

## Problem Set 12

1. An urn initially contains 5 black balls and 5 white balls. The following experiment is repeated indefinitely: A ball is drawn from the urn at random: if the ball is white it is put back in the urn, otherwise it is left out. Let  $X_n$  be the number of black balls remaining in the urn after  $n$  draws.
  - a) Is  $X_n$  a Markov Chain? Why? If so list the states.
  - b) Draw the state diagram and find the transition probabilities.
  - c) Do the transition probabilities depend on  $n$ ?
  - d) Give the transition probability matrix.
  - e) List the communicating classes of the Markov Chain
  - f) Is the Markov Chain Irreducible?
  - g) List the Transient classes.
  - h) List the recurrent classes.
  - i) Find the steady state probability vector  $\mathbf{p}_\infty$ .
  
2. Consider an urn with 2 black balls and 2 white balls. The following experiment is repeated indefinitely: We draw a ball at random and with probability  $a$  we change the colour of the ball and put it back in the urn, otherwise we put the ball back without change. Let  $X_n$  be the number of black balls in the urn at time  $n$ .
  - a) Is  $X_n$  a Markov Chain? Why? If so list the states.
  - b) Draw the state diagram
  - c) Find the state transition probabilities
  - d) Do the transition probabilities depend on  $n$ ?
  - e) Give the transition probability matrix.
  - f) List the communicating classes of the Markov Chain
  - g) Is the Markov Chain irreducible?
  - h) List the Transient classes.
  - i) List the recurrent classes.
  - j) Assuming  $a = \frac{1}{2}$ , find the state probability vector at the times  $n = 1, 2, 3, 10, 100, 1000$ . Use the computer. Repeat for  $a = \frac{3}{4}$ .
  - k) Assume that  $a = \frac{1}{2}$ , is the Markov chain periodic? Show your work.
  - l) Assuming that  $a = 1$ , modify the state diagram. Is the Markov Chain periodic
  - m) Find the steady state probability vector  $\mathbf{p}_\infty$  in the case that  $a = 1/2$  using manual computation.