REINFORCEMENT LEARNING

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Lecture 7:
Basic RL algorithms

Basic RL algorithms

1. building blocks of RL methods
2. policy evaluation
   - Monte Carlo
   - temporal difference learning
3. policy evaluation and learning
   - SARSA
   - Q-learning

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EPILOGUE

from
Dynamic Programming
to
Reinforcement Learning

FROM DYNAMIC PROGRAMMING TO VALUE-BASED REINFORCEMENT LEARNING

Policy iteration:

$$V_k$$

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Policy iteration:

$$V_k \xrightarrow[\pi_k = GV_k]{\text{improve policy}} \pi_k$$

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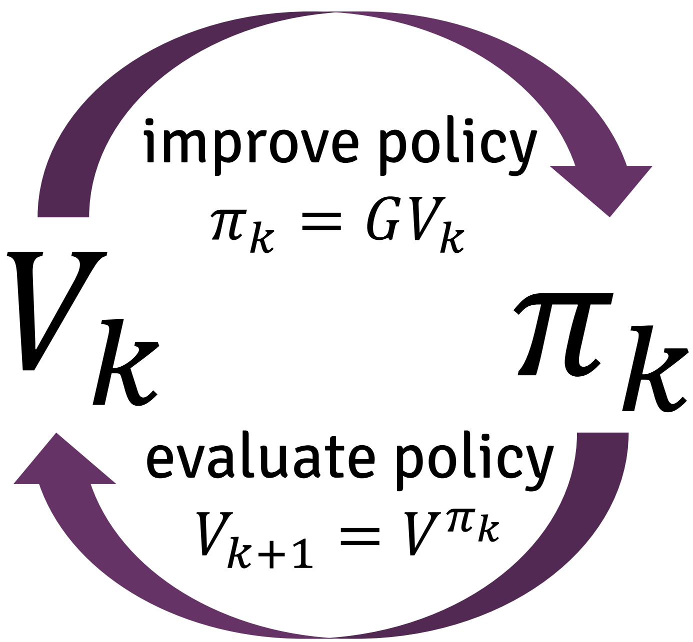

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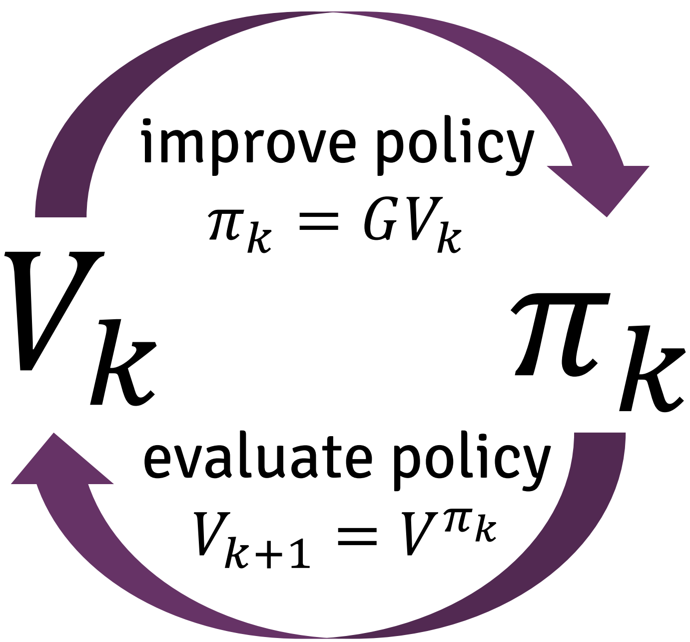
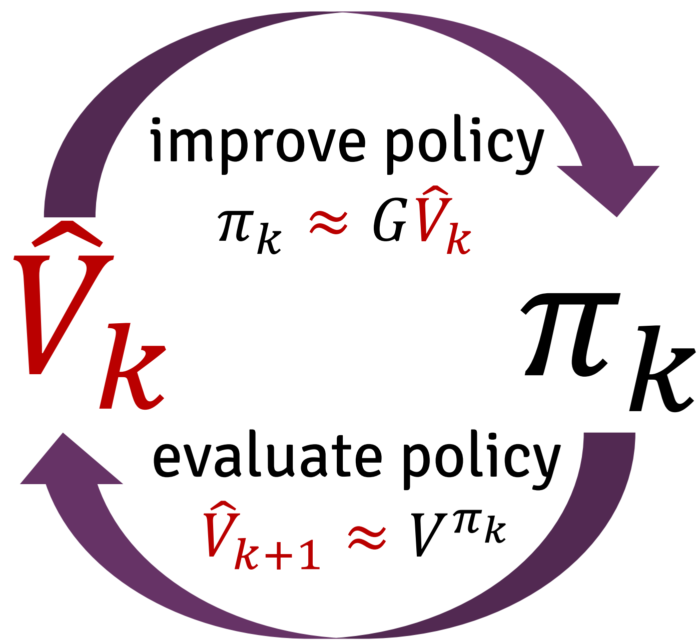


Approximate policy iteration:



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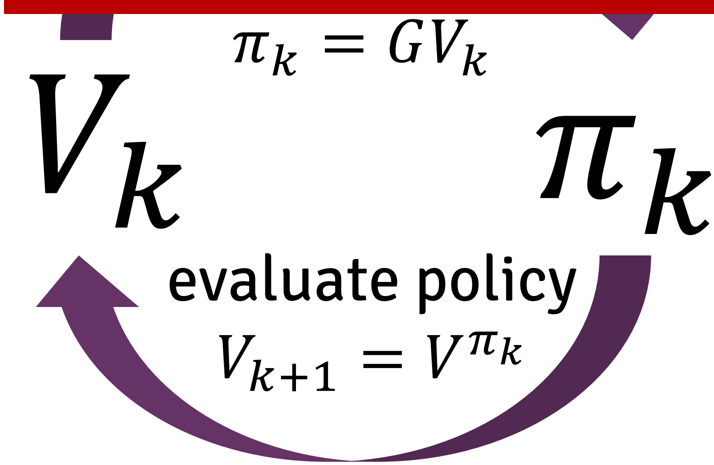
Fundamental RL tasks:

- Policy evaluation
- Policy improvement



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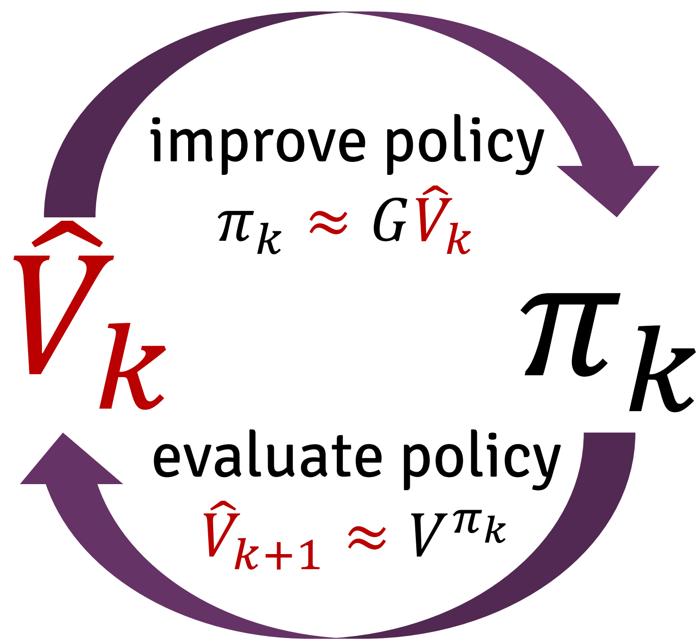

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Challenges in RL:

- Unknown transition and reward functions ⇒ have to learn from sample access only
- State/action space can be large ⇒ V^* and π^* cannot be stored in memory

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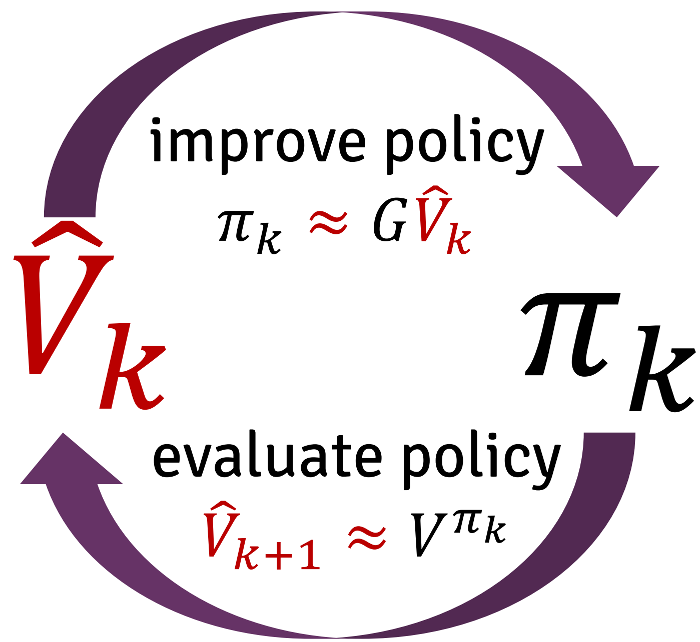

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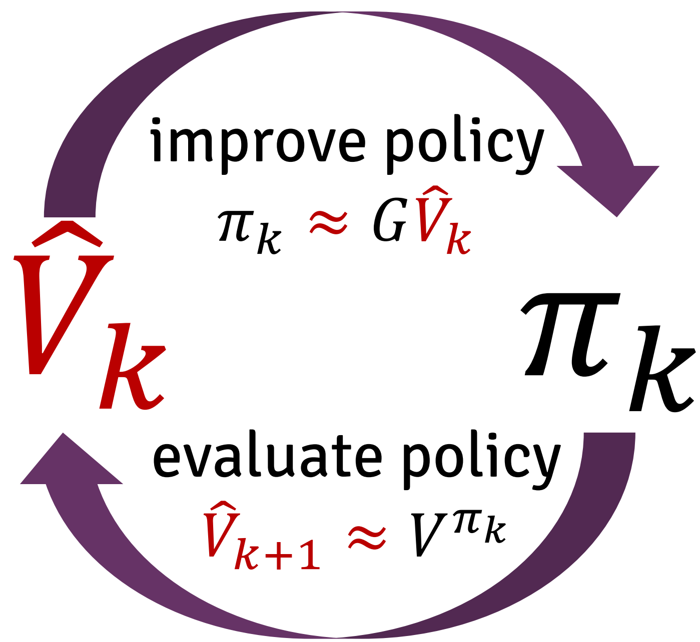

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LEVELS OF SAMPLE ACCESS

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Full knowledge of P
⇒ Planning (not RL)

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Generative model:
Full sample access to $P(\cdot \mid x, a)$ for any (x, a)

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Samples from full trajectories
+ reset action or save states

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Idea: approximate V^* and/or π^* in a computationally tractable way!

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Approximating V^*: linear function approximation

- Define a set of d features:
$$\phi_i : X \to \mathbf{R}$$
- Parametrize value functions as
$$V_\theta(x) = \theta^\top \phi(x)$$
- Learning $V^* \Leftrightarrow$ Learning a good θ_*
$$V_{\theta^*} \approx V^*$$

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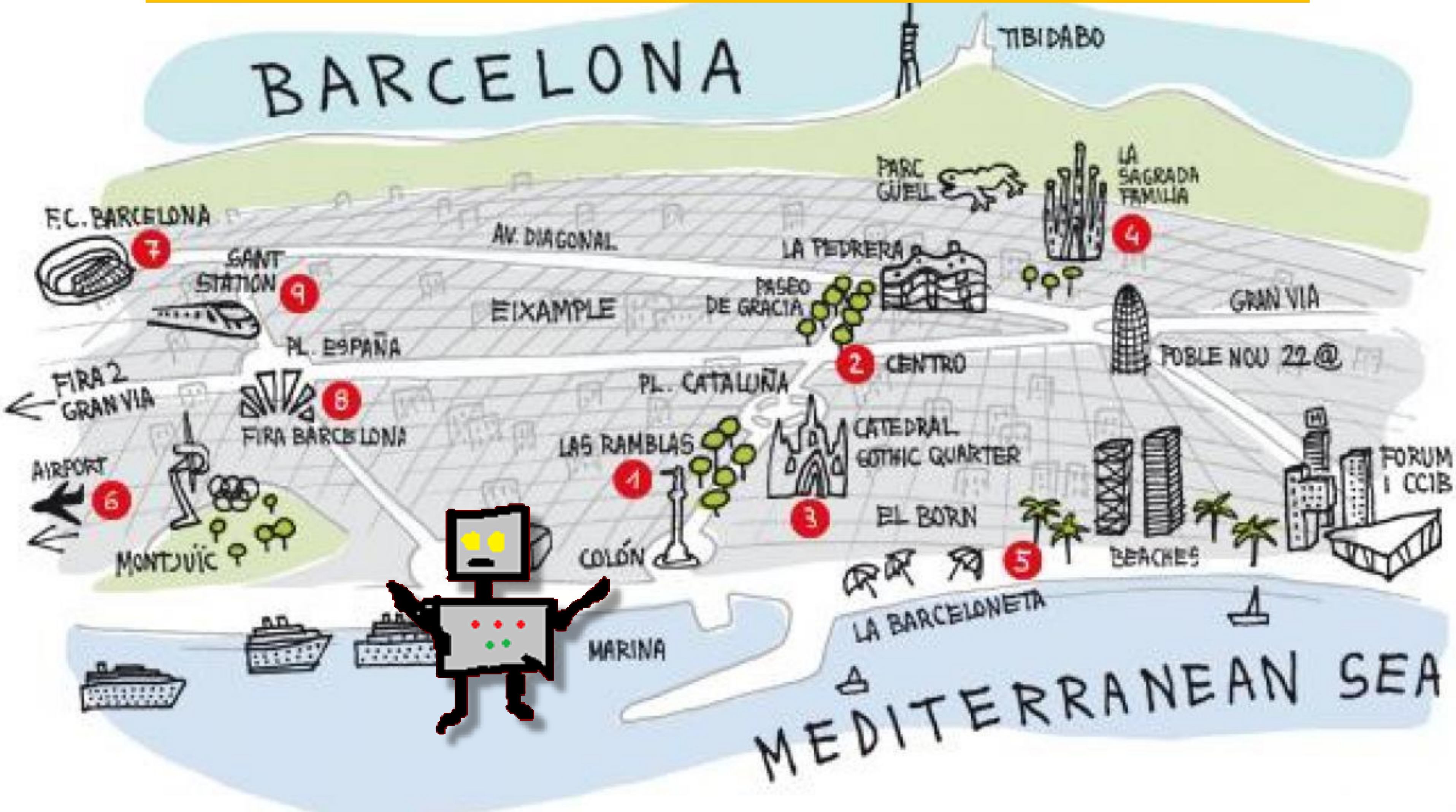
FEATURE MAP EXAMPLE



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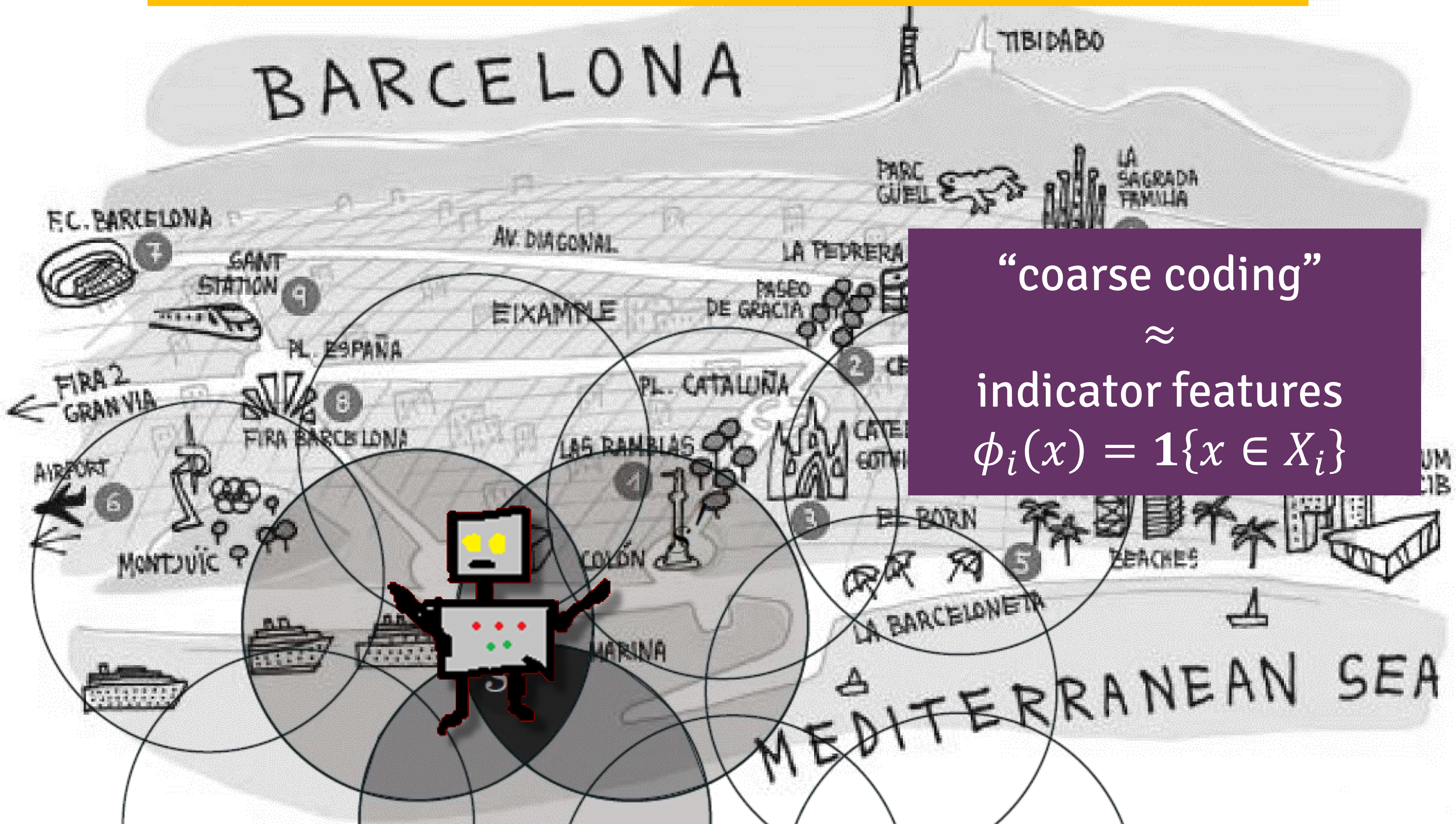


“coarse coding”

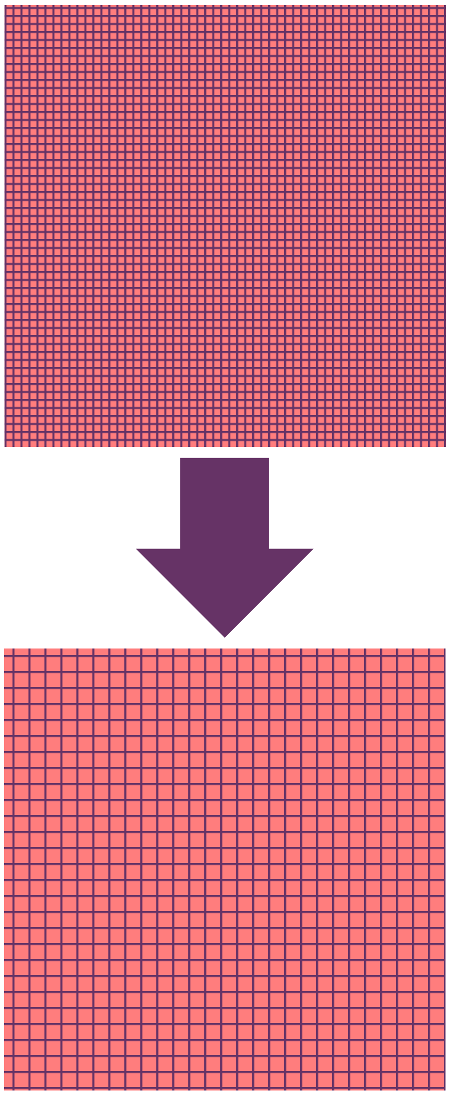
$\approx$

indicator features

$\phi_i(x) = \mathbf{1}\{x \in X_i\}$

STATE AGGREGATION

- Partition the state space X into d sets:
$$X = C_1 \cup C_2 \cup \cdots \cup C_d$$
with $C_i \cap C_j = \emptyset$ when $i \neq j$
- Define feature i as $\phi_i(x) = \mathbf{1}\{x \in C_i\}$



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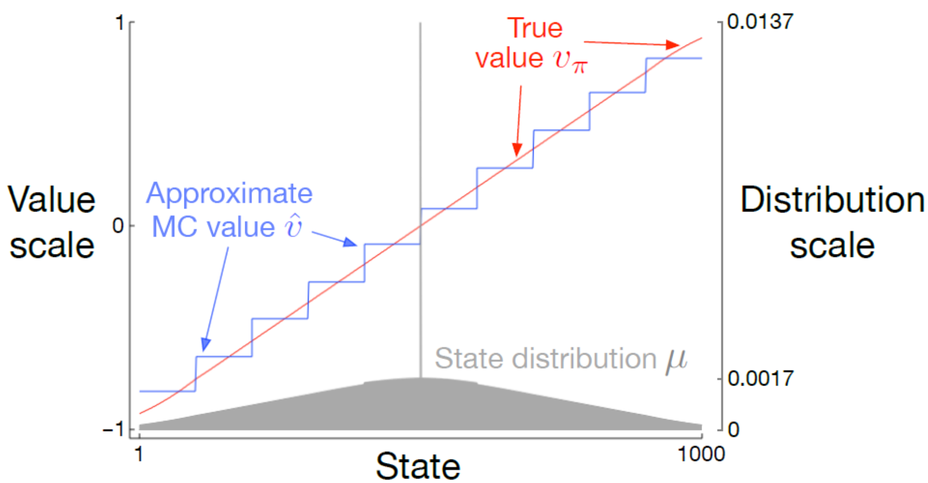
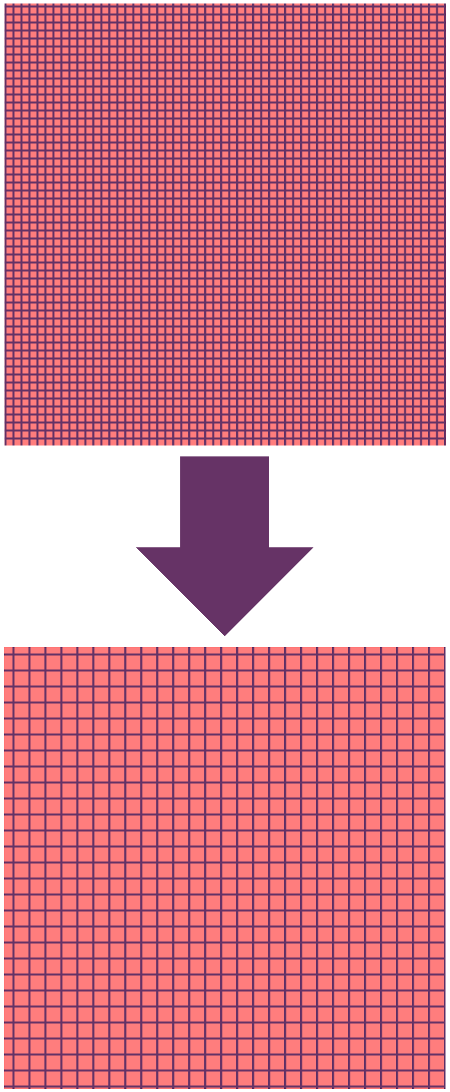
- Parametrizes piecewise constant functions





STATE AGGREGATION EXAMPLE: “PROST” FEATURES FOR ATARI GAMES

High-dimensional observations: 192×160 pixels

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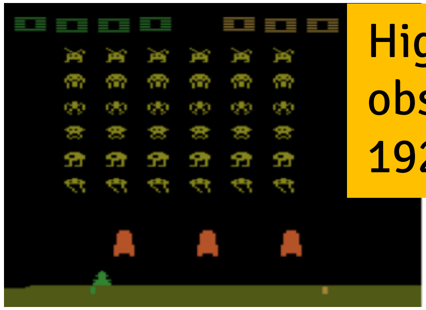

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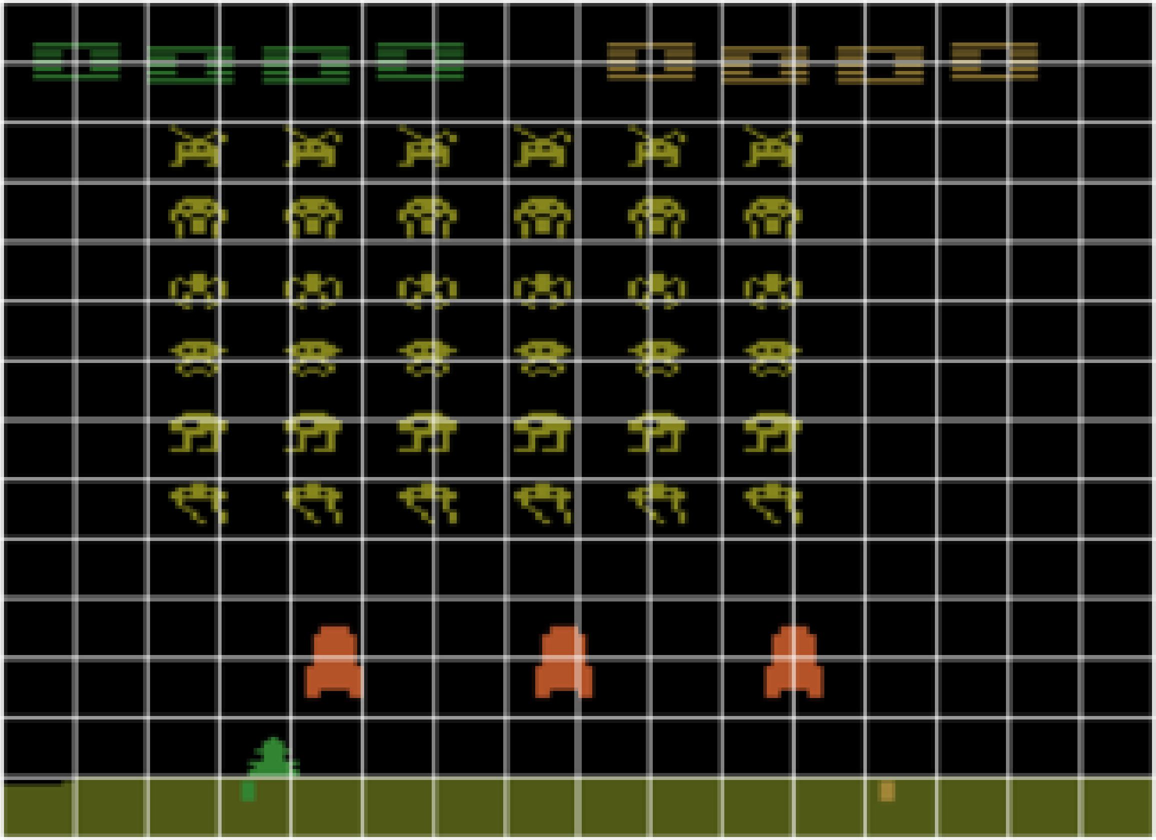

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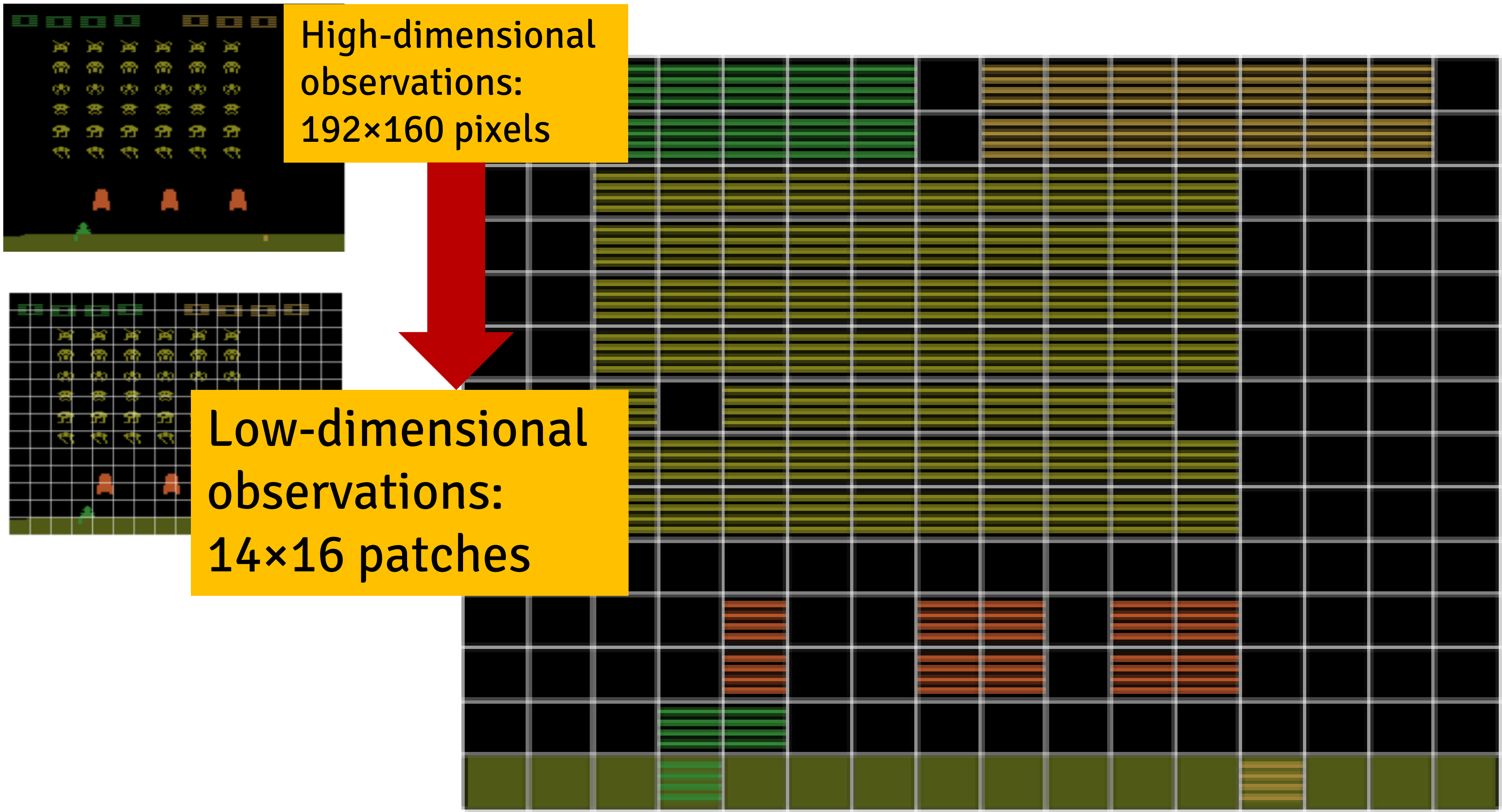

TRIVIAL STATE AGGREGATION: THE TABULAR CASE

Strictly generalizes the basic setting:
for each state $x \in X$, introduce a feature ϕ_x as

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Value function $\approx$ lookup table

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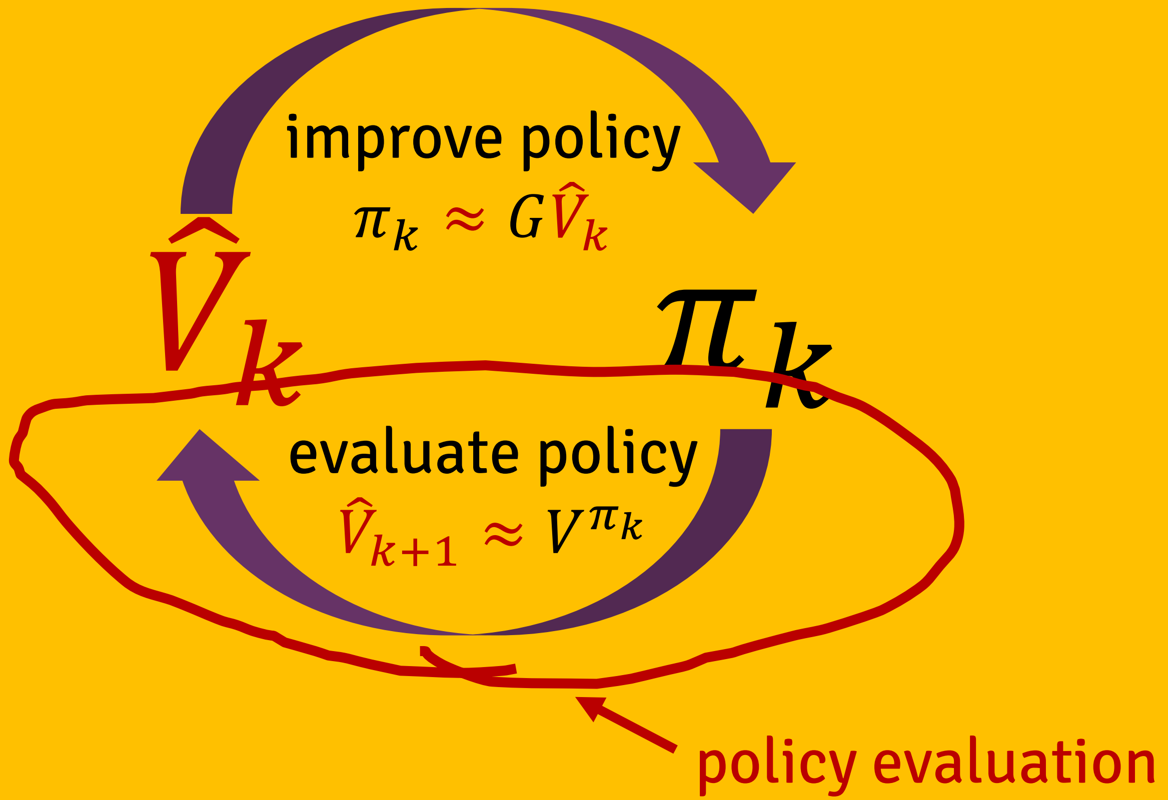

METHODS FOR POLICY EVALUATION

A GENTLE START: MONTE CARLO

Observe:

Policy evaluation = estimating V^{π}:

$$V^{\pi}(x) = \mathbf{E}_{\pi}[\sum_{t=0}^{\infty} \gamma^{t} r(x_t, a_t) \,|x_0 = x]$$

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Idea:
approximate $\mathbf{E}_{\pi}[\cdot]$ by sample averages!

- Simulate N trajectories using policy π
- For every state x that appears in the trajectories, let

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Average of i.i.d. random variables: $\lim_{N\to\infty} \hat{V}_N = V^\pi$

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MONTE CARLO WITH FEATURES

Monte Carlo policy evaluation

Input:

N trajectories $\sim \pi$, feature map $\phi : X \to \mathbb{R}^d$

Output:

$$\hat{V}_N = \arg\min_{\theta \in \mathbb{R}^d} \mathbf{E}_x \left[\sum_{i=1}^{N} \left(\theta^\top \phi(x) - R_i(x) \right)^2 \right]$$

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Least-squares fit of discounted returns

Recovers the previous definition:

$$\arg\min_v \sum_i \left(v(x) - R_i(x) \right)^2 = \operatorname{avg}\left(R_{1:N}(x) \right)$$

MONTE CARLO EXAMPLE: BLACKJACK

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In each episode:

- Start with 2 cards
- Observe 1 showing card of dealer
- Decide to either hit (draw another card) or stick
- Episode ends when you stick or go bust (sum over 21)
- Outcome:
    - Dealer’s sum $<$ your sum $\Rightarrow$ win
    - Dealer’s sum $=$ your sum $\Rightarrow$ draw
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MDP formulation:

- State description:
- current sum of player (12-21)
- dealer's showing card (1-10)
- presence of usable ace

- Actions:
- hit
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- Rewards:
- lose: -1
- draw: 0
- win: 1

- Transition dynamics:
- random initial state
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Transition function P is complicated to represent ☹

BUT

sample episodes are easy to generate! ☺

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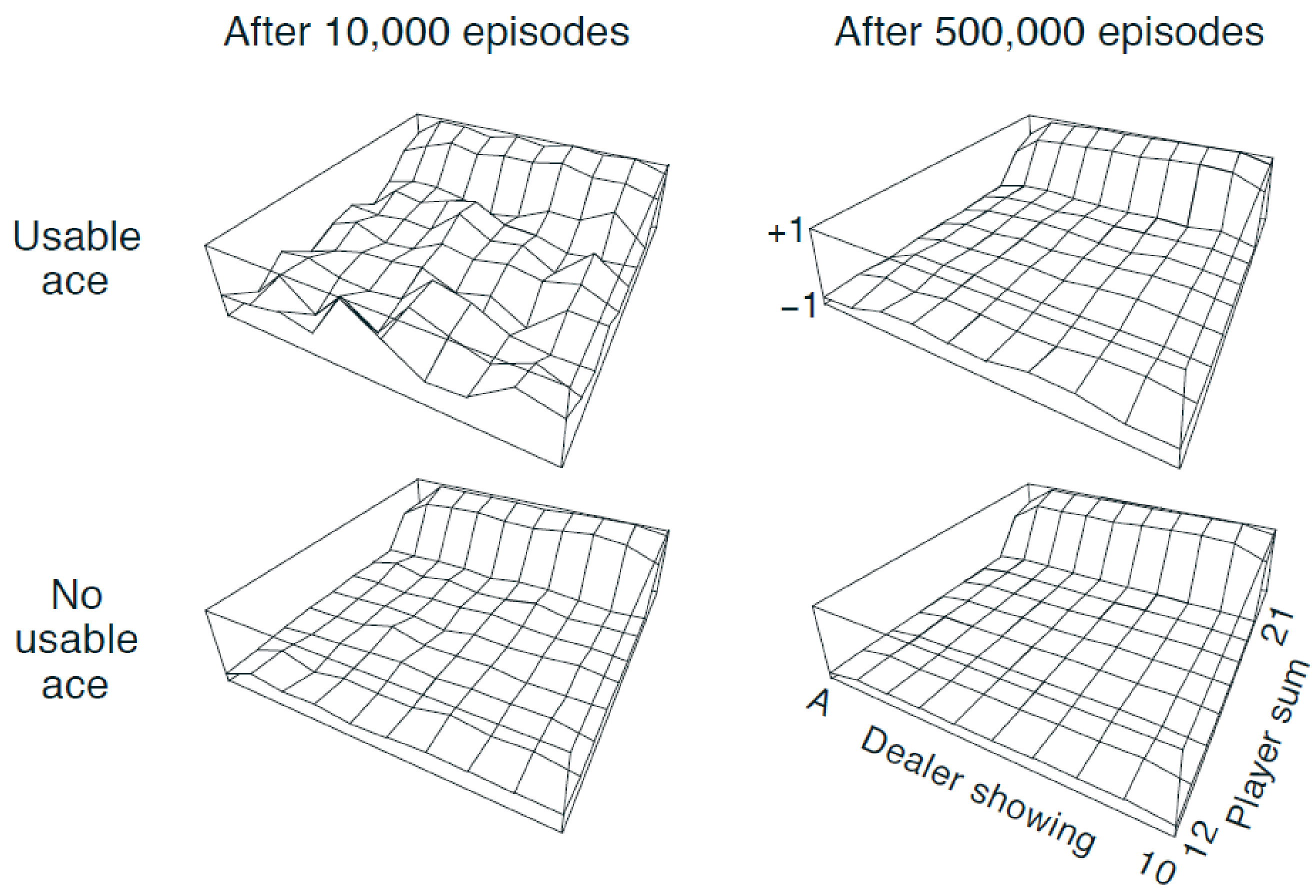
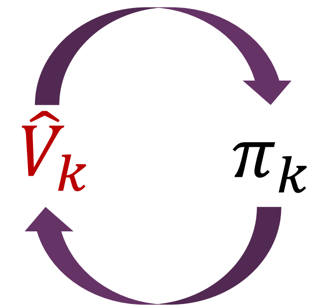
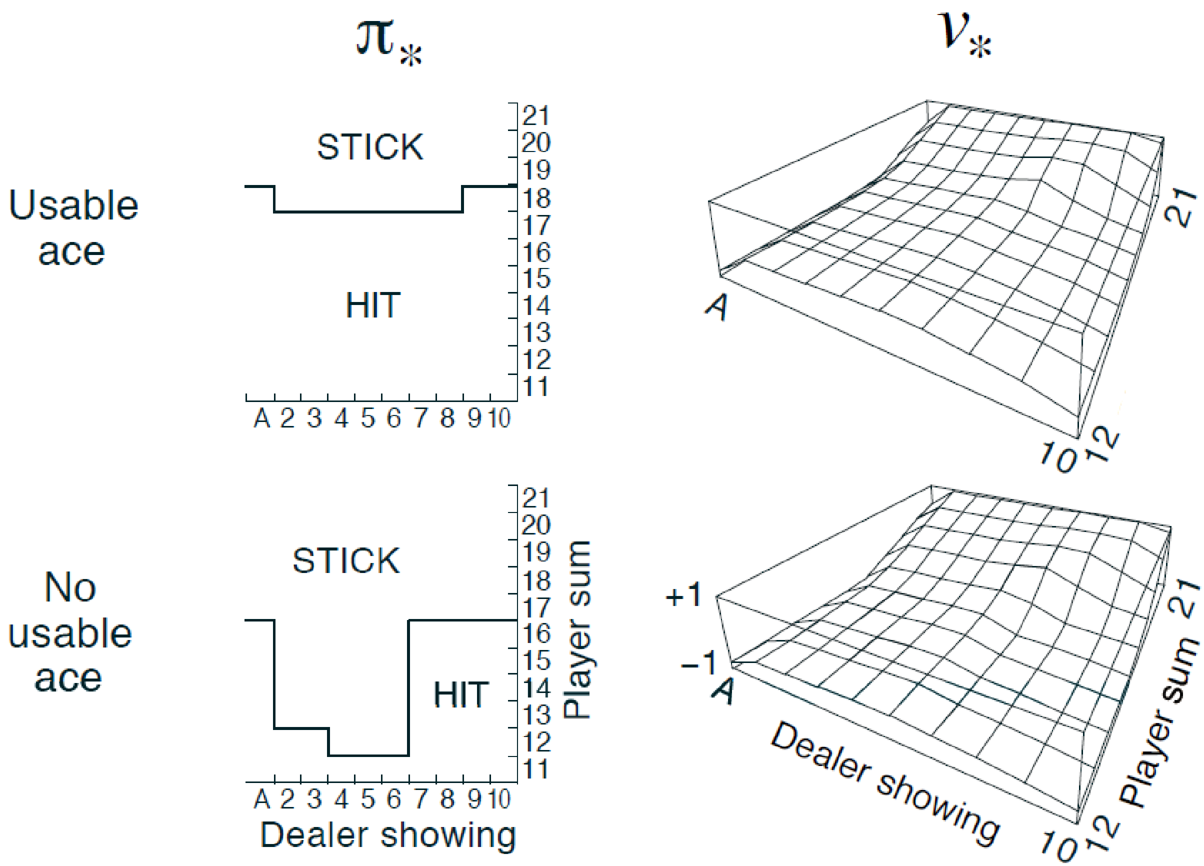
Value of policy that always hits below 20



MONTE CARLO POLICY ITERATION FOR BLACKJACK





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The Bellman error

$$\Delta_V(x) = T^\pi V(x) - V(x)$$

Idea: use stochastic approximation to find $\hat{V}$ such that $\mathbf{E}_{x\sim\mu}[\Delta_{\hat{V}}(x)] \approx 0$

TEMPORAL DIFFERENCE LEARNING

TD(0)

Input: arbitrary function $\hat{V}_0 : X \to \mathbf{R}$

For $t = 0,1, \ldots,$

$$\delta_t = r_t + \gamma \hat{V}_t(x_{t+1}) - \hat{V}_t(x_t)$$
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$\delta_t =$ "what I saw" $-$ "what I expected to see"
"target" $-$ "current prediction"

"temporal difference"

THE CONVERGENCE OF TD(0)

Sequence of estimates $(\hat{V}_1, \hat{V}_2, \ldots)$ is a dynamical system

Converges to equilibrium if step-sizes satisfy

$$\sum_{t=0}^{\infty} \alpha_t = \infty \quad \text{and} \quad \sum_{t=0}^{\infty} \alpha_t^2 < \infty$$

Equilibrium:

$$\mathbf{E}[\delta_t] = \mathbf{E}\left[r_t + \gamma \hat{V}_t(x_{t+1}) - \hat{V}_t(x_t)\right] = 0$$

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Intuition: for small stepsizes, $\hat{V}_t$ tracks the path of the ODE

$$\frac{\mathrm{d}\tilde{V}(t)}{\mathrm{d}t} = (T^\pi - I)\tilde{V}(t)$$

which is globally asymptotically stable with equilibrium $V^\pi = T^\pi V^\pi$

TD(0) WITH LINEAR FUNCTION APPROXIMATION

Let $\phi: X \to \mathbf{R}^d$ be a feature vector

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Approximating $V^\pi(x) \approx \theta^\top \phi(x)$ by TD(0):

TD(0) with LFA

Input: arbitrary param. vector $\theta_0 \in \mathbf{R}^d$

For $t = 0,1, \ldots,$

$$\delta_t = r_t + \gamma \theta_t^\top \phi(x_{t+1}) - \theta_t^\top \phi(x_t)$$

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$$\theta_{t+1} = \theta_t + \alpha_t \delta_t \phi(x_t)$$

This still converges!!!

to the projection of V^π to the span of the features

TD(0) VS. SGD FOR LINEAR REGRESSION

Linear least-squares regression:

- Data set: $(\phi_t, y_t)_{t=1}^{n}$ drawn from distribution $\mathcal{D}$ with feature vectors $\phi_t \in \mathbb{R}^d$, and outputs y_t
- Goal: find θ^* minimizing
  $L(\theta) = \mathbb{E}_{\phi,y\sim\mathcal{D}}[\ell(\theta;\phi,y)] = \mathbb{E}_{\phi,y\sim\mathcal{D}}[(\phi^\top\theta - y)^2]$

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SGD for linear regression

Input: arbitrary param. vector $\theta_0 \in \mathbf{R}^d$

For $t = 0,1,\dots,$

$$\begin{aligned}\theta_{t+1} &= \theta_t + \alpha_t \nabla\ell(\theta_t;\phi_t,y_t)\\ &= \theta_t + \alpha_t(\phi_t^\top\theta_t - y_t)\phi_t\end{aligned}$$

TD(0) VS. SGD FOR LINEAR REGRESSION

Linear least-squares regression:

- Data set: $(\phi_t, y_t)_{t=1}^{n}$ drawn from distribution $\mathcal{D}$ with feature vectors $\phi_t \in \mathbb{R}^d$, and outputs y_t
- Goal: find θ^* minimizing $L(\theta) = \mathbb{E}_{\phi,y\sim\mathcal{D}}[\ell(\theta;\phi,y)] = \mathbb{E}_{\phi,y\sim\mathcal{D}}[(\phi^\top\theta - y)^2]$

SGD for linear regression

Input: arbitrary param. vector $\theta_0 \in \mathbf{R}^d$

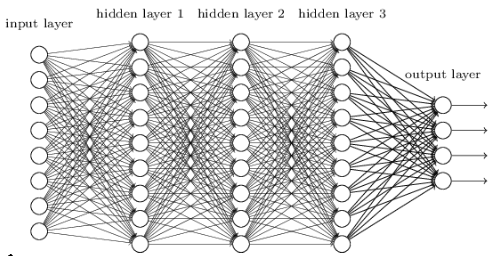
For $t = 0,1,\ldots,$

$$\begin{aligned}\theta_{t+1} &= \theta_t + \alpha_t \nabla\ell(\theta_t;\phi_t,y_t)\\ &= \theta_t + \alpha_t(\phi_t^\top\theta_t - y_t)\phi_t\end{aligned}$$

$\delta_t = (\phi_t^\top\theta_t - y_t) =$ "prediction error"

TD(0) WITH NONLINEAR FUNCTION APPROXIMATION

Let $V_\theta: X \to R$ be a parametric class of functions (e.g., deep neural network)



Approximating $V^\pi(x) \approx V_\theta(x)$ by TD(0):

<u>TD(0) with general FA</u>

Input: arbitrary param. vector $\theta_0 \in \mathbf{R}^d$

For $t = 0, 1, \ldots,$

$$\delta_t = r_t + \gamma V_{\theta_t}(x_{t+1}) - V_{\theta_t}(x_t)$$

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TD(0) WITH NONLINEAR FUNCTION APPROXIMATION

Let $V_\theta : X \to R$ be a p
functions (e.g., deep

Not much is known about convergence ☹

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PROPERTIES OF TD(0)

☺ Value estimates converge to true values ☺

☺ Doesn't need prior knowledge of P or r ☺

☺ Based on the concept of Bellman error ☺

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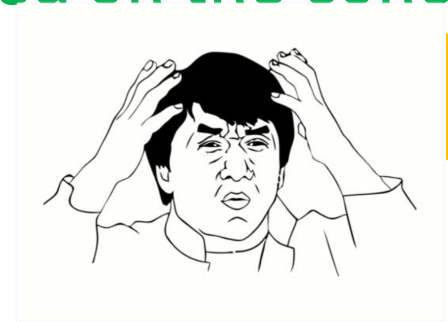


= "bootstrapping"

PROPERTIES OF TD(0)

☺ Value estimates converge to true values ☺

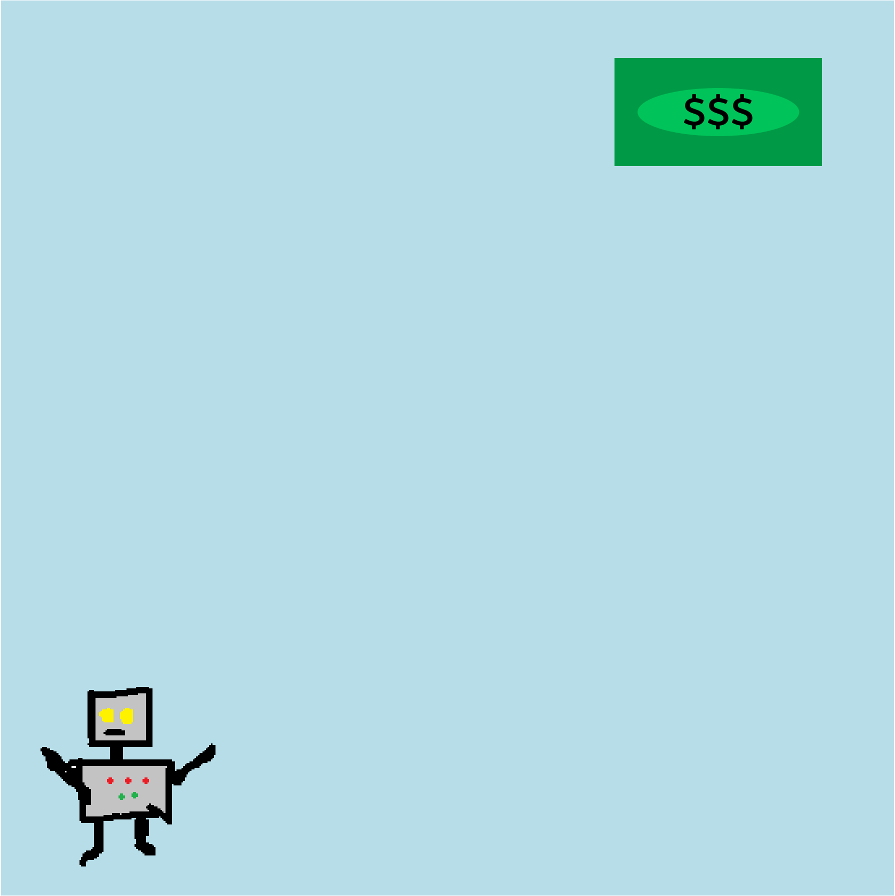
☺ … r ☺

+ new problem:
updates propagate very slowly!

☺ Based on the concept of Bellman error ☺

= "bootstrapping"

EXAMPLE



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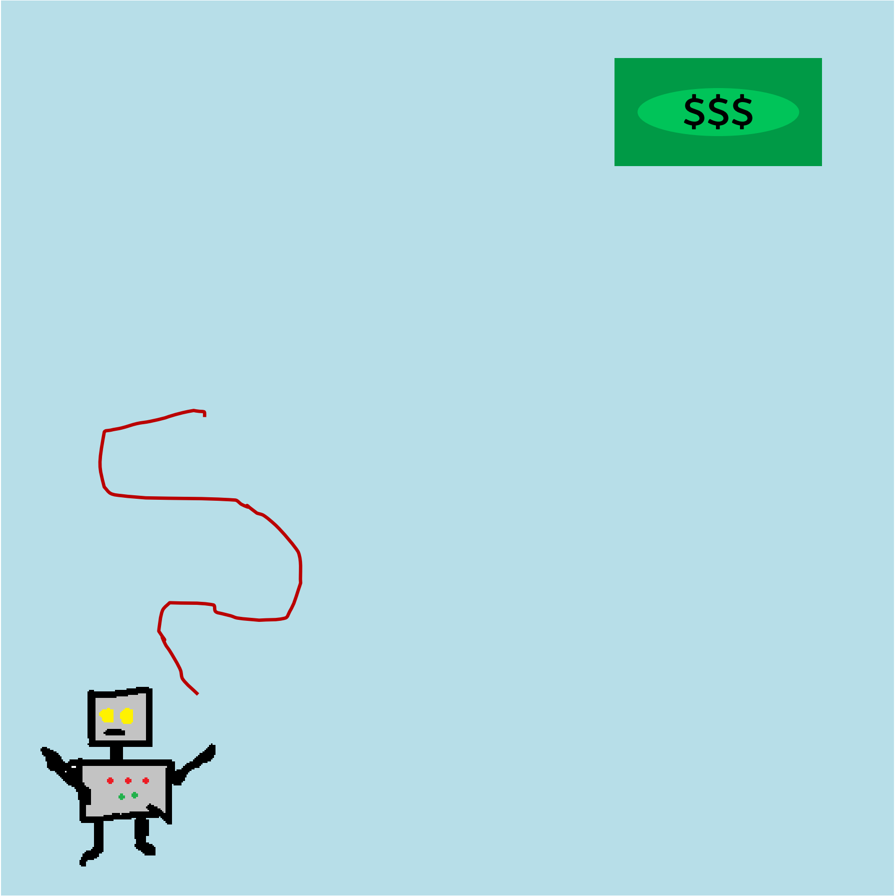

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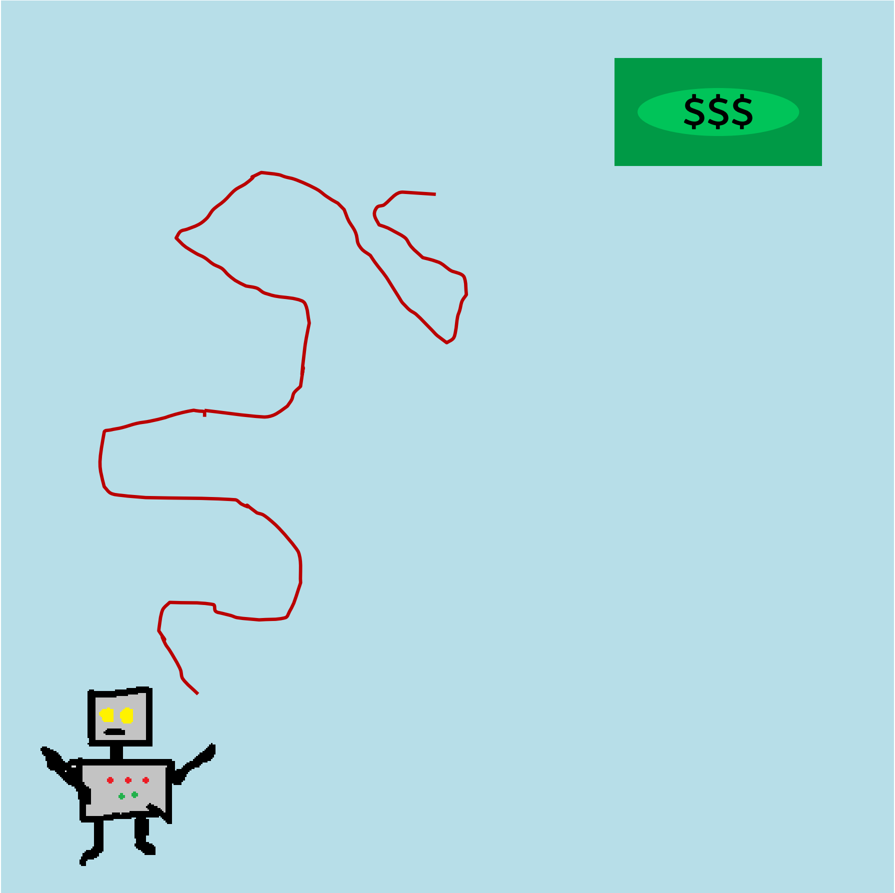

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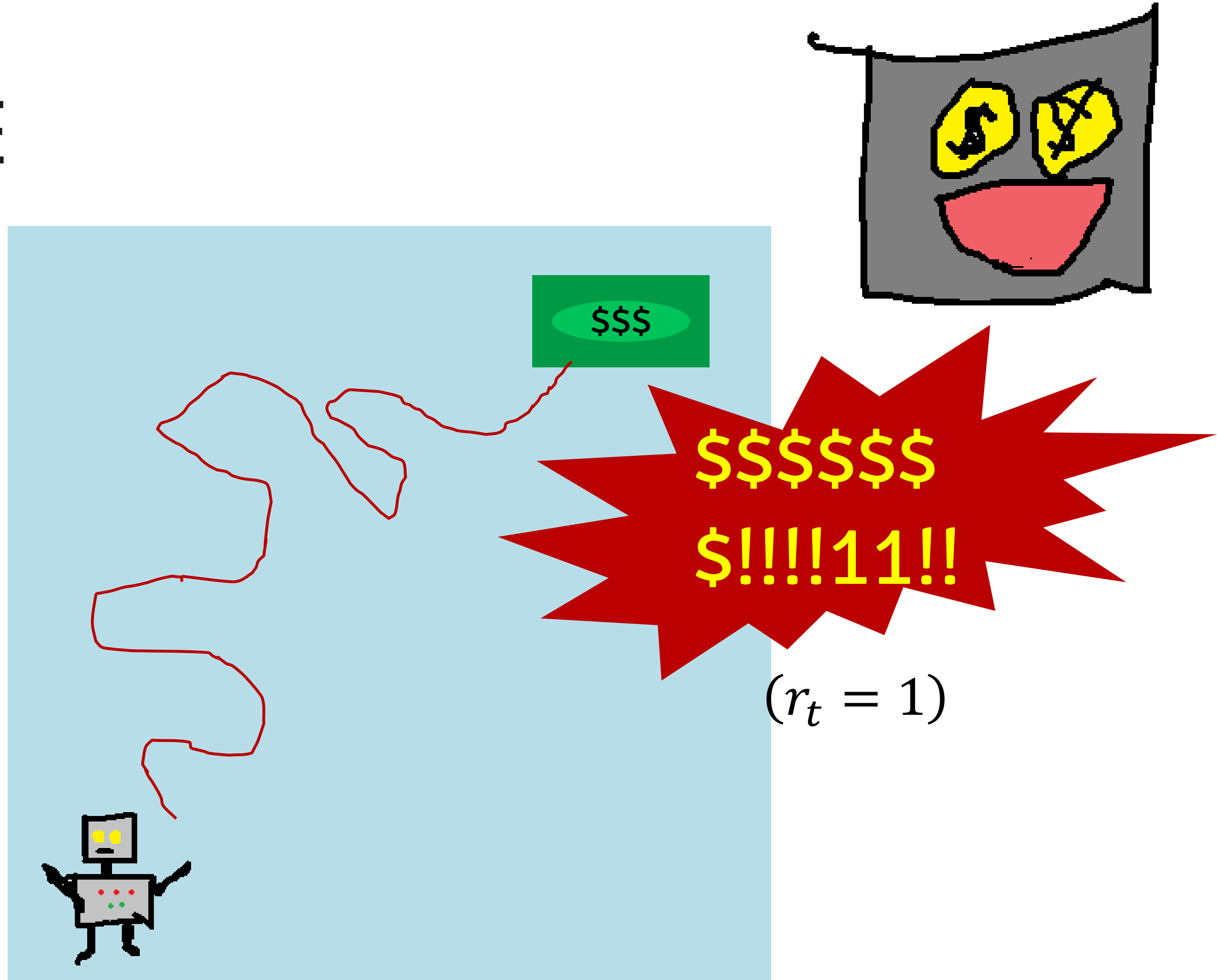
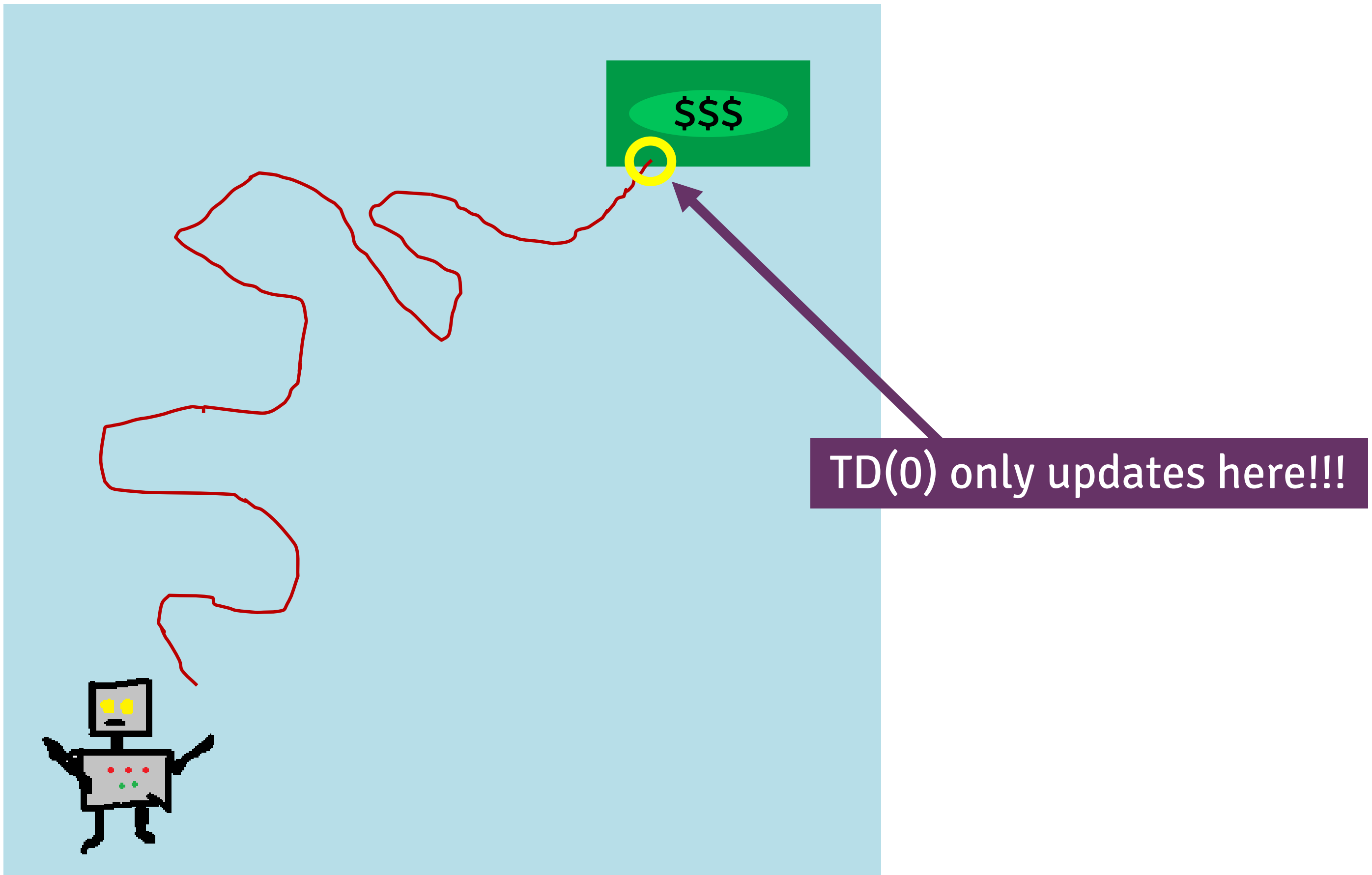


$(r_t = 1)$

EXAMPLE



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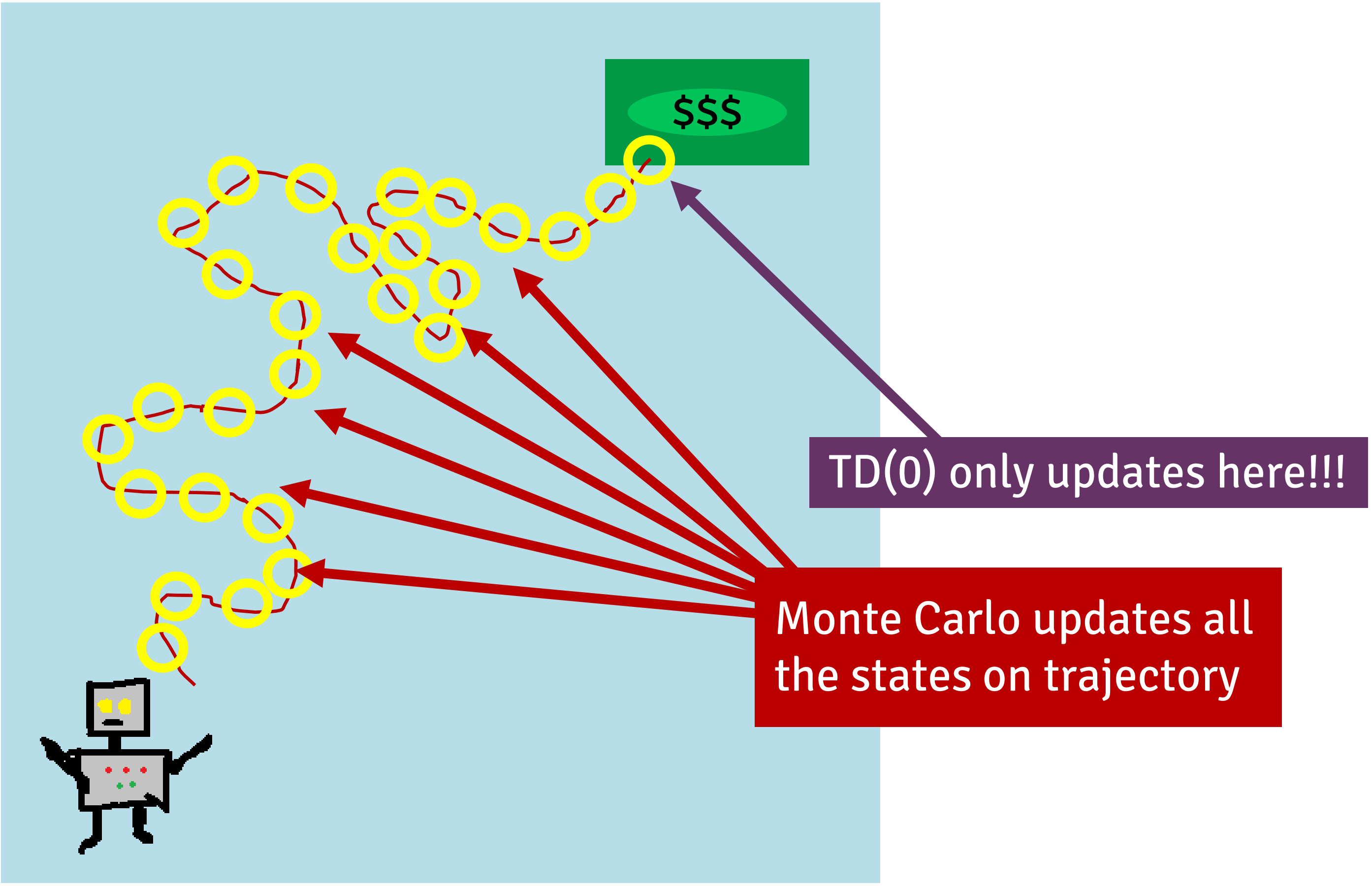
$$$
TD(0) only updates here!!!
Monte Carlo updates all the states on trajectory

BETWEEN MONTE CARLO AND TD(0): TD(λ)

Idea: propagate the updates δ_t to all previous states on the trajectory

Between Monte Carlo and TD(0): TD(λ)

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- Update in x_t: $\hat{V}_{t+1}(x_t) = \hat{V}_t(x_t) + \alpha_t \delta_t$

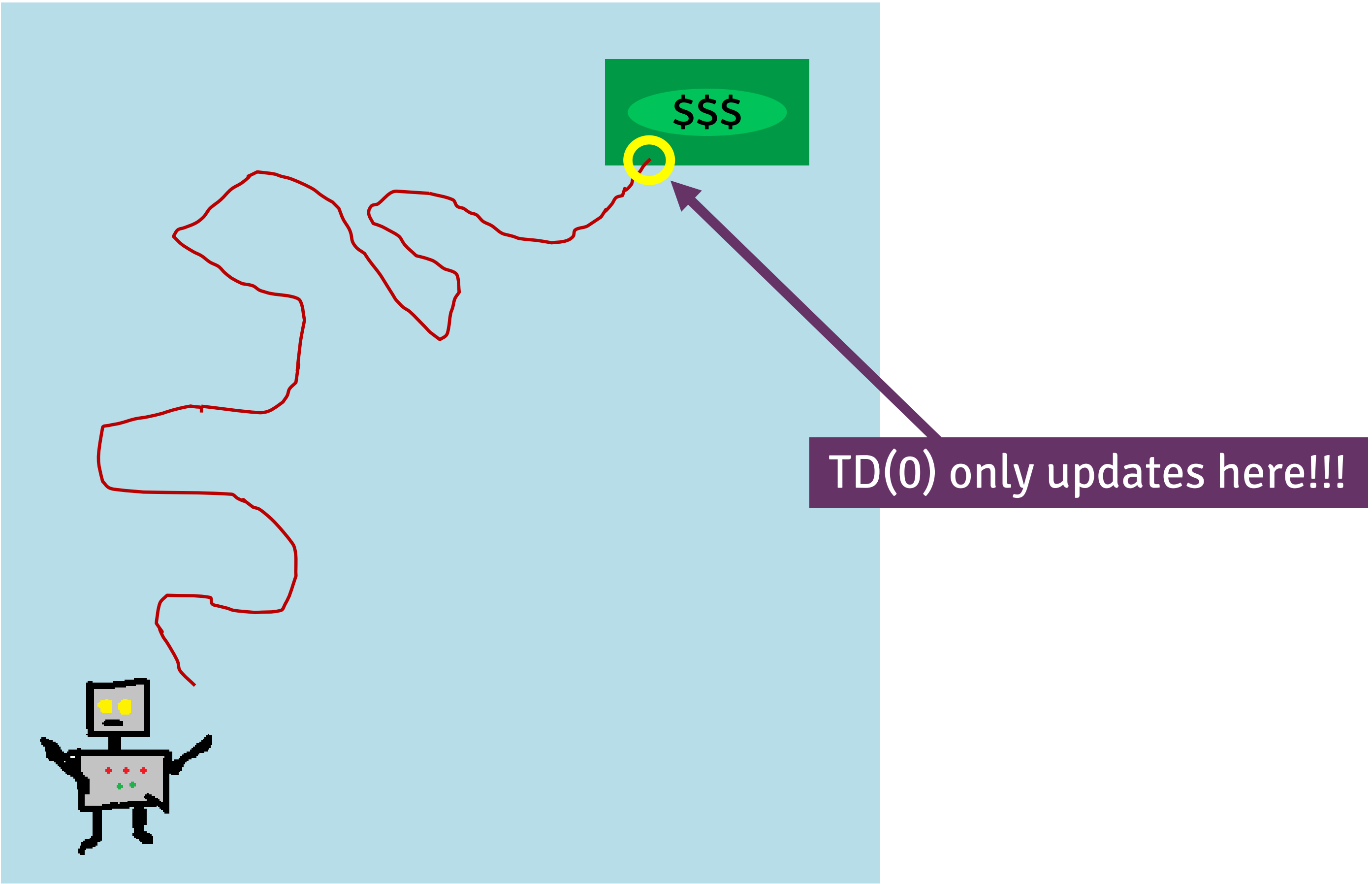
BETWEEN MONTE CARLO AND TD(0): TD(λ)

Idea: propagate the updates δ_t to all previous states on the trajectory

- Update in x_t: $\hat{V}_{t+1}(x_t) = \hat{V}_t(x_t) + \alpha_t \delta_t$
- Update in x_{t-1}: $\hat{V}_{t+1}(x_{t-1}) = \hat{V}_t(x_{t-1}) + \alpha_t \gamma\lambda \delta_t$
- Update in x_{t-2}: $\hat{V}_{t+1}(x_{t-2}) = \hat{V}_t(x_{t-2}) + \alpha_t (\gamma\lambda)^2 \delta_t$
- …
- Update in x_{t-k}: $\hat{V}_{t+1}(x_{t-k}) = \hat{V}_t(x_{t-k}) + \alpha_t (\gamma\lambda)^k \delta_t$

$$\lambda \in [0,1]$$

EXAMPLE



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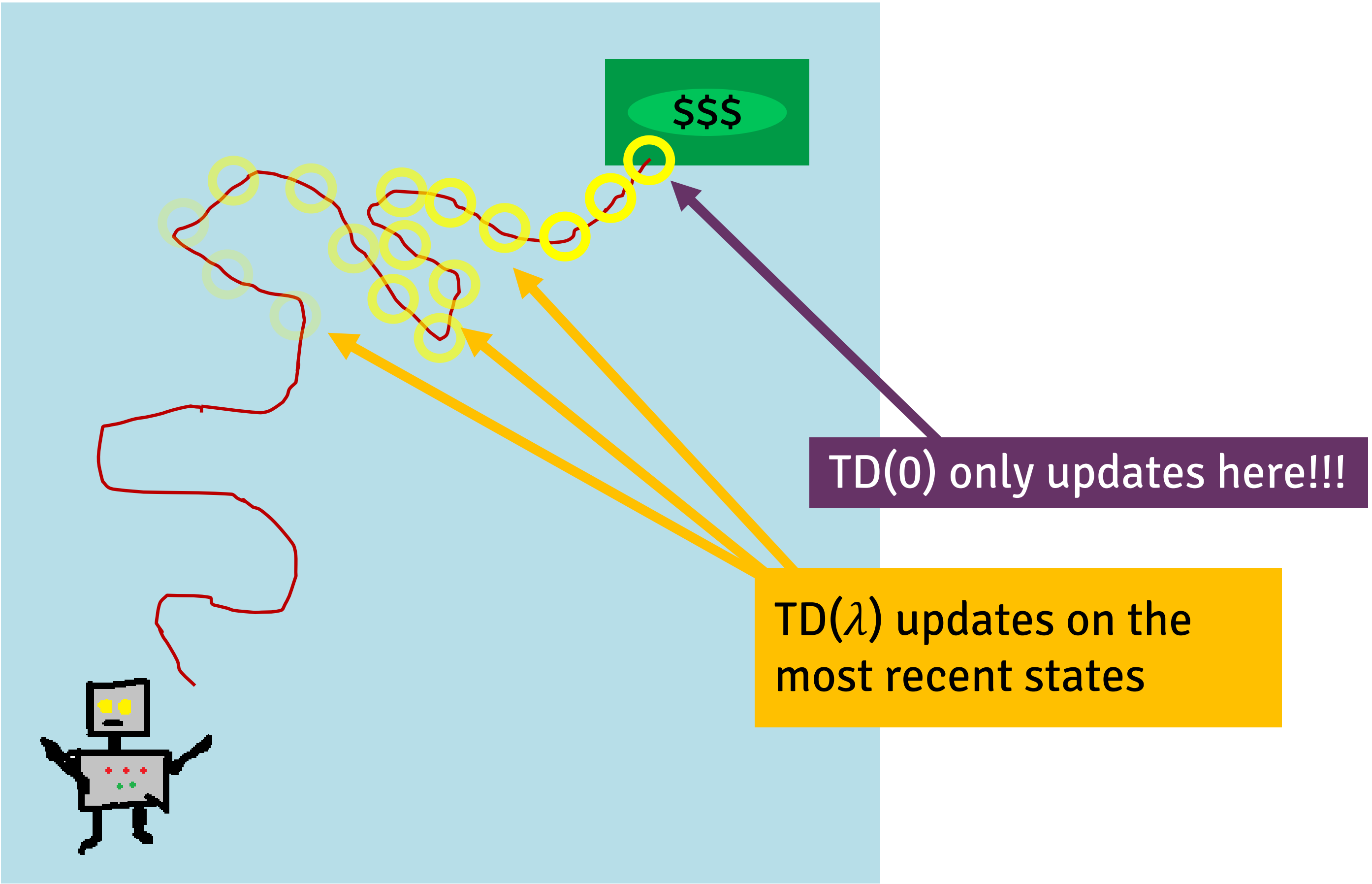
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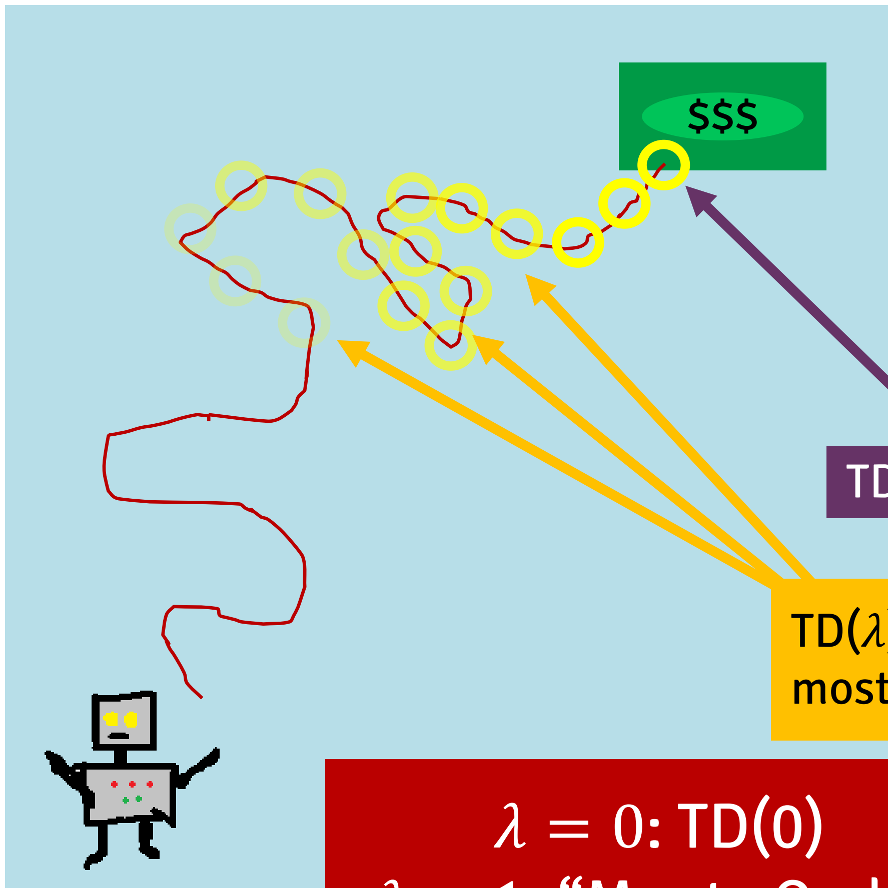

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TD(λ) updates on the most recent states

$\lambda = 0$: TD(0)
$\lambda = 1$: "Monte Carlo"

TD(λ) WITH FUNCTION APPROXIMATION

TD(0) with LFA

Input: arbitrary param. vector $\theta_0 \in \mathbf{R}^d$

For $t = 0,1, \dots,$

$$\delta_t = r_t + \gamma\theta_t^\top \phi(x_{t+1}) - \theta_t^\top \phi(x_t)$$
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TD(λ) WITH FUNCTION APPROXIMATION

TD(λ) with LFA

Input: arbitrary param. vector $\theta_0 \in \mathbf{R}^d$

For $t = 0, 1, \ldots,$

$$z_t = \gamma\lambda z_{t-1} + \phi(x_t)$$

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z_t: "eligibility trace"

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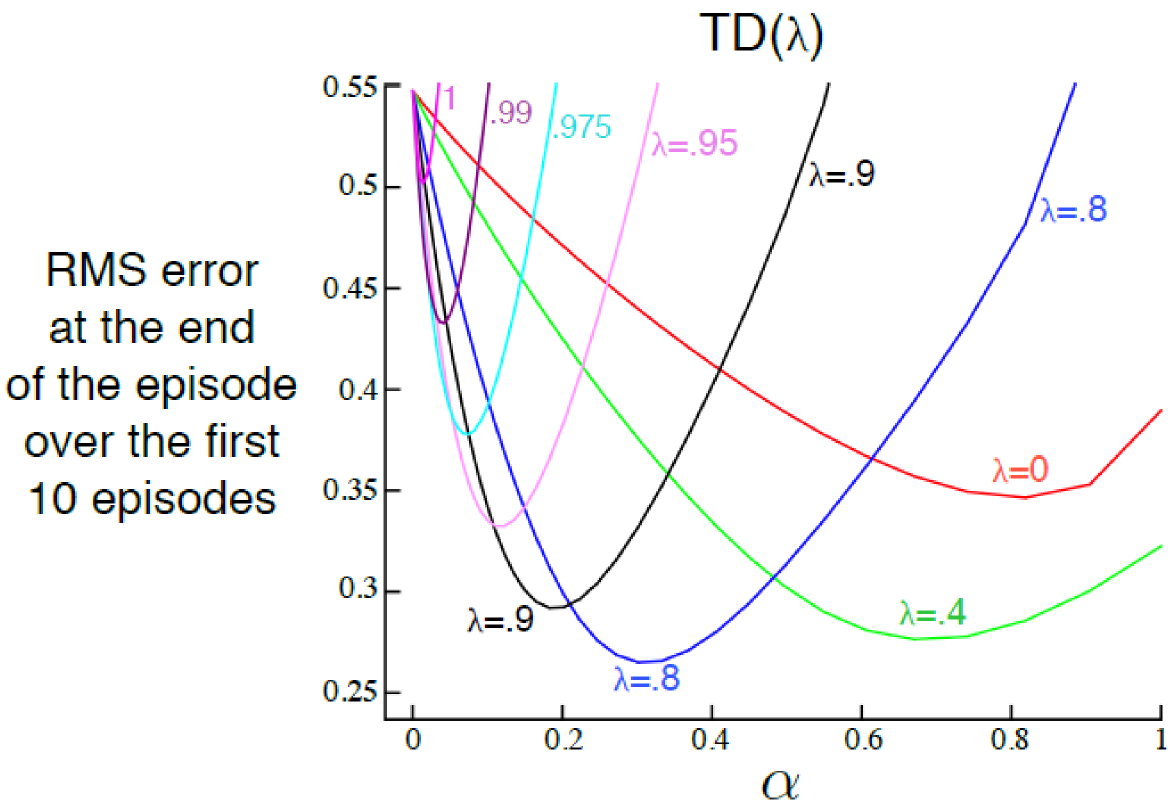
Equilibrium:
$\mathbf{E}[\delta_t z_t] = 0$

WHAT IS THE RIGHT CHOICE OF λ?



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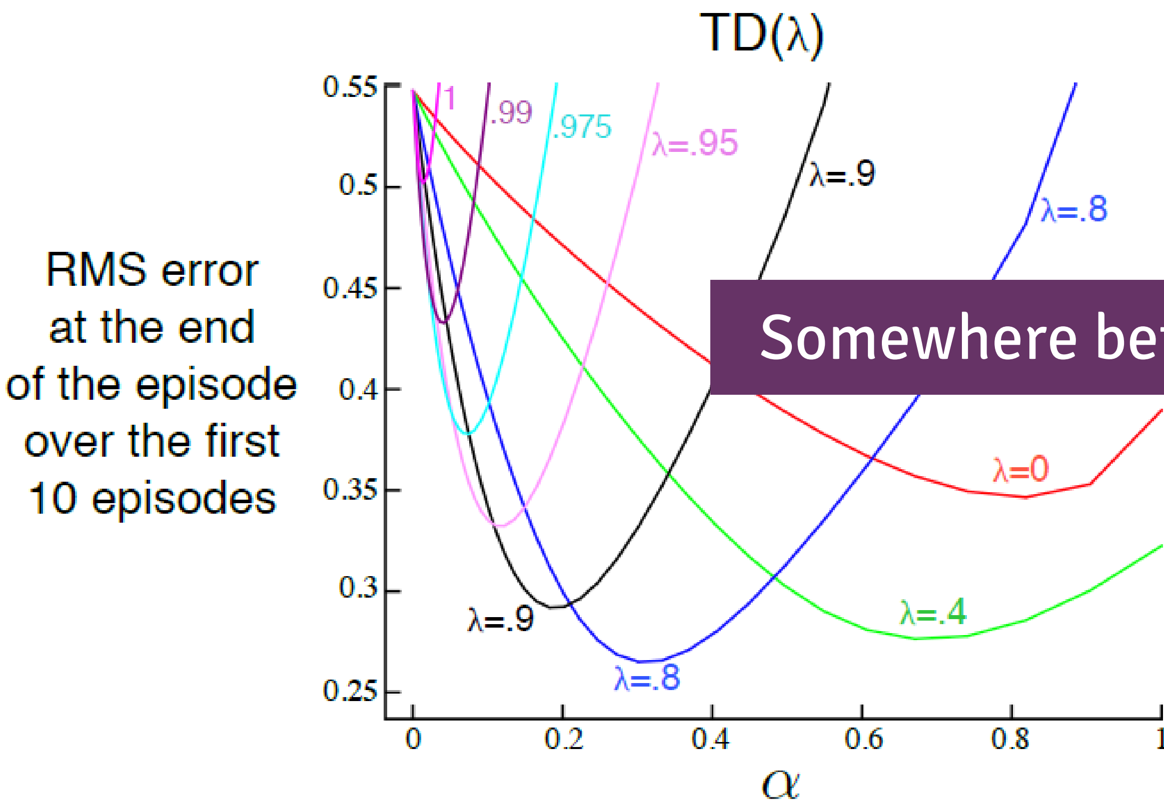

Somewhere between 0 and 1 ☺

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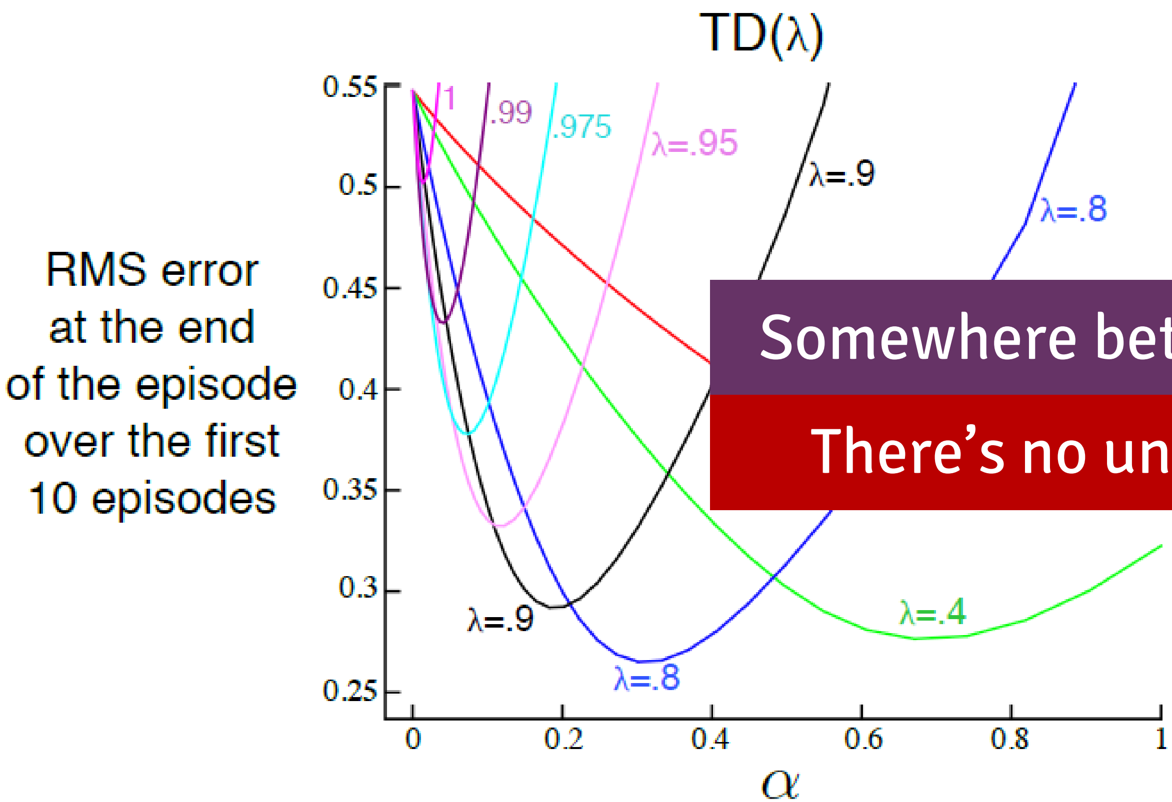
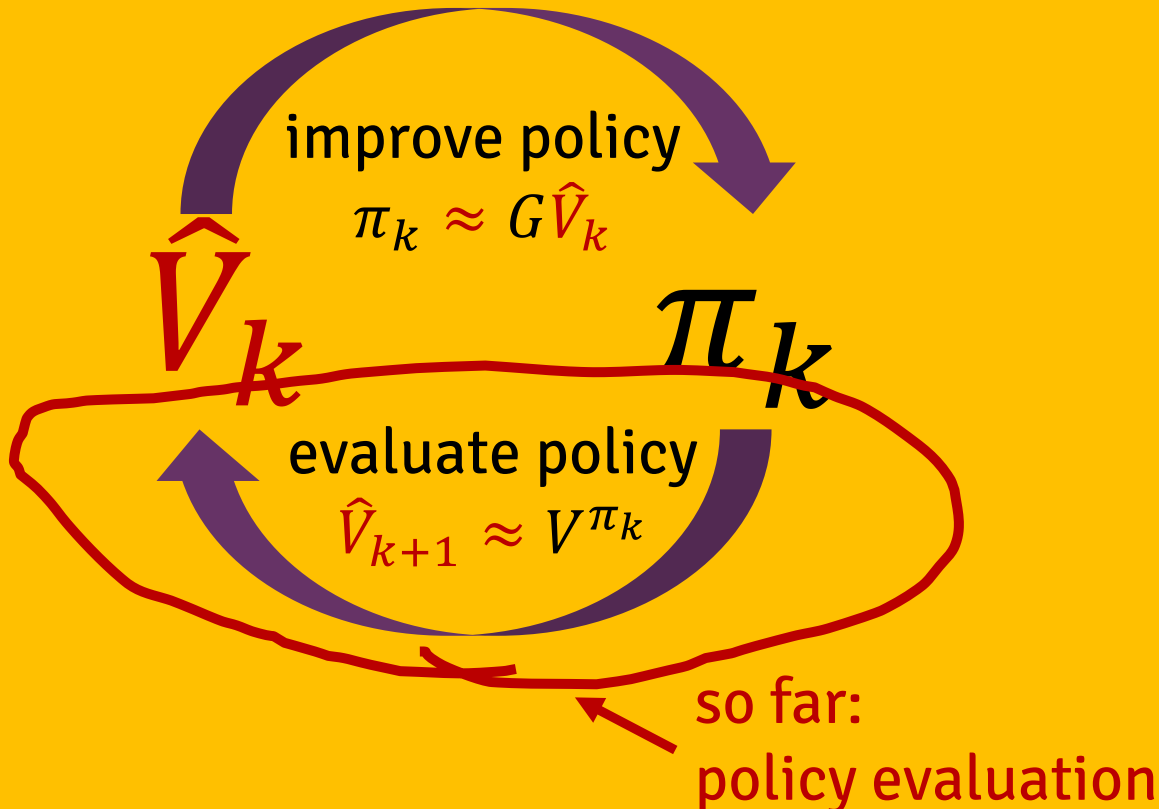


Somewhere between 0 and 1 ☺

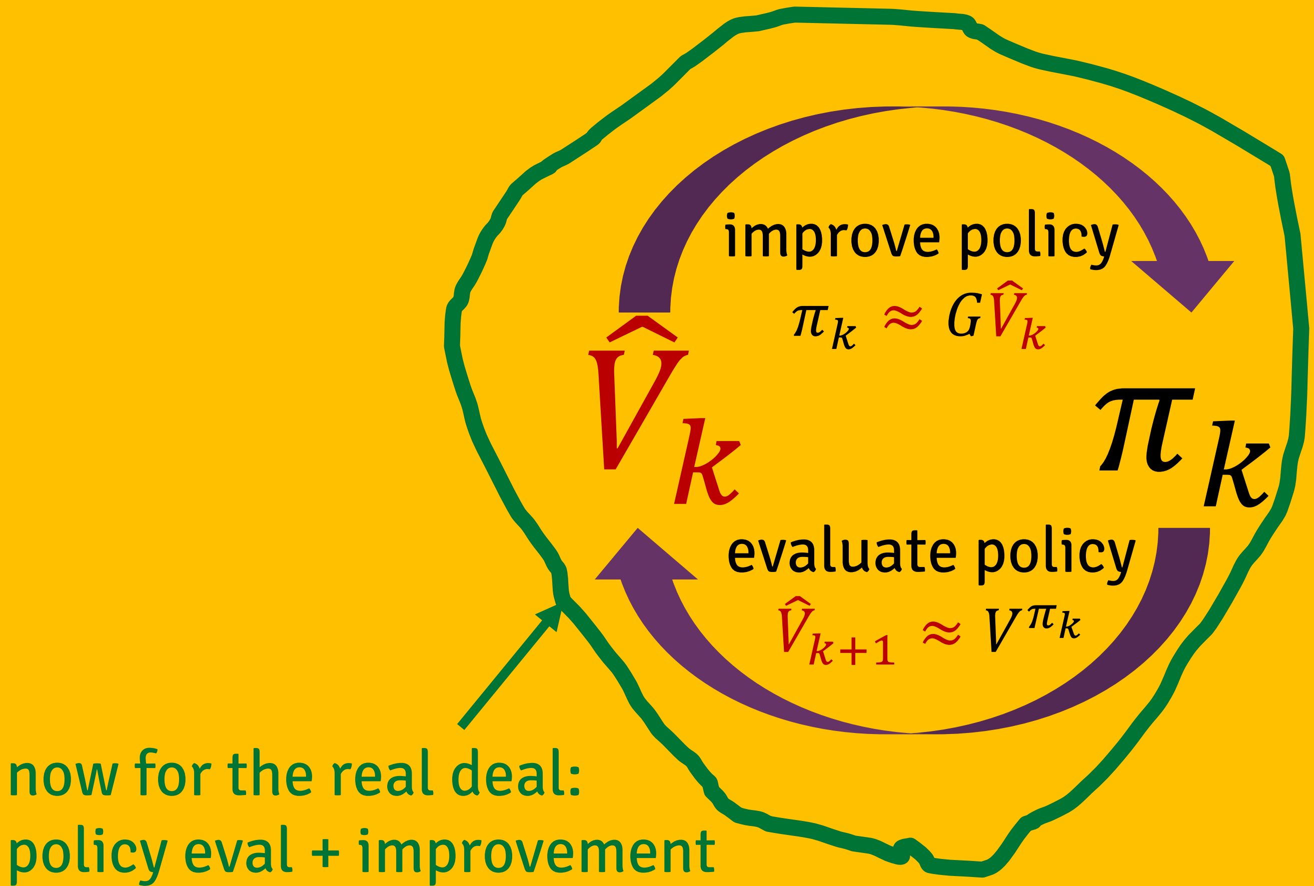
There's no universal recipe!

Basic RL algorithms

1. building blocks of RL methods
2. policy evaluation
   - Monte Carlo
   - temporal difference learning
3. policy evaluation and learning
   - SARSA
   - Q-learning



FROM POLICY EVALUATION POLICY IMPROVEMENT



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CONTROL WHILE LEARNING: SARSA

Idea: Let's try to improve the policy on the fly, based on the current value estimates!

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Changes to the TD recipe:

- Estimate the Q-function instead of V
- Compute ε-greedy policy w.r.t. $\hat{Q}_t$:

$$\pi_t(x) = \begin{cases} \arg\max \hat{Q}_t(x,a)\,, & \text{w.p. } 1-\varepsilon \\ \text{uniform random action,} & \text{w.p. } \varepsilon \end{cases}$$

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- Optimize the Bellman error

$$\Delta_t = \mathbf{E}\left[r_t + \gamma \hat{Q}_t(x_{t+1}, \pi_t(x_{t+1}))\right] - \hat{Q}_t(x_t, a_t)$$

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Drifting target policy!

CONTROL WHILE LEARNING: SARSA

SARSA

Input: arbitrary $\hat{Q}_0 : X \times A \to \mathbf{R}$

For $t = 0,1, \dots,$

- Choose action $a_t \sim \varepsilon$-greedy w.r.t. $\hat{Q}_t$
- Observe r_t, x_{t+1}, a'_{t+1}
- Compute

  $\delta_t = r_t + \gamma \hat{Q}_t(x_{t+1}, a'_{t+1}) - \hat{Q}_t(x_t, a_t)$

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SARSA = $(s_t, a_t, r_t, s_{t+1}, a'_{t+1})$

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SARSA ~ XARXA

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TRACKING A DRIFTING POLICY

Bellman error "optimized" by SARSA:

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target policy $\pi_t =$ behavior policy π_t

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On-policy learning:

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Multiple challenges for analysis:

- Data distribution drifts as policy keeps changing
- The target function itself drifts as well!
- Behavior policy needs to be increasingly greedy to converge to optimum ($\varepsilon \to 0$ for large t)

OFF-POLICY CONTROL: Q-LEARNING

Idea: Let's try to

- directly learn about Q^*, and
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ON-POLICY VS. OFF-POLICY

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ON-POLICY VS. OFF-POLICY

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Multiple challenges for analysis:

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- Behavior policy needs to be increasingly greedy to converge to optimum – a little less

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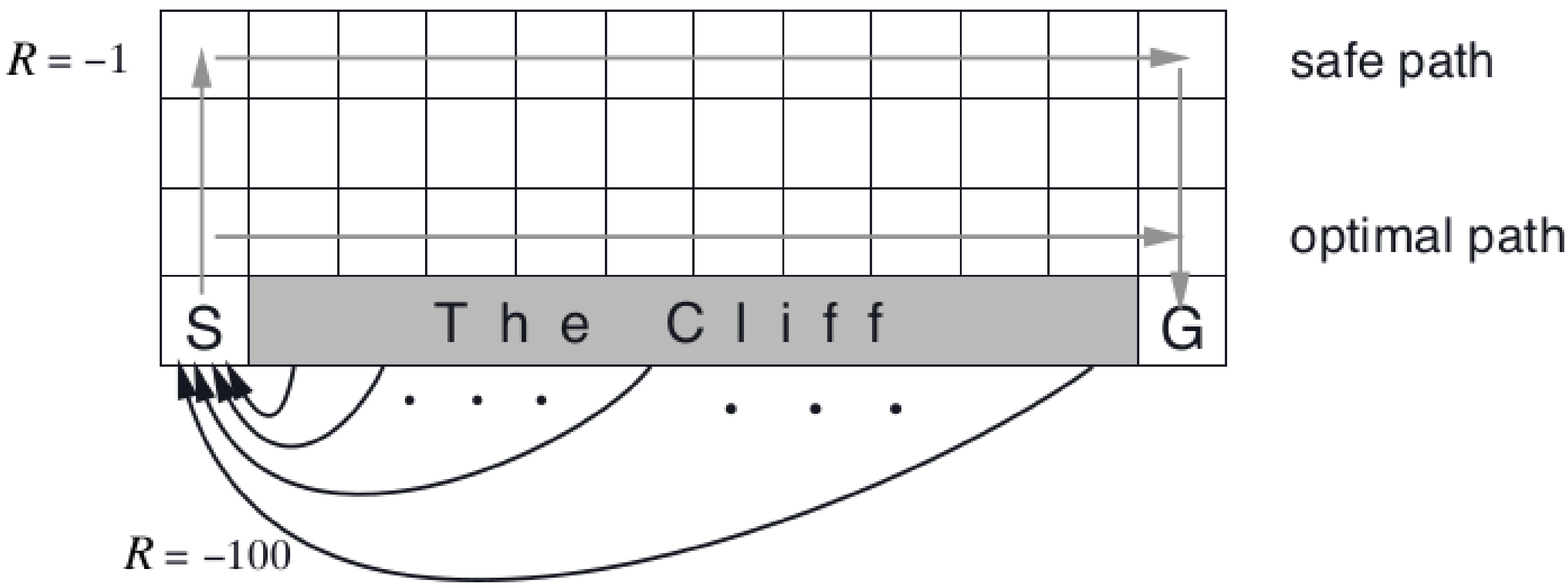
EXAMPLE: Q-LEARNING VS. SARSA

The cliff-walking task:



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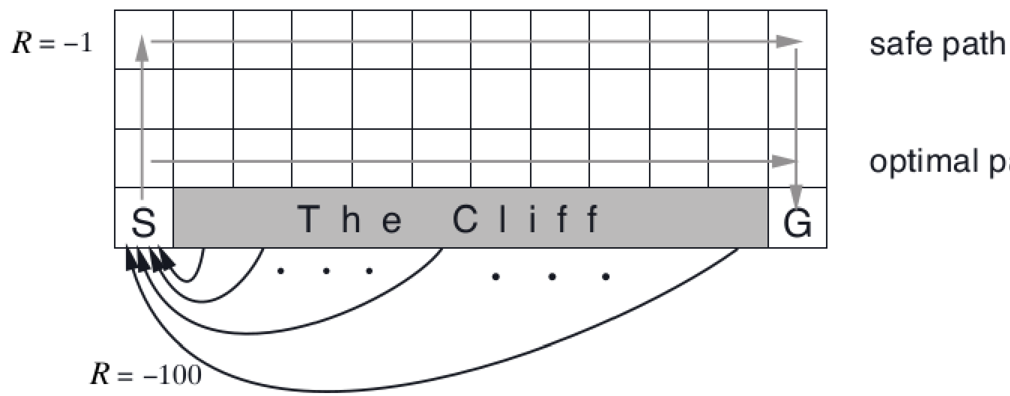

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SARSA

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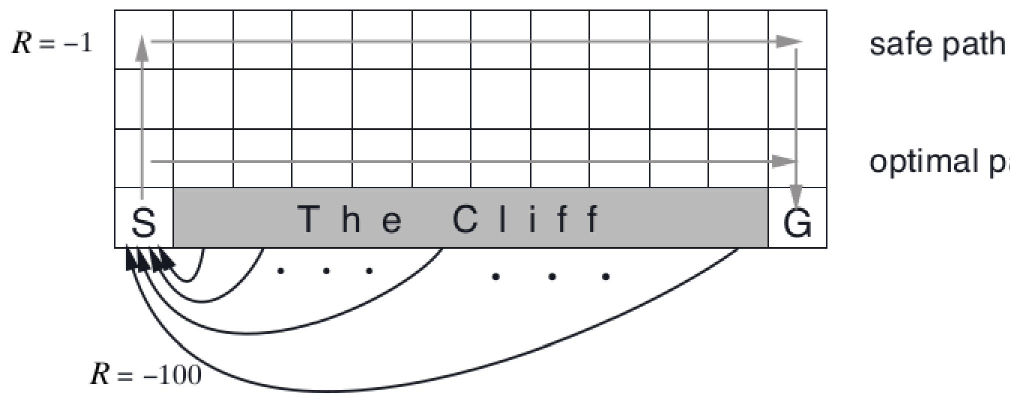

Q-learning

- Gathers less reward ☹
- Learns the optimal policy ☺☺☺

SARSA

- Gathers more reward ☺
- Learns the longer path ☹☹☹

Q-LEARNING VS. SARSA WITH FUNCTION APPROXIMATION

- Both algorithms can be adapted to linear and non-linear FA by using the update rule
$$\theta_{t+1} = \theta_t + \alpha_t \delta_t \nabla_\theta Q_\theta(x_t, a_t)$$
- Ditto with eligibility traces: SARSA(λ), $Q(\lambda)$

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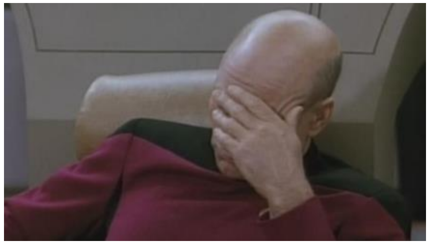
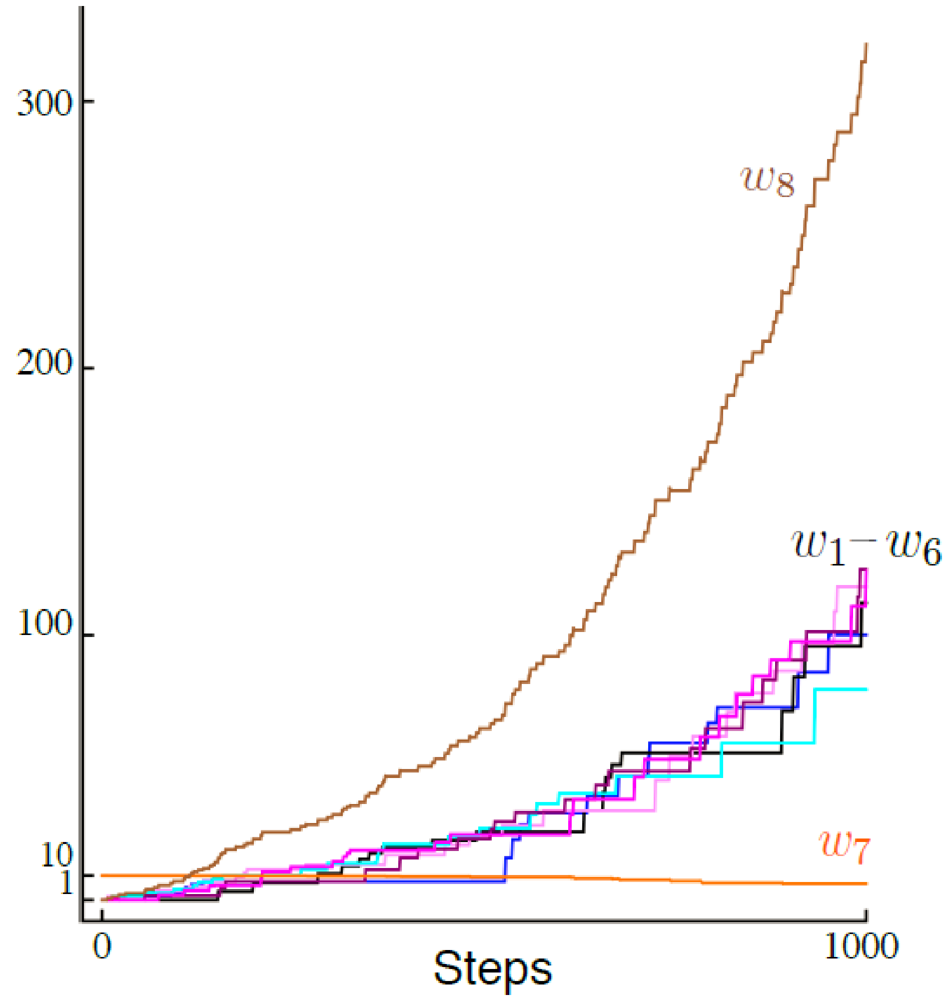
  - Practical solution: tune it until it works



DIVERGENCE OF OFF-POLICY TD LEARNING

The "deadly triad":

- Function approximation
- Bootstrapping
- Off-policy learning



DIVERGENCE OF OFF-POLICY TD LEARNING

The "deadly triad":

- Function approximation
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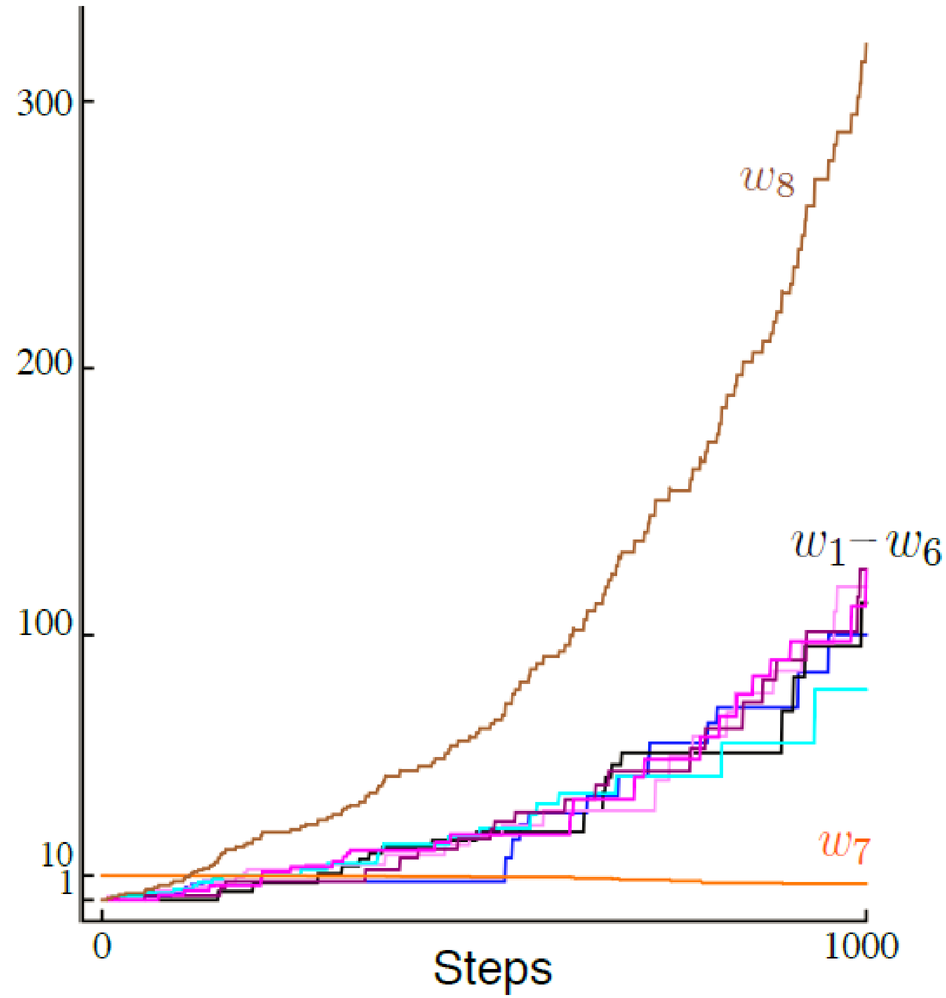
BUT

Divergence is typically not too extreme when behavior policy is close to evaluation policy and FA is linear



Next week:
Batch RL and
policy optimization