ECE537

Random Processes

Problem Set 4

1. Let U_1, U_2, and U_3 be three independent zero-mean Gaussian random variables with unit variance. Let $X = U_1$, $Y = U_1 + U_2$, and $Z = U_1 + U_2 + U_3$.

   (a) Find $K_{X,Y,Z}$ their covariance matrix.
   (b) Find the joint PDF of (X, Y, Z).
   (c) Find $f_{(Y,Z|X)}(y, z|x)$
   (d) Find $f_{(Z|X,Y)}(z|x, y)$

2. The number of domestic, international and visiting students that have joined the ECE department of UofT by time t are independent Poisson random variables with expectations $\lambda_i t$, $i \in \{1,2,3\}$. Let N_i, $i \in \{1,2,3\}$ denote the number of joined students in each category by time T, where T is exponentially distributed with mean α^{-1}. Find the covariance matrix of (N_1, N_2, N_3).

3. If X and Y are independent exponential random variables with parameters λ_1 and λ_2 respectively, find the PDF of $Z = X/Y$.

4. If $\boldsymbol{X} = [X_1, X_2, X_3]^T$ is a jointly Gaussian random vector with mean $m_{\boldsymbol{X}} = 0$, and covariance matrix $K = I_{3\times3}$, where $I_{3\times3}$ is the identity matrix, find the PDF of $Y = \max(X_1, X_2, X_3)$.

5. The covariance matrix of the Jointly Gaussian random vector $X = [X_1, X_2, X_3\}^T$ is

$$K = \begin{bmatrix} 1 & -0.5 & -0.5 \\ -0.5 & 1 & -0.5 \\ -0.5 & -0.5 & 1 \end{bmatrix}$$

   Find the matrix A such that the elements of $Y = AX$ and independent jointly Gaussian random variables.

6. The PDF of a Cauchy random variable with parameter $\gamma > 0$ is $f_X(x) = \frac{\gamma}{\pi(\gamma^2 + x^2)}$ and its characteristic function is $\Phi_X(\omega) = e^{-\gamma|\omega|}$. Find the PDF of $Y = \sum_{i=1}^{n} X_i$, where the X_i's are independent Cauchy random variables with parameter γ_i.

7. In the food industry, we determine the popularity of a product by asking a large number of people to taste a little of it and feedback their reviews as 0 (awful) or 1 (awesome). Suppose by the kind genie, we know that the popularity of the Trump Vodka is 0.1. At least how many persons should take a test shot to make sure that with probability 99%, we see 0.89 up to 0.91 of the participants unsatisfied?

8. In the context of Problem 7, we have 1000 participants. By using the Chernoff bound, find an upper bound for the probability that at least half of them are satisfied.