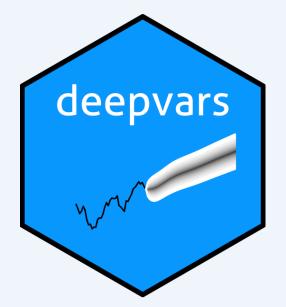
# Deep Vector Autoregression for Macroeconomic Data





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## Can we leverage the power of deep learning in VAR models?

We propose **Deep VAR**: a novel approach towards VAR that leverages the power of deep learning in order to model non-linear relationships.

#### Key contributions

- Simple methodology close in spirit to conventional benchmark
- Significant improvement in predictive performance
- Open source R package to facilitate reproducibility

## Less is more

#### A simple methodology

We developed our idea under the following premise: maximise performance of an existing and trusted framework under minimal intervention. We relax the assumption of linearity through one simple step: estimate each system equation through a recurrent neural network.

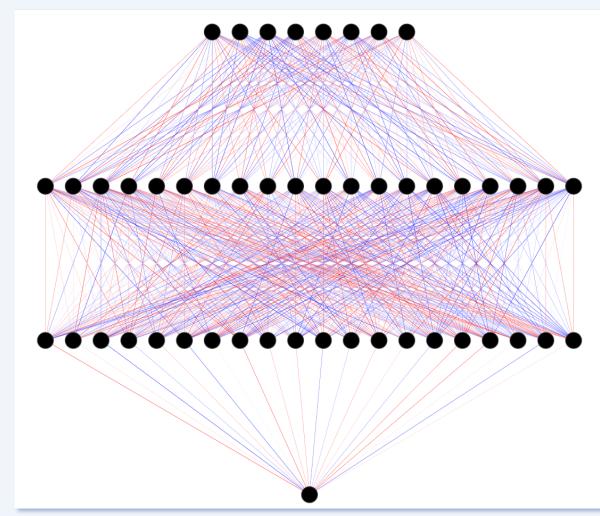


Figure 1: Neural Network Architecture. Inputs are lags of all variables. Output is variable of interest in time t.

## Significant performance gains

#### Empirical evidence

To evaluate our proposed methodology empirically we use the FRED-MD data base to collect a sample of monthly US data on output (IP), unemployment (UR), inflation (CPI) and interest rates (FFR). Our sample spans the period from January 1959 through March 2021.

## Significantly reduced in-sample error ...

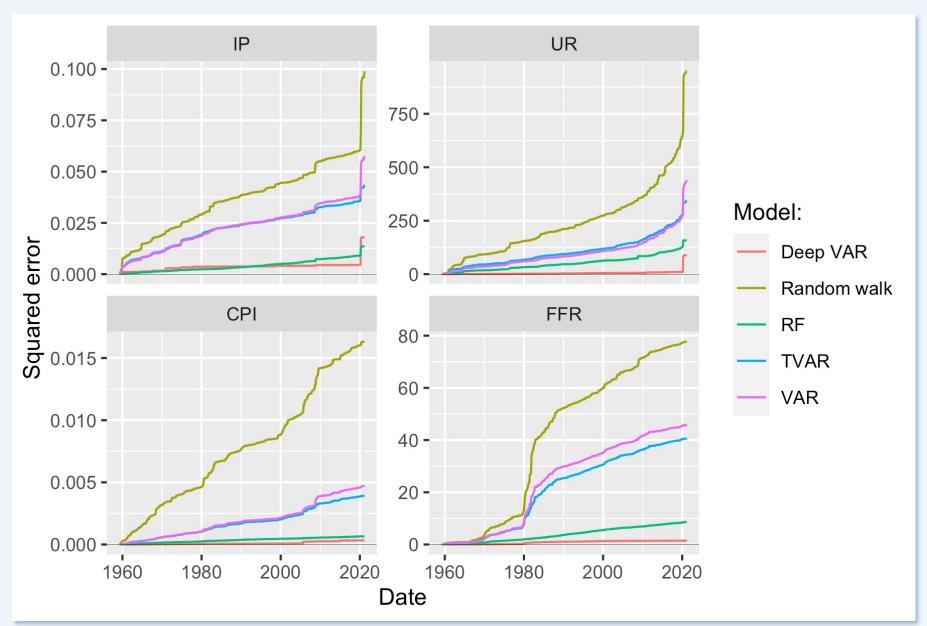


Figure 2: Comparison of cumulative loss over the entire sample period for Deep VAR and benchmarks.

## ... out-of-sample error ...

Variable	DVAR	VAR	Ratio (DVAR / VAR)
IP	0.00494	0.01484	0.33253
UR	0.94542	1.65170	0.57240
CPI	0.00231	0.00342	0.67642
FFR	0.17494	0.23974	0.72972

Table 1: Test root mean squared error (RMSE) for the two models across variables.

## ... and multi-step-ahead forecasting error.

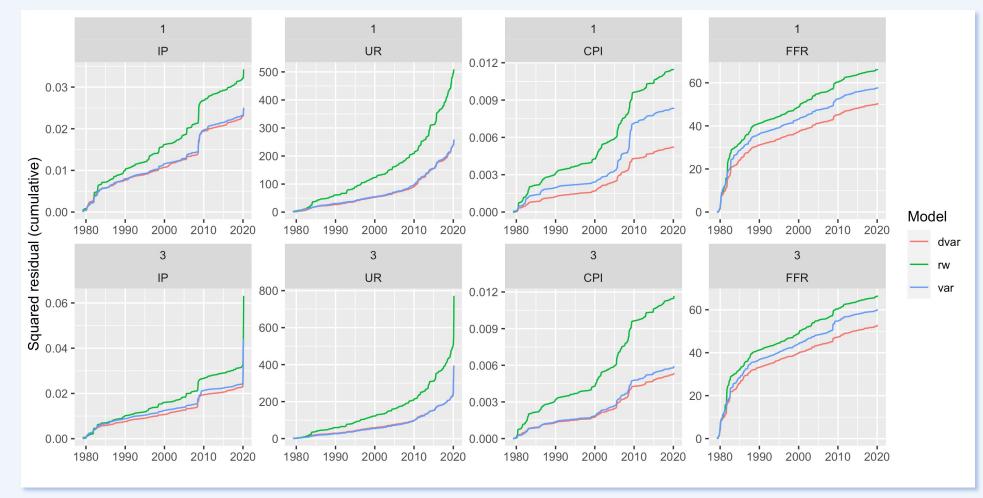


Figure 3: Cumulative rolling-window forecasting error for the 1-month and 3-month horizon.

## Have we merely scratched the surface?

#### Hyperparameter tuning

For a our baseline comparison we keep things simple: for example, we let the conventional VAR guide our search for optimal lag length. A short exercise in hyperparameter tuning demonstrates that the Deep VAR is less prone to overfitting with respect to the number of lags among other things.

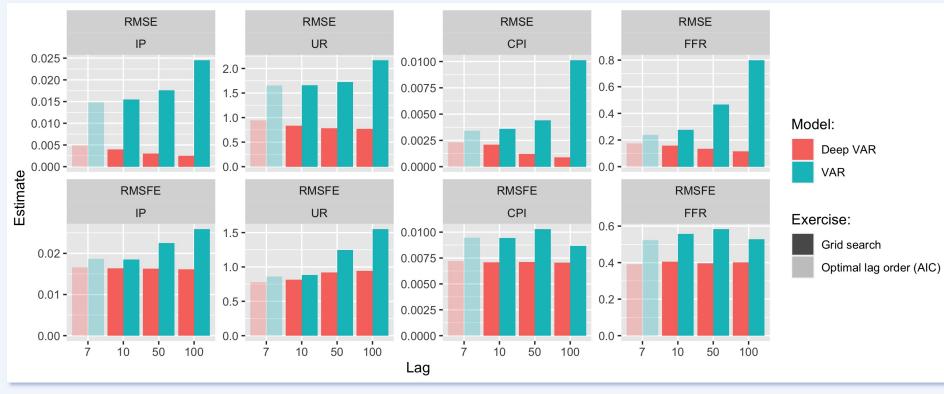


Figure 4:: Pseudo out-of-sample RMSE and RMSFE for VAR and Deep VAR across different lag choices.

## Where to go from here

#### Planned developments

- From deterministic to Bayesian deep learning: this should aid with interpretability and uncertainty quantification.
- Recent work by Daxberger et al. (2021) shows that Laplace
   Approximation is a promising way forward.
- Can the existing toolbox (IRFs, variance decomposition, policy counterfactuals, ...) be derived for Deep VAR?
- Add other benchmark models.
- Structural identification how to proceed? Verstyuk (2020) works with Cholesky decomposition as in conventional VAR.
- Deep VAR as a monitoring tool for detecting non-linearities?

#### References

Daxberger et al. (2021). "Laplace Redux-Effortless Bayesian Deep Learning.". In: Advances in Neural Information Processing Systems 34

Kilian and Luetkepohl (2017). "Structural Vector Autoregressive Analysis.". In: Cambridge University Press.

Verstyuk (2020). "Modeling Multivariate Time Series in Economics: From Auto-Regressions to Recurrent Neural Networks.". In: Available at SSRN 3589337.

