

Deterministic models and optimization 2020

Homework on Combinatorial Optimization

Each algorithm proposed should contain pseudo-code briefly commented; a brief proof of correctness; and a derivation of the complexity in terms of the number of operations.

1. Let a_1, \dots, a_n be a sequence of distinct real numbers. An inversion is a pair of indexes $i < j$ such that $a_i > a_j$. Design an algorithm that, given a sequence of n numbers, computes the number of inversions of the sequence in $O(n \log n)$ time. The size of the input is n .
2. Given two strings of text X and Y , there we wish to measure by how much X and Y differ. Consider the following three operations on a string:
 - D: Deletion of a character.
 - I: Insertion of a character.
 - S: Substitution of a character.

The edit distance $d(X, Y)$ is the minimum number of operations $\{D, I, S\}$ needed to perform on X to produce Y .

- (a) Design an efficient algorithm that, given strings X and Y , computes the edit distance between X and Y . The algorithm should also provide the optimal sequence of operations transforming X into Y . The size of the input is $|X| + |Y|$.
- (b) Modify the previous algorithm with a penalty cost function: operations D and I have unit cost 2, whereas operation S has unit cost 1.