ECE537

Random Processes

Problem Set 3

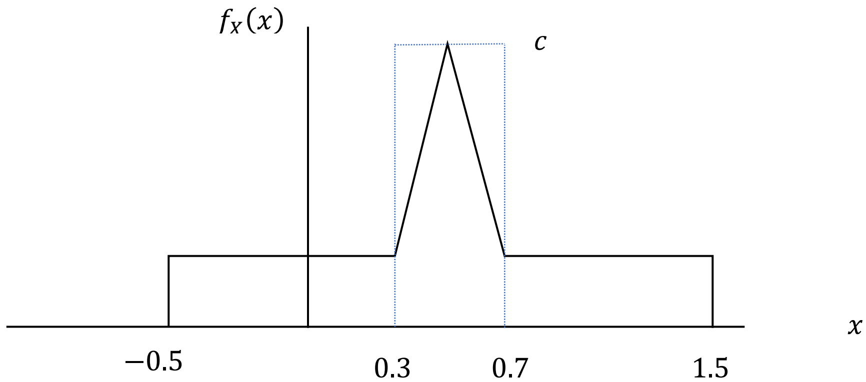
1. Assume that X is a r.v. uniformly distributed over $\{-4,-3,\cdots,3,4\}$. Find the mean and variance of $Y = \sin^2\left(\frac{\pi X}{4}\right)$.

2. Let M be a geometric random variable, i.e. $P(M = k) = p^k(1-p)$, for $k = 0,1,2,\cdots$ Show that M satisfies the memoriless property, i.e.

$$P(M \geq k + j)/M \geq j + 1) = P(M \geq k) \qquad \forall\ k, j$$

3. Find the mean of a Pareto distributed random variable with $\alpha = 1$, and $x_m = 1$, where $f_X(x;\alpha,x_m) = \frac{\alpha x_m^\alpha}{x^{\alpha+1}}\ \ x \geq x_m$, and zero else where.

4. The PDF of the random variable X is shown in the Figure
   a) Find the value of c.
   b) Find the mean and variance of X.
   c) Find the characteristic function of X, defined as $\Phi_X(\omega) = \mathcal{E}\left(e^{j\omega X}\right)$.



5. By using the derivative method (textbook page 179) show that if X is a Cauchy random variable with parameter α, then $Y = X^{-1}$ is a Cauchy random variable with parameter α^{-1}. (Note $f_X(x) = \frac{\alpha/\pi}{x^2+\alpha^2}$).
6. Let U be a random variable with uniform pdf on [0,1] and $X = -\ln\left(\left|1-\frac{1}{U}\right|\right)$. Find $f_X(x)$.
7. Let X be a Gaussian random variable with mean μ and variance σ^2. Find the PDF of

$$Y = \begin{cases} 0 & X \leq 0 \\ X & X > 0 \end{cases}$$

8. Show that the covariance matrix of a random vector is positive semidefinite.

9. If μ and K are the mean vector and covariance matrix of the random variable X $(n \times 1)$, find the mean and the covariance matrix for the random vector $Y = AX + b$, where A is a matrix $(m \times n)$, and b is a vector $(m \times 1)$.