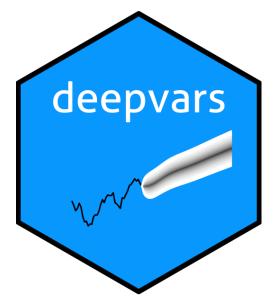
Deep Vector Autoregression for Macroeconomic Data





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Can we leverage the power of deep learning in VAR models?

We propose Deep VAR: a novel approach towards VAR that leverages the power of deep learning in order to model non-linear relationships. By modelling each equation of the VAR system as a deep neural network, our proposed extension outperforms modern benchmarks in terms of insample fit, out-of- sample fit and point forecasting accuracy. In particular, we find that the Deep VAR is able to better capture the structural economic changes during periods of uncertainty and recession. By staying methodologically as close as possible to the original benchmark, we hope that our approach is more likely to find acceptance in the economics domain.

Key contributions

- Simple methodology close in spirit to conventional benchmark
- Significant improvement in predictive performance
- Open source R package to facilitate reproducibility

Less is more

A simple methodology

We developed our idea under the following premise: maximise performance of an existing and trusted framework under minimal intervention. VAR has become a standard tool for practitioners to construct economic forecasts, but the assumption of linearity through time and variables is restrictive. We relax that assumption through one simple step: estimate each system equation through a recurrent neural network.

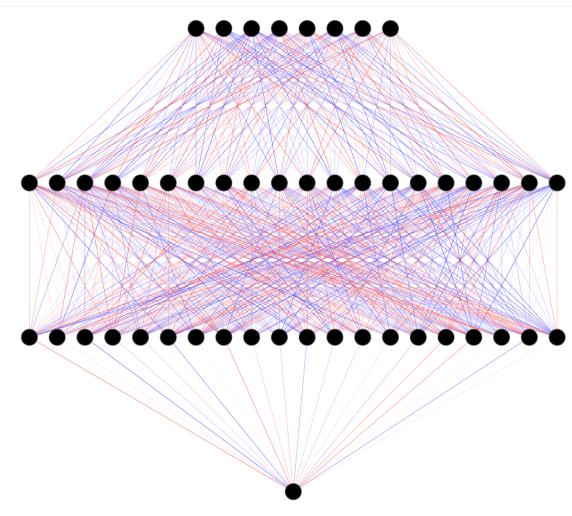


Figure 1: Neural Network Architecture. Inputs are lags of all variables. Output is variable of interest in time t.

Significant performance gains

Empirical evidence

To evaluate our proposed methodology empirically we use the **FRED-MD** data base to collect a sample of monthly US data on output (IP), unemployment (UR), inflation (CPI) and interest rates (FFR). Our sample spans the period from January 1959 through March 2021.

Our findings show a **consistent and significant improvement in predictive performance**: the Deep VAR incurs much lower loss than the conventional benchmark. It also outperforms another popular approach towards VAR models that addresses non-linearity (Threshold VAR).

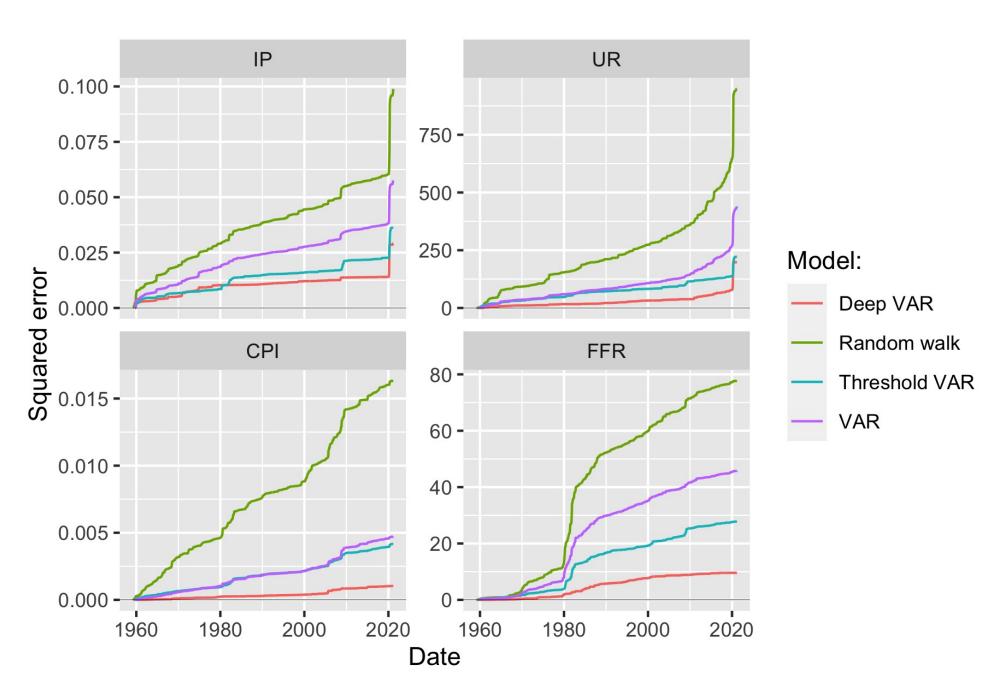


Figure 2: Comparison of cumulative loss over the entire sample period for Deep VAR and benchmarks.

We also test model performance with respect to a test sample: future realizations arrive and we compute 1-step ahead predictions without retraining (Table 1). In the paper, we also present evidence that the Deep VAR outperforms on *n*-step ahead forecasts.

Variable	DVAR	VAR	Ratio (DVAR / VAR)
IP	0.00494	0.01484	0.33253
UR	0.94542	1.65170	0.57240
CPI	0.00231	0.00342	0.67642
FFR	0.17494	0.23974	0.72972

Table 1: Test root mean squared error (RMSE) for the two models across variables.

Have we merely scratched the surface?

Hyperparameter tuning

For a our baseline comparison we keep things simple: for example, we let the conventional VAR guide our search for optimal lag length. A short exercise in hyperparameter tuning demonstrates that the Deep VAR is less prone to overfitting with respect to the number of lags among other things.

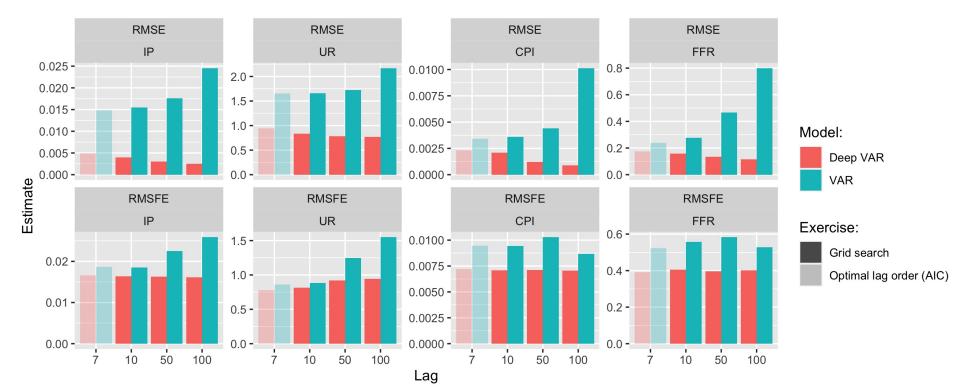


Figure 3: Pseudo out-of-sample RMSE and RMSFE for VAR and Deep VAR across different lag choices.

Where to go from here

Recent work

- We have added Threshold VAR for comparison (Figure 2).
- Progress on uncertainty quantification through Bayesian deep learning
 MC dropout (Gal and Ghahramani 2016). Recent work by Daxberger et al. (2021) shows that Laplace Approximation is a promising way forward.

Open questions

- What other benchmark models should we consider?
- Structural identification how to proceed? Verstyuk (2020) works with Cholesky decomposition as in conventional VAR.
- Can the existing toolbox (IRFs, variance decomposition, policy counterfactuals, ...) be derived for Deep VAR?
- Deep VAR as a tool for detecting non-linearities?

References

Daxberger et al. (2021). "Laplace Redux-Effortless Bayesian Deep Learning.". In: Advances in Neural Information Processing Systems 34

Gal and Ghaharamani (2016). "Dropout as a Bayesian Approximation: Representing Model Uncertainty in Deep Learning.". In: International Conference on Machine Learning, 1050–59. PMLR.

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