Inverse Kinematics for Human Fingers 2018

This practical exercise focuses on the kinematics of a human finger. The objective is to be able to pose the finger in such a way that the fingertip ends up at a desired spot or follows a given trajectory.

Anatomy of a Human Finger

A human hand consists of a wrist, a palm, and five fingers. A finger is a bony structure with multiple joints allowing it to bend. The finger bones are referred to as phalanges. The index finger, middle finger, ring finger, and little finger have three phalanges; the thumb has two phalanges.

- The *proximal phalanx (PP)* is the finger bone closest to the palm. Each proximal phalanx is connected to a so-called metacarpal in the palm.
- The *intermediate phalanx (IP)* is the middle finger bone. The thumb has no intermediate phalanx.
- The *distal phalanx (DP)* is the finger bone furthest from the palm.

The following table lists the average lengths of the phalanges per finger.

Finger	Proximal	Intermediate	Distal
Thumb	31.6	-	21.7
Index finger	39.8	22.4	15.8
Middle finger	44.6	26.3	17.4
Ring finger	41.4	25.7	17.3
Little finger	32.7	18.1	16.0

Table 1: Average lengths (in mm) of the phalanges of all five fingers, taken from [1].

A finger has three 1-DOF joints that allow the phalanges to rotate relative to each other or to a metacarpal. All axes of rotation of the joints of a single finger are parallel.

- The *metacarpophalangeal (MCP) joint* connects the proximal phalanx to the metacarpal.
- The *proximal interphalangeal (PIP) joint* connects the intermediate phalanx to the proximal phalanx.
- The *distal interphalangeal (DIP) joint* connects the distal phalanx to the intermediate phalanx.

Contrary to the other four fingers the thumb has only one interphalangeal joint because it lacks an intermediate phalanx.

Simplification and Setting

The thumb is not considered in the current study. To further simplify the analysis, the phalanges are represented by line segments and the joints by points. Assume that the metacarpophalangeal joint is anchored at the origin of our coordinate frame and that all joint axes are parallel to the z-axis. As a consequence, all motions of the finger take place within the (x,y)-plane. Let θ_M be the metacarpophalangeal joint angle, which is defined as the counterclockwise angle between the proximal phalanx and the x-axis. Let θ_P be the proximal interphalangeal joint angle and define it as the counterclockwise angle between the intermediate phalanx and the extension of the proximal phalanx. Similarly, let θ_D be the distal interphalangeal joint angle and define it as the counterclockwise angle between the distal phalanx and the extension of the intermediate phalanx. Note that if $\theta_M = \theta_P = \theta_D = 0$ the finger is aligned with the *x*-axis.

The space of the finger contains a single unbounded object $O = \{ (x,y,z) \in R \mid y + 2 \leq 0 \}$. The goal of the exercises below is to place the fingertip in contact with *O*. You are free to extend your research into forward and inverse kinematics of a human finger beyond the questions listed below.

Unconstrained Joint Angles

First consider the situation in which no dependencies between the joint angles exist. Assume that $-\pi/3 \leq \theta_M \leq \pi/3$, $-2\pi/3 \leq \theta_P \leq 0$, and $-2\pi/3 \leq \theta_D \leq 0$ and neglect potential intersections of the phalanges.

Derive the forward kinematics equations that express position of the fingertip (and orientation of the fingertip) as a function of the joint angles for a designated finger using the average lengths of the phalanges listed in Table 1. (*Hint*: Use simple trigonometry for this inherently two-dimensional scenario instead of the essentially three-dimensional Denavit-Hartenberg algorithm.) Investigate the kinematics of the finger by thoroughly studying the relation between θ_M, θ_P, and θ_D and positions *p* of the fingertip on *O*. Explore the set of finger configurations that place the fingertip close to a specific point *p* on *O*. Can you find a way to place the distal phalanx vertically, or the proximal phalanx horizontally? What is the set of finger configurations that reach *p* under such additional requirements? Consider various fingertip placements on *O*, and study the same questions. Also explore the set of reachable positions of the fingertip (and orientations of the distal phalanx) for varying θ_M, θ_P, and θ_D, ignoring the obstacle *O*.

Constrained Joint Angles

It turns out that the rotations of the proximal and distal interphalangeal joints are not independent but instead satisfy the fixed ratio $\theta_D = 2\theta_P/3$. Adjust the forward kinematics equations to reflect the fixed ratio.

Develop an iterative inverse kinematics solver to determine the joint angles that put the fingertip in a given position. Pick a point on the surface of the object *O* and run your inverse kinematics solver to place the fingertip at that point. Study the decrease of the error as a function of the number of iterations, and consider different initial guesses for your iterative solver. Terminate the process if you consider the error small enough. Explore extreme placements of the fingertip on *O*, and also anywhere in space (that is, not necessarily on *O*). Does the solver perform equally well in all cases? Can you find cases in which the solver struggles?

Sample a sequence of points at regular small distances on the surface of *O* and solve the inverse kinematics problem to generate the illusion of a moving finger with its tip sliding along the object surface. Is there a smart choice for the initial solution for every next point?

Report

Write a report of ideally 3000-5000 words that explains your mathematical model, your solvers, and the findings of your experiments. The quality of your work will be assessed according to the following four criteria, all four weighing equally:

- thoroughness and extensiveness of the research (modeling, research questions),

- quality of the solver and results,
- clarity of the exposition in the report, and
- general appearance (structure, layout) of the report.

Hand in your report on paper before Wednesday October 24 at 15:15. After that, you will need to sign up for a short meeting with Ioannis Nemparis in which you answer questions about your work and demo your solver. You should bring your own laptop to the meeting to save time.

Reference:

[1] A. Buryanov and V. Kotiuk, Proportions of hand segments, Int. J. Morphol. **28**(3) (2010), pp. 755-758.