**NETWORKS: CONCEPTS AND ALGORITHMS - BGSE 2021**
**Problem Set 1**

**Deadline:** You have time until Friday, March 5. I am going to upload a second problem before that date.

Problem 1 Let's consider here some properties of degrees in undirected and unweighted networks without self-loops. We are going to use the following notation: $n$ number of nodes; $L$ number of undirected links. Prove the following properties:

a) The number of nodes with odd degree is even.

b) There are always at least two nodes with same degree.

c) (You don't have to hand in a solution for this one.) Finally, here there is a curious math puzzle:

Shaking Hands Problem - Video

There is the solution in the same video but I hope you may find it intriguing and may want to give it a try before checking the solution.

Problem 2 Given the adjacency matrix $G$ of a directed and unweighted network, provide an interpretation for the entries in the matrices $GG^T$ and $G^TG$, where $G^T$ is the transpose of $G$. Make use of a small (not more than ten nodes) directed network to illustrate numerically the differences between both matrices.

Problem 3 Read the section on bipartite networks in Barabási, chapter 2, and solve the two problems on bipartite networks at the end of the chapter.

Problem 4 Consider the star network, in which there are $n$ nodes with node 1, the hub, connected to all other nodes, the spokes, and the spokes are only connected to the hub. There are no self-loops.

a) Without using matrices try to answer the following questions:

i) how many paths of length $k \geq 1$ there are starting at 1 and ending at 2?

ii) how many cycles of length $k \geq 1$ there are starting at 1 and ending at 1?

b) Write down the adjacency matrix $G$ of this network.

c) Compute a few powers of the adjacency matrix and obtain a general expression(s) for all powers $G^k$, $k \geq 1$.

d) Use the previous section to compute the Katz-Bonacich centrality of each node for general values of $\alpha$ and $\beta$.

e) Now compute again the Katz-Bonacich centrality of each node by directly solving the linear system equations that characterizes it, and check that you obtain the same result as in the previous section.

Problem 5 Consider a network with the following adjacency matrix

$$G = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

a) How many components there are in this network?

b) How many paths of length 7 there are from node 1 to node 5? And from node 1 to node 8?

c) Given parameters $\alpha$ and $\beta$, compute the Katz-Bonacich centrality of each node.

d) Order nodes according to Katz-Bonacich centrality for the particular case in which $\alpha = 2$ and $\beta = 0.25$. Would it change anything in this order if we change $\alpha = 2$ for some other strictly positive number?

e) Compute the eigenvector centrality of each node. You can use a computer to make computations but you cannot directly use some network analysis software with a build function to compute this directly. Explain the steps you follow.

Problem 6 Find the definition of diameter of a network. Then, answer the following questions:

a) Consider the following network, that we are going to call a circle: node 1 is connected to node $n$ and node 2, node 2 is connected to node 1 and node 3, node 3 is connected to node 2 and node 4,..., node $n$ is connected to node *n-1* and node 1. What is the diameter of that network? Is that network bipartite?

b) Find a network with largest possible diameter within the set of networks that have $n$ nodes and just one single component.

c) Find a network with smallest possible diameter within the set of networks that have $n$ nodes and just one single component.

d) Find an undirected network with one single component in which the diameter is 4 times larger than the average distance.

Problem 7 Find the dataset about jazz musicians, from the paper P.Gleiser and L. Danon , Adv. Complex Syst.6, 565 (2003). Using some software for network analysis:

a) Compute the degree centrality of each node.

b) Compute the eigenvector centrality of each node.

c) Compare the top 10 nodes according to the previous two centrality measures.

d) Compute the individual clustering coefficients of each node, and the average clustering coefficient. Compare this average clustering to the ratio $L/\binom{n}{2}$ of that network.