STOCHASTIC MODELS AND OPTIMIZATION

Gergely Neu
(Univ. Pompeu Fabra)
Hrvoje Stojic
(University College London / Prowler.io)

INTRODUCTION

Gergely Neu [ˈgɛrgɛj nɔɣ] / Gergo / Gergő

2003-2008: MSc in Electrical Engineering
Budapest Univ. of Technology and Economics

2008-2013: PhD in Computer Science
Budapest Univ. of Technology and Economics
(12 months at University of Alberta, Canada)

2013-2015: INRIA Lille, France

2015-: UPF DTIC

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2013-2015: postdoc @ INRIA Lille, France

2015-: postdoc / asst. prof. / etc. @ UPF DTIC

Working in reinforcement learning since 2006

COURSE HISTORY

–2019

- focus on Dynamic Programming
- applications in management science
- by Mihalis Markakis

- focus on Reinforcement Learning
- from the broader (?) perspective of contemporary AI / Machine Learning

2020–

THE COURSE SINCE 2020

Two main parts:

Reinforcement Learning
in Markov decision processes
by Gergely

Bandit problems
by Hrvoje

MORE SPECIFICALLY…

1. Reinforcement learning preliminaries
2. Dynamic programming basics
3. Multi-armed bandits
4. Multi-armed bandits: extensions and applications
5. Contextual multi-armed bandits
6. Bayesian optimization and contextual bandits applications
7. Basic RL algorithms
8. Foundations of deep RL
9. Regularization in RL
10. Exploration vs. exploitation in RL

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by Hrvoje

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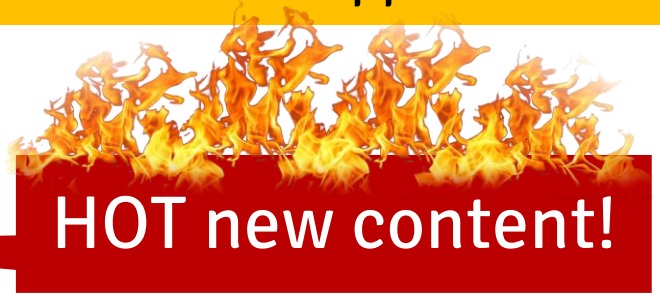
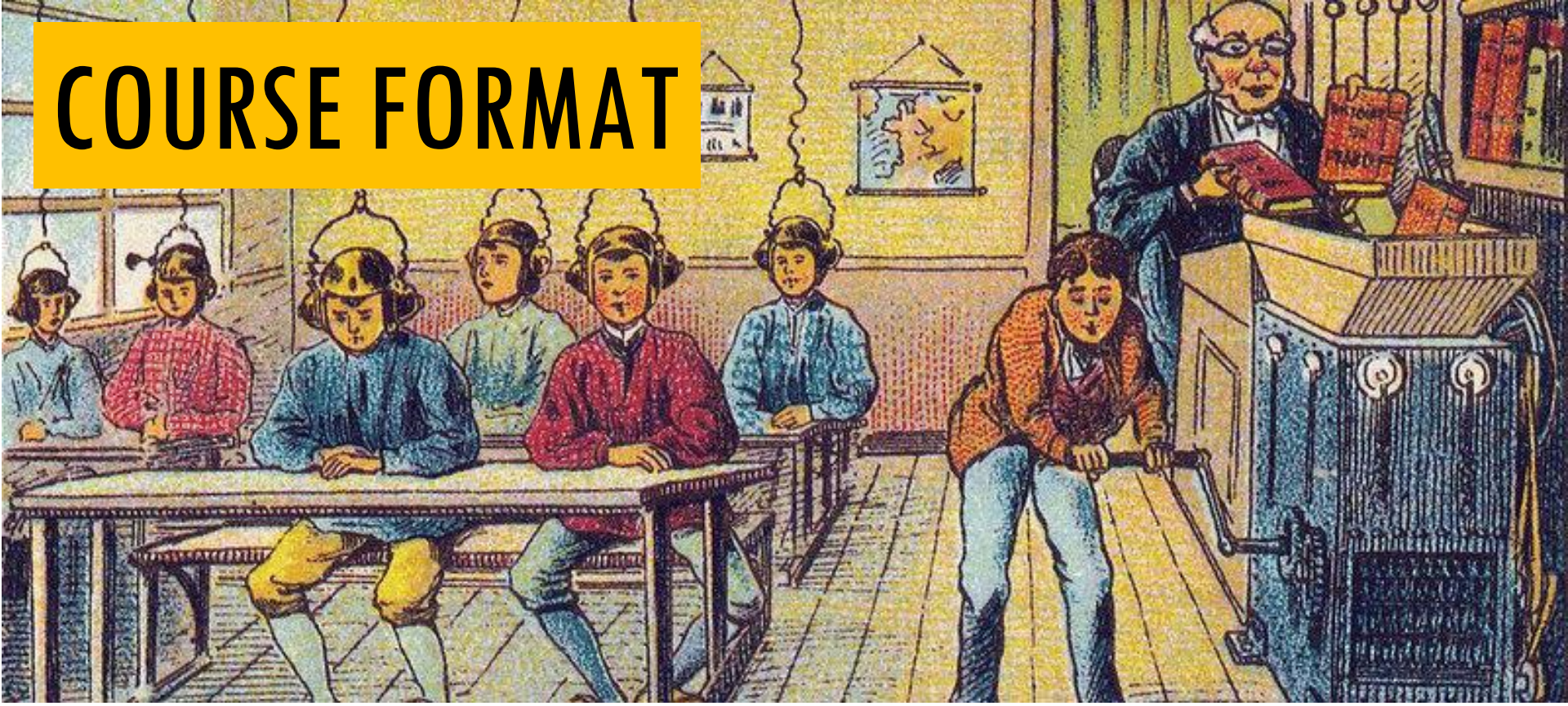
by Hrvoje



COURSE FORMAT



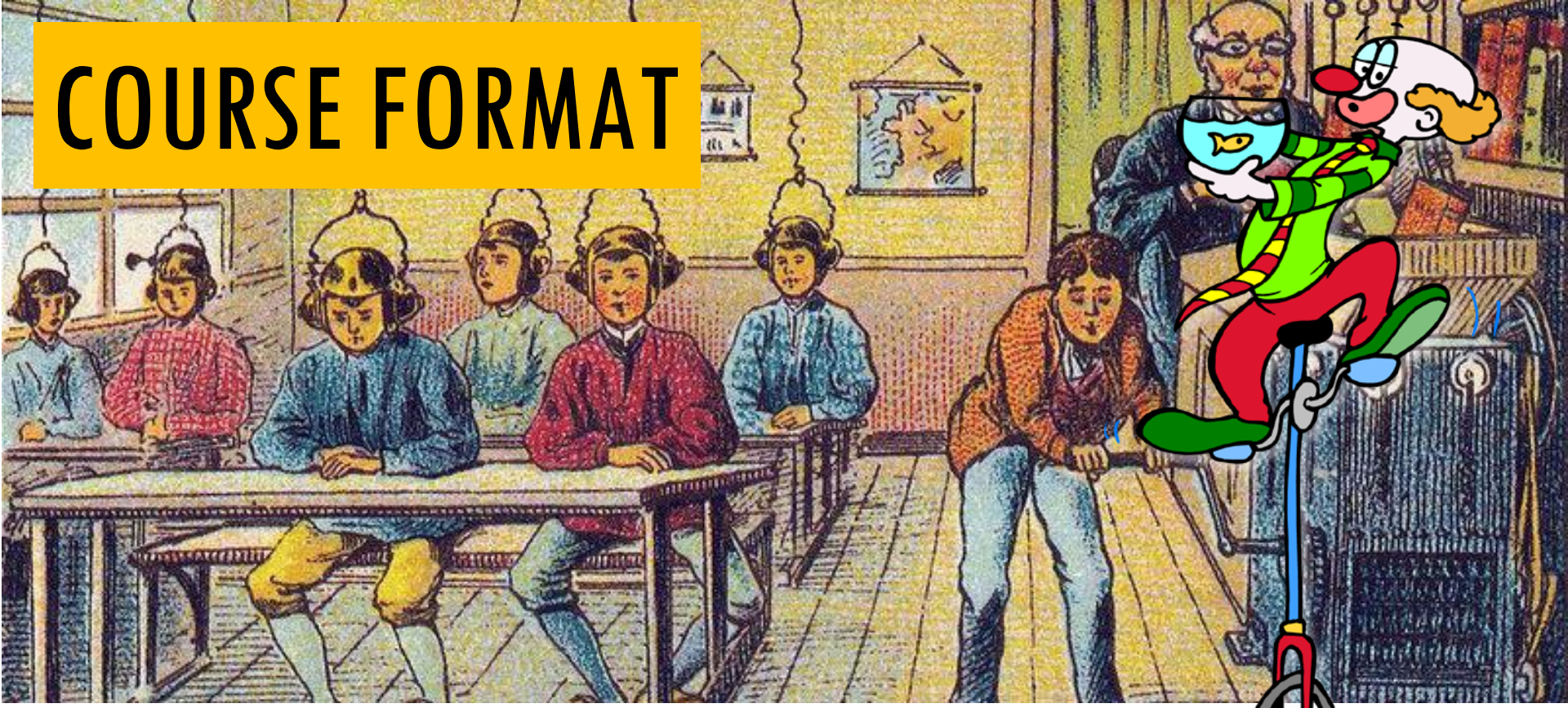
Face-to-face with Gergely (hopefully)

Online with Hrvoje

For further questions:

- gergely.neu@gmail.com
- hrvoje.stojic@protonmail.com

COURSE FORMAT



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- hrvoje.stojic@protonmail.com

COURSE REQUIREMENTS

50% of score: problem sets

- individual work
- 3 from Gergely, 2 from Hrvoje

50% of score: final project

- programming (e.g., implementing an RL algorithm on a benchmark) or literature review
- in groups of at most 3 students

NECESSARY BACKGROUND

Mathematics:

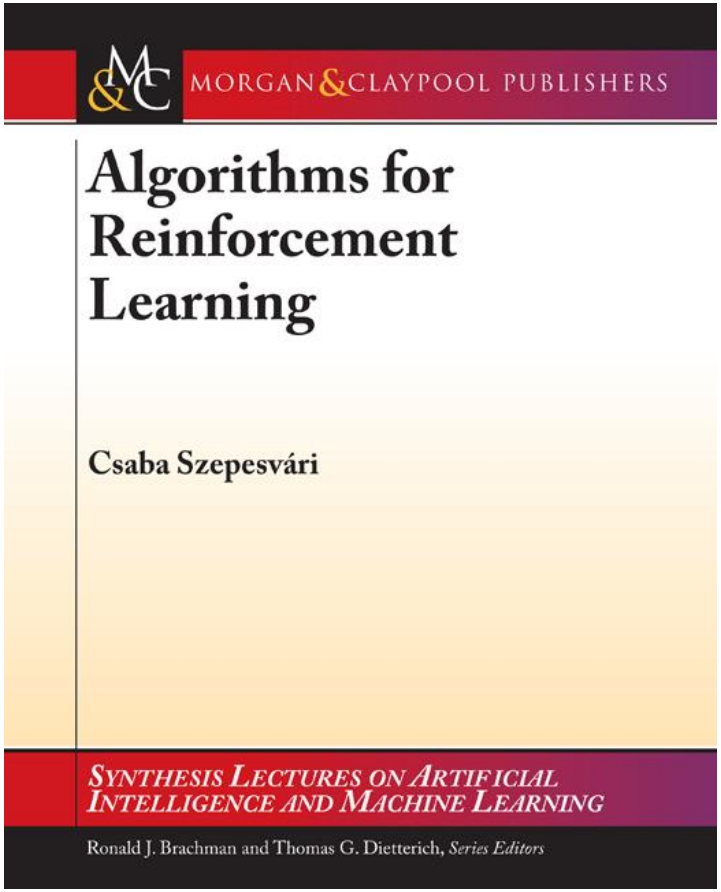
- Important: basic probability, linear algebra
- Useful: stochastic processes, multivariate calculus, constrained optimization…
- Advice: prepare for tons of definitions!

Coding:

- Problem sets: simple coding exercises
- Final project: more complex, but there's a lot of good material online

RECOMMENDED READING ON RL

- Richard Sutton and Andrew Barto (2018): "Reinforcement Learning: An Introduction (2nd edition)"
  - For an enjoyable (but not very rigorous) introduction
- Dimitri Bertsekas (2012): "Dynamic Programming and Optimal Control"
  - For a rigorous treatment of the basics
- Csaba Szepesvári (2012): "Algorithms for RL"
  - For a rigorous description of basic RL algorithms



WANT TO LEARN MORE?

Edit profile

RL Theory Virtual Seminars

@RLtheory

Virtual seminar series featuring the latest advances in theoretical reinforcement learning. Seminars (approximately) every Tuesday at 5pm UTC.

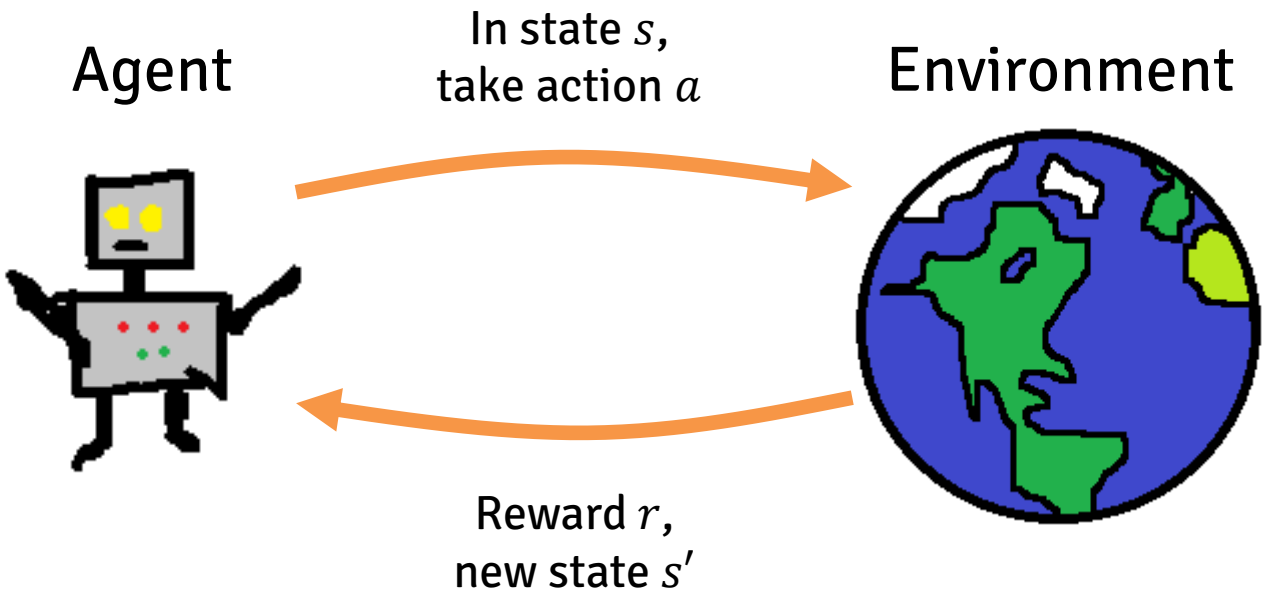
sites.google.com/view/rltheorys... Joined April 2020

0 Following **3,003** Followers

REINFORCEMENT LEARNING

Gergely Neu
Univ. Pompeu Fabra

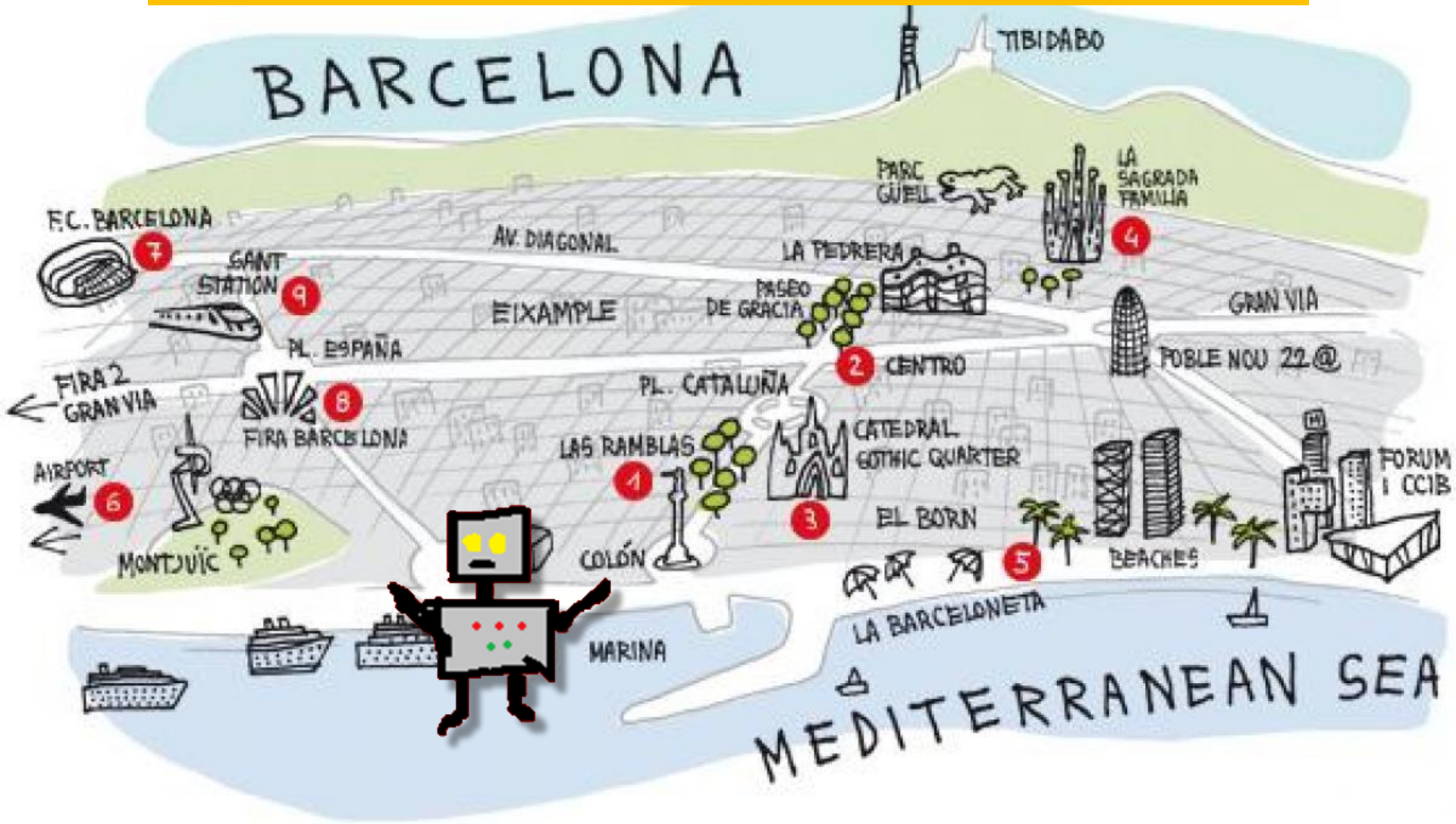
WHAT IS REINFORCEMENT LEARNING?



Learning to

- maximize reward
- in a reactive environment
- under partial feedback

RL EXAMPLE 0.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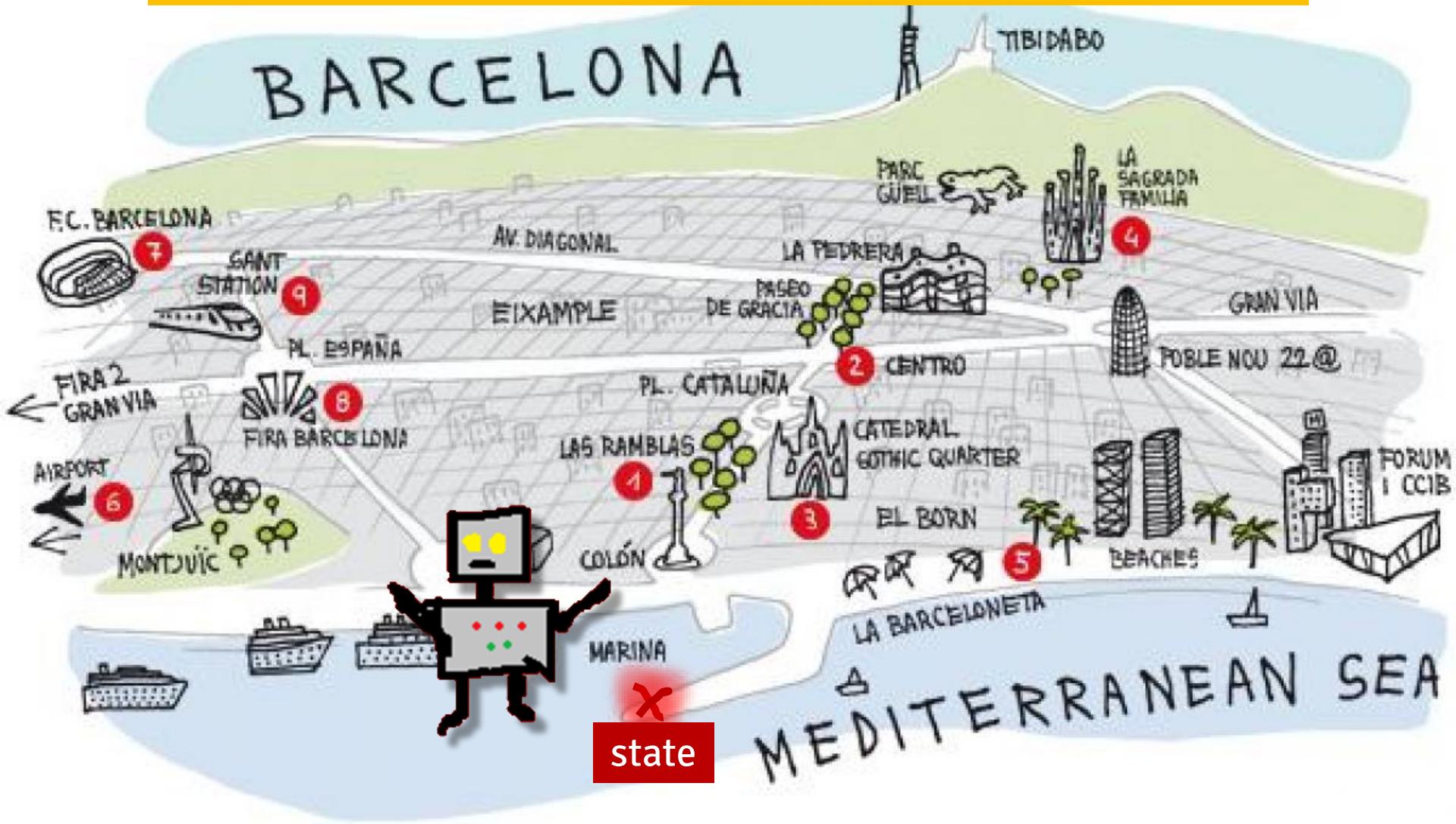
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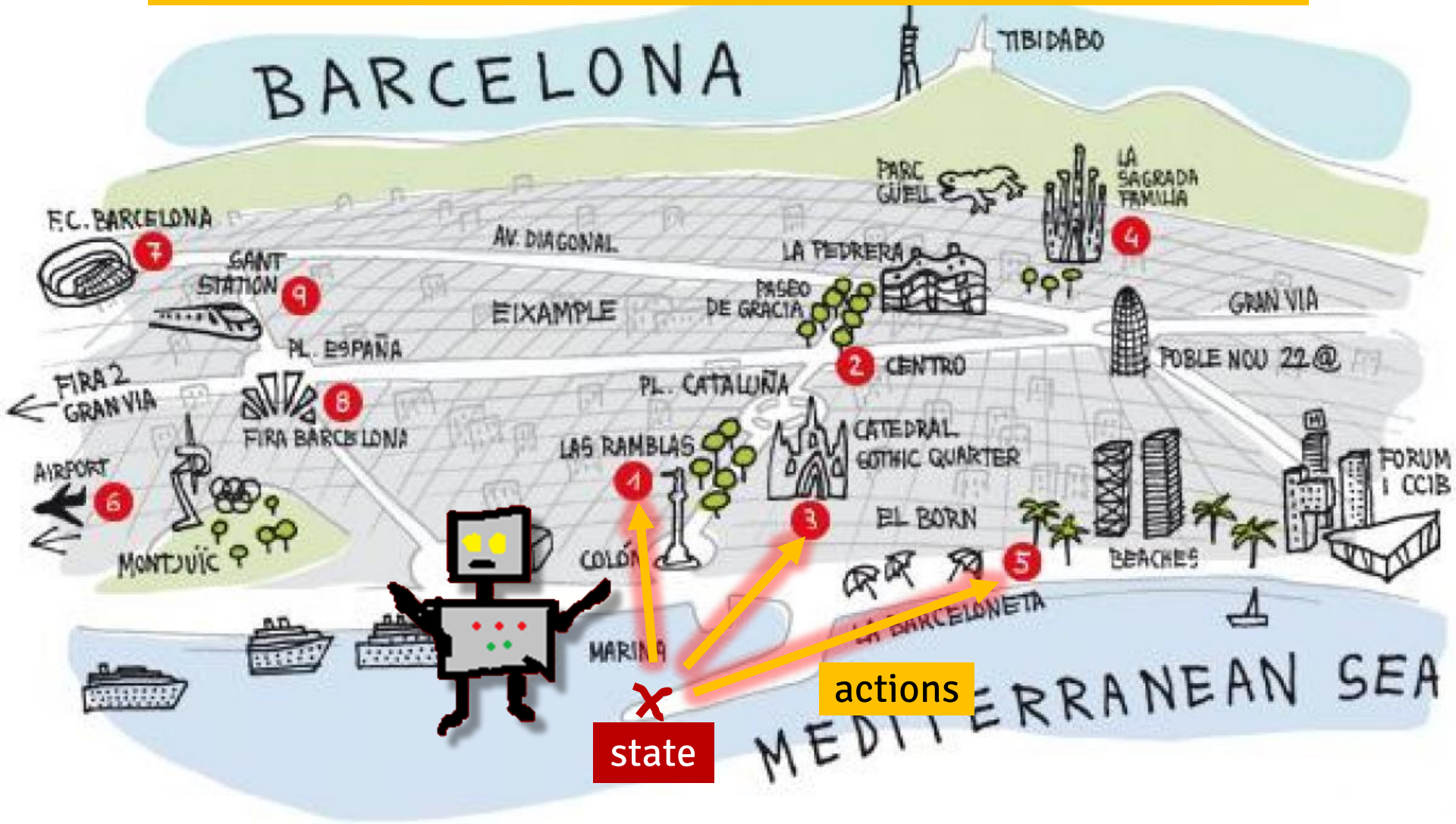
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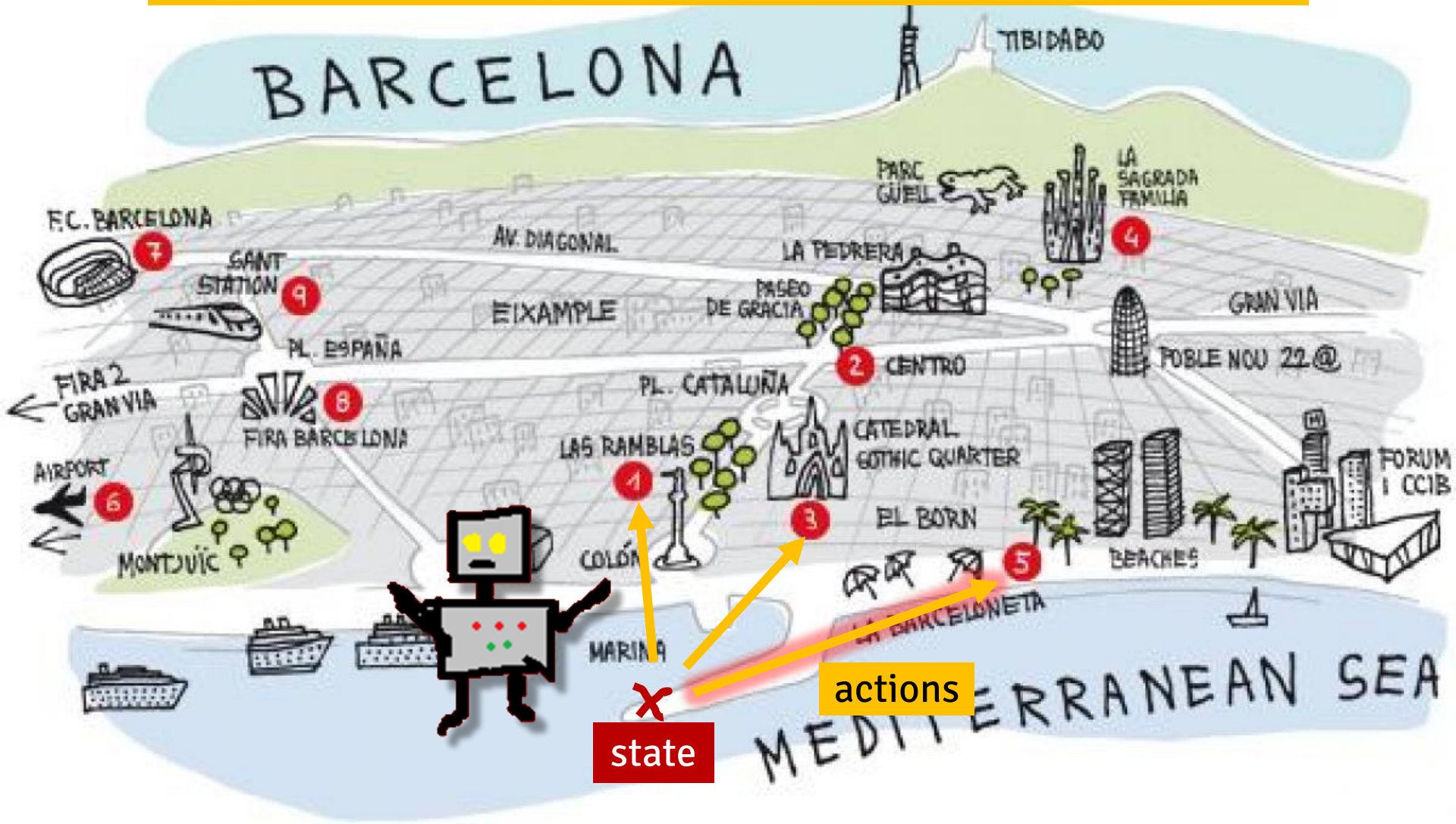
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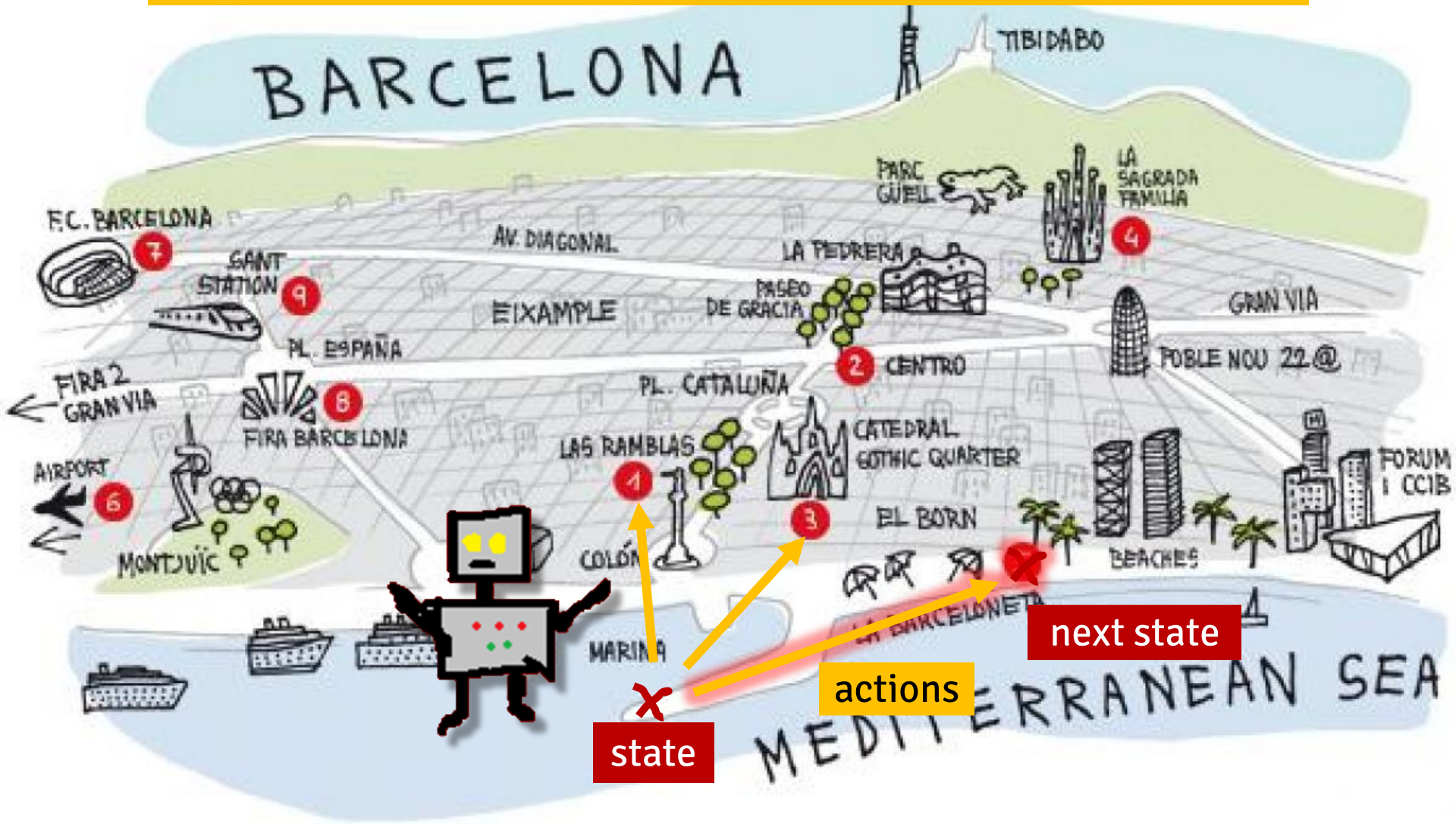
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RL EXAMPLE 0.


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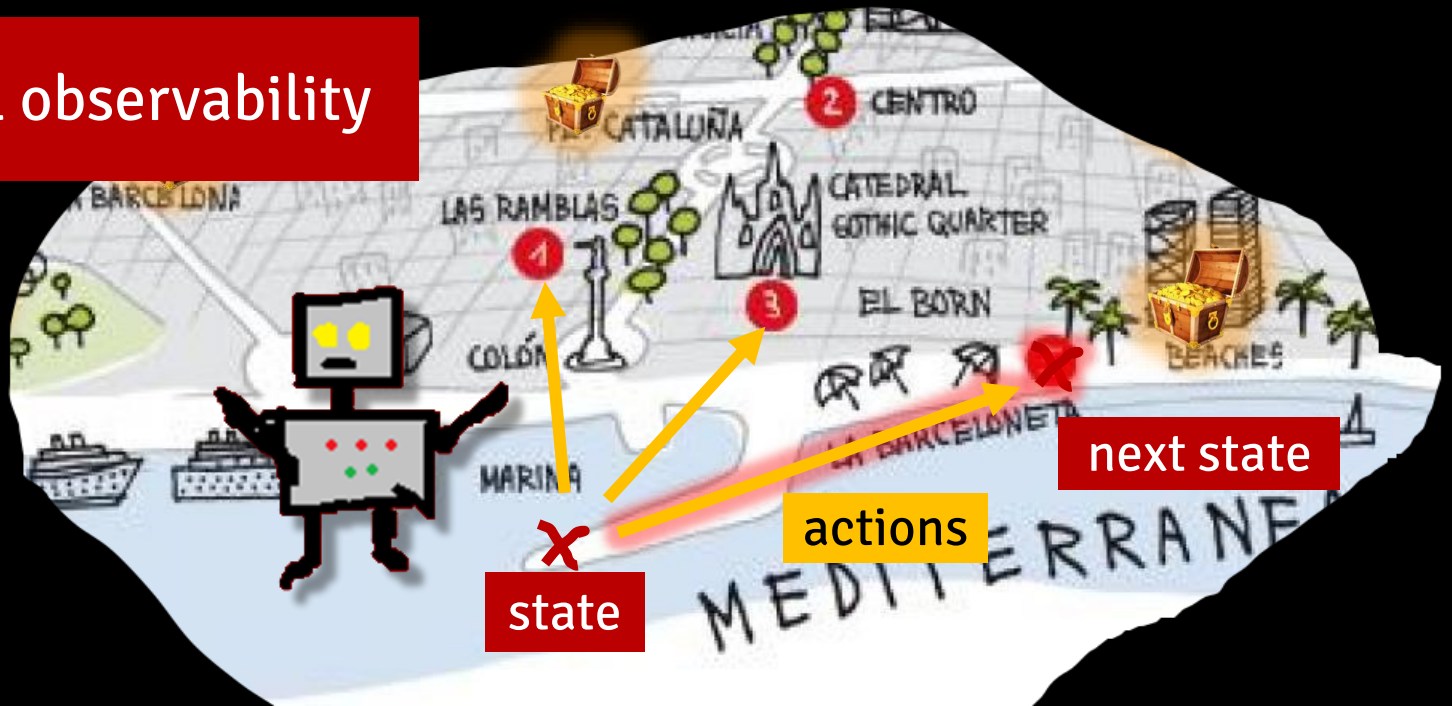
RL EXAMPLE 0.

partial observability



RL EXAMPLE 0.

reward?

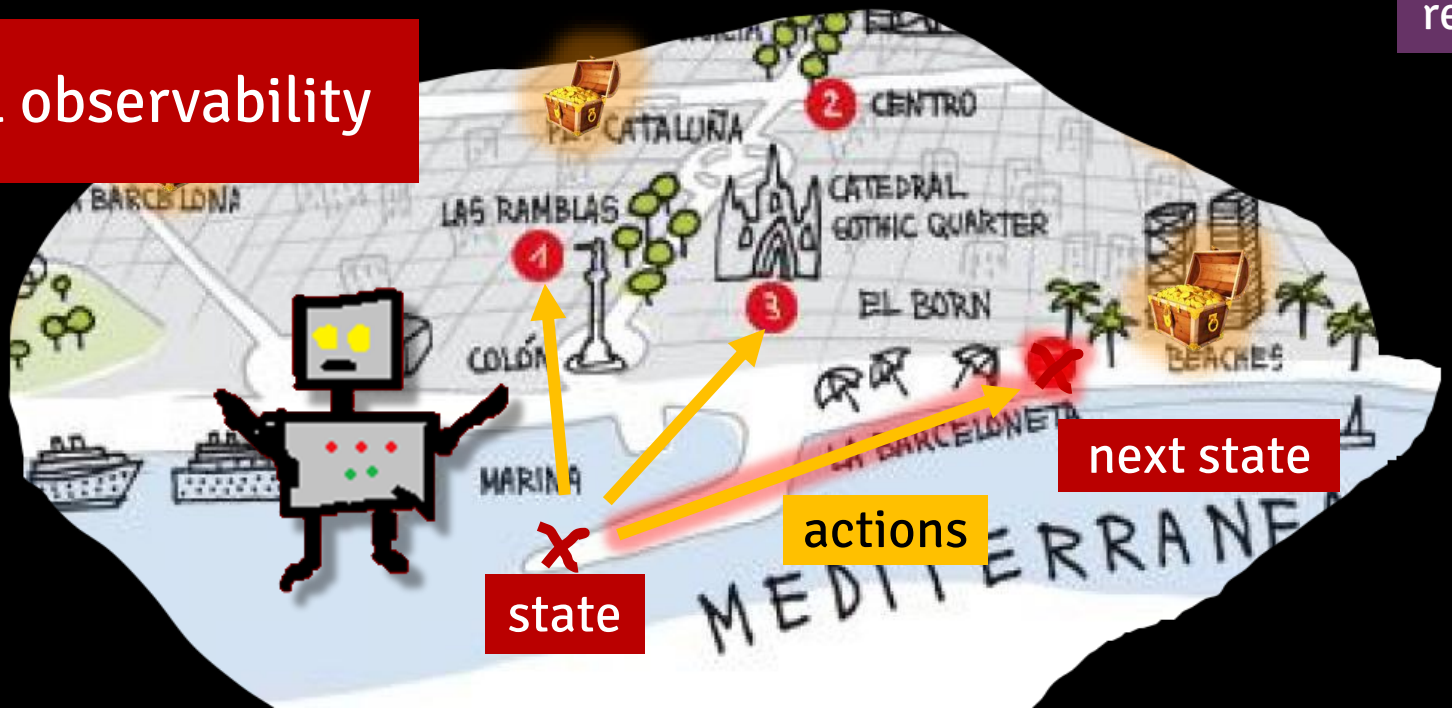
reward?

reward?

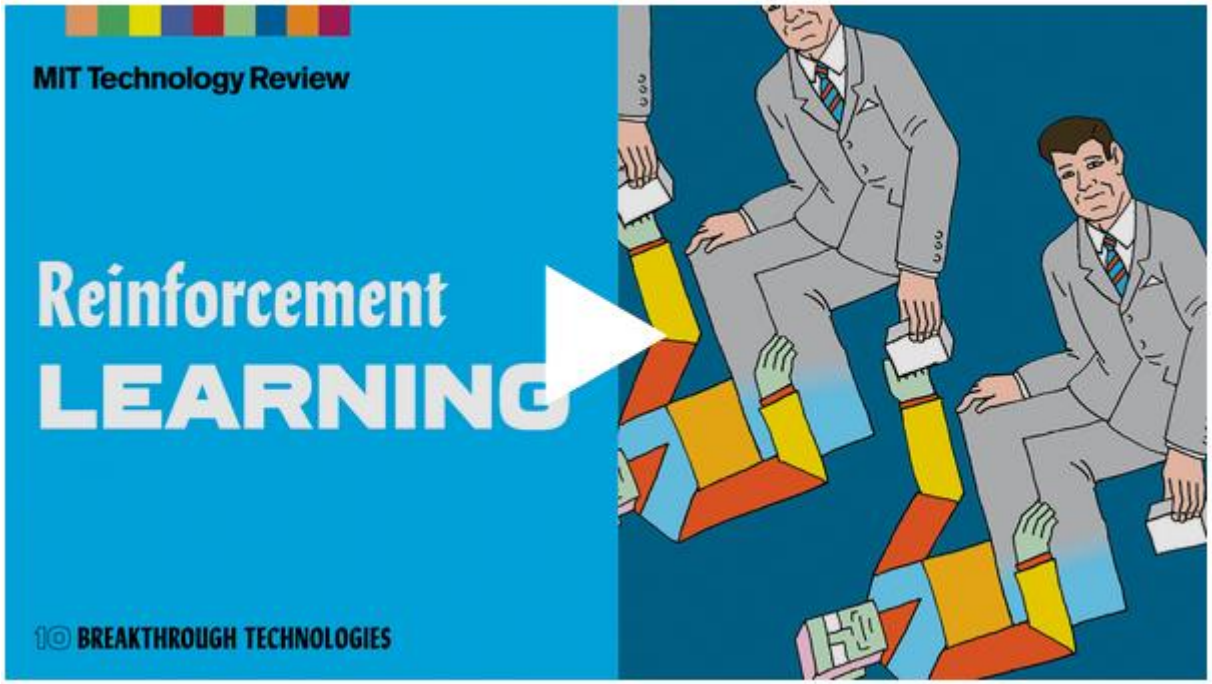
reward?

reward?

partial observability



WHY SHOULD I CARE?



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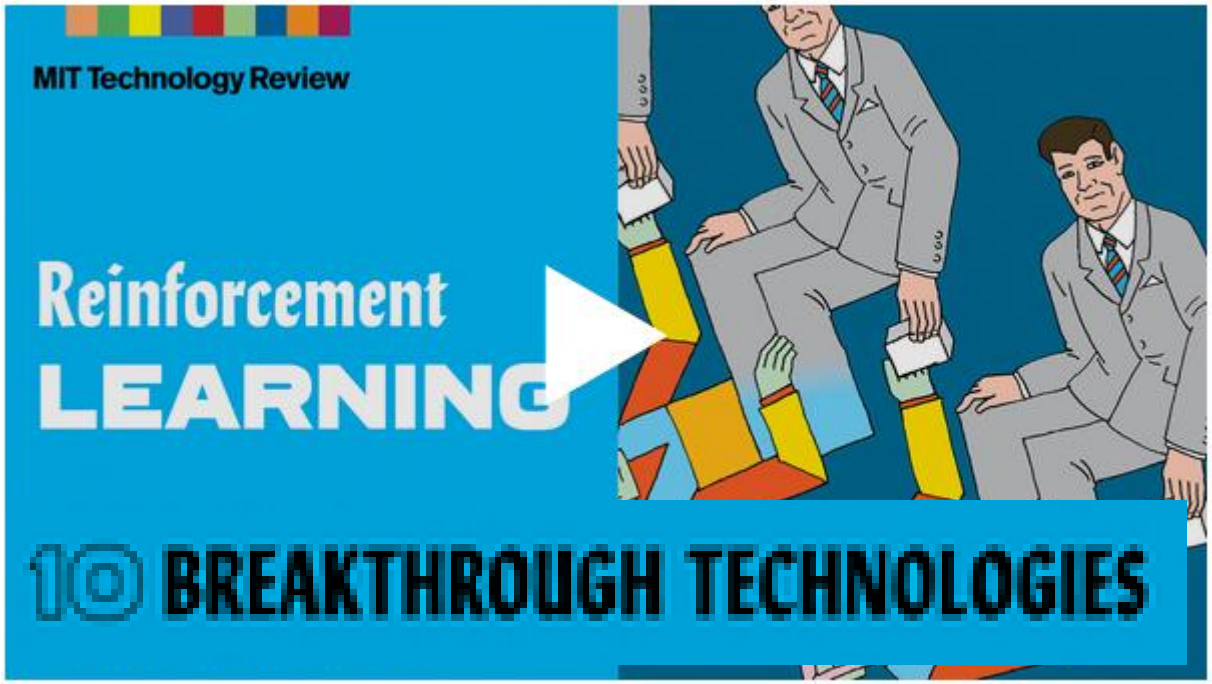

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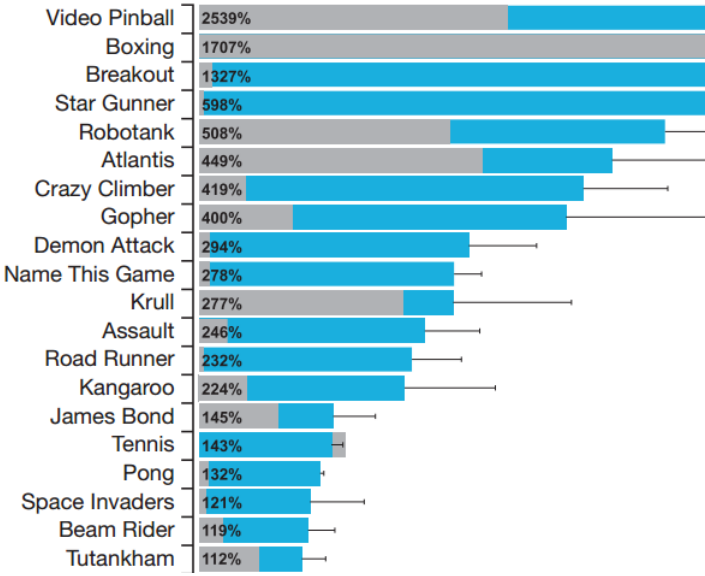
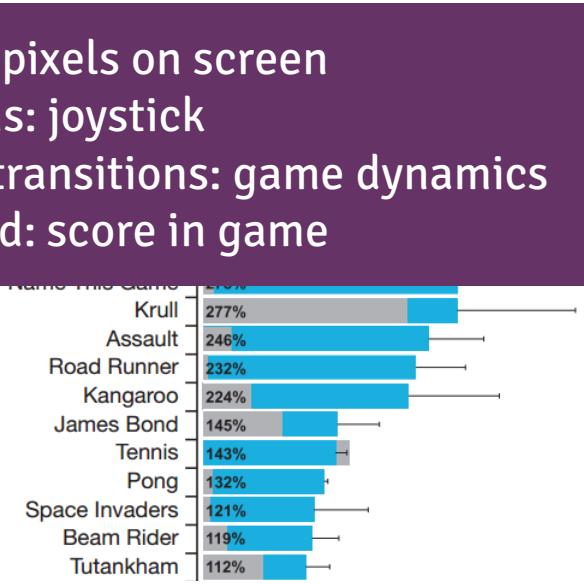


WHY SHOULD I CARE?

- State: pixels on screen
- Actions: joystick
- State transitions: game dynamics
- Reward: score in game

Breakthrough in Atari game playing



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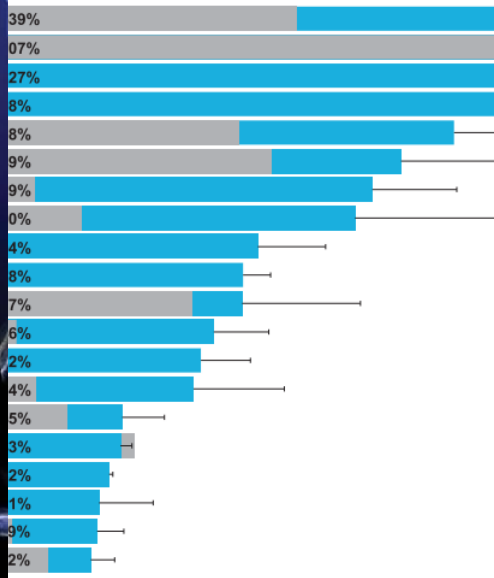
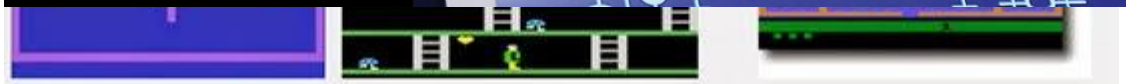


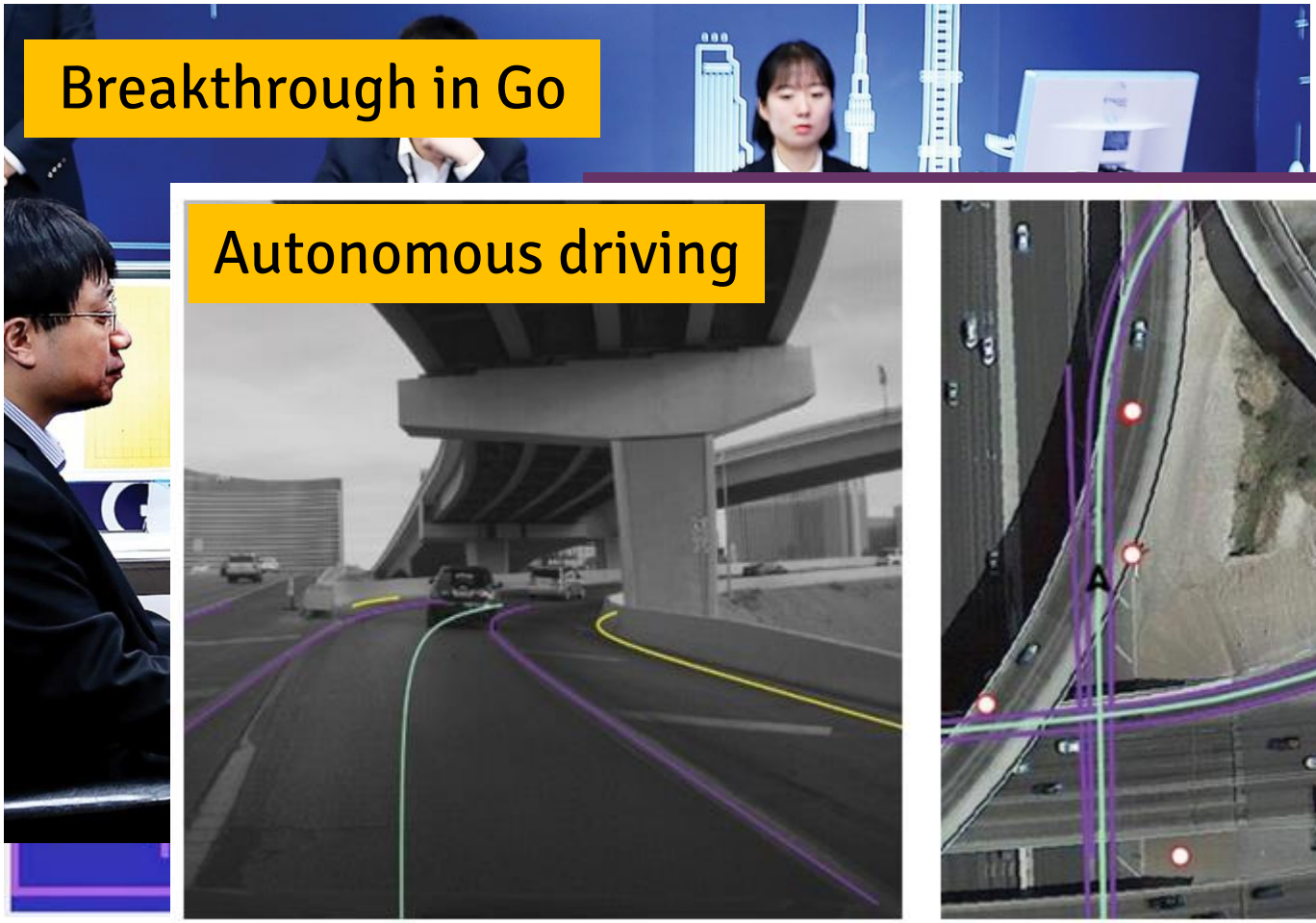
WHY SHOULD I CARE?

- State: stones currently on board
- Actions: place stone on board
- State transitions: own move + adversary's move
- Reward: +1 for winning the game



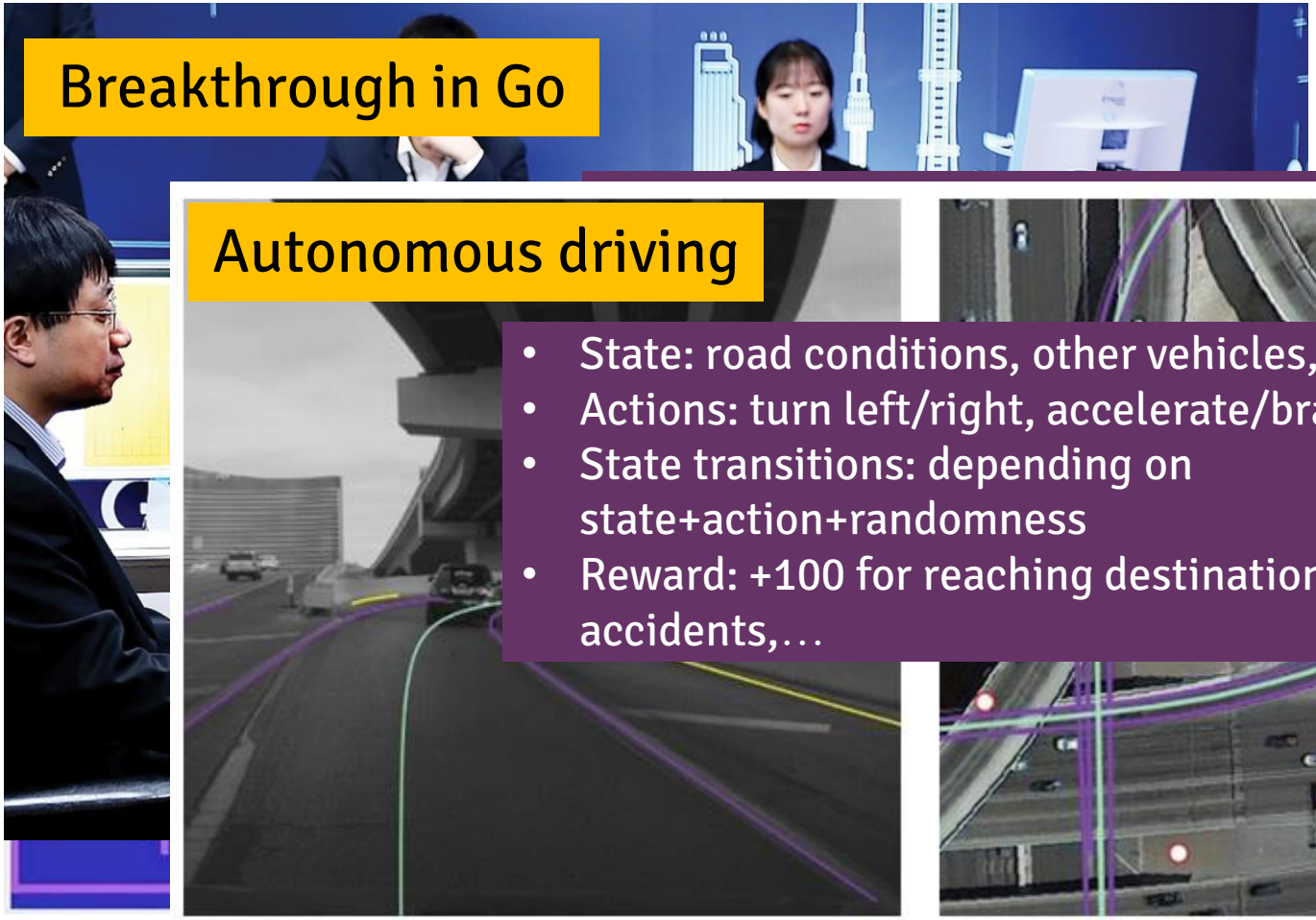
WHY SHOULD I CARE?

Breakthrough in Go



Autonomous driving

WHY SHOULD I CARE?

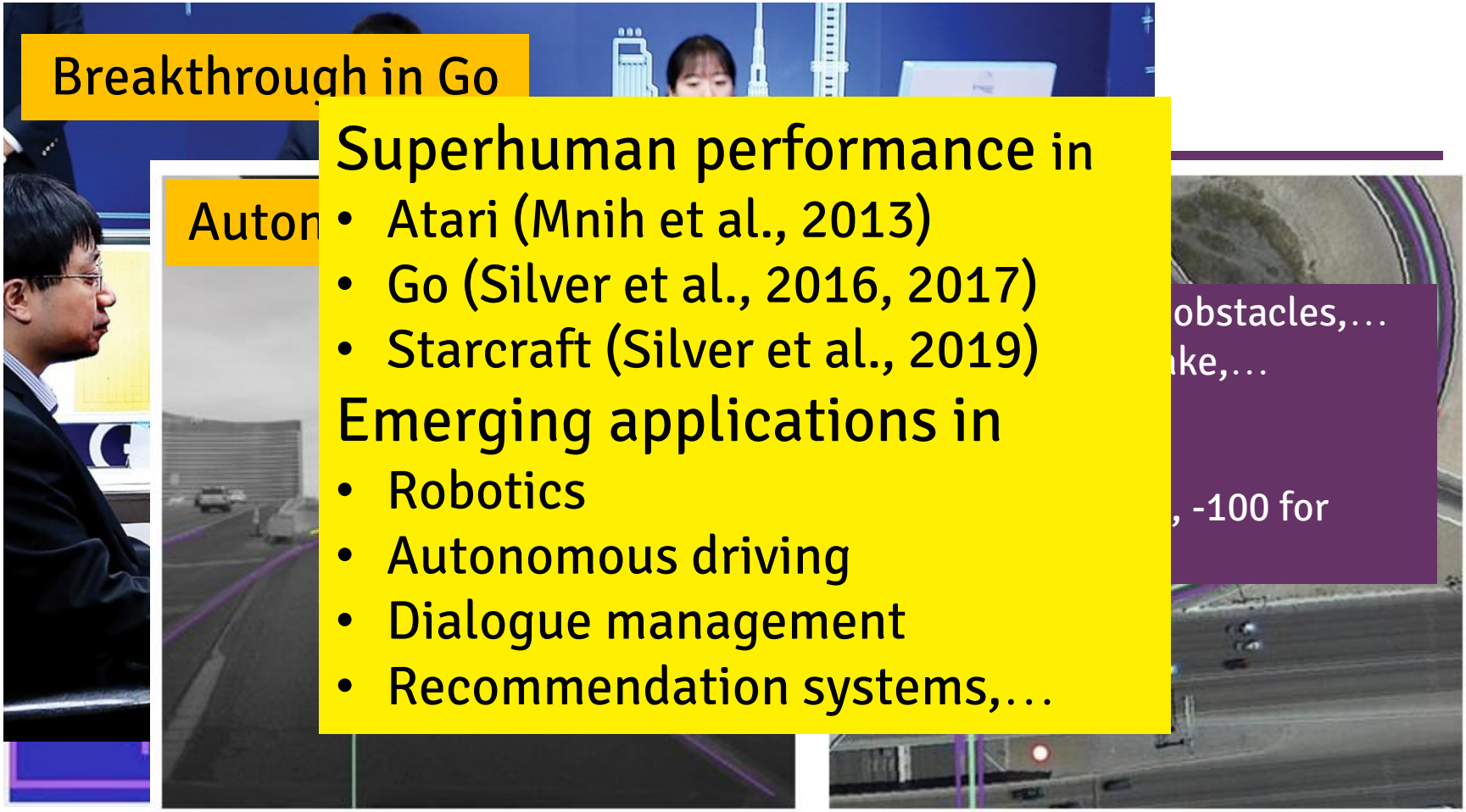


Breakthrough in Go

Autonomous driving

- State: road conditions, other vehicles, obstacles,…
- Actions: turn left/right, accelerate/brake,…
- State transitions: depending on state+action+randomness
- Reward: +100 for reaching destination, -100 for accidents,…

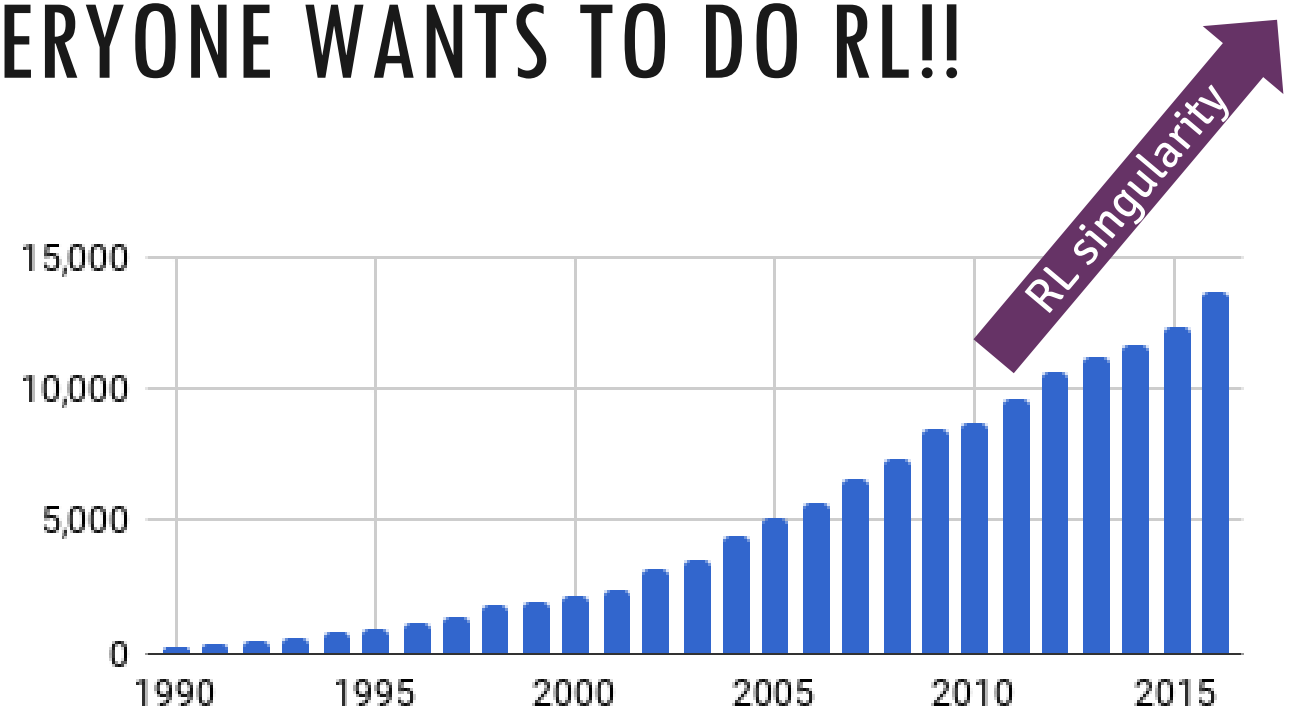
WHY SHOULD I CARE?



Superhuman performance in
- Atari (Mnih et al., 2013)
- Go (Silver et al., 2016, 2017)
- Starcraft (Silver et al., 2019)

Emerging applications in
- Robotics
- Autonomous driving
- Dialogue management
- Recommendation systems,…

EVERYONE WANTS TO DO RL!!



# of RL papers / year

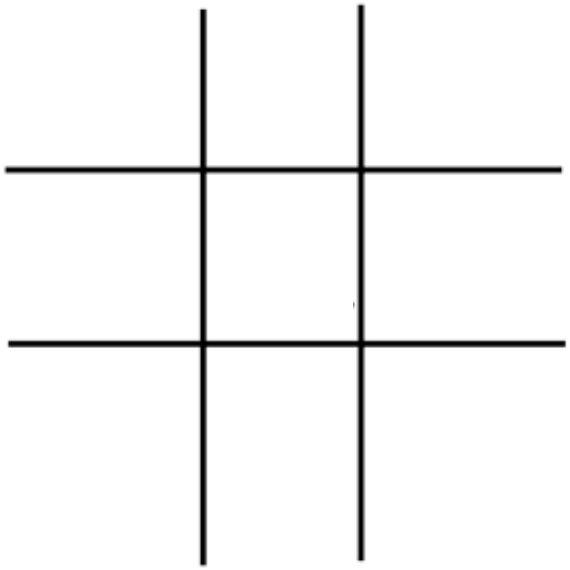
Facebook AI Research

DeepMind

OpenAI

Microsoft Research

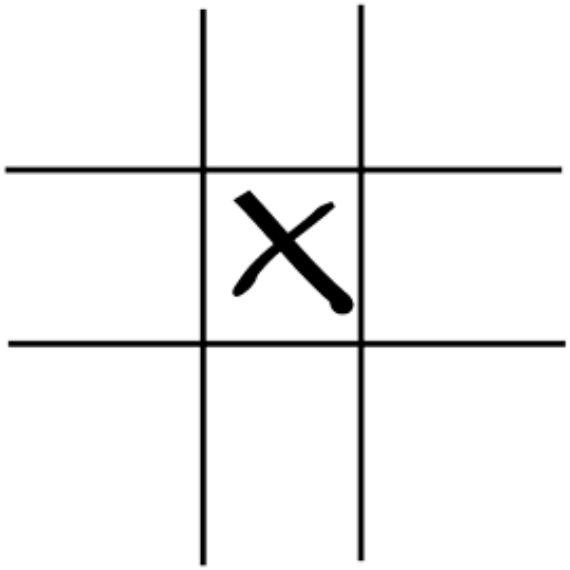
A CONCRETE EXAMPLE: TIC-TAC-TOE



In a sequence of games

- observe state of the board

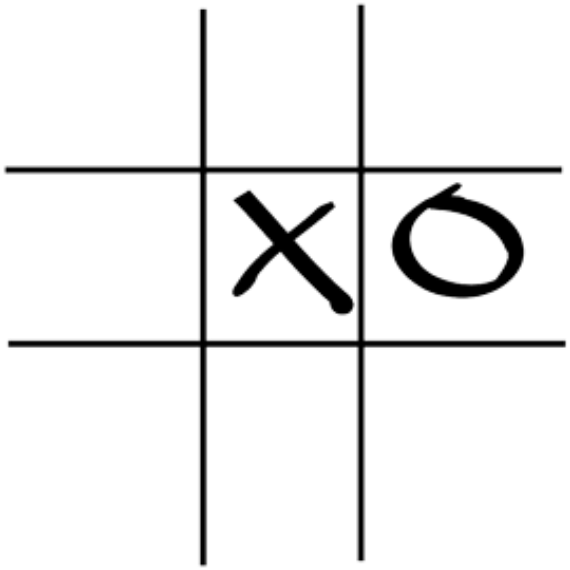
A CONCRETE EXAMPLE: TIC-TAC-TOE



In a sequence of games

- observe state of the board
- take action

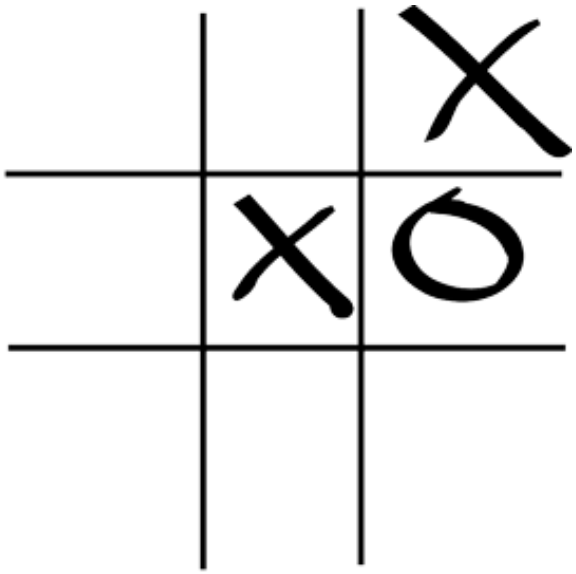
A CONCRETE EXAMPLE: TIC-TAC-TOE



In a sequence of games

- observe state of the board
- take action
- observe opponent's move

A CONCRETE EXAMPLE: TIC-TAC-TOE



In a sequence of games

- observe state of the board
- take action
- observe opponent's move
- … repeat…

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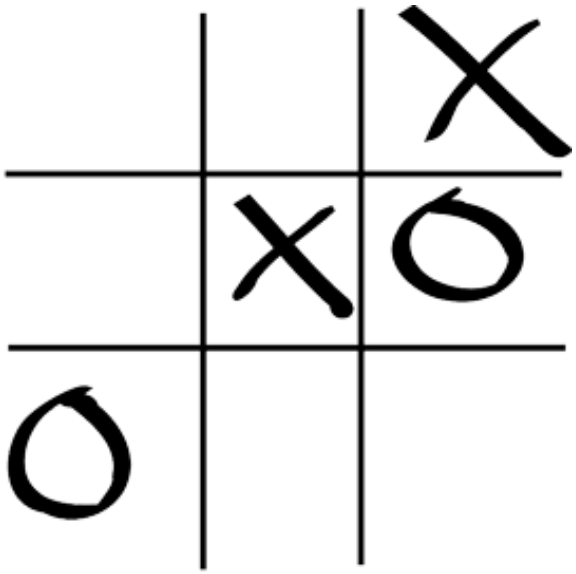

In a sequence of games

- observe state of the board
- take action
- observe opponent's move
- … repeat…

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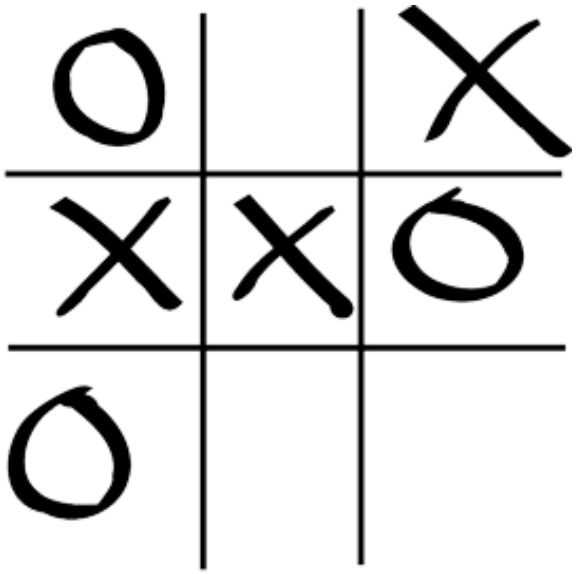

In a sequence of games

- observe state of the board
- take action
- observe opponent's move
- … repeat…

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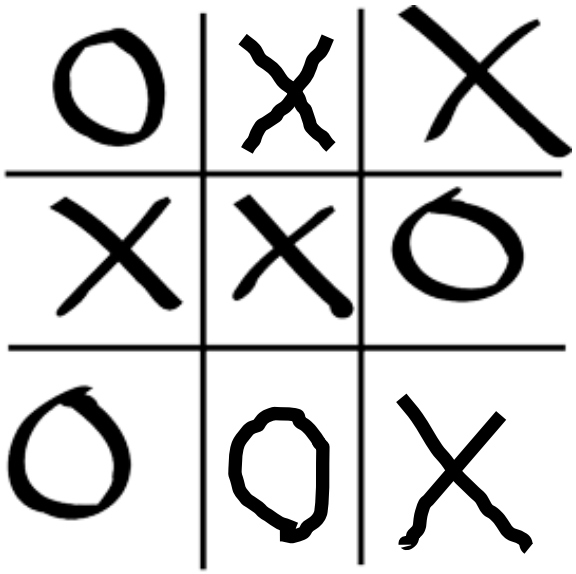
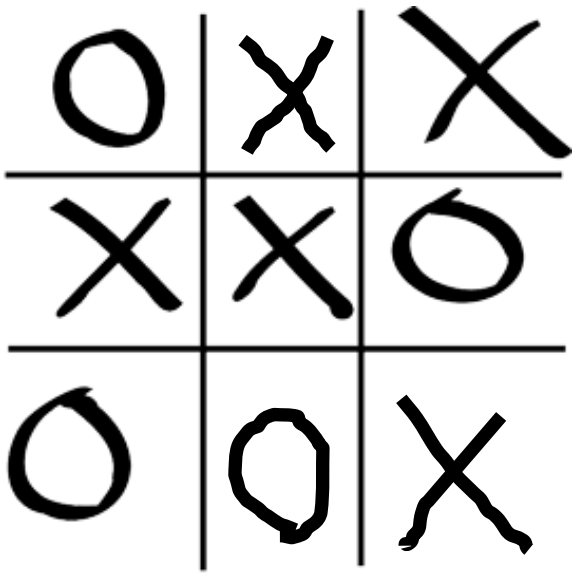


In a sequence of games

- observe state of the board
- take action
- observe opponent's move
- … repeat…
- until game is over and obtain reward

$$R = \begin{cases} +1 \text{ if win} \\ -1 \text{ if lose} \\ 0 \text{ if draw} \end{cases}$$

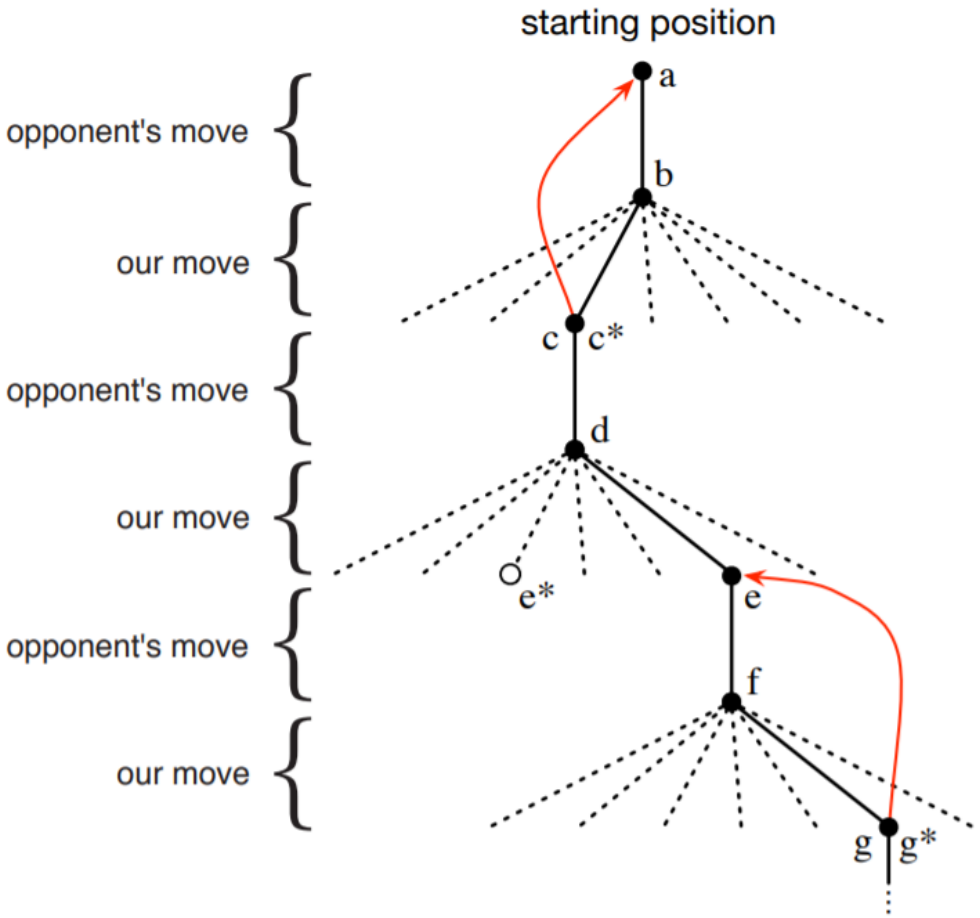
A CONCRETE EXAMPLE: TIC-TAC-TOE



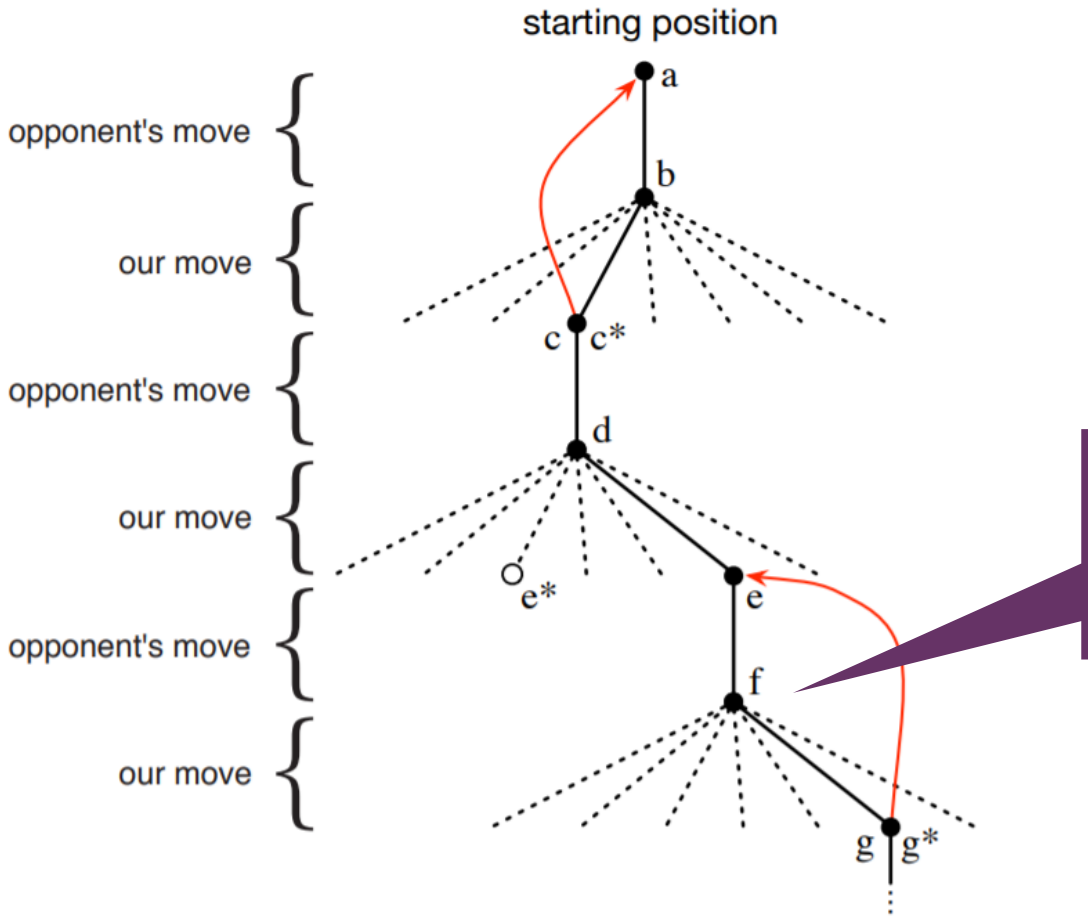
Two challenges:

1. each action influences the future evolution of the game
2. we never find out what would have happened if we took a different action

TIC-TAC-TOE DISSECTED

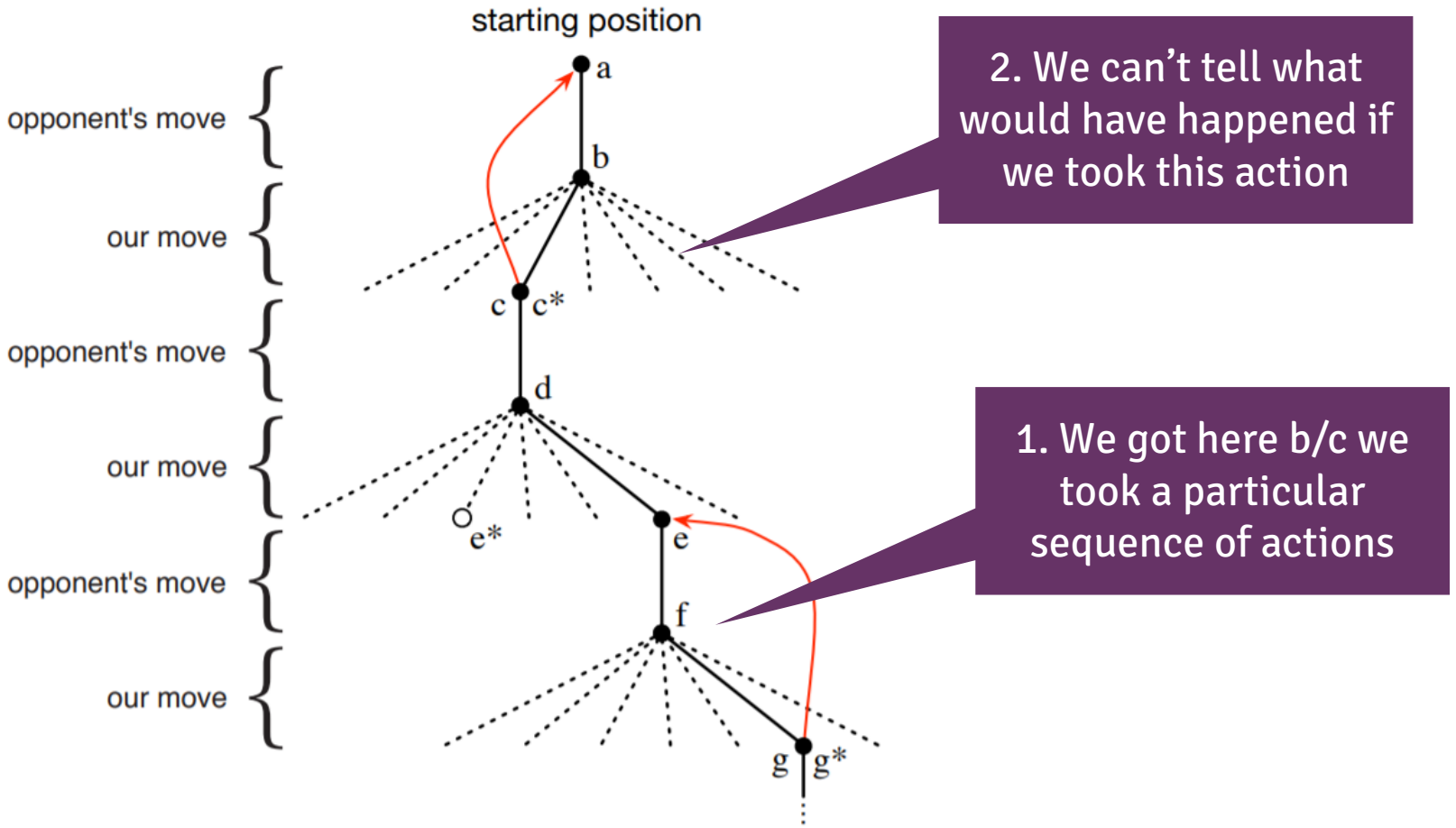


TIC-TAC-TOE DISSECTED



1. We got here b/c we took a particular sequence of actions

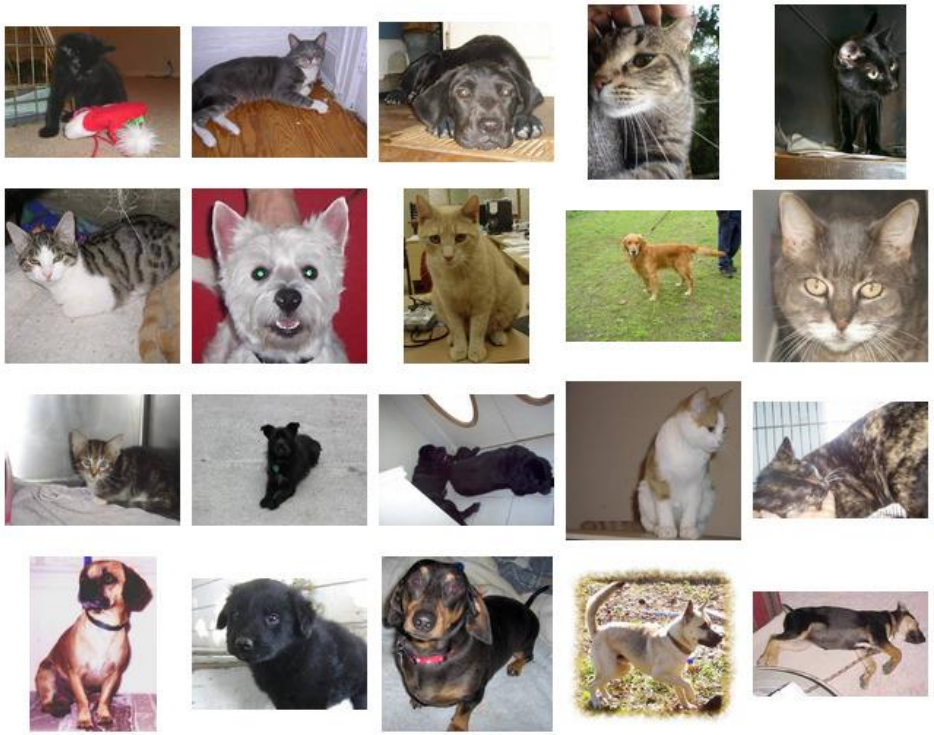
TIC-TAC-TOE DISSECTED



CONTRAST WITH SUPERVISED LEARNING

Supervised learning:

"given a labeled data set $\{x_n, y_n\}_{n=1}^{N}$, produce a function f such that $f(x) \approx y$ on unseen data (x, y)"



labeled set of cat and dog images

image classifier

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Two challenges of RL:

1. each action influences the future evolution of the game
2. we never find out what would have happened if we took a different action

THE TWO KEY CHALLENGES OF REINFORCEMENT LEARNING

1. dealing with long-term effects of actions

2. dealing with uncertainty due to partial feedback

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Addressed by Dynamic Programming

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DYNAMIC PROGRAMMING ≈

"constructing solutions to a computational problem by breaking it into subproblems"

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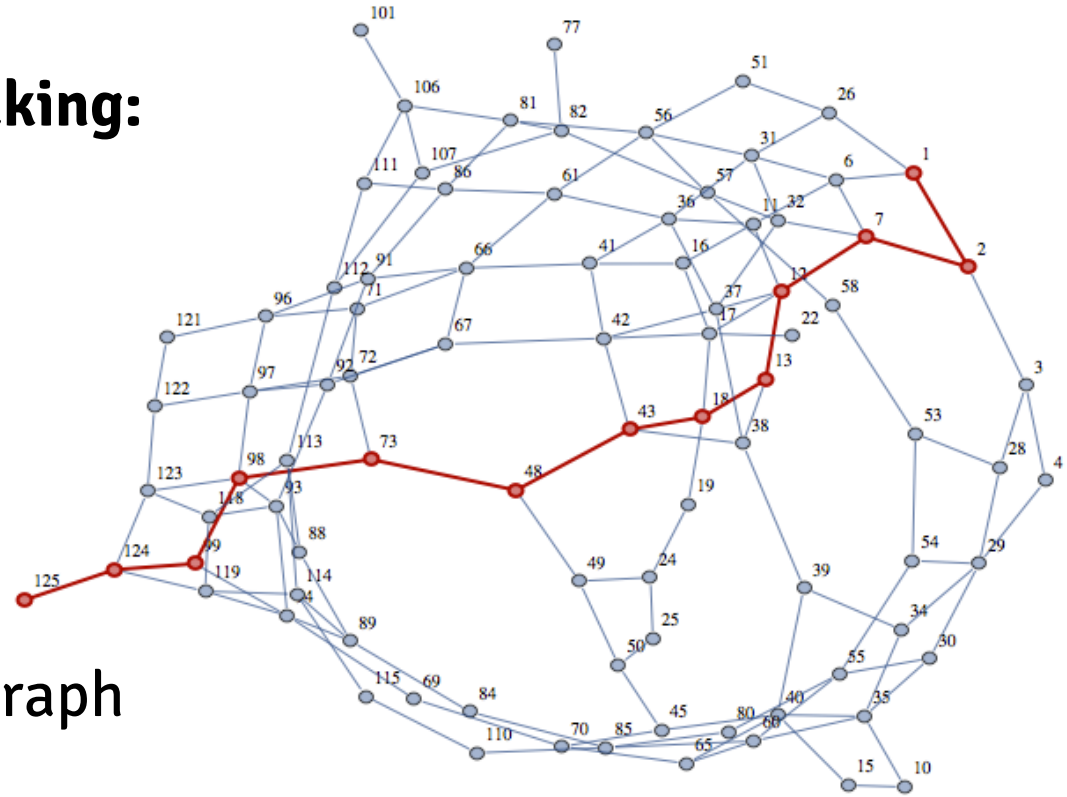
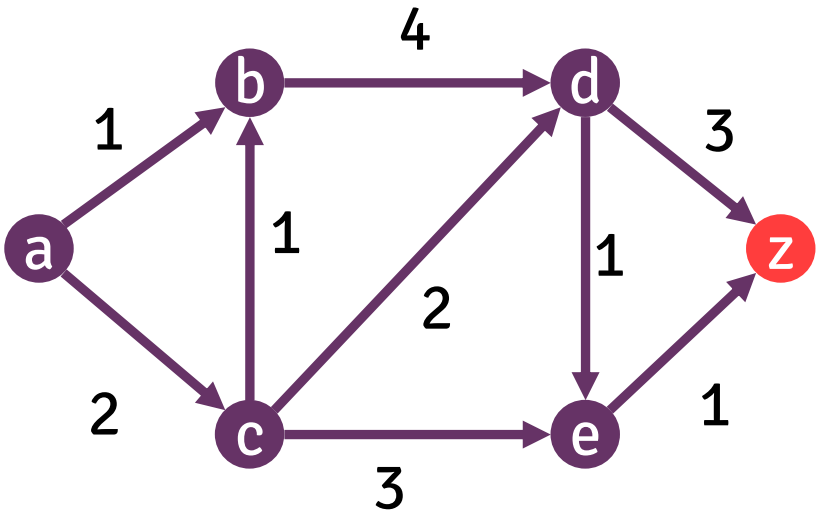
for sequential decision-making:
"exploiting the sequential nature of the problem to decompose computation"

prime example:
Dijkstra's algorithm for computing shortest paths between two vertices in a graph



THE KEY IDEA OF DYNAMIC PROGRAMMING



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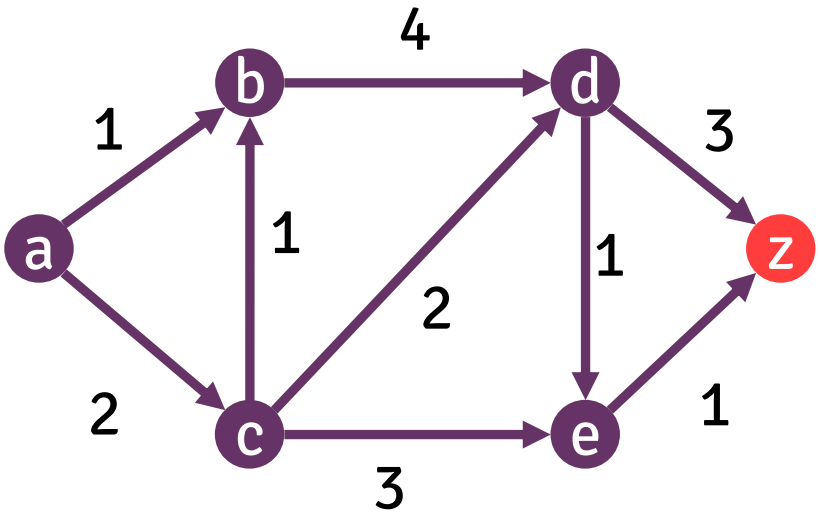
Shortest path decomposes:

- optimal path always ends with an optimal “sub-path”



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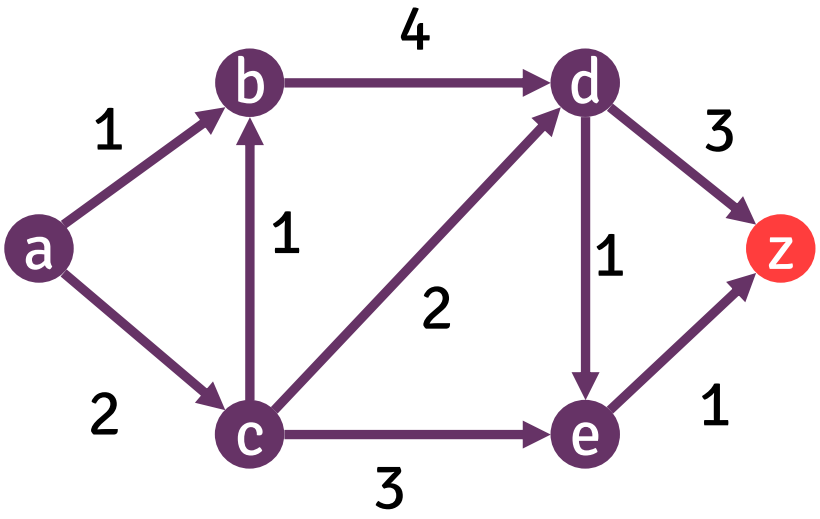

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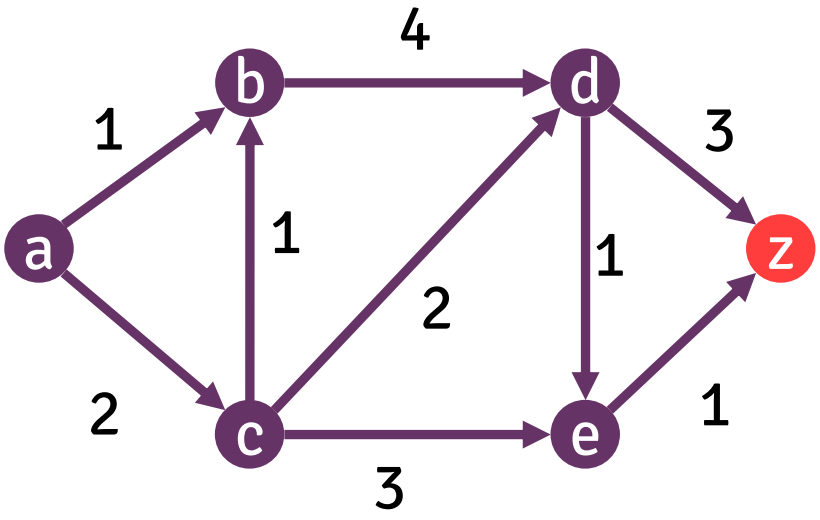

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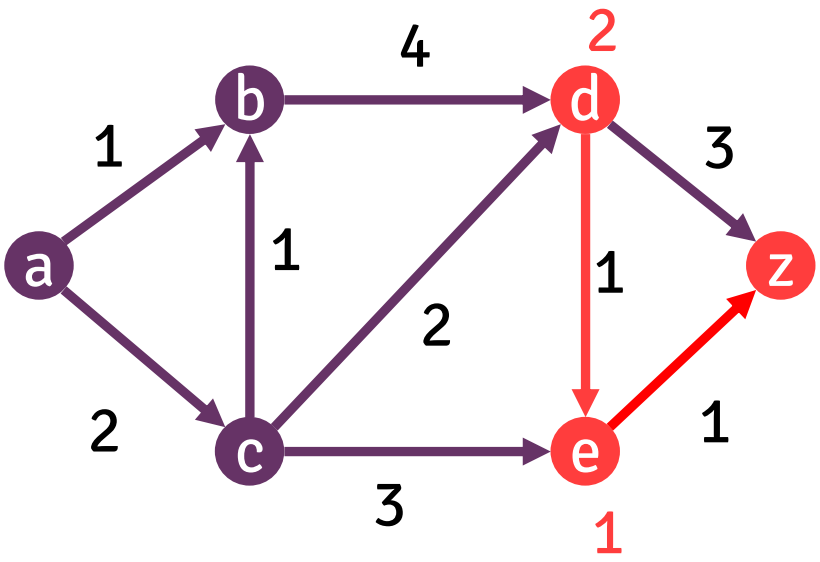

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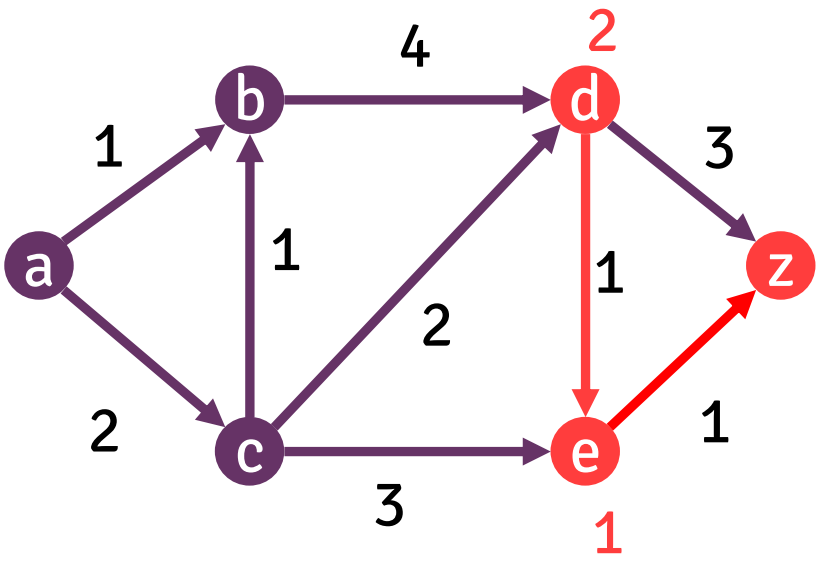

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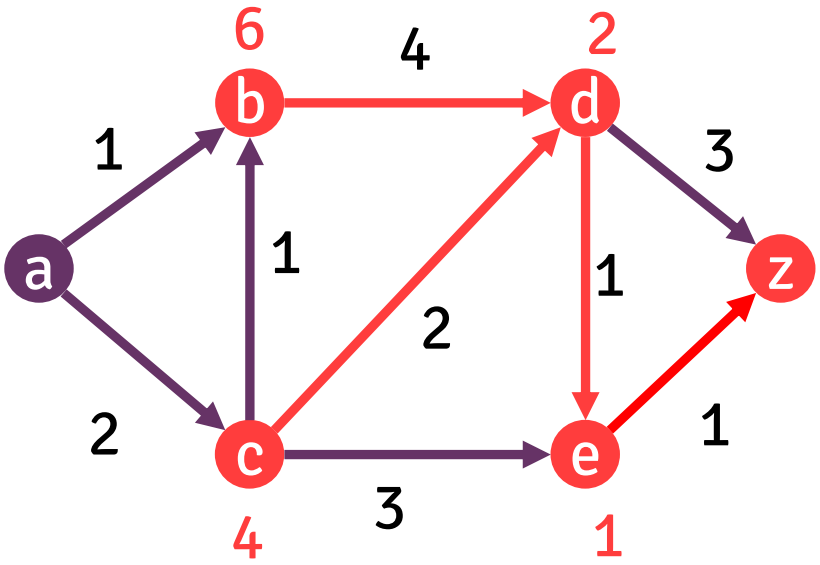

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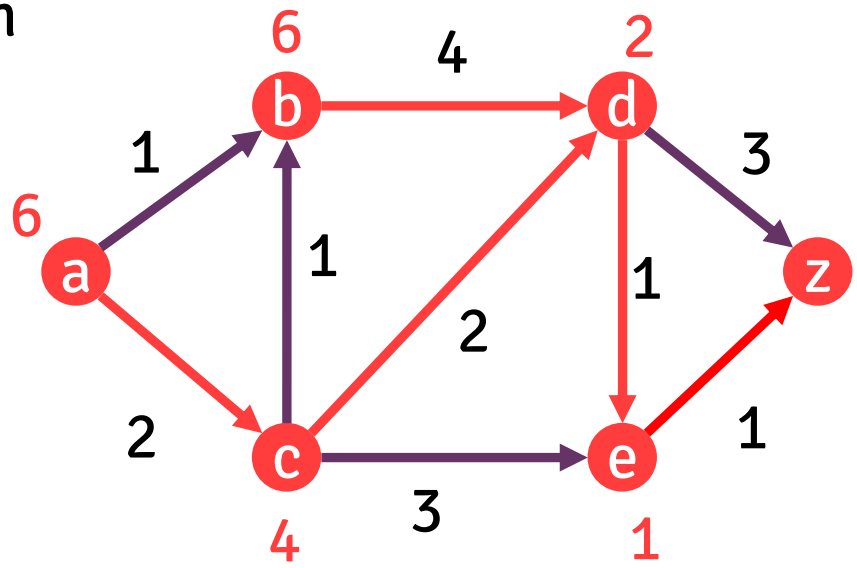

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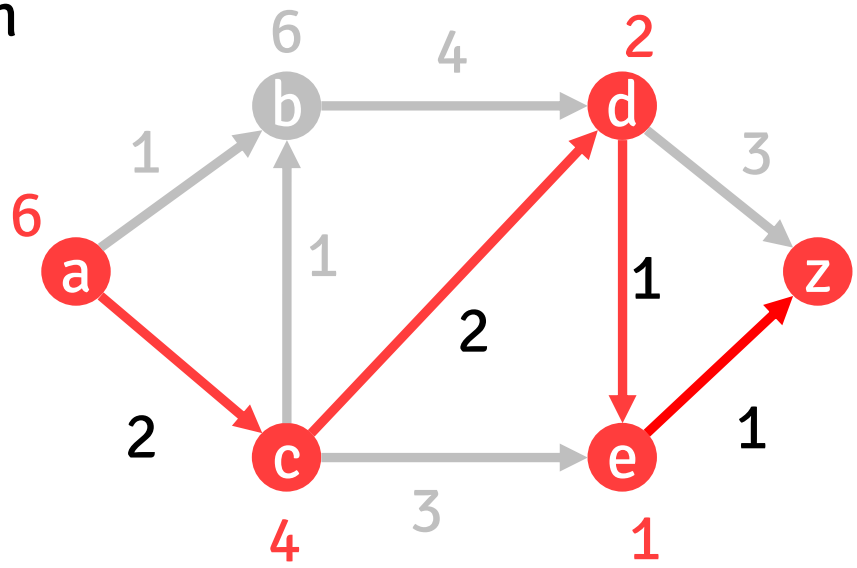
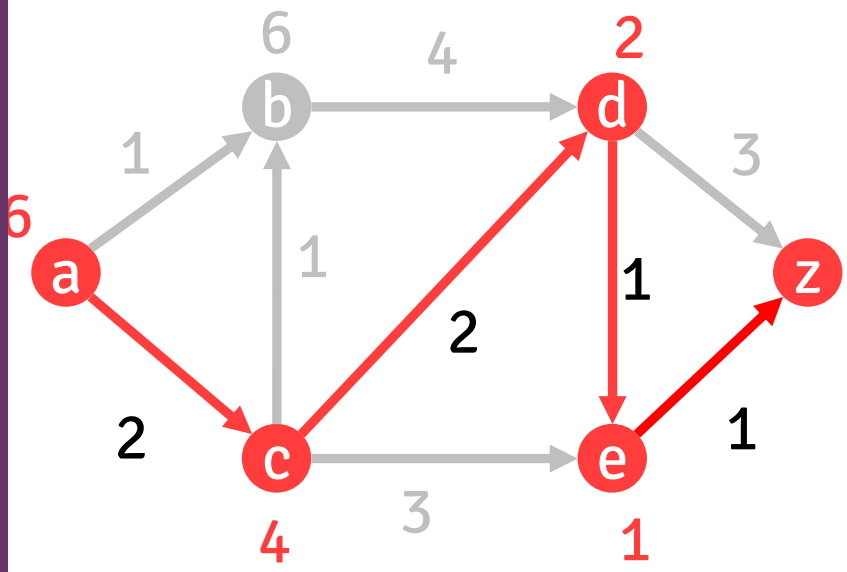
  - Do the same with b and c
  - Do the same with a
  - Extract optimal path



THE KEY IDEA OF DYNAMIC PROGRAMMING

We'll study a generalized version of this method that

1. can deal with infinite decision-making horizons,
2. can deal with randomness,
3. mixes well with machine learning techniques.



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Addressed by Dynamic Programming

2. dealing with uncertainty due to partial feedback

Addressed by Multi-armed bandit theory


THE BANDIT PROBLEM
Walk into the casino, play slot machines forever, try to maximize gain
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Exploration vs. exploitation

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MULTI-ARMED BANDIT THEORY

- winning probabilities $p_1, p_2, \dots, p_K$ unknown
- can only learn by taking actions and observing sample rewards
- we never observe what would have happened if we played a different action

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p_1 p_2 p_3 $\dots$ p_K

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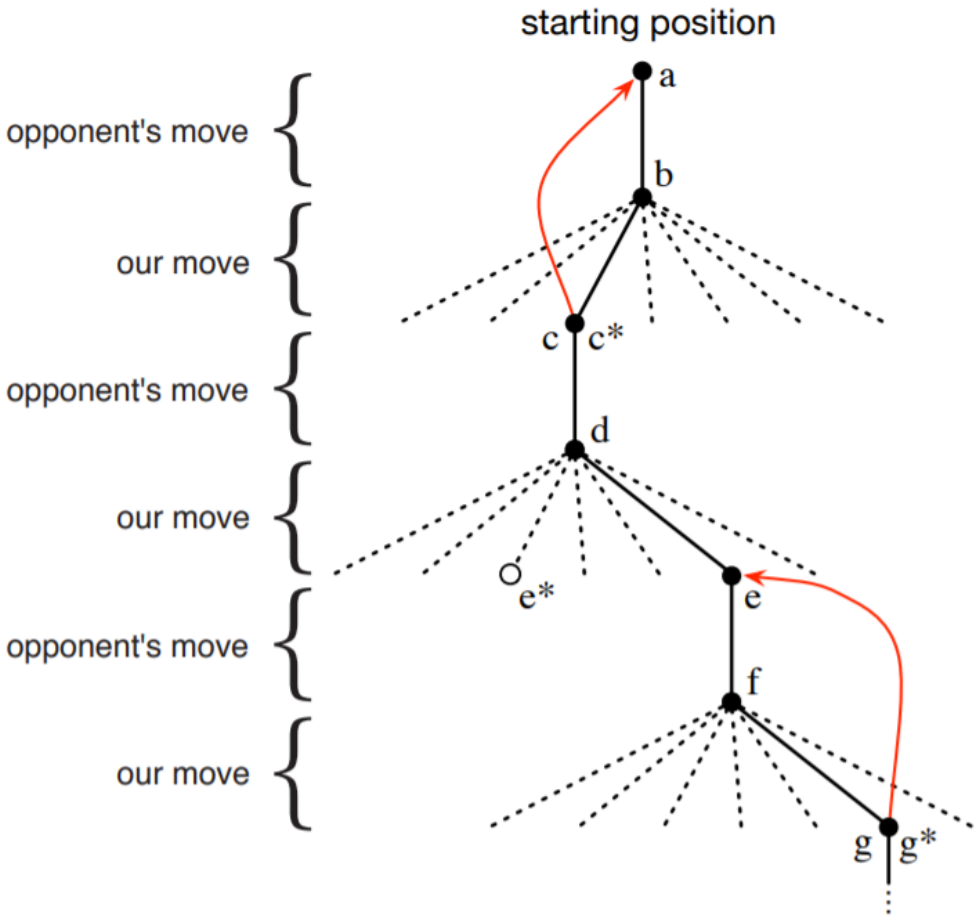
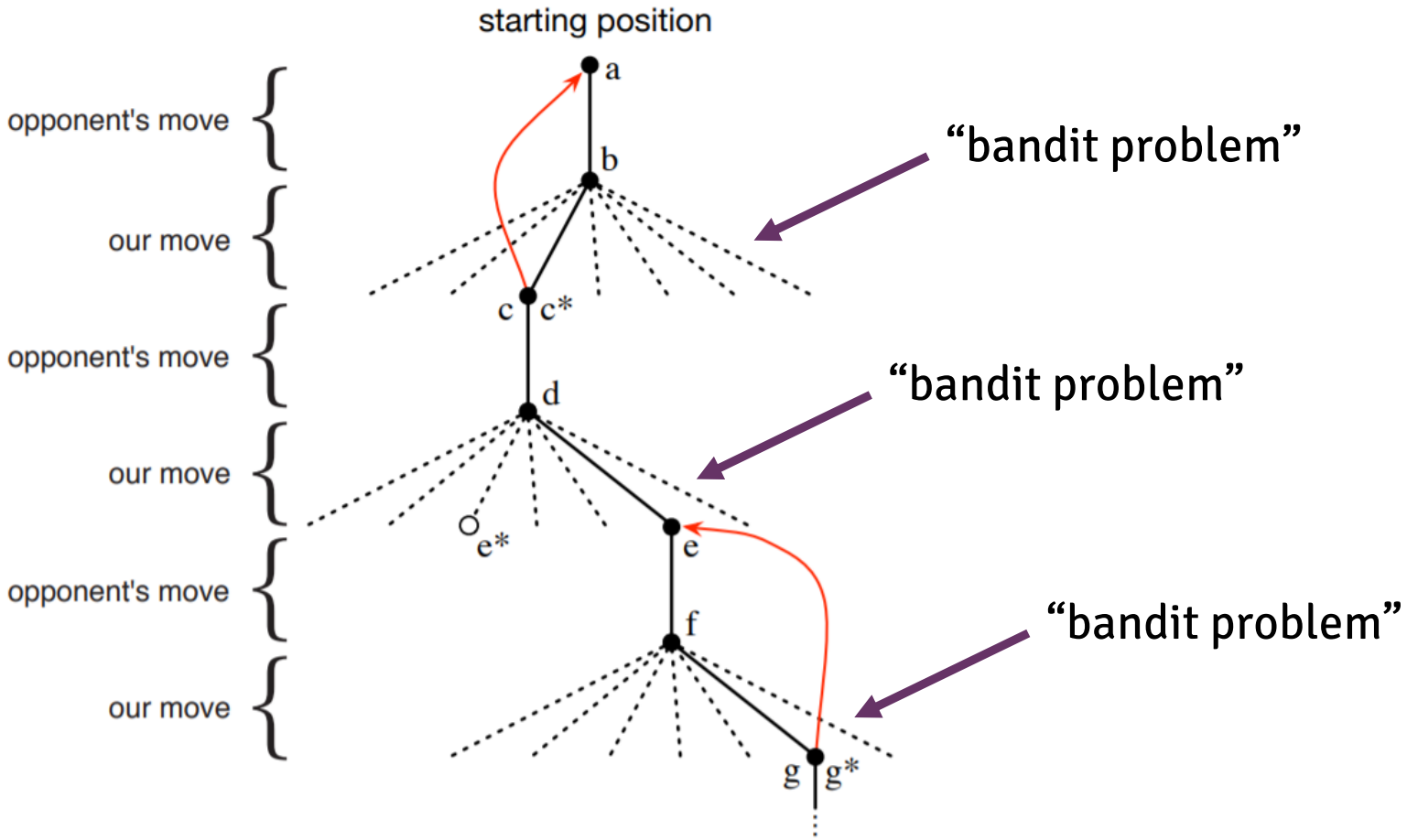
Just like in RL!!

REINFORCEMENT LEARNING AND THE BANDIT PROBLEM

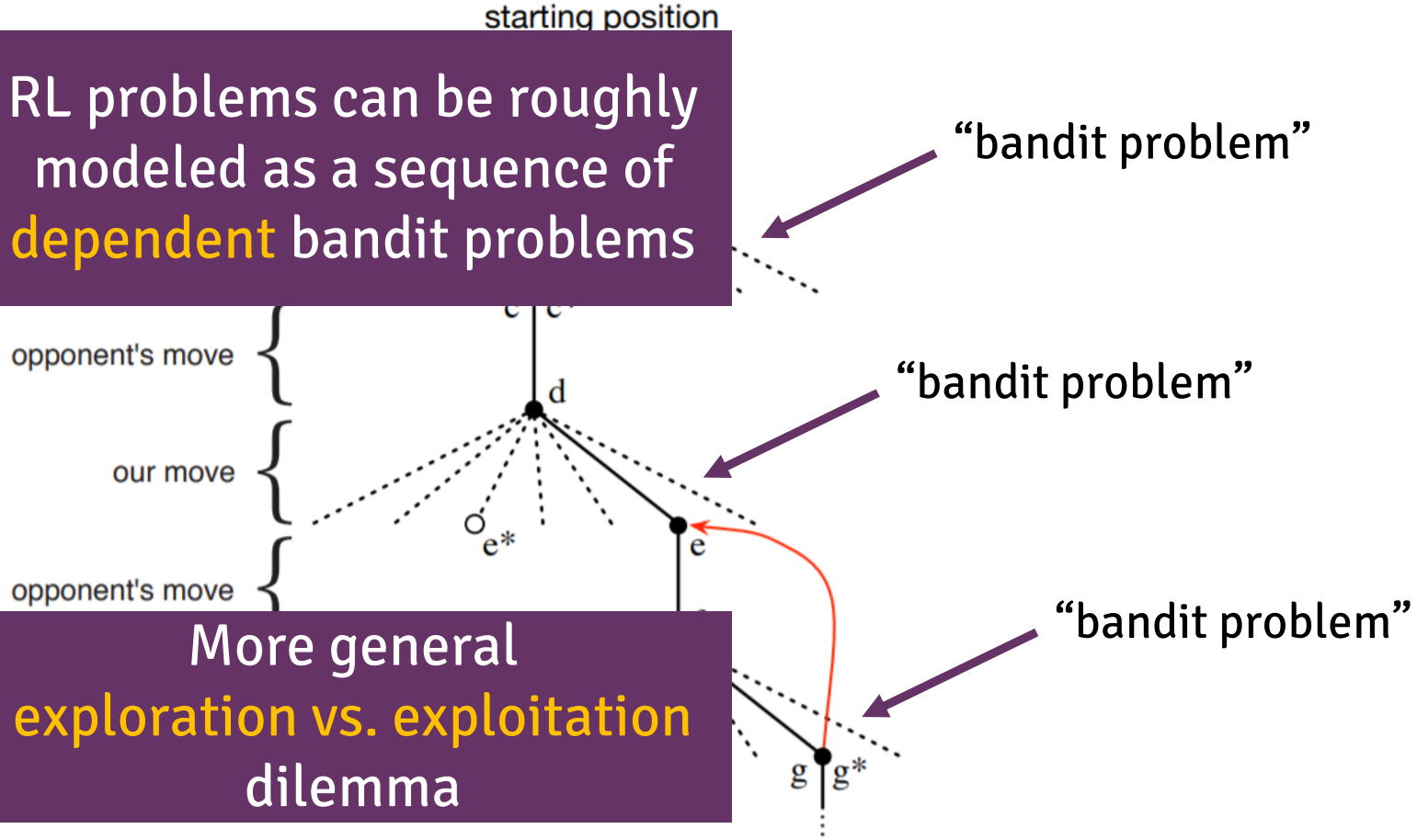


REINFORCEMENT LEARNING AND THE BANDIT PROBLEM



REINFORCEMENT LEARNING AND THE BANDIT PROBLEM

RL problems can be roughly modeled as a sequence of dependent bandit problems



More general exploration vs. exploitation dilemma

WHY MULTI-ARMED BANDIT THEORY?

Provides a comprehensive understanding of the exploration-exploitation tradeoff:

- algorithmic ideas to treat uncertainty
- precise characterization of performance limits
- extensible in numerous ways (e.g., contextual bandits, combinatorial bandits)

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Limitation:
cannot properly model long-term effects of actions

RECIPES FOR REINFORCEMENT LEARNING

A theorist's recipe:
dynamic programming + bandit theory

A practitioner's recipe:
dynamic programming + ML tricks for scalability

The ultimate recipe:
dynamic programming + bandit theory
+ ML tricks for scalability

Next week: dynamic programming