Modelling & Simulation: Colonising a Zombie Infested Island

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1. Context

A recent discovery has revealed the existence of an "uninhabited" island with an abundance of precious resources, presenting an enticing opportunity for human colonisation. However, preliminary surveillance of this island indicates that it is home to a population of zombies, posing a significant threat to any colonisation plans.

To ensure the success of the mission and minimise human losses, modelling and simulation methods are employed to design effective strategies and predict their outcomes. The primary aim of this project is to develop a mathematical and computational framework to model the interactions between humans and zombies on the island. This model will be used to simulate different strategies for human colonisation and assess their effectiveness under various conditions. Ultimately, this report will provide actionable insights that maximise the chances of human survival and successful colonisation of the island.

There are several assumptions and constraints to consider:

- **Timeframe:** The mission is desired to occur over a short timescale, with no human births or natural deaths occurring during the operation.
- **Initial Population:** A finite number of humans will be deployed as the initial colonisers, with no reinforcements or external support available during the mission.
- **Geographical Isolation:** The island is surrounded by jagged cliffs, making it inaccessible by boat. Humans will be deployed via parachute, and escape is not possible during the operation.

The report begins by constructing a basic mathematical model of the human-zombie system as a foundation. Incrementally, additional features and complexities will be introduced to better capture the dynamics of the system, including potential interventions such as human aggression strategies and the role of resurrected "turned" zombie states. By applying numerical and computational methods, the system will be analysed for key properties such as fixed points, stability, and overall complexity.

2. Programming

The code used for this project is accessible in a GitHub repository (https://github.com/SaianPatel/Modelling_and_Simulation).

In each iteration, Mathematica's NDSolve solver was used. NDSolve is distinctly advantageous given that it automatically selects a numerical method to use based on the problem, and automatically adjusts the step sizes during solving. NDSolve is also easy to implement. This mitigates any potential errors when using custom written solvers such as Euler Forward or RKF45.

3. HZR Model

Inspired by the basic SIR model [1], the basis model for this report considers 3 populations:

- Human (H)
- Zombie (Z)
- Removed (R)

The dynamics of the system are listed below:

- Humans can undergo zombification after an encounter with a zombie through mass-action transmission, governed by the transmission parameter β_Z. Mass-action incidence specifies that an average member of an infected population (which is Z in this case) makes sufficient contact to transmit infectious disease with βN others per unit time, where N is the total population without infection. The probability that a zombie and a human randomly contact is given by $\frac{H}{N}$ and therefore the new zombies per unit time is given by $(\beta_Z N)\left(\frac{H}{N}\right)Z = \beta_Z HZ$.

- Zombies are naturally removed from the system, governed by a removal rate δ_Z. The physical meaning of such removal can be described by starvation, or other trivial causes such as a zombie falling off a cliff at the edge of the island – all of which happen independent of the human population.

This basic system can be summarised by the equations:

$$H' = -\beta_Z HZ$$

$$Z' = \beta_Z HZ - \delta_Z Z$$

$$R' = \delta_Z Z$$

This model is shown in Figure 1.

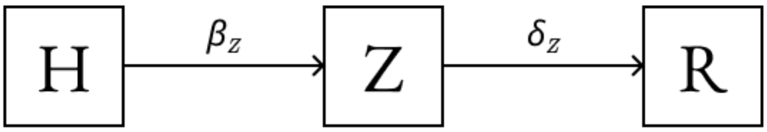

Figure 1: System diagram for the basic HZR model.

3.1. Non-Dimensionalisation

The system can be non-dimensionalised using a characteristic time scale. This approach ensures that all parameters and variables are dimensionless, which improves numerical stability and facilitates easier analysis of system dynamics. In this system, the characteristic time scale will be governed by the zombification rate:

$$t_C = \frac{1}{\beta_Z}$$

This means that dimensionless time ($\tilde{t}$) is given by:

$$\tilde{t} = \frac{t}{t_C} = \beta_Z t$$

We can also scale the population based on the initial human population (H_0):

$$\widetilde{H} = \frac{H}{H_0}, \tilde{Z} = \frac{Z}{H_0}, \widetilde{H} = \frac{R}{H_0}$$

These dimensionless parameters and variables can be substituted into the original system equations:

$$\widetilde{H}' = -\widetilde{H}\tilde{Z}$$

$$\tilde{Z}' = \widetilde{H}\tilde{Z} - \widetilde{\delta_Z}\tilde{Z}$$
$$\widetilde{R}' = \widetilde{\delta_Z}\tilde{Z}$$

From this point, the tildes will be dropped for simplicity, so the non-dimensional equations will be:

$$H' = -HZ$$

$$Z' = HZ - \delta_Z Z$$

$$R' = \delta_Z Z$$

3.2. Fixed Points

The fixed points for this system can be found by equating these differential equations to 0:

$$H' = -HZ = 0$$

$$Z' = HZ - \delta_Z Z = 0$$

$$R' = \delta_Z Z = 0$$

From the first equation, either $H = 0$ or $Z = 0$, and the latter is always true due to the equation for R' (assuming $\delta_Z > 0$), therefore we get 2 fixed points:

- **Case 1:** $(\bar{H}, \bar{Z}, \bar{R}) = (0,0,R)$. This represents the apocalyptic scenario where there are no humans or zombies in the system, just corpses in the removed population.
- **Case 2:** $(\bar{H}, \bar{Z}, \bar{R}) = (H,0,R)$. This represents the zombie-free scenario where humans have managed to outlast the zombies.

Both fixed points demonstrate that human-zombie coexistence is not possible on the island, irrespective of stability. The time evolution for the system for both cases are shown in Figure 2.

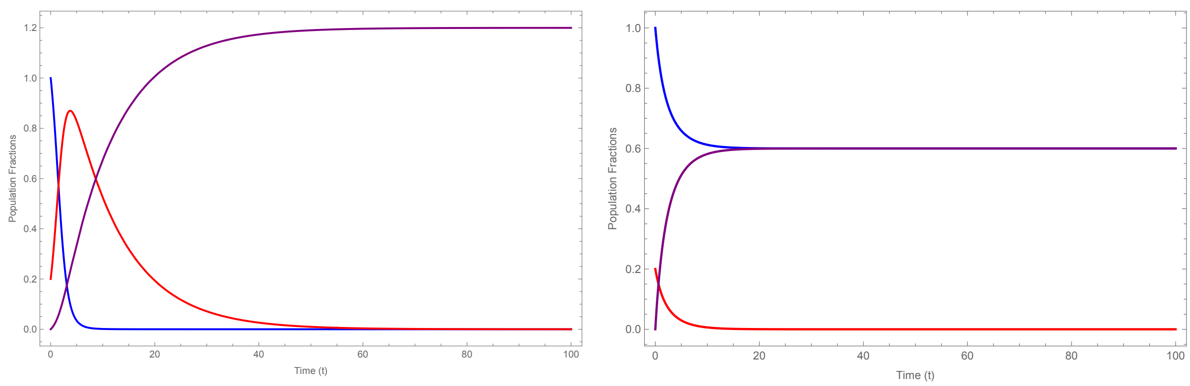

Figure 2: Time evolution of the system for Case 1 (left) and Case 2 (right).

3.3. Stability Analysis

The stability of each of the above fixed points can be analysed through observing the determinant and trace of the Jacobian matrix and through eigen analysis. First, the Jacobian matrix for the system is computed:

$$J(H,Z,R) = \begin{bmatrix} -Z & -H & 0 \\ Z & H - \delta_Z & 0 \\ 0 & \delta_Z & 0 \end{bmatrix}$$

Trace-Determinant Diagram

The trace and determinant of the Jacobian can be evaluated and viewed in the trace-determinant plane. Generally, for this system, the trace and determinant are defined below.

$$\mathrm{Tr}(J) = H - Z - \delta_Z$$

$$\det(J) = 0$$

The determinant, which is the product of the eigenvalues, is always 0 which means that at least one of the eigenvalues is 0. As such, this system will have a direction in which there is no exponential growth or decay, which implies a neutral dynamic in said direction. This means that this system shows the 'borderline' case in the trace-determinant plane and requires further analysis.

The trace, however, is not 0 and differs for the 2 fixed points.

- **Case 1:** $(\bar{H}, \bar{Z}, \bar{R}) = (0,0,R)$, the trace becomes $\mathrm{Tr}(J) = -\delta_Z$. Since $\delta_Z > 0$ this means that the trace is always negative in this case.
- **Case 2:** $(\bar{H}, \bar{Z}, \bar{R}) = (H, 0, R)$, the trace becomes $\mathrm{Tr}(J) = H - \delta_Z$.

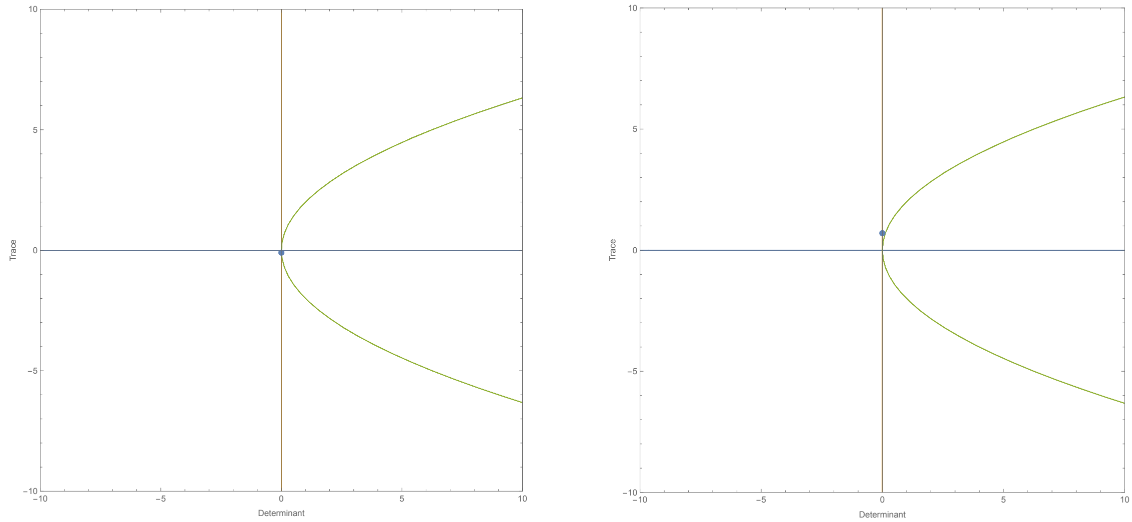

Figure 3: Trace-Determinant diagram for Case 1 (left) and Case 2 (right), where the trace varies with the determinant always at 0.

Eigen-analysis

To find the eigenvalues for the system, we solve:

$$\det(J - \lambda I) = 0$$

$$\det(J - \lambda I) = \begin{bmatrix} -Z - \lambda & -H & 0 \\ Z & H - \delta_Z - \lambda & 0 \\ 0 & \delta_Z & -\lambda \end{bmatrix} = 0$$

$$\lambda_1 = 0, \lambda_{2,3} = \frac{H - Z - \delta_Z}{2} \pm \frac{\sqrt{H^2 - 2HZ - 2H\delta_Z + Z^2 - 2Z\delta_Z + \delta_Z^2}}{2}$$

Table 1: Summary of eigenvalues per case for the basic model.

Case	λ_1	λ_2	λ_3
$(\bar{H}, \bar{Z}, \bar{R}) = (H, 0, R)$	0	$H - \delta_Z$	0
$(\bar{H}, \bar{Z}, \bar{R}) = (0,0, R)$	0	$-\delta_Z$	0

For Case 1, the sign of the eigenvalue depends on the values of H and δ_Z. $H < 1$ when there is at least one zombie in the system initially therefore if $\delta_Z = 1$ the sign of the eigenvalue is negative at the fixed point which suggests stability along this direction. Conversely, if $\delta_Z < 1$ then the eigenvalue could be positive or negative. Similarly, in Case 2, the second eigenvalue is always negative which suggests stability in this direction.

While these eigenvalues provide some insight into the system stability, the stability of the fixed point can't be determined solely from the eigenvalues due to the existence of the zero eigenvalues which indicate a marginal case. In both cases, the first eigenvalue is 0 and the behaviour in that direction is driven by nonlinear terms in the system which suggests that further analysis is required, and likewise for the third eigenvalue.

3.4. Phase Space

The system can be analysed in phase space by considering phase plots describing how the populations evolve over time with varying δ_Z values.

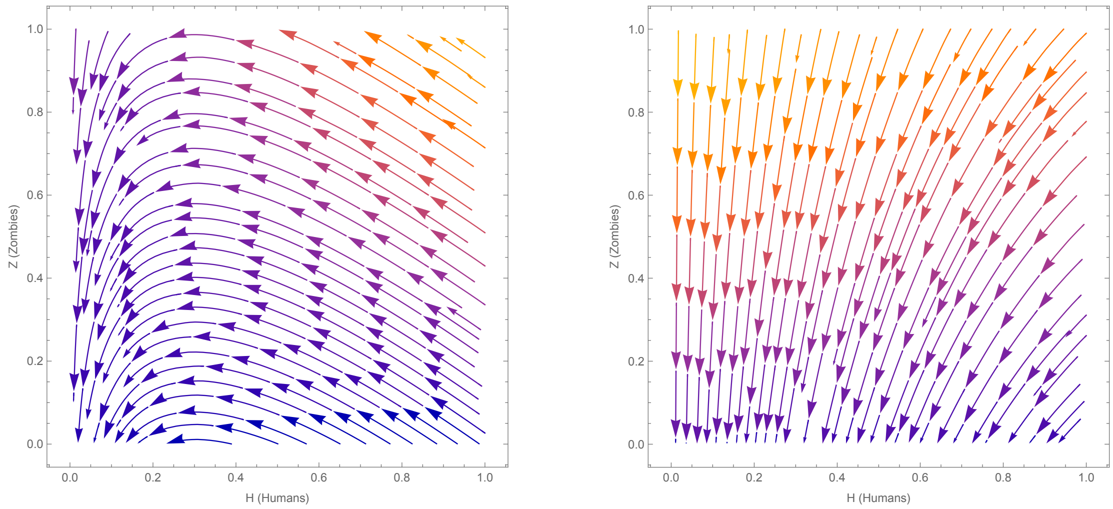

Figure 4: Phase plots for the system human and zombie populations with low δ_Z value (left) and a higher δ_Z value (right).

Low δ_Z (0.1)

As shown in Figure 4, the trajectories in the phase plot predominantly flow toward smaller values of H and Z, indicating that both populations decline over time. This behaviour is consistent with the dynamics of the system, where humans are zombified after interacting with zombies and zombies are continually removed from the system over time. The phase plot exhibits the presence of a global attractor at $(H, Z) = (0,0)$ as all trajectories converge to this point, irrespective of the initial conditions for H and Z. There are clearly no other fixed points of the system at this δ_Z value which further supports the notion that $(H, Z) = (0,0)$ is the global attractor of the system. Examining behaviour near the axes, it can be seen that the trajectories are directed vertically downward near $H = 0$ which indicates that the zombie population decreases aggressively in the absence of humans given that the zombie population can't grow via zombification of humans. The line $Z = 0$ acts as a nullcline along which humans can persist in the system under certain initial conditions as there are no zombies to infect them. The phase plot shows that zombie persistence is not possible in this parameter regime. While both the human and zombie populations decline globally, that of the zombies is much more prominent when human numbers are small, leading to eventual extinction.

High δ_Z (1.5)

The trajectories in the phase plot for a higher value of δ_Z show a slightly different behaviour compared to the previous case. With a high dimensionless zombie removal rate, the trajectories tend to flow toward smaller values of zombies while the human population stabilises at higher values; there is a convergence to a human-dominated state. Contrary to the previous case, this indicates that humans survive in this system while the zombie population is driven to extinction. Along the Z axis, the trajectories are directed downwards which suggests that zombies decrease when there are no humans present to zombify. This result highlights that it is possible for humans to outlast the zombies on the island.

Overall, these phase plots show that the system experiences a shift in behaviour depending on the value of δ_Z.

3.5. Model Evaluation

For humans to survive the zombies on this island it is clear that dimensionless δ_Z must be maximised. However, the humans will have no influence over this within the current description of the model and rather will have to passively hope that the dimensionless δ_Z is large enough to permit their survival. Additionally, the original formation of δ_Z suggests that zombies can die naturally which is flawed on two fronts: 1) If zombies die naturally, irrespective of human interaction, then the optimal strategy would be to observe the island from afar and wait for the zombies to die out naturally before trying to conquer the island, and 2) Zombies are coined the 'undead' and therefore shouldn't die off naturally. Going forward, the model should be refined to have the human influence directly influence the removal of zombies.

4. Adding Human Attack

To address the key limitation of the previous model, this iteration adapts the removal dynamics of the zombies to be dependent on the number of humans. As such, δ_Z now describes the rate by which humans can attack and kill zombies when interacting with them. Consequently, zombies no longer die out naturally in this system and are only removed by humans. In this model, it is assumed that a human can avoid zombification during an interaction with a zombie by attacking and defeating the zombie. Each human is capable of resisting zombification through attacks at the rate δ_Z. As such, the same mass-action concept can be applied to provide the equation which governs this interaction: $(\delta_Z N)\left(\frac{Z}{N}\right)H = \delta_Z HZ$.

This updated system can be summarised by the equations:

$$H' = -\beta_Z HZ$$

$$Z' = \beta_Z HZ - \delta_Z HZ = (\beta_Z - \delta_Z)HZ$$

$$R' = \delta_Z HZ$$

This model is similar to Munz's basic SZR model [2], except it doesn't include resurrection of the zombies, and builds on the basic SIR model by introducing a second mass-action transmission dynamic.

4.1. Non-Dimensionalisation

Like the previous iteration, this system can be non-dimensionalised using a characteristic time scale. Again, the zombification rate (β_Z) will be used to define this:

$$t_C = \frac{1}{\beta_Z}$$

This means that dimensionless time ($\tilde{t}$) is given by:

$$\tilde{t} = \frac{t}{t_C} = \beta_Z t$$

The populations will also be scaled based on the initial human population (H_0):

$$\tilde{H} = \frac{H}{H_0}, \tilde{Z} = \frac{Z}{H_0}, \tilde{H} = \frac{R}{H_0}$$

These dimensionless parameters and variables can be substituted into the original system equations:

$$\tilde{H}' = -\tilde{H}\tilde{Z}$$

$$\tilde{Z}' = \tilde{H}\tilde{Z} - \widetilde{\delta_Z}\tilde{H}\tilde{Z}$$

$$\tilde{R}' = \widetilde{\delta_Z}\tilde{H}\tilde{Z}$$

Again, from this point the tildes will be dropped for simplicity, so the equations for the updated system are:

$$H' = -HZ$$

$$Z' = HZ(1 - \delta_Z)$$

$$R' = \delta_Z HZ$$

4.2. Fixed Points

The fixed points for this system can be found by equating these differential equations to 0:

$$H' = -HZ = 0$$

$$Z' = HZ(1 - \delta_Z) = 0$$

$$R' = \delta_Z HZ = 0$$

We get 3 fixed points:

- **Case 1:** $(\bar{H}, \bar{Z}, \bar{R}) = (0, Z, R)$. This represents the scenario where there are no humans in the system and zombies dominate the island.
- **Case 2:** $(\bar{H}, \bar{Z}, \bar{R}) = (H, 0, R)$. This represents the zombie-free scenario where zombies have been successfully defeated from the island.
- **Case 3**: $(\bar{H}, \bar{Z}, \bar{R}) = (0,0,R)$. This fixed point is when both zombies and humans die out. This point is only possible if there is a particular balance between the initial number of zombies, humans and the dimensionless parameter δ_Z. This case occurs when $\delta_Z = 1 + \frac{Z_0}{H_0}$.

All fixed points again demonstrate that human-zombie coexistence is not possible on the island in the updated system.

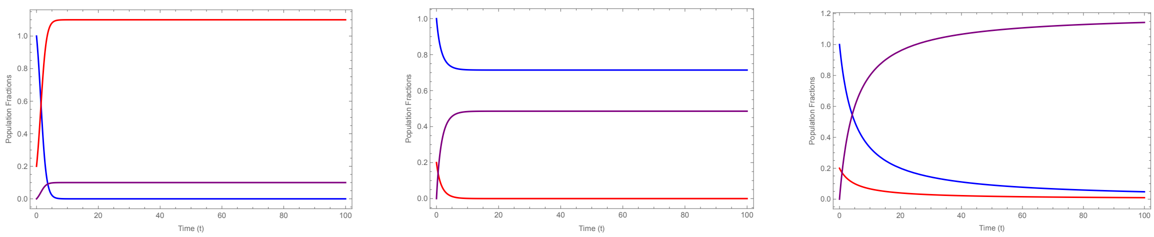

Figure 5: Time evolution of the system for Case 1 (left) and Case 2 (middle) and Case 3 (right).

4.3. Stability Analysis

The Jacobian matrix for the system is given by:

$$J(H,Z,R) = \begin{bmatrix} -Z & -H & 0 \\ (1-\delta_Z)Z & (1-\delta_Z)H & 0 \\ \delta_Z Z & \delta_Z H & 0 \end{bmatrix}$$

Trace-Determinant Diagram

The trace and determinant for this system are given by:

$$\mathrm{Tr}(J) = (1-\delta_Z)H - Z$$
$$\det(J) = 0$$

As before, the determinant is always 0, however, the trace varies for each of the fixed points as shown in Table 2.

Table 2: Summary of eigenvalues per case for the human attack model.

Case	$\det(J)$	$\mathrm{Tr}(J)$
$(\bar{H},\bar{Z},\bar{R}) = (0,Z,R)$	0	$-Z$
$(\bar{H},\bar{Z},\bar{R}) = (H,0,R)$	0	$(1-\delta_Z)H$
$(\bar{H},\bar{Z},\bar{R}) = (0,0,R)$	0	0

4.4. Phase Space

From the phase plots shown in Figure 6, it is clear that varying δ_Z dictates the gradient and direction of the trajectories.

When $\delta_Z < 1,\ Z' > 0$ which results in the trajectories pointing upwards and to the left. This behaviour is indicative of ineffective human attacks causing the zombie population to inevitably grow and overpower the human population.

When $\delta_Z = 1, Z' = 0$ which results in the horizontal trajectories shown in Figure 6; Z does not change regardless of the value of H, however, H still reduces due to the presence of Z. The physical interpretation of this is that $\delta_Z = 1$ signifies a balance for the zombie population between the human attack and zombification processes.

When $\delta_Z > 1,\ Z' < 0$ which results in the trajectories flowing downwards and to the left. This suggests that the human attacks become more effective than the zombification process; zombies are removed which increases the persistence of humans in the system.

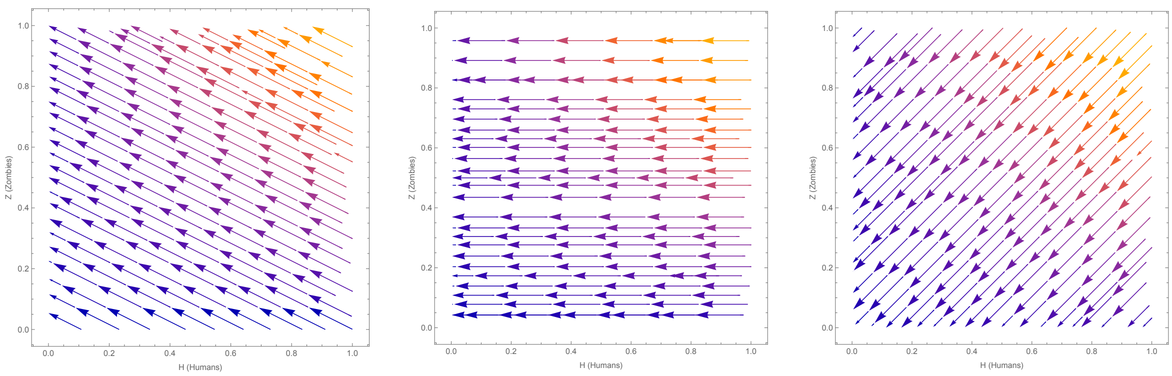

Figure 6: Phase plots for the system at varied values for δ_Z: $\delta_Z < 1$ (left), $\delta_Z = 1$ (middle), $\delta_Z > 1$ (right).

4.5. Bifurcation Analysis

As previously stated, there is a critical value of δ_Z where both the human and zombie populations collapse to zero. This critical value is where $\delta_Z = 1 + \frac{Z_0}{H_0}$ and as shown in Figure 7, this marks the critical point where the system transitions from zombie dominance to human dominance. This can be explained by considering the initial conditions.

When $t = 0$, the equations are:

$$H' = -H_0 Z_0$$

$$Z' = H_0 Z_0 (1 - \delta_Z)$$

$$R' = \delta_Z H_0 Z_0$$

When $\delta_Z = 1 + \frac{Z_0}{H_0}$ this gives:

$$Z' = H_0 Z_0 \left(1 - 1 - \frac{Z_0}{H_0}\right)$$

$$Z' = H_0 Z_0 \left(-\frac{Z_0}{H_0}\right)$$

$$Z' = -Z_0^2$$

This means that, initially, the zombification and human attack removal terms are balanced but the quadratic decay term ensures that the zombie population begins to decay immediately. Furthermore, as humans are zombified at a rate proportional to HZ this ultimately leads to a simultaneous decline of the two populations, leading to the fixed point $(0,0,R)$.

Therefore, $\delta_Z = 1 + \frac{Z_0}{H_0}$ signifies a bifurcation in the system's dynamics (shown in Figure 7).

- $\delta_Z < 1 + \frac{Z_0}{H_0}$ leads to a zombie-dominated state as the system converges to $(0, Z, R)$.
- $\delta_Z > 1 + \frac{Z_0}{H_0}$ leads to a human-dominated state as the system converges to $(H, 0, R)$.
- $\delta_Z = 1 + \frac{Z_0}{H_0}$ leads to a total extinction state as the system converges to $(0,0,R)$.

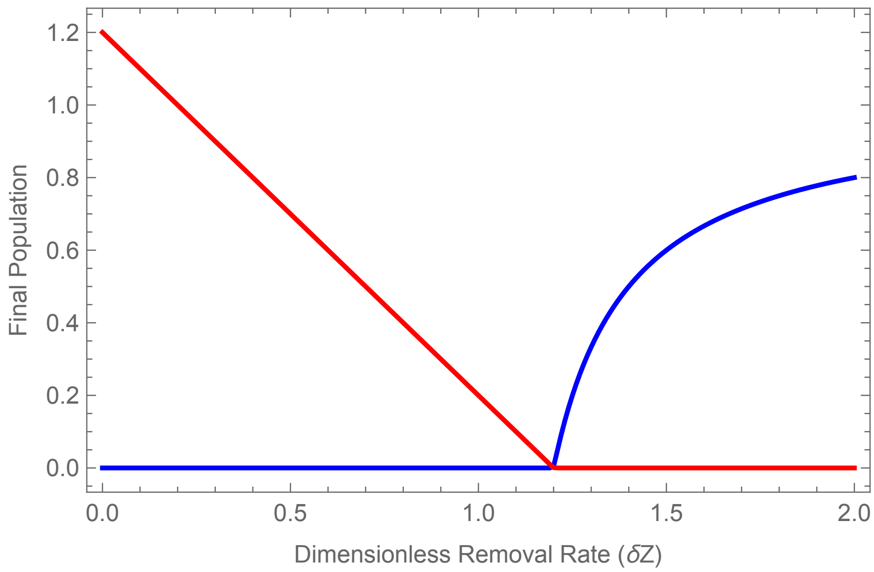

Figure 7: A diagram of how the final human (blue) and zombie (red) populations change as the dimensionless removal rate δ_Z is increased. This diagram shows the transition in system regimes, through the critical point $\delta_Z = 1 + \frac{Z_0}{H_0}$, from a zombie dominated state to a human dominated state.

4.6. Model Evaluation

This updated model effectively captures human intervention through direct zombie attacks and highlights that the best strategy for human survival is to adopt an aggressive approach to eliminating zombies. The key objective is to outpace the zombification process, which can be achieved by equipping humans with the necessary tools, such as weapons, to efficiently remove zombies. While the model successfully demonstrates the direct interactions between humans and zombies, it places little emphasis on the role of the removed population. Additionally, the overall dynamics of the system remain relatively simple. Future improvements could involve assigning greater significance to the removed population or incorporating entirely new effects into the model to enhance its complexity and realism.

5. Turned Zombies – Forced Resurrection

It has been discovered that a strain of zombies, termed "Turned" zombies (T), can resurrect from the removed population. Turned zombies attack normal zombies and eventually recover back into humans, creating a closed-loop system as illustrated in Figure 8.

$$
\begin{tikzcd}
 & T \arrow[dl, "\delta_T"'] & \\
H \arrow[r, "\beta_Z"] & Z \arrow[r, "\delta_{ZT}", "\delta_{ZH}"'] & R \arrow[ul, "\beta_T"']
\end{tikzcd}
$$

Figure 8: System diagram for the updated system involving a Turned population (T).

The updated system is described by the following equations:

$$H' = -\beta_Z HZ + \delta_T T$$

$$Z' = \beta_{ZH}HZ - \delta_{ZH}HZ - \delta_T TZ = (\beta_Z - \delta_{ZH})HZ - \delta_{ZT}TZ$$

$$T' = \beta_T R - \delta_T T$$

$$R' = \delta_{ZH}HZ + \delta_{ZT}TZ - \beta_T R$$

There are several new parameters introduced:

- δ_{ZH}: This is equivalent to the previous δ_Z and denotes the removal of zombies by human attacks.
- δ_{ZT}: The removal of zombies by attack from turned zombies.
- β_T: The resurrection rate of corpses into turned zombies.
- δ_T: The recovery of turned zombies back into healthy humans.

5.1. Non-Dimensionalisation

As previously, this system can be non-dimensionalised using a characteristic time scale. Again, the zombification rate (β_Z) will be used to define this:

$$t_C = \frac{1}{\beta_Z}$$

The populations will also be scaled based on the initial human population (H_0). This gives rise to the following equations:

$$H' = -HZ + \delta_T T$$

$$Z' = (1 - \delta_{ZH})HZ - \delta_{ZT}TZ$$

$$T' = \beta_T R - \delta_T T$$

$$R' = \delta_{ZH}HZ + \delta_{ZT}TZ - \beta_T R$$

5.2. Example Scenario

With the 4 populations and additional nonlinear terms, the system becomes very difficult to solve analytically and thus we can rely on numerical simulations to examine the behaviour of the system. As such, an example scenario is shown below.

It is seen that all the populations experience some damped oscillation before stabilising at non-zero values. This shows that coexistence between the populations is possible in this system which is a feature that was not seen in previous models.

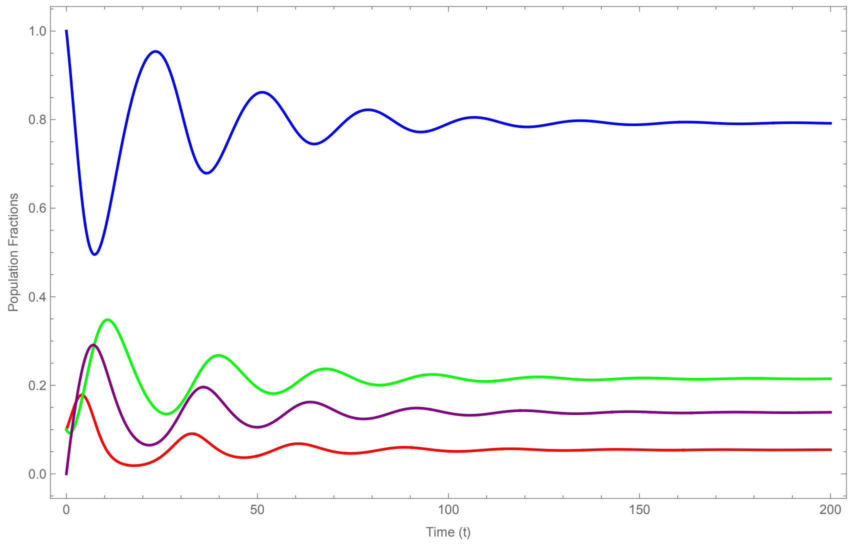

Figure 9: Example scenario for the turned zombie system using $\delta_{ZT} = 1.36, \delta_{ZH} = 0.631, \beta_T = 0.31, \delta_T = 0.2,$ $H_0 = 500, Z_0 = 50, T_0 = 50, R_0 = 0$. The human population is represented by the blue trace, the zombie population by the red trace, the turned population by the green trace and the removed population by the purple trace.

5.2.1. Phase Space

The phase plot illustrating the interaction between human and zombie populations reveals a spiral sink, with trajectories converging towards a stable fixed point where both populations stabilise at non-zero values. This spiral behaviour indicates oscillatory dynamics as the system approaches equilibrium, suggesting a damped feedback loop where changes in one population influence the other before stabilising. This is expected as the oscillations in the time evolution plot are damped.

The phase plot for human and turned populations demonstrates convergence to a stable fixed point where both human and turned populations coexist at non-zero values. Unlike the H-Z plot, the trajectories here do not exhibit spiral behaviour, instead showing a more direct path towards the fixed point.

The phase plot for zombie and turned populations exhibits distinct dynamical regions. A critical line around the stabilised T value ($T \approx 0.2$) appears to act as a threshold: above this line, trajectories flow downward and to the left, while below it, trajectories move upward and to the right. This dynamic behaviour aligns with the oscillations observed in the time evolution plot (Figure 9). When the turned population exceeds its equilibrium value, both Z and T decrease, and conversely when it falls below equilibrium, both populations increase, creating cyclic behaviour before the fixed point is inevitably reached.

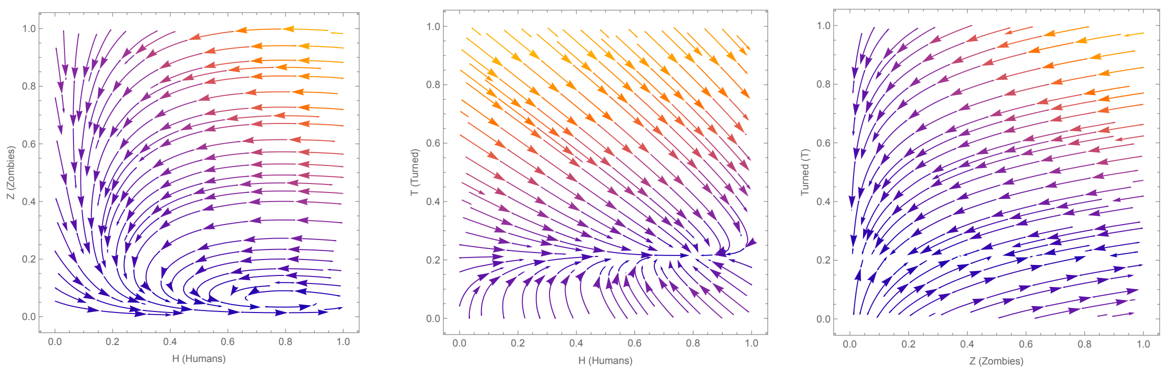

Figure 10: Phase plots for the system considering different populations: H- Z (left), H- T (middle), Z- T (right),

5.3. Stability Analysis

The Jacobian matrix for the system is given by:

$$J(H,Z,T,R) = \begin{bmatrix} -Z & -H & \delta_T & 0 \\ (1-\delta_{ZH})Z & (1-\delta_{ZH})H - \delta_{ZT}T & -\delta_{ZT}Z & 0 \\ 0 & 0 & -\delta_T & \beta_T \\ \delta_{ZH}Z & \delta_{ZH}H + \delta_{ZT}T & \delta_{ZT}Z & -\beta_T \end{bmatrix}$$

Trace-Determinant Diagram

The trace and determinant for this system are given by:

$$\mathrm{Tr}(J) = (1-\delta_{ZH})H - Z - \delta_{ZT}T - \delta_T - \beta_T$$
$$\det(J) = 0$$

As previously, the determinant is always 0.

5.4. Bifurcation Analysis

In this closed-loop system, parameter balance significantly impacts dynamics. A bifurcation analysis was conducted on the human and turned attack rates, which are the key survival mechanisms in this system. Each parameter was varied individually while others were fixed, using values from the previous scenario.

5.4.1. Varying Human Attack Rate

The system transitions from coexistence to human dominance as the human attack rate (δ_{ZH}) is increased. The critical point appears to be at $\delta_{ZH} = 1$ beyond which the human attack rate is high enough, with the addition of zombie removal via turned zombies, that the humans can successfully eradicate the normal zombies.

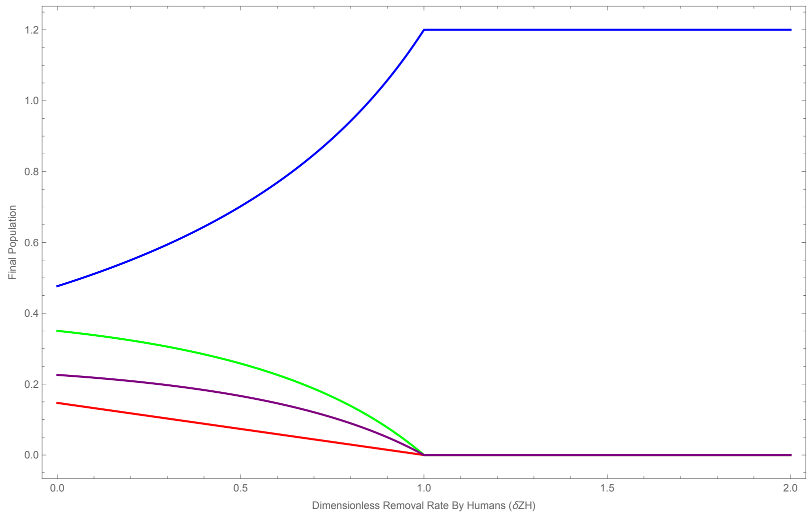

Figure 11: Varying the human attack rate δ_{ZH} and observing the effect on the final population values for each population.

5.4.2. Varying Turned Attack Rate

As the turned attack rate (δ_{ZT}) is increased, the system switches from zombie dominance to coexistence. There is a critical value beyond which the turned zombies enable coexistence; this appears to be around $\delta_{ZT} = 0.18$.

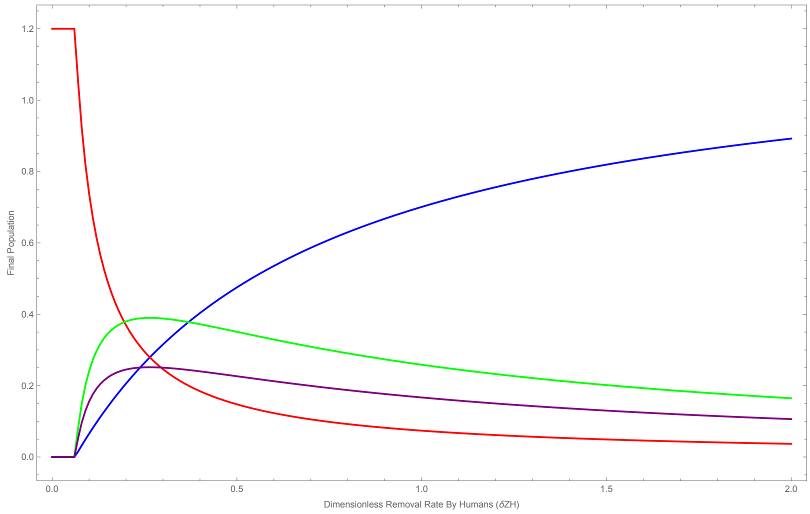

Figure 12: Varying the turned attack rate δ_{ZT} and observing the effect on the final population values for each population.

5.5. Ensemble Simulations

5.5.1 Sensitivity to Initial Conditions

Within the context of this model, the populations are initially passed in as absolute values before being divided by H_0. This means that the resolution for a human should be 0.5, in absolute terms. As such, an ensemble simulation was performed, varying the initial conditions from 0.5 above and below the desired initial population values. As shown in Figure 13, the system is not very sensitive to changes in the initial conditions.

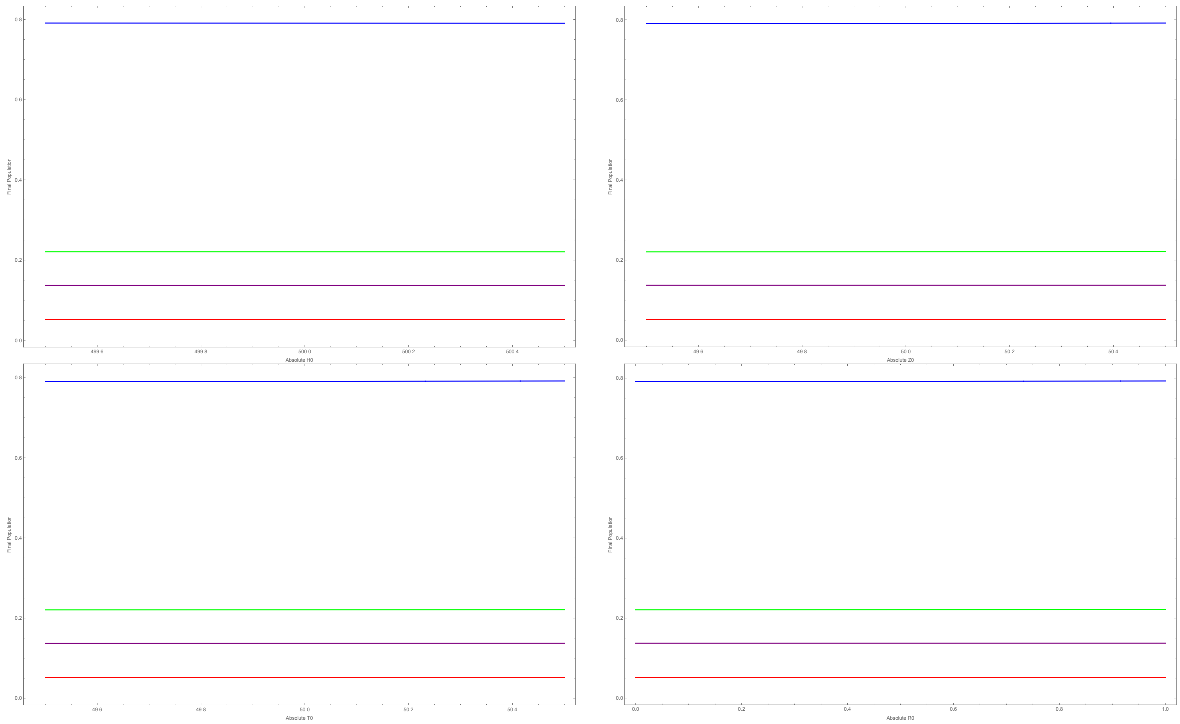

Figure 13: Varying the initial condition values for the system to examine the sensitivity.

5.5.2 Sensitivity to Parameter Values

The model's sensitivity to the parameters was also studied by examining the final population values as each condition was varied between $\pm 1\%$ of the desired value. Each of the parameter sweeps are shown in Figure 14. The system does not appear to be overly sensitive to the values of these parameters.

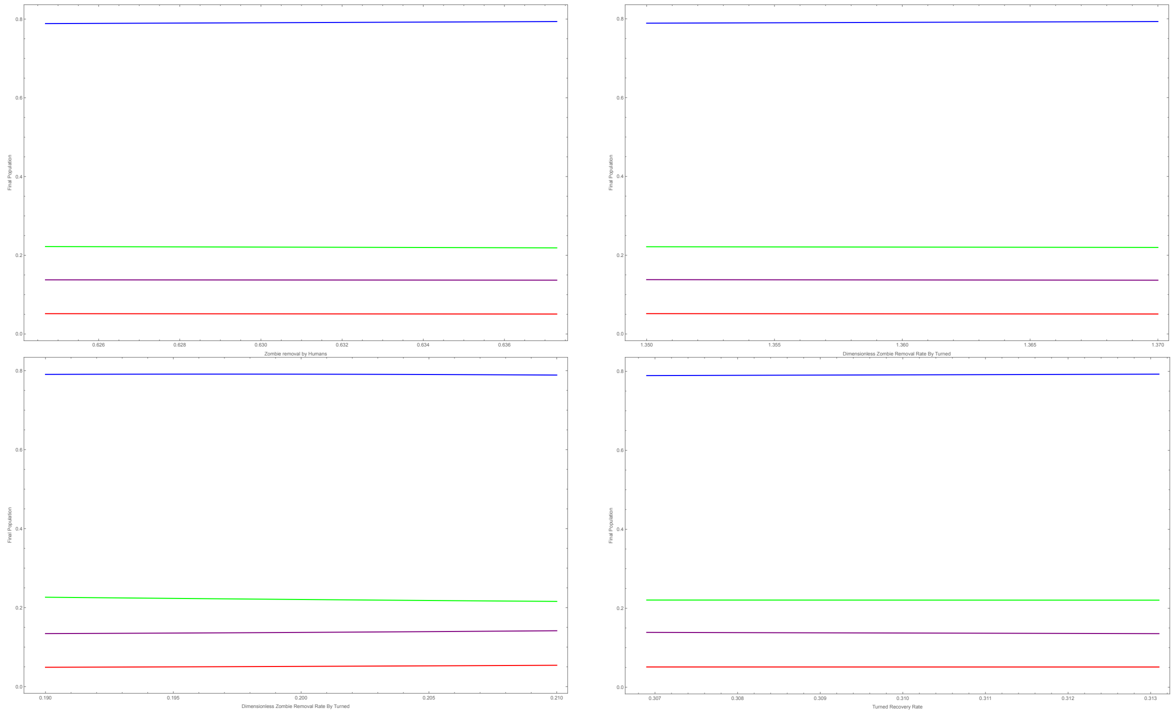

Figure 14: Varying the parameter values for the system to examine the sensitivity.

5.6. Model Evaluation

As was the case with the initial two models, this updated model demonstrates that human colonisation of the island is achievable. However, unlike the previous models, this system allows for the possibility of coexistence. The closed-loop dynamics of the system, coupled with the regulating effect of the turned zombie population, create conditions where both populations can stabilise at non-zero values. The presence of turned zombies plays a critical role in modulating the overall system, acting as an intermediary that balances the population dynamics.

Strategically, it remains evident that the most effective approach for humans would be to adopt a hostile stance. By maximising their attack rate, humans can efficiently eliminate zombies and convert them into turned zombies, which eventually transition back into the human population. This closed cycle minimises losses while facilitating rapid human dominance. For this strategy to succeed, the human and turned attack rates must be sufficiently high. From a physical perspective, this translates to humans arriving on the island prepared and determined to act decisively, ensuring swift colonisation with minimal human casualties.

While this updated model introduces more complex behaviours, it is important to note that it doesn't, and can't, exhibit chaotic dynamics. Chaos is precluded in this system due to several factors: the level of nonlinearity and feedback is insufficient, the dissipation of driving forces leads the system toward stability, and there are no recursive interactions that amplify small perturbations. Consequently, the system is inherently stable, with its dynamics settling into predictable and stable patterns over time.

6. Conclusion

After developing and analysing 3 distinct zombie apocalypse models for the island, it is evident that strategic preparation and an aggressive approach are essential for the success of the human colonisation plan.

The modelling and simulation efforts highlighted the dynamics between human, zombie, and turned populations, offering a detailed understanding of the potential outcomes under varying conditions. Each model incrementally added complexity, revealing key factors that influence the system's stability and highlighting the optimal strategies for achieving human dominance on the island. The key findings from this report can be grouped into 3 categories:

1. **Effectiveness of Aggression**: Across all models, a clear trend prevailed: the more prepared and aggressive the humans are upon arrival, the greater their chances of success. An aggressive strategy ensures the rapid elimination of zombies, reducing their ability to multiply and spread. As such, the humans should be provided with the necessary weaponry to eradicate and armour to defend themselves from zombies. No expense should be spared here as the greater the effectivity of zombie removal by humans, the quicker the operation is complete, minimising human life loss.

2. **Role of Turned Zombies**: The introduction of turned zombies in the third model added an additional layer of complexity. This intermediary population serves as a stabilising factor between the populations in the system. By converting zombies into turned zombies, who eventually transition back into the human population, the system allows for potential coexistence and long-term stabilisation. In this case, it must be questioned whether coexistence is at all desirable otherwise, as stated above, the humans will need to be as aggressive as possible and can't rely on the turned zombies to regulate their population.
3. **Constraints and Challenges**: The isolated nature of the island, combined with the finite number of humans and absence of reinforcements, means the strategy must be executed with precision. There is little opportunity for conventional tactics to work in this case and so the humans will need to be inventive to ensure their survival in the extreme conditions.

While the models presented in this report provide good initial understanding of the dynamics of the human-zombie interaction, they are not exhaustive. The simplicity of the models allows for clear analysis but inevitably omits certain real-world complexities, such as more nuanced zombie behaviours, resource limitations, or environmental factors. Future iterations could build upon this foundation by incorporating additional variables, refining assumptions, and exploring more sophisticated dynamics, such as chaotic behaviour, probabilistic elements or even spatial dynamics. Making such improvements to the models would afford greater understanding of the system under more realistic conditions and ultimately provide better predictive power of the models, enabling a more effective colonisation plan to be formulated. Although the current models are sufficient for outlining broad strategies, further development is crucial to provide a robust framework for decision-making in similar high-stakes scenarios.

7. References

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