Ejercicio 2

Nilson

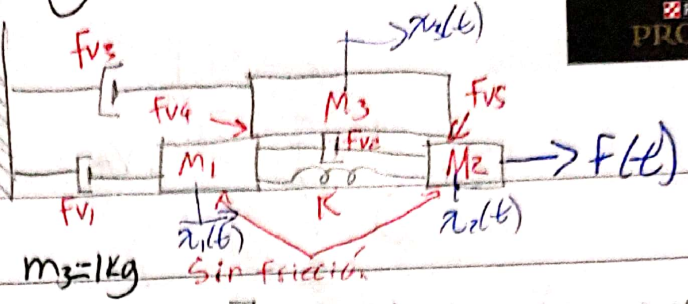
Fecha: ____ / ____

3.4

$F_{v1} = 1\ N\frac{s}{m}$ $\quad F_{v4} = 1\ N\frac{s}{m}$

$F_{v2} = 1\ N\frac{s}{m}$ $\quad F_{v5} = 1\ N\frac{s}{m}$

$F_{v3} = 1\ N\frac{s}{m}$ $\quad K = 2$



$m_1 = 2Kg \quad m_2 = 1Kg \quad m_3 = 1Kg$

M_1: $X_1(s)[M_1 s^2 + s(F_{v1}+F_{v4}+F_{v2}) + K] - (K+F_{v2})X_2(s) - (F_{v4} s)X_3(s) = 0$

M_2: $X_2(s)[M_2 s^2 + s(F_{v5}+F_{v2}) + K] - (K+F_{v2})X_1(s) - (F_{v5} s)X_3(s) = f(s)$

M_3: $X_3(s)[M_3 s^2 + s(F_{v4}+F_{v3}+F_{v5})] - (sF_{v4})X_1(s) - (sF_{v5})X_2(s) = 0$

$\Rightarrow$ Laplace

$$\begin{cases} (2s^2+3s+2)X_1(s) - (s+2)X_2(s) - sX_3(s) = 0 \\ -(s+2)X_1(s) + (s^2+2s+2)X_2(s) - sX_3(s) = F(s) \\ -sX_1(s) - sX_2(s) + (s^2+3s)X_3(s) = 0 \end{cases}$$

$\Rightarrow$ Diferencial

$$\begin{cases} 2\ddot{x}_1 + 3\dot{x}_1 + 2x_1 - \dot{x}_2 - 2x_2 - \dot{x}_3 = 0 \\ -\dot{x}_1 - 2x_1 + \ddot{x}_2 + 2\dot{x}_2 + 2x_2 - \dot{x}_3 = f(t) \\ -\dot{x}_1 - \dot{x}_2 + \ddot{x}_3 + 3\dot{x}_3 = 0 \end{cases}$$

$\Rightarrow$ Asignar variables

$z_1 = x_1,\ z_2 = \underset{=\dot{z}_1}{\dot{x}_1},\ z_3 = x_2,\ z_4 = \underset{=\dot{z}_3}{\dot{x}_2},\ z_5 = x_3,\ z_6 = \underset{=\dot{z}_5}{\dot{x}_3}$

$\Rightarrow$ Sustituir

$$\begin{cases} \dot{z}_1 = z_2 & \dot{z}_2 = -z_1 - \frac{3}{2}z_2 + z_3 + \frac{1}{2}z_4 + \frac{1}{2}z_6 \\ \dot{z}_3 = z_4 & \dot{z}_4 = 2z_1 + z_2 - 2z_3 - 2z_4 + z_6 + f(t) \\ \dot{z}_5 = z_6 & \dot{z}_6 = z_2 + z_4 - 3z_6 \end{cases}$$

$\Rightarrow$

$$\dot{z} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & -1.5 & 1 & 0.5 & 0 & 0.5 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & -2 & -2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & -3 \end{bmatrix} z + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} f(t) \ ; \ y = [0\,0\,0\,0\,1\,0]\, z$$

325

Fecha: ____ / ____

$\dot{x}_0 = a_{00} x_0 + a_{02} x_2 + d_0$

$\dot{x}_1 = a_{10} x_0 + a_{11} x_1 + a_{12} x_2$

$\dot{x}_2 = a_{20} x_0 + a_{21} x_1 + a_{22} x_2 + a_{23} x_3 + a_{24} x_4$

$\dot{x}_3 = a_{32} x_2 + a_{33} x_3$

$\dot{x}_4 = a_{42} x_2 + a_{44} x_4$

Entonces:

$\dot{x} = Ax + Bu$

$$\Rightarrow \dot{x} = \begin{bmatrix} a_{00} & 0 & a_{02} & 0 & 0 \\ a_{10} & a_{11} & a_{12} & 0 & 0 \\ a_{20} & a_{21} & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{32} & a_{33} & 0 \\ 0 & 0 & a_{42} & 0 & a_{44} \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} d_0$$

$* \; x = [x_0 \; x_1 \; x_2 \; x_3 \; x_4]^T$

5

Polos = ?

$$R(s) \rightarrow \boxed{\frac{s^2+4s-3}{s^4+4s^3+8s^2+20s+15}} \rightarrow c(s)$$

$s = -1, -3, \pm\sqrt{5}\,i$

∴ 2gw, 2lhp

Routh table

s^4	1^{a_4}	8^{a_2}	15^{a_0}
s^3	4^{a_3}	20^{a_1}	0
s^2	5^{b_1}	15^{b_2}	0
s^1	0^{c_1}	0	6
s^0	NB^{d_1}	0	0

Hagpanian

1 $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}'$

$$\dot{x} = f(x,u) = \begin{bmatrix} x_2 \\ g/l \sin x_1 - b/(ml^2)\, x_2 + \frac{1}{ml^2}\, \text{sat}(u) \end{bmatrix}$$

$y = g(x,u) = x_1$

a) linearized point (x^{eq}, u^{eq}) $\qquad \delta x = x - x^{eq}$

$$\delta \dot{x} = \left.\frac{\partial f(x,u)}{\partial x}\right|_{\substack{x=x^{eq}\\u=u^{eq}}} \delta x + \left.\frac{\partial f(x,u)}{\partial u}\right|_{\substack{x=x^{eq}\\u=u^{eq}}} \delta u$$

$$= \begin{bmatrix} 0 & 1 \\ \frac{g}{l}\cos(x_1^{eq}) & -\frac{b}{ml^2} \end{bmatrix} \delta x + \begin{bmatrix} 0 \\ \frac{1}{ml^2} \frac{d}{du}\text{sat}(u^{eq}) \end{bmatrix} \delta u$$

$\delta u = u - u^{eq}$

$$\delta y = \frac{\partial g(x,u)}{\partial x}\bigg|_{\substack{x=x^{eq}\\u=u^{eq}}} \delta x + \frac{\partial g(x,u)}{\partial u}\bigg|_{\substack{x=x^{eq}\\u=u^{eq}}} \delta u$$

$$= [1 \; 0]\,\delta x$$

$\Rightarrow x_1^{eq} = \theta = 0 \qquad x_2^{eq} = \dot\theta = 0 \qquad u^{eq} = 0$

$$\Rightarrow \delta\dot x = \begin{bmatrix} 0 & 1 \\ g/l & -b/(ml^2) \end{bmatrix} \delta x + \begin{bmatrix} 0 \\ 1/ml^2 \end{bmatrix} \delta u$$

$$\delta y = [1 \; 0]\,\delta x$$

b) $x_1^{eq} = \theta = \pi \qquad x_2^{eq} = \dot\theta = 0 \qquad u^{eq} = 0$

$$\delta\dot x = \begin{bmatrix} 0 & 1 \\ -g/l & -b/ml^2 \end{bmatrix} \delta x + \begin{bmatrix} 0 \\ 1/ml^2 \end{bmatrix} \delta u$$

$$\delta y = [1 \; 0]\,\delta x$$

c) $x_1^{eq} = \theta = \pi/4 \rightarrow$ necesito $0 = mgl\,\text{sen}\left(\frac{\pi}{4}\right) + \text{sat}(u_1^{eq})$

$\Rightarrow mgl \frac{\sqrt2}{2} \in [-1, 1]$

$$\delta\dot x = \begin{bmatrix} 0 & 1 \\ \frac{g}{l}\frac{\sqrt2}{2} & -b/ml^2 \end{bmatrix} \delta x + \begin{bmatrix} 0 \\ 1/ml^2 \end{bmatrix} \delta u$$

$$\delta y = [1 \; 0]\,\delta x$$

d) $\theta(t) = t \qquad \dot\theta(t) = 1 \qquad \ddot\theta(t) = 0 \qquad \forall t \geq 0$

NEWTON

$0 = mgl\,\text{sen}(t) - b + T(t)$

$\Rightarrow T(t) = b - mgl\,\text{sen}(t)$

$= \frac{1}{2} - \frac{1}{4}\text{sen}(t) \in \left[\frac14, \frac34\right] \qquad \forall t \geq 0$

Linealizar aqui:

$$\delta\dot x = \begin{bmatrix} 0 & 1 \\ (g/l)\cos(t) & -1/ml^2 \end{bmatrix} \delta x + \begin{bmatrix} 0 \\ 1/ml^2 \end{bmatrix} \delta y$$

$$\delta y = [1 \; 0]\,\delta x$$

Fecha: ____ / ____ . a) $\dot{x}_1 = -x_1 + x_1(x_1^2 + x_2^2)$

8.7 $\dot{x}_2 = -x_2 + x_2(x_1^2 + x_2^2)$

$x_1 = x_2 = 0$

$$\text{dynamics} = \begin{bmatrix} -x_1 + x_1(x_1^2 + x_2^2) \\ -x_2 + x_2(x_1^2 + x_2^2) \end{bmatrix}$$

$$\text{eig}(A1) = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \therefore \text{Localmente estable}$$

b) $\ddot{\omega} + g(\omega)\dot{\omega} + \omega = 0$

$\omega = x_1 \qquad \dot{\omega} = x_2$

$\dot{x}_1 = \dot{\omega} = x_2 \qquad \dot{x}_2 = \ddot{\omega}$

$\Rightarrow \ddot{\omega} = -\omega - g(\omega)\dot{\omega}$

$\Rightarrow \dot{x}_2 = -x_1 - g(x_1)x_2$

$$\Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -x_1 - g(x_1)x_2 \end{bmatrix} = \begin{bmatrix} \dot{\omega} \\ -\omega - g(\omega)\dot{\omega} \end{bmatrix}$$

$\text{eig}(A) = -0.5 \pm 0.86\,i$

$g(0) = 1$ Localmente asintóticamente estable

$g(0) > 1$ Localmente estable

$g(0) = 0$ marginalmente estable