Problems 9

1. Let $X[n]$ be a wide sense stationary, discrete random process with autocorrelation function $R_{XX}[n]$, and let c be a constant.
   (a) Find the autocorrelation function for the discrete random process $Y[n] = X[n] + c$.
   (b) Are $X[n]$ and $Y[n]$ independent? Uncorrelated? Orthogonal?
2. A wide sense stationary, discrete random process $X[n]$ has an autocorrelation function of $R_{XX}[k]$. Find the expected value of $Y[n] = (X[n+m] - X[n-m])^2$, where m is an arbitrary integer.
3. Show by example that the random process $Z(t) = X(t) + Y(t)$ may be a wide sense stationary process even though the random processes $X(t)$ and $Y(t)$ are not.
   Hint: Let $A(t)$ and $B(t)$ be independent, wide sense stationary random processes with zero means and identical autocorrelation functions. Then let $X(t) = A(t)\sin(t)$ and $Y(t) = B(t)\cos(t)$. Show that $X(t)$ and $Y(t)$ are not wide sense stationary. Then show that $Z(t)$ is wide sense stationary.
4. Let $X(t) = A(t)\cos(\omega_0 t + \theta)$, where $A(t)$ is a wide sense stationary random process independent of θ, and let θ be a random variable distributed uniformly over $[0, 2\pi)$. Define a related process $Y(t) = A(t)\cos((\omega_0 + \omega_1)t + \theta)$. Show that $X(t)$ and $Y(t)$ are stationary in the wide sense but that the cross correlation $R_{XY}(t, t+\tau)$ between $X(t)$ and $Y(t)$, is not a function of τ only and, hence, $Z(t) = X(t) + Y(t)$ is not stationary in the wide sense.
5. Let $X(t)$ and $Y(t)$ be two jointly wide sense stationary Gaussian random processes with zero means and with auto-correlation and cross-correlation functions denoted as $R_{XX}(\tau), R_{YY}(\tau), R_{XY}(\tau)$. Determine the cross-correlation function between $X^2(t)$ and $Y^2(t)$.
6. Problem 9.82 from the textbook
7. Problem 9.83 from the textbook
8. Problem 9.84 from the textbook
9. Problem 9.85 from the textbook
10. Problem 9.88 from the textbook