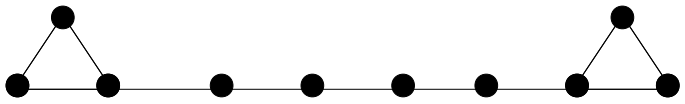
Networks: Concepts and Algorithms - BGSE 2021
Problem Set 2

Deadline: Friday, March 12.

Problem 1 Check whether the networks in Figures 14.19, 14.20, and 14.21 in Easley and Kleinberg, ch. 14, are strongly connected and aperiodic. For the ones that satisfy both properties, find the PageRank equilibrium values of all nodes (without the use of a computer; explain the steps you follow).

Problem 2 We define a clique as a subset of nodes in the network such that any pair of nodes in the clique is connected, i.e, it is a subset $S \subset N$ such that $g_{ij} = 0$ for every pair $i, j \in S$. A maximal clique is a clique that is maximal according to the previous property.

i) Find all the maximal cliques in the following network. Explain how can you find this list without making use of a computer.



ii) Consider Zachary's karate club dataset. With the use of a computer, find a list of all its maximal cliques. Find as well a clique comembership matrix, explaining how to interpret the entries in that matrix.

Problem 3 Solve all problems from Barabási, ch.9, except the initial one.

Problem 4 Consider the same network as in problem 2.

a) Apply the Girvan-Newman method of community detection and draw your resulting dendrogram. You should do this in two ways: with a computer, and without a computer, making computations by hand.

b) Use modularity to choose one particular layer in the dendrogram.

c) Apply an agglomerative method of community detection with Jaccard's distance and using *complete linkage* clustering. Compute the modularity in each step of the algorithm and check in which one modularity is maximal. Again, you have to do this in two different ways: with a computer, and making computations by hand.