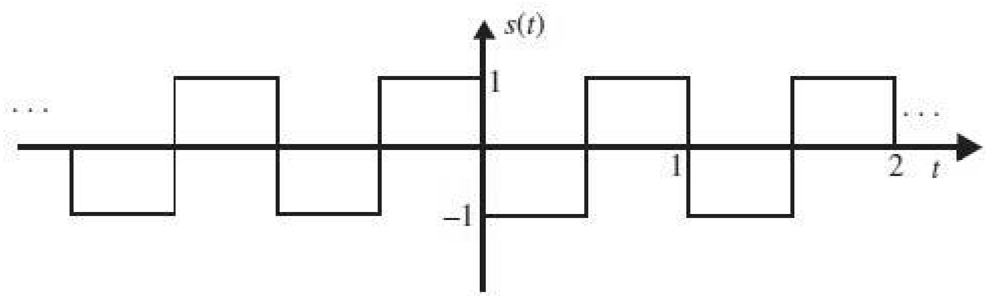
PS 7

1. A random process $X(t)$ consists of three sample paths: $x_1(t) = 1$, $x_2(t) = -3$, and $x_3(t) = \sin(2\pi t)$. Each member function occurs with equal probability.
   (a) Find the mean function, $\mu_X(t)$.
   (b) Find the autocorrelation function, $R_{X,X}(t_1, t_2)$.
2. A random process $X(t)$ has the following sample paths: $x_1(t) = -2\cos(t)$, $x_2(t) = -2\sin(t)$, $x_3(t) = 2[\cos(t) + \sin(t)]$, $x_4(t) = [\cos(t) - \sin(t)]$, and$x_5(t) = [\sin(t) - \cos(t)]$.
   (a) Find the mean function, $\mu_X(t)$.
   (b) Find the autocorrelation function, $R_{X,X}(t_1, t_2)$.
3. Let a discrete random process $X[n]$ be generated by repeated tosses of a fair die. Let the values of the random process be equal to the results of each toss.
   (a) Find the mean function, $\mu_X[n]$.
   (b) Find the autocorrelation function, $R_{X,X}(k_1, k_2)$.
4. A discrete random process, $X[n]$, is generated by repeated tosses of a coin. Let the occurrence of a head be denoted by 1 and that of a tail by -1. A new discrete random process is generated by $Y[2n] = X[n]$ for $n = 0, \pm 1, \pm 2, \cdots$ and $Y[n] = X[n+1]$ for $\pm n$ for n odd. Find the autocorrelation function for $Y[n]$.
5. A random process is given by $X(t) = A\cos(\omega t) + B\sin(\omega t)$, where A and B are independent zero mean random variables.
   (a) Find the mean function, $\mu_X(t)$.
   (b) Find the autocorrelation function, $R_{X,X}(t_1, t_2)$.
6. Let $s(t)$ be a periodic square wave as illustrated in the accompanying figure. Suppose a random process is created according to$X(t) = s(t - T)$, where T is a random variable uniformly distributed over $(0, 1)$.

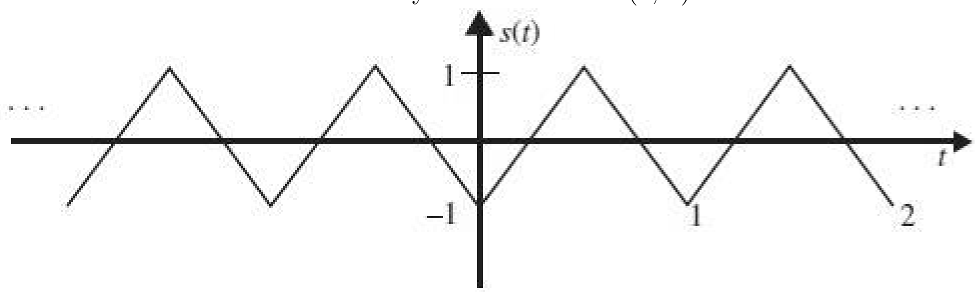


(a) Find the probability mass function of $X(t)$.

(b) Find the mean function, $\mu_X(t)$.

(c) Find the autocorrelation function, $R_{X,X}(t_1, t_2)$.

7. Let $s(t)$ be a periodic triangle wave as illustrated in the accompanying figure. Suppose a random process is created according to $X(t) = s(t-T)$, where T is a random variable uniformly distributed over (0, 1).



(a) Find the probability density function of$X(t)$.

(b) Find the mean function, $\mu_X(t)$.

(c) Find the autocorrelation function, $R_{X,X}(t_1, t_2)$.

8. Let a random process consist of a sequence of pulses with the following properties: (i) the pulses are rectangular and of equal duration Δ (with no dead space in between pulses), (ii) the pulse amplitudes are equally likely to be ± 1, (iii) all pulse amplitudes are statistically independent, and (iv) the various members of the ensemble are not synchronized.

(a) Find the mean function, $\mu_X(t)$.

(b) Find the autocorrelation function, $R_{X,X}(t_1, t_2)$.

9. A random process is defined by $X(t) = \exp(-At)u(t)$, where A is a random variable with PDF $f_A(a)$.

(a) Find the PDF of $X(t)$ in terms of $f_A(a)$.

(b) If A is an exponential random variable, with $f_A(a) = e^{-a}u(a)$, find $\mu_X(t)$ and $R_{X,X}(t_1, t_2)$.

10. Two zero mean discrete random processes, $X[n]$ and $Y[n]$, are statistically independent. Let a new random process be $Z[n] = X[n] + Y[n]$. Let the autocorrelation functions for $X[n]$ and $Y[n]$ be

$$R_{XX}[k] = \left(\frac{1}{2}\right)^{|k|}, \quad R_{YY}[k] = \left(\frac{1}{3}\right)^{|k|}$$

Find $R_{ZZ}[k]$.