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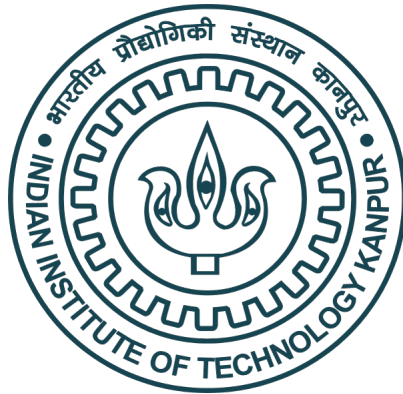
# AE 681 COMPOSITE MATERIALS PROJECT REPORT

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*by*

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**Objective:** To find the effective properties of the composite using:

1. Strength of Materials Approach
2. Hill's Concentration Factors Approach
  - Voigt Approximation
  - Reuss Approximation
3. Concentric Cylinder Assemblage Model
4. Self-consistent Method
5. Mori-Tanaka Method
6. Halpin-Tsai method
7. Hashin Shtrikman Bounds

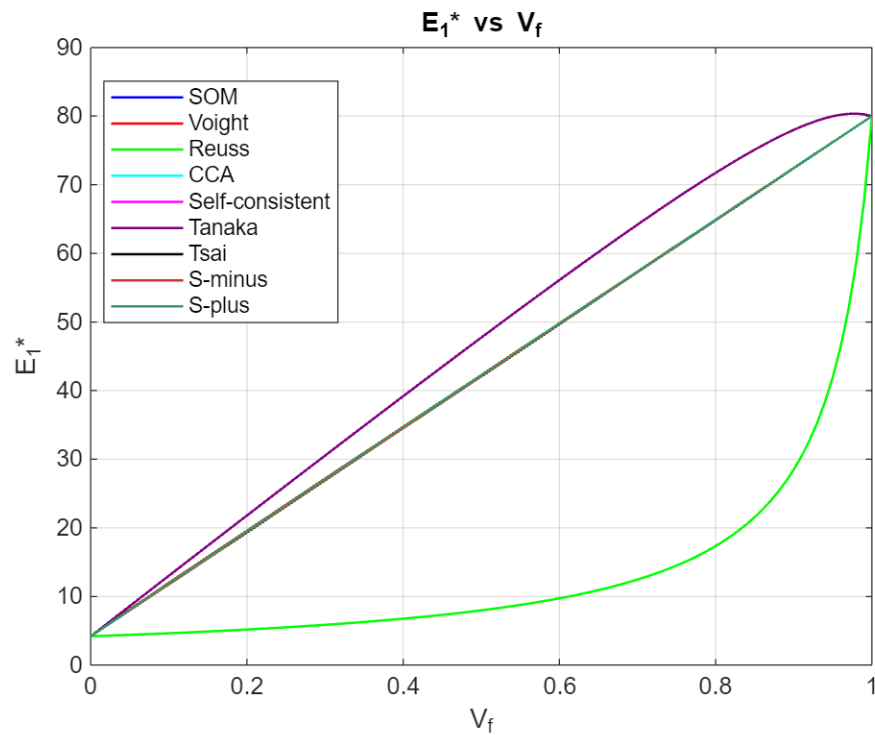
## Given Materials:

Fiber type	E-glass 21xK43 Gevetex
Longitudinal modulus, $E_1$ (GPa)	80
Transverse modulus, $E_2$ (GPa)	80
In-plane shear modulus, $G_{12}$ (GPa)	33.33
Major Poisson's ratio, $\nu_{12}$	0.2
Transverse shear modulus, $G_{23}$ (GPa)	33.33
Longitudinal thermal coefficient, $\alpha_1$ ( $10^{-6} / ^\circ\text{C}$ )	4.9
Transverse thermal coefficient, $\alpha_2$ ( $10^{-6} / ^\circ\text{C}$ )	4.9

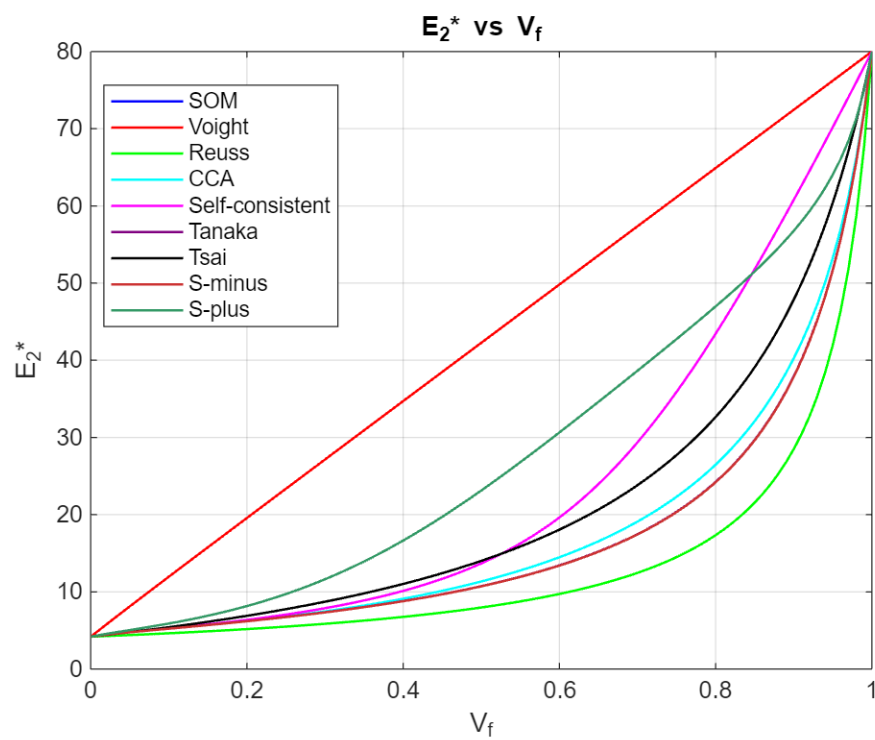
Matrix type	3501-6 epoxy
Modulus, $E_m$ (GPa)	4.2
Poisson's ratio, $\nu_m$	0.34
Thermal coefficient, $\alpha_m$ ( $10^{-6} / ^\circ\text{C}$ )	45

## Results: (All Methods Together)

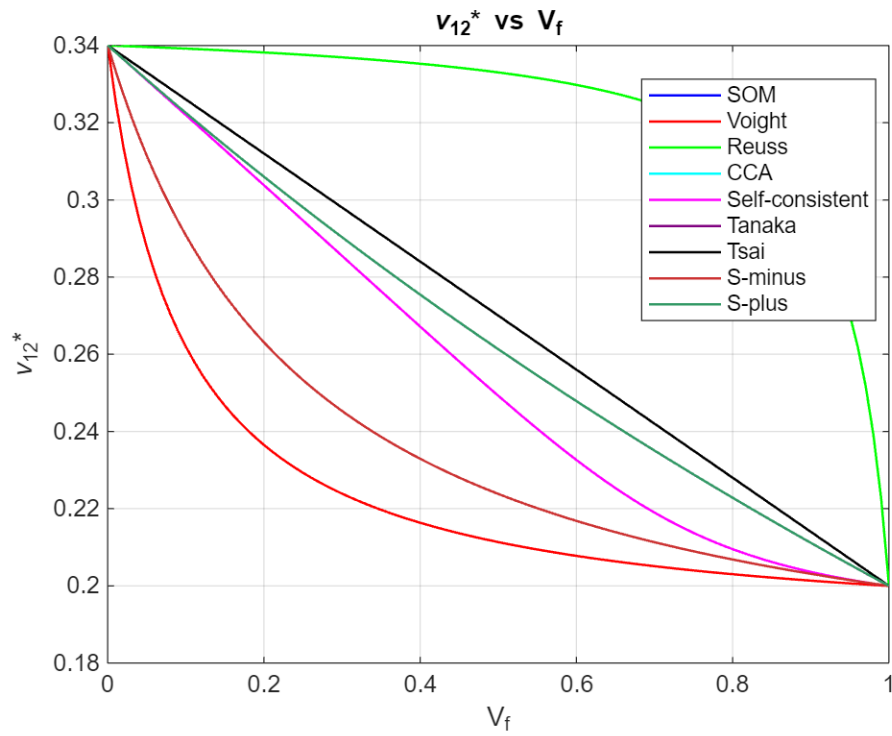
Effective Axial Modulus,  $E_1^*$  :



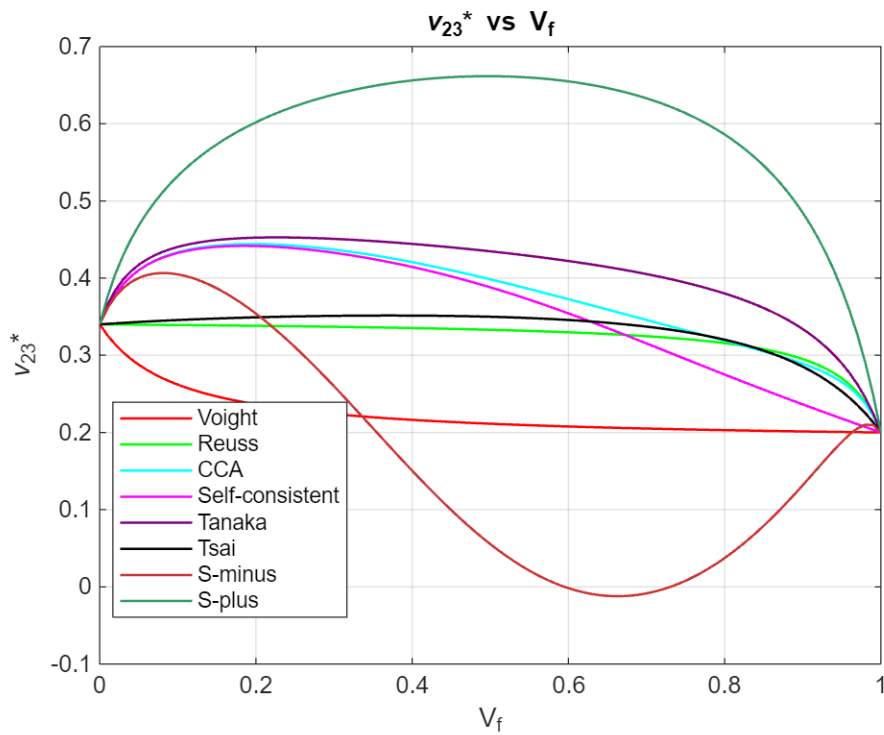
Effective Transverse Modulus,  $E_2^*$  :



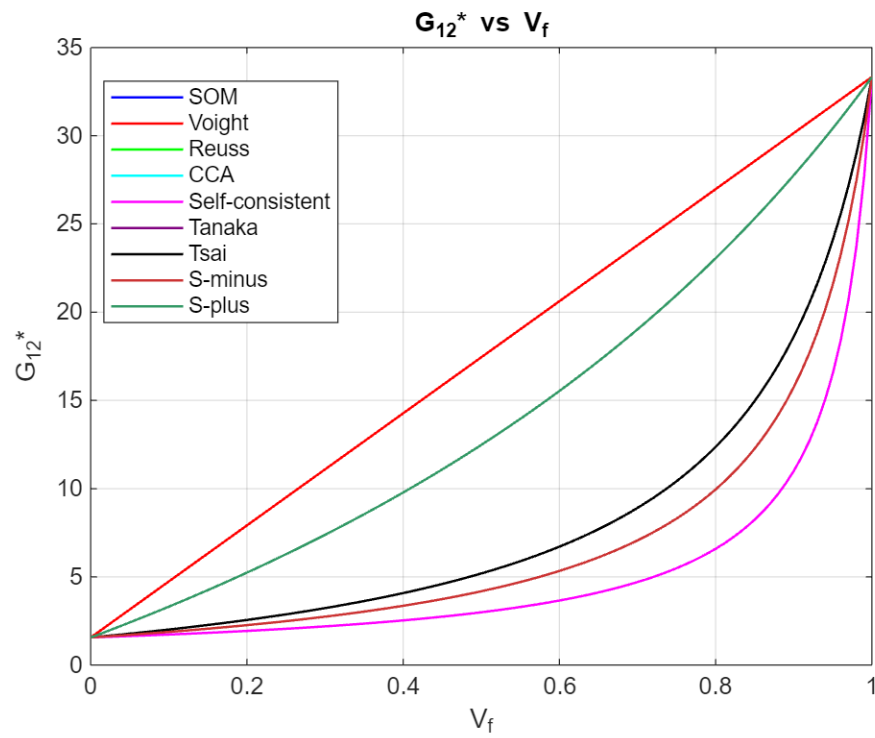
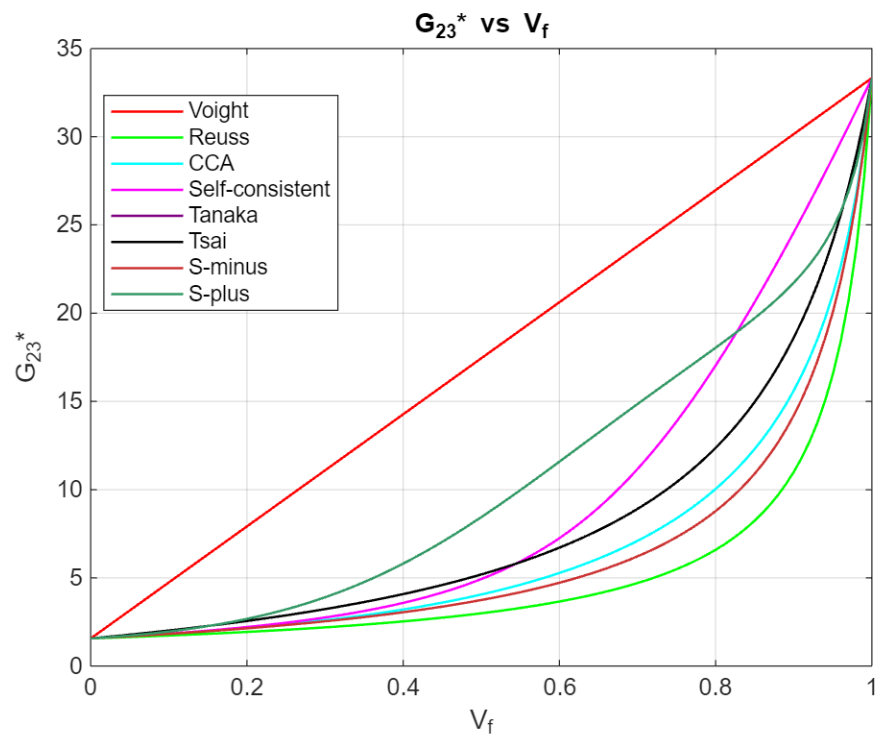
Effective Axial (Major) Poison's Ratio,  $\nu_{12}^*$  :



Effective Transverse Poison's Ratio,  $\nu_{23}^*$  :

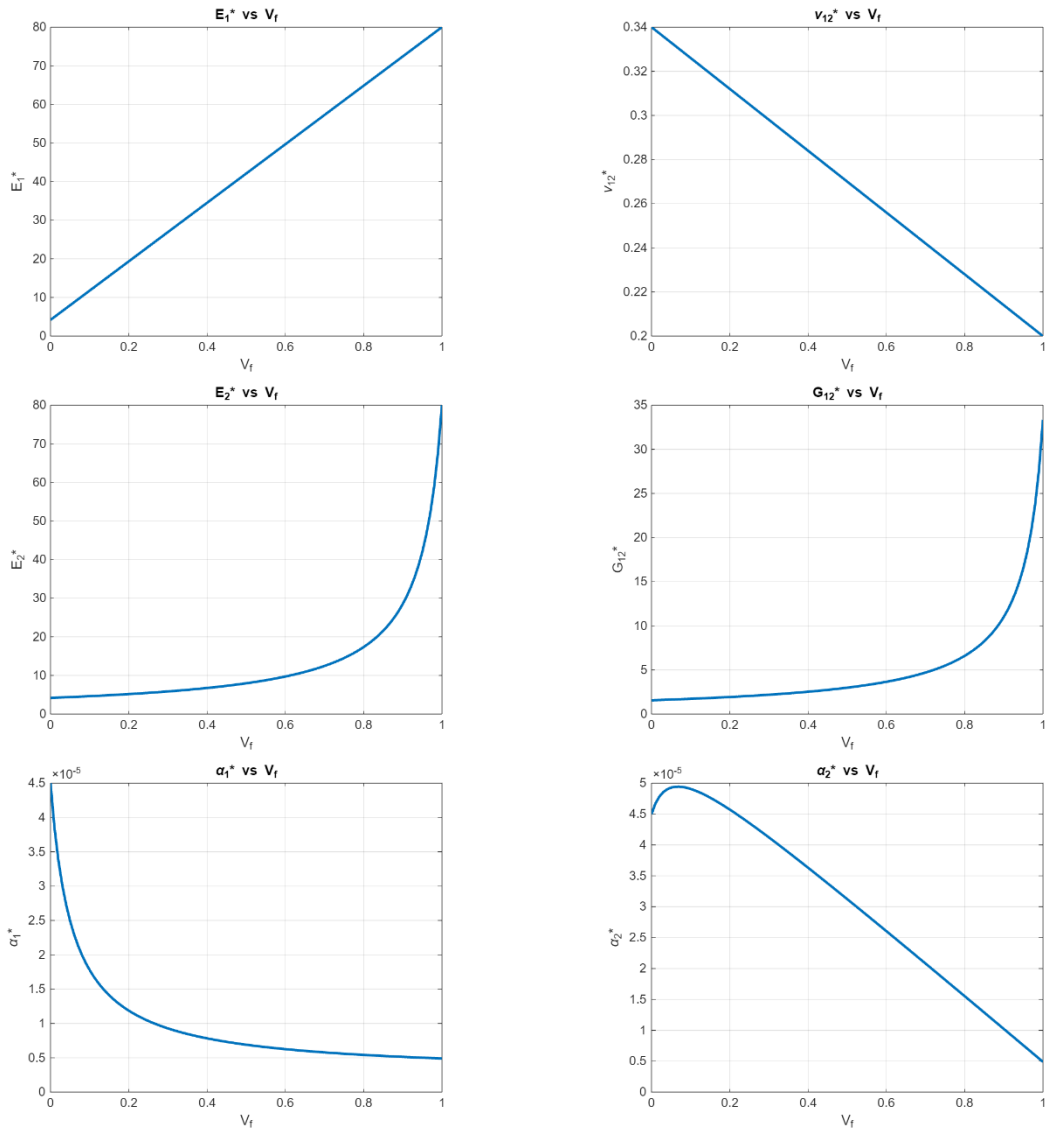


Effective Axial Shear Modulus,  $G_{12}^*$  :

Effective Transverse Shear Modulus,  $G_{23}^*$  :

# Observation:

## 1. Strength of Materials Approach:



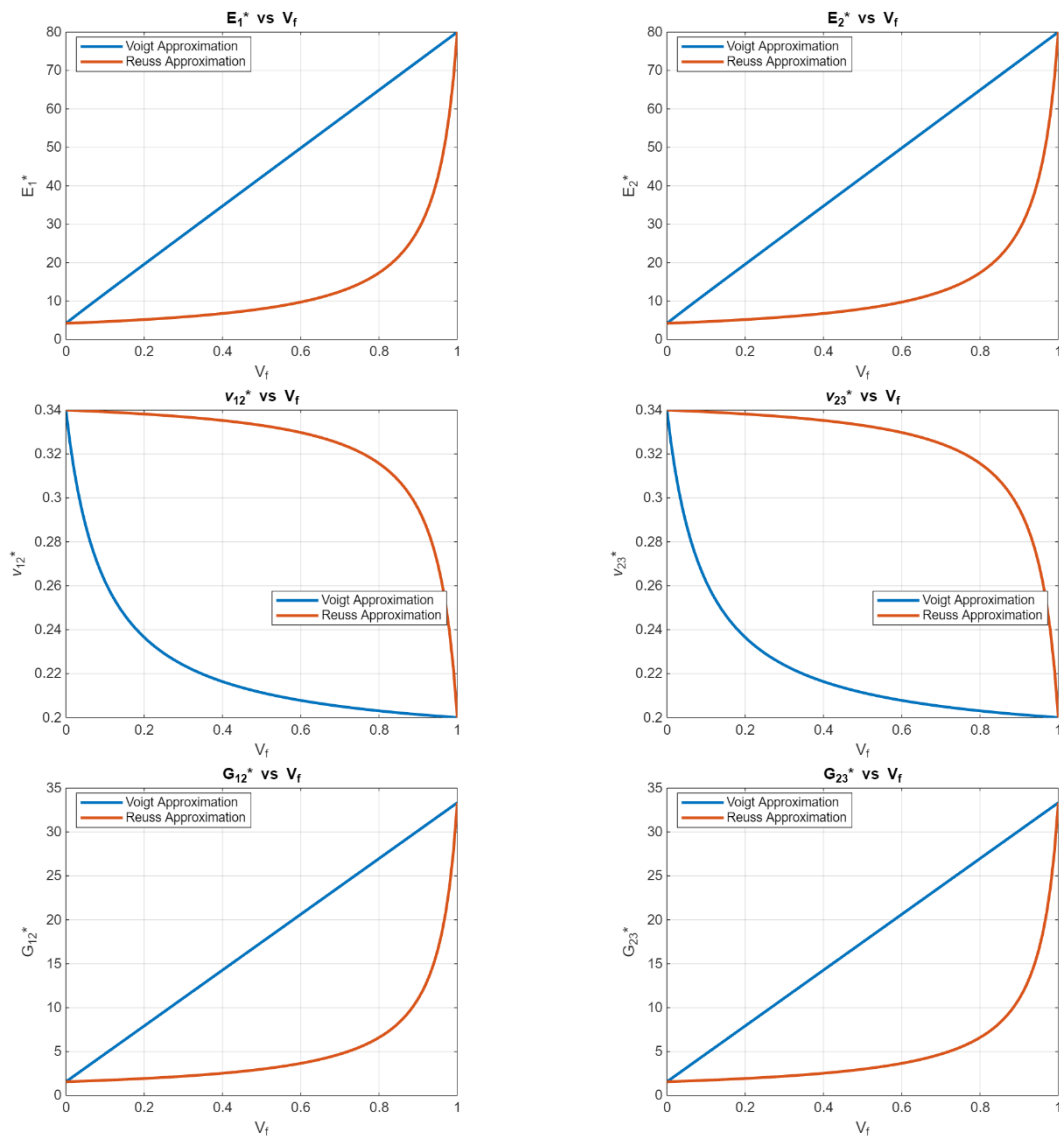
## Advantages:

- Simple and straightforward implementation.
- Can estimate key in-plane effective properties such as  $E_1^*$ ,  $E_2^*$ ,  $\nu_{12}^*$ , and  $G_{12}^*$ .
- Capable of determining effective thermal properties, which other methods may not address.

## Limitations:

- Unable to determine certain effective properties such as  $\nu_{23}^*$  and  $G_{23}^*$ .

## 2. Hill's Concentration Factors Approach:



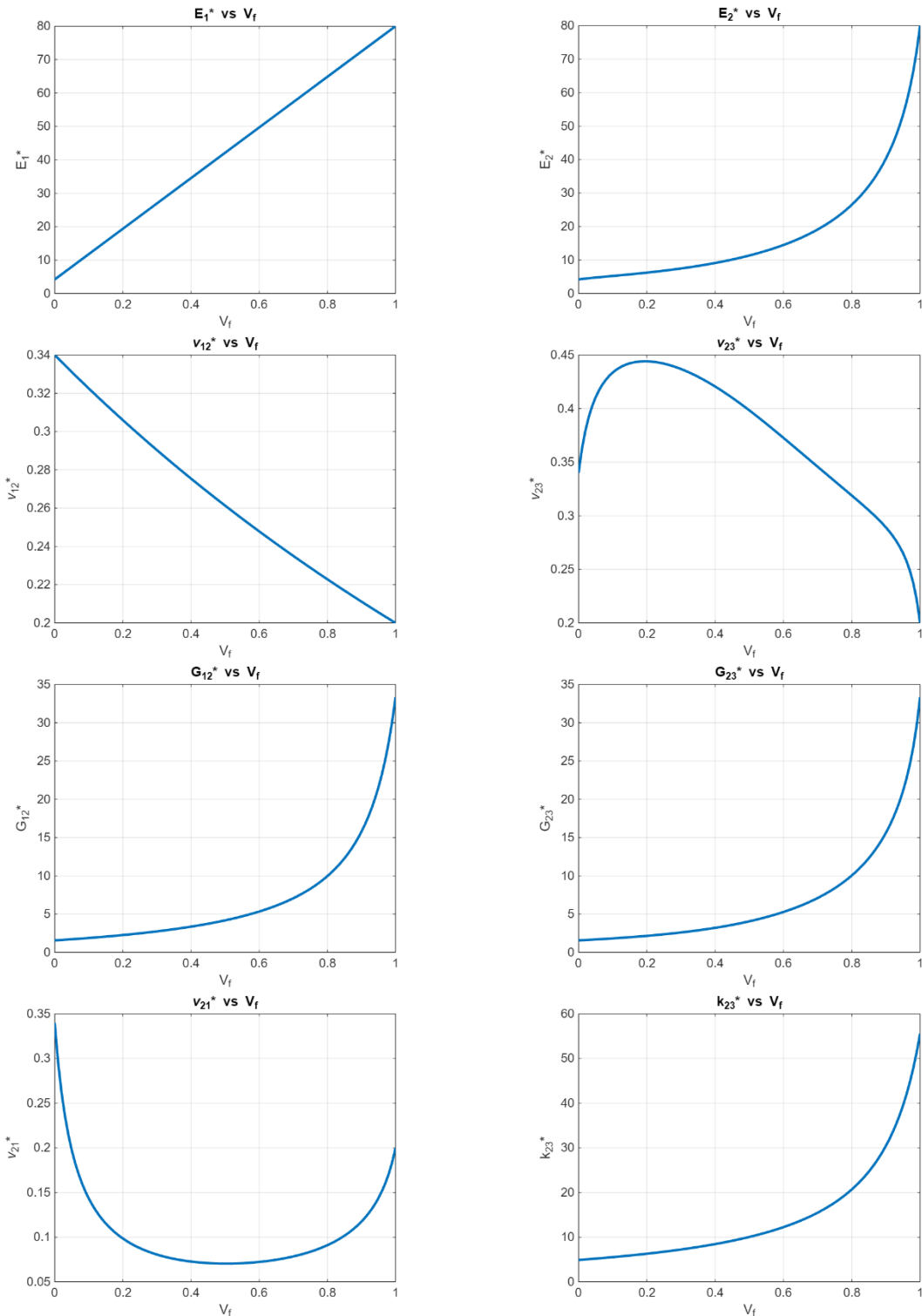
### Advantages:

- Simple and straightforward implementation.
- We can find all effective properties  $E_1^*$ ,  $E_2^*$ ,  $\nu_{12}^*$ ,  $\nu_{23}^*$ ,  $G_{12}^*$  and  $G_{23}^*$ .
- Provides both upper (Voigt) and lower (Reuss) bounds, offering a useful range for property estimation.

### Limitations:

- Lacks accuracy in some cases, as it assumes the composite behaves as an effective **isotropic** material when both fibre and matrix are isotropic. This is not realistic for unidirectional composites, which should exhibit **transversely isotropic** behaviour rather than fully isotropic behaviour.

### 3. Concentric Cylinder Assemblage Model:



#### Advantages:

- Capable of predicting all major effective properties of a composite, including typically difficult parameters such as  $\nu_{21}^*$  and  $k_{23}^*$ .
- Provides a reasonably accurate representation of longitudinal and transverse elastic moduli  $E_1^*$ ,  $E_2^*$  as well as Poisson's ratios  $\nu_{12}^*$ ,  $\nu_{23}^*$ .



**Limitations:**

- The model often yields nearly equal values for  $G_{12}^*$  and  $G_{23}^*$ , which is not physically accurate for transversely isotropic composites where in-plane and out-of-plane shear moduli should differ. This reflects a limitation in capturing the anisotropic shear behaviour accurately.

***Problem faced while implementation:***

The direct formula given for  $G_{23}^*$  was not giving accurate results as the shear modulus at  $V_f = 0, 1$  were not matching to the values of matrix and fibre respectively.

$$\frac{G_{23}^*}{G_m} = 1 + \frac{V_f}{\frac{G_m}{(G_{23}^{(f)} - G_m)} + \frac{(k_m + \frac{7}{3}G_m)}{(2k_m + \frac{8}{3}G_m)}}$$

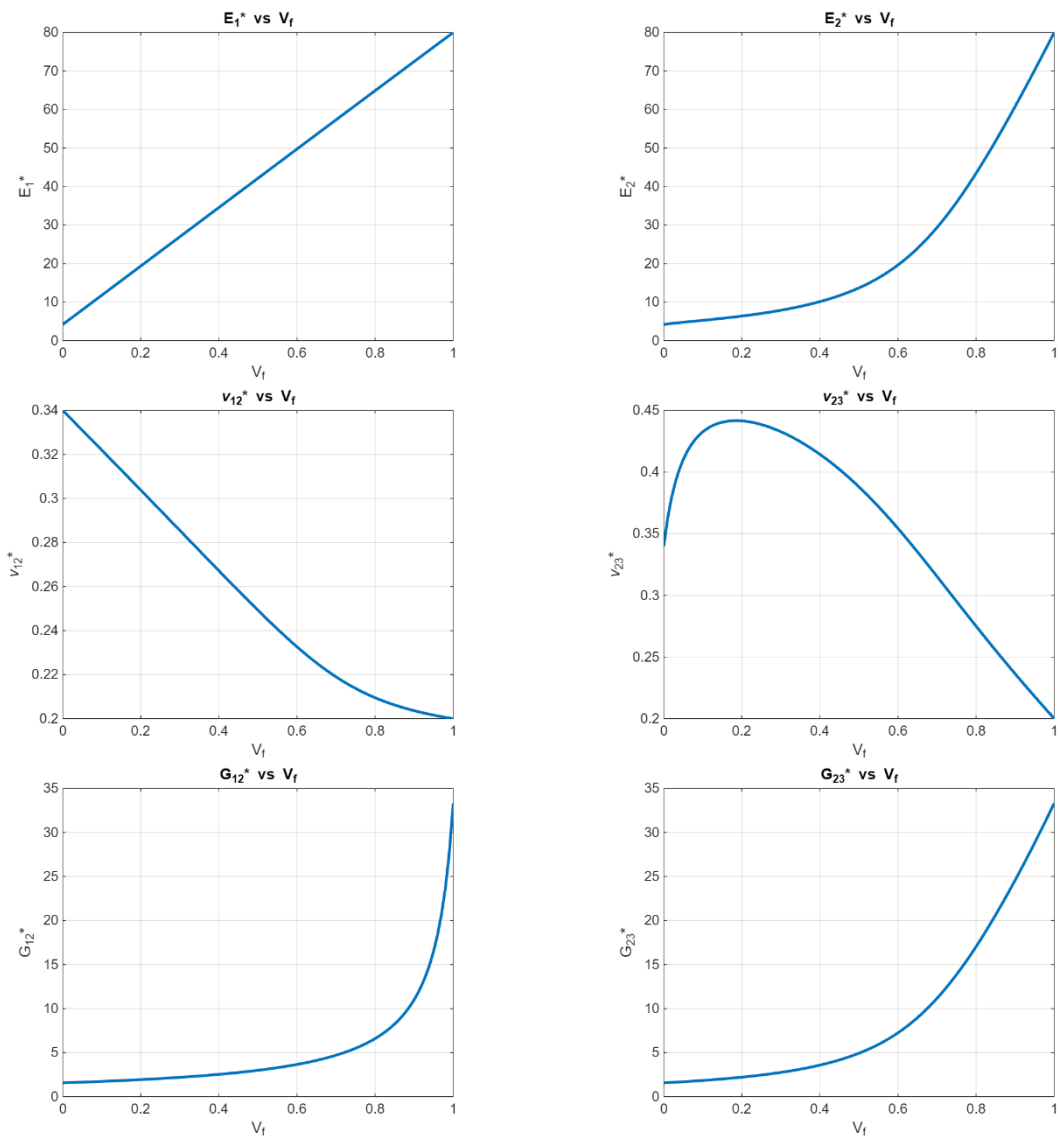
***Fix:***

To resolve this issue, we adopted a more general and accurate approach. Instead of relying on the direct formula, we used a quadratic equation of the form:

$$A \left( \frac{G_{23}^*}{G_{(m)}} \right)^2 + B \left( \frac{G_{23}^*}{G_{(m)}} \right) + D = 0$$

Solving this quadratic equation gave better and more accurate results for  $G_{23}^*$ .

#### 4. Self-consistent Method:



#### Advantages:

- Provides a highly accurate prediction of all effective composite properties.
- Accurately captures transverse isotropy, even when both the fibre and matrix materials are isotropic.

#### Limitations:

- Computationally more involved, as it requires solving nonlinear equations, making implementation more complex compared to simpler analytical models.

***Problem faced while implementation:***

When solving the nonlinear equations in the self-consistent approach for composite materials, the solution array contains non-unique values, including physically unrealistic imaginary and negative solutions.

$$\frac{V_f k_f}{k_f + m^*} + \frac{V_m k_m}{k_m + m^*} = 2 \left[ \frac{V_f m_m}{m_m - m^*} + \frac{V_m m_f}{m_f - m^*} \right]$$

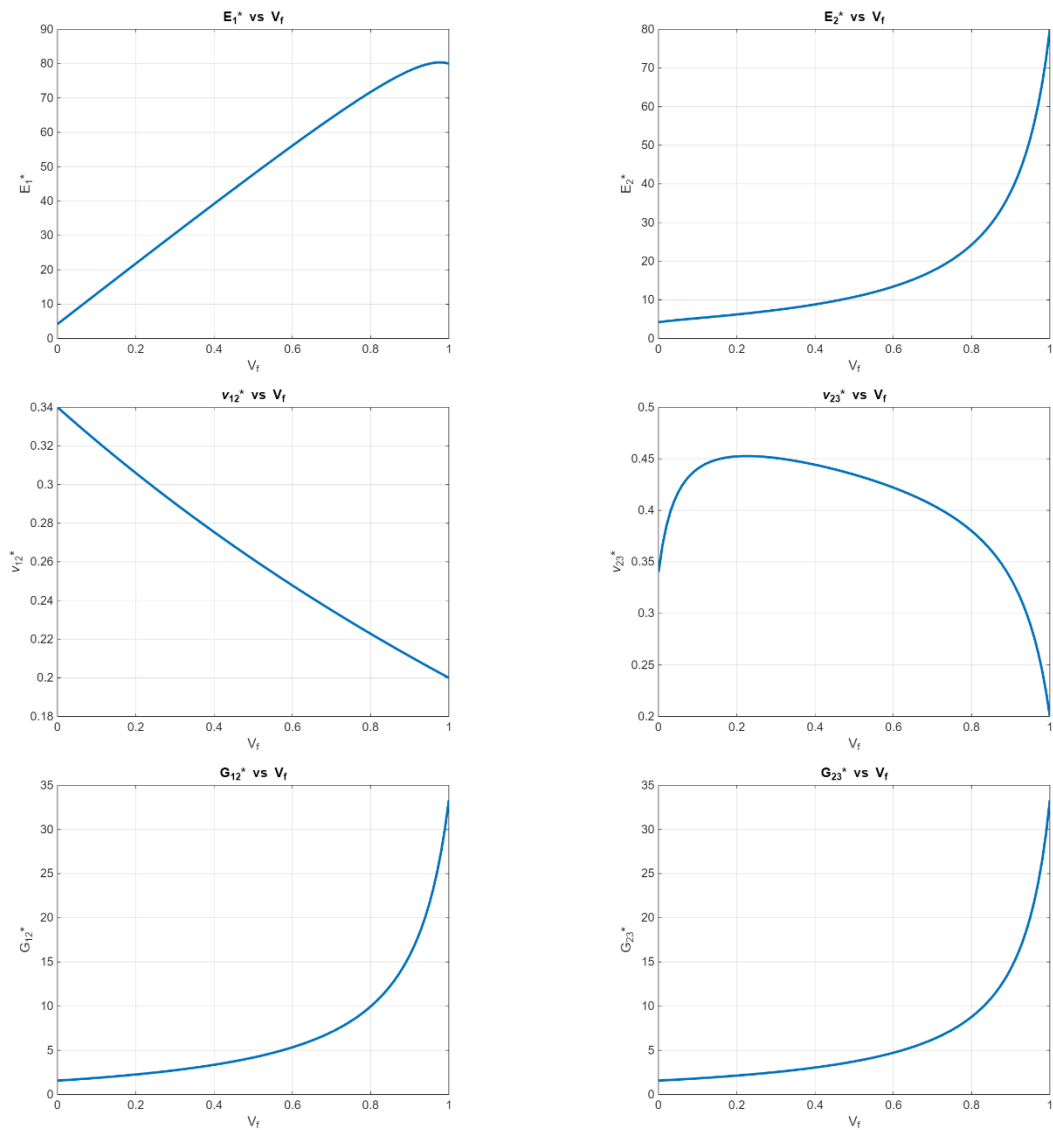
$$\frac{1}{2p^*} = \frac{V_f}{p^* - p_m} + \frac{V_m}{p^* - p_f}$$

$$\frac{1}{k^* + m^*} = \frac{V_f}{k_f + m^*} + \frac{V_m}{k_m + m^*}$$

***Fix:***

To address this issue, we should implement a filtering step that selects only the real and positive solutions from the computed  $6 \times 1$  solution array. These physically meaningful solutions align with the underlying material properties of composites and ensure proper convergence of the self-consistent homogenization scheme.

## 5. Mori-Tanaka Method:



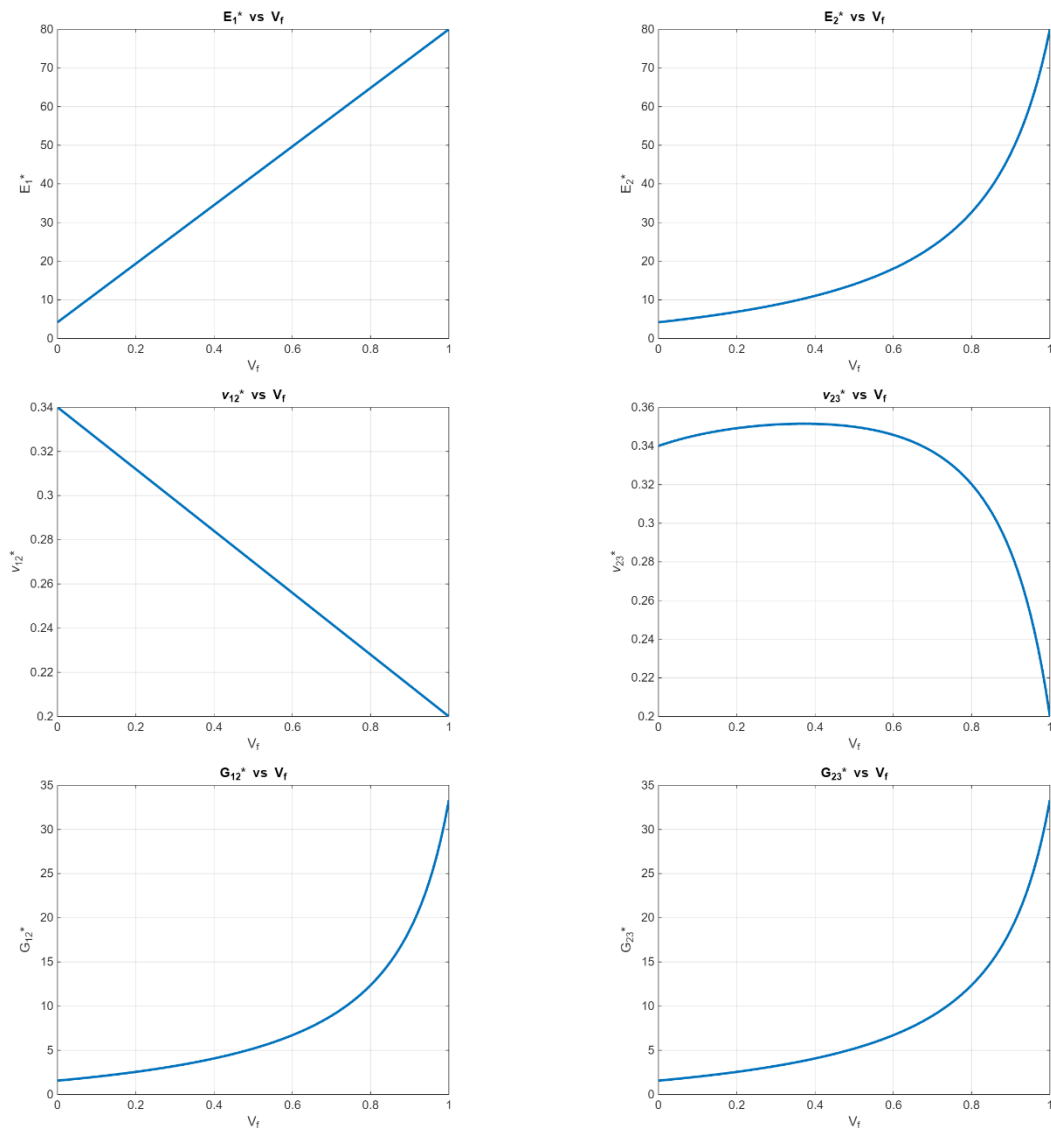
### Advantages:

- We can find all effective properties  $E_1^*$ ,  $E_2^*$ ,  $\nu_{12}^*$ ,  $\nu_{23}^*$ ,  $G_{12}^*$  and  $G_{23}^*$ .
- Accurately captures transverse isotropy, even when both the fibre and matrix materials are isotropic.

### Limitations:

- Not much variation between  $G_{12}^*$  and  $G_{23}^*$ .

## 6. Halpin-Tsai method:



We assume  $\zeta = 2$  which is widely accepted in literature for unidirectional composites with continuous circular fibres.

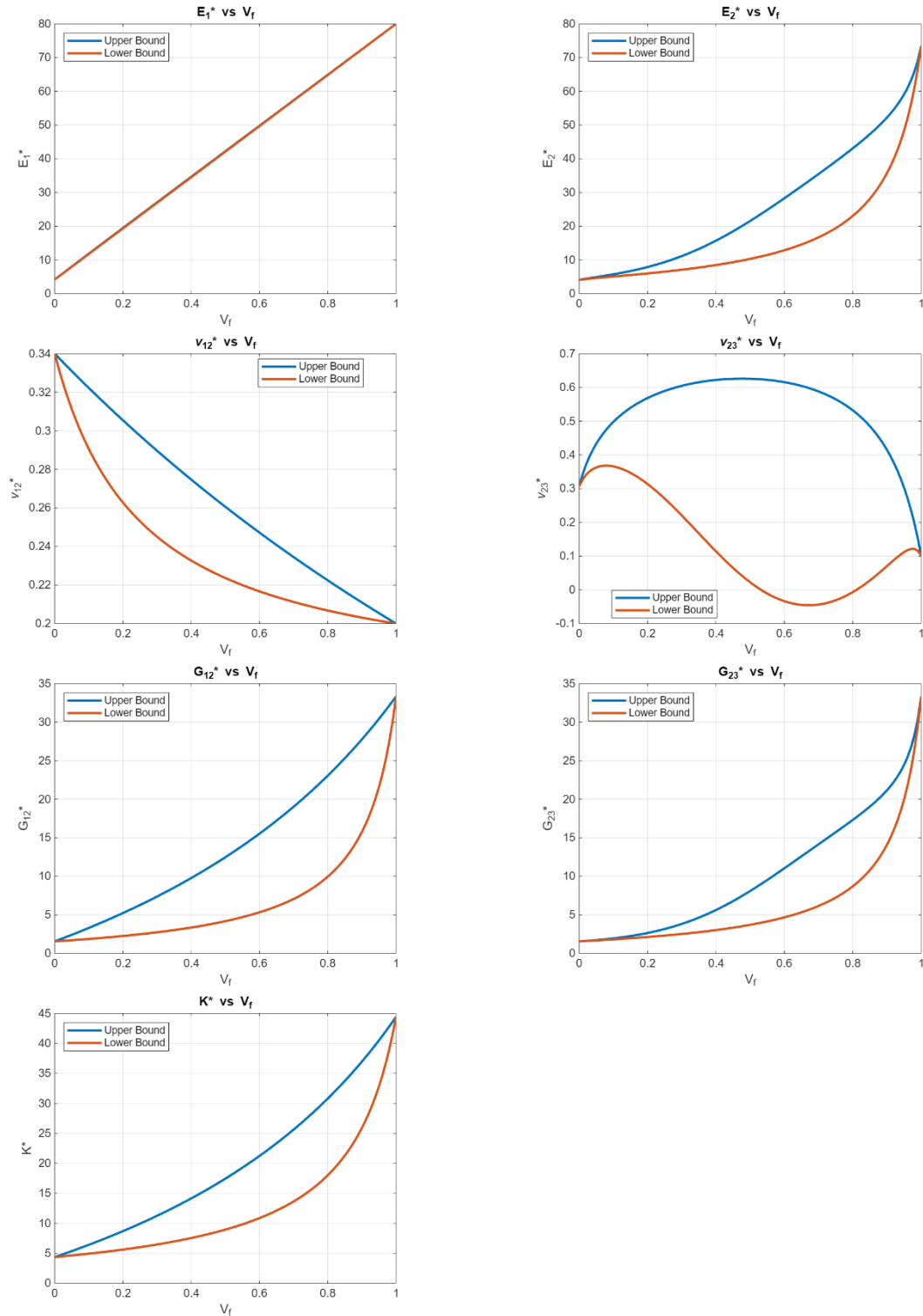
### Advantages:

- Provides a reasonably accurate representation of longitudinal and transverse elastic moduli  $E_1^*$ ,  $E_2^*$  as well as Poisson's ratios  $\nu_{12}^*$ ,  $\nu_{23}^*$ .
- Accurately captures transverse isotropy, even when both the fibre and matrix materials are isotropic.

### Limitations:

- Not much variation between  $G_{12}^*$  and  $G_{23}^*$ .

## 7. Hashin Shtrikman Bounds:



### Advantages:

- Straightforward implementation requiring minimal computational resources.
- Provides rigorous upper and lower bounds on effective properties, establishing a clear range of possible values.

- Delivers accurate predictions for all effective mechanical properties.
- Explicitly determines the bulk modulus ( $K$ ) along with other essential parameters.

**Limitations:**

- Bounds may become too wide to be practically useful.

# Conclusion:

Method	Complexity	Accuracy
Strength of Materials Approach	Very Low	Moderate
Voigt Approximation	Low	Low
Reuss Approximation	Low	Low
Concentric Cylinder Assemblage (CCA)	Moderate	High for aligned continuous fibres
Self-consistent Method	High	High
Mori-Tanaka Method	Moderate	High
Halpin-Tsai Method	Low to Moderate	Moderate
Hashin-Shtrikman Bounds	Moderate	Moderate to High

For general composite systems, the Concentric Cylinder Assemblage (CCA) method, Self-consistent method, and Mori-Tanaka method typically provide the most accurate predictions of effective material properties.

However, in our specific case involving isotropic fibre and matrix materials, the Self-consistent method yields the most accurate results.