can also form autocatalytic loops.

4.5.1 The Brusselator

In 1968 Ilya Prigogine (1917–2003) proposed what has become known as the Brusselator model to describe the Belousov-Zhabotinsky (BZ) reaction. The story behind the discovery of the reaction will be told part in this chapter and part in Chap. 7. The basic chemical form is

$$\begin{aligned} A &\longrightarrow X \\ 2X + Y &\longrightarrow 3X \\ B + X &\longrightarrow Y + D \\ X &\longrightarrow E \end{aligned}$$

that can be represented (at least in part) by the differential equations

$$\begin{aligned} \frac{dX}{dt} &= A + X^2Y - BX - X \\ \frac{dY}{dt} &= BX - X^2Y \end{aligned}$$

The equilibrium point of this system is

$$\begin{aligned} X &= A \\ Y &= \frac{B}{A} \end{aligned}$$

and becomes unstable when

$$B > 1 + A^2$$

when the dynamics become an oscillatory limit cycle. The reaction is often created using potassium bromate ($KBrO_3$), malonic acid ($CH_2(COOH)_2$), and manganese sulfate ($MnSO_4$) in a heated solution of sulfuric acid (H_2SO_4). The reason for this mix is because the oscillations can be seen as colorful changes as the concentrations shift back and forth.

Eventually one or more of the reactants will be used up. In this case, the BZ reaction will be a damped oscillation that spirals down to a final concentration of X and Y. In phase space, this will look something like the left side of Fig. 4.19.

In the equations above, we assume that the solution is well mixed. That means that X and Y are the concentrations throughout the entire solution. In media that is not well mixed, for example, in a still and shallow dish, regions can only share chemicals with nearby regions, much in the same way as a cellular automaton. This can be modeled using the reaction-diffusion equation, where $f(u)$ is replaced by the Brusselator. The dynamics can now become much more complex as regions

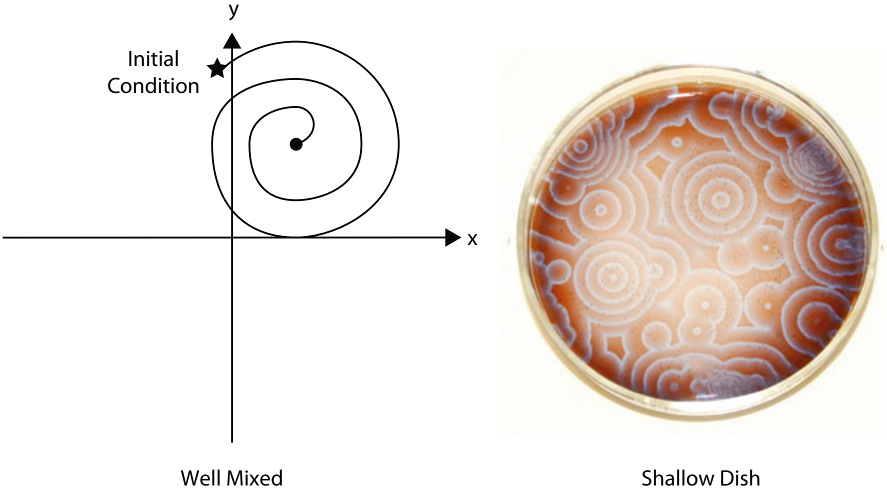


Fig. 4.19 The BZ (Brusselator) reaction in phase space (left) when well mixed. The BZ reaction can display spirals (right) in a shallow dish when not well mixed

that may have higher concentration of X will spread outward through the process of diffusion. Likewise regions that have a higher concentration of Y will move through diffusion into regions with lower concentrations of Y. But reactions can also transform X into Y and Y into X at any given location. Given the right conditions, such a system can in fact generate beautiful colored spirals as in the right side of Fig. 4.19.

4.5.2 Autocatalytic Memory

The push and pull between reactions are just one example of autocatalysis. Examples can come from sociopolitical and cultural systems, technological innovation, and species in an ecosystem. Any system that can support self-closing functions can become autocatalytic. Consider the proposal by the mathematician John Conway (the same person who invented the Game of Life) for how stable memories might be autocatalytic. Keep in mind that Conway's definition of memory was very broad and could span human and digital memory as well as DNA and laws.

Imagine a number sequence such as

$$r = 3321111355$$

The sequence can be broken up into repeated numbers

$$r = 33 - 2 - 1111 - 3 - 55$$

and then transformed to a new sequence—two 3s, one 2, four 1s, one 3, and two 5s or

$$s = 2312411325$$

A function, δ, can be hypothesized that relates r and s

$$s = \delta(r)$$

that will transform any r to the corresponding s. It is important to note that the function δ is a rudimentary version of what happens in most compression algorithms, like the jpeg or mpeg formats, where redundant information is squeezed out.

Now consider the following sequences:

$$a = 111312211312111322 21\ldots$$
$$b = 3113112221131112311332\ldots$$
$$c = 1321132132211331121132123\ldots$$

A little checking will reveal that

$$b = \delta(a)$$
$$c = \delta(b)$$
$$a = \delta(c)$$

a, b, c, and the operation δ form a logically closed system that is similar to Fig. 1.4. This is an abstract autocatalytic system and is robust in the sense that each of the elements could be generated from all of the other elements. Of course we have created a trio, but other sequences could be generated with much more complex interrelationships and degrees of robustness.

4.6 Patterns in Other Spaces

We explored patterns that can form in space, time, and phase space. But there are many spaces that are not as easy to describe and analyze. Below we highlight two, evolutionary patterns and the pattern of the self, that span across time and space.

4.6.1 Evolutionary Patterns

Ever since the theory of evolution was proposed, there have been deep questions about how order can arise out of randomness. The order in evolution appears to have progressed from reproducing autocatalytic chemical reactions to more stable chains of molecules (such as RNA) to even more robust chains (such as DNA), then cells, multicellular organisms, and societies of organisms. In this chapter, we explored the Ising model which is a simple system where an undirected external biasing force (the temperature) can control the formation of wide-scale patterns. We also explored how causal loops might sustain and control internal signals. Combining these two ideas together opens up the possibility that a system might emerge on its own—to have its cause be itself and have some measure of control over its own order parameters. Perhaps evolution is one grand pattern, continuously unfolding and becoming more complex over time, that uses autocatalytic loops at multiple spatial and temporal levels to turn randomness into order. Future chapters will continue to address this speculative proposal.

In considering evolution as a system, interactions with the external nonorganic environment matter as well. As an example consider a simple thought experiment. If an organism from the early earth were to be transported to our time, it would die instantly—likely poisoned by our environment. Likewise, if you were transported to the early earth, you would die very quickly due to the lack of oxygen. All evolved systems (e.g. technology, law, economy, transportation) develop in and are dependent upon, their environment. At the same time, many of these systems can also change their environment. So they are both changing and changed by their surroundings in multiple interacting causal loops. We will explore this idea much further in Chap. 7.

4.6.2 The Pattern of "I"

Much has been written about the nature of a self. At some level, nearly all deep thinkers on the topic have come up with some version of a self as a pattern that is self-sustaining over time, even in the face of perturbations from the outside. To some this pattern is independent of the physical being, taking the form of a unique "soul." To others it is a complex and multidimensional physical entity distributed throughout the brain and body. The concept of a self will come up again, but for now, we will follow the idea that a self is a particular kind of pattern.

Much of Douglas Hofstadter's (1945–) 1979 Pulitzer Prize-winning work *Godel Escher Bach: An Eternal Golden Braid* explores how cognition, consciousness, and self emerge from causal loops and self-reference. Although it is not stated explicitly, the argument resembles a more elaborate version of autocatalytic memory; a series of causal loops, at multiple levels of a hierarchy, all self-reinforce one another. The book is long and technical in some places, and Hofstadter in fact complained that many readers missed the point. The general concept was simplified in his later book, *I Am a Strange Loop*. In a deeply personal section of the book, Hofstadter argues that in some ways his wife, who had recently died, is still alive because the pattern that was "her" is continuing to reverberate in the works she produced and minds she touched. Hofstadter was in fact expressing a more technical version of a quote that has been attributed to many sources—"You die twice. One time when you stop breathing, and a second time a bit later on, when somebody says your name for the last time." A similar idea is expressed in the Disney movie Coco, where a person dies a second time in the land of the dead when all of the living have forgotten them.

The case of Johann Sebastian Bach (1685–1750) is a provocative illustration of how a self might be contained as much out in the world as inside of a body. In his day, Bach was a reasonably well-known composer and gifted organist. After his death, however, his compositions slipped into obscurity for over a century. It was in the 1830s that the composer and conductor Felix Mendelssohn (1809–1847) "rediscovered" Bach and began orchestrating his works for contemporary instruments. This revival of his works continues to this day, and we now consider Bach's compositions to be among the pillars of Western music. Did Bach's "self" disappear only to reemerge, perhaps even more strongly after he was medically dead for 100 years? There are all manner of these kinds of stories where a person essentially becomes a part of history. When the history is rewritten, or reinterpreted, does the pattern of that particular self change?

How we define a self has important consequences. For example, there is a debate about whether a person arrives into the world as a *blank slate* or whether they come prewired with some very basic functions. On one side of this argument is the idea that a person arrives as a blank slate but then undergoes a series of symmetry breaks that leads to the development of behavioral patterns. Over time a unique pattern of a self emerges from the blank slate. The result is a "differentiated self." When a self does not differentiate properly, it could lead to problems in personality that show up later in life. In fact this is exactly the way many schools of psychology think of a wide range of personality disorders. Alternatively, a person may inherit many self-building functions through the genome that serve as a seed. These basic functions are unchangeable and will be recombined throughout a lifetime to form a more and more complex self.

Both theories have their supporters, and so far there have been no definitive proofs or experiments. The varying theories do sit at the bedrock of some very difficult issues. For example, is a person a person before they are born? Reframed, this is a question about whether or not an unborn human is a self (or proto-self). Likewise at the end of life, we could question at what point someone is no longer a self and how that relates to being a person both legally and medically. To take this even farther, what responsibility might family have to care for the reputation of the deceased? Even more provocative claims about the self will be explored in Chaps. 9 and 10.

4.7 Questions

- Wine and other alcohols, when swirled around a glass will leave small trails, known as “legs.” This is due to the Marangoni effect. Research this effect, and speculate on how it relates to topics in pattern formation.
- Interview a friend or family member about something that was surprising to them. Listen to their story for moments when they bring up something that displays a pattern. Dig deeper by questioning them about it.
- In what ways do architects break symmetries? Can you give some examples. Why do you think they broke the symmetry? Was it because of the form or the function of the building?
- Wander somewhere until you find something that has a fractal-like geometry. Can you speculate on how this fractal came into being? Example how might you measure its fractal dimension.
- Listen to two or three jazz (no vocals) songs. During the solos, how do the players (both the soloists and the accompanists) create and break symmetries in the music? In what ways are the following some rules while breaking others?
- There are many different species in an ecosystem. In some ecosystems, there are several species that take on the important function of a vulture (e.g., a catfish, fungi, bacteria), cleaning up and recycling the waste products of other species. They are all part of an autocatalytic loop, whereby energy (food) is moved from one species to another, to another, and finally back around again. Can you find other ecosystems that are human-made that behave in a similar way? What are the equivalent of “vultures” in this system?
- Can you find real-life examples where two phenomena seem to be out of phase with one another? Explain the cycles of each and then why you think they are out of phase.
- Most jokes are in one way or another a break in a pattern. Expectations are created to get the audience thinking one way, and then the comic will flip the pattern. This is known more formally as the *incongruous juxtaposition theory*. Study a joke, and look for ways in which it breaks some assumption or pattern.
- Memories were discussed as repeatable pattern within a system. What are some cultural types of memories? What mechanisms within a culture act to reinforce the memory of particular historical events, processes, or policies? In what ways are these mechanisms autocatalytic?
- Many of our mental, physical, and perhaps even emotional functions can be thought of in the same way gait was described—essentially a series of interacting patterns that might excite or inhibit one another. As an example, try the following experiment. Sit down and begin moving your right foot clockwise. Continue that motion and now extend your right index finger. Try to draw a “6.” What you will likely experience is that your foot tries to reverse directions. There are some deep neurological reasons for this phenomenon that involve the intertwining of various motors circuits. You may have notice that you instinctively close your eyes when you sneeze. Pigeons bob their heads when they walk, and if you try to stop the

bobbing, they cannot walk. The brain and body are filled with these quirks. And it is likely that all systems, both human-made and natural, contain them as well. Can you identify another example and discuss it?

- Identify a tension in a political system. How has that tension resulted in particular social patterns or policies?
- Symmetry breaking often appears when there are two equally likely outcomes, yet one scenario eventually dominates. A classic example is encountered by physicists trying to explain why we see more matter than antimatter in the universe. According to the current thoughts about the origins of the universe, it would seem that equal parts matter and antimatter should have been created. If that were true a series of annihilations should have left the universe with no matter at all. A similar idea is seen in biology where nearly all organic molecules have an inherent left-handedness, known as life's *chirality*, even though all known biological processes could theoretically be based upon right-handed molecules. Perform some research on one of these two imbalances, outlining the various theories, and then explain which one you think is most plausible and why.
- The concept of a triangle is a structural and functional pattern that will appear several times throughout this text. Civil engineers often use triangles as a simple stable structure that can be repeated over and over again. Likewise the idea of modern concept of sustainability has three elements—ecosystem, environment, and economy (the three Es)—also known as the three Ps (people, planet, profit) or triple bottom line. The idea of a triple bottom line can be applied more personally as the balance between health and fitness, career, and social/family life. How do you balance this personal triangle in your own life? How do you react when your life is out of balance?
- Repetition is perhaps the simplest kind of pattern. Many teachers, politicians, parents, and musicians use repetition to great effect. Find a specific example (e.g., poem, speech, lyrics), and analyze the use of repetition. Are there other patterns that are being used as well?
- There are several ways that symmetries can be broken, but only a few were hinted at in the chapter. For example the concept of *nucleation* is helpful in understanding how water condenses around a small particle to form a rain drop. The general idea is that a small seed can become the source of a much wider pattern. Explain a societal pattern that has been created by a particular nucleation point (perhaps a person or event or both) that has grown into a much wider pattern.
- Synchronization had a positive connotation throughout the chapter as a helpful phenomenon in complex systems. Synchronization of two or more cycles, however, might also lock together two detrimental cycles or habits that reinforce one another and make both cycles harder to break. Give an example of two (or more) human-created systems that have synchronized in some way that is harmful. Be explicit about what has synchronized. Explain how you would propose uncoupling the two systems (perhaps only partially) so that they are less synchronized.

- Some people seem to be pattern creators. Others are pattern breakers. A particular individual is certainly not entirely one or the other. Rather each person can be thought of as being along a spectrum between pattern creator and breaker. Discuss where you think you are on this spectrum.
- Benard cells were explored in the chapter as a break in symmetry that occurs due to an instability. Speculate on the instability that gives rise to sun spots and solar flares.
- Sports are often dominated by particular patterns that an opponent can read and gain some information on possible future moves or strategies. A classic example from the world of boxing is "the tell"—which is a small sign of what punch is coming next. It is the reason why boxers (and many other individual and team sports) spend time watching video of their opponent. Similar ideas can be applied in business or poker. Discuss a pattern that you use to predict the future moves of someone else.
- Retreats, both personal and group, are meant to break the patterns of daily life. During a retreat, meta-reflection can allow one to consider existing patterns and seed new patterns. Take a 30-minute silent retreat during which you consider new patterns that you might try to establish in your life. Explain your findings and what next steps you might take.
- In the book *Guns, Germs, and Steel*, Jared Diamond (1937–) makes an interdisciplinary argument as to why Eurasian and North African civilizations flourished and conquered other civilizations. Rather than base his premise on some intellectual or genetic superiority, as has been done by some past thinkers, he bases his argument on patterns in the environment that endowed some civilizations with more than others. He explores the types of proto-crops, animal sources of food and labor, soil and weather, growing seasons, disease propagation, and other factors. He goes on to explore how technologies were invented and then used to spread populations and ideals more broadly. He even makes the argument that the axis of a continent (north-south versus east-west) can enable this spread without the need to change crops or customs (e.g., an east-west axis allows for easier long-distance spreading). Diamond's follow-up book *Collapse: Why Societies Choose to Fail or Succeed* deals with the sustainable future of societies. Together these two books explore the societal patterns of growth and decay. Where else do you imagine a similar argument being made? What other systems may be embedded in an environment that is patterned in such a way that some they gain an advantage? Be explicit about what these advantages are and how positive feedback cycles might bolster the successful while weakening the less successful?
- *Moire patterns* are formed by superimposing two relatively simple visual patterns on top of one another. The result is a much more complex pattern that can be changed, sometimes radically, by slight changes in the orientation of simpler patterns. A similar idea applies to video where a camera is pointed at its own

display screen. The result is a semi-infinite feedback loop that can turn a very simple initial image into something much more complex. Find examples of artists or designers who have used Moire patterns in their work. Discuss how they are generating their patterns.
- It is often the case that we pick up patterns from others. A pop phenomenon, attributed to Jim Rohn (1930–), is the idea that you are some average of the five people you spend the most time with. Make a list of the people you spend the most time with (do not worry if it is the five or not). What patterns or habits do you think you have picked up from them? What habits do you think they have picked up from you?

4.8 Resources and Further Reading

A wonderful introduction to patterns can be found in Philip Ball's *Patterns in Nature*. The many photos are stunning and contain many patterns (e.g., Chladni figures, Kelvin-Helmholtz instabilities, bubbles, cracking) not discussed in this chapter. Philip Ball has published three other texts that are more detailed. *The Geometry of Time* by Arthur Winfree (1942–2002) is a comprehensive summary of how viewing rhythms as limit cycles (and other shapes in phase space) can lead to breakthroughs. Winfree in fact was responsible for uncovering the mathematical source of many patterns using the tools of non-linear dynamics from Chap. 3. *Rhythms of the Brain* by Gyorgy Buzaki (1949–) is an excellent application of complex systems thinking as applied to the brain and mind. The unifying theme is that various rhythms make up conscious and unconscious mental functions. A third book *From Clocks to Chaos: The Rhythms of Life* by Leon Glass (1943–) and Michael Mackey (1942–) presents a unifying vision of how rhythms dominate a wide range of natural phenomenon. The importance of central pattern generators is discussed in detail in *Spikes, Decisions, and Actions* by Hugh Wilson (1943–2018).

Fractals: The Patterns of Chaos by John Biggs is a very nice introduction to fractals using mostly graphical arguments with a small dose of algebra. Many of the books referenced in Chap. 3 also contain chapters on fractals and their relationship to chaos and strange attractors. The simulation software NetLogo, mentioned in Chap. 2, allows for creating many of the patterns mentioned in this chapter. Spontaneous pattern formation can also be explored in Ian Stewart's *What Shape is a Snowflake* and Brian Goodwin's *How the Leopard Changed its Spots*.

Chapter 5
Networks

Imagine placing the names of everyone you have ever met on individual sticky notes. Then place these on a whiteboard and begin drawing lines between the people who know one another. A completed diagram would map out your social network, with you at the center. You might imagine that just by looking at the network, you could identify particular phases in your life, including family, elementary school, high school clubs, past, and current co-workers. You might also be able to identify isolated chance meetings—those individuals who are not connected to anyone else in your network. Now imagine highlighting the connections that you still maintain regularly in red. This is your current social network, and it is the source of much of your information about the world. What might such a network look like, and how would you analyze it?

All complex systems contain an internal structure that constrains what parts can talk to what other parts. In a society the structure is which people talk to one another. In a cell it might be which reactions can take place. In a business it might be the formal and informal opportunities for people to contact one another and share information.

The field graph theory, from which network theory is derived, was first explored by the most prolific mathematician of all time, Leonhard Euler (1707–1783). Euler was in the German city of Konigsberg (now Kaliningrad, Russia) on the Pregel River. Within the river are two islands, and over time seven bridges were built to connect various parts of the city, as shown in Fig. 5.1. A question at the time was if it was possible to find a pathway that would cross each bridge only once but allow one to visit each part of the city. Euler proved that it was not possible by inventing an abstraction of the problem that represented each landmass as a dot and each bridge as a line connecting them. Euler's abstraction established the graphical nature of network theory.

In this chapter we will introduce network science as the study of interconnected parts. We will explore some of how these structures might change over time but will save the study of dynamics on networks for Chap. 9.

J. Tranquillo, *An Introduction to Complex Systems*,
https://doi.org/10.1007/978-3-030-02589-2_5

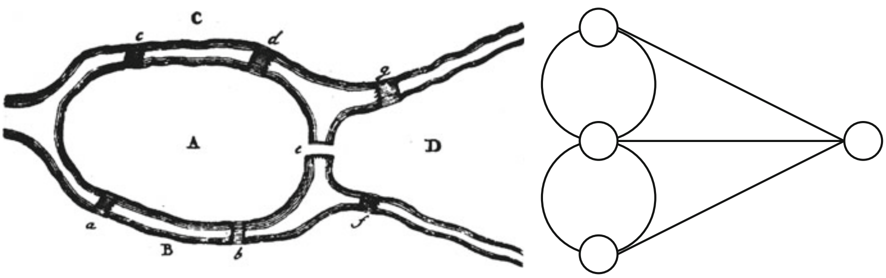


Fig. 5.1 Euler's original network analysis (right) of bridges of Konigsberg (left)

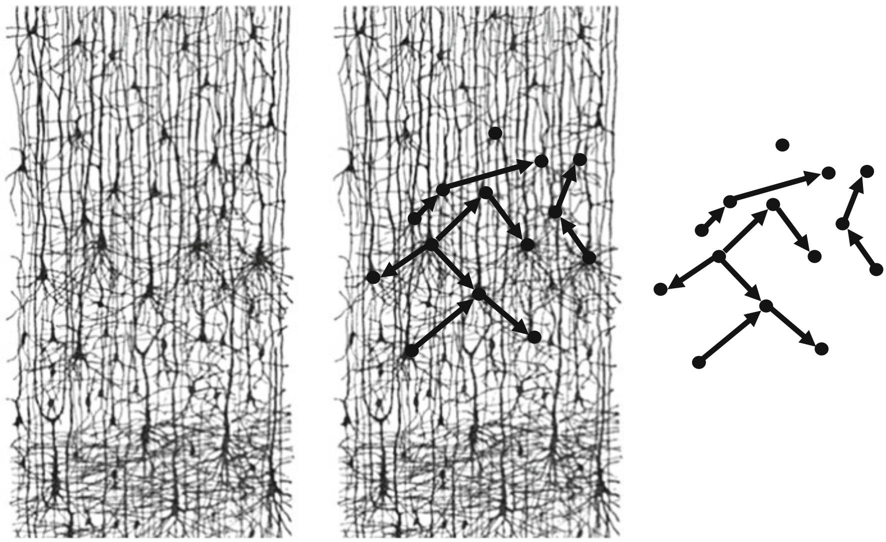


Fig. 5.2 Network analysis of a drawing by Santiago Ramón y Cajal. The left diagram is one of Cajal's drawings. The middle diagram places dots where neurons are and arrows to show connections. The right diagram has the image removed to show the abstracted connections

5.1 Theory of Networks

The general concept of a network is very simple. A system is composed of several parts, which will be represented by dots (sometimes called nodes, units, vertices, sites, actors, or agents depending on the field). The relationships between these dots are then represented by lines (sometimes called edges, bonds, links, ties, or connections, again depending on the field). For example, the left-hand side of Fig. 5.2 shows a drawing of a slice from the brain by the Nobel Prize winner Santiago Ramón y Cajal (1852–1934), considered to be the father of modern neuroscience.

In the middle panel of Fig. 5.2, dots have been placed on the neural bodies, and lines show how these neural bodies are connected together with axons. Once the dots and lines have been laid down, we can take away the original picture and study just the structure of the system. In this abstract form, the diagram might describe cities connected by roads, a group of friends, or metabolic reactions in a cell. What we will build up in this chapter are the tools for analyzing the structural properties of complex systems.

Although a network constrains the flow of material, energy, information, or people, it does not completely determine these flows. If presented with an abstract network structure, there are some questions that network science aims to answer. For example, how many unique pathways are there from one point to another? If there are many pathways, which are the shortest? What is the average number of connections that must be traveled to get from any one point to any other point?

5.1.1 Simple Networks and Neighbor Connectivity

One of the most basic networks is a line, as shown in Fig. 5.3. Each node is connected to its left and right neighbor, much the way the Wolfram automaton was in Chap. 2. The two end nodes only have one neighbor. This is known in mathematics as a boundary condition, a simple structural rule that defines connectivity at the edges of a system.

A line of nodes is in fact how electrical engineers model the transmission of electricity along a cable, and how physiologists model muscle contraction. Each muscle cell only contracts a tiny amount, but they are all connected together end-to-end. This is known as the sliding filament theory which was first proposed by Andrew Huxley (1917–2012) around 1953. The coordinated contraction of the muscle cells causes the entire line to shorten by a significant amount, leading to large-scale motion of the bones to which the muscle is attached.

A related network can be created by connecting the two endpoints of a line to form a ring, as in the right side of Fig. 5.3. A ring has periodic boundary conditions. This single connection can have some significant implications. For

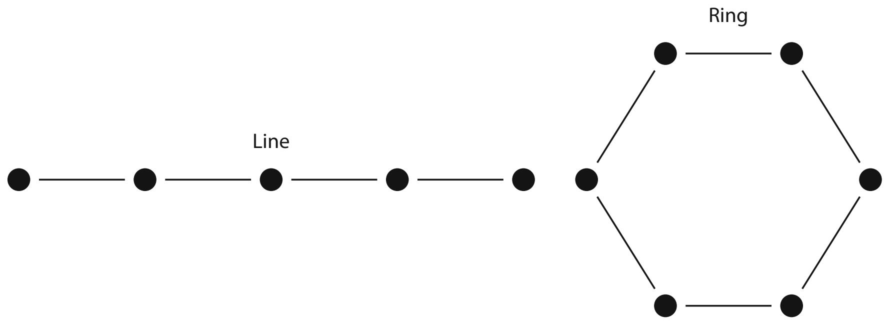


Fig. 5.3 Line (left) and ring (right) networks

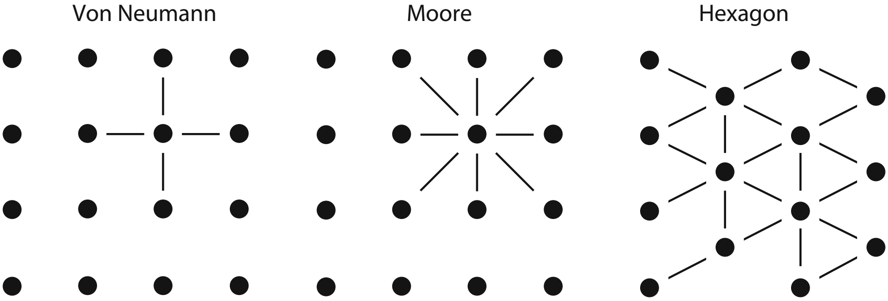


Fig. 5.4 Three types of lattice networks, von Neumann, Moore, and hexagon. The complete connectivity is not shown, only an example pattern of connectivity for each node

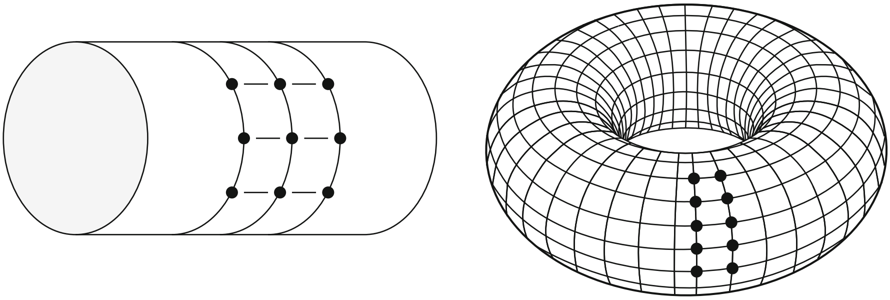


Fig. 5.5 Folding of a lattice network to form a cylinder (left) and torus (right)

example, imagine a signal that moves down a line. When it arrives at the end, it will have nowhere else to go. When that same signal, however, is moving in a ring, it may continue to circulate around and around, in what is known as *reentry*. Structurally, a ring is therefore a way to support a repeating rhythm in a way that is fundamentally different than the limit cycles introduced in Chap. 3. In a limit cycle, the rhythm was a product of non-linear interactions. In a ring it is the structure that gives rise to a rhythm. This is a generalization of the central pattern generators discussed in Chap. 4, where no single node is responsible for the repeated pattern.

A line can be extended in two dimensions to form a grid of points. Each node can be connected to neighbors, as in Conway's Game of Life introduced in Chap. 2. As shown in Fig. 5.4, there are many ways to connect neighbors. Neighbors might be connected up, down, left, and right, known as a von Neumann neighborhood, through the diagonals, called a Moore neighborhood, or through hexagonal connections.

The same trick of folding a line to become a ring can be applied to the 2D grid. Folding the two ends together can form a cylinder, as shown on the left side of Fig. 5.5. Then folding the remaining two edges forms a torus, as on the right side of Fig. 5.5.

These basic network shapes can be bent into all sorts of other shapes but will still retain the same connectivity. This is one of the critical abstractions of network theory—physically, the shape of the network may change (e.g., a sheet may be stretched or a torus twisted in some way), but the connectivity remains the same. A mathematician would say that the underlying *topology* has not changed. It is this concept that allows comparisons to be made between the structure of networks of neurons, genes, highways, power grids, or channels for moving money. What we will find, again a theme in complex systems, is that there are some underlying patterns to how complex systems are connected together that transcend scale (in time and space) and exact physical instantiation.

5.1.2 Trees and Hierarchies

Another dot-and-line pattern that occurs in many systems is the *branch*, where one unit connects to two (or more) others. This is known as a *motif* in network theory and is a small pattern of connectivity that may be repeated over and over again. When a branch is iterated upon, the result is a tree network. Starting at the base, a first branch is formed and then higher up another branch and another, with each branch potentially having many subbranches. Trees can be used to disperse a message or resource (e.g., arterial blood supply in the vascular system, electricity in a power grid) or collect it (e.g., a river tributary system, venus blood return system). Linguists use trees to show the grammar of a sentence and evolutionary biologists use a taxonomy tree to show evolutionary relationships between species. Yet another type of tree network is the traditional organization chart, shown in Fig. 5.6 that describes the formal reporting structure in many companies.

5.1.3 Properties of Nodes and Edges

Network diagrams can graphically capture more information about connectivity. As can be seen in the network of neurons, the connections are *directed*, meaning that messages that move from one node to another may only proceed in one direction. The left and middle panels of Fig. 5.7 show the same network with directed (flows can only occur in one direction) and undirected (flows occur in either direction) connections. The constraint of directionality can impact the possible behaviors a complex system can express.

Another property that may be added is to indicate the relative strength of the connection between nodes, generally represented by a number as in the right side of Fig. 5.7. Not all connections are the same. Some neurons are more tightly connected than others. You talk to your friends more than others. Some computers have more bandwidth than other computers. The number of cars commuting on the roads connecting two cities may vary over time. In graph theory, these differences in

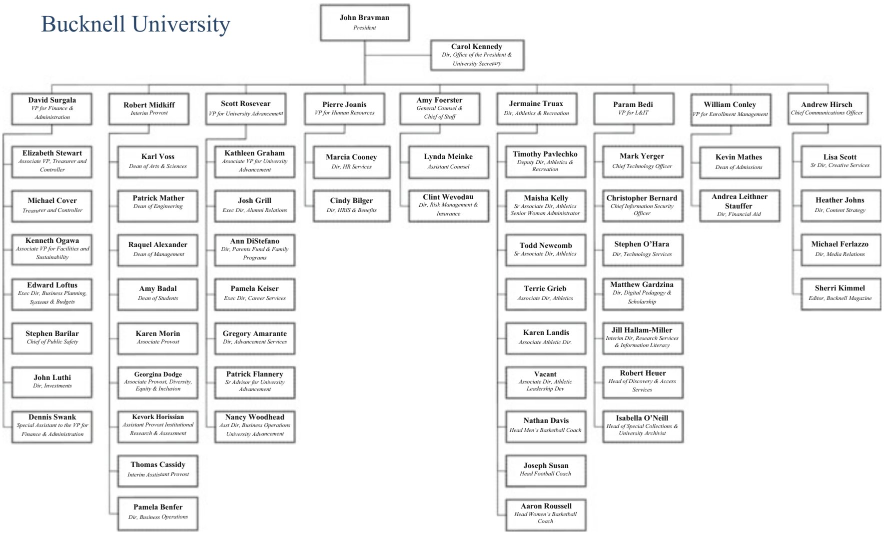


Fig. 5.6 Organizational chart for Bucknell University

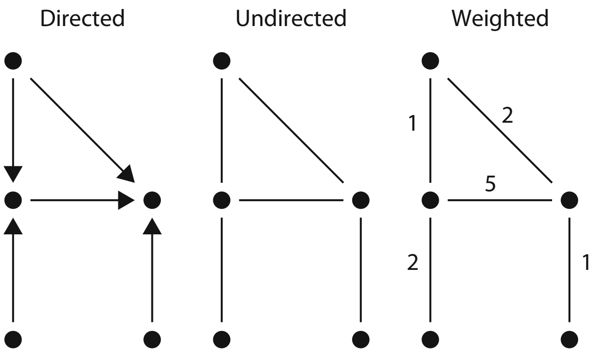


Fig. 5.7 Example of directed (left), undirected (middle), and weighted (right) networks

the connection strength form what are called *weighted* networks. Weighting is also at the core of the PageRank algorithm used by an early version of the Google search engine to score the relative importance of pages based upon how many other webpages link to them. Without going into the details, the significance of a page can be scored by a combination of how many webpages link to that page but also how well scored those linked pages are.

Nodes can also have properties attached to them. For example, they might store color, bandwidth, emotion, skills, or fitness. These node properties, often called *affinities* or *attributes*, may give additional information on how information flows through that particular node. For example, it may be that one person has high bandwidth for getting work done, while another person may not. Such variability

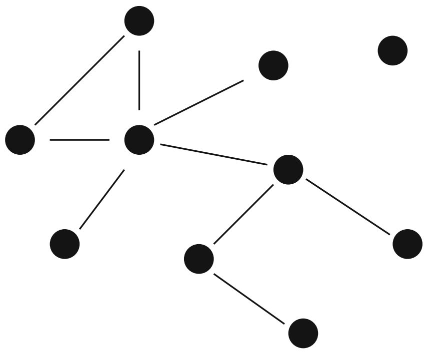


Fig. 5.8 Example of a random network

in a node property may also change the overall flows within the network. We will explore these kinds of dynamics on networks in Chap. 9.

5.1.4 Random and Orderly Networks

Regular networks are based upon a repeating motif. The opposite extreme was explored by Paul Erdos (1913–1996) and Alfred Renyi (1921–1970) in a classic paper on random networks. The stories about Paul Erdos, the second most published mathematician (1525 publications) of all time behind Euler, are legendary. He was known to simply show up with a suitcase, anywhere in the world, work with a group for a week or two, and then move on to another group. He was essentially homeless. But everywhere he went, he produced. He made breakthroughs in an astounding array of subdisciplines of mathematics. His story is told because he is the source of a famous measurement in the mathematics community that will be discussed below.

There are many ways to generate a random network, but we will consider just one. Begin by drawing 100 dots and numbering them 1 to 100. Then using a random number generator, list two numbers between 1 and 100. If the numbers are 34 and 79, then the dots corresponding to these numbers will be connected. This process is repeated over and over again to form a random network. A simple example is shown in Fig. 5.8. What is special about random networks is that they can be used to explain the phenomenon of *six degrees of separation* in a social network.

5.1.5 Six Degrees of Separation

In 1967 Stanley Milgram (1933–1984) performed one of the all-time great (in the sense of influence) social experiments. He gave several hundred people in Omaha instructions to try to get a letter to a particular person in Boston. They were not to look this person up but rather send the letter, with a copy of the instructions, to someone they thought might be able to get it closer to that person. This person would do the same. Each person who received the letter and then sent it out again was to record their name in the next letter. Some of the letters did make it to the target person in Boston, and the results were surprising. On average the letters passed through the hands of a bit less than six people. Milgram's conclusion was that everyone is approximately six degrees from everyone else.

There are a number of methodological issues that have been pointed out with Milgram's study, and it is worth mentioning at least some of them. First, the sample size of the letters returned was likely too small to draw concrete conclusions. Second, it is not clear what happened to all the other letters—only some were returned, and it was from this select sample that Milgram drew his conclusions. Third, Omaha and Boston are, in this global age, seemingly close together. But in Milgram's defense, it was much less likely for someone in Omaha to know a person in Boston in the 1960s. Despite these flaws, the term "Six Degrees" stuck.

One of the early websites was the Oracle of Kevin Bacon, where actors were scored based on how far removed they are from Kevin Bacon. The metric is that Kevin Bacon is of degree 0. Anyone who has been in a movie with him is of degree 1. And anyone who has been in a movie with a degree 1 actor is themselves a degree 2 actor and so on. In fact, the originators of Internet Movie Database (IMDB.com) were inspired by this story to create one of the first prototypes of their website.

The same idea has been adopted by the mathematics community as a measure of distance from Paul Erdos. The Erdos number is how many steps one is from having published work with Erdos. Because he published with so many people, over 500 researchers have an Erdos number of 1. Many of Erdos' collaborators were already well published themselves, earning their collaborators an Erdos number of 2 (i.e., Einstein is of degree 2). For a full list, visit https://oakland.edu/enp/.

In random networks, the size of the network (what we will define later as the diameter) grows much more slowly than the number of nodes. This means that once a network is connected, it can grow and continue to have approximately the same size. So adding several thousand new actors to a network will not significantly impact the degrees of separation from Kevin Bacon. Stated another way, information flows around a random network are not significantly impeded by adding new units, thus giving rise to the six degrees phenomena. This will become important for networks that grow over time.

5.1.6 Small-World Networks

A network can be tuned to be between ordered (e.g., grid, ring) and random. For example, imagine that the roads in a city are initially laid out as a perfect grid. For historical reasons, however, new streets may be put down that do not conform to the grid pattern. Likewise streets might be removed to make way for buildings, or some streets might be made one-way. Over time the city streets will become a mix of order (from the grid) and disorder (from the changes).

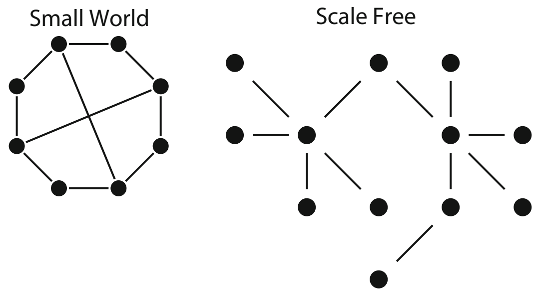


Fig. 5.9 Examples of small-world (left) and scale-free (right) networks. A small-world network is created by starting with a regular network (e.g., a ring) and then iteratively cutting a regular connection and adding back in a random connection. The generation of a scale-free network is shown in Fig. 5.22

This idea was formalized by Duncan Watts (1971–) and Steven Strogatz (1959–) in a 1999 paper that helped bring about a resurgence in network science research. They performed a very simple and intuitive change to a ring network. In a ring, if you want to send information from one node to another, you must go through all of the nodes between them. Watts and Strogaz proposed a slight modification, whereby one connection would be removed, but then in its place, a new link would be made between two random nodes. The number of nodes and connections always stays the same. It is only the structure that changes. The process starts with an orderly ring of connections. On each iteration, it will become more and more disordered, eventually becoming a random network. Using this method, it is easy to study the transition in network properties as the structure gradually changes from regular to random.

We will explore how to compute network properties later in this chapter. For now, we can make an intuitive argument as to the functions that can arise in networks that exist between ordered and disordered. Imagine a regular ring network as shown on the left side of Fig. 5.3. To get from node 1 to node 6 will require moving through four other nodes. Now consider the left-hand side of Fig. 5.9 where two regular connections have been replaced by two random connections. The number of connections has remained the same, but the effective distance between nodes has drastically changed. It now only requires movement through a few other nodes to get anywhere in the network. Just two connections that cut across the network can create a functionally smaller network. This effect is in fact magnified in larger networks.

What Watts and Strogatz showed was that when it comes to social networks, the origin of the six degrees lies in that most people know people who are geographically close by. They likely know, however, at least a few people who are not close by. It is these nonlocal connections that build a small-world network that gives rise to six degrees of separation.

5.1.7 Scale-Free Networks

Around the same time that Watts and Strogaz were exploring small-world networks, Albert Barabasi (1967–) was conducting studies on his university's email traffic. What he found was surprising. Many of the ideas of small worlds were present—no one seemed to be that far removed from anyone else, despite there being many people at the school. But it was in a way that was very different from the network structure described by Watts and Strogatz. Barabasi's networks contained what are known as *hubs*, shown in a simple example on the right side of Fig. 5.9. In the email network, these were the few individuals who were very well connected, making the network seem small. What is more these networks followed a power law of connectivity, first encountered in Chap. 4 in the context of scale-free distributions.

Scale-free networks retained a hierarchy but in a different way than the tree structure. Flows can circulate around the network but often pass through hubs. Some of the most creative companies such as Gore, IDEO, and InnoCentive have purposely structured their companies (either formally or informally) to enable a more free flow of ideas and information.

An important property of scale-free networks is that they are robust against random changes or attacks. If one node is chosen at random, it is very unlikely for it to be one of the hubs. If a non-hub is attacked, it largely goes unnoticed by the rest of the network. This property, however, comes at a cost. A coordinated attack, one that can find and take out a hub, will in one step take out a lot of the network connectivity. Hubs are the Achilles Heel of the system. On the other hand, hubs in a network can serve as leverage points to remove something unwanted, whether it is a cancer, political leader, terrorist network, or virus. Needless to say, Barabasi, having published his work just prior to September 11, 2001, has since been working with the US government to understand these phenomena in more detail.

The hub and spoke model used by many airline companies provides an interesting case study. Because most aircraft will fly through these hubs, certain time and resource efficiencies arise. For example, all of the servicing, cleaning, and staffing can be handled at a few hubs. The result is that passengers who fly Delta might very often be routed through Atlanta, while passengers who fly United Airlines will be routed through Chicago. The downside is that if a hub is not sending flights in or out, perhaps due to snow or a thunderstorm, it will stall the entire system.

5.1.8 Ecosystems

Many of the original thinkers in network theory were trying to understand food webs, and in particular energy flows between species, as in Fig. 5.10. These interactions could be predator-prey (e.g., salmon and bears), a host-virus relationship (e.g., human and Ebola), or some kind of symbiotic relationship (e.g., lichen as a relationship between algae and fungi). An impact on one species will often

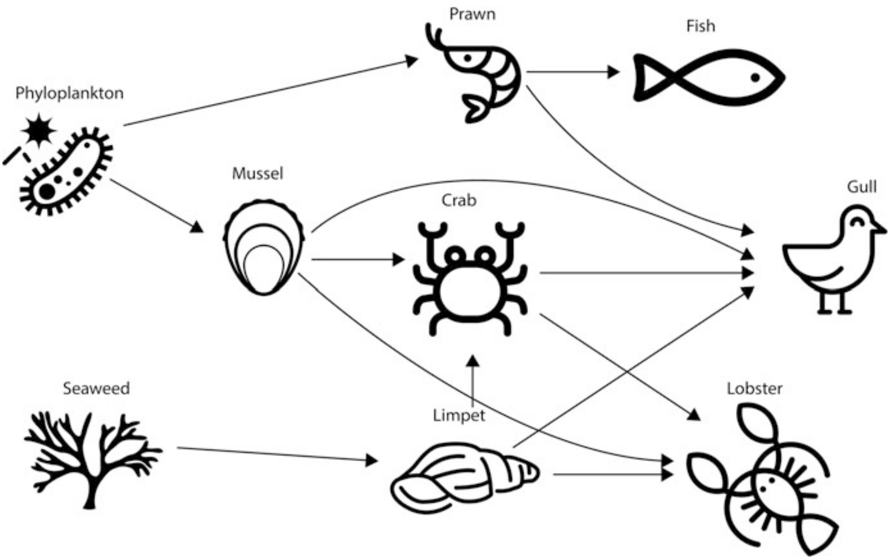


Fig. 5.10 A food web in an ecosystem represented by a network. Similar networks can be drawn to trace energy, information, money, power, or other resources within a system

directly or indirectly impact the entire web by disrupting the flow of energy through the ecosystem. A common finding from a network analysis of ecosystems is the identification of a keystone species, those species through which much of the ecosystem energy flows. Bees are food for some species but also pollinate flowers that form a food source and shelter for other insects. Ivory tree coral, in creating a reef, creates a landscape in which many other species live and interact. Sugar maples bring water from the ground upward to insects, birds, and small mammals as well as to provide many species with a refuge from predators. Each of these organisms acts as hubs of their respective ecosystems. As such, an ecosystem is very sensitive to any changes in its keystone species. We will explore ecosystems in more detail in Chap. 8.

5.1.9 Gene and Cell Networks

The past few decades have brought about a revolution in cell and molecular biology, largely brought about by the ability to systematically map out relationships of the "omes" (e.g., genome, transcriptome, proteome, metabolome). There are a wide variety of techniques for mapping out the interactions between units in these systems, for example, DNA microarrays. Mapping biological networks has led to

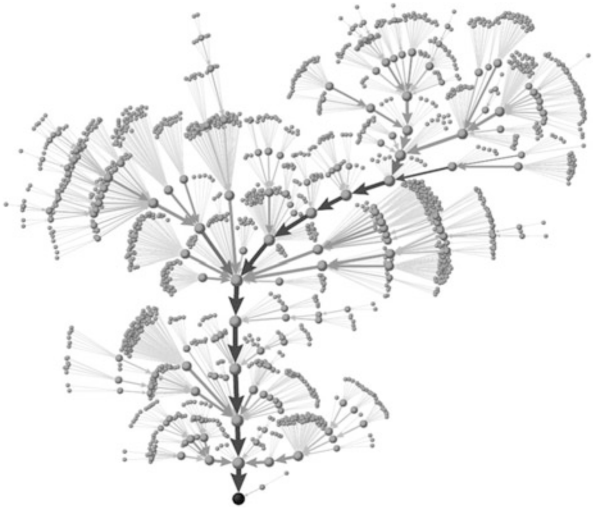


Fig. 5.11 A protein network for yeast showing 1764 states. Arrows indicate the attractor of the G1 phase. Image from Li, Fangting et al., *The yeast cell-cycle network is robustly designed.* PNAS 101 14 (2004): 4781-6. Copyright (2004) National Academy of Sciences

the rise of *systems biology*. In general systems biology is a subset of mathematical biology. In the past, however, math in biology was generally relegated to analysis of data using statistical methods or the building of simple models that would represent phenomena. With systems biology, the goal is much wider—to study biology as a system of interactions and derive basic "laws"—in the same way a physicist would study forces or matter. In fact, much of the work in systems biology is being performed by physicist working in collaboration with biologists. To learn more, an excellent introduction is *An Introduction to Systems Biology* by Uri Alon (1969–). A less technical introduction can be found in Dennis Bray's (1938–) *Wetwear*.

To take one example, the metabolic reactions inside of a cell can be thought of as a network, a sort of ecosystem of reactions. There might be molecules that act as hubs, for example, ATP, that are necessary for many reactions to take place. And there may be secondary hubs as well, often enzymes that catalyze reactions. And finally there are outlying metabolites that might only participate in a few reactions. When these networks are mapped out, they can reveal some deep patterns. For example, the 1764 interactions between proteins were mapped out for the yeast cell cycle, as shown in Fig. 5.11. What comes directly out of this network are the various modes in which the cell might operate (e.g., the S, G0, G1, G2 phases of growth and development). On this network, these growth phases are flows of reactions that can be turned on or off. Mapping out other molecular networks (e.g., genes, proteins) has revealed other motifs that have been conserved throughout evolution and might be present in species ranging from jellyfish and oak trees to bacteria and humans.

5.2 Adjacency Matrices and Network Measures

The mathematical study of networks, known as graph theory, is a collection of theorems and quantitative measures, both local and global that characterize various types of structures.

5.2.1 *Adjacency Matrices*

Graphically networks are represented by dots and lines. When numerically analyzing structure, however, connectivity is usually represented by an adjacency matrix. Once in this form, matrix operations can be used to uncover properties of the network structure.

The purpose of this text is not to detail the matrix approach, but it can be helpful to know a little about how some measurements are made. Figure 5.12 shows a small undirected network. To translate this diagram into an adjacency matrix, each row will represent a node. Node 1 will be represented by the first row, node 2 by the second row, and so on. The matrix representing this structure is

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

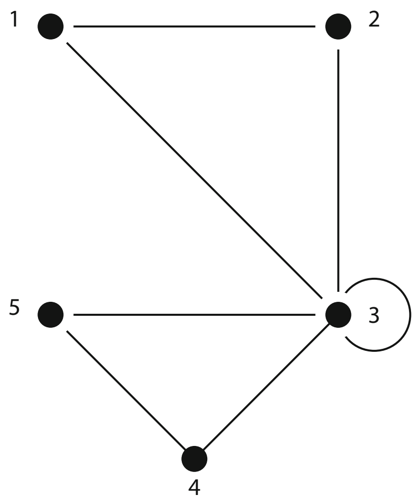


Fig. 5.12 An undirected network to demonstration the formation of an adjacency matrix

In each row, a 1 will be placed if that node is connected to the node that occupies that column. In Fig. 5.12, node 1 is connected to nodes 2 and 3, and so there is a 1 in each of these locations in row 1. Likewise node 2 is connected to nodes 1 and 3. In this way, the matrix will capture the entire connectivity of the network.

A few points are worth mentioning. First, this matrix captures the undirected network. In an undirected network, the adjacency matrix will be symmetric, meaning that nodes are mutually connected to one another, with the same pattern, reflected above and below the matrix diagonal. These types of patterns are easy to detect using matrix analysis techniques. Second, node 3 has the interesting property of being connected to itself. In some systems this is not allowed, and in others it is an important feature. The matrix can capture this structure easily by placing a 1 on the diagonal of the matrix (at location 3,3). Third, when the network is directed (indicated with an arrow as in Fig. 5.2), it is possible to have a flow go from node 1 to node 2, but not the opposite direction. This situation can also be represented in a non-symmetric adjacency matrix. Fourth, an adjacency matrix can also represent weighted connections (as on the right side of Fig. 5.7) simply by replacing the values of 1, with the weight.

As an exercise, it is interesting to consider what the adjacency matrices would look like for the various network structures discussed above. For example, how does a line differ from a ring? What would the adjacency matrix look like for a random network or for a scale-free network?

5.2.2 Node Degree and Histograms

One of the simplest properties of a node is its *degree*, defined as the number of other nodes to which it is connected. In Fig. 5.12, the degree of node 2 is two, because it is connected to two other nodes. Likewise, the degree of node 3 is five. This is an example of a *local* measure, one that represents properties of a particular node.

The measure of node degree can also be used to define two global properties of a network. The first is the *average degree* of nodes in the network. As a single number, average degree does not provide much information about the actual structure of the network. A more comprehensive representation is to plot a degree histogram. The degree of every node is computed. Then a count is made of all nodes with degree one, then all nodes with degree two, and so on. The result of this binning can be plotted (degree versus node count) in a histogram. Figure 5.13 shows a degree histogram for a few types of networks.

Regular networks, where the pattern of connectivity is repeated, will reveal a pattern in a degree histogram (e.g., top two panels of Fig. 5.13). For example, in a ring, each node is connected to its two neighbors, and so there is a single spike in the histogram. As soon as the network is irregular, the degree distribution can be described statistically. For example, a network with random connections might have a bell-shaped histogram centered around some average degree, as shown in the

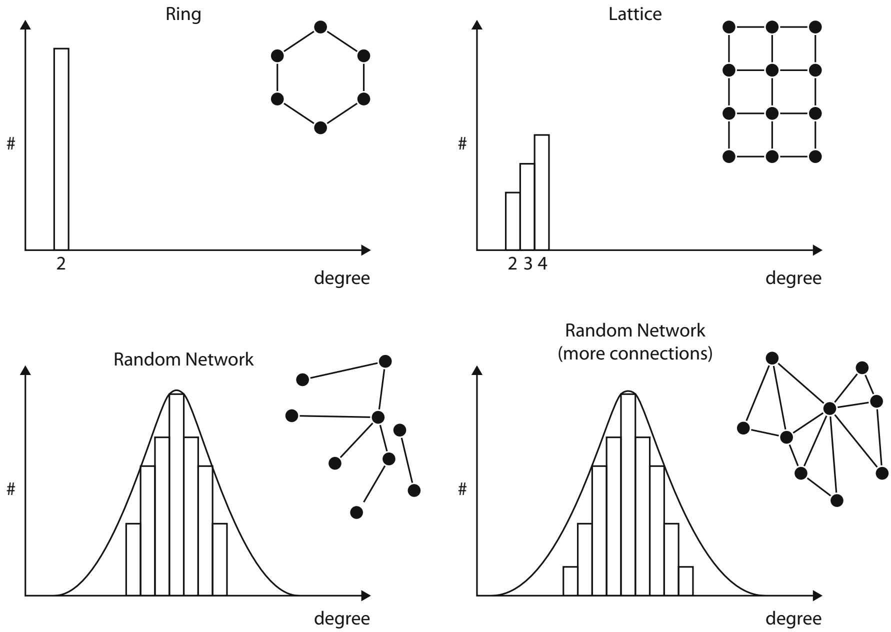


Fig. 5.13 Degree histograms for ring (top left), lattice (top right), and random networks (bottom). The bottom random network histograms show a random network that is less (left) and more (right) connected. The distribution stays the same as the mean shifts to the right

bottom two panels of Fig. 5.13. The mean might change as more connections are added, but the distribution would retain other statistical properties.

5.2.3 Centrality

Degree histograms capture important properties of a network, but they do a somewhat poor job of understanding how connected nodes are in relation to the size of the network. For example, if a particular node has a degree of 15, the meaning of that number is very different if the total size of the network is 20, versus if the total size is 2000. To overcome this sensitivity to the size of the network, *centrality* can be computed to gain insight into the importance of a node within the network. For example, *degree centrality* measures the relative degree as

$$\frac{\text{degree of node}}{\text{number of nodes} - 1}$$

Likewise we can compute *closeness centrality* as how close a node is to all other nodes in the network

$$\frac{\text{number of nodes} - 1}{\text{sum of distance from a node to all other nodes}}$$

Betweenness centrality computes how well situated a node is in terms of the flow paths within the network. What is important is that there are various measured of how "important" a node is within the network, and these are all termed centrality.

5.2.4 Pathlength and Diameter

A simple measure of flow in a network is the *pathlength* between two nodes. This is the shortest number of connections that are needed to get from a start node to an end node. It is important that the measure is only the shortest path because there may be many ways to get from a start to an end node.

As with degree, we can compute an average pathlength to measure the relative closeness of nodes within the network. Remember that in network theory, this does not mean physically close but rather close in terms of connectivity. New York and Tokyo are "close" in the sense that there are direct flights between the two cities. New York and Zanesville, Ohio, on the other hand are far apart because multiple flights would be required to travel between the two locations.

Based on the idea of pathlength, the *diameter* of a network can be defined as the longest shortest path that exists in the network. A few important caveats should be mentioned here. First, the "longest shortest" may sound strange, but it is very important. We only consider the shortest pathways between two nodes, and of those, the diameter will be the longest. The meaning is that the diameter is the farthest any two nodes could possibly be away from one another. Second, the diameter only really makes sense for a connected network, where all nodes can be reached by all other nodes in one way or another. If two nodes cannot be reached from one another, then the network is disconnected, and the diameter is infinite.

The pathlength is important in terms of how efficiently information can move throughout a network. For example, imagine that at each unit, there is a delay between receiving information and then broadcasting it back out. In general, the larger the diameter, the slower the system response time will be to a perturbation.

In 1999 the diameter of the Internet was measured to be 19. This means that all websites that are reachable on the web were at most 19 clicks apart. This is astounding given how many website there were in 1999. No one has yet updated this number, but estimates, despite the enormous growth since 1999, are that the Internet diameter is likely to be 20–23.

5.2.5 *Clustering and Clustering Coefficient*

How tightly nodes are connected to others can be measured by a variety of *clustering* measures that typically range between 0 and 1. A low clustering coefficient means that the network is not very dense—most nodes are connected to only a few of the possible nodes in the network. Likewise, a high clustering coefficient means that most nodes are connected to most others nodes. A nice demonstration is to recall the iterative creation of a random network. At first there is no clustering because no nodes are connected. As more connections are made, the clustering coefficient will rise. Eventually the network will become what is known as *full connected*, where every node is connected to every other node. In this case the clustering coefficient becomes 1.

Without going into the exact mathematics, clustering coefficients come in both global and local flavors. The local clustering of an individual node can be computed as

$$C_l = \frac{\text{number of triangle connected to the node}}{\text{number of triangle centered at the node}}$$

What this really measures, if we imagine a friend network, is the extent to which two friends of mine happen to be friends with each other. For the global flavor, the formal definition is based upon triangles of connected nodes.

$$C_g = \frac{3 \times \text{number of triangles in the network}}{\text{number of connected triples of nodes}}$$

This measure will be between 0 and 1 and gives an indication of how densely connected the network is. There are many flavors of clustering coefficients, all with slight variations on the theme of giving a relative idea of the connection density.

5.2.6 *Applications to Small-World Networks*

In analyzing the small-world model, Watts and Strogatz were tracking two measures of network structure as they deleted ordered connections and added random ones. One variable was the diameter of the network, a property that random networks do well at minimizing. Remember that the origin of the six degrees lies in the addition of long-range connections that decrease the diameter of the network. The other measure was the clustering coefficient, which measures the density of local connections. Local clustering is important because structural motifs which lead to coherent functions generally will only be repeated locally—they are not composed of random long-range connections. What Watts and Strogatz found was that there was a sweet spot. As an ordered network becomes more random, there is a point where it retains most of its local motifs, but overall the network has a small diameter. Small-world networks were found in power grids, the neural network of

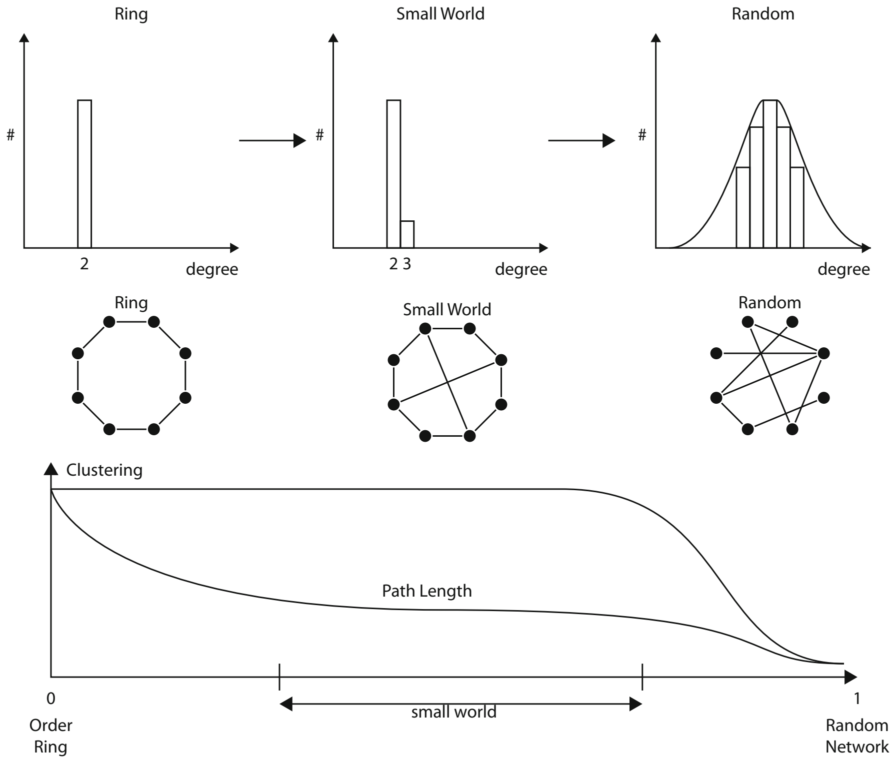


Fig. 5.14 Degree histogram during the generation of a small-world network. A small-world network is between an orderly ring and a random network histogram. Clustering and pathlength (bottom) changing at different rates as the network moves from ordered to disordered

C. elegans and the interconnections between movie actors. Figure 5.14 shows these two measures related to the degree of disorder in the network.

5.2.7 Maintaining Connections and Redundancy

Redundancy of flow pathways is a common property of many natural and human-made systems. For example, in a line network if one connection goes away, the entire network splits into two parts, effectively becoming disconnected. In a lattice, if one connection goes away (or even several), there will still be a way to get from one node to another by a longer path. The vasculature of leaves generally has a branching tree-like structure. If something blocks a branch, the flow will stop to all points downstream. To counteract this potential problem, leaves contain small loops that can divert the flow if needed. These are known as *collateral pathways* and give an intuitive feel for robustness—the ability to follow alternative pathways if one happens to be blocked. Similar arguments can be made for a wide range of human-made networks that aim to distribute resources, from the flow of money throughout a company and passengers in the various transportation networks to emergency response notifications and the hiring of second- and third-string quarterbacks by American football teams.

Alternative pathways are created by extra connections. In most natural and designed systems, however, it will cost the system something (e.g., resources, delay times) to create and maintain a connection. This is most certainly true for rail, airline, and neural connections. Furthermore, as a network develops, resource constraints will generally act to only retain critical network connections. An efficient network would contain only those connections that would be necessary. Such a perfectly efficient network, however, would be vulnerable to the loss of a single connection.

The balance between efficiency and robustness is a very complex subject and one that manifests itself differently in different networks. In fact some have called this the Goldilocks phenomenon (after the parable of Goldilocks wanting porridge that is "just right") of networks. Systems can have just the right amount of robustness without sacrificing much in the way of efficiency. We will explore this balance more in Chap. 10.

There is an entire branch of mathematics, called *percolation theory*, that studies what happens when connections (or in some cases nodes) are removed from a network. This is another way to study redundancy. The field has some driving questions that are important for all complex systems. For example, how many connections (or nodes) can be removed before the network begins to become nonfunctional? Does the execution of that function suddenly disappear or only lose effectiveness as connections are cut? Are there particular nodes (or connections) that have a greater relative impact on the flows in the network if they are cut? Another concept that is important is how to make what is known as a *minimal cut* (the smallest number of cuts) that will disconnect the network. This is important when trying to stop a rumor, virus, or other flow that is undesirable.

As a final example, the Internet has a surprisingly small diameter yet also has a great deal of robustness. The Internet was after all a product of thinking about the robustness of communication networks. During the Cold War (1947–1991), the existential threat was that for the first time in history, weapons existed that could completely knock out any ability to retaliate. A war could be won not through a series of smaller battles but rather by one coordinated attack in a matter of minutes. There are a variety of stories about the origin of the Internet. One is that the Advance Research Projects Agency (known as ARPA, the precursor of DARPA) funded a special project to build a communication network that would be robust against a coordinated attack. The engineering term for this is that it would *degrade gracefully* as lines of communication were cut. Building this sort of network would make it impossible to knock out one key target and bring down the entire communication network. The architects of the original ARPANET (as it was called) showed that networks could be built with any desired degree of redundancy.

5.2.8 *Nested and Overlapping Networks*

Complex systems are sometimes composed of a network of networks. To a physiologist, a cell or a tissue is a network. But to an economist, a human is a unit. In other words, networks might be *nested* inside of one another. Such is the relationship of ants to an ant colony, countries in a global economy, or departments in a company.

Networks may overlap in ways that are not hierarchically nested, sometimes called *multipartite graphs*. For example, several networks may occupy the same space, but their structure may support the flow of different types of information or material. In a cell there is a protein network, a gene network, a metabolic network, and a transcription network, all occupying the same physical space. They use different types of molecules, interaction rules, and connections. But they do have points of overlap in particular molecules or processes. Likewise, there are at least three networks that exist in your brain; the neural network, the glial network, and the vascular network. The vascular network distributes resources, the glial network regulates concentrations of various molecules (as well as other functions), and the neural network passes electrical impulses. These three networks must work together to perform the functions of the brain. A disruption to any one of the networks will impact the entire system. As another example we can consider social networks. Within an elementary school, we might have the network that exists within a classroom. Certain children form friendship bonds on the playground, others sit next to one another, and still others might be in learning groups. All of these interactions create a complex, classroom network. And there will certainly be children who will have connections across the boundaries of their class. These may have been from previous classrooms, extracurricular activities, or simply that their parents are friends.

5.2.9 *Business Networks and Innovation*

In the business world, efficiency is often associate with the ridged tree-like structure of an organizational chart. Policies and procedures ensure that information propagates up and down throughout the hierarchy in a predictable and orderly way. Innovation, on the other hand, seems to be associated with a nonhierarchical network (sometimes call a flat hierarchy) that encourages information and ideas to flow and be recombined more freely. Efficiency and innovation appear to be at odds with one another from a structural point of view. This conflict was in fact the central idea in Clayton Christensen's (1952–) classic business book *The Innovator's Dilemma*. The problem is that the nonhierarchical start-up company that is very creative will begin to look for ways to increase efficiency as they become more successful. They will do so by putting in place processes that will remove some redundancies to gain economies of scale but will likely reduce chance meetings, the primary driver of the free flow of information. Over time, these processes will become embedded in job descriptions, hierarchies, and formal positions, which will over time kill off creativity. Furthermore, it is rare for an entrepreneur to successfully transition to CEO of a company—the structure, culture, and skill sets needed during the start-up phase is often very different than those needed to develop and maintain a sustainable company.

There are a number of business strategies that have been derived over the years to balance efficiency and innovation. One of the classics is the Skunk Works. Lockheed Martin recognized that truly forward-thinking technologies might be developed by assembling a group of very smart people, working far from the oversight of bureaucratic administers who were looking for constant forward progress. The Lockheed Skunk Works was a secretive group with their own budget and limited reporting structure. That original group, organized in 1943, created some of the most groundbreaking military technology, such as the U-2, SR-7 Blackbird, F-117 Nighthawk, and F-22 Raptor. Other companies followed suit. When something innovative was required, it was passed off to the Skunk Works team. Incidentally, the term Skunk Works was used because the original Lockheed site was next to a plastic manufacturing company. The smell inspired the engineers to borrow the term "Skonk Works" from a popular comic strip at the time and the name stuck.

A problem was found in most companies, however, with isolating a group of people from the main activities of the company. First, from a network perspective, they become disconnected. Over time, if the personnel remained in the Skunk Works for too long, they could easily lose sight of their critical role in the company. Second, it is too tempting for a new manager to try to meddle in the work, asking for updates, questioning expenditures, and perhaps even dismantling the group. Third, truly brilliant work coming from the Skunk Works might not appear to the higher ups to be relevant or important. The canonical example is the creation of the first windowing operating system and mouse by Xerox PARC. The PARC research group was a division of Xerox that was purposely moved out to California, away from the company headquarters in New York. But, in the creation of a graphical operating system, coupled to a mouse, they were so far ahead that Xerox didn't take notice. The result is that they gave the technology away to Apple (and later Microsoft).

Over the past few decades, the balance between efficiency and innovation has been achieved in a new way. Google, 3M, and other companies have implemented variations of an Innovation Everywhere for Everyone program. Everyone in the company is tasked with innovating in at least two ways. First is to discover ways to do their own job better. When someone finds a new way to do something, they are encouraged to share it with others. Second is to actively engage in projects that are new. This has resulted in the Google 20% rule. At Google, and other companies, 80% of an employee's effort is in support of their job description. But they also have 20% of their time and effort dedicated to pushing the boundaries—often by working on projects that radically cross divisions. The company is counting on a return on that 20% in the form of new programs, products, or processes.

Yet another concept relies on the idea of overlapping networks. John Kotter (1947–), one of the gurus of business thinking, has recently argued in his book

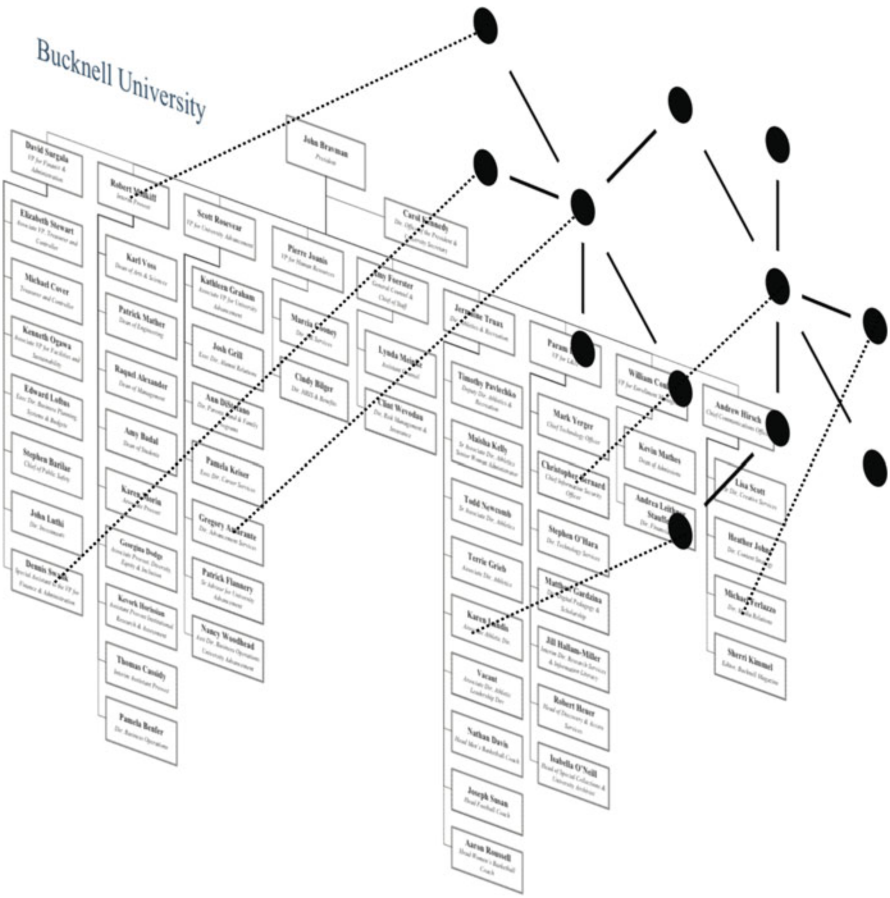


Fig. 5.15 John Kotter's traditional business hierarchy for management tasks (foreground) with ad hoc flat hierarchy for innovation (background). Individuals may play roles in both networks

Accelerate for a two-network system as shown in Fig. 5.15. The running of the company and concern about profits and efficiency is the domain of management. As such management tasks are best supported by the traditional tree-like organizational chart. When leadership and innovation are required, this should be passed off to a different organizational structure. Kotter says that this organization should be temporary (e.g., only put together for a particular task), composed of people with a diverse range of skill sets and levels within the traditional organization (e.g., create a flat hierarchy), and be given little oversight about how they accomplish the task (e.g., the group can make their own rules, within reason). Kotter's point is that perhaps these two different kinds of networks can coexist, one being efficient and the other being innovative.

5.3 Scale-Free Networks

With some network measures in hand, we can return to scale-free networks from an analytical perspective.

5.3.1 Histogram of the Scale-Free Network

The hub and spoke network in Fig. 5.9 can be analyzed using a histogram. This histogram, shown in Fig. 5.16, reflects that there are many nodes, n, with small degrees, d, and a few nodes with very high degree. It may be tempting to look at this graph and assume that it is best described by an exponential decay. In other words that the relationship between d and n is something like $n = e^{-ad}$, where a is a constant. Barabasi's analysis, however, was clear that the relationship was not exponential. It was of the same form as the power law from Chap. 4.

$$n(d) = d^{-a}$$

The main difference between the exponential and scale-free distributions is what happens toward the far right side of the plot, where high-degree nodes are represented. In an exponential distribution, the rate of decay keeps decreasing at

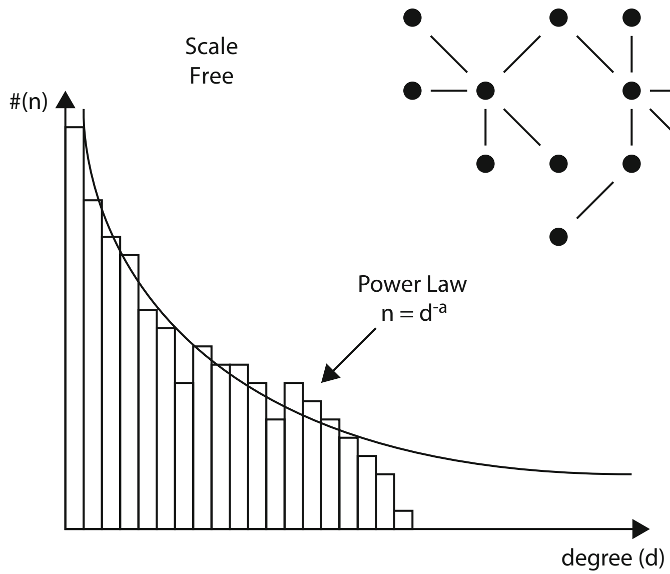


Fig. 5.16 Degree histogram for a scale-free network. The network is called scale-free because it follows a power law distribution in degree

the same rate—that is in fact the definition of exponential decay. Any phenomenon toward the right-hand side becomes less and less likely, at a faster and faster rate, and quickly goes to zero. This type of rapid decay does not occur with a power law. In fact, the rate of decrease becomes smaller farther to the right. The result is that while phenomenon may become rare, their probability does not go to zero. Some high-degree nodes do in fact exist.

The same idea was present when we explored, although in graphical form, the self-similarity of fractals. Fractals were defined as having a length (or area) that depends on the size of the measurement tool. When the size of the tool and the total measurement are plotted against one another, the result was a power law. In a sense, the connectivity of a scale-free network is a kind of fractal. Remember that networks are not representing physical location but rather relationships. What this means is that a hub plays the same role whether it is a local hub or a global hub.

To make this idea more concrete, remember the exercise from the introduction of this chapter where you imagined your social network. You likely know some individuals who are very well connected and others who are much less connected than you. To those who are less well connected, you are their hub. If you ask people who are better connected than you what they see, they will likely report knowing people who are even better connected. In a theoretical scale-free network, every point in the network will appear to be about the same. In this way, all people in a scale-free social network are self-similar.

5.3.2 Power Laws in Language and Wealth

Power laws were discovered in systems long before they were studied by network scientists. George Zipf (1902–1950) pointed out that words in common English texts followed a power law, regardless of the time period or style of literature. For example, if one were to take a representative body of work in the English literature, for example, the works of Hemingway, and plot the frequency of word distribution in a histogram, the distribution would be roughly a power law. In a large body of text, words such as “the” and “a” and “if” would appear very often, while other words such as “green” and “culture” might be less well used, with “astrobleme” or “croze” being used very rarely. The distribution for this chapter can be seen in Fig. 5.17.

Zipf also went on to show a similar power law underlying city sizes and income distributions. In fact, the idea of a power law goes back even farther to the nineteenth-century economist Vilfredo Pareto (1848–1923). He was the originator of the 80/20 rule, sometimes called *Pareto’s law*. The simplistic version of the law is that 80% of the wealth is in the hands of 20% of the people. But, Pareto then said that if you take those top 20% earners and create a distribution of them, you would get the same distribution—80% of the wealth among the wealthy are held by the 20% who are super-wealthy. This is another example of self-similarity.

5.3.3 The Long Tail and the Black Swan

A power law is also sometimes called a *fat tail* or *long tail*, where nodes that are very well connected come to dominate the dynamics. Two popular books have explored the implications of power laws in society. *The Black Swan* by Nassim Taleb (1960–) and *The Long Tail* by Chris Anderson (1961–) both provide many examples of how power law distributions intersect society.

Rare or even onetime events can dominate the future direction of a system. Consider a culture that has only encountered white swans. They would take as a fact that swans cannot be black. An encounter with a black swan, however, would require a rethinking of a previous assumption. The *black swan effect* is directly related to power laws, most specifically the idea that very rare events are also the ones that often have major impacts. For example, we have no record of a magnitude 10 earthquake. There may only be a few that have happened in the history of the earth, but they would most certainly have an enormous impact. The same can be said about major volcanic eruptions or asteroid strikes.

In society, rare events can set up *path dependencies* that shape the future. Large failures in a system, such as an economic crisis, prompt a change in rules in an attempt to prevent that same (or similar) failures in the future. Major events could also be personal and shape a person for the rest of their life. A crisis leaves a kind of scar or record. The same, however, can go for luck and opportunities. A chance

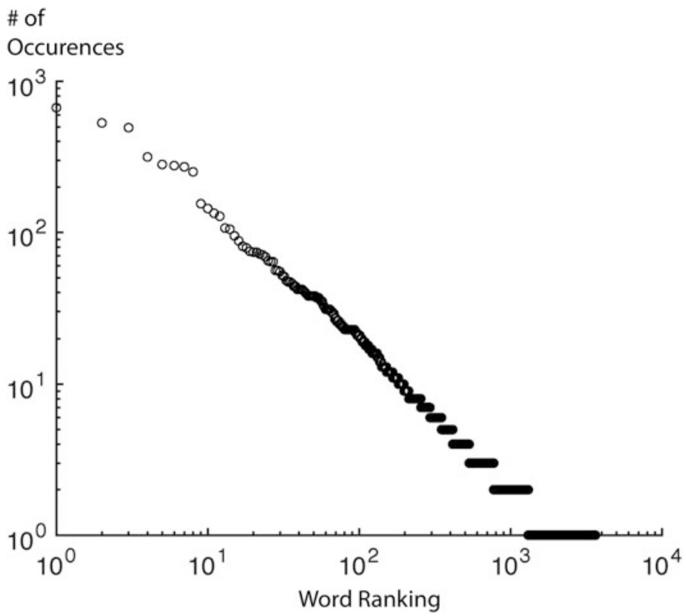


Fig. 5.17 A histogram of word usage in this chapter, demonstrating Zipf's Law

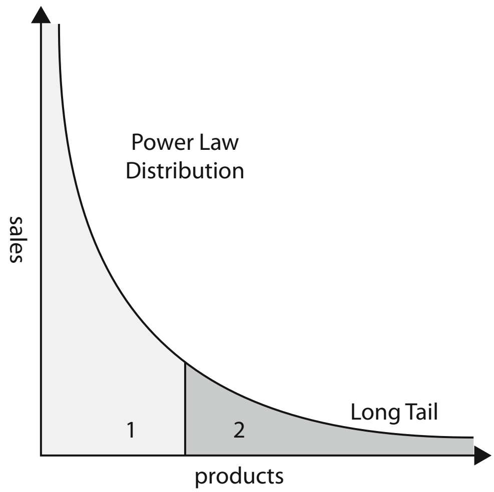


Fig. 5.18 Scale-free histogram distribution, sometimes called a long tail. In some distributions, there may be more area under the tail (less popular, region 2) than in high volume (more popular, region 1)

event, although rare, may reverberate through a life, ecosystem, or political system for a very long time.

The idea of a *long tail* is another idea that is derived from a power law. As shown in Fig. 5.18, some distributions have more area under the right-hand side of the curve (the long tail, region 2) than in the steeper left side of the curve (region 1). The message behind Anderson's book is that the Internet has prompted a fundamental shift in the distribution of products. Before the Internet, only a few very popular products in a particular market segment could survive (those in region 1). The products that were on the long tail, although interesting, would eventually die because they either could not find enough customers or they would be bought out by larger companies. This would resemble an exponential curve where less popular products effectively do not exist. The Internet enabled smaller business and their niche products, to survive by helping them find customers who might be spread out across the globe. The result is that while there might still be a few products that are enormously popular, they coexist with many products in the same market that are less popular. What is more, Anderson's claim is that in a growing number of markets, more profit is being made in the many products on the long tail than by the few very popular products.

5.3.4 Universality of Scale-Free Distributions

In Chap. 4 we learned that fractals (in space), bursts (in time), and strange attractors and chaos (in phase space) all display scale-free behavior. In this chapter scale-free networks were found to describe the connection patterns in economic, gene, and neural systems, as well as the structure of DNA and the size of earthquakes and other disasters. There are many in the complex systems world who hypothesize that a power law is a fundamental property of complex emergent systems. The basic argument is that as structure and function co-emerge together, the system will naturally become scale-free in the distributions of connections and behaviors. We will explore this idea more deeply in Chap. 10.

5.4 Growing Networks

Some systems grow and change over time. Connections will be created or cut, perhaps governed by simple rules. In this chapter we have already explored some examples of simple rules that can grow or modify networks over time, for example, Erdos' way of creating a random network and the Watts and Strogatz method of rewiring a ring network to become a small world. These rules, however, were created by mathematicians to study networks and were not meant to represent how actual networks grow. In reality changes in structure are most likely driven by balancing efficiency and robustness. In this section we will explore a few ways in which complex systems may more plausibly grow their own structure. The idea of structures that can change will be a theme of Chaps. 7 and 10.

5.4.1 L-Systems

A beautiful and scientifically interesting example of growing a branching structure is Lindenmayer systems (sometimes called L-systems). They were introduced in 1968 by Aristid Lindenmayer (1925–1989), a plant biologist. Lindenmayer hypothesized that there was a kind of grammar that would guide how plants branch and grow. He introduced a system of rewrite rules, based upon earlier work by mathematician Emil Post (1897–1954), to describe three growth principles in L-systems. There is an alphabet of symbols that represent something. There are initial strings made up of symbols in the alphabet, known mathematically as axioms. Lastly there are a set of rules, sometimes called production rules, which transform one set of symbols to another set. As a simple example, imagine that the only two symbols are A and B and that the axiom (initial symbol) is A. We can then specify some production rules.

$$A \longrightarrow B$$
$$B \longrightarrow AB$$

Given these simple rules and an initial condition, we can iterate upon the rules.

$$A$$
$$B$$

$$AB$$
$$BAB$$
$$ABBAB$$
$$BABABBAB$$
$$ABBABBABABBAB$$

Counting the number of characters in each line leads to the Fibonacci series (1,1,2,3,5,8,13, ...), a well-known series first formally described in 1202 by the Italian mathematician Leonardo of Pisa (1175–1250, also known as Fibonacci). The series potentially has an earlier origin in India and has been connected to the way leaves and branches spiral around a stem, sunflower seed spirals, and the golden ratio that has a long history in design, architecture, art, and music.

The example above was provided only as an introduction. Because the rules operate on symbols, these symbols can stand for anything. In L-systems they stand for rules for drawing instructions. Below is an example of how an alphabet can be mapped to drawing rules:

F: Draw a line segment of length d, in the current direction
$+$: Turn left (counterclockwise) by the angle θ
$-$: Turn left (clockwise) by the angle θ
[: Remember the current state of the drawing tool
]: Return to the old location

The two symbols [and] allow for a drawing tool to be picked up and placed somewhere else but then to remember where to return later. An example of a production rule would then be

$$F \longrightarrow FF$$
$$X \longrightarrow F[-X]F[+X] - X$$

With the value of $\theta = 22°$ and an initial string of X, a series of drawings can be created that are iterations of the graphical substitution rules. Figure 5.19 shows the first, third, and sixth iteration of the L-system grammar with these rules.

Using his system, Lindenmayer was able to write down rules (with corresponding angles and initial conditions) for particular plants. He then was able to determine, based on simple changes to the rules, which plants must be closely related. In his experiments, he in fact identified some plants that his grammar would suggest were misclassified in the taxonomic tree at that time. It was only decades later that his reclassifications were confirmed by DNA analysis.

L-system rules can be written that will generate other types of fractal structures, including the Cantor set, Koch curve, Sierpinski triangle, and Fibonacci series. They can also be easily extended into three dimensions. Others have introduced probabilistic rules that will generate plants that are not identical but easily recognized as

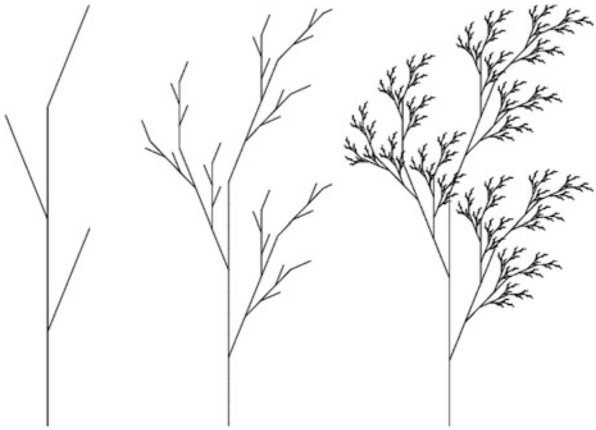


Fig. 5.19 Example of L-system plant generation. Iterations shown are 1 (left), 3 (middle), and 6 (right). Such systems operate on the principle of *recursion*, when a basic generative pattern is repeated at multiple levels in time or space

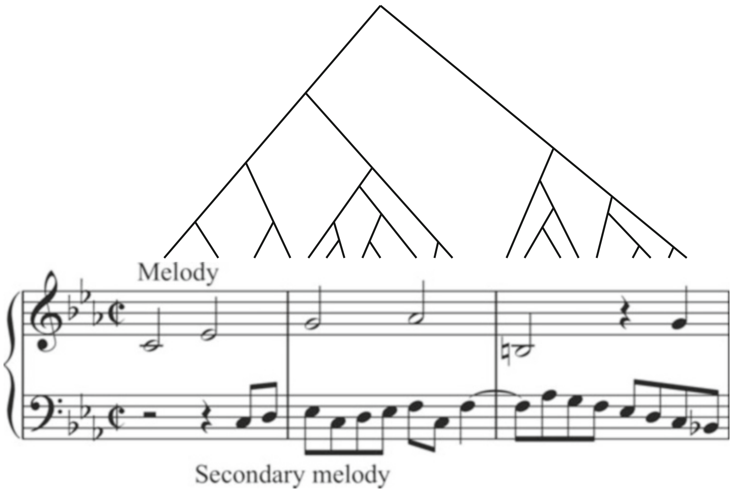


Fig. 5.20 Musical grammar as a tree structure

belonging to the same family. Because the symbols can stand for anything, there have been many extensions of L-systems from the generation of language (explored more in Chap. 6) to computerized music (shown in Fig. 5.20).

It is worth mentioning that the pioneer of studying biological structures using mathematics was the Scottish scientist D'Arcy Thompson (1860–1948). In his 1917 book *On Growth and Form*, he introduced the idea of morphogenesis as mathematical patterns that would be expressed in plants and animals as they develop. Thompson's aim was to link evolution with physical laws (specifically mechanics). His idea had many similarities with L-systems, where a slight change in mathematical rules could transform an animal or plant, as shown in Fig. 5.21. Thompson's work, although later shown to be incorrect, was very influential in demonstrating that mathematics could be applied to biology.

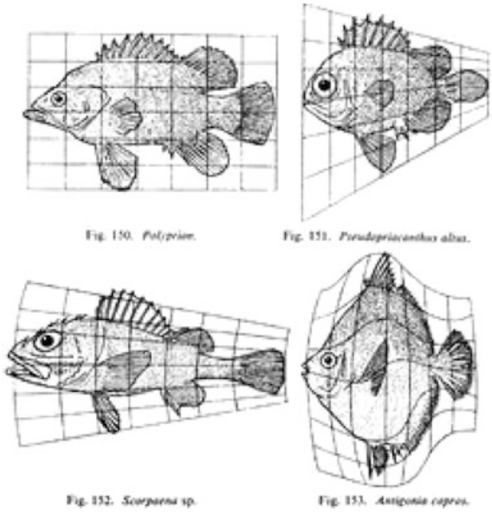


Fig. 5.21 Geometric transforms of one species to another according to D'Arcy Thompson. Thompson imagined that these transformations to be governed by a sort of mathematical grammar

5.4.2 Rich Get Richer

When Barabasi characterized scale-free networks, he quickly pointed out that power law distributions can be generated using some simple growth rules. In a follow-up paper, he presented a mathematical version of a *rich get richer* algorithm for forming connections, sometimes called the St. Matthew principle after the biblical quote (Matthew 25:29), "For to everyone who has will more be given." Barabasi's model uses *preferential attachment* to grow from a few connected nodes. When a new node is added, it is connected to already existing nodes. Barabasi's algorithm simply states that these new connections will not occur at random. Instead a new node will connect to already well-connected nodes with a higher probability. This process is demonstrated in Fig. 5.22. Over time, well-connected nodes will become even more well connected. But the algorithm is probabilistic, so nodes with lower degrees will still occasionally gain new connections.

There are a number of insights that can be gained from this growth model. The early nodes that form the seed often become the hubs of the larger network for two reasons. First, they are in the network from the beginning, so they have had more time to form connections. In the scientific world, this is similar to the fact that foundational papers published many years ago have had more time to be cited, whereas more recently published papers have not had time to build up citations. Second, the first new nodes that are added to the network have no other choice but to connect to the seed nodes. This means that they are biased very early on to be relatively well connected. In urban development, it is often the case that the location of the first settlements and roads turn into city centers and main transportation pathways. The growth of some technologies also follow this rule in the marketplace if they arrive first, sometimes called the *first-mover advantage*. Likewise, families

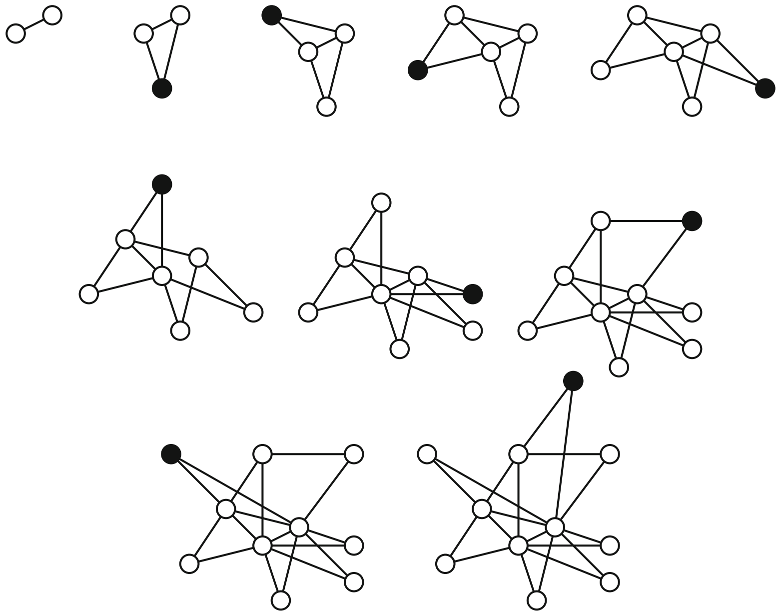


Fig. 5.22 The generation of a scale-free network based upon preferential attachment. Nodes that are more highly connected will more strongly attract the connections of newly added nodes

who have been living in an area for generations often have become hubs of the community because newcomers will more likely gravitate to them for advice and friendship.

5.4.3 Tiling and Pruning

The field of artificial neural networks has created another network growth algorithm called tiling. The idea is based upon the assumption that a desired function can be represented by some particular network configuration and flow on that network. The goal is therefore to find a particular network architecture that will produce that particular function. As it is not clear what the size or make-up of the network should be at the onset, an iterative process starts with only a few nodes. Connections between these few nodes are added, sometimes with changes in the weights along the connections. At each iteration, the ability to reproduce the function is tested. If the structure reproduces the function, no more iteration is necessary. If the function cannot be reproduced within some margin of error, then more nodes are added, and the process is repeated. Some claim that this process is what all developing and learning organisms, whether human-made or natural, undergo to some degree.

The opposite of tiling is pruning—the cutting of connections that are unnecessary. For example, it is well documented that the brains of human newborns are overconnected. The emergence of coherent functions can be viewed as the balance between formation and pruning of connections. We will explore this idea more in Chap. 10.

5.4.4 Network Phase Transitions

In Barabasi's model, when a node is added to the network, it necessarily must connect to other nodes. The result is that the network will always be connected; no node will end up being on its own, and flows will therefore be able to reach the entire system. This is not always how networks arise. Sometimes (perhaps often in some systems) the creation of a node is an entirely separate process from the forming of connections.

Stuart Kauffman (1939–) proposed a simple analogy to understand how fully connected networks may arise suddenly by adding connections once nodes already exist. Imagine 100 buttons on a table. You pick two random buttons, and tie them together with a bit of string. Then you randomly select a button, and pick it up. On the first try, it is very likely that only one button will be picked up. At most two buttons will be picked up but even that would be very unlikely. You repeat this two-step operation over and over again, tying together two random buttons and then picking up a random button. As this algorithm is repeated, you will eventually start picking up several buttons at once, due to connections that were made earlier. If you keep going with this process, you will eventually connect enough buttons that everything will be connected, and you will pick up the entire network.

Kauffman's question was about how networks transition from being completely unconnected to completely connected. What he proved was a bit striking. For early iterations, you will find yourself picking up small groups of one or two or maybe four buttons. Then, all of a sudden, over just a few iterations, you will begin picking up much larger groups of 10 or 20 or more. And with a few more iterations, you will be picking up the entire network. The basic mechanism underlying this phenomenon is that larger groups join together faster. Once a few larger groups have formed, there is a runaway process by which the entire network becomes connected very quickly. What is more, this transition to a fully connected system always seems to happen when the number of connections is approximately half of the number of nodes. In the example this would be when 50 bits of string have been used. This is a another example of a phase transition, as shown in Fig. 5.23. This particular transition is known as a "second-order" transition and will be discussed in greater detail in future chapters.

5.4.5 Structurally Adaptive Networks

We have assumed that complex systems could adapt but that they were *functionally adaptive*. These functions were already stored in some preprogrammed way within

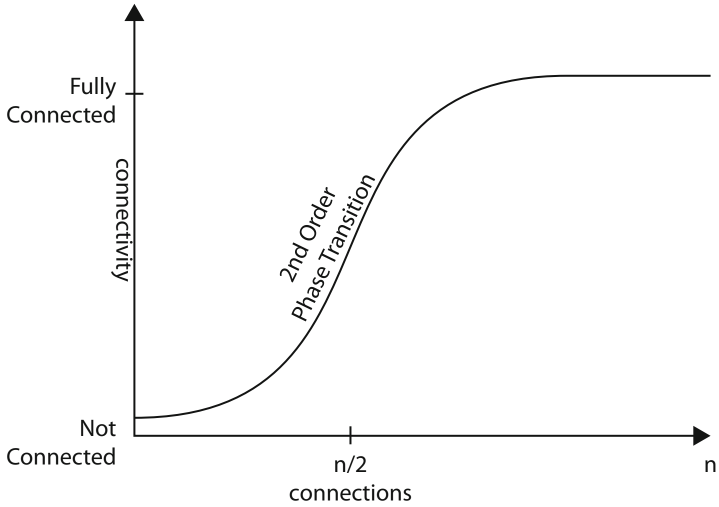


Fig. 5.23 Second-order phase transition in Kauffman's network growth model

the system. For example, they might be separated by some separatix, as in Chap. 3. The flexibility comes in how and when they are *expressed* due to some internal or external trigger, as in the expression of a gene. The ability to grow a network, however, implies a fundamentally different way for a system to adapt. A complex system can potentially repair a problem when it is structurally injured. If a connection is cut, for example, if a key employee retires or a species goes extinct, the system can rewire itself. Likewise, the network might alter its structure to better balance efficiency and robustness, by adding or deleting nodes or connections.

The ability to change both structurally and functionally is a property of many complex adaptive systems. The deep implication is that a complex adaptive system has a convoluted causal relationship between structure and function. In a system that can change its own structure, it is not possible to claim that structure dictates function, because one of the functions of the system is to change its own structure.

5.4.6 The Emergence of Functions

Rewiring of a network can be a pathway to discovering new functions. A way to think about Kauffman's button analogy is that as the network becomes connected, more and more potential functions are being created. With each new connection, there is the possibility of new emergent behaviors. In fact, Kauffman was first trained as a medical doctor, and his network idea was meant to represent his view of evolution; various genes (and their expression in proteins) may have arisen (nodes), but as they became more and more connected, there was the emergence of complex biological functions.

The idea of functions arising due to connectivity may also, at least partially, explain how autocatalysis (discussed in Chap. 4) may arise. When there is only one chemical reaction, the reactants must be supplied from the outside to keep the reaction going. When there are multiple reactions, the right connective reaction may result in an autocatalytic loop. A similar idea can be used to explain how a group of unconnected neurons would never continuously generate electrical patterns, but once connected, they could generate a complex cycle of firings. Likewise, when we claim that a team is more than the sum of their parts, we are acknowledging that a function has emerged that could not be achieved by any particular person.

5.4.7 Evolution and Hierarchies

Motifs, structural patterns of connectivity that appear over and over again, are found throughout complex systems. Herbert Simon (1916–2001) came up with a clever way of illustrating how systems, both natural and human-made, might develop motifs and modularity in a classic book *The Sciences of the Artificial*. Imagine two watchmakers who produce identical quality watches. Watchmaker one can create a watch from start to finish in 60 minutes. Watchmaker two has decided to decompose the watch into ten parts. Each part takes 7 minutes to create, and then it is trivial to recombine them to form the functional watch. Who will be more productive in the long run, watchmaker one or two? Some simple math shows that watchmaker one creates a watch in 60 minutes, while watchmaker two creates one in 70 minutes. Clearly watchmaker one will be able to fill more orders.

Simon, however, throws in an important consideration. What if the watchmakers also need to tend to other parts of their business, for example, taking orders? What if they need to take time out to make repairs? In other words, what happens when a system needs to be multifunctional? Simon framed the problem; each time a watchmaker is interrupted, they lose all of their previous work and must start over. It is possible that watchmaker one could lose up to 59 minutes of work if a customer comes knocking toward the end of the assembly. Watchmaker two is not as susceptible to this problem because at worst, 6 minutes will be lost. Consider now what happens as both watchmakers become popular and have more watches to make, as well as more orders and repairs. Watchmaker one will initially win out, but becoming more productive also means being interrupted more often. At some point watchmaker two gains an advantage.

In this story we find two related insights that are relevant to the study of complex systems. The first is that a modular design *scales*, meaning that it is designed to quickly create many copies, what biologists would call reproductive fitness. And it is the “quickly” that matters. In a world of finite resources (e.g., food, water, customers, attention), the system that can be reproduced more quickly will eventually win out. Modularity accelerates reproduction. Furthermore, in a limited resource setting, a complex system will only be able to express some small percentage of the total possible functions at any one time. Functions will constantly be interrupting one another, leading to even more pressure to modularize. As a result modularity is found in biological systems from genomes to societies and human-created systems from products to business models.

There is a second powerful argument as to why watchmaker two will win. Simon's argument was for watchmakers who never change their design. But we know that this would likely not be the case, as it is in the history of watchmaking and nearly all technological systems. A slightly changed module will likely still be compatible with other functions already in the system. As a result, a new motif can spread quickly to the entire system. If a modular business structure is in place, the discovery of a way to reduce parts, labor, time, or some other variable can more immediately be used elsewhere. Perhaps more importantly, small changes can occur to a single module without requiring changes to the entire system. As a result more possible changes can be tried in a shorter amount of time, increasing the rate of evolution.

Underlying the argument for modularity and iteration is the idea that evolution, and perhaps the mechanisms of natural selection, can be applied to all complex systems. Just as a spiral is a pattern that can arise in systems with particular properties, perhaps evolution is a mechanism that most easily arises and thrives in modular systems.

5.4.8 Humans and Technological Becoming

There are two striking examples of connectivity leading to emergence of new functions that can serve as case studies. The first is the claim by Michael Gazzaniga (1939–), one of the founders of cognitive neuroscience that the striking abilities of a human emerges from the recombination of existing functions. In this regard, recombination is achieved by adding connections. As he explains in *Who's in Charge*, out of this recombination new functions emerge that did not exist before. Previous evolutionary transitions may have been the origin of the cell, the origin of multicellular organisms, and so on. What Gazzaniga claims is that humans are on the steep slope of a phase transition. Martin Nowak (1965–) would agree and is even more specific that our ability to manipulate all kinds of language, natural and symbolic, as a tool that has pushed up through this phase transition. We really are different in kind (not simply degree) from other entities and perhaps are not just another species on this planet. It is not so much about the number or kind of functions we can express, which are very likely shared with other living organisms, but rather about the way in which they are connected and expressed together. Of course this could also be the arrogant self-justification of a species attempting to assert their superiority.

The second striking example is from a speculation about where technological advancement may be taking us. There are many dystopian visions of technological takeovers, from robots and artificial intelligence to mutated viruses and horrific biological creations. In many cases, however, the claim has been that as the creators, we are in control and would put in place fail-safe rules that would prevent widespread takeover. In this view, sliding into a dystopian world is entirely of our own making.

With the idea of phase transitions, however, it is possible to see how we might not be able to "pull the plug" soon enough. Throughout the history of our species, we have created more and more complex and multifunctional tools. We have also learned new ways to exploit natural laws, which has led not only to new tools, but new ways to recombine those tools. Robots, computers, and the creation of synthetic biological organisms are all due to the recombination of previously created tools. Is it possible that in recombining tools we will inadvertently push technology through a phase transition? Such a technology would be of a different kind, much as Gazzaniga claims that we are of a different kind. And, because phase transitions happen suddenly, we would have little warning.

To present one possible scenario, consider the recent trend of the *Internet of Things*. We have created a wide range of products that help us with our daily functions. Then we put some minimal "smarts" into them. With the Internet of Things, we are beginning to connect smart devices together. Imagine that your car knows that you are on your way home and will trigger your refrigerator and stove to start preparing dinner, chosen based upon your past preferences. Your phone recognized that in conversations earlier that day your voice indicated that you had a hard day, so an alcoholic drink has been prepared for you. In addition, an email was sent out to a friend of yours to come over to your house to watch a game that night. Their car automatically drove them to your house after they got out of work. In such a world, who is in control? What would it take for such an interconnected technological system to "wake up"?

5.5 Questions

- Some educators use *concept maps* to explain how topics in a discipline are related to one another. In some classes, the instructor will even ask students to create their own concept maps. A concept map is simply a network with concepts being the dots and lines representing the relationships. Interview a friend or family member about some area with which they are familiar. Once they have laid out the main ideas, introduce them to the idea of a network, and how they can organize their field into a concept map. Stay with them while they work it out. Note that many conversations can be thought of as filling in an internal concept map—in this exercise you have simply make the map visible. Turn in the concept map that you created together along with who you interviewed.
- Open up a newspaper or news website. Read over the headlines and perhaps the first paragraph of each article. Can you create a network of each of these articles and how they might be related to one another? Use the topic of the articles as dots and then when you make a line connecting two articles, label it (with words)

to indicate how they might be related. Note that there may be some articles that are not connected to any other articles (at least that you can imagine).

- It is a key hypothesis of Richard Ogle that creative groups of people, in one way or another, form a scale-free network. He lays out his argument in *Smart World* and describes a scale-free network as the right balance between order and disorder, feedback, robustness, and continuous development. What do you think about this? Can you think of creative groups that you have been a part of? Were they (or were they not) scale-free in some way?
- Vannevar Bush (1890–1974), an administrator for the Manhattan Project, founder of Raytheon, and one of the chief architects of the National Science Foundation, published *As We May Think* in 1945. In the book he proposed what he called memex, essentially the idea of hypertext. He predicted that "wholly new forms of encyclopedias will appear, ready made with a mesh of associative trails running through them, ready to be dropped into the memex and there amplified." Go to a Wikipedia page that is of interest to you. Write down the topic in the middle of a page with a circle around it. This will serve as the hub of a network diagram. Then map out (at least in part) the links that you could follow to create a concept map (see the first question above). You should go at least a few levels deep, following a few links off of your main topic. Do not try to map out everything! Your goal is simply to begin mapping out your concept of interest.
- Related to the previous exercise you may play WikiGame (also known as WikiRace) with some friends. The idea is to start with a randomly selected starting and ending Wikipage. The goal is to find the shortest click pathway from start to end in a set amount of time. Play the game with friends. Afterward discuss your strategy, and take notes on the various approaches. Turn in these notes.
- Architects sometimes map out the flows within a home or building by treating rooms as nodes and the passageways between rooms as edges. Find a relatively complex space, and map it out as a network. Discuss (or perhaps draw out) changes you would make if you were the architect, and explain your rationale.
- The idea of robustness through structural adaptation in a network was framed as a positive aspect of a network. It is often a way for a network to heal in the face of injury. But robustness can be a double-edged sword. Sometimes robustness can make something undesirable hard to remove, for example, terrorists networks or cancer. Because of a loose organizational structure that is constantly amorphous, it is not possible to simply knock out some key component. Pick an example of a system that is harmful but hard to change. Discuss how this system's network connections make it robust.
- Many sports teams can be thought of as an ecosystem where various plays flow from player to player. In many sports, one (or a few) players will act in a similar way to a keystone species—plays often move through these players. In some cases this is due to that particular player's position, while in others, it is more dependent upon their particular set of abilities or the calls of a coach. Choose a specific sports team, and dissect it as an ecosystem. Are their keystone players? What makes them the keystone of their team?

- Networks of concepts in the brain are thought to be responsible for creating context. When we talk to another person, there is a great deal that is left unsaid because we assume that there is a shared network of context. The result is that we can assume many things in a conversation. Likewise, some great teachers, spiritual leaders, parents, and speakers leave things unsaid, to be filled in by the listener. Have a conversation with someone, and try to notice the things that are unsaid that allow you to have the conversation. What concepts are part of your shared network of ideas?
- Leaves were discussed as having redundancy to continue the flow of a resource when pathways are disabled. The Internet was also discussed as having redundancy as a way to continue the flow of information. What other systems seem to contain redundancy? How does that system achieve redundancy?
- Networks can be used to map out a goal complex—some collection of interrelated goals that one hopes to achieve. The structure of this network could take any form, from a simple hierarchical line or tree to a much more complex structure. Map out your own goal complex. Where is there harmony between your goals? Where might there be conflicts? Are there bottlenecks (i.e., critical goals that must be achieved)?
- In Chap. 1 it was discussed that Plato advocated "cutting nature at its joints, like a good butcher." That often means cutting a network where there are few network connections leading in or out—in fact, this is exactly the idea of a minimal cut in a network. What is an example of a system where one cut will disconnect the overall network?
- We are all part of multiple overlapping networks. For example, you may be involved in both a social group and a sports team. Give a specific example of how you spread information from one of your networks to another. Could that information have reached the receiving network through another person?
- A wide variety of genetic modification tools now allow for humans to essentially hack the genome of an organism. From a systems perspective, changing the food (input) or waste (output) of an organism could have major ecosystem effects. For example, genetic modifications to certain bacteria can be used to metabolize oil (input food source) or alternatively create insulin (output waste product). Write a short story about the ecological and societal impacts of a new change to either the input or output of a single cell organism. Show evidence of network concepts in your story.
- Path dependencies were briefly mentioned and will be considered in more depth in Chap. 7. Technology is certainly one of the sociocultural elements that can forever change the trajectory of history. Pick a technology, and then explain what would have happened if it had been available much earlier. For example, what would have happened if the Ancient Greeks had discovered some version of the telegraph? Pick your own example, and explain your hypothesized consequences.
- *Deja vu* is the feeling that something is familiar. A more powerful form is *deja vecu* which is the feeling that you have already lived through something. There is much speculation as to the neural origins, ranging from it serving some function to it being an accidental glitch of neural evolution. Given the explanation of

networks in this chapter, what do you think might be the difference between the usual recall of information and the experience of deja vu?

- A network of actions that must take place is a common way for a project manager to break down a complex task. Some networks of actions form a line—one action follows the next, follows the next, in sequence. Others are cyclical in some way. Other action networks might be scale-free. In any of these network structures, there are often bottlenecks—functions that hold up other functions from happening, often because of delays, resources, or information. These functions are along what are known as the *critical path*. Map out the actions you take during a typical day. What are the actions that are bottlenecks for you? Why are these actions bottlenecks?
- The term *intersectionality*, explored in modern terms by Kimberle Crenshaw (1959–), is a framework to understand an individual as being a complex mix of identities. In the most simplistic example, Crenshaw explains how being a black women cannot be understood by considering gender and race separately. This example can be extended to include not only race and gender but sexuality, political affiliation, socioeconomic class, cultural status, disability, age, and other dimensions of identity. No one, or even small collection, of these attributes of identity can completely describe you. Create a network map of your identity. Which are easy to identify by someone else simply by looking at you? Which might give hints at other attributes of your identity? For example, by talking to you, might someone guess from the way you speak where you grew up? Likewise the content of your speech might reveal your socioeconomic class, religious beliefs, or political affiliation. What aspects of your identify are less obvious? Which do you hide? Why?
- The exact mapping of neural connections has been known for the 302 neurons in the brain of the *C. elegans* since 1986. The Connectome Project aims to map out connections in the entire human brain. This project also intersects the National Academy of Engineering Grand Challenge to Reverse Engineer the Brain. What other complicated networks might productively be mapped? Can you speculate on what might be learned?
- As Euler showed, locations can be the dots, and walking pathways can be the lines. Think of a party that you have had and the layout of the space. Make a dot for the various locations, and then make lines where you suspect most people walk between. Use weights, directed or undirected lines, or perhaps some attributes of nodes to indicate network properties. After making this diagram, discuss your rationale. For example, do you think this picture is only valid during certain phases of the party? Does the network map evolve during the course of the party?

5.6 Resources and Further Reading

There has been a massive rise over the past two decades in works explaining networks to both the lay person and to scientists working outside of network theory. Some very nice lay works are Barabasi's *Linked: The New Science of Networks*, Nicholas Christakis' *Connected*, and Watt's *Six Degrees: The Science of a Connected Age*. Two teaching-oriented books are Barabasi's *Network Science* and David Easley and Jon Kleinberg's *Networks, Crowds, and Markets*. For those who would like a more technical treatment, *The Structure and Dynamics of Networks* by Newman, Barabasi, and Watts is a collection of seminal papers in network science along with commentary. Some highly cited review articles are also good starting points, especially Newman's SIAM review "The structure and function of complex networks," Strogatz's "Exploring Complex Networks" in Nature and Barabasi's "Network biology: understanding the cell's functional organization."

Chapter 6
Information Flow

If one were to take a single human and split them up into their basic elements, we could compute how much the raw materials would be worth on the market. We are made of some very basic elements: 65% oxygen, 19% carbon, 10% hydrogen, 3% nitrogen, and so forth. Depending on the source and purity of the elements, estimates are approximately $4.50 per human. This kind of calculation seems to go back to at least the 1920s when Dr. Charles Mayo (one of the founders of the Mayo Clinic) calculated the price to be $0.84. This calculation is far from the value we would place on a human life because a human is much more than the sum of their parts. You are not a well-mixed combination of elements but rather a sophisticated pattern. A similar argument would hold for the raw materials of other objects, such as bridges, medical devices, and smartphones. The elements in many computers would add up to slightly more than $4.50. Patterns, both functional and structural, have value to us because they contain information.

Information is a term that has one meaning in natural language but a more precise meaning in science. For example, in regular language information is often treated as a single data point. Information theory, on the other hand, studies how information flow between a sender and receiver reduces the uncertainty of a message. Many fields, such as physics, mathematics, psychology, linguistics, biology, and computer science, have adopted information theory by considering various parts of the system as senders and receivers of messages.

In this chapter we will explore how information flows can lead to value, meaning, and knowledge within a wider system. The ideas of computation will be touched upon as well as what it means for something to be unknowable. Information theory also can help connect together and quantify the view from the small-scale perspective of a system (e.g., molecules, ants, neurons) and the larger-scale patterns that emerge. As such it can serve as a generalization of the ideas from Chap. 2—individual agents follow simple rules, and when they interact, new phenomena emerge. Information theory will also serve as a jumping off point for how energy flows can be transformed into work in Chap. 7.

J. Tranquillo, *An Introduction to Complex Systems*,
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6.1 Information Theory

Claude Shannon (1916–2001), an AT & T scientist, was nearly single-handedly responsible for what has become known as information theory. In the 1940s, the sending and receiving of messages was of particular interest to AT & T as they were exploring several forms of communication technology beyond traditional telephones. Shannon's critical insight was that information could be considered abstractly as uncertainty in the future.

6.1.1 Probabilities

Understanding Shannon's definition of information requires knowing a bit about probability. Imagine a pack of playing cards, fresh from the store. In this state the cards are uniquely ordered, with all suits separated and arranged in numerical order. As the deck is shuffled, symmetries are being broken such that the new configuration of the deck is more heterogeneous. After the first shuffle, that configuration may still not be that far away from the original order. With more shuffles, however, the order of the deck will move farther and farther away from the original order. At some point the cards will have reached complete asymmetry, and any additional shuffling will simply yield a different random sequence. You can think of shuffling as moving the "trajectory" of the deck toward an attractor. Unlike the deterministic attractors explored in Chap. 3 (e.g., points, limit cycles, strange attractors), shuffling moves the deck toward a random state.

To understand the connection between information and uncertainty, consider the fresh deck and the fully shuffled deck. As you draw cards from the fresh deck, eventually you will be able to guess, with greater and greater certainty, what the next card will be. At some point, you may not even need to turn over anymore cards and you could fairly confidently write down the order of the entire rest of the deck. In the world of information theory, one would say that flipping over additional cards does not gain you any more information. On the other hand, if you were to perform the same guessing game with a fully shuffled deck, you would need to keep flipping over cards. Each card gives you new information that you didn't have before. Of course, it is possible that when dealing out cards, patterns will arise, but that is what makes most card games so much fun. It is also why poker players know the probability of getting certain combinations of cards, with the less likely combinations being worth more in the game.

There is a catch. When you flip over a card, and therefore ensure that you won't flip over that card again, the information contained in the following cards goes down slightly. In the extreme case when there is one card left, you will know which card it must be. This idea of eliminating cards that have already been drawn from a shuffled deck is the origin of the idea of "counting cards," which (although not simple to do in practice) is illegal in casinos.

What is the probability of drawing a spade from a randomly ordered deck? We first have the total number of possible outcomes if we draw one card. That is simply 52 (the number of cards). Then there are 13 ways we can draw a spade (p_{spade}). So the probability is

$$p_{\text{spade}} = \frac{13}{52} = \frac{1}{4}$$

We can extend this same idea to other probabilities in general as

$$p_{\text{event}} = \frac{\text{chance of this event}}{\text{all possible events}}$$

Using this general formula, a one-card draw can be extended to multiple card draws. For example, what are the chances of drawing a Black King, followed by any card that is a Heart? In the first draw, the probability of drawing a Black King is 2/52. In the second draw we only have 51 cards left, so the probability of the next card being a Heart is 13/51. The nice part about probabilities is that they multiply. So the total probability is

$$p_{\text{Black King,Heart}} = \frac{2}{52}\frac{13}{51}$$

What are the odds of flipping over all 52 cards from a shuffled deck and having them exactly back in the order in which they came from the store?

$$p_{\text{perfectorder}} = \frac{1}{52}\frac{1}{51}\frac{1}{50}\cdots = \frac{1}{52!}$$

which is a very very small probability. The connection between this probability and uncertainty is that $\frac{1}{52!}$ are the odds of you picking every card correctly from a randomly ordered deck. But, if you know something about the symmetries in the deck, your odds will greatly improve.

6.1.2 Bayes' Law

Descriptive statistical techniques are applied to real data with the intention of learning something about the underlying system that produced that data. When applied to signals from a complex system, statistical analysis can shed light on the type of system that produced the signal. It is an inductive process that is always backward looking and requires real data to get started. A simple example is an average—one must have already gathered the data to compute the average.

Inferential statistics on the other hand uses past data to make predictions of future data or events. One of the key mathematical findings in this area is Bayes' law, derived by Thomas Bayes (1701–1761). The general idea is that a history of system events will build up over time. These events can then be used to determine the most likely events looking into the future. Bayes' law is a way to continuously update predictions of future events (called a *Bayesian inference*) as events unfold. For example, if handed a deck of cards and asked to predict the next card, you would start by flipping over cards. After flipping over maybe five or six cards, you may use what you have seen so far to help guide how you will make future predictions. If the cards seem to be ordered in some way, you will use the past reveals to make better guesses in the future. Patterns that are discovered by watching a complex system over time allow for better and better probabilistic predictions of what events might occur in the future.

The formulation of Bayes' law is based upon how two events, A and B, might depend upon one another:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

$P(A)$ and $P(B)$ are the probabilities of the two events occurring independently of one another. But, it is possible that they depend upon one another with some probability. The interpretation of $P(B|A)$ is the probability of B occurring if we know that A has already happened. The line (|) is known as a conditional probability. Therefore $P(A|B)$ is the probability of A happening if we know that B has occurred.

Bayes' law has a number of real-world applications. For example, consider the probability of contracting a rare but deadly disease that impacts 1 out of every 10,000 people or $P(A) = 0.0001$. Also consider that there is a test that can determine if you have the disease or not. But the test is not perfect. For every 100 people who test positive, only 99 actually have the disease, and for every 100 people who test negative, 1 of them actually has the disease. We might say that the "false positive rate" and "false negative rate" are both 1%. This means $P(B|A)$, or the probability of getting a positive test result if you have the disease, is 0.99. It also means, however, that your probability of getting a positive test result, regardless of whether you have the disease or not, is almost 1 out of 100. Because of a very small degree of inaccuracy in the test, your risk of testing positive is far higher than the risk of the disease itself. As a patient, you want to know $P(A|B)$ or the probability that you have the disease given a positive test result. A simple application of Bayes' formula above shows you that receiving a positive test result means your probability of having the disease has increased to nearly 0.01. This 0.01 probability is higher than your original $P(A)$ of 0.0001, but there is still only a small chance that you have this rare disease, despite testing positive. Another way to state the same idea is that because the disease is rare, the number of false positives will outnumber the people who truly have the disease.

Conditional probability is filled with surprising and counterintuitive examples that can have real consequences in the world. One example is the *prosecutor's fallacy*, whereby the probability of matching a set of characteristics is assumed to equate to a defendant being guilty. The 1968 People v. Collins case of a robbery highlights the problem. A witness described a black male with a beard and a mustache and a white female with a blonde ponytail leaving the crime scene of the robbery in a yellow car. The prosecutor asked the jury to consider the odds as

Black man with a beard (0.1)
Man with a mustache (0.25)
White woman with a pony tail (0.1)
White woman with blonde hair (0.33)
Yellow car (0.1)
Interracial couple (0.001)

The argument made was that the individual probabilities multiplied together yields a probability of 8×10^{-8} or 100 million to 1. The jury gave back a guilty verdict. The ruling, however, was overturned by the California Supreme Court due to ignoring conditional probability. For example, many men with a beard also have a mustache. Other legal cases have centered around DNA matches to crime databases which are finite in size and can have false positives.

The Monty Hall problem has become a classic puzzle to illustrate the flaws in our intuition about probabilities. Imagine you are on a game show and asked to choose between three doors. Behind one door is a new car and behind the other two doors are goats. According to probability you have a one third chance of picking the right door. You choose a door, but it is not opened yet. The game show host, Monty, knows which door has the car, and in a tantalizing twist, opens one of the doors you did not pick to reveal a goat. He offers for you to pick again, essentially asking you if you would like to stick with your original door or switch to the other remaining closed door. Should you stay or should you switch?

On the surface it seems as though it would make no difference if you stay with your original guess or switch. In fact, it pays to switch with a two-thirds probability. To understand the rationale consider the possibilities for your first choice

Car
Goat 1
Goat 2

So there is a 1/3 chance of winning the car as you would expect. After you make your first choice, the combinations of the remaining two doors are

Goat 1 and Goat 2
Car and Goat 1
Car and Goat 2

If Monty always picks a goat, then the remaining unopened door (not the one you originally picked or the one he *must* open) could only contain

Goat 1 or 2
Car
Car

What is critical is that Monty is not picking which door to show you at random. He is forced to pick a door with a goat, and so in the two out of three possible scenarios where you have already picked a goat, he is forced to show you the other door with the goat. In both of those scenarios (i.e., two out of three times the game is played), the unclaimed door will contain a car, and therefore in two out of three times, you will win by switching to the last remaining door. Of course you still have the original 1/3 chance that you picked correctly the first time and have now switched away from that correct pick. But you double your odds by switching.

The Monty Hall problem is an interesting illustration of how our rationality is not tuned to consider the delicate balance between past history and future odds. To go beyond the Monty Hall problem, it is often the case that a seemingly unimportant bit of information can reveal something that was not known before—even if it is to negate a possibility.

Predicting future events given the past almost always relies in one way or another on Bayes' law. The self-professed modern master is Nate Silver (1978–), the force behind the website www.fivethirtyeight.com that gives predictions about everything from sports and politics to economics and cultural trends. His book *The Signal and the Noise* is essentially a modern-day rehashing of much of Shannon's information theory. Prediction is essentially about separating true signal from noise and then using the past history of that signal, with an understanding of the underlying system, to predict where the signal will go next. In the book, Silver dissects several different systems (e.g., sports, weather, the economy) to show what it is about the systems or measurements that make the future difficult to predict. Sometimes it is the noise, and other times it is about how the system is put together. It may also be the reliability of the measurement techniques or how intertwined the predicted system is with other systems. The basic conclusion is obvious and profound at the same time—some systems are easier to predict than others, but unpredictable systems can be unpredictable in a variety of ways.

A critical problem arises when making predictions about a changing or emergent system—Bayes' law begins to breaks down when the system can change over time. Imagine trying to make predictions from a deck of cards when the number of cards is constantly being changed, with new cards being interjected or being removed over time. In the context of emergence, sometimes a new structure or phenomena may arise. With a deck of cards this would be like gaining an entirely new type of card (e.g., a zero card).

Conditional probability, $P(B|A)$, is a prediction of a future event (B) given an event that has already happened (A). Abduction, which was introduced in Chap. 2, is a sort of mirror image in time. Recall that in abduction

The surprising fact C is observed
But if A were true, C would be a matter of course
Hence, there is reason to *suspect* that A is true.

Based upon an observation (C) we make a hypothesis about the cause (A) that is very likely. This can be restated as $P(A|C)$, or the probability that A is the cause of C. It is important to note that in conditional probability the event A has already happened, whereas in abduction we are guessing that *A* was a cause. Didier Dubois (1952–) and David Poole have called this Bayesian abduction and have proposed that it is a way that any entity compares and tests competing models of the world.

6.1.3 Shannon's Information Theory

We can now explore more fully Shannon's formulation of information theory as the sender and a receiver shown in Fig. 6.1. Note that he included the possibility that the message could be corrupted in some way, either due to transmission errors, malicious intent, or perhaps misinterpretation. Shannon's key insight was that information could be measured as the change in uncertainty before and after a transmission. He went even further to propose that a reduction in uncertainty was similar to the measurement of *entropy* that will be discussed more in Chap. 7 in the context of energy flow.

To gain a sense for Shannon's idea, imagine a sequence of numbers "1 2 3 4 1 2 3 ?." If asked to predict the value ?, you could be fairly sure, although not 100% certain, that it will be a "4." We would say that the information in this last digit is low because we are almost certain it will be a "4." The higher the certainty of an event before it occurs, the lower the information of the event after it has occurred. On the other hand, if you had the following numbers "3 9 2 -1 18 27 8 ?," you would be much less certain of the next number. When that next number arrives, you expect that it will have a high degree of information.

Computer scientists use this principle in many ways, for example, in compression (e.g., zip files, .jpeg image format). In these formats, redundant information is taken out so that the signal has high information content. If your photo has mostly blue sky, a compression algorithm can remove much of that redundant information. Another example is the common abbreviations from the texting world (e.g., "lol," "omg," "thx," "k") that are either acronyms or have unnecessary letters removed. And there

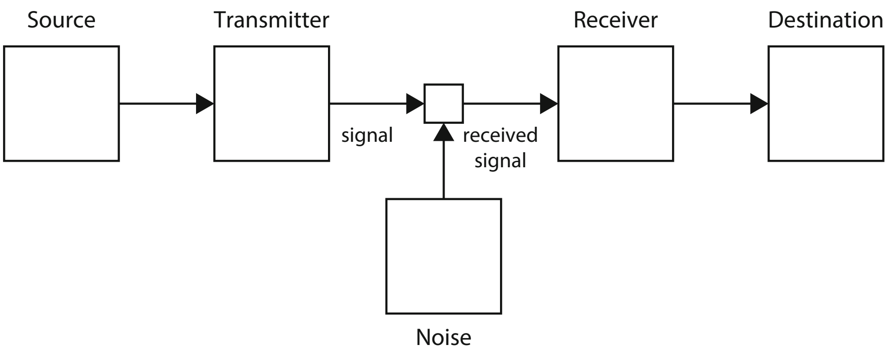

Fig. 6.1 Shannon's view of communication between a sender and receiver

are some classic linguistic example known as *disemvoweling*, or the removal of vowels from a sentence. For example, the sentence

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The quick brown fox jumps over the lazy dog
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becomes

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Th qck brwn fx jmps vr th lzy dg
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To most native English speakers, a little effort would be necessary, but they would be able to decipher the disenvoweled sentence. The interpretation of these sentences can go back to Fig. 6.1 where the sender can eliminate unnecessary elements of a signal, given what they know about how the receiver will interpret them.

An important realization that Shannon had was that information is always based on a difference. For example, in the digital logic that underlies many of our high-tech devices, there must be, at a minimum, the ability to distinguish between 0s and 1s. Likewise in (most) artwork or music, there must be some differences, either in color, texture, or sound. At a philosophical level, it could even be the difference between the presence and absence of some quality. In the visual arts, it is the difference between figure and ground. What is critical is that information is gained when some symmetry is broken to establish a pattern.

6.1.4 Measures of Information

There are a number of variations on how to measure information (I), which we will not explore here. The most basic is

$$I = -\sum_{i} p_i log_2(p_i)$$

In this equation the assumption is that a signal is composed of a sequence of values of length i with the probability of being able to predict the next value being p_i. If a signal of values is entirely known (say all 0s), then $p_i = 1$, meaning we can predict with certainty what the next value will be. But in this case, $log_2(p_i) = 0$, so the total information in that sequence is in fact 0. Now we can explore the other end of the spectrum, when the sequence is entirely random (the next value could be with equal probability either a 0 or a 1). In this case $p_i = 1/2$ (as it would be in flipping a coin). But $log_2(1/2) = -1$, so $I = 1/2$. It turns out that this is the maximum that it can be. And in fact, this makes a great deal of sense because when a fair coin is flipped, the probability of it turning up heads is 1/2. Each flip of the coin adds the maximum amount of information. Somewhat paradoxically signals that carry the most information are also the most random.

The terms order and disorder are often used in discussing the patterns that can emerge from complex systems. Generally, order means more predictable and disorder means less predictable. It turns out, however, that order and disorder are difficult to precisely define. To understand why, consider modifying Shannon's diagram so that there are two receivers that both record the *same signal*—"agt4fns5ifyiu6hz90d." One of these receivers has a key that will allow for the signal to be decoded, while the other receiver does not. To the receiver without the key, this code appears to be a random sequence of numbers and letters. As it is random, they would describe it as disordered, unpredictable, and containing a great deal of information. To the receiver with the key, however, they decode the message as "Hello, how are you?." To this second receiver, the message is orderly and relatively predictable. A similar phenomenon is encountered when trying to communicate with someone who does not speak the same languages as you. The terms order and disorder are often relative to the first-person perception of the receiver and not an inherent property of the signal.

6.1.5 What Can Carry Information

One of Shannon's main goals was to provide a theory that would be independent of the message, sender or receiver, or even the nature of the what might carry information (e.g., 0s and 1s, letters, DNA). With such a theoretical formulation, some mathematicians and philosophers asked if perhaps information could exist separate from physical reality. In other words, is it possible to have "pure information" that does not require a physical medium? In a 1991 article in *Physics Today*, Rolf Landauer (1927–1999) argued that all information must manifest itself in physical form. In all examples we can think of, signals (as defined by marketers, engineers, and economists) are a flow of information through a channel. In engineering the channel might be a wire, and in business it might be mass media (billboards, TV spots). The most familiar flow of information is human language. In this case information moves from a mind of the sender to vibrations in their larynx and mouth, through the air as vibrations, and then to the ears and mind of the receiver. All parts of this information pathway are supported by physical structures. Landauer's proposition is that information must be a flow of something and that flow must take place on a physical medium. Even in the quantum realm, where the meaning of information becomes fuzzy, it is real particles that carry the information.

Due to the Internet, we have become disconnected from the physical reality of information, using terms such "the cloud" to describe some nonphysical storage place. An apocryphal story illustrates this point. It was around 2010 in the early days of data storage in the "cloud." A woman in Silicon Valley was waiting in line behind a group of 20-somethings with a shopping cart full of hard drives. She asks what they are doing with all of that storage space and why they aren't simply "storing all their data in the cloud." Their answer was, "We are the cloud." What this story highlights is that despite our cultural ideas of raw information, at some level all information is physical.

To extend this idea to other systems, we can consider that information flows from one place in a system to another. Along the way the information may even be transformed from one carrier to another. For example, variations in pH in the body may be transformed into ionic currents in neurons, which might be then transformed into commands to alter breathing rate and the expulsion of carbon dioxide. There might be many steps in sending some messages back and forth across many systems. Whenever there is a medium change, there is a *transduction* of information from one type to another. This is simply Shannon's diagram shown in Fig. 6.1, repeated many times. Every time there is a transduction of information from one type to another, there is also the possibility of errors. In fact, many system diseases can be thought of as introducing errors into the signaling pathways within a wider system.

6.2 Applications to Communication, Genes, and Language

There are a wide range of applications of information theory. In this section we will begin with Shannon's original reason for studying information in the first place and then move on to examples from language, music, and gene encoding.

6.2.1 Communication and Noise

The definition of noise to many is subjective in that it is anything that is undesirable. To your parents, your music might be considered noise. In information theory, however, noise has a very particular connotation. It is the introduction of randomness into a signal that was not intended by the sender. The effect is to add uncertainty to the receiver in what was sent by the sender. The dilemma, as pointed out by Shannon, is that the most compressed (and efficient to send) signal would also be random. So how might a sender-receiver pair be able to tell the difference between a very efficient signal (with high information content and therefore random) and a signal that looks random because of added noise? Both are received as an information-rich signal. The answer is that the difference is really in the relationship between the sender and receiver. The same signal could either be viewed as unpredictable or as very information rich, depending on how much the sender and receiver know about one another and the channel along which they are communicating. In a perfect system there would be no added noise, and the sender and receiver would share the exact same encoding and decoding mechanism. In this kind of perfect system, the signal will appear to be entirely random to an outside of observer (e.g., have high information content) but would be perfectly understandable by both the sender and receiver.

In real systems there is always noise and errors. The only way to counter imperfections would be to increase the degree of redundancy in a signal—what he called *error checking* (or *parity*) that would enable the receiver to detect if there is a problem with the signal. Adding more redundancy allows for better and better detection of errors and potentially even the ability to make corrections. But this comes at the cost of making the signal less efficient (e.g., more data must be sent). What Shannon was able to show was that the degree of error checking could be tuned. An even better option, however, is often *consilience*—two signals from independent sources that lead to the same conclusion or meaning. A similar idea is called triangulation by scientists, journalists, and detectives.

Consider that in a complex system, different parts must communicate with one another. Information theory applies regardless of the type of system or medium, so this is a common problem in all complex systems—how to reliably send signals from one part of the system to another. As such, all complex systems must overcome the problem of noise versus signal. For example, in a company what the boss says may be misinterpreted. The way around this is to put in place safeguards and redundancy that will ensure the message as sent is begin received as intended.

Noise in a complex system often can come from within the system itself as competing signals cross contaminate one another. For example, in your heart, strong electrical signals are generated that coordinate the timing of each beat. But to your brain, which also acts on electrical impulses that are an order of magnitude smaller, these heart-related electrical pulses are noise. Similar cross-contamination occurs in the blood which contains thousands of messages carried by a variety of organic and nonorganic molecules. It is amazing that a body can function with so much cross-noise.

To reliably send a signal, the sender must know something about how much noise will occur along the pathway. The more noise that will be added, the more redundant the sent signal must be. But, then the question arises as to the cost/benefit of adding in more redundancy. For example, assume that by adding 2% more to a signal, we can overcome 90% of the noise. Practically this means that the sender adds 2% more to the signals to enable the receiver to decipher the intended signal 90% of the time. Perhaps by adding 5% more to the signal, 95% of the noise many be overcome. Remember that adding more to a signal will also mean that both the sender and receiver pay some sort of penalty in time delays or resources or both. The level of redundancy is therefore a sort of negotiation between the sender and receiver of what error is tolerable.

The type of noise is often characterized by the frequency of the random distribution, often associate with colors. For example, there is white noise, pink noise, brown noise, and so on. In complex systems one type of noise shows up again and again. Perhaps not surprisingly at this point in the text, the distribution is a power law, known as flicker or 1/f-noise ("f" stands for frequency). The meaning is that there is more lower frequency content with a drop off as the frequency gets higher. Unlike an exponential, the rate of that drop off becomes smaller and smaller. A hypothesis is that 1/f noise is a signature of a complex system. Perhaps the sender and receiver would coevolve, reaching a sort of optimum balance between efficiency and redundancy. When many signals are present, the cross-contamination would require adding in redundancy. Somehow reaching this optimum results in 1/f noise. We will explore this idea more in Chap. 10 when we discuss how two coupled systems will begin to mirror one another over time. A fun introduction to the many 1/f patterns in our world can be found in John Barrow's *The Artful Universe*.

6.2.2 Genetic Information

Information theory was originally developed to consider analog and digital electrical signals. Shannon, however, purposely developed his mathematical formulation so that it could apply to any kind of system or signal. Consider the genetic information contained in DNA. DNA is a chain of molecules, called bases, that are linked together by chemical bonds. There are only four bases (adenine, thymine, cytosine, and guanine), but they come together in triplets (e.g., ATG) that code for amino acids, the building blocks of all proteins. Although there are many possible three base combinations (theoretically 64), there are only 20 amino acids that exist in nature. Therefore, there is a degree of redundancy at this level, with multiple triplets coding for the same amino acid. The 20 amino acids serve as a kind of alphabet that can be recombined to form an enormous array of proteins. It is the unique expression of these proteins that make one organism different from another.

The string of DNA base pairs can be thought of as a message with noise being added by mutations. For example, AGC-CTA is a code for two amino acids, serine and leucine, respectively. If there is a change, it might code for a different amino acid. How redundant is the DNA strand? We would expect at least some redundancy so that errors can be detected and possibly fixed (known as *kinetic proofreading*). To begin computing the information content, we want to compare a real DNA sequence to a statistically random sequence. For example,

AACTAGTTAGCCTGCG

This segment is of course too short to really be used to compute probabilities, but we can use it to demonstrate the concept.

$$I = -\sum_i p_i log_2(p_i) = -[P(A)log_2(P(A)) + P(G)log_2(P(G)) \\ + P(C)log_2(P(C)) + P(T)log_2(P(T))] \quad (6.1)$$

Then in place of the probabilities, we enter the statistical values for the sequence above. This example was set up so that each base appears four times, and therefore all the probabilities are 1/4. When the calculations are done, $I = 2$. This is in fact the maximum that it could be. But what if the bases were not of equal probability?

AATGACTATTAAAGTATA

The same calculation will give a value for I of 1.61 meaning that there is less information in this second code or that there is more redundancy. There is, however, a problem with this method that can be illustrated by the following example.

AAAACCCCTTTTGGGG

If we use the same computation as above, this would have the same value for redundancy as the first code ($I = 2$). Yet there does seem to be a pattern. Instead, consider pairs of values (such as AA or TG), and then compute their probabilities.

$$I = -[P(AA)log_2(P(AA)) + P(AG)log_2(P(AG)) \\ +P(AC)log_2(P(AC)) + P(AT)log_2(P(AT))\dots] \tag{6.2}$$

Using this method we arrive at an important conclusion. On the surface, the first sequence will retain its relatively high information content, while the information content of the third sequence will fall. The second sequence, however, may actually look like it has more information content, when comparing single values to pairs. It also will matter where we start and stop reading the sequence too. For example the two sequences below will yield different measures of information.

AA-CT-AG-TT-AG-CC-TG-CG
A-AC-TA-GT-TA-GC-CT-GC-G

Lastly, we know (from experiments) that bases are read off as triplets. So, to compute the actual information of DNA would require computing the probabilities for AAA, AAG, AAC, AAT, and so on.

At a much deeper level, this line of reasoning reinforces the claim that information can only be considered in the context of the overall sender-receiver system. In the case of DNA, to interpret the information content, we must know that it is important to study triplets. But this is a function of how the information is used by the molecular reading and transcribing systems within a cell. It is entirely possible that some other genetic encoding (perhaps based upon electrical laws) could be devised that works on codes of 2 or 4 or 5. Understanding genetic encoding using information theory is a rich area of research, both for biological and non-biological encoding. For example, it has been shown that some areas of the biological genome have more information (less redundancy) than others. There are also some areas that are more impacted by noise in the form of mutations.

As a biological species, we seem to have an innate drive to spread out. This is evident on our planet, but it also is beginning to apply to distant worlds as well. A quick look at some popular books and movies will provide many examples. For example, imagine sometime in the future that we contact an alien species. It is quickly realized that our ability to engage in two-way communication is severely limited by the speed of light. A solution (explored by many science fiction writers) would be to send a signal at the speed of light that contains our DNA sequence. An even more provocative idea is to send the information in some way through a spinning black hole to short-cut across vast spaces and perhaps even times. Then the aliens could grow a human and directly talk to a member of our species. Of course they could also have beamed to us the assembly codes (likely not DNA) of their species.

Such a scenario presents at least two dilemmas. First, a DNA sequence can only be read out by a biological organism from this planet (the proverbial chicken and egg problem). The same problem would almost certainly apply to the alien genetic code. Second, such a representative of our species would develop separated from a human culture. It is very likely that DNA, grown into a human on a distant planet, would be fundamentally different physically, emotionally, psychologically, and socially than a human born on earth.

A more radical alternative would be to allow a human to develop on earth but then use some (likely destructive) method of encoding their entire body (potentially atom-by-atom). An alien species would therefore be able to reconstruct a human using only the laws of physics and the data received. This would be much like the thought experiment that began this chapter—the information contained in a human would not be based upon the elements but rather in the complex way in which they are put together. Of course it would be very important to make sure that good error checking was performed to ensure that this person would show up on the other side the same or at least very similar. Although this is all science fiction (at least for now), it illustrates how important information theory is in any form of encoding, transmitting information, and then decoding on the other end.

6.2.3 Language

In the English language we can perform a similar study. First words are made up of letters. At the level of individual letters, we have already discussed the removal of letters or substitutions as a way to compress language. How much information is contained in the next letter in a word? If the letters were totally random, then the probability of a particular letter occurring would always be 1/26. But if you were presented with "Th?" and asked to guess what "?" would be, you would almost certainly guess that it will be a vowel, and even then it would likely be "e." Of course we can use the same trick of letter combinations that were used in DNA analysis to study the probabilities of doubles (e.g., high probability of "ed" for past tense words), triples (e.g., "est" to establish relations as in "fastest"), or other patterns.

At higher levels we can explore how words are put together to express meaning. We already encountered Zipf's law in Chap. 5, where some words (and word pairs) occur more often than others. At another level we could explore how much information content is in a sentence or paragraph. A clearer way to demonstrate this idea is to ask, "how far into this paragraph did you need to read before you got the point?." In fact at this point in the paragraph, you (the reader), very likely are not gaining much more information. But I (the writer) should add some redundancy just to be sure, in the form of an example. Consider a conversation that intends to share some particular content. The sender is a teenager. The first receiver is another teenager, and the second receiver is the teenager's parent. The sending teenager will almost certainly use a different signal to share the same content because they will consider the nature of the channel and receiver. Which conversation do you think will require fewer words?

Until now we have assumed that the roles of sender and receiver do not change. In such a situation the conversation can only be one sided, as it regrettably is in a textbook. What makes spoken language (and several other forms of communication) different is the possibility of flipping the roles of sender and receiver. This enables a way to perform a different kind of redundant error checking. When a message is received and its content is in question, the receiver can become a sender and ask for confirmation of some kind. In everyday terms this is the act of asking a clarifying question. We will not explore all of the possibilities here, but this is the foundation of active listening. In active listening, one hears what the sender is saying and then summarizes back to them what was received. The sender can then determine if the message was received as intended or not. This process can continue on in an iterative fashion. In digital message passing this is known as a "handshake." Complex systems often use this kind of two-way error checking, sometimes in very intricate ways, to ensure that messages are being sent and received properly.

6.2.4 Information in Structures

The above sections may imply that information theory is only useful for signals that occur in time (music, language, the reading of DNA). But information theory can also be applied to structures, either as they occur in space (as in Chap. 4) or in relationships of parts (as in Chap. 5). For example, if given only a partial pattern in space, could the incomplete parts be filled in? If so, that would indicate the pattern has relatively low information. A similar argument can be made for patterns of relationships. As demonstrated by Barabasi, networks patterns can be repeated *motifs* of three or four or five nodes that occur much more often than random. In fact, this was how Barabasi performed his statistical analysis, by comparing randomly generated networks with networks from real functioning systems. The implications are that a motif can do something interesting and do it in a stable/robust way. We will explore this more in Chap. 9. Likewise, Mark Newman has explored methods that, given an incomplete network structure, can predict how much more of the network remains hidden.

6.3 The Theory of Computing

Information has become associated very strongly with computers and the Internet. We use the terms "information age" and "computer age" (and the associated ideas of the Internet, cloud computing, smartphones, etc.) almost interchangeably. That was not always the case. Modern computing and information theory arose roughly around the same time but were not explicitly tied together. In this section, some definitions of computing will be given, and then we will apply those definitions to the study of artificial intelligence as one of the places where computation and information overlap.

6.3.1 A Short History of Computing

The idea of computing goes back a very long way. The original term was used to characterize a person who would make calculations. Many problems in mathematics could only be approximated using pencil and paper by iteratively applying rules that would yield a result. This was how the calculations for the first atomic bomb were carried out, almost entirely by groups of women, double and triple checking each other's work.

The mechanistic view of computing began with Charles Babbage (1791–1871) and his plans for the difference engine. Although it was entirely mechanical, its operation as a general calculating machine was essentially the same as a modern computer. What made it more than a calculator is that it could be programmed. While Babbage focused on the hardware, the software programmer was Ada Lovelace (1815–1852), considered to be the originator of algorithmic programming.

Babbage's difference engine was in fact only a theoretical device until the mechanical device was built out of over 8,000 parts in 2002 to demonstrate that it would work. In the 1940s, however, Babbage's idea was much more efficiently expressed using electronics, first using cathode ray tubes and later with transistors. The general architecture was one that would *gate* information in some selective way to execute an algorithm.

The person credited with describing general rules for how information is gated to form a computing device was Alan Turing. Like Shannon, his goal was to describe computing in a way that would be independent of the medium. He created a thought experiment by which an extraordinarily simple computer could be created using a paper strip and some simple rules. On the strip would be some form of cells upon which 0s and 1s could be written or overwritten. The paper would be housed in a machine that could skip to any part of the strip and read or write (or overwrite) 0s and 1s according to the rules.

What Turing showed was that this simple computer could execute *any* algorithmic computation, no matter how complex, as long as it had an infinitely long tape and as much time as needed. This system is now known as a *Turing machine* and refers to any general purpose computing device.

6.3.2 Other Forms of Computation

Turing's abstraction of computing meant that a computer could be created out of just about anything. Groups have created simple computers out of tinker toys, Wolfram's Class IV rules, and Conway's Game of Life. One critical point is that any real physical computers will have finite memory and so will not be the same as a theoretical Turing machine. This limitation, however, is only encountered in cases where extremely large amounts of memory are required.

Turing machines are generally described as *serial* machines, meaning that they can only perform one operation at a time. There is such a thing as *parallel* computing, whereby a number of individual computers work together. What can be shown, however, is that if a problem takes X computations, and is then split between several computers, it still takes at least X computations. The time to perform the computations might decrease because the problem has been divided up, but there is no gain (in fact often there is a loss) in the number of operations that are performed. In other words, any computation that can be done in parallel can in theory be done in serial, it just might take longer.

This idea counters some propositions that a brain is fundamentally different than a computer because it operates in parallel. That is not the right line of argument because parallel computations can be simulated on a serial machine; they simply will occur much more slowly. At the level of human interactions, this idea has some additional interesting implications with regard to delegation of work on a team. If you have a job of size X, are there cases where splitting it up could actually cause less overall work to be done? Note that this is different than the question usually being asked which is what is the most efficient way to perform the work.

Some have questioned if it might be possible for a Turing machine to emerge in nature. The only requirement would be to gate information flow in the right ways. Is it possible to have organic computers? There have been at least two research threads that have demonstrated the possibilities. The first is the switching on and off of neurons in the brain, and the second is the switching on and off of genes during development.

The idea of thought being a sort of computer, governed by laws of logic, has been proposed in a variety of forms since at least the Ancient Greeks. The first person to make a very clear case, however, was George Boole (1815–1864), about whom we will learn more below. His ideas were expanded upon in 1943 by the neuroscientist Warren McCulloch (1898–1969) and logician Walter Pitts (1923–1969). In their work, they demonstrated how neurons could act as very simple logic gates. A hypothetical neuron, called a *perceptron*, would take in inputs (perhaps weighed in some way) and sum them together. If the sum came out to be larger than some threshold, the neuron would output a 1. If the sum was not over the threshold, the neuron would output a 0. The biological interpretation was more or less accurate, with the dendritic tree of a neuron collecting (and weighting) inputs, while the neural body summed outputs. In a real neuron, when the threshold is exceeded, the neuron creates an action potential, a brief burst of electrical activity, that was represented by a 1 in the McCulloch-Pitts model. By changing the incoming weights and threshold, different logic gates (such as AND, OR, and NOT) could be created. What McCulloch and Pitts demonstrated was that in principle, neurons could be used to create a computational device.

In 1958 the next step was taken by Frank Rosenblatt (1928–1971). He began stringing together perceptions to form networks where the output of one set of perceptrons became the input to more perceptrons. Furthermore, he demonstrated that these networks could match patterns simply by tuning the weights. This was building upon the idea put forward by Donald Hebb (1904–1984) in 1949 that learning is simply the tuning of synaptic weights. What was more, Hebb proposed a tuning rule that has become known as "fire together, wire together." In other words neurons that tend to fire at (or close) to the same time will become more tightly connected together.

A network of perceptions with tuning rules that adjust weights is an *artificial neural network*. Various structures and learning rules would enable a network to learn to operate as a very sophisticated computer, even one that could work in parallel and detect patterns. Artificial neural networks have gone through several iterations since the 1940s, sometimes showing great promise and other times being crushed by some demonstration of their limitations.

A similar concept was advocated in the 1960s by Jacques Monod (1910–1976), Francois Jacob (1920–2013), and Andre Lwoff (1902–1994). The trio won the Nobel Prize in Medicine in 1965 for describing how proteins are expressed (or repressed). This was a fundamental question at the time because it was clear that not all proteins are expressed by the DNA at one time. They are seemingly switched on and off by some mechanism, the timing of which seemed to coincide with environmental cues as well as when other genes were being turned on and off. Understanding the basic mechanisms of this genetic switching was therefore the key to understanding growth and development as the differential expression of particular proteins.

The switching on and off of a gene is controlled by a *promotor* region that appears just before the gene code. Monod's work in the *lac* promotor in *E. coli* was the first glimpse of a much more complex *transcription* regulation system. Transcription is simply how the DNA is read out to form a protein. It is the regulation system that turns on and off genes. In a complex cell, a *transcription network* describes the relationship between proteins that can turn on (or inhibit) the expression of one or more other genes. When taken all together, as in Fig. 5.11, a transcription network can be viewed as a complex computational device. Uri Alon's (1969–) *An Introduction to Systems Biology* goes into much more depth about the nuances of how gene networks can form logic gates and other higher-order computational functions. We will explore this idea further in Chap. 9.

6.4 Artificial Intelligence

The idea that the brain is a computer prompted some to ask if a computer might be programmed to be intelligent. Artificial intelligence (AI) includes artificial neural networks but has diverged to include many other ways of building smarts into a machine. In this section we will explore the goals of artificial intelligence as well as some of the paradoxes that arise.

6.4.1 The Goals of Artificial Intelligence

The founders of AI had two related goals in mind. The first was to follow along with a quote from Alfred North Whitehead (1861–1947) "Civilization advances by extending the number of important operations which we can perform without thinking about them." One practical motivation of AI was to automate thinking. Just as the industrial revolution moved some of the physical burden of work to machines, so too the AI revolution would move some cognitive burdens to computers. There were even attempts to write programs that would solve general problems and play games. For example, Herbert Simon and Allen Newel (1927–1992) wrote a very good checker-playing program that could in fact beat the programmers. There was also the General Problem Solver that was meant to take in raw data and spit out abstract insights. As a demonstration, the program developed a version of Johannes Kepler's (1571–1630) laws of planetary motion from the data of Tycho Brahe (1546–1601). More recently computers have beaten human chess champions. This thread has continued to the present day and is perhaps best captured in such creations as the Google search engine and big data mining algorithms that rely on *deep learning*. In fact, Google announced the first computer to master the game of GO which was thought to be impossible for a computer. This application-driven and narrow approach is sometimes called *soft AI* because it does not aim to create a true intelligence. Rather useful features of intelligence are applied to some narrow domain of expertise.

The second major goal of AI was to shed light on the nature of intelligence. Philosophers have struggled with the nature of intelligence and rationality throughout recorded history. The early proponents of AI viewed their work as clarifying what we mean by the words "thought," "intelligence," and "mind." In fact, George Boole, the logician who mathematically developed the logical operations of computation, called his master work "*The Laws of Thought*". Rather than sit in a room and think about intelligence, the originators of AI took a more active approach—to try to build an intelligence. Even if they did not succeed, they would most certainly learn a lot about what intelligence is (or is not). This more philosophical quest to create a truly intelligent entity is often called *strong AI*. A strong AI would in fact possess general purpose intelligence that could be applied out in the real world.

6.4.2 Creating an AI

Complex systems theory intersects AI in how one might go about creating a genuinely intelligent entity. The overwhelmingly popular approach has been to use a top-down approach to program as many rules of behavior as possible. In fact there are research projects dedicated to building common sense knowledge databases (the most well-known being the Cyc project) that could then be uploaded into the "brain" of an AI. If an intelligence emerges from a complex system, however, it likely would evolve through a bottom-up process. Such an intelligence would develop from a few basic rules and through interactions with a real or virtual world would create its own internal rules. These internal rules would then recombine in such a way as to push the entity through a phase transition that would result in intelligence. It is important to note that in this more emergent form of intelligence, the programmer simply starts the process and then leaves the system largely alone to become intelligent on its own.

An outstanding example of this second approach comes from the lab of Rodney Brooks (1954–), an emeritus robotics professor at MIT and founder and CTO of iRobot. Brooks advocated for two important elements that he felt were missing from most approaches to AI in a very readable 1990 article, "Elephants Don't Play Chess." The first was the idea of emergence and learning. The second was that an intelligence would necessarily need to be *embodied*, meaning that it must be able to interact with an outside world that is rich in stimuli. Brooks took the extraordinary step of having students in his lab interact with the robots they were building. Over years and in some cases even decades, some of these robots have shown signs of displaying what we might consider to be real contextual intelligence.

6.4.3 The Turing Test

Artificial intelligence has been cleverly defined by Pamela McCorduck (1940–) as "science fiction that hasn't happened yet." Once we achieve some technical computing feat, we seem to discover that it is not really intelligent. For example, if someone from 1900 came across the Internet, Google, and IBM Watson, they would likely declare the combination to be intelligent. This highlights how our ideas about what is (or isn't) defined as intelligence has changed over time. For example, in the 1950s, when artificial intelligence came of age, intelligence was thought of as recall and manipulation of information in the form of symbols. Intelligence tests were therefore almost entirely geared toward pattern recognition and memory recall.

In 1950 Alan Turing published a very readable article called "Computing Machinery and Intelligence." In the paper he muses about the nature of intelligence and what it would mean for a computer to be intelligent. He puts his finger on a very tricky subject that still echoes today throughout philosophical discussions of intelligence and mind. How would we know if an entity, not just a computer, possessed general intelligence? His conclusion was that the word “think” is in many ways an internal state that can only be known by the entity possessing it. In other words, it is a first-person property. He recognized that intelligence would likely be a holistic property of a complex system, and as such it would not be possible to simply take it apart and look for the “intelligence.” There would be no magical place that would determine whether an entity was intelligent or not. As such thinking would be a subjective experience that could not really be known to some third party observer. In the absence of any theoretical way to determine intelligence from a third party perspective, he proposed an empirical way to make a very good guess as to the intelligence of a system.

Turing’s test was originally called the Imitation Game (upon which the popular movie was based), later renamed the Turing test. In the test the subject to be tested is placed in a room, while a human is placed in the room next door. Then a human evaluator is brought in. This evaluator will interact with both the human and the subject (e.g., computer) for some duration (say 5 minutes) and carry on a conversation that can be about whatever the evaluator would like. To keep the test fair, the evaluator can only use a keyboard and monitor for interaction. At the end of both interviews, the evaluator is asked to guess which is the human. Of course we can bring in several evaluators and run the test several times. The subject (computer) has passed the test if the evaluator cannot tell the difference between the human and computer. In the original version, Turing even gave the computer the benefit of the doubt, saying that the computer need only fool evaluators 30% of the time to pass the test.

There are several criticisms and modifications of the Turing test. The main criticism is the assumption that an intelligence must be human-like to pass the test. For example, it is entirely possible that a super intelligent alien would fail the test simply because they were incapable of dumbing it down enough to act human. In terms of modifications, one of the most iconic is shown in the movie *Blade Runner*. In the movie there is a very strict version of a Turning test conducted on a replicant—a nearly perfect human-like being created by a company. The test involves not only a face-to-face conversation but also physiological measurements. Another interesting example is also demonstrated in the movie *Ex Machina*, where a being that clearly has been created by humans has its own interests and motivations and is in many ways more capable than its creators.

6.5 Meaning as Information Networks

Information theory was designed specifically to be context-free and therefore reveals little about how meaning and value might arise. Other fields, however, have a variety of views about the relationship between information and meaning. We will explore some of these ideas in this section.

6.5.1 Networks of Information and Models of the World

Many theories in the disciplines of philosophy, neuroscience, and psychology view meaning and knowledge as emerging from networks of information. Ludwig Wittgenstein (1889–1951) described the structure of knowledge as a network of interrelated information. For example, Fig. 6.2 shows a simplified network of concepts that surround the idea of a "cat." Notice that what it means to be a cat is defined not as a thing in and of itself but rather in terms of the other concepts to which it is related. Likewise, a term such as "claws" will have its own rich network of connections, endowing it with meaning. It is also of note that the network may have some interesting connections that are formed either through unique experiences or second-order connections (e.g., "cat" and "Ferrari" are related because they share the concept "purr").

When a more complex network of events, actors, rules, and environments are taken in total, it forms an internal *model of the world*. To know something therefore means to be able to place it within a network. This idea has had a strong influence on how cognitive neuroscientists think of neural networks storing information, how psychologists think of schemata, how linguists think of grammar, and how educators think of framing and framebreaking.

The network view of knowledge may explain both how knowledge is used and how it is acquired. Using knowledge, sometimes referred to as recall, is simply

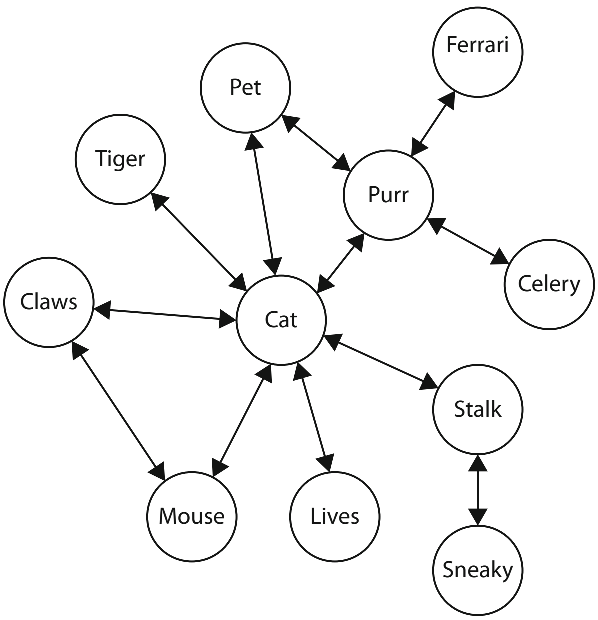


Fig. 6.2 A concept map for the idea of a "cat"

activating or energizing some part of the network. In a brain, the exact neural mechanism is not entirely clear, but neuroscientists would point to the coordinated activation of particular neural circuits. At any one time, a person is therefore only activating a particular part of their overall knowledge network. With our understanding of non-linear dynamics, we can think of this as a series of attractors, where an initial condition (or perturbation) can send thoughts spiraling into a particular area of the network. This view was explored by John Hopfield (1933–) in what is now known as a Hopfield network, a kind of artificial neural network where memories are the dynamic attractors of the system.

A memory is form of information. As such it must be stored in some physical medium, generally a network. An idea can be stored in a mind or a book, but both require some physical means of storage—synaptic weights and protein configurations for the brain and lines on paper for a book. Likewise in computers there are registers that hold numbers. In finance there are account balances. In each case, the memory is retrieved when a particular part of a network is activated.

Memory also forms the basis for *perception*—comparing a sensed state now to some previous state. It is the difference that forms the perception. In other words, memory allows for comparisons to be made across time—a current state with a previous state. For example, bacteria cannot measure an absolute concentration of food or a toxin. Rather as it swims it measures a concentration and compares that concentration to what it measured at some previous time. It is the previous concentration that must be stored in some sort of memory.

Recall of information from a knowledge network also partially explains the psychological phenomena of *priming*. When one bit of information is activated, it will naturally trigger related concepts to be activated, making them easier to recall. Although priming can be helpful in many circumstances, as it serves a feedforward function, it also likely underlies a wide range of biases, some of which seem to be buried deep within our unconscious. These biases are implicit in the network and are a function of how information is connected together. Another interesting phenomenon that can be explained is the *tip of the tongue* experience, whereby some bit of information is only partially retrieved. The experience is one of knowing that you know the information, but not being able to gain access to it in that exact moment.

Knowledge acquisition is a change to an existing network that may take the form of new information (nodes), new connections (edges), or both. To get a network started generally requires a seed. Most neuroscientists and psychologists agree that at least some part of this seed is inherited, although they disagree on exactly what is innate and what is learned. Over time new information and connections are added, giving rise to a constantly evolving model of the world. In a feedback cycle, this model of the world in fact impacts how the world is viewed, how information is processed, and how the model is updated.

Given the network view of a model of the world, we can speculate on some properties that may follow. First, and perhaps the most profound conclusion, is that each knowledge network will be unique to the individual, based upon their first-person perspective of events in the world. Second, a dynamic knowledge network may evolve different structures from very disconnected to very interconnected. When considering robustness and efficiency, the model likely will evolve toward some balance between underconnected and overconnected. These kinds of balances will be explored in Chap. 10. Third, during acquisition, concepts that are activated together will become interconnected with one another. If you learn something in a particular context (e.g., it is structurally embedded in a particular way in your knowledge network), it is most likely to be activated when that context arises again. The timing of observations and path dependencies within the existing network can have a strong influence (again leading to priming) on how the network changes during learning. Lastly, the development of a robust knowledge network necessarily requires rich observations. As Brooks pointed out, being embedded in a sterile world would lead to a relatively weak and static knowledge network.

6.5.2 Consciousness, Mind, Story, and Self

The nature of intelligence, meaning, knowledge, and value seem to be intertwined with what it means to possess consciousness, mind, and self—all topics we will touch upon more in future chapters. Many have written about the interrelationships. For example, a distinction is often made, thanks to Descartes, between the physical structure of a brain and the functions of a mind. Some argue that physical flows on a physical brain result in the physical functions of the mind. There is no mystery to be solved. Others claim that a physical brain is necessary for a mind to arise but that something additional is required. The relationship between consciousness and mind is another tricky relationship. A good introduction to consciousness is provided in a 1995 article by David Chalmers called "Facing up to the Problem of Consciousness." In the article Chalmers differentiates between what he calls the *soft problem* which is the relationship between consciousness and the biophysical mechanisms underlying thoughts and actions and the *hard problem* which is the internal experience of being present in a moment. This again is another example of the difference between the third person (soft problem) and first person (hard problem).

Some have taken a different approach—to list the properties or features that a mind or a consciousness will possess. As one example, Gregory Bateson claimed that story (at an abstract level, not necessarily the way a novel is written) is one of the defining features of a mind. In his view, a story is an efficient way to organize facts and events. For that reason, a story would be the most powerful and memorable way to move information from one mind to another. His claim is convincing in that humans are attracted to stories (e.g., parables, urban legend) and good storytellers (e.g., teachers, marketers, politicians, parents). Another related provocative claim from Daniel Dennett is that a self is really just the protagonist in an internal story that is created to manufacture the illusion of purpose and meaning that drive decisions and actions.

6.6 The Unknowable

Information theory and the related idea of computation were purposely context independent. Neither field ventured into truth, judgment, or decision-making. Yet, we very often endow information with the property of truth. Discovering information is often associated with uncovering the truth. Suppose we were to draw a large circle of everything that is true (assume for now the usual definition of truth). Then draw several smaller circles inside of the larger circle, all scattered about. Inside these smaller circles are all the truths that can never be known. "Never" is meant in a strong way—the smaller circles exist because of something more fundamental, not simply due to some human or technological limitation. What do you think this picture would look like? How many little circles would it contain? Would there be any? If there are some, would they all be clustered in a few places? What would be the percentage if you added up all of the little circles relative to the larger circle? Are there limits to what can be known?

Many would argue that there are things we can never know but only because of the limitations of being human such as our size, time perceptions, memory, or other organic limitations. To take one example, because all of our experience and intuition is built up in a world of relatively large objects that move at relatively slow speeds, we do not have a good feel for relativity or quantum mechanics. Both theories seem counterintuitive but that is only because we tend to see nature over a small range of time and space scales. We cannot directly observe either the very small or very large, and we often miss events that occur over a long time scale or are very very brief. Our technologies have allowed us to overcome some of those limitations, but the results still seem to go against our usual experience.

The 1900s may be looked back on as the century when we discovered the limitations on what can be known. In this section we will explore some of the arguments.

6.6.1 Gödel's Proof

One of the most incredible proofs in all of mathematics concerns what cannot be known. The story begins long before the actual proof, dating back to the Greeks and the formal study of logic. The Greeks studied logic as it appears in language, with the classic example being, "If all men are mortal, and Socrates is a man, then Socrates is mortal." Logic was applied to many arenas for 2000 years, but the first major breakthrough in logical thought came when George Boole mapped linguistic statements to formal mathematical language using symbols. He was able to show that logical conclusions could be made based upon a rigorous method of proof. Others around the same time, most notably Giuseppe Peano (1858–1932), were able to show that the rules of basic arithmetic could be derived from a few axioms and the laws of logic.

In 1899 David Hilbert (1862–1943), announced 23 unsolved problems in mathematics that he challenged his colleagues to solve over the next 100 years. The second problem on that list was to prove the axioms of arithmetic to be consistent. This might sound obvious to a non-mathematician, but what Hilbert was really after was a deeper version of what Peano demonstrated—he was essentially asking if mathematics is internally consistent.

Between 1910 and 1913 Alfred North Whitehead (1861–1947) and Bertrand Russell (1872–1970) published a three-volume book, *Principia Mathematica*, that attempted to work from a very small list of axioms and reduce all of mathematics to symbolic logic. They were not simply answering Hilbert's challenge but making the much stronger claim that all true statements in mathematics were provable in a self-consistent system. Until 1931 it seemed that they had shown how every branch of mathematics could be derived from one logical framework.

In 1931 a 19-page paper appeared by Kurt Gödel (1906–1978), a 25-year-old Hungarian mathematician, showing that the system Whitehead and Russell has devised was in fact not self-consistent and that any attempt to fix the system would always leave some hole that could never be repaired. The basic form of the logical problem has been known for a long time, hidden in statements like "This Sentence Is False." If the proposition is true, then it asserts its own falsehood. And if it is false, then it is true. The problem is that some statements are self-referential. Whitehead and Russell were aware of the problem of self-reference, which can be summed up nicely in the *Barber contradiction*: The barber is someone who shaves everyone who does not shave themselves. Does the barber shave himself? Trying to answer this question results in a contradiction, and Russell and Whitehead swept it under the logical carpet.

What Gödel did was to make a mathematical version of this kind of sentence and then see how a consistent mathematical system would deal with it. The outline of the proof is presented below but is expertly told and interpreted in *Gödel's Proof* by Ernest Nagel (1901–1985), James Newman (1907–1966), and Douglas Hofstadter (1945–). As a first step, Gödel introduced a numbering system, for example,

Logical symbol	Gödel number	Logical meaning
$\sim$	1	Not
$\wedge$	2	And
$\Rightarrow$	3	Implies
$\exists$	4	Exists
$=$	5	Equals
0	6	Zero
s	7	Successor
(	8	Open group
)	9	Closed group
,	10	Separator

And we can represent variables as

x	11
y	13
z	15

Using this scheme we can code a logical statement. For example,

$$(\exists x)(x = sy)$$

This statement means, there exists a number x $(\exists x)$, such that x has a successor y, $(x = sy)$. This is a fundamental idea in math that any integer number can be incremented by 1 to yield a new number. The code for a logical statement like this is

$$8411981157139$$

To achieve a unique Gödel number, we raise the sequential primes to the powers of the code

$$m = 2^8 \times 3^4 \times 5^{11} \times 7^9 \times 11^8 \times 13^{11} \times 17^5 \times 19^7 \times 23^{13} \times 29^9$$

This is a huge number (m), but it will uniquely represent any logical statement. What is more, we can create sequences of statements.

$$(\exists x)(x = sy) \rightarrow m$$
$$(\exists x)(x = s0) \rightarrow n$$

where n was derived in the same way as m. But we can furthermore define

$$k = 2^m \times 3^n$$

Systems of equations can be uniquely coded as a single number too.

The reason Gödel chose this numbering scheme was due to the fundamental theorem of arithmetic that says there is a unique factorization for any integer. That means that if given a Gödel number, we could in principle arrive at the corresponding logical statement. For example,

$$\begin{aligned} 243,000,000 \\ 64 \times 243 \times 15,625 \\ 2^6 \times 3^5 \times 5^6 \\ 656 \\ 0 = 0 \end{aligned}$$

What is needed next is a way for statements to talk about themselves. As a first step Gödel introduced what is known as "meta-mathematics." How would we create the sentence "is '$\sim$' the first symbol in the equation $\sim(0 = 0)$?." The statement $\sim(0 = 0)$ is false. But the claim that "$\sim$" is the first symbol is true. This is the core of meta-math—math that can talk about itself.

For example, how might we construct the mathematical sentence "is the exponent of the smallest prime factor of Gödel number a, a 1 ?." Logically this looks like

$$(\exists z)(sss \ldots sss0) = z \times ss0)$$

or read more literally, there exists a number z such that $2 \times z$ is equal to a (represented by $sss \ldots sss0$). In this way mathematical statements can talk about mathematics.

The next step is to build up the idea of what it means to prove something. Imagine that we have a sequence of logical operations that form a proof.

$$equation1 \rightarrow a$$

$$equation2 \rightarrow b$$

$$equation3 \rightarrow c$$

$$equation4 \rightarrow z$$

where equations 1–3 prove equation 4. We can construct the Gödel number of equations 1–3 as before.

$$x = 2^a \times 3^b \times 5^c$$

and remember that we can also form

$$x \implies z$$

This is a very common form for a proof. To avoid writing this form over and over again, Gödel created a mathematical function.

$$dem(x, z)$$

that means "the Gödel number x is a sequence of equations that proves the equation with Gödel number z."

Likewise, you could construct other logical propositions such as $\sim dem(x, z)$, meaning the equations x do not prove z. It will not be shown here, but Gödel demonstrated that this function can be created using prime factorizations.

The next step is to define a new function called sub. Remember that

$$(\exists x)(x = sy) \rightarrow m$$

What if we replace the variable with the Gödel number 13 (i.e., a y) with m

$$(\exists x)(x = ssss \ldots sss0)$$

where there are $m + 1$ s's. This new formula has a different Gödel number (it is no longer m). There is of course a circularity in substituting a string's own Gödel number into that string, and this is what the sub-function will do.

$$sub(x, 13, x)$$

which means take the formula with Gödel number x and whenever you see a "y" (i.e., 13), substitute in $ssss \ldots sss0$. So the example above would be

$$sub(m, 13, m)$$

Again, Gödel showed that the sub function can be created using prime factorizations.

We now come to the last and critical step.

$$(\exists x) dem(x, z)$$

which means "the formula with Gödel number z is demonstrable by the equations x." And likewise, we could imagine the opposite

$$\sim (\exists x) dem(x, z)$$

meaning "there is no x that can demonstrate z" and therefore z is not provable. Next we can form a special case of this equation. According to logic, both statements cannot be true—either z is or is not provable.

$$\sim (\exists x) dem(x, sub(y, 13, y)) \rightarrow n$$

where $sub(y, 13, y)$ is the Gödel number of the formula obtained by substituting for the valuable number 13 (all occurrences of "y") the *numeral* for y. This means, the formula with Gödel number $sub(y, 13, y)$ is not demonstrable. Remember, however, that y is still a variable. So we can substitute in any number we want. The last step is to substitute in n (the Gödel number from above) for all occurrences of y.

$$\sim (\exists x) dem(x, sub(n, 13, n)) \rightarrow g$$

which means the formula with Gödel number $sub(n, 13, n)$ is not demonstrable. But what is the value of this statement itself? A little reflection will show that $g = sub(n, 13, n)$ (just compare the previous two equations).

The key is that there was a substitution ($y = n$), which is in fact the definition of the *sub* function. In other words, we substituted in all occurrences of the 13 symbol ("y") with n. But, the final equation says that there does not exist proof of g, but this statement *is* g. So what Gödel did was to show how to construct a version of "This sentence is false" inside of mathematics. But he did not stop there. Gödel went on to the next possibility. What if a mathematician tried to fix the system by making g an axiom? It turns out that no matter how many axioms you added, a new statement will always appear that will have the same self-referential properties. Gödel's conclusion was that either mathematics was incomplete (i.e., not able to reach all true statements) or inconsistent (i.e., would assign truth to both a statement and its negation).

6.6.2 The Halting Problem and Traveling Salesmen

In 1936, while laying down the groundwork for a theory of computation, Turing came across a major problem. When a program is entered into a computer and set in motion, how long will it take to complete? How do we know that the program won't simply run forever? At the time, there were plenty of programs known that seemed to run forever. Remember that Turing was assuming infinite resources (e.g., the program won't stop because it consumes too much energy, a user presses an escape key, or other such external ways to stop the program). It is easy to create a simple program that will go into an infinite loop. But for more complex problems one will not know ahead of time if it will ever stop. Turing proved that there is no foolproof way, simply by looking at a program, to know if it will halt or not. This problem is known as the *halting problem*.

The classic example of a halting problem is the *traveling salesman problem*. In this scenario we imagine a salesperson who will travel to various cities. The goal is to find the shortest overall pathway that will allow the salesperson to visit each city only once. If we consider five cities, the shortest route can be found by trying all of the options. A quick calculation will show that there are five ways to visit the first city, then four possibilities after that, and so on, resulting in $5! = 120$ possible pathways. If we then increase to 20 cities, we find that there is an explosion in the pathways that would need to be checked. Is there any algorithm that can be sure to find the shortest pathway, no matter the number of cities, without trying every single pathway?

It is important to note that when abstracted, the traveling salesman problem is really about navigating any type of network in a way that will visit each node only once. For that reason, it can be applied to all sorts of problems in the real world. In fact, whenever mathematicians, decision scientists, or computer scientists encounter a problem of this sort, they often will call it a traveling salesman problem.

Even more generally we want to know how much worse a problem will become as it scales. Some problems do not become any worse as we scale them. Others are worse but scale linearly. Then there are problems that get worse but in terms of some polynomial. In other words the problem becomes x^2 times worse or perhaps x^5 worse. What can be shown is that the traveling salesman problem gets worse even faster than any polynomial expression. For that reason it is referred to as an NP-hard (for nondeterministic polynomial time) problem. What Turing in fact proved was that problems that are of type P (polynomial time) can be solved algorithmically. But when it came to NP problems, there would be no way to write an algorithm that could do better than simply trying all of the possibilities.

Around the same time Alonzo Church (1903–1995) was working with a field of mathematics known as lambda calculus, similar to the logical system developed by Whitehead and Russell. Church showed another variation of the problem of algorithmic proof. Another mathematician working around the same time was Emile Post (1897–1954) who created the recursive method that formed the basis for the L-systems in Chap. 4. Perhaps recursion would be a way around solving NP-hard problems. It turned out that recursion was not a way out of the halting problem either. What is even more, Turing and Church went on to prove that all of these problems were the computational equivalent of Gödel's incompleteness theorem. Both are about the undecidability of questions through either algorithms or proofs.

6.6.3 Quantum Information and Computing

Quantum theory, and more specifically the uncertainty principle first derived by Werner Heisenberg (1901–1976), shows that at nature's smallest scales, we cannot know some of the most basic information. More specifically we cannot at the same time know the position and velocity of a particle. There are several interpretations of this idea, but it seems that quantum theory says something very deep about the reality of the world, independently of what can be proven in a mathematical sense. It claims that to know something means to be able to measure it and that the act of measuring (the act of knowing) can actually change reality. In other words, there is no such thing as passively knowing something—the act of knowing changes what can be known. As such there is a self-referential loop between the world and an observer.

Quantum theory can be exploited to create a new kind of computing, known as quantum computing. Although quantum computing seems to be relatively far off, there have been some successes, both in terms of physically creating hardware and devising algorithms that would run on a theoretical quantum computer. Just as Babbage and Lovelace worked out the hardware and software from traditional computing devices, the pioneers of quantum computing are making early theoretical progress on the hardware and software that will execute quantum algorithms.

It is not entirely clear where quantum computing is headed. But one important question is, will a quantum computer be able to turn NP-hard problems, the kind that don't scale well and lead to the halting problem, into some other type of problem that does scale better? Would a quantum computer be a sort of super-Turing machine (also known as hypercomputing), with the implication that it will be able to solve problems that are different in kind, not just degree? The consensus at this time is that a quantum computer will likely solve the traveling salesman problem faster but that it will still be on non-polynomial time (e.g., still an NP problem) and still require trying all of the possibilities. That means that quantum computing will not be able to solve any problems that are currently unsolvable through mathematical proof or Turing-type computing.

6.6.4 Sending Codes

Much of this section discussed lofty theories about what is and what is or is not knowable. An easy counter is that the things that are unknowable really do not matter in the real world. They are either too small, or happen too slowly, or will never affect us. In some ways these criticisms may be well founded. Likewise, some claim that these arguments only apply to systems that follow the rules of logic or are bound by physical law. If a human mind does not operate solely on these principles, then perhaps we can escape from the traps laid by Turing and Gödel. Maybe we can know truths that a physical or logical system never can.

There are some real-world implications to the unknowable or uncomputable. For example the field of cryptography is concerned with building codes that enable senders and receivers to have their message remain private, even if someone else was able to listen in on the signal. The usual way to accomplish this task is for the sender and receiver to share a *key* that only they know. A secret receiver without the key would have no algorithmic way to crack the code and leave them with no other option but to try every possibility. The growth of computing power enables such a brute force approach, but keys and codes can easily be created that would require centuries, if not longer, to crack. Even quantum computing, with the ability to simultaneously try out many possibilities at once, would only enable faster trials.

6.7 Privacy

The idea of privacy is grounded in how information flows within a system. The dilemma we seem to face is that there are often advantages to giving up information in return for something. Take a family, for example. One of the defining features of a family unit is that they share certain experiences that only they know about (e.g., trips, meals, medical histories, inside jokes, painful conflicts) that become the unique aspects of their family that help socially bind the members together. A similar idea applies outside of families as well. Consider two people who are dating and begin to learn more about one another. Their bidirectional sharing of information (decreasing their individual privacy) draws them closer together.

At a societal level, something similar happens. For example, when you tell your medical doctor or priest or lawyer something private, you are most likely doing so because of the better service they can then provide you. There are laws that ensure that these parties will keep that information to themselves. In network terms, they will not spread your information throughout the social network. You share with them, but they do not respond in kind by sharing information with you.

Concerns regarding privacy are not new. The exchange of information for a service, however, has taken on a new dimension in the Information/Internet Age. We are willing to give up privacy in exchange for information about others. This is the driving force behind much of social media. Participating in Facebook means that you reveal information about yourself and in return you can learn gossip about others. Functionally this is the same as the sharing that occurs within a family unit. What is different, however, is the scale upon which social media enables this sharing beyond what could ever happen face-to-face. Many of the ramifications are related to the idea of emergence—with this open flow of information new phenomena can emerge. To cite one simple example, my department was interviewing faculty candidates. On one memorable occasion after an interview, we were alerted (though Google's notification software) that a candidate had blogged about their interview with us. It was human nature to read the blog post, which was very nice but gave us much more information about the candidate (in good and bad ways). This post was clearly not something the candidate intentionally shared with us, rather they assumed we would never find the post. Such phenomena would never have occurred without the Internet.

The erosion of privacy is often discussed as a negative, but that may not necessarily be the case. Perhaps privacy is a human invention that helped us through some societal/evolutionary bottleneck in our development as a species. We needed it to retain some degree of individuality, which helped grow society and civilization to a particular point. As the size of human groups grew, privacy emerged. And as the groups grew even larger, privacy became layered, with more private aspects of life being shared with a few and less private aspects of life being shared with many. Perhaps, like other human inventions, privacy will be shed (another phase transition in the evolution of humans) once it has lost its purpose.

6.8 Questions

- In what ways do you think privacy is an emergent phenomenon? Will it eventually disappear as an important factor in our lives?
- How do you think artificial intelligence will evolve? How far do you think it will progress in your lifetime? What form will it take? What emergent phenomenon do you think will occur in AI?

- Try making a concept map of some particular concept that you feel you understand well, as in Fig. 6.2. Put the concept in the middle and then build outward from there. What surprises do you find as you build the network outward?
- Given that a brain, a gene network, and tinker toys can form a computer, are there perhaps other networks that can be thought of as computers? For example, could a city or ecosystem be thought of as a computer? What about a cloud formation? What other systems might “compute”? What are they computing?
- In *Wisdom of Crowds* by James Surowiecki (1967–), the idea is put forward that a correct answer might emerge from a group despite none of the individual members having the right answer. To illustrate, imagine many people guessing the number of jelly beans in a jar. No one may guess the right number, but for every person who guesses 100 over the right answer, there will be another person who guesses 100 under the right answer. What results is a kind of averaging of errors, such that the group’s averaged guess is very close to the reality. The key is that the guesses must be made independently of one another. This can be contrasted with the example from Chap. 2 of mob formation, where individuals are not independent and engage in groupthink. Where could you imagine taking advantage of the wisdom of crowds to arrive at a better answer than any one person?
- Gregory Bateson thought of a story as the non-linear and complex glue that links together events and agents. Our senses, however, stream in new information as it becomes available from the environment. Bateson’s view of storytelling is therefore necessarily an emergent process—we create and update the story as we go. The composer and lyricist Stephen Sondheim credits much of the success of his work to laying out stories that can be enjoyed the first time as a coherent linear thread but then later for the hidden interconnections. His goal was to not tax the listener by requiring them to recall very far back in the past to make sense of the current lyric or scene. Movies provide another nice case study. In suspenseful movies such as *The Usual Suspects*, it is in fact the obscured interconnections, in both the timing of events and relationships, that keep the watcher guessing. The ending of such movies is often some revelation that helps the watcher complete the causal connections as they really happened. There are many creative ways in which information can be revealed in movies and books, for example, in *Memento* the story is told backward. Quentin Tarantino is known for his non-linear narratives. Likewise M. Night Shyamalan is known for supernatural movies that have a twist at the end. These types of stories may take several viewings to “get it.” Choose one of your favorite stories and deconstruct how the author managed when readers gain particular information.
- Many systems contain within them particular signals that reveal some dynamic that is about to unfold. In economics these are known as *leading indicators*. In health care they are early symptoms. In mining it is the canary in the coal mine that would give an early warning of dangerous gasses. Most often these signals are used to detect some impending bad dynamic. Can you give some examples where an early indictor signals a good or helpful emerging pattern?

- Some leaders use asymmetry in information to stay in power. Parents also do this with their children (e.g., not burdening them with the family finances). What are other examples where someone might create an asymmetry in information? In what ways might this be helpful or not?
- Statistical analysis has become an important tool in determining when an observed correlation has a high probability of being a causal connection. The term *statistical significance* indicates a level of certainty that a connection is in fact causal. For example, there are some who claim that poverty is correlated with crime. But is it correlated enough to be causal? This question arises in almost all fields that aim to make causal arguments. Statistical arguments, however, can go wrong when they do not take into account hidden human biases. An amusing case comes from the methods of functional magnetic resonance imaging (fMRI) that created vivid 3D pictures of brain activity. To make a point, a group at Dartmouth College placed a dead salmon into a scanner and asked it questions. By following the usual methods and statistical procedures, they were able to show correlations between the questions and "neural" activities. Of course any rational scientist would recognize that there is no way the questions should illicit neural activity in the brain of a dead salmon. The study in fact won an Ig Nobel Prize, the anti-Nobel award, for unusual research that sounds silly but also makes one think more deeply. Where do you see a causal argument being made that you don't necessarily think is true? What do you think has gone wrong in the methodology? Could you propose a test that would reveal the gap in logic?
- Imagine two social circles that overlap. The nature of that overlap means that information can flow between the two groups, usually because they share a common member or two. These individuals act as conduits of information. Are you a connector of two social worlds? Given an example of information that you have transmitted from one group to the other, do you possess any degree of power due to your position between these two networks?
- Noam Chomsky (1928–) revolutionized the way we think of language. In the 1950s he proposed that underlying all human languages was some master grammar, similar to the simple rules discussed in Chap. 2, that was built into what it means to be human. For example the idea of nouns and verbs are built into essentially all languages. Likewise most languages use verb tense to express causal connections between events. This universal grammar serves as a framework that every human inherits. When learning a particular language, humans are simply tuning various settings of this universal grammar to become fluent in a particular language. Therefore the traditional grammar of a particular language is really just one instance of a more basic set of language rules that are shared by all humans. In the context of information theory, the sender and receiver necessarily must share at least some basic similarities to establish communication. Chomsky's claim has been challenged by some real data but remains the core of modern linguistics. Are there other basic ways of communicating that you believe are so basic that they are inherited and enable any two humans, regardless of their background, to communicate? Do you think a similar logic applies to communication with animals?

- Steven Strogatz and some of his collaborators found a way for two chaotic oscillators to agree upon a common phase. This was somewhat surprising because chaotic systems are generally very sensitive to perturbations, and their "phases" are infinitely complex (as opposed to a simple limit cycle). Strogatz in fact received a patent for the use of such a phenomenon to send and receive information. In this case the "key" is actually the rules of the oscillator, which only two parties may have. What might some applications be for such a unique kind of cryptography? A hint is that such communication systems would be sensitive to outside perturbations.
- Wittgenstein focused a great deal on language and its relationship to knowledge because he argued that anything that was thinkable would be expressible in language. Perhaps thinking and language are really just the same thing. That someone cannot express themselves clearly is perhaps not as much about the thought in the sender but rather in the error-prone channels and nature of the receiver. Do you think that anything that is thinkable is also expressible? Why or why not?
- The Voyager 1 and 2 spacecraft launched in 1977 are still functional and now well beyond our solar system. Both contain two golden phonograph records that contain a welcome message to any entity that might happen upon the spacecrafts. A fundamental question when the Voyager missions were developing was how to send a message to some unknown receiver. If you were in charge of selecting the content of the welcome message, what would you select and why? How would you encode a message in such a way that some other intelligent being would be able to decode it? It may help to think about what we might share in common with some unknown alien. After answering, look up what was sent and how it was encoded. How does your solution and the actual solution compare?
- The psychologist John Bargh (1955–) claims that embedded within our language are the visceral feelings that we experience. An example is how we as a species will gravitate toward warm places and generally move away from cold places. In our language we therefore will refer to someone as being either warm or cold. We do not mean that these people are in actuality warmer or cooler, but rather than they are more or less desirable to be around. Bargh has gone on to show that similar linguistic phenomenon appear across a wide range of cultures. What are similar examples of privative desires being reflected in our language?
- In past democratic elections, individuals would vote, and then the results would be revealed all at once at some later time. The implication was that each individual was truly making their own decision. Over time, however, the media and Internet connectivity have allowed for real-time data to be reported before polls have closed. This is combined with pre-polling that gives early indicators of the results of an election. What do you think are the possible implications of such early (an incomplete) information?
- Models of the world were discussed in this chapter as networks of interrelated information. Related to this idea was the idea of bias. Some would claim that it is not possible to possess a model of the world that is free from bias. Do you agree or disagree? Can you support your claim using concepts from the chapter?

- Engineers often talk about signal-to-noise ratio as measure of how much "important" information is present. A high signal-to-noise ratio means the information content is high relative to the noise. The opposite is known as *swamping*—when the noise is so great that it is overwhelming. In some cases a system will purposely engage in swamping to cover up information. Magicians, politicians, and parents of small children are often experts at this technique. Describe a situation in which you have purposely engaged in swamping? What noise were you introducing? What was the signal you were attempting to cover up?
- Several thinkers have challenged the idea that intelligence, in the usual academic way in which it is understood, is the only pathway to success. For example, the idea of emotional intelligence was first discussed by Michael Beldoch but then gained greater popularity with the 1995 publication of *Emotional Intelligence* by Daniel Goleman (1946–). A broader view of intelligence was put forward by Howard Gardner (1943–) in his book *Multiple Intelligences*. Gardner's original claim was that there were at least six different types of intelligence, later expanded to be at least nine. Everyone is some mix of all of the types of intelligence, with some expressed or repressed more than others. Success can therefore be achieved through multiple pathways, both by being extraordinary at one type of intelligence or through some complex combination of intelligences. Read a review of Gardner's multiple intelligences and try to guess which forms of intelligence you most readily express. Then find one of the many online multiple intelligence tests and take it. What was expected? What surprised you?
- Kinetic proofreading was mentioned as a means by which DNA can self-correct itself or at least detect errors. List other non-biological systems that engage in self-correction. Pick one and then explain how this system detects and potentially corrects its own errors.
- Nodes within a network can often take on the role of both a source and sink of information. For example, *The Wall Street Journal* can be thought of as a source of information because it has many outgoing connections. Information diverges from these sources. Alternatively, a node might have many incoming connections and be a sink for information. This information can then be synthesized into something new—essentially a second-order form of information. A detective works in much this way. Of course, few (if any) network nodes will be pure sources or sinks in a system. Sources and sinks for information align well with Malcolm Gladwell's (1963–) discussion of mavens, connectors, and salespeople in his book *The Tipping Point*. In short, mavens are the people who have the information and are often the builders of new ideas. Connectors are the social hubs because they know everyone and enjoy introducing them to one another. Salespeople are the integrators of ideas and storytellers. Which role or roles do you feel you play in the networks in which you are an actor? Do you always play the same role or do you sometimes take on different personas?
- Some believe that everything is measurable and therefore assessable. The idea seems to work well when the measured quantity is of only a few dimensions (e.g., position and velocity of a baseball). When trying to measure a multidimensional quality, for example, happiness, the measurements become less clear. Perhaps

combining together various indicators might provide a more complete picture of some higher-level attribute or state. Such is the claim made in *The Leading Indicators: A Short History of the Numbers That Rule Our World* by Zachary Karabell (1967–). In the book Karabell explains that economic measurements such as the gross domestic product (GDP) were created at a time when these crude measurements made more sense but that they no longer reflect what we what to know now. Toward the end of the book, he explains how some newer and multidimensional measures of overall happiness are being used by some countries to make policy decisions. His claim is that all too often we make the simplistic measurements that we can but then mistake the movement of those measurements over time as a sign of progress. Examine some areas in your life where you might be driven by a measurement that is far too simplistic and not reflective of some overall goal. What new multidimensional measures might you adopt?

- An interesting intersection between computing and information theory is Bremermann's limit, calculated by Hans-Joachim Bremermann (1926–1996), as the theoretical limit of computational speed. A similar idea for memory is the *oblivion effect*—essentially that no physical system can have infinite memory. These limits are based on the idea that all information is constrained by the flow of some material and as such it must obey physical laws. For example, information cannot flow faster than the speed of light. Write a short story about a culture (human or nonhuman) that is bumping up against the oblivion effect of memory or Bremermann's limit of computation. What things can they do with their incredible computing speed? What dilemmas do they face in knowing that they cannot computer faster or store more memories?
- In 1980 John Searl (1932–) proposed the Chinese room experiment. In his thought experiment you observe slips of paper written in Chinese entering into room. You also see slips of paper being passed back out, also written in Chinese. Over time, you see that there is a real conversation going on between what appears to be two intelligent minds that can converse in Chinese. What Searl then reveals is that the room contains a person who knows no Chinese but has access to a large book containing a list of symbol manipulations. When the person inside the room receives a string of symbols, they write down another string of symbols based upon the rules contained in the book and then pass the answer back out of the room. What Searl wants to know is does the room (as a system containing the person and book) literally understand Chinese, or is it merely simulating the ability to understand and converse in Chinese? This is very much related to the difference between first- and third-person perspectives as well as Chalmer's distinction between the hard and soft problem of consciousness. What do you think about this thought experiment? Does the room as a system understand Chinese? Is it possible for a system as a whole to understand something that no individual part can understand?
- Lists are a common means of keeping information readily available for one to consult. This could be the ordering of steps in a process, a shopping list, a "to do" list, or a checklist. The purpose is often to serve as a secondary memory

source to ensure that one does not forget an item or step. A nice introduction can be found in Atul Gawande's (1952–) *The Checklist Manifesto* that contains stories and advice on how to make the most of lists. In what ways do you use lists or checklists? What information do they contain? Where do you keep them? How detailed are they? When have they served you well? When have they not and why?

- There have been several times in this text that self-references have appeared. For example, Fig. 5.17 is a histogram of word usage in Chap. 5, yet the figure caption for this figure was been counted in the histogram. This is a common tactic in many fields. In the postmodern artistic idea of breaking the fourth wall, the author and actors acknowledge that they are engaging in self-reference of the work itself. Some people might refer to themselves in the third person, as if they were someone else reference the person they are. Find another example of self-reference at work in the world. What is the intention (if any) of such a self-reference? What is the impact on those who observe this self-reference?
- Consciousness and intelligence were both mentioned in this chapter. How do you think they are related? Is it possible to have consciousness without intelligence? Do you think it is possible to be intelligent without being consciousness?
- By some estimates up to 80% of our genome is never directly expressed. The unexpressed parts of the genome, often called introns, might be "junk DNA" left over from our convoluted evolutionary trajectory or might play an indirect role in transcription and translation. Genes that are never turned on will not influence fitness but can continue to propagate, playing the genetic equivalent of the appendix. There is an interesting series of stories by the science fiction writer Greg Bear (1951–) called *Darwin's Radio* that follows the reexpression of a long dormant part of our DNA that only kicks in to modulate radical changes in evolution. A similar argument could be made for cultural memes. What remnants of our historical trajectory might be dormant? What might wake them up to be reexpressed? What might be the consequences?
- No real computing device has an infinite memory. All must therefore adopt a particular *forgetting factor*—the rate at which information will be purged to make way for newly created or incoming information. Choose a social system and deconstruct what you think the forgetting factor is. Are there different rates of forgetting for different kinds of information?

6.9 Resources and Further Reading

There are not many lay books specifically on information theory. Most books on information theory focus specifically on the digital revolution as a technological or algorithmic rather than a theoretical breakthrough. Charles Seife's *Decoding the Universe* is an exception and contains not only an overview of the history and meaning of information theory. It does, however, provide speculation on how information theory may help unite several areas of science. James Gleick's *The*

Information is another popular book that specifically targets information theory. For a wonderful summary of Shannon's work at Bell Labs, Jon Gertner's *The Idea Factory: Bell Labs and the Great Age of American Innovation* contains many interesting anecdotes about Shannon and others who started the information revolution. Although not about information theory, Bateson's *Mind and Nature* and *Steps to an Ecology of Mind* have many ideas about information and knowledge as it pertains to minds and brains. *The Mind's Eye*, edited by Dennett and Hofstadter, is an older collection of seminal essays on the mind, many of which incorporate elements of information theory. On the technical side, Shannon's original work *The Mathematical Theory of Communication* is a nice place to start. *An Introduction to Information Theory* by John Pierce (1910–2002) is another classic. Some nice works on language can be found in Maryanne Wolf's (1960–) *Proust and the Squid: The Story and Science of the Reading Brain*, as well as Steven Pinker's (1954–) *The Stuff of Thought* and *The Blank Slate*. A more technical but still readable introduction to linguistics can be found in *The Ascent of Babel* by Gerry Altmann (1960–).

Chapter 7
Thermodynamics of Complex Systems

Imagine you run a farm that raises free-range livestock. The more livestock you raise, the more money you make. Yet, the more livestock you have, the more time and labor it requires to manage the herd. For example, more of your time and energy are being taken up transporting livestock that have wandered off of your property. To gain some efficiency, you decide to build a large fence that will still allow for roaming but better enable you to manage your herd. Due to the size of the fence, it will be a big job. You will expend a great deal of time, money, and labor, but once completed, it will quickly pay for itself in returned time, money, and labor. The dilemma is that you cannot simply drop all the other functions that are needed to keep your business going. So, you will build the fence a bit at a time, perhaps cashing in favors with friends to help speed up the job.

All complex systems must keep existing functions going while still having enough reserves to learn, grow, develop, and heal. Both maintenance and growth require various flows on a structure. Until this point we have assumed that flows simply happen. This is far from the case—all systems require some driving forces to keep flows going. The catch is that there is always a cost in energy to switching tasks and expending valuable resources. This is the domain of thermodynamics and will be the focus of this chapter.

Thermodynamics is a word that can send even the most mathematically inclined to scatter. The convoluted definitions of heat, work, energy, order, disorder, and so on make it a field that feels impenetrable. In *The Two Cultures*, C. P. Snow (1905–1980) suggested that in the realm of science, being able to describe the second law of thermodynamics is the equivalent in the arts of being able to discuss the works of Shakespeare. Likewise, Albert Einstein (1879–1955) claimed that thermodynamics "is the only physical theory of universal content, which I am convinced, that within the framework of applicability of its basic concepts will never be overthrown."

The concepts of thermodynamics are in fact fairly intuitive. In the past chapters, we established that functions arise from flows on a structure and that gradients (often manifest in patterns) are the driving force behind these flows. Thermodynamics,

J. Tranquillo, *An Introduction to Complex Systems*,
https://doi.org/10.1007/978-3-030-02589-2_7

being about much more than the flow of heat, will help unite these ideas. We will first explore the origin of thermodynamics with its focus on heat, temperature, and entropy. We will then consider how energy and matter flow through systems that are open to their environment and how these systems can perform work. The final section will explore how thermodynamics can be applied more broadly to the ways in which complex systems can direct work and energy inward to heal, learn, and grow.

7.1 Irreversibility and the Origin of Thermodynamics

In the 1700 and 1800, the steam engine was one of the drivers of the industrial revolution. The principles of heat flow upon which the steam engine was based became a major topic for scientific investigation. Sadi Carnot (1796–1832), one of the early pioneers of thermodynamics, put his finger on a paradox. According to the laws of physics, planets could easily circle in the opposite direction. If one watched billiard balls moving around on a table, it would not be possible to know which order (forward or backward in time) the ricochets were occurring. Perhaps all physical processes were *reversible*, meaning that the laws of physics would work if $-t$ were used in all equations of motion and flow. To use the terminology of Chap. 3, trajectories in phase space could be reversed. When it came to heat flow, however, reactions only seemed to go in one direction.

To illustrate the concept, consider two cases of placing two blocks in contact with one another. In the first case (top of Fig. 7.1), the two blocks have different temperatures, T_h and T_c. If we assume box h is warmer than box c, then there will be a flow of heat (F_{hc}) from h to c. A little thought will reveal that the driving force for this flow is the difference in temperature (called a temperature gradient) that will over time equilibrate the two boxes to some intermediate temperature. Once this temperature is reached, the two blocks will remain at that temperature forever (known as a thermodynamic equilibrium). This first case should seem intuitive to you, as it describes why a cold beverage will warm up on a hot day and why a frozen turkey will thaw when taken out of the freezer.

In the second case, assume that the blocks are already at the same temperature, $T_h = T_c$ (bottom of Fig. 7.1). But then somehow, block h becomes hotter, and block c becomes cooler, due to some flow of heat, F_{ch}. This should *not* seem intuitive at all, and in fact, you should never have seen this happen in a real situation.

The two cases are simply time-reversed versions of one another. Case 1 is known as *spontaneous flow*, meaning it will occur without any influence from the outside. What bothered physicists was that none of the laws of physics up until that point disallowed Case 2. The directionality of time, sometime called the *arrow of time*, is not limited to heat flow. If you were shown a video of a destructive process, such as an egg falling off of a counter and cracking, you would know if the recording was being run in forward or reverse. Cracked eggs cannot jump back on the table whole again. The directionality of time is the basis for causality and often acts

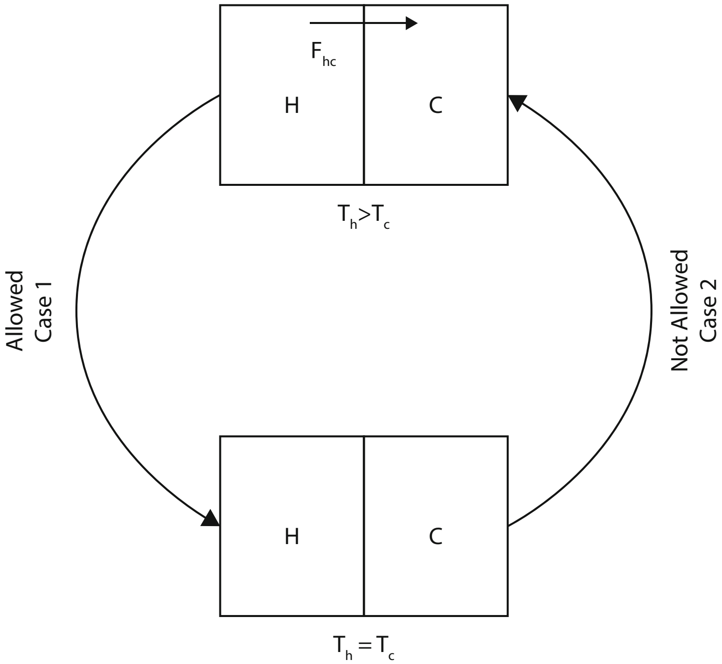


Fig. 7.1 Thermodynamic demonstration of the irreversibility of some processes. Case 1 tends to spontaneously equilibrate gradients and is allowed. The reverse (the creation of a gradient as in Case 2) will not spontaneously occur

to homogenize gradients over time. Another way to think of irreversibility is that patterns that may have been in an initial condition (e.g., like the gradient on the top of Fig. 7.1) are erased over time.

7.1.1 Entropy

The term *entropy* was coined in 1865 by Rudolf Clausius (1822–1888) to mathematically account for the irreversibility of time. Clausius came up with the idea that something must be changing in a one-way direction that allows Case 1, but not Case 2. Unlike some other physics terms, entropy is not strictly measurable, like the mass of an electron, but rather is a mathematical abstraction.

Entropy change, ΔS, occurs when a system exchanges some amount of heat with another system.

$$\Delta S = \frac{\Delta Q}{T}$$

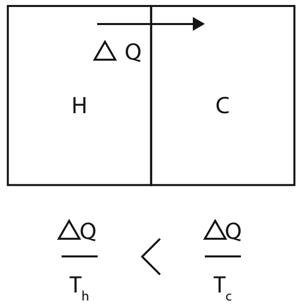


Fig. 7.2 The flow of heat resulting in a difference in entropy between hot (H) and cold (C) blocks

where ΔQ (positive when entering, negative when leaving) represents a flow of heat energy and T is the temperature of the system. This allows one to consider the flow of heat ($\Delta Q = F_{hc}$) between the two blocks in Fig. 7.2. Suppose a small amount of heat leaves (ΔQ is negative) the hot block. The entropy of the hot block is reduced by $-\Delta Q/T_h$. But, that same ΔQ enters the cooler block (now $+\Delta Q$). The resulting increase in entropy of the cooler block is $\Delta Q/T_c$.

The entropy goes down in the hot block that is cooling, and entropy goes up in the cool block that is warming. But the amount of change in entropy is unequal because $\Delta Q/T_c$ is greater than $\Delta Q/T_h$. When considering both blocks, the entropy of both blocks together has increase by

$$\Delta S_{\text{total}} = \frac{\Delta Q}{\frac{1}{T_h} - \frac{1}{T_c}}$$

even though the total heat energy of the two blocks has remained the same.

7.1.2 The Molecular Meaning of Temperature

We have been using the terms temperature and heat without clearly defining what they are. This was largely the work of Ludwig Boltzmann (1844–1906), several decades after Clausius and Carnot. Boltzmann described heat as a type of energy, more specifically the energy contained in the collective motion of many small particles. Boltzmann's molecular theory of heat is analogous to billiard balls all moving around a table, colliding with one another, and the rails of the table. The total heat energy, often called *enthalpy* of the system, is simply the sum of all of the individual energies. The more energetic the particles, the more heat energy there will be in the system.

Temperature is a *measurement* of heat. Particles collide with a detector, as in Fig. 7.3. The more energetic the particles and the more often they collide with the detector, the higher the measured temperature. To gain some intuition, imagine a system that contains a particular heat energy and a temperature detector that

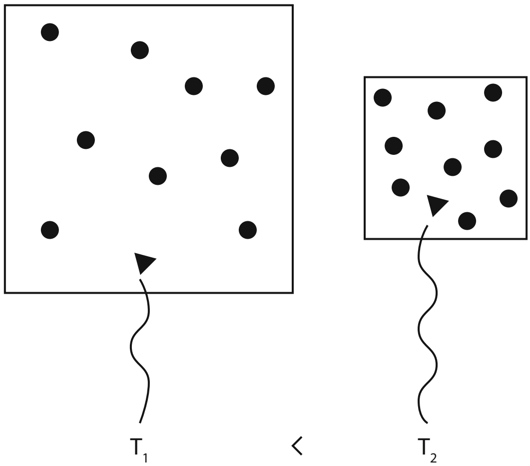


Fig. 7.3 The measurement of temperature as particle (dots) collisions with a detector (triangle). Greater force and frequency of collisions will result in the measurement of a higher temperature

measures a temperature of T_1. If the volume of the system suddenly became smaller, the internal energy would remained the same, but the particles would collide with the detector more often, and a higher temperature (T_2) would be measured. Although temperature is a measure of the heat energy in a system, the two are not equivalent.

Clausius' original formulation of entropy was a convenient mathematical trick that showed how energy could be conserved, yet processes might be irreversible. Boltzmann's critical insight was that collectively molecules would move toward a system state that would be most probable. At the macro level, this would be a stable equilibrium that would attract all nearby states. This attracting state would be homogeneous (e.g., containing no gradients), as in the top of Fig. 7.1, and once reached would not change on the macroscale. This led Josiah Gibbs (1839–1903) to extend Boltzmann's ideas to redefine entropy as the distribution of particles in the system:

$$S = -k_b \sum p_i ln(p_i)$$

where k_b is a constant and p_i is the probability of finding a particle in a particular location. Entropy was no longer a mathematical trick to describe the arrow of time but rather a quantity that arose from the molecular theory of heat.

The connection between complex systems and thermodynamics is twofold. First, thermodynamics historically studied heat as a type of energy. The central tenets apply, however, to any kind of energy and the flows that result when gradients exist. As such, thermodynamic principles generalize to any system that contains flows and gradients. Second, the mathematical formulation underlying thermodynamics, called *statistical mechanics*, connects micro-level particles in motion to macro-level phenomena and can be applied to any multi-bodied system. In the context of complex systems, this idea is expressed by John Miller and Scott Page (1963–) in their text *Complex Adaptive Systems*. "In many social scenarios, differences nicely cancel one another out. For example, consider tracking the behavior of a swarm of bees. If you observe any one bee in the swarm its behavior is pretty erratic, making an exact prediction of that bee's next location is nearly impossible; however, keep your eye on the center of the swarm—the average—and you will detect a fairly predictable pattern." This is an example of how a micro-level behavior may be hard to predict, but averages become much more tractable. Furthermore, using ideas from thermodynamics, we could begin to numerically compare the "temperature" or "entropy", reflected in the kinetic motion of the bees, of a beehive during the day and at night. Similar ideas can be applied to the Boids model of flocking, cars in traffic flow, or the molecular flows inside of a cell.

7.1.3 The First Law of Thermodynamics

The declaration by Carnot that the total energy in a system cannot change, called *conservation of energy* or the *first law of thermodynamics*, applies to all forms of energy. Sometimes this goes by the phrase, "there is no such thing as a free lunch." Energy cannot disappear or appear. Energy can, however, be transformed from one type (mechanical, electrical, heat) to another. When the total energy is considered, however, no energy will be left unaccounted for.

The ability to move energy around within a system has an important consequence. A gradient of one type of energy can be used to drive another type of energy against its natural gradient. Imagine a frictionless roller coaster. Potential energy (denoted as mgh, where m is mass, g is the gravitational constant, and h is height) is built up by moving the cars of the coaster up a hill. That potential is transformed into kinetic energy (denoted as $\frac{1}{2}mv^2$, where v is velocity) when the coaster is moving. At any point in time,

$$mgh + \frac{1}{2}mv^2 = C$$

where C is the total energy of the coaster. Every time there is a decrease in height (h), the coaster goes faster (v), as in Fig. 7.4. Every time the coaster goes up a hill (increase in h), it will necessarily lose speed (decrease in v). A good roller coaster designer learns to balance potential and kinetic energy.

7.1.4 Work on a System

How did the roller coaster become endowed with energy in the first place? Physicists will often talk about the work (W) that is performed on some body. In this case work must be performed on the coaster to get it moving up the hill. The traditional definition of work is

$$W = F\Delta d$$

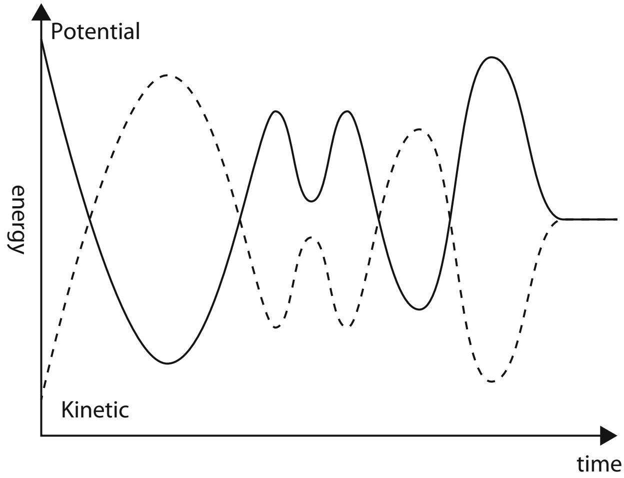


Fig. 7.4 The oscillation of kinetic (dashed line) and potential (solid line) energy during a roller coaster ride. It is assumed that this closed system is endowed with potential energy at the beginning (the ride starts at the top of the hill), and no energy leaves the system (the ride ends with the coaster continuing to coast)

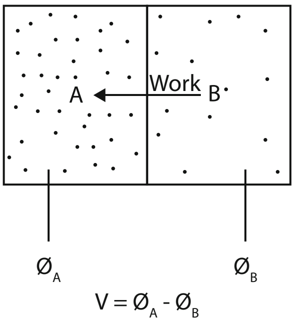


Fig. 7.5 Electrical work moves charge against an electrical gradient (measured as voltage). The resulting flow of current will make the gradient larger, placing more charge in compartment A

where F is some force applied over some distance, Δd, to move an object. In the case of the roller coaster, W is the work done to drag the coaster up the hill. More generally, work is movement against a gradient of some sort. Work can just as easily be defined as work against an electrical, chemical, or heat gradient. In the case of the roller coaster, it is work against a gravitational gradient, usually called a gravitational potential.

Work can also be performed to reverse an electrical gradient as well. In Fig. 7.5 there is a buildup of charge in A compared to B that makes the gradient larger. Charge will naturally flow from A to B as the like charges repel one another. Moving a charge from B to A will require work. This is summed up in the equation for electrical work

$$W = Vi\Delta t$$

where V is the gradient of charge between A and B and i is the flow of charge. $i\Delta t$ is the amount of charge that flows from B to A.

In Figs. 7.4 and 7.5, the work is being done by something that is not shown in the system. For the roller coaster, it is likely from an electric or gasoline motor. In the electrical example, it may come from some externally applied electric or magnetic field. In these cases, one system is running down its own energy to supply another system. As will be discussed below, in complex systems that contain multiple compartments, energy in one part of the system might be used to do work on another part of the system.

7.1.5 The Second Law of Thermodynamics

Given the definition of work, we can return to the idea of entropy to gain some additional insight. Returning to Fig. 7.1, it is possible to force Case 2 to occur by performing work on the system. One gradient can be homogenized to make another gradient larger. Likewise, it might be possible to put a cracked egg back together. However, more energy will be expended in forcing Case 2 to occur than will be released through the spontaneous pathway of Case 1. In other words, once a gradient has been run down to perform some work, it will require more work to recharge that gradient. This concept is known as the *second law of thermodynamics*.

The second law holds whenever there is a flow across a boundary or energy is used to do work. For example, in the roller coaster example, we can imagine an electric battery (energy manifest in an electrical gradient) being used to drive the coaster up the hill (energy manifest in a gravitational gradient). The work done by the electrical gradient will never be entirely returned, even after the energy stored in the gravitational gradient has been expended at the end of the ride. The second law acts to homogenize gradients, resulting in less ability to perform work. On a much grander scale, some astrophysicists argue that our entire universe will ultimately become homogenized (e.g., no planets, stars, or other patterns).

7.1.6 Not All Energy Can Do Work

The practical meaning of the second law of thermodynamics is that not all energy can do work. The canonical example is the difference between the top and bottom of Fig. 7.1. Both have the same amount of total heat energy stored in the motion of particles. The top of Fig. 7.1, however, contains a gradient that can be used to perform work, whereas the system in the bottom panel does not. The qualitative terms “high-quality” energy and “low-quality” energy are often used to distinguish how much work can be extracted from a gradient. In general, the larger the gradient, and the more high quality the energy, the more potential there is to do work. A reframing of the second law is therefore that work necessarily means turning high-quality energy into lower-quality energy.

Josiah Gibbs again found a way to mathematically connect together ideas in the first and second law of thermodynamics with the idea of high- and low-quality energy. Consider the total energy of a system to be composed of the portion that is usable for work and the portion that is not usable.

$$H = W_{\text{useful}} + W_{\text{not useful}}$$

$$H = G + ST$$

The second equation is more specific in that H is the total energy of the system, S is entropy, T is temperature, and G is the energy available to do work. The usual way to write Gibbs relation is

$$G = H - ST$$

G is called the *free energy* of the system. There are a few different formulations of free energy, and this one is known as *Gibbs energy*, which assumes a constant volume and pressure. When G is high, that means a significant amount of the energy in the system is available to do work. The most useful formulation, however, is to compare the values of these terms before and after some work has been done:

$$\Delta G = \Delta H - \Delta(ST)$$

In closed systems, H does not change so $\Delta H = 0$. Therefore, as work is performed and gradients are run down, ΔS rises (for a constant temperature), and therefore the energy available to do work decreases. G will decrease over time, and a spontaneous flow in a system will mean that ΔG will be negative. In fact, in chemistry Gibbs equation is used to determine if a reaction will occur spontaneously or not. If a process has a negative change in free energy, then the reaction will occur spontaneously. It is important to point out, however, that G does not always go to zero—reactions do not always go to completion. This will become important when we discuss open systems.

Different types of energy have come to be associated with being high or low quality. For example, the random kinetic movements of molecules that characterize heat energy have greater entropy and less ability to do work. Heat is often associated with the lowest-quality kind of energy. On the other hand, electricity is often associated with being high-quality energy. Electricity, however, is a flow of electrons down a wire due to the potential gradient across the wire. It is the wire (the structure of the system) that enables the electrical gradient to be released—a random collection of charges scattered throughout a space would be considered low-quality energy. In many ways the structure of a system constraints how flows of higher quality energy are transformed into lower quality energy.

The broad application of thermodynamics to complex systems comes with a caveat. For example, a person's (or company's) net worth is not always a good reflection of how much financial capital they can use, often called liquid assets. Some amount of wealth is almost always locked up in properties, bonds, or other long-term investments. Such might be an economic version of Gibbs relation. It is tempting to think of gradients of money (differences in wealth) as driving the economic system. In such an analogy, high-quality wealth would be constantly degraded into lower-quality wealth. A little thought will reveal that this is not strictly the case and in some cases is exactly the opposite of a real economic system.

7.1.7 Maxwell's Demon

A thought experiment proposed by James Clerk Maxwell (1831–1879) raised a challenge to the second law of thermodynamics. Suppose the situation shown in Fig. 7.6 where there is a little demon between two compartments. This demon is in control of a frictionless door that can be opened or closed. When a high-speed (high-energy) molecule from the right compartment approaches the boundary, the demon lets it pass to the left compartment. Over time there would be a separation between high- and low-energy molecules that would become a difference in energy (and temperature). The demon would seemingly violate the second law of thermodynamics—a heat gradient would be created using no energy at all. The practical meaning was that perhaps perpetual motion machines might be possible and used to do work for free.

Over the next 100 years, clever arguments were made to show that such a demon could never exist, but each were found to have a logical or physical flaw. For example, one could claim that there is no such thing as a frictionless door. But

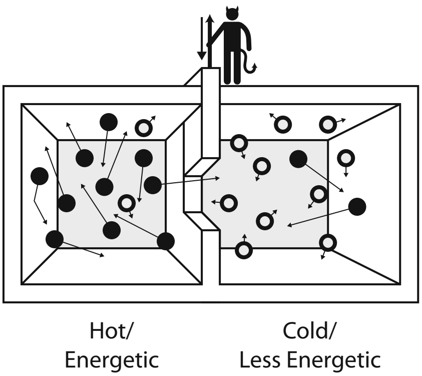


Fig. 7.6 Maxwell's demon creating a gradient while expending no energy. Closed dots represent high-energy particles, while open dots represent low-energy particles. The demon will only open the pathway when a high-energy particle from the right is moving toward the left. As shown, the demon would quickly close the opening. Over time the demon will create a gradient with the left side being a higher temperature than the right side

there are frictionless things in the world (e.g., superconductivity), and so it might be possible in practice to create such a door. It would also be tempting to claim that the demon would require energy. Again, in theory, it may be possible for some theoretical demon to operate without using energy. Maxwell's paradox was not fully resolved until the 1990s when it was shown using information theory that the demon, in its ability to detect the state of the particles, would require memory. Making measurements and storing them in a memory in fact could in principle be performed without expending energy. It is the erasing of information to make way for new information that uses energy. The proof builds upon work by Rolf Landauer (1927–1999) in the early 1960s who showed that erasing memory will necessarily require a dissipation of low-quality energy. This is now known as Landauer's principle. There really is no way to get energy for free.

7.1.8 Approaches to Equilibrium

Gibbs relation explains a wide range of system dynamics whereby free energy is exhausted, no more spontaneous flows may occur, and an equilibrium is reached. For the first century of thermodynamics, it was assumed that reactions and flows would proceed in one direction as fast as allowable by the system. In other words, a system would move directly toward the equilibrium. In some systems, this is in fact not true as illustrated in the case below.

Breakthroughs are sometimes not recognized until long after they occur. Such is the case of Boris Belousov (1893–1970), a chemist working behind the Soviet Union Iron Curtain in the 1950s. Belousov was working with a mix of potassium bromate, cerium sulfate, propanedioic acid, and citric acid, all mixed in sulfuric acid. What he found was that the solution would alternate between yellow and colorless. This was a very striking result because chemical reactions were supposed to proceed from one state to the next, with each state having lowering free energy, eventually reaching a thermodynamic minimum equilibrium. Once a system reached that minimum, it should not spontaneously reverse and go back in the other direction. When Belousov tried to publish his result, the editors of the journal rejected his paper because his finding seemed to violate the second law of thermodynamics. It was suspected that his result was due to a methodological error. He did eventually publish his work in a non-peer-reviewed journal.

Anatoly Zhabotinsky (1938–2008) learned of Belousov's reaction and publicized it a decade later at a conference in 1968. This sparked curiosity and prompted some chemists to try to reproduce this strange result. To their surprise, they were able to find a whole family of reactions that behaved in a similar manner. These are now known collectively as the Belousov-Zhabotinsky (BZ) reaction that we encountered in Chap. 4.

The BZ reaction in fact obeys all of the laws of thermodynamics—free energy is constantly being reduced. The flaw was in considering the concentrations of the reactants individually. In coupled equations and reactions, the path to reducing energy might be through an oscillation of reactants that gets smaller and smaller each time. As the reactants that drive one reaction get low, they trigger another reaction that might act to resupply the low reactants. These two reactions can then oscillate about one another, losing free energy in the process. The BZ reaction will eventually stop oscillating and settle down to an equilibrium; it simply takes a seemingly unusual path to reach that equilibrium. It was unusual to see this in a chemical reaction, but it is essentially what occurs when a spring undergoes a series of decaying oscillations, passing through the eventual equilibrium several times, before finally coming to rest.

7.1.9 Autocatalytic Reactions (Revisited)

The BZ reaction was historically important because it demonstrated that it is not the individual reactions that must have their energy reduced monotonically. Instead it is the entire family of interrelated reactions that must reduce Gibbs energy. A generic *autocatalytic* set of reactions may take the form

$$A \rightleftharpoons B$$
$$B \rightleftharpoons C$$
$$A \rightleftharpoons C$$

The product of one reaction serves as the reactant of another reaction. To understand how these kinds of systems can oscillate, assume that we begin with a high concentration of A; then

- most of A is converted into B, leaving a little bit of A unconverted.
- most of B is converted into C, leaving a little bit of B unconverted.
- most of C is converted into A, leaving a little bit of C unconverted.
- this cycle repeats, with the unconverted material building up over time until the system converges to equilibrium concentrations.

The concentrations of A, B, and C can be plotted against one another as in Chap. 3. The three-dimensional spiral in phase space is shown in Fig. 7.7. The hypothetical stable equilibrium (final concentrations) is centered on 1,1,1. Any perturbation from this concentration will be attracted to this point and spiral inward. But, because the plane along which the spiral occurs is tilted in phase space, the reduction of one reactant will push another reactant higher.

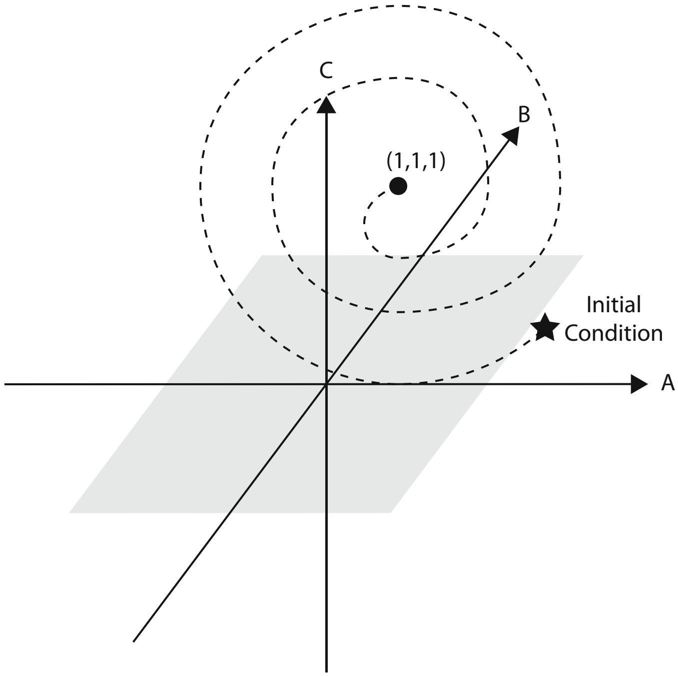


Fig. 7.7 The non-linear spiraling of concentrations in three dimensions of an autocatalytic system. The initial concentrations will not monotonically decrease but rather react as a series of decaying oscillations until reaching the equilibrium point. The free energy, however, does decrease monotonically

7.1.10 The "Problem" of Order and Complexity

Information and entropy are often mentioned together in the complex system literature because the equations look similar. This was in fact due to Claude Shannon. An apocryphal story was the advice that John von Neumann was claimed to have given Shannon when he derived the measure of information in Chap. 6: "You should call it entropy for two reasons: first because that is what the formula is in statistical mechanics but second and more important, as nobody knows what entropy is, whenever you use the term you will always be at an advantage." That these terms remain shrouded in mystery is not simply the math. Rather it is the confusing way information and entropy are taught in relation to order and disorder.

Historically *order* and *disorder* were introduced by Boltzmann. Disorder implied higher entropy. This was powerful because the everyday meaning of order and disorder aligned with how entropy was manifest in Boltzmann's thought experiments. Over 100 years of textbooks propagated that disorder and entropy were synonymous. For example, a common way to explain entropy is using the order or disorder of a room. The claim is that there is only one way for a room to be ordered (a low entropy state) but many ways for that room to be disordered (a high entropy state). As the analogy goes, the probability of finding the room orderly is very small in comparison to the number of ways it can be disordered. The farther from ordered the room is, the more entropy it has. In fact, this argument was made by one of the greatest minds of the past century, Stephen Hawking (1942–2018), in the best-selling physics book of all time *A Brief History of Time*.

Information theory inherited ideas about order and disorder and expanded upon them. The ability to predict patterns in time (e.g., repeated choruses in music), space (e.g., repeated motifs in artwork), and phase space (e.g., limit cycles) implied order. Prediction and probability (upon which measures of entropy and information are based) seemed to be based upon the degree of order.

The equating of order with low entropy and disorder with high entropy exposes a fundamental mismatch in the difference between a first- and third-person perspective. In Chap. 6, two signals were compared—"agt4fns5ifyiu6hz90d" and "Hello, how are you?." One appears to be ordered and the other disordered. They in fact mean the same thing. The only difference is that one is in English and the other needs to be decoded by a key. The first-person perspective (whether you have the key or don't) will determine whether or not you consider these signals to be ordered or disordered. To someone with the key, they are identical and have the same degree of order. On the other hand, the mathematical definition of entropy would give a clear third-person perspective. The most simplistic way to see this is that the English statement contains 10 unique symbols and the code contains 18 unique symbols. Knowing nothing about the languages used, the code would objectively have higher entropy. Likewise an "ordered room" only has meaning in context and is very much in the eye of the beholder. No matter how messy or clean a room might be, the various parts of a room do not form gradients and therefore cannot do any work or send meaningful signals. The analogies used to connect disorder to entropy and information are not always wrong, but sometimes they are misleading.

7.1.11 Semipermeable Barriers and Network Entropy

To further cause confusion, there are several ways (generally framed as mathematical formulas) to compute entropy. For example, there are Shannon entropy, Kolmogorov-Sinai entropy, network flow entropy, multiscale entropy, dynamical entropy, and so on. Each is essentially comparing a particular configuration or state to the possible configurations or states that could exist. These measures often apply well to some complex systems but not to others.

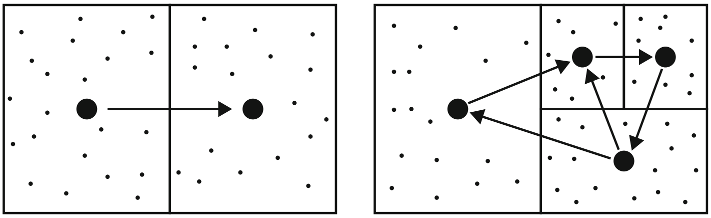


Fig. 7.8 The increase in the potential for gradients and flows as structural barriers are added to a system. Such structures can easily be mapped to the network diagram of dots and lines explored in Chap. 5

A helpful way to understand these varied measures is to distinguish between the structural and functional properties of a system. Functional entropy measures consider the flows that occur within a system and quantify the gradients that are present, the work that is done, and the information that is sent. Much of Chap. 6 and this chapter have focused on functional flows.

Structural measures of entropy, on the other hand, consider the possibilities for gradients, flows, work, and the sending of information within a system. To gain some sense of structural entropy, consider Fig. 7.8. The left- and right-hand sides have the same energy, and because there are no gradients, there are no flows. The right-hand side, however, has more potential for gradients. Given two compartments, only one flow can exist. With four compartments, there are many more flows that could exist. As more compartments are added, the number of possible gradients and flows will explode very quickly. Each of these flows also possesses the ability to perform work that enables functions of the system. The network measures explored in Chap. 5 give some idea of the structural entropy inherent in a system.

The connection between structural and functional entropy can be revealed in considering that flows on a structure are needed for a system to perform work and execute functions. If the barriers between compartments are entirely open, some process (likely a form of diffusion) would quickly homogenize any initial gradient. On the other hand, if the barrier could never be let down (e.g., two disconnected compartments), there would be no way for flows to occur. Practically what is needed is for the barriers to be semipermeable. As such the practical structure of a system can be thought of as a series of semipermeable boundaries that can support gradients and through which information, money, material, or energy can flow.

7.2 Open Systems

Dynamic systems wind down over time, eventually becoming homogenized and unable to perform any more work. So how do complex systems sustain themselves? It is possible that systems, such as human beings, are endowed with a great deal of charged gradients that slowly are erased over time. A wound watch works in this way. But a human, and many other systems, seems to grow, develop, and heal themselves. In this section we will explore open systems that can “feed” off of high-quality energy from the environment and dispel “waste,” or lower-quality energy, into the environment.

Open systems still obey conservation of energy, but now the energy inside the system (H_{sys}) and the environment (H_{ext}) must be taken into account. Any energy that exits the system (ΔH_{sys}) must show up in the environment (ΔH_{ext}) and vice versa.

$$H_{sys} + H_{ext} = C$$

$$\Delta H_{sys} + \Delta H_{ext} = 0$$

A similar kind of argument can be made for the second law of thermodynamics.

$$\Delta S_{sys} + \Delta S_{ext} \geq 0$$

which means that when considering how entropy changes in open systems, the system and its environment must both be taken into account. It is in fact possible for entropy to decrease within a system at the expense of the surrounding environment. In the following sections, we will consider how high-quality energy steaming into a system, with the output of lower-quality energy into the environment, can enable an open system to maintain, and perhaps even grow, its functions over time.

7.2.1 The Input of High-Quality Energy

Open systems can accept high-quality energy from the environment that can be used internally to do work. The roller coaster energy profile shown in Fig. 7.4 assumed a closed system endowed with a great deal of potential energy at the beginning of the ride. In reality a coaster is an open system that receives energy from an outside source (usually a motor attached to a drive chain) that drags the cars against a gravitational gradient at the beginning of each ride. The system initially contains very low energy (before the hill), as in Fig. 7.9, to which energy is slowly added.

Another example of an open system is a refrigerator. Case 2 in Fig. 7.1 is essentially the idea of a refrigerator or air conditioner—to create a gradient between inside and outside by doing work. A high-quality electrical gradient, usually coming from a plug, is used to drive the heat gradient in the opposite direction. In other words, the outside world is performing work on the refrigerator.

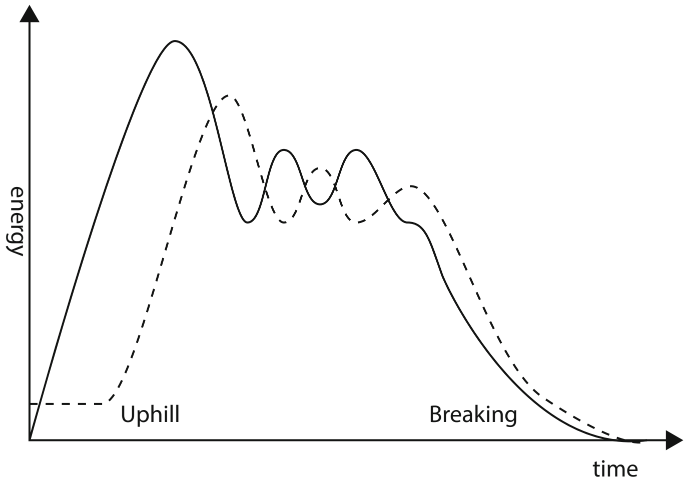


Fig. 7.9 An update to Fig. 7.4 to include high-quality energy input (motor to move the coaster up the hill) and lower-quality energy output (due to braking). The dashed line is potential energy. The dotted line is kinetic energy

7.2.2 The Dissipation of Low-Quality Energy

Open systems can also release lower-quality energy into the environment. In the roller coaster example, the friction between the wheels and the track will generate heat that is released into the environment around the wheels. Likewise a braking mechanism will bring the car to rest (e.g., no more kinetic energy), as on the right side of Fig. 7.9. Likewise, the transfer of energy across multiple compartments and parts in a refrigerator will lead to the generation of heat that exits the unit. *Dissipative systems* are those that can expel lower-quality energy into the environment.

7.2.3 System Metabolism

In open systems, high-quality energy can be accepted and used to move gradients against their natural flow, and then those recharged gradients can be used to perform useful work all the while discarding lower-quality energy. This is the definition of a *metabolism*. We generally do not think of a refrigerator as having a metabolism, but its function is in many ways similar to biological metabolism. Of course there are some significant differences in how a biological organism came into being and how a refrigerator came into being. An organic organism and a refrigerator also use different sources of energy and expel different kinds of waste. At a high level, however, they are both systems that exploit high-quality energy from an environment to perform internal work.

Consider how total free energy changes in metabolizing systems.

$$\Delta G_{\mathrm{tot}} = \Delta G_{\mathrm{in}} - \Delta G_{\mathrm{out}}$$

$$\Delta G_{\mathrm{tot}} = \Delta H_{\mathrm{in}} - T_{\mathrm{in}} \Delta S_{\mathrm{in}} - [\Delta H_{\mathrm{out}} - T_{\mathrm{out}} \Delta S_{\mathrm{out}}]$$

In a closed system, the last two terms are zero, and we are back to the usual formulation of Gibbs. The total energy must remain the same, so $\Delta H_{\mathrm{in}} = \Delta H_{\mathrm{out}}$. Any energy lost (or gained) is accounted for.

$$\Delta G_{\mathrm{tot}} = \Delta G_{\mathrm{in}} - \Delta G_{\mathrm{out}} = -T_{\mathrm{in}} \Delta S_{\mathrm{in}} - T_{\mathrm{out}} \Delta S_{\mathrm{out}}$$

The internal free energy (ability to do work, G_{in}) can in fact go up as long as the entropy leaving the system is higher than the entropy entering the system ($\Delta G_{\mathrm{in}} < \Delta G_{\mathrm{out}}$).

The ability for a system to dissipate energy now takes on an even more important meaning. It is the mechanism by which a system can dispel lower-quality energy. Some other examples of dissipative systems, and the kinds of patterns that can arise, are hurricanes and tornados. Both are spirals that are the result of the unequal heating of different regions of the atmosphere by the sun. The cycle of energy through an ecosystem is another example, powered almost exclusively by the sun. Of course, in total, the energy coming from the sun is degrading the sun itself, so the net entropy of the solar system is always rising.

Metabolizing systems can also shed some light on the nature of the various types of attractors discussed in Chap. 3. It is the dissipation of some form of energy or homogenization of a gradient that moves a transient to stable points. Dissipation acts to contract trajectories in phase space, keeping them bounded within some volume. Yet some systems we studied mathematically in Chap. 3 also contained expansion terms and led to limit cycles and strange attractors. In the real world, these expansion terms will require some work to be done and energy to sustain—there is no such thing as a perpetual motion machine or a limitless limit cycle.

7.2.4 *Snowflakes and Crystal Formation*

The balance between the input and output of energy in an open system can lead to some seemingly paradoxical phenomena. A classic example occurs in crystal formation, whereby a system is cooled by expelling heat into the environment. A good example is the formation of a snowflake as a liquid water droplet is cooled. Gibbs energy is constantly decreasing,

$$\Delta G = \Delta H - S \Delta T$$

but in an open system, this can occur because all terms (including ΔH) are decreasing. Temperature becomes important in these cases because it determines how the system goes about decreasing free energy. At a high temperature, the dominate way to decrease free energy is by increasing entropy, as the ST term dominates changes in G. At a low temperature, the ST term becomes less important. As such entropy can in fact spontaneously decrease (e.g., for the formation of crystals) as G, S, H, and T all decrease. A cool snowflake, despite seeming to be more orderly, has lower free energy than a warmer water droplet.

7.2.5 Phase Transitions

As systems accept or release energy to or from their environment, they can undergo rearrangements of their internal parts that can be characterized as changes in entropy. These changes can sometimes occur rapidly in *phase transitions*. One of the most familiar phase transitions is between ice and liquid water. If we ignore changes in pressure, we can demonstrate the idea of a phase transition from ice (point 1) to liquid water (point 2) as in Fig. 7.10. At T_0, the freezing point, there is a discontinuous jump in the entropy of the system. Consider a block of ice (to the left of T_0) to which we are adding heat energy. Usually energy coming into this system shows up as an increase in temperature. But remember that heat (energy) is different than temperature (a measurement). At the critical temperature T_0, heat energy begins to break molecular van der Waals bonds that keep the ice together. The added heat energy, often called *latent heat*, goes into breaking those bonds rather than increasing the temperature.

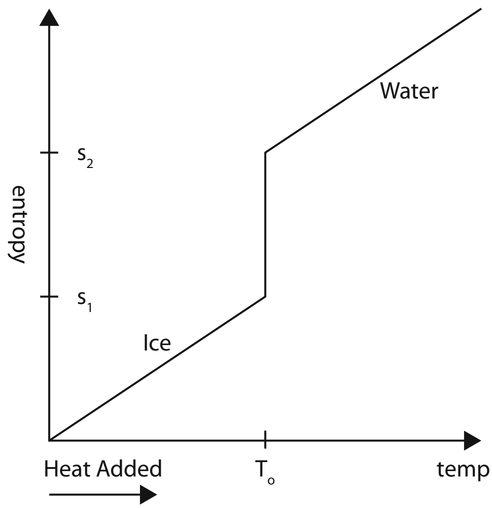


Fig. 7.10 The abrupt change in entropy during a first-order phase transition. At T_0 the entropy jumps without a change in temperature

The abrupt jump in Fig. 7.10 can be captured mathematically by considering the equation

$$\Delta S = S_2 - S_1 = -\left[\frac{\Delta G}{\Delta T}\right] > 0$$

which shows how entropy changes as a function of free energy and temperature changes. The key point is that the derivative ($\frac{\Delta G}{\Delta T}$) changes with a constant derivative before and after the transition. But right at the transition, there is a discontinuity that is known as a *first-order phase transition*. The temperature (T_0) at which this jump occurs is called a *critical point*. It is at the critical point that some instability appears which results in a difference in how a system reacts to changes in energy (either increased or decreased). Generally this instability appears due to a shift in how agents or particles interact. In the water example, it was the van der Waals forces that are overcome.

In the water-ice example, the critical point is not truly a single point but rather a more complex shape that is determined by temperature, pressure, and volume. This is the reason pressure cookers can run at a higher temperature and why boiling occurs at a lower temperature at a higher altitude. This is common in complex systems where several variables may determine a complicated surface or higher-dimensional *critical* shape that forms the border between phase transitions.

The sudden switching of system behavior was also described in Chap. 3, with bifurcations and separatrices that demarcate fundamentally different kinds of behavior. Bifurcations and separatrices, however, are properties of lower-dimensional systems (e.g., only a few coupled equations). Phase transitions on the other hand are an emergent property of a system with a virtually uncountable number of degrees of freedom. In fact, another way to think of a phase transition is that the degrees of freedom of the system drastically change. Often on one side of a transition, each element behaves independently (e.g., gas state), whereas on the other side of the transition, the entire system acts as one large element (e.g., solid state). Such differences are the reason a gas will fill its container, whereas a solid will not.

7.2.6 Traffic and Second-Order Phase Transitions

In the study of complex systems, second-order phase transitions (a discontinuity in the second derivative) seem to appear very often. There is a more gradual transition between phases rather than a critical point. We have already seen an example of a second-order phase transition in Kauffman's button network from Chap. 5 (Fig. 5.23).

A real-life example can arise in traffic flow. When there are only a few cars on the road, flow is fairly regular and often seems to be slightly above the posted speed limit. Rather than inject heat into this system, we can inject more cars, increasing the car density. In this hypothetical example, we can imagine that as more cars are added, the speed will stay constant at first, but as the density becomes greater, the speeds will begin to slow. Adding cars will eventually result in a

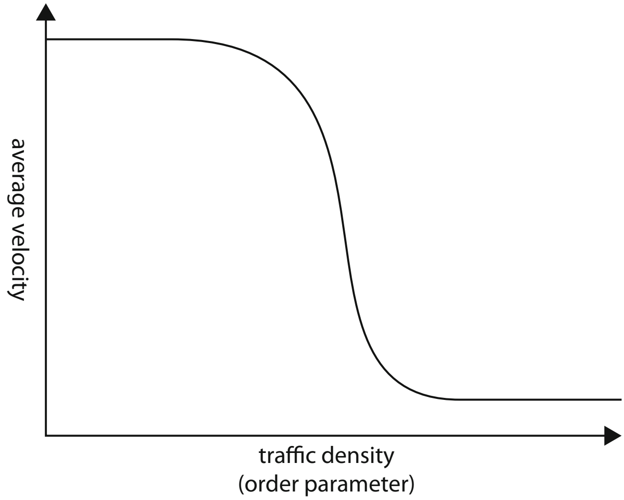


Fig. 7.11 Smooth change in entropy during a second-order phase transition. The example given is of the average velocity of traffic flow as the density of cars is increased. When few cars are on the road, the velocity is high. As more cars are added, there is a range over which the velocity decreases until eventually there is a traffic jam

traffic jam, essentially a frozen or crystalized state. But, unlike the water example, traffic will not immediately switch from regular flow to a traffic jam all at once. Rather, there is a range of densities over which regular traffic flow transitions to a traffic jam, as shown in Fig. 7.11. There are of course other factors, such as construction, road signs, weather, police, and accidents, that will impact the range of this transition. These kinds of flow dynamics occur in many systems and are known more generically as the *jamming transition*, at which flow comes to a standstill.

Because second-order transitions occur over a range, there is no critical point. Instead we can define an *order parameter* that serves a similar function. Generally the order parameter is a global property of the system (like car density) that when varied moves the system through a phase transition. Using an order parameter, we can more precisely define how wide the range is over which the transition takes place. For example, you may have noticed that on some highways, the transition range may be very large (e.g., able to absorb a great density without locking up completely), while for others it may be very small. An interesting question is how much of this range is determined by the road and how much is a function of the habits of the drivers.

A possible origin of second-order transitions will be explored in greater detail in Chap. 10. Here we will speculate on a functional reason why many complex systems seem to operate in the transition region of a second-order phase transition. Phase transition changes how individual elements within a system can relate to one another. On one side of the transition (similar to a solid or crystal state), elements can only communicate with nearest neighbors. Messages can be sent throughout the system but not through long-range pathways. On the other side of the phase transition (similar to a gas state), any element could potentially communicate with any other element in the system—each element acts independently, and there is no overall system structure. The system truly is different on either side of the transition, obeying different rules and having the ability to express different functions. The range of message passing can be measured as a *correlation length*. In a first-order phase transition, the correlation length jumps abruptly, and so the system can only have two modes to its internal structure. Furthermore, it is more difficult to find and remain at the single critical point of a first-order phase transition. In a second-order phase transition, however, the system gains the ability to send messages on any length scale and has the potential to exist in many structural configurations. The order parameter of a system is a way to control the length over which messages can be passed, and because it occurs over a range, it enables a degree of tuning. It is also within this range that a wider variety of rules and behaviors can coexist as well as a mix of local and long-range communication. This is similar to the balance found in small-world and scale-free networks described in Chap. 5.

7.2.7 Nonequilibrium Thermodynamics

It is important to note that the BZ reaction occurs in a closed system. The initial conditions can be thought of as an initial perturbation, from which the system will reduce free energy through a spiral in phase space. Eventually the oscillations will die out as free energy is exhausted. It is like watching a windup toy that will dance for some time but eventually run out of energy.

Consider, however, what would happen if the BZ reaction was an open system with a control mechanism that would periodically squirt bits of a reactant into the system. In this way it would be getting the necessary chemicals to remain far from equilibrium, moving outward in phase space rather than inward. What would be more amazing would be if that control system was contained within the reaction dish itself—sensing when concentrations were low and then activating actuators to get the necessary chemicals. Some have asked if this hypothetical system would be considered alive. The result would be a sustained oscillation *far from equilibrium*. We will explore this idea in more detail in Chap. 10.

The behavior of systems far from equilibrium was the life work of Ilya Prigogine (1917–2003), the 1977 Nobel Prize winner in chemistry. More specifically, Prigogine explored how open dissipative systems can sustain being away from their natural (closed-system) equilibrium points. The mechanism of staying far from equilibrium has already been explored above—a system accepts in high-quality matter, energy, or work from the outside and then dissipates waste as lower-quality matter or energy. All it takes is the right perturbations from the outside to avoid settling into an equilibrium state. A nice summary of Prigogine's work can be found in the accessible *Order Out of Chaos* and the more technical *From Being to Becoming*.

7.2.8 Instability and Patterns Far from Equilibrium

Prigogine was most interested in the unique behaviors that could only arise when a closed system was opened up to its environment. In Chap. 4, the phenomena of Benard cells were discussed. Briefly a liquid is heated from the bottom, and at some point, an instability appears that creates hexagonal patterns. Benard cells are in fact the canonical example of a system that, when pushed far from equilibrium, establishes an entirely new behavior that cannot exist close to equilibrium. It is thought that these kinds of systems must be dissipative and must accept some source of energy that pushes them far from equilibrium. In the case of Benard cells, energy is dissipated at the top of the plate, while energy enters through the heat source at the bottom.

Patterns can form far from equilibrium because of an instability in how energy moves throughout the system before and after a phase transition. In Benard cells, this occurs when the flow of heat through conduction is not enough to dissipate heat from the top of the plate. The system switches to convection, giving rise to the large-scale motion of fluid of the hexagonal cells. The balance between energy streaming in and out serves as an order parameter, pushing the entire system through a phase transition. When the rate of energy flowing in is low, the system remains close to equilibrium, and the system uses only local pathways for energy flow. As the rate of energy is increased, the system is moved far from equilibrium, and long-range pathways for energy flow are opened up. From a functional perspective, the reordering of the system means that different functions (flows) occur when a system is close to or far from equilibrium.

7.2.9 Doing Work on the Environment

Metabolism was discussed above as a system that accepts high-quality energy from the environment and expels lower-quality energy back into the environment. Systems, however, might also use that high-quality energy to do work on the environment. This is most simply seen when a motor turns a gear that drives a device that is outside of the system. Such was the case of the motor in the roller coaster example. Likewise, the production of “waste” from one system, when released into the environment, might become the high-quality energy for another system, leading to all manner of energy ecosystems. The work done on the environment, however, can become much more permanent. Such is the case of building houses, ant hills, bird and bee nests, beaver dams, and many other modifications to the environment. While an environment might endow a system with high-quality energy, that same environment might in return have work performed on it.

In some systems this phenomenon has a degree of intention, allowing a system to alter its environment to be more favorable to itself. This is the purpose of homes, irrigation, agriculture, cities, tools, and many other interactions and modifications of the environment. A system (e.g., a human or group of humans) performs work on the environment to make it more favorable. The meaning of "favorable" is to optimize in some way how high-quality energy goes into the system and how low-quality energy is expelled, being careful not to mix the two together in the environment. We will explore this idea when we discuss autopoiesis in Chap. 9.

7.2.10 Functionally Adaptive Machines

Machines, whether they are mechanical, chemical, electrical, thermal, or some combination, often express a wide range of functions. Networks and patterns establish semipermeable boundaries that enable gradients to be built up, flows to occur, and work to be done. If these machines are open to the environment and express different functions close to and far from equilibrium, they might also be able to slide back and forth across a second-order phase transition, thereby gaining the ability to express an even wider range of functions. What is more, a complex system might be viewed as a collection of nested subsystems that all talk to one another, pushing and pulling work out of gradients in a sort of internal multistage metabolism. Such systems will be able to flexibly express (or repress) latent behaviors in response to external perturbations. They are *functionally adaptive* and can respond to external (and perhaps internal) perturbations in sophisticated and sometimes surprising ways.

7.3 Structural Adaptation

Functionally adaptive systems can flexibly express functions that already exist within the system and perform work on their environment. In this section we will explore how a system can use available energy to do work on its own internal structures. Structures, whether they are physical or relational, constrain and bias how flows can occur and therefore determine what functions are possible. A system that can change its own structure is therefore one that can change its own functions. Such a system has a feedback loop between structure and function. They can grow and develop, heal by repairing damage, reorder internal patterns to become more efficient, and learn by expanding the range of functions that are possible. *Structurally adaptive systems* go well beyond the self-correcting mechanisms of feedback loops of traditional "machines" to more closely mimic systems that might be considered alive.

Many systems we have already encountered are structurally adaptive. In the brain an example of these bidirectional relationships between structure and function is found in the changing of synaptic weights—there are molecular functions by which the brain can change its own structure, thereby changing its function. In a culture, laws may be enacted, which change behaviors that then might give rise to new laws.

In an ecosystem, defense mechanisms might evolve. In these examples, it should be noted that the structural change might cross from one level of a hierarchy to another. For example, in the brain, molecular level changes at a synapse will alter electrical propagation in the larger neural network.

As a system grows, the number of possible flows grows much more quickly than the number of parts. This phenomena was encountered in Chap. 5 when adding nodes to a network resulted in an explosion in the number of ways to connect those nodes together. The more semipermeable barriers there are, the more potential for gradients, the kinds of work that can be done, and therefore the range of functions that might be expressed. Over time a system can accumulate these semipermeable boundaries. The reverse is also true—systems may remove semipermeable boundaries to simplify the system in some way.

7.3.1 What Complex Systems Build

In Chap. 4, a limit cycle, and its function as a clock or timer, was proposed to be one of a few basic machines that most complex systems seem to contain. In considering systems that build new internal functions, we revisit this idea by considering four more basic functions: energy storage, funnels, sensors/gatekeepers, and actuators. These basic machines are discussed further in Chap. 9.

External energy can push a system away from equilibrium and that may in fact be where a system must remain to be fully functional. But, that also means the system is then dependent on a constant flow of energy. Realistically the environment rarely will present a system with such a constant flow. One basic element a complex system may build is the equivalent of an internal *battery*. Consider your electronic devices. Some are tethered to the wall—they must be plugged in to function. Others are mobile and have their own internal power supply. When rechargeable, they simply have a way to "feed" every once in a while, after which they can continue to operate for some period of time away from an external power source. Generically, a battery is a large internal gradient. The larger the gradient, the longer the system can function without an input of high-quality energy. Some systems may even use a battery all the time as a reliable and constant supply of energy.

Energy typically will not flow into a storage device by accident. Likewise, energy will not be distributed throughout a system. In both cases energy flow must be managed inside the system. In collecting energy from a spatially and temporally heterogeneous environment, there is a degree of *funneling* of energy that may take place to direct it to the energy storage. Funneling is inherently a convergent process and is a way of moving high-quality energy that may come from many sources into a single gradient. Energy from that storage, however, must be distributed to the functions within the system that need it to perform work. This requires some delivery of energy, sometimes called fanout. Delivery is inherently a divergent process.

Using the idea of networks, we can imagine a tree-like structure that collects energy from many sources and consolidates it. Another tree-like structure, with the energy source at the trunk, may distribute energy through a variety of branches and subbranches. The dual processes of funneling and distribution allow all types of flows (e.g., energy, resources, messages, waste) that can keep a system functioning. For example, the circulatory system of many animals is composed of a venous system (funneling of blood and return back to the heart) and arterial system (distribution of blood from the heart).

Open systems can communicate with the environment through the bidirectional flow of energy. Systems can also send information out to the world or accept information into the system. When accepting information from the environment, a system will use a *sensor*, sometimes also called a detector or sniffer. The function is to measure some state in the environment. Of course there are the familiar human senses provided by our ears, eyes, nose, tongue, and skin. But bacteria and ants can detect chemical gradients, and economies can sense bull or bear trends. No sensor is perfect. They are limited by the structures that make up the sensing function. For example, our eyes can only detect colors within certain ranges.

Sensors can in fact function anywhere inside a system as well. The monitoring of internal variables is the critical information that is needed to complete the various feedback loops that support homeostasis. Most biological organisms have a range of internal sensors that detect temperature, pH, concentrations, and other variables that are entirely contained within their boundaries. Likewise, political decisions are often made by considering the measurements made by economists. Business executives decide if an employee will stay or leave a company based upon performance metrics. Someone in the company, often with the title of analyst, fulfills the function of collecting and synthesizing these metrics.

The movement of energy (or mass) across a boundary, whether with an outside environment or compartments inside of a system, may not happen by simple diffusion. It may in fact be regulated by *gatekeepers* that can bias or switch flow pathways. Another way to think about this is that the permeability of a boundary may be "smart." A *filter* is a kind of nuanced gatekeeper that enables information or energy to pass but with changes. They exist in between two extremes: either allowing all information to pass or entirely blocking all information. Some filters remove noise. Others can amplify small- or short-lived signals or reduce strong and persistent signals. In some cases, information will be preserved. In other cases, information will practically be lost.

Gatekeepers and filters become very powerful when coupled to sensors. In fact, the combination of a sensor and gatekeeper is exactly the makeup of Maxwell's demon. The only difference is the acknowledgment that sensing and gatekeeping are functions that require free energy from the system. The combination of sensing, gatekeeping, and funneling allows a complex system to extract useful energy, mass, and information from the environment, even if that environment is entirely random. Such a system can passively wait for high-quality energy to come along and then selectively accept it into the system. Likewise, the system can selectively channel lower-quality energy out of the system. Such a system works like a mechanical ratchet, moving high-quality energy in one direction (inward) and low-quality energy in the other direction (outward).

Systems can be passive and opportunistic, capitalizing on good fortune, or they can actively gather energy or information. There are many ways a system could become more active, but the most obvious is a system that is mobile and can go where the energy or information is. Doing so, especially if energy and information are ephemeral, is an enormous advantage. The structure that can be built to provide this more active approach is an *actuator*. Plants have a relatively predictable source of energy from the sun and so can afford to stay in one place. Animals, on the other hand, typically need to move to the resource (often a plant) to gain high-quality energy. But on closer examination, most plants have actuators too. The example of a sunflower provides a perfect example. Although rooting in one place, a sunflower can change its orientation to face the sun. Likewise, the weapons used by countries to invade other countries can be thought of as actuators.

Actuators, like sensors, can also face inward. In fact gatekeepers are a kind of internal actuator. In humans, our fingers are outward-facing actuators, but we also possess many internal actuators in the form of valves, muscles, and ion channels. In fact, internal actuators are the means by which a system can change its own internal structure, completing the feedback loop between structure and function.

Internal actuators can lead to many important generative functions that enable a system to become structurally adaptive. An example is the ability for a system to copy the structure and function of an existing function. First, copies can ensure that even if one particular structure is damaged, there is a degree of robustness within the system. Second, multiple copies of the same function might enable emergent behaviors, similar to the Boids model of flocking, that can only occur through interactions. Third, copies often increase the bandwidth of information or resource flow within a system such that non-linear economies of scale may be gained. Fourth, having multiple copies of the same function can enable the original version to continue its current function, so the copies can be used for recombination. You may recall that recombination is often the means by which new and more complex functions can arise. Lastly, a copy can be mutated or changed in some way and then functionally compared to the original. Such a situation could establish competition between the variants (discussed further in Chap. 8), leading to internal selective pressures.

James Miller (1916–2002) in *Living Systems* presented a long and detailed proposal for the 20 functions that must be present in any living system. He split these functions into those that processed matter and energy and those that processed information. The matter/energy processors are ingestor, distributor, converter, producer, storage, extruder, motor, and supporter. The information processors are input transducer, internal transducer, channel and net, timer (added later), decoder, associator, memory, decider, encoder, and output transducer. Although these basic functions do not entirely overlap with the basic functions proposed above, they may serve as an additional way to explore functions that are necessary to support the development of complex systems.

7.3.2 Balancing an Energy Budget

An important theme has been that complex systems are more than the sum of their parts. Individual functions, driven by their own gradients, can communicate with one another through energy flows and the work that they can perform on one another. Higher-level functions may emerge that could never be achieved simply by making a single gradient larger. The "waste" of one function might become the fuel for another function. For example, expelling waste in a particular direction could be used for locomotion, as a toxin, or to attract a mate. More functions provide more possibilities for recombination and the discovery of new functions. It would seem that the more functions, the better. There is a counterbalancing force—all real systems operate in a resource-limited environment and therefore cannot multiply functions *ad infinitum*.

Consider the two simple cases presented in Fig. 7.8. On the left side, there is the potential for a large gradient that can be used to do a great deal of work. Having only one flow will result in a very simple function. On the right side of Fig. 7.8, there are more semipermeable boundaries and therefore the potential for more gradients and a more complex function. Anytime a gradient is used to do work, however, there will be a decrease in free energy. All things being equal, the more flow pathways and boundary crossings, the more the system will lose free energy. Likewise, energy crossing the system boundary to refresh a gradient will also result in free energy losses.

These simple examples demonstrate that a sort of energy budget that must be balanced in growing and changing systems. Some free energy will be used to continue current functions, recharging gradients as they are used to do work. Creating a new function will require semipermeable boundaries to be built by the actuators within the system, requiring free energy above and beyond that required for the regular functioning of the system. Likewise, new boundaries mean new gradients that need to be maintained. It will also mean more gradients that will expend free energy more readily. A new function will itself become part of the baseline energy budget of the system. There is an assumption made that enough future energy will be available for maintenance. The problem is that a system may go through a lean time and begin to lose functionality. This is likely what happens when a system begins to die—it simply cannot maintain the gradients it needs to function. The more gradients that need to be maintained, the more dependent a system will become on a regular source of energy. A new function, however, might pay off over a longer period of time relative to what the maintenance costs would have been. The system is more efficient with the new function, and the free energy saved in the budget can then be used for other tasks. For example, it will take energy to build a new road, but then that road will need to be maintained. Over its lifetime, maintenance of the road may in fact equal or exceed the building costs. The creation of that road, however, may have eliminated the need for other roads in which case it was a good energy investment.

The idea of an energy budget can in fact be made for any resource of the system and in fact can become very complicated when multiple types of budgets are being balanced. Building a new road requires consideration of money, time, materials, traffic flow during and after construction and political ramification, all of which must be balanced against one another. In biological systems homeostasis is a sort of budget balancing. It is one of the reasons why a system often will fight back against change, an idea first encountered in Chap. 5 in the form of the Innovator's Dilemma. The way in which a system develops through structural adaptations will lead to path dependencies. These dependencies exist because collections of functions have become dependent upon one another as the system optimized how various budgets would be balanced. Changing one part of the system might be advantageous to one budget but not to another. Such interconnectedness of a complex system is one of the reasons they often fight back against change to maintain a status quo.

7.3.3 Learning

Systems that can direct their own structural adaptation can learn. For a system to learn something is by definition about how that system will act differently when presented with the same (or similar) stimulus before and after the learning has taken place. For this functional change to be reproducible, something physical must change. An adaptive learning system may change itself on long or short time scales that may or may not result in easy-to-observe changes. For this reason, Ken Bain defines learning in humans as occurring when we "think, act or feel differently." Learning, when defined as a structural change that gates information differently, is broader than the simple recall and pattern matching that is stressed in many formal classrooms.

It is important to distinguish between systems that structurally change due to some external forces and those that direct these changes from within. We would not say that a car that gets some new tricked out motor has "learned" even though its capabilities may have expanded. The reason is that it was not the car, but rather some human mechanic, that made the change. Likewise we would not call a computer a learning machine unless it could somehow change its own hardware or software on its own.

It would be a very rare instance, perhaps impossible, for a system to use all of its available energy to learn (change structurally). This would be a city in which all people, materials, and energy are continuously building. Rather, most systems balance the use of available energy between maintenance and structural adaptations (e.g., growth, healing). A city will "learn" through structural adaptations, but most of the energy likely will go toward maintaining existing functions.

7.3.4 The Discovery and Encoding of New Functions

How does a system "know" which constraints to put in place such that new functions will arise? There are varying answers to this question, ranging from blind trial and error to entirely directed. A middle ground which will be explored more in Chap. 10 is that a system might bias the formation of semipermeable barriers such that they often lead to useful functions. Perhaps there is a grammar for building constraints.

Structural adaptations likely do not occur all at once. Many of them begin as functional adaptations that serve as an initial prototype in which utility can be estimated. This functional adaptation becomes the template that will then be encoded into the structure of the system. Such is the case of an employee who discovers a process that helps them in their job, which only later becomes an institutionalized practice. The transition from functional to structural adaptation is to take a chance pathway for information and make it more likely that the system will follow that path in the future. This may in fact be a primary mechanism of learning—trying out solutions until one works and then encoding it more permanently into the system. In this regard, the structure of a system can be viewed as a past history of problems that have been solved.

Two related growth heuristics may form part of the grammar of change. First, a form of Hebbian learning (discussed in Chap. 6 in a neural context as "fire together, wire together") may apply to other systems. Practice burns in pathways that are easier to follow in the future, known in the learning community as *grooving*. Usually this would involve some balance between strengthening (excitation or promotion) and weakening (inhibition or repression) rules. Due to energy budgets, the creation of a new function or pathway might necessarily require the destruction of another function or pathway. Such a heuristic could apply to the addition and deletion of nodes and connections, as discussed in Chap. 5. Within a system such dynamics might even establish a sort of internal survival of the fittest.

A second possible change heuristic would be based upon the information content of signals. Internal signals, based upon flows across semipermeable boundaries, should have high information content yet still have enough redundancy to be received by other parts of the system. This must all occur in the context of simultaneous flows all cross-contaminating one another. There seems to be a sort of sweet spot where a system can support multiple gradients that can do work, as well as send and receive signals, yet not become a noisy system. In future chapters, we will explore how systems might find this sweet spot.

7.3.5 The Adjacent Possible

The cyberneticists explored feedback loops but limited their study them primarily to functionally adaptive systems. This way of thinking is often called *first-order*

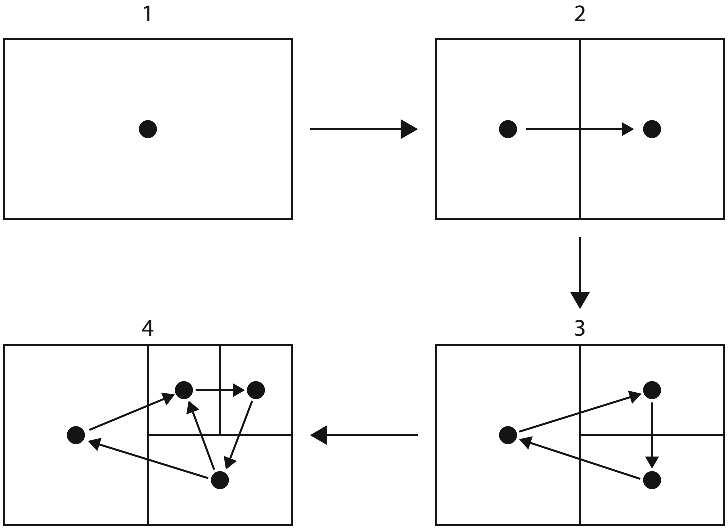


Fig. 7.12 A system evolving as it adds semipermeable structural barriers that realize adjacent possibilities

cybernetics. Structural adaptation enables a system to change itself. Such systems are *nonstationary* in that their abilities are constantly in flux. This way of thinking is often called *second-order cybernetics*.

Consider Fig. 7.12 as showing the intermediate steps in the evolution of the system shown in Fig. 7.8. Each step adds one more semipermeable barrier and the ability to sustain more gradients. The key is the last step when one more barrier adds the possibility of two new signals. As this system evolves, future functions may emerge as more structures are added.

When structures, rules, and functions can all change, there are many unrealized future functions that could arise should two (or more) existing functions happen to interact. A nice term that is used to sum up this idea is the *adjacent possible*, coined by Stuart Kauffman. When an adjacent possibility has been realized, it becomes a structural adaptation. In the creation of that new function, however, a new set of adjacent possibilities arises that did not exist before. The adjacent possible is an ever-expanding edge of possibility and future development.

7.3.6 Life and Mind

Much of this chapter was focused on taking systems apart to understanding flows of energy. But, we can think about the reverse—how to intentionally design systems that are functionally and structurally open and adaptive. To an engineer, this is about how to create a nonstationary system that can modify itself in some directed way.

There are a handful of revolutionary thinkers who are proposing that computers, buildings, planes, and other technologies might be able to change their internal structures to adapt to an environment. In these cases the system is directing its own changes and might be considered alive. Ideas on what it means to be living will be discussed in more depth in Chap. 9.

The functions of sensing and observation that may have arisen to be outward facing might over time be turned inward. When a human applies the powers of observation to their own internal processes, the experience is one of self-reflection. As mentioned in Chap. 3, Minsky's *The Society of Mind* and Fodor's *Modularity of Mind* go one step farther to claim that a mind is a series of competing and cooperating subconsciousness functions all feeding back on one another. And yet another step is taken by Nicholas Humphrey (1943–) who in his book *Soul Dust* claims that consciousness is achieved when one can reflect on one's own cognitive processes. These ideas will be explored more in Chaps. 9 and 10.

7.4 Questions

- A major result of thermodynamics is that there seems to be a favored direction of time. Yet some have proposed that time is an illusion. In *A New Kind of Science*, Wolfram proposes that perhaps time is not actually a dimension. Rather it is the manifestation of entropy increasing that results as the world *unfolds* according to some Class IV rule. Agents within the system perceive entropy increasing and attribute this to the variable of time. What we experience as the dimension of time is actually an emergent phenomenon. This is similar to asking the question, "Do you age because time passes, or do you experience time passing because you are aging and collecting memories as you go?" The difficulty in such questions is that the arguments and equations generally rely on causal thinking, yet the question itself is about the nature of time. It is self-referential in a similar way as discussed in Chap. 6. Do you think it is possible that time is an emergent phenomenon? Are there any ways to prove your hypothesis?
- "Goldfish expand to fill the size of their bowl" is a phrase that represents what is known as the *island effect*, also sometimes called *Foster's rule*. The general idea is that an organism will grow larger when resources are plentiful. This effect has been observed in a wide range of species. It can be seen in some human-created systems as well. For example, as computing speed and memory have increased, computer programs have become larger. Find another human-created system that has undergone an island effect when resources have become more available. Be specific about the resource and along what dimension the system has grown. What would happen if the resource was reallocated or taken away?
- In childhood development, there are periods of time when new behaviors arise. Can you map a few of these shifts to phase transitions? Are there some that are first order (occur abruptly)? Are there others that occur more smoothly but over

some time range (second order)? In what ways do these transitions map to other ideas in this chapter?

- Structural adaptation is very often much slower than functional adaptation. It takes time to build new internal structures because it requires a greater expenditure of resources. It is why often systems will first respond with a functional adaptation. Only later will a structural change occur. What is an example of a functional adaptation that became a structural adaptation? What new structures or laws must be put in place to make the change more permanent?
- Charles Darwin (1809–1882) explored the idea that a particular function may change its meaning to an organism as other functions became available. This idea was greatly expanded upon by Stephen Jay Gould (1942–2002) in 1982 when he coined the term *exaptation*. A function may have evolved for one purpose which is only later co-opted by the organism for another reason. Birds are thought to have begun to evolve feathers as a sort of fur for the purposes of thermal insulation. Over time feathers were co-opted to help with flight. A similar idea was expressed by Steve Jobs in his 2005 address to Stanford, when he spoke of "connecting the dots backward." His meaning was that we take actions in our lives, and events happen around us that have one meaning when they happen. But on looking back at them later, they may connect to other events and take on a new meaning. What are some examples you see from your life where this has happened? In what ways has it opened up possibilities? Alternatively when have path dependencies appeared to constrain your future possibilities?
- In balancing an energy budget, most systems rarely, if ever, operate at their peak metabolic rate. That is because the peak metabolism burns energy at a far greater rate than the input of high-quality energy, and usually with less efficiency. The consequence is that a system can engage in short bursts of work, followed by a period of recharging the expended gradients. Choose a system, and explain an event or activity that metabolizes energy faster than the input of energy. What are the relative time scales of the event and the recharge rate? What happens if the system is required to engage in the event again before being fully recharged?
- What is an example of when you have put in energy to build something in your life that eventually paid off?
- Lev Vygotsky (1896–1934) was a Russian psychologist who developed a theory of teaching and learning known as the *zone of proximal development*. The general way of expressing this zone is with three circles embedded within one another. In the center of the circle is what a learner can do on their own with no help. In the outermost circle is what the learner cannot yet do. In the middle circle, known as the zone of proximal development, is what the learner can do with help or guidance. The theory is that being a teacher is to help someone grow their zone of proximal development outward in such a way that they can do even more without help from others. This is sometimes known as the "growing edge." As a learner, what is something that is at your growing edge? Who might you reach out to for guidance? As a teacher, are there people around you that you can help with their growing edge? How might you best help them?

- Creativity researchers often make the claim that novelty is nothing more than recombination. This is the origin of the phrase "there is nothing new under the sun." New things are simply old parts put together in new ways, and those parts might be physical, artistic, or ideological. New creations can set up the adjacent possible for another creations. The "ah ha" moment is really just a phase transition—parts have been slowly recombining in the background all the while. What is an example from your life when you feel as though work was put in the background that led to an "ah ha" moment? Be explicit about the preparation that led up to your creative insight.
- Resources (like $ and power) are broadly defined as free energy that can be consumed, fought over, traded, or stolen. They enable a kind of work (defined more broadly than in physics) to be done. Organized crime often arises to fill some unmet need (e.g., cleaning up a crime-ridden or depressed area) using less than legal means. It is only later that the power, resources, and networks are used for more selfish reasons. What are other human-made or natural systems that collect or store resources, over which others will fight for?
- A city can be thought of as a multi-box system, with a variety of networks and flows that keep it running. Some of these are physical (e.g., roads), while others are more conceptual (e.g., economies and laws). Explore how a city is an open system, with its own metabolism that may be far from equilibrium. What are the various boxes and flows between these boxes?
- What are some path dependencies in your own upbringing? How do they influence you today?
- It is a strategy of many new business leaders to change something about the way information flows throughout their company. This may be creating a new process, building a new department, or more radically restructuring an entire company. Most good leaders know that a system can be forever changed by external events. And so another tactic of a leader is to either change the external environment or give the appearance that the external environment has changed. What examples have you seen of leaders making some internal change in response to a real (or perceived) external change?
- Being mobile and going where the resources are can be a useful strategy. We can generalize this idea to include all sorts of environmental navigation that can extend to many types of systems. For example, one country will invade another country to gain its resources. Likewise, some geographic areas talk about brain drain (the loss of talented people). We can see this in countries where there may be a selective flow of people into or out of the country. What are other examples, historical or current, where a system will become mobile to secure a resource?
- An idea related to the adjacent possible is the concept of *affordances*. Originally expressed in 1977 by the psychologist James Gibson (1904–1979), an affordance was originally defined as all that an environment offers a species. It has since spread to other fields to include objects, thoughts, and perceptions. Affordances are separate from what is actually expressed—like an adjacent possible, it is the

set of possible futures. Learning to perceive affordances is to Gibson a critical developmental step that enables a human to see the world around them as filled with unrealized value. A next step is for a developing human to modify their environment so as to set up even more affordances in the future. Spend 15–20 minutes listing some affordances that you have around you. Which of these are available to everyone around you, and which have you created for yourself? What have these affordances enabled you to do so far? What might you be able to do in the future?

- The phrase "necessity is the mother of invention" implies that a system may encounter a problem that it cannot solve with the existing structures within itself. So it modifies something internally to enable a solution to be generated. In the process of making this modification, however, the system may become imbalanced in some way, creating a new problem. Explain an example where a "fix" to a system has resulted in a new problem that was in fact worse than the original problem.
- The development of most systems occurs within the context of the constraints that have been established at some early point in development. For example, when a new city is developing, it likely must conform to regional or national codes that were developed for other cities. The same can be said about the genome and the development of an organism. These are long-term historical trends that determine the rules by which something may develop. This is a way to preserve constraints that have worked in the past. Explore another example of a system that is in the process of developing. What constraints has it inherited from its past history? Which of these constraints do you think will productively guide future development? Which do you feel are impeding development? Why?
- We often think of learning as occurring in formal environments such as classrooms. By some estimates, however, the majority of learning occurs in informal settings and without the explicit intention of learning. Tell a story of how you learned something simply by being in the right place at the right time.
- Generally when physicists talk of energy, they are referring to the flow down a potential gradient. Likewise, money is a form of economic potential. But we can also think about social or political capital, trust, or favors as a sort of energy that can be stored and used at some later time. What are some examples of potential that you store up with the expectation that you will "cashed in" to achieve some later function or goal?
- Companies take great interest in reducing energy, time, and resources as much as possible. Often consultants are hired to identify where excess may be cut and then make recommendations on restructuring. Likewise, some methodologies such as Six Sigma and Lean Manufacturing are meant to optimize internal processes. What is an example of a system that you feel could optimize and is an important resource? What changes would you recommend? Will it pay to make changes to the system now in return for better optimization in the future?
- Actuators can serve many purposes in a complex system, both to mediate information flow within the system and to impact the outside world. An excellent example is the flagella used by bacteria to move within an environment. When

the flagella move in a counterclockwise motion, the bacteria will be propelled forward. When the flagella move clockwise, the bacteria will thrash about in what is called tumbling. By alternating between these two basic modes of travel, a bacterium can swim toward food sources and away from noxious chemicals. These two modes, directed forward motion toward a goal and random exploration, can often characterize phases in a person's life as well. Give examples of how you have alternated between goal-directed and exploratory behaviors.

- In Chap. 6, the ideas of intelligence and consciousness were discussed. In this chapter first- and second-order phase transitions were explored. In the course of evolution, do you think that intelligence has been first (emerging suddenly) or second (emerging over several species) order? What about consciousness? Do you think it emerges in a first- or second-order transition? Explain your rationale.
- In this chapter the relationship between structure and function was proposed as a way to back-trace the history of a system. This is well known in some fields. In archaeology discovered artifacts are used to construct a plausible past history of a culture. Paleontologists, geologists, and some anthropologists do the same with ancient animals, geologic forces, and cultures with no written records. Engineers often take apart a device to understand how it might work, but they can also engage in product archaeology—speculating on the decisions of past engineers. Likewise the field of cognitive archaeology aims to understand the evolution of mental abilities by studying the kinds of artifacts left behind by ancient people. Find a system that you know little about, and based upon your observations, speculate on that system's past history.
- Some believe that life is a very complex set of autocatalytic reactions. Some go even further to suggest that life got its start as some very simple organic autocatalytic reactions that could sustain themselves against perturbations from the outside. We will explore this idea more in chapters to come. What do you think are some minimum conditions for an organic system to maintain itself? How can such an early organic system be simultaneously open to energy but also have selectively permeable barriers to the outside world?
- The idea that patterns could self-organize in systems far from equilibrium was explored as early as the 1960s by Hermann Haken (1927–). Haken was the founder of *synergetics*, an interdisciplinary study that mirrors many of the topics that will be discussed in Chap. 10. Essentially order parameters in self-organizing systems are a result of a massive reduction in the degrees of freedom that a system could potentially have. Because an order parameter controls the relationship between elements in a system, it can be used as a sort of master system controller, a principle called *slaving* by Haken. Synergetics was first applied to the development of lasers, for which Haken was a pioneer, but quickly expanded out to pattern formation in physics, chemistry, and biology. For more exploration, Haken's books *Synergetics: An Introduction* and *Advanced Synergetics* will provide much more insight. Choose a human-created system (e.g., economics, law, politics, power grids), and explore the possible intersections between

synergetic and this system. What do you speculate is the order parameter? What happens as that parameter is varied? What is in control of the parameter?

- Open systems accept inputs from the outside, while structurally adaptive systems have the potential to learn. When these two properties are put together, the possibility of learning due to outside influence becomes possible. In humans, and perhaps some other systems such as animals, organizations, and countries, it becomes possible to learn by watching another similar system. Describe two lessons that you believe were learned in this way: one that you learned simply by watching someone else and a second of a group that you are a part of that learned by watching another group.
- Some interesting examples of functionally adaptive machines are self-driving cars and buildings that can "breathe" as they adapt to changing external conditions. What are some other examples of machines that are functionally adaptive? What might be added to them so that they are structurally adaptive as well?
- A battery, essentially a gradient in space, is a way to store energy. A similar idea in the realm of information is the ability to store a memory. Chapters 3 and 5 discussed how memories could be stored in functional limit cycles or reentrant cycles embedded within a network structure. Chapter 6 characterized memory as an autocatalytic loop. The purpose of storing a memory in the first place, however, is so that a previous system state can be compared to a current (or perhaps future) system state. It is this gradient of information between past and present that makes a memory powerful. What are the ways in which you believe your brain stores memories? What does it mean to forget or misremember something?
- Many systems undergo transitions from one state to another. If the transition happens slowly enough, the system might not even be able to detect the change. The classic parable used to describe this phenomenon is the frog in the boiling water. As the story goes, if you drop a frog into a pot of boiling water, it will hop out as fast as it can. But if you put a frog into a cool pot and slowly turn up the temperature, it will stay in the water even when it boils, getting cooked in the process. What system change (good or bad) might be happening that is hard to detect because the change is happening on a long time scale? What do you think are the consequences?

7.5 Resources and Further Reading

Thermodynamics is a topic that all too often is taught only from the perspective of heat flow and using mathematical arguments. A quick tour of a technical library or an online site will reveal many excellent introductions. For some less technical reads, it may be helpful to read *Into the Cool: Energy Flow, Thermodynamics, and Life* by Eric Schneider (1938–) and Dorion Sagan (1959–) or *The Emergence of Everything* by Harold Morowitz (1927–2016). *Information, Entropy, Life and the Universe* by Arieh Ben-Naim (1934–) and *Decoding the Universe* by Charles Seife are nice introductions to the connections between thermodynamics and information theory. Various books by Nick Lane (1967–) discuss the origins of life from a thermodynamic perspective and are very readable, most especially *The Vital Question*. Peter Hoffmann's book *Life's Ratchet: How Molecular Machines Extract Order from Chaos* is a great introduction to open systems that metabolize. For more on the irreversibility of time, a nice place to start is *The Arrow of Time* by Peter Coveney and Roger Highfield (1958–) or Prigogine's *The End of Certainty*.

Chapter 8
Game Theory

Imagine you enter a maze made of many nondescript rooms. It is very dark and your goal is to find the exit. How would you proceed? At first you may stumble around, feeling walls, following contours, trying to make a map of the space in your mind. “Have I been here before? How might I ensure that I don’t keep visiting the same room over and over again?”

At some point you hear a door slam off to your right, far in the distance. With this information you guess that it might be an exit. After getting hopelessly lost, you hear the door slam again. This time it is in front of you. How many exits are there? Maybe there are several. Or it could be that you lost track of where the first door was. You continue on.

After wandering for a while, you think you hear footsteps in the next room. You call out and a woman’s voice answers. You meet between rooms and learn that she is trying to find a way out too. After a short conversation, you agree that the best course of action is to split up but to remain in constant voice contact.

Together you explore many rooms and realize that the maze is huge. Sometimes you hear other voices in the distance. Eventually a small group forms and you learn some facts. There are many others in the maze and it is a sort of a contest to see who will get out first. The rewards (or penalties) for being the first out are unknown. There seem to be some people who are giving false (or at least conflicting) information. There might be many exits, but over time they seem to be shifting around, with only one exit being open at any one time. Should you cooperate with others that you meet? Is it better to cooperate with every group you may meet or to distrust everyone at first? Maybe you could try to remember which groups misled you and which seemed to help.

The game just described is similar to the science fiction movie *The Cube*. Although a dramatization, it illustrates how almost everything we do requires making decisions when we do not have as much information as we would like. These situations, and the strategies one might adopt to navigate them, were first formally studied in the 1940s by John von Neumann and Oskar Morgenstern. In *Theory of*

J. Tranquillo, *An Introduction to Complex Systems*,
https://doi.org/10.1007/978-3-030-02589-2_8

Games and Economic Behavior they explored the strategies one might use when many agents are competing or cooperating for the same limited resource. Although initially applied to human decision-making, we will explore in this chapter how game theory can be applied broadly to biological evolution, ecosystems, immune systems, economies, technological advancements, and geopolitical systems.

8.1 Zero-Sum Games and Game Trees

Many real-world decisions involve a finite resource that every actor wants or needs. Actors adopt a strategy that leads them to take actions that will help them gain the resource (e.g., money, attention, energy, customers). Game theory is a framework for comparing strategies to determine which will pay off in the end. In this section we will consider resources that are conserved, meaning that for an actor to win, some other actor must lose. Any quantity that enters the game will stay in the game. These games are like a basement game of poker—the group will collectively leave the basement with the same amount of money, only redistributed. For this reason, these games are known as *zero-sum*.

8.1.1 Game Trees

The primary way to explore zero-sum games is to map out all possible combinations of legal moves allowed by the rules in a *game tree*. The tree reflects that the possible moves at any point in the game have been limited by previous moves, a concept referred to as *path dependencies* in previous chapters. A simple example is the game of tic-tac-toe. The tree starts with the first move, perhaps player 1 placing an "X" in the center. Player 2 can now only play into the remaining eight blank spaces. This narrowing continues until the game is over. The game tree graphically shows all possible plays for both players. A portion of such a game tree is shown in Fig. 8.1. Although the game trees for checkers and chess are enormous, they are in principle finite. The move of each player will push the game down branches and sub-branches until a terminal point is reached and the game is over, at which point it can be determined who won and lost or if it was a draw.

8.1.2 Rational Players

Most game theory analyses assume that both players are "rational." Practically this means that given the same information, players will apply a strategy that will probabilistically be best for them. In the tic-tac-toe example, when faced with six possible places to place an "X," a player will navigate down the tree in a way that is expected to benefit them the most.

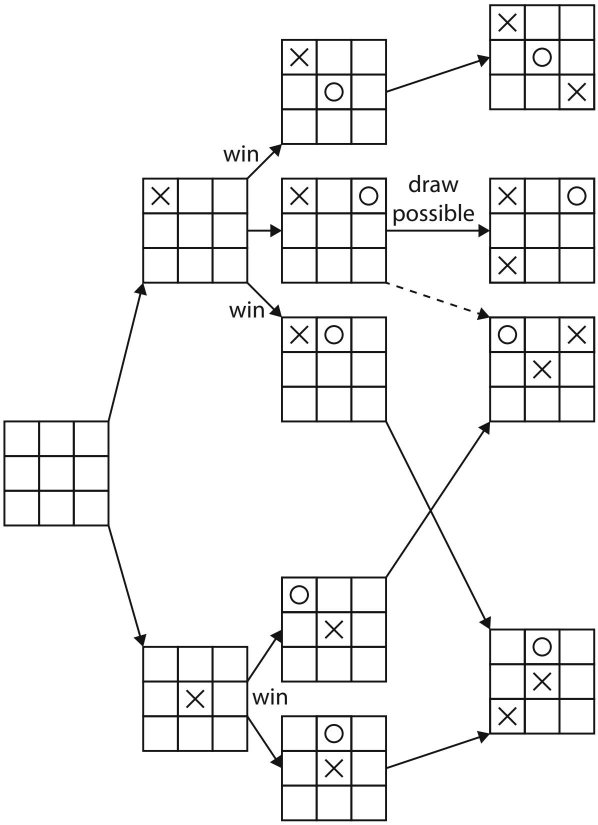


Fig. 8.1 A partial game tree for tic-tac-toe

There are several reasons why someone might not act rationally. First, they may not have the ability to think beyond some number of moves in the future. You may have experiences playing tic-tac-toe against someone (perhaps a young child) who cannot see past the most immediate move. Second, an adult might purposely lose to a child to help teach them or give them confidence. In this case the adult and child are not really playing the same game. Likewise, the child may respond in kind and let their parent win when they realize what the parent is doing. Again, this "game" is now no longer about winning at tic-tac-toe but rather a social/emotional bonding moment between a parent and child. They are playing a very different game than tic-tac-toe. Third, sometimes multiple games are being played at once. For example, politicians are often playing several games, often called brinksmanship, and are willing to lose some to win at others. Fourth, a strategy might be to fain weakness as pool sharks will do, who fake their real talent for a later payoff. Such a strategy is based upon an asymmetry of information and the time scale of the payoff. For the purposes of much of this chapter, we will assume that the players are only playing one game at a time and that they are rational. In the real world, however, players are most often balancing several games at once and do not always behave rationally.

The first reason a child might lose at tic-tac-toe is that they do not have the mental capacity to look ahead in the game. They wait until the other player has made their move and then decide what to do next. In some games, this can work. In others, like chess, a player who is only reactive will quickly fall into traps laid by their opponent. Good chess players will proactively think several moves ahead.

Most strategies attempt to balance being reactive and proactive. Being reactive does not require much memory or rationality—a simple feedback loop may work. Being proactive, on the other hand, requires looking ahead and hypothesizing what might happen farther down the game tree. This is the classic, "if I move here, they will move there, and then I will move there." The catch is that the options in many game trees explode as one tries to look deeper down the tree. Looking to future possibilities therefore requires more and more computation and memory. Any enacted strategy must be within the cognitive abilities of the players. At some point, the player will become overwhelmed and not be able to compute the entire game tree. This is especially true for games like chess where the game tree is enormous. In these cases it likely does not pay to proactively compute too far ahead.

To understand how computing a game tree can give a player an advantage, consider the following two players. Player 1 can compute ahead three steps in the game tree. Player 2 can compute ahead four steps. The way for Player 2 to win is to find areas of the game tree where three moves ahead look like a good opportunity, but four moves away results in losses. Player 2 can then bait Player 1 to go down a pathway on the game tree that they know (four moves away) will be a win for them. Both players have the uncertainty of exactly where that part of the game tree eventually leads, but Player 2 has more information because throughout the game they can see farther ahead.

When the computer Deep Blue beat the reigning chess champion Gary Kasparov (1963–) in 1997, it made front page news. Some criticized Kasparov for playing the wrong way. But beating a human in chess seemed to be an inevitable step for a computer. Advances were allowing computers to compute more and more of the game tree in less and less time. It was only a matter of time before a human competitor would be overwhelmed by the computer's ability to see more of the tree. The real criticism was that Deep Blue did not play chess with "intelligence"—in other words, it did not use human strategies to play.

8.1.3 Minimax

In the absence of complete information, when one cannot see or compute the entire tree, having a strategy becomes critical. It was proven that in any zero-sum game, the ideal strategy is known as *minimax*. In this strategy you look ahead as far as you can and then attempt to go down pathways that probabilistically will decrease your losses. More precisely you minimize your maximum losses. This works because any time you minimize losses in a zero-sum game, you are preventing your adversary from winning too much. Minimax was originally created for two player games but has since been extended to multiplayer games.

8.1.4 Emergent and Infinite Game Trees

Some games have rigid rules such that there are only so many possible moves, and the game tree is finite. These games, once started, have a clear ending. Not all game trees, however, are predetermined ahead of time. In many real-world situations, the rules may be evolving as the game is being played. In other games the rules allow for an infinite number of possible moves. In both cases the game tree emerges over time as moves open up new possibilities but also destroy other possibilities. This is another instance of Stuart Kauffman's *adjacent possible*, discussed in Chap. 7.

Consider the release of a new technology such as the Apple iPhone. The presence of that iPhone opens up new possibilities that did not exist beforehand (e.g., apps, holders and cases, add-ons like Square). Existing companies such as Skype, Facebook, Twitter, Instagram, and Pandora all went mobile. The iPhone, and other flavors of smart phones, opened up parts of the technological game tree but in the process eroded technology associated with landline phones, such as answering machines. Joseph Schumpeter (1883–1950), building upon earlier work by Karl Marx, called this phenomenon *creative destruction*. Every new innovation necessarily kills off some older innovation. Digital cameras all but killed the film industry, and buggy manufactures went out of business when cars became affordable.

An emerging game tree can also be applied to the legal system. What is within the bounds of the law today may not have been 100 years ago. And what will be possible 100 years from now may limit (or encourage) new possibilities. One law will often simultaneously create new opportunities and destroy others—a sort of adjacent destruction that runs in parallel to Kauffman's adjacent possible. The legal and technological game trees are also intertwined. For example, the laws that are coming to allow for self-driving cars may open up a whole range of possibilities for new products, laws, and social behaviors. But those same laws might at some time in the future outlaw human-driven cars. In this sense, the game tree cannot be known because the rules by which it is evolving are changing as players make their moves.

The card game Mau was created to demonstrate the social implications of an emerging game tree. In the traditional playing of the game, a dealer knows a set number of rules that are in place at the beginning of the game. No one else knows the rules. Through a series of penalties and rewards, other players begin to discover the rules. The catch is that as the game is played (much like Uno), players have moments when they can create new rules. These new rules they can keep secret, but often they have consequences on other players who will receive rewards or penalties.

The game can continue on, virtually indefinitely, as the rules become more and more complex, perhaps with no one knowing the entire full set of rules. Rules might even begin to interact or contradict each other, leading to new rules. Some call these situations *infinite games* because there is no bottom of the game tree.

The types of strategies that are used in finite and infinite games are often different. Imagine playing a finite game like chess where the goal is to get to the bottom of the game tree as the winner as fast as possible. You can expect that your competitors are doing the same. This is especially important if there is no prize for second place. In an infinite game, there is never a clear winner or loser because there is no bottom of the game tree. Players may lose interest, give up, run out of some resource (like money or time), turn to a different game, or die.

8.2 Non-Zero-Sum Games

In a zero-sum game (with rational actors and a finite resource), everyone defects because your win is necessarily another's loss. Non-zero-sum games lift the restriction that the resource must be conserved. In addition to win-loss, there may also be win-win and lose-lose situations. Furthermore there can be gradations of these outcomes, for example, both win but one player wins more than the other. Likewise, in a lose-lose situation, one player might lose less. The range of possible strategies grows as now it may make rational sense use some combination of cooperation and defection.

8.2.1 Prisoner's Dilemma and Nash Equilibrium

Almost all fields have a particular system that is simple enough to study yet captures the most interesting phenomenon of that field. For example, biologists will study *E. coli*, fruit flies, and mice as model systems. To game theorists, the *prisoner's dilemma* serves as the model system. It is simple enough to be studied in detail, but also displays a range of strategies that balance cooperation and defection.

The most common way to explore a non-zero-sum game is with a *payoff matrix* that describes what players win or lose. To create a payoff matrix, we list the moves of Player A on one axis and Player B on the other axis. In the simplest case, Player A and B can either Cooperate (C) or Defect (D), yielding a 2×2 matrix, as in Fig. 8.2.

The Prisoner's Dilemma was created by Merrill Flood (1908–1991) and Melvin Dresher (1911–1992) and begins with two suspects being brought in for separate questioning about a crime that has been committed. Let us assume, from our third-person perspective, that both suspects are innocent. They are explained the rules and then asked to make a choice, not knowing what the other suspect will do. From a first-person perspective, if you confess to the crime (cooperate) and the other person points the finger at you (defect), you get 5 years in prison and they go free. If you

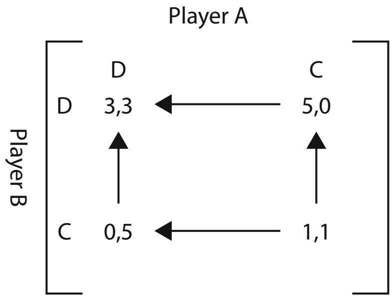


Fig. 8.2 The Nash equilibrium for a prisoner's dilemma. Arrows indicate how the two players would minimize their jail time but become trapped in a less than ideal equilibrium

both confess you each are sentenced to 1 year in prison. If you both point the finger at each other you both get 3 years. The payoff matrix in Fig. 8.2 summarizes the rules of the situation. Not knowing the other suspect, would you risk confessing?

There is a clear (rational) solution that minimizes the overall jail time (each confess and go to jail for 1 year). The problem is that you would each need to cooperate, and if you cooperate and the other person does not, you go to jail for 5 years. The safe thing to do is to plead guilty and guarantee that you can do no worse than 3 years. John Nash (1928–2015), the mathematician who won the Nobel prize in economics and was the subject of the big budget movie *A Beautiful Mind*, proved that these kinds of payoff matrices, shown in Fig. 8.2, have an unstable equilibrium for both cooperating and a stable equilibrium for both defecting. The arrows in the figure show how when making decisions a player might make their prospects better by changing their strategy. To make your prospects better than (1,1) you would move to (5,0). But remember that you cannot get to 5,0 on your own—this would require the other player to plead guilty. The same idea can be said to apply to your partner. And so you are both attracted to (3,3) because it is the most stable solution for the two of you combined. This attractor is called a *Nash equilibrium* and will be arrived at by following a minimax strategy whereby one minimizes the maximum jail time. Furthermore, Nash proved that such equilibria exist in all finite non-zero-sum games.

The idea of a Nash equilibrium was also found independently by the mathematical biologist John Maynard Smith (1920–2004) and termed an *evolutionary stable strategy*. In this case a species is playing other species for resources in the environment. The strategies used are the functions that can be expressed. The stable strategies are those that will enable a species to survive even when another species tries to gain a greater share of the resource. Survival of the fittest is the stable strategy that results, which essentially is to always defect. We will explore this idea further below.

8.2.2 The Ultimatum Game

The ultimatum game has become a second model system because it maps well to concepts of social justice and fairness. One player is promised a sum of money, say \$10. They are then instructed to give some percentage of that money to a second player. For example, the first player could split it \$5-\$5, or \$7-\$3, keep all of it, or give it all away. The catch is that the second player can decide whether or not to accept the offer. If the second player does not accept the offer then neither player gets any money. In this way, the second player has a degree of power in that they can refuse an unfair split and penalize the other player for being greedy.

The ultimatum game has been played with varying age groups, cultures and professions. The rules have been tweaked, and the rewards have varied from trivial to fairly large. There are even studies using twins. Likewise, it has been played (in a modified form) with primates. A nearly consistent finding is that when the second player receives a percentage of the total that is lower than a certain amount, they give up any small reward to themselves in favor of penalizing the other player. In many studies that percentage seems to be around 30% of the total. There are many economists and game theorists who use the ultimatum game to understand how humans make decisions in situations where fairness is involved.

8.2.3 Other Games

The wins and losses in the payoff matrix can vary. We can generalize the payoff matrix as

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{bmatrix} a & b \\ c & d \end{bmatrix} \end{array}$$

When both players corporate, they both get a. When they both defect, they both get d. But when they do not adopt the same strategy then the defector gets c and the cooperator gets b. Depending on the values of $a - d$ the dynamics of the game can change. Game theorists have a number of ways to classify different types of games, many of them with clever names such as Snow Drift, Staghunt and Chicken. For example, in the game of chicken the payoffs can be mixed (both positive and negative):

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{bmatrix} -c & b \\ 0 & b/2 \end{bmatrix} \end{array}$$

In this case cooperation means neither player backs down and they both lose. If they both defect, meaning they both back down, they only have a partial win (a sort of draw) whereby they split any possible gains. When the strategies don't match, one wins and the other gains nothing. In this case it pays to adopt the opposite strategy of your opponent.

In the prisoner's dilemma, these moves are limited to simply cooperate or defect. But we can imagine other games where there are multiple options (e.g., yes, no, abstain). A classic example is the game of Rock Paper Scissor where the dynamics are not simply cooperating or defecting. A payoff matrix looks like.

$$\left[\begin{array}{cccc} & R & P & S \\ R & 0 & -a_2 & b_3 \\ P & b_1 & 0 & -a_3 \\ S & -a_1 & b_2 & 0 \end{array}\right]$$

Notice that along the diagonals, where both players play the same, it is a draw. But if the strategies disagree, one player will win, and the other player will lose. In the usual way to play the game all of the positive values are 1, and the negative values are -1. More complex version of the game may have a non-symmetric matrix. For example rock losing to scissor might be different than scissor winning over paper.

8.2.4 Repeated Games and the Evolution of Cooperation

The Nash equilibrium paints an unhopeful picture of selfish actors all defecting to avoid worse case scenarios. Sustained cooperation would be unlikely to arise in any situation, natural or human-made. There are a few possible ways out. The most simple is if somehow the rules of the game are such that everyone could win. This would be the same as saying that the stable strategies lead to win-win. In some narrow domains, this might in fact be possible, but it is unlikely to account for the degree of cooperation that is observed in the real world. The second way for cooperation to arise is to escape from the Nash equilibrium through repeated play.

In the early 1980s, Robert Axelrod (1943–), a political scientist and game theorist, was studying what is known as the repeated prisoner's dilemma, whereby players play against one another several times. Axelrod's intuition, based on some early thinking by W.D. Hamilton (1936–2000), was that if Player A cooperated all the time and happened to play against Player B who also cooperated, they would help each other reach a win-win. Essentially Players A and B would develop trust. But if Player A then met Player C who defected all the time, Player A would begin to also defect. The question Axelrod asked was how to adopt a flexible strategy that could be used in repeated interactions with any player. This being the early era of personal computing, Axelrod proposed a computer tournament where others could enter a particular strategy—always cooperate, always defect, or make a random determination were already included in the tournament by Axelrod.

Only eight entries came in for the first tournament, some very simple and some very elaborate. Each entry was paired off with every other entry and several rounds of play (approximately 200). At the end of the tournament, the totals came in with which strategy paid off the best. It turned out that a very simple strategy won, submitted by another game theorist, Anatol Rapoport (1911–2007). His strategy was called "tit-for-tat"—cooperate on the first turn and then simply mirror the other player's move in the future. If on the first turn you cooperated but the other player defected, you would defect on the next turn. In this regard, tit-for-tat is an adaptive strategy. If you are playing against a cooperator, then the best strategy is to cooperate. But if you are playing against a defector, the strategy prevents you from repeatedly losing.

Axelrod ran a larger tournament with 63 entries, included tit-for-tat. Again, tit-for-tat won. Once the results were revealed, the race was on to try to find a better solution than the simple tit-for-tat. There were two considerations. First a general strategy was required. Even in Axelrod's tournaments, there were sometimes cases where two strategies would do very well against one another but then fail miserably when put head to head with some other strategy. To win the overall tournament, a strategy would need to work well against a wide range of other strategies. Second, a more complex strategy might be able to take advantage of patterns of play but would require a long string of historical data, significant memory, and an algorithm to make sense of new data. This can be contrasted with tit-for-tat that only needs to know one move in the past and can be much more reactive.

Over time a new champion was found, what has been called "generous tit-for-tat." This strategy is the same as tit-for-tat but will forgive a single defection. It pays off (depending upon the payoff matrix) to forgive, at least once. This modified strategy also comes with an added benefit. Some tournaments were run where mistakes could be made, for example, what if five percent of the time you intend to cooperate (or defect) but instead do the opposite. If this happens with tit-for-tat, the players can get locked in a never ending oscillation of cooperate-defect. Generous tit-for-tat can break that cycle and is therefore more robust against mistakes.

8.3 Multiple Players

Most all real-world games are played between multiple players with multiple options beyond simple cooperate or defect. For example, our ancestors lived in small groups where interactions were frequent, reputation mattered, and everyone at some level had direct contact with everyone else. They had common resources that needed to be shared, which could easily become a source of tension and perceptions of unfairness. In this section, we will explore the dynamics of multiplayer games.

Axelrod reported his findings in a book called *The Evolution of Cooperation*, and he did not stop with the tournament. He asked what would happen if a population that had adopted one particular strategy was invaded by some other strategy. For example, what would happen if there was an entire population that had adopted the random strategy and then suddenly encountered some number of tit-for-tat players? Axelrod conducted a sort of cellular automaton on a 2D grid but where the rules were governed by the prisoner's dilemma. If you saw a neighbor winning more than you, you would adopt their strategy. Some strategies were very good at invading, but were easily invaded themselves. Others were not good at invading but if established were hard to displace. Again, tit-for-tat won out in multiplayer games.

8.3.1 Tragedy of the Commons

Another form of multiplayer game theory occurs when all players share a common resource. The classic example was outlined in a now famous 1968 article by Garrett Hardin (1915–2003) in *Science* called *Tragedy of the Commons*. In the article, Hardin retells a story written in 1833 by the economist William Forester Lloyd (1794–1852). In the original formulation, a small agricultural community shares a central plot of land for grazing livestock. Each farmer is allowed to have some number, say 50 animals, graze on that plot of land. If a farmer allowed 51 of their animals to graze, however, no one will know the difference. The problem is that if everyone does this the land will be overgrazed. Because everyone takes just a little more than they are assigned, and not caught, everyone begins doing it, and the entire community eventually suffers. Hardin extended this idea to any kind of common resource (e.g., a communal cookie jar, hazardous waste, taxes). In many cases a resource is accumulated through some collective effort, with the idea that there will be equal access to the payoff. A small bit of selfishness, however, can eat away at the payoff for all. A similar idea is the *free rider problem*, where a lazy team member gets the rewards of a well-functioning team.

8.3.2 Fitness Functions

The ultimate shared experience is when multiple players come together into an ecosystem. In almost every ecosystem, whether it is technical, business, biological, or social, there are opportunities to cooperate and defect. What is more, an individual agent could cooperate (or defect) with members of its own type (e.g., intraspecies cooperation) and at the same time defect (or cooperate) with another type (e.g., interspecies competition). What is often established are patterns of interaction that can be characterized as flows of matter, energy, or information on a network. When game theory is included in these network flows, a *fitness landscape* may emerge. The idea of a fitness landscape was first introduced by Sewall Wright (1889–1988) in 1932. Because many of these ideas were first developed to study biological ecosystems, we will use them as an example. Keep in mind, however, that fitness functions apply to almost all complex systems.

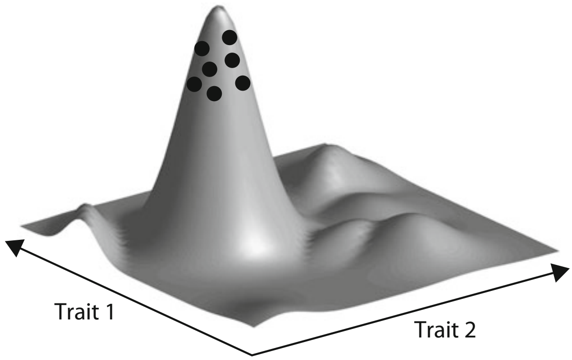


Fig. 8.3 The fitness peak on a fitness landscape for individuals of the same species

A fitness landscape is a visual representation of a hypothetical *fitness function*. In biological terms, an organism is an interconnected series of structures, usually thought of as partially determined by the DNA sequence that defines the species. These structures endow the organism with particular flows of matter and energy that provide them with the combination of functions that are unique to that species. Due to differences in the genome, some individuals within that species might have slightly "better" flows than others, meaning that they can function in a way that provides an advantage. These relative advantages are given the term *fitness*.

Figure 8.3 shows a hypothetical fitness landscape for the combination of two traits. When these functions are plotted against each other, the height (z-axis) represents fitness. Of course most species have many functions, and so theoretically there are many axes, and the surface is multidimensional. What is more, the fitness function is often a complex and non-linear combination of functions. Graphically this means that the landscape (as a combination of expressed functions) can take on many shapes and perhaps even have multiple peaks.

A peak, sometimes called a *niche*, on the landscape corresponds to an ideal fitness. It is unlikely, however, that any particular individual will actually be at a peak. Instead, as shown in Fig. 8.3, there is a cloud of individuals, each with their own unique variations in the functions expressed, surrounding the peak. The spread of this cloud is a measure of the diversity within a species, a topic that will be expanded upon below.

Survival of the fittest is the term used to describe how random mutations across generations can lead to greater fitness. On a fitness landscape, the evolution of a species has a visual interpretation. It is the movement of the average cloud higher up the peak. As a species, the cloud "senses" which way to climb up the fitness landscape.

The term fitness (graphically represented as the height of the landscape) is charged with debate. The most common way to define fitness is reproductive capability—individuals that happen to be higher on the fitness landscape will reproduce at a higher rate. Even a slight advantage will eventually lead to the survival of the individuals that reproduce the most.

If reproductive fitness were the only measure, we would need to admit that many insects, bacteria and other quickly reproducing organisms are more fit than an elephant or blue whale. It would seem to disadvantage species that do not multiply quickly. For example, an individual bristlecone pine can live to be over 5000 years old, and a hydra is essentially immortal. The ability to reproduce is simply one proxy of fitness.

There are many other proxies for fitness. For example fitness may be the percentage of available energy, land or other resource that a particular individual or species owns or uses. Likewise it might be the efficiency of maximizing function while minimizing energy used. For example, it may be a wonderful defense for a worm or fish to be able to create a short pulse of electricity (as in an electric eel). This requires a significant amount of energy to be expended, not simply to release the shock but to maintain all of the protein ion channels that it requires. Rather than traverse this portion of the evolutionary tree, a worm uses other defenses which are not as effective but require much less energy.

Measures of fitness bring up two other considerations. First, the usual way to classify a species is through sexual reproduction. Two individuals that can breed are defined as being from the same species. This idea has many problems because many organisms do not reproduce through sexual reproduction, and there is evidence that even those species that do, can sometimes cross-breed or share genetic materials with other species. A more nuanced way to think of a species is that it is composed of a clustering of individuals around a fitness peak. This also helps when thinking about other complex systems. For example, the fitness of a company is not measured by how much it can reproduce itself, but rather by the relative share of the market, as compared to other companies that are competing for the same resource (e.g., money, loyalty, attention, and other features of a customer).

Second, an individual organism usually goes through some development process. It therefore must be able to survive every phase of growth and development, often pouring energy into building structures that will be temporary. A new born calf is not very fit. In this light, the evolution of eggs as well as gestation and parenting can be viewed as an adaption to protect (as best as possible) a new organism past the point where it is most vulnerable. Human-made systems are no different. A new team, a new country, a new mind, or a new company are all vulnerable early in their development. Some even require temporary assistance. For example, business incubators are an effort to temporarily nurture new companies, rather than throw them out into the cut-throat world unprepared.

8.4 Interspecies Interactions

A single landscape may have multiple ways to achieve high fitness, represented by multiple peaks. Figure 8.4 shows the two interacting species of frogs and salamanders. Both are a unique combination of structures and functions that are fit in their own way. The cloud of dots on the landscape represent different species

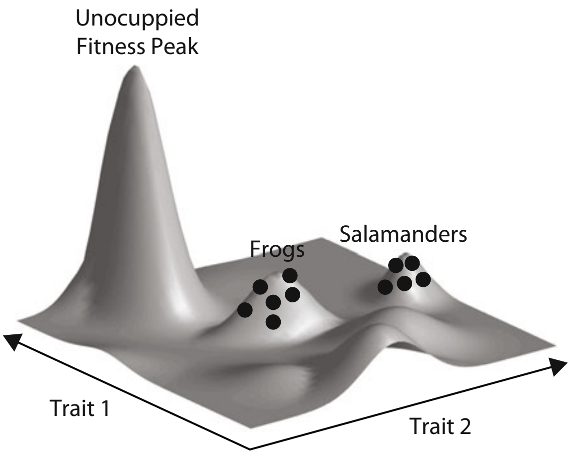


Fig. 8.4 The fitness peaks of frogs and salamanders. Each dot represents an entire population of a species. There are many ways to be a frog. There also may be unoccupied peaks in the ecosystem that would provide a hypothetical species with high fitness. Stuart Kauffman proposed a mathematical way to tune such a landscape known as the NK model

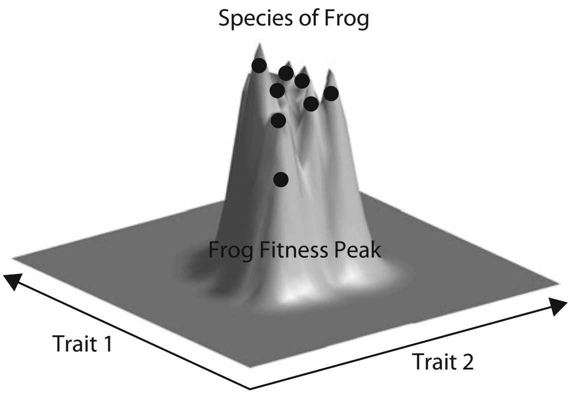


Fig. 8.5 A zoomed-in view of the frog peak in Fig. 8.4 showing a rugged landscape of many closely spaced types of frogs

of frogs, as in Fig. 8.5. But if we zoom in even more to any particular species dot, it would be its own mini-cloud of individuals. In other words, there is an overall peak that represents a way of surviving in an ecosystem, for example, as a frog. But then there are also multiple ways of being a particular species of frog.

Evolutionary dynamics, how species and individuals climb on the fitness landscape, will depend upon the nature of the landscape. Landscapes that are smooth are relatively easy to climb—heading in the direction of the steepest ascent will over time allow a species to crawl to higher peaks. They also are often more forgiving and allow a wider spread of the cloud and more diversity. On the other hand, *rugged landscapes*, where several peaks are very closely spaced together, are much more difficult to climb. On a rugged landscape, a population or individual would need to descend into a valley to reach a new (perhaps higher) peak. As a result the population will cluster tightly around the peak, resulting in less diversity.

In the case of frogs and salamanders, fitness landscapes can also help understand how ecosystems form. Both frogs and salamanders overlap in their food sources and habitats. A species is not fit on its own. The peak that it is climbing was created by

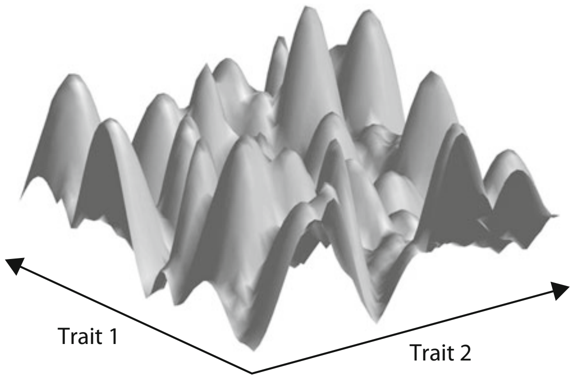


Fig. 8.6 A rugged landscape representing an ecosystem of competing and cooperating species. Incremental evolutionary processes do not work well on a rugged landscape because movement in any direction from a peak is likely to result in a sharp drop in fitness

the environment and the other species. In many ways the definition of an ecosystem is one where the location, as well as the height and ruggedness of peaks in the landscape are determined by a combination of the environment and other species. Traditional causal chains break down. A species adapts to its environment but that environment, and the other species within that environment, changes as a result of the adaptation. The result is a multidimensional fitness landscape, represented in simplified form in the rugged landscape in Fig. 8.6.

8.4.1 The Red Queen and Ecosystems

The description so far is of a static fitness landscape. An entire species would climb up a peak and eventually become a more or less homogeneous "winner" at the top. This is known as the *competitive excursion principle*—two species competing for the same resource cannot coexist forever. Mutations might occur, but the population as a whole would remain on the peak. The population clouds would become less and less spread out and diverse. Yet this is rarely what we observe in the real world. One explanation is that we simply have not waited long enough—over time it will happen. The other explanation, and the one that has a great deal of evidence, is that most fitness landscapes are dynamic.

The explanation can be summed up in an anecdote from Stuart Kauffman. The frogs with the fastest tongues will catch more flies, giving them an advantage, and eventually all frogs will have faster tongues. But individual flies with a slightly wider angle of vision will get away more often and be able to propagate their genes more readily. Then frogs that can camouflage better will gain reproductive fitness. But flies that have small barbs on their backs will gain a slight advantage because the frog tongues won't stick to them as easily. And then the frogs with slightly sticky tongues will gain an advantage. This kind of "leapfrogging" can continue forever in a sort of biological arms race. In social systems the same idea is summed up in the phrase "keeping up with the Joneses." This idea has been expressed by many biologists but has gotten the name Red Queen Effect first coined by Leigh Van Valen (1935–2010) in 1973 to describe a kind of antagonistic coevolution. The term comes from the character of the Red Queen in Lewis Carroll's (1832–1898) *Through the Looking Glass* when she says, "it takes all the running you can do, to keep in the same place." Peaks and valleys on the landscape are coupled together such that the hill climbing of one species may impact the location and height of the peak of many other species. The result is that all species are constantly chasing a moving peak.

The critical insight is that the location and shape of a fitness peak is dependent upon other species, as well as other environmental factors. Kauffman calls them "dancing" landscapes. Symbiosis, parasitic relationships, mimicry, and predator-prey are only some of the ways species may become intertwined. The two species might not even directly interact. The waste of one species may be the food of another. One species might change the microclimate such that another species benefits (or is harmed). Likewise, the availability of resources may change. Sunlight may be lowered due to a volcano, or water may be more or less plentiful in a given year. Some environmental phenomena seem to run in cycles (e.g., ice ages), while others seem to be more chaotic (e.g., earthquakes). What arises in an ecosystem is a much more nuanced version of cooperation and competition.

The dynamic nature of fitness landscapes has some consequences. First, there may be peaks that are not reachable, and therefore not populated with a species cloud, because there is no dynamic way to get from any current peak to that peak. That could change quickly however if a new player is introduced from the outside. For example, an invasive species could enter the ecosystem and occupy that peak. But the presence of that species may unbalance the ecosystem, directly competing with other species for resources or perhaps create a new source of food or waste. These direct and indirect changes can fundamentally alter the landscape, moving existing peaks, changing their heights, or potentially creating new niches. This is a classic case of the adjacent possible but at an ecosystem level. Second, fitness is relative in both time and space. An individual with the same genome may be fit now but not as fit at some later time. Third, it is hard to truly compare the fitness of two peaks—they achieve fitness in very different ways. Consider that you and your grandparents are likely "fit" in very different ways because the socio-eco-cultural systems you inhabit are very hard to compare.

The mechanisms of how new species come into being have been a question since before Darwin's publication of *Origin of the Species*. The simplest explanation is that physical barriers such as rivers, mountains, or island chains separate members of a single species, sending the seed population off in different genetic directions. A second is based upon human interventions, as in plant and animal breeding. Fitness landscapes, however, reveal another possible mechanism. Parts of the fitness landscape might become more rugged, a process that could drive what is known as *speciation*, a term coined by Orator Cook (1867–1949) in 1906. One could imagine that when zooming in to the fitness landscape of a particular species that it does not have one sharp peak, but rather a series of closely spaced peaks. Even in a well-mixed and interbreeding population, these peaks may begin to separate due to the

Red Queen effect and sexual selection. The result is that the two peaks move farther and farther apart, leading to two distinct species.

8.4.2 Individual or Group Fitness

There is a debate that surfaces every so often in evolutionary biology over the level at which selection takes place in living systems. The basic unit of evolution is generally considered to be the genome. The individual that possesses a certain genome will be "scored" by the environment/ecosystem in such a way that more fit individuals will reproduce more often or own more of the energy resources. On a long time scale, the most fit genomes will survive and climb higher on the fitness landscape. Under this model, all changes begin with mutations at the gene level and then propagate upward toward the ecological. Feedback on fitness then propagates backward from the ecological to the genome. This view is known as the *central dogma of molecular biology*, first proposed by Francis Crick (1916–2004) in 1958. He explained that the flow of information progresses linearly from the DNA in a gene to RNA to the expression of a protein and finally to function. There is no backward flow of information from functions back to the DNA except indirectly through relative fitness.

A linear flow of genetic information is perhaps best expressed in Richard Dawkins' (1941–) idea of a *Selfish Gene* (after a book by the same name) that the individual exists only as a mechanism of propagating genes. In this view individual genes compete with one another to express a particular function. We hear echoes of the selfish gene in the news almost every night—some scientist has "found" the gene that controls obesity or heart disease or autism. The idea of "one gene-one protein" has been shown to be the exception, not the rule. The hope for a magic cure to disease by simply fixing a single genetic mutation will only work in a few select cases. Most diseases are a much more complex mix of multiple genes mixed with environmental conditions during development.

There is a modification to this picture, known as group selection, that has been in an out of favor and become more nuanced over the past several decades. This is the life work of E.O. Wilson (1928–). To illustrate the idea of group selection, imagine a proto-fish swimming in the ocean. Some of these proto-fish develop a mutation by which they school together. Others do not have the mutation and stay loners. Over time these two populations diverge (occupy different peaks on the fitness landscape). What is argued by those in the group selection camp is that an individual in the schooling group may in fact be genetically less fit as an individual but that as a school they reproduce more often than the loners. The argument is that the group is selected for at a different level of the ecological fitness hierarchy (that of the group). What is more, it may be that the school can adapt faster to changes in the ecosystem than an individual genome due to the inherent diversity of the group. This is a tricky debate, with many logical twists and turns.

The factor that has made this debate even more interesting is that we now know that not everything is set by the genes. The genes certainly place bounds on fitness but where an individual falls within these bounds is determined by other factors, often called *epigenetics*. Essentially epigenetics, a term coined by Conrad Waddington (1905–1975), is any factor that is encoded outside the genome. One of the most studied molecular mechanisms is DNA methylation, whereby methyl groups are added to the DNA, typically repressing expression of that particular gene. What is more, methylation can be induced by external means such as exposure to poisons or other environmental factors. In some studies methylation can in fact be transmitted to future generations, thereby creating inherited modifications to the genome.

From a systems perspective, epigenetics endows biological systems with the ability to adapt over some range of time scales of information flows (e.g., expression of functions). The Dawkins/Wilson debate therefore is at least in part about the relative importance of epigenetic factors in determining fitness. The more general debate, about how much internal versus external factors matter, occurs well outside of the biological realm. For example, in the corporate world, which is more important—the vision of the leader (internal) or brand recognition (external)? Both are important, but at different times or in different environments, one might become more important than the other.

8.4.3 Symbiosis

A classic example of win-win cooperation in the biological world is *symbiosis*. An important component of symbiosis is that it works in a very different way than the incremental, mutation-driven, form of evolution. It works by putting together existing parts that have evolved separately and only later merged together. It is very much related to the idea of *exaptation*, encountered in a question in Chap. 7. It is also an example of Herbert Simon's Watchmaker analogy, creating a modular structure that can be put together later in ways that may not have been anticipated. Below various levels of symbiosis are explored.

Genes cooperate with one another to create more complex functions. There are a few cases of "one gene-one protein" functions, but this is not true for most biological functions. Genes cluster together in non-linear ways and experience selective pressures together.

A second level of symbiosis occurs at the level of cells and is sometimes called *endosymbiosis*. Lynn Maragolis (1938–2011) proposed, and then demonstrated, that early cells likely formed through the accumulation of stand-alone organelles. The classic example is the mitochondria which at some point in the past was a stand-alone organism with its own reproductive capabilities. It was only later in evolutionary history that mitochondria formed a symbiotic relationship with cells.

A third level of symbiosis occurs in the development of multicellular organisms, whereby individual cells co-occupy a similar space, often triggering differentiation and a variety of cellular roles. One of the early evolutionary pioneers were the slime molds explored in Chap. 4. Other animals and plants went on to have much more defined coordination of cell types.

A fourth level of symbiosis occurs at the level of species, whereby members of the same species will work together. This is seen in the development of groups, packs, flocks, and other coordinated groups of the same species that was touched upon in Chap. 2. Much of game theory in fact uses these types of groups (e.g., interactions between humans in games) as models. The interactions between members of the same species may become very complex—especially in social, economic, or cultural interactions between players.

A fifth level of symbiosis occurs across species. There are many examples that include the relationship between some alligators and birds in the animal kingdom and the formation of lichens (symbiosis of algae and fungus). One can think of these interactions as co-adaptations of two peaks on the fitness landscape. In some cases the peaks have become so tightly tied together as to be critically dependent upon one another.

The last level of symbiosis occurs at the level of multiple interactions between species to form an ecosystem. Examples are wolves thinning a herd of their weakest members, but the herd also needing enough grazing land to support its population. This interaction is entwined with many others that support varying cycles of nutrients and energy. It is at this level that more complex predator-prey, parasite-host, and other mixes of competition and cooperation may arise. We will expand upon this view of symbiosis more in Chap. 9.

8.4.4 The Evolution of Technology, Economies, and Cultures

The idea of a fitness landscape can be applied to human-made systems. Nearly all new technologies are recombinations of previously invented technologies. Laws are also often built upon previous legal precedents. Many cultural phenomena are the accumulation and recombination of previous events and cultural elements. The non-biological elements that are being passed on are known as *memes*, a term coined by Richard Dawkins in 1976. Just as the recombination of genes interacting with the environment form biological fitness landscapes, the recombination of memes, sometimes called a meme complex, can form their own landscapes. For example the invisible hand in economic systems explored in Chap. 1 is an economic emergent phenomenon whereby selfish players co-evolve as they climbing fitness peaks in the economy.

Companies (and other human-made systems) have a few unique features. First, a company may consume very little resources but have an enormous impact on the fitness landscape. This is the case with many high-tech companies. They rely, however, on existing infrastructure that enables their existence. Second, some short-lived companies can continue to have an impact even after they are out of business. While Google lays claim to the largest share of searches on the Internet, a number of companies that came before them (Yahoo, AOL, AltaVista, and others) had at least as large an impact because they created the legal, political, cultural, and technological infrastructure of the search landscape.

Cultural memes also can help explain how a person who was not necessarily biologically fit can be fit in other ways. Vincent van Gogh (1853–1890) struggled with mental illness and committed suicide at 37. Marcel Proust (1871–1922) was sick most of his life and did much of his best work while in bed. Alan Turing (1912–1954) was a very accomplished marathoner, but his admission of being a homosexual led the British government force him to undergo hormone treatment. He later committed suicide at the age of 41. Elizabeth I (1533–1603) and Isaac Newton (1643–1727) never married, and both were allegedly virgins when they died. These historical figures, rather than climb a biological fitness landscape, made major contributions to other landscapes that were the creation of humans.

8.4.5 Diversity

In many realms of society, we value diversity of background, opinion, language, experience, origin, attitude, and much more. A financial advisor may discuss how to diversify investments. Companies think about how to build a diverse portfolio. There are some that will maintain that diversity is important for ethical reasons. Others will claim that diversity is simply more interesting and makes life richer. Yet others take a pragmatic approach—diversity is the engine that drives forward novelty and change. It can also be a form of insurance that underlies long-term stability. For example, the term immunodiversity is a measure of how robust an organism is to a wide range of pathogens. It is not the purpose of this text to weigh in on equity, inclusion, institutional barriers or social justice. There are, however, some lessons that might be learned from fitness landscapes and populations clouds.

Diversity is functionally critical in volatile ecosystems where the fitness landscape is moving. When the landscape moves, some in the cloud will be closer to the new peak than before and gain a slight advantage. Likewise, others will lose their advantage. In this way, a population cloud can holistically "sense" the direction that the landscape is moving. For this reason evolutionary biologists often use the diversity of a species as a measure of how robustly that species has colonized a niche and how stable it might be when presented with challenges. Diversity is the key to the long-term sustainability of a dynamic ecosystem.

8.4.6 Genetic Algorithms and Evolutionary Computing

Solving problems is often attributed to an individual agent making top-down rational decisions. Evolution uses a different pathway—to generate many possible solutions (e.g., individuals of a species) that are in competition with one another. In a bottom-up fashion, good solutions emerge. The environment becomes the game space and judge of the competitions. If it were only survival of the fittest, the result would be a winner take all, with the best or strongest initial solution eventually winning. But evolution has other mechanisms to iteratively generate new solutions. For example, when solutions can reproduce and then mutate, closely related solutions might coexist.

Evolutionary computing and genetic algorithms extend this approach to problem solving outside of the biological realm. First pioneered by John Holland (1929–2015), each problem is assigned a particular fitness measure. Scoring well by this fitness measure, which may be a combination of many factors, defines how solutions will compete. A fitness function can be as simple as an equation or as complex as scoring how many "points" or "resources" have been harvested in a given amount of time.

Each genetic algorithm begins with the generation of initial solutions with the aim of populating as much of the fitness landscape as possible. The intuitive next step would be to apply the fitness function, find the solution that scores the best, and declare that solution the winner. A genetic algorithm, however, takes a more iterative approach—the score is only used to increase or decrease the chances of that solution surviving. This means that a solution that is initially poor still has a chance of surviving. The reasoning is that sometimes a solution that appears to be poor at first is simply missing some critical component that would make it a great solution.

There are three mechanisms for adding a missing critical component. These are known as *genetic operators* and determine what changes will take place to the agents. To keep track of the changes, a solution at one time point is called a *parent* and any new solutions derived from that parent are known as *children*. In this regard each iteration is like moving forward by one generation. In some genetic algorithms the parent solution is removed once a child has been created. Another way to think about this is that the parent has simply become the child. In other implementations the parent and child solutions can coexist.

The first genetic operator is *reproduction*, which describes how parents are selected for breeding. This can range from every solution reproducing to some very selective procedure (e.g., only the top ten percent of solutions can breed). The second genetic operator is *mutation*, either random or targeted. The mutation rate (e.g., probability of a mutation) and size (e.g., the degree of the change) can be varied. The third genetic operator is *recombination* or *crossover* of genetic material between two or more parents to create a child. All manner of directed or random crossover, essentially a generalized sexual reproduction, might occur in generating a new child.

A specific genetic algorithm is the mix of genetic operators, fitness functions that reflect the environment, makeup of the individual agents, as well as how iterations take place to move to the next generation. Unlike biological operators which are constrained to work a particular way by the chemistry of life, genetic algorithms can take a wide variety of forms. For example, it might be that seven parents result in a

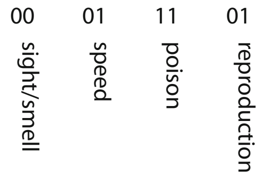


Fig. 8.7 The digital DNA of a simple ant. Each trait is allocated two bits of the DNA, resulting in four possible levels of that trait. The makeup of an individual ant will determine its abilities and therefore its fitness

child with some weighted recombination of genes such that higher scoring parents give proportionally more genetic material.

To make the concept of genetic algorithms more concrete, imagine a group of ants where each ant is driven to eat food and avoid poison in a two-dimensional environment. Ants that eat more than four poison pellets die, while those with ten or more food pellets will be allowed to be parents in the next generation. As breeding takes energy, the food pellet number goes back down to zero after a successful breeding. Poison, however, accumulates forever and when it reaches four that ant will die.

Each ant contains within it a crude sort of digital DNA, shown in Fig. 8.7 that determines how it will navigate the environment. This DNA will consist of:

2 Bits for Sight/Smell
2 Bits for Movement Speed
2 Bits for Poison Detection
2 Bits for Reproduction Propensity

As an example, the two bits for sight/smell mean there are four different sensing abilities. A zero (binary 00) means the ant is entirely blind and will follow a random walk. A one (binary 01) means the ant can see or smell one step ahead in the 2D grid. When sensing food one step away the ant will follow a more directed path to harvest food pellets. Likewise a two (binary 10) and three (binary 11) for sight/smell means that the ant can detect food two or three steps ahead on the grid. A similar kind of thinking could govern the other components of the digital DNA. Therefore, an ant with the DNA

$$00101101$$

has zero level of sight (00), the second level of speed (10), the third level of poison detection (11), and the first level (01) for reproductive propensity. Let us assume that this ant has over time avoided poison and accumulated 13 food points, meaning it can breed with another ant. We might assume that at each time step, all of the possible ants that could breed are randomly selected to pair with one another and generate a child. Through this process the ant above is paired with another ant with the DNA

$$01111001$$

At this point there are several possibilities for how a child DNA might be created. A simplistic method would be to take the first four bits of one parent and the second four bits of the other parent, resulting in a child with the DNA

01111101

If this concept is scaled, the dynamics can become quite complex, with ants dying, being born, breeding, and generally navigating a 2D space in a way that to an outsider looks a lot like real ants. In this example, the ideal solution was already known, a “super ant” with a DNA of all ones. Genetic algorithms, however, are usually applied to cases where there is not one way to be fit. For example, the ant DNA might have an additional set of bits that enable an ant to follow another ant toward poison or away from poison. This would open up another way to survive and breed—follow a very fit ant. Adding more of these bits, especially if they open up new behaviors, will lead to a diversity of strategies. Such complexity can begin to resemble the idea of artificial life introduced in Chap. 2.

The example above was still modeled on biological evolution, but genetic algorithms can be tuned to solve problems that are not easy to solve through traditional analytic or serial algorithmic means. A classic example is the traveling salesman problem introduced in Chap. 6. The challenge is to find the shortest pathway that visits each city only once. This was deemed an NP-hard problem which means that the only way to find this shortest pathway is to exhaustively try all pathways. In many real situations, however, getting close to the shortest pathway is good enough. There is no guarantee to get the ideal solution, but using a genetic algorithm, a very good solution can be found quickly. For example, two solutions might breed by taking the first half of one solution and recombining it with the second half of another solution. If mutation is also included, the result over time will be better and better solutions.

Problems that are inherited from an environment might change over time. For example, imagine how much harder the traveling salesman problem would become if the list of cities was constantly being updated, with some cities disappearing and others being added over time. Likewise, for the ants there might be some new competitor that suddenly appears and eats the same food. Alternatively there might be a new kind of bug that appears that eats the poison. All of these environmental changes, with the accompanying dynamic fitness landscape, will alter the way in which solutions can be fit. Although genetic algorithms were not originally designed for such problems, they in fact perform quiet well even when the fitness landscape is changing. What is more, genetic algorithms are one example of *evolutionary computing*, essentially the use of biological algorithms to solve hard problems. An evolutionary algorithm might weave together elements of genetic algorithms with other kinds of algorithms, such as L-systems (Chap. 5), swarms, and the leaking buckets approach (Chap. 2).

8.5 Game Theory in the Real World

The picture painted of games and evolution is grim. It seems that cooperation may come about but only when it directly benefits an individual agent. Everyone is an opportunistic cooperator. A fundamental question in both biological and societal dynamics therefore is how *altruism* might come about. What is meant by altruism here is essentially giving up some degree of value or power, perhaps even fitness, to another, without knowing if there will ever be a return. The idea can be framed in many ways. For example, in Axelrod's tournaments there is a certain degree of reciprocal trust required. This idea is known as *direct reciprocity*, and in common language it is "If you scratch my back, I will scratch yours." Of course the timing and value of the return is dependent on the system and agents involved. A politician might help out another politician in exchange for help passing their own bill at some later date. This is how coalitions can form between countries as well. Direct reciprocity requires both trust and memory.

In biological systems there seems to be another way to overcome barriers to cooperation. From an evolutionary perspective, one might show a degree of altruistic behavior toward a relative. You share some percentage of your genes, at least probabilistically, and therefore you might help them. The concept fits very well with Dawkins' idea of the selfish gene, building on earlier work by George C. Williams (1926–2010). It can help explain some very striking displays of altruism such as a parent or sibling giving up their life for someone in their family. This idea has become known as *kin selection* and immortalized in a famous 1932 quip by J.B.S Haldane (1892–1964) that he would willingly die for two brothers or eight cousins. In fact, from the perspective of genes, his odds were exactly right.

The idea of kin selection seems to also be at work beyond simple blood relationships. Militaries, gangs, corporations, and other organizations (perhaps even ones you belong to) have learned how to hack kin selection. Militaries march in units and their synchronous motion has been shown to promote bonding. Some gangs, including fraternities and sororities, call each other brothers and sisters. Corporations often have holiday parties, as if they were a family. It is not necessarily about laying down one's life for a co-worker (although that has been known to happen), but a family-like bond can allow a group to more tightly trust one another and therefore move out of the Nash equilibrium of defection.

Yet another way that cooperation can get going is for there to be forms of punishment. Some values in the payoff matrix might be negative. In other words, some actions are not simply tolerated (e.g., their payoff score is zero), they may actually incur a penalty (e.g., payoff score is negative). Punishment can be observed in many biological and social situations. Sometimes groups will punish an offender even at their own expense (both parties receive a negative in the payoff matrix). Practically this means that the group is willing to collectively lose to punish an individual (or subgroup). The intent is to send a message that an action is not tolerated by the group.

Punishment and reward can also tie into *social contract theory*, where an individual, by accepting to live in a group (or by signing a contract), agrees to abide by the rules of the group. When an individual breaks the rules, they are punished in some way, often at the expense of the good of the group. A great example is that in the United States, we pay taxes that support state and federal prisons, giving a portion of our wages to punish those who have broken laws. What is perhaps most important about punishment is that as a society we are willing to punish people who have not directly harmed us, and we often have never met. We trust the assessment of a formal or informal system of justice that may be implemented by people we do not know. The idea that you might trust someone simply based upon their position, or rumors about them, is called *indirect reciprocity*. It is also important that punishment (and reward) can take many forms, from physical or emotional to social or economic. For example, shame is a kind of emotional punishment that may be exacted by a co-worker, parent, friend, or teacher.

Reputation is another form of indirect reciprocity. Like punishment, it requires trust and memory and goes well beyond direct interactions. We often will trust someone simply based upon a recommendation. This concept has taken on a whole new meaning in the age of the internet and social media. Sites like Amazon, Angie's List, Craigslist, eBay, Airbnb, and others are go-to places mainly because customers can read the reviews of other customers, helping to build a reputation of particular sellers. What they have created is an economy of reputation. In fact, sellers can become the hub of a network based almost solely upon their reputation. Likewise the reputation of a company may be their most valuable asset, more powerful than money or property. Reputations, however, are asymmetric—they generally take a long time to build but can be destroyed by a single act.

8.5.1 Spatial Game Theory and Stereotypes

In the standard framing of multiplayer games, the population is assumed to be well mixed, meaning that every agent has an equal likelihood of meeting every other agent. This is clearly not the case in real populations. We only interact with certain individuals, defined by our various social networks, and in a large society, it is not possible to interact with everyone. In some sense this is the purpose of indirect reciprocity—it allows us to make decisions about how to interact with individuals with whom we have never interacted before and might never interact with again. This situation is almost identical to the original one-off version of the prisoner's dilemma. What indirect reciprocity provides is a bit more information that is critical in making decisions. Knowing that an agent has been a cooperator with others in the past does not mean they will cooperate with you, but it does give you some trust that they are likely to cooperate.

Spatial game theory allows for interactions between some agents to be governed by a network of relationships. These could be family ties, location in space, websites to which you subscribe, or your church. You play games with some people much more frequently than others. This slightly more complex form of reciprocity is known as *network reciprocity*, and it is an extension of indirect reciprocity. There have been some incredible insights gained by combining game theory and network theory with regard to spatial games. Some seminal contributions have come from Martin Nowak (1965–), the author of several technical works such as *Evolutionary Dynamics* and the popular book *Supercooperators*. One of Nowak's most highly cited articles is "Winner's Don't Punish" where he proves mathematically that under a wide range of circumstances if you are the winner, in the long run, it pays to show some mercy on the loser.

A more disturbing consequence of game theory is how it plays into the building and perpetuation of stereotypes. Much of altruism and punishment relies on trust (or lack of trust) and memory. If we have had an experience before (good or bad) with a particular agent, game theory says that we should take that into account in future dealings with that agent. But we seem to abstract this concept beyond the level of individuals—if someone reminds us of a past agent we may respond to them as if they were that agent. This is a double-edged sword. If you had a trusting and caring grandmother, you will likely trust other grandmothers. On the other hand, if you had an abusive uncle, you may distrust all uncles. These are cases where indirect reciprocity meets direct reciprocity. What makes these cases all the more problematic is that they often extend to network reciprocity—in other words there can be guilt (or praise) simply by association. The way to break stereotypes seems to be to collect more data or become more conscious that not all uncles (or grandmothers) are the same. Then again, the network structure often makes it difficult or even impossible for that to occur.

8.5.2 Bounded Rationality and Intuition

Much of game theory assumes rational players who will make the best possible decision for themselves. This assumption makes the mathematical analysis easier, a theme we have seen in other studies of complex systems. There are several researchers, both past and current, who have studied game theory under *bounded rationality* a term coined by Herbert Simon (1916–2001). Just because an agent might have access to complete information does not mean that all of it will be used (or used equally) to make a decision. There could simply be too much information to remember or process before the next "move" is required. Likewise, games in the real world are different than the scripted games that are studied in theory. They are occurring between multiple agents, playing several games for many resources, at the same time, on different networks that may overlap in complicated ways. In other words, once game theory gets out of the lab and into the real world, it is virtually impossible for an agent to adopt a truly "rational" strategy.

Some environments are predictable, meaning that patterns can be exploited to calculate outcomes ahead of time. A plant or farmer can be certain that the sun will come up each day. Other arenas are far less predictable, but still might contain patterns that enable a probabilistic guess. It is in these volatile and less predictable situations that bounded rationality becomes *intuition*. As an agent tries and succeeds, or tries and fails, or observes others doing the same, they notice patterns. These backward-looking trends (similar to descriptive statistics or stories) can be encoded as probabilistic decision-making rules, as in Bayes' Law from Chap. 6. Backward-looking pattern detection and probabilistic prediction is how most agents operate in fuzzy environments. The simple rules that help make good probabilistic decisions are often called *heuristics* better known as a "rule of thumb."

Intuition and heuristics are built up through experience and keep the focus on pertinent information, enabling an agent to make quick, but generally good, decisions. Intuition is what allows agents to function in new environments that they may not have been in before and interact with agents that they have not met before. Furthermore, simple heuristics seem to work best in complex and dynamic environments, because they quickly trim the game tree to the most promising pathways.

8.5.3 The Origin of Bounded Rationality

Simon was led to bounded rationality through a variety of routes, but his most famous was the study of novice and expert chess players. In the study, Simon presented both groups with a snapshot of a chess board in the middle of a game. He allowed them some time to memorize the location of the pieces. At a later time, the subjects were asked to reproduce the locations on another board. Not unexpectedly, the expert chess players were able to memorize the board faster and could more accurately reproduce the board than the novices. It would be simple to explain this result as being a function of how much more familiar experts are at memorizing chess locations. But in the second part of the study, Simon randomized the pieces. In other words the location was not the result of an actual game of chess. In this study the novices and experts did about the same. The conclusion was that in both studies, the novices were operating the same way—to simply memorize piece by piece—whereas the experts were using two very different strategies. When asked how they could memorize the board in the first study and reproduce it so quickly, the experts cited intuition as their method. In the second case, however, these same experts reverted to the novice strategy of simply memorizing the locations of individual pieces.

The study revealed what the psychological literature calls *chunking*. Rather than remember individual bits of information, we remember patterns of interconnected bits. Another version of this same idea was explored in Chap. 5 when we described memories as being stored and recalled in network structures and Chap. 6 in the context of stories. Bits of information always come packaged together in a complex system, whether it is how concepts are stored in the brain, how genes are packaged or how roads are clustered together. When we pick up one bit of information, others come with it. This is in fact, the origin of what is known as *priming* in the literature.

For example, if you hear the word “red,” there are a number of other words that will come to mind (e.g., “Christmas,” “apple,” “blood,” “car”). Of course the exact words will vary depending on the person. But the combination of chucking and priming means that we do rarely need to consciously memorize individual things.

The ability to chunk, and in many cases ignore, information reduces what is known as *cognitive load* and is a way to make decisions when information is either incomplete or coming so fast that some of it must be ignored. One particular strategy that humans use to reduce information overload is the well-documented phenomenon known as *selective attention*. Essentially only the information to which we put our conscious attention will be processed. All other information is assumed to be extraneous, even when there might be a drastic change in the environment. The result is all manner of *cognitive illusions*, where the mind can be fooled, just as in a visual illusion, to miss or misinterpret reality. It is suspected that any system which must detect many signals over a wide dynamic range will be susceptible to these sorts of blind spots. A nice catalog of distortions, errors, and biases can be found in John Warfield’s (1925–2009) *A Science of Generic Design* in which he discusses 25 mindbugs.

8.5.4 Fast and Slow Systems

It would be easy to have many good rules that would help in make decisions over a wide range of situations. With a long list of rules, it is inevitable that some situation will arise that will bring two or more rules in conflict. On the other hand, having one (or a few) simple and compact rules (heuristics) will allow reasonable decisions to be made very quickly. What comes from this tension is a balance between having many good rules to cover all (or nearly all) situations and having a few very simple and compact rules that can act quickly.

Some systems have found a clever way to balance heuristics and more complex contextual strategies. They build two parallel systems, one that is fast (composed of heuristics) and one that is slow but more intentional. A great example is the balance between the innate and adaptive immune systems. The innate immune system is an evolutionarily ancient set of responses to injury. We experience this as the swelling, itching, and clotting mechanisms that occur soon after an injury or invasion by some outside entity. The innate immune system acts very quickly, is meant to minimize damage in most all situations, and helps kick start the healing process. But, it is a blunt instrument. It is a one-size-fits-all response. The adaptive immune system, on the other hand, takes time to mount a more targeted response. For example, on encountering a virus, an antibody is created with which the virus can be detected on a mass scale. This combinatorial process occurs with a very limited set of internal compounds. Only after a solution is found in this model is it broadcast out to the rest of the body. B- and T-cell receptors (on white blood cells) use the antibody to recognize invaders and destroy them. It is more like a precision surgical tool. Similar ideas can be found in the corporate world, government, and clean up after a crisis.

A similar argument is made by Daniel Kahneman (1934–) in his book *Thinking, Fast and Slow*. He proposes, and several psychological studies agree, that the mind has a fast system composed of instinct, emotion, intuition, and other unconscious processes. The mind also contains a slow system composed of rational and algorithmic thinking. The book is an exploration of the work of Kahneman and his collaborator Amos Tversky (1937–1996) that is known as *prospect theory*. The idea behind prospect theory is how a system with bounded rationality will weight losses (risks) against gains (benefits). The work effectively launched the still developing field of behavioral economics. The basic idea can be summed up in a simple experiment. Which one of these would you choose:

95% change to win $10,000, or 100% chance to win $9499?

Nearly everyone chooses to take the sure thing (100% chance of winning $9499). Now compare this to:

95% change of losing $10,000, or 100% chance to lose $9499?

In this second case, most people take the 95% chance of losing $10,000. When considered probabilistically, the two cases are identical, yet you change your interpretation of probabilities depending upon whether you are considering winnings or losses. There are two important points. First, wins and losses are not viewed the same by most people. We tend to focus more on what could be lost and weight losing more heavily in our decision-making. Second, most people do not look at the value of a probability uniformly from 0 to 1. Lower probabilities tend to be overweighted, while high probabilities are underweighted. We effectively squash the true probability values to be in some smaller range. For example, most people treat 90 and 99% as virtually the same, when in fact they are very different.

8.5.5 Emotions and Bounded Rationality

Bounded rationality and heuristics may have an interesting relationship to the origin and function of emotions. Emotions are a kind of heuristic, similar to the innate immune response. They allow us to unconsciously make a decision very quickly or at least bias the direction that the slow line of reasoning will pursue. A quick decision could be overruled later, but emotions seem to be able to coax reasoning down particular pathways of a decision or game tree. From an evolutionary perspective, an emotion, especially in a species without any form of reasoning, is a way to point an organism in the right direction. Emotions are a one size (or perhaps a few sizes) fits all response, while rationality is a way to fine tune decisions. A more complex way to view bounded rationality is therefore the interplay between rationality (slow) and emotion (fast), with all the cooperation and defection that can occur between these two systems within an individual agent.

There are some lines of philosophy that view an enlightened human as an agent that, given all information and plenty of time to apply their reasoning, would arrive at “the best” decision. This is the ideal actor that is studied in classic game theory and economics. But as several studies have shown, such an ideal actor does not seem to exist. Some of the most provocative studies have been conducted by Antonio Damasio (1944–). In his early work, he studied patients with a rare disorder whereby they have extremely low emotional responses. They are so extreme that they do not even trigger physiological responses (e.g., quickened heart rate, pupil changes) that signal an emotionally charged state. On the surface, one would think that these patients would be ideal rational actors and be able to make the most logical decisions for themselves and others. But in fact, they are virtually paralyzed with indecision most of their lives.

Damasio has gone on to study the interplay of emotions and decision-making through a variety of modern neuroscientific techniques. One of his most seminal demonstrations, however, was perhaps the most simple. A number of participants were asked to play a game of poker, while being attached to a galvanic skin response device (similar to a lie detector) to measure their emotional responses. They were told that as they played they could choose to draw from one of two decks of cards. Unbeknown to them, the deck on the right had a statistically better chance of giving them high-value hands. The question was at what point in the game they would emotionally and rationally understand that they should draw from the deck on the right. The results were conclusive. All participants eventually could verbalize that the deck of cards on the right was better to draw from. But, approximately 50 cards before they could verbalize this, they began to statistically (still unbeknown to them) draw from the right deck more often, and this timing was correlated with a heightened emotional response. The conclusion is that the emotional self knew which deck to draw from before the rational self knew. This idea has grown into the *somatic marker hypothesis*, which is a biologically based theory about how decisions are made using emotions.

It is important to note that this is different than the original way of thinking of an agent as “bounded” simply because they are not able to marshal the internal resources of attention and memory. Rather, they are in fact bounded by the complexity of the problems and decisions, where these is no “best” anymore. A purely rational actor, when faced with a tie, will simply halt or make a random decision (which is itself a heuristic). Emotions may be a way to break out of a kind of halting problem (as discussed in Chap. 6) that can limit formal logical systems.

8.5.6 Impossible Spaces

There may be areas of a game tree that exist according to the rules of the game but that no rational actor would ever visit. In other words, logic will never reach these situations. Bounded rationality allows for more of the game tree to be explored. Might there still be parts of a game tree that cannot be reached? Are there some regions of a fitness landscape that have high fitness but can never be reached? Or stated another way, are all organisms possible? Below are a few considerations.

On the most basic level, some areas of a fitness landscape may not be reachable through the incremental approach taken by evolution. Fitness barriers (valleys in the fitness landscape) could make certain areas practically unreachable. The invention of sex, and other forms of recombination, by biological organisms was a clever way to accelerate the incremental nature of evolution. Although limited to being within a species, the recombination of functions across generations can enable jumps in fitness. The concept of sex can also be applied to culture, music, politics, technology, and languages. Matt Ridley (1958–) has in fact called innovation "sex with ideas." Perhaps some systems can use a version of sex to reach spaces that seem impossible.

A fitness landscape may also be dynamic but only within the ranges allowed by the constraints of physical laws. A fitness landscape, however, is based upon function not structure. It is entirely possible that an area of the landscape could be reached through various structures that exploit the law of physics, only in a different way (e.g., achieving the function electrically rather than mechanically). This may not be possible in the biological fitness landscape, which is bound by organic chemistry (at least so far on this planet). But remember that fitness landscapes can apply to technology, politics, culture, and many other disciplines.

It is still not clear if there truly are impossible spaces that can never be reached. What do you think?

8.5.7 Trust and Further Considerations

We have so far assumed that information could be trusted. What happens when an agent can intentionally share incorrect information? Below are a few of the phenomena that can arise when game theory is extended to include delays and misinformation.

It is not always clear whether a game was won or lost. There are delays in almost all complex systems such that it is not always easy to connect an action taken now to a reward (or penalty) later. In the interim you might need to be making other decisions (and actions) that could potentially affect the outcome. Likewise, there are generally several games being played between the same players simultaneously. A single action might be rewarding in one way but penalizing in others. In other words, it is often not clear which game you have been playing or who is winning or losing and on what time scale.

Agents may also be changing over time in their strategies, definition of winning and losing, or even which games they are playing. They may also change their network, meaning which other agents they are playing against. This of course does not invalidate the fundamental findings of game theory. It does, however, add on different layers that may allow new strategies to emerge.

In most game theory analyses, there can be both privileged information (only certain agents know about it) and public information. If an agent has information, their decision is either to share it or not share it with another agent. In reality, the situation is much more complex because information can be used to deceive another agent. The idea is to interject false information into a network of agents, making it difficult to tell the difference between true information, false information, and a mistake.

We can consider the case where false information is presented for strategic reasons. Here the signal to other agents (or perhaps some select group of agents) is intentionally corrupted. Examples abound, from showing signs of wealth that you don't actually have (e.g., a cool car you cannot really afford) to outright lying about where you were the previous night (e.g., throw a suspicious friend or the police off of your case). There are in fact all manner of deceit and lies. An agent can share accurate information with some and share inaccurate (or incomplete) information with others for the purposes of instigating confusion. Likewise, they could share variations of inaccurate information with everyone. What this reveals is that in more complex games, especially with multiple agents, once we admit that not all information is the same, the possibility of deceit may arise.

War strategy has considered these types of cases for a very long time. Sun Tzu's (545–470 BC) *The Art of War* is full of advice on how to feed an enemy incorrect information. One of the most devious is to send out spies with false information who you know will likely get caught. They will be tortured and give up what they think is true information, but is in reality false. Likewise some political writers, for example, Niccolo Machiavelli (1469–1527), advocate for the careful and strategic use of information. One of Alan Turing's most infamous tasks was creating a device that could decrypt the German Enigma cipher, thereby enabling the Allies to know where bombs would be falling. But in an incredible show of restraint, the allies allowed places to be bombed, knowing innocent civilians would die, so as not to reveal that they had broken the code. Determining the reliability of information has become all that much more complex in the age of digital information, with the associated pathways for flow of information via the internet. This has opened up the possibility of cyber-sleuthing, hacking, cyber-warfare, and online spying that further complicates what information is available, to whom, and how reliable it may be.

What is really changing in each of these cases is how, when, and what information is available to the players, how much they share with others, and the impact it may have on their strategies. Relationships between a teacher and learner, worker and boss, or two people falling in love can be viewed as a complex entanglement of information and constantly shifting strategies.

A final example is the games that are played between parents and their children. When children are young, their parents essentially play all of their games for them. As children age, they begin to become more and more sophisticated agents, shifting their strategies based upon new learning and information about the world and themselves. There are many shocks in store for parents, such as the first time they realize their "innocent" child is capable of lying. Parents then need to adopt new strategies that they may not have used in the past (e.g., checking in with other parents, looking at text messages or social media). The strategies of both the children and parents are in continual flux. Likewise children, as they become adults, sometimes recognize that they are adopting, often to their dismay, the strategies they saw their parents use.

8.6 Questions

- The card game Mau was mentioned as a game where the rules are constantly changing and only some select few (initially the dealer) know the rules. Some have used this game to demonstrate how frustrating it can be to have changing institutional, linguistic, legal, cultural, and other barriers that prevent success. We also discussed how the makeup of social networks, some of which are also embedded in institutional practices, can reinforce stereotypes. When combined together these two factors can lead to all manner of vicious cycles and institutional inequities. Social justice thinking has gone through a sort of evolution in this regard. Initially the call was for fairness—the same distribution or access for all. Later efforts focused on equity—everyone gets what they need to be equally successful. A more recent focus has been on removing the institutional barriers—to change the system in such a way that there is no longer any need for different pathways to success. From a game theory perspective, this is changing the game tree so that everyone can play the same game. These efforts often go under the heading of *universal design*. Find a long-standing injustice and explain the historical and current social justice efforts. Based upon your knowledge of systems, what is at the heart of the problem? Can you use ideas of game theory to suggest a new or modified approach?
- Heuristics have been studied in a variety of fields. For example, Kathleen Eisenhardt (1947–) has written several high-profile business articles on how companies can use simple rules to guide their employees in making decisions without the need to constantly check in with higher-ups. For example, an early corporate heuristic at Google was "Don't Be Evil." It was removed in 2015, but the idea was for all employees to think of an action they were taking and ask if it might do harm. Heuristics are also what help many people through their day—small slogans that they live by that help to navigate hard situations. If you were going to give a heuristic that you use, what would it be? Can you give an instance where it helped you make a decision?
- Diversity was discussed as a protection against perturbations of a fitness landscape. There are a variety of factors that can lead to a lack of diversity, for example, the segregation model introduced in Chap. 2. Another is the *founder effect* which can occur when a small population with limited diversity multiplies very quickly. The result is a large population that is very uniform. Choose a human-created system, and deconstruct how it may be susceptible to the founder's effect. What is (or might be) the consequences of a lack of diversity in this system?
- Ethical dilemmas can often be framed in terms of game theory—a decision must be made that will have both positive and negative effects, and the dilemma is how to weigh these effects relative to one another. If there was an *E. coli*-type model system for ethics, it would be the *trolley problem*. There are many variations, but one version is as follows. A trolley with five people on it is headed down the tracks toward a horrific disaster in which everyone aboard will certainly die. You

are above them on a bridge and next to you is a rather larger person. You know that if you pushed this single person onto the tracks, the trolley would stop, and the disaster would be avoided. If you take this action one person will die. If you do not, all five people on the trolley will die. The question is what would you do and why? Would it change your decision if you knew one or more of the people involved? What if one is a well-known public figure?

- When we work on teams we are cooperating with the other members of the team. In a somewhat pure form your team may be competing for something that other teams are also working on. In this regard you are competing with them. But there are other more complex cases where competition within a team can make that team better and therefore better able to compete against other teams. Give an example of when the tension between cooperation and competition within a team led to a good outcome. Also give a counterexample, when the balance was not right (either too much cooperation or too much competition).
- Seeking out happiness and pleasure as a very natural heuristic. Likewise we recoil from pain as a sign that something is not good for us. But sometimes we are fooled—seeking pleasure in the short-term could become a long-term problem (e.g., addictions), whereas pain in the short-term might pay off in the end. Identify a pleasure and pain of a culture of which you are a member. In what ways does this culture become healthy or sick by following its pleasures and pains? Dissect the culture using some principles of game theory.
- We often talk of evolution as being about increasing physical, or perhaps mental, fitness. With humans, however, we often seem to try to increase our social fitness. Explain a situation where you have been playing multiple games (physical, mental, social, perhaps others) where a move has led to a win in one arena but perhaps a loss in another arena. Use ideas from the chapter to build your analysis.
- Dissect the idea of branding using game theory. Identify the players, how they compete or cooperate, why they win or lose, and what information is being shared (and with whom).
- Animals often send information out into the environment to broadcast their intentions. The warning of a rattlesnake's rattle is an example. In fact, the current reigning champion strategy in game theory, even better than generous tit-for-tat, is a sort of messaging between agents before they make their move. What are examples of human-made systems where one agent warns another agent about their most likely next move. What is the impact of this warning on how decisions are made?
- Groups and cultures have a variety of mechanisms for voting. It is one of the foundational principles of a democracy but often times is not implemented as a simple majority wins. Chose a country other than your own and explore how they engage in the voting process. How is this voting process different from how your friends or family might vote on where to go out to dinner? Why do you think there is a difference?
- Some animals will use the idea of mimicry to send false signals. A classic example is an animal that will try to look like a different poisonous species,

simply by having some red coloring (from an evolutionary perspective, often a sign of poison). In some ways these fakers are hijacking the defense system of another species. Are there examples in social or other human-made systems of this kind of mimicry?

- You may have found yourself adapting you driving habits based upon where you are. Driving in the country (leisurely) versus driving in New York (defensive but relatively orderly) versus driving in Naples or New Delhi (a free-for-all) are very different experiences. It is not the road that are necessarily different but rather the other drivers with which you interact. Find another example of a real-world system in which your strategies vary depending upon the context and other players. Explain your strategies and what it is that prompts you to vary them.
- When cell phones first appeared on the market, there were a number of strange phenomena that began to take shape. One of those took place in Brazil where fake wooden phones appeared. People would be driving their cars or walking down the streets having fake conversations on their fake phones. Why would someone do this? In what ways are they gaining fitness? Are there other examples of this kind of behavior?
- Heuristics were introduced in the context of making decisions with incomplete information. A heuristic can also be thought of as simple formula that works most of the time. For example, there is a formula to creating a perfume commercial. You get two models, one female and one male, both very scantily dressed, and you put them on the beach. You have them kiss or nearly kiss. The focus of the camera is soft, and there is a voice-over that must be extremely pretentious. At the end of the commercial, the perfume name is finally revealed. What other formulaic scripts can you identify in the world? You can be as sarcastic as you wish to be.
- Pick an individual artist (musician, writer, or actor). Create a map of all of the people involved in the success of that particular artist. In what ways have these actors cooperated (or defect) with that artist during their career?
- Big data aims to overcome the problem of having too much information. In an effort to overcome bounded rationality, computer algorithms, endowed with massive memory and speed, can cut through the overload of data. What might be missing from a big data approach and how would you propose to fix it?
- What is a landscape upon which you are playing that you feel is moving faster than you can make decisions? What is causing that landscape to change? What does it mean to win or lose? How are others doing? Use the ideas from the chapter to support your analysis.
- The Pixar–Disney movie *Inside Out* follows the 11-year-old girl Riley as she is coming to terms with complex emotions. Throughout the movie the main characters represent the emotions that she has inside her head. They are simplistic and one-dimensional—Joy, Sadness, Anger, Fear, and Disgust. These are all in some sense strategies for dealing with particular situations. As Riley learns throughout the movie, as the situations become more complex so do the emotions that arise. What is a situation when you have experienced two or more emotions

that have been in conflict with one another? Did one "win" or was there some kind of a compromise? Did your rationality come into play to mediate? If so, how?

- Competition and cooperation were major themes of this chapter, usually framed in terms of wins and losses. The term *commensalism* is a middle ground—when two entities coexist with no benefit or harm to either. Choose a living and a nonliving system that coexist but do not appear to impact one another either positively or negatively. Explain why you believe this is an example of commensalism. Are there situations, or perhaps future changes, that would change this relationship, either for benefit or harm?
- An idea only briefly mentioned in the chapter was that an agent could not only play against other agents but also understand their strategies. When an agent sees another agent winning more than they are, a higher-level strategy is to adopt their strategy. There are in fact tournaments (like Axelrod's) where agents are allowed to adopt the strategy of one of their neighbors if it seems to be working better. Are there strategies that you have adopted from others? Are there cases where that has been a good thing? Are there other cases where this meta-strategy has not worked out well?
- Watch a team sport where strategy is involved. Listen to the announcers, and record the various ways in which their analysis maps to game theory. When do they talk about smart strategies that will yield short- or long-term gains, versus when they talk about bad strategies? Notice cases where the announcers either missed the long-term strategy or were wrong about the payoff in the end.
- In playing any sort of game against another agent, it is common to create a simplified model of that agent. Doing so, usually based on past data, will help you better predict their moves. Name an agent with which you interact (it doesn't necessarily need to be a person) for which you feel you have created a fairly robust internal model. How do you use this internal model to make predictions? Are your predictions ever wrong? If so, did you update your internal model?
- Fitness might be measured on multiple levels of a hierarchy. For example, when considering economic systems, fitness rewards (and punishments) might be directed toward an individual, family, city, region, or nation. Likewise, what is good for the human species might not always be good for the larger ecosystem. The result is a tension (of rewards and punishments) that cross hierarchical levels. Pick an example of a cross-level tension and deconstruct the tension using concepts and vocabulary from this chapter.
- Pick a favorite poem. Deconstruct how words or phrases form either synergistic groupings (cooperate) or are in tension (defect). How has the author used one, or the other, or both?
- Imagine that you inherited a major professional sports team. Due to events before you acquired the team, you need a head coach. Who should you choose? Is it best to pick someone who has deep knowledge of the game and other teams, as well as a proven management style? Or should you pick the player-coach who is well-known by the press, will be immediately respected by the current players, and may bring in fans? What information would you try to collect? Who would you

talk to? Can you think of other situations outside of sports where similar types of decisions must be made?

- Game theory often is framed as being about creating a cycle of winning such that the rich get richer. The down side, especially in zero-sum games, is that as the rich get richer, the poor get poorer. Many social justice movements aim to level the playing field and break this cycle. Choose a social justice issue and dissect the resource allocations, strategies, and heuristics that are propagating the disparity. What new strategies might be implemented that would rebalance the disparity?
- In 1960 Theodore Levitt introduced the term *marketing myopia*, radical at the time, that companies should focus on pleasing their customers rather than selling as many products as possible. Levitt's claim was that in the long run, such a strategy would pay off. He claimed that company should broaden the way they think of their competition. For example, consider the competition for a luxury car company such as Lamborghini. On the surface their competitors would seem to be other luxury cars such as Ferrari, McLaren, Porsche, Bugatti, and Aston Martin. The competition is in fact much wider, including many high-end luxury items such as vacation houses, properties, experiences, and so on. Both of Levitt's points are about broadening scope—either in the time scale of one's strategies or the breadth of who one considers to be a competitor. Is there an organization that you think should broaden their viewpoint? Do they have a problem of time scale, or competition, or both? What new strategy do you suggest they adopt?
- When game theory is applied to repeated interactions between multiple members of a society, some claim that moral reasoning and development emerges. There are a variety of theories of moral systems and development, but the most prominent is Lawrence Kohlberg's (1925–1987) six stages of moral development. The stages are:

  1. Obedience (avoid punishment)
  2. Self-interest (payment and benefits)
  3. Interpersonal social norms (staying popular)
  4. Authority and social order (obeying personal laws and duties)
  5. Social contracts (personal and cultural respect and justice)
  6. Universal ethical principles (principled conscience)

  The progression through these stages can be viewed as driven by two parallel development processes. First, cognitive development enables a person to move from simply reactive, to having more and more cognitive capacity which enables more and more sophisticated strategies. Second, the view of the self expands outward to include genuine care and empathy, resulting in more and more people being considered in one's strategies. Kohlberg's claim was that most adults reach stage five, but there are only a few people in history who have truly reached stage six.

  Although influential, there are several criticisms of Kohlberg's Stages. First, an individual often compartmentalize aspects of their life and may express one stage in some arenas of their life and other stages in other contexts. Second, the idea of stages can imply moral superiority that simplistically reinforces male-dominated Western ideas of justice. Third, some conservatives warn that the stages imply a liberal view of the world. Pick one of these criticisms and analyze it using the ideas presented in this chapter.

- An interesting thought experiment was created in 1994 by Brian Arthur (1946–) and is known as the *El Farol Bar problem*, after a popular bar in Santa Fe, New Mexico. Imagine that every Friday night everyone in Santa Fe must decide whether or not to go to the bar or not. The problem is that an empty bar is no fun to be at, but if everyone goes to the bar then there will be a long line to get in. Everyone needs to decide whether or not to go to the bar around the same time. The issue is that if everyone uses the same strategy (e.g., stay at home because it is assumed the lines will be long), the balance will be tipped to one of the extremes (the bar will in fact be empty). What are some other examples of group decisions that are similar to the El Farol Bar problem? What are some of the dynamics in your example?
- The term sometimes used to describe an integrated biological community is a *biome*. One of the more unusual is the diverse biome of microorganisms living in the human gut. These organisms have evolved to survive in a harsh environment and are in fact very helpful, acting as a barrier to invaders, fermenting some fibers, synthesizing vitamins B and K, and metabolizing a variety of acids. This unique ecosystem emerges around the age of two but then will adapt to changes in diet and health. Some antibiotics can in fact wipe out large populations of these helpful microorganisms, leaving the gastrointestinal system vulnerable to other pathogens. An interesting exploration can be found in *The Mind-Gut Connection* by Emeran Mayer (1950–). The idea of a biome can also be applied to human-created systems as well. Choose a human-created system and explain the various actors in its biome. Are there any players that play a similar role to the helpful bacteria that populate your gut?
- A mantra of the innovator, entrepreneur, and change agent is to *fail forward*. The general idea is to learn what does not work as quickly as possible, narrowing down to the few good ideas that may actually work. Often this means trying out an idea in a real situation, what is generally called *prototyping*. Prototyping can take many forms, both tangible and intangible, and in some situations is an excellent strategy. When you practice a presentation to a small audience as a way to rehearse, you are engaging in prototyping. In other situations failing forward can be costly, illegal, or unethical. For example, it is illegal in the United States to test a medical device on patients without a long and costly formal clinical trial. In your own life, what are examples when you have created a sort of prototype so that you could fail forward? On the other hand, what is an example of something you would like to prototype but cannot or should not?
- Biological fitness landscapes are typically difficult to study because they change on the time scale of evolution and are dependent on many factors. Most studies therefore attempt to deconstruct available historical data that is incomplete. That has not stopped some scientists from designing controlled experiments that alter the fitness landscape on which an organism is evolving. An article by Santiago Elena (1967–) and Richard Lenski (1956–) "Evolution Experiments

with Microorganisms: The Dynamics and Genetic Bases of Adaptation" reviews how a genetically identical population of bacteria can be split, exposed to different conditions (e.g., sugar solutions, temperature, etc.), and watched over tens of thousands of generations. As predicted by evolutionary theory, the populations diverge in a way that makes them more fit in a given environment. These changes were not only achieved through the genetic expression of latent functions but also through new mutations. Even the rate of mutation was found to change in some experiments—unfit populations ramped up their mutation rate to expand diversity, thereby enabling better sensing of the location of the new fitness peak. Technological, cultural, economic, and political systems also can be thought of as evolving on a fitness landscape. What kinds of controlled experiments can you imagine designing to explore evolution on these human-created fitness landscapes? Be as specific as you can be about the population sizes, resources needed, and collaborators you would want to recruit to help you run such a study.

- Many types of auctions exist whereby individuals bid on a particular item or service. Find an online bidding site, and classify the types of auctions they allow—note that you may need to do some general research into the types of auctions first. Then describe a strategy that over time you suspect would pay off as you bid on several different items.
- We explored the idea that a strategy might be hard to displace by other strategies. The same could be said of ideas. What idea that has entered the world relatively recently (e.g., the last 30 years) do you think will not be displaced? Explain your rationale.
- Exaptation can be used in daily life, for example, when you use a book to keep a door open. But regular objects can sometimes be used in hilarious ways in an improvisational game. Gather a few friends in an environment that is rich with objects. Take turns picking up an object and then quickly improvising other ways you could use it. You can do this either in individual turns or as a group. What were some of the funniest exaptations? What made them so funny?

8.7 Resources and Further Reading

In addition to the works mentioned in the body of the chapter, there are some excellent introductions to game theory. *Game Theory: A Nontechnical Introduction to the Analysis of Strategy* by Roger McCain is a nice place to start. An even more fun version is Len Fisher's (1942–) *Rock, Paper, Scissors: Game Theory in Everyday Life* where the author attempts, sometimes in very amusing ways, to use game theory in his life. There is a particularly interesting set of meta-strategies that Fisher proposes toward the end of the text that can help almost anyone build a strategy. Another fun introduction is Haim Shapira's (1962) *Gladiators, Pirates and Games of Trust*. Robert Trivers (1943–), a pioneer of game theory as it applies to biological systems, has written *The Folly of Fools: The Logic of Deceit and Self-Deception* *in Human Life*. It is a wonderfully honest tour of game theory in human life, with Trivers sharing many personal stories from his own life. Matt Ridley's *The Origins of Virtue: Human Instincts and the Evolution of Cooperation* lies at the intersection between game theory, evolution, and morality. For applications to business, it is worth exploring *The Art of Strategy* by Avinash Dixit (1944–). Although not focused on game theory, John Holland's *Hidden Order* contains a sort of blueprint for how agents that can operate in an environment might emerge and begin to adopt strategies.

Chapter 9
Dynamics of Networks

Imagine that you wish to drink a cup of coffee. First you must open your fingers. This action is controlled by a series of neural circuits—one small subnetwork within your brain. That circuit sends information, encoded in a sequence of electrical impulses, to the appropriate place in your motor cortex. The motor cortex will recode the information to send it down your spinal cord, and then out to the nerves in your hand. Second, you must reach out your arm. This is a different neural circuit that will send a signal to the appropriate place in your motor cortex and out to your arm. Then the first circuit is re-activated, but this time in "reverse" to cause your fingers to close around the cup. Then the second circuit is reactivated (again in reverse) to move your arm to your mouth. Lastly, a third circuit must be activated to tip the cup the right amount so that a slow and steady stream of coffee will enter your mouth.

In reality, this sequence has been greatly simplified in at least three important ways. First, the motion of your fingers and arms are controlled by many circuits, not just one, that must work together. The same goes for the arm motions. Second, there are other motions that we have not covered, such as moving muscles of the mouth and eyes to provide the feedback necessary to know where your arm and fingers are in relation to the cup. Third, the neuroscientist Eva Marder (1948–) showed that the same network of neural connections could produce different functions.

In general, the structure of a complex system constrains a flow (e.g., money, ions, energy, people, neural impulses), and it is this flow that dictates the functions that are expressed. Neural control is just one of many examples. Previous chapters have focused on either structure or function. Chapters 2 and 3 focused on the function of individual agents and nodes, while Chaps. 5 and 6 where mainly concerned with how structures create constraints. In Chaps. 7 and 8, structure and function began to blur. In this chapter we will expand upon the functions that emerge when there are flows on networks.

J. Tranquillo, *An Introduction to Complex Systems*,
https://doi.org/10.1007/978-3-030-02589-2_9

9.1 Network Effects

There are a range of behaviors that can arise in networks when the nodes can display their own behavior. Some of these have already been explored in previous chapters, most especially the section on agent based modeling in Chap. 2 and multi-player games in Chap. 8. In this section some new behaviors and properties of network dynamics are introduced.

9.1.1 Diffusion of Innovations

Everett Rogers (1931–2004) developed a theory of how ideas and products spread on networks. He considered three elements; the innovation, the potential adopters and the network. People gain information that they use to assess the value of adopting an innovation. The value itself is roughly the ratio of benefit/cost. As a person gains more information they are simply becoming more certain of the benefits and costs. Rogers' master work, *Diffusion of Innovations*, which he revised several times throughout his lifetime, has become the theory upon which much of modern marketing is based.

Figure 9.1 represents the way in which an individual moves from awareness that some idea or product exists, to either adopting it or not. During each of the five phases, the individual is weighing the expected value against unknowns. In some sense they are playing a sort of game with the innovation, gaining information through an iterative strategy to determine if they want to play longer.

A potential adopter can exit the adoption pathway at any phase, but likewise they might reenter again at some later point in time. The time it takes an individual to move along the pathway is highly variable. This timing is influenced by a variety of factors, which will be discussed below. Rogers purposely separated the decision phase (a mental phenomenon) from the implementation phase (an action) because it is between these two steps that an adopter often stalls—Many of us decide to do something, but then it takes a while to get around to actually doing it.

The phases of adoption have several consequences when taking place on a network. Figure 9.1 shows the *channels* that individuals typically use to gain information at each phase. Some information is available to all (or most) in the form of mass media, while other sources of information are through personal observation or conversation. In many ways marketing has traditionally been thought of as mass media, whereby information is actively sent out, sometimes to particular individuals who are most likely to adopt. To enhance targeting, many marketers will create *archetypes* of potential adopters so that they are not wasting their resources trying to reach everyone. The more personal sources of information will be discussed below when we explore how rumors spread.

Another feature of the diffusion model is how an individual moves from one stage to the next. At each phase, a person is becoming more certain of the value of

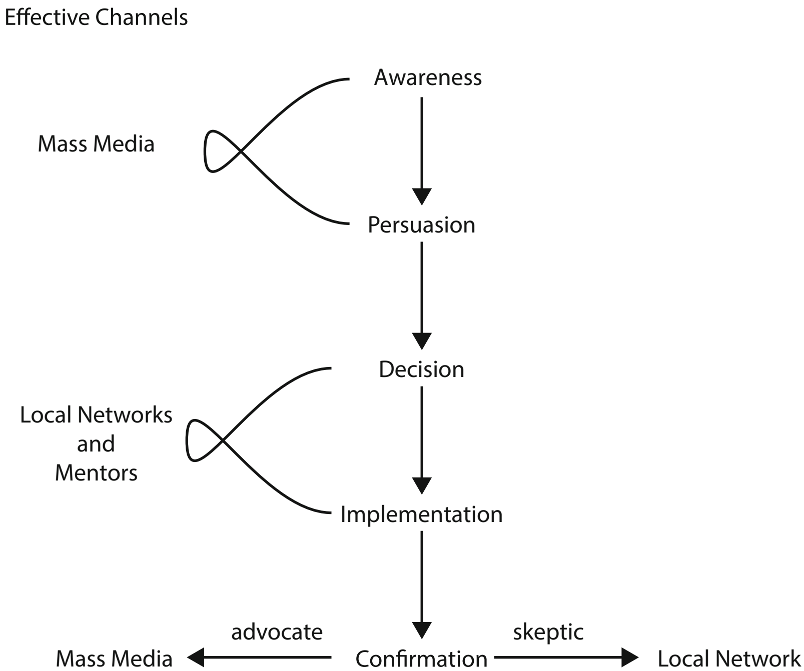


Fig. 9.1 Rogers' decision making flow in an individual adopter that progress through five stages. Moving from one stage to the next is triggered by a reduction in the uncertainty in the value of adoption. Various information sources (channels) may aid in this reduction, with some being more influential than others at particular stages

Table 9.1 Stages in Rogers' diffusion of innovation framework

Stage	Description
Awareness	First contact with the innovation
Persuasion	Information is collected on the value (benefit/cost) of the innovation
Decision	Convinced that the value is high, and uncertainty is low
Implementation	An action is taken to adopt the innovation
Confirmation	Adopter either fully adopts the innovation (advocate) or not (skeptic)

the innovation. Moving on to the next phase occurs when a particular threshold of clarity is achieved in the value (e.g. benefits and costs). In other words, moving from one phase to the next occurs when some minimum tolerable threshold in uncertainty is the surpassed. Because different people have different thresholds for adoption, similar to the mob formation idea in Chap. 2, there will be some who do not require much certainty on the value they will gain. Rogers coined the term *early adopters* to describe people with lower thresholds who move through the adopter sequence relatively quickly.

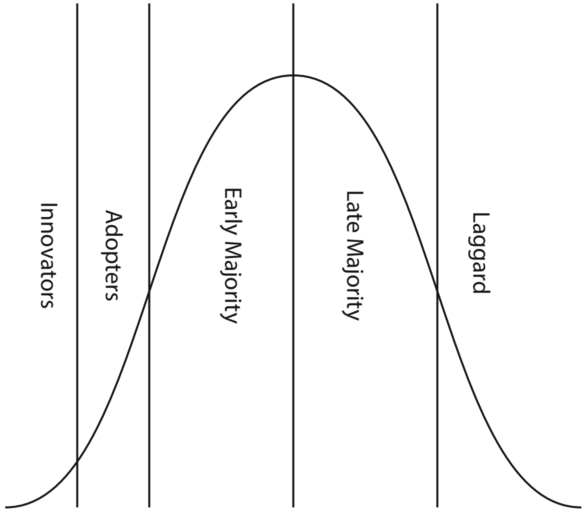


Fig. 9.2 Rogers' bell-shaped curve for the proportion of adopters in a population. Each region has a particular archetype that is generally defined by how uncertain they are willing to be before adopting an innovation. The earlier the adoption the less certainty is needed

The consequence of the variability within a population led Rogers to hypothesize, and later to fit to data, a "bell-shaped" curve of adoption, as shown in Fig. 9.2. The curve, representing a population not an individual, is split into five sections, with the area in each section representing the approximate percentage of the population that might adopt the innovation. On the far left are the innovators who have developed the idea or product. There are generally very few of them and because they created the innovation they can most clearly understand the value. The next to adopt are the early adopters who, as stated earlier, are willing to tolerate a great deal of uncertainty. In general there are more early adopters than innovators, but there are still not very many people in this group. The bulk of the population of potential adopters, perhaps up to 70% of the population, fall into either the early majority or late majority. At the far end of the curve are the laggards who will resist adoption until non-adoption becomes socially unacceptable or perhaps even mandated by law or employers.

Another way to view the adoption curve is to sum up the areas over time to form the S-shaped curve in Fig. 9.3. Adoption starts off slowly with only the innovators and the early adopters. The curve becomes much steeper as the early majority adopts, and continues to steepen as the late majority adopts. Eventually the innovation *saturates* when there are no more potential adopters left. It is not a coincidence that this curve is essentially the same as a second order phase transition.

In 1991 Geoffrey Moore (1946–) published *Crossing the Chasm*, in which he explored the idea that there is a critical gap between the early adopters and the early majority. Jumping over this gap, capturing the early majority, is the key to an innovation taking off. It is a sort of tipping point or bifurcation in the adoption that corresponds to the steep part of the S-shaped adoption curve. Like the activation energy in a chemical reaction, once crossed the network will spread the innovation as a sort of spontaneous reaction. If the innovation is a product, this also will often correspond to the financial break-even point.

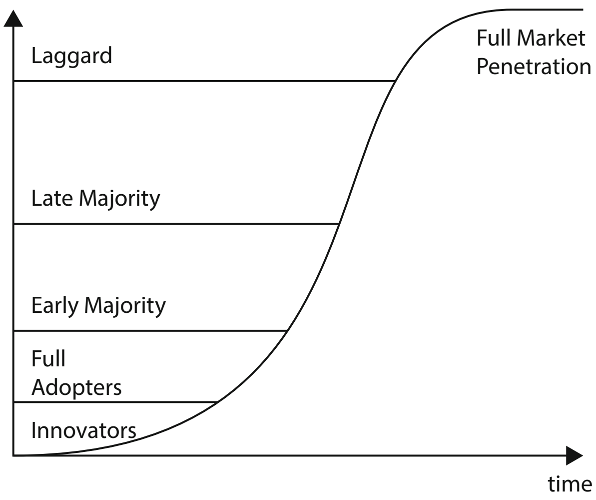


Fig. 9.3 Rogers' S-shaped curve over time, as an innovation diffuses into a population. The S-shape is the area under the curve of Fig. 9.2

In Chap. 5 Chris Anderson's idea of the long tail was introduced as a phenomenon brought about by the internet, whereby a company can find a small market that might be scattered among the population. Using Rogers' idea of diffusion, it may be possible to survive in a niche market by only capturing early adopters if you can find enough of them. The strategy of the company is in fact not to diffuse the innovation more broadly, but rather to maintain a "cool" status among a smaller number of customers. Similar concepts apply to niche music, art house movie theaters and radical fashion.

An individual does not always play the same role on a network (e.g., early adopter or laggard), as these roles are relative to the innovation. For example, an individual may be an early adopter with regard to fashion, but a laggard when it comes to high tech devices. Likewise, an individual might change over time, perhaps being an early adopter of technology early in life, but later in life becoming a later adopter. Superimposed upon these dynamics are the many ways in which individuals gain information from varying sources, ranging from mass media to local friend networks. What is more, not all of the information they gain will necessarily align, which could add to uncertainty and delays in their movement toward adoption.

Knowing how a past innovation has spread on a network can give some insights into how a new innovation might spread on that same network. Keeping track of the individuals who adopted and which channels worked can enable better targeting and timing of messages. Some marketers have even taken to building networks of early adopters so that future innovations will spread more easily.

Rogers created his theory before social media existed. But Fig. 9.1 can be used to understand just how disruptive Facebook, Pinterest, Instagram, Snapchat and other companies, as well as sites that allows for crowd-sourced voting such as Amazon

Table 9.2 Rogers' elements of an innovation

Innovation elements	Description
Simplicity	Ease with which an individual can understand the innovation value
Compatibility	How the innovation fits in with already adopted innovations
Trialability	A potential adopter can try out an innovation
Observability	How apparent it is that someone is an adopter
Relative advantage	How the innovation compares to similar innovations

and Angi's List, can be. Before these apps and sites, when someone adopted an innovation they could only tell their local network about it. Likewise if they became a skeptic they could only complain to those in their local social network. This has changed with social media.

9.1.2 Properties of Innovations and Metcalfe's Law

The spread of ideas on a network can depend greatly on the type of idea or technology that is being spread. Rogers classified five elements of an innovation, shown in Table 9.2, that would make it more likely to diffuse quickly and deeply into a network.

Innovations that score high on these attributes will be more likely to spread. Each attribute, however, is the *perception* of an individual; What scores high on simplicity to some may not appear simple to others. These five attributes may be used when developing an innovation to make it more *sticky*. For example, a variety of products will claim that they have a "30 day money back guarantee." This is amplifying the perception of trialability.

Some innovations become more powerful as they are adopted by others. For example, when there were only a handful of telephones, the value in having a telephone was fairly small. As more and more people began to buy and install phones in their homes, it encouraged even more people to get a phone.

This virtuous cycle was articulated by Robert Metcalfe (1946–) to describe the adoption of the Ethernet, which he co-invented in 1973 and now forms the backbone of internet communication protocols. In general, technologies that make connections will follow *Metcalfe's Law*. Classic examples include the telegraph, fax machine, email, cell phones, internet-enabled computers and smart devices. A single person with one of these devices would find it to be completely useless. Like the Kauffman's button network described in Chap. 5, Metcalfe's Law follows a second order phase transition.

One downside is when an innovation begins to die off, such as happened with fax machines. A technology's death can be just as rapid as its adoption. In other words, un-adoption can follow a similar kind of curve but in reverse. A second downside is that some innovations become less valuable as they spread because the allure is their uniqueness. This is certainly the case with many movements, such as fashion and lifestyles. For example, many of the original hippies, yuppies, and hipsters gave up their ways when too many others latched onto their way of life.

9.1.3 The Network Power of Weak Ties

In 1973 Mark Granovetter (1943–) published "The Strength of Weak Ties," in which he explored how recent graduates find their jobs. His thesis was that everyone is embedded in multiple networks of who they know. In some sense they are the hub of their own personal network that includes their family, friends, classmates, colleagues and so on. But each person with whom they are connected is also the hub of their own network of connections.

When a new graduate is looking for a job they are engaged in a sort of game with employers where each is looking for a win-win. As we explored in Chap. 8, achieving a win-win requires trust. Interviews, reference letters, and other forms of communication between an employer and potential employee are about gaining information that will build trust that a hire will result in a win-win.

What Granovetter explored was the very first step in this process, what Rogers would call awareness. Often a third party will introduce an employer to a potential employee (remember this was in the 1970s before internet job search sites). But who makes the recommendation matters. We generally trust information when it is coming from a source that has nothing to gain (or lose) in sharing information. An unbiased source can be trusted. When your advocate is a parent, they have a perceived (perhaps real) bias, whereas the same recommendation coming from a virtual stranger with nothing to gain, is more trusted. It is not who you know, but rather the strength of your second-order ties (those with nothing to gain) that often matters the most.

9.1.4 Inhibition

Much of network theory is focused on connections that spread information or create some sort of activation. Network connections might also inhibit (or sometimes turn off entirely) a communication channel or function of the system. You can think of this as one part of the system telling another part of the system not to receive (or send) information. For example one function might turn off other functions so they do not consume resources. Inhibition may be very targeted (e.g., assassination of a key leader, synaptic inhibition in the brain, personal insult) or global (e.g., spread of a drug or neurotransmitter into the bloodstream).

While inhibition is a basic characteristic of many types of systems, there are a number of ways that inhibition can be manifest in a system. Perhaps most straightforward is if some action requires a resource to function. It might be money, an ion concentration, or power source. The lack of distribution of that resource can inhibit activity. This is the case in the brain, where the vascular network supplies blood. If neurons begin to fire quickly and use up a resource such as glucose, they will effectively become inhibited.

A poison, either targeted or systemic, can inhibit too. Its presence means that the system cannot function or perhaps has some sort of downgraded function. Inhibitory neurotransmitters work in this way by changing the thresholds for neural firing. A similar idea occurs in cold blooded animals—lowering their internal temperature will slow down metabolism to conserve energy. Likewise, information from a friend might inhibit you from taking certain actions.

The properties of the network itself might also inhibit certain behaviors. For example a pathway for flow may simply be limited in size, what is often called a *bottleneck*. When it rains very hard, rivers and tributaries may not be large enough to support all of that water, resulting in a flood. The functional result is a type of inhibition of normal function of a river in efficiently moving water to the sea. A similar idea occurs in traffic flow when many cars must pass through a narrow roadway, bridge or construction site.

9.1.5 Network Balance

The mix of activating and inhibiting connections between nodes leads to the idea that some network configurations may be inherently stable or unstable. A nice analogy is in a friend/enemy network. If you and I share a common enemy, is a stable network configuration. Likewise, and more pessimistically, I could be the outsider, with two enemies, which is also stable. But what happens when you are friends with two people who dislike one another? This is often an unstable situation. On the other hand a network where three people all dislike one another is stable. These possible network characteristics are shown in Fig. 9.4. What can be proven is that as long as there are one or three positive links, this small network will be stable.

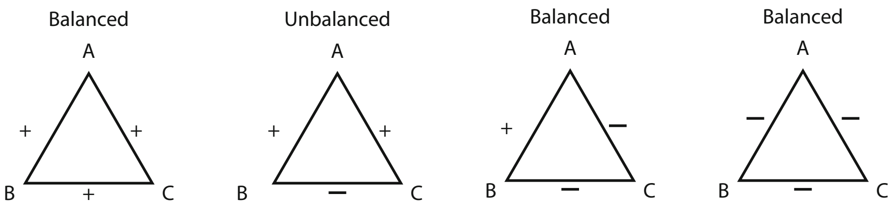


Fig. 9.4 Balanced and unbalanced connection patterns for a triad of nodes. In general an odd number of positive connections will result in a balanced network

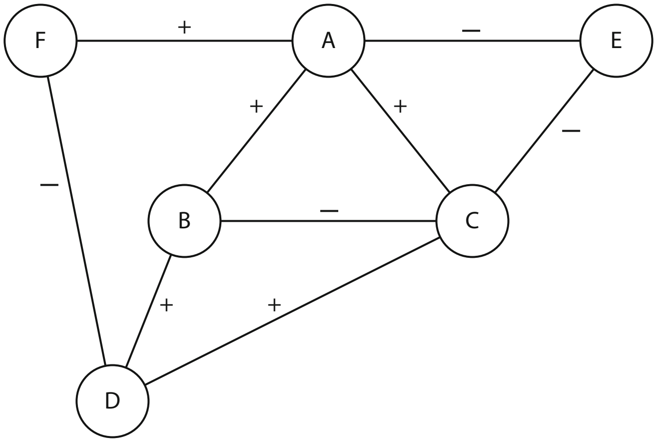


Fig. 9.5 A complex network that is between balanced and unbalanced. Some triads are balanced while others are not. Likewise some nodes are more balanced than others

These effects can become more complex as more nodes are added. There are in fact theorems that explain how to calculate whether a set of weighted relationships will be stable or not. In some networks the connections may be weighted, dynamic and perhaps even switch signs. This means that someone might be your friend or ally some of the time but your enemy at other times. This is captured perfectly in the term “frienemies.”

Some complex systems are partially balanced, as in Fig. 9.5. There are regions of balance and regions of imbalance. This partial instability is enhanced when the network is structurally adaptive, with nodes and connections coming and going. Remove a few units (or add a few) and the system may become more or less stable. Almost all complex systems will have some degree of latent instability. For example, in the brain there is a balance between excitatory and inhibitory synapses. Likewise, John Holland discussed a hypothetical triangle ecosystem, where no two species can exist without the other two. One can imagine bimolecular pathways, economies, social networks and distributions chains between companies that form triangles or other more complex networks, that may exist between stable and unstable. The idea that complex system necessarily contain some internal tension will be explore in Chap. 10.

9.1.6 Spread of Viruses, Rumors and Fashion

A popular model of diffusion on a network is derived from the spread of viruses, known as the SIR model. The S-term is for susceptible, meaning that an individual can be infected. The I-term is for infected, meaning that the individual has acquired a contagion and can spread it through contact with others. The R-term is recovered, meaning that the individual is no longer infected but spends some amount of time when they are not susceptible. The SIR model, and its variants, have been used to simulate rumors, fashions or memes that might spread throughout a population.

An individual can go through a cycle of being susceptible, then infected, then recovered, and back to being susceptible. In some derivative models, immunity is added such that once an individual is recovered they cannot be infected again. Other models can also include vaccinations whereby an individual can be inoculated against a virus.

When considering a no-growth population of N people that is well mixed (e.g., everyone could potential meet everyone else)

$$N = S(t) + I(t) + R(t)$$

This means that the total population will stay the same but the number of people that are in the states of S, I or R can change. The equations used to describe the number of people with states S, I and R are

$$\frac{dS}{dt} = -\beta\frac{SI}{N}$$
$$\frac{dI}{dt} = -\beta\frac{SI}{N} - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$

In this compartment model, β controls the rate at which people move from being Susceptible to Infected and γ controls the rate at which people move from being Infected to Recovered. These parameters are often properties of the virus such as how easily it spreads as well as how long an individual remains contagious. Figure 9.6 shows example time courses of the rise and fall of a contagion.

As described above there are many other possible variations that involve changing or adding to the SIR model. For example it is relatively easy to explore the effect of removing people from the population, either through quarantine or death, as a decrease in the value of N. Such numerical analysis can help epidemiologists make predictions about how a virus will spread, as well as who should be inoculated or quarantined.

The general formulation of the SIR model is for a mixed population. In this case the values of S, I and R at any point in time can be thought of as the number of individuals in that state. The same equations, however, can also be used to form an agent-based model for an individual. Here the value of S is a relative measure of how susceptible the agent is. Likewise I is a relative level of infection and R is how well recovered an individual is. When this adjustment is made, the SIR model can become an agent-based model that plays out on a network. Individuals are the nodes, and once a node becomes infected, it can potentially infect the nodes to which it is connected. This is the opposite of a well-mixed population because an

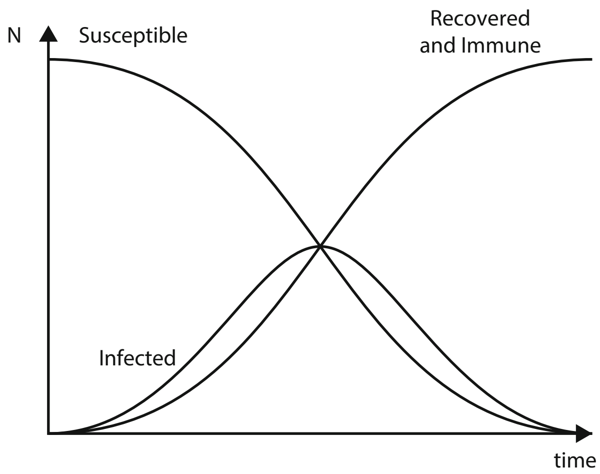


Fig. 9.6 The rise and fall of a contagion in a fixed, but well-mixed, population

individual will only be in contact with its immediate connections. The spread of a contagion is now governed by both the parameters of the virus (γ and β parameters) and the structure of the network. A rich area of continued research are the emergent behaviors that can arise from the SIR model on various types of networks.

9.2 Network Motifs and Functions

In Chap. 7 we explored the general idea that flows on small structures within a larger network can perform basic functions. These might include batteries (internal energy sources), communication and energy management networks, sensors, clocks, memory, amplifiers, filters and actuators. The recombination of these simple parts can create higher level functions. In this section we will explore more deeply a few examples of how these kinds of simple functions can be created within small networks.

9.2.1 Motifs

A *motif* is a small functional network of connections. To explore the idea, we will follow the work of Uri Alon (1969–) in how genes are expressed. In general there is a section of DNA that contains the gene, but just prior to that gene is a small region known as the *promoter*. The promotor takes in a signal, X, known as a *transcription factor*. When X is present it will either *activate* or *represess* the rate of mRNA

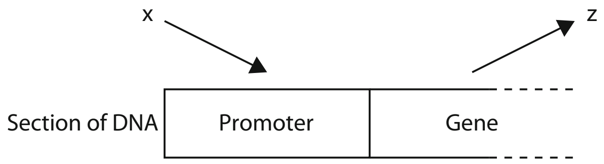


Fig. 9.7 Transcription factor, X, binds to a gene promotor region, which triggers the production of the protein Z

production, which is responsible for producing the protein, Z, using the information in that particular gene sequence. The level of the input signal, X will determine how much of the protein is expressed (or how strongly it is repressed). Figure 9.7 is graphical representation of the promotor/gene relationship.

Mathematically we can capture this idea as

$$\frac{dZ}{dt} = f(X) - \alpha Z$$

where α is the rate at which Z is degraded. The Hill function is used for $f(X)$. As an activator the Hill function is

$$f(X) = \frac{\beta X^n}{K^n + X^n}$$

and for a repressor

$$f(X) = \frac{\beta}{1 + (X/K)^n}$$

where β is the maximum expression, K is the activation (or repression) coefficient, and n is known as the Hill coefficient. Example plots of $f(X)$ can be found in Fig. 9.8 for activation. As n becomes larger, $f(X)$ will more closely represent a step function. This was the idea expressed in Chap. 6 when discussing how genes can act as logic gates. Another way to think about the Hill function is that it represents a kind of bifurcation in the behavior of the system.

The output of a particular protein, Z, could be used to create a part of an ion channel or some other component of a cell. Z could also be a signal that is used to communicate within the cell or a transcription factor for the activation (or repression) of another gene. In this latter regard a complex network can emerge, whereby the proteins generated from various genes can activate (or repress) one another.

Alon's lab studied the network structure of a number of different systems, including gene networks, but also neural connections in the worm *C. elegans*. In each of these systems they define groups of three connected units as a triad. There are 13 different ways three nodes can be connected (both directed and undirected),

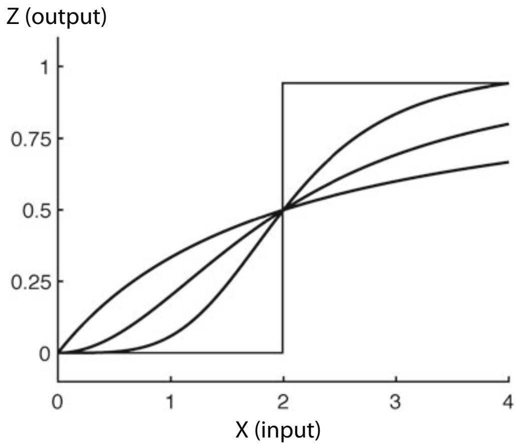


Fig. 9.8 The Hill function, $f(X)$, for varying N. As N increases, the function becomes more and more of an on-off switch. $N = \infty$ generates a sharp step function

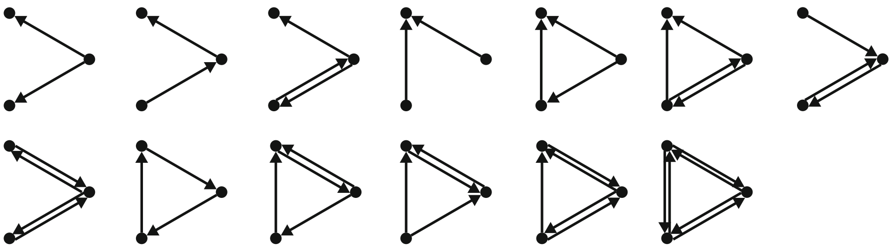


Fig. 9.9 Thirteen possible connection patterns for three nodes. Some motifs have been shown to appear more often in a larger network, presumably because that triad produces a useful function

as shown in Fig. 9.9. If all of these patterns in a large network were equally likely then Alon expected to see all 13 connection patterns with about the same frequency. Surprisingly, his group found that in most networks particular triads were found much more often than others. What was more, the two or three most observed motifs were different depending on whether they were gene networks or neural networks. They repeated the study for groups of four units (where there are even more possible connections) and found the same striking pattern. Alon later went on to show how these structural patterns could lead to useful and stable functions.

The idea of repeated motifs supports the concepts of modularity. For example, in the cognitive world the mind has been described as modular, and in biology the combination of genes and proteins in the Kreb Cycle has been conserved throughout evolution. The idea of modularity also highlights that complex functions, as combinations of simpler functions, are only fit (in the sense of a fitness landscape) within a particular environment. Searching for a particular gene, or perhaps even a small set of genes, that govern a complex function such as intelligence or criminality, almost always fails.

9.2.2 Networks as Logic Gates

Alon studied a variety of the commonly found motifs. The coherent feedforward motif that expresses $Z = f(X \text{ AND } Y)$, is shown in Fig. 9.10. Z turns on after a delay in Y, when Y crosses some threshold. But Z then turns off immediately due to the signal X. Such dynamics form a low pass filter and can protect against brief input fluctuations. It can be used as a basic persistence detector—Z will only go up if X is present for a long period of time.

The same network structure, however, can act in a different way if the coherent feedforward motif expresses $Z = f(X \text{ OR } Y)$, shown in Fig. 9.11. Z turns on immediately when X is present, but then has a delayed turn off when Y is present. The functional result is protection against the loss of an input signal. In a real

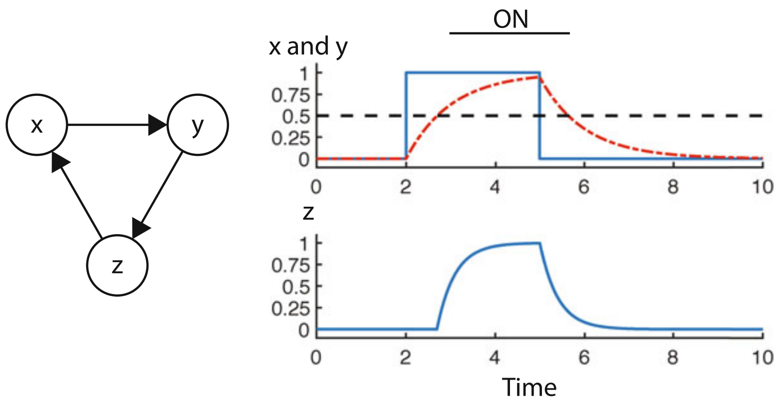


Fig. 9.10 The coherent feedforward motif that computes $Z = f(X \text{ AND } Y)$

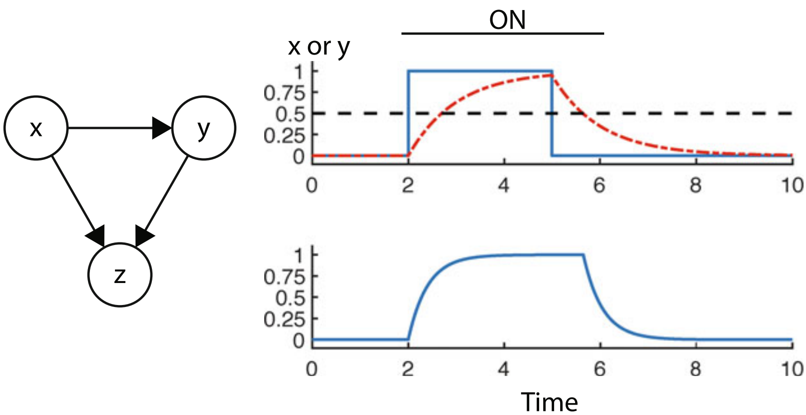


Fig. 9.11 The coherent feedforward motif that computes $Z = f(XORY)$

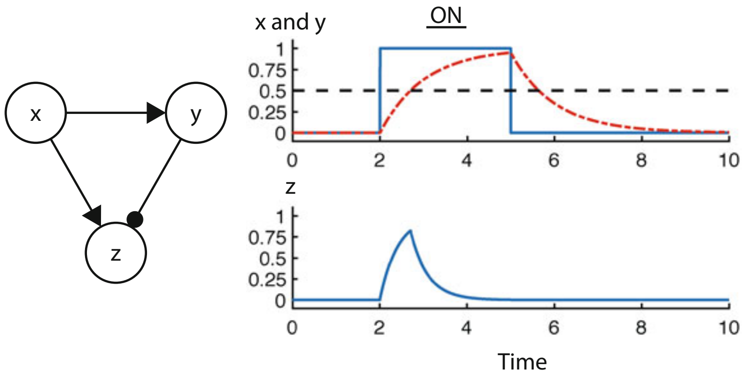
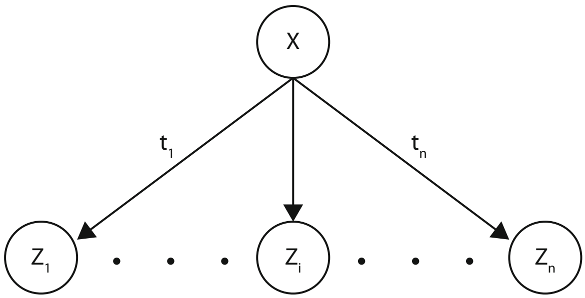


Fig. 9.12 The incoherent feedforward motif. The connection between Y and Z (denoted by a line with a dot at the end) is inhibitory

Fig. 9.13 A master transcription factor, X, that switches on expression of several proteins, $Z_1 \dots Z_n$, with delays $t_1 \dots t_n$



network this kind of motif can be used as a warning signal—Z will go up even when X is present for a brief period of time.

Because the combined function $f(X, Y)$ can be represented as a Hill function, the parameters K, β and n will determine the exact timing and thresholds of how Z is expressed. What is more, these parameters can be tuned over time to yield the proper function. More on this kind of tuning will be explored in Chap. 10.

The signals X and Y can be any internal or external signal. This means that they can be used in sensors that are outward facing, inward facing or perhaps even some combination. Likewise, Z can act as a transcription factor that activates or suppresses other motifs. For example, in Fig. 9.12, Y acts as a repressor of Z, indicated by the line that ends in a dot rather than an arrow. This incoherent feedforward motif turns on with a rise in X, but off when Y rises. The result is a pulse in Z.

There are a wide range of other possible networks. For example, in Fig. 9.13 one transcription factor could activate (or repress) several genes. In this case X is a sort of master transcription factor, and might even have different delays or thresholds for turning on the various genes ($Z_1 \dots$). Likewise, a wide range of other simple functions, can arise from simple networks. Examples are the clocks

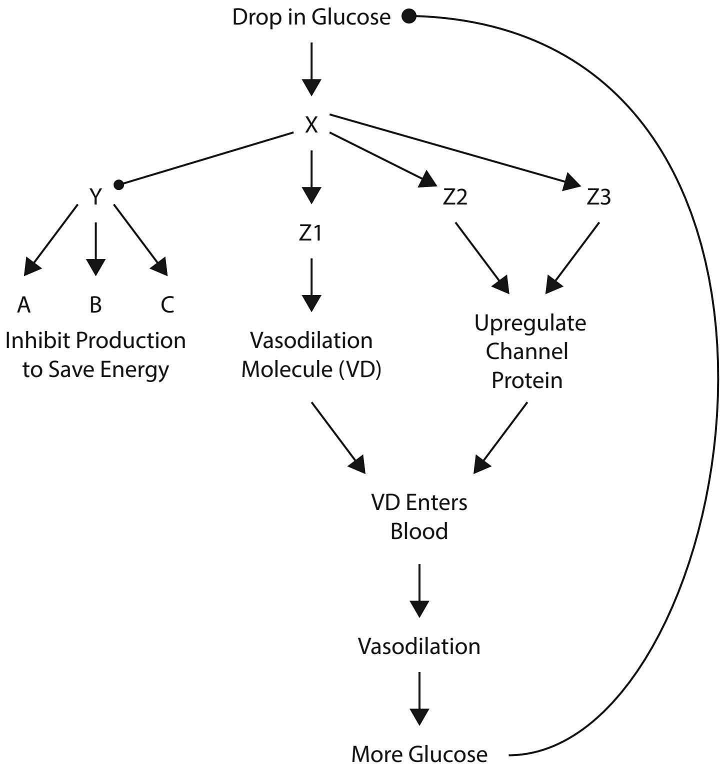


Fig. 9.14 A hypothetical network of transcription factors and gene products (proteins) for glucose regulation

that underly locomotion, financial and ecological cycles (Chap. 4), actuators and sensors (Chap. 7) and memory storage (Chap. 6).

As a demonstration, consider the hypothetical glucose dynamics shown in Fig. 9.14. When a cell has a drop in glucose it must signal for more glucose to be sent to it by the blood stream. Consider that a drop in glucose could be the transcription factor to produce X. But X causes the expression of Y, $Z1$, $Z2$ and $Z3$. The presence of $Z1$ triggers the upregulation of a vasodilation molecule, which when released into the blood stream would act to increase blood flow to the cell. Because the molecule is inside the cell it cannot have the intended function. $Z2$ and $Z3$, however, act together to upregulate cell channel proteins that transport the vasodilation molecule out of the cell. In the background Y is a master transcription factor (similar to Fig. 9.13) that represses the expression of many other genes to conserve energy while the cell waits for more glucose. When the cell finally does receive more glucose, it will turn off production of X.

9.2.3 Recombination and Hierarchies

The recombination of existing structural motifs opens up a combinatorial explosion of possibilities for higher-level functions. A system might need to "try" all of these possibilities blindly. Motifs, however, have a clear input and output, such that motif-to-motif communication must obey certain rules. This drastically narrows the possibilities for recombination. The utility and stability of a higher-level function means that it can also serve as a motif at some higher level of the system. A cell is a node when considering an organism. But an organism is a node when considering an ecosystem. Hierarchies of networks can form (Fig. 9.15) as functions are layered and nested.

In all functional motifs, there are delays inherent in the physical flows on the structure. Generally fewer units will result in a motif that can respond more quickly. Larger groups have longer delays. For example, cells can generally react much more quickly than organisms which can react more quickly than ecosystems. The idea of feedforward was briefly introduced in Chap. 1 as a way to prime a latent function that may be needed in the near future. When internal feedback, feedforward, clocks and inhibition are added, a complex system can bring latent functions into the foreground when they are needed, and repress them when not needed. Such a system will become meta-stable, as explored in Chap. 3.

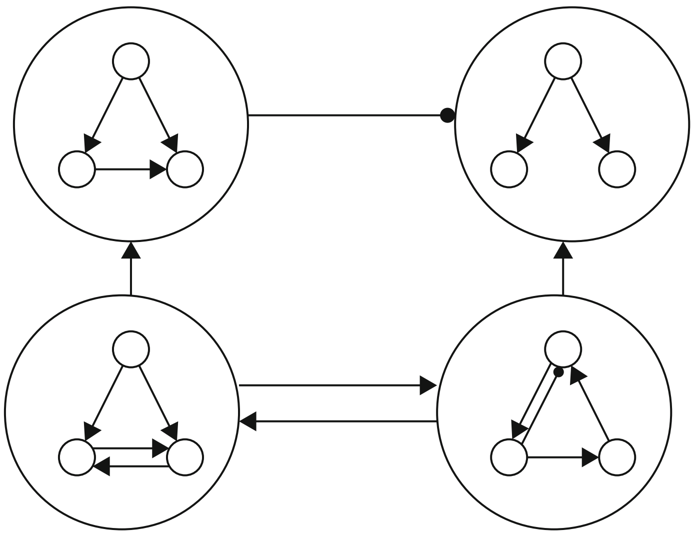


Fig. 9.15 Example of motifs forming nodes at the next higher level of a network hierarchy. Such an arrangement forms a nested network of networks

9.2.4 Pathways to Disease

System diseases can be understood as an incompatibility or imbalance in how parts have been put together. The neural disorder of epilepsy highlights the many ways in which this may occur. The basic functional problem in epilepsy is that neurons synchronize in mass, to the point where large portions of the brain fire together. This is dangerous because it means that neurons are firing as one in a sort of dynamic crystallization or neuronal groupthink. A synchronized network cannot process information. The result in epileptic patients is that they generally do not remember anything that happened during an episode—they are temporarily cut off from the world. Whereas Chap. 4 discussed how various parts of a system might synchronize to perform some function, in epilepsy a cure would be to have some mechanism of desynchronization.

There are many possible causes of epilepsy. Neurons may be over-connected such that signals can converge, likely on hubs, which then act as the pace setters. Such hubs might form naturally due to either a genetic disorder or a physical injury. This is a structural pathway to synchronization. But, the brain also contains a balance between excitation and inhibition at the level of synapses. If this balance is thrown off, for example by there being too little inhibition, neurons will also synchronize. Likewise, a drug, hormone or metabolic byproduct could disrupt the sensitivity of individual neurons, making them more prone to synchronization. In fact, the oldest documented anti-epileptic remedy, used by the ancient Egyptians, was a ketogenic diet that reduces available energy. There is even evidence that a fever can alter the sensitive dynamics of ion channels in such a way that neurons become more susceptible to epilepsy. Epilepsy can arise from a disruption in structure, function or a combination.

Similarly, an economy where everyone did the same thing would not remain a functioning economy for very long. A random economy would be dysfunctional as well. Likewise, if species were constantly coming and going or if the fitness landscape was so rugged that nothing could evolve, the ecosystem would fall apart. A complex system seems to operate most effectively in some middle ground, that is at or near a phase transition. Deviations from this middle ground can be viewed broadly as a disease of the system. We will explore this idea in more detail in Chap. 10.

9.2.5 Learning Revisited

In Chap. 7 we speculated that a learning system is one that can support both functional and structural adaptation. Furthermore, a useful functional adaptation can be somehow turned into a structural adaptation. Another way to restate this is that discoveries are made through a functional adaptation, often as smaller functions are selectively turned on and off and happen to interact with one another. Functional adaptation enables recombinations to be tested out. Learning is the ability of the system to recognize a useful recombination and then change the structure to bias that same combination of functions to occur in the same (or similar) way in the future. Newly discovered functional adaptations can become encoded into the structure, often through practice. Furthermore, internal actuators, discussed in Chap. 7, can use available energy to make these structural changes.

We are also left with the question of how a newly discovered functional adaptation might be recognized and turned into a structural adaptation. How does a system "decide" when a new function is worthwhile enough to encode in its own internal structure? Strategic decisions, where resources must be allocated, is the province of game theory. We can image that a system may contain internal fitness functions that guide whether to retain or discard a newly discovered function.

An important learning consideration is how to assign credit or blame to a particular element of a system when that element was one of many along the flow pathway. Anyone who has worked on a team has encountered this phenomenon—the measure of team success often does not accurately reflect the contributions of the individuals. In system terms this is often framed as a problem of error backpropagation. When the output of a system is less than desirable, can the system itself identify where targeted learning should occur? Such is the case when companies fire or reward employees, teachers and parents praise or reprimand, or counties impose tariffs or sanctions on another countries. In a dynamic network, the assignment of credit or blame will often only be guesses.

9.2.6 Action Networks and Expertice

In 2008 Hugh Dubberly published *How do You Design? A Compendium of Models*, in which he summarized various academic and industry design processes. Over 100 diagrams of design processes are presented, ranging from linear (sometimes called waterfall or stage-gate methods) to spirals and helices, to those derived from feedback control systems, and even those inspired by biological models. Each captures a particular designer's (or company's) view of the process that they follow and is likely tuned to the type of end products and environments of the designers.

A focus on process is common in many fields. The scientific process, the structure of a good essay, and the recipes in a cookbook are all examples. A set of actions are placed in an orderly sequence, such that if someone were to follow the same steps they would be able to reproduce a desired result. Following the process does not guarantee a good outcome, but increases the chances that something useful will result. As such, the real value of articulating a process is to communicate a process that has worked in the past to others. In this regard a process diagram is a teaching tool, framework or heuristic that is handed down from an expert to a novice.

There are some critical differences between a novice and a master. In the 1980s the philosopher brothers Stuart and Hubert Dreyfus (1929–2017) proposed a model of skill acquisition as progressing through five steps: Novice, Competent, Proficient, Expert, and Master. A novice will attempt to adhere rigidly to rules or plans, small decisions take thought and time, and progress can be easily derailed. For this reason a simple process diagram is almost always a good place to learn what in reality may be a complex set of instructions. As one progresses to become a master, rigid plans disappear or at least are not consciously followed. A more holistic view of the task at hand develops. Tricks and heuristics become guiding principles, intuition based upon experience becomes the driving force in decision making, and improvisation becomes the rule rather than the exception. In fact, a master may not even be able to communicate how they do what they do.

In the context of this chapter, most processes are a set of actions that when put together in a particular way achieve a desired result. But an expert or master will tune the precise nature of those actions and their ordering as they go, perhaps skipping a step or jumping back to a previous step. They essentially contain within them a collection of actions that they could perform and then adaptively express them as needed. The same could be said of a company that contains core competencies that can be expressed in delivering on a value proposition. Innovation is the recombination of already existing processes, materials, information and core competencies. This idea is similar to how a biological organism contains within it the ability to inhibit or activate the expression of particular structures that will achieve a needed function.

The diagrams that appear in textbooks and on corporate websites are therefore not necessarily the exact process followed by the master, but rather are the common pathways through an *action network*. A designer (or company, chef or scientist) collect skills that enable them to execute particular actions. As they build up experience putting these actions together into coherent processes they develop motifs of actions that often go together. If a master has had good results, they inevitably are asked to communicate their processes to others. Rather than confuse a novice with all of their contextual knowledge, they simplify the actual process down to only the most common pathway through their design action network, usually removing all of the contextual decisions they make along with way. For this reason, masters in the same discipline may draw different process diagrams either due to differences in the end results they aim to achieve or their own history and experience working in the discipline.

9.3 Origins and Autopoiesis

As stated in the introduction, we see complex systems all around us. How do these kinds of complex systems get started? Do they come into being fully formed or do they emerge from simpler systems? What might the seeds of such simpler systems be like?

A related idea is what it means for a system to be “alive.” This may seem to be a simple question and throughout much of recorded history the answer seemed to be obvious. Over time, however, the definition of a living organism has blurred. For example, is a virus really a living organism? Is it possible for a silicon-based entity to be alive? Could an intricate series of clouds of interstellar gas be alive? Some science fiction writers have even proposed that plasma flows inside of a star could display many of the structures and functions of a living entity. In this section we will explore the idea of origins and a proposal for the definition of life.

9.3.1 The Origin of Complex Systems

Origin is a loaded word. It touches on many big philosophical questions. What is the origin of free will, life, minds, morality, economies, ecosystems and the universe? It is not the intent here to answer these questions. Instead we will focus more narrowly on a particular way to view origins. The origin of complex systems is the study of how, beginning from some basic parts, structures can spontaneously organize that will use energy/information to create more complex structures that will ensure the long-term sustainability of the system. The last clause ("ensure the long-term sustainability of the system") is critical. The seed of a system must provide an initial building block from which more complexity can emerge and maintain itself as something distinct from its surrounding environment.

What separates a random jumble of parts from a seed seems to be that the seed is self-consistent. Each part takes on some meaning in the system that is beyond its meaning when on its own. The parts must also stick together and perform a function that no individual part on its own could perform. Such a system does not need to reproduce, only remain together despite perturbations from the environment. We can even go one step farther and state that there is likely some ratio of internal communication to external communication—a coherent system generally has a high ratio of internal to external information flow. Such a system may even have all necessarily functions within its own boundaries, in which case it is *functionally closed*, sometimes also referred to as *operationally closed*. These are not meant to be precise definitions.

The simplest seed would be one that executes just a single function. Of all the types of simple functions to pick from, clock or autocatalytic set is the most likely. There are at least a few logical reasons for this position. First, a clock has a very clear boundary between what is happening inside and outside. A simple recognizable pattern is established, relative to the outside environment. Second, the movement from one state to the next to the next and back to the beginning, naturally forms a self-contained and self-sustaining function that needs no inputs or outputs, expect perhaps energy or materials from the outside. Third, and as stated in previous chapters, other functions can easily attach to a clock and be executed when the clock reaches a particular state. In this sense, a clock can form the initial hub of a more complex system. Lastly, a clock is one of the simplest structures that can generate a gradient of some resource—as a sort of pump—that can manage the use of energy, matter or information. Autocatalytic sets are argued to be the theoretical underpinning of the origin of life, consciousness, mind and perhaps someday in the future, artificial life and intelligence.

Regardless of the exact form of the initial seed, there are at least three different possible origins of a seed. First, it could come simply from random interactions. In fact, simple autocatalytic loops have been observed to spontaneously form due to random interactions of molecules. Second, a seed could be intentionally designed by some other system. For example, it is likely that the first wheel (a sort of mechanical autocatalytic set) was created by some unknown person long ago. Once the wheel was created, it opened the door to many other possible inventions. Third, a seed could be unintentionally created by another system. It might be a byproduct, similar to the glider gun discussed in the context of the Game of Life, or it might be some emergent behavior that rises from the interaction of many agents, as in the flocking of Boids.

9.3.2 Mechanisms of Complexification

A seed might grow in its complexity over time, by adding new structures. There are at least three mechanisms by which this may occur. First, an initial seed might simply wait until some other seed has grown and then a recombination may occur. This is logically the idea of symbiosis explored in Chap. 8. Essentially two independent systems merge in a win-win situation. The best mergers and acquisitions in the business world occur in this way. And it could be argued that the conquering of one country by another is a similar kind of dynamic, although perhaps less synergistic recombination.

Second, as new functions become encoded in the structure of a system, those same functions might be copied and used for other purposes. This is the concept of *exaptation* explored in Chap. 7. Several functions might arise independently to perform functions within a wider system. Due to a functional adaptation, they may come together to form a new higher level function. If this new function is useful it may become a structural adaptation. Complex parts, such as the eye, may have come into being this way. Likewise some cognitive scientists claim that our language abilities emerged out of the recombination of latent functions that developed for other purposes. In the realm of technology and business, Gorilla Glass was developed in the 1960s by Corning for military applications. The glass remained relatively dormant until around 2005, when Apple was looking for a strong, scratch-resistant covering for its smart phones. A glass that was developed for one purpose became one of the key components in a device that could not even have been imagined in the 1960s.

Third, complexification may occur through what Brian Arthur (1946–) has called *structural deepening*. Arthur points out that fitness gains can be made by adding new functions. But sometimes fitness is gained by finding a more efficient way to perform an existing function. The meaning of efficiency in this sense is that the same function is achieved using less avaliable energy. The savings can be used elsewhere in the system.

In human-made systems, structural deepening can take on a slightly different form. To highlight the point consider the old children's toy of two tin cans connected by a string. One child can speak into one end of the can and the signal is propagated down the string to be listened to by another child on the other end. We will compare how a biological system and a human-made system would engage in structural deepening of this basic function.

If a biological system discovered this mechanism of sending a signal, as it has done in nerve propagation down an axon, it would be based in organic chemistry. As such, it would be governed by particular rules of chemistry. Mutations and modifications could be made, but they would remain in the realm of organic chemistry. The cans might change their shape, or the string might be made thicker, but the basic materials and physical principles would remain the same.

On the other hand, the invention of the telephone has engaged in structural deepening in a much more holistic and powerful way. Instead of being limited to a particular physical domain, inventors can pick and choose from any law of nature. The basic mechanics of collecting sound were replaced by electronic collection. Digitization of the signal moved the phenomena outside of the realm of mechanical vibrations down a string. And with this change, the possibility of sending a signal with less losses became possible by exploiting the laws of electromagnetism. It also opened the door to transmitting through fiber optics and without a wire in the form of radio signals.

Similar arguments can be made for other inventions such as heavier than air flight. Birds evolved a variety of body designs to fly faster, or longer or more efficiently. But these modifications have been constrained to stay within the realm of the biological. Human-made systems are not under these constraints and so we have exploited whatever laws of nature would best perform each individual function. Likewise, the basic concept of an economy may have started with simple bartering, which is in fact observed in nonhuman species. In humans the idea of an economy has become more complex over time as it has become more intertwined with technology, politics and culture.

9.3.3 Autopoietic Systems

Musings about the nature of life have been around for at least as long as recorded history. With the rise of scientific approaches to big questions, there were many who developed principles of what it means to be alive. Historically these have involved interactions between organic molecules. The only examples that could obviously be classified as being alive were organic forms. The rise of computation, and the hint that artificial intelligence might be possible, prompted a reevaluation of the definition of life. Perhaps a life form could be based in silicon, or some other material. Likewise some astrobiologists demonstrated that at least in principle an intelligence might function on radically small or large time or space scales, and exist in environments that would never allow for organic organisms. Such organisms may be very difficult for us to detect and even more difficult to communicate with.

These concepts broadened the idea of life, and prompted an abstract functional approach. The definition of life would be grounded in the kinds of patterns and flows that could occur within the system, rather than the materials of the physical structure or temporal/spatial scales of the functions. Some initial ideas were proposed by the cyberneticists in the form of feedback loops. Perhaps a life form is one that has particular types of feedback loops that enable that system to tune its internal parameters. Homeostasis would be a hallmark of life. Likewise, a life form would maintain an internal consistency in the face of perturbations from the outside. Another key idea, discussed in Chap. 7, is that a living system would metabolize: taking in high quality energy to perform internal work and releasing lower quality energy back into the environment. Ross Ashby (1903–1972) described such systems as, "open to energy but closed to information."

In 1973 these ideas were built upon in *Autopoiesis and Cognition: The Realization of the Living* by Humberto Maturana (1928–) and Francisco Varela (1946–2001). Because both scientists were working in Santiago, Chile, their theory is sometimes referred to as *Santiago Theory*. They began with the idea of an *allopoietic* system—one that produces something that is different than the system itself. A classic example is a car factory. The factory (the system) is very different than its product (the cars). An *autopoietic* system is one that produces itself, or in other words, is its own product. This does not necessarily mean the system reproduces, it must only emerge and sustain itself in its environment.

To be more precise an autopoietic system must be operationally closed. This means that it contains within it all of the functions that are needed to sustain itself. The result of this operational closure is a logical boundary (not necessarily a physical boundary) that determines self versus non-self, or system versus environment. The boundary, however, is permeable in such a way that metabolism (high quality energy in, low quality energy out) can occur. The system, however, can alter the permeability of its own boundary such that it can regulate its own internal metabolism. This broad definition does not specify the energy source, nature of the internal functions, time or space scales, or even what the system must be made from.

There are some deep philosophical implications of such a definition. An allopoietic system seems to exist for the purpose of producing its product. An autopoietic system, on the other hand, seems to exist to continue its own existence. It is self-referential in the ways discussed in Chap. 6. It may display what we commonly call goals or desires. In other words, it's purpose is directed inward. Logically this is the definition of an independent, perhaps selfish, agent capable of making decisions about its own future.

9.3.4 Artificial Life and Viruses

Scientists are attempting to create life using the definition of autopoiesis. The first approach is known as *synthetic biology*. In the early days of the field, roughly the 1990s, synthetic life was essentially the genetic recombination of known biological functions. A new sudo-species could be created out of parts from a variety of other species at the genetic level. More recently, however, genetic techniques have enabled new organic life forms to be created from scratch. Consider that early in evolution all aerobic organisms locked into using the Kreb Cycle to generate ATP, the energy source used to fuel nearly all the various functions of a cell. Synthetic biology could in principle design a better or more efficient version of this cycle and embed it in a designed organism. Such an organism would be aerobic in a new way.

The second major exploration of designing a life form is in digital media, known as *artificial life*. These life forms would live inside of a simulated environment, feeding off of and perhaps competing for the resource of computational cycles. They could mutate their internal code to test out better ways to compete with other digital life forms. Entire ecosystems could emerge, developing their own languages, cultures and beliefs about their origins. Several science fiction writers and the movie *The Thirteenth Floor* explore this idea.

One aspect of an autopoietic system that may seem to be missing is the idea of reproduction. This was an intentional choice on the part of Maturana and Varela. First, a mule, the offspring of a donkey and a horse, cannot reproduce but it would certainly be considered alive. That is because it fits the description of an autopoietic system. Likewise a human who cannot reproduce would still be considered alive. Second, DNA most certainly reproduces itself, but it is a molecule and does not metabolize. It cannot exist on its own and only can reproduce by producing an organism that has the proper machinery for reading and expressing the genome. Another example is the vast diversity of viruses. They cannot maintain themselves through their own metabolism and must enter into another autopoietic entity and hijack its metabolism. So while a virus can reproduce, it is not operationally closed. By this definition a virus, while it certainly has some properties of life, would not be considered a living organism. The same could be said of computer viruses—without hardware and an operating system to run on, it is just lines of meaningless code.

Any system that could regenerate itself indefinitely would have no need to reproduce. Due to degradation of patterns over time most organisms at some point lose their ability to metabolize. In organic life forms we would associate this with death. In this light, reproduction (sexual or otherwise) is just a special case of regeneration. A system generates a new system with the same or similar internal patterns, such that this new system will persist after its own death. In this way reproduction, genetic or otherwise, is a way for an individual organism to retain some measure of immortality, not of its own self, but of its patterns.

9.3.5 Gaia and Detecting ET

Maturana and Varela initially made it clear that their theory was aimed at developing a rigorous definition of cellular life. That did not stop others, including later on Varela, from speculating that other systems might fit the description of an autopoietic system. For example, Evan Thompson (1962–) in his book *Mind in Life* argues that a mind is a kind of autopoietic system. Similar arguments might be made for economies or technological systems when considered broadly. One of the more controversial applications is the *Gaia hypothesis* proposed by James Lovelock (1919–).

Lovelock was working for NASA in the 1960s on the problem of how to detect life on other planets. He realized that perhaps it was not possible to actually find a living organism, but that some signature could be detected that would indicate a metabolizing system. As a chemist, he was able to give some concrete ideas about what kinds of byproducts might result that would not likely be created through other processes.

In his investigations, however, Lovelock came to a much broader conclusion that perhaps the constituents of an entire planet's atmosphere might provide a signature that life must exist on that planet. After all, a sampling of the earth's atmosphere would reveal that some sort of metabolism was taking place. It would not be possible to know exactly what was living on the planet, but the presence of life might be detected. When applied to other planets, this idea opened the door to the possibility that we might identify which planets outside of the earth might harbor life simply by knowing the make up of the atmosphere. This idea is actively being pursued by astrobiologists both within our solar system and in the many newly discovered extrasolar planets.

Lovelock's conclusion also led him to the controversial *Gaia hypothesis*—that the entire earth is a living organism. This claim was later expanded upon and refined by Lynn Margalis and Dorian Sagan (1959–). Because reproduction is not a criteria for autopoietic life, the earth can be viewed as a self-consistent pattern that sustains itself. It has a clear physical boundary. It takes in high quality energy from the sun and expels (primarily) low quality heat energy out into space. As an operationally closed entity, the earth also changes its own internal flows of energy to adapt to changes in its outside environment. This could include acute perturbations from the outside such as asteroid strikes, after which the earth will essentially heal itself. But it can also take into account more persistent changes. A classic example is the well documented fact that the energy from the sun has been decreasing for several billion years. Yet, the temperature of the earth has remained relatively constant during this time. This is, in itself, a nice demonstration of a sort of global homeostasis.

To illustrate the point, Lovelock and Andrew Watson (1952–) created the simulated agent base model known as Daisyworld in 1983. In Daisyworld there are two species of daisies, one black and one white. Black daisies absorb more light which warms the planet, while white daisies reflect light and cool the planet. There is competition between these daisies, however, and the growth rates of each are dependent on temperature; white daisies like it a bit on the warmer side and black daisies like it a bit cooler. The simulated energy from the sun can be varied over time. What results is a mix of white and black daisies that normalizes the temperature over a fairly wide range of solar energy input. When the suns energy is increased more white daisies grow and reflect more light, creating a cooling effect. When the suns energy is decreased more black daisies grow and absorb more, warming the planet back up. Later versions of Daisyworld included an ecosystem of species that were found to further increase the stability of a homeostatic setpoint of multiple variables, including the salinity of the ocean and concentration of oxygen in the atmosphere.

Another approach to artificial life allows digital organisms to metabolize computing cycles as if they were energy. Each organism is competing with other organisms for the resources of CPU time. Tierra, created by Thomas Ray (1954–), is an artificial life simulation that is built upon this idea. The software platform Avida is an entire software platform in which programmers can create their own environments and ecosystems. The digital organisms within these worlds can then evolve over time. These simulated environments and organisms reveal just how different living organisms might be. In fact it is hard to imagine being able to have a meaningful conversation with an organism that not only speaks an entirely different language, but inhabits a different world and likely has an entirely different viewpoint on the meaning of life.

9.4 Questions

- List out a variety of products, ideas or other innovations that you have adopted. In your estimation were you an early adopter, early majority, late majority or laggard? Are there any themes that you can identify in your patterns of adoption? Use Rogers language to talk about your observations.
- Give an personal example of when adding a person has changed the balance of a social network. Was the addition of that person stabilizing or not? Can you pinpoint the chain of smaller instabilities that may have led to the network instability?
- Is there a time that you were the beneficiary of a weak tie? What was the connection? Can you dissect why you think that tie worked the way it did?
- Are there motifs that you think underlie cultural phenomenon? You can draw your examples from technology, business, politics, ethics or other systems.
- Explain to another person the idea of a diseased system. Then ask them what systems they think are diseased. Identify one of these systems and engage in a discussion of the various causes of the disease. After the conversation record your joint thoughts.
- Discuss the concept of origins with a friend studying a discipline other than your own. Ask them what is known about the origin of their discipline. What core idea started that discipline? In your conversation can you dissect what it was about that particular idea that led to it being the spark of a new discipline?

- Are there any examples of autopoietic systems in your own field? Can you draw out the properties? What is metabolized? What are the boundaries? How does the system sustain itself against perturbations?
- Movie trailers are meant to hook a potential audience member. This necessarily means revealing some information about the movie, but not so much that it would be pointless to pay to see it in the theater. Choose a few movies that have made it big in the theater and review the trailers. How much do you think the trailer contributed to the success of the movie? How much do you think it was other elements? Can you map out this success using Roger's Diffusion of Innovation framework?
- One important concept in complex systems is the interplay between the time scale on which the system functions and the time scale on which the system can change. For example, in molecular biology there is a large difference in the time scales required for proteins to interact, the expression (up or down regulation) of the protein, and the evolution over many generations of the gene that provides the template for the protein. The multi-scale nature of complex systems has come up many times throughout the text. Chose a human-made system and explore the various time scales of action (function) and change (structure).
- We can reconsider the nature of analogies and isomorphisms that were introduced in Chap. 1. A perfect isomorphism exists between two systems that have the same structure and function, meaning they have the same network structure and flow patterns. Note that this does not mean that they work on exactly the same principles, only that both have the same structure and potential for the same function. An analogy, on the other hand may share only some elements, perhaps only the structure or only the flows. Of course isomorphisms and analogies blur together when comparing real systems. For example, we might compare the branching of a tree to the branching that takes place in the respiratory system. Both are treelike structures and both distribute something to extremities—alveoli in the lungs and leaves in trees. But the physical processes by which they do this are different. It becomes less clear if they are isomorphic or analogies of one another. Pick two systems and discuss whether they are isomorphic (having both structure and function in common) or analogies (having one or the other in common but not both).
- The Toyoda Production System (TPS) is a widely adopted strategy for manufacturing products. Also known as lean or "just-in-time" manufacturing, it focuses on a holistic approach to minimizing waste, time, space, errors, work load and other dimensions of manufacturing. In many ways it is more of a philosophy of how to adapt structures, processes, teams, problems and value propositions, rather than an explicit focus on the product. At its heart, TPS is a way to create a production network that is constantly learning and optimizing flows. Read up on the Toyoda Production System. Then identify a system that you are part of that you believe would benefit from adopting this type of methodology. How might you convince this system to adopt the Toyoda Production System?
- There are some who theorize that any ecosystem will eventually give rise to some player that fulfills the role of a virus—a shortcut that enables some incomplete

member of the ecosystem to thrive at the expense of other members of the ecosystem. Do you think this is true? Can you think of any counter examples of ecosystem that do not contain a virus-like member?

- Conflicting information was discussed in the context of Roger's Diffusion of Innovation framework. This is exactly the case in Gregory Bateson's (1904–1980) *double bind theory* that explores the emotional dilemma resulting from two or more conflicting messages that originate from different levels of a hierarchy. For example, what if a respected person asks (or orders) you to commit an act that is either impossible or directly conflicts with some social norm or law. In this sense a double bind is not simply a contradiction, which occurs within a particular logic level, but rather is a tension across logic levels or hierarchies. For example, what happens when an order from an individual conflicts with a rule of a culture? We encounter the double bind quite naturally in many situations, including in humor, poetry and regular language. It can, however, also be used as a weapon against others, especially by those in power, to create confusion. Discuss an example when you have been in a double bind. Identify the levels from which you were receiving information and the nature of the conflict. What was the resolution?
- Weak ties were discussed in a positive light, as a way for trusted information to be conveyed through a relatively neutral third party. But information only has meaning in context. It is equally likely that a weak tie could share information that is damaging to someone's reputation. Give an example from your life, history or current pop culture, where a weak tie has shared damaging information.
- Rubbernecking is the morbid curiosity about events where someone has been injured. This occurs on highways when there is a crash and most cars slow down to find out what happened. The result on traffic is often a backup, even if lanes are not closed. What are other examples where some relatively small collective habit can magnify an effect? Are there delays involved? Or perhaps some shared resource or other network effect?
- Bottlenecks can appear in a system through both function (e.g., critical slow down from Chap. 3) and structure (e.g., hierarchies and hub networks from Chap. 5). Bottlenecks are places in a system where a great deal of flow enters and exists. As such when one aims to change a system, bottlenecks are often a good place to start. Identify a system you would like to change. What are the bottlenecks in delays, resources or flows? What might you do to impact the system at this bottleneck?
- Dreams are sometimes claimed to be the neural reconsolidation of memories—short term memories are being repackaged to become long term memories. The mechanism of this translation is necessarily a replaying of an event or experience but in a form that can be stored more permanently. For this reason some advocate that keeping a dream journal can help one gain insight into how the unconscious mind is processing experiences. What is the most recent dream you had? Speculate on how this dream might be placing a new memory into your existing neural network.

- Some systems engage in self-handicapping. A classic example is the male peacock which expends enormous resources to create a beautiful outward display of colorful plumage. The rationale is that any system that can maintain such a costly outwardly display must be internally fit and therefore desirable. Some have argued that humans engage in this type of behavior as well but not always targeted at sexual selection. What are some examples of an individual human handicapping themselves in some way to display their cultural fitness? Do you engage in this behavior?
- The Barbie Doll was designed by Ruth Handler (1916–2002) and released by Mattel in 1959. Over the years Barbie has been praised and ridiculed for a variety of reasons. The marketing of Barbie, however, recognized a critical supplement to Roger's ideas of how innovations spread. In the classic diffusion of innovation framework the adopter is also the user. For a Barbie that is not true—generally it is a parent who pays for the doll (they are the adopter) but it is the child who plays with the doll (they are the user). What is another product where the adopter is not the same as the user? How is that product marketed?
- The book *Adaption-Innovation in the Context of Diversity and Change* by Michael Kirton contrasts two models of problem solving. An adaptor is someone who has an impact by making critical changes to an existing idea or physical realization. An historical example is Thomas Edison (1847–1931) who greatly improved the existing concept of the light bulb. An innovator is someone who finds a new solution. Albert Einstein is an example, in that his General Theory of Relatively completely reframing how we imagine gravity to work. Kirton's claim is that good teams should have both and that understanding the make-up of the team is critical. An individual will not be purely an adaptor or innovator, but rather exist somewhere on a spectrum between these two extremes. If you needed to assign yourself a percentage of Adaptor and a percentage of Innovator what would it be? Are there teams you are working on now? What do you think their makeup is? Back up your claim with examples.
- One sign of an efficient and healthy ecosystem is on that generate minimal waste. Flows are organized in such a way that the waste of one component becomes a resource to some other component. What waste do you see around you that is not being used? Is there some innovative way to turn that waste into a resource?

9.5 Resources and Further Reading

In general, most texts that cover networks will also discuss the the dynamics of functions on networks. As such, many of the suggestions from Chap. 5 are good sources to explore the dynamics of networks. *Networks of the Brain* by Olaf Sporns (1963–) and *Dynamic Patterns* by Scott Kelso (1947–), when paired together, demonstrate how much we know about how neural connections support brain function. *An Introduction to Systems Biology* by Uri Alon does that same but for molecular and genetic networks. A similar intent is captured in David Easley (1950–) and Jon Kleinberg's (1971–) *Networks, Crowds, and Markets* for human created systems.

Chapter 10
The Edge

Imagine that you are the co-founder of a small company that has had your first hit product. You are now being pushed by investors to scale up the product—to expand the market and potentially go international. To fulfill this request you are beginning to hire your first employees through a rigorous hiring process to fill formal job descriptions. As a founder, however, you enjoy the start-up culture and mindset. You have several ideas for new products that you could add to the company's portfolio. But there is only so much the company can do with the resources they have (e.g., time, money, energy, attention). How will you balance the allocation of these resources toward scaling your first hit and exciting future products?

Now imagine that you are the head of a massive multinational company. You have a large portfolio of products that span a diverse range of markets. There is no way you can know everything that happens in your company and it certainly is not possible to know all of the employees. Although the company is financially secure, a few recent events have caused the board to encourage you to bring back the entrepreneurial spirit that long ago served the company well. How might you inject some innovation into an otherwise rigid and machinelike organization?

Both of these companies are struggling with how to balance the competing forces of innovation and risk on one hand versus efficiency and long-term security on the other hand. Making decisions going forward will necessarily require a careful balance of resources and messaging. In achieving this balance, both companies will become complex adaptive systems.

Complex adaptive systems can do some amazing things. They heal when they are injured. They make decisions with incomplete information. The rules and structures that created them can be destroyed and replaced with new rules. They develop over time, but in a way that after the fact appears planned. They are ordered enough to have repeatable functions but also have enough disorder that they can be flexible. They can tune the internal degree of order to mirror the degree of order in the environment. If the environment changes, the internal structures and flows will change too. Somehow complex adaptive systems find their way to the steep slope

J. Tranquillo, *An Introduction to Complex Systems*,
https://doi.org/10.1007/978-3-030-02589-2_10

of a second-order phase transition—they self-organize to this critical state. In this chapter we will examine some ways in which complex adaptive systems are self-tuning.

10.1 Reaching Criticality

Phase transitions were encountered in various forms throughout Chaps. 4–7. First-order transitions are sharp, occurring suddenly as a parameter of the system is changed. Second-order transitions are more gradual. Typically on one side of the transition, interactions are local, and information is not correlated across long distances or times. These correlation lengths are measured by how far information can travel in the system. The other side of the transition is fluid or gas-like. Information can flow freely within the system but without any direction. In between, over the range of a second-order phase transition, is where complex patterns of information flow can exist that are mostly local but have the potential to move across long distances within the system. Stability and short-range correlations exist alongside instability and long-range correlations. It is along the steep part of a second-order phase transition that system functions can be flexibly expressed as latent functions are "woken up" as needed yet still remain a coherent whole. In this section we will explore some ways in which a complex system might find and maintain a homeostasis on the steep edge of a second-order phase transition.

10.1.1 Self-Controlling Order Parameters

In Chap. 7, the idea of an order parameter was introduced as a single variable that could move a system through a phase transition. The example used was for the density of traffic versus the flow of traffic. When the density is low, the velocity of the cars is greatest. As the density rises, there is a transition, over which travel velocity decreases. At some point the density is so great that the flow freezes in the form of a traffic jam.

In some complex systems, an order parameter might be under the control of the system itself. When the system moves too far away from the transition, other variables will snap the system back to the transition region. For example, the input of energy might be the order parameter. As a complex system may have control over the flow of energy, materials, and information across its own boundary, it can tune itself to a transition region.

10.1.2 *Self-Organization of Internal Rules and Constraints*

Rules can be either rigid and unchanging or flexible and adaptive. Rigid rules are often, but not always, set from the outside (e.g., a programmer, a god, rules of physics), and the system must conform to them. Many of the products we use fall into this category—a designer endowed the product with a specific set of functions which the user awakens. Likewise, a chemical reaction is bound by the rules of chemistry and physics and will proceed based upon rigid physical rules. Some systems, most especially systems that are considered to be alive, can alter their own internal pathways for information flow and thus can change their own rules from within. Such structural adaptation is the basis for development, growth, and learning.

Another way to view rules is that they act as constraints. An analogy given by Stuart Kauffman highlights the idea. Energy, materials, and design are used to create a piston that might go into a car. Once the piston is made, it can harness and channel energy in a direction. But the piston does not create itself, and the constraints for energy flow were put in place by the designer. Self-organizing systems, on the other hand, can use disorganized energy to organize internal structures that constrain energy to flow in coordinated directions to produce useful work. The implication is that complex adaptive systems can create their own constraints and rules for energy and information flow.

In an adaptive system, the rules themselves are moving toward a phase transition. They are not so rigid as to create the same patterns and behaviors every time, but they are also not so flexible as to yield no patterns. In Chap. 2, the Wolfram Cellular Automaton rules were explored. It was found that Class IV automaton displayed the kind of behavior observed within a phase transition—patterned and chaotic behavior can coexist. In the late 1980s, Chris Langdon (1948–) pointed out that all Class IV rules were exactly at the border of 50% alive and 50% dead. In other words the rules were balanced between creating and killing cells on the next iteration. It is important to note that Langdon was not proposing a phase change in behavior. Rather it was a balance in the rules which govern the behavior. Langdon explored other automaton, such as the Game of Life, and found that there was a delicate balance between competing rules.

The Wolfram automaton and other automaton rules were created by people. Wolfram in fact exhaustively checked all of the possible sets of rules to find those that were Class IV. What was learned from this exercise was that the rules are not clustered in one area of the rule space. In the absence of a designer, how might an adaptive system find these hidden rule sets? What kinds of algorithms do complex adaptive systems use to ensure that their rules are balanced? This question is all that much more intriguing when considering that most real complex systems have multiple variables (unlike a simple automaton), that all may be competing and cooperating with one another.

10.1.3 Tuned Chaos

Must of the excitement surrounding chaos theory in the 1970–1990s was the demonstration that a system with simple deterministic rules could exhibit very complex and unpredictable behavior. But it was pointed out by several thinkers that the transition to chaos through a sequence of bifurcations generally occurred over a very small parameter range. To take fluid flow as an example, there is regular (known as laminar) flow, then a transition region, and then chaotic (known as turbulent) flow. The sweet spot for a complex system would be in the transition region. But, chaos theory gave no explanation as to how a complex system might tune itself to this regime. The simple existence of a transition region does not tell us how a complex system might find and then stay in that range. Why would the homeostasis of a system gravitate toward a transition to chaos?

To illustrate the point, consider that, given fluid properties and the diameter of the tubing, it is the flow rate that determines what regime the system will be in (laminar, transition, or turbulent). In a designed system, that often means that the rate of fluid entering (and exiting) the system will determine the regime of behavior—it is some external influence that determines whether or not the system will be in the turbulent range. But, now consider what happens if the system can adapt to the flow rate. It might be able to adjust the diameter of its own internal tubing or perhaps even change the viscosity of the fluid. These internal adjustments would mean that a very large range of flow rates coming into the system might result in turbulent behavior. Likewise, the system may flexibly control how open the boundaries are to the outside environment. A similar behavior was explored in the Daisyworld system described in Chap. 9 where temperature is stabilized. Likewise, the same kind of behavior might be demonstrated in an economy if we substitute money for fluid flow. In each case, the adaptive system is tuning itself to a transitional region that lies between chaos and some more simple behavior. Chaos theory shows that such regions exist, but not how or why a system will land on the sharp edge of chaotic behavior.

10.1.4 Power Laws and 1/f Noise

A single limit cycle will result in a characteristic frequency. One way to think of a complex or chaotic system is that it is the combination of many of these limit cycles all superimposed upon one another. There is a difference, however, in the distribution of frequencies in complex and chaotic systems. A fully chaotic system has a flat distribution of frequencies, meaning that all frequencies are present to roughly the same degree. There is no privileged or dominant limit cycle. This can be contrasted with complex systems, which often have a power law distribution—lower frequencies are present much more than higher frequencies. This is known as 1/f behavior, with the f standing for frequency.

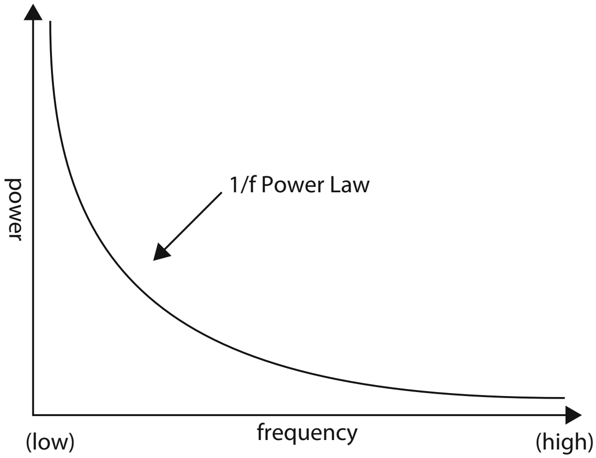


Fig. 10.1 Frequency distribution of music often follows a power law

The 1/f distribution is seen in a range of systems both human-made and natural. Zipf's law describes the frequency of words used in language. Pareto's law represents the distribution of wealth income in a society. Power laws were also found to govern the structures seen in scale-free networks and objects with fractal dimensions. In Chap. 3 we discussed how music could be mapped to a strange attractor. It turns out that most music also has a 1/f distribution, as shown in Fig. 10.1. Extinction events in evolution seem to also follow a power law too. Likewise earthquakes, floods, tornadoes, and asteroid strikes also seem to fit well to a 1/f distribution in both the time between events and magnitude. Many of our senses also register magnitude based upon a power law. This phenomenon, known as Weber's law, measures magnitudes in decibels which are based on a power law distribution. It seems unlikely that power laws would be observed across such different systems. It is therefore hypothesized that a power law distribution is a signature of a complex system balanced on the edge of a second-order phase transition.

10.1.5 Tensegrity

The concept of balancing competing forces is a core element of the idea of *tensegrity* (also known as tensional integrity or floating compression). The most tangible demonstration is manifest in physical structures composed of solid beams in compression combined with wires in tension. The result is a sculpture that looks impossibly balanced as in Fig. 10.2. Every member (beam or wire) is unstable on its own, but together they form a remarkably stable structure. Because there are no bending moments, the structures can maintain their tension and compression even

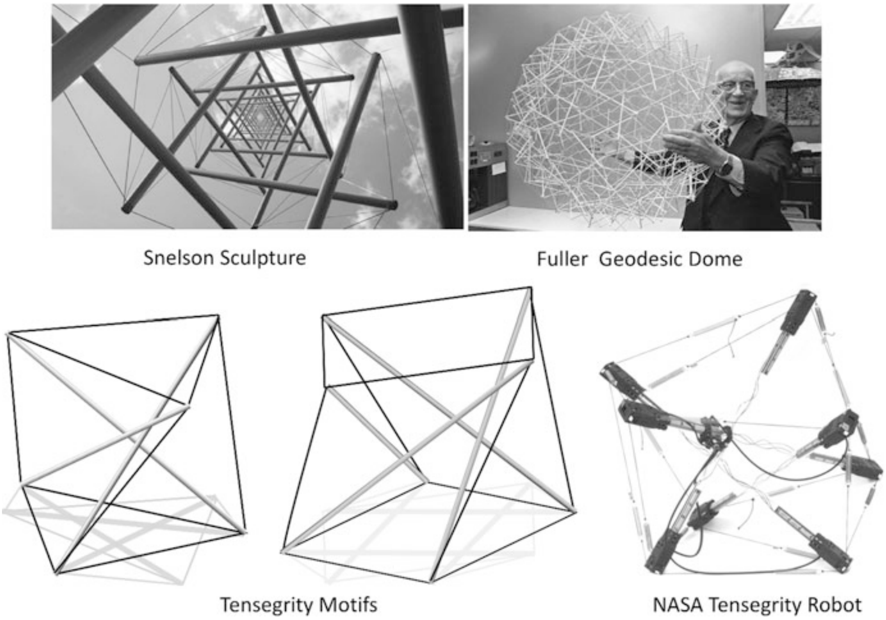


Fig. 10.2 Examples of human-created tensegrity structures. Larger structures can be created by recombining a range of tensegrity motifs

under extreme loads. There is simultaneously a great deal of potential for instability, yet the structure is balanced. In some tensegrity structures, beams or wires can even be removed, and the rest of the structure will redistribute loads to the remaining beams and wires. Tensegrity has been used to create sculptures and geodesic domes, typified by Disney's Epcot Center in Orlando, Florida, and has even been proposed as pop-up shelters or tents to be used by backpackers and astronauts exploring other planets.

It is important to note that there are some basic structural motifs (combinations of beams and wires) that balance tension and compression. These can be repeated over and over again to form larger structures. These motifs are a sort of structural grammar that can be used to design larger and more complex structures that will naturally display tensional integrity.

Tensegrity may also be applied to function. In chaos theory, there is a necessary balance between positive (causing growth of a trajectory in phase space) and negative (causing contraction of a trajectory in phase space) feedback. Gyorgy Buzsaki has proposed that neural networks develop a functional tensegrity that balances excitation and inhibition. The metabolism of regions of the brain and glial cell networks might also co-develop based upon principles of tensegrity. A more broad application of tensegrity to biological systems can be found in Graham Scarr's *Biotensegrity: The Structural Basis of Life*.

The origin of tensegrity ironically contains its own tension. Kenneth Snelson (1927–2016), a sculptor, and Buckminster Fuller (1895–1983), a scientist, both claimed credit. Snelson has a patent on the idea, but Fuller demonstrated the power of the technique by proposing larger carbon balls (the first identified nanoparticles). These hollowed out structures, known as Bucky Balls, are being used in applications ranging from drug delivery to the design of flexible robotic appendages. Fuller also proposed the geodesic dome, which is very similar to the structure of a Bucky Ball but on a larger scale. Despite the fact that Snelson and Fuller are no longer alive, their supporters have carried on the tension, each claiming ownership for their respective fields.

10.2 Self-Organized Criticality

Complex adaptive systems seem to organize in such a way that the critical region is an attractor. Internal counterbalancing forces act to return any deviation back to the transition of a second-order transition. Such tuning is known as *self-organized criticality*.

There are a few theories about possible mechanisms. The cyberneticists imagined a developing system adding and subtracting positive and negative feedback loops until a balance was achieved. Such development would amount to adding and subtracting counterbalancing rules. How does the system "know" that it needs to add another negative feedback loop? Doing so would become particularly difficult as the system becomes very complex—it is not as if the system can globally assess all of the feedback loops, especially when those loops may be weighted and perhaps not always expressed. No plausible mechanism was proposed other than blind trial and error or an elaborate hierarchy of control mechanisms.

The term self-organized criticality was first introduced by Per Bak (1948–2002) who proposed an alternative mechanism that would enable a tuning to a critical point to emerge from an agent-based system. Bak's model, however, is different from other agent-based models such as Boids or the Game of Life in that the system is open to the environment. It takes in something (energy, information, material) and gives off "waste." It has a connection with its surrounding environment yet maintains a functionally dynamic boundary such that it can absorb (or let out) more or less. In this section we will explore Bak's model of self-organized criticality.

10.2.1 Sandpiles and Avalanches

Bak and his colleagues searched for a simple model that would highlight the key attributes of a system that could self-tune to a critical point. After considering various model systems, they settled on an agent-based sandpile that was simple and intuitive. Imagine a table onto which someone is slowly dropping grains of sand, as in Fig. 10.3. When the sand is first dropped onto the table, a cone shape will appear and begin to grow larger and larger until it fills the table. At that point,

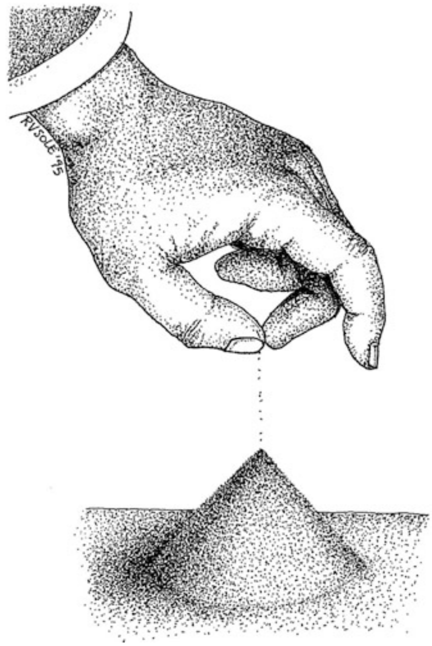


Fig. 10.3 Sandpile analogy of self-organized criticality used by Per Bak to build intuition of the buildup of tension that is released by avalanches of all sizes. Figure created by Ricard Solé

adding more sand will not grow the sandpile any larger. The sandpile has reached a sort of "maturity."

Bak intentionally built into the sandpile model some important rules. First, grains of sand in a pile only communicate with their immediate neighbors through mechanical forces. Any global behavior would therefore necessarily be an emergent phenomenon. Second, as the sandpile grows there is a particular angle that the cone of sand makes that is maintained throughout development. This same angle will be maintained even after the sandpile has reached maturity. It is a universal and emergent property of the sandpile.

Bak's mathematical model takes place on a lattice, shown in Fig. 10.4, where each cell can communicate with four neighbors (up, down, left, and right). Each cell can take on one of four states (0, 1, 2, or 3), corresponding to the number of "grains" in that cell. Imagine an initial condition where each cell has some random number of grains between 0 and 3. A simulated "grain" is then added to a random cell. If the previous state of that cell was 1, afterward it is 2. This process is then repeated over and over again.

When a cell is at a state of three grains and has one more grain added, an *avalanche* occurs. During an avalanche, a cell distributes its four grains (counting the one it just received) to each of its four neighbors. At the end of this avalanche, the receiving cell is left with a state of zero and its neighbors each gain one more grain. But, what happens if one of the neighbors is also at 3? It will also undergo an avalanche that could potentially trigger another avalanche and so on. A single small avalanche can spread to some larger area of the lattice.

2	3	0
2	1	1
3	0	2

Fig. 10.4 The simulated model of self-organized criticality on a grid. Each cell contains some number of "grains" ranging between zero and three. A grains falls on a random location from outside of the system. If any cell collects four grains, that cell will undergo an avalanche—sharing its grains with its four neighbors and reset back to zero

The model is not meant to simulate the building of a physical sandpile because the highest a single cell can become is three grains high. But it is meant to demonstrate what Bak thought was the most important feature—internal tension is slowly built up in a system, pushing it farther and farther from equilibrium, until that tension is released in a sudden avalanche of activity. An avalanche is a correction that acts to tune the system.

10.2.2 Pennies and Avalanches

An even simpler game can be played with a group of people. Imagine 12 people in a line, each holding a unique playing card from a poker deck in one hand. They also hold a penny in their other hand. They flip the coin, and it will land either heads or tails up. This is the initial condition. A dealer (a 13th person) has the remainder of the poker deck and their own penny. The dealer flips their penny and flips over a card. What is important at this point is that all of these occurrences (card draws and penny flips) are random.

The pennies and cards work in the following way. Let us assume the card was an 8; this will identify a particular person at random in the lineup. The state of number 8's penny, and the dealer's penny, will determine what happens next. If the dealer flipped a tails, nothing happens, and the round is over. But, if the dealer flips heads, the dynamics become more interesting. That heads will be "added" to number 8's state. If number 8 has a tails, they will add the dealers heads to their tails, and their new state will be heads (e.g., tails + heads = heads). The round is over. If both the dealer and player number 8 have a heads, an avalanche will occur—Player 8 will give up their two heads to their right and left neighbor (e.g., heads + heads = tails + avalanche). The round is not yet over, because the redistribution of heads might propagate the avalanche.

The critical point in this simple system is 50% heads and 50% tails. What this means is that the overall state of the system may deviate from 50/50 but that over time it will snap back to this point. To understand why, we need to see how two forces interact—one adding heads, the other absorbing them. Consider the unlikely initial condition of all tails. Any new tails from the dealer will not change the state, but any new heads will. This means that there will be a slow buildup of heads. Consider, however, an initial condition of all heads. On the dealer's first heads, there will be a large avalanche. In fact any initial condition will oscillate in an unpredictable way about a 50/50 critical state.

10.2.3 Insights from the Sandpile Model

There are several phenomena that emerge from Bak's sandpile model of self-organized criticality (SOC). First, the system has a large range of tensions that can be built up over time. The degree of the tension will be roughly proportional to the size of the restoring avalanche. The farther the system strays from this point, the more strongly the pull back to the critical point will be and the larger the avalanche. On the other hand, if there is very little tension in the system, it will absorb heads (or grains of sand) coming from the outside to build up more tension.

Second, the distribution of events (avalanches) fits a power law distribution. Small avalanches are very common with larger avalanches being more rare. In other words they follow a long tail where large events are rare but do happen. What is more, they also display a 1/f frequency distribution—the time between avalanche events also follows a power law. Furthermore, one cannot predict either the timing or magnitude of the next avalanche. So, it is not possible to make statements such as "there hasn't been a large avalanche for a while so one is due soon." Such statements are often uttered by prognosticators. For example, sportscasters often will claim that a star player is "due" to hit or score because statistically they are below their average numbers.

Chaos arrives at unpredictability due to sensitivity to initial conditions and perturbations. Self-organizing systems, on the other hand, are not sensitive to initial conditions or perturbations—in fact, they will tend to wipe them out through repeated avalanches. Instead they are unpredictable because many agents are interacting, all of which stored and release energy and tension.

Third, the mechanisms for generating large and small events are exactly the same. This may seem obvious given the sandpile and penny model systems. But it is in fact counter to the practices of many scientific disciplines. It is common to remove extreme events from data, assuming that they are generated by some other mechanisms—so-called outliers. The advantage of removing outliers is that often the results are mathematically tractable. Large events are studied using a separate set of assumptions, as if they are part of some completely different system. An example is found in the study of creativity. For a long time, the study of everyday creativity was studied in a very different way than "genius" creativity. A speculative implication of self-organized criticality is that both mundane and genius creativity are driven by the same mechanism. We will explore this idea in more detail below.

Fourth, during an avalanche the correlations in space can be of all sizes. Although all interactions are local, an avalanche may spread some distance throughout the system. In the largest avalanches, the entire system might be involved. The pathway of an avalanche can be considered to be a brief flow of information from one part of the system to another. It turns out that the length over which information can spread in a sandpile is also a power law in space. Bak made the tentative claim that self-organized criticality may be the origin of some kinds of fractals and scale-free networks that are observed out in the world. Furthermore, a pathway discovered during an avalanche might turn out to be a useful combination of flows and functions. Such a discovery could then be strengthened, leading to a structural adaptation that would make that flow more probable in the future. Because the length scales of functional flows during an avalanche follow a power law, structural adaptation would over time lead to a structure that also follows a power law.

Fifth, Bak performed many variations on the sandpile model to demonstrate that self-organized criticality is not sensitive to the particulars of the system or incoming stream of sand. In one study he made the value at which sand would topple random. This would be like changing the capacity of various places in a system for storing tension. He also varied the size of the particles. In another study he explored what would happen if sand entered the system at the same cell in the lattice every time. Bak also explored intermittent sand entry as well as the placement of barriers in the system such that avalanches would be biased to move in particular directions. In all of these cases, he found that the system would settle to a critical point where avalanches followed a power law.

One particular observation from these studies stands out. In most systems we tend to think that if we put in a constant input then the output will be constant, or at least repeating in some way. In Bak's experiment, however, a regular deterministic input is transformed into a power law. Likewise, a random input could also become a power law. And finally a chaotic input of sand will result in a power law. Any input that builds up internal tension will be organized by the system to a point of criticality and display avalanches.

Sixth, some avalanches might reach the boundaries of the system. In Bak's model, the grains or heads will fall off of the edge into a "waste" pile. With this simple rule, there will be an overall conservation of grains or heads—the materials that enter the system will either stay in the system or eventually leave the system boundary. At a microscale, studies have labeled individual grains to explore the time between a grain entering and exiting the pile. In theory, every grain will eventually leave the pile if the pile lasts long enough. What is found is that some grains enter and leave very quickly, while others stay for a very long time.

Seventh, if one were to measure the sand that is output from the system, that too would follow a power law. Sometimes large amounts of sand fall off of the table all at once. Other times it is a trickle. There may be long periods of time when nothing leaves the table, or there might be bursts. What this means is that if an observer were outside the system watching, there would be clues that the system was self-organized to a critical point. This is in fact part of the argument that James Lovelock made in determining if there was life on another planet. Systems that take in a constant stream of energy and output a power law would indicate that something on the planet may be self-organized.

Eighth, the terms large and small, rare and common, are relative. The absolute magnitudes and frequencies of the avalanches depend upon the makeup of the network and the amount of energy being poured into the system. For example, pouring more energy (or sand) into a system builds up tension at a faster rate and therefore may increase the rate of avalanches. But, it does little to control the upper size limit of the avalanches—that seems to be dependent upon the makeup of the system. When it comes to magnitude, there seems to be at least two determinants. The number of interacting parts places a limit on the size of the largest possible event. The more interacting parts, the larger the possible avalanche. Another determinant is the "springiness" of the system, determined by how much energy can be stored in the system before an avalanche is triggered. The more energy that can be stored, the larger the possible avalanche. This is essentially a measure of how far the system can stray from a critical region before being returned through an avalanche. A somewhat simple measure of how much energy can be stored can be found in the power law—the greater the slope, the more energy being stored.

10.2.4 Avalanches in Other Systems

The concept of an avalanche is very old and can be summed up in the phrase, "the straw that broke the camel's back." A system absorbs something (e.g., energy, tension, water) until something gives way, and the system releases some of the tension. In a real complex system, pennies or grains would likely flow on a hierarchical network system. The simple heads/tails states would be replaced by many possible states. And it might not just be pennies or gains flowing. Likewise the flows might become non-linear. The important point is that even in a simple system, avalanches may help balance about some critical state.

The most obvious examples of self-organization are real earthquakes and avalanches. On a snowy slope, sunlight introduces heat that breaks the bonds between ice crystals. Because the system is robust, it will absorb this breaking of bonds but only to a point. Once enough bonds are broken, a chain reaction will occur, and an avalanche of snow will result. A similar process appears to occur in the buildup stresses in the earth's crust which are released in the form of an earthquake. The same kind of analogy can be used, and some argue that it is not an analogy but reality, to describe the release of stresses in financial markets, ecological systems, neural information flow, and traffic dynamics. Each of these systems has complex dynamics that can result in seemingly unpredictable cycles of tension building followed by a relatively sudden release. Below we explore a few other systems where the ideas of self-organized criticality can yield new insights.

10.2.5 Evolution, Extinction, and Missing Links

The evolution of species and ecosystems was explored in Chap. 8 as dancing fitness landscapes. The usual view of evolution, however, is of gradual change over a long period of time. Such a gradual change should leave behind traces that can be found by paleontologists. Yet we do not always find “missing links” between species. Perhaps evolution is not as gradual as it appears. This is the idea of *punctuated equilibria*, first proposed by Niles Eldredge and Stephen Jay Gould (1941–2002) in 1972. In this context a particular species is the equilibrium, a stable collection of genes and functions that leads to relatively high fitness. The punctuated part means that the evolution of that species occurs in fits and starts rather than some smooth progression over long periods of time. Eldredge and Gould were not proposing that these changes would occur in a single generation but rather that the rate of evolution might not be constant over time. Although it was proposed before Bak’s avalanche model, it has many of the same features when applied to the evolutionary time scale.

Consider a huge population of species A. This species over time mutates into a subspecies B which mutates into subspecies C and so on down the line. At the end of this chain, we have what would be classified as a new species Z. It may be possible that all of these species compete for resources, but A and Z are best adapted to the current environment. Although species B-Y may still remain, they do so with very small populations. The chances of finding a fossil from these missing links are very small. Essentially getting from species A to Z (both of which have larger populations) required going through the bottleneck of the species B-Y, all of which have smaller populations. Furthermore, if there is an extinction event (e.g., flood, asteroid), the larger populations of A and Z (along with perhaps more diversity in these populations) may enable them to survive the event. The observation in the fossil record would be many remains of A and Z, but very few of the “missing links” in between.

We can contrast this thought experiment with the possibility that perhaps species A does gradually mutate into species B and B outcompetes A. But then B mutates into C, which outcompetes B. This cycle can continue and results in the more traditional and more incremental evolution from A to Z. The result will likely be that there will be a fossil record of the gradual changes.

Eldredge and Gould’s point was that both incremental and punctuated evolution could occur at the same time in different species. A particular species might undergo long periods of incremental change, interspersed with short bursts of rapid (on an evolutionary time scale) change. For a long time, despite the punctuation observed in the fossil record, it was assumed this was due to sampling error—we simply did not have enough data to see the gradual changes. Bak in fact worked with Gould to fit his fossil data to a power law, demonstrating that at least in principle the evolution of species might also be a variation of self-organized criticality. As such evolution might generate species in intermittent avalanches of all sizes and time scales. Of course, a problem with this hypothesis is that it is descriptive of onetime events that are unpredictable and therefore cannot be systematically tested. The main point, however, was that stops and starts in evolution might happen due to natural processes and not some external event.

The picture becomes more convoluted when "artificial" extinction events are possible. In this context, artificial means from outside of the system (e.g., asteroid strikes). The artificial extinction of a particular species will likely leave a particular niche (a fit peak) empty. For example, a particular frog may have become extinct due to some intervention. For the sake of the thought experiment, we can imagine that this particular frog ate a particular fly that other frogs could not eat. With that particular frog out of the picture, there is a role that could be played in the ecosystem to "harvest" the energy in those flies. It would be simple to imagine that another frog might fill this role. But it is also possible that an entirely new species might adapt to eat this kind of fly, for example, a spider or lizard. In some sense the first species to mutate in a way that would allow it to take advantage of the new energy source would likely win, at least until evolution changes the game through the Red Queen Effect. This readjustment could cause other ripples in a sort of metabolic avalanche. For example, because the waste products of frogs, lizards, and spiders are different, the impact on the environment will also be different. If the entire ecosystem is at a critical point, such a relatively small change could set off an avalanche that would cause many species to adapt or go extinct.

10.2.6 Technological Disruption and Innovation

To continue on with the theme of evolution, sometimes a new innovation prompts an explosion in new recombinations. The "discovery" of multicellularism seems to be one important example in the evolution of life on this planet. Once it was possible for cells to bind together, and later on to differentiate into different cell types, there was an explosion of new life-forms known as the Cambrian explosion. Technological innovation follows a similar pattern—a new generative tool comes along and opens the door to many new ideas and products. In the language of innovation, this generative innovation would be considered *disruptive*. Both the human-made and biological cases are examples of the adjacent possible leading to the opening up of entirely new areas of the fitness landscape.

In Chap. 8 we explored the evolution of technology using a fitness landscape. The idea of the sandpile can yield additional insights. One particular technology might spend its time in the "pile" (the technological landscape), interacting with other technologies. But then an avalanche could sweep it off the landscape. Sometimes these avalanches are small and isolated to particular kinds of technologies. Other times they are massive, and an entirely new set of technologies replace several older technologies. Some would argue that the information revolution that we are currently in is one of these massive avalanches and that we are in fact still on the leading edge of what is to come. All technologies will eventually become obsolete. Even the most fundamental tools, such as the wheel, might go away someday.

Technologies are often thought of as solving problems—resolving some tension in the wider system. But their presence creates tension somewhere else. In fact, an innovative entity could be viewed as simultaneously releasing and creating tension. This was the basis of the work of Kathy Eisenhardt (1947–) and Shona Brown (1966–), described in *Competing on the Edge*. They document a variety of cases where companies were able to remain far from equilibrium and balance at or near a critical point. At a critical point is where the company will be at its optimum in terms of balancing robustness and flexibility. Such a balance is a precondition for generating avalanches of new products. Although avalanches cannot be forced, a company can build up productive internal tension that will increase the rate and magnitude of disruptive innovations.

10.2.7 Relationship Between CAS and SOC

Some complex systems theorists suspect that there is a relationship between self-organized criticality and complex adaptive systems. Do all self-organizing systems gravitate toward a critical point? Does a homeostatic system necessarily land on the edge of a second-order phase transition? Do fractals, chaos, and power laws emerge from systems that are at a critical point? These relationships are not entirely clear, but some speculations are below.

Self-organized criticality is a description of how a system functionally adapts to the buildup of tension. A critical state might be achieved through functional adaptation. Such is the case of the sandpile—it is the canonical example of self-organized criticality, but it is only functionally adaptive. The reason is that the sandpile cannot change its own internal rules. It was argued above, however, that perhaps there are critical points in the rules of a system that can be reached through structural adaptations. If the rules stray too far in one direction (perhaps toward too much inhibition), an avalanche of structural adaptation would bring the rules of the system back into balance. As such, self-organized criticality could be the basis for both functional and structural adaptation.

10.2.8 Healing, Biodynamics, and the Economy

Complex adaptive systems can heal themselves when injured. The healed system, however, might be somewhat different than the system before the injury or perturbation. In other words, sometimes the definition of healing is to find a new homeostasis. Cancer, the decay of cities, ant colony disruption, business collapse, and other injuries can prompt the system to stray from a critical point. An avalanche may be disruptive, but it can be viewed as a healing response of a system that is out of balance. From this viewpoint, extinction of a species or death of an individual is the inability to a find a new homeostasis or critical point.

By definition a system that is at a point of criticality is always on some edge, generally due to tension, where its functions are stable and repeatable but still flexible. To be at a critical point necessarily means variability will be seen in that system. There are some fascinating studies in detecting variability in biomarkers such as heart rate. Somewhat against intuition, variability is a hallmark of a heathy system. A perfectly regular heart rate is a warning sign of disease. What we know from the theory of critical states is that this variability has a very unique signature called 1/f noise. This noise is not generated by the environment, or the measuring device, but is a product of the system itself. Some modern diagnostic devices in fact use this well-documented clinical variability as a way to detect the health (or disease) of the body. A nice summary can be found in *Biodynamics: Why the Wirewalker Doesn't Fall* by Bruce West (1941–) and Lori Griffin.

In economic theory there is a debate that still rages on between the followers of John Maynard Keynes (1883–1947) and Friedrich Hayek (1899–1992). Keynes is considered to be the founder of modern macroeconomics. He articulated that in a free market, there would be natural cycles but that these cycles could at times swing too widely, with huge booms followed by major busts. There would be some buildup of economic prosperity during good times, all the while building up tensions that would be released during a crash. Keynes argued that regulations, systems of checks and balances, and other means could be used to temper the wide swings.

Hayek on the other hand viewed the economy as a complex adaptive system that was robust enough to heal itself. In his view, perturbations from a government would perhaps fix a downturn in the short term but only lead to a greater crash later. Avalanches are simply the price to be paid for a long-term efficient system. In Hayek's view the government should stay out of the way and let the market heal itself.

10.3 Systems That Tune to Their Environment

A critical difference between Bak's sandpile model and many other agent-based models is that the sandpile is an open system. There is an interface with the environment and a trading of some quantity across the boundary. Consider an agent that has everything it needs internally to function but requires an energy source. Let's suppose it finds an energy source that is always to its upper right. Because the source can be counted upon, the system will over time adapt its own internal energy management system to take advantage of this regularity. In some sense the "mouth" of the system would face toward the right. This would be a natural structural adaptation to having a reliable energy source.

We can now consider what would happen if that energy source is the sun. The sun is not in the same place and is not always present, but it follows a repeating pattern. A simple solution will be for the agent to have a simple control mechanism, combined with sensors and actuators, that would always face the "mouth" toward the sun. Solar panels and many plants track the sun throughout the day and shut down non-vital functions during the night.

The most complex case is if the energy source does not follow an easily discernible pattern in space or time. A solution to having an unreliable energy source is to combine movement with energy management such as batteries or reservoirs. There are certainly businesses that are opportunistic in this way, gaining income from a variety of sources when they are available. Another way to say this is that they have diversified their revenue stream.

These thought experiments reveal that open agents will over time tune to the energy sources upon which they depend. The nature of these energy sources will impact the nature and expression of internal functions such as sensing, filtering, funneling, and moving. A similar argument can be made for material, information, and other inputs and outputs. In this section we will explore some of the consequences of how the external environment and internal organization of an agent are tuned to one another.

10.3.1 Conway's Law

In 1968 Mel Conway introduced an idea that has come to be known as *Conway's law*. His claim was that "organizations which design systems are constrained to produce designs which are copies of the communication structures of these organizations." In other words, the organizational structure of a system will be reflected in the outputs of that system. For example, if a company makes extremely complex products with many interrelated parts, its internal processes and flows (if it survives) will become similarly complex and interrelated. On the other hand, a simple modular product will likely result in simple streamlined processes. Conway's law implies that if you want to know about the internal organization of a car company, look at its cars.

We can consider a modified version of a sandpile, one that can adapt its own internal structure and thus become a complex adaptive system. Although an adaptive system cannot predict the size or timing of an avalanche, it can put in place structural barriers that will build up gradients of tension and release those gradients during an avalanche in a somewhat predictable way. Imagine what would happen if there was a stream of sand coming into a particular cell. Over time the adaptive sandpile would organize internally in such a way as to channel the resulting avalanches to perform particular functions. It is likely that the internal structure of such a system would be relatively simple. In other words, a simple and consistent input from the environment bends an adaptive system to be simple too. On the other hand, if there was little pattern to the input, the system would need to build some structural complexity.

A nice example comes from the work on the dynamics of the brain. In 2003 John Beggs demonstrated neural avalanches of information in neural tissue grown in a dish. The size of these avalanches followed a power law. Other neuroscientists were able to show similar behavior in slices and whole brains. The implication is that the brain is a complex adaptive system that is self-organized to a critical point. Based in part on this work, Dante Chialvo (1956–) demonstrated that our senses, which are a combination of sensors and processors, are in fact tuned to our environment. They have evolved over time to detect the things in the environment that matter. Some claim that our outward-facing sensors, coupled to our internal processing, have tuned our perceptions to the power laws observed in music, speech, and art. The result is that power laws are how we can most easily send and receive information-rich signals to one another.

Ross Ashby (1902–1972) was an English psychiatrist and one of the original cyberneticists. He made a wide range of contributions to systems thinking including popularizing the term cybernetics and creating a homeostat in 1948—the first machine that could adapt to its environment through a rudimentary learning algorithm. Another of his contributions was the *Law of Requisite Variety*, described in the text *Design for a Brain*. When a system is trying to control another system, it will necessarily contain within it a model of the system being controlled. For example, a teacher is generally trying to direct their students toward a particular learning goal. To do so will generally mean the teacher must contain within themselves a model of how students should learn. This model can be conscious or subconscious but guides a teacher's pedagogical decisions. In this way the teacher and student over time will mirror one another more and more closely.

A related idea is found in the difference between an *r-strategy* and a *K-strategy*. The distinction comes from exponential growth that is bounded by some carrying capacity, as introduced in Fig. 2.2. When a system is faced with an unstable and unpredictable environment (the exponential part of growth), it pays to expend resources to be in constant growth mode. The strategy is that those that reproduce and adapt quickly will win out in the end. This can be contrasted with the K-strategy (the asymptotic part of growth) that a successful system will adopt when the environment is relatively stable and predictable. Generally the K-strategy is to maintain a status quo and perhaps look to optimize functions. In both cases, the system strategy mirrors the nature of the environment.

10.3.2 Coadapting Agents and Environments

Imagine an ecosystem where the waste of one species is the food for another. If the first species is at a critical point, it will generate waste that is a power law in time and perhaps in space. When that waste is the input to another species, it will over time help tune that species to a critical point as well. When scaled up to an entire ecosystem, such a mechanism may move the entire ecosystem toward a critical point.

Natural nonorganic systems often seem to be at critical point too. Avalanches, earthquakes, floods, asteroid strikes, and volcanic eruptions are some of the most obvious. These all will impact biological ecosystems, pushing them to self-organize to critical points. In human-made systems, political, economic, and technological dynamics all seem to operate at critical points and will shape the individuals that exist in those systems. What is more human-made and natural systems can be open to one another. Food ultimately comes from the sun and nutrients in the air and soil, but humans modify the landscape through agriculture.

The relationship between an open system and its environment is bidirectional. Many systems fundamentally change their environment. For example, coral is a tiny organism that builds reefs the size of cities or even countries. Ants move massive amounts of material to create their hills. Beavers create dams that fundamentally change the environment for themselves and many other species that depend on the pool of water that is created. This is the concept of *stigmergy*—the ability of a system to reshape its environment to make its own fitness better. The environment can become what Bateson called an *extraregulator*—an external force that contributes to homeostasis. Humans are experts at this. There are some obvious examples, such as our cities. But human organizations also engage in stigmergy. For example, companies use lobbying to changing laws to make their own business more fit in the marketplace.

10.3.3 Paradigm Shifts: Evolution vs Revolution

The scientific process is often taught to the public as a steady march toward the truth—from the unknown to the known. Scientific disciplines incrementally evolve as they add new ideas, processes, and tools. In 1962 Thomas Kuhn (1922–1996) published *The Structure of Scientific Revolutions* in which he introduced the idea of a *paradigm shift* as a revolutionary restructuring of a discipline.

In Kuhn's theory, societies and disciplines build up a network of knowledge that explains how various facts, events, and observations relate to one another. A theory is the framework for this knowledge network, what Kuhn called a paradigm. This paradigm, however, is socially constructed by people and is a representation of a perceived reality. As such, theories and knowledge networks can be overturned and replaced by a new organization in a paradigm shift.

Kuhn described the usual scientific process as a slow incremental gathering of information that is placed within an existing knowledge network. Over time, however, some new information does not fit well into the current organization. This dissonance builds up tension. At first, the tension can be ignored or explained away. Just as in the sandpile, eventually the tension reaches the point where an avalanche occurs. During the avalanche, there is a scramble to take the same parts (knowledge, tools, ideas, and observations) and rearrange them in a way that has less tension. The result is a new network structure or theory that explains what is currently known.

In Kuhn's view, fields progress by a slow incremental evolution, during which tension builds, followed by brief and disruptive revolutions, during which tension is released. This cycle plays out again and again in every field, in a similar way to punctuated equilibrium in evolution. Others have built on Kuhn's idea of paradigm shifts to explain the dynamics of social, cultural, economic, political, and technological systems.

10.3.4 Tangled Hierarchies

Emergent behaviors can be nested, whereby emergence at one level becomes the basic parts for the next level of emergence. Causality flows from the bottom level rules up through the various levels. Molecules emerge out of atomic forces. Proteins and cells emerge from molecules. Multicellular tissues arise from cells and so on up the hierarchy. The rules at the bottom level constrain what is possible at the emergent level above. This is known as *upward causation*.

We know that upward causation is incomplete. There is also *downward causation*, where the higher levels of a system can influence lower levels. Downward causation was first coined by the social scientist Donald Campbell (1916–1996) to describe a feedback loop between levels of a hierarchy. For example, humans in some cultures are lactose tolerant, meaning that they can continue to drink milk into adulthood without side effects. Because of our Western viewpoint, we might consider someone to be "lactose intolerant" as if it were a disorder, but in fact it is due to a mutation—the natural evolved state of mammals is to only drink milk as newborns. There is ample evidence that the human mutation at the genetic level was selected for due to cultural forces. This is an example of a very high level of a hierarchy (culture) driving changes at a very low level (genes).

A similar kind of feedback occurs across levels in the economy. There are numbers, such as gross domestic product and mean household income that indicate an overall state of the system. These measures, however, are the result of how individuals make decisions on how they spend their money. These decisions may themselves be based upon bounded rationality that is derived from evolutionary imperatives that arose over many generations. The economics numbers, along with the media reporting these numbers, can then impact the economic behavior of individuals. Similar arguments could be made for the feedback between local laws and sociocultural phenomena. Functionally there is no true top or bottom hierarchy, as all levels have their own privileged position. The result is what Douglas Hofstadter (1945–) called *tangled hierarchies* or strange loops, whereby upward and downward causations cross back and forth between levels of a system. It should be noted that there are virtually no ideas about how such tangled hierarchies emerge. Likewise, because measurement must necessarily span time and space scales, there are not many scientific tools that explicitly are designed to study tangled hierarchies. There are, however, some interesting questions. For example, it is possible that the tangled loops across levels enable a nested system to tune its own critical points.

10.4 Creative Systems

Creativity is not easy to define or study. Yet there is a huge body of literature on the topic that spans across many fields. In Chap. 9 we discussed the origin of complex systems from an initial seed. In discussing creativity, we are in some sense discussing the origin of origins—how new things come into being.

A central idea in the study of creativity, perhaps the only agreed upon idea, is that new things come out of the recombination of existing parts or ideas. A functional adaptation might discover a helpful recombination, which if found to be of value to the system could be imprinted within the system as a structural adaptation. Below we will focus on the missing step—how a functional adaptation might be generated through the recombination of existing parts.

10.4.1 Dissolved Networks and Discovery

Structure constrains the flows on a network that give rise to function. In complex systems there is a tuning of both the structure and function so that the system is metastable. Practically this means that there is always a *latent instability* in the system that is never completely satisfied. In the language of this chapter, tension is built up that is released in avalanches of all sizes. An avalanche, however, is a functional event. After it is over, the system will go on functioning largely as it had before.

During an avalanche, long-range communication may occur within a system. Motifs that would normally never interact suddenly can. Unusual combinations of functions might suddenly be expressed together and recombine. An avalanche is a sort of temporary mixing bowl, enabling flows that generally would not occur. What is more, the avalanches are of all sizes, so over a long period of time, almost any two (or more) functions within the system might be briefly in contact with one another. The process of bringing together disparate elements was called *bisociation* by Arthur Koestler (1905–1983) in his 1964 book *Act of Creation*.

The eureka or "ah ha" moment may be our conscious experience of an avalanche. There was a lot of work that went into building up motifs and tension beforehand. Creative insight is when it all comes together and some tension is released. This view also highlights why many argue that one should keep an insight book. Because the new insights are only functional, they can quickly be lost. Writing down an insight in the moment is a way to capture the new idea (recombination) to be reviewed later.

10.4.2 Intermittent Creativity

In 2011 Barabasi's lab published an article in Nature of a "flavor network," shown in Fig. 10.5. The network was created from hundreds of cookbooks from a wide range of cultures. There are some clear themes and ingredients that can be picked out from this network that roughly correspond to styles of cooking.

On a normal day, you might follow a traditional recipe, only exploring one small part of this network. Every so often you create a recipe from scratch. Even these ad hoc recipes, however, likely conform to some small and well-known areas of the ingredient and process networks. The result might be a dish that is easily identified

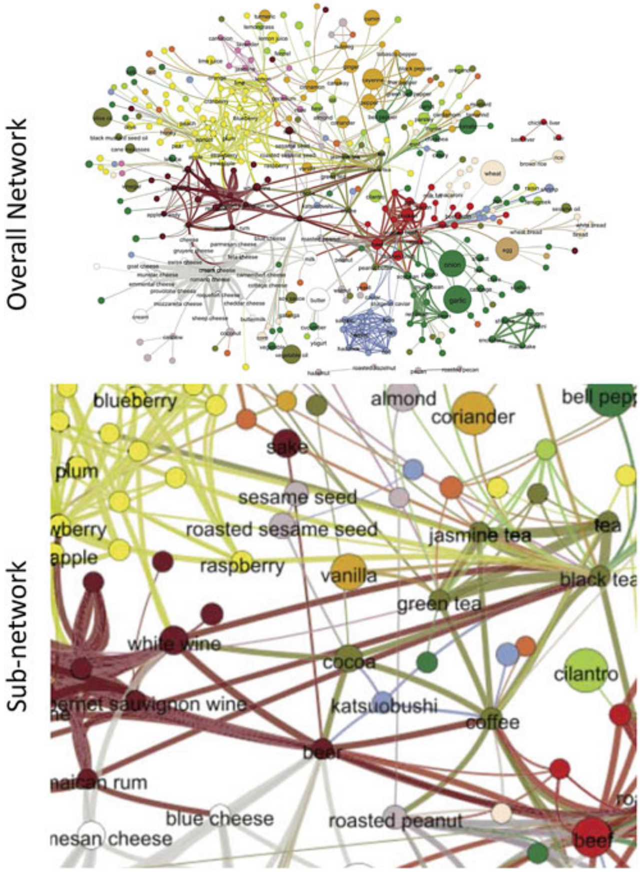


Fig. 10.5 Barbasi's flavor network derived from many cookbooks (top) with a zoom in to one part of the network (bottom). Creating a recipe is a recombination of the ingredients within the network. Original image appears in Ahn et al.'s *Flavor Network and the Principles of Food Pairing* Nature Scientific Reports 1:196(2011)

as inspired by Italian, Mexican, or Japanese cuisine. An avalanche does not need to take over the entire system. It may be relegated to certain types of motifs or potentially one level of a hierarchy.

Depending upon your experience and adventurousness, you may sometimes truly experiment. In these times you may mix and match ingredients from all over the flavor network. This is similar to what professional chefs are asked to do on TV shows such as Iron Chef. You may also need to modify the processes to adjust to the recombination of different ingredients. If a dish has too much liquid, it could either be drained, or the chef might search for something else in the flavor network that absorbs liquid. These experimentations are similar to avalanches—a functional dissolving of the flavor network enabling unique ingredients and processes to interact in a single dish.

In general radical experimentation does not happen very often. When it does, however, the cook will get feedback on if the experiment worked. If the new recipe was a hit, it might be added to the family cookbook. A similar argument could be made for musical compositions, scientific discoveries, cocktails, new species in an ecosystem, or economic policies. Often a creative work arises from a system that can temporarily break its traditional hierarchy of functions.

What we have described is strikingly similar to what Steven Johnson (1968–) has called *fluid networks* in his book *Where Good Ideas Come From: The Natural History of Innovation*. Johnson gives the impression, however, that the network is fluid all of the time. How can a truly fluid network still function? One possible answer, and the one suggested by Bak's avalanches, is that they in fact are only fluid in short bursts. A system that is constantly trying to do novel things would not last very long. Relatively "boring" periods of stability are punctuated by periods of creativity. In the language of non-linear dynamics, what we need is metastability—to maintain stable patterns most of the time but allow for short bursts of utter chaos.

10.4.3 Building Tension

In all systems that undergo avalanches, tension is necessary. If creativity is the result of avalanches, then it would mean that tension is required. What is more, the more tension that can be stored, the greater the frequency and magnitude of avalanches. A geological analogy is instructive—along fault lines there are not only larger earthquakes but also more of them.

In the creativity literature, there exists data to support the idea of tension that leads to creative avalanches. Mihaly Csikszentmihalyi (1934–) in *Creativity: Flow and the Psychology of Discovery and Invention* describes his findings from interviews of a wide range of outstandingly creative people. One of the more profound revelations was that almost every one of them had some dimension of their personality that was split between two extremes. They might display tension between male and female, or laziness and being a workaholic, or being extremely extroverted sometimes and introvert at other times. Tension is often a way of life for creative people. They are personalities far from equilibrium, channeling creative tension to generate novelty.

It has also been well documented that creative people tend to not only have major creative hits but also many smaller displays of creativity. Even after they pass away, it is often discovered that they generated outstanding works that were not known during their lifetime. It has not been determined if these events follow a power law, but it seems plausible. There is also data emerging that groups display these characteristics as well. The most creative teams, companies, countries, and so on all seem to have inner tensions that drive them and create all magnitudes of creative breakthroughs from every day to world-changing. The works of Keith Sawyer (1960–), summed up in *Group Genius*, *Explaining Creativity*, and *Zig Zag: The Surprising Path to Greater Creativity*, contain many insights into team creativity.

Lastly creativity might actually be a kind of pattern, like a spiral. It may be manifest in any kind of medium, on any time or space scale, as long as the system is put together the right way. A culture, business, ecosystem, team, or cell might be creative if the flows are right. A nice historical example is explored in Frans Johansson's *The Medici Effect* about the diverse ecosystem of players during the rise of the Italian Renaissance. Others have explored how the positions of water coolers, coffee machines, and bathrooms in the workplace can foster the patterns of mixing that will generate creative interactions. Creative systems are not one-hit wonders.

Spirals form in reaction-diffusion systems. Unlike a spiral, there likely is not a master equation or algorithm for creativity. Rather, creative systems might be those that balance on a critical phase transition and undergo periodic avalanches. Such systems can have "guided accidents"—the serendipity of being in the right state to take advantage of chance events.

Much work on creativity has separated genius creativity from mundane creativity. The implication is that an Einstein or Da Vinci is creative in a fundamentally different way than you or I when we take a detour to avoid traffic. A systems view of creativity, however, implies that all creative systems, spanning not only humans but all complex adaptive systems that build up tension and release it in avalanches, are creative in essentially the same way. That society attributes a magnitude to a creative breakthrough is less a reflection of the internal working of the person or team who created the work or idea and more a reflection of external judges.

10.5 Being Human

What it means to be a human has frustrated many thinkers throughout the ages. Definitions range from the biological to the mystical. A common theme is to regard a person as a complex pattern. The disagreements seem to be on what these patterns are and how they arise. Possible candidates are the patterns within our genes, minds, tools, language, moral systems, and ethereal souls. In this section we will explore how complex systems theory might add some perspective.

10.5.1 What Is a Self

In Chap. 4 the idea was introduced that a self might be a self-reinforcing multilevel network, out of which emerges a new kind of behavior that is manifest in a self. In this chapter the concept of a strange loop was suggested as a possible structure for this multilevel network. What can be added here is that a self might be a dynamic autopoietic system that, when healthy, self-organizes to a critical point. As such it is ever changing and contains all necessary functions within. These include but are not limited to models of the world, ideas, memories, habits, skills, and likely much more. One word of warning is necessary. Much of what has been written about the self, and what is shared below, is a Western view. There are many cultures that do not share this kind of "independent" version of the self.

As an autopoietic system, a self would necessarily be open and require information and energy to be streaming in and out. This is compatible with various theories of a self, such as Abraham Maslow's (1908–1970) hierarchy of needs, first articulated in a 1943 paper, "A Theory of Human Motivation." The essential idea is that a person must have a set of "vital" inputs to continue to be a self. These include, moving up the hierarchy, physiological needs, safety and security, and social relationships. Without these, a person cannot achieve full self-hood or become a truly independent agent. In Maslow's view once these conditions are met, it is possible for self-esteem (e.g., confidence) and self-actualization (e.g., creativity) to emerge. Such a way of thinking has influenced clinical and abnormal psychology as they consider how a unique individual develops and interacts with the world.

A related idea is that a self is not a physical thing at all, but rather a series of purposes that emerge and intertwine over time. These purposes begin as reflexes, simple reactions to inputs. As outlined in Chap. 7, internal structures become more and more complex over time, leading to more and more sophisticated reflexes. The ability to anticipate and make predictions, however, brings with it the organization of resources and expression of internal functions needed to plan for the future. Such a system will have a tension between being reactive and proactive. Out of this tension can come purposes that originate within—a proactive plan is created and executed upon until a roadblock is encountered, at which point a reactive strategy is used to get back to the original plan. Of course such a system could become distracted, abandoning or forgetting the original plan in favor of some new plan. Such a trajectory could in fact be thought of as a metastable system that is constantly being perturbed from one basin of attraction to another.

There are some intriguing ideas that arise when viewing a self as a purpose-driven entity. Here we will only discuss one idea. A Ulysses Contract is a psychological construct named after the story of Ulysses (Odysseus in Greek) and the Sirens from Homer's *Odyssey*. In the story, Ulysses is sailing toward the Sirenum Scopuli islands which are infamous for the Sirens who live there. When a ship sails by the islands, the Sirens sing their song, and the men on board are so entranced that they crash the ship into the island, never to be heard from again. In many popular accounts, the Sirens are sexualized, but in Homer's account, they are offering knowledge. Ulysses, however, cleverly asks his men to block up their ears, tie him to the mast, and then not pay attention to any of his pleas until they are well clear of the island. In such a situation, the ship passes safely past the island, and Ulysses becomes the first human to hear the Siren song and live. He was able to predict that his future self would take an action (crash the ship into the island) that his present self does not desire, so he takes an action in the present to prevent it. In the context of Chap. 8, Ulysses has played a sort of game with his future self, constraining his decisions in the future, such that the payoff will be better. The idea of a Ulysses Contract has a modern interpretation. Recovering addicts are trained to constrain their decisions such as removing the cues that might cause them to relapse. This is one way in which a self may be simultaneously reactive and proactive.

10.5.2 Free Will and Consciousness

Two of the thorniest problems related to being human are free will and consciousness. Both topics have been touched upon throughout the text without resolution. For example, in Chap. 7, we discussed the idea put forward by Nicholas Humphrey (1943–) in *Soul Dust* that consciousness is a turning inward of powers that previously were targeted toward the outside world. Sensations, thoughts, and actions originated to survive in the world. To make proactive decisions in the world requires some model of the world with which to make predictions. And this model of the world is constantly being refined as new information becomes available. When the ability to make a model is focused inward, the result is a model of the self that can be just as rich in detail and equally dynamic. The definition of consciousness has been debated for a long time, but an operational definition of consciousness may simply be a model of the world that has been turned inward to include the model of the maker. A self is therefore self-referential in the same manner as discussed in Chap. 6.

The evolutionary purpose of consciousness is also a difficult question. Various thinkers have espoused a range of reasons from it being accidental, as expressed by the neuroscientist David Linden (1961–) in his book *The Accidental Mind*, to driven by some higher power, as in the analogy of God being a potter who is shaping us as if we were clay (Isaiah 64:8). Others claim that consciousness is an inevitable emergent property of evolution.

A critical question is whether or not animals, especially primates, possess some form of consciousness. The answer to such a question has many implications related to our treatment of animals as pets, workers, and a source of food. Empirical arguments for animal consciousness range from some primates and bird's abilities to use rudimentary language and tools to documented perceptions of fairness when some animals play the ultimatum game. One of the most convincing set of studies involves whether or not an animal can recognize itself in a mirror. The list of animals that can recognize themselves is long and growing, including elephants, apes, dolphins, whales, and magpies, potentially some ants and manta rays. Many of these studies have concluded that consciousness is not one particular thing and that it likely comes in various levels or strengths. Such a view is consistent with the idea that consciousness may fall along a second-order phase transition.

Often connected in some way to consciousness is free will. Thinkers ranging across philosophy, religion, mathematics, and neuroscience have mused on this topic throughout recorded history, with conclusions that are equally varied. In the context of this chapter, an autopoietic system contains within itself all of the functions necessary to make decisions about future actions. Such an independent agent that is influenced by the outside world but not driven by it does seem to fit the cultural meaning of free will, if not the formal definitions developed by philosophers. A cell, however, is an autopoietic system, but most would not attribute free will to a liver cell. Perhaps being an autopoietic system is a necessary, but not sufficient, condition for possessing free will, which would mean that some other systemic organization is also necessary.

Empirical approaches to studying free will build upon causality, namely, that thoughts (the cause) precede actions (the effect). The experimentalist Benjamin Libet (1916–2007) in fact performed a variety of experiments asking participants to become conscious of when they decide to act and when the act actually occurs. The results of his experiments seemed to show that if there is a causal relationship between the two, the time window must be very small, on the order of a few hundred milliseconds. Libet's conclusion, summed up in his book *Mind Time*, was that we do not have free will in the way we suppose, but rather *free won't*. Options are presented to us by our unconscious mind. All our conscious mind can do is intervene at the last possible moment to veto the action or at least slow it down enough so that the unconscious mind can develop an alternative action.

These studies have gone even deeper into the brain using functional magnetic resonance imaging. Researchers can watch as areas of the brain light up, preparing for some action, before the participant says they have decided. The latest estimate is that the time window between unconscious brain activity and conscious perception of a decision can be up to 10 seconds. In some cases researchers can see the neural patterns of an action several seconds before the subject consciously knows they are planning the action.

The model that is emerging from these types of studies is that our unconscious self is making the vast majority of choices for us, upon which we unconsciously act. In a small percentage of cases, an unconscious decision cannot be reached, and the conscious self becomes involved. In this view the true bounds on free will (as a property of consciousness) seem to be very narrow, but at least our conscious self would have a say in decisions that truly matter. This hopeful proposition is debated in an article by John Bargh (1955–) and Tanya Chartrand, "The Unbearable Automaticity of Being." In the first half of the article, they cite a wide range of evidence that narrows free will to a very small corner of consciousness. We mostly seem to react to situations as they happen. In the second half of the argument, they take on a stickier argument made by others—perhaps our conscious mind can train our unconscious mind to react in certain ways. We may not be free in the moment, but on a long time scale, we can plan for our future, and on this time scale, we do have free will. Essentially we use our conscious minds to build unconscious habits. Bargh and Chartrand point to a growing body of literature, from both the nature and nurture ends of the spectrum, that even our long-term desires and goals are not our own. Rather they are so influenced by the environment as to not be attributable to an individual. Again, their conclusion is that if the conscious mind is free, even on long time scales, it must be so only over a very small range of situations.

Michael Gazzaniga (1939–), who was introduced in Chap. 5, has proposed an alternative view of free will that aligns well with complex systems theory. He points out that free will only matters when other people are involved. Ethics, justice, social contracts, and laws only seem to apply when there are interactions with other agents. This to Gazzaniga is a clue that free will is not located in individuals but rather is a phenomenon that emerges in a second-order phase transition when groups coexist. In this view free will is a social phenomenon that emerges in much the same that a "V" pattern emerges in flocking birds.

10.5.3 Moral Systems and Stories

Humans seem to be the only entity with a moral compass. Phrased another way, we have risen above the "Nature, red in tooth and claw" view of evolution expressed by the poet Alfred Lord Tennyson (1809–1892). As with other dimensions of being human, there are many origin stories and interpretations. A starting point for a systems analysis is to assume that a moral system is composed of various moral elements. In a particular society at any given time, the balance and interrelations between these elements might be different. Perhaps there is not one way to achieve a moral balance. This idea has been debated throughout history. Is there one master system of morals that can be derive from human nature or are moral systems relative to a culture and time period? There seems to be evidence on both sides. For example, almost every culture has some version of the Golden Rule (e.g., Do unto others as you want them to do to you). On the other hand, what is taboo in one culture may be perfectly acceptable in another (e.g., ritualistic cannibalism).

Perhaps there are only a few moral elements, derived from human nature, that are in tension. Each culture tunes the balance of these elements to create a moral system that matches their local needs at that time. This is a kind of moral modularity. A modern thinker who expresses such a view is the psychologist Jonathan Haidt (1963–), who built upon the work of anthropologist Richard Shweder (1945–). In his book *The Righteous Mind: Why Good People Are Divided by Politics and Religion*, Haidt outlines five moral elements: Care/Harm, Fairness/Cheating, Loyalty/Betrayal, Authority/Subversion, and Sanctity/Degradation. We encounter situations every day where two or more of these dimensions are in conflict with one another. For example, if someone has done something that is clearly their own fault, should you help them out (expressing Care) or chalk it up to justice (expressing Fairness)? Some have even claimed that these elements are innate, encoded in some way in our genes, but then our environment fine tunes the balance. Because there are multiple possible stable combinations, perhaps mapped to a sort of moral fitness landscape, there can be competing moral systems.

When groups with a different balance encounter one another, they may view what is "right" differently. The conflict that ensues is not a rational battle but a moral battle. For example, when it comes to welfare programs, the moral element of care may be in tension with fairness. As Haidt points out, a particular society will often have in place laws and social norms that are some compromise between competing moral systems.

The function of legal, political, educational, and religious systems may be in part a way to encode and disseminate aspects of these moral compromises. When a group is small, direct reciprocity can keep the moral compromises in check because everyone has the possibility of having repeated interactions with everyone else. As the size of a group becomes larger, direct reciprocity begins to fails. The solution is to develop and enforce a variety of social contracts.

In many cultures parables and folktales are used to communicate information about the balance of morals as well as taboos. Haidt has constructed some modern day stories that are meant to expose how particular taboos may have been created for practical reasons. Consider one of Haidt's scenarios:

> Julie and Mark are brother and sister. They are traveling together in France on summer vacation from college. One night they are staying alone in a cabin near the beach. They decide that it would be interesting and fun if they tried making love. At the very least, it would be a new experience for each of them. Julie was already taking birth control pills, but Mark uses a condom too, just to be safe. They both enjoy making love, but they decide never to do it again. They keep that night as a special secret, which makes them feel even closer to each other. What do you think about that? Was it ok for them to make love?

Most would say "no way" to this story, and Haidt has documented that almost everyone would agree, even across cultures. But the study was not about how people answered but about their rationale. When pressed to give a reason why this scenario is morally wrong, almost everyone becomes stuck. The idea of genetic defects from inbreeding has carefully been countered in the story—they will not get pregnant. Likewise, the social-emotional disruption to a sibling relationship has been countered—they don't share it with anyone, and their relationship becomes stronger. Furthermore, it never happens again. Lastly, some might say that it is illegal. That may be true in most cultures, but that is Haidt's point—the law is a cultural creation and could vary depending on culture (in fact in France, it is not illegal) and could be changed by politicians at some later date.

Of course Haidt is not advocating for incest. He created stories that were meant to evoke initial reactions of disgust or shame but then to have no (or little) rational basis. His point is that morals are somehow distinct from logic. Others have gone much farther to claim that morals emerged for practical reasons from evolutionary pressures. Before birth control, incest could lead to birth defects. Eating particular animals might become taboo for practical reasons but are encoded in the moral and legal systems of that culture. In these situations, when the original practical reason for the morale principle has become irrelevant, might a social taboo change?

10.5.4 Tools and the Extended Mind

It is an assumption of the West that each person is an independent entity. Throughout history, however, there have been numerous thinkers who have pointed out that an individual is an open system. A Westernization of this idea is to imagine that mental, physical, and social tools extend what it means to be a member of the human species.

Humans have used tools throughout the history of our species to amplify the power of our physical bodies. We are not bigger or stronger or faster than other species. We might claim that our advantage lies in our more developed brain, but a human cannot "think" and have food arrive or fight off an attack from a tiger. Our real advantage, perhaps derived from the structure and function of our brain, is our use of tools. As portrayed beautifully in the opening scene of the movie *2001 A Space Odyssey*, this process may have begun with found objects such as bones and stones. A next step may have been the modification of found objects—a found stick might be too long for a specific purpose, so it is broken to be a more appropriate length. A further step might be to combine together two or more tools to form a more complex tool. Of course this is a "just so" story that comes with all of the warnings from Chap. 1.

A growing number of anthropologists and evolutionary biologists claim that grammar originally evolved to create tools. The precise shaping of a rock or stick, or the building of a more complex tool, required a particular sequencing of actions. These sequences were likely stored in the cortical area associated with hand motions. A next step would have been the emergence of a protolanguage of hand signals, followed by the co-opting of motor areas needed for vocalizations and the origin of spoken language. What may have begun as a set of mental commands for toolmaking were co-opted for other mental processes. Regardless of the exact origin, grammar may be a mental constraint that also applies to the human inventions of logic, mathematics, music, and art.

The concept of a self has been mentioned previously. Even putting aside any metaphysical or cultural component of the self, some question if a self must be entirely contained within a body. The self could be more networked, with the body only serving as a sort of hub around which the other parts of a self revolve. The philosophers Andy Clark (1957–) and David Chalmers (1966–) wrote an article called "The Extended Mind" in 1998 in which they proposed that a self is not really contained within the body. They argue that many functions of a self are in fact being played more and more by the technology that we have created. For example, the written word can be viewed as an externalization of our biological memory. As a mundane example, Clark and Chalmers compare two people, both of whom are trying to find a particular location. One uses their biological memory, while the other uses previous sketches in a notebook. Clark and Chalmer's point is that there is no logical difference between these two people in how they find the place, only that for one person the system is the body and for the other person it is the body plus the notebook. The argument is that we are pushing more and more of our "self" out into the world, as expressed in *Natural-Born Cyborgs* and *Supersizing the Mind*, both by Clark. How much of "you" is contained in your cell phone? Has technology allowed the pattern of "you" to become larger and more far reaching in both time and space? What is more, the pushing out of functions to external objects does not end with the physical and mental. The more recent explosion in social media is a technological augmentation to our social abilities.

The allure of technology may be explained using the concept of fitness landscapes introduced in Chap. 8. Because humans can create and iterate upon tools, we can think of these tools as evolving on their own fitness landscape. Our collective iterations and redesigns are simply changes to the structure and function of a tool such that it will have greater fitness. Furthermore, we are the arbiters of whether a tool is fit or not. When we push a function out of our bodies and minds, we essentially have moved the evolution of that function to a different fitness landscape, the one of tools. No longer does that function need to go through the bottleneck of the genome or the constraints of biochemistry for changes to occur. Changes to a tool can happen at any level of a hierarchy, so technology designed for one purpose can more easily be co-opted by another technology. This kind of exaptation, as introduced in Chaps. 7 and 9, also extends to the way in which a function is achieved. If an optical solution works better than an electrical one, the change to the tool is relatively easy to make. The result is that a function that has been pushed out to a tool can evolve in many more directions and much more rapidly than if it remained trapped in a biological organism.

Perhaps a defining feature of being human is our unique ability to create and design new fitness landscapes (e.g., tools, languages, cities). It might even be claimed that a human stripped of technology, language, or community would not actually be a human. Then again, this statement does not consider the rights we grant to babies, the elderly and sick, or disabled humans.

10.5.5 The Singularity

The disparity in the rates of change on the technological and biological landscapes has prompted a number of scientists, science fiction writers, and philosophers to conclude that it is inevitable that biological life and mind will eventually be outpaced by non-biological entities. Such a view was expressed in Chap. 5 with regard to the Internet of Things becoming a massively interconnected network of smart devices. We are seeding our environment with functions that do not necessarily need us to continue on. When enough connections are made, the probability that these devices will collectively "wake up" will dramatically rise. The claim is that there are various accelerations that are converging. Kevin Kelly (1952–) has detailed some of these, mostly focused on the digital revolution, in his book *The Inevitable*. Two technological luminaries, Elon Musk (1971–) and Stephen Hawking (1942–2018), both agree that our greatest existential threat is the rise of non-biological species, not climate change or nuclear winter. We perhaps unknowingly are sowing the seeds of our own demise.

The inventor and futurist Ray Kurzweil (1948–) has made a career out of predicting technological trends. His writings, typified by *Age of Intelligent Machines*, *Age of Spiritual Machines*, and *The Singularity is Near*, outline a vision for what society will be like in the future. It is highly biased toward digital technology and most specifically artificial intelligence, but the conclusion Kurzweil comes to is that we are approaching *The Singularity*, a term made popular by Vernor Vinge (1944–) in 1993. The claim is that we have been accelerating past the biological for some time. Rather than disappear, biological systems will simply merge with technology systems. The popularized portrayal of cyborgs as part human and part machine is a crude vision. We will become something else altogether from the finest levels of life and mind to the most grand. In Kurzweil's vision, this new form of life and mind will radiate outward to the planet, solar system, galaxy, and eventually universe, perhaps even merging with other entities that are coevolving in other parts of our universe.

10.5.6 Staying Human

The threat of a non-biological takeover is perhaps overblown or so far off in time that it will not impact anyone living today. But the acceleration of technology has already profoundly impacted our lives and our collective perceptions of who we are. A recent take, along with a healthy dose of complex systems, can be found in Thomas Friedman's (1953–) book *Thank You for Being Late: An Optimist's Guide to Thriving in the Age of Accelerations*.

How will we retain what it is to be human? This is not a new question and one that we, as a species conscious of itself, will continue to ask. All the while we will continue to change, necessitating a constant redefinition of what it means to be human. Throughout much of this text complex systems theory has served as a framework for understanding the changing world around us. When pointed inward, it may also help reveal who we are and predict who we might become in the future.

10.6 Questions

- Identify a recent "ah ha" moment that you have had. Can you recall what might have been recombined to lead to this moment? What connection was made? Was there a tension that may have brought about that connection?
- There are a few radical scientists that have speculated that the fundamental rules of nature are in actually changing and potentially even emerging over time. What we experience is simply a snapshot of the rules at this time. For example, there is some evidence that the speed of light may be slowly changing over time. Even more radical is that there might be something like a fourth law of thermodynamics (which creates order) to counter the second law (which creates disorder). Perhaps the universe is a self-organizing system that can adapt its own internal rules. It would be its own emergent phenomenon. What are your thoughts about this very speculative claim?
- Pick two systems and explain how they are intertwined with one another. How might they push one another to coadapt to their own critical points? Are either of your examples tangled hierarchies? What tensions exist between the two?

- Is there a system at your university or place of work that you feel is sick or diseased in some way? Can you use the concept of self-organized criticality to identify the source of the problem? Can you prescribe a possible solution? Might it require an avalanche or can the solution be reached in a more incremental way?
- List the major problems facing the world at this time. Then recruit a few friends and make a network map of the problems, connecting problems that are interrelated. For example, poor education might have an impact on poverty which can influence ideas about climate change and energy. Make a network diagram to map out the world's problems. Where do you see bottlenecks? What inertial elements of the systems involved will make these problems difficult to solve?
- Politicians and others often debate and criticize one another over their respective moral systems. In some cases the conflict is over contradictions or misalignments in particular moral system. This is a philosophical difference. A more practical criticism is that a particular person or group has not lived by their moral system. Find a news article of one person (or group) attaching another and dissect in what ways their attack is based upon differing moral systems.
- The terms bifurcation and phase transition have been used throughout the text in a somewhat fuzzy way. In general phase transitions are the term used in statistical mechanics to describe fundamental changes in behavior that result when many particles or agents interact. These are inherently high dimensional systems, meaning they have many components. Bifurcations on the other hand are in the domain of non-linear systems, usually described with a few equations and therefore of a low dimension. Where the two sometimes meet is when a higher dimensional system is reduced (or averaged in some way) to create a description or model that is lower dimensional. Some fields call this *mean field theory* which reduces the complexity of a many-bodied or dimensioned system to a few dimensions. An order parameter is an example of how a single important variable might represent the collective behavior of many interacting parts. List some other measures in other systems that are derived from mean field theory. What do they tell about the system? What can be missed in the averaging? What important dimensions might be missed?
- Interview a person from a different discipline. Ask them what the major debates are in their field. Where are the tensions? Can you map their response to ideas from complex systems theory? Do you think their disciple is too ordered, too disordered, or at a critical point?
- Much has been written about what it means to learn. One practical sign of learning, attributed to Ken Bain, is whenever someone changes how they think, act, or feel. We could, in fact, encode this in a differential equation as

  $$\text{learning rate} = \frac{d(\text{think})}{dt} + \frac{d(\text{act})}{dt} + \frac{d(\text{feel})}{dt}$$

  Discuss how complex systems theory concepts might map to one or more of the following educational concepts:

1. *Cognitive dissonance*, proposed by Leon Festinger (1919–1989) in 1957, is the state of mental discomfort that builds up when real data contradicts a person's view of the world or themselves. A person who feels this tension will either ignore it or if it becomes too discomforting find some way to resolve it. Learning is one possible result of resolving the tension.
2. *Self-determination theory*, proposed by Edward Deci (1942–) and Richard Ryan (1953–), proposes a balance between intrinsic and extrinsic motivation that is driven by three needs: autonomy, competence, and relatedness. Autonomy is the feeling of having some degree of causal control over the world around you, competence is the feeling of mastering some particular aspect of the world, and relatedness is the feeling of working toward some higher purpose that will impact others.
3. *Social constructivism* developed in several steps, principally by Jean Piaget (1896–1980), Jerome Bruner, Albert Bandura (1925–), Perter Berger, and Thomas Luckmann, views the mind and learning as more than information processing. Knowledge is situated in a real environment and co-created by social groups. The theory suggests that observing and imitating others, often in communities of practice that are interacting with real environments, is how individuals construct (and therefore learn about) reality.

- A range of online music stations aim to create an internal model of your musical preferences using a list of your favorite artists. By voting up or down particular songs, over time, they iteratively create a sort of musical DNA for you. What other services might also use the same concept of building a model of your preferences?
- In this and previous chapters, the ideas of partial or looped causality were discussed. In a strict sense, these do not violate the basic tenets of causality, only that when various forms of feedback become intertwined, it becomes more and more difficult to determine singular causes of events that occur. At a high level, this is the case in the feedback between structural and functional adaptations. Some, however, claim that there is such a thing as backward causality, sometimes called retro-causation. The essential idea is that in some circumstances (usually assumed to be very limited), future events can impact the present. As discussed in Chap. 7, as long as the laws of thermodynamics are not violated, all other physical laws that we know of are indistinguishable going forward or backward in time. Likewise, there are some particles that seem to travel backward in time. It is important to note that backward causation is not time travel. Rather it is relaxing the assumption that causes and effects are bound by the arrow of time. That we experience causes and effects flowing only in one direction could be due to the macroscopic scale of our experience. What do you think of this claim? Does it matter if there is such a thing as backward causation? How might you go about proving, or disproving, that it exists?
- It was speculated that sustained creativity may be a property of systems that balance at the edge of a phase transition and regularly undergo avalanches. If that is true, it may be possible to diagnose and treat creative diseases, in people,

teams, and institutions. Select some system that you do not feel is particularly creative and dissect the possible causes using systems theory ideas. What would you prescribe for this system?

- The psychologist Erik Erikson (1902–1994) created one of the more influential theories of human development. In popular culture he is probably best known for coining the phrase *identity crisis*. He viewed a developing human as progressing through roughly eight stages that described the particular kind of crisis faced at that particular stage. For example, very early in life, there is a crisis over what can be trusted and what cannot be trusted. Failure to develop trust in something at this stage will lead to developmental problems later on. Later stages have their own tensions, between intimacy and isolation or being generative versus stagnation. The eighth stage is essentially one of achieved wisdom. In the context of this chapter it is as if within each stage a tension is building up and must be released. The release of this tension brings about an internal paradigm shift, after which the system is different and begins to build up a new sort of tension. What fundamental tension or identity crisis are you facing? What event or paradigm shift would release tension? What might a resolution look like?
- In Chap. 8 Daniel Kahneman's ideas of a fast and slow mental systems were introduced. A related idea, also introduced by Kahneman, is of the *remembered self* versus the *experienced self*. The remembered self is your model of who you think you are, including your memories and habits. It is accumulated and continually modified throughout your lifetime. Your experienced self is a sliding window in time of perhaps 10–20 seconds that stores your perceptions in the moment. Kahneman's reason for splitting these two is that your experienced self is only indirectly connected to your remembered self. In other words, your experiences in the moment are not simply dumped directly into your remembered self. They are very heavily processed in such a way that they will fit into your existing model of yourself. This can lead to all manner of interesting phenomenon. For example, we often can better remember a first encounter because it is what establishes a new part of our mental model. Future encounters only modify our model of that first experience. Likewise, we can easily misremember an event if we have experienced similar events in the past. This is on full display in the work of Elizabeth Loftus (1944–) who has systematically demonstrated how flawed witness testimony can be in a court of law. Trace back some event or idea in your life that has undergone revision through repeated encounters. Can you remember your first encounter? How about the repeats? Can you identify any disjunctions between your experienced and remembered self?
- In this chapter we explored how balanced rules might emerge, leading to complex systems. Furthermore, humans and human-created systems can observe, dissect, and question these rules. In questioning rules, it is natural to ask what would be different if the rules were different. Scientists ranging from physicists to evolutionary biologists have been able to demonstrate slight changes in the rules or historical events would prevent anything like us humans from arising. For example, if the charge of the electron was slightly higher or lower, it would not

allow for the type of organic chemistry that gives rise to life. Likewise, if the early origins of life did not expel oxygen into the atmosphere, the energetic reactions that power multicellular life would not have been possible. Without these kinds of finely tuned parameters, it seems that no entities could exist that could even ask these types of questions. This kind of broad claim is known as the *anthropic principle*—that we can ask the question implies that the rules necessarily needed to be the way they are. It is a variation of the question, "if a tree falls in the woods and no one is there to hear or see it, did the tree fall?" There are many variations of the anthropic principle, but almost all of them concern carbon-based-conscious life. Choose a complex nonorganic technology and explain how "it" might argue a similar kind of principle (both rules and path-dependent histories) that describe how the technological fitness landscape necessarily must be the way it is.

- A number of human-created systems and policies aim to regulate our impact on the natural ecosystem. Deconstruct one of these using the concepts of self-organization, Conway's law, and critical points.
- The board of directors for a company is generally charged with watching out for the long-term health and sustainability of a company and helping to make critical decisions. It is considered good practice to have a diversity of opinions that will balance each other. In a similar manner, a professional development technique is to create your own personal board of directors. If you were to create a personal board of directors composed of people you know, remembering to keep a balance of voices and opinions, who would you choose and why?
- In *What does a Martian Look Like?*, Jack Cohen (1933–) and Ian Stewart (1945–) explore the science behind alien life-forms. Throughout the book, they balance three interesting perspectives: (1) the variety of life on our own planet and the somewhat surprising "aliens" living among us; (2) the various possible forms of life that could emerge, develop, become intelligent, and form cultures and societies; and (3) the many science fiction writers who have explored the space of alien life and culture. Choose an environment that has within it a natural energy source or gradient. Then build up a plausible autocatalytic system that might metabolism based upon this energy source. Keep exploring how this system might continue to develop into a variety of organisms, perhaps eventually untethering from the energy source. Keep following as long as possible to include ecosystems, societies, and cultures.
- Stigmergy was introduced as the changing of the environment by the collective action of a group of agents. In an example of feedback, these structures then change the way the individual agents behave. In this way, one channel for communication between agents is in fact the structures that they are collectively building. The term is generally used to describe how biological organisms change their environment. In 2014 a group of researchers at Harvard created termite-inspired robots that operate on simple rules and work together to construct structures. They are fully autonomous, and there is no explicit plan for the structures that are built. Write a short story, taking place in the future, where most if not all of the technologies we see around us, from our sneakers and phones to bridges and buildings, are created by swarms of autonomous robots.

- David Kirsh (1954–) has discussed how cooks place utensils and ingredients such that they facilitate the order of steps in the cocking process. This is another example of the extended mind, but in this case as an aid to a process. The placement of the various steps in the process can be a reminder of what comes next. Find an example of some way that you have externalized a process. How did your method arise? How has it evolved over time? What would happen if someone disrupted your externalized method?
- Tangled and hierarchical networks might self-organize to critical points that span across levels of a system. It may therefore be possible that multilevel avalanches could occur. For example, an avalanche on one level might trigger an avalanche at another level. And these two avalanches may be occurring simultaneously on different time and space scales. Can you find a system in which this may be the case? Be clear about the levels, the avalanches, and how an avalanche might cross from one level to another.

10.7 Resources and Further Reading

As this chapter tied together ideas from previous chapters, many of the references and further reading from previous chapters could also double as further reading for this chapter as well. David Engstrom and Scott Kelso's (1947–) *The Complementary Nature* is an interesting, although sometimes dense, proposal for how to handle conflicting contraries. It resonates well with the idea of self-organized criticality but from a philosophical perspective. For a brief overview of self-organized criticality, a nice place to start is Per Bak's *How Nature Works*. In the realm of minds and brains, Dante Chialvo's "Critical brain networks" and "Emergent complex neural dynamics" are nice follow-ups to the claim that our brain operates at a critical point. Scott Camazine's *Self-Organization in Biological Systems* has some excellent insights. Evan Thompson's *Mind in Life* brings in an interesting philosophical perspective. A slightly outdated but still wonderful book on some major controversies and tensions in science is John Casti's (1943–) *Paradigms Lost*. Each chapter covers the history behind a major question from the origins of life, consciousness, and language to extraterrestrials, artificial intelligence, and how much we can know the real world.

Chapter 11
Criticisms and Future Directions

An introductory text aims to provide a taste of a field and arm one with the tools, definitions, historical context, and direction to learn more. The purpose of this final chapter is a bit different. First, there are some unignorable criticisms of both the tools and general approach of complex systems theory. A theme throughout the text is that the structures that are built up for some outward-facing function can be turned inward. We can do the same with complex systems theory—turn its thinking inward to the field itself. Second, it is perhaps unwise to reveal the weak points of a field, but I believe an awareness of the problems of complex systems theory will help you be a better explorer and reveal fertile ground for future exploration.

11.1 Criticisms of Complex Systems Theory

The criticisms below will help you become aware of how complex systems theory is sometimes viewed by others. You would be wise to keep these in mind when having conversations with others who have not read this text.

11.1.1 Complex Systems Theory Is Not New

As discussed in Chap. 1, various thinkers throughout history have discovered and rediscovered key concepts of complex systems. For example, Kant described emergent systems, and Heraclitus of Ephesus captured the essence of dynamic systems in his quote "You cannot step twice into the same river." Likewise, Jane Jacob described a city as a metabolizing entity many decades before Geoffrey West

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laid down mathematical expressions for the same idea. Leonardo da Vinci in fact noted that city design should follow biological principles of resource distribution and waste elimination.

Some fields also have adopted or independently discovered topics that fall under complex systems theory. For example, biology has studied the structure and function of growing organisms for several hundred years. Somewhere along the way, however, biologists followed the path of physics in the search for basic building blocks. The field became reductionistic and made claims of “one gene–one function.” This idea has been largely overturned and replaced by gene networks and the rise of subfields such as systems biology, but only recently. Ironically biology has always been the study of systems. It simply found that being reductionistic was in the ethos and the availability of tools of the times was yielding discoveries. Likewise, other fields have oscillated between being reductionistic and holistic in the nature of the questions they ask, the tools they use, and what counts as an answer.

Complex systems theory has arisen largely as a reaction to the Western tradition of reductionism and individualism, stressing holism and interconnectedness. For millennia, however, Eastern traditions, as well as some pre-Columbian and African cultures, have advocated for similar approaches to understanding the world. To these traditions complex systems thinking is obvious and simply is the way the world works.

Complex systems theory aims to provide a general framework upon which ideas and concepts can be placed, regardless of their disciplinary origin. When applied to a particular discipline, the reordering can result in an avalanche that may (or may not) bring new insights. Thomas Kuhn’s regular science is the slow buildup of tension while his paradigm shift is the avalanche that releases the tension and reorders the discipline. What is more typical, however, is a second-order phase transition. Complex systems ideas and tools creep into a disciple because one or a few people in that field map ideas to the framework of complex system. This mapping, accompanied by a new viewpoint, might attract others, and the field as a whole begins to adopt this viewpoint. Rather than a sudden jump, complex systems theory becomes absorbed into the normal way in which this particular field operates. In some cases it is barely even noticed. This is an idealistic view of how complex systems theory can help other fields. In reality, such transitions are not always welcomed.

11.1.2 Models, Stories, and Predictability

Many studies in complexity, and those that were used as examples throughout this text, are stripped-down versions of reality known by physicists as *toy models*. The inventor of the model removed anything that they felt was not necessary to reproduce a certain behavior. In this process their hope was to do accomplish three tasks: (1) identify the key components that give rise to the phenomenon, (2) create a simple model in which to systematically study the behavior in more detail, and (3) explain to others their findings in a compact form. For example, Per Bak’s model of self-organized criticality in a sandpile was meant to demonstrate criticality, not a real avalanche. The problem comes when the model is taken to be how real avalanches, earthquakes, evolution, and the stock market, work in the real world. It is hard for some fields to swallow that a model points to general phenomena of systems rather than the details of their own particular system.

Complex systems theories often use stories as a way to illustrate core principles. While a story might weave together a coherent narrative that explains all of the current facts, to be considered scientific, it should generate predictions and falsifiable hypotheses. The dilemma is that many of the phenomena studied by complex systems are onetime events that cannot be repeated. Statistical measures can be used, but then complex systems throws in that some phenomena are sensitive to initial conditions, or have path dependencies. When asked to predict the next earthquake, the size of the next stock market crash, or how good a season or game a ball player will have, the field of complex systems skirts around making these kinds of event-level predictions. When faced with these criticisms, the response is often, in the jargon of the field, that such phenomena emerge from systems that are non-linear, networked, self-organizing, and path-dependent phenomenon. They therefore cannot be predicted. To a scientist looking for rigor, such excuses have the appearance of pseudoscience.

The theory of evolution is instructive. It only works at macro spatial and temporal scales. As such evolution will not have predictive power at the level of individuals or particular species. It does, however, afford for general predictions and measurements to be made on some scales. For example, when an invasive species enters a new ecosystem, some general patterns could be predicted about what populations will have the biodiversity to evolve in response. Energy flows can even help predict which populations will be most impacted. The form of the response (e.g., longer beaks, shorter legs, faster digestive capabilities), however, will not be predictable. The same scale-dependent predictive capability also applies to the sandpile—predictions cannot be made about individual grains or the size of an avalanche, but the buildup of tension can be measured. In short, if we care holistically about sandpiles over long periods of time, complex systems theory can be predictive. However, if the goal is to make predictions at the level of sand grains, complex systems theory will fall short.

For many of these reasons, Roman Frigg (1972–) has suggested that "theory" does not accurately capture much of complex systems thinking. Rather he suggests that it is a sort of calculus—a series of tools that are helpful in taking apart phenomena out in the world and advancing other disciplines. As a collection of tools and methods, perhaps we should not expect complex systems theory to make predictions and generate refutable hypotheses.

11.1.3 Definitions Are Murky at Best

One characteristic of a distinct field is that it has its own language, means of disseminating peer-reviewed ideas, way of framing questions, and tools for finding answers. By that definition, the study of complex systems seems to be a field. The definition of *complex*, however, is not entirely clear. The literature abounds with different, and sometimes conflicting, definitions. To some it is structural complexity—counting the number of parts, number of interconnections, degree of modularity and redundancy, and so on. To others it is functional complexity—cataloging the distinct types of behaviors that can be expressed and how the system switches between them. What is more, you may have noticed that each chapter had a slightly different meaning of complexity. In some chapters it is purely structural or functional, while in others it is about what can emerge from the system—complexity to a game theorist means something different than it does to a thermodynamicist.

Layered on top of these considerations are the various attributes of a system. Does a system need to be structurally adaptive to be considered complex, or is functional adaptation sufficient? Do hierarchies or modularity need to be present? Do these need to be nested in some way? Is some form of emergence required? Perhaps an evolutionary trajectory on a fitness landscape endows a system with complexity. Can we even compare complexity across systems? For example, which is more complex, a cell, a person, or an economy? Because the earth contains living organisms, does that mean it must be alive as well?

Emergence, tension, and other complex systems terms are susceptible to these same definitional problems. The field could be paralyzed by a lack of definitions. Yet complex systems theory recognizes that some definitions will be third person (agreed upon by many), while others may be first person, or shared by only some subset of the field. It was promised early on in this text that definitions would emerge over time as terms gained greater and greater interconnectivity with other terms. But the emergence of these definitions was meant to be personal. Your definition of terms from this text will almost certainly be different than mine.

11.1.4 Promises, Transcendence, and Arrogance

Newer fields often are tempted to make bold claims to show that they add value to the wider enterprise of understanding the world. Complex systems is no different. Sometimes the argument for complex systems is made so strongly that it takes on the nature of dogma. This only invites critics to search for ways in which complex systems is being blinded by its own tenets. It is not easy to admit that an idea is incomplete or incorrect when someone else holds a countering viewpoint.

A key way to gain recognition for a new field is to make promises. For example, chaos and fractals initially promised to revolutionize the approach to many existing disciplines. After much anticipation, we are still waiting for fractals to explain the stock market. Likewise, Per Bak called his book *How Nature Works*, and before that, Rene Thom claimed that catastrophe theory would explain essentially all unexpected jumps in a system's behavior. Network theory, when combined with big data and artificial intelligence, is making similar promises today. These kinds of bold promises attract headlines in the short-term, but can sour critics in the long-term. In reality complex systems theory has not proposed a universal rule that has risen to the level of Newton's laws or the laws of thermodynamics.

The people who carry the torch for complex systems are often extraordinarily smart and have distinguished themselves in a well-established discipline. When they first began exploring complex systems theory, they were in search of something grander than their own narrow corner of the academic arena. As such, the field has attracted bright minds who have strayed from, or outright left, their home discipline behind. This does not always sit well in academic realms.

A scholar from one field should be respectful when venturing into the territory of other thinkers. Yet, some complex systems researchers claim they transcend particular systems and disciplines—"We see things that you don't see because we are searching for universal principles. To a mature disciple, say politics, such claims from an outsider can come off as arrogant. Arguing from universal system principles can come off as minimizing disciplinary study. Sadly when there is pushback, some in the complex systems field revert to math, make analogies to other systems, or give murky definitions in their defense. We have also been known to use one of the principles of complex systems theory in debates—to adjust and layer our definitions over time. Again, this does not sit well in most academic realms.

A related problem can also occur when a person firmly committed to a specific discipline encounters a system thinker. It can be annoying when someone jumps from ants to economies to brains and then to legal systems, all the while intending to demonstrate a universal principle. At best it can come off as bragging about how much one knows. At worst it might appear to be the product of a scattered and disconnected mind.

Not helping the perception of arrogance is that complex systems has been developed primarily by aging Western white males. You likely noticed this in reading the text. After all, I am an aging Western white male. The New England Complex Systems Institute (NECSI) lists two women among its 33 members. The tide is changing slowly, as places like the Santa Fe Institute include more women and acknowledge the great work of Eastern thinkers. Likewise, NECSI has a much more diverse group of students and visitors that promise to seed the field in the future. As argued in the last several chapters, diversity and inclusion are signs of a robust system.

Lastly, the field of complex systems is sometimes a victim of its own success. When a breakthrough happens in an interdisciplinary field, it is not always easy for journalists and the public to assign credit. In these cases it is much simpler to focus on a known academic discipline that studies a specific real-world phenomenon rather than the more general principles underlying the breakthrough. The result is that the contributions from complex systems are all too often minimized.

11.2 Future Directions

The previous section turned a critical eye toward complex systems. This section will offer a small sampling of the possible theoretical and practical contributions that might help drive the field forward.

11.2.1 Complex Machines

The industrial revolution and information revolution were sparked by a deep understanding of some aspect of the world. For example, early pioneers of flight started off trying to mimic birds. The breakthroughs happened, however, when inventors began to abstract the principles of flight. This intuitive leap led to flying machines that did not fly like birds. Future iterations were made even better by taking full advantage of the principles of aerodynamics.

Understanding complex systems may serve as a precursor to a new wave of technological innovation. Machines created using the principles of complex systems will not stay machines in the traditional sense. They will develop, adapt to their environment, heal, and perhaps die in the same way as organic life-forms. They may even display drives, ambitions, emotions, and purposes. In the final chapters, we explored the potentially terrifying (or hopeful) prospect of creating agents (either on purpose or by accident) that could replace us.

Some initial movements in this direction fall under the term *biomimetics*—interjecting biological principles into human-made tools and processes. Most often these approaches have focused on functional adaptation only. A second wave is coming where machines can change their own internal rules and structures. Computer science may show the way. Some applications now call for their own updates. Others can self-generate their own updates to fix functional problems. Yet others can replicate or even author new programs without the need for a human programmer. At some point, these same ideas may spill over into physical hardware, resulting in organic machines that can develop, heal, learn, and replicate.

11.2.2 Applications to New Fields

As argued above, complex systems can be thought of as a series of tools and perspectives that can be applied to other fields. An analogy may help to clarify how this may come about. Imagine a discipline as a dynamically moving set of ideas and people. The triangle game introduced in Chap. 2 can serve as a good model for the dynamics of disciplinary rules, interactions, and emergent phenomena. The people within this field are the agents. Some will remain fairly close to the ideas that form the core of the discipline. There are those, however, who are a bit more adventurous.

A disciplinary thinker who is wandering may accidentally bump into a wanderer from another discipline. They may interact briefly but have so little in common that they do not stay together very long. The history of many new interdisciplinary fields, however, has started when two disciplinary thinkers have stuck together, usually united by a common question, tool, or process. This intellectual seed might grow and form the basis for a new field that will attract others.

Wandering thinkers often meet in an *interstitial space* between disciplines. If they simply used their respective languages, they would likely talk past one another. Complex systems can serve as a common language that can plant intellectual seeds in unexplored interstitial spaces. To become a robust new field will require much more, but the initial seed is critical.

In a world that is changing very rapidly, fields will likely emerge, grow, merge with other fields, and die more quickly. The ability to develop a new field or discipline will become a skill. Those who can work in interstitial spaces, who can find ways to communicate, and who productively work with others outside of their discipline will become the leaders of the next new wave of discoveries.

11.2.3 Education and Complex Systems Thinking

Learning systems have been touched upon several times throughout the text. What was not explicitly discussed is the potential value of teaching complex systems thinking. As expressed above, one reason may be to accelerate the exploration of interstitial spaces. Another may be that complex systems could be considered part of the modern-day liberal arts. The core content of the liberal arts has always been dynamic. Yet the constant has been to practice the habits of thought and mind that any educated citizen-leader will need to better the world. Exactly how to achieve this would be left up to that particular individual. Complex systems thinking is a framework that can helps one act in the world. There will always be a need to learn a discipline, but as with the liberal arts, complex systems theory is a general perspective that can be helpful in almost any environment.

As mentioned in the preface, courses can and do exist at the undergraduate and graduate level in complex systems thinking. These concepts, however, go well beyond the college classroom. It may sound radical, but most if not all of the concepts presented in this text could be explained to kindergarteners. To take a simple example, imagine challenging a group of children to build a model of a tree house out of blocks. Such an exercise could focus on ideas of modularity, robustness, hierarchies, or how a structure can emerge without pre-existing plans. Likewise the same exercise could be used to introduce basic ideas of heuristics, decision-making, ecosystems, and homeostasis. Many of the simple games introduced in Chap. 8 have been played with very young children. A game of telephone could illustrate many of the principles of information theory. I have tried some of these techniques, and oftentimes children get the ideas before adults.

The aging population could be another target. Many older adults find that the specific skills they learned and practiced over a career are no longer relevant. Many still have energy and the desire to have an impact. Reeducation in a new discipline may feel daunting. Yet life experience often teaches the general lessons of complex systems. Many older adults find it to be a very natural way of thinking. The reinvigoration of learning something new may be exactly what is needed to spark one to take on a new project later in life.

Hopefully reading this text has challenged you to meet every complex system you encounter, including other people, as *becoming*. A system you have come to know well might suddenly reveal parts of its nature that have only recently emerged. Approaching the world in this way achieves one of the central aims of this text to help instill an enduring sense of curiosity.

11.2.4 Problems and Policies

Complex systems may be a framework for approaching the types of complex worldwide problems that we face. Consider the 17 United Nations Sustainable Development Goals that were adopted by the 192 member council in 2015. The goals range from reducing hunger, poverty, and inequalities to better access to clean water and education to focusing on better infrastructure, responsible consumption, and clean affordable energy. The sub-goals underneath each main goal reinforce that there will be no silver bullets of policy or technology. Solving these problems will require a systems approach. To further illustrate the point, the 17th goal is "Partnerships for the Goals." This goal recognizes that the goals are interdependent, that progress on one goal should not come at the expense of another goal, and that searching for synergies might point the way to better resource allocation.

Uniting around a common cause or goal is usually not easy. Philosophies, politics, pride, and path dependences can all get in the way. Debates can range from social and economic policy to where to take the next family vacation. They play out everywhere—in the news, government, academy, and family dinner table. This is part of being human and part of being a citizen. The point of a good debate is not necessarily to resolve a tension but rather to clarify and stake out the bounds. Complex systems thinking can in fact help diagnose the nature of the disagreement. It could be a vicious feedback cycle or a difference in the level of hierarchy of a system. It may be about the nature of non-linearities or the organization of a network. It is from a common language that models of the world can be dissected, analogies compared and contrasted, and assumptions made explicit. There is no promise that understanding a problem will fix the problem, but it is almost always a first critical step.

11.2.5 Complex Systems Alphabet

A clever idea that has been pitched by multiple groups and thinkers is that complex systems topics are like an alphabet. Each concept, for example, positive feedback, would have a particular symbol. A system could then be represented in some way using these symbols. A simple glance would sum up the key elements of the system.

There is already some precedent for this idea. Some Eastern languages use complex kanji characters that are recombinations of more simple characters. The postmodern dance form contact improvisation has a series of graphics known as the Underscore, created by one of the form founders Nancy Stark Smith. The vocabulary can describe the various forms that emerge over time as two (or more) bodies interact. Some symbols represent individual interactions, while other symbols represent various phases or moods of an entire space. The postmodern composer John Cage developed a variety of graphical languages to indicate directions to musicians that would serve as a framework for their playing. The exact interpretation was left to them. Perhaps a universal graphical language of complex systems could be developed. I have challenged my students to develop such an alphabet, and they indicated that it has changed the way they view the systems around them.

11.3 Waiting for the Integrator

The opening epigram was a dialog between Marco Polo and Kublai Khan discussing whether to focus on the form of the arch or the individual stones. It was promised that this text would present the stones of complex systems but that over time an arch-like structure of complex systems would emerge. In that regard, this text has failed. No clear, coherent, and agreed-upon whole has truly emerged. Like its subject matter, complex systems theory is still emerging. The arch is still in the process of becoming. The key thinkers mentioned throughout the text are the equivalents of Ptolemy, Copernicus, Kepler, and Galileo—finding a particular interesting general principle but not quite connecting it to the larger whole. The field of complex systems is waiting for the equivalent of Newton who can unify the various theories, processes, and tools. Are you up to the challenge?

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