ECE537
Random Processes
Quiz 2
Nov 29, 2023

a) Consider the random process $X(t)$. Give the definition, i.e. the condition, for the process to have a derivative in the mean square sense at the point $t = t_0$.

b) Consider a Poisson process $N(t)$ with parameter $\lambda$ arrivals per unit time. Are the sample functions continuous at all points in time $t_0$? Prove that the process is continuous in the mean square sense at an arbitrary point in time $t_0$.

c) Consider a wide sense stationary process $X(t)$ with autocorrelation function $R_X(\tau) = R_0 e^{-\left|\frac{\tau}{\tau_0}\right|}$. This process is input to a linear system that is an ideal low-pass filter, i.e. $H(f) = 1$ if $|f| < B$, and 0 otherwise. Determine the power spectral density of the random process at the filter output.

d) Let $X(t) = U(t - T)$, where $U(\cdot)$ is the step function and $T$ is a random variable with exponential distribution with mean $T_0$. Find the auto-correlation function for the process $X(t)$. Is it wide-sense stationary?