ECE537
Random Processes
Quiz 1

1. A standard die has 6 sides numbered 1 to 6. However we will modify the die so that the sides numbered 4, 5, and 6 are all renumbered to 3. Consider an experiment where we throw the modified die twice to obtain the outcome (x, y), where x is the outcome of the first throw, and y is the outcome of the second throw.

   a) List the points (outcomes) in the sample space. How many points are there?
   b) Assume that we wish to calculate probabilities for all possible events that we can specify involving the two throws of the modified die. We need to specify a sigma-field. What is the number of events in this sigma-field?
   c) What is the probability of the event $E = \{$one of the throws shows 3 and the other shows 1$\}$
   d) Consider the two events $A = \{$first throw shows 1 and second throw shows 2$\}$, and $B = \{$one of the throws shows 3$\}$. Find the smallest sigma-field containing these two events (list all elements, i.e. events). Describe each event in this sigma-field in words.

2. Consider the above experiment where we throw the modified die twice. Define a random variable X, where X is equal to the sum of the two outcomes.

   a) Determine the CDF for X.
   b) Determine the PDF for X.
   c) Determine the expected value for X.
   d) Determine the variance for X (give the expression, no need for the full calculation).

3. Consider two independent random variables U, V with uniform distribution on the interval $[-1,1]$. Define the random vector $X = (X_1, X_2) = (U, U + V/2)$. Determine the correlation matrix for X