ECE537 Random Processes

Programming Assignment 1

Question 1 - Let Z be a random variable with the following CDF.

$$P(Z \leq z) = \begin{cases} 0 & z < 0 \\ 0.5z & 0 \leq z < 1 \\ 0.25 + 0.25z & 1 \leq z < 3 \\ 1 & 3 \leq z \end{cases}$$

(a) Generate $N = 100$ independent samples of Z and plot the histogram.

(b) Find the average of the samples and compare it to the expected value.

(c) Find the empirical variance and compare it to the true variance.

(d) For $N = 100, 200, 300, 400, 500, 1000, 2000, 5000$ samples of Z find the average and variance and plot them as a function of N. Do they converge as N increases?

Question 2 - Consider two biased dice each with the following PMF:

$$P(X = 1) = P(X = 2) = 0.25$$

and

$$P(X = 3) = P(X = 4) = P(X = 5) = P(X = 6) = 0.125$$

(a) Generate $N = 100$ independent samples of the two dice tosses denoted by (X, Y), and show the joint empirical PMF in a table.

(b) Using the simulated data, study whether the two random variables X and Y are independent.

(c) Define

$$\begin{aligned} Z_1 &= X + Y \\ Z_2 &= X - Y \end{aligned}$$

Using the simulated data, study whether Z_1 and Z_2 are independent?