Ecu. Diferencia

① $h[n] = 2.5(0.5^n)u[n] - 1.5(0.3^n)u[n]$

$$H(z) = 2.5\frac{z}{z-0.5} - 1.5\frac{z}{z-0.3}$$

Orden 2

$$= \frac{z^2}{(z-0.5)(z-0.3)} = \frac{z^2}{z^2-0.8z+0.15} = \frac{1}{1-0.8z^{-1}+0.}$$

$$\Rightarrow y[n] = x[n] + 0.8y[n-1] - 0.15y[n-2]$$

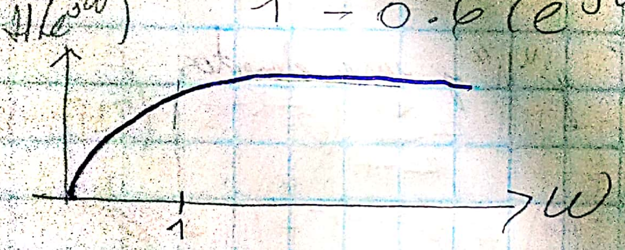
② $y[n] = \overset{b_0}{\lambda} x[n] - \overset{b_1}{\lambda} x[n-1] + \overset{a_0}{\lambda} y[n-1]$; $\lambda = 0.6$ respuesta frecuencia

$$H(z) = \frac{\lambda - \lambda z^{-1}}{1-\lambda z^{-1}} = \frac{0.6-0.6z^{-1}}{1-0.6z^{-1}}$$

$$\Rightarrow H(e^{j\omega}) = \frac{0.6-0.6(e^{-j\omega})}{1-0.6(e^{-j\omega})}$$

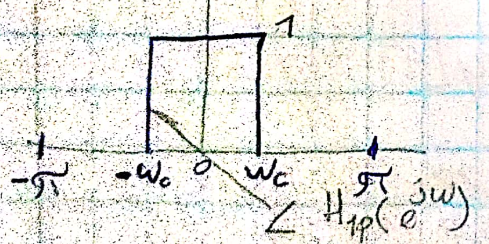
$\cos(\omega)=1$

$$\begin{cases} \text{Magnitud} = 0.6820 \\ \text{Ángulo} = 0.9292 \text{ rad} \end{cases}$$



Pasa altas (HPF)

③



$$H(e^{j\omega}) = \begin{cases} 1 & \text{si } |\omega| \le \omega_c \\ 0 & \text{si } \omega_c < |\omega| \le \pi \end{cases}$$

$$h[n] = \frac{1}{2\pi}\int H(e^{j\omega})e^{j\omega n}\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega n}\,d\omega = \frac{1}{2\pi}\left(\frac{e^{j\omega n}}{jn}\right)\Bigg|_{-\omega_c}^{\omega_c} = \frac{1}{2\pi n j}\left(e^{j\omega_c n} - e^{-j\omega_c n}\right)$$

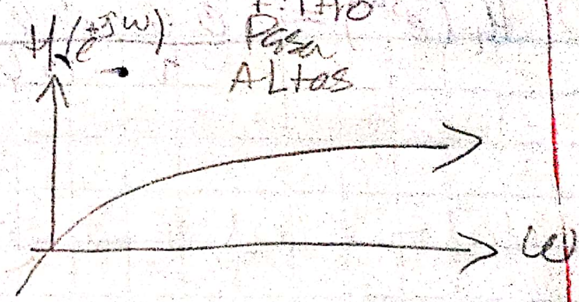
$$= \frac{1}{\pi n}\left(\frac{e^{j\omega_c n} - e^{-j\omega_c n}}{2j}\right) = \frac{\operatorname{sen}(\omega_c n)}{\pi n}$$

∴ No es causal, tampoco es implementable

④ $y[n] = \frac{1}{2} x[n] - \frac{1}{2} x[n-1]$

$H(z) = 1/2 - 1/2\, z^{-1}$

$H(e^{j\omega}) = \frac{1}{2} - \frac{1}{2}(e^{-j\omega}) \rightarrow$



⑤ ① $\omega[n] = 0.75\,\omega[n-1] - 0.125\,\omega[n-2] + x[n]$

② $y[n] = \omega[n] - \omega[n-1]$ {Ec. dif?}

$\mathcal{Z}\{①\}$

$\Rightarrow W(z)(0.75 z^{-1} + 0.125 z^{-2} + 1) = X(z)$ ③

$\mathcal{Z}\{②\}$

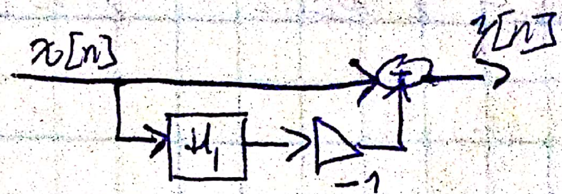
$\Rightarrow Y(z) = W(z)(1 - z^{-1})$ ④

$\Rightarrow$ ③ / ④

$\hookrightarrow \left(\frac{1 - 0.75 z^{-1} + 0.125 z^{-2}}{1 - z^{-1}}\right)^{-1} = H(z)$

$\mathcal{Z}^{-1}\{H(z)\} \Rightarrow y[n] = x[n] + 0.75\,y[n-1] - 0.125\,y[n-2] - x[n-1]$

⑥ $H_1(e^{j\omega}) = \dfrac{0.6 - 0.6(e^{-j\omega})}{1 - 0.6(e^{-j\omega})}$



$y[n] = x[n] + \mathcal{Z}^{-1}\{-H_1\}$

$y[n] = x[n] - 0.6x[n] + 0.6x[n-1] - 0.6y[n-1]$

$y[n] = 0.4x[n] + 0.6x[n-1] - 0.6y[n-1]$

$H(z) = \dfrac{0.4 + 0.6z^{-1}}{1 + 0.6z^{-1}}$ Pasa bajos

⑦ ① $H_2(e^{j\omega}) = \frac{1}{2} - \frac{1}{2}(e^{-j\omega})$ ② $y[n] = (-1)^n\left(\frac{1}{2}x[n] - \frac{1}{2}x[n-1]\right)$

② $y[n] = \dfrac{(-1)^n}{2}x[n] - \dfrac{(-1)^n}{2}x[n-1]$

$= \dfrac{\delta[n](-1)^n}{2} - \dfrac{\delta[n-1](-1)^n}{2}$

$\Rightarrow \frac{1}{2}\sum \delta[n](-1)^n z^{-n} - \frac{1}{2}\sum(-1)^n\delta[n-1]z^{-n}$

$= \frac{1}{2} - \frac{1}{2}z^{-1}$

$\Rightarrow x[n] \rightarrow \boxed{H_1} \rightarrow \boxed{H_2} \rightarrow y[n]$ ∴ Solo incremento el orden, Pasa alta lineal

9 FIR $\quad H(z) = 1 - \sqrt{2}\, z^{-1} - z^{-2}$

a) Fase lineal?
Al ser FIR es perfectamente lineal, además, cumple que no es recursiva

b) $H(e^{j\omega}) = 1 - \sqrt{2}\,(e^{-j\omega}) - (e^{-2j\omega})$

c) $|H(e^{j\omega})| = ?$

$$H(e^{j\omega}) = 1 - \sqrt{2}\,(\cos(\omega) - j\,\mathrm{sen}(\omega)) - (\cos(2\omega) - j\,\mathrm{sen}(2\omega))$$

$$\Rightarrow |H(e^{j\omega})| = \sqrt{[1 - \sqrt{2}\cos(\omega) - \cos(2\omega)]^2 + [\mathrm{sen}(\omega) - \mathrm{sen}(2\omega)]^2}$$

$$\measuredangle H(e^{j\omega}) = \tan^{-1}\left(\frac{-\sqrt{2}\,\mathrm{sen}(\omega) - \mathrm{sen}(2\omega)}{1 - \sqrt{2}\cos(\omega) - \cos(2\omega)}\right)$$

d) $\dfrac{-d}{d\omega}\left(\measuredangle H(e^{j\omega})\right)$ $\qquad$ DUPA ...

$$= \frac{1}{\left(\frac{-\sqrt{2}\,\mathrm{sen}(\omega) - \mathrm{sen}(2\omega)}{1 - \sqrt{2}\cos(\omega) - \cos(2\omega)}\right)^2 + 1} \cdot$$

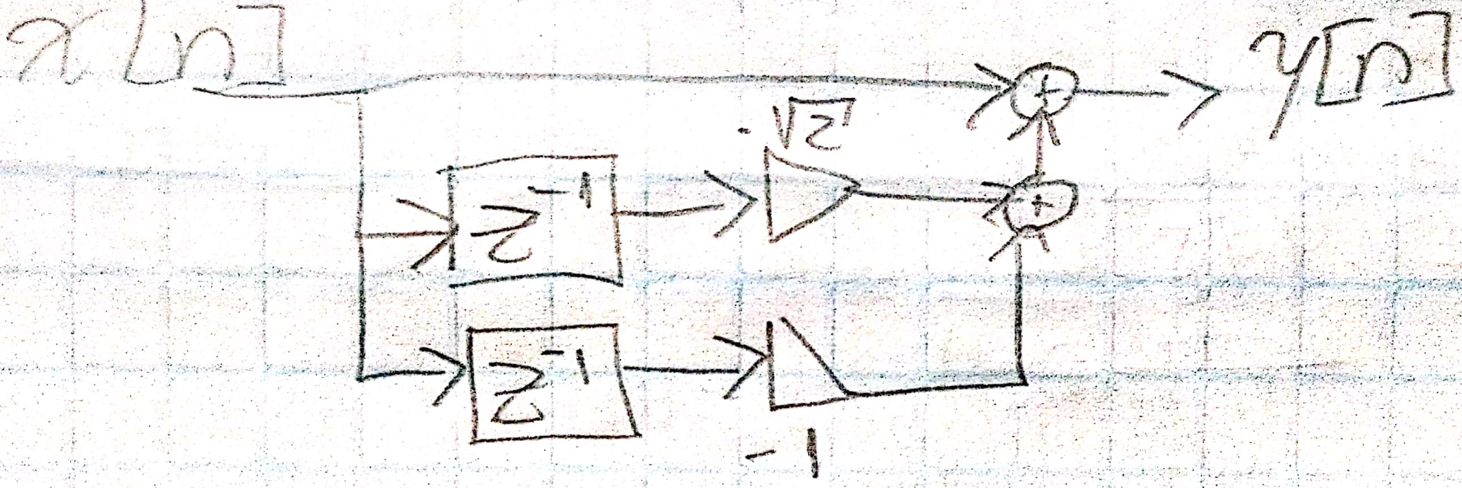
$$\frac{[-\sqrt{2}\cos(\omega) - 2\cos(2\omega)][1 - \sqrt{2}\cos(\omega) - \cos(2\omega)] - [\sqrt{2}\,\mathrm{sen}(\omega) + 2\,\mathrm{sen}(2\omega)][-\sqrt{2}\,\mathrm{sen}(\omega) - \mathrm{sen}(2\omega)]}{(1 - \sqrt{2}\cos(\omega) - \cos(2\omega))^2}$$

10

$$\frac{\left[-\sqrt{2}\cos(\omega)-2\cos(2\omega)\right]\left[1-\sqrt{2}\cos(\omega)-\cos(2\omega)\right]-\left[\sqrt{2}\,\mathrm{sen}(\omega)+2\,\mathrm{sen}(2\omega)\right]\left[-\sqrt{\ }\right.}{\left(1-\sqrt{2}\cos(\omega)-\cos(2\omega)\right)^2}$$

10

$$y[n] = x[n] - \sqrt{2}\,x[n-1] - x[n-2]$$



1) $A = 10$ $F = 100\,Hz$ $f_s = 1KHz$

$$\Omega_0 = 2\pi f = 2\pi(100Hz) = 200\pi \frac{rad}{seg}$$

Para evitar aliasing $\rightarrow$ $F_s \geq 2F \Rightarrow \Omega_s \geq 2\Omega_m$

$$1KHz \geq 200\,Hz \checkmark$$

$$\omega = \Omega \cdot T_s \Rightarrow \left(200\pi \frac{rad}{seg}\right)\left(\frac{1}{1000\,Hz}\right) = \omega = 0.2\pi$$

$$H(z) = \frac{(z - e^{j\omega})(z - e^{-j\omega})}{(z - r e^{j\omega})(z - r e^{-j\omega})}$$ Asumo $r = 0.99$

$$= \frac{z^2 - z e^{-j0.2\pi} - z e^{j0.2\pi} + 1}{z^2 - e^{-j0.2\pi} z r - e^{j0.2\pi} z r + r^2}$$

$$= \frac{z^2 - z\left(e^{-0.2\pi j} + e^{0.2\pi j}\right) + 1}{z^2 - z r\left(e^{-j0.2\pi} + e^{j0.2\pi}\right) + r^2}$$

$$= \frac{z^2 - z\left[\cos(+0.2\pi) - \cancel{j\,\text{sen}(0.2\pi)} + \cos(0.2\pi) \pm \cancel{j\,\text{sen}(0.2\pi)}\right] + 1}{z^2 - z r\left[\cos(0.2\pi) + \cos(0.2\pi)\right] + r^2}$$

$$= \frac{z^2 - z\left[2\cos(0.2\pi)\right] + 1}{z^2 - z r\left[2\cos(0.2\pi)\right] + r^2} = \frac{z^2 - z(1.618) + 1}{z^2 - 0.99 z(1.618) + (0.99)^2}$$

$$H(z) = \frac{z^2 - 1.618z + 1}{z^2 - 1.601z + 0.98} \Rightarrow H(z) = -9\left(H(z)\right) + 10$$

$$H(z) = \frac{z^2 - 1.618z + 0.98}{z^2 - 1.601z + 0.998}$$

(12) LPF IIR digital $\omega_c = 10\,T_s \ \frac{rad}{muestra}$ $F_s = 80 \ \frac{rad}{s}$

$$\omega_c = 10\left(\frac{1}{80 \frac{rad}{s}}\right) = 0.125 \ \frac{rad}{muestra}$$

1.91.38

$$\omega_c = \Omega_c T_s = \frac{2\pi f_c}{F_s} \Rightarrow \frac{2\pi f_c}{80} = 0.125 \Rightarrow f_c = 1.591 \text{ Hz}$$

⇒ Filter Designer

* Butterworth * Orden 6 *

$$* \ H_1(z) = \left[\frac{1 + 2z^{-1} + z^{-2}}{1 - 1.92z^{-1} + 0.93z^{-2}}\right] 0.00377$$

$$H_2(z) = \left[\frac{1 + 2z^{-1} + z^{-2}}{1 - 1.82z^{-1} + 0.838z^{-2}}\right] 0.00358$$

$$H_3(z) = \left[\frac{1 + 2z^{-1} + z^{-2}}{1 - 1.77z^{-1} + 0.785z^{-2}}\right] 0.00348$$

$$H(z) = H_1(z)\,H_2(z)\,H_3(z)$$

$$= \frac{(1 + 2z^{-1} + z^{-2})^3 \, 0.0000000469}{(1 - 1.92z^{-1} + 0.93z^{-2})(1 - 1.82z^{-1} + 0.838z^{-2})(1 - 1.77z^{-1} + 0.775z^{-2})}$$

DUPA

$$= \frac{4.69\times10^{-8} + 2.81\times10^{-7}z^{-1} + 7.03\times10^{-7}z^{-2} + 9.38\times10^{-7}z^{-3} + 7.03\times10^{-7}z^{-4} + 2.81\times10^{-7}z^{-5} + 4.69\times10^{-8}z^{-6}}{1 - 5.51z^{-1} + 12.65z^{-2} - 15.5z^{-3} + 10.69z^{-4} - 3.92z^{-5} + 0.602z^{-6}}$$

$$y[n] = 4.69\times10^{-8}x[n] + 2.81\times10^{-7}x[n-1] + 7.03\times10^{-7}x[n-2] + 9.38\times10^{-7}x[n-3] + 7.03\times10^{-7}x[n-4] + 2.81\times10^{-7}x[n-5] + 4.69\times10^{-8}x[n-6] + 5.51y[n-1] - 12.65y[n-2] + 15.5y[n-3] - 10.69y[n-4] + 3.92y[n-5] - 0.60y[n-6]$$

13 IIR BPF digital, Butterworth

$$H_{LPF}(s) = \frac{1}{s+1} \qquad H_{HPF}(s) = \frac{s}{s+1}$$

$$\frac{w_l}{T_s} = 0.15\,\Omega_s \qquad \frac{w_u}{T_s} = 0.35\,\Omega_s \qquad A: \frac{w}{T_s} = \frac{\Omega_s}{4}$$

$$w_l = 0.15\,T_s\,\Omega_s = 0.3\pi \ \frac{rad}{muestra}$$

$$w_u = 0.35\,T_s\,\Omega_s = 0.7\pi \ \frac{rad}{muestra}$$

$$w = \frac{T_s\,\Omega_s}{4} = \frac{2}{T_s}$$

$\Rightarrow$ LPF

$$\Omega'_c = \frac{2}{T_s}\tan\left(\frac{0.7\pi}{2}\right) = \frac{3.925}{T_s}$$

$$\Rightarrow H_{LPF}(s) = \frac{1}{s+1} = \frac{1}{Ts/3.925 + 1}$$

DUPA

$$H_{LPF}(z) = \frac{3.925(z+1)}{2(z-1) + 3.925(z+1)}$$

$\Rightarrow H_{PF}$

$$\Omega' = \frac{2}{T_s}\tan\left(\frac{0.3\pi}{2}\right) \Rightarrow s = \frac{1.091}{T_s}$$

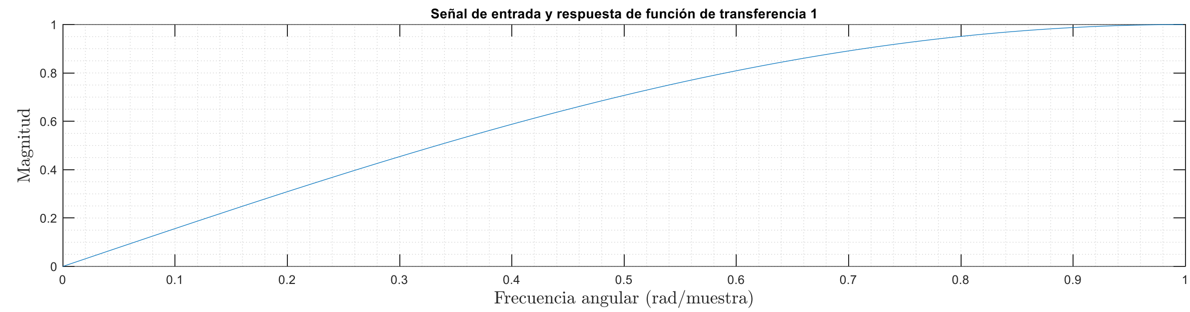
$$\Rightarrow H_{PF}(z) = \frac{1}{0.50955\frac{(z+1)}{(z-1)} + 1} = \frac{z-1}{1.50955z - 0.49095}$$

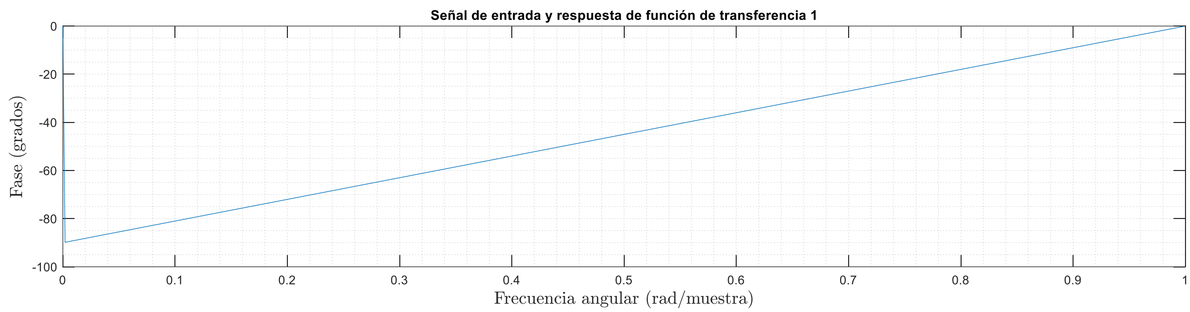
$$\Rightarrow \frac{z^2 - 1}{2z + 8.6z^2 - 0.2405}$$

$$1 = H(z) = \frac{K\,(z^2 - 1)}{2.2786 z^2 - 0.2405} \Rightarrow K = 0.7939$$

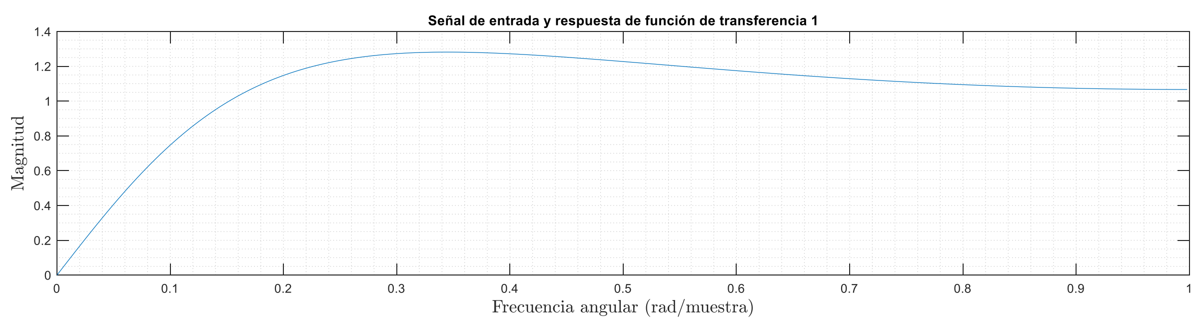
$$\therefore \; H(z) = 1.2596 \left(\frac{z^2 - 1}{2.2786 z^2 - 0.2405} \right)$$

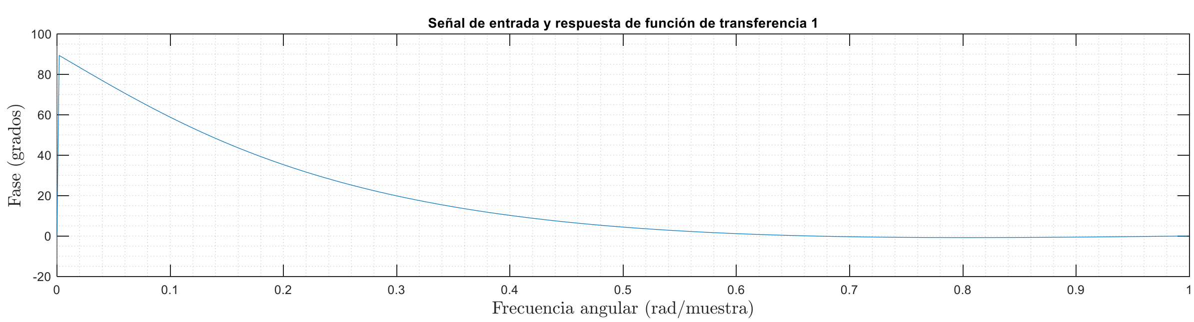
Gráfica ejercicio 4



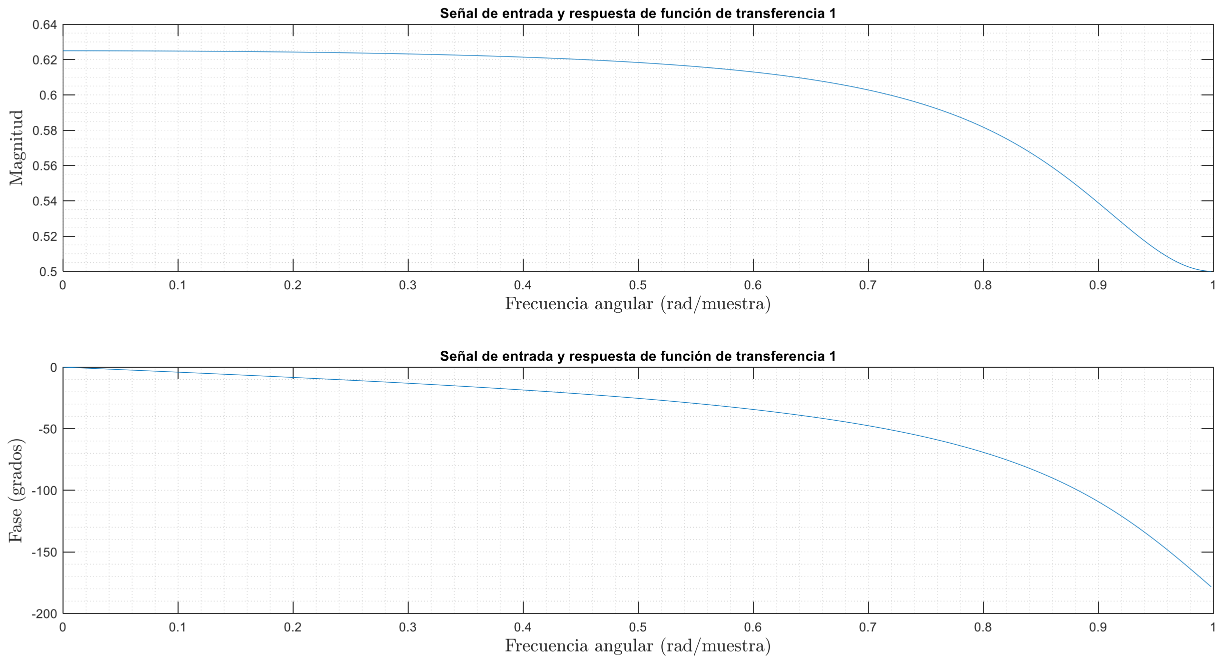


Gráfica ejercicio 5





Gráfica ejercicio 6



Gráfica ejercicio 7