Problems 10

1. Consider a discrete time wide sense stationary random processes whose auto-correlation function is of the form

$$R_{XX}[k] = a^{|k|}, \quad \text{where } |a| < 1$$

Assume this process has zero-mean. Is the process ergodic in the mean?

2. Let $X(t)$ be a wide sense stationary random process that is ergodic in the mean and the auto-correlation. However, $X(t)$ is not zero-mean. Let $Y(t) = CX(t)$, where C is a random variable independent of $X(t)$ and C is not zero-mean. Show that $Y(t)$ is not ergodic in the mean.

3. Consider a random process of the form

$$X(t) = b\cos(2\pi\Psi t + \Theta)$$

where b is a constant, Θ is a uniform random variable over $[0, 2\pi)$, and Φ is a random variable that is independent of Θ and has a PDF, $f_\psi(\psi)$. Find the PSD $S_{XX}(f)$ in terms of $f_\psi(\psi)$. In so doing, prove that for any $S(f)$ that is a valid PSD function, we can always construct a random process with PSD equal to $S(f)$.

4. Let $X(t) = \sum_{n=1}^{N} a_n \cos(\omega_n t + \theta_n)$, where all of the ω_n are nonzero constants, the a_n are constants, and the θ_n are IID random variables, each uniformly distributed over $[0, 2\pi)$.

   (a) Determine the autocorrelation function of $X(t)$.

   (b) Determine the power spectral density of $X(t)$.

5. A sinusoidal signal of the form $X(t) = b\cos(2\pi f_0 t + \Theta)$ is transmitted from a fixed platform. The signal is received by an antenna on a mobile platform that is in motion relative to the transmitter, with a velocity of V relative to the direction of signal propagation between the transmitter and receiver. Hence, the received signal experiences a Doppler shift and (ignoring noise in the receiver) is of the form.

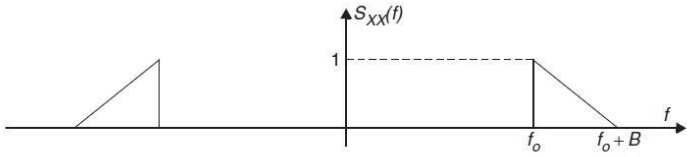
$$Y(t) = b\cos\left(2\pi f_0\left(1 + \frac{V}{c}\right)t + \Theta\right)$$

where c is the speed of light. Find the PSD of the received signal if V is uniformly distributed over $(-v_0, v_0)$. Qualitatively, what does the Doppler effect do to the PSD of the sinusoidal signal?

6. Let $X(t)$ be a random process whose PSD is shown in the accompanying figure. A new process is formed by multiplying $X(t)$ by a carrier to produce

$$Y(t) = X(t)\cos(\omega_0 t + \Theta)$$

where Θ is uniform over $[0, 2\pi)$ and independent of $X(t)$. Find and sketch the PSD of the process $Y(t)$.



7. A binary phase shift keying signal is defined according to

$$X(t) = \cos\left(2\pi f_c t + B[n]\frac{\pi}{2}\right) \quad \text{for} \quad nT \leq t < (n+1)T$$

for all n, and $B[n]$ is a discrete time, Bernoulli random process that has values of $+1$ or -1.

   (a) Determine the autocorrelation function for the random process $X(t)$. Is the process WSS?

   (b) Determine the power spectral density of $X(t)$.

8. Suppose $X(t)$ is a stationary, zero-mean Gaussian random process with PSD, $S_{XX}(f)$.

   (a) Find the PSD of $Y(t) = X^2(t)$ in terms of $S_{XX}(f)$.

   (b) Sketch the resulting PSD if $S_{XX}(f) = \text{rect}\left(\frac{f}{2B}\right)$.

   (c) Is $Y(t)$ WSS?

9. Problem 10.31 from the textbook.

10. Problem 10.32 from the textbook.