ECE537
Random Processes
Midterm 2
Nov 15, 2023
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1. A random process has the following sample functions $X(t) = 2 + A\cos(\omega t + \theta) + B\sin(\omega t + \theta)$, where ω is a constant, A, B, are independent zero mean random variables, and θ has a PDF that is uniform in the interval $[0,2\pi]$.

    a) Find the mean of the process.
    b) Find the auto-correlation function of the process.
    c) Find the auto-covariance function of the process.
    d) Find the average power of the process.
    e) Now, set $\theta = 0$, and assume that A and B are independent random variables with uniform distribution in $[-1,1]$. Find the PDF for $X(t)$ at $t = \frac{\pi}{4\omega}$.

2. Consider a Poisson process with average arrival rate equal to $\lambda = 5$ arrivals per hour.

    a) Determine the probability of 3 arrivals in the time interval from 2-4 PM.
    b) We start observing the process at 12 noon. Find the probability that the first observed arrival will take longer then 2 hours.
    c) Let $N(t)$ be the Poisson process where $N(t)$ is the number of arrivals from the origin up to time t, and the average number of arrivals per unit time is λ. Find the mean and auto-covariance of the process $N(t)$.
    d) Let T_i be the time for the i^{th} arrival in c). Determine the probability density function for T_1 and also for T_2.
    e) Consider a process $Z(t)$, where $Z(t)$ is derived from a Poisson process by discarding the odd arrivals, i.e. arrivals 1, 3, 5, … Consider the distribution for the time of the first arrival in $Z(t)$. Is $Z(t)$ a Poisson process? Or not? Justify your answer.

3. Consider a Binomial Distribution with parameters $n = 200$ and $p = 1/4$. Consider a random variable k with Binomial probability mass function.

    a) What is the expected value of k?
    b) What is the variance of k?
    c) Use the Central limit theorem to find the PDF of a distribution that can approximate the distribution for the random variable k.
    d) Use the approximate distribution in c) to evaluate the probability $P(40 \leq k \leq 60)$. Note that you may require the evaluation of special functions. You can leave the result in terms of these special functions.
    e) Consider a Bernoulli process where a Bernoulli trial occurs every second with $P(Z = 1) = 1/1000$. Use the Poisson distribution to evaluate the probability that two successes occur (i.e. two outcomes with $Z = 1$) in an hour.

4. Consider a Wiener process $W(t)$. At time $t = 1$ the random variable $W(1)$ has a variance equal to 2.
    a) Determine the variance of $W(t)$ at time $t = 10$.
    b) Determine the correlation for the two random variable $W(3)$ and $W(10)$.