

**NETWORKS: MODELS AND APPLICATIONS**  
**Homework 1 - BGSE 2021**

**Deadline:** The section on social multipliers is due for Wednesday, May 5, before the beginning of the class. The report on the paper is due for Friday, May 7. I may upload new homework before these dates.

## On Social Multipliers

Let's explore a bit more what happens with social multipliers in three different models.

### A Linear-in-Sums Model

Consider the game with strategic complementarities in an undirected network with adjacency matrix  $G$  we studied in class, in which utilities were

$$u_i(e_i, e_{-i}) = \alpha e_i - \frac{1}{2} e_i^2 + \beta \sum_{j=1}^n g_{ij} e_i e_j \quad i = 1, \dots, n.$$

We continue assuming that  $\alpha > 0$  and  $\beta > 0$ .

- a) Suppose first, as a benchmark, that we study an empty network, i.e.  $g_{ij} = 0$  for all  $i, j$ . If  $\alpha$  increases by  $\Delta$ , what is the effect in equilibrium of that change in individual and aggregate efforts?
- b) Consider now a complete network, i.e.  $g_{ij} = 1$  for all  $i \neq j$  (let's assume there are no self-loops, so  $g_{ii} = 0$  for all  $i$ ). If  $\alpha$  increases by  $\Delta$ , what is the effect in equilibrium of that change in individual and aggregate efforts?
- c) Consider now a star network, with node 1 being the hub and the rest of nodes being spokes, i.e.  $g_{1j} = g_{j1} = 1$  for all  $j = 2, \dots, n$  and  $g_{ij} = 0$  for all other pairs. If  $\alpha$  increases by  $\Delta$ , what is the effect in equilibrium of that change in individual and aggregate efforts?

### A Linear-in-Means Model

Suppose now we are considering a linear-in-means model. Usually these immediately refer to an econometric model of the kind

$$y_i = \mu + \beta \sum_{j=1}^n \tilde{g}_{ij} y_j + \gamma c_i \delta \sum_{j=1}^n \tilde{g}_{ij} c_j + \varepsilon_i \quad i = 1, \dots, n$$

with  $\tilde{g}_{ij} = g_{ij}/d_i$ . Since the network is undirected  $d_i$  refers at the same time to the in- and out- degree of node  $i$ .

- a) Show that we can understand the linear-in-means structure as, for example, the best-replies of a game with utility function

$$u_i(y_i, y_{-i}) = \alpha_i + \delta \sum_{j=1}^n \tilde{g}_{ij} y_j - \frac{1}{2} y_i^2 + \beta y_i \sum_{j=1}^n \tilde{g}_{ij} y_j \quad i = 1, \dots, n.$$

- b) Study the effects in individual and aggregate equilibrium efforts of an increase  $\Delta$  in  $\mu$  in the empty, complete, and star network. Compare it to the case of a linear-in-sums model from the previous section.
- c) Study the effects in individual and aggregate equilibrium efforts of a common increase  $\Delta$  in the characteristic  $c_i$  of all nodes in the empty, complete, and star network.

## A Model of Peer Pressure

Strategic complementarities don't follow necessarily from the kind of explanation we gave in class (*if my friend studies more it is easier for me to study more*). An alternative explanation could be that students face *peer pressure*: if my friends study more (resp. less) I feel the pressure to behave like them and I want to study more (resp. less). We can model this alternative situation with, for example, utility functions of the form:

$$u_i(e_i, e_{-i}) = \alpha e_i - \frac{1}{2} e_i^2 - \kappa (e_i - \bar{e}_i)^2 \quad i = 1, \dots, n.$$

where  $\bar{e}_i = \sum_{j=1}^n \tilde{g}_{ij} e_j$  (same meaning of  $\tilde{g}_{ij}$  as in previous section) and  $\alpha > 0$  and  $\kappa > 0$ .

- a) Explain why this utility function represents the peer pressure effect.
- b) Study the effects in individual and aggregate equilibrium efforts of an increase  $\Delta$  in  $\alpha$  in the empty, complete, and star network. Compare it to the case of a linear-in-sums model.

## Reading on Methodology

Read the paper (I don't expect a fully detailed reading, just a reading that is deep enough for you to get the main messages from the paper)

Alan Griffith, (2020) Name Your Friends, but Only Five? The Importance of Censoring in Peer Effects Estimates using Social Network Data , working paper

and write a short report on it. This report should include a brief explanation in your own words of the main ideas/results of the paper and your thoughts about it.