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Putu Ayu Sudyanti

The current algorithm poses some mixing problem in which the chain failed to explore other modes in the posterior. To fix this, an additional Metropolis-Hastings step is needed to guarantee that other possible solutions are considered. There are 2 different ways in which we can attempt this procedure; with independent or dependent transition probability.

1. Independent transition probability
   The first method assumes that sampling of $(\theta_i^n \theta_i^t)$ and ζ^i are done independently in the transition probability of the Metropolis-Hastings step. The algorithm can be written as follows:

   i. Randomly sample $\theta_i^* = (\theta_i^{n^*}, \theta_i^{t^*})$ uniformly each with probability $\dfrac{1}{9}$

   ii. Pick $P(\zeta_i^*|\theta_i^*)$ based on the following:
      - If ζ_i is not a singleton:
        $\zeta_i^* = \zeta_{c^*}$ with probability $\dfrac{n_{c^*}}{N-1}$ where $c^* = 1, 2, \cdots, K$
      - Otherwise:
        $\zeta_i^* \sim H$ with probability 1

   iii. Determine the acceptance probability of the Metropolis-Hastings by:

$$
\begin{aligned}
r &= \frac{P(\zeta_i^*, \theta_i^*|y) \times q(\zeta_i, \theta_i|\zeta_i^*, \theta_i^*)}{P(\zeta_i, \theta_i|y) \times q(\zeta_i^*, \theta_i^{n*}, \theta_i^{t*}|\zeta_i, \theta_i^n, \theta_i^t)} \\
&= \frac{\dfrac{1}{9}\dfrac{n_{c*}}{n+\alpha-1} \times P(y_i|\zeta_i^*, \theta_i^*)\ \dfrac{1}{9}\dfrac{n_c}{N-1}}{\dfrac{1}{9}\dfrac{n_c}{n+\alpha-1} \times P(y_i|\zeta_i, \theta_i)\ \dfrac{1}{9}\dfrac{n_{c^*}}{N-1}} \\
&= \frac{P(y_i|\zeta_i^*, \theta_i^*)}{P(y_i|\zeta_i, \theta_i)}
\end{aligned}
$$

2. Dependent transition
   This second method assumes that the choice of ζ_i should depend on the sample of (θ_i^n, θ_i^t) in the transition probability of the algorithm. The pseudo-code is as follows:

   i. Propose $\theta^* = (\theta_i^{n*}, \theta_i^{t*})$ uniformly with probability $\dfrac{1}{9}$

   ii. For nonsingleton:
      Set $\zeta^* = \zeta_{c^*}$ for an existing cluster $c^* = 1, 2, \cdots, K$ with probability

$$
q(\zeta^* = \zeta_{c^*}|\theta^*) \propto \frac{n_{c^*} P(y_i|\zeta_{c^*}, \theta^*)}{z(\theta^*)}
$$

      where $z(\theta^*) = \sum_{j=1}^{K} n_j P(y_i|\zeta_j, \theta^*)$. The acceptance rate will be:

$$
\begin{aligned}
r &= \frac{P(y_i, \theta^*, \zeta^*)}{P(y_i, \theta, \zeta)} \times \frac{q(\theta, \zeta)}{q(\theta^*, \zeta^*)} \\
&= \frac{\frac{1}{9}\frac{n_{c^*}}{N+\alpha-1}P(y_i|\theta^*, \zeta_{c^*})}{\frac{1}{9}\frac{n_c}{N+\alpha-1}P(y_i|\theta, \zeta)} \times \frac{\frac{1}{9}\frac{n_c P(y_i|\zeta_c, \theta)}{z(\theta)}}{\frac{1}{9}\frac{n_{c^*} P(y_i|\zeta_{c^*}, \theta^*)}{z(\theta^*)}} \\
&= \frac{z(\theta^*)}{z(\theta)}
\end{aligned}
$$

iii. For singleton:
The transition probability is $q(\zeta^*) = p(\zeta^*)$ then the acceptance rate becomes:

$$
\begin{aligned}
r &= \frac{P(y_i, \theta^*, \zeta^*)}{P(y_i, \theta, \zeta)} \times \frac{q(\theta, \zeta)}{q(\theta^*, \zeta^*)} \\
&= \frac{\frac{1}{9}P(\zeta^*)P(y_i|\theta^*, \zeta_{c^*})}{\frac{1}{9}P(\zeta)P(y_i|\theta, \zeta)} \times \frac{\frac{1}{9}P(\zeta)}{\frac{1}{9}P(\zeta^*)} \\
&= \frac{P(y_i|\zeta^*, \theta_*)}{P(y_i|\zeta, \theta)}
\end{aligned}
$$